Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.6-Improper-linear-binomial/83-1.1.6.5

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**275**]. This is test number [83].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (275)	0.00 (0)
Rubi	98.91 (272)	1.09 (3)
Fricas	93.45 (257)	6.55 (18)
Maple	88.36 (243)	11.64 (32)
Reduce	80.73 (222)	19.27 (53)
Giac	57.82 (159)	42.18 (116)
Maxima	40.36 (111)	59.64 (164)
Mupad	21.82 (60)	78.18 (215)
Sympy	10.18 (28)	89.82 (247)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

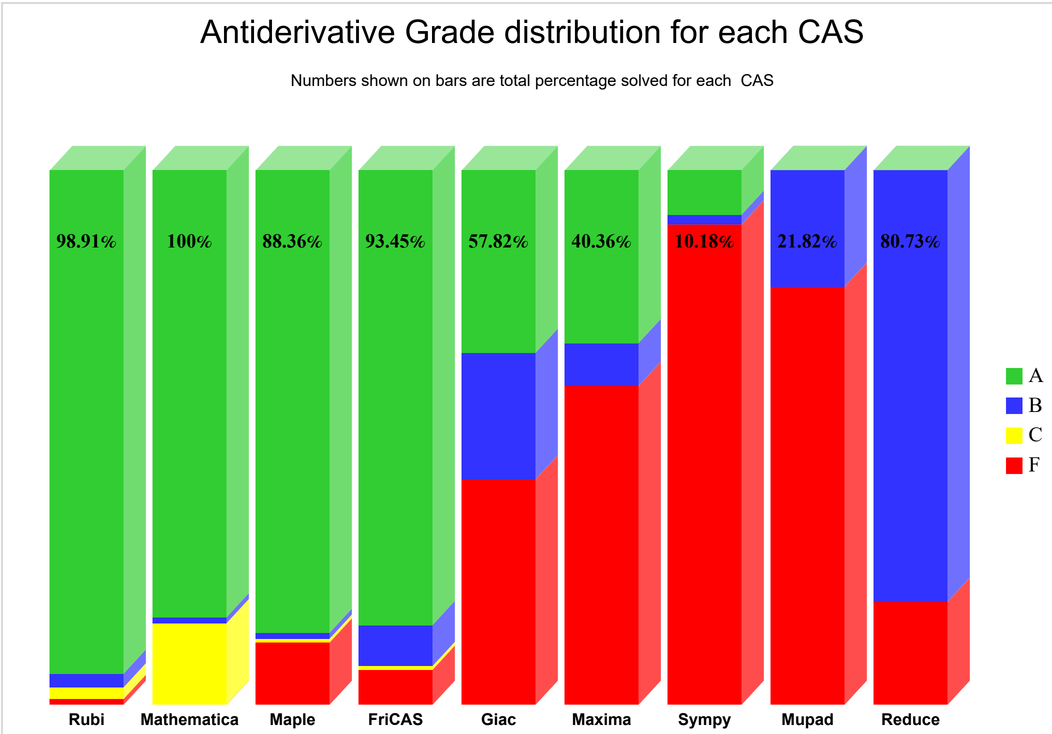
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

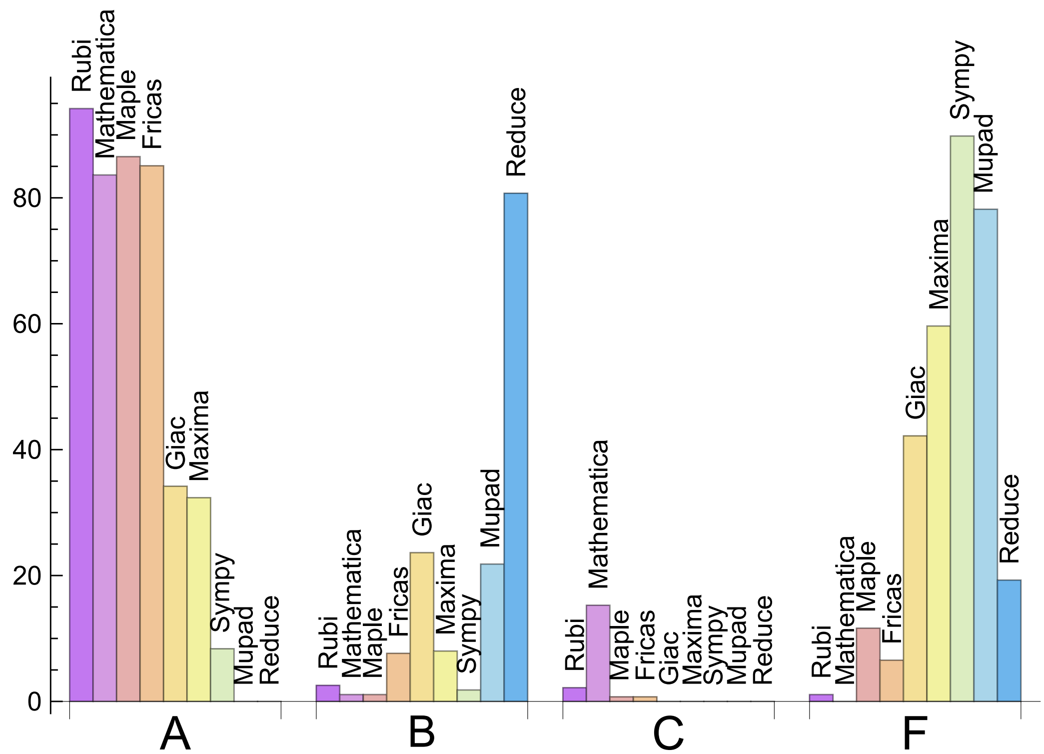
System	% A grade	% B grade	% C grade	% F grade
Rubi	94.182	2.545	2.182	1.091
Maple	86.545	1.091	0.727	11.636
Fricas	85.091	7.636	0.727	6.545
Mathematica	83.636	1.091	15.273	0.000
Giac	34.182	23.636	0.000	42.182
Maxima	32.364	8.000	0.000	59.636
Sympy	8.364	1.818	0.000	89.818
Mupad	0.000	21.818	0.000	78.182
Reduce	0.000	80.727	0.000	19.273

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Rubi	3	100.00	0.00	0.00
Fricas	18	100.00	0.00	0.00
Maple	32	100.00	0.00	0.00
Reduce	53	100.00	0.00	0.00
Giac	116	52.59	16.38	31.03
Maxima	164	76.83	1.83	21.34
Mupad	215	0.00	100.00	0.00
Sympy	247	97.57	2.43	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.04
Fricas	0.15
Giac	0.46
Rubi	0.59
Maple	0.69
Sympy	0.78
Reduce	1.96
Mathematica	2.33
Mupad	10.07

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	148.87	0.85	123.00	0.80
Rubi	175.67	1.09	155.50	1.01
Mupad	205.00	1.29	163.50	1.13
Mathematica	207.24	1.32	146.00	0.98
Maxima	263.26	1.54	227.00	1.39
Giac	321.38	2.02	235.00	1.36
Reduce	400.29	2.39	266.50	1.59
Sympy	460.43	1.81	367.00	1.66
Fricas	468.24	2.60	344.00	2.00

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

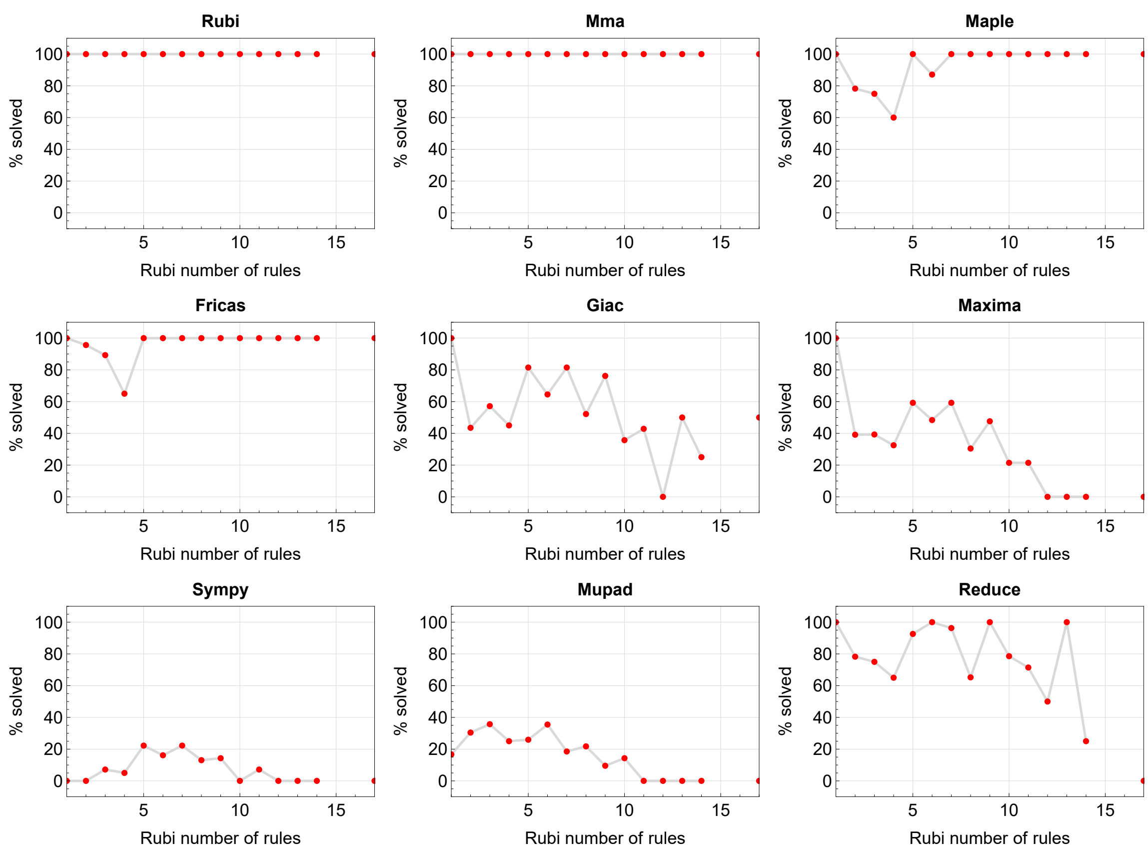


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

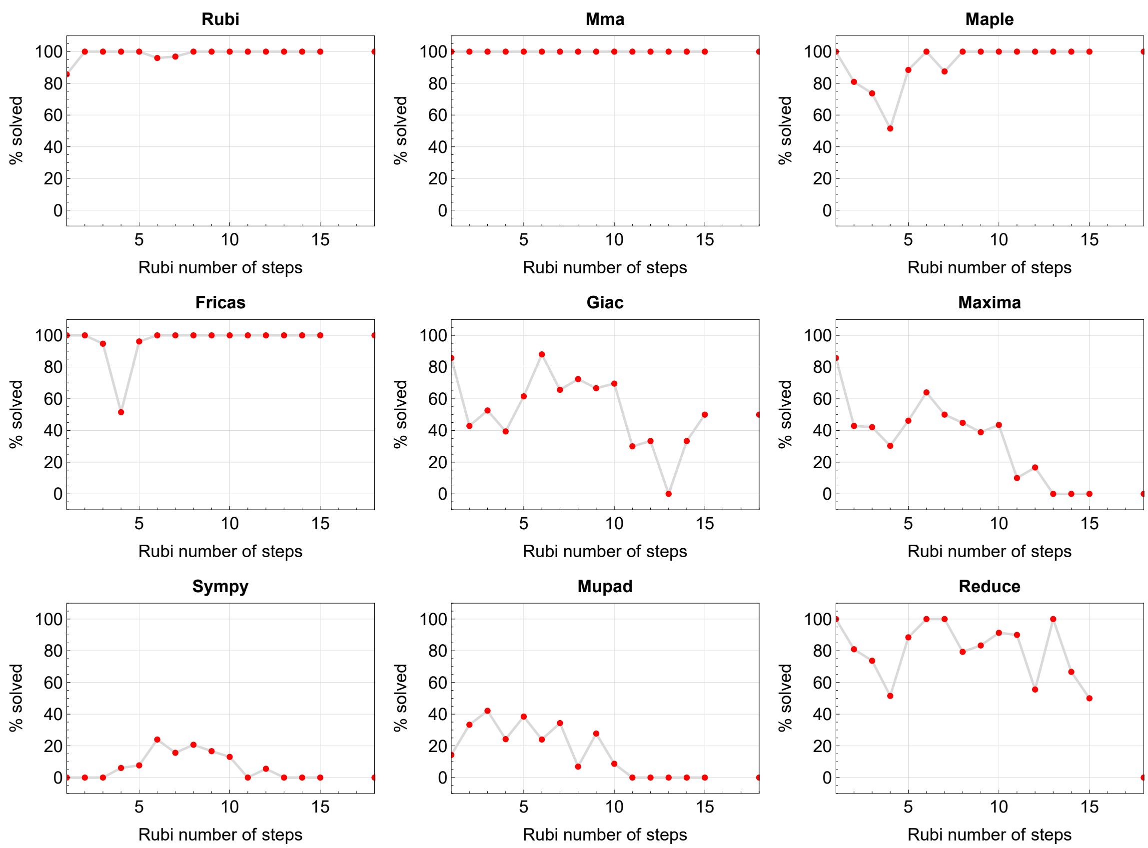


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

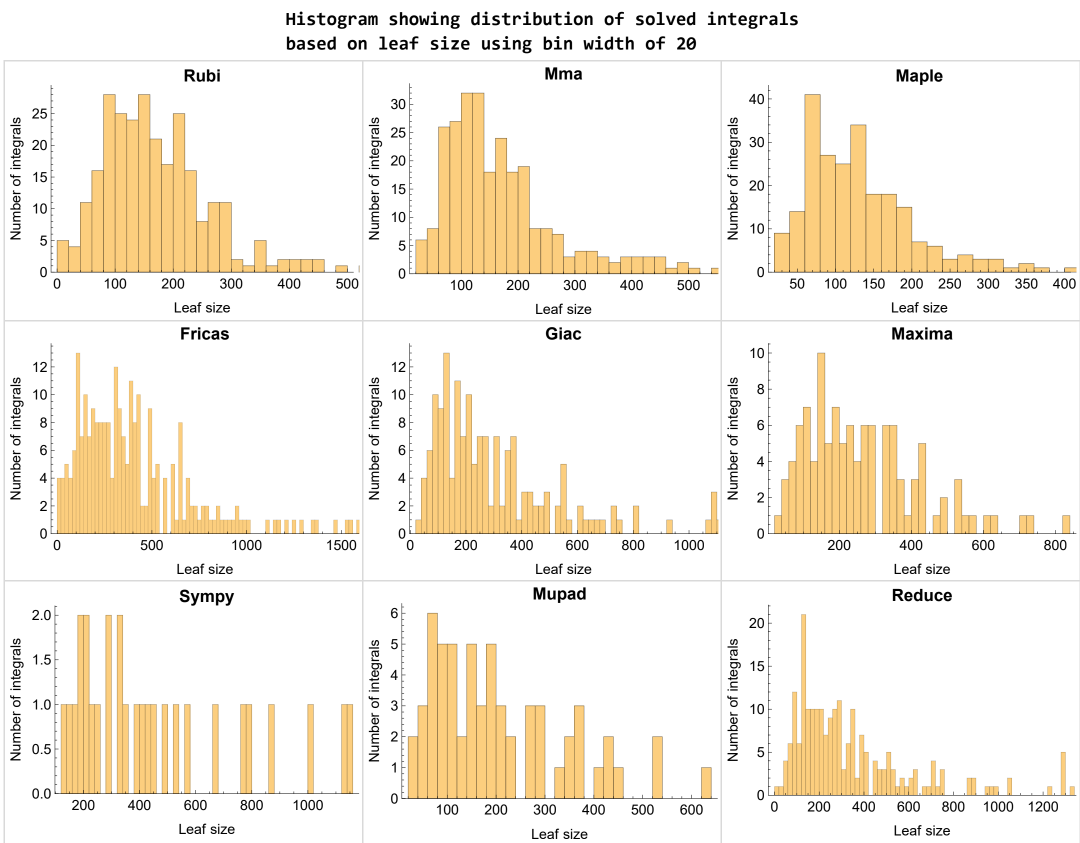


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

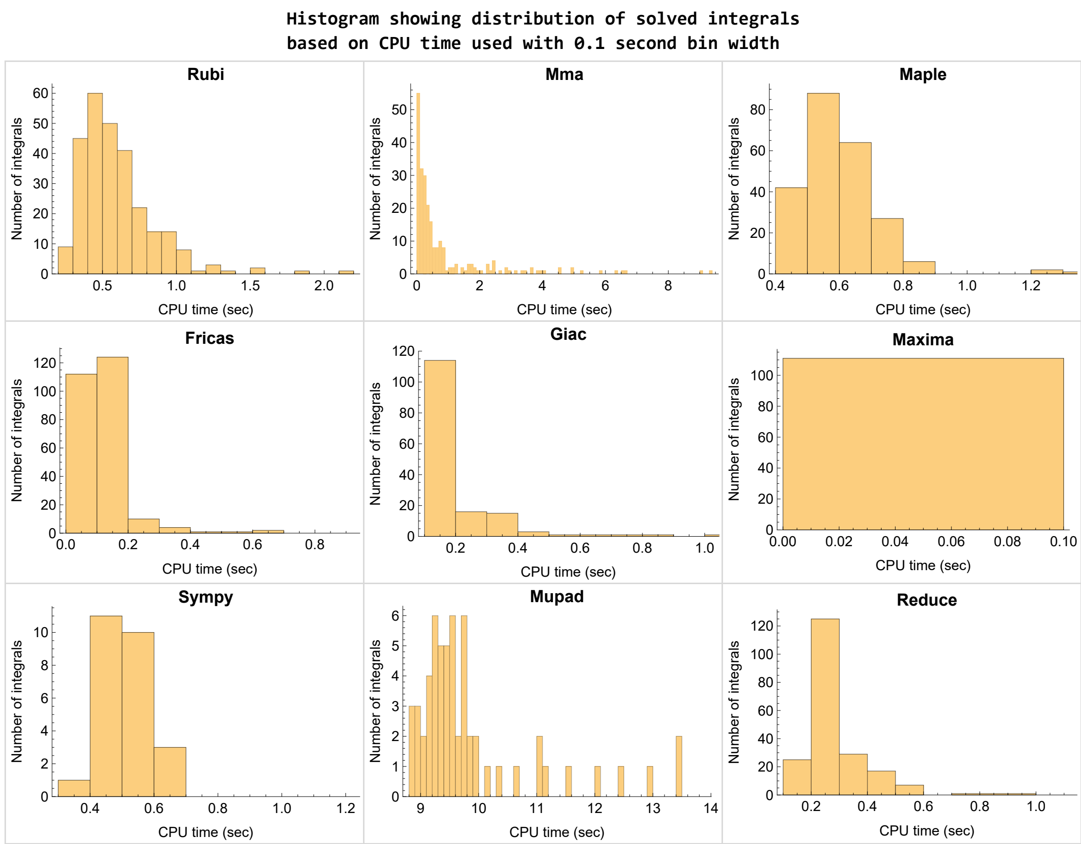


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

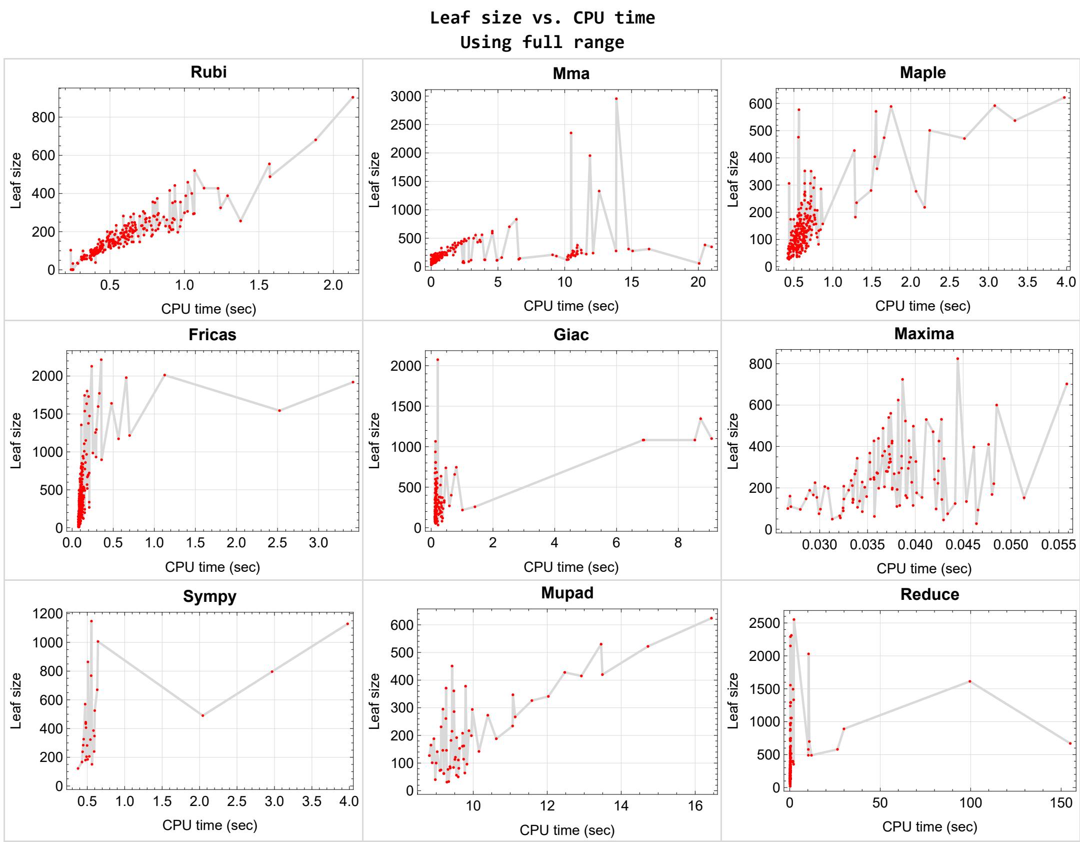


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {178}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

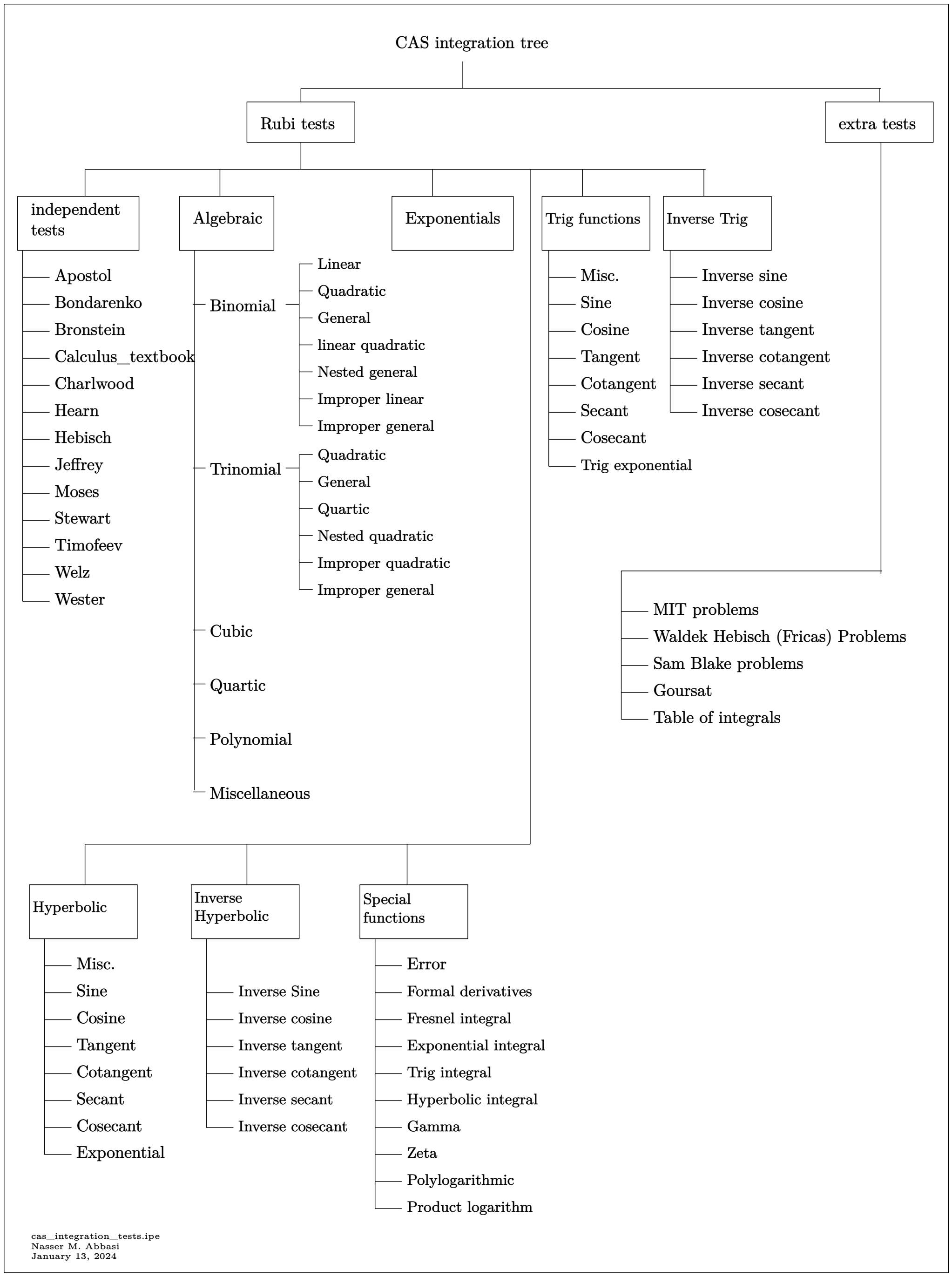
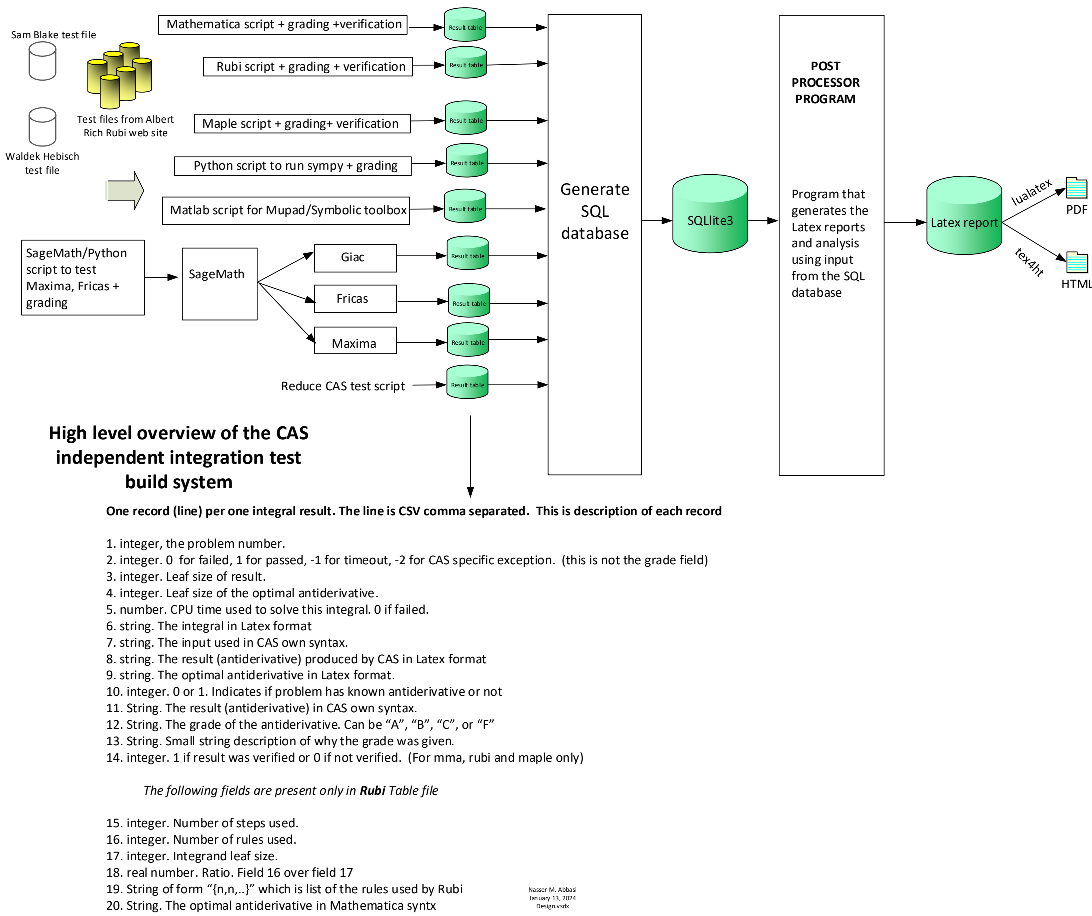


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 212, 214, 215, 216, 219, 220, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 267, 268, 269, 270, 271, 272, 273, 274, 275 }

B grade { 96, 104, 111, 121, 198, 210, 213 }

C grade { 178, 217, 218, 221, 222, 266 }

F normal fail { 54, 177, 179 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 33, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 95, 98, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110, 111, 112, 113, 114, 119, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 205, 206, 212, 213, 214, 215, 216, 219, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275 }

B grade { 49, 51, 210 }

C grade { 30, 31, 32, 34, 42, 92, 93, 94, 96, 97, 105, 106, 115, 116, 117, 118, 120, 122, 143, 144, 145, 152, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 207, 208, 209, 211, 217, 218, 220, 221, 222, 223 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 206, 208, 209, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 258 }

B grade { 198, 205, 207 }

C grade { 210, 211 }

F normal fail { 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 255, 256, 257, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 197, 200, 201, 202, 203, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 262, 263, 264, 265, 266, 267, 268, 269 }

B grade { 46, 47, 48, 49, 50, 51, 110, 111, 112, 124, 194, 198, 199, 204, 205, 206, 207, 208, 209, 260, 261 }

C grade { 210, 211 }

F normal fail { 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 270, 271, 272, 273, 274, 275 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 16, 18, 19, 20, 21, 22, 23, 26, 29, 56, 57, 58, 59, 60, 61, 62, 63, 67, 68, 69, 70, 71, 72, 73, 79, 80, 81, 82, 83, 84, 85, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 181, 182, 183, 184, 224, 225, 226, 232, 233, 234, 258 }

B grade { 7, 17, 27, 28, 64, 65, 66, 74, 75, 76, 77, 78, 86, 87, 88, 89, 90, 91, 177, 178, 179, 180 }

C grade { }

F normal fail { 34, 35, 36, 37, 41, 42, 43, 44, 49, 50, 51, 95, 96, 97, 98, 99, 103, 104, 105, 106, 107, 111, 112, 113, 114, 118, 119, 120, 121, 122, 123, 124, 147, 148, 149, 154, 155, 156, 157, 158, 159, 160, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 227, 228, 229, 230, 231, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275 }

F(-1) timedout fail { 15, 24, 25 }

F(-2) exception fail { 30, 31, 32, 33, 38, 39, 40, 45, 46, 47, 48, 52, 53, 54, 55, 92, 93, 94, 100, 101, 102, 108, 109, 110, 115, 116, 117, 143, 144, 145, 146, 150, 151, 152, 153 }

Giac

A grade { 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 24, 34, 35, 56, 57, 58, 59, 60, 61, 62, 67, 68, 69, 70, 71, 72, 73, 79, 80, 81, 82, 83, 84, 85, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 161, 162, 163, 164, 168, 169, 170, 171, 172, 177, 181, 191, 192, 193, 194, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 243, 244 }

B grade { 6, 7, 8, 9, 16, 17, 18, 19, 25, 26, 27, 28, 29, 36, 37, 41, 42, 43, 45, 46, 47, 48, 49, 63, 64, 65, 66, 74, 75, 76, 77, 78, 86, 87, 88, 89, 90, 91, 97, 98, 99, 105, 106, 108, 109, 110, 111, 112, 122, 123, 124, 152, 154, 155, 157, 158, 178, 179, 185, 186, 187, 188, 189, 190, 251 }

C grade { }

F normal fail { 165, 166, 167, 173, 174, 175, 176, 180, 182, 183, 184, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275 }

F(-1) timedout fail { 38, 39, 100, 101, 102, 103, 150, 240, 241, 242, 245, 246, 247, 248, 249, 250, 252, 253, 254 }

F(-2) exception fail { 30, 31, 32, 33, 40, 44, 50, 51, 52, 53, 54, 55, 92, 93, 94, 95, 96, 104, 107, 113, 114, 115, 116, 117, 118, 119, 120, 121, 143, 144, 145, 151, 153, 156, 159, 160 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 27, 28, 29, 58, 64, 65, 66, 76, 77, 78, 89, 90, 91, 128, 129, 130, 131, 132, 133, 140, 141, 142, 163, 164, 165, 166, 167, 172, 173, 174, 175, 176, 180, 181, 182, 183, 184, 224, 225, 226, 232, 233, 234 }

C grade { }

F normal fail { }

F(-1) timedout fail { 5, 6, 14, 15, 16, 23, 24, 25, 26, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 63, 67, 68, 69, 70, 71, 72, 73, 74, 75, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 134, 135, 136, 137, 138, 139, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 168, 169, 170, 171, 177, 178, 179, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 227, 228, 229, 230, 231, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 10, 11, 12, 20, 21, 56, 57, 58, 59, 68, 69, 70, 82, 125, 126, 127, 134, 135, 136, 137 }

B grade { 67, 79, 80, 81, 128 }

C grade { }

F normal fail { 4, 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 60, 61, 62, 63, 64, 65, 66, 71, 72, 73, 74, 75, 76, 77, 78, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 129, 130, 131, 132, 133, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269 }

F(-1) timedout fail { 270, 271, 272, 273, 274, 275 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261 }

C grade { }

F normal fail { 45, 51, 79, 100, 108, 109, 114, 115, 116, 124, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	160	201	114	242	302	238	157	217	215
N.S.	1	0.81	1.02	0.58	1.23	1.53	1.21	0.80	1.10	1.09
time (sec)	N/A	0.522	0.192	0.490	0.035	0.097	0.438	0.154	0.223	9.427

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	112	137	97	198	255	206	129	177	165
N.S.	1	0.70	0.86	0.61	1.24	1.59	1.29	0.81	1.11	1.03
time (sec)	N/A	0.400	0.175	0.461	0.036	0.102	0.528	0.120	0.254	8.858

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	98	122	85	154	206	167	102	137	127
N.S.	1	0.79	0.98	0.69	1.24	1.66	1.35	0.82	1.10	1.02
time (sec)	N/A	0.383	0.101	0.448	0.030	0.104	0.430	0.132	0.222	8.815

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	83	87	66	109	157	0	76	98	101
N.S.	1	0.88	0.93	0.70	1.16	1.67	0.00	0.81	1.04	1.07
time (sec)	N/A	0.403	0.048	0.439	0.027	0.103	0.000	0.126	0.205	8.892

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	77	82	57	89	139	0	80	98	0
N.S.	1	1.10	1.17	0.81	1.27	1.99	0.00	1.14	1.40	0.00
time (sec)	N/A	0.394	0.040	0.447	0.032	0.092	0.000	0.350	0.205	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	74	91	61	85	142	0	140	91	0
N.S.	1	1.01	1.25	0.84	1.16	1.95	0.00	1.92	1.25	0.00
time (sec)	N/A	0.392	0.024	0.443	0.034	0.099	0.000	0.341	0.222	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	36	37	100	56	0	191	106	100
N.S.	1	1.00	0.63	0.65	1.75	0.98	0.00	3.35	1.86	1.75
time (sec)	N/A	0.344	0.011	0.435	0.027	0.085	0.000	0.308	0.223	8.998

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	89	57	54	146	81	0	251	148	146
N.S.	1	0.99	0.63	0.60	1.62	0.90	0.00	2.79	1.64	1.62
time (sec)	N/A	0.407	0.015	0.455	0.033	0.083	0.000	0.335	0.216	9.277

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	121	73	70	192	105	0	311	188	192
N.S.	1	0.97	0.58	0.56	1.54	0.84	0.00	2.49	1.50	1.54
time (sec)	N/A	0.502	0.023	0.488	0.038	0.082	0.000	0.341	0.198	9.551

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	217	279	189	427	490	406	255	386	371
N.S.	1	0.74	0.95	0.64	1.45	1.67	1.38	0.87	1.31	1.26
time (sec)	N/A	0.732	1.419	0.591	0.036	0.138	0.484	0.245	0.290	9.266

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	184	204	159	355	415	348	213	321	295
N.S.	1	0.74	0.83	0.64	1.44	1.68	1.41	0.86	1.30	1.19
time (sec)	N/A	0.592	1.031	0.546	0.037	0.095	0.597	0.175	0.278	9.182

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	155	173	132	283	338	284	171	256	231
N.S.	1	0.79	0.89	0.68	1.45	1.73	1.46	0.88	1.31	1.18
time (sec)	N/A	0.523	0.804	0.527	0.034	0.102	0.446	0.129	0.226	9.132

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	136	133	105	211	261	0	129	193	182
N.S.	1	0.89	0.88	0.69	1.39	1.72	0.00	0.85	1.27	1.20
time (sec)	N/A	0.571	0.229	0.507	0.033	0.114	0.000	0.136	0.205	9.400

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	129	128	95	166	223	0	119	180	0
N.S.	1	0.92	0.91	0.68	1.19	1.59	0.00	0.85	1.29	0.00
time (sec)	N/A	0.532	0.438	0.508	0.029	0.097	0.000	0.143	0.251	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	116	98	85	0	215	0	172	173	0
N.S.	1	1.16	0.98	0.85	0.00	2.15	0.00	1.72	1.73	0.00
time (sec)	N/A	0.560	0.313	0.514	0.000	0.107	0.000	0.148	0.188	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	1	111	97	152	228	0	266	180	0
N.S.	1	0.01	1.00	0.87	1.37	2.05	0.00	2.40	1.62	0.00
time (sec)	N/A	0.251	0.300	0.525	0.039	0.085	0.000	0.336	0.226	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	138	70	68	199	108	0	351	215	199
N.S.	1	1.34	0.68	0.66	1.93	1.05	0.00	3.41	2.09	1.93
time (sec)	N/A	0.534	0.223	0.514	0.038	0.088	0.000	0.173	0.220	9.952

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	172	99	90	273	146	0	446	282	273
N.S.	1	1.04	0.60	0.55	1.65	0.88	0.00	2.70	1.71	1.65
time (sec)	N/A	0.600	0.301	0.554	0.038	0.075	0.000	0.159	0.238	10.395

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	204	131	124	347	185	0	541	347	347
N.S.	1	1.03	0.66	0.62	1.74	0.93	0.00	2.72	1.74	1.74
time (sec)	N/A	0.668	0.293	0.574	0.040	0.079	0.000	0.144	0.230	11.073

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	269	330	228	540	603	525	311	490	451
N.S.	1	0.77	0.95	0.65	1.55	1.73	1.50	0.89	1.40	1.29
time (sec)	N/A	0.924	1.598	0.602	0.037	0.108	0.600	0.156	11.998	9.428

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	194	281	188	439	498	434	255	400	361
N.S.	1	0.71	1.02	0.68	1.60	1.81	1.58	0.93	1.45	1.31
time (sec)	N/A	0.653	1.287	0.569	0.036	0.090	0.480	0.128	1.671	9.476

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	206	189	150	340	395	0	199	312	286
N.S.	1	0.92	0.84	0.67	1.52	1.76	0.00	0.89	1.39	1.28
time (sec)	N/A	0.807	0.350	0.552	0.037	0.106	0.000	0.125	0.413	9.482

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	184	172	136	270	329	0	172	294	0
N.S.	1	0.92	0.86	0.68	1.34	1.64	0.00	0.86	1.46	0.00
time (sec)	N/A	0.786	0.651	0.529	0.036	0.100	0.000	0.130	0.418	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	176	160	126	0	309	0	211	279	0
N.S.	1	1.14	1.03	0.81	0.00	1.99	0.00	1.36	1.80	0.00
time (sec)	N/A	0.779	0.346	0.518	0.000	0.090	0.000	0.294	0.206	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	182	130	121	0	309	0	298	267	0
N.S.	1	1.31	0.94	0.87	0.00	2.22	0.00	2.14	1.92	0.00
time (sec)	N/A	0.728	0.498	0.535	0.000	0.091	0.000	0.321	0.237	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	1	159	139	241	332	0	426	289	0
N.S.	1	0.01	0.99	0.86	1.50	2.06	0.00	2.65	1.80	0.00
time (sec)	N/A	0.239	0.442	0.557	0.042	0.086	0.000	0.144	0.219	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	229	112	110	326	174	0	546	349	326
N.S.	1	1.58	0.77	0.76	2.25	1.20	0.00	3.77	2.41	2.25
time (sec)	N/A	0.873	0.340	0.573	0.037	0.081	0.000	0.131	0.243	11.586

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	263	156	149	428	227	0	676	441	428
N.S.	1	1.18	0.70	0.67	1.93	1.02	0.00	3.05	1.99	1.93
time (sec)	N/A	0.932	0.437	0.608	0.039	0.118	0.000	0.124	0.234	12.474

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	295	197	188	530	279	0	806	531	530
N.S.	1	1.02	0.68	0.65	1.83	0.96	0.00	2.78	1.83	1.83
time (sec)	N/A	1.066	0.482	0.677	0.041	0.086	0.000	0.148	0.250	13.457

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	277	484	182	0	860	0	0	344	0
N.S.	1	1.25	2.18	0.82	0.00	3.87	0.00	0.00	1.55	0.00
time (sec)	N/A	0.861	2.493	1.289	0.000	0.162	0.000	0.000	0.337	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	176	438	131	0	639	0	0	245	0
N.S.	1	1.07	2.65	0.79	0.00	3.87	0.00	0.00	1.48	0.00
time (sec)	N/A	0.690	3.691	0.602	0.000	0.113	0.000	0.000	0.297	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	138	402	99	0	482	0	0	168	0
N.S.	1	1.22	3.56	0.88	0.00	4.27	0.00	0.00	1.49	0.00
time (sec)	N/A	0.555	2.274	0.586	0.000	0.118	0.000	0.000	0.309	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	146	131	77	0	396	0	0	131	0
N.S.	1	1.68	1.51	0.89	0.00	4.55	0.00	0.00	1.51	0.00
time (sec)	N/A	0.648	0.284	0.658	0.000	0.120	0.000	0.000	0.232	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	100	346	69	0	172	0	97	126	0
N.S.	1	1.35	4.68	0.93	0.00	2.32	0.00	1.31	1.70	0.00
time (sec)	N/A	0.400	1.224	0.604	0.000	0.084	0.000	0.155	0.241	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	131	133	88	0	216	0	181	186	0
N.S.	1	1.18	1.20	0.79	0.00	1.95	0.00	1.63	1.68	0.00
time (sec)	N/A	0.444	0.276	0.619	0.000	0.088	0.000	0.291	0.280	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	196	159	131	0	303	0	307	277	0
N.S.	1	1.21	0.98	0.81	0.00	1.87	0.00	1.90	1.71	0.00
time (sec)	N/A	0.596	0.449	0.648	0.000	0.085	0.000	0.290	0.280	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	270	205	167	0	406	0	467	387	0
N.S.	1	1.20	0.91	0.74	0.00	1.80	0.00	2.08	1.72	0.00
time (sec)	N/A	0.745	0.673	0.682	0.000	0.101	0.000	0.141	0.350	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	250	279	180	0	1107	0	0	888	0
N.S.	1	1.24	1.38	0.89	0.00	5.48	0.00	0.00	4.40	0.00
time (sec)	N/A	0.740	10.596	0.768	0.000	0.147	0.000	0.000	0.324	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	161	192	141	0	797	0	0	714	0
N.S.	1	1.09	1.30	0.95	0.00	5.39	0.00	0.00	4.82	0.00
time (sec)	N/A	0.614	10.467	0.640	0.000	0.129	0.000	0.000	0.258	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	146	159	120	0	839	0	0	603	0
N.S.	1	1.21	1.31	0.99	0.00	6.93	0.00	0.00	4.98	0.00
time (sec)	N/A	0.535	0.497	0.599	0.000	0.132	0.000	0.000	0.258	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	106	108	65	0	251	0	313	266	0
N.S.	1	1.38	1.40	0.84	0.00	3.26	0.00	4.06	3.45	0.00
time (sec)	N/A	0.375	0.438	0.602	0.000	0.098	0.000	0.424	0.264	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	152	1330	100	0	346	0	746	949	0
N.S.	1	1.39	12.20	0.92	0.00	3.17	0.00	6.84	8.71	0.00
time (sec)	N/A	0.454	12.575	0.628	0.000	0.097	0.000	0.814	0.298	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	193	152	133	0	489	0	1100	743	0
N.S.	1	1.30	1.03	0.90	0.00	3.30	0.00	7.43	5.02	0.00
time (sec)	N/A	0.575	0.777	0.674	0.000	0.120	0.000	9.076	0.362	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	248	203	167	0	642	0	0	1332	0
N.S.	1	1.25	1.03	0.84	0.00	3.24	0.00	0.00	6.73	0.00
time (sec)	N/A	0.666	1.657	0.727	0.000	0.122	0.000	0.000	2.169	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	351	292	290	0	2216	0	573	24	0
N.S.	1	1.13	0.94	0.93	0.00	7.13	0.00	1.84	0.08	0.00
time (sec)	N/A	0.920	11.230	0.753	0.000	0.355	0.000	0.164	200.044	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	288	226	242	0	1730	0	527	2552	0
N.S.	1	1.22	0.95	1.02	0.00	7.30	0.00	2.22	10.77	0.00
time (sec)	N/A	0.820	10.920	0.719	0.000	0.201	0.000	0.301	2.308	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	214	198	212	0	1802	0	494	2292	0
N.S.	1	1.15	1.06	1.14	0.00	9.69	0.00	2.66	12.32	0.00
time (sec)	N/A	0.725	10.770	0.662	0.000	0.183	0.000	0.197	0.360	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	127	122	94	0	474	0	407	1056	0
N.S.	1	0.95	0.92	0.71	0.00	3.56	0.00	3.06	7.94	0.00
time (sec)	N/A	0.442	10.158	0.628	0.000	0.101	0.000	0.190	0.514	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	172	2352	113	0	573	0	419	1555	0
N.S.	1	1.25	17.04	0.82	0.00	4.15	0.00	3.04	11.27	0.00
time (sec)	N/A	0.485	10.471	0.632	0.000	0.119	0.000	0.154	0.349	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	217	200	185	0	732	0	0	2030	0
N.S.	1	1.09	1.01	0.93	0.00	3.68	0.00	0.00	10.20	0.00
time (sec)	N/A	0.609	10.279	0.713	0.000	0.117	0.000	0.000	10.385	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	281	2955	208	0	943	0	0	24	0
N.S.	1	1.08	11.41	0.80	0.00	3.64	0.00	0.00	0.09	0.00
time (sec)	N/A	0.700	13.864	0.770	0.000	0.133	0.000	0.000	200.115	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	146	131	77	0	396	0	0	131	0
N.S.	1	1.68	1.51	0.89	0.00	4.55	0.00	0.00	1.51	0.00
time (sec)	N/A	0.603	0.045	0.569	0.000	0.121	0.000	0.000	0.430	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	146	131	77	0	396	0	0	131	0
N.S.	1	1.68	1.51	0.89	0.00	4.55	0.00	0.00	1.51	0.00
time (sec)	N/A	0.657	0.002	0.543	0.000	0.117	0.000	0.000	0.485	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	0	131	77	0	396	0	0	131	0
N.S.	1	0.00	1.51	0.89	0.00	4.55	0.00	0.00	1.51	0.00
time (sec)	N/A	0.000	0.002	0.546	0.000	0.124	0.000	0.000	0.368	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	146	131	77	0	396	0	0	131	0
N.S.	1	1.68	1.51	0.89	0.00	4.55	0.00	0.00	1.51	0.00
time (sec)	N/A	0.700	0.001	0.545	0.000	0.118	0.000	0.000	0.315	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	197	236	148	324	401	444	220	297	0
N.S.	1	0.75	0.89	0.56	1.23	1.52	1.68	0.83	1.12	0.00
time (sec)	N/A	0.610	0.329	0.513	0.035	0.110	0.480	0.143	0.426	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	149	219	131	280	350	386	191	257	0
N.S.	1	0.66	0.96	0.58	1.23	1.54	1.70	0.84	1.13	0.00
time (sec)	N/A	0.476	0.236	0.484	0.037	0.109	0.583	0.153	0.415	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	135	167	116	236	303	326	163	217	208
N.S.	1	0.71	0.87	0.61	1.24	1.59	1.71	0.85	1.14	1.09
time (sec)	N/A	0.420	0.211	0.468	0.033	0.112	0.453	0.146	0.322	9.713

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	124	181	97	189	256	490	136	177	0
N.S.	1	0.78	1.14	0.61	1.19	1.61	3.08	0.86	1.11	0.00
time (sec)	N/A	0.440	0.141	0.486	0.033	0.107	2.043	0.325	0.331	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	120	110	85	147	209	0	108	137	0
N.S.	1	0.97	0.89	0.69	1.19	1.69	0.00	0.87	1.10	0.00
time (sec)	N/A	0.445	0.065	0.460	0.029	0.119	0.000	0.120	0.373	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	103	101	73	129	187	0	107	138	0
N.S.	1	0.98	0.96	0.70	1.23	1.78	0.00	1.02	1.31	0.00
time (sec)	N/A	0.421	0.063	0.462	0.033	0.101	0.000	0.154	0.302	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	106	112	78	146	171	0	170	122	0
N.S.	1	1.07	1.13	0.79	1.47	1.73	0.00	1.72	1.23	0.00
time (sec)	N/A	0.438	0.066	0.483	0.033	0.107	0.000	0.254	0.381	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	97	113	78	158	189	0	259	134	0
N.S.	1	0.99	1.15	0.80	1.61	1.93	0.00	2.64	1.37	0.00
time (sec)	N/A	0.404	0.042	0.479	0.035	0.106	0.000	0.158	0.382	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	36	39	176	79	0	311	148	142
N.S.	1	1.00	0.63	0.68	3.09	1.39	0.00	5.46	2.60	2.49
time (sec)	N/A	0.327	0.014	0.471	0.040	0.084	0.000	0.151	0.429	10.155

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	89	57	56	222	105	0	371	188	188
N.S.	1	0.99	0.63	0.62	2.47	1.17	0.00	4.12	2.09	2.09
time (sec)	N/A	0.399	0.019	0.494	0.042	0.096	0.000	0.363	0.381	10.624

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	121	76	73	268	130	0	431	228	234
N.S.	1	0.97	0.61	0.58	2.14	1.04	0.00	3.45	1.82	1.87
time (sec)	N/A	0.437	0.020	0.539	0.038	0.094	0.000	0.126	0.395	11.065

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	254	341	244	560	645	768	345	516	0
N.S.	1	0.65	0.87	0.62	1.42	1.64	1.95	0.88	1.31	0.00
time (sec)	N/A	0.765	1.853	0.637	0.037	0.115	0.552	0.127	0.782	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	221	311	215	488	568	670	304	451	0
N.S.	1	0.63	0.89	0.62	1.40	1.63	1.92	0.87	1.29	0.00
time (sec)	N/A	0.638	1.702	0.590	0.037	0.182	0.634	0.124	0.582	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	192	280	188	416	491	570	261	386	0
N.S.	1	0.65	0.94	0.63	1.40	1.65	1.92	0.88	1.30	0.00
time (sec)	N/A	0.571	1.284	0.548	0.038	0.156	0.472	0.128	0.392	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	178	248	160	343	416	796	220	321	0
N.S.	1	0.72	1.01	0.65	1.39	1.69	3.24	0.89	1.30	0.00
time (sec)	N/A	0.618	1.082	0.542	0.034	0.129	2.968	0.127	0.493	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	171	165	133	273	341	0	178	256	0
N.S.	1	0.84	0.81	0.65	1.34	1.67	0.00	0.87	1.25	0.00
time (sec)	N/A	0.588	0.302	0.530	0.036	0.118	0.000	0.128	0.524	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	155	164	123	227	301	0	165	257	0
N.S.	1	0.85	0.90	0.67	1.24	1.64	0.00	0.90	1.40	0.00
time (sec)	N/A	0.559	0.757	0.533	0.040	0.133	0.000	0.132	0.381	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	157	138	113	228	271	0	214	226	0
N.S.	1	0.88	0.78	0.63	1.28	1.52	0.00	1.20	1.27	0.00
time (sec)	N/A	0.625	0.500	0.532	0.034	0.125	0.000	0.137	0.406	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	140	145	115	241	271	0	331	227	0
N.S.	1	1.07	1.11	0.88	1.84	2.07	0.00	2.53	1.73	0.00
time (sec)	N/A	0.555	0.351	0.551	0.036	0.112	0.000	0.140	0.335	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	1	143	125	279	303	0	454	247	0
N.S.	1	0.01	1.04	0.91	2.02	2.20	0.00	3.29	1.79	0.00
time (sec)	N/A	0.251	0.439	0.586	0.037	0.099	0.000	0.154	0.450	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	138	70	70	323	146	0	541	282	267
N.S.	1	1.34	0.68	0.68	3.14	1.42	0.00	5.25	2.74	2.59
time (sec)	N/A	0.534	0.322	0.561	0.037	0.086	0.000	0.148	0.370	11.135

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	172	101	98	397	185	0	636	347	341
N.S.	1	1.02	0.60	0.58	2.36	1.10	0.00	3.79	2.07	2.03
time (sec)	N/A	0.594	0.410	0.615	0.046	0.092	0.000	0.320	0.443	12.031

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	204	131	126	471	224	0	731	412	415
N.S.	1	1.03	0.66	0.63	2.37	1.13	0.00	3.67	2.07	2.09
time (sec)	N/A	0.659	0.541	0.655	0.042	0.089	0.000	0.137	0.408	12.924

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	540	325	454	351	824	915	1148	484	24	0
N.S.	1	0.60	0.84	0.65	1.53	1.69	2.13	0.90	0.04	0.00
time (sec)	N/A	1.243	2.623	0.720	0.044	0.138	0.557	0.139	200.035	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	306	413	306	724	810	1006	427	670	0
N.S.	1	0.64	0.87	0.64	1.52	1.70	2.11	0.90	1.41	0.00
time (sec)	N/A	1.021	2.201	0.638	0.039	0.111	0.644	0.148	155.057	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	231	370	266	624	707	864	372	580	0
N.S.	1	0.59	0.95	0.68	1.60	1.81	2.21	0.95	1.48	0.00
time (sec)	N/A	0.711	1.818	0.635	0.038	0.108	0.509	0.136	26.378	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	247	330	234	523	602	1129	317	490	0
N.S.	1	0.71	0.95	0.67	1.50	1.72	3.23	0.91	1.40	0.00
time (sec)	N/A	0.917	1.461	0.596	0.039	0.108	3.977	0.256	10.269	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	221	230	197	426	499	0	261	400	0
N.S.	1	0.81	0.84	0.72	1.56	1.83	0.00	0.96	1.47	0.00
time (sec)	N/A	0.902	0.481	0.569	0.038	0.122	0.000	0.156	1.737	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	210	253	175	353	429	0	232	386	0
N.S.	1	0.82	0.99	0.68	1.38	1.68	0.00	0.91	1.51	0.00
time (sec)	N/A	0.912	0.982	0.574	0.039	0.108	0.000	0.148	1.758	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	211	203	161	327	385	0	271	353	0
N.S.	1	0.87	0.84	0.66	1.35	1.58	0.00	1.12	1.45	0.00
time (sec)	N/A	0.923	0.765	0.578	0.040	0.118	0.000	0.151	2.136	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	198	167	150	323	379	0	373	341	0
N.S.	1	1.10	0.93	0.83	1.79	2.11	0.00	2.07	1.89	0.00
time (sec)	N/A	0.896	0.654	0.559	0.037	0.106	0.000	0.150	0.251	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	214	174	162	362	383	0	525	342	0
N.S.	1	1.26	1.02	0.95	2.13	2.25	0.00	3.09	2.01	0.00
time (sec)	N/A	0.823	0.725	0.680	0.037	0.112	0.000	0.151	0.257	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	1	198	178	426	433	0	684	381	0
N.S.	1	0.01	1.05	0.95	2.27	2.30	0.00	3.64	2.03	0.00
time (sec)	N/A	0.252	0.652	0.665	0.042	0.110	0.000	0.182	0.246	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	229	114	112	498	226	0	806	441	420
N.S.	1	1.58	0.79	0.77	3.43	1.56	0.00	5.56	3.04	2.90
time (sec)	N/A	0.899	0.600	0.639	0.040	0.107	0.000	0.140	0.263	13.493

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	261	156	151	600	279	0	936	531	522
N.S.	1	1.18	0.70	0.68	2.70	1.26	0.00	4.22	2.39	2.35
time (sec)	N/A	0.978	0.771	0.667	0.049	0.108	0.000	0.137	0.274	14.724

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	293	197	190	702	331	0	1066	621	624
N.S.	1	1.02	0.69	0.66	2.45	1.16	0.00	3.73	2.17	2.18
time (sec)	N/A	1.060	0.238	0.761	0.056	0.111	0.000	0.141	0.312	16.441

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	299	555	261	0	1172	0	0	579	0
N.S.	1	1.01	1.87	0.88	0.00	3.95	0.00	0.00	1.95	0.00
time (sec)	N/A	1.001	3.323	0.642	0.000	0.565	0.000	0.000	10.250	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	235	500	184	0	896	0	0	456	0
N.S.	1	1.10	2.34	0.86	0.00	4.19	0.00	0.00	2.13	0.00
time (sec)	N/A	0.829	2.814	0.671	0.000	0.358	0.000	0.000	0.266	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	202	449	130	0	669	0	0	352	0
N.S.	1	1.23	2.74	0.79	0.00	4.08	0.00	0.00	2.15	0.00
time (sec)	N/A	0.660	2.264	0.629	0.000	0.186	0.000	0.000	0.254	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	151	162	104	0	523	0	0	279	0
N.S.	1	1.32	1.42	0.91	0.00	4.59	0.00	0.00	2.45	0.00
time (sec)	N/A	0.525	0.378	0.630	0.000	0.147	0.000	0.000	0.226	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	427	418	115	0	523	0	0	269	0
N.S.	1	3.92	3.83	1.06	0.00	4.80	0.00	0.00	2.47	0.00
time (sec)	N/A	1.226	1.769	0.619	0.000	0.137	0.000	0.000	0.244	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	134	383	100	0	225	0	196	282	0
N.S.	1	1.28	3.65	0.95	0.00	2.14	0.00	1.87	2.69	0.00
time (sec)	N/A	0.451	1.618	0.626	0.000	0.101	0.000	0.125	0.254	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	165	160	126	0	318	0	355	392	0
N.S.	1	1.04	1.01	0.79	0.00	2.00	0.00	2.23	2.47	0.00
time (sec)	N/A	0.486	0.567	0.661	0.000	0.089	0.000	0.139	0.303	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	264	203	157	0	433	0	550	509	0
N.S.	1	1.18	0.91	0.70	0.00	1.93	0.00	2.46	2.27	0.00
time (sec)	N/A	0.738	0.813	0.783	0.000	0.096	0.000	0.143	0.383	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	400	334	306	0	1772	0	0	24	0
N.S.	1	1.18	0.98	0.90	0.00	5.21	0.00	0.00	0.07	0.00
time (sec)	N/A	1.049	10.909	0.711	0.000	0.332	0.000	0.000	200.028	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	262	267	236	0	1376	0	0	891	0
N.S.	1	1.02	1.03	0.91	0.00	5.33	0.00	0.00	3.45	0.00
time (sec)	N/A	0.882	10.692	0.681	0.000	0.198	0.000	0.000	29.943	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	210	205	185	0	1008	0	0	719	0
N.S.	1	1.07	1.05	0.94	0.00	5.14	0.00	0.00	3.67	0.00
time (sec)	N/A	0.785	10.510	0.650	0.000	0.148	0.000	0.000	0.348	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	193	166	156	0	767	0	0	601	0
N.S.	1	1.28	1.10	1.03	0.00	5.08	0.00	0.00	3.98	0.00
time (sec)	N/A	0.578	10.441	0.658	0.000	0.121	0.000	0.000	0.272	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	555	176	139	0	684	0	0	530	0
N.S.	1	4.24	1.34	1.06	0.00	5.22	0.00	0.00	4.05	0.00
time (sec)	N/A	1.570	0.662	0.618	0.000	0.116	0.000	0.000	0.246	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	146	832	109	0	246	0	658	624	0
N.S.	1	1.42	8.08	1.06	0.00	2.39	0.00	6.39	6.06	0.00
time (sec)	N/A	0.437	6.379	0.661	0.000	0.091	0.000	0.761	0.297	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	197	1953	143	0	339	0	1345	980	0
N.S.	1	1.21	11.98	0.88	0.00	2.08	0.00	8.25	6.01	0.00
time (sec)	N/A	0.502	11.885	0.695	0.000	0.088	0.000	8.721	0.548	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	273	209	173	0	468	0	0	749	0
N.S.	1	1.28	0.98	0.81	0.00	2.19	0.00	0.00	3.50	0.00
time (sec)	N/A	0.766	0.866	0.770	0.000	0.118	0.000	0.000	0.511	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	428	377	327	0	2129	0	610	24	0
N.S.	1	1.29	1.14	0.98	0.00	6.41	0.00	1.84	0.07	0.00
time (sec)	N/A	1.132	10.950	0.764	0.000	0.238	0.000	0.198	200.039	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	245	273	258	0	1635	0	548	22	0
N.S.	1	0.92	1.03	0.97	0.00	6.15	0.00	2.06	0.08	0.00
time (sec)	N/A	0.835	10.941	0.720	0.000	0.177	0.000	0.191	200.041	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	216	237	221	0	1745	0	497	2312	0
N.S.	1	0.99	1.08	1.01	0.00	7.97	0.00	2.27	10.56	0.00
time (sec)	N/A	0.726	10.886	0.717	0.000	0.152	0.000	0.179	0.896	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	681	189	195	0	1538	0	475	2151	0
N.S.	1	3.80	1.06	1.09	0.00	8.59	0.00	2.65	12.02	0.00
time (sec)	N/A	1.881	10.786	0.641	0.000	0.154	0.000	0.167	0.355	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	143	126	87	0	401	0	388	1056	0
N.S.	1	1.23	1.09	0.75	0.00	3.46	0.00	3.34	9.10	0.00
time (sec)	N/A	0.439	10.272	0.638	0.000	0.098	0.000	0.150	0.960	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	189	173	166	0	562	0	0	1613	0
N.S.	1	1.15	1.05	1.01	0.00	3.41	0.00	0.00	9.78	0.00
time (sec)	N/A	0.500	10.213	0.736	0.000	0.102	0.000	0.000	99.580	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	282	215	207	0	710	0	0	24	0
N.S.	1	1.29	0.98	0.95	0.00	3.24	0.00	0.00	0.11	0.00
time (sec)	N/A	0.793	10.505	0.800	0.000	0.101	0.000	0.000	200.035	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	493	488	702	577	0	1920	0	0	22	0
N.S.	1	0.99	1.42	1.17	0.00	3.89	0.00	0.00	0.04	0.00
time (sec)	N/A	1.575	5.854	0.565	0.000	3.418	0.000	0.000	200.038	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	388	625	476	0	1544	0	0	21	0
N.S.	1	1.02	1.64	1.25	0.00	4.06	0.00	0.00	0.06	0.00
time (sec)	N/A	1.289	4.579	0.558	0.000	2.524	0.000	0.000	200.042	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	355	560	275	0	1216	0	0	699	0
N.S.	1	1.22	1.92	0.95	0.00	4.18	0.00	0.00	2.40	0.00
time (sec)	N/A	0.973	3.804	0.642	0.000	0.700	0.000	0.000	10.785	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	263	499	180	0	932	0	0	570	0
N.S.	1	1.20	2.28	0.82	0.00	4.26	0.00	0.00	2.60	0.00
time (sec)	N/A	0.816	3.180	0.668	0.000	0.291	0.000	0.000	0.289	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	204	191	133	0	722	0	0	477	0
N.S.	1	1.23	1.15	0.80	0.00	4.35	0.00	0.00	2.87	0.00
time (sec)	N/A	0.631	0.504	0.684	0.000	0.208	0.000	0.000	0.266	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	185	443	148	0	685	0	0	466	0
N.S.	1	1.26	3.01	1.01	0.00	4.66	0.00	0.00	3.17	0.00
time (sec)	N/A	0.624	2.430	0.702	0.000	0.193	0.000	0.000	0.289	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	904	174	135	0	698	0	0	471	0
N.S.	1	6.37	1.23	0.95	0.00	4.92	0.00	0.00	3.32	0.00
time (sec)	N/A	2.131	0.579	0.730	0.000	0.197	0.000	0.000	0.285	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	168	421	132	0	327	0	372	507	0
N.S.	1	1.22	3.05	0.96	0.00	2.37	0.00	2.70	3.67	0.00
time (sec)	N/A	0.482	2.033	0.819	0.000	0.087	0.000	0.141	0.408	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	199	209	157	0	448	0	600	631	0
N.S.	1	0.90	0.95	0.71	0.00	2.04	0.00	2.73	2.87	0.00
time (sec)	N/A	0.515	0.755	0.870	0.000	0.086	0.000	0.135	0.512	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	282	238	235	0	1356	0	2074	24	0
N.S.	1	1.09	0.92	0.91	0.00	5.24	0.00	8.01	0.09	0.00
time (sec)	N/A	0.590	10.588	1.304	0.000	0.112	0.000	0.213	200.033	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	150	188	97	206	256	204	134	177	0
N.S.	1	0.93	1.16	0.60	1.27	1.58	1.26	0.83	1.09	0.00
time (sec)	N/A	0.516	0.085	0.497	0.031	0.090	0.491	0.152	0.189	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	118	125	80	160	209	180	106	137	0
N.S.	1	0.93	0.98	0.63	1.26	1.65	1.42	0.83	1.08	0.00
time (sec)	N/A	0.449	0.110	0.482	0.027	0.089	0.476	0.131	0.199	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	75	108	75	115	158	151	80	99	0
N.S.	1	0.82	1.17	0.82	1.25	1.72	1.64	0.87	1.08	0.00
time (sec)	N/A	0.345	0.057	0.434	0.040	0.089	0.563	0.142	0.195	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	91	46	75	119	122	59	64	77
N.S.	1	1.00	1.65	0.84	1.36	2.16	2.22	1.07	1.16	1.40
time (sec)	N/A	0.309	0.044	0.431	0.030	0.081	0.377	0.155	0.198	9.319

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	74	49	49	117	0	59	55	50
N.S.	1	1.00	1.42	0.94	0.94	2.25	0.00	1.13	1.06	0.96
time (sec)	N/A	0.312	0.020	0.424	0.031	0.093	0.000	0.173	0.210	9.595

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	35	32	62	35	0	76	64	33
N.S.	1	1.00	0.61	0.56	1.09	0.61	0.00	1.33	1.12	0.58
time (sec)	N/A	0.329	0.013	0.421	0.036	0.076	0.000	0.146	0.261	9.338

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	89	56	49	106	58	0	133	106	56
N.S.	1	0.99	0.62	0.54	1.18	0.64	0.00	1.48	1.18	0.62
time (sec)	N/A	0.382	0.016	0.433	0.034	0.085	0.000	0.121	0.209	9.547

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	121	76	66	152	82	0	191	148	113
N.S.	1	0.97	0.61	0.53	1.22	0.66	0.00	1.53	1.18	0.90
time (sec)	N/A	0.447	0.018	0.464	0.051	0.077	0.000	0.341	0.212	9.505

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	153	95	83	198	106	0	251	188	146
N.S.	1	0.96	0.59	0.52	1.24	0.66	0.00	1.57	1.18	0.91
time (sec)	N/A	0.510	0.194	0.473	0.031	0.082	0.000	0.161	0.209	9.178

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	207	244	160	368	416	323	217	321	0
N.S.	1	0.83	0.98	0.64	1.48	1.67	1.30	0.87	1.29	0.00
time (sec)	N/A	0.748	0.874	0.623	0.035	0.089	0.543	0.125	0.231	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	175	184	132	294	341	282	175	256	0
N.S.	1	0.87	0.91	0.65	1.46	1.69	1.40	0.87	1.27	0.00
time (sec)	N/A	0.648	0.782	0.537	0.039	0.090	0.498	0.152	0.207	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	147	153	104	220	264	240	134	193	0
N.S.	1	0.95	0.99	0.67	1.42	1.70	1.55	0.86	1.25	0.00
time (sec)	N/A	0.519	0.552	0.507	0.048	0.090	0.592	0.136	0.204	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	113	109	79	154	191	185	96	135	0
N.S.	1	1.05	1.01	0.73	1.43	1.77	1.71	0.89	1.25	0.00
time (sec)	N/A	0.447	0.126	0.488	0.041	0.098	0.506	0.161	0.206	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	92	102	74	103	173	0	90	122	0
N.S.	1	1.12	1.24	0.90	1.26	2.11	0.00	1.10	1.49	0.00
time (sec)	N/A	0.453	0.252	0.490	0.032	0.085	0.000	0.136	0.249	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	1	100	70	97	176	0	118	114	0
N.S.	1	0.01	1.11	0.78	1.08	1.96	0.00	1.31	1.27	0.00
time (sec)	N/A	0.246	0.157	0.501	0.030	0.084	0.000	0.374	0.203	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	138	68	63	136	73	0	167	149	75
N.S.	1	1.34	0.66	0.61	1.32	0.71	0.00	1.62	1.45	0.73
time (sec)	N/A	0.518	0.168	0.492	0.034	0.077	0.000	0.246	0.198	9.128

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	172	100	91	208	109	0	258	215	141
N.S.	1	1.04	0.61	0.55	1.26	0.66	0.00	1.56	1.30	0.85
time (sec)	N/A	0.579	0.213	0.510	0.033	0.087	0.000	1.416	0.201	9.027

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	204	130	119	282	148	0	351	282	188
N.S.	1	1.03	0.65	0.60	1.42	0.74	0.00	1.76	1.42	0.94
time (sec)	N/A	0.659	0.238	0.534	0.042	0.085	0.000	0.125	0.225	8.938

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	217	469	125	0	696	0	0	334	0
N.S.	1	1.29	2.79	0.74	0.00	4.14	0.00	0.00	1.99	0.00
time (sec)	N/A	0.638	3.638	0.633	0.000	0.132	0.000	0.000	0.232	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	156	431	95	0	534	0	0	237	0
N.S.	1	1.36	3.75	0.83	0.00	4.64	0.00	0.00	2.06	0.00
time (sec)	N/A	0.485	3.478	0.641	0.000	0.102	0.000	0.000	0.224	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	104	380	69	0	432	0	0	166	0
N.S.	1	1.20	4.37	0.79	0.00	4.97	0.00	0.00	1.91	0.00
time (sec)	N/A	0.430	1.985	0.547	0.000	0.101	0.000	0.000	0.201	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	68	92	42	0	129	0	58	101	0
N.S.	1	1.31	1.77	0.81	0.00	2.48	0.00	1.12	1.94	0.00
time (sec)	N/A	0.308	0.134	0.539	0.000	0.087	0.000	0.138	0.199	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	104	110	79	0	216	0	89	168	0
N.S.	1	1.33	1.41	1.01	0.00	2.77	0.00	1.14	2.15	0.00
time (sec)	N/A	0.371	0.238	0.575	0.000	0.080	0.000	0.128	0.229	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	151	132	109	0	300	0	145	263	0
N.S.	1	1.29	1.13	0.93	0.00	2.56	0.00	1.24	2.25	0.00
time (sec)	N/A	0.450	0.359	0.627	0.000	0.089	0.000	0.148	0.256	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	205	167	130	0	405	0	235	372	0
N.S.	1	1.23	1.01	0.78	0.00	2.44	0.00	1.42	2.24	0.00
time (sec)	N/A	0.542	0.440	0.635	0.000	0.088	0.000	0.385	0.330	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	236	199	179	0	1151	0	0	873	0
N.S.	1	1.32	1.11	1.00	0.00	6.43	0.00	0.00	4.88	0.00
time (sec)	N/A	0.670	10.635	0.705	0.000	0.166	0.000	0.000	0.321	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	187	175	146	0	851	0	0	709	0
N.S.	1	1.42	1.33	1.11	0.00	6.45	0.00	0.00	5.37	0.00
time (sec)	N/A	0.534	0.833	0.621	0.000	0.118	0.000	0.000	0.319	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	104	590	73	0	297	0	379	293	0
N.S.	1	1.22	6.94	0.86	0.00	3.49	0.00	4.46	3.45	0.00
time (sec)	N/A	0.393	4.574	0.563	0.000	0.096	0.000	0.287	0.223	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	117	137	83	0	344	0	0	518	0
N.S.	1	1.21	1.41	0.86	0.00	3.55	0.00	0.00	5.34	0.00
time (sec)	N/A	0.412	0.364	0.589	0.000	0.091	0.000	0.000	0.206	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	177	157	132	0	488	0	739	712	0
N.S.	1	1.22	1.08	0.91	0.00	3.37	0.00	5.10	4.91	0.00
time (sec)	N/A	0.491	0.804	0.671	0.000	0.095	0.000	0.476	0.267	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	232	204	174	0	646	0	1082	1291	0
N.S.	1	1.12	0.99	0.84	0.00	3.12	0.00	5.23	6.24	0.00
time (sec)	N/A	0.607	1.180	0.698	0.000	0.096	0.000	6.882	0.467	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	306	260	214	0	787	0	0	1494	0
N.S.	1	1.09	0.93	0.76	0.00	2.80	0.00	0.00	5.32	0.00
time (sec)	N/A	0.724	1.793	0.706	0.000	0.108	0.000	0.000	1.813	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	232	204	174	0	646	0	1082	1291	0
N.S.	1	1.12	0.99	0.84	0.00	3.12	0.00	5.23	6.24	0.00
time (sec)	N/A	0.598	0.135	0.632	0.000	0.096	0.000	6.860	0.508	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	232	204	174	0	646	0	1082	1291	0
N.S.	1	1.12	0.99	0.84	0.00	3.12	0.00	5.23	6.24	0.00
time (sec)	N/A	0.652	0.004	0.590	0.000	0.102	0.000	8.533	0.481	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	232	204	174	0	646	0	0	1291	0
N.S.	1	1.12	0.99	0.84	0.00	3.12	0.00	0.00	6.24	0.00
time (sec)	N/A	0.717	0.003	0.499	0.000	0.097	0.000	0.000	0.459	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	232	204	174	0	646	0	0	1291	0
N.S.	1	1.12	0.99	0.84	0.00	3.12	0.00	0.00	6.24	0.00
time (sec)	N/A	0.777	0.004	0.458	0.000	0.105	0.000	0.000	0.463	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	122	125	125	163	260	0	135	152	0
N.S.	1	1.01	1.03	1.03	1.35	2.15	0.00	1.12	1.26	0.00
time (sec)	N/A	0.593	0.096	0.465	0.039	0.107	0.000	0.156	0.193	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	84	103	68	115	205	0	107	122	0
N.S.	1	1.02	1.26	0.83	1.40	2.50	0.00	1.30	1.49	0.00
time (sec)	N/A	0.395	0.062	0.658	0.038	0.095	0.000	0.143	0.190	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	77	63	65	166	0	80	83	64
N.S.	1	1.00	1.28	1.05	1.08	2.77	0.00	1.33	1.38	1.07
time (sec)	N/A	0.321	0.027	0.552	0.032	0.107	0.000	0.206	0.256	9.774

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	33	30	30	55	44	0	33	68	31
N.S.	1	0.69	0.62	0.62	1.15	0.92	0.00	0.69	1.42	0.65
time (sec)	N/A	0.251	0.009	0.423	0.032	0.093	0.000	0.224	0.187	9.289

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	60	54	49	96	69	0	0	93	62
N.S.	1	0.70	0.63	0.57	1.12	0.80	0.00	0.00	1.08	0.72
time (sec)	N/A	0.314	0.013	0.590	0.028	0.080	0.000	0.000	0.200	9.203

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	92	75	66	142	94	0	0	124	87
N.S.	1	0.75	0.61	0.54	1.16	0.77	0.00	0.00	1.02	0.71
time (sec)	N/A	0.372	0.017	0.454	0.033	0.087	0.000	0.000	0.200	9.371

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	124	94	83	188	118	0	0	154	161
N.S.	1	0.79	0.60	0.53	1.20	0.75	0.00	0.00	0.98	1.03
time (sec)	N/A	0.434	0.019	0.520	0.029	0.086	0.000	0.000	0.200	9.711

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	256	246	170	377	503	0	245	326	0
N.S.	1	1.01	0.97	0.67	1.48	1.98	0.00	0.96	1.28	0.00
time (sec)	N/A	1.377	0.859	0.698	0.037	0.095	0.000	0.144	0.226	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	197	181	139	299	420	0	204	275	0
N.S.	1	0.96	0.88	0.68	1.46	2.05	0.00	1.00	1.34	0.00
time (sec)	N/A	0.956	0.705	0.592	0.037	0.128	0.000	0.146	0.197	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	136	151	115	225	337	0	162	226	0
N.S.	1	0.86	0.96	0.73	1.42	2.13	0.00	1.03	1.43	0.00
time (sec)	N/A	0.614	0.455	0.555	0.030	0.208	0.000	0.234	0.194	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	105	125	144	267	0	122	172	0
N.S.	1	1.00	1.17	1.39	1.60	2.97	0.00	1.36	1.91	0.00
time (sec)	N/A	0.422	0.198	0.540	0.043	0.209	0.000	0.135	0.196	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	103	100	82	110	243	0	88	148	96
N.S.	1	1.13	1.10	0.90	1.21	2.67	0.00	0.97	1.63	1.05
time (sec)	N/A	0.430	0.164	0.816	0.038	0.154	0.000	0.143	0.235	9.831

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	105	67	63	124	85	0	0	149	81
N.S.	1	1.07	0.68	0.64	1.27	0.87	0.00	0.00	1.52	0.83
time (sec)	N/A	0.468	0.209	0.526	0.044	0.104	0.000	0.000	0.191	9.611

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	143	100	91	193	121	0	0	184	121
N.S.	1	0.88	0.61	0.56	1.18	0.74	0.00	0.00	1.13	0.74
time (sec)	N/A	0.525	0.216	0.519	0.038	0.138	0.000	0.000	0.206	9.520

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	175	131	121	267	159	0	0	234	217
N.S.	1	0.89	0.67	0.62	1.36	0.81	0.00	0.00	1.19	1.11
time (sec)	N/A	0.585	0.252	0.572	0.034	0.117	0.000	0.000	0.203	9.884

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	202	160	149	341	197	0	0	283	261
N.S.	1	0.89	0.70	0.65	1.50	0.86	0.00	0.00	1.24	1.14
time (sec)	N/A	0.654	0.347	0.579	0.043	0.153	0.000	0.000	0.197	9.261

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	B	A	F	A	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	0	180	157	531	511	0	341	406	0
N.S.	1	0.00	0.89	0.78	2.63	2.53	0.00	1.69	2.01	0.00
time (sec)	N/A	0.000	0.557	0.598	0.043	0.142	0.000	0.160	0.200	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	A	F	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	33	145	120	410	399	0	298	304	0
N.S.	1	0.27	1.19	0.98	3.36	3.27	0.00	2.44	2.49	0.00
time (sec)	N/A	0.285	0.486	0.622	0.048	0.143	0.000	0.209	0.188	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	B	A	F	B	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	0	111	112	295	309	0	235	219	0
N.S.	1	0.00	1.03	1.04	2.73	2.86	0.00	2.18	2.03	0.00
time (sec)	N/A	0.000	0.213	0.510	0.039	0.127	0.000	0.257	0.207	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	66	68	205	94	0	0	216	82
N.S.	1	1.00	1.02	1.05	3.15	1.45	0.00	0.00	3.32	1.26
time (sec)	N/A	0.324	0.171	0.497	0.037	0.104	0.000	0.000	0.192	9.381

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	78	95	94	203	129	0	110	264	111
N.S.	1	0.46	0.56	0.56	1.20	0.76	0.00	0.65	1.56	0.66
time (sec)	N/A	0.329	0.188	0.553	0.035	0.125	0.000	0.185	0.253	9.598

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	143	130	128	250	171	0	0	299	162
N.S.	1	0.76	0.70	0.68	1.34	0.91	0.00	0.00	1.60	0.87
time (sec)	N/A	0.550	0.344	0.517	0.036	0.122	0.000	0.000	0.217	9.746

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	178	161	156	324	208	0	0	351	294
N.S.	1	0.75	0.68	0.66	1.36	0.87	0.00	0.00	1.47	1.24
time (sec)	N/A	0.596	0.329	0.560	0.039	0.131	0.000	0.000	0.214	9.975

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	210	188	184	400	247	0	0	400	378
N.S.	1	0.73	0.65	0.64	1.38	0.85	0.00	0.00	1.38	1.31
time (sec)	N/A	0.677	0.385	0.551	0.037	0.124	0.000	0.000	0.218	9.794

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	227	160	306	0	372	0	341	215	0
N.S.	1	1.14	0.80	1.54	0.00	1.87	0.00	1.71	1.08	0.00
time (sec)	N/A	0.971	0.172	0.438	0.000	0.133	0.000	0.225	0.182	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	172	168	167	0	539	0	378	220	0
N.S.	1	0.93	0.91	0.90	0.00	2.91	0.00	2.04	1.19	0.00
time (sec)	N/A	0.631	0.337	0.582	0.000	0.138	0.000	0.252	0.179	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	128	129	151	0	422	0	278	150	0
N.S.	1	0.89	0.90	1.05	0.00	2.93	0.00	1.93	1.04	0.00
time (sec)	N/A	0.501	0.182	0.539	0.000	0.131	0.000	0.180	0.251	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	107	103	110	0	337	0	210	102	0
N.S.	1	0.98	0.94	1.01	0.00	3.09	0.00	1.93	0.94	0.00
time (sec)	N/A	0.407	0.096	0.528	0.000	0.132	0.000	0.164	0.225	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	99	89	103	0	294	0	204	87	0
N.S.	1	0.97	0.87	1.01	0.00	2.88	0.00	2.00	0.85	0.00
time (sec)	N/A	0.390	0.075	0.536	0.000	0.138	0.000	0.142	0.274	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	157	146	211	0	967	0	373	261	0
N.S.	1	1.01	0.94	1.36	0.00	6.24	0.00	2.41	1.68	0.00
time (sec)	N/A	0.504	0.225	0.560	0.000	0.176	0.000	0.318	0.254	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	210	194	189	0	1294	0	218	393	0
N.S.	1	1.02	0.94	0.92	0.00	6.28	0.00	1.06	1.91	0.00
time (sec)	N/A	0.651	0.456	0.757	0.000	0.296	0.000	1.013	0.217	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	280	254	231	0	1640	0	269	556	0
N.S.	1	0.99	0.90	0.82	0.00	5.80	0.00	0.95	1.96	0.00
time (sec)	N/A	0.800	0.816	0.703	0.000	0.480	0.000	0.590	0.209	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	358	328	286	0	2013	0	401	721	0
N.S.	1	0.98	0.89	0.78	0.00	5.49	0.00	1.09	1.96	0.00
time (sec)	N/A	0.938	0.767	0.847	0.000	1.126	0.000	0.651	0.220	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	74	46	52	0	134	0	52	90	0
N.S.	1	1.80	1.12	1.27	0.00	3.27	0.00	1.27	2.20	0.00
time (sec)	N/A	0.336	0.046	0.537	0.000	0.127	0.000	0.147	0.212	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	374	310	571	0	517	0	0	873	0
N.S.	1	0.99	0.82	1.52	0.00	1.38	0.00	0.00	2.32	0.00
time (sec)	N/A	0.818	16.309	1.554	0.000	0.185	0.000	0.000	3.144	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	293	238	427	0	425	0	0	487	0
N.S.	1	0.99	0.80	1.44	0.00	1.44	0.00	0.00	1.65	0.00
time (sec)	N/A	0.659	12.117	1.276	0.000	0.141	0.000	0.000	1.835	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	233	208	352	0	356	0	0	469	0
N.S.	1	1.11	1.00	1.68	0.00	1.70	0.00	0.00	2.24	0.00
time (sec)	N/A	0.542	9.087	0.640	0.000	0.114	0.000	0.000	1.509	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	231	160	280	0	356	0	0	40	0
N.S.	1	2.57	1.78	3.11	0.00	3.96	0.00	0.00	0.44	0.00
time (sec)	N/A	0.619	5.283	1.490	0.000	0.084	0.000	0.000	0.791	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	243	186	277	0	379	0	0	43	0
N.S.	1	1.15	0.88	1.31	0.00	1.79	0.00	0.00	0.20	0.00
time (sec)	N/A	0.626	9.375	2.068	0.000	0.103	0.000	0.000	1.569	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	356	221	501	0	482	0	0	45	0
N.S.	1	1.17	0.73	1.65	0.00	1.59	0.00	0.00	0.15	0.00
time (sec)	N/A	0.788	11.612	2.243	0.000	0.115	0.000	0.000	2.963	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	459	276	592	0	600	0	0	45	0
N.S.	1	1.19	0.72	1.54	0.00	1.56	0.00	0.00	0.12	0.00
time (sec)	N/A	1.023	15.081	3.078	0.000	0.099	0.000	0.000	3.540	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	442	382	589	0	641	0	0	873	0
N.S.	1	1.09	0.94	1.45	0.00	1.58	0.00	0.00	2.15	0.00
time (sec)	N/A	0.936	20.494	1.747	0.000	0.125	0.000	0.000	3.459	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	352	310	474	0	532	0	0	487	0
N.S.	1	1.11	0.97	1.49	0.00	1.67	0.00	0.00	1.53	0.00
time (sec)	N/A	0.802	14.776	1.658	0.000	0.127	0.000	0.000	1.966	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	277	240	404	0	439	0	0	62	0
N.S.	1	1.14	0.99	1.66	0.00	1.81	0.00	0.00	0.26	0.00
time (sec)	N/A	0.639	11.253	1.539	0.000	0.117	0.000	0.000	0.972	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	159	145	360	0	382	0	0	61	0
N.S.	1	0.79	0.72	1.79	0.00	1.90	0.00	0.00	0.30	0.00
time (sec)	N/A	0.428	6.639	1.566	0.000	0.087	0.000	0.000	1.140	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	152	130	218	0	382	0	0	71	0
N.S.	1	0.74	0.63	1.06	0.00	1.86	0.00	0.00	0.35	0.00
time (sec)	N/A	0.501	6.559	2.178	0.000	0.086	0.000	0.000	1.923	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	296	220	471	0	493	0	0	329	0
N.S.	1	1.20	0.89	1.91	0.00	2.00	0.00	0.00	1.33	0.00
time (sec)	N/A	0.734	10.948	2.689	0.000	0.084	0.000	0.000	1.729	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	416	276	537	0	616	0	0	741	0
N.S.	1	1.30	0.87	1.68	0.00	1.93	0.00	0.00	2.32	0.00
time (sec)	N/A	0.901	13.839	3.337	0.000	0.098	0.000	0.000	2.996	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	520	348	622	0	743	0	0	1136	0
N.S.	1	1.28	0.86	1.53	0.00	1.83	0.00	0.00	2.80	0.00
time (sec)	N/A	1.068	20.992	3.971	0.000	0.125	0.000	0.000	4.678	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	52	57	52	0	20	0	0	28	0
N.S.	1	2.48	2.71	2.48	0.00	0.95	0.00	0.00	1.33	0.00
time (sec)	N/A	0.374	20.058	0.568	0.000	0.082	0.000	0.000	0.239	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	38	115	66	0	20	0	0	27	0
N.S.	1	1.81	5.48	3.14	0.00	0.95	0.00	0.00	1.29	0.00
time (sec)	N/A	0.403	2.990	0.429	0.000	0.079	0.000	0.000	0.737	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	0	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.382	0.228	0.000	0.000	0.000	0.000	0.000	1.282	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	209	78	0	0	0	0	0	0	0
N.S.	1	2.68	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.585	0.169	0.000	0.000	0.000	0.000	0.000	0.556	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	103	76	0	0	0	0	0	0	0
N.S.	1	1.36	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.237	0.141	0.000	0.000	0.000	0.000	0.000	0.289	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	0	0	0	0	0	1580	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	22.25	0.00
time (sec)	N/A	0.371	0.150	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	76	0	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.377	0.174	0.000	0.000	0.000	0.000	0.000	0.311	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	232	80	80	0	0	0	0	0	0	0
N.S.	1	0.34	0.34	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.384	0.203	0.000	0.000	0.000	0.000	0.000	0.565	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	219	80	80	0	0	0	0	0	0	0
N.S.	1	0.37	0.37	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.386	0.195	0.000	0.000	0.000	0.000	0.000	0.341	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	166	200	132	0	0	0	0	0	0	0
N.S.	1	1.20	0.80	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.482	0.125	0.000	0.000	0.000	0.000	0.000	0.234	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	99	78	0	0	0	0	0	0	0
N.S.	1	1.21	0.95	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.319	0.139	0.000	0.000	0.000	0.000	0.000	0.224	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	165	73	73	0	0	0	0	0	495	0
N.S.	1	0.44	0.44	0.00	0.00	0.00	0.00	0.00	3.00	0.00
time (sec)	N/A	0.367	0.150	0.000	0.000	0.000	0.000	0.000	0.232	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	217	81	78	0	0	0	0	0	0	0
N.S.	1	0.37	0.36	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.400	0.163	0.000	0.000	0.000	0.000	0.000	0.239	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	57	0	0	0	0	0	0	0
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.373	0.154	0.000	0.000	0.000	0.000	0.000	0.222	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	124	75	75	98	81	0	120	69	84
N.S.	1	0.96	0.58	0.58	0.76	0.63	0.00	0.93	0.53	0.65
time (sec)	N/A	0.525	0.022	0.684	0.042	0.093	0.000	0.128	0.218	9.468

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	90	55	56	75	56	0	93	45	62
N.S.	1	0.98	0.60	0.61	0.82	0.61	0.00	1.01	0.49	0.67
time (sec)	N/A	0.420	0.013	0.645	0.043	0.100	0.000	0.129	0.216	9.207

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	36	36	45	34	0	65	24	40
N.S.	1	1.00	0.62	0.62	0.78	0.59	0.00	1.12	0.41	0.69
time (sec)	N/A	0.340	0.008	0.539	0.043	0.086	0.000	0.130	0.215	8.976

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	56	66	27	0	137	0	77	51	0
N.S.	1	0.97	1.14	0.47	0.00	2.36	0.00	1.33	0.88	0.00
time (sec)	N/A	0.376	0.017	0.438	0.000	0.094	0.000	0.134	0.225	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	62	68	38	0	148	0	60	90	0
N.S.	1	0.97	1.06	0.59	0.00	2.31	0.00	0.94	1.41	0.00
time (sec)	N/A	0.365	0.021	0.488	0.000	0.092	0.000	0.145	0.206	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	97	93	43	0	197	0	115	132	0
N.S.	1	0.92	0.89	0.41	0.00	1.88	0.00	1.10	1.26	0.00
time (sec)	N/A	0.434	0.033	0.494	0.000	0.088	0.000	0.153	0.213	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	131	116	64	0	246	0	131	170	0
N.S.	1	0.92	0.82	0.45	0.00	1.73	0.00	0.92	1.20	0.00
time (sec)	N/A	0.537	0.049	0.504	0.000	0.090	0.000	0.163	0.229	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	165	135	82	0	295	0	180	206	0
N.S.	1	0.92	0.75	0.46	0.00	1.65	0.00	1.01	1.15	0.00
time (sec)	N/A	0.603	0.050	0.523	0.000	0.092	0.000	0.173	0.207	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	169	126	136	168	147	0	200	139	153
N.S.	1	0.90	0.67	0.72	0.89	0.78	0.00	1.06	0.74	0.81
time (sec)	N/A	0.553	0.091	0.842	0.048	0.076	0.000	0.131	0.217	9.632

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	129	96	108	134	108	0	163	98	114
N.S.	1	0.93	0.69	0.78	0.96	0.78	0.00	1.17	0.71	0.82
time (sec)	N/A	0.493	0.087	0.799	0.045	0.080	0.000	0.135	0.217	9.741

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	95	67	72	93	73	0	122	61	73
N.S.	1	0.99	0.70	0.75	0.97	0.76	0.00	1.27	0.64	0.76
time (sec)	N/A	0.413	0.047	0.559	0.047	0.075	0.000	0.129	0.229	9.100

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	113	87	63	0	188	0	138	91	0
N.S.	1	1.16	0.90	0.65	0.00	1.94	0.00	1.42	0.94	0.00
time (sec)	N/A	0.506	0.097	0.523	0.000	0.094	0.000	0.117	0.207	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	105	85	57	0	197	0	81	123	0
N.S.	1	1.13	0.91	0.61	0.00	2.12	0.00	0.87	1.32	0.00
time (sec)	N/A	0.410	0.123	0.535	0.000	0.090	0.000	0.148	0.214	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	129	107	64	0	229	0	133	191	0
N.S.	1	1.07	0.88	0.53	0.00	1.89	0.00	1.10	1.58	0.00
time (sec)	N/A	0.455	0.183	0.527	0.000	0.092	0.000	0.152	0.222	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	155	140	79	0	302	0	208	251	0
N.S.	1	0.91	0.82	0.46	0.00	1.78	0.00	1.22	1.48	0.00
time (sec)	N/A	0.492	0.232	0.550	0.000	0.100	0.000	0.189	0.204	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	184	174	104	0	379	0	278	312	0
N.S.	1	0.83	0.79	0.47	0.00	1.71	0.00	1.26	1.41	0.00
time (sec)	N/A	0.531	0.348	0.570	0.000	0.095	0.000	0.191	0.211	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	160	137	138	0	420	0	0	212	0
N.S.	1	0.94	0.81	0.81	0.00	2.47	0.00	0.00	1.25	0.00
time (sec)	N/A	0.567	0.225	0.707	0.000	0.089	0.000	0.000	0.210	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	120	105	102	0	316	0	0	134	0
N.S.	1	0.99	0.87	0.84	0.00	2.61	0.00	0.00	1.11	0.00
time (sec)	N/A	0.491	0.204	0.622	0.000	0.092	0.000	0.000	0.206	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	92	88	75	0	234	0	0	79	0
N.S.	1	1.10	1.05	0.89	0.00	2.79	0.00	0.00	0.94	0.00
time (sec)	N/A	0.386	0.085	0.582	0.000	0.093	0.000	0.000	0.203	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	72	70	54	0	144	0	79	50	0
N.S.	1	1.29	1.25	0.96	0.00	2.57	0.00	1.41	0.89	0.00
time (sec)	N/A	0.325	0.045	0.582	0.000	0.089	0.000	0.126	0.199	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	103	114	37	0	455	0	152	122	0
N.S.	1	1.08	1.20	0.39	0.00	4.79	0.00	1.60	1.28	0.00
time (sec)	N/A	0.461	0.094	0.569	0.000	0.106	0.000	0.133	0.219	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	143	156	60	0	604	0	0	216	0
N.S.	1	1.12	1.22	0.47	0.00	4.72	0.00	0.00	1.69	0.00
time (sec)	N/A	0.497	0.211	0.609	0.000	0.118	0.000	0.000	0.215	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	202	197	85	0	774	0	0	346	0
N.S.	1	1.09	1.06	0.46	0.00	4.18	0.00	0.00	1.87	0.00
time (sec)	N/A	0.635	0.441	0.664	0.000	0.156	0.000	0.000	0.213	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	266	247	116	0	985	0	0	489	0
N.S.	1	1.06	0.99	0.46	0.00	3.94	0.00	0.00	1.96	0.00
time (sec)	N/A	0.742	0.639	0.724	0.000	0.253	0.000	0.000	0.214	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	157	134	111	0	503	0	0	329	0
N.S.	1	1.16	0.99	0.82	0.00	3.73	0.00	0.00	2.44	0.00
time (sec)	N/A	0.513	0.380	0.656	0.000	0.117	0.000	0.000	0.208	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	113	117	94	0	378	0	0	244	0
N.S.	1	1.08	1.11	0.90	0.00	3.60	0.00	0.00	2.32	0.00
time (sec)	N/A	0.433	0.231	0.611	0.000	0.094	0.000	0.000	0.211	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	101	107	77	0	333	0	0	156	0
N.S.	1	1.09	1.15	0.83	0.00	3.58	0.00	0.00	1.68	0.00
time (sec)	N/A	0.369	0.173	0.598	0.000	0.091	0.000	0.000	0.206	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	170	142	68	0	941	0	334	516	0
N.S.	1	1.17	0.98	0.47	0.00	6.49	0.00	2.30	3.56	0.00
time (sec)	N/A	0.553	0.517	0.609	0.000	0.130	0.000	0.424	0.214	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	222	169	99	0	1256	0	0	743	0
N.S.	1	1.14	0.87	0.51	0.00	6.44	0.00	0.00	3.81	0.00
time (sec)	N/A	0.671	0.636	0.710	0.000	0.284	0.000	0.000	0.200	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	296	240	123	0	1598	0	0	971	0
N.S.	1	1.10	0.89	0.46	0.00	5.92	0.00	0.00	3.60	0.00
time (sec)	N/A	0.826	1.142	0.789	0.000	0.317	0.000	0.000	0.247	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	388	307	155	0	1979	0	0	1238	0
N.S.	1	1.07	0.85	0.43	0.00	5.47	0.00	0.00	3.42	0.00
time (sec)	N/A	1.009	1.699	0.756	0.000	0.659	0.000	0.000	0.269	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	246	202	170	0	0	563	0	0	299	0
N.S.	1	0.82	0.69	0.00	0.00	2.29	0.00	0.00	1.22	0.00
time (sec)	N/A	0.610	0.296	0.000	0.000	0.110	0.000	0.000	0.294	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	172	141	116	0	0	410	0	0	173	0
N.S.	1	0.82	0.67	0.00	0.00	2.38	0.00	0.00	1.01	0.00
time (sec)	N/A	0.494	0.257	0.000	0.000	0.111	0.000	0.000	0.246	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	94	83	0	0	289	0	0	80	0
N.S.	1	0.80	0.70	0.00	0.00	2.45	0.00	0.00	0.68	0.00
time (sec)	N/A	0.391	0.056	0.000	0.000	0.096	0.000	0.000	0.207	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	32	33	27	46	0	0	18	0
N.S.	1	1.00	0.89	0.92	0.75	1.28	0.00	0.00	0.50	0.00
time (sec)	N/A	0.281	0.010	0.460	0.046	0.077	0.000	0.000	0.222	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	116	110	0	0	439	0	0	103	0
N.S.	1	0.84	0.80	0.00	0.00	3.18	0.00	0.00	0.75	0.00
time (sec)	N/A	0.446	0.081	0.000	0.000	0.098	0.000	0.000	0.212	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	210	200	158	0	0	845	0	0	377	0
N.S.	1	0.95	0.75	0.00	0.00	4.02	0.00	0.00	1.80	0.00
time (sec)	N/A	0.659	0.389	0.000	0.000	0.119	0.000	0.000	0.208	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	299	248	218	0	0	1473	0	0	865	0
N.S.	1	0.83	0.73	0.00	0.00	4.93	0.00	0.00	2.89	0.00
time (sec)	N/A	0.649	0.763	0.000	0.000	0.212	0.000	0.000	0.225	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	100	111	0	0	108	0	0	42	0
N.S.	1	1.01	1.12	0.00	0.00	1.09	0.00	0.00	0.42	0.00
time (sec)	N/A	0.405	4.917	0.000	0.000	0.082	0.000	0.000	0.307	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	113	0	0	109	0	0	45	0
N.S.	1	1.00	1.14	0.00	0.00	1.10	0.00	0.00	0.45	0.00
time (sec)	N/A	0.405	4.939	0.000	0.000	0.076	0.000	0.000	0.311	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	120	0	0	109	0	0	45	0
N.S.	1	1.00	1.21	0.00	0.00	1.10	0.00	0.00	0.45	0.00
time (sec)	N/A	0.405	4.031	0.000	0.000	0.079	0.000	0.000	0.312	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	122	0	0	110	0	0	46	0
N.S.	1	1.00	1.22	0.00	0.00	1.10	0.00	0.00	0.46	0.00
time (sec)	N/A	0.405	3.995	0.000	0.000	0.085	0.000	0.000	0.331	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	64	90	0	0	11	0	0	33	0
N.S.	1	1.10	1.55	0.00	0.00	0.19	0.00	0.00	0.57	0.00
time (sec)	N/A	0.320	2.818	0.000	0.000	0.080	0.000	0.000	0.244	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	73	0	0	11	0	0	35	0
N.S.	1	1.00	1.28	0.00	0.00	0.19	0.00	0.00	0.61	0.00
time (sec)	N/A	0.332	2.370	0.000	0.000	0.083	0.000	0.000	0.315	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	94	0	0	11	0	0	35	0
N.S.	1	1.00	1.62	0.00	0.00	0.19	0.00	0.00	0.60	0.00
time (sec)	N/A	0.329	2.408	0.000	0.000	0.076	0.000	0.000	0.316	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	68	0	0	11	0	0	33	0
N.S.	1	1.00	1.19	0.00	0.00	0.19	0.00	0.00	0.58	0.00
time (sec)	N/A	0.321	2.458	0.000	0.000	0.084	0.000	0.000	0.249	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	92	0	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.426	0.393	0.000	0.000	0.000	0.000	0.000	0.776	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	86	0	0	0	0	0	28	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.413	0.259	0.000	0.000	0.000	0.000	0.000	200.028	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	86	0	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.403	0.218	0.000	0.000	0.000	0.000	0.000	44.533	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	84	0	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.413	0.193	0.000	0.000	0.000	0.000	0.000	0.398	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	80	0	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.390	0.185	0.000	0.000	0.000	0.000	0.000	0.264	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	84	0	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.405	0.207	0.000	0.000	0.000	0.000	0.000	0.434	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [100] had the largest ratio of [.7083329999999999990]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	6	0.81	22	0.273
2	A	5	4	0.70	20	0.200
3	A	5	4	0.79	19	0.211
4	A	5	4	0.88	22	0.182
5	A	5	4	1.10	22	0.182
6	A	7	6	1.01	22	0.273
7	A	2	2	1.00	22	0.091
8	A	3	3	0.99	22	0.136
9	A	4	4	0.97	22	0.182
10	A	9	8	0.74	24	0.333
11	A	7	6	0.74	22	0.273
12	A	7	6	0.79	21	0.286
13	A	7	6	0.89	24	0.250
14	A	7	6	0.92	24	0.250
15	A	9	8	1.16	24	0.333
16	A	1	1	0.01	24	0.042
17	A	5	5	1.34	24	0.208
18	A	6	6	1.04	24	0.250
19	A	7	7	1.03	24	0.292
20	A	9	8	0.77	22	0.364
21	A	7	6	0.71	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	9	8	0.92	24	0.333
23	A	9	8	0.92	24	0.333
24	A	11	10	1.14	24	0.417
25	A	4	4	1.31	24	0.167
26	A	1	1	0.01	24	0.042
27	A	8	8	1.58	24	0.333
28	A	9	9	1.18	24	0.375
29	A	10	10	1.02	24	0.417
30	A	13	12	1.25	24	0.500
31	A	8	7	1.07	22	0.318
32	A	7	6	1.22	21	0.286
33	A	2	2	1.68	24	0.083
34	A	5	4	1.35	24	0.167
35	A	6	5	1.18	24	0.208
36	A	10	9	1.21	24	0.375
37	A	12	11	1.20	24	0.458
38	A	14	13	1.24	24	0.542
39	A	7	6	1.09	22	0.273
40	A	7	6	1.21	21	0.286
41	A	5	4	1.38	24	0.167
42	A	6	5	1.39	24	0.208
43	A	10	9	1.30	24	0.375
44	A	12	11	1.25	24	0.458
45	A	15	14	1.13	24	0.583
46	A	14	13	1.22	24	0.542
47	A	8	7	1.15	22	0.318
48	A	4	3	0.95	21	0.143
49	A	6	5	1.25	24	0.208
50	A	10	9	1.09	24	0.375
51	A	12	11	1.08	24	0.458
52	A	2	2	1.68	24	0.083
53	A	3	3	1.68	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	F	0	0	N/A	0.000	N/A
55	A	4	4	1.68	23	0.174
56	A	8	7	0.75	22	0.318
57	A	6	5	0.66	20	0.250
58	A	6	5	0.71	19	0.263
59	A	6	5	0.78	22	0.227
60	A	6	5	0.97	22	0.227
61	A	6	5	0.98	22	0.227
62	A	8	7	1.07	22	0.318
63	A	8	7	0.99	22	0.318
64	A	2	2	1.00	22	0.091
65	A	3	3	0.99	22	0.136
66	A	4	4	0.97	22	0.182
67	A	10	9	0.65	24	0.375
68	A	8	7	0.63	22	0.318
69	A	8	7	0.65	21	0.333
70	A	8	7	0.72	24	0.292
71	A	8	7	0.84	24	0.292
72	A	8	7	0.85	24	0.292
73	A	10	9	0.88	24	0.375
74	A	10	9	1.07	24	0.375
75	A	1	1	0.01	24	0.042
76	A	5	5	1.34	24	0.208
77	A	6	6	1.02	24	0.250
78	A	7	7	1.03	24	0.292
79	A	12	11	0.60	24	0.458
80	A	10	9	0.64	22	0.409
81	A	8	7	0.59	21	0.333
82	A	10	9	0.71	24	0.375
83	A	10	9	0.81	24	0.375
84	A	10	9	0.82	24	0.375
85	A	12	11	0.87	24	0.458

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	12	11	1.10	24	0.458
87	A	4	4	1.26	24	0.167
88	A	1	1	0.01	24	0.042
89	A	8	8	1.58	24	0.333
90	A	9	9	1.18	24	0.375
91	A	10	10	1.02	24	0.417
92	A	10	9	1.01	22	0.409
93	A	9	8	1.10	21	0.381
94	A	11	10	1.23	24	0.417
95	A	9	8	1.32	24	0.333
96	B	2	2	3.92	24	0.083
97	A	6	5	1.28	24	0.208
98	A	7	6	1.04	24	0.250
99	A	12	11	1.18	24	0.458
100	A	18	17	1.18	24	0.708
101	A	9	8	1.02	22	0.364
102	A	10	9	1.07	21	0.429
103	A	11	10	1.28	24	0.417
104	B	2	2	4.24	24	0.083
105	A	6	5	1.42	24	0.208
106	A	7	6	1.21	24	0.250
107	A	12	11	1.28	24	0.458
108	A	18	17	1.29	24	0.708
109	A	9	8	0.92	22	0.364
110	A	8	7	0.99	21	0.333
111	B	2	2	3.80	24	0.083
112	A	6	5	1.23	24	0.208
113	A	7	6	1.15	24	0.250
114	A	12	11	1.29	24	0.458
115	A	12	11	0.99	22	0.500
116	A	11	10	1.02	21	0.476
117	A	15	14	1.22	24	0.583

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	13	12	1.20	24	0.500
119	A	11	10	1.23	24	0.417
120	A	11	10	1.26	24	0.417
121	B	2	2	6.37	24	0.083
122	A	7	6	1.22	24	0.250
123	A	8	7	0.90	24	0.292
124	A	9	8	1.09	24	0.333
125	A	7	6	0.93	22	0.273
126	A	6	5	0.93	22	0.227
127	A	4	3	0.82	20	0.150
128	A	4	3	1.00	19	0.158
129	A	4	3	1.00	22	0.136
130	A	2	2	1.00	22	0.091
131	A	3	3	0.99	22	0.136
132	A	4	4	0.97	22	0.182
133	A	5	5	0.96	22	0.227
134	A	9	8	0.83	24	0.333
135	A	8	7	0.87	24	0.292
136	A	6	5	0.95	22	0.227
137	A	6	5	1.05	21	0.238
138	A	6	5	1.12	24	0.208
139	A	1	1	0.01	24	0.042
140	A	5	5	1.34	24	0.208
141	A	6	6	1.04	24	0.250
142	A	7	7	1.03	24	0.292
143	A	11	10	1.29	24	0.417
144	A	9	8	1.36	24	0.333
145	A	6	5	1.20	22	0.227
146	A	3	2	1.31	21	0.095
147	A	5	4	1.33	24	0.167
148	A	8	7	1.29	24	0.292
149	A	10	9	1.23	24	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	11	10	1.32	24	0.417
151	A	9	8	1.42	24	0.333
152	A	4	3	1.22	22	0.136
153	A	4	3	1.21	21	0.143
154	A	8	7	1.22	24	0.292
155	A	10	9	1.12	24	0.375
156	A	12	11	1.09	24	0.458
157	A	10	9	1.12	24	0.375
158	A	11	10	1.12	22	0.455
159	A	13	12	1.12	38	0.316
160	A	13	12	1.12	36	0.333
161	A	8	7	1.01	22	0.318
162	A	5	4	1.02	22	0.182
163	A	5	4	1.00	20	0.200
164	A	1	1	0.69	19	0.053
165	A	2	2	0.70	22	0.091
166	A	3	3	0.75	22	0.136
167	A	4	4	0.79	22	0.182
168	A	11	10	1.01	24	0.417
169	A	10	9	0.96	24	0.375
170	A	7	6	0.86	24	0.250
171	A	6	5	1.00	22	0.227
172	A	7	6	1.13	21	0.286
173	A	4	4	1.07	24	0.167
174	A	5	5	0.88	24	0.208
175	A	6	6	0.89	24	0.250
176	A	7	7	0.89	24	0.292
177	F	0	0	N/A	0.000	N/A
178	C	2	2	0.27	24	0.083
179	F	0	0	N/A	0.000	N/A
180	A	2	2	1.00	22	0.091
181	A	2	2	0.46	21	0.095

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	5	5	0.76	24	0.208
183	A	6	6	0.75	24	0.250
184	A	7	7	0.73	24	0.292
185	A	9	8	1.14	28	0.286
186	A	3	3	0.93	28	0.107
187	A	3	3	0.89	28	0.107
188	A	5	4	0.98	28	0.143
189	A	5	4	0.97	28	0.143
190	A	8	7	1.01	28	0.250
191	A	10	9	1.02	28	0.321
192	A	12	11	0.99	28	0.393
193	A	14	13	0.98	28	0.464
194	A	7	6	1.80	26	0.231
195	A	12	12	0.99	26	0.462
196	A	10	10	0.99	26	0.385
197	A	8	8	1.11	26	0.308
198	B	8	8	2.57	24	0.333
199	A	8	8	1.15	23	0.348
200	A	12	12	1.17	26	0.462
201	A	14	14	1.19	26	0.538
202	A	12	12	1.09	26	0.462
203	A	10	10	1.11	26	0.385
204	A	8	8	1.14	26	0.308
205	A	5	5	0.79	26	0.192
206	A	5	5	0.74	24	0.208
207	A	8	8	1.20	23	0.348
208	A	12	12	1.30	26	0.462
209	A	14	14	1.28	26	0.538
210	B	8	7	2.48	20	0.350
211	A	9	8	1.81	22	0.364
212	A	4	4	1.00	22	0.182
213	B	4	3	2.68	20	0.150

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	3	2	1.36	19	0.105
215	A	4	4	1.00	22	0.182
216	A	4	4	1.00	22	0.182
217	C	4	4	0.34	23	0.174
218	C	4	4	0.37	23	0.174
219	A	5	4	1.20	21	0.190
220	A	4	3	1.21	20	0.150
221	C	4	4	0.44	23	0.174
222	C	4	4	0.37	23	0.174
223	A	4	3	1.00	20	0.150
224	A	4	4	0.96	24	0.167
225	A	3	3	0.98	24	0.125
226	A	2	2	1.00	22	0.091
227	A	2	2	0.97	21	0.095
228	A	4	3	0.97	24	0.125
229	A	5	4	0.92	24	0.167
230	A	6	5	0.92	24	0.208
231	A	7	6	0.92	24	0.250
232	A	3	3	0.90	26	0.115
233	A	3	3	0.93	26	0.115
234	A	3	3	0.99	24	0.125
235	A	2	2	1.16	23	0.087
236	A	7	6	1.13	26	0.231
237	A	7	6	1.07	26	0.231
238	A	8	7	0.91	26	0.269
239	A	9	8	0.83	26	0.308
240	A	3	3	0.94	26	0.115
241	A	3	3	0.99	26	0.115
242	A	5	4	1.10	26	0.154
243	A	4	3	1.29	24	0.125
244	A	6	5	1.08	23	0.217
245	A	8	7	1.12	26	0.269

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	10	9	1.09	26	0.346
247	A	12	11	1.06	26	0.423
248	A	7	6	1.16	26	0.231
249	A	5	4	1.08	26	0.154
250	A	5	4	1.09	24	0.167
251	A	8	7	1.17	23	0.304
252	A	10	9	1.14	26	0.346
253	A	12	11	1.10	26	0.423
254	A	14	13	1.07	26	0.500
255	A	7	6	0.82	36	0.167
256	A	7	6	0.82	36	0.167
257	A	5	4	0.80	34	0.118
258	A	2	2	1.00	27	0.074
259	A	5	4	0.84	36	0.111
260	A	7	6	0.95	36	0.167
261	A	7	6	0.83	36	0.167
262	A	3	3	1.01	36	0.083
263	A	3	3	1.00	37	0.081
264	A	3	3	1.00	37	0.081
265	A	3	3	1.00	38	0.079
266	C	2	2	1.10	36	0.056
267	A	2	2	1.00	36	0.056
268	A	2	2	1.00	36	0.056
269	A	2	2	1.00	36	0.056
270	A	4	4	1.00	28	0.143
271	A	4	4	1.00	26	0.154
272	A	4	4	1.00	24	0.167
273	A	4	4	1.00	23	0.174
274	A	4	4	1.00	26	0.154
275	A	4	4	1.00	26	0.154

CHAPTER 3

Listing of integrals

3.1 $\int x^2(c+dx)\sqrt{ax+bx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 197

$$\int x^2(c+dx)\sqrt{ax+bx^2}\,dx = \frac{a^3(10bc-7ad)\sqrt{ax+bx^2}}{128b^4} - \frac{a^2(10bc-7ad)x\sqrt{ax+bx^2}}{192b^3} + \frac{a(10bc-7ad)x^2\sqrt{ax+bx^2}}{240b^2} + \frac{(10bc-7ad)x^3\sqrt{ax+bx^2}}{40b} + \frac{dx^2(ax+bx^2)^{3/2}}{5b} - \frac{a^4(10bc-7ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{128b^{9/2}}$$

output
```
1/128*a^3*(-7*a*d+10*b*c)*(b*x^2+a*x)^(1/2)/b^4-1/192*a^2*(-7*a*d+10*b*c)*
x*(b*x^2+a*x)^(1/2)/b^3+1/240*a*(-7*a*d+10*b*c)*x^2*(b*x^2+a*x)^(1/2)/b^2+
1/40*(-7*a*d+10*b*c)*x^3*(b*x^2+a*x)^(1/2)/b+1/5*d*x^2*(b*x^2+a*x)^(3/2)/b
-1/128*a^4*(-7*a*d+10*b*c)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.02

$$\int x^2(c+dx)\sqrt{ax+bx^2}\,dx$$
$$=\frac{\sqrt{x}\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\sqrt{a+bx}(-105a^4d+16ab^3x^2(5c+3dx)+96b^4x^3(5c+4dx)+10a^3b(15c+7dx)-4\right.}{1920b^{9/2}\sqrt{x(a+bx)}}$$

input
```
Integrate[x^2*(c + d*x)*Sqrt[a*x + b*x^2],x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-105*a^4*d + 16*a*b
^3*x^2*(5*c + 3*d*x) + 96*b^4*x^3*(5*c + 4*d*x) + 10*a^3*b*(15*c + 7*d*x)
- 4*a^2*b^2*x*(25*c + 14*d*x)) + 300*a^4*b*c*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sq
rt[a] - Sqrt[a + b*x])] + 210*a^5*d*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] +
Sqrt[a + b*x])]))/(1920*b^(9/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1221, 1134, 1160, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{ax+bx^2}(c+dx)\,dx$$
$$\Big\downarrow \text{1221}$$
$$\frac{(10bc-7ad)\int x^2\sqrt{bx^2+ax}dx}{10b}+\frac{dx^2\left(ax+bx^2\right)^{3/2}}{5b}$$
$$\Big\downarrow \text{1134}$$
$$\frac{(10bc-7ad)\left(\frac{x\left(ax+bx^2\right)^{3/2}}{4b}-\frac{5a\int x\sqrt{bx^2+ax}dx}{8b}\right)}{10b}+\frac{dx^2\left(ax+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{1160}$$

$$\frac{(10bc-7ad)\left(\frac{x\left(ax+bx^2\right)^{3/2}}{4b}-\frac{5a\left(\frac{\left(ax+bx^2\right)^{3/2}}{3b}-\frac{a\int\sqrt{bx^2+ax}dx}{2b}\right)}{8b}\right)}{10b}+\frac{dx^2\left(ax+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(10bc-7ad)\left(\frac{x\left(ax+bx^2\right)^{3/2}}{4b}-\frac{5a\left(\frac{\left(ax+bx^2\right)^{3/2}}{3b}-\frac{a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)}{2b}\right)}{8b}\right)}{10b}+\frac{dx^2\left(ax+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{1091}$$

$$\frac{(10bc-7ad)\left(\frac{x\left(ax+bx^2\right)^{3/2}}{4b}-\frac{5a\left(\frac{\left(ax+bx^2\right)^{3/2}}{3b}-\frac{a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)}{2b}\right)}{8b}\right)}{10b}+\frac{dx^2\left(ax+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(\frac{x(ax+bx^2)^{3/2}}{4b}-\frac{5a\left(\frac{(ax+bx^2)^{3/2}}{3b}-\frac{a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)}{2b}\right)}{8b}\right)(10bc-7ad)}{10b}+\frac{dx^2(ax+bx^2)^{3/2}}{5b}$$

input
```
Int[x^2*(c + d*x)*Sqrt[a*x + b*x^2],x]
```

output
```
(d*x^2*(a*x + b*x^2)^(3/2))/(5*b) + ((10*b*c - 7*a*d)*((x*(a*x + b*x^2)^(3
/2))/(4*b) - (5*a*((a*x + b*x^2)^(3/2)/(3*b) - (a*(((a + 2*b*x)*Sqrt[a*x +
 b*x^2])/(4*b) - (a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*b^(3/2)))
)/(2*b)))/(8*b)))/(10*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.58

method	result
pseudoelliptic	$\frac{\frac{7a^4\left(ad-\frac{10bc}{7}\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{128}-\frac{7\left(-\frac{10\left(\frac{7dx}{15}+c\right)a^3b^{\frac{3}{2}}}{7}+\frac{20xa^2\left(\frac{14dx}{25}+c\right)b^{\frac{5}{2}}}{21}-\frac{16x^2\left(\frac{3dx}{5}+c\right)ab^{\frac{7}{2}}}{21}-\frac{32x^3\left(\frac{4dx}{5}+c\right)b^{\frac{9}{2}}}{7}+\sqrt{b}\,a\right.}{128}}{b^{\frac{9}{2}}}$
risch	$-\frac{(-384dx^4b^4-48ab^3dx^3-480b^4cx^3+56a^2b^2dx^2-80ab^3cx^2-70a^3bdx+100a^2b^2cx+105a^4d-150a^3bc)x(bx+a)}{1920b^4\sqrt{x(bx+a)}}+\frac{a^4}{}$
default	$c\left(\frac{x(bx^2+ax)^{\frac{3}{2}}}{4b}-\frac{5a\left(\frac{(bx^2+ax)^{\frac{3}{2}}}{3b}-\frac{a\left(\frac{(2bx+a)\sqrt{bx^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{bx^2+ax}\right)}{8b^{\frac{3}{2}}}\right)}{2b}\right)}{8b}\right)+d\left(\frac{x^2(bx^2+ax)^{\frac{3}{2}}}{5b}-\right.$

input
```
int(x^2*(d*x+c)*(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
7/128/b^(9/2)*(a^4*(a*d-10/7*b*c)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-(-1
0/7*(7/15*d*x+c)*a^3*b^(3/2)+20/21*x*a^2*(14/25*d*x+c)*b^(5/2)-16/21*x^2*(
3/5*d*x+c)*a*b^(7/2)-32/7*x^3*(4/5*d*x+c)*b^(9/2)+b^(1/2)*a^4*d)*(x*(b*x+a
))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.53

$$\int x^2(c+dx)\sqrt{ax+bx^2}\,dx$$
$$=\left[-\frac{15\,(10\,a^4bc-7\,a^5d)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,(384\,b^5dx^4+150\,a^3b^2c-105\,a^4bd+48\,(}{3840\,b^5}\right.$$

input
```
integrate(x^2*(d*x+c)*(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/3840*(15*(10*a^4*b*c - 7*a^5*d)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 +
 a*x)*sqrt(b)) - 2*(384*b^5*d*x^4 + 150*a^3*b^2*c - 105*a^4*b*d + 48*(10*b
^5*c + a*b^4*d)*x^3 + 8*(10*a*b^4*c - 7*a^2*b^3*d)*x^2 - 10*(10*a^2*b^3*c
- 7*a^3*b^2*d)*x)*sqrt(b*x^2 + a*x))/b^5, 1/1920*(15*(10*a^4*b*c - 7*a^5*d
)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (384*b^5*d*x^4 +
 150*a^3*b^2*c - 105*a^4*b*d + 48*(10*b^5*c + a*b^4*d)*x^3 + 8*(10*a*b^4*c
 - 7*a^2*b^3*d)*x^2 - 10*(10*a^2*b^3*c - 7*a^3*b^2*d)*x)*sqrt(b*x^2 + a*x)
)/b^5]
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.21

$$\int x^2(c+dx)\sqrt{ax+bx^2}\,dx$$
$$=\begin{cases}-\frac{5a^3\left(ac-\frac{7a\left(\frac{ad}{10}+bc\right)}{8b}\right)\left(\begin{cases}\frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ \frac{a^2}{b}\neq 0\\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{16b^3}+\sqrt{ax+bx^2}\cdot\left(\frac{5a^2\left(ac-\frac{7a\left(\frac{ad}{10}+bc\right)}{8b}\right)}{8b^3}-\frac{5ax\left(ac-\right.}{}\right. \\ \frac{2\left(\frac{c(ax)^{\frac{7}{2}}}{7}+\frac{d(ax)^{\frac{9}{2}}}{9a}\right)}{a^3} \\ 0\end{cases}$$

input
```
integrate(x**2*(d*x+c)*(b*x**2+a*x)**(1/2),x)
```

output
```
Piecewise((-5*a**3*(a*c - 7*a*(a*d/10 + b*c)/(8*b))*Piecewise((log(a + 2*s
qrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)
*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2), True))/(16*b**3) + sqrt(a*x +
b*x**2)*(5*a**2*(a*c - 7*a*(a*d/10 + b*c)/(8*b))/(8*b**3) - 5*a*x*(a*c - 7
*a*(a*d/10 + b*c)/(8*b))/(12*b**2) + d*x**4/5 + x**3*(a*d/10 + b*c)/(4*b)
+ x**2*(a*c - 7*a*(a*d/10 + b*c)/(8*b))/(3*b)), Ne(b, 0)), (2*(c*(a*x)**(7
/2)/7 + d*(a*x)**(9/2)/(9*a))/a**3, Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.23

$$\begin{aligned}\int x^2(c+dx)\sqrt{ax+bx^2}\,dx &= \frac{(bx^2+ax)^{\frac{3}{2}}dx^2}{5\,b} + \frac{5\,\sqrt{bx^2+ax}a^2cx}{32\,b^2} + \frac{(bx^2+ax)^{\frac{3}{2}}cx}{4\,b} \\ &\quad - \frac{7\,\sqrt{bx^2+ax}a^3dx}{64\,b^3} - \frac{7\,(bx^2+ax)^{\frac{3}{2}}adx}{40\,b^2} \\ &\quad - \frac{5\,a^4c\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{7}{2}}} \\ &\quad + \frac{7\,a^5d\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{256\,b^{\frac{9}{2}}} \\ &\quad + \frac{5\,\sqrt{bx^2+ax}a^3c}{64\,b^3} - \frac{5\,(bx^2+ax)^{\frac{3}{2}}ac}{24\,b^2} \\ &\quad - \frac{7\,\sqrt{bx^2+ax}a^4d}{128\,b^4} + \frac{7\,(bx^2+ax)^{\frac{3}{2}}a^2d}{48\,b^3}\end{aligned}$$

input
```
integrate(x^2*(d*x+c)*(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
1/5*(b*x^2 + a*x)^(3/2)*d*x^2/b + 5/32*sqrt(b*x^2 + a*x)*a^2*c*x/b^2 + 1/4
*(b*x^2 + a*x)^(3/2)*c*x/b - 7/64*sqrt(b*x^2 + a*x)*a^3*d*x/b^3 - 7/40*(b*
x^2 + a*x)^(3/2)*a*d*x/b^2 - 5/128*a^4*c*log(2*b*x + a + 2*sqrt(b*x^2 + a*
x)*sqrt(b))/b^(7/2) + 7/256*a^5*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt
(b))/b^(9/2) + 5/64*sqrt(b*x^2 + a*x)*a^3*c/b^3 - 5/24*(b*x^2 + a*x)^(3/2)
*a*c/b^2 - 7/128*sqrt(b*x^2 + a*x)*a^4*d/b^4 + 7/48*(b*x^2 + a*x)^(3/2)*a^
2*d/b^3
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.80

$$\int x^2(c+dx)\sqrt{ax+bx^2}\,dx$$
$$=\frac{1}{1920}\sqrt{bx^2+ax}\left(2\left(4\left(6\left(8\,dx+\frac{10\,b^4c+ab^3d}{b^4}\right)x+\frac{10\,ab^3c-7\,a^2b^2d}{b^4}\right)x-\frac{5\,(10\,a^2b^2c-7\,a^3bd)}{b^4}\right)x+\right.$$
$$+\frac{(10\,a^4bc-7\,a^5d)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{256\,b^{\frac{9}{2}}}$$

input
```
integrate(x^2*(d*x+c)*(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/1920*sqrt(b*x^2 + a*x)*(2*(4*(6*(8*d*x + (10*b^4*c + a*b^3*d)/b^4)*x + (
10*a*b^3*c - 7*a^2*b^2*d)/b^4)*x - 5*(10*a^2*b^2*c - 7*a^3*b*d)/b^4)*x + 1
5*(10*a^3*b*c - 7*a^4*d)/b^4) + 1/256*(10*a^4*b*c - 7*a^5*d)*log(abs(2*(sq
rt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(9/2)
```

Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.09

$$\int x^2(c+dx)\sqrt{ax+bx^2}\,dx$$
$$= \frac{c\,x\,\left(b\,x^2+a\,x\right)^{3/2}}{4\,b} - \frac{5\,a\,c\,\left(\frac{a^3\,\ln\left(\frac{a+2\,b\,x}{\sqrt{b}}+2\,\sqrt{b\,x^2+a\,x}\right)}{16\,b^{5/2}}+\frac{\sqrt{b\,x^2+a\,x}\,\left(-3\,a^2+2\,a\,b\,x+8\,b^2\,x^2\right)}{24\,b^2}\right)}{8\,b} - \frac{7\,a\,d\,\left(\frac{x\,\left(b\,x^2+a\,x\right)^{3/2}}{4\,b}-\frac{5\,a\,\left(\frac{a^3\,\ln\left(\frac{a+2\,b\,x}{\sqrt{b}}+2\,\sqrt{b\,x^2+a\,x}\right)}{16\,b^{5/2}}+\frac{\sqrt{b\,x^2+a\,x}\,\left(-3\,a^2+2\,a\,b\,x+8\,b^2\,x^2\right)}{24\,b^2}\right)}{8\,b}\right)}{10\,b} + \frac{d\,x^2\,\left(b\,x^2+a\,x\right)^{3/2}}{5\,b}$$

input
```
int(x^2*(a*x + b*x^2)^(1/2)*(c + d*x),x)
```

output
```
(c*x*(a*x + b*x^2)^(3/2))/(4*b) - (5*a*c*((a^3*log((a + 2*b*x)/b^(1/2) + 2
*(a*x + b*x^2)^(1/2)))/(16*b^(5/2)) + ((a*x + b*x^2)^(1/2)*(8*b^2*x^2 - 3*
a^2 + 2*a*b*x))/(24*b^2)))/(8*b) - (7*a*d*((x*(a*x + b*x^2)^(3/2))/(4*b) -
 (5*a*((a^3*log((a + 2*b*x)/b^(1/2) + 2*(a*x + b*x^2)^(1/2)))/(16*b^(5/2))
 + ((a*x + b*x^2)^(1/2)*(8*b^2*x^2 - 3*a^2 + 2*a*b*x))/(24*b^2)))/(8*b)))/
(10*b) + (d*x^2*(a*x + b*x^2)^(3/2))/(5*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.10

$$\int x^2(c+dx)\sqrt{ax+bx^2}\,dx$$
$$= \frac{-105\sqrt{x}\,\sqrt{bx+a}\,a^4bd+150\sqrt{x}\,\sqrt{bx+a}\,a^3b^2c+70\sqrt{x}\,\sqrt{bx+a}\,a^3b^2dx-100\sqrt{x}\,\sqrt{bx+a}\,a^2b^3cx-56\sqrt{}}{}$$

input
```
int(x^2*(d*x+c)*(b*x^2+a*x)^(1/2),x)
```

output
```
( - 105*sqrt(x)*sqrt(a + b*x)*a**4*b*d + 150*sqrt(x)*sqrt(a + b*x)*a**3*b*
*2*c + 70*sqrt(x)*sqrt(a + b*x)*a**3*b**2*d*x - 100*sqrt(x)*sqrt(a + b*x)*
a**2*b**3*c*x - 56*sqrt(x)*sqrt(a + b*x)*a**2*b**3*d*x**2 + 80*sqrt(x)*sqr
t(a + b*x)*a*b**4*c*x**2 + 48*sqrt(x)*sqrt(a + b*x)*a*b**4*d*x**3 + 480*sq
rt(x)*sqrt(a + b*x)*b**5*c*x**3 + 384*sqrt(x)*sqrt(a + b*x)*b**5*d*x**4 +
105*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**5*d - 150*sq
rt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4*b*c)/(1920*b**5)
```

3.2 $\int x(c+dx)\sqrt{ax+bx^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 160

$$\int x(c+dx)\sqrt{ax+bx^2}\,dx = -\frac{a^2(8bc-5ad)\sqrt{ax+bx^2}}{64b^3}+\frac{a(8bc-5ad)x\sqrt{ax+bx^2}}{96b^2}+\frac{(8bc-5ad)x^2\sqrt{ax+bx^2}}{24b}+\frac{dx(ax+bx^2)^{3/2}}{4b}+\frac{a^3(8bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{64b^{7/2}}$$

output
```
-1/64*a^2*(-5*a*d+8*b*c)*(b*x^2+a*x)^(1/2)/b^3+1/96*a*(-5*a*d+8*b*c)*x*(b*
x^2+a*x)^(1/2)/b^2+1/24*(-5*a*d+8*b*c)*x^2*(b*x^2+a*x)^(1/2)/b+1/4*d*x*(b*
x^2+a*x)^(3/2)/b+1/64*a^3*(-5*a*d+8*b*c)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/
2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.86

$$\int x(c+dx)\sqrt{ax+bx^2}\,dx$$
$$=\frac{\sqrt{x(a+bx)}\left(\sqrt{b}(15a^3d+8ab^2x(2c+dx)+16b^3x^2(4c+3dx)-2a^2b(12c+5dx))+\frac{6a^3(-8bc+5ad)\text{arctanh}}{\sqrt{x}\sqrt{a+bx}}\right.}{192b^{7/2}}$$

input
```
Integrate[x*(c + d*x)*Sqrt[a*x + b*x^2],x]
```

output
```
(Sqrt[x*(a + b*x)]*(Sqrt[b]*(15*a^3*d + 8*a*b^2*x*(2*c + d*x) + 16*b^3*x^2
*(4*c + 3*d*x) - 2*a^2*b*(12*c + 5*d*x)) + (6*a^3*(-8*b*c + 5*a*d)*ArcTanh
[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])])/(Sqrt[x]*Sqrt[a + b*x])))/(
192*b^(7/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.70, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1225, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{ax+bx^2}(c+dx)\,dx$$
$$\Big\downarrow \text{1225}$$
$$\frac{(ax+bx^2)^{3/2}(-5ad+8bc+6bdx)}{24b^2}-\frac{a(8bc-5ad)\int\sqrt{bx^2+ax}dx}{16b^2}$$
$$\Big\downarrow \text{1087}$$
$$\frac{(ax+bx^2)^{3/2}(-5ad+8bc+6bdx)}{24b^2}-\frac{a(8bc-5ad)\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)}{16b^2}$$
$$\Big\downarrow \text{1091}$$

$$\frac{(ax+bx^2)^{3/2}(-5ad+8bc+6bdx)}{24b^2}-\frac{a(8bc-5ad)\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)}{16b^2}$$

$$\Big\downarrow \; 219$$

$$\frac{(ax+bx^2)^{3/2}(-5ad+8bc+6bdx)}{24b^2}-\frac{a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)(8bc-5ad)}{16b^2}$$

input
```
Int[x*(c + d*x)*Sqrt[a*x + b*x^2],x]
```

output
```
((8*b*c - 5*a*d + 6*b*d*x)*(a*x + b*x^2)^(3/2))/(24*b^2) - (a*(8*b*c - 5*a
*d)*(((a + 2*b*x)*Sqrt[a*x + b*x^2])/(4*b) - (a^2*ArcTanh[(Sqrt[b]*x)/Sqrt
[a*x + b*x^2]])/(4*b^(3/2))))/(16*b^2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$-\frac{5\left(a^{3}\left(ad-\frac{8bc}{5}\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)-\sqrt{x(bx+a)}\left(-\frac{8\left(\frac{5dx}{12}+c\right)a^{2}b^{\frac{3}{2}}}{5}+\frac{16x\left(\frac{dx}{2}+c\right)a\,b^{\frac{5}{2}}}{15}+\frac{64x^{2}\left(\frac{3dx}{4}+c\right)b^{\frac{7}{2}}}{15}+\sqrt{b}\,a^{3}d\right)\right)}{64b^{\frac{7}{2}}}$
risch	$\frac{\left(48b^{3}d\,x^{3}+8a\,b^{2}d\,x^{2}+64b^{3}c\,x^{2}-10a^{2}bdx+16a\,b^{2}cx+15a^{3}d-24c\,a^{2}b\right)x(bx+a)}{192b^{3}\sqrt{x(bx+a)}}-\frac{a^{3}(5ad-8bc)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{128b^{\frac{7}{2}}}$
default	$c\left(\frac{\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{3b}-\frac{a\left(\frac{(2bx+a)\sqrt{b\,x^{2}+ax}}{4b}-\frac{a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8b^{\frac{3}{2}}}\right)}{2b}\right)+d\left(\frac{x\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{4b}-\frac{5a\left(\frac{\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{3b}-\cdots\right.}{\cdots}\right.$

input
```
int(x*(d*x+c)*(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-5/64/b^(7/2)*(a^3*(a*d-8/5*b*c)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-(x*(
b*x+a))^(1/2)*(-8/5*(5/12*d*x+c)*a^2*b^(3/2)+16/15*x*(1/2*d*x+c)*a*b^(5/2)
+64/15*x^2*(3/4*d*x+c)*b^(7/2)+b^(1/2)*a^3*d))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.59

$$\int x(c+dx)\sqrt{ax+bx^2}\,dx$$
$$= \left[-\frac{3\,(8\,a^3bc-5\,a^4d)\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,(48\,b^4dx^3-24\,a^2b^2c+15\,a^3bd+8\,(8\,b^4c+\right.}{384\,b^4}$$
$$-\frac{3\,(8\,a^3bc-5\,a^4d)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(48\,b^4dx^3-24\,a^2b^2c+15\,a^3bd+8\,(8\,b^4c+ab^3d)x^2+2}{192\,b^4}$$

input
```
integrate(x*(d*x+c)*(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/384*(3*(8*a^3*b*c - 5*a^4*d)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*
x)*sqrt(b)) - 2*(48*b^4*d*x^3 - 24*a^2*b^2*c + 15*a^3*b*d + 8*(8*b^4*c + a
*b^3*d)*x^2 + 2*(8*a*b^3*c - 5*a^2*b^2*d)*x)*sqrt(b*x^2 + a*x))/b^4, -1/19
2*(3*(8*a^3*b*c - 5*a^4*d)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x
 + a)) - (48*b^4*d*x^3 - 24*a^2*b^2*c + 15*a^3*b*d + 8*(8*b^4*c + a*b^3*d)
*x^2 + 2*(8*a*b^3*c - 5*a^2*b^2*d)*x)*sqrt(b*x^2 + a*x))/b^4]
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.29

$$\int x(c+dx)\sqrt{ax+bx^2}\,dx$$
$$= \begin{cases} \frac{3a^2\left(ac-\frac{5a\left(\frac{ad}{8}+bc\right)}{6b}\right)\left(\begin{cases}\frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0\\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{8b^2}+\sqrt{ax+bx^2}\left(-\frac{3a\left(ac-\frac{5a\left(\frac{ad}{8}+bc\right)}{6b}\right)}{4b^2}+\frac{dx^3}{4}+\frac{x^2(}{}\right. \\ \frac{2\left(\frac{c(ax)^{\frac{5}{2}}}{5}+\frac{d(ax)^{\frac{7}{2}}}{7a}\right)}{a^2} \\ 0 \end{cases}$$

input
```
integrate(x*(d*x+c)*(b*x**2+a*x)**(1/2),x)
```

output
```
Piecewise((3*a**2*(a*c - 5*a*(a*d/8 + b*c)/(6*b))*Piecewise((log(a + 2*sqr
t(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*l
og(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2), True))/(8*b**2) + sqrt(a*x + b*x
**2)*(-3*a*(a*c - 5*a*(a*d/8 + b*c)/(6*b))/(4*b**2) + d*x**3/4 + x**2*(a*d
/8 + b*c)/(3*b) + x*(a*c - 5*a*(a*d/8 + b*c)/(6*b))/(2*b)), Ne(b, 0)), (2*
(c*(a*x)**(5/2)/5 + d*(a*x)**(7/2)/(7*a))/a**2, Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.24

$$\int x(c+dx)\sqrt{ax+bx^2}\,dx = -\frac{\sqrt{bx^2+ax}acx}{4\,b} + \frac{5\,\sqrt{bx^2+ax}a^2dx}{32\,b^2} + \frac{(bx^2+ax)^{\frac{3}{2}}dx}{4\,b} + \frac{a^3c\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{5}{2}}} - \frac{5\,a^4d\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{7}{2}}} - \frac{\sqrt{bx^2+ax}a^2c}{8\,b^2} + \frac{(bx^2+ax)^{\frac{3}{2}}c}{3\,b} + \frac{5\,\sqrt{bx^2+ax}a^3d}{64\,b^3} - \frac{5\,(bx^2+ax)^{\frac{3}{2}}ad}{24\,b^2}$$

input
```
integrate(x*(d*x+c)*(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*sqrt(b*x^2 + a*x)*a*c*x/b + 5/32*sqrt(b*x^2 + a*x)*a^2*d*x/b^2 + 1/4*
(b*x^2 + a*x)^(3/2)*d*x/b + 1/16*a^3*c*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)
*sqrt(b))/b^(5/2) - 5/128*a^4*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b
))/b^(7/2) - 1/8*sqrt(b*x^2 + a*x)*a^2*c/b^2 + 1/3*(b*x^2 + a*x)^(3/2)*c/b
 + 5/64*sqrt(b*x^2 + a*x)*a^3*d/b^3 - 5/24*(b*x^2 + a*x)^(3/2)*a*d/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.81

$$\int x(c+dx)\sqrt{ax+bx^2}\,dx = \frac{1}{192}\sqrt{bx^2+ax}\left(2\left(4\left(6\,dx+\frac{8\,b^3c+ab^2d}{b^3}\right)x+\frac{8\,ab^2c-5\,a^2bd}{b^3}\right)x-\frac{3\,(8\,a^2bc-5\,a^3d)}{b^3}\right) - \frac{(8\,a^3bc-5\,a^4d)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{128\,b^{\frac{7}{2}}}$$

input
```
integrate(x*(d*x+c)*(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/192*sqrt(b*x^2 + a*x)*(2*(4*(6*d*x + (8*b^3*c + a*b^2*d)/b^3)*x + (8*a*b
^2*c - 5*a^2*b*d)/b^3)*x - 3*(8*a^2*b*c - 5*a^3*d)/b^3) - 1/128*(8*a^3*b*c
 - 5*a^4*d)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(7/2
)
```

Mupad [B] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.03

$$\int x(c+dx)\sqrt{ax+bx^2}\,dx = \frac{a^3\,c\,\ln\left(\frac{a+2\,b\,x}{\sqrt{b}}+2\,\sqrt{b\,x^2+a\,x}\right)}{16\,b^{5/2}} + \frac{c\,\sqrt{b\,x^2+a\,x}\,\left(-3\,a^2+2\,a\,b\,x+8\,b^2\,x^2\right)}{24\,b^2} + \frac{d\,x\,{\left(b\,x^2+a\,x\right)}^{3/2}}{4\,b} - \frac{5\,a\,d\,\left(\frac{a^3\,\ln\left(\frac{a+2\,b\,x}{\sqrt{b}}+2\,\sqrt{b\,x^2+a\,x}\right)}{16\,b^{5/2}}+\frac{\sqrt{b\,x^2+a\,x}\,\left(-3\,a^2+2\,a\,b\,x+8\,b^2\,x^2\right)}{24\,b^2}\right)}{8\,b}$$

input
```
int(x*(a*x + b*x^2)^(1/2)*(c + d*x),x)
```

output
```
(a^3*c*log((a + 2*b*x)/b^(1/2) + 2*(a*x + b*x^2)^(1/2)))/(16*b^(5/2)) + (c
*(a*x + b*x^2)^(1/2)*(8*b^2*x^2 - 3*a^2 + 2*a*b*x))/(24*b^2) + (d*x*(a*x +
 b*x^2)^(3/2))/(4*b) - (5*a*d*((a^3*log((a + 2*b*x)/b^(1/2) + 2*(a*x + b*x
^2)^(1/2)))/(16*b^(5/2)) + ((a*x + b*x^2)^(1/2)*(8*b^2*x^2 - 3*a^2 + 2*a*b
*x))/(24*b^2)))/(8*b)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.11

$$\int x(c+dx)\sqrt{ax+bx^2}\,dx$$

$$= \frac{15\sqrt{x}\,\sqrt{bx+a}\,a^3bd - 24\sqrt{x}\,\sqrt{bx+a}\,a^2b^2c - 10\sqrt{x}\,\sqrt{bx+a}\,a^2b^2dx + 16\sqrt{x}\,\sqrt{bx+a}\,a\,b^3cx + 8\sqrt{x}\,\sqrt{bx}}{}$$

input
```
int(x*(d*x+c)*(b*x^2+a*x)^(1/2),x)
```

output
```
(15*sqrt(x)*sqrt(a + b*x)*a**3*b*d - 24*sqrt(x)*sqrt(a + b*x)*a**2*b**2*c
- 10*sqrt(x)*sqrt(a + b*x)*a**2*b**2*d*x + 16*sqrt(x)*sqrt(a + b*x)*a*b**3
*c*x + 8*sqrt(x)*sqrt(a + b*x)*a*b**3*d*x**2 + 64*sqrt(x)*sqrt(a + b*x)*b*
*4*c*x**2 + 48*sqrt(x)*sqrt(a + b*x)*b**4*d*x**3 - 15*sqrt(b)*log((sqrt(a
+ b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4*d + 24*sqrt(b)*log((sqrt(a + b*x)
+ sqrt(x)*sqrt(b))/sqrt(a))*a**3*b*c)/(192*b**4)
```

3.3 $\int (c+dx)\sqrt{ax+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 124

$$\int (c+dx)\sqrt{ax+bx^2}\,dx = \frac{a(2bc-ad)\sqrt{ax+bx^2}}{8b^2}+\frac{(2bc-ad)x\sqrt{ax+bx^2}}{4b} + \frac{d(ax+bx^2)^{3/2}}{3b}-\frac{a^2(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{8b^{5/2}}$$

output
```
1/8*a*(-a*d+2*b*c)*(b*x^2+a*x)^(1/2)/b^2+1/4*(-a*d+2*b*c)*x*(b*x^2+a*x)^(1
/2)/b+1/3*d*(b*x^2+a*x)^(3/2)/b-1/8*a^2*(-a*d+2*b*c)*arctanh(b^(1/2)*x/(b*
x^2+a*x)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

$$\int (c+dx)\sqrt{ax+bx^2}\,dx = \frac{\sqrt{x(a+bx)}\left(\sqrt{b}\sqrt{x}(-3a^2d+2ab(3c+dx)+4b^2x(3c+2dx))+\frac{6a^2(-2bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{\sqrt{a+bx}}\right)}{24b^{5/2}\sqrt{x}}$$

input
```
Integrate[(c + d*x)*Sqrt[a*x + b*x^2],x]
```

output
```
(Sqrt[x*(a + b*x)]*(Sqrt[b]*Sqrt[x]*(-3*a^2*d + 2*a*b*(3*c + d*x) + 4*b^2*
x*(3*c + 2*d*x)) + (6*a^2*(-2*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[
a] + Sqrt[a + b*x])])/Sqrt[a + b*x]))/(24*b^(5/2)*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.79, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1160, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ax+bx^2}(c+dx)\,dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{(2bc-ad)\int \sqrt{bx^2+ax}dx}{2b}+\frac{d\left(ax+bx^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(2bc-ad)\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)}{2b}+\frac{d\left(ax+bx^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \text{1091}$$

$$\frac{(2bc-ad)\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)}{2b}+\frac{d\left(ax+bx^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)(2bc-ad)}{2b}+\frac{d\left(ax+bx^2\right)^{3/2}}{3b}$$

input
```
Int[(c + d*x)*Sqrt[a*x + b*x^2],x]
```

output
```
(d*(a*x + b*x^2)^(3/2))/(3*b) + ((2*b*c - a*d)*(((a + 2*b*x)*Sqrt[a*x + b*
x^2])/(4*b) - (a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*b^(3/2))))/(
2*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$\frac{(a^3d-2c\,a^2b)\,\mathrm{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)-\left(-2\left(\frac{dx}{3}+c\right)a\,b^{\frac{3}{2}}+\left(-\frac{8}{3}d\,x^2-4cx\right)b^{\frac{5}{2}}+\sqrt{b}\,a^2d\right)\sqrt{x(bx+a)}}{8b^{\frac{5}{2}}}$
risch	$-\frac{(-8b^2d\,x^2-2abdx-12b^2cx+3a^2d-6abc)x(bx+a)}{24b^2\sqrt{x(bx+a)}}+\frac{a^2(ad-2bc)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{16b^{\frac{5}{2}}}$
default	$c\left(\frac{(2bx+a)\sqrt{b\,x^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8b^{\frac{3}{2}}}\right)+d\left(\frac{(b\,x^2+ax)^{\frac{3}{2}}}{3b}-\frac{a\left(\frac{(2bx+a)\sqrt{b\,x^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{\right.}\right.}{8b^{\frac{3}{2}}}\right.}{2b}\right.$

input
```
int((d*x+c)*(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/b^(5/2)*((a^3*d-2*a^2*b*c)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-(-2*(1
/3*d*x+c)*a*b^(3/2)+(-8/3*d*x^2-4*c*x)*b^(5/2)+b^(1/2)*a^2*d)*(x*(b*x+a))^
(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.66

$$\int (c+dx)\sqrt{ax+bx^2}\,dx$$
$$=\left[-\frac{3\,(2\,a^2bc-a^3d)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,(8\,b^3dx^2+6\,ab^2c-3\,a^2bd+2\,(6\,b^3c+ab^2d)}{48\,b^3}\right.$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(2*a^2*b*c - a^3*d)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*
sqrt(b)) - 2*(8*b^3*d*x^2 + 6*a*b^2*c - 3*a^2*b*d + 2*(6*b^3*c + a*b^2*d)*
x)*sqrt(b*x^2 + a*x))/b^3, 1/24*(3*(2*a^2*b*c - a^3*d)*sqrt(-b)*arctan(sqr
t(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (8*b^3*d*x^2 + 6*a*b^2*c - 3*a^2*b*d
+ 2*(6*b^3*c + a*b^2*d)*x)*sqrt(b*x^2 + a*x))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.35

$$\int (c+dx)\sqrt{ax+bx^2}\,dx$$
$$= \begin{cases} -\frac{a\left(ac-\frac{3a\left(\frac{ad}{6}+bc\right)}{4b}\right)\left(\begin{cases}\frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0\\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{2b}+\sqrt{ax+bx^2}\left(\frac{dx^2}{3}+\frac{x\left(\frac{ad}{6}+bc\right)}{2b}+\frac{ac-\frac{3a\left(\frac{ad}{6}+bc\right)}{4b}}{b}\right) \\ \frac{2\left(\frac{c(ax)^{\frac{3}{2}}}{3}+\frac{d(ax)^{\frac{5}{2}}}{5a}\right)}{a} \\ 0 \end{cases}$$

input
```
integrate((d*x+c)*(b*x**2+a*x)**(1/2),x)
```

output
```
Piecewise((-a*(a*c - 3*a*(a*d/6 + b*c)/(4*b))*Piecewise((log(a + 2*sqrt(b)
*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a
/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2), True))/(2*b) + sqrt(a*x + b*x**2)*(d
*x**2/3 + x*(a*d/6 + b*c)/(2*b) + (a*c - 3*a*(a*d/6 + b*c)/(4*b))/b), Ne(b
, 0)), (2*(c*(a*x)**(3/2)/3 + d*(a*x)**(5/2)/(5*a))/a, Ne(a, 0)), (0, True
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.24

$$\int (c+dx)\sqrt{ax+bx^2}\,dx = \frac{1}{2}\sqrt{bx^2+ax}cx - \frac{\sqrt{bx^2+ax}adx}{4b} - \frac{a^2c\log\left(2bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{8b^{\frac{3}{2}}} + \frac{a^3d\log\left(2bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{16b^{\frac{5}{2}}} + \frac{\sqrt{bx^2+ax}ac}{4b} - \frac{\sqrt{bx^2+ax}a^2d}{8b^2} + \frac{(bx^2+ax)^{\frac{3}{2}}d}{3b}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a*x)*c*x - 1/4*sqrt(b*x^2 + a*x)*a*d*x/b - 1/8*a^2*c*log(
2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(3/2) + 1/16*a^3*d*log(2*b*x +
a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5/2) + 1/4*sqrt(b*x^2 + a*x)*a*c/b - 1
/8*sqrt(b*x^2 + a*x)*a^2*d/b^2 + 1/3*(b*x^2 + a*x)^(3/2)*d/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

$$\int (c+dx)\sqrt{ax+bx^2}\,dx = \frac{1}{24}\sqrt{bx^2+ax}\left(2\left(4dx+\frac{6b^2c+abd}{b^2}\right)x+\frac{3(2abc-a^2d)}{b^2}\right) + \frac{(2a^2bc-a^3d)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{16b^{\frac{5}{2}}}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(b*x^2 + a*x)*(2*(4*d*x + (6*b^2*c + a*b*d)/b^2)*x + 3*(2*a*b*c -
 a^2*d)/b^2) + 1/16*(2*a^2*b*c - a^3*d)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2
+ a*x))*sqrt(b) + a))/b^(5/2)
```

Mupad [B] (verification not implemented)

Time = 8.81 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

$$\int (c+dx)\sqrt{ax+bx^2}\,dx = c\sqrt{b\,x^2+a\,x}\,\left(\frac{x}{2}+\frac{a}{4\,b}\right)-\frac{a^2\,c\,\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{8\,b^{3/2}}+\frac{a^3\,d\,\ln\left(\frac{a+2\,b\,x}{\sqrt{b}}+2\,\sqrt{b\,x^2+a\,x}\right)}{16\,b^{5/2}}+\frac{d\,\sqrt{b\,x^2+a\,x}\,\left(-3\,a^2+2\,a\,b\,x+8\,b^2\,x^2\right)}{24\,b^2}$$

input
```
int((a*x + b*x^2)^(1/2)*(c + d*x),x)
```

output
```
c*(a*x + b*x^2)^(1/2)*(x/2 + a/(4*b)) - (a^2*c*log((a/2 + b*x)/b^(1/2) + (
a*x + b*x^2)^(1/2)))/(8*b^(3/2)) + (a^3*d*log((a + 2*b*x)/b^(1/2) + 2*(a*x
 + b*x^2)^(1/2)))/(16*b^(5/2)) + (d*(a*x + b*x^2)^(1/2)*(8*b^2*x^2 - 3*a^2
 + 2*a*b*x))/(24*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10

$$\int (c+dx)\sqrt{ax+bx^2}\,dx = \frac{-3\sqrt{x}\,\sqrt{bx+a}\,a^2bd+6\sqrt{x}\,\sqrt{bx+a}\,a\,b^2c+2\sqrt{x}\,\sqrt{bx+a}\,a\,b^2dx+12\sqrt{x}\,\sqrt{bx+a}\,b^3cx+8\sqrt{x}\,\sqrt{bx+a}\,}{24b^3}$$

input
```
int((d*x+c)*(b*x^2+a*x)^(1/2),x)
```

output
```
( - 3*sqrt(x)*sqrt(a + b*x)*a**2*b*d + 6*sqrt(x)*sqrt(a + b*x)*a*b**2*c +
2*sqrt(x)*sqrt(a + b*x)*a*b**2*d*x + 12*sqrt(x)*sqrt(a + b*x)*b**3*c*x + 8
*sqrt(x)*sqrt(a + b*x)*b**3*d*x**2 + 3*sqrt(b)*log((sqrt(a + b*x) + sqrt(x
)*sqrt(b))/sqrt(a))*a**3*d - 6*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b
))/sqrt(a))*a**2*b*c)/(24*b**3)
```

3.4 $\int \frac{(c+dx)\sqrt{ax+bx^2}}{x} dx$

Optimal result

Integrand size = 22, antiderivative size = 94

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x} dx = \frac{(4bc-ad)\sqrt{ax+bx^2}}{4b} + \frac{d(ax+bx^2)^{3/2}}{2bx} + \frac{a(4bc-ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}$$

output
```
1/4*(-a*d+4*b*c)*(b*x^2+a*x)^(1/2)/b+1/2*d*(b*x^2+a*x)^(3/2)/b/x+1/4*a*(-a
*d+4*b*c)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x} dx = \frac{\sqrt{x(a+bx)}\left(\sqrt{b}(4bc+ad+2bdx) + \frac{a(-4bc+ad)\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{x}\sqrt{a+bx}}\right)}{4b^{3/2}}$$

input
```
Integrate[((c + d*x)*Sqrt[a*x + b*x^2])/x,x]
```

output
```
(Sqrt[x*(a + b*x)]*(Sqrt[b]*(4*b*c + a*d + 2*b*d*x) + (a*(-4*b*c + a*d)*Lo
g[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[x]*Sqrt[a + b*x])))/(4*b^(3/2
))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1221, 1131, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}(c+dx)}{x}\,dx$$

$$\Big\downarrow \ 1221$$

$$\frac{(4bc-ad)\int \frac{\sqrt{bx^2+ax}}{x}dx}{4b}+\frac{d\left(ax+bx^2\right)^{3/2}}{2bx}$$

$$\Big\downarrow \ 1131$$

$$\frac{(4bc-ad)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+ax}}dx+\sqrt{ax+bx^2}\right)}{4b}+\frac{d\left(ax+bx^2\right)^{3/2}}{2bx}$$

$$\Big\downarrow \ 1091$$

$$\frac{(4bc-ad)\left(a\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}+\sqrt{ax+bx^2}\right)}{4b}+\frac{d\left(ax+bx^2\right)^{3/2}}{2bx}$$

$$\Big\downarrow \ 219$$

$$\frac{\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}+\sqrt{ax+bx^2}\right)(4bc-ad)}{4b}+\frac{d\left(ax+bx^2\right)^{3/2}}{2bx}$$

input
```
Int[((c + d*x)*Sqrt[a*x + b*x^2])/x,x]
```

output
```
(d*(a*x + b*x^2)^(3/2))/(2*b*x) + ((4*b*c - a*d)*(Sqrt[a*x + b*x^2] + (a*A
rcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]))/(4*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.70

method	result	size
pseudoelliptic	$-\frac{(a^{2}d-4abc)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)-\left((2dx+4c)b^{\frac{3}{2}}+\sqrt{b}\,ad\right)\sqrt{x(bx+a)}}{4b^{\frac{3}{2}}}$	66
risch	$\frac{(2bdx+ad+4bc)x(bx+a)}{4b\sqrt{x(bx+a)}}-\frac{a(ad-4bc)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8b^{\frac{3}{2}}}$	73
default	$d\left(\frac{(2bx+a)\sqrt{b\,x^{2}+ax}}{4b}-\frac{a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8b^{\frac{3}{2}}}\right)+c\left(\sqrt{b\,x^{2}+ax}+\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2\sqrt{b}}\right)$	103

input
```
int((d*x+c)*(b*x^2+a*x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/4/b^(3/2)*((a^2*d-4*a*b*c)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-((2*d*x
+4*c)*b^(3/2)+b^(1/2)*a*d)*(x*(b*x+a))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.67

$$\int\frac{(c+dx)\sqrt{ax+bx^2}}{x}\,dx$$
$$=\left[-\frac{(4\,abc-a^2d)\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,(2\,b^2dx+4\,b^2c+abd)\sqrt{bx^2+ax}}{8\,b^2},\right.$$
$$\left.-\frac{(4\,abc-a^2d)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(2\,b^2dx+4\,b^2c+abd)\sqrt{bx^2+ax}}{4\,b^2}\right]$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(1/2)/x,x, algorithm="fricas")
```

output
```
[-1/8*((4*a*b*c - a^2*d)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(
b)) - 2*(2*b^2*d*x + 4*b^2*c + a*b*d)*sqrt(b*x^2 + a*x))/b^2, -1/4*((4*a*b
*c - a^2*d)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (2*b^2
*d*x + 4*b^2*c + a*b*d)*sqrt(b*x^2 + a*x))/b^2]
```

Sympy [F]

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x}\,dx = \int \frac{\sqrt{x\,(a+bx)}(c+dx)}{x}\,dx$$

input
```
integrate((d*x+c)*(b*x**2+a*x)**(1/2)/x,x)
```

output
```
Integral(sqrt(x*(a + b*x))*(c + d*x)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.16

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x}\,dx = \frac{1}{2}\sqrt{bx^2+ax}dx + \frac{ac\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{2\,\sqrt{b}} - \frac{a^2d\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{3}{2}}} + \sqrt{bx^2+ax}c + \frac{\sqrt{bx^2+ax}ad}{4\,b}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(1/2)/x,x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a*x)*d*x + 1/2*a*c*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sq
rt(b))/sqrt(b) - 1/8*a^2*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^
(3/2) + sqrt(b*x^2 + a*x)*c + 1/4*sqrt(b*x^2 + a*x)*a*d/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.81

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x}\,dx = \frac{1}{4}\sqrt{bx^2+ax}\left(2\,dx+\frac{4\,bc+ad}{b}\right) - \frac{(4\,abc-a^2d)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,b^{\frac{3}{2}}}$$

input

```
integrate((d*x+c)*(b*x^2+a*x)^(1/2)/x,x, algorithm="giac")
```

output

```
1/4*sqrt(b*x^2 + a*x)*(2*d*x + (4*b*c + a*d)/b) - 1/8*(4*a*b*c - a^2*d)*lo
g(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.07

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x}\,dx = c\sqrt{b\,x^2+a\,x}+d\sqrt{b\,x^2+a\,x}\left(\frac{x}{2}+\frac{a}{4\,b}\right) - \frac{a^2\,d\,\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{8\,b^{3/2}} + \frac{a\,c\,\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{2\,\sqrt{b}}$$

input

```
int(((a*x + b*x^2)^(1/2)*(c + d*x))/x,x)
```

output

```
c*(a*x + b*x^2)^(1/2) + d*(a*x + b*x^2)^(1/2)*(x/2 + a/(4*b)) - (a^2*d*log
((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2)))/(8*b^(3/2)) + (a*c*log((a/2 +
 b*x)/b^(1/2) + (a*x + b*x^2)^(1/2)))/(2*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x}\,dx = \frac{\sqrt{x}\,\sqrt{bx+a}\,abd+4\sqrt{x}\,\sqrt{bx+a}\,b^2c+2\sqrt{x}\,\sqrt{bx+a}\,b^2dx-\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2d+4\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)abc}{4b^2}$$

input
```
int((d*x+c)*(b*x^2+a*x)^(1/2)/x,x)
```

output
```
(sqrt(x)*sqrt(a + b*x)*a*b*d + 4*sqrt(x)*sqrt(a + b*x)*b**2*c + 2*sqrt(x)*
sqrt(a + b*x)*b**2*d*x - sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqr
t(a))*a**2*d + 4*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*
b*c)/(4*b**2)
```

3.5 $\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^2} dx = d\sqrt{ax+bx^2} - \frac{2c\sqrt{ax+bx^2}}{x} + \frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

output
```
d*(b*x^2+a*x)^(1/2)-2*c*(b*x^2+a*x)^(1/2)/x+(a*d+2*b*c)*arctanh(b^(1/2)*x/
(b*x^2+a*x)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^2} dx = \frac{\sqrt{x(a+bx)}\left(-2c+dx+\frac{2(2bc+ad)\sqrt{x}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{\sqrt{b}\sqrt{a+bx}}\right)}{x}$$

input
```
Integrate[((c + d*x)*Sqrt[a*x + b*x^2])/x^2,x]
```

output
```
(Sqrt[x*(a + b*x)]*(-2*c + d*x + (2*(2*b*c + a*d)*Sqrt[x]*ArcTanh[(Sqrt[b]
*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(Sqrt[b]*Sqrt[a + b*x])))/x
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1220, 1131, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}(c+dx)}{x^2} \, dx$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{(ad+2bc)\int \frac{\sqrt{bx^2+ax}}{x}dx}{a}-\frac{2c\left(ax+bx^2\right)^{3/2}}{ax^2}$$

$$\Big\downarrow \text{ 1131}$$

$$\frac{(ad+2bc)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+ax}}dx+\sqrt{ax+bx^2}\right)}{a}-\frac{2c\left(ax+bx^2\right)^{3/2}}{ax^2}$$

$$\Big\downarrow \text{ 1091}$$

$$\frac{(ad+2bc)\left(a\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}+\sqrt{ax+bx^2}\right)}{a}-\frac{2c\left(ax+bx^2\right)^{3/2}}{ax^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}+\sqrt{ax+bx^2}\right)(ad+2bc)}{a}-\frac{2c\left(ax+bx^2\right)^{3/2}}{ax^2}$$

input
```
Int[((c + d*x)*Sqrt[a*x + b*x^2])/x^2,x]
```

output
```
(-2*c*(a*x + b*x^2)^(3/2))/(a*x^2) + ((2*b*c + a*d)*(Sqrt[a*x + b*x^2] + (
a*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]))/a
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$-\frac{2\left(-\frac{(ad+2bc)\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)x}{2}+\left(-\frac{dx}{2}+c\right)\sqrt{b}\,\sqrt{x(bx+a)}\right)}{\sqrt{b}\,x}$
risch	$-\frac{(bx+a)(-dx+2c)}{\sqrt{x(bx+a)}}+\frac{\left(\frac{ad}{2}+bc\right)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$
default	$c\left(-\frac{2(b\,x^2+ax)^{\frac{3}{2}}}{a\,x^2}+\frac{2b\left(\sqrt{b\,x^2+ax}+\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2\sqrt{b}}\right)}{a}\right)+d\left(\sqrt{b\,x^2+ax}+\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right.}{2\sqrt{b}}\right.$

input
```
int((d*x+c)*(b*x^2+a*x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2/b^(1/2)*(-1/2*(a*d+2*b*c)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*x+(-1/2*
d*x+c)*b^(1/2)*(x*(b*x+a))^(1/2))/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.99

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^2}\,dx = \left[\frac{(2\,bc+ad)\sqrt{b}x\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(bdx-2\,bc)\sqrt{bx^2+ax}}{2\,bx},\ -\frac{(2\,bc+ad)\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(bdx-2\,bc)\sqrt{bx^2+ax}}{bx}\right]$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/2*((2*b*c + a*d)*sqrt(b)*x*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))
 + 2*(b*d*x - 2*b*c)*sqrt(b*x^2 + a*x))/(b*x), -((2*b*c + a*d)*sqrt(-b)*x*
arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (b*d*x - 2*b*c)*sqrt(b*x^2
+ a*x))/(b*x)]
```

Sympy [F]

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^2}\,dx = \int \frac{\sqrt{x\,(a+bx)}(c+dx)}{x^2}\,dx$$

input
```
integrate((d*x+c)*(b*x**2+a*x)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(x*(a + b*x))*(c + d*x)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^2}\,dx = \sqrt{b}c\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) + \frac{ad\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{2\,\sqrt{b}} + \sqrt{bx^2+ax}d - \frac{2\,\sqrt{bx^2+ax}c}{x}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
sqrt(b)*c*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 1/2*a*d*log(2*b*x
 + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b) + sqrt(b*x^2 + a*x)*d - 2*sqrt
(b*x^2 + a*x)*c/x
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.14

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^2}\,dx = \sqrt{bx^2+ax}d - \frac{(2\,bc+ad)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{2\sqrt{b}} + \frac{2\,ac}{\sqrt{b}x-\sqrt{bx^2+ax}}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(1/2)/x^2,x, algorithm="giac")
```

output
```
sqrt(b*x^2 + a*x)*d - 1/2*(2*b*c + a*d)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2
+ a*x))*sqrt(b) + a))/sqrt(b) + 2*a*c/(sqrt(b)*x - sqrt(b*x^2 + a*x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^2}\,dx = \int \frac{\sqrt{b\,x^2+a\,x}\,(c+d\,x)}{x^2}\,dx$$

input
```
int(((a*x + b*x^2)^(1/2)*(c + d*x))/x^2,x)
```

output
```
int(((a*x + b*x^2)^(1/2)*(c + d*x))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.40

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^2}\,dx = \frac{-8\sqrt{x}\,\sqrt{bx+a}\,bc+4\sqrt{x}\,\sqrt{bx+a}\,bdx+4\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)adx+8\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)bcx-\sqrt{b}}{4bx}$$

input
```
int((d*x+c)*(b*x^2+a*x)^(1/2)/x^2,x)
```

output
```
( - 8*sqrt(x)*sqrt(a + b*x)*b*c + 4*sqrt(x)*sqrt(a + b*x)*b*d*x + 4*sqrt(b
)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*d*x + 8*sqrt(b)*log((sq
rt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b*c*x - sqrt(b)*a*d*x - 8*sqrt(b)*
b*c*x)/(4*b*x)
```

3.6 $\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^3}\,dx = -\frac{2d\sqrt{ax+bx^2}}{x} - \frac{2c(ax+bx^2)^{3/2}}{3ax^3} + 2\sqrt{b}d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)$$

output
```
-2*d*(b*x^2+a*x)^(1/2)/x-2/3*c*(b*x^2+a*x)^(3/2)/a/x^3+2*b^(1/2)*d*arctanh
(b^(1/2)*x/(b*x^2+a*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^3}\,dx = -\frac{2\sqrt{x(a+bx)}(ac+bcx+3adx)}{3ax^2} - \frac{2\sqrt{b}d\sqrt{x(a+bx)}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{x}\sqrt{a+bx}}$$

input
```
Integrate[((c + d*x)*Sqrt[a*x + b*x^2])/x^3,x]
```

output
```
(-2*Sqrt[x*(a + b*x)]*(a*c + b*c*x + 3*a*d*x))/(3*a*x^2) - (2*Sqrt[b]*d*Sq
rt[x*(a + b*x)]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[x]*Sqrt[a +
 b*x])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1220, 1125, 25, 27, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}(c+dx)}{x^3}\,dx$$

$$\Big\downarrow \text{1220}$$

$$d\int \frac{\sqrt{bx^2+ax}}{x^2}dx-\frac{2c\left(ax+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{1125}$$

$$d\left(-\int -\frac{b}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2c\left(ax+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{25}$$

$$d\left(\int \frac{b}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2c\left(ax+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{27}$$

$$d\left(b\int \frac{1}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2c\left(ax+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{1091}$$

$$d\left(2b\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2c\left(ax+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{219}$$

$$d\left(2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2c(ax+bx^2)^{3/2}}{3ax^3}$$

input
```
Int[((c + d*x)*Sqrt[a*x + b*x^2])/x^3,x]
```

output
```
(-2*c*(a*x + b*x^2)^(3/2))/(3*a*x^3) + d*((-2*Sqrt[a*x + b*x^2])/x + 2*Sqr
t[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{6 d a \sqrt{b}\, \operatorname{arctanh}\left(\frac{\sqrt{x \left(b x +a \right)}}{x \sqrt{b}}\right) x^{2}-2 \sqrt{x \left(b x +a \right)}\, \left(\left(3 d x +c \right) a +c b x \right)}{3 a \,x^{2}}$	61
risch	$-\frac{2 \left(b x +a \right) \left(3 a d x +c b x +a c \right)}{3 x \sqrt{x \left(b x +a \right)}\, a}+\sqrt{b}\, d \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b \,x^{2}+a x}\right)$	66
default	$-\frac{2 c \left(b \,x^{2}+a x \right)^{\frac{3}{2}}}{3 a \,x^{3}}+d \left(-\frac{2 \left(b \,x^{2}+a x \right)^{\frac{3}{2}}}{a \,x^{2}}+\frac{2 b \left(\sqrt{b \,x^{2}+a x}+\frac{a \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b \,x^{2}+a x}\right)}{2 \sqrt{b}}\right)}{a}\right)$	92

input
```
int((d*x+c)*(b*x^2+a*x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*(6*d*a*b^(1/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*x^2-2*(x*(b*x+a))^
(1/2)*((3*d*x+c)*a+c*b*x))/a/x^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.95

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^3}\,dx = \left[\frac{3\,a\sqrt{b}dx^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,\sqrt{bx^2+ax}(ac+(bc+3\,ad)x)}{3\,ax^2}, -\frac{2\left(3\,a\sqrt{-b}dx^2\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+\sqrt{bx^2+ax}(ac+(bc+3\,ad)x)\right)}{3\,ax^2}\right]$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/3*(3*a*sqrt(b)*d*x^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*s
qrt(b*x^2 + a*x)*(a*c + (b*c + 3*a*d)*x))/(a*x^2), -2/3*(3*a*sqrt(-b)*d*x^
2*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + sqrt(b*x^2 + a*x)*(a*c +
(b*c + 3*a*d)*x))/(a*x^2)]
```

Sympy [F]

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^3}\,dx = \int \frac{\sqrt{x\,(a+bx)}(c+dx)}{x^3}\,dx$$

input
```
integrate((d*x+c)*(b*x**2+a*x)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(x*(a + b*x))*(c + d*x)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.16

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^3}\,dx = \left(\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-\frac{2\,\sqrt{bx^2+ax}}{x}\right)d - \frac{2}{3}\,c\left(\frac{\sqrt{bx^2+ax}b}{ax}+\frac{\sqrt{bx^2+ax}}{x^2}\right)$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
(sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*sqrt(b*x^2 + a*x
)/x)*d - 2/3*c*(sqrt(b*x^2 + a*x)*b/(a*x) + sqrt(b*x^2 + a*x)/x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(61) = 122.

Time = 0.34 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.92

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^3}\,dx = -\sqrt{b}d\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right) + \frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2bc+3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2ad+3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)a\sqrt{b}c+a^2c\right)}{3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(1/2)/x^3,x, algorithm="giac")
```

output
```
-sqrt(b)*d*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)) + 2/3*(
3*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*b*c + 3*(sqrt(b)*x - sqrt(b*x^2 + a*x)
)^2*a*d + 3*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a*sqrt(b)*c + a^2*c)/(sqrt(b)*
x - sqrt(b*x^2 + a*x))^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^3}\,dx = \int \frac{\sqrt{b\,x^2+a\,x}\,(c+d\,x)}{x^3}\,dx$$

input
```
int(((a*x + b*x^2)^(1/2)*(c + d*x))/x^3,x)
```

output
```
int(((a*x + b*x^2)^(1/2)*(c + d*x))/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^3}\,dx$$
$$= \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,ac}{3} - 2\sqrt{x}\,\sqrt{bx+a}\,adx - \frac{2\sqrt{x}\,\sqrt{bx+a}\,bcx}{3} + 2\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)ad\,x^2 + \frac{2\sqrt{b}\,ad\,x^2}{3} - \frac{2\sqrt{b}\,bc\,x^2}{3}}{a\,x^2}$$

input
```
int((d*x+c)*(b*x^2+a*x)^(1/2)/x^3,x)
```

output
```
(2*( - sqrt(x)*sqrt(a + b*x)*a*c - 3*sqrt(x)*sqrt(a + b*x)*a*d*x - sqrt(x)
*sqrt(a + b*x)*b*c*x + 3*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqr
t(a))*a*d*x**2 + sqrt(b)*a*d*x**2 - sqrt(b)*b*c*x**2))/(3*a*x**2)
```

3.7 $\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 57

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^4} dx = -\frac{2c(ax+bx^2)^{3/2}}{5ax^4} + \frac{2(2bc-5ad)(ax+bx^2)^{3/2}}{15a^2x^3}$$

output
```
-2/5*c*(b*x^2+a*x)^(3/2)/a/x^4+2/15*(-5*a*d+2*b*c)*(b*x^2+a*x)^(3/2)/a^2/x
^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.63

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^4} dx = -\frac{2(x(a+bx))^{3/2}(3ac-2bcx+5adx)}{15a^2x^4}$$

input
```
Integrate[((c + d*x)*Sqrt[a*x + b*x^2])/x^4,x]
```

output
```
(-2*(x*(a + b*x))^(3/2)*(3*a*c - 2*b*c*x + 5*a*d*x))/(15*a^2*x^4)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1220, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}(c+dx)}{x^4} \, dx$$

$$\Big\downarrow \text{ 1220}$$

$$-\frac{(2bc-5ad)\int \frac{\sqrt{bx^2+ax}}{x^3}dx}{5a}-\frac{2c\left(ax+bx^2\right)^{3/2}}{5ax^4}$$

$$\Big\downarrow \text{ 1123}$$

$$\frac{2\left(ax+bx^2\right)^{3/2}(2bc-5ad)}{15a^2x^3}-\frac{2c\left(ax+bx^2\right)^{3/2}}{5ax^4}$$

input
```
Int[((c + d*x)*Sqrt[a*x + b*x^2])/x^4,x]
```

output
```
(-2*c*(a*x + b*x^2)^(3/2))/(5*a*x^4) + (2*(2*b*c - 5*a*d)*(a*x + b*x^2)^(3
/2))/(15*a^2*x^3)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.65

method	result	size
pseudoelliptic	$-\frac{2\sqrt{x(bx+a)}\,(bx+a)\left(\left(\frac{5dx}{3}+c\right)a-\frac{2cbx}{3}\right)}{5x^{3}a^{2}}$	37
gosper	$-\frac{2(bx+a)(5adx-2cbx+3ac)\sqrt{b\,x^{2}+ax}}{15a^{2}x^{3}}$	40
orering	$-\frac{2(bx+a)(5adx-2cbx+3ac)\sqrt{b\,x^{2}+ax}}{15a^{2}x^{3}}$	40
trager	$-\frac{2\left(5abd\,x^{2}-2b^{2}c\,x^{2}+5a^{2}dx+abcx+3a^{2}c\right)\sqrt{b\,x^{2}+ax}}{15a^{2}x^{3}}$	56
risch	$-\frac{2(bx+a)\left(5abd\,x^{2}-2b^{2}c\,x^{2}+5a^{2}dx+abcx+3a^{2}c\right)}{15x^{2}\sqrt{x(bx+a)}\,a^{2}}$	59
default	$c\left(-\frac{2\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{5a\,x^{4}}+\frac{4b\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{15a^{2}x^{3}}\right)-\frac{2d\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{3a\,x^{3}}$	64

input
```
int((d*x+c)*(b*x^2+a*x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/5*(x*(b*x+a))^(1/2)*(b*x+a)*((5/3*d*x+c)*a-2/3*c*b*x)/x^3/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^4}\,dx = -\frac{2\,(3\,a^2c-(2\,b^2c-5\,abd)x^2+(abc+5\,a^2d)x)\sqrt{bx^2+ax}}{15\,a^2x^3}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-2/15*(3*a^2*c - (2*b^2*c - 5*a*b*d)*x^2 + (a*b*c + 5*a^2*d)*x)*sqrt(b*x^2
 + a*x)/(a^2*x^3)
```

Sympy [F]

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^4}\,dx = \int \frac{\sqrt{x\,(a+bx)}(c+dx)}{x^4}\,dx$$

input
```
integrate((d*x+c)*(b*x**2+a*x)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(x*(a + b*x))*(c + d*x)/x**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(49) = 98.

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.75

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^4}\,dx = \frac{4\,\sqrt{bx^2+ax}b^2c}{15\,a^2x} - \frac{2\,\sqrt{bx^2+ax}bd}{3\,ax} - \frac{2\,\sqrt{bx^2+ax}bc}{15\,ax^2} - \frac{2\,\sqrt{bx^2+ax}d}{3\,x^2} - \frac{2\,\sqrt{bx^2+ax}c}{5\,x^3}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
4/15*sqrt(b*x^2 + a*x)*b^2*c/(a^2*x) - 2/3*sqrt(b*x^2 + a*x)*b*d/(a*x) - 2
/15*sqrt(b*x^2 + a*x)*b*c/(a*x^2) - 2/3*sqrt(b*x^2 + a*x)*d/x^2 - 2/5*sqrt
(b*x^2 + a*x)*c/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(49) = 98$.

Time = 0.31 (sec) , antiderivative size = 191, normalized size of antiderivative = 3.35

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^4}\,dx = \frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4bd+15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3b^{\frac{3}{2}}c+15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3a\sqrt{b}d+25\left(\sqrt{\right.}\right.}{15\left(\sqrt{b}x-\sqrt{bx^2}\right.}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(1/2)/x^4,x, algorithm="giac")
```

output
```
2/15*(15*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*b*d + 15*(sqrt(b)*x - sqrt(b*x^
2 + a*x))^3*b^(3/2)*c + 15*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a*sqrt(b)*d +
 25*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a*b*c + 5*(sqrt(b)*x - sqrt(b*x^2 +
a*x))^2*a^2*d + 15*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^2*sqrt(b)*c + 3*a^3*c
)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^5
```

Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.75

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^4}\,dx = \frac{4\,b^2\,c\,\sqrt{b\,x^2+a\,x}}{15\,a^2\,x} - \frac{2\,d\,\sqrt{b\,x^2+a\,x}}{3\,x^2} - \frac{2\,b\,c\,\sqrt{b\,x^2+a\,x}}{15\,a\,x^2} - \frac{2\,b\,d\,\sqrt{b\,x^2+a\,x}}{3\,a\,x} - \frac{2\,c\,\sqrt{b\,x^2+a\,x}}{5\,x^3}$$

input
```
int(((a*x + b*x^2)^(1/2)*(c + d*x))/x^4,x)
```

output
```
(4*b^2*c*(a*x + b*x^2)^(1/2))/(15*a^2*x) - (2*d*(a*x + b*x^2)^(1/2))/(3*x^
2) - (2*b*c*(a*x + b*x^2)^(1/2))/(15*a*x^2) - (2*b*d*(a*x + b*x^2)^(1/2))/
(3*a*x) - (2*c*(a*x + b*x^2)^(1/2))/(5*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.86

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^4}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2c}{5}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2dx}{3}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,abcx}{15}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,abd\,x^2}{3}+\frac{4\sqrt{x}\,\sqrt{bx+a}\,b^2c\,x^2}{15}-\frac{2\sqrt{b}\,abd\,x^3}{15}-\frac{4\sqrt{b}\,b^2c\,x^3}{15}}{a^2x^3}$$

input
```
int((d*x+c)*(b*x^2+a*x)^(1/2)/x^4,x)
```

output
```
(2*( - 3*sqrt(x)*sqrt(a + b*x)*a**2*c - 5*sqrt(x)*sqrt(a + b*x)*a**2*d*x -
 sqrt(x)*sqrt(a + b*x)*a*b*c*x - 5*sqrt(x)*sqrt(a + b*x)*a*b*d*x**2 + 2*sq
rt(x)*sqrt(a + b*x)*b**2*c*x**2 - sqrt(b)*a*b*d*x**3 - 2*sqrt(b)*b**2*c*x*
*3))/(15*a**2*x**3)
```

3.8 $\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^5}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^5}\,dx = -\frac{2c(ax+bx^2)^{3/2}}{7ax^5} + \frac{2(4bc-7ad)\,(ax+bx^2)^{3/2}}{35a^2x^4} - \frac{4b(4bc-7ad)\,(ax+bx^2)^{3/2}}{105a^3x^3}$$

output
```
-2/7*c*(b*x^2+a*x)^(3/2)/a/x^5+2/35*(-7*a*d+4*b*c)*(b*x^2+a*x)^(3/2)/a^2/x
^4-4/105*b*(-7*a*d+4*b*c)*(b*x^2+a*x)^(3/2)/a^3/x^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.63

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^5}\,dx = \frac{2(x(a+bx))^{3/2}\,(-8b^2cx^2-3a^2(5c+7dx)+2abx(6c+7dx))}{105a^3x^5}$$

input
```
Integrate[((c + d*x)*Sqrt[a*x + b*x^2])/x^5,x]
```

output
```
(2*(x*(a + b*x))^(3/2)*(-8*b^2*c*x^2 - 3*a^2*(5*c + 7*d*x) + 2*a*b*x*(6*c
+ 7*d*x)))/(105*a^3*x^5)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1220, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}(c+dx)}{x^5} \, dx$$

$$\Big\downarrow \text{1220}$$

$$-\frac{(4bc-7ad)\int \frac{\sqrt{bx^2+ax}}{x^4}dx}{7a}-\frac{2c\left(ax+bx^2\right)^{3/2}}{7ax^5}$$

$$\Big\downarrow \text{1129}$$

$$-\frac{(4bc-7ad)\left(-\frac{2b\int \frac{\sqrt{bx^2+ax}}{x^3}dx}{5a}-\frac{2\left(ax+bx^2\right)^{3/2}}{5ax^4}\right)}{7a}-\frac{2c\left(ax+bx^2\right)^{3/2}}{7ax^5}$$

$$\Big\downarrow \text{1123}$$

$$-\frac{\left(\frac{4b\left(ax+bx^2\right)^{3/2}}{15a^2x^3}-\frac{2\left(ax+bx^2\right)^{3/2}}{5ax^4}\right)(4bc-7ad)}{7a}-\frac{2c\left(ax+bx^2\right)^{3/2}}{7ax^5}$$

input
```
Int[((c + d*x)*Sqrt[a*x + b*x^2])/x^5,x]
```

output
```
(-2*c*(a*x + b*x^2)^(3/2))/(7*a*x^5) - ((4*b*c - 7*a*d)*((-2*(a*x + b*x^2)
^(3/2))/(5*a*x^4) + (4*b*(a*x + b*x^2)^(3/2))/(15*a^2*x^3)))/(7*a)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.60

method	result	size
pseudoelliptic	$-\frac{2\sqrt{x(bx+a)}\,(bx+a)\left(\left(\frac{7dx}{5}+c\right)a^{2}-\frac{4x\left(\frac{7dx}{6}+c\right)ba}{5}+\frac{8b^{2}c\,x^{2}}{15}\right)}{7x^{4}a^{3}}$	54
gosper	$-\frac{2(bx+a)\left(-14abd\,x^{2}+8b^{2}c\,x^{2}+21a^{2}dx-12abcx+15a^{2}c\right)\sqrt{b\,x^{2}+ax}}{105x^{4}a^{3}}$	62
orering	$-\frac{2(bx+a)\left(-14abd\,x^{2}+8b^{2}c\,x^{2}+21a^{2}dx-12abcx+15a^{2}c\right)\sqrt{b\,x^{2}+ax}}{105x^{4}a^{3}}$	62
trager	$-\frac{2\left(-14a\,b^{2}d\,x^{3}+8b^{3}c\,x^{3}+7a^{2}bd\,x^{2}-4a\,b^{2}c\,x^{2}+21a^{3}dx+3a^{2}bcx+15c\,a^{3}\right)\sqrt{b\,x^{2}+ax}}{105x^{4}a^{3}}$	81
risch	$-\frac{2(bx+a)\left(-14a\,b^{2}d\,x^{3}+8b^{3}c\,x^{3}+7a^{2}bd\,x^{2}-4a\,b^{2}c\,x^{2}+21a^{3}dx+3a^{2}bcx+15c\,a^{3}\right)}{105x^{3}\sqrt{x(bx+a)}\,a^{3}}$	84
default	$c\left(-\frac{2\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{7a\,x^{5}}-\frac{4b\left(-\frac{2\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{5a\,x^{4}}+\frac{4b\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{15a^{2}x^{3}}\right)}{7a}\right)+d\left(-\frac{2\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{5a\,x^{4}}+\frac{4b\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{15a^{2}x^{3}}\right)$	112

input
```
int((d*x+c)*(b*x^2+a*x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-2/7*(x*(b*x+a))^(1/2)*(b*x+a)*((7/5*d*x+c)*a^2-4/5*x*(7/6*d*x+c)*b*a+8/15
*b^2*c*x^2)/x^4/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^5}\,dx = -\frac{2\left(15\,a^3c+2\left(4\,b^3c-7\,ab^2d\right)x^3-\left(4\,ab^2c-7\,a^2bd\right)x^2+3\left(a^2bc+7\,a^3d\right)x\right)\sqrt{bx^2+ax}}{105\,a^3x^4}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
-2/105*(15*a^3*c + 2*(4*b^3*c - 7*a*b^2*d)*x^3 - (4*a*b^2*c - 7*a^2*b*d)*x
^2 + 3*(a^2*b*c + 7*a^3*d)*x)*sqrt(b*x^2 + a*x)/(a^3*x^4)
```

Sympy [F]

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^5}\,dx = \int \frac{\sqrt{x\,(a+bx)}(c+dx)}{x^5}\,dx$$

input
```
integrate((d*x+c)*(b*x**2+a*x)**(1/2)/x**5,x)
```

output
```
Integral(sqrt(x*(a + b*x))*(c + d*x)/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.62

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^5}\,dx = -\frac{16\sqrt{bx^2+ax}b^3c}{105\,a^3x} + \frac{4\sqrt{bx^2+ax}b^2d}{15\,a^2x} + \frac{8\sqrt{bx^2+ax}b^2c}{105\,a^2x^2} - \frac{2\sqrt{bx^2+ax}bd}{15\,ax^2} - \frac{2\sqrt{bx^2+ax}bc}{35\,ax^3} - \frac{2\sqrt{bx^2+ax}d}{5\,x^3} - \frac{2\sqrt{bx^2+ax}c}{7\,x^4}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-16/105*sqrt(b*x^2 + a*x)*b^3*c/(a^3*x) + 4/15*sqrt(b*x^2 + a*x)*b^2*d/(a^
2*x) + 8/105*sqrt(b*x^2 + a*x)*b^2*c/(a^2*x^2) - 2/15*sqrt(b*x^2 + a*x)*b*
d/(a*x^2) - 2/35*sqrt(b*x^2 + a*x)*b*c/(a*x^3) - 2/5*sqrt(b*x^2 + a*x)*d/x
^3 - 2/7*sqrt(b*x^2 + a*x)*c/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(78) = 156.

Time = 0.33 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.79

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^5}\,dx$$
$$= \frac{2\left(105\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5 b^{\frac{3}{2}}d+140\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4 b^2c+175\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4 abd+315\right.}{}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(1/2)/x^5,x, algorithm="giac")
```

output
```
2/105*(105*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*b^(3/2)*d + 140*(sqrt(b)*x -
sqrt(b*x^2 + a*x))^4*b^2*c + 175*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a*b*d +
 315*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a*b^(3/2)*c + 105*(sqrt(b)*x - sqrt
(b*x^2 + a*x))^3*a^2*sqrt(b)*d + 273*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2
*b*c + 21*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^3*d + 105*(sqrt(b)*x - sqrt(
b*x^2 + a*x))*a^3*sqrt(b)*c + 15*a^4*c)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^7
```

Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.62

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^5}\,dx = \frac{8\,b^2\,c\,\sqrt{b\,x^2+a\,x}}{105\,a^2\,x^2}-\frac{2\,d\,\sqrt{b\,x^2+a\,x}}{5\,x^3}-\frac{2\,b\,c\,\sqrt{b\,x^2+a\,x}}{35\,a\,x^3} -\frac{2\,b\,d\,\sqrt{b\,x^2+a\,x}}{15\,a\,x^2}-\frac{2\,c\,\sqrt{b\,x^2+a\,x}}{7\,x^4} -\frac{16\,b^3\,c\,\sqrt{b\,x^2+a\,x}}{105\,a^3\,x}+\frac{4\,b^2\,d\,\sqrt{b\,x^2+a\,x}}{15\,a^2\,x}$$

input
```
int(((a*x + b*x^2)^(1/2)*(c + d*x))/x^5,x)
```

output
```
(8*b^2*c*(a*x + b*x^2)^(1/2))/(105*a^2*x^2) - (2*d*(a*x + b*x^2)^(1/2))/(5
*x^3) - (2*b*c*(a*x + b*x^2)^(1/2))/(35*a*x^3) - (2*b*d*(a*x + b*x^2)^(1/2
))/(15*a*x^2) - (2*c*(a*x + b*x^2)^(1/2))/(7*x^4) - (16*b^3*c*(a*x + b*x^2
)^(1/2))/(105*a^3*x) + (4*b^2*d*(a*x + b*x^2)^(1/2))/(15*a^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.64

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^5}\,dx$$
$$= \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3c}{7} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3dx}{5} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2bcx}{35} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2bd\,x^2}{15} + \frac{8\sqrt{x}\,\sqrt{bx+a}\,a\,b^2c\,x^2}{105} + \frac{4\sqrt{x}\,\sqrt{bx+a}\,a\,b^2d\,x^3}{15} -}{a^3x^4}$$

input
```
int((d*x+c)*(b*x^2+a*x)^(1/2)/x^5,x)
```

output
```
(2*( - 15*sqrt(x)*sqrt(a + b*x)*a**3*c - 21*sqrt(x)*sqrt(a + b*x)*a**3*d*x
 - 3*sqrt(x)*sqrt(a + b*x)*a**2*b*c*x - 7*sqrt(x)*sqrt(a + b*x)*a**2*b*d*x
**2 + 4*sqrt(x)*sqrt(a + b*x)*a*b**2*c*x**2 + 14*sqrt(x)*sqrt(a + b*x)*a*b
**2*d*x**3 - 8*sqrt(x)*sqrt(a + b*x)*b**3*c*x**3 - 14*sqrt(b)*a*b**2*d*x**
4 + 8*sqrt(b)*b**3*c*x**4))/(105*a**3*x**4)
```

3.9 $\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^6}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 125

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^6}\,dx = -\frac{2c(ax+bx^2)^{3/2}}{9ax^6}+\frac{2(2bc-3ad)\,(ax+bx^2)^{3/2}}{21a^2x^5} - \frac{8b(2bc-3ad)\,(ax+bx^2)^{3/2}}{105a^3x^4} + \frac{16b^2(2bc-3ad)\,(ax+bx^2)^{3/2}}{315a^4x^3}$$

output
```
-2/9*c*(b*x^2+a*x)^(3/2)/a/x^6+2/21*(-3*a*d+2*b*c)*(b*x^2+a*x)^(3/2)/a^2/x
^5-8/105*b*(-3*a*d+2*b*c)*(b*x^2+a*x)^(3/2)/a^3/x^4+16/315*b^2*(-3*a*d+2*b
*c)*(b*x^2+a*x)^(3/2)/a^4/x^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.58

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^6}\,dx = -\frac{2(x(a+bx))^{3/2}\left(-16b^3cx^3+24ab^2x^2(c+dx)-6a^2bx(5c+6dx)+5a^3(7c+9dx)\right)}{315a^4x^6}$$

input
```
Integrate[((c + d*x)*Sqrt[a*x + b*x^2])/x^6,x]
```

output
```
(-2*(x*(a + b*x))^(3/2)*(-16*b^3*c*x^3 + 24*a*b^2*x^2*(c + d*x) - 6*a^2*b*
x*(5*c + 6*d*x) + 5*a^3*(7*c + 9*d*x)))/(315*a^4*x^6)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1220, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}(c+dx)}{x^6}\,dx$$

$$\Big\downarrow \text{1220}$$

$$-\frac{(2bc-3ad)\int \frac{\sqrt{bx^2+ax}}{x^5}dx}{3a}-\frac{2c\left(ax+bx^2\right)^{3/2}}{9ax^6}$$

$$\Big\downarrow \text{1129}$$

$$-\frac{(2bc-3ad)\left(-\frac{4b\int \frac{\sqrt{bx^2+ax}}{x^4}dx}{7a}-\frac{2\left(ax+bx^2\right)^{3/2}}{7ax^5}\right)}{3a}-\frac{2c\left(ax+bx^2\right)^{3/2}}{9ax^6}$$

$$\Big\downarrow \text{1129}$$

$$-\frac{(2bc-3ad)\left(-\frac{4b\left(-\frac{2b\int \frac{\sqrt{bx^2+ax}}{x^3}dx}{5a}-\frac{2\left(ax+bx^2\right)^{3/2}}{5ax^4}\right)}{7a}-\frac{2\left(ax+bx^2\right)^{3/2}}{7ax^5}\right)}{3a}-\frac{2c\left(ax+bx^2\right)^{3/2}}{9ax^6}$$

$$\Big\downarrow \text{1123}$$

$$-\frac{\left(-\frac{4b\left(\frac{4b\left(ax+bx^2\right)^{3/2}}{15a^2x^3}-\frac{2\left(ax+bx^2\right)^{3/2}}{5ax^4}\right)}{7a}-\frac{2\left(ax+bx^2\right)^{3/2}}{7ax^5}\right)(2bc-3ad)}{3a}-\frac{2c\left(ax+bx^2\right)^{3/2}}{9ax^6}$$

input
```
Int[((c + d*x)*Sqrt[a*x + b*x^2])/x^6,x]
```

output
```
(-2*c*(a*x + b*x^2)^(3/2))/(9*a*x^6) - ((2*b*c - 3*a*d)*((-2*(a*x + b*x^2)
^(3/2))/(7*a*x^5) - (4*b*((-2*(a*x + b*x^2)^(3/2))/(5*a*x^4) + (4*b*(a*x +
 b*x^2)^(3/2))/(15*a^2*x^3)))/(7*a)))/(3*a)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.56

method	result
pseudoelliptic	$-\frac{2\left(\left(\frac{9dx}{7}+c\right)a^{3}-\frac{6xb\left(\frac{6dx}{5}+c\right)a^{2}}{7}+\frac{24b^{2}x^{2}(dx+c)a}{35}-\frac{16b^{3}c\,x^{3}}{35}\right)\sqrt{x(bx+a)}\,(bx+a)}{9x^{5}a^{4}}$
gosper	$-\frac{2(bx+a)\left(24a\,b^{2}d\,x^{3}-16b^{3}c\,x^{3}-36a^{2}bd\,x^{2}+24a\,b^{2}c\,x^{2}+45a^{3}dx-30a^{2}bcx+35c\,a^{3}\right)\sqrt{b\,x^{2}+ax}}{315x^{5}a^{4}}$
orering	$-\frac{2(bx+a)\left(24a\,b^{2}d\,x^{3}-16b^{3}c\,x^{3}-36a^{2}bd\,x^{2}+24a\,b^{2}c\,x^{2}+45a^{3}dx-30a^{2}bcx+35c\,a^{3}\right)\sqrt{b\,x^{2}+ax}}{315x^{5}a^{4}}$
trager	$-\frac{2\left(24x^{4}a\,b^{3}d-16x^{4}b^{4}c-12a^{2}b^{2}d\,x^{3}+8a\,b^{3}c\,x^{3}+9a^{3}bd\,x^{2}-6a^{2}b^{2}c\,x^{2}+45a^{4}dx+5a^{3}bcx+35c\,a^{4}\right)\sqrt{b\,x^{2}+ax}}{315x^{5}a^{4}}$
risch	$-\frac{2(bx+a)\left(24x^{4}a\,b^{3}d-16x^{4}b^{4}c-12a^{2}b^{2}d\,x^{3}+8a\,b^{3}c\,x^{3}+9a^{3}bd\,x^{2}-6a^{2}b^{2}c\,x^{2}+45a^{4}dx+5a^{3}bcx+35c\,a^{4}\right)}{315x^{4}\sqrt{x(bx+a)}\,a^{4}}$
default	$c\left(-\frac{2\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{9a\,x^{6}}-\frac{2b\left(-\frac{2\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{7a\,x^{5}}-\frac{4b\left(-\frac{2\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{5a\,x^{4}}+\frac{4b\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{15a^{2}x^{3}}\right)}{7a}\right)}{3a}\right)+d\left(-\frac{2\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{7a\,x^{5}}-\frac{4b(}{}\right.$

input
```
int((d*x+c)*(b*x^2+a*x)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-2/9*((9/7*d*x+c)*a^3-6/7*x*b*(6/5*d*x+c)*a^2+24/35*b^2*x^2*(d*x+c)*a-16/3
5*b^3*c*x^3)*(x*(b*x+a))^(1/2)*(b*x+a)/x^5/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^6}\,dx = -\frac{2\left(35\,a^4c-8\,(2\,b^4c-3\,ab^3d)x^4+4\,(2\,ab^3c-3\,a^2b^2d)x^3-3\,(2\,a^2b^2c-3\,a^3bd)x^2+5\,(a^3bc+9\,a^4d)x\right)\sqrt{}}{315\,a^4x^5}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
-2/315*(35*a^4*c - 8*(2*b^4*c - 3*a*b^3*d)*x^4 + 4*(2*a*b^3*c - 3*a^2*b^2*
d)*x^3 - 3*(2*a^2*b^2*c - 3*a^3*b*d)*x^2 + 5*(a^3*b*c + 9*a^4*d)*x)*sqrt(b
*x^2 + a*x)/(a^4*x^5)
```

Sympy [F]

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^6}\,dx = \int \frac{\sqrt{x\,(a+bx)}(c+dx)}{x^6}\,dx$$

input
```
integrate((d*x+c)*(b*x**2+a*x)**(1/2)/x**6,x)
```

output
```
Integral(sqrt(x*(a + b*x))*(c + d*x)/x**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.54

$$\begin{aligned}\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^6}\,dx = {} & \frac{32\sqrt{bx^2+ax}b^4c}{315\,a^4x} - \frac{16\sqrt{bx^2+ax}b^3d}{105\,a^3x} - \frac{16\sqrt{bx^2+ax}b^3c}{315\,a^3x^2} \\ & + \frac{8\sqrt{bx^2+ax}b^2d}{105\,a^2x^2} + \frac{4\sqrt{bx^2+ax}b^2c}{105\,a^2x^3} - \frac{2\sqrt{bx^2+ax}bd}{35\,ax^3} \\ & - \frac{2\sqrt{bx^2+ax}bc}{63\,ax^4} - \frac{2\sqrt{bx^2+ax}d}{7\,x^4} - \frac{2\sqrt{bx^2+ax}c}{9\,x^5}\end{aligned}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
32/315*sqrt(b*x^2 + a*x)*b^4*c/(a^4*x) - 16/105*sqrt(b*x^2 + a*x)*b^3*d/(a
^3*x) - 16/315*sqrt(b*x^2 + a*x)*b^3*c/(a^3*x^2) + 8/105*sqrt(b*x^2 + a*x)
*b^2*d/(a^2*x^2) + 4/105*sqrt(b*x^2 + a*x)*b^2*c/(a^2*x^3) - 2/35*sqrt(b*x
^2 + a*x)*b*d/(a*x^3) - 2/63*sqrt(b*x^2 + a*x)*b*c/(a*x^4) - 2/7*sqrt(b*x^
2 + a*x)*d/x^4 - 2/9*sqrt(b*x^2 + a*x)*c/x^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(109) = 218$.

Time = 0.34 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.49

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^6}\,dx = \frac{2\left(420\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6b^2d+630\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5b^{\frac{5}{2}}c+945\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5ab^{\frac{3}{2}}d+17\right.}{}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(1/2)/x^6,x, algorithm="giac")
```

output
```
2/315*(420*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*b^2*d + 630*(sqrt(b)*x - sqrt
(b*x^2 + a*x))^5*b^(5/2)*c + 945*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a*b^(3/
2)*d + 1764*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a*b^2*c + 819*(sqrt(b)*x - s
qrt(b*x^2 + a*x))^4*a^2*b*d + 1995*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^2*b
^(3/2)*c + 315*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^3*sqrt(b)*d + 1125*(sqr
t(b)*x - sqrt(b*x^2 + a*x))^2*a^3*b*c + 45*(sqrt(b)*x - sqrt(b*x^2 + a*x))
^2*a^4*d + 315*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^4*sqrt(b)*c + 35*a^5*c)/(
sqrt(b)*x - sqrt(b*x^2 + a*x))^9
```

Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.54

$$\begin{aligned}\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^6}\,dx &= \frac{4\,b^2\,c\,\sqrt{b\,x^2+a\,x}}{105\,a^2\,x^3}-\frac{2\,d\,\sqrt{b\,x^2+a\,x}}{7\,x^4}-\frac{2\,b\,c\,\sqrt{b\,x^2+a\,x}}{63\,a\,x^4}\\ &\quad-\frac{2\,b\,d\,\sqrt{b\,x^2+a\,x}}{35\,a\,x^3}-\frac{2\,c\,\sqrt{b\,x^2+a\,x}}{9\,x^5}\\ &\quad-\frac{16\,b^3\,c\,\sqrt{b\,x^2+a\,x}}{315\,a^3\,x^2}+\frac{32\,b^4\,c\,\sqrt{b\,x^2+a\,x}}{315\,a^4\,x}\\ &\quad+\frac{8\,b^2\,d\,\sqrt{b\,x^2+a\,x}}{105\,a^2\,x^2}-\frac{16\,b^3\,d\,\sqrt{b\,x^2+a\,x}}{105\,a^3\,x}\end{aligned}$$

input
```
int(((a*x + b*x^2)^(1/2)*(c + d*x))/x^6,x)
```

output
```
(4*b^2*c*(a*x + b*x^2)^(1/2))/(105*a^2*x^3) - (2*d*(a*x + b*x^2)^(1/2))/(7
*x^4) - (2*b*c*(a*x + b*x^2)^(1/2))/(63*a*x^4) - (2*b*d*(a*x + b*x^2)^(1/2
))/(35*a*x^3) - (2*c*(a*x + b*x^2)^(1/2))/(9*x^5) - (16*b^3*c*(a*x + b*x^2
)^(1/2))/(315*a^3*x^2) + (32*b^4*c*(a*x + b*x^2)^(1/2))/(315*a^4*x) + (8*b
^2*d*(a*x + b*x^2)^(1/2))/(105*a^2*x^2) - (16*b^3*d*(a*x + b*x^2)^(1/2))/(
105*a^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.50

$$\int \frac{(c+dx)\sqrt{ax+bx^2}}{x^6}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4c}{9} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4dx}{7} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3bcx}{63} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3bd\,x^2}{35} + \frac{4\sqrt{x}\,\sqrt{bx+a}\,a^2b^2c\,x^2}{105} + \frac{8\sqrt{x}\,\sqrt{bx+a}\,a^2b^2d\,x^3}{105} -}{a^4x^5}$$

input
```
int((d*x+c)*(b*x^2+a*x)^(1/2)/x^6,x)
```

output
```
(2*( - 35*sqrt(x)*sqrt(a + b*x)*a**4*c - 45*sqrt(x)*sqrt(a + b*x)*a**4*d*x
 - 5*sqrt(x)*sqrt(a + b*x)*a**3*b*c*x - 9*sqrt(x)*sqrt(a + b*x)*a**3*b*d*x
**2 + 6*sqrt(x)*sqrt(a + b*x)*a**2*b**2*c*x**2 + 12*sqrt(x)*sqrt(a + b*x)*
a**2*b**2*d*x**3 - 8*sqrt(x)*sqrt(a + b*x)*a*b**3*c*x**3 - 24*sqrt(x)*sqrt
(a + b*x)*a*b**3*d*x**4 + 16*sqrt(x)*sqrt(a + b*x)*b**4*c*x**4 + 24*sqrt(b
)*a*b**3*d*x**5 - 16*sqrt(b)*b**4*c*x**5))/(315*a**4*x**5)
```

3.10 $\int x^2(c+dx)^2\sqrt{ax+bx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 294

$$\begin{aligned}\int x^2(c+dx)^2\sqrt{ax+bx^2}\,dx &= \frac{a^3(40b^2c^2-7ad(8bc-3ad))\sqrt{ax+bx^2}}{512b^5} \\ &- \frac{a^2(40b^2c^2-7ad(8bc-3ad))\,x\sqrt{ax+bx^2}}{768b^4} \\ &+ \frac{a(40b^2c^2-7ad(8bc-3ad))\,x^2\sqrt{ax+bx^2}}{960b^3} \\ &+ \frac{1}{160}\left(40c^2-\frac{7ad(8bc-3ad)}{b^2}\right)x^3\sqrt{ax+bx^2} \\ &+ \frac{d(8bc-3ad)x^2(ax+bx^2)^{3/2}}{20b^2} + \frac{d^2x^3(ax+bx^2)^{3/2}}{6b} \\ &- \frac{a^4(40b^2c^2-7ad(8bc-3ad))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{512b^{11/2}}\end{aligned}$$

output
```
1/512*a^3*(40*b^2*c^2-7*a*d*(-3*a*d+8*b*c))*(b*x^2+a*x)^(1/2)/b^5-1/768*a^
2*(40*b^2*c^2-7*a*d*(-3*a*d+8*b*c))*x*(b*x^2+a*x)^(1/2)/b^4+1/960*a*(40*b^
2*c^2-7*a*d*(-3*a*d+8*b*c))*x^2*(b*x^2+a*x)^(1/2)/b^3+1/160*(40*c^2-7*a*d*
(-3*a*d+8*b*c)/b^2)*x^3*(b*x^2+a*x)^(1/2)+1/20*d*(-3*a*d+8*b*c)*x^2*(b*x^2
+a*x)^(3/2)/b^2+1/6*d^2*x^3*(b*x^2+a*x)^(3/2)/b-1/512*a^4*(40*b^2*c^2-7*a*
d*(-3*a*d+8*b*c))*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.95

$$\int x^2(c+dx)^2\sqrt{ax+bx^2}\,dx$$
$$= \frac{\sqrt{x(a+bx)}(600a^3b^2c^2-840a^4bcd+315a^5d^2-400a^2b^3c^2x+560a^3b^2cdx-210a^4bd^2x+320ab^4c^2x^2-}{}$$
$$-\frac{a^4(40b^2c^2-56abcd+21a^2d^2)\sqrt{x(a+bx)}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{256b^{11/2}\sqrt{x}\sqrt{a+bx}}$$

input
```
Integrate[x^2*(c + d*x)^2*Sqrt[a*x + b*x^2],x]
```

output
```
(Sqrt[x*(a + b*x)]*(600*a^3*b^2*c^2 - 840*a^4*b*c*d + 315*a^5*d^2 - 400*a^
2*b^3*c^2*x + 560*a^3*b^2*c*d*x - 210*a^4*b*d^2*x + 320*a*b^4*c^2*x^2 - 44
8*a^2*b^3*c*d*x^2 + 168*a^3*b^2*d^2*x^2 + 1920*b^5*c^2*x^3 + 384*a*b^4*c*d
*x^3 - 144*a^2*b^3*d^2*x^3 + 3072*b^5*c*d*x^4 + 128*a*b^4*d^2*x^4 + 1280*b
^5*d^2*x^5))/(7680*b^5) - (a^4*(40*b^2*c^2 - 56*a*b*c*d + 21*a^2*d^2)*Sqrt
[x*(a + b*x)]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(256*
b^(11/2)*Sqrt[x]*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.74, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1262, 27, 1221, 1134, 1160, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{ax+bx^2}(c+dx)^2\,dx$$

$$\Big\downarrow \text{1262}$$

$$\frac{\int \frac{3}{2}x^2\left(4bc^2+d(8bc-3ad)x\right)\sqrt{bx^2+ax}dx}{6b}+\frac{d^2x^3\left(ax+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int x^2\left(4bc^2+d(8bc-3ad)x\right)\sqrt{bx^2+ax}dx}{4b}+\frac{d^2x^3\left(ax+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \text{ 1221}$$

$$\frac{\frac{\left(21a^2d^2-56abcd+40b^2c^2\right)\int x^2\sqrt{bx^2+ax}dx}{10b}+\frac{dx^2\left(ax+bx^2\right)^{3/2}(8bc-3ad)}{5b}}{4b}+\frac{d^2x^3\left(ax+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \text{ 1134}$$

$$\frac{\frac{\left(21a^2d^2-56abcd+40b^2c^2\right)\left(\frac{x\left(ax+bx^2\right)^{3/2}}{4b}-\frac{5a\int x\sqrt{bx^2+ax}dx}{8b}\right)}{10b}+\frac{dx^2\left(ax+bx^2\right)^{3/2}(8bc-3ad)}{5b}}{4b}+\frac{d^2x^3\left(ax+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{\frac{\left(21a^2d^2-56abcd+40b^2c^2\right)\left(\frac{x\left(ax+bx^2\right)^{3/2}}{4b}-\frac{5a\left(\frac{\left(ax+bx^2\right)^{3/2}}{3b}-\frac{a\int\sqrt{bx^2+ax}dx}{2b}\right)}{8b}\right)}{10b}+\frac{dx^2\left(ax+bx^2\right)^{3/2}(8bc-3ad)}{5b}}{4b}+\frac{d^2x^3\left(ax+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{\frac{\left(21a^2d^2-56abcd+40b^2c^2\right)\left(\frac{x\left(ax+bx^2\right)^{3/2}}{4b}-\frac{5a\left(\frac{\left(ax+bx^2\right)^{3/2}}{3b}-\frac{a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)}{2b}\right)}{8b}\right)}{10b}+\frac{dx^2\left(ax+bx^2\right)^{3/2}(8bc-3ad)}{5b}}{4b}+\frac{d^2x^3\left(ax+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \text{ 1091}$$

$$\frac{\frac{(21a^2d^2-56abcd+40b^2c^2)\left(\frac{x\left(ax+bx^2\right)^{3/2}}{4b}-\frac{5a\left(\frac{\left(ax+bx^2\right)^{3/2}}{3b}-\frac{a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)}{2b}\right)}{8b}\right)}{10b}+\frac{dx^2\left(ax+bx^2\right)^{3/2}(8bc}{5b}}{4b}$$

$$\frac{d^2x^3\left(ax+bx^2\right)^{3/2}}{6b}$$

$\downarrow$ 219

$$\frac{\frac{\left(\frac{x\left(ax+bx^2\right)^{3/2}}{4b}-\frac{5a\left(\frac{\left(ax+bx^2\right)^{3/2}}{3b}-\frac{a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)}{2b}\right)}{8b}\right)(21a^2d^2-56abcd+40b^2c^2)}{10b}+\frac{dx^2\left(ax+bx^2\right)^{3/2}(8bc-}{5b}}{4b}$$

$$\frac{d^2x^3\left(ax+bx^2\right)^{3/2}}{6b}$$

input

```
Int[x^2*(c + d*x)^2*Sqrt[a*x + b*x^2],x]
```

output

```
(d^2*x^3*(a*x + b*x^2)^(3/2))/(6*b) + ((d*(8*b*c - 3*a*d)*x^2*(a*x + b*x^2
)^(3/2))/(5*b) + ((40*b^2*c^2 - 56*a*b*c*d + 21*a^2*d^2)*((x*(a*x + b*x^2)
^(3/2))/(4*b) - (5*a*((a*x + b*x^2)^(3/2)/(3*b) - (a*(((a + 2*b*x)*Sqrt[a*
x + b*x^2])/(4*b) - (a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*b^(3/2
))))/(2*b)))/(8*b)))/(10*b))/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$-\frac{21\left(a^4\left(a^2d^2-\frac{8}{3}abcd+\frac{40}{21}b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)-\sqrt{x(bx+a)}\left(\frac{128x^3\left(\frac{2}{3}d^2x^2+\frac{8}{5}cdx+c^2\right)b^{\frac{11}{2}}}{21}+\left(\frac{40\left(\frac{7}{25}d^2x^2+\frac{14}{15}cdx\right.}{21}\right.\right.\right.}{512b^{\frac{11}{2}}}$
risch	$\frac{(1280b^5d^2x^5+128a\,b^4d^2x^4+3072b^5cd\,x^4-144a^2b^3d^2x^3+384a\,b^4cd\,x^3+1920b^5c^2x^3+168a^3b^2d^2x^2-448a^2b^3cd\,x^2+320a\,b}{7680b^5\sqrt{x(bx+a)}}$
default	$c^2\left(\frac{x(b\,x^2+ax)^{\frac{3}{2}}}{4b}-\frac{5a\left(\frac{(b\,x^2+ax)^{\frac{3}{2}}}{3b}-\frac{a\left(\frac{(2bx+a)\sqrt{b\,x^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8b^{\frac{3}{2}}}\right)}{2b}\right)}{8b}\right)+d^2\left(\frac{x^3(b\,x^2+ax)^{\frac{3}{2}}}{6b}\right.$

input
```
int(x^2*(d*x+c)^2*(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-21/512/b^(11/2)*(a^4*(a^2*d^2-8/3*a*b*c*d+40/21*b^2*c^2)*arctanh((x*(b*x+
a))^(1/2)/x/b^(1/2))-(x*(b*x+a))^(1/2)*(128/21*x^3*(2/3*d^2*x^2+8/5*c*d*x+
c^2)*b^(11/2)+(40/21*(7/25*d^2*x^2+14/15*c*d*x+c^2)*a^2*b^(5/2)-80/63*x*a*
(9/25*d^2*x^2+28/25*c*d*x+c^2)*b^(7/2)+64/63*x^2*(2/5*d^2*x^2+6/5*c*d*x+c^
2)*b^(9/2)+d*(2/3*(-d*x-4*c)*b^(3/2)+b^(1/2)*a*d)*a^3)*a))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.67

$$\int x^2(c+dx)^2\sqrt{ax+bx^2}\,dx$$
$$= \left[\frac{15\left(40\,a^4b^2c^2-56\,a^5bcd+21\,a^6d^2\right)\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\left(1280\,b^6d^2x^5+600\,a^3b^3c^2\right.}{}\right.$$

input
```
integrate(x^2*(d*x+c)^2*(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/15360*(15*(40*a^4*b^2*c^2 - 56*a^5*b*c*d + 21*a^6*d^2)*sqrt(b)*log(2*b*
x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(1280*b^6*d^2*x^5 + 600*a^3*b^3*c
^2 - 840*a^4*b^2*c*d + 315*a^5*b*d^2 + 128*(24*b^6*c*d + a*b^5*d^2)*x^4 +
48*(40*b^6*c^2 + 8*a*b^5*c*d - 3*a^2*b^4*d^2)*x^3 + 8*(40*a*b^5*c^2 - 56*a
^2*b^4*c*d + 21*a^3*b^3*d^2)*x^2 - 10*(40*a^2*b^4*c^2 - 56*a^3*b^3*c*d + 2
1*a^4*b^2*d^2)*x)*sqrt(b*x^2 + a*x))/b^6, 1/7680*(15*(40*a^4*b^2*c^2 - 56*
a^5*b*c*d + 21*a^6*d^2)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x +
a)) + (1280*b^6*d^2*x^5 + 600*a^3*b^3*c^2 - 840*a^4*b^2*c*d + 315*a^5*b*d^
2 + 128*(24*b^6*c*d + a*b^5*d^2)*x^4 + 48*(40*b^6*c^2 + 8*a*b^5*c*d - 3*a^
2*b^4*d^2)*x^3 + 8*(40*a*b^5*c^2 - 56*a^2*b^4*c*d + 21*a^3*b^3*d^2)*x^2 -
10*(40*a^2*b^4*c^2 - 56*a^3*b^3*c*d + 21*a^4*b^2*d^2)*x)*sqrt(b*x^2 + a*x)
)/b^6]
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.38

$$\int x^2(c+dx)^2\sqrt{ax+bx^2}\,dx$$

$$= \begin{cases} -\frac{5a^3\left(ac^2-\frac{7a\left(2acd-\frac{9a\left(\frac{ad^2}{12}+2bcd\right)}{10b}+bc^2\right)}{8b}\right)\left(\begin{cases}\frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0\\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{16b^3}+\sqrt{ax+bx^2}\cdot\left(\frac{5a^2\left(ac^2-\frac{7a(\cdots}{\cdots}\right.}{\cdots}\right. \\ \frac{2\left(\frac{c^2(ax)^{\frac{7}{2}}}{7}+\frac{2cd(ax)^{\frac{9}{2}}}{9a}+\frac{d^2(ax)^{\frac{11}{2}}}{11a^2}\right)}{a^3} \\ 0 \end{cases}$$

input
```
integrate(x**2*(d*x+c)**2*(b*x**2+a*x)**(1/2),x)
```

output
```
Piecewise((-5*a**3*(a*c**2 - 7*a*(2*a*c*d - 9*a*(a*d**2/12 + 2*b*c*d)/(10*
b) + b*c**2)/(8*b))*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*
x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b
) + x)**2), True))/(16*b**3) + sqrt(a*x + b*x**2)*(5*a**2*(a*c**2 - 7*a*(2
*a*c*d - 9*a*(a*d**2/12 + 2*b*c*d)/(10*b) + b*c**2)/(8*b))/(8*b**3) - 5*a*
x*(a*c**2 - 7*a*(2*a*c*d - 9*a*(a*d**2/12 + 2*b*c*d)/(10*b) + b*c**2)/(8*b
))/(12*b**2) + d**2*x**5/6 + x**4*(a*d**2/12 + 2*b*c*d)/(5*b) + x**3*(2*a*
c*d - 9*a*(a*d**2/12 + 2*b*c*d)/(10*b) + b*c**2)/(4*b) + x**2*(a*c**2 - 7*
a*(2*a*c*d - 9*a*(a*d**2/12 + 2*b*c*d)/(10*b) + b*c**2)/(8*b))/(3*b)), Ne(
b, 0)), (2*(c**2*(a*x)**(7/2)/7 + 2*c*d*(a*x)**(9/2)/(9*a) + d**2*(a*x)**(
11/2)/(11*a**2))/a**3, Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.45

$$\int x^2(c+dx)^2\sqrt{ax+bx^2}\,dx = \frac{(bx^2+ax)^{\frac{3}{2}}d^2x^3}{6\,b} + \frac{2\,(bx^2+ax)^{\frac{3}{2}}cdx^2}{5\,b} - \frac{3\,(bx^2+ax)^{\frac{3}{2}}ad^2x^2}{20\,b^2} + \frac{5\,\sqrt{bx^2+ax}a^2c^2x}{32\,b^2} + \frac{(bx^2+ax)^{\frac{3}{2}}c^2x}{4\,b} - \frac{7\,\sqrt{bx^2+ax}a^3cdx}{32\,b^3} - \frac{7\,(bx^2+ax)^{\frac{3}{2}}acdx}{20\,b^2} + \frac{21\,\sqrt{bx^2+ax}a^4d^2x}{256\,b^4} + \frac{21\,(bx^2+ax)^{\frac{3}{2}}a^2d^2x}{160\,b^3} - \frac{5\,a^4c^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{7}{2}}} + \frac{7\,a^5cd\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{9}{2}}} - \frac{21\,a^6d^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{1024\,b^{\frac{11}{2}}} + \frac{5\,\sqrt{bx^2+ax}a^3c^2}{64\,b^3} - \frac{5\,(bx^2+ax)^{\frac{3}{2}}ac^2}{24\,b^2} - \frac{7\,\sqrt{bx^2+ax}a^4cd}{64\,b^4} + \frac{7\,(bx^2+ax)^{\frac{3}{2}}a^2cd}{24\,b^3} + \frac{21\,\sqrt{bx^2+ax}a^5d^2}{512\,b^5} - \frac{7\,(bx^2+ax)^{\frac{3}{2}}a^3d^2}{64\,b^4}$$

input
```
integrate(x^2*(d*x+c)^2*(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a*x)^(3/2)*d^2*x^3/b + 2/5*(b*x^2 + a*x)^(3/2)*c*d*x^2/b - 3/
20*(b*x^2 + a*x)^(3/2)*a*d^2*x^2/b^2 + 5/32*sqrt(b*x^2 + a*x)*a^2*c^2*x/b^
2 + 1/4*(b*x^2 + a*x)^(3/2)*c^2*x/b - 7/32*sqrt(b*x^2 + a*x)*a^3*c*d*x/b^3
 - 7/20*(b*x^2 + a*x)^(3/2)*a*c*d*x/b^2 + 21/256*sqrt(b*x^2 + a*x)*a^4*d^2
*x/b^4 + 21/160*(b*x^2 + a*x)^(3/2)*a^2*d^2*x/b^3 - 5/128*a^4*c^2*log(2*b*
x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(7/2) + 7/128*a^5*c*d*log(2*b*x + a
 + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(9/2) - 21/1024*a^6*d^2*log(2*b*x + a +
2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(11/2) + 5/64*sqrt(b*x^2 + a*x)*a^3*c^2/b^3
 - 5/24*(b*x^2 + a*x)^(3/2)*a*c^2/b^2 - 7/64*sqrt(b*x^2 + a*x)*a^4*c*d/b^4
 + 7/24*(b*x^2 + a*x)^(3/2)*a^2*c*d/b^3 + 21/512*sqrt(b*x^2 + a*x)*a^5*d^2
/b^5 - 7/64*(b*x^2 + a*x)^(3/2)*a^3*d^2/b^4
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.87

$$\int x^2(c+dx)^2\sqrt{ax+bx^2}\,dx$$
$$=\frac{1}{7680}\sqrt{bx^2+ax}\left(2\left(4\left(2\left(8\left(10\,d^2x+\frac{24\,b^5cd+ab^4d^2}{b^5}\right)x+\frac{3\,(40\,b^5c^2+8\,ab^4cd-3\,a^2b^3d^2)}{b^5}\right)x+\frac{40\,a\right.\right.\right.\right.$$
$$+\frac{(40\,a^4b^2c^2-56\,a^5bcd+21\,a^6d^2)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{1024\,b^{\frac{11}{2}}}$$

input
```
integrate(x^2*(d*x+c)^2*(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/7680*sqrt(b*x^2 + a*x)*(2*(4*(2*(8*(10*d^2*x + (24*b^5*c*d + a*b^4*d^2)/
b^5)*x + 3*(40*b^5*c^2 + 8*a*b^4*c*d - 3*a^2*b^3*d^2)/b^5)*x + (40*a*b^4*c
^2 - 56*a^2*b^3*c*d + 21*a^3*b^2*d^2)/b^5)*x - 5*(40*a^2*b^3*c^2 - 56*a^3*
b^2*c*d + 21*a^4*b*d^2)/b^5)*x + 15*(40*a^3*b^2*c^2 - 56*a^4*b*c*d + 21*a^
5*d^2)/b^5) + 1/1024*(40*a^4*b^2*c^2 - 56*a^5*b*c*d + 21*a^6*d^2)*log(abs(
2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(11/2)
```

Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.26

$$\int x^2(c+dx)^2\sqrt{ax+bx^2}\,dx$$

$$= \frac{d^2\,x^3\,\left(b\,x^2+a\,x\right)^{3/2}}{6\,b} - \frac{5\,a\,c^2\,\left(\frac{a^3\,\ln\left(\frac{a+2\,b\,x}{\sqrt{b}}+2\,\sqrt{b\,x^2+a\,x}\right)}{16\,b^{5/2}}+\frac{\sqrt{b\,x^2+a\,x}\,\left(-3\,a^2+2\,a\,b\,x+8\,b^2\,x^2\right)}{24\,b^2}\right)}{8\,b}$$
$$+ \frac{3\,a\,d^2\,\left(\frac{7\,a\,\left(\frac{x\,\left(b\,x^2+a\,x\right)^{3/2}}{4\,b}-\frac{5\,a\,\left(\frac{a^3\,\ln\left(\frac{a+2\,b\,x}{\sqrt{b}}+2\,\sqrt{b\,x^2+a\,x}\right)}{16\,b^{5/2}}+\frac{\sqrt{b\,x^2+a\,x}\,\left(-3\,a^2+2\,a\,b\,x+8\,b^2\,x^2\right)}{24\,b^2}\right)}{8\,b}\right)}{10\,b}-\frac{x^2\,\left(b\,x^2+a\,x\right)^{3/2}}{5\,b}\right)}{4\,b}$$
$$+ \frac{c^2\,x\,\left(b\,x^2+a\,x\right)^{3/2}}{4\,b}$$
$$- \frac{7\,a\,c\,d\,\left(\frac{x\,\left(b\,x^2+a\,x\right)^{3/2}}{4\,b}-\frac{5\,a\,\left(\frac{a^3\,\ln\left(\frac{a+2\,b\,x}{\sqrt{b}}+2\,\sqrt{b\,x^2+a\,x}\right)}{16\,b^{5/2}}+\frac{\sqrt{b\,x^2+a\,x}\,\left(-3\,a^2+2\,a\,b\,x+8\,b^2\,x^2\right)}{24\,b^2}\right)}{8\,b}\right)}{5\,b}$$
$$+ \frac{2\,c\,d\,x^2\,\left(b\,x^2+a\,x\right)^{3/2}}{5\,b}$$

input
```
int(x^2*(a*x + b*x^2)^(1/2)*(c + d*x)^2,x)
```

output
```
(d^2*x^3*(a*x + b*x^2)^(3/2))/(6*b) - (5*a*c^2*((a^3*log((a + 2*b*x)/b^(1/
2) + 2*(a*x + b*x^2)^(1/2)))/(16*b^(5/2)) + ((a*x + b*x^2)^(1/2)*(8*b^2*x^
2 - 3*a^2 + 2*a*b*x))/(24*b^2)))/(8*b) + (3*a*d^2*((7*a*((x*(a*x + b*x^2)^
(3/2))/(4*b) - (5*a*((a^3*log((a + 2*b*x)/b^(1/2) + 2*(a*x + b*x^2)^(1/2))
)/(16*b^(5/2)) + ((a*x + b*x^2)^(1/2)*(8*b^2*x^2 - 3*a^2 + 2*a*b*x))/(24*b
^2)))/(8*b)))/(10*b) - (x^2*(a*x + b*x^2)^(3/2))/(5*b)))/(4*b) + (c^2*x*(a
*x + b*x^2)^(3/2))/(4*b) - (7*a*c*d*((x*(a*x + b*x^2)^(3/2))/(4*b) - (5*a*
((a^3*log((a + 2*b*x)/b^(1/2) + 2*(a*x + b*x^2)^(1/2)))/(16*b^(5/2)) + ((a
*x + b*x^2)^(1/2)*(8*b^2*x^2 - 3*a^2 + 2*a*b*x))/(24*b^2)))/(8*b)))/(5*b)
+ (2*c*d*x^2*(a*x + b*x^2)^(3/2))/(5*b)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.31

$$\int x^2(c+dx)^2\sqrt{ax+bx^2}\,dx$$
$$= \frac{315\sqrt{x}\,\sqrt{bx+a}\,a^5b\,d^2-840\sqrt{x}\,\sqrt{bx+a}\,a^4b^2cd-210\sqrt{x}\,\sqrt{bx+a}\,a^4b^2d^2x+600\sqrt{x}\,\sqrt{bx+a}\,a^3b^3c^2+5}{}$$

input
```
int(x^2*(d*x+c)^2*(b*x^2+a*x)^(1/2),x)
```

output
```
(315*sqrt(x)*sqrt(a + b*x)*a**5*b*d**2 - 840*sqrt(x)*sqrt(a + b*x)*a**4*b*
*2*c*d - 210*sqrt(x)*sqrt(a + b*x)*a**4*b**2*d**2*x + 600*sqrt(x)*sqrt(a +
 b*x)*a**3*b**3*c**2 + 560*sqrt(x)*sqrt(a + b*x)*a**3*b**3*c*d*x + 168*sqr
t(x)*sqrt(a + b*x)*a**3*b**3*d**2*x**2 - 400*sqrt(x)*sqrt(a + b*x)*a**2*b*
*4*c**2*x - 448*sqrt(x)*sqrt(a + b*x)*a**2*b**4*c*d*x**2 - 144*sqrt(x)*sqr
t(a + b*x)*a**2*b**4*d**2*x**3 + 320*sqrt(x)*sqrt(a + b*x)*a*b**5*c**2*x**
2 + 384*sqrt(x)*sqrt(a + b*x)*a*b**5*c*d*x**3 + 128*sqrt(x)*sqrt(a + b*x)*
a*b**5*d**2*x**4 + 1920*sqrt(x)*sqrt(a + b*x)*b**6*c**2*x**3 + 3072*sqrt(x
)*sqrt(a + b*x)*b**6*c*d*x**4 + 1280*sqrt(x)*sqrt(a + b*x)*b**6*d**2*x**5
 - 315*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**6*d**2 + 8
40*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**5*b*c*d - 600
*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4*b**2*c**2)/(7
680*b**6)
```

3.11 $\int x(c+dx)^2\sqrt{ax+bx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 247

$$\begin{aligned}\int x(c+dx)^2\sqrt{ax+bx^2}\,dx = &-\frac{a^2(16b^2c^2-20abcd+7a^2d^2)\sqrt{ax+bx^2}}{128b^4}\\ &+\frac{a(16b^2c^2-20abcd+7a^2d^2)\,x\sqrt{ax+bx^2}}{192b^3}\\ &+\frac{1}{48}\left(16c^2-\frac{ad(20bc-7ad)}{b^2}\right)x^2\sqrt{ax+bx^2}\\ &+\frac{d(20bc-7ad)x(ax+bx^2)^{3/2}}{40b^2}+\frac{d^2x^2(ax+bx^2)^{3/2}}{5b}\\ &+\frac{a^3(16b^2c^2-20abcd+7a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{128b^{9/2}}\end{aligned}$$

output

```
-1/128*a^2*(7*a^2*d^2-20*a*b*c*d+16*b^2*c^2)*(b*x^2+a*x)^(1/2)/b^4+1/192*a
*(7*a^2*d^2-20*a*b*c*d+16*b^2*c^2)*x*(b*x^2+a*x)^(1/2)/b^3+1/48*(16*c^2-a*
d*(-7*a*d+20*b*c)/b^2)*x^2*(b*x^2+a*x)^(1/2)+1/40*d*(-7*a*d+20*b*c)*x*(b*x
^2+a*x)^(3/2)/b^2+1/5*d^2*x^2*(b*x^2+a*x)^(3/2)/b+1/128*a^3*(7*a^2*d^2-20*
a*b*c*d+16*b^2*c^2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.83

$$\int x(c+dx)^2\sqrt{ax+bx^2}\,dx$$
$$=\frac{\sqrt{x(a+bx)}\Big(\sqrt{b}(-105a^4d^2+10a^3bd(30c+7dx)+16ab^3x(10c^2+10cdx+3d^2x^2)+64b^4x^2(10c^2+15c}{1920b^{9/2}}$$

input
```
Integrate[x*(c + d*x)^2*Sqrt[a*x + b*x^2],x]
```

output
```
(Sqrt[x*(a + b*x)]*(Sqrt[b]*(-105*a^4*d^2 + 10*a^3*b*d*(30*c + 7*d*x) + 16
*a*b^3*x*(10*c^2 + 10*c*d*x + 3*d^2*x^2) + 64*b^4*x^2*(10*c^2 + 15*c*d*x +
 6*d^2*x^2) - 8*a^2*b^2*(30*c^2 + 25*c*d*x + 7*d^2*x^2)) + (30*a^3*(16*b^2
*c^2 - 20*a*b*c*d + 7*a^2*d^2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[
a + b*x])])/(Sqrt[x]*Sqrt[a + b*x])))/(1920*b^(9/2))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.74, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1262, 27, 1225, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{ax+bx^2}(c+dx)^2\,dx$$
$$\Big\downarrow \text{1262}$$
$$\frac{\int \frac{1}{2}x\left(10bc^2+d(20bc-7ad)x\right)\sqrt{bx^2+ax}dx}{5b}+\frac{d^2x^2\left(ax+bx^2\right)^{3/2}}{5b}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int x\left(10bc^2+d(20bc-7ad)x\right)\sqrt{bx^2+ax}dx}{10b}+\frac{d^2x^2\left(ax+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{1225}$$

$$\frac{\frac{(ax+bx^2)^{3/2}(5(16b^2c^2-ad(20bc-7ad))+6bdx(20bc-7ad))}{24b^2}-\frac{5a(16b^2c^2-ad(20bc-7ad))\int\sqrt{bx^2+ax}dx}{16b^2}}{10b}+\frac{d^2x^2(ax+bx^2)^{3/2}}{5b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\frac{(ax+bx^2)^{3/2}(5(16b^2c^2-ad(20bc-7ad))+6bdx(20bc-7ad))}{24b^2}-\frac{5a(16b^2c^2-ad(20bc-7ad))\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)}{16b^2}}{10b}+\frac{d^2x^2(ax+bx^2)^{3/2}}{5b}$$

$$\Big\downarrow \text{1091}$$

$$\frac{\frac{(ax+bx^2)^{3/2}(5(16b^2c^2-ad(20bc-7ad))+6bdx(20bc-7ad))}{24b^2}-\frac{5a(16b^2c^2-ad(20bc-7ad))\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)}{16b^2}}{10b}+\frac{d^2x^2(ax+bx^2)^{3/2}}{5b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{(ax+bx^2)^{3/2}(5(16b^2c^2-ad(20bc-7ad))+6bdx(20bc-7ad))}{24b^2}-\frac{5a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)(16b^2c^2-ad(20bc-7ad))}{16b^2}}{10b}+\frac{d^2x^2(ax+bx^2)^{3/2}}{5b}$$

input
```
Int[x*(c + d*x)^2*Sqrt[a*x + b*x^2],x]
```

output
```
(d^2*x^2*(a*x + b*x^2)^(3/2))/(5*b) + (((5*(16*b^2*c^2 - a*d*(20*b*c - 7*a
*d)) + 6*b*d*(20*b*c - 7*a*d)*x)*(a*x + b*x^2)^(3/2))/(24*b^2) - (5*a*(16*
b^2*c^2 - a*d*(20*b*c - 7*a*d))*(((a + 2*b*x)*Sqrt[a*x + b*x^2])/(4*b) - (
a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*b^(3/2))))/(16*b^2))/(10*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{\frac{7a^3\left(a^2d^2-\frac{20}{7}abcd+\frac{16}{7}b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{128}-\frac{7\left(\frac{16a^2\left(\frac{7}{30}d^2x^2+\frac{5}{6}cdx+c^2\right)b^{\frac{5}{2}}}{7}-\frac{32x\left(\frac{3}{10}d^2x^2+cdx+c^2\right)ab^{\frac{7}{2}}}{21}-\frac{128x^2\left(\frac{3}{5}d^2x^2 \cdots\right.}{\cdots}\right.}{128}}{b^{\frac{9}{2}}}$
risch	$-\frac{\left(-384b^4d^2x^4-48a\,b^3d^2x^3-960b^4cd\,x^3+56a^2b^2d^2x^2-160a\,b^3cd\,x^2-640c^2x^2b^4-70a^3b\,d^2x+200a^2b^2cdx-160a\,b^3c^2x+\cdots\right.}{1920b^4\sqrt{x(bx+a)}}$
default	$c^2\left(\frac{(b\,x^2+ax)^{\frac{3}{2}}}{3b}-\frac{a\left(\frac{(2bx+a)\sqrt{b\,x^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8b^{\frac{3}{2}}}\right)}{2b}\right)+d^2\left(\frac{x^2(b\,x^2+ax)^{\frac{3}{2}}}{5b}-\frac{7a\left(\frac{x\left(b\,x^2+ax\right)\cdots}{4b}\cdots\right.}{\cdots}\right.$

input

```
int(x*(d*x+c)^2*(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
7/128*(a^3*(a^2*d^2-20/7*a*b*c*d+16/7*b^2*c^2)*arctanh((x*(b*x+a))^(1/2)/x
/b^(1/2))-(16/7*a^2*(7/30*d^2*x^2+5/6*c*d*x+c^2)*b^(5/2)-32/21*x*(3/10*d^2
*x^2+c*d*x+c^2)*a*b^(7/2)-128/21*x^2*(3/5*d^2*x^2+3/2*c*d*x+c^2)*b^(9/2)+d
*((-2/3*d*x-20/7*c)*b^(3/2)+b^(1/2)*a*d)*a^3)*(x*(b*x+a))^(1/2))/b^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.68

$$\int x(c+dx)^2\sqrt{ax+bx^2}\,dx$$
$$=\left[\frac{15\,(16\,a^3b^2c^2-20\,a^4bcd+7\,a^5d^2)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(384\,b^5d^2x^4-240\,a^2b^3c^2+3}{}\right.$$
$$-\frac{15\,(16\,a^3b^2c^2-20\,a^4bcd+7\,a^5d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(384\,b^5d^2x^4-240\,a^2b^3c^2+300\,a^3b^2cd}{}$$

input
```
integrate(x*(d*x+c)^2*(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/3840*(15*(16*a^3*b^2*c^2 - 20*a^4*b*c*d + 7*a^5*d^2)*sqrt(b)*log(2*b*x
+ a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(384*b^5*d^2*x^4 - 240*a^2*b^3*c^2
+ 300*a^3*b^2*c*d - 105*a^4*b*d^2 + 48*(20*b^5*c*d + a*b^4*d^2)*x^3 + 8*(8
0*b^5*c^2 + 20*a*b^4*c*d - 7*a^2*b^3*d^2)*x^2 + 10*(16*a*b^4*c^2 - 20*a^2*
b^3*c*d + 7*a^3*b^2*d^2)*x)*sqrt(b*x^2 + a*x))/b^5, -1/1920*(15*(16*a^3*b^
2*c^2 - 20*a^4*b*c*d + 7*a^5*d^2)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-
b)/(b*x + a)) - (384*b^5*d^2*x^4 - 240*a^2*b^3*c^2 + 300*a^3*b^2*c*d - 105
*a^4*b*d^2 + 48*(20*b^5*c*d + a*b^4*d^2)*x^3 + 8*(80*b^5*c^2 + 20*a*b^4*c*
d - 7*a^2*b^3*d^2)*x^2 + 10*(16*a*b^4*c^2 - 20*a^2*b^3*c*d + 7*a^3*b^2*d^2
)*x)*sqrt(b*x^2 + a*x))/b^5]
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.41

$$\int x(c+dx)^2\sqrt{ax+bx^2}\,dx$$

$$= \begin{cases} \frac{3a^2\left(ac^2-\frac{5a\left(2acd-\frac{7a\left(\frac{ad^2}{10}+2bcd\right)}{8b}+bc^2\right)}{6b}\right)\left(\begin{cases}\frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0\\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{8b^2}+\sqrt{ax+bx^2}\left(-\frac{3a\left(ac^2-\frac{5a\left(2acd\cdots\right.}{\cdots}\right.}{\cdots}\right. \\ \frac{2\left(\frac{c^2(ax)^{\frac{5}{2}}}{5}+\frac{2cd(ax)^{\frac{7}{2}}}{7a}+\frac{d^2(ax)^{\frac{9}{2}}}{9a^2}\right)}{a^2} \\ 0 \end{cases}$$

input
```
integrate(x*(d*x+c)**2*(b*x**2+a*x)**(1/2),x)
```

output
```
Piecewise((3*a**2*(a*c**2 - 5*a*(2*a*c*d - 7*a*(a*d**2/10 + 2*b*c*d)/(8*b)
 + b*c**2)/(6*b))*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)
/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b)
+ x)**2), True))/(8*b**2) + sqrt(a*x + b*x**2)*(-3*a*(a*c**2 - 5*a*(2*a*c*
d - 7*a*(a*d**2/10 + 2*b*c*d)/(8*b) + b*c**2)/(6*b))/(4*b**2) + d**2*x**4/
5 + x**3*(a*d**2/10 + 2*b*c*d)/(4*b) + x**2*(2*a*c*d - 7*a*(a*d**2/10 + 2*
b*c*d)/(8*b) + b*c**2)/(3*b) + x*(a*c**2 - 5*a*(2*a*c*d - 7*a*(a*d**2/10 +
 2*b*c*d)/(8*b) + b*c**2)/(6*b))/(2*b)), Ne(b, 0)), (2*(c**2*(a*x)**(5/2)/
5 + 2*c*d*(a*x)**(7/2)/(7*a) + d**2*(a*x)**(9/2)/(9*a**2))/a**2, Ne(a, 0))
, (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.44

$$\int x(c+dx)^2\sqrt{ax+bx^2}\,dx = \frac{(bx^2+ax)^{\frac{3}{2}}d^2x^2}{5\,b} - \frac{\sqrt{bx^2+ax}ac^2x}{4\,b} + \frac{5\,\sqrt{bx^2+ax}a^2cdx}{16\,b^2} + \frac{(bx^2+ax)^{\frac{3}{2}}cdx}{2\,b} - \frac{7\,\sqrt{bx^2+ax}a^3d^2x}{64\,b^3} - \frac{7\,(bx^2+ax)^{\frac{3}{2}}ad^2x}{40\,b^2} + \frac{a^3c^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{5}{2}}} - \frac{5\,a^4cd\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{64\,b^{\frac{7}{2}}} + \frac{7\,a^5d^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{256\,b^{\frac{9}{2}}} - \frac{\sqrt{bx^2+ax}a^2c^2}{8\,b^2} + \frac{(bx^2+ax)^{\frac{3}{2}}c^2}{3\,b} + \frac{5\,\sqrt{bx^2+ax}a^3cd}{32\,b^3} - \frac{5\,(bx^2+ax)^{\frac{3}{2}}acd}{12\,b^2} - \frac{7\,\sqrt{bx^2+ax}a^4d^2}{128\,b^4} + \frac{7\,(bx^2+ax)^{\frac{3}{2}}a^2d^2}{48\,b^3}$$

input
```
integrate(x*(d*x+c)^2*(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
1/5*(b*x^2 + a*x)^(3/2)*d^2*x^2/b - 1/4*sqrt(b*x^2 + a*x)*a*c^2*x/b + 5/16
*sqrt(b*x^2 + a*x)*a^2*c*d*x/b^2 + 1/2*(b*x^2 + a*x)^(3/2)*c*d*x/b - 7/64*
sqrt(b*x^2 + a*x)*a^3*d^2*x/b^3 - 7/40*(b*x^2 + a*x)^(3/2)*a*d^2*x/b^2 + 1
/16*a^3*c^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5/2) - 5/64*a^
4*c*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(7/2) + 7/256*a^5*d^2
*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(9/2) - 1/8*sqrt(b*x^2 + a
*x)*a^2*c^2/b^2 + 1/3*(b*x^2 + a*x)^(3/2)*c^2/b + 5/32*sqrt(b*x^2 + a*x)*a
^3*c*d/b^3 - 5/12*(b*x^2 + a*x)^(3/2)*a*c*d/b^2 - 7/128*sqrt(b*x^2 + a*x)*
a^4*d^2/b^4 + 7/48*(b*x^2 + a*x)^(3/2)*a^2*d^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.86

$$\int x(c+dx)^2\sqrt{ax+bx^2}\,dx$$
$$=\frac{1}{1920}\sqrt{bx^2+ax}\left(2\left(4\left(6\left(8\,d^2x+\frac{20\,b^4cd+ab^3d^2}{b^4}\right)x+\frac{80\,b^4c^2+20\,ab^3cd-7\,a^2b^2d^2}{b^4}\right)x+\frac{5\,(16\,ab^3c^2\right.\right.$$
$$-\frac{(16\,a^3b^2c^2-20\,a^4bcd+7\,a^5d^2)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{256\,b^{\frac{9}{2}}}$$

input
```
integrate(x*(d*x+c)^2*(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/1920*sqrt(b*x^2 + a*x)*(2*(4*(6*(8*d^2*x + (20*b^4*c*d + a*b^3*d^2)/b^4)
*x + (80*b^4*c^2 + 20*a*b^3*c*d - 7*a^2*b^2*d^2)/b^4)*x + 5*(16*a*b^3*c^2
- 20*a^2*b^2*c*d + 7*a^3*b*d^2)/b^4)*x - 15*(16*a^2*b^2*c^2 - 20*a^3*b*c*d
 + 7*a^4*d^2)/b^4) - 1/256*(16*a^3*b^2*c^2 - 20*a^4*b*c*d + 7*a^5*d^2)*log
(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(9/2)
```

Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.19

$$\int x(c+dx)^2\sqrt{ax+bx^2}\,dx$$
$$=\frac{d^2\,x^2\,{\left(b\,x^2+a\,x\right)}^{3/2}}{5\,b}$$
$$-\frac{7\,a\,d^2\left(\frac{x\,{\left(b\,x^2+a\,x\right)}^{3/2}}{4\,b}-\frac{5\,a\left(\frac{a^3\,\ln\left(\frac{a+2\,b\,x}{\sqrt{b}}+2\,\sqrt{b\,x^2+a\,x}\right)}{16\,b^{5/2}}+\frac{\sqrt{b\,x^2+a\,x}\,\left(-3\,a^2+2\,a\,b\,x+8\,b^2\,x^2\right)}{24\,b^2}\right)}{8\,b}\right)}{10\,b}$$
$$+\frac{a^3\,c^2\,\ln\left(\frac{a+2\,b\,x}{\sqrt{b}}+2\,\sqrt{b\,x^2+a\,x}\right)}{16\,b^{5/2}}+\frac{c^2\,\sqrt{b\,x^2+a\,x}\,\left(-3\,a^2+2\,a\,b\,x+8\,b^2\,x^2\right)}{24\,b^2}$$
$$-\frac{5\,a\,c\,d\left(\frac{a^3\,\ln\left(\frac{a+2\,b\,x}{\sqrt{b}}+2\,\sqrt{b\,x^2+a\,x}\right)}{16\,b^{5/2}}+\frac{\sqrt{b\,x^2+a\,x}\,\left(-3\,a^2+2\,a\,b\,x+8\,b^2\,x^2\right)}{24\,b^2}\right)}{4\,b}+\frac{c\,d\,x\,{\left(b\,x^2+a\,x\right)}^{3/2}}{2\,b}$$

input
```
int(x*(a*x + b*x^2)^(1/2)*(c + d*x)^2,x)
```

output
```
(d^2*x^2*(a*x + b*x^2)^(3/2))/(5*b) - (7*a*d^2*((x*(a*x + b*x^2)^(3/2))/(4
*b) - (5*a*((a^3*log((a + 2*b*x)/b^(1/2) + 2*(a*x + b*x^2)^(1/2)))/(16*b^(
5/2)) + ((a*x + b*x^2)^(1/2)*(8*b^2*x^2 - 3*a^2 + 2*a*b*x))/(24*b^2)))/(8*
b)))/(10*b) + (a^3*c^2*log((a + 2*b*x)/b^(1/2) + 2*(a*x + b*x^2)^(1/2)))/(
16*b^(5/2)) + (c^2*(a*x + b*x^2)^(1/2)*(8*b^2*x^2 - 3*a^2 + 2*a*b*x))/(24*
b^2) - (5*a*c*d*((a^3*log((a + 2*b*x)/b^(1/2) + 2*(a*x + b*x^2)^(1/2)))/(1
6*b^(5/2)) + ((a*x + b*x^2)^(1/2)*(8*b^2*x^2 - 3*a^2 + 2*a*b*x))/(24*b^2))
)/(4*b) + (c*d*x*(a*x + b*x^2)^(3/2))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.30

$$\int x(c+dx)^2\sqrt{ax+bx^2}\,dx = \frac{-105\sqrt{x}\,\sqrt{bx+a}\,a^4b\,d^2+300\sqrt{x}\,\sqrt{bx+a}\,a^3b^2cd+70\sqrt{x}\,\sqrt{bx+a}\,a^3b^2d^2x-240\sqrt{x}\,\sqrt{bx+a}\,a^2b^3c^2-2}{}$$

input
```
int(x*(d*x+c)^2*(b*x^2+a*x)^(1/2),x)
```

output
```
( - 105*sqrt(x)*sqrt(a + b*x)*a**4*b*d**2 + 300*sqrt(x)*sqrt(a + b*x)*a**3
*b**2*c*d + 70*sqrt(x)*sqrt(a + b*x)*a**3*b**2*d**2*x - 240*sqrt(x)*sqrt(a
 + b*x)*a**2*b**3*c**2 - 200*sqrt(x)*sqrt(a + b*x)*a**2*b**3*c*d*x - 56*sq
rt(x)*sqrt(a + b*x)*a**2*b**3*d**2*x**2 + 160*sqrt(x)*sqrt(a + b*x)*a*b**4
*c**2*x + 160*sqrt(x)*sqrt(a + b*x)*a*b**4*c*d*x**2 + 48*sqrt(x)*sqrt(a +
b*x)*a*b**4*d**2*x**3 + 640*sqrt(x)*sqrt(a + b*x)*b**5*c**2*x**2 + 960*sqr
t(x)*sqrt(a + b*x)*b**5*c*d*x**3 + 384*sqrt(x)*sqrt(a + b*x)*b**5*d**2*x**
4 + 105*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**5*d**2 -
 300*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4*b*c*d + 2
40*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*b**2*c**2)/
(1920*b**5)
```

3.12 $\int (c+dx)^2\sqrt{ax+bx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 195

$$\begin{aligned}\int (c+dx)^2\sqrt{ax+bx^2}\,dx &= \frac{a(16b^2c^2-16abcd+5a^2d^2)\sqrt{ax+bx^2}}{64b^3} \\ &+ \frac{1}{32}\left(16c^2-\frac{ad(16bc-5ad)}{b^2}\right)x\sqrt{ax+bx^2} \\ &+ \frac{d(16bc-5ad)\left(ax+bx^2\right)^{3/2}}{24b^2}+\frac{d^2x\left(ax+bx^2\right)^{3/2}}{4b} \\ &- \frac{a^2(16b^2c^2-16abcd+5a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{64b^{7/2}}\end{aligned}$$

output
```
1/64*a*(5*a^2*d^2-16*a*b*c*d+16*b^2*c^2)*(b*x^2+a*x)^(1/2)/b^3+1/32*(16*c^
2-a*d*(-5*a*d+16*b*c)/b^2)*x*(b*x^2+a*x)^(1/2)+1/24*d*(-5*a*d+16*b*c)*(b*x
^2+a*x)^(3/2)/b^2+1/4*d^2*x*(b*x^2+a*x)^(3/2)/b-1/64*a^2*(5*a^2*d^2-16*a*b
*c*d+16*b^2*c^2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.89

$$\int (c+dx)^2\sqrt{ax+bx^2}\,dx$$
$$=\frac{\sqrt{x(a+bx)}\left(\sqrt{b}(15a^3d^2-2a^2bd(24c+5dx)+8ab^2(6c^2+4cdx+d^2x^2)+16b^3x(6c^2+8cdx+3d^2x^2))\right.}{192b^{7/2}}$$

input
```
Integrate[(c + d*x)^2*Sqrt[a*x + b*x^2],x]
```

output
```
(Sqrt[x*(a + b*x)]*(Sqrt[b]*(15*a^3*d^2 - 2*a^2*b*d*(24*c + 5*d*x) + 8*a*b
^2*(6*c^2 + 4*c*d*x + d^2*x^2) + 16*b^3*x*(6*c^2 + 8*c*d*x + 3*d^2*x^2)) +
 (6*a^2*(16*b^2*c^2 - 16*a*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(S
qrt[a] - Sqrt[a + b*x])])/(Sqrt[x]*Sqrt[a + b*x])))/(192*b^(7/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.79, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1166, 27, 1160, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ax+bx^2}(c+dx)^2\,dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{\int \frac{1}{2}(c(8bc-3ad)+5d(2bc-ad)x)\sqrt{bx^2+ax}dx}{4b}+\frac{d\left(ax+bx^2\right)^{3/2}(c+dx)}{4b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (c(8bc-3ad)+5d(2bc-ad)x)\sqrt{bx^2+ax}dx}{8b}+\frac{d\left(ax+bx^2\right)^{3/2}(c+dx)}{4b}$$

$$\Big\downarrow \text{1160}$$

$$\frac{\frac{\left(5a^2d^2-16abcd+16b^2c^2\right)\int \sqrt{bx^2+ax}dx}{2b}+\frac{5d\left(ax+bx^2\right)^{3/2}(2bc-ad)}{3b}}{8b}+\frac{d\left(ax+bx^2\right)^{3/2}(c+dx)}{4b}$$

$\Big\downarrow$ 1087

$$\frac{\frac{\left(5a^2d^2-16abcd+16b^2c^2\right)\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)}{2b}+\frac{5d\left(ax+bx^2\right)^{3/2}(2bc-ad)}{3b}}{8b}+\frac{d\left(ax+bx^2\right)^{3/2}(c+dx)}{4b}$$

$\Big\downarrow$ 1091

$$\frac{\frac{\left(5a^2d^2-16abcd+16b^2c^2\right)\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)}{2b}+\frac{5d\left(ax+bx^2\right)^{3/2}(2bc-ad)}{3b}}{8b}+\frac{d\left(ax+bx^2\right)^{3/2}(c+dx)}{4b}$$

$\Big\downarrow$ 219

$$\frac{\frac{\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)\left(5a^2d^2-16abcd+16b^2c^2\right)}{2b}+\frac{5d\left(ax+bx^2\right)^{3/2}(2bc-ad)}{3b}}{8b}+\frac{d\left(ax+bx^2\right)^{3/2}(c+dx)}{4b}$$

input
```
Int[(c + d*x)^2*Sqrt[a*x + b*x^2],x]
```

output
```
(d*(c + d*x)*(a*x + b*x^2)^(3/2))/(4*b) + ((5*d*(2*b*c - a*d)*(a*x + b*x^2
)^(3/2))/(3*b) + ((16*b^2*c^2 - 16*a*b*c*d + 5*a^2*d^2)*(((a + 2*b*x)*Sqrt
[a*x + b*x^2])/(4*b) - (a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*b^(
3/2))))/(2*b))/(8*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$-\frac{5\left(a^2\left(a^2d^2-\frac{16}{5}abcd+\frac{16}{5}b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)-\sqrt{x(bx+a)}\left(\frac{16\left(\frac{1}{6}d^2x^2+\frac{2}{3}cdx+c^2\right)a\,b^{\frac{5}{2}}}{5}+\frac{32\left(\frac{1}{2}d^2x^2+\frac{4}{3}cdx+c^2\right)x\,b}{5}\right.\right.}{64b^{\frac{7}{2}}}$
risch	$\frac{\left(48b^3d^2x^3+8a\,b^2d^2x^2+128b^3cd\,x^2-10a^2b\,d^2x+32a\,b^2cdx+96b^3c^2x+15a^3d^2-48a^2bcd+48a\,c^2b^2\right)x(bx+a)}{192b^3\sqrt{x(bx+a)}}-\frac{a^2\left(5a^2d^2-\right.}{}$
default	$c^2\left(\frac{(2bx+a)\sqrt{b\,x^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8b^{\frac{3}{2}}}\right)+d^2\left(\frac{x\left(b\,x^2+ax\right)^{\frac{3}{2}}}{4b}-\frac{5a\left(\frac{\left(b\,x^2+ax\right)^{\frac{3}{2}}}{3b}-\frac{a\left(\frac{(2bx+a)\sqrt{b\,x^2}}{4b}\right.}{}\right.}{8}\right.$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-5/64/b^(7/2)*(a^2*(a^2*d^2-16/5*a*b*c*d+16/5*b^2*c^2)*arctanh((x*(b*x+a))
^(1/2)/x/b^(1/2))-(x*(b*x+a))^(1/2)*(16/5*(1/6*d^2*x^2+2/3*c*d*x+c^2)*a*b^
(5/2)+32/5*(1/2*d^2*x^2+4/3*c*d*x+c^2)*x*b^(7/2)+d*((-2/3*d*x-16/5*c)*b^(3
/2)+b^(1/2)*a*d)*a^2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.73

$$\int (c+dx)^2\sqrt{ax+bx^2}\,dx$$

$$=\left[\frac{3\left(16a^2b^2c^2-16a^3bcd+5a^4d^2\right)\sqrt{b}\log\left(2bx+a-2\sqrt{bx^2+ax}\sqrt{b}\right)+2\left(48b^4d^2x^3+48ab^3c^2-48a^2\right.}{384b^4}\right.$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/384*(3*(16*a^2*b^2*c^2 - 16*a^3*b*c*d + 5*a^4*d^2)*sqrt(b)*log(2*b*x +
a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(48*b^4*d^2*x^3 + 48*a*b^3*c^2 - 48*a
^2*b^2*c*d + 15*a^3*b*d^2 + 8*(16*b^4*c*d + a*b^3*d^2)*x^2 + 2*(48*b^4*c^2
 + 16*a*b^3*c*d - 5*a^2*b^2*d^2)*x)*sqrt(b*x^2 + a*x))/b^4, 1/192*(3*(16*a
^2*b^2*c^2 - 16*a^3*b*c*d + 5*a^4*d^2)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*s
qrt(-b)/(b*x + a)) + (48*b^4*d^2*x^3 + 48*a*b^3*c^2 - 48*a^2*b^2*c*d + 15*
a^3*b*d^2 + 8*(16*b^4*c*d + a*b^3*d^2)*x^2 + 2*(48*b^4*c^2 + 16*a*b^3*c*d
- 5*a^2*b^2*d^2)*x)*sqrt(b*x^2 + a*x))/b^4]
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.46

$$\int (c+dx)^2\sqrt{ax+bx^2}\,dx$$

$$= \begin{cases} -\frac{a\left(ac^2-\frac{3a\left(2acd-\frac{5a\left(\frac{ad^2}{8}+2bcd\right)}{6b}+bc^2\right)}{4b}\right)\left(\begin{cases}\frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0\\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{2b}+\sqrt{ax+bx^2}\left(\frac{d^2x^3}{4}+\frac{x^2\left(\frac{ad^2}{8}+2\right.}{3b}\right. \\ \frac{2\left(\frac{c^2(ax)^{\frac{3}{2}}}{3}+\frac{2cd(ax)^{\frac{5}{2}}}{5a}+\frac{d^2(ax)^{\frac{7}{2}}}{7a^2}\right)}{a} \\ 0 \end{cases}$$

input
```
integrate((d*x+c)**2*(b*x**2+a*x)**(1/2),x)
```

output
```
Piecewise((-a*(a*c**2 - 3*a*(2*a*c*d - 5*a*(a*d**2/8 + 2*b*c*d)/(6*b) + b*
c**2)/(4*b))*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt
(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)*
*2), True))/(2*b) + sqrt(a*x + b*x**2)*(d**2*x**3/4 + x**2*(a*d**2/8 + 2*b
*c*d)/(3*b) + x*(2*a*c*d - 5*a*(a*d**2/8 + 2*b*c*d)/(6*b) + b*c**2)/(2*b)
+ (a*c**2 - 3*a*(2*a*c*d - 5*a*(a*d**2/8 + 2*b*c*d)/(6*b) + b*c**2)/(4*b))
/b), Ne(b, 0)), (2*(c**2*(a*x)**(3/2)/3 + 2*c*d*(a*x)**(5/2)/(5*a) + d**2*
(a*x)**(7/2)/(7*a**2))/a, Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.45

$$\int (c+dx)^2\sqrt{ax+bx^2}\,dx = \frac{1}{2}\sqrt{bx^2+ax}c^2x - \frac{\sqrt{bx^2+ax}acdx}{2\,b} + \frac{5\sqrt{bx^2+ax}a^2d^2x}{32\,b^2} + \frac{(bx^2+ax)^{\frac{3}{2}}d^2x}{4\,b} - \frac{a^2c^2\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{3}{2}}} + \frac{a^3cd\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{5}{2}}} - \frac{5\,a^4d^2\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{7}{2}}} + \frac{\sqrt{bx^2+ax}ac^2}{4\,b} - \frac{\sqrt{bx^2+ax}a^2cd}{4\,b^2} + \frac{2\,(bx^2+ax)^{\frac{3}{2}}cd}{3\,b} + \frac{5\sqrt{bx^2+ax}a^3d^2}{64\,b^3} - \frac{5\,(bx^2+ax)^{\frac{3}{2}}ad^2}{24\,b^2}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a*x)*c^2*x - 1/2*sqrt(b*x^2 + a*x)*a*c*d*x/b + 5/32*sqrt(
b*x^2 + a*x)*a^2*d^2*x/b^2 + 1/4*(b*x^2 + a*x)^(3/2)*d^2*x/b - 1/8*a^2*c^2
*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(3/2) + 1/8*a^3*c*d*log(2*
b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5/2) - 5/128*a^4*d^2*log(2*b*x +
 a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(7/2) + 1/4*sqrt(b*x^2 + a*x)*a*c^2/b
- 1/4*sqrt(b*x^2 + a*x)*a^2*c*d/b^2 + 2/3*(b*x^2 + a*x)^(3/2)*c*d/b + 5/64
*sqrt(b*x^2 + a*x)*a^3*d^2/b^3 - 5/24*(b*x^2 + a*x)^(3/2)*a*d^2/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.88

$$\int (c+dx)^2\sqrt{ax+bx^2}\,dx$$
$$=\frac{1}{192}\sqrt{bx^2+ax}\left(2\left(4\left(6\,d^2x+\frac{16\,b^3cd+ab^2d^2}{b^3}\right)x+\frac{48\,b^3c^2+16\,ab^2cd-5\,a^2bd^2}{b^3}\right)x+\frac{3\,(16\,ab^2c^2-16\right.$$
$$+\frac{(16\,a^2b^2c^2-16\,a^3bcd+5\,a^4d^2)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{128\,b^{\frac{7}{2}}}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/192*sqrt(b*x^2 + a*x)*(2*(4*(6*d^2*x + (16*b^3*c*d + a*b^2*d^2)/b^3)*x +
 (48*b^3*c^2 + 16*a*b^2*c*d - 5*a^2*b*d^2)/b^3)*x + 3*(16*a*b^2*c^2 - 16*a
^2*b*c*d + 5*a^3*d^2)/b^3) + 1/128*(16*a^2*b^2*c^2 - 16*a^3*b*c*d + 5*a^4*
d^2)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(7/2)
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.18

$$\int (c+dx)^2\sqrt{ax+bx^2}\,dx$$
$$=c^2\sqrt{b\,x^2+a\,x}\left(\frac{x}{2}+\frac{a}{4\,b}\right)$$
$$-\frac{5\,a\,d^2\left(\frac{a^3\,\ln\left(\frac{a+2\,b\,x}{\sqrt{b}}+2\,\sqrt{b\,x^2+a\,x}\right)}{16\,b^{5/2}}+\frac{\sqrt{b\,x^2+a\,x}\,(-3\,a^2+2\,a\,b\,x+8\,b^2\,x^2)}{24\,b^2}\right)}{8\,b}$$
$$-\frac{a^2\,c^2\,\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{8\,b^{3/2}}+\frac{d^2\,x\,(b\,x^2+a\,x)^{3/2}}{4\,b}$$
$$+\frac{a^3\,c\,d\,\ln\left(\frac{a+2\,b\,x}{\sqrt{b}}+2\,\sqrt{b\,x^2+a\,x}\right)}{8\,b^{5/2}}+\frac{c\,d\,\sqrt{b\,x^2+a\,x}\,(-3\,a^2+2\,a\,b\,x+8\,b^2\,x^2)}{12\,b^2}$$

input
```
int((a*x + b*x^2)^(1/2)*(c + d*x)^2,x)
```

output
```
c^2*(a*x + b*x^2)^(1/2)*(x/2 + a/(4*b)) - (5*a*d^2*((a^3*log((a + 2*b*x)/b
^(1/2) + 2*(a*x + b*x^2)^(1/2)))/(16*b^(5/2)) + ((a*x + b*x^2)^(1/2)*(8*b^
2*x^2 - 3*a^2 + 2*a*b*x))/(24*b^2)))/(8*b) - (a^2*c^2*log((a/2 + b*x)/b^(1
/2) + (a*x + b*x^2)^(1/2)))/(8*b^(3/2)) + (d^2*x*(a*x + b*x^2)^(3/2))/(4*b
) + (a^3*c*d*log((a + 2*b*x)/b^(1/2) + 2*(a*x + b*x^2)^(1/2)))/(8*b^(5/2))
 + (c*d*(a*x + b*x^2)^(1/2)*(8*b^2*x^2 - 3*a^2 + 2*a*b*x))/(12*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.31

$$\int (c+dx)^2\sqrt{ax+bx^2}\,dx$$

$$= \frac{15\sqrt{x}\,\sqrt{bx+a}\,a^3b\,d^2-48\sqrt{x}\,\sqrt{bx+a}\,a^2b^2cd-10\sqrt{x}\,\sqrt{bx+a}\,a^2b^2d^2x+48\sqrt{x}\,\sqrt{bx+a}\,a\,b^3c^2+32\sqrt{x}\cdots}{}$$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(1/2),x)
```

output
```
(15*sqrt(x)*sqrt(a + b*x)*a**3*b*d**2 - 48*sqrt(x)*sqrt(a + b*x)*a**2*b**2
*c*d - 10*sqrt(x)*sqrt(a + b*x)*a**2*b**2*d**2*x + 48*sqrt(x)*sqrt(a + b*x
)*a*b**3*c**2 + 32*sqrt(x)*sqrt(a + b*x)*a*b**3*c*d*x + 8*sqrt(x)*sqrt(a +
 b*x)*a*b**3*d**2*x**2 + 96*sqrt(x)*sqrt(a + b*x)*b**4*c**2*x + 128*sqrt(x
)*sqrt(a + b*x)*b**4*c*d*x**2 + 48*sqrt(x)*sqrt(a + b*x)*b**4*d**2*x**3 -
15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4*d**2 + 48*s
qrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*b*c*d - 48*sqrt
(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*b**2*c**2)/(192*b*
*4)
```

3.13 $\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 152

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x}\,dx = \frac{(8b^2c^2-4abcd+a^2d^2)\sqrt{ax+bx^2}}{8b^2} + \frac{d^2(ax+bx^2)^{3/2}}{3b} + \frac{d(4bc-ad)(ax+bx^2)^{3/2}}{4b^2x} + \frac{a(8b^2c^2-4abcd+a^2d^2)\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{8b^{5/2}}$$

output
```
1/8*(a^2*d^2-4*a*b*c*d+8*b^2*c^2)*(b*x^2+a*x)^(1/2)/b^2+1/3*d^2*(b*x^2+a*x
)^(3/2)/b+1/4*d*(-a*d+4*b*c)*(b*x^2+a*x)^(3/2)/b^2/x+1/8*a*(a^2*d^2-4*a*b*
c*d+8*b^2*c^2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x}\,dx = \frac{\sqrt{x(a+bx)}\left(\sqrt{b}(-3a^2d^2+2abd(6c+dx)+8b^2(3c^2+3cdx+d^2x^2)) - \frac{3a(8b^2c^2-4abcd+a^2d^2)\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a-\right.}}{\sqrt{x}\sqrt{a+bx}}\right.}{24b^{5/2}}$$

input
```
Integrate[((c + d*x)^2*Sqrt[a*x + b*x^2])/x,x]
```

output
```
(Sqrt[x*(a + b*x)]*(Sqrt[b]*(-3*a^2*d^2 + 2*a*b*d*(6*c + d*x) + 8*b^2*(3*c
^2 + 3*c*d*x + d^2*x^2)) - (3*a*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*Log[-(Sq
rt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[x]*Sqrt[a + b*x])))/(24*b^(5/2))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1262, 27, 1221, 1131, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}(c+dx)^2}{x}dx$$

$$\Big\downarrow \text{1262}$$

$$\frac{\int \frac{3(2bc^2+d(4bc-ad)x)\sqrt{bx^2+ax}}{2x}dx}{3b}+\frac{d^2\left(ax+bx^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(2bc^2+d(4bc-ad)x)\sqrt{bx^2+ax}}{x}dx}{2b}+\frac{d^2\left(ax+bx^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \text{1221}$$

$$\frac{\frac{(a^2d^2-4abcd+8b^2c^2)\int \frac{\sqrt{bx^2+ax}}{x}dx}{4b}+\frac{d\left(ax+bx^2\right)^{3/2}(4bc-ad)}{2bx}}{2b}+\frac{d^2\left(ax+bx^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \text{1131}$$

$$\frac{\frac{(a^2d^2-4abcd+8b^2c^2)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+ax}}dx+\sqrt{ax+bx^2}\right)}{4b}+\frac{d\left(ax+bx^2\right)^{3/2}(4bc-ad)}{2bx}}{2b}+\frac{d^2\left(ax+bx^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \text{1091}$$

$$\frac{\frac{(a^2d^2-4abcd+8b^2c^2)\left(a\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}+\sqrt{ax+bx^2}\right)}{4b}+\frac{d(ax+bx^2)^{3/2}(4bc-ad)}{2bx}}{2b}+\frac{d^2(ax+bx^2)^{3/2}}{3b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}+\sqrt{ax+bx^2}\right)(a^2d^2-4abcd+8b^2c^2)}{4b}+\frac{d(ax+bx^2)^{3/2}(4bc-ad)}{2bx}}{2b}+\frac{d^2(ax+bx^2)^{3/2}}{3b}$$

input
```
Int[((c + d*x)^2*Sqrt[a*x + b*x^2])/x,x]
```

output
```
(d^2*(a*x + b*x^2)^(3/2))/(3*b) + ((d*(4*b*c - a*d)*(a*x + b*x^2)^(3/2))/(
2*b*x) + ((8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*(Sqrt[a*x + b*x^2] + (a*ArcTan
h[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]))/(4*b))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$\frac{a\left(a^{2} d^{2}-4 a b c d+8 b^{2} c^{2}\right) \operatorname{arctanh}\left(\frac{\sqrt{x (b x+a)}}{x \sqrt{b}}\right)-\left(\left(-\frac{8}{3} d^{2} x^{2}-8 c d x-8 c^{2}\right) b^{\frac{5}{2}}+d a \left(\left(-\frac{2 d x}{3}-4 c \right) b^{\frac{3}{2}}+\sqrt{b}\, a d \right)\right) \sqrt{x (b x+a)}}{8 b^{\frac{5}{2}}}$
risch	$-\frac{\left(-8 b^{2} d^{2} x^{2}-2 a b \,d^{2} x-24 b^{2} c x d+3 a^{2} d^{2}-12 a b c d-24 b^{2} c^{2}\right) x (b x+a)}{24 b^{2} \sqrt{x (b x+a)}}+\frac{a \left(a^{2} d^{2}-4 a b c d+8 b^{2} c^{2}\right) \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b \,x^{2}+a x}\right)}{16 b^{\frac{5}{2}}}$
default	$c^{2} \left(\sqrt{b \,x^{2}+a x}+\frac{a \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b \,x^{2}+a x}\right)}{2 \sqrt{b}}\right)+d^{2} \left(\frac{\left(b \,x^{2}+a x \right)^{\frac{3}{2}}}{3 b}-\frac{a \left(\frac{(2 b x+a) \sqrt{b \,x^{2}+a x}}{4 b}-\frac{a^{2} \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b \,x^{2}} \right.}{8 b^{\frac{3}{2}}} \right.}{2 b} \right.$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/8/b^(5/2)*(a*(a^2*d^2-4*a*b*c*d+8*b^2*c^2)*arctanh((x*(b*x+a))^(1/2)/x/b
^(1/2))-((-8/3*d^2*x^2-8*c*d*x-8*c^2)*b^(5/2)+d*a*((-2/3*d*x-4*c)*b^(3/2)+
b^(1/2)*a*d))*(x*(b*x+a))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.72

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x}\,dx = \left[\frac{3\,(8\,ab^2c^2-4\,a^2bcd+a^3d^2)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(8\,b^3d^2x^2+24\,b^3c^2+12\,ab^2cd-3}{48\,b^3}\right.$$
$$-\frac{3\,(8\,ab^2c^2-4\,a^2bcd+a^3d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(8\,b^3d^2x^2+24\,b^3c^2+12\,ab^2cd-3\,a^2bd^2+2}{24\,b^3}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/48*(3*(8*a*b^2*c^2 - 4*a^2*b*c*d + a^3*d^2)*sqrt(b)*log(2*b*x + a + 2*s
qrt(b*x^2 + a*x)*sqrt(b)) + 2*(8*b^3*d^2*x^2 + 24*b^3*c^2 + 12*a*b^2*c*d -
 3*a^2*b*d^2 + 2*(12*b^3*c*d + a*b^2*d^2)*x)*sqrt(b*x^2 + a*x))/b^3, -1/24
*(3*(8*a*b^2*c^2 - 4*a^2*b*c*d + a^3*d^2)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x
)*sqrt(-b)/(b*x + a)) - (8*b^3*d^2*x^2 + 24*b^3*c^2 + 12*a*b^2*c*d - 3*a^2
*b*d^2 + 2*(12*b^3*c*d + a*b^2*d^2)*x)*sqrt(b*x^2 + a*x))/b^3]
```

Sympy [F]

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x}\,dx = \int \frac{\sqrt{x\,(a+bx)}\,(c+dx)^2}{x}\,dx$$

input
```
integrate((d*x+c)**2*(b*x**2+a*x)**(1/2)/x,x)
```

output
```
Integral(sqrt(x*(a + b*x))*(c + d*x)**2/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.39

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x}\,dx = \sqrt{bx^2+ax}cdx - \frac{\sqrt{bx^2+ax}ad^2x}{4\,b} + \frac{ac^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{2\,\sqrt{b}} - \frac{a^2cd\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{4\,b^{\frac{3}{2}}} + \frac{a^3d^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{5}{2}}} + \sqrt{bx^2+ax}c^2 + \frac{\sqrt{bx^2+ax}acd}{2\,b} - \frac{\sqrt{bx^2+ax}a^2d^2}{8\,b^2} + \frac{(bx^2+ax)^{\frac{3}{2}}d^2}{3\,b}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(1/2)/x,x, algorithm="maxima")
```

output
```
sqrt(b*x^2 + a*x)*c*d*x - 1/4*sqrt(b*x^2 + a*x)*a*d^2*x/b + 1/2*a*c^2*log(
2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b) - 1/4*a^2*c*d*log(2*b*x +
 a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(3/2) + 1/16*a^3*d^2*log(2*b*x + a + 2
*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5/2) + sqrt(b*x^2 + a*x)*c^2 + 1/2*sqrt(b*x
^2 + a*x)*a*c*d/b - 1/8*sqrt(b*x^2 + a*x)*a^2*d^2/b^2 + 1/3*(b*x^2 + a*x)^
(3/2)*d^2/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x}\,dx = \frac{1}{24}\,\sqrt{bx^2+ax}\left(2\left(4\,d^2x+\frac{12\,b^2cd+abd^2}{b^2}\right)x+\frac{3\,(8\,b^2c^2+4\,abcd-a^2d^2)}{b^2}\right) - \frac{(8\,ab^2c^2-4\,a^2bcd+a^3d^2)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{16\,b^{\frac{5}{2}}}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(1/2)/x,x, algorithm="giac")
```

output
```
1/24*sqrt(b*x^2 + a*x)*(2*(4*d^2*x + (12*b^2*c*d + a*b*d^2)/b^2)*x + 3*(8*
b^2*c^2 + 4*a*b*c*d - a^2*d^2)/b^2) - 1/16*(8*a*b^2*c^2 - 4*a^2*b*c*d + a^
3*d^2)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(5/2)
```

Mupad [B] (verification not implemented)

Time = 9.40 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.20

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x}\,dx = c^2\,\sqrt{b\,x^2+a\,x}+\frac{a\,c^2\,\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{2\,\sqrt{b}} + 2\,c\,d\,\sqrt{b\,x^2+a\,x}\,\left(\frac{x}{2}+\frac{a}{4\,b}\right) + \frac{a^3\,d^2\,\ln\left(\frac{a+2\,b\,x}{\sqrt{b}}+2\,\sqrt{b\,x^2+a\,x}\right)}{16\,b^{5/2}} + \frac{d^2\,\sqrt{b\,x^2+a\,x}\,\left(-3\,a^2+2\,a\,b\,x+8\,b^2\,x^2\right)}{24\,b^2} - \frac{a^2\,c\,d\,\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{4\,b^{3/2}}$$

input
```
int(((a*x + b*x^2)^(1/2)*(c + d*x)^2)/x,x)
```

output
```
c^2*(a*x + b*x^2)^(1/2) + (a*c^2*log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(
1/2)))/(2*b^(1/2)) + 2*c*d*(a*x + b*x^2)^(1/2)*(x/2 + a/(4*b)) + (a^3*d^2*
log((a + 2*b*x)/b^(1/2) + 2*(a*x + b*x^2)^(1/2)))/(16*b^(5/2)) + (d^2*(a*x
 + b*x^2)^(1/2)*(8*b^2*x^2 - 3*a^2 + 2*a*b*x))/(24*b^2) - (a^2*c*d*log((a/
2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2)))/(4*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x}\,dx$$
$$= \frac{-3\sqrt{x}\,\sqrt{bx+a}\,a^2b\,d^2+12\sqrt{x}\,\sqrt{bx+a}\,a\,b^2cd+2\sqrt{x}\,\sqrt{bx+a}\,a\,b^2d^2x+24\sqrt{x}\,\sqrt{bx+a}\,b^3c^2+24\sqrt{x}\,\sqrt{bx}}{}$$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(1/2)/x,x)
```

output
```
( - 3*sqrt(x)*sqrt(a + b*x)*a**2*b*d**2 + 12*sqrt(x)*sqrt(a + b*x)*a*b**2*
c*d + 2*sqrt(x)*sqrt(a + b*x)*a*b**2*d**2*x + 24*sqrt(x)*sqrt(a + b*x)*b**
3*c**2 + 24*sqrt(x)*sqrt(a + b*x)*b**3*c*d*x + 8*sqrt(x)*sqrt(a + b*x)*b**
3*d**2*x**2 + 3*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**
3*d**2 - 12*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*b*
c*d + 24*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b**2*c**
2)/(24*b**3)
```

3.14 $\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 140

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^2}\,dx = \frac{1}{4}\left(\frac{8bc^2}{a}+8cd-\frac{ad^2}{b}\right)\sqrt{ax+bx^2} - \frac{2c^2(ax+bx^2)^{3/2}}{ax^2}+\frac{d^2(ax+bx^2)^{3/2}}{2bx} - \frac{(a^2d^2-8bc(bc+ad))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}$$

output
```
1/4*(8*b*c^2/a+8*c*d-a*d^2/b)*(b*x^2+a*x)^(1/2)-2*c^2*(b*x^2+a*x)^(3/2)/a/
x^2+1/2*d^2*(b*x^2+a*x)^(3/2)/b/x-1/4*(a^2*d^2-8*b*c*(a*d+b*c))*arctanh(b^
(1/2)*x/(b*x^2+a*x)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^2}\,dx = \frac{\sqrt{x(a+bx)}\left(\sqrt{b}(ad^2x+b(-8c^2+8cdx+2d^2x^2))+\frac{2(8b^2c^2+8abcd-a^2d^2)\sqrt{x}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{\sqrt{a+bx}}\right)}{4b^{3/2}x}$$

input
```
Integrate[((c + d*x)^2*Sqrt[a*x + b*x^2])/x^2,x]
```

output
```
(Sqrt[x*(a + b*x)]*(Sqrt[b]*(a*d^2*x + b*(-8*c^2 + 8*c*d*x + 2*d^2*x^2)) +
 (2*(8*b^2*c^2 + 8*a*b*c*d - a^2*d^2)*Sqrt[x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-
Sqrt[a] + Sqrt[a + b*x]])/Sqrt[a + b*x]))/(4*b^(3/2)*x)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1262, 27, 1220, 1131, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}(c+dx)^2}{x^2}dx$$

$$\Big\downarrow \text{1262}$$

$$\frac{\int \frac{(4bc^2+d(8bc-ad)x)\sqrt{bx^2+ax}}{2x^2}dx}{2b}+\frac{d^2\left(ax+bx^2\right)^{3/2}}{2bx}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(4bc^2+d(8bc-ad)x)\sqrt{bx^2+ax}}{x^2}dx}{4b}+\frac{d^2\left(ax+bx^2\right)^{3/2}}{2bx}$$

$$\Big\downarrow \text{1220}$$

$$\frac{\frac{(-a^2d^2+8abcd+8b^2c^2)\int \frac{\sqrt{bx^2+ax}}{x}dx}{a}-\frac{8bc^2\left(ax+bx^2\right)^{3/2}}{ax^2}}{4b}+\frac{d^2\left(ax+bx^2\right)^{3/2}}{2bx}$$

$$\Big\downarrow \text{1131}$$

$$\frac{\frac{(-a^2d^2+8abcd+8b^2c^2)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+ax}}dx+\sqrt{ax+bx^2}\right)}{a}-\frac{8bc^2\left(ax+bx^2\right)^{3/2}}{ax^2}}{4b}+\frac{d^2\left(ax+bx^2\right)^{3/2}}{2bx}$$

$$\Big\downarrow \text{1091}$$

$$\frac{\frac{(-a^2d^2+8abcd+8b^2c^2)\left(a\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}+\sqrt{ax+bx^2}\right)}{a}-\frac{8bc^2(ax+bx^2)^{3/2}}{ax^2}}{4b}+\frac{d^2(ax+bx^2)^{3/2}}{2bx}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}+\sqrt{ax+bx^2}\right)(-a^2d^2+8abcd+8b^2c^2)}{a}-\frac{8bc^2(ax+bx^2)^{3/2}}{ax^2}}{4b}+\frac{d^2(ax+bx^2)^{3/2}}{2bx}$$

input
```
Int[((c + d*x)^2*Sqrt[a*x + b*x^2])/x^2,x]
```

output
```
(d^2*(a*x + b*x^2)^(3/2))/(2*b*x) + ((-8*b*c^2*(a*x + b*x^2)^(3/2))/(a*x^2
) + ((8*b^2*c^2 + 8*a*b*c*d - a^2*d^2)*(Sqrt[a*x + b*x^2] + (a*ArcTanh[(Sq
rt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]))/a)/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$-\frac{x\left(a^{2} d^{2}-8 a b c d-8 b^{2} c^{2}\right) \operatorname{arctanh}\left(\frac{\sqrt{x \left(b x+a\right)}}{x \sqrt{b}}\right)-\left(\left(2 d^{2} x^{2}+8 c d x-8 c^{2}\right) b^{\frac{3}{2}}+\sqrt{b}\, a\, d^{2} x\right) \sqrt{x \left(b x+a\right)}}{4 b^{\frac{3}{2}} x}$
risch	$\frac{\left(b x+a\right) \left(2 b\, x^{2} d^{2}+a\, d^{2} x+8 b c d x-8 b\, c^{2}\right)}{4 b \sqrt{x \left(b x+a\right)}}-\frac{\left(a^{2} d^{2}-8 a b c d-8 b^{2} c^{2}\right) \ln\left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\, x^{2}+a x}\right)}{8 b^{\frac{3}{2}}}$
default	$d^{2} \left(\frac{\left(2 b x+a\right) \sqrt{b\, x^{2}+a x}}{4 b}-\frac{a^{2} \ln\left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\, x^{2}+a x}\right)}{8 b^{\frac{3}{2}}}\right)+c^{2} \left(-\frac{2 \left(b\, x^{2}+a x\right)^{\frac{3}{2}}}{a\, x^{2}}+\frac{2 b \left(\sqrt{b\, x^{2}+a x}+\frac{a \ln\left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b}\right.}{2 \sqrt{b}}\right.}{a}\right.$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(x*(a^2*d^2-8*a*b*c*d-8*b^2*c^2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))
-((2*d^2*x^2+8*c*d*x-8*c^2)*b^(3/2)+b^(1/2)*a*d^2*x)*(x*(b*x+a))^(1/2))/b^
(3/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.59

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^2}\,dx$$
$$= \left[-\frac{(8\,b^2c^2+8\,abcd-a^2d^2)\sqrt{b}x\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,(2\,b^2d^2x^2-8\,b^2c^2+(8\,b^2cd+abd^2}{8\,b^2x}\right.$$
$$-\frac{(8\,b^2c^2+8\,abcd-a^2d^2)\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(2\,b^2d^2x^2-8\,b^2c^2+(8\,b^2cd+abd^2)x)\sqrt{bx^2+a}}{4\,b^2x}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[-1/8*((8*b^2*c^2 + 8*a*b*c*d - a^2*d^2)*sqrt(b)*x*log(2*b*x + a - 2*sqrt(
b*x^2 + a*x)*sqrt(b)) - 2*(2*b^2*d^2*x^2 - 8*b^2*c^2 + (8*b^2*c*d + a*b*d^
2)*x)*sqrt(b*x^2 + a*x))/(b^2*x), -1/4*((8*b^2*c^2 + 8*a*b*c*d - a^2*d^2)*
sqrt(-b)*x*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (2*b^2*d^2*x^2 -
 8*b^2*c^2 + (8*b^2*c*d + a*b*d^2)*x)*sqrt(b*x^2 + a*x))/(b^2*x)]
```

Sympy [F]

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^2}\,dx=\int \frac{\sqrt{x\,(a+bx)}\,(c+dx)^2}{x^2}\,dx$$

input
```
integrate((d*x+c)**2*(b*x**2+a*x)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(x*(a + b*x))*(c + d*x)**2/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^2}\,dx = \frac{1}{2}\sqrt{bx^2+ax}d^2x + \sqrt{b}c^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) + \frac{acd\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}} - \frac{a^2d^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{3}{2}}} + 2\,\sqrt{bx^2+ax}cd + \frac{\sqrt{bx^2+ax}ad^2}{4\,b} - \frac{2\,\sqrt{bx^2+ax}c^2}{x}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a*x)*d^2*x + sqrt(b)*c^2*log(2*b*x + a + 2*sqrt(b*x^2 + a
*x)*sqrt(b)) + a*c*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
- 1/8*a^2*d^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(3/2) + 2*sqr
t(b*x^2 + a*x)*c*d + 1/4*sqrt(b*x^2 + a*x)*a*d^2/b - 2*sqrt(b*x^2 + a*x)*c
^2/x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^2}\,dx = \frac{2\,ac^2}{\sqrt{b}x-\sqrt{bx^2+ax}} + \frac{1}{4}\left(2\,d^2x+\frac{8\,bcd+ad^2}{b}\right)\sqrt{bx^2+ax} - \frac{(8\,b^2c^2+8\,abcd-a^2d^2)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^2,x, algorithm="giac")
```

output
```
2*a*c^2/(sqrt(b)*x - sqrt(b*x^2 + a*x)) + 1/4*(2*d^2*x + (8*b*c*d + a*d^2)
/b)*sqrt(b*x^2 + a*x) - 1/8*(8*b^2*c^2 + 8*a*b*c*d - a^2*d^2)*log(abs(2*(s
qrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^2}\,dx = \int \frac{\sqrt{b\,x^2+a\,x}\,(c+d\,x)^2}{x^2}\,dx$$

input
```
int(((a*x + b*x^2)^(1/2)*(c + d*x)^2)/x^2,x)
```

output
```
int(((a*x + b*x^2)^(1/2)*(c + d*x)^2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^2}\,dx = \frac{\sqrt{x}\,\sqrt{bx+a}\,ab\,d^2x-8\sqrt{x}\,\sqrt{bx+a}\,b^2c^2+8\sqrt{x}\,\sqrt{bx+a}\,b^2cdx+2\sqrt{x}\,\sqrt{bx+a}\,b^2d^2x^2-\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{\cdots}}{\sqrt{a}}\cdots\right.}{4b^2\cdots}$$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^2,x)
```

output
```
(sqrt(x)*sqrt(a + b*x)*a*b*d**2*x - 8*sqrt(x)*sqrt(a + b*x)*b**2*c**2 + 8*
sqrt(x)*sqrt(a + b*x)*b**2*c*d*x + 2*sqrt(x)*sqrt(a + b*x)*b**2*d**2*x**2
- sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*d**2*x + 8*s
qrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b*c*d*x + 8*sqrt(b
)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b**2*c**2*x - 2*sqrt(b)*a
*b*c*d*x - 8*sqrt(b)*b**2*c**2*x)/(4*b**2*x)
```

3.15 $\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 100

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^3}\,dx = d^2\sqrt{ax+bx^2} - \frac{4cd\sqrt{ax+bx^2}}{x} - \frac{2c^2(ax+bx^2)^{3/2}}{3ax^3} + \frac{d(4bc+ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

output
```
d^2*(b*x^2+a*x)^(1/2)-4*c*d*(b*x^2+a*x)^(1/2)/x-2/3*c^2*(b*x^2+a*x)^(3/2)/
a/x^3+d*(a*d+4*b*c)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^3}\,dx = \frac{\sqrt{x(a+bx)}\left(-12cdx + 3d^2x^2 - \frac{2c^2(a+bx)}{a} - \frac{3d(4bc+ad)x^{3/2}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{a+bx}}\right)}{3x^2}$$

input
```
Integrate[((c + d*x)^2*Sqrt[a*x + b*x^2])/x^3,x]
```

output
```
(Sqrt[x*(a + b*x)]*(-12*c*d*x + 3*d^2*x^2 - (2*c^2*(a + b*x))/a - (3*d*(4*
b*c + a*d)*x^(3/2)*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]*Sqrt[
a + b*x])))/(3*x^2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1262, 27, 1220, 1125, 25, 27, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}(c+dx)^2}{x^3} \, dx$$

$$\Big\downarrow \text{1262}$$

$$\frac{\int \frac{(2bc^2+d(4bc+ad)x)\sqrt{bx^2+ax}}{2x^3}dx}{b}+\frac{d^2\left(ax+bx^2\right)^{3/2}}{bx^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(2bc^2+d(4bc+ad)x)\sqrt{bx^2+ax}}{x^3}dx}{2b}+\frac{d^2\left(ax+bx^2\right)^{3/2}}{bx^2}$$

$$\Big\downarrow \text{1220}$$

$$\frac{d(ad+4bc)\int \frac{\sqrt{bx^2+ax}}{x^2}dx-\frac{4bc^2\left(ax+bx^2\right)^{3/2}}{3ax^3}}{2b}+\frac{d^2\left(ax+bx^2\right)^{3/2}}{bx^2}$$

$$\Big\downarrow \text{1125}$$

$$\frac{d(ad+4bc)\left(-\int -\frac{b}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{4bc^2\left(ax+bx^2\right)^{3/2}}{3ax^3}}{2b}+\frac{d^2\left(ax+bx^2\right)^{3/2}}{bx^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{d(ad+4bc)\left(\int \frac{b}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{4bc^2\left(ax+bx^2\right)^{3/2}}{3ax^3}}{2b}+\frac{d^2\left(ax+bx^2\right)^{3/2}}{bx^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{d(ad+4bc)\left(b\int \frac{1}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{4bc^2(ax+bx^2)^{3/2}}{3ax^3}}{2b}+\frac{d^2(ax+bx^2)^{3/2}}{bx^2}$$

$$\Big\downarrow \text{1091}$$

$$\frac{d(ad+4bc)\left(2b\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{4bc^2(ax+bx^2)^{3/2}}{3ax^3}}{2b}+\frac{d^2(ax+bx^2)^{3/2}}{bx^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\left(2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)-\frac{2\sqrt{ax+bx^2}}{x}\right)(ad+4bc)-\frac{4bc^2(ax+bx^2)^{3/2}}{3ax^3}}{2b}+\frac{d^2(ax+bx^2)^{3/2}}{bx^2}$$

input
```
Int[((c + d*x)^2*Sqrt[a*x + b*x^2])/x^3,x]
```

output
```
(d^2*(a*x + b*x^2)^(3/2))/(b*x^2) + ((-4*b*c^2*(a*x + b*x^2)^(3/2))/(3*a*x
^3) + d*(4*b*c + a*d)*((-2*Sqrt[a*x + b*x^2])/x + 2*Sqrt[b]*ArcTanh[(Sqrt[
b]*x)/Sqrt[a*x + b*x^2]]))/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.85

method	result
pseudoelliptic	$\frac{a d\,x^{2} (a d+4 b c) \operatorname{arctanh}\left(\frac{\sqrt{x (b x+a)}}{x \sqrt{b}}\right)-\frac{2 \left(b^{\frac{3}{2}} c^{2} x+\sqrt{b}\, a \left(-\frac{3}{2} d^{2} x^{2}+6 c d x+c^{2}\right)\right) \sqrt{x (b x+a)}}{3}}{\sqrt{b}\, x^{2} a}$
risch	$-\frac{(b x+a) \left(-3 a\,d^{2} x^{2}+12 a d x c+2 c^{2} b x+2 a\,c^{2}\right)}{3 x \sqrt{x (b x+a)}\, a}+\frac{(a d+4 b c) d \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\,x^{2}+a x}\right)}{2 \sqrt{b}}$
default	$-\frac{2 c^{2} \left(b\,x^{2}+a x\right)^{\frac{3}{2}}}{3 a\,x^{3}}+d^{2} \left(\sqrt{b\,x^{2}+a x}+\frac{a \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\,x^{2}+a x}\right)}{2 \sqrt{b}}\right)+2 c d \left(-\frac{2 \left(b\,x^{2}+a x\right)^{\frac{3}{2}}}{a\,x^{2}}+\frac{2 b \left(\sqrt{b\,x^{2}+\right.}{}\right.$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^(1/2)*(a*d*x^2*(a*d+4*b*c)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-2/3*(b
^(3/2)*c^2*x+b^(1/2)*a*(-3/2*d^2*x^2+6*c*d*x+c^2))*(x*(b*x+a))^(1/2))/x^2/
a
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.15

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^3}\,dx$$
$$= \left[\frac{3\,(4\,abcd+a^2d^2)\sqrt{b}x^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(3\,abd^2x^2-2\,abc^2-2\,(b^2c^2+6\,abcd)x)\sqrt{b}}{6\,abx^2}\right.$$
$$\left.-\frac{3\,(4\,abcd+a^2d^2)\sqrt{-b}x^2\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(3\,abd^2x^2-2\,abc^2-2\,(b^2c^2+6\,abcd)x)\sqrt{bx^2+ax}}{3\,abx^2}\right]$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/6*(3*(4*a*b*c*d + a^2*d^2)*sqrt(b)*x^2*log(2*b*x + a + 2*sqrt(b*x^2 + a
*x)*sqrt(b)) + 2*(3*a*b*d^2*x^2 - 2*a*b*c^2 - 2*(b^2*c^2 + 6*a*b*c*d)*x)*s
qrt(b*x^2 + a*x))/(a*b*x^2), -1/3*(3*(4*a*b*c*d + a^2*d^2)*sqrt(-b)*x^2*ar
ctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (3*a*b*d^2*x^2 - 2*a*b*c^2 -
2*(b^2*c^2 + 6*a*b*c*d)*x)*sqrt(b*x^2 + a*x))/(a*b*x^2)]
```

Sympy [F]

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^3}\,dx=\int \frac{\sqrt{x\,(a+bx)}\,(c+dx)^2}{x^3}\,dx$$

input
```
integrate((d*x+c)**2*(b*x**2+a*x)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(x*(a + b*x))*(c + d*x)**2/x**3, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^3}\,dx=\text{Timed out}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.72

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^3}\,dx$$
$$=\sqrt{bx^2+ax}d^2-\frac{(4\,bcd+ad^2)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{2\sqrt{b}}$$
$$+\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2bc^2+6\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2acd+3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)a\sqrt{b}c^2+a^2c^2\right)}{3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^3,x, algorithm="giac")
```

output
```
sqrt(b*x^2 + a*x)*d^2 - 1/2*(4*b*c*d + a*d^2)*log(abs(2*(sqrt(b)*x - sqrt(
b*x^2 + a*x))*sqrt(b) + a))/sqrt(b) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a*x
))^2*b*c^2 + 6*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a*c*d + 3*(sqrt(b)*x - sq
rt(b*x^2 + a*x))*a*sqrt(b)*c^2 + a^2*c^2)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^
3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^3}\,dx=\int \frac{\sqrt{b\,x^2+a\,x}\,{\left(c+d\,x\right)}^2}{x^3}\,dx$$

input
```
int(((a*x + b*x^2)^(1/2)*(c + d*x)^2)/x^3,x)
```

output
```
int(((a*x + b*x^2)^(1/2)*(c + d*x)^2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.73

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^3}\,dx$$
$$= \frac{-4\sqrt{x}\,\sqrt{bx+a}\,ab\,c^2 - 24\sqrt{x}\,\sqrt{bx+a}\,abcdx + 6\sqrt{x}\,\sqrt{bx+a}\,ab\,d^2x^2 - 4\sqrt{x}\,\sqrt{bx+a}\,b^2c^2x + 6\sqrt{b}\log\Big(\surd}{6ab\,x^2}$$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^3,x)
```

output
```
( - 4*sqrt(x)*sqrt(a + b*x)*a*b*c**2 - 24*sqrt(x)*sqrt(a + b*x)*a*b*c*d*x
+ 6*sqrt(x)*sqrt(a + b*x)*a*b*d**2*x**2 - 4*sqrt(x)*sqrt(a + b*x)*b**2*c**
2*x + 6*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*d**2*x
**2 + 24*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b*c*d*x*
*2 + sqrt(b)*a**2*d**2*x**2 + 8*sqrt(b)*a*b*c*d*x**2 - 4*sqrt(b)*b**2*c**2
*x**2)/(6*a*b*x**2)
```

3.16 $\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^4} dx$

Optimal result

Integrand size = 24, antiderivative size = 111

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^4}\,dx = -\frac{2d^2\sqrt{ax+bx^2}}{x} - \frac{2c^2(ax+bx^2)^{3/2}}{5ax^4} + \frac{4c(bc-5ad)\,(ax+bx^2)^{3/2}}{15a^2x^3} + 2\sqrt{b}d^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)$$

output
```
-2*d^2*(b*x^2+a*x)^(1/2)/x-2/5*c^2*(b*x^2+a*x)^(3/2)/a/x^4+4/15*c*(-5*a*d+
b*c)*(b*x^2+a*x)^(3/2)/a^2/x^3+2*b^(1/2)*d^2*arctanh(b^(1/2)*x/(b*x^2+a*x)
^(1/2))
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^4}\,dx = \frac{2\sqrt{x(a+bx)}\left(-15d^2x^2 - \frac{10cdx(a+bx)}{a} + c^2\left(-3-\frac{bx}{a}+\frac{2b^2x^2}{a^2}\right) - \frac{15\sqrt{b}d^2x^{5/2}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{a+bx}}\right)}{15x^3}$$

input
```
Integrate[((c + d*x)^2*Sqrt[a*x + b*x^2])/x^4,x]
```

output
```
(2*Sqrt[x*(a + b*x)]*(-15*d^2*x^2 - (10*c*d*x*(a + b*x))/a + c^2*(-3 - (b*
x)/a + (2*b^2*x^2)/a^2) - (15*Sqrt[b]*d^2*x^(5/2)*Log[-(Sqrt[b]*Sqrt[x]) +
 Sqrt[a + b*x]])/Sqrt[a + b*x]))/(15*x^3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{1290\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}(c+dx)^2}{x^4}\,dx$$

$$\Big\downarrow \text{ 1290}$$

$$\text{Indeterminate}$$

input
```
Int[((c + d*x)^2*Sqrt[a*x + b*x^2])/x^4,x]
```

output
```
Indeterminate
```

Defintions of rubi rules used

rule 1290
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(f + g*x)^
n, d + e*x, x], R = PolynomialRemainder[(f + g*x)^n, d + e*x, x]}, Simp[(e*
R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*
e^2)), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1
)*(a + b*x + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R
*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 1] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$\frac{2a^2\sqrt{b}\,d^2\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)x^3-\frac{2\sqrt{x(bx+a)}\left(\left(5d^2x^2+\frac{10}{3}cdx+c^2\right)a^2+\frac{bcx(10dx+c)a}{3}-\frac{2b^2c^2x^2}{3}\right)}{5}}{a^2x^3}$
risch	$-\frac{2(bx+a)\left(15a^2d^2x^2+10abcd\,x^2-2b^2c^2x^2+10a^2cdx+ab\,c^2x+3a^2c^2\right)}{15x^2\sqrt{x(bx+a)}\,a^2}+\sqrt{b}\,d^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)$
default	$c^2\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{5a\,x^4}+\frac{4b\left(b\,x^2+ax\right)^{\frac{3}{2}}}{15a^2x^3}\right)+d^2\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{a\,x^2}+\frac{2b\left(\sqrt{b\,x^2+ax}+\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2\sqrt{b}}\right)}{a}\right)$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
2/5*(5*a^2*b^(1/2)*d^2*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*x^3-(x*(b*x+a)
)^(1/2)*((5*d^2*x^2+10/3*c*d*x+c^2)*a^2+1/3*b*c*x*(10*d*x+c)*a-2/3*b^2*c^2
*x^2))/a^2/x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.05

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^4}\,dx$$
$$= \left[\frac{15\,a^2\sqrt{b}d^2x^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,(3\,a^2c^2-(2\,b^2c^2-10\,abcd-15\,a^2d^2)x^2+(abc^2+10\right.}{15\,a^2x^3}$$
$$-\frac{2\left(15\,a^2\sqrt{-b}d^2x^3\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+(3\,a^2c^2-(2\,b^2c^2-10\,abcd-15\,a^2d^2)x^2+(abc^2+10\,a^2cd)x\right.}{15\,a^2x^3}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[1/15*(15*a^2*sqrt(b)*d^2*x^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))
 - 2*(3*a^2*c^2 - (2*b^2*c^2 - 10*a*b*c*d - 15*a^2*d^2)*x^2 + (a*b*c^2 + 1
0*a^2*c*d)*x)*sqrt(b*x^2 + a*x))/(a^2*x^3), -2/15*(15*a^2*sqrt(-b)*d^2*x^3
*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (3*a^2*c^2 - (2*b^2*c^2 -
10*a*b*c*d - 15*a^2*d^2)*x^2 + (a*b*c^2 + 10*a^2*c*d)*x)*sqrt(b*x^2 + a*x)
)/(a^2*x^3)]
```

Sympy [F]

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^4}\,dx=\int \frac{\sqrt{x\,(a+bx)}(c+dx)^2}{x^4}\,dx$$

input
```
integrate((d*x+c)**2*(b*x**2+a*x)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(x*(a + b*x))*(c + d*x)**2/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.37

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^4}\,dx = \left(\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-\frac{2\,\sqrt{bx^2+ax}}{x}\right)d^2 - \frac{4}{3}\,cd\left(\frac{\sqrt{bx^2+ax}b}{ax}+\frac{\sqrt{bx^2+ax}}{x^2}\right) + \frac{2}{15}\,c^2\left(\frac{2\,\sqrt{bx^2+ax}b^2}{a^2x}-\frac{\sqrt{bx^2+ax}b}{ax^2}-\frac{3\,\sqrt{bx^2+ax}}{x^3}\right)$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
(sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*sqrt(b*x^2 + a*x
)/x)*d^2 - 4/3*c*d*(sqrt(b*x^2 + a*x)*b/(a*x) + sqrt(b*x^2 + a*x)/x^2) + 2
/15*c^2*(2*sqrt(b*x^2 + a*x)*b^2/(a^2*x) - sqrt(b*x^2 + a*x)*b/(a*x^2) - 3
*sqrt(b*x^2 + a*x)/x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(95) = 190.

Time = 0.34 (sec) , antiderivative size = 266, normalized size of antiderivative = 2.40

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^4}\,dx = -\sqrt{b}d^2\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right) + \frac{2\left(30\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4 bcd+15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4 ad^2+15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3 b^{\frac{3}{2}}c^2+30\left(\right.}{}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^4,x, algorithm="giac")
```

output
```
-sqrt(b)*d^2*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)) + 2/1
5*(30*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*b*c*d + 15*(sqrt(b)*x - sqrt(b*x^2
 + a*x))^4*a*d^2 + 15*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*b^(3/2)*c^2 + 30*(
sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a*sqrt(b)*c*d + 25*(sqrt(b)*x - sqrt(b*x^
2 + a*x))^2*a*b*c^2 + 10*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2*c*d + 15*(s
qrt(b)*x - sqrt(b*x^2 + a*x))*a^2*sqrt(b)*c^2 + 3*a^3*c^2)/(sqrt(b)*x - sq
rt(b*x^2 + a*x))^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^4}\,dx=\int \frac{\sqrt{b\,x^2+a\,x}\,{\left(c+d\,x\right)}^2}{x^4}\,dx$$

input
```
int(((a*x + b*x^2)^(1/2)*(c + d*x)^2)/x^4,x)
```

output
```
int(((a*x + b*x^2)^(1/2)*(c + d*x)^2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.62

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^4}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2c^2}{5}-\frac{4\sqrt{x}\,\sqrt{bx+a}\,a^2cdx}{3}-2\sqrt{x}\,\sqrt{bx+a}\,a^2d^2x^2-\frac{2\sqrt{x}\,\sqrt{bx+a}\,ab\,c^2x}{15}-\frac{4\sqrt{x}\,\sqrt{bx+a}\,abcd\,x^2}{3}+\frac{4\sqrt{x}\,\sqrt{bx+a}\,b^{2}}{15}}{a^2x^3}$$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^4,x)
```

output
```
(2*( - 3*sqrt(x)*sqrt(a + b*x)*a**2*c**2 - 10*sqrt(x)*sqrt(a + b*x)*a**2*c
*d*x - 15*sqrt(x)*sqrt(a + b*x)*a**2*d**2*x**2 - sqrt(x)*sqrt(a + b*x)*a*b
*c**2*x - 10*sqrt(x)*sqrt(a + b*x)*a*b*c*d*x**2 + 2*sqrt(x)*sqrt(a + b*x)*
b**2*c**2*x**2 + 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))
*a**2*d**2*x**3 + 9*sqrt(b)*a**2*d**2*x**3 - 2*sqrt(b)*a*b*c*d*x**3 - 2*sq
rt(b)*b**2*c**2*x**3))/(15*a**2*x**3)
```

3.17 $\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^5} dx$

Optimal result

Integrand size = 24, antiderivative size = 103

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^5}\,dx = \frac{8c(bc-ad)\left(ax+bx^2\right)^{3/2}}{35a^2x^4} - \frac{8(2bc-5ad)(bc-ad)\left(ax+bx^2\right)^{3/2}}{105a^3x^3} - \frac{2(c+dx)^2\left(ax+bx^2\right)^{3/2}}{7ax^5}$$

output
```
8/35*c*(-a*d+b*c)*(b*x^2+a*x)^(3/2)/a^2/x^4-8/105*(-5*a*d+2*b*c)*(-a*d+b*c
)*(b*x^2+a*x)^(3/2)/a^3/x^3-2/7*(d*x+c)^2*(b*x^2+a*x)^(3/2)/a/x^5
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.68

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^5}\,dx = -\frac{2(x(a+bx))^{3/2}\left(8b^2c^2x^2-4abcx(3c+7dx)+a^2\left(15c^2+42cdx+35d^2x^2\right)\right)}{105a^3x^5}$$

input
```
Integrate[((c + d*x)^2*Sqrt[a*x + b*x^2])/x^5,x]
```

output
```
(-2*(x*(a + b*x))^(3/2)*(8*b^2*c^2*x^2 - 4*a*b*c*x*(3*c + 7*d*x) + a^2*(15
*c^2 + 42*c*d*x + 35*d^2*x^2)))/(105*a^3*x^5)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.34, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1262, 27, 1220, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}(c+dx)^2}{x^5} \, dx$$

$$\Big\downarrow \text{1262}$$

$$-\frac{\int -\frac{(2bc^2+d(4bc-5ad)x)\sqrt{bx^2+ax}}{2x^5}dx}{b}-\frac{d^2\left(ax+bx^2\right)^{3/2}}{bx^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(2bc^2+d(4bc-5ad)x)\sqrt{bx^2+ax}}{x^5}dx}{2b}-\frac{d^2\left(ax+bx^2\right)^{3/2}}{bx^4}$$

$$\Big\downarrow \text{1220}$$

$$\frac{-\frac{(35a^2d^2-28abcd+8b^2c^2)\int \frac{\sqrt{bx^2+ax}}{x^4}dx}{7a}-\frac{4bc^2(ax+bx^2)^{3/2}}{7ax^5}}{2b}-\frac{d^2\left(ax+bx^2\right)^{3/2}}{bx^4}$$

$$\Big\downarrow \text{1129}$$

$$\frac{-\frac{(35a^2d^2-28abcd+8b^2c^2)\left(-\frac{2b\int \frac{\sqrt{bx^2+ax}}{x^3}dx}{5a}-\frac{2\left(ax+bx^2\right)^{3/2}}{5ax^4}\right)}{7a}-\frac{4bc^2(ax+bx^2)^{3/2}}{7ax^5}}{2b}-\frac{d^2\left(ax+bx^2\right)^{3/2}}{bx^4}$$

$$\Big\downarrow \text{1123}$$

$$\frac{-\frac{\left(\frac{4b\left(ax+bx^2\right)^{3/2}}{15a^2x^3}-\frac{2\left(ax+bx^2\right)^{3/2}}{5ax^4}\right)(35a^2d^2-28abcd+8b^2c^2)}{7a}-\frac{4bc^2(ax+bx^2)^{3/2}}{7ax^5}}{2b}-\frac{d^2\left(ax+bx^2\right)^{3/2}}{bx^4}$$

input
```
Int[((c + d*x)^2*Sqrt[a*x + b*x^2])/x^5,x]
```

output
```
-((d^2*(a*x + b*x^2)^(3/2))/(b*x^4)) + ((-4*b*c^2*(a*x + b*x^2)^(3/2))/(7*
a*x^5) - ((8*b^2*c^2 - 28*a*b*c*d + 35*a^2*d^2)*((-2*(a*x + b*x^2)^(3/2))/
(5*a*x^4) + (4*b*(a*x + b*x^2)^(3/2))/(15*a^2*x^3)))/(7*a))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$-\frac{2\sqrt{x(bx+a)}\,(bx+a)\left(\left(\frac{7}{3}d^2x^2+\frac{14}{5}cdx+c^2\right)a^2-\frac{4xb\left(\frac{7dx}{3}+c\right)ca}{5}+\frac{8b^2c^2x^2}{15}\right)}{7x^4a^3}$
gosper	$-\frac{2(bx+a)\left(35a^2d^2x^2-28abcd\,x^2+8b^2c^2x^2+42a^2cdx-12ab\,c^2x+15a^2c^2\right)\sqrt{b\,x^2+ax}}{105x^4a^3}$
orering	$-\frac{2(bx+a)\left(35a^2d^2x^2-28abcd\,x^2+8b^2c^2x^2+42a^2cdx-12ab\,c^2x+15a^2c^2\right)\sqrt{b\,x^2+ax}}{105x^4a^3}$
trager	$-\frac{2\left(35d^2x^3a^2b-28a\,b^2cd\,x^3+8b^3c^2x^3+35a^3d^2x^2+14x^2a^2bcd-4a\,b^2c^2x^2+42a^3cdx+3a^2b\,c^2x+15c^2a^3\right)\sqrt{b\,x^2+ax}}{105x^4a^3}$
risch	$-\frac{2(bx+a)\left(35d^2x^3a^2b-28a\,b^2cd\,x^3+8b^3c^2x^3+35a^3d^2x^2+14x^2a^2bcd-4a\,b^2c^2x^2+42a^3cdx+3a^2b\,c^2x+15c^2a^3\right)}{105x^3\sqrt{x(bx+a)}\,a^3}$
default	$c^2\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{7a\,x^5}-\frac{4b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{5a\,x^4}+\frac{4b\left(b\,x^2+ax\right)^{\frac{3}{2}}}{15a^2x^3}\right)}{7a}\right)-\frac{2d^2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{3a\,x^3}+2cd\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{5a\,x^4}+\right.$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-2/7*(x*(b*x+a))^(1/2)*(b*x+a)*((7/3*d^2*x^2+14/5*c*d*x+c^2)*a^2-4/5*x*b*(
7/3*d*x+c)*c*a+8/15*b^2*c^2*x^2)/x^4/a^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.05

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^5}\,dx = -\frac{2\,(15\,a^3c^2+(8\,b^3c^2-28\,ab^2cd+35\,a^2bd^2)x^3-(4\,ab^2c^2-14\,a^2bcd-35\,a^3d^2)x^2+3\,(a^2bc^2+14\,a^3cd)}{105\,a^3x^4}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
-2/105*(15*a^3*c^2 + (8*b^3*c^2 - 28*a*b^2*c*d + 35*a^2*b*d^2)*x^3 - (4*a*
b^2*c^2 - 14*a^2*b*c*d - 35*a^3*d^2)*x^2 + 3*(a^2*b*c^2 + 14*a^3*c*d)*x)*s
qrt(b*x^2 + a*x)/(a^3*x^4)
```

Sympy [F]

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^5}\,dx = \int \frac{\sqrt{x\,(a+bx)}(c+dx)^2}{x^5}\,dx$$

input
```
integrate((d*x+c)**2*(b*x**2+a*x)**(1/2)/x**5,x)
```

output
```
Integral(sqrt(x*(a + b*x))*(c + d*x)**2/x**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(91) = 182.

Time = 0.04 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.93

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^5}\,dx = -\frac{16\,\sqrt{bx^2+ax}b^3c^2}{105\,a^3x}+\frac{8\,\sqrt{bx^2+ax}b^2cd}{15\,a^2x}-\frac{2\,\sqrt{bx^2+ax}bd^2}{3\,ax}+\frac{8\,\sqrt{bx^2+ax}b^2c^2}{105\,a^2x^2}-\frac{4\,\sqrt{bx^2+ax}bcd}{15\,ax^2}-\frac{2\,\sqrt{bx^2+ax}d^2}{3\,x^2}-\frac{2\,\sqrt{bx^2+ax}bc^2}{35\,ax^3}-\frac{4\,\sqrt{bx^2+ax}cd}{5\,x^3}-\frac{2\,\sqrt{bx^2+ax}c^2}{7\,x^4}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-16/105*sqrt(b*x^2 + a*x)*b^3*c^2/(a^3*x) + 8/15*sqrt(b*x^2 + a*x)*b^2*c*d
/(a^2*x) - 2/3*sqrt(b*x^2 + a*x)*b*d^2/(a*x) + 8/105*sqrt(b*x^2 + a*x)*b^2
*c^2/(a^2*x^2) - 4/15*sqrt(b*x^2 + a*x)*b*c*d/(a*x^2) - 2/3*sqrt(b*x^2 + a
*x)*d^2/x^2 - 2/35*sqrt(b*x^2 + a*x)*b*c^2/(a*x^3) - 4/5*sqrt(b*x^2 + a*x)
*c*d/x^3 - 2/7*sqrt(b*x^2 + a*x)*c^2/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(91) = 182$.

Time = 0.17 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.41

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^5}\,dx = \frac{2\left(105\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6bd^2+210\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5b^{\frac{3}{2}}cd+105\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5a\sqrt{b}d^2+\right.}{}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^5,x, algorithm="giac")
```

output
```
2/105*(105*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*b*d^2 + 210*(sqrt(b)*x - sqrt
(b*x^2 + a*x))^5*b^(3/2)*c*d + 105*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a*sqr
t(b)*d^2 + 140*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*b^2*c^2 + 350*(sqrt(b)*x
- sqrt(b*x^2 + a*x))^4*a*b*c*d + 35*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^2*
d^2 + 315*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a*b^(3/2)*c^2 + 210*(sqrt(b)*x
 - sqrt(b*x^2 + a*x))^3*a^2*sqrt(b)*c*d + 273*(sqrt(b)*x - sqrt(b*x^2 + a*
x))^2*a^2*b*c^2 + 42*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^3*c*d + 105*(sqrt
(b)*x - sqrt(b*x^2 + a*x))*a^3*sqrt(b)*c^2 + 15*a^4*c^2)/(sqrt(b)*x - sqrt
(b*x^2 + a*x))^7
```

Mupad [B] (verification not implemented)

Time = 9.95 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.93

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^5}\,dx = \frac{8\,b^2\,c^2\,\sqrt{b\,x^2+a\,x}}{105\,a^2\,x^2} - \frac{2\,d^2\,\sqrt{b\,x^2+a\,x}}{3\,x^2} - \frac{4\,c\,d\,\sqrt{b\,x^2+a\,x}}{5\,x^3} - \frac{2\,c^2\,\sqrt{b\,x^2+a\,x}}{7\,x^4} - \frac{16\,b^3\,c^2\,\sqrt{b\,x^2+a\,x}}{105\,a^3\,x} - \frac{2\,b\,c^2\,\sqrt{b\,x^2+a\,x}}{35\,a\,x^3} - \frac{2\,b\,d^2\,\sqrt{b\,x^2+a\,x}}{3\,a\,x} + \frac{8\,b^2\,c\,d\,\sqrt{b\,x^2+a\,x}}{15\,a^2\,x} - \frac{4\,b\,c\,d\,\sqrt{b\,x^2+a\,x}}{15\,a\,x^2}$$

input
```
int(((a*x + b*x^2)^(1/2)*(c + d*x)^2)/x^5,x)
```

output
```
(8*b^2*c^2*(a*x + b*x^2)^(1/2))/(105*a^2*x^2) - (2*d^2*(a*x + b*x^2)^(1/2)
)/(3*x^2) - (4*c*d*(a*x + b*x^2)^(1/2))/(5*x^3) - (2*c^2*(a*x + b*x^2)^(1/
2))/(7*x^4) - (16*b^3*c^2*(a*x + b*x^2)^(1/2))/(105*a^3*x) - (2*b*c^2*(a*x
 + b*x^2)^(1/2))/(35*a*x^3) - (2*b*d^2*(a*x + b*x^2)^(1/2))/(3*a*x) + (8*b
^2*c*d*(a*x + b*x^2)^(1/2))/(15*a^2*x) - (4*b*c*d*(a*x + b*x^2)^(1/2))/(15
*a*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.09

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^5}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3c^2}{7} - \frac{4\sqrt{x}\,\sqrt{bx+a}\,a^3cdx}{5} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3d^2x^2}{3} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2b\,c^2x}{35} - \frac{4\sqrt{x}\,\sqrt{bx+a}\,a^2bcd\,x^2}{15} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2b\,d^2x}{3}}{a^3x^4}$$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^5,x)
```

output
```
(2*( - 15*sqrt(x)*sqrt(a + b*x)*a**3*c**2 - 42*sqrt(x)*sqrt(a + b*x)*a**3*
c*d*x - 35*sqrt(x)*sqrt(a + b*x)*a**3*d**2*x**2 - 3*sqrt(x)*sqrt(a + b*x)*
a**2*b*c**2*x - 14*sqrt(x)*sqrt(a + b*x)*a**2*b*c*d*x**2 - 35*sqrt(x)*sqrt
(a + b*x)*a**2*b*d**2*x**3 + 4*sqrt(x)*sqrt(a + b*x)*a*b**2*c**2*x**2 + 28
*sqrt(x)*sqrt(a + b*x)*a*b**2*c*d*x**3 - 8*sqrt(x)*sqrt(a + b*x)*b**3*c**2
*x**3 + 5*sqrt(b)*a**2*b*d**2*x**4 - 28*sqrt(b)*a*b**2*c*d*x**4 + 8*sqrt(b
)*b**3*c**2*x**4))/(105*a**3*x**4)
```

3.18 $\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^6}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 165

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^6}\,dx = -\frac{8(bc-ad)(2bc+ad)\left(ax+bx^2\right)^{3/2}}{105a^3x^4} + \frac{8(2bc-5ad)(bc-ad)(2bc+ad)\left(ax+bx^2\right)^{3/2}}{315a^4cx^3} + \frac{2(2bc+ad)(c+dx)^2\left(ax+bx^2\right)^{3/2}}{21a^2cx^5} - \frac{2(c+dx)^3\left(ax+bx^2\right)^{3/2}}{9acx^6}$$

output
```
-8/105*(-a*d+b*c)*(a*d+2*b*c)*(b*x^2+a*x)^(3/2)/a^3/x^4+8/315*(-5*a*d+2*b*
c)*(-a*d+b*c)*(a*d+2*b*c)*(b*x^2+a*x)^(3/2)/a^4/c/x^3+2/21*(a*d+2*b*c)*(d*
x+c)^2*(b*x^2+a*x)^(3/2)/a^2/c/x^5-2/9*(d*x+c)^3*(b*x^2+a*x)^(3/2)/a/c/x^6
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.60

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^6}\,dx$$
$$= \frac{2(x(a+bx))^{3/2}\left(16b^3c^2x^3-24ab^2cx^2(c+2dx)+6a^2bx(5c^2+12cdx+7d^2x^2)-a^3(35c^2+90cdx+63d\right.}{315a^4x^6}$$

input
```
Integrate[((c + d*x)^2*Sqrt[a*x + b*x^2])/x^6,x]
```

output
```
(2*(x*(a + b*x))^(3/2)*(16*b^3*c^2*x^3 - 24*a*b^2*c*x^2*(c + 2*d*x) + 6*a^
2*b*x*(5*c^2 + 12*c*d*x + 7*d^2*x^2) - a^3*(35*c^2 + 90*c*d*x + 63*d^2*x^2
)))/(315*a^4*x^6)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1262, 27, 1220, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}(c+dx)^2}{x^6}\,dx$$

$$\Big\downarrow \; 1262$$

$$-\frac{\int -\frac{(4bc^2+d(8bc-7ad)x)\sqrt{bx^2+ax}}{2x^6}dx}{2b}-\frac{d^2\left(ax+bx^2\right)^{3/2}}{2bx^5}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(4bc^2+d(8bc-7ad)x)\sqrt{bx^2+ax}}{x^6}dx}{4b}-\frac{d^2\left(ax+bx^2\right)^{3/2}}{2bx^5}$$

$$\Big\downarrow \; 1220$$

$$\frac{-\frac{\left(21a^2d^2-24abcd+8b^2c^2\right)\int \frac{\sqrt{bx^2+ax}}{x^5}dx}{3a}-\frac{8bc^2\left(ax+bx^2\right)^{3/2}}{9ax^6}}{4b}-\frac{d^2\left(ax+bx^2\right)^{3/2}}{2bx^5}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{-\frac{\left(21a^2d^2-24abcd+8b^2c^2\right)\left(-\frac{4b\int \frac{\sqrt{bx^2+ax}}{x^4}dx}{7a}-\frac{2\left(ax+bx^2\right)^{3/2}}{7ax^5}\right)}{3a}-\frac{8bc^2\left(ax+bx^2\right)^{3/2}}{9ax^6}}{4b}-\frac{d^2\left(ax+bx^2\right)^{3/2}}{2bx^5}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{-\frac{\left(21a^2d^2-24abcd+8b^2c^2\right)\left(-\frac{4b\left(-\frac{2b\int \frac{\sqrt{bx^2+ax}}{x^3}dx}{5a}-\frac{2\left(ax+bx^2\right)^{3/2}}{5ax^4}\right)}{7a}-\frac{2\left(ax+bx^2\right)^{3/2}}{7ax^5}\right)}{3a}-\frac{8bc^2\left(ax+bx^2\right)^{3/2}}{9ax^6}}{4b}-\frac{d^2\left(ax+bx^2\right)^{3/2}}{2bx^5}$$

$$\Big\downarrow \text{ 1123}$$

$$\frac{-\frac{\left(-\frac{4b\left(\frac{4b\left(ax+bx^2\right)^{3/2}}{15a^2x^3}-\frac{2\left(ax+bx^2\right)^{3/2}}{5ax^4}\right)}{7a}-\frac{2\left(ax+bx^2\right)^{3/2}}{7ax^5}\right)\left(21a^2d^2-24abcd+8b^2c^2\right)}{3a}-\frac{8bc^2\left(ax+bx^2\right)^{3/2}}{9ax^6}}{4b}-\frac{d^2\left(ax+bx^2\right)^{3/2}}{2bx^5}$$

input
```
Int[((c + d*x)^2*Sqrt[a*x + b*x^2])/x^6,x]
```

output
```
-1/2*(d^2*(a*x + b*x^2)^(3/2))/(b*x^5) + ((-8*b*c^2*(a*x + b*x^2)^(3/2))/(
9*a*x^6) - ((8*b^2*c^2 - 24*a*b*c*d + 21*a^2*d^2)*((-2*(a*x + b*x^2)^(3/2)
)/(7*a*x^5) - (4*b*((-2*(a*x + b*x^2)^(3/2))/(5*a*x^4) + (4*b*(a*x + b*x^2
)^(3/2))/(15*a^2*x^3)))/(7*a)))/(3*a))/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.55

method	result
pseudoelliptic	$-\frac{2\sqrt{x(bx+a)}\,(bx+a)\left(\left(\frac{9}{5}d^2x^2+\frac{18}{7}cdx+c^2\right)a^3-\frac{6(dx+c)\left(\frac{7dx}{5}+c\right)xb\,a^2}{7}+\frac{24b^2c\,x^2(2dx+c)a}{35}-\frac{16b^3c^2x^3}{35}\right)}{9x^5a^4}$
gosper	$-\frac{2(bx+a)(-42d^2x^3a^2b+48a\,b^2cd\,x^3-16b^3c^2x^3+63a^3d^2x^2-72x^2a^2bcd+24a\,b^2c^2x^2+90a^3cdx-30a^2b\,c^2x+35c^2a^3)\sqrt{b\,x}}{315x^5a^4}$
orering	$-\frac{2(bx+a)(-42d^2x^3a^2b+48a\,b^2cd\,x^3-16b^3c^2x^3+63a^3d^2x^2-72x^2a^2bcd+24a\,b^2c^2x^2+90a^3cdx-30a^2b\,c^2x+35c^2a^3)\sqrt{b\,x}}{315x^5a^4}$
trager	$-\frac{2(-42a^2b^2d^2x^4+48a\,b^3cd\,x^4-16b^4c^2x^4+21a^3b\,d^2x^3-24a^2b^2cd\,x^3+8a\,b^3c^2x^3+63a^4d^2x^2+18a^3dcb\,x^2-6a^2b^2c^2x^2+90}{315x^5a^4}$
risch	$-\frac{2(bx+a)(-42a^2b^2d^2x^4+48a\,b^3cd\,x^4-16b^4c^2x^4+21a^3b\,d^2x^3-24a^2b^2cd\,x^3+8a\,b^3c^2x^3+63a^4d^2x^2+18a^3dcb\,x^2-6a^2b^2c^2}{315x^4\sqrt{x(bx+a)}\,a^4}$
default	$c^2\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{9a\,x^6}-\frac{2b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{7a\,x^5}-\frac{4b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{5a\,x^4}+\frac{4b\left(b\,x^2+ax\right)^{\frac{3}{2}}}{15a^2x^3}\right)}{7a}\right)}{3a}\right)+d^2\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{5a\,x^4}+4$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-2/9*(x*(b*x+a))^(1/2)*(b*x+a)*((9/5*d^2*x^2+18/7*c*d*x+c^2)*a^3-6/7*(d*x+
c)*(7/5*d*x+c)*x*b*a^2+24/35*b^2*c*x^2*(2*d*x+c)*a-16/35*b^3*c^2*x^3)/x^5/
a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^6}\,dx =$$

$$-\frac{2\,(35\,a^4c^2-2\,(8\,b^4c^2-24\,ab^3cd+21\,a^2b^2d^2)x^4+(8\,ab^3c^2-24\,a^2b^2cd+21\,a^3bd^2)x^3-3\,(2\,a^2b^2c^2-6}{315\,a^4x^5}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
-2/315*(35*a^4*c^2 - 2*(8*b^4*c^2 - 24*a*b^3*c*d + 21*a^2*b^2*d^2)*x^4 + (
8*a*b^3*c^2 - 24*a^2*b^2*c*d + 21*a^3*b*d^2)*x^3 - 3*(2*a^2*b^2*c^2 - 6*a^
3*b*c*d - 21*a^4*d^2)*x^2 + 5*(a^3*b*c^2 + 18*a^4*c*d)*x)*sqrt(b*x^2 + a*x
)/(a^4*x^5)
```

Sympy [F]

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^6}\,dx = \int \frac{\sqrt{x\,(a+bx)}(c+dx)^2}{x^6}\,dx$$

input
```
integrate((d*x+c)**2*(b*x**2+a*x)**(1/2)/x**6,x)
```

output
```
Integral(sqrt(x*(a + b*x))*(c + d*x)**2/x**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.65

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^6}\,dx = \frac{32\sqrt{bx^2+ax}b^4c^2}{315\,a^4x} - \frac{32\sqrt{bx^2+ax}b^3cd}{105\,a^3x} + \frac{4\sqrt{bx^2+ax}b^2d^2}{15\,a^2x} - \frac{16\sqrt{bx^2+ax}b^3c^2}{315\,a^3x^2} + \frac{16\sqrt{bx^2+ax}b^2cd}{105\,a^2x^2} - \frac{2\sqrt{bx^2+ax}bd^2}{15\,ax^2} + \frac{4\sqrt{bx^2+ax}b^2c^2}{105\,a^2x^3} - \frac{4\sqrt{bx^2+ax}bcd}{35\,ax^3} - \frac{2\sqrt{bx^2+ax}d^2}{5\,x^3} - \frac{2\sqrt{bx^2+ax}bc^2}{63\,ax^4} - \frac{4\sqrt{bx^2+ax}cd}{7\,x^4} - \frac{2\sqrt{bx^2+ax}c^2}{9\,x^5}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
32/315*sqrt(b*x^2 + a*x)*b^4*c^2/(a^4*x) - 32/105*sqrt(b*x^2 + a*x)*b^3*c*
d/(a^3*x) + 4/15*sqrt(b*x^2 + a*x)*b^2*d^2/(a^2*x) - 16/315*sqrt(b*x^2 + a
*x)*b^3*c^2/(a^3*x^2) + 16/105*sqrt(b*x^2 + a*x)*b^2*c*d/(a^2*x^2) - 2/15*
sqrt(b*x^2 + a*x)*b*d^2/(a*x^2) + 4/105*sqrt(b*x^2 + a*x)*b^2*c^2/(a^2*x^3
) - 4/35*sqrt(b*x^2 + a*x)*b*c*d/(a*x^3) - 2/5*sqrt(b*x^2 + a*x)*d^2/x^3 -
 2/63*sqrt(b*x^2 + a*x)*b*c^2/(a*x^4) - 4/7*sqrt(b*x^2 + a*x)*c*d/x^4 - 2/
9*sqrt(b*x^2 + a*x)*c^2/x^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(149) = 298$.

Time = 0.16 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.70

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^6}\,dx = \frac{2\left(315\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^7 b^{\frac{3}{2}}d^2+840\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6 b^2cd+525\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6 abd^2+6\right.}{}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^6,x, algorithm="giac")
```

output
```
2/315*(315*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*b^(3/2)*d^2 + 840*(sqrt(b)*x
- sqrt(b*x^2 + a*x))^6*b^2*c*d + 525*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a*b
*d^2 + 630*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*b^(5/2)*c^2 + 1890*(sqrt(b)*x
 - sqrt(b*x^2 + a*x))^5*a*b^(3/2)*c*d + 315*(sqrt(b)*x - sqrt(b*x^2 + a*x)
)^5*a^2*sqrt(b)*d^2 + 1764*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a*b^2*c^2 + 1
638*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^2*b*c*d + 63*(sqrt(b)*x - sqrt(b*x
^2 + a*x))^4*a^3*d^2 + 1995*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^2*b^(3/2)*
c^2 + 630*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^3*sqrt(b)*c*d + 1125*(sqrt(b
)*x - sqrt(b*x^2 + a*x))^2*a^3*b*c^2 + 90*(sqrt(b)*x - sqrt(b*x^2 + a*x))^
2*a^4*c*d + 315*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^4*sqrt(b)*c^2 + 35*a^5*c
^2)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^9
```

Mupad [B] (verification not implemented)

Time = 10.39 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.65

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^6}\,dx = \frac{4\,b^2\,c^2\,\sqrt{b\,x^2+a\,x}}{105\,a^2\,x^3}-\frac{2\,d^2\,\sqrt{b\,x^2+a\,x}}{5\,x^3}-\frac{4\,c\,d\,\sqrt{b\,x^2+a\,x}}{7\,x^4}-\frac{2\,c^2\,\sqrt{b\,x^2+a\,x}}{9\,x^5}-\frac{16\,b^3\,c^2\,\sqrt{b\,x^2+a\,x}}{315\,a^3\,x^2}+\frac{32\,b^4\,c^2\,\sqrt{b\,x^2+a\,x}}{315\,a^4\,x}+\frac{4\,b^2\,d^2\,\sqrt{b\,x^2+a\,x}}{15\,a^2\,x}-\frac{2\,b\,c^2\,\sqrt{b\,x^2+a\,x}}{63\,a\,x^4}-\frac{2\,b\,d^2\,\sqrt{b\,x^2+a\,x}}{15\,a\,x^2}+\frac{16\,b^2\,c\,d\,\sqrt{b\,x^2+a\,x}}{105\,a^2\,x^2}-\frac{32\,b^3\,c\,d\,\sqrt{b\,x^2+a\,x}}{105\,a^3\,x}-\frac{4\,b\,c\,d\,\sqrt{b\,x^2+a\,x}}{35\,a\,x^3}$$

input
```
int(((a*x + b*x^2)^(1/2)*(c + d*x)^2)/x^6,x)
```

output
```
(4*b^2*c^2*(a*x + b*x^2)^(1/2))/(105*a^2*x^3) - (2*d^2*(a*x + b*x^2)^(1/2)
)/(5*x^3) - (4*c*d*(a*x + b*x^2)^(1/2))/(7*x^4) - (2*c^2*(a*x + b*x^2)^(1/
2))/(9*x^5) - (16*b^3*c^2*(a*x + b*x^2)^(1/2))/(315*a^3*x^2) + (32*b^4*c^2
*(a*x + b*x^2)^(1/2))/(315*a^4*x) + (4*b^2*d^2*(a*x + b*x^2)^(1/2))/(15*a^
2*x) - (2*b*c^2*(a*x + b*x^2)^(1/2))/(63*a*x^4) - (2*b*d^2*(a*x + b*x^2)^(
1/2))/(15*a*x^2) + (16*b^2*c*d*(a*x + b*x^2)^(1/2))/(105*a^2*x^2) - (32*b^
3*c*d*(a*x + b*x^2)^(1/2))/(105*a^3*x) - (4*b*c*d*(a*x + b*x^2)^(1/2))/(35
*a*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.71

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^6}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4c^2}{9}-\frac{4\sqrt{x}\,\sqrt{bx+a}\,a^4cdx}{7}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4d^2x^2}{5}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3b\,c^2x}{63}-\frac{4\sqrt{x}\,\sqrt{bx+a}\,a^3bcd\,x^2}{35}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3b\,d^2x}{15}}{}$$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^6,x)
```

output
```
(2*( - 35*sqrt(x)*sqrt(a + b*x)*a**4*c**2 - 90*sqrt(x)*sqrt(a + b*x)*a**4*
c*d*x - 63*sqrt(x)*sqrt(a + b*x)*a**4*d**2*x**2 - 5*sqrt(x)*sqrt(a + b*x)*
a**3*b*c**2*x - 18*sqrt(x)*sqrt(a + b*x)*a**3*b*c*d*x**2 - 21*sqrt(x)*sqrt
(a + b*x)*a**3*b*d**2*x**3 + 6*sqrt(x)*sqrt(a + b*x)*a**2*b**2*c**2*x**2 +
 24*sqrt(x)*sqrt(a + b*x)*a**2*b**2*c*d*x**3 + 42*sqrt(x)*sqrt(a + b*x)*a*
*2*b**2*d**2*x**4 - 8*sqrt(x)*sqrt(a + b*x)*a*b**3*c**2*x**3 - 48*sqrt(x)*
sqrt(a + b*x)*a*b**3*c*d*x**4 + 16*sqrt(x)*sqrt(a + b*x)*b**4*c**2*x**4 -
42*sqrt(b)*a**2*b**2*d**2*x**5 + 48*sqrt(b)*a*b**3*c*d*x**5 - 16*sqrt(b)*b
**4*c**2*x**5))/(315*a**4*x**5)
```

3.19 $\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^7}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 199

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^7}\,dx = -\frac{2c^2(ax+bx^2)^{3/2}}{11ax^7}+\frac{4c(4bc-11ad)(ax+bx^2)^{3/2}}{99a^2x^6} - \frac{2\left(33d^2+\frac{4bc(4bc-11ad)}{a^2}\right)(ax+bx^2)^{3/2}}{231ax^5} + \frac{8b(33a^2d^2+4bc(4bc-11ad))(ax+bx^2)^{3/2}}{1155a^4x^4} - \frac{16b^2(33a^2d^2+4bc(4bc-11ad))(ax+bx^2)^{3/2}}{3465a^5x^3}$$

output

```
-2/11*c^2*(b*x^2+a*x)^(3/2)/a/x^7+4/99*c*(-11*a*d+4*b*c)*(b*x^2+a*x)^(3/2)
/a^2/x^6-2/231*(33*d^2+4*b*c*(-11*a*d+4*b*c)/a^2)*(b*x^2+a*x)^(3/2)/a/x^5+
8/1155*b*(33*a^2*d^2+4*b*c*(-11*a*d+4*b*c))*(b*x^2+a*x)^(3/2)/a^4/x^4-16/3
465*b^2*(33*a^2*d^2+4*b*c*(-11*a*d+4*b*c))*(b*x^2+a*x)^(3/2)/a^5/x^3
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.66

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^7}\,dx = -\frac{2(x(a+bx))^{3/2}\left(128b^4c^2x^4-32ab^3cx^3(6c+11dx)+24a^2b^2x^2(10c^2+22cdx+11d^2x^2)+5a^4(63c^2+\right.}{3465a^5x^7}$$

input
```
Integrate[((c + d*x)^2*Sqrt[a*x + b*x^2])/x^7,x]
```

output
```
(-2*(x*(a + b*x))^(3/2)*(128*b^4*c^2*x^4 - 32*a*b^3*c*x^3*(6*c + 11*d*x) +
 24*a^2*b^2*x^2*(10*c^2 + 22*c*d*x + 11*d^2*x^2) + 5*a^4*(63*c^2 + 154*c*d
*x + 99*d^2*x^2) - 4*a^3*b*x*(70*c^2 + 165*c*d*x + 99*d^2*x^2)))/(3465*a^5
*x^7)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1262, 27, 1220, 1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}(c+dx)^2}{x^7}\,dx$$

$$\Big\downarrow \text{1262}$$

$$-\frac{\int -\frac{3(2bc^2+d(4bc-3ad)x)\sqrt{bx^2+ax}}{2x^7}dx}{3b}-\frac{d^2\left(ax+bx^2\right)^{3/2}}{3bx^6}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(2bc^2+d(4bc-3ad)x)\sqrt{bx^2+ax}}{x^7}dx}{2b}-\frac{d^2\left(ax+bx^2\right)^{3/2}}{3bx^6}$$

$$\Big\downarrow \text{1220}$$

$$\frac{-\frac{(33a^2d^2-44abcd+16b^2c^2)\int \frac{\sqrt{bx^2+ax}}{x^6}dx}{11a}-\frac{4bc^2(ax+bx^2)^{3/2}}{11ax^7}}{2b}-\frac{d^2(ax+bx^2)^{3/2}}{3bx^6}$$

$\downarrow$ 1129

$$\frac{-\frac{(33a^2d^2-44abcd+16b^2c^2)\left(-\frac{2b\int \frac{\sqrt{bx^2+ax}}{x^5}dx}{3a}-\frac{2(ax+bx^2)^{3/2}}{9ax^6}\right)}{11a}-\frac{4bc^2(ax+bx^2)^{3/2}}{11ax^7}}{2b}-\frac{d^2(ax+bx^2)^{3/2}}{3bx^6}$$

$\downarrow$ 1129

$$\frac{-\frac{(33a^2d^2-44abcd+16b^2c^2)\left(-\frac{2b\left(-\frac{4b\int \frac{\sqrt{bx^2+ax}}{x^4}dx}{7a}-\frac{2(ax+bx^2)^{3/2}}{7ax^5}\right)}{3a}-\frac{2(ax+bx^2)^{3/2}}{9ax^6}\right)}{11a}-\frac{4bc^2(ax+bx^2)^{3/2}}{11ax^7}}{2b}-\frac{d^2(ax+bx^2)^{3/2}}{3bx^6}$$

$\downarrow$ 1129

$$\frac{-\frac{(33a^2d^2-44abcd+16b^2c^2)\left(-\frac{2b\left(-\frac{4b\left(-\frac{2b\int \frac{\sqrt{bx^2+ax}}{x^3}dx}{5a}-\frac{2(ax+bx^2)^{3/2}}{5ax^4}\right)}{7a}-\frac{2(ax+bx^2)^{3/2}}{7ax^5}\right)}{3a}-\frac{2(ax+bx^2)^{3/2}}{9ax^6}\right)}{11a}-\frac{4bc^2(ax+bx^2)^{3/2}}{11ax^7}}{2b}-\frac{d^2(ax+bx^2)^{3/2}}{3bx^6}$$

$\downarrow$ 1123

$$\frac{-\frac{\left(-\frac{2b\left(-\frac{4b\left(\frac{4b\left(ax+bx^2\right)^{3/2}}{15a^2x^3}-\frac{2\left(ax+bx^2\right)^{3/2}}{5ax^4}\right)}{7a}-\frac{2\left(ax+bx^2\right)^{3/2}}{7ax^5}\right)}{3a}-\frac{2\left(ax+bx^2\right)^{3/2}}{9ax^6}\right)\left(33a^2d^2-44abcd+16b^2c^2\right)}{11a}-\frac{4bc^2\left(ax+bx^2\right)^{3/2}}{11ax^7}}{2b}-\frac{d^2\left(ax+bx^2\right)^{3/2}}{3bx^6}$$

input
```
Int[((c + d*x)^2*Sqrt[a*x + b*x^2])/x^7,x]
```

output
```
-1/3*(d^2*(a*x + b*x^2)^(3/2))/(b*x^6) + ((-4*b*c^2*(a*x + b*x^2)^(3/2))/(
11*a*x^7) - ((16*b^2*c^2 - 44*a*b*c*d + 33*a^2*d^2)*((-2*(a*x + b*x^2)^(3/
2))/(9*a*x^6) - (2*b*((-2*(a*x + b*x^2)^(3/2))/(7*a*x^5) - (4*b*((-2*(a*x
+ b*x^2)^(3/2))/(5*a*x^4) + (4*b*(a*x + b*x^2)^(3/2))/(15*a^2*x^3)))/(7*a)
))/(3*a)))/(11*a))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$-\frac{2\sqrt{x(bx+a)}\,(bx+a)\left(\left(\frac{11}{7}d^2x^2+\frac{22}{9}cdx+c^2\right)a^4-\frac{8\left(\frac{99}{70}d^2x^2+\frac{33}{14}cdx+c^2\right)xb\,a^3}{9}+\frac{16x^2b^2\left(\frac{11}{10}d^2x^2+\frac{11}{5}cdx+c^2\right)a^2}{21}-\frac{64x^3b^3\left(\frac{11}{6}\right.}{105}\right.}{11x^6a^5}$
gosper	$-\frac{2(bx+a)\left(264a^2b^2d^2x^4-352a\,b^3cd\,x^4+128b^4c^2x^4-396a^3b\,d^2x^3+528a^2b^2cd\,x^3-192a\,b^3c^2x^3+495a^4d^2x^2-660a^3dcb\,x^2\right.}{3465x^6a^5}$
orering	$-\frac{2(bx+a)\left(264a^2b^2d^2x^4-352a\,b^3cd\,x^4+128b^4c^2x^4-396a^3b\,d^2x^3+528a^2b^2cd\,x^3-192a\,b^3c^2x^3+495a^4d^2x^2-660a^3dcb\,x^2\right.}{3465x^6a^5}$
trager	$-\frac{2\left(264a^2b^3d^2x^5-352a\,b^4cd\,x^5+128b^5c^2x^5-132a^3b^2d^2x^4+176a^2b^3cd\,x^4-64a\,b^4c^2x^4+99a^4b\,d^2x^3-132a^3b^2cd\,x^3+48a^2\right.}{3465x^6a^5}$
risch	$-\frac{2(bx+a)\left(264a^2b^3d^2x^5-352a\,b^4cd\,x^5+128b^5c^2x^5-132a^3b^2d^2x^4+176a^2b^3cd\,x^4-64a\,b^4c^2x^4+99a^4b\,d^2x^3-132a^3b^2cd\,x^3\right.}{3465x^5\sqrt{x(bx+a)}\,a^5}$
default	$c^2\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{11a\,x^7}-\frac{8b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{9a\,x^6}-\frac{2b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{7a\,x^5}-\frac{4b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{5a\,x^4}+\frac{4b\left(b\,x^2+ax\right)^{\frac{3}{2}}}{15a^2x^3}\right)}{7a}\right)}{3a}\right)}{11a}\right)+d^2\left(\right.$

input

```
int((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output

```
-2/11*(x*(b*x+a))^(1/2)*(b*x+a)*((11/7*d^2*x^2+22/9*c*d*x+c^2)*a^4-8/9*(99
/70*d^2*x^2+33/14*c*d*x+c^2)*x*b*a^3+16/21*x^2*b^2*(11/10*d^2*x^2+11/5*c*d
*x+c^2)*a^2-64/105*x^3*b^3*(11/6*d*x+c)*c*a+128/315*b^4*c^2*x^4)/x^6/a^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^7}\,dx = -\frac{2\,(315\,a^5c^2 + 8\,(16\,b^5c^2 - 44\,ab^4cd + 33\,a^2b^3d^2)x^5 - 4\,(16\,ab^4c^2 - 44\,a^2b^3cd + 33\,a^3b^2d^2)x^4 + 3\,(16\,a^2b}{3465}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
-2/3465*(315*a^5*c^2 + 8*(16*b^5*c^2 - 44*a*b^4*c*d + 33*a^2*b^3*d^2)*x^5
- 4*(16*a*b^4*c^2 - 44*a^2*b^3*c*d + 33*a^3*b^2*d^2)*x^4 + 3*(16*a^2*b^3*c
^2 - 44*a^3*b^2*c*d + 33*a^4*b*d^2)*x^3 - 5*(8*a^3*b^2*c^2 - 22*a^4*b*c*d
- 99*a^5*d^2)*x^2 + 35*(a^4*b*c^2 + 22*a^5*c*d)*x)*sqrt(b*x^2 + a*x)/(a^5*
x^6)
```

Sympy [F]

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^7}\,dx = \int \frac{\sqrt{x\,(a+bx)}\,(c+dx)^2}{x^7}\,dx$$

input
```
integrate((d*x+c)**2*(b*x**2+a*x)**(1/2)/x**7,x)
```

output
```
Integral(sqrt(x*(a + b*x))*(c + d*x)**2/x**7, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.74

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^7}\,dx = -\frac{256\sqrt{bx^2+ax}b^5c^2}{3465\,a^5x} + \frac{64\sqrt{bx^2+ax}b^4cd}{315\,a^4x} - \frac{16\sqrt{bx^2+ax}b^3d^2}{105\,a^3x} + \frac{128\sqrt{bx^2+ax}b^4c^2}{3465\,a^4x^2} - \frac{32\sqrt{bx^2+ax}b^3cd}{315\,a^3x^2} + \frac{8\sqrt{bx^2+ax}b^2d^2}{105\,a^2x^2} - \frac{32\sqrt{bx^2+ax}b^3c^2}{1155\,a^3x^3} + \frac{8\sqrt{bx^2+ax}b^2cd}{105\,a^2x^3} - \frac{2\sqrt{bx^2+ax}bd^2}{35\,ax^3} + \frac{16\sqrt{bx^2+ax}b^2c^2}{693\,a^2x^4} - \frac{4\sqrt{bx^2+ax}bcd}{63\,ax^4} - \frac{2\sqrt{bx^2+ax}d^2}{7\,x^4} - \frac{2\sqrt{bx^2+ax}bc^2}{99\,ax^5} - \frac{4\sqrt{bx^2+ax}cd}{9\,x^5} - \frac{2\sqrt{bx^2+ax}c^2}{11\,x^6}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
-256/3465*sqrt(b*x^2 + a*x)*b^5*c^2/(a^5*x) + 64/315*sqrt(b*x^2 + a*x)*b^4
*c*d/(a^4*x) - 16/105*sqrt(b*x^2 + a*x)*b^3*d^2/(a^3*x) + 128/3465*sqrt(b*
x^2 + a*x)*b^4*c^2/(a^4*x^2) - 32/315*sqrt(b*x^2 + a*x)*b^3*c*d/(a^3*x^2)
+ 8/105*sqrt(b*x^2 + a*x)*b^2*d^2/(a^2*x^2) - 32/1155*sqrt(b*x^2 + a*x)*b^
3*c^2/(a^3*x^3) + 8/105*sqrt(b*x^2 + a*x)*b^2*c*d/(a^2*x^3) - 2/35*sqrt(b*
x^2 + a*x)*b*d^2/(a*x^3) + 16/693*sqrt(b*x^2 + a*x)*b^2*c^2/(a^2*x^4) - 4/
63*sqrt(b*x^2 + a*x)*b*c*d/(a*x^4) - 2/7*sqrt(b*x^2 + a*x)*d^2/x^4 - 2/99*
sqrt(b*x^2 + a*x)*b*c^2/(a*x^5) - 4/9*sqrt(b*x^2 + a*x)*c*d/x^5 - 2/11*sqr
t(b*x^2 + a*x)*c^2/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(179) = 358$.

Time = 0.14 (sec) , antiderivative size = 541, normalized size of antiderivative = 2.72

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^7}\,dx$$
$$= \frac{2\left(4620\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^8b^2d^2+13860\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^7b^{\frac{5}{2}}cd+10395\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^7ab\right.}{}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^7,x, algorithm="giac")
```

output
```
2/3465*(4620*(sqrt(b)*x - sqrt(b*x^2 + a*x))^8*b^2*d^2 + 13860*(sqrt(b)*x
- sqrt(b*x^2 + a*x))^7*b^(5/2)*c*d + 10395*(sqrt(b)*x - sqrt(b*x^2 + a*x))
^7*a*b^(3/2)*d^2 + 11088*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*b^3*c^2 + 38808
*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a*b^2*c*d + 9009*(sqrt(b)*x - sqrt(b*x^
2 + a*x))^6*a^2*b*d^2 + 36960*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a*b^(5/2)*
c^2 + 43890*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a^2*b^(3/2)*c*d + 3465*(sqrt
(b)*x - sqrt(b*x^2 + a*x))^5*a^3*sqrt(b)*d^2 + 51480*(sqrt(b)*x - sqrt(b*x
^2 + a*x))^4*a^2*b^2*c^2 + 24750*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^3*b*c
*d + 495*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^4*d^2 + 38115*(sqrt(b)*x - sq
rt(b*x^2 + a*x))^3*a^3*b^(3/2)*c^2 + 6930*(sqrt(b)*x - sqrt(b*x^2 + a*x))^
3*a^4*sqrt(b)*c*d + 15785*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^4*b*c^2 + 77
0*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^5*c*d + 3465*(sqrt(b)*x - sqrt(b*x^2
 + a*x))*a^5*sqrt(b)*c^2 + 315*a^6*c^2)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^11
```

Mupad [B] (verification not implemented)

Time = 11.07 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.74

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^7}\,dx = \frac{16\,b^2\,c^2\,\sqrt{b\,x^2+a\,x}}{693\,a^2\,x^4}-\frac{2\,d^2\,\sqrt{b\,x^2+a\,x}}{7\,x^4}-\frac{4\,c\,d\,\sqrt{b\,x^2+a\,x}}{9\,x^5}-\frac{2\,c^2\,\sqrt{b\,x^2+a\,x}}{11\,x^6}-\frac{32\,b^3\,c^2\,\sqrt{b\,x^2+a\,x}}{1155\,a^3\,x^3}+\frac{128\,b^4\,c^2\,\sqrt{b\,x^2+a\,x}}{3465\,a^4\,x^2}-\frac{256\,b^5\,c^2\,\sqrt{b\,x^2+a\,x}}{3465\,a^5\,x}+\frac{8\,b^2\,d^2\,\sqrt{b\,x^2+a\,x}}{105\,a^2\,x^2}-\frac{16\,b^3\,d^2\,\sqrt{b\,x^2+a\,x}}{105\,a^3\,x}-\frac{2\,b\,c^2\,\sqrt{b\,x^2+a\,x}}{99\,a\,x^5}-\frac{2\,b\,d^2\,\sqrt{b\,x^2+a\,x}}{35\,a\,x^3}+\frac{8\,b^2\,c\,d\,\sqrt{b\,x^2+a\,x}}{105\,a^2\,x^3}-\frac{32\,b^3\,c\,d\,\sqrt{b\,x^2+a\,x}}{315\,a^3\,x^2}+\frac{64\,b^4\,c\,d\,\sqrt{b\,x^2+a\,x}}{315\,a^4\,x}-\frac{4\,b\,c\,d\,\sqrt{b\,x^2+a\,x}}{63\,a\,x^4}$$

input
```
int(((a*x + b*x^2)^(1/2)*(c + d*x)^2)/x^7,x)
```

output
```
(16*b^2*c^2*(a*x + b*x^2)^(1/2))/(693*a^2*x^4) - (2*d^2*(a*x + b*x^2)^(1/2
))/(7*x^4) - (4*c*d*(a*x + b*x^2)^(1/2))/(9*x^5) - (2*c^2*(a*x + b*x^2)^(1
/2))/(11*x^6) - (32*b^3*c^2*(a*x + b*x^2)^(1/2))/(1155*a^3*x^3) + (128*b^4
*c^2*(a*x + b*x^2)^(1/2))/(3465*a^4*x^2) - (256*b^5*c^2*(a*x + b*x^2)^(1/2
))/(3465*a^5*x) + (8*b^2*d^2*(a*x + b*x^2)^(1/2))/(105*a^2*x^2) - (16*b^3*
d^2*(a*x + b*x^2)^(1/2))/(105*a^3*x) - (2*b*c^2*(a*x + b*x^2)^(1/2))/(99*a
*x^5) - (2*b*d^2*(a*x + b*x^2)^(1/2))/(35*a*x^3) + (8*b^2*c*d*(a*x + b*x^2
)^(1/2))/(105*a^2*x^3) - (32*b^3*c*d*(a*x + b*x^2)^(1/2))/(315*a^3*x^2) +
(64*b^4*c*d*(a*x + b*x^2)^(1/2))/(315*a^4*x) - (4*b*c*d*(a*x + b*x^2)^(1/2
))/(63*a*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.74

$$\int \frac{(c+dx)^2\sqrt{ax+bx^2}}{x^7}\,dx$$
$$= \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^5c^2}{11}-\frac{4\sqrt{x}\,\sqrt{bx+a}\,a^5cdx}{9}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^5d^2x^2}{7}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4b\,c^2x}{99}-\frac{4\sqrt{x}\,\sqrt{bx+a}\,a^4bcd\,x^2}{63}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4b\,d^2x}{35}}{}$$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(1/2)/x^7,x)
```

output
```
(2*( - 315*sqrt(x)*sqrt(a + b*x)*a**5*c**2 - 770*sqrt(x)*sqrt(a + b*x)*a**
5*c*d*x - 495*sqrt(x)*sqrt(a + b*x)*a**5*d**2*x**2 - 35*sqrt(x)*sqrt(a + b
*x)*a**4*b*c**2*x - 110*sqrt(x)*sqrt(a + b*x)*a**4*b*c*d*x**2 - 99*sqrt(x)
*sqrt(a + b*x)*a**4*b*d**2*x**3 + 40*sqrt(x)*sqrt(a + b*x)*a**3*b**2*c**2*
x**2 + 132*sqrt(x)*sqrt(a + b*x)*a**3*b**2*c*d*x**3 + 132*sqrt(x)*sqrt(a +
 b*x)*a**3*b**2*d**2*x**4 - 48*sqrt(x)*sqrt(a + b*x)*a**2*b**3*c**2*x**3 -
 176*sqrt(x)*sqrt(a + b*x)*a**2*b**3*c*d*x**4 - 264*sqrt(x)*sqrt(a + b*x)*
a**2*b**3*d**2*x**5 + 64*sqrt(x)*sqrt(a + b*x)*a*b**4*c**2*x**4 + 352*sqrt
(x)*sqrt(a + b*x)*a*b**4*c*d*x**5 - 128*sqrt(x)*sqrt(a + b*x)*b**5*c**2*x*
*5 + 264*sqrt(b)*a**2*b**3*d**2*x**6 - 352*sqrt(b)*a*b**4*c*d*x**6 + 128*s
qrt(b)*b**5*c**2*x**6))/(3465*a**5*x**6)
```

3.20 $\int x(c+dx)^3\sqrt{ax+bx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 349

$$
\begin{aligned}
&\int x(c+dx)^3\sqrt{ax+bx^2}\,dx \\
&= -\frac{a^2(64b^3c^3-3ad(40b^2c^2-28abcd+7a^2d^2))\sqrt{ax+bx^2}}{512b^5} \\
&\quad+\frac{a(64b^3c^3-3ad(40b^2c^2-28abcd+7a^2d^2))\,x\sqrt{ax+bx^2}}{768b^4} \\
&\quad+\frac{1}{192}\left(64c^3-\frac{3ad(40b^2c^2-28abcd+7a^2d^2)}{b^3}\right)x^2\sqrt{ax+bx^2} \\
&\quad+\frac{3d(40b^2c^2-28abcd+7a^2d^2)\,x(ax+bx^2)^{3/2}}{160b^3} \\
&\quad+\frac{3d^2(4bc-ad)x^2(ax+bx^2)^{3/2}}{20b^2}+\frac{d^3x^3(ax+bx^2)^{3/2}}{6b} \\
&\quad+\frac{a^3(64b^3c^3-3ad(40b^2c^2-28abcd+7a^2d^2))\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{512b^{11/2}}
\end{aligned}
$$

output
```
-1/512*a^2*(64*b^3*c^3-3*a*d*(7*a^2*d^2-28*a*b*c*d+40*b^2*c^2))*(b*x^2+a*x
)^(1/2)/b^5+1/768*a*(64*b^3*c^3-3*a*d*(7*a^2*d^2-28*a*b*c*d+40*b^2*c^2))*x
*(b*x^2+a*x)^(1/2)/b^4+1/192*(64*c^3-3*a*d*(7*a^2*d^2-28*a*b*c*d+40*b^2*c^
2)/b^3)*x^2*(b*x^2+a*x)^(1/2)+3/160*d*(7*a^2*d^2-28*a*b*c*d+40*b^2*c^2)*x*
(b*x^2+a*x)^(3/2)/b^3+3/20*d^2*(-a*d+4*b*c)*x^2*(b*x^2+a*x)^(3/2)/b^2+1/6*
d^3*x^3*(b*x^2+a*x)^(3/2)/b+1/512*a^3*(64*b^3*c^3-3*a*d*(7*a^2*d^2-28*a*b*
c*d+40*b^2*c^2))*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.95

$$\int x(c+dx)^3\sqrt{ax+bx^2}\,dx$$
$$=\frac{\sqrt{x}\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\sqrt{a+bx}(315a^5d^3-210a^4bd^2(6c+dx)+24a^3b^2d(75c^2+35cdx+7d^2x^2)+64ab^4x(\right.}{}$$

input
```
Integrate[x*(c + d*x)^3*Sqrt[a*x + b*x^2],x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(315*a^5*d^3 - 210*a
^4*b*d^2*(6*c + d*x) + 24*a^3*b^2*d*(75*c^2 + 35*c*d*x + 7*d^2*x^2) + 64*a
*b^4*x*(10*c^3 + 15*c^2*d*x + 9*c*d^2*x^2 + 2*d^3*x^3) - 48*a^2*b^3*(20*c^
3 + 25*c^2*d*x + 14*c*d^2*x^2 + 3*d^3*x^3) + 128*b^5*x^2*(20*c^3 + 45*c^2*
d*x + 36*c*d^2*x^2 + 10*d^3*x^3)) + 90*a^4*d*(40*b^2*c^2 + 7*a^2*d^2)*ArcT
anh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])] + 120*a^3*b*c*(16*b^2*c^2
 + 21*a^2*d^2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(76
80*b^(11/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.77, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1262, 27, 2169, 27, 1225, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{ax+bx^2}(c+dx)^3\,dx$$

$$\Big\downarrow \; 1262$$

$$\frac{\int \frac{3}{2}x\sqrt{bx^2+ax}\left(4bc^3+12bdxc^2+3d^2(4bc-ad)x^2\right)dx}{6b}+\frac{d^3x^3\left(ax+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \; 27$$

$$\frac{\int x\sqrt{bx^2+ax}\left(4bc^3+12bdxc^2+3d^2(4bc-ad)x^2\right)dx}{4b}+\frac{d^3x^3\left(ax+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \; 2169$$

$$\frac{\frac{\int \frac{1}{2}x\left(40b^2c^3+3d\left(40b^2c^2-28abdc+7a^2d^2\right)x\right)\sqrt{bx^2+ax}dx}{5b}+\frac{3d^2x^2\left(ax+bx^2\right)^{3/2}(4bc-ad)}{5b}}{4b}+\frac{d^3x^3\left(ax+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int x\left(40b^2c^3+3d\left(40b^2c^2-28abdc+7a^2d^2\right)x\right)\sqrt{bx^2+ax}dx}{10b}+\frac{3d^2x^2\left(ax+bx^2\right)^{3/2}(4bc-ad)}{5b}}{4b}+\frac{d^3x^3\left(ax+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \; 1225$$

$$\frac{\frac{\frac{\left(ax+bx^2\right)^{3/2}\left(18bdx\left(7a^2d^2-28abcd+40b^2c^2\right)+5\left(64b^3c^3-3ad\left(7a^2d^2-28abcd+40b^2c^2\right)\right)\right)}{24b^2}-\frac{5a\left(64b^3c^3-3ad\left(7a^2d^2-28abcd+40b^2c^2\right)\right)\int \sqrt{bx^2+ax}dx}{16b^2}}{10b}+3d}{4b}$$

$$\frac{d^3x^3\left(ax+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \; 1087$$

$$\frac{\frac{\frac{\left(ax+bx^2\right)^{3/2}\left(18bdx\left(7a^2d^2-28abcd+40b^2c^2\right)+5\left(64b^3c^3-3ad\left(7a^2d^2-28abcd+40b^2c^2\right)\right)\right)}{24b^2}-\frac{5a\left(64b^3c^3-3ad\left(7a^2d^2-28abcd+40b^2c^2\right)\right)\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\right.}{16b^2}}{10b}}{4b}$$

$$\frac{d^3x^3\left(ax+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \; 1091$$

$$\frac{\frac{\frac{\left(ax+bx^2\right)^{3/2}\left(18bdx\left(7a^2d^2-28abcd+40b^2c^2\right)+5\left(64b^3c^3-3ad\left(7a^2d^2-28abcd+40b^2c^2\right)\right)\right)}{24b^2}-\frac{5a\left(64b^3c^3-3ad\left(7a^2d^2-28abcd+40b^2c^2\right)\right)\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\right.}{16b^2}}{10b}}{4b}$$

$$\frac{d^3x^3\left(ax+bx^2\right)^{3/2}}{6b}$$

$\downarrow$ 219

$$\frac{\frac{\frac{\left(ax+bx^2\right)^{3/2}\left(18bdx\left(7a^2d^2-28abcd+40b^2c^2\right)+5\left(64b^3c^3-3ad\left(7a^2d^2-28abcd+40b^2c^2\right)\right)\right)}{24b^2}-\frac{5a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)\left(64b^3c^3-\right.}{16b^2}}{10b}}{4b}$$

$$\frac{d^3x^3\left(ax+bx^2\right)^{3/2}}{6b}$$

input
```
Int[x*(c + d*x)^3*Sqrt[a*x + b*x^2],x]
```

output
```
(d^3*x^3*(a*x + b*x^2)^(3/2))/(6*b) + ((3*d^2*(4*b*c - a*d)*x^2*(a*x + b*x
^2)^(3/2))/(5*b) + (((5*(64*b^3*c^3 - 3*a*d*(40*b^2*c^2 - 28*a*b*c*d + 7*a
^2*d^2)) + 18*b*d*(40*b^2*c^2 - 28*a*b*c*d + 7*a^2*d^2)*x)*(a*x + b*x^2)^(
3/2))/(24*b^2) - (5*a*(64*b^3*c^3 - 3*a*d*(40*b^2*c^2 - 28*a*b*c*d + 7*a^2
*d^2))*(((a + 2*b*x)*Sqrt[a*x + b*x^2])/(4*b) - (a^2*ArcTanh[(Sqrt[b]*x)/S
qrt[a*x + b*x^2]])/(4*b^(3/2))))/(16*b^2))/(10*b))/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

rule 2169
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d -
b*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2
, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$-\frac{21\left(a^{3}\left(a^{3} d^{3}-4 a^{2} b c d^{2}+\frac{40}{7} a b^{2} c^{2} d-\frac{64}{21} b^{3} c^{3}\right) \operatorname{arctanh}\left(\frac{\sqrt{x(b x+a)}}{x \sqrt{b}}\right)-\left(\frac{512\left(\frac{1}{2} d^{3} x^{3}+\frac{9}{5} c d^{2} x^{2}+\frac{9}{4} c^{2} d x+c^{3}\right) x^{2} b^{\frac{11}{2}}}{63}+\left(-\frac{64(}{}\right.\right.\right.}{}$
risch	$\frac{(1280 b^{5} d^{3} x^{5}+128 a b^{4} d^{3} x^{4}+4608 b^{5} c d^{2} x^{4}-144 a^{2} b^{3} d^{3} x^{3}+576 a b^{4} c d^{2} x^{3}+5760 b^{5} c^{2} d x^{3}+168 a^{3} b^{2} d^{3} x^{2}-672 a^{2} b^{3} c d^{2} x^{2}+96}{}$
default	$c^{3}\left(\frac{\left(b x^{2}+a x\right)^{\frac{3}{2}}}{3 b}-\frac{a\left(\frac{(2 b x+a) \sqrt{b x^{2}+a x}}{4 b}-\frac{a^{2} \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b x^{2}+a x}\right)}{8 b^{\frac{3}{2}}}\right)}{2 b}\right)+d^{3}\left(\frac{x^{3}\left(b x^{2}+a x\right)^{\frac{3}{2}}}{6 b}-\frac{3 a\left(\frac{x^{2}\left(b x^{2}+a x\right)}{5 b}\right.}{}\right.$

input
```
int(x*(d*x+c)^3*(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-21/512/b^(11/2)*(a^3*(a^3*d^3-4*a^2*b*c*d^2+40/7*a*b^2*c^2*d-64/21*b^3*c^
3)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-(512/63*(1/2*d^3*x^3+9/5*c*d^2*x^2
+9/4*c^2*d*x+c^3)*x^2*b^(11/2)+(-64/21*(3/20*d^3*x^3+7/10*c*d^2*x^2+5/4*c^
2*d*x+c^3)*a*b^(7/2)+128/63*(2/5*d^2*x^2+c*d*x+c^2)*x*(1/2*d*x+c)*b^(9/2)+
d*a^2*((8/15*d^2*x^2+8/3*c*d*x+40/7*c^2)*b^(5/2)+d*a*((-2/3*d*x-4*c)*b^(3/
2)+b^(1/2)*a*d)))*a)*(x*(b*x+a))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.73

$$\int x(c+dx)^3\sqrt{ax+bx^2}\,dx$$

$$= \left[-\frac{15\,(64\,a^3b^3c^3-120\,a^4b^2c^2d+84\,a^5bcd^2-21\,a^6d^3)\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,(1280\,b^6d^3\cdots}{\cdots}\right.$$

$$-\frac{15\,(64\,a^3b^3c^3-120\,a^4b^2c^2d+84\,a^5bcd^2-21\,a^6d^3)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(1280\,b^6d^3x^5-960\,a^2\cdots}{\cdots}$$

input
```
integrate(x*(d*x+c)^3*(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/15360*(15*(64*a^3*b^3*c^3 - 120*a^4*b^2*c^2*d + 84*a^5*b*c*d^2 - 21*a^
6*d^3)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*(1280*b^6*
d^3*x^5 - 960*a^2*b^4*c^3 + 1800*a^3*b^3*c^2*d - 1260*a^4*b^2*c*d^2 + 315*
a^5*b*d^3 + 128*(36*b^6*c*d^2 + a*b^5*d^3)*x^4 + 144*(40*b^6*c^2*d + 4*a*b
^5*c*d^2 - a^2*b^4*d^3)*x^3 + 8*(320*b^6*c^3 + 120*a*b^5*c^2*d - 84*a^2*b^
4*c*d^2 + 21*a^3*b^3*d^3)*x^2 + 10*(64*a*b^5*c^3 - 120*a^2*b^4*c^2*d + 84*
a^3*b^3*c*d^2 - 21*a^4*b^2*d^3)*x)*sqrt(b*x^2 + a*x))/b^6, -1/7680*(15*(64
*a^3*b^3*c^3 - 120*a^4*b^2*c^2*d + 84*a^5*b*c*d^2 - 21*a^6*d^3)*sqrt(-b)*a
rctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (1280*b^6*d^3*x^5 - 960*a^2*
b^4*c^3 + 1800*a^3*b^3*c^2*d - 1260*a^4*b^2*c*d^2 + 315*a^5*b*d^3 + 128*(3
6*b^6*c*d^2 + a*b^5*d^3)*x^4 + 144*(40*b^6*c^2*d + 4*a*b^5*c*d^2 - a^2*b^4
*d^3)*x^3 + 8*(320*b^6*c^3 + 120*a*b^5*c^2*d - 84*a^2*b^4*c*d^2 + 21*a^3*b
^3*d^3)*x^2 + 10*(64*a*b^5*c^3 - 120*a^2*b^4*c^2*d + 84*a^3*b^3*c*d^2 - 21
*a^4*b^2*d^3)*x)*sqrt(b*x^2 + a*x))/b^6]
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.50

$$\int x(c+dx)^3\sqrt{ax+bx^2}\,dx$$

$$= \begin{cases} \frac{3a^2\left(ac^3 - \frac{5a\left(3ac^2d - \frac{7a\left(3acd^2 - \frac{9a\left(\frac{ad^3}{12}+3bcd^2\right)}{10b} + 3bc^2d\right)}{8b} + bc^3\right)}{6b}\right)\left(\begin{cases}\frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{8b^2} + \sqrt{ax+bx^2} \\ \frac{2\left(\frac{c^3(ax)^{\frac{5}{2}}}{5} + \frac{3c^2d(ax)^{\frac{7}{2}}}{7a} + \frac{cd^2(ax)^{\frac{9}{2}}}{3a^2} + \frac{d^3(ax)^{\frac{11}{2}}}{11a^3}\right)}{a^2} \\ 0 \end{cases}$$

input
```
integrate(x*(d*x+c)**3*(b*x**2+a*x)**(1/2),x)
```

output
```
Piecewise((3*a**2*(a*c**3 - 5*a*(3*a*c**2*d - 7*a*(3*a*c*d**2 - 9*a*(a*d**
3/12 + 3*b*c*d**2)/(10*b) + 3*b*c**2*d)/(8*b) + b*c**3)/(6*b))*Piecewise((
log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(a**2/b, 0)), ((a
/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2), True))/(8*b**2) + s
qrt(a*x + b*x**2)*(-3*a*(a*c**3 - 5*a*(3*a*c**2*d - 7*a*(3*a*c*d**2 - 9*a*
(a*d**3/12 + 3*b*c*d**2)/(10*b) + 3*b*c**2*d)/(8*b) + b*c**3)/(6*b))/(4*b*
*2) + d**3*x**5/6 + x**4*(a*d**3/12 + 3*b*c*d**2)/(5*b) + x**3*(3*a*c*d**2
 - 9*a*(a*d**3/12 + 3*b*c*d**2)/(10*b) + 3*b*c**2*d)/(4*b) + x**2*(3*a*c**
2*d - 7*a*(3*a*c*d**2 - 9*a*(a*d**3/12 + 3*b*c*d**2)/(10*b) + 3*b*c**2*d)/
(8*b) + b*c**3)/(3*b) + x*(a*c**3 - 5*a*(3*a*c**2*d - 7*a*(3*a*c*d**2 - 9*
a*(a*d**3/12 + 3*b*c*d**2)/(10*b) + 3*b*c**2*d)/(8*b) + b*c**3)/(6*b))/(2*
b)), Ne(b, 0)), (2*(c**3*(a*x)**(5/2)/5 + 3*c**2*d*(a*x)**(7/2)/(7*a) + c*
d**2*(a*x)**(9/2)/(3*a**2) + d**3*(a*x)**(11/2)/(11*a**3))/a**2, Ne(a, 0))
, (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.55

$$\int x(c+dx)^3\sqrt{ax+bx^2}\,dx = \frac{(bx^2+ax)^{\frac{3}{2}}d^3x^3}{6\,b} + \frac{3\,(bx^2+ax)^{\frac{3}{2}}cd^2x^2}{5\,b} - \frac{3\,(bx^2+ax)^{\frac{3}{2}}ad^3x^2}{20\,b^2} - \frac{\sqrt{bx^2+ax}ac^3x}{4\,b} + \frac{15\,\sqrt{bx^2+ax}a^2c^2dx}{32\,b^2} + \frac{3\,(bx^2+ax)^{\frac{3}{2}}c^2dx}{4\,b} - \frac{21\,\sqrt{bx^2+ax}a^3cd^2x}{64\,b^3} - \frac{21\,(bx^2+ax)^{\frac{3}{2}}acd^2x}{40\,b^2} + \frac{21\,\sqrt{bx^2+ax}a^4d^3x}{256\,b^4} + \frac{21\,(bx^2+ax)^{\frac{3}{2}}a^2d^3x}{160\,b^3} + \frac{a^3c^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{5}{2}}} - \frac{15\,a^4c^2d\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{7}{2}}} + \frac{21\,a^5cd^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{256\,b^{\frac{9}{2}}} - \frac{21\,a^6d^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{1024\,b^{\frac{11}{2}}} - \frac{\sqrt{bx^2+ax}a^2c^3}{8\,b^2} + \frac{(bx^2+ax)^{\frac{3}{2}}c^3}{3\,b} + \frac{15\,\sqrt{bx^2+ax}a^3c^2d}{64\,b^3} - \frac{5\,(bx^2+ax)^{\frac{3}{2}}ac^2d}{8\,b^2} - \frac{21\,\sqrt{bx^2+ax}a^4cd^2}{128\,b^4} + \frac{7\,(bx^2+ax)^{\frac{3}{2}}a^2cd^2}{16\,b^3} + \frac{21\,\sqrt{bx^2+ax}a^5d^3}{512\,b^5} - \frac{7\,(bx^2+ax)^{\frac{3}{2}}a^3d^3}{64\,b^4}$$

input
```
integrate(x*(d*x+c)^3*(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a*x)^(3/2)*d^3*x^3/b + 3/5*(b*x^2 + a*x)^(3/2)*c*d^2*x^2/b -
3/20*(b*x^2 + a*x)^(3/2)*a*d^3*x^2/b^2 - 1/4*sqrt(b*x^2 + a*x)*a*c^3*x/b +
 15/32*sqrt(b*x^2 + a*x)*a^2*c^2*d*x/b^2 + 3/4*(b*x^2 + a*x)^(3/2)*c^2*d*x
/b - 21/64*sqrt(b*x^2 + a*x)*a^3*c*d^2*x/b^3 - 21/40*(b*x^2 + a*x)^(3/2)*a
*c*d^2*x/b^2 + 21/256*sqrt(b*x^2 + a*x)*a^4*d^3*x/b^4 + 21/160*(b*x^2 + a*
x)^(3/2)*a^2*d^3*x/b^3 + 1/16*a^3*c^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*
sqrt(b))/b^(5/2) - 15/128*a^4*c^2*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sq
rt(b))/b^(7/2) + 21/256*a^5*c*d^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt
(b))/b^(9/2) - 21/1024*a^6*d^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)
)/b^(11/2) - 1/8*sqrt(b*x^2 + a*x)*a^2*c^3/b^2 + 1/3*(b*x^2 + a*x)^(3/2)*c
^3/b + 15/64*sqrt(b*x^2 + a*x)*a^3*c^2*d/b^3 - 5/8*(b*x^2 + a*x)^(3/2)*a*c
^2*d/b^2 - 21/128*sqrt(b*x^2 + a*x)*a^4*c*d^2/b^4 + 7/16*(b*x^2 + a*x)^(3/
2)*a^2*c*d^2/b^3 + 21/512*sqrt(b*x^2 + a*x)*a^5*d^3/b^5 - 7/64*(b*x^2 + a*
x)^(3/2)*a^3*d^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.89

$$\int x(c+dx)^3\sqrt{ax+bx^2}\,dx$$
$$=\frac{1}{7680}\sqrt{bx^2+ax}\left(2\left(4\left(2\left(8\left(10\,d^3x+\frac{36\,b^5cd^2+ab^4d^3}{b^5}\right)x+\frac{9\,(40\,b^5c^2d+4\,ab^4cd^2-a^2b^3d^3)}{b^5}\right)x+\right.\right.\right.\right.$$
$$-\frac{(64\,a^3b^3c^3-120\,a^4b^2c^2d+84\,a^5bcd^2-21\,a^6d^3)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{1024\,b^{\frac{11}{2}}}$$

input
```
integrate(x*(d*x+c)^3*(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/7680*sqrt(b*x^2 + a*x)*(2*(4*(2*(8*(10*d^3*x + (36*b^5*c*d^2 + a*b^4*d^3
)/b^5)*x + 9*(40*b^5*c^2*d + 4*a*b^4*c*d^2 - a^2*b^3*d^3)/b^5)*x + (320*b^
5*c^3 + 120*a*b^4*c^2*d - 84*a^2*b^3*c*d^2 + 21*a^3*b^2*d^3)/b^5)*x + 5*(6
4*a*b^4*c^3 - 120*a^2*b^3*c^2*d + 84*a^3*b^2*c*d^2 - 21*a^4*b*d^3)/b^5)*x
 - 15*(64*a^2*b^3*c^3 - 120*a^3*b^2*c^2*d + 84*a^4*b*c*d^2 - 21*a^5*d^3)/b^
5) - 1/1024*(64*a^3*b^3*c^3 - 120*a^4*b^2*c^2*d + 84*a^5*b*c*d^2 - 21*a^6*
d^3)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(11/2)
```

Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.29

$$\int x(c+dx)^3\sqrt{ax+bx^2}\,dx = \frac{d^3\,x^3\,\left(b\,x^2+a\,x\right)^{3/2}}{6\,b} + \frac{a^3\,c^3\,\ln\left(\frac{a+2\,b\,x}{\sqrt{b}}+2\,\sqrt{b\,x^2+a\,x}\right)}{16\,b^{5/2}} + \frac{c^3\,\sqrt{b\,x^2+a\,x}\,\left(-3\,a^2+2\,a\,b\,x+8\,b^2\,x^2\right)}{24\,b^2} + \frac{3\,a\,d^3\,\left(\frac{7\,a\,\left(\frac{x\,\left(b\,x^2+a\,x\right)^{3/2}}{4\,b}-\frac{5\,a\,\left(\frac{a^3\,\ln\left(\frac{a+2\,b\,x}{\sqrt{b}}+2\,\sqrt{b\,x^2+a\,x}\right)}{16\,b^{5/2}}+\frac{\sqrt{b\,x^2+a\,x}\,\left(-3\,a^2+2\,a\,b\,x+8\,b^2\,x^2\right)}{24\,b^2}\right)}{8\,b}\right)}{10\,b}-\frac{x^2\,\left(b\,x^2+a\,x\right)^{3/2}}{5\,b}\right)}{4\,b} + \frac{3\,c^2\,d\,x\,\left(b\,x^2+a\,x\right)^{3/2}}{4\,b} - \frac{15\,a\,c^2\,d\,\left(\frac{a^3\,\ln\left(\frac{a+2\,b\,x}{\sqrt{b}}+2\,\sqrt{b\,x^2+a\,x}\right)}{16\,b^{5/2}}+\frac{\sqrt{b\,x^2+a\,x}\,\left(-3\,a^2+2\,a\,b\,x+8\,b^2\,x^2\right)}{24\,b^2}\right)}{8\,b} - \frac{21\,a\,c\,d^2\,\left(\frac{x\,\left(b\,x^2+a\,x\right)^{3/2}}{4\,b}-\frac{5\,a\,\left(\frac{a^3\,\ln\left(\frac{a+2\,b\,x}{\sqrt{b}}+2\,\sqrt{b\,x^2+a\,x}\right)}{16\,b^{5/2}}+\frac{\sqrt{b\,x^2+a\,x}\,\left(-3\,a^2+2\,a\,b\,x+8\,b^2\,x^2\right)}{24\,b^2}\right)}{8\,b}\right)}{10\,b} + \frac{3\,c\,d^2\,x^2\,\left(b\,x^2+a\,x\right)^{3/2}}{5\,b}$$

input
```
int(x*(a*x + b*x^2)^(1/2)*(c + d*x)^3,x)
```

output
```
(d^3*x^3*(a*x + b*x^2)^(3/2))/(6*b) + (a^3*c^3*log((a + 2*b*x)/b^(1/2) + 2
*(a*x + b*x^2)^(1/2)))/(16*b^(5/2)) + (c^3*(a*x + b*x^2)^(1/2)*(8*b^2*x^2
- 3*a^2 + 2*a*b*x))/(24*b^2) + (3*a*d^3*((7*a*((x*(a*x + b*x^2)^(3/2))/(4*
b) - (5*a*((a^3*log((a + 2*b*x)/b^(1/2) + 2*(a*x + b*x^2)^(1/2)))/(16*b^(5
/2)) + ((a*x + b*x^2)^(1/2)*(8*b^2*x^2 - 3*a^2 + 2*a*b*x))/(24*b^2)))/(8*b
)))/(10*b) - (x^2*(a*x + b*x^2)^(3/2))/(5*b)))/(4*b) + (3*c^2*d*x*(a*x + b
*x^2)^(3/2))/(4*b) - (15*a*c^2*d*((a^3*log((a + 2*b*x)/b^(1/2) + 2*(a*x +
b*x^2)^(1/2)))/(16*b^(5/2)) + ((a*x + b*x^2)^(1/2)*(8*b^2*x^2 - 3*a^2 + 2*
a*b*x))/(24*b^2)))/(8*b) - (21*a*c*d^2*((x*(a*x + b*x^2)^(3/2))/(4*b) - (5
*a*((a^3*log((a + 2*b*x)/b^(1/2) + 2*(a*x + b*x^2)^(1/2)))/(16*b^(5/2)) +
((a*x + b*x^2)^(1/2)*(8*b^2*x^2 - 3*a^2 + 2*a*b*x))/(24*b^2)))/(8*b)))/(10
*b) + (3*c*d^2*x^2*(a*x + b*x^2)^(3/2))/(5*b)
```

Reduce [B] (verification not implemented)

Time = 12.00 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.40

$$\int x(c+dx)^3\sqrt{ax+bx^2}\,dx$$

$$= \frac{315\sqrt{x}\,\sqrt{bx+a}\,a^5b\,d^3 - 1260\sqrt{x}\,\sqrt{bx+a}\,a^4b^2c\,d^2 - 210\sqrt{x}\,\sqrt{bx+a}\,a^4b^2d^3x + 1800\sqrt{x}\,\sqrt{bx+a}\,a^3b^3c^2}{}$$

input
```
int(x*(d*x+c)^3*(b*x^2+a*x)^(1/2),x)
```

output

```
(315*sqrt(x)*sqrt(a + b*x)*a**5*b*d**3 - 1260*sqrt(x)*sqrt(a + b*x)*a**4*b
**2*c*d**2 - 210*sqrt(x)*sqrt(a + b*x)*a**4*b**2*d**3*x + 1800*sqrt(x)*sqr
t(a + b*x)*a**3*b**3*c**2*d + 840*sqrt(x)*sqrt(a + b*x)*a**3*b**3*c*d**2*x
 + 168*sqrt(x)*sqrt(a + b*x)*a**3*b**3*d**3*x**2 - 960*sqrt(x)*sqrt(a + b*
x)*a**2*b**4*c**3 - 1200*sqrt(x)*sqrt(a + b*x)*a**2*b**4*c**2*d*x - 672*sq
rt(x)*sqrt(a + b*x)*a**2*b**4*c*d**2*x**2 - 144*sqrt(x)*sqrt(a + b*x)*a**2
*b**4*d**3*x**3 + 640*sqrt(x)*sqrt(a + b*x)*a*b**5*c**3*x + 960*sqrt(x)*sq
rt(a + b*x)*a*b**5*c**2*d*x**2 + 576*sqrt(x)*sqrt(a + b*x)*a*b**5*c*d**2*x
**3 + 128*sqrt(x)*sqrt(a + b*x)*a*b**5*d**3*x**4 + 2560*sqrt(x)*sqrt(a + b
*x)*b**6*c**3*x**2 + 5760*sqrt(x)*sqrt(a + b*x)*b**6*c**2*d*x**3 + 4608*sq
rt(x)*sqrt(a + b*x)*b**6*c*d**2*x**4 + 1280*sqrt(x)*sqrt(a + b*x)*b**6*d**
3*x**5 - 315*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**6*d
**3 + 1260*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**5*b*c
*d**2 - 1800*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4*b
**2*c**2*d + 960*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*
*3*b**3*c**3)/(7680*b**6)
```

3.21 $\int (c+dx)^3\sqrt{ax+bx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 275

$$\begin{aligned}\int (c+dx)^3\sqrt{ax+bx^2}\,dx &= \frac{a(2bc-ad)\left(16b^2c^2-16abcd+7a^2d^2\right)\sqrt{ax+bx^2}}{128b^4} \\ &+ \frac{(2bc-ad)\left(16b^2c^2-16abcd+7a^2d^2\right)x\sqrt{ax+bx^2}}{64b^3} \\ &+ \frac{d(48b^2c^2-30abcd+7a^2d^2)\left(ax+bx^2\right)^{3/2}}{48b^3} \\ &+ \frac{d^2(30bc-7ad)x(ax+bx^2)^{3/2}}{40b^2} + \frac{d^3x^2(ax+bx^2)^{3/2}}{5b} \\ &- \frac{a^2(2bc-ad)\left(16b^2c^2-16abcd+7a^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{128b^{9/2}}\end{aligned}$$

output
```
1/128*a*(-a*d+2*b*c)*(7*a^2*d^2-16*a*b*c*d+16*b^2*c^2)*(b*x^2+a*x)^(1/2)/b
^4+1/64*(-a*d+2*b*c)*(7*a^2*d^2-16*a*b*c*d+16*b^2*c^2)*x*(b*x^2+a*x)^(1/2)
/b^3+1/48*d*(7*a^2*d^2-30*a*b*c*d+48*b^2*c^2)*(b*x^2+a*x)^(3/2)/b^3+1/40*d
^2*(-7*a*d+30*b*c)*x*(b*x^2+a*x)^(3/2)/b^2+1/5*d^3*x^2*(b*x^2+a*x)^(3/2)/b
-1/128*a^2*(-a*d+2*b*c)*(7*a^2*d^2-16*a*b*c*d+16*b^2*c^2)*arctanh(b^(1/2)*
x/(b*x^2+a*x)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.02

$$\int (c+dx)^3\sqrt{ax+bx^2}\,dx$$
$$= \frac{\sqrt{x(a+bx)}(480ab^3c^3-720a^2b^2c^2d+450a^3bcd^2-105a^4d^3+960b^4c^3x+480ab^3c^2dx-300a^2b^2cd^2x+}{1920b^4}$$
$$+\frac{a^2(-32b^3c^3+48ab^2c^2d-30a^2bcd^2+7a^3d^3)\sqrt{x(a+bx)}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{64b^{9/2}\sqrt{x}\sqrt{a+bx}}$$

input
```
Integrate[(c + d*x)^3*Sqrt[a*x + b*x^2],x]
```

output
```
(Sqrt[x*(a + b*x)]*(480*a*b^3*c^3 - 720*a^2*b^2*c^2*d + 450*a^3*b*c*d^2 -
105*a^4*d^3 + 960*b^4*c^3*x + 480*a*b^3*c^2*d*x - 300*a^2*b^2*c*d^2*x + 70
*a^3*b*d^3*x + 1920*b^4*c^2*d*x^2 + 240*a*b^3*c*d^2*x^2 - 56*a^2*b^2*d^3*x
^2 + 1440*b^4*c*d^2*x^3 + 48*a*b^3*d^3*x^3 + 384*b^4*d^3*x^4))/(1920*b^4)
+ (a^2*(-32*b^3*c^3 + 48*a*b^2*c^2*d - 30*a^2*b*c*d^2 + 7*a^3*d^3)*Sqrt[x*
(a + b*x)]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(64*b^(9
/2)*Sqrt[x]*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.71, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1166, 27, 1225, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ax+bx^2}(c+dx)^3\,dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{\int \frac{1}{2}(c+dx)(c(10bc-3ad)+7d(2bc-ad)x)\sqrt{bx^2+ax}dx}{5b}+\frac{d\left(ax+bx^2\right)^{3/2}(c+dx)^2}{5b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (c+dx)(c(10bc-3ad)+7d(2bc-ad)x)\sqrt{bx^2+ax}dx}{10b}+\frac{d\left(ax+bx^2\right)^{3/2}(c+dx)^2}{5b}$$

$$\Big\downarrow \text{1225}$$

$$\frac{\frac{5(2bc-ad)\left(7a^2d^2-16abcd+16b^2c^2\right)\int \sqrt{bx^2+ax}dx}{16b^2}+\frac{d\left(ax+bx^2\right)^{3/2}\left(35a^2d^2+42bdx(2bc-ad)-150abcd+192b^2c^2\right)}{24b^2}}{10b}+\frac{d\left(ax+bx^2\right)^{3/2}(c+dx)^2}{5b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\frac{5(2bc-ad)\left(7a^2d^2-16abcd+16b^2c^2\right)\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)}{16b^2}+\frac{d\left(ax+bx^2\right)^{3/2}\left(35a^2d^2+42bdx(2bc-ad)-150abcd+192b^2c^2\right)}{24b^2}}{10b}+\frac{d\left(ax+bx^2\right)^{3/2}(c+dx)^2}{5b}$$

$$\Big\downarrow \text{1091}$$

$$\frac{\frac{5(2bc-ad)\left(7a^2d^2-16abcd+16b^2c^2\right)\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)}{16b^2}+\frac{d\left(ax+bx^2\right)^{3/2}\left(35a^2d^2+42bdx(2bc-ad)-150abcd+192b^2c^2\right)}{24b^2}}{10b}+\frac{d\left(ax+bx^2\right)^{3/2}(c+dx)^2}{5b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{5\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)(2bc-ad)\left(7a^2d^2-16abcd+16b^2c^2\right)}{16b^2}+\frac{d\left(ax+bx^2\right)^{3/2}\left(35a^2d^2+42bdx(2bc-ad)-150abcd+192b^2c^2\right)}{24b^2}}{10b}+\frac{d\left(ax+bx^2\right)^{3/2}(c+dx)^2}{5b}$$

input
```
Int[(c + d*x)^3*Sqrt[a*x + b*x^2],x]
```

output
```
(d*(c + d*x)^2*(a*x + b*x^2)^(3/2))/(5*b) + ((d*(192*b^2*c^2 - 150*a*b*c*d
 + 35*a^2*d^2 + 42*b*d*(2*b*c - a*d)*x)*(a*x + b*x^2)^(3/2))/(24*b^2) + (5
*(2*b*c - a*d)*(16*b^2*c^2 - 16*a*b*c*d + 7*a^2*d^2)*(((a + 2*b*x)*Sqrt[a*
x + b*x^2])/(4*b) - (a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*b^(3/2
))))/(16*b^2))/(10*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$\frac{\frac{7\left(a^2d^2-\frac{16}{7}abcd+\frac{16}{7}b^2c^2\right)(ad-2bc)a^2\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{128}-\frac{7\left(-\frac{32\left(\frac{1}{10}d^3x^3+\frac{1}{2}cd^2x^2+c^2dx+c^3\right)ab^{\frac{7}{2}}}{7}-\frac{64x\left(\frac{2}{5}d^3x^3+\frac{3}{2}cd^2x^2+2\right.}{7}\right.}{}}{b^{\frac{9}{2}}}$
risch	$-\frac{(-384b^4d^3x^4-48ab^3d^3x^3-1440b^4cd^2x^3+56a^2b^2d^3x^2-240ab^3cd^2x^2-1920b^4c^2dx^2-70a^3bd^3x+300a^2b^2cd^2x-480a}{1920b^4\sqrt{x(bx+a)}}$
default	$c^3\left(\frac{(2bx+a)\sqrt{bx^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{bx^2+ax}\right)}{8b^{\frac{3}{2}}}\right)+d^3\left(\frac{x^2\left(bx^2+ax\right)^{\frac{3}{2}}}{5b}-\frac{7a\left(\frac{x\left(bx^2+ax\right)^{\frac{3}{2}}}{4b}-\frac{5a\left(\frac{\left(bx^2+ax\right)}{3b}\right.}{}\right.}{}\right.$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
7/128/b^(9/2)*((a^2*d^2-16/7*a*b*c*d+16/7*b^2*c^2)*(a*d-2*b*c)*a^2*arctanh
((x*(b*x+a))^(1/2)/x/b^(1/2))-(-32/7*(1/10*d^3*x^3+1/2*c*d^2*x^2+c^2*d*x+c
^3)*a*b^(7/2)-64/7*x*(2/5*d^3*x^3+3/2*c*d^2*x^2+2*c^2*d*x+c^3)*b^(9/2)+d*(
(8/15*d^2*x^2+20/7*c*d*x+48/7*c^2)*b^(5/2)+d*((-2/3*d*x-30/7*c)*b^(3/2)+b^
(1/2)*a*d)*a)*a^2)*(x*(b*x+a))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.81

$$\int (c+dx)^3\sqrt{ax+bx^2}\,dx$$
$$= \left[-\frac{15\,(32\,a^2b^3c^3-48\,a^3b^2c^2d+30\,a^4bcd^2-7\,a^5d^3)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,(384\,b^5d^3x^4\right.$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/3840*(15*(32*a^2*b^3*c^3 - 48*a^3*b^2*c^2*d + 30*a^4*b*c*d^2 - 7*a^5*d
^3)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*(384*b^5*d^3*
x^4 + 480*a*b^4*c^3 - 720*a^2*b^3*c^2*d + 450*a^3*b^2*c*d^2 - 105*a^4*b*d^
3 + 48*(30*b^5*c*d^2 + a*b^4*d^3)*x^3 + 8*(240*b^5*c^2*d + 30*a*b^4*c*d^2
- 7*a^2*b^3*d^3)*x^2 + 10*(96*b^5*c^3 + 48*a*b^4*c^2*d - 30*a^2*b^3*c*d^2
+ 7*a^3*b^2*d^3)*x)*sqrt(b*x^2 + a*x))/b^5, 1/1920*(15*(32*a^2*b^3*c^3 - 4
8*a^3*b^2*c^2*d + 30*a^4*b*c*d^2 - 7*a^5*d^3)*sqrt(-b)*arctan(sqrt(b*x^2 +
 a*x)*sqrt(-b)/(b*x + a)) + (384*b^5*d^3*x^4 + 480*a*b^4*c^3 - 720*a^2*b^3
*c^2*d + 450*a^3*b^2*c*d^2 - 105*a^4*b*d^3 + 48*(30*b^5*c*d^2 + a*b^4*d^3)
*x^3 + 8*(240*b^5*c^2*d + 30*a*b^4*c*d^2 - 7*a^2*b^3*d^3)*x^2 + 10*(96*b^5
*c^3 + 48*a*b^4*c^2*d - 30*a^2*b^3*c*d^2 + 7*a^3*b^2*d^3)*x)*sqrt(b*x^2 +
a*x))/b^5]
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.58

$$\int (c+dx)^3\sqrt{ax+bx^2}\,dx$$

$$= \begin{cases} -\frac{a\left(ac^3-\frac{3a\left(3ac^2d-\frac{5a\left(3acd^2-\frac{7a\left(\frac{ad^3}{10}+3bcd^2\right)}{8b}+3bc^2d\right)}{6b}+bc^3\right)}{4b}\right)\left(\begin{cases}\frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0\\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{2b} + \sqrt{ax+bx^2} \\ \frac{2\left(\frac{c^3(ax)^{\frac{3}{2}}}{3}+\frac{3c^2d(ax)^{\frac{5}{2}}}{5a}+\frac{3cd^2(ax)^{\frac{7}{2}}}{7a^2}+\frac{d^3(ax)^{\frac{9}{2}}}{9a^3}\right)}{a} \\ 0 \end{cases}$$

input
```
integrate((d*x+c)**3*(b*x**2+a*x)**(1/2),x)
```

output
```
Piecewise((-a*(a*c**3 - 3*a*(3*a*c**2*d - 5*a*(3*a*c*d**2 - 7*a*(a*d**3/10
 + 3*b*c*d**2)/(8*b) + 3*b*c**2*d)/(6*b) + b*c**3)/(4*b))*Piecewise((log(a
 + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b
) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2), True))/(2*b) + sqrt(a*x
+ b*x**2)*(d**3*x**4/5 + x**3*(a*d**3/10 + 3*b*c*d**2)/(4*b) + x**2*(3*a*c
*d**2 - 7*a*(a*d**3/10 + 3*b*c*d**2)/(8*b) + 3*b*c**2*d)/(3*b) + x*(3*a*c*
*2*d - 5*a*(3*a*c*d**2 - 7*a*(a*d**3/10 + 3*b*c*d**2)/(8*b) + 3*b*c**2*d)/
(6*b) + b*c**3)/(2*b) + (a*c**3 - 3*a*(3*a*c**2*d - 5*a*(3*a*c*d**2 - 7*a*
(a*d**3/10 + 3*b*c*d**2)/(8*b) + 3*b*c**2*d)/(6*b) + b*c**3)/(4*b))/b), Ne
(b, 0)), (2*(c**3*(a*x)**(3/2)/3 + 3*c**2*d*(a*x)**(5/2)/(5*a) + 3*c*d**2*
(a*x)**(7/2)/(7*a**2) + d**3*(a*x)**(9/2)/(9*a**3))/a, Ne(a, 0)), (0, True
))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.60

$$
\begin{aligned}
\int (c+dx)^3\sqrt{ax+bx^2}\,dx &= \frac{(bx^2+ax)^{\frac{3}{2}}d^3x^2}{5\,b} + \frac{1}{2}\,\sqrt{bx^2+ax}c^3x - \frac{3\,\sqrt{bx^2+ax}ac^2dx}{4\,b} \\
&+ \frac{15\,\sqrt{bx^2+ax}a^2cd^2x}{32\,b^2} + \frac{3\,(bx^2+ax)^{\frac{3}{2}}cd^2x}{4\,b} \\
&- \frac{7\,\sqrt{bx^2+ax}a^3d^3x}{64\,b^3} - \frac{7\,(bx^2+ax)^{\frac{3}{2}}ad^3x}{40\,b^2} \\
&- \frac{a^2c^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{3}{2}}} \\
&+ \frac{3\,a^3c^2d\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{5}{2}}} \\
&- \frac{15\,a^4cd^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{7}{2}}} \\
&+ \frac{7\,a^5d^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{256\,b^{\frac{9}{2}}} \\
&+ \frac{\sqrt{bx^2+ax}ac^3}{4\,b} - \frac{3\,\sqrt{bx^2+ax}a^2c^2d}{8\,b^2} + \frac{(bx^2+ax)^{\frac{3}{2}}c^2d}{b} \\
&+ \frac{15\,\sqrt{bx^2+ax}a^3cd^2}{64\,b^3} - \frac{5\,(bx^2+ax)^{\frac{3}{2}}acd^2}{8\,b^2} \\
&- \frac{7\,\sqrt{bx^2+ax}a^4d^3}{128\,b^4} + \frac{7\,(bx^2+ax)^{\frac{3}{2}}a^2d^3}{48\,b^3}
\end{aligned}
$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
1/5*(b*x^2 + a*x)^(3/2)*d^3*x^2/b + 1/2*sqrt(b*x^2 + a*x)*c^3*x - 3/4*sqrt
(b*x^2 + a*x)*a*c^2*d*x/b + 15/32*sqrt(b*x^2 + a*x)*a^2*c*d^2*x/b^2 + 3/4*
(b*x^2 + a*x)^(3/2)*c*d^2*x/b - 7/64*sqrt(b*x^2 + a*x)*a^3*d^3*x/b^3 - 7/4
0*(b*x^2 + a*x)^(3/2)*a*d^3*x/b^2 - 1/8*a^2*c^3*log(2*b*x + a + 2*sqrt(b*x
^2 + a*x)*sqrt(b))/b^(3/2) + 3/16*a^3*c^2*d*log(2*b*x + a + 2*sqrt(b*x^2 +
 a*x)*sqrt(b))/b^(5/2) - 15/128*a^4*c*d^2*log(2*b*x + a + 2*sqrt(b*x^2 + a
*x)*sqrt(b))/b^(7/2) + 7/256*a^5*d^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*s
qrt(b))/b^(9/2) + 1/4*sqrt(b*x^2 + a*x)*a*c^3/b - 3/8*sqrt(b*x^2 + a*x)*a^
2*c^2*d/b^2 + (b*x^2 + a*x)^(3/2)*c^2*d/b + 15/64*sqrt(b*x^2 + a*x)*a^3*c*
d^2/b^3 - 5/8*(b*x^2 + a*x)^(3/2)*a*c*d^2/b^2 - 7/128*sqrt(b*x^2 + a*x)*a^
4*d^3/b^4 + 7/48*(b*x^2 + a*x)^(3/2)*a^2*d^3/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.93

$$\int (c+dx)^3\sqrt{ax+bx^2}\,dx = \frac{1}{1920}\sqrt{bx^2+ax}\left(2\left(4\left(6\left(8\,d^3x+\frac{30\,b^4cd^2+ab^3d^3}{b^4}\right)x+\frac{240\,b^4c^2d+30\,ab^3cd^2-7\,a^2b^2d^3}{b^4}\right)x+\frac{5\,(96\,b\right.\right.\right.$$
$$+\frac{\left(32\,a^2b^3c^3-48\,a^3b^2c^2d+30\,a^4bcd^2-7\,a^5d^3\right)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{256\,b^{\frac{9}{2}}}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/1920*sqrt(b*x^2 + a*x)*(2*(4*(6*(8*d^3*x + (30*b^4*c*d^2 + a*b^3*d^3)/b^
4)*x + (240*b^4*c^2*d + 30*a*b^3*c*d^2 - 7*a^2*b^2*d^3)/b^4)*x + 5*(96*b^4
*c^3 + 48*a*b^3*c^2*d - 30*a^2*b^2*c*d^2 + 7*a^3*b*d^3)/b^4)*x + 15*(32*a*
b^3*c^3 - 48*a^2*b^2*c^2*d + 30*a^3*b*c*d^2 - 7*a^4*d^3)/b^4) + 1/256*(32*
a^2*b^3*c^3 - 48*a^3*b^2*c^2*d + 30*a^4*b*c*d^2 - 7*a^5*d^3)*log(abs(2*(sq
rt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(9/2)
```

Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.31

$$\int (c+dx)^3\sqrt{ax+bx^2}\,dx$$
$$= c^3\,\sqrt{b\,x^2+a\,x}\,\left(\frac{x}{2}+\frac{a}{4\,b}\right) - \frac{7\,a\,d^3\,\left(\frac{x\,{\left(b\,x^2+a\,x\right)}^{3/2}}{4\,b}-\frac{5\,a\,\left(\frac{a^3\,\ln\left(\frac{a+2\,b\,x}{\sqrt{b}}+2\,\sqrt{b\,x^2+a\,x}\right)}{16\,b^{5/2}}+\frac{\sqrt{b\,x^2+a\,x}\,\left(-3\,a^2+2\,a\,b\,x+8\,b^2\,x^2\right)}{24\,b^2}\right)}{8\,b}\right)}{10\,b}$$
$$+ \frac{d^3\,x^2\,{\left(b\,x^2+a\,x\right)}^{3/2}}{5\,b} - \frac{a^2\,c^3\,\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{8\,b^{3/2}} + \frac{3\,c\,d^2\,x\,{\left(b\,x^2+a\,x\right)}^{3/2}}{4\,b}$$
$$- \frac{15\,a\,c\,d^2\,\left(\frac{a^3\,\ln\left(\frac{a+2\,b\,x}{\sqrt{b}}+2\,\sqrt{b\,x^2+a\,x}\right)}{16\,b^{5/2}}+\frac{\sqrt{b\,x^2+a\,x}\,\left(-3\,a^2+2\,a\,b\,x+8\,b^2\,x^2\right)}{24\,b^2}\right)}{8\,b}$$
$$+ \frac{3\,a^3\,c^2\,d\,\ln\left(\frac{a+2\,b\,x}{\sqrt{b}}+2\,\sqrt{b\,x^2+a\,x}\right)}{16\,b^{5/2}} + \frac{c^2\,d\,\sqrt{b\,x^2+a\,x}\,\left(-3\,a^2+2\,a\,b\,x+8\,b^2\,x^2\right)}{8\,b^2}$$

input
```
int((a*x + b*x^2)^(1/2)*(c + d*x)^3,x)
```

output
```
c^3*(a*x + b*x^2)^(1/2)*(x/2 + a/(4*b)) - (7*a*d^3*((x*(a*x + b*x^2)^(3/2)
)/(4*b) - (5*a*((a^3*log((a + 2*b*x)/b^(1/2) + 2*(a*x + b*x^2)^(1/2)))/(16
*b^(5/2)) + ((a*x + b*x^2)^(1/2)*(8*b^2*x^2 - 3*a^2 + 2*a*b*x))/(24*b^2)))
/(8*b)))/(10*b) + (d^3*x^2*(a*x + b*x^2)^(3/2))/(5*b) - (a^2*c^3*log((a/2
 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2)))/(8*b^(3/2)) + (3*c*d^2*x*(a*x + b*x
^2)^(3/2))/(4*b) - (15*a*c*d^2*((a^3*log((a + 2*b*x)/b^(1/2) + 2*(a*x + b*
x^2)^(1/2)))/(16*b^(5/2)) + ((a*x + b*x^2)^(1/2)*(8*b^2*x^2 - 3*a^2 + 2*a*
b*x))/(24*b^2))))/(8*b) + (3*a^3*c^2*d*log((a + 2*b*x)/b^(1/2) + 2*(a*x + b
*x^2)^(1/2)))/(16*b^(5/2)) + (c^2*d*(a*x + b*x^2)^(1/2)*(8*b^2*x^2 - 3*a^2
 + 2*a*b*x))/(8*b^2)
```

Reduce [B] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.45

$$\int (c+dx)^3\sqrt{ax+bx^2}\,dx$$
$$= \frac{-105\sqrt{x}\,\sqrt{bx+a}\,a^4b\,d^3+450\sqrt{x}\,\sqrt{bx+a}\,a^3b^2c\,d^2+70\sqrt{x}\,\sqrt{bx+a}\,a^3b^2d^3x-720\sqrt{x}\,\sqrt{bx+a}\,a^2b^3c^2d}{}$$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(1/2),x)
```

output
```
( - 105*sqrt(x)*sqrt(a + b*x)*a**4*b*d**3 + 450*sqrt(x)*sqrt(a + b*x)*a**3
*b**2*c*d**2 + 70*sqrt(x)*sqrt(a + b*x)*a**3*b**2*d**3*x - 720*sqrt(x)*sqr
t(a + b*x)*a**2*b**3*c**2*d - 300*sqrt(x)*sqrt(a + b*x)*a**2*b**3*c*d**2*x
 - 56*sqrt(x)*sqrt(a + b*x)*a**2*b**3*d**3*x**2 + 480*sqrt(x)*sqrt(a + b*x
)*a*b**4*c**3 + 480*sqrt(x)*sqrt(a + b*x)*a*b**4*c**2*d*x + 240*sqrt(x)*sq
rt(a + b*x)*a*b**4*c*d**2*x**2 + 48*sqrt(x)*sqrt(a + b*x)*a*b**4*d**3*x**3
 + 960*sqrt(x)*sqrt(a + b*x)*b**5*c**3*x + 1920*sqrt(x)*sqrt(a + b*x)*b**5
*c**2*d*x**2 + 1440*sqrt(x)*sqrt(a + b*x)*b**5*c*d**2*x**3 + 384*sqrt(x)*s
qrt(a + b*x)*b**5*d**3*x**4 + 105*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqr
t(b))/sqrt(a))*a**5*d**3 - 450*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b
))/sqrt(a))*a**4*b*c*d**2 + 720*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(
b))/sqrt(a))*a**3*b**2*c**2*d - 480*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*s
qrt(b))/sqrt(a))*a**2*b**3*c**3)/(1920*b**5)
```

3.22 $\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 224

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x}\,dx = \frac{1}{64}\left(64c^3-\frac{ad(48b^2c^2-24abcd+5a^2d^2)}{b^3}\right)\sqrt{ax+bx^2} + \frac{d^2(24bc-5ad)\,(ax+bx^2)^{3/2}}{24b^2} + \frac{d(48b^2c^2-24abcd+5a^2d^2)\,(ax+bx^2)^{3/2}}{32b^3x} + \frac{d^3x(ax+bx^2)^{3/2}}{4b} + \frac{a(64b^3c^3-ad(48b^2c^2-24abcd+5a^2d^2))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{64b^{7/2}}$$

output
```
1/64*(64*c^3-a*d*(5*a^2*d^2-24*a*b*c*d+48*b^2*c^2)/b^3)*(b*x^2+a*x)^(1/2)+
1/24*d^2*(-5*a*d+24*b*c)*(b*x^2+a*x)^(3/2)/b^2+1/32*d*(5*a^2*d^2-24*a*b*c*
d+48*b^2*c^2)*(b*x^2+a*x)^(3/2)/b^3/x+1/4*d^3*x*(b*x^2+a*x)^(3/2)/b+1/64*a
*(64*b^3*c^3-a*d*(5*a^2*d^2-24*a*b*c*d+48*b^2*c^2))*arctanh(b^(1/2)*x/(b*x
^2+a*x)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x}\,dx$$
$$= \frac{\sqrt{x(a+bx)}\Big(\sqrt{b}(15a^3d^3-2a^2bd^2(36c+5dx)+8ab^2d(18c^2+6cdx+d^2x^2)+48b^3(4c^3+6c^2dx+4cd^2x}{192b^{7/2}}$$

input
```
Integrate[((c + d*x)^3*Sqrt[a*x + b*x^2])/x,x]
```

output
```
(Sqrt[x*(a + b*x)]*(Sqrt[b]*(15*a^3*d^3 - 2*a^2*b*d^2*(36*c + 5*d*x) + 8*a
*b^2*d*(18*c^2 + 6*c*d*x + d^2*x^2) + 48*b^3*(4*c^3 + 6*c^2*d*x + 4*c*d^2*
x^2 + d^3*x^3)) + (3*a*(-64*b^3*c^3 + 48*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 5*
a^3*d^3)*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[x]*Sqrt[a + b*x]))
)/(192*b^(7/2))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1262, 27, 2169, 27, 1221, 1131, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}(c+dx)^3}{x}\,dx$$
$$\Big\downarrow \; 1262$$
$$\frac{\int \frac{\sqrt{bx^2+ax}\left(8bc^3+24bdxc^2+d^2(24bc-5ad)x^2\right)}{2x}dx}{4b}+\frac{d^3x\left(ax+bx^2\right)^{3/2}}{4b}$$
$$\Big\downarrow \; 27$$
$$\frac{\int \frac{\sqrt{bx^2+ax}\left(8bc^3+24bdxc^2+d^2(24bc-5ad)x^2\right)}{x}dx}{8b}+\frac{d^3x\left(ax+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{ 2169}$$

$$\frac{\frac{\int \frac{3\left(16b^2c^3+d\left(48b^2c^2-24abdc+5a^2d^2\right)x\right)\sqrt{bx^2+ax}}{2x}dx}{3b}+\frac{d^2\left(ax+bx^2\right)^{3/2}(24bc-5ad)}{3b}}{8b}+\frac{d^3x\left(ax+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{\left(16b^2c^3+d\left(48b^2c^2-24abdc+5a^2d^2\right)x\right)\sqrt{bx^2+ax}}{x}dx}{2b}+\frac{d^2\left(ax+bx^2\right)^{3/2}(24bc-5ad)}{3b}}{8b}+\frac{d^3x\left(ax+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{ 1221}$$

$$\frac{\frac{\frac{\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)\int \frac{\sqrt{bx^2+ax}}{x}dx}{4b}+\frac{d\left(ax+bx^2\right)^{3/2}\left(5a^2d^2-24abcd+48b^2c^2\right)}{2bx}}{2b}+\frac{d^2\left(ax+bx^2\right)^{3/2}(24bc-5ad)}{3b}}{8b}+\frac{d^3x\left(ax+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{ 1131}$$

$$\frac{\frac{\frac{\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+ax}}dx+\sqrt{ax+bx^2}\right)}{4b}+\frac{d\left(ax+bx^2\right)^{3/2}\left(5a^2d^2-24abcd+48b^2c^2\right)}{2bx}}{2b}+\frac{d^2\left(ax+bx^2\right)^{3/2}(24bc-5ad)}{3b}}{8b}+\frac{d^3x\left(ax+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{ 1091}$$

$$\frac{\frac{\frac{\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)\left(a\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}+\sqrt{ax+bx^2}\right)}{4b}+\frac{d\left(ax+bx^2\right)^{3/2}\left(5a^2d^2-24abcd+48b^2c^2\right)}{2bx}}{2b}+\frac{d^2\left(ax+bx^2\right)^{3/2}(24bc-5ad)}{3b}}{8b}+\frac{d^3x\left(ax+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\frac{d\left(ax+bx^2\right)^{3/2}\left(5a^2d^2-24abcd+48b^2c^2\right)}{2bx}+\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}+\sqrt{ax+bx^2}\right)\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)}{4b}}{2b}+\frac{d^2\left(ax+bx^2\right)^{3/2}(24bc-5ad)}{3b}}{8b}+\frac{d^3x\left(ax+bx^2\right)^{3/2}}{4b}$$

input
```
Int[((c + d*x)^3*Sqrt[a*x + b*x^2])/x,x]
```

output
```
(d^3*x*(a*x + b*x^2)^(3/2))/(4*b) + ((d^2*(24*b*c - 5*a*d)*(a*x + b*x^2)^(
3/2))/(3*b) + ((d*(48*b^2*c^2 - 24*a*b*c*d + 5*a^2*d^2)*(a*x + b*x^2)^(3/2
))/(2*b*x) + ((64*b^3*c^3 - 48*a*b^2*c^2*d + 24*a^2*b*c*d^2 - 5*a^3*d^3)*(
Sqrt[a*x + b*x^2] + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]))/(
4*b))/(2*b))/(8*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

rule 2169
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d -
b*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2
, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$-\frac{5\left(a\left(a^3d^3-\frac{24}{5}a^2bc\,d^2+\frac{48}{5}a\,b^2c^2d-\frac{64}{5}b^3c^3\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)-\sqrt{x(bx+a)}\left(\frac{64\left(\frac{dx}{2}+c\right)\left(\frac{1}{2}d^2x^2+cdx+c^2\right)b^{\frac{7}{2}}}{5}+da\left(\left(\frac{8}{1}\right.\right.\right.\right.}{64b^{\frac{7}{2}}}$
risch	$\frac{\left(48b^3d^3x^3+8a\,b^2d^3x^2+192b^3c\,d^2x^2-10a^2b\,d^3x+48a\,b^2c\,d^2x+288b^3c^2dx+15a^3d^3-72a^2bc\,d^2+144a\,b^2c^2d+192b^3c^3\right)x(bx}{192b^3\sqrt{x(bx+a)}}$
default	$c^3\left(\sqrt{b\,x^2+ax}+\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2\sqrt{b}}\right)+d^3\left(\frac{x(b\,x^2+ax)^{\frac{3}{2}}}{4b}-\frac{5a\left(\frac{(b\,x^2+ax)^{\frac{3}{2}}}{3b}-\frac{a\left(\frac{(2bx+a)\sqrt{b\,x^2+ax}}{4b}\right.}{}\right)}{8b}\right.$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-5/64*(a*(a^3*d^3-24/5*a^2*b*c*d^2+48/5*a*b^2*c^2*d-64/5*b^3*c^3)*arctanh(
(x*(b*x+a))^(1/2)/x/b^(1/2))-(x*(b*x+a))^(1/2)*(64/5*(1/2*d*x+c)*(1/2*d^2*
x^2+c*d*x+c^2)*b^(7/2)+d*a*((8/15*d^2*x^2+16/5*c*d*x+48/5*c^2)*b^(5/2)+d*(
(-2/3*d*x-24/5*c)*b^(3/2)+b^(1/2)*a*d)*a)))/b^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.76

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x}\,dx$$
$$=\left[-\frac{3\left(64\,ab^3c^3-48\,a^2b^2c^2d+24\,a^3bcd^2-5\,a^4d^3\right)\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\left(48\,b^4d^3x^3+1\right.}{}\right.$$
$$-\frac{3\left(64\,ab^3c^3-48\,a^2b^2c^2d+24\,a^3bcd^2-5\,a^4d^3\right)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-\left(48\,b^4d^3x^3+192\,b^4c^3+14\right.}{}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/x,x, algorithm="fricas")
```

output
```
[-1/384*(3*(64*a*b^3*c^3 - 48*a^2*b^2*c^2*d + 24*a^3*b*c*d^2 - 5*a^4*d^3)*
sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*(48*b^4*d^3*x^3 +
 192*b^4*c^3 + 144*a*b^3*c^2*d - 72*a^2*b^2*c*d^2 + 15*a^3*b*d^3 + 8*(24*b
^4*c*d^2 + a*b^3*d^3)*x^2 + 2*(144*b^4*c^2*d + 24*a*b^3*c*d^2 - 5*a^2*b^2*
d^3)*x)*sqrt(b*x^2 + a*x))/b^4, -1/192*(3*(64*a*b^3*c^3 - 48*a^2*b^2*c^2*d
 + 24*a^3*b*c*d^2 - 5*a^4*d^3)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/
(b*x + a)) - (48*b^4*d^3*x^3 + 192*b^4*c^3 + 144*a*b^3*c^2*d - 72*a^2*b^2*
c*d^2 + 15*a^3*b*d^3 + 8*(24*b^4*c*d^2 + a*b^3*d^3)*x^2 + 2*(144*b^4*c^2*d
 + 24*a*b^3*c*d^2 - 5*a^2*b^2*d^3)*x)*sqrt(b*x^2 + a*x))/b^4]
```

Sympy [F]

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x}\,dx = \int \frac{\sqrt{x\,(a+bx)}(c+dx)^3}{x}\,dx$$

input
```
integrate((d*x+c)**3*(b*x**2+a*x)**(1/2)/x,x)
```

output
```
Integral(sqrt(x*(a + b*x))*(c + d*x)**3/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.52

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x}\,dx = \frac{3}{2}\sqrt{bx^2+ax}c^2dx - \frac{3\sqrt{bx^2+ax}acd^2x}{4\,b} + \frac{5\sqrt{bx^2+ax}a^2d^3x}{32\,b^2} + \frac{(bx^2+ax)^{\frac{3}{2}}d^3x}{4\,b} + \frac{ac^3\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{2\sqrt{b}} - \frac{3\,a^2c^2d\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{3}{2}}} + \frac{3\,a^3cd^2\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{5}{2}}} - \frac{5\,a^4d^3\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{7}{2}}} + \sqrt{bx^2+ax}c^3 + \frac{3\sqrt{bx^2+ax}ac^2d}{4\,b} - \frac{3\sqrt{bx^2+ax}a^2cd^2}{8\,b^2} + \frac{(bx^2+ax)^{\frac{3}{2}}cd^2}{b} + \frac{5\sqrt{bx^2+ax}a^3d^3}{64\,b^3} - \frac{5\,(bx^2+ax)^{\frac{3}{2}}ad^3}{24\,b^2}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/x,x, algorithm="maxima")
```

output
```
3/2*sqrt(b*x^2 + a*x)*c^2*d*x - 3/4*sqrt(b*x^2 + a*x)*a*c*d^2*x/b + 5/32*s
qrt(b*x^2 + a*x)*a^2*d^3*x/b^2 + 1/4*(b*x^2 + a*x)^(3/2)*d^3*x/b + 1/2*a*c
^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b) - 3/8*a^2*c^2*d*lo
g(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(3/2) + 3/16*a^3*c*d^2*log(2*
b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5/2) - 5/128*a^4*d^3*log(2*b*x +
 a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(7/2) + sqrt(b*x^2 + a*x)*c^3 + 3/4*sq
rt(b*x^2 + a*x)*a*c^2*d/b - 3/8*sqrt(b*x^2 + a*x)*a^2*c*d^2/b^2 + (b*x^2 +
 a*x)^(3/2)*c*d^2/b + 5/64*sqrt(b*x^2 + a*x)*a^3*d^3/b^3 - 5/24*(b*x^2 + a
*x)^(3/2)*a*d^3/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x}\,dx$$
$$= \frac{1}{192}\sqrt{bx^2+ax}\left(2\left(4\left(6\,d^3x+\frac{24\,b^3cd^2+ab^2d^3}{b^3}\right)x+\frac{144\,b^3c^2d+24\,ab^2cd^2-5\,a^2bd^3}{b^3}\right)x+\frac{3\,(64\,b^3c^3+\right.$$
$$-\frac{(64\,ab^3c^3-48\,a^2b^2c^2d+24\,a^3bcd^2-5\,a^4d^3)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{128\,b^{\frac{7}{2}}}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/x,x, algorithm="giac")
```

output
```
1/192*sqrt(b*x^2 + a*x)*(2*(4*(6*d^3*x + (24*b^3*c*d^2 + a*b^2*d^3)/b^3)*x
 + (144*b^3*c^2*d + 24*a*b^2*c*d^2 - 5*a^2*b*d^3)/b^3)*x + 3*(64*b^3*c^3 +
 48*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 5*a^3*d^3)/b^3) - 1/128*(64*a*b^3*c^3 -
 48*a^2*b^2*c^2*d + 24*a^3*b*c*d^2 - 5*a^4*d^3)*log(abs(2*(sqrt(b)*x - sqr
t(b*x^2 + a*x))*sqrt(b) + a))/b^(7/2)
```

Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.28

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x}\,dx$$
$$= c^3\sqrt{b\,x^2+a\,x}+\frac{a\,c^3\,\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{2\,\sqrt{b}}$$
$$-\frac{5\,a\,d^3\left(\frac{a^3\,\ln\left(\frac{a+2\,b\,x}{\sqrt{b}}+2\,\sqrt{b\,x^2+a\,x}\right)}{16\,b^{5/2}}+\frac{\sqrt{b\,x^2+a\,x}\,(-3\,a^2+2\,a\,b\,x+8\,b^2\,x^2)}{24\,b^2}\right)}{8\,b}$$
$$+3\,c^2\,d\sqrt{b\,x^2+a\,x}\left(\frac{x}{2}+\frac{a}{4\,b}\right)+\frac{d^3\,x\,(b\,x^2+a\,x)^{3/2}}{4\,b}$$
$$-\frac{3\,a^2\,c^2\,d\,\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{8\,b^{3/2}}+\frac{3\,a^3\,c\,d^2\,\ln\left(\frac{a+2\,b\,x}{\sqrt{b}}+2\,\sqrt{b\,x^2+a\,x}\right)}{16\,b^{5/2}}$$
$$+\frac{c\,d^2\,\sqrt{b\,x^2+a\,x}\,(-3\,a^2+2\,a\,b\,x+8\,b^2\,x^2)}{8\,b^2}$$

input
```
int(((a*x + b*x^2)^(1/2)*(c + d*x)^3)/x,x)
```

output
```
c^3*(a*x + b*x^2)^(1/2) + (a*c^3*log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(
1/2)))/(2*b^(1/2)) - (5*a*d^3*((a^3*log((a + 2*b*x)/b^(1/2) + 2*(a*x + b*x
^2)^(1/2)))/(16*b^(5/2)) + ((a*x + b*x^2)^(1/2)*(8*b^2*x^2 - 3*a^2 + 2*a*b
*x))/(24*b^2)))/(8*b) + 3*c^2*d*(a*x + b*x^2)^(1/2)*(x/2 + a/(4*b)) + (d^3
*x*(a*x + b*x^2)^(3/2))/(4*b) - (3*a^2*c^2*d*log((a/2 + b*x)/b^(1/2) + (a*
x + b*x^2)^(1/2)))/(8*b^(3/2)) + (3*a^3*c*d^2*log((a + 2*b*x)/b^(1/2) + 2*
(a*x + b*x^2)^(1/2)))/(16*b^(5/2)) + (c*d^2*(a*x + b*x^2)^(1/2)*(8*b^2*x^2
 - 3*a^2 + 2*a*b*x))/(8*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.39

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x}\,dx$$
$$= \frac{15\sqrt{x}\,\sqrt{bx+a}\,a^3b\,d^3 - 72\sqrt{x}\,\sqrt{bx+a}\,a^2b^2c\,d^2 - 10\sqrt{x}\,\sqrt{bx+a}\,a^2b^2d^3x + 144\sqrt{x}\,\sqrt{bx+a}\,a\,b^3c^2d + 48}{}$$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(1/2)/x,x)
```

output
```
(15*sqrt(x)*sqrt(a + b*x)*a**3*b*d**3 - 72*sqrt(x)*sqrt(a + b*x)*a**2*b**2
*c*d**2 - 10*sqrt(x)*sqrt(a + b*x)*a**2*b**2*d**3*x + 144*sqrt(x)*sqrt(a +
 b*x)*a*b**3*c**2*d + 48*sqrt(x)*sqrt(a + b*x)*a*b**3*c*d**2*x + 8*sqrt(x)
*sqrt(a + b*x)*a*b**3*d**3*x**2 + 192*sqrt(x)*sqrt(a + b*x)*b**4*c**3 + 28
8*sqrt(x)*sqrt(a + b*x)*b**4*c**2*d*x + 192*sqrt(x)*sqrt(a + b*x)*b**4*c*d
**2*x**2 + 48*sqrt(x)*sqrt(a + b*x)*b**4*d**3*x**3 - 15*sqrt(b)*log((sqrt(
a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4*d**3 + 72*sqrt(b)*log((sqrt(a +
b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*b*c*d**2 - 144*sqrt(b)*log((sqrt(a +
 b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*b**2*c**2*d + 192*sqrt(b)*log((sqrt
(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b**3*c**3)/(192*b**4)
```

3.23 $\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 201

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^2}\,dx = -\frac{\left(\frac{a^2d^2(6bc-ad)}{b^2}-8c^2(2bc+3ad)\right)\sqrt{ax+bx^2}}{8a} + \frac{d^3(ax+bx^2)^{3/2}}{3b} - \frac{2c^3(ax+bx^2)^{3/2}}{ax^2} + \frac{d^2(6bc-ad)\,(ax+bx^2)^{3/2}}{4b^2x} - \frac{(a^2d^2(6bc-ad)-8b^2c^2(2bc+3ad))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{8b^{5/2}}$$

output
```
-1/8*(a^2*d^2*(-a*d+6*b*c)/b^2-8*c^2*(3*a*d+2*b*c))*(b*x^2+a*x)^(1/2)/a+1/
3*d^3*(b*x^2+a*x)^(3/2)/b-2*c^3*(b*x^2+a*x)^(3/2)/a/x^2+1/4*d^2*(-a*d+6*b*
c)*(b*x^2+a*x)^(3/2)/b^2/x-1/8*(a^2*d^2*(-a*d+6*b*c)-8*b^2*c^2*(3*a*d+2*b*
c))*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^2}\,dx$$
$$= \frac{\sqrt{x(a+bx)}\left(\sqrt{b}(-3a^2d^3x+2abd^2x(9c+dx)+b^2(-48c^3+72c^2dx+36cd^2x^2+8d^3x^3))+\frac{6(16b^3c^3+24ab^2\ldots}{\ldots}\right.}{24b^{5/2}x}$$

input
```
Integrate[((c + d*x)^3*Sqrt[a*x + b*x^2])/x^2,x]
```

output
```
(Sqrt[x*(a + b*x)]*(Sqrt[b]*(-3*a^2*d^3*x + 2*a*b*d^2*x*(9*c + d*x) + b^2*
(-48*c^3 + 72*c^2*d*x + 36*c*d^2*x^2 + 8*d^3*x^3)) + (6*(16*b^3*c^3 + 24*a
*b^2*c^2*d - 6*a^2*b*c*d^2 + a^3*d^3)*Sqrt[x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-
Sqrt[a] + Sqrt[a + b*x])])/Sqrt[a + b*x]))/(24*b^(5/2)*x)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1262, 27, 2169, 27, 1220, 1131, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}(c+dx)^3}{x^2}\,dx$$

$$\Big\downarrow \text{1262}$$

$$\frac{\int \frac{3\sqrt{bx^2+ax}(2bc^3+6bdxc^2+d^2(6bc-ad)x^2)}{2x^2}dx}{3b}+\frac{d^3(ax+bx^2)^{3/2}}{3b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{bx^2+ax}(2bc^3+6bdxc^2+d^2(6bc-ad)x^2)}{x^2}dx}{2b}+\frac{d^3(ax+bx^2)^{3/2}}{3b}$$

$$\Big\downarrow \text{2169}$$

$$\frac{\frac{\int \frac{\left(8b^2c^3+d\left(24b^2c^2-6abdc+a^2d^2\right)x\right)\sqrt{bx^2+ax}}{2x^2}dx}{2b}+\frac{d^2\left(ax+bx^2\right)^{3/2}(6bc-ad)}{2bx}}{2b}+\frac{d^3\left(ax+bx^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(8b^2c^3+d\left(24b^2c^2-6abdc+a^2d^2\right)x\right)\sqrt{bx^2+ax}}{x^2}dx}{4b}+\frac{d^2\left(ax+bx^2\right)^{3/2}(6bc-ad)}{2bx}}{2b}+\frac{d^3\left(ax+bx^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \; 1220$$

$$\frac{\frac{\frac{\left(a^3d^3-6a^2bcd^2+24ab^2c^2d+16b^3c^3\right)\int \frac{\sqrt{bx^2+ax}}{x}dx}{a}-\frac{16b^2c^3\left(ax+bx^2\right)^{3/2}}{ax^2}}{4b}+\frac{d^2\left(ax+bx^2\right)^{3/2}(6bc-ad)}{2bx}}{2b}+\frac{d^3\left(ax+bx^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \; 1131$$

$$\frac{\frac{\frac{\left(a^3d^3-6a^2bcd^2+24ab^2c^2d+16b^3c^3\right)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+ax}}dx+\sqrt{ax+bx^2}\right)}{a}-\frac{16b^2c^3\left(ax+bx^2\right)^{3/2}}{ax^2}}{4b}+\frac{d^2\left(ax+bx^2\right)^{3/2}(6bc-ad)}{2bx}}{2b}+\frac{d^3\left(ax+bx^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \; 1091$$

$$\frac{\frac{\frac{\left(a^3d^3-6a^2bcd^2+24ab^2c^2d+16b^3c^3\right)\left(a\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}+\sqrt{ax+bx^2}\right)}{a}-\frac{16b^2c^3\left(ax+bx^2\right)^{3/2}}{ax^2}}{4b}+\frac{d^2\left(ax+bx^2\right)^{3/2}(6bc-ad)}{2bx}}{2b}+\frac{d^3\left(ax+bx^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}+\sqrt{ax+bx^2}\right)\left(a^3d^3-6a^2bcd^2+24ab^2c^2d+16b^3c^3\right)}{a}-\frac{16b^2c^3\left(ax+bx^2\right)^{3/2}}{ax^2}}{4b}+\frac{d^2\left(ax+bx^2\right)^{3/2}(6bc-ad)}{2bx}}{2b}+\frac{d^3\left(ax+bx^2\right)^{3/2}}{3b}$$

input
```
Int[((c + d*x)^3*Sqrt[a*x + b*x^2])/x^2,x]
```

output
```
(d^3*(a*x + b*x^2)^(3/2))/(3*b) + ((d^2*(6*b*c - a*d)*(a*x + b*x^2)^(3/2))
/(2*b*x) + ((-16*b^2*c^3*(a*x + b*x^2)^(3/2))/(a*x^2) + ((16*b^3*c^3 + 24*
a*b^2*c^2*d - 6*a^2*b*c*d^2 + a^3*d^3)*(Sqrt[a*x + b*x^2] + (a*ArcTanh[(Sq
rt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]))/a)/(4*b))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

rule 2169
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d -
b*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2
, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$\frac{x\left(a^{3} d^{3}-6 a^{2} b c\, d^{2}+24 a\, b^{2} c^{2} d+16 b^{3} c^{3}\right) \operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)-\left(\left(-\frac{8}{3} d^{3} x^{3}-12 c\, d^{2} x^{2}-24 c^{2} d x+16 c^{3}\right) b^{\frac{5}{2}}+d^{2} x\left(\left(-\frac{2 d x}{3}-6 c\right)\right.\right.}{8 b^{\frac{5}{2}} x}$
risch	$-\frac{(bx+a)\left(-8 b^{2} d^{3} x^{3}-2 a b\, d^{3} x^{2}-36 b^{2} c\, d^{2} x^{2}+3 a^{2} d^{3} x-18 a b c\, d^{2} x-72 b^{2} c^{2} d x+48 c^{3} b^{2}\right)}{24 b^{2} \sqrt{x(bx+a)}}+\frac{\left(a^{3} d^{3}-6 a^{2} b c\, d^{2}+24 a\, b^{2} c^{2} d+16\right.}{16 b}$
default	$c^{3}\left(-\frac{2\left(b\,x^{2}+a x\right)^{\frac{3}{2}}}{a\,x^{2}}+\frac{2 b\left(\sqrt{b\,x^{2}+a x}+\frac{a \ln\left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\,x^{2}+a x}\right)}{2\sqrt{b}}\right)}{a}\right)+d^{3}\left(\frac{\left(b\,x^{2}+a x\right)^{\frac{3}{2}}}{3 b}-\frac{a\left(\frac{(2 b x+a)\sqrt{b\,x^{2}+a x}}{4 b}-\right.}{}\right.$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/8/b^(5/2)*(x*(a^3*d^3-6*a^2*b*c*d^2+24*a*b^2*c^2*d+16*b^3*c^3)*arctanh((
x*(b*x+a))^(1/2)/x/b^(1/2))-((-8/3*d^3*x^3-12*c*d^2*x^2-24*c^2*d*x+16*c^3)
*b^(5/2)+d^2*x*((-2/3*d*x-6*c)*b^(3/2)+b^(1/2)*a*d)*a)*(x*(b*x+a))^(1/2))/
x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.64

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^2}\,dx$$
$$= \left[\frac{3\left(16\,b^3c^3+24\,ab^2c^2d-6\,a^2bcd^2+a^3d^3\right)\sqrt{b}x\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\left(8\,b^3d^3x^3-48\,b^3c^3\right.}{48\,b^3x}\right.$$
$$-\frac{3\left(16\,b^3c^3+24\,ab^2c^2d-6\,a^2bcd^2+a^3d^3\right)\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-\left(8\,b^3d^3x^3-48\,b^3c^3+2\left(18\,b^3c\right.\right.}{24\,b^3x}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/48*(3*(16*b^3*c^3 + 24*a*b^2*c^2*d - 6*a^2*b*c*d^2 + a^3*d^3)*sqrt(b)*x
*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(8*b^3*d^3*x^3 - 48*b^3*
c^3 + 2*(18*b^3*c*d^2 + a*b^2*d^3)*x^2 + 3*(24*b^3*c^2*d + 6*a*b^2*c*d^2 -
 a^2*b*d^3)*x)*sqrt(b*x^2 + a*x))/(b^3*x), -1/24*(3*(16*b^3*c^3 + 24*a*b^2
*c^2*d - 6*a^2*b*c*d^2 + a^3*d^3)*sqrt(-b)*x*arctan(sqrt(b*x^2 + a*x)*sqrt
(-b)/(b*x + a)) - (8*b^3*d^3*x^3 - 48*b^3*c^3 + 2*(18*b^3*c*d^2 + a*b^2*d^
3)*x^2 + 3*(24*b^3*c^2*d + 6*a*b^2*c*d^2 - a^2*b*d^3)*x)*sqrt(b*x^2 + a*x)
)/(b^3*x)]
```

Sympy [F]

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^2}\,dx = \int \frac{\sqrt{x\,(a+bx)}(c+dx)^3}{x^2}\,dx$$

input
```
integrate((d*x+c)**3*(b*x**2+a*x)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(x*(a + b*x))*(c + d*x)**3/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.34

$$\begin{aligned}\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^2}\,dx &= \frac{3}{2}\sqrt{bx^2+ax}cd^2x - \frac{\sqrt{bx^2+ax}ad^3x}{4\,b} \\ &+ \sqrt{b}c^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) \\ &+ \frac{3\,ac^2d\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{2\,\sqrt{b}} \\ &- \frac{3\,a^2cd^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{3}{2}}} \\ &+ \frac{a^3d^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{5}{2}}} \\ &+ 3\,\sqrt{bx^2+ax}c^2d + \frac{3\,\sqrt{bx^2+ax}acd^2}{4\,b} \\ &- \frac{\sqrt{bx^2+ax}a^2d^3}{8\,b^2} + \frac{(bx^2+ax)^{\frac{3}{2}}d^3}{3\,b} - \frac{2\,\sqrt{bx^2+ax}c^3}{x}\end{aligned}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
3/2*sqrt(b*x^2 + a*x)*c*d^2*x - 1/4*sqrt(b*x^2 + a*x)*a*d^3*x/b + sqrt(b)*
c^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 3/2*a*c^2*d*log(2*b*x +
 a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b) - 3/8*a^2*c*d^2*log(2*b*x + a +
2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(3/2) + 1/16*a^3*d^3*log(2*b*x + a + 2*sqrt
(b*x^2 + a*x)*sqrt(b))/b^(5/2) + 3*sqrt(b*x^2 + a*x)*c^2*d + 3/4*sqrt(b*x^
2 + a*x)*a*c*d^2/b - 1/8*sqrt(b*x^2 + a*x)*a^2*d^3/b^2 + 1/3*(b*x^2 + a*x)
^(3/2)*d^3/b - 2*sqrt(b*x^2 + a*x)*c^3/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^2}\,dx$$
$$= \frac{2\,ac^3}{\sqrt{b}x-\sqrt{bx^2+ax}} + \frac{1}{24}\sqrt{bx^2+ax}\left(2\left(4\,d^3x+\frac{18\,b^2cd^2+abd^3}{b^2}\right)x+\frac{3\,(24\,b^2c^2d+6\,abcd^2-a^2d^3)}{b^2}\right) - \frac{(16\,b^3c^3+24\,ab^2c^2d-6\,a^2bcd^2+a^3d^3)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{16\,b^{\frac{5}{2}}}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^2,x, algorithm="giac")
```

output
```
2*a*c^3/(sqrt(b)*x - sqrt(b*x^2 + a*x)) + 1/24*sqrt(b*x^2 + a*x)*(2*(4*d^3
*x + (18*b^2*c*d^2 + a*b*d^3)/b^2)*x + 3*(24*b^2*c^2*d + 6*a*b*c*d^2 - a^2
*d^3)/b^2) - 1/16*(16*b^3*c^3 + 24*a*b^2*c^2*d - 6*a^2*b*c*d^2 + a^3*d^3)*
log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^2}\,dx = \int \frac{\sqrt{b\,x^2+a\,x}\,(c+d\,x)^3}{x^2}\,dx$$

input
```
int(((a*x + b*x^2)^(1/2)*(c + d*x)^3)/x^2,x)
```

output
```
int(((a*x + b*x^2)^(1/2)*(c + d*x)^3)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.46

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^2}\,dx$$
$$= \frac{-24\sqrt{x}\,\sqrt{bx+a}\,a^2b\,d^3x+144\sqrt{x}\,\sqrt{bx+a}\,a\,b^2c\,d^2x+16\sqrt{x}\,\sqrt{bx+a}\,a\,b^2d^3x^2-384\sqrt{x}\,\sqrt{bx+a}\,b^3c^3+}{}$$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^2,x)
```

output
```
( - 24*sqrt(x)*sqrt(a + b*x)*a**2*b*d**3*x + 144*sqrt(x)*sqrt(a + b*x)*a*b
**2*c*d**2*x + 16*sqrt(x)*sqrt(a + b*x)*a*b**2*d**3*x**2 - 384*sqrt(x)*sqr
t(a + b*x)*b**3*c**3 + 576*sqrt(x)*sqrt(a + b*x)*b**3*c**2*d*x + 288*sqrt(
x)*sqrt(a + b*x)*b**3*c*d**2*x**2 + 64*sqrt(x)*sqrt(a + b*x)*b**3*d**3*x**
3 + 24*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*d**3*x
- 144*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*b*c*d**2
*x + 576*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b**2*c**
2*d*x + 384*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b**3*c*
*3*x + 3*sqrt(b)*a**3*d**3*x - 144*sqrt(b)*a*b**2*c**2*d*x - 384*sqrt(b)*b
**3*c**3*x)/(192*b**3*x)
```

3.24 $\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 155

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^3}\,dx = \frac{d^2(12bc+ad)\sqrt{ax+bx^2}}{4b} - \frac{6c^2d\sqrt{ax+bx^2}}{x} + \frac{1}{2}d^3x\sqrt{ax+bx^2} - \frac{2c^3(ax+bx^2)^{3/2}}{3ax^3} + \frac{d(24b^2c^2+12abcd-a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}$$

output
```
1/4*d^2*(a*d+12*b*c)*(b*x^2+a*x)^(1/2)/b-6*c^2*d*(b*x^2+a*x)^(1/2)/x+1/2*d
^3*x*(b*x^2+a*x)^(1/2)-2/3*c^3*(b*x^2+a*x)^(3/2)/a/x^3+1/4*d*(-a^2*d^2+12*
a*b*c*d+24*b^2*c^2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^3}\,dx = \frac{\sqrt{x(a+bx)}\left(\sqrt{b}\sqrt{a+bx}(-8b^2c^3x+3a^2d^3x^2+2ab(-4c^3-36c^2dx+18cd^2x^2+3d^3x^3))+3ad(-24b^2c^2\right.}{12ab^{3/2}x^2\sqrt{a+bx}}$$

input
```
Integrate[((c + d*x)^3*Sqrt[a*x + b*x^2])/x^3,x]
```

output
```
(Sqrt[x*(a + b*x)]*(Sqrt[b]*Sqrt[a + b*x]*(-8*b^2*c^3*x + 3*a^2*d^3*x^2 +
2*a*b*(-4*c^3 - 36*c^2*d*x + 18*c*d^2*x^2 + 3*d^3*x^3)) + 3*a*d*(-24*b^2*c
^2 - 12*a*b*c*d + a^2*d^2)*x^(3/2)*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]
))/(12*a*b^(3/2)*x^2*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1262, 27, 2169, 27, 1220, 1125, 25, 27, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}(c+dx)^3}{x^3} \, dx$$

$$\Big\downarrow \text{1262}$$

$$\frac{\int \frac{\sqrt{bx^2+ax}(4bc^3+12bdxc^2+d^2(12bc-ad)x^2)}{2x^3}dx}{2b}+\frac{d^3\left(ax+bx^2\right)^{3/2}}{2bx}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{bx^2+ax}(4bc^3+12bdxc^2+d^2(12bc-ad)x^2)}{x^3}dx}{4b}+\frac{d^3\left(ax+bx^2\right)^{3/2}}{2bx}$$

$$\Big\downarrow \text{2169}$$

$$\frac{\frac{\int \frac{\left(8b^2c^3+d\left(24b^2c^2+12abdc-a^2d^2\right)x\right)\sqrt{bx^2+ax}}{2x^3}dx}{b}+\frac{d^2\left(ax+bx^2\right)^{3/2}(12bc-ad)}{bx^2}}{4b}+\frac{d^3\left(ax+bx^2\right)^{3/2}}{2bx}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\left(8b^2c^3+d\left(24b^2c^2+12abdc-a^2d^2\right)x\right)\sqrt{bx^2+ax}}{x^3}dx}{2b}+\frac{d^2\left(ax+bx^2\right)^{3/2}(12bc-ad)}{bx^2}}{4b}+\frac{d^3\left(ax+bx^2\right)^{3/2}}{2bx}$$

$$\Big\downarrow \text{1220}$$

$$\frac{\frac{d(-a^2d^2+12abcd+24b^2c^2)\int \frac{\sqrt{bx^2+ax}}{x^2}dx-\frac{16b^2c^3\left(ax+bx^2\right)^{3/2}}{3ax^3}}{2b}+\frac{d^2\left(ax+bx^2\right)^{3/2}(12bc-ad)}{bx^2}}{4b}+\frac{d^3\left(ax+bx^2\right)^{3/2}}{2bx}$$

$$\Big\downarrow \text{ 1125}$$

$$\frac{\frac{d(-a^2d^2+12abcd+24b^2c^2)\left(-\int -\frac{b}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{16b^2c^3\left(ax+bx^2\right)^{3/2}}{3ax^3}}{2b}+\frac{d^2\left(ax+bx^2\right)^{3/2}(12bc-ad)}{bx^2}}{4b}+\frac{d^3\left(ax+bx^2\right)^{3/2}}{2bx}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{d(-a^2d^2+12abcd+24b^2c^2)\left(\int \frac{b}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{16b^2c^3\left(ax+bx^2\right)^{3/2}}{3ax^3}}{2b}+\frac{d^2\left(ax+bx^2\right)^{3/2}(12bc-ad)}{bx^2}}{4b}+\frac{d^3\left(ax+bx^2\right)^{3/2}}{2bx}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{d(-a^2d^2+12abcd+24b^2c^2)\left(b\int \frac{1}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{16b^2c^3\left(ax+bx^2\right)^{3/2}}{3ax^3}}{2b}+\frac{d^2\left(ax+bx^2\right)^{3/2}(12bc-ad)}{bx^2}}{4b}+\frac{d^3\left(ax+bx^2\right)^{3/2}}{2bx}$$

$$\Big\downarrow \text{ 1091}$$

$$\frac{\frac{d(-a^2d^2+12abcd+24b^2c^2)\left(2b\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{16b^2c^3\left(ax+bx^2\right)^{3/2}}{3ax^3}}{2b}+\frac{d^2\left(ax+bx^2\right)^{3/2}(12bc-ad)}{bx^2}}{4b}+\frac{d^3\left(ax+bx^2\right)^{3/2}}{2bx}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{d\left(2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)-\frac{2\sqrt{ax+bx^2}}{x}\right)(-a^2d^2+12abcd+24b^2c^2)-\frac{16b^2c^3\left(ax+bx^2\right)^{3/2}}{3ax^3}}{2b}+\frac{d^2\left(ax+bx^2\right)^{3/2}(12bc-ad)}{bx^2}}{4b}+\frac{d^3\left(ax+bx^2\right)^{3/2}}{2bx}$$

input
```
Int[((c + d*x)^3*Sqrt[a*x + b*x^2])/x^3,x]
```

output
```
(d^3*(a*x + b*x^2)^(3/2))/(2*b*x) + ((d^2*(12*b*c - a*d)*(a*x + b*x^2)^(3/
2))/(b*x^2) + ((-16*b^2*c^3*(a*x + b*x^2)^(3/2))/(3*a*x^3) + d*(24*b^2*c^2
 + 12*a*b*c*d - a^2*d^2)*((-2*Sqrt[a*x + b*x^2])/x + 2*Sqrt[b]*ArcTanh[(Sq
rt[b]*x)/Sqrt[a*x + b*x^2]]))/(2*b))/(4*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

rule 2169
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d -
b*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2
, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$-\frac{a d\,x^{2} \left(a^{2} d^{2}-12 a b c d-24 b^{2} c^{2}\right) \operatorname{arctanh}\left(\frac{\sqrt{x (b x+a)}}{x \sqrt{b}}\right)-\left(-\frac{8 \left(-\frac{3}{4} d^{3} x^{3}-\frac{9}{2} c\,d^{2} x^{2}+9 c^{2} d x+c^{3}\right) a\,b^{\frac{3}{2}}}{3}+x \left(a^{2} d^{3} x \sqrt{b}-\frac{8 b^{\frac{5}{2}} c^{3}}{3}\right)\right)}{4 b^{\frac{3}{2}} a\,x^{2}}$
risch	$\frac{\left(b x+a\right) \left(6 a b\,d^{3} x^{3}+3 a^{2} d^{3} x^{2}+36 a b c\,d^{2} x^{2}-72 a b\,c^{2} d x-8 c^{3} b^{2} x-8 a b\,c^{3}\right)}{12 a \sqrt{x \left(b x+a\right)}\, x b}-\frac{\left(a^{2} d^{2}-12 a b c d-24 b^{2} c^{2}\right) d \ln\left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\,x^{2}+a x}\right)}{8 b^{\frac{3}{2}}}$
default	$d^{3} \left(\frac{\left(2 b x+a\right) \sqrt{b\,x^{2}+a x}}{4 b}-\frac{a^{2} \ln\left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\,x^{2}+a x}\right)}{8 b^{\frac{3}{2}}}\right)-\frac{2 c^{3} \left(b\,x^{2}+a x\right)^{\frac{3}{2}}}{3 a\,x^{3}}+3 c\,d^{2} \left(\sqrt{b\,x^{2}+a x}+\frac{a \ln\left(\frac{\frac{a}{2}+b x}{\sqrt{b}} \cdots\right.}{\cdots}\right.$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/b^(3/2)*(a*d*x^2*(a^2*d^2-12*a*b*c*d-24*b^2*c^2)*arctanh((x*(b*x+a))^
(1/2)/x/b^(1/2))-(-8/3*(-3/4*d^3*x^3-9/2*c*d^2*x^2+9*c^2*d*x+c^3)*a*b^(3/2
)+x*(a^2*d^3*x*b^(1/2)-8/3*b^(5/2)*c^3))*(x*(b*x+a))^(1/2))/a/x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.99

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^3}\,dx$$
$$= \left[-\frac{3\left(24\,ab^2c^2d+12\,a^2bcd^2-a^3d^3\right)\sqrt{b}x^2\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\left(6\,ab^2d^3x^3-8\,ab^2c^3+3\left(\cdots\right.\right.}{24\,ab^2x^2}\right.$$
$$-\frac{3\left(24\,ab^2c^2d+12\,a^2bcd^2-a^3d^3\right)\sqrt{-b}x^2\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-\left(6\,ab^2d^3x^3-8\,ab^2c^3+3\left(12\,ab^2cd^2+\cdots\right.\right.}{12\,ab^2x^2}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[-1/24*(3*(24*a*b^2*c^2*d + 12*a^2*b*c*d^2 - a^3*d^3)*sqrt(b)*x^2*log(2*b*
x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*(6*a*b^2*d^3*x^3 - 8*a*b^2*c^3 +
3*(12*a*b^2*c*d^2 + a^2*b*d^3)*x^2 - 8*(b^3*c^3 + 9*a*b^2*c^2*d)*x)*sqrt(b
*x^2 + a*x))/(a*b^2*x^2), -1/12*(3*(24*a*b^2*c^2*d + 12*a^2*b*c*d^2 - a^3*
d^3)*sqrt(-b)*x^2*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (6*a*b^2*
d^3*x^3 - 8*a*b^2*c^3 + 3*(12*a*b^2*c*d^2 + a^2*b*d^3)*x^2 - 8*(b^3*c^3 +
9*a*b^2*c^2*d)*x)*sqrt(b*x^2 + a*x))/(a*b^2*x^2)]
```

Sympy [F]

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^3}\,dx = \int \frac{\sqrt{x\,(a+bx)}\,(c+dx)^3}{x^3}\,dx$$

input
```
integrate((d*x+c)**3*(b*x**2+a*x)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(x*(a + b*x))*(c + d*x)**3/x**3, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.36

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^3}\,dx = \frac{1}{4}\left(2\,d^3x+\frac{12\,bcd^2+ad^3}{b}\right)\sqrt{bx^2+ax} - \frac{\left(24\,b^2c^2d+12\,abcd^2-a^2d^3\right)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,b^{\frac{3}{2}}} + \frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2bc^3+9\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2ac^2d+3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)a\sqrt{b}c^3+a^2c^3\right)}{3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^3,x, algorithm="giac")
```

output
```
1/4*(2*d^3*x + (12*b*c*d^2 + a*d^3)/b)*sqrt(b*x^2 + a*x) - 1/8*(24*b^2*c^2
*d + 12*a*b*c*d^2 - a^2*d^3)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqr
t(b) + a))/b^(3/2) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*b*c^3 + 9*(s
qrt(b)*x - sqrt(b*x^2 + a*x))^2*a*c^2*d + 3*(sqrt(b)*x - sqrt(b*x^2 + a*x)
)*a*sqrt(b)*c^3 + a^2*c^3)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^3}\,dx = \int \frac{\sqrt{b\,x^2+a\,x}\,{\left(c+d\,x\right)}^3}{x^3}\,dx$$

input
```
int(((a*x + b*x^2)^(1/2)*(c + d*x)^3)/x^3,x)
```

output
```
int(((a*x + b*x^2)^(1/2)*(c + d*x)^3)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.80

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^3}\,dx$$
$$= \frac{24\sqrt{x}\,\sqrt{bx+a}\,a^2b\,d^3x^2-64\sqrt{x}\,\sqrt{bx+a}\,a\,b^2c^3-576\sqrt{x}\,\sqrt{bx+a}\,a\,b^2c^2dx+288\sqrt{x}\,\sqrt{bx+a}\,a\,b^2c\,d^2x^2 +}{}$$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^3,x)
```

output
```
(24*sqrt(x)*sqrt(a + b*x)*a**2*b*d**3*x**2 - 64*sqrt(x)*sqrt(a + b*x)*a*b*
*2*c**3 - 576*sqrt(x)*sqrt(a + b*x)*a*b**2*c**2*d*x + 288*sqrt(x)*sqrt(a +
 b*x)*a*b**2*c*d**2*x**2 + 48*sqrt(x)*sqrt(a + b*x)*a*b**2*d**3*x**3 - 64*
sqrt(x)*sqrt(a + b*x)*b**3*c**3*x - 24*sqrt(b)*log((sqrt(a + b*x) + sqrt(x
)*sqrt(b))/sqrt(a))*a**3*d**3*x**2 + 288*sqrt(b)*log((sqrt(a + b*x) + sqrt
(x)*sqrt(b))/sqrt(a))*a**2*b*c*d**2*x**2 + 576*sqrt(b)*log((sqrt(a + b*x)
+ sqrt(x)*sqrt(b))/sqrt(a))*a*b**2*c**2*d*x**2 - sqrt(b)*a**3*d**3*x**2 +
48*sqrt(b)*a**2*b*c*d**2*x**2 + 192*sqrt(b)*a*b**2*c**2*d*x**2 - 64*sqrt(b
)*b**3*c**3*x**2)/(96*a*b**2*x**2)
```

3.25 $\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 139

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^4}\,dx = d^3\sqrt{ax+bx^2} - \frac{6cd^2\sqrt{ax+bx^2}}{x} - \frac{2c^3(ax+bx^2)^{3/2}}{5ax^4} + \frac{2c^2(2bc-15ad)\,(ax+bx^2)^{3/2}}{15a^2x^3} + \frac{d^2(6bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

output
```
d^3*(b*x^2+a*x)^(1/2)-6*c*d^2*(b*x^2+a*x)^(1/2)/x-2/5*c^3*(b*x^2+a*x)^(3/2
)/a/x^4+2/15*c^2*(-15*a*d+2*b*c)*(b*x^2+a*x)^(3/2)/a^2/x^3+d^2*(a*d+6*b*c)
*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^4}\,dx$$
$$= \frac{\sqrt{x(a+bx)}\left(-90cd^2x^2+15d^3x^3-\frac{30c^2dx(a+bx)}{a}+c^3\left(-6-\frac{2bx}{a}+\frac{4b^2x^2}{a^2}\right)-\frac{15d^2(6bc+ad)x^{5/2}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right.}{\sqrt{b}\sqrt{a+bx}}\right.}{15x^3}$$

input
```
Integrate[((c + d*x)^3*Sqrt[a*x + b*x^2])/x^4,x]
```

output
```
(Sqrt[x*(a + b*x)]*(-90*c*d^2*x^2 + 15*d^3*x^3 - (30*c^2*d*x*(a + b*x))/a
+ c^3*(-6 - (2*b*x)/a + (4*b^2*x^2)/a^2) - (15*d^2*(6*b*c + a*d)*x^(5/2)*L
og[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]*Sqrt[a + b*x])))/(15*x^3)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1262, 27, 2167, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}(c+dx)^3}{x^4}\,dx$$

$$\Big\downarrow \text{1262}$$

$$\frac{\int \frac{\sqrt{bx^2+ax}(2bc^3+6bdxc^2+d^2(6bc+ad)x^2)}{2x^4}dx}{b}+\frac{d^3\left(ax+bx^2\right)^{3/2}}{bx^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{bx^2+ax}(2bc^3+6bdxc^2+d^2(6bc+ad)x^2)}{x^4}dx}{2b}+\frac{d^3\left(ax+bx^2\right)^{3/2}}{bx^2}$$

$$\Big\downarrow \text{2167}$$

$$\frac{\int \left(\frac{2b\sqrt{bx^2+ax}c^3}{x^4}+\frac{6bd\sqrt{bx^2+ax}c^2}{x^3}+\frac{d^2(6bc+ad)\sqrt{bx^2+ax}}{x^2}\right)dx}{2b}+\frac{d^3\left(ax+bx^2\right)^{3/2}}{bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{8b^2c^3\left(ax+bx^2\right)^{3/2}}{15a^2x^3}+2\sqrt{b}d^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(ad+6bc)-\frac{4bc^3\left(ax+bx^2\right)^{3/2}}{5ax^4}-\frac{4bc^2d\left(ax+bx^2\right)^{3/2}}{ax^3}-\frac{2d^2\sqrt{ax+bx^2}(ad+6bc)}{x}}{2b}+\frac{d^3\left(ax+bx^2\right)^{3/2}}{bx^2}$$

input
```
Int[((c + d*x)^3*Sqrt[a*x + b*x^2])/x^4,x]
```

output
```
(d^3*(a*x + b*x^2)^(3/2))/(b*x^2) + ((-2*d^2*(6*b*c + a*d)*Sqrt[a*x + b*x^
2])/x - (4*b*c^3*(a*x + b*x^2)^(3/2))/(5*a*x^4) + (8*b^2*c^3*(a*x + b*x^2)
^(3/2))/(15*a^2*x^3) - (4*b*c^2*d*(a*x + b*x^2)^(3/2))/(a*x^3) + 2*Sqrt[b]
*d^2*(6*b*c + a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2167
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*x + c*x^2)^p, (d + e*x)^m*Pq, x]
, x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&
 EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + Expon[Pq, x] + 2*p + 1, 0] && ILt
Q[m, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$\frac{a^2 d^2 x^3 (ad+6bc)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)-\frac{2\left(\frac{a c^2 x(15dx+c) b^{\frac{3}{2}}}{3}-\frac{2 b^{\frac{5}{2}} c^3 x^2}{3}+\sqrt{b}\, a^2\left(-\frac{5}{2} d^3 x^3+15 c\, d^2 x^2+5 c^2 d x+c^3\right)\right)\sqrt{x(bx+a)}}{5}}{\sqrt{b}\, x^3 a^2}$
risch	$-\frac{(bx+a)\left(-15a^2d^3x^3+90a^2c\,d^2x^2+30ab\,c^2x^2d-4b^2c^3x^2+30a^2c^2dx+2ab\,c^3x+6a^2c^3\right)}{15x^2\sqrt{x(bx+a)}\,a^2}+\frac{(ad+6bc)d^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2}\right.}{2\sqrt{b}}$
default	$c^3\left(-\frac{2(b\,x^2+ax)^{\frac{3}{2}}}{5a\,x^4}+\frac{4b(b\,x^2+ax)^{\frac{3}{2}}}{15a^2x^3}\right)+d^3\left(\sqrt{b\,x^2+ax}+\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2\sqrt{b}}\right)+3c\,d^2\left(-\frac{2(b\right.$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/b^(1/2)*(a^2*d^2*x^3*(a*d+6*b*c)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-2/
5*(1/3*a*c^2*x*(15*d*x+c)*b^(3/2)-2/3*b^(5/2)*c^3*x^2+b^(1/2)*a^2*(-5/2*d^
3*x^3+15*c*d^2*x^2+5*c^2*d*x+c^3))*(x*(b*x+a))^(1/2))/x^3/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.22

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^4}\,dx$$
$$= \left[\frac{15\left(6\,a^2bcd^2+a^3d^3\right)\sqrt{b}x^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\left(15\,a^2bd^3x^3-6\,a^2bc^3+2\left(2\,b^3c^3-15\,a\right.\right.}{30\,a^2bx^3}\right.$$
$$-\frac{15\left(6\,a^2bcd^2+a^3d^3\right)\sqrt{-b}x^3\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-\left(15\,a^2bd^3x^3-6\,a^2bc^3+2\left(2\,b^3c^3-15\,ab^2c^2d-45\right.\right.}{15\,a^2bx^3}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[1/30*(15*(6*a^2*b*c*d^2 + a^3*d^3)*sqrt(b)*x^3*log(2*b*x + a + 2*sqrt(b*x
^2 + a*x)*sqrt(b)) + 2*(15*a^2*b*d^3*x^3 - 6*a^2*b*c^3 + 2*(2*b^3*c^3 - 15
*a*b^2*c^2*d - 45*a^2*b*c*d^2)*x^2 - 2*(a*b^2*c^3 + 15*a^2*b*c^2*d)*x)*sqr
t(b*x^2 + a*x))/(a^2*b*x^3), -1/15*(15*(6*a^2*b*c*d^2 + a^3*d^3)*sqrt(-b)*
x^3*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (15*a^2*b*d^3*x^3 - 6*a
^2*b*c^3 + 2*(2*b^3*c^3 - 15*a*b^2*c^2*d - 45*a^2*b*c*d^2)*x^2 - 2*(a*b^2*
c^3 + 15*a^2*b*c^2*d)*x)*sqrt(b*x^2 + a*x))/(a^2*b*x^3)]
```

Sympy [F]

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^4}\,dx = \int \frac{\sqrt{x\,(a+bx)}(c+dx)^3}{x^4}\,dx$$

input
```
integrate((d*x+c)**3*(b*x**2+a*x)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(x*(a + b*x))*(c + d*x)**3/x**4, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(121) = 242$.

Time = 0.32 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.14

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^4}\,dx$$
$$= \sqrt{bx^2+ax}d^3 - \frac{(6\,bcd^2+ad^3)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{2\sqrt{b}}$$
$$+ \frac{2\left(45\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4bc^2d+45\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4acd^2+15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3b^{\frac{3}{2}}c^3+45\right.}{}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^4,x, algorithm="giac")
```

output
```
sqrt(b*x^2 + a*x)*d^3 - 1/2*(6*b*c*d^2 + a*d^3)*log(abs(2*(sqrt(b)*x - sqr
t(b*x^2 + a*x))*sqrt(b) + a))/sqrt(b) + 2/15*(45*(sqrt(b)*x - sqrt(b*x^2 +
 a*x))^4*b*c^2*d + 45*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a*c*d^2 + 15*(sqrt
(b)*x - sqrt(b*x^2 + a*x))^3*b^(3/2)*c^3 + 45*(sqrt(b)*x - sqrt(b*x^2 + a*
x))^3*a*sqrt(b)*c^2*d + 25*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a*b*c^3 + 15*
(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2*c^2*d + 15*(sqrt(b)*x - sqrt(b*x^2 +
 a*x))*a^2*sqrt(b)*c^3 + 3*a^3*c^3)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^4}\,dx = \int \frac{\sqrt{b\,x^2+a\,x}\,{(c+d\,x)}^3}{x^4}\,dx$$

input
```
int(((a*x + b*x^2)^(1/2)*(c + d*x)^3)/x^4,x)
```

output
```
int(((a*x + b*x^2)^(1/2)*(c + d*x)^3)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.92

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^4}\,dx$$
$$= \frac{-24\sqrt{x}\,\sqrt{bx+a}\,a^2b\,c^3-120\sqrt{x}\,\sqrt{bx+a}\,a^2b\,c^2dx-360\sqrt{x}\,\sqrt{bx+a}\,a^2bc\,d^2x^2+60\sqrt{x}\,\sqrt{bx+a}\,a^2b\,d^3x^3}{}$$

input

```
int((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^4,x)
```

output

```
( - 24*sqrt(x)*sqrt(a + b*x)*a**2*b*c**3 - 120*sqrt(x)*sqrt(a + b*x)*a**2*
b*c**2*d*x - 360*sqrt(x)*sqrt(a + b*x)*a**2*b*c*d**2*x**2 + 60*sqrt(x)*sqr
t(a + b*x)*a**2*b*d**3*x**3 - 8*sqrt(x)*sqrt(a + b*x)*a*b**2*c**3*x - 120*
sqrt(x)*sqrt(a + b*x)*a*b**2*c**2*d*x**2 + 16*sqrt(x)*sqrt(a + b*x)*b**3*c
**3*x**2 + 60*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*
d**3*x**3 + 360*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**
2*b*c*d**2*x**3 + 27*sqrt(b)*a**3*d**3*x**3 + 216*sqrt(b)*a**2*b*c*d**2*x*
*3 - 24*sqrt(b)*a*b**2*c**2*d*x**3 - 16*sqrt(b)*b**3*c**3*x**3)/(60*a**2*b
*x**3)
```

3.26 $\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 161

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^5}\,dx = -\frac{2d^3\sqrt{ax+bx^2}}{x}-\frac{2c^3(ax+bx^2)^{3/2}}{7ax^5}+\frac{2c^2(4bc-21ad)\,(ax+bx^2)^{3/2}}{35a^2x^4}-\frac{2c(8b^2c^2-42abcd+105a^2d^2)\,(ax+bx^2)^{3/2}}{105a^3x^3}+2\sqrt{b}d^3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)$$

output

```
-2*d^3*(b*x^2+a*x)^(1/2)/x-2/7*c^3*(b*x^2+a*x)^(3/2)/a/x^5+2/35*c^2*(-21*a
*d+4*b*c)*(b*x^2+a*x)^(3/2)/a^2/x^4-2/105*c*(105*a^2*d^2-42*a*b*c*d+8*b^2*
c^2)*(b*x^2+a*x)^(3/2)/a^3/x^3+2*b^(1/2)*d^3*arctanh(b^(1/2)*x/(b*x^2+a*x)
^(1/2))
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^5}\,dx$$
$$= \frac{2\sqrt{x(a+bx)}\left(-\frac{8b^3c^3x^3}{a^3}+\frac{2b^2c^2x^2(2c+21dx)}{a^2}-\frac{3bcx(c^2+7cdx+35d^2x^2)}{a}-3(5c^3+21c^2dx+35cd^2x^2+35d^3x^3)-\right.}{105x^4}$$

input
```
Integrate[((c + d*x)^3*Sqrt[a*x + b*x^2])/x^5,x]
```

output
```
(2*Sqrt[x*(a + b*x)]*((-8*b^3*c^3*x^3)/a^3 + (2*b^2*c^2*x^2*(2*c + 21*d*x)
)/a^2 - (3*b*c*x*(c^2 + 7*c*d*x + 35*d^2*x^2))/a - 3*(5*c^3 + 21*c^2*d*x +
 35*c*d^2*x^2 + 35*d^3*x^3) - (105*Sqrt[b]*d^3*x^(7/2)*Log[-(Sqrt[b]*Sqrt[
x]) + Sqrt[a + b*x]])/Sqrt[a + b*x]))/(105*x^4)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1290}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}(c+dx)^3}{x^5}\,dx$$
$$\Big\downarrow \; 1290$$
$$\text{Indeterminate}$$

input
```
Int[((c + d*x)^3*Sqrt[a*x + b*x^2])/x^5,x]
```

output
```
Indeterminate
```

Defintions of rubi rules used

rule 1290
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(f + g*x)^
n, d + e*x, x], R = PolynomialRemainder[(f + g*x)^n, d + e*x, x]}, Simp[(e*
R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*
e^2)), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1
)*(a + b*x + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R
*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 1] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$\frac{2a^3\sqrt{b}\,d^3\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)x^4-\frac{2\sqrt{x(bx+a)}\left(\left(7d^3x^3+7c\,d^2x^2+\frac{21}{5}c^2dx+c^3\right)a^3+\frac{bcx\left(35d^2x^2+7cdx+c^2\right)a^2}{5}-\frac{4x^2b^2c^2\left(\frac{21dx}{2}+\right)}{15}\right)}{7}}{a^3x^4}$
risch	$-\frac{2(bx+a)(105a^3d^3x^3+105a^2bc\,d^2x^3-42a\,b^2c^2d\,x^3+8b^3c^3x^3+105a^3c\,d^2x^2+21a^2b\,c^2x^2d-4a\,b^2c^3x^2+63a^3c^2dx+3a^2b\,c^3)}{105x^3\sqrt{x(bx+a)}\,a^3}$
default	$c^3\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{7a\,x^5}-\frac{4b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{5a\,x^4}+\frac{4b\left(b\,x^2+ax\right)^{\frac{3}{2}}}{15a^2x^3}\right)}{7a}\right)+d^3\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{a\,x^2}+\frac{2b\left(\sqrt{b\,x^2+ax}+\frac{a\ln\left(\frac{a}{2}\right)}{}\right)}{a}\right)$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
2/7*(7*a^3*b^(1/2)*d^3*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*x^4-(x*(b*x+a)
)^(1/2)*((7*d^3*x^3+7*c*d^2*x^2+21/5*c^2*d*x+c^3)*a^3+1/5*b*c*x*(35*d^2*x^
2+7*c*d*x+c^2)*a^2-4/15*x^2*b^2*c^2*(21/2*d*x+c)*a+8/15*b^3*c^3*x^3))/a^3/
x^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.06

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^5}\,dx = \left[\frac{105\,a^3\sqrt{b}d^3x^4\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,(15\,a^3c^3+(8\,b^3c^3-42\,ab^2c^2d+105\,a^2bcd^2+105\,a\right.}{105\,a^3x^4}$$

$$-\frac{2\left(105\,a^3\sqrt{-b}d^3x^4\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+(15\,a^3c^3+(8\,b^3c^3-42\,ab^2c^2d+105\,a^2bcd^2+105\,a^3d^3)x^3\right.}{105\,a^3x^4}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
[1/105*(105*a^3*sqrt(b)*d^3*x^4*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b
)) - 2*(15*a^3*c^3 + (8*b^3*c^3 - 42*a*b^2*c^2*d + 105*a^2*b*c*d^2 + 105*a
^3*d^3)*x^3 - (4*a*b^2*c^3 - 21*a^2*b*c^2*d - 105*a^3*c*d^2)*x^2 + 3*(a^2*
b*c^3 + 21*a^3*c^2*d)*x)*sqrt(b*x^2 + a*x))/(a^3*x^4), -2/105*(105*a^3*sqr
t(-b)*d^3*x^4*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (15*a^3*c^3 +
 (8*b^3*c^3 - 42*a*b^2*c^2*d + 105*a^2*b*c*d^2 + 105*a^3*d^3)*x^3 - (4*a*b
^2*c^3 - 21*a^2*b*c^2*d - 105*a^3*c*d^2)*x^2 + 3*(a^2*b*c^3 + 21*a^3*c^2*d
)*x)*sqrt(b*x^2 + a*x))/(a^3*x^4)]
```

Sympy [F]

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^5}\,dx = \int \frac{\sqrt{x\,(a+bx)}(c+dx)^3}{x^5}\,dx$$

input
```
integrate((d*x+c)**3*(b*x**2+a*x)**(1/2)/x**5,x)
```

output
```
Integral(sqrt(x*(a + b*x))*(c + d*x)**3/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.50

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^5}\,dx$$
$$= \left(\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-\frac{2\,\sqrt{bx^2+ax}}{x}\right)d^3$$
$$-2\,cd^2\left(\frac{\sqrt{bx^2+ax}b}{ax}+\frac{\sqrt{bx^2+ax}}{x^2}\right)$$
$$+\frac{2}{5}\,c^2d\left(\frac{2\,\sqrt{bx^2+ax}b^2}{a^2x}-\frac{\sqrt{bx^2+ax}b}{ax^2}-\frac{3\,\sqrt{bx^2+ax}}{x^3}\right)$$
$$-\frac{2}{105}\,c^3\left(\frac{8\,\sqrt{bx^2+ax}b^3}{a^3x}-\frac{4\,\sqrt{bx^2+ax}b^2}{a^2x^2}+\frac{3\,\sqrt{bx^2+ax}b}{ax^3}+\frac{15\,\sqrt{bx^2+ax}}{x^4}\right)$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
(sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*sqrt(b*x^2 + a*x
)/x)*d^3 - 2*c*d^2*(sqrt(b*x^2 + a*x)*b/(a*x) + sqrt(b*x^2 + a*x)/x^2) + 2
/5*c^2*d*(2*sqrt(b*x^2 + a*x)*b^2/(a^2*x) - sqrt(b*x^2 + a*x)*b/(a*x^2) -
3*sqrt(b*x^2 + a*x)/x^3) - 2/105*c^3*(8*sqrt(b*x^2 + a*x)*b^3/(a^3*x) - 4*
sqrt(b*x^2 + a*x)*b^2/(a^2*x^2) + 3*sqrt(b*x^2 + a*x)*b/(a*x^3) + 15*sqrt(
b*x^2 + a*x)/x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. 2(141) = 282.

Time = 0.14 (sec) , antiderivative size = 426, normalized size of antiderivative = 2.65

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^5}\,dx = -\sqrt{b}d^3\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)$$
$$+\frac{2\left(315\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6bcd^2+105\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6ad^3+315\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5b^{\frac{3}{2}}c^2d-\cdots\right.}{\cdots}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^5,x, algorithm="giac")
```

output
```
-sqrt(b)*d^3*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)) + 2/1
05*(315*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*b*c*d^2 + 105*(sqrt(b)*x - sqrt(
b*x^2 + a*x))^6*a*d^3 + 315*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*b^(3/2)*c^2*
d + 315*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a*sqrt(b)*c*d^2 + 140*(sqrt(b)*x
 - sqrt(b*x^2 + a*x))^4*b^2*c^3 + 525*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a*
b*c^2*d + 105*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^2*c*d^2 + 315*(sqrt(b)*x
 - sqrt(b*x^2 + a*x))^3*a*b^(3/2)*c^3 + 315*(sqrt(b)*x - sqrt(b*x^2 + a*x)
)^3*a^2*sqrt(b)*c^2*d + 273*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2*b*c^3 +
63*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^3*c^2*d + 105*(sqrt(b)*x - sqrt(b*x
^2 + a*x))*a^3*sqrt(b)*c^3 + 15*a^4*c^3)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^7
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^5}\,dx = \int \frac{\sqrt{b\,x^2+a\,x}\,{\left(c+d\,x\right)}^3}{x^5} \,dx$$

input
```
int(((a*x + b*x^2)^(1/2)*(c + d*x)^3)/x^5,x)
```

output
```
int(((a*x + b*x^2)^(1/2)*(c + d*x)^3)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.80

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^5}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3c^3}{7}-\frac{6\sqrt{x}\,\sqrt{bx+a}\,a^3c^2dx}{5}-2\sqrt{x}\,\sqrt{bx+a}\,a^3c\,d^2x^2-2\sqrt{x}\,\sqrt{bx+a}\,a^3d^3x^3-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2b\,c^3x}{35}-2\sqrt{}}{}$$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^5,x)
```

output

```
(2*( - 15*sqrt(x)*sqrt(a + b*x)*a**3*c**3 - 63*sqrt(x)*sqrt(a + b*x)*a**3*
c**2*d*x - 105*sqrt(x)*sqrt(a + b*x)*a**3*c*d**2*x**2 - 105*sqrt(x)*sqrt(a
 + b*x)*a**3*d**3*x**3 - 3*sqrt(x)*sqrt(a + b*x)*a**2*b*c**3*x - 21*sqrt(x
)*sqrt(a + b*x)*a**2*b*c**2*d*x**2 - 105*sqrt(x)*sqrt(a + b*x)*a**2*b*c*d*
*2*x**3 + 4*sqrt(x)*sqrt(a + b*x)*a*b**2*c**3*x**2 + 42*sqrt(x)*sqrt(a + b
*x)*a*b**2*c**2*d*x**3 - 8*sqrt(x)*sqrt(a + b*x)*b**3*c**3*x**3 + 105*sqrt
(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*d**3*x**4 + 75*sqr
t(b)*a**3*d**3*x**4 + 15*sqrt(b)*a**2*b*c*d**2*x**4 - 42*sqrt(b)*a*b**2*c*
*2*d*x**4 + 8*sqrt(b)*b**3*c**3*x**4))/(105*a**3*x**4)
```

3.27 $\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^6}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^6}\,dx = -\frac{16c(bc-ad)^2\left(ax+bx^2\right)^{3/2}}{105a^3x^4} + \frac{16(2bc-5ad)(bc-ad)^2\left(ax+bx^2\right)^{3/2}}{315a^4x^3} + \frac{4(bc-ad)(c+dx)^2\left(ax+bx^2\right)^{3/2}}{21a^2x^5} - \frac{2(c+dx)^3\left(ax+bx^2\right)^{3/2}}{9ax^6}$$

output
```
-16/105*c*(-a*d+b*c)^2*(b*x^2+a*x)^(3/2)/a^3/x^4+16/315*(-5*a*d+2*b*c)*(-a
*d+b*c)^2*(b*x^2+a*x)^(3/2)/a^4/x^3+4/21*(-a*d+b*c)*(d*x+c)^2*(b*x^2+a*x)^
(3/2)/a^2/x^5-2/9*(d*x+c)^3*(b*x^2+a*x)^(3/2)/a/x^6
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.77

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^6}\,dx = -\frac{2(x(a+bx))^{3/2}\left(-16b^3c^3x^3+24ab^2c^2x^2(c+3dx)-6a^2bcx(5c^2+18cdx+21d^2x^2)+a^3(35c^3+135c^2\right.}{315a^4x^6}$$

input
```
Integrate[((c + d*x)^3*Sqrt[a*x + b*x^2])/x^6,x]
```

output
```
(-2*(x*(a + b*x))^(3/2)*(-16*b^3*c^3*x^3 + 24*a*b^2*c^2*x^2*(c + 3*d*x) -
6*a^2*b*c*x*(5*c^2 + 18*c*d*x + 21*d^2*x^2) + a^3*(35*c^3 + 135*c^2*d*x +
189*c*d^2*x^2 + 105*d^3*x^3)))/(315*a^4*x^6)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.58, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1262, 27, 2169, 27, 1220, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}(c+dx)^3}{x^6}\,dx$$

$$\Big\downarrow \text{ 1262}$$

$$-\frac{\int -\frac{\sqrt{bx^2+ax}\left(2bc^3+6bdxc^2+d^2(6bc-5ad)x^2\right)}{2x^6}dx}{b}-\frac{d^3\left(ax+bx^2\right)^{3/2}}{bx^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sqrt{bx^2+ax}\left(2bc^3+6bdxc^2+d^2(6bc-5ad)x^2\right)}{x^6}dx}{2b}-\frac{d^3\left(ax+bx^2\right)^{3/2}}{bx^4}$$

$$\Big\downarrow \text{ 2169}$$

$$-\frac{-\frac{\int -\frac{\left(8b^2c^3+d\left(24b^2c^2-42abdc+35a^2d^2\right)x\right)\sqrt{bx^2+ax}}{2x^6}dx}{2b}-\frac{d^2\left(ax+bx^2\right)^{3/2}(6bc-5ad)}{2bx^5}}{2b}-\frac{d^3\left(ax+bx^2\right)^{3/2}}{bx^4}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{\left(8b^2c^3+d\left(24b^2c^2-42abdc+35a^2d^2\right)x\right)\sqrt{bx^2+ax}}{x^6}dx}{4b}-\frac{d^2\left(ax+bx^2\right)^{3/2}(6bc-5ad)}{2bx^5}}{2b}-\frac{d^3\left(ax+bx^2\right)^{3/2}}{bx^4}$$

$\downarrow$ 1220

$$\frac{\frac{-\frac{\left(-105a^3d^3+126a^2bcd^2-72ab^2c^2d+16b^3c^3\right)\int \frac{\sqrt{bx^2+ax}}{x^5}dx}{3a}-\frac{16b^2c^3\left(ax+bx^2\right)^{3/2}}{9ax^6}}{4b}-\frac{d^2\left(ax+bx^2\right)^{3/2}(6bc-5ad)}{2bx^5}}{2b}-\frac{d^3\left(ax+bx^2\right)^{3/2}}{bx^4}$$

$\downarrow$ 1129

$$\frac{\frac{-\frac{\left(-105a^3d^3+126a^2bcd^2-72ab^2c^2d+16b^3c^3\right)\left(-\frac{4b\int \frac{\sqrt{bx^2+ax}}{x^4}dx}{7a}-\frac{2\left(ax+bx^2\right)^{3/2}}{7ax^5}\right)}{3a}-\frac{16b^2c^3\left(ax+bx^2\right)^{3/2}}{9ax^6}}{4b}-\frac{d^2\left(ax+bx^2\right)^{3/2}(6bc-5ad)}{2bx^5}}{2b}-\frac{d^3\left(ax+bx^2\right)^{3/2}}{bx^4}$$

$\downarrow$ 1129

$$\frac{\frac{-\frac{\left(-105a^3d^3+126a^2bcd^2-72ab^2c^2d+16b^3c^3\right)\left(-\frac{4b\left(-\frac{2b\int \frac{\sqrt{bx^2+ax}}{x^3}dx}{5a}-\frac{2\left(ax+bx^2\right)^{3/2}}{5ax^4}\right)}{7a}-\frac{2\left(ax+bx^2\right)^{3/2}}{7ax^5}\right)}{3a}-\frac{16b^2c^3\left(ax+bx^2\right)^{3/2}}{9ax^6}}{4b}-\frac{d^2\left(ax+bx^2\right)^{3/2}}{2bx^5}}{2b}$$

$$\frac{d^3\left(ax+bx^2\right)^{3/2}}{bx^4}$$

$\downarrow$ 1123

$$\frac{-\frac{-\frac{\left(-\frac{4b\left(\frac{4b\left(ax+bx^2\right)^{3/2}}{15a^2x^3}-\frac{2\left(ax+bx^2\right)^{3/2}}{5ax^4}\right)}{7a}-\frac{2\left(ax+bx^2\right)^{3/2}}{7ax^5}\right)\left(-105a^3d^3+126a^2bcd^2-72ab^2c^2d+16b^3c^3\right)}{3a}-\frac{16b^2c^3\left(ax+bx^2\right)^{3/2}}{9ax^6}}{4b}-\frac{d^2\left(ax+bx^2\right)^{3/2}(6}{2bx^5}}{2b}$$

$$\frac{d^3\left(ax+bx^2\right)^{3/2}}{bx^4}$$

input
```
Int[((c + d*x)^3*Sqrt[a*x + b*x^2])/x^6,x]
```

output
```
-((d^3*(a*x + b*x^2)^(3/2))/(b*x^4)) + (-1/2*(d^2*(6*b*c - 5*a*d)*(a*x + b
*x^2)^(3/2))/(b*x^5) + ((-16*b^2*c^3*(a*x + b*x^2)^(3/2))/(9*a*x^6) - ((16
*b^3*c^3 - 72*a*b^2*c^2*d + 126*a^2*b*c*d^2 - 105*a^3*d^3)*((-2*(a*x + b*x
^2)^(3/2))/(7*a*x^5) - (4*b*((-2*(a*x + b*x^2)^(3/2))/(5*a*x^4) + (4*b*(a*
x + b*x^2)^(3/2))/(15*a^2*x^3)))/(7*a)))/(3*a))/(4*b))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

rule 2169
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d -
b*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2
, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$-\frac{2\sqrt{x(bx+a)}\,(bx+a)\left(\left(3d^3x^3+\frac{27}{5}c\,d^2x^2+\frac{27}{7}c^2dx+c^3\right)a^3-\frac{6\left(\frac{21}{5}d^2x^2+\frac{18}{5}cdx+c^2\right)xbc\,a^2}{7}+\frac{24b^2c^2x^2(3dx+c)a}{35}-\frac{16b^3c^3x^3}{35}\right)}{9x^5a^4}$
gosper	$-\frac{2(bx+a)\left(105a^3d^3x^3-126a^2bc\,d^2x^3+72a\,b^2c^2d\,x^3-16b^3c^3x^3+189a^3c\,d^2x^2-108a^2b\,c^2x^2d+24a\,b^2c^3x^2+135a^3c^2dx-30\right.}{315x^5a^4}$
orering	$-\frac{2(bx+a)\left(105a^3d^3x^3-126a^2bc\,d^2x^3+72a\,b^2c^2d\,x^3-16b^3c^3x^3+189a^3c\,d^2x^2-108a^2b\,c^2x^2d+24a\,b^2c^3x^2+135a^3c^2dx-30\right.}{315x^5a^4}$
trager	$-\frac{2\left(105a^3b\,d^3x^4-126a^2b^2c\,d^2x^4+72a\,b^3c^2d\,x^4-16b^4c^3x^4+105a^4d^3x^3+63a^3bc\,d^2x^3-36a^2b^2c^2d\,x^3+8a\,b^3c^3x^3+189a^4c\right.}{315x^5a^4}$
risch	$-\frac{2(bx+a)\left(105a^3b\,d^3x^4-126a^2b^2c\,d^2x^4+72a\,b^3c^2d\,x^4-16b^4c^3x^4+105a^4d^3x^3+63a^3bc\,d^2x^3-36a^2b^2c^2d\,x^3+8a\,b^3c^3x^3+1\right.}{315x^4\sqrt{x(bx+a)}\,a^4}$
default	$c^3\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{9a\,x^6}-\frac{2b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{7a\,x^5}-\frac{4b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{5a\,x^4}+\frac{4b\left(b\,x^2+ax\right)^{\frac{3}{2}}}{15a^2x^3}\right)}{7a}\right)}{3a}\right)-\frac{2d^3\left(b\,x^2+ax\right)^{\frac{3}{2}}}{3a\,x^3}+3c\,d^2($

input
```
int((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-2/9*(x*(b*x+a))^(1/2)*(b*x+a)*((3*d^3*x^3+27/5*c*d^2*x^2+27/7*c^2*d*x+c^3
)*a^3-6/7*(21/5*d^2*x^2+18/5*c*d*x+c^2)*x*b*c*a^2+24/35*b^2*c^2*x^2*(3*d*x
+c)*a-16/35*b^3*c^3*x^3)/x^5/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.20

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^6}\,dx = -\frac{2\left(35\,a^4c^3-\left(16\,b^4c^3-72\,ab^3c^2d+126\,a^2b^2cd^2-105\,a^3bd^3\right)x^4+\left(8\,ab^3c^3-36\,a^2b^2c^2d+63\,a^3bcd^2+1\right.}{315\,a^4x^5}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
-2/315*(35*a^4*c^3 - (16*b^4*c^3 - 72*a*b^3*c^2*d + 126*a^2*b^2*c*d^2 - 10
5*a^3*b*d^3)*x^4 + (8*a*b^3*c^3 - 36*a^2*b^2*c^2*d + 63*a^3*b*c*d^2 + 105*
a^4*d^3)*x^3 - 3*(2*a^2*b^2*c^3 - 9*a^3*b*c^2*d - 63*a^4*c*d^2)*x^2 + 5*(a
^3*b*c^3 + 27*a^4*c^2*d)*x)*sqrt(b*x^2 + a*x)/(a^4*x^5)
```

Sympy [F]

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^6}\,dx = \int \frac{\sqrt{x\,(a+bx)}(c+dx)^3}{x^6}\,dx$$

input
```
integrate((d*x+c)**3*(b*x**2+a*x)**(1/2)/x**6,x)
```

output
```
Integral(sqrt(x*(a + b*x))*(c + d*x)**3/x**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(129) = 258$.

Time = 0.04 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.25

$$\begin{aligned}\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^6}\,dx = {}& \frac{32\sqrt{bx^2+ax}b^4c^3}{315\,a^4x} - \frac{16\sqrt{bx^2+ax}b^3c^2d}{35\,a^3x} \\ &+ \frac{4\sqrt{bx^2+ax}b^2cd^2}{5\,a^2x} - \frac{2\sqrt{bx^2+ax}bd^3}{3\,ax} - \frac{16\sqrt{bx^2+ax}b^3c^3}{315\,a^3x^2} \\ &+ \frac{8\sqrt{bx^2+ax}b^2c^2d}{35\,a^2x^2} - \frac{2\sqrt{bx^2+ax}bcd^2}{5\,ax^2} - \frac{2\sqrt{bx^2+ax}d^3}{3\,x^2} \\ &+ \frac{4\sqrt{bx^2+ax}b^2c^3}{105\,a^2x^3} - \frac{6\sqrt{bx^2+ax}bc^2d}{35\,ax^3} - \frac{6\sqrt{bx^2+ax}cd^2}{5\,x^3} \\ &- \frac{2\sqrt{bx^2+ax}bc^3}{63\,ax^4} - \frac{6\sqrt{bx^2+ax}c^2d}{7\,x^4} - \frac{2\sqrt{bx^2+ax}c^3}{9\,x^5}\end{aligned}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
32/315*sqrt(b*x^2 + a*x)*b^4*c^3/(a^4*x) - 16/35*sqrt(b*x^2 + a*x)*b^3*c^2
*d/(a^3*x) + 4/5*sqrt(b*x^2 + a*x)*b^2*c*d^2/(a^2*x) - 2/3*sqrt(b*x^2 + a*
x)*b*d^3/(a*x) - 16/315*sqrt(b*x^2 + a*x)*b^3*c^3/(a^3*x^2) + 8/35*sqrt(b*
x^2 + a*x)*b^2*c^2*d/(a^2*x^2) - 2/5*sqrt(b*x^2 + a*x)*b*c*d^2/(a*x^2) - 2
/3*sqrt(b*x^2 + a*x)*d^3/x^2 + 4/105*sqrt(b*x^2 + a*x)*b^2*c^3/(a^2*x^3) -
 6/35*sqrt(b*x^2 + a*x)*b*c^2*d/(a*x^3) - 6/5*sqrt(b*x^2 + a*x)*c*d^2/x^3
- 2/63*sqrt(b*x^2 + a*x)*b*c^3/(a*x^4) - 6/7*sqrt(b*x^2 + a*x)*c^2*d/x^4 -
 2/9*sqrt(b*x^2 + a*x)*c^3/x^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 546 vs. $2(129) = 258$.

Time = 0.13 (sec) , antiderivative size = 546, normalized size of antiderivative = 3.77

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^6}\,dx$$
$$= \frac{2\left(315\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^8bd^3+945\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^7b^{\frac{3}{2}}cd^2+315\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^7a\sqrt{b}d^3+\right.}{}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^6,x, algorithm="giac")
```

output
```
2/315*(315*(sqrt(b)*x - sqrt(b*x^2 + a*x))^8*b*d^3 + 945*(sqrt(b)*x - sqrt
(b*x^2 + a*x))^7*b^(3/2)*c*d^2 + 315*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*a*s
qrt(b)*d^3 + 1260*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*b^2*c^2*d + 1575*(sqrt
(b)*x - sqrt(b*x^2 + a*x))^6*a*b*c*d^2 + 105*(sqrt(b)*x - sqrt(b*x^2 + a*x
))^6*a^2*d^3 + 630*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*b^(5/2)*c^3 + 2835*(s
qrt(b)*x - sqrt(b*x^2 + a*x))^5*a*b^(3/2)*c^2*d + 945*(sqrt(b)*x - sqrt(b*
x^2 + a*x))^5*a^2*sqrt(b)*c*d^2 + 1764*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a
*b^2*c^3 + 2457*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^2*b*c^2*d + 189*(sqrt(
b)*x - sqrt(b*x^2 + a*x))^4*a^3*c*d^2 + 1995*(sqrt(b)*x - sqrt(b*x^2 + a*x
))^3*a^2*b^(3/2)*c^3 + 945*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^3*sqrt(b)*c
^2*d + 1125*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^3*b*c^3 + 135*(sqrt(b)*x -
 sqrt(b*x^2 + a*x))^2*a^4*c^2*d + 315*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^4*
sqrt(b)*c^3 + 35*a^5*c^3)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^9
```

Mupad [B] (verification not implemented)

Time = 11.59 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.25

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^6}\,dx = \frac{4\,b^2\,c^3\,\sqrt{b\,x^2+a\,x}}{105\,a^2\,x^3} - \frac{2\,d^3\,\sqrt{b\,x^2+a\,x}}{3\,x^2} - \frac{6\,c\,d^2\,\sqrt{b\,x^2+a\,x}}{5\,x^3} - \frac{6\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{7\,x^4} - \frac{2\,c^3\,\sqrt{b\,x^2+a\,x}}{9\,x^5} - \frac{16\,b^3\,c^3\,\sqrt{b\,x^2+a\,x}}{315\,a^3\,x^2} + \frac{32\,b^4\,c^3\,\sqrt{b\,x^2+a\,x}}{315\,a^4\,x} - \frac{2\,b\,c^3\,\sqrt{b\,x^2+a\,x}}{63\,a\,x^4} - \frac{2\,b\,d^3\,\sqrt{b\,x^2+a\,x}}{3\,a\,x} - \frac{2\,b\,c\,d^2\,\sqrt{b\,x^2+a\,x}}{5\,a\,x^2} - \frac{6\,b\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{35\,a\,x^3} + \frac{4\,b^2\,c\,d^2\,\sqrt{b\,x^2+a\,x}}{5\,a^2\,x} + \frac{8\,b^2\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{35\,a^2\,x^2} - \frac{16\,b^3\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{35\,a^3\,x}$$

input
```
int(((a*x + b*x^2)^(1/2)*(c + d*x)^3)/x^6,x)
```

output
```
(4*b^2*c^3*(a*x + b*x^2)^(1/2))/(105*a^2*x^3) - (2*d^3*(a*x + b*x^2)^(1/2)
)/(3*x^2) - (6*c*d^2*(a*x + b*x^2)^(1/2))/(5*x^3) - (6*c^2*d*(a*x + b*x^2)
^(1/2))/(7*x^4) - (2*c^3*(a*x + b*x^2)^(1/2))/(9*x^5) - (16*b^3*c^3*(a*x +
 b*x^2)^(1/2))/(315*a^3*x^2) + (32*b^4*c^3*(a*x + b*x^2)^(1/2))/(315*a^4*x
) - (2*b*c^3*(a*x + b*x^2)^(1/2))/(63*a*x^4) - (2*b*d^3*(a*x + b*x^2)^(1/2
))/(3*a*x) - (2*b*c*d^2*(a*x + b*x^2)^(1/2))/(5*a*x^2) - (6*b*c^2*d*(a*x +
 b*x^2)^(1/2))/(35*a*x^3) + (4*b^2*c*d^2*(a*x + b*x^2)^(1/2))/(5*a^2*x) +
(8*b^2*c^2*d*(a*x + b*x^2)^(1/2))/(35*a^2*x^2) - (16*b^3*c^2*d*(a*x + b*x^
2)^(1/2))/(35*a^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.41

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^6}\,dx$$
$$= \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4c^3}{9}-\frac{6\sqrt{x}\,\sqrt{bx+a}\,a^4c^2dx}{7}-\frac{6\sqrt{x}\,\sqrt{bx+a}\,a^4c\,d^2x^2}{5}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4d^3x^3}{3}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3b\,c^3x}{63}-\frac{6\sqrt{x}\,\sqrt{bx+a}\,a^3b\,c^2}{35}}{}$$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^6,x)
```

output
```
(2*( - 35*sqrt(x)*sqrt(a + b*x)*a**4*c**3 - 135*sqrt(x)*sqrt(a + b*x)*a**4
*c**2*d*x - 189*sqrt(x)*sqrt(a + b*x)*a**4*c*d**2*x**2 - 105*sqrt(x)*sqrt(
a + b*x)*a**4*d**3*x**3 - 5*sqrt(x)*sqrt(a + b*x)*a**3*b*c**3*x - 27*sqrt(
x)*sqrt(a + b*x)*a**3*b*c**2*d*x**2 - 63*sqrt(x)*sqrt(a + b*x)*a**3*b*c*d*
*2*x**3 - 105*sqrt(x)*sqrt(a + b*x)*a**3*b*d**3*x**4 + 6*sqrt(x)*sqrt(a +
b*x)*a**2*b**2*c**3*x**2 + 36*sqrt(x)*sqrt(a + b*x)*a**2*b**2*c**2*d*x**3
+ 126*sqrt(x)*sqrt(a + b*x)*a**2*b**2*c*d**2*x**4 - 8*sqrt(x)*sqrt(a + b*x
)*a*b**3*c**3*x**3 - 72*sqrt(x)*sqrt(a + b*x)*a*b**3*c**2*d*x**4 + 16*sqrt
(x)*sqrt(a + b*x)*b**4*c**3*x**4 + 35*sqrt(b)*a**3*b*d**3*x**5 - 126*sqrt(
b)*a**2*b**2*c*d**2*x**5 + 72*sqrt(b)*a*b**3*c**2*d*x**5 - 16*sqrt(b)*b**4
*c**3*x**5))/(315*a**4*x**5)
```

3.28 $\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^7}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 222

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^7}\,dx = \frac{16(bc-ad)^2(8bc+3ad)\left(ax+bx^2\right)^{3/2}}{1155a^4x^4} - \frac{16(2bc-5ad)(bc-ad)^2(8bc+3ad)\left(ax+bx^2\right)^{3/2}}{3465a^5cx^3} - \frac{4(bc-ad)(8bc+3ad)(c+dx)^2\left(ax+bx^2\right)^{3/2}}{231a^3cx^5} + \frac{2(8bc+3ad)(c+dx)^3\left(ax+bx^2\right)^{3/2}}{99a^2cx^6} - \frac{2(c+dx)^4\left(ax+bx^2\right)^{3/2}}{11acx^7}$$

output
```
16/1155*(-a*d+b*c)^2*(3*a*d+8*b*c)*(b*x^2+a*x)^(3/2)/a^4/x^4-16/3465*(-5*a
*d+2*b*c)*(-a*d+b*c)^2*(3*a*d+8*b*c)*(b*x^2+a*x)^(3/2)/a^5/c/x^3-4/231*(-a
*d+b*c)*(3*a*d+8*b*c)*(d*x+c)^2*(b*x^2+a*x)^(3/2)/a^3/c/x^5+2/99*(3*a*d+8*
b*c)*(d*x+c)^3*(b*x^2+a*x)^(3/2)/a^2/c/x^6-2/11*(d*x+c)^4*(b*x^2+a*x)^(3/2
)/a/c/x^7
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.70

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^7}\,dx$$
$$= \frac{2(x(a+bx))^{3/2}\left(-128b^4c^3x^4+48ab^3c^2x^3(4c+11dx)-24a^2b^2cx^2(10c^2+33cdx+33d^2x^2)-3a^4(105c^3\right.}{3465a^5x^7}$$

input
```
Integrate[((c + d*x)^3*Sqrt[a*x + b*x^2])/x^7,x]
```

output
```
(2*(x*(a + b*x))^(3/2)*(-128*b^4*c^3*x^4 + 48*a*b^3*c^2*x^3*(4*c + 11*d*x)
 - 24*a^2*b^2*c*x^2*(10*c^2 + 33*c*d*x + 33*d^2*x^2) - 3*a^4*(105*c^3 + 38
5*c^2*d*x + 495*c*d^2*x^2 + 231*d^3*x^3) + 2*a^3*b*x*(140*c^3 + 495*c^2*d*
x + 594*c*d^2*x^2 + 231*d^3*x^3)))/(3465*a^5*x^7)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1262, 27, 2169, 27, 1220, 1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}(c+dx)^3}{x^7}\,dx$$

$$\Big\downarrow \text{1262}$$

$$-\frac{\int -\frac{\sqrt{bx^2+ax}\left(4bc^3+12bdxc^2+d^2(12bc-7ad)x^2\right)}{2x^7}dx}{2b}-\frac{d^3\left(ax+bx^2\right)^{3/2}}{2bx^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{bx^2+ax}\left(4bc^3+12bdxc^2+d^2(12bc-7ad)x^2\right)}{x^7}dx}{4b}-\frac{d^3\left(ax+bx^2\right)^{3/2}}{2bx^5}$$

$$\Big\downarrow \text{2169}$$

$$-\frac{\frac{\int -\frac{3\left(8b^2c^3+3d\left(8b^2c^2-12abdc+7a^2d^2\right)x\right)\sqrt{bx^2+ax}}{2x^7}dx}{3b}-\frac{d^2\left(ax+bx^2\right)^{3/2}(12bc-7ad)}{3bx^6}}{4b}-\frac{d^3\left(ax+bx^2\right)^{3/2}}{2bx^5}$$

$\big\downarrow$ 27

$$\frac{\frac{\int \frac{\left(8b^2c^3+3d\left(8b^2c^2-12abdc+7a^2d^2\right)x\right)\sqrt{bx^2+ax}}{x^7}dx}{2b}-\frac{d^2\left(ax+bx^2\right)^{3/2}(12bc-7ad)}{3bx^6}}{4b}-\frac{d^3\left(ax+bx^2\right)^{3/2}}{2bx^5}$$

$\big\downarrow$ 1220

$$\frac{\frac{-\frac{\left(-231a^3d^3+396a^2bcd^2-264ab^2c^2d+64b^3c^3\right)\int \frac{\sqrt{bx^2+ax}}{x^6}dx}{11a}-\frac{16b^2c^3\left(ax+bx^2\right)^{3/2}}{11ax^7}}{2b}-\frac{d^2\left(ax+bx^2\right)^{3/2}(12bc-7ad)}{3bx^6}}{4b}-\frac{d^3\left(ax+bx^2\right)^{3/2}}{2bx^5}$$

$\big\downarrow$ 1129

$$\frac{\frac{-\frac{\left(-231a^3d^3+396a^2bcd^2-264ab^2c^2d+64b^3c^3\right)\left(-\frac{2b\int \frac{\sqrt{bx^2+ax}}{x^5}dx}{3a}-\frac{2\left(ax+bx^2\right)^{3/2}}{9ax^6}\right)}{11a}-\frac{16b^2c^3\left(ax+bx^2\right)^{3/2}}{11ax^7}}{2b}-\frac{d^2\left(ax+bx^2\right)^{3/2}(12bc-7ad)}{3bx^6}}{4b}-\frac{d^3\left(ax+bx^2\right)^{3/2}}{2bx^5}$$

$\big\downarrow$ 1129

$$\frac{\frac{-\frac{\left(-231a^3d^3+396a^2bcd^2-264ab^2c^2d+64b^3c^3\right)\left(-\frac{2b\left(-\frac{4b\int \frac{\sqrt{bx^2+ax}}{x^4}dx}{7a}-\frac{2\left(ax+bx^2\right)^{3/2}}{7ax^5}\right)}{3a}-\frac{2\left(ax+bx^2\right)^{3/2}}{9ax^6}\right)}{11a}-\frac{16b^2c^3\left(ax+bx^2\right)^{3/2}}{11ax^7}}{2b}-\frac{d^2\left(ax+bx^2\right)^{3/}}{3b}}{4b}$$

$$\frac{d^3\left(ax+bx^2\right)^{3/2}}{2bx^5}$$

$\big\downarrow$ 1129

$$\frac{-\frac{-\frac{\left(-231a^3d^3+396a^2bcd^2-264ab^2c^2d+64b^3c^3\right)\left(-\frac{2b\left(-\frac{4b\left(-\frac{2b\int \frac{\sqrt{bx^2+ax}}{x^3}dx}{5a}-\frac{2\left(ax+bx^2\right)^{3/2}}{5ax^4}\right)}{7a}-\frac{2\left(ax+bx^2\right)^{3/2}}{7ax^5}\right)}{3a}-\frac{2\left(ax+bx^2\right)^{3/2}}{9ax^6}\right)}{11a}-\frac{16b^2c^3\left(ax+bx^2\cdots\right.}{11ax^7}\cdots}{2b}\cdots}{4b}$$

$$\frac{d^3\left(ax+bx^2\right)^{3/2}}{2bx^5}$$

$$\Big\downarrow \; 1123$$

$$\frac{-\frac{-\frac{\left(-\frac{2b\left(-\frac{4b\left(\frac{4b\left(ax+bx^2\right)^{3/2}}{15a^2x^3}-\frac{2\left(ax+bx^2\right)^{3/2}}{5ax^4}\right)}{7a}-\frac{2\left(ax+bx^2\right)^{3/2}}{7ax^5}\right)}{3a}-\frac{2\left(ax+bx^2\right)^{3/2}}{9ax^6}\right)\left(-231a^3d^3+396a^2bcd^2-264ab^2c^2d+64b^3c^3\right)}{11a}-\frac{16b^2c^3\left(ax+bx^2\right)^{3\cdots}}{11ax^7}\cdots}{2b}\cdots}{4b}$$

$$\frac{d^3\left(ax+bx^2\right)^{3/2}}{2bx^5}$$

input

```
Int[((c + d*x)^3*Sqrt[a*x + b*x^2])/x^7,x]
```

output

```
-1/2*(d^3*(a*x + b*x^2)^(3/2))/(b*x^5) + (-1/3*(d^2*(12*b*c - 7*a*d)*(a*x
+ b*x^2)^(3/2))/(b*x^6) + ((-16*b^2*c^3*(a*x + b*x^2)^(3/2))/(11*a*x^7) -
((64*b^3*c^3 - 264*a*b^2*c^2*d + 396*a^2*b*c*d^2 - 231*a^3*d^3)*((-2*(a*x
+ b*x^2)^(3/2))/(9*a*x^6) - (2*b*((-2*(a*x + b*x^2)^(3/2))/(7*a*x^5) - (4*
b*((-2*(a*x + b*x^2)^(3/2))/(5*a*x^4) + (4*b*(a*x + b*x^2)^(3/2))/(15*a^2*
x^3)))/(7*a)))/(3*a)))/(11*a))/(2*b))/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

rule 2169
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d -
b*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2
, 0]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$-\frac{2\sqrt{x(bx+a)}\,(bx+a)\left(\left(\frac{11}{5}d^3x^3+\frac{33}{7}c\,d^2x^2+\frac{11}{3}c^2dx+c^3\right)a^4-\frac{8x\left(\frac{33}{20}d^3x^3+\frac{297}{70}c\,d^2x^2+\frac{99}{28}c^2dx+c^3\right)b\,a^3}{9}+\frac{16\left(\frac{33}{10}d^2x^2+\frac{33}{10}cdx\right.}{21}\right.}{11x^6a^5}$
gosper	$-\frac{2(bx+a)(-462a^3b\,d^3x^4+792a^2b^2c\,d^2x^4-528a\,b^3c^2d\,x^4+128b^4c^3x^4+693a^4d^3x^3-1188a^3bc\,d^2x^3+792a^2b^2c^2d\,x^3-192a}{3465x^6a^5}$
orering	$-\frac{2(bx+a)(-462a^3b\,d^3x^4+792a^2b^2c\,d^2x^4-528a\,b^3c^2d\,x^4+128b^4c^3x^4+693a^4d^3x^3-1188a^3bc\,d^2x^3+792a^2b^2c^2d\,x^3-192a}{3465x^6a^5}$
trager	$-\frac{2(-462a^3b^2d^3x^5+792a^2b^3c\,d^2x^5-528a\,b^4c^2d\,x^5+128b^5c^3x^5+231a^4b\,d^3x^4-396a^3b^2c\,d^2x^4+264a^2b^3c^2d\,x^4-64a\,b^4c^3x}{}$
risch	$-\frac{2(bx+a)(-462a^3b^2d^3x^5+792a^2b^3c\,d^2x^5-528a\,b^4c^2d\,x^5+128b^5c^3x^5+231a^4b\,d^3x^4-396a^3b^2c\,d^2x^4+264a^2b^3c^2d\,x^4-64}{}$
default	$c^3\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{11a\,x^7}-\frac{8b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{9a\,x^6}-\frac{2b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{7a\,x^5}-\frac{4b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{5a\,x^4}+\frac{4b\left(b\,x^2+ax\right)^{\frac{3}{2}}}{15a^2x^3}\right)}{7a}\right)}{3a}\right)}{11a}\right)+d^3\Big($

input
```
int((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-2/11*(x*(b*x+a))^(1/2)*(b*x+a)*((11/5*d^3*x^3+33/7*c*d^2*x^2+11/3*c^2*d*x
+c^3)*a^4-8/9*x*(33/20*d^3*x^3+297/70*c*d^2*x^2+99/28*c^2*d*x+c^3)*b*a^3+1
6/21*(33/10*d^2*x^2+33/10*c*d*x+c^2)*x^2*b^2*c*a^2-64/105*(11/4*d*x+c)*x^3
*b^3*c^2*a+128/315*b^4*c^3*x^4)/x^6/a^5
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^7}\,dx = -\frac{2\,(315\,a^5c^3 + 2\,(64\,b^5c^3 - 264\,ab^4c^2d + 396\,a^2b^3cd^2 - 231\,a^3b^2d^3)x^5 - (64\,ab^4c^3 - 264\,a^2b^3c^2d + 396\,a^3 \cdots}{\cdots}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
-2/3465*(315*a^5*c^3 + 2*(64*b^5*c^3 - 264*a*b^4*c^2*d + 396*a^2*b^3*c*d^2
 - 231*a^3*b^2*d^3)*x^5 - (64*a*b^4*c^3 - 264*a^2*b^3*c^2*d + 396*a^3*b^2*
c*d^2 - 231*a^4*b*d^3)*x^4 + 3*(16*a^2*b^3*c^3 - 66*a^3*b^2*c^2*d + 99*a^4
*b*c*d^2 + 231*a^5*d^3)*x^3 - 5*(8*a^3*b^2*c^3 - 33*a^4*b*c^2*d - 297*a^5*
c*d^2)*x^2 + 35*(a^4*b*c^3 + 33*a^5*c^2*d)*x)*sqrt(b*x^2 + a*x)/(a^5*x^6)
```

Sympy [F]

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^7}\,dx = \int \frac{\sqrt{x\,(a+bx)}(c+dx)^3}{x^7}\,dx$$

input
```
integrate((d*x+c)**3*(b*x**2+a*x)**(1/2)/x**7,x)
```

output
```
Integral(sqrt(x*(a + b*x))*(c + d*x)**3/x**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(202) = 404$.

Time = 0.04 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.93

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^7}\,dx = -\frac{256\sqrt{bx^2+ax}b^5c^3}{3465\,a^5x} + \frac{32\sqrt{bx^2+ax}b^4c^2d}{105\,a^4x} - \frac{16\sqrt{bx^2+ax}b^3cd^2}{35\,a^3x} + \frac{4\sqrt{bx^2+ax}b^2d^3}{15\,a^2x} + \frac{128\sqrt{bx^2+ax}b^4c^3}{3465\,a^4x^2} - \frac{16\sqrt{bx^2+ax}b^3c^2d}{105\,a^3x^2} + \frac{8\sqrt{bx^2+ax}b^2cd^2}{35\,a^2x^2} - \frac{2\sqrt{bx^2+ax}bd^3}{15\,ax^2} - \frac{32\sqrt{bx^2+ax}b^3c^3}{1155\,a^3x^3} + \frac{4\sqrt{bx^2+ax}b^2c^2d}{35\,a^2x^3} - \frac{6\sqrt{bx^2+ax}bcd^2}{35\,ax^3} - \frac{2\sqrt{bx^2+ax}d^3}{5\,x^3} + \frac{16\sqrt{bx^2+ax}b^2c^3}{693\,a^2x^4} - \frac{2\sqrt{bx^2+ax}bc^2d}{21\,ax^4} - \frac{6\sqrt{bx^2+ax}cd^2}{7\,x^4} - \frac{2\sqrt{bx^2+ax}bc^3}{99\,ax^5} - \frac{2\sqrt{bx^2+ax}c^2d}{3\,x^5} - \frac{2\sqrt{bx^2+ax}c^3}{11\,x^6}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
-256/3465*sqrt(b*x^2 + a*x)*b^5*c^3/(a^5*x) + 32/105*sqrt(b*x^2 + a*x)*b^4
*c^2*d/(a^4*x) - 16/35*sqrt(b*x^2 + a*x)*b^3*c*d^2/(a^3*x) + 4/15*sqrt(b*x
^2 + a*x)*b^2*d^3/(a^2*x) + 128/3465*sqrt(b*x^2 + a*x)*b^4*c^3/(a^4*x^2) -
 16/105*sqrt(b*x^2 + a*x)*b^3*c^2*d/(a^3*x^2) + 8/35*sqrt(b*x^2 + a*x)*b^2
*c*d^2/(a^2*x^2) - 2/15*sqrt(b*x^2 + a*x)*b*d^3/(a*x^2) - 32/1155*sqrt(b*x
^2 + a*x)*b^3*c^3/(a^3*x^3) + 4/35*sqrt(b*x^2 + a*x)*b^2*c^2*d/(a^2*x^3) -
 6/35*sqrt(b*x^2 + a*x)*b*c*d^2/(a*x^3) - 2/5*sqrt(b*x^2 + a*x)*d^3/x^3 +
16/693*sqrt(b*x^2 + a*x)*b^2*c^3/(a^2*x^4) - 2/21*sqrt(b*x^2 + a*x)*b*c^2*
d/(a*x^4) - 6/7*sqrt(b*x^2 + a*x)*c*d^2/x^4 - 2/99*sqrt(b*x^2 + a*x)*b*c^3
/(a*x^5) - 2/3*sqrt(b*x^2 + a*x)*c^2*d/x^5 - 2/11*sqrt(b*x^2 + a*x)*c^3/x^
6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 676 vs. 2(202) = 404.

Time = 0.12 (sec) , antiderivative size = 676, normalized size of antiderivative = 3.05

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^7}\,dx$$

$$= \frac{2\left(3465\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^9 b^{\frac{3}{2}}d^3+13860\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^8 b^2cd^2+5775\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^8 ab\right.}{}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^7,x, algorithm="giac")
```

output
```
2/3465*(3465*(sqrt(b)*x - sqrt(b*x^2 + a*x))^9*b^(3/2)*d^3 + 13860*(sqrt(b
)*x - sqrt(b*x^2 + a*x))^8*b^2*c*d^2 + 5775*(sqrt(b)*x - sqrt(b*x^2 + a*x)
)^8*a*b*d^3 + 20790*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*b^(5/2)*c^2*d + 3118
5*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*a*b^(3/2)*c*d^2 + 3465*(sqrt(b)*x - sq
rt(b*x^2 + a*x))^7*a^2*sqrt(b)*d^3 + 11088*(sqrt(b)*x - sqrt(b*x^2 + a*x))
^6*b^3*c^3 + 58212*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a*b^2*c^2*d + 27027*(
sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a^2*b*c*d^2 + 693*(sqrt(b)*x - sqrt(b*x^2
 + a*x))^6*a^3*d^3 + 36960*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a*b^(5/2)*c^3
 + 65835*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a^2*b^(3/2)*c^2*d + 10395*(sqrt
(b)*x - sqrt(b*x^2 + a*x))^5*a^3*sqrt(b)*c*d^2 + 51480*(sqrt(b)*x - sqrt(b
*x^2 + a*x))^4*a^2*b^2*c^3 + 37125*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^3*b
*c^2*d + 1485*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^4*c*d^2 + 38115*(sqrt(b)
*x - sqrt(b*x^2 + a*x))^3*a^3*b^(3/2)*c^3 + 10395*(sqrt(b)*x - sqrt(b*x^2
+ a*x))^3*a^4*sqrt(b)*c^2*d + 15785*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^4*
b*c^3 + 1155*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^5*c^2*d + 3465*(sqrt(b)*x
 - sqrt(b*x^2 + a*x))*a^5*sqrt(b)*c^3 + 315*a^6*c^3)/(sqrt(b)*x - sqrt(b*x
^2 + a*x))^11
```

Mupad [B] (verification not implemented)

Time = 12.47 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.93

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^7}\,dx = \frac{16\,b^2\,c^3\,\sqrt{b\,x^2+a\,x}}{693\,a^2\,x^4}-\frac{2\,d^3\,\sqrt{b\,x^2+a\,x}}{5\,x^3}-\frac{6\,c\,d^2\,\sqrt{b\,x^2+a\,x}}{7\,x^4}-\frac{2\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{3\,x^5}-\frac{2\,c^3\,\sqrt{b\,x^2+a\,x}}{11\,x^6}-\frac{32\,b^3\,c^3\,\sqrt{b\,x^2+a\,x}}{1155\,a^3\,x^3}+\frac{128\,b^4\,c^3\,\sqrt{b\,x^2+a\,x}}{3465\,a^4\,x^2}-\frac{256\,b^5\,c^3\,\sqrt{b\,x^2+a\,x}}{3465\,a^5\,x}+\frac{4\,b^2\,d^3\,\sqrt{b\,x^2+a\,x}}{15\,a^2\,x}-\frac{2\,b\,c^3\,\sqrt{b\,x^2+a\,x}}{99\,a\,x^5}-\frac{2\,b\,d^3\,\sqrt{b\,x^2+a\,x}}{15\,a\,x^2}-\frac{6\,b\,c\,d^2\,\sqrt{b\,x^2+a\,x}}{35\,a\,x^3}-\frac{2\,b\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{21\,a\,x^4}+\frac{8\,b^2\,c\,d^2\,\sqrt{b\,x^2+a\,x}}{35\,a^2\,x^2}+\frac{4\,b^2\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{35\,a^2\,x^3}-\frac{16\,b^3\,c\,d^2\,\sqrt{b\,x^2+a\,x}}{35\,a^3\,x}-\frac{16\,b^3\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{105\,a^3\,x^2}+\frac{32\,b^4\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{105\,a^4\,x}$$

input
```
int(((a*x + b*x^2)^(1/2)*(c + d*x)^3)/x^7,x)
```

output
```
(16*b^2*c^3*(a*x + b*x^2)^(1/2))/(693*a^2*x^4) - (2*d^3*(a*x + b*x^2)^(1/2
))/(5*x^3) - (6*c*d^2*(a*x + b*x^2)^(1/2))/(7*x^4) - (2*c^2*d*(a*x + b*x^2
)^(1/2))/(3*x^5) - (2*c^3*(a*x + b*x^2)^(1/2))/(11*x^6) - (32*b^3*c^3*(a*x
 + b*x^2)^(1/2))/(1155*a^3*x^3) + (128*b^4*c^3*(a*x + b*x^2)^(1/2))/(3465*
a^4*x^2) - (256*b^5*c^3*(a*x + b*x^2)^(1/2))/(3465*a^5*x) + (4*b^2*d^3*(a*
x + b*x^2)^(1/2))/(15*a^2*x) - (2*b*c^3*(a*x + b*x^2)^(1/2))/(99*a*x^5) -
(2*b*d^3*(a*x + b*x^2)^(1/2))/(15*a*x^2) - (6*b*c*d^2*(a*x + b*x^2)^(1/2))
/(35*a*x^3) - (2*b*c^2*d*(a*x + b*x^2)^(1/2))/(21*a*x^4) + (8*b^2*c*d^2*(a
*x + b*x^2)^(1/2))/(35*a^2*x^2) + (4*b^2*c^2*d*(a*x + b*x^2)^(1/2))/(35*a^
2*x^3) - (16*b^3*c*d^2*(a*x + b*x^2)^(1/2))/(35*a^3*x) - (16*b^3*c^2*d*(a*
x + b*x^2)^(1/2))/(105*a^3*x^2) + (32*b^4*c^2*d*(a*x + b*x^2)^(1/2))/(105*
a^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.99

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^7}\,dx$$
$$= \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^5c^3}{11}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^5c^2dx}{3}-\frac{6\sqrt{x}\,\sqrt{bx+a}\,a^5c\,d^2x^2}{7}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^5d^3x^3}{5}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4b\,c^3x}{99}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4b\,c^2}{21}}{}$$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^7,x)
```

output
```
(2*( - 315*sqrt(x)*sqrt(a + b*x)*a**5*c**3 - 1155*sqrt(x)*sqrt(a + b*x)*a*
*5*c**2*d*x - 1485*sqrt(x)*sqrt(a + b*x)*a**5*c*d**2*x**2 - 693*sqrt(x)*sq
rt(a + b*x)*a**5*d**3*x**3 - 35*sqrt(x)*sqrt(a + b*x)*a**4*b*c**3*x - 165*
sqrt(x)*sqrt(a + b*x)*a**4*b*c**2*d*x**2 - 297*sqrt(x)*sqrt(a + b*x)*a**4*
b*c*d**2*x**3 - 231*sqrt(x)*sqrt(a + b*x)*a**4*b*d**3*x**4 + 40*sqrt(x)*sq
rt(a + b*x)*a**3*b**2*c**3*x**2 + 198*sqrt(x)*sqrt(a + b*x)*a**3*b**2*c**2
*d*x**3 + 396*sqrt(x)*sqrt(a + b*x)*a**3*b**2*c*d**2*x**4 + 462*sqrt(x)*sq
rt(a + b*x)*a**3*b**2*d**3*x**5 - 48*sqrt(x)*sqrt(a + b*x)*a**2*b**3*c**3*
x**3 - 264*sqrt(x)*sqrt(a + b*x)*a**2*b**3*c**2*d*x**4 - 792*sqrt(x)*sqrt(
a + b*x)*a**2*b**3*c*d**2*x**5 + 64*sqrt(x)*sqrt(a + b*x)*a*b**4*c**3*x**4
 + 528*sqrt(x)*sqrt(a + b*x)*a*b**4*c**2*d*x**5 - 128*sqrt(x)*sqrt(a + b*x
)*b**5*c**3*x**5 - 462*sqrt(b)*a**3*b**2*d**3*x**6 + 792*sqrt(b)*a**2*b**3
*c*d**2*x**6 - 528*sqrt(b)*a*b**4*c**2*d*x**6 + 128*sqrt(b)*b**5*c**3*x**6
))/(3465*a**5*x**6)
```

3.29 $\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 290

$$\begin{aligned}\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^8}\,dx &= -\frac{2c^3(ax+bx^2)^{3/2}}{13ax^8}+\frac{2c^2(10bc-39ad)\,(ax+bx^2)^{3/2}}{143a^2x^7}\\
&\quad-\frac{2c(80b^2c^2-312abcd+429a^2d^2)\,(ax+bx^2)^{3/2}}{1287a^3x^6}\\
&\quad-\frac{2\left(429d^3-\frac{2bc(80b^2c^2-312abcd+429a^2d^2)}{a^3}\right)(ax+bx^2)^{3/2}}{3003ax^5}\\
&\quad+\frac{8b(429a^3d^3-2bc(80b^2c^2-312abcd+429a^2d^2))\,(ax+bx^2)^{3/2}}{15015a^5x^4}\\
&\quad-\frac{16b^2(429a^3d^3-2bc(80b^2c^2-312abcd+429a^2d^2))\,(ax+bx^2)^{3/2}}{45045a^6x^3}\end{aligned}$$

output
```
-2/13*c^3*(b*x^2+a*x)^(3/2)/a/x^8+2/143*c^2*(-39*a*d+10*b*c)*(b*x^2+a*x)^(
3/2)/a^2/x^7-2/1287*c*(429*a^2*d^2-312*a*b*c*d+80*b^2*c^2)*(b*x^2+a*x)^(3/
2)/a^3/x^6-2/3003*(429*d^3-2*b*c*(429*a^2*d^2-312*a*b*c*d+80*b^2*c^2)/a^3)
*(b*x^2+a*x)^(3/2)/a/x^5+8/15015*b*(429*a^3*d^3-2*b*c*(429*a^2*d^2-312*a*b
*c*d+80*b^2*c^2))*(b*x^2+a*x)^(3/2)/a^5/x^4-16/45045*b^2*(429*a^3*d^3-2*b*
c*(429*a^2*d^2-312*a*b*c*d+80*b^2*c^2))*(b*x^2+a*x)^(3/2)/a^6/x^3
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.68

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^8}\,dx = -\frac{2(x(a+bx))^{3/2}\left(-1280b^5c^3x^5+384ab^4c^2x^4(5c+13dx)-48a^2b^3cx^3(50c^2+156cdx+143d^2x^2)+15a\right.}{}$$

input
```
Integrate[((c + d*x)^3*Sqrt[a*x + b*x^2])/x^8,x]
```

output
```
(-2*(x*(a + b*x))^(3/2)*(-1280*b^5*c^3*x^5 + 384*a*b^4*c^2*x^4*(5*c + 13*d
*x) - 48*a^2*b^3*c*x^3*(50*c^2 + 156*c*d*x + 143*d^2*x^2) + 15*a^5*(231*c^
3 + 819*c^2*d*x + 1001*c*d^2*x^2 + 429*d^3*x^3) + 8*a^3*b^2*x^2*(350*c^3 +
 1170*c^2*d*x + 1287*c*d^2*x^2 + 429*d^3*x^3) - 6*a^4*b*x*(525*c^3 + 1820*
c^2*d*x + 2145*c*d^2*x^2 + 858*d^3*x^3)))/(45045*a^6*x^8)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1262, 27, 2169, 27, 1220, 1129, 1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}(c+dx)^3}{x^8}\,dx$$

$$\Big\downarrow \text{1262}$$

$$-\frac{\int -\frac{3\sqrt{bx^2+ax}\left(2bc^3+6bdxc^2+3d^2(2bc-ad)x^2\right)}{2x^8}dx}{3b}-\frac{d^3\left(ax+bx^2\right)^{3/2}}{3bx^6}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{bx^2+ax}\left(2bc^3+6bdxc^2+3d^2(2bc-ad)x^2\right)}{x^8}dx}{2b}-\frac{d^3\left(ax+bx^2\right)^{3/2}}{3bx^6}$$

$$\Big\downarrow \text{2169}$$

$$\frac{-\frac{\int -\frac{\left(16b^2c^3+3d\left(16b^2c^2-22abdc+11a^2d^2\right)x\right)\sqrt{bx^2+ax}}{2x^8}dx}{4b}-\frac{3d^2\left(ax+bx^2\right)^{3/2}(2bc-ad)}{4bx^7}}{2b}-\frac{d^3\left(ax+bx^2\right)^{3/2}}{3bx^6}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\left(16b^2c^3+3d\left(16b^2c^2-22abdc+11a^2d^2\right)x\right)\sqrt{bx^2+ax}}{x^8}dx}{8b}-\frac{3d^2\left(ax+bx^2\right)^{3/2}(2bc-ad)}{4bx^7}}{2b}-\frac{d^3\left(ax+bx^2\right)^{3/2}}{3bx^6}$$

$\Big\downarrow$ 1220

$$\frac{-\frac{\frac{\left(-429a^3d^3+858a^2bcd^2-624ab^2c^2d+160b^3c^3\right)\int \frac{\sqrt{bx^2+ax}}{x^7}dx}{13a}-\frac{32b^2c^3\left(ax+bx^2\right)^{3/2}}{13ax^8}}{8b}-\frac{3d^2\left(ax+bx^2\right)^{3/2}(2bc-ad)}{4bx^7}}{2b}-\frac{d^3\left(ax+bx^2\right)^{3/2}}{3bx^6}$$

$\Big\downarrow$ 1129

$$\frac{-\frac{\frac{\left(-429a^3d^3+858a^2bcd^2-624ab^2c^2d+160b^3c^3\right)\left(-\frac{8b\int \frac{\sqrt{bx^2+ax}}{x^6}dx}{11a}-\frac{2\left(ax+bx^2\right)^{3/2}}{11ax^7}\right)}{13a}-\frac{32b^2c^3\left(ax+bx^2\right)^{3/2}}{13ax^8}}{8b}-\frac{3d^2\left(ax+bx^2\right)^{3/2}(2bc-ad)}{4bx^7}}{2b}-\frac{d^3\left(ax+bx^2\right)^{3/2}}{3bx^6}$$

$\Big\downarrow$ 1129

$$\frac{-\frac{\frac{\left(-429a^3d^3+858a^2bcd^2-624ab^2c^2d+160b^3c^3\right)\left(-\frac{8b\left(-\frac{2b\int \frac{\sqrt{bx^2+ax}}{x^5}dx}{3a}-\frac{2\left(ax+bx^2\right)^{3/2}}{9ax^6}\right)}{11a}-\frac{2\left(ax+bx^2\right)^{3/2}}{11ax^7}\right)}{13a}-\frac{32b^2c^3\left(ax+bx^2\right)^{3/2}}{13ax^8}}{8b}-\frac{3d^2\left(ax+bx^2\right)^{3/2}(2bc-ad)}{4bx^7}}{2b}-\frac{d^3\left(ax+bx^2\right)^{3/2}}{3bx^6}$$

$\Big\downarrow$ 1129

$$-\frac{-\frac{\left(-429a^3d^3+858a^2bcd^2-624ab^2c^2d+160b^3c^3\right)\left(-\frac{8b\left(-\frac{2b\left(-\frac{4b\int \frac{\sqrt{bx^2+ax}}{x^4}dx}{7a}-\frac{2\left(ax+bx^2\right)^{3/2}}{7ax^5}\right)}{3a}-\frac{2\left(ax+bx^2\right)^{3/2}}{9ax^6}\right)}{11a}-\frac{2\left(ax+bx^2\right)^{3/2}}{11ax^7}\right)}{13a}-\frac{32b^2c^3\left(ax+bx\right.}{13ax^8}}{8b}}{2b}$$

$$\frac{d^3\left(ax+bx^2\right)^{3/2}}{3bx^6}$$

$\downarrow$ 1129

$$-\frac{-\frac{\left(-429a^3d^3+858a^2bcd^2-624ab^2c^2d+160b^3c^3\right)\left(-\frac{8b\left(-\frac{2b\left(-\frac{4b\left(-\frac{2b\int \frac{\sqrt{bx^2+ax}}{x^3}dx}{5a}-\frac{2\left(ax+bx^2\right)^{3/2}}{5ax^4}\right)}{7a}-\frac{2\left(ax+bx^2\right)^{3/2}}{7ax^5}\right)}{3a}-\frac{2\left(ax+bx^2\right)^{3/2}}{9ax^6}\right)}{11a}-\frac{2\left(ax+bx\right.}{11ax}\right.}{13a}}{8b}}{2b}$$

$$\frac{d^3\left(ax+bx^2\right)^{3/2}}{3bx^6}$$

$\downarrow$ 1123

$$-\frac{-\frac{8b\left(-\frac{2b\left(-\frac{4b\left(\frac{4b(ax+bx^2)^{3/2}}{15a^2x^3}-\frac{2(ax+bx^2)^{3/2}}{5ax^4}\right)}{7a}-\frac{2(ax+bx^2)^{3/2}}{7ax^5}\right)}{3a}-\frac{2(ax+bx^2)^{3/2}}{9ax^6}\right)}{11a}-\frac{2(ax+bx^2)^{3/2}}{11ax^7}}{13a}\left(-429a^3d^3+858a^2bcd^2-624ab^2c^2d+160\right.}{8b}}{2b}$$

$$\frac{d^3(ax+bx^2)^{3/2}}{3bx^6}$$

input
```
Int[((c + d*x)^3*Sqrt[a*x + b*x^2])/x^8,x]
```

output
```
-1/3*(d^3*(a*x + b*x^2)^(3/2))/(b*x^6) + ((-3*d^2*(2*b*c - a*d)*(a*x + b*x
^2)^(3/2))/(4*b*x^7) + ((-32*b^2*c^3*(a*x + b*x^2)^(3/2))/(13*a*x^8) - ((1
60*b^3*c^3 - 624*a*b^2*c^2*d + 858*a^2*b*c*d^2 - 429*a^3*d^3)*((-2*(a*x +
b*x^2)^(3/2))/(11*a*x^7) - (8*b*((-2*(a*x + b*x^2)^(3/2))/(9*a*x^6) - (2*b
*((-2*(a*x + b*x^2)^(3/2))/(7*a*x^5) - (4*b*((-2*(a*x + b*x^2)^(3/2))/(5*a
*x^4) + (4*b*(a*x + b*x^2)^(3/2))/(15*a^2*x^3)))/(7*a)))/(3*a)))/(11*a)))/
(13*a))/(8*b))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

rule 2169
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d -
b*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2
, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$-\frac{2\left(\left(\frac{13}{7}d^3x^3+\frac{13}{3}c\,d^2x^2+\frac{39}{11}c^2dx+c^3\right)a^5-\frac{10xb\left(\frac{286}{175}d^3x^3+\frac{143}{35}c\,d^2x^2+\frac{52}{15}c^2dx+c^3\right)a^4}{11}+\frac{80x^2b^2\left(\frac{429}{350}d^3x^3+\frac{1287}{350}c\,d^2x^2+\frac{117}{35}c^2\right.}{99}\right.}{13x^7a^6}$
gosper	$-\frac{2(bx+a)\left(3432a^3b^2d^3x^5-6864a^2b^3c\,d^2x^5+4992a\,b^4c^2d\,x^5-1280b^5c^3x^5-5148a^4b\,d^3x^4+10296a^3b^2c\,d^2x^4-7488a^2b^3c^2\right.}{}$
orering	$-\frac{2(bx+a)\left(3432a^3b^2d^3x^5-6864a^2b^3c\,d^2x^5+4992a\,b^4c^2d\,x^5-1280b^5c^3x^5-5148a^4b\,d^3x^4+10296a^3b^2c\,d^2x^4-7488a^2b^3c^2\right.}{}$
trager	$-\frac{2\left(3432a^3b^3d^3x^6-6864a^2b^4c\,d^2x^6+4992a\,b^5c^2d\,x^6-1280b^6c^3x^6-1716a^4b^2d^3x^5+3432a^3b^3c\,d^2x^5-2496a^2b^4c^2d\,x^5+64\right.}{}$
risch	$-\frac{2(bx+a)\left(3432a^3b^3d^3x^6-6864a^2b^4c\,d^2x^6+4992a\,b^5c^2d\,x^6-1280b^6c^3x^6-1716a^4b^2d^3x^5+3432a^3b^3c\,d^2x^5-2496a^2b^4c^2d\right.}{}$
default	$c^3\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{13a\,x^8}-\frac{10b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{11a\,x^7}-\frac{8b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{9a\,x^6}-\frac{2b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{7a\,x^5}-\frac{4b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{5a\,x^4}+\frac{4b\left(b\,x^2+ax\right)}{15a^2x^3}\right.}{7a}\right.}{3a}\right.}{11a}\right.}{13a}\right.$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-2/13*((13/7*d^3*x^3+13/3*c*d^2*x^2+39/11*c^2*d*x+c^3)*a^5-10/11*x*b*(286/
175*d^3*x^3+143/35*c*d^2*x^2+52/15*c^2*d*x+c^3)*a^4+80/99*x^2*b^2*(429/350
*d^3*x^3+1287/350*c*d^2*x^2+117/35*c^2*d*x+c^3)*a^3-160/231*x^3*b^3*(143/5
0*d^2*x^2+78/25*c*d*x+c^2)*c*a^2+128/231*(13/5*d*x+c)*x^4*b^4*c^2*a-256/69
3*b^5*c^3*x^5)*(x*(b*x+a))^(1/2)*(b*x+a)/x^7/a^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.96

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^8}\,dx =$$
$$-\frac{2\,(3465\,a^6c^3 - 8\,(160\,b^6c^3 - 624\,ab^5c^2d + 858\,a^2b^4cd^2 - 429\,a^3b^3d^3)x^6 + 4\,(160\,ab^5c^3 - 624\,a^2b^4c^2d + 8}{}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^8,x, algorithm="fricas")
```

output
```
-2/45045*(3465*a^6*c^3 - 8*(160*b^6*c^3 - 624*a*b^5*c^2*d + 858*a^2*b^4*c*
d^2 - 429*a^3*b^3*d^3)*x^6 + 4*(160*a*b^5*c^3 - 624*a^2*b^4*c^2*d + 858*a^
3*b^3*c*d^2 - 429*a^4*b^2*d^3)*x^5 - 3*(160*a^2*b^4*c^3 - 624*a^3*b^3*c^2*
d + 858*a^4*b^2*c*d^2 - 429*a^5*b*d^3)*x^4 + 5*(80*a^3*b^3*c^3 - 312*a^4*b
^2*c^2*d + 429*a^5*b*c*d^2 + 1287*a^6*d^3)*x^3 - 35*(10*a^4*b^2*c^3 - 39*a
^5*b*c^2*d - 429*a^6*c*d^2)*x^2 + 315*(a^5*b*c^3 + 39*a^6*c^2*d)*x)*sqrt(b
*x^2 + a*x)/(a^6*x^7)
```

Sympy [F]

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^8}\,dx = \int \frac{\sqrt{x\,(a+bx)}\,(c+dx)^3}{x^8}\,dx$$

input
```
integrate((d*x+c)**3*(b*x**2+a*x)**(1/2)/x**8,x)
```

output
```
Integral(sqrt(x*(a + b*x))*(c + d*x)**3/x**8, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.83

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^8}\,dx = \frac{512\sqrt{bx^2+ax}b^6c^3}{9009\,a^6x} - \frac{256\sqrt{bx^2+ax}b^5c^2d}{1155\,a^5x} + \frac{32\sqrt{bx^2+ax}b^4cd^2}{105\,a^4x} - \frac{16\sqrt{bx^2+ax}b^3d^3}{105\,a^3x} - \frac{256\sqrt{bx^2+ax}b^5c^3}{9009\,a^5x^2} + \frac{128\sqrt{bx^2+ax}b^4c^2d}{1155\,a^4x^2} - \frac{16\sqrt{bx^2+ax}b^3cd^2}{105\,a^3x^2} + \frac{8\sqrt{bx^2+ax}b^2d^3}{105\,a^2x^2} + \frac{64\sqrt{bx^2+ax}b^4c^3}{3003\,a^4x^3} - \frac{32\sqrt{bx^2+ax}b^3c^2d}{385\,a^3x^3} + \frac{4\sqrt{bx^2+ax}b^2cd^2}{35\,a^2x^3} - \frac{2\sqrt{bx^2+ax}bd^3}{35\,ax^3} - \frac{160\sqrt{bx^2+ax}b^3c^3}{9009\,a^3x^4} + \frac{16\sqrt{bx^2+ax}b^2c^2d}{231\,a^2x^4} - \frac{2\sqrt{bx^2+ax}bcd^2}{21\,ax^4} - \frac{2\sqrt{bx^2+ax}d^3}{7\,x^4} + \frac{20\sqrt{bx^2+ax}b^2c^3}{1287\,a^2x^5} - \frac{2\sqrt{bx^2+ax}bc^2d}{33\,ax^5} - \frac{2\sqrt{bx^2+ax}cd^2}{3\,x^5} - \frac{2\sqrt{bx^2+ax}bc^3}{143\,ax^6} - \frac{6\sqrt{bx^2+ax}c^2d}{11\,x^6} - \frac{2\sqrt{bx^2+ax}c^3}{13\,x^7}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^8,x, algorithm="maxima")
```

output
```
512/9009*sqrt(b*x^2 + a*x)*b^6*c^3/(a^6*x) - 256/1155*sqrt(b*x^2 + a*x)*b^
5*c^2*d/(a^5*x) + 32/105*sqrt(b*x^2 + a*x)*b^4*c*d^2/(a^4*x) - 16/105*sqrt
(b*x^2 + a*x)*b^3*d^3/(a^3*x) - 256/9009*sqrt(b*x^2 + a*x)*b^5*c^3/(a^5*x^
2) + 128/1155*sqrt(b*x^2 + a*x)*b^4*c^2*d/(a^4*x^2) - 16/105*sqrt(b*x^2 +
a*x)*b^3*c*d^2/(a^3*x^2) + 8/105*sqrt(b*x^2 + a*x)*b^2*d^3/(a^2*x^2) + 64/
3003*sqrt(b*x^2 + a*x)*b^4*c^3/(a^4*x^3) - 32/385*sqrt(b*x^2 + a*x)*b^3*c^
2*d/(a^3*x^3) + 4/35*sqrt(b*x^2 + a*x)*b^2*c*d^2/(a^2*x^3) - 2/35*sqrt(b*x
^2 + a*x)*b*d^3/(a*x^3) - 160/9009*sqrt(b*x^2 + a*x)*b^3*c^3/(a^3*x^4) + 1
6/231*sqrt(b*x^2 + a*x)*b^2*c^2*d/(a^2*x^4) - 2/21*sqrt(b*x^2 + a*x)*b*c*d
^2/(a*x^4) - 2/7*sqrt(b*x^2 + a*x)*d^3/x^4 + 20/1287*sqrt(b*x^2 + a*x)*b^2
*c^3/(a^2*x^5) - 2/33*sqrt(b*x^2 + a*x)*b*c^2*d/(a*x^5) - 2/3*sqrt(b*x^2 +
 a*x)*c*d^2/x^5 - 2/143*sqrt(b*x^2 + a*x)*b*c^3/(a*x^6) - 6/11*sqrt(b*x^2
+ a*x)*c^2*d/x^6 - 2/13*sqrt(b*x^2 + a*x)*c^3/x^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(266) = 532$.

Time = 0.15 (sec) , antiderivative size = 806, normalized size of antiderivative = 2.78

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^8}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^8,x, algorithm="giac")
```

output

```
2/45045*(60060*(sqrt(b)*x - sqrt(b*x^2 + a*x))^10*b^2*d^3 + 270270*(sqrt(b
)*x - sqrt(b*x^2 + a*x))^9*b^(5/2)*c*d^2 + 135135*(sqrt(b)*x - sqrt(b*x^2
+ a*x))^9*a*b^(3/2)*d^3 + 432432*(sqrt(b)*x - sqrt(b*x^2 + a*x))^8*b^3*c^2
*d + 756756*(sqrt(b)*x - sqrt(b*x^2 + a*x))^8*a*b^2*c*d^2 + 117117*(sqrt(b
)*x - sqrt(b*x^2 + a*x))^8*a^2*b*d^3 + 240240*(sqrt(b)*x - sqrt(b*x^2 + a*
x))^7*b^(7/2)*c^3 + 1441440*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*a*b^(5/2)*c^
2*d + 855855*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*a^2*b^(3/2)*c*d^2 + 45045*(
sqrt(b)*x - sqrt(b*x^2 + a*x))^7*a^3*sqrt(b)*d^3 + 926640*(sqrt(b)*x - sqr
t(b*x^2 + a*x))^6*a*b^3*c^3 + 2007720*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a^
2*b^2*c^2*d + 482625*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a^3*b*c*d^2 + 6435*
(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a^4*d^3 + 1531530*(sqrt(b)*x - sqrt(b*x^
2 + a*x))^5*a^2*b^(5/2)*c^3 + 1486485*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a^
3*b^(3/2)*c^2*d + 135135*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a^4*sqrt(b)*c*d
^2 + 1401400*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^3*b^2*c^3 + 615615*(sqrt(
b)*x - sqrt(b*x^2 + a*x))^4*a^4*b*c^2*d + 15015*(sqrt(b)*x - sqrt(b*x^2 +
a*x))^4*a^5*c*d^2 + 765765*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^4*b^(3/2)*c
^3 + 135135*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^5*sqrt(b)*c^2*d + 249795*(
sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^5*b*c^3 + 12285*(sqrt(b)*x - sqrt(b*x^2
 + a*x))^2*a^6*c^2*d + 45045*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^6*sqrt(b)*c
^3 + 3465*a^7*c^3)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^13
```

Mupad [B] (verification not implemented)

Time = 13.46 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.83

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^8}\,dx = \frac{20\,b^2\,c^3\,\sqrt{b\,x^2+a\,x}}{1287\,a^2\,x^5}-\frac{2\,d^3\,\sqrt{b\,x^2+a\,x}}{7\,x^4}-\frac{2\,c\,d^2\,\sqrt{b\,x^2+a\,x}}{3\,x^5}-\frac{6\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{11\,x^6}-\frac{2\,c^3\,\sqrt{b\,x^2+a\,x}}{13\,x^7}-\frac{160\,b^3\,c^3\,\sqrt{b\,x^2+a\,x}}{9009\,a^3\,x^4}+\frac{64\,b^4\,c^3\,\sqrt{b\,x^2+a\,x}}{3003\,a^4\,x^3}-\frac{256\,b^5\,c^3\,\sqrt{b\,x^2+a\,x}}{9009\,a^5\,x^2}+\frac{512\,b^6\,c^3\,\sqrt{b\,x^2+a\,x}}{9009\,a^6\,x}+\frac{8\,b^2\,d^3\,\sqrt{b\,x^2+a\,x}}{105\,a^2\,x^2}-\frac{16\,b^3\,d^3\,\sqrt{b\,x^2+a\,x}}{105\,a^3\,x}-\frac{2\,b\,c^3\,\sqrt{b\,x^2+a\,x}}{143\,a\,x^6}-\frac{2\,b\,d^3\,\sqrt{b\,x^2+a\,x}}{35\,a\,x^3}-\frac{2\,b\,c\,d^2\,\sqrt{b\,x^2+a\,x}}{21\,a\,x^4}-\frac{2\,b\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{33\,a\,x^5}+\frac{4\,b^2\,c\,d^2\,\sqrt{b\,x^2+a\,x}}{35\,a^2\,x^3}+\frac{16\,b^2\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{231\,a^2\,x^4}-\frac{16\,b^3\,c\,d^2\,\sqrt{b\,x^2+a\,x}}{105\,a^3\,x^2}-\frac{32\,b^3\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{385\,a^3\,x^3}+\frac{32\,b^4\,c\,d^2\,\sqrt{b\,x^2+a\,x}}{105\,a^4\,x}+\frac{128\,b^4\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{1155\,a^4\,x^2}-\frac{256\,b^5\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{1155\,a^5\,x}$$

input
```
int(((a*x + b*x^2)^(1/2)*(c + d*x)^3)/x^8,x)
```

output
```
(20*b^2*c^3*(a*x + b*x^2)^(1/2))/(1287*a^2*x^5) - (2*d^3*(a*x + b*x^2)^(1/
2))/(7*x^4) - (2*c*d^2*(a*x + b*x^2)^(1/2))/(3*x^5) - (6*c^2*d*(a*x + b*x^
2)^(1/2))/(11*x^6) - (2*c^3*(a*x + b*x^2)^(1/2))/(13*x^7) - (160*b^3*c^3*(
a*x + b*x^2)^(1/2))/(9009*a^3*x^4) + (64*b^4*c^3*(a*x + b*x^2)^(1/2))/(300
3*a^4*x^3) - (256*b^5*c^3*(a*x + b*x^2)^(1/2))/(9009*a^5*x^2) + (512*b^6*c
^3*(a*x + b*x^2)^(1/2))/(9009*a^6*x) + (8*b^2*d^3*(a*x + b*x^2)^(1/2))/(10
5*a^2*x^2) - (16*b^3*d^3*(a*x + b*x^2)^(1/2))/(105*a^3*x) - (2*b*c^3*(a*x
+ b*x^2)^(1/2))/(143*a*x^6) - (2*b*d^3*(a*x + b*x^2)^(1/2))/(35*a*x^3) - (
2*b*c*d^2*(a*x + b*x^2)^(1/2))/(21*a*x^4) - (2*b*c^2*d*(a*x + b*x^2)^(1/2)
)/(33*a*x^5) + (4*b^2*c*d^2*(a*x + b*x^2)^(1/2))/(35*a^2*x^3) + (16*b^2*c^
2*d*(a*x + b*x^2)^(1/2))/(231*a^2*x^4) - (16*b^3*c*d^2*(a*x + b*x^2)^(1/2)
)/(105*a^3*x^2) - (32*b^3*c^2*d*(a*x + b*x^2)^(1/2))/(385*a^3*x^3) + (32*b
^4*c*d^2*(a*x + b*x^2)^(1/2))/(105*a^4*x) + (128*b^4*c^2*d*(a*x + b*x^2)^(
1/2))/(1155*a^4*x^2) - (256*b^5*c^2*d*(a*x + b*x^2)^(1/2))/(1155*a^5*x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.83

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{x^8}\,dx$$

$$= \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^6d^3x^3}{7}+\frac{512\sqrt{x}\,\sqrt{bx+a}\,b^6c^3x^6}{9009}+\frac{16\sqrt{b}\,a^3b^3d^3x^7}{105}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^6c^3}{13}-\frac{512\sqrt{b}\,b^6c^3x^7}{9009}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^5bc^2dx^2}{33}-2}{}$$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(1/2)/x^8,x)
```

output
```
(2*( - 3465*sqrt(x)*sqrt(a + b*x)*a**6*c**3 - 12285*sqrt(x)*sqrt(a + b*x)*
a**6*c**2*d*x - 15015*sqrt(x)*sqrt(a + b*x)*a**6*c*d**2*x**2 - 6435*sqrt(x
)*sqrt(a + b*x)*a**6*d**3*x**3 - 315*sqrt(x)*sqrt(a + b*x)*a**5*b*c**3*x -
 1365*sqrt(x)*sqrt(a + b*x)*a**5*b*c**2*d*x**2 - 2145*sqrt(x)*sqrt(a + b*x
)*a**5*b*c*d**2*x**3 - 1287*sqrt(x)*sqrt(a + b*x)*a**5*b*d**3*x**4 + 350*s
qrt(x)*sqrt(a + b*x)*a**4*b**2*c**3*x**2 + 1560*sqrt(x)*sqrt(a + b*x)*a**4
*b**2*c**2*d*x**3 + 2574*sqrt(x)*sqrt(a + b*x)*a**4*b**2*c*d**2*x**4 + 171
6*sqrt(x)*sqrt(a + b*x)*a**4*b**2*d**3*x**5 - 400*sqrt(x)*sqrt(a + b*x)*a*
*3*b**3*c**3*x**3 - 1872*sqrt(x)*sqrt(a + b*x)*a**3*b**3*c**2*d*x**4 - 343
2*sqrt(x)*sqrt(a + b*x)*a**3*b**3*c*d**2*x**5 - 3432*sqrt(x)*sqrt(a + b*x)
*a**3*b**3*d**3*x**6 + 480*sqrt(x)*sqrt(a + b*x)*a**2*b**4*c**3*x**4 + 249
6*sqrt(x)*sqrt(a + b*x)*a**2*b**4*c**2*d*x**5 + 6864*sqrt(x)*sqrt(a + b*x)
*a**2*b**4*c*d**2*x**6 - 640*sqrt(x)*sqrt(a + b*x)*a*b**5*c**3*x**5 - 4992
*sqrt(x)*sqrt(a + b*x)*a*b**5*c**2*d*x**6 + 1280*sqrt(x)*sqrt(a + b*x)*b**
6*c**3*x**6 + 3432*sqrt(b)*a**3*b**3*d**3*x**7 - 6864*sqrt(b)*a**2*b**4*c*
d**2*x**7 + 4992*sqrt(b)*a*b**5*c**2*d*x**7 - 1280*sqrt(b)*b**6*c**3*x**7)
)/(45045*a**6*x**7)
```

3.30 $\int \frac{x^2\sqrt{ax+bx^2}}{c+dx}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 222

$$\int \frac{x^2\sqrt{ax+bx^2}}{c+dx}\,dx = \frac{(2bc-ad)(4bc+ad)\sqrt{ax+bx^2}}{8b^2d^3} - \frac{(6bc-ad)x\sqrt{ax+bx^2}}{12bd^2} + \frac{x^2\sqrt{ax+bx^2}}{3d} - \frac{(16b^3c^3-8ab^2c^2d-2a^2bcd^2-a^3d^3)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{8b^{5/2}d^4} + \frac{2c^{5/2}\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{d^4}$$

output

```
1/8*(-a*d+2*b*c)*(a*d+4*b*c)*(b*x^2+a*x)^(1/2)/b^2/d^3-1/12*(-a*d+6*b*c)*x
*(b*x^2+a*x)^(1/2)/b/d^2+1/3*x^2*(b*x^2+a*x)^(1/2)/d-1/8*(-a^3*d^3-2*a^2*b
*c*d^2-8*a*b^2*c^2*d+16*b^3*c^3)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(5
/2)/d^4+2*c^(5/2)*(-a*d+b*c)^(1/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x
^2+a*x)^(1/2))/d^4
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.49 (sec) , antiderivative size = 484, normalized size of antiderivative = 2.18

$$\int \frac{x^2\sqrt{ax+bx^2}}{c+dx}\,dx$$
$$= \frac{\sqrt{x}\sqrt{a+bx}\left(\sqrt{b}d\sqrt{x}\sqrt{a+bx}(-3a^2d^2+2abd(-3c+dx)+4b^2(6c^2-3cdx+2d^2x^2))+48b^{3/2}c^{3/2}\left(bc-\right.\right.}{}$$

input
```
Integrate[(x^2*Sqrt[a*x + b*x^2])/(c + d*x),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(Sqrt[b]*d*Sqrt[x]*Sqrt[a + b*x]*(-3*a^2*d^2 + 2*a*
b*d*(-3*c + d*x) + 4*b^2*(6*c^2 - 3*c*d*x + 2*d^2*x^2)) + 48*b^(3/2)*c^(3/
2)*(b*c - a*d - I*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d])*Sqrt[-(b*c) + 2*a*d - (
2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*ArcTan[(Sqrt[-(b*c) + 2*a*d - (2*I)*
Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*(Sqrt[a] - Sqrt[a + b*x
]))] + 48*b^(3/2)*c^(3/2)*(b*c - a*d + I*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d])*
Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*ArcTan[(Sqrt[
-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*
(Sqrt[a] - Sqrt[a + b*x]))] + 6*(16*b^3*c^3 - 8*a*b^2*c^2*d - 2*a^2*b*c*d^
2 - a^3*d^3)*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])]))/(24*b^
(5/2)*d^4*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1261, 112, 27, 171, 27, 171, 27, 175, 65, 104, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{ax+bx^2}}{c+dx}\,dx$$

$$\Big\downarrow \ 1261$$

$$\frac{\sqrt{ax+bx^2}\int \frac{x^{5/2}\sqrt{a+bx}}{c+dx}dx}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \ 112$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{x^{5/2}\sqrt{a+bx}}{3d}-\frac{\int \frac{x^{3/2}(5ac+(6bc-ad)x)}{2\sqrt{a+bx}(c+dx)}dx}{3d}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{x^{5/2}\sqrt{a+bx}}{3d}-\frac{\int \frac{x^{3/2}(5ac+(6bc-ad)x)}{\sqrt{a+bx}(c+dx)}dx}{6d}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \ 171$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{x^{5/2}\sqrt{a+bx}}{3d}-\frac{\frac{\int -\frac{3\sqrt{x}(ac(6bc-ad)+(2bc-ad)(4bc+ad)x)}{2\sqrt{a+bx}(c+dx)}dx}{2bd}+\frac{x^{3/2}\sqrt{a+bx}(6bc-ad)}{2bd}}{6d}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{x^{5/2}\sqrt{a+bx}}{3d}-\frac{\frac{x^{3/2}\sqrt{a+bx}(6bc-ad)}{2bd}-\frac{3\int \frac{\sqrt{x}(ac(6bc-ad)+(2bc-ad)(4bc+ad)x)}{\sqrt{a+bx}(c+dx)}dx}{4bd}}{6d}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \ 171$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{x^{5/2}\sqrt{a+bx}}{3d}-\frac{\frac{x^{3/2}\sqrt{a+bx}(6bc-ad)}{2bd}-\frac{3\left(\frac{\int -\frac{ac(2bc-ad)(4bc+ad)+\left(16b^3c^3-8ab^2dc^2-2a^2bd^2c-a^3d^3\right)x}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{bd}+\frac{\sqrt{x}\sqrt{a+bx}(2bc-ad)(ad+4b\cdots)}{bd}\right)}{4bd}}{6d}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{x^{5/2}\sqrt{a+bx}}{3d}-\frac{\frac{x^{3/2}\sqrt{a+bx}(6bc-ad)}{2bd}-\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}(2bc-ad)(ad+4bc)}{bd}-\frac{\int \frac{ac(2bc-ad)(4bc+ad)+\left(16b^3c^3-8ab^2dc^2-2a^2bd^2c-a^3d^3\right)x}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{2bd}\right.}{4bd}}{6d}\right.}{\sqrt{x}\sqrt{a+bx}}$$

$\downarrow$ 175

$$\frac{\sqrt{ax+bx^2}\left(\frac{x^{5/2}\sqrt{a+bx}}{3d}-\frac{\frac{x^{3/2}\sqrt{a+bx}(6bc-ad)}{2bd}-\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}(2bc-ad)(ad+4bc)}{bd}-\frac{\frac{\left(-a^3d^3-2a^2bcd^2-8ab^2c^2d+16b^3c^3\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{d}-\frac{16b^2c^3(b}{}}{2bd}\right.}{4bd}}{6d}\right.}{\sqrt{x}\sqrt{a+bx}}$$

$\downarrow$ 65

$$\frac{\sqrt{ax+bx^2}\left(\frac{x^{5/2}\sqrt{a+bx}}{3d}-\frac{\frac{x^{3/2}\sqrt{a+bx}(6bc-ad)}{2bd}-\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}(2bc-ad)(ad+4bc)}{bd}-\frac{\frac{2\left(-a^3d^3-2a^2bcd^2-8ab^2c^2d+16b^3c^3\right)\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}-\frac{16}{}}{2bd}\right.}{4bd}}{6d}\right.}{\sqrt{x}\sqrt{a+bx}}$$

$\downarrow$ 104

$$\frac{\sqrt{ax+bx^2}\left(\frac{x^{5/2}\sqrt{a+bx}}{3d}-\frac{\frac{x^{3/2}\sqrt{a+bx}(6bc-ad)}{2bd}-\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}(2bc-ad)(ad+4bc)}{bd}-\frac{\frac{2\left(-a^3d^3-2a^2bcd^2-8ab^2c^2d+16b^3c^3\right)\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}-\frac{32}{}}{2bd}\right.}{4bd}}{6d}\right.}{\sqrt{x}\sqrt{a+bx}}$$

$\downarrow$ 219

$$\frac{\sqrt{ax+bx^2}\left(\frac{x^{5/2}\sqrt{a+bx}}{3d}-\frac{\frac{x^{3/2}\sqrt{a+bx}(6bc-ad)}{2bd}-\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}(2bc-ad)(ad+4bc)}{bd}-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)\left(-a^3d^3-2a^2bcd^2-8ab^2c^2d+16b^3c^3\right)}{\sqrt{b}d}-\cdots}{2bd}\right)}{4bd}}{6d}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$\Big\downarrow$ 221

$$\frac{\sqrt{ax+bx^2}\left(\frac{x^{5/2}\sqrt{a+bx}}{3d}-\frac{\frac{x^{3/2}\sqrt{a+bx}(6bc-ad)}{2bd}-\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}(2bc-ad)(ad+4bc)}{bd}-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)\left(-a^3d^3-2a^2bcd^2-8ab^2c^2d+16b^3c^3\right)}{\sqrt{b}d}-\cdots}{2bd}\right)}{4bd}}{6d}\right)}{\sqrt{x}\sqrt{a+bx}}$$

input
```
Int[(x^2*Sqrt[a*x + b*x^2])/(c + d*x),x]
```

output
```
(Sqrt[a*x + b*x^2]*((x^(5/2)*Sqrt[a + b*x])/(3*d) - (((6*b*c - a*d)*x^(3/2
)*Sqrt[a + b*x])/(2*b*d) - (3*(((2*b*c - a*d)*(4*b*c + a*d)*Sqrt[x]*Sqrt[a
 + b*x])/(b*d) - ((2*(16*b^3*c^3 - 8*a*b^2*c^2*d - 2*a^2*b*c*d^2 - a^3*d^3
)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(Sqrt[b]*d) - (32*b^2*c^(5/2)*
Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])]
)/d)/(2*b*d)))/(4*b*d))/(6*d)))/(Sqrt[x]*Sqrt[a + b*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 112
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*(m + n +
p + 1))), x] - Simp[1/(f*(m + n + p + 1))   Int[(a + b*x)^(m - 1)*(c + d*x)
^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*(b*e - a
*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] &&
GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m, 2*n, 2*p
] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$-\frac{\frac{d\sqrt{x(bx+a)}\left(-8b^2d^2x^2-2ab\,d^2x+12b^2cxd+3a^2d^2+6abcd-24b^2c^2\right)}{24b^2}-\frac{\left(a^3d^3+2a^2bc\,d^2+8a\,b^2c^2d-16b^3c^3\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{8b^{\frac{5}{2}}}-}{d^4}$
risch	$-\frac{\left(-8b^2d^2x^2-2ab\,d^2x+12b^2cxd+3a^2d^2+6abcd-24b^2c^2\right)x(bx+a)}{24b^2d^3\sqrt{x(bx+a)}}+\frac{\frac{\left(a^3d^3+2a^2bc\,d^2+8a\,b^2c^2d-16b^3c^3\right)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2\right)}{d\sqrt{b}}}{}$
default	$\frac{\frac{\left(b\,x^2+ax\right)^{\frac{3}{2}}}{3b}-\frac{a\left(\frac{(2bx+a)\sqrt{b\,x^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8b^{\frac{3}{2}}}\right)}{2b}}{d}+\frac{c^2\left(\sqrt{b(x+\frac{c}{d})^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}+\frac{(ad-2bc}{}\right.}{}$

input
```
int(x^2*(b*x^2+a*x)^(1/2)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/d^4*(1/24*d*(x*(b*x+a))^(1/2)*(-8*b^2*d^2*x^2-2*a*b*d^2*x+12*b^2*c*d*x+
3*a^2*d^2+6*a*b*c*d-24*b^2*c^2)/b^2-1/8*(a^3*d^3+2*a^2*b*c*d^2+8*a*b^2*c^2
*d-16*b^3*c^3)/b^(5/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-2*(a*d-b*c)*c^
3/(c*(a*d-b*c))^(1/2)*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 860, normalized size of antiderivative = 3.87

$$\int \frac{x^2\sqrt{ax+bx^2}}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x^2+a*x)^(1/2)/(d*x+c),x, algorithm="fricas")
```

output
```
[1/48*(48*sqrt(b*c^2 - a*c*d)*b^3*c^2*log((a*c + (2*b*c - a*d)*x + 2*sqrt(
b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) - 3*(16*b^3*c^3 - 8*a*b^2*c^2
*d - 2*a^2*b*c*d^2 - a^3*d^3)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*
sqrt(b)) + 2*(8*b^3*d^3*x^2 + 24*b^3*c^2*d - 6*a*b^2*c*d^2 - 3*a^2*b*d^3 -
 2*(6*b^3*c*d^2 - a*b^2*d^3)*x)*sqrt(b*x^2 + a*x))/(b^3*d^4), -1/48*(96*sq
rt(-b*c^2 + a*c*d)*b^3*c^2*arctan(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(
b*c*x + a*c)) + 3*(16*b^3*c^3 - 8*a*b^2*c^2*d - 2*a^2*b*c*d^2 - a^3*d^3)*s
qrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*(8*b^3*d^3*x^2 + 2
4*b^3*c^2*d - 6*a*b^2*c*d^2 - 3*a^2*b*d^3 - 2*(6*b^3*c*d^2 - a*b^2*d^3)*x)
*sqrt(b*x^2 + a*x))/(b^3*d^4), 1/24*(24*sqrt(b*c^2 - a*c*d)*b^3*c^2*log((a
*c + (2*b*c - a*d)*x + 2*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c))
 + 3*(16*b^3*c^3 - 8*a*b^2*c^2*d - 2*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b)*arcta
n(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (8*b^3*d^3*x^2 + 24*b^3*c^2*d -
6*a*b^2*c*d^2 - 3*a^2*b*d^3 - 2*(6*b^3*c*d^2 - a*b^2*d^3)*x)*sqrt(b*x^2 +
a*x))/(b^3*d^4), -1/24*(48*sqrt(-b*c^2 + a*c*d)*b^3*c^2*arctan(sqrt(-b*c^2
 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) - 3*(16*b^3*c^3 - 8*a*b^2*c^2*d
 - 2*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*
x + a)) - (8*b^3*d^3*x^2 + 24*b^3*c^2*d - 6*a*b^2*c*d^2 - 3*a^2*b*d^3 - 2*
(6*b^3*c*d^2 - a*b^2*d^3)*x)*sqrt(b*x^2 + a*x))/(b^3*d^4)]
```

Sympy [F]

$$\int \frac{x^2\sqrt{ax+bx^2}}{c+dx}\,dx = \int \frac{x^2\sqrt{x\,(a+bx)}}{c+dx}\,dx$$

input
```
integrate(x**2*(b*x**2+a*x)**(1/2)/(d*x+c),x)
```

output
```
Integral(x**2*sqrt(x*(a + b*x))/(c + d*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{ax+bx^2}}{c+dx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(b*x^2+a*x)^(1/2)/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-2*b*c>0)', see `assume?` for
 more deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{ax+bx^2}}{c+dx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(b*x^2+a*x)^(1/2)/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{ax+bx^2}}{c+dx}\,dx=\int \frac{x^2\,\sqrt{b\,x^2+a\,x}}{c+d\,x}\,dx$$

input
```
int((x^2*(a*x + b*x^2)^(1/2))/(c + d*x),x)
```

output
```
int((x^2*(a*x + b*x^2)^(1/2))/(c + d*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.55

$$\int \frac{x^2\sqrt{ax+bx^2}}{c+dx}\,dx = \frac{48\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)b^3c^2+48\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}+\sqrt{d}\,\sqrt{bx+a}+\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)b}{}$$

input
```
int(x^2*(b*x^2+a*x)^(1/2)/(d*x+c),x)
```

output
```
(48*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x)
- sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**3*c**2 + 48*sqrt(c)*sqrt(
a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)
*sqrt(b))/(sqrt(c)*sqrt(b)))*b**3*c**2 - 3*sqrt(x)*sqrt(a + b*x)*a**2*b*d*
*3 - 6*sqrt(x)*sqrt(a + b*x)*a*b**2*c*d**2 + 2*sqrt(x)*sqrt(a + b*x)*a*b**
2*d**3*x + 24*sqrt(x)*sqrt(a + b*x)*b**3*c**2*d - 12*sqrt(x)*sqrt(a + b*x)
*b**3*c*d**2*x + 8*sqrt(x)*sqrt(a + b*x)*b**3*d**3*x**2 + 3*sqrt(b)*log((s
qrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*d**3 + 6*sqrt(b)*log((sqrt(a
 + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*b*c*d**2 + 24*sqrt(b)*log((sqrt(a
 + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b**2*c**2*d - 48*sqrt(b)*log((sqrt(a
 + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b**3*c**3)/(24*b**3*d**4)
```

3.31 $\int \frac{x\sqrt{ax+bx^2}}{c+dx} dx$

Optimal result

Integrand size = 22, antiderivative size = 165

$$\int \frac{x\sqrt{ax+bx^2}}{c+dx} dx = -\frac{(4bc-ad)\sqrt{ax+bx^2}}{4bd^2} + \frac{x\sqrt{ax+bx^2}}{2d} + \frac{(8b^2c^2-4abcd-a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}d^3} - \frac{2c^{3/2}\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{d^3}$$

output
```
-1/4*(-a*d+4*b*c)*(b*x^2+a*x)^(1/2)/b/d^2+1/2*x*(b*x^2+a*x)^(1/2)/d+1/4*(-
a^2*d^2-4*a*b*c*d+8*b^2*c^2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(3/2)/
d^3-2*c^(3/2)*(-a*d+b*c)^(1/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a
*x)^(1/2))/d^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.69 (sec) , antiderivative size = 438, normalized size of antiderivative = 2.65

$$\int \frac{x\sqrt{ax+bx^2}}{c+dx}\,dx$$
$$= \frac{\sqrt{x}\sqrt{a+bx}\left(\sqrt{b}d\sqrt{x}\sqrt{a+bx}(-4bc+ad+2bdx)+8\sqrt{b}\sqrt{c}\left(bc-ad-i\sqrt{a}\sqrt{d}\sqrt{bc-ad}\right)\sqrt{-bc+2ad}\right.}{}$$

input
```
Integrate[(x*Sqrt[a*x + b*x^2])/(c + d*x),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(Sqrt[b]*d*Sqrt[x]*Sqrt[a + b*x]*(-4*b*c + a*d + 2*
b*d*x) + 8*Sqrt[b]*Sqrt[c]*(b*c - a*d - I*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d])
*Sqrt[-(b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*ArcTan[(Sqrt
[-(b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]
*(-Sqrt[a] + Sqrt[a + b*x]))] + 8*Sqrt[b]*Sqrt[c]*(b*c - a*d + I*Sqrt[a]*S
qrt[d]*Sqrt[b*c - a*d])*Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b
*c - a*d]]*ArcTan[(Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c -
a*d]]*Sqrt[x])/(Sqrt[c]*(-Sqrt[a] + Sqrt[a + b*x]))] + 2*(8*b^2*c^2 - 4*a*
b*c*d - a^2*d^2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(
4*b^(3/2)*d^3*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1231, 27, 1269, 1091, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{ax+bx^2}}{c+dx}\,dx$$
$$\Big\downarrow \text{1231}$$

$$-\frac{\int -\frac{ac(4bc-ad)+(8b^2c^2-4abdc-a^2d^2)x}{2(c+dx)\sqrt{bx^2+ax}}dx}{4bd^2}-\frac{\sqrt{ax+bx^2}(-ad+4bc-2bdx)}{4bd^2}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{ac(4bc-ad)+(8b^2c^2-4abdc-a^2d^2)x}{(c+dx)\sqrt{bx^2+ax}}dx}{8bd^2}-\frac{\sqrt{ax+bx^2}(-ad+4bc-2bdx)}{4bd^2}$$

$\Big\downarrow$ 1269

$$\frac{\frac{(-a^2d^2-4abcd+8b^2c^2)\int \frac{1}{\sqrt{bx^2+ax}}dx}{d}-\frac{8bc^2(bc-ad)\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{8bd^2}-\frac{\sqrt{ax+bx^2}(-ad+4bc-2bdx)}{4bd^2}$$

$\Big\downarrow$ 1091

$$\frac{\frac{2(-a^2d^2-4abcd+8b^2c^2)\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{d}-\frac{8bc^2(bc-ad)\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{8bd^2}-\frac{\sqrt{ax+bx^2}(-ad+4bc-2bdx)}{4bd^2}$$

$\Big\downarrow$ 219

$$\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(-a^2d^2-4abcd+8b^2c^2)}{\sqrt{b}d}-\frac{8bc^2(bc-ad)\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{8bd^2}-\frac{\sqrt{ax+bx^2}(-ad+4bc-2bdx)}{4bd^2}$$

$\Big\downarrow$ 1154

$$\frac{\frac{16bc^2(bc-ad)\int \frac{1}{4c(bc-ad)-\frac{(ac+(2bc-ad)x)^2}{bx^2+ax}}d\left(-\frac{ac+(2bc-ad)x}{\sqrt{bx^2+ax}}\right)}{d}+\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(-a^2d^2-4abcd+8b^2c^2)}{\sqrt{b}d}}{8bd^2}-\frac{\sqrt{ax+bx^2}(-ad+4bc-2bdx)}{4bd^2}$$

$\Big\downarrow$ 219

$$\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(-a^2d^2-4abcd+8b^2c^2)}{\sqrt{b}d}-\frac{8bc^{3/2}\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{d}}{8bd^2}-\frac{\sqrt{ax+bx^2}(-ad+4bc-2bdx)}{4bd^2}$$

input

```
Int[(x*Sqrt[a*x + b*x^2])/(c + d*x),x]
```

output
```
-1/4*((4*b*c - a*d - 2*b*d*x)*Sqrt[a*x + b*x^2])/(b*d^2) + ((2*(8*b^2*c^2
- 4*a*b*c*d - a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(Sqrt[b]*d)
 - (8*b*c^(3/2)*Sqrt[b*c - a*d]*ArcTanh[(a*c + (2*b*c - a*d)*x)/(2*Sqrt[c]
*Sqrt[b*c - a*d]*Sqrt[a*x + b*x^2])])/d)/(8*b*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$-\frac{-\frac{d\sqrt{x(bx+a)}\,(2bdx+ad-4bc)}{4b}+\frac{\left(a^2d^2+4abcd-8b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{4b^{\frac{3}{2}}}+\frac{2c^2(ad-bc)\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{\sqrt{c(ad-bc)}}}{d^3}$
risch	$\frac{(2bdx+ad-4bc)x(bx+a)}{4b\,d^2\sqrt{x(bx+a)}}-\frac{\frac{\left(a^2d^2+4abcd-8b^2c^2\right)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{d\sqrt{b}}+\frac{8c^2(ad-bc)b\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\cdots}{\cdots}\right)}{d^2\sqrt{\cdots}}}{8b\,d^2}$
default	$\frac{\frac{(2bx+a)\sqrt{b\,x^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8b^{\frac{3}{2}}}}{d}-\frac{c\left(\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}+\frac{(ad-2bc)\ln\left(\frac{\frac{ad-2bc}{2d}+b\left(x+\frac{c}{d}\right)}{\sqrt{b}}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$

input
```
int(x*(b*x^2+a*x)^(1/2)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/d^3*(-1/4*d*(x*(b*x+a))^(1/2)*(2*b*d*x+a*d-4*b*c)/b+1/4*(a^2*d^2+4*a*b*
c*d-8*b^2*c^2)/b^(3/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))+2*c^2*(a*d-b*c
)/(c*(a*d-b*c))^(1/2)*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 639, normalized size of antiderivative = 3.87

$$\int \frac{x\sqrt{ax+bx^2}}{c+dx}\,dx$$
$$= \left[\frac{8\,\sqrt{bc^2-acd}b^2c\log\left(\frac{ac+(2\,bc-ad)x-2\,\sqrt{bc^2-acd}\sqrt{bx^2+ax}}{dx+c}\right)-(8\,b^2c^2-4\,abcd-a^2d^2)\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{b}\right.}{8\,b^2d^3}\right.$$

input
```
integrate(x*(b*x^2+a*x)^(1/2)/(d*x+c),x, algorithm="fricas")
```

output
```
[1/8*(8*sqrt(b*c^2 - a*c*d)*b^2*c*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*c^
2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) - (8*b^2*c^2 - 4*a*b*c*d - a^2*d^
2)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(2*b^2*d^2*x -
 4*b^2*c*d + a*b*d^2)*sqrt(b*x^2 + a*x))/(b^2*d^3), 1/8*(16*sqrt(-b*c^2 +
a*c*d)*b^2*c*arctan(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c))
- (8*b^2*c^2 - 4*a*b*c*d - a^2*d^2)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 +
 a*x)*sqrt(b)) + 2*(2*b^2*d^2*x - 4*b^2*c*d + a*b*d^2)*sqrt(b*x^2 + a*x))/
(b^2*d^3), 1/4*(4*sqrt(b*c^2 - a*c*d)*b^2*c*log((a*c + (2*b*c - a*d)*x - 2
*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) - (8*b^2*c^2 - 4*a*b*c*
d - a^2*d^2)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (2*b^
2*d^2*x - 4*b^2*c*d + a*b*d^2)*sqrt(b*x^2 + a*x))/(b^2*d^3), 1/4*(8*sqrt(-
b*c^2 + a*c*d)*b^2*c*arctan(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x
+ a*c)) - (8*b^2*c^2 - 4*a*b*c*d - a^2*d^2)*sqrt(-b)*arctan(sqrt(b*x^2 + a
*x)*sqrt(-b)/(b*x + a)) + (2*b^2*d^2*x - 4*b^2*c*d + a*b*d^2)*sqrt(b*x^2 +
 a*x))/(b^2*d^3)]
```

Sympy [F]

$$\int \frac{x\sqrt{ax+bx^2}}{c+dx}\,dx = \int \frac{x\sqrt{x\,(a+bx)}}{c+dx}\,dx$$

input
```
integrate(x*(b*x**2+a*x)**(1/2)/(d*x+c),x)
```

output
```
Integral(x*sqrt(x*(a + b*x))/(c + d*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{ax+bx^2}}{c+dx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(b*x^2+a*x)^(1/2)/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-2*b*c>0)', see `assume?` for
 more deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{ax+bx^2}}{c+dx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(b*x^2+a*x)^(1/2)/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{ax+bx^2}}{c+dx}\,dx = \int \frac{x\,\sqrt{b\,x^2+a\,x}}{c+d\,x}\,dx$$

input
```
int((x*(a*x + b*x^2)^(1/2))/(c + d*x),x)
```

output
```
int((x*(a*x + b*x^2)^(1/2))/(c + d*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.48

$$\int \frac{x\sqrt{ax+bx^2}}{c+dx}\,dx$$
$$= \frac{-8\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)b^2c-8\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}+\sqrt{d}\,\sqrt{bx+a}+\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)b^2}{}$$

input
```
int(x*(b*x^2+a*x)^(1/2)/(d*x+c),x)
```

output
```
( - 8*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x
) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**2*c - 8*sqrt(c)*sqrt(a*
d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*s
qrt(b))/(sqrt(c)*sqrt(b)))*b**2*c + sqrt(x)*sqrt(a + b*x)*a*b*d**2 - 4*sqr
t(x)*sqrt(a + b*x)*b**2*c*d + 2*sqrt(x)*sqrt(a + b*x)*b**2*d**2*x - sqrt(b
)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*d**2 - 4*sqrt(b)*log
((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b*c*d + 8*sqrt(b)*log((sqrt(
a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b**2*c**2)/(4*b**2*d**3)
```

3.32 $\int \frac{\sqrt{ax+bx^2}}{c+dx}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 113

$$\int \frac{\sqrt{ax+bx^2}}{c+dx}\,dx = \frac{\sqrt{ax+bx^2}}{d} - \frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}d^2} + \frac{2\sqrt{c}\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{d^2}$$

output
```
(b*x^2+a*x)^(1/2)/d-(-a*d+2*b*c)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(1
/2)/d^2+2*c^(1/2)*(-a*d+b*c)^(1/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x
^2+a*x)^(1/2))/d^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.27 (sec) , antiderivative size = 402, normalized size of antiderivative = 3.56

$$\int \frac{\sqrt{ax+bx^2}}{c+dx}\,dx = \frac{\sqrt{x}\sqrt{a+bx}\left(b\sqrt{c}d\sqrt{x}\sqrt{a+bx}+2\left(bc-ad-i\sqrt{a}\sqrt{d}\sqrt{bc-ad}\right)\sqrt{-bc+2ad-2i\sqrt{a}\sqrt{d}\sqrt{bc-ad}}\,\text{arct}}{}$$

input
```
Integrate[Sqrt[a*x + b*x^2]/(c + d*x),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(b*Sqrt[c]*d*Sqrt[x]*Sqrt[a + b*x] + 2*(b*c - a*d -
 I*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d])*Sqrt[-(b*c) + 2*a*d - (2*I)*Sqrt[a]*Sq
rt[d]*Sqrt[b*c - a*d]]*ArcTan[(Sqrt[-(b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]
*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*(Sqrt[a] - Sqrt[a + b*x]))] + 2*(b*c -
 a*d + I*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d])*Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt
[a]*Sqrt[d]*Sqrt[b*c - a*d]]*ArcTan[(Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*S
qrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*(Sqrt[a] - Sqrt[a + b*x]))] + 2*
Sqrt[b]*Sqrt[c]*(2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a
+ b*x])]))/(b*Sqrt[c]*d^2*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1162, 1269, 1091, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{c+dx}\,dx$$

$$\Big\downarrow \ 1162$$

$$\frac{\sqrt{ax+bx^2}}{d}-\frac{\int \frac{ac+(2bc-ad)x}{(c+dx)\sqrt{bx^2+ax}}dx}{2d}$$

$$\Big\downarrow \ 1269$$

$$\frac{\sqrt{ax+bx^2}}{d}-\frac{\frac{(2bc-ad)\int \frac{1}{\sqrt{bx^2+ax}}dx}{d}-\frac{2c(bc-ad)\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{2d}$$

$$\Big\downarrow \ 1091$$

$$\frac{\sqrt{ax+bx^2}}{d}-\frac{\frac{2(2bc-ad)\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{d}-\frac{2c(bc-ad)\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{2d}$$

$$\Big\downarrow \ 219$$

$$\frac{\sqrt{ax+bx^2}}{d}-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(2bc-ad)}{\sqrt{b}d}-\frac{2c(bc-ad)\int\frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{2d}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\sqrt{ax+bx^2}}{d}-\frac{\frac{4c(bc-ad)\int\frac{1}{4c(bc-ad)-\frac{(ac+(2bc-ad)x)^2}{bx^2+ax}}d\left(-\frac{ac+(2bc-ad)x}{\sqrt{bx^2+ax}}\right)}{d}+\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(2bc-ad)}{\sqrt{b}d}}{2d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{ax+bx^2}}{d}-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(2bc-ad)}{\sqrt{b}d}-\frac{2\sqrt{c}\sqrt{bc-ad}\text{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{d}}{2d}$$

input
```
Int[Sqrt[a*x + b*x^2]/(c + d*x),x]
```

output
```
Sqrt[a*x + b*x^2]/d - ((2*(2*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x
^2]])/(Sqrt[b]*d) - (2*Sqrt[c]*Sqrt[b*c - a*d]*ArcTanh[(a*c + (2*b*c - a*d
)*x)/(2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[a*x + b*x^2])])/d)/(2*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$-\frac{-d\sqrt{x(bx+a)}-\frac{(ad-2bc)\,\text{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{\sqrt{b}}-\frac{2c(ad-bc)\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{\sqrt{c(ad-bc)}}}{d^2}$
risch	$\frac{x(bx+a)}{d\sqrt{x(bx+a)}}+\frac{\frac{(ad-2bc)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{d\sqrt{b}}+\frac{2c(ad-bc)\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(a}{}}}{x+\frac{c}{d}}\right.}{d^2\sqrt{-\frac{c(ad-bc)}{d^2}}}}{2d}$
default	$\frac{\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}+\frac{(ad-2bc)\ln\left(\frac{\frac{ad-2bc}{2d}+b\left(x+\frac{c}{d}\right)}{\sqrt{b}}+\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}\right)}{2d\sqrt{b}}+\frac{c(ad-b}{}}{d}$

input
```
int((b*x^2+a*x)^(1/2)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/d^2*(-d*(x*(b*x+a))^(1/2)-(a*d-2*b*c)/b^(1/2)*arctanh((x*(b*x+a))^(1/2)
/x/b^(1/2))-2*c*(a*d-b*c)/(c*(a*d-b*c))^(1/2)*arctan((x*(b*x+a))^(1/2)/x*c
/(c*(a*d-b*c))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 482, normalized size of antiderivative = 4.27

$$\int \frac{\sqrt{ax+bx^2}}{c+dx}\,dx$$
$$= \left[\frac{2\sqrt{bx^2+ax}bd-(2bc-ad)\sqrt{b}\log\left(2bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)+2\sqrt{bc^2-acd}b\log\left(\frac{ac+(2bc-ad)x+2\sqrt{\cdot}}{dx+\cdot}\right)}{2bd^2}\right.$$

input
```
integrate((b*x^2+a*x)^(1/2)/(d*x+c),x, algorithm="fricas")
```

output
```
[1/2*(2*sqrt(b*x^2 + a*x)*b*d - (2*b*c - a*d)*sqrt(b)*log(2*b*x + a + 2*sq
rt(b*x^2 + a*x)*sqrt(b)) + 2*sqrt(b*c^2 - a*c*d)*b*log((a*c + (2*b*c - a*d
)*x + 2*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)))/(b*d^2), 1/2*(2
*sqrt(b*x^2 + a*x)*b*d - 4*sqrt(-b*c^2 + a*c*d)*b*arctan(sqrt(-b*c^2 + a*c
*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) - (2*b*c - a*d)*sqrt(b)*log(2*b*x + a
 + 2*sqrt(b*x^2 + a*x)*sqrt(b)))/(b*d^2), (sqrt(b*x^2 + a*x)*b*d + (2*b*c
- a*d)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + sqrt(b*c^2
- a*c*d)*b*log((a*c + (2*b*c - a*d)*x + 2*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 +
 a*x))/(d*x + c)))/(b*d^2), (sqrt(b*x^2 + a*x)*b*d - 2*sqrt(-b*c^2 + a*c*d
)*b*arctan(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) + (2*b*c
- a*d)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)))/(b*d^2)]
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2}}{c+dx}\,dx = \int \frac{\sqrt{x(a+bx)}}{c+dx}\,dx$$

input
```
integrate((b*x**2+a*x)**(1/2)/(d*x+c),x)
```

output
```
Integral(sqrt(x*(a + b*x))/(c + d*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax+bx^2}}{c+dx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a*x)^(1/2)/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax+bx^2}}{c+dx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a*x)^(1/2)/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^2}}{c+dx}\,dx = \int \frac{\sqrt{b\,x^2+a\,x}}{c+d\,x}\,dx$$

input
```
int((a*x + b*x^2)^(1/2)/(c + d*x),x)
```

output
```
int((a*x + b*x^2)^(1/2)/(c + d*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{ax+bx^2}}{c+dx}\,dx$$
$$= \frac{2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)b+2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}+\sqrt{d}\,\sqrt{bx+a}+\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)b+\sqrt{}}{b\,d^2}$$

input
```
int((b*x^2+a*x)^(1/2)/(d*x+c),x)
```

output
```
(2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) -
 sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b + 2*sqrt(c)*sqrt(a*d - b*c)
*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/
(sqrt(c)*sqrt(b)))*b + sqrt(x)*sqrt(a + b*x)*b*d + sqrt(b)*log((sqrt(a + b
*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*d - 2*sqrt(b)*log((sqrt(a + b*x) + sqrt(
x)*sqrt(b))/sqrt(a))*b*c)/(b*d**2)
```

3.33 $\int \frac{\sqrt{ax+bx^2}}{x(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)}\,dx = \frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{d} - \frac{2\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{\sqrt{c}d}$$

output
```
2*b^(1/2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/d-2*(-a*d+b*c)^(1/2)*arctan
h((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)}\,dx = -\frac{2\sqrt{x}\sqrt{a+bx}\left(\sqrt{-bc+ad}\arctan\left(\frac{-d\sqrt{x}\sqrt{a+bx}+\sqrt{b}(c+dx)}{\sqrt{c}\sqrt{-bc+ad}}\right)+\sqrt{b}\sqrt{c}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)\right)}{\sqrt{c}d\sqrt{x(a+bx)}}$$

input
```
Integrate[Sqrt[a*x + b*x^2]/(x*(c + d*x)),x]
```

output
```
(-2*Sqrt[x]*Sqrt[a + b*x]*(Sqrt[-(b*c) + a*d]*ArcTan[(-(d*Sqrt[x]*Sqrt[a +
 b*x]) + Sqrt[b]*(c + d*x))/(Sqrt[c]*Sqrt[-(b*c) + a*d])] + Sqrt[b]*Sqrt[c
]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]))/(Sqrt[c]*d*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.68, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1260, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)}\,dx$$

$$\Big\downarrow \text{1260}$$

$$\int \left(\frac{\sqrt{ax+bx^2}}{cx}-\frac{d\sqrt{ax+bx^2}}{c(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(2bc-ad)}{\sqrt{b}cd}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{\sqrt{c}d}+\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}c}$$

input
```
Int[Sqrt[a*x + b*x^2]/(x*(c + d*x)),x]
```

output
```
(a*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(Sqrt[b]*c) + ((2*b*c - a*d)*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(Sqrt[b]*c*d) - (Sqrt[b*c - a*d]*Arc
Tanh[(a*c + (2*b*c - a*d)*x)/(2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[a*x + b*x^2])
])/(Sqrt[c]*d)
```

Defintions of rubi rules used

rule 1260
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x + c*x^2)^p
, (d + e*x)^m*(f + g*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ
[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + n + 2*p + 1, 0] && ILtQ[m, 0] && ILtQ
[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$-\frac{2\left(-\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)+\frac{(ad-bc)\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{\sqrt{c(ad-bc)}}\right)}{d}$
default	$\frac{\sqrt{b\,x^2+ax}+\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2\sqrt{b}}}{c}-\frac{\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}+\frac{(ad-2bc)\ln\left(\frac{\frac{ad-2bc}{2d}+b\left(x+\frac{c}{d}\right)}{\sqrt{b}}+\sqrt{b\left(x+\frac{c}{d}\right.}\right.}{2d\sqrt{b}}}{}$

input
```
int((b*x^2+a*x)^(1/2)/x/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(-b^(1/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))+(a*d-b*c)/(c*(a*d-b*c)
)^(1/2)*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 396, normalized size of antiderivative = 4.55

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)}\,dx = \left[\frac{\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)x-2\,\sqrt{bx^2+ax}c\sqrt{\frac{bc-ad}{c}}}{dx+c}\right)}{d}, -\frac{2\,\sqrt{-\frac{bc-ad}{c}}\arctan\left(-\frac{\sqrt{bx^2+ax}c\sqrt{-\frac{bc-ad}{c}}}{(bc-ad)x}\right)-\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{d}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)x-2\,\sqrt{bx^2+ax}c\sqrt{\frac{bc-ad}{c}}}{dx+c}\right)}{d}, -\frac{2\left(\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+\sqrt{-\frac{bc-ad}{c}}\arctan\left(-\frac{\sqrt{bx^2+ax}c\sqrt{-\frac{bc-ad}{c}}}{(bc-ad)x}\right)\right)}{d}\right]$$

input
```
integrate((b*x^2+a*x)^(1/2)/x/(d*x+c),x, algorithm="fricas")
```

output
```
[(sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + sqrt((b*c - a*d)/
c)*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)*c*sqrt((b*c - a*d)/c))
/(d*x + c)))/d, -(2*sqrt(-(b*c - a*d)/c)*arctan(-sqrt(b*x^2 + a*x)*c*sqrt(
-(b*c - a*d)/c)/((b*c - a*d)*x)) - sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 +
a*x)*sqrt(b)))/d, -(2*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)
) - sqrt((b*c - a*d)/c)*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)*c
*sqrt((b*c - a*d)/c))/(d*x + c)))/d, -2*(sqrt(-b)*arctan(sqrt(b*x^2 + a*x)
*sqrt(-b)/(b*x + a)) + sqrt(-(b*c - a*d)/c)*arctan(-sqrt(b*x^2 + a*x)*c*sq
rt(-(b*c - a*d)/c)/((b*c - a*d)*x)))/d]
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)}\,dx = \int \frac{\sqrt{x\,(a+bx)}}{x\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a*x)**(1/2)/x/(d*x+c),x)
```

output
```
Integral(sqrt(x*(a + b*x))/(x*(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a*x)^(1/2)/x/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a*x)^(1/2)/x/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)}\,dx = \int \frac{\sqrt{b\,x^2+a\,x}}{x\,(c+d\,x)}\,dx$$

input
```
int((a*x + b*x^2)^(1/2)/(x*(c + d*x)),x)
```

output
```
int((a*x + b*x^2)^(1/2)/(x*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)}\,dx = \frac{-2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)-2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}+\sqrt{d}\,\sqrt{bx+a}+\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)+2\sqrt{}}{cd}$$

input
```
int((b*x^2+a*x)^(1/2)/x/(d*x+c),x)
```

output
```
(2*( - sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*
x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b))) - sqrt(c)*sqrt(a*d - b*c)
*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/
(sqrt(c)*sqrt(b))) + sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)
)*c))/(c*d)
```

3.34 $\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)}\,dx = -\frac{2\sqrt{ax+bx^2}}{cx}+\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{3/2}}$$

output
```
-2*(b*x^2+a*x)^(1/2)/c/x+2*(-a*d+b*c)^(1/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(
1/2)/(b*x^2+a*x)^(1/2))/c^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 346, normalized size of antiderivative = 4.68

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)}\,dx$$
$$= \frac{2\sqrt{a+bx}\left(-bc^{3/2}\sqrt{a+bx}+\left(bc-ad-i\sqrt{a}\sqrt{d}\sqrt{bc-ad}\right)\sqrt{-bc+2ad-2i\sqrt{a}\sqrt{d}\sqrt{bc-ad}}\sqrt{x}\,\text{arctan}}{}$$

input
```
Integrate[Sqrt[a*x + b*x^2]/(x^2*(c + d*x)),x]
```

output
```
(2*Sqrt[a + b*x]*(-(b*c^(3/2)*Sqrt[a + b*x]) + (b*c - a*d - I*Sqrt[a]*Sqrt
[d]*Sqrt[b*c - a*d])*Sqrt[-(b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c
- a*d]]*Sqrt[x]*ArcTan[(Sqrt[-(b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b
*c - a*d]]*Sqrt[x])/(Sqrt[c]*(Sqrt[a] - Sqrt[a + b*x]))] + (b*c - a*d + I*
Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d])*Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[
d]*Sqrt[b*c - a*d]]*Sqrt[x]*ArcTan[(Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sq
rt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*(Sqrt[a] - Sqrt[a + b*x]))]))/(b*
c^(5/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1261, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)}dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{\sqrt{ax+bx^2}\int \frac{\sqrt{a+bx}}{x^{3/2}(c+dx)}dx}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{105}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{(bc-ad)\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{c}-\frac{2\sqrt{a+bx}}{c\sqrt{x}}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{104}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{2(bc-ad)\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}-\frac{2\sqrt{a+bx}}{c\sqrt{x}}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{2\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}}-\frac{2\sqrt{a+bx}}{c\sqrt{x}}\right)}{\sqrt{x}\sqrt{a+bx}}$$

input
```
Int[Sqrt[a*x + b*x^2]/(x^2*(c + d*x)),x]
```

output
```
(Sqrt[a*x + b*x^2]*((-2*Sqrt[a + b*x])/(c*Sqrt[x]) + (2*Sqrt[b*c - a*d]*Ar
cTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/c^(3/2)))/(Sqrt[
x]*Sqrt[a + b*x])
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$\frac{-\frac{2\sqrt{x(bx+a)}}{x}+\frac{2(ad-bc)\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{\sqrt{c(ad-bc)}}}{c}$
risch	$-\frac{2(bx+a)}{c\sqrt{x(bx+a)}}+\frac{(ad-bc)\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{x+\frac{c}{d}}\right)}{dc\sqrt{-\frac{c(ad-bc)}{d^2}}}$
default	$\frac{-\frac{2\left(bx^2+ax\right)^{\frac{3}{2}}}{a\,x^2}+\frac{2b\left(\sqrt{bx^2+ax}+\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{bx^2+ax}\right)}{2\sqrt{b}}\right)}{a}}{c}+\frac{d\left(\sqrt{b(x+\frac{c}{d})^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}+\frac{(ad-2bc)\ln\left(\ldots\right.}{\ldots}\right.}{\ldots}$

input
```
int((b*x^2+a*x)^(1/2)/x^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
2/c*(-(x*(b*x+a))^(1/2)/x+(a*d-b*c)/(c*(a*d-b*c))^(1/2)*arctan((x*(b*x+a))
^(1/2)/x*c/(c*(a*d-b*c))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.32

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)}\,dx$$
$$=\left[\frac{x\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)x+2\,\sqrt{bx^2+ax}c\sqrt{\frac{bc-ad}{c}}}{dx+c}\right)-2\,\sqrt{bx^2+ax}}{cx},\ \frac{2\left(x\sqrt{-\frac{bc-ad}{c}}\arctan\left(-\frac{\sqrt{bx^2+ax}c\sqrt{-\frac{bc-ad}{c}}}{(bc-ad)x}\right.\right.}{cx}\right.$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^2/(d*x+c),x, algorithm="fricas")
```

output
```
[(x*sqrt((b*c - a*d)/c)*log((a*c + (2*b*c - a*d)*x + 2*sqrt(b*x^2 + a*x)*c
*sqrt((b*c - a*d)/c))/(d*x + c)) - 2*sqrt(b*x^2 + a*x))/(c*x), 2*(x*sqrt(-
(b*c - a*d)/c)*arctan(-sqrt(b*x^2 + a*x)*c*sqrt(-(b*c - a*d)/c)/((b*c - a*
d)*x)) - sqrt(b*x^2 + a*x))/(c*x)]
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)}\,dx = \int \frac{\sqrt{x\,(a+bx)}}{x^2\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a*x)**(1/2)/x**2/(d*x+c),x)
```

output
```
Integral(sqrt(x*(a + b*x))/(x**2*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)}\,dx = \int \frac{\sqrt{bx^2+ax}}{(dx+c)x^2}\,dx$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^2/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a*x)/((d*x + c)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)}\,dx = \frac{2\,(bc-ad)\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)d+\sqrt{b}c}{\sqrt{-bc^2+acd}}\right)}{\sqrt{-bc^2+acd}c} + \frac{2\,a}{\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)c}$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^2/(d*x+c),x, algorithm="giac")
```

output
```
2*(b*c - a*d)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a*x))*d + sqrt(b)*c)/sqrt
(-b*c^2 + a*c*d))/(sqrt(-b*c^2 + a*c*d)*c) + 2*a/((sqrt(b)*x - sqrt(b*x^2
+ a*x))*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)}\,dx = \int \frac{\sqrt{b\,x^2+a\,x}}{x^2\,\left(c+d\,x\right)}\,dx$$

input
```
int((a*x + b*x^2)^(1/2)/(x^2*(c + d*x)),x)
```

output
```
int((a*x + b*x^2)^(1/2)/(x^2*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)}\,dx = \frac{2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)x+2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}+\sqrt{d}\,\sqrt{bx+a}+\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)x-2}{c^2x}$$

input
```
int((b*x^2+a*x)^(1/2)/x^2/(d*x+c),x)
```

output
```
(2*(sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x)
- sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*x + sqrt(c)*sqrt(a*d - b*c)*
atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(
sqrt(c)*sqrt(b)))*x - sqrt(x)*sqrt(a + b*x)*c - sqrt(b)*c*x))/(c**2*x)
```

3.35 $\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 111

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)}\,dx = -\frac{2\sqrt{ax+bx^2}}{3cx^2}-\frac{2(bc-3ad)\sqrt{ax+bx^2}}{3ac^2x} - \frac{2d\sqrt{bc-ad}\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{5/2}}$$

output
```
-2/3*(b*x^2+a*x)^(1/2)/c/x^2-2/3*(-3*a*d+b*c)*(b*x^2+a*x)^(1/2)/a/c^2/x-2*
d*(-a*d+b*c)^(1/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c
^(5/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)}\,dx = -\frac{2\sqrt{x(a+bx)}\left(\sqrt{c}\sqrt{a+bx}(bcx+a(c-3dx))+3ad\sqrt{-bc+ad}\,x^{3/2}\arctan\left(\frac{-d\sqrt{x}\sqrt{a+bx}+\sqrt{b}(c+dx)}{\sqrt{c}\sqrt{-bc+ad}}\right)\right)}{3ac^{5/2}x^2\sqrt{a+bx}}$$

input
```
Integrate[Sqrt[a*x + b*x^2]/(x^3*(c + d*x)),x]
```

output
```
(-2*Sqrt[x*(a + b*x)]*(Sqrt[c]*Sqrt[a + b*x]*(b*c*x + a*(c - 3*d*x)) + 3*a
*d*Sqrt[-(b*c) + a*d]*x^(3/2)*ArcTan[(-(d*Sqrt[x]*Sqrt[a + b*x]) + Sqrt[b]
*(c + d*x))/(Sqrt[c]*Sqrt[-(b*c) + a*d])]))/(3*a*c^(5/2)*x^2*Sqrt[a + b*x]
)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1261, 107, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)}\,dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{\sqrt{ax+bx^2}\int \frac{\sqrt{a+bx}}{x^{5/2}(c+dx)}dx}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{107}$$

$$\frac{\sqrt{ax+bx^2}\left(-\frac{d\int \frac{\sqrt{a+bx}}{x^{3/2}(c+dx)}dx}{c}-\frac{2(a+bx)^{3/2}}{3acx^{3/2}}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{105}$$

$$\frac{\sqrt{ax+bx^2}\left(-\frac{d\left(\frac{(bc-ad)\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{c}-\frac{2\sqrt{a+bx}}{c\sqrt{x}}\right)}{c}-\frac{2(a+bx)^{3/2}}{3acx^{3/2}}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{104}$$

$$\frac{\sqrt{ax+bx^2}\left(-\frac{d\left(\frac{2(bc-ad)\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}-\frac{2\sqrt{a+bx}}{c\sqrt{x}}\right)}{c}-\frac{2(a+bx)^{3/2}}{3acx^{3/2}}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 221$$

$$\frac{\sqrt{ax+bx^2}\left(-\frac{d\left(\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}}-\frac{2\sqrt{a+bx}}{c\sqrt{x}}\right)}{c}-\frac{2(a+bx)^{3/2}}{3acx^{3/2}}\right)}{\sqrt{x}\sqrt{a+bx}}$$

input
```
Int[Sqrt[a*x + b*x^2]/(x^3*(c + d*x)),x]
```

output
```
(Sqrt[a*x + b*x^2]*((-2*(a + b*x)^(3/2))/(3*a*c*x^(3/2)) - (d*((-2*Sqrt[a
+ b*x])/(c*Sqrt[x]) + (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])
/(Sqrt[c]*Sqrt[a + b*x])])/c^(3/2)))/c))/(Sqrt[x]*Sqrt[a + b*x])
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{-\frac{2\sqrt{x(bx+a)}\,(-3adx+cbx+ac)}{3x^2}-\frac{2(ad-bc)ad\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{\sqrt{c(ad-bc)}}}{a\,c^2}$
risch	$-\frac{2(bx+a)(-3adx+cbx+ac)}{3a\,c^2\sqrt{x(bx+a)}\,x}-\frac{(ad-bc)\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-c\cdots}}{x+\frac{c}{d}}\right.}{c^2\sqrt{-\frac{c(ad-bc)}{d^2}}}\cdots$
default	$-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{3ca\,x^3}+\frac{d^2\left(\sqrt{b\,x^2+ax}+\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2\sqrt{b}}\right)}{c^3}-\frac{d\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{a\,x^2}+\frac{2b\left(\sqrt{b\,x^2+ax}+\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2\cdots}\right)}{2\sqrt{b}}\cdots\right)}{a}\cdots\right)}{c^2}\cdots$

input
```
int((b*x^2+a*x)^(1/2)/x^3/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
2/c^2*(-1/3*(x*(b*x+a))^(1/2)/x^2*(-3*a*d*x+b*c*x+a*c)-(a*d-b*c)*a*d/(c*(a
*d-b*c))^(1/2)*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2)))/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.95

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)}\,dx = \left[\frac{3\,adx^2\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)x-2\,\sqrt{bx^2+ax}c\sqrt{\frac{bc-ad}{c}}}{dx+c}\right)-2\,\sqrt{bx^2+ax}(ac+(bc-3\,ad)x)}{3\,ac^2x^2}, -\frac{2\left(3\,adx^2\sqrt{-\frac{bc-ad}{c}}\arctan\left(-\frac{\sqrt{bx^2+ax}c\sqrt{-\frac{bc-ad}{c}}}{(bc-ad)x}\right)+\sqrt{bx^2+ax}(ac+(bc-3\,ad)x)\right)}{3\,ac^2x^2}\right]$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^3/(d*x+c),x, algorithm="fricas")
```

output
```
[1/3*(3*a*d*x^2*sqrt((b*c - a*d)/c)*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*
x^2 + a*x)*c*sqrt((b*c - a*d)/c))/(d*x + c)) - 2*sqrt(b*x^2 + a*x)*(a*c +
(b*c - 3*a*d)*x))/(a*c^2*x^2), -2/3*(3*a*d*x^2*sqrt(-(b*c - a*d)/c)*arctan
(-sqrt(b*x^2 + a*x)*c*sqrt(-(b*c - a*d)/c)/((b*c - a*d)*x)) + sqrt(b*x^2 +
 a*x)*(a*c + (b*c - 3*a*d)*x))/(a*c^2*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)}\,dx = \int \frac{\sqrt{x\,(a+bx)}}{x^3\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a*x)**(1/2)/x**3/(d*x+c),x)
```

output
```
Integral(sqrt(x*(a + b*x))/(x**3*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)}\,dx = \int \frac{\sqrt{bx^2+ax}}{(dx+c)x^3}\,dx$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^3/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a*x)/((d*x + c)*x^3), x)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.63

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)}\,dx = -\frac{2\,(bcd-ad^2)\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)d+\sqrt{b}c}{\sqrt{-bc^2+acd}}\right)}{\sqrt{-bc^2+acd}c^2} + \frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2bc-3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2ad+3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)a\sqrt{b}c+a^2c\right)}{3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3c^2}$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^3/(d*x+c),x, algorithm="giac")
```

output
```
-2*(b*c*d - a*d^2)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a*x))*d + sqrt(b)*c)
/sqrt(-b*c^2 + a*c*d))/(sqrt(-b*c^2 + a*c*d)*c^2) + 2/3*(3*(sqrt(b)*x - sq
rt(b*x^2 + a*x))^2*b*c - 3*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a*d + 3*(sqrt
(b)*x - sqrt(b*x^2 + a*x))*a*sqrt(b)*c + a^2*c)/((sqrt(b)*x - sqrt(b*x^2 +
 a*x))^3*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)}\,dx=\int \frac{\sqrt{b\,x^2+a\,x}}{x^3\,\left(c+d\,x\right)}\,dx$$

input
```
int((a*x + b*x^2)^(1/2)/(x^3*(c + d*x)),x)
```

output
```
int((a*x + b*x^2)^(1/2)/(x^3*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)}\,dx = \frac{-2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)ad\,x^2-2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}+\sqrt{d}\,\sqrt{bx+a}+\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)}{a\,c^3x^2}$$

input
```
int((b*x^2+a*x)^(1/2)/x^3/(d*x+c),x)
```

output
```
(2*( - 3*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a +
b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*d*x**2 - 3*sqrt(c)*sq
rt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt
(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*d*x**2 - sqrt(x)*sqrt(a + b*x)*a*c**2 +
3*sqrt(x)*sqrt(a + b*x)*a*c*d*x - sqrt(x)*sqrt(a + b*x)*b*c**2*x - sqrt(b)
*a*c*d*x**2 - sqrt(b)*b*c**2*x**2))/(3*a*c**3*x**2)
```

3.36 $\int \frac{\sqrt{ax+bx^2}}{x^4(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 162

$$\int \frac{\sqrt{ax+bx^2}}{x^4(c+dx)}\,dx = -\frac{2\sqrt{ax+bx^2}}{5cx^3} - \frac{2(bc-5ad)\sqrt{ax+bx^2}}{15ac^2x^2} + \frac{2(2b^2c^2+5abcd-15a^2d^2)\sqrt{ax+bx^2}}{15a^2c^3x} + \frac{2d^2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{7/2}}$$

output
```
-2/5*(b*x^2+a*x)^(1/2)/c/x^3-2/15*(-5*a*d+b*c)*(b*x^2+a*x)^(1/2)/a/c^2/x^2
+2/15*(-15*a^2*d^2+5*a*b*c*d+2*b^2*c^2)*(b*x^2+a*x)^(1/2)/a^2/c^3/x+2*d^2*
(-a*d+b*c)^(1/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(
7/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{ax+bx^2}}{x^4(c+dx)}\,dx = \frac{2\sqrt{x(a+bx)}\left(\frac{\sqrt{c}(2b^2c^2x^2-abcx(c-5dx)+a^2(-3c^2+5cdx-15d^2x^2))}{a^2}+\frac{15d^2\sqrt{-bc+ad}x^{5/2}\arctan\left(\frac{-d\sqrt{x}\sqrt{a+bx}+\sqrt{b}(c+dx)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{a+bx}}\right)}{15c^{7/2}x^3}$$

input
```
Integrate[Sqrt[a*x + b*x^2]/(x^4*(c + d*x)),x]
```

output
```
(2*Sqrt[x*(a + b*x)]*((Sqrt[c]*(2*b^2*c^2*x^2 - a*b*c*x*(c - 5*d*x) + a^2*
(-3*c^2 + 5*c*d*x - 15*d^2*x^2)))/a^2 + (15*d^2*Sqrt[-(b*c) + a*d]*x^(5/2)
*ArcTan[(-(d*Sqrt[x]*Sqrt[a + b*x]) + Sqrt[b]*(c + d*x))/(Sqrt[c]*Sqrt[-(b
*c) + a*d])])/Sqrt[a + b*x]))/(15*c^(7/2)*x^3)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1261, 110, 27, 169, 27, 169, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{x^4(c+dx)}\,dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{\sqrt{ax+bx^2}\int \frac{\sqrt{a+bx}}{x^{7/2}(c+dx)}dx}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{110}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{2\int \frac{bc-5ad-4bdx}{2x^{5/2}\sqrt{a+bx}(c+dx)}dx}{5c}-\frac{2\sqrt{a+bx}}{5cx^{5/2}}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\int \frac{bc-5ad-4bdx}{x^{5/2}\sqrt{a+bx}(c+dx)}dx}{5c}-\frac{2\sqrt{a+bx}}{5cx^{5/2}}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 169$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{-\frac{2\int \frac{2b^2c^2+5abdc-15a^2d^2+2bd(bc-5ad)x}{2x^{3/2}\sqrt{a+bx}(c+dx)}dx}{3ac}-\frac{2\sqrt{a+bx}(bc-5ad)}{3acx^{3/2}}}{5c}-\frac{2\sqrt{a+bx}}{5cx^{5/2}}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{-\frac{\int \frac{2b^2c^2+5abdc-15a^2d^2+2bd(bc-5ad)x}{x^{3/2}\sqrt{a+bx}(c+dx)}dx}{3ac}-\frac{2\sqrt{a+bx}(bc-5ad)}{3acx^{3/2}}}{5c}-\frac{2\sqrt{a+bx}}{5cx^{5/2}}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 169$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{-\frac{-\frac{2\int \frac{15a^2d^2(bc-ad)}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{ac}-\frac{2\sqrt{a+bx}\left(\frac{2b^2c}{a}-\frac{15ad^2}{c}+5bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(bc-5ad)}{3acx^{3/2}}}{5c}-\frac{2\sqrt{a+bx}}{5cx^{5/2}}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{-\frac{-\frac{15ad^2(bc-ad)\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{c}-\frac{2\sqrt{a+bx}\left(\frac{2b^2c}{a}-\frac{15ad^2}{c}+5bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(bc-5ad)}{3acx^{3/2}}}{5c}-\frac{2\sqrt{a+bx}}{5cx^{5/2}}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 104$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{-\frac{-\frac{30ad^2(bc-ad)\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}-\frac{2\sqrt{a+bx}\left(\frac{2b^2c}{a}-\frac{15ad^2}{c}+5bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(bc-5ad)}{3acx^{3/2}}}{5c}-\frac{2\sqrt{a+bx}}{5cx^{5/2}}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 221$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{-\frac{-\frac{30ad^2\sqrt{bc-ad}\,\mathrm{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}}-\frac{2\sqrt{a+bx}\left(\frac{2b^2c}{a}-\frac{15ad^2}{c}+5bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(bc-5ad)}{3acx^{3/2}}}{5c}-\frac{2\sqrt{a+bx}}{5cx^{5/2}}\right)}{\sqrt{x}\sqrt{a+bx}}$$

input
```
Int[Sqrt[a*x + b*x^2]/(x^4*(c + d*x)),x]
```

output
```
(Sqrt[a*x + b*x^2]*((-2*Sqrt[a + b*x])/(5*c*x^(5/2)) + ((-2*(b*c - 5*a*d)*
Sqrt[a + b*x])/(3*a*c*x^(3/2)) - ((-2*((2*b^2*c)/a + 5*b*d - (15*a*d^2)/c)
*Sqrt[a + b*x])/Sqrt[x] - (30*a*d^2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b*c - a*
d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/c^(3/2))/(3*a*c))/(5*c)))/(Sqrt[x]*S
qrt[a + b*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$-\frac{2\left(\left(\left(bx+a\right)\left(-\frac{2bx}{3}+a\right)c^{2}-\frac{5\left(bx+a\right)acdx}{3}+5a^{2}d^{2}x^{2}\right)\sqrt{c\left(ad-bc\right)}\,\sqrt{x\left(bx+a\right)}-5a^{2}d^{2}x^{3}\left(ad-bc\right)\arctan\left(\frac{\sqrt{x\left(bx+a\right)}\,c}{x\sqrt{c\left(ad-bc\right)}}\right)\right)}{5\sqrt{c\left(ad-bc\right)}\,c^{3}x^{3}a^{2}}$
risch	$-\frac{2\left(bx+a\right)\left(15a^{2}d^{2}x^{2}-5abcd\,x^{2}-2b^{2}c^{2}x^{2}-5a^{2}cdx+ab\,c^{2}x+3a^{2}c^{2}\right)}{15a^{2}c^{3}\sqrt{x\left(bx+a\right)}\,x^{2}}+\frac{\left(ad-bc\right)d\ln\left(\frac{-\frac{2c\left(ad-bc\right)}{d^{2}}+\frac{\left(ad-2bc\right)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\cdots}}{\cdots}\right)}{c^{3}\sqrt{-\cdots}}$
default	$\frac{-\frac{2\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{5a\,x^{4}}+\frac{4b\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{15a^{2}x^{3}}}{c}+\frac{d^{3}\left(\sqrt{b\left(x+\frac{c}{d}\right)^{2}+\frac{\left(ad-2bc\right)\left(x+\frac{c}{d}\right)}{d}-\frac{c\left(ad-bc\right)}{d^{2}}}+\frac{\left(ad-2bc\right)\ln\left(\frac{\frac{ad-2bc}{2d}+b\left(x+\frac{c}{d}\right)}{\sqrt{b}}+\sqrt{b\left(x+\frac{c}{d}\right)\cdots}\right)}{2d\sqrt{b}}\cdots\right)}{\cdots}$

input
```
int((b*x^2+a*x)^(1/2)/x^4/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2/5/(c*(a*d-b*c))^(1/2)*(((b*x+a)*(-2/3*b*x+a)*c^2-5/3*(b*x+a)*a*c*d*x+5*
a^2*d^2*x^2)*(c*(a*d-b*c))^(1/2)*(x*(b*x+a))^(1/2)-5*a^2*d^2*x^3*(a*d-b*c)
*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2)))/c^3/x^3/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.87

$$\int \frac{\sqrt{ax+bx^2}}{x^4(c+dx)}\,dx = \left[\frac{15\,a^2d^2x^3\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)x+2\,\sqrt{bx^2+ax}c\sqrt{\frac{bc-ad}{c}}}{dx+c}\right) - 2\,(3\,a^2c^2 - (2\,b^2c^2 + 5\,abcd - 15\,a^2d^2)x^2 + (abc}{15\,a^2c^3x^3}\right.$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^4/(d*x+c),x, algorithm="fricas")
```

output
```
[1/15*(15*a^2*d^2*x^3*sqrt((b*c - a*d)/c)*log((a*c + (2*b*c - a*d)*x + 2*s
qrt(b*x^2 + a*x)*c*sqrt((b*c - a*d)/c))/(d*x + c)) - 2*(3*a^2*c^2 - (2*b^2
*c^2 + 5*a*b*c*d - 15*a^2*d^2)*x^2 + (a*b*c^2 - 5*a^2*c*d)*x)*sqrt(b*x^2 +
 a*x))/(a^2*c^3*x^3), 2/15*(15*a^2*d^2*x^3*sqrt(-(b*c - a*d)/c)*arctan(-sq
rt(b*x^2 + a*x)*c*sqrt(-(b*c - a*d)/c)/((b*c - a*d)*x)) - (3*a^2*c^2 - (2*
b^2*c^2 + 5*a*b*c*d - 15*a^2*d^2)*x^2 + (a*b*c^2 - 5*a^2*c*d)*x)*sqrt(b*x^
2 + a*x))/(a^2*c^3*x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2}}{x^4(c+dx)}\,dx = \int \frac{\sqrt{x\,(a+bx)}}{x^4\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a*x)**(1/2)/x**4/(d*x+c),x)
```

output
```
Integral(sqrt(x*(a + b*x))/(x**4*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^2}}{x^4(c+dx)}\,dx = \int \frac{\sqrt{bx^2+ax}}{(dx+c)x^4}\,dx$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^4/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a*x)/((d*x + c)*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. 2(140) = 280.

Time = 0.29 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.90

$$\int \frac{\sqrt{ax+bx^2}}{x^4(c+dx)}\,dx = \frac{2\,(bcd^2-ad^3)\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)d+\sqrt{b}c}{\sqrt{-bc^2+acd}}\right)}{\sqrt{-bc^2+acd}c^3} - \frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4bcd-15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4ad^2-15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3b^{\frac{3}{2}}c^2+15\left(\right.\right.}{}$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^4/(d*x+c),x, algorithm="giac")
```

output
```
2*(b*c*d^2 - a*d^3)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a*x))*d + sqrt(b)*c
)/sqrt(-b*c^2 + a*c*d))/(sqrt(-b*c^2 + a*c*d)*c^3) - 2/15*(15*(sqrt(b)*x -
 sqrt(b*x^2 + a*x))^4*b*c*d - 15*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a*d^2 -
 15*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*b^(3/2)*c^2 + 15*(sqrt(b)*x - sqrt(b
*x^2 + a*x))^3*a*sqrt(b)*c*d - 25*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a*b*c^
2 + 5*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2*c*d - 15*(sqrt(b)*x - sqrt(b*x
^2 + a*x))*a^2*sqrt(b)*c^2 - 3*a^3*c^2)/((sqrt(b)*x - sqrt(b*x^2 + a*x))^5
*c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^2}}{x^4(c+dx)}\,dx=\int \frac{\sqrt{b\,x^2+a\,x}}{x^4\,\left(c+d\,x\right)}\,dx$$

input
```
int((a*x + b*x^2)^(1/2)/(x^4*(c + d*x)),x)
```

output
```
int((a*x + b*x^2)^(1/2)/(x^4*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{ax+bx^2}}{x^4(c+dx)}\,dx$$
$$=\frac{2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)a^2d^2x^3+2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}+\sqrt{d}\,\sqrt{bx+a}+\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)\cdots}{}$$

input
```
int((b*x^2+a*x)^(1/2)/x^4/(d*x+c),x)
```

output
```
(2*(15*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*
x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*d**2*x**3 + 15*sqrt(
c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)
*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*d**2*x**3 - 3*sqrt(x)*sqrt(a + b
*x)*a**2*c**3 + 5*sqrt(x)*sqrt(a + b*x)*a**2*c**2*d*x - 15*sqrt(x)*sqrt(a
+ b*x)*a**2*c*d**2*x**2 - sqrt(x)*sqrt(a + b*x)*a*b*c**3*x + 5*sqrt(x)*sqr
t(a + b*x)*a*b*c**2*d*x**2 + 2*sqrt(x)*sqrt(a + b*x)*b**2*c**3*x**2 + 9*sq
rt(b)*a**2*c*d**2*x**3 + sqrt(b)*a*b*c**2*d*x**3 - 2*sqrt(b)*b**2*c**3*x**
3))/(15*a**2*c**4*x**3)
```

3.37 $\int \frac{\sqrt{ax+bx^2}}{x^5(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 225

$$\int \frac{\sqrt{ax+bx^2}}{x^5(c+dx)}\,dx = -\frac{2\sqrt{ax+bx^2}}{7cx^4} - \frac{2(bc-7ad)\sqrt{ax+bx^2}}{35ac^2x^3} + \frac{2(4b^2c^2+7abcd-35a^2d^2)\sqrt{ax+bx^2}}{105a^2c^3x^2} - \frac{2(8b^3c^3+14ab^2c^2d+35a^2bcd^2-105a^3d^3)\sqrt{ax+bx^2}}{105a^3c^4x} - \frac{2d^3\sqrt{bc-ad}\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{9/2}}$$

output
```
-2/7*(b*x^2+a*x)^(1/2)/c/x^4-2/35*(-7*a*d+b*c)*(b*x^2+a*x)^(1/2)/a/c^2/x^3
+2/105*(-35*a^2*d^2+7*a*b*c*d+4*b^2*c^2)*(b*x^2+a*x)^(1/2)/a^2/c^3/x^2-2/1
05*(-105*a^3*d^3+35*a^2*b*c*d^2+14*a*b^2*c^2*d+8*b^3*c^3)*(b*x^2+a*x)^(1/2
)/a^3/c^4/x-2*d^3*(-a*d+b*c)^(1/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x
^2+a*x)^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{ax+bx^2}}{x^5(c+dx)}\,dx$$
$$= \frac{2\sqrt{x(a+bx)}\left(-\frac{\sqrt{c}(8b^3c^3x^3+2ab^2c^2x^2(-2c+7dx)+a^2bcx(3c^2-7cdx+35d^2x^2)+a^3(15c^3-21c^2dx+35cd^2x^2-105d^3x^3))}{a^3}-\frac{105d^3\sqrt{-}\cdots}{}\right.}{105c^{9/2}x^4}$$

input
```
Integrate[Sqrt[a*x + b*x^2]/(x^5*(c + d*x)),x]
```

output
```
(2*Sqrt[x*(a + b*x)]*(-((Sqrt[c]*(8*b^3*c^3*x^3 + 2*a*b^2*c^2*x^2*(-2*c +
7*d*x) + a^2*b*c*x*(3*c^2 - 7*c*d*x + 35*d^2*x^2) + a^3*(15*c^3 - 21*c^2*d
*x + 35*c*d^2*x^2 - 105*d^3*x^3)))/a^3) - (105*d^3*Sqrt[-(b*c) + a*d]*x^(7
/2)*ArcTan[(-(d*Sqrt[x]*Sqrt[a + b*x]) + Sqrt[b]*(c + d*x))/(Sqrt[c]*Sqrt[
-(b*c) + a*d])])/Sqrt[a + b*x]))/(105*c^(9/2)*x^4)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1261, 110, 27, 169, 27, 169, 27, 169, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{x^5(c+dx)}\,dx$$

$$\Big\downarrow \ 1261$$

$$\frac{\sqrt{ax+bx^2}\int \frac{\sqrt{a+bx}}{x^{9/2}(c+dx)}dx}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \ 110$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{2\int \frac{bc-7ad-6bdx}{2x^{7/2}\sqrt{a+bx}(c+dx)}dx}{7c}-\frac{2\sqrt{a+bx}}{7cx^{7/2}}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$\downarrow$ 27

$$\frac{\sqrt{ax+bx^2}\left(\frac{\int \frac{bc-7ad-6bdx}{x^{7/2}\sqrt{a+bx}(c+dx)}dx}{7c}-\frac{2\sqrt{a+bx}}{7cx^{7/2}}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$\downarrow$ 169

$$\frac{\sqrt{ax+bx^2}\left(\frac{-\frac{2\int \frac{4b^2c^2+7abdc-35a^2d^2+4bd(bc-7ad)x}{2x^{5/2}\sqrt{a+bx}(c+dx)}dx}{5ac}-\frac{2\sqrt{a+bx}(bc-7ad)}{5acx^{5/2}}}{7c}-\frac{2\sqrt{a+bx}}{7cx^{7/2}}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$\downarrow$ 27

$$\frac{\sqrt{ax+bx^2}\left(\frac{-\frac{\int \frac{4b^2c^2+7abdc-35a^2d^2+4bd(bc-7ad)x}{x^{5/2}\sqrt{a+bx}(c+dx)}dx}{5ac}-\frac{2\sqrt{a+bx}(bc-7ad)}{5acx^{5/2}}}{7c}-\frac{2\sqrt{a+bx}}{7cx^{7/2}}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$\downarrow$ 169

$$\frac{\sqrt{ax+bx^2}\left(\frac{-\frac{-\frac{2\int \frac{8b^3c^3+14ab^2dc^2+35a^2bd^2c-105a^3d^3+2bd\left(4b^2c^2+7abdc-35a^2d^2\right)x}{2x^{3/2}\sqrt{a+bx}(c+dx)}dx}{3ac}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{35ad^2}{c}+7bd\right)}{3x^{3/2}}}{5ac}-\frac{2\sqrt{a+bx}(bc-7ad)}{5acx^{5/2}}}{7c}-\frac{2\sqrt{a+bx}}{7cx^{7/2}}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$\downarrow$ 27

$$\frac{\sqrt{ax+bx^2}\left(\frac{-\frac{-\frac{\int \frac{8b^3c^3+14ab^2dc^2+35a^2bd^2c-105a^3d^3+2bd\left(4b^2c^2+7abdc-35a^2d^2\right)x}{x^{3/2}\sqrt{a+bx}(c+dx)}dx}{3ac}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{35ad^2}{c}+7bd\right)}{3x^{3/2}}}{5ac}-\frac{2\sqrt{a+bx}(bc-7ad)}{5acx^{5/2}}}{7c}-\frac{2\sqrt{a+bx}}{7cx^{7/2}}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$\downarrow$ 169

$$\frac{\sqrt{ax+bx^2}\left(-\frac{-\frac{-\frac{2\int \frac{105a^3d^3(bc-ad)}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{ac}-\frac{2\sqrt{a+bx}\left(-105a^3d^3+35a^2bcd^2+14ab^2c^2d+8b^3c^3\right)}{ac\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{35ad^2}{c}+7bd\right)}{3x^{3/2}}}{5ac}-\frac{2\sqrt{a+bx}(bc-7ad)}{5acx^{5/2}}}{7c} -$$

$$\sqrt{x}\sqrt{a+bx}$$

$\big\downarrow$ 27

$$\frac{\sqrt{ax+bx^2}\left(-\frac{-\frac{-\frac{105a^2d^3(bc-ad)\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{c}-\frac{2\sqrt{a+bx}\left(-105a^3d^3+35a^2bcd^2+14ab^2c^2d+8b^3c^3\right)}{ac\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{35ad^2}{c}+7bd\right)}{3x^{3/2}}}{5ac}-\frac{2\sqrt{a+b}}{5a}}{7c}}{\sqrt{x}\sqrt{a+bx}}$$

$\big\downarrow$ 104

$$\frac{\sqrt{ax+bx^2}\left(-\frac{-\frac{-\frac{210a^2d^3(bc-ad)\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}-\frac{2\sqrt{a+bx}\left(-105a^3d^3+35a^2bcd^2+14ab^2c^2d+8b^3c^3\right)}{ac\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{35ad^2}{c}+7bd\right)}{3x^{3/2}}}{5ac}-\frac{2\sqrt{a}}{}}{7c}}{\sqrt{x}\sqrt{a+bx}}$$

$\big\downarrow$ 221

$$\frac{\sqrt{ax+bx^2}\left(-\frac{-\frac{-\frac{210a^2d^3\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}}-\frac{2\sqrt{a+bx}\left(-105a^3d^3+35a^2bcd^2+14ab^2c^2d+8b^3c^3\right)}{ac\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{35ad^2}{c}+7bd\right)}{3x^{3/2}}}{5ac}-2\sqrt{}}{7c}}{\sqrt{x}\sqrt{a+bx}}$$

input
```
Int[Sqrt[a*x + b*x^2]/(x^5*(c + d*x)),x]
```

output
```
(Sqrt[a*x + b*x^2]*((-2*Sqrt[a + b*x])/(7*c*x^(7/2)) + ((-2*(b*c - 7*a*d)*
Sqrt[a + b*x])/(5*a*c*x^(5/2)) - ((-2*((4*b^2*c)/a + 7*b*d - (35*a*d^2)/c)
*Sqrt[a + b*x])/(3*x^(3/2)) - ((-2*(8*b^3*c^3 + 14*a*b^2*c^2*d + 35*a^2*b*
c*d^2 - 105*a^3*d^3)*Sqrt[a + b*x])/(a*c*Sqrt[x]) - (210*a^2*d^3*Sqrt[b*c
- a*d]*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/c^(3/2)
)/(3*a*c))/(5*a*c))/(7*c)))/(Sqrt[x]*Sqrt[a + b*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] && !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$-\frac{2\left(\left(\left(bx+a\right)\left(\frac{8}{15}b^2x^2-\frac{4}{5}abx+a^2\right)c^3-\frac{7dx(bx+a)a\left(-\frac{2bx}{3}+a\right)c^2}{5}+\frac{7(bx+a)a^2cd^2x^2}{3}-7a^3d^3x^3\right)\sqrt{c(ad-bc)}\sqrt{x(bx+a)}+7a^3\right.}{7\sqrt{c(ad-bc)}\,c^4x^4a^3}$
risch	$-\frac{2(bx+a)\left(-105a^3d^3x^3+35a^2bc\,d^2x^3+14a\,b^2c^2d\,x^3+8b^3c^3x^3+35a^3c\,d^2x^2-7a^2b\,c^2x^2d-4a\,b^2c^3x^2-21a^3c^2dx+3a^2b\,c^3x\right.}{105a^3c^4\sqrt{x(bx+a)}\,x^3}$
default	$\frac{-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{7a\,x^5}-\frac{4b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{5a\,x^4}+\frac{4b\left(b\,x^2+ax\right)^{\frac{3}{2}}}{15a^2x^3}\right)}{7a}}{c}+\frac{d^4\left(\sqrt{b\,x^2+ax}+\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2\sqrt{b}}\right)}{c^5}-\frac{2d^2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{3c^3a\,x^3}$

input
```
int((b*x^2+a*x)^(1/2)/x^5/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2/7*(((b*x+a)*(8/15*b^2*x^2-4/5*a*b*x+a^2)*c^3-7/5*d*x*(b*x+a)*a*(-2/3*b*
x+a)*c^2+7/3*(b*x+a)*a^2*c*d^2*x^2-7*a^3*d^3*x^3)*(c*(a*d-b*c))^(1/2)*(x*(
b*x+a))^(1/2)+7*a^3*d^3*x^4*(a*d-b*c)*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d
-b*c))^(1/2)))/(c*(a*d-b*c))^(1/2)/c^4/x^4/a^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{ax+bx^2}}{x^5(c+dx)}\,dx$$
$$= \left[\frac{105\,a^3d^3x^4\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)x-2\,\sqrt{bx^2+ax}c\sqrt{\frac{bc-ad}{c}}}{dx+c}\right)-2\,(15\,a^3c^3+(8\,b^3c^3+14\,ab^2c^2d+35\,a^2bcd^2-}{105\,a^3c^4x^4}\right.$$
$$-\frac{2\left(105\,a^3d^3x^4\sqrt{-\frac{bc-ad}{c}}\arctan\left(-\frac{\sqrt{bx^2+ax}c\sqrt{-\frac{bc-ad}{c}}}{(bc-ad)x}\right)+(15\,a^3c^3+(8\,b^3c^3+14\,ab^2c^2d+35\,a^2bcd^2-10}{105\,a^3c^4x^4}$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^5/(d*x+c),x, algorithm="fricas")
```

output
```
[1/105*(105*a^3*d^3*x^4*sqrt((b*c - a*d)/c)*log((a*c + (2*b*c - a*d)*x - 2
*sqrt(b*x^2 + a*x)*c*sqrt((b*c - a*d)/c))/(d*x + c)) - 2*(15*a^3*c^3 + (8*
b^3*c^3 + 14*a*b^2*c^2*d + 35*a^2*b*c*d^2 - 105*a^3*d^3)*x^3 - (4*a*b^2*c^
3 + 7*a^2*b*c^2*d - 35*a^3*c*d^2)*x^2 + 3*(a^2*b*c^3 - 7*a^3*c^2*d)*x)*sqr
t(b*x^2 + a*x))/(a^3*c^4*x^4), -2/105*(105*a^3*d^3*x^4*sqrt(-(b*c - a*d)/c
)*arctan(-sqrt(b*x^2 + a*x)*c*sqrt(-(b*c - a*d)/c)/((b*c - a*d)*x)) + (15*
a^3*c^3 + (8*b^3*c^3 + 14*a*b^2*c^2*d + 35*a^2*b*c*d^2 - 105*a^3*d^3)*x^3
- (4*a*b^2*c^3 + 7*a^2*b*c^2*d - 35*a^3*c*d^2)*x^2 + 3*(a^2*b*c^3 - 7*a^3*
c^2*d)*x)*sqrt(b*x^2 + a*x))/(a^3*c^4*x^4)]
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2}}{x^5(c+dx)}\,dx=\int \frac{\sqrt{x\,(a+bx)}}{x^5\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a*x)**(1/2)/x**5/(d*x+c),x)
```

output
```
Integral(sqrt(x*(a + b*x))/(x**5*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^2}}{x^5(c+dx)}\,dx = \int \frac{\sqrt{bx^2+ax}}{(dx+c)x^5}\,dx$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^5/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a*x)/((d*x + c)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(199) = 398$.

Time = 0.14 (sec) , antiderivative size = 467, normalized size of antiderivative = 2.08

$$\int \frac{\sqrt{ax+bx^2}}{x^5(c+dx)}\,dx = -\frac{2\,(bcd^3-ad^4)\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)d+\sqrt{b}c}{\sqrt{-bc^2+acd}}\right)}{\sqrt{-bc^2+acd}\,c^4} + \frac{2\left(105\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6 bcd^2 - 105\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6 ad^3 - 105\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5 b^{\frac{3}{2}}c^2d\right.}{}$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^5/(d*x+c),x, algorithm="giac")
```

output
```
-2*(b*c*d^3 - a*d^4)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a*x))*d + sqrt(b)*
c)/sqrt(-b*c^2 + a*c*d))/(sqrt(-b*c^2 + a*c*d)*c^4) + 2/105*(105*(sqrt(b)*
x - sqrt(b*x^2 + a*x))^6*b*c*d^2 - 105*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a
*d^3 - 105*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*b^(3/2)*c^2*d + 105*(sqrt(b)*
x - sqrt(b*x^2 + a*x))^5*a*sqrt(b)*c*d^2 + 140*(sqrt(b)*x - sqrt(b*x^2 + a
*x))^4*b^2*c^3 - 175*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a*b*c^2*d + 35*(sqr
t(b)*x - sqrt(b*x^2 + a*x))^4*a^2*c*d^2 + 315*(sqrt(b)*x - sqrt(b*x^2 + a*
x))^3*a*b^(3/2)*c^3 - 105*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^2*sqrt(b)*c^
2*d + 273*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2*b*c^3 - 21*(sqrt(b)*x - sq
rt(b*x^2 + a*x))^2*a^3*c^2*d + 105*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^3*sqr
t(b)*c^3 + 15*a^4*c^3)/((sqrt(b)*x - sqrt(b*x^2 + a*x))^7*c^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^2}}{x^5(c+dx)}\,dx = \int \frac{\sqrt{b\,x^2+a\,x}}{x^5\,\left(c+d\,x\right)}\,dx$$

input
```
int((a*x + b*x^2)^(1/2)/(x^5*(c + d*x)),x)
```

output
```
int((a*x + b*x^2)^(1/2)/(x^5*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.72

$$\int \frac{\sqrt{ax+bx^2}}{x^5(c+dx)}\,dx = \frac{-2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)a^3d^3x^4-2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}+\sqrt{d}\,\sqrt{bx+a}+\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)}{}$$

input
```
int((b*x^2+a*x)^(1/2)/x^5/(d*x+c),x)
```

output
```
(2*( - 105*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a
+ b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*d**3*x**4 - 105*
sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sq
rt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*d**3*x**4 - 15*sqrt(x)*sqrt
(a + b*x)*a**3*c**4 + 21*sqrt(x)*sqrt(a + b*x)*a**3*c**3*d*x - 35*sqrt(x)*
sqrt(a + b*x)*a**3*c**2*d**2*x**2 + 105*sqrt(x)*sqrt(a + b*x)*a**3*c*d**3*
x**3 - 3*sqrt(x)*sqrt(a + b*x)*a**2*b*c**4*x + 7*sqrt(x)*sqrt(a + b*x)*a**
2*b*c**3*d*x**2 - 35*sqrt(x)*sqrt(a + b*x)*a**2*b*c**2*d**2*x**3 + 4*sqrt(
x)*sqrt(a + b*x)*a*b**2*c**4*x**2 - 14*sqrt(x)*sqrt(a + b*x)*a*b**2*c**3*d
*x**3 - 8*sqrt(x)*sqrt(a + b*x)*b**3*c**4*x**3 - 75*sqrt(b)*a**3*c*d**3*x*
*4 + 5*sqrt(b)*a**2*b*c**2*d**2*x**4 + 14*sqrt(b)*a*b**2*c**3*d*x**4 + 8*s
qrt(b)*b**3*c**4*x**4))/(105*a**3*c**5*x**4)
```

3.38 $\int \frac{x^2\sqrt{ax+bx^2}}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 202

$$\int \frac{x^2\sqrt{ax+bx^2}}{(c+dx)^2}\,dx = -\frac{(12bc-ad)\sqrt{ax+bx^2}}{4bd^3}+\frac{3x\sqrt{ax+bx^2}}{2d^2}-\frac{x^2\sqrt{ax+bx^2}}{d(c+dx)} + \frac{(24b^2c^2-8abcd-a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}d^4} - \frac{c^{3/2}(6bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{d^4\sqrt{bc-ad}}$$

output
```
-1/4*(-a*d+12*b*c)*(b*x^2+a*x)^(1/2)/b/d^3+3/2*x*(b*x^2+a*x)^(1/2)/d^2-x^2
*(b*x^2+a*x)^(1/2)/d/(d*x+c)+1/4*(-a^2*d^2-8*a*b*c*d+24*b^2*c^2)*arctanh(b
^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(3/2)/d^4-c^(3/2)*(-5*a*d+6*b*c)*arctanh((-a
*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/d^4/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 10.60 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.38

$$\int \frac{x^2\sqrt{ax+bx^2}}{(c+dx)^2}\,dx = \frac{\sqrt{x}\left(-\left((24b^3c^3-32ab^2c^2d+7a^2bcd^2+a^3d^3)(a+bx)(c+dx)\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\right)+\sqrt{a}\sqrt{b}\sqrt{1+\frac{bx}{a}}\left(d(-b\right.\right.}{4\sqrt{a}b^{3/2}d^4(-b}$$

input
```
Integrate[(x^2*Sqrt[a*x + b*x^2])/(c + d*x)^2,x]
```

output
```
(Sqrt[x]*(-((24*b^3*c^3 - 32*a*b^2*c^2*d + 7*a^2*b*c*d^2 + a^3*d^3)*(a + b
*x)*(c + d*x)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]) + Sqrt[a]*Sqrt[b]*Sqrt[1
 + (b*x)/a]*(d*(-(b*c) + a*d)*Sqrt[x]*(a + b*x)*(a*d*(c + d*x) - 2*b*(6*c^
2 + 3*c*d*x - d^2*x^2)) + 4*b*c^(3/2)*(6*b*c - 5*a*d)*Sqrt[b*c - a*d]*Sqrt
[a + b*x]*(c + d*x)*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*
x])])))/(4*Sqrt[a]*b^(3/2)*d^4*(-(b*c) + a*d)*Sqrt[x*(a + b*x)]*Sqrt[1 + (
b*x)/a]*(c + d*x))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.24, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {1261, 108, 27, 171, 25, 27, 171, 27, 175, 65, 104, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{ax+bx^2}}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{ 1261}$$

$$\frac{\sqrt{ax+bx^2}\int \frac{x^{5/2}\sqrt{a+bx}}{(c+dx)^2}dx}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 108}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\int \frac{x^{3/2}(5a+6bx)}{2\sqrt{a+bx}(c+dx)}dx}{d}-\frac{x^{5/2}\sqrt{a+bx}}{d(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\int \frac{x^{3/2}(5a+6bx)}{\sqrt{a+bx}(c+dx)}dx}{2d}-\frac{x^{5/2}\sqrt{a+bx}}{d(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 171$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\int -\frac{b\sqrt{x}(9ac+(12bc-ad)x)}{\sqrt{a+bx}(c+dx)}dx}{2bd}+\frac{3x^{3/2}\sqrt{a+bx}}{d}}{2d}-\frac{x^{5/2}\sqrt{a+bx}}{d(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{3x^{3/2}\sqrt{a+bx}}{d}-\frac{\int \frac{b\sqrt{x}(9ac+(12bc-ad)x)}{\sqrt{a+bx}(c+dx)}dx}{2bd}}{2d}-\frac{x^{5/2}\sqrt{a+bx}}{d(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{3x^{3/2}\sqrt{a+bx}}{d}-\frac{\int \frac{\sqrt{x}(9ac+(12bc-ad)x)}{\sqrt{a+bx}(c+dx)}dx}{2d}}{2d}-\frac{x^{5/2}\sqrt{a+bx}}{d(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 171$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{3x^{3/2}\sqrt{a+bx}}{d}-\frac{\frac{\int -\frac{ac(12bc-ad)+\left(24b^2c^2-8abdc-a^2d^2\right)x}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{bd}+\frac{\sqrt{x}\sqrt{a+bx}(12bc-ad)}{bd}}{2d}}{2d}-\frac{x^{5/2}\sqrt{a+bx}}{d(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{3x^{3/2}\sqrt{a+bx}}{d}-\frac{\frac{\sqrt{x}\sqrt{a+bx}(12bc-ad)}{bd}-\frac{\int \frac{ac(12bc-ad)+\left(24b^2c^2-8abdc-a^2d^2\right)x}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{2bd}}{2d}}{2d}-\frac{x^{5/2}\sqrt{a+bx}}{d(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$\big\downarrow$ 175

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{3x^{3/2}\sqrt{a+bx}}{d}-\frac{\frac{\sqrt{x}\sqrt{a+bx}(12bc-ad)}{bd}-\frac{\frac{\left(-a^2d^2-8abcd+24b^2c^2\right)\int\frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{d}-\frac{4bc^2(6bc-5ad)\int\frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}}{2bd}}{2d}}{2d}-\frac{x^{5/2}\sqrt{a+bx}}{d(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$\big\downarrow$ 65

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{3x^{3/2}\sqrt{a+bx}}{d}-\frac{\frac{\sqrt{x}\sqrt{a+bx}(12bc-ad)}{bd}-\frac{\frac{2\left(-a^2d^2-8abcd+24b^2c^2\right)\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}-\frac{4bc^2(6bc-5ad)\int\frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}}{2bd}}{2d}}{2d}-\frac{x^{5/2}\sqrt{a+bx}}{d(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$\big\downarrow$ 104

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{3x^{3/2}\sqrt{a+bx}}{d}-\frac{\frac{\sqrt{x}\sqrt{a+bx}(12bc-ad)}{bd}-\frac{\frac{2\left(-a^2d^2-8abcd+24b^2c^2\right)\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}-\frac{8bc^2(6bc-5ad)\int\frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}}{2bd}}{2d}}{2d}-\frac{x^{5/2}\sqrt{a+bx}}{d(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$\big\downarrow$ 219

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{3x^{3/2}\sqrt{a+bx}}{d}-\frac{\frac{\sqrt{x}\sqrt{a+bx}(12bc-ad)}{bd}-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)\left(-a^2d^2-8abcd+24b^2c^2\right)}{\sqrt{b}d}-\frac{8bc^2(6bc-5ad)\int\frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}}{2bd}}{2d}}{2d}-\frac{x^{5/2}\sqrt{a+bx}}{d(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$\big\downarrow$ 221

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{3x^{3/2}\sqrt{a+bx}}{d}-\frac{\frac{\sqrt{x}\sqrt{a+bx}(12bc-ad)}{bd}-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)\left(-a^2d^2-8abcd+24b^2c^2\right)}{\sqrt{b}d}-\frac{8bc^{3/2}(6bc-5ad)\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{d\sqrt{bc-ad}}}{2bd}}{2d}}{2d}-\right.}{\sqrt{x}\sqrt{a+bx}}$$

input
```
Int[(x^2*Sqrt[a*x + b*x^2])/(c + d*x)^2,x]
```

output
```
(Sqrt[a*x + b*x^2]*(-((x^(5/2)*Sqrt[a + b*x])/(d*(c + d*x))) + ((3*x^(3/2)
*Sqrt[a + b*x])/d - (((12*b*c - a*d)*Sqrt[x]*Sqrt[a + b*x])/(b*d) - ((2*(2
4*b^2*c^2 - 8*a*b*c*d - a^2*d^2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])
/(Sqrt[b]*d) - (8*b*c^(3/2)*(6*b*c - 5*a*d)*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[
x])/(Sqrt[c]*Sqrt[a + b*x])])/(d*Sqrt[b*c - a*d]))/(2*b*d))/(2*d))/(2*d)))
/(Sqrt[x]*Sqrt[a + b*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$-\frac{3\left(-2(dx+c)c^{2}\left(bc-\frac{5ad}{6}\right)b^{\frac{5}{2}}\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)+\sqrt{c(ad-bc)}\left(\frac{b\left(a^{2}d^{2}+8abcd-24b^{2}c^{2}\right)(dx+c)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{12}+c\right.\right.}{\sqrt{c(ad-bc)}\,b^{\frac{5}{2}}d^{4}(dx+c)}$
risch	$\frac{(2bdx+ad-8bc)x(bx+a)}{4b\,d^{3}\sqrt{x(bx+a)}}-\frac{\frac{\left(a^{2}d^{2}+8abcd-24b^{2}c^{2}\right)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{d\sqrt{b}}+\frac{8b\,c^{2}(3ad-4bc)\ln\left(\frac{-\frac{2c(ad-bc)}{d^{2}}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}}{\cdots}\right.}{d^{2}}}{\cdots}$
default	$\frac{\frac{(2bx+a)\sqrt{b\,x^{2}+ax}}{4b}-\frac{a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8b^{\frac{3}{2}}}}{d^{2}}+\frac{c^{2}\left(\frac{d^{2}\left(b\left(x+\frac{c}{d}\right)^{2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^{2}}\right)^{\frac{3}{2}}}{c(ad-bc)\left(x+\frac{c}{d}\right)}-\frac{(ad-2bc)d\left(\sqrt{b\left(x+\frac{c}{d}\right)^{2}\cdots}\right.}{\cdots}\right.}{\cdots}$

input
```
int(x^2*(b*x^2+a*x)^(1/2)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-3*(-2*(d*x+c)*c^2*(b*c-5/6*a*d)*b^(5/2)*arctan((x*(b*x+a))^(1/2)/x*c/(c*(
a*d-b*c))^(1/2))+(c*(a*d-b*c))^(1/2)*(1/12*b*(a^2*d^2+8*a*b*c*d-24*b^2*c^2
)*(d*x+c)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))+d*(x*(b*x+a))^(1/2)*(b*c^2-
1/12*d*(-6*b*x+a)*c-1/12*d^2*x*(2*b*x+a))*b^(3/2)))/(c*(a*d-b*c))^(1/2)/b^
(5/2)/d^4/(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1107, normalized size of antiderivative = 5.48

$$\int \frac{x^2\sqrt{ax+bx^2}}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x^2+a*x)^(1/2)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[-1/8*((24*b^2*c^3 - 8*a*b*c^2*d - a^2*c*d^2 + (24*b^2*c^2*d - 8*a*b*c*d^2
 - a^2*d^3)*x)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 4*(6
*b^3*c^3 - 5*a*b^2*c^2*d + (6*b^3*c^2*d - 5*a*b^2*c*d^2)*x)*sqrt(c/(b*c -
a*d))*log((a*c + (2*b*c - a*d)*x + 2*sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(c/
(b*c - a*d)))/(d*x + c)) - 2*(2*b^2*d^3*x^2 - 12*b^2*c^2*d + a*b*c*d^2 - (
6*b^2*c*d^2 - a*b*d^3)*x)*sqrt(b*x^2 + a*x))/(b^2*d^5*x + b^2*c*d^4), -1/8
*(8*(6*b^3*c^3 - 5*a*b^2*c^2*d + (6*b^3*c^2*d - 5*a*b^2*c*d^2)*x)*sqrt(-c/
(b*c - a*d))*arctan(-sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(-c/(b*c - a*d))/(b
*c*x + a*c)) + (24*b^2*c^3 - 8*a*b*c^2*d - a^2*c*d^2 + (24*b^2*c^2*d - 8*a
*b*c*d^2 - a^2*d^3)*x)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)
) - 2*(2*b^2*d^3*x^2 - 12*b^2*c^2*d + a*b*c*d^2 - (6*b^2*c*d^2 - a*b*d^3)*
x)*sqrt(b*x^2 + a*x))/(b^2*d^5*x + b^2*c*d^4), -1/4*((24*b^2*c^3 - 8*a*b*c
^2*d - a^2*c*d^2 + (24*b^2*c^2*d - 8*a*b*c*d^2 - a^2*d^3)*x)*sqrt(-b)*arct
an(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + 2*(6*b^3*c^3 - 5*a*b^2*c^2*d +
(6*b^3*c^2*d - 5*a*b^2*c*d^2)*x)*sqrt(c/(b*c - a*d))*log((a*c + (2*b*c - a
*d)*x + 2*sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(c/(b*c - a*d)))/(d*x + c)) -
(2*b^2*d^3*x^2 - 12*b^2*c^2*d + a*b*c*d^2 - (6*b^2*c*d^2 - a*b*d^3)*x)*sqr
t(b*x^2 + a*x))/(b^2*d^5*x + b^2*c*d^4), -1/4*(4*(6*b^3*c^3 - 5*a*b^2*c^2*
d + (6*b^3*c^2*d - 5*a*b^2*c*d^2)*x)*sqrt(-c/(b*c - a*d))*arctan(-sqrt(b*x
^2 + a*x)*(b*c - a*d)*sqrt(-c/(b*c - a*d))/(b*c*x + a*c)) + (24*b^2*c^3...
```

Sympy [F]

$$\int \frac{x^2\sqrt{ax+bx^2}}{(c+dx)^2}\,dx = \int \frac{x^2\sqrt{x(a+bx)}}{(c+dx)^2}\,dx$$

input
```
integrate(x**2*(b*x**2+a*x)**(1/2)/(d*x+c)**2,x)
```

output
```
Integral(x**2*sqrt(x*(a + b*x))/(c + d*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{ax+bx^2}}{(c+dx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(b*x^2+a*x)^(1/2)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-2*b*c>0)', see `assume?` for
 more deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{ax+bx^2}}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(b*x^2+a*x)^(1/2)/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{ax+bx^2}}{(c+dx)^2}\,dx=\int \frac{x^2\,\sqrt{b\,x^2+a\,x}}{{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((x^2*(a*x + b*x^2)^(1/2))/(c + d*x)^2,x)
```

output
```
int((x^2*(a*x + b*x^2)^(1/2))/(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 888, normalized size of antiderivative = 4.40

$$\int \frac{x^2\sqrt{ax+bx^2}}{(c+dx)^2}\,dx=\text{Too large to display}$$

input
```
int(x^2*(b*x^2+a*x)^(1/2)/(d*x+c)^2,x)
```

output
```
( - 20*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*
x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c**2*d - 20*sqrt(c
)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*
sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c*d**2*x + 24*sqrt(c)*sqrt(a*d
- b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqr
t(b))/(sqrt(c)*sqrt(b)))*b**3*c**3 + 24*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt
(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sq
rt(b)))*b**3*c**2*d*x - 20*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) +
 sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**
2*c**2*d - 20*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt
(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c*d**2*x +
24*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) +
 sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**3*c**3 + 24*sqrt(c)*sqrt(a
*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*
sqrt(b))/(sqrt(c)*sqrt(b)))*b**3*c**2*d*x + sqrt(x)*sqrt(a + b*x)*a**2*b*c
*d**3 + sqrt(x)*sqrt(a + b*x)*a**2*b*d**4*x - 13*sqrt(x)*sqrt(a + b*x)*a*b
**2*c**2*d**2 - 7*sqrt(x)*sqrt(a + b*x)*a*b**2*c*d**3*x + 2*sqrt(x)*sqrt(a
 + b*x)*a*b**2*d**4*x**2 + 12*sqrt(x)*sqrt(a + b*x)*b**3*c**3*d + 6*sqrt(x
)*sqrt(a + b*x)*b**3*c**2*d**2*x - 2*sqrt(x)*sqrt(a + b*x)*b**3*c*d**3*x**
2 - sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*c*d**3 ...
```

3.39 $\int \frac{x\sqrt{ax+bx^2}}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 148

$$\int \frac{x\sqrt{ax+bx^2}}{(c+dx)^2}\,dx = \frac{2\sqrt{ax+bx^2}}{d^2} - \frac{x\sqrt{ax+bx^2}}{d(c+dx)} - \frac{(4bc-ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}d^3} + \frac{\sqrt{c}(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{d^3\sqrt{bc-ad}}$$

output
```
2*(b*x^2+a*x)^(1/2)/d^2-x*(b*x^2+a*x)^(1/2)/d/(d*x+c)-(-a*d+4*b*c)*arctanh
(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(1/2)/d^3+c^(1/2)*(-3*a*d+4*b*c)*arctanh((
-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/d^3/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 10.47 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.30

$$\int \frac{x\sqrt{ax+bx^2}}{(c+dx)^2}\,dx = \frac{\sqrt{x(a+bx)}\left(\frac{d(-bc+ad)\sqrt{x}(2c+dx)}{c+dx} + \frac{(4b^2c^2-5abcd+a^2d^2)\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{1+\frac{bx}{a}}} - \frac{\sqrt{c}(4bc-3ad)\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{bc-ad}\sqrt{x}}{\sqrt{c}\sqrt{a+bx}}\right)}{\sqrt{a+bx}}\right)}{d^3(-bc+ad)\sqrt{x}}$$

input
```
Integrate[(x*Sqrt[a*x + b*x^2])/(c + d*x)^2,x]
```

output
```
(Sqrt[x*(a + b*x)]*((d*(-(b*c) + a*d)*Sqrt[x]*(2*c + d*x))/(c + d*x) + ((4
*b^2*c^2 - 5*a*b*c*d + a^2*d^2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[
a]*Sqrt[b]*Sqrt[1 + (b*x)/a]) - (Sqrt[c]*(4*b*c - 3*a*d)*Sqrt[b*c - a*d]*A
rcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/Sqrt[a + b*x]))
/(d^3*(-(b*c) + a*d)*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1230, 1269, 1091, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{ax+bx^2}}{(c+dx)^2}dx$$

$$\Big\downarrow \text{1230}$$

$$\frac{\sqrt{ax+bx^2}(2c+dx)}{d^2(c+dx)}-\frac{\int \frac{2ac+(4bc-ad)x}{(c+dx)\sqrt{bx^2+ax}}dx}{2d^2}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\sqrt{ax+bx^2}(2c+dx)}{d^2(c+dx)}-\frac{\frac{(4bc-ad)\int \frac{1}{\sqrt{bx^2+ax}}dx}{d}-\frac{c(4bc-3ad)\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{2d^2}$$

$$\Big\downarrow \text{1091}$$

$$\frac{\sqrt{ax+bx^2}(2c+dx)}{d^2(c+dx)}-\frac{\frac{2(4bc-ad)\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{d}-\frac{c(4bc-3ad)\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{2d^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{ax+bx^2}(2c+dx)}{d^2(c+dx)}-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(4bc-ad)}{\sqrt{b}d}-\frac{c(4bc-3ad)\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{2d^2}$$

$$\Big\downarrow \; 1154$$

$$\frac{\sqrt{ax+bx^2}(2c+dx)}{d^2(c+dx)}-\frac{\frac{2c(4bc-3ad)\int \frac{1}{4c(bc-ad)-\frac{(ac+(2bc-ad)x)^2}{bx^2+ax}}d\left(-\frac{ac+(2bc-ad)x}{\sqrt{bx^2+ax}}\right)}{d}+\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(4bc-ad)}{\sqrt{b}d}}{2d^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{ax+bx^2}(2c+dx)}{d^2(c+dx)}-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(4bc-ad)}{\sqrt{b}d}-\frac{\sqrt{c}(4bc-3ad)\text{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{d\sqrt{bc-ad}}}{2d^2}$$

input

```
Int[(x*Sqrt[a*x + b*x^2])/(c + d*x)^2,x]
```

output

```
((2*c + d*x)*Sqrt[a*x + b*x^2])/(d^2*(c + d*x)) - ((2*(4*b*c - a*d)*ArcTan
h[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(Sqrt[b]*d) - (Sqrt[c]*(4*b*c - 3*a*d)*A
rcTanh[(a*c + (2*b*c - a*d)*x)/(2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[a*x + b*x^2
])])/(d*Sqrt[b*c - a*d]))/(2*d^2)
```

Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1154

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$-\frac{4\left((dx+c)\left(bc-\frac{3ad}{4}\right)\sqrt{b}\,c\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)-\frac{\left(\frac{(dx+c)(ad-4bc)\,\text{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{2}+d\left(\frac{dx}{2}+c\right)\sqrt{x(bx+a)}\sqrt{b}\right)\sqrt{c(ad-}}{2}}{\sqrt{b}\sqrt{c(ad-bc)}\,d^3(dx+c)}$
risch	$\frac{x(bx+a)}{d^2\sqrt{x(bx+a)}}+\frac{\frac{(ad-4bc)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{d\sqrt{b}}+\frac{2c(2ad-3bc)\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2}}{x+\frac{c}{d}}\right)}{d^2\sqrt{-\frac{c(ad-bc)}{d^2}}}}{}$
default	$\frac{\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}+\frac{(ad-2bc)\ln\left(\frac{\frac{ad-2bc}{2d}+b\left(x+\frac{c}{d}\right)}{\sqrt{b}}+\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}\right)}{2d\sqrt{b}}+\frac{c(ad-b}{}}{d^2}$

input

```
int(x*(b*x^2+a*x)^(1/2)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output

```
-4/b^(1/2)/(c*(a*d-b*c))^(1/2)*((d*x+c)*(b*c-3/4*a*d)*b^(1/2)*c*arctan((x*
(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))-1/2*(1/2*(d*x+c)*(a*d-4*b*c)*arcta
nh((x*(b*x+a))^(1/2)/x/b^(1/2))+d*(1/2*d*x+c)*(x*(b*x+a))^(1/2)*b^(1/2))*(
c*(a*d-b*c))^(1/2))/d^3/(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 797, normalized size of antiderivative = 5.39

$$\int \frac{x\sqrt{ax+bx^2}}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(b*x^2+a*x)^(1/2)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[-1/2*((4*b*c^2 - a*c*d + (4*b*c*d - a*d^2)*x)*sqrt(b)*log(2*b*x + a + 2*s
qrt(b*x^2 + a*x)*sqrt(b)) + (4*b^2*c^2 - 3*a*b*c*d + (4*b^2*c*d - 3*a*b*d^
2)*x)*sqrt(c/(b*c - a*d))*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)
*(b*c - a*d)*sqrt(c/(b*c - a*d)))/(d*x + c)) - 2*(b*d^2*x + 2*b*c*d)*sqrt(
b*x^2 + a*x))/(b*d^4*x + b*c*d^3), 1/2*(2*(4*b^2*c^2 - 3*a*b*c*d + (4*b^2*
c*d - 3*a*b*d^2)*x)*sqrt(-c/(b*c - a*d))*arctan(-sqrt(b*x^2 + a*x)*(b*c -
a*d)*sqrt(-c/(b*c - a*d))/(b*c*x + a*c)) - (4*b*c^2 - a*c*d + (4*b*c*d - a
*d^2)*x)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(b*d^2*x
 + 2*b*c*d)*sqrt(b*x^2 + a*x))/(b*d^4*x + b*c*d^3), 1/2*(2*(4*b*c^2 - a*c*
d + (4*b*c*d - a*d^2)*x)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x +
 a)) - (4*b^2*c^2 - 3*a*b*c*d + (4*b^2*c*d - 3*a*b*d^2)*x)*sqrt(c/(b*c - a
*d))*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(c/(
b*c - a*d)))/(d*x + c)) + 2*(b*d^2*x + 2*b*c*d)*sqrt(b*x^2 + a*x))/(b*d^4*
x + b*c*d^3), ((4*b^2*c^2 - 3*a*b*c*d + (4*b^2*c*d - 3*a*b*d^2)*x)*sqrt(-c
/(b*c - a*d))*arctan(-sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(-c/(b*c - a*d))/(
b*c*x + a*c)) + (4*b*c^2 - a*c*d + (4*b*c*d - a*d^2)*x)*sqrt(-b)*arctan(sq
rt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (b*d^2*x + 2*b*c*d)*sqrt(b*x^2 + a*x
))/(b*d^4*x + b*c*d^3)]
```

Sympy [F]

$$\int \frac{x\sqrt{ax+bx^2}}{(c+dx)^2}\,dx = \int \frac{x\sqrt{x\,(a+bx)}}{(c+dx)^2}\,dx$$

input
```
integrate(x*(b*x**2+a*x)**(1/2)/(d*x+c)**2,x)
```

output
```
Integral(x*sqrt(x*(a + b*x))/(c + d*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{ax+bx^2}}{(c+dx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(b*x^2+a*x)^(1/2)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [F(-1)]

Timed out.

$$\int \frac{x\sqrt{ax+bx^2}}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(b*x^2+a*x)^(1/2)/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{ax+bx^2}}{(c+dx)^2}\,dx = \int \frac{x\,\sqrt{b\,x^2+a\,x}}{{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((x*(a*x + b*x^2)^(1/2))/(c + d*x)^2,x)
```

output
```
int((x*(a*x + b*x^2)^(1/2))/(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 714, normalized size of antiderivative = 4.82

$$\int \frac{x\sqrt{ax+bx^2}}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int(x*(b*x^2+a*x)^(1/2)/(d*x+c)^2,x)
```

output
```
(3*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) -
 sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b*c*d + 3*sqrt(c)*sqrt(a*d
- b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqr
t(b))/(sqrt(c)*sqrt(b)))*a*b*d**2*x - 4*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt
(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sq
rt(b)))*b**2*c**2 - 4*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt
(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**2*c*d*x
 + 3*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x)
 + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b*c*d + 3*sqrt(c)*sqrt(a*
d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*s
qrt(b))/(sqrt(c)*sqrt(b)))*a*b*d**2*x - 4*sqrt(c)*sqrt(a*d - b*c)*atan((sq
rt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*
sqrt(b)))*b**2*c**2 - 4*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sq
rt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**2*c*d
*x + 2*sqrt(x)*sqrt(a + b*x)*a*b*c*d**2 + sqrt(x)*sqrt(a + b*x)*a*b*d**3*x
 - 2*sqrt(x)*sqrt(a + b*x)*b**2*c**2*d - sqrt(x)*sqrt(a + b*x)*b**2*c*d**2
*x + sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*c*d**2 +
sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*d**3*x - 5*sqr
t(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b*c**2*d - 5*sqrt(b)
*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b*c*d**2*x + 4*sqrt(b...
```

3.40 $\int \frac{\sqrt{ax+bx^2}}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 121

$$\int \frac{\sqrt{ax+bx^2}}{(c+dx)^2}\,dx = -\frac{\sqrt{ax+bx^2}}{d(c+dx)} + \frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{d^2} - \frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{\sqrt{c}d^2\sqrt{bc-ad}}$$

output
```
-(b*x^2+a*x)^(1/2)/d/(d*x+c)+2*b^(1/2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2)
)/d^2-(-a*d+2*b*c)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c
^(1/2)/d^2/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{ax+bx^2}}{(c+dx)^2}\,dx = \frac{\sqrt{x(a+bx)}\left(-\frac{d}{c+dx} + \frac{(2bc-ad)\arctan\left(\frac{-d\sqrt{x}\sqrt{a+bx}+\sqrt{b}(c+dx)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{c}\sqrt{-bc+ad}\sqrt{x}\sqrt{a+bx}} - \frac{2\sqrt{b}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{x}\sqrt{a+bx}}\right)}{d^2}$$

input
```
Integrate[Sqrt[a*x + b*x^2]/(c + d*x)^2,x]
```

output
```
(Sqrt[x*(a + b*x)]*(-(d/(c + d*x)) + ((2*b*c - a*d)*ArcTan[(-(d*Sqrt[x]*Sq
rt[a + b*x]) + Sqrt[b]*(c + d*x))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(Sqrt[c]*
Sqrt[-(b*c) + a*d]*Sqrt[x]*Sqrt[a + b*x]) - (2*Sqrt[b]*Log[-(Sqrt[b]*Sqrt[
x]) + Sqrt[a + b*x]])/(Sqrt[x]*Sqrt[a + b*x])))/d^2
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1161, 1269, 1091, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{(c+dx)^2}dx$$

$$\Big\downarrow \text{1161}$$

$$\frac{\int \frac{a+2bx}{(c+dx)\sqrt{bx^2+ax}}dx}{2d}-\frac{\sqrt{ax+bx^2}}{d(c+dx)}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\frac{2b\int \frac{1}{\sqrt{bx^2+ax}}dx}{d}-\frac{(2bc-ad)\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{2d}-\frac{\sqrt{ax+bx^2}}{d(c+dx)}$$

$$\Big\downarrow \text{1091}$$

$$\frac{\frac{4b\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{d}-\frac{(2bc-ad)\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{2d}-\frac{\sqrt{ax+bx^2}}{d(c+dx)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{4\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{d}-\frac{(2bc-ad)\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{2d}-\frac{\sqrt{ax+bx^2}}{d(c+dx)}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\frac{2(2bc-ad)\int \frac{1}{4c(bc-ad)-\frac{(ac+(2bc-ad)x)^2}{bx^2+ax}}d\left(-\frac{ac+(2bc-ad)x}{\sqrt{bx^2+ax}}\right)}{d}+\frac{4\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{d}}{2d}-\frac{\sqrt{ax+bx^2}}{d(c+dx)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{4\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{d}-\frac{(2bc-ad)\text{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{\sqrt{c}d\sqrt{bc-ad}}}{2d}-\frac{\sqrt{ax+bx^2}}{d(c+dx)}$$

input
```
Int[Sqrt[a*x + b*x^2]/(c + d*x)^2,x]
```

output
```
-(Sqrt[a*x + b*x^2]/(d*(c + d*x))) + ((4*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[
a*x + b*x^2]])/d - ((2*b*c - a*d)*ArcTanh[(a*c + (2*b*c - a*d)*x)/(2*Sqrt[
c]*Sqrt[b*c - a*d]*Sqrt[a*x + b*x^2])])/(Sqrt[c]*d*Sqrt[b*c - a*d]))/(2*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$-\frac{(dx+c)(ad-2bc)\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)-2\left(\sqrt{b}\,(dx+c)\,\text{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)-\frac{d\sqrt{x(bx+a)}}{2}\right)\sqrt{c(ad-bc)}}{\sqrt{c(ad-bc)}\,d^2(dx+c)}$
default	$\frac{\frac{d^2\left(b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}\right)^{\frac{3}{2}}}{c(ad-bc)\left(x+\frac{c}{d}\right)}-\frac{(ad-2bc)d\left(\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}+\frac{(ad-2bc)\ln\left(\frac{\frac{ad-2bc}{2d}+b(}{\sqrt{b}}\right.}{}\right.}{}}{}$

input
```
int((b*x^2+a*x)^(1/2)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-((d*x+c)*(a*d-2*b*c)*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))-2*
(b^(1/2)*(d*x+c)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-1/2*d*(x*(b*x+a))^(1
/2))*(c*(a*d-b*c))^(1/2))/(c*(a*d-b*c))^(1/2)/d^2/(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 839, normalized size of antiderivative = 6.93

$$\int \frac{\sqrt{ax+bx^2}}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a*x)^(1/2)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[1/2*(2*(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x)*sqrt(b)*log(2*b*x + a +
2*sqrt(b*x^2 + a*x)*sqrt(b)) - (2*b*c^2 - a*c*d + (2*b*c*d - a*d^2)*x)*sqr
t(b*c^2 - a*c*d)*log((a*c + (2*b*c - a*d)*x + 2*sqrt(b*c^2 - a*c*d)*sqrt(b
*x^2 + a*x))/(d*x + c)) - 2*(b*c^2*d - a*c*d^2)*sqrt(b*x^2 + a*x))/(b*c^3*
d^2 - a*c^2*d^3 + (b*c^2*d^3 - a*c*d^4)*x), ((2*b*c^2 - a*c*d + (2*b*c*d -
 a*d^2)*x)*sqrt(-b*c^2 + a*c*d)*arctan(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a
*x)/(b*c*x + a*c)) + (b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x)*sqrt(b)*log
(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - (b*c^2*d - a*c*d^2)*sqrt(b*x^2
 + a*x))/(b*c^3*d^2 - a*c^2*d^3 + (b*c^2*d^3 - a*c*d^4)*x), -1/2*(4*(b*c^3
 - a*c^2*d + (b*c^2*d - a*c*d^2)*x)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt
(-b)/(b*x + a)) + (2*b*c^2 - a*c*d + (2*b*c*d - a*d^2)*x)*sqrt(b*c^2 - a*c
*d)*log((a*c + (2*b*c - a*d)*x + 2*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/
(d*x + c)) + 2*(b*c^2*d - a*c*d^2)*sqrt(b*x^2 + a*x))/(b*c^3*d^2 - a*c^2*d
^3 + (b*c^2*d^3 - a*c*d^4)*x), ((2*b*c^2 - a*c*d + (2*b*c*d - a*d^2)*x)*sq
rt(-b*c^2 + a*c*d)*arctan(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x +
a*c)) - 2*(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x)*sqrt(-b)*arctan(sqrt(b
*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (b*c^2*d - a*c*d^2)*sqrt(b*x^2 + a*x))/(
b*c^3*d^2 - a*c^2*d^3 + (b*c^2*d^3 - a*c*d^4)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2}}{(c+dx)^2}\,dx = \int \frac{\sqrt{x\,(a+bx)}}{(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a*x)**(1/2)/(d*x+c)**2,x)
```

output
```
Integral(sqrt(x*(a + b*x))/(c + d*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax+bx^2}}{(c+dx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a*x)^(1/2)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax+bx^2}}{(c+dx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a*x)^(1/2)/(d*x+c)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^2}}{(c+dx)^2}\,dx = \int \frac{\sqrt{b\,x^2+a\,x}}{{(c+d\,x)}^2}\,dx$$

input
```
int((a*x + b*x^2)^(1/2)/(c + d*x)^2,x)
```

output
```
int((a*x + b*x^2)^(1/2)/(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 603, normalized size of antiderivative = 4.98

$$\int \frac{\sqrt{ax+bx^2}}{(c+dx)^2}\,dx = \frac{-\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)acd-\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)a\,d^2}{}$$

input
```
int((b*x^2+a*x)^(1/2)/(d*x+c)^2,x)
```

output
```
( - sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x)
- sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*c*d - sqrt(c)*sqrt(a*d - b
*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b
))/(sqrt(c)*sqrt(b)))*a*d**2*x + 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d
- b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)
))*b*c**2 + 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt
(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b*c*d*x - sqrt(c)*
sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sq
rt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*c*d - sqrt(c)*sqrt(a*d - b*c)*atan((sq
rt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*
sqrt(b)))*a*d**2*x + 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqr
t(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b*c**2 +
2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) +
sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b*c*d*x - sqrt(x)*sqrt(a + b*x
)*a*c*d**2 + sqrt(x)*sqrt(a + b*x)*b*c**2*d + 2*sqrt(b)*log((sqrt(a + b*x)
 + sqrt(x)*sqrt(b))/sqrt(a))*a*c**2*d + 2*sqrt(b)*log((sqrt(a + b*x) + sqr
t(x)*sqrt(b))/sqrt(a))*a*c*d**2*x - 2*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)
*sqrt(b))/sqrt(a))*b*c**3 - 2*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b)
)/sqrt(a))*b*c**2*d*x)/(c*d**2*(a*c*d + a*d**2*x - b*c**2 - b*c*d*x))
```

3.41 $\int \frac{\sqrt{ax+bx^2}}{x(c+dx)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)^2}\,dx = \frac{\sqrt{ax+bx^2}}{c(c+dx)} + \frac{a\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{3/2}\sqrt{bc-ad}}$$

output
```
(b*x^2+a*x)^(1/2)/c/(d*x+c)+a*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*
x)^(1/2))/c^(3/2)/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)^2}\,dx = \frac{\sqrt{x(a+bx)}\left(\frac{\sqrt{c}}{c+dx} - \frac{a\arctan\left(\frac{-d\sqrt{x}\sqrt{a+bx}+\sqrt{b}(c+dx)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}\sqrt{x}\sqrt{a+bx}}\right)}{c^{3/2}}$$

input
```
Integrate[Sqrt[a*x + b*x^2]/(x*(c + d*x)^2),x]
```

output
```
(Sqrt[x*(a + b*x)]*(Sqrt[c]/(c + d*x) - (a*ArcTan[(-(d*Sqrt[x]*Sqrt[a + b*
x]) + Sqrt[b]*(c + d*x))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(Sqrt[-(b*c) + a*d
]*Sqrt[x]*Sqrt[a + b*x])))/c^(3/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.38, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1261, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)^2}\,dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{\sqrt{ax+bx^2}\int \frac{\sqrt{a+bx}}{\sqrt{x}(c+dx)^2}dx}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{105}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{2c}+\frac{\sqrt{x}\sqrt{a+bx}}{c(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{104}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{a\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}+\frac{\sqrt{x}\sqrt{a+bx}}{c(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{x}\sqrt{a+bx}}{c(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

input
```
Int[Sqrt[a*x + b*x^2]/(x*(c + d*x)^2),x]
```

output
```
(Sqrt[a*x + b*x^2]*((Sqrt[x]*Sqrt[a + b*x])/(c*(c + d*x)) + (a*ArcTanh[(Sq
rt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(c^(3/2)*Sqrt[b*c - a*d])
))/(Sqrt[x]*Sqrt[a + b*x])
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{\frac{\sqrt{x(bx+a)}}{dx+c}-\frac{a\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{\sqrt{c(ad-bc)}}}{c}$
default	$\frac{\sqrt{b\,x^2+ax}+\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2\sqrt{b}}}{c^2}-\frac{\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}+\frac{(ad-2bc)\ln\left(\frac{\frac{ad-2bc}{2d}+b\left(x+\frac{c}{d}\right)}{\sqrt{b}}+\sqrt{b\left(x+\frac{c}{d}\right.}\ \ldots\right.}{2d\sqrt{b}}}{}$

input
```
int((b*x^2+a*x)^(1/2)/x/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*((x*(b*x+a))^(1/2)/(d*x+c)-a/(c*(a*d-b*c))^(1/2)*arctan((x*(b*x+a))^(1
/2)/x*c/(c*(a*d-b*c))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 251, normalized size of antiderivative = 3.26

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)^2}\,dx$$
$$=\left[\frac{\sqrt{bc^2-acd}(adx+ac)\log\left(\frac{ac+(2\,bc-ad)x+2\,\sqrt{bc^2-acd}\sqrt{bx^2+ax}}{dx+c}\right)+2\,(bc^2-acd)\sqrt{bx^2+ax}}{2\,(bc^4-ac^3d+(bc^3d-ac^2d^2)x)},\right.$$
$$\left.-\frac{\sqrt{-bc^2+acd}(adx+ac)\arctan\left(\frac{\sqrt{-bc^2+acd}\sqrt{bx^2+ax}}{bcx+ac}\right)-(bc^2-acd)\sqrt{bx^2+ax}}{bc^4-ac^3d+(bc^3d-ac^2d^2)x}\right]$$

input
```
integrate((b*x^2+a*x)^(1/2)/x/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b*c^2 - a*c*d)*(a*d*x + a*c)*log((a*c + (2*b*c - a*d)*x + 2*sqr
t(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) + 2*(b*c^2 - a*c*d)*sqrt(b*
x^2 + a*x))/(b*c^4 - a*c^3*d + (b*c^3*d - a*c^2*d^2)*x), -(sqrt(-b*c^2 + a
*c*d)*(a*d*x + a*c)*arctan(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x +
 a*c)) - (b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(b*c^4 - a*c^3*d + (b*c^3*d -
a*c^2*d^2)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)^2}\,dx = \int \frac{\sqrt{x\,(a+bx)}}{x\,(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a*x)**(1/2)/x/(d*x+c)**2,x)
```

output
```
Integral(sqrt(x*(a + b*x))/(x*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)^2}\,dx = \int \frac{\sqrt{bx^2+ax}}{(dx+c)^2x}\,dx$$

input
```
integrate((b*x^2+a*x)^(1/2)/x/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a*x)/((d*x + c)^2*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(65) = 130$.

Time = 0.42 (sec) , antiderivative size = 313, normalized size of antiderivative = 4.06

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)^2}\,dx = -\frac{1}{2}d^3\left(\frac{a\log\left(\left|2\,bcd-ad^2-2\sqrt{bc^2-acd}\left(\sqrt{b-\frac{2\,bc}{dx+c}+\frac{bc^2}{(dx+c)^2}+\frac{ad}{dx+c}-\frac{acd}{(dx+c)^2}}+\frac{\sqrt{bc^2d^2-acd^3}}{(dx+c)d}\right)|d|\right|\right)\mathrm{sg}}{\sqrt{bc^2-acd}cd^2|d|}\right.$$

input
```
integrate((b*x^2+a*x)^(1/2)/x/(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/2*d^3*(a*log(abs(2*b*c*d - a*d^2 - 2*sqrt(b*c^2 - a*c*d)*(sqrt(b - 2*b*
c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d/(d*x + c) - a*c*d/(d*x + c)^2) + sqr
t(b*c^2*d^2 - a*c*d^3)/((d*x + c)*d))*abs(d)))*sgn(1/(d*x + c))*sgn(d)/(sq
rt(b*c^2 - a*c*d)*c*d^2*abs(d)) - 2*sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x
+ c)^2 + a*d/(d*x + c) - a*c*d/(d*x + c)^2)*sgn(1/(d*x + c))*sgn(d)/(c*d^4
) - (a*d^2*log(abs(2*b*c*d - a*d^2 - 2*sqrt(b*c^2 - a*c*d)*sqrt(b)*abs(d))
) - 2*sqrt(b*c^2 - a*c*d)*sqrt(b)*abs(d))*sgn(1/(d*x + c))*sgn(d)/(sqrt(b*
c^2 - a*c*d)*c*d^4*abs(d)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)^2}\,dx = \int \frac{\sqrt{b\,x^2+a\,x}}{x\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((a*x + b*x^2)^(1/2)/(x*(c + d*x)^2),x)
```

output
```
int((a*x + b*x^2)^(1/2)/(x*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.45

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)^2}\,dx$$
$$= \frac{-\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)ac-\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)adx-}{}$$

input
```
int((b*x^2+a*x)^(1/2)/x/(d*x+c)^2,x)
```

output
```
( - sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x)
- sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*c - sqrt(c)*sqrt(a*d - b*c
)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))
/(sqrt(c)*sqrt(b)))*a*d*x - sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c)
+ sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*c
- sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) +
sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*d*x + sqrt(x)*sqrt(a + b*x)*
a*c*d - sqrt(x)*sqrt(a + b*x)*b*c**2)/(c**2*(a*c*d + a*d**2*x - b*c**2 - b
*c*d*x))
```

3.42 $\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)^2}\,dx = -\frac{3\sqrt{ax+bx^2}}{c^2x}+\frac{\sqrt{ax+bx^2}}{cx(c+dx)}+\frac{(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{5/2}\sqrt{bc-ad}}$$

output
```
-3*(b*x^2+a*x)^(1/2)/c^2/x+(b*x^2+a*x)^(1/2)/c/x/(d*x+c)+(-3*a*d+2*b*c)*ar
ctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(5/2)/(-a*d+b*c)^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.57 (sec) , antiderivative size = 1330, normalized size of antiderivative = 12.20

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[a*x + b*x^2]/(x^2*(c + d*x)^2),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*((-2*a*c^(3/2))/(Sqrt[x]*(c + d*x)*(-Sqrt[a] + Sqrt
[a + b*x])) - (2*b*c^(3/2)*Sqrt[x])/((c + d*x)*(-Sqrt[a] + Sqrt[a + b*x]))
 - (3*a*Sqrt[c]*d*Sqrt[x])/((c + d*x)*(-Sqrt[a] + Sqrt[a + b*x])) - (3*b*S
qrt[c]*d*x^(3/2))/((c + d*x)*(-Sqrt[a] + Sqrt[a + b*x])) + (2*Sqrt[a]*c^(3
/2)*Sqrt[a + b*x])/(Sqrt[x]*(c + d*x)*(-Sqrt[a] + Sqrt[a + b*x])) + (3*Sqr
t[a]*Sqrt[c]*d*Sqrt[x]*Sqrt[a + b*x])/((c + d*x)*(-Sqrt[a] + Sqrt[a + b*x]
)) + (a*d*((5*I)*Sqrt[a]*b*c*Sqrt[d] + 5*b*c*Sqrt[b*c - a*d] - 3*a*d*Sqrt[
b*c - a*d])*ArcTan[(Sqrt[-(b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c -
 a*d]]*Sqrt[x])/(Sqrt[c]*(Sqrt[a] - Sqrt[a + b*x]))]])/((b*c - a*d)^(3/2)*S
qrt[-(b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]) + (((2*I)*Sqr
t[a]*b^2*c^2*Sqrt[d] + (3*I)*a^(5/2)*d^(5/2) + 5*a*b*c*d*Sqrt[b*c - a*d] -
 3*a^2*d^2*Sqrt[b*c - a*d])*ArcTan[(Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sq
rt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*(Sqrt[a] - Sqrt[a + b*x]))]])/((b*
c - a*d)^(3/2)*Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]
]) + ((2*I)*Sqrt[a]*b^2*c^2*Sqrt[d]*ArcTan[(Sqrt[-(b*c) + 2*a*d - (2*I)*Sq
rt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*(-Sqrt[a] + Sqrt[a + b*x]
))]])/((b*c - a*d)^(3/2)*Sqrt[-(b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b
*c - a*d]]) + ((3*I)*a^(5/2)*d^(5/2)*ArcTan[(Sqrt[-(b*c) + 2*a*d - (2*I)*S
qrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*(-Sqrt[a] + Sqrt[a + b*x
]))]])/((b*c - a*d)^(3/2)*Sqrt[-(b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sq...
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.39, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1261, 107, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)^2}\,dx$$

$$\Big\downarrow \text{ 1261}$$

$$\frac{\sqrt{ax+bx^2}\int \frac{\sqrt{a+bx}}{x^{3/2}(c+dx)^2}dx}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 107}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{(2bc-3ad)\int \frac{\sqrt{a+bx}}{\sqrt{x}(c+dx)^2}dx}{ac}-\frac{2(a+bx)^{3/2}}{ac\sqrt{x}(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{105}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{(2bc-3ad)\left(\frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{2c}+\frac{\sqrt{x}\sqrt{a+bx}}{c(c+dx)}\right)}{ac}-\frac{2(a+bx)^{3/2}}{ac\sqrt{x}(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{104}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{(2bc-3ad)\left(\frac{a\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}+\frac{\sqrt{x}\sqrt{a+bx}}{c(c+dx)}\right)}{ac}-\frac{2(a+bx)^{3/2}}{ac\sqrt{x}(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{(2bc-3ad)\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{x}\sqrt{a+bx}}{c(c+dx)}\right)}{ac}-\frac{2(a+bx)^{3/2}}{ac\sqrt{x}(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

input
```
Int[Sqrt[a*x + b*x^2]/(x^2*(c + d*x)^2),x]
```

output
```
(Sqrt[a*x + b*x^2]*((-2*(a + b*x)^(3/2))/(a*c*Sqrt[x]*(c + d*x)) + ((2*b*c
 - 3*a*d)*((Sqrt[x]*Sqrt[a + b*x])/(c*(c + d*x)) + (a*ArcTanh[(Sqrt[b*c -
a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(c^(3/2)*Sqrt[b*c - a*d])))/(a*c))
)/(Sqrt[x]*Sqrt[a + b*x])
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.92

method	result
pseudoelliptic	$\frac{-2\sqrt{x(bx+a)}\,\sqrt{c(ad-bc)}\left(\frac{3dx}{2}+c\right)+x\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)(dx+c)(3ad-2bc)}{\sqrt{c(ad-bc)}\,c^{2}x(dx+c)}$
risch	$-\frac{2(bx+a)}{c^{2}\sqrt{x(bx+a)}}-\frac{-\frac{a\ln\left(\frac{-\frac{2c(ad-bc)}{d^{2}}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^{2}}}\sqrt{b\left(x+\frac{c}{d}\right)^{2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^{2}}}}{x+\frac{c}{d}}\right)}{\sqrt{-\frac{c(ad-bc)}{d^{2}}}}+\frac{c(ad-b}{}}{}$
default	Expression too large to display

input
```
int((b*x^2+a*x)^(1/2)/x^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
(-2*(x*(b*x+a))^(1/2)*(c*(a*d-b*c))^(1/2)*(3/2*d*x+c)+x*arctan((x*(b*x+a))
^(1/2)/x*c/(c*(a*d-b*c))^(1/2))*(d*x+c)*(3*a*d-2*b*c))/(c*(a*d-b*c))^(1/2)
/c^2/x/(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 346, normalized size of antiderivative = 3.17

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)^2}\,dx$$
$$= \left[-\frac{\sqrt{bc^2-acd}((2\,bcd-3\,ad^2)x^2+(2\,bc^2-3\,acd)x)\log\left(\frac{ac+(2\,bc-ad)x-2\,\sqrt{bc^2-acd}\sqrt{bx^2+ax}}{dx+c}\right)+2\,(2\,bc^3-2\,a}{2\,((bc^4d-ac^3d^2)x^2+(bc^5-ac^4d)x)}\right.$$
$$-\frac{\sqrt{-bc^2+acd}((2\,bcd-3\,ad^2)x^2+(2\,bc^2-3\,acd)x)\arctan\left(\frac{\sqrt{-bc^2+acd}\sqrt{bx^2+ax}}{bcx+ac}\right)+(2\,bc^3-2\,ac^2d+3\,(b}{(bc^4d-ac^3d^2)x^2+(bc^5-ac^4d)x}$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(b*c^2 - a*c*d)*((2*b*c*d - 3*a*d^2)*x^2 + (2*b*c^2 - 3*a*c*d)*
x)*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(
d*x + c)) + 2*(2*b*c^3 - 2*a*c^2*d + 3*(b*c^2*d - a*c*d^2)*x)*sqrt(b*x^2 +
 a*x))/((b*c^4*d - a*c^3*d^2)*x^2 + (b*c^5 - a*c^4*d)*x), -(sqrt(-b*c^2 +
a*c*d)*((2*b*c*d - 3*a*d^2)*x^2 + (2*b*c^2 - 3*a*c*d)*x)*arctan(sqrt(-b*c^
2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) + (2*b*c^3 - 2*a*c^2*d + 3*(b*
c^2*d - a*c*d^2)*x)*sqrt(b*x^2 + a*x))/((b*c^4*d - a*c^3*d^2)*x^2 + (b*c^5
 - a*c^4*d)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)^2}\,dx = \int \frac{\sqrt{x\,(a+bx)}}{x^2\,(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a*x)**(1/2)/x**2/(d*x+c)**2,x)
```

output
```
Integral(sqrt(x*(a + b*x))/(x**2*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)^2}\,dx = \int \frac{\sqrt{bx^2+ax}}{(dx+c)^2x^2}\,dx$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a*x)/((d*x + c)^2*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 746 vs. $2(95) = 190$.

Time = 0.81 (sec) , antiderivative size = 746, normalized size of antiderivative = 6.84

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/6*d^2*((2*b*c - 3*a*d)*log(abs(-2*(b*c^3 - a*c^2*d)*(sqrt(b - 2*b*c/(d*
x + c) + b*c^2/(d*x + c)^2 + a*d/(d*x + c) - a*c*d/(d*x + c)^2) + sqrt(b*c
^2*d^2 - a*c*d^3)/((d*x + c)*d))^3*abs(d) + (6*b*c^2*d - 5*a*c*d^2)*sqrt(b
*c^2 - a*c*d)*(sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d/(d*x + c
) - a*c*d/(d*x + c)^2) + sqrt(b*c^2*d^2 - a*c*d^3)/((d*x + c)*d))^2 - 2*(3
*b^2*c^3 - 5*a*b*c^2*d + 2*a^2*c*d^2)*(sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d
*x + c)^2 + a*d/(d*x + c) - a*c*d/(d*x + c)^2) + sqrt(b*c^2*d^2 - a*c*d^3)
/((d*x + c)*d))*abs(d) + (2*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*sqrt(b*c^2
- a*c*d)))*sgn(1/(d*x + c))*sgn(d)/(sqrt(b*c^2 - a*c*d)*c^2*d*abs(d)) + 6*
sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d/(d*x + c) - a*c*d/(d*x
+ c)^2)*sgn(1/(d*x + c))*sgn(d)/(c^2*d^2) - (2*b*c*d*log(abs(-8*b^(5/2)*c^
3*abs(d) + 12*a*b^(3/2)*c^2*d*abs(d) - 4*a^2*sqrt(b)*c*d^2*abs(d) + 8*sqrt
(b*c^2 - a*c*d)*b^2*c^2*d - 8*sqrt(b*c^2 - a*c*d)*a*b*c*d^2 + sqrt(b*c^2 -
 a*c*d)*a^2*d^3)) - 3*a*d^2*log(abs(-8*b^(5/2)*c^3*abs(d) + 12*a*b^(3/2)*c
^2*d*abs(d) - 4*a^2*sqrt(b)*c*d^2*abs(d) + 8*sqrt(b*c^2 - a*c*d)*b^2*c^2*d
 - 8*sqrt(b*c^2 - a*c*d)*a*b*c*d^2 + sqrt(b*c^2 - a*c*d)*a^2*d^3)) + 6*sqr
t(b*c^2 - a*c*d)*sqrt(b)*abs(d))*sgn(1/(d*x + c))*sgn(d)/(sqrt(b*c^2 - a*c
*d)*c^2*d^2*abs(d)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)^2}\,dx = \int \frac{\sqrt{b\,x^2+a\,x}}{x^2\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((a*x + b*x^2)^(1/2)/(x^2*(c + d*x)^2),x)
```

output
```
int((a*x + b*x^2)^(1/2)/(x^2*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 949, normalized size of antiderivative = 8.71

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a*x)^(1/2)/x^2/(d*x+c)^2,x)
```

output
```
(9*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) -
 sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*c*d**2*x + 9*sqrt(c)*sqr
t(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(
d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*d**3*x**2 - 18*sqrt(c)*sqrt(a*d - b*c)
*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/
(sqrt(c)*sqrt(b)))*a*b*c**2*d*x - 18*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*
d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(
b)))*a*b*c*d**2*x**2 + 8*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - s
qrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**2*c*
*3*x + 8*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a +
b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**2*c**2*d*x**2 + 9*sq
rt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt
(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*c*d**2*x + 9*sqrt(c)*sqrt(a*d
 - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sq
rt(b))/(sqrt(c)*sqrt(b)))*a**2*d**3*x**2 - 18*sqrt(c)*sqrt(a*d - b*c)*atan
((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt
(c)*sqrt(b)))*a*b*c**2*d*x - 18*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b
*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*
a*b*c*d**2*x**2 + 8*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d
)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**2*c**3...
```

3.43 $\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 148

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)^2}\,dx = -\frac{5\sqrt{ax+bx^2}}{3c^2x^2} - \frac{(2bc-15ad)\sqrt{ax+bx^2}}{3ac^3x} + \frac{\sqrt{ax+bx^2}}{cx^2(c+dx)} - \frac{d(4bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{7/2}\sqrt{bc-ad}}$$

output
```
-5/3*(b*x^2+a*x)^(1/2)/c^2/x^2-1/3*(-15*a*d+2*b*c)*(b*x^2+a*x)^(1/2)/a/c^3
/x+(b*x^2+a*x)^(1/2)/c/x^2/(d*x+c)-d*(-5*a*d+4*b*c)*arctanh((-a*d+b*c)^(1/
2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(7/2)/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)^2}\,dx = \frac{\sqrt{x(a+bx)}\left(\sqrt{c}\left(-\frac{2bcx}{a}+\frac{-2c^2+10cdx+15d^2x^2}{c+dx}\right)+\frac{3d(4bc-5ad)x^{3/2}\arctan\left(\frac{-d\sqrt{x}\sqrt{a+bx}+\sqrt{b}(c+dx)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}\sqrt{a+bx}}\right)}{3c^{7/2}x^2}$$

input
```
Integrate[Sqrt[a*x + b*x^2]/(x^3*(c + d*x)^2),x]
```

output
```
(Sqrt[x*(a + b*x)]*(Sqrt[c]*((-2*b*c*x)/a + (-2*c^2 + 10*c*d*x + 15*d^2*x^
2)/(c + d*x)) + (3*d*(4*b*c - 5*a*d)*x^(3/2)*ArcTan[(-(d*Sqrt[x]*Sqrt[a +
b*x]) + Sqrt[b]*(c + d*x))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(Sqrt[-(b*c) + a
*d]*Sqrt[a + b*x])))/(3*c^(7/2)*x^2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.30, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1261, 110, 27, 168, 27, 169, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)^2}dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{\sqrt{ax+bx^2}\int \frac{\sqrt{a+bx}}{x^{5/2}(c+dx)^2}dx}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{110}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{2\int \frac{bc-5ad-4bdx}{2x^{3/2}\sqrt{a+bx}(c+dx)^2}dx}{3c}-\frac{2\sqrt{a+bx}}{3cx^{3/2}(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\int \frac{bc-5ad-4bdx}{x^{3/2}\sqrt{a+bx}(c+dx)^2}dx}{3c}-\frac{2\sqrt{a+bx}}{3cx^{3/2}(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{168}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{-\frac{\int -\frac{(bc-ad)(2bc-15ad-10bdx)}{2x^{3/2}\sqrt{a+bx}(c+dx)}dx}{c(bc-ad)}-\frac{5d\sqrt{a+bx}}{c\sqrt{x}(c+dx)}}{3c}-\frac{2\sqrt{a+bx}}{3cx^{3/2}(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\int \frac{2bc-15ad-10bdx}{x^{3/2}\sqrt{a+bx}(c+dx)}dx}{2c}-\frac{5d\sqrt{a+bx}}{c\sqrt{x}(c+dx)}}{3c}-\frac{2\sqrt{a+bx}}{3cx^{3/2}(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 169$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{-\frac{2\int \frac{3ad(4bc-5ad)}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{ac}-\frac{2\sqrt{a+bx}(2bc-15ad)}{ac\sqrt{x}}}{2c}-\frac{5d\sqrt{a+bx}}{c\sqrt{x}(c+dx)}}{3c}-\frac{2\sqrt{a+bx}}{3cx^{3/2}(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{-\frac{3d(4bc-5ad)\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{c}-\frac{2\sqrt{a+bx}(2bc-15ad)}{ac\sqrt{x}}}{2c}-\frac{5d\sqrt{a+bx}}{c\sqrt{x}(c+dx)}}{3c}-\frac{2\sqrt{a+bx}}{3cx^{3/2}(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 104$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{-\frac{6d(4bc-5ad)\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}-\frac{2\sqrt{a+bx}(2bc-15ad)}{ac\sqrt{x}}}{2c}-\frac{5d\sqrt{a+bx}}{c\sqrt{x}(c+dx)}}{3c}-\frac{2\sqrt{a+bx}}{3cx^{3/2}(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 221$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{-\frac{6d(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{a+bx}(2bc-15ad)}{ac\sqrt{x}}}{2c}-\frac{5d\sqrt{a+bx}}{c\sqrt{x}(c+dx)}}{3c}-\frac{2\sqrt{a+bx}}{3cx^{3/2}(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

input
```
Int[Sqrt[a*x + b*x^2]/(x^3*(c + d*x)^2),x]
```

output
```
(Sqrt[a*x + b*x^2]*((-2*Sqrt[a + b*x])/(3*c*x^(3/2)*(c + d*x)) + ((-5*d*Sq
rt[a + b*x])/(c*Sqrt[x]*(c + d*x)) + ((-2*(2*b*c - 15*a*d)*Sqrt[a + b*x])/
(a*c*Sqrt[x]) - (6*d*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sq
rt[c]*Sqrt[a + b*x])])/(c^(3/2)*Sqrt[b*c - a*d]))/(2*c))/(3*c)))/(Sqrt[x]*
Sqrt[a + b*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$\frac{-2\sqrt{x(bx+a)}\left(c^2(bx+a)-5d\left(-\frac{bx}{5}+a\right)xc-\frac{15a\,d^2x^2}{2}\right)\sqrt{c(ad-bc)}-3d\,x^2a\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)(dx+c)(5ad-4bc)}{3\sqrt{c(ad-bc)}\,c^3x^2(dx+c)a}$
risch	$-\frac{2(bx+a)(-6adx+cbx+ac)}{3a\,c^3\sqrt{x(bx+a)}\,x}+\frac{d\left(-\frac{(2ad-bc)\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}\cdots}}{x+\frac{c}{d}}\right)}{d\sqrt{-\frac{c(ad-bc)}{d^2}}}\cdots\right)}{\cdots}$
default	Expression too large to display

input
```
int((b*x^2+a*x)^(1/2)/x^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-2*(x*(b*x+a))^(1/2)*(c^2*(b*x+a)-5*d*(-1/5*b*x+a)*x*c-15/2*a*d^2*x^2
)*(c*(a*d-b*c))^(1/2)-3*d*x^2*a*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))
^(1/2))*(d*x+c)*(5*a*d-4*b*c))/(c*(a*d-b*c))^(1/2)/c^3/x^2/(d*x+c)/a
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 489, normalized size of antiderivative = 3.30

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)^2}\,dx$$
$$= \left[-\frac{3\left((4\,abcd^2-5\,a^2d^3)x^3+(4\,abc^2d-5\,a^2cd^2)x^2\right)\sqrt{bc^2-acd}\log\left(\frac{ac+(2\,bc-ad)x+2\,\sqrt{bc^2-acd}\sqrt{bx^2+ax}}{dx+c}\right)+2\,(2}{6\left((abc^5d-a^2c^4d^2)x^3+\right.}\right.$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[-1/6*(3*((4*a*b*c*d^2 - 5*a^2*d^3)*x^3 + (4*a*b*c^2*d - 5*a^2*c*d^2)*x^2)
*sqrt(b*c^2 - a*c*d)*log((a*c + (2*b*c - a*d)*x + 2*sqrt(b*c^2 - a*c*d)*sq
rt(b*x^2 + a*x))/(d*x + c)) + 2*(2*a*b*c^4 - 2*a^2*c^3*d + (2*b^2*c^3*d -
17*a*b*c^2*d^2 + 15*a^2*c*d^3)*x^2 + 2*(b^2*c^4 - 6*a*b*c^3*d + 5*a^2*c^2*
d^2)*x)*sqrt(b*x^2 + a*x))/((a*b*c^5*d - a^2*c^4*d^2)*x^3 + (a*b*c^6 - a^2
*c^5*d)*x^2), 1/3*(3*((4*a*b*c*d^2 - 5*a^2*d^3)*x^3 + (4*a*b*c^2*d - 5*a^2
*c*d^2)*x^2)*sqrt(-b*c^2 + a*c*d)*arctan(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 +
 a*x)/(b*c*x + a*c)) - (2*a*b*c^4 - 2*a^2*c^3*d + (2*b^2*c^3*d - 17*a*b*c^
2*d^2 + 15*a^2*c*d^3)*x^2 + 2*(b^2*c^4 - 6*a*b*c^3*d + 5*a^2*c^2*d^2)*x)*s
qrt(b*x^2 + a*x))/((a*b*c^5*d - a^2*c^4*d^2)*x^3 + (a*b*c^6 - a^2*c^5*d)*x
^2)]
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)^2}\,dx = \int \frac{\sqrt{x\,(a+bx)}}{x^3\,(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a*x)**(1/2)/x**3/(d*x+c)**2,x)
```

output
```
Integral(sqrt(x*(a + b*x))/(x**3*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)^2}\,dx = \int \frac{\sqrt{bx^2+ax}}{(dx+c)^2x^3}\,dx$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a*x)/((d*x + c)^2*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1100 vs. $2(128) = 256$.

Time = 9.08 (sec) , antiderivative size = 1100, normalized size of antiderivative = 7.43

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/10*d^2*((4*b*c - 5*a*d)*log(abs(-2*(b*c^4 - a*c^3*d)*(sqrt(b - 2*b*c/(d*
x + c) + b*c^2/(d*x + c)^2 + a*d/(d*x + c) - a*c*d/(d*x + c)^2) + sqrt(b*c
^2*d^2 - a*c*d^3)/((d*x + c)*d))^5*abs(d) + (10*b*c^3*d - 9*a*c^2*d^2)*sqr
t(b*c^2 - a*c*d)*(sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d/(d*x
+ c) - a*c*d/(d*x + c)^2) + sqrt(b*c^2*d^2 - a*c*d^3)/((d*x + c)*d))^4 - 4
*(5*b^2*c^4 - 9*a*b*c^3*d + 4*a^2*c^2*d^2)*(sqrt(b - 2*b*c/(d*x + c) + b*c
^2/(d*x + c)^2 + a*d/(d*x + c) - a*c*d/(d*x + c)^2) + sqrt(b*c^2*d^2 - a*c
*d^3)/((d*x + c)*d))^3*abs(d) + 2*(10*b^2*c^3*d - 17*a*b*c^2*d^2 + 7*a^2*c
*d^3)*sqrt(b*c^2 - a*c*d)*(sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 +
a*d/(d*x + c) - a*c*d/(d*x + c)^2) + sqrt(b*c^2*d^2 - a*c*d^3)/((d*x + c)*
d))^2 - 2*(5*b^3*c^4 - 13*a*b^2*c^3*d + 11*a^2*b*c^2*d^2 - 3*a^3*c*d^3)*(s
qrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d/(d*x + c) - a*c*d/(d*x +
 c)^2) + sqrt(b*c^2*d^2 - a*c*d^3)/((d*x + c)*d))*abs(d) + (2*b^3*c^3*d -
5*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3 - a^3*d^4)*sqrt(b*c^2 - a*c*d)))*sgn(1/(d*
x + c))*sgn(d)/(sqrt(b*c^2 - a*c*d)*c^3*abs(d)) + 10*sqrt(b - 2*b*c/(d*x +
 c) + b*c^2/(d*x + c)^2 + a*d/(d*x + c) - a*c*d/(d*x + c)^2)*sgn(1/(d*x +
c))*sgn(d)/(c^3*d) - (4*b*c*d*log(abs(-32*b^(7/2)*c^4*abs(d) + 64*a*b^(5/2
)*c^3*d*abs(d) - 38*a^2*b^(3/2)*c^2*d^2*abs(d) + 6*a^3*sqrt(b)*c*d^3*abs(d
) + 32*sqrt(b*c^2 - a*c*d)*b^3*c^3*d - 48*sqrt(b*c^2 - a*c*d)*a*b^2*c^2*d^
2 + 18*sqrt(b*c^2 - a*c*d)*a^2*b*c*d^3 - sqrt(b*c^2 - a*c*d)*a^3*d^4)) ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)^2}\,dx=\int \frac{\sqrt{b\,x^2+a\,x}}{x^3\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((a*x + b*x^2)^(1/2)/(x^3*(c + d*x)^2),x)
```

output
```
int((a*x + b*x^2)^(1/2)/(x^3*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 743, normalized size of antiderivative = 5.02

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)^2}\,dx = \text{Too large to display}$$

input

```
int((b*x^2+a*x)^(1/2)/x^3/(d*x+c)^2,x)
```

output

```
( - 15*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*
x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*c*d**2*x**2 - 15*sqr
t(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(
x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*d**3*x**3 + 12*sqrt(c)*sqrt(a*
d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*s
qrt(b))/(sqrt(c)*sqrt(b)))*a*b*c**2*d*x**2 + 12*sqrt(c)*sqrt(a*d - b*c)*at
an((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sq
rt(c)*sqrt(b)))*a*b*c*d**2*x**3 - 15*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*
d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(
b)))*a**2*c*d**2*x**2 - 15*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) +
 sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*
d**3*x**3 + 12*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqr
t(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b*c**2*d*x**2 +
 12*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x)
+ sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b*c*d**2*x**3 - 2*sqrt(x)*
sqrt(a + b*x)*a**2*c**3*d + 10*sqrt(x)*sqrt(a + b*x)*a**2*c**2*d**2*x + 15
*sqrt(x)*sqrt(a + b*x)*a**2*c*d**3*x**2 + 2*sqrt(x)*sqrt(a + b*x)*a*b*c**4
 - 12*sqrt(x)*sqrt(a + b*x)*a*b*c**3*d*x - 17*sqrt(x)*sqrt(a + b*x)*a*b*c*
*2*d**2*x**2 + 2*sqrt(x)*sqrt(a + b*x)*b**2*c**4*x + 2*sqrt(x)*sqrt(a + b*
x)*b**2*c**3*d*x**2 - 9*sqrt(b)*a**2*c**2*d**2*x**2 - 9*sqrt(b)*a**2*c*...
```

3.44 $\int \frac{\sqrt{ax+bx^2}}{x^4(c+dx)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 198

$$\int \frac{\sqrt{ax+bx^2}}{x^4(c+dx)^2}\,dx = -\frac{7\sqrt{ax+bx^2}}{5c^2x^3} - \frac{(2bc-35ad)\sqrt{ax+bx^2}}{15ac^3x^2} + \frac{(4b^2c^2+20abcd-105a^2d^2)\sqrt{ax+bx^2}}{15a^2c^4x} + \frac{\sqrt{ax+bx^2}}{cx^3(c+dx)} + \frac{d^2(6bc-7ad)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{9/2}\sqrt{bc-ad}}$$

output
```
-7/5*(b*x^2+a*x)^(1/2)/c^2/x^3-1/15*(-35*a*d+2*b*c)*(b*x^2+a*x)^(1/2)/a/c^
3/x^2+1/15*(-105*a^2*d^2+20*a*b*c*d+4*b^2*c^2)*(b*x^2+a*x)^(1/2)/a^2/c^4/x
+(b*x^2+a*x)^(1/2)/c/x^3/(d*x+c)+d^2*(-7*a*d+6*b*c)*arctanh((-a*d+b*c)^(1/
2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(9/2)/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{ax+bx^2}}{x^4(c+dx)^2}\,dx = \frac{\sqrt{x(a+bx)}\left(\frac{\sqrt{c}(4b^2c^2x^2(c+dx)-2abcx(c^2-9cdx-10d^2x^2)-a^2(6c^3-14c^2dx+70cd^2x^2+105d^3x^3))}{a^2(c+dx)}+\frac{15d^2(-6bc+7ad)x^{5/2}\arctan\left(\frac{-}{}\right.}{\sqrt{-bc+ad}\sqrt{a}}\right.}{15c^{9/2}x^3}$$

input
```
Integrate[Sqrt[a*x + b*x^2]/(x^4*(c + d*x)^2),x]
```

output
```
(Sqrt[x*(a + b*x)]*((Sqrt[c]*(4*b^2*c^2*x^2*(c + d*x) - 2*a*b*c*x*(c^2 - 9
*c*d*x - 10*d^2*x^2) - a^2*(6*c^3 - 14*c^2*d*x + 70*c*d^2*x^2 + 105*d^3*x^
3)))/(a^2*(c + d*x)) + (15*d^2*(-6*b*c + 7*a*d)*x^(5/2)*ArcTan[(-(d*Sqrt[x
]*Sqrt[a + b*x]) + Sqrt[b]*(c + d*x))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(Sqrt
[-(b*c) + a*d]*Sqrt[a + b*x])))/(15*c^(9/2)*x^3)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.25, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1261, 110, 27, 168, 27, 169, 27, 169, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{x^4(c+dx)^2}\,dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{\sqrt{ax+bx^2}\int \frac{\sqrt{a+bx}}{x^{7/2}(c+dx)^2}dx}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{110}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{2\int \frac{bc-7ad-6bdx}{2x^{5/2}\sqrt{a+bx}(c+dx)^2}dx}{5c}-\frac{2\sqrt{a+bx}}{5cx^{5/2}(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\int \frac{bc-7ad-6bdx}{x^{5/2}\sqrt{a+bx}(c+dx)^2}dx}{5c}-\frac{2\sqrt{a+bx}}{5cx^{5/2}(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{168}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{-\frac{\int -\frac{(bc-ad)(2bc-35ad-28bdx)}{2x^{5/2}\sqrt{a+bx}(c+dx)}dx}{c(bc-ad)}-\frac{7d\sqrt{a+bx}}{cx^{3/2}(c+dx)}}{5c}-\frac{2\sqrt{a+bx}}{5cx^{5/2}(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\int \frac{2bc-35ad-28bdx}{x^{5/2}\sqrt{a+bx}(c+dx)}dx}{2c}-\frac{7d\sqrt{a+bx}}{cx^{3/2}(c+dx)}}{5c}-\frac{2\sqrt{a+bx}}{5cx^{5/2}(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{169}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{-\frac{2\int \frac{4b^2c^2+20abdc-105a^2d^2+2bd(2bc-35ad)x}{2x^{3/2}\sqrt{a+bx}(c+dx)}dx}{3ac}-\frac{2\sqrt{a+bx}(2bc-35ad)}{3acx^{3/2}}}{2c}-\frac{7d\sqrt{a+bx}}{cx^{3/2}(c+dx)}}{5c}-\frac{2\sqrt{a+bx}}{5cx^{5/2}(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{-\frac{\int \frac{4b^2c^2+20abdc-105a^2d^2+2bd(2bc-35ad)x}{x^{3/2}\sqrt{a+bx}(c+dx)}dx}{3ac}-\frac{2\sqrt{a+bx}(2bc-35ad)}{3acx^{3/2}}}{2c}-\frac{7d\sqrt{a+bx}}{cx^{3/2}(c+dx)}}{5c}-\frac{2\sqrt{a+bx}}{5cx^{5/2}(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{169}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{-\frac{-\frac{2\int \frac{15a^2d^2(6bc-7ad)}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{ac}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{105ad^2}{c}+20bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(2bc-35ad)}{3acx^{3/2}}}{2c}-\frac{7d\sqrt{a+bx}}{cx^{3/2}(c+dx)}}{5c}-\frac{2\sqrt{a+bx}}{5cx^{5/2}(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$\big\downarrow$ 27

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{-\frac{-\frac{15ad^2(6bc-7ad)\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{c}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{105ad^2}{c}+20bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(2bc-35ad)}{3acx^{3/2}}}{2c}-\frac{7d\sqrt{a+bx}}{cx^{3/2}(c+dx)}}{5c}-\frac{2\sqrt{a+bx}}{5cx^{5/2}(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$\big\downarrow$ 104

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{-\frac{-\frac{30ad^2(6bc-7ad)\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{105ad^2}{c}+20bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(2bc-35ad)}{3acx^{3/2}}}{2c}-\frac{7d\sqrt{a+bx}}{cx^{3/2}(c+dx)}}{5c}-\frac{2\sqrt{a+bx}}{5cx^{5/2}(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$\big\downarrow$ 221

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{-\frac{-\frac{30ad^2(6bc-7ad)\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{105ad^2}{c}+20bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(2bc-35ad)}{3acx^{3/2}}}{2c}-\frac{7d\sqrt{a+bx}}{cx^{3/2}(c+dx)}}{5c}-\frac{2\sqrt{a+bx}}{5cx^{5/2}(c+dx)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

input
```
Int[Sqrt[a*x + b*x^2]/(x^4*(c + d*x)^2),x]
```

output
```
(Sqrt[a*x + b*x^2]*((-2*Sqrt[a + b*x])/(5*c*x^(5/2)*(c + d*x)) + ((-7*d*Sq
rt[a + b*x])/(c*x^(3/2)*(c + d*x)) + ((-2*(2*b*c - 35*a*d)*Sqrt[a + b*x])/
(3*a*c*x^(3/2)) - ((-2*((4*b^2*c)/a + 20*b*d - (105*a*d^2)/c)*Sqrt[a + b*x
])/Sqrt[x] - (30*a*d^2*(6*b*c - 7*a*d)*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(
Sqrt[c]*Sqrt[a + b*x])])/(c^(3/2)*Sqrt[b*c - a*d]))/(3*a*c))/(2*c))/(5*c))
)/(Sqrt[x]*Sqrt[a + b*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$-\frac{2\left(\left(bx+a\right)\left(-\frac{2bx}{3}+a\right)c^{3}-\frac{7\left(\frac{2bx}{7}+a\right)dx\left(bx+a\right)c^{2}}{3}+\frac{35d^{2}x^{2}\left(-\frac{2bx}{7}+a\right)ac}{3}+\frac{35a^{2}d^{3}x^{3}}{2}\right)\sqrt{c\left(ad-bc\right)}\,\sqrt{x\left(bx+a\right)}-35\left(dx+c\right)d^{2}\cdots}{5\sqrt{c\left(ad-bc\right)}\,c^{4}x^{3}\left(dx+c\right)a^{2}}$
risch	$-\frac{2\left(bx+a\right)\left(45a^{2}d^{2}x^{2}-10abcd\,x^{2}-2b^{2}c^{2}x^{2}-10a^{2}cdx+ab\,c^{2}x+3a^{2}c^{2}\right)}{15a^{2}c^{4}\sqrt{x\left(bx+a\right)}\,x^{2}}-\frac{d^{2}\left(-\frac{\left(3ad-2bc\right)\ln\left(\frac{-\frac{2c\left(ad-bc\right)}{d^{2}}+\frac{\left(ad-2bc\right)\left(x+\frac{c}{d}\right)}{d}\cdots}{\cdots}\right)}{\cdots}\cdots\right)}{\cdots}$
default	Expression too large to display

input
```
int((b*x^2+a*x)^(1/2)/x^4/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/5/(c*(a*d-b*c))^(1/2)*(2*((b*x+a)*(-2/3*b*x+a)*c^3-7/3*(2/7*b*x+a)*d*x*
(b*x+a)*c^2+35/3*d^2*x^2*(-2/7*b*x+a)*a*c+35/2*a^2*d^3*x^3)*(c*(a*d-b*c))^
(1/2)*(x*(b*x+a))^(1/2)-35*(d*x+c)*d^2*x^3*(a*d-6/7*b*c)*a^2*arctan((x*(b*
x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2)))/c^4/x^3/(d*x+c)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 642, normalized size of antiderivative = 3.24

$$\int \frac{\sqrt{ax+bx^2}}{x^4(c+dx)^2}\,dx$$

$$= \left[-\frac{15\,((6\,a^2bcd^3-7\,a^3d^4)x^4+(6\,a^2bc^2d^2-7\,a^3cd^3)x^3)\sqrt{bc^2-acd}\log\left(\frac{ac+(2\,bc-ad)x-2\,\sqrt{bc^2-acd}\sqrt{bx^2+ax}}{dx+c}\right)+\cdots}{\cdots}\right.$$

$$-\frac{15\,((6\,a^2bcd^3-7\,a^3d^4)x^4+(6\,a^2bc^2d^2-7\,a^3cd^3)x^3)\sqrt{-bc^2+acd}\arctan\left(\frac{\sqrt{-bc^2+acd}\sqrt{bx^2+ax}}{bcx+ac}\right)+(6\,a^2bc\cdots}{\cdots}$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^4/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[-1/30*(15*((6*a^2*b*c*d^3 - 7*a^3*d^4)*x^4 + (6*a^2*b*c^2*d^2 - 7*a^3*c*d
^3)*x^3)*sqrt(b*c^2 - a*c*d)*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*c^2 - a
*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) + 2*(6*a^2*b*c^5 - 6*a^3*c^4*d - (4*b^
3*c^4*d + 16*a*b^2*c^3*d^2 - 125*a^2*b*c^2*d^3 + 105*a^3*c*d^4)*x^3 - 2*(2
*b^3*c^5 + 7*a*b^2*c^4*d - 44*a^2*b*c^3*d^2 + 35*a^3*c^2*d^3)*x^2 + 2*(a*b
^2*c^5 - 8*a^2*b*c^4*d + 7*a^3*c^3*d^2)*x)*sqrt(b*x^2 + a*x))/((a^2*b*c^6*
d - a^3*c^5*d^2)*x^4 + (a^2*b*c^7 - a^3*c^6*d)*x^3), -1/15*(15*((6*a^2*b*c
*d^3 - 7*a^3*d^4)*x^4 + (6*a^2*b*c^2*d^2 - 7*a^3*c*d^3)*x^3)*sqrt(-b*c^2 +
 a*c*d)*arctan(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) + (6*
a^2*b*c^5 - 6*a^3*c^4*d - (4*b^3*c^4*d + 16*a*b^2*c^3*d^2 - 125*a^2*b*c^2*
d^3 + 105*a^3*c*d^4)*x^3 - 2*(2*b^3*c^5 + 7*a*b^2*c^4*d - 44*a^2*b*c^3*d^2
 + 35*a^3*c^2*d^3)*x^2 + 2*(a*b^2*c^5 - 8*a^2*b*c^4*d + 7*a^3*c^3*d^2)*x)*
sqrt(b*x^2 + a*x))/((a^2*b*c^6*d - a^3*c^5*d^2)*x^4 + (a^2*b*c^7 - a^3*c^6
*d)*x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2}}{x^4(c+dx)^2}\,dx = \int \frac{\sqrt{x\,(a+bx)}}{x^4\,(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a*x)**(1/2)/x**4/(d*x+c)**2,x)
```

output
```
Integral(sqrt(x*(a + b*x))/(x**4*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^2}}{x^4(c+dx)^2}\,dx = \int \frac{\sqrt{bx^2+ax}}{(dx+c)^2x^4}\,dx$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^4/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a*x)/((d*x + c)^2*x^4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax+bx^2}}{x^4(c+dx)^2}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^4/(d*x+c)^2,x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: 2859
40382 icas_eval sage2285.953 NTL factor begin285.953 NTL factor end285.953
 NTL factor begin285.953 NTL factor endPsr 9.14286, Mod 191.109, Heu 11764
.9, Min9.14286GCD d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^2}}{x^4(c+dx)^2}\,dx=\int \frac{\sqrt{b\,x^2+a\,x}}{x^4\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((a*x + b*x^2)^(1/2)/(x^4*(c + d*x)^2),x)
```

output
```
int((a*x + b*x^2)^(1/2)/(x^4*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 1332, normalized size of antiderivative = 6.73

$$\int \frac{\sqrt{ax+bx^2}}{x^4(c+dx)^2}\,dx=\text{Too large to display}$$

input
```
int((b*x^2+a*x)^(1/2)/x^4/(d*x+c)^2,x)
```

output

```
(735*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x)
 - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**4*c*d**4*x**3 + 735*sqrt
(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x
)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**4*d**5*x**4 - 1050*sqrt(c)*sqrt(a
*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*
sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*b*c**2*d**3*x**3 - 1050*sqrt(c)*sqrt(a*d
- b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqr
t(b))/(sqrt(c)*sqrt(b)))*a**3*b*c*d**4*x**4 + 360*sqrt(c)*sqrt(a*d - b*c)*
atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(
sqrt(c)*sqrt(b)))*a**2*b**2*c**3*d**2*x**3 + 360*sqrt(c)*sqrt(a*d - b*c)*a
tan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(s
qrt(c)*sqrt(b)))*a**2*b**2*c**2*d**3*x**4 + 735*sqrt(c)*sqrt(a*d - b*c)*at
an((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sq
rt(c)*sqrt(b)))*a**4*c*d**4*x**3 + 735*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(
a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqr
t(b)))*a**4*d**5*x**4 - 1050*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c)
 + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**
3*b*c**2*d**3*x**3 - 1050*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) +
sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*b
*c*d**4*x**4 + 360*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt...
```

3.45 $\int \frac{x^3\sqrt{ax+bx^2}}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 311

$$\begin{aligned}\int \frac{x^3\sqrt{ax+bx^2}}{(c+dx)^3}\,dx = & -\frac{(24b^2c^2-24abcd+a^2d^2)\sqrt{ax+bx^2}}{4bd^4(bc-ad)} \\ & +\frac{(12bc-11ad)x\sqrt{ax+bx^2}}{4d^3(bc-ad)} \\ & -\frac{x^3\sqrt{ax+bx^2}}{2d(c+dx)^2}-\frac{(8bc-7ad)x^2\sqrt{ax+bx^2}}{4d^2(bc-ad)(c+dx)} \\ & +\frac{(48b^2c^2-12abcd-a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}d^5} \\ & -\frac{c^{3/2}(48b^2c^2-84abcd+35a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{4d^5(bc-ad)^{3/2}}\end{aligned}$$

output

```
-1/4*(a^2*d^2-24*a*b*c*d+24*b^2*c^2)*(b*x^2+a*x)^(1/2)/b/d^4/(-a*d+b*c)+1/
4*(-11*a*d+12*b*c)*x*(b*x^2+a*x)^(1/2)/d^3/(-a*d+b*c)-1/2*x^3*(b*x^2+a*x)^
(1/2)/d/(d*x+c)^2-1/4*(-7*a*d+8*b*c)*x^2*(b*x^2+a*x)^(1/2)/d^2/(-a*d+b*c)/
(d*x+c)+1/4*(-a^2*d^2-12*a*b*c*d+48*b^2*c^2)*arctanh(b^(1/2)*x/(b*x^2+a*x)
^(1/2))/b^(3/2)/d^5-1/4*c^(3/2)*(35*a^2*d^2-84*a*b*c*d+48*b^2*c^2)*arctanh
((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/d^5/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 11.23 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.94

$$\int \frac{x^3\sqrt{ax+bx^2}}{(c+dx)^3}\,dx$$
$$= \frac{\sqrt{x(a+bx)}\left(-\frac{\sqrt{b}d\sqrt{x}(-a^2d^2(c+dx)^2+abd(24c^3+37c^2dx+9cd^2x^2-2d^3x^3)-2b^2c(12c^3+18c^2dx+4cd^2x^2-d^3x^3))}{(-bc+ad)(c+dx)^2}+\frac{(48b^2c^2-12abcd}{}\right.}{4b^{3/2}d^5\sqrt{x}}$$

input
```
Integrate[(x^3*Sqrt[a*x + b*x^2])/(c + d*x)^3,x]
```

output
```
(Sqrt[x*(a + b*x)]*(-((Sqrt[b]*d*Sqrt[x]*(-(a^2*d^2*(c + d*x)^2) + a*b*d*(
24*c^3 + 37*c^2*d*x + 9*c*d^2*x^2 - 2*d^3*x^3) - 2*b^2*c*(12*c^3 + 18*c^2*
d*x + 4*c*d^2*x^2 - d^3*x^3)))/((-(b*c) + a*d)*(c + d*x)^2)) + ((48*b^2*c^
2 - 12*a*b*c*d - a^2*d^2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqr
t[1 + (b*x)/a]) - (b^(3/2)*c^(3/2)*(48*b^2*c^2 - 84*a*b*c*d + 35*a^2*d^2)*
ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/((b*c - a*d)^(
3/2)*Sqrt[a + b*x])))/(4*b^(3/2)*d^5*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.13, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {1261, 108, 27, 166, 27, 171, 27, 171, 25, 175, 65, 104, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{ax+bx^2}}{(c+dx)^3}\,dx$$
$$\Big\downarrow \text{1261}$$
$$\frac{\sqrt{ax+bx^2}\int \frac{x^{7/2}\sqrt{a+bx}}{(c+dx)^3}dx}{\sqrt{x}\sqrt{a+bx}}$$
$$\Big\downarrow \text{108}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\int \frac{x^{5/2}(7a+8bx)}{2\sqrt{a+bx}(c+dx)^2}dx}{2d}-\frac{x^{7/2}\sqrt{a+bx}}{2d(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\int \frac{x^{5/2}(7a+8bx)}{\sqrt{a+bx}(c+dx)^2}dx}{4d}-\frac{x^{7/2}\sqrt{a+bx}}{2d(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 166$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{-\frac{\int -\frac{x^{3/2}(5a(8bc-7ad)+4b(12bc-11ad)x)}{2\sqrt{a+bx}(c+dx)}dx}{d(bc-ad)}-\frac{x^{5/2}\sqrt{a+bx}(8bc-7ad)}{d(c+dx)(bc-ad)}}{4d}-\frac{x^{7/2}\sqrt{a+bx}}{2d(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\int \frac{x^{3/2}(5a(8bc-7ad)+4b(12bc-11ad)x)}{\sqrt{a+bx}(c+dx)}dx}{2d(bc-ad)}-\frac{x^{5/2}\sqrt{a+bx}(8bc-7ad)}{d(c+dx)(bc-ad)}}{4d}-\frac{x^{7/2}\sqrt{a+bx}}{2d(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 171$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\frac{\int -\frac{2b\sqrt{x}\left(3ac(12bc-11ad)+2\left(24b^2c^2-24abdc+a^2d^2\right)x\right)}{\sqrt{a+bx}(c+dx)}dx}{2bd}+\frac{2x^{3/2}\sqrt{a+bx}(12bc-11ad)}{d}}{2d(bc-ad)}-\frac{x^{5/2}\sqrt{a+bx}(8bc-7ad)}{d(c+dx)(bc-ad)}}{4d}-\frac{x^{7/2}\sqrt{a+bx}}{2d(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\frac{2x^{3/2}\sqrt{a+bx}(12bc-11ad)}{d}-\frac{\int \frac{\sqrt{x}\left(3ac(12bc-11ad)+2\left(24b^2c^2-24abdc+a^2d^2\right)x\right)}{\sqrt{a+bx}(c+dx)}dx}{d}}{2d(bc-ad)}-\frac{x^{5/2}\sqrt{a+bx}(8bc-7ad)}{d(c+dx)(bc-ad)}}{4d}-\frac{x^{7/2}\sqrt{a+bx}}{2d(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 171$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\frac{2x^{3/2}\sqrt{a+bx}(12bc-11ad)}{d}-\frac{\frac{\int -\frac{ac\left(24b^2c^2-24abdc+a^2d^2\right)+(bc-ad)\left(48b^2c^2-12abdc-a^2d^2\right)x}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{bd}-2\sqrt{x}\sqrt{a+bx}\left(-\frac{a^2d}{b}+24ac-\frac{24bc^2}{d}\right)}{d}}{2d(bc-ad)}-\frac{x^5\cdots}{\cdots}}{4d}\cdots\right)}{\sqrt{x}\sqrt{a+bx}}$$

$\downarrow$ 25

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\frac{2x^{3/2}\sqrt{a+bx}(12bc-11ad)}{d}-\frac{-\frac{\int \frac{ac\left(24b^2c^2-24abdc+a^2d^2\right)+(bc-ad)\left(48b^2c^2-12abdc-a^2d^2\right)x}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{bd}-2\sqrt{x}\sqrt{a+bx}\left(-\frac{a^2d}{b}+24ac-\frac{24bc^2}{d}\right)}{d}}{2d(bc-ad)}-\frac{x^5\cdots}{\cdots}}{4d}\cdots\right)}{\sqrt{x}\sqrt{a+bx}}$$

$\downarrow$ 175

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\frac{2x^{3/2}\sqrt{a+bx}(12bc-11ad)}{d}-\frac{-\frac{\frac{(bc-ad)\left(-a^2d^2-12abcd+48b^2c^2\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{d}-\frac{bc^2\left(35a^2d^2-84abcd+48b^2c^2\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}}{bd}-2\cdots}{d}}{2d(bc-ad)}\cdots}{4d}\cdots\right)}{\sqrt{x}\sqrt{a+bx}}$$

$\downarrow$ 65

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\frac{2x^{3/2}\sqrt{a+bx}(12bc-11ad)}{d}-\frac{-\frac{\frac{2(bc-ad)\left(-a^2d^2-12abcd+48b^2c^2\right)\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}-\frac{bc^2\left(35a^2d^2-84abcd+48b^2c^2\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}\cdots}{d}}{bd}\cdots}{d}}{2d(bc-ad)}\cdots}{4d}\cdots\right)}{\sqrt{x}\sqrt{a+bx}}$$

$\downarrow$ 104

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{2x^{3/2}\sqrt{a+bx}(12bc-11ad)}{d}-\frac{-\frac{\frac{2(bc-ad)\left(-a^2d^2-12abcd+48b^2c^2\right)\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}-\frac{2bc^2\left(35a^2d^2-84abcd+48b^2c^2\right)\int\frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{\cdots}}{}}{d}}{bd}}{d}}{2d(bc-ad)}}{4d}\right.}{\sqrt{x}\sqrt{a+bx}}$$

$\downarrow$ 219

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{2x^{3/2}\sqrt{a+bx}(12bc-11ad)}{d}-\frac{-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)(bc-ad)\left(-a^2d^2-12abcd+48b^2c^2\right)}{\sqrt{b}d}-\frac{2bc^2\left(35a^2d^2-84abcd+48b^2c^2\right)\int\frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{}{\sqrt{\cdots}}}{d}}{bd}}{d}}{2d(bc-ad)}}{4d}\right.}{\sqrt{x}\sqrt{a+bx}}$$

$\downarrow$ 221

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{2x^{3/2}\sqrt{a+bx}(12bc-11ad)}{d}-\frac{-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)(bc-ad)\left(-a^2d^2-12abcd+48b^2c^2\right)}{\sqrt{b}d}-\frac{2bc^{3/2}\left(35a^2d^2-84abcd+48b^2c^2\right)\text{arctanh}\left(\frac{\sqrt{\cdots}}{\sqrt{\cdots}}\right)}{d\sqrt{bc-ad}}}{bd}}{d}}{2d(bc-ad)}}{4d}\right.}{\sqrt{x}\sqrt{a+bx}}$$

input
```
Int[(x^3*Sqrt[a*x + b*x^2])/(c + d*x)^3,x]
```

output
```
(Sqrt[a*x + b*x^2]*(-1/2*(x^(7/2)*Sqrt[a + b*x])/(d*(c + d*x)^2) + (-(((8*
b*c - 7*a*d)*x^(5/2)*Sqrt[a + b*x])/(d*(b*c - a*d)*(c + d*x))) + ((2*(12*b
*c - 11*a*d)*x^(3/2)*Sqrt[a + b*x])/d - (-2*(24*a*c - (24*b*c^2)/d - (a^2*
d)/b)*Sqrt[x]*Sqrt[a + b*x] - ((2*(b*c - a*d)*(48*b^2*c^2 - 12*a*b*c*d - a
^2*d^2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(Sqrt[b]*d) - (2*b*c^(3/
2)*(48*b^2*c^2 - 84*a*b*c*d + 35*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x]
)/(Sqrt[c]*Sqrt[a + b*x])])/(d*Sqrt[b*c - a*d]))/(b*d))/d)/(2*d*(b*c - a*d
)))/(4*d)))/(Sqrt[x]*Sqrt[a + b*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$-\frac{3\left(4(dx+c)^2 b^{\frac{5}{2}} x\left(b^2c^2-\frac{7}{4}abcd+\frac{35}{48}a^2d^2\right)a\,c^2\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)+\left(\frac{abx(dx+c)^2(ad-bc)\left(a^2d^2+12abcd-48b^2c^2\right)\operatorname{arctanh}}{12}\right.\right.}{}$
risch	$\frac{(2bdx+ad-12bc)x(bx+a)}{4b\,d^4\sqrt{x(bx+a)}}-\frac{\frac{\left(a^2d^2+12abcd-48b^2c^2\right)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{d\sqrt{b}}+\frac{16b\,c^2(3ad-5bc)\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\right.}{d}}{}\right.}{}}{}$
default	Expression too large to display

input
```
int(x^3*(b*x^2+a*x)^(1/2)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-3*(4*(d*x+c)^2*b^(5/2)*x*(b^2*c^2-7/4*a*b*c*d+35/48*a^2*d^2)*a*c^2*arctan
((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))+(1/12*a*b*x*(d*x+c)^2*(a*d-b*c
)*(a^2*d^2+12*a*b*c*d-48*b^2*c^2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))+b^(
3/2)*d*((-b^2*c^4+11/12*a*b*c^3*d)*(x*(b*x+a))^(3/2)-1/12*(x*(b*x+a))^(1/2
)*x*(a*d-b*c)*(-12*(-b*x+a)*b*c^3+d*a*(-35*b*x+a)*c^2+2*a*d^2*x*(-4*b*x+a)
*c+a*d^3*x^2*(2*b*x+a))))*(c*(a*d-b*c))^(1/2))/(c*(a*d-b*c))^(1/2)/a/b^(5/
2)/d^5/x/(d*x+c)^2/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 2216, normalized size of antiderivative = 7.13

$$\int \frac{x^3\sqrt{ax+bx^2}}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(b*x^2+a*x)^(1/2)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
[-1/8*((48*b^3*c^5 - 60*a*b^2*c^4*d + 11*a^2*b*c^3*d^2 + a^3*c^2*d^3 + (48
*b^3*c^3*d^2 - 60*a*b^2*c^2*d^3 + 11*a^2*b*c*d^4 + a^3*d^5)*x^2 + 2*(48*b^
3*c^4*d - 60*a*b^2*c^3*d^2 + 11*a^2*b*c^2*d^3 + a^3*c*d^4)*x)*sqrt(b)*log(
2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) + (48*b^4*c^5 - 84*a*b^3*c^4*d +
35*a^2*b^2*c^3*d^2 + (48*b^4*c^3*d^2 - 84*a*b^3*c^2*d^3 + 35*a^2*b^2*c*d^4
)*x^2 + 2*(48*b^4*c^4*d - 84*a*b^3*c^3*d^2 + 35*a^2*b^2*c^2*d^3)*x)*sqrt(c
/(b*c - a*d))*log((a*c + (2*b*c - a*d)*x + 2*sqrt(b*x^2 + a*x)*(b*c - a*d)
*sqrt(c/(b*c - a*d)))/(d*x + c)) + 2*(24*b^3*c^4*d - 24*a*b^2*c^3*d^2 + a^
2*b*c^2*d^3 - 2*(b^3*c*d^4 - a*b^2*d^5)*x^3 + (8*b^3*c^2*d^3 - 9*a*b^2*c*d
^4 + a^2*b*d^5)*x^2 + (36*b^3*c^3*d^2 - 37*a*b^2*c^2*d^3 + 2*a^2*b*c*d^4)*
x)*sqrt(b*x^2 + a*x))/(b^3*c^3*d^5 - a*b^2*c^2*d^6 + (b^3*c*d^7 - a*b^2*d^
8)*x^2 + 2*(b^3*c^2*d^6 - a*b^2*c*d^7)*x), -1/8*(2*(48*b^4*c^5 - 84*a*b^3*
c^4*d + 35*a^2*b^2*c^3*d^2 + (48*b^4*c^3*d^2 - 84*a*b^3*c^2*d^3 + 35*a^2*b
^2*c*d^4)*x^2 + 2*(48*b^4*c^4*d - 84*a*b^3*c^3*d^2 + 35*a^2*b^2*c^2*d^3)*x
)*sqrt(-c/(b*c - a*d))*arctan(-sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(-c/(b*c
- a*d))/(b*c*x + a*c)) + (48*b^3*c^5 - 60*a*b^2*c^4*d + 11*a^2*b*c^3*d^2 +
 a^3*c^2*d^3 + (48*b^3*c^3*d^2 - 60*a*b^2*c^2*d^3 + 11*a^2*b*c*d^4 + a^3*d
^5)*x^2 + 2*(48*b^3*c^4*d - 60*a*b^2*c^3*d^2 + 11*a^2*b*c^2*d^3 + a^3*c*d^
4)*x)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(24*b^3*c^4
*d - 24*a*b^2*c^3*d^2 + a^2*b*c^2*d^3 - 2*(b^3*c*d^4 - a*b^2*d^5)*x^3 +...
```

Sympy [F]

$$\int \frac{x^3\sqrt{ax+bx^2}}{(c+dx)^3}\,dx = \int \frac{x^3\sqrt{x\,(a+bx)}}{(c+dx)^3}\,dx$$

input
```
integrate(x**3*(b*x**2+a*x)**(1/2)/(d*x+c)**3,x)
```

output
```
Integral(x**3*sqrt(x*(a + b*x))/(c + d*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{ax+bx^2}}{(c+dx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(b*x^2+a*x)^(1/2)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-2*b*c>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(275) = 550$.

Time = 0.16 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.84

$$\int \frac{x^3\sqrt{ax+bx^2}}{(c+dx)^3}\,dx = \frac{1}{4}\sqrt{bx^2+ax}\left(\frac{2\,x}{d^3} - \frac{12\,bcd^8 - ad^9}{bd^{12}}\right)$$
$$- \frac{(48\,b^2c^4 - 84\,abc^3d + 35\,a^2c^2d^2)\arctan\left(-\frac{\left(\sqrt{b}x - \sqrt{bx^2+ax}\right)d + \sqrt{b}c}{\sqrt{-bc^2+acd}}\right)}{4\,(bcd^5 - ad^6)\sqrt{-bc^2+acd}}$$
$$- \frac{32\left(\sqrt{b}x - \sqrt{bx^2+ax}\right)^3 b^2c^4d - 44\left(\sqrt{b}x - \sqrt{bx^2+ax}\right)^3 abc^3d^2 + 13\left(\sqrt{b}x - \sqrt{bx^2+ax}\right)^3 a^2c^2d^3 + 5}{}$$
$$- \frac{(48\,b^2c^2 - 12\,abcd - a^2d^2)\log\left(\left|2\left(\sqrt{b}x - \sqrt{bx^2+ax}\right)\sqrt{b} + a\right|\right)}{8\,b^{\frac{3}{2}}d^5}$$

input
```
integrate(x^3*(b*x^2+a*x)^(1/2)/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/4*sqrt(b*x^2 + a*x)*(2*x/d^3 - (12*b*c*d^8 - a*d^9)/(b*d^12)) - 1/4*(48*
b^2*c^4 - 84*a*b*c^3*d + 35*a^2*c^2*d^2)*arctan(-((sqrt(b)*x - sqrt(b*x^2
+ a*x))*d + sqrt(b)*c)/sqrt(-b*c^2 + a*c*d))/((b*c*d^5 - a*d^6)*sqrt(-b*c^
2 + a*c*d)) - 1/4*(32*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*b^2*c^4*d - 44*(sq
rt(b)*x - sqrt(b*x^2 + a*x))^3*a*b*c^3*d^2 + 13*(sqrt(b)*x - sqrt(b*x^2 +
a*x))^3*a^2*c^2*d^3 + 56*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*b^(5/2)*c^5 - 6
0*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a*b^(3/2)*c^4*d + 7*(sqrt(b)*x - sqrt(
b*x^2 + a*x))^2*a^2*sqrt(b)*c^3*d^2 + 56*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a
*b^2*c^5 - 64*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^2*b*c^4*d + 11*(sqrt(b)*x
- sqrt(b*x^2 + a*x))*a^3*c^3*d^2 + 14*a^2*b^(3/2)*c^5 - 13*a^3*sqrt(b)*c^4
*d)/((b*c*d^5 - a*d^6)*((sqrt(b)*x - sqrt(b*x^2 + a*x))^2*d + 2*(sqrt(b)*x
 - sqrt(b*x^2 + a*x))*sqrt(b)*c + a*c)^2) - 1/8*(48*b^2*c^2 - 12*a*b*c*d -
 a^2*d^2)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/(b^(3/2)
*d^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{ax+bx^2}}{(c+dx)^3}\,dx = \int \frac{x^3\,\sqrt{b\,x^2+a\,x}}{{\left(c+d\,x\right)}^3}\,dx$$

input
```
int((x^3*(a*x + b*x^2)^(1/2))/(c + d*x)^3,x)
```

output
```
int((x^3*(a*x + b*x^2)^(1/2))/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{x^3\sqrt{ax+bx^2}}{(c+dx)^3}\,dx = \int \frac{x^3\sqrt{b\,x^2+ax}}{(dx+c)^3}dx$$

input
```
int(x^3*(b*x^2+a*x)^(1/2)/(d*x+c)^3,x)
```

output
```
int(x^3*(b*x^2+a*x)^(1/2)/(d*x+c)^3,x)
```

3.46 $\int \frac{x^2\sqrt{ax+bx^2}}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 237

$$\begin{aligned}\int \frac{x^2\sqrt{ax+bx^2}}{(c+dx)^3}\,dx &= \frac{(12bc-11ad)\sqrt{ax+bx^2}}{4d^3(bc-ad)} - \frac{x^2\sqrt{ax+bx^2}}{2d(c+dx)^2} \\ &\quad - \frac{(6bc-5ad)x\sqrt{ax+bx^2}}{4d^2(bc-ad)(c+dx)} - \frac{(6bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}d^4} \\ &\quad + \frac{\sqrt{c}(24b^2c^2-40abcd+15a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{4d^4(bc-ad)^{3/2}}\end{aligned}$$

output

```
1/4*(-11*a*d+12*b*c)*(b*x^2+a*x)^(1/2)/d^3/(-a*d+b*c)-1/2*x^2*(b*x^2+a*x)^
(1/2)/d/(d*x+c)^2-1/4*(-5*a*d+6*b*c)*x*(b*x^2+a*x)^(1/2)/d^2/(-a*d+b*c)/(d
*x+c)-(-a*d+6*b*c)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(1/2)/d^4+1/4*c^
(1/2)*(15*a^2*d^2-40*a*b*c*d+24*b^2*c^2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2
)/(b*x^2+a*x)^(1/2))/d^4/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 10.92 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.95

$$\int \frac{x^2\sqrt{ax+bx^2}}{(c+dx)^3}\,dx$$
$$= \frac{\sqrt{x(a+bx)}\left(\frac{d\sqrt{x}(-2bc(6c^2+9cdx+2d^2x^2)+ad(11c^2+17cdx+4d^2x^2))}{(-bc+ad)(c+dx)^2}+\frac{4(-6bc+ad)\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{1+\frac{bx}{a}}}+\frac{\sqrt{c}(24b^2c^2-40abcd+15a}{(bc-aa}\right.}{4d^4\sqrt{x}}$$

input
```
Integrate[(x^2*Sqrt[a*x + b*x^2])/(c + d*x)^3,x]
```

output
```
(Sqrt[x*(a + b*x)]*((d*Sqrt[x]*(-2*b*c*(6*c^2 + 9*c*d*x + 2*d^2*x^2) + a*d
*(11*c^2 + 17*c*d*x + 4*d^2*x^2)))/((-(b*c) + a*d)*(c + d*x)^2) + (4*(-6*b
*c + a*d)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[1 + (b
*x)/a]) + (Sqrt[c]*(24*b^2*c^2 - 40*a*b*c*d + 15*a^2*d^2)*ArcTanh[(Sqrt[b*
c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/((b*c - a*d)^(3/2)*Sqrt[a + b*
x])))/(4*d^4*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.22, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {1261, 108, 27, 166, 27, 171, 25, 27, 175, 65, 104, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{ax+bx^2}}{(c+dx)^3}\,dx$$
$$\Big\downarrow \text{ 1261}$$
$$\frac{\sqrt{ax+bx^2}\int \frac{x^{5/2}\sqrt{a+bx}}{(c+dx)^3}dx}{\sqrt{x}\sqrt{a+bx}}$$
$$\Big\downarrow \text{ 108}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\int \frac{x^{3/2}(5a+6bx)}{2\sqrt{a+bx}(c+dx)^2}dx}{2d}-\frac{x^{5/2}\sqrt{a+bx}}{2d(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\int \frac{x^{3/2}(5a+6bx)}{\sqrt{a+bx}(c+dx)^2}dx}{4d}-\frac{x^{5/2}\sqrt{a+bx}}{2d(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 166}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{-\frac{\int -\frac{\sqrt{x}(3a(6bc-5ad)+2b(12bc-11ad)x)}{2\sqrt{a+bx}(c+dx)}dx}{d(bc-ad)}-\frac{x^{3/2}\sqrt{a+bx}(6bc-5ad)}{d(c+dx)(bc-ad)}}{4d}-\frac{x^{5/2}\sqrt{a+bx}}{2d(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\int \frac{\sqrt{x}(3a(6bc-5ad)+2b(12bc-11ad)x)}{\sqrt{a+bx}(c+dx)}dx}{2d(bc-ad)}-\frac{x^{3/2}\sqrt{a+bx}(6bc-5ad)}{d(c+dx)(bc-ad)}}{4d}-\frac{x^{5/2}\sqrt{a+bx}}{2d(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 171}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\frac{\int -\frac{b(ac(12bc-11ad)+4(bc-ad)(6bc-ad)x)}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{bd}+\frac{2\sqrt{x}\sqrt{a+bx}(12bc-11ad)}{d}}{2d(bc-ad)}-\frac{x^{3/2}\sqrt{a+bx}(6bc-5ad)}{d(c+dx)(bc-ad)}}{4d}-\frac{x^{5/2}\sqrt{a+bx}}{2d(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\frac{2\sqrt{x}\sqrt{a+bx}(12bc-11ad)}{d}-\frac{\int \frac{b(ac(12bc-11ad)+4(bc-ad)(6bc-ad)x)}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{bd}}{2d(bc-ad)}-\frac{x^{3/2}\sqrt{a+bx}(6bc-5ad)}{d(c+dx)(bc-ad)}}{4d}-\frac{x^{5/2}\sqrt{a+bx}}{2d(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\frac{2\sqrt{x}\sqrt{a+bx}(12bc-11ad)}{d}-\frac{\int \frac{ac(12bc-11ad)+4(bc-ad)(6bc-ad)x}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}}{2d(bc-ad)}-\frac{x^{3/2}\sqrt{a+bx}(6bc-5ad)}{d(c+dx)(bc-ad)}}{4d}-\frac{x^{5/2}\sqrt{a+bx}}{2d(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{175}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\frac{2\sqrt{x}\sqrt{a+bx}(12bc-11ad)}{d}-\frac{\frac{4(bc-ad)(6bc-ad)\int\frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{d}-\frac{c\left(15a^2d^2-40abcd+24b^2c^2\right)\int\frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}}{d}}{2d(bc-ad)}-\frac{x^{3/2}\sqrt{a+bx}(6bc-5ad)}{d(c+dx)(bc-ad)}}{4d}\cdots\right.}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{65}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\frac{2\sqrt{x}\sqrt{a+bx}(12bc-11ad)}{d}-\frac{\frac{8(bc-ad)(6bc-ad)\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}-\frac{c\left(15a^2d^2-40abcd+24b^2c^2\right)\int\frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}}{d}}{2d(bc-ad)}-\frac{x^{3/2}\sqrt{a+bx}(6bc-\cdots}{d(c+dx)(bc-ad\cdots}}{4d}\cdots\right.}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{104}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\frac{2\sqrt{x}\sqrt{a+bx}(12bc-11ad)}{d}-\frac{\frac{8(bc-ad)(6bc-ad)\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}-\frac{2c\left(15a^2d^2-40abcd+24b^2c^2\right)\int\frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}}{d}}{2d(bc-ad)}-\frac{x^{3/2}\sqrt{a+bx}(6\cdots}{d(c+dx)(bc\cdots}}{4d}\cdots\right.}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\frac{2\sqrt{x}\sqrt{a+bx}(12bc-11ad)}{d}-\frac{\frac{8\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)(bc-ad)(6bc-ad)}{\sqrt{b}d}-\frac{2c\left(15a^2d^2-40abcd+24b^2c^2\right)\int\frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}}{d}}{2d(bc-ad)}-\frac{x^{3/2}\sqrt{a+bx}(\cdots}{d(c+dx)(b\cdots}}{4d}\cdots\right.}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\frac{2\sqrt{x}\sqrt{a+bx}(12bc-11ad)}{d}-\frac{\frac{8\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)(bc-ad)(6bc-ad)}{\sqrt{b}d}-\frac{2\sqrt{c}\left(15a^2d^2-40abcd+24b^2c^2\right)\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{d\sqrt{bc-ad}}}{d}}{2d(bc-ad)}-\frac{x^{3/2}\sqrt{a+\cdots}}{d(c+d\cdots}}{4d}\right.}{\sqrt{x}\sqrt{a+bx}}$$

input
```
Int[(x^2*Sqrt[a*x + b*x^2])/(c + d*x)^3,x]
```

output
```
(Sqrt[a*x + b*x^2]*(-1/2*(x^(5/2)*Sqrt[a + b*x])/(d*(c + d*x)^2) + (-(((6*
b*c - 5*a*d)*x^(3/2)*Sqrt[a + b*x])/(d*(b*c - a*d)*(c + d*x))) + ((2*(12*b
*c - 11*a*d)*Sqrt[x]*Sqrt[a + b*x])/d - ((8*(b*c - a*d)*(6*b*c - a*d)*ArcT
anh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(Sqrt[b]*d) - (2*Sqrt[c]*(24*b^2*c^2
 - 40*a*b*c*d + 15*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqr
t[a + b*x])])/(d*Sqrt[b*c - a*d]))/d)/(2*d*(b*c - a*d)))/(4*d)))/(Sqrt[x]*
Sqrt[a + b*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{6(dx+c)^2x\sqrt{b}\left(b^2c^2-\frac{5}{3}abcd+\frac{5}{8}a^2d^2\right)ac\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)+2\left(\frac{ax(dx+c)^2(ad-bc)(ad-6bc)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{2}+d\sqrt{b}\left(\cdots\right.\right.}{a\sqrt{b}\,d^4x(dx+c)^2(ad-bc)\sqrt{c(ad\cdots}}$
risch	$\frac{x(bx+a)}{d^3\sqrt{x(bx+a)}}+\frac{\frac{(ad-6bc)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{d\sqrt{b}}+\frac{6c(ad-2bc)\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2+\cdots}}{x+\frac{c}{d}}\right.}{d^2\sqrt{-\frac{c(ad-bc)}{d^2}}}\cdots}{\cdots}$
default	Expression too large to display

input
```
int(x^2*(b*x^2+a*x)^(1/2)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
2/b^(1/2)*(3*(d*x+c)^2*x*b^(1/2)*(b^2*c^2-5/3*a*b*c*d+5/8*a^2*d^2)*a*c*arc
tan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))+(1/2*a*x*(d*x+c)^2*(a*d-b*c
)*(a*d-6*b*c)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))+d*b^(1/2)*((-b*c^3+7/8*
a*c^2*d)*(x*(b*x+a))^(3/2)+1/2*x*(a*d-b*c)*(x*(b*x+a))^(1/2)*((-2*b*x+a)*c
^2+17/4*a*d*x*c+a*d^2*x^2)))*(c*(a*d-b*c))^(1/2))/(c*(a*d-b*c))^(1/2)/a/d^
4/x/(d*x+c)^2/(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(207) = 414$.

Time = 0.20 (sec) , antiderivative size = 1730, normalized size of antiderivative = 7.30

$$\int \frac{x^2\sqrt{ax+bx^2}}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x^2+a*x)^(1/2)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
[-1/8*(4*(6*b^2*c^4 - 7*a*b*c^3*d + a^2*c^2*d^2 + (6*b^2*c^2*d^2 - 7*a*b*c
*d^3 + a^2*d^4)*x^2 + 2*(6*b^2*c^3*d - 7*a*b*c^2*d^2 + a^2*c*d^3)*x)*sqrt(
b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + (24*b^3*c^4 - 40*a*b^2*c
^3*d + 15*a^2*b*c^2*d^2 + (24*b^3*c^2*d^2 - 40*a*b^2*c*d^3 + 15*a^2*b*d^4)
*x^2 + 2*(24*b^3*c^3*d - 40*a*b^2*c^2*d^2 + 15*a^2*b*c*d^3)*x)*sqrt(c/(b*c
 - a*d))*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt
(c/(b*c - a*d)))/(d*x + c)) - 2*(12*b^2*c^3*d - 11*a*b*c^2*d^2 + 4*(b^2*c*
d^3 - a*b*d^4)*x^2 + (18*b^2*c^2*d^2 - 17*a*b*c*d^3)*x)*sqrt(b*x^2 + a*x))
/(b^2*c^3*d^4 - a*b*c^2*d^5 + (b^2*c*d^6 - a*b*d^7)*x^2 + 2*(b^2*c^2*d^5 -
 a*b*c*d^6)*x), 1/4*((24*b^3*c^4 - 40*a*b^2*c^3*d + 15*a^2*b*c^2*d^2 + (24
*b^3*c^2*d^2 - 40*a*b^2*c*d^3 + 15*a^2*b*d^4)*x^2 + 2*(24*b^3*c^3*d - 40*a
*b^2*c^2*d^2 + 15*a^2*b*c*d^3)*x)*sqrt(-c/(b*c - a*d))*arctan(-sqrt(b*x^2
+ a*x)*(b*c - a*d)*sqrt(-c/(b*c - a*d))/(b*c*x + a*c)) - 2*(6*b^2*c^4 - 7*
a*b*c^3*d + a^2*c^2*d^2 + (6*b^2*c^2*d^2 - 7*a*b*c*d^3 + a^2*d^4)*x^2 + 2*
(6*b^2*c^3*d - 7*a*b*c^2*d^2 + a^2*c*d^3)*x)*sqrt(b)*log(2*b*x + a + 2*sqr
t(b*x^2 + a*x)*sqrt(b)) + (12*b^2*c^3*d - 11*a*b*c^2*d^2 + 4*(b^2*c*d^3 -
a*b*d^4)*x^2 + (18*b^2*c^2*d^2 - 17*a*b*c*d^3)*x)*sqrt(b*x^2 + a*x))/(b^2*
c^3*d^4 - a*b*c^2*d^5 + (b^2*c*d^6 - a*b*d^7)*x^2 + 2*(b^2*c^2*d^5 - a*b*c
*d^6)*x), 1/8*(8*(6*b^2*c^4 - 7*a*b*c^3*d + a^2*c^2*d^2 + (6*b^2*c^2*d^2 -
 7*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(6*b^2*c^3*d - 7*a*b*c^2*d^2 + a^2*c*d^...
```

Sympy [F]

$$\int \frac{x^2\sqrt{ax+bx^2}}{(c+dx)^3}\,dx = \int \frac{x^2\sqrt{x\,(a+bx)}}{(c+dx)^3}\,dx$$

input
```
integrate(x**2*(b*x**2+a*x)**(1/2)/(d*x+c)**3,x)
```

output
```
Integral(x**2*sqrt(x*(a + b*x))/(c + d*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{ax+bx^2}}{(c+dx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(b*x^2+a*x)^(1/2)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(207) = 414$.

Time = 0.30 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.22

$$\int \frac{x^2\sqrt{ax+bx^2}}{(c+dx)^3}\,dx = -\frac{(24\,b^2c^3-40\,abc^2d+15\,a^2cd^2)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)d+\sqrt{b}c}{\sqrt{-bc^2+acd}}\right)}{4\,(bcd^4-ad^5)\sqrt{-bc^2+acd}}$$
$$+\frac{24\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3b^2c^3d-32\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3abc^2d^2+9\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3a^2cd^3+40}{}$$
$$+\frac{\sqrt{bx^2+ax}}{d^3}+\frac{(6\,bc-ad)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{2\,\sqrt{b}d^4}$$

input
```
integrate(x^2*(b*x^2+a*x)^(1/2)/(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/4*(24*b^2*c^3 - 40*a*b*c^2*d + 15*a^2*c*d^2)*arctan(((sqrt(b)*x - sqrt(
b*x^2 + a*x))*d + sqrt(b)*c)/sqrt(-b*c^2 + a*c*d))/((b*c*d^4 - a*d^5)*sqrt
(-b*c^2 + a*c*d)) + 1/4*(24*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*b^2*c^3*d -
32*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a*b*c^2*d^2 + 9*(sqrt(b)*x - sqrt(b*x
^2 + a*x))^3*a^2*c*d^3 + 40*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*b^(5/2)*c^4
- 40*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a*b^(3/2)*c^3*d + 3*(sqrt(b)*x - sq
rt(b*x^2 + a*x))^2*a^2*sqrt(b)*c^2*d^2 + 40*(sqrt(b)*x - sqrt(b*x^2 + a*x)
)*a*b^2*c^4 - 44*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^2*b*c^3*d + 7*(sqrt(b)*
x - sqrt(b*x^2 + a*x))*a^3*c^2*d^2 + 10*a^2*b^(3/2)*c^4 - 9*a^3*sqrt(b)*c^
3*d)/((b*c*d^4 - a*d^5)*((sqrt(b)*x - sqrt(b*x^2 + a*x))^2*d + 2*(sqrt(b)*
x - sqrt(b*x^2 + a*x))*sqrt(b)*c + a*c)^2) + sqrt(b*x^2 + a*x)/d^3 + 1/2*(
6*b*c - a*d)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/(sqrt
(b)*d^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{ax+bx^2}}{(c+dx)^3}\,dx = \int \frac{x^2\,\sqrt{b\,x^2+a\,x}}{{\left(c+d\,x\right)}^3}\,dx$$

input
```
int((x^2*(a*x + b*x^2)^(1/2))/(c + d*x)^3,x)
```

output
```
int((x^2*(a*x + b*x^2)^(1/2))/(c + d*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 2552, normalized size of antiderivative = 10.77

$$\int \frac{x^2\sqrt{ax+bx^2}}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int(x^2*(b*x^2+a*x)^(1/2)/(d*x+c)^3,x)
```

output
```
(60*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x)
- sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*b*c**2*d**3 + 120*sqrt(
c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)
*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*b*c*d**4*x + 60*sqrt(c)*sqrt(a*d
 - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sq
rt(b))/(sqrt(c)*sqrt(b)))*a**3*b*d**5*x**2 - 280*sqrt(c)*sqrt(a*d - b*c)*a
tan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(s
qrt(c)*sqrt(b)))*a**2*b**2*c**3*d**2 - 560*sqrt(c)*sqrt(a*d - b*c)*atan((s
qrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)
*sqrt(b)))*a**2*b**2*c**2*d**3*x - 280*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(
a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqr
t(b)))*a**2*b**2*c*d**4*x**2 + 416*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d
- b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)
))*a*b**3*c**4*d + 832*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqr
t(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**3*c*
*3*d**2*x + 416*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sq
rt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**3*c**2*d**3
*x**2 - 192*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a
 + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**4*c**5 - 384*sqrt
(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqr...
```

3.47 $\int \frac{x\sqrt{ax+bx^2}}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 186

$$\int \frac{x\sqrt{ax+bx^2}}{(c+dx)^3}\,dx = -\frac{x\sqrt{ax+bx^2}}{2d(c+dx)^2} - \frac{(4bc-3ad)\sqrt{ax+bx^2}}{4d^2(bc-ad)(c+dx)} + \frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{d^3} - \frac{(8b^2c^2-12abcd+3a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{4\sqrt{c}d^3(bc-ad)^{3/2}}$$

output
```
-1/2*x*(b*x^2+a*x)^(1/2)/d/(d*x+c)^2-1/4*(-3*a*d+4*b*c)*(b*x^2+a*x)^(1/2)/
d^2/(-a*d+b*c)/(d*x+c)+2*b^(1/2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/d^3-
1/4*(3*a^2*d^2-12*a*b*c*d+8*b^2*c^2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b
*x^2+a*x)^(1/2))/c^(1/2)/d^3/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 10.77 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.06

$$\int \frac{x\sqrt{ax+bx^2}}{(c+dx)^3}\,dx = \frac{\sqrt{x(a+bx)}\left(-\frac{d\sqrt{x}(-2bc(2c+3dx)+ad(3c+5dx))}{(-bc+ad)(c+dx)^2} + \frac{8\sqrt{b}\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1+\frac{bx}{a}}} - \frac{(8b^2c^2-12abcd+3a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\sqrt{x}}{\sqrt{c}\sqrt{a+bx}}\right)}{\sqrt{c}(bc-ad)^{3/2}\sqrt{a+bx}}\right)}{4d^3\sqrt{x}}$$

input
```
Integrate[(x*Sqrt[a*x + b*x^2])/(c + d*x)^3,x]
```

output
```
(Sqrt[x*(a + b*x)]*(-((d*Sqrt[x]*(-2*b*c*(2*c + 3*d*x) + a*d*(3*c + 5*d*x)
))/((-(b*c) + a*d)*(c + d*x)^2)) + (8*Sqrt[b]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sq
rt[a]])/(Sqrt[a]*Sqrt[1 + (b*x)/a]) - ((8*b^2*c^2 - 12*a*b*c*d + 3*a^2*d^2
)*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(Sqrt[c]*(b*
c - a*d)^(3/2)*Sqrt[a + b*x])))/(4*d^3*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1229, 27, 1269, 1091, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{ax+bx^2}}{(c+dx)^3}dx$$

$$\Big\downarrow \text{ 1229}$$

$$-\frac{\int -\frac{c(a(4bc-3ad)+8b(bc-ad)x)}{2(c+dx)\sqrt{bx^2+ax}}dx}{4cd^2(bc-ad)}-\frac{\sqrt{ax+bx^2}(dx(6bc-5ad)+c(4bc-3ad))}{4d^2(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{a(4bc-3ad)+8b(bc-ad)x}{(c+dx)\sqrt{bx^2+ax}}dx}{8d^2(bc-ad)}-\frac{\sqrt{ax+bx^2}(dx(6bc-5ad)+c(4bc-3ad))}{4d^2(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{\frac{8b(bc-ad)\int \frac{1}{\sqrt{bx^2+ax}}dx}{d}-\frac{(3a^2d^2-12abcd+8b^2c^2)\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{8d^2(bc-ad)}-\frac{\sqrt{ax+bx^2}(dx(6bc-5ad)+c(4bc-3ad))}{4d^2(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{ 1091}$$

$$\frac{\frac{16b(bc-ad)\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{d}-\frac{(3a^2d^2-12abcd+8b^2c^2)\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{8d^2(bc-ad)}-\frac{\sqrt{ax+bx^2}(dx(6bc-5ad)+c(4bc-3ad))}{4d^2(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{16\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(bc-ad)}{d}-\frac{(3a^2d^2-12abcd+8b^2c^2)\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{8d^2(bc-ad)}-\frac{\sqrt{ax+bx^2}(dx(6bc-5ad)+c(4bc-3ad))}{4d^2(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{2(3a^2d^2-12abcd+8b^2c^2)\int \frac{1}{4c(bc-ad)-\frac{(ac+(2bc-ad)x)^2}{bx^2+ax}}d\left(-\frac{ac+(2bc-ad)x}{\sqrt{bx^2+ax}}\right)}{d}+\frac{16\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(bc-ad)}{d}}{8d^2(bc-ad)}-\frac{\sqrt{ax+bx^2}(dx(6bc-5ad)+c(4bc-3ad))}{4d^2(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{16\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(bc-ad)}{d}-\frac{(3a^2d^2-12abcd+8b^2c^2)\text{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{\sqrt{c}d\sqrt{bc-ad}}}{8d^2(bc-ad)}-\frac{\sqrt{ax+bx^2}(dx(6bc-5ad)+c(4bc-3ad))}{4d^2(c+dx)^2(bc-ad)}$$

input

```
Int[(x*Sqrt[a*x + b*x^2])/(c + d*x)^3,x]
```

output

```
-1/4*((c*(4*b*c - 3*a*d) + d*(6*b*c - 5*a*d)*x)*Sqrt[a*x + b*x^2])/(d^2*(b
*c - a*d)*(c + d*x)^2) + ((16*Sqrt[b]*(b*c - a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt
[a*x + b*x^2]])/d - ((8*b^2*c^2 - 12*a*b*c*d + 3*a^2*d^2)*ArcTanh[(a*c + (
2*b*c - a*d)*x)/(2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[a*x + b*x^2])])/(Sqrt[c]*d
*Sqrt[b*c - a*d]))/(8*d^2*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$-\frac{2\left(\frac{3(dx+c)^2 xa\left(a^2d^2-4abcd+\frac{8}{3}b^2c^2\right)\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{8}+\sqrt{c(ad-bc)}\left((dx+c)^2x\left(b^{\frac{3}{2}}c-\sqrt{b}\,ad\right)a\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)\right.\right.}{\sqrt{c(ad-bc)}\,a\,d^3x(dx+c)^2(ad-bc)}$
default	Expression too large to display

input
```
int(x*(b*x^2+a*x)^(1/2)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*(3/8*(d*x+c)^2*x*a*(a^2*d^2-4*a*b*c*d+8/3*b^2*c^2)*arctan((x*(b*x+a))^(
1/2)/x*c/(c*(a*d-b*c))^(1/2))+(c*(a*d-b*c))^(1/2)*((d*x+c)^2*x*(b^(3/2)*c-
b^(1/2)*a*d)*a*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))+5/8*(1/5*(3*a*c*d-4*b*
c^2)*(x*(b*x+a))^(3/2)+(x*(b*x+a))^(1/2)*(a*d-4/5*b*c)*x^2*(a*d-b*c))*d))/
(c*(a*d-b*c))^(1/2)/a/d^3/x/(d*x+c)^2/(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. 2(160) = 320.

Time = 0.18 (sec) , antiderivative size = 1802, normalized size of antiderivative = 9.69

$$\int \frac{x\sqrt{ax+bx^2}}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(b*x^2+a*x)^(1/2)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
[1/8*(8*(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^3*d^2 - 2*a*b*c^2*d^
3 + a^2*c*d^4)*x^2 + 2*(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x)*sqrt(b
)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - (8*b^2*c^4 - 12*a*b*c^3*d
 + 3*a^2*c^2*d^2 + (8*b^2*c^2*d^2 - 12*a*b*c*d^3 + 3*a^2*d^4)*x^2 + 2*(8*b
^2*c^3*d - 12*a*b*c^2*d^2 + 3*a^2*c*d^3)*x)*sqrt(b*c^2 - a*c*d)*log((a*c +
 (2*b*c - a*d)*x + 2*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) - 2
*(4*b^2*c^4*d - 7*a*b*c^3*d^2 + 3*a^2*c^2*d^3 + (6*b^2*c^3*d^2 - 11*a*b*c^
2*d^3 + 5*a^2*c*d^4)*x)*sqrt(b*x^2 + a*x))/(b^2*c^5*d^3 - 2*a*b*c^4*d^4 +
a^2*c^3*d^5 + (b^2*c^3*d^5 - 2*a*b*c^2*d^6 + a^2*c*d^7)*x^2 + 2*(b^2*c^4*d
^4 - 2*a*b*c^3*d^5 + a^2*c^2*d^6)*x), 1/4*((8*b^2*c^4 - 12*a*b*c^3*d + 3*a
^2*c^2*d^2 + (8*b^2*c^2*d^2 - 12*a*b*c*d^3 + 3*a^2*d^4)*x^2 + 2*(8*b^2*c^3
*d - 12*a*b*c^2*d^2 + 3*a^2*c*d^3)*x)*sqrt(-b*c^2 + a*c*d)*arctan(sqrt(-b*
c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) + 4*(b^2*c^5 - 2*a*b*c^4*d +
 a^2*c^3*d^2 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2 + 2*(b^2*c^4*
d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 +
 a*x)*sqrt(b)) - (4*b^2*c^4*d - 7*a*b*c^3*d^2 + 3*a^2*c^2*d^3 + (6*b^2*c^3
*d^2 - 11*a*b*c^2*d^3 + 5*a^2*c*d^4)*x)*sqrt(b*x^2 + a*x))/(b^2*c^5*d^3 -
2*a*b*c^4*d^4 + a^2*c^3*d^5 + (b^2*c^3*d^5 - 2*a*b*c^2*d^6 + a^2*c*d^7)*x^
2 + 2*(b^2*c^4*d^4 - 2*a*b*c^3*d^5 + a^2*c^2*d^6)*x), -1/8*(16*(b^2*c^5 -
2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x...
```

Sympy [F]

$$\int \frac{x\sqrt{ax+bx^2}}{(c+dx)^3}\,dx=\int \frac{x\sqrt{x\,(a+bx)}}{(c+dx)^3}\,dx$$

input
```
integrate(x*(b*x**2+a*x)**(1/2)/(d*x+c)**3,x)
```

output
```
Integral(x*sqrt(x*(a + b*x))/(c + d*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{ax+bx^2}}{(c+dx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(b*x^2+a*x)^(1/2)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(160) = 320$.

Time = 0.20 (sec) , antiderivative size = 494, normalized size of antiderivative = 2.66

$$\int \frac{x\sqrt{ax+bx^2}}{(c+dx)^3}\,dx = -\frac{(8\,b^2c^2 - 12\,abcd + 3\,a^2d^2)\arctan\left(-\frac{\left(\sqrt{b}x - \sqrt{bx^2+ax}\right)d + \sqrt{b}c}{\sqrt{-bc^2+acd}}\right)}{4\,(bcd^3 - ad^4)\sqrt{-bc^2+acd}} - \frac{\sqrt{b}\log\left(\left|2\left(\sqrt{b}x - \sqrt{bx^2+ax}\right)\sqrt{b} + a\right|\right)}{d^3} - \frac{16\left(\sqrt{b}x - \sqrt{bx^2+ax}\right)^3 b^2c^2d - 20\left(\sqrt{b}x - \sqrt{bx^2+ax}\right)^3 abcd^2 + 5\left(\sqrt{b}x - \sqrt{bx^2+ax}\right)^3 a^2d^3 + 24\,(}{}$$

input
```
integrate(x*(b*x^2+a*x)^(1/2)/(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/4*(8*b^2*c^2 - 12*a*b*c*d + 3*a^2*d^2)*arctan(-((sqrt(b)*x - sqrt(b*x^2
 + a*x))*d + sqrt(b)*c)/sqrt(-b*c^2 + a*c*d))/((b*c*d^3 - a*d^4)*sqrt(-b*c
^2 + a*c*d)) - sqrt(b)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) +
 a))/d^3 - 1/4*(16*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*b^2*c^2*d - 20*(sqrt(
b)*x - sqrt(b*x^2 + a*x))^3*a*b*c*d^2 + 5*(sqrt(b)*x - sqrt(b*x^2 + a*x))^
3*a^2*d^3 + 24*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*b^(5/2)*c^3 - 20*(sqrt(b)
*x - sqrt(b*x^2 + a*x))^2*a*b^(3/2)*c^2*d - (sqrt(b)*x - sqrt(b*x^2 + a*x)
)^2*a^2*sqrt(b)*c*d^2 + 24*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a*b^2*c^3 - 24*
(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^2*b*c^2*d + 3*(sqrt(b)*x - sqrt(b*x^2 +
a*x))*a^3*c*d^2 + 6*a^2*b^(3/2)*c^3 - 5*a^3*sqrt(b)*c^2*d)/((b*c*d^3 - a*d
^4)*((sqrt(b)*x - sqrt(b*x^2 + a*x))^2*d + 2*(sqrt(b)*x - sqrt(b*x^2 + a*x
))*sqrt(b)*c + a*c)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{ax+bx^2}}{(c+dx)^3}\,dx=\int \frac{x\,\sqrt{b\,x^2+a\,x}}{{\left(c+d\,x\right)}^3} \,dx$$

input
```
int((x*(a*x + b*x^2)^(1/2))/(c + d*x)^3,x)
```

output
```
int((x*(a*x + b*x^2)^(1/2))/(c + d*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 2292, normalized size of antiderivative = 12.32

$$\int \frac{x\sqrt{ax+bx^2}}{(c+dx)^3}\,dx=\text{Too large to display}$$

input
```
int(x*(b*x^2+a*x)^(1/2)/(d*x+c)^3,x)
```

output

```
( - 6*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x
) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*c**2*d**3 - 12*sqrt(c
)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*
sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*c*d**4*x - 6*sqrt(c)*sqrt(a*d - b
*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b
))/(sqrt(c)*sqrt(b)))*a**3*d**5*x**2 + 36*sqrt(c)*sqrt(a*d - b*c)*atan((sq
rt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*
sqrt(b)))*a**2*b*c**3*d**2 + 72*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b
*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*
a**2*b*c**2*d**3*x + 36*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sq
rt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*b*c
*d**4*x**2 - 64*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sq
rt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c**4*d -
128*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x)
- sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c**3*d**2*x - 64*sqrt
(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x
)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c**2*d**3*x**2 + 32*sqrt(c)*s
qrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqr
t(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**3*c**5 + 64*sqrt(c)*sqrt(a*d - b*c)*at
an((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/...
```

3.48 $\int \frac{\sqrt{ax+bx^2}}{(c+dx)^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 133

$$\int \frac{\sqrt{ax+bx^2}}{(c+dx)^3} \, dx = -\frac{a\sqrt{ax+bx^2}}{4c(bc-ad)(c+dx)} + \frac{(ax+bx^2)^{3/2}}{2(bc-ad)x(c+dx)^2} - \frac{a^2 \operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{4c^{3/2}(bc-ad)^{3/2}}$$

output
```
-1/4*a*(b*x^2+a*x)^(1/2)/c/(-a*d+b*c)/(d*x+c)+1/2*(b*x^2+a*x)^(3/2)/(-a*d+
b*c)/x/(d*x+c)^2-1/4*a^2*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1
/2))/c^(3/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 10.16 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{ax+bx^2}}{(c+dx)^3} \, dx = \frac{\sqrt{x(a+bx)}\left(\frac{\sqrt{c}(2bcx+a(c-dx))}{(bc-ad)(c+dx)^2} - \frac{a^2 \operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\sqrt{x}}{\sqrt{c}\sqrt{a+bx}}\right)}{(bc-ad)^{3/2}\sqrt{x}\sqrt{a+bx}}\right)}{4c^{3/2}}$$

input
```
Integrate[Sqrt[a*x + b*x^2]/(c + d*x)^3,x]
```

output
```
(Sqrt[x*(a + b*x)]*((Sqrt[c]*(2*b*c*x + a*(c - d*x)))/((b*c - a*d)*(c + d*
x)^2) - (a^2*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(
(b*c - a*d)^(3/2)*Sqrt[x]*Sqrt[a + b*x])))/(4*c^(3/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{(c+dx)^3}dx$$

$$\Big\downarrow \text{ 1152}$$

$$\frac{\sqrt{ax+bx^2}(x(2bc-ad)+ac)}{4c(c+dx)^2(bc-ad)}-\frac{a^2\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{8c(bc-ad)}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{a^2\int \frac{1}{4c(bc-ad)-\frac{(ac+(2bc-ad)x)^2}{bx^2+ax}}d\left(-\frac{ac+(2bc-ad)x}{\sqrt{bx^2+ax}}\right)}{4c(bc-ad)}+\frac{\sqrt{ax+bx^2}(x(2bc-ad)+ac)}{4c(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{ax+bx^2}(x(2bc-ad)+ac)}{4c(c+dx)^2(bc-ad)}-\frac{a^2\text{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{8c^{3/2}(bc-ad)^{3/2}}$$

input
```
Int[Sqrt[a*x + b*x^2]/(c + d*x)^3,x]
```

output
```
((a*c + (2*b*c - a*d)*x)*Sqrt[a*x + b*x^2])/(4*c*(b*c - a*d)*(c + d*x)^2)
- (a^2*ArcTanh[(a*c + (2*b*c - a*d)*x)/(2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[a*x
 + b*x^2])])/(8*c^(3/2)*(b*c - a*d)^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.71

method	result	size
pseudoelliptic	$-\frac{a^{2}\left(\frac{\sqrt{x(bx+a)}\,(-adx+2cbx+ac)}{a^{2}(dx+c)^{2}}+\frac{\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{\sqrt{c(ad-bc)}}\right)}{4(ad-bc)c}$	94
default	Expression too large to display	986

input
```
int((b*x^2+a*x)^(1/2)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2/(a*d-b*c)/c*((x*(b*x+a))^(1/2)*(-a*d*x+2*b*c*x+a*c)/a^2/(d*x+c)^2
+1/(c*(a*d-b*c))^(1/2)*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(113) = 226$.

Time = 0.10 (sec) , antiderivative size = 474, normalized size of antiderivative = 3.56

$$\int \frac{\sqrt{ax+bx^2}}{(c+dx)^3}\,dx$$
$$= \left[-\frac{(a^2d^2x^2+2\,a^2cdx+a^2c^2)\sqrt{bc^2-acd}\log\left(\frac{ac+(2\,bc-ad)x+2\,\sqrt{bc^2-acd}\sqrt{bx^2+ax}}{dx+c}\right)-2\,(abc^3-a^2c^2d+(2\,b^2c^3}{8\,(b^2c^6-2\,abc^5d+a^2c^4d^2+(b^2c^4d^2-2\,abc^3d^3+a^2c^2d^4)x^2+2\,(b^2c^5d-2\,abc^4d^2}\right.$$

input
```
integrate((b*x^2+a*x)^(1/2)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
[-1/8*((a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2)*sqrt(b*c^2 - a*c*d)*log((a*c
+ (2*b*c - a*d)*x + 2*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) -
2*(a*b*c^3 - a^2*c^2*d + (2*b^2*c^3 - 3*a*b*c^2*d + a^2*c*d^2)*x)*sqrt(b*x
^2 + a*x))/(b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c^4*d^2 - 2*a*b*c^3
*d^3 + a^2*c^2*d^4)*x^2 + 2*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x),
1/4*((a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2)*sqrt(-b*c^2 + a*c*d)*arctan(sqr
t(-b*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) + (a*b*c^3 - a^2*c^2*d
+ (2*b^2*c^3 - 3*a*b*c^2*d + a^2*c*d^2)*x)*sqrt(b*x^2 + a*x))/(b^2*c^6 - 2
*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*x^2
 + 2*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2}}{(c+dx)^3}\,dx = \int \frac{\sqrt{x\,(a+bx)}}{(c+dx)^3}\,dx$$

input
```
integrate((b*x**2+a*x)**(1/2)/(d*x+c)**3,x)
```

output
```
Integral(sqrt(x*(a + b*x))/(c + d*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax+bx^2}}{(c+dx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a*x)^(1/2)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(113) = 226.

Time = 0.19 (sec) , antiderivative size = 407, normalized size of antiderivative = 3.06

$$\int \frac{\sqrt{ax+bx^2}}{(c+dx)^3}\,dx = -\frac{a^2\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)d+\sqrt{b}c}{\sqrt{-bc^2+acd}}\right)}{4\,(bc^2-acd)\sqrt{-bc^2+acd}} + \frac{8\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3 b^2c^2d - 8\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3 abcd^2 + \left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3 a^2d^3 + 8\left(\sqrt{b}x-\right.}{4\,(bc^2}$$

input
```
integrate((b*x^2+a*x)^(1/2)/(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/4*a^2*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a*x))*d + sqrt(b)*c)/sqrt(-b*c
^2 + a*c*d))/((b*c^2 - a*c*d)*sqrt(-b*c^2 + a*c*d)) + 1/4*(8*(sqrt(b)*x -
sqrt(b*x^2 + a*x))^3*b^2*c^2*d - 8*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a*b*c
*d^2 + (sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^2*d^3 + 8*(sqrt(b)*x - sqrt(b*x
^2 + a*x))^2*b^(5/2)*c^3 - 5*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2*sqrt(b)
*c*d^2 + 8*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a*b^2*c^3 - 4*(sqrt(b)*x - sqrt
(b*x^2 + a*x))*a^2*b*c^2*d - (sqrt(b)*x - sqrt(b*x^2 + a*x))*a^3*c*d^2 + 2
*a^2*b^(3/2)*c^3 - a^3*sqrt(b)*c^2*d)/((b*c^2*d^2 - a*c*d^3)*((sqrt(b)*x -
 sqrt(b*x^2 + a*x))^2*d + 2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b)*c + a*
c)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^2}}{(c+dx)^3}\,dx=\int \frac{\sqrt{b\,x^2+a\,x}}{{\left(c+d\,x\right)}^3}\,dx$$

input
```
int((a*x + b*x^2)^(1/2)/(c + d*x)^3,x)
```

output
```
int((a*x + b*x^2)^(1/2)/(c + d*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 1056, normalized size of antiderivative = 7.94

$$\int \frac{\sqrt{ax+bx^2}}{(c+dx)^3}\,dx=\text{Too large to display}$$

input
```
int((b*x^2+a*x)^(1/2)/(d*x+c)^3,x)
```

output
```
( - 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x
) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*c**2*d - 4*sqrt(c)*sq
rt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt
(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*c*d**2*x - 2*sqrt(c)*sqrt(a*d - b*c)*
atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(
sqrt(c)*sqrt(b)))*a**3*d**3*x**2 + 4*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*
d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(
b)))*a**2*b*c**3 + 8*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(
d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*b*c**2
*d*x + 4*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a +
b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*b*c*d**2*x**2 - 2*
sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sq
rt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*c**2*d - 4*sqrt(c)*sqrt(a*d
 - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sq
rt(b))/(sqrt(c)*sqrt(b)))*a**3*c*d**2*x - 2*sqrt(c)*sqrt(a*d - b*c)*atan((
sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c
)*sqrt(b)))*a**3*d**3*x**2 + 4*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*
c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a
**2*b*c**3 + 8*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqr
t(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*b*c**2*d*...
```

3.49 $\int \frac{\sqrt{ax+bx^2}}{x(c+dx)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 138

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)^3}\,dx = \frac{\sqrt{ax+bx^2}}{2c(c+dx)^2} + \frac{(2bc-3ad)\sqrt{ax+bx^2}}{4c^2(bc-ad)(c+dx)} + \frac{a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{4c^{5/2}(bc-ad)^{3/2}}$$

output
```
1/2*(b*x^2+a*x)^(1/2)/c/(d*x+c)^2+1/4*(-3*a*d+2*b*c)*(b*x^2+a*x)^(1/2)/c^2
/(-a*d+b*c)/(d*x+c)+1/4*a*(-3*a*d+4*b*c)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2
)/(b*x^2+a*x)^(1/2))/c^(5/2)/(-a*d+b*c)^(3/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2352 vs. 2(138) = 276.

Time = 10.47 (sec) , antiderivative size = 2352, normalized size of antiderivative = 17.04

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[a*x + b*x^2]/(x*(c + d*x)^3),x]
```

output
```
(Sqrt[x*(a + b*x)]*(5*a^9*c*d^2*Sqrt[x] + 3*a^9*d^3*x^(3/2) + 32768*b^9*c^
3*x^(15/2) - 32768*b^(17/2)*c^3*x^7*Sqrt[a + b*x] + Sqrt[b]*Sqrt[a + b*x]*
(3*a^8*c^2*d - 69*a^8*c*d^2*x - 42*a^8*d^3*x^2) + b*(-49*a^8*c^2*d*Sqrt[x]
 + 473*a^8*c*d^2*x^(3/2) + 294*a^8*d^3*x^(5/2)) + b^(3/2)*Sqrt[a + b*x]*(-
2*a^7*c^3 + 392*a^7*c^2*d*x - 2100*a^7*c*d^2*x^2 - 1344*a^7*d^3*x^3) + b^2
*(30*a^7*c^3*Sqrt[x] - 2072*a^7*c^2*d*x^(3/2) + 7084*a^7*c*d^2*x^(5/2) + 4
704*a^7*d^3*x^(7/2)) + b^(5/2)*Sqrt[a + b*x]*(-224*a^6*c^3*x + 7840*a^6*c^
2*d*x^2 - 17248*a^6*c*d^2*x^3 - 12096*a^6*d^3*x^4) + b^3*(1120*a^6*c^3*x^(
3/2) - 24416*a^6*c^2*d*x^(5/2) + 37856*a^6*c*d^2*x^(7/2) + 28224*a^6*d^3*x
^(9/2)) + b^(7/2)*Sqrt[a + b*x]*(-4032*a^5*c^3*x^2 + 56448*a^5*c^2*d*x^3 -
 55296*a^5*c*d^2*x^4 - 46080*a^5*d^3*x^5) + b^4*(12096*a^5*c^3*x^(5/2) - 1
24032*a^5*c^2*d*x^(7/2) + 86784*a^5*c*d^2*x^(9/2) + 80640*a^5*d^3*x^(11/2)
) + b^(9/2)*Sqrt[a + b*x]*(-26880*a^4*c^3*x^3 + 188160*a^4*c^2*d*x^4 - 678
40*a^4*c*d^2*x^5 - 84480*a^4*d^3*x^6) + b^5*(57600*a^4*c^3*x^(7/2) - 31872
0*a^4*c^2*d*x^(9/2) + 72960*a^4*c*d^2*x^(11/2) + 118272*a^4*d^3*x^(13/2))
+ b^(11/2)*Sqrt[a + b*x]*(-84480*a^3*c^3*x^4 + 315392*a^3*c^2*d*x^5 + 1024
*a^3*c*d^2*x^6 - 73728*a^3*d^3*x^7) + b^6*(140800*a^3*c^3*x^(9/2) - 434176
*a^3*c^2*d*x^(11/2) - 27648*a^3*c*d^2*x^(13/2) + 86016*a^3*d^3*x^(15/2)) +
 b^(15/2)*Sqrt[a + b*x]*(-106496*a*c^3*x^6 + 81920*a*c^2*d*x^7 + 32768*a*c
*d^2*x^8) + b^(13/2)*Sqrt[a + b*x]*(-135168*a^2*c^3*x^5 + 258048*a^2*c^...
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1261, 107, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)^3}dx$$

$$\Big\downarrow \text{ 1261}$$

$$\frac{\sqrt{ax+bx^2}\int \frac{\sqrt{a+bx}}{\sqrt{x}(c+dx)^3}dx}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 107}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{(4bc-3ad)\int \frac{\sqrt{a+bx}}{\sqrt{x}(c+dx)^2}dx}{4c(bc-ad)}-\frac{d\sqrt{x}(a+bx)^{3/2}}{2c(c+dx)^2(bc-ad)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 105$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{(4bc-3ad)\left(\frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{2c}+\frac{\sqrt{x}\sqrt{a+bx}}{c(c+dx)}\right)}{4c(bc-ad)}-\frac{d\sqrt{x}(a+bx)^{3/2}}{2c(c+dx)^2(bc-ad)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 104$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{(4bc-3ad)\left(\frac{a\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}+\frac{\sqrt{x}\sqrt{a+bx}}{c(c+dx)}\right)}{4c(bc-ad)}-\frac{d\sqrt{x}(a+bx)^{3/2}}{2c(c+dx)^2(bc-ad)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 221$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{(4bc-3ad)\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{x}\sqrt{a+bx}}{c(c+dx)}\right)}{4c(bc-ad)}-\frac{d\sqrt{x}(a+bx)^{3/2}}{2c(c+dx)^2(bc-ad)}\right)}{\sqrt{x}\sqrt{a+bx}}$$

input

```
Int[Sqrt[a*x + b*x^2]/(x*(c + d*x)^3),x]
```

output

```
(Sqrt[a*x + b*x^2]*(-1/2*(d*Sqrt[x]*(a + b*x)^(3/2))/(c*(b*c - a*d)*(c + d
*x)^2) + ((4*b*c - 3*a*d)*((Sqrt[x]*Sqrt[a + b*x])/(c*(c + d*x)) + (a*ArcT
anh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(c^(3/2)*Sqrt[b*c
- a*d])))/(4*c*(b*c - a*d))))/(Sqrt[x]*Sqrt[a + b*x])
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.82

method	result	size
pseudoelliptic	$-\frac{a\left(-\frac{\sqrt{x(bx+a)}\,(3a\,d^2x-2bcdx+5acd-4b\,c^2)}{a(dx+c)^2}+\frac{(3ad-4bc)\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{\sqrt{c(ad-bc)}}\right)}{4(ad-bc)c^2}$	113
default	Expression too large to display	1913

input
```
int((b*x^2+a*x)^(1/2)/x/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/(a*d-b*c)/c^2*(-(x*(b*x+a))^(1/2)/a*(3*a*d^2*x-2*b*c*d*x+5*a*c*d-4*
b*c^2)/(d*x+c)^2+(3*a*d-4*b*c)/(c*(a*d-b*c))^(1/2)*arctan((x*(b*x+a))^(1/2
)/x*c/(c*(a*d-b*c))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(118) = 236$.

Time = 0.12 (sec) , antiderivative size = 573, normalized size of antiderivative = 4.15

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)^3}\,dx$$
$$= \left[\frac{(4\,abc^3-3\,a^2c^2d+(4\,abcd^2-3\,a^2d^3)x^2+2\,(4\,abc^2d-3\,a^2cd^2)x)\sqrt{bc^2-acd}\log\left(\frac{ac+(2\,bc-ad)x+2\,\sqrt{bc^2-ac}}{dx+c}\right.}{8\,(b^2c^7-2\,abc^6d+a^2c^5d^2+(b^2c^5d^2-2\,abc^4d^3+a^2c^3}\right.$$
$$-\frac{(4\,abc^3-3\,a^2c^2d+(4\,abcd^2-3\,a^2d^3)x^2+2\,(4\,abc^2d-3\,a^2cd^2)x)\sqrt{-bc^2+acd}\arctan\left(\frac{\sqrt{-bc^2+acd}\sqrt{bx^2-}}{bcx+ac}\right.}{4\,(b^2c^7-2\,abc^6d+a^2c^5d^2+(b^2c^5d^2-2\,abc^4d^3+a^2c^3d^4)}$$

input
```
integrate((b*x^2+a*x)^(1/2)/x/(d*x+c)^3,x, algorithm="fricas")
```

output
```
[1/8*((4*a*b*c^3 - 3*a^2*c^2*d + (4*a*b*c*d^2 - 3*a^2*d^3)*x^2 + 2*(4*a*b*
c^2*d - 3*a^2*c*d^2)*x)*sqrt(b*c^2 - a*c*d)*log((a*c + (2*b*c - a*d)*x + 2
*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) + 2*(4*b^2*c^4 - 9*a*b*
c^3*d + 5*a^2*c^2*d^2 + (2*b^2*c^3*d - 5*a*b*c^2*d^2 + 3*a^2*c*d^3)*x)*sqr
t(b*x^2 + a*x))/(b^2*c^7 - 2*a*b*c^6*d + a^2*c^5*d^2 + (b^2*c^5*d^2 - 2*a*
b*c^4*d^3 + a^2*c^3*d^4)*x^2 + 2*(b^2*c^6*d - 2*a*b*c^5*d^2 + a^2*c^4*d^3)
*x), -1/4*((4*a*b*c^3 - 3*a^2*c^2*d + (4*a*b*c*d^2 - 3*a^2*d^3)*x^2 + 2*(4
*a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(-b*c^2 + a*c*d)*arctan(sqrt(-b*c^2 + a*c
*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) - (4*b^2*c^4 - 9*a*b*c^3*d + 5*a^2*c^
2*d^2 + (2*b^2*c^3*d - 5*a*b*c^2*d^2 + 3*a^2*c*d^3)*x)*sqrt(b*x^2 + a*x))/
(b^2*c^7 - 2*a*b*c^6*d + a^2*c^5*d^2 + (b^2*c^5*d^2 - 2*a*b*c^4*d^3 + a^2*
c^3*d^4)*x^2 + 2*(b^2*c^6*d - 2*a*b*c^5*d^2 + a^2*c^4*d^3)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)^3}\,dx = \int \frac{\sqrt{x\,(a+bx)}}{x\,(c+dx)^3}\,dx$$

input
```
integrate((b*x**2+a*x)**(1/2)/x/(d*x+c)**3,x)
```

output
```
Integral(sqrt(x*(a + b*x))/(x*(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)^3}\,dx = \int \frac{\sqrt{bx^2+ax}}{(dx+c)^3x}\,dx$$

input
```
integrate((b*x^2+a*x)^(1/2)/x/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a*x)/((d*x + c)^3*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(118) = 236$.

Time = 0.15 (sec) , antiderivative size = 419, normalized size of antiderivative = 3.04

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)^3}\,dx = -\frac{(4\,abc-3\,a^2d)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)d+\sqrt{b}c}{\sqrt{-bc^2+acd}}\right)}{4\,(bc^3-ac^2d)\sqrt{-bc^2+acd}} - \frac{4\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3abcd^2-3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3a^2d^3-8\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2b^{\frac{5}{2}}c^3+20\left(\sqrt{b}x\right.}{4}$$

input
```
integrate((b*x^2+a*x)^(1/2)/x/(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/4*(4*a*b*c - 3*a^2*d)*arctan(((sqrt(b)*x - sqrt(b*x^2 + a*x))*d + sqrt(
b)*c)/sqrt(-b*c^2 + a*c*d))/((b*c^3 - a*c^2*d)*sqrt(-b*c^2 + a*c*d)) - 1/4
*(4*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a*b*c*d^2 - 3*(sqrt(b)*x - sqrt(b*x^
2 + a*x))^3*a^2*d^3 - 8*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*b^(5/2)*c^3 + 20
*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a*b^(3/2)*c^2*d - 9*(sqrt(b)*x - sqrt(b
*x^2 + a*x))^2*a^2*sqrt(b)*c*d^2 - 8*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a*b^2
*c^3 + 16*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^2*b*c^2*d - 5*(sqrt(b)*x - sqr
t(b*x^2 + a*x))*a^3*c*d^2 - 2*a^2*b^(3/2)*c^3 + 3*a^3*sqrt(b)*c^2*d)/((b*c
^3*d - a*c^2*d^2)*((sqrt(b)*x - sqrt(b*x^2 + a*x))^2*d + 2*(sqrt(b)*x - sq
rt(b*x^2 + a*x))*sqrt(b)*c + a*c)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)^3}\,dx = \int \frac{\sqrt{b\,x^2+a\,x}}{x\,{\left(c+d\,x\right)}^3}\,dx$$

input
```
int((a*x + b*x^2)^(1/2)/(x*(c + d*x)^3),x)
```

output
```
int((a*x + b*x^2)^(1/2)/(x*(c + d*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 1555, normalized size of antiderivative = 11.27

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)^3}\,dx = \text{Too large to display}$$

input

```
int((b*x^2+a*x)^(1/2)/x/(d*x+c)^3,x)
```

output

```
( - 6*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x
) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*c**2*d**3 - 12*sqrt(c
)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*
sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*c*d**4*x - 6*sqrt(c)*sqrt(a*d - b
*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b
))/(sqrt(c)*sqrt(b)))*a**3*d**5*x**2 + 20*sqrt(c)*sqrt(a*d - b*c)*atan((sq
rt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*
sqrt(b)))*a**2*b*c**3*d**2 + 40*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b
*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*
a**2*b*c**2*d**3*x + 20*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sq
rt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*b*c
*d**4*x**2 - 16*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sq
rt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c**4*d -
32*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) -
 sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c**3*d**2*x - 16*sqrt(
c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)
*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c**2*d**3*x**2 - 6*sqrt(c)*sqr
t(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(
d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*c**2*d**3 - 12*sqrt(c)*sqrt(a*d - b*c)
*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b...
```

3.50 $\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 199

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)^3}\,dx = -\frac{(14bc-15ad)\sqrt{ax+bx^2}}{4c^3(bc-ad)x} + \frac{\sqrt{ax+bx^2}}{2cx(c+dx)^2} + \frac{(4bc-5ad)\sqrt{ax+bx^2}}{4c^2(bc-ad)x(c+dx)} + \frac{(8b^2c^2-24abcd+15a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{4c^{7/2}(bc-ad)^{3/2}}$$

output
```
-1/4*(-15*a*d+14*b*c)*(b*x^2+a*x)^(1/2)/c^3/(-a*d+b*c)/x+1/2*(b*x^2+a*x)^(
1/2)/c/x/(d*x+c)^2+1/4*(-5*a*d+4*b*c)*(b*x^2+a*x)^(1/2)/c^2/(-a*d+b*c)/x/(
d*x+c)+1/4*(15*a^2*d^2-24*a*b*c*d+8*b^2*c^2)*arctanh((-a*d+b*c)^(1/2)*x/c^
(1/2)/(b*x^2+a*x)^(1/2))/c^(7/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 10.28 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)^3}\,dx = \frac{\sqrt{x(a+bx)}\left(2d(a+bx)+\frac{(c+dx)\left(c^{3/2}d(6bc-5ad)(a+bx)^{3/2}+(8b^2c^2-24abcd+15a^2d^2)(c+dx)\left(\sqrt{c}\sqrt{a+bx}-\sqrt{bc-ad}\sqrt{x}\,\text{arctanh}\right.\right.}{c^{5/2}(bc-ad)\sqrt{a+bx}}\right.}{4c(-bc+ad)x(c+dx)^2}$$

input
```
Integrate[Sqrt[a*x + b*x^2]/(x^2*(c + d*x)^3),x]
```

output
```
(Sqrt[x*(a + b*x)]*(2*d*(a + b*x) + ((c + d*x)*(c^(3/2)*d*(6*b*c - 5*a*d)*
(a + b*x)^(3/2) + (8*b^2*c^2 - 24*a*b*c*d + 15*a^2*d^2)*(c + d*x)*(Sqrt[c]
*Sqrt[a + b*x] - Sqrt[b*c - a*d]*Sqrt[x]*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])
/(Sqrt[c]*Sqrt[a + b*x])])))/(c^(5/2)*(b*c - a*d)*Sqrt[a + b*x])))/(4*c*(-
(b*c) + a*d)*x*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1261, 110, 27, 168, 27, 168, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)^3}\,dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{\sqrt{ax+bx^2}\int \frac{\sqrt{a+bx}}{x^{3/2}(c+dx)^3}dx}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{110}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{2\int \frac{bc-5ad-4bdx}{2\sqrt{x}\sqrt{a+bx}(c+dx)^3}dx}{c}-\frac{2\sqrt{a+bx}}{c\sqrt{x}(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\int \frac{bc-5ad-4bdx}{\sqrt{x}\sqrt{a+bx}(c+dx)^3}dx}{c}-\frac{2\sqrt{a+bx}}{c\sqrt{x}(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{168}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{-\frac{\int -\frac{(bc-ad)(4bc-15ad-10bdx)}{2\sqrt{x}\sqrt{a+bx}(c+dx)^2}dx}{2c(bc-ad)}-\frac{5d\sqrt{x}\sqrt{a+bx}}{2c(c+dx)^2}}{c}-\frac{2\sqrt{a+bx}}{c\sqrt{x}(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\int \frac{4bc-15ad-10bdx}{\sqrt{x}\sqrt{a+bx}(c+dx)^2}dx}{4c}-\frac{5d\sqrt{x}\sqrt{a+bx}}{2c(c+dx)^2}}{c}-\frac{2\sqrt{a+bx}}{c\sqrt{x}(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{168}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{-\frac{\int -\frac{8b^2c^2-24abdc+15a^2d^2}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{c(bc-ad)}-\frac{d\sqrt{x}\sqrt{a+bx}(14bc-15ad)}{c(c+dx)(bc-ad)}}{4c}-\frac{5d\sqrt{x}\sqrt{a+bx}}{2c(c+dx)^2}}{c}-\frac{2\sqrt{a+bx}}{c\sqrt{x}(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\frac{(15a^2d^2-24abcd+8b^2c^2)\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{2c(bc-ad)}-\frac{d\sqrt{x}\sqrt{a+bx}(14bc-15ad)}{c(c+dx)(bc-ad)}}{4c}-\frac{5d\sqrt{x}\sqrt{a+bx}}{2c(c+dx)^2}}{c}-\frac{2\sqrt{a+bx}}{c\sqrt{x}(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{104}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\frac{(15a^2d^2-24abcd+8b^2c^2)\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c(bc-ad)}-\frac{d\sqrt{x}\sqrt{a+bx}(14bc-15ad)}{c(c+dx)(bc-ad)}}{4c}-\frac{5d\sqrt{x}\sqrt{a+bx}}{2c(c+dx)^2}}{c}-\frac{2\sqrt{a+bx}}{c\sqrt{x}(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\frac{(15a^2d^2-24abcd+8b^2c^2)\operatorname{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}(bc-ad)^{3/2}}-\frac{d\sqrt{x}\sqrt{a+bx}(14bc-15ad)}{c(c+dx)(bc-ad)}}{4c}-\frac{5d\sqrt{x}\sqrt{a+bx}}{2c(c+dx)^2}}{c}-\frac{2\sqrt{a+bx}}{c\sqrt{x}(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

input
```
Int[Sqrt[a*x + b*x^2]/(x^2*(c + d*x)^3),x]
```

output
```
(Sqrt[a*x + b*x^2]*((-2*Sqrt[a + b*x])/(c*Sqrt[x]*(c + d*x)^2) + ((-5*d*Sq
rt[x]*Sqrt[a + b*x])/(2*c*(c + d*x)^2) + (-((d*(14*b*c - 15*a*d)*Sqrt[x]*S
qrt[a + b*x])/(c*(b*c - a*d)*(c + d*x))) + ((8*b^2*c^2 - 24*a*b*c*d + 15*a
^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(c^(3/
2)*(b*c - a*d)^(3/2)))/(4*c))/c))/(Sqrt[x]*Sqrt[a + b*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$-\frac{2\left(-\frac{15(dx+c)^2x\left(a^2d^2-\frac{8}{5}abcd+\frac{8}{15}b^2c^2\right)a\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{8}+\sqrt{c(ad-bc)}\left(\left(\frac{9}{8}a\,d^2c-b\,c^2d\right)(x(bx+a))^{\frac{3}{2}}+\sqrt{x(bx+a)}\,\left(\right.\right.\right.}{\sqrt{c(ad-bc)}\,c^3x(dx+c)^2(ad-bc)a}$ $(ad-bc$
risch	$-\frac{2(bx+a)}{c^3\sqrt{x(bx+a)}}-\frac{-\frac{a\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{x+\frac{c}{d}}\right)}{\sqrt{-\frac{c(ad-bc)}{d^2}}}+}{}$
default	Expression too large to display

input
```
int((b*x^2+a*x)^(1/2)/x^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/(c*(a*d-b*c))^(1/2)*(-15/8*(d*x+c)^2*x*(a^2*d^2-8/5*a*b*c*d+8/15*b^2*c^
2)*a*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))+(c*(a*d-b*c))^(1/2)
*((9/8*a*d^2*c-b*c^2*d)*(x*(b*x+a))^(3/2)+(x*(b*x+a))^(1/2)*(a*c^2+2*d*x*(
-1/2*b*x+a)*c+15/8*a*d^2*x^2)*(a*d-b*c)))/c^3/x/(d*x+c)^2/(a*d-b*c)/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(175) = 350$.

Time = 0.12 (sec) , antiderivative size = 732, normalized size of antiderivative = 3.68

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)^3}\,dx$$
$$= \left[-\frac{((8\,b^2c^2d^2 - 24\,abcd^3 + 15\,a^2d^4)x^3 + 2\,(8\,b^2c^3d - 24\,abc^2d^2 + 15\,a^2cd^3)x^2 + (8\,b^2c^4 - 24\,abc^3d + 15\,a}{8\,((b^2c^6d^2}\right.$$
$$-\frac{((8\,b^2c^2d^2 - 24\,abcd^3 + 15\,a^2d^4)x^3 + 2\,(8\,b^2c^3d - 24\,abc^2d^2 + 15\,a^2cd^3)x^2 + (8\,b^2c^4 - 24\,abc^3d + 15\,a^2}{4\,((b^2c^6d^2 - 2\,a}$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^2/(d*x+c)^3,x, algorithm="fricas")
```

output
```
[-1/8*(((8*b^2*c^2*d^2 - 24*a*b*c*d^3 + 15*a^2*d^4)*x^3 + 2*(8*b^2*c^3*d -
 24*a*b*c^2*d^2 + 15*a^2*c*d^3)*x^2 + (8*b^2*c^4 - 24*a*b*c^3*d + 15*a^2*c
^2*d^2)*x)*sqrt(b*c^2 - a*c*d)*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*c^2 -
 a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) + 2*(8*b^2*c^5 - 16*a*b*c^4*d + 8*a^
2*c^3*d^2 + (14*b^2*c^3*d^2 - 29*a*b*c^2*d^3 + 15*a^2*c*d^4)*x^2 + (24*b^2
*c^4*d - 49*a*b*c^3*d^2 + 25*a^2*c^2*d^3)*x)*sqrt(b*x^2 + a*x))/((b^2*c^6*
d^2 - 2*a*b*c^5*d^3 + a^2*c^4*d^4)*x^3 + 2*(b^2*c^7*d - 2*a*b*c^6*d^2 + a^
2*c^5*d^3)*x^2 + (b^2*c^8 - 2*a*b*c^7*d + a^2*c^6*d^2)*x), -1/4*(((8*b^2*c
^2*d^2 - 24*a*b*c*d^3 + 15*a^2*d^4)*x^3 + 2*(8*b^2*c^3*d - 24*a*b*c^2*d^2
+ 15*a^2*c*d^3)*x^2 + (8*b^2*c^4 - 24*a*b*c^3*d + 15*a^2*c^2*d^2)*x)*sqrt(
-b*c^2 + a*c*d)*arctan(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c
)) + (8*b^2*c^5 - 16*a*b*c^4*d + 8*a^2*c^3*d^2 + (14*b^2*c^3*d^2 - 29*a*b*
c^2*d^3 + 15*a^2*c*d^4)*x^2 + (24*b^2*c^4*d - 49*a*b*c^3*d^2 + 25*a^2*c^2*
d^3)*x)*sqrt(b*x^2 + a*x))/((b^2*c^6*d^2 - 2*a*b*c^5*d^3 + a^2*c^4*d^4)*x^
3 + 2*(b^2*c^7*d - 2*a*b*c^6*d^2 + a^2*c^5*d^3)*x^2 + (b^2*c^8 - 2*a*b*c^7
*d + a^2*c^6*d^2)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)^3}\,dx = \int \frac{\sqrt{x\,(a+bx)}}{x^2\,(c+dx)^3}\,dx$$

input
```
integrate((b*x**2+a*x)**(1/2)/x**2/(d*x+c)**3,x)
```

output
```
Integral(sqrt(x*(a + b*x))/(x**2*(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)^3}\,dx = \int \frac{\sqrt{bx^2+ax}}{(dx+c)^3x^2}\,dx$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^2/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a*x)/((d*x + c)^3*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^2/(d*x+c)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)^3}\,dx=\int \frac{\sqrt{b\,x^2+a\,x}}{x^2\,{\left(c+d\,x\right)}^3} \,d x$$

input
```
int((a*x + b*x^2)^(1/2)/(x^2*(c + d*x)^3),x)
```

output
```
int((a*x + b*x^2)^(1/2)/(x^2*(c + d*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 10.38 (sec) , antiderivative size = 2030, normalized size of antiderivative = 10.20

$$\int \frac{\sqrt{ax+bx^2}}{x^2(c+dx)^3}\,dx=\text{Too large to display}$$

input
```
int((b*x^2+a*x)^(1/2)/x^2/(d*x+c)^3,x)
```

output
```
(75*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x)
- sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*c**2*d**3*x + 150*sqrt(
c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)
*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*c*d**4*x**2 + 75*sqrt(c)*sqrt(a*
d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*s
qrt(b))/(sqrt(c)*sqrt(b)))*a**3*d**5*x**3 - 240*sqrt(c)*sqrt(a*d - b*c)*at
an((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sq
rt(c)*sqrt(b)))*a**2*b*c**3*d**2*x - 480*sqrt(c)*sqrt(a*d - b*c)*atan((sqr
t(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*s
qrt(b)))*a**2*b*c**2*d**3*x**2 - 240*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*
d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(
b)))*a**2*b*c*d**4*x**3 + 232*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c
) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*
b**2*c**4*d*x + 464*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d
)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c**3*
d**2*x**2 + 232*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sq
rt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c**2*d**3
*x**3 - 64*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a
+ b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**3*c**5*x - 128*sqr
t(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sq...
```

3.51 $\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 259

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)^3} dx = -\frac{(32bc-35ad)\sqrt{ax+bx^2}}{12c^3(bc-ad)x^2} - \frac{(8b^2c^2-110abcd+105a^2d^2)\sqrt{ax+bx^2}}{12ac^4(bc-ad)x} + \frac{\sqrt{ax+bx^2}}{2cx^2(c+dx)^2} + \frac{(6bc-7ad)\sqrt{ax+bx^2}}{4c^2(bc-ad)x^2(c+dx)} - \frac{d(24b^2c^2-60abcd+35a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{4c^{9/2}(bc-ad)^{3/2}}$$

output
```
-1/12*(-35*a*d+32*b*c)*(b*x^2+a*x)^(1/2)/c^3/(-a*d+b*c)/x^2-1/12*(105*a^2*
d^2-110*a*b*c*d+8*b^2*c^2)*(b*x^2+a*x)^(1/2)/a/c^4/(-a*d+b*c)/x+1/2*(b*x^2
+a*x)^(1/2)/c/x^2/(d*x+c)^2+1/4*(-7*a*d+6*b*c)*(b*x^2+a*x)^(1/2)/c^2/(-a*d
+b*c)/x^2/(d*x+c)-1/4*d*(35*a^2*d^2-60*a*b*c*d+24*b^2*c^2)*arctanh((-a*d+b
*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(9/2)/(-a*d+b*c)^(3/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2955 vs. 2(259) = 518.

Time = 13.86 (sec) , antiderivative size = 2955, normalized size of antiderivative = 11.41

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[a*x + b*x^2]/(x^3*(c + d*x)^3),x]
```

output
```
(Sqrt[x*(a + b*x)]*(-8*a^7*c^3*d + 56*a^7*c^2*d^2*x + 175*a^7*c*d^3*x^2 +
105*a^7*d^4*x^3 + Sqrt[b]*Sqrt[a + b*x]*(88*a^6*c^3*d*Sqrt[x] - 511*a^6*c^
2*d^2*x^(3/2) - 1715*a^6*c*d^3*x^(5/2) - 1050*a^6*d^4*x^(7/2)) + b*(8*a^6*
c^4 - 552*a^6*c^3*d*x + 2073*a^6*c^2*d^2*x^2 + 8255*a^6*c*d^3*x^3 + 5250*a
^6*d^4*x^4) + b^(3/2)*Sqrt[a + b*x]*(-88*a^5*c^4*Sqrt[x] + 2354*a^5*c^3*d*
x^(3/2) - 4172*a^5*c^2*d^2*x^(5/2) - 24800*a^5*c*d^3*x^(7/2) - 16800*a^5*d
^4*x^(9/2)) + b^2*(496*a^5*c^4*x - 7638*a^5*c^3*d*x^2 + 2580*a^5*c^2*d^2*x
^3 + 56800*a^5*c*d^3*x^4 + 42000*a^5*d^4*x^5) + b^(5/2)*Sqrt[a + b*x]*(-18
40*a^4*c^4*x^(3/2) + 17176*a^4*c^3*d*x^(5/2) + 17888*a^4*c^2*d^2*x^(7/2) -
 80400*a^4*c*d^3*x^(9/2) - 70560*a^4*d^4*x^(11/2)) + b^3*(5360*a^4*c^4*x^2
 - 33496*a^4*c^3*d*x^3 - 65408*a^4*c^2*d^2*x^4 + 108880*a^4*c*d^3*x^5 + 11
7600*a^4*d^4*x^6) + b^(7/2)*Sqrt[a + b*x]*(-11136*a^3*c^4*x^(5/2) + 37216*
a^3*c^3*d*x^(7/2) + 137600*a^3*c^2*d^2*x^(9/2) - 42560*a^3*c*d^3*x^(11/2)
- 107520*a^3*d^4*x^(13/2)) + b^4*(22016*a^3*c^4*x^3 - 41376*a^3*c^3*d*x^4
- 234240*a^3*c^2*d^2*x^5 + 2240*a^3*c*d^3*x^6 + 134400*a^3*d^4*x^7) + b^(1
3/2)*Sqrt[a + b*x]*(-12288*c^4*x^(11/2) - 55296*c^3*d*x^(13/2) - 36864*c^2
*d^2*x^(15/2)) + b^(11/2)*Sqrt[a + b*x]*(-30720*a*c^4*x^(9/2) - 81408*a*c^
3*d*x^(11/2) + 64512*a*c^2*d^2*x^(13/2) + 92160*a*c*d^3*x^(15/2)) + b^(9/2
)*Sqrt[a + b*x]*(-27904*a^2*c^4*x^(7/2) - 5120*a^2*c^3*d*x^(9/2) + 214016*
a^2*c^2*d^2*x^(11/2) + 103680*a^2*c*d^3*x^(13/2) - 53760*a^2*d^4*x^(15/...
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {1261, 110, 27, 168, 27, 168, 27, 169, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)^3}dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{\sqrt{ax+bx^2}\int \frac{\sqrt{a+bx}}{x^{5/2}(c+dx)^3}dx}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{110}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{2\int \frac{bc-7ad-6bdx}{2x^{3/2}\sqrt{a+bx}(c+dx)^3}dx}{3c}-\frac{2\sqrt{a+bx}}{3cx^{3/2}(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\int \frac{bc-7ad-6bdx}{x^{3/2}\sqrt{a+bx}(c+dx)^3}dx}{3c}-\frac{2\sqrt{a+bx}}{3cx^{3/2}(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{168}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{-\frac{\int -\frac{(bc-ad)(4bc-35ad-28bdx)}{2x^{3/2}\sqrt{a+bx}(c+dx)^2}dx}{2c(bc-ad)}-\frac{7d\sqrt{a+bx}}{2c\sqrt{x}(c+dx)^2}}{3c}-\frac{2\sqrt{a+bx}}{3cx^{3/2}(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\int \frac{4bc-35ad-28bdx}{x^{3/2}\sqrt{a+bx}(c+dx)^2}dx}{4c}-\frac{7d\sqrt{a+bx}}{2c\sqrt{x}(c+dx)^2}}{3c}-\frac{2\sqrt{a+bx}}{3cx^{3/2}(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{168}$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{-\frac{\int -\frac{8b^2c^2-110abdc+105a^2d^2-2bd(32bc-35ad)x}{2x^{3/2}\sqrt{a+bx}(c+dx)}dx}{c(bc-ad)}-\frac{d\sqrt{a+bx}(32bc-35ad)}{c\sqrt{x}(c+dx)(bc-ad)}}{4c}-\frac{7d\sqrt{a+bx}}{2c\sqrt{x}(c+dx)^2}}{3c}-\frac{2\sqrt{a+bx}}{3cx^{3/2}(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\frac{\int \frac{8b^2c^2-110abdc+105a^2d^2-2bd(32bc-35ad)x}{x^{3/2}\sqrt{a+bx}(c+dx)}dx}{2c(bc-ad)}-\frac{d\sqrt{a+bx}(32bc-35ad)}{c\sqrt{x}(c+dx)(bc-ad)}}{4c}-\frac{7d\sqrt{a+bx}}{2c\sqrt{x}(c+dx)^2}}{3c}-\frac{2\sqrt{a+bx}}{3cx^{3/2}(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 169$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\frac{-\frac{2\int \frac{3ad\left(24b^2c^2-60abdc+35a^2d^2\right)}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{ac}-\frac{2\sqrt{a+bx}\left(\frac{8b^2c}{a}+\frac{105ad^2}{c}-110bd\right)}{\sqrt{x}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}(32bc-35ad)}{c\sqrt{x}(c+dx)(bc-ad)}}{4c}-\frac{7d\sqrt{a+bx}}{2c\sqrt{x}(c+dx)^2}}{3c}-\frac{2\sqrt{a+bx}}{3cx^{3/2}(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{\frac{-\frac{3d\left(35a^2d^2-60abcd+24b^2c^2\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{c}-\frac{2\sqrt{a+bx}\left(\frac{8b^2c}{a}+\frac{105ad^2}{c}-110bd\right)}{\sqrt{x}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}(32bc-35ad)}{c\sqrt{x}(c+dx)(bc-ad)}}{4c}-\frac{7d\sqrt{a+bx}}{2c\sqrt{x}(c+dx)^2}}{3c}-\frac{2\sqrt{a+bx}}{3cx^{3/2}(c+dx)^2}\right)}{\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 104$$

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{-\frac{\frac{6d(35a^2d^2-60abcd+24b^2c^2)\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}-\frac{2\sqrt{a+bx}\left(\frac{8b^2c}{a}+\frac{105ad^2}{c}-110bd\right)}{\sqrt{x}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}(32bc-35ad)}{c\sqrt{x}(c+dx)(bc-ad)}}{4c}-\frac{7d\sqrt{a+bx}}{2c\sqrt{x}(c+dx)^2}}{3c}-\frac{}{3c\ldots}\right.}{\sqrt{x}\sqrt{a+bx}}$$

$\downarrow$ 221

$$\frac{\sqrt{ax+bx^2}\left(\frac{\frac{-\frac{\frac{6d(35a^2d^2-60abcd+24b^2c^2)\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{a+bx}\left(\frac{8b^2c}{a}+\frac{105ad^2}{c}-110bd\right)}{\sqrt{x}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}(32bc-35ad)}{c\sqrt{x}(c+dx)(bc-ad)}}{4c}-\frac{7d\sqrt{a+bx}}{2c\sqrt{x}(c+dx)^2}}{3c}-\frac{}{3\ldots}\right.}{\sqrt{x}\sqrt{a+bx}}$$

input
```
Int[Sqrt[a*x + b*x^2]/(x^3*(c + d*x)^3),x]
```

output
```
(Sqrt[a*x + b*x^2]*((-2*Sqrt[a + b*x])/(3*c*x^(3/2)*(c + d*x)^2) + ((-7*d*
Sqrt[a + b*x])/(2*c*Sqrt[x]*(c + d*x)^2) + (-((d*(32*b*c - 35*a*d)*Sqrt[a
+ b*x])/(c*(b*c - a*d)*Sqrt[x]*(c + d*x))) + ((-2*((8*b^2*c)/a - 110*b*d +
 (105*a*d^2)/c)*Sqrt[a + b*x])/Sqrt[x] - (6*d*(24*b^2*c^2 - 60*a*b*c*d + 3
5*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(c^
(3/2)*Sqrt[b*c - a*d]))/(2*c*(b*c - a*d)))/(4*c))/(3*c)))/(Sqrt[x]*Sqrt[a
+ b*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$-\frac{2\left(\frac{105(dx+c)^2 d\,x^2 a\left(a^2d^2-\frac{12}{7}abcd+\frac{24}{35}b^2c^2\right)\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{8}+\sqrt{c(ad-bc)}\left(-\frac{39d^2x\left(ad-\frac{12bc}{13}\right)c(x(bx+a))^{\frac{3}{2}}}{8}+\sqrt{x(bx\right.\right.}{3\sqrt{c(ad-bc)}\,c^4x^2(dx+c)^2(ad-bc)a}$
risch	$-\frac{2(bx+a)(-9adx+cbx+ac)}{3a\,c^4\sqrt{x(bx+a)}\,x}+\frac{\left(d\left(-\frac{(3ad-bc)\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right.}{d}}}{x+\frac{c}{d}}\right)}{d\sqrt{-\frac{c(ad-bc)}{d^2}}}\right.\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a*x)^(1/2)/x^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(105/8*(d*x+c)^2*d*x^2*a*(a^2*d^2-12/7*a*b*c*d+24/35*b^2*c^2)*arctan(
(x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))+(c*(a*d-b*c))^(1/2)*(-39/8*d^2*
x*(a*d-12/13*b*c)*c*(x*(b*x+a))^(3/2)+(x*(b*x+a))^(1/2)*(a*d-b*c)*((b*x+a)
*c^3-7*d*x*(-2/7*b*x+a)*c^2-17*d^2*x^2*(-11/34*b*x+a)*c-105/8*a*x^3*d^3)))
/(c*(a*d-b*c))^(1/2)/c^4/x^2/(d*x+c)^2/(a*d-b*c)/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(231) = 462$.

Time = 0.13 (sec) , antiderivative size = 943, normalized size of antiderivative = 3.64

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^3/(d*x+c)^3,x, algorithm="fricas")
```

output
```
[-1/24*(3*((24*a*b^2*c^2*d^3 - 60*a^2*b*c*d^4 + 35*a^3*d^5)*x^4 + 2*(24*a*
b^2*c^3*d^2 - 60*a^2*b*c^2*d^3 + 35*a^3*c*d^4)*x^3 + (24*a*b^2*c^4*d - 60*
a^2*b*c^3*d^2 + 35*a^3*c^2*d^3)*x^2)*sqrt(b*c^2 - a*c*d)*log((a*c + (2*b*c
 - a*d)*x + 2*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) + 2*(8*a*b
^2*c^6 - 16*a^2*b*c^5*d + 8*a^3*c^4*d^2 + (8*b^3*c^4*d^2 - 118*a*b^2*c^3*d
^3 + 215*a^2*b*c^2*d^4 - 105*a^3*c*d^5)*x^3 + (16*b^3*c^5*d - 204*a*b^2*c^
4*d^2 + 363*a^2*b*c^3*d^3 - 175*a^3*c^2*d^4)*x^2 + 8*(b^3*c^6 - 9*a*b^2*c^
5*d + 15*a^2*b*c^4*d^2 - 7*a^3*c^3*d^3)*x)*sqrt(b*x^2 + a*x))/((a*b^2*c^7*
d^2 - 2*a^2*b*c^6*d^3 + a^3*c^5*d^4)*x^4 + 2*(a*b^2*c^8*d - 2*a^2*b*c^7*d^
2 + a^3*c^6*d^3)*x^3 + (a*b^2*c^9 - 2*a^2*b*c^8*d + a^3*c^7*d^2)*x^2), 1/1
2*(3*((24*a*b^2*c^2*d^3 - 60*a^2*b*c*d^4 + 35*a^3*d^5)*x^4 + 2*(24*a*b^2*c
^3*d^2 - 60*a^2*b*c^2*d^3 + 35*a^3*c*d^4)*x^3 + (24*a*b^2*c^4*d - 60*a^2*b
*c^3*d^2 + 35*a^3*c^2*d^3)*x^2)*sqrt(-b*c^2 + a*c*d)*arctan(sqrt(-b*c^2 +
a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) - (8*a*b^2*c^6 - 16*a^2*b*c^5*d +
8*a^3*c^4*d^2 + (8*b^3*c^4*d^2 - 118*a*b^2*c^3*d^3 + 215*a^2*b*c^2*d^4 - 1
05*a^3*c*d^5)*x^3 + (16*b^3*c^5*d - 204*a*b^2*c^4*d^2 + 363*a^2*b*c^3*d^3
- 175*a^3*c^2*d^4)*x^2 + 8*(b^3*c^6 - 9*a*b^2*c^5*d + 15*a^2*b*c^4*d^2 - 7
*a^3*c^3*d^3)*x)*sqrt(b*x^2 + a*x))/((a*b^2*c^7*d^2 - 2*a^2*b*c^6*d^3 + a^
3*c^5*d^4)*x^4 + 2*(a*b^2*c^8*d - 2*a^2*b*c^7*d^2 + a^3*c^6*d^3)*x^3 + (a*
b^2*c^9 - 2*a^2*b*c^8*d + a^3*c^7*d^2)*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)^3}\,dx = \int \frac{\sqrt{x\,(a+bx)}}{x^3\,(c+dx)^3}\,dx$$

input
```
integrate((b*x**2+a*x)**(1/2)/x**3/(d*x+c)**3,x)
```

output
```
Integral(sqrt(x*(a + b*x))/(x**3*(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)^3}\,dx = \int \frac{\sqrt{bx^2+ax}}{(dx+c)^3x^3}\,dx$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^3/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a*x)/((d*x + c)^3*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a*x)^(1/2)/x^3/(d*x+c)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)^3}\,dx = \int \frac{\sqrt{b\,x^2+a\,x}}{x^3\,{\left(c+d\,x\right)}^3}\,dx$$

input
```
int((a*x + b*x^2)^(1/2)/(x^3*(c + d*x)^3),x)
```

output
```
int((a*x + b*x^2)^(1/2)/(x^3*(c + d*x)^3), x)
```

Reduce [F]

$$\int \frac{\sqrt{ax+bx^2}}{x^3(c+dx)^3}\,dx = \int \frac{\sqrt{b\,x^2+ax}}{x^3\,(dx+c)^3}dx$$

input
```
int((b*x^2+a*x)^(1/2)/x^3/(d*x+c)^3,x)
```

output
```
int((b*x^2+a*x)^(1/2)/x^3/(d*x+c)^3,x)
```

3.52 $\int \frac{\sqrt{ax+bx^2}}{x(c+dx)} dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)}\,dx = \frac{2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{d} - \frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{\sqrt{c}d}$$

output
```
2*b^(1/2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/d-2*(-a*d+b*c)^(1/2)*arctan
h((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)}\,dx = -\frac{2\sqrt{x}\sqrt{a+bx}\left(\sqrt{-bc+ad}\arctan\left(\frac{-d\sqrt{x}\sqrt{a+bx}+\sqrt{b}(c+dx)}{\sqrt{c}\sqrt{-bc+ad}}\right)+\sqrt{b}\sqrt{c}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)\right)}{\sqrt{c}d\sqrt{x(a+bx)}}$$

input
```
Integrate[Sqrt[a*x + b*x^2]/(x*(c + d*x)),x]
```

output
```
(-2*Sqrt[x]*Sqrt[a + b*x]*(Sqrt[-(b*c) + a*d]*ArcTan[(-(d*Sqrt[x]*Sqrt[a +
 b*x]) + Sqrt[b]*(c + d*x))/(Sqrt[c]*Sqrt[-(b*c) + a*d])] + Sqrt[b]*Sqrt[c
]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]))/(Sqrt[c]*d*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.68, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1260, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)}\,dx$$

$$\Big\downarrow \ 1260$$

$$\int \left(\frac{\sqrt{ax+bx^2}}{cx}-\frac{d\sqrt{ax+bx^2}}{c(c+dx)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(2bc-ad)}{\sqrt{b}cd}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{\sqrt{c}d}+\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}c}$$

input
```
Int[Sqrt[a*x + b*x^2]/(x*(c + d*x)),x]
```

output
```
(a*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(Sqrt[b]*c) + ((2*b*c - a*d)*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(Sqrt[b]*c*d) - (Sqrt[b*c - a*d]*Arc
Tanh[(a*c + (2*b*c - a*d)*x)/(2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[a*x + b*x^2])
])/(Sqrt[c]*d)
```

Defintions of rubi rules used

rule 1260
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x + c*x^2)^p
, (d + e*x)^m*(f + g*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ
[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + n + 2*p + 1, 0] && ILtQ[m, 0] && ILtQ
[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$-\frac{2\left(-\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)+\frac{(ad-bc)\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{\sqrt{c(ad-bc)}}\right)}{d}$
default	$\frac{\sqrt{b\,x^{2}+ax}+\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2\sqrt{b}}}{c}-\frac{\sqrt{b\left(x+\frac{c}{d}\right)^{2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^{2}}}+\frac{(ad-2bc)\ln\left(\frac{\frac{ad-2bc}{2d}+b\left(x+\frac{c}{d}\right)}{\sqrt{b}}+\sqrt{b\left(x+\frac{c}{d}\right.}\right.}{2d\sqrt{b}}}{}$

input
```
int((b*x^2+a*x)^(1/2)/x/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(-b^(1/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))+(a*d-b*c)/(c*(a*d-b*c)
)^(1/2)*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 396, normalized size of antiderivative = 4.55

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)}\,dx = \left[\frac{\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)x-2\,\sqrt{bx^2+ax}c\sqrt{\frac{bc-ad}{c}}}{dx+c}\right)}{d}, -\frac{2\,\sqrt{-\frac{bc-ad}{c}}\arctan\left(-\frac{\sqrt{bx^2+ax}c\sqrt{-\frac{bc-ad}{c}}}{(bc-ad)x}\right)-\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{d}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)x-2\,\sqrt{bx^2+ax}c\sqrt{\frac{bc-ad}{c}}}{dx+c}\right)}{d}, -\frac{2\left(\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+\sqrt{-\frac{bc-ad}{c}}\arctan\left(-\frac{\sqrt{bx^2+ax}c\sqrt{-\frac{bc-ad}{c}}}{(bc-ad)x}\right)\right)}{d}\right]$$

input
```
integrate((b*x^2+a*x)^(1/2)/x/(d*x+c),x, algorithm="fricas")
```

output
```
[(sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + sqrt((b*c - a*d)/
c)*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)*c*sqrt((b*c - a*d)/c))
/(d*x + c)))/d, -(2*sqrt(-(b*c - a*d)/c)*arctan(-sqrt(b*x^2 + a*x)*c*sqrt(
-(b*c - a*d)/c)/((b*c - a*d)*x)) - sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 +
a*x)*sqrt(b)))/d, -(2*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)
) - sqrt((b*c - a*d)/c)*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)*c
*sqrt((b*c - a*d)/c))/(d*x + c)))/d, -2*(sqrt(-b)*arctan(sqrt(b*x^2 + a*x)
*sqrt(-b)/(b*x + a)) + sqrt(-(b*c - a*d)/c)*arctan(-sqrt(b*x^2 + a*x)*c*sq
rt(-(b*c - a*d)/c)/((b*c - a*d)*x)))/d]
```

Sympy [F]

$$\int \frac{\sqrt{ax + bx^2}}{x(c + dx)} \, dx = \int \frac{\sqrt{x (a + bx)}}{x (c + dx)} \, dx$$

input
```
integrate((b*x**2+a*x)**(1/2)/x/(d*x+c),x)
```

output
```
Integral(sqrt(x*(a + b*x))/(x*(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax + bx^2}}{x(c + dx)} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a*x)^(1/2)/x/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax + bx^2}}{x(c + dx)} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a*x)^(1/2)/x/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)}\,dx = \int \frac{\sqrt{b\,x^2+a\,x}}{x\,(c+d\,x)}\,dx$$

input
```
int((a*x + b*x^2)^(1/2)/(x*(c + d*x)),x)
```

output
```
int((a*x + b*x^2)^(1/2)/(x*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)}\,dx = \frac{-2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)-2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}+\sqrt{d}\,\sqrt{bx+a}+\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)+2\sqrt{}}{cd}$$

input
```
int((b*x^2+a*x)^(1/2)/x/(d*x+c),x)
```

output
```
(2*( - sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*
x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b))) - sqrt(c)*sqrt(a*d - b*c)
*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/
(sqrt(c)*sqrt(b))) + sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)
)*c))/(c*d)
```

3.53 $\int \frac{\sqrt{x(a+bx)}}{x(c+dx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{\sqrt{x(a+bx)}}{x(c+dx)}\,dx = \frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{d} - \frac{2\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{\sqrt{c}d}$$

output
```
2*b^(1/2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/d-2*(-a*d+b*c)^(1/2)*arctan
h((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{x(a+bx)}}{x(c+dx)}\,dx = -\frac{2\sqrt{x}\sqrt{a+bx}\left(\sqrt{-bc+ad}\arctan\left(\frac{-d\sqrt{x}\sqrt{a+bx}+\sqrt{b}(c+dx)}{\sqrt{c}\sqrt{-bc+ad}}\right)+\sqrt{b}\sqrt{c}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)\right)}{\sqrt{c}d\sqrt{x(a+bx)}}$$

input
```
Integrate[Sqrt[x*(a + b*x)]/(x*(c + d*x)),x]
```

output
```
(-2*Sqrt[x]*Sqrt[a + b*x]*(Sqrt[-(b*c) + a*d]*ArcTan[(-(d*Sqrt[x]*Sqrt[a +
 b*x]) + Sqrt[b]*(c + d*x))/(Sqrt[c]*Sqrt[-(b*c) + a*d])] + Sqrt[b]*Sqrt[c
]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]))/(Sqrt[c]*d*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.68, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2048, 1260, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x(a+bx)}}{x(c+dx)}dx$$
$$\Big\downarrow \text{2048}$$
$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)}dx$$
$$\Big\downarrow \text{1260}$$
$$\int \left(\frac{\sqrt{ax+bx^2}}{cx}-\frac{d\sqrt{ax+bx^2}}{c(c+dx)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(2bc-ad)}{\sqrt{b}cd}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{\sqrt{c}d}+\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}c}$$

input
```
Int[Sqrt[x*(a + b*x)]/(x*(c + d*x)),x]
```

output
```
(a*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(Sqrt[b]*c) + ((2*b*c - a*d)*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(Sqrt[b]*c*d) - (Sqrt[b*c - a*d]*Arc
Tanh[(a*c + (2*b*c - a*d)*x)/(2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[a*x + b*x^2])
])/(Sqrt[c]*d)
```

Defintions of rubi rules used

rule 1260
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x + c*x^2)^p
, (d + e*x)^m*(f + g*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ
[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + n + 2*p + 1, 0] && ILtQ[m, 0] && ILtQ
[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$-\frac{2\left(-\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)+\frac{(ad-bc)\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{\sqrt{c(ad-bc)}}\right)}{d}$
default	$\frac{\sqrt{b\,x^{2}+ax}+\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2\sqrt{b}}}{c}-\frac{\sqrt{b\left(x+\frac{c}{d}\right)^{2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^{2}}}+\frac{(ad-2bc)\ln\left(\frac{\frac{ad-2bc}{2d}+b\left(x+\frac{c}{d}\right)}{\sqrt{b}}+\sqrt{b\left(x+\frac{c}{d}\right.}\right.}{2d\sqrt{b}}}{}$

input
```
int((x*(b*x+a))^(1/2)/x/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(-b^(1/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))+(a*d-b*c)/(c*(a*d-b*c)
)^(1/2)*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 396, normalized size of antiderivative = 4.55

$$\int \frac{\sqrt{x(a+bx)}}{x(c+dx)}\,dx = \left[\frac{\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)x-2\,\sqrt{bx^2+ax}c\sqrt{\frac{bc-ad}{c}}}{dx+c}\right)}{d},\right.$$
$$-\frac{2\,\sqrt{-\frac{bc-ad}{c}}\arctan\left(-\frac{\sqrt{bx^2+ax}c\sqrt{-\frac{bc-ad}{c}}}{(bc-ad)x}\right)-\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{d},$$
$$-\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)x-2\,\sqrt{bx^2+ax}c\sqrt{\frac{bc-ad}{c}}}{dx+c}\right)}{d},$$
$$\left.-\frac{2\left(\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+\sqrt{-\frac{bc-ad}{c}}\arctan\left(-\frac{\sqrt{bx^2+ax}c\sqrt{-\frac{bc-ad}{c}}}{(bc-ad)x}\right)\right)}{d}\right]$$

input
```
integrate((x*(b*x+a))^(1/2)/x/(d*x+c),x, algorithm="fricas")
```

output
```
[(sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + sqrt((b*c - a*d)/
c)*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)*c*sqrt((b*c - a*d)/c))
/(d*x + c)))/d, -(2*sqrt(-(b*c - a*d)/c)*arctan(-sqrt(b*x^2 + a*x)*c*sqrt(
-(b*c - a*d)/c)/((b*c - a*d)*x)) - sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 +
a*x)*sqrt(b)))/d, -(2*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)
) - sqrt((b*c - a*d)/c)*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)*c
*sqrt((b*c - a*d)/c))/(d*x + c)))/d, -2*(sqrt(-b)*arctan(sqrt(b*x^2 + a*x)
*sqrt(-b)/(b*x + a)) + sqrt(-(b*c - a*d)/c)*arctan(-sqrt(b*x^2 + a*x)*c*sq
rt(-(b*c - a*d)/c)/((b*c - a*d)*x)))/d]
```

Sympy [F]

$$\int \frac{\sqrt{x(a+bx)}}{x(c+dx)}\,dx = \int \frac{\sqrt{x\,(a+bx)}}{x\,(c+dx)}\,dx$$

input
```
integrate((x*(b*x+a))**(1/2)/x/(d*x+c),x)
```

output
```
Integral(sqrt(x*(a + b*x))/(x*(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{x(a+bx)}}{x(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((x*(b*x+a))^(1/2)/x/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{x(a+bx)}}{x(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((x*(b*x+a))^(1/2)/x/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx)::;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x(a+bx)}}{x(c+dx)}\,dx = \int \frac{\sqrt{x\,(a+b\,x)}}{x\,(c+d\,x)}\,dx$$

input
```
int((x*(a + b*x))^(1/2)/(x*(c + d*x)),x)
```

output
```
int((x*(a + b*x))^(1/2)/(x*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{x(a+bx)}}{x(c+dx)}\,dx$$
$$= \frac{-2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right) - 2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}+\sqrt{d}\,\sqrt{bx+a}+\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right) + 2\sqrt{}}{cd}$$

input
```
int((x*(b*x+a))^(1/2)/x/(d*x+c),x)
```

output
```
(2*( - sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*
x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b))) - sqrt(c)*sqrt(a*d - b*c)
*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/
(sqrt(c)*sqrt(b))) + sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)
)*c))/(c*d)
```

3.54 $\int \frac{\sqrt{ax+bx^2}}{cx+dx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 87

$$\int \frac{\sqrt{ax+bx^2}}{cx+dx^2}\,dx=\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{d}-\frac{2\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{\sqrt{c}d}$$

output
```
2*b^(1/2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/d-2*(-a*d+b*c)^(1/2)*arctan
h((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{ax+bx^2}}{cx+dx^2}\,dx = -\frac{2\sqrt{x}\sqrt{a+bx}\left(\sqrt{-bc+ad}\arctan\left(\frac{-d\sqrt{x}\sqrt{a+bx}+\sqrt{b}(c+dx)}{\sqrt{c}\sqrt{-bc+ad}}\right)+\sqrt{b}\sqrt{c}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)\right)}{\sqrt{c}d\sqrt{x(a+bx)}}$$

input
```
Integrate[Sqrt[a*x + b*x^2]/(c*x + d*x^2),x]
```

output
```
(-2*Sqrt[x]*Sqrt[a + b*x]*(Sqrt[-(b*c) + a*d]*ArcTan[(-(d*Sqrt[x]*Sqrt[a +
 b*x]) + Sqrt[b]*(c + d*x))/(Sqrt[c]*Sqrt[-(b*c) + a*d])] + Sqrt[b]*Sqrt[c
]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]))/(Sqrt[c]*d*Sqrt[x*(a + b*x)])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{cx+dx^2}dx$$
$$\Big\downarrow \; 1325$$
$$\int \frac{\sqrt{ax+bx^2}}{cx+dx^2}dx$$

input
```
Int[Sqrt[a*x + b*x^2]/(c*x + d*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1325
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Unintegrable[(a + b*x + c*x^2)^p*(d + e*x + f*x^2)
^q, x] /; FreeQ[{a, b, c, d, e, f, p, q}, x] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0
]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$-\frac{2\left(-\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)+\frac{(ad-bc)\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{\sqrt{c(ad-bc)}}\right)}{d}$
default	$\frac{\sqrt{b\,x^{2}+ax}+\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2\sqrt{b}}}{c}-\frac{\sqrt{b\left(x+\frac{c}{d}\right)^{2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^{2}}}+\frac{(ad-2bc)\ln\left(\frac{\frac{ad-2bc}{2d}+b\left(x+\frac{c}{d}\right)}{\sqrt{b}}+\sqrt{b\left(x+\frac{c}{d}\right.}\right.}{2d\sqrt{b}}}{}$

input

```
int((b*x^2+a*x)^(1/2)/(d*x^2+c*x),x,method=_RETURNVERBOSE)
```

output

```
-2/d*(-b^(1/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))+(a*d-b*c)/(c*(a*d-b*c)
)^(1/2)*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 396, normalized size of antiderivative = 4.55

$$\int \frac{\sqrt{ax+bx^2}}{cx+dx^2}\,dx$$

$$= \left[\frac{\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)x-2\,\sqrt{bx^2+ax}c\sqrt{\frac{bc-ad}{c}}}{dx+c}\right)}{d},\right.$$

$$-\frac{2\,\sqrt{-\frac{bc-ad}{c}}\arctan\left(-\frac{\sqrt{bx^2+ax}c\sqrt{-\frac{bc-ad}{c}}}{(bc-ad)x}\right)-\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{d},$$

$$-\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)x-2\,\sqrt{bx^2+ax}c\sqrt{\frac{bc-ad}{c}}}{dx+c}\right)}{d},$$

$$\left.-\frac{2\left(\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+\sqrt{-\frac{bc-ad}{c}}\arctan\left(-\frac{\sqrt{bx^2+ax}c\sqrt{-\frac{bc-ad}{c}}}{(bc-ad)x}\right)\right)}{d}\right]$$

input
```
integrate((b*x^2+a*x)^(1/2)/(d*x^2+c*x),x, algorithm="fricas")
```

output
```
[(sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + sqrt((b*c - a*d)/
c)*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)*c*sqrt((b*c - a*d)/c))
/(d*x + c)))/d, -(2*sqrt(-(b*c - a*d)/c)*arctan(-sqrt(b*x^2 + a*x)*c*sqrt(
-(b*c - a*d)/c)/((b*c - a*d)*x)) - sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 +
a*x)*sqrt(b)))/d, -(2*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)
) - sqrt((b*c - a*d)/c)*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)*c
*sqrt((b*c - a*d)/c))/(d*x + c)))/d, -2*(sqrt(-b)*arctan(sqrt(b*x^2 + a*x)
*sqrt(-b)/(b*x + a)) + sqrt(-(b*c - a*d)/c)*arctan(-sqrt(b*x^2 + a*x)*c*sq
rt(-(b*c - a*d)/c)/((b*c - a*d)*x)))/d]
```

Sympy [F]

$$\int \frac{\sqrt{ax + bx^2}}{cx + dx^2}\, dx = \int \frac{\sqrt{x\,(a + bx)}}{x\,(c + dx)}\, dx$$

input
```
integrate((b*x**2+a*x)**(1/2)/(d*x**2+c*x),x)
```

output
```
Integral(sqrt(x*(a + b*x))/(x*(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax + bx^2}}{cx + dx^2}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a*x)^(1/2)/(d*x^2+c*x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax+bx^2}}{cx+dx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a*x)^(1/2)/(d*x^2+c*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^2}}{cx+dx^2}\,dx = \int \frac{\sqrt{b\,x^2+a\,x}}{d\,x^2+c\,x}\,dx$$

input
```
int((a*x + b*x^2)^(1/2)/(c*x + d*x^2),x)
```

output
```
int((a*x + b*x^2)^(1/2)/(c*x + d*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{ax+bx^2}}{cx+dx^2}\,dx = \frac{-2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)-2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}+\sqrt{d}\,\sqrt{bx+a}+\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)+2\sqrt{}}{cd}$$

input
```
int((b*x^2+a*x)^(1/2)/(d*x^2+c*x),x)
```

output

```
(2*( - sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*
x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b))) - sqrt(c)*sqrt(a*d - b*c)
*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/
(sqrt(c)*sqrt(b))) + sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)
)*c))/(c*d)
```

3.55 $\int \frac{\sqrt{x(a+bx)}}{cx+dx^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{\sqrt{x(a+bx)}}{cx+dx^2} dx = \frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{d} - \frac{2\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{\sqrt{c}d}$$

output
```
2*b^(1/2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/d-2*(-a*d+b*c)^(1/2)*arctan
h((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{x(a+bx)}}{cx+dx^2} dx = -\frac{2\sqrt{x}\sqrt{a+bx}\left(\sqrt{-bc+ad}\arctan\left(\frac{-d\sqrt{x}\sqrt{a+bx}+\sqrt{b}(c+dx)}{\sqrt{c}\sqrt{-bc+ad}}\right)+\sqrt{b}\sqrt{c}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)\right)}{\sqrt{c}d\sqrt{x(a+bx)}}$$

input
```
Integrate[Sqrt[x*(a + b*x)]/(c*x + d*x^2),x]
```

output
```
(-2*Sqrt[x]*Sqrt[a + b*x]*(Sqrt[-(b*c) + a*d]*ArcTan[(-(d*Sqrt[x]*Sqrt[a +
 b*x]) + Sqrt[b]*(c + d*x))/(Sqrt[c]*Sqrt[-(b*c) + a*d])] + Sqrt[b]*Sqrt[c
]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]))/(Sqrt[c]*d*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.68, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2026, 2048, 1260, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x(a+bx)}}{cx+dx^2}\,dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{\sqrt{x(a+bx)}}{x(c+dx)}dx$$

$$\Big\downarrow \text{2048}$$

$$\int \frac{\sqrt{ax+bx^2}}{x(c+dx)}dx$$

$$\Big\downarrow \text{1260}$$

$$\int \left(\frac{\sqrt{ax+bx^2}}{cx}-\frac{d\sqrt{ax+bx^2}}{c(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(2bc-ad)}{\sqrt{b}cd}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{\sqrt{c}d}+\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}c}$$

input
```
Int[Sqrt[x*(a + b*x)]/(c*x + d*x^2),x]
```

output
```
(a*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(Sqrt[b]*c) + ((2*b*c - a*d)*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(Sqrt[b]*c*d) - (Sqrt[b*c - a*d]*Arc
Tanh[(a*c + (2*b*c - a*d)*x)/(2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[a*x + b*x^2])
])/(Sqrt[c]*d)
```

Defintions of rubi rules used

rule 1260
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x + c*x^2)^p
, (d + e*x)^m*(f + g*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ
[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + n + 2*p + 1, 0] && ILtQ[m, 0] && ILtQ
[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$-\frac{2\left(-\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)+\frac{(ad-bc)\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{\sqrt{c(ad-bc)}}\right)}{d}$
default	$\frac{\sqrt{b\,x^2+ax}+\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2\sqrt{b}}}{c}-\frac{\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}+\frac{(ad-2bc)\ln\left(\frac{\frac{ad-2bc}{2d}+b\left(x+\frac{c}{d}\right)}{\sqrt{b}}+\sqrt{b\left(x+\frac{c}{d}\right.}\right.}{2d\sqrt{b}}}{}$

input
```
int((x*(b*x+a))^(1/2)/(d*x^2+c*x),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(-b^(1/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))+(a*d-b*c)/(c*(a*d-b*c)
)^(1/2)*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 396, normalized size of antiderivative = 4.55

$$\int \frac{\sqrt{x(a+bx)}}{cx+dx^2}\,dx = \left[\frac{\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)x-2\,\sqrt{bx^2+ax}c\sqrt{\frac{bc-ad}{c}}}{dx+c}\right)}{d},\right.$$
$$-\frac{2\sqrt{-\frac{bc-ad}{c}}\arctan\left(-\frac{\sqrt{bx^2+ax}c\sqrt{-\frac{bc-ad}{c}}}{(bc-ad)x}\right)-\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{d},$$
$$-\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)x-2\,\sqrt{bx^2+ax}c\sqrt{\frac{bc-ad}{c}}}{dx+c}\right)}{d},$$
$$\left.-\frac{2\left(\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+\sqrt{-\frac{bc-ad}{c}}\arctan\left(-\frac{\sqrt{bx^2+ax}c\sqrt{-\frac{bc-ad}{c}}}{(bc-ad)x}\right)\right)}{d}\right]$$

input
```
integrate((x*(b*x+a))^(1/2)/(d*x^2+c*x),x, algorithm="fricas")
```

output
```
[(sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + sqrt((b*c - a*d)/
c)*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)*c*sqrt((b*c - a*d)/c))
/(d*x + c)))/d, -(2*sqrt(-(b*c - a*d)/c)*arctan(-sqrt(b*x^2 + a*x)*c*sqrt(
-(b*c - a*d)/c)/((b*c - a*d)*x)) - sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 +
a*x)*sqrt(b)))/d, -(2*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)
) - sqrt((b*c - a*d)/c)*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)*c
*sqrt((b*c - a*d)/c))/(d*x + c)))/d, -2*(sqrt(-b)*arctan(sqrt(b*x^2 + a*x)
*sqrt(-b)/(b*x + a)) + sqrt(-(b*c - a*d)/c)*arctan(-sqrt(b*x^2 + a*x)*c*sq
rt(-(b*c - a*d)/c)/((b*c - a*d)*x)))/d]
```

Sympy [F]

$$\int \frac{\sqrt{x(a+bx)}}{cx+dx^2}\,dx = \int \frac{\sqrt{x\,(a+bx)}}{x\,(c+dx)}\,dx$$

input
```
integrate((x*(b*x+a))**(1/2)/(d*x**2+c*x),x)
```

output
```
Integral(sqrt(x*(a + b*x))/(x*(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{x(a+bx)}}{cx+dx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((x*(b*x+a))^(1/2)/(d*x^2+c*x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{x(a+bx)}}{cx+dx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((x*(b*x+a))^(1/2)/(d*x^2+c*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x(a+bx)}}{cx+dx^2}\,dx = \int \frac{\sqrt{x\,(a+b\,x)}}{d\,x^2+c\,x}\,dx$$

input
```
int((x*(a + b*x))^(1/2)/(c*x + d*x^2),x)
```

output
```
int((x*(a + b*x))^(1/2)/(c*x + d*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{x(a+bx)}}{cx+dx^2}\,dx$$
$$= \frac{-2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)-2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}+\sqrt{d}\,\sqrt{bx+a}+\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)+2\sqrt{}}{cd}$$

input
```
int((x*(b*x+a))^(1/2)/(d*x^2+c*x),x)
```

output
```
(2*( - sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*
x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b))) - sqrt(c)*sqrt(a*d - b*c)
*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/
(sqrt(c)*sqrt(b))) + sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)
)*c))/(c*d)
```

3.56 $\int x^2(c+dx)\left(ax+bx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 264

$$\begin{aligned}\int x^2(c+dx)\left(ax+bx^2\right)^{3/2}\,dx &= -\frac{a^5(14bc-9ad)\sqrt{ax+bx^2}}{1024b^5} \\ &+ \frac{a^4(14bc-9ad)x\sqrt{ax+bx^2}}{1536b^4} - \frac{a^3(14bc-9ad)x^2\sqrt{ax+bx^2}}{1920b^3} \\ &+ \frac{a^2(14bc-9ad)x^3\sqrt{ax+bx^2}}{2240b^2} + \frac{13a(14bc-9ad)x^4\sqrt{ax+bx^2}}{840b} \\ &+ \frac{1}{84}(14bc-9ad)x^5\sqrt{ax+bx^2} + \frac{dx^2\left(ax+bx^2\right)^{5/2}}{7b} \\ &+ \frac{a^6(14bc-9ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{1024b^{11/2}}\end{aligned}$$

output
```
-1/1024*a^5*(-9*a*d+14*b*c)*(b*x^2+a*x)^(1/2)/b^5+1/1536*a^4*(-9*a*d+14*b*
c)*x*(b*x^2+a*x)^(1/2)/b^4-1/1920*a^3*(-9*a*d+14*b*c)*x^2*(b*x^2+a*x)^(1/2
)/b^3+1/2240*a^2*(-9*a*d+14*b*c)*x^3*(b*x^2+a*x)^(1/2)/b^2+13/840*a*(-9*a*
d+14*b*c)*x^4*(b*x^2+a*x)^(1/2)/b+1/84*(-9*a*d+14*b*c)*x^5*(b*x^2+a*x)^(1/
2)+1/7*d*x^2*(b*x^2+a*x)^(5/2)/b+1/1024*a^6*(-9*a*d+14*b*c)*arctanh(b^(1/2
)*x/(b*x^2+a*x)^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.89

$$\int x^2(c+dx)\left(ax+bx^2\right)^{3/2}\,dx=\frac{(x(a+bx))^{3/2}\left(-1470a^5bc+945a^6d+980a^4b^2cx-630a^5bdx-784a^3b^3cx^2+504a^4b^2dx^2+\right.}{1075}$$

$$-\frac{a^6(-14bc+9ad)(x(a+bx))^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{512b^{11/2}x^{3/2}(a+bx)^{3/2}}$$

input
```
Integrate[x^2*(c + d*x)*(a*x + b*x^2)^(3/2),x]
```

output
```
((x*(a + b*x))^(3/2)*(-1470*a^5*b*c + 945*a^6*d + 980*a^4*b^2*c*x - 630*a^
5*b*d*x - 784*a^3*b^3*c*x^2 + 504*a^4*b^2*d*x^2 + 672*a^2*b^4*c*x^3 - 432*
a^3*b^3*d*x^3 + 23296*a*b^5*c*x^4 + 384*a^2*b^4*d*x^4 + 17920*b^6*c*x^5 +
19200*a*b^5*d*x^5 + 15360*b^6*d*x^6))/(107520*b^5*x*(a + b*x)) - (a^6*(-14
*b*c + 9*a*d)*(x*(a + b*x))^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sq
rt[a + b*x])])/(512*b^(11/2)*x^(3/2)*(a + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.75, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1221, 1134, 1160, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(ax+bx^2\right)^{3/2}(c+dx)\,dx$$

$$\Big\downarrow \text{ 1221}$$

$$\frac{(14bc-9ad)\int x^2\left(bx^2+ax\right)^{3/2}dx}{14b}+\frac{dx^2\left(ax+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \text{ 1134}$$

$$\frac{(14bc-9ad)\left(\frac{x\left(ax+bx^2\right)^{5/2}}{6b}-\frac{7a\int x\left(bx^2+ax\right)^{3/2}dx}{12b}\right)}{14b}+\frac{dx^2\left(ax+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \text{1160}$$

$$\frac{(14bc-9ad)\left(\frac{x\left(ax+bx^2\right)^{5/2}}{6b}-\frac{7a\left(\frac{\left(ax+bx^2\right)^{5/2}}{5b}-\frac{a\int\left(bx^2+ax\right)^{3/2}dx}{2b}\right)}{12b}\right)}{14b}+\frac{dx^2\left(ax+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(14bc-9ad)\left(\frac{x\left(ax+bx^2\right)^{5/2}}{6b}-\frac{7a\left(\frac{\left(ax+bx^2\right)^{5/2}}{5b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\int\sqrt{bx^2+ax}dx}{16b}\right)}{2b}\right)}{12b}\right)}{14b}+\frac{dx^2\left(ax+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(14bc-9ad)\left(\frac{x\left(ax+bx^2\right)^{5/2}}{6b}-\frac{7a\left(\frac{\left(ax+bx^2\right)^{5/2}}{5b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)}{16b}\right)}{2b}\right)}{12b}\right)}{14b}+\frac{dx^2\left(ax+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \ 1091$$

$$\frac{(14bc-9ad)\left(\frac{x\left(ax+bx^2\right)^{5/2}}{6b}-\frac{7a\left(\frac{\left(ax+bx^2\right)^{5/2}}{5b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)}{16b}\right)}{2b}\right)}{12b}\right)}{14b}+\frac{dx^2\left(ax+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \ 219$$

$$\frac{\left(\frac{x(ax+bx^2)^{5/2}}{6b}-\frac{7a\left(\frac{(ax+bx^2)^{5/2}}{5b}-\frac{a\left(\frac{(a+2bx)(ax+bx^2)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)}{16b}\right)}{2b}\right)}{12b}\right)(14bc-9ad)}{14b}+\frac{dx^2(ax+bx^2)^{5/2}}{7b}$$

input
```
Int[x^2*(c + d*x)*(a*x + b*x^2)^(3/2),x]
```

output
```
(d*x^2*(a*x + b*x^2)^(5/2))/(7*b) + ((14*b*c - 9*a*d)*((x*(a*x + b*x^2)^(5
/2))/(6*b) - (7*a*((a*x + b*x^2)^(5/2)/(5*b) - (a*(((a + 2*b*x)*(a*x + b*x
^2)^(3/2))/(8*b) - (3*a^2*(((a + 2*b*x)*Sqrt[a*x + b*x^2])/(4*b) - (a^2*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*b^(3/2))))/(16*b)))/(2*b)))/(12*b
)))/(14*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.56

method	result
pseudoelliptic	$-\frac{9\left(a^6\left(ad-\frac{14bc}{9}\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)-\left(\frac{3328x^4\left(\frac{75dx}{91}+c\right)a\,b^{\frac{11}{2}}}{135}+\frac{512\left(\frac{6dx}{7}+c\right)x^5b^{\frac{13}{2}}}{27}+a^2\left(-\frac{14\left(\frac{3dx}{7}+c\right)a^3b^{\frac{3}{2}}}{9}+\frac{28x\left(\frac{18}{3}\right.}{}\right.\right.\right.}{1024b^{\frac{11}{2}}}$
risch	$\frac{(15360b^6d\,x^6+19200a\,b^5d\,x^5+17920b^6c\,x^5+384a^2b^4d\,x^4+23296a\,b^5c\,x^4-432a^3b^3d\,x^3+672a^2b^4c\,x^3+504a^4b^2d\,x^2-784a^3}{107520b^5\sqrt{x(bx+a)}}$
default	$c\left(\frac{x(b\,x^2+ax)^{\frac{5}{2}}}{6b}-\frac{7a\left(\frac{(b\,x^2+ax)^{\frac{5}{2}}}{5b}-\frac{a\left(\frac{(2bx+a)(b\,x^2+ax)^{\frac{3}{2}}}{8b}-\frac{3a^2\left(\frac{(2bx+a)\sqrt{b\,x^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8b^{\frac{3}{2}}}\right)}{16b}\right)}{2b}\right)}{12b}\right.$

input
```
int(x^2*(d*x+c)*(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-9/1024*(a^6*(a*d-14/9*b*c)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-(3328/135
*x^4*(75/91*d*x+c)*a*b^(11/2)+512/27*(6/7*d*x+c)*x^5*b^(13/2)+a^2*(-14/9*(
3/7*d*x+c)*a^3*b^(3/2)+28/27*x*(18/35*d*x+c)*a^2*b^(5/2)-112/135*x^2*(27/4
9*d*x+c)*a*b^(7/2)+32/45*x^3*(4/7*d*x+c)*b^(9/2)+b^(1/2)*a^4*d))*(x*(b*x+a
))^(1/2))/b^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.52

$$\int x^2(c+dx)\left(ax+bx^2\right)^{3/2}\,dx=\left[-\frac{105\left(14\,a^6bc-9\,a^7d\right)\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\left(15360\,b^7dx^6-1470\,a^5b^2c\right.}{}\right.$$

$$-\frac{105\left(14\,a^6bc-9\,a^7d\right)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-\left(15360\,b^7dx^6-1470\,a^5b^2c+945\,a^6bd+1280\left(14\,b^7c+\right.\right.}{}$$

input
```
integrate(x^2*(d*x+c)*(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/215040*(105*(14*a^6*b*c - 9*a^7*d)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^
2 + a*x)*sqrt(b)) - 2*(15360*b^7*d*x^6 - 1470*a^5*b^2*c + 945*a^6*b*d + 12
80*(14*b^7*c + 15*a*b^6*d)*x^5 + 128*(182*a*b^6*c + 3*a^2*b^5*d)*x^4 + 48*
(14*a^2*b^5*c - 9*a^3*b^4*d)*x^3 - 56*(14*a^3*b^4*c - 9*a^4*b^3*d)*x^2 + 7
0*(14*a^4*b^3*c - 9*a^5*b^2*d)*x)*sqrt(b*x^2 + a*x))/b^6, -1/107520*(105*(
14*a^6*b*c - 9*a^7*d)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)
) - (15360*b^7*d*x^6 - 1470*a^5*b^2*c + 945*a^6*b*d + 1280*(14*b^7*c + 15*
a*b^6*d)*x^5 + 128*(182*a*b^6*c + 3*a^2*b^5*d)*x^4 + 48*(14*a^2*b^5*c - 9*
a^3*b^4*d)*x^3 - 56*(14*a^3*b^4*c - 9*a^4*b^3*d)*x^2 + 70*(14*a^4*b^3*c -
9*a^5*b^2*d)*x)*sqrt(b*x^2 + a*x))/b^6]
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.68

$$\int x^2(c+dx)\left(ax+bx^2\right)^{3/2}\,dx=\begin{cases}\frac{35a^4\left(a^2c-\frac{9a\left(a^2d+2abc-\frac{11a\left(\frac{15abd}{14}+b^2c\right)}{12b}\right)}{10b}\right)\left(\begin{cases}\frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0\\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{128b^4}+\sqrt{ax+bx^2}\Bigg(\\ \frac{2\left(\frac{c(ax)^{\frac{9}{2}}}{9}+\frac{d(ax)^{\frac{11}{2}}}{11a}\right)}{a^3} \\ 0\end{cases}$$

input
```
integrate(x**2*(d*x+c)*(b*x**2+a*x)**(3/2),x)
```

output
```
Piecewise((35*a**4*(a**2*c - 9*a*(a**2*d + 2*a*b*c - 11*a*(15*a*b*d/14 + b
**2*c)/(12*b))/(10*b))*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2
*b*x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(
2*b) + x)**2), True))/(128*b**4) + sqrt(a*x + b*x**2)*(-35*a**3*(a**2*c -
9*a*(a**2*d + 2*a*b*c - 11*a*(15*a*b*d/14 + b**2*c)/(12*b))/(10*b))/(64*b*
*4) + 35*a**2*x*(a**2*c - 9*a*(a**2*d + 2*a*b*c - 11*a*(15*a*b*d/14 + b**2
*c)/(12*b))/(10*b))/(96*b**3) - 7*a*x**2*(a**2*c - 9*a*(a**2*d + 2*a*b*c -
 11*a*(15*a*b*d/14 + b**2*c)/(12*b))/(10*b))/(24*b**2) + b*d*x**6/7 + x**5
*(15*a*b*d/14 + b**2*c)/(6*b) + x**4*(a**2*d + 2*a*b*c - 11*a*(15*a*b*d/14
 + b**2*c)/(12*b))/(5*b) + x**3*(a**2*c - 9*a*(a**2*d + 2*a*b*c - 11*a*(15
*a*b*d/14 + b**2*c)/(12*b))/(10*b))/(4*b)), Ne(b, 0)), (2*(c*(a*x)**(9/2)/
9 + d*(a*x)**(11/2)/(11*a))/a**3, Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.23

$$\int x^2(c+dx)\left(ax+bx^2\right)^{3/2}\,dx=\frac{(bx^2+ax)^{\frac{5}{2}}dx^2}{7\,b}-\frac{7\,\sqrt{bx^2+ax}a^4cx}{256\,b^3}+\frac{7\,(bx^2+ax)^{\frac{3}{2}}a^2cx}{96\,b^2}+\frac{(bx^2+ax)^{\frac{5}{2}}cx}{6\,b}+\frac{9\,\sqrt{bx^2+ax}a^5dx}{512\,b^4}-\frac{3\,(bx^2+ax)^{\frac{3}{2}}a^3dx}{64\,b^3}-\frac{3\,(bx^2+ax)^{\frac{5}{2}}adx}{28\,b^2}+\frac{7\,a^6c\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{1024\,b^{\frac{9}{2}}}-\frac{9\,a^7d\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{2048\,b^{\frac{11}{2}}}-\frac{7\,\sqrt{bx^2+ax}a^5c}{512\,b^4}+\frac{7\,(bx^2+ax)^{\frac{3}{2}}a^3c}{192\,b^3}-\frac{7\,(bx^2+ax)^{\frac{5}{2}}ac}{60\,b^2}+\frac{9\,\sqrt{bx^2+ax}a^6d}{1024\,b^5}-\frac{3\,(bx^2+ax)^{\frac{3}{2}}a^4d}{128\,b^4}+\frac{3\,(bx^2+ax)^{\frac{5}{2}}a^2d}{40\,b^3}$$

input
```
integrate(x^2*(d*x+c)*(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
1/7*(b*x^2 + a*x)^(5/2)*d*x^2/b - 7/256*sqrt(b*x^2 + a*x)*a^4*c*x/b^3 + 7/
96*(b*x^2 + a*x)^(3/2)*a^2*c*x/b^2 + 1/6*(b*x^2 + a*x)^(5/2)*c*x/b + 9/512
*sqrt(b*x^2 + a*x)*a^5*d*x/b^4 - 3/64*(b*x^2 + a*x)^(3/2)*a^3*d*x/b^3 - 3/
28*(b*x^2 + a*x)^(5/2)*a*d*x/b^2 + 7/1024*a^6*c*log(2*b*x + a + 2*sqrt(b*x
^2 + a*x)*sqrt(b))/b^(9/2) - 9/2048*a^7*d*log(2*b*x + a + 2*sqrt(b*x^2 + a
*x)*sqrt(b))/b^(11/2) - 7/512*sqrt(b*x^2 + a*x)*a^5*c/b^4 + 7/192*(b*x^2 +
 a*x)^(3/2)*a^3*c/b^3 - 7/60*(b*x^2 + a*x)^(5/2)*a*c/b^2 + 9/1024*sqrt(b*x
^2 + a*x)*a^6*d/b^5 - 3/128*(b*x^2 + a*x)^(3/2)*a^4*d/b^4 + 3/40*(b*x^2 +
a*x)^(5/2)*a^2*d/b^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.83

$$\int x^2(c+dx)\left(ax+bx^2\right)^{3/2}\,dx=\frac{1}{107520}\,\sqrt{bx^2+ax}\left(2\left(4\left(2\left(8\left(10\left(12\,bdx+\frac{14\,b^7c+15\,ab^6d}{b^6}\right)x+\frac{182\,ab^6c+3\,a^2b^5d}{b^6}\right)x\right.\right.\right.\right.\right.-\frac{(14\,a^6bc-9\,a^7d)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{2048\,b^{\frac{11}{2}}}$$

input
```
integrate(x^2*(d*x+c)*(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
1/107520*sqrt(b*x^2 + a*x)*(2*(4*(2*(8*(10*(12*b*d*x + (14*b^7*c + 15*a*b^
6*d)/b^6)*x + (182*a*b^6*c + 3*a^2*b^5*d)/b^6)*x + 3*(14*a^2*b^5*c - 9*a^3
*b^4*d)/b^6)*x - 7*(14*a^3*b^4*c - 9*a^4*b^3*d)/b^6)*x + 35*(14*a^4*b^3*c
- 9*a^5*b^2*d)/b^6)*x - 105*(14*a^5*b^2*c - 9*a^6*b*d)/b^6) - 1/2048*(14*a
^6*b*c - 9*a^7*d)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/
b^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(c+dx)\left(ax+bx^2\right)^{3/2}\,dx=\int x^2\left(b\,x^2+a\,x\right)^{3/2}(c+d\,x)\,dx$$

input
```
int(x^2*(a*x + b*x^2)^(3/2)*(c + d*x),x)
```

output
```
int(x^2*(a*x + b*x^2)^(3/2)*(c + d*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.12

$$\int x^2(c+dx)\left(ax+bx^2\right)^{3/2}\,dx=\frac{945\sqrt{x}\,\sqrt{bx+a}\,a^6bd-1470\sqrt{x}\,\sqrt{bx+a}\,a^5b^2c-630\sqrt{x}\,\sqrt{bx+a}\,a^5b^2dx+980\sqrt{x}\,\sqrt{bx+a}}{}$$

input
```
int(x^2*(d*x+c)*(b*x^2+a*x)^(3/2),x)
```

output
```
(945*sqrt(x)*sqrt(a + b*x)*a**6*b*d - 1470*sqrt(x)*sqrt(a + b*x)*a**5*b**2
*c - 630*sqrt(x)*sqrt(a + b*x)*a**5*b**2*d*x + 980*sqrt(x)*sqrt(a + b*x)*a
**4*b**3*c*x + 504*sqrt(x)*sqrt(a + b*x)*a**4*b**3*d*x**2 - 784*sqrt(x)*sq
rt(a + b*x)*a**3*b**4*c*x**2 - 432*sqrt(x)*sqrt(a + b*x)*a**3*b**4*d*x**3
+ 672*sqrt(x)*sqrt(a + b*x)*a**2*b**5*c*x**3 + 384*sqrt(x)*sqrt(a + b*x)*a
**2*b**5*d*x**4 + 23296*sqrt(x)*sqrt(a + b*x)*a*b**6*c*x**4 + 19200*sqrt(x
)*sqrt(a + b*x)*a*b**6*d*x**5 + 17920*sqrt(x)*sqrt(a + b*x)*b**7*c*x**5 +
15360*sqrt(x)*sqrt(a + b*x)*b**7*d*x**6 - 945*sqrt(b)*log((sqrt(a + b*x) +
 sqrt(x)*sqrt(b))/sqrt(a))*a**7*d + 1470*sqrt(b)*log((sqrt(a + b*x) + sqrt
(x)*sqrt(b))/sqrt(a))*a**6*b*c)/(107520*b**6)
```

3.57 $\int x(c+dx)\left(ax+bx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 227

$$\begin{aligned}\int x(c+dx)\left(ax+bx^2\right)^{3/2}\,dx =& \frac{a^4(12bc-7ad)\sqrt{ax+bx^2}}{512b^4}\\ &-\frac{a^3(12bc-7ad)x\sqrt{ax+bx^2}}{768b^3}+\frac{a^2(12bc-7ad)x^2\sqrt{ax+bx^2}}{960b^2}\\ &+\frac{11a(12bc-7ad)x^3\sqrt{ax+bx^2}}{480b}+\frac{1}{60}(12bc-7ad)x^4\sqrt{ax+bx^2}\\ &+\frac{dx(ax+bx^2)^{5/2}}{6b}-\frac{a^5(12bc-7ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{512b^{9/2}}\end{aligned}$$

output

```
1/512*a^4*(-7*a*d+12*b*c)*(b*x^2+a*x)^(1/2)/b^4-1/768*a^3*(-7*a*d+12*b*c)*
x*(b*x^2+a*x)^(1/2)/b^3+1/960*a^2*(-7*a*d+12*b*c)*x^2*(b*x^2+a*x)^(1/2)/b^
2+11/480*a*(-7*a*d+12*b*c)*x^3*(b*x^2+a*x)^(1/2)/b+1/60*(-7*a*d+12*b*c)*x^
4*(b*x^2+a*x)^(1/2)+1/6*d*x*(b*x^2+a*x)^(5/2)/b-1/512*a^5*(-7*a*d+12*b*c)*
arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.96

$$\int x(c+dx)\left(ax+bx^2\right)^{3/2}\,dx=\frac{\sqrt{x}\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\sqrt{a+bx}(-105a^5d+48a^2b^3x^2(2c+dx)+256b^5x^4(6c+5dx)-8a^3b^2x(\right.}{}$$

input
```
Integrate[x*(c + d*x)*(a*x + b*x^2)^(3/2),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-105*a^5*d + 48*a^2
*b^3*x^2*(2*c + d*x) + 256*b^5*x^4*(6*c + 5*d*x) - 8*a^3*b^2*x*(15*c + 7*d
*x) + 10*a^4*b*(18*c + 7*d*x) + 64*a*b^4*x^3*(33*c + 26*d*x)) + 360*a^5*b*
c*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])] + 210*a^6*d*ArcTanh
[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(7680*b^(9/2)*Sqrt[x*(a +
 b*x)])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.66, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1225, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax+bx^2\right)^{3/2}(c+dx)\,dx$$

$$\Big\downarrow \text{1225}$$

$$\frac{\left(ax+bx^2\right)^{5/2}(-7ad+12bc+10bdx)}{60b^2}-\frac{a(12bc-7ad)\int\left(bx^2+ax\right)^{3/2}dx}{24b^2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\left(ax+bx^2\right)^{5/2}(-7ad+12bc+10bdx)}{60b^2}-\frac{a(12bc-7ad)\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\int\sqrt{bx^2+ax}dx}{16b}\right)}{24b^2}$$

$$\Big\downarrow \; 1087$$

$$\frac{(ax+bx^2)^{5/2}(-7ad+12bc+10bdx)}{60b^2}-\frac{a(12bc-7ad)\left(\frac{(a+2bx)(ax+bx^2)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)}{16b}\right)}{24b^2}$$

$$\Big\downarrow \; 1091$$

$$\frac{(ax+bx^2)^{5/2}(-7ad+12bc+10bdx)}{60b^2}-\frac{a(12bc-7ad)\left(\frac{(a+2bx)(ax+bx^2)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)}{16b}\right)}{24b^2}$$

$$\Big\downarrow \; 219$$

$$\frac{(ax+bx^2)^{5/2}(-7ad+12bc+10bdx)}{60b^2}-\frac{a\left(\frac{(a+2bx)(ax+bx^2)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)}{16b}\right)(12bc-7ad)}{24b^2}$$

input

```
Int[x*(c + d*x)*(a*x + b*x^2)^(3/2),x]
```

output

```
((12*b*c - 7*a*d + 10*b*d*x)*(a*x + b*x^2)^(5/2))/(60*b^2) - (a*(12*b*c -
7*a*d)*(((a + 2*b*x)*(a*x + b*x^2)^(3/2))/(8*b) - (3*a^2*(((a + 2*b*x)*Sqr
t[a*x + b*x^2])/(4*b) - (a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*b^
(3/2))))/(16*b)))/(24*b^2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.58

method	result
pseudoelliptic	$\frac{\frac{7a^5\left(ad-\frac{12bc}{7}\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{512}-\frac{7\left(-\frac{512x^4\left(\frac{5dx}{6}+c\right)b^{\frac{11}{2}}}{35}+a\left(-\frac{12\left(\frac{7dx}{18}+c\right)a^3b^{\frac{3}{2}}}{7}+\frac{8xa^2\left(\frac{7dx}{15}+c\right)b^{\frac{5}{2}}}{7}-\frac{32x^2\left(\frac{dx}{2}+c\right)ab^{\frac{7}{2}}}{35}-\cdots\right.\right.}{512}}{b^{\frac{9}{2}}}$
risch	$-\frac{\left(-1280dx^5b^5-1664ab^4dx^4-1536b^5cx^4-48a^2b^3dx^3-2112ab^4cx^3+56a^3b^2dx^2-96a^2b^3cx^2-70a^4bdx+120a^3b^2cx+1\cdots\right.}{7680b^4\sqrt{x(bx+a)}}$
default	$c\left(\frac{\left(bx^2+ax\right)^{\frac{5}{2}}}{5b}-\frac{a\left(\frac{(2bx+a)\left(bx^2+ax\right)^{\frac{3}{2}}}{8b}-\frac{3a^2\left(\frac{(2bx+a)\sqrt{bx^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{bx^2+ax}\right)}{8b^{\frac{3}{2}}}\right)}{16b}\right)}{2b}\right)+d\left(\frac{x\left(bx^2+ax\cdots\right.}{6b}\cdots\right.$

input
```
int(x*(d*x+c)*(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
7/512/b^(9/2)*(a^5*(a*d-12/7*b*c)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-(-5
12/35*x^4*(5/6*d*x+c)*b^(11/2)+a*(-12/7*(7/18*d*x+c)*a^3*b^(3/2)+8/7*x*a^2
*(7/15*d*x+c)*b^(5/2)-32/35*x^2*(1/2*d*x+c)*a*b^(7/2)-704/35*x^3*(26/33*d*
x+c)*b^(9/2)+b^(1/2)*a^4*d))*(x*(b*x+a))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.54

$$\int x(c+dx)\left(ax+bx^2\right)^{3/2}\,dx = \left[-\frac{15\left(12\,a^5bc-7\,a^6d\right)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\left(1280\,b^6dx^5+180\,a^4b^2c-1\right.}{}\right.$$

input
```
integrate(x*(d*x+c)*(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/15360*(15*(12*a^5*b*c - 7*a^6*d)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2
+ a*x)*sqrt(b)) - 2*(1280*b^6*d*x^5 + 180*a^4*b^2*c - 105*a^5*b*d + 128*(1
2*b^6*c + 13*a*b^5*d)*x^4 + 48*(44*a*b^5*c + a^2*b^4*d)*x^3 + 8*(12*a^2*b^
4*c - 7*a^3*b^3*d)*x^2 - 10*(12*a^3*b^3*c - 7*a^4*b^2*d)*x)*sqrt(b*x^2 + a
*x))/b^5, 1/7680*(15*(12*a^5*b*c - 7*a^6*d)*sqrt(-b)*arctan(sqrt(b*x^2 + a
*x)*sqrt(-b)/(b*x + a)) + (1280*b^6*d*x^5 + 180*a^4*b^2*c - 105*a^5*b*d +
128*(12*b^6*c + 13*a*b^5*d)*x^4 + 48*(44*a*b^5*c + a^2*b^4*d)*x^3 + 8*(12*
a^2*b^4*c - 7*a^3*b^3*d)*x^2 - 10*(12*a^3*b^3*c - 7*a^4*b^2*d)*x)*sqrt(b*x
^2 + a*x))/b^5]
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.70

$$\int x(c+dx)\left(ax+bx^2\right)^{3/2}\,dx = \begin{cases} -\frac{5a^3\left(a^2c-\frac{7a\left(a^2d+2abc-\frac{9a\left(\frac{13abd}{12}+b^2c\right)}{10b}\right)}{8b}\right)\left(\begin{cases}\frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ \frac{a^2}{b}\neq 0\\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{16b^3}+\sqrt{ax+bx^2}\cdot \\ \frac{2\left(\frac{c(ax)^{\frac{7}{2}}}{7}+\frac{d(ax)^{\frac{9}{2}}}{9a}\right)}{a^2} \\ 0 \end{cases}$$

input
```
integrate(x*(d*x+c)*(b*x**2+a*x)**(3/2),x)
```

output
```
Piecewise((-5*a**3*(a**2*c - 7*a*(a**2*d + 2*a*b*c - 9*a*(13*a*b*d/12 + b*
*2*c)/(10*b))/(8*b))*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b
*x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*
b) + x)**2), True))/(16*b**3) + sqrt(a*x + b*x**2)*(5*a**2*(a**2*c - 7*a*(
a**2*d + 2*a*b*c - 9*a*(13*a*b*d/12 + b**2*c)/(10*b))/(8*b))/(8*b**3) - 5*
a*x*(a**2*c - 7*a*(a**2*d + 2*a*b*c - 9*a*(13*a*b*d/12 + b**2*c)/(10*b))/(
8*b))/(12*b**2) + b*d*x**5/6 + x**4*(13*a*b*d/12 + b**2*c)/(5*b) + x**3*(a
**2*d + 2*a*b*c - 9*a*(13*a*b*d/12 + b**2*c)/(10*b))/(4*b) + x**2*(a**2*c
- 7*a*(a**2*d + 2*a*b*c - 9*a*(13*a*b*d/12 + b**2*c)/(10*b))/(8*b))/(3*b))
, Ne(b, 0)), (2*(c*(a*x)**(7/2)/7 + d*(a*x)**(9/2)/(9*a))/a**2, Ne(a, 0)),
 (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.23

$$\int x(c+dx)\left(ax+bx^2\right)^{3/2}\,dx = \frac{3\sqrt{bx^2+ax}a^3cx}{64\,b^2} - \frac{(bx^2+ax)^{\frac{3}{2}}acx}{8\,b} - \frac{7\sqrt{bx^2+ax}a^4dx}{256\,b^3} + \frac{7\,(bx^2+ax)^{\frac{3}{2}}a^2dx}{96\,b^2} + \frac{(bx^2+ax)^{\frac{5}{2}}dx}{6\,b} - \frac{3\,a^5c\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{256\,b^{\frac{7}{2}}} + \frac{7\,a^6d\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{1024\,b^{\frac{9}{2}}} + \frac{3\sqrt{bx^2+ax}a^4c}{128\,b^3} - \frac{(bx^2+ax)^{\frac{3}{2}}a^2c}{16\,b^2} + \frac{(bx^2+ax)^{\frac{5}{2}}c}{5\,b} - \frac{7\sqrt{bx^2+ax}a^5d}{512\,b^4} + \frac{7\,(bx^2+ax)^{\frac{3}{2}}a^3d}{192\,b^3} - \frac{7\,(bx^2+ax)^{\frac{5}{2}}ad}{60\,b^2}$$

input
```
integrate(x*(d*x+c)*(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
3/64*sqrt(b*x^2 + a*x)*a^3*c*x/b^2 - 1/8*(b*x^2 + a*x)^(3/2)*a*c*x/b - 7/2
56*sqrt(b*x^2 + a*x)*a^4*d*x/b^3 + 7/96*(b*x^2 + a*x)^(3/2)*a^2*d*x/b^2 +
1/6*(b*x^2 + a*x)^(5/2)*d*x/b - 3/256*a^5*c*log(2*b*x + a + 2*sqrt(b*x^2 +
 a*x)*sqrt(b))/b^(7/2) + 7/1024*a^6*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*
sqrt(b))/b^(9/2) + 3/128*sqrt(b*x^2 + a*x)*a^4*c/b^3 - 1/16*(b*x^2 + a*x)^
(3/2)*a^2*c/b^2 + 1/5*(b*x^2 + a*x)^(5/2)*c/b - 7/512*sqrt(b*x^2 + a*x)*a^
5*d/b^4 + 7/192*(b*x^2 + a*x)^(3/2)*a^3*d/b^3 - 7/60*(b*x^2 + a*x)^(5/2)*a
*d/b^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.84

$$\int x(c+dx)\left(ax+bx^2\right)^{3/2}\,dx=\frac{1}{7680}\sqrt{bx^2+ax}\left(2\left(4\left(2\left(8\left(10\,bdx+\frac{12\,b^6c+13\,ab^5d}{b^5}\right)x+\frac{3\,(44\,ab^5c+a^2b^4d)}{b^5}\right)x+12\right.\right.\right.\right.$$
$$+\frac{(12\,a^5bc-7\,a^6d)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{1024\,b^{\frac{9}{2}}}$$

input
```
integrate(x*(d*x+c)*(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
1/7680*sqrt(b*x^2 + a*x)*(2*(4*(2*(8*(10*b*d*x + (12*b^6*c + 13*a*b^5*d)/b
^5)*x + 3*(44*a*b^5*c + a^2*b^4*d)/b^5)*x + (12*a^2*b^4*c - 7*a^3*b^3*d)/b
^5)*x - 5*(12*a^3*b^3*c - 7*a^4*b^2*d)/b^5)*x + 15*(12*a^4*b^2*c - 7*a^5*b
*d)/b^5) + 1/1024*(12*a^5*b*c - 7*a^6*d)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2
 + a*x))*sqrt(b) + a))/b^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int x(c+dx)\left(ax+bx^2\right)^{3/2}\,dx=\int x\,{\left(b\,x^2+a\,x\right)}^{3/2}\,\left(c+d\,x\right)\,dx$$

input
```
int(x*(a*x + b*x^2)^(3/2)*(c + d*x),x)
```

output
```
int(x*(a*x + b*x^2)^(3/2)*(c + d*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.13

$$\int x(c+dx)\left(ax+bx^2\right)^{3/2}\,dx=\frac{-105\sqrt{x}\,\sqrt{bx+a}\,a^5bd+180\sqrt{x}\,\sqrt{bx+a}\,a^4b^2c+70\sqrt{x}\,\sqrt{bx+a}\,a^4b^2dx-120\sqrt{x}\,\sqrt{bx+a}\,a}{}$$

input
```
int(x*(d*x+c)*(b*x^2+a*x)^(3/2),x)
```

output
```
( - 105*sqrt(x)*sqrt(a + b*x)*a**5*b*d + 180*sqrt(x)*sqrt(a + b*x)*a**4*b*
*2*c + 70*sqrt(x)*sqrt(a + b*x)*a**4*b**2*d*x - 120*sqrt(x)*sqrt(a + b*x)*
a**3*b**3*c*x - 56*sqrt(x)*sqrt(a + b*x)*a**3*b**3*d*x**2 + 96*sqrt(x)*sqr
t(a + b*x)*a**2*b**4*c*x**2 + 48*sqrt(x)*sqrt(a + b*x)*a**2*b**4*d*x**3 +
2112*sqrt(x)*sqrt(a + b*x)*a*b**5*c*x**3 + 1664*sqrt(x)*sqrt(a + b*x)*a*b*
*5*d*x**4 + 1536*sqrt(x)*sqrt(a + b*x)*b**6*c*x**4 + 1280*sqrt(x)*sqrt(a +
 b*x)*b**6*d*x**5 + 105*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt
(a))*a**6*d - 180*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a
**5*b*c)/(7680*b**5)
```

3.58 $\int (c+dx)\left(ax+bx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 191

$$\int (c+dx)\left(ax+bx^2\right)^{3/2}\,dx = -\frac{3a^3(2bc-ad)\sqrt{ax+bx^2}}{128b^3} + \frac{a^2(2bc-ad)x\sqrt{ax+bx^2}}{64b^2} + \frac{3a(2bc-ad)x^2\sqrt{ax+bx^2}}{16b} + \frac{1}{8}(2bc-ad)x^3\sqrt{ax+bx^2} + \frac{d\left(ax+bx^2\right)^{5/2}}{5b} + \frac{3a^4(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{128b^{7/2}}$$

output

```
-3/128*a^3*(-a*d+2*b*c)*(b*x^2+a*x)^(1/2)/b^3+1/64*a^2*(-a*d+2*b*c)*x*(b*x
^2+a*x)^(1/2)/b^2+3/16*a*(-a*d+2*b*c)*x^2*(b*x^2+a*x)^(1/2)/b+1/8*(-a*d+2*
b*c)*x^3*(b*x^2+a*x)^(1/2)+1/5*d*(b*x^2+a*x)^(5/2)/b+3/128*a^4*(-a*d+2*b*c
)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.87

$$\int (c+dx)\left(ax+bx^2\right)^{3/2}\,dx=\frac{(x(a+bx))^{3/2}\left(\frac{\sqrt{b}\sqrt{x}(15a^4d-10a^3b(3c+dx)+4a^2b^2x(5c+2dx)+32b^4x^3(5c+4dx)+16ab^3x^2(15c+11dx))}{a+bx}+\frac{30a^4(-2b}{}\right.}{640b^{7/2}x^{3/2}}$$

input
```
Integrate[(c + d*x)*(a*x + b*x^2)^(3/2),x]
```

output
```
((x*(a + b*x))^(3/2)*((Sqrt[b]*Sqrt[x]*(15*a^4*d - 10*a^3*b*(3*c + d*x) +
4*a^2*b^2*x*(5*c + 2*d*x) + 32*b^4*x^3*(5*c + 4*d*x) + 16*a*b^3*x^2*(15*c
+ 11*d*x)))/(a + b*x) + (30*a^4*(-2*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[x])/(
Sqrt[a] - Sqrt[a + b*x])])/(a + b*x)^(3/2)))/(640*b^(7/2)*x^(3/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.71, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1160, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^2\right)^{3/2}(c+dx)\,dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{(2bc-ad)\int \left(bx^2+ax\right)^{3/2}dx}{2b}+\frac{d\left(ax+bx^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(2bc-ad)\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\int \sqrt{bx^2+ax}dx}{16b}\right)}{2b}+\frac{d\left(ax+bx^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(2bc-ad)\left(\frac{(a+2bx)(ax+bx^2)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)}{16b}\right)}{2b}+\frac{d(ax+bx^2)^{5/2}}{5b}$$

$$\Big\downarrow \; 1091$$

$$\frac{(2bc-ad)\left(\frac{(a+2bx)(ax+bx^2)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)}{16b}\right)}{2b}+\frac{d(ax+bx^2)^{5/2}}{5b}$$

$$\Big\downarrow \; 219$$

$$\frac{\left(\frac{(a+2bx)(ax+bx^2)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)}{16b}\right)(2bc-ad)}{2b}+\frac{d(ax+bx^2)^{5/2}}{5b}$$

input
```
Int[(c + d*x)*(a*x + b*x^2)^(3/2),x]
```

output
```
(d*(a*x + b*x^2)^(5/2))/(5*b) + ((2*b*c - a*d)*(((a + 2*b*x)*(a*x + b*x^2)
^(3/2))/(8*b) - (3*a^2*(((a + 2*b*x)*Sqrt[a*x + b*x^2])/(4*b) - (a^2*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*b^(3/2))))/(16*b)))/(2*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$-\frac{3\left(\left(a^{5} d-2 a^{4} b c\right) \operatorname{arctanh}\left(\frac{\sqrt{x(b x+a)}}{x \sqrt{b}}\right)-\sqrt{x(b x+a)}\left(-2\left(\frac{d x}{3}+c\right) a^{3} b^{\frac{3}{2}}+\frac{4 x\left(\frac{2 d x}{5}+c\right) a^{2} b^{\frac{5}{2}}}{3}+16 x^{2} a\left(\frac{11 d x}{15}+c\right) b^{\frac{7}{2}}+\frac{32 x^{3}\left(\right.}{}\right.\right.}{128 b^{\frac{7}{2}}}$
risch	$\frac{\left(128 d x^{4} b^{4}+176 a b^{3} d x^{3}+160 b^{4} c x^{3}+8 a^{2} b^{2} d x^{2}+240 a b^{3} c x^{2}-10 a^{3} b d x+20 a^{2} b^{2} c x+15 a^{4} d-30 a^{3} b c\right) x(b x+a)}{640 b^{3} \sqrt{x(b x+a)}}-\frac{3 a^{4}(a d-}{}$
default	$c\left(\frac{(2 b x+a)\left(b x^{2}+a x\right)^{\frac{3}{2}}}{8 b}-\frac{3 a^{2}\left(\frac{(2 b x+a) \sqrt{b x^{2}+a x}}{4 b}-\frac{a^{2} \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b x^{2}+a x}\right)}{8 b^{\frac{3}{2}}}\right)}{16 b}\right)+d\left(\frac{\left(b x^{2}+a x\right)^{\frac{5}{2}}}{5 b}-\frac{a\left(\frac{(2 b x+a)}{}\right.}{}\right.$

input
```
int((d*x+c)*(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-3/128/b^(7/2)*((a^5*d-2*a^4*b*c)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-(x*
(b*x+a))^(1/2)*(-2*(1/3*d*x+c)*a^3*b^(3/2)+4/3*x*(2/5*d*x+c)*a^2*b^(5/2)+1
6*x^2*a*(11/15*d*x+c)*b^(7/2)+32/3*x^3*(4/5*d*x+c)*b^(9/2)+b^(1/2)*a^4*d))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.59

$$\int (c+dx)\left(ax+bx^2\right)^{3/2} dx = \left[-\frac{15\left(2 a^{4} b c-a^{5} d\right) \sqrt{b} \log \left(2 b x+a-2 \sqrt{b x^{2}+a x} \sqrt{b}\right)-2\left(128 b^{5} d x^{4}-30 a^{3} b^{2} c+15 a^{4} b\right.}{1280}\right.$$

$$-\frac{15\left(2 a^{4} b c-a^{5} d\right) \sqrt{-b} \arctan \left(\frac{\sqrt{b x^{2}+a x} \sqrt{-b}}{b x+a}\right)-\left(128 b^{5} d x^{4}-30 a^{3} b^{2} c+15 a^{4} b d+16\left(10 b^{5} c+11 a b^{4} d\right) x^{3}\right.}{640 b^{4}}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/1280*(15*(2*a^4*b*c - a^5*d)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*
x)*sqrt(b)) - 2*(128*b^5*d*x^4 - 30*a^3*b^2*c + 15*a^4*b*d + 16*(10*b^5*c
+ 11*a*b^4*d)*x^3 + 8*(30*a*b^4*c + a^2*b^3*d)*x^2 + 10*(2*a^2*b^3*c - a^3
*b^2*d)*x)*sqrt(b*x^2 + a*x))/b^4, -1/640*(15*(2*a^4*b*c - a^5*d)*sqrt(-b)
*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (128*b^5*d*x^4 - 30*a^3*b^
2*c + 15*a^4*b*d + 16*(10*b^5*c + 11*a*b^4*d)*x^3 + 8*(30*a*b^4*c + a^2*b^
3*d)*x^2 + 10*(2*a^2*b^3*c - a^3*b^2*d)*x)*sqrt(b*x^2 + a*x))/b^4]
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.71

$$\int (c+dx)\left(ax+bx^2\right)^{3/2}\,dx=\begin{cases}\frac{3a^2\left(a^2c-\frac{5a\left(a^2d+2abc-\frac{7a\left(\frac{11abd}{10}+b^2c\right)}{8b}\right)}{6b}\right)\left(\begin{cases}\frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0\\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{8b^2}+\sqrt{ax+bx^2}\left(-\right.\\ \frac{2\left(\frac{c(ax)^{\frac{5}{2}}}{5}+\frac{d(ax)^{\frac{7}{2}}}{7a}\right)}{a}\\ 0\end{cases}$$

input
```
integrate((d*x+c)*(b*x**2+a*x)**(3/2),x)
```

output
```
Piecewise((3*a**2*(a**2*c - 5*a*(a**2*d + 2*a*b*c - 7*a*(11*a*b*d/10 + b**
2*c)/(8*b))/(6*b))*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x
)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b)
 + x)**2), True))/(8*b**2) + sqrt(a*x + b*x**2)*(-3*a*(a**2*c - 5*a*(a**2*
d + 2*a*b*c - 7*a*(11*a*b*d/10 + b**2*c)/(8*b))/(6*b))/(4*b**2) + b*d*x**4
/5 + x**3*(11*a*b*d/10 + b**2*c)/(4*b) + x**2*(a**2*d + 2*a*b*c - 7*a*(11*
a*b*d/10 + b**2*c)/(8*b))/(3*b) + x*(a**2*c - 5*a*(a**2*d + 2*a*b*c - 7*a*
(11*a*b*d/10 + b**2*c)/(8*b))/(6*b))/(2*b)), Ne(b, 0)), (2*(c*(a*x)**(5/2)
/5 + d*(a*x)**(7/2)/(7*a))/a, Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.24

$$\int (c+dx)\left(ax+bx^2\right)^{3/2}\,dx=\frac{1}{4}\left(bx^2+ax\right)^{\frac{3}{2}}cx-\frac{3\sqrt{bx^2+ax}a^2cx}{32\,b}+\frac{3\sqrt{bx^2+ax}a^3dx}{64\,b^2}-\frac{\left(bx^2+ax\right)^{\frac{3}{2}}adx}{8\,b}+\frac{3\,a^4c\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{5}{2}}}-\frac{3\,a^5d\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{256\,b^{\frac{7}{2}}}-\frac{3\sqrt{bx^2+ax}a^3c}{64\,b^2}+\frac{\left(bx^2+ax\right)^{\frac{3}{2}}ac}{8\,b}+\frac{3\sqrt{bx^2+ax}a^4d}{128\,b^3}-\frac{\left(bx^2+ax\right)^{\frac{3}{2}}a^2d}{16\,b^2}+\frac{\left(bx^2+ax\right)^{\frac{5}{2}}d}{5\,b}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a*x)^(3/2)*c*x - 3/32*sqrt(b*x^2 + a*x)*a^2*c*x/b + 3/64*sqrt
(b*x^2 + a*x)*a^3*d*x/b^2 - 1/8*(b*x^2 + a*x)^(3/2)*a*d*x/b + 3/128*a^4*c*
log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5/2) - 3/256*a^5*d*log(2*b
*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(7/2) - 3/64*sqrt(b*x^2 + a*x)*a^3
*c/b^2 + 1/8*(b*x^2 + a*x)^(3/2)*a*c/b + 3/128*sqrt(b*x^2 + a*x)*a^4*d/b^3
 - 1/16*(b*x^2 + a*x)^(3/2)*a^2*d/b^2 + 1/5*(b*x^2 + a*x)^(5/2)*d/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.85

$$\int (c+dx)\left(ax+bx^2\right)^{3/2}\,dx=\frac{1}{640}\sqrt{bx^2+ax}\left(2\left(4\left(2\left(8\,bdx+\frac{10\,b^5c+11\,ab^4d}{b^4}\right)x+\frac{30\,ab^4c+a^2b^3d}{b^4}\right)x+\frac{5\,(2\,a^2b^3c-}{b^4}\right.\right.$$
$$-\frac{3\,(2\,a^4bc-a^5d)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{256\,b^{\frac{7}{2}}}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
1/640*sqrt(b*x^2 + a*x)*(2*(4*(2*(8*b*d*x + (10*b^5*c + 11*a*b^4*d)/b^4)*x
 + (30*a*b^4*c + a^2*b^3*d)/b^4)*x + 5*(2*a^2*b^3*c - a^3*b^2*d)/b^4)*x -
15*(2*a^3*b^2*c - a^4*b*d)/b^4) - 3/256*(2*a^4*b*c - a^5*d)*log(abs(2*(sqr
t(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(7/2)
```

Mupad [B] (verification not implemented)

Time = 9.71 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.09

$$\int (c+dx)\left(ax+bx^2\right)^{3/2}\,dx=\frac{d\,{\left(b\,x^2+a\,x\right)}^{5/2}}{5\,b}-\frac{3\,a^2\,c\,\left(\sqrt{b\,x^2+a\,x}\,\left(\frac{x}{2}+\frac{a}{4\,b}\right)-\frac{a^2\,\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{8\,b^{3/2}}\right)}{16\,b}-\frac{a\,d\,\left(\frac{x\,{\left(b\,x^2+a\,x\right)}^{3/2}}{4}+\frac{a\,{\left(b\,x^2+a\,x\right)}^{3/2}}{8\,b}-\frac{3\,a^2\,\left(\frac{\sqrt{b\,x^2+a\,x}\,\left(a+2\,b\,x\right)}{4\,b}-\frac{a^2\,\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{8\,b^{3/2}}\right)}{16\,b}\right)}{2\,b}+\frac{c\,{\left(b\,x^2+a\,x\right)}^{3/2}\,\left(\frac{a}{2}+b\,x\right)}{4\,b}$$

input
```
int((a*x + b*x^2)^(3/2)*(c + d*x),x)
```

output
```
(d*(a*x + b*x^2)^(5/2))/(5*b) - (3*a^2*c*((a*x + b*x^2)^(1/2)*(x/2 + a/(4*
b)) - (a^2*log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2)))/(8*b^(3/2))))/(
16*b) - (a*d*((x*(a*x + b*x^2)^(3/2))/4 + (a*(a*x + b*x^2)^(3/2))/(8*b) -
(3*a^2*(((a*x + b*x^2)^(1/2)*(a + 2*b*x))/(4*b) - (a^2*log((a/2 + b*x)/b^(
1/2) + (a*x + b*x^2)^(1/2)))/(8*b^(3/2))))/(16*b)))/(2*b) + (c*(a*x + b*x^
2)^(3/2)*(a/2 + b*x))/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.14

$$\int (c+dx)\left(ax+bx^2\right)^{3/2}\,dx = \frac{15\sqrt{x}\,\sqrt{bx+a}\,a^4bd-30\sqrt{x}\,\sqrt{bx+a}\,a^3b^2c-10\sqrt{x}\,\sqrt{bx+a}\,a^3b^2dx+20\sqrt{x}\,\sqrt{bx+a}\,a^2b^3ca}{}$$

input
```
int((d*x+c)*(b*x^2+a*x)^(3/2),x)
```

output
```
(15*sqrt(x)*sqrt(a + b*x)*a**4*b*d - 30*sqrt(x)*sqrt(a + b*x)*a**3*b**2*c
- 10*sqrt(x)*sqrt(a + b*x)*a**3*b**2*d*x + 20*sqrt(x)*sqrt(a + b*x)*a**2*b
**3*c*x + 8*sqrt(x)*sqrt(a + b*x)*a**2*b**3*d*x**2 + 240*sqrt(x)*sqrt(a +
b*x)*a*b**4*c*x**2 + 176*sqrt(x)*sqrt(a + b*x)*a*b**4*d*x**3 + 160*sqrt(x)
*sqrt(a + b*x)*b**5*c*x**3 + 128*sqrt(x)*sqrt(a + b*x)*b**5*d*x**4 - 15*sq
rt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**5*d + 30*sqrt(b)*l
og((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4*b*c)/(640*b**4)
```

3.59 $\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x} dx$

Optimal result

Integrand size = 22, antiderivative size = 159

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x} dx = \frac{a^2(8bc-3ad)\sqrt{ax+bx^2}}{64b^2} + \frac{7a(8bc-3ad)x\sqrt{ax+bx^2}}{96b} + \frac{1}{24}(8bc-3ad)x^2\sqrt{ax+bx^2} + \frac{d\left(ax+bx^2\right)^{5/2}}{4bx} - \frac{a^3(8bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{64b^{5/2}}$$

output
```
1/64*a^2*(-3*a*d+8*b*c)*(b*x^2+a*x)^(1/2)/b^2+7/96*a*(-3*a*d+8*b*c)*x*(b*x
^2+a*x)^(1/2)/b+1/24*(-3*a*d+8*b*c)*x^2*(b*x^2+a*x)^(1/2)+1/4*d*(b*x^2+a*x
)^(5/2)/b/x-1/64*a^3*(-3*a*d+8*b*c)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b
^(5/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.14

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x}\,dx = \frac{\sqrt{x}\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\sqrt{a+bx}(-9a^3d+6a^2b(4c+dx)+16b^3x^2(4c+3dx)+\right.}{192b^{5/2}}$$

input
```
Integrate[((c + d*x)*(a*x + b*x^2)^(3/2))/x,x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-9*a^3*d + 6*a^2*b*
(4*c + d*x) + 16*b^3*x^2*(4*c + 3*d*x) + 8*a*b^2*x*(14*c + 9*d*x)) + 48*a^
3*b*c*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])] + 18*a^4*d*ArcT
anh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(192*b^(5/2)*Sqrt[x*(a
 + b*x)])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1221, 1131, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{3/2}(c+dx)}{x}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{(8bc-3ad)\int \frac{\left(bx^2+ax\right)^{3/2}}{x}dx}{8b}+\frac{d\left(ax+bx^2\right)^{5/2}}{4bx}$$

$$\Big\downarrow \text{1131}$$

$$\frac{(8bc-3ad)\left(\frac{1}{2}a\int \sqrt{bx^2+ax}dx+\frac{1}{3}\left(ax+bx^2\right)^{3/2}\right)}{8b}+\frac{d\left(ax+bx^2\right)^{5/2}}{4bx}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(8bc-3ad)\left(\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)+\frac{1}{3}\left(ax+bx^2\right)^{3/2}\right)}{8b}+\frac{d\left(ax+bx^2\right)^{5/2}}{4bx}$$

$$\Big\downarrow \text{ 1091}$$

$$\frac{(8bc-3ad)\left(\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)+\frac{1}{3}\left(ax+bx^2\right)^{3/2}\right)}{8b}+\frac{d\left(ax+bx^2\right)^{5/2}}{4bx}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\left(\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)+\frac{1}{3}\left(ax+bx^2\right)^{3/2}\right)(8bc-3ad)}{8b}+\frac{d\left(ax+bx^2\right)^{5/2}}{4bx}$$

input
```
Int[((c + d*x)*(a*x + b*x^2)^(3/2))/x,x]
```

output
```
(d*(a*x + b*x^2)^(5/2))/(4*b*x) + ((8*b*c - 3*a*d)*((a*x + b*x^2)^(3/2)/3
+ (a*(((a + 2*b*x)*Sqrt[a*x + b*x^2])/(4*b) - (a^2*ArcTanh[(Sqrt[b]*x)/Sqr
t[a*x + b*x^2]])/(4*b^(3/2))))/2))/(8*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$\frac{\frac{3 a^{3} \left(a d-\frac{8 b c}{3}\right) \operatorname{arctanh}\left(\frac{\sqrt{x \left(b x+a\right)}}{x \sqrt{b}}\right)}{64}-\frac{3 \sqrt{x \left(b x+a\right)}\, \left(-\frac{8 \left(\frac{d x}{4}+c\right) a^{2} b^{\frac{3}{2}}}{3}-\frac{112 x \left(\frac{9 d x}{14}+c\right) a\, b^{\frac{5}{2}}}{9}-\frac{64 x^{2} \left(\frac{3 d x}{4}+c\right) b^{\frac{7}{2}}}{9}+\sqrt{b}\, a^{3} d\right)}{64}}{b^{\frac{5}{2}}}$
risch	$-\frac{\left(-48 b^{3} d\,x^{3}-72 a\,b^{2} d\,x^{2}-64 b^{3} c\,x^{2}-6 a^{2} b d x-112 a\,b^{2} c x+9 a^{3} d-24 c\,a^{2} b\right) x \left(b x+a\right)}{192 b^{2} \sqrt{x \left(b x+a\right)}}+\frac{a^{3} \left(3 a d-8 b c\right) \ln\left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\,x^{2}+a x}\right.}{128 b^{\frac{5}{2}}}$
default	$d \left(\frac{\left(2 b x+a\right) \left(b\,x^{2}+a x\right)^{\frac{3}{2}}}{8 b}-\frac{3 a^{2} \left(\frac{\left(2 b x+a\right) \sqrt{b\,x^{2}+a x}}{4 b}-\frac{a^{2} \ln\left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\,x^{2}+a x}\right)}{8 b^{\frac{3}{2}}}\right)}{16 b}\right)+c \left(\frac{\left(b\,x^{2}+a x\right)^{\frac{3}{2}}}{3}+\frac{a \left(\frac{\left(2 b x+a\right)}{}\right.}{}\right.$

input
```
int((d*x+c)*(b*x^2+a*x)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
3/64/b^(5/2)*(a^3*(a*d-8/3*b*c)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-(x*(b
*x+a))^(1/2)*(-8/3*(1/4*d*x+c)*a^2*b^(3/2)-112/9*x*(9/14*d*x+c)*a*b^(5/2)-
64/9*x^2*(3/4*d*x+c)*b^(7/2)+b^(1/2)*a^3*d))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.61

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x}\,dx = \left[-\frac{3\left(8\,a^3bc-3\,a^4d\right)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\left(48\,b^4dx^3+24\right.}{38}\right.$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(3/2)/x,x, algorithm="fricas")
```

output
```
[-1/384*(3*(8*a^3*b*c - 3*a^4*d)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*
x)*sqrt(b)) - 2*(48*b^4*d*x^3 + 24*a^2*b^2*c - 9*a^3*b*d + 8*(8*b^4*c + 9*
a*b^3*d)*x^2 + 2*(56*a*b^3*c + 3*a^2*b^2*d)*x)*sqrt(b*x^2 + a*x))/b^3, 1/1
92*(3*(8*a^3*b*c - 3*a^4*d)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*
x + a)) + (48*b^4*d*x^3 + 24*a^2*b^2*c - 9*a^3*b*d + 8*(8*b^4*c + 9*a*b^3*
d)*x^2 + 2*(56*a*b^3*c + 3*a^2*b^2*d)*x)*sqrt(b*x^2 + a*x))/b^3]
```

Sympy [A] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 490, normalized size of antiderivative = 3.08

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x**2+a*x)**(3/2)/x,x)
```

output
```
a*c*Piecewise((-a**2*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b
*x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*
b) + x)**2), True))/(8*b) + (a/(4*b) + x/2)*sqrt(a*x + b*x**2), Ne(b, 0)),
 (2*(a*x)**(3/2)/(3*a), Ne(a, 0)), (0, True)) + a*d*Piecewise((a**3*Piecew
ise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(a**2/b, 0))
, ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2), True))/(16*b**
2) + sqrt(a*x + b*x**2)*(-a**2/(8*b**2) + a*x/(12*b) + x**2/3), Ne(b, 0)),
 (2*(a*x)**(5/2)/(5*a**2), Ne(a, 0)), (0, True)) + b*c*Piecewise((a**3*Pie
cewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(a**2/b,
0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2), True))/(16*
b**2) + sqrt(a*x + b*x**2)*(-a**2/(8*b**2) + a*x/(12*b) + x**2/3), Ne(b, 0
)), (2*(a*x)**(5/2)/(5*a**2), Ne(a, 0)), (0, True)) + b*d*Piecewise((-5*a*
*4*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(a*
*2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2), True)
)/(128*b**3) + sqrt(a*x + b*x**2)*(5*a**3/(64*b**3) - 5*a**2*x/(96*b**2) +
 a*x**2/(24*b) + x**3/4), Ne(b, 0)), (2*(a*x)**(7/2)/(7*a**3), Ne(a, 0)),
(0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.19

$$
\begin{aligned}
\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x}\,dx &= \frac{1}{4}\sqrt{bx^2+ax}acx + \frac{1}{4}\left(bx^2+ax\right)^{\frac{3}{2}}dx \\
&- \frac{3\sqrt{bx^2+ax}a^2dx}{32\,b} - \frac{a^3c\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{3}{2}}} \\
&+ \frac{3\,a^4d\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{5}{2}}} + \frac{1}{3}\left(bx^2+ax\right)^{\frac{3}{2}}c \\
&+ \frac{\sqrt{bx^2+ax}a^2c}{8\,b} - \frac{3\sqrt{bx^2+ax}a^3d}{64\,b^2} + \frac{\left(bx^2+ax\right)^{\frac{3}{2}}ad}{8\,b}
\end{aligned}
$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(3/2)/x,x, algorithm="maxima")
```

output
```
1/4*sqrt(b*x^2 + a*x)*a*c*x + 1/4*(b*x^2 + a*x)^(3/2)*d*x - 3/32*sqrt(b*x^
2 + a*x)*a^2*d*x/b - 1/16*a^3*c*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b
))/b^(3/2) + 3/128*a^4*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5
/2) + 1/3*(b*x^2 + a*x)^(3/2)*c + 1/8*sqrt(b*x^2 + a*x)*a^2*c/b - 3/64*sqr
t(b*x^2 + a*x)*a^3*d/b^2 + 1/8*(b*x^2 + a*x)^(3/2)*a*d/b
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x}\,dx = \frac{1}{192}\sqrt{bx^2+ax}\left(2\left(4\left(6\,bdx+\frac{8\,b^4c+9\,ab^3d}{b^3}\right)x+\frac{56\,ab^3c+3\,a^2b^2d}{b^3}\right)x+ \right. \\ + \frac{\left(8\,a^3bc-3\,a^4d\right)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{128\,b^{\frac{5}{2}}}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(3/2)/x,x, algorithm="giac")
```

output
```
1/192*sqrt(b*x^2 + a*x)*(2*(4*(6*b*d*x + (8*b^4*c + 9*a*b^3*d)/b^3)*x + (5
6*a*b^3*c + 3*a^2*b^2*d)/b^3)*x + 3*(8*a^2*b^2*c - 3*a^3*b*d)/b^3) + 1/128
*(8*a^3*b*c - 3*a^4*d)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) +
 a))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x}\,dx = \int \frac{\left(b\,x^2+a\,x\right)^{3/2}\,\left(c+d\,x\right)}{x}\,dx$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x))/x,x)
```

output
```
int(((a*x + b*x^2)^(3/2)*(c + d*x))/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.11

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x}\,dx = \frac{-9\sqrt{x}\,\sqrt{bx+a}\,a^3bd+24\sqrt{x}\,\sqrt{bx+a}\,a^2b^2c+6\sqrt{x}\,\sqrt{bx+a}\,a^2b^2dx+112\sqrt{}}{}$$

input
```
int((d*x+c)*(b*x^2+a*x)^(3/2)/x,x)
```

output
```
( - 9*sqrt(x)*sqrt(a + b*x)*a**3*b*d + 24*sqrt(x)*sqrt(a + b*x)*a**2*b**2*
c + 6*sqrt(x)*sqrt(a + b*x)*a**2*b**2*d*x + 112*sqrt(x)*sqrt(a + b*x)*a*b*
*3*c*x + 72*sqrt(x)*sqrt(a + b*x)*a*b**3*d*x**2 + 64*sqrt(x)*sqrt(a + b*x)
*b**4*c*x**2 + 48*sqrt(x)*sqrt(a + b*x)*b**4*d*x**3 + 9*sqrt(b)*log((sqrt(
a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4*d - 24*sqrt(b)*log((sqrt(a + b*x
) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*b*c)/(192*b**3)
```

3.60 $\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 124

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^2}\,dx=\frac{5a(6bc-ad)\sqrt{ax+bx^2}}{24b}+\frac{1}{12}(6bc-ad)x\sqrt{ax+bx^2}+\frac{d\left(ax+bx^2\right)^{5/2}}{3bx^2}+\frac{a^2(6bc-ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{8b^{3/2}}$$

output
```
5/24*a*(-a*d+6*b*c)*(b*x^2+a*x)^(1/2)/b+1/12*(-a*d+6*b*c)*x*(b*x^2+a*x)^(1
/2)+1/3*d*(b*x^2+a*x)^(5/2)/b/x^2+1/8*a^2*(-a*d+6*b*c)*arctanh(b^(1/2)*x/(
b*x^2+a*x)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^2}\,dx=\frac{\sqrt{x(a+bx)}\left(\sqrt{b}\left(3a^2d+4b^2x(3c+2dx)+2ab(15c+7dx)\right)+\frac{3a^2(-6bc+ad)\log}{\sqrt{x}}\right.}{24b^{3/2}}$$

input
```
Integrate[((c + d*x)*(a*x + b*x^2)^(3/2))/x^2,x]
```

output
```
(Sqrt[x*(a + b*x)]*(Sqrt[b]*(3*a^2*d + 4*b^2*x*(3*c + 2*d*x) + 2*a*b*(15*c
 + 7*d*x)) + (3*a^2*(-6*b*c + a*d)*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]
)/(Sqrt[x]*Sqrt[a + b*x])))/(24*b^(3/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1220, 1131, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{3/2}(c+dx)}{x^2}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{2c\left(ax+bx^2\right)^{5/2}}{ax^2}-\frac{(6bc-ad)\int \frac{\left(bx^2+ax\right)^{3/2}}{x}dx}{a}$$

$$\Big\downarrow \text{1131}$$

$$\frac{2c\left(ax+bx^2\right)^{5/2}}{ax^2}-\frac{(6bc-ad)\left(\frac{1}{2}a\int \sqrt{bx^2+ax}dx+\frac{1}{3}\left(ax+bx^2\right)^{3/2}\right)}{a}$$

$$\Big\downarrow \text{1087}$$

$$\frac{2c\left(ax+bx^2\right)^{5/2}}{ax^2}-\frac{(6bc-ad)\left(\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)+\frac{1}{3}\left(ax+bx^2\right)^{3/2}\right)}{a}$$

$$\Big\downarrow \text{1091}$$

$$\frac{2c\left(ax+bx^2\right)^{5/2}}{ax^2}-\frac{(6bc-ad)\left(\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)+\frac{1}{3}\left(ax+bx^2\right)^{3/2}\right)}{a}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{2c(ax+bx^2)^{5/2}}{ax^2}-\left(\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)+\frac{1}{3}(ax+bx^2)^{3/2}\right)(6bc-ad)}{a}$$

input
```
Int[((c + d*x)*(a*x + b*x^2)^(3/2))/x^2,x]
```

output
```
(2*c*(a*x + b*x^2)^(5/2))/(a*x^2) - ((6*b*c - a*d)*((a*x + b*x^2)^(3/2)/3
+ (a*(((a + 2*b*x)*Sqrt[a*x + b*x^2])/(4*b) - (a^2*ArcTanh[(Sqrt[b]*x)/Sqr
t[a*x + b*x^2]])/(4*b^(3/2))))/2))/a
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$-\frac{\left(a^{3} d-6 c \,a^{2} b\right) \operatorname{arctanh}\left(\frac{\sqrt{x \left(b x +a \right)}}{x \sqrt{b}}\right)-\left(10 a \left(\frac{7 d x}{15}+c \right) b^{\frac{3}{2}}+\left(\frac{8}{3} d \,x^{2}+4 c x \right) b^{\frac{5}{2}}+\sqrt{b}\, a^{2} d \right) \sqrt{x \left(b x +a \right)}}{8 b^{\frac{3}{2}}}$
risch	$\frac{\left(8 b^{2} d \,x^{2}+14 a b d x +12 b^{2} c x +3 a^{2} d +30 a b c \right) x \left(b x +a \right)}{24 b \sqrt{x \left(b x +a \right)}}-\frac{a^{2} \left(a d -6 b c \right) \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b \,x^{2}+a x}\right)}{16 b^{\frac{3}{2}}}$
default	$c \left(\frac{2 \left(b \,x^{2}+a x \right)^{\frac{5}{2}}}{a \,x^{2}}-\frac{6 b \left(\frac{\left(b \,x^{2}+a x \right)^{\frac{3}{2}}}{3}+\frac{a \left(\frac{\left(2 b x +a \right) \sqrt{b \,x^{2}+a x}}{4 b}-\frac{a^{2} \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b \,x^{2}+a x}\right)}{8 b^{\frac{3}{2}}}\right)}{2}\right)}{a}\right)+d \left(\frac{\left(b \,x^{2}+a x \right)^{\frac{3}{2}}}{3}+\right.$

input
```
int((d*x+c)*(b*x^2+a*x)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8/b^(3/2)*((a^3*d-6*a^2*b*c)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-(10*a
*(7/15*d*x+c)*b^(3/2)+(8/3*d*x^2+4*c*x)*b^(5/2)+b^(1/2)*a^2*d)*(x*(b*x+a))
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.69

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^2}\,dx = \left[-\frac{3\left(6\,a^2bc-a^3d\right)\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\left(8\,b^3dx^2+30\,a\right.}{48\,b^2}\right.$$

$$-\frac{3\left(6\,a^2bc-a^3d\right)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-\left(8\,b^3dx^2+30\,ab^2c+3\,a^2bd+2\left(6\,b^3c+7\,ab^2d\right)x\right)\sqrt{bx^2+ax}}{24\,b^2}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
[-1/48*(3*(6*a^2*b*c - a^3*d)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*
sqrt(b)) - 2*(8*b^3*d*x^2 + 30*a*b^2*c + 3*a^2*b*d + 2*(6*b^3*c + 7*a*b^2*
d)*x)*sqrt(b*x^2 + a*x))/b^2, -1/24*(3*(6*a^2*b*c - a^3*d)*sqrt(-b)*arctan
(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (8*b^3*d*x^2 + 30*a*b^2*c + 3*a^2
*b*d + 2*(6*b^3*c + 7*a*b^2*d)*x)*sqrt(b*x^2 + a*x))/b^2]
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^2}\,dx = \int \frac{\left(x(a+bx)\right)^{\frac{3}{2}}(c+dx)}{x^2}\,dx$$

input
```
integrate((d*x+c)*(b*x**2+a*x)**(3/2)/x**2,x)
```

output
```
Integral((x*(a + b*x))**(3/2)*(c + d*x)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^2}\,dx = \frac{1}{4}\sqrt{bx^2+ax}adx + \frac{3a^2c\log\left(2bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{8\sqrt{b}} - \frac{a^3d\log\left(2bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{16b^{\frac{3}{2}}} + \frac{3}{4}\sqrt{bx^2+ax}ac + \frac{1}{3}\left(bx^2+ax\right)^{\frac{3}{2}}d + \frac{\sqrt{bx^2+ax}a^2d}{8b} + \frac{\left(bx^2+ax\right)^{\frac{3}{2}}c}{2x}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
1/4*sqrt(b*x^2 + a*x)*a*d*x + 3/8*a^2*c*log(2*b*x + a + 2*sqrt(b*x^2 + a*x
)*sqrt(b))/sqrt(b) - 1/16*a^3*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b
))/b^(3/2) + 3/4*sqrt(b*x^2 + a*x)*a*c + 1/3*(b*x^2 + a*x)^(3/2)*d + 1/8*s
qrt(b*x^2 + a*x)*a^2*d/b + 1/2*(b*x^2 + a*x)^(3/2)*c/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^2}\,dx = \frac{1}{24}\sqrt{bx^2+ax}\left(2\left(4bdx+\frac{6b^3c+7ab^2d}{b^2}\right)x+\frac{3\left(10ab^2c+a^2bd\right)}{b^2}\right) - \frac{\left(6a^2bc-a^3d\right)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{16b^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(3/2)/x^2,x, algorithm="giac")
```

output
```
1/24*sqrt(b*x^2 + a*x)*(2*(4*b*d*x + (6*b^3*c + 7*a*b^2*d)/b^2)*x + 3*(10*
a*b^2*c + a^2*b*d)/b^2) - 1/16*(6*a^2*b*c - a^3*d)*log(abs(2*(sqrt(b)*x -
sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^2}\,dx = \int \frac{\left(b\,x^2+a\,x\right)^{3/2}\,(c+d\,x)}{x^2}\,dx$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x))/x^2,x)
```

output
```
int(((a*x + b*x^2)^(3/2)*(c + d*x))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^2}\,dx = \frac{3\sqrt{x}\,\sqrt{bx+a}\,a^2bd+30\sqrt{x}\,\sqrt{bx+a}\,a\,b^2c+14\sqrt{x}\,\sqrt{bx+a}\,a\,b^2dx+12\sqrt{x}\,\sqrt{}}{}$$

input
```
int((d*x+c)*(b*x^2+a*x)^(3/2)/x^2,x)
```

output
```
(3*sqrt(x)*sqrt(a + b*x)*a**2*b*d + 30*sqrt(x)*sqrt(a + b*x)*a*b**2*c + 14
*sqrt(x)*sqrt(a + b*x)*a*b**2*d*x + 12*sqrt(x)*sqrt(a + b*x)*b**3*c*x + 8*
sqrt(x)*sqrt(a + b*x)*b**3*d*x**2 - 3*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)
*sqrt(b))/sqrt(a))*a**3*d + 18*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b
))/sqrt(a))*a**2*b*c)/(24*b**2)
```

3.61 $\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^3}\,dx = \frac{3}{4}(4bc+ad)\sqrt{ax+bx^2} - \frac{2c\left(ax+bx^2\right)^{3/2}}{x^2} + \frac{d\left(ax+bx^2\right)^{3/2}}{2x} + \frac{3a(4bc+ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4\sqrt{b}}$$

output
```
3/4*(a*d+4*b*c)*(b*x^2+a*x)^(1/2)-2*c*(b*x^2+a*x)^(3/2)/x^2+1/2*d*(b*x^2+a
*x)^(3/2)/x+3/4*a*(a*d+4*b*c)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.96

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^3}\,dx = \frac{\sqrt{x(a+bx)}\left(2bx(2c+dx)+a(-8c+5dx)+\frac{6a(4bc+ad)\sqrt{x}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+b}}\right.}{\sqrt{b}\sqrt{a+bx}}\right.}{4x}$$

input
```
Integrate[((c + d*x)*(a*x + b*x^2)^(3/2))/x^3,x]
```

output
```
(Sqrt[x*(a + b*x)]*(2*b*x*(2*c + d*x) + a*(-8*c + 5*d*x) + (6*a*(4*b*c + a
*d)*Sqrt[x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(Sqrt[b
]*Sqrt[a + b*x])))/(4*x)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1220, 1131, 1131, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}(c+dx)}{x^3}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(ad+4bc)\int \frac{(bx^2+ax)^{3/2}}{x^2}dx}{a}-\frac{2c(ax+bx^2)^{5/2}}{ax^3}$$

$$\Big\downarrow \text{1131}$$

$$\frac{(ad+4bc)\left(\frac{3}{4}a\int \frac{\sqrt{bx^2+ax}}{x}dx+\frac{(ax+bx^2)^{3/2}}{2x}\right)}{a}-\frac{2c(ax+bx^2)^{5/2}}{ax^3}$$

$$\Big\downarrow \text{1131}$$

$$\frac{(ad+4bc)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+ax}}dx+\sqrt{ax+bx^2}\right)+\frac{(ax+bx^2)^{3/2}}{2x}\right)}{a}-\frac{2c(ax+bx^2)^{5/2}}{ax^3}$$

$$\Big\downarrow \text{1091}$$

$$\frac{(ad+4bc)\left(\frac{3}{4}a\left(a\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}+\sqrt{ax+bx^2}\right)+\frac{(ax+bx^2)^{3/2}}{2x}\right)}{a}-\frac{2c(ax+bx^2)^{5/2}}{ax^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}+\sqrt{ax+bx^2}\right)+\frac{(ax+bx^2)^{3/2}}{2x}\right)(ad+4bc)}{a}-\frac{2c(ax+bx^2)^{5/2}}{ax^3}$$

input
```
Int[((c + d*x)*(a*x + b*x^2)^(3/2))/x^3,x]
```

output
```
(-2*c*(a*x + b*x^2)^(5/2))/(a*x^3) + ((4*b*c + a*d)*((a*x + b*x^2)^(3/2)/(
2*x) + (3*a*(Sqrt[a*x + b*x^2] + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]]
)/Sqrt[b]))/4))/a
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$\frac{\frac{3ax(ad+4bc)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{4}-2\left(-\frac{x\left(\frac{dx}{2}+c\right)b^{\frac{3}{2}}}{2}+\sqrt{b}\,a\left(-\frac{5dx}{8}+c\right)\right)\sqrt{x(bx+a)}}{\sqrt{b}\,x}$
risch	$-\frac{(bx+a)\left(-2bd\,x^{2}-5adx-4cbx+8ac\right)}{4\sqrt{x(bx+a)}}+\frac{3(ad+4bc)a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8\sqrt{b}}$
default	$c\left(-\frac{2\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{a\,x^{3}}+\frac{4b\left(\frac{2\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{a\,x^{2}}-\frac{6b\left(\frac{\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{3}+\frac{a\left(\frac{(2bx+a)\sqrt{b\,x^{2}+ax}}{4b}-\frac{a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8b^{\frac{3}{2}}}\right)}{2}\right)}{a}\right)}{a}\right)+$

input
```
int((d*x+c)*(b*x^2+a*x)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
3/4/b^(1/2)*(a*x*(a*d+4*b*c)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-8/3*(-1/
2*x*(1/2*d*x+c)*b^(3/2)+b^(1/2)*a*(-5/8*d*x+c))*(x*(b*x+a))^(1/2))/x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.78

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^3}\,dx = \left[\frac{3\,(4\,abc+a^2d)\sqrt{b}x\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(2\,b^2dx^2-8\,abc+}{8\,bx}\right.$$

$$\left.-\frac{3\,(4\,abc+a^2d)\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(2\,b^2dx^2-8\,abc+(4\,b^2c+5\,abd)x)\sqrt{bx^2+ax}}{4\,bx}\right]$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
[1/8*(3*(4*a*b*c + a^2*d)*sqrt(b)*x*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sq
rt(b)) + 2*(2*b^2*d*x^2 - 8*a*b*c + (4*b^2*c + 5*a*b*d)*x)*sqrt(b*x^2 + a*
x))/(b*x), -1/4*(3*(4*a*b*c + a^2*d)*sqrt(-b)*x*arctan(sqrt(b*x^2 + a*x)*s
qrt(-b)/(b*x + a)) - (2*b^2*d*x^2 - 8*a*b*c + (4*b^2*c + 5*a*b*d)*x)*sqrt(
b*x^2 + a*x))/(b*x)]
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^3}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}\,(c+dx)}{x^3}\,dx$$

input
```
integrate((d*x+c)*(b*x**2+a*x)**(3/2)/x**3,x)
```

output
```
Integral((x*(a + b*x))**(3/2)*(c + d*x)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.23

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^3}\,dx = \frac{3}{2}\,a\sqrt{b}c\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) + \frac{3\,a^2d\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,\sqrt{b}} + \frac{3}{4}\,\sqrt{bx^2+ax}ad - \frac{3\,\sqrt{bx^2+ax}ac}{x} + \frac{(bx^2+ax)^{\frac{3}{2}}d}{2\,x} + \frac{(bx^2+ax)^{\frac{3}{2}}c}{x^2}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
3/2*a*sqrt(b)*c*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 3/8*a^2*d*l
og(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b) + 3/4*sqrt(b*x^2 + a*x
)*a*d - 3*sqrt(b*x^2 + a*x)*a*c/x + 1/2*(b*x^2 + a*x)^(3/2)*d/x + (b*x^2 +
 a*x)^(3/2)*c/x^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^3}\,dx = \frac{2\,a^2c}{\sqrt{b}x-\sqrt{bx^2+ax}} + \frac{1}{4}\left(2\,bdx+\frac{4\,b^2c+5\,abd}{b}\right)\sqrt{bx^2+ax} - \frac{3\,(4\,abc+a^2d)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,\sqrt{b}}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(3/2)/x^3,x, algorithm="giac")
```

output
```
2*a^2*c/(sqrt(b)*x - sqrt(b*x^2 + a*x)) + 1/4*(2*b*d*x + (4*b^2*c + 5*a*b*
d)/b)*sqrt(b*x^2 + a*x) - 3/8*(4*a*b*c + a^2*d)*log(abs(2*(sqrt(b)*x - sqr
t(b*x^2 + a*x))*sqrt(b) + a))/sqrt(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^3}\,dx=\int \frac{{\left(b\,x^2+a\,x\right)}^{3/2}\,\left(c+d\,x\right)}{x^3} \,d x$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x))/x^3,x)
```

output
```
int(((a*x + b*x^2)^(3/2)*(c + d*x))/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.31

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^3}\,dx=\frac{-8\sqrt{x}\,\sqrt{bx+a}\,abc+5\sqrt{x}\,\sqrt{bx+a}\,abdx+4\sqrt{x}\,\sqrt{bx+a}\,b^2cx+2\sqrt{x}\,\sqrt{bx}$$

input
```
int((d*x+c)*(b*x^2+a*x)^(3/2)/x^3,x)
```

output
```
( - 8*sqrt(x)*sqrt(a + b*x)*a*b*c + 5*sqrt(x)*sqrt(a + b*x)*a*b*d*x + 4*sq
rt(x)*sqrt(a + b*x)*b**2*c*x + 2*sqrt(x)*sqrt(a + b*x)*b**2*d*x**2 + 3*sqr
t(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*d*x + 12*sqrt(b)*
log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b*c*x - sqrt(b)*a**2*d*x
- 9*sqrt(b)*a*b*c*x)/(4*b*x)
```

3.62 $\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 99

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^4}\,dx = bd\sqrt{ax+bx^2}-\frac{2(bc+ad)\sqrt{ax+bx^2}}{x} -\frac{2c(ax+bx^2)^{3/2}}{3x^3}+\sqrt{b}(2bc+3ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)$$

output
```
b*d*(b*x^2+a*x)^(1/2)-2*(a*d+b*c)*(b*x^2+a*x)^(1/2)/x-2/3*c*(b*x^2+a*x)^(3
/2)/x^3+b^(1/2)*(3*a*d+2*b*c)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^4}\,dx = \frac{\sqrt{x(a+bx)}\left(-\sqrt{a+bx}(bx(8c-3dx)+2a(c+3dx))+6\sqrt{b}(2bc+3ad)x^{3/}\right.}{3x^2\sqrt{a+bx}}$$

input
```
Integrate[((c + d*x)*(a*x + b*x^2)^(3/2))/x^4,x]
```

output
```
(Sqrt[x*(a + b*x)]*(-(Sqrt[a + b*x]*(b*x*(8*c - 3*d*x) + 2*a*(c + 3*d*x)))
 + 6*Sqrt[b]*(2*b*c + 3*a*d)*x^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] +
 Sqrt[a + b*x])]))/(3*x^2*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1220, 1125, 25, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}(c+dx)}{x^4}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(3ad+2bc)\int \frac{(bx^2+ax)^{3/2}}{x^3}dx}{3a}-\frac{2c(ax+bx^2)^{5/2}}{3ax^4}$$

$$\Big\downarrow \text{1125}$$

$$\frac{(3ad+2bc)\left(-\int -\frac{b(2a+bx)}{\sqrt{bx^2+ax}}dx-\frac{2a\sqrt{ax+bx^2}}{x}\right)}{3a}-\frac{2c(ax+bx^2)^{5/2}}{3ax^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{(3ad+2bc)\left(\int \frac{b(2a+bx)}{\sqrt{bx^2+ax}}dx-\frac{2a\sqrt{ax+bx^2}}{x}\right)}{3a}-\frac{2c(ax+bx^2)^{5/2}}{3ax^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{(3ad+2bc)\left(b\int \frac{2a+bx}{\sqrt{bx^2+ax}}dx-\frac{2a\sqrt{ax+bx^2}}{x}\right)}{3a}-\frac{2c(ax+bx^2)^{5/2}}{3ax^4}$$

$$\Big\downarrow \text{1160}$$

$$\frac{(3ad+2bc)\left(b\left(\frac{3}{2}a\int \frac{1}{\sqrt{bx^2+ax}}dx+\sqrt{ax+bx^2}\right)-\frac{2a\sqrt{ax+bx^2}}{x}\right)}{3a}-\frac{2c(ax+bx^2)^{5/2}}{3ax^4}$$

$$\Big\downarrow \text{1091}$$

$$\frac{(3ad+2bc)\left(b\left(3a\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}+\sqrt{ax+bx^2}\right)-\frac{2a\sqrt{ax+bx^2}}{x}\right)}{3a}-\frac{2c(ax+bx^2)^{5/2}}{3ax^4}$$

$$\Big\downarrow \; 219$$

$$\frac{\left(b\left(\frac{3a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}+\sqrt{ax+bx^2}\right)-\frac{2a\sqrt{ax+bx^2}}{x}\right)(3ad+2bc)}{3a}-\frac{2c(ax+bx^2)^{5/2}}{3ax^4}$$

input
```
Int[((c + d*x)*(a*x + b*x^2)^(3/2))/x^4,x]
```

output
```
(-2*c*(a*x + b*x^2)^(5/2))/(3*a*x^4) + ((2*b*c + 3*a*d)*((-2*a*Sqrt[a*x +
b*x^2])/x + b*(Sqrt[a*x + b*x^2] + (3*a*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x
^2]])/Sqrt[b])))/(3*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{3b x^2 \left(ad+\frac{2bc}{3}\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)-\frac{2\left(\left(-\frac{3}{2}d x^2+4cx\right)b^{\frac{3}{2}}+a\sqrt{b}\,(3dx+c)\right)\sqrt{x(bx+a)}}{3}}{\sqrt{b}\,x^2}$
risch	$-\frac{(bx+a)(-3bd x^2+6adx+8cbx+2ac)}{3x\sqrt{x(bx+a)}}+\frac{(3ad+2bc)\sqrt{b}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b x^2+ax}\right)}{2}$
default	$c\left(-\frac{2(b x^2+ax)^{\frac{5}{2}}}{3a x^4}+\frac{2b\left(-\frac{2\left(b x^2+ax\right)^{\frac{5}{2}}}{a x^3}+\frac{4b\left(\frac{2\left(b x^2+ax\right)^{\frac{5}{2}}}{a x^2}-\frac{6b\left(\frac{\left(b x^2+ax\right)^{\frac{3}{2}}}{3}+\frac{a\left(\frac{(2bx+a)\sqrt{b x^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b}\right.}{8b^{\frac{3}{2}}}\right.}{2}\right.}{a}\right.}{a}\right.}{3a}\right.$

input
```
int((d*x+c)*(b*x^2+a*x)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
3/b^(1/2)*(b*x^2*(a*d+2/3*b*c)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-2/9*((
-3/2*d*x^2+4*c*x)*b^(3/2)+a*b^(1/2)*(3*d*x+c))*(x*(b*x+a))^(1/2))/x^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.73

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^4}\,dx = \left[\frac{3\,(2\,bc+3\,ad)\sqrt{b}x^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(3\,bdx^2-2\,ac-}{6\,x^2}\right.$$

$$\left.-\frac{3\,(2\,bc+3\,ad)\sqrt{-b}x^2\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-\left(3\,bdx^2-2\,ac-2\,(4\,bc+3\,ad)x\right)\sqrt{bx^2+ax}}{3\,x^2}\right]$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[1/6*(3*(2*b*c + 3*a*d)*sqrt(b)*x^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sq
rt(b)) + 2*(3*b*d*x^2 - 2*a*c - 2*(4*b*c + 3*a*d)*x)*sqrt(b*x^2 + a*x))/x^
2, -1/3*(3*(2*b*c + 3*a*d)*sqrt(-b)*x^2*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/
(b*x + a)) - (3*b*d*x^2 - 2*a*c - 2*(4*b*c + 3*a*d)*x)*sqrt(b*x^2 + a*x))/
x^2]
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^4}\,dx = \int \frac{\left(x(a+bx)\right)^{\frac{3}{2}}(c+dx)}{x^4}\,dx$$

input
```
integrate((d*x+c)*(b*x**2+a*x)**(3/2)/x**4,x)
```

output
```
Integral((x*(a + b*x))**(3/2)*(c + d*x)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.47

$$\int \frac{(c+dx)(ax+bx^2)^{3/2}}{x^4}\,dx = b^{\frac{3}{2}}c\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) + \frac{3}{2}\,a\sqrt{b}d\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) - \frac{7\,\sqrt{bx^2+ax}bc}{3\,x} - \frac{3\,\sqrt{bx^2+ax}ad}{x} - \frac{\sqrt{bx^2+ax}ac}{3\,x^2} + \frac{(bx^2+ax)^{\frac{3}{2}}d}{x^2} - \frac{(bx^2+ax)^{\frac{3}{2}}c}{3\,x^3}$$

input

```
integrate((d*x+c)*(b*x^2+a*x)^(3/2)/x^4,x, algorithm="maxima")
```

output

```
b^(3/2)*c*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 3/2*a*sqrt(b)*d*l
og(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 7/3*sqrt(b*x^2 + a*x)*b*c/x
- 3*sqrt(b*x^2 + a*x)*a*d/x - 1/3*sqrt(b*x^2 + a*x)*a*c/x^2 + (b*x^2 + a*x
)^(3/2)*d/x^2 - 1/3*(b*x^2 + a*x)^(3/2)*c/x^3
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.72

$$\int \frac{(c+dx)(ax+bx^2)^{3/2}}{x^4}\,dx = \sqrt{bx^2+ax}bd - \frac{(2\,b^2c+3\,abd)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{2\,\sqrt{b}} + \frac{2\left(6\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2abc+3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2a^2d+3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)a^2\sqrt{b}c+a^3c\right)}{3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3}$$

input

```
integrate((d*x+c)*(b*x^2+a*x)^(3/2)/x^4,x, algorithm="giac")
```

output
```
sqrt(b*x^2 + a*x)*b*d - 1/2*(2*b^2*c + 3*a*b*d)*log(abs(2*(sqrt(b)*x - sqr
t(b*x^2 + a*x))*sqrt(b) + a))/sqrt(b) + 2/3*(6*(sqrt(b)*x - sqrt(b*x^2 + a
*x))^2*a*b*c + 3*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2*d + 3*(sqrt(b)*x -
sqrt(b*x^2 + a*x))*a^2*sqrt(b)*c + a^3*c)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^
3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^4}\,dx=\int \frac{\left(b\,x^2+a\,x\right)^{3/2}\,\left(c+d\,x\right)}{x^4}\,dx$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x))/x^4,x)
```

output
```
int(((a*x + b*x^2)^(3/2)*(c + d*x))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.23

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^4}\,dx=\frac{-4\sqrt{x}\,\sqrt{bx+a}\,ac-12\sqrt{x}\,\sqrt{bx+a}\,adx-16\sqrt{x}\,\sqrt{bx+a}\,bcx+6\sqrt{x}\,\sqrt{bx+}}{}$$

input
```
int((d*x+c)*(b*x^2+a*x)^(3/2)/x^4,x)
```

output
```
( - 4*sqrt(x)*sqrt(a + b*x)*a*c - 12*sqrt(x)*sqrt(a + b*x)*a*d*x - 16*sqrt
(x)*sqrt(a + b*x)*b*c*x + 6*sqrt(x)*sqrt(a + b*x)*b*d*x**2 + 18*sqrt(b)*lo
g((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*d*x**2 + 12*sqrt(b)*log((sq
rt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b*c*x**2 + 5*sqrt(b)*a*d*x**2)/(6*
x**2)
```

3.63 $\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^5} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^5} \, dx = -\frac{2ad\sqrt{ax+bx^2}}{3x^2}-\frac{8bd\sqrt{ax+bx^2}}{3x} -\frac{2c(ax+bx^2)^{5/2}}{5ax^5}+2b^{3/2}d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)$$

output
```
-2/3*a*d*(b*x^2+a*x)^(1/2)/x^2-8/3*b*d*(b*x^2+a*x)^(1/2)/x-2/5*c*(b*x^2+a*
x)^(5/2)/a/x^5+2*b^(3/2)*d*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.15

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^5} \, dx = -\frac{2\sqrt{x(a+bx)}\left(\sqrt{a+bx}(3b^2cx^2+a^2(3c+5dx)+2abx(3c+10dx))+15ab^{3/2}dx^{5/2}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+\right.\right.}}{15ax^3\sqrt{a+bx}}$$

input
```
Integrate[((c + d*x)*(a*x + b*x^2)^(3/2))/x^5,x]
```

output
```
(-2*Sqrt[x*(a + b*x)]*(Sqrt[a + b*x]*(3*b^2*c*x^2 + a^2*(3*c + 5*d*x) + 2*
a*b*x*(3*c + 10*d*x)) + 15*a*b^(3/2)*d*x^(5/2)*Log[-(Sqrt[b]*Sqrt[x]) + Sq
rt[a + b*x]]))/(15*a*x^3*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1220, 1130, 1125, 25, 27, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}(c+dx)}{x^5}dx$$

$$\Big\downarrow \text{ 1220}$$

$$d\int \frac{(bx^2+ax)^{3/2}}{x^4}dx-\frac{2c(ax+bx^2)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{ 1130}$$

$$d\left(b\int \frac{\sqrt{bx^2+ax}}{x^2}dx-\frac{2(ax+bx^2)^{3/2}}{3x^3}\right)-\frac{2c(ax+bx^2)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{ 1125}$$

$$d\left(b\left(-\int -\frac{b}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2(ax+bx^2)^{3/2}}{3x^3}\right)-\frac{2c(ax+bx^2)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{ 25}$$

$$d\left(b\left(\int \frac{b}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2(ax+bx^2)^{3/2}}{3x^3}\right)-\frac{2c(ax+bx^2)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{ 27}$$

$$d\left(b\left(b\int \frac{1}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2(ax+bx^2)^{3/2}}{3x^3}\right)-\frac{2c(ax+bx^2)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{ 1091}$$

$$d\left(b\left(2b\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2\left(ax+bx^2\right)^{3/2}}{3x^3}\right)-\frac{2c\left(ax+bx^2\right)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{219}$$

$$d\left(b\left(2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2\left(ax+bx^2\right)^{3/2}}{3x^3}\right)-\frac{2c\left(ax+bx^2\right)^{5/2}}{5ax^5}$$

input
```
Int[((c + d*x)*(a*x + b*x^2)^(3/2))/x^5,x]
```

output
```
(-2*c*(a*x + b*x^2)^(5/2))/(5*a*x^5) + d*((-2*(a*x + b*x^2)^(3/2))/(3*x^3)
 + b*((-2*Sqrt[a*x + b*x^2])/x + 2*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x +
b*x^2]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{2da\,b^{\frac{3}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)x^{3}-\frac{2\left(\left(\frac{5dx}{3}+c\right)a^{2}+2\left(\frac{10dx}{3}+c\right)xba+b^{2}c\,x^{2}\right)\sqrt{x(bx+a)}}{5}}{a\,x^{3}}$
risch	$-\frac{2(bx+a)\left(20abd\,x^{2}+3b^{2}c\,x^{2}+5a^{2}dx+6abcx+3a^{2}c\right)}{15x^{2}\sqrt{x(bx+a)}\,a}+b^{\frac{3}{2}}d\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)$
default	$-\frac{2c\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{5a\,x^{5}}+d\left(-\frac{2\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{3a\,x^{4}}+\frac{2b\left(-\frac{2\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{a\,x^{3}}+\frac{4b\left(\frac{2\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{a\,x^{2}}-\frac{6b\left(\frac{\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{3}+\frac{a\left(\frac{(2bx+a)\sqrt{b\,x^{2}\cdots}}{4b}\cdots\right)}{\cdots}\right)}{\cdots}\right)}{a}\right)}{3a}\cdots\right)$

input
```
int((d*x+c)*(b*x^2+a*x)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
2/5*(5*d*a*b^(3/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*x^3-((5/3*d*x+c)*a
^2+2*(10/3*d*x+c)*x*b*a+b^2*c*x^2)*(x*(b*x+a))^(1/2))/a/x^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.93

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^5}\,dx = \left[\frac{15\,ab^{\frac{3}{2}}dx^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,(3\,a^2c+(3\,b^2c+20\,abd)x^2 \cdots}{15\,ax^3}\right.$$
$$\left.-\frac{2\left(15\,a\sqrt{-b}bdx^3\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+(3\,a^2c+(3\,b^2c+20\,abd)x^2+(6\,abc+5\,a^2d)x)\sqrt{bx^2+ax}\right)}{15\,ax^3}\right]$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
[1/15*(15*a*b^(3/2)*d*x^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2
*(3*a^2*c + (3*b^2*c + 20*a*b*d)*x^2 + (6*a*b*c + 5*a^2*d)*x)*sqrt(b*x^2 +
 a*x))/(a*x^3), -2/15*(15*a*sqrt(-b)*b*d*x^3*arctan(sqrt(b*x^2 + a*x)*sqrt
(-b)/(b*x + a)) + (3*a^2*c + (3*b^2*c + 20*a*b*d)*x^2 + (6*a*b*c + 5*a^2*d
)*x)*sqrt(b*x^2 + a*x))/(a*x^3)]
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^5}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}\,(c+dx)}{x^5}\,dx$$

input
```
integrate((d*x+c)*(b*x**2+a*x)**(3/2)/x**5,x)
```

output
```
Integral((x*(a + b*x))**(3/2)*(c + d*x)/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.61

$$\int \frac{(c+dx)(ax+bx^2)^{3/2}}{x^5}\,dx = b^{\frac{3}{2}}d\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) - \frac{2\,\sqrt{bx^2+ax}b^2c}{5\,ax} - \frac{7\,\sqrt{bx^2+ax}bd}{3\,x} + \frac{\sqrt{bx^2+ax}bc}{5\,x^2} - \frac{\sqrt{bx^2+ax}ad}{3\,x^2} + \frac{3\,\sqrt{bx^2+ax}ac}{5\,x^3} - \frac{(bx^2+ax)^{\frac{3}{2}}d}{3\,x^3} - \frac{(bx^2+ax)^{\frac{3}{2}}c}{x^4}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
b^(3/2)*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2/5*sqrt(b*x^2 +
a*x)*b^2*c/(a*x) - 7/3*sqrt(b*x^2 + a*x)*b*d/x + 1/5*sqrt(b*x^2 + a*x)*b*c
/x^2 - 1/3*sqrt(b*x^2 + a*x)*a*d/x^2 + 3/5*sqrt(b*x^2 + a*x)*a*c/x^3 - 1/3
*(b*x^2 + a*x)^(3/2)*d/x^3 - (b*x^2 + a*x)^(3/2)*c/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(80) = 160.

Time = 0.16 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.64

$$\int \frac{(c+dx)(ax+bx^2)^{3/2}}{x^5}\,dx = -b^{\frac{3}{2}}d\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right) + \frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4b^2c+30\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4abd+30\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3ab^{\frac{3}{2}}c+15\left(\sqrt{b}\right.\right.}{1}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(3/2)/x^5,x, algorithm="giac")
```

output
```
-b^(3/2)*d*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)) + 2/15*
(15*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*b^2*c + 30*(sqrt(b)*x - sqrt(b*x^2 +
 a*x))^4*a*b*d + 30*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a*b^(3/2)*c + 15*(sq
rt(b)*x - sqrt(b*x^2 + a*x))^3*a^2*sqrt(b)*d + 30*(sqrt(b)*x - sqrt(b*x^2
+ a*x))^2*a^2*b*c + 5*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^3*d + 15*(sqrt(b
)*x - sqrt(b*x^2 + a*x))*a^3*sqrt(b)*c + 3*a^4*c)/(sqrt(b)*x - sqrt(b*x^2
+ a*x))^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^5}\,dx=\int \frac{{\left(b\,x^2+a\,x\right)}^{3/2}\,\left(c+d\,x\right)}{x^5}\,dx$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x))/x^5,x)
```

output
```
int(((a*x + b*x^2)^(3/2)*(c + d*x))/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.37

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^5}\,dx=\frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2c}{5}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2dx}{3}-\frac{4\sqrt{x}\,\sqrt{bx+a}\,abcx}{5}-\frac{8\sqrt{x}\,\sqrt{bx+a}\,abd\,x^2}{3}-\frac{2\sqrt{x}\,\sqrt{bx+a}}{5}}{a\,x^3}$$

input
```
int((d*x+c)*(b*x^2+a*x)^(3/2)/x^5,x)
```

output
```
(2*( - 3*sqrt(x)*sqrt(a + b*x)*a**2*c - 5*sqrt(x)*sqrt(a + b*x)*a**2*d*x -
 6*sqrt(x)*sqrt(a + b*x)*a*b*c*x - 20*sqrt(x)*sqrt(a + b*x)*a*b*d*x**2 - 3
*sqrt(x)*sqrt(a + b*x)*b**2*c*x**2 + 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(
x)*sqrt(b))/sqrt(a))*a*b*d*x**3 + 8*sqrt(b)*a*b*d*x**3 - 3*sqrt(b)*b**2*c*
x**3))/(15*a*x**3)
```

3.64 $\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^6} dx$

Optimal result

Integrand size = 22, antiderivative size = 57

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^6} dx = -\frac{2c\left(ax+bx^2\right)^{5/2}}{7ax^6} + \frac{2(2bc-7ad)\left(ax+bx^2\right)^{5/2}}{35a^2x^5}$$

output
```
-2/7*c*(b*x^2+a*x)^(5/2)/a/x^6+2/35*(-7*a*d+2*b*c)*(b*x^2+a*x)^(5/2)/a^2/x
^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.63

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^6} dx = -\frac{2(x(a+bx))^{5/2}(5ac-2bcx+7adx)}{35a^2x^6}$$

input
```
Integrate[((c + d*x)*(a*x + b*x^2)^(3/2))/x^6,x]
```

output
```
(-2*(x*(a + b*x))^(5/2)*(5*a*c - 2*b*c*x + 7*a*d*x))/(35*a^2*x^6)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1220, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}(c+dx)}{x^6}dx$$

$$\Big\downarrow \text{ 1220}$$

$$-\frac{(2bc-7ad)\int \frac{(bx^2+ax)^{3/2}}{x^5}dx}{7a}-\frac{2c(ax+bx^2)^{5/2}}{7ax^6}$$

$$\Big\downarrow \text{ 1123}$$

$$\frac{2(ax+bx^2)^{5/2}(2bc-7ad)}{35a^2x^5}-\frac{2c(ax+bx^2)^{5/2}}{7ax^6}$$

input
```
Int[((c + d*x)*(a*x + b*x^2)^(3/2))/x^6,x]
```

output
```
(-2*c*(a*x + b*x^2)^(5/2))/(7*a*x^6) + (2*(2*b*c - 7*a*d)*(a*x + b*x^2)^(5
/2))/(35*a^2*x^5)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.68

method	result	size
pseudoelliptic	$-\frac{2\sqrt{x(bx+a)}\,(bx+a)^{2}\left(\left(\frac{7dx}{5}+c\right)a-\frac{2cbx}{5}\right)}{7x^{4}a^{2}}$	39
gosper	$-\frac{2(bx+a)(7adx-2cbx+5ac)\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{35a^{2}x^{5}}$	40
orering	$-\frac{2(bx+a)(7adx-2cbx+5ac)\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{35a^{2}x^{5}}$	40
default	$c\left(-\frac{2\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{7a\,x^{6}}+\frac{4b\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{35a^{2}x^{5}}\right)-\frac{2d\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{5a\,x^{5}}$	64
trager	$-\frac{2\left(7a\,b^{2}d\,x^{3}-2b^{3}c\,x^{3}+14a^{2}bd\,x^{2}+a\,b^{2}c\,x^{2}+7a^{3}dx+8a^{2}bcx+5c\,a^{3}\right)\sqrt{b\,x^{2}+ax}}{35x^{4}a^{2}}$	80
risch	$-\frac{2(bx+a)\left(7a\,b^{2}d\,x^{3}-2b^{3}c\,x^{3}+14a^{2}bd\,x^{2}+a\,b^{2}c\,x^{2}+7a^{3}dx+8a^{2}bcx+5c\,a^{3}\right)}{35x^{3}\sqrt{x(bx+a)}\,a^{2}}$	83

input
```
int((d*x+c)*(b*x^2+a*x)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-2/7*(x*(b*x+a))^(1/2)*(b*x+a)^2*((7/5*d*x+c)*a-2/5*c*b*x)/x^4/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.39

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^6}\,dx = -\frac{2\left(5\,a^3c-(2\,b^3c-7\,ab^2d)x^3+(ab^2c+14\,a^2bd)x^2+(8\,a^2bc+7\,a^3d)x\right)\sqrt{bx^2+ax}}{35\,a^2x^4}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
-2/35*(5*a^3*c - (2*b^3*c - 7*a*b^2*d)*x^3 + (a*b^2*c + 14*a^2*b*d)*x^2 +
(8*a^2*b*c + 7*a^3*d)*x)*sqrt(b*x^2 + a*x)/(a^2*x^4)
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^6}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}\,(c+dx)}{x^6}\,dx$$

input
```
integrate((d*x+c)*(b*x**2+a*x)**(3/2)/x**6,x)
```

output
```
Integral((x*(a + b*x))**(3/2)*(c + d*x)/x**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(49) = 98.

Time = 0.04 (sec) , antiderivative size = 176, normalized size of antiderivative = 3.09

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^6}\,dx = \frac{4\,\sqrt{bx^2+ax}b^3c}{35\,a^2x} - \frac{2\,\sqrt{bx^2+ax}b^2d}{5\,ax} - \frac{2\,\sqrt{bx^2+ax}b^2c}{35\,ax^2} + \frac{\sqrt{bx^2+ax}bd}{5\,x^2} + \frac{3\,\sqrt{bx^2+ax}bc}{70\,x^3} + \frac{3\,\sqrt{bx^2+ax}ad}{5\,x^3} + \frac{3\,\sqrt{bx^2+ax}ac}{14\,x^4} - \frac{(bx^2+ax)^{\frac{3}{2}}d}{x^4} - \frac{(bx^2+ax)^{\frac{3}{2}}c}{2\,x^5}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
4/35*sqrt(b*x^2 + a*x)*b^3*c/(a^2*x) - 2/5*sqrt(b*x^2 + a*x)*b^2*d/(a*x) -
 2/35*sqrt(b*x^2 + a*x)*b^2*c/(a*x^2) + 1/5*sqrt(b*x^2 + a*x)*b*d/x^2 + 3/
70*sqrt(b*x^2 + a*x)*b*c/x^3 + 3/5*sqrt(b*x^2 + a*x)*a*d/x^3 + 3/14*sqrt(b
*x^2 + a*x)*a*c/x^4 - (b*x^2 + a*x)^(3/2)*d/x^4 - 1/2*(b*x^2 + a*x)^(3/2)*
c/x^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(49) = 98$.

Time = 0.15 (sec) , antiderivative size = 311, normalized size of antiderivative = 5.46

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^6}\,dx = \frac{2\left(35\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6b^2d+35\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5b^{\frac{5}{2}}c+70\left(\sqrt{b}x-\right.\right.}{}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(3/2)/x^6,x, algorithm="giac")
```

output
```
2/35*(35*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*b^2*d + 35*(sqrt(b)*x - sqrt(b*
x^2 + a*x))^5*b^(5/2)*c + 70*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a*b^(3/2)*d
 + 105*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a*b^2*c + 70*(sqrt(b)*x - sqrt(b*
x^2 + a*x))^4*a^2*b*d + 140*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^2*b^(3/2)*
c + 35*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^3*sqrt(b)*d + 98*(sqrt(b)*x - s
qrt(b*x^2 + a*x))^2*a^3*b*c + 7*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^4*d +
35*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^4*sqrt(b)*c + 5*a^5*c)/(sqrt(b)*x - s
qrt(b*x^2 + a*x))^7
```

Mupad [B] (verification not implemented)

Time = 10.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.49

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^6}\,dx = \frac{4\,b^3\,c\,\sqrt{b\,x^2+a\,x}}{35\,a^2\,x} - \frac{2\,a\,d\,\sqrt{b\,x^2+a\,x}}{5\,x^3} - \frac{16\,b\,c\,\sqrt{b\,x^2+a\,x}}{35\,x^3} - \frac{4\,b\,d\,\sqrt{b\,x^2+a\,x}}{5\,x^2} - \frac{2\,b^2\,c\,\sqrt{b\,x^2+a\,x}}{35\,a\,x^2} - \frac{2\,a\,c\,\sqrt{b\,x^2+a\,x}}{7\,x^4} - \frac{2\,b^2\,d\,\sqrt{b\,x^2+a\,x}}{5\,a\,x}$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x))/x^6,x)
```

output
```
(4*b^3*c*(a*x + b*x^2)^(1/2))/(35*a^2*x) - (2*a*d*(a*x + b*x^2)^(1/2))/(5*
x^3) - (16*b*c*(a*x + b*x^2)^(1/2))/(35*x^3) - (4*b*d*(a*x + b*x^2)^(1/2))
/(5*x^2) - (2*b^2*c*(a*x + b*x^2)^(1/2))/(35*a*x^2) - (2*a*c*(a*x + b*x^2)
^(1/2))/(7*x^4) - (2*b^2*d*(a*x + b*x^2)^(1/2))/(5*a*x)
```

Reduce [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.60

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^6}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3c}{7} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3dx}{5} - \frac{16\sqrt{x}\,\sqrt{bx+a}\,a^2bcx}{35} - \frac{4\sqrt{x}\,\sqrt{bx+a}\,a^2bdx^2}{5} - \frac{2\sqrt{x}\,\sqrt{b}}{}}{a^2x^4}$$

input
```
int((d*x+c)*(b*x^2+a*x)^(3/2)/x^6,x)
```

output
```
(2*( - 5*sqrt(x)*sqrt(a + b*x)*a**3*c - 7*sqrt(x)*sqrt(a + b*x)*a**3*d*x -
 8*sqrt(x)*sqrt(a + b*x)*a**2*b*c*x - 14*sqrt(x)*sqrt(a + b*x)*a**2*b*d*x*
*2 - sqrt(x)*sqrt(a + b*x)*a*b**2*c*x**2 - 7*sqrt(x)*sqrt(a + b*x)*a*b**2*
d*x**3 + 2*sqrt(x)*sqrt(a + b*x)*b**3*c*x**3 - 3*sqrt(b)*a*b**2*d*x**4 - 2
*sqrt(b)*b**3*c*x**4))/(35*a**2*x**4)
```

3.65 $\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^7} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^7} \, dx = -\frac{2c(ax+bx^2)^{5/2}}{9ax^7} + \frac{2(4bc-9ad)\left(ax+bx^2\right)^{5/2}}{63a^2x^6} - \frac{4b(4bc-9ad)\left(ax+bx^2\right)^{5/2}}{315a^3x^5}$$

output
```
-2/9*c*(b*x^2+a*x)^(5/2)/a/x^7+2/63*(-9*a*d+4*b*c)*(b*x^2+a*x)^(5/2)/a^2/x
^6-4/315*b*(-9*a*d+4*b*c)*(b*x^2+a*x)^(5/2)/a^3/x^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.63

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^7} \, dx = \frac{2(x(a+bx))^{5/2}\left(-8b^2cx^2-5a^2(7c+9dx)+2abx(10c+9dx)\right)}{315a^3x^7}$$

input
```
Integrate[((c + d*x)*(a*x + b*x^2)^(3/2))/x^7,x]
```

output
```
(2*(x*(a + b*x))^(5/2)*(-8*b^2*c*x^2 - 5*a^2*(7*c + 9*d*x) + 2*a*b*x*(10*c
 + 9*d*x)))/(315*a^3*x^7)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1220, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}(c+dx)}{x^7}dx$$

$$\Big\downarrow \text{1220}$$

$$-\frac{(4bc-9ad)\int \frac{(bx^2+ax)^{3/2}}{x^6}dx}{9a}-\frac{2c(ax+bx^2)^{5/2}}{9ax^7}$$

$$\Big\downarrow \text{1129}$$

$$-\frac{(4bc-9ad)\left(-\frac{2b\int \frac{(bx^2+ax)^{3/2}}{x^5}dx}{7a}-\frac{2(ax+bx^2)^{5/2}}{7ax^6}\right)}{9a}-\frac{2c(ax+bx^2)^{5/2}}{9ax^7}$$

$$\Big\downarrow \text{1123}$$

$$-\frac{\left(\frac{4b(ax+bx^2)^{5/2}}{35a^2x^5}-\frac{2(ax+bx^2)^{5/2}}{7ax^6}\right)(4bc-9ad)}{9a}-\frac{2c(ax+bx^2)^{5/2}}{9ax^7}$$

input
```
Int[((c + d*x)*(a*x + b*x^2)^(3/2))/x^7,x]
```

output
```
(-2*c*(a*x + b*x^2)^(5/2))/(9*a*x^7) - ((4*b*c - 9*a*d)*((-2*(a*x + b*x^2)
^(5/2))/(7*a*x^6) + (4*b*(a*x + b*x^2)^(5/2))/(35*a^2*x^5)))/(9*a)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

method	result	size
pseudoelliptic	$-\frac{2\sqrt{x(bx+a)}\,(bx+a)^2\left(\left(\frac{9dx}{7}+c\right)a^2-\frac{4x\left(\frac{9dx}{10}+c\right)ba}{7}+\frac{8b^2c\,x^2}{35}\right)}{9x^5a^3}$	56
gosper	$-\frac{2(bx+a)(-18abd\,x^2+8b^2c\,x^2+45a^2dx-20abcx+35a^2c)(b\,x^2+ax)^{\frac{3}{2}}}{315x^6a^3}$	62
orering	$-\frac{2(bx+a)(-18abd\,x^2+8b^2c\,x^2+45a^2dx-20abcx+35a^2c)(b\,x^2+ax)^{\frac{3}{2}}}{315x^6a^3}$	62
trager	$-\frac{2(-18x^4a\,b^3d+8x^4b^4c+9a^2b^2d\,x^3-4a\,b^3c\,x^3+72a^3bd\,x^2+3a^2b^2c\,x^2+45a^4dx+50a^3bcx+35c\,a^4)\sqrt{b\,x^2+ax}}{315a^3x^5}$	105
risch	$-\frac{2(bx+a)(-18x^4a\,b^3d+8x^4b^4c+9a^2b^2d\,x^3-4a\,b^3c\,x^3+72a^3bd\,x^2+3a^2b^2c\,x^2+45a^4dx+50a^3bcx+35c\,a^4)}{315x^4\sqrt{x(bx+a)}\,a^3}$	108
default	$c\left(-\frac{2(b\,x^2+ax)^{\frac{5}{2}}}{9a\,x^7}-\frac{4b\left(-\frac{2(b\,x^2+ax)^{\frac{5}{2}}}{7a\,x^6}+\frac{4b(b\,x^2+ax)^{\frac{5}{2}}}{35a^2x^5}\right)}{9a}\right)+d\left(-\frac{2(b\,x^2+ax)^{\frac{5}{2}}}{7a\,x^6}+\frac{4b(b\,x^2+ax)^{\frac{5}{2}}}{35a^2x^5}\right)$	112

input
```
int((d*x+c)*(b*x^2+a*x)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-2/9*(x*(b*x+a))^(1/2)*(b*x+a)^2*((9/7*d*x+c)*a^2-4/7*x*(9/10*d*x+c)*b*a+8
/35*b^2*c*x^2)/x^5/a^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.17

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^7}\,dx = -\frac{2\left(35\,a^4c+2\,(4\,b^4c-9\,ab^3d)x^4-(4\,ab^3c-9\,a^2b^2d)x^3+3\,(a^2b^2c+24\,a^3bd)x^2+5\,(10\,a^3bc+9\,a^4d)x\right)\sqrt{b\ldots}}{315\,a^3x^5}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
-2/315*(35*a^4*c + 2*(4*b^4*c - 9*a*b^3*d)*x^4 - (4*a*b^3*c - 9*a^2*b^2*d)
*x^3 + 3*(a^2*b^2*c + 24*a^3*b*d)*x^2 + 5*(10*a^3*b*c + 9*a^4*d)*x)*sqrt(b
*x^2 + a*x)/(a^3*x^5)
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^7}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}\,(c+dx)}{x^7}\,dx$$

input
```
integrate((d*x+c)*(b*x**2+a*x)**(3/2)/x**7,x)
```

output
```
Integral((x*(a + b*x))**(3/2)*(c + d*x)/x**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(78) = 156.

Time = 0.04 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.47

$$\begin{aligned}\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^7}\,dx &= -\frac{16\,\sqrt{bx^2+ax}b^4c}{315\,a^3x} + \frac{4\,\sqrt{bx^2+ax}b^3d}{35\,a^2x} \\ &+ \frac{8\,\sqrt{bx^2+ax}b^3c}{315\,a^2x^2} - \frac{2\,\sqrt{bx^2+ax}b^2d}{35\,ax^2} - \frac{2\,\sqrt{bx^2+ax}b^2c}{105\,ax^3} + \frac{3\,\sqrt{bx^2+ax}bd}{70\,x^3} \\ &+ \frac{\sqrt{bx^2+ax}bc}{63\,x^4} + \frac{3\,\sqrt{bx^2+ax}ad}{14\,x^4} + \frac{\sqrt{bx^2+ax}ac}{9\,x^5} - \frac{(bx^2+ax)^{\frac{3}{2}}d}{2\,x^5} - \frac{(bx^2+ax)^{\frac{3}{2}}c}{3\,x^6}\end{aligned}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
-16/315*sqrt(b*x^2 + a*x)*b^4*c/(a^3*x) + 4/35*sqrt(b*x^2 + a*x)*b^3*d/(a^
2*x) + 8/315*sqrt(b*x^2 + a*x)*b^3*c/(a^2*x^2) - 2/35*sqrt(b*x^2 + a*x)*b^
2*d/(a*x^2) - 2/105*sqrt(b*x^2 + a*x)*b^2*c/(a*x^3) + 3/70*sqrt(b*x^2 + a*
x)*b*d/x^3 + 1/63*sqrt(b*x^2 + a*x)*b*c/x^4 + 3/14*sqrt(b*x^2 + a*x)*a*d/x
^4 + 1/9*sqrt(b*x^2 + a*x)*a*c/x^5 - 1/2*(b*x^2 + a*x)^(3/2)*d/x^5 - 1/3*(
b*x^2 + a*x)^(3/2)*c/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(78) = 156$.

Time = 0.36 (sec) , antiderivative size = 371, normalized size of antiderivative = 4.12

$$\int \frac{(c+dx)\,(ax+bx^2)^{3/2}}{x^7}\,dx = \frac{2\left(315\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^7 b^{\frac{5}{2}}d+420\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6 b^3c+945\left(\sqrt{}\right.\right.}{}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(3/2)/x^7,x, algorithm="giac")
```

output
```
2/315*(315*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*b^(5/2)*d + 420*(sqrt(b)*x -
sqrt(b*x^2 + a*x))^6*b^3*c + 945*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a*b^2*d
 + 1575*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a*b^(5/2)*c + 1260*(sqrt(b)*x -
sqrt(b*x^2 + a*x))^5*a^2*b^(3/2)*d + 2583*(sqrt(b)*x - sqrt(b*x^2 + a*x))^
4*a^2*b^2*c + 882*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^3*b*d + 2310*(sqrt(b
)*x - sqrt(b*x^2 + a*x))^3*a^3*b^(3/2)*c + 315*(sqrt(b)*x - sqrt(b*x^2 + a
*x))^3*a^4*sqrt(b)*d + 1170*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^4*b*c + 45
*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^5*d + 315*(sqrt(b)*x - sqrt(b*x^2 + a
*x))*a^5*sqrt(b)*c + 35*a^6*c)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^9
```

Mupad [B] (verification not implemented)

Time = 10.62 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.09

$$\begin{aligned}\int \frac{(c+dx)\,(ax+bx^2)^{3/2}}{x^7}\,dx &= \frac{8\,b^3\,c\,\sqrt{b\,x^2+a\,x}}{315\,a^2\,x^2}-\frac{2\,a\,d\,\sqrt{b\,x^2+a\,x}}{7\,x^4}\\ &\quad-\frac{20\,b\,c\,\sqrt{b\,x^2+a\,x}}{63\,x^4}-\frac{16\,b\,d\,\sqrt{b\,x^2+a\,x}}{35\,x^3}-\frac{2\,b^2\,c\,\sqrt{b\,x^2+a\,x}}{105\,a\,x^3}-\frac{2\,a\,c\,\sqrt{b\,x^2+a\,x}}{9\,x^5}\\ &\quad-\frac{16\,b^4\,c\,\sqrt{b\,x^2+a\,x}}{315\,a^3\,x}-\frac{2\,b^2\,d\,\sqrt{b\,x^2+a\,x}}{35\,a\,x^2}+\frac{4\,b^3\,d\,\sqrt{b\,x^2+a\,x}}{35\,a^2\,x}\end{aligned}$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x))/x^7,x)
```

output
```
(8*b^3*c*(a*x + b*x^2)^(1/2))/(315*a^2*x^2) - (2*a*d*(a*x + b*x^2)^(1/2))/
(7*x^4) - (20*b*c*(a*x + b*x^2)^(1/2))/(63*x^4) - (16*b*d*(a*x + b*x^2)^(1
/2))/(35*x^3) - (2*b^2*c*(a*x + b*x^2)^(1/2))/(105*a*x^3) - (2*a*c*(a*x +
b*x^2)^(1/2))/(9*x^5) - (16*b^4*c*(a*x + b*x^2)^(1/2))/(315*a^3*x) - (2*b^
2*d*(a*x + b*x^2)^(1/2))/(35*a*x^2) + (4*b^3*d*(a*x + b*x^2)^(1/2))/(35*a^
2*x)
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.09

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^7}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4c}{9} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4dx}{7} - \frac{20\sqrt{x}\,\sqrt{bx+a}\,a^3bcx}{63} - \frac{16\sqrt{x}\,\sqrt{bx+a}\,a^3bd\,x^2}{35} - 2\sqrt{x}\,\sqrt{}}{}$$

input
```
int((d*x+c)*(b*x^2+a*x)^(3/2)/x^7,x)
```

output
```
(2*( - 35*sqrt(x)*sqrt(a + b*x)*a**4*c - 45*sqrt(x)*sqrt(a + b*x)*a**4*d*x
 - 50*sqrt(x)*sqrt(a + b*x)*a**3*b*c*x - 72*sqrt(x)*sqrt(a + b*x)*a**3*b*d
*x**2 - 3*sqrt(x)*sqrt(a + b*x)*a**2*b**2*c*x**2 - 9*sqrt(x)*sqrt(a + b*x)
*a**2*b**2*d*x**3 + 4*sqrt(x)*sqrt(a + b*x)*a*b**3*c*x**3 + 18*sqrt(x)*sqr
t(a + b*x)*a*b**3*d*x**4 - 8*sqrt(x)*sqrt(a + b*x)*b**4*c*x**4 - 18*sqrt(b
)*a*b**3*d*x**5 + 8*sqrt(b)*b**4*c*x**5))/(315*a**3*x**5)
```

3.66 $\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^8}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 125

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^8}\,dx = -\frac{2c\left(ax+bx^2\right)^{5/2}}{11ax^8}+\frac{2(6bc-11ad)\left(ax+bx^2\right)^{5/2}}{99a^2x^7} - \frac{8b(6bc-11ad)\left(ax+bx^2\right)^{5/2}}{693a^3x^6}+\frac{16b^2(6bc-11ad)\left(ax+bx^2\right)^{5/2}}{3465a^4x^5}$$

output
```
-2/11*c*(b*x^2+a*x)^(5/2)/a/x^8+2/99*(-11*a*d+6*b*c)*(b*x^2+a*x)^(5/2)/a^2
/x^7-8/693*b*(-11*a*d+6*b*c)*(b*x^2+a*x)^(5/2)/a^3/x^6+16/3465*b^2*(-11*a*
d+6*b*c)*(b*x^2+a*x)^(5/2)/a^4/x^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.61

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^8}\,dx = -\frac{2(x(a+bx))^{5/2}\left(-48b^3cx^3+35a^3(9c+11dx)+8ab^2x^2(15c+11dx)-10a^2bx(21c+22dx)\right)}{3465a^4x^8}$$

input
```
Integrate[((c + d*x)*(a*x + b*x^2)^(3/2))/x^8,x]
```

output
```
(-2*(x*(a + b*x))^(5/2)*(-48*b^3*c*x^3 + 35*a^3*(9*c + 11*d*x) + 8*a*b^2*x
^2*(15*c + 11*d*x) - 10*a^2*b*x*(21*c + 22*d*x)))/(3465*a^4*x^8)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1220, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}(c+dx)}{x^8}dx$$

$$\Big\downarrow \text{1220}$$

$$-\frac{(6bc-11ad)\int \frac{(bx^2+ax)^{3/2}}{x^7}dx}{11a}-\frac{2c(ax+bx^2)^{5/2}}{11ax^8}$$

$$\Big\downarrow \text{1129}$$

$$-\frac{(6bc-11ad)\left(-\frac{4b\int \frac{(bx^2+ax)^{3/2}}{x^6}dx}{9a}-\frac{2(ax+bx^2)^{5/2}}{9ax^7}\right)}{11a}-\frac{2c(ax+bx^2)^{5/2}}{11ax^8}$$

$$\Big\downarrow \text{1129}$$

$$-\frac{(6bc-11ad)\left(-\frac{4b\left(-\frac{2b\int \frac{(bx^2+ax)^{3/2}}{x^5}dx}{7a}-\frac{2(ax+bx^2)^{5/2}}{7ax^6}\right)}{9a}-\frac{2(ax+bx^2)^{5/2}}{9ax^7}\right)}{11a}-\frac{2c(ax+bx^2)^{5/2}}{11ax^8}$$

$$\Big\downarrow \text{1123}$$

$$-\frac{\left(-\frac{4b\left(\frac{4b(ax+bx^2)^{5/2}}{35a^2x^5}-\frac{2(ax+bx^2)^{5/2}}{7ax^6}\right)}{9a}-\frac{2(ax+bx^2)^{5/2}}{9ax^7}\right)(6bc-11ad)}{11a}-\frac{2c(ax+bx^2)^{5/2}}{11ax^8}$$

input
```
Int[((c + d*x)*(a*x + b*x^2)^(3/2))/x^8,x]
```

output
```
(-2*c*(a*x + b*x^2)^(5/2))/(11*a*x^8) - ((6*b*c - 11*a*d)*((-2*(a*x + b*x^
2)^(5/2))/(9*a*x^7) - (4*b*((-2*(a*x + b*x^2)^(5/2))/(7*a*x^6) + (4*b*(a*x
 + b*x^2)^(5/2))/(35*a^2*x^5)))/(9*a)))/(11*a)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.58

method	result
pseudoelliptic	$-\frac{2\sqrt{x(bx+a)}\,(bx+a)^2\left(\left(\frac{11dx}{9}+c\right)a^3-\frac{2x\left(\frac{22dx}{21}+c\right)b\,a^2}{3}+\frac{8x^2b^2\left(\frac{11dx}{15}+c\right)a}{21}-\frac{16b^3c\,x^3}{105}\right)}{11x^6a^4}$
gosper	$-\frac{2(bx+a)\left(88a\,b^2d\,x^3-48b^3c\,x^3-220a^2bd\,x^2+120a\,b^2c\,x^2+385a^3dx-210a^2bcx+315c\,a^3\right)\left(b\,x^2+ax\right)^{\frac{3}{2}}}{3465x^7a^4}$
orering	$-\frac{2(bx+a)\left(88a\,b^2d\,x^3-48b^3c\,x^3-220a^2bd\,x^2+120a\,b^2c\,x^2+385a^3dx-210a^2bcx+315c\,a^3\right)\left(b\,x^2+ax\right)^{\frac{3}{2}}}{3465x^7a^4}$
trager	$-\frac{2\left(88a\,b^4d\,x^5-48b^5c\,x^5-44x^4a^2b^3d+24x^4a\,b^4c+33a^3b^2d\,x^3-18a^2b^3c\,x^3+550a^4bd\,x^2+15a^3b^2c\,x^2+385a^5dx+420a^4bcx\right.}{3465a^4x^6}$
risch	$-\frac{2(bx+a)\left(88a\,b^4d\,x^5-48b^5c\,x^5-44x^4a^2b^3d+24x^4a\,b^4c+33a^3b^2d\,x^3-18a^2b^3c\,x^3+550a^4bd\,x^2+15a^3b^2c\,x^2+385a^5dx+4\right.}{3465x^5\sqrt{x(bx+a)}\,a^4}$
default	$c\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{11a\,x^8}-\frac{6b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{9a\,x^7}-\frac{4b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{7a\,x^6}+\frac{4b\left(b\,x^2+ax\right)^{\frac{5}{2}}}{35a^2x^5}\right)}{9a}\right)}{11a}\right)+d\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{9a\,x^7}-\frac{4b\left(\right.}{}\right.$

input
```
int((d*x+c)*(b*x^2+a*x)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-2/11*(x*(b*x+a))^(1/2)*(b*x+a)^2*((11/9*d*x+c)*a^3-2/3*x*(22/21*d*x+c)*b*
a^2+8/21*x^2*b^2*(11/15*d*x+c)*a-16/105*b^3*c*x^3)/x^6/a^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^8}\,dx =$$

$$-\frac{2\left(315\,a^5c-8\left(6\,b^5c-11\,ab^4d\right)x^5+4\left(6\,ab^4c-11\,a^2b^3d\right)x^4-3\left(6\,a^2b^3c-11\,a^3b^2d\right)x^3+5\left(3\,a^3b^2c+110\right.\right.}{3465\,a^4x^6}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
-2/3465*(315*a^5*c - 8*(6*b^5*c - 11*a*b^4*d)*x^5 + 4*(6*a*b^4*c - 11*a^2*
b^3*d)*x^4 - 3*(6*a^2*b^3*c - 11*a^3*b^2*d)*x^3 + 5*(3*a^3*b^2*c + 110*a^4
*b*d)*x^2 + 35*(12*a^4*b*c + 11*a^5*d)*x)*sqrt(b*x^2 + a*x)/(a^4*x^6)
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^8}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}\,(c+dx)}{x^8}\,dx$$

input
```
integrate((d*x+c)*(b*x**2+a*x)**(3/2)/x**8,x)
```

output
```
Integral((x*(a + b*x))**(3/2)*(c + d*x)/x**8, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(109) = 218.

Time = 0.04 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.14

$$\begin{aligned}\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^8}\,dx &= \frac{32\,\sqrt{bx^2+ax}b^5c}{1155\,a^4x} - \frac{16\,\sqrt{bx^2+ax}b^4d}{315\,a^3x} \\ &- \frac{16\,\sqrt{bx^2+ax}b^4c}{1155\,a^3x^2} + \frac{8\,\sqrt{bx^2+ax}b^3d}{315\,a^2x^2} + \frac{4\,\sqrt{bx^2+ax}b^3c}{385\,a^2x^3} \\ &- \frac{2\,\sqrt{bx^2+ax}b^2d}{105\,ax^3} - \frac{2\,\sqrt{bx^2+ax}b^2c}{231\,ax^4} + \frac{\sqrt{bx^2+ax}bd}{63\,x^4} + \frac{\sqrt{bx^2+ax}bc}{132\,x^5} \\ &+ \frac{\sqrt{bx^2+ax}ad}{9\,x^5} + \frac{3\,\sqrt{bx^2+ax}ac}{44\,x^6} - \frac{\left(bx^2+ax\right)^{\frac{3}{2}}d}{3\,x^6} - \frac{\left(bx^2+ax\right)^{\frac{3}{2}}c}{4\,x^7}\end{aligned}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
32/1155*sqrt(b*x^2 + a*x)*b^5*c/(a^4*x) - 16/315*sqrt(b*x^2 + a*x)*b^4*d/(
a^3*x) - 16/1155*sqrt(b*x^2 + a*x)*b^4*c/(a^3*x^2) + 8/315*sqrt(b*x^2 + a*
x)*b^3*d/(a^2*x^2) + 4/385*sqrt(b*x^2 + a*x)*b^3*c/(a^2*x^3) - 2/105*sqrt(
b*x^2 + a*x)*b^2*d/(a*x^3) - 2/231*sqrt(b*x^2 + a*x)*b^2*c/(a*x^4) + 1/63*
sqrt(b*x^2 + a*x)*b*d/x^4 + 1/132*sqrt(b*x^2 + a*x)*b*c/x^5 + 1/9*sqrt(b*x
^2 + a*x)*a*d/x^5 + 3/44*sqrt(b*x^2 + a*x)*a*c/x^6 - 1/3*(b*x^2 + a*x)^(3/
2)*d/x^6 - 1/4*(b*x^2 + a*x)^(3/2)*c/x^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(109) = 218$.

Time = 0.13 (sec) , antiderivative size = 431, normalized size of antiderivative = 3.45

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^8}\,dx = \frac{2\left(4620\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^8 b^3d + 6930\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^7 b^{\frac{7}{2}}c + 1732\right.}{}$$

input
```
integrate((d*x+c)*(b*x^2+a*x)^(3/2)/x^8,x, algorithm="giac")
```

output
```
2/3465*(4620*(sqrt(b)*x - sqrt(b*x^2 + a*x))^8*b^3*d + 6930*(sqrt(b)*x - s
qrt(b*x^2 + a*x))^7*b^(7/2)*c + 17325*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*a*
b^(5/2)*d + 30492*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a*b^3*c + 28413*(sqrt(
b)*x - sqrt(b*x^2 + a*x))^6*a^2*b^2*d + 58905*(sqrt(b)*x - sqrt(b*x^2 + a*
x))^5*a^2*b^(5/2)*c + 25410*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a^3*b^(3/2)*
d + 63855*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^3*b^2*c + 12870*(sqrt(b)*x -
 sqrt(b*x^2 + a*x))^4*a^4*b*d + 41580*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^
4*b^(3/2)*c + 3465*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^5*sqrt(b)*d + 16170
*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^5*b*c + 385*(sqrt(b)*x - sqrt(b*x^2 +
 a*x))^2*a^6*d + 3465*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^6*sqrt(b)*c + 315*
a^7*c)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^11
```

Mupad [B] (verification not implemented)

Time = 11.07 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.87

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^8}\,dx = \frac{4\,b^3\,c\,\sqrt{b\,x^2+a\,x}}{385\,a^2\,x^3}-\frac{2\,a\,d\,\sqrt{b\,x^2+a\,x}}{9\,x^5}-\frac{8\,b\,c\,\sqrt{b\,x^2+a\,x}}{33\,x^5}-\frac{20\,b\,d\,\sqrt{b\,x^2+a\,x}}{63\,x^4}-\frac{2\,b^2\,c\,\sqrt{b\,x^2+a\,x}}{231\,a\,x^4}-\frac{2\,a\,c\,\sqrt{b\,x^2+a\,x}}{11\,x^6}-\frac{16\,b^4\,c\,\sqrt{b\,x^2+a\,x}}{1155\,a^3\,x^2}+\frac{32\,b^5\,c\,\sqrt{b\,x^2+a\,x}}{1155\,a^4\,x}-\frac{2\,b^2\,d\,\sqrt{b\,x^2+a\,x}}{105\,a\,x^3}+\frac{8\,b^3\,d\,\sqrt{b\,x^2+a\,x}}{315\,a^2\,x^2}-\frac{16\,b^4\,d\,\sqrt{b\,x^2+a\,x}}{315\,a^3\,x}$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x))/x^8,x)
```

output
```
(4*b^3*c*(a*x + b*x^2)^(1/2))/(385*a^2*x^3) - (2*a*d*(a*x + b*x^2)^(1/2))/
(9*x^5) - (8*b*c*(a*x + b*x^2)^(1/2))/(33*x^5) - (20*b*d*(a*x + b*x^2)^(1/
2))/(63*x^4) - (2*b^2*c*(a*x + b*x^2)^(1/2))/(231*a*x^4) - (2*a*c*(a*x + b
*x^2)^(1/2))/(11*x^6) - (16*b^4*c*(a*x + b*x^2)^(1/2))/(1155*a^3*x^2) + (3
2*b^5*c*(a*x + b*x^2)^(1/2))/(1155*a^4*x) - (2*b^2*d*(a*x + b*x^2)^(1/2))/
(105*a*x^3) + (8*b^3*d*(a*x + b*x^2)^(1/2))/(315*a^2*x^2) - (16*b^4*d*(a*x
 + b*x^2)^(1/2))/(315*a^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.82

$$\int \frac{(c+dx)\left(ax+bx^2\right)^{3/2}}{x^8}\,dx = -\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^5c}{11}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^5dx}{9}-\frac{8\sqrt{x}\,\sqrt{bx+a}\,a^4bcx}{33}-\frac{20\sqrt{x}\,\sqrt{bx+a}\,a^4bd\,x^2}{63}-2\sqrt{x}\,\sqrt{b\cdots}$$

input
```
int((d*x+c)*(b*x^2+a*x)^(3/2)/x^8,x)
```

output
```
(2*( - 315*sqrt(x)*sqrt(a + b*x)*a**5*c - 385*sqrt(x)*sqrt(a + b*x)*a**5*d
*x - 420*sqrt(x)*sqrt(a + b*x)*a**4*b*c*x - 550*sqrt(x)*sqrt(a + b*x)*a**4
*b*d*x**2 - 15*sqrt(x)*sqrt(a + b*x)*a**3*b**2*c*x**2 - 33*sqrt(x)*sqrt(a
+ b*x)*a**3*b**2*d*x**3 + 18*sqrt(x)*sqrt(a + b*x)*a**2*b**3*c*x**3 + 44*s
qrt(x)*sqrt(a + b*x)*a**2*b**3*d*x**4 - 24*sqrt(x)*sqrt(a + b*x)*a*b**4*c*
x**4 - 88*sqrt(x)*sqrt(a + b*x)*a*b**4*d*x**5 + 48*sqrt(x)*sqrt(a + b*x)*b
**5*c*x**5 + 88*sqrt(b)*a*b**4*d*x**6 - 48*sqrt(b)*b**5*c*x**6))/(3465*a**
4*x**6)
```

3.67 $\int x^2(c+dx)^2 \left(ax+bx^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 393

$$\begin{aligned}\int x^2(c+dx)^2 \left(ax+bx^2\right)^{3/2} \, dx = & -\frac{a^5(224b^2c^2-9ad(32bc-11ad))\sqrt{ax+bx^2}}{16384b^6} \\ & +\frac{a^4(224b^2c^2-9ad(32bc-11ad))\,x\sqrt{ax+bx^2}}{24576b^5} \\ & -\frac{a^3(224b^2c^2-9ad(32bc-11ad))\,x^2\sqrt{ax+bx^2}}{30720b^4} \\ & +\frac{a^2(224b^2c^2-9ad(32bc-11ad))\,x^3\sqrt{ax+bx^2}}{35840b^3} \\ & +\frac{13a(224b^2c^2-9ad(32bc-11ad))\,x^4\sqrt{ax+bx^2}}{13440b^2} \\ & +\frac{(224b^2c^2-9ad(32bc-11ad))\,x^5\sqrt{ax+bx^2}}{1344b} \\ & +\frac{d(32bc-11ad)x^2\left(ax+bx^2\right)^{5/2}}{112b^2}+\frac{d^2x^3\left(ax+bx^2\right)^{5/2}}{8b} \\ & +\frac{a^6(224b^2c^2-9ad(32bc-11ad))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{16384b^{13/2}}\end{aligned}$$

output
```
-1/16384*a^5*(224*b^2*c^2-9*a*d*(-11*a*d+32*b*c))*(b*x^2+a*x)^(1/2)/b^6+1/
24576*a^4*(224*b^2*c^2-9*a*d*(-11*a*d+32*b*c))*x*(b*x^2+a*x)^(1/2)/b^5-1/3
0720*a^3*(224*b^2*c^2-9*a*d*(-11*a*d+32*b*c))*x^2*(b*x^2+a*x)^(1/2)/b^4+1/
35840*a^2*(224*b^2*c^2-9*a*d*(-11*a*d+32*b*c))*x^3*(b*x^2+a*x)^(1/2)/b^3+1
3/13440*a*(224*b^2*c^2-9*a*d*(-11*a*d+32*b*c))*x^4*(b*x^2+a*x)^(1/2)/b^2+1
/1344*(224*b^2*c^2-9*a*d*(-11*a*d+32*b*c))*x^5*(b*x^2+a*x)^(1/2)/b+1/112*d
*(-11*a*d+32*b*c)*x^2*(b*x^2+a*x)^(5/2)/b^2+1/8*d^2*x^3*(b*x^2+a*x)^(5/2)/
b+1/16384*a^6*(224*b^2*c^2-9*a*d*(-11*a*d+32*b*c))*arctanh(b^(1/2)*x/(b*x^
2+a*x)^(1/2))/b^(13/2)
```

Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.87

$$\int x^2(c+dx)^2\left(ax+bx^2\right)^{3/2}\,dx=\frac{\sqrt{x}\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\sqrt{a+bx}(-10395a^7d^2+630a^6bd(48c+11dx)+768a^2b^5x^3(14c^2+16cdx\right.}{}$$

input
```
Integrate[x^2*(c + d*x)^2*(a*x + b*x^2)^(3/2),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-10395*a^7*d^2 + 63
0*a^6*b*d*(48*c + 11*d*x) + 768*a^2*b^5*x^3*(14*c^2 + 16*c*d*x + 5*d^2*x^2
) + 10240*b^7*x^5*(28*c^2 + 48*c*d*x + 21*d^2*x^2) - 128*a^3*b^4*x^2*(98*c
^2 + 108*c*d*x + 33*d^2*x^2) - 168*a^5*b^2*(140*c^2 + 120*c*d*x + 33*d^2*x
^2) + 1024*a*b^6*x^4*(364*c^2 + 600*c*d*x + 255*d^2*x^2) + 16*a^4*b^3*x*(9
80*c^2 + 1008*c*d*x + 297*d^2*x^2)) + 60480*a^7*b*c*d*ArcTanh[(Sqrt[b]*Sqr
t[x])/(Sqrt[a] - Sqrt[a + b*x])] + 210*a^6*(224*b^2*c^2 + 99*a^2*d^2)*ArcT
anh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(1720320*b^(13/2)*Sqrt
[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.65, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1262, 27, 1221, 1134, 1160, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(ax+bx^2\right)^{3/2} (c+dx)^2 \, dx$$

$$\Big\downarrow \text{ 1262}$$

$$\frac{\int \frac{1}{2}x^2\left(16bc^2+d(32bc-11ad)x\right)\left(bx^2+ax\right)^{3/2}dx}{8b}+\frac{d^2x^3\left(ax+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int x^2\left(16bc^2+d(32bc-11ad)x\right)\left(bx^2+ax\right)^{3/2}dx}{16b}+\frac{d^2x^3\left(ax+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \text{ 1221}$$

$$\frac{\frac{\left(99a^2d^2-288abcd+224b^2c^2\right)\int x^2\left(bx^2+ax\right)^{3/2}dx}{14b}+\frac{dx^2\left(ax+bx^2\right)^{5/2}(32bc-11ad)}{7b}}{16b}+\frac{d^2x^3\left(ax+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \text{ 1134}$$

$$\frac{\frac{\left(99a^2d^2-288abcd+224b^2c^2\right)\left(\frac{x\left(ax+bx^2\right)^{5/2}}{6b}-\frac{7a\int x\left(bx^2+ax\right)^{3/2}dx}{12b}\right)}{14b}+\frac{dx^2\left(ax+bx^2\right)^{5/2}(32bc-11ad)}{7b}}{16b}+\frac{d^2x^3\left(ax+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{\frac{\left(99a^2d^2-288abcd+224b^2c^2\right)\left(\frac{x\left(ax+bx^2\right)^{5/2}}{6b}-\frac{7a\left(\frac{\left(ax+bx^2\right)^{5/2}}{5b}-\frac{a\int\left(bx^2+ax\right)^{3/2}dx}{2b}\right)}{12b}\right)}{14b}+\frac{dx^2\left(ax+bx^2\right)^{5/2}(32bc-11ad)}{7b}}{16b}+\frac{d^2x^3\left(ax+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{\frac{\left(99a^2d^2-288abcd+224b^2c^2\right)\left(\frac{x\left(ax+bx^2\right)^{5/2}}{6b}-\frac{7a\left(\frac{\left(ax+bx^2\right)^{5/2}}{5b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\int\sqrt{bx^2+ax}dx}{16b}\right)}{2b}\right)}{12b}\right)}{14b}+\frac{dx^2\left(ax+bx^2\right)^{5/2}(32bc-1}{7b}}{16b}$$

$$\frac{d^2x^3\left(ax+bx^2\right)^{5/2}}{8b}$$

$\downarrow$ 1087

$$\frac{\frac{\left(99a^2d^2-288abcd+224b^2c^2\right)\left(\frac{x\left(ax+bx^2\right)^{5/2}}{6b}-\frac{7a\left(\frac{\left(ax+bx^2\right)^{5/2}}{5b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)}{16b}\right)}{2b}\right)}{12b}\right)}{14b}+}{16b}$$

$$\frac{d^2x^3\left(ax+bx^2\right)^{5/2}}{8b}$$

$\downarrow$ 1091

$$\frac{\frac{\left(99a^2d^2-288abcd+224b^2c^2\right)\left(\frac{x\left(ax+bx^2\right)^{5/2}}{6b}-\frac{7a\left(\frac{\left(ax+bx^2\right)^{5/2}}{5b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)}{16b}\right)}{2b}\right)}{12b}\right)}{14b}}{16b}$$

$$\frac{d^2x^3\left(ax+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\left(\frac{x\left(ax+bx^2\right)^{5/2}}{6b}-\frac{7a\left(\frac{\left(ax+bx^2\right)^{5/2}}{5b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)}{16b}\right)}{2b}\right)}{12b}\right)\left(99a^2d^2-288abcd+224b\right.}{14b}}{16b}$$

$$\frac{d^2x^3\left(ax+bx^2\right)^{5/2}}{8b}$$

input
```
Int[x^2*(c + d*x)^2*(a*x + b*x^2)^(3/2),x]
```

output
```
(d^2*x^3*(a*x + b*x^2)^(5/2))/(8*b) + ((d*(32*b*c - 11*a*d)*x^2*(a*x + b*x
^2)^(5/2))/(7*b) + ((224*b^2*c^2 - 288*a*b*c*d + 99*a^2*d^2)*((x*(a*x + b*
x^2)^(5/2))/(6*b) - (7*a*((a*x + b*x^2)^(5/2)/(5*b) - (a*(((a + 2*b*x)*(a*
x + b*x^2)^(3/2))/(8*b) - (3*a^2*(((a + 2*b*x)*Sqrt[a*x + b*x^2])/(4*b) -
(a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*b^(3/2))))/(16*b)))/(2*b))
)/(12*b)))/(14*b))/(16*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$\frac{\frac{99a^{6}\left(a^{2}d^{2}-\frac{32}{11}abcd+\frac{224}{99}b^{2}c^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{16384}-\frac{99\left(-\frac{512\left(\frac{5}{14}d^{2}x^{2}+\frac{8}{7}cdx+c^{2}\right)x^{3}a^{2}b^{\frac{11}{2}}}{495}-\frac{53248\left(\frac{255}{364}d^{2}x^{2}+\frac{150}{91}cdx+c^{2}\right)x^{4}}{1485}\right.}{}}{}$
risch	$-\frac{\left(-215040b^{7}d^{2}x^{7}-261120a\,b^{6}d^{2}x^{6}-491520b^{7}cd\,x^{6}-3840a^{2}b^{5}d^{2}x^{5}-614400a\,b^{6}cd\,x^{5}-286720b^{7}c^{2}x^{5}+4224a^{3}b^{4}d^{2}x^{4}-\right.}{}$
default	$c^{2}\left(\frac{x\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{6b}-\frac{7a\left(\frac{\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{5b}-\frac{a\left(\frac{(2bx+a)\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{8b}-\frac{3a^{2}\left(\frac{(2bx+a)\sqrt{b\,x^{2}+ax}}{4b}-\frac{a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8b^{\frac{3}{2}}}\right)}{16b}\right)}{2b}\right)}{12b}\right.$

input
```
int(x^2*(d*x+c)^2*(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
99/16384/b^(13/2)*(a^6*(a^2*d^2-32/11*a*b*c*d+224/99*b^2*c^2)*arctanh((x*(
b*x+a))^(1/2)/x/b^(1/2))-(-512/495*(5/14*d^2*x^2+8/7*c*d*x+c^2)*x^3*a^2*b^
(11/2)-53248/1485*(255/364*d^2*x^2+150/91*c*d*x+c^2)*x^4*a*b^(13/2)-8192/2
97*x^5*(3/4*d^2*x^2+12/7*c*d*x+c^2)*b^(15/2)+(224/99*(33/140*d^2*x^2+6/7*c
*d*x+c^2)*a^2*b^(5/2)-448/297*x*(297/980*d^2*x^2+36/35*c*d*x+c^2)*a*b^(7/2
)+1792/1485*(33/98*d^2*x^2+54/49*c*d*x+c^2)*x^2*b^(9/2)+d*a^3*((-2/3*d*x-3
2/11*c)*b^(3/2)+b^(1/2)*a*d))*a^3)*(x*(b*x+a))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.64

$$\int x^2(c+dx)^2 \left(ax+bx^2\right)^{3/2} dx = \left[\frac{105 \left(224 a^6b^2c^2 - 288 a^7bcd + 99 a^8d^2\right)\sqrt{b} \log \left(2 bx + a + 2 \sqrt{bx^2+ax}\sqrt{b}\right) + 2 \left(215040 b^8 d\right.}{} \right.$$

$$- \frac{105 \left(224 a^6b^2c^2 - 288 a^7bcd + 99 a^8d^2\right)\sqrt{-b} \arctan \left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right) - \left(215040 b^8d^2x^7 - 23520 a^5b^3c^2 + 302\right.}{}$$

input
```
integrate(x^2*(d*x+c)^2*(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[1/3440640*(105*(224*a^6*b^2*c^2 - 288*a^7*b*c*d + 99*a^8*d^2)*sqrt(b)*log
(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(215040*b^8*d^2*x^7 - 23520*
a^5*b^3*c^2 + 30240*a^6*b^2*c*d - 10395*a^7*b*d^2 + 15360*(32*b^8*c*d + 17
*a*b^7*d^2)*x^6 + 1280*(224*b^8*c^2 + 480*a*b^7*c*d + 3*a^2*b^6*d^2)*x^5 +
 128*(2912*a*b^7*c^2 + 96*a^2*b^6*c*d - 33*a^3*b^5*d^2)*x^4 + 48*(224*a^2*
b^6*c^2 - 288*a^3*b^5*c*d + 99*a^4*b^4*d^2)*x^3 - 56*(224*a^3*b^5*c^2 - 28
8*a^4*b^4*c*d + 99*a^5*b^3*d^2)*x^2 + 70*(224*a^4*b^4*c^2 - 288*a^5*b^3*c*
d + 99*a^6*b^2*d^2)*x)*sqrt(b*x^2 + a*x))/b^7, -1/1720320*(105*(224*a^6*b^
2*c^2 - 288*a^7*b*c*d + 99*a^8*d^2)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt
(-b)/(b*x + a)) - (215040*b^8*d^2*x^7 - 23520*a^5*b^3*c^2 + 30240*a^6*b^2*
c*d - 10395*a^7*b*d^2 + 15360*(32*b^8*c*d + 17*a*b^7*d^2)*x^6 + 1280*(224*
b^8*c^2 + 480*a*b^7*c*d + 3*a^2*b^6*d^2)*x^5 + 128*(2912*a*b^7*c^2 + 96*a^
2*b^6*c*d - 33*a^3*b^5*d^2)*x^4 + 48*(224*a^2*b^6*c^2 - 288*a^3*b^5*c*d +
99*a^4*b^4*d^2)*x^3 - 56*(224*a^3*b^5*c^2 - 288*a^4*b^4*c*d + 99*a^5*b^3*d
^2)*x^2 + 70*(224*a^4*b^4*c^2 - 288*a^5*b^3*c*d + 99*a^6*b^2*d^2)*x)*sqrt(
b*x^2 + a*x))/b^7]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 768 vs. $2(382) = 764$.

Time = 0.55 (sec) , antiderivative size = 768, normalized size of antiderivative = 1.95

$$\int x^2(c+dx)^2\left(ax+bx^2\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(d*x+c)**2*(b*x**2+a*x)**(3/2),x)
```

output
```
Piecewise((35*a**4*(a**2*c**2 - 9*a*(2*a**2*c*d + 2*a*b*c**2 - 11*a*(a**2*
d**2 + 4*a*b*c*d - 13*a*(17*a*b*d**2/16 + 2*b**2*c*d)/(14*b) + b**2*c**2)/
(12*b))/(10*b))*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/s
qrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) +
x)**2), True))/(128*b**4) + sqrt(a*x + b*x**2)*(-35*a**3*(a**2*c**2 - 9*a*
(2*a**2*c*d + 2*a*b*c**2 - 11*a*(a**2*d**2 + 4*a*b*c*d - 13*a*(17*a*b*d**2
/16 + 2*b**2*c*d)/(14*b) + b**2*c**2)/(12*b))/(10*b))/(64*b**4) + 35*a**2*
x*(a**2*c**2 - 9*a*(2*a**2*c*d + 2*a*b*c**2 - 11*a*(a**2*d**2 + 4*a*b*c*d
- 13*a*(17*a*b*d**2/16 + 2*b**2*c*d)/(14*b) + b**2*c**2)/(12*b))/(10*b))/(
96*b**3) - 7*a*x**2*(a**2*c**2 - 9*a*(2*a**2*c*d + 2*a*b*c**2 - 11*a*(a**2
*d**2 + 4*a*b*c*d - 13*a*(17*a*b*d**2/16 + 2*b**2*c*d)/(14*b) + b**2*c**2)
/(12*b))/(10*b))/(24*b**2) + b*d**2*x**7/8 + x**6*(17*a*b*d**2/16 + 2*b**2
*c*d)/(7*b) + x**5*(a**2*d**2 + 4*a*b*c*d - 13*a*(17*a*b*d**2/16 + 2*b**2*
c*d)/(14*b) + b**2*c**2)/(6*b) + x**4*(2*a**2*c*d + 2*a*b*c**2 - 11*a*(a**
2*d**2 + 4*a*b*c*d - 13*a*(17*a*b*d**2/16 + 2*b**2*c*d)/(14*b) + b**2*c**2
)/(12*b))/(5*b) + x**3*(a**2*c**2 - 9*a*(2*a**2*c*d + 2*a*b*c**2 - 11*a*(a
**2*d**2 + 4*a*b*c*d - 13*a*(17*a*b*d**2/16 + 2*b**2*c*d)/(14*b) + b**2*c*
*2)/(12*b))/(10*b))/(4*b)), Ne(b, 0)), (2*(c**2*(a*x)**(9/2)/9 + 2*c*d*(a*
x)**(11/2)/(11*a) + d**2*(a*x)**(13/2)/(13*a**2))/a**3, Ne(a, 0)), (0, Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.42

$$\int x^2(c+dx)^2\left(ax+bx^2\right)^{3/2}\,dx = \frac{(bx^2+ax)^{\frac{5}{2}}d^2x^3}{8\,b}+\frac{2\,(bx^2+ax)^{\frac{5}{2}}cdx^2}{7\,b} - \frac{11\,(bx^2+ax)^{\frac{5}{2}}ad^2x^2}{112\,b^2}-\frac{7\,\sqrt{bx^2+ax}a^4c^2x}{256\,b^3}+\frac{7\,(bx^2+ax)^{\frac{3}{2}}a^2c^2x}{96\,b^2} + \frac{(bx^2+ax)^{\frac{5}{2}}c^2x}{6\,b}+\frac{9\,\sqrt{bx^2+ax}a^5cdx}{256\,b^4}-\frac{3\,(bx^2+ax)^{\frac{3}{2}}a^3cdx}{32\,b^3} - \frac{3\,(bx^2+ax)^{\frac{5}{2}}acdx}{14\,b^2}-\frac{99\,\sqrt{bx^2+ax}a^6d^2x}{8192\,b^5}+\frac{33\,(bx^2+ax)^{\frac{3}{2}}a^4d^2x}{1024\,b^4} + \frac{33\,(bx^2+ax)^{\frac{5}{2}}a^2d^2x}{448\,b^3}+\frac{7\,a^6c^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{1024\,b^{\frac{9}{2}}} - \frac{9\,a^7cd\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{1024\,b^{\frac{11}{2}}} + \frac{99\,a^8d^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{32768\,b^{\frac{13}{2}}}-\frac{7\,\sqrt{bx^2+ax}a^5c^2}{512\,b^4} + \frac{7\,(bx^2+ax)^{\frac{3}{2}}a^3c^2}{192\,b^3}-\frac{7\,(bx^2+ax)^{\frac{5}{2}}ac^2}{60\,b^2}+\frac{9\,\sqrt{bx^2+ax}a^6cd}{512\,b^5} - \frac{3\,(bx^2+ax)^{\frac{3}{2}}a^4cd}{64\,b^4}+\frac{3\,(bx^2+ax)^{\frac{5}{2}}a^2cd}{20\,b^3}-\frac{99\,\sqrt{bx^2+ax}a^7d^2}{16384\,b^6} + \frac{33\,(bx^2+ax)^{\frac{3}{2}}a^5d^2}{2048\,b^5}-\frac{33\,(bx^2+ax)^{\frac{5}{2}}a^3d^2}{640\,b^4}$$

input
```
integrate(x^2*(d*x+c)^2*(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
1/8*(b*x^2 + a*x)^(5/2)*d^2*x^3/b + 2/7*(b*x^2 + a*x)^(5/2)*c*d*x^2/b - 11
/112*(b*x^2 + a*x)^(5/2)*a*d^2*x^2/b^2 - 7/256*sqrt(b*x^2 + a*x)*a^4*c^2*x
/b^3 + 7/96*(b*x^2 + a*x)^(3/2)*a^2*c^2*x/b^2 + 1/6*(b*x^2 + a*x)^(5/2)*c^
2*x/b + 9/256*sqrt(b*x^2 + a*x)*a^5*c*d*x/b^4 - 3/32*(b*x^2 + a*x)^(3/2)*a
^3*c*d*x/b^3 - 3/14*(b*x^2 + a*x)^(5/2)*a*c*d*x/b^2 - 99/8192*sqrt(b*x^2 +
 a*x)*a^6*d^2*x/b^5 + 33/1024*(b*x^2 + a*x)^(3/2)*a^4*d^2*x/b^4 + 33/448*(
b*x^2 + a*x)^(5/2)*a^2*d^2*x/b^3 + 7/1024*a^6*c^2*log(2*b*x + a + 2*sqrt(b
*x^2 + a*x)*sqrt(b))/b^(9/2) - 9/1024*a^7*c*d*log(2*b*x + a + 2*sqrt(b*x^2
 + a*x)*sqrt(b))/b^(11/2) + 99/32768*a^8*d^2*log(2*b*x + a + 2*sqrt(b*x^2
+ a*x)*sqrt(b))/b^(13/2) - 7/512*sqrt(b*x^2 + a*x)*a^5*c^2/b^4 + 7/192*(b*
x^2 + a*x)^(3/2)*a^3*c^2/b^3 - 7/60*(b*x^2 + a*x)^(5/2)*a*c^2/b^2 + 9/512*
sqrt(b*x^2 + a*x)*a^6*c*d/b^5 - 3/64*(b*x^2 + a*x)^(3/2)*a^4*c*d/b^4 + 3/2
0*(b*x^2 + a*x)^(5/2)*a^2*c*d/b^3 - 99/16384*sqrt(b*x^2 + a*x)*a^7*d^2/b^6
 + 33/2048*(b*x^2 + a*x)^(3/2)*a^5*d^2/b^5 - 33/640*(b*x^2 + a*x)^(5/2)*a^
3*d^2/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.88

$$\int x^2(c+dx)^2\,(ax+bx^2)^{3/2}\,dx = \frac{1}{1720320}\sqrt{bx^2+ax}\left(2\left(4\left(2\left(8\left(10\left(12\left(14\,bd^2x+\frac{32\,b^8cd+17\,ab^7d^2}{b^7}\right)x+\frac{224\,b^8c^2+48\cdots}{}\right.\right.\right.\right.\right.\right.$$
$$-\frac{(224\,a^6b^2c^2-288\,a^7bcd+99\,a^8d^2)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{32768\,b^{\frac{13}{2}}}$$

input
```
integrate(x^2*(d*x+c)^2*(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
1/1720320*sqrt(b*x^2 + a*x)*(2*(4*(2*(8*(10*(12*(14*b*d^2*x + (32*b^8*c*d
+ 17*a*b^7*d^2)/b^7)*x + (224*b^8*c^2 + 480*a*b^7*c*d + 3*a^2*b^6*d^2)/b^7
)*x + (2912*a*b^7*c^2 + 96*a^2*b^6*c*d - 33*a^3*b^5*d^2)/b^7)*x + 3*(224*a
^2*b^6*c^2 - 288*a^3*b^5*c*d + 99*a^4*b^4*d^2)/b^7)*x - 7*(224*a^3*b^5*c^2
 - 288*a^4*b^4*c*d + 99*a^5*b^3*d^2)/b^7)*x + 35*(224*a^4*b^4*c^2 - 288*a^
5*b^3*c*d + 99*a^6*b^2*d^2)/b^7)*x - 105*(224*a^5*b^3*c^2 - 288*a^6*b^2*c*
d + 99*a^7*b*d^2)/b^7) - 1/32768*(224*a^6*b^2*c^2 - 288*a^7*b*c*d + 99*a^8
*d^2)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(13/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(c+dx)^2 \left(ax+bx^2\right)^{3/2} \, dx = \int x^2 \left(b\,x^2+a\,x\right)^{3/2} (c+d\,x)^2 \, dx$$

input
```
int(x^2*(a*x + b*x^2)^(3/2)*(c + d*x)^2,x)
```

output
```
int(x^2*(a*x + b*x^2)^(3/2)*(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.31

$$\int x^2(c+dx)^2 \left(ax+bx^2\right)^{3/2} \, dx = \frac{-10395\sqrt{x}\,\sqrt{bx+a}\,a^7 b\,d^2+30240\sqrt{x}\,\sqrt{bx+a}\,a^6 b^2 cd+6930\sqrt{x}\,\sqrt{bx+a}\,a^6 b^2 d^2 x-23520}{}$$

input
```
int(x^2*(d*x+c)^2*(b*x^2+a*x)^(3/2),x)
```

output
```
( - 10395*sqrt(x)*sqrt(a + b*x)*a**7*b*d**2 + 30240*sqrt(x)*sqrt(a + b*x)*
a**6*b**2*c*d + 6930*sqrt(x)*sqrt(a + b*x)*a**6*b**2*d**2*x - 23520*sqrt(x
)*sqrt(a + b*x)*a**5*b**3*c**2 - 20160*sqrt(x)*sqrt(a + b*x)*a**5*b**3*c*d
*x - 5544*sqrt(x)*sqrt(a + b*x)*a**5*b**3*d**2*x**2 + 15680*sqrt(x)*sqrt(a
 + b*x)*a**4*b**4*c**2*x + 16128*sqrt(x)*sqrt(a + b*x)*a**4*b**4*c*d*x**2
+ 4752*sqrt(x)*sqrt(a + b*x)*a**4*b**4*d**2*x**3 - 12544*sqrt(x)*sqrt(a +
b*x)*a**3*b**5*c**2*x**2 - 13824*sqrt(x)*sqrt(a + b*x)*a**3*b**5*c*d*x**3
- 4224*sqrt(x)*sqrt(a + b*x)*a**3*b**5*d**2*x**4 + 10752*sqrt(x)*sqrt(a +
b*x)*a**2*b**6*c**2*x**3 + 12288*sqrt(x)*sqrt(a + b*x)*a**2*b**6*c*d*x**4
+ 3840*sqrt(x)*sqrt(a + b*x)*a**2*b**6*d**2*x**5 + 372736*sqrt(x)*sqrt(a +
 b*x)*a*b**7*c**2*x**4 + 614400*sqrt(x)*sqrt(a + b*x)*a*b**7*c*d*x**5 + 26
1120*sqrt(x)*sqrt(a + b*x)*a*b**7*d**2*x**6 + 286720*sqrt(x)*sqrt(a + b*x)
*b**8*c**2*x**5 + 491520*sqrt(x)*sqrt(a + b*x)*b**8*c*d*x**6 + 215040*sqrt
(x)*sqrt(a + b*x)*b**8*d**2*x**7 + 10395*sqrt(b)*log((sqrt(a + b*x) + sqrt
(x)*sqrt(b))/sqrt(a))*a**8*d**2 - 30240*sqrt(b)*log((sqrt(a + b*x) + sqrt(
x)*sqrt(b))/sqrt(a))*a**7*b*c*d + 23520*sqrt(b)*log((sqrt(a + b*x) + sqrt(
x)*sqrt(b))/sqrt(a))*a**6*b**2*c**2)/(1720320*b**7)
```

3.68 $\int x(c+dx)^2 \left(ax+bx^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 349

$$\int x(c+dx)^2 \left(ax+bx^2\right)^{3/2} \, dx = \frac{a^4(24b^2c^2-28abcd+9a^2d^2)\sqrt{ax+bx^2}}{1024b^5} - \frac{a^3(24b^2c^2-28abcd+9a^2d^2)\,x\sqrt{ax+bx^2}}{1536b^4} + \frac{a^2(24b^2c^2-28abcd+9a^2d^2)\,x^2\sqrt{ax+bx^2}}{1920b^3} + \frac{11a(24b^2c^2-28abcd+9a^2d^2)\,x^3\sqrt{ax+bx^2}}{960b^2} + \frac{(24b^2c^2-28abcd+9a^2d^2)\,x^4\sqrt{ax+bx^2}}{120b} + \frac{d(28bc-9ad)x(ax+bx^2)^{5/2}}{84b^2} + \frac{d^2x^2(ax+bx^2)^{5/2}}{7b} - \frac{a^5(24b^2c^2-28abcd+9a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{1024b^{11/2}}$$

output
```
1/1024*a^4*(9*a^2*d^2-28*a*b*c*d+24*b^2*c^2)*(b*x^2+a*x)^(1/2)/b^5-1/1536*
a^3*(9*a^2*d^2-28*a*b*c*d+24*b^2*c^2)*x*(b*x^2+a*x)^(1/2)/b^4+1/1920*a^2*(
9*a^2*d^2-28*a*b*c*d+24*b^2*c^2)*x^2*(b*x^2+a*x)^(1/2)/b^3+11/960*a*(9*a^2
*d^2-28*a*b*c*d+24*b^2*c^2)*x^3*(b*x^2+a*x)^(1/2)/b^2+1/120*(9*a^2*d^2-28*
a*b*c*d+24*b^2*c^2)*x^4*(b*x^2+a*x)^(1/2)/b+1/84*d*(-9*a*d+28*b*c)*x*(b*x^
2+a*x)^(5/2)/b^2+1/7*d^2*x^2*(b*x^2+a*x)^(5/2)/b-1/1024*a^5*(9*a^2*d^2-28*
a*b*c*d+24*b^2*c^2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.89

$$\int x(c+dx)^2\left(ax+bx^2\right)^{3/2}\,dx=\frac{\sqrt{x}\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\sqrt{a+bx}(945a^6d^2-210a^5bd(14c+3dx)+192a^2b^4x^2(7c^2+7cdx+2d^2x\right.}{}$$

input
```
Integrate[x*(c + d*x)^2*(a*x + b*x^2)^(3/2),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(945*a^6*d^2 - 210*a
^5*b*d*(14*c + 3*d*x) + 192*a^2*b^4*x^2*(7*c^2 + 7*c*d*x + 2*d^2*x^2) + 56
*a^4*b^2*(45*c^2 + 35*c*d*x + 9*d^2*x^2) + 1024*b^6*x^4*(21*c^2 + 35*c*d*x
 + 15*d^2*x^2) - 16*a^3*b^3*x*(105*c^2 + 98*c*d*x + 27*d^2*x^2) + 128*a*b^
5*x^3*(231*c^2 + 364*c*d*x + 150*d^2*x^2)) + 630*a^5*(8*b^2*c^2 + 3*a^2*d^
2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])] + 5880*a^6*b*c*d*A
rcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(107520*b^(11/2)*Sq
rt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.63, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1262, 27, 1225, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax+bx^2\right)^{3/2}(c+dx)^2\,dx$$

$$\Big\downarrow \ 1262$$

$$\frac{\int \frac{1}{2}x\left(14bc^2+d(28bc-9ad)x\right)\left(bx^2+ax\right)^{3/2}dx}{7b}+\frac{d^2x^2\left(ax+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \ 27$$

$$\frac{\int x\left(14bc^2+d(28bc-9ad)x\right)\left(bx^2+ax\right)^{3/2}dx}{14b}+\frac{d^2x^2\left(ax+bx^2\right)^{5/2}}{7b}$$

$\big\downarrow$ 1225

$$\frac{\frac{\left(ax+bx^2\right)^{5/2}\left(7\left(24b^2c^2-ad(28bc-9ad)\right)+10bdx(28bc-9ad)\right)}{60b^2}-\frac{7a\left(24b^2c^2-ad(28bc-9ad)\right)\int\left(bx^2+ax\right)^{3/2}dx}{24b^2}}{14b}+\frac{d^2x^2\left(ax+bx^2\right)^{5/2}}{7b}$$

$\big\downarrow$ 1087

$$\frac{\frac{\left(ax+bx^2\right)^{5/2}\left(7\left(24b^2c^2-ad(28bc-9ad)\right)+10bdx(28bc-9ad)\right)}{60b^2}-\frac{7a\left(24b^2c^2-ad(28bc-9ad)\right)\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\int\sqrt{bx^2+ax}dx}{16b}\right)}{24b^2}}{14b}+\frac{d^2x^2\left(ax+bx^2\right)^{5/2}}{7b}$$

$\big\downarrow$ 1087

$$\frac{\frac{\left(ax+bx^2\right)^{5/2}\left(7\left(24b^2c^2-ad(28bc-9ad)\right)+10bdx(28bc-9ad)\right)}{60b^2}-\frac{7a\left(24b^2c^2-ad(28bc-9ad)\right)\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2}{\cdots}\right.}{16b}\right.}{24b^2}}{14b}$$

$$\frac{d^2x^2\left(ax+bx^2\right)^{5/2}}{7b}$$

$\big\downarrow$ 1091

$$\frac{\frac{\left(ax+bx^2\right)^{5/2}\left(7\left(24b^2c^2-ad(28bc-9ad)\right)+10bdx(28bc-9ad)\right)}{60b^2}-\frac{7a\left(24b^2c^2-ad(28bc-9ad)\right)\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2}{\cdots}\right.}{16\cdots}\right.}{24b^2}}{14b}$$

$$\frac{d^2x^2\left(ax+bx^2\right)^{5/2}}{7b}$$

$\big\downarrow$ 219

$$\frac{\frac{(ax+bx^2)^{5/2}(7(24b^2c^2-ad(28bc-9ad))+10bdx(28bc-9ad))}{60b^2}-\frac{7a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)}{16b}\right)}{24b^2}}{14b}$$

$$\frac{d^2x^2\left(ax+bx^2\right)^{5/2}}{7b}$$

input
```
Int[x*(c + d*x)^2*(a*x + b*x^2)^(3/2),x]
```

output
```
(d^2*x^2*(a*x + b*x^2)^(5/2))/(7*b) + (((7*(24*b^2*c^2 - a*d*(28*b*c - 9*a
*d)) + 10*b*d*(28*b*c - 9*a*d)*x)*(a*x + b*x^2)^(5/2))/(60*b^2) - (7*a*(24
*b^2*c^2 - a*d*(28*b*c - 9*a*d))*(((a + 2*b*x)*(a*x + b*x^2)^(3/2))/(8*b)
- (3*a^2*(((a + 2*b*x)*Sqrt[a*x + b*x^2])/(4*b) - (a^2*ArcTanh[(Sqrt[b]*x)
/Sqrt[a*x + b*x^2]])/(4*b^(3/2))))/(16*b)))/(24*b^2))/(14*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$-\frac{9\left(a^{5}\left(a^{2}d^{2}-\frac{28}{9}abcd+\frac{8}{3}b^{2}c^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)-\left(\frac{1408\left(\frac{50}{77}d^{2}x^{2}+\frac{52}{33}cdx+c^{2}\right)x^{3}ab^{\frac{11}{2}}}{45}+\frac{1024x^{4}\left(\frac{5}{7}d^{2}x^{2}+\frac{5}{3}cdx+c^{2}\right)b^{\frac{13}{2}}}{45}\right.\right.}{}$
risch	$\frac{(15360b^{6}d^{2}x^{6}+19200a\,b^{5}d^{2}x^{5}+35840b^{6}cd\,x^{5}+384a^{2}b^{4}d^{2}x^{4}+46592a\,b^{5}cd\,x^{4}+21504b^{6}c^{2}x^{4}-432a^{3}b^{3}d^{2}x^{3}+1344a^{2}b^{4}cd\,x}{}$
default	$c^{2}\left(\frac{\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{5b}-\frac{a\left(\frac{(2bx+a)\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{8b}-\frac{3a^{2}\left(\frac{(2bx+a)\sqrt{b\,x^{2}+ax}}{4b}-\frac{a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8b^{\frac{3}{2}}}\right)}{16b}\right)}{2b}\right)+d^{2}\left(\frac{x^{2}\left(b\,x^{2}\right.}{7}\right.$

input
```
int(x*(d*x+c)^2*(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-9/1024/b^(11/2)*(a^5*(a^2*d^2-28/9*a*b*c*d+8/3*b^2*c^2)*arctanh((x*(b*x+a
))^(1/2)/x/b^(1/2))-(1408/45*(50/77*d^2*x^2+52/33*c*d*x+c^2)*x^3*a*b^(11/2
)+1024/45*x^4*(5/7*d^2*x^2+5/3*c*d*x+c^2)*b^(13/2)+(8/3*(1/5*d^2*x^2+7/9*c
*d*x+c^2)*a^2*b^(5/2)-16/9*x*a*(9/35*d^2*x^2+14/15*c*d*x+c^2)*b^(7/2)+64/4
5*x^2*(2/7*d^2*x^2+c*d*x+c^2)*b^(9/2)+d*((-2/3*d*x-28/9*c)*b^(3/2)+b^(1/2)
*a*d)*a^3)*a^2)*(x*(b*x+a))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.63

$$\int x(c+dx)^2 \left(ax+bx^2\right)^{3/2} dx = \left[\frac{105\left(24\,a^5b^2c^2-28\,a^6bcd+9\,a^7d^2\right)\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\left(15360\,b^7d^2x^6\right.}{}\right.$$

input
```
integrate(x*(d*x+c)^2*(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[1/215040*(105*(24*a^5*b^2*c^2 - 28*a^6*b*c*d + 9*a^7*d^2)*sqrt(b)*log(2*b
*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(15360*b^7*d^2*x^6 + 2520*a^4*b^
3*c^2 - 2940*a^5*b^2*c*d + 945*a^6*b*d^2 + 1280*(28*b^7*c*d + 15*a*b^6*d^2
)*x^5 + 128*(168*b^7*c^2 + 364*a*b^6*c*d + 3*a^2*b^5*d^2)*x^4 + 48*(616*a*
b^6*c^2 + 28*a^2*b^5*c*d - 9*a^3*b^4*d^2)*x^3 + 56*(24*a^2*b^5*c^2 - 28*a^
3*b^4*c*d + 9*a^4*b^3*d^2)*x^2 - 70*(24*a^3*b^4*c^2 - 28*a^4*b^3*c*d + 9*a
^5*b^2*d^2)*x)*sqrt(b*x^2 + a*x))/b^6, 1/107520*(105*(24*a^5*b^2*c^2 - 28*
a^6*b*c*d + 9*a^7*d^2)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a
)) + (15360*b^7*d^2*x^6 + 2520*a^4*b^3*c^2 - 2940*a^5*b^2*c*d + 945*a^6*b*
d^2 + 1280*(28*b^7*c*d + 15*a*b^6*d^2)*x^5 + 128*(168*b^7*c^2 + 364*a*b^6*
c*d + 3*a^2*b^5*d^2)*x^4 + 48*(616*a*b^6*c^2 + 28*a^2*b^5*c*d - 9*a^3*b^4*
d^2)*x^3 + 56*(24*a^2*b^5*c^2 - 28*a^3*b^4*c*d + 9*a^4*b^3*d^2)*x^2 - 70*(
24*a^3*b^4*c^2 - 28*a^4*b^3*c*d + 9*a^5*b^2*d^2)*x)*sqrt(b*x^2 + a*x))/b^6
]
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 670, normalized size of antiderivative = 1.92

$$\int x(c+dx)^2\left(ax+bx^2\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(x*(d*x+c)**2*(b*x**2+a*x)**(3/2),x)
```

output
```
Piecewise((-5*a**3*(a**2*c**2 - 7*a*(2*a**2*c*d + 2*a*b*c**2 - 9*a*(a**2*d
**2 + 4*a*b*c*d - 11*a*(15*a*b*d**2/14 + 2*b**2*c*d)/(12*b) + b**2*c**2)/(
10*b))/(8*b))*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqr
t(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)
**2), True))/(16*b**3) + sqrt(a*x + b*x**2)*(5*a**2*(a**2*c**2 - 7*a*(2*a*
*2*c*d + 2*a*b*c**2 - 9*a*(a**2*d**2 + 4*a*b*c*d - 11*a*(15*a*b*d**2/14 +
2*b**2*c*d)/(12*b) + b**2*c**2)/(10*b))/(8*b))/(8*b**3) - 5*a*x*(a**2*c**2
 - 7*a*(2*a**2*c*d + 2*a*b*c**2 - 9*a*(a**2*d**2 + 4*a*b*c*d - 11*a*(15*a*
b*d**2/14 + 2*b**2*c*d)/(12*b) + b**2*c**2)/(10*b))/(8*b))/(12*b**2) + b*d
**2*x**6/7 + x**5*(15*a*b*d**2/14 + 2*b**2*c*d)/(6*b) + x**4*(a**2*d**2 +
4*a*b*c*d - 11*a*(15*a*b*d**2/14 + 2*b**2*c*d)/(12*b) + b**2*c**2)/(5*b) +
 x**3*(2*a**2*c*d + 2*a*b*c**2 - 9*a*(a**2*d**2 + 4*a*b*c*d - 11*a*(15*a*b
*d**2/14 + 2*b**2*c*d)/(12*b) + b**2*c**2)/(10*b))/(4*b) + x**2*(a**2*c**2
 - 7*a*(2*a**2*c*d + 2*a*b*c**2 - 9*a*(a**2*d**2 + 4*a*b*c*d - 11*a*(15*a*
b*d**2/14 + 2*b**2*c*d)/(12*b) + b**2*c**2)/(10*b))/(8*b))/(3*b)), Ne(b, 0
)), (2*(c**2*(a*x)**(7/2)/7 + 2*c*d*(a*x)**(9/2)/(9*a) + d**2*(a*x)**(11/2
)/(11*a**2))/a**2, Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.40

$$\int x(c+dx)^2\left(ax+bx^2\right)^{3/2}\,dx=\frac{(bx^2+ax)^{\frac{5}{2}}d^2x^2}{7\,b}+\frac{3\,\sqrt{bx^2+ax}a^3c^2x}{64\,b^2}-\frac{(bx^2+ax)^{\frac{3}{2}}ac^2x}{8\,b}-\frac{7\,\sqrt{bx^2+ax}a^4cdx}{128\,b^3}+\frac{7\,(bx^2+ax)^{\frac{3}{2}}a^2cdx}{48\,b^2}+\frac{(bx^2+ax)^{\frac{5}{2}}cdx}{3\,b}+\frac{9\,\sqrt{bx^2+ax}a^5d^2x}{512\,b^4}-\frac{3\,(bx^2+ax)^{\frac{3}{2}}a^3d^2x}{64\,b^3}-\frac{3\,(bx^2+ax)^{\frac{5}{2}}ad^2x}{28\,b^2}-\frac{3\,a^5c^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{256\,b^{\frac{7}{2}}}+\frac{7\,a^6cd\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{512\,b^{\frac{9}{2}}}-\frac{9\,a^7d^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{2048\,b^{\frac{11}{2}}}+\frac{3\,\sqrt{bx^2+ax}a^4c^2}{128\,b^3}-\frac{(bx^2+ax)^{\frac{3}{2}}a^2c^2}{16\,b^2}+\frac{(bx^2+ax)^{\frac{5}{2}}c^2}{5\,b}-\frac{7\,\sqrt{bx^2+ax}a^5cd}{256\,b^4}+\frac{7\,(bx^2+ax)^{\frac{3}{2}}a^3cd}{96\,b^3}-\frac{7\,(bx^2+ax)^{\frac{5}{2}}acd}{30\,b^2}+\frac{9\,\sqrt{bx^2+ax}a^6d^2}{1024\,b^5}-\frac{3\,(bx^2+ax)^{\frac{3}{2}}a^4d^2}{128\,b^4}+\frac{3\,(bx^2+ax)^{\frac{5}{2}}a^2d^2}{40\,b^3}$$

input
```
integrate(x*(d*x+c)^2*(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
1/7*(b*x^2 + a*x)^(5/2)*d^2*x^2/b + 3/64*sqrt(b*x^2 + a*x)*a^3*c^2*x/b^2 -
 1/8*(b*x^2 + a*x)^(3/2)*a*c^2*x/b - 7/128*sqrt(b*x^2 + a*x)*a^4*c*d*x/b^3
 + 7/48*(b*x^2 + a*x)^(3/2)*a^2*c*d*x/b^2 + 1/3*(b*x^2 + a*x)^(5/2)*c*d*x/
b + 9/512*sqrt(b*x^2 + a*x)*a^5*d^2*x/b^4 - 3/64*(b*x^2 + a*x)^(3/2)*a^3*d
^2*x/b^3 - 3/28*(b*x^2 + a*x)^(5/2)*a*d^2*x/b^2 - 3/256*a^5*c^2*log(2*b*x
+ a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(7/2) + 7/512*a^6*c*d*log(2*b*x + a +
 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(9/2) - 9/2048*a^7*d^2*log(2*b*x + a + 2*s
qrt(b*x^2 + a*x)*sqrt(b))/b^(11/2) + 3/128*sqrt(b*x^2 + a*x)*a^4*c^2/b^3 -
 1/16*(b*x^2 + a*x)^(3/2)*a^2*c^2/b^2 + 1/5*(b*x^2 + a*x)^(5/2)*c^2/b - 7/
256*sqrt(b*x^2 + a*x)*a^5*c*d/b^4 + 7/96*(b*x^2 + a*x)^(3/2)*a^3*c*d/b^3 -
 7/30*(b*x^2 + a*x)^(5/2)*a*c*d/b^2 + 9/1024*sqrt(b*x^2 + a*x)*a^6*d^2/b^5
 - 3/128*(b*x^2 + a*x)^(3/2)*a^4*d^2/b^4 + 3/40*(b*x^2 + a*x)^(5/2)*a^2*d^
2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.87

$$\int x(c+dx)^2\left(ax+bx^2\right)^{3/2}\,dx = \frac{1}{107520}\sqrt{bx^2+ax}\left(2\left(4\left(2\left(8\left(10\left(12\,bd^2x+\frac{28\,b^7cd+15\,ab^6d^2}{b^6}\right)x+\frac{168\,b^7c^2+364\,ab^6\ldots}{b^6}\right.\right.\right.\right.\right.$$
$$+\frac{\left(24\,a^5b^2c^2-28\,a^6bcd+9\,a^7d^2\right)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{2048\,b^{\frac{11}{2}}}$$

input
```
integrate(x*(d*x+c)^2*(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
1/107520*sqrt(b*x^2 + a*x)*(2*(4*(2*(8*(10*(12*b*d^2*x + (28*b^7*c*d + 15*
a*b^6*d^2)/b^6)*x + (168*b^7*c^2 + 364*a*b^6*c*d + 3*a^2*b^5*d^2)/b^6)*x +
 3*(616*a*b^6*c^2 + 28*a^2*b^5*c*d - 9*a^3*b^4*d^2)/b^6)*x + 7*(24*a^2*b^5
*c^2 - 28*a^3*b^4*c*d + 9*a^4*b^3*d^2)/b^6)*x - 35*(24*a^3*b^4*c^2 - 28*a^
4*b^3*c*d + 9*a^5*b^2*d^2)/b^6)*x + 105*(24*a^4*b^3*c^2 - 28*a^5*b^2*c*d +
 9*a^6*b*d^2)/b^6) + 1/2048*(24*a^5*b^2*c^2 - 28*a^6*b*c*d + 9*a^7*d^2)*lo
g(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int x(c+dx)^2 \left(ax+bx^2\right)^{3/2} dx = \int x \left(b x^2 + a x\right)^{3/2} (c + d x)^2 dx$$

input
```
int(x*(a*x + b*x^2)^(3/2)*(c + d*x)^2,x)
```

output
```
int(x*(a*x + b*x^2)^(3/2)*(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.29

$$\int x(c+dx)^2 \left(ax+bx^2\right)^{3/2} dx = \frac{945\sqrt{x}\,\sqrt{bx+a}\,a^6 b\,d^2 - 2940\sqrt{x}\,\sqrt{bx+a}\,a^5 b^2 cd - 630\sqrt{x}\,\sqrt{bx+a}\,a^5 b^2 d^2 x + 2520\sqrt{x}\,\sqrt{bx}}{}$$

input
```
int(x*(d*x+c)^2*(b*x^2+a*x)^(3/2),x)
```

output
```
(945*sqrt(x)*sqrt(a + b*x)*a**6*b*d**2 - 2940*sqrt(x)*sqrt(a + b*x)*a**5*b
**2*c*d - 630*sqrt(x)*sqrt(a + b*x)*a**5*b**2*d**2*x + 2520*sqrt(x)*sqrt(a
 + b*x)*a**4*b**3*c**2 + 1960*sqrt(x)*sqrt(a + b*x)*a**4*b**3*c*d*x + 504*
sqrt(x)*sqrt(a + b*x)*a**4*b**3*d**2*x**2 - 1680*sqrt(x)*sqrt(a + b*x)*a**
3*b**4*c**2*x - 1568*sqrt(x)*sqrt(a + b*x)*a**3*b**4*c*d*x**2 - 432*sqrt(x
)*sqrt(a + b*x)*a**3*b**4*d**2*x**3 + 1344*sqrt(x)*sqrt(a + b*x)*a**2*b**5
*c**2*x**2 + 1344*sqrt(x)*sqrt(a + b*x)*a**2*b**5*c*d*x**3 + 384*sqrt(x)*s
qrt(a + b*x)*a**2*b**5*d**2*x**4 + 29568*sqrt(x)*sqrt(a + b*x)*a*b**6*c**2
*x**3 + 46592*sqrt(x)*sqrt(a + b*x)*a*b**6*c*d*x**4 + 19200*sqrt(x)*sqrt(a
 + b*x)*a*b**6*d**2*x**5 + 21504*sqrt(x)*sqrt(a + b*x)*b**7*c**2*x**4 + 35
840*sqrt(x)*sqrt(a + b*x)*b**7*c*d*x**5 + 15360*sqrt(x)*sqrt(a + b*x)*b**7
*d**2*x**6 - 945*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*
*7*d**2 + 2940*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**6
*b*c*d - 2520*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**5*
b**2*c**2)/(107520*b**6)
```

3.69 $\int (c+dx)^2 \left(ax+bx^2\right)^{3/2} dx$

Optimal result

Integrand size = 21, antiderivative size = 297

$$\int (c+dx)^2 \left(ax+bx^2\right)^{3/2} dx = -\frac{a^3(24b^2c^2-24abcd+7a^2d^2)\sqrt{ax+bx^2}}{512b^4} + \frac{a^2(24b^2c^2-24abcd+7a^2d^2)\,x\sqrt{ax+bx^2}}{768b^3} + \frac{a(24b^2c^2-24abcd+7a^2d^2)\,x^2\sqrt{ax+bx^2}}{64b^2} + \frac{(24b^2c^2-24abcd+7a^2d^2)\,x^3\sqrt{ax+bx^2}}{96b} + \frac{d(24bc-7ad)\left(ax+bx^2\right)^{5/2}}{60b^2} + \frac{d^2x(ax+bx^2)^{5/2}}{6b} + \frac{a^4(24b^2c^2-24abcd+7a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{512b^{9/2}}$$

output
```
-1/512*a^3*(7*a^2*d^2-24*a*b*c*d+24*b^2*c^2)*(b*x^2+a*x)^(1/2)/b^4+1/768*a
^2*(7*a^2*d^2-24*a*b*c*d+24*b^2*c^2)*x*(b*x^2+a*x)^(1/2)/b^3+1/64*a*(7*a^2
*d^2-24*a*b*c*d+24*b^2*c^2)*x^2*(b*x^2+a*x)^(1/2)/b^2+1/96*(7*a^2*d^2-24*a
*b*c*d+24*b^2*c^2)*x^3*(b*x^2+a*x)^(1/2)/b+1/60*d*(-7*a*d+24*b*c)*(b*x^2+a
*x)^(5/2)/b^2+1/6*d^2*x*(b*x^2+a*x)^(5/2)/b+1/512*a^4*(7*a^2*d^2-24*a*b*c*
d+24*b^2*c^2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.94

$$\int (c+dx)^2 \left(ax+bx^2\right)^{3/2} dx = \frac{\sqrt{x}\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\sqrt{a+bx}(-105a^5d^2+10a^4bd(36c+7dx)+48a^2b^3x(5c^2+4cdx+d^2x^2)\right.}{}$$

input
```
Integrate[(c + d*x)^2*(a*x + b*x^2)^(3/2),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-105*a^5*d^2 + 10*a
^4*b*d*(36*c + 7*d*x) + 48*a^2*b^3*x*(5*c^2 + 4*c*d*x + d^2*x^2) - 8*a^3*b
^2*(45*c^2 + 30*c*d*x + 7*d^2*x^2) + 128*b^5*x^3*(15*c^2 + 24*c*d*x + 10*d
^2*x^2) + 64*a*b^4*x^2*(45*c^2 + 66*c*d*x + 26*d^2*x^2)) + 720*a^5*b*c*d*A
rcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])] + 30*a^4*(24*b^2*c^2 +
 7*a^2*d^2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(7680*
b^(9/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.65, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1166, 27, 1160, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^2\right)^{3/2} (c+dx)^2 \, dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{\int \frac{1}{2}(c(12bc-5ad)+7d(2bc-ad)x)\left(bx^2+ax\right)^{3/2}dx}{6b}+\frac{d\left(ax+bx^2\right)^{5/2}(c+dx)}{6b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \left(c(12bc-5ad)+7d(2bc-ad)x\right)\left(bx^2+ax\right)^{3/2}dx}{12b}+\frac{d\left(ax+bx^2\right)^{5/2}(c+dx)}{6b}$$

$\Big\downarrow$ 1160

$$\frac{\frac{\left(7a^2d^2-24abcd+24b^2c^2\right)\int \left(bx^2+ax\right)^{3/2}dx}{2b}+\frac{7d\left(ax+bx^2\right)^{5/2}(2bc-ad)}{5b}}{12b}+\frac{d\left(ax+bx^2\right)^{5/2}(c+dx)}{6b}$$

$\Big\downarrow$ 1087

$$\frac{\frac{\left(7a^2d^2-24abcd+24b^2c^2\right)\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\int \sqrt{bx^2+ax}dx}{16b}\right)}{2b}+\frac{7d\left(ax+bx^2\right)^{5/2}(2bc-ad)}{5b}}{12b}+\frac{d\left(ax+bx^2\right)^{5/2}(c+dx)}{6b}$$

$\Big\downarrow$ 1087

$$\frac{\frac{\left(7a^2d^2-24abcd+24b^2c^2\right)\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)}{16b}\right)}{2b}+\frac{7d\left(ax+bx^2\right)^{5/2}(2bc-ad)}{5b}}{12b}+\frac{d\left(ax+bx^2\right)^{5/2}(c+dx)}{6b}$$

$\Big\downarrow$ 1091

$$\frac{\frac{\left(7a^2d^2-24abcd+24b^2c^2\right)\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)}{16b}\right)}{2b}+\frac{7d\left(ax+bx^2\right)^{5/2}(2bc-ad)}{5b}}{12b}+\frac{d\left(ax+bx^2\right)^{5/2}(c+dx)}{6b}$$

$\Big\downarrow$ 219

$$\frac{\frac{\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)}{16b}\right)(7a^2d^2-24abcd+24b^2c^2)}{2b}+\frac{7d\left(ax+bx^2\right)^{5/2}(2bc-ad)}{5b}}{12b}+\frac{d\left(ax+bx^2\right)^{5/2}(c+dx)}{6b}$$

input
```
Int[(c + d*x)^2*(a*x + b*x^2)^(3/2),x]
```

output
```
(d*(c + d*x)*(a*x + b*x^2)^(5/2))/(6*b) + ((7*d*(2*b*c - a*d)*(a*x + b*x^2
)^(5/2))/(5*b) + ((24*b^2*c^2 - 24*a*b*c*d + 7*a^2*d^2)*(((a + 2*b*x)*(a*x
 + b*x^2)^(3/2))/(8*b) - (3*a^2*(((a + 2*b*x)*Sqrt[a*x + b*x^2])/(4*b) - (
a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*b^(3/2))))/(16*b)))/(2*b))/
(12*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$\frac{\frac{7a^4\left(a^2d^2-\frac{24}{7}abcd+\frac{24}{7}b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{512}-\frac{7\sqrt{x(bx+a)}\left(-\frac{128x^3\left(\frac{2}{3}d^2x^2+\frac{8}{5}cdx+c^2\right)b^{\frac{11}{2}}}{7}+\left(\frac{24\left(\frac{7}{45}d^2x^2+\frac{2}{3}cdx+c^2\right)a^2b^{\frac{5}{2}}}{7}\right.\right.}{}}{b^{\frac{9}{2}}}$
risch	$-\frac{\left(-1280b^5d^2x^5-1664a\,b^4d^2x^4-3072b^5cd\,x^4-48a^2b^3d^2x^3-4224a\,b^4cd\,x^3-1920b^5c^2x^3+56a^3b^2d^2x^2-192a^2b^3cd\,x^2-28\right.}{7680b^4\sqrt{x(bx+a)}}$
default	$c^2\left(\frac{(2bx+a)\left(b\,x^2+ax\right)^{\frac{3}{2}}}{8b}-\frac{3a^2\left(\frac{(2bx+a)\sqrt{b\,x^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8b^{\frac{3}{2}}}\right)}{16b}\right)+d^2\left(\frac{x\left(b\,x^2+ax\right)^{\frac{5}{2}}}{6b}-\frac{7a\left(\frac{(b}{}\right.}{}\right.$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
7/512/b^(9/2)*(a^4*(a^2*d^2-24/7*a*b*c*d+24/7*b^2*c^2)*arctanh((x*(b*x+a))
^(1/2)/x/b^(1/2))-(x*(b*x+a))^(1/2)*(-128/7*x^3*(2/3*d^2*x^2+8/5*c*d*x+c^2
)*b^(11/2)+(24/7*(7/45*d^2*x^2+2/3*c*d*x+c^2)*a^2*b^(5/2)-16/7*x*a*(1/5*d^
2*x^2+4/5*c*d*x+c^2)*b^(7/2)-192/7*x^2*(26/45*d^2*x^2+22/15*c*d*x+c^2)*b^(
9/2)+d*((-2/3*d*x-24/7*c)*b^(3/2)+b^(1/2)*a*d)*a^3)*a))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.65

$$\int (c+dx)^2 (ax+bx^2)^{3/2}\, dx = \left[\frac{15\,(24\,a^4b^2c^2 - 24\,a^5bcd + 7\,a^6d^2)\sqrt{b}\log\left(2\,bx + a + 2\,\sqrt{bx^2+ax}\sqrt{b}\right) + 2\,(1280\,b^6d^2x^5 - \dots}{\dots}, -\frac{15\,(24\,a^4b^2c^2 - 24\,a^5bcd + 7\,a^6d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right) - (1280\,b^6d^2x^5 - 360\,a^3b^3c^2 + 360\,a^4b^2cd - \dots}{\dots}\right.$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[1/15360*(15*(24*a^4*b^2*c^2 - 24*a^5*b*c*d + 7*a^6*d^2)*sqrt(b)*log(2*b*x
 + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(1280*b^6*d^2*x^5 - 360*a^3*b^3*c^
2 + 360*a^4*b^2*c*d - 105*a^5*b*d^2 + 128*(24*b^6*c*d + 13*a*b^5*d^2)*x^4
+ 48*(40*b^6*c^2 + 88*a*b^5*c*d + a^2*b^4*d^2)*x^3 + 8*(360*a*b^5*c^2 + 24
*a^2*b^4*c*d - 7*a^3*b^3*d^2)*x^2 + 10*(24*a^2*b^4*c^2 - 24*a^3*b^3*c*d +
7*a^4*b^2*d^2)*x)*sqrt(b*x^2 + a*x))/b^5, -1/7680*(15*(24*a^4*b^2*c^2 - 24
*a^5*b*c*d + 7*a^6*d^2)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x +
a)) - (1280*b^6*d^2*x^5 - 360*a^3*b^3*c^2 + 360*a^4*b^2*c*d - 105*a^5*b*d^
2 + 128*(24*b^6*c*d + 13*a*b^5*d^2)*x^4 + 48*(40*b^6*c^2 + 88*a*b^5*c*d +
a^2*b^4*d^2)*x^3 + 8*(360*a*b^5*c^2 + 24*a^2*b^4*c*d - 7*a^3*b^3*d^2)*x^2
+ 10*(24*a^2*b^4*c^2 - 24*a^3*b^3*c*d + 7*a^4*b^2*d^2)*x)*sqrt(b*x^2 + a*x
))/b^5]
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.92

$$\int (c+dx)^2 \left(ax+bx^2\right)^{3/2}\,dx = \begin{cases} \frac{3a^2\left(a^2c^2-\frac{5a\left(2a^2cd+2abc^2-\frac{7a\left(a^2d^2+4abcd-\frac{9a\left(\frac{13abd^2}{12}+2b^2cd\right)}{10b}+b^2c^2\right)}{8b}\right)}{6b}\right)\left(\begin{cases}\frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b} \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherw} \end{cases}\right.}{8b^2} \\ \frac{2\left(\frac{c^2(ax)^{\frac{5}{2}}}{5}+\frac{2cd(ax)^{\frac{7}{2}}}{7a}+\frac{d^2(ax)^{\frac{9}{2}}}{9a^2}\right)}{a} \\ 0 \end{cases}$$

input
```
integrate((d*x+c)**2*(b*x**2+a*x)**(3/2),x)
```

output
```
Piecewise((3*a**2*(a**2*c**2 - 5*a*(2*a**2*c*d + 2*a*b*c**2 - 7*a*(a**2*d*
*2 + 4*a*b*c*d - 9*a*(13*a*b*d**2/12 + 2*b**2*c*d)/(10*b) + b**2*c**2)/(8*
b))/(6*b))*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b
), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2
), True))/(8*b**2) + sqrt(a*x + b*x**2)*(-3*a*(a**2*c**2 - 5*a*(2*a**2*c*d
 + 2*a*b*c**2 - 7*a*(a**2*d**2 + 4*a*b*c*d - 9*a*(13*a*b*d**2/12 + 2*b**2*
c*d)/(10*b) + b**2*c**2)/(8*b))/(6*b))/(4*b**2) + b*d**2*x**5/6 + x**4*(13
*a*b*d**2/12 + 2*b**2*c*d)/(5*b) + x**3*(a**2*d**2 + 4*a*b*c*d - 9*a*(13*a
*b*d**2/12 + 2*b**2*c*d)/(10*b) + b**2*c**2)/(4*b) + x**2*(2*a**2*c*d + 2*
a*b*c**2 - 7*a*(a**2*d**2 + 4*a*b*c*d - 9*a*(13*a*b*d**2/12 + 2*b**2*c*d)/
(10*b) + b**2*c**2)/(8*b))/(3*b) + x*(a**2*c**2 - 5*a*(2*a**2*c*d + 2*a*b*
c**2 - 7*a*(a**2*d**2 + 4*a*b*c*d - 9*a*(13*a*b*d**2/12 + 2*b**2*c*d)/(10*
b) + b**2*c**2)/(8*b))/(6*b))/(2*b)), Ne(b, 0)), (2*(c**2*(a*x)**(5/2)/5 +
 2*c*d*(a*x)**(7/2)/(7*a) + d**2*(a*x)**(9/2)/(9*a**2))/a, Ne(a, 0)), (0,
True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.40

$$\int (c+dx)^2 \left(ax+bx^2\right)^{3/2} \, dx = \frac{1}{4} \left(bx^2+ax\right)^{\frac{3}{2}} c^2 x - \frac{3 \sqrt{bx^2+ax} a^2 c^2 x}{32 \, b} + \frac{3 \sqrt{bx^2+ax} a^3 c d x}{32 \, b^2} - \frac{\left(bx^2+ax\right)^{\frac{3}{2}} a c d x}{4 \, b} - \frac{7 \sqrt{bx^2+ax} a^4 d^2 x}{256 \, b^3} + \frac{7 \left(bx^2+ax\right)^{\frac{3}{2}} a^2 d^2 x}{96 \, b^2} + \frac{\left(bx^2+ax\right)^{\frac{5}{2}} d^2 x}{6 \, b} + \frac{3 \, a^4 c^2 \log \left(2 \, bx+a+2 \sqrt{bx^2+ax} \sqrt{b}\right)}{128 \, b^{\frac{5}{2}}} - \frac{3 \, a^5 c d \log \left(2 \, bx+a+2 \sqrt{bx^2+ax} \sqrt{b}\right)}{128 \, b^{\frac{7}{2}}} + \frac{7 \, a^6 d^2 \log \left(2 \, bx+a+2 \sqrt{bx^2+ax} \sqrt{b}\right)}{1024 \, b^{\frac{9}{2}}} - \frac{3 \sqrt{bx^2+ax} a^3 c^2}{64 \, b^2} + \frac{\left(bx^2+ax\right)^{\frac{3}{2}} a c^2}{8 \, b} + \frac{3 \sqrt{bx^2+ax} a^4 c d}{64 \, b^3} - \frac{\left(bx^2+ax\right)^{\frac{3}{2}} a^2 c d}{8 \, b^2} + \frac{2 \left(bx^2+ax\right)^{\frac{5}{2}} c d}{5 \, b} - \frac{7 \sqrt{bx^2+ax} a^5 d^2}{512 \, b^4} + \frac{7 \left(bx^2+ax\right)^{\frac{3}{2}} a^3 d^2}{192 \, b^3} - \frac{7 \left(bx^2+ax\right)^{\frac{5}{2}} a d^2}{60 \, b^2}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a*x)^(3/2)*c^2*x - 3/32*sqrt(b*x^2 + a*x)*a^2*c^2*x/b + 3/32*
sqrt(b*x^2 + a*x)*a^3*c*d*x/b^2 - 1/4*(b*x^2 + a*x)^(3/2)*a*c*d*x/b - 7/25
6*sqrt(b*x^2 + a*x)*a^4*d^2*x/b^3 + 7/96*(b*x^2 + a*x)^(3/2)*a^2*d^2*x/b^2
 + 1/6*(b*x^2 + a*x)^(5/2)*d^2*x/b + 3/128*a^4*c^2*log(2*b*x + a + 2*sqrt(
b*x^2 + a*x)*sqrt(b))/b^(5/2) - 3/128*a^5*c*d*log(2*b*x + a + 2*sqrt(b*x^2
 + a*x)*sqrt(b))/b^(7/2) + 7/1024*a^6*d^2*log(2*b*x + a + 2*sqrt(b*x^2 + a
*x)*sqrt(b))/b^(9/2) - 3/64*sqrt(b*x^2 + a*x)*a^3*c^2/b^2 + 1/8*(b*x^2 + a
*x)^(3/2)*a*c^2/b + 3/64*sqrt(b*x^2 + a*x)*a^4*c*d/b^3 - 1/8*(b*x^2 + a*x)
^(3/2)*a^2*c*d/b^2 + 2/5*(b*x^2 + a*x)^(5/2)*c*d/b - 7/512*sqrt(b*x^2 + a*
x)*a^5*d^2/b^4 + 7/192*(b*x^2 + a*x)^(3/2)*a^3*d^2/b^3 - 7/60*(b*x^2 + a*x
)^(5/2)*a*d^2/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.88

$$\int (c+dx)^2 \left(ax+bx^2\right)^{3/2} \, dx = \frac{1}{7680}\sqrt{bx^2+ax}\left(2\left(4\left(2\left(8\left(10\,bd^2x+\frac{24\,b^6cd+13\,ab^5d^2}{b^5}\right)x+\frac{3\left(40\,b^6c^2+88\,ab^5cd+a^2\right.}{b^5}\right.\right.\right.\right. - \frac{\left(24\,a^4b^2c^2-24\,a^5bcd+7\,a^6d^2\right)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{1024\,b^{\frac{9}{2}}}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
1/7680*sqrt(b*x^2 + a*x)*(2*(4*(2*(8*(10*b*d^2*x + (24*b^6*c*d + 13*a*b^5*
d^2)/b^5)*x + 3*(40*b^6*c^2 + 88*a*b^5*c*d + a^2*b^4*d^2)/b^5)*x + (360*a*
b^5*c^2 + 24*a^2*b^4*c*d - 7*a^3*b^3*d^2)/b^5)*x + 5*(24*a^2*b^4*c^2 - 24*
a^3*b^3*c*d + 7*a^4*b^2*d^2)/b^5)*x - 15*(24*a^3*b^3*c^2 - 24*a^4*b^2*c*d
+ 7*a^5*b*d^2)/b^5) - 1/1024*(24*a^4*b^2*c^2 - 24*a^5*b*c*d + 7*a^6*d^2)*l
og(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \left(ax+bx^2\right)^{3/2}\,dx = \int \left(b\,x^2+a\,x\right)^{3/2}\,{\left(c+d\,x\right)}^2 \,dx$$

input
```
int((a*x + b*x^2)^(3/2)*(c + d*x)^2,x)
```

output
```
int((a*x + b*x^2)^(3/2)*(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.30

$$\int (c+dx)^2 \left(ax+bx^2\right)^{3/2}\,dx = \frac{-105\sqrt{x}\,\sqrt{bx+a}\,a^5 b\,d^2+360\sqrt{x}\,\sqrt{bx+a}\,a^4 b^2 cd+70\sqrt{x}\,\sqrt{bx+a}\,a^4 b^2 d^2 x-360\sqrt{x}\,\sqrt{bx+}}{}$$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(3/2),x)
```

output
```
( - 105*sqrt(x)*sqrt(a + b*x)*a**5*b*d**2 + 360*sqrt(x)*sqrt(a + b*x)*a**4
*b**2*c*d + 70*sqrt(x)*sqrt(a + b*x)*a**4*b**2*d**2*x - 360*sqrt(x)*sqrt(a
 + b*x)*a**3*b**3*c**2 - 240*sqrt(x)*sqrt(a + b*x)*a**3*b**3*c*d*x - 56*sq
rt(x)*sqrt(a + b*x)*a**3*b**3*d**2*x**2 + 240*sqrt(x)*sqrt(a + b*x)*a**2*b
**4*c**2*x + 192*sqrt(x)*sqrt(a + b*x)*a**2*b**4*c*d*x**2 + 48*sqrt(x)*sqr
t(a + b*x)*a**2*b**4*d**2*x**3 + 2880*sqrt(x)*sqrt(a + b*x)*a*b**5*c**2*x*
*2 + 4224*sqrt(x)*sqrt(a + b*x)*a*b**5*c*d*x**3 + 1664*sqrt(x)*sqrt(a + b*
x)*a*b**5*d**2*x**4 + 1920*sqrt(x)*sqrt(a + b*x)*b**6*c**2*x**3 + 3072*sqr
t(x)*sqrt(a + b*x)*b**6*c*d*x**4 + 1280*sqrt(x)*sqrt(a + b*x)*b**6*d**2*x*
*5 + 105*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**6*d**2
- 360*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**5*b*c*d +
360*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4*b**2*c**2)
/(7680*b**5)
```

3.70 $\int \frac{(c+dx)^2(ax+bx^2)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 246

$$\int \frac{(c+dx)^2(ax+bx^2)^{3/2}}{x}\,dx = \frac{a^2(16b^2c^2-3ad(4bc-ad))\sqrt{ax+bx^2}}{128b^3} + \frac{7a(16b^2c^2-3ad(4bc-ad))\,x\sqrt{ax+bx^2}}{192b^2} + \frac{(16b^2c^2-3ad(4bc-ad))\,x^2\sqrt{ax+bx^2}}{48b} + \frac{d^2(ax+bx^2)^{5/2}}{5b} + \frac{d(4bc-ad)\,(ax+bx^2)^{5/2}}{8b^2x} - \frac{a^3(16b^2c^2-3ad(4bc-ad))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{128b^{7/2}}$$

output
```
1/128*a^2*(16*b^2*c^2-3*a*d*(-a*d+4*b*c))*(b*x^2+a*x)^(1/2)/b^3+7/192*a*(1
6*b^2*c^2-3*a*d*(-a*d+4*b*c))*x*(b*x^2+a*x)^(1/2)/b^2+1/48*(16*b^2*c^2-3*a
*d*(-a*d+4*b*c))*x^2*(b*x^2+a*x)^(1/2)/b+1/5*d^2*(b*x^2+a*x)^(5/2)/b+1/8*d
*(-a*d+4*b*c)*(b*x^2+a*x)^(5/2)/b^2/x-1/128*a^3*(16*b^2*c^2-3*a*d*(-a*d+4*
b*c))*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx)^2 \left(ax+bx^2\right)^{3/2}}{x} \, dx = \frac{(x(a+bx))^{3/2} \left(240a^2b^2c^2 - 180a^3bcd + 45a^4d^2 + 1120ab^3c^2x + 120a^2b^2cd\right.}{}$$

$$- \frac{a^3(16b^2c^2 - 12abcd + 3a^2d^2) (x(a+bx))^{3/2} \text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{64b^{7/2}x^{3/2}(a+bx)^{3/2}}$$

input
```
Integrate[((c + d*x)^2*(a*x + b*x^2)^(3/2))/x,x]
```

output
```
((x*(a + b*x))^(3/2)*(240*a^2*b^2*c^2 - 180*a^3*b*c*d + 45*a^4*d^2 + 1120*
a*b^3*c^2*x + 120*a^2*b^2*c*d*x - 30*a^3*b*d^2*x + 640*b^4*c^2*x^2 + 1440*
a*b^3*c*d*x^2 + 24*a^2*b^2*d^2*x^2 + 960*b^4*c*d*x^3 + 528*a*b^3*d^2*x^3 +
 384*b^4*d^2*x^4))/(1920*b^3*x*(a + b*x)) - (a^3*(16*b^2*c^2 - 12*a*b*c*d
+ 3*a^2*d^2)*(x*(a + b*x))^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqr
t[a + b*x])])/(64*b^(7/2)*x^(3/2)*(a + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.72, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1262, 27, 1221, 1131, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{3/2} (c+dx)^2}{x} \, dx$$

$$\Big\downarrow \text{1262}$$

$$\frac{\int \frac{5\left(2bc^2+d(4bc-ad)x\right)\left(bx^2+ax\right)^{3/2}}{2x}dx}{5b} + \frac{d^2\left(ax+bx^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(2bc^2+d(4bc-ad)x)(bx^2+ax)^{3/2}}{x}dx}{2b}+\frac{d^2(ax+bx^2)^{5/2}}{5b}$$

$\Big\downarrow$ 1221

$$\frac{\frac{(3a^2d^2-12abcd+16b^2c^2)\int \frac{(bx^2+ax)^{3/2}}{x}dx}{8b}+\frac{d(ax+bx^2)^{5/2}(4bc-ad)}{4bx}}{2b}+\frac{d^2(ax+bx^2)^{5/2}}{5b}$$

$\Big\downarrow$ 1131

$$\frac{\frac{(3a^2d^2-12abcd+16b^2c^2)\left(\frac{1}{2}a\int \sqrt{bx^2+ax}dx+\frac{1}{3}(ax+bx^2)^{3/2}\right)}{8b}+\frac{d(ax+bx^2)^{5/2}(4bc-ad)}{4bx}}{2b}+\frac{d^2(ax+bx^2)^{5/2}}{5b}$$

$\Big\downarrow$ 1087

$$\frac{\frac{(3a^2d^2-12abcd+16b^2c^2)\left(\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)+\frac{1}{3}(ax+bx^2)^{3/2}\right)}{8b}+\frac{d(ax+bx^2)^{5/2}(4bc-ad)}{4bx}}{2b}+\frac{d^2(ax+bx^2)^{5/2}}{5b}$$

$\Big\downarrow$ 1091

$$\frac{\frac{(3a^2d^2-12abcd+16b^2c^2)\left(\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)+\frac{1}{3}(ax+bx^2)^{3/2}\right)}{8b}+\frac{d(ax+bx^2)^{5/2}(4bc-ad)}{4bx}}{2b}+\frac{d^2(ax+bx^2)^{5/2}}{5b}$$

$\Big\downarrow$ 219

$$\frac{\frac{\left(\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)+\frac{1}{3}(ax+bx^2)^{3/2}\right)(3a^2d^2-12abcd+16b^2c^2)}{8b}+\frac{d(ax+bx^2)^{5/2}(4bc-ad)}{4bx}}{2b}+\frac{d^2(ax+bx^2)^{5/2}}{5b}$$

input
```
Int[((c + d*x)^2*(a*x + b*x^2)^(3/2))/x,x]
```

output
```
(d^2*(a*x + b*x^2)^(5/2))/(5*b) + ((d*(4*b*c - a*d)*(a*x + b*x^2)^(5/2))/(
4*b*x) + ((16*b^2*c^2 - 12*a*b*c*d + 3*a^2*d^2)*((a*x + b*x^2)^(3/2)/3 + (
a*(((a + 2*b*x)*Sqrt[a*x + b*x^2])/(4*b) - (a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a
*x + b*x^2]])/(4*b^(3/2))))/2))/(8*b))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$-\frac{3\left(a^{3}\left(a^{2}d^{2}-4abcd+\frac{16}{3}b^{2}c^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)-\left(\frac{16\left(\frac{1}{10}d^{2}x^{2}+\frac{1}{2}cdx+c^{2}\right)a^{2}b^{\frac{5}{2}}}{3}+\frac{224\left(\frac{33}{70}d^{2}x^{2}+\frac{9}{7}cdx+c^{2}\right)xab^{\frac{7}{2}}}{9}+\frac{128x}{}\right.\right.}{128b^{\frac{7}{2}}}$
risch	$\frac{(384b^{4}d^{2}x^{4}+528ab^{3}d^{2}x^{3}+960b^{4}cdx^{3}+24a^{2}b^{2}d^{2}x^{2}+1440ab^{3}cdx^{2}+640c^{2}x^{2}b^{4}-30a^{3}bd^{2}x+120a^{2}b^{2}cdx+1120ab^{3}c^{2}x+4}{1920b^{3}\sqrt{x(bx+a)}}$
default	$c^{2}\left(\frac{(bx^{2}+ax)^{\frac{3}{2}}}{3}+\frac{a\left(\frac{(2bx+a)\sqrt{bx^{2}+ax}}{4b}-\frac{a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{bx^{2}+ax}\right)}{8b^{\frac{3}{2}}}\right)}{2}\right)+d^{2}\left(\frac{(bx^{2}+ax)^{\frac{5}{2}}}{5b}-\frac{a\left(\frac{(2bx+a)\left(bx^{2}+a\right.}{8b}\right.}{}\right.$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-3/128/b^(7/2)*(a^3*(a^2*d^2-4*a*b*c*d+16/3*b^2*c^2)*arctanh((x*(b*x+a))^(
1/2)/x/b^(1/2))-(16/3*(1/10*d^2*x^2+1/2*c*d*x+c^2)*a^2*b^(5/2)+224/9*(33/7
0*d^2*x^2+9/7*c*d*x+c^2)*x*a*b^(7/2)+128/9*x^2*(3/5*d^2*x^2+3/2*c*d*x+c^2)
*b^(9/2)+d*((-2/3*d*x-4*c)*b^(3/2)+b^(1/2)*a*d)*a^3)*(x*(b*x+a))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.69

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x} dx = \left[\frac{15 (16 a^3b^2c^2 - 12 a^4bcd + 3 a^5d^2)\sqrt{b}\log\left(2 bx + a - 2 \sqrt{bx^2 + ax}\sqrt{b}\right) +}{} \right.$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/3840*(15*(16*a^3*b^2*c^2 - 12*a^4*b*c*d + 3*a^5*d^2)*sqrt(b)*log(2*b*x
+ a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(384*b^5*d^2*x^4 + 240*a^2*b^3*c^2
- 180*a^3*b^2*c*d + 45*a^4*b*d^2 + 48*(20*b^5*c*d + 11*a*b^4*d^2)*x^3 + 8*
(80*b^5*c^2 + 180*a*b^4*c*d + 3*a^2*b^3*d^2)*x^2 + 10*(112*a*b^4*c^2 + 12*
a^2*b^3*c*d - 3*a^3*b^2*d^2)*x)*sqrt(b*x^2 + a*x))/b^4, 1/1920*(15*(16*a^3
*b^2*c^2 - 12*a^4*b*c*d + 3*a^5*d^2)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqr
t(-b)/(b*x + a)) + (384*b^5*d^2*x^4 + 240*a^2*b^3*c^2 - 180*a^3*b^2*c*d +
45*a^4*b*d^2 + 48*(20*b^5*c*d + 11*a*b^4*d^2)*x^3 + 8*(80*b^5*c^2 + 180*a*
b^4*c*d + 3*a^2*b^3*d^2)*x^2 + 10*(112*a*b^4*c^2 + 12*a^2*b^3*c*d - 3*a^3*
b^2*d^2)*x)*sqrt(b*x^2 + a*x))/b^4]
```

Sympy [A] (verification not implemented)

Time = 2.97 (sec) , antiderivative size = 796, normalized size of antiderivative = 3.24

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x} dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**2*(b*x**2+a*x)**(3/2)/x,x)
```

output
```
a*c**2*Piecewise((-a**2*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) +
2*b*x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/
(2*b) + x)**2), True))/(8*b) + (a/(4*b) + x/2)*sqrt(a*x + b*x**2), Ne(b, 0
)), (2*(a*x)**(3/2)/(3*a), Ne(a, 0)), (0, True)) + 2*a*c*d*Piecewise((a**3
*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(a**2
/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2), True))/
(16*b**2) + sqrt(a*x + b*x**2)*(-a**2/(8*b**2) + a*x/(12*b) + x**2/3), Ne(
b, 0)), (2*(a*x)**(5/2)/(5*a**2), Ne(a, 0)), (0, True)) + a*d**2*Piecewise
((-5*a**4*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b)
, Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2)
, True))/(128*b**3) + sqrt(a*x + b*x**2)*(5*a**3/(64*b**3) - 5*a**2*x/(96*
b**2) + a*x**2/(24*b) + x**3/4), Ne(b, 0)), (2*(a*x)**(7/2)/(7*a**3), Ne(a
, 0)), (0, True)) + b*c**2*Piecewise((a**3*Piecewise((log(a + 2*sqrt(b)*sq
rt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2
*b) + x)/sqrt(b*(a/(2*b) + x)**2), True))/(16*b**2) + sqrt(a*x + b*x**2)*(
-a**2/(8*b**2) + a*x/(12*b) + x**2/3), Ne(b, 0)), (2*(a*x)**(5/2)/(5*a**2)
, Ne(a, 0)), (0, True)) + 2*b*c*d*Piecewise((-5*a**4*Piecewise((log(a + 2*
sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x
)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2), True))/(128*b**3) + sqrt(a*x
+ b*x**2)*(5*a**3/(64*b**3) - 5*a**2*x/(96*b**2) + a*x**2/(24*b) + x**3...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.39

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x} dx = \frac{1}{4}\sqrt{bx^2+ax}ac^2x + \frac{1}{2}(bx^2+ax)^{\frac{3}{2}}cdx - \frac{3\sqrt{bx^2+ax}a^2cdx}{16b} + \frac{3\sqrt{bx^2+ax}a^3d^2x}{64b^2} - \frac{(bx^2+ax)^{\frac{3}{2}}ad^2x}{8b} - \frac{a^3c^2\log\left(2bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{16b^{\frac{3}{2}}} + \frac{3a^4cd\log\left(2bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{64b^{\frac{5}{2}}} - \frac{3a^5d^2\log\left(2bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{256b^{\frac{7}{2}}} + \frac{1}{3}(bx^2+ax)^{\frac{3}{2}}c^2 + \frac{\sqrt{bx^2+ax}a^2c^2}{8b} - \frac{3\sqrt{bx^2+ax}a^3cd}{32b^2} + \frac{(bx^2+ax)^{\frac{3}{2}}acd}{4b} + \frac{3\sqrt{bx^2+ax}a^4d^2}{128b^3} - \frac{(bx^2+ax)^{\frac{3}{2}}a^2d^2}{16b^2} + \frac{(bx^2+ax)^{\frac{5}{2}}d^2}{5b}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x,x, algorithm="maxima")
```

output
```
1/4*sqrt(b*x^2 + a*x)*a*c^2*x + 1/2*(b*x^2 + a*x)^(3/2)*c*d*x - 3/16*sqrt(
b*x^2 + a*x)*a^2*c*d*x/b + 3/64*sqrt(b*x^2 + a*x)*a^3*d^2*x/b^2 - 1/8*(b*x
^2 + a*x)^(3/2)*a*d^2*x/b - 1/16*a^3*c^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*
x)*sqrt(b))/b^(3/2) + 3/64*a^4*c*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqr
t(b))/b^(5/2) - 3/256*a^5*d^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))
/b^(7/2) + 1/3*(b*x^2 + a*x)^(3/2)*c^2 + 1/8*sqrt(b*x^2 + a*x)*a^2*c^2/b -
 3/32*sqrt(b*x^2 + a*x)*a^3*c*d/b^2 + 1/4*(b*x^2 + a*x)^(3/2)*a*c*d/b + 3/
128*sqrt(b*x^2 + a*x)*a^4*d^2/b^3 - 1/16*(b*x^2 + a*x)^(3/2)*a^2*d^2/b^2 +
 1/5*(b*x^2 + a*x)^(5/2)*d^2/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x}\,dx = \frac{1}{1920}\sqrt{bx^2+ax}\left(2\left(4\left(6\left(8\,bd^2x+\frac{20\,b^5cd+11\,ab^4d^2}{b^4}\right)x+\frac{80\,b^5c^2+18\cdots}{\cdots}\right.\right.\right.$$
$$+\frac{(16\,a^3b^2c^2-12\,a^4bcd+3\,a^5d^2)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{256\,b^{\frac{7}{2}}}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x,x, algorithm="giac")
```

output
```
1/1920*sqrt(b*x^2 + a*x)*(2*(4*(6*(8*b*d^2*x + (20*b^5*c*d + 11*a*b^4*d^2)
/b^4)*x + (80*b^5*c^2 + 180*a*b^4*c*d + 3*a^2*b^3*d^2)/b^4)*x + 5*(112*a*b
^4*c^2 + 12*a^2*b^3*c*d - 3*a^3*b^2*d^2)/b^4)*x + 15*(16*a^2*b^3*c^2 - 12*
a^3*b^2*c*d + 3*a^4*b*d^2)/b^4) + 1/256*(16*a^3*b^2*c^2 - 12*a^4*b*c*d + 3
*a^5*d^2)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x}\,dx = \int \frac{{\left(b\,x^2+a\,x\right)}^{3/2}\,{\left(c+d\,x\right)}^2}{x}\,dx$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^2)/x,x)
```

output
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.30

$$\int \frac{(c+dx)^2 \left(ax+bx^2\right)^{3/2}}{x}\,dx = \frac{45\sqrt{x}\,\sqrt{bx+a}\,a^4b\,d^2-180\sqrt{x}\,\sqrt{bx+a}\,a^3b^2cd-30\sqrt{x}\,\sqrt{bx+a}\,a^3b^2d^2x+}{}$$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(3/2)/x,x)
```

output
```
(45*sqrt(x)*sqrt(a + b*x)*a**4*b*d**2 - 180*sqrt(x)*sqrt(a + b*x)*a**3*b**
2*c*d - 30*sqrt(x)*sqrt(a + b*x)*a**3*b**2*d**2*x + 240*sqrt(x)*sqrt(a + b
*x)*a**2*b**3*c**2 + 120*sqrt(x)*sqrt(a + b*x)*a**2*b**3*c*d*x + 24*sqrt(x
)*sqrt(a + b*x)*a**2*b**3*d**2*x**2 + 1120*sqrt(x)*sqrt(a + b*x)*a*b**4*c*
*2*x + 1440*sqrt(x)*sqrt(a + b*x)*a*b**4*c*d*x**2 + 528*sqrt(x)*sqrt(a + b
*x)*a*b**4*d**2*x**3 + 640*sqrt(x)*sqrt(a + b*x)*b**5*c**2*x**2 + 960*sqrt
(x)*sqrt(a + b*x)*b**5*c*d*x**3 + 384*sqrt(x)*sqrt(a + b*x)*b**5*d**2*x**4
 - 45*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**5*d**2 + 1
80*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4*b*c*d - 240
*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*b**2*c**2)/(1
920*b**4)
```

3.71 $\int \frac{(c+dx)^2(ax+bx^2)^{3/2}}{x^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 204

$$\int \frac{(c+dx)^2(ax+bx^2)^{3/2}}{x^2} dx = \frac{5a(48b^2c^2-16abcd+3a^2d^2)\sqrt{ax+bx^2}}{192b^2} + \frac{(48b^2c^2-16abcd+3a^2d^2)x\sqrt{ax+bx^2}}{96b} + \frac{d(16bc-3ad)(ax+bx^2)^{5/2}}{24b^2x^2} + \frac{d^2(ax+bx^2)^{5/2}}{4bx} + \frac{a^2(48b^2c^2-16abcd+3a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{64b^{5/2}}$$

output
```
5/192*a*(3*a^2*d^2-16*a*b*c*d+48*b^2*c^2)*(b*x^2+a*x)^(1/2)/b^2+1/96*(3*a^
2*d^2-16*a*b*c*d+48*b^2*c^2)*x*(b*x^2+a*x)^(1/2)/b+1/24*d*(-3*a*d+16*b*c)*
(b*x^2+a*x)^(5/2)/b^2/x^2+1/4*d^2*(b*x^2+a*x)^(5/2)/b/x+1/64*a^2*(3*a^2*d^
2-16*a*b*c*d+48*b^2*c^2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.81

$$\int \frac{(c+dx)^2\left(ax+bx^2\right)^{3/2}}{x^2}\,dx = \frac{\sqrt{x(a+bx)}\left(\sqrt{b}(-9a^3d^2+6a^2bd(8c+dx)+16b^3x(6c^2+8cdx+3d^2x^2)\right.}{192}$$

input
```
Integrate[((c + d*x)^2*(a*x + b*x^2)^(3/2))/x^2,x]
```

output
```
(Sqrt[x*(a + b*x)]*(Sqrt[b]*(-9*a^3*d^2 + 6*a^2*b*d*(8*c + d*x) + 16*b^3*x
*(6*c^2 + 8*c*d*x + 3*d^2*x^2) + 8*a*b^2*(30*c^2 + 28*c*d*x + 9*d^2*x^2))
- (3*a^2*(48*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*Log[-(Sqrt[b]*Sqrt[x]) + Sq
rt[a + b*x]])/(Sqrt[x]*Sqrt[a + b*x])))/(192*b^(5/2))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.84, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1262, 27, 1220, 1131, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{3/2}(c+dx)^2}{x^2}dx$$

$$\Big\downarrow \text{1262}$$

$$\frac{\int \frac{\left(8bc^2+d(16bc-3ad)x\right)\left(bx^2+ax\right)^{3/2}}{2x^2}dx}{4b}+\frac{d^2\left(ax+bx^2\right)^{5/2}}{4bx}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(8bc^2+d(16bc-3ad)x\right)\left(bx^2+ax\right)^{3/2}}{x^2}dx}{8b}+\frac{d^2\left(ax+bx^2\right)^{5/2}}{4bx}$$

$$\Big\downarrow \text{1220}$$

$$\frac{\frac{16bc^2(ax+bx^2)^{5/2}}{ax^2}-\frac{(3a^2d^2-16abcd+48b^2c^2)\int\frac{(bx^2+ax)^{3/2}}{x}dx}{a}}{8b}+\frac{d^2(ax+bx^2)^{5/2}}{4bx}$$

$$\Big\downarrow \ 1131$$

$$\frac{\frac{16bc^2(ax+bx^2)^{5/2}}{ax^2}-\frac{(3a^2d^2-16abcd+48b^2c^2)\left(\frac{1}{2}a\int\sqrt{bx^2+ax}dx+\frac{1}{3}(ax+bx^2)^{3/2}\right)}{a}}{8b}+\frac{d^2(ax+bx^2)^{5/2}}{4bx}$$

$$\Big\downarrow \ 1087$$

$$\frac{\frac{16bc^2(ax+bx^2)^{5/2}}{ax^2}-\frac{(3a^2d^2-16abcd+48b^2c^2)\left(\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)+\frac{1}{3}(ax+bx^2)^{3/2}\right)}{a}}{8b}+\frac{d^2(ax+bx^2)^{5/2}}{4bx}$$

$$\Big\downarrow \ 1091$$

$$\frac{\frac{16bc^2(ax+bx^2)^{5/2}}{ax^2}-\frac{(3a^2d^2-16abcd+48b^2c^2)\left(\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)+\frac{1}{3}(ax+bx^2)^{3/2}\right)}{a}}{8b}+\frac{d^2(ax+bx^2)^{5/2}}{4bx}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{16bc^2(ax+bx^2)^{5/2}}{ax^2}-\frac{\left(\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)+\frac{1}{3}(ax+bx^2)^{3/2}\right)(3a^2d^2-16abcd+48b^2c^2)}{a}}{8b}+\frac{d^2(ax+bx^2)^{5/2}}{4bx}$$

input
```
Int[((c + d*x)^2*(a*x + b*x^2)^(3/2))/x^2,x]
```

output
```
(d^2*(a*x + b*x^2)^(5/2))/(4*b*x) + ((16*b*c^2*(a*x + b*x^2)^(5/2))/(a*x^2
) - ((48*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*((a*x + b*x^2)^(3/2)/3 + (a*(((
a + 2*b*x)*Sqrt[a*x + b*x^2])/(4*b) - (a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x +
b*x^2]])/(4*b^(3/2))))/2))/a)/(8*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$\frac{\frac{3a^2\left(a^2d^2-\frac{16}{3}abcd+16b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{64}-\frac{3\left(-\frac{80\left(\frac{3}{10}d^2x^2+\frac{14}{15}cdx+c^2\right)a\,b^{\frac{5}{2}}}{3}-\frac{32\left(\frac{1}{2}d^2x^2+\frac{4}{3}cdx+c^2\right)x\,b^{\frac{7}{2}}}{3}+d\left(\frac{2(-dx-8c}{3}\cdots\right.\right.}{64}}{b^{\frac{5}{2}}}$
risch	$-\frac{\left(-48b^3d^2x^3-72a\,b^2d^2x^2-128b^3c\,x^2d-6a^2b\,d^2x-224a\,b^2cdx-96b^3c^2x+9a^3d^2-48a^2bcd-240a\,c^2b^2\right)x(bx+a)}{192b^2\sqrt{x(bx+a)}}+\frac{a^2(3\cdots}{\cdots}$
default	$d^2\left(\frac{(2bx+a)\left(b\,x^2+ax\right)^{\frac{3}{2}}}{8b}-\frac{3a^2\left(\frac{(2bx+a)\sqrt{b\,x^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8b^{\frac{3}{2}}}\right)}{16b}\right)+c^2\left(\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{a\,x^2}-\frac{6b\left(\cdots\right.}{\cdots}\right.$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
3/64*(a^2*(a^2*d^2-16/3*a*b*c*d+16*b^2*c^2)*arctanh((x*(b*x+a))^(1/2)/x/b^
(1/2))-(-80/3*(3/10*d^2*x^2+14/15*c*d*x+c^2)*a*b^(5/2)-32/3*(1/2*d^2*x^2+4
/3*c*d*x+c^2)*x*b^(7/2)+d*(2/3*(-d*x-8*c)*b^(3/2)+b^(1/2)*a*d)*a^2)*(x*(b*
x+a))^(1/2))/b^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.67

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^2}\,dx = \left[\frac{3\,(48\,a^2b^2c^2 - 16\,a^3bcd + 3\,a^4d^2)\sqrt{b}\log\left(2\,bx + a + 2\,\sqrt{bx^2+ax}\sqrt{b}\right) + 2}{}\right.$$

$$-\frac{3\,(48\,a^2b^2c^2 - 16\,a^3bcd + 3\,a^4d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right) - (48\,b^4d^2x^3 + 240\,ab^3c^2 + 48\,a^2b^2cd - 9\,a^3bc}{192\,b^3}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
[1/384*(3*(48*a^2*b^2*c^2 - 16*a^3*b*c*d + 3*a^4*d^2)*sqrt(b)*log(2*b*x +
a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(48*b^4*d^2*x^3 + 240*a*b^3*c^2 + 48*
a^2*b^2*c*d - 9*a^3*b*d^2 + 8*(16*b^4*c*d + 9*a*b^3*d^2)*x^2 + 2*(48*b^4*c
^2 + 112*a*b^3*c*d + 3*a^2*b^2*d^2)*x)*sqrt(b*x^2 + a*x))/b^3, -1/192*(3*(
48*a^2*b^2*c^2 - 16*a^3*b*c*d + 3*a^4*d^2)*sqrt(-b)*arctan(sqrt(b*x^2 + a*
x)*sqrt(-b)/(b*x + a)) - (48*b^4*d^2*x^3 + 240*a*b^3*c^2 + 48*a^2*b^2*c*d
- 9*a^3*b*d^2 + 8*(16*b^4*c*d + 9*a*b^3*d^2)*x^2 + 2*(48*b^4*c^2 + 112*a*b
^3*c*d + 3*a^2*b^2*d^2)*x)*sqrt(b*x^2 + a*x))/b^3]
```

Sympy [F]

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^2}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}\,(c+dx)^2}{x^2}\,dx$$

input
```
integrate((d*x+c)**2*(b*x**2+a*x)**(3/2)/x**2,x)
```

output
```
Integral((x*(a + b*x))**(3/2)*(c + d*x)**2/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.34

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^2} dx = \frac{1}{2} \sqrt{bx^2+ax}acdx + \frac{1}{4} (bx^2+ax)^{\frac{3}{2}} d^2x - \frac{3 \sqrt{bx^2+ax}a^2d^2x}{32 b} + \frac{3 a^2c^2 \log \left(2 bx + a + 2 \sqrt{bx^2+ax}\sqrt{b}\right)}{8 \sqrt{b}} - \frac{a^3cd \log \left(2 bx + a + 2 \sqrt{bx^2+ax}\sqrt{b}\right)}{8 b^{\frac{3}{2}}} + \frac{3 a^4d^2 \log \left(2 bx + a + 2 \sqrt{bx^2+ax}\sqrt{b}\right)}{128 b^{\frac{5}{2}}} + \frac{3}{4} \sqrt{bx^2+ax}ac^2 + \frac{2}{3} (bx^2+ax)^{\frac{3}{2}} cd + \frac{\sqrt{bx^2+ax}a^2cd}{4 b} - \frac{3 \sqrt{bx^2+ax}a^3d^2}{64 b^2} + \frac{(bx^2+ax)^{\frac{3}{2}} ad^2}{8 b} + \frac{(bx^2+ax)^{\frac{3}{2}} c^2}{2 x}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a*x)*a*c*d*x + 1/4*(b*x^2 + a*x)^(3/2)*d^2*x - 3/32*sqrt(
b*x^2 + a*x)*a^2*d^2*x/b + 3/8*a^2*c^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)
*sqrt(b))/sqrt(b) - 1/8*a^3*c*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b
))/b^(3/2) + 3/128*a^4*d^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^
(5/2) + 3/4*sqrt(b*x^2 + a*x)*a*c^2 + 2/3*(b*x^2 + a*x)^(3/2)*c*d + 1/4*sq
rt(b*x^2 + a*x)*a^2*c*d/b - 3/64*sqrt(b*x^2 + a*x)*a^3*d^2/b^2 + 1/8*(b*x^
2 + a*x)^(3/2)*a*d^2/b + 1/2*(b*x^2 + a*x)^(3/2)*c^2/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^2} dx = \frac{1}{192} \sqrt{bx^2+ax} \left(2 \left(4 \left(6 bd^2x + \frac{16 b^4cd + 9 ab^3d^2}{b^3}\right)x + \frac{48 b^4c^2 + 112 ab^3c}{b^3}\right.\right. - \frac{(48 a^2b^2c^2 - 16 a^3bcd + 3 a^4d^2) \log \left(\left|2 \left(\sqrt{b}x - \sqrt{bx^2+ax}\right)\sqrt{b} + a\right|\right)}{128 b^{\frac{5}{2}}}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^2,x, algorithm="giac")
```

output
```
1/192*sqrt(b*x^2 + a*x)*(2*(4*(6*b*d^2*x + (16*b^4*c*d + 9*a*b^3*d^2)/b^3)
*x + (48*b^4*c^2 + 112*a*b^3*c*d + 3*a^2*b^2*d^2)/b^3)*x + 3*(80*a*b^3*c^2
 + 16*a^2*b^2*c*d - 3*a^3*b*d^2)/b^3) - 1/128*(48*a^2*b^2*c^2 - 16*a^3*b*c
*d + 3*a^4*d^2)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^
(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\left(ax+bx^2\right)^{3/2}}{x^2}\,dx=\int \frac{\left(b\,x^2+a\,x\right)^{3/2}\,{\left(c+d\,x\right)}^2}{x^2}\,dx$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^2)/x^2,x)
```

output
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.25

$$\int \frac{(c+dx)^2\left(ax+bx^2\right)^{3/2}}{x^2}\,dx=\frac{-9\sqrt{x}\,\sqrt{bx+a}\,a^3b\,d^2+48\sqrt{x}\,\sqrt{bx+a}\,a^2b^2cd+6\sqrt{x}\,\sqrt{bx+a}\,a^2b^2d^2x+2}{}$$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^2,x)
```

output
```
( - 9*sqrt(x)*sqrt(a + b*x)*a**3*b*d**2 + 48*sqrt(x)*sqrt(a + b*x)*a**2*b*
*2*c*d + 6*sqrt(x)*sqrt(a + b*x)*a**2*b**2*d**2*x + 240*sqrt(x)*sqrt(a + b
*x)*a*b**3*c**2 + 224*sqrt(x)*sqrt(a + b*x)*a*b**3*c*d*x + 72*sqrt(x)*sqrt
(a + b*x)*a*b**3*d**2*x**2 + 96*sqrt(x)*sqrt(a + b*x)*b**4*c**2*x + 128*sq
rt(x)*sqrt(a + b*x)*b**4*c*d*x**2 + 48*sqrt(x)*sqrt(a + b*x)*b**4*d**2*x**
3 + 9*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4*d**2 - 4
8*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*b*c*d + 144*
sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*b**2*c**2)/(19
2*b**3)
```

3.72 $\int \frac{(c+dx)^2(ax+bx^2)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 183

$$\int \frac{(c+dx)^2(ax+bx^2)^{3/2}}{x^3}\,dx = -\frac{5(a^2d^2-12bc(2bc+ad))\sqrt{ax+bx^2}}{24b} - \frac{(a^2d^2-12bc(2bc+ad))\,x\sqrt{ax+bx^2}}{12a} - \frac{2c^2(ax+bx^2)^{5/2}}{ax^3} + \frac{d^2(ax+bx^2)^{5/2}}{3bx^2} - \frac{a(a^2d^2-12bc(2bc+ad))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{8b^{3/2}}$$

output
```
-5/24*(a^2*d^2-12*b*c*(a*d+2*b*c))*(b*x^2+a*x)^(1/2)/b-1/12*(a^2*d^2-12*b*
c*(a*d+2*b*c))*x*(b*x^2+a*x)^(1/2)/a-2*c^2*(b*x^2+a*x)^(5/2)/a/x^3+1/3*d^2
*(b*x^2+a*x)^(5/2)/b/x^2-1/8*a*(a^2*d^2-12*b*c*(a*d+2*b*c))*arctanh(b^(1/2
)*x/(b*x^2+a*x)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^3} dx = \frac{(x(a+bx))^{3/2} \left(\frac{\sqrt{b}(3a^2d^2x+8b^2x(3c^2+3cdx+d^2x^2)+2ab(-24c^2+30cdx+7d^2x^2))}{a+bx} + \frac{6a(-24}{}\right.}{24b^{3/2}x^2}$$

input
```
Integrate[((c + d*x)^2*(a*x + b*x^2)^(3/2))/x^3,x]
```

output
```
((x*(a + b*x))^(3/2)*((Sqrt[b]*(3*a^2*d^2*x + 8*b^2*x*(3*c^2 + 3*c*d*x + d
^2*x^2) + 2*a*b*(-24*c^2 + 30*c*d*x + 7*d^2*x^2)))/(a + b*x) + (6*a*(-24*b
^2*c^2 - 12*a*b*c*d + a^2*d^2)*Sqrt[x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a]
- Sqrt[a + b*x])])/(a + b*x)^(3/2)))/(24*b^(3/2)*x^2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.85, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1262, 27, 1220, 1131, 1131, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2} (c+dx)^2}{x^3} dx$$

$$\Big\downarrow \ 1262$$

$$\frac{\int \frac{(6bc^2+d(12bc-ad)x)(bx^2+ax)^{3/2}}{2x^3} dx}{3b} + \frac{d^2(ax+bx^2)^{5/2}}{3bx^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(6bc^2+d(12bc-ad)x)(bx^2+ax)^{3/2}}{x^3} dx}{6b} + \frac{d^2(ax+bx^2)^{5/2}}{3bx^2}$$

$$\Big\downarrow \ 1220$$

$$\frac{\frac{\left(-a^2d^2+12abcd+24b^2c^2\right)\int \frac{\left(bx^2+ax\right)^{3/2}}{x^2}dx}{a}-\frac{12bc^2\left(ax+bx^2\right)^{5/2}}{ax^3}}{6b}+\frac{d^2\left(ax+bx^2\right)^{5/2}}{3bx^2}$$

$$\Big\downarrow \; 1131$$

$$\frac{\frac{\left(-a^2d^2+12abcd+24b^2c^2\right)\left(\frac{3}{4}a\int \frac{\sqrt{bx^2+ax}}{x}dx+\frac{\left(ax+bx^2\right)^{3/2}}{2x}\right)}{a}-\frac{12bc^2\left(ax+bx^2\right)^{5/2}}{ax^3}}{6b}+\frac{d^2\left(ax+bx^2\right)^{5/2}}{3bx^2}$$

$$\Big\downarrow \; 1131$$

$$\frac{\frac{\left(-a^2d^2+12abcd+24b^2c^2\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+ax}}dx+\sqrt{ax+bx^2}\right)+\frac{\left(ax+bx^2\right)^{3/2}}{2x}\right)}{a}-\frac{12bc^2\left(ax+bx^2\right)^{5/2}}{ax^3}}{6b}+\frac{d^2\left(ax+bx^2\right)^{5/2}}{3bx^2}$$

$$\Big\downarrow \; 1091$$

$$\frac{\frac{\left(-a^2d^2+12abcd+24b^2c^2\right)\left(\frac{3}{4}a\left(a\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}+\sqrt{ax+bx^2}\right)+\frac{\left(ax+bx^2\right)^{3/2}}{2x}\right)}{a}-\frac{12bc^2\left(ax+bx^2\right)^{5/2}}{ax^3}}{6b}+\frac{d^2\left(ax+bx^2\right)^{5/2}}{3bx^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}+\sqrt{ax+bx^2}\right)+\frac{\left(ax+bx^2\right)^{3/2}}{2x}\right)\left(-a^2d^2+12abcd+24b^2c^2\right)}{a}-\frac{12bc^2\left(ax+bx^2\right)^{5/2}}{ax^3}}{6b}+\frac{d^2\left(ax+bx^2\right)^{5/2}}{3bx^2}$$

input
```
Int[((c + d*x)^2*(a*x + b*x^2)^(3/2))/x^3,x]
```

output
```
(d^2*(a*x + b*x^2)^(5/2))/(3*b*x^2) + ((-12*b*c^2*(a*x + b*x^2)^(5/2))/(a*
x^3) + ((24*b^2*c^2 + 12*a*b*c*d - a^2*d^2)*((a*x + b*x^2)^(3/2)/(2*x) + (
3*a*(Sqrt[a*x + b*x^2] + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b
]))/4))/a)/(6*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$-\frac{ax\left(a^2d^2-12abcd-24b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)-\left(-16\left(-\frac{7}{24}d^2x^2-\frac{5}{4}cdx+c^2\right)a\,b^{\frac{3}{2}}+x\left(\left(\frac{8}{3}d^2x^2+8cdx+8c^2\right)b^{\frac{5}{2}}+a^2d^2\sqrt{}\right.\right.}{8b^{\frac{3}{2}}x}$
risch	$\frac{(bx+a)\left(8x^3b^2d^2+14ab\,d^2x^2+24b^2c\,x^2d+3a^2d^2x+60abcdx+24x\,b^2c^2-48ab\,c^2\right)}{24b\sqrt{x(bx+a)}}-\frac{\left(a^2d^2-12abcd-24b^2c^2\right)a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\right.}{16b^{\frac{3}{2}}}$
default	$c^2\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{a\,x^3}+\frac{4b\left(\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{a\,x^2}-\frac{6b\left(\frac{\left(b\,x^2+ax\right)^{\frac{3}{2}}}{3}+\frac{a\left(\frac{(2bx+a)\sqrt{b\,x^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8b^{\frac{3}{2}}}\right)}{2}\right)}{a}\right)}{a}\right)$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8/b^(3/2)*(a*x*(a^2*d^2-12*a*b*c*d-24*b^2*c^2)*arctanh((x*(b*x+a))^(1/2
)/x/b^(1/2))-(-16*(-7/24*d^2*x^2-5/4*c*d*x+c^2)*a*b^(3/2)+x*((8/3*d^2*x^2+
8*c*d*x+8*c^2)*b^(5/2)+a^2*d^2*b^(1/2)))*(x*(b*x+a))^(1/2))/x
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.64

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^3}\,dx = \left[-\frac{3\,(24\,ab^2c^2+12\,a^2bcd-a^3d^2)\sqrt{b}x\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)-\cdots}{\cdots}\right.$$

$$-\frac{3\,(24\,ab^2c^2+12\,a^2bcd-a^3d^2)\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(8\,b^3d^2x^3-48\,ab^2c^2+2\,(12\,b^3cd+7\,ab^2d^2)x\cdots}{24\,b^2x}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
[-1/48*(3*(24*a*b^2*c^2 + 12*a^2*b*c*d - a^3*d^2)*sqrt(b)*x*log(2*b*x + a
- 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*(8*b^3*d^2*x^3 - 48*a*b^2*c^2 + 2*(12*b
^3*c*d + 7*a*b^2*d^2)*x^2 + 3*(8*b^3*c^2 + 20*a*b^2*c*d + a^2*b*d^2)*x)*sq
rt(b*x^2 + a*x))/(b^2*x), -1/24*(3*(24*a*b^2*c^2 + 12*a^2*b*c*d - a^3*d^2)
*sqrt(-b)*x*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (8*b^3*d^2*x^3
- 48*a*b^2*c^2 + 2*(12*b^3*c*d + 7*a*b^2*d^2)*x^2 + 3*(8*b^3*c^2 + 20*a*b^
2*c*d + a^2*b*d^2)*x)*sqrt(b*x^2 + a*x))/(b^2*x)]
```

Sympy [F]

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^3}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}\,(c+dx)^2}{x^3}\,dx$$

input
```
integrate((d*x+c)**2*(b*x**2+a*x)**(3/2)/x**3,x)
```

output
```
Integral((x*(a + b*x))**(3/2)*(c + d*x)**2/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.24

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^3} dx = \frac{1}{4}\sqrt{bx^2+ax}ad^2x + \frac{3}{2}a\sqrt{b}c^2 \log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) + \frac{3\,a^2cd\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{4\,\sqrt{b}} - \frac{a^3d^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{3}{2}}} + \frac{3}{2}\sqrt{bx^2+ax}acd + \frac{1}{3}\left(bx^2+ax\right)^{\frac{3}{2}}d^2 + \frac{\sqrt{bx^2+ax}a^2d^2}{8\,b} - \frac{3\,\sqrt{bx^2+ax}ac^2}{x} + \frac{(bx^2+ax)^{\frac{3}{2}}cd}{x} + \frac{(bx^2+ax)^{\frac{3}{2}}c^2}{x^2}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
1/4*sqrt(b*x^2 + a*x)*a*d^2*x + 3/2*a*sqrt(b)*c^2*log(2*b*x + a + 2*sqrt(b
*x^2 + a*x)*sqrt(b)) + 3/4*a^2*c*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqr
t(b))/sqrt(b) - 1/16*a^3*d^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/
b^(3/2) + 3/2*sqrt(b*x^2 + a*x)*a*c*d + 1/3*(b*x^2 + a*x)^(3/2)*d^2 + 1/8*
sqrt(b*x^2 + a*x)*a^2*d^2/b - 3*sqrt(b*x^2 + a*x)*a*c^2/x + (b*x^2 + a*x)^
(3/2)*c*d/x + (b*x^2 + a*x)^(3/2)*c^2/x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^3} dx = \frac{2\,a^2c^2}{\sqrt{b}x-\sqrt{bx^2+ax}} + \frac{1}{24}\sqrt{bx^2+ax}\left(2\left(4\,bd^2x+\frac{12\,b^3cd+7\,ab^2d^2}{b^2}\right)x+\frac{3\,(8\,b^3c^2+20\,ab^2cd+a^2bd^2)}{b^2}\right) - \frac{(24\,ab^2c^2+12\,a^2bcd-a^3d^2)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{16\,b^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^3,x, algorithm="giac")
```

output
```
2*a^2*c^2/(sqrt(b)*x - sqrt(b*x^2 + a*x)) + 1/24*sqrt(b*x^2 + a*x)*(2*(4*b
*d^2*x + (12*b^3*c*d + 7*a*b^2*d^2)/b^2)*x + 3*(8*b^3*c^2 + 20*a*b^2*c*d +
 a^2*b*d^2)/b^2) - 1/16*(24*a*b^2*c^2 + 12*a^2*b*c*d - a^3*d^2)*log(abs(2*
(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^3}\,dx=\int \frac{(b\,x^2+a\,x)^{3/2}\,(c+d\,x)^2}{x^3}\,dx$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^2)/x^3,x)
```

output
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.40

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^3}\,dx=\frac{24\sqrt{x}\,\sqrt{bx+a}\,a^2b\,d^2x-384\sqrt{x}\,\sqrt{bx+a}\,a\,b^2c^2+480\sqrt{x}\,\sqrt{bx+a}\,a\,b^2cdx}{}$$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^3,x)
```

output
```
(24*sqrt(x)*sqrt(a + b*x)*a**2*b*d**2*x - 384*sqrt(x)*sqrt(a + b*x)*a*b**2
*c**2 + 480*sqrt(x)*sqrt(a + b*x)*a*b**2*c*d*x + 112*sqrt(x)*sqrt(a + b*x)
*a*b**2*d**2*x**2 + 192*sqrt(x)*sqrt(a + b*x)*b**3*c**2*x + 192*sqrt(x)*sq
rt(a + b*x)*b**3*c*d*x**2 + 64*sqrt(x)*sqrt(a + b*x)*b**3*d**2*x**3 - 24*s
qrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*d**2*x + 288*sq
rt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*b*c*d*x + 576*sq
rt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b**2*c**2*x + 3*sqr
t(b)*a**3*d**2*x - 96*sqrt(b)*a**2*b*c*d*x - 432*sqrt(b)*a*b**2*c**2*x)/(1
92*b**2*x)
```

3.73 $\int \frac{(c+dx)^2\left(ax+bx^2\right)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 178

$$\int \frac{(c+dx)^2\left(ax+bx^2\right)^{3/2}}{x^4}\,dx = \frac{(8b^2c^2+24abcd+3a^2d^2)\sqrt{ax+bx^2}}{4a} - \frac{4c(bc+3ad)\left(ax+bx^2\right)^{3/2}}{3ax^2} + \frac{d^2\left(ax+bx^2\right)^{3/2}}{2x} - \frac{2c^2\left(ax+bx^2\right)^{5/2}}{3ax^4} + \frac{(8b^2c^2+24abcd+3a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4\sqrt{b}}$$

output
```
1/4*(3*a^2*d^2+24*a*b*c*d+8*b^2*c^2)*(b*x^2+a*x)^(1/2)/a-4/3*c*(3*a*d+b*c)
*(b*x^2+a*x)^(3/2)/a/x^2+1/2*d^2*(b*x^2+a*x)^(3/2)/x-2/3*c^2*(b*x^2+a*x)^(
5/2)/a/x^4+1/4*(3*a^2*d^2+24*a*b*c*d+8*b^2*c^2)*arctanh(b^(1/2)*x/(b*x^2+a
*x)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.78

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^4} \, dx = \frac{\sqrt{x(a+bx)} \Big(2bx(-16c^2+12cdx+3d^2x^2)+a(-8c^2-48cdx+15d^2x^2)+}{12x^2}$$

input
```
Integrate[((c + d*x)^2*(a*x + b*x^2)^(3/2))/x^4,x]
```

output
```
(Sqrt[x*(a + b*x)]*(2*b*x*(-16*c^2 + 12*c*d*x + 3*d^2*x^2) + a*(-8*c^2 - 4
8*c*d*x + 15*d^2*x^2) + (6*(8*b^2*c^2 + 24*a*b*c*d + 3*a^2*d^2)*x^(3/2)*Ar
cTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(Sqrt[b]*Sqrt[a + b*x
])))/(12*x^2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.88, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1262, 27, 1220, 1125, 25, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2} (c+dx)^2}{x^4} \, dx$$

$$\Big\downarrow \text{1262}$$

$$\frac{\int \frac{(4bc^2+d(8bc+ad)x)(bx^2+ax)^{3/2}}{2x^4}dx}{2b} + \frac{d^2 (ax+bx^2)^{5/2}}{2bx^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(4bc^2+d(8bc+ad)x)(bx^2+ax)^{3/2}}{x^4}dx}{4b} + \frac{d^2 (ax+bx^2)^{5/2}}{2bx^3}$$

$$\Big\downarrow \text{1220}$$

$$\frac{\frac{(3a^2d^2+24abcd+8b^2c^2)\int \frac{(bx^2+ax)^{3/2}}{x^3}dx}{3a}-\frac{8bc^2(ax+bx^2)^{5/2}}{3ax^4}}{4b}+\frac{d^2(ax+bx^2)^{5/2}}{2bx^3}$$

$$\Big\downarrow \; 1125$$

$$\frac{\frac{(3a^2d^2+24abcd+8b^2c^2)\left(-\int -\frac{b(2a+bx)}{\sqrt{bx^2+ax}}dx-\frac{2a\sqrt{ax+bx^2}}{x}\right)}{3a}-\frac{8bc^2(ax+bx^2)^{5/2}}{3ax^4}}{4b}+\frac{d^2(ax+bx^2)^{5/2}}{2bx^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{(3a^2d^2+24abcd+8b^2c^2)\left(\int \frac{b(2a+bx)}{\sqrt{bx^2+ax}}dx-\frac{2a\sqrt{ax+bx^2}}{x}\right)}{3a}-\frac{8bc^2(ax+bx^2)^{5/2}}{3ax^4}}{4b}+\frac{d^2(ax+bx^2)^{5/2}}{2bx^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{(3a^2d^2+24abcd+8b^2c^2)\left(b\int \frac{2a+bx}{\sqrt{bx^2+ax}}dx-\frac{2a\sqrt{ax+bx^2}}{x}\right)}{3a}-\frac{8bc^2(ax+bx^2)^{5/2}}{3ax^4}}{4b}+\frac{d^2(ax+bx^2)^{5/2}}{2bx^3}$$

$$\Big\downarrow \; 1160$$

$$\frac{\frac{(3a^2d^2+24abcd+8b^2c^2)\left(b\left(\frac{3}{2}a\int \frac{1}{\sqrt{bx^2+ax}}dx+\sqrt{ax+bx^2}\right)-\frac{2a\sqrt{ax+bx^2}}{x}\right)}{3a}-\frac{8bc^2(ax+bx^2)^{5/2}}{3ax^4}}{4b}+\frac{d^2(ax+bx^2)^{5/2}}{2bx^3}$$

$$\Big\downarrow \; 1091$$

$$\frac{\frac{(3a^2d^2+24abcd+8b^2c^2)\left(b\left(3a\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}+\sqrt{ax+bx^2}\right)-\frac{2a\sqrt{ax+bx^2}}{x}\right)}{3a}-\frac{8bc^2(ax+bx^2)^{5/2}}{3ax^4}}{4b}+\frac{d^2(ax+bx^2)^{5/2}}{2bx^3}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\left(b\left(\frac{3a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}+\sqrt{ax+bx^2}\right)-\frac{2a\sqrt{ax+bx^2}}{x}\right)(3a^2d^2+24abcd+8b^2c^2)}{3a}-\frac{8bc^2(ax+bx^2)^{5/2}}{3ax^4}}{4b}+\frac{d^2(ax+bx^2)^{5/2}}{2bx^3}$$

input
```
Int[((c + d*x)^2*(a*x + b*x^2)^(3/2))/x^4,x]
```

output
```
(d^2*(a*x + b*x^2)^(5/2))/(2*b*x^3) + ((-8*b*c^2*(a*x + b*x^2)^(5/2))/(3*a
*x^4) + ((8*b^2*c^2 + 24*a*b*c*d + 3*a^2*d^2)*((-2*a*Sqrt[a*x + b*x^2])/x
+ b*(Sqrt[a*x + b*x^2] + (3*a*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt
[b])))/(3*a))/(4*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$\frac{\frac{3x^{2}\left(a^{2}d^{2}+8abcd+\frac{8}{3}b^{2}c^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{4}-\frac{2\left(\left(-\frac{3}{4}d^{2}x^{3}-3cd\,x^{2}+4c^{2}x\right)b^{\frac{3}{2}}+a\sqrt{b}\left(-\frac{15}{8}d^{2}x^{2}+6cdx+c^{2}\right)\right)\sqrt{x(bx+a)}}{3}}{\sqrt{b}\,x^{2}}$
risch	$-\frac{(bx+a)\left(-6b\,d^{2}x^{3}-15a\,d^{2}x^{2}-24bcd\,x^{2}+48adxc+32c^{2}bx+8a\,c^{2}\right)}{12x\sqrt{x(bx+a)}}+\frac{\left(\frac{3}{8}a^{2}d^{2}+3abcd+b^{2}c^{2}\right)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{\sqrt{b}}$
default	$c^{2}\left(-\frac{2\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{3a\,x^{4}}+\frac{2b\left(-\frac{2\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{a\,x^{3}}+\frac{4b\left(\frac{2\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{a\,x^{2}}-\frac{6b\left(\frac{\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{3}+\frac{a\left(\frac{(2bx+a)\sqrt{b\,x^{2}+ax}}{4b}-\frac{a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{\cdots}\right)}{8b^{\frac{3}{2}}}\cdots\right)}{2}\cdots\right)}{a}\cdots\right)}{a}\cdots\right)}{3a}\cdots\right.$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
3/4/b^(1/2)*(x^2*(a^2*d^2+8*a*b*c*d+8/3*b^2*c^2)*arctanh((x*(b*x+a))^(1/2)
/x/b^(1/2))-8/9*((-3/4*d^2*x^3-3*c*d*x^2+4*c^2*x)*b^(3/2)+a*b^(1/2)*(-15/8
*d^2*x^2+6*c*d*x+c^2))*(x*(b*x+a))^(1/2))/x^2
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.52

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^4} dx = \left[\frac{3 (8 b^2c^2 + 24 abcd + 3 a^2d^2)\sqrt{b}x^2 \log \left(2 bx + a + 2 \sqrt{bx^2 + ax}\sqrt{b}\right) + 2 (6}{24} \right.$$

$$- \frac{3 (8 b^2c^2 + 24 abcd + 3 a^2d^2)\sqrt{-b}x^2 \arctan \left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right) - (6 b^2d^2x^3 - 8 abc^2 + 3 (8 b^2cd + 5 abd^2)x^2 - 1}{12 bx^2}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[1/24*(3*(8*b^2*c^2 + 24*a*b*c*d + 3*a^2*d^2)*sqrt(b)*x^2*log(2*b*x + a +
2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(6*b^2*d^2*x^3 - 8*a*b*c^2 + 3*(8*b^2*c*d
 + 5*a*b*d^2)*x^2 - 16*(2*b^2*c^2 + 3*a*b*c*d)*x)*sqrt(b*x^2 + a*x))/(b*x^
2), -1/12*(3*(8*b^2*c^2 + 24*a*b*c*d + 3*a^2*d^2)*sqrt(-b)*x^2*arctan(sqrt
(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (6*b^2*d^2*x^3 - 8*a*b*c^2 + 3*(8*b^2*
c*d + 5*a*b*d^2)*x^2 - 16*(2*b^2*c^2 + 3*a*b*c*d)*x)*sqrt(b*x^2 + a*x))/(b
*x^2)]
```

Sympy [F]

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^4} dx = \int \frac{(x(a+bx))^{\frac{3}{2}} (c+dx)^2}{x^4} dx$$

input
```
integrate((d*x+c)**2*(b*x**2+a*x)**(3/2)/x**4,x)
```

output
```
Integral((x*(a + b*x))**(3/2)*(c + d*x)**2/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.28

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^4} dx = b^{\frac{3}{2}} c^2 \log \left(2 bx + a + 2 \sqrt{bx^2 + ax}\sqrt{b}\right) + 3 a\sqrt{b}cd \log \left(2 bx + a + 2 \sqrt{bx^2 + ax}\sqrt{b}\right) + \frac{3 a^2 d^2 \log \left(2 bx + a + 2 \sqrt{bx^2 + ax}\sqrt{b}\right)}{8 \sqrt{b}} + \frac{3}{4} \sqrt{bx^2 + ax} a d^2 - \frac{7 \sqrt{bx^2 + ax} b c^2}{3 x} - \frac{6 \sqrt{bx^2 + ax} a c d}{x} + \frac{(bx^2 + ax)^{\frac{3}{2}} d^2}{2 x} - \frac{\sqrt{bx^2 + ax} a c^2}{3 x^2} + \frac{2 (bx^2 + ax)^{\frac{3}{2}} c d}{x^2} - \frac{(bx^2 + ax)^{\frac{3}{2}} c^2}{3 x^3}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
b^(3/2)*c^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 3*a*sqrt(b)*c*d
*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 3/8*a^2*d^2*log(2*b*x + a
+ 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b) + 3/4*sqrt(b*x^2 + a*x)*a*d^2 - 7/3
*sqrt(b*x^2 + a*x)*b*c^2/x - 6*sqrt(b*x^2 + a*x)*a*c*d/x + 1/2*(b*x^2 + a*
x)^(3/2)*d^2/x - 1/3*sqrt(b*x^2 + a*x)*a*c^2/x^2 + 2*(b*x^2 + a*x)^(3/2)*c
*d/x^2 - 1/3*(b*x^2 + a*x)^(3/2)*c^2/x^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.20

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^4} dx = \frac{1}{4}\left(2\,bd^2x + \frac{8\,b^2cd+5\,abd^2}{b}\right)\sqrt{bx^2+ax} - \frac{(8\,b^2c^2+24\,abcd+3\,a^2d^2)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\sqrt{b}} + \frac{2\left(6\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2abc^2+6\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2a^2cd+3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)a^2\sqrt{b}c^2+a^3c^2\right)}{3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^4,x, algorithm="giac")
```

output
```
1/4*(2*b*d^2*x + (8*b^2*c*d + 5*a*b*d^2)/b)*sqrt(b*x^2 + a*x) - 1/8*(8*b^2
*c^2 + 24*a*b*c*d + 3*a^2*d^2)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*s
qrt(b) + a))/sqrt(b) + 2/3*(6*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a*b*c^2 +
6*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2*c*d + 3*(sqrt(b)*x - sqrt(b*x^2 +
a*x))*a^2*sqrt(b)*c^2 + a^3*c^2)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^4} dx = \int \frac{(b\,x^2+a\,x)^{3/2}\,(c+d\,x)^2}{x^4} dx$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^2)/x^4,x)
```

output
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^4} dx = \frac{-64\sqrt{x}\sqrt{bx+a}\,ab\,c^2 - 384\sqrt{x}\sqrt{bx+a}\,abcdx + 120\sqrt{x}\sqrt{bx+a}\,ab\,d^2x^2 -$$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^4,x)
```

output
```
( - 64*sqrt(x)*sqrt(a + b*x)*a*b*c**2 - 384*sqrt(x)*sqrt(a + b*x)*a*b*c*d*
x + 120*sqrt(x)*sqrt(a + b*x)*a*b*d**2*x**2 - 256*sqrt(x)*sqrt(a + b*x)*b*
*2*c**2*x + 192*sqrt(x)*sqrt(a + b*x)*b**2*c*d*x**2 + 48*sqrt(x)*sqrt(a +
b*x)*b**2*d**2*x**3 + 72*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqr
t(a))*a**2*d**2*x**2 + 576*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/s
qrt(a))*a*b*c*d*x**2 + 192*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/s
qrt(a))*b**2*c**2*x**2 + 15*sqrt(b)*a**2*d**2*x**2 + 160*sqrt(b)*a*b*c*d*x
**2)/(96*b*x**2)
```

3.74 $\int \frac{(c+dx)^2(ax+bx^2)^{3/2}}{x^5} dx$

Optimal result

Integrand size = 24, antiderivative size = 131

$$\int \frac{(c+dx)^2(ax+bx^2)^{3/2}}{x^5} dx = bd^2\sqrt{ax+bx^2} - \frac{2d(2bc+ad)\sqrt{ax+bx^2}}{x} - \frac{4cd(ax+bx^2)^{3/2}}{3x^3} - \frac{2c^2(ax+bx^2)^{5/2}}{5ax^5} + \sqrt{b}d(4bc+3ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)$$

output
```
b*d^2*(b*x^2+a*x)^(1/2)-2*d*(a*d+2*b*c)*(b*x^2+a*x)^(1/2)/x-4/3*c*d*(b*x^2
+a*x)^(3/2)/x^3-2/5*c^2*(b*x^2+a*x)^(5/2)/a/x^5+b^(1/2)*d*(3*a*d+4*b*c)*ar
ctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.11

$$\int \frac{(c+dx)^2(ax+bx^2)^{3/2}}{x^5} dx = -\frac{\sqrt{x(a+bx)}\left(\sqrt{a+bx}(6b^2c^2x^2+abx(12c^2+80cdx-15d^2x^2)+a^2(6c^2+20cdx+30d^2x^2))+15a\sqrt{b}d(4\right.}{15ax^3\sqrt{a+bx}}$$

input
```
Integrate[((c + d*x)^2*(a*x + b*x^2)^(3/2))/x^5,x]
```

output
```
-1/15*(Sqrt[x*(a + b*x)]*(Sqrt[a + b*x]*(6*b^2*c^2*x^2 + a*b*x*(12*c^2 + 8
0*c*d*x - 15*d^2*x^2) + a^2*(6*c^2 + 20*c*d*x + 30*d^2*x^2)) + 15*a*Sqrt[b
]*d*(4*b*c + 3*a*d)*x^(5/2)*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]))/(a*x
^3*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1262, 27, 1220, 1130, 1125, 25, 27, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}(c+dx)^2}{x^5}dx$$

$$\Big\downarrow \text{1262}$$

$$\frac{\int \frac{(2bc^2+d(4bc+3ad)x)(bx^2+ax)^{3/2}}{2x^5}dx}{b}+\frac{d^2(ax+bx^2)^{5/2}}{bx^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(2bc^2+d(4bc+3ad)x)(bx^2+ax)^{3/2}}{x^5}dx}{2b}+\frac{d^2(ax+bx^2)^{5/2}}{bx^4}$$

$$\Big\downarrow \text{1220}$$

$$\frac{d(3ad+4bc)\int \frac{(bx^2+ax)^{3/2}}{x^4}dx-\frac{4bc^2(ax+bx^2)^{5/2}}{5ax^5}}{2b}+\frac{d^2(ax+bx^2)^{5/2}}{bx^4}$$

$$\Big\downarrow \text{1130}$$

$$\frac{d(3ad+4bc)\left(b\int \frac{\sqrt{bx^2+ax}}{x^2}dx-\frac{2(ax+bx^2)^{3/2}}{3x^3}\right)-\frac{4bc^2(ax+bx^2)^{5/2}}{5ax^5}}{2b}+\frac{d^2(ax+bx^2)^{5/2}}{bx^4}$$

$$\Big\downarrow \text{1125}$$

$$\frac{d(3ad+4bc)\left(b\left(-\int -\frac{b}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2(ax+bx^2)^{3/2}}{3x^3}\right)-\frac{4bc^2(ax+bx^2)^{5/2}}{5ax^5}}{2b}+\frac{d^2(ax+bx^2)^{5/2}}{bx^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{d(3ad+4bc)\left(b\left(\int \frac{b}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2(ax+bx^2)^{3/2}}{3x^3}\right)-\frac{4bc^2(ax+bx^2)^{5/2}}{5ax^5}}{2b}+\frac{d^2(ax+bx^2)^{5/2}}{bx^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{d(3ad+4bc)\left(b\left(b\int \frac{1}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2(ax+bx^2)^{3/2}}{3x^3}\right)-\frac{4bc^2(ax+bx^2)^{5/2}}{5ax^5}}{2b}+\frac{d^2(ax+bx^2)^{5/2}}{bx^4}$$

$$\Big\downarrow \text{1091}$$

$$\frac{d(3ad+4bc)\left(b\left(2b\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2(ax+bx^2)^{3/2}}{3x^3}\right)-\frac{4bc^2(ax+bx^2)^{5/2}}{5ax^5}}{2b}+\frac{d^2(ax+bx^2)^{5/2}}{bx^4}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\left(b\left(2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2(ax+bx^2)^{3/2}}{3x^3}\right)(3ad+4bc)-\frac{4bc^2(ax+bx^2)^{5/2}}{5ax^5}}{2b}+\frac{d^2(ax+bx^2)^{5/2}}{bx^4}$$

input
```
Int[((c + d*x)^2*(a*x + b*x^2)^(3/2))/x^5,x]
```

output
```
(d^2*(a*x + b*x^2)^(5/2))/(b*x^4) + ((-4*b*c^2*(a*x + b*x^2)^(5/2))/(5*a*x
^5) + d*(4*b*c + 3*a*d)*((-2*(a*x + b*x^2)^(3/2))/(3*x^3) + b*((-2*Sqrt[a*
x + b*x^2])/x + 2*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])))/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$-\frac{2\left(-\frac{15 d \left(a d+\frac{4 b c}{3}\right) x^{3} b a \,\mathrm{arctanh}\left(\frac{\sqrt{x (b x+a)}}{x \sqrt{b}}\right)}{2}+\left(2 x \left(-\frac{5}{4} d^{2} x^{2}+\frac{20}{3} c d x+c^{2}\right) a \,b^{\frac{3}{2}}+c^{2} x^{2} b^{\frac{5}{2}}+a^{2} \sqrt{b}\,\left(5 d^{2} x^{2}+\frac{10}{3} c d x+c^{2}\right)\right) \sqrt{x\cdots}\right)}{5 \sqrt{b}\, x^{3} a}$
risch	$-\frac{(bx+a)\left(-15d^{2}x^{3}ab+30a^{2}d^{2}x^{2}+80abcd\,x^{2}+6b^{2}c^{2}x^{2}+20a^{2}cdx+12ab\,c^{2}x+6a^{2}c^{2}\right)}{15x^{2}\sqrt{x(bx+a)}\,a}+\frac{(3ad+4bc)\sqrt{b}\,d\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}\cdots}\right)}{2}$
default	$-\frac{2c^{2}\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{5a\,x^{5}}+d^{2}\left(-\frac{2\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{a\,x^{3}}+\frac{4b\left(\frac{2\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{a\,x^{2}}-\frac{6b\left(\frac{\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{3}+\frac{a\left(\frac{(2bx+a)\sqrt{b\,x^{2}+ax}}{4b}-\frac{a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}\cdots\right)}{8b\cdots}\right)}{2}\cdots\right)}{a}\cdots\right)}{a}\cdots\right)$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-15/2*d*(a*d+4/3*b*c)*x^3*b*a*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))+(
2*x*(-5/4*d^2*x^2+20/3*c*d*x+c^2)*a*b^(3/2)+c^2*x^2*b^(5/2)+a^2*b^(1/2)*(5
*d^2*x^2+10/3*c*d*x+c^2))*(x*(b*x+a))^(1/2))/b^(1/2)/x^3/a
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.07

$$\int \frac{(c+dx)^2\,(ax+bx^2)^{3/2}}{x^5}\,dx = \left[\frac{15\,(4\,abcd+3\,a^2d^2)\sqrt{b}x^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(15\,abd^2x^3 \ldots}{3 \ldots}\right.$$

$$-\frac{15\,(4\,abcd+3\,a^2d^2)\sqrt{-b}x^3\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(15\,abd^2x^3-6\,a^2c^2-2\,(3\,b^2c^2+40\,abcd+15\,a^2d^2)x^2 \ldots}{15\,ax^3}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
[1/30*(15*(4*a*b*c*d + 3*a^2*d^2)*sqrt(b)*x^3*log(2*b*x + a + 2*sqrt(b*x^2
 + a*x)*sqrt(b)) + 2*(15*a*b*d^2*x^3 - 6*a^2*c^2 - 2*(3*b^2*c^2 + 40*a*b*c
*d + 15*a^2*d^2)*x^2 - 4*(3*a*b*c^2 + 5*a^2*c*d)*x)*sqrt(b*x^2 + a*x))/(a*
x^3), -1/15*(15*(4*a*b*c*d + 3*a^2*d^2)*sqrt(-b)*x^3*arctan(sqrt(b*x^2 + a
*x)*sqrt(-b)/(b*x + a)) - (15*a*b*d^2*x^3 - 6*a^2*c^2 - 2*(3*b^2*c^2 + 40*
a*b*c*d + 15*a^2*d^2)*x^2 - 4*(3*a*b*c^2 + 5*a^2*c*d)*x)*sqrt(b*x^2 + a*x)
)/(a*x^3)]
```

Sympy [F]

$$\int \frac{(c+dx)^2\,(ax+bx^2)^{3/2}}{x^5}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}\,(c+dx)^2}{x^5}\,dx$$

input
```
integrate((d*x+c)**2*(b*x**2+a*x)**(3/2)/x**5,x)
```

output
```
Integral((x*(a + b*x))**(3/2)*(c + d*x)**2/x**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(113) = 226.

Time = 0.04 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.84

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^5}\,dx = 2\,b^{\frac{3}{2}}cd\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) + \frac{3}{2}\,a\sqrt{b}d^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) - \frac{2\,\sqrt{bx^2+ax}b^2c^2}{5\,ax} - \frac{14\,\sqrt{bx^2+ax}bcd}{3\,x} - \frac{3\,\sqrt{bx^2+ax}ad^2}{x} + \frac{\sqrt{bx^2+ax}bc^2}{5\,x^2} - \frac{2\,\sqrt{bx^2+ax}acd}{3\,x^2} + \frac{(bx^2+ax)^{\frac{3}{2}}d^2}{x^2} + \frac{3\,\sqrt{bx^2+ax}ac^2}{5\,x^3} - \frac{2\,(bx^2+ax)^{\frac{3}{2}}cd}{3\,x^3} - \frac{(bx^2+ax)^{\frac{3}{2}}c^2}{x^4}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
2*b^(3/2)*c*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 3/2*a*sqrt(b)
*d^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2/5*sqrt(b*x^2 + a*x)*
b^2*c^2/(a*x) - 14/3*sqrt(b*x^2 + a*x)*b*c*d/x - 3*sqrt(b*x^2 + a*x)*a*d^2
/x + 1/5*sqrt(b*x^2 + a*x)*b*c^2/x^2 - 2/3*sqrt(b*x^2 + a*x)*a*c*d/x^2 + (
b*x^2 + a*x)^(3/2)*d^2/x^2 + 3/5*sqrt(b*x^2 + a*x)*a*c^2/x^3 - 2/3*(b*x^2
+ a*x)^(3/2)*c*d/x^3 - (b*x^2 + a*x)^(3/2)*c^2/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(113) = 226.

Time = 0.14 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.53

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^5} dx = \sqrt{bx^2+ax}bd^2 - \frac{(4b^2cd+3abd^2)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{2\sqrt{b}} + \frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4 b^2c^2 + 60\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4 abcd + 15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4 a^2d^2 + 30\left(\sqrt{}\right.\right.}{}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^5,x, algorithm="giac")
```

output
```
sqrt(b*x^2 + a*x)*b*d^2 - 1/2*(4*b^2*c*d + 3*a*b*d^2)*log(abs(2*(sqrt(b)*x
 - sqrt(b*x^2 + a*x))*sqrt(b) + a))/sqrt(b) + 2/15*(15*(sqrt(b)*x - sqrt(b
*x^2 + a*x))^4*b^2*c^2 + 60*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a*b*c*d + 15
*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^2*d^2 + 30*(sqrt(b)*x - sqrt(b*x^2 +
a*x))^3*a*b^(3/2)*c^2 + 30*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^2*sqrt(b)*c
*d + 30*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2*b*c^2 + 10*(sqrt(b)*x - sqrt
(b*x^2 + a*x))^2*a^3*c*d + 15*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^3*sqrt(b)*
c^2 + 3*a^4*c^2)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^5} dx = \int \frac{(b\,x^2+a\,x)^{3/2}\,(c+d\,x)^2}{x^5} dx$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^2)/x^5,x)
```

output
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^2)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.73

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^5} dx = \frac{-24\sqrt{x}\sqrt{bx+a}\,a^2c^2 - 80\sqrt{x}\sqrt{bx+a}\,a^2cdx - 120\sqrt{x}\sqrt{bx+a}\,a^2d^2x^2 - 4}{}$$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^5,x)
```

output
```
( - 24*sqrt(x)*sqrt(a + b*x)*a**2*c**2 - 80*sqrt(x)*sqrt(a + b*x)*a**2*c*d
*x - 120*sqrt(x)*sqrt(a + b*x)*a**2*d**2*x**2 - 48*sqrt(x)*sqrt(a + b*x)*a
*b*c**2*x - 320*sqrt(x)*sqrt(a + b*x)*a*b*c*d*x**2 + 60*sqrt(x)*sqrt(a + b
*x)*a*b*d**2*x**3 - 24*sqrt(x)*sqrt(a + b*x)*b**2*c**2*x**2 + 180*sqrt(b)*
log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*d**2*x**3 + 240*sqrt(b
)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b*c*d*x**3 + 99*sqrt(b)
*a**2*d**2*x**3 + 128*sqrt(b)*a*b*c*d*x**3 - 24*sqrt(b)*b**2*c**2*x**3)/(6
0*a*x**3)
```

3.75 $\int \frac{(c+dx)^2(ax+bx^2)^{3/2}}{x^6}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 138

$$\int \frac{(c+dx)^2(ax+bx^2)^{3/2}}{x^6}\,dx = -\frac{2ad^2\sqrt{ax+bx^2}}{3x^2} - \frac{8bd^2\sqrt{ax+bx^2}}{3x} - \frac{2c^2(ax+bx^2)^{5/2}}{7ax^6} + \frac{4c(bc-7ad)(ax+bx^2)^{5/2}}{35a^2x^5} + 2b^{3/2}d^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)$$

output
```
-2/3*a*d^2*(b*x^2+a*x)^(1/2)/x^2-8/3*b*d^2*(b*x^2+a*x)^(1/2)/x-2/7*c^2*(b*
x^2+a*x)^(5/2)/a/x^6+4/35*c*(-7*a*d+b*c)*(b*x^2+a*x)^(5/2)/a^2/x^5+2*b^(3/
2)*d^2*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx)^2(ax+bx^2)^{3/2}}{x^6}\,dx = \frac{2\sqrt{x(a+bx)}\left(\frac{6b^3c^2x^3}{a^2} - \frac{3b^2cx^2(c+14dx)}{a} - 4bx(6c^2+21cdx+35d^2x^2) - a(15\right.}{105x^4}$$

input
```
Integrate[((c + d*x)^2*(a*x + b*x^2)^(3/2))/x^6,x]
```

output
```
(2*Sqrt[x*(a + b*x)]*((6*b^3*c^2*x^3)/a^2 - (3*b^2*c*x^2*(c + 14*d*x))/a -
 4*b*x*(6*c^2 + 21*c*d*x + 35*d^2*x^2) - a*(15*c^2 + 42*c*d*x + 35*d^2*x^2
) - (105*b^(3/2)*d^2*x^(7/2)*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/Sqrt
[a + b*x]))/(105*x^4)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{1290\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}(c+dx)^2}{x^6}\,dx$$

$$\Big\downarrow \text{1290}$$

$$\text{Indeterminate}$$

input
```
Int[((c + d*x)^2*(a*x + b*x^2)^(3/2))/x^6,x]
```

output
```
Indeterminate
```

Defintions of rubi rules used

rule 1290
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(f + g*x)^
n, d + e*x, x], R = PolynomialRemainder[(f + g*x)^n, d + e*x, x]}, Simp[(e*
R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*
e^2)), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1
)*(a + b*x + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R
*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 1] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

method	result
pseudoelliptic	$\frac{2a^2b^{\frac{3}{2}}d^2\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)x^4-\frac{2\sqrt{x(bx+a)}\left(\left(\frac{7}{3}d^2x^2+\frac{14}{5}cdx+c^2\right)a^3+\frac{8xb\left(\frac{35}{6}d^2x^2+\frac{7}{2}cdx+c^2\right)a^2}{5}+\frac{b^2cx^2(14dx+c)a}{5}-\frac{2b^3c^2}{5}\right.}{7}}{x^4a^2}$
risch	$-\frac{2(bx+a)(140d^2x^3a^2b+42a\,b^2cd\,x^3-6b^3c^2x^3+35a^3d^2x^2+84x^2a^2bcd+3a\,b^2c^2x^2+42a^3cdx+24a^2b\,c^2x+15c^2a^3)}{105x^3\sqrt{x(bx+a)}\,a^2}+b^{\frac{3}{2}}$
default	$c^2\left(-\frac{2(b\,x^2+ax)^{\frac{5}{2}}}{7a\,x^6}+\frac{4b(b\,x^2+ax)^{\frac{5}{2}}}{35a^2x^5}\right)+d^2\left(-\frac{2(b\,x^2+ax)^{\frac{5}{2}}}{3a\,x^4}+\frac{2b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{a\,x^3}+\frac{4b\left(\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{a\,x^2}-\frac{6b\left(\left(b\,x\right.\right.}{}\right)}{}\right)}{}\right.$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
2/7*(7*a^2*b^(3/2)*d^2*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*x^4-(x*(b*x+a)
)^(1/2)*((7/3*d^2*x^2+14/5*c*d*x+c^2)*a^3+8/5*x*b*(35/6*d^2*x^2+7/2*c*d*x+
c^2)*a^2+1/5*b^2*c*x^2*(14*d*x+c)*a-2/5*b^3*c^2*x^3))/x^4/a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.20

$$\int \frac{(c+dx)^2\,(ax+bx^2)^{3/2}}{x^6}\,dx = \left[\frac{105\,a^2b^{\frac{3}{2}}d^2x^4\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,(15\,a^3c^2-2\,(3\,b^3c^2-\right.$$

$$-\frac{2\left(105\,a^2\sqrt{-b}bd^2x^4\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+(15\,a^3c^2-2\,(3\,b^3c^2-21\,ab^2cd-70\,a^2bd^2)x^3+(3\,ab^2c^2+84\right.}{105\,a^2x^4}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
[1/105*(105*a^2*b^(3/2)*d^2*x^4*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b
)) - 2*(15*a^3*c^2 - 2*(3*b^3*c^2 - 21*a*b^2*c*d - 70*a^2*b*d^2)*x^3 + (3*
a*b^2*c^2 + 84*a^2*b*c*d + 35*a^3*d^2)*x^2 + 6*(4*a^2*b*c^2 + 7*a^3*c*d)*x
)*sqrt(b*x^2 + a*x))/(a^2*x^4), -2/105*(105*a^2*sqrt(-b)*b*d^2*x^4*arctan(
sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (15*a^3*c^2 - 2*(3*b^3*c^2 - 21*a*
b^2*c*d - 70*a^2*b*d^2)*x^3 + (3*a*b^2*c^2 + 84*a^2*b*c*d + 35*a^3*d^2)*x^
2 + 6*(4*a^2*b*c^2 + 7*a^3*c*d)*x)*sqrt(b*x^2 + a*x))/(a^2*x^4)]
```

Sympy [F]

$$\int \frac{(c+dx)^2\,(ax+bx^2)^{3/2}}{x^6}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}\,(c+dx)^2}{x^6}\,dx$$

input
```
integrate((d*x+c)**2*(b*x**2+a*x)**(3/2)/x**6,x)
```

output
```
Integral((x*(a + b*x))**(3/2)*(c + d*x)**2/x**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(116) = 232.

Time = 0.04 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.02

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^6} dx = b^{\frac{3}{2}} d^2 \log \left(2 bx + a + 2 \sqrt{bx^2 + ax}\sqrt{b}\right) + \frac{4 \sqrt{bx^2 + ax} b^3 c^2}{35 a^2 x} - \frac{4 \sqrt{bx^2 + ax} b^2 cd}{5 ax} - \frac{7 \sqrt{bx^2 + ax} bd^2}{3 x} - \frac{2 \sqrt{bx^2 + ax} b^2 c^2}{35 ax^2} + \frac{2 \sqrt{bx^2 + ax} bcd}{5 x^2} - \frac{\sqrt{bx^2 + ax} ad^2}{3 x^2} + \frac{3 \sqrt{bx^2 + ax} bc^2}{70 x^3} + \frac{6 \sqrt{bx^2 + ax} acd}{5 x^3} - \frac{(bx^2 + ax)^{\frac{3}{2}} d^2}{3 x^3} + \frac{3 \sqrt{bx^2 + ax} ac^2}{14 x^4} - \frac{2 (bx^2 + ax)^{\frac{3}{2}} cd}{x^4} - \frac{(bx^2 + ax)^{\frac{3}{2}} c^2}{2 x^5}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
b^(3/2)*d^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 4/35*sqrt(b*x^2
 + a*x)*b^3*c^2/(a^2*x) - 4/5*sqrt(b*x^2 + a*x)*b^2*c*d/(a*x) - 7/3*sqrt(b
*x^2 + a*x)*b*d^2/x - 2/35*sqrt(b*x^2 + a*x)*b^2*c^2/(a*x^2) + 2/5*sqrt(b*
x^2 + a*x)*b*c*d/x^2 - 1/3*sqrt(b*x^2 + a*x)*a*d^2/x^2 + 3/70*sqrt(b*x^2 +
 a*x)*b*c^2/x^3 + 6/5*sqrt(b*x^2 + a*x)*a*c*d/x^3 - 1/3*(b*x^2 + a*x)^(3/2
)*d^2/x^3 + 3/14*sqrt(b*x^2 + a*x)*a*c^2/x^4 - 2*(b*x^2 + a*x)^(3/2)*c*d/x
^4 - 1/2*(b*x^2 + a*x)^(3/2)*c^2/x^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. 2(116) = 232.

Time = 0.15 (sec) , antiderivative size = 454, normalized size of antiderivative = 3.29

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^6} dx = -b^{\frac{3}{2}} d^2 \log \left(\left|2 \left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)\sqrt{b} + a\right|\right) + \frac{2 \left(210 \left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)^6 b^2 cd + 210 \left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)^6 abd^2 + 105 \left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)^5 b^{\frac{5}{2}} c^2 + 42\right.}{}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^6,x, algorithm="giac")
```

output
```
-b^(3/2)*d^2*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)) + 2/1
05*(210*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*b^2*c*d + 210*(sqrt(b)*x - sqrt(
b*x^2 + a*x))^6*a*b*d^2 + 105*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*b^(5/2)*c^
2 + 420*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a*b^(3/2)*c*d + 105*(sqrt(b)*x -
 sqrt(b*x^2 + a*x))^5*a^2*sqrt(b)*d^2 + 315*(sqrt(b)*x - sqrt(b*x^2 + a*x)
)^4*a*b^2*c^2 + 420*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^2*b*c*d + 35*(sqrt
(b)*x - sqrt(b*x^2 + a*x))^4*a^3*d^2 + 420*(sqrt(b)*x - sqrt(b*x^2 + a*x))
^3*a^2*b^(3/2)*c^2 + 210*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^3*sqrt(b)*c*d
 + 294*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^3*b*c^2 + 42*(sqrt(b)*x - sqrt(
b*x^2 + a*x))^2*a^4*c*d + 105*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^4*sqrt(b)*
c^2 + 15*a^5*c^2)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^7
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2(ax+bx^2)^{3/2}}{x^6}\,dx = \int \frac{(b\,x^2+a\,x)^{3/2}\,(c+d\,x)^2}{x^6}\,dx$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^2)/x^6,x)
```

output
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^2)/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.79

$$\int \frac{(c+dx)^2(ax+bx^2)^{3/2}}{x^6}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3c^2}{7}-\frac{4\sqrt{x}\,\sqrt{bx+a}\,a^3cdx}{5}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3d^2x^2}{3}-\frac{16\sqrt{x}\,\sqrt{bx+a}\,a^2b\,c^2x}{35}-8\sqrt{}\cdots}{}$$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^6,x)
```

output
```
(2*( - 15*sqrt(x)*sqrt(a + b*x)*a**3*c**2 - 42*sqrt(x)*sqrt(a + b*x)*a**3*
c*d*x - 35*sqrt(x)*sqrt(a + b*x)*a**3*d**2*x**2 - 24*sqrt(x)*sqrt(a + b*x)
*a**2*b*c**2*x - 84*sqrt(x)*sqrt(a + b*x)*a**2*b*c*d*x**2 - 140*sqrt(x)*sq
rt(a + b*x)*a**2*b*d**2*x**3 - 3*sqrt(x)*sqrt(a + b*x)*a*b**2*c**2*x**2 -
42*sqrt(x)*sqrt(a + b*x)*a*b**2*c*d*x**3 + 6*sqrt(x)*sqrt(a + b*x)*b**3*c*
*2*x**3 + 105*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*
b*d**2*x**4 + 80*sqrt(b)*a**2*b*d**2*x**4 - 18*sqrt(b)*a*b**2*c*d*x**4 - 6
*sqrt(b)*b**3*c**2*x**4))/(105*a**2*x**4)
```

3.76 $\int \frac{(c+dx)^2\left(ax+bx^2\right)^{3/2}}{x^7}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 103

$$\int \frac{(c+dx)^2\,(ax+bx^2)^{3/2}}{x^7}\,dx = \frac{8c(bc-ad)\,(ax+bx^2)^{5/2}}{63a^2x^6} - \frac{8(2bc-7ad)(bc-ad)\,(ax+bx^2)^{5/2}}{315a^3x^5} - \frac{2(c+dx)^2\,(ax+bx^2)^{5/2}}{9ax^7}$$

output
```
8/63*c*(-a*d+b*c)*(b*x^2+a*x)^(5/2)/a^2/x^6-8/315*(-7*a*d+2*b*c)*(-a*d+b*c
)*(b*x^2+a*x)^(5/2)/a^3/x^5-2/9*(d*x+c)^2*(b*x^2+a*x)^(5/2)/a/x^7
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.68

$$\int \frac{(c+dx)^2\,(ax+bx^2)^{3/2}}{x^7}\,dx = -\frac{2(x(a+bx))^{5/2}\,(8b^2c^2x^2-4abcx(5c+9dx)+a^2(35c^2+90cdx+63d^2x^2))}{315a^3x^7}$$

input
```
Integrate[((c + d*x)^2*(a*x + b*x^2)^(3/2))/x^7,x]
```

output
```
(-2*(x*(a + b*x))^(5/2)*(8*b^2*c^2*x^2 - 4*a*b*c*x*(5*c + 9*d*x) + a^2*(35
*c^2 + 90*c*d*x + 63*d^2*x^2)))/(315*a^3*x^7)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.34, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1262, 27, 1220, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}(c+dx)^2}{x^7}\,dx$$

$$\Big\downarrow \text{1262}$$

$$-\frac{\int -\frac{(2bc^2+d(4bc-7ad)x)(bx^2+ax)^{3/2}}{2x^7}dx}{b}-\frac{d^2(ax+bx^2)^{5/2}}{bx^6}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(2bc^2+d(4bc-7ad)x)(bx^2+ax)^{3/2}}{x^7}dx}{2b}-\frac{d^2(ax+bx^2)^{5/2}}{bx^6}$$

$$\Big\downarrow \text{1220}$$

$$\frac{-\frac{(63a^2d^2-36abcd+8b^2c^2)\int \frac{(bx^2+ax)^{3/2}}{x^6}dx}{9a}-\frac{4bc^2(ax+bx^2)^{5/2}}{9ax^7}}{2b}-\frac{d^2(ax+bx^2)^{5/2}}{bx^6}$$

$$\Big\downarrow \text{1129}$$

$$\frac{-\frac{(63a^2d^2-36abcd+8b^2c^2)\left(-\frac{2b\int \frac{(bx^2+ax)^{3/2}}{x^5}dx}{7a}-\frac{2(ax+bx^2)^{5/2}}{7ax^6}\right)}{9a}-\frac{4bc^2(ax+bx^2)^{5/2}}{9ax^7}}{2b}-\frac{d^2(ax+bx^2)^{5/2}}{bx^6}$$

$$\Big\downarrow \text{1123}$$

$$\frac{-\frac{\left(\frac{4b(ax+bx^2)^{5/2}}{35a^2x^5}-\frac{2(ax+bx^2)^{5/2}}{7ax^6}\right)(63a^2d^2-36abcd+8b^2c^2)}{9a}-\frac{4bc^2(ax+bx^2)^{5/2}}{9ax^7}}{2b}-\frac{d^2(ax+bx^2)^{5/2}}{bx^6}$$

input
```
Int[((c + d*x)^2*(a*x + b*x^2)^(3/2))/x^7,x]
```

output
```
-((d^2*(a*x + b*x^2)^(5/2))/(b*x^6)) + ((-4*b*c^2*(a*x + b*x^2)^(5/2))/(9*
a*x^7) - ((8*b^2*c^2 - 36*a*b*c*d + 63*a^2*d^2)*((-2*(a*x + b*x^2)^(5/2))/
(7*a*x^6) + (4*b*(a*x + b*x^2)^(5/2))/(35*a^2*x^5)))/(9*a))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$-\frac{2\sqrt{x(bx+a)}\,(bx+a)^{2}\left(\left(\frac{9}{5}d^{2}x^{2}+\frac{18}{7}cdx+c^{2}\right)a^{2}-\frac{4xbc\left(\frac{9dx}{5}+c\right)a}{7}+\frac{8b^{2}c^{2}x^{2}}{35}\right)}{9x^{5}a^{3}}$
gosper	$-\frac{2(bx+a)\left(63a^{2}d^{2}x^{2}-36abcd\,x^{2}+8b^{2}c^{2}x^{2}+90a^{2}cdx-20ab\,c^{2}x+35a^{2}c^{2}\right)\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{315x^{6}a^{3}}$
orering	$-\frac{2(bx+a)\left(63a^{2}d^{2}x^{2}-36abcd\,x^{2}+8b^{2}c^{2}x^{2}+90a^{2}cdx-20ab\,c^{2}x+35a^{2}c^{2}\right)\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{315x^{6}a^{3}}$
default	$c^{2}\left(-\frac{2\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{9a\,x^{7}}-\frac{4b\left(-\frac{2\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{7a\,x^{6}}+\frac{4b\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{35a^{2}x^{5}}\right)}{9a}\right)-\frac{2d^{2}\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{5a\,x^{5}}+2cd\left(-\frac{2\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{7a\,x^{6}}+\right.$
trager	$-\frac{2(63a^{2}b^{2}d^{2}x^{4}-36a\,b^{3}cd\,x^{4}+8b^{4}c^{2}x^{4}+126a^{3}b\,d^{2}x^{3}+18a^{2}b^{2}cd\,x^{3}-4a\,b^{3}c^{2}x^{3}+63a^{4}d^{2}x^{2}+144a^{3}dcb\,x^{2}+3a^{2}b^{2}c^{2}x^{2}+90a}{315a^{3}x^{5}}$
risch	$-\frac{2(bx+a)(63a^{2}b^{2}d^{2}x^{4}-36a\,b^{3}cd\,x^{4}+8b^{4}c^{2}x^{4}+126a^{3}b\,d^{2}x^{3}+18a^{2}b^{2}cd\,x^{3}-4a\,b^{3}c^{2}x^{3}+63a^{4}d^{2}x^{2}+144a^{3}dcb\,x^{2}+3a^{2}b^{2}c^{2}}{315x^{4}\sqrt{x(bx+a)}\,a^{3}}$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-2/9*(x*(b*x+a))^(1/2)*(b*x+a)^2*((9/5*d^2*x^2+18/7*c*d*x+c^2)*a^2-4/7*x*b
*c*(9/5*d*x+c)*a+8/35*b^2*c^2*x^2)/x^5/a^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.42

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^7} \, dx =$$
$$-\frac{2 \left(35 \, a^4 c^2 + (8 \, b^4 c^2 - 36 \, a b^3 c d + 63 \, a^2 b^2 d^2) x^4 - 2 \, (2 \, a b^3 c^2 - 9 \, a^2 b^2 c d - 63 \, a^3 b d^2) x^3 + 3 \, (a^2 b^2 c^2 + 48 \, a^3 b c\right.}{315 \, a^3 x^5}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
-2/315*(35*a^4*c^2 + (8*b^4*c^2 - 36*a*b^3*c*d + 63*a^2*b^2*d^2)*x^4 - 2*(
2*a*b^3*c^2 - 9*a^2*b^2*c*d - 63*a^3*b*d^2)*x^3 + 3*(a^2*b^2*c^2 + 48*a^3*
b*c*d + 21*a^4*d^2)*x^2 + 10*(5*a^3*b*c^2 + 9*a^4*c*d)*x)*sqrt(b*x^2 + a*x
)/(a^3*x^5)
```

Sympy [F]

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^7} \, dx = \int \frac{(x(a+bx))^{\frac{3}{2}} (c+dx)^2}{x^7} \, dx$$

input
```
integrate((d*x+c)**2*(b*x**2+a*x)**(3/2)/x**7,x)
```

output
```
Integral((x*(a + b*x))**(3/2)*(c + d*x)**2/x**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(91) = 182.

Time = 0.04 (sec) , antiderivative size = 323, normalized size of antiderivative = 3.14

$$\int \frac{(c+dx)^2\,(ax+bx^2)^{3/2}}{x^7}\,dx = -\frac{16\sqrt{bx^2+ax}b^4c^2}{315\,a^3x} + \frac{8\sqrt{bx^2+ax}b^3cd}{35\,a^2x} - \frac{2\sqrt{bx^2+ax}b^2d^2}{5\,ax} + \frac{8\sqrt{bx^2+ax}b^3c^2}{315\,a^2x^2} - \frac{4\sqrt{bx^2+ax}b^2cd}{35\,ax^2} + \frac{\sqrt{bx^2+ax}bd^2}{5\,x^2} - \frac{2\sqrt{bx^2+ax}b^2c^2}{105\,ax^3} + \frac{3\sqrt{bx^2+ax}bcd}{35\,x^3} + \frac{3\sqrt{bx^2+ax}ad^2}{5\,x^3} + \frac{\sqrt{bx^2+ax}bc^2}{63\,x^4} + \frac{3\sqrt{bx^2+ax}acd}{7\,x^4} - \frac{(bx^2+ax)^{\frac{3}{2}}d^2}{x^4} + \frac{\sqrt{bx^2+ax}ac^2}{9\,x^5} - \frac{(bx^2+ax)^{\frac{3}{2}}cd}{x^5} - \frac{(bx^2+ax)^{\frac{3}{2}}c^2}{3\,x^6}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
-16/315*sqrt(b*x^2 + a*x)*b^4*c^2/(a^3*x) + 8/35*sqrt(b*x^2 + a*x)*b^3*c*d
/(a^2*x) - 2/5*sqrt(b*x^2 + a*x)*b^2*d^2/(a*x) + 8/315*sqrt(b*x^2 + a*x)*b
^3*c^2/(a^2*x^2) - 4/35*sqrt(b*x^2 + a*x)*b^2*c*d/(a*x^2) + 1/5*sqrt(b*x^2
 + a*x)*b*d^2/x^2 - 2/105*sqrt(b*x^2 + a*x)*b^2*c^2/(a*x^3) + 3/35*sqrt(b*
x^2 + a*x)*b*c*d/x^3 + 3/5*sqrt(b*x^2 + a*x)*a*d^2/x^3 + 1/63*sqrt(b*x^2 +
 a*x)*b*c^2/x^4 + 3/7*sqrt(b*x^2 + a*x)*a*c*d/x^4 - (b*x^2 + a*x)^(3/2)*d^
2/x^4 + 1/9*sqrt(b*x^2 + a*x)*a*c^2/x^5 - (b*x^2 + a*x)^(3/2)*c*d/x^5 - 1/
3*(b*x^2 + a*x)^(3/2)*c^2/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. 2(91) = 182.

Time = 0.15 (sec) , antiderivative size = 541, normalized size of antiderivative = 5.25

$$\int \frac{(c+dx)^2\,(ax+bx^2)^{3/2}}{x^7}\,dx = \frac{2\left(315\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^8 b^2d^2 + 630\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^7 b^{\frac{5}{2}}cd + 630\right.}{}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^7,x, algorithm="giac")
```

output
```
2/315*(315*(sqrt(b)*x - sqrt(b*x^2 + a*x))^8*b^2*d^2 + 630*(sqrt(b)*x - sq
rt(b*x^2 + a*x))^7*b^(5/2)*c*d + 630*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*a*b
^(3/2)*d^2 + 420*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*b^3*c^2 + 1890*(sqrt(b)
*x - sqrt(b*x^2 + a*x))^6*a*b^2*c*d + 630*(sqrt(b)*x - sqrt(b*x^2 + a*x))^
6*a^2*b*d^2 + 1575*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a*b^(5/2)*c^2 + 2520*
(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a^2*b^(3/2)*c*d + 315*(sqrt(b)*x - sqrt(
b*x^2 + a*x))^5*a^3*sqrt(b)*d^2 + 2583*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a
^2*b^2*c^2 + 1764*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^3*b*c*d + 63*(sqrt(b
)*x - sqrt(b*x^2 + a*x))^4*a^4*d^2 + 2310*(sqrt(b)*x - sqrt(b*x^2 + a*x))^
3*a^3*b^(3/2)*c^2 + 630*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^4*sqrt(b)*c*d
+ 1170*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^4*b*c^2 + 90*(sqrt(b)*x - sqrt(
b*x^2 + a*x))^2*a^5*c*d + 315*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^5*sqrt(b)*
c^2 + 35*a^6*c^2)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^9
```

Mupad [B] (verification not implemented)

Time = 11.13 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.59

$$\int \frac{(c+dx)^2\,(ax+bx^2)^{3/2}}{x^7}\,dx = \frac{8\,b^3\,c^2\,\sqrt{b\,x^2+a\,x}}{315\,a^2\,x^2} - \frac{2\,a\,d^2\,\sqrt{b\,x^2+a\,x}}{5\,x^3} - \frac{20\,b\,c^2\,\sqrt{b\,x^2+a\,x}}{63\,x^4} - \frac{4\,b\,d^2\,\sqrt{b\,x^2+a\,x}}{5\,x^2} - \frac{2\,b^2\,c^2\,\sqrt{b\,x^2+a\,x}}{105\,a\,x^3} - \frac{2\,a\,c^2\,\sqrt{b\,x^2+a\,x}}{9\,x^5} - \frac{16\,b^4\,c^2\,\sqrt{b\,x^2+a\,x}}{315\,a^3\,x} - \frac{2\,b^2\,d^2\,\sqrt{b\,x^2+a\,x}}{5\,a\,x} - \frac{4\,a\,c\,d\,\sqrt{b\,x^2+a\,x}}{7\,x^4} - \frac{32\,b\,c\,d\,\sqrt{b\,x^2+a\,x}}{35\,x^3} - \frac{4\,b^2\,c\,d\,\sqrt{b\,x^2+a\,x}}{35\,a\,x^2} + \frac{8\,b^3\,c\,d\,\sqrt{b\,x^2+a\,x}}{35\,a^2\,x}$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^2)/x^7,x)
```

output
```
(8*b^3*c^2*(a*x + b*x^2)^(1/2))/(315*a^2*x^2) - (2*a*d^2*(a*x + b*x^2)^(1/
2))/(5*x^3) - (20*b*c^2*(a*x + b*x^2)^(1/2))/(63*x^4) - (4*b*d^2*(a*x + b*
x^2)^(1/2))/(5*x^2) - (2*b^2*c^2*(a*x + b*x^2)^(1/2))/(105*a*x^3) - (2*a*c
^2*(a*x + b*x^2)^(1/2))/(9*x^5) - (16*b^4*c^2*(a*x + b*x^2)^(1/2))/(315*a^
3*x) - (2*b^2*d^2*(a*x + b*x^2)^(1/2))/(5*a*x) - (4*a*c*d*(a*x + b*x^2)^(1
/2))/(7*x^4) - (32*b*c*d*(a*x + b*x^2)^(1/2))/(35*x^3) - (4*b^2*c*d*(a*x +
 b*x^2)^(1/2))/(35*a*x^2) + (8*b^3*c*d*(a*x + b*x^2)^(1/2))/(35*a^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.74

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^7}\,dx = -\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4c^2}{9} - \frac{4\sqrt{x}\,\sqrt{bx+a}\,a^4cdx}{7} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4d^2x^2}{5} - \frac{20\sqrt{x}\,\sqrt{bx+a}\,a^3b\,c^2x}{63} - \frac{32\sqrt{}}{}$$

input

```
int((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^7,x)
```

output

```
(2*( - 35*sqrt(x)*sqrt(a + b*x)*a**4*c**2 - 90*sqrt(x)*sqrt(a + b*x)*a**4*
c*d*x - 63*sqrt(x)*sqrt(a + b*x)*a**4*d**2*x**2 - 50*sqrt(x)*sqrt(a + b*x)
*a**3*b*c**2*x - 144*sqrt(x)*sqrt(a + b*x)*a**3*b*c*d*x**2 - 126*sqrt(x)*s
qrt(a + b*x)*a**3*b*d**2*x**3 - 3*sqrt(x)*sqrt(a + b*x)*a**2*b**2*c**2*x**
2 - 18*sqrt(x)*sqrt(a + b*x)*a**2*b**2*c*d*x**3 - 63*sqrt(x)*sqrt(a + b*x)
*a**2*b**2*d**2*x**4 + 4*sqrt(x)*sqrt(a + b*x)*a*b**3*c**2*x**3 + 36*sqrt(
x)*sqrt(a + b*x)*a*b**3*c*d*x**4 - 8*sqrt(x)*sqrt(a + b*x)*b**4*c**2*x**4
- 7*sqrt(b)*a**2*b**2*d**2*x**5 - 36*sqrt(b)*a*b**3*c*d*x**5 + 8*sqrt(b)*b
**4*c**2*x**5))/(315*a**3*x**5)
```

3.77 $\int \frac{(c+dx)^2(ax+bx^2)^{3/2}}{x^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 168

$$\int \frac{(c+dx)^2(ax+bx^2)^{3/2}}{x^8}\,dx = -\frac{8(bc-ad)(6bc+5ad)(ax+bx^2)^{5/2}}{693a^3x^6} + \frac{8(2bc-7ad)(bc-ad)(6bc+5ad)(ax+bx^2)^{5/2}}{3465a^4cx^5} + \frac{2(6bc+5ad)(c+dx)^2(ax+bx^2)^{5/2}}{99a^2cx^7} - \frac{2(c+dx)^3(ax+bx^2)^{5/2}}{11acx^8}$$

output
```
-8/693*(-a*d+b*c)*(5*a*d+6*b*c)*(b*x^2+a*x)^(5/2)/a^3/x^6+8/3465*(-7*a*d+2
*b*c)*(-a*d+b*c)*(5*a*d+6*b*c)*(b*x^2+a*x)^(5/2)/a^4/c/x^5+2/99*(5*a*d+6*b
*c)*(d*x+c)^2*(b*x^2+a*x)^(5/2)/a^2/c/x^7-2/11*(d*x+c)^3*(b*x^2+a*x)^(5/2)
/a/c/x^8
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.60

$$\int \frac{(c+dx)^2 \left(ax+bx^2\right)^{3/2}}{x^8} \, dx = \frac{2(x(a+bx))^{5/2} \left(48b^3c^2x^3 - 8ab^2cx^2(15c+22dx) - 5a^3(63c^2+154cdx+9\right.}{3465a^4x^8}$$

input
```
Integrate[((c + d*x)^2*(a*x + b*x^2)^(3/2))/x^8,x]
```

output
```
(2*(x*(a + b*x))^(5/2)*(48*b^3*c^2*x^3 - 8*a*b^2*c*x^2*(15*c + 22*d*x) - 5
*a^3*(63*c^2 + 154*c*d*x + 99*d^2*x^2) + 2*a^2*b*x*(105*c^2 + 220*c*d*x +
99*d^2*x^2)))/(3465*a^4*x^8)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1262, 27, 1220, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{3/2} (c+dx)^2}{x^8} \, dx$$

$$\Big\downarrow \text{1262}$$

$$-\frac{\int -\frac{\left(4bc^2+d(8bc-9ad)x\right)\left(bx^2+ax\right)^{3/2}}{2x^8}dx}{2b} - \frac{d^2\left(ax+bx^2\right)^{5/2}}{2bx^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(4bc^2+d(8bc-9ad)x\right)\left(bx^2+ax\right)^{3/2}}{x^8}dx}{4b} - \frac{d^2\left(ax+bx^2\right)^{5/2}}{2bx^7}$$

$$\Big\downarrow \text{1220}$$

$$\frac{-\frac{\left(99a^2d^2-88abcd+24b^2c^2\right)\int \frac{\left(bx^2+ax\right)^{3/2}}{x^7}dx}{11a} - \frac{8bc^2\left(ax+bx^2\right)^{5/2}}{11ax^8}}{4b} - \frac{d^2\left(ax+bx^2\right)^{5/2}}{2bx^7}$$

$$\Big\downarrow \ 1129$$

$$-\frac{\frac{(99a^2d^2-88abcd+24b^2c^2)\left(-\frac{4b\int \frac{(bx^2+ax)^{3/2}}{x^6}dx}{9a}-\frac{2(ax+bx^2)^{5/2}}{9ax^7}\right)}{11a}-\frac{8bc^2(ax+bx^2)^{5/2}}{11ax^8}}{4b}-\frac{d^2(ax+bx^2)^{5/2}}{2bx^7}$$

$$\Big\downarrow \ 1129$$

$$-\frac{\frac{(99a^2d^2-88abcd+24b^2c^2)\left(-\frac{4b\left(-\frac{2b\int \frac{(bx^2+ax)^{3/2}}{x^5}dx}{7a}-\frac{2(ax+bx^2)^{5/2}}{7ax^6}\right)}{9a}-\frac{2(ax+bx^2)^{5/2}}{9ax^7}\right)}{11a}-\frac{8bc^2(ax+bx^2)^{5/2}}{11ax^8}}{4b}-\frac{d^2(ax+bx^2)^{5/2}}{2bx^7}$$

$$\Big\downarrow \ 1123$$

$$-\frac{\frac{\left(-\frac{4b\left(\frac{4b(ax+bx^2)^{5/2}}{35a^2x^5}-\frac{2(ax+bx^2)^{5/2}}{7ax^6}\right)}{9a}-\frac{2(ax+bx^2)^{5/2}}{9ax^7}\right)(99a^2d^2-88abcd+24b^2c^2)}{11a}-\frac{8bc^2(ax+bx^2)^{5/2}}{11ax^8}}{4b}-\frac{d^2(ax+bx^2)^{5/2}}{2bx^7}$$

input

```
Int[((c + d*x)^2*(a*x + b*x^2)^(3/2))/x^8,x]
```

output

```
-1/2*(d^2*(a*x + b*x^2)^(5/2))/(b*x^7) + ((-8*b*c^2*(a*x + b*x^2)^(5/2))/(
11*a*x^8) - ((24*b^2*c^2 - 88*a*b*c*d + 99*a^2*d^2)*((-2*(a*x + b*x^2)^(5/
2))/(9*a*x^7) - (4*b*((-2*(a*x + b*x^2)^(5/2))/(7*a*x^6) + (4*b*(a*x + b*x
^2)^(5/2))/(35*a^2*x^5)))/(9*a)))/(11*a))/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.58

method	result
pseudoelliptic	$-\frac{2\left(\left(\frac{11}{7}d^2x^2+\frac{22}{9}cdx+c^2\right)a^3-\frac{2x\left(\frac{33}{35}d^2x^2+\frac{44}{21}cdx+c^2\right)b\,a^2}{3}+\frac{8x^2\left(\frac{22dx}{15}+c\right)b^2ca}{21}-\frac{16b^3c^2x^3}{105}\right)\sqrt{x(bx+a)}\,(bx+a)^2}{11x^6a^4}$
gosper	$-\frac{2(bx+a)\left(-198d^2x^3a^2b+176a\,b^2cd\,x^3-48b^3c^2x^3+495a^3d^2x^2-440x^2a^2bcd+120a\,b^2c^2x^2+770a^3cdx-210a^2b\,c^2x+315c\right.}{3465x^7a^4}$
orering	$-\frac{2(bx+a)\left(-198d^2x^3a^2b+176a\,b^2cd\,x^3-48b^3c^2x^3+495a^3d^2x^2-440x^2a^2bcd+120a\,b^2c^2x^2+770a^3cdx-210a^2b\,c^2x+315c\right.}{3465x^7a^4}$
trager	$-\frac{2\left(-198a^2b^3d^2x^5+176a\,b^4cd\,x^5-48b^5c^2x^5+99a^3b^2d^2x^4-88a^2b^3cd\,x^4+24a\,b^4c^2x^4+792a^4b\,d^2x^3+66a^3b^2cd\,x^3-18a^2b^3\right.}{3465a^4x^6}$
risch	$-\frac{2(bx+a)\left(-198a^2b^3d^2x^5+176a\,b^4cd\,x^5-48b^5c^2x^5+99a^3b^2d^2x^4-88a^2b^3cd\,x^4+24a\,b^4c^2x^4+792a^4b\,d^2x^3+66a^3b^2cd\,x^3-\right.}{3465x^5\sqrt{x(bx+a)}\,a^4}$
default	$c^2\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{11a\,x^8}-\frac{6b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{9a\,x^7}-\frac{4b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{7a\,x^6}+\frac{4b\left(b\,x^2+ax\right)^{\frac{5}{2}}}{35a^2x^5}\right)}{9a}\right)}{11a}\right)+d^2\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{7a\,x^6}+\frac{4\ldots}{}\right.$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-2/11*((11/7*d^2*x^2+22/9*c*d*x+c^2)*a^3-2/3*x*(33/35*d^2*x^2+44/21*c*d*x+
c^2)*b*a^2+8/21*x^2*(22/15*d*x+c)*b^2*c*a-16/105*b^3*c^2*x^3)*(x*(b*x+a))^
(1/2)*(b*x+a)^2/x^6/a^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^2\,(ax+bx^2)^{3/2}}{x^8}\,dx =$$

$$-\frac{2\,(315\,a^5c^2-2\,(24\,b^5c^2-88\,ab^4cd+99\,a^2b^3d^2)x^5+(24\,ab^4c^2-88\,a^2b^3cd+99\,a^3b^2d^2)x^4-6\,(3\,a^2b^3c^2-\ldots}{3465\,a\ldots}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
-2/3465*(315*a^5*c^2 - 2*(24*b^5*c^2 - 88*a*b^4*c*d + 99*a^2*b^3*d^2)*x^5
+ (24*a*b^4*c^2 - 88*a^2*b^3*c*d + 99*a^3*b^2*d^2)*x^4 - 6*(3*a^2*b^3*c^2
- 11*a^3*b^2*c*d - 132*a^4*b*d^2)*x^3 + 5*(3*a^3*b^2*c^2 + 220*a^4*b*c*d +
 99*a^5*d^2)*x^2 + 70*(6*a^4*b*c^2 + 11*a^5*c*d)*x)*sqrt(b*x^2 + a*x)/(a^4
*x^6)
```

Sympy [F]

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^8}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}} (c+dx)^2}{x^8}\,dx$$

input
```
integrate((d*x+c)**2*(b*x**2+a*x)**(3/2)/x**8,x)
```

output
```
Integral((x*(a + b*x))**(3/2)*(c + d*x)**2/x**8, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. 2(152) = 304.

Time = 0.05 (sec) , antiderivative size = 397, normalized size of antiderivative = 2.36

$$\begin{aligned}\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^8}\,dx &= \frac{32\sqrt{bx^2+ax}\,b^5c^2}{1155\,a^4x} - \frac{32\sqrt{bx^2+ax}\,b^4cd}{315\,a^3x} \\ &+ \frac{4\sqrt{bx^2+ax}\,b^3d^2}{35\,a^2x} - \frac{16\sqrt{bx^2+ax}\,b^4c^2}{1155\,a^3x^2} + \frac{16\sqrt{bx^2+ax}\,b^3cd}{315\,a^2x^2} - \frac{2\sqrt{bx^2+ax}\,b^2d^2}{35\,ax^2} \\ &+ \frac{4\sqrt{bx^2+ax}\,b^3c^2}{385\,a^2x^3} - \frac{4\sqrt{bx^2+ax}\,b^2cd}{105\,ax^3} + \frac{3\sqrt{bx^2+ax}\,bd^2}{70\,x^3} - \frac{2\sqrt{bx^2+ax}\,b^2c^2}{231\,ax^4} \\ &+ \frac{2\sqrt{bx^2+ax}\,bcd}{63\,x^4} + \frac{3\sqrt{bx^2+ax}\,ad^2}{14\,x^4} + \frac{\sqrt{bx^2+ax}\,bc^2}{132\,x^5} + \frac{2\sqrt{bx^2+ax}\,acd}{9\,x^5} \\ &- \frac{(bx^2+ax)^{\frac{3}{2}}d^2}{2\,x^5} + \frac{3\sqrt{bx^2+ax}\,ac^2}{44\,x^6} - \frac{2\,(bx^2+ax)^{\frac{3}{2}}cd}{3\,x^6} - \frac{(bx^2+ax)^{\frac{3}{2}}c^2}{4\,x^7}\end{aligned}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
32/1155*sqrt(b*x^2 + a*x)*b^5*c^2/(a^4*x) - 32/315*sqrt(b*x^2 + a*x)*b^4*c
*d/(a^3*x) + 4/35*sqrt(b*x^2 + a*x)*b^3*d^2/(a^2*x) - 16/1155*sqrt(b*x^2 +
 a*x)*b^4*c^2/(a^3*x^2) + 16/315*sqrt(b*x^2 + a*x)*b^3*c*d/(a^2*x^2) - 2/3
5*sqrt(b*x^2 + a*x)*b^2*d^2/(a*x^2) + 4/385*sqrt(b*x^2 + a*x)*b^3*c^2/(a^2
*x^3) - 4/105*sqrt(b*x^2 + a*x)*b^2*c*d/(a*x^3) + 3/70*sqrt(b*x^2 + a*x)*b
*d^2/x^3 - 2/231*sqrt(b*x^2 + a*x)*b^2*c^2/(a*x^4) + 2/63*sqrt(b*x^2 + a*x
)*b*c*d/x^4 + 3/14*sqrt(b*x^2 + a*x)*a*d^2/x^4 + 1/132*sqrt(b*x^2 + a*x)*b
*c^2/x^5 + 2/9*sqrt(b*x^2 + a*x)*a*c*d/x^5 - 1/2*(b*x^2 + a*x)^(3/2)*d^2/x
^5 + 3/44*sqrt(b*x^2 + a*x)*a*c^2/x^6 - 2/3*(b*x^2 + a*x)^(3/2)*c*d/x^6 -
1/4*(b*x^2 + a*x)^(3/2)*c^2/x^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 636 vs. $2(152) = 304$.

Time = 0.32 (sec) , antiderivative size = 636, normalized size of antiderivative = 3.79

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^8}\,dx = \frac{2\left(3465\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^9 b^{\frac{5}{2}}d^2+9240\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^8 b^3cd+10\right.}{}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^8,x, algorithm="giac")
```

output
```
2/3465*(3465*(sqrt(b)*x - sqrt(b*x^2 + a*x))^9*b^(5/2)*d^2 + 9240*(sqrt(b)
*x - sqrt(b*x^2 + a*x))^8*b^3*c*d + 10395*(sqrt(b)*x - sqrt(b*x^2 + a*x))^
8*a*b^2*d^2 + 6930*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*b^(7/2)*c^2 + 34650*(
sqrt(b)*x - sqrt(b*x^2 + a*x))^7*a*b^(5/2)*c*d + 13860*(sqrt(b)*x - sqrt(b
*x^2 + a*x))^7*a^2*b^(3/2)*d^2 + 30492*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a
*b^3*c^2 + 56826*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a^2*b^2*c*d + 9702*(sqr
t(b)*x - sqrt(b*x^2 + a*x))^6*a^3*b*d^2 + 58905*(sqrt(b)*x - sqrt(b*x^2 +
a*x))^5*a^2*b^(5/2)*c^2 + 50820*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a^3*b^(3
/2)*c*d + 3465*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a^4*sqrt(b)*d^2 + 63855*(
sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^3*b^2*c^2 + 25740*(sqrt(b)*x - sqrt(b*x
^2 + a*x))^4*a^4*b*c*d + 495*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^5*d^2 + 4
1580*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^4*b^(3/2)*c^2 + 6930*(sqrt(b)*x -
 sqrt(b*x^2 + a*x))^3*a^5*sqrt(b)*c*d + 16170*(sqrt(b)*x - sqrt(b*x^2 + a*
x))^2*a^5*b*c^2 + 770*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^6*c*d + 3465*(sq
rt(b)*x - sqrt(b*x^2 + a*x))*a^6*sqrt(b)*c^2 + 315*a^7*c^2)/(sqrt(b)*x - s
qrt(b*x^2 + a*x))^11
```

Mupad [B] (verification not implemented)

Time = 12.03 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.03

$$\int \frac{(c+dx)^2\,(ax+bx^2)^{3/2}}{x^8}\,dx = \frac{4\,b^3\,c^2\,\sqrt{b\,x^2+a\,x}}{385\,a^2\,x^3} - \frac{2\,a\,d^2\,\sqrt{b\,x^2+a\,x}}{7\,x^4} - \frac{8\,b\,c^2\,\sqrt{b\,x^2+a\,x}}{33\,x^5} - \frac{16\,b\,d^2\,\sqrt{b\,x^2+a\,x}}{35\,x^3} - \frac{2\,b^2\,c^2\,\sqrt{b\,x^2+a\,x}}{231\,a\,x^4} - \frac{2\,a\,c^2\,\sqrt{b\,x^2+a\,x}}{11\,x^6} - \frac{16\,b^4\,c^2\,\sqrt{b\,x^2+a\,x}}{1155\,a^3\,x^2} + \frac{32\,b^5\,c^2\,\sqrt{b\,x^2+a\,x}}{1155\,a^4\,x} - \frac{2\,b^2\,d^2\,\sqrt{b\,x^2+a\,x}}{35\,a\,x^2} + \frac{4\,b^3\,d^2\,\sqrt{b\,x^2+a\,x}}{35\,a^2\,x} - \frac{4\,a\,c\,d\,\sqrt{b\,x^2+a\,x}}{9\,x^5} - \frac{40\,b\,c\,d\,\sqrt{b\,x^2+a\,x}}{63\,x^4} - \frac{4\,b^2\,c\,d\,\sqrt{b\,x^2+a\,x}}{105\,a\,x^3} + \frac{16\,b^3\,c\,d\,\sqrt{b\,x^2+a\,x}}{315\,a^2\,x^2} - \frac{32\,b^4\,c\,d\,\sqrt{b\,x^2+a\,x}}{315\,a^3\,x}$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^2)/x^8,x)
```

output
```
(4*b^3*c^2*(a*x + b*x^2)^(1/2))/(385*a^2*x^3) - (2*a*d^2*(a*x + b*x^2)^(1/
2))/(7*x^4) - (8*b*c^2*(a*x + b*x^2)^(1/2))/(33*x^5) - (16*b*d^2*(a*x + b*
x^2)^(1/2))/(35*x^3) - (2*b^2*c^2*(a*x + b*x^2)^(1/2))/(231*a*x^4) - (2*a*
c^2*(a*x + b*x^2)^(1/2))/(11*x^6) - (16*b^4*c^2*(a*x + b*x^2)^(1/2))/(1155
*a^3*x^2) + (32*b^5*c^2*(a*x + b*x^2)^(1/2))/(1155*a^4*x) - (2*b^2*d^2*(a*
x + b*x^2)^(1/2))/(35*a*x^2) + (4*b^3*d^2*(a*x + b*x^2)^(1/2))/(35*a^2*x)
- (4*a*c*d*(a*x + b*x^2)^(1/2))/(9*x^5) - (40*b*c*d*(a*x + b*x^2)^(1/2))/(
63*x^4) - (4*b^2*c*d*(a*x + b*x^2)^(1/2))/(105*a*x^3) + (16*b^3*c*d*(a*x +
 b*x^2)^(1/2))/(315*a^2*x^2) - (32*b^4*c*d*(a*x + b*x^2)^(1/2))/(315*a^3*x
)
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.07

$$\int \frac{(c+dx)^2\,(ax+bx^2)^{3/2}}{x^8}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^5c^2}{11} - \frac{4\sqrt{x}\,\sqrt{bx+a}\,a^5cdx}{9} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,a^5d^2x^2}{7} - \frac{8\sqrt{x}\,\sqrt{bx+a}\,a^4bc^2x}{33} - \frac{40\sqrt{}}{}}{}$$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^8,x)
```

output
```
(2*( - 315*sqrt(x)*sqrt(a + b*x)*a**5*c**2 - 770*sqrt(x)*sqrt(a + b*x)*a**
5*c*d*x - 495*sqrt(x)*sqrt(a + b*x)*a**5*d**2*x**2 - 420*sqrt(x)*sqrt(a +
b*x)*a**4*b*c**2*x - 1100*sqrt(x)*sqrt(a + b*x)*a**4*b*c*d*x**2 - 792*sqrt
(x)*sqrt(a + b*x)*a**4*b*d**2*x**3 - 15*sqrt(x)*sqrt(a + b*x)*a**3*b**2*c*
*2*x**2 - 66*sqrt(x)*sqrt(a + b*x)*a**3*b**2*c*d*x**3 - 99*sqrt(x)*sqrt(a
+ b*x)*a**3*b**2*d**2*x**4 + 18*sqrt(x)*sqrt(a + b*x)*a**2*b**3*c**2*x**3
+ 88*sqrt(x)*sqrt(a + b*x)*a**2*b**3*c*d*x**4 + 198*sqrt(x)*sqrt(a + b*x)*
a**2*b**3*d**2*x**5 - 24*sqrt(x)*sqrt(a + b*x)*a*b**4*c**2*x**4 - 176*sqrt
(x)*sqrt(a + b*x)*a*b**4*c*d*x**5 + 48*sqrt(x)*sqrt(a + b*x)*b**5*c**2*x**
5 - 198*sqrt(b)*a**2*b**3*d**2*x**6 + 176*sqrt(b)*a*b**4*c*d*x**6 - 48*sqr
t(b)*b**5*c**2*x**6))/(3465*a**4*x**6)
```

3.78 $\int \frac{(c+dx)^2\left(ax+bx^2\right)^{3/2}}{x^9}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 199

$$\int \frac{(c+dx)^2\,(ax+bx^2)^{3/2}}{x^9}\,dx = -\frac{2c^2(ax+bx^2)^{5/2}}{13ax^9} + \frac{4c(4bc-13ad)\,(ax+bx^2)^{5/2}}{143a^2x^8} - \frac{2\left(143d^2+\frac{12bc(4bc-13ad)}{a^2}\right)(ax+bx^2)^{5/2}}{1287ax^7} + \frac{8b(143a^2d^2+12bc(4bc-13ad))\,(ax+bx^2)^{5/2}}{9009a^4x^6} - \frac{16b^2(143a^2d^2+12bc(4bc-13ad))\,(ax+bx^2)^{5/2}}{45045a^5x^5}$$

output
```
-2/13*c^2*(b*x^2+a*x)^(5/2)/a/x^9+4/143*c*(-13*a*d+4*b*c)*(b*x^2+a*x)^(5/2
)/a^2/x^8-2/1287*(143*d^2+12*b*c*(-13*a*d+4*b*c)/a^2)*(b*x^2+a*x)^(5/2)/a/
x^7+8/9009*b*(143*a^2*d^2+12*b*c*(-13*a*d+4*b*c))*(b*x^2+a*x)^(5/2)/a^4/x^
6-16/45045*b^2*(143*a^2*d^2+12*b*c*(-13*a*d+4*b*c))*(b*x^2+a*x)^(5/2)/a^5/
x^5
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.66

$$\int \frac{(c+dx)^2 \left(ax+bx^2\right)^{3/2}}{x^9} \, dx =$$
$$-\frac{2(x(a+bx))^{5/2} \left(384b^4c^2x^4 - 96ab^3cx^3(10c+13dx) + 35a^4(99c^2+234cdx+143d^2x^2) - 20a^3bx(126c^2+\right.}{45045a^5x^9}$$

input
```
Integrate[((c + d*x)^2*(a*x + b*x^2)^(3/2))/x^9,x]
```

output
```
(-2*(x*(a + b*x))^(5/2)*(384*b^4*c^2*x^4 - 96*a*b^3*c*x^3*(10*c + 13*d*x)
+ 35*a^4*(99*c^2 + 234*c*d*x + 143*d^2*x^2) - 20*a^3*b*x*(126*c^2 + 273*c*
d*x + 143*d^2*x^2) + 8*a^2*b^2*x^2*(210*c^2 + 390*c*d*x + 143*d^2*x^2)))/(
45045*a^5*x^9)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1262, 27, 1220, 1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{3/2} (c+dx)^2}{x^9} \, dx$$

$$\Big\downarrow \text{ 1262}$$

$$-\frac{\int -\frac{\left(6bc^2+d(12bc-11ad)x\right)\left(bx^2+ax\right)^{3/2}}{2x^9}dx}{3b} - \frac{d^2\left(ax+bx^2\right)^{5/2}}{3bx^8}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\left(6bc^2+d(12bc-11ad)x\right)\left(bx^2+ax\right)^{3/2}}{x^9}dx}{6b} - \frac{d^2\left(ax+bx^2\right)^{5/2}}{3bx^8}$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{-\frac{(143a^2d^2-156abcd+48b^2c^2)\int \frac{(bx^2+ax)^{3/2}}{x^8}dx}{13a}-\frac{12bc^2(ax+bx^2)^{5/2}}{13ax^9}}{6b}-\frac{d^2(ax+bx^2)^{5/2}}{3bx^8}$$

$\downarrow$ 1129

$$\frac{-\frac{(143a^2d^2-156abcd+48b^2c^2)\left(-\frac{6b\int \frac{(bx^2+ax)^{3/2}}{x^7}dx}{11a}-\frac{2(ax+bx^2)^{5/2}}{11ax^8}\right)}{13a}-\frac{12bc^2(ax+bx^2)^{5/2}}{13ax^9}}{6b}-\frac{d^2(ax+bx^2)^{5/2}}{3bx^8}$$

$\downarrow$ 1129

$$\frac{-\frac{(143a^2d^2-156abcd+48b^2c^2)\left(-\frac{6b\left(-\frac{4b\int \frac{(bx^2+ax)^{3/2}}{x^6}dx}{9a}-\frac{2(ax+bx^2)^{5/2}}{9ax^7}\right)}{11a}-\frac{2(ax+bx^2)^{5/2}}{11ax^8}\right)}{13a}-\frac{12bc^2(ax+bx^2)^{5/2}}{13ax^9}}{6b}-\frac{d^2(ax+bx^2)^{5/2}}{3bx^8}$$

$\downarrow$ 1129

$$\frac{-\frac{(143a^2d^2-156abcd+48b^2c^2)\left(-\frac{6b\left(-\frac{4b\left(-\frac{2b\int \frac{(bx^2+ax)^{3/2}}{x^5}dx}{7a}-\frac{2(ax+bx^2)^{5/2}}{7ax^6}\right)}{9a}-\frac{2(ax+bx^2)^{5/2}}{9ax^7}\right)}{11a}-\frac{2(ax+bx^2)^{5/2}}{11ax^8}\right)}{13a}-\frac{12bc^2(ax+bx^2)^{5/2}}{13ax^9}}{6b}-\frac{d^2(ax+bx^2)^{5/2}}{3bx^8}$$

$\downarrow$ 1123

$$-\frac{-\frac{\left(-\frac{6b\left(-\frac{4b\left(\frac{4b\left(ax+bx^2\right)^{5/2}}{35a^2x^5}-\frac{2\left(ax+bx^2\right)^{5/2}}{7ax^6}\right)}{9a}-\frac{2\left(ax+bx^2\right)^{5/2}}{9ax^7}\right)}{11a}-\frac{2\left(ax+bx^2\right)^{5/2}}{11ax^8}\right)\left(143a^2d^2-156abcd+48b^2c^2\right)}{13a}-\frac{12bc^2\left(ax+bx^2\right)^{5/2}}{13ax^9}}{6b}-\frac{d^2\left(ax+bx^2\right)^{5/2}}{3bx^8}$$

input
```
Int[((c + d*x)^2*(a*x + b*x^2)^(3/2))/x^9,x]
```

output
```
-1/3*(d^2*(a*x + b*x^2)^(5/2))/(b*x^8) + ((-12*b*c^2*(a*x + b*x^2)^(5/2))/
(13*a*x^9) - ((48*b^2*c^2 - 156*a*b*c*d + 143*a^2*d^2)*((-2*(a*x + b*x^2)^
(5/2))/(11*a*x^8) - (6*b*((-2*(a*x + b*x^2)^(5/2))/(9*a*x^7) - (4*b*((-2*(
a*x + b*x^2)^(5/2))/(7*a*x^6) + (4*b*(a*x + b*x^2)^(5/2))/(35*a^2*x^5)))/(
9*a)))/(11*a)))/(13*a))/(6*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$-\frac{2\sqrt{x(bx+a)}\,(bx+a)^2\left(\left(\frac{13}{9}d^2x^2+\frac{26}{11}cdx+c^2\right)a^4-\frac{8xb\left(\frac{143}{126}d^2x^2+\frac{13}{6}cdx+c^2\right)a^3}{11}+\frac{16x^2\left(\frac{143}{210}d^2x^2+\frac{13}{7}cdx+c^2\right)b^2a^2}{33}-\frac{64x^3b^3c}{}\right.}{13x^7a^5}$
gosper	$-\frac{2(bx+a)\left(1144a^2b^2d^2x^4-1248a\,b^3cd\,x^4+384b^4c^2x^4-2860a^3b\,d^2x^3+3120a^2b^2cd\,x^3-960a\,b^3c^2x^3+5005a^4d^2x^2-5460a^3\right.}{45045x^8a^5}$
orering	$-\frac{2(bx+a)\left(1144a^2b^2d^2x^4-1248a\,b^3cd\,x^4+384b^4c^2x^4-2860a^3b\,d^2x^3+3120a^2b^2cd\,x^3-960a\,b^3c^2x^3+5005a^4d^2x^2-5460a^3\right.}{45045x^8a^5}$
trager	$-\frac{2\left(1144a^2b^4d^2x^6-1248a\,b^5cd\,x^6+384b^6c^2x^6-572a^3b^3d^2x^5+624a^2b^4cd\,x^5-192a\,b^5c^2x^5+429a^4b^2d^2x^4-468a^3b^3cd\,x^4+\right.}{}$
risch	$-\frac{2(bx+a)\left(1144a^2b^4d^2x^6-1248a\,b^5cd\,x^6+384b^6c^2x^6-572a^3b^3d^2x^5+624a^2b^4cd\,x^5-192a\,b^5c^2x^5+429a^4b^2d^2x^4-468a^3b\right.}{}$
default	$c^2\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{13a\,x^9}-\frac{8b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{11a\,x^8}-\frac{6b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{9a\,x^7}-\frac{4b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{7a\,x^6}+\frac{4b\left(b\,x^2+ax\right)^{\frac{5}{2}}}{35a^2x^5}\right)}{9a}\right)}{11a}\right)}{13a}\right)+d^2\left(\right.$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-2/13*(x*(b*x+a))^(1/2)*(b*x+a)^2*((13/9*d^2*x^2+26/11*c*d*x+c^2)*a^4-8/11
*x*b*(143/126*d^2*x^2+13/6*c*d*x+c^2)*a^3+16/33*x^2*(143/210*d^2*x^2+13/7*
c*d*x+c^2)*b^2*a^2-64/231*x^3*b^3*c*(13/10*d*x+c)*a+128/1155*b^4*c^2*x^4)/
x^7/a^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^9} dx =$$
$$-\frac{2\,(3465\,a^6c^2+8\,(48\,b^6c^2-156\,ab^5cd+143\,a^2b^4d^2)x^6-4\,(48\,ab^5c^2-156\,a^2b^4cd+143\,a^3b^3d^2)x^5+3\,(48$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
-2/45045*(3465*a^6*c^2 + 8*(48*b^6*c^2 - 156*a*b^5*c*d + 143*a^2*b^4*d^2)*
x^6 - 4*(48*a*b^5*c^2 - 156*a^2*b^4*c*d + 143*a^3*b^3*d^2)*x^5 + 3*(48*a^2
*b^4*c^2 - 156*a^3*b^3*c*d + 143*a^4*b^2*d^2)*x^4 - 10*(12*a^3*b^3*c^2 - 3
9*a^4*b^2*c*d - 715*a^5*b*d^2)*x^3 + 35*(3*a^4*b^2*c^2 + 312*a^5*b*c*d + 1
43*a^6*d^2)*x^2 + 630*(7*a^5*b*c^2 + 13*a^6*c*d)*x)*sqrt(b*x^2 + a*x)/(a^5
*x^7)
```

Sympy [F]

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^9} dx = \int \frac{(x(a+bx))^{\frac{3}{2}} (c+dx)^2}{x^9} dx$$

input
```
integrate((d*x+c)**2*(b*x**2+a*x)**(3/2)/x**9,x)
```

output
```
Integral((x*(a + b*x))**(3/2)*(c + d*x)**2/x**9, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. 2(179) = 358.

Time = 0.04 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.37

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^9}\,dx = -\frac{256\sqrt{bx^2+ax}b^6c^2}{15015\,a^5x} + \frac{64\sqrt{bx^2+ax}b^5cd}{1155\,a^4x} - \frac{16\sqrt{bx^2+ax}b^4d^2}{315\,a^3x} + \frac{128\sqrt{bx^2+ax}b^5c^2}{15015\,a^4x^2} - \frac{32\sqrt{bx^2+ax}b^4cd}{1155\,a^3x^2} + \frac{8\sqrt{bx^2+ax}b^3d^2}{315\,a^2x^2} - \frac{32\sqrt{bx^2+ax}b^4c^2}{5005\,a^3x^3} + \frac{8\sqrt{bx^2+ax}b^3cd}{385\,a^2x^3} - \frac{2\sqrt{bx^2+ax}b^2d^2}{105\,ax^3} + \frac{16\sqrt{bx^2+ax}b^3c^2}{3003\,a^2x^4} - \frac{4\sqrt{bx^2+ax}b^2cd}{231\,ax^4} + \frac{\sqrt{bx^2+ax}bd^2}{63\,x^4} - \frac{2\sqrt{bx^2+ax}b^2c^2}{429\,ax^5} + \frac{\sqrt{bx^2+ax}bcd}{66\,x^5} + \frac{\sqrt{bx^2+ax}ad^2}{9\,x^5} + \frac{3\sqrt{bx^2+ax}bc^2}{715\,x^6} + \frac{3\sqrt{bx^2+ax}acd}{22\,x^6} - \frac{(bx^2+ax)^{\frac{3}{2}}d^2}{3\,x^6} + \frac{3\sqrt{bx^2+ax}ac^2}{65\,x^7} - \frac{(bx^2+ax)^{\frac{3}{2}}cd}{2\,x^7} - \frac{(bx^2+ax)^{\frac{3}{2}}c^2}{5\,x^8}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
-256/15015*sqrt(b*x^2 + a*x)*b^6*c^2/(a^5*x) + 64/1155*sqrt(b*x^2 + a*x)*b
^5*c*d/(a^4*x) - 16/315*sqrt(b*x^2 + a*x)*b^4*d^2/(a^3*x) + 128/15015*sqrt
(b*x^2 + a*x)*b^5*c^2/(a^4*x^2) - 32/1155*sqrt(b*x^2 + a*x)*b^4*c*d/(a^3*x
^2) + 8/315*sqrt(b*x^2 + a*x)*b^3*d^2/(a^2*x^2) - 32/5005*sqrt(b*x^2 + a*x
)*b^4*c^2/(a^3*x^3) + 8/385*sqrt(b*x^2 + a*x)*b^3*c*d/(a^2*x^3) - 2/105*sq
rt(b*x^2 + a*x)*b^2*d^2/(a*x^3) + 16/3003*sqrt(b*x^2 + a*x)*b^3*c^2/(a^2*x
^4) - 4/231*sqrt(b*x^2 + a*x)*b^2*c*d/(a*x^4) + 1/63*sqrt(b*x^2 + a*x)*b*d
^2/x^4 - 2/429*sqrt(b*x^2 + a*x)*b^2*c^2/(a*x^5) + 1/66*sqrt(b*x^2 + a*x)*
b*c*d/x^5 + 1/9*sqrt(b*x^2 + a*x)*a*d^2/x^5 + 3/715*sqrt(b*x^2 + a*x)*b*c^
2/x^6 + 3/22*sqrt(b*x^2 + a*x)*a*c*d/x^6 - 1/3*(b*x^2 + a*x)^(3/2)*d^2/x^6
 + 3/65*sqrt(b*x^2 + a*x)*a*c^2/x^7 - 1/2*(b*x^2 + a*x)^(3/2)*c*d/x^7 - 1/
5*(b*x^2 + a*x)^(3/2)*c^2/x^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(179) = 358$.

Time = 0.14 (sec) , antiderivative size = 731, normalized size of antiderivative = 3.67

$$\int \frac{(c+dx)^2 \left(ax+bx^2\right)^{3/2}}{x^9} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^9,x, algorithm="giac")
```

output
```
2/45045*(60060*(sqrt(b)*x - sqrt(b*x^2 + a*x))^10*b^3*d^2 + 180180*(sqrt(b
)*x - sqrt(b*x^2 + a*x))^9*b^(7/2)*c*d + 225225*(sqrt(b)*x - sqrt(b*x^2 +
a*x))^9*a*b^(5/2)*d^2 + 144144*(sqrt(b)*x - sqrt(b*x^2 + a*x))^8*b^4*c^2 +
 792792*(sqrt(b)*x - sqrt(b*x^2 + a*x))^8*a*b^3*c*d + 369369*(sqrt(b)*x -
sqrt(b*x^2 + a*x))^8*a^2*b^2*d^2 + 720720*(sqrt(b)*x - sqrt(b*x^2 + a*x))^
7*a*b^(7/2)*c^2 + 1531530*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*a^2*b^(5/2)*c*
d + 330330*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*a^3*b^(3/2)*d^2 + 1595880*(sq
rt(b)*x - sqrt(b*x^2 + a*x))^6*a^2*b^3*c^2 + 1660230*(sqrt(b)*x - sqrt(b*x
^2 + a*x))^6*a^3*b^2*c*d + 167310*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a^4*b*
d^2 + 2027025*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a^3*b^(5/2)*c^2 + 1081080*
(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a^4*b^(3/2)*c*d + 45045*(sqrt(b)*x - sqr
t(b*x^2 + a*x))^5*a^5*sqrt(b)*d^2 + 1606605*(sqrt(b)*x - sqrt(b*x^2 + a*x)
)^4*a^4*b^2*c^2 + 420420*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^5*b*c*d + 500
5*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^6*d^2 + 810810*(sqrt(b)*x - sqrt(b*x
^2 + a*x))^3*a^5*b^(3/2)*c^2 + 90090*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^6
*sqrt(b)*c*d + 253890*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^6*b*c^2 + 8190*(
sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^7*c*d + 45045*(sqrt(b)*x - sqrt(b*x^2 +
 a*x))*a^7*sqrt(b)*c^2 + 3465*a^8*c^2)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^13
```

Mupad [B] (verification not implemented)

Time = 12.92 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.09

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^9} dx = \frac{16 b^3 c^2 \sqrt{b x^2 + a x}}{3003 a^2 x^4} - \frac{2 a d^2 \sqrt{b x^2 + a x}}{9 x^5} - \frac{28 b c^2 \sqrt{b x^2 + a x}}{143 x^6} - \frac{20 b d^2 \sqrt{b x^2 + a x}}{63 x^4} - \frac{2 b^2 c^2 \sqrt{b x^2 + a x}}{429 a x^5} - \frac{2 a c^2 \sqrt{b x^2 + a x}}{13 x^7} - \frac{32 b^4 c^2 \sqrt{b x^2 + a x}}{5005 a^3 x^3} + \frac{128 b^5 c^2 \sqrt{b x^2 + a x}}{15015 a^4 x^2} - \frac{256 b^6 c^2 \sqrt{b x^2 + a x}}{15015 a^5 x} - \frac{2 b^2 d^2 \sqrt{b x^2 + a x}}{105 a x^3} + \frac{8 b^3 d^2 \sqrt{b x^2 + a x}}{315 a^2 x^2} - \frac{16 b^4 d^2 \sqrt{b x^2 + a x}}{315 a^3 x} - \frac{4 a c d \sqrt{b x^2 + a x}}{11 x^6} - \frac{16 b c d \sqrt{b x^2 + a x}}{33 x^5} - \frac{4 b^2 c d \sqrt{b x^2 + a x}}{231 a x^4} + \frac{8 b^3 c d \sqrt{b x^2 + a x}}{385 a^2 x^3} - \frac{32 b^4 c d \sqrt{b x^2 + a x}}{1155 a^3 x^2} + \frac{64 b^5 c d \sqrt{b x^2 + a x}}{1155 a^4 x}$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^2)/x^9,x)
```

output
```
(16*b^3*c^2*(a*x + b*x^2)^(1/2))/(3003*a^2*x^4) - (2*a*d^2*(a*x + b*x^2)^(
1/2))/(9*x^5) - (28*b*c^2*(a*x + b*x^2)^(1/2))/(143*x^6) - (20*b*d^2*(a*x
+ b*x^2)^(1/2))/(63*x^4) - (2*b^2*c^2*(a*x + b*x^2)^(1/2))/(429*a*x^5) - (
2*a*c^2*(a*x + b*x^2)^(1/2))/(13*x^7) - (32*b^4*c^2*(a*x + b*x^2)^(1/2))/(
5005*a^3*x^3) + (128*b^5*c^2*(a*x + b*x^2)^(1/2))/(15015*a^4*x^2) - (256*b
^6*c^2*(a*x + b*x^2)^(1/2))/(15015*a^5*x) - (2*b^2*d^2*(a*x + b*x^2)^(1/2)
)/(105*a*x^3) + (8*b^3*d^2*(a*x + b*x^2)^(1/2))/(315*a^2*x^2) - (16*b^4*d^
2*(a*x + b*x^2)^(1/2))/(315*a^3*x) - (4*a*c*d*(a*x + b*x^2)^(1/2))/(11*x^6
) - (16*b*c*d*(a*x + b*x^2)^(1/2))/(33*x^5) - (4*b^2*c*d*(a*x + b*x^2)^(1/
2))/(231*a*x^4) + (8*b^3*c*d*(a*x + b*x^2)^(1/2))/(385*a^2*x^3) - (32*b^4*
c*d*(a*x + b*x^2)^(1/2))/(1155*a^3*x^2) + (64*b^5*c*d*(a*x + b*x^2)^(1/2))
/(1155*a^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.07

$$\int \frac{(c+dx)^2 (ax+bx^2)^{3/2}}{x^9} dx = -\frac{2\sqrt{x}\sqrt{bx+a}\,a^6c^2}{13} - \frac{4\sqrt{x}\sqrt{bx+a}\,a^6cdx}{11} - \frac{2\sqrt{x}\sqrt{bx+a}\,a^6d^2x^2}{9} - \frac{28\sqrt{x}\sqrt{bx+a}\,a^5bc^2x}{143} - \frac{16\sqrt{}}{}$$

input
```
int((d*x+c)^2*(b*x^2+a*x)^(3/2)/x^9,x)
```

output
```
(2*( - 3465*sqrt(x)*sqrt(a + b*x)*a**6*c**2 - 8190*sqrt(x)*sqrt(a + b*x)*a
**6*c*d*x - 5005*sqrt(x)*sqrt(a + b*x)*a**6*d**2*x**2 - 4410*sqrt(x)*sqrt(
a + b*x)*a**5*b*c**2*x - 10920*sqrt(x)*sqrt(a + b*x)*a**5*b*c*d*x**2 - 715
0*sqrt(x)*sqrt(a + b*x)*a**5*b*d**2*x**3 - 105*sqrt(x)*sqrt(a + b*x)*a**4*
b**2*c**2*x**2 - 390*sqrt(x)*sqrt(a + b*x)*a**4*b**2*c*d*x**3 - 429*sqrt(x
)*sqrt(a + b*x)*a**4*b**2*d**2*x**4 + 120*sqrt(x)*sqrt(a + b*x)*a**3*b**3*
c**2*x**3 + 468*sqrt(x)*sqrt(a + b*x)*a**3*b**3*c*d*x**4 + 572*sqrt(x)*sqr
t(a + b*x)*a**3*b**3*d**2*x**5 - 144*sqrt(x)*sqrt(a + b*x)*a**2*b**4*c**2*
x**4 - 624*sqrt(x)*sqrt(a + b*x)*a**2*b**4*c*d*x**5 - 1144*sqrt(x)*sqrt(a
+ b*x)*a**2*b**4*d**2*x**6 + 192*sqrt(x)*sqrt(a + b*x)*a*b**5*c**2*x**5 +
1248*sqrt(x)*sqrt(a + b*x)*a*b**5*c*d*x**6 - 384*sqrt(x)*sqrt(a + b*x)*b**
6*c**2*x**6 + 1144*sqrt(b)*a**2*b**4*d**2*x**7 - 1248*sqrt(b)*a*b**5*c*d*x
**7 + 384*sqrt(b)*b**6*c**2*x**7))/(45045*a**5*x**7)
```

3.79 $\int x^2(c+dx)^3 \left(ax+bx^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 540

$$
\begin{aligned}
\int x^2(c+dx)^3 \left(ax+bx^2\right)^{3/2} \, dx = \\
& -\frac{a^5(448b^3c^3-ad(864b^2c^2-594abcd+143a^2d^2))\sqrt{ax+bx^2}}{32768b^7} \\
& +\frac{a^4(448b^3c^3-ad(864b^2c^2-594abcd+143a^2d^2))\,x\sqrt{ax+bx^2}}{49152b^6} \\
& -\frac{a^3(448b^3c^3-ad(864b^2c^2-594abcd+143a^2d^2))\,x^2\sqrt{ax+bx^2}}{61440b^5} \\
& +\frac{a^2(448b^3c^3-ad(864b^2c^2-594abcd+143a^2d^2))\,x^3\sqrt{ax+bx^2}}{71680b^4} \\
& +\frac{13a(448b^3c^3-ad(864b^2c^2-594abcd+143a^2d^2))\,x^4\sqrt{ax+bx^2}}{26880b^3} \\
& +\frac{(448b^3c^3-ad(864b^2c^2-594abcd+143a^2d^2))\,x^5\sqrt{ax+bx^2}}{2688b^2} \\
& +\frac{d(864b^2c^2-594abcd+143a^2d^2)\,x^2(ax+bx^2)^{5/2}}{2016b^3} \\
& +\frac{d^2(54bc-13ad)x^3(ax+bx^2)^{5/2}}{144b^2}+\frac{d^3x^4(ax+bx^2)^{5/2}}{9b} \\
& +\frac{a^6(448b^3c^3-ad(864b^2c^2-594abcd+143a^2d^2))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{32768b^{15/2}}
\end{aligned}
$$

output
```
-1/32768*a^5*(448*b^3*c^3-a*d*(143*a^2*d^2-594*a*b*c*d+864*b^2*c^2))*(b*x^
2+a*x)^(1/2)/b^7+1/49152*a^4*(448*b^3*c^3-a*d*(143*a^2*d^2-594*a*b*c*d+864
*b^2*c^2))*x*(b*x^2+a*x)^(1/2)/b^6-1/61440*a^3*(448*b^3*c^3-a*d*(143*a^2*d
^2-594*a*b*c*d+864*b^2*c^2))*x^2*(b*x^2+a*x)^(1/2)/b^5+1/71680*a^2*(448*b^
3*c^3-a*d*(143*a^2*d^2-594*a*b*c*d+864*b^2*c^2))*x^3*(b*x^2+a*x)^(1/2)/b^4
+13/26880*a*(448*b^3*c^3-a*d*(143*a^2*d^2-594*a*b*c*d+864*b^2*c^2))*x^4*(b
*x^2+a*x)^(1/2)/b^3+1/2688*(448*b^3*c^3-a*d*(143*a^2*d^2-594*a*b*c*d+864*b
^2*c^2))*x^5*(b*x^2+a*x)^(1/2)/b^2+1/2016*d*(143*a^2*d^2-594*a*b*c*d+864*b
^2*c^2)*x^2*(b*x^2+a*x)^(5/2)/b^3+1/144*d^2*(-13*a*d+54*b*c)*x^3*(b*x^2+a*
x)^(5/2)/b^2+1/9*d^3*x^4*(b*x^2+a*x)^(5/2)/b+1/32768*a^6*(448*b^3*c^3-a*d*
(143*a^2*d^2-594*a*b*c*d+864*b^2*c^2))*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2)
)/b^(15/2)
```

Mathematica [A] (verified)

Time = 2.62 (sec) , antiderivative size = 454, normalized size of antiderivative = 0.84

$$\int x^2(c+dx)^3\left(ax+bx^2\right)^{3/2}\,dx=\frac{\sqrt{x}\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\sqrt{a+bx}(45045a^8d^3-2310a^7bd^2(81c+13dx)+84a^6b^2d(3240c^2+1485\right.}{}$$

input
```
Integrate[x^2*(c + d*x)^3*(a*x + b*x^2)^(3/2),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(45045*a^8*d^3 - 231
0*a^7*b*d^2*(81*c + 13*d*x) + 84*a^6*b^2*d*(3240*c^2 + 1485*c*d*x + 286*d^
2*x^2) + 1536*a^2*b^6*x^3*(42*c^3 + 72*c^2*d*x + 45*c*d^2*x^2 + 10*d^3*x^3
) + 20480*b^8*x^5*(84*c^3 + 216*c^2*d*x + 189*c*d^2*x^2 + 56*d^3*x^3) - 25
6*a^3*b^5*x^2*(294*c^3 + 486*c^2*d*x + 297*c*d^2*x^2 + 65*d^3*x^3) - 144*a
^5*b^3*(980*c^3 + 1260*c^2*d*x + 693*c*d^2*x^2 + 143*d^3*x^3) + 32*a^4*b^4
*x*(2940*c^3 + 4536*c^2*d*x + 2673*c*d^2*x^2 + 572*d^3*x^3) + 2048*a*b^7*x
^4*(1092*c^3 + 2700*c^2*d*x + 2295*c*d^2*x^2 + 665*d^3*x^3)) + 630*a^7*d*(
864*b^2*c^2 + 143*a^2*d^2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b
*x])] + 1260*a^6*b*c*(224*b^2*c^2 + 297*a^2*d^2)*ArcTanh[(Sqrt[b]*Sqrt[x])
/(-Sqrt[a] + Sqrt[a + b*x])]))/(10321920*b^(15/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.60, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {1262, 27, 2169, 27, 1221, 1134, 1160, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(ax+bx^2\right)^{3/2} (c+dx)^3 \, dx$$

$$\Big\downarrow \text{ 1262}$$

$$\frac{\int \frac{1}{2} x^2 \left(bx^2+ax\right)^{3/2} \left(18bc^3+54bdxc^2+d^2(54bc-13ad)x^2\right)dx}{9b}+\frac{d^3x^4\left(ax+bx^2\right)^{5/2}}{9b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int x^2 \left(bx^2+ax\right)^{3/2} \left(18bc^3+54bdxc^2+d^2(54bc-13ad)x^2\right)dx}{18b}+\frac{d^3x^4\left(ax+bx^2\right)^{5/2}}{9b}$$

$$\Big\downarrow \text{ 2169}$$

$$\frac{\frac{\int \frac{1}{2}x^2\left(288b^2c^3+d\left(864b^2c^2-594abdc+143a^2d^2\right)x\right)\left(bx^2+ax\right)^{3/2}dx}{8b}+\frac{d^2x^3\left(ax+bx^2\right)^{5/2}(54bc-13ad)}{8b}}{18b}+\frac{d^3x^4\left(ax+bx^2\right)^{5/2}}{9b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int x^2\left(288b^2c^3+d\left(864b^2c^2-594abdc+143a^2d^2\right)x\right)\left(bx^2+ax\right)^{3/2}dx}{16b}+\frac{d^2x^3\left(ax+bx^2\right)^{5/2}(54bc-13ad)}{8b}}{18b}+\frac{d^3x^4\left(ax+bx^2\right)^{5/2}}{9b}$$

$$\Big\downarrow \text{ 1221}$$

$$\frac{\frac{\frac{9\left(-143a^3d^3+594a^2bcd^2-864ab^2c^2d+448b^3c^3\right)\int x^2\left(bx^2+ax\right)^{3/2}dx}{14b}+\frac{dx^2\left(ax+bx^2\right)^{5/2}\left(143a^2d^2-594abcd+864b^2c^2\right)}{7b}}{16b}+\frac{d^2x^3\left(ax+bx^2\right)^{5/2}(54bc-13ad)}{8b}}{18b}$$

$$\frac{d^3x^4\left(ax+bx^2\right)^{5/2}}{9b}$$

$$\Big\downarrow \text{ 1134}$$

$$\frac{\frac{\frac{9\left(-143a^3d^3+594a^2bcd^2-864ab^2c^2d+448b^3c^3\right)\left(\frac{x\left(ax+bx^2\right)^{5/2}}{6b}-\frac{7a\int x\left(bx^2+ax\right)^{3/2}dx}{12b}\right)}{14b}+\frac{dx^2\left(ax+bx^2\right)^{5/2}\left(143a^2d^2-594abcd+864b^2c^2\right)}{7b}}{16b}+\frac{d^2x^3(ax+}{}}{18b}$$

$$\frac{d^3x^4\left(ax+bx^2\right)^{5/2}}{9b}$$

$\downarrow$ 1160

$$\frac{\frac{\frac{9\left(-143a^3d^3+594a^2bcd^2-864ab^2c^2d+448b^3c^3\right)\left(\frac{x\left(ax+bx^2\right)^{5/2}}{6b}-\frac{7a\left(\frac{\left(ax+bx^2\right)^{5/2}}{5b}-\frac{a\int\left(bx^2+ax\right)^{3/2}dx}{2b}\right)}{12b}\right)}{14b}+\frac{dx^2\left(ax+bx^2\right)^{5/2}\left(143a^2d^2-594abcd+864b^{}\right.}{7b}}{16b}}{18b}$$

$$\frac{d^3x^4\left(ax+bx^2\right)^{5/2}}{9b}$$

$\downarrow$ 1087

$$\frac{\frac{\frac{9\left(-143a^3d^3+594a^2bcd^2-864ab^2c^2d+448b^3c^3\right)\left(\frac{x\left(ax+bx^2\right)^{5/2}}{6b}-\frac{7a\left(\frac{\left(ax+bx^2\right)^{5/2}}{5b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\int\sqrt{bx^2+ax}dx}{16b}\right)}{2b}\right)}{12b}\right)}{14b}+\frac{dx^2\left(ax+bx^2\right)^{}}{}}{16b}}{18b}$$

$$\frac{d^3x^4\left(ax+bx^2\right)^{5/2}}{9b}$$

$\downarrow$ 1087

$$\frac{\dfrac{\dfrac{9\left(-143a^3d^3+594a^2bcd^2-864ab^2c^2d+448b^3c^3\right)\left(\dfrac{x\left(ax+bx^2\right)^{5/2}}{6b}-\dfrac{7a\left(\dfrac{\left(ax+bx^2\right)^{5/2}}{5b}-\dfrac{a\left(\dfrac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\dfrac{3a^2\left(\dfrac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\dfrac{a^2\int\frac{1}{\sqrt{bx^2+ax}}\cdots}{8b}\cdots\right)}{16b}\cdots\right)}{2b}\cdots\right)}{12b}\cdots\right)}{14b}\cdots}{16b}\cdots}{18b}$$

$$\frac{d^3x^4\left(ax+bx^2\right)^{5/2}}{9b}$$

$\downarrow$ 1091

$$a^2 \int \frac{1}{1-\frac{bx^2}{bx^2+ax}}$$

$$3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b} - \frac{}{4b}\right.$$

$$a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b} - \frac{}{16b}\right.$$

$$7a\left(\frac{\left(ax+bx^2\right)^{5/2}}{5b} - \frac{}{2b}\right.$$

$$9\left(-143a^3d^3+594a^2bcd^2-864ab^2c^2d+448b^3c^3\right)\left(\frac{x\left(ax+bx^2\right)^{5/2}}{6b} - \frac{}{12b}\right.$$

$$\frac{}{14b}$$

$$\frac{}{16b}$$

$$\frac{}{18b}$$

$$\frac{d^3x^4\left(ax+bx^2\right)^{5/2}}{9b}$$

$\downarrow$ 219

$$\frac{\frac{dx^2\left(ax+bx^2\right)^{5/2}\left(143a^2d^2-594abcd+864b^2c^2\right)}{7b}+\frac{9\left(\frac{x\left(ax+bx^2\right)^{5/2}}{6b}-\frac{7a\left(\frac{\left(ax+bx^2\right)^{5/2}}{5b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctan}}{4}\right.}{16b}\right.}{2b}\right.}{12b}\right.}{14b}}{16b}}{18b}$$

$$\frac{d^3x^4\left(ax+bx^2\right)^{5/2}}{9b}$$

input
```
Int[x^2*(c + d*x)^3*(a*x + b*x^2)^(3/2),x]
```

output
```
(d^3*x^4*(a*x + b*x^2)^(5/2))/(9*b) + ((d^2*(54*b*c - 13*a*d)*x^3*(a*x + b
*x^2)^(5/2))/(8*b) + ((d*(864*b^2*c^2 - 594*a*b*c*d + 143*a^2*d^2)*x^2*(a*
x + b*x^2)^(5/2))/(7*b) + (9*(448*b^3*c^3 - 864*a*b^2*c^2*d + 594*a^2*b*c*
d^2 - 143*a^3*d^3)*((x*(a*x + b*x^2)^(5/2))/(6*b) - (7*a*((a*x + b*x^2)^(5
/2)/(5*b) - (a*(((a + 2*b*x)*(a*x + b*x^2)^(3/2))/(8*b) - (3*a^2*(((a + 2*
b*x)*Sqrt[a*x + b*x^2])/(4*b) - (a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]
])/(4*b^(3/2))))/(16*b)))/(2*b)))/(12*b)))/(14*b))/(16*b))/(18*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

rule 2169
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d -
b*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2
, 0]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$-\frac{143\left(a^6\left(\frac{864}{143}a\,b^2c^2d-\frac{54}{13}a^2bc\,d^2-\frac{448}{143}b^3c^3+a^3d^3\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)-\left(-\frac{3584x^2a^3\left(\frac{65}{294}d^3x^3+\frac{99}{98}c\,d^2x^2+\frac{81}{49}c^2dx+c^3\right)}{2145}\right.\right.}{}$
risch	$\frac{(1146880b^8d^3x^8+1361920a\,b^7d^3x^7+3870720b^8c\,d^2x^7+15360a^2b^6d^3x^6+4700160a\,b^7c\,d^2x^6+4423680b^8c^2d\,x^6-16640a^3b^5}{}$

input
```
int(x^2*(d*x+c)^3*(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-143/32768/b^(15/2)*(a^6*(864/143*a*b^2*c^2*d-54/13*a^2*b*c*d^2-448/143*b^
3*c^3+a^3*d^3)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-(-3584/2145*x^2*a^3*(6
5/294*d^3*x^3+99/98*c*d^2*x^2+81/49*c^2*d*x+c^3)*b^(11/2)+1024/715*x^3*(5/
21*d^3*x^3+15/14*c*d^2*x^2+12/7*c^2*d*x+c^3)*a^2*b^(13/2)+8192/165*x^4*a*(
95/156*d^3*x^3+765/364*c*d^2*x^2+225/91*c^2*d*x+c^3)*b^(15/2)+16384/429*x^
5*(2/3*d^3*x^3+9/4*c*d^2*x^2+18/7*c^2*d*x+c^3)*b^(17/2)+(-448/143*(143/980
*d^3*x^3+99/140*c*d^2*x^2+9/7*c^2*d*x+c^3)*a*b^(7/2)+896/429*x*(143/735*d^
3*x^3+891/980*c*d^2*x^2+54/35*c^2*d*x+c^3)*b^(9/2)+d*((8/15*d^2*x^2+864/14
3*c^2+36/13*c*d*x)*b^(5/2)+d*((-2/3*d*x-54/13*c)*b^(3/2)+b^(1/2)*a*d)*a)*a
^2)*a^4)*(x*(b*x+a))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 915, normalized size of antiderivative = 1.69

$$\int x^2(c+dx)^3\left(ax+bx^2\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(d*x+c)^3*(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/20643840*(315*(448*a^6*b^3*c^3 - 864*a^7*b^2*c^2*d + 594*a^8*b*c*d^2 -
 143*a^9*d^3)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*(11
46880*b^9*d^3*x^8 - 141120*a^5*b^4*c^3 + 272160*a^6*b^3*c^2*d - 187110*a^7
*b^2*c*d^2 + 45045*a^8*b*d^3 + 71680*(54*b^9*c*d^2 + 19*a*b^8*d^3)*x^7 + 1
5360*(288*b^9*c^2*d + 306*a*b^8*c*d^2 + a^2*b^7*d^3)*x^6 + 1280*(1344*b^9*
c^3 + 4320*a*b^8*c^2*d + 54*a^2*b^7*c*d^2 - 13*a^3*b^6*d^3)*x^5 + 128*(174
72*a*b^8*c^3 + 864*a^2*b^7*c^2*d - 594*a^3*b^6*c*d^2 + 143*a^4*b^5*d^3)*x^
4 + 144*(448*a^2*b^7*c^3 - 864*a^3*b^6*c^2*d + 594*a^4*b^5*c*d^2 - 143*a^5
*b^4*d^3)*x^3 - 168*(448*a^3*b^6*c^3 - 864*a^4*b^5*c^2*d + 594*a^5*b^4*c*d
^2 - 143*a^6*b^3*d^3)*x^2 + 210*(448*a^4*b^5*c^3 - 864*a^5*b^4*c^2*d + 594
*a^6*b^3*c*d^2 - 143*a^7*b^2*d^3)*x)*sqrt(b*x^2 + a*x))/b^8, -1/10321920*(
315*(448*a^6*b^3*c^3 - 864*a^7*b^2*c^2*d + 594*a^8*b*c*d^2 - 143*a^9*d^3)*
sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (1146880*b^9*d^3*x
^8 - 141120*a^5*b^4*c^3 + 272160*a^6*b^3*c^2*d - 187110*a^7*b^2*c*d^2 + 45
045*a^8*b*d^3 + 71680*(54*b^9*c*d^2 + 19*a*b^8*d^3)*x^7 + 15360*(288*b^9*c
^2*d + 306*a*b^8*c*d^2 + a^2*b^7*d^3)*x^6 + 1280*(1344*b^9*c^3 + 4320*a*b^
8*c^2*d + 54*a^2*b^7*c*d^2 - 13*a^3*b^6*d^3)*x^5 + 128*(17472*a*b^8*c^3 +
864*a^2*b^7*c^2*d - 594*a^3*b^6*c*d^2 + 143*a^4*b^5*d^3)*x^4 + 144*(448*a^
2*b^7*c^3 - 864*a^3*b^6*c^2*d + 594*a^4*b^5*c*d^2 - 143*a^5*b^4*d^3)*x^3 -
 168*(448*a^3*b^6*c^3 - 864*a^4*b^5*c^2*d + 594*a^5*b^4*c*d^2 - 143*a^6...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1148 vs. $2(529) = 1058$.

Time = 0.56 (sec) , antiderivative size = 1148, normalized size of antiderivative = 2.13

$$\int x^2(c+dx)^3 \left(ax+bx^2\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(x**2*(d*x+c)**3*(b*x**2+a*x)**(3/2),x)
```

output
```
Piecewise((35*a**4*(a**2*c**3 - 9*a*(3*a**2*c**2*d + 2*a*b*c**3 - 11*a*(3*
a**2*c*d**2 + 6*a*b*c**2*d - 13*a*(a**2*d**3 + 6*a*b*c*d**2 - 15*a*(19*a*b
*d**3/18 + 3*b**2*c*d**2)/(16*b) + 3*b**2*c**2*d)/(14*b) + b**2*c**3)/(12*
b))/(10*b))*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(
b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**
2), True))/(128*b**4) + sqrt(a*x + b*x**2)*(-35*a**3*(a**2*c**3 - 9*a*(3*a
**2*c**2*d + 2*a*b*c**3 - 11*a*(3*a**2*c*d**2 + 6*a*b*c**2*d - 13*a*(a**2*
d**3 + 6*a*b*c*d**2 - 15*a*(19*a*b*d**3/18 + 3*b**2*c*d**2)/(16*b) + 3*b**
2*c**2*d)/(14*b) + b**2*c**3)/(12*b))/(10*b))/(64*b**4) + 35*a**2*x*(a**2*
c**3 - 9*a*(3*a**2*c**2*d + 2*a*b*c**3 - 11*a*(3*a**2*c*d**2 + 6*a*b*c**2*
d - 13*a*(a**2*d**3 + 6*a*b*c*d**2 - 15*a*(19*a*b*d**3/18 + 3*b**2*c*d**2)
/(16*b) + 3*b**2*c**2*d)/(14*b) + b**2*c**3)/(12*b))/(10*b))/(96*b**3) - 7
*a*x**2*(a**2*c**3 - 9*a*(3*a**2*c**2*d + 2*a*b*c**3 - 11*a*(3*a**2*c*d**2
 + 6*a*b*c**2*d - 13*a*(a**2*d**3 + 6*a*b*c*d**2 - 15*a*(19*a*b*d**3/18 +
3*b**2*c*d**2)/(16*b) + 3*b**2*c**2*d)/(14*b) + b**2*c**3)/(12*b))/(10*b))
/(24*b**2) + b*d**3*x**8/9 + x**7*(19*a*b*d**3/18 + 3*b**2*c*d**2)/(8*b) +
 x**6*(a**2*d**3 + 6*a*b*c*d**2 - 15*a*(19*a*b*d**3/18 + 3*b**2*c*d**2)/(1
6*b) + 3*b**2*c**2*d)/(7*b) + x**5*(3*a**2*c*d**2 + 6*a*b*c**2*d - 13*a*(a
**2*d**3 + 6*a*b*c*d**2 - 15*a*(19*a*b*d**3/18 + 3*b**2*c*d**2)/(16*b) + 3
*b**2*c**2*d)/(14*b) + b**2*c**3)/(6*b) + x**4*(3*a**2*c**2*d + 2*a*b*c...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 824, normalized size of antiderivative = 1.53

$$\int x^2(c+dx)^3\left(ax+bx^2\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(d*x+c)^3*(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
1/9*(b*x^2 + a*x)^(5/2)*d^3*x^4/b + 3/8*(b*x^2 + a*x)^(5/2)*c*d^2*x^3/b -
13/144*(b*x^2 + a*x)^(5/2)*a*d^3*x^3/b^2 + 3/7*(b*x^2 + a*x)^(5/2)*c^2*d*x
^2/b - 33/112*(b*x^2 + a*x)^(5/2)*a*c*d^2*x^2/b^2 + 143/2016*(b*x^2 + a*x)
^(5/2)*a^2*d^3*x^2/b^3 - 7/256*sqrt(b*x^2 + a*x)*a^4*c^3*x/b^3 + 7/96*(b*x
^2 + a*x)^(3/2)*a^2*c^3*x/b^2 + 1/6*(b*x^2 + a*x)^(5/2)*c^3*x/b + 27/512*s
qrt(b*x^2 + a*x)*a^5*c^2*d*x/b^4 - 9/64*(b*x^2 + a*x)^(3/2)*a^3*c^2*d*x/b^
3 - 9/28*(b*x^2 + a*x)^(5/2)*a*c^2*d*x/b^2 - 297/8192*sqrt(b*x^2 + a*x)*a^
6*c*d^2*x/b^5 + 99/1024*(b*x^2 + a*x)^(3/2)*a^4*c*d^2*x/b^4 + 99/448*(b*x^
2 + a*x)^(5/2)*a^2*c*d^2*x/b^3 + 143/16384*sqrt(b*x^2 + a*x)*a^7*d^3*x/b^6
 - 143/6144*(b*x^2 + a*x)^(3/2)*a^5*d^3*x/b^5 - 143/2688*(b*x^2 + a*x)^(5/
2)*a^3*d^3*x/b^4 + 7/1024*a^6*c^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt
(b))/b^(9/2) - 27/2048*a^7*c^2*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(
b))/b^(11/2) + 297/32768*a^8*c*d^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqr
t(b))/b^(13/2) - 143/65536*a^9*d^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqr
t(b))/b^(15/2) - 7/512*sqrt(b*x^2 + a*x)*a^5*c^3/b^4 + 7/192*(b*x^2 + a*x)
^(3/2)*a^3*c^3/b^3 - 7/60*(b*x^2 + a*x)^(5/2)*a*c^3/b^2 + 27/1024*sqrt(b*x
^2 + a*x)*a^6*c^2*d/b^5 - 9/128*(b*x^2 + a*x)^(3/2)*a^4*c^2*d/b^4 + 9/40*(
b*x^2 + a*x)^(5/2)*a^2*c^2*d/b^3 - 297/16384*sqrt(b*x^2 + a*x)*a^7*c*d^2/b
^6 + 99/2048*(b*x^2 + a*x)^(3/2)*a^5*c*d^2/b^5 - 99/640*(b*x^2 + a*x)^(5/2
)*a^3*c*d^2/b^4 + 143/32768*sqrt(b*x^2 + a*x)*a^8*d^3/b^7 - 143/12288*(...
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 484, normalized size of antiderivative = 0.90

$$\int x^2(c+dx)^3 (ax+bx^2)^{3/2} \, dx = \frac{1}{10321920} \sqrt{bx^2+ax}\left(2\left(4\left(2\left(8\left(10\left(4\left(14\left(16\,bd^3x+\frac{54\,b^9cd^2+19\,ab^8d^3}{b^8}\right)x+\frac{3\,(288\,b\right.\right.\right.\right.\right.\right.\right.$$
$$-\frac{(448\,a^6b^3c^3-864\,a^7b^2c^2d+594\,a^8bcd^2-143\,a^9d^3)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{65536\,b^{\frac{15}{2}}}$$

input
```
integrate(x^2*(d*x+c)^3*(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
1/10321920*sqrt(b*x^2 + a*x)*(2*(4*(2*(8*(10*(4*(14*(16*b*d^3*x + (54*b^9*
c*d^2 + 19*a*b^8*d^3)/b^8)*x + 3*(288*b^9*c^2*d + 306*a*b^8*c*d^2 + a^2*b^
7*d^3)/b^8)*x + (1344*b^9*c^3 + 4320*a*b^8*c^2*d + 54*a^2*b^7*c*d^2 - 13*a
^3*b^6*d^3)/b^8)*x + (17472*a*b^8*c^3 + 864*a^2*b^7*c^2*d - 594*a^3*b^6*c*
d^2 + 143*a^4*b^5*d^3)/b^8)*x + 9*(448*a^2*b^7*c^3 - 864*a^3*b^6*c^2*d + 5
94*a^4*b^5*c*d^2 - 143*a^5*b^4*d^3)/b^8)*x - 21*(448*a^3*b^6*c^3 - 864*a^4
*b^5*c^2*d + 594*a^5*b^4*c*d^2 - 143*a^6*b^3*d^3)/b^8)*x + 105*(448*a^4*b^
5*c^3 - 864*a^5*b^4*c^2*d + 594*a^6*b^3*c*d^2 - 143*a^7*b^2*d^3)/b^8)*x -
315*(448*a^5*b^4*c^3 - 864*a^6*b^3*c^2*d + 594*a^7*b^2*c*d^2 - 143*a^8*b*d
^3)/b^8) - 1/65536*(448*a^6*b^3*c^3 - 864*a^7*b^2*c^2*d + 594*a^8*b*c*d^2
- 143*a^9*d^3)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(
15/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(c+dx)^3\left(ax+bx^2\right)^{3/2}\,dx=\int x^2\left(b\,x^2+a\,x\right)^{3/2}(c+d\,x)^3\,dx$$

input
```
int(x^2*(a*x + b*x^2)^(3/2)*(c + d*x)^3,x)
```

output
```
int(x^2*(a*x + b*x^2)^(3/2)*(c + d*x)^3, x)
```

Reduce [F]

$$\int x^2(c+dx)^3\left(ax+bx^2\right)^{3/2}\,dx=\int x^2(dx+c)^3\left(b\,x^2+ax\right)^{\frac{3}{2}}dx$$

input
```
int(x^2*(d*x+c)^3*(b*x^2+a*x)^(3/2),x)
```

output
```
int(x^2*(d*x+c)^3*(b*x^2+a*x)^(3/2),x)
```

3.80 $\int x(c+dx)^3 \left(ax+bx^2\right)^{3/2} dx$

Optimal result

Integrand size = 22, antiderivative size = 476

$$\begin{aligned}\int x(c+dx)^3 \left(ax+bx^2\right)^{3/2} dx &= \frac{3a^4(128b^3c^3-ad(224b^2c^2-144abcd+33a^2d^2))\sqrt{ax+bx^2}}{16384b^6} \\ &- \frac{a^3(128b^3c^3-ad(224b^2c^2-144abcd+33a^2d^2))\,x\sqrt{ax+bx^2}}{8192b^5} \\ &+ \frac{a^2(128b^3c^3-ad(224b^2c^2-144abcd+33a^2d^2))\,x^2\sqrt{ax+bx^2}}{10240b^4} \\ &+ \frac{11a(128b^3c^3-ad(224b^2c^2-144abcd+33a^2d^2))\,x^3\sqrt{ax+bx^2}}{5120b^3} \\ &+ \frac{(128b^3c^3-ad(224b^2c^2-144abcd+33a^2d^2))\,x^4\sqrt{ax+bx^2}}{640b^2} \\ &+ \frac{d(224b^2c^2-144abcd+33a^2d^2)\,x(ax+bx^2)^{5/2}}{448b^3} \\ &+ \frac{d^2(48bc-11ad)x^2(ax+bx^2)^{5/2}}{112b^2} + \frac{d^3x^3(ax+bx^2)^{5/2}}{8b} \\ &- \frac{3a^5(128b^3c^3-ad(224b^2c^2-144abcd+33a^2d^2))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{16384b^{13/2}}\end{aligned}$$

output
```
3/16384*a^4*(128*b^3*c^3-a*d*(33*a^2*d^2-144*a*b*c*d+224*b^2*c^2))*(b*x^2+
a*x)^(1/2)/b^6-1/8192*a^3*(128*b^3*c^3-a*d*(33*a^2*d^2-144*a*b*c*d+224*b^2
*c^2))*x*(b*x^2+a*x)^(1/2)/b^5+1/10240*a^2*(128*b^3*c^3-a*d*(33*a^2*d^2-14
4*a*b*c*d+224*b^2*c^2))*x^2*(b*x^2+a*x)^(1/2)/b^4+11/5120*a*(128*b^3*c^3-a
*d*(33*a^2*d^2-144*a*b*c*d+224*b^2*c^2))*x^3*(b*x^2+a*x)^(1/2)/b^3+1/640*(
128*b^3*c^3-a*d*(33*a^2*d^2-144*a*b*c*d+224*b^2*c^2))*x^4*(b*x^2+a*x)^(1/2
)/b^2+1/448*d*(33*a^2*d^2-144*a*b*c*d+224*b^2*c^2)*x*(b*x^2+a*x)^(5/2)/b^3
+1/112*d^2*(-11*a*d+48*b*c)*x^2*(b*x^2+a*x)^(5/2)/b^2+1/8*d^3*x^3*(b*x^2+a
*x)^(5/2)/b-3/16384*a^5*(128*b^3*c^3-a*d*(33*a^2*d^2-144*a*b*c*d+224*b^2*c
^2))*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(13/2)
```

Mathematica [A] (verified)

Time = 2.20 (sec) , antiderivative size = 413, normalized size of antiderivative = 0.87

$$\int x(c+dx)^3\left(ax+bx^2\right)^{3/2}\,dx=\frac{\sqrt{x}\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\sqrt{a+bx}(-3465a^7d^3+210a^6bd^2(72c+11dx)-168a^5b^2d(140c^2+60cdx$$

input
```
Integrate[x*(c + d*x)^3*(a*x + b*x^2)^(3/2),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-3465*a^7*d^3 + 210
*a^6*b*d^2*(72*c + 11*d*x) - 168*a^5*b^2*d*(140*c^2 + 60*c*d*x + 11*d^2*x^
2) + 256*a^2*b^5*x^2*(28*c^3 + 42*c^2*d*x + 24*c*d^2*x^2 + 5*d^3*x^3) - 12
8*a^3*b^4*x*(70*c^3 + 98*c^2*d*x + 54*c*d^2*x^2 + 11*d^3*x^3) + 2048*b^7*x
^4*(56*c^3 + 140*c^2*d*x + 120*c*d^2*x^2 + 35*d^3*x^3) + 1024*a*b^6*x^3*(1
54*c^3 + 364*c^2*d*x + 300*c*d^2*x^2 + 85*d^3*x^3) + 16*a^4*b^3*(840*c^3 +
 980*c^2*d*x + 504*c*d^2*x^2 + 99*d^3*x^3)) + 3360*a^5*b*c*(8*b^2*c^2 + 9*
a^2*d^2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])] + 210*a^6*d*
(224*b^2*c^2 + 33*a^2*d^2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a +
b*x])]))/(573440*b^(13/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.64, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1262, 27, 2169, 27, 1225, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax+bx^2\right)^{3/2}(c+dx)^3\,dx$$

$$\Big\downarrow \; 1262$$

$$\frac{\int \frac{1}{2}x\left(bx^2+ax\right)^{3/2}\left(16bc^3+48bdxc^2+d^2(48bc-11ad)x^2\right)dx}{8b}+\frac{d^3x^3\left(ax+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \; 27$$

$$\frac{\int x\left(bx^2+ax\right)^{3/2}\left(16bc^3+48bdxc^2+d^2(48bc-11ad)x^2\right)dx}{16b}+\frac{d^3x^3\left(ax+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \; 2169$$

$$\frac{\frac{\int \frac{1}{2}x\left(224b^2c^3+3d\left(224b^2c^2-144abdc+33a^2d^2\right)x\right)\left(bx^2+ax\right)^{3/2}dx}{7b}+\frac{d^2x^2\left(ax+bx^2\right)^{5/2}(48bc-11ad)}{7b}}{16b}+\frac{d^3x^3\left(ax+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int x\left(224b^2c^3+3d\left(224b^2c^2-144abdc+33a^2d^2\right)x\right)\left(bx^2+ax\right)^{3/2}dx}{14b}+\frac{d^2x^2\left(ax+bx^2\right)^{5/2}(48bc-11ad)}{7b}}{16b}+\frac{d^3x^3\left(ax+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \; 1225$$

$$\frac{\frac{\frac{\left(ax+bx^2\right)^{5/2}\left(10bdx\left(33a^2d^2-144abcd+224b^2c^2\right)+7\left(128b^3c^3-ad\left(33a^2d^2-144abcd+224b^2c^2\right)\right)\right)}{20b^2}-\frac{7a\left(128b^3c^3-ad\left(33a^2d^2-144abcd+224b^2c^2\right)\right)\int\left(bx^2+ax\right)}{8b^2}}{14b}}{16b}$$

$$\frac{d^3x^3\left(ax+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \; 1087$$

$$\frac{\frac{\frac{(ax+bx^2)^{5/2}\left(10bdx\left(33a^2d^2-144abcd+224b^2c^2\right)+7\left(128b^3c^3-ad\left(33a^2d^2-144abcd+224b^2c^2\right)\right)\right)}{20b^2}-\frac{7a\left(128b^3c^3-ad\left(33a^2d^2-144abcd+224b^2c^2\right)\right)\left(\frac{(a+2bx)(\cdots}{\cdots}\right.\cdots}{8b^2}}{14b}}{16b}\cdots$$

$$\frac{d^3x^3\left(ax+bx^2\right)^{5/2}}{8b}$$

$\Big\downarrow$ 1087

$$\frac{\frac{\frac{(ax+bx^2)^{5/2}\left(10bdx\left(33a^2d^2-144abcd+224b^2c^2\right)+7\left(128b^3c^3-ad\left(33a^2d^2-144abcd+224b^2c^2\right)\right)\right)}{20b^2}-\frac{7a\left(128b^3c^3-ad\left(33a^2d^2-144abcd+224b^2c^2\right)\right)\left(\frac{(a+2bx)(\cdots}{\cdots}\right.\cdots}{14b}}{16b}\cdots$$

$$\frac{d^3x^3\left(ax+bx^2\right)^{5/2}}{8b}$$

$\Big\downarrow$ 1091

$$\frac{\frac{\frac{(ax+bx^2)^{5/2}\left(10bdx\left(33a^2d^2-144abcd+224b^2c^2\right)+7\left(128b^3c^3-ad\left(33a^2d^2-144abcd+224b^2c^2\right)\right)\right)}{20b^2}-\frac{7a\left(128b^3c^3-ad\left(33a^2d^2-144abcd+224b^2c^2\right)\right)\left(\frac{(a+2bx)(\cdots}{\cdots}\right.\cdots}{14b}}{16b}\cdots$$

$$\frac{d^3x^3\left(ax+bx^2\right)^{5/2}}{8b}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{(ax+bx^2)^{5/2}\left(10bdx\left(33a^2d^2-144abcd+224b^2c^2\right)+7\left(128b^3c^3-ad\left(33a^2d^2-144abcd+224b^2c^2\right)\right)\right)}{20b^2}-\frac{7a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\cdots}{\cdots}\right.\cdots}{16b\cdots}\right.\cdots}{14b}}{16b}\cdots$$

$$\frac{d^3x^3\left(ax+bx^2\right)^{5/2}}{8b}$$

input
```
Int[x*(c + d*x)^3*(a*x + b*x^2)^(3/2),x]
```

output
```
(d^3*x^3*(a*x + b*x^2)^(5/2))/(8*b) + ((d^2*(48*b*c - 11*a*d)*x^2*(a*x + b
*x^2)^(5/2))/(7*b) + (((7*(128*b^3*c^3 - a*d*(224*b^2*c^2 - 144*a*b*c*d +
33*a^2*d^2)) + 10*b*d*(224*b^2*c^2 - 144*a*b*c*d + 33*a^2*d^2)*x)*(a*x + b
*x^2)^(5/2))/(20*b^2) - (7*a*(128*b^3*c^3 - a*d*(224*b^2*c^2 - 144*a*b*c*d
 + 33*a^2*d^2))*(((a + 2*b*x)*(a*x + b*x^2)^(3/2))/(8*b) - (3*a^2*(((a + 2
*b*x)*Sqrt[a*x + b*x^2])/(4*b) - (a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2
]])/(4*b^(3/2))))/(16*b)))/(8*b^2))/(14*b))/(16*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

rule 2169
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d -
b*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2
, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{99a^5\left(-\frac{48}{11}a^2bc\,d^2+\frac{224}{33}a\,b^2c^2d-\frac{128}{33}b^3c^3+a^3d^3\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{16384}-\frac{99\left(-\frac{1024x^2\left(\frac{dx}{2}+c\right)\left(\frac{5}{14}d^2x^2+cdx+c^2\right)a^2b^{\frac{11}{2}}}{495}-\frac{2048\left(\frac{85}{15}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$
risch	$-\frac{\left(-71680b^7d^3x^7-87040a\,b^6d^3x^6-245760b^7c\,d^2x^6-1280a^2b^5d^3x^5-307200a\,b^6c\,d^2x^5-286720b^7c^2d\,x^5+1408a^3b^4d^3x^4\cdots\right.}{\cdots}$
default	$c^3\left(\frac{\left(b\,x^2+ax\right)^{\frac{5}{2}}}{5b}-\frac{a\left(\frac{(2bx+a)\left(b\,x^2+ax\right)^{\frac{3}{2}}}{8b}-\frac{3a^2\left(\frac{(2bx+a)\sqrt{b\,x^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8b^{\frac{3}{2}}}\right)}{16b}\right)}{2b}\right)+d^3\left(\frac{x^3\left(b\,x^2\cdots\right.}{8\cdots}\cdots\right.$

input
```
int(x*(d*x+c)^3*(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
99/16384/b^(13/2)*(a^5*(-48/11*a^2*b*c*d^2+224/33*a*b^2*c^2*d-128/33*b^3*c
^3+a^3*d^3)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-(-1024/495*x^2*(1/2*d*x+c
)*(5/14*d^2*x^2+c*d*x+c^2)*a^2*b^(11/2)-2048/45*(85/154*d^3*x^3+150/77*c*d
^2*x^2+26/11*c^2*d*x+c^3)*x^3*a*b^(13/2)-16384/495*(5/8*d^3*x^3+15/7*c*d^2
*x^2+5/2*c^2*d*x+c^3)*x^4*b^(15/2)+a^3*(-128/33*(33/280*d^3*x^3+3/5*c*d^2*
x^2+7/6*c^2*d*x+c^3)*a*b^(7/2)+256/99*(11/70*d^3*x^3+27/35*c*d^2*x^2+7/5*c
^2*d*x+c^3)*x*b^(9/2)+d*a^2*((32/11*c*d*x+224/33*c^2+8/15*d^2*x^2)*b^(5/2)
+d*((-48/11*c-2/3*d*x)*b^(3/2)+b^(1/2)*a*d)*a)))*(x*(b*x+a))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 810, normalized size of antiderivative = 1.70

$$\int x(c+dx)^3\left(ax+bx^2\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(d*x+c)^3*(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/1146880*(105*(128*a^5*b^3*c^3 - 224*a^6*b^2*c^2*d + 144*a^7*b*c*d^2 -
33*a^8*d^3)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*(7168
0*b^8*d^3*x^7 + 13440*a^4*b^4*c^3 - 23520*a^5*b^3*c^2*d + 15120*a^6*b^2*c*
d^2 - 3465*a^7*b*d^3 + 5120*(48*b^8*c*d^2 + 17*a*b^7*d^3)*x^6 + 1280*(224*
b^8*c^2*d + 240*a*b^7*c*d^2 + a^2*b^6*d^3)*x^5 + 128*(896*b^8*c^3 + 2912*a
*b^7*c^2*d + 48*a^2*b^6*c*d^2 - 11*a^3*b^5*d^3)*x^4 + 16*(9856*a*b^7*c^3 +
 672*a^2*b^6*c^2*d - 432*a^3*b^5*c*d^2 + 99*a^4*b^4*d^3)*x^3 + 56*(128*a^2
*b^6*c^3 - 224*a^3*b^5*c^2*d + 144*a^4*b^4*c*d^2 - 33*a^5*b^3*d^3)*x^2 - 7
0*(128*a^3*b^5*c^3 - 224*a^4*b^4*c^2*d + 144*a^5*b^3*c*d^2 - 33*a^6*b^2*d^
3)*x)*sqrt(b*x^2 + a*x))/b^7, 1/573440*(105*(128*a^5*b^3*c^3 - 224*a^6*b^2
*c^2*d + 144*a^7*b*c*d^2 - 33*a^8*d^3)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*s
qrt(-b)/(b*x + a)) + (71680*b^8*d^3*x^7 + 13440*a^4*b^4*c^3 - 23520*a^5*b^
3*c^2*d + 15120*a^6*b^2*c*d^2 - 3465*a^7*b*d^3 + 5120*(48*b^8*c*d^2 + 17*a
*b^7*d^3)*x^6 + 1280*(224*b^8*c^2*d + 240*a*b^7*c*d^2 + a^2*b^6*d^3)*x^5 +
 128*(896*b^8*c^3 + 2912*a*b^7*c^2*d + 48*a^2*b^6*c*d^2 - 11*a^3*b^5*d^3)*
x^4 + 16*(9856*a*b^7*c^3 + 672*a^2*b^6*c^2*d - 432*a^3*b^5*c*d^2 + 99*a^4*
b^4*d^3)*x^3 + 56*(128*a^2*b^6*c^3 - 224*a^3*b^5*c^2*d + 144*a^4*b^4*c*d^2
 - 33*a^5*b^3*d^3)*x^2 - 70*(128*a^3*b^5*c^3 - 224*a^4*b^4*c^2*d + 144*a^5
*b^3*c*d^2 - 33*a^6*b^2*d^3)*x)*sqrt(b*x^2 + a*x))/b^7]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1006 vs. $2(469) = 938$.

Time = 0.64 (sec) , antiderivative size = 1006, normalized size of antiderivative = 2.11

$$\int x(c+dx)^3 \left(ax+bx^2\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(x*(d*x+c)**3*(b*x**2+a*x)**(3/2),x)
```

output
```
Piecewise((-5*a**3*(a**2*c**3 - 7*a*(3*a**2*c**2*d + 2*a*b*c**3 - 9*a*(3*a
**2*c*d**2 + 6*a*b*c**2*d - 11*a*(a**2*d**3 + 6*a*b*c*d**2 - 13*a*(17*a*b*
d**3/16 + 3*b**2*c*d**2)/(14*b) + 3*b**2*c**2*d)/(12*b) + b**2*c**3)/(10*b
))/(8*b))*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b)
, Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2)
, True))/(16*b**3) + sqrt(a*x + b*x**2)*(5*a**2*(a**2*c**3 - 7*a*(3*a**2*c
**2*d + 2*a*b*c**3 - 9*a*(3*a**2*c*d**2 + 6*a*b*c**2*d - 11*a*(a**2*d**3 +
 6*a*b*c*d**2 - 13*a*(17*a*b*d**3/16 + 3*b**2*c*d**2)/(14*b) + 3*b**2*c**2
*d)/(12*b) + b**2*c**3)/(10*b))/(8*b))/(8*b**3) - 5*a*x*(a**2*c**3 - 7*a*(
3*a**2*c**2*d + 2*a*b*c**3 - 9*a*(3*a**2*c*d**2 + 6*a*b*c**2*d - 11*a*(a**
2*d**3 + 6*a*b*c*d**2 - 13*a*(17*a*b*d**3/16 + 3*b**2*c*d**2)/(14*b) + 3*b
**2*c**2*d)/(12*b) + b**2*c**3)/(10*b))/(8*b))/(12*b**2) + b*d**3*x**7/8 +
 x**6*(17*a*b*d**3/16 + 3*b**2*c*d**2)/(7*b) + x**5*(a**2*d**3 + 6*a*b*c*d
**2 - 13*a*(17*a*b*d**3/16 + 3*b**2*c*d**2)/(14*b) + 3*b**2*c**2*d)/(6*b)
+ x**4*(3*a**2*c*d**2 + 6*a*b*c**2*d - 11*a*(a**2*d**3 + 6*a*b*c*d**2 - 13
*a*(17*a*b*d**3/16 + 3*b**2*c*d**2)/(14*b) + 3*b**2*c**2*d)/(12*b) + b**2*
c**3)/(5*b) + x**3*(3*a**2*c**2*d + 2*a*b*c**3 - 9*a*(3*a**2*c*d**2 + 6*a*
b*c**2*d - 11*a*(a**2*d**3 + 6*a*b*c*d**2 - 13*a*(17*a*b*d**3/16 + 3*b**2*
c*d**2)/(14*b) + 3*b**2*c**2*d)/(12*b) + b**2*c**3)/(10*b))/(4*b) + x**2*(
a**2*c**3 - 7*a*(3*a**2*c**2*d + 2*a*b*c**3 - 9*a*(3*a**2*c*d**2 + 6*a*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 724, normalized size of antiderivative = 1.52

$$\int x(c+dx)^3\left(ax+bx^2\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(d*x+c)^3*(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
1/8*(b*x^2 + a*x)^(5/2)*d^3*x^3/b + 3/7*(b*x^2 + a*x)^(5/2)*c*d^2*x^2/b -
11/112*(b*x^2 + a*x)^(5/2)*a*d^3*x^2/b^2 + 3/64*sqrt(b*x^2 + a*x)*a^3*c^3*
x/b^2 - 1/8*(b*x^2 + a*x)^(3/2)*a*c^3*x/b - 21/256*sqrt(b*x^2 + a*x)*a^4*c
^2*d*x/b^3 + 7/32*(b*x^2 + a*x)^(3/2)*a^2*c^2*d*x/b^2 + 1/2*(b*x^2 + a*x)^
(5/2)*c^2*d*x/b + 27/512*sqrt(b*x^2 + a*x)*a^5*c*d^2*x/b^4 - 9/64*(b*x^2 +
 a*x)^(3/2)*a^3*c*d^2*x/b^3 - 9/28*(b*x^2 + a*x)^(5/2)*a*c*d^2*x/b^2 - 99/
8192*sqrt(b*x^2 + a*x)*a^6*d^3*x/b^5 + 33/1024*(b*x^2 + a*x)^(3/2)*a^4*d^3
*x/b^4 + 33/448*(b*x^2 + a*x)^(5/2)*a^2*d^3*x/b^3 - 3/256*a^5*c^3*log(2*b*
x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(7/2) + 21/1024*a^6*c^2*d*log(2*b*x
 + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(9/2) - 27/2048*a^7*c*d^2*log(2*b*x
+ a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(11/2) + 99/32768*a^8*d^3*log(2*b*x +
 a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(13/2) + 3/128*sqrt(b*x^2 + a*x)*a^4*c
^3/b^3 - 1/16*(b*x^2 + a*x)^(3/2)*a^2*c^3/b^2 + 1/5*(b*x^2 + a*x)^(5/2)*c^
3/b - 21/512*sqrt(b*x^2 + a*x)*a^5*c^2*d/b^4 + 7/64*(b*x^2 + a*x)^(3/2)*a^
3*c^2*d/b^3 - 7/20*(b*x^2 + a*x)^(5/2)*a*c^2*d/b^2 + 27/1024*sqrt(b*x^2 +
a*x)*a^6*c*d^2/b^5 - 9/128*(b*x^2 + a*x)^(3/2)*a^4*c*d^2/b^4 + 9/40*(b*x^2
 + a*x)^(5/2)*a^2*c*d^2/b^3 - 99/16384*sqrt(b*x^2 + a*x)*a^7*d^3/b^6 + 33/
2048*(b*x^2 + a*x)^(3/2)*a^5*d^3/b^5 - 33/640*(b*x^2 + a*x)^(5/2)*a^3*d^3/
b^4
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 427, normalized size of antiderivative = 0.90

$$\int x(c+dx)^3 (ax+bx^2)^{3/2}\, dx = \frac{1}{573440}\sqrt{bx^2+ax}\left(2\left(4\left(2\left(8\left(10\left(4\left(14\,bd^3x + \frac{48\,b^8cd^2+17\,ab^7d^3}{b^7}\right)x + \frac{224\,b^8c^2d+24}{}\right.\right.\right.\right.\right.\right. + \frac{3\,(128\,a^5b^3c^3 - 224\,a^6b^2c^2d + 144\,a^7bcd^2 - 33\,a^8d^3)\log\left(\left|2\left(\sqrt{b}x - \sqrt{bx^2+ax}\right)\sqrt{b} + a\right|\right)}{32768\,b^{\frac{13}{2}}}$$

input
```
integrate(x*(d*x+c)^3*(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
1/573440*sqrt(b*x^2 + a*x)*(2*(4*(2*(8*(10*(4*(14*b*d^3*x + (48*b^8*c*d^2
+ 17*a*b^7*d^3)/b^7)*x + (224*b^8*c^2*d + 240*a*b^7*c*d^2 + a^2*b^6*d^3)/b
^7)*x + (896*b^8*c^3 + 2912*a*b^7*c^2*d + 48*a^2*b^6*c*d^2 - 11*a^3*b^5*d^
3)/b^7)*x + (9856*a*b^7*c^3 + 672*a^2*b^6*c^2*d - 432*a^3*b^5*c*d^2 + 99*a
^4*b^4*d^3)/b^7)*x + 7*(128*a^2*b^6*c^3 - 224*a^3*b^5*c^2*d + 144*a^4*b^4*
c*d^2 - 33*a^5*b^3*d^3)/b^7)*x - 35*(128*a^3*b^5*c^3 - 224*a^4*b^4*c^2*d +
 144*a^5*b^3*c*d^2 - 33*a^6*b^2*d^3)/b^7)*x + 105*(128*a^4*b^4*c^3 - 224*a
^5*b^3*c^2*d + 144*a^6*b^2*c*d^2 - 33*a^7*b*d^3)/b^7) + 3/32768*(128*a^5*b
^3*c^3 - 224*a^6*b^2*c^2*d + 144*a^7*b*c*d^2 - 33*a^8*d^3)*log(abs(2*(sqrt
(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(13/2)
```

Mupad [F(-1)]

Timed out.

$$\int x(c+dx)^3 \left(ax+bx^2\right)^{3/2} dx = \int x \left(b\,x^2+a\,x\right)^{3/2} (c+d\,x)^3 \,dx$$

input
```
int(x*(a*x + b*x^2)^(3/2)*(c + d*x)^3,x)
```

output
```
int(x*(a*x + b*x^2)^(3/2)*(c + d*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 155.06 (sec) , antiderivative size = 670, normalized size of antiderivative = 1.41

$$\int x(c+dx)^3 \left(ax+bx^2\right)^{3/2} dx = \frac{3465\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\sqrt{b}}{\sqrt{a}}\right)a^8d^3 + 15120\sqrt{x}\sqrt{bx+a}\,a^6b^2c\,d^2 + 2310\sqrt{x}\sqrt{bx+a}\,a^6b^2d^3x - \cdots}{\cdots}$$

input
```
int(x*(d*x+c)^3*(b*x^2+a*x)^(3/2),x)
```

output
```
( - 3465*sqrt(x)*sqrt(a + b*x)*a**7*b*d**3 + 15120*sqrt(x)*sqrt(a + b*x)*a
**6*b**2*c*d**2 + 2310*sqrt(x)*sqrt(a + b*x)*a**6*b**2*d**3*x - 23520*sqrt
(x)*sqrt(a + b*x)*a**5*b**3*c**2*d - 10080*sqrt(x)*sqrt(a + b*x)*a**5*b**3
*c*d**2*x - 1848*sqrt(x)*sqrt(a + b*x)*a**5*b**3*d**3*x**2 + 13440*sqrt(x)
*sqrt(a + b*x)*a**4*b**4*c**3 + 15680*sqrt(x)*sqrt(a + b*x)*a**4*b**4*c**2
*d*x + 8064*sqrt(x)*sqrt(a + b*x)*a**4*b**4*c*d**2*x**2 + 1584*sqrt(x)*sqr
t(a + b*x)*a**4*b**4*d**3*x**3 - 8960*sqrt(x)*sqrt(a + b*x)*a**3*b**5*c**3
*x - 12544*sqrt(x)*sqrt(a + b*x)*a**3*b**5*c**2*d*x**2 - 6912*sqrt(x)*sqrt
(a + b*x)*a**3*b**5*c*d**2*x**3 - 1408*sqrt(x)*sqrt(a + b*x)*a**3*b**5*d**
3*x**4 + 7168*sqrt(x)*sqrt(a + b*x)*a**2*b**6*c**3*x**2 + 10752*sqrt(x)*sq
rt(a + b*x)*a**2*b**6*c**2*d*x**3 + 6144*sqrt(x)*sqrt(a + b*x)*a**2*b**6*c
*d**2*x**4 + 1280*sqrt(x)*sqrt(a + b*x)*a**2*b**6*d**3*x**5 + 157696*sqrt(
x)*sqrt(a + b*x)*a*b**7*c**3*x**3 + 372736*sqrt(x)*sqrt(a + b*x)*a*b**7*c*
*2*d*x**4 + 307200*sqrt(x)*sqrt(a + b*x)*a*b**7*c*d**2*x**5 + 87040*sqrt(x
)*sqrt(a + b*x)*a*b**7*d**3*x**6 + 114688*sqrt(x)*sqrt(a + b*x)*b**8*c**3*
x**4 + 286720*sqrt(x)*sqrt(a + b*x)*b**8*c**2*d*x**5 + 245760*sqrt(x)*sqrt
(a + b*x)*b**8*c*d**2*x**6 + 71680*sqrt(x)*sqrt(a + b*x)*b**8*d**3*x**7 +
3465*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**8*d**3 - 15
120*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**7*b*c*d**2 +
 23520*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**6*b**2...
```

3.81 $\int (c+dx)^3 \left(ax+bx^2\right)^{3/2} dx$

Optimal result

Integrand size = 21, antiderivative size = 391

$$\begin{aligned}
\int (c+dx)^3 \left(ax+bx^2\right)^{3/2} dx =
&-\frac{3a^3(2bc-ad)\left(8b^2c^2-8abcd+3a^2d^2\right)\sqrt{ax+bx^2}}{1024b^5} \\
&+\frac{a^2(2bc-ad)\left(8b^2c^2-8abcd+3a^2d^2\right)x\sqrt{ax+bx^2}}{512b^4} \\
&+\frac{3a(2bc-ad)\left(8b^2c^2-8abcd+3a^2d^2\right)x^2\sqrt{ax+bx^2}}{128b^3} \\
&+\frac{(2bc-ad)\left(8b^2c^2-8abcd+3a^2d^2\right)x^3\sqrt{ax+bx^2}}{64b^2} \\
&+\frac{d\left(24b^2c^2-14abcd+3a^2d^2\right)\left(ax+bx^2\right)^{5/2}}{40b^3} \\
&+\frac{d^2(14bc-3ad)x\left(ax+bx^2\right)^{5/2}}{28b^2}+\frac{d^3x^2\left(ax+bx^2\right)^{5/2}}{7b} \\
&+\frac{3a^4(2bc-ad)\left(8b^2c^2-8abcd+3a^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{1024b^{11/2}}
\end{aligned}$$

output
```
-3/1024*a^3*(-a*d+2*b*c)*(3*a^2*d^2-8*a*b*c*d+8*b^2*c^2)*(b*x^2+a*x)^(1/2)
/b^5+1/512*a^2*(-a*d+2*b*c)*(3*a^2*d^2-8*a*b*c*d+8*b^2*c^2)*x*(b*x^2+a*x)^
(1/2)/b^4+3/128*a*(-a*d+2*b*c)*(3*a^2*d^2-8*a*b*c*d+8*b^2*c^2)*x^2*(b*x^2+
a*x)^(1/2)/b^3+1/64*(-a*d+2*b*c)*(3*a^2*d^2-8*a*b*c*d+8*b^2*c^2)*x^3*(b*x^
2+a*x)^(1/2)/b^2+1/40*d*(3*a^2*d^2-14*a*b*c*d+24*b^2*c^2)*(b*x^2+a*x)^(5/2
)/b^3+1/28*d^2*(-3*a*d+14*b*c)*x*(b*x^2+a*x)^(5/2)/b^2+1/7*d^3*x^2*(b*x^2+
a*x)^(5/2)/b+3/1024*a^4*(-a*d+2*b*c)*(3*a^2*d^2-8*a*b*c*d+8*b^2*c^2)*arcta
nh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.95

$$\int (c+dx)^3 \left(ax+bx^2\right)^{3/2} dx = \frac{\sqrt{x}\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\sqrt{a+bx}(315a^6d^3-210a^5bd^2(7c+dx)+28a^4b^2d(90c^2+35cdx+6d^2x^2\right.}{}$$

input
```
Integrate[(c + d*x)^3*(a*x + b*x^2)^(3/2),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(315*a^6*d^3 - 210*a
^5*b*d^2*(7*c + d*x) + 28*a^4*b^2*d*(90*c^2 + 35*c*d*x + 6*d^2*x^2) + 32*a
^2*b^4*x*(35*c^3 + 42*c^2*d*x + 21*c*d^2*x^2 + 4*d^3*x^3) - 16*a^3*b^3*(10
5*c^3 + 105*c^2*d*x + 49*c*d^2*x^2 + 9*d^3*x^3) + 256*b^6*x^3*(35*c^3 + 84
*c^2*d*x + 70*c*d^2*x^2 + 20*d^3*x^3) + 128*a*b^5*x^2*(105*c^3 + 231*c^2*d
*x + 182*c*d^2*x^2 + 50*d^3*x^3)) + 630*a^5*d*(8*b^2*c^2 + a^2*d^2)*ArcTan
h[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])] + 420*a^4*b*c*(8*b^2*c^2 +
7*a^2*d^2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(35840*
b^(11/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.59, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1166, 27, 1225, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^2\right)^{3/2} (c+dx)^3 \, dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{\int \frac{1}{2}(c+dx)(c(14bc-5ad)+9d(2bc-ad)x)\left(bx^2+ax\right)^{3/2}dx}{7b}+\frac{d\left(ax+bx^2\right)^{5/2}(c+dx)^2}{7b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (c+dx)(c(14bc-5ad)+9d(2bc-ad)x)\left(bx^2+ax\right)^{3/2}dx}{14b}+\frac{d\left(ax+bx^2\right)^{5/2}(c+dx)^2}{7b}$$

$$\Big\downarrow \text{1225}$$

$$\frac{\frac{7(2bc-ad)\left(3a^2d^2-8abcd+8b^2c^2\right)\int \left(bx^2+ax\right)^{3/2}dx}{8b^2}+\frac{d\left(ax+bx^2\right)^{5/2}\left(21a^2d^2+30bdx(2bc-ad)-98abcd+128b^2c^2\right)}{20b^2}}{14b}+\frac{d\left(ax+bx^2\right)^{5/2}(c+dx)^2}{7b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\frac{7(2bc-ad)\left(3a^2d^2-8abcd+8b^2c^2\right)\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\int \sqrt{bx^2+ax}dx}{16b}\right)}{8b^2}+\frac{d\left(ax+bx^2\right)^{5/2}\left(21a^2d^2+30bdx(2bc-ad)-98abcd+128b^2c^2\right)}{20b^2}}{14b}+\frac{d\left(ax+bx^2\right)^{5/2}(c+dx)^2}{7b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\frac{7(2bc-ad)\left(3a^2d^2-8abcd+8b^2c^2\right)\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)}{16b}\right)}{8b^2}+\frac{d\left(ax+bx^2\right)^{5/2}\left(21a^2d^2+30bdx(2bc-\right.}{20b^2}}{14b}$$

$$\frac{d\left(ax+bx^2\right)^{5/2}(c+dx)^2}{7b}$$

$\Big\downarrow$ 1091

$$\frac{\frac{7(2bc-ad)\left(3a^2d^2-8abcd+8b^2c^2\right)\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)}{16b}\right)}{8b^2}+\frac{d\left(ax+bx^2\right)^{5/2}\left(21a^2d^2+30\right.}{}}{14b}$$

$$\frac{d\left(ax+bx^2\right)^{5/2}(c+dx)^2}{7b}$$

$\Big\downarrow$ 219

$$\frac{\frac{7\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)}{16b}\right)(2bc-ad)\left(3a^2d^2-8abcd+8b^2c^2\right)}{8b^2}+\frac{d\left(ax+bx^2\right)^{5/2}\left(21a^2d^2+30b\right.}{2}}{14b}$$

$$\frac{d\left(ax+bx^2\right)^{5/2}(c+dx)^2}{7b}$$

input

```
Int[(c + d*x)^3*(a*x + b*x^2)^(3/2),x]
```

output

```
(d*(c + d*x)^2*(a*x + b*x^2)^(5/2))/(7*b) + ((d*(128*b^2*c^2 - 98*a*b*c*d
+ 21*a^2*d^2 + 30*b*d*(2*b*c - a*d)*x)*(a*x + b*x^2)^(5/2))/(20*b^2) + (7*
(2*b*c - a*d)*(8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*(((a + 2*b*x)*(a*x + b*x
^2)^(3/2))/(8*b) - (3*a^2*(((a + 2*b*x)*Sqrt[a*x + b*x^2])/(4*b) - (a^2*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*b^(3/2))))/(16*b)))/(8*b^2))/(14*
b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$-\frac{9\left(\left(a^2d^2-\frac{8}{3}abcd+\frac{8}{3}b^2c^2\right)(ad-2bc)a^4\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)-\left(\frac{128\left(\frac{10}{21}d^3x^3+\frac{26}{15}c\,d^2x^2+\frac{11}{5}c^2dx+c^3\right)x^2a\,b^{\frac{11}{2}}}{3}+\frac{256x^3\left(\frac{4}{7}d^3\right.}{}\right.\right.}{}$
risch	$\frac{(5120b^6d^3x^6+6400a\,b^5d^3x^5+17920b^6c\,d^2x^5+128a^2b^4d^3x^4+23296a\,b^5c\,d^2x^4+21504b^6c^2d\,x^4-144a^3b^3d^3x^3+672a^2b^4c\,d^2}{}$
default	$c^3\left(\frac{(2bx+a)\left(b\,x^2+ax\right)^{\frac{3}{2}}}{8b}-\frac{3a^2\left(\frac{(2bx+a)\sqrt{b\,x^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8b^{\frac{3}{2}}}\right)}{16b}\right)+d^3\left(\frac{x^2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{7b}-\frac{9a\left(\right.}{}\right.$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-9/1024/b^(11/2)*((a^2*d^2-8/3*a*b*c*d+8/3*b^2*c^2)*(a*d-2*b*c)*a^4*arctan
h((x*(b*x+a))^(1/2)/x/b^(1/2))-(128/3*(10/21*d^3*x^3+26/15*c*d^2*x^2+11/5*
c^2*d*x+c^3)*x^2*a*b^(11/2)+256/9*x^3*(4/7*d^3*x^3+2*c*d^2*x^2+12/5*c^2*d*
x+c^3)*b^(13/2)+(-16/3*a*(3/35*d^3*x^3+7/15*c*d^2*x^2+c^2*d*x+c^3)*b^(7/2)
+32/9*x*(4/35*d^3*x^3+3/5*c*d^2*x^2+6/5*c^2*d*x+c^3)*b^(9/2)+d*((8/15*d^2*
x^2+28/9*c*d*x+8*c^2)*b^(5/2)+d*((-2/3*d*x-14/3*c)*b^(3/2)+b^(1/2)*a*d)*a)
*a^2)*a^2)*(x*(b*x+a))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 707, normalized size of antiderivative = 1.81

$$\int (c+dx)^3 \left(ax+bx^2\right)^{3/2}\,dx = \left[-\frac{105\left(16\,a^4b^3c^3-24\,a^5b^2c^2d+14\,a^6bcd^2-3\,a^7d^3\right)\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)-}{}\right.$$

$$-\frac{105\left(16\,a^4b^3c^3-24\,a^5b^2c^2d+14\,a^6bcd^2-3\,a^7d^3\right)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-\left(5120\,b^7d^3x^6-1680\,a^3b^4c^3\right.}{}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/71680*(105*(16*a^4*b^3*c^3 - 24*a^5*b^2*c^2*d + 14*a^6*b*c*d^2 - 3*a^7
*d^3)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*(5120*b^7*d
^3*x^6 - 1680*a^3*b^4*c^3 + 2520*a^4*b^3*c^2*d - 1470*a^5*b^2*c*d^2 + 315*
a^6*b*d^3 + 1280*(14*b^7*c*d^2 + 5*a*b^6*d^3)*x^5 + 128*(168*b^7*c^2*d + 1
82*a*b^6*c*d^2 + a^2*b^5*d^3)*x^4 + 16*(560*b^7*c^3 + 1848*a*b^6*c^2*d + 4
2*a^2*b^5*c*d^2 - 9*a^3*b^4*d^3)*x^3 + 56*(240*a*b^6*c^3 + 24*a^2*b^5*c^2*
d - 14*a^3*b^4*c*d^2 + 3*a^4*b^3*d^3)*x^2 + 70*(16*a^2*b^5*c^3 - 24*a^3*b^
4*c^2*d + 14*a^4*b^3*c*d^2 - 3*a^5*b^2*d^3)*x)*sqrt(b*x^2 + a*x))/b^6, -1/
35840*(105*(16*a^4*b^3*c^3 - 24*a^5*b^2*c^2*d + 14*a^6*b*c*d^2 - 3*a^7*d^3
)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (5120*b^7*d^3*x^
6 - 1680*a^3*b^4*c^3 + 2520*a^4*b^3*c^2*d - 1470*a^5*b^2*c*d^2 + 315*a^6*b
*d^3 + 1280*(14*b^7*c*d^2 + 5*a*b^6*d^3)*x^5 + 128*(168*b^7*c^2*d + 182*a*
b^6*c*d^2 + a^2*b^5*d^3)*x^4 + 16*(560*b^7*c^3 + 1848*a*b^6*c^2*d + 42*a^2
*b^5*c*d^2 - 9*a^3*b^4*d^3)*x^3 + 56*(240*a*b^6*c^3 + 24*a^2*b^5*c^2*d - 1
4*a^3*b^4*c*d^2 + 3*a^4*b^3*d^3)*x^2 + 70*(16*a^2*b^5*c^3 - 24*a^3*b^4*c^2
*d + 14*a^4*b^3*c*d^2 - 3*a^5*b^2*d^3)*x)*sqrt(b*x^2 + a*x))/b^6]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 864 vs. $2(388) = 776$.

Time = 0.51 (sec) , antiderivative size = 864, normalized size of antiderivative = 2.21

$$\int (c+dx)^3 \left(ax+bx^2\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3*(b*x**2+a*x)**(3/2),x)
```

output
```
Piecewise((3*a**2*(a**2*c**3 - 5*a*(3*a**2*c**2*d + 2*a*b*c**3 - 7*a*(3*a*
*2*c*d**2 + 6*a*b*c**2*d - 9*a*(a**2*d**3 + 6*a*b*c*d**2 - 11*a*(15*a*b*d*
*3/14 + 3*b**2*c*d**2)/(12*b) + 3*b**2*c**2*d)/(10*b) + b**2*c**3)/(8*b))/
(6*b))*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), N
e(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2), T
rue))/(8*b**2) + sqrt(a*x + b*x**2)*(-3*a*(a**2*c**3 - 5*a*(3*a**2*c**2*d
+ 2*a*b*c**3 - 7*a*(3*a**2*c*d**2 + 6*a*b*c**2*d - 9*a*(a**2*d**3 + 6*a*b*
c*d**2 - 11*a*(15*a*b*d**3/14 + 3*b**2*c*d**2)/(12*b) + 3*b**2*c**2*d)/(10
*b) + b**2*c**3)/(8*b))/(6*b))/(4*b**2) + b*d**3*x**6/7 + x**5*(15*a*b*d**
3/14 + 3*b**2*c*d**2)/(6*b) + x**4*(a**2*d**3 + 6*a*b*c*d**2 - 11*a*(15*a*
b*d**3/14 + 3*b**2*c*d**2)/(12*b) + 3*b**2*c**2*d)/(5*b) + x**3*(3*a**2*c*
d**2 + 6*a*b*c**2*d - 9*a*(a**2*d**3 + 6*a*b*c*d**2 - 11*a*(15*a*b*d**3/14
 + 3*b**2*c*d**2)/(12*b) + 3*b**2*c**2*d)/(10*b) + b**2*c**3)/(4*b) + x**2
*(3*a**2*c**2*d + 2*a*b*c**3 - 7*a*(3*a**2*c*d**2 + 6*a*b*c**2*d - 9*a*(a*
*2*d**3 + 6*a*b*c*d**2 - 11*a*(15*a*b*d**3/14 + 3*b**2*c*d**2)/(12*b) + 3*
b**2*c**2*d)/(10*b) + b**2*c**3)/(8*b))/(3*b) + x*(a**2*c**3 - 5*a*(3*a**2
*c**2*d + 2*a*b*c**3 - 7*a*(3*a**2*c*d**2 + 6*a*b*c**2*d - 9*a*(a**2*d**3
+ 6*a*b*c*d**2 - 11*a*(15*a*b*d**3/14 + 3*b**2*c*d**2)/(12*b) + 3*b**2*c**
2*d)/(10*b) + b**2*c**3)/(8*b))/(6*b))/(2*b)), Ne(b, 0)), (2*(c**3*(a*x)**
(5/2)/5 + 3*c**2*d*(a*x)**(7/2)/(7*a) + c*d**2*(a*x)**(9/2)/(3*a**2) + ...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.60

$$\int (c+dx)^3 \left(ax+bx^2\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
1/7*(b*x^2 + a*x)^(5/2)*d^3*x^2/b + 1/4*(b*x^2 + a*x)^(3/2)*c^3*x - 3/32*s
qrt(b*x^2 + a*x)*a^2*c^3*x/b + 9/64*sqrt(b*x^2 + a*x)*a^3*c^2*d*x/b^2 - 3/
8*(b*x^2 + a*x)^(3/2)*a*c^2*d*x/b - 21/256*sqrt(b*x^2 + a*x)*a^4*c*d^2*x/b
^3 + 7/32*(b*x^2 + a*x)^(3/2)*a^2*c*d^2*x/b^2 + 1/2*(b*x^2 + a*x)^(5/2)*c*
d^2*x/b + 9/512*sqrt(b*x^2 + a*x)*a^5*d^3*x/b^4 - 3/64*(b*x^2 + a*x)^(3/2)
*a^3*d^3*x/b^3 - 3/28*(b*x^2 + a*x)^(5/2)*a*d^3*x/b^2 + 3/128*a^4*c^3*log(
2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5/2) - 9/256*a^5*c^2*d*log(2*b
*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(7/2) + 21/1024*a^6*c*d^2*log(2*b*
x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(9/2) - 9/2048*a^7*d^3*log(2*b*x +
a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(11/2) - 3/64*sqrt(b*x^2 + a*x)*a^3*c^3
/b^2 + 1/8*(b*x^2 + a*x)^(3/2)*a*c^3/b + 9/128*sqrt(b*x^2 + a*x)*a^4*c^2*d
/b^3 - 3/16*(b*x^2 + a*x)^(3/2)*a^2*c^2*d/b^2 + 3/5*(b*x^2 + a*x)^(5/2)*c^
2*d/b - 21/512*sqrt(b*x^2 + a*x)*a^5*c*d^2/b^4 + 7/64*(b*x^2 + a*x)^(3/2)*
a^3*c*d^2/b^3 - 7/20*(b*x^2 + a*x)^(5/2)*a*c*d^2/b^2 + 9/1024*sqrt(b*x^2 +
 a*x)*a^6*d^3/b^5 - 3/128*(b*x^2 + a*x)^(3/2)*a^4*d^3/b^4 + 3/40*(b*x^2 +
a*x)^(5/2)*a^2*d^3/b^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.95

$$\int (c+dx)^3 \left(ax+bx^2\right)^{3/2} dx = \frac{1}{35840}\sqrt{bx^2+ax}\left(2\left(4\left(2\left(8\left(10\left(4\,bd^3x+\frac{14\,b^7cd^2+5\,ab^6d^3}{b^6}\right)x+\frac{168\,b^7c^2d+182\,ab^6c\cdots}{b^6}\right.\right.\right.\right.\right.$$
$$-\frac{3\left(16\,a^4b^3c^3-24\,a^5b^2c^2d+14\,a^6bcd^2-3\,a^7d^3\right)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{2048\,b^{\frac{11}{2}}}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
1/35840*sqrt(b*x^2 + a*x)*(2*(4*(2*(8*(10*(4*b*d^3*x + (14*b^7*c*d^2 + 5*a
*b^6*d^3)/b^6)*x + (168*b^7*c^2*d + 182*a*b^6*c*d^2 + a^2*b^5*d^3)/b^6)*x
+ (560*b^7*c^3 + 1848*a*b^6*c^2*d + 42*a^2*b^5*c*d^2 - 9*a^3*b^4*d^3)/b^6)
*x + 7*(240*a*b^6*c^3 + 24*a^2*b^5*c^2*d - 14*a^3*b^4*c*d^2 + 3*a^4*b^3*d^
3)/b^6)*x + 35*(16*a^2*b^5*c^3 - 24*a^3*b^4*c^2*d + 14*a^4*b^3*c*d^2 - 3*a
^5*b^2*d^3)/b^6)*x - 105*(16*a^3*b^4*c^3 - 24*a^4*b^3*c^2*d + 14*a^5*b^2*c
*d^2 - 3*a^6*b*d^3)/b^6) - 3/2048*(16*a^4*b^3*c^3 - 24*a^5*b^2*c^2*d + 14*
a^6*b*c*d^2 - 3*a^7*d^3)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b)
 + a))/b^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \left(ax+bx^2\right)^{3/2} \, dx = \int \left(b\,x^2+a\,x\right)^{3/2}\,{\left(c+d\,x\right)}^3 \,dx$$

input
```
int((a*x + b*x^2)^(3/2)*(c + d*x)^3,x)
```

output
```
int((a*x + b*x^2)^(3/2)*(c + d*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 26.38 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.48

$$\int (c+dx)^3 \left(ax+bx^2\right)^{3/2} \, dx = \frac{-1470\sqrt{x}\,\sqrt{bx+a}\,a^5b^2c\,d^2-210\sqrt{x}\,\sqrt{bx+a}\,a^5b^2d^3x+2520\sqrt{x}\,\sqrt{bx+a}\,a^4b^3c^2d+168\sqrt{x}}{}$$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(3/2),x)
```

output
```
(315*sqrt(x)*sqrt(a + b*x)*a**6*b*d**3 - 1470*sqrt(x)*sqrt(a + b*x)*a**5*b
**2*c*d**2 - 210*sqrt(x)*sqrt(a + b*x)*a**5*b**2*d**3*x + 2520*sqrt(x)*sqr
t(a + b*x)*a**4*b**3*c**2*d + 980*sqrt(x)*sqrt(a + b*x)*a**4*b**3*c*d**2*x
 + 168*sqrt(x)*sqrt(a + b*x)*a**4*b**3*d**3*x**2 - 1680*sqrt(x)*sqrt(a + b
*x)*a**3*b**4*c**3 - 1680*sqrt(x)*sqrt(a + b*x)*a**3*b**4*c**2*d*x - 784*s
qrt(x)*sqrt(a + b*x)*a**3*b**4*c*d**2*x**2 - 144*sqrt(x)*sqrt(a + b*x)*a**
3*b**4*d**3*x**3 + 1120*sqrt(x)*sqrt(a + b*x)*a**2*b**5*c**3*x + 1344*sqrt
(x)*sqrt(a + b*x)*a**2*b**5*c**2*d*x**2 + 672*sqrt(x)*sqrt(a + b*x)*a**2*b
**5*c*d**2*x**3 + 128*sqrt(x)*sqrt(a + b*x)*a**2*b**5*d**3*x**4 + 13440*sq
rt(x)*sqrt(a + b*x)*a*b**6*c**3*x**2 + 29568*sqrt(x)*sqrt(a + b*x)*a*b**6*
c**2*d*x**3 + 23296*sqrt(x)*sqrt(a + b*x)*a*b**6*c*d**2*x**4 + 6400*sqrt(x
)*sqrt(a + b*x)*a*b**6*d**3*x**5 + 8960*sqrt(x)*sqrt(a + b*x)*b**7*c**3*x*
*3 + 21504*sqrt(x)*sqrt(a + b*x)*b**7*c**2*d*x**4 + 17920*sqrt(x)*sqrt(a +
 b*x)*b**7*c*d**2*x**5 + 5120*sqrt(x)*sqrt(a + b*x)*b**7*d**3*x**6 - 315*s
qrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**7*d**3 + 1470*sqr
t(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**6*b*c*d**2 - 2520*s
qrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**5*b**2*c**2*d + 1
680*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4*b**3*c**3)
/(35840*b**6)
```

3.82 $\int \frac{(c+dx)^3(ax+bx^2)^{3/2}}{x} dx$

Optimal result

Integrand size = 24, antiderivative size = 349

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x} dx = \frac{a^2(64b^3c^3 - ad(72b^2c^2 - 36abcd + 7a^2d^2))\sqrt{ax+bx^2}}{512b^4} + \frac{7a(64b^3c^3 - ad(72b^2c^2 - 36abcd + 7a^2d^2))x\sqrt{ax+bx^2}}{768b^3} + \frac{(64b^3c^3 - ad(72b^2c^2 - 36abcd + 7a^2d^2))x^2\sqrt{ax+bx^2}}{192b^2} + \frac{d^2(36bc - 7ad)(ax+bx^2)^{5/2}}{60b^2} + \frac{d(72b^2c^2 - 36abcd + 7a^2d^2)(ax+bx^2)^{5/2}}{96b^3x} + \frac{d^3x(ax+bx^2)^{5/2}}{6b} - \frac{a^3(64b^3c^3 - ad(72b^2c^2 - 36abcd + 7a^2d^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{512b^{9/2}}$$

output
```
1/512*a^2*(64*b^3*c^3-a*d*(7*a^2*d^2-36*a*b*c*d+72*b^2*c^2))*(b*x^2+a*x)^(
1/2)/b^4+7/768*a*(64*b^3*c^3-a*d*(7*a^2*d^2-36*a*b*c*d+72*b^2*c^2))*x*(b*x
^2+a*x)^(1/2)/b^3+1/192*(64*b^3*c^3-a*d*(7*a^2*d^2-36*a*b*c*d+72*b^2*c^2))
*x^2*(b*x^2+a*x)^(1/2)/b^2+1/60*d^2*(-7*a*d+36*b*c)*(b*x^2+a*x)^(5/2)/b^2+
1/96*d*(7*a^2*d^2-36*a*b*c*d+72*b^2*c^2)*(b*x^2+a*x)^(5/2)/b^3/x+1/6*d^3*x
*(b*x^2+a*x)^(5/2)/b-1/512*a^3*(64*b^3*c^3-a*d*(7*a^2*d^2-36*a*b*c*d+72*b^
2*c^2))*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x} dx = \frac{\sqrt{x}\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\sqrt{a+bx}(-105a^5d^3+10a^4bd^2(54c+7dx)-8a^3b^2d(13\right.}{}$$

input
```
Integrate[((c + d*x)^3*(a*x + b*x^2)^(3/2))/x,x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-105*a^5*d^3 + 10*a
^4*b*d^2*(54*c + 7*d*x) - 8*a^3*b^2*d*(135*c^2 + 45*c*d*x + 7*d^2*x^2) + 4
8*a^2*b^3*(20*c^3 + 15*c^2*d*x + 6*c*d^2*x^2 + d^3*x^3) + 128*b^5*x^2*(20*
c^3 + 45*c^2*d*x + 36*c*d^2*x^2 + 10*d^3*x^3) + 64*a*b^4*x*(70*c^3 + 135*c
^2*d*x + 99*c*d^2*x^2 + 26*d^3*x^3)) + 120*a^3*b*c*(16*b^2*c^2 + 9*a^2*d^2
)*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])] + 30*a^4*d*(72*b^2*
c^2 + 7*a^2*d^2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(
7680*b^(9/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.71, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1262, 27, 2169, 27, 1221, 1131, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2} (c+dx)^3}{x} dx$$

$$\Big\downarrow \text{1262}$$

$$\frac{\int \frac{(bx^2+ax)^{3/2}(12bc^3+36bdxc^2+d^2(36bc-7ad)x^2)}{2x}dx}{6b}+\frac{d^3x(ax+bx^2)^{5/2}}{6b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(bx^2+ax)^{3/2}(12bc^3+36bdxc^2+d^2(36bc-7ad)x^2)}{x}dx}{12b}+\frac{d^3x(ax+bx^2)^{5/2}}{6b}$$

$\downarrow$ 2169

$$\frac{\frac{\int \frac{5\left(24b^2c^3+d\left(72b^2c^2-36abdc+7a^2d^2\right)x\right)\left(bx^2+ax\right)^{3/2}}{2x}dx}{5b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(36bc-7ad)}{5b}}{12b}+\frac{d^3x\left(ax+bx^2\right)^{5/2}}{6b}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{\left(24b^2c^3+d\left(72b^2c^2-36abdc+7a^2d^2\right)x\right)\left(bx^2+ax\right)^{3/2}}{x}dx}{2b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(36bc-7ad)}{5b}}{12b}+\frac{d^3x\left(ax+bx^2\right)^{5/2}}{6b}$$

$\downarrow$ 1221

$$\frac{\frac{\frac{3\left(-7a^3d^3+36a^2bcd^2-72ab^2c^2d+64b^3c^3\right)\int \frac{\left(bx^2+ax\right)^{3/2}}{x}dx}{8b}+\frac{d\left(ax+bx^2\right)^{5/2}\left(7a^2d^2-36abcd+72b^2c^2\right)}{4bx}}{2b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(36bc-7ad)}{5b}}{12b}+\frac{d^3x\left(ax+bx^2\right)^{5/2}}{6b}$$

$\downarrow$ 1131

$$\frac{\frac{\frac{3\left(-7a^3d^3+36a^2bcd^2-72ab^2c^2d+64b^3c^3\right)\left(\frac{1}{2}a\int \sqrt{bx^2+ax}dx+\frac{1}{3}\left(ax+bx^2\right)^{3/2}\right)}{8b}+\frac{d\left(ax+bx^2\right)^{5/2}\left(7a^2d^2-36abcd+72b^2c^2\right)}{4bx}}{2b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(36bc-7ad)}{5b}}{12b}+\frac{d^3x\left(ax+bx^2\right)^{5/2}}{6b}$$

$\downarrow$ 1087

$$\frac{\frac{\frac{3\left(-7a^3d^3+36a^2bcd^2-72ab^2c^2d+64b^3c^3\right)\left(\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)+\frac{1}{3}\left(ax+bx^2\right)^{3/2}\right)}{8b}+\frac{d\left(ax+bx^2\right)^{5/2}\left(7a^2d^2-36abcd+72b^2c^2\right)}{4bx}}{2b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(36bc-7ad)}{5b}}{12b}+\frac{d^3x\left(ax+bx^2\right)^{5/2}}{6b}$$

$\downarrow$ 1091

$$\frac{\frac{\frac{3\left(-7a^3d^3+36a^2bcd^2-72ab^2c^2d+64b^3c^3\right)\left(\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)+\frac{1}{3}\left(ax+bx^2\right)^{3/2}\right)}{8b}+\frac{d\left(ax+bx^2\right)^{5/2}\left(7a^2d^2-36abcd+72b^2c^2\right)}{4bx}}{2b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(36bc-7ad)}{5b}}{12b}+\frac{d^3x\left(ax+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{d\left(ax+bx^2\right)^{5/2}\left(7a^2d^2-36abcd+72b^2c^2\right)}{4bx}+\frac{3\left(\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)+\frac{1}{3}\left(ax+bx^2\right)^{3/2}\right)\left(-7a^3d^3+36a^2bcd^2-72ab^2c^2d+64b^3\right.}{8b}}{2b}}{12b}$$

$$\frac{d^3x\left(ax+bx^2\right)^{5/2}}{6b}$$

input
```
Int[((c + d*x)^3*(a*x + b*x^2)^(3/2))/x,x]
```

output
```
(d^3*x*(a*x + b*x^2)^(5/2))/(6*b) + ((d^2*(36*b*c - 7*a*d)*(a*x + b*x^2)^(
5/2))/(5*b) + ((d*(72*b^2*c^2 - 36*a*b*c*d + 7*a^2*d^2)*(a*x + b*x^2)^(5/2
))/(4*b*x) + (3*(64*b^3*c^3 - 72*a*b^2*c^2*d + 36*a^2*b*c*d^2 - 7*a^3*d^3)
*((a*x + b*x^2)^(3/2)/3 + (a*(((a + 2*b*x)*Sqrt[a*x + b*x^2])/(4*b) - (a^2
*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*b^(3/2))))/2))/(8*b))/(2*b))/(
12*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

rule 2169
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d -
b*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2
, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$\frac{\frac{7a^3\left(a^3d^3-\frac{36}{7}a^2bc\,d^2+\frac{72}{7}a\,b^2c^2d-\frac{64}{7}b^3c^3\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{512}-\frac{7\left(-\frac{512\left(\frac{1}{2}d^3x^3+\frac{9}{5}c\,d^2x^2+\frac{9}{4}c^2dx+c^3\right)x^2b^{\frac{11}{2}}}{21}+a\left(-\frac{64\left(\frac{1}{20}d^3x\right.}{}\right.\right.}{}}{}$
risch	$-\frac{\left(-1280b^5d^3x^5-1664a\,b^4d^3x^4-4608b^5c\,d^2x^4-48a^2b^3d^3x^3-6336a\,b^4c\,d^2x^3-5760b^5c^2d\,x^3+56a^3b^2d^3x^2-288a^2b^3c\,d^2x\right.}{}$
default	$c^3\left(\frac{\left(b\,x^2+ax\right)^{\frac{3}{2}}}{3}+\frac{a\left(\frac{(2bx+a)\sqrt{b\,x^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8b^{\frac{3}{2}}}\right)}{2}\right)+d^3\left(\frac{x\left(b\,x^2+ax\right)^{\frac{5}{2}}}{6b}-\frac{7a\left(\frac{\left(b\,x^2+ax\right)^{\frac{5}{2}}}{5b}-\right.}{}\right.$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
7/512*(a^3*(a^3*d^3-36/7*a^2*b*c*d^2+72/7*a*b^2*c^2*d-64/7*b^3*c^3)*arctan
h((x*(b*x+a))^(1/2)/x/b^(1/2))-(-512/21*(1/2*d^3*x^3+9/5*c*d^2*x^2+9/4*c^2
*d*x+c^3)*x^2*b^(11/2)+a*(-64/7*(1/20*d^3*x^3+3/10*c*d^2*x^2+3/4*c^2*d*x+c
^3)*a*b^(7/2)-128/3*x*(13/35*d^3*x^3+99/70*c*d^2*x^2+27/14*c^2*d*x+c^3)*b^
(9/2)+d*((8/15*d^2*x^2+24/7*c*d*x+72/7*c^2)*b^(5/2)+d*a*((-2/3*d*x-36/7*c)
*b^(3/2)+b^(1/2)*a*d))*a^2))*(x*(b*x+a))^(1/2))/b^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.72

$$\int \frac{(c+dx)^3\,(ax+bx^2)^{3/2}}{x}\,dx = \left[-\frac{15\,(64\,a^3b^3c^3-72\,a^4b^2c^2d+36\,a^5bcd^2-7\,a^6d^3)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{\right.$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x,x, algorithm="fricas")
```

output
```
[-1/15360*(15*(64*a^3*b^3*c^3 - 72*a^4*b^2*c^2*d + 36*a^5*b*c*d^2 - 7*a^6*
d^3)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*(1280*b^6*d^
3*x^5 + 960*a^2*b^4*c^3 - 1080*a^3*b^3*c^2*d + 540*a^4*b^2*c*d^2 - 105*a^5
*b*d^3 + 128*(36*b^6*c*d^2 + 13*a*b^5*d^3)*x^4 + 48*(120*b^6*c^2*d + 132*a
*b^5*c*d^2 + a^2*b^4*d^3)*x^3 + 8*(320*b^6*c^3 + 1080*a*b^5*c^2*d + 36*a^2
*b^4*c*d^2 - 7*a^3*b^3*d^3)*x^2 + 10*(448*a*b^5*c^3 + 72*a^2*b^4*c^2*d - 3
6*a^3*b^3*c*d^2 + 7*a^4*b^2*d^3)*x)*sqrt(b*x^2 + a*x))/b^5, 1/7680*(15*(64
*a^3*b^3*c^3 - 72*a^4*b^2*c^2*d + 36*a^5*b*c*d^2 - 7*a^6*d^3)*sqrt(-b)*arc
tan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (1280*b^6*d^3*x^5 + 960*a^2*b^
4*c^3 - 1080*a^3*b^3*c^2*d + 540*a^4*b^2*c*d^2 - 105*a^5*b*d^3 + 128*(36*b
^6*c*d^2 + 13*a*b^5*d^3)*x^4 + 48*(120*b^6*c^2*d + 132*a*b^5*c*d^2 + a^2*b
^4*d^3)*x^3 + 8*(320*b^6*c^3 + 1080*a*b^5*c^2*d + 36*a^2*b^4*c*d^2 - 7*a^3
*b^3*d^3)*x^2 + 10*(448*a*b^5*c^3 + 72*a^2*b^4*c^2*d - 36*a^3*b^3*c*d^2 +
7*a^4*b^2*d^3)*x)*sqrt(b*x^2 + a*x))/b^5]
```

Sympy [A] (verification not implemented)

Time = 3.98 (sec) , antiderivative size = 1129, normalized size of antiderivative = 3.23

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x} dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3*(b*x**2+a*x)**(3/2)/x,x)
```

output
```
a*c**3*Piecewise((-a**2*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) +
2*b*x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/
(2*b) + x)**2), True))/(8*b) + (a/(4*b) + x/2)*sqrt(a*x + b*x**2), Ne(b, 0
)), (2*(a*x)**(3/2)/(3*a), Ne(a, 0)), (0, True)) + 3*a*c**2*d*Piecewise((a
**3*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(a
**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2), True
))/(16*b**2) + sqrt(a*x + b*x**2)*(-a**2/(8*b**2) + a*x/(12*b) + x**2/3),
Ne(b, 0)), (2*(a*x)**(5/2)/(5*a**2), Ne(a, 0)), (0, True)) + 3*a*c*d**2*Pi
ecewise((-5*a**4*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/
sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) +
 x)**2), True))/(128*b**3) + sqrt(a*x + b*x**2)*(5*a**3/(64*b**3) - 5*a**2
*x/(96*b**2) + a*x**2/(24*b) + x**3/4), Ne(b, 0)), (2*(a*x)**(7/2)/(7*a**3
), Ne(a, 0)), (0, True)) + a*d**3*Piecewise((7*a**5*Piecewise((log(a + 2*s
qrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)
*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2), True))/(256*b**4) + sqrt(a*x +
 b*x**2)*(-7*a**4/(128*b**4) + 7*a**3*x/(192*b**3) - 7*a**2*x**2/(240*b**2
) + a*x**3/(40*b) + x**4/5), Ne(b, 0)), (2*(a*x)**(9/2)/(9*a**4), Ne(a, 0)
), (0, True)) + b*c**3*Piecewise((a**3*Piecewise((log(a + 2*sqrt(b)*sqrt(a
*x + b*x**2) + 2*b*x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b)
+ x)/sqrt(b*(a/(2*b) + x)**2), True))/(16*b**2) + sqrt(a*x + b*x**2)*(-...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.50

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x}\,dx = \frac{1}{4}\sqrt{bx^2+ax}ac^3x + \frac{3}{4}(bx^2+ax)^{\frac{3}{2}}c^2dx - \frac{9\sqrt{bx^2+ax}a^2c^2dx}{32\,b} + \frac{9\sqrt{bx^2+ax}a^3cd^2x}{64\,b^2} - \frac{3(bx^2+ax)^{\frac{3}{2}}acd^2x}{8\,b} - \frac{7\sqrt{bx^2+ax}a^4d^3x}{256\,b^3} + \frac{7(bx^2+ax)^{\frac{3}{2}}a^2d^3x}{96\,b^2} + \frac{(bx^2+ax)^{\frac{5}{2}}d^3x}{6\,b} - \frac{a^3c^3\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{3}{2}}} + \frac{9\,a^4c^2d\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{5}{2}}} - \frac{9\,a^5cd^2\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{256\,b^{\frac{7}{2}}} + \frac{7\,a^6d^3\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{1024\,b^{\frac{9}{2}}} + \frac{1}{3}(bx^2+ax)^{\frac{3}{2}}c^3 + \frac{\sqrt{bx^2+ax}a^2c^3}{8\,b} - \frac{9\sqrt{bx^2+ax}a^3c^2d}{64\,b^2} + \frac{3(bx^2+ax)^{\frac{3}{2}}ac^2d}{8\,b} + \frac{9\sqrt{bx^2+ax}a^4cd^2}{128\,b^3} - \frac{3(bx^2+ax)^{\frac{3}{2}}a^2cd^2}{16\,b^2} + \frac{3(bx^2+ax)^{\frac{5}{2}}cd^2}{5\,b} - \frac{7\sqrt{bx^2+ax}a^5d^3}{512\,b^4} + \frac{7(bx^2+ax)^{\frac{3}{2}}a^3d^3}{192\,b^3} - \frac{7(bx^2+ax)^{\frac{5}{2}}ad^3}{60\,b^2}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x,x, algorithm="maxima")
```

output
```
1/4*sqrt(b*x^2 + a*x)*a*c^3*x + 3/4*(b*x^2 + a*x)^(3/2)*c^2*d*x - 9/32*sqr
t(b*x^2 + a*x)*a^2*c^2*d*x/b + 9/64*sqrt(b*x^2 + a*x)*a^3*c*d^2*x/b^2 - 3/
8*(b*x^2 + a*x)^(3/2)*a*c*d^2*x/b - 7/256*sqrt(b*x^2 + a*x)*a^4*d^3*x/b^3
+ 7/96*(b*x^2 + a*x)^(3/2)*a^2*d^3*x/b^2 + 1/6*(b*x^2 + a*x)^(5/2)*d^3*x/b
 - 1/16*a^3*c^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(3/2) + 9/1
28*a^4*c^2*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5/2) - 9/256*
a^5*c*d^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(7/2) + 7/1024*a^
6*d^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(9/2) + 1/3*(b*x^2 +
a*x)^(3/2)*c^3 + 1/8*sqrt(b*x^2 + a*x)*a^2*c^3/b - 9/64*sqrt(b*x^2 + a*x)*
a^3*c^2*d/b^2 + 3/8*(b*x^2 + a*x)^(3/2)*a*c^2*d/b + 9/128*sqrt(b*x^2 + a*x
)*a^4*c*d^2/b^3 - 3/16*(b*x^2 + a*x)^(3/2)*a^2*c*d^2/b^2 + 3/5*(b*x^2 + a*
x)^(5/2)*c*d^2/b - 7/512*sqrt(b*x^2 + a*x)*a^5*d^3/b^4 + 7/192*(b*x^2 + a*
x)^(3/2)*a^3*d^3/b^3 - 7/60*(b*x^2 + a*x)^(5/2)*a*d^3/b^2
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx)^3\left(ax+bx^2\right)^{3/2}}{x}\,dx = \frac{1}{7680}\sqrt{bx^2+ax}\left(2\left(4\left(2\left(8\left(10\,bd^3x+\frac{36\,b^6cd^2+13\,ab^5d^3}{b^5}\right)x+\frac{3\,(120\,\cdots\right.\right.\right.\right.$$
$$+\frac{\left(64\,a^3b^3c^3-72\,a^4b^2c^2d+36\,a^5bcd^2-7\,a^6d^3\right)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{1024\,b^{\frac{9}{2}}}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x,x, algorithm="giac")
```

output
```
1/7680*sqrt(b*x^2 + a*x)*(2*(4*(2*(8*(10*b*d^3*x + (36*b^6*c*d^2 + 13*a*b^
5*d^3)/b^5)*x + 3*(120*b^6*c^2*d + 132*a*b^5*c*d^2 + a^2*b^4*d^3)/b^5)*x +
 (320*b^6*c^3 + 1080*a*b^5*c^2*d + 36*a^2*b^4*c*d^2 - 7*a^3*b^3*d^3)/b^5)*
x + 5*(448*a*b^5*c^3 + 72*a^2*b^4*c^2*d - 36*a^3*b^3*c*d^2 + 7*a^4*b^2*d^3
)/b^5)*x + 15*(64*a^2*b^4*c^3 - 72*a^3*b^3*c^2*d + 36*a^4*b^2*c*d^2 - 7*a^
5*b*d^3)/b^5) + 1/1024*(64*a^3*b^3*c^3 - 72*a^4*b^2*c^2*d + 36*a^5*b*c*d^2
 - 7*a^6*d^3)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(9
/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x} \, dx = \int \frac{(b\,x^2+a\,x)^{3/2}\,(c+d\,x)^3}{x} \, dx$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^3)/x,x)
```

output
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^3)/x, x)
```

Reduce [B] (verification not implemented)

Time = 10.27 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.40

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x} \, dx = \frac{-105\sqrt{x}\,\sqrt{bx+a}\,a^5 b\,d^3 + 540\sqrt{x}\,\sqrt{bx+a}\,a^4 b^2 c\,d^2 + 70\sqrt{x}\,\sqrt{bx+a}\,a^4 b^2 d$$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(3/2)/x,x)
```

output
```
( - 105*sqrt(x)*sqrt(a + b*x)*a**5*b*d**3 + 540*sqrt(x)*sqrt(a + b*x)*a**4
*b**2*c*d**2 + 70*sqrt(x)*sqrt(a + b*x)*a**4*b**2*d**3*x - 1080*sqrt(x)*sq
rt(a + b*x)*a**3*b**3*c**2*d - 360*sqrt(x)*sqrt(a + b*x)*a**3*b**3*c*d**2*
x - 56*sqrt(x)*sqrt(a + b*x)*a**3*b**3*d**3*x**2 + 960*sqrt(x)*sqrt(a + b*
x)*a**2*b**4*c**3 + 720*sqrt(x)*sqrt(a + b*x)*a**2*b**4*c**2*d*x + 288*sqr
t(x)*sqrt(a + b*x)*a**2*b**4*c*d**2*x**2 + 48*sqrt(x)*sqrt(a + b*x)*a**2*b
**4*d**3*x**3 + 4480*sqrt(x)*sqrt(a + b*x)*a*b**5*c**3*x + 8640*sqrt(x)*sq
rt(a + b*x)*a*b**5*c**2*d*x**2 + 6336*sqrt(x)*sqrt(a + b*x)*a*b**5*c*d**2*
x**3 + 1664*sqrt(x)*sqrt(a + b*x)*a*b**5*d**3*x**4 + 2560*sqrt(x)*sqrt(a +
 b*x)*b**6*c**3*x**2 + 5760*sqrt(x)*sqrt(a + b*x)*b**6*c**2*d*x**3 + 4608*
sqrt(x)*sqrt(a + b*x)*b**6*c*d**2*x**4 + 1280*sqrt(x)*sqrt(a + b*x)*b**6*d
**3*x**5 + 105*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**6
*d**3 - 540*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**5*b*
c*d**2 + 1080*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4*
b**2*c**2*d - 960*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a
**3*b**3*c**3)/(7680*b**5)
```

3.83 $\int \frac{(c+dx)^3(ax+bx^2)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 273

$$\int \frac{(c+dx)^3(ax+bx^2)^{3/2}}{x^2}\,dx = \frac{5a(4bc-ad)(8b^2c^2-2abcd+a^2d^2)\sqrt{ax+bx^2}}{128b^3} + \frac{(4bc-ad)(8b^2c^2-2abcd+a^2d^2)x\sqrt{ax+bx^2}}{64b^2} + \frac{d^3(ax+bx^2)^{5/2}}{5b} + \frac{d(16b^2c^2-6abcd+a^2d^2)(ax+bx^2)^{5/2}}{16b^3x^2} + \frac{d^2(6bc-ad)(ax+bx^2)^{5/2}}{8b^2x} + \frac{3a^2(4bc-ad)(8b^2c^2-2abcd+a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{128b^{7/2}}$$

output

```
5/128*a*(-a*d+4*b*c)*(a^2*d^2-2*a*b*c*d+8*b^2*c^2)*(b*x^2+a*x)^(1/2)/b^3+1
/64*(-a*d+4*b*c)*(a^2*d^2-2*a*b*c*d+8*b^2*c^2)*x*(b*x^2+a*x)^(1/2)/b^2+1/5
*d^3*(b*x^2+a*x)^(5/2)/b+1/16*d*(a^2*d^2-6*a*b*c*d+16*b^2*c^2)*(b*x^2+a*x)
^(5/2)/b^3/x^2+1/8*d^2*(-a*d+6*b*c)*(b*x^2+a*x)^(5/2)/b^2/x+3/128*a^2*(-a*
d+4*b*c)*(a^2*d^2-2*a*b*c*d+8*b^2*c^2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2)
)/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx)^3 \left(ax+bx^2\right)^{3/2}}{x^2}\,dx = \frac{\sqrt{x(a+bx)}\left(\sqrt{b}(15a^4d^3-10a^3bd^2(9c+dx)+4a^2b^2d(60c^2+15cdx+2d^2\right.}{}$$

input
```
Integrate[((c + d*x)^3*(a*x + b*x^2)^(3/2))/x^2,x]
```

output
```
(Sqrt[x*(a + b*x)]*(Sqrt[b]*(15*a^4*d^3 - 10*a^3*b*d^2*(9*c + d*x) + 4*a^2
*b^2*d*(60*c^2 + 15*c*d*x + 2*d^2*x^2) + 32*b^4*x*(10*c^3 + 20*c^2*d*x + 1
5*c*d^2*x^2 + 4*d^3*x^3) + 16*a*b^3*(50*c^3 + 70*c^2*d*x + 45*c*d^2*x^2 +
11*d^3*x^3)) + (15*a^2*(-32*b^3*c^3 + 16*a*b^2*c^2*d - 6*a^2*b*c*d^2 + a^3
*d^3)*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[x]*Sqrt[a + b*x])))/(
640*b^(7/2))
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.81, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1262, 27, 2169, 27, 1220, 1131, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{3/2}(c+dx)^3}{x^2}dx$$

$$\Big\downarrow \; 1262$$

$$\frac{\int \frac{5\left(bx^2+ax\right)^{3/2}\left(2bc^3+6bdxc^2+d^2(6bc-ad)x^2\right)}{2x^2}dx}{5b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(bx^2+ax\right)^{3/2}\left(2bc^3+6bdxc^2+d^2(6bc-ad)x^2\right)}{x^2}dx}{2b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \; 2169$$

$$\frac{\frac{\int \frac{\left(16b^2c^3+3d\left(16b^2c^2-6abdc+a^2d^2\right)x\right)\left(bx^2+ax\right)^{3/2}}{2x^2}dx}{4b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(6bc-ad)}{4bx}}{2b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(16b^2c^3+3d\left(16b^2c^2-6abdc+a^2d^2\right)x\right)\left(bx^2+ax\right)^{3/2}}{x^2}dx}{8b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(6bc-ad)}{4bx}}{2b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \; 1220$$

$$\frac{\frac{\frac{32b^2c^3\left(ax+bx^2\right)^{5/2}}{ax^2}-\frac{3(4bc-ad)\left(a^2d^2-2abcd+8b^2c^2\right)\int \frac{\left(bx^2+ax\right)^{3/2}}{x}dx}{a}}{8b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(6bc-ad)}{4bx}}{2b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \; 1131$$

$$\frac{\frac{\frac{32b^2c^3\left(ax+bx^2\right)^{5/2}}{ax^2}-\frac{3(4bc-ad)\left(a^2d^2-2abcd+8b^2c^2\right)\left(\frac{1}{2}a\int \sqrt{bx^2+ax}dx+\frac{1}{3}\left(ax+bx^2\right)^{3/2}\right)}{a}}{8b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(6bc-ad)}{4bx}}{2b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \; 1087$$

$$\frac{\frac{\frac{32b^2c^3\left(ax+bx^2\right)^{5/2}}{ax^2}-\frac{3(4bc-ad)\left(a^2d^2-2abcd+8b^2c^2\right)\left(\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)+\frac{1}{3}\left(ax+bx^2\right)^{3/2}\right)}{a}}{8b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(6bc-ad)}{4bx}}{2b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \; 1091$$

$$\frac{\frac{\frac{32b^2c^3\left(ax+bx^2\right)^{5/2}}{ax^2}-\frac{3(4bc-ad)\left(a^2d^2-2abcd+8b^2c^2\right)\left(\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)+\frac{1}{3}\left(ax+bx^2\right)^{3/2}\right)}{a}}{8b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(6}{4bx}}{2b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{32b^2c^3\left(ax+bx^2\right)^{5/2}}{ax^2}-\frac{3\left(\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)+\frac{1}{3}\left(ax+bx^2\right)^{3/2}\right)(4bc-ad)\left(a^2d^2-2abcd+8b^2c^2\right)}{a}}{8b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(6b}{4bx}}{2b}$$

$$\frac{d^3\left(ax+bx^2\right)^{5/2}}{5b}$$

input
```
Int[((c + d*x)^3*(a*x + b*x^2)^(3/2))/x^2,x]
```

output
```
(d^3*(a*x + b*x^2)^(5/2))/(5*b) + ((d^2*(6*b*c - a*d)*(a*x + b*x^2)^(5/2))
/(4*b*x) + ((32*b^2*c^3*(a*x + b*x^2)^(5/2))/(a*x^2) - (3*(4*b*c - a*d)*(8
*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((a*x + b*x^2)^(3/2)/3 + (a*(((a + 2*b*x)*
Sqrt[a*x + b*x^2])/(4*b) - (a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4
*b^(3/2))))/2))/a)/(8*b))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

rule 2169
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d -
b*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2
, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$-\frac{3\left(\left(a^{5}d^{3}-6a^{4}bc\,d^{2}+16a^{3}b^{2}c^{2}d-32a^{2}b^{3}c^{3}\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)-\left(\frac{160\left(\frac{11}{50}d^{3}x^{3}+\frac{9}{10}c\,d^{2}x^{2}+\frac{7}{5}c^{2}dx+c^{3}\right)a\,b^{\frac{7}{2}}}{3}+\frac{64x\left(\frac{2}{5}d^{3} \cdots\right.}{\cdots}\right.\right.}{128b^{\frac{7}{2}}}$
risch	$\frac{(128b^{4}d^{3}x^{4}+176a\,b^{3}d^{3}x^{3}+480b^{4}c\,d^{2}x^{3}+8a^{2}b^{2}d^{3}x^{2}+720a\,b^{3}c\,d^{2}x^{2}+640b^{4}c^{2}d\,x^{2}-10a^{3}b\,d^{3}x+60a^{2}b^{2}c\,d^{2}x+1120a\,b^{3}c^{2}dx \cdots}{640b^{3}\sqrt{x(bx+a)}}$
default	$c^{3}\left(\frac{2\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{a\,x^{2}}-\frac{6b\left(\frac{\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{3}+\frac{a\left(\frac{(2bx+a)\sqrt{b\,x^{2}+ax}}{4b}-\frac{a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8b^{\frac{3}{2}}}\right)}{2}\right)}{a}\right)+d^{3}\left(\frac{\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{5b} \cdots\right.$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-3/128*((a^5*d^3-6*a^4*b*c*d^2+16*a^3*b^2*c^2*d-32*a^2*b^3*c^3)*arctanh((x
*(b*x+a))^(1/2)/x/b^(1/2))-(160/3*(11/50*d^3*x^3+9/10*c*d^2*x^2+7/5*c^2*d*
x+c^3)*a*b^(7/2)+64/3*x*(2/5*d^3*x^3+3/2*c*d^2*x^2+2*c^2*d*x+c^3)*b^(9/2)+
d*((8/15*d^2*x^2+4*c*d*x+16*c^2)*b^(5/2)+d*((-2/3*d*x-6*c)*b^(3/2)+b^(1/2)
*a*d)*a)*a^2)*(x*(b*x+a))^(1/2))/b^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.83

$$\int \frac{(c+dx)^3\,(ax+bx^2)^{3/2}}{x^2}\,dx = \left[-\frac{15\,(32\,a^2b^3c^3-16\,a^3b^2c^2d+6\,a^4bcd^2-a^5d^3)\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2 \cdots}\right.}{\cdots}\right.$$

$$-\frac{15\,(32\,a^2b^3c^3-16\,a^3b^2c^2d+6\,a^4bcd^2-a^5d^3)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(128\,b^5d^3x^4+800\,ab^4c^3+240 \cdots}{\cdots}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
[-1/1280*(15*(32*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 6*a^4*b*c*d^2 - a^5*d^3)
*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*(128*b^5*d^3*x^4
 + 800*a*b^4*c^3 + 240*a^2*b^3*c^2*d - 90*a^3*b^2*c*d^2 + 15*a^4*b*d^3 + 1
6*(30*b^5*c*d^2 + 11*a*b^4*d^3)*x^3 + 8*(80*b^5*c^2*d + 90*a*b^4*c*d^2 + a
^2*b^3*d^3)*x^2 + 10*(32*b^5*c^3 + 112*a*b^4*c^2*d + 6*a^2*b^3*c*d^2 - a^3
*b^2*d^3)*x)*sqrt(b*x^2 + a*x))/b^4, -1/640*(15*(32*a^2*b^3*c^3 - 16*a^3*b
^2*c^2*d + 6*a^4*b*c*d^2 - a^5*d^3)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt
(-b)/(b*x + a)) - (128*b^5*d^3*x^4 + 800*a*b^4*c^3 + 240*a^2*b^3*c^2*d - 9
0*a^3*b^2*c*d^2 + 15*a^4*b*d^3 + 16*(30*b^5*c*d^2 + 11*a*b^4*d^3)*x^3 + 8*
(80*b^5*c^2*d + 90*a*b^4*c*d^2 + a^2*b^3*d^3)*x^2 + 10*(32*b^5*c^3 + 112*a
*b^4*c^2*d + 6*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x)*sqrt(b*x^2 + a*x))/b^4]
```

Sympy [F]

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^2}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}} (c+dx)^3}{x^2}\,dx$$

input
```
integrate((d*x+c)**3*(b*x**2+a*x)**(3/2)/x**2,x)
```

output
```
Integral((x*(a + b*x))**(3/2)*(c + d*x)**3/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.56

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^2}\,dx = \frac{3}{4}\sqrt{bx^2+ax}ac^2dx + \frac{3}{4}\left(bx^2+ax\right)^{\frac{3}{2}}cd^2x - \frac{9\sqrt{bx^2+ax}a^2cd^2x}{32\,b} + \frac{3\sqrt{bx^2+ax}a^3d^3x}{64\,b^2} - \frac{\left(bx^2+ax\right)^{\frac{3}{2}}ad^3x}{8\,b} + \frac{3\,a^2c^3\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{8\sqrt{b}} - \frac{3\,a^3c^2d\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{3}{2}}} + \frac{9\,a^4cd^2\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{5}{2}}} - \frac{3\,a^5d^3\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{256\,b^{\frac{7}{2}}} + \frac{3}{4}\sqrt{bx^2+ax}ac^3 + \left(bx^2+ax\right)^{\frac{3}{2}}c^2d + \frac{3\sqrt{bx^2+ax}a^2c^2d}{8\,b} - \frac{9\sqrt{bx^2+ax}a^3cd^2}{64\,b^2} + \frac{3\left(bx^2+ax\right)^{\frac{3}{2}}acd^2}{8\,b} + \frac{3\sqrt{bx^2+ax}a^4d^3}{128\,b^3} - \frac{\left(bx^2+ax\right)^{\frac{3}{2}}a^2d^3}{16\,b^2} + \frac{\left(bx^2+ax\right)^{\frac{5}{2}}d^3}{5\,b} + \frac{\left(bx^2+ax\right)^{\frac{3}{2}}c^3}{2\,x}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
3/4*sqrt(b*x^2 + a*x)*a*c^2*d*x + 3/4*(b*x^2 + a*x)^(3/2)*c*d^2*x - 9/32*s
qrt(b*x^2 + a*x)*a^2*c*d^2*x/b + 3/64*sqrt(b*x^2 + a*x)*a^3*d^3*x/b^2 - 1/
8*(b*x^2 + a*x)^(3/2)*a*d^3*x/b + 3/8*a^2*c^3*log(2*b*x + a + 2*sqrt(b*x^2
 + a*x)*sqrt(b))/sqrt(b) - 3/16*a^3*c^2*d*log(2*b*x + a + 2*sqrt(b*x^2 + a
*x)*sqrt(b))/b^(3/2) + 9/128*a^4*c*d^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)
*sqrt(b))/b^(5/2) - 3/256*a^5*d^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt
(b))/b^(7/2) + 3/4*sqrt(b*x^2 + a*x)*a*c^3 + (b*x^2 + a*x)^(3/2)*c^2*d + 3
/8*sqrt(b*x^2 + a*x)*a^2*c^2*d/b - 9/64*sqrt(b*x^2 + a*x)*a^3*c*d^2/b^2 +
3/8*(b*x^2 + a*x)^(3/2)*a*c*d^2/b + 3/128*sqrt(b*x^2 + a*x)*a^4*d^3/b^3 -
1/16*(b*x^2 + a*x)^(3/2)*a^2*d^3/b^2 + 1/5*(b*x^2 + a*x)^(5/2)*d^3/b + 1/2
*(b*x^2 + a*x)^(3/2)*c^3/x
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.96

$$\int \frac{(c+dx)^3 \left(ax+bx^2\right)^{3/2}}{x^2}\,dx = \frac{1}{640}\sqrt{bx^2+ax}\left(2\left(4\left(2\left(8\,bd^3x+\frac{30\,b^5cd^2+11\,ab^4d^3}{b^4}\right)x+\frac{80\,b^5c^2d+9\cdots}{\cdots}\right.\right.\right.$$
$$-\frac{3\left(32\,a^2b^3c^3-16\,a^3b^2c^2d+6\,a^4bcd^2-a^5d^3\right)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{256\,b^{\frac{7}{2}}}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^2,x, algorithm="giac")
```

output
```
1/640*sqrt(b*x^2 + a*x)*(2*(4*(2*(8*b*d^3*x + (30*b^5*c*d^2 + 11*a*b^4*d^3
)/b^4)*x + (80*b^5*c^2*d + 90*a*b^4*c*d^2 + a^2*b^3*d^3)/b^4)*x + 5*(32*b^
5*c^3 + 112*a*b^4*c^2*d + 6*a^2*b^3*c*d^2 - a^3*b^2*d^3)/b^4)*x + 5*(160*a
*b^4*c^3 + 48*a^2*b^3*c^2*d - 18*a^3*b^2*c*d^2 + 3*a^4*b*d^3)/b^4) - 3/256
*(32*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 6*a^4*b*c*d^2 - a^5*d^3)*log(abs(2*(
sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3 \left(ax+bx^2\right)^{3/2}}{x^2}\,dx = \int \frac{{\left(b\,x^2+a\,x\right)}^{3/2}\,{\left(c+d\,x\right)}^3}{x^2}\,dx$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^3)/x^2,x)
```

output
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^3)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.47

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^2} dx = \frac{15\sqrt{x}\,\sqrt{bx+a}\,a^4b\,d^3 - 90\sqrt{x}\,\sqrt{bx+a}\,a^3b^2c\,d^2 - 10\sqrt{x}\,\sqrt{bx+a}\,a^3b^2d^3x +}{}$$

input

```
int((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^2,x)
```

output

```
(15*sqrt(x)*sqrt(a + b*x)*a**4*b*d**3 - 90*sqrt(x)*sqrt(a + b*x)*a**3*b**2
*c*d**2 - 10*sqrt(x)*sqrt(a + b*x)*a**3*b**2*d**3*x + 240*sqrt(x)*sqrt(a +
 b*x)*a**2*b**3*c**2*d + 60*sqrt(x)*sqrt(a + b*x)*a**2*b**3*c*d**2*x + 8*s
qrt(x)*sqrt(a + b*x)*a**2*b**3*d**3*x**2 + 800*sqrt(x)*sqrt(a + b*x)*a*b**
4*c**3 + 1120*sqrt(x)*sqrt(a + b*x)*a*b**4*c**2*d*x + 720*sqrt(x)*sqrt(a +
 b*x)*a*b**4*c*d**2*x**2 + 176*sqrt(x)*sqrt(a + b*x)*a*b**4*d**3*x**3 + 32
0*sqrt(x)*sqrt(a + b*x)*b**5*c**3*x + 640*sqrt(x)*sqrt(a + b*x)*b**5*c**2*
d*x**2 + 480*sqrt(x)*sqrt(a + b*x)*b**5*c*d**2*x**3 + 128*sqrt(x)*sqrt(a +
 b*x)*b**5*d**3*x**4 - 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sq
rt(a))*a**5*d**3 + 90*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a
))*a**4*b*c*d**2 - 240*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(
a))*a**3*b**2*c**2*d + 480*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/s
qrt(a))*a**2*b**3*c**3)/(640*b**4)
```

3.84 $\int \frac{(c+dx)^3(ax+bx^2)^{3/2}}{x^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 256

$$\begin{aligned}\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^3} dx = &-\frac{5}{64}\left(\frac{a^2d^2(8bc-ad)}{b^2}-16c^2(4bc+3ad)\right)\sqrt{ax+bx^2}\\ &+\frac{1}{32}\left(\frac{64b^2c^3}{a}+48bc^2d-8acd^2+\frac{a^2d^3}{b}\right)x\sqrt{ax+bx^2}\\ &-\frac{2c^3(ax+bx^2)^{5/2}}{ax^3}+\frac{d^2(8bc-ad)(ax+bx^2)^{5/2}}{8b^2x^2}+\frac{d^3(ax+bx^2)^{5/2}}{4bx}\\ &-\frac{3a(a^2d^2(8bc-ad)-16b^2c^2(4bc+3ad))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{64b^{5/2}}\end{aligned}$$

output
```
-5/64*(a^2*d^2*(-a*d+8*b*c)/b^2-16*c^2*(3*a*d+4*b*c))*(b*x^2+a*x)^(1/2)+1/
32*(64*b^2*c^3/a+48*b*c^2*d-8*a*c*d^2+a^2*d^3/b)*x*(b*x^2+a*x)^(1/2)-2*c^3
*(b*x^2+a*x)^(5/2)/a/x^3+1/8*d^2*(-a*d+8*b*c)*(b*x^2+a*x)^(5/2)/b^2/x^2+1/
4*d^3*(b*x^2+a*x)^(5/2)/b/x-3/64*a*(a^2*d^2*(-a*d+8*b*c)-16*b^2*c^2*(3*a*d
+4*b*c))*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx)^3\left(ax+bx^2\right)^{3/2}}{x^3}\,dx = \frac{(x(a+bx))^{3/2}\left(-128ab^2c^3+64b^3c^3x+240ab^2c^2dx+24a^2bcd^2x-3a^3d^3x\right.}{64b^2x}$$
$$+\frac{3a\left(64b^3c^3+48ab^2c^2d-8a^2bcd^2+a^3d^3\right)(x(a+bx))^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{32b^{5/2}x^{3/2}(a+bx)^{3/2}}$$

input
```
Integrate[((c + d*x)^3*(a*x + b*x^2)^(3/2))/x^3,x]
```

output
```
((x*(a + b*x))^(3/2)*(-128*a*b^2*c^3 + 64*b^3*c^3*x + 240*a*b^2*c^2*d*x +
24*a^2*b*c*d^2*x - 3*a^3*d^3*x + 96*b^3*c^2*d*x^2 + 112*a*b^2*c*d^2*x^2 +
2*a^2*b*d^3*x^2 + 64*b^3*c*d^2*x^3 + 24*a*b^2*d^3*x^3 + 16*b^3*d^3*x^4))/(
64*b^2*x^2*(a + b*x)) + (3*a*(64*b^3*c^3 + 48*a*b^2*c^2*d - 8*a^2*b*c*d^2
+ a^3*d^3)*(x*(a + b*x))^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[
a + b*x])])/(32*b^(5/2)*x^(3/2)*(a + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.82, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1262, 27, 2169, 27, 1220, 1131, 1131, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{3/2}(c+dx)^3}{x^3}\,dx$$

$$\Big\downarrow \; 1262$$

$$\frac{\int \frac{\left(bx^2+ax\right)^{3/2}\left(8bc^3+24bdxc^2+3d^2(8bc-ad)x^2\right)}{2x^3}dx}{4b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{4bx}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(bx^2+ax)^{3/2}(8bc^3+24bdxc^2+3d^2(8bc-ad)x^2)}{x^3}dx}{8b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{4bx}$$

$\Big\downarrow$ 2169

$$\frac{\frac{\int \frac{3\left(16b^2c^3+d\left(48b^2c^2-8abdc+a^2d^2\right)x\right)\left(bx^2+ax\right)^{3/2}}{2x^3}dx}{3b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(8bc-ad)}{bx^2}}{8b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{4bx}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\left(16b^2c^3+d\left(48b^2c^2-8abdc+a^2d^2\right)x\right)\left(bx^2+ax\right)^{3/2}}{x^3}dx}{2b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(8bc-ad)}{bx^2}}{8b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{4bx}$$

$\Big\downarrow$ 1220

$$\frac{\frac{\frac{\left(a^3d^3-8a^2bcd^2+48ab^2c^2d+64b^3c^3\right)\int \frac{\left(bx^2+ax\right)^{3/2}}{x^2}dx}{a}-\frac{32b^2c^3\left(ax+bx^2\right)^{5/2}}{ax^3}}{2b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(8bc-ad)}{bx^2}}{8b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{4bx}$$

$\Big\downarrow$ 1131

$$\frac{\frac{\frac{\left(a^3d^3-8a^2bcd^2+48ab^2c^2d+64b^3c^3\right)\left(\frac{3}{4}a\int \frac{\sqrt{bx^2+ax}}{x}dx+\frac{\left(ax+bx^2\right)^{3/2}}{2x}\right)}{a}-\frac{32b^2c^3\left(ax+bx^2\right)^{5/2}}{ax^3}}{2b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(8bc-ad)}{bx^2}}{8b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{4bx}$$

$\Big\downarrow$ 1131

$$\frac{\frac{\frac{\left(a^3d^3-8a^2bcd^2+48ab^2c^2d+64b^3c^3\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+ax}}dx+\sqrt{ax+bx^2}\right)+\frac{\left(ax+bx^2\right)^{3/2}}{2x}\right)}{a}-\frac{32b^2c^3\left(ax+bx^2\right)^{5/2}}{ax^3}}{2b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(8bc-ad)}{bx^2}}{8b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{4bx}$$

$\Big\downarrow$ 1091

$$\frac{\frac{\frac{\left(a^3d^3-8a^2bcd^2+48ab^2c^2d+64b^3c^3\right)\left(\frac{3}{4}a\left(a\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}+\sqrt{ax+bx^2}\right)+\frac{\left(ax+bx^2\right)^{3/2}}{2x}\right)}{a}-\frac{32b^2c^3\left(ax+bx^2\right)^{5/2}}{ax^3}}{2b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(8bc-ad)}{bx^2}}{8b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{4bx}$$

$\downarrow$ 219

$$\frac{\frac{\frac{\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}+\sqrt{ax+bx^2}\right)+\frac{\left(ax+bx^2\right)^{3/2}}{2x}\right)\left(a^3d^3-8a^2bcd^2+48ab^2c^2d+64b^3c^3\right)}{a}-\frac{32b^2c^3\left(ax+bx^2\right)^{5/2}}{ax^3}}{2b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(8bc-ad)}{bx^2}}{8b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{4bx}$$

input
```
Int[((c + d*x)^3*(a*x + b*x^2)^(3/2))/x^3,x]
```

output
```
(d^3*(a*x + b*x^2)^(5/2))/(4*b*x) + ((d^2*(8*b*c - a*d)*(a*x + b*x^2)^(5/2
))/(b*x^2) + ((-32*b^2*c^3*(a*x + b*x^2)^(5/2))/(a*x^3) + ((64*b^3*c^3 + 4
8*a*b^2*c^2*d - 8*a^2*b*c*d^2 + a^3*d^3)*((a*x + b*x^2)^(3/2)/(2*x) + (3*a
*(Sqrt[a*x + b*x^2] + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]))
/4))/a)/(2*b))/(8*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

rule 2169
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d -
b*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2
, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$\frac{\frac{3ax\left(a^3d^3-8a^2bc\,d^2+48a\,b^2c^2d+64b^3c^3\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{64}-\frac{3\sqrt{x(bx+a)}\left(\frac{128a\left(-\frac{3}{16}d^3x^3-\frac{7}{8}c\,d^2x^2-\frac{15}{8}c^2dx+c^3\right)b^{\frac{5}{2}}}{3}+x\left(\left(-\frac{1}{3}\right.\right.\right.}{}}{b^{\frac{5}{2}}x}$
risch	$-\frac{(bx+a)\left(-16b^3d^3x^4-24a\,b^2d^3x^3-64b^3c\,x^3d^2-2a^2b\,d^3x^2-112a\,b^2c\,d^2x^2-96b^3c^2d\,x^2+3a^3d^3x-24a^2bc\,d^2x-240a\,b^2c^2d\right.}{64b^2\sqrt{x(bx+a)}}$
default	$d^3\left(\frac{(2bx+a)\left(b\,x^2+ax\right)^{\frac{3}{2}}}{8b}-\frac{3a^2\left(\frac{(2bx+a)\sqrt{b\,x^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8b^{\frac{3}{2}}}\right)}{16b}\right)+c^3\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{a\,x^3}+\frac{4b\left(\right.}{}\right.$

input

```
int((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output

```
3/64/b^(5/2)*(a*x*(a^3*d^3-8*a^2*b*c*d^2+48*a*b^2*c^2*d+64*b^3*c^3)*arctan
h((x*(b*x+a))^(1/2)/x/b^(1/2))-(x*(b*x+a))^(1/2)*(128/3*a*(-3/16*d^3*x^3-7
/8*c*d^2*x^2-15/8*c^2*d*x+c^3)*b^(5/2)+x*((-16/3*d^3*x^3-64/3*c*d^2*x^2-32
*c^2*d*x-64/3*c^3)*b^(7/2)+d^2*((-2/3*d*x-8*c)*b^(3/2)+b^(1/2)*a*d)*a^2)))
/x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.68

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^3} \, dx = \left[\frac{3 (64 ab^3c^3 + 48 a^2b^2c^2d - 8 a^3bcd^2 + a^4d^3)\sqrt{b}x \log \left(2 bx + a + 2 \sqrt{bx^2 + \right.}{} \right.$$

$$- \frac{3 (64 ab^3c^3 + 48 a^2b^2c^2d - 8 a^3bcd^2 + a^4d^3)\sqrt{-b}x \arctan \left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right) - (16 b^4d^3x^4 - 128 ab^3c^3 + 8 (8 b^4}{}$$

input

```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^3,x, algorithm="fricas")
```

output

```
[1/128*(3*(64*a*b^3*c^3 + 48*a^2*b^2*c^2*d - 8*a^3*b*c*d^2 + a^4*d^3)*sqrt
(b)*x*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(16*b^4*d^3*x^4 - 1
28*a*b^3*c^3 + 8*(8*b^4*c*d^2 + 3*a*b^3*d^3)*x^3 + 2*(48*b^4*c^2*d + 56*a*
b^3*c*d^2 + a^2*b^2*d^3)*x^2 + (64*b^4*c^3 + 240*a*b^3*c^2*d + 24*a^2*b^2*
c*d^2 - 3*a^3*b*d^3)*x)*sqrt(b*x^2 + a*x))/(b^3*x), -1/64*(3*(64*a*b^3*c^3
 + 48*a^2*b^2*c^2*d - 8*a^3*b*c*d^2 + a^4*d^3)*sqrt(-b)*x*arctan(sqrt(b*x^
2 + a*x)*sqrt(-b)/(b*x + a)) - (16*b^4*d^3*x^4 - 128*a*b^3*c^3 + 8*(8*b^4*
c*d^2 + 3*a*b^3*d^3)*x^3 + 2*(48*b^4*c^2*d + 56*a*b^3*c*d^2 + a^2*b^2*d^3)
*x^2 + (64*b^4*c^3 + 240*a*b^3*c^2*d + 24*a^2*b^2*c*d^2 - 3*a^3*b*d^3)*x)*
sqrt(b*x^2 + a*x))/(b^3*x)]
```

Sympy [F]

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^3} \, dx = \int \frac{(x(a+bx))^{\frac{3}{2}} (c+dx)^3}{x^3} \, dx$$

input

```
integrate((d*x+c)**3*(b*x**2+a*x)**(3/2)/x**3,x)
```

output

```
Integral((x*(a + b*x))**(3/2)*(c + d*x)**3/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.38

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^3} dx = \frac{3}{4}\sqrt{bx^2+ax}acd^2x + \frac{1}{4}\left(bx^2+ax\right)^{\frac{3}{2}}d^3x - \frac{3\sqrt{bx^2+ax}a^2d^3x}{32\,b} + \frac{3}{2}a\sqrt{b}c^3\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right) + \frac{9\,a^2c^2d\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{8\sqrt{b}} - \frac{3\,a^3cd^2\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{3}{2}}} + \frac{3\,a^4d^3\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{5}{2}}} + \frac{9}{4}\sqrt{bx^2+ax}ac^2d + \left(bx^2+ax\right)^{\frac{3}{2}}cd^2 + \frac{3\sqrt{bx^2+ax}a^2cd^2}{8\,b} - \frac{3\sqrt{bx^2+ax}a^3d^3}{64\,b^2} + \frac{\left(bx^2+ax\right)^{\frac{3}{2}}ad^3}{8\,b} - \frac{3\sqrt{bx^2+ax}ac^3}{x} + \frac{3\left(bx^2+ax\right)^{\frac{3}{2}}c^2d}{2\,x} + \frac{\left(bx^2+ax\right)^{\frac{3}{2}}c^3}{x^2}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
3/4*sqrt(b*x^2 + a*x)*a*c*d^2*x + 1/4*(b*x^2 + a*x)^(3/2)*d^3*x - 3/32*sqr
t(b*x^2 + a*x)*a^2*d^3*x/b + 3/2*a*sqrt(b)*c^3*log(2*b*x + a + 2*sqrt(b*x^
2 + a*x)*sqrt(b)) + 9/8*a^2*c^2*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt
(b))/sqrt(b) - 3/16*a^3*c*d^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))
/b^(3/2) + 3/128*a^4*d^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5
/2) + 9/4*sqrt(b*x^2 + a*x)*a*c^2*d + (b*x^2 + a*x)^(3/2)*c*d^2 + 3/8*sqrt
(b*x^2 + a*x)*a^2*c*d^2/b - 3/64*sqrt(b*x^2 + a*x)*a^3*d^3/b^2 + 1/8*(b*x^
2 + a*x)^(3/2)*a*d^3/b - 3*sqrt(b*x^2 + a*x)*a*c^3/x + 3/2*(b*x^2 + a*x)^(
3/2)*c^2*d/x + (b*x^2 + a*x)^(3/2)*c^3/x^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^3} dx = \frac{2 a^2 c^3}{\sqrt{b}x - \sqrt{bx^2+ax}} + \frac{1}{64} \sqrt{bx^2+ax} \left(2 \left(4 \left(2 bd^3 x + \frac{8 b^4 cd^2 + 3 ab^3 d^3}{b^3}\right) x + \frac{48 b^4 c^2 d + 56 ab^3 cd^2 + a^2 b^2 d^3}{b^3}\right) x + \frac{64 b^4 c^3 + 240 a}{}\right. - \frac{3 (64 ab^3 c^3 + 48 a^2 b^2 c^2 d - 8 a^3 bcd^2 + a^4 d^3) \log \left(\left|2 \left(\sqrt{b}x - \sqrt{bx^2+ax}\right) \sqrt{b} + a\right|\right)}{128 b^{\frac{5}{2}}}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^3,x, algorithm="giac")
```

output
```
2*a^2*c^3/(sqrt(b)*x - sqrt(b*x^2 + a*x)) + 1/64*sqrt(b*x^2 + a*x)*(2*(4*(
2*b*d^3*x + (8*b^4*c*d^2 + 3*a*b^3*d^3)/b^3)*x + (48*b^4*c^2*d + 56*a*b^3*
c*d^2 + a^2*b^2*d^3)/b^3)*x + (64*b^4*c^3 + 240*a*b^3*c^2*d + 24*a^2*b^2*c
*d^2 - 3*a^3*b*d^3)/b^3) - 3/128*(64*a*b^3*c^3 + 48*a^2*b^2*c^2*d - 8*a^3*
b*c*d^2 + a^4*d^3)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))
/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^3} dx = \int \frac{(b x^2 + a x)^{3/2} (c + d x)^3}{x^3} dx$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^3)/x^3,x)
```

output
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^3)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.51

$$\int \frac{(c+dx)^3\,(ax+bx^2)^{3/2}}{x^3}\,dx = \frac{-3\sqrt{x}\,\sqrt{bx+a}\,a^3b\,d^3x+24\sqrt{x}\,\sqrt{bx+a}\,a^2b^2c\,d^2x+2\sqrt{x}\,\sqrt{bx+a}\,a^2b^2d^3x}{}$$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^3,x)
```

output
```
( - 3*sqrt(x)*sqrt(a + b*x)*a**3*b*d**3*x + 24*sqrt(x)*sqrt(a + b*x)*a**2*
b**2*c*d**2*x + 2*sqrt(x)*sqrt(a + b*x)*a**2*b**2*d**3*x**2 - 128*sqrt(x)*
sqrt(a + b*x)*a*b**3*c**3 + 240*sqrt(x)*sqrt(a + b*x)*a*b**3*c**2*d*x + 11
2*sqrt(x)*sqrt(a + b*x)*a*b**3*c*d**2*x**2 + 24*sqrt(x)*sqrt(a + b*x)*a*b*
*3*d**3*x**3 + 64*sqrt(x)*sqrt(a + b*x)*b**4*c**3*x + 96*sqrt(x)*sqrt(a +
b*x)*b**4*c**2*d*x**2 + 64*sqrt(x)*sqrt(a + b*x)*b**4*c*d**2*x**3 + 16*sqr
t(x)*sqrt(a + b*x)*b**4*d**3*x**4 + 3*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)
*sqrt(b))/sqrt(a))*a**4*d**3*x - 24*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*s
qrt(b))/sqrt(a))*a**3*b*c*d**2*x + 144*sqrt(b)*log((sqrt(a + b*x) + sqrt(x
)*sqrt(b))/sqrt(a))*a**2*b**2*c**2*d*x + 192*sqrt(b)*log((sqrt(a + b*x) +
sqrt(x)*sqrt(b))/sqrt(a))*a*b**3*c**3*x + 3*sqrt(b)*a**3*b*c*d**2*x - 48*s
qrt(b)*a**2*b**2*c**2*d*x - 144*sqrt(b)*a*b**3*c**3*x)/(64*b**3*x)
```

3.85 $\int \frac{(c+dx)^3(ax+bx^2)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 243

$$\int \frac{(c+dx)^3(ax+bx^2)^{3/2}}{x^4}\,dx = \frac{(16b^3c^3+72ab^2c^2d+18a^2bcd^2-a^3d^3)\sqrt{ax+bx^2}}{8ab} + \frac{1}{3}d^3(ax+bx^2)^{3/2} - \frac{2c^2(2bc+9ad)(ax+bx^2)^{3/2}}{3ax^2} + \frac{d^2(6bc+ad)(ax+bx^2)^{3/2}}{4bx} - \frac{2c^3(ax+bx^2)^{5/2}}{3ax^4} + \frac{(16b^3c^3+72ab^2c^2d+18a^2bcd^2-a^3d^3)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{8b^{3/2}}$$

output

```
1/8*(-a^3*d^3+18*a^2*b*c*d^2+72*a*b^2*c^2*d+16*b^3*c^3)*(b*x^2+a*x)^(1/2)/
a/b+1/3*d^3*(b*x^2+a*x)^(3/2)-2/3*c^2*(9*a*d+2*b*c)*(b*x^2+a*x)^(3/2)/a/x^
2+1/4*d^2*(a*d+6*b*c)*(b*x^2+a*x)^(3/2)/b/x-2/3*c^3*(b*x^2+a*x)^(5/2)/a/x^
4+1/8*(-a^3*d^3+18*a^2*b*c*d^2+72*a*b^2*c^2*d+16*b^3*c^3)*arctanh(b^(1/2)*
x/(b*x^2+a*x)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^4} \, dx = \frac{(x(a+bx))^{3/2} \left(\frac{\sqrt{b}(3a^2d^3x^2-2ab(8c^3+72c^2dx-45cd^2x^2-7d^3x^3)+4b^2x(-16c^3+18c^2dx+9cd^2}{a+bx} \right.}{24b^{3/2}x^3}$$

input
```
Integrate[((c + d*x)^3*(a*x + b*x^2)^(3/2))/x^4,x]
```

output
```
((x*(a + b*x))^(3/2)*((Sqrt[b]*(3*a^2*d^3*x^2 - 2*a*b*(8*c^3 + 72*c^2*d*x
- 45*c*d^2*x^2 - 7*d^3*x^3) + 4*b^2*x*(-16*c^3 + 18*c^2*d*x + 9*c*d^2*x^2
+ 2*d^3*x^3)))/(a + b*x) + (6*(16*b^3*c^3 + 72*a*b^2*c^2*d + 18*a^2*b*c*d^
2 - a^3*d^3)*x^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]
)/(a + b*x)^(3/2)))/(24*b^(3/2)*x^3)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.87, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1262, 27, 2169, 27, 1220, 1125, 25, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2} (c+dx)^3}{x^4} \, dx$$

$$\Big\downarrow \text{1262}$$

$$\frac{\int \frac{(bx^2+ax)^{3/2}(6bc^3+18bdxc^2+d^2(18bc-ad)x^2)}{2x^4}dx}{3b}+\frac{d^3(ax+bx^2)^{5/2}}{3bx^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(bx^2+ax)^{3/2}(6bc^3+18bdxc^2+d^2(18bc-ad)x^2)}{x^4}dx}{6b}+\frac{d^3(ax+bx^2)^{5/2}}{3bx^2}$$

$$\Big\downarrow \text{2169}$$

$$\frac{\frac{\int \frac{\left(24b^2c^3+d\left(72b^2c^2+18abdc-a^2d^2\right)x\right)\left(bx^2+ax\right)^{3/2}}{2x^4}dx}{2b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(18bc-ad)}{2bx^3}}{6b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{3bx^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\left(24b^2c^3+d\left(72b^2c^2+18abdc-a^2d^2\right)x\right)\left(bx^2+ax\right)^{3/2}}{x^4}dx}{4b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(18bc-ad)}{2bx^3}}{6b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{3bx^2}$$

$$\Big\downarrow \text{1220}$$

$$\frac{\frac{\frac{\left(-a^3d^3+18a^2bcd^2+72ab^2c^2d+16b^3c^3\right)\int \frac{\left(bx^2+ax\right)^{3/2}}{x^3}dx}{a}-\frac{16b^2c^3\left(ax+bx^2\right)^{5/2}}{ax^4}}{4b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(18bc-ad)}{2bx^3}}{6b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{3bx^2}$$

$$\Big\downarrow \text{1125}$$

$$\frac{\frac{\frac{\left(-a^3d^3+18a^2bcd^2+72ab^2c^2d+16b^3c^3\right)\left(-\int -\frac{b(2a+bx)}{\sqrt{bx^2+ax}}dx-\frac{2a\sqrt{ax+bx^2}}{x}\right)}{a}-\frac{16b^2c^3\left(ax+bx^2\right)^{5/2}}{ax^4}}{4b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(18bc-ad)}{2bx^3}}{6b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{3bx^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\frac{\left(-a^3d^3+18a^2bcd^2+72ab^2c^2d+16b^3c^3\right)\left(\int \frac{b(2a+bx)}{\sqrt{bx^2+ax}}dx-\frac{2a\sqrt{ax+bx^2}}{x}\right)}{a}-\frac{16b^2c^3\left(ax+bx^2\right)^{5/2}}{ax^4}}{4b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(18bc-ad)}{2bx^3}}{6b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{3bx^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\left(-a^3d^3+18a^2bcd^2+72ab^2c^2d+16b^3c^3\right)\left(b\int \frac{2a+bx}{\sqrt{bx^2+ax}}dx-\frac{2a\sqrt{ax+bx^2}}{x}\right)}{a}-\frac{16b^2c^3\left(ax+bx^2\right)^{5/2}}{ax^4}}{4b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(18bc-ad)}{2bx^3}}{6b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{3bx^2}$$

$$\Big\downarrow \text{1160}$$

$$\frac{\frac{\frac{\left(-a^3d^3+18a^2bcd^2+72ab^2c^2d+16b^3c^3\right)\left(b\left(\frac{3}{2}a\int\frac{1}{\sqrt{bx^2+ax}}dx+\sqrt{ax+bx^2}\right)-\frac{2a\sqrt{ax+bx^2}}{x}\right)}{a}-\frac{16b^2c^3\left(ax+bx^2\right)^{5/2}}{ax^4}}{4b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(18bc-ad)}{2bx^3}}{6b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{3bx^2}$$

$$\Big\downarrow \text{1091}$$

$$\frac{\frac{\frac{\left(-a^3d^3+18a^2bcd^2+72ab^2c^2d+16b^3c^3\right)\left(b\left(3a\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}+\sqrt{ax+bx^2}\right)-\frac{2a\sqrt{ax+bx^2}}{x}\right)}{a}-\frac{16b^2c^3\left(ax+bx^2\right)^{5/2}}{ax^4}}{4b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(18bc-ad)}{2bx^3}}{6b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{3bx^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{\left(b\left(\frac{3a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}+\sqrt{ax+bx^2}\right)-\frac{2a\sqrt{ax+bx^2}}{x}\right)\left(-a^3d^3+18a^2bcd^2+72ab^2c^2d+16b^3c^3\right)}{a}-\frac{16b^2c^3\left(ax+bx^2\right)^{5/2}}{ax^4}}{4b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(18bc-ad)}{2bx^3}}{6b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{3bx^2}$$

input
```
Int[((c + d*x)^3*(a*x + b*x^2)^(3/2))/x^4,x]
```

output
```
(d^3*(a*x + b*x^2)^(5/2))/(3*b*x^2) + ((d^2*(18*b*c - a*d)*(a*x + b*x^2)^(
5/2))/(2*b*x^3) + ((-16*b^2*c^3*(a*x + b*x^2)^(5/2))/(a*x^4) + ((16*b^3*c^
3 + 72*a*b^2*c^2*d + 18*a^2*b*c*d^2 - a^3*d^3)*((-2*a*Sqrt[a*x + b*x^2])/x
 + b*(Sqrt[a*x + b*x^2] + (3*a*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqr
t[b])))/a)/(4*b))/(6*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

rule 2169
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d -
b*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2
, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$-\frac{x^2\left(a^3d^3-18a^2bc\,d^2-72a\,b^2c^2d-16b^3c^3\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)-\sqrt{x(bx+a)}\left(-\frac{16\left(-\frac{7}{8}d^3x^3-\frac{45}{8}c\,d^2x^2+9c^2dx+c^3\right)a\,b^{\frac{3}{2}}}{3}+\cdots\right.}{8b^{\frac{3}{2}}x^2}$
risch	$\frac{(bx+a)\left(8b^2x^4d^3+14ab\,d^3x^3+36b^2c\,x^3d^2+3a^2d^3x^2+90abc\,d^2x^2+72b^2c^2d\,x^2-144ab\,c^2dx-64c^3b^2x-16ab\,c^3\right)}{24\sqrt{x(bx+a)}\,xb}-\frac{\left(a^3d^3-\cdots\right.}{\cdots}$
default	$c^3\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{3a\,x^4}+\frac{2b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{a\,x^3}+\frac{4b\left(\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{a\,x^2}-\frac{6b\left(\frac{\left(b\,x^2+ax\right)^{\frac{3}{2}}}{3}+\frac{a\left(\frac{(2bx+a)\sqrt{b\,x^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{\cdots}\right)}{8b^{\frac{3}{2}}}\cdots\right)}{2}\cdots\right)}{a}\cdots\right)}{a}\cdots\right)}{3a}\cdots\right.$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/8/b^(3/2)*(x^2*(a^3*d^3-18*a^2*b*c*d^2-72*a*b^2*c^2*d-16*b^3*c^3)*arcta
nh((x*(b*x+a))^(1/2)/x/b^(1/2))-(x*(b*x+a))^(1/2)*(-16/3*(-7/8*d^3*x^3-45/
8*c*d^2*x^2+9*c^2*d*x+c^3)*a*b^(3/2)+((8/3*d^3*x^3+12*c*d^2*x^2+24*c^2*d*x
-64/3*c^3)*b^(5/2)+a^2*d^3*x*b^(1/2))*x))/x^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.58

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^4}\,dx = \left[-\frac{3\,(16\,b^3c^3+72\,ab^2c^2d+18\,a^2bcd^2-a^3d^3)\sqrt{b}x^2\log\left(2\,bx+a-2\,\sqrt{bx^2}\right.}{} \right.$$

$$-\frac{3\,(16\,b^3c^3+72\,ab^2c^2d+18\,a^2bcd^2-a^3d^3)\sqrt{-b}x^2\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(8\,b^3d^3x^4-16\,ab^2c^3+2\,(18\,b^3c}{24\,b^2x^2}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[-1/48*(3*(16*b^3*c^3 + 72*a*b^2*c^2*d + 18*a^2*b*c*d^2 - a^3*d^3)*sqrt(b)
*x^2*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*(8*b^3*d^3*x^4 - 16*
a*b^2*c^3 + 2*(18*b^3*c*d^2 + 7*a*b^2*d^3)*x^3 + 3*(24*b^3*c^2*d + 30*a*b^
2*c*d^2 + a^2*b*d^3)*x^2 - 16*(4*b^3*c^3 + 9*a*b^2*c^2*d)*x)*sqrt(b*x^2 +
a*x))/(b^2*x^2), -1/24*(3*(16*b^3*c^3 + 72*a*b^2*c^2*d + 18*a^2*b*c*d^2 -
a^3*d^3)*sqrt(-b)*x^2*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (8*b^
3*d^3*x^4 - 16*a*b^2*c^3 + 2*(18*b^3*c*d^2 + 7*a*b^2*d^3)*x^3 + 3*(24*b^3*
c^2*d + 30*a*b^2*c*d^2 + a^2*b*d^3)*x^2 - 16*(4*b^3*c^3 + 9*a*b^2*c^2*d)*x
)*sqrt(b*x^2 + a*x))/(b^2*x^2)]
```

Sympy [F]

$$\int \frac{(c+dx)^3 \left(ax+bx^2\right)^{3/2}}{x^4}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}\,(c+dx)^3}{x^4}\,dx$$

input
```
integrate((d*x+c)**3*(b*x**2+a*x)**(3/2)/x**4,x)
```

output
```
Integral((x*(a + b*x))**(3/2)*(c + d*x)**3/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.35

$$\begin{aligned}\int \frac{(c+dx)^3 \left(ax+bx^2\right)^{3/2}}{x^4}\,dx &= \frac{1}{4}\sqrt{bx^2+ax}ad^3x \\ &+ b^{\frac{3}{2}}c^3\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right) \\ &+ \frac{9}{2}\,a\sqrt{b}c^2d\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right) \\ &+ \frac{9\,a^2cd^2\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{8\sqrt{b}} \\ &- \frac{a^3d^3\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{3}{2}}} + \frac{9}{4}\sqrt{bx^2+ax}acd^2 \\ &+ \frac{1}{3}\left(bx^2+ax\right)^{\frac{3}{2}}d^3 + \frac{\sqrt{bx^2+ax}a^2d^3}{8\,b} - \frac{7\sqrt{bx^2+ax}bc^3}{3\,x} - \frac{9\sqrt{bx^2+ax}ac^2d}{x} \\ &+ \frac{3\left(bx^2+ax\right)^{\frac{3}{2}}cd^2}{2\,x} - \frac{\sqrt{bx^2+ax}ac^3}{3\,x^2} + \frac{3\left(bx^2+ax\right)^{\frac{3}{2}}c^2d}{x^2} - \frac{\left(bx^2+ax\right)^{\frac{3}{2}}c^3}{3\,x^3}\end{aligned}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
1/4*sqrt(b*x^2 + a*x)*a*d^3*x + b^(3/2)*c^3*log(2*b*x + a + 2*sqrt(b*x^2 +
 a*x)*sqrt(b)) + 9/2*a*sqrt(b)*c^2*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*s
qrt(b)) + 9/8*a^2*c*d^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(
b) - 1/16*a^3*d^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(3/2) + 9
/4*sqrt(b*x^2 + a*x)*a*c*d^2 + 1/3*(b*x^2 + a*x)^(3/2)*d^3 + 1/8*sqrt(b*x^
2 + a*x)*a^2*d^3/b - 7/3*sqrt(b*x^2 + a*x)*b*c^3/x - 9*sqrt(b*x^2 + a*x)*a
*c^2*d/x + 3/2*(b*x^2 + a*x)^(3/2)*c*d^2/x - 1/3*sqrt(b*x^2 + a*x)*a*c^3/x
^2 + 3*(b*x^2 + a*x)^(3/2)*c^2*d/x^2 - 1/3*(b*x^2 + a*x)^(3/2)*c^3/x^3
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^4} dx = \frac{1}{24} \sqrt{bx^2+ax} \left(2 \left(4 bd^3x + \frac{18 b^3cd^2 + 7 ab^2d^3}{b^2}\right)x + \frac{3 (24 b^3c^2d + 30 ab^2cd}{b^2}\right.$$
$$- \frac{(16 b^3c^3 + 72 ab^2c^2d + 18 a^2bcd^2 - a^3d^3) \log \left(\left|2 \left(\sqrt{b}x - \sqrt{bx^2+ax}\right)\sqrt{b} + a\right|\right)}{16 b^{\frac{3}{2}}}$$
$$+ \frac{2 \left(6 \left(\sqrt{b}x - \sqrt{bx^2+ax}\right)^2 abc^3 + 9 \left(\sqrt{b}x - \sqrt{bx^2+ax}\right)^2 a^2c^2d + 3 \left(\sqrt{b}x - \sqrt{bx^2+ax}\right) a^2\sqrt{b}c^3 + a^3c^3\right)}{3 \left(\sqrt{b}x - \sqrt{bx^2+ax}\right)^3}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^4,x, algorithm="giac")
```

output
```
1/24*sqrt(b*x^2 + a*x)*(2*(4*b*d^3*x + (18*b^3*c*d^2 + 7*a*b^2*d^3)/b^2)*x
 + 3*(24*b^3*c^2*d + 30*a*b^2*c*d^2 + a^2*b*d^3)/b^2) - 1/16*(16*b^3*c^3 +
 72*a*b^2*c^2*d + 18*a^2*b*c*d^2 - a^3*d^3)*log(abs(2*(sqrt(b)*x - sqrt(b*
x^2 + a*x))*sqrt(b) + a))/b^(3/2) + 2/3*(6*(sqrt(b)*x - sqrt(b*x^2 + a*x))
^2*a*b*c^3 + 9*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2*c^2*d + 3*(sqrt(b)*x
- sqrt(b*x^2 + a*x))*a^2*sqrt(b)*c^3 + a^3*c^3)/(sqrt(b)*x - sqrt(b*x^2 +
a*x))^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^4} \, dx = \int \frac{(b\,x^2+a\,x)^{3/2}\,(c+d\,x)^3}{x^4} \, dx$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^3)/x^4,x)
```

output
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^3)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.45

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^4} \, dx = \frac{24\sqrt{x}\,\sqrt{bx+a}\,a^2b\,d^3x^2 - 128\sqrt{x}\,\sqrt{bx+a}\,a\,b^2c^3 - 1152\sqrt{x}\,\sqrt{bx+a}\,a\,b^2c^2}{}$$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^4,x)
```

output
```
(24*sqrt(x)*sqrt(a + b*x)*a**2*b*d**3*x**2 - 128*sqrt(x)*sqrt(a + b*x)*a*b
**2*c**3 - 1152*sqrt(x)*sqrt(a + b*x)*a*b**2*c**2*d*x + 720*sqrt(x)*sqrt(a
 + b*x)*a*b**2*c*d**2*x**2 + 112*sqrt(x)*sqrt(a + b*x)*a*b**2*d**3*x**3 -
512*sqrt(x)*sqrt(a + b*x)*b**3*c**3*x + 576*sqrt(x)*sqrt(a + b*x)*b**3*c**
2*d*x**2 + 288*sqrt(x)*sqrt(a + b*x)*b**3*c*d**2*x**3 + 64*sqrt(x)*sqrt(a
+ b*x)*b**3*d**3*x**4 - 24*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/s
qrt(a))*a**3*d**3*x**2 + 432*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))
/sqrt(a))*a**2*b*c*d**2*x**2 + 1728*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*s
qrt(b))/sqrt(a))*a*b**2*c**2*d*x**2 + 384*sqrt(b)*log((sqrt(a + b*x) + sqr
t(x)*sqrt(b))/sqrt(a))*b**3*c**3*x**2 - 3*sqrt(b)*a**3*d**3*x**2 + 90*sqrt
(b)*a**2*b*c*d**2*x**2 + 480*sqrt(b)*a*b**2*c**2*d*x**2)/(192*b**2*x**2)
```

3.86 $\int \frac{(c+dx)^3(ax+bx^2)^{3/2}}{x^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 180

$$\int \frac{(c+dx)^3\left(ax+bx^2\right)^{3/2}}{x^5}\,dx = \frac{1}{4}d^2(12bc+5ad)\sqrt{ax+bx^2} - \frac{6cd(bc+ad)\sqrt{ax+bx^2}}{x} + \frac{1}{2}bd^3x\sqrt{ax+bx^2} - \frac{2c^2d(ax+bx^2)^{3/2}}{x^3} - \frac{2c^3(ax+bx^2)^{5/2}}{5ax^5} + \frac{3d(8b^2c^2+12abcd+a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4\sqrt{b}}$$

output

```
1/4*d^2*(5*a*d+12*b*c)*(b*x^2+a*x)^(1/2)-6*c*d*(a*d+b*c)*(b*x^2+a*x)^(1/2)
/x+1/2*b*d^3*x*(b*x^2+a*x)^(1/2)-2*c^2*d*(b*x^2+a*x)^(3/2)/x^3-2/5*c^3*(b*
x^2+a*x)^(5/2)/a/x^5+3/4*d*(a^2*d^2+12*a*b*c*d+8*b^2*c^2)*arctanh(b^(1/2)*
x/(b*x^2+a*x)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx)^3\left(ax+bx^2\right)^{3/2}}{x^5}\,dx = \frac{\sqrt{x(a+bx)}\left(-\frac{8b^2c^3x^2}{a}-a(8c^3+40c^2dx+120cd^2x^2-25d^3x^3)+2bx(-8c\right.}{2}$$

input
```
Integrate[((c + d*x)^3*(a*x + b*x^2)^(3/2))/x^5,x]
```

output
```
(Sqrt[x*(a + b*x)]*((-8*b^2*c^3*x^2)/a - a*(8*c^3 + 40*c^2*d*x + 120*c*d^2
*x^2 - 25*d^3*x^3) + 2*b*x*(-8*c^3 - 80*c^2*d*x + 30*c*d^2*x^2 + 5*d^3*x^3
) - (15*d*(8*b^2*c^2 + 12*a*b*c*d + a^2*d^2)*x^(5/2)*Log[-(Sqrt[b]*Sqrt[x]
) + Sqrt[a + b*x]])/(Sqrt[b]*Sqrt[a + b*x])))/(20*x^3)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1262, 27, 2169, 27, 1220, 1130, 1125, 25, 27, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{3/2}(c+dx)^3}{x^5}\,dx$$

$$\Big\downarrow \text{1262}$$

$$\frac{\int \frac{\left(bx^2+ax\right)^{3/2}\left(4bc^3+12bdxc^2+d^2(12bc+ad)x^2\right)}{2x^5}dx}{2b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{2bx^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(bx^2+ax\right)^{3/2}\left(4bc^3+12bdxc^2+d^2(12bc+ad)x^2\right)}{x^5}dx}{4b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{2bx^3}$$

$$\Big\downarrow \text{2169}$$

$$\frac{\frac{\int \frac{\left(8b^2c^3+3d\left(8b^2c^2+12abdc+a^2d^2\right)x\right)\left(bx^2+ax\right)^{3/2}}{2x^5}dx}{b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(ad+12bc)}{bx^4}}{4b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{2bx^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{\left(8b^2c^3+3d\left(8b^2c^2+12abdc+a^2d^2\right)x\right)\left(bx^2+ax\right)^{3/2}}{x^5}dx}{2b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(ad+12bc)}{bx^4}}{4b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{2bx^3}$$

$$\Big\downarrow \ 1220$$

$$\frac{\frac{3d(a^2d^2+12abcd+8b^2c^2)\int \frac{\left(bx^2+ax\right)^{3/2}}{x^4}dx-\frac{16b^2c^3\left(ax+bx^2\right)^{5/2}}{5ax^5}}{2b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(ad+12bc)}{bx^4}}{4b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{2bx^3}$$

$$\Big\downarrow \ 1130$$

$$\frac{\frac{3d(a^2d^2+12abcd+8b^2c^2)\left(b\int \frac{\sqrt{bx^2+ax}}{x^2}dx-\frac{2\left(ax+bx^2\right)^{3/2}}{3x^3}\right)-\frac{16b^2c^3\left(ax+bx^2\right)^{5/2}}{5ax^5}}{2b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(ad+12bc)}{bx^4}}{4b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{2bx^3}$$

$$\Big\downarrow \ 1125$$

$$\frac{\frac{3d(a^2d^2+12abcd+8b^2c^2)\left(b\left(-\int -\frac{b}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2\left(ax+bx^2\right)^{3/2}}{3x^3}\right)-\frac{16b^2c^3\left(ax+bx^2\right)^{5/2}}{5ax^5}}{2b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(ad+12bc)}{bx^4}}{4b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{2bx^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{3d(a^2d^2+12abcd+8b^2c^2)\left(b\left(\int \frac{b}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2\left(ax+bx^2\right)^{3/2}}{3x^3}\right)-\frac{16b^2c^3\left(ax+bx^2\right)^{5/2}}{5ax^5}}{2b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(ad+12bc)}{bx^4}}{4b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{2bx^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{3d(a^2d^2+12abcd+8b^2c^2)\left(b\left(b\int \frac{1}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2\left(ax+bx^2\right)^{3/2}}{3x^3}\right)-\frac{16b^2c^3\left(ax+bx^2\right)^{5/2}}{5ax^5}}{2b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(ad+12bc)}{bx^4}}{4b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{2bx^3}$$

$$\Big\downarrow \; 1091$$

$$\frac{\frac{3d(a^2d^2+12abcd+8b^2c^2)\left(b\left(2b\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2\left(ax+bx^2\right)^{3/2}}{3x^3}\right)-\frac{16b^2c^3\left(ax+bx^2\right)^{5/2}}{5ax^5}}{2b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(ad+12bc)}{bx^4}}{4b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{2bx^3}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{3d\left(b\left(2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2\left(ax+bx^2\right)^{3/2}}{3x^3}\right)(a^2d^2+12abcd+8b^2c^2)-\frac{16b^2c^3\left(ax+bx^2\right)^{5/2}}{5ax^5}}{2b}+\frac{d^2\left(ax+bx^2\right)^{5/2}(ad+12bc)}{bx^4}}{4b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{2bx^3}$$

input
```
Int[((c + d*x)^3*(a*x + b*x^2)^(3/2))/x^5,x]
```

output
```
(d^3*(a*x + b*x^2)^(5/2))/(2*b*x^3) + ((d^2*(12*b*c + a*d)*(a*x + b*x^2)^(
5/2))/(b*x^4) + ((-16*b^2*c^3*(a*x + b*x^2)^(5/2))/(5*a*x^5) + 3*d*(8*b^2*
c^2 + 12*a*b*c*d + a^2*d^2)*((-2*(a*x + b*x^2)^(3/2))/(3*x^3) + b*((-2*Sqr
t[a*x + b*x^2])/x + 2*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])))/(2
*b))/(4*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

rule 2169
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d -
b*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2
, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{\frac{3ad\,x^{3}\left(a^{2}d^{2}+12abcd+8b^{2}c^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{4}-\frac{2\sqrt{x(bx+a)}\left(2x\left(-\frac{5}{8}d^{3}x^{3}-\frac{15}{4}c\,d^{2}x^{2}+10c^{2}dx+c^{3}\right)a\,b^{\frac{3}{2}}+b^{\frac{5}{2}}c^{3}x^{2}+a^{2}\sqrt{b}\left(-\frac{25}{8}\right.\right.}{5}}{x^{3}a\sqrt{b}}$
risch	$-\frac{(bx+a)\left(-10ab\,d^{3}x^{4}-25a^{2}d^{3}x^{3}-60abc\,d^{2}x^{3}+120a^{2}c\,d^{2}x^{2}+160ab\,c^{2}x^{2}d+8b^{2}c^{3}x^{2}+40a^{2}c^{2}dx+16ab\,c^{3}x+8a^{2}c^{3}\right)}{20x^{2}\sqrt{x(bx+a)}\,a}+\frac{3(}{}$
default	$-\frac{2c^{3}\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{5a\,x^{5}}+d^{3}\left(\frac{2\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{a\,x^{2}}-\frac{6b\left(\frac{\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{3}+\frac{a\left(\frac{(2bx+a)\sqrt{b\,x^{2}+ax}}{4b}-\frac{a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8b^{\frac{3}{2}}}\right)}{2}\right)}{a}\right)-$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
3/4/b^(1/2)*(a*d*x^3*(a^2*d^2+12*a*b*c*d+8*b^2*c^2)*arctanh((x*(b*x+a))^(1
/2)/x/b^(1/2))-8/15*(x*(b*x+a))^(1/2)*(2*x*(-5/8*d^3*x^3-15/4*c*d^2*x^2+10
*c^2*d*x+c^3)*a*b^(3/2)+b^(5/2)*c^3*x^2+a^2*b^(1/2)*(-25/8*d^3*x^3+15*c*d^
2*x^2+5*c^2*d*x+c^3)))/x^3/a
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.11

$$\int \frac{(c+dx)^3\,(ax+bx^2)^{3/2}}{x^5}\,dx = \left[\frac{15\,(8\,ab^2c^2d+12\,a^2bcd^2+a^3d^3)\sqrt{b}x^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+\dots}{\dots}\right.$$

$$-\frac{15\,(8\,ab^2c^2d+12\,a^2bcd^2+a^3d^3)\sqrt{-b}x^3\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(10\,ab^2d^3x^4-8\,a^2bc^3+5\,(12\,ab^2cd^2+5\,\dots}{20\,abx^3}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
[1/40*(15*(8*a*b^2*c^2*d + 12*a^2*b*c*d^2 + a^3*d^3)*sqrt(b)*x^3*log(2*b*x
 + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(10*a*b^2*d^3*x^4 - 8*a^2*b*c^3 +
5*(12*a*b^2*c*d^2 + 5*a^2*b*d^3)*x^3 - 8*(b^3*c^3 + 20*a*b^2*c^2*d + 15*a^
2*b*c*d^2)*x^2 - 8*(2*a*b^2*c^3 + 5*a^2*b*c^2*d)*x)*sqrt(b*x^2 + a*x))/(a*
b*x^3), -1/20*(15*(8*a*b^2*c^2*d + 12*a^2*b*c*d^2 + a^3*d^3)*sqrt(-b)*x^3*
arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (10*a*b^2*d^3*x^4 - 8*a^2*b
*c^3 + 5*(12*a*b^2*c*d^2 + 5*a^2*b*d^3)*x^3 - 8*(b^3*c^3 + 20*a*b^2*c^2*d
+ 15*a^2*b*c*d^2)*x^2 - 8*(2*a*b^2*c^3 + 5*a^2*b*c^2*d)*x)*sqrt(b*x^2 + a*
x))/(a*b*x^3)]
```

Sympy [F]

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^5}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}} (c+dx)^3}{x^5}\,dx$$

input
```
integrate((d*x+c)**3*(b*x**2+a*x)**(3/2)/x**5,x)
```

output
```
Integral((x*(a + b*x))**(3/2)*(c + d*x)**3/x**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(156) = 312.

Time = 0.04 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.79

$$\begin{aligned}\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^5}\,dx &= 3\,b^{\frac{3}{2}}c^2d\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) \\ &+ \frac{9}{2}\,a\sqrt{b}cd^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) \\ &+ \frac{3\,a^2d^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,\sqrt{b}} + \frac{3}{4}\,\sqrt{bx^2+ax}ad^3 \\ &- \frac{2\,\sqrt{bx^2+ax}b^2c^3}{5\,ax} - \frac{7\,\sqrt{bx^2+ax}bc^2d}{x} - \frac{9\,\sqrt{bx^2+ax}acd^2}{x} \\ &+ \frac{(bx^2+ax)^{\frac{3}{2}}d^3}{2\,x} + \frac{\sqrt{bx^2+ax}bc^3}{5\,x^2} - \frac{\sqrt{bx^2+ax}ac^2d}{x^2} + \frac{3\,(bx^2+ax)^{\frac{3}{2}}cd^2}{x^2} \\ &+ \frac{3\,\sqrt{bx^2+ax}ac^3}{5\,x^3} - \frac{(bx^2+ax)^{\frac{3}{2}}c^2d}{x^3} - \frac{(bx^2+ax)^{\frac{3}{2}}c^3}{x^4}\end{aligned}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
3*b^(3/2)*c^2*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 9/2*a*sqrt(
b)*c*d^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 3/8*a^2*d^3*log(2*
b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b) + 3/4*sqrt(b*x^2 + a*x)*a*d
^3 - 2/5*sqrt(b*x^2 + a*x)*b^2*c^3/(a*x) - 7*sqrt(b*x^2 + a*x)*b*c^2*d/x -
 9*sqrt(b*x^2 + a*x)*a*c*d^2/x + 1/2*(b*x^2 + a*x)^(3/2)*d^3/x + 1/5*sqrt(
b*x^2 + a*x)*b*c^3/x^2 - sqrt(b*x^2 + a*x)*a*c^2*d/x^2 + 3*(b*x^2 + a*x)^(
3/2)*c*d^2/x^2 + 3/5*sqrt(b*x^2 + a*x)*a*c^3/x^3 - (b*x^2 + a*x)^(3/2)*c^2
*d/x^3 - (b*x^2 + a*x)^(3/2)*c^3/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(156) = 312$.

Time = 0.15 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.07

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^5} dx = \frac{1}{4} \left(2 bd^3x + \frac{12 b^2cd^2 + 5 abd^3}{b}\right) \sqrt{bx^2+ax} - \frac{3 (8 b^2c^2d + 12 abcd^2 + a^2d^3) \log \left(\left|2 \left(\sqrt{b}x - \sqrt{bx^2+ax}\right) \sqrt{b} + a\right|\right)}{8 \sqrt{b}} + \frac{2 \left(5 \left(\sqrt{b}x - \sqrt{bx^2+ax}\right)^4 b^2c^3 + 30 \left(\sqrt{b}x - \sqrt{bx^2+ax}\right)^4 abc^2d + 15 \left(\sqrt{b}x - \sqrt{bx^2+ax}\right)^4 a^2cd^2 + 10 \left(\right.}{}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^5,x, algorithm="giac")
```

output
```
1/4*(2*b*d^3*x + (12*b^2*c*d^2 + 5*a*b*d^3)/b)*sqrt(b*x^2 + a*x) - 3/8*(8*
b^2*c^2*d + 12*a*b*c*d^2 + a^2*d^3)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*
x))*sqrt(b) + a))/sqrt(b) + 2/5*(5*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*b^2*c
^3 + 30*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a*b*c^2*d + 15*(sqrt(b)*x - sqrt
(b*x^2 + a*x))^4*a^2*c*d^2 + 10*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a*b^(3/2
)*c^3 + 15*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^2*sqrt(b)*c^2*d + 10*(sqrt(
b)*x - sqrt(b*x^2 + a*x))^2*a^2*b*c^3 + 5*(sqrt(b)*x - sqrt(b*x^2 + a*x))^
2*a^3*c^2*d + 5*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^3*sqrt(b)*c^3 + a^4*c^3)
/(sqrt(b)*x - sqrt(b*x^2 + a*x))^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^5} dx = \int \frac{(b x^2 + a x)^{3/2} (c + d x)^3}{x^5} dx$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^3)/x^5,x)
```

output
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^3)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.89

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^5} dx = \frac{-32\sqrt{x}\,\sqrt{bx+a}\,a^2 b\,c^3 - 160\sqrt{x}\,\sqrt{bx+a}\,a^2 b\,c^2 dx - 480\sqrt{x}\,\sqrt{bx+a}\,a^2 bc\,c}{}$$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^5,x)
```

output
```
( - 32*sqrt(x)*sqrt(a + b*x)*a**2*b*c**3 - 160*sqrt(x)*sqrt(a + b*x)*a**2*
b*c**2*d*x - 480*sqrt(x)*sqrt(a + b*x)*a**2*b*c*d**2*x**2 + 100*sqrt(x)*sq
rt(a + b*x)*a**2*b*d**3*x**3 - 64*sqrt(x)*sqrt(a + b*x)*a*b**2*c**3*x - 64
0*sqrt(x)*sqrt(a + b*x)*a*b**2*c**2*d*x**2 + 240*sqrt(x)*sqrt(a + b*x)*a*b
**2*c*d**2*x**3 + 40*sqrt(x)*sqrt(a + b*x)*a*b**2*d**3*x**4 - 32*sqrt(x)*s
qrt(a + b*x)*b**3*c**3*x**2 + 60*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt
(b))/sqrt(a))*a**3*d**3*x**3 + 720*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sq
rt(b))/sqrt(a))*a**2*b*c*d**2*x**3 + 480*sqrt(b)*log((sqrt(a + b*x) + sqrt
(x)*sqrt(b))/sqrt(a))*a*b**2*c**2*d*x**3 + 31*sqrt(b)*a**3*d**3*x**3 + 396
*sqrt(b)*a**2*b*c*d**2*x**3 + 256*sqrt(b)*a*b**2*c**2*d*x**3 - 32*sqrt(b)*
b**3*c**3*x**3)/(80*a*b*x**3)
```

3.87 $\int \frac{(c+dx)^3(ax+bx^2)^{3/2}}{x^6}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 170

$$\int \frac{(c+dx)^3(ax+bx^2)^{3/2}}{x^6}\,dx = bd^3\sqrt{ax+bx^2} - \frac{2d^2(3bc+ad)\sqrt{ax+bx^2}}{x} - \frac{2cd^2(ax+bx^2)^{3/2}}{x^3} - \frac{2c^3(ax+bx^2)^{5/2}}{7ax^6} + \frac{2c^2(2bc-21ad)(ax+bx^2)^{5/2}}{35a^2x^5} + 3\sqrt{b}d^2(2bc+ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)$$

output
```
b*d^3*(b*x^2+a*x)^(1/2)-2*d^2*(a*d+3*b*c)*(b*x^2+a*x)^(1/2)/x-2*c*d^2*(b*x
^2+a*x)^(3/2)/x^3-2/7*c^3*(b*x^2+a*x)^(5/2)/a/x^6+2/35*c^2*(-21*a*d+2*b*c)
*(b*x^2+a*x)^(5/2)/a^2/x^5+3*b^(1/2)*d^2*(a*d+2*b*c)*arctanh(b^(1/2)*x/(b*
x^2+a*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx)^3 \left(ax+bx^2\right)^{3/2}}{x^6}\,dx = \frac{\sqrt{x(a+bx)}\left(\frac{4b^3c^3x^3}{a^2}-\frac{2b^2c^2x^2(c+21dx)}{a}+bx(-16c^3-84c^2dx-280cd^2x^2+3\right.}{}$$

input
```
Integrate[((c + d*x)^3*(a*x + b*x^2)^(3/2))/x^6,x]
```

output
```
(Sqrt[x*(a + b*x)]*((4*b^3*c^3*x^3)/a^2 - (2*b^2*c^2*x^2*(c + 21*d*x))/a +
 b*x*(-16*c^3 - 84*c^2*d*x - 280*c*d^2*x^2 + 35*d^3*x^3) - 2*a*(5*c^3 + 21
*c^2*d*x + 35*c*d^2*x^2 + 35*d^3*x^3) - (105*Sqrt[b]*d^2*(2*b*c + a*d)*x^(
7/2)*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/Sqrt[a + b*x]))/(35*x^4)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.26, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1262, 27, 2167, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{3/2}(c+dx)^3}{x^6}\,dx$$

$$\Big\downarrow \text{1262}$$

$$\frac{\int \frac{(bx^2+ax)^{3/2}(2bc^3+6bdxc^2+3d^2(2bc+ad)x^2)}{2x^6}dx}{b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{bx^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(bx^2+ax)^{3/2}(2bc^3+6bdxc^2+3d^2(2bc+ad)x^2)}{x^6}dx}{2b}+\frac{d^3\left(ax+bx^2\right)^{5/2}}{bx^4}$$

$$\Big\downarrow \text{2167}$$

$$\frac{\int \left(\frac{2b(bx^2+ax)^{3/2}c^3}{x^6}+\frac{6bd(bx^2+ax)^{3/2}c^2}{x^5}+\frac{3d^2(2bc+ad)(bx^2+ax)^{3/2}}{x^4}\right)dx}{2b}+\frac{d^3(ax+bx^2)^{5/2}}{bx^4}$$

$\Big\downarrow$ 2009

$$\frac{\frac{8b^2c^3(ax+bx^2)^{5/2}}{35a^2x^5}+6b^{3/2}d^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(ad+2bc)-\frac{4bc^3(ax+bx^2)^{5/2}}{7ax^6}-\frac{12bc^2d(ax+bx^2)^{5/2}}{5ax^5}-\frac{6bd^2\sqrt{ax+bx^2}(ad+2bc}{x}}{2b}$$
$$\frac{d^3(ax+bx^2)^{5/2}}{bx^4}$$

input
```
Int[((c + d*x)^3*(a*x + b*x^2)^(3/2))/x^6,x]
```

output
```
(d^3*(a*x + b*x^2)^(5/2))/(b*x^4) + ((-6*b*d^2*(2*b*c + a*d)*Sqrt[a*x + b*
x^2])/x - (2*d^2*(2*b*c + a*d)*(a*x + b*x^2)^(3/2))/x^3 - (4*b*c^3*(a*x +
b*x^2)^(5/2))/(7*a*x^6) + (8*b^2*c^3*(a*x + b*x^2)^(5/2))/(35*a^2*x^5) - (
12*b*c^2*d*(a*x + b*x^2)^(5/2))/(5*a*x^5) + 6*b^(3/2)*d^2*(2*b*c + a*d)*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2167
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*x + c*x^2)^p, (d + e*x)^m*Pq, x]
, x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&
 EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + Expon[Pq, x] + 2*p + 1, 0] && ILt
Q[m, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$-\frac{2\left(-\frac{21a^{2}b\,d^{2}x^{4}(ad+2bc)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{2}+\left(\frac{8\left(-\frac{35}{16}d^{3}x^{3}+\frac{35}{2}c\,d^{2}x^{2}+\frac{21}{4}c^{2}dx+c^{3}\right)x\,a^{2}b^{\frac{3}{2}}}{5}+\frac{a\,c^{2}x^{2}(21dx+c)b^{\frac{5}{2}}}{5}-\frac{2b^{\frac{7}{2}}c^{3}x^{3}\cdots}{5}\cdots\right.\right.}{7\sqrt{b}\,x^{4}a^{2}\cdots}$
risch	$-\frac{(bx+a)\left(-35a^{2}b\,d^{3}x^{4}+70a^{3}d^{3}x^{3}+280a^{2}bc\,d^{2}x^{3}+42a\,b^{2}c^{2}d\,x^{3}-4b^{3}c^{3}x^{3}+70a^{3}c\,d^{2}x^{2}+84a^{2}b\,c^{2}x^{2}d+2a\,b^{2}c^{3}x^{2}+42a^{3}c^{2}\cdots\right.}{35x^{3}\sqrt{x(bx+a)}\,a^{2}\cdots}$
default	$c^{3}\left(-\frac{2(b\,x^{2}+ax)^{\frac{5}{2}}}{7a\,x^{6}}+\frac{4b(b\,x^{2}+ax)^{\frac{5}{2}}}{35a^{2}x^{5}}\right)+d^{3}\left(-\frac{2(b\,x^{2}+ax)^{\frac{5}{2}}}{a\,x^{3}}+\frac{4b\left(\frac{2\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{a\,x^{2}}-\frac{6b\left(\frac{\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{3}+\frac{a\left(\frac{(2bx+a\cdots}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}\right)}{a\cdots}\cdots\right.$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-2/7/b^(1/2)*(-21/2*a^2*b*d^2*x^4*(a*d+2*b*c)*arctanh((x*(b*x+a))^(1/2)/x/
b^(1/2))+(8/5*(-35/16*d^3*x^3+35/2*c*d^2*x^2+21/4*c^2*d*x+c^3)*x*a^2*b^(3/
2)+1/5*a*c^2*x^2*(21*d*x+c)*b^(5/2)-2/5*b^(7/2)*c^3*x^3+a^3*b^(1/2)*(7*d^3
*x^3+7*c*d^2*x^2+21/5*c^2*d*x+c^3))*(x*(b*x+a))^(1/2))/x^4/a^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.25

$$\int \frac{(c+dx)^3\,(ax+bx^2)^{3/2}}{x^6}\,dx = \left[\frac{105\,(2\,a^2bcd^2+a^3d^3)\sqrt{b}x^4\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(35\,a^2bd^3 \cdots}{\cdots}\right.$$
$$-\frac{105\,(2\,a^2bcd^2+a^3d^3)\sqrt{-b}x^4\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(35\,a^2bd^3x^4-10\,a^3c^3+2\,(2\,b^3c^3-21\,ab^2c^2d-140 \cdots}{35\,a^2x^4 \cdots}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
[1/70*(105*(2*a^2*b*c*d^2 + a^3*d^3)*sqrt(b)*x^4*log(2*b*x + a + 2*sqrt(b*
x^2 + a*x)*sqrt(b)) + 2*(35*a^2*b*d^3*x^4 - 10*a^3*c^3 + 2*(2*b^3*c^3 - 21
*a*b^2*c^2*d - 140*a^2*b*c*d^2 - 35*a^3*d^3)*x^3 - 2*(a*b^2*c^3 + 42*a^2*b
*c^2*d + 35*a^3*c*d^2)*x^2 - 2*(8*a^2*b*c^3 + 21*a^3*c^2*d)*x)*sqrt(b*x^2
+ a*x))/(a^2*x^4), -1/35*(105*(2*a^2*b*c*d^2 + a^3*d^3)*sqrt(-b)*x^4*arcta
n(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (35*a^2*b*d^3*x^4 - 10*a^3*c^3 +
 2*(2*b^3*c^3 - 21*a*b^2*c^2*d - 140*a^2*b*c*d^2 - 35*a^3*d^3)*x^3 - 2*(a*
b^2*c^3 + 42*a^2*b*c^2*d + 35*a^3*c*d^2)*x^2 - 2*(8*a^2*b*c^3 + 21*a^3*c^2
*d)*x)*sqrt(b*x^2 + a*x))/(a^2*x^4)]
```

Sympy [F]

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^6}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}} (c+dx)^3}{x^6}\,dx$$

input
```
integrate((d*x+c)**3*(b*x**2+a*x)**(3/2)/x**6,x)
```

output
```
Integral((x*(a + b*x))**(3/2)*(c + d*x)**3/x**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(150) = 300$.

Time = 0.04 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.13

$$\begin{aligned}\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^6}\,dx &= 3\,b^{\frac{3}{2}}cd^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)\\ &+\frac{3}{2}\,a\sqrt{b}d^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+\frac{4\,\sqrt{bx^2+ax}b^3c^3}{35\,a^2x}\\ &-\frac{6\,\sqrt{bx^2+ax}b^2c^2d}{5\,ax}-\frac{7\,\sqrt{bx^2+ax}bcd^2}{x}-\frac{3\,\sqrt{bx^2+ax}ad^3}{x}\\ &-\frac{2\,\sqrt{bx^2+ax}b^2c^3}{35\,ax^2}+\frac{3\,\sqrt{bx^2+ax}bc^2d}{5\,x^2}-\frac{\sqrt{bx^2+ax}acd^2}{x^2}\\ &+\frac{(bx^2+ax)^{\frac{3}{2}}d^3}{x^2}+\frac{3\,\sqrt{bx^2+ax}bc^3}{70\,x^3}+\frac{9\,\sqrt{bx^2+ax}ac^2d}{5\,x^3}-\frac{(bx^2+ax)^{\frac{3}{2}}cd^2}{x^3}\\ &+\frac{3\,\sqrt{bx^2+ax}ac^3}{14\,x^4}-\frac{3\,(bx^2+ax)^{\frac{3}{2}}c^2d}{x^4}-\frac{(bx^2+ax)^{\frac{3}{2}}c^3}{2\,x^5}\end{aligned}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
3*b^(3/2)*c*d^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 3/2*a*sqrt(
b)*d^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 4/35*sqrt(b*x^2 + a*
x)*b^3*c^3/(a^2*x) - 6/5*sqrt(b*x^2 + a*x)*b^2*c^2*d/(a*x) - 7*sqrt(b*x^2
+ a*x)*b*c*d^2/x - 3*sqrt(b*x^2 + a*x)*a*d^3/x - 2/35*sqrt(b*x^2 + a*x)*b^
2*c^3/(a*x^2) + 3/5*sqrt(b*x^2 + a*x)*b*c^2*d/x^2 - sqrt(b*x^2 + a*x)*a*c*
d^2/x^2 + (b*x^2 + a*x)^(3/2)*d^3/x^2 + 3/70*sqrt(b*x^2 + a*x)*b*c^3/x^3 +
 9/5*sqrt(b*x^2 + a*x)*a*c^2*d/x^3 - (b*x^2 + a*x)^(3/2)*c*d^2/x^3 + 3/14*
sqrt(b*x^2 + a*x)*a*c^3/x^4 - 3*(b*x^2 + a*x)^(3/2)*c^2*d/x^4 - 1/2*(b*x^2
 + a*x)^(3/2)*c^3/x^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(150) = 300$.

Time = 0.15 (sec) , antiderivative size = 525, normalized size of antiderivative = 3.09

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^6} \, dx = \sqrt{bx^2+ax}bd^3 - \frac{3(2b^2cd^2+abd^3)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{2\sqrt{b}} + \frac{2\left(105\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6 b^2c^2d + 210\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6 abcd^2 + 35\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6 a^2d^3 + 3\right.}{}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^6,x, algorithm="giac")
```

output
```
sqrt(b*x^2 + a*x)*b*d^3 - 3/2*(2*b^2*c*d^2 + a*b*d^3)*log(abs(2*(sqrt(b)*x
 - sqrt(b*x^2 + a*x))*sqrt(b) + a))/sqrt(b) + 2/35*(105*(sqrt(b)*x - sqrt(
b*x^2 + a*x))^6*b^2*c^2*d + 210*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a*b*c*d^
2 + 35*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a^2*d^3 + 35*(sqrt(b)*x - sqrt(b*
x^2 + a*x))^5*b^(5/2)*c^3 + 210*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a*b^(3/2
)*c^2*d + 105*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a^2*sqrt(b)*c*d^2 + 105*(s
qrt(b)*x - sqrt(b*x^2 + a*x))^4*a*b^2*c^3 + 210*(sqrt(b)*x - sqrt(b*x^2 +
a*x))^4*a^2*b*c^2*d + 35*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^3*c*d^2 + 140
*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^2*b^(3/2)*c^3 + 105*(sqrt(b)*x - sqrt
(b*x^2 + a*x))^3*a^3*sqrt(b)*c^2*d + 98*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*
a^3*b*c^3 + 21*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^4*c^2*d + 35*(sqrt(b)*x
 - sqrt(b*x^2 + a*x))*a^4*sqrt(b)*c^3 + 5*a^5*c^3)/(sqrt(b)*x - sqrt(b*x^2
 + a*x))^7
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3\,(ax+bx^2)^{3/2}}{x^6}\,dx = \int \frac{(b\,x^2+a\,x)^{3/2}\,(c+d\,x)^3}{x^6}\,dx$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^3)/x^6,x)
```

output
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^3)/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.01

$$\int \frac{(c+dx)^3\,(ax+bx^2)^{3/2}}{x^6}\,dx = \frac{-10\sqrt{x}\,\sqrt{bx+a}\,a^3c^3-42\sqrt{x}\,\sqrt{bx+a}\,a^3c^2dx-70\sqrt{x}\,\sqrt{bx+a}\,a^3c\,d^2x^2-}{}$$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^6,x)
```

output
```
( - 10*sqrt(x)*sqrt(a + b*x)*a**3*c**3 - 42*sqrt(x)*sqrt(a + b*x)*a**3*c**
2*d*x - 70*sqrt(x)*sqrt(a + b*x)*a**3*c*d**2*x**2 - 70*sqrt(x)*sqrt(a + b*
x)*a**3*d**3*x**3 - 16*sqrt(x)*sqrt(a + b*x)*a**2*b*c**3*x - 84*sqrt(x)*sq
rt(a + b*x)*a**2*b*c**2*d*x**2 - 280*sqrt(x)*sqrt(a + b*x)*a**2*b*c*d**2*x
**3 + 35*sqrt(x)*sqrt(a + b*x)*a**2*b*d**3*x**4 - 2*sqrt(x)*sqrt(a + b*x)*
a*b**2*c**3*x**2 - 42*sqrt(x)*sqrt(a + b*x)*a*b**2*c**2*d*x**3 + 4*sqrt(x)
*sqrt(a + b*x)*b**3*c**3*x**3 + 105*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*s
qrt(b))/sqrt(a))*a**3*d**3*x**4 + 210*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)
*sqrt(b))/sqrt(a))*a**2*b*c*d**2*x**4 + 75*sqrt(b)*a**3*d**3*x**4 + 160*sq
rt(b)*a**2*b*c*d**2*x**4 - 18*sqrt(b)*a*b**2*c**2*d*x**4 - 4*sqrt(b)*b**3*
c**3*x**4)/(35*a**2*x**4)
```

3.88 $\int \frac{(c+dx)^3 \left(ax+bx^2\right)^{3/2}}{x^7} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 188

$$\int \frac{(c+dx)^3 \left(ax+bx^2\right)^{3/2}}{x^7} \, dx = -\frac{2ad^3\sqrt{ax+bx^2}}{3x^2}-\frac{8bd^3\sqrt{ax+bx^2}}{3x} -\frac{2c^3(ax+bx^2)^{5/2}}{9ax^7}+\frac{2c^2(4bc-27ad)\left(ax+bx^2\right)^{5/2}}{63a^2x^6} -\frac{2c(8b^2c^2-54abcd+189a^2d^2)\left(ax+bx^2\right)^{5/2}}{315a^3x^5}+2b^{3/2}d^3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)$$

output

```
-2/3*a*d^3*(b*x^2+a*x)^(1/2)/x^2-8/3*b*d^3*(b*x^2+a*x)^(1/2)/x-2/9*c^3*(b*
x^2+a*x)^(5/2)/a/x^7+2/63*c^2*(-27*a*d+4*b*c)*(b*x^2+a*x)^(5/2)/a^2/x^6-2/
315*c*(189*a^2*d^2-54*a*b*c*d+8*b^2*c^2)*(b*x^2+a*x)^(5/2)/a^3/x^5+2*b^(3/
2)*d^3*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.05

$$\int \frac{(c+dx)^3\left(ax+bx^2\right)^{3/2}}{x^7}\,dx = \frac{2\sqrt{x(a+bx)}\left(-\frac{8b^4c^3x^4}{a^3}+\frac{2b^3c^2x^3(2c+27dx)}{a^2}-\frac{3b^2cx^2(c^2+9cdx+63d^2x^2)}{a}-a(35c^3+\right.}{}$$

input
```
Integrate[((c + d*x)^3*(a*x + b*x^2)^(3/2))/x^7,x]
```

output
```
(2*Sqrt[x*(a + b*x)]*((-8*b^4*c^3*x^4)/a^3 + (2*b^3*c^2*x^3*(2*c + 27*d*x)
)/a^2 - (3*b^2*c*x^2*(c^2 + 9*c*d*x + 63*d^2*x^2))/a - a*(35*c^3 + 135*c^2
*d*x + 189*c*d^2*x^2 + 105*d^3*x^3) - 2*b*x*(25*c^3 + 108*c^2*d*x + 189*c*
d^2*x^2 + 210*d^3*x^3) - (315*b^(3/2)*d^3*x^(9/2)*Log[-(Sqrt[b]*Sqrt[x]) +
 Sqrt[a + b*x]])/Sqrt[a + b*x]))/(315*x^5)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{1290\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{3/2}(c+dx)^3}{x^7}\,dx$$

$$\Big\downarrow \; 1290$$

$$\text{Indeterminate}$$

input
```
Int[((c + d*x)^3*(a*x + b*x^2)^(3/2))/x^7,x]
```

output
```
Indeterminate
```

Defintions of rubi rules used

rule 1290
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(f + g*x)^
n, d + e*x, x], R = PolynomialRemainder[(f + g*x)^n, d + e*x, x]}, Simp[(e*
R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*
e^2)), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1
)*(a + b*x + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R
*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 1] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$\frac{2a^3b^{\frac{3}{2}}d^3\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)x^5-\frac{2\left(\left(3d^3x^3+\frac{27}{5}c\,d^2x^2+\frac{27}{7}c^2dx+c^3\right)a^4+\frac{10xb\left(\frac{42}{5}d^3x^3+\frac{189}{25}c\,d^2x^2+\frac{108}{25}c^2dx+c^3\right)a^3}{7}+\frac{3b^2c\,x^2}{}\right.}{9}}{a^3x^5}$
risch	$-\frac{2(bx+a)(420a^3b\,d^3x^4+189a^2b^2c\,d^2x^4-54a\,b^3c^2d\,x^4+8b^4c^3x^4+105a^4d^3x^3+378a^3bc\,d^2x^3+27a^2b^2c^2d\,x^3-4a\,b^3c^3x^3+1}{315x^4\sqrt{x(bx+a)}\,a^3}$
default	$c^3\left(-\frac{2(b\,x^2+ax)^{\frac{5}{2}}}{9a\,x^7}-\frac{4b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{7a\,x^6}+\frac{4b\left(b\,x^2+ax\right)^{\frac{5}{2}}}{35a^2x^5}\right)}{9a}\right)+d^3\left(-\frac{2(b\,x^2+ax)^{\frac{5}{2}}}{3a\,x^4}+\frac{2b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{a\,x^3}+- \quad 4b\right.}{}\right.$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
2/9*(9*a^3*b^(3/2)*d^3*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*x^5-((3*d^3*x^
3+27/5*c*d^2*x^2+27/7*c^2*d*x+c^3)*a^4+10/7*x*b*(42/5*d^3*x^3+189/25*c*d^2
*x^2+108/25*c^2*d*x+c^3)*a^3+3/35*b^2*c*x^2*(63*d^2*x^2+9*c*d*x+c^2)*a^2-4
/35*x^3*(27/2*d*x+c)*b^3*c^2*a+8/35*b^4*c^3*x^4)*(x*(b*x+a))^(1/2))/a^3/x^
5
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.30

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^7}\,dx = \left[\frac{315\,a^3b^{\frac{3}{2}}d^3x^5\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,(35\,a^4c^3+(8\,b^4c^3-54\ldots}{\ldots}\right.$$

$$-\frac{2\left(315\,a^3\sqrt{-b}bd^3x^5\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+(35\,a^4c^3+(8\,b^4c^3-54\,ab^3c^2d+189\,a^2b^2cd^2+420\,a^3bd^3)x^4\ldots\right.}{\ldots}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
[1/315*(315*a^3*b^(3/2)*d^3*x^5*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b
)) - 2*(35*a^4*c^3 + (8*b^4*c^3 - 54*a*b^3*c^2*d + 189*a^2*b^2*c*d^2 + 420
*a^3*b*d^3)*x^4 - (4*a*b^3*c^3 - 27*a^2*b^2*c^2*d - 378*a^3*b*c*d^2 - 105*
a^4*d^3)*x^3 + 3*(a^2*b^2*c^3 + 72*a^3*b*c^2*d + 63*a^4*c*d^2)*x^2 + 5*(10
*a^3*b*c^3 + 27*a^4*c^2*d)*x)*sqrt(b*x^2 + a*x))/(a^3*x^5), -2/315*(315*a^
3*sqrt(-b)*b*d^3*x^5*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (35*a^
4*c^3 + (8*b^4*c^3 - 54*a*b^3*c^2*d + 189*a^2*b^2*c*d^2 + 420*a^3*b*d^3)*x
^4 - (4*a*b^3*c^3 - 27*a^2*b^2*c^2*d - 378*a^3*b*c*d^2 - 105*a^4*d^3)*x^3
+ 3*(a^2*b^2*c^3 + 72*a^3*b*c^2*d + 63*a^4*c*d^2)*x^2 + 5*(10*a^3*b*c^3 +
27*a^4*c^2*d)*x)*sqrt(b*x^2 + a*x))/(a^3*x^5)]
```

Sympy [F]

$$\int \frac{(c+dx)^3\left(ax+bx^2\right)^{3/2}}{x^7}\,dx=\int \frac{(x(a+bx))^{\frac{3}{2}}\,(c+dx)^3}{x^7}\,dx$$

input
```
integrate((d*x+c)**3*(b*x**2+a*x)**(3/2)/x**7,x)
```

output
```
Integral((x*(a + b*x))**(3/2)*(c + d*x)**3/x**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. 2(162) = 324.

Time = 0.04 (sec) , antiderivative size = 426, normalized size of antiderivative = 2.27

$$\begin{aligned}\int \frac{(c+dx)^3\left(ax+bx^2\right)^{3/2}}{x^7}\,dx &= b^{\frac{3}{2}}d^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)\\
&-\frac{16\,\sqrt{bx^2+ax}b^4c^3}{315\,a^3x}+\frac{12\,\sqrt{bx^2+ax}b^3c^2d}{35\,a^2x}-\frac{6\,\sqrt{bx^2+ax}b^2cd^2}{5\,ax}\\
&-\frac{7\,\sqrt{bx^2+ax}bd^3}{3\,x}+\frac{8\,\sqrt{bx^2+ax}b^3c^3}{315\,a^2x^2}-\frac{6\,\sqrt{bx^2+ax}b^2c^2d}{35\,ax^2}\\
&+\frac{3\,\sqrt{bx^2+ax}bcd^2}{5\,x^2}-\frac{\sqrt{bx^2+ax}ad^3}{3\,x^2}-\frac{2\,\sqrt{bx^2+ax}b^2c^3}{105\,ax^3}+\frac{9\,\sqrt{bx^2+ax}bc^2d}{70\,x^3}\\
&+\frac{9\,\sqrt{bx^2+ax}acd^2}{5\,x^3}-\frac{(bx^2+ax)^{\frac{3}{2}}d^3}{3\,x^3}+\frac{\sqrt{bx^2+ax}bc^3}{63\,x^4}+\frac{9\,\sqrt{bx^2+ax}ac^2d}{14\,x^4}\\
&-\frac{3\,(bx^2+ax)^{\frac{3}{2}}cd^2}{x^4}+\frac{\sqrt{bx^2+ax}ac^3}{9\,x^5}-\frac{3\,(bx^2+ax)^{\frac{3}{2}}c^2d}{2\,x^5}-\frac{(bx^2+ax)^{\frac{3}{2}}c^3}{3\,x^6}\end{aligned}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
b^(3/2)*d^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 16/315*sqrt(b*x
^2 + a*x)*b^4*c^3/(a^3*x) + 12/35*sqrt(b*x^2 + a*x)*b^3*c^2*d/(a^2*x) - 6/
5*sqrt(b*x^2 + a*x)*b^2*c*d^2/(a*x) - 7/3*sqrt(b*x^2 + a*x)*b*d^3/x + 8/31
5*sqrt(b*x^2 + a*x)*b^3*c^3/(a^2*x^2) - 6/35*sqrt(b*x^2 + a*x)*b^2*c^2*d/(
a*x^2) + 3/5*sqrt(b*x^2 + a*x)*b*c*d^2/x^2 - 1/3*sqrt(b*x^2 + a*x)*a*d^3/x
^2 - 2/105*sqrt(b*x^2 + a*x)*b^2*c^3/(a*x^3) + 9/70*sqrt(b*x^2 + a*x)*b*c^
2*d/x^3 + 9/5*sqrt(b*x^2 + a*x)*a*c*d^2/x^3 - 1/3*(b*x^2 + a*x)^(3/2)*d^3/
x^3 + 1/63*sqrt(b*x^2 + a*x)*b*c^3/x^4 + 9/14*sqrt(b*x^2 + a*x)*a*c^2*d/x^
4 - 3*(b*x^2 + a*x)^(3/2)*c*d^2/x^4 + 1/9*sqrt(b*x^2 + a*x)*a*c^3/x^5 - 3/
2*(b*x^2 + a*x)^(3/2)*c^2*d/x^5 - 1/3*(b*x^2 + a*x)^(3/2)*c^3/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(162) = 324$.

Time = 0.18 (sec) , antiderivative size = 684, normalized size of antiderivative = 3.64

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^7}\,dx = -b^{\frac{3}{2}}d^3\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)$$

$$+\frac{2\left(945\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^8 b^2cd^2+630\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^8 abd^3+945\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^7 b^{\frac{5}{2}}c^2d+\right.}{}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^7,x, algorithm="giac")
```

output
```
-b^(3/2)*d^3*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)) + 2/3
15*(945*(sqrt(b)*x - sqrt(b*x^2 + a*x))^8*b^2*c*d^2 + 630*(sqrt(b)*x - sqr
t(b*x^2 + a*x))^8*a*b*d^3 + 945*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*b^(5/2)*
c^2*d + 1890*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*a*b^(3/2)*c*d^2 + 315*(sqrt
(b)*x - sqrt(b*x^2 + a*x))^7*a^2*sqrt(b)*d^3 + 420*(sqrt(b)*x - sqrt(b*x^2
 + a*x))^6*b^3*c^3 + 2835*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a*b^2*c^2*d +
1890*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a^2*b*c*d^2 + 105*(sqrt(b)*x - sqrt
(b*x^2 + a*x))^6*a^3*d^3 + 1575*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a*b^(5/2
)*c^3 + 3780*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a^2*b^(3/2)*c^2*d + 945*(sq
rt(b)*x - sqrt(b*x^2 + a*x))^5*a^3*sqrt(b)*c*d^2 + 2583*(sqrt(b)*x - sqrt(
b*x^2 + a*x))^4*a^2*b^2*c^3 + 2646*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^3*b
*c^2*d + 189*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^4*c*d^2 + 2310*(sqrt(b)*x
 - sqrt(b*x^2 + a*x))^3*a^3*b^(3/2)*c^3 + 945*(sqrt(b)*x - sqrt(b*x^2 + a*
x))^3*a^4*sqrt(b)*c^2*d + 1170*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^4*b*c^3
 + 135*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^5*c^2*d + 315*(sqrt(b)*x - sqrt
(b*x^2 + a*x))*a^5*sqrt(b)*c^3 + 35*a^6*c^3)/(sqrt(b)*x - sqrt(b*x^2 + a*x
))^9
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3\,(ax+bx^2)^{3/2}}{x^7}\,dx = \int \frac{(b\,x^2+a\,x)^{3/2}\,(c+d\,x)^3}{x^7}\,dx$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^3)/x^7,x)
```

output
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^3)/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.03

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^7} dx = \frac{-\frac{2\sqrt{x}\sqrt{bx+a}\,a^4c^3}{9} - \frac{6\sqrt{x}\sqrt{bx+a}\,a^4c^2dx}{7} - \frac{6\sqrt{x}\sqrt{bx+a}\,a^4c\,d^2x^2}{5} - \frac{2\sqrt{x}\sqrt{bx+a}\,a^4d^3x^3}{3} - 20}{}$$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^7,x)
```

output
```
(2*( - 35*sqrt(x)*sqrt(a + b*x)*a**4*c**3 - 135*sqrt(x)*sqrt(a + b*x)*a**4
*c**2*d*x - 189*sqrt(x)*sqrt(a + b*x)*a**4*c*d**2*x**2 - 105*sqrt(x)*sqrt(
a + b*x)*a**4*d**3*x**3 - 50*sqrt(x)*sqrt(a + b*x)*a**3*b*c**3*x - 216*sqr
t(x)*sqrt(a + b*x)*a**3*b*c**2*d*x**2 - 378*sqrt(x)*sqrt(a + b*x)*a**3*b*c
*d**2*x**3 - 420*sqrt(x)*sqrt(a + b*x)*a**3*b*d**3*x**4 - 3*sqrt(x)*sqrt(a
 + b*x)*a**2*b**2*c**3*x**2 - 27*sqrt(x)*sqrt(a + b*x)*a**2*b**2*c**2*d*x*
*3 - 189*sqrt(x)*sqrt(a + b*x)*a**2*b**2*c*d**2*x**4 + 4*sqrt(x)*sqrt(a +
b*x)*a*b**3*c**3*x**3 + 54*sqrt(x)*sqrt(a + b*x)*a*b**3*c**2*d*x**4 - 8*sq
rt(x)*sqrt(a + b*x)*b**4*c**3*x**4 + 315*sqrt(b)*log((sqrt(a + b*x) + sqrt
(x)*sqrt(b))/sqrt(a))*a**3*b*d**3*x**5 + 280*sqrt(b)*a**3*b*d**3*x**5 - 21
*sqrt(b)*a**2*b**2*c*d**2*x**5 - 54*sqrt(b)*a*b**3*c**2*d*x**5 + 8*sqrt(b)
*b**4*c**3*x**5))/(315*a**3*x**5)
```

3.89 $\int \frac{(c+dx)^3\left(ax+bx^2\right)^{3/2}}{x^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{(c+dx)^3\left(ax+bx^2\right)^{3/2}}{x^8}\,dx = -\frac{16c(bc-ad)^2\left(ax+bx^2\right)^{5/2}}{231a^3x^6} + \frac{16(2bc-7ad)(bc-ad)^2\left(ax+bx^2\right)^{5/2}}{1155a^4x^5} + \frac{4(bc-ad)(c+dx)^2\left(ax+bx^2\right)^{5/2}}{33a^2x^7} - \frac{2(c+dx)^3\left(ax+bx^2\right)^{5/2}}{11ax^8}$$

output
```
-16/231*c*(-a*d+b*c)^2*(b*x^2+a*x)^(5/2)/a^3/x^6+16/1155*(-7*a*d+2*b*c)*(-
a*d+b*c)^2*(b*x^2+a*x)^(5/2)/a^4/x^5+4/33*(-a*d+b*c)*(d*x+c)^2*(b*x^2+a*x)
^(5/2)/a^2/x^7-2/11*(d*x+c)^3*(b*x^2+a*x)^(5/2)/a/x^8
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.79

$$\int \frac{(c+dx)^3\left(ax+bx^2\right)^{3/2}}{x^8}\,dx = -\frac{2(x(a+bx))^{5/2}\left(-16b^3c^3x^3+8ab^2c^2x^2(5c+11dx)-2a^2bcx(35c^2+110cdx+99d^2x^2)+a^3(105c^3+385\right.}{1155a^4x^8}$$

input
```
Integrate[((c + d*x)^3*(a*x + b*x^2)^(3/2))/x^8,x]
```

output
```
(-2*(x*(a + b*x))^(5/2)*(-16*b^3*c^3*x^3 + 8*a*b^2*c^2*x^2*(5*c + 11*d*x)
- 2*a^2*b*c*x*(35*c^2 + 110*c*d*x + 99*d^2*x^2) + a^3*(105*c^3 + 385*c^2*d
*x + 495*c*d^2*x^2 + 231*d^3*x^3)))/(1155*a^4*x^8)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.58, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1262, 27, 2169, 27, 1220, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}(c+dx)^3}{x^8}dx$$

$$\Big\downarrow \text{1262}$$

$$-\frac{\int -\frac{(bx^2+ax)^{3/2}(2bc^3+6bdxc^2+d^2(6bc-7ad)x^2)}{2x^8}dx}{b}-\frac{d^3(ax+bx^2)^{5/2}}{bx^6}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(bx^2+ax)^{3/2}(2bc^3+6bdxc^2+d^2(6bc-7ad)x^2)}{x^8}dx}{2b}-\frac{d^3(ax+bx^2)^{5/2}}{bx^6}$$

$$\Big\downarrow \text{2169}$$

$$\frac{-\frac{\int -\frac{(8b^2c^3+3d(8b^2c^2-18abdc+21a^2d^2)x)(bx^2+ax)^{3/2}}{2x^8}dx}{2b}-\frac{d^2(ax+bx^2)^{5/2}(6bc-7ad)}{2bx^7}}{2b}-\frac{d^3(ax+bx^2)^{5/2}}{bx^6}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{(8b^2c^3+3d(8b^2c^2-18abdc+21a^2d^2)x)(bx^2+ax)^{3/2}}{x^8}dx}{4b}-\frac{d^2(ax+bx^2)^{5/2}(6bc-7ad)}{2bx^7}}{2b}-\frac{d^3(ax+bx^2)^{5/2}}{bx^6}$$

$$\Big\downarrow \text{1220}$$

$$\frac{\frac{-\frac{3\left(-231a^3d^3+198a^2bcd^2-88ab^2c^2d+16b^3c^3\right)\int \frac{\left(bx^2+ax\right)^{3/2}}{x^7}dx}{11a}-\frac{16b^2c^3\left(ax+bx^2\right)^{5/2}}{11ax^8}}{4b}-\frac{d^2\left(ax+bx^2\right)^{5/2}(6bc-7ad)}{2bx^7}}{2b}-\frac{d^3\left(ax+bx^2\right)^{5/2}}{bx^6}$$

$\downarrow$ 1129

$$\frac{\frac{-\frac{3\left(-231a^3d^3+198a^2bcd^2-88ab^2c^2d+16b^3c^3\right)\left(-\frac{4b\int \frac{\left(bx^2+ax\right)^{3/2}}{x^6}dx}{9a}-\frac{2\left(ax+bx^2\right)^{5/2}}{9ax^7}\right)}{11a}-\frac{16b^2c^3\left(ax+bx^2\right)^{5/2}}{11ax^8}}{4b}-\frac{d^2\left(ax+bx^2\right)^{5/2}(6bc-7ad)}{2bx^7}}{2b}-\frac{d^3\left(ax+bx^2\right)^{5/2}}{bx^6}$$

$\downarrow$ 1129

$$\frac{\frac{-\frac{3\left(-231a^3d^3+198a^2bcd^2-88ab^2c^2d+16b^3c^3\right)\left(-\frac{4b\left(-\frac{2b\int \frac{\left(bx^2+ax\right)^{3/2}}{x^5}dx}{7a}-\frac{2\left(ax+bx^2\right)^{5/2}}{7ax^6}\right)}{9a}-\frac{2\left(ax+bx^2\right)^{5/2}}{9ax^7}\right)}{11a}-\frac{16b^2c^3\left(ax+bx^2\right)^{5/2}}{11ax^8}}{4b}-\frac{d^2\left(ax+bx^2\right)^{5/2}(6bc-7ad)}{2bx^7}}{2b}-\frac{d^3\left(ax+bx^2\right)^{5/2}}{bx^6}$$

$\downarrow$ 1123

$$\frac{\frac{-\frac{3\left(-\frac{4b\left(\frac{4b\left(ax+bx^2\right)^{5/2}}{35a^2x^5}-\frac{2\left(ax+bx^2\right)^{5/2}}{7ax^6}\right)}{9a}-\frac{2\left(ax+bx^2\right)^{5/2}}{9ax^7}\right)\left(-231a^3d^3+198a^2bcd^2-88ab^2c^2d+16b^3c^3\right)}{11a}-\frac{16b^2c^3\left(ax+bx^2\right)^{5/2}}{11ax^8}}{4b}-\frac{d^2\left(ax+bx^2\right)^{5/2}(6bc-7ad)}{2bx^7}}{2b}-\frac{d^3\left(ax+bx^2\right)^{5/2}}{bx^6}$$

input
```
Int[((c + d*x)^3*(a*x + b*x^2)^(3/2))/x^8,x]
```

output
```
-((d^3*(a*x + b*x^2)^(5/2))/(b*x^6)) + (-1/2*(d^2*(6*b*c - 7*a*d)*(a*x + b
*x^2)^(5/2))/(b*x^7) + ((-16*b^2*c^3*(a*x + b*x^2)^(5/2))/(11*a*x^8) - (3*
(16*b^3*c^3 - 88*a*b^2*c^2*d + 198*a^2*b*c*d^2 - 231*a^3*d^3)*((-2*(a*x +
b*x^2)^(5/2))/(9*a*x^7) - (4*b*((-2*(a*x + b*x^2)^(5/2))/(7*a*x^6) + (4*b*
(a*x + b*x^2)^(5/2))/(35*a^2*x^5)))/(9*a)))/(11*a))/(4*b))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

rule 2169
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d -
b*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2
, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.77

method	result
pseudoelliptic	$-\frac{2\sqrt{x(bx+a)}\,(bx+a)^2\left(\left(\frac{11}{5}d^3x^3+\frac{33}{7}c\,d^2x^2+\frac{11}{3}c^2dx+c^3\right)a^3-\frac{2xbc\left(\frac{99}{35}d^2x^2+\frac{22}{7}cdx+c^2\right)a^2}{3}+\frac{8x^2\left(\frac{11dx}{5}+c\right)b^2c^2a}{21}-\frac{16b^3c^3x}{105}\right.}{11x^6a^4}$
gosper	$-\frac{2(bx+a)(231a^3d^3x^3-198a^2bc\,d^2x^3+88a\,b^2c^2d\,x^3-16b^3c^3x^3+495a^3c\,d^2x^2-220a^2b\,c^2x^2d+40a\,b^2c^3x^2+385a^3c^2dx-70}{1155x^7a^4}$
orering	$-\frac{2(bx+a)(231a^3d^3x^3-198a^2bc\,d^2x^3+88a\,b^2c^2d\,x^3-16b^3c^3x^3+495a^3c\,d^2x^2-220a^2b\,c^2x^2d+40a\,b^2c^3x^2+385a^3c^2dx-70}{1155x^7a^4}$
default	$c^3\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{11a\,x^8}-\frac{6b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{9a\,x^7}-\frac{4b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{7a\,x^6}+\frac{4b\left(b\,x^2+ax\right)^{\frac{5}{2}}}{35a^2x^5}\right)}{9a}\right)}{11a}\right)-\frac{2d^3\left(b\,x^2+ax\right)^{\frac{5}{2}}}{5a\,x^5}+3c\,d^2($
trager	$-\frac{2(231a^3b^2d^3x^5-198a^2b^3c\,d^2x^5+88a\,b^4c^2d\,x^5-16b^5c^3x^5+462a^4b\,d^3x^4+99a^3b^2c\,d^2x^4-44a^2b^3c^2d\,x^4+8a\,b^4c^3x^4+231a}{}$
risch	$-\frac{2(bx+a)(231a^3b^2d^3x^5-198a^2b^3c\,d^2x^5+88a\,b^4c^2d\,x^5-16b^5c^3x^5+462a^4b\,d^3x^4+99a^3b^2c\,d^2x^4-44a^2b^3c^2d\,x^4+8a\,b^4c^3x}{115}$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-2/11*(x*(b*x+a))^(1/2)*(b*x+a)^2*((11/5*d^3*x^3+33/7*c*d^2*x^2+11/3*c^2*d
*x+c^3)*a^3-2/3*x*b*c*(99/35*d^2*x^2+22/7*c*d*x+c^2)*a^2+8/21*x^2*(11/5*d*
x+c)*b^2*c^2*a-16/105*b^3*c^3*x^3)/x^6/a^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.56

$$\int \frac{(c+dx)^3\left(ax+bx^2\right)^{3/2}}{x^8}\,dx =$$

$$-\frac{2\left(105\,a^5c^3-\left(16\,b^5c^3-88\,ab^4c^2d+198\,a^2b^3cd^2-231\,a^3b^2d^3\right)x^5+\left(8\,ab^4c^3-44\,a^2b^3c^2d+99\,a^3b^2cd^2+\right.\right.}{}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
-2/1155*(105*a^5*c^3 - (16*b^5*c^3 - 88*a*b^4*c^2*d + 198*a^2*b^3*c*d^2 -
231*a^3*b^2*d^3)*x^5 + (8*a*b^4*c^3 - 44*a^2*b^3*c^2*d + 99*a^3*b^2*c*d^2
+ 462*a^4*b*d^3)*x^4 - 3*(2*a^2*b^3*c^3 - 11*a^3*b^2*c^2*d - 264*a^4*b*c*d
^2 - 77*a^5*d^3)*x^3 + 5*(a^3*b^2*c^3 + 110*a^4*b*c^2*d + 99*a^5*c*d^2)*x^
2 + 35*(4*a^4*b*c^3 + 11*a^5*c^2*d)*x)*sqrt(b*x^2 + a*x)/(a^4*x^6)
```

Sympy [F]

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^8} \, dx = \int \frac{(x(a+bx))^{\frac{3}{2}} (c+dx)^3}{x^8} \, dx$$

input
```
integrate((d*x+c)**3*(b*x**2+a*x)**(3/2)/x**8,x)
```

output
```
Integral((x*(a + b*x))**(3/2)*(c + d*x)**3/x**8, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(129) = 258$.

Time = 0.04 (sec) , antiderivative size = 498, normalized size of antiderivative = 3.43

$$\begin{aligned}\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^8} \, dx &= \frac{32 \sqrt{bx^2+ax} b^5 c^3}{1155 \, a^4 x} - \frac{16 \sqrt{bx^2+ax} b^4 c^2 d}{105 \, a^3 x} \\ &+ \frac{12 \sqrt{bx^2+ax} b^3 c d^2}{35 \, a^2 x} - \frac{2 \sqrt{bx^2+ax} b^2 d^3}{5 \, ax} - \frac{16 \sqrt{bx^2+ax} b^4 c^3}{1155 \, a^3 x^2} \\ &+ \frac{8 \sqrt{bx^2+ax} b^3 c^2 d}{105 \, a^2 x^2} - \frac{6 \sqrt{bx^2+ax} b^2 c d^2}{35 \, ax^2} + \frac{\sqrt{bx^2+ax} b d^3}{5 \, x^2} \\ &+ \frac{4 \sqrt{bx^2+ax} b^3 c^3}{385 \, a^2 x^3} - \frac{2 \sqrt{bx^2+ax} b^2 c^2 d}{35 \, ax^3} + \frac{9 \sqrt{bx^2+ax} b c d^2}{70 \, x^3} \\ &+ \frac{3 \sqrt{bx^2+ax} a d^3}{5 \, x^3} - \frac{2 \sqrt{bx^2+ax} b^2 c^3}{231 \, ax^4} + \frac{\sqrt{bx^2+ax} b c^2 d}{21 \, x^4} \\ &+ \frac{9 \sqrt{bx^2+ax} a c d^2}{14 \, x^4} - \frac{(bx^2+ax)^{\frac{3}{2}} d^3}{x^4} + \frac{\sqrt{bx^2+ax} b c^3}{132 \, x^5} + \frac{\sqrt{bx^2+ax} a c^2 d}{3 \, x^5} \\ &- \frac{3 (bx^2+ax)^{\frac{3}{2}} c d^2}{2 \, x^5} + \frac{3 \sqrt{bx^2+ax} a c^3}{44 \, x^6} - \frac{(bx^2+ax)^{\frac{3}{2}} c^2 d}{x^6} - \frac{(bx^2+ax)^{\frac{3}{2}} c^3}{4 \, x^7}\end{aligned}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
32/1155*sqrt(b*x^2 + a*x)*b^5*c^3/(a^4*x) - 16/105*sqrt(b*x^2 + a*x)*b^4*c
^2*d/(a^3*x) + 12/35*sqrt(b*x^2 + a*x)*b^3*c*d^2/(a^2*x) - 2/5*sqrt(b*x^2
+ a*x)*b^2*d^3/(a*x) - 16/1155*sqrt(b*x^2 + a*x)*b^4*c^3/(a^3*x^2) + 8/105
*sqrt(b*x^2 + a*x)*b^3*c^2*d/(a^2*x^2) - 6/35*sqrt(b*x^2 + a*x)*b^2*c*d^2/
(a*x^2) + 1/5*sqrt(b*x^2 + a*x)*b*d^3/x^2 + 4/385*sqrt(b*x^2 + a*x)*b^3*c^
3/(a^2*x^3) - 2/35*sqrt(b*x^2 + a*x)*b^2*c^2*d/(a*x^3) + 9/70*sqrt(b*x^2 +
 a*x)*b*c*d^2/x^3 + 3/5*sqrt(b*x^2 + a*x)*a*d^3/x^3 - 2/231*sqrt(b*x^2 + a
*x)*b^2*c^3/(a*x^4) + 1/21*sqrt(b*x^2 + a*x)*b*c^2*d/x^4 + 9/14*sqrt(b*x^2
 + a*x)*a*c*d^2/x^4 - (b*x^2 + a*x)^(3/2)*d^3/x^4 + 1/132*sqrt(b*x^2 + a*x
)*b*c^3/x^5 + 1/3*sqrt(b*x^2 + a*x)*a*c^2*d/x^5 - 3/2*(b*x^2 + a*x)^(3/2)*
c*d^2/x^5 + 3/44*sqrt(b*x^2 + a*x)*a*c^3/x^6 - (b*x^2 + a*x)^(3/2)*c^2*d/x
^6 - 1/4*(b*x^2 + a*x)^(3/2)*c^3/x^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(129) = 258$.

Time = 0.14 (sec) , antiderivative size = 806, normalized size of antiderivative = 5.56

$$\int \frac{(c+dx)^3 \left(ax+bx^2\right)^{3/2}}{x^8} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^8,x, algorithm="giac")
```

output
```
2/1155*(1155*(sqrt(b)*x - sqrt(b*x^2 + a*x))^10*b^2*d^3 + 3465*(sqrt(b)*x
- sqrt(b*x^2 + a*x))^9*b^(5/2)*c*d^2 + 2310*(sqrt(b)*x - sqrt(b*x^2 + a*x)
)^9*a*b^(3/2)*d^3 + 4620*(sqrt(b)*x - sqrt(b*x^2 + a*x))^8*b^3*c^2*d + 103
95*(sqrt(b)*x - sqrt(b*x^2 + a*x))^8*a*b^2*c*d^2 + 2310*(sqrt(b)*x - sqrt(
b*x^2 + a*x))^8*a^2*b*d^3 + 2310*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*b^(7/2)
*c^3 + 17325*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*a*b^(5/2)*c^2*d + 13860*(sq
rt(b)*x - sqrt(b*x^2 + a*x))^7*a^2*b^(3/2)*c*d^2 + 1155*(sqrt(b)*x - sqrt(
b*x^2 + a*x))^7*a^3*sqrt(b)*d^3 + 10164*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*
a*b^3*c^3 + 28413*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a^2*b^2*c^2*d + 9702*(
sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a^3*b*c*d^2 + 231*(sqrt(b)*x - sqrt(b*x^2
 + a*x))^6*a^4*d^3 + 19635*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a^2*b^(5/2)*c
^3 + 25410*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a^3*b^(3/2)*c^2*d + 3465*(sqr
t(b)*x - sqrt(b*x^2 + a*x))^5*a^4*sqrt(b)*c*d^2 + 21285*(sqrt(b)*x - sqrt(
b*x^2 + a*x))^4*a^3*b^2*c^3 + 12870*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^4*
b*c^2*d + 495*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^5*c*d^2 + 13860*(sqrt(b)
*x - sqrt(b*x^2 + a*x))^3*a^4*b^(3/2)*c^3 + 3465*(sqrt(b)*x - sqrt(b*x^2 +
 a*x))^3*a^5*sqrt(b)*c^2*d + 5390*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^5*b*
c^3 + 385*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^6*c^2*d + 1155*(sqrt(b)*x -
sqrt(b*x^2 + a*x))*a^6*sqrt(b)*c^3 + 105*a^7*c^3)/(sqrt(b)*x - sqrt(b*x^2
+ a*x))^11
```

Mupad [B] (verification not implemented)

Time = 13.49 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.90

$$\int \frac{(c+dx)^3\left(ax+bx^2\right)^{3/2}}{x^8}\,dx = \frac{4\,b^3\,c^3\,\sqrt{b\,x^2+a\,x}}{385\,a^2\,x^3} - \frac{2\,a\,d^3\,\sqrt{b\,x^2+a\,x}}{5\,x^3} - \frac{8\,b\,c^3\,\sqrt{b\,x^2+a\,x}}{33\,x^5} - \frac{4\,b\,d^3\,\sqrt{b\,x^2+a\,x}}{5\,x^2} - \frac{2\,b^2\,c^3\,\sqrt{b\,x^2+a\,x}}{231\,a\,x^4} - \frac{2\,a\,c^3\,\sqrt{b\,x^2+a\,x}}{11\,x^6} - \frac{16\,b^4\,c^3\,\sqrt{b\,x^2+a\,x}}{1155\,a^3\,x^2} + \frac{32\,b^5\,c^3\,\sqrt{b\,x^2+a\,x}}{1155\,a^4\,x} - \frac{2\,b^2\,d^3\,\sqrt{b\,x^2+a\,x}}{5\,a\,x} - \frac{6\,a\,c\,d^2\,\sqrt{b\,x^2+a\,x}}{7\,x^4} - \frac{2\,a\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{3\,x^5} - \frac{48\,b\,c\,d^2\,\sqrt{b\,x^2+a\,x}}{35\,x^3} - \frac{20\,b\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{21\,x^4} - \frac{6\,b^2\,c\,d^2\,\sqrt{b\,x^2+a\,x}}{35\,a\,x^2} - \frac{2\,b^2\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{35\,a\,x^3} + \frac{12\,b^3\,c\,d^2\,\sqrt{b\,x^2+a\,x}}{35\,a^2\,x} + \frac{8\,b^3\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{105\,a^2\,x^2} - \frac{16\,b^4\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{105\,a^3\,x}$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^3)/x^8,x)
```

output
```
(4*b^3*c^3*(a*x + b*x^2)^(1/2))/(385*a^2*x^3) - (2*a*d^3*(a*x + b*x^2)^(1/
2))/(5*x^3) - (8*b*c^3*(a*x + b*x^2)^(1/2))/(33*x^5) - (4*b*d^3*(a*x + b*x
^2)^(1/2))/(5*x^2) - (2*b^2*c^3*(a*x + b*x^2)^(1/2))/(231*a*x^4) - (2*a*c^
3*(a*x + b*x^2)^(1/2))/(11*x^6) - (16*b^4*c^3*(a*x + b*x^2)^(1/2))/(1155*a
^3*x^2) + (32*b^5*c^3*(a*x + b*x^2)^(1/2))/(1155*a^4*x) - (2*b^2*d^3*(a*x
+ b*x^2)^(1/2))/(5*a*x) - (6*a*c*d^2*(a*x + b*x^2)^(1/2))/(7*x^4) - (2*a*c
^2*d*(a*x + b*x^2)^(1/2))/(3*x^5) - (48*b*c*d^2*(a*x + b*x^2)^(1/2))/(35*x
^3) - (20*b*c^2*d*(a*x + b*x^2)^(1/2))/(21*x^4) - (6*b^2*c*d^2*(a*x + b*x^
2)^(1/2))/(35*a*x^2) - (2*b^2*c^2*d*(a*x + b*x^2)^(1/2))/(35*a*x^3) + (12*
b^3*c*d^2*(a*x + b*x^2)^(1/2))/(35*a^2*x) + (8*b^3*c^2*d*(a*x + b*x^2)^(1/
2))/(105*a^2*x^2) - (16*b^4*c^2*d*(a*x + b*x^2)^(1/2))/(105*a^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 441, normalized size of antiderivative = 3.04

$$\int \frac{(c+dx)^3\,(ax+bx^2)^{3/2}}{x^8}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^5c^3}{11} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,a^5c^2dx}{3} - \frac{6\sqrt{x}\,\sqrt{bx+a}\,a^5cd^2x^2}{7} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,a^5d^3x^3}{5} - \frac{8\sqrt{}}{}}{}$$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^8,x)
```

output
```
(2*( - 105*sqrt(x)*sqrt(a + b*x)*a**5*c**3 - 385*sqrt(x)*sqrt(a + b*x)*a**
5*c**2*d*x - 495*sqrt(x)*sqrt(a + b*x)*a**5*c*d**2*x**2 - 231*sqrt(x)*sqrt
(a + b*x)*a**5*d**3*x**3 - 140*sqrt(x)*sqrt(a + b*x)*a**4*b*c**3*x - 550*s
qrt(x)*sqrt(a + b*x)*a**4*b*c**2*d*x**2 - 792*sqrt(x)*sqrt(a + b*x)*a**4*b
*c*d**2*x**3 - 462*sqrt(x)*sqrt(a + b*x)*a**4*b*d**3*x**4 - 5*sqrt(x)*sqrt
(a + b*x)*a**3*b**2*c**3*x**2 - 33*sqrt(x)*sqrt(a + b*x)*a**3*b**2*c**2*d*
x**3 - 99*sqrt(x)*sqrt(a + b*x)*a**3*b**2*c*d**2*x**4 - 231*sqrt(x)*sqrt(a
 + b*x)*a**3*b**2*d**3*x**5 + 6*sqrt(x)*sqrt(a + b*x)*a**2*b**3*c**3*x**3
+ 44*sqrt(x)*sqrt(a + b*x)*a**2*b**3*c**2*d*x**4 + 198*sqrt(x)*sqrt(a + b*
x)*a**2*b**3*c*d**2*x**5 - 8*sqrt(x)*sqrt(a + b*x)*a*b**4*c**3*x**4 - 88*s
qrt(x)*sqrt(a + b*x)*a*b**4*c**2*d*x**5 + 16*sqrt(x)*sqrt(a + b*x)*b**5*c*
*3*x**5 + 21*sqrt(b)*a**3*b**2*d**3*x**6 - 198*sqrt(b)*a**2*b**3*c*d**2*x*
*6 + 88*sqrt(b)*a*b**4*c**2*d*x**6 - 16*sqrt(b)*b**5*c**3*x**6))/(1155*a**
4*x**6)
```

3.90 $\int \frac{(c+dx)^3(ax+bx^2)^{3/2}}{x^9} dx$

Optimal result

Integrand size = 24, antiderivative size = 222

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^9} dx = \frac{16(bc-ad)^2(8bc+5ad) (ax+bx^2)^{5/2}}{3003a^4x^6} - \frac{16(2bc-7ad)(bc-ad)^2(8bc+5ad) (ax+bx^2)^{5/2}}{15015a^5cx^5} - \frac{4(bc-ad)(8bc+5ad)(c+dx)^2 (ax+bx^2)^{5/2}}{429a^3cx^7} + \frac{2(8bc+5ad)(c+dx)^3 (ax+bx^2)^{5/2}}{143a^2cx^8} - \frac{2(c+dx)^4 (ax+bx^2)^{5/2}}{13acx^9}$$

output
```
16/3003*(-a*d+b*c)^2*(5*a*d+8*b*c)*(b*x^2+a*x)^(5/2)/a^4/x^6-16/15015*(-7*
a*d+2*b*c)*(-a*d+b*c)^2*(5*a*d+8*b*c)*(b*x^2+a*x)^(5/2)/a^5/c/x^5-4/429*(-
a*d+b*c)*(5*a*d+8*b*c)*(d*x+c)^2*(b*x^2+a*x)^(5/2)/a^3/c/x^7+2/143*(5*a*d+
8*b*c)*(d*x+c)^3*(b*x^2+a*x)^(5/2)/a^2/c/x^8-2/13*(d*x+c)^4*(b*x^2+a*x)^(5
/2)/a/c/x^9
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.70

$$\int \frac{(c+dx)^3 \left(ax+bx^2\right)^{3/2}}{x^9} \, dx = \frac{2(x(a+bx))^{5/2} \left(-128b^4c^3x^4+16ab^3c^2x^3(20c+39dx)-8a^2b^2cx^2(70c^2+\right.}{}$$

input
```
Integrate[((c + d*x)^3*(a*x + b*x^2)^(3/2))/x^9,x]
```

output
```
(2*(x*(a + b*x))^(5/2)*(-128*b^4*c^3*x^4 + 16*a*b^3*c^2*x^3*(20*c + 39*d*x
) - 8*a^2*b^2*c*x^2*(70*c^2 + 195*c*d*x + 143*d^2*x^2) - 5*a^4*(231*c^3 +
819*c^2*d*x + 1001*c*d^2*x^2 + 429*d^3*x^3) + 2*a^3*b*x*(420*c^3 + 1365*c^
2*d*x + 1430*c*d^2*x^2 + 429*d^3*x^3)))/(15015*a^5*x^9)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1262, 27, 2169, 27, 1220, 1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{3/2} (c+dx)^3}{x^9} \, dx$$

$$\Big\downarrow \text{1262}$$

$$-\frac{\int -\frac{\left(bx^2+ax\right)^{3/2} \left(4bc^3+12bdxc^2+3d^2(4bc-3ad)x^2\right)}{2x^9}dx}{2b}-\frac{d^3\left(ax+bx^2\right)^{5/2}}{2bx^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(bx^2+ax\right)^{3/2} \left(4bc^3+12bdxc^2+3d^2(4bc-3ad)x^2\right)}{x^9}dx}{4b}-\frac{d^3\left(ax+bx^2\right)^{5/2}}{2bx^7}$$

$$\Big\downarrow \text{2169}$$

$$-\frac{\frac{\int -\frac{3\left(8b^2c^3+d\left(24b^2c^2-44abdc+33a^2d^2\right)x\right)\left(bx^2+ax\right)^{3/2}}{2x^9}dx}{3b}-\frac{d^2\left(ax+bx^2\right)^{5/2}(4bc-3ad)}{bx^8}}{4b}-\frac{d^3\left(ax+bx^2\right)^{5/2}}{2bx^7}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\left(8b^2c^3+d\left(24b^2c^2-44abdc+33a^2d^2\right)x\right)\left(bx^2+ax\right)^{3/2}}{x^9}dx}{2b}-\frac{d^2\left(ax+bx^2\right)^{5/2}(4bc-3ad)}{bx^8}}{4b}-\frac{d^3\left(ax+bx^2\right)^{5/2}}{2bx^7}$$

$\Big\downarrow$ 1220

$$\frac{\frac{-\frac{\left(-429a^3d^3+572a^2bcd^2-312ab^2c^2d+64b^3c^3\right)\int \frac{\left(bx^2+ax\right)^{3/2}}{x^8}dx}{13a}-\frac{16b^2c^3\left(ax+bx^2\right)^{5/2}}{13ax^9}}{2b}-\frac{d^2\left(ax+bx^2\right)^{5/2}(4bc-3ad)}{bx^8}}{4b}-\frac{d^3\left(ax+bx^2\right)^{5/2}}{2bx^7}$$

$\Big\downarrow$ 1129

$$\frac{\frac{-\frac{\left(-429a^3d^3+572a^2bcd^2-312ab^2c^2d+64b^3c^3\right)\left(-\frac{6b\int \frac{\left(bx^2+ax\right)^{3/2}}{x^7}dx}{11a}-\frac{2\left(ax+bx^2\right)^{5/2}}{11ax^8}\right)}{13a}-\frac{16b^2c^3\left(ax+bx^2\right)^{5/2}}{13ax^9}}{2b}-\frac{d^2\left(ax+bx^2\right)^{5/2}(4bc-3ad)}{bx^8}}{4b}-\frac{d^3\left(ax+bx^2\right)^{5/2}}{2bx^7}$$

$\Big\downarrow$ 1129

$$\frac{\frac{-\frac{\left(-429a^3d^3+572a^2bcd^2-312ab^2c^2d+64b^3c^3\right)\left(-\frac{6b\left(-\frac{4b\int \frac{\left(bx^2+ax\right)^{3/2}}{x^6}dx}{9a}-\frac{2\left(ax+bx^2\right)^{5/2}}{9ax^7}\right)}{11a}-\frac{2\left(ax+bx^2\right)^{5/2}}{11ax^8}\right)}{13a}-\frac{16b^2c^3\left(ax+bx^2\right)^{5/2}}{13ax^9}}{2b}-\frac{d^2\left(ax+bx\right.}{}}{4b}$$

$$\frac{d^3\left(ax+bx^2\right)^{5/2}}{2bx^7}$$

$\Big\downarrow$ 1129

$$\frac{-\frac{\left(-429a^3d^3+572a^2bcd^2-312ab^2c^2d+64b^3c^3\right)\left(-\frac{6b\left(-\frac{4b\left(-\frac{2b\int\frac{\left(bx^2+ax\right)^{3/2}}{x^5}dx}{7a}-\frac{2\left(ax+bx^2\right)^{5/2}}{7ax^6}\right)}{9a}-\frac{2\left(ax+bx^2\right)^{5/2}}{9ax^7}\right)}{11a}-\frac{2\left(ax+bx^2\right)^{5/2}}{11ax^8}\right)}{13a}-\frac{16b^2c^3\left(ax+\cdots\right.}{13ax\cdots}}{2b}}{4b}$$

$$\frac{d^3\left(ax+bx^2\right)^{5/2}}{2bx^7}$$

$$\Big\downarrow \ 1123$$

$$\frac{-\frac{\left(-\frac{6b\left(-\frac{4b\left(\frac{4b\left(ax+bx^2\right)^{5/2}}{35a^2x^5}-\frac{2\left(ax+bx^2\right)^{5/2}}{7ax^6}\right)}{9a}-\frac{2\left(ax+bx^2\right)^{5/2}}{9ax^7}\right)}{11a}-\frac{2\left(ax+bx^2\right)^{5/2}}{11ax^8}\right)\left(-429a^3d^3+572a^2bcd^2-312ab^2c^2d+64b^3c^3\right)}{13a}-\frac{16b^2c^3\left(ax+bx^2\right)^{5/\cdots}}{13ax^9}}{2b}}{4b}$$

$$\frac{d^3\left(ax+bx^2\right)^{5/2}}{2bx^7}$$

input
```
Int[((c + d*x)^3*(a*x + b*x^2)^(3/2))/x^9,x]
```

output
```
-1/2*(d^3*(a*x + b*x^2)^(5/2))/(b*x^7) + (-((d^2*(4*b*c - 3*a*d)*(a*x + b*
x^2)^(5/2))/(b*x^8)) + ((-16*b^2*c^3*(a*x + b*x^2)^(5/2))/(13*a*x^9) - ((6
4*b^3*c^3 - 312*a*b^2*c^2*d + 572*a^2*b*c*d^2 - 429*a^3*d^3)*((-2*(a*x + b
*x^2)^(5/2))/(11*a*x^8) - (6*b*((-2*(a*x + b*x^2)^(5/2))/(9*a*x^7) - (4*b*
((-2*(a*x + b*x^2)^(5/2))/(7*a*x^6) + (4*b*(a*x + b*x^2)^(5/2))/(35*a^2*x^
5)))/(9*a)))/(11*a)))/(13*a))/(2*b))/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

rule 2169
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d -
b*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2
, 0]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$-\frac{2\sqrt{x(bx+a)}\,(bx+a)^2\left(\left(\frac{13}{7}d^3x^3+\frac{13}{3}c\,d^2x^2+\frac{39}{11}c^2dx+c^3\right)a^4-\frac{8\left(\frac{143}{140}d^3x^3+\frac{143}{42}c\,d^2x^2+\frac{13}{4}c^2dx+c^3\right)xb\,a^3}{11}+\frac{16\left(\frac{143}{70}d^2x^2+\frac{39}{14}\cdots\right.}{3\cdots}\right.}{13x^7a^5}$
gosper	$-\frac{2(bx+a)\left(-858a^3b\,d^3x^4+1144a^2b^2c\,d^2x^4-624a\,b^3c^2d\,x^4+128b^4c^3x^4+2145a^4d^3x^3-2860a^3bc\,d^2x^3+1560a^2b^2c^2d\,x^3-3\cdots\right.}{15015x^8a^5\cdots}$
orering	$-\frac{2(bx+a)\left(-858a^3b\,d^3x^4+1144a^2b^2c\,d^2x^4-624a\,b^3c^2d\,x^4+128b^4c^3x^4+2145a^4d^3x^3-2860a^3bc\,d^2x^3+1560a^2b^2c^2d\,x^3-3\cdots\right.}{15015x^8a^5\cdots}$
trager	$-\frac{2\left(-858a^3b^3d^3x^6+1144a^2b^4c\,d^2x^6-624a\,b^5c^2d\,x^6+128b^6c^3x^6+429a^4b^2d^3x^5-572a^3b^3c\,d^2x^5+312a^2b^4c^2d\,x^5-64a\,b^5c^3\cdots\right.}{\cdots}$
risch	$-\frac{2(bx+a)\left(-858a^3b^3d^3x^6+1144a^2b^4c\,d^2x^6-624a\,b^5c^2d\,x^6+128b^6c^3x^6+429a^4b^2d^3x^5-572a^3b^3c\,d^2x^5+312a^2b^4c^2d\,x^5-\cdots\right.}{\cdots}$
default	$c^3\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{13a\,x^9}-\frac{8b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{11a\,x^8}-\frac{6b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{9a\,x^7}-\frac{4b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{7a\,x^6}+\frac{4b\left(b\,x^2+ax\right)^{\frac{5}{2}}}{35a^2x^5}\right)}{9a}\right)}{11a}\right)}{13a}\right)+d^3\Big(\cdots$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-2/13*(x*(b*x+a))^(1/2)*(b*x+a)^2*((13/7*d^3*x^3+13/3*c*d^2*x^2+39/11*c^2*
d*x+c^3)*a^4-8/11*(143/140*d^3*x^3+143/42*c*d^2*x^2+13/4*c^2*d*x+c^3)*x*b*
a^3+16/33*(143/70*d^2*x^2+39/14*c*d*x+c^2)*x^2*b^2*c*a^2-64/231*(39/20*d*x
+c)*x^3*b^3*c^2*a+128/1155*b^4*c^3*x^4)/x^7/a^5
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.26

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^9}\,dx =$$
$$-\frac{2\,(1155\,a^6c^3+2\,(64\,b^6c^3-312\,ab^5c^2d+572\,a^2b^4cd^2-429\,a^3b^3d^3)x^6-(64\,ab^5c^3-312\,a^2b^4c^2d+572\,a^3b$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
-2/15015*(1155*a^6*c^3 + 2*(64*b^6*c^3 - 312*a*b^5*c^2*d + 572*a^2*b^4*c*d
^2 - 429*a^3*b^3*d^3)*x^6 - (64*a*b^5*c^3 - 312*a^2*b^4*c^2*d + 572*a^3*b^
3*c*d^2 - 429*a^4*b^2*d^3)*x^5 + 3*(16*a^2*b^4*c^3 - 78*a^3*b^3*c^2*d + 14
3*a^4*b^2*c*d^2 + 1144*a^5*b*d^3)*x^4 - 5*(8*a^3*b^3*c^3 - 39*a^4*b^2*c^2*
d - 1430*a^5*b*c*d^2 - 429*a^6*d^3)*x^3 + 35*(a^4*b^2*c^3 + 156*a^5*b*c^2*
d + 143*a^6*c*d^2)*x^2 + 105*(14*a^5*b*c^3 + 39*a^6*c^2*d)*x)*sqrt(b*x^2 +
 a*x)/(a^5*x^7)
```

Sympy [F]

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^9}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}} (c+dx)^3}{x^9}\,dx$$

input
```
integrate((d*x+c)**3*(b*x**2+a*x)**(3/2)/x**9,x)
```

output
```
Integral((x*(a + b*x))**(3/2)*(c + d*x)**3/x**9, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. 2(202) = 404.

Time = 0.05 (sec) , antiderivative size = 600, normalized size of antiderivative = 2.70

$$\int \frac{(c+dx)^3 \left(ax+bx^2\right)^{3/2}}{x^9}\,dx = -\frac{256\sqrt{bx^2+ax}b^6c^3}{15015\,a^5x} + \frac{32\sqrt{bx^2+ax}b^5c^2d}{385\,a^4x} - \frac{16\sqrt{bx^2+ax}b^4cd^2}{105\,a^3x} + \frac{4\sqrt{bx^2+ax}b^3d^3}{35\,a^2x} + \frac{128\sqrt{bx^2+ax}b^5c^3}{15015\,a^4x^2} - \frac{16\sqrt{bx^2+ax}b^4c^2d}{385\,a^3x^2} + \frac{8\sqrt{bx^2+ax}b^3cd^2}{105\,a^2x^2} - \frac{2\sqrt{bx^2+ax}b^2d^3}{35\,ax^2} - \frac{32\sqrt{bx^2+ax}b^4c^3}{5005\,a^3x^3} + \frac{12\sqrt{bx^2+ax}b^3c^2d}{385\,a^2x^3} - \frac{2\sqrt{bx^2+ax}b^2cd^2}{35\,ax^3} + \frac{3\sqrt{bx^2+ax}bd^3}{70\,x^3} + \frac{16\sqrt{bx^2+ax}b^3c^3}{3003\,a^2x^4} - \frac{2\sqrt{bx^2+ax}b^2c^2d}{77\,ax^4} + \frac{\sqrt{bx^2+ax}bcd^2}{21\,x^4} + \frac{3\sqrt{bx^2+ax}ad^3}{14\,x^4} - \frac{2\sqrt{bx^2+ax}b^2c^3}{429\,ax^5} + \frac{\sqrt{bx^2+ax}bc^2d}{44\,x^5} + \frac{\sqrt{bx^2+ax}acd^2}{3\,x^5} - \frac{(bx^2+ax)^{\frac{3}{2}}d^3}{2\,x^5} + \frac{3\sqrt{bx^2+ax}bc^3}{715\,x^6} + \frac{9\sqrt{bx^2+ax}ac^2d}{44\,x^6} - \frac{(bx^2+ax)^{\frac{3}{2}}cd^2}{x^6} + \frac{3\sqrt{bx^2+ax}ac^3}{65\,x^7} - \frac{3\,(bx^2+ax)^{\frac{3}{2}}c^2d}{4\,x^7} - \frac{(bx^2+ax)^{\frac{3}{2}}c^3}{5\,x^8}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
-256/15015*sqrt(b*x^2 + a*x)*b^6*c^3/(a^5*x) + 32/385*sqrt(b*x^2 + a*x)*b^
5*c^2*d/(a^4*x) - 16/105*sqrt(b*x^2 + a*x)*b^4*c*d^2/(a^3*x) + 4/35*sqrt(b
*x^2 + a*x)*b^3*d^3/(a^2*x) + 128/15015*sqrt(b*x^2 + a*x)*b^5*c^3/(a^4*x^2
) - 16/385*sqrt(b*x^2 + a*x)*b^4*c^2*d/(a^3*x^2) + 8/105*sqrt(b*x^2 + a*x)
*b^3*c*d^2/(a^2*x^2) - 2/35*sqrt(b*x^2 + a*x)*b^2*d^3/(a*x^2) - 32/5005*sq
rt(b*x^2 + a*x)*b^4*c^3/(a^3*x^3) + 12/385*sqrt(b*x^2 + a*x)*b^3*c^2*d/(a^
2*x^3) - 2/35*sqrt(b*x^2 + a*x)*b^2*c*d^2/(a*x^3) + 3/70*sqrt(b*x^2 + a*x)
*b*d^3/x^3 + 16/3003*sqrt(b*x^2 + a*x)*b^3*c^3/(a^2*x^4) - 2/77*sqrt(b*x^2
 + a*x)*b^2*c^2*d/(a*x^4) + 1/21*sqrt(b*x^2 + a*x)*b*c*d^2/x^4 + 3/14*sqrt
(b*x^2 + a*x)*a*d^3/x^4 - 2/429*sqrt(b*x^2 + a*x)*b^2*c^3/(a*x^5) + 1/44*s
qrt(b*x^2 + a*x)*b*c^2*d/x^5 + 1/3*sqrt(b*x^2 + a*x)*a*c*d^2/x^5 - 1/2*(b*
x^2 + a*x)^(3/2)*d^3/x^5 + 3/715*sqrt(b*x^2 + a*x)*b*c^3/x^6 + 9/44*sqrt(b
*x^2 + a*x)*a*c^2*d/x^6 - (b*x^2 + a*x)^(3/2)*c*d^2/x^6 + 3/65*sqrt(b*x^2
+ a*x)*a*c^3/x^7 - 3/4*(b*x^2 + a*x)^(3/2)*c^2*d/x^7 - 1/5*(b*x^2 + a*x)^(
3/2)*c^3/x^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 936 vs. $2(202) = 404$.

Time = 0.14 (sec) , antiderivative size = 936, normalized size of antiderivative = 4.22

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^9} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^9,x, algorithm="giac")
```

output
```
2/15015*(15015*(sqrt(b)*x - sqrt(b*x^2 + a*x))^11*b^(5/2)*d^3 + 60060*(sqr
t(b)*x - sqrt(b*x^2 + a*x))^10*b^3*c*d^2 + 45045*(sqrt(b)*x - sqrt(b*x^2 +
 a*x))^10*a*b^2*d^3 + 90090*(sqrt(b)*x - sqrt(b*x^2 + a*x))^9*b^(7/2)*c^2*
d + 225225*(sqrt(b)*x - sqrt(b*x^2 + a*x))^9*a*b^(5/2)*c*d^2 + 60060*(sqrt
(b)*x - sqrt(b*x^2 + a*x))^9*a^2*b^(3/2)*d^3 + 48048*(sqrt(b)*x - sqrt(b*x
^2 + a*x))^8*b^4*c^3 + 396396*(sqrt(b)*x - sqrt(b*x^2 + a*x))^8*a*b^3*c^2*
d + 369369*(sqrt(b)*x - sqrt(b*x^2 + a*x))^8*a^2*b^2*c*d^2 + 42042*(sqrt(b
)*x - sqrt(b*x^2 + a*x))^8*a^3*b*d^3 + 240240*(sqrt(b)*x - sqrt(b*x^2 + a*
x))^7*a*b^(7/2)*c^3 + 765765*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*a^2*b^(5/2)
*c^2*d + 330330*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*a^3*b^(3/2)*c*d^2 + 1501
5*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*a^4*sqrt(b)*d^3 + 531960*(sqrt(b)*x -
sqrt(b*x^2 + a*x))^6*a^2*b^3*c^3 + 830115*(sqrt(b)*x - sqrt(b*x^2 + a*x))^
6*a^3*b^2*c^2*d + 167310*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a^4*b*c*d^2 + 2
145*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a^5*d^3 + 675675*(sqrt(b)*x - sqrt(b
*x^2 + a*x))^5*a^3*b^(5/2)*c^3 + 540540*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*
a^4*b^(3/2)*c^2*d + 45045*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a^5*sqrt(b)*c*
d^2 + 535535*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^4*b^2*c^3 + 210210*(sqrt(
b)*x - sqrt(b*x^2 + a*x))^4*a^5*b*c^2*d + 5005*(sqrt(b)*x - sqrt(b*x^2 + a
*x))^4*a^6*c*d^2 + 270270*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^5*b^(3/2)*c^
3 + 45045*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^6*sqrt(b)*c^2*d + 84630*(...
```

Mupad [B] (verification not implemented)

Time = 14.72 (sec) , antiderivative size = 522, normalized size of antiderivative = 2.35

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^9} dx = \frac{16 b^3 c^3 \sqrt{b x^2 + a x}}{3003 a^2 x^4} - \frac{2 a d^3 \sqrt{b x^2 + a x}}{7 x^4} - \frac{28 b c^3 \sqrt{b x^2 + a x}}{143 x^6} - \frac{16 b d^3 \sqrt{b x^2 + a x}}{35 x^3} - \frac{2 b^2 c^3 \sqrt{b x^2 + a x}}{429 a x^5} - \frac{2 a c^3 \sqrt{b x^2 + a x}}{13 x^7} - \frac{32 b^4 c^3 \sqrt{b x^2 + a x}}{5005 a^3 x^3} + \frac{128 b^5 c^3 \sqrt{b x^2 + a x}}{15015 a^4 x^2} - \frac{256 b^6 c^3 \sqrt{b x^2 + a x}}{15015 a^5 x} - \frac{2 b^2 d^3 \sqrt{b x^2 + a x}}{35 a x^2} + \frac{4 b^3 d^3 \sqrt{b x^2 + a x}}{35 a^2 x} - \frac{2 a c d^2 \sqrt{b x^2 + a x}}{3 x^5} - \frac{6 a c^2 d \sqrt{b x^2 + a x}}{11 x^6} - \frac{20 b c d^2 \sqrt{b x^2 + a x}}{21 x^4} - \frac{8 b c^2 d \sqrt{b x^2 + a x}}{11 x^5} - \frac{2 b^2 c d^2 \sqrt{b x^2 + a x}}{35 a x^3} - \frac{2 b^2 c^2 d \sqrt{b x^2 + a x}}{77 a x^4} + \frac{8 b^3 c d^2 \sqrt{b x^2 + a x}}{105 a^2 x^2} + \frac{12 b^3 c^2 d \sqrt{b x^2 + a x}}{385 a^2 x^3} - \frac{16 b^4 c d^2 \sqrt{b x^2 + a x}}{105 a^3 x} - \frac{16 b^4 c^2 d \sqrt{b x^2 + a x}}{385 a^3 x^2} + \frac{32 b^5 c^2 d \sqrt{b x^2 + a x}}{385 a^4 x}$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^3)/x^9,x)
```

output
```
(16*b^3*c^3*(a*x + b*x^2)^(1/2))/(3003*a^2*x^4) - (2*a*d^3*(a*x + b*x^2)^(
1/2))/(7*x^4) - (28*b*c^3*(a*x + b*x^2)^(1/2))/(143*x^6) - (16*b*d^3*(a*x
+ b*x^2)^(1/2))/(35*x^3) - (2*b^2*c^3*(a*x + b*x^2)^(1/2))/(429*a*x^5) - (
2*a*c^3*(a*x + b*x^2)^(1/2))/(13*x^7) - (32*b^4*c^3*(a*x + b*x^2)^(1/2))/(
5005*a^3*x^3) + (128*b^5*c^3*(a*x + b*x^2)^(1/2))/(15015*a^4*x^2) - (256*b
^6*c^3*(a*x + b*x^2)^(1/2))/(15015*a^5*x) - (2*b^2*d^3*(a*x + b*x^2)^(1/2)
)/(35*a*x^2) + (4*b^3*d^3*(a*x + b*x^2)^(1/2))/(35*a^2*x) - (2*a*c*d^2*(a*
x + b*x^2)^(1/2))/(3*x^5) - (6*a*c^2*d*(a*x + b*x^2)^(1/2))/(11*x^6) - (20
*b*c*d^2*(a*x + b*x^2)^(1/2))/(21*x^4) - (8*b*c^2*d*(a*x + b*x^2)^(1/2))/(
11*x^5) - (2*b^2*c*d^2*(a*x + b*x^2)^(1/2))/(35*a*x^3) - (2*b^2*c^2*d*(a*x
 + b*x^2)^(1/2))/(77*a*x^4) + (8*b^3*c*d^2*(a*x + b*x^2)^(1/2))/(105*a^2*x
^2) + (12*b^3*c^2*d*(a*x + b*x^2)^(1/2))/(385*a^2*x^3) - (16*b^4*c*d^2*(a*
x + b*x^2)^(1/2))/(105*a^3*x) - (16*b^4*c^2*d*(a*x + b*x^2)^(1/2))/(385*a^
3*x^2) + (32*b^5*c^2*d*(a*x + b*x^2)^(1/2))/(385*a^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 531, normalized size of antiderivative = 2.39

$$\int \frac{(c+dx)^3\,(ax+bx^2)^{3/2}}{x^9}\,dx = -\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^6d^3x^3}{7} - \frac{256\sqrt{x}\,\sqrt{bx+a}\,b^6c^3x^6}{15015} - \frac{4\sqrt{b}\,a^3b^3d^3x^7}{35} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,a^6c^3}{13} + \frac{256\sqrt{b}\,b}{150}$$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^9,x)
```

output
```
(2*( - 1155*sqrt(x)*sqrt(a + b*x)*a**6*c**3 - 4095*sqrt(x)*sqrt(a + b*x)*a
**6*c**2*d*x - 5005*sqrt(x)*sqrt(a + b*x)*a**6*c*d**2*x**2 - 2145*sqrt(x)*
sqrt(a + b*x)*a**6*d**3*x**3 - 1470*sqrt(x)*sqrt(a + b*x)*a**5*b*c**3*x -
5460*sqrt(x)*sqrt(a + b*x)*a**5*b*c**2*d*x**2 - 7150*sqrt(x)*sqrt(a + b*x)
*a**5*b*c*d**2*x**3 - 3432*sqrt(x)*sqrt(a + b*x)*a**5*b*d**3*x**4 - 35*sqr
t(x)*sqrt(a + b*x)*a**4*b**2*c**3*x**2 - 195*sqrt(x)*sqrt(a + b*x)*a**4*b*
*2*c**2*d*x**3 - 429*sqrt(x)*sqrt(a + b*x)*a**4*b**2*c*d**2*x**4 - 429*sqr
t(x)*sqrt(a + b*x)*a**4*b**2*d**3*x**5 + 40*sqrt(x)*sqrt(a + b*x)*a**3*b**
3*c**3*x**3 + 234*sqrt(x)*sqrt(a + b*x)*a**3*b**3*c**2*d*x**4 + 572*sqrt(x
)*sqrt(a + b*x)*a**3*b**3*c*d**2*x**5 + 858*sqrt(x)*sqrt(a + b*x)*a**3*b**
3*d**3*x**6 - 48*sqrt(x)*sqrt(a + b*x)*a**2*b**4*c**3*x**4 - 312*sqrt(x)*s
qrt(a + b*x)*a**2*b**4*c**2*d*x**5 - 1144*sqrt(x)*sqrt(a + b*x)*a**2*b**4*
c*d**2*x**6 + 64*sqrt(x)*sqrt(a + b*x)*a*b**5*c**3*x**5 + 624*sqrt(x)*sqrt
(a + b*x)*a*b**5*c**2*d*x**6 - 128*sqrt(x)*sqrt(a + b*x)*b**6*c**3*x**6 -
858*sqrt(b)*a**3*b**3*d**3*x**7 + 1144*sqrt(b)*a**2*b**4*c*d**2*x**7 - 624
*sqrt(b)*a*b**5*c**2*d*x**7 + 128*sqrt(b)*b**6*c**3*x**7))/(15015*a**5*x**
7)
```

3.91 $\int \frac{(c+dx)^3(ax+bx^2)^{3/2}}{x^{10}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 286

$$\int \frac{(c+dx)^3(ax+bx^2)^{3/2}}{x^{10}}\,dx = -\frac{2c^3(ax+bx^2)^{5/2}}{15ax^{10}} + \frac{2c^2(2bc-9ad)(ax+bx^2)^{5/2}}{39a^2x^9} - \frac{2c(117a^2d^2+8bc(2bc-9ad))(ax+bx^2)^{5/2}}{429a^3x^8} - \frac{2\left(143d^3-\frac{2bc(117a^2d^2+8bc(2bc-9ad))}{a^3}\right)(ax+bx^2)^{5/2}}{1287ax^7} + \frac{8b(143a^3d^3-2bc(117a^2d^2+8bc(2bc-9ad)))(ax+bx^2)^{5/2}}{9009a^5x^6} - \frac{16b^2(143a^3d^3-2bc(117a^2d^2+8bc(2bc-9ad)))(ax+bx^2)^{5/2}}{45045a^6x^5}$$

output
```
-2/15*c^3*(b*x^2+a*x)^(5/2)/a/x^10+2/39*c^2*(-9*a*d+2*b*c)*(b*x^2+a*x)^(5/
2)/a^2/x^9-2/429*c*(117*a^2*d^2+8*b*c*(-9*a*d+2*b*c))*(b*x^2+a*x)^(5/2)/a^
3/x^8-2/1287*(143*d^3-2*b*c*(117*a^2*d^2+8*b*c*(-9*a*d+2*b*c))/a^3)*(b*x^2
+a*x)^(5/2)/a/x^7+8/9009*b*(143*a^3*d^3-2*b*c*(117*a^2*d^2+8*b*c*(-9*a*d+2
*b*c)))*(b*x^2+a*x)^(5/2)/a^5/x^6-16/45045*b^2*(143*a^3*d^3-2*b*c*(117*a^2
*d^2+8*b*c*(-9*a*d+2*b*c)))*(b*x^2+a*x)^(5/2)/a^6/x^5
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.69

$$\int \frac{(c+dx)^3 \left(ax+bx^2\right)^{3/2}}{x^{10}}\,dx =$$
$$-\frac{2(x(a+bx))^{5/2}\left(-256b^5c^3x^5+128ab^4c^2x^4(5c+9dx)-16a^2b^3cx^3\left(70c^2+180cdx+117d^2x^2\right)+8a^3b^2x^2\right.}{}$$

input
```
Integrate[((c + d*x)^3*(a*x + b*x^2)^(3/2))/x^10,x]
```

output
```
(-2*(x*(a + b*x))^(5/2)*(-256*b^5*c^3*x^5 + 128*a*b^4*c^2*x^4*(5*c + 9*d*x
) - 16*a^2*b^3*c*x^3*(70*c^2 + 180*c*d*x + 117*d^2*x^2) + 8*a^3*b^2*x^2*(2
10*c^3 + 630*c^2*d*x + 585*c*d^2*x^2 + 143*d^3*x^3) - 10*a^4*b*x*(231*c^3
+ 756*c^2*d*x + 819*c*d^2*x^2 + 286*d^3*x^3) + 7*a^5*(429*c^3 + 1485*c^2*d
*x + 1755*c*d^2*x^2 + 715*d^3*x^3)))/(45045*a^6*x^10)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1262, 27, 2169, 27, 1220, 1129, 1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{3/2}(c+dx)^3}{x^{10}}\,dx$$
$$\Big\downarrow \text{1262}$$
$$-\frac{\int -\frac{\left(bx^2+ax\right)^{3/2}\left(6bc^3+18bdxc^2+d^2(18bc-11ad)x^2\right)}{2x^{10}}dx}{3b}-\frac{d^3\left(ax+bx^2\right)^{5/2}}{3bx^8}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{\left(bx^2+ax\right)^{3/2}\left(6bc^3+18bdxc^2+d^2(18bc-11ad)x^2\right)}{x^{10}}dx}{6b}-\frac{d^3\left(ax+bx^2\right)^{5/2}}{3bx^8}$$

$$\Big\downarrow \ 2169$$

$$\frac{-\frac{\int -\frac{\left(48b^2c^3+d\left(144b^2c^2-234abdc+143a^2d^2\right)x\right)\left(bx^2+ax\right)^{3/2}}{2x^{10}}dx}{4b}-\frac{d^2\left(ax+bx^2\right)^{5/2}(18bc-11ad)}{4bx^9}}{6b}-\frac{d^3\left(ax+bx^2\right)^{5/2}}{3bx^8}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{\left(48b^2c^3+d\left(144b^2c^2-234abdc+143a^2d^2\right)x\right)\left(bx^2+ax\right)^{3/2}}{x^{10}}dx}{8b}-\frac{d^2\left(ax+bx^2\right)^{5/2}(18bc-11ad)}{4bx^9}}{6b}-\frac{d^3\left(ax+bx^2\right)^{5/2}}{3bx^8}$$

$$\Big\downarrow \ 1220$$

$$\frac{-\frac{\frac{\left(-143a^3d^3+234a^2bcd^2-144ab^2c^2d+32b^3c^3\right)\int \frac{\left(bx^2+ax\right)^{3/2}}{x^9}dx}{a}-\frac{32b^2c^3\left(ax+bx^2\right)^{5/2}}{5ax^{10}}}{8b}-\frac{d^2\left(ax+bx^2\right)^{5/2}(18bc-11ad)}{4bx^9}}{6b}-\frac{d^3\left(ax+bx^2\right)^{5/2}}{3bx^8}$$

$$\Big\downarrow \ 1129$$

$$\frac{-\frac{\frac{\left(-143a^3d^3+234a^2bcd^2-144ab^2c^2d+32b^3c^3\right)\left(-\frac{8b\int \frac{\left(bx^2+ax\right)^{3/2}}{x^8}dx}{13a}-\frac{2\left(ax+bx^2\right)^{5/2}}{13ax^9}\right)}{a}-\frac{32b^2c^3\left(ax+bx^2\right)^{5/2}}{5ax^{10}}}{8b}-\frac{d^2\left(ax+bx^2\right)^{5/2}(18bc-11ad)}{4bx^9}}{6b}-\frac{d^3\left(ax+bx^2\right)^{5/2}}{3bx^8}$$

$$\Big\downarrow \ 1129$$

$$\frac{-\frac{\frac{\left(-143a^3d^3+234a^2bcd^2-144ab^2c^2d+32b^3c^3\right)\left(-\frac{8b\left(-\frac{6b\int \frac{\left(bx^2+ax\right)^{3/2}}{x^7}dx}{11a}-\frac{2\left(ax+bx^2\right)^{5/2}}{11ax^8}\right)}{13a}-\frac{2\left(ax+bx^2\right)^{5/2}}{13ax^9}\right)}{a}-\frac{32b^2c^3\left(ax+bx^2\right)^{5/2}}{5ax^{10}}}{8b}-\frac{d^2\left(ax+bx\right.}{}}{6b}$$

$$\frac{d^3\left(ax+bx^2\right)^{5/2}}{3bx^8}$$

$$\Big\downarrow \ 1129$$

$$-\frac{\frac{\frac{\left(-143a^3d^3+234a^2bcd^2-144ab^2c^2d+32b^3c^3\right)\left(-\frac{8b\left(-\frac{6b\left(-\frac{4b\int\frac{\left(bx^2+ax\right)^{3/2}}{x^6}dx}{9a}-\frac{2\left(ax+bx^2\right)^{5/2}}{9ax^7}\right)}{11a}-\frac{2\left(ax+bx^2\right)^{5/2}}{11ax^8}\right)}{13a}-\frac{2\left(ax+bx^2\right)^{5/2}}{13ax^9}\right)}{a}-\frac{32b^2c^3\left(ax+\cdots\right.}{5ax\cdots}}{8b}\cdots}{6b}\cdots$$

$$\frac{d^3\left(ax+bx^2\right)^{5/2}}{3bx^8}$$

$\Big\downarrow$ 1129

$$-\frac{\frac{\frac{\left(-143a^3d^3+234a^2bcd^2-144ab^2c^2d+32b^3c^3\right)\left(-\frac{8b\left(-\frac{6b\left(-\frac{4b\left(-\frac{2b\int\frac{\left(bx^2+ax\right)^{3/2}}{x^5}dx}{7a}-\frac{2\left(ax+bx^2\right)^{5/2}}{7ax^6}\right)}{9a}-\frac{2\left(ax+bx^2\right)^{5/2}}{9ax^7}\right)}{11a}-\frac{2\left(ax+bx^2\right)^{5/2}}{11ax^8}\right)}{13a}-\frac{2\left(ax-\cdots\right.}{1\cdots}\cdots\right.}{a}\cdots}{8b}\cdots}{6b}\cdots$$

$$\frac{d^3\left(ax+bx^2\right)^{5/2}}{3bx^8}$$

$\big\downarrow$ 1123

$$-\frac{-\frac{\left(-\frac{8b\left(-\frac{6b\left(-\frac{4b\left(\frac{4b\left(ax+bx^2\right)^{5/2}}{35a^2x^5}-\frac{2\left(ax+bx^2\right)^{5/2}}{7ax^6}\right)}{9a}-\frac{2\left(ax+bx^2\right)^{5/2}}{9ax^7}\right)}{11a}-\frac{2\left(ax+bx^2\right)^{5/2}}{11ax^8}\right)}{13a}-\frac{2\left(ax+bx^2\right)^{5/2}}{13ax^9}\right)\left(-143a^3d^3+234a^2bcd^2-144ab^2c^2d+32b}{a}}{8b}}{6b}$$

$$\frac{d^3\left(ax+bx^2\right)^{5/2}}{3bx^8}$$

input
```
Int[((c + d*x)^3*(a*x + b*x^2)^(3/2))/x^10,x]
```

output
```
-1/3*(d^3*(a*x + b*x^2)^(5/2))/(b*x^8) + (-1/4*(d^2*(18*b*c - 11*a*d)*(a*x
 + b*x^2)^(5/2))/(b*x^9) + ((-32*b^2*c^3*(a*x + b*x^2)^(5/2))/(5*a*x^10) -
 ((32*b^3*c^3 - 144*a*b^2*c^2*d + 234*a^2*b*c*d^2 - 143*a^3*d^3)*((-2*(a*x
 + b*x^2)^(5/2))/(13*a*x^9) - (8*b*((-2*(a*x + b*x^2)^(5/2))/(11*a*x^8) -
(6*b*((-2*(a*x + b*x^2)^(5/2))/(9*a*x^7) - (4*b*((-2*(a*x + b*x^2)^(5/2))/
(7*a*x^6) + (4*b*(a*x + b*x^2)^(5/2))/(35*a^2*x^5)))/(9*a)))/(11*a)))/(13*
a)))/a)/(8*b))/(6*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

rule 2169
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d -
b*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2
, 0]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$-\frac{2\sqrt{x(bx+a)}\,(bx+a)^2\left(\left(\frac{5}{3}d^3x^3+\frac{45}{11}c\,d^2x^2+\frac{45}{13}c^2dx+c^3\right)a^5-\frac{10\left(\frac{26}{21}d^3x^3+\frac{39}{11}c\,d^2x^2+\frac{36}{11}c^2dx+c^3\right)xb\,a^4}{13}+\frac{80\left(\frac{143}{210}d^3x^3+\frac{39}{14}c\,c\right.}{}\right.}{15x^8a^6}$
gosper	$-\frac{2(bx+a)\left(1144a^3b^2d^3x^5-1872a^2b^3c\,d^2x^5+1152a\,b^4c^2d\,x^5-256b^5c^3x^5-2860a^4b\,d^3x^4+4680a^3b^2c\,d^2x^4-2880a^2b^3c^2d\,x\right.}{}$
orering	$-\frac{2(bx+a)\left(1144a^3b^2d^3x^5-1872a^2b^3c\,d^2x^5+1152a\,b^4c^2d\,x^5-256b^5c^3x^5-2860a^4b\,d^3x^4+4680a^3b^2c\,d^2x^4-2880a^2b^3c^2d\,x\right.}{}$
trager	$-\frac{2\left(1144a^3b^4d^3x^7-1872a^2b^5c\,d^2x^7+1152a\,b^6c^2d\,x^7-256b^7c^3x^7-572a^4b^3d^3x^6+936a^3b^4c\,d^2x^6-576a^2b^5c^2d\,x^6+128a\,b^6\right.}{}$
risch	$-\frac{2(bx+a)\left(1144a^3b^4d^3x^7-1872a^2b^5c\,d^2x^7+1152a\,b^6c^2d\,x^7-256b^7c^3x^7-572a^4b^3d^3x^6+936a^3b^4c\,d^2x^6-576a^2b^5c^2d\,x^6+\right.}{}$
default	$c^3\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{15a\,x^{10}}-\frac{2b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{13a\,x^9}-\frac{8b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{11a\,x^8}-\frac{6b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{9a\,x^7}-\frac{4b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{7a\,x^6}+\frac{4b\left(b\,x^2+ax\right)^{\frac{5}{2}}}{35a^2x^5}\right.}{9a}\right.}{11a}\right.}{13a}\right.}{3a}\right.$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-2/15*(x*(b*x+a))^(1/2)*(b*x+a)^2*((5/3*d^3*x^3+45/11*c*d^2*x^2+45/13*c^2*
d*x+c^3)*a^5-10/13*(26/21*d^3*x^3+39/11*c*d^2*x^2+36/11*c^2*d*x+c^3)*x*b*a
^4+80/143*(143/210*d^3*x^3+39/14*c*d^2*x^2+3*c^2*d*x+c^3)*x^2*b^2*a^3-160/
429*(117/70*d^2*x^2+18/7*c*d*x+c^2)*x^3*b^3*c*a^2+640/3003*(9/5*d*x+c)*x^4
*b^4*c^2*a-256/3003*b^5*c^3*x^5)/x^8/a^6
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.16

$$\int \frac{(c+dx)^3\,(ax+bx^2)^{3/2}}{x^{10}}\,dx = -\frac{2\,(3003\,a^7c^3 - 8\,(32\,b^7c^3 - 144\,ab^6c^2d + 234\,a^2b^5cd^2 - 143\,a^3b^4d^3)x^7 + 4\,(32\,ab^6c^3 - 144\,a^2b^5c^2d + 234\,a^3 \ldots}{}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^10,x, algorithm="fricas")
```

output
```
-2/45045*(3003*a^7*c^3 - 8*(32*b^7*c^3 - 144*a*b^6*c^2*d + 234*a^2*b^5*c*d
^2 - 143*a^3*b^4*d^3)*x^7 + 4*(32*a*b^6*c^3 - 144*a^2*b^5*c^2*d + 234*a^3*
b^4*c*d^2 - 143*a^4*b^3*d^3)*x^6 - 3*(32*a^2*b^5*c^3 - 144*a^3*b^4*c^2*d +
 234*a^4*b^3*c*d^2 - 143*a^5*b^2*d^3)*x^5 + 5*(16*a^3*b^4*c^3 - 72*a^4*b^3
*c^2*d + 117*a^5*b^2*c*d^2 + 1430*a^6*b*d^3)*x^4 - 35*(2*a^4*b^3*c^3 - 9*a
^5*b^2*c^2*d - 468*a^6*b*c*d^2 - 143*a^7*d^3)*x^3 + 63*(a^5*b^2*c^3 + 210*
a^6*b*c^2*d + 195*a^7*c*d^2)*x^2 + 231*(16*a^6*b*c^3 + 45*a^7*c^2*d)*x)*sq
rt(b*x^2 + a*x)/(a^6*x^8)
```

Sympy [F]

$$\int \frac{(c+dx)^3\,(ax+bx^2)^{3/2}}{x^{10}}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}\,(c+dx)^3}{x^{10}}\,dx$$

input
```
integrate((d*x+c)**3*(b*x**2+a*x)**(3/2)/x**10,x)
```

output
```
Integral((x*(a + b*x))**(3/2)*(c + d*x)**3/x**10, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 702 vs. 2(262) = 524.

Time = 0.06 (sec) , antiderivative size = 702, normalized size of antiderivative = 2.45

$$\begin{aligned}\int \frac{(c+dx)^3\,(ax+bx^2)^{3/2}}{x^{10}}\,dx =& \frac{512\,\sqrt{bx^2+ax}b^7c^3}{45045\,a^6x} \\ &- \frac{256\,\sqrt{bx^2+ax}b^6c^2d}{5005\,a^5x} + \frac{32\,\sqrt{bx^2+ax}b^5cd^2}{385\,a^4x} - \frac{16\,\sqrt{bx^2+ax}b^4d^3}{315\,a^3x} \\ &- \frac{256\,\sqrt{bx^2+ax}b^6c^3}{45045\,a^5x^2} + \frac{128\,\sqrt{bx^2+ax}b^5c^2d}{5005\,a^4x^2} - \frac{16\,\sqrt{bx^2+ax}b^4cd^2}{385\,a^3x^2} \\ &+ \frac{8\,\sqrt{bx^2+ax}b^3d^3}{315\,a^2x^2} + \frac{64\,\sqrt{bx^2+ax}b^5c^3}{15015\,a^4x^3} - \frac{96\,\sqrt{bx^2+ax}b^4c^2d}{5005\,a^3x^3} \\ &+ \frac{12\,\sqrt{bx^2+ax}b^3cd^2}{385\,a^2x^3} - \frac{2\,\sqrt{bx^2+ax}b^2d^3}{105\,ax^3} - \frac{32\,\sqrt{bx^2+ax}b^4c^3}{9009\,a^3x^4} \\ &+ \frac{16\,\sqrt{bx^2+ax}b^3c^2d}{1001\,a^2x^4} - \frac{2\,\sqrt{bx^2+ax}b^2cd^2}{77\,ax^4} + \frac{\sqrt{bx^2+ax}bd^3}{63\,x^4} \\ &+ \frac{4\,\sqrt{bx^2+ax}b^3c^3}{1287\,a^2x^5} - \frac{2\,\sqrt{bx^2+ax}b^2c^2d}{143\,ax^5} + \frac{\sqrt{bx^2+ax}bcd^2}{44\,x^5} \\ &+ \frac{\sqrt{bx^2+ax}ad^3}{9\,x^5} - \frac{2\,\sqrt{bx^2+ax}b^2c^3}{715\,ax^6} + \frac{9\,\sqrt{bx^2+ax}bc^2d}{715\,x^6} \\ &+ \frac{9\,\sqrt{bx^2+ax}acd^2}{44\,x^6} - \frac{(bx^2+ax)^{\frac{3}{2}}d^3}{3\,x^6} + \frac{\sqrt{bx^2+ax}bc^3}{390\,x^7} + \frac{9\,\sqrt{bx^2+ax}ac^2d}{65\,x^7} \\ &- \frac{3\,(bx^2+ax)^{\frac{3}{2}}cd^2}{4\,x^7} + \frac{\sqrt{bx^2+ax}ac^3}{30\,x^8} - \frac{3\,(bx^2+ax)^{\frac{3}{2}}c^2d}{5\,x^8} - \frac{(bx^2+ax)^{\frac{3}{2}}c^3}{6\,x^9}\end{aligned}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^10,x, algorithm="maxima")
```

output
```
512/45045*sqrt(b*x^2 + a*x)*b^7*c^3/(a^6*x) - 256/5005*sqrt(b*x^2 + a*x)*b
^6*c^2*d/(a^5*x) + 32/385*sqrt(b*x^2 + a*x)*b^5*c*d^2/(a^4*x) - 16/315*sqr
t(b*x^2 + a*x)*b^4*d^3/(a^3*x) - 256/45045*sqrt(b*x^2 + a*x)*b^6*c^3/(a^5*
x^2) + 128/5005*sqrt(b*x^2 + a*x)*b^5*c^2*d/(a^4*x^2) - 16/385*sqrt(b*x^2
+ a*x)*b^4*c*d^2/(a^3*x^2) + 8/315*sqrt(b*x^2 + a*x)*b^3*d^3/(a^2*x^2) + 6
4/15015*sqrt(b*x^2 + a*x)*b^5*c^3/(a^4*x^3) - 96/5005*sqrt(b*x^2 + a*x)*b^
4*c^2*d/(a^3*x^3) + 12/385*sqrt(b*x^2 + a*x)*b^3*c*d^2/(a^2*x^3) - 2/105*s
qrt(b*x^2 + a*x)*b^2*d^3/(a*x^3) - 32/9009*sqrt(b*x^2 + a*x)*b^4*c^3/(a^3*
x^4) + 16/1001*sqrt(b*x^2 + a*x)*b^3*c^2*d/(a^2*x^4) - 2/77*sqrt(b*x^2 + a
*x)*b^2*c*d^2/(a*x^4) + 1/63*sqrt(b*x^2 + a*x)*b*d^3/x^4 + 4/1287*sqrt(b*x
^2 + a*x)*b^3*c^3/(a^2*x^5) - 2/143*sqrt(b*x^2 + a*x)*b^2*c^2*d/(a*x^5) +
1/44*sqrt(b*x^2 + a*x)*b*c*d^2/x^5 + 1/9*sqrt(b*x^2 + a*x)*a*d^3/x^5 - 2/7
15*sqrt(b*x^2 + a*x)*b^2*c^3/(a*x^6) + 9/715*sqrt(b*x^2 + a*x)*b*c^2*d/x^6
 + 9/44*sqrt(b*x^2 + a*x)*a*c*d^2/x^6 - 1/3*(b*x^2 + a*x)^(3/2)*d^3/x^6 +
1/390*sqrt(b*x^2 + a*x)*b*c^3/x^7 + 9/65*sqrt(b*x^2 + a*x)*a*c^2*d/x^7 - 3
/4*(b*x^2 + a*x)^(3/2)*c*d^2/x^7 + 1/30*sqrt(b*x^2 + a*x)*a*c^3/x^8 - 3/5*
(b*x^2 + a*x)^(3/2)*c^2*d/x^8 - 1/6*(b*x^2 + a*x)^(3/2)*c^3/x^9
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1066 vs. $2(262) = 524$.

Time = 0.14 (sec) , antiderivative size = 1066, normalized size of antiderivative = 3.73

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^{10}} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^10,x, algorithm="giac")
```

output
```
2/45045*(60060*(sqrt(b)*x - sqrt(b*x^2 + a*x))^12*b^3*d^3 + 270270*(sqrt(b
)*x - sqrt(b*x^2 + a*x))^11*b^(7/2)*c*d^2 + 225225*(sqrt(b)*x - sqrt(b*x^2
 + a*x))^11*a*b^(5/2)*d^3 + 432432*(sqrt(b)*x - sqrt(b*x^2 + a*x))^10*b^4*
c^2*d + 1189188*(sqrt(b)*x - sqrt(b*x^2 + a*x))^10*a*b^3*c*d^2 + 369369*(s
qrt(b)*x - sqrt(b*x^2 + a*x))^10*a^2*b^2*d^3 + 240240*(sqrt(b)*x - sqrt(b*
x^2 + a*x))^9*b^(9/2)*c^3 + 2162160*(sqrt(b)*x - sqrt(b*x^2 + a*x))^9*a*b^
(7/2)*c^2*d + 2297295*(sqrt(b)*x - sqrt(b*x^2 + a*x))^9*a^2*b^(5/2)*c*d^2
+ 330330*(sqrt(b)*x - sqrt(b*x^2 + a*x))^9*a^3*b^(3/2)*d^3 + 1338480*(sqrt
(b)*x - sqrt(b*x^2 + a*x))^8*a*b^4*c^3 + 4787640*(sqrt(b)*x - sqrt(b*x^2 +
 a*x))^8*a^2*b^3*c^2*d + 2490345*(sqrt(b)*x - sqrt(b*x^2 + a*x))^8*a^3*b^2
*c*d^2 + 167310*(sqrt(b)*x - sqrt(b*x^2 + a*x))^8*a^4*b*d^3 + 3333330*(sqr
t(b)*x - sqrt(b*x^2 + a*x))^7*a^2*b^(7/2)*c^3 + 6081075*(sqrt(b)*x - sqrt(
b*x^2 + a*x))^7*a^3*b^(5/2)*c^2*d + 1621620*(sqrt(b)*x - sqrt(b*x^2 + a*x)
)^7*a^4*b^(3/2)*c*d^2 + 45045*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*a^5*sqrt(b
)*d^3 + 4844840*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a^3*b^3*c^3 + 4819815*(s
qrt(b)*x - sqrt(b*x^2 + a*x))^6*a^4*b^2*c^2*d + 630630*(sqrt(b)*x - sqrt(b
*x^2 + a*x))^6*a^5*b*c*d^2 + 5005*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a^6*d^
3 + 4513509*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a^4*b^(5/2)*c^3 + 2432430*(s
qrt(b)*x - sqrt(b*x^2 + a*x))^5*a^5*b^(3/2)*c^2*d + 135135*(sqrt(b)*x - sq
rt(b*x^2 + a*x))^5*a^6*sqrt(b)*c*d^2 + 2788695*(sqrt(b)*x - sqrt(b*x^2 ...
```

Mupad [B] (verification not implemented)

Time = 16.44 (sec) , antiderivative size = 624, normalized size of antiderivative = 2.18

$$\int \frac{(c+dx)^3 (ax+bx^2)^{3/2}}{x^{10}}\,dx = \frac{4\,b^3\,c^3\,\sqrt{b\,x^2+a\,x}}{1287\,a^2\,x^5} - \frac{2\,a\,d^3\,\sqrt{b\,x^2+a\,x}}{9\,x^5} - \frac{32\,b\,c^3\,\sqrt{b\,x^2+a\,x}}{195\,x^7} - \frac{20\,b\,d^3\,\sqrt{b\,x^2+a\,x}}{63\,x^4} - \frac{2\,b^2\,c^3\,\sqrt{b\,x^2+a\,x}}{715\,a\,x^6} - \frac{2\,a\,c^3\,\sqrt{b\,x^2+a\,x}}{15\,x^8} - \frac{32\,b^4\,c^3\,\sqrt{b\,x^2+a\,x}}{9009\,a^3\,x^4} + \frac{64\,b^5\,c^3\,\sqrt{b\,x^2+a\,x}}{15015\,a^4\,x^3} - \frac{256\,b^6\,c^3\,\sqrt{b\,x^2+a\,x}}{45045\,a^5\,x^2} + \frac{512\,b^7\,c^3\,\sqrt{b\,x^2+a\,x}}{45045\,a^6\,x} - \frac{2\,b^2\,d^3\,\sqrt{b\,x^2+a\,x}}{105\,a\,x^3} + \frac{8\,b^3\,d^3\,\sqrt{b\,x^2+a\,x}}{315\,a^2\,x^2} - \frac{16\,b^4\,d^3\,\sqrt{b\,x^2+a\,x}}{315\,a^3\,x} - \frac{6\,a\,c\,d^2\,\sqrt{b\,x^2+a\,x}}{11\,x^6} - \frac{6\,a\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{13\,x^7} - \frac{8\,b\,c\,d^2\,\sqrt{b\,x^2+a\,x}}{11\,x^5} - \frac{84\,b\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{143\,x^6} - \frac{2\,b^2\,c\,d^2\,\sqrt{b\,x^2+a\,x}}{77\,a\,x^4} - \frac{2\,b^2\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{143\,a\,x^5} + \frac{12\,b^3\,c\,d^2\,\sqrt{b\,x^2+a\,x}}{385\,a^2\,x^3} + \frac{16\,b^3\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{1001\,a^2\,x^4} - \frac{16\,b^4\,c\,d^2\,\sqrt{b\,x^2+a\,x}}{385\,a^3\,x^2} - \frac{96\,b^4\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{5005\,a^3\,x^3} + \frac{32\,b^5\,c\,d^2\,\sqrt{b\,x^2+a\,x}}{385\,a^4\,x} + \frac{128\,b^5\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{5005\,a^4\,x^2} - \frac{256\,b^6\,c^2\,d\,\sqrt{b\,x^2+a\,x}}{5005\,a^5\,x}$$

input
```
int(((a*x + b*x^2)^(3/2)*(c + d*x)^3)/x^10,x)
```

output
```
(4*b^3*c^3*(a*x + b*x^2)^(1/2))/(1287*a^2*x^5) - (2*a*d^3*(a*x + b*x^2)^(1
/2))/(9*x^5) - (32*b*c^3*(a*x + b*x^2)^(1/2))/(195*x^7) - (20*b*d^3*(a*x +
 b*x^2)^(1/2))/(63*x^4) - (2*b^2*c^3*(a*x + b*x^2)^(1/2))/(715*a*x^6) - (2
*a*c^3*(a*x + b*x^2)^(1/2))/(15*x^8) - (32*b^4*c^3*(a*x + b*x^2)^(1/2))/(9
009*a^3*x^4) + (64*b^5*c^3*(a*x + b*x^2)^(1/2))/(15015*a^4*x^3) - (256*b^6
*c^3*(a*x + b*x^2)^(1/2))/(45045*a^5*x^2) + (512*b^7*c^3*(a*x + b*x^2)^(1/
2))/(45045*a^6*x) - (2*b^2*d^3*(a*x + b*x^2)^(1/2))/(105*a*x^3) + (8*b^3*d
^3*(a*x + b*x^2)^(1/2))/(315*a^2*x^2) - (16*b^4*d^3*(a*x + b*x^2)^(1/2))/(
315*a^3*x) - (6*a*c*d^2*(a*x + b*x^2)^(1/2))/(11*x^6) - (6*a*c^2*d*(a*x +
b*x^2)^(1/2))/(13*x^7) - (8*b*c*d^2*(a*x + b*x^2)^(1/2))/(11*x^5) - (84*b*
c^2*d*(a*x + b*x^2)^(1/2))/(143*x^6) - (2*b^2*c*d^2*(a*x + b*x^2)^(1/2))/(
77*a*x^4) - (2*b^2*c^2*d*(a*x + b*x^2)^(1/2))/(143*a*x^5) + (12*b^3*c*d^2*
(a*x + b*x^2)^(1/2))/(385*a^2*x^3) + (16*b^3*c^2*d*(a*x + b*x^2)^(1/2))/(1
001*a^2*x^4) - (16*b^4*c*d^2*(a*x + b*x^2)^(1/2))/(385*a^3*x^2) - (96*b^4*
c^2*d*(a*x + b*x^2)^(1/2))/(5005*a^3*x^3) + (32*b^5*c*d^2*(a*x + b*x^2)^(1
/2))/(385*a^4*x) + (128*b^5*c^2*d*(a*x + b*x^2)^(1/2))/(5005*a^4*x^2) - (2
56*b^6*c^2*d*(a*x + b*x^2)^(1/2))/(5005*a^5*x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 621, normalized size of antiderivative = 2.17

$$\int \frac{(c+dx)^3 \left(ax+bx^2\right)^{3/2}}{x^{10}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^7d^3x^3}{9}+\frac{512\sqrt{x}\,\sqrt{bx+a}\,b^7c^3x^7}{45045}+\frac{16\sqrt{b}\,a^3b^4d^3x^8}{315}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^7c^3}{15}-\frac{512\sqrt{b}}{45}$$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(3/2)/x^10,x)
```

output
```
(2*( - 3003*sqrt(x)*sqrt(a + b*x)*a**7*c**3 - 10395*sqrt(x)*sqrt(a + b*x)*
a**7*c**2*d*x - 12285*sqrt(x)*sqrt(a + b*x)*a**7*c*d**2*x**2 - 5005*sqrt(x
)*sqrt(a + b*x)*a**7*d**3*x**3 - 3696*sqrt(x)*sqrt(a + b*x)*a**6*b*c**3*x
- 13230*sqrt(x)*sqrt(a + b*x)*a**6*b*c**2*d*x**2 - 16380*sqrt(x)*sqrt(a +
b*x)*a**6*b*c*d**2*x**3 - 7150*sqrt(x)*sqrt(a + b*x)*a**6*b*d**3*x**4 - 63
*sqrt(x)*sqrt(a + b*x)*a**5*b**2*c**3*x**2 - 315*sqrt(x)*sqrt(a + b*x)*a**
5*b**2*c**2*d*x**3 - 585*sqrt(x)*sqrt(a + b*x)*a**5*b**2*c*d**2*x**4 - 429
*sqrt(x)*sqrt(a + b*x)*a**5*b**2*d**3*x**5 + 70*sqrt(x)*sqrt(a + b*x)*a**4
*b**3*c**3*x**3 + 360*sqrt(x)*sqrt(a + b*x)*a**4*b**3*c**2*d*x**4 + 702*sq
rt(x)*sqrt(a + b*x)*a**4*b**3*c*d**2*x**5 + 572*sqrt(x)*sqrt(a + b*x)*a**4
*b**3*d**3*x**6 - 80*sqrt(x)*sqrt(a + b*x)*a**3*b**4*c**3*x**4 - 432*sqrt(
x)*sqrt(a + b*x)*a**3*b**4*c**2*d*x**5 - 936*sqrt(x)*sqrt(a + b*x)*a**3*b*
*4*c*d**2*x**6 - 1144*sqrt(x)*sqrt(a + b*x)*a**3*b**4*d**3*x**7 + 96*sqrt(
x)*sqrt(a + b*x)*a**2*b**5*c**3*x**5 + 576*sqrt(x)*sqrt(a + b*x)*a**2*b**5
*c**2*d*x**6 + 1872*sqrt(x)*sqrt(a + b*x)*a**2*b**5*c*d**2*x**7 - 128*sqrt
(x)*sqrt(a + b*x)*a*b**6*c**3*x**6 - 1152*sqrt(x)*sqrt(a + b*x)*a*b**6*c**
2*d*x**7 + 256*sqrt(x)*sqrt(a + b*x)*b**7*c**3*x**7 + 1144*sqrt(b)*a**3*b*
*4*d**3*x**8 - 1872*sqrt(b)*a**2*b**5*c*d**2*x**8 + 1152*sqrt(b)*a*b**6*c*
*2*d*x**8 - 256*sqrt(b)*b**7*c**3*x**8))/(45045*a**6*x**8)
```

3.92 $\int \frac{x(ax+bx^2)^{3/2}}{c+dx}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 297

$$\int \frac{x(ax+bx^2)^{3/2}}{c+dx}\,dx = -\frac{(64b^3c^3-80ab^2c^2d+8a^2bcd^2+3a^3d^3)\sqrt{ax+bx^2}}{64b^2d^4} - \frac{\left(56ac-\frac{48bc^2}{d}-\frac{3a^2d}{b}\right)x\sqrt{ax+bx^2}}{96d^2} - \frac{(8bc-3ad)x^2\sqrt{ax+bx^2}}{24d^2} + \frac{x(ax+bx^2)^{3/2}}{4d} + \frac{(128b^4c^4-192ab^3c^3d+48a^2b^2c^2d^2+8a^3bcd^3+3a^4d^4)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{64b^{5/2}d^5} - \frac{2c^{5/2}(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{d^5}$$

output
```
-1/64*(3*a^3*d^3+8*a^2*b*c*d^2-80*a*b^2*c^2*d+64*b^3*c^3)*(b*x^2+a*x)^(1/2
)/b^2/d^4-1/96*(56*a*c-48*b*c^2/d-3*a^2*d/b)*x*(b*x^2+a*x)^(1/2)/d^2-1/24*
(-3*a*d+8*b*c)*x^2*(b*x^2+a*x)^(1/2)/d^2+1/4*x*(b*x^2+a*x)^(3/2)/d+1/64*(3
*a^4*d^4+8*a^3*b*c*d^3+48*a^2*b^2*c^2*d^2-192*a*b^3*c^3*d+128*b^4*c^4)*arc
tanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(5/2)/d^5-2*c^(5/2)*(-a*d+b*c)^(3/2)*a
rctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/d^5
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.32 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.87

$$\int \frac{x(ax+bx^2)^{3/2}}{c+dx}\,dx = \frac{(x(a+bx))^{3/2}\left(\sqrt{b}d\sqrt{x}\sqrt{a+bx}(-9a^3d^3+6a^2bd^2(-4c+dx)+8ab^2d(30c^2-14cd\right.}{}$$

input
```
Integrate[(x*(a*x + b*x^2)^(3/2))/(c + d*x),x]
```

output
```
((x*(a + b*x))^(3/2)*(Sqrt[b]*d*Sqrt[x]*Sqrt[a + b*x]*(-9*a^3*d^3 + 6*a^2*
b*d^2*(-4*c + d*x) + 8*a*b^2*d*(30*c^2 - 14*c*d*x + 9*d^2*x^2) - 16*b^3*(1
2*c^3 - 6*c^2*d*x + 4*c*d^2*x^2 - 3*d^3*x^3)) + 384*b^(3/2)*c^(3/2)*(b*c -
 a*d)*(b*c - a*d - I*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d])*Sqrt[-(b*c) + 2*a*d
- (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*ArcTan[(Sqrt[-(b*c) + 2*a*d - (2*
I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*(-Sqrt[a] + Sqrt[a +
 b*x]))] + 384*b^(3/2)*c^(3/2)*(b*c - a*d)*(b*c - a*d + I*Sqrt[a]*Sqrt[d]*
Sqrt[b*c - a*d])*Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*
d]]*ArcTan[(Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*S
qrt[x])/(Sqrt[c]*(-Sqrt[a] + Sqrt[a + b*x]))] + 6*(128*b^4*c^4 - 192*a*b^3
*c^3*d + 48*a^2*b^2*c^2*d^2 + 8*a^3*b*c*d^3 + 3*a^4*d^4)*ArcTanh[(Sqrt[b]*
Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(192*b^(5/2)*d^5*x^(3/2)*(a + b*x)^
(3/2))
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1231, 27, 1231, 27, 1269, 1091, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(ax+bx^2\right)^{3/2}}{c+dx}\,dx$$

$\Big\downarrow$ 1231

$$-\frac{\int -\frac{(ac(8bc-3ad)+(4bc-3ad)(4bc+ad)x)\sqrt{bx^2+ax}}{2(c+dx)}dx}{8bd^2}-\frac{(ax+bx^2)^{3/2}(-3ad+8bc-6bdx)}{24bd^2}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{(ac(8bc-3ad)+(4bc-3ad)(4bc+ad)x)\sqrt{bx^2+ax}}{c+dx}dx}{16bd^2}-\frac{(ax+bx^2)^{3/2}(-3ad+8bc-6bdx)}{24bd^2}$$

$\Big\downarrow$ 1231

$$\frac{-\frac{\int -\frac{ac\left(64b^3c^3-80ab^2dc^2+8a^2bd^2c+3a^3d^3\right)+\left(128b^4c^4-192ab^3dc^3+48a^2b^2d^2c^2+8a^3bd^3c+3a^4d^4\right)x}{2(c+dx)\sqrt{bx^2+ax}}dx}{4bd^2}-\frac{\sqrt{ax+bx^2}(3a^3d^3+8a^2bcd^2-80ab^2c^2d-2bc}{4bd^2}}{16bd^2}$$
$$\frac{(ax+bx^2)^{3/2}(-3ad+8bc-6bdx)}{24bd^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{ac\left(64b^3c^3-80ab^2dc^2+8a^2bd^2c+3a^3d^3\right)+\left(128b^4c^4-192ab^3dc^3+48a^2b^2d^2c^2+8a^3bd^3c+3a^4d^4\right)x}{(c+dx)\sqrt{bx^2+ax}}dx}{8bd^2}-\frac{\sqrt{ax+bx^2}(3a^3d^3+8a^2bcd^2-80ab^2c^2d-2bdx(4}{4bd^2}}{16bd^2}$$
$$\frac{(ax+bx^2)^{3/2}(-3ad+8bc-6bdx)}{24bd^2}$$

$\Big\downarrow$ 1269

$$\frac{\frac{\frac{\left(3a^4d^4+8a^3bcd^3+48a^2b^2c^2d^2-192ab^3c^3d+128b^4c^4\right)\int \frac{1}{\sqrt{bx^2+ax}}dx}{d}-\frac{128b^2c^3(bc-ad)^2\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{8bd^2}-\frac{\sqrt{ax+bx^2}(3a^3d^3+8a^2bcd^2-80ab^2c^2d}{4bd}}{16bd^2}$$
$$\frac{(ax+bx^2)^{3/2}(-3ad+8bc-6bdx)}{24bd^2}$$

$\Big\downarrow$ 1091

$$\frac{\frac{\frac{2\left(3a^4d^4+8a^3bcd^3+48a^2b^2c^2d^2-192ab^3c^3d+128b^4c^4\right)\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{d}-\frac{128b^2c^3(bc-ad)^2\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{8bd^2}-\frac{\sqrt{ax+bx^2}(3a^3d^3+8a^2bcd^2}{}}{16bd^2}$$
$$\frac{(ax+bx^2)^{3/2}(-3ad+8bc-6bdx)}{24bd^2}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)\left(3a^4d^4+8a^3bcd^3+48a^2b^2c^2d^2-192ab^3c^3d+128b^4c^4\right)}{\sqrt{b}d}-\frac{128b^2c^3(bc-ad)^2\int\frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{8bd^2}-\sqrt{ax+bx^2}\left(3a^3d^3+8a^2bcd^2-\right.}{16bd^2}$$

$$\frac{\left(ax+bx^2\right)^{3/2}(-3ad+8bc-6bdx)}{24bd^2}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{\frac{\frac{256b^2c^3(bc-ad)^2\int\frac{1}{4c(bc-ad)-\frac{(ac+(2bc-ad)x)^2}{bx^2+ax}}d\left(-\frac{ac+(2bc-ad)x}{\sqrt{bx^2+ax}}\right)}{d}+\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)\left(3a^4d^4+8a^3bcd^3+48a^2b^2c^2d^2-192ab^3c^3d+128b^4c^4\right)}{\sqrt{b}d}}{8bd^2}-}{16bd^2}$$

$$\frac{\left(ax+bx^2\right)^{3/2}(-3ad+8bc-6bdx)}{24bd^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)\left(3a^4d^4+8a^3bcd^3+48a^2b^2c^2d^2-192ab^3c^3d+128b^4c^4\right)}{\sqrt{b}d}-\frac{128b^2c^{5/2}(bc-ad)^{3/2}\text{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{d}}{8bd^2}-\sqrt{ax+bx^2}\left(\right.}{16bd^2}$$

$$\frac{\left(ax+bx^2\right)^{3/2}(-3ad+8bc-6bdx)}{24bd^2}$$

input

```
Int[(x*(a*x + b*x^2)^(3/2))/(c + d*x),x]
```

output

```
-1/24*((8*b*c - 3*a*d - 6*b*d*x)*(a*x + b*x^2)^(3/2))/(b*d^2) + (-1/4*((64
*b^3*c^3 - 80*a*b^2*c^2*d + 8*a^2*b*c*d^2 + 3*a^3*d^3 - 2*b*d*(4*b*c - 3*a
*d)*(4*b*c + a*d)*x)*Sqrt[a*x + b*x^2])/(b*d^2) + ((2*(128*b^4*c^4 - 192*a
*b^3*c^3*d + 48*a^2*b^2*c^2*d^2 + 8*a^3*b*c*d^3 + 3*a^4*d^4)*ArcTanh[(Sqrt
[b]*x)/Sqrt[a*x + b*x^2]])/(Sqrt[b]*d) - (128*b^2*c^(5/2)*(b*c - a*d)^(3/2
)*ArcTanh[(a*c + (2*b*c - a*d)*x)/(2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[a*x + b*
x^2])])/d)/(8*b*d^2))/(16*b*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$\frac{2c^3\left(a^2 b^{\frac{5}{2}} d^2-2a\,b^{\frac{7}{2}} c d+b^{\frac{9}{2}} c^2\right)\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)+\frac{3\left(\left(a^4 d^4+\frac{8}{3}a^3 b c\,d^3+16a^2 b^2 c^2 d^2-64a\,b^3 c^3 d+\frac{128}{3}c^4 b^4\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx\cdots}}{x\sqrt{b\cdots}}\right)\cdots\right.}{\cdots}}{b^{\frac{5}{2}} d^5\cdots}$
risch	$-\frac{\left(-48b^3 d^3 x^3-72a\,b^2 d^3 x^2+64b^3 c\,d^2 x^2-6a^2 b\,d^3 x+112a\,b^2 c\,d^2 x-96b^3 c^2 d x+9a^3 d^3+24a^2 b c\,d^2-240a\,b^2 c^2 d+192b^3 c^3\right)x(\cdots}{192b^2 d^4\sqrt{x(bx+a)}}$
default	$\frac{\frac{(2bx+a)\left(b\,x^2+ax\right)^{\frac{3}{2}}}{8b}-\frac{3a^2\left(\frac{(2bx+a)\sqrt{b\,x^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8b^{\frac{3}{2}}}\right)}{16b}}{d}-\frac{c\left(\frac{\left(b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}\right)^{\frac{3}{2}}}{3}\cdots\right.}{\cdots}$

input
```
int(x*(b*x^2+a*x)^(3/2)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
3/64/b^(5/2)/(c*(a*d-b*c))^(1/2)*(128/3*c^3*(a^2*b^(5/2)*d^2-2*a*b^(7/2)*c
*d+b^(9/2)*c^2)*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))+((a^4*d^
4+8/3*a^3*b*c*d^3+16*a^2*b^2*c^2*d^2-64*a*b^3*c^3*d+128/3*c^4*b^4)*arctanh
((x*(b*x+a))^(1/2)/x/b^(1/2))-d*(16/3*(-d^3*x^3+4/3*c*d^2*x^2-2*c^2*d*x+4*
c^3)*b^(7/2)+d*a*(8*(-d^2*x^2+14/9*c*d*x-10/3*c^2)*b^(5/2)+d*a*(2/3*(-d*x+
4*c)*b^(3/2)+b^(1/2)*a*d)))*(x*(b*x+a))^(1/2))*(c*(a*d-b*c))^(1/2))/d^5
```

Fricas [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 1172, normalized size of antiderivative = 3.95

$$\int \frac{x(ax+bx^2)^{3/2}}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(x*(b*x^2+a*x)^(3/2)/(d*x+c),x, algorithm="fricas")
```

output
```
[1/384*(3*(128*b^4*c^4 - 192*a*b^3*c^3*d + 48*a^2*b^2*c^2*d^2 + 8*a^3*b*c*
d^3 + 3*a^4*d^4)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 38
4*(b^4*c^3 - a*b^3*c^2*d)*sqrt(b*c^2 - a*c*d)*log((a*c + (2*b*c - a*d)*x +
 2*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) + 2*(48*b^4*d^4*x^3 -
 192*b^4*c^3*d + 240*a*b^3*c^2*d^2 - 24*a^2*b^2*c*d^3 - 9*a^3*b*d^4 - 8*(8
*b^4*c*d^3 - 9*a*b^3*d^4)*x^2 + 2*(48*b^4*c^2*d^2 - 56*a*b^3*c*d^3 + 3*a^2
*b^2*d^4)*x)*sqrt(b*x^2 + a*x))/(b^3*d^5), 1/384*(768*(b^4*c^3 - a*b^3*c^2
*d)*sqrt(-b*c^2 + a*c*d)*arctan(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*
c*x + a*c)) + 3*(128*b^4*c^4 - 192*a*b^3*c^3*d + 48*a^2*b^2*c^2*d^2 + 8*a^
3*b*c*d^3 + 3*a^4*d^4)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)
) + 2*(48*b^4*d^4*x^3 - 192*b^4*c^3*d + 240*a*b^3*c^2*d^2 - 24*a^2*b^2*c*d
^3 - 9*a^3*b*d^4 - 8*(8*b^4*c*d^3 - 9*a*b^3*d^4)*x^2 + 2*(48*b^4*c^2*d^2 -
 56*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*x)*sqrt(b*x^2 + a*x))/(b^3*d^5), -1/192*(
3*(128*b^4*c^4 - 192*a*b^3*c^3*d + 48*a^2*b^2*c^2*d^2 + 8*a^3*b*c*d^3 + 3*
a^4*d^4)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + 192*(b^4*
c^3 - a*b^3*c^2*d)*sqrt(b*c^2 - a*c*d)*log((a*c + (2*b*c - a*d)*x + 2*sqrt
(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) - (48*b^4*d^4*x^3 - 192*b^4*
c^3*d + 240*a*b^3*c^2*d^2 - 24*a^2*b^2*c*d^3 - 9*a^3*b*d^4 - 8*(8*b^4*c*d^
3 - 9*a*b^3*d^4)*x^2 + 2*(48*b^4*c^2*d^2 - 56*a*b^3*c*d^3 + 3*a^2*b^2*d^4)
*x)*sqrt(b*x^2 + a*x))/(b^3*d^5), 1/192*(384*(b^4*c^3 - a*b^3*c^2*d)*sq...
```

Sympy [F]

$$\int \frac{x(ax+bx^2)^{3/2}}{c+dx}\,dx = \int \frac{x(x(a+bx))^{\frac{3}{2}}}{c+dx}\,dx$$

input
```
integrate(x*(b*x**2+a*x)**(3/2)/(d*x+c),x)
```

output
```
Integral(x*(x*(a + b*x))**(3/2)/(c + d*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(ax+bx^2)^{3/2}}{c+dx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(b*x^2+a*x)^(3/2)/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-2*b*c>0)', see `assume?` for
 more deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(ax+bx^2)^{3/2}}{c+dx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(b*x^2+a*x)^(3/2)/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(ax+bx^2)^{3/2}}{c+dx}\,dx = \int \frac{x\,(b\,x^2+a\,x)^{3/2}}{c+d\,x}\,dx$$

input
```
int((x*(a*x + b*x^2)^(3/2))/(c + d*x),x)
```

output
```
int((x*(a*x + b*x^2)^(3/2))/(c + d*x), x)
```

Reduce [B] (verification not implemented)

Time = 10.25 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.95

$$\int \frac{x(ax+bx^2)^{3/2}}{c+dx}\,dx = \frac{384\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)a\,b^3c^2d-384\sqrt{c}\,\sqrt{ad-bc}\,atan\Big(}{}$$

input
```
int(x*(b*x^2+a*x)^(3/2)/(d*x+c),x)
```

output
```
(384*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x)
 - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**3*c**2*d - 384*sqrt(c)
*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*s
qrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**4*c**3 + 384*sqrt(c)*sqrt(a*d - b*c)
*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/
(sqrt(c)*sqrt(b)))*a*b**3*c**2*d - 384*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(
a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqr
t(b)))*b**4*c**3 - 9*sqrt(x)*sqrt(a + b*x)*a**3*b*d**4 - 24*sqrt(x)*sqrt(a
 + b*x)*a**2*b**2*c*d**3 + 6*sqrt(x)*sqrt(a + b*x)*a**2*b**2*d**4*x + 240*
sqrt(x)*sqrt(a + b*x)*a*b**3*c**2*d**2 - 112*sqrt(x)*sqrt(a + b*x)*a*b**3*
c*d**3*x + 72*sqrt(x)*sqrt(a + b*x)*a*b**3*d**4*x**2 - 192*sqrt(x)*sqrt(a
+ b*x)*b**4*c**3*d + 96*sqrt(x)*sqrt(a + b*x)*b**4*c**2*d**2*x - 64*sqrt(x
)*sqrt(a + b*x)*b**4*c*d**3*x**2 + 48*sqrt(x)*sqrt(a + b*x)*b**4*d**4*x**3
 + 9*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4*d**4 + 24
*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*b*c*d**3 + 14
4*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*b**2*c**2*d*
*2 - 576*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b**3*c**
3*d + 384*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b**4*c**4
)/(192*b**3*d**5)
```

3.93 $\int \frac{(ax+bx^2)^{3/2}}{c+dx} dx$

Optimal result

Integrand size = 21, antiderivative size = 214

$$\int \frac{(ax+bx^2)^{3/2}}{c+dx} dx = -\frac{\left(10ac-\frac{8bc^2}{d}-\frac{a^2d}{b}\right)\sqrt{ax+bx^2}}{8d^2} - \frac{(2bc-ad)x\sqrt{ax+bx^2}}{4d^2} + \frac{(ax+bx^2)^{3/2}}{3d} - \frac{(2bc-ad)(8b^2c^2-8abcd-a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{8b^{3/2}d^4} + \frac{2c^{3/2}(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{d^4}$$

output

```
-1/8*(10*a*c-8*b*c^2/d-a^2*d/b)*(b*x^2+a*x)^(1/2)/d^2-1/4*(-a*d+2*b*c)*x*(
b*x^2+a*x)^(1/2)/d^2+1/3*(b*x^2+a*x)^(3/2)/d-1/8*(-a*d+2*b*c)*(-a^2*d^2-8*
a*b*c*d+8*b^2*c^2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(3/2)/d^4+2*c^(3
/2)*(-a*d+b*c)^(3/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))
/d^4
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.81 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.34

$$\int \frac{(ax+bx^2)^{3/2}}{c+dx}\,dx = \frac{(x(a+bx))^{3/2}\left(\sqrt{b}d\sqrt{x}\sqrt{a+bx}(3a^2d^2+2abd(-15c+7dx)+4b^2(6c^2-3cdx+2d\right.}{}$$

input
```
Integrate[(a*x + b*x^2)^(3/2)/(c + d*x),x]
```

output
```
((x*(a + b*x))^(3/2)*(Sqrt[b]*d*Sqrt[x]*Sqrt[a + b*x]*(3*a^2*d^2 + 2*a*b*d
*(-15*c + 7*d*x) + 4*b^2*(6*c^2 - 3*c*d*x + 2*d^2*x^2)) + 48*Sqrt[b]*Sqrt[
c]*(b*c - a*d)*(b*c - a*d - I*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d])*Sqrt[-(b*c)
 + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*ArcTan[(Sqrt[-(b*c) + 2*
a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*(Sqrt[a] -
Sqrt[a + b*x]))] + 48*Sqrt[b]*Sqrt[c]*(b*c - a*d)*(b*c - a*d + I*Sqrt[a]*S
qrt[d]*Sqrt[b*c - a*d])*Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b
*c - a*d]]*ArcTan[(Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c -
a*d]]*Sqrt[x])/(Sqrt[c]*(Sqrt[a] - Sqrt[a + b*x]))] + 6*(16*b^3*c^3 - 24*a
*b^2*c^2*d + 6*a^2*b*c*d^2 + a^3*d^3)*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] -
 Sqrt[a + b*x])]))/(24*b^(3/2)*d^4*x^(3/2)*(a + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1162, 1231, 27, 1269, 1091, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}}{c+dx}\,dx$$

$$\Big\downarrow \text{1162}$$

$$\frac{\left(ax+bx^2\right)^{3/2}}{3d}-\frac{\int \frac{(ac+(2bc-ad)x)\sqrt{bx^2+ax}}{c+dx}dx}{2d}$$

$$\Big\downarrow \; 1231$$

$$\frac{\left(ax+bx^2\right)^{3/2}}{3d}-\frac{-\frac{\int -\frac{ac\left(8b^2c^2-10abdc+a^2d^2\right)+(2bc-ad)\left(8b^2c^2-8abdc-a^2d^2\right)x}{2(c+dx)\sqrt{bx^2+ax}}dx}{4bd^2}-\frac{\sqrt{ax+bx^2}\left(a^2d^2-2bdx(2bc-ad)-10abcd+8b^2c^2\right)}{4bd^2}}{2d}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(ax+bx^2\right)^{3/2}}{3d}-\frac{\frac{\int \frac{ac\left(8b^2c^2-10abdc+a^2d^2\right)+(2bc-ad)\left(8b^2c^2-8abdc-a^2d^2\right)x}{(c+dx)\sqrt{bx^2+ax}}dx}{8bd^2}-\frac{\sqrt{ax+bx^2}\left(a^2d^2-2bdx(2bc-ad)-10abcd+8b^2c^2\right)}{4bd^2}}{2d}$$

$$\Big\downarrow \; 1269$$

$$\frac{\left(ax+bx^2\right)^{3/2}}{3d}-\frac{\frac{\frac{(2bc-ad)\left(-a^2d^2-8abcd+8b^2c^2\right)\int \frac{1}{\sqrt{bx^2+ax}}dx}{d}-\frac{16bc^2(bc-ad)^2\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{8bd^2}-\frac{\sqrt{ax+bx^2}\left(a^2d^2-2bdx(2bc-ad)-10abcd+8b^2c^2\right)}{4bd^2}}{2d}$$

$$\Big\downarrow \; 1091$$

$$\frac{\left(ax+bx^2\right)^{3/2}}{3d}-\frac{\frac{\frac{2(2bc-ad)\left(-a^2d^2-8abcd+8b^2c^2\right)\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{d}-\frac{16bc^2(bc-ad)^2\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{8bd^2}-\frac{\sqrt{ax+bx^2}\left(a^2d^2-2bdx(2bc-ad)-10abcd+8b^2c^2\right)}{4bd^2}}{2d}$$

$$\Big\downarrow \; 219$$

$$\frac{\left(ax+bx^2\right)^{3/2}}{3d}-\frac{\frac{\frac{2\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(2bc-ad)\left(-a^2d^2-8abcd+8b^2c^2\right)}{\sqrt{b}d}-\frac{16bc^2(bc-ad)^2\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{8bd^2}-\frac{\sqrt{ax+bx^2}\left(a^2d^2-2bdx(2bc-ad)-10abcd+8b^2c^2\right)}{4bd^2}}{2d}$$

$$\Big\downarrow \; 1154$$

$$\frac{(ax+bx^2)^{3/2}}{3d}-\frac{\frac{\frac{32bc^2(bc-ad)^2\int \frac{1}{4c(bc-ad)-\frac{(ac+(2bc-ad)x)^2}{bx^2+ax}}d\left(-\frac{ac+(2bc-ad)x}{\sqrt{bx^2+ax}}\right)}{d}+\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(2bc-ad)\left(-a^2d^2-8abcd+8b^2c^2\right)}{\sqrt{b}d}}{8bd^2}-\frac{\sqrt{ax+bx^2}\left(a^2d^2-2bc\cdots\right.}{}}{2d}$$

$$\Big\downarrow \; 219$$

$$\frac{(ax+bx^2)^{3/2}}{3d}-\frac{\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(2bc-ad)\left(-a^2d^2-8abcd+8b^2c^2\right)}{\sqrt{b}d}-\frac{16bc^{3/2}(bc-ad)^{3/2}\text{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{d}}{8bd^2}-\frac{\sqrt{ax+bx^2}\left(a^2d^2-2bdx(2bc-ad)\cdots\right.}{4bd^2}}{2d}$$

input
```
Int[(a*x + b*x^2)^(3/2)/(c + d*x),x]
```

output
```
(a*x + b*x^2)^(3/2)/(3*d) - (-1/4*((8*b^2*c^2 - 10*a*b*c*d + a^2*d^2 - 2*b
*d*(2*b*c - a*d)*x)*Sqrt[a*x + b*x^2])/(b*d^2) + ((2*(2*b*c - a*d)*(8*b^2*
c^2 - 8*a*b*c*d - a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(Sqrt[b
]*d) - (16*b*c^(3/2)*(b*c - a*d)^(3/2)*ArcTanh[(a*c + (2*b*c - a*d)*x)/(2*
Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[a*x + b*x^2])])/d)/(8*b*d^2))/(2*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$-\frac{-\frac{d\sqrt{x(bx+a)}\,\left(8b^2d^2x^2+14ab\,d^2x-12b^2cxd+3a^2d^2-30abcd+24b^2c^2\right)}{24b}+\frac{\left(a^3d^3+6a^2bc\,d^2-24a\,b^2c^2d+16b^3c^3\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right.}{8b^{\frac{3}{2}}}}{d^4}$
risch	$\frac{(8b^2d^2x^2+14ab\,d^2x-12b^2cxd+3a^2d^2-30abcd+24b^2c^2)x(bx+a)}{24b\,d^3\sqrt{x(bx+a)}}-\frac{\frac{\left(a^3d^3+6a^2bc\,d^2-24a\,b^2c^2d+16b^3c^3\right)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+\right.}{d\sqrt{b}}}{}$
default	$\frac{\frac{\left(b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}\right)^{\frac{3}{2}}}{3}+\frac{(ad-2bc)\left(\frac{\left(2b\left(x+\frac{c}{d}\right)+\frac{ad-2bc}{d}\right)\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{4b}+\frac{\left(-\frac{4bc}{}\right.}{}\right.}{}}{}$

input

```
int((b*x^2+a*x)^(3/2)/(d*x+c),x,method=_RETURNVERBOSE)
```

output

```
-1/d^4*(-1/24*d*(x*(b*x+a))^(1/2)*(8*b^2*d^2*x^2+14*a*b*d^2*x-12*b^2*c*d*x
+3*a^2*d^2-30*a*b*c*d+24*b^2*c^2)/b+1/8*(a^3*d^3+6*a^2*b*c*d^2-24*a*b^2*c^
2*d+16*b^3*c^3)/b^(3/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))+2*(a*d-b*c)^2
*c^2/(c*(a*d-b*c))^(1/2)*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))
)
```

Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 896, normalized size of antiderivative = 4.19

$$\int \frac{(ax+bx^2)^{3/2}}{c+dx}\,dx = \text{Too large to display}$$

input

```
integrate((b*x^2+a*x)^(3/2)/(d*x+c),x, algorithm="fricas")
```

output
```
[1/48*(3*(16*b^3*c^3 - 24*a*b^2*c^2*d + 6*a^2*b*c*d^2 + a^3*d^3)*sqrt(b)*l
og(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 48*(b^3*c^2 - a*b^2*c*d)*sqr
t(b*c^2 - a*c*d)*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*c^2 - a*c*d)*sqrt(b
*x^2 + a*x))/(d*x + c)) + 2*(8*b^3*d^3*x^2 + 24*b^3*c^2*d - 30*a*b^2*c*d^2
 + 3*a^2*b*d^3 - 2*(6*b^3*c*d^2 - 7*a*b^2*d^3)*x)*sqrt(b*x^2 + a*x))/(b^2*
d^4), -1/48*(96*(b^3*c^2 - a*b^2*c*d)*sqrt(-b*c^2 + a*c*d)*arctan(sqrt(-b*
c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) - 3*(16*b^3*c^3 - 24*a*b^2*c
^2*d + 6*a^2*b*c*d^2 + a^3*d^3)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x
)*sqrt(b)) - 2*(8*b^3*d^3*x^2 + 24*b^3*c^2*d - 30*a*b^2*c*d^2 + 3*a^2*b*d^
3 - 2*(6*b^3*c*d^2 - 7*a*b^2*d^3)*x)*sqrt(b*x^2 + a*x))/(b^2*d^4), 1/24*(3
*(16*b^3*c^3 - 24*a*b^2*c^2*d + 6*a^2*b*c*d^2 + a^3*d^3)*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - 24*(b^3*c^2 - a*b^2*c*d)*sqrt(b*c^2
 - a*c*d)*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 +
a*x))/(d*x + c)) + (8*b^3*d^3*x^2 + 24*b^3*c^2*d - 30*a*b^2*c*d^2 + 3*a^2*
b*d^3 - 2*(6*b^3*c*d^2 - 7*a*b^2*d^3)*x)*sqrt(b*x^2 + a*x))/(b^2*d^4), -1/
24*(48*(b^3*c^2 - a*b^2*c*d)*sqrt(-b*c^2 + a*c*d)*arctan(sqrt(-b*c^2 + a*c
*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) - 3*(16*b^3*c^3 - 24*a*b^2*c^2*d + 6*
a^2*b*c*d^2 + a^3*d^3)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a
)) - (8*b^3*d^3*x^2 + 24*b^3*c^2*d - 30*a*b^2*c*d^2 + 3*a^2*b*d^3 - 2*(6*b
^3*c*d^2 - 7*a*b^2*d^3)*x)*sqrt(b*x^2 + a*x))/(b^2*d^4)]
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/2}}{c+dx}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}}{c+dx}\,dx$$

input
```
integrate((b*x**2+a*x)**(3/2)/(d*x+c),x)
```

output
```
Integral((x*(a + b*x))**(3/2)/(c + d*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ax+bx^2)^{3/2}}{c+dx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a*x)^(3/2)/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ax+bx^2)^{3/2}}{c+dx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a*x)^(3/2)/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{c+dx}\,dx = \int \frac{(b\,x^2+a\,x)^{3/2}}{c+d\,x}\,dx$$

input
```
int((a*x + b*x^2)^(3/2)/(c + d*x),x)
```

output
```
int((a*x + b*x^2)^(3/2)/(c + d*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.13

$$\int \frac{(ax+bx^2)^{3/2}}{c+dx}\,dx = \frac{-48\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)a\,b^2cd+48\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad}}{}\right.}{}$$

input
```
int((b*x^2+a*x)^(3/2)/(d*x+c),x)
```

output
```
( - 48*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*
x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c*d + 48*sqrt(c)*s
qrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqr
t(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**3*c**2 - 48*sqrt(c)*sqrt(a*d - b*c)*at
an((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sq
rt(c)*sqrt(b)))*a*b**2*c*d + 48*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b
*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*
b**3*c**2 + 3*sqrt(x)*sqrt(a + b*x)*a**2*b*d**3 - 30*sqrt(x)*sqrt(a + b*x)
*a*b**2*c*d**2 + 14*sqrt(x)*sqrt(a + b*x)*a*b**2*d**3*x + 24*sqrt(x)*sqrt(
a + b*x)*b**3*c**2*d - 12*sqrt(x)*sqrt(a + b*x)*b**3*c*d**2*x + 8*sqrt(x)*
sqrt(a + b*x)*b**3*d**3*x**2 - 3*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt
(b))/sqrt(a))*a**3*d**3 - 18*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))
/sqrt(a))*a**2*b*c*d**2 + 72*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))
/sqrt(a))*a*b**2*c**2*d - 48*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))
/sqrt(a))*b**3*c**3)/(24*b**2*d**4)
```

3.94 $\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 164

$$\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)} \, dx = -\frac{(4bc-3ad)\sqrt{ax+bx^2}}{4d^2} + \frac{(ax+bx^2)^{3/2}}{2dx} + \frac{(8b^2c^2-12abcd+3a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4\sqrt{b}d^3} - \frac{2\sqrt{c}(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{d^3}$$

output
```
-1/4*(-3*a*d+4*b*c)*(b*x^2+a*x)^(1/2)/d^2+1/2*(b*x^2+a*x)^(3/2)/d/x+1/4*(3
*a^2*d^2-12*a*b*c*d+8*b^2*c^2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(1/2
)/d^3-2*c^(1/2)*(-a*d+b*c)^(3/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2
+a*x)^(1/2))/d^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.26 (sec) , antiderivative size = 449, normalized size of antiderivative = 2.74

$$\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)}\,dx = \frac{(x(a+bx))^{3/2}\left(b\sqrt{c}d\sqrt{x}\sqrt{a+bx}(-4bc+5ad+2bdx)+8(bc-ad)\left(bc-ad-i\sqrt{c}\right.\right.}{}$$

input
```
Integrate[(a*x + b*x^2)^(3/2)/(x*(c + d*x)),x]
```

output
```
((x*(a + b*x))^(3/2)*(b*Sqrt[c]*d*Sqrt[x]*Sqrt[a + b*x]*(-4*b*c + 5*a*d +
2*b*d*x) + 8*(b*c - a*d)*(b*c - a*d - I*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d])*S
qrt[-(b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*ArcTan[(Sqrt[-
(b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*(
-Sqrt[a] + Sqrt[a + b*x]))] + 8*(b*c - a*d)*(b*c - a*d + I*Sqrt[a]*Sqrt[d]
*Sqrt[b*c - a*d])*Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a
*d]]*ArcTan[(Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*
Sqrt[x])/(Sqrt[c]*(-Sqrt[a] + Sqrt[a + b*x]))] + 2*Sqrt[b]*Sqrt[c]*(8*b^2*
c^2 - 12*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a
 + b*x])]))/(4*b*Sqrt[c]*d^3*x^(3/2)*(a + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.23, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1261, 112, 27, 171, 27, 175, 65, 104, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)}\,dx$$

$$\Big\downarrow \; 1261$$

$$\frac{(ax+bx^2)^{3/2}\int \frac{\sqrt{x}(a+bx)^{3/2}}{c+dx}dx}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{112}$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\sqrt{x}(a+bx)^{3/2}}{2d}-\frac{\int \frac{\sqrt{a+bx}(ac+(4bc-3ad)x)}{2\sqrt{x}(c+dx)}dx}{2d}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\sqrt{x}(a+bx)^{3/2}}{2d}-\frac{\int \frac{\sqrt{a+bx}(ac+(4bc-3ad)x)}{\sqrt{x}(c+dx)}dx}{4d}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{171}$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\sqrt{x}(a+bx)^{3/2}}{2d}-\frac{\frac{\int -\frac{ac(4bc-5ad)+\left(8b^2c^2-12abdc+3a^2d^2\right)x}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}+\frac{\sqrt{x}\sqrt{a+bx}(4bc-3ad)}{d}}{4d}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\sqrt{x}(a+bx)^{3/2}}{2d}-\frac{\frac{\sqrt{x}\sqrt{a+bx}(4bc-3ad)}{d}-\frac{\int \frac{ac(4bc-5ad)+\left(8b^2c^2-12abdc+3a^2d^2\right)x}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{2d}}{4d}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{175}$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\sqrt{x}(a+bx)^{3/2}}{2d}-\frac{\frac{\sqrt{x}\sqrt{a+bx}(4bc-3ad)}{d}-\frac{\frac{\left(3a^2d^2-12abcd+8b^2c^2\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{d}-\frac{8c(bc-ad)^2\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}}{2d}}{4d}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{65}$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\sqrt{x}(a+bx)^{3/2}}{2d}-\frac{\frac{\sqrt{x}\sqrt{a+bx}(4bc-3ad)}{d}-\frac{\frac{2\left(3a^2d^2-12abcd+8b^2c^2\right)\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}-\frac{8c(bc-ad)^2\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}}{2d}}{4d}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{104}$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\sqrt{x}(a+bx)^{3/2}}{2d}-\frac{\frac{\sqrt{x}\sqrt{a+bx}(4bc-3ad)}{d}-\frac{\frac{2\left(3a^2d^2-12abcd+8b^2c^2\right)\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}-\frac{16c(bc-ad)^2\int\frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}}{2d}}{4d}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$\Big\downarrow$ 219

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\sqrt{x}(a+bx)^{3/2}}{2d}-\frac{\frac{\sqrt{x}\sqrt{a+bx}(4bc-3ad)}{d}-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)\left(3a^2d^2-12abcd+8b^2c^2\right)}{\sqrt{b}d}-\frac{16c(bc-ad)^2\int\frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}}{2d}}{4d}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$\Big\downarrow$ 221

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\sqrt{x}(a+bx)^{3/2}}{2d}-\frac{\frac{\sqrt{x}\sqrt{a+bx}(4bc-3ad)}{d}-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)\left(3a^2d^2-12abcd+8b^2c^2\right)}{\sqrt{b}d}-\frac{16\sqrt{c}(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{d}}{2d}}{4d}\right)}{x^{3/2}(a+bx)^{3/2}}$$

input
```
Int[(a*x + b*x^2)^(3/2)/(x*(c + d*x)),x]
```

output
```
((a*x + b*x^2)^(3/2)*((Sqrt[x]*(a + b*x)^(3/2))/(2*d) - (((4*b*c - 3*a*d)*
Sqrt[x]*Sqrt[a + b*x])/d - ((2*(8*b^2*c^2 - 12*a*b*c*d + 3*a^2*d^2)*ArcTan
h[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(Sqrt[b]*d) - (16*Sqrt[c]*(b*c - a*d)^
(3/2)*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/d)/(2*d)
)/(4*d)))/(x^(3/2)*(a + b*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 112
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*(m + n +
p + 1))), x] - Simp[1/(f*(m + n + p + 1))   Int[(a + b*x)^(m - 1)*(c + d*x)
^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*(b*e - a
*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] &&
GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m, 2*n, 2*p
] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] && !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$-\frac{-\frac{d\sqrt{x(bx+a)}\,(2bdx+5ad-4bc)}{4}-\frac{\left(3a^{2}d^{2}-12abcd+8b^{2}c^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{4\sqrt{b}}-\frac{2(ad-bc)^{2}c\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{\sqrt{c(ad-bc)}}}{d^{3}}$
risch	$\frac{(2bdx+5ad-4bc)x(bx+a)}{4d^{2}\sqrt{x(bx+a)}}+\frac{\frac{\left(3a^{2}d^{2}-12abcd+8b^{2}c^{2}\right)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{d\sqrt{b}}+\frac{8c\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\ln\left(\frac{-\frac{2c(ad-bc)}{d^{2}}+\frac{(ad-2}{}}{}\right.}{}}{8d^{2}}$
default	$\frac{\frac{\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{3}+\frac{a\left(\frac{(2bx+a)\sqrt{b\,x^{2}+ax}}{4b}-\frac{a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8b^{\frac{3}{2}}}\right)}{2}}{c}-\frac{\frac{\left(b\left(x+\frac{c}{d}\right)^{2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^{2}}\right)^{\frac{3}{2}}}{3}+\frac{(ad-2bc)}{}}{}$

input
```
int((b*x^2+a*x)^(3/2)/x/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/d^3*(-1/4*d*(x*(b*x+a))^(1/2)*(2*b*d*x+5*a*d-4*b*c)-1/4*(3*a^2*d^2-12*a
*b*c*d+8*b^2*c^2)/b^(1/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-2*(a*d-b*c)
^2*c/(c*(a*d-b*c))^(1/2)*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))
)
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 669, normalized size of antiderivative = 4.08

$$\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)}\,dx = \left[\frac{(8\,b^2c^2-12\,abcd+3\,a^2d^2)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-8\,(b^2c-abd)\sqrt{bc}}{8}\right.$$

$$-\frac{(8\,b^2c^2-12\,abcd+3\,a^2d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+4\,(b^2c-abd)\sqrt{bc^2-acd}\log\left(\frac{ac+(2\,bc-ad)x+2\,\sqrt{bc^2-}}{dx+c}\right.}{4\,bd^3}$$

input
```
integrate((b*x^2+a*x)^(3/2)/x/(d*x+c),x, algorithm="fricas")
```

output
```
[1/8*((8*b^2*c^2 - 12*a*b*c*d + 3*a^2*d^2)*sqrt(b)*log(2*b*x + a + 2*sqrt(
b*x^2 + a*x)*sqrt(b)) - 8*(b^2*c - a*b*d)*sqrt(b*c^2 - a*c*d)*log((a*c + (
2*b*c - a*d)*x + 2*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) + 2*(
2*b^2*d^2*x - 4*b^2*c*d + 5*a*b*d^2)*sqrt(b*x^2 + a*x))/(b*d^3), 1/8*(16*(
b^2*c - a*b*d)*sqrt(-b*c^2 + a*c*d)*arctan(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2
 + a*x)/(b*c*x + a*c)) + (8*b^2*c^2 - 12*a*b*c*d + 3*a^2*d^2)*sqrt(b)*log(
2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(2*b^2*d^2*x - 4*b^2*c*d + 5*
a*b*d^2)*sqrt(b*x^2 + a*x))/(b*d^3), -1/4*((8*b^2*c^2 - 12*a*b*c*d + 3*a^2
*d^2)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + 4*(b^2*c - a
*b*d)*sqrt(b*c^2 - a*c*d)*log((a*c + (2*b*c - a*d)*x + 2*sqrt(b*c^2 - a*c*
d)*sqrt(b*x^2 + a*x))/(d*x + c)) - (2*b^2*d^2*x - 4*b^2*c*d + 5*a*b*d^2)*s
qrt(b*x^2 + a*x))/(b*d^3), 1/4*(8*(b^2*c - a*b*d)*sqrt(-b*c^2 + a*c*d)*arc
tan(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) - (8*b^2*c^2 - 1
2*a*b*c*d + 3*a^2*d^2)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a
)) + (2*b^2*d^2*x - 4*b^2*c*d + 5*a*b*d^2)*sqrt(b*x^2 + a*x))/(b*d^3)]
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}}{x\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a*x)**(3/2)/x/(d*x+c),x)
```

output
```
Integral((x*(a + b*x))**(3/2)/(x*(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a*x)^(3/2)/x/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a*x)^(3/2)/x/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)}\,dx=\int \frac{{\left(b\,x^2+a\,x\right)}^{3/2}}{x\,\left(c+d\,x\right)}\,dx$$

input
```
int((a*x + b*x^2)^(3/2)/(x*(c + d*x)),x)
```

output
```
int((a*x + b*x^2)^(3/2)/(x*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.15

$$\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)}\,dx=\frac{8\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)abd-8\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}}{}\right.}{}$$

input
```
int((b*x^2+a*x)^(3/2)/x/(d*x+c),x)
```

output
```
(8*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) -
 sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b*d - 8*sqrt(c)*sqrt(a*d -
b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(
b))/(sqrt(c)*sqrt(b)))*b**2*c + 8*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d -
 b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b))
)*a*b*d - 8*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a
 + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**2*c + 5*sqrt(x)*s
qrt(a + b*x)*a*b*d**2 - 4*sqrt(x)*sqrt(a + b*x)*b**2*c*d + 2*sqrt(x)*sqrt(
a + b*x)*b**2*d**2*x + 3*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqr
t(a))*a**2*d**2 - 12*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)
)*a*b*c*d + 8*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b**2*
c**2)/(4*b*d**3)
```

3.95 $\int \frac{\left(ax+bx^2\right)^{3/2}}{x^2(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 114

$$\int \frac{(ax+bx^2)^{3/2}}{x^2(c+dx)}\,dx = \frac{b\sqrt{ax+bx^2}}{d} - \frac{\sqrt{b}(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{d^2} + \frac{2(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{\sqrt{c}d^2}$$

output
```
b*(b*x^2+a*x)^(1/2)/d-b^(1/2)*(-3*a*d+2*b*c)*arctanh(b^(1/2)*x/(b*x^2+a*x)
^(1/2))/d^2+2*(-a*d+b*c)^(3/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a
*x)^(1/2))/c^(1/2)/d^2
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.42

$$\int \frac{(ax+bx^2)^{3/2}}{x^2(c+dx)}\,dx = \frac{\sqrt{x}\sqrt{a+bx}\left(b\sqrt{c}d\sqrt{x}\sqrt{a+bx} - 2(-bc+ad)^{3/2}\arctan\left(\frac{-d\sqrt{x}\sqrt{a+bx}+\sqrt{b}(c+dx)}{\sqrt{c}\sqrt{-bc+ad}}\right) + \sqrt{b}\cdots}{\sqrt{c}d^2\sqrt{x(a+bx)}}$$

input
```
Integrate[(a*x + b*x^2)^(3/2)/(x^2*(c + d*x)),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(b*Sqrt[c]*d*Sqrt[x]*Sqrt[a + b*x] - 2*(-(b*c) + a*
d)^(3/2)*ArcTan[(-(d*Sqrt[x]*Sqrt[a + b*x]) + Sqrt[b]*(c + d*x))/(Sqrt[c]*
Sqrt[-(b*c) + a*d])] + Sqrt[b]*Sqrt[c]*(2*b*c - 3*a*d)*Log[-(Sqrt[b]*Sqrt[
x]) + Sqrt[a + b*x]]))/(Sqrt[c]*d^2*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.32, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1261, 113, 27, 175, 65, 104, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}}{x^2(c+dx)}dx$$

$$\Big\downarrow \text{ 1261}$$

$$\frac{(ax+bx^2)^{3/2}\int \frac{(a+bx)^{3/2}}{\sqrt{x}(c+dx)}dx}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{ 113}$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\int -\frac{a(bc-2ad)+b(2bc-3ad)x}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}+\frac{b\sqrt{x}\sqrt{a+bx}}{d}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{b\sqrt{x}\sqrt{a+bx}}{d}-\frac{\int \frac{a(bc-2ad)+b(2bc-3ad)x}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{2d}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{ 175}$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{b\sqrt{x}\sqrt{a+bx}}{d}-\frac{\frac{b(2bc-3ad)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{d}-\frac{2(bc-ad)^2\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}}{2d}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{ 65}$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{b\sqrt{x}\sqrt{a+bx}}{d}-\frac{\frac{2b(2bc-3ad)\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}-\frac{2(bc-ad)^2\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}}{2d}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \; 104$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{b\sqrt{x}\sqrt{a+bx}}{d}-\frac{\frac{2b(2bc-3ad)\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}-\frac{4(bc-ad)^2\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}}{2d}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{b\sqrt{x}\sqrt{a+bx}}{d}-\frac{\frac{2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)(2bc-3ad)}{d}-\frac{4(bc-ad)^2\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}}{2d}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{b\sqrt{x}\sqrt{a+bx}}{d}-\frac{\frac{2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)(2bc-3ad)}{d}-\frac{4(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{\sqrt{c}d}}{2d}\right)}{x^{3/2}(a+bx)^{3/2}}$$

input
```
Int[(a*x + b*x^2)^(3/2)/(x^2*(c + d*x)),x]
```

output
```
((a*x + b*x^2)^(3/2)*((b*Sqrt[x]*Sqrt[a + b*x])/d - ((2*Sqrt[b]*(2*b*c - 3
*a*d)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/d - (4*(b*c - a*d)^(3/2)*A
rcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(Sqrt[c]*d))/(2
*d)))/(x^(3/2)*(a + b*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 113
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.91

method	result
pseudoelliptic	$-\frac{b\left(-d\sqrt{x(bx+a)}-\frac{(3ad-2bc)\,\mathrm{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{\sqrt{b}}\right)+\frac{2(ad-bc)^2\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{\sqrt{c(ad-bc)}}}{d^2}$
risch	$\frac{x(bx+a)b}{d\sqrt{x(bx+a)}}+\frac{\frac{\sqrt{b}\,(3ad-2bc)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{d}-\frac{2\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^2}}}{x+\frac{c}{d}}\right)}{d^2\sqrt{-\frac{c(ad-bc)}{d^2}}}}{2d}$
default	$\frac{\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{a\,x^2}-\frac{6b\left(\frac{\left(b\,x^2+ax\right)^{\frac{3}{2}}}{3}+\frac{a\left(\frac{(2bx+a)\sqrt{b\,x^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8b^{\frac{3}{2}}}\right)}{2}\right)}{a}}{c}+\frac{d\left(\frac{\left(b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\right.}{3}\right.}{}$

input
```
int((b*x^2+a*x)^(3/2)/x^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/d^2*(b*(-d*(x*(b*x+a))^(1/2)-(3*a*d-2*b*c)/b^(1/2)*arctanh((x*(b*x+a))^
(1/2)/x/b^(1/2)))+2*(a*d-b*c)^2/(c*(a*d-b*c))^(1/2)*arctan((x*(b*x+a))^(1/
2)/x*c/(c*(a*d-b*c))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 523, normalized size of antiderivative = 4.59

$$\int \frac{(ax+bx^2)^{3/2}}{x^2(c+dx)}\,dx = \left[\frac{2\sqrt{bx^2+ax}bd-(2\,bc-3\,ad)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,(bc-ad)\sqrt{}}{2\,d^2}\right.$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^2/(d*x+c),x, algorithm="fricas")
```

output
```
[1/2*(2*sqrt(b*x^2 + a*x)*b*d - (2*b*c - 3*a*d)*sqrt(b)*log(2*b*x + a + 2*
sqrt(b*x^2 + a*x)*sqrt(b)) - 2*(b*c - a*d)*sqrt((b*c - a*d)/c)*log((a*c +
(2*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)*c*sqrt((b*c - a*d)/c))/(d*x + c)))/d
^2, 1/2*(2*sqrt(b*x^2 + a*x)*b*d + 4*(b*c - a*d)*sqrt(-(b*c - a*d)/c)*arct
an(-sqrt(b*x^2 + a*x)*c*sqrt(-(b*c - a*d)/c)/((b*c - a*d)*x)) - (2*b*c - 3
*a*d)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)))/d^2, (sqrt(b*x
^2 + a*x)*b*d + (2*b*c - 3*a*d)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)
/(b*x + a)) - (b*c - a*d)*sqrt((b*c - a*d)/c)*log((a*c + (2*b*c - a*d)*x -
 2*sqrt(b*x^2 + a*x)*c*sqrt((b*c - a*d)/c))/(d*x + c)))/d^2, (sqrt(b*x^2 +
 a*x)*b*d + (2*b*c - 3*a*d)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*
x + a)) + 2*(b*c - a*d)*sqrt(-(b*c - a*d)/c)*arctan(-sqrt(b*x^2 + a*x)*c*s
qrt(-(b*c - a*d)/c)/((b*c - a*d)*x)))/d^2]
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x^2(c+dx)}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}}{x^2\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a*x)**(3/2)/x**2/(d*x+c),x)
```

output
```
Integral((x*(a + b*x))**(3/2)/(x**2*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x^2(c+dx)}\,dx = \int \frac{(bx^2+ax)^{\frac{3}{2}}}{(dx+c)x^2}\,dx$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^2/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(3/2)/((d*x + c)*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ax+bx^2)^{3/2}}{x^2(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^2/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{x^2(c+dx)}\,dx = \int \frac{(b\,x^2+a\,x)^{3/2}}{x^2\,(c+d\,x)}\,dx$$

input
```
int((a*x + b*x^2)^(3/2)/(x^2*(c + d*x)),x)
```

output
```
int((a*x + b*x^2)^(3/2)/(x^2*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.45

$$\int \frac{(ax+bx^2)^{3/2}}{x^2(c+dx)}\,dx = \frac{-2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)ad+2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{}}{}\right.}{}$$

input
```
int((b*x^2+a*x)^(3/2)/x^2/(d*x+c),x)
```

output
```
( - 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x
) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*d + 2*sqrt(c)*sqrt(a*d -
 b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt
(b))/(sqrt(c)*sqrt(b)))*b*c - 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b
*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*
a*d + 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b
*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b*c + sqrt(x)*sqrt(a + b
*x)*b*c*d + 3*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*c*d
 - 2*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b*c**2)/(c*d**
2)
```

3.96 $\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)} dx$

Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)} dx = -\frac{2a\sqrt{ax+bx^2}}{cx} + \frac{2b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{d} - \frac{2(bc-ad)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{3/2}d}$$

output
```
-2*a*(b*x^2+a*x)^(1/2)/c/x+2*b^(3/2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/
d-2*(-a*d+b*c)^(3/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))
/c^(3/2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.77 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.83

$$\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)} dx = \frac{2(x(a+bx))^{3/2}\left(-abc^{3/2}d\sqrt{a+bx} + (bc-ad)\left(bc-ad-i\sqrt{a}\sqrt{d}\sqrt{bc-ad}\right)\sqrt{-b\ldots}\right.}{\ldots}$$

input
```
Integrate[(a*x + b*x^2)^(3/2)/(x^3*(c + d*x)),x]
```

output
```
(2*(x*(a + b*x))^(3/2)*(-(a*b*c^(3/2)*d*Sqrt[a + b*x]) + (b*c - a*d)*(b*c
- a*d - I*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d])*Sqrt[-(b*c) + 2*a*d - (2*I)*Sqr
t[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x]*ArcTan[(Sqrt[-(b*c) + 2*a*d - (2*I)*
Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*(-Sqrt[a] + Sqrt[a + b*
x]))] + (b*c - a*d)*(b*c - a*d + I*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d])*Sqrt[-
(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x]*ArcTan[(Sqr
t[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c
]*(-Sqrt[a] + Sqrt[a + b*x]))] + 2*b^(5/2)*c^(5/2)*Sqrt[x]*ArcTanh[(Sqrt[b
]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(b*c^(5/2)*d*x^2*(a + b*x)^(3/2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 427 vs. $2(109) = 218$.

Time = 1.23 (sec) , antiderivative size = 427, normalized size of antiderivative = 3.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1260, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)} \, dx$$

$$\Big\downarrow \text{ 1260}$$

$$\int \left(-\frac{d^3(ax+bx^2)^{3/2}}{c^3(c+dx)}+\frac{d^2(ax+bx^2)^{3/2}}{c^3x}-\frac{d(ax+bx^2)^{3/2}}{c^2x^2}+\frac{(ax+bx^2)^{3/2}}{cx^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^3d^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{8b^{3/2}c^3}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(2bc-ad)\left(-a^2d^2-8abcd+8b^2c^2\right)}{8b^{3/2}c^3d}-\frac{3a^2d\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4\sqrt{b}c^2}-\frac{\sqrt{ax+bx^2}\left(a^2d^2-2bdx(2bc-ad)-10abcd+8b^2c^2\right)}{8bc^3}-\frac{(bc-ad)^{3/2}\text{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{c^{3/2}d}+\frac{3a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{c}+\frac{ad^2(a+2bx)\sqrt{ax+bx^2}}{8bc^3}-\frac{3ad\sqrt{ax+bx^2}}{4c^2}-\frac{d\left(ax+bx^2\right)^{3/2}}{2c^2x}-\frac{2a\sqrt{ax+bx^2}}{cx}+\frac{b\sqrt{ax+bx^2}}{c}$$

input
```
Int[(a*x + b*x^2)^(3/2)/(x^3*(c + d*x)),x]
```

output
```
(b*Sqrt[a*x + b*x^2])/c - (3*a*d*Sqrt[a*x + b*x^2])/(4*c^2) - (2*a*Sqrt[a*
x + b*x^2])/(c*x) + (a*d^2*(a + 2*b*x)*Sqrt[a*x + b*x^2])/(8*b*c^3) - ((8*
b^2*c^2 - 10*a*b*c*d + a^2*d^2 - 2*b*d*(2*b*c - a*d)*x)*Sqrt[a*x + b*x^2])
/(8*b*c^3) - (d*(a*x + b*x^2)^(3/2))/(2*c^2*x) + (3*a*Sqrt[b]*ArcTanh[(Sqr
t[b]*x)/Sqrt[a*x + b*x^2]])/c - (3*a^2*d*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*
x^2]])/(4*Sqrt[b]*c^2) - (a^3*d^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/
(8*b^(3/2)*c^3) + ((2*b*c - a*d)*(8*b^2*c^2 - 8*a*b*c*d - a^2*d^2)*ArcTanh
[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(8*b^(3/2)*c^3*d) - ((b*c - a*d)^(3/2)*Ar
cTanh[(a*c + (2*b*c - a*d)*x)/(2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[a*x + b*x^2]
)])/(c^(3/2)*d)
```

Defintions of rubi rules used

rule 1260
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x + c*x^2)^p
, (d + e*x)^m*(f + g*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ
[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + n + 2*p + 1, 0] && ILtQ[m, 0] && ILtQ
[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$-\frac{2\left(-x(ad-bc)^2\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)+\left(-b^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)cx+\sqrt{x(bx+a)}\,ad\right)\sqrt{c(ad-bc)}\right)}{\sqrt{c(ad-bc)}\,cdx}$
risch	$-\frac{2a(bx+a)}{c\sqrt{x(bx+a)}}+\frac{\frac{c\,b^{\frac{3}{2}}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{d}+\frac{\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^2}}\sqrt{b\left(x+\cdots\right.}}{x+\frac{c}{d}}\cdots\right)}{d^2\sqrt{-\frac{c(ad-bc)}{d^2}}}\cdots}{c}$
default	$\frac{-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{a\,x^3}+\frac{4b\left(\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{a\,x^2}-\frac{6b\left(\frac{\left(b\,x^2+ax\right)^{\frac{3}{2}}}{3}+\frac{a\left(\frac{(2bx+a)\sqrt{b\,x^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8b^{\frac{3}{2}}}\right)}{2}\right)}{a}\right)}{a}}{c}+\frac{d^2\left(\frac{\left(b\,x^2+\cdots\right.}{\cdots}\cdots\right)}{\cdots}$

input
```
int((b*x^2+a*x)^(3/2)/x^3/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2/(c*(a*d-b*c))^(1/2)*(-x*(a*d-b*c)^2*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*
d-b*c))^(1/2))+(-b^(3/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*c*x+(x*(b*x+
a))^(1/2)*a*d)*(c*(a*d-b*c))^(1/2))/c/d/x
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 523, normalized size of antiderivative = 4.80

$$\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)}\,dx = \left[\frac{b^{\frac{3}{2}}cx\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-(bc-ad)x\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)x+2\,\sqrt{bx^2}}{dx+c}\right.}{cdx}\right.$$

$$-\frac{2\,\sqrt{-b}bcx\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+(bc-ad)x\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)x+2\,\sqrt{bx^2+ax}c\sqrt{\frac{bc-ad}{c}}}{dx+c}\right)+2\,\sqrt{bx^2+ax}a}{cdx}$$

$$\left.-\frac{2\left(\sqrt{-b}bcx\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+(bc-ad)x\sqrt{-\frac{bc-ad}{c}}\arctan\left(-\frac{\sqrt{bx^2+ax}c\sqrt{-\frac{bc-ad}{c}}}{(bc-ad)x}\right)+\sqrt{bx^2+ax}ad\right)}{cdx}\right]$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^3/(d*x+c),x, algorithm="fricas")
```

output
```
[(b^(3/2)*c*x*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - (b*c - a*d)*x
*sqrt((b*c - a*d)/c)*log((a*c + (2*b*c - a*d)*x + 2*sqrt(b*x^2 + a*x)*c*sq
rt((b*c - a*d)/c))/(d*x + c)) - 2*sqrt(b*x^2 + a*x)*a*d)/(c*d*x), (b^(3/2)
*c*x*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*(b*c - a*d)*x*sqrt(-
(b*c - a*d)/c)*arctan(-sqrt(b*x^2 + a*x)*c*sqrt(-(b*c - a*d)/c)/((b*c - a*
d)*x)) - 2*sqrt(b*x^2 + a*x)*a*d)/(c*d*x), -(2*sqrt(-b)*b*c*x*arctan(sqrt(
b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (b*c - a*d)*x*sqrt((b*c - a*d)/c)*log((
a*c + (2*b*c - a*d)*x + 2*sqrt(b*x^2 + a*x)*c*sqrt((b*c - a*d)/c))/(d*x +
c)) + 2*sqrt(b*x^2 + a*x)*a*d)/(c*d*x), -2*(sqrt(-b)*b*c*x*arctan(sqrt(b*x
^2 + a*x)*sqrt(-b)/(b*x + a)) + (b*c - a*d)*x*sqrt(-(b*c - a*d)/c)*arctan(
-sqrt(b*x^2 + a*x)*c*sqrt(-(b*c - a*d)/c)/((b*c - a*d)*x)) + sqrt(b*x^2 +
a*x)*a*d)/(c*d*x)]
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}}{x^3\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a*x)**(3/2)/x**3/(d*x+c),x)
```

output
```
Integral((x*(a + b*x))**(3/2)/(x**3*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)}\,dx = \int \frac{(bx^2+ax)^{\frac{3}{2}}}{(dx+c)x^3}\,dx$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^3/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(3/2)/((d*x + c)*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^3/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)}\,dx=\int \frac{(b\,x^2+a\,x)^{3/2}}{x^3\,(c+d\,x)}\,dx$$

input
```
int((a*x + b*x^2)^(3/2)/(x^3*(c + d*x)),x)
```

output
```
int((a*x + b*x^2)^(3/2)/(x^3*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.47

$$\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)}\,dx=\frac{2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)adx-2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{c}\cdots}{\cdots}\right.\cdots}{\cdots}$$

input
```
int((b*x^2+a*x)^(3/2)/x^3/(d*x+c),x)
```

output
```
(2*(sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x)
- sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*d*x - sqrt(c)*sqrt(a*d - b
*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b
))/(sqrt(c)*sqrt(b)))*b*c*x + sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c
) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*
d*x - sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x
) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b*c*x - sqrt(x)*sqrt(a + b
*x)*a*c*d + sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b*c**2*
x - sqrt(b)*a*c*d*x))/(c**2*d*x)
```

3.97 $\int \frac{(ax+bx^2)^{3/2}}{x^4(c+dx)} dx$

Optimal result

Integrand size = 24, antiderivative size = 105

$$\int \frac{(ax+bx^2)^{3/2}}{x^4(c+dx)} dx = -\frac{2(bc-ad)\sqrt{ax+bx^2}}{c^2x} - \frac{2(ax+bx^2)^{3/2}}{3cx^3} + \frac{2(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{5/2}}$$

output
```
-2*(-a*d+b*c)*(b*x^2+a*x)^(1/2)/c^2/x-2/3*(b*x^2+a*x)^(3/2)/c/x^3+2*(-a*d+
b*c)^(3/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(5/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.62 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.65

$$\int \frac{(ax+bx^2)^{3/2}}{x^4(c+dx)} dx = \frac{2(x(a+bx))^{3/2}\left(-bc^{3/2}\sqrt{a+bx}(4bcx+a(c-3dx))+3(bc-ad)\left(bc-ad-i\sqrt{a}\sqrt{}\right.\right.}{}$$

input
```
Integrate[(a*x + b*x^2)^(3/2)/(x^4*(c + d*x)),x]
```

output
```
(2*(x*(a + b*x))^(3/2)*(-(b*c^(3/2)*Sqrt[a + b*x]*(4*b*c*x + a*(c - 3*d*x)
)) + 3*(b*c - a*d)*(b*c - a*d - I*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d])*Sqrt[-(
b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*x^(3/2)*ArcTan[(Sqrt
[-(b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]
*(Sqrt[a] - Sqrt[a + b*x]))] + 3*(b*c - a*d)*(b*c - a*d + I*Sqrt[a]*Sqrt[d
]*Sqrt[b*c - a*d])*Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c -
a*d]]*x^(3/2)*ArcTan[(Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c
 - a*d]]*Sqrt[x])/(Sqrt[c]*(Sqrt[a] - Sqrt[a + b*x]))]))/(3*b*c^(7/2)*x^3*
(a + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1261, 105, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{3/2}}{x^4(c+dx)}dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{\left(ax+bx^2\right)^{3/2}\int \frac{(a+bx)^{3/2}}{x^{5/2}(c+dx)}dx}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{105}$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{(bc-ad)\int \frac{\sqrt{a+bx}}{x^{3/2}(c+dx)}dx}{c}-\frac{2(a+bx)^{3/2}}{3cx^{3/2}}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{105}$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{(bc-ad)\left(\frac{(bc-ad)\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{c}-\frac{2\sqrt{a+bx}}{c\sqrt{x}}\right)}{c}-\frac{2(a+bx)^{3/2}}{3cx^{3/2}}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{104}$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{(bc-ad)\left(\frac{2(bc-ad)\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}-\frac{2\sqrt{a+bx}}{c\sqrt{x}}\right)}{c}-\frac{2(a+bx)^{3/2}}{3cx^{3/2}}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{(bc-ad)\left(\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}}-\frac{2\sqrt{a+bx}}{c\sqrt{x}}\right)}{c}-\frac{2(a+bx)^{3/2}}{3cx^{3/2}}\right)}{x^{3/2}(a+bx)^{3/2}}$$

input
```
Int[(a*x + b*x^2)^(3/2)/(x^4*(c + d*x)),x]
```

output
```
((a*x + b*x^2)^(3/2)*((-2*(a + b*x)^(3/2))/(3*c*x^(3/2)) + ((b*c - a*d)*((
-2*Sqrt[a + b*x])/(c*Sqrt[x]) + (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b*c - a*d
]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/c^(3/2)))/c))/(x^(3/2)*(a + b*x)^(3/2
))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$-\frac{2\left(3x^{2}(ad-bc)^{2}\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)+\sqrt{x(bx+a)}\,((4bx+a)c-3adx)\sqrt{c(ad-bc)}\right)}{3\sqrt{c(ad-bc)}\,c^{2}x^{2}}$
risch	$-\frac{2(bx+a)(-3adx+4cbx+ac)}{3c^{2}\sqrt{x(bx+a)}\,x}-\frac{(a^{2}d^{2}-2abcd+b^{2}c^{2})\ln\left(\frac{-\frac{2c(ad-bc)}{d^{2}}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^{2}}}\sqrt{b\left(x+\frac{c}{d}\right)^{2}+\frac{(ad-}{}}}{x+\frac{c}{d}}\right)}{d\,c^{2}\sqrt{-\frac{c(ad-bc)}{d^{2}}}}$
default	Expression too large to display

input
```
int((b*x^2+a*x)^(3/2)/x^4/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(3*x^2*(a*d-b*c)^2*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))+
(x*(b*x+a))^(1/2)*((4*b*x+a)*c-3*a*d*x)*(c*(a*d-b*c))^(1/2))/(c*(a*d-b*c))
^(1/2)/c^2/x^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.14

$$\int \frac{(ax+bx^2)^{3/2}}{x^4(c+dx)}\,dx = \left[-\frac{3\,(bc-ad)x^2\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)x-2\,\sqrt{bx^2+ax}c\sqrt{\frac{bc-ad}{c}}}{dx+c}\right)+2\,\sqrt{bx^2+ax}(ac+(4}{3\,c^2x^2}\right.$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^4/(d*x+c),x, algorithm="fricas")
```

output
```
[-1/3*(3*(b*c - a*d)*x^2*sqrt((b*c - a*d)/c)*log((a*c + (2*b*c - a*d)*x -
2*sqrt(b*x^2 + a*x)*c*sqrt((b*c - a*d)/c))/(d*x + c)) + 2*sqrt(b*x^2 + a*x
)*(a*c + (4*b*c - 3*a*d)*x))/(c^2*x^2), 2/3*(3*(b*c - a*d)*x^2*sqrt(-(b*c
- a*d)/c)*arctan(-sqrt(b*x^2 + a*x)*c*sqrt(-(b*c - a*d)/c)/((b*c - a*d)*x)
) - sqrt(b*x^2 + a*x)*(a*c + (4*b*c - 3*a*d)*x))/(c^2*x^2)]
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x^4(c+dx)}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}}{x^4\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a*x)**(3/2)/x**4/(d*x+c),x)
```

output
```
Integral((x*(a + b*x))**(3/2)/(x**4*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(ax + bx^2)^{3/2}}{x^4(c + dx)}\,dx = \int \frac{(bx^2 + ax)^{\frac{3}{2}}}{(dx + c)x^4}\,dx$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^4/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(3/2)/((d*x + c)*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(89) = 178.

Time = 0.13 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.87

$$\int \frac{(ax + bx^2)^{3/2}}{x^4(c + dx)}\,dx = \frac{2\,(b^2c^2 - 2\,abcd + a^2d^2)\arctan\left(-\frac{\left(\sqrt{b}x - \sqrt{bx^2+ax}\right)d + \sqrt{b}c}{\sqrt{-bc^2+acd}}\right)}{\sqrt{-bc^2 + acd}c^2} + \frac{2\left(6\left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)^2 abc - 3\left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)^2 a^2d + 3\left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)a^2\sqrt{b}c + a^3c\right)}{3\left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)^3 c^2}$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^4/(d*x+c),x, algorithm="giac")
```

output
```
2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a*x))
*d + sqrt(b)*c)/sqrt(-b*c^2 + a*c*d))/(sqrt(-b*c^2 + a*c*d)*c^2) + 2/3*(6*
(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a*b*c - 3*(sqrt(b)*x - sqrt(b*x^2 + a*x)
)^2*a^2*d + 3*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^2*sqrt(b)*c + a^3*c)/((sqr
t(b)*x - sqrt(b*x^2 + a*x))^3*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{x^4(c+dx)}\,dx = \int \frac{(b\,x^2+a\,x)^{3/2}}{x^4\,(c+d\,x)}\,dx$$

input
```
int((a*x + b*x^2)^(3/2)/(x^4*(c + d*x)),x)
```

output
```
int((a*x + b*x^2)^(3/2)/(x^4*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.69

$$\int \frac{(ax+bx^2)^{3/2}}{x^4(c+dx)}\,dx = \frac{-2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)ad\,x^2+2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}}{}\right.}{}$$

input
```
int((b*x^2+a*x)^(3/2)/x^4/(d*x+c),x)
```

output
```
(2*( - 3*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a +
b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*d*x**2 + 3*sqrt(c)*sq
rt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt
(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b*c*x**2 - 3*sqrt(c)*sqrt(a*d - b*c)*atan(
(sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(
c)*sqrt(b)))*a*d*x**2 + 3*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) +
sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b*c*x*
*2 - sqrt(x)*sqrt(a + b*x)*a*c**2 + 3*sqrt(x)*sqrt(a + b*x)*a*c*d*x - 4*sq
rt(x)*sqrt(a + b*x)*b*c**2*x - sqrt(b)*a*c*d*x**2))/(3*c**3*x**2)
```

3.98 $\int \frac{(ax+bx^2)^{3/2}}{x^5(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 159

$$\int \frac{(ax+bx^2)^{3/2}}{x^5(c+dx)}\,dx = -\frac{2a\sqrt{ax+bx^2}}{5cx^3} - \frac{2(6bc-5ad)\sqrt{ax+bx^2}}{15c^2x^2} - \frac{2(3b^2c^2-20abcd+15a^2d^2)\sqrt{ax+bx^2}}{15ac^3x} - \frac{2d(bc-ad)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{7/2}}$$

output
```
-2/5*a*(b*x^2+a*x)^(1/2)/c/x^3-2/15*(-5*a*d+6*b*c)*(b*x^2+a*x)^(1/2)/c^2/x
^2-2/15*(15*a^2*d^2-20*a*b*c*d+3*b^2*c^2)*(b*x^2+a*x)^(1/2)/a/c^3/x-2*d*(-
a*d+b*c)^(3/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(7/
2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.01

$$\int \frac{(ax+bx^2)^{3/2}}{x^5(c+dx)}\,dx = \frac{2\sqrt{x(a+bx)}\left(-\frac{\sqrt{c}(3b^2c^2x^2+2abcx(3c-10dx)+a^2(3c^2-5cdx+15d^2x^2))}{a} + \frac{15d(-bc+ad)^{3/2}x^{5/2}\arctan\left(\cdots\right.}{\sqrt{a+bx}\cdots}\right.}{15c^{7/2}x^3}$$

input
```
Integrate[(a*x + b*x^2)^(3/2)/(x^5*(c + d*x)),x]
```

output
```
(2*Sqrt[x*(a + b*x)]*(-((Sqrt[c]*(3*b^2*c^2*x^2 + 2*a*b*c*x*(3*c - 10*d*x)
 + a^2*(3*c^2 - 5*c*d*x + 15*d^2*x^2)))/a) + (15*d*(-(b*c) + a*d)^(3/2)*x^
(5/2)*ArcTan[(-(d*Sqrt[x]*Sqrt[a + b*x]) + Sqrt[b]*(c + d*x))/(Sqrt[c]*Sqr
t[-(b*c) + a*d])])/Sqrt[a + b*x]))/(15*c^(7/2)*x^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1261, 107, 105, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}}{x^5(c+dx)}dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{(ax+bx^2)^{3/2}\int \frac{(a+bx)^{3/2}}{x^{7/2}(c+dx)}dx}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{107}$$

$$\frac{(ax+bx^2)^{3/2}\left(-\frac{d\int \frac{(a+bx)^{3/2}}{x^{5/2}(c+dx)}dx}{c}-\frac{2(a+bx)^{5/2}}{5acx^{5/2}}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{105}$$

$$\frac{(ax+bx^2)^{3/2}\left(-\frac{d\left(\frac{(bc-ad)\int \frac{\sqrt{a+bx}}{x^{3/2}(c+dx)}dx}{c}-\frac{2(a+bx)^{3/2}}{3cx^{3/2}}\right)}{c}-\frac{2(a+bx)^{5/2}}{5acx^{5/2}}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{105}$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(-\frac{d\left(\frac{(bc-ad)\left(\frac{(bc-ad)\int\frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{c}-\frac{2\sqrt{a+bx}}{c\sqrt{x}}\right)}{c}-\frac{2(a+bx)^{3/2}}{3cx^{3/2}}\right)}{c}-\frac{2(a+bx)^{5/2}}{5acx^{5/2}}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$\Big\downarrow$ 104

$$\frac{\left(ax+bx^2\right)^{3/2}\left(-\frac{d\left(\frac{(bc-ad)\left(\frac{2(bc-ad)\int\frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}-\frac{2\sqrt{a+bx}}{c\sqrt{x}}\right)}{c}-\frac{2(a+bx)^{3/2}}{3cx^{3/2}}\right)}{c}-\frac{2(a+bx)^{5/2}}{5acx^{5/2}}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$\Big\downarrow$ 221

$$\frac{\left(ax+bx^2\right)^{3/2}\left(-\frac{d\left(\frac{(bc-ad)\left(\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}}-\frac{2\sqrt{a+bx}}{c\sqrt{x}}\right)}{c}-\frac{2(a+bx)^{3/2}}{3cx^{3/2}}\right)}{c}-\frac{2(a+bx)^{5/2}}{5acx^{5/2}}\right)}{x^{3/2}(a+bx)^{3/2}}$$

input
```
Int[(a*x + b*x^2)^(3/2)/(x^5*(c + d*x)),x]
```

output
```
((a*x + b*x^2)^(3/2)*((-2*(a + b*x)^(5/2))/(5*a*c*x^(5/2)) - (d*((-2*(a +
b*x)^(3/2))/(3*c*x^(3/2)) + ((b*c - a*d)*((-2*Sqrt[a + b*x]])/(c*Sqrt[x]) +
 (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*
x])])/c^(3/2)))/c))/c))/(x^(3/2)*(a + b*x)^(3/2))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$-\frac{2\left(\left(c^2(bx+a)^2-\frac{5adx(4bx+a)c}{3}+5a^2d^2x^2\right)\sqrt{c(ad-bc)}\,\sqrt{x(bx+a)}-5da\,x^3(ad-bc)^2\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)\right)}{5\sqrt{c(ad-bc)}\,c^3x^3a}$
risch	$-\frac{2(bx+a)(15a^2d^2x^2-20abcd\,x^2+3b^2c^2x^2-5a^2cdx+6ab\,c^2x+3a^2c^2)}{15a\,c^3\sqrt{x(bx+a)}\,x^2}+\frac{(a^2d^2-2abcd+b^2c^2)\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)}{d}}{}\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a*x)^(3/2)/x^5/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2/5/(c*(a*d-b*c))^(1/2)*((c^2*(b*x+a)^2-5/3*a*d*x*(4*b*x+a)*c+5*a^2*d^2*x
^2)*(c*(a*d-b*c))^(1/2)*(x*(b*x+a))^(1/2)-5*d*a*x^3*(a*d-b*c)^2*arctan((x*
(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2)))/c^3/x^3/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.00

$$\int \frac{(ax+bx^2)^{3/2}}{x^5(c+dx)}\,dx = \left[-\frac{15\,(abcd-a^2d^2)x^3\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)x+2\,\sqrt{bx^2+ax}c\sqrt{\frac{bc-ad}{c}}}{dx+c}\right)+2\,(3\,a^2c^2+(3\,b^2}{15\,ac^3x^3}\right.$$

$$-\frac{2\left(15\,(abcd-a^2d^2)x^3\sqrt{-\frac{bc-ad}{c}}\arctan\left(-\frac{\sqrt{bx^2+ax}c\sqrt{-\frac{bc-ad}{c}}}{(bc-ad)x}\right)+(3\,a^2c^2+(3\,b^2c^2-20\,abcd+15\,a^2d^2)x^2\right.}{15\,ac^3x^3}$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^5/(d*x+c),x, algorithm="fricas")
```

output
```
[-1/15*(15*(a*b*c*d - a^2*d^2)*x^3*sqrt((b*c - a*d)/c)*log((a*c + (2*b*c -
 a*d)*x + 2*sqrt(b*x^2 + a*x)*c*sqrt((b*c - a*d)/c))/(d*x + c)) + 2*(3*a^2
*c^2 + (3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*x^2 + (6*a*b*c^2 - 5*a^2*c*d)
*x)*sqrt(b*x^2 + a*x))/(a*c^3*x^3), -2/15*(15*(a*b*c*d - a^2*d^2)*x^3*sqrt
(-(b*c - a*d)/c)*arctan(-sqrt(b*x^2 + a*x)*c*sqrt(-(b*c - a*d)/c)/((b*c -
a*d)*x)) + (3*a^2*c^2 + (3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*x^2 + (6*a*b
*c^2 - 5*a^2*c*d)*x)*sqrt(b*x^2 + a*x))/(a*c^3*x^3)]
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x^5(c+dx)}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}}{x^5\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a*x)**(3/2)/x**5/(d*x+c),x)
```

output
```
Integral((x*(a + b*x))**(3/2)/(x**5*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x^5(c+dx)}\,dx = \int \frac{(bx^2+ax)^{\frac{3}{2}}}{(dx+c)x^5}\,dx$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^5/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(3/2)/((d*x + c)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(137) = 274.

Time = 0.14 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.23

$$\int \frac{(ax+bx^2)^{3/2}}{x^5(c+dx)}\,dx = -\frac{2\,(b^2c^2d-2\,abcd^2+a^2d^3)\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)d+\sqrt{b}c}{\sqrt{-bc^2+acd}}\right)}{\sqrt{-bc^2+acd}c^3} + \frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4b^2c^2-30\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4abcd+15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4a^2d^2+30\left(\sqrt{}\right.\right.}{}$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^5/(d*x+c),x, algorithm="giac")
```

output
```
-2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*arctan(-((sqrt(b)*x - sqrt(b*x^2 +
a*x))*d + sqrt(b)*c)/sqrt(-b*c^2 + a*c*d))/(sqrt(-b*c^2 + a*c*d)*c^3) + 2/
15*(15*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*b^2*c^2 - 30*(sqrt(b)*x - sqrt(b*
x^2 + a*x))^4*a*b*c*d + 15*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^2*d^2 + 30*
(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a*b^(3/2)*c^2 - 15*(sqrt(b)*x - sqrt(b*x
^2 + a*x))^3*a^2*sqrt(b)*c*d + 30*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2*b*
c^2 - 5*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^3*c*d + 15*(sqrt(b)*x - sqrt(b
*x^2 + a*x))*a^3*sqrt(b)*c^2 + 3*a^4*c^2)/((sqrt(b)*x - sqrt(b*x^2 + a*x))
^5*c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{x^5(c+dx)}\,dx = \int \frac{(b\,x^2+a\,x)^{3/2}}{x^5\,(c+d\,x)}\,dx$$

input
```
int((a*x + b*x^2)^(3/2)/(x^5*(c + d*x)),x)
```

output
```
int((a*x + b*x^2)^(3/2)/(x^5*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.47

$$\int \frac{(ax+bx^2)^{3/2}}{x^5(c+dx)}\,dx = \frac{2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)a^2d^2x^3 - 2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}\cdots}{}\right.}{}$$

input
```
int((b*x^2+a*x)^(3/2)/x^5/(d*x+c),x)
```

output
```
(2*(15*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*
x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*d**2*x**3 - 15*sqrt(
c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)
*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b*c*d*x**3 + 15*sqrt(c)*sqrt(a*d -
b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(
b))/(sqrt(c)*sqrt(b)))*a**2*d**2*x**3 - 15*sqrt(c)*sqrt(a*d - b*c)*atan((s
qrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)
*sqrt(b)))*a*b*c*d*x**3 - 3*sqrt(x)*sqrt(a + b*x)*a**2*c**3 + 5*sqrt(x)*sq
rt(a + b*x)*a**2*c**2*d*x - 15*sqrt(x)*sqrt(a + b*x)*a**2*c*d**2*x**2 - 6*
sqrt(x)*sqrt(a + b*x)*a*b*c**3*x + 20*sqrt(x)*sqrt(a + b*x)*a*b*c**2*d*x**
2 - 3*sqrt(x)*sqrt(a + b*x)*b**2*c**3*x**2 + 9*sqrt(b)*a**2*c*d**2*x**3 -
8*sqrt(b)*a*b*c**2*d*x**3 - 3*sqrt(b)*b**2*c**3*x**3))/(15*a*c**4*x**3)
```

3.99 $\int \frac{(ax+bx^2)^{3/2}}{x^6(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 224

$$\int \frac{(ax+bx^2)^{3/2}}{x^6(c+dx)}\,dx = -\frac{2a\sqrt{ax+bx^2}}{7cx^4} - \frac{2(8bc-7ad)\sqrt{ax+bx^2}}{35c^2x^3} - \frac{2(3b^2c^2-42abcd+35a^2d^2)\sqrt{ax+bx^2}}{105ac^3x^2} + \frac{2(6b^3c^3+21ab^2c^2d-140a^2bcd^2+105a^3d^3)\sqrt{ax+bx^2}}{105a^2c^4x} + \frac{2d^2(bc-ad)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{9/2}}$$

output
```
-2/7*a*(b*x^2+a*x)^(1/2)/c/x^4-2/35*(-7*a*d+8*b*c)*(b*x^2+a*x)^(1/2)/c^2/x
^3-2/105*(35*a^2*d^2-42*a*b*c*d+3*b^2*c^2)*(b*x^2+a*x)^(1/2)/a/c^3/x^2+2/1
05*(105*a^3*d^3-140*a^2*b*c*d^2+21*a*b^2*c^2*d+6*b^3*c^3)*(b*x^2+a*x)^(1/2
)/a^2/c^4/x+2*d^2*(-a*d+b*c)^(3/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x
^2+a*x)^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.91

$$\int \frac{\left(ax+bx^2\right)^{3/2}}{x^6(c+dx)}\,dx = \frac{2\sqrt{x(a+bx)}\left(\frac{\sqrt{c}\left(6b^3c^3x^3-3ab^2c^2x^2(c-7dx)-2a^2bcx\left(12c^2-21cdx+70d^2x^2\right)+a^3\left(-15c^3+21c^2dx-35cd^2x^2\right.\right.}{a^2}\right.}{105c^{9/2}x^4}$$

input
```
Integrate[(a*x + b*x^2)^(3/2)/(x^6*(c + d*x)),x]
```

output
```
(2*Sqrt[x*(a + b*x)]*((Sqrt[c]*(6*b^3*c^3*x^3 - 3*a*b^2*c^2*x^2*(c - 7*d*x
) - 2*a^2*b*c*x*(12*c^2 - 21*c*d*x + 70*d^2*x^2) + a^3*(-15*c^3 + 21*c^2*d
*x - 35*c*d^2*x^2 + 105*d^3*x^3)))/a^2 - (105*d^2*(-(b*c) + a*d)^(3/2)*x^(
7/2)*ArcTan[(-(d*Sqrt[x]*Sqrt[a + b*x]) + Sqrt[b]*(c + d*x))/(Sqrt[c]*Sqrt
[-(b*c) + a*d])])/Sqrt[a + b*x]))/(105*c^(9/2)*x^4)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.18, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1261, 109, 27, 169, 27, 169, 27, 169, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{3/2}}{x^6(c+dx)}\,dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{\left(ax+bx^2\right)^{3/2}\int \frac{(a+bx)^{3/2}}{x^{9/2}(c+dx)}dx}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{109}$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(-\frac{2\int -\frac{a(8bc-7ad)+b(7bc-6ad)x}{2x^{7/2}\sqrt{a+bx}(c+dx)}dx}{7c}-\frac{2a\sqrt{a+bx}}{7cx^{7/2}}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$\downarrow$ 27

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\int \frac{a(8bc-7ad)+b(7bc-6ad)x}{x^{7/2}\sqrt{a+bx}(c+dx)}dx}{7c}-\frac{2a\sqrt{a+bx}}{7cx^{7/2}}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$\downarrow$ 169

$$\frac{(ax+bx^2)^{3/2}\left(\frac{-\frac{2\int -\frac{a\left(3b^2c^2-42abdc+35a^2d^2-4bd(8bc-7ad)x\right)}{2x^{5/2}\sqrt{a+bx}(c+dx)}dx}{5ac}-\frac{2\sqrt{a+bx}(8bc-7ad)}{5cx^{5/2}}}{7c}-\frac{2a\sqrt{a+bx}}{7cx^{7/2}}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$\downarrow$ 27

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\frac{\int \frac{3b^2c^2-42abdc+35a^2d^2-4bd(8bc-7ad)x}{x^{5/2}\sqrt{a+bx}(c+dx)}dx}{5c}-\frac{2\sqrt{a+bx}(8bc-7ad)}{5cx^{5/2}}}{7c}-\frac{2a\sqrt{a+bx}}{7cx^{7/2}}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$\downarrow$ 169

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\frac{-\frac{2\int \frac{6b^3c^3+21ab^2dc^2-140a^2bd^2c+105a^3d^3+2bd\left(3b^2c^2-42abdc+35a^2d^2\right)x}{2x^{3/2}\sqrt{a+bx}(c+dx)}dx}{3ac}-\frac{2\sqrt{a+bx}\left(\frac{3b^2c}{a}+\frac{35ad^2}{c}-42bd\right)}{3x^{3/2}}}{5c}-\frac{2\sqrt{a+bx}(8bc-7ad)}{5cx^{5/2}}}{7c}}{x^{3/2}(a+bx)^{3/2}}$$

$\downarrow$ 27

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\frac{-\frac{\int \frac{6b^3c^3+21ab^2dc^2-140a^2bd^2c+105a^3d^3+2bd\left(3b^2c^2-42abdc+35a^2d^2\right)x}{x^{3/2}\sqrt{a+bx}(c+dx)}dx}{3ac}-\frac{2\sqrt{a+bx}\left(\frac{3b^2c}{a}+\frac{35ad^2}{c}-42bd\right)}{3x^{3/2}}}{5c}-\frac{2\sqrt{a+bx}(8bc-7ad)}{5cx^{5/2}}}{7c}}{x^{3/2}(a+bx)^{3/2}}$$

$\downarrow$ 169

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{-\frac{-\frac{-\frac{2\int \frac{105a^2d^2(bc-ad)^2}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{ac}-\frac{2\sqrt{a+bx}\left(105a^3d^3-140a^2bcd^2+21ab^2c^2d+6b^3c^3\right)}{ac\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}\left(\frac{3b^2c}{a}+\frac{35ad^2}{c}-42bd\right)}{3x^{3/2}}}{5c}-\frac{2\sqrt{a+bx}(8bc-7ac}{5cx^{5/2}}}{7c}\right.}{x^{3/2}(a+bx)^{3/2}}$$

$\downarrow$ 27

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{-\frac{-\frac{-\frac{105ad^2(bc-ad)^2\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{c}-\frac{2\sqrt{a+bx}\left(105a^3d^3-140a^2bcd^2+21ab^2c^2d+6b^3c^3\right)}{ac\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}\left(\frac{3b^2c}{a}+\frac{35ad^2}{c}-42bd\right)}{3x^{3/2}}}{5c}-2\sqrt{}}{7c}\right.}{x^{3/2}(a+bx)^{3/2}}$$

$\downarrow$ 104

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{-\frac{-\frac{-\frac{210ad^2(bc-ad)^2\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}-\frac{2\sqrt{a+bx}\left(105a^3d^3-140a^2bcd^2+21ab^2c^2d+6b^3c^3\right)}{ac\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}\left(\frac{3b^2c}{a}+\frac{35ad^2}{c}-42bd\right)}{3x^{3/2}}}{5c}-2}{7c}\right.}{x^{3/2}(a+bx)^{3/2}}$$

$\downarrow$ 221

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{-\frac{-\frac{-\frac{2\sqrt{a+bx}\left(105a^3d^3-140a^2bcd^2+21ab^2c^2d+6b^3c^3\right)}{ac\sqrt{x}}-\frac{210ad^2(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}}}{3ac}-\frac{2\sqrt{a+bx}\left(\frac{3b^2c}{a}+\frac{35ad^2}{c}-42bd\right)}{3x^{3/2}}}{5c}}{7c}\right.}{x^{3/2}(a+bx)^{3/2}}$$

input

```
Int[(a*x + b*x^2)^(3/2)/(x^6*(c + d*x)),x]
```

output
```
((a*x + b*x^2)^(3/2)*((-2*a*Sqrt[a + b*x])/(7*c*x^(7/2)) + ((-2*(8*b*c - 7
*a*d)*Sqrt[a + b*x])/(5*c*x^(5/2)) + ((-2*((3*b^2*c)/a - 42*b*d + (35*a*d^
2)/c)*Sqrt[a + b*x])/(3*x^(3/2)) - ((-2*(6*b^3*c^3 + 21*a*b^2*c^2*d - 140*
a^2*b*c*d^2 + 105*a^3*d^3)*Sqrt[a + b*x])/(a*c*Sqrt[x]) - (210*a*d^2*(b*c
- a*d)^(3/2)*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/c
^(3/2))/(3*a*c))/(5*c))/(7*c)))/(x^(3/2)*(a + b*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$-\frac{2\left(\sqrt{c(ad-bc)}\,\left((bx+a)^{2}\left(-\frac{2bx}{5}+a\right)c^{3}-\frac{7adx(bx+a)^{2}c^{2}}{5}+\frac{7a^{2}d^{2}x^{2}(4bx+a)c}{3}-7a^{3}d^{3}x^{3}\right)\sqrt{x(bx+a)}+7a^{2}d^{2}x^{4}(ad-bc)^{2}\,\text{ar}}{7\sqrt{c(ad-bc)}\,c^{4}x^{4}a^{2}}$
risch	$-\frac{2(bx+a)\left(-105a^{3}d^{3}x^{3}+140a^{2}bc\,d^{2}x^{3}-21a\,b^{2}c^{2}d\,x^{3}-6b^{3}c^{3}x^{3}+35a^{3}c\,d^{2}x^{2}-42a^{2}b\,c^{2}x^{2}d+3a\,b^{2}c^{3}x^{2}-21a^{3}c^{2}dx+24a^{2}b\right.}{105a^{2}c^{4}\sqrt{x(bx+a)}\,x^{3}}$
default	Expression too large to display

input
```
int((b*x^2+a*x)^(3/2)/x^6/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2/7/(c*(a*d-b*c))^(1/2)*((c*(a*d-b*c))^(1/2)*((b*x+a)^2*(-2/5*b*x+a)*c^3-
7/5*a*d*x*(b*x+a)^2*c^2+7/3*a^2*d^2*x^2*(4*b*x+a)*c-7*a^3*d^3*x^3)*(x*(b*x
+a))^(1/2)+7*a^2*d^2*x^4*(a*d-b*c)^2*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-
b*c))^(1/2)))/c^4/x^4/a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.93

$$\int \frac{(ax+bx^2)^{3/2}}{x^6(c+dx)}\,dx = \left[-\frac{105\,(a^2bcd^2-a^3d^3)x^4\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)x-2\,\sqrt{bx^2+ax}c\sqrt{\frac{bc-ad}{c}}}{dx+c}\right)+2\,(15\,a^3c^3-\right.$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^6/(d*x+c),x, algorithm="fricas")
```

output
```
[-1/105*(105*(a^2*b*c*d^2 - a^3*d^3)*x^4*sqrt((b*c - a*d)/c)*log((a*c + (2
*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)*c*sqrt((b*c - a*d)/c))/(d*x + c)) + 2*
(15*a^3*c^3 - (6*b^3*c^3 + 21*a*b^2*c^2*d - 140*a^2*b*c*d^2 + 105*a^3*d^3)
*x^3 + (3*a*b^2*c^3 - 42*a^2*b*c^2*d + 35*a^3*c*d^2)*x^2 + 3*(8*a^2*b*c^3
- 7*a^3*c^2*d)*x)*sqrt(b*x^2 + a*x))/(a^2*c^4*x^4), 2/105*(105*(a^2*b*c*d^
2 - a^3*d^3)*x^4*sqrt(-(b*c - a*d)/c)*arctan(-sqrt(b*x^2 + a*x)*c*sqrt(-(b
*c - a*d)/c)/((b*c - a*d)*x)) - (15*a^3*c^3 - (6*b^3*c^3 + 21*a*b^2*c^2*d
- 140*a^2*b*c*d^2 + 105*a^3*d^3)*x^3 + (3*a*b^2*c^3 - 42*a^2*b*c^2*d + 35*
a^3*c*d^2)*x^2 + 3*(8*a^2*b*c^3 - 7*a^3*c^2*d)*x)*sqrt(b*x^2 + a*x))/(a^2*
c^4*x^4)]
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x^6(c+dx)}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}}{x^6\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a*x)**(3/2)/x**6/(d*x+c),x)
```

output
```
Integral((x*(a + b*x))**(3/2)/(x**6*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x^6(c+dx)}\,dx=\int \frac{(bx^2+ax)^{\frac{3}{2}}}{(dx+c)x^6}\,dx$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^6/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(3/2)/((d*x + c)*x^6), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(198) = 396.

Time = 0.14 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.46

$$\int \frac{(ax+bx^2)^{3/2}}{x^6(c+dx)}\,dx=\frac{2\,(b^2c^2d^2-2\,abcd^3+a^2d^4)\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)d+\sqrt{b}c}{\sqrt{-bc^2+acd}}\right)}{\sqrt{-bc^2+acd}c^4}$$
$$-\frac{2\left(105\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6b^2c^2d-210\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6abcd^2+105\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6a^2d^3-\right.}{}$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^6/(d*x+c),x, algorithm="giac")
```

output
```
2*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*arctan(-((sqrt(b)*x - sqrt(b*x^2 +
 a*x))*d + sqrt(b)*c)/sqrt(-b*c^2 + a*c*d))/(sqrt(-b*c^2 + a*c*d)*c^4) - 2
/105*(105*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*b^2*c^2*d - 210*(sqrt(b)*x - s
qrt(b*x^2 + a*x))^6*a*b*c*d^2 + 105*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a^2*
d^3 - 105*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*b^(5/2)*c^3 + 210*(sqrt(b)*x -
 sqrt(b*x^2 + a*x))^5*a*b^(3/2)*c^2*d - 105*(sqrt(b)*x - sqrt(b*x^2 + a*x)
)^5*a^2*sqrt(b)*c*d^2 - 315*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a*b^2*c^3 +
210*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^2*b*c^2*d - 35*(sqrt(b)*x - sqrt(b
*x^2 + a*x))^4*a^3*c*d^2 - 420*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^2*b^(3/
2)*c^3 + 105*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^3*sqrt(b)*c^2*d - 294*(sq
rt(b)*x - sqrt(b*x^2 + a*x))^2*a^3*b*c^3 + 21*(sqrt(b)*x - sqrt(b*x^2 + a*
x))^2*a^4*c^2*d - 105*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^4*sqrt(b)*c^3 - 15
*a^5*c^3)/((sqrt(b)*x - sqrt(b*x^2 + a*x))^7*c^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{x^6(c+dx)}\,dx = \int \frac{\left(b\,x^2+a\,x\right)^{3/2}}{x^6\,\left(c+d\,x\right)}\,dx$$

input
```
int((a*x + b*x^2)^(3/2)/(x^6*(c + d*x)),x)
```

output
```
int((a*x + b*x^2)^(3/2)/(x^6*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 509, normalized size of antiderivative = 2.27

$$\int \frac{(ax+bx^2)^{3/2}}{x^6(c+dx)}\,dx = \frac{-2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)a^3d^3x^4+2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-}}{}\right.}{}$$

input
```
int((b*x^2+a*x)^(3/2)/x^6/(d*x+c),x)
```

output
```
(2*( - 105*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a
+ b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*d**3*x**4 + 105*
sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sq
rt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*b*c*d**2*x**4 - 105*sqrt(c)
*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*s
qrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*d**3*x**4 + 105*sqrt(c)*sqrt(a*d -
 b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt
(b))/(sqrt(c)*sqrt(b)))*a**2*b*c*d**2*x**4 - 15*sqrt(x)*sqrt(a + b*x)*a**3
*c**4 + 21*sqrt(x)*sqrt(a + b*x)*a**3*c**3*d*x - 35*sqrt(x)*sqrt(a + b*x)*
a**3*c**2*d**2*x**2 + 105*sqrt(x)*sqrt(a + b*x)*a**3*c*d**3*x**3 - 24*sqrt
(x)*sqrt(a + b*x)*a**2*b*c**4*x + 42*sqrt(x)*sqrt(a + b*x)*a**2*b*c**3*d*x
**2 - 140*sqrt(x)*sqrt(a + b*x)*a**2*b*c**2*d**2*x**3 - 3*sqrt(x)*sqrt(a +
 b*x)*a*b**2*c**4*x**2 + 21*sqrt(x)*sqrt(a + b*x)*a*b**2*c**3*d*x**3 + 6*s
qrt(x)*sqrt(a + b*x)*b**3*c**4*x**3 - 75*sqrt(b)*a**3*c*d**3*x**4 + 80*sqr
t(b)*a**2*b*c**2*d**2*x**4 + 9*sqrt(b)*a*b**2*c**3*d*x**4 - 6*sqrt(b)*b**3
*c**4*x**4))/(105*a**2*c**5*x**4)
```

3.100 $\int \frac{x^2(ax+bx^2)^{3/2}}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 340

$$\begin{aligned}\int \frac{x^2(ax+bx^2)^{3/2}}{(c+dx)^2}\,dx &= -\frac{(320b^3c^3-304ab^2c^2d+16a^2bcd^2+3a^3d^3)\sqrt{ax+bx^2}}{64b^2d^5}\\ &+\frac{(240b^2c^2-208abcd+3a^2d^2)\,x\sqrt{ax+bx^2}}{96bd^4}\\ &-\frac{(40bc-33ad)x^2\sqrt{ax+bx^2}}{24d^3}+\frac{5bx^3\sqrt{ax+bx^2}}{4d^2}-\frac{x^2(ax+bx^2)^{3/2}}{d(c+dx)}\\ &+\frac{(640b^4c^4-768ab^3c^3d+144a^2b^2c^2d^2+16a^3bcd^3+3a^4d^4)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{64b^{5/2}d^6}\\ &-\frac{c^{5/2}(10bc-7ad)\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{d^6}\end{aligned}$$

output

```
-1/64*(3*a^3*d^3+16*a^2*b*c*d^2-304*a*b^2*c^2*d+320*b^3*c^3)*(b*x^2+a*x)^(
1/2)/b^2/d^5+1/96*(3*a^2*d^2-208*a*b*c*d+240*b^2*c^2)*x*(b*x^2+a*x)^(1/2)/
b/d^4-1/24*(-33*a*d+40*b*c)*x^2*(b*x^2+a*x)^(1/2)/d^3+5/4*b*x^3*(b*x^2+a*x
)^(1/2)/d^2-x^2*(b*x^2+a*x)^(3/2)/d/(d*x+c)+1/64*(3*a^4*d^4+16*a^3*b*c*d^3
+144*a^2*b^2*c^2*d^2-768*a*b^3*c^3*d+640*b^4*c^4)*arctanh(b^(1/2)*x/(b*x^2
+a*x)^(1/2))/b^(5/2)/d^6-c^(5/2)*(-7*a*d+10*b*c)*(-a*d+b*c)^(1/2)*arctanh(
(-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/d^6
```

Mathematica [A] (verified)

Time = 10.91 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.98

$$\int \frac{x^2(ax+bx^2)^{3/2}}{(c+dx)^2}\,dx = \frac{\sqrt{x(a+bx)}\left(\frac{\sqrt{b}d\sqrt{x}(-9a^3d^3(c+dx)+6a^2bd^2(-8c^2-7cdx+d^2x^2)+8ab^2d(114c^3+62c^2dx-19cd^2x^2+9d^3x}{c+dx}\right.}{}$$

input
```
Integrate[(x^2*(a*x + b*x^2)^(3/2))/(c + d*x)^2,x]
```

output
```
(Sqrt[x*(a + b*x)]*((Sqrt[b]*d*Sqrt[x]*(-9*a^3*d^3*(c + d*x) + 6*a^2*b*d^2
*(-8*c^2 - 7*c*d*x + d^2*x^2) + 8*a*b^2*d*(114*c^3 + 62*c^2*d*x - 19*c*d^2
*x^2 + 9*d^3*x^3) - 16*b^3*(60*c^4 + 30*c^3*d*x - 10*c^2*d^2*x^2 + 5*c*d^3
*x^3 - 3*d^4*x^4)))/(c + d*x) + (3*(640*b^4*c^4 - 768*a*b^3*c^3*d + 144*a^
2*b^2*c^2*d^2 + 16*a^3*b*c*d^3 + 3*a^4*d^4)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt
[a]])/(Sqrt[a]*Sqrt[1 + (b*x)/a]) - (192*b^(5/2)*c^(5/2)*(10*b*c - 7*a*d)*
Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])]
)/Sqrt[a + b*x]))/(192*b^(5/2)*d^6*Sqrt[x])
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.18, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.708$, Rules used = {1261, 108, 27, 171, 25, 27, 171, 27, 171, 27, 171, 27, 175, 65, 104, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(ax+bx^2)^{3/2}}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{ 1261}$$

$$\frac{(ax+bx^2)^{3/2}\int \frac{x^{7/2}(a+bx)^{3/2}}{(c+dx)^2}dx}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{ 108}$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\int \frac{x^{5/2}\sqrt{a+bx}(7a+10bx)}{2(c+dx)}dx}{d}-\frac{x^{7/2}(a+bx)^{3/2}}{d(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\int \frac{x^{5/2}\sqrt{a+bx}(7a+10bx)}{c+dx}dx}{2d}-\frac{x^{7/2}(a+bx)^{3/2}}{d(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \; 171$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\frac{\int -\frac{bx^{3/2}\sqrt{a+bx}(25ac+(40bc-3ad)x)}{c+dx}dx}{4bd}+\frac{5x^{5/2}(a+bx)^{3/2}}{2d}}{2d}-\frac{x^{7/2}(a+bx)^{3/2}}{d(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\frac{5x^{5/2}(a+bx)^{3/2}}{2d}-\frac{\int \frac{bx^{3/2}\sqrt{a+bx}(25ac+(40bc-3ad)x)}{c+dx}dx}{4bd}}{2d}-\frac{x^{7/2}(a+bx)^{3/2}}{d(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\frac{5x^{5/2}(a+bx)^{3/2}}{2d}-\frac{\int \frac{x^{3/2}\sqrt{a+bx}(25ac+(40bc-3ad)x)}{c+dx}dx}{4d}}{2d}-\frac{x^{7/2}(a+bx)^{3/2}}{d(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \; 171$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\frac{5x^{5/2}(a+bx)^{3/2}}{2d}-\frac{\frac{\int -\frac{3\sqrt{x}\sqrt{a+bx}\left(ac(40bc-3ad)+\left(80b^2c^2-16abdc-3a^2d^2\right)x\right)}{2(c+dx)}dx}{3bd}+\frac{x^{3/2}(a+bx)^{3/2}(40bc-3ad)}{3bd}}{4d}}{2d}-\frac{x^{7/2}(a+bx)^{3/2}}{d(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\frac{5x^{5/2}(a+bx)^{3/2}}{2d}-\frac{\frac{x^{3/2}(a+bx)^{3/2}(40bc-3ad)}{3bd}-\frac{\int \frac{\sqrt{x}\sqrt{a+bx}\left(ac(40bc-3ad)+\left(80b^2c^2-16abdc-3a^2d^2\right)x\right)}{c+dx}dx}{2bd}}{4d}}{2d}-\frac{x^{7/2}(a+bx)^{3/2}}{d(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$\Big\downarrow$ 171

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\frac{5x^{5/2}(a+bx)^{3/2}}{2d}-\frac{\frac{x^{3/2}(a+bx)^{3/2}(40bc-3ad)}{3bd}-\frac{\frac{\frac{\int -\frac{\sqrt{a+bx}\left(ac\left(80b^2c^2-16abdc-3a^2d^2\right)+\left(320b^3c^3-144ab^2dc^2-16a^2bd^2c-3a^3d^3\right)x\right)}{2\sqrt{x}(c+dx)}\cdots}{2bd}\cdots}{2bd}\cdots}{4d}}{2d}\cdots\right)}{x^{3/2}(a+bx)^{3/2}\cdots}$$

$\Big\downarrow$ 27

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\frac{5x^{5/2}(a+bx)^{3/2}}{2d}-\frac{\frac{x^{3/2}(a+bx)^{3/2}(40bc-3ad)}{3bd}-\frac{-\frac{\int \frac{\sqrt{a+bx}\left(ac\left(80b^2c^2-16abdc-3a^2d^2\right)+\left(320b^3c^3-144ab^2dc^2-16a^2bd^2c-3a^3d^3\right)x\right)}{\sqrt{x}(c+dx)}\cdots}{4bd}\cdots}{2bd}\cdots}{4d}}{2d}\cdots\right)}{x^{3/2}(a+bx)^{3/2}\cdots}$$

$\Big\downarrow$ 171

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\frac{5x^{5/2}(a+bx)^{3/2}}{2d}-\frac{\frac{x^{3/2}(a+bx)^{3/2}(40bc-3ad)}{3bd}-\frac{-\frac{\frac{\int -\frac{ac\left(320b^3c^3-304ab^2dc^2+16a^2bd^2c+3a^3d^3\right)+\left(640b^4c^4-768ab^3dc^3+144a^2b^2d^2c\cdots\right.}{2\sqrt{x}\sqrt{a+bx}(c+dx)}\cdots}{d}\cdots}{4bd\cdots}\cdots}{\cdots}\cdots}{2d}\cdots\right)}{x^{3/2}(a+bx\cdots}$$

$\Big\downarrow$ 27

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\frac{5x^{5/2}(a+bx)^{3/2}}{2d}-\frac{\frac{x^{3/2}(a+bx)^{3/2}(40bc-3ad)}{3bd}-\frac{-\frac{\frac{\sqrt{x}\sqrt{a+bx}\left(-3a^3d^3-16a^2bcd^2-144ab^2c^2d+320b^3c^3\right)}{d}-\frac{\int \frac{ac\left(320b^3c^3-304ab^2dc^2+\cdots\right.}{\cdots}}{\cdots}}{4bd}\cdots}{\cdots}\cdots}{2d}\cdots\right)}{x^{3/2}(a+bx)\cdots}$$

$\Big\downarrow$ 175

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{5x^{5/2}(a+bx)^{3/2}}{2d}-\frac{\frac{x^{3/2}(a+bx)^{3/2}(40bc-3ad)}{3bd}-\frac{-\frac{\frac{\sqrt{x}\sqrt{a+bx}\left(-3a^3d^3-16a^2bcd^2-144ab^2c^2d+320b^3c^3\right)}{d}-\frac{\left(3a^4d^4+16a^3bcd^3+144a^2b^2\cdots\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}\right.}{x^{3/2}(a\cdots}$$

$$\Big\downarrow \; 65$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{5x^{5/2}(a+bx)^{3/2}}{2d}-\frac{\frac{x^{3/2}(a+bx)^{3/2}(40bc-3ad)}{3bd}-\frac{-\frac{\frac{\sqrt{x}\sqrt{a+bx}\left(-3a^3d^3-16a^2bcd^2-144ab^2c^2d+320b^3c^3\right)}{d}-\frac{2\left(3a^4d^4+16a^3bcd^3+144a^2b\cdots\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}\right.}{x^{3/2}\cdots}$$

$$\Big\downarrow \; 104$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{5x^{5/2}(a+bx)^{3/2}}{2d}-\frac{\frac{x^{3/2}(a+bx)^{3/2}(40bc-3ad)}{3bd}-\frac{-\frac{\frac{\sqrt{x}\sqrt{a+bx}\left(-3a^3d^3-16a^2bcd^2-144ab^2c^2d+320b^3c^3\right)}{d}-\frac{2\left(3a^4d^4+16a^3bcd^3+144a^2b\cdots\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}\right.}{x^{3/\cdots}}$$

$$\Big\downarrow \; 219$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{5x^{5/2}(a+bx)^{3/2}}{2d}-\frac{\frac{x^{3/2}(a+bx)^{3/2}(40bc-3ad)}{3bd}-\frac{-\frac{\frac{\sqrt{x}\sqrt{a+bx}\left(-3a^3d^3-16a^2bcd^2-144ab^2c^2d+320b^3c^3\right)}{d}-\frac{2\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)\left(3a^4\cdots\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}\right.}{x^{3/\cdots}}$$

$$\Big\downarrow \; 221$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{5x^{5/2}(a+bx)^{3/2}}{2d}-\frac{\frac{x^{3/2}(a+bx)^{3/2}(40bc-3ad)}{3bd}-\frac{-\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\left(\frac{3a^2d}{b}+16ac-\frac{80bc^2}{d}\right)-\frac{\sqrt{x}\sqrt{a+bx}\left(-3a^3d^3-16a^2bcd^2-144ab^2c^2d+}{d}}{}}{}\right.}{x}$$

input
```
Int[(x^2*(a*x + b*x^2)^(3/2))/(c + d*x)^2,x]
```

output
```
((a*x + b*x^2)^(3/2)*(-((x^(7/2)*(a + b*x)^(3/2))/(d*(c + d*x))) + ((5*x^(
5/2)*(a + b*x)^(3/2))/(2*d) - (((40*b*c - 3*a*d)*x^(3/2)*(a + b*x)^(3/2))/
(3*b*d) - (-1/2*((16*a*c - (80*b*c^2)/d + (3*a^2*d)/b)*Sqrt[x]*(a + b*x)^(
3/2)) - (((320*b^3*c^3 - 144*a*b^2*c^2*d - 16*a^2*b*c*d^2 - 3*a^3*d^3)*Sqr
t[x]*Sqrt[a + b*x])/d - ((2*(640*b^4*c^4 - 768*a*b^3*c^3*d + 144*a^2*b^2*c
^2*d^2 + 16*a^3*b*c*d^3 + 3*a^4*d^4)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*
x]])/(Sqrt[b]*d) - (128*b^2*c^(5/2)*(10*b*c - 7*a*d)*Sqrt[b*c - a*d]*ArcTa
nh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/d)/(2*d))/(4*b*d))/
(2*b*d))/(4*d))/(2*d)))/(x^(3/2)*(a + b*x)^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$\frac{7(dx+c)\left(a^2 b^{\frac{5}{2}} d^2-\frac{17a\,b^{\frac{7}{2}}\,cd}{7}+\frac{10b^{\frac{9}{2}}\,c^2}{7}\right)c^3\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)+\frac{3\left(\left(a^4d^4+\frac{16}{3}a^3bc\,d^3+48a^2b^2c^2d^2-256a\,b^3c^3d+\frac{640}{3}c^4b^4\right)(c\right.}{}}{}$
risch	$-\frac{\left(-48b^3d^3x^3-72a\,b^2d^3x^2+128b^3c\,d^2x^2-6a^2b\,d^3x+224a\,b^2c\,d^2x-288b^3c^2dx+9a^3d^3+48a^2bc\,d^2-720a\,b^2c^2d+768b^3c^3\right)}{192b^2d^5\sqrt{x(bx+a)}}$
default	Expression too large to display

input
```
int(x^2*(b*x^2+a*x)^(3/2)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
3/64/b^(5/2)/(c*(a*d-b*c))^(1/2)*(448/3*(d*x+c)*(a^2*b^(5/2)*d^2-17/7*a*b^
(7/2)*c*d+10/7*b^(9/2)*c^2)*c^3*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))
^(1/2))+((a^4*d^4+16/3*a^3*b*c*d^3+48*a^2*b^2*c^2*d^2-256*a*b^3*c^3*d+640/
3*c^4*b^4)*(d*x+c)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-d*(x*(b*x+a))^(1/2
)*(16/3*(-d^4*x^4+5/3*c*d^3*x^3-10/3*d^2*c^2*x^2+10*c^3*d*x+20*c^4)*b^(7/2
)+d*a*(8*(-d^3*x^3+19/9*c*d^2*x^2-62/9*c^2*d*x-38/3*c^3)*b^(5/2)+(d*x+c)*d
*(2/3*(-d*x+8*c)*b^(3/2)+b^(1/2)*a*d)*a))*(c*(a*d-b*c))^(1/2))/d^6/(d*x+c
)
```

Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 1772, normalized size of antiderivative = 5.21

$$\int \frac{x^2(ax+bx^2)^{3/2}}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x^2+a*x)^(3/2)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[1/384*(3*(640*b^4*c^5 - 768*a*b^3*c^4*d + 144*a^2*b^2*c^3*d^2 + 16*a^3*b*
c^2*d^3 + 3*a^4*c*d^4 + (640*b^4*c^4*d - 768*a*b^3*c^3*d^2 + 144*a^2*b^2*c
^2*d^3 + 16*a^3*b*c*d^4 + 3*a^4*d^5)*x)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x
^2 + a*x)*sqrt(b)) - 192*(10*b^4*c^4 - 7*a*b^3*c^3*d + (10*b^4*c^3*d - 7*a
*b^3*c^2*d^2)*x)*sqrt(b*c^2 - a*c*d)*log((a*c + (2*b*c - a*d)*x + 2*sqrt(b
*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) + 2*(48*b^4*d^5*x^4 - 960*b^4*
c^4*d + 912*a*b^3*c^3*d^2 - 48*a^2*b^2*c^2*d^3 - 9*a^3*b*c*d^4 - 8*(10*b^4
*c*d^4 - 9*a*b^3*d^5)*x^3 + 2*(80*b^4*c^2*d^3 - 76*a*b^3*c*d^4 + 3*a^2*b^2
*d^5)*x^2 - (480*b^4*c^3*d^2 - 496*a*b^3*c^2*d^3 + 42*a^2*b^2*c*d^4 + 9*a^
3*b*d^5)*x)*sqrt(b*x^2 + a*x))/(b^3*d^7*x + b^3*c*d^6), 1/384*(384*(10*b^4
*c^4 - 7*a*b^3*c^3*d + (10*b^4*c^3*d - 7*a*b^3*c^2*d^2)*x)*sqrt(-b*c^2 + a
*c*d)*arctan(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) + 3*(64
0*b^4*c^5 - 768*a*b^3*c^4*d + 144*a^2*b^2*c^3*d^2 + 16*a^3*b*c^2*d^3 + 3*a
^4*c*d^4 + (640*b^4*c^4*d - 768*a*b^3*c^3*d^2 + 144*a^2*b^2*c^2*d^3 + 16*a
^3*b*c*d^4 + 3*a^4*d^5)*x)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqr
t(b)) + 2*(48*b^4*d^5*x^4 - 960*b^4*c^4*d + 912*a*b^3*c^3*d^2 - 48*a^2*b^2
*c^2*d^3 - 9*a^3*b*c*d^4 - 8*(10*b^4*c*d^4 - 9*a*b^3*d^5)*x^3 + 2*(80*b^4*
c^2*d^3 - 76*a*b^3*c*d^4 + 3*a^2*b^2*d^5)*x^2 - (480*b^4*c^3*d^2 - 496*a*b
^3*c^2*d^3 + 42*a^2*b^2*c*d^4 + 9*a^3*b*d^5)*x)*sqrt(b*x^2 + a*x))/(b^3*d^
7*x + b^3*c*d^6), -1/192*(3*(640*b^4*c^5 - 768*a*b^3*c^4*d + 144*a^2*b^...
```

Sympy [F]

$$\int \frac{x^2(ax+bx^2)^{3/2}}{(c+dx)^2}\,dx = \int \frac{x^2(x(a+bx))^{\frac{3}{2}}}{(c+dx)^2}\,dx$$

input
```
integrate(x**2*(b*x**2+a*x)**(3/2)/(d*x+c)**2,x)
```

output
```
Integral(x**2*(x*(a + b*x))**(3/2)/(c + d*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(ax+bx^2)^{3/2}}{(c+dx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(b*x^2+a*x)^(3/2)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-2*b*c>0)', see `assume?` for
 more deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^2(ax+bx^2)^{3/2}}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(b*x^2+a*x)^(3/2)/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(ax+bx^2)^{3/2}}{(c+dx)^2}\,dx=\int \frac{x^2\,{\left(b\,x^2+a\,x\right)}^{3/2}}{{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((x^2*(a*x + b*x^2)^(3/2))/(c + d*x)^2,x)
```

output
```
int((x^2*(a*x + b*x^2)^(3/2))/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{x^2(ax+bx^2)^{3/2}}{(c+dx)^2}\,dx=\int \frac{x^2(b\,x^2+ax)^{\frac{3}{2}}}{(dx+c)^2}dx$$

input
```
int(x^2*(b*x^2+a*x)^(3/2)/(d*x+c)^2,x)
```

output
```
int(x^2*(b*x^2+a*x)^(3/2)/(d*x+c)^2,x)
```

3.101 $\int \frac{x\left(ax+bx^2\right)^{3/2}}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 258

$$\begin{aligned}\int \frac{x(ax+bx^2)^{3/2}}{(c+dx)^2}\,dx &= \frac{(32b^2c^2-28abcd+a^2d^2)\sqrt{ax+bx^2}}{8bd^4}\\ &-\frac{(24bc-19ad)x\sqrt{ax+bx^2}}{12d^3}+\frac{4bx^2\sqrt{ax+bx^2}}{3d^2}-\frac{x(ax+bx^2)^{3/2}}{d(c+dx)}\\ &-\frac{(64b^3c^3-72ab^2c^2d+12a^2bcd^2+a^3d^3)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{8b^{3/2}d^5}\\ &+\frac{c^{3/2}(8bc-5ad)\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{d^5}\end{aligned}$$

output
```
1/8*(a^2*d^2-28*a*b*c*d+32*b^2*c^2)*(b*x^2+a*x)^(1/2)/b/d^4-1/12*(-19*a*d+
24*b*c)*x*(b*x^2+a*x)^(1/2)/d^3+4/3*b*x^2*(b*x^2+a*x)^(1/2)/d^2-x*(b*x^2+a
*x)^(3/2)/d/(d*x+c)-1/8*(a^3*d^3+12*a^2*b*c*d^2-72*a*b^2*c^2*d+64*b^3*c^3)
*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(3/2)/d^5+c^(3/2)*(-5*a*d+8*b*c)*(
-a*d+b*c)^(1/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/d^5
```

Mathematica [A] (verified)

Time = 10.69 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.03

$$\int \frac{x(ax+bx^2)^{3/2}}{(c+dx)^2}\,dx = \frac{\sqrt{x(a+bx)}\left(\frac{\sqrt{b}d\sqrt{x}(3a^2d^2(c+dx)+2abd(-42c^2-23cdx+7d^2x^2)+8b^2(12c^3+6c^2dx-2cd^2x^2+d^3x^3))}{c+dx}-\right.}{24b^3}$$

input
```
Integrate[(x*(a*x + b*x^2)^(3/2))/(c + d*x)^2,x]
```

output
```
(Sqrt[x*(a + b*x)]*((Sqrt[b]*d*Sqrt[x]*(3*a^2*d^2*(c + d*x) + 2*a*b*d*(-42
*c^2 - 23*c*d*x + 7*d^2*x^2) + 8*b^2*(12*c^3 + 6*c^2*d*x - 2*c*d^2*x^2 + d
^3*x^3)))/(c + d*x) - (3*(64*b^3*c^3 - 72*a*b^2*c^2*d + 12*a^2*b*c*d^2 + a
^3*d^3)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[1 + (b*x)/a]) +
(24*b^(3/2)*c^(3/2)*(8*b*c - 5*a*d)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b*c - a*
d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/Sqrt[a + b*x]))/(24*b^(3/2)*d^5*Sqrt
[x])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1230, 1231, 27, 1269, 1091, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(ax+bx^2\right)^{3/2}}{(c+dx)^2}dx$$

$$\Big\downarrow \text{1230}$$

$$\frac{\left(ax+bx^2\right)^{3/2}(4c+dx)}{3d^2(c+dx)}-\frac{\int \frac{(4ac+(8bc-ad)x)\sqrt{bx^2+ax}}{c+dx}dx}{2d^2}$$

$$\Big\downarrow \text{1231}$$

$$\frac{(ax+bx^2)^{3/2}(4c+dx)}{3d^2(c+dx)}-\frac{-\frac{\int -\frac{ac(32b^2c^2-28abdc+a^2d^2)+(64b^3c^3-72ab^2dc^2+12a^2bd^2c+a^3d^3)x}{2(c+dx)\sqrt{bx^2+ax}}dx}{4bd^2}-\frac{\sqrt{ax+bx^2}(a^2d^2-2bdx(8bc-ad)-28abcd+32b^2c^2)}{4bd^2}}{2d^2}$$

$\big\downarrow$ 27

$$\frac{(ax+bx^2)^{3/2}(4c+dx)}{3d^2(c+dx)}-\frac{\frac{\int \frac{ac(32b^2c^2-28abdc+a^2d^2)+(64b^3c^3-72ab^2dc^2+12a^2bd^2c+a^3d^3)x}{(c+dx)\sqrt{bx^2+ax}}dx}{8bd^2}-\frac{\sqrt{ax+bx^2}(a^2d^2-2bdx(8bc-ad)-28abcd+32b^2c^2)}{4bd^2}}{2d^2}$$

$\big\downarrow$ 1269

$$\frac{(ax+bx^2)^{3/2}(4c+dx)}{3d^2(c+dx)}-\frac{\frac{\frac{(a^3d^3+12a^2bcd^2-72ab^2c^2d+64b^3c^3)\int \frac{1}{\sqrt{bx^2+ax}}dx}{d}-\frac{8bc^2(8bc-5ad)(bc-ad)\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{8bd^2}-\frac{\sqrt{ax+bx^2}(a^2d^2-2bdx(8bc-ad)-28abcd+32b^2c}{4bd^2}}{2d^2}$$

$\big\downarrow$ 1091

$$\frac{(ax+bx^2)^{3/2}(4c+dx)}{3d^2(c+dx)}-\frac{\frac{\frac{2(a^3d^3+12a^2bcd^2-72ab^2c^2d+64b^3c^3)\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{d}-\frac{8bc^2(8bc-5ad)(bc-ad)\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{8bd^2}-\frac{\sqrt{ax+bx^2}(a^2d^2-2bdx(8bc-ad)-28a}{4bd^2}}{2d^2}$$

$\big\downarrow$ 219

$$\frac{(ax+bx^2)^{3/2}(4c+dx)}{3d^2(c+dx)}-\frac{\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(a^3d^3+12a^2bcd^2-72ab^2c^2d+64b^3c^3)}{\sqrt{b}d}-\frac{8bc^2(8bc-5ad)(bc-ad)\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{8bd^2}-\frac{\sqrt{ax+bx^2}(a^2d^2-2bdx(8bc-ad)-28ab}{4bd^2}}{2d^2}$$

$\big\downarrow$ 1154

$$\frac{(ax+bx^2)^{3/2}(4c+dx)}{3d^2(c+dx)}-\frac{\frac{\frac{16bc^2(8bc-5ad)(bc-ad)\int \frac{1}{4c(bc-ad)-\frac{(ac+(2bc-ad)x)^2}{bx^2+ax}}d\left(-\frac{ac+(2bc-ad)x}{\sqrt{bx^2+ax}}\right)}{d}+\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(a^3d^3+12a^2bcd^2-72ab^2c^2d+64b^3c^3)}{\sqrt{b}d}}{8bd^2}-\sqrt{ax+b}}{2d^2}$$

$$\Big\downarrow \; 219$$

$$\frac{(ax+bx^2)^{3/2}(4c+dx)}{3d^2(c+dx)}-\frac{\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(a^3d^3+12a^2bcd^2-72ab^2c^2d+64b^3c^3)}{\sqrt{b}d}-\frac{8bc^{3/2}(8bc-5ad)\sqrt{bc-ad}\text{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{d}}{8bd^2}-\frac{\sqrt{ax+bx^2}(a^2d^2-2bdx}{}}{2d^2}$$

input
```
Int[(x*(a*x + b*x^2)^(3/2))/(c + d*x)^2,x]
```

output
```
((4*c + d*x)*(a*x + b*x^2)^(3/2))/(3*d^2*(c + d*x)) - (-1/4*((32*b^2*c^2 -
 28*a*b*c*d + a^2*d^2 - 2*b*d*(8*b*c - a*d)*x)*Sqrt[a*x + b*x^2])/(b*d^2)
+ ((2*(64*b^3*c^3 - 72*a*b^2*c^2*d + 12*a^2*b*c*d^2 + a^3*d^3)*ArcTanh[(Sq
rt[b]*x)/Sqrt[a*x + b*x^2]])/(Sqrt[b]*d) - (8*b*c^(3/2)*(8*b*c - 5*a*d)*Sq
rt[b*c - a*d]*ArcTanh[(a*c + (2*b*c - a*d)*x)/(2*Sqrt[c]*Sqrt[b*c - a*d]*S
qrt[a*x + b*x^2])])/d)/(8*b*d^2))/(2*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.91

method	result
pseudoelliptic	$-\frac{8\left((dx+c)\left(bc-\frac{5ad}{8}\right)b^{\frac{5}{2}}c^{2}(-ad+bc)\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)-\frac{\sqrt{c(ad-bc)}\left(-\frac{b\left(a^{3}d^{3}+12a^{2}bc\,d^{2}-72a\,b^{2}c^{2}d+64b^{3}c^{3}\right)(dx+c)\,\text{ar}}{32}\right.}{}\right.}{\sqrt{c(ad-bc)}\,b^{\frac{5}{2}}\,c}$
risch	$\frac{\left(8b^{2}d^{2}x^{2}+14ab\,d^{2}x-24b^{2}cxd+3a^{2}d^{2}-60abcd+72b^{2}c^{2}\right)x(bx+a)}{24b\,d^{4}\sqrt{x(bx+a)}}-\frac{\frac{\left(a^{3}d^{3}+12a^{2}bc\,d^{2}-72a\,b^{2}c^{2}d+64b^{3}c^{3}\right)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}}\right.}{d\sqrt{b}}}{}$
default	Expression too large to display

input
```
int(x*(b*x^2+a*x)^(3/2)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-8*((d*x+c)*(b*c-5/8*a*d)*b^(5/2)*c^2*(-a*d+b*c)*arctan((x*(b*x+a))^(1/2)/
x*c/(c*(a*d-b*c))^(1/2))-1/2*(c*(a*d-b*c))^(1/2)*(-1/32*b*(a^3*d^3+12*a^2*
b*c*d^2-72*a*b^2*c^2*d+64*b^3*c^3)*(d*x+c)*arctanh((x*(b*x+a))^(1/2)/x/b^(
1/2))+d*(x*(b*x+a))^(1/2)*(c^3*b^2-7/8*d*b*(-4/7*b*x+a)*c^2+1/32*d^2*(-16/
3*b^2*x^2-46/3*a*b*x+a^2)*c+1/32*(4*b*x+a)*d^3*x*(2/3*b*x+a))*b^(3/2)))/(c
*(a*d-b*c))^(1/2)/b^(5/2)/d^5/(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1376, normalized size of antiderivative = 5.33

$$\int \frac{x(ax+bx^2)^{3/2}}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(b*x^2+a*x)^(3/2)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[1/48*(3*(64*b^3*c^4 - 72*a*b^2*c^3*d + 12*a^2*b*c^2*d^2 + a^3*c*d^3 + (64
*b^3*c^3*d - 72*a*b^2*c^2*d^2 + 12*a^2*b*c*d^3 + a^3*d^4)*x)*sqrt(b)*log(2
*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 24*(8*b^3*c^3 - 5*a*b^2*c^2*d +
(8*b^3*c^2*d - 5*a*b^2*c*d^2)*x)*sqrt(b*c^2 - a*c*d)*log((a*c + (2*b*c - a
*d)*x - 2*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) + 2*(8*b^3*d^4
*x^3 + 96*b^3*c^3*d - 84*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - 2*(8*b^3*c*d^3 -
7*a*b^2*d^4)*x^2 + (48*b^3*c^2*d^2 - 46*a*b^2*c*d^3 + 3*a^2*b*d^4)*x)*sqrt
(b*x^2 + a*x))/(b^2*d^6*x + b^2*c*d^5), -1/48*(48*(8*b^3*c^3 - 5*a*b^2*c^2
*d + (8*b^3*c^2*d - 5*a*b^2*c*d^2)*x)*sqrt(-b*c^2 + a*c*d)*arctan(sqrt(-b*
c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) - 3*(64*b^3*c^4 - 72*a*b^2*c
^3*d + 12*a^2*b*c^2*d^2 + a^3*c*d^3 + (64*b^3*c^3*d - 72*a*b^2*c^2*d^2 + 1
2*a^2*b*c*d^3 + a^3*d^4)*x)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sq
rt(b)) - 2*(8*b^3*d^4*x^3 + 96*b^3*c^3*d - 84*a*b^2*c^2*d^2 + 3*a^2*b*c*d^
3 - 2*(8*b^3*c*d^3 - 7*a*b^2*d^4)*x^2 + (48*b^3*c^2*d^2 - 46*a*b^2*c*d^3 +
 3*a^2*b*d^4)*x)*sqrt(b*x^2 + a*x))/(b^2*d^6*x + b^2*c*d^5), 1/24*(3*(64*b
^3*c^4 - 72*a*b^2*c^3*d + 12*a^2*b*c^2*d^2 + a^3*c*d^3 + (64*b^3*c^3*d - 7
2*a*b^2*c^2*d^2 + 12*a^2*b*c*d^3 + a^3*d^4)*x)*sqrt(-b)*arctan(sqrt(b*x^2
+ a*x)*sqrt(-b)/(b*x + a)) - 12*(8*b^3*c^3 - 5*a*b^2*c^2*d + (8*b^3*c^2*d
- 5*a*b^2*c*d^2)*x)*sqrt(b*c^2 - a*c*d)*log((a*c + (2*b*c - a*d)*x - 2*sqr
t(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) + (8*b^3*d^4*x^3 + 96*b^...
```

Sympy [F]

$$\int \frac{x(ax+bx^2)^{3/2}}{(c+dx)^2}\,dx = \int \frac{x(x(a+bx))^{\frac{3}{2}}}{(c+dx)^2}\,dx$$

input
```
integrate(x*(b*x**2+a*x)**(3/2)/(d*x+c)**2,x)
```

output
```
Integral(x*(x*(a + b*x))**(3/2)/(c + d*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(ax+bx^2)^{3/2}}{(c+dx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(b*x^2+a*x)^(3/2)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [F(-1)]

Timed out.

$$\int \frac{x(ax+bx^2)^{3/2}}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(b*x^2+a*x)^(3/2)/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(ax+bx^2)^{3/2}}{(c+dx)^2}\,dx = \int \frac{x\,(b\,x^2+a\,x)^{3/2}}{(c+d\,x)^2}\,dx$$

input
```
int((x*(a*x + b*x^2)^(3/2))/(c + d*x)^2,x)
```

output
```
int((x*(a*x + b*x^2)^(3/2))/(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 29.94 (sec) , antiderivative size = 891, normalized size of antiderivative = 3.45

$$\int \frac{x(ax+bx^2)^{3/2}}{(c+dx)^2}\,dx = \text{Too large to display}$$

input

```
int(x*(b*x^2+a*x)^(3/2)/(d*x+c)^2,x)
```

output

```
( - 120*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b
*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c**2*d - 120*sqrt
(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x
)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c*d**2*x + 192*sqrt(c)*sqrt(a
*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*
sqrt(b))/(sqrt(c)*sqrt(b)))*b**3*c**3 + 192*sqrt(c)*sqrt(a*d - b*c)*atan((
sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c
)*sqrt(b)))*b**3*c**2*d*x - 120*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b
*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*
a*b**2*c**2*d - 120*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d
)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c*d**
2*x + 192*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a +
 b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**3*c**3 + 192*sqrt(c
)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*
sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**3*c**2*d*x + 3*sqrt(x)*sqrt(a + b*x
)*a**2*b*c*d**3 + 3*sqrt(x)*sqrt(a + b*x)*a**2*b*d**4*x - 84*sqrt(x)*sqrt(
a + b*x)*a*b**2*c**2*d**2 - 46*sqrt(x)*sqrt(a + b*x)*a*b**2*c*d**3*x + 14*
sqrt(x)*sqrt(a + b*x)*a*b**2*d**4*x**2 + 96*sqrt(x)*sqrt(a + b*x)*b**3*c**
3*d + 48*sqrt(x)*sqrt(a + b*x)*b**3*c**2*d**2*x - 16*sqrt(x)*sqrt(a + b*x)
*b**3*c*d**3*x**2 + 8*sqrt(x)*sqrt(a + b*x)*b**3*d**4*x**3 - 3*sqrt(b)*...
```

3.102 $\int \frac{(ax+bx^2)^{3/2}}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 196

$$\int \frac{(ax+bx^2)^{3/2}}{(c+dx)^2}\,dx = -\frac{3(4bc-3ad)\sqrt{ax+bx^2}}{4d^3}+\frac{3bx\sqrt{ax+bx^2}}{2d^2} - \frac{(ax+bx^2)^{3/2}}{d(c+dx)}+\frac{3(8b^2c^2-8abcd+a^2d^2)\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4\sqrt{b}d^4} - \frac{3\sqrt{c}\sqrt{bc-ad}(2bc-ad)\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{d^4}$$

output

```
-3/4*(-3*a*d+4*b*c)*(b*x^2+a*x)^(1/2)/d^3+3/2*b*x*(b*x^2+a*x)^(1/2)/d^2-(b
*x^2+a*x)^(3/2)/d/(d*x+c)+3/4*(a^2*d^2-8*a*b*c*d+8*b^2*c^2)*arctanh(b^(1/2
)*x/(b*x^2+a*x)^(1/2))/b^(1/2)/d^4-3*c^(1/2)*(-a*d+b*c)^(1/2)*(-a*d+2*b*c)
*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/d^4
```

Mathematica [A] (verified)

Time = 10.51 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.05

$$\int \frac{(ax+bx^2)^{3/2}}{(c+dx)^2}\,dx = \frac{\sqrt{x(a+bx)}\left(\frac{d\sqrt{x}(ad(9c+5dx)-2b(6c^2+3cdx-d^2x^2))}{c+dx} + \frac{3(8b^2c^2-8abcd+a^2d^2)\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{1+\frac{bx}{a}}} - 12\cdots\right.}{4d^4\sqrt{x}}$$

input
```
Integrate[(a*x + b*x^2)^(3/2)/(c + d*x)^2,x]
```

output
```
(Sqrt[x*(a + b*x)]*((d*Sqrt[x]*(a*d*(9*c + 5*d*x) - 2*b*(6*c^2 + 3*c*d*x -
 d^2*x^2)))/(c + d*x) + (3*(8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*ArcSinh[(Sqrt
[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[1 + (b*x)/a]) - (12*Sqrt[c]*S
qrt[b*c - a*d]*(2*b*c - a*d)*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sq
rt[a + b*x])])/Sqrt[a + b*x]))/(4*d^4*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1161, 1231, 25, 27, 1269, 1091, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}}{(c+dx)^2}dx$$

$$\Big\downarrow \text{1161}$$

$$\frac{3\int \frac{(a+2bx)\sqrt{bx^2+ax}}{c+dx}dx}{2d} - \frac{(ax+bx^2)^{3/2}}{d(c+dx)}$$

$$\Big\downarrow \text{1231}$$

$$\frac{3\left(-\frac{\int -\frac{b\left(ac(4bc-3ad)+\left(8b^2c^2-8abdc+a^2d^2\right)x\right)}{(c+dx)\sqrt{bx^2+ax}}dx}{4bd^2} - \frac{\sqrt{ax+bx^2}(-3ad+4bc-2bdx)}{2d^2}\right)}{2d} - \frac{(ax+bx^2)^{3/2}}{d(c+dx)}$$

$$\Big\downarrow 25$$

$$\frac{3\left(\frac{\int \frac{b\left(ac(4bc-3ad)+\left(8b^2c^2-8abdc+a^2d^2\right)x\right)}{(c+dx)\sqrt{bx^2+ax}}dx}{4bd^2}-\frac{\sqrt{ax+bx^2}(-3ad+4bc-2bdx)}{2d^2}\right)}{2d}-\frac{\left(ax+bx^2\right)^{3/2}}{d(c+dx)}$$

$$\Big\downarrow 27$$

$$\frac{3\left(\frac{\int \frac{ac(4bc-3ad)+\left(8b^2c^2-8abdc+a^2d^2\right)x}{(c+dx)\sqrt{bx^2+ax}}dx}{4d^2}-\frac{\sqrt{ax+bx^2}(-3ad+4bc-2bdx)}{2d^2}\right)}{2d}-\frac{\left(ax+bx^2\right)^{3/2}}{d(c+dx)}$$

$$\Big\downarrow 1269$$

$$\frac{3\left(\frac{\frac{\left(a^2d^2-8abcd+8b^2c^2\right)\int \frac{1}{\sqrt{bx^2+ax}}dx}{d}-\frac{4c(bc-ad)(2bc-ad)\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{4d^2}-\frac{\sqrt{ax+bx^2}(-3ad+4bc-2bdx)}{2d^2}\right)}{2d}-\frac{\left(ax+bx^2\right)^{3/2}}{d(c+dx)}$$

$$\Big\downarrow 1091$$

$$\frac{3\left(\frac{\frac{2\left(a^2d^2-8abcd+8b^2c^2\right)\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{d}-\frac{4c(bc-ad)(2bc-ad)\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{4d^2}-\frac{\sqrt{ax+bx^2}(-3ad+4bc-2bdx)}{2d^2}\right)}{2d}-\frac{\left(ax+bx^2\right)^{3/2}}{d(c+dx)}$$

$$\Big\downarrow 219$$

$$\frac{3\left(\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)\left(a^2d^2-8abcd+8b^2c^2\right)}{\sqrt{b}d}-\frac{4c(bc-ad)(2bc-ad)\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{4d^2}-\frac{\sqrt{ax+bx^2}(-3ad+4bc-2bdx)}{2d^2}\right)}{2d}-\frac{\left(ax+bx^2\right)^{3/2}}{d(c+dx)}$$

$$\Big\downarrow 1154$$

$$\frac{3\left(\frac{\frac{8c(bc-ad)(2bc-ad)\int \frac{1}{4c(bc-ad)-\frac{(ac+(2bc-ad)x)^2}{bx^2+ax}}d\left(-\frac{ac+(2bc-ad)x}{\sqrt{bx^2+ax}}\right)}{d}+\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)\left(a^2d^2-8abcd+8b^2c^2\right)}{\sqrt{b}d}}{4d^2}-\frac{\sqrt{ax+bx^2}(-3ad+4bc-}{2d^2}\right.}{2d}$$

$$\frac{\left(ax+bx^2\right)^{3/2}}{d(c+dx)}$$

$\Big\downarrow$ 219

$$\frac{3\left(\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)\left(a^2d^2-8abcd+8b^2c^2\right)}{\sqrt{b}d}-\frac{4\sqrt{c}\sqrt{bc-ad}(2bc-ad)\text{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{d}}{4d^2}-\frac{\sqrt{ax+bx^2}(-3ad+4bc-2bdx)}{2d^2}\right)}{2d}-$$

$$\frac{\left(ax+bx^2\right)^{3/2}}{d(c+dx)}$$

input
```
Int[(a*x + b*x^2)^(3/2)/(c + d*x)^2,x]
```

output
```
-((a*x + b*x^2)^(3/2)/(d*(c + d*x))) + (3*(-1/2*((4*b*c - 3*a*d - 2*b*d*x)
*Sqrt[a*x + b*x^2])/d^2 + ((2*(8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*ArcTanh[(S
qrt[b]*x)/Sqrt[a*x + b*x^2]])/(Sqrt[b]*d) - (4*Sqrt[c]*Sqrt[b*c - a*d]*(2*
b*c - a*d)*ArcTanh[(a*c + (2*b*c - a*d)*x)/(2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt
[a*x + b*x^2])])/d)/(4*d^2)))/(2*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$\frac{6(dx+c)(-ad+bc)\left(bc-\frac{ad}{2}\right)\sqrt{b}\,c\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)-3\left(-\frac{\left(a^2d^2-8abcd+8b^2c^2\right)(dx+c)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{4}+d\left(b\,c^2-\frac{3d}{}\right.\right.}{\sqrt{b}\,d^4(dx+c)\sqrt{c(ad-bc)}}$
risch	$\frac{(2bdx+5ad-8bc)x(bx+a)}{4d^3\sqrt{x(bx+a)}}+\frac{\frac{3\left(a^2d^2-8abcd+8b^2c^2\right)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{d\sqrt{b}}+\frac{16c\left(a^2d^2-3abcd+2b^2c^2\right)\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-}{}}{}\right.}{}}{}$
default	Expression too large to display

input
```
int((b*x^2+a*x)^(3/2)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
6/b^(1/2)/(c*(a*d-b*c))^(1/2)*((d*x+c)*(-a*d+b*c)*(b*c-1/2*a*d)*b^(1/2)*c*
arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))-1/2*(-1/4*(a^2*d^2-8*a*b
*c*d+8*b^2*c^2)*(d*x+c)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))+d*(b*c^2-3/4*
d*(-2/3*b*x+a)*c-5/12*d^2*x*(2/5*b*x+a))*b^(1/2)*(x*(b*x+a))^(1/2))*(c*(a*
d-b*c))^(1/2))/d^4/(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1008, normalized size of antiderivative = 5.14

$$\int \frac{\left(ax+bx^2\right)^{3/2}}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a*x)^(3/2)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[1/8*(3*(8*b^2*c^3 - 8*a*b*c^2*d + a^2*c*d^2 + (8*b^2*c^2*d - 8*a*b*c*d^2
+ a^2*d^3)*x)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 12*(2
*b^2*c^2 - a*b*c*d + (2*b^2*c*d - a*b*d^2)*x)*sqrt(b*c^2 - a*c*d)*log((a*c
 + (2*b*c - a*d)*x + 2*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) +
 2*(2*b^2*d^3*x^2 - 12*b^2*c^2*d + 9*a*b*c*d^2 - (6*b^2*c*d^2 - 5*a*b*d^3)
*x)*sqrt(b*x^2 + a*x))/(b*d^5*x + b*c*d^4), 1/8*(24*(2*b^2*c^2 - a*b*c*d +
 (2*b^2*c*d - a*b*d^2)*x)*sqrt(-b*c^2 + a*c*d)*arctan(sqrt(-b*c^2 + a*c*d)
*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) + 3*(8*b^2*c^3 - 8*a*b*c^2*d + a^2*c*d^2
 + (8*b^2*c^2*d - 8*a*b*c*d^2 + a^2*d^3)*x)*sqrt(b)*log(2*b*x + a + 2*sqrt
(b*x^2 + a*x)*sqrt(b)) + 2*(2*b^2*d^3*x^2 - 12*b^2*c^2*d + 9*a*b*c*d^2 - (
6*b^2*c*d^2 - 5*a*b*d^3)*x)*sqrt(b*x^2 + a*x))/(b*d^5*x + b*c*d^4), -1/4*(
3*(8*b^2*c^3 - 8*a*b*c^2*d + a^2*c*d^2 + (8*b^2*c^2*d - 8*a*b*c*d^2 + a^2*
d^3)*x)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + 6*(2*b^2*c
^2 - a*b*c*d + (2*b^2*c*d - a*b*d^2)*x)*sqrt(b*c^2 - a*c*d)*log((a*c + (2*
b*c - a*d)*x + 2*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) - (2*b^
2*d^3*x^2 - 12*b^2*c^2*d + 9*a*b*c*d^2 - (6*b^2*c*d^2 - 5*a*b*d^3)*x)*sqrt
(b*x^2 + a*x))/(b*d^5*x + b*c*d^4), 1/4*(12*(2*b^2*c^2 - a*b*c*d + (2*b^2*
c*d - a*b*d^2)*x)*sqrt(-b*c^2 + a*c*d)*arctan(sqrt(-b*c^2 + a*c*d)*sqrt(b*
x^2 + a*x)/(b*c*x + a*c)) - 3*(8*b^2*c^3 - 8*a*b*c^2*d + a^2*c*d^2 + (8*b^
2*c^2*d - 8*a*b*c*d^2 + a^2*d^3)*x)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*s...
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/2}}{(c+dx)^2}\,dx=\int \frac{(x(a+bx))^{\frac{3}{2}}}{(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a*x)**(3/2)/(d*x+c)**2,x)
```

output
```
Integral((x*(a + b*x))**(3/2)/(c + d*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ax+bx^2)^{3/2}}{(c+dx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a*x)^(3/2)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a*x)^(3/2)/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{(c+dx)^2}\,dx = \int \frac{(b\,x^2+a\,x)^{3/2}}{(c+d\,x)^2}\,dx$$

input
```
int((a*x + b*x^2)^(3/2)/(c + d*x)^2,x)
```

output
```
int((a*x + b*x^2)^(3/2)/(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 719, normalized size of antiderivative = 3.67

$$\int \frac{(ax+bx^2)^{3/2}}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a*x)^(3/2)/(d*x+c)^2,x)
```

output
```
(12*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x)
- sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b*c*d + 12*sqrt(c)*sqrt(a*
d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*s
qrt(b))/(sqrt(c)*sqrt(b)))*a*b*d**2*x - 24*sqrt(c)*sqrt(a*d - b*c)*atan((s
qrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)
*sqrt(b)))*b**2*c**2 - 24*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) -
sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**2*c
*d*x + 12*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a +
 b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b*c*d + 12*sqrt(c)*s
qrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqr
t(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b*d**2*x - 24*sqrt(c)*sqrt(a*d - b*c)*a
tan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(s
qrt(c)*sqrt(b)))*b**2*c**2 - 24*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b
*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*
b**2*c*d*x + 9*sqrt(x)*sqrt(a + b*x)*a*b*c*d**2 + 5*sqrt(x)*sqrt(a + b*x)*
a*b*d**3*x - 12*sqrt(x)*sqrt(a + b*x)*b**2*c**2*d - 6*sqrt(x)*sqrt(a + b*x
)*b**2*c*d**2*x + 2*sqrt(x)*sqrt(a + b*x)*b**2*d**3*x**2 + 3*sqrt(b)*log((
sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*c*d**2 + 3*sqrt(b)*log((sqr
t(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*d**3*x - 24*sqrt(b)*log((sqrt(
a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b*c**2*d - 24*sqrt(b)*log((sqrt(...
```

3.103 $\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 151

$$\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)^2}\,dx = \frac{2b\sqrt{ax+bx^2}}{d^2} - \frac{(ax+bx^2)^{3/2}}{dx(c+dx)} - \frac{\sqrt{b}(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{d^3} + \frac{\sqrt{bc-ad}(4bc-ad)\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{\sqrt{c}d^3}$$

output
```
2*b*(b*x^2+a*x)^(1/2)/d^2-(b*x^2+a*x)^(3/2)/d/x/(d*x+c)-b^(1/2)*(-3*a*d+4*
b*c)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/d^3+(-a*d+b*c)^(1/2)*(-a*d+4*b*c
)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(1/2)/d^3
```

Mathematica [A] (verified)

Time = 10.44 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.10

$$\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)^2}\,dx = \frac{\sqrt{x(a+bx)}\left(\frac{d\sqrt{x}(2bc-ad+bdx)}{c+dx} + \frac{\sqrt{b}(-4bc+3ad)\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1+\frac{bx}{a}}} + \frac{\sqrt{bc-ad}(4bc-ad)\text{arctanh}(\cdots}{\sqrt{c}\sqrt{a+bx}}\right.}{d^3\sqrt{x}}$$

input
```
Integrate[(a*x + b*x^2)^(3/2)/(x*(c + d*x)^2),x]
```

output
```
(Sqrt[x*(a + b*x)]*((d*Sqrt[x]*(2*b*c - a*d + b*d*x))/(c + d*x) + (Sqrt[b]
*(-4*b*c + 3*a*d)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[1 + (b
*x)/a]) + (Sqrt[b*c - a*d]*(4*b*c - a*d)*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])
/(Sqrt[c]*Sqrt[a + b*x])])/(Sqrt[c]*Sqrt[a + b*x])))/(d^3*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.28, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1261, 108, 27, 171, 25, 175, 65, 104, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)^2}dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{(ax+bx^2)^{3/2}\int \frac{\sqrt{x}(a+bx)^{3/2}}{(c+dx)^2}dx}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{108}$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\int \frac{\sqrt{a+bx}(a+4bx)}{2\sqrt{x}(c+dx)}dx}{d}-\frac{\sqrt{x}(a+bx)^{3/2}}{d(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\int \frac{\sqrt{a+bx}(a+4bx)}{\sqrt{x}(c+dx)}dx}{2d}-\frac{\sqrt{x}(a+bx)^{3/2}}{d(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{171}$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\frac{\int -\frac{a(2bc-ad)+b(4bc-3ad)x}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}+\frac{4b\sqrt{x}\sqrt{a+bx}}{d}}{2d}-\frac{\sqrt{x}(a+bx)^{3/2}}{d(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\frac{4b\sqrt{x}\sqrt{a+bx}}{d}-\frac{\int\frac{a(2bc-ad)+b(4bc-3ad)x}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}}{2d}-\frac{\sqrt{x}(a+bx)^{3/2}}{d(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \ 175$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\frac{4b\sqrt{x}\sqrt{a+bx}}{d}-\frac{\frac{b(4bc-3ad)\int\frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{d}-\frac{(bc-ad)(4bc-ad)\int\frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}}{d}}{2d}-\frac{\sqrt{x}(a+bx)^{3/2}}{d(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \ 65$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\frac{4b\sqrt{x}\sqrt{a+bx}}{d}-\frac{\frac{2b(4bc-3ad)\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}-\frac{(bc-ad)(4bc-ad)\int\frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}}{d}}{2d}-\frac{\sqrt{x}(a+bx)^{3/2}}{d(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \ 104$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\frac{4b\sqrt{x}\sqrt{a+bx}}{d}-\frac{\frac{2b(4bc-3ad)\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}-\frac{2(bc-ad)(4bc-ad)\int\frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}}{d}}{2d}-\frac{\sqrt{x}(a+bx)^{3/2}}{d(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \ 219$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\frac{4b\sqrt{x}\sqrt{a+bx}}{d}-\frac{\frac{2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)(4bc-3ad)}{d}-\frac{2(bc-ad)(4bc-ad)\int\frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}}{d}}{2d}-\frac{\sqrt{x}(a+bx)^{3/2}}{d(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \ 221$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\frac{4b\sqrt{x}\sqrt{a+bx}}{d}-\frac{\frac{2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)(4bc-3ad)}{d}-\frac{2\sqrt{bc-ad}(4bc-ad)\,\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{\sqrt{c}d}}{d}}{2d}-\frac{\sqrt{x}(a+bx)^{3/2}}{d(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

input
```
Int[(a*x + b*x^2)^(3/2)/(x*(c + d*x)^2),x]
```

output
```
((a*x + b*x^2)^(3/2)*(-((Sqrt[x]*(a + b*x)^(3/2))/(d*(c + d*x))) + ((4*b*S
qrt[x]*Sqrt[a + b*x])/d - ((2*Sqrt[b]*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqr
t[x])/Sqrt[a + b*x]])/d - (2*Sqrt[b*c - a*d]*(4*b*c - a*d)*ArcTanh[(Sqrt[b
*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(Sqrt[c]*d))/d)/(2*d)))/(x^(3
/2)*(a + b*x)^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{-4(dx+c)\sqrt{b}\left(bc-\frac{ad}{4}\right)(-ad+bc)\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)+2\left(\frac{3(dx+c)b\left(ad-\frac{4bc}{3}\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{2}+\sqrt{b}\,d\left(bc-\frac{d(-bx+a)}{2}\right)\right.}{\sqrt{b}\,d^3(dx+c)\sqrt{c(ad-bc)}}$
risch	$\frac{x(bx+a)b}{d^2\sqrt{x(bx+a)}}+\frac{\frac{\sqrt{b}\,(3ad-4bc)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{d}-\frac{2\left(a^2d^2-4abcd+3b^2c^2\right)\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-b}{d^2}}}{x+}\right.}{d^2\sqrt{-\frac{c(ad-bc)}{d^2}}}}{}$
default	Expression too large to display

input
```
int((b*x^2+a*x)^(3/2)/x/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2/b^(1/2)/(c*(a*d-b*c))^(1/2)*(-2*(d*x+c)*b^(1/2)*(b*c-1/4*a*d)*(-a*d+b*c)
*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))+(3/2*(d*x+c)*b*(a*d-4/3
*b*c)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))+b^(1/2)*d*(b*c-1/2*d*(-b*x+a))*
(x*(b*x+a))^(1/2))*(c*(a*d-b*c))^(1/2))/d^3/(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 767, normalized size of antiderivative = 5.08

$$\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a*x)^(3/2)/x/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[-1/2*((4*b*c^2 - 3*a*c*d + (4*b*c*d - 3*a*d^2)*x)*sqrt(b)*log(2*b*x + a +
 2*sqrt(b*x^2 + a*x)*sqrt(b)) + (4*b*c^2 - a*c*d + (4*b*c*d - a*d^2)*x)*sq
rt((b*c - a*d)/c)*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)*c*sqrt(
(b*c - a*d)/c))/(d*x + c)) - 2*(b*d^2*x + 2*b*c*d - a*d^2)*sqrt(b*x^2 + a*
x))/(d^4*x + c*d^3), 1/2*(2*(4*b*c^2 - a*c*d + (4*b*c*d - a*d^2)*x)*sqrt(-
(b*c - a*d)/c)*arctan(-sqrt(b*x^2 + a*x)*c*sqrt(-(b*c - a*d)/c)/((b*c - a*
d)*x)) - (4*b*c^2 - 3*a*c*d + (4*b*c*d - 3*a*d^2)*x)*sqrt(b)*log(2*b*x + a
 + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(b*d^2*x + 2*b*c*d - a*d^2)*sqrt(b*x^2
 + a*x))/(d^4*x + c*d^3), 1/2*(2*(4*b*c^2 - 3*a*c*d + (4*b*c*d - 3*a*d^2)*
x)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (4*b*c^2 - a*c*
d + (4*b*c*d - a*d^2)*x)*sqrt((b*c - a*d)/c)*log((a*c + (2*b*c - a*d)*x -
2*sqrt(b*x^2 + a*x)*c*sqrt((b*c - a*d)/c))/(d*x + c)) + 2*(b*d^2*x + 2*b*c
*d - a*d^2)*sqrt(b*x^2 + a*x))/(d^4*x + c*d^3), ((4*b*c^2 - 3*a*c*d + (4*b
*c*d - 3*a*d^2)*x)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) +
 (4*b*c^2 - a*c*d + (4*b*c*d - a*d^2)*x)*sqrt(-(b*c - a*d)/c)*arctan(-sqrt
(b*x^2 + a*x)*c*sqrt(-(b*c - a*d)/c)/((b*c - a*d)*x)) + (b*d^2*x + 2*b*c*d
 - a*d^2)*sqrt(b*x^2 + a*x))/(d^4*x + c*d^3)]
```

Sympy [F]

$$\int \frac{(ax + bx^2)^{3/2}}{x(c + dx)^2}\, dx = \int \frac{(x(a + bx))^{\frac{3}{2}}}{x\,(c + dx)^2}\, dx$$

input
```
integrate((b*x**2+a*x)**(3/2)/x/(d*x+c)**2,x)
```

output
```
Integral((x*(a + b*x))**(3/2)/(x*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{(ax + bx^2)^{3/2}}{x(c + dx)^2}\, dx = \int \frac{(bx^2 + ax)^{\frac{3}{2}}}{(dx + c)^2 x}\, dx$$

input
```
integrate((b*x^2+a*x)^(3/2)/x/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(3/2)/((d*x + c)^2*x), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a*x)^(3/2)/x/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)^2}\,dx = \int \frac{{\left(b\,x^2+a\,x\right)}^{3/2}}{x\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((a*x + b*x^2)^(3/2)/(x*(c + d*x)^2),x)
```

output
```
int((a*x + b*x^2)^(3/2)/(x*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 601, normalized size of antiderivative = 3.98

$$\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)^2}\,dx = \frac{-\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right) acd-\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}}{\ldots}\right.}{\ldots}$$

input
```
int((b*x^2+a*x)^(3/2)/x/(d*x+c)^2,x)
```

output
```
( - sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x)
- sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*c*d - sqrt(c)*sqrt(a*d - b
*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b
))/(sqrt(c)*sqrt(b)))*a*d**2*x + 4*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d
- b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)
))*b*c**2 + 4*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt
(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b*c*d*x - sqrt(c)*
sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sq
rt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*c*d - sqrt(c)*sqrt(a*d - b*c)*atan((sq
rt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*
sqrt(b)))*a*d**2*x + 4*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqr
t(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b*c**2 +
4*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) +
sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b*c*d*x - sqrt(x)*sqrt(a + b*x
)*a*c*d**2 + 2*sqrt(x)*sqrt(a + b*x)*b*c**2*d + sqrt(x)*sqrt(a + b*x)*b*c*
d**2*x + 3*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*c**2*d
 + 3*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*c*d**2*x - 4
*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b*c**3 - 4*sqrt(b)
*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b*c**2*d*x)/(c*d**3*(c + d
*x))
```

3.104 $\int \frac{(ax+bx^2)^{3/2}}{x^2(c+dx)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 131

$$\int \frac{(ax+bx^2)^{3/2}}{x^2(c+dx)^2}\,dx = -\frac{(bc-ad)\sqrt{ax+bx^2}}{cd(c+dx)} + \frac{2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{d^2} - \frac{\sqrt{bc-ad}(2bc+ad)\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{3/2}d^2}$$

output
```
-(-a*d+b*c)*(b*x^2+a*x)^(1/2)/c/d/(d*x+c)+2*b^(3/2)*arctanh(b^(1/2)*x/(b*x
^2+a*x)^(1/2))/d^2-(-a*d+b*c)^(1/2)*(a*d+2*b*c)*arctanh((-a*d+b*c)^(1/2)*x
/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(3/2)/d^2
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.34

$$\int \frac{(ax+bx^2)^{3/2}}{x^2(c+dx)^2}\,dx = \frac{(x(a+bx))^{3/2}\left(\frac{d(-bc+ad)\sqrt{x}}{c(a+bx)(c+dx)} - \frac{\sqrt{-bc+ad}(2bc+ad)\arctan\left(\frac{-d\sqrt{x}\sqrt{a+bx}+\sqrt{b}(c+dx)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{c^{3/2}(a+bx)^{3/2}} - \frac{2b^{3/2}\log\left(-\sqrt{b}\right.}{(a+bx}\right.}{d^2x^{3/2}}$$

input
```
Integrate[(a*x + b*x^2)^(3/2)/(x^2*(c + d*x)^2),x]
```

output
```
((x*(a + b*x))^(3/2)*((d*(-(b*c) + a*d)*Sqrt[x])/(c*(a + b*x)*(c + d*x)) -
 (Sqrt[-(b*c) + a*d]*(2*b*c + a*d)*ArcTan[(-(d*Sqrt[x]*Sqrt[a + b*x]) + Sq
rt[b]*(c + d*x))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(c^(3/2)*(a + b*x)^(3/2))
- (2*b^(3/2)*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(a + b*x)^(3/2)))/(d
^2*x^(3/2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 555 vs. 2(131) = 262.

Time = 1.57 (sec) , antiderivative size = 555, normalized size of antiderivative = 4.24, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1260, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}}{x^2(c+dx)^2}\,dx$$

$$\Big\downarrow \text{1260}$$

$$\int \left(\frac{2d^2(ax+bx^2)^{3/2}}{c^3(c+dx)}-\frac{2d(ax+bx^2)^{3/2}}{c^3x}+\frac{d^2(ax+bx^2)^{3/2}}{c^2(c+dx)^2}+\frac{(ax+bx^2)^{3/2}}{c^2x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}c^3}+\frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(a^2d^2-8abcd+8b^2c^2)}{4\sqrt{b}c^2d^2}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(2bc-ad)(-a^2d^2-8abcd+8b^2c^2)}{4b^{3/2}c^3d^2}+\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4\sqrt{b}c^2}+\frac{\sqrt{ax+bx^2}(a^2d^2-2bdx(2bc-ad)-10abcd+8b^2c^2)}{4bc^3d}+\frac{2(bc-ad)^{3/2}\text{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{c^{3/2}d^2}-\frac{3\sqrt{bc-ad}(2bc-ad)\text{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{2c^{3/2}d^2}-\frac{ad(a+2bx)\sqrt{ax+bx^2}}{4bc^3}-\frac{d(ax+bx^2)^{3/2}}{c^2(c+dx)}-\frac{3\sqrt{ax+bx^2}(-3ad+4bc-2bdx)}{4c^2d}+\frac{3a\sqrt{ax+bx^2}}{4c^2}+\frac{(ax+bx^2)^{3/2}}{2c^2x}$$

input
```
Int[(a*x + b*x^2)^(3/2)/(x^2*(c + d*x)^2),x]
```

output
```
(3*a*Sqrt[a*x + b*x^2])/(4*c^2) - (a*d*(a + 2*b*x)*Sqrt[a*x + b*x^2])/(4*b
*c^3) - (3*(4*b*c - 3*a*d - 2*b*d*x)*Sqrt[a*x + b*x^2])/(4*c^2*d) + ((8*b^
2*c^2 - 10*a*b*c*d + a^2*d^2 - 2*b*d*(2*b*c - a*d)*x)*Sqrt[a*x + b*x^2])/(
4*b*c^3*d) + (a*x + b*x^2)^(3/2)/(2*c^2*x) - (d*(a*x + b*x^2)^(3/2))/(c^2*
(c + d*x)) + (3*a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*Sqrt[b]*c^2
) + (a^3*d*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*b^(3/2)*c^3) - ((2*b
*c - a*d)*(8*b^2*c^2 - 8*a*b*c*d - a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x +
 b*x^2]])/(4*b^(3/2)*c^3*d^2) + (3*(8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*Sqrt[b]*c^2*d^2) + (2*(b*c - a*d)^(3
/2)*ArcTanh[(a*c + (2*b*c - a*d)*x)/(2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[a*x +
b*x^2]])])/(c^(3/2)*d^2) - (3*Sqrt[b*c - a*d]*(2*b*c - a*d)*ArcTanh[(a*c +
(2*b*c - a*d)*x)/(2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[a*x + b*x^2]])])/(2*c^(3/2
)*d^2)
```

Defintions of rubi rules used

rule 1260
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x + c*x^2)^p
, (d + e*x)^m*(f + g*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ
[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + n + 2*p + 1, 0] && ILtQ[m, 0] && ILtQ
[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$\frac{-(ad+2bc)(ad-bc)(dx+c)\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)+\left(2c\,b^{\frac{3}{2}}(dx+c)\,\mathrm{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)+d\sqrt{x(bx+a)}\,(ad-bc)\right)\sqrt{c(ad-bc}}{\sqrt{c(ad-bc)}\,d^2(dx+c)c}$
default	Expression too large to display

input
```
int((b*x^2+a*x)^(3/2)/x^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
(-(a*d+2*b*c)*(a*d-b*c)*(d*x+c)*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))
^(1/2))+(2*c*b^(3/2)*(d*x+c)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))+d*(x*(b*
x+a))^(1/2)*(a*d-b*c))*(c*(a*d-b*c))^(1/2))/(c*(a*d-b*c))^(1/2)/d^2/(d*x+c
)/c
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 684, normalized size of antiderivative = 5.22

$$\int \frac{(ax+bx^2)^{3/2}}{x^2(c+dx)^2}\,dx = \left[\frac{2\,(bcdx+bc^2)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+(2\,bc^2+acd+(2\,bcd+ad^2)x}{2\,(cd^3x+c^2d}\right.$$

$$-\frac{(2\,bc^2+acd+(2\,bcd+ad^2)x)\sqrt{-\frac{bc-ad}{c}}\arctan\left(-\frac{\sqrt{bx^2+ax}c\sqrt{-\frac{bc-ad}{c}}}{(bc-ad)x}\right)-(bcdx+bc^2)\sqrt{b}\log\left(2\,bx+a+2\right.}{cd^3x+c^2d^2}$$

$$-\frac{4\,(bcdx+bc^2)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(2\,bc^2+acd+(2\,bcd+ad^2)x)\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)x-2\,\sqrt{bx^2}}{dx+c}\right.}{2\,(cd^3x+c^2d^2)}$$

$$-\frac{2\,(bcdx+bc^2)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+(2\,bc^2+acd+(2\,bcd+ad^2)x)\sqrt{-\frac{bc-ad}{c}}\arctan\left(-\frac{\sqrt{bx^2+ax}c\sqrt{-}}{(bc-ad)}\right.}{cd^3x+c^2d^2}$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[1/2*(2*(b*c*d*x + b*c^2)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt
(b)) + (2*b*c^2 + a*c*d + (2*b*c*d + a*d^2)*x)*sqrt((b*c - a*d)/c)*log((a*
c + (2*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)*c*sqrt((b*c - a*d)/c))/(d*x + c)
) - 2*(b*c*d - a*d^2)*sqrt(b*x^2 + a*x))/(c*d^3*x + c^2*d^2), -((2*b*c^2 +
 a*c*d + (2*b*c*d + a*d^2)*x)*sqrt(-(b*c - a*d)/c)*arctan(-sqrt(b*x^2 + a*
x)*c*sqrt(-(b*c - a*d)/c)/((b*c - a*d)*x)) - (b*c*d*x + b*c^2)*sqrt(b)*log
(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + (b*c*d - a*d^2)*sqrt(b*x^2 + a
*x))/(c*d^3*x + c^2*d^2), -1/2*(4*(b*c*d*x + b*c^2)*sqrt(-b)*arctan(sqrt(b
*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (2*b*c^2 + a*c*d + (2*b*c*d + a*d^2)*x)*
sqrt((b*c - a*d)/c)*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)*c*sqr
t((b*c - a*d)/c))/(d*x + c)) + 2*(b*c*d - a*d^2)*sqrt(b*x^2 + a*x))/(c*d^3
*x + c^2*d^2), -(2*(b*c*d*x + b*c^2)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqr
t(-b)/(b*x + a)) + (2*b*c^2 + a*c*d + (2*b*c*d + a*d^2)*x)*sqrt(-(b*c - a*
d)/c)*arctan(-sqrt(b*x^2 + a*x)*c*sqrt(-(b*c - a*d)/c)/((b*c - a*d)*x)) +
(b*c*d - a*d^2)*sqrt(b*x^2 + a*x))/(c*d^3*x + c^2*d^2)]
```

Sympy [F]

$$\int \frac{(ax + bx^2)^{3/2}}{x^2(c + dx)^2} \, dx = \int \frac{(x(a + bx))^{\frac{3}{2}}}{x^2 (c + dx)^2} \, dx$$

input
```
integrate((b*x**2+a*x)**(3/2)/x**2/(d*x+c)**2,x)
```

output
```
Integral((x*(a + b*x))**(3/2)/(x**2*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{(ax + bx^2)^{3/2}}{x^2(c + dx)^2} \, dx = \int \frac{(bx^2 + ax)^{\frac{3}{2}}}{(dx + c)^2 x^2} \, dx$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(3/2)/((d*x + c)^2*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ax+bx^2)^{3/2}}{x^2(c+dx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{x^2(c+dx)^2}\,dx = \int \frac{(b\,x^2+a\,x)^{3/2}}{x^2\,(c+d\,x)^2}\,dx$$

input
```
int((a*x + b*x^2)^(3/2)/(x^2*(c + d*x)^2),x)
```

output
```
int((a*x + b*x^2)^(3/2)/(x^2*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 530, normalized size of antiderivative = 4.05

$$\int \frac{(ax+bx^2)^{3/2}}{x^2(c+dx)^2}\,dx = \frac{-\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)acd-\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}}{}\right.}{}$$

input
```
int((b*x^2+a*x)^(3/2)/x^2/(d*x+c)^2,x)
```

output
```
( - sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x)
- sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*c*d - sqrt(c)*sqrt(a*d - b
*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b
))/(sqrt(c)*sqrt(b)))*a*d**2*x - 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d
- b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)
))*b*c**2 - 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt
(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b*c*d*x - sqrt(c)*
sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sq
rt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*c*d - sqrt(c)*sqrt(a*d - b*c)*atan((sq
rt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*
sqrt(b)))*a*d**2*x - 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqr
t(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b*c**2 -
2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) +
sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b*c*d*x + sqrt(x)*sqrt(a + b*x
)*a*c*d**2 - sqrt(x)*sqrt(a + b*x)*b*c**2*d + 2*sqrt(b)*log((sqrt(a + b*x)
 + sqrt(x)*sqrt(b))/sqrt(a))*b*c**3 + 2*sqrt(b)*log((sqrt(a + b*x) + sqrt(
x)*sqrt(b))/sqrt(a))*b*c**2*d*x)/(c**2*d**2*(c + d*x))
```

3.105 $\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 103

$$\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)^2}\,dx = -\frac{3a\sqrt{ax+bx^2}}{c^2x} + \frac{(ax+bx^2)^{3/2}}{cx^2(c+dx)} + \frac{3a\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{5/2}}$$

output
```
-3*a*(b*x^2+a*x)^(1/2)/c^2/x+(b*x^2+a*x)^(3/2)/c/x^2/(d*x+c)+3*a*(-a*d+b*c
)^(1/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(5/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.38 (sec) , antiderivative size = 832, normalized size of antiderivative = 8.08

$$\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)^2}\,dx = \frac{(x(a+bx))^{3/2}\left(\frac{\sqrt{c}(-2ac+bcx-3adx)\left(a+bx-\sqrt{a}\sqrt{a+bx}\right)}{\sqrt{x}(c+dx)\left(-\sqrt{a}+\sqrt{a+bx}\right)} + \frac{3a^2d\arctan\left(\frac{\sqrt{-bc+2ad-2i\sqrt{a}\sqrt{d}\sqrt{bc-ad}}\sqrt{x}}{\sqrt{c}\left(\sqrt{a}-\sqrt{a+bx}\right)}\right)}{\sqrt{-bc+2ad-2i\sqrt{a}\sqrt{d}\sqrt{bc-ad}}}\right.}{}$$

input
```
Integrate[(a*x + b*x^2)^(3/2)/(x^3*(c + d*x)^2),x]
```

output
```
((x*(a + b*x))^(3/2)*((Sqrt[c]*(-2*a*c + b*c*x - 3*a*d*x)*(a + b*x - Sqrt[
a]*Sqrt[a + b*x]))/(Sqrt[x]*(c + d*x)*(-Sqrt[a] + Sqrt[a + b*x])) + (3*a^2
*d*ArcTan[(Sqrt[-(b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sq
rt[x])/(Sqrt[c]*(Sqrt[a] - Sqrt[a + b*x]))])/Sqrt[-(b*c) + 2*a*d - (2*I)*S
qrt[a]*Sqrt[d]*Sqrt[b*c - a*d]] + (3*a^2*d*ArcTan[(Sqrt[-(b*c) + 2*a*d + (
2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*(Sqrt[a] - Sqrt[a
+ b*x]))])/Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]] +
((3*I)*a^(3/2)*Sqrt[d]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[-(b*c) + 2*a*d + (2*I)
*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*(Sqrt[a] - Sqrt[a + b*
x]))])/Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]] + (3*a
*b*c*ArcTan[(Sqrt[-(b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*
Sqrt[x])/(Sqrt[c]*(-Sqrt[a] + Sqrt[a + b*x]))])/Sqrt[-(b*c) + 2*a*d - (2*I
)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]] + ((3*I)*a^(3/2)*Sqrt[d]*Sqrt[b*c - a*d
]*ArcTan[(Sqrt[-(b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqr
t[x])/(Sqrt[c]*(-Sqrt[a] + Sqrt[a + b*x]))])/Sqrt[-(b*c) + 2*a*d - (2*I)*S
qrt[a]*Sqrt[d]*Sqrt[b*c - a*d]] + (3*a*b*c*ArcTan[(Sqrt[-(b*c) + 2*a*d + (
2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*(-Sqrt[a] + Sqrt[a
 + b*x]))])/Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]))
/(c^(5/2)*x^(3/2)*(a + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.42, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1261, 105, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)^2}\,dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{(ax+bx^2)^{3/2}\int \frac{(a+bx)^{3/2}}{x^{3/2}(c+dx)^2}dx}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{105}$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{3(bc-ad)\int\frac{\sqrt{a+bx}}{\sqrt{x}(c+dx)^2}dx}{c}-\frac{2(a+bx)^{3/2}}{c\sqrt{x}(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \textbf{105}$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{3(bc-ad)\left(\frac{a\int\frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{2c}+\frac{\sqrt{x}\sqrt{a+bx}}{c(c+dx)}\right)}{c}-\frac{2(a+bx)^{3/2}}{c\sqrt{x}(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \textbf{104}$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{3(bc-ad)\left(\frac{a\int\frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}+\frac{\sqrt{x}\sqrt{a+bx}}{c(c+dx)}\right)}{c}-\frac{2(a+bx)^{3/2}}{c\sqrt{x}(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{3(bc-ad)\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{x}\sqrt{a+bx}}{c(c+dx)}\right)}{c}-\frac{2(a+bx)^{3/2}}{c\sqrt{x}(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

input
```
Int[(a*x + b*x^2)^(3/2)/(x^3*(c + d*x)^2),x]
```

output
```
((a*x + b*x^2)^(3/2)*((-2*(a + b*x)^(3/2))/(c*Sqrt[x]*(c + d*x)) + (3*(b*c
 - a*d)*((Sqrt[x]*Sqrt[a + b*x])/(c*(c + d*x)) + (a*ArcTanh[(Sqrt[b*c - a*
d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(c^(3/2)*Sqrt[b*c - a*d])))/c))/(x^(
3/2)*(a + b*x)^(3/2))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$\frac{-2\sqrt{x(bx+a)}\,\sqrt{c(ad-bc)}\left(\left(-\frac{bx}{2}+a\right)c+\frac{3adx}{2}\right)+3ax(dx+c)(ad-bc)\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{\sqrt{c(ad-bc)}\,c^2x(dx+c)}$
risch	$-\frac{2a(bx+a)}{c^2\sqrt{x(bx+a)}}-\frac{-\frac{(a^2d^2-b^2c^2)\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{x+\frac{c}{d}}\right.}{d^2\sqrt{-\frac{c(ad-bc)}{d^2}}}}{}$
default	Expression too large to display

input
```
int((b*x^2+a*x)^(3/2)/x^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
(-2*(x*(b*x+a))^(1/2)*(c*(a*d-b*c))^(1/2)*((-1/2*b*x+a)*c+3/2*a*d*x)+3*a*x
*(d*x+c)*(a*d-b*c)*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2)))/(c*(
a*d-b*c))^(1/2)/c^2/x/(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.39

$$\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)^2}\,dx = \left[\frac{3\,(adx^2+acx)\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)x+2\,\sqrt{bx^2+ax}c\sqrt{\frac{bc-ad}{c}}}{dx+c}\right)-2\,\sqrt{bx^2+ax}(2\,ac-(}{2\,(c^2dx^2+c^3x)}\right.$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[1/2*(3*(a*d*x^2 + a*c*x)*sqrt((b*c - a*d)/c)*log((a*c + (2*b*c - a*d)*x +
 2*sqrt(b*x^2 + a*x)*c*sqrt((b*c - a*d)/c))/(d*x + c)) - 2*sqrt(b*x^2 + a*
x)*(2*a*c - (b*c - 3*a*d)*x))/(c^2*d*x^2 + c^3*x), (3*(a*d*x^2 + a*c*x)*sq
rt(-(b*c - a*d)/c)*arctan(-sqrt(b*x^2 + a*x)*c*sqrt(-(b*c - a*d)/c)/((b*c
- a*d)*x)) - sqrt(b*x^2 + a*x)*(2*a*c - (b*c - 3*a*d)*x))/(c^2*d*x^2 + c^3
*x)]
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)^2}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}}{x^3\,(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a*x)**(3/2)/x**3/(d*x+c)**2,x)
```

output
```
Integral((x*(a + b*x))**(3/2)/(x**3*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)^2}\,dx = \int \frac{(bx^2+ax)^{\frac{3}{2}}}{(dx+c)^2x^3}\,dx$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(3/2)/((d*x + c)^2*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(89) = 178$.

Time = 0.76 (sec) , antiderivative size = 658, normalized size of antiderivative = 6.39

$$\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/2*d^2*(sqrt(b*c^2 - a*c*d)*a*log(abs(-2*(b*c^3 - a*c^2*d)*(sqrt(b - 2*b
*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d/(d*x + c) - a*c*d/(d*x + c)^2) + sq
rt(b*c^2*d^2 - a*c*d^3)/((d*x + c)*d))^3*abs(d) + (6*b*c^2*d - 5*a*c*d^2)*
sqrt(b*c^2 - a*c*d)*(sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d/(d
*x + c) - a*c*d/(d*x + c)^2) + sqrt(b*c^2*d^2 - a*c*d^3)/((d*x + c)*d))^2
- 2*(3*b^2*c^3 - 5*a*b*c^2*d + 2*a^2*c*d^2)*(sqrt(b - 2*b*c/(d*x + c) + b*
c^2/(d*x + c)^2 + a*d/(d*x + c) - a*c*d/(d*x + c)^2) + sqrt(b*c^2*d^2 - a*
c*d^3)/((d*x + c)*d))*abs(d) + (2*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*sqrt(
b*c^2 - a*c*d)))*sgn(1/(d*x + c))*sgn(d)/(c^3*d*abs(d)) + (2*b^(3/2)*c^2*a
bs(d) - 2*a*sqrt(b)*c*d*abs(d) - sqrt(b*c^2 - a*c*d)*a*d^2*log(abs(-8*b^(5
/2)*c^3*abs(d) + 12*a*b^(3/2)*c^2*d*abs(d) - 4*a^2*sqrt(b)*c*d^2*abs(d) +
8*sqrt(b*c^2 - a*c*d)*b^2*c^2*d - 8*sqrt(b*c^2 - a*c*d)*a*b*c*d^2 + sqrt(b
*c^2 - a*c*d)*a^2*d^3)))*sgn(1/(d*x + c))*sgn(d)/(c^3*d^3*abs(d)) - 2*(b*c
^3*d^2*sgn(1/(d*x + c))*sgn(d) - a*c^2*d^3*sgn(1/(d*x + c))*sgn(d))*sqrt(b
 - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d/(d*x + c) - a*c*d/(d*x + c)^2
)/(c^4*d^5))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)^2}\,dx = \int \frac{(b\,x^2+a\,x)^{3/2}}{x^3\,(c+d\,x)^2}\,dx$$

input
```
int((a*x + b*x^2)^(3/2)/(x^3*(c + d*x)^2),x)
```

output
```
int((a*x + b*x^2)^(3/2)/(x^3*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 624, normalized size of antiderivative = 6.06

$$\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)^2}\,dx = \frac{9\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)a^2cdx+9\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\cdots}{\cdots}\right.\cdots}{\cdots}$$

input
```
int((b*x^2+a*x)^(3/2)/x^3/(d*x+c)^2,x)
```

output
```
(9*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) -
 sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*c*d*x + 9*sqrt(c)*sqrt(a
*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*
sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*d**2*x**2 - 12*sqrt(c)*sqrt(a*d - b*c)*at
an((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sq
rt(c)*sqrt(b)))*a*b*c**2*x - 12*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b
*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*
a*b*c*d*x**2 + 9*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*s
qrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*c*d*x + 9*
sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sq
rt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*d**2*x**2 - 12*sqrt(c)*sqrt
(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d
)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b*c**2*x - 12*sqrt(c)*sqrt(a*d - b*c)*atan
((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt
(c)*sqrt(b)))*a*b*c*d*x**2 - 6*sqrt(x)*sqrt(a + b*x)*a**2*c**2*d - 9*sqrt(
x)*sqrt(a + b*x)*a**2*c*d**2*x + 8*sqrt(x)*sqrt(a + b*x)*a*b*c**3 + 15*sqr
t(x)*sqrt(a + b*x)*a*b*c**2*d*x - 4*sqrt(x)*sqrt(a + b*x)*b**2*c**3*x + 3*
sqrt(b)*a**2*c**2*d*x + 3*sqrt(b)*a**2*c*d**2*x**2 - 9*sqrt(b)*a*b*c**3*x
- 9*sqrt(b)*a*b*c**2*d*x**2)/(c**3*x*(3*a*c*d + 3*a*d**2*x - 4*b*c**2 - 4*
b*c*d*x))
```

3.106 $\int \frac{(ax+bx^2)^{3/2}}{x^4(c+dx)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 163

$$\int \frac{(ax+bx^2)^{3/2}}{x^4(c+dx)^2}\,dx = -\frac{(11bc-15ad)\sqrt{ax+bx^2}}{3c^3x} - \frac{2a\sqrt{ax+bx^2}}{3cx^2(c+dx)} + \frac{(3bc-5ad)\sqrt{ax+bx^2}}{3c^2x(c+dx)} + \frac{(2bc-5ad)\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{7/2}}$$

output
```
-1/3*(-15*a*d+11*b*c)*(b*x^2+a*x)^(1/2)/c^3/x-2/3*a*(b*x^2+a*x)^(1/2)/c/x^
2/(d*x+c)+1/3*(-5*a*d+3*b*c)*(b*x^2+a*x)^(1/2)/c^2/x/(d*x+c)+(-5*a*d+2*b*c
)*(-a*d+b*c)^(1/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c
^(7/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.89 (sec) , antiderivative size = 1953, normalized size of antiderivative = 11.98

$$\int \frac{(ax+bx^2)^{3/2}}{x^4(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a*x + b*x^2)^(3/2)/(x^4*(c + d*x)^2),x]
```

output
```
((x*(a + b*x))^(3/2)*(-((Sqrt[c]*(262144*a^10 + 1376256*a^9*b*x + 3080192*
a^8*b^2*x^2 + 3829760*a^7*b^3*x^3 + 2888704*a^6*b^4*x^4 + 1354496*a^5*b^5*
x^5 + 388608*a^4*b^6*x^6 + 64416*a^3*b^7*x^7 + 5460*a^2*b^8*x^8 + 181*a*b^
9*x^9 + b^10*x^10 - 262144*a^(19/2)*Sqrt[a + b*x] - 1245184*a^(17/2)*b*x*S
qrt[a + b*x] - 2490368*a^(15/2)*b^2*x^2*Sqrt[a + b*x] - 2723840*a^(13/2)*b
^3*x^3*Sqrt[a + b*x] - 1770496*a^(11/2)*b^4*x^4*Sqrt[a + b*x] - 695552*a^(
9/2)*b^5*x^5*Sqrt[a + b*x] - 160512*a^(7/2)*b^6*x^6*Sqrt[a + b*x] - 20064*
a^(5/2)*b^7*x^7*Sqrt[a + b*x] - 1140*a^(3/2)*b^8*x^8*Sqrt[a + b*x] - 19*Sq
rt[a]*b^9*x^9*Sqrt[a + b*x])*(b*c*x*(8*c + 11*d*x) + a*(2*c^2 - 10*c*d*x -
 15*d^2*x^2)))/(x^(3/2)*(c + d*x)*(-262144*a^(19/2) - 1245184*a^(17/2)*b*x
 - 2490368*a^(15/2)*b^2*x^2 - 2723840*a^(13/2)*b^3*x^3 - 1770496*a^(11/2)*
b^4*x^4 - 695552*a^(9/2)*b^5*x^5 - 160512*a^(7/2)*b^6*x^6 - 20064*a^(5/2)*
b^7*x^7 - 1140*a^(3/2)*b^8*x^8 - 19*Sqrt[a]*b^9*x^9 + 262144*a^9*Sqrt[a +
b*x] + 1114112*a^8*b*x*Sqrt[a + b*x] + 1966080*a^7*b^2*x^2*Sqrt[a + b*x] +
 1863680*a^6*b^3*x^3*Sqrt[a + b*x] + 1025024*a^5*b^4*x^4*Sqrt[a + b*x] + 3
29472*a^4*b^5*x^5*Sqrt[a + b*x] + 59136*a^3*b^6*x^6*Sqrt[a + b*x] + 5280*a
^2*b^7*x^7*Sqrt[a + b*x] + 180*a*b^8*x^8*Sqrt[a + b*x] + b^9*x^9*Sqrt[a +
b*x]))) + (21*a*b*c*d*ArcTan[(Sqrt[-(b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*
Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*(Sqrt[a] - Sqrt[a + b*x]))])/Sqrt[-(b*c
) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]] + ((15*I)*a^(3/2)*d^...
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1261, 107, 105, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{3/2}}{x^4(c+dx)^2}\,dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{\left(ax+bx^2\right)^{3/2}\int \frac{(a+bx)^{3/2}}{x^{5/2}(c+dx)^2}dx}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \; 107$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{(2bc-5ad)\int \frac{(a+bx)^{3/2}}{x^{3/2}(c+dx)^2}dx}{3ac}-\frac{2(a+bx)^{5/2}}{3acx^{3/2}(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \; 105$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{(2bc-5ad)\left(\frac{3(bc-ad)\int \frac{\sqrt{a+bx}}{\sqrt{x}(c+dx)^2}dx}{c}-\frac{2(a+bx)^{3/2}}{c\sqrt{x}(c+dx)}\right)}{3ac}-\frac{2(a+bx)^{5/2}}{3acx^{3/2}(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \; 105$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{(2bc-5ad)\left(\frac{3(bc-ad)\left(\frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{2c}+\frac{\sqrt{x}\sqrt{a+bx}}{c(c+dx)}\right)}{c}-\frac{2(a+bx)^{3/2}}{c\sqrt{x}(c+dx)}\right)}{3ac}-\frac{2(a+bx)^{5/2}}{3acx^{3/2}(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \; 104$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{(2bc-5ad)\left(\frac{3(bc-ad)\left(\frac{a\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}+\frac{\sqrt{x}\sqrt{a+bx}}{c(c+dx)}\right)}{c}-\frac{2(a+bx)^{3/2}}{c\sqrt{x}(c+dx)}\right)}{3ac}-\frac{2(a+bx)^{5/2}}{3acx^{3/2}(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{(2bc-5ad)\left(\frac{3(bc-ad)\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{x}\sqrt{a+bx}}{c(c+dx)}\right)}{c}-\frac{2(a+bx)^{3/2}}{c\sqrt{x}(c+dx)}\right)}{3ac}-\frac{2(a+bx)^{5/2}}{3acx^{3/2}(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

input
```
Int[(a*x + b*x^2)^(3/2)/(x^4*(c + d*x)^2),x]
```

output
```
((a*x + b*x^2)^(3/2)*((-2*(a + b*x)^(5/2))/(3*a*c*x^(3/2)*(c + d*x)) + ((2
*b*c - 5*a*d)*((-2*(a + b*x)^(3/2))/(c*Sqrt[x]*(c + d*x)) + (3*(b*c - a*d)
*((Sqrt[x]*Sqrt[a + b*x])/(c*(c + d*x)) + (a*ArcTanh[(Sqrt[b*c - a*d]*Sqrt
[x])/(Sqrt[c]*Sqrt[a + b*x])])/(c^(3/2)*Sqrt[b*c - a*d])))/c))/(3*a*c)))/(
x^(3/2)*(a + b*x)^(3/2))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$\frac{-2\sqrt{c(ad-bc)}\,\sqrt{x(bx+a)}\left((4bx+a)c^2-5dx\left(-\frac{11bx}{10}+a\right)c-\frac{15a\,d^2x^2}{2}\right)-3x^2\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)(dx+c)\left(5a^2d^2-7abcd+2\right.}{3\sqrt{c(ad-bc)}\,c^3x^2(dx+c)}$
risch	$-\frac{2(bx+a)(-6adx+4cbx+ac)}{3c^3\sqrt{x(bx+a)}\,x}+\frac{c\left(a^2d^2-2abcd+b^2c^2\right)\left(\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{c(ad-bc)\left(x+\frac{c}{d}\right)}-\frac{(ad-2bc)d\ln\left(-\frac{2c(ad-}{d^2}\right.}{}\right.}{d^2}$
default	Expression too large to display

input
```
int((b*x^2+a*x)^(3/2)/x^4/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-2*(c*(a*d-b*c))^(1/2)*(x*(b*x+a))^(1/2)*((4*b*x+a)*c^2-5*d*x*(-11/10
*b*x+a)*c-15/2*a*d^2*x^2)-3*x^2*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))
^(1/2))*(d*x+c)*(5*a^2*d^2-7*a*b*c*d+2*b^2*c^2))/(c*(a*d-b*c))^(1/2)/c^3/x
^2/(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.08

$$\int \frac{(ax+bx^2)^{3/2}}{x^4(c+dx)^2}\,dx = \left[-\frac{3\left((2\,bcd-5\,ad^2)x^3+(2\,bc^2-5\,acd)x^2\right)\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)x-2\,\sqrt{bx^2+ax}c\sqrt{\frac{bc-ad}{c}}}{dx+c}\right.}{6\,(c^3dx^3+c^4x^2}\right.$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^4/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[-1/6*(3*((2*b*c*d - 5*a*d^2)*x^3 + (2*b*c^2 - 5*a*c*d)*x^2)*sqrt((b*c - a
*d)/c)*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)*c*sqrt((b*c - a*d)
/c))/(d*x + c)) + 2*(2*a*c^2 + (11*b*c*d - 15*a*d^2)*x^2 + 2*(4*b*c^2 - 5*
a*c*d)*x)*sqrt(b*x^2 + a*x))/(c^3*d*x^3 + c^4*x^2), 1/3*(3*((2*b*c*d - 5*a
*d^2)*x^3 + (2*b*c^2 - 5*a*c*d)*x^2)*sqrt(-(b*c - a*d)/c)*arctan(-sqrt(b*x
^2 + a*x)*c*sqrt(-(b*c - a*d)/c)/((b*c - a*d)*x)) - (2*a*c^2 + (11*b*c*d -
 15*a*d^2)*x^2 + 2*(4*b*c^2 - 5*a*c*d)*x)*sqrt(b*x^2 + a*x))/(c^3*d*x^3 +
c^4*x^2)]
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x^4(c+dx)^2}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}}{x^4\,(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a*x)**(3/2)/x**4/(d*x+c)**2,x)
```

output
```
Integral((x*(a + b*x))**(3/2)/(x**4*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x^4(c+dx)^2}\,dx = \int \frac{(bx^2+ax)^{\frac{3}{2}}}{(dx+c)^2x^4}\,dx$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^4/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(3/2)/((d*x + c)^2*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1345 vs. $2(141) = 282$.

Time = 8.72 (sec) , antiderivative size = 1345, normalized size of antiderivative = 8.25

$$\int \frac{(ax+bx^2)^{3/2}}{x^4(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^4/(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/10*d^2*(2*(2*sqrt(b*c^2 - a*c*d)*b^2*c^7*d^2*abs(d)*sgn(1/(d*x + c))*sg
n(d) - 7*sqrt(b*c^2 - a*c*d)*a*b*c^6*d^3*abs(d)*sgn(1/(d*x + c))*sgn(d) +
5*sqrt(b*c^2 - a*c*d)*a^2*c^5*d^4*abs(d)*sgn(1/(d*x + c))*sgn(d))*log(abs(
b*c^2 - a*c*d))/(b*c^10*d^5 - a*c^9*d^6) + sqrt(b*c^2 - a*c*d)*(2*b*c - 5*
a*d)*log(abs(-2*(b*c^4 - a*c^3*d)*(sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x +
 c)^2 + a*d/(d*x + c) - a*c*d/(d*x + c)^2) + sqrt(b*c^2*d^2 - a*c*d^3)/((d
*x + c)*d))^5*abs(d) + (10*b*c^3*d - 9*a*c^2*d^2)*sqrt(b*c^2 - a*c*d)*(sqr
t(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d/(d*x + c) - a*c*d/(d*x + c
)^2) + sqrt(b*c^2*d^2 - a*c*d^3)/((d*x + c)*d))^4 - 4*(5*b^2*c^4 - 9*a*b*c
^3*d + 4*a^2*c^2*d^2)*(sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d/
(d*x + c) - a*c*d/(d*x + c)^2) + sqrt(b*c^2*d^2 - a*c*d^3)/((d*x + c)*d))^
3*abs(d) + 2*(10*b^2*c^3*d - 17*a*b*c^2*d^2 + 7*a^2*c*d^3)*sqrt(b*c^2 - a*
c*d)*(sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d/(d*x + c) - a*c*d
/(d*x + c)^2) + sqrt(b*c^2*d^2 - a*c*d^3)/((d*x + c)*d))^2 - 2*(5*b^3*c^4
- 13*a*b^2*c^3*d + 11*a^2*b*c^2*d^2 - 3*a^3*c*d^3)*(sqrt(b - 2*b*c/(d*x +
c) + b*c^2/(d*x + c)^2 + a*d/(d*x + c) - a*c*d/(d*x + c)^2) + sqrt(b*c^2*d
^2 - a*c*d^3)/((d*x + c)*d))*abs(d) + (2*b^3*c^3*d - 5*a*b^2*c^2*d^2 + 4*a
^2*b*c*d^3 - a^3*d^4)*sqrt(b*c^2 - a*c*d)))*sgn(1/(d*x + c))*sgn(d)/(c^4*d
*abs(d)) - (10*b^(3/2)*c^2*abs(d) - 10*a*sqrt(b)*c*d*abs(d) + 2*sqrt(b*c^2
 - a*c*d)*b*c*d*log(abs(-32*b^(7/2)*c^4*abs(d) + 64*a*b^(5/2)*c^3*d*abs...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{x^4(c+dx)^2}\,dx = \int \frac{(b\,x^2+a\,x)^{3/2}}{x^4\,(c+d\,x)^2}\,dx$$

input
```
int((a*x + b*x^2)^(3/2)/(x^4*(c + d*x)^2),x)
```

output
```
int((a*x + b*x^2)^(3/2)/(x^4*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 980, normalized size of antiderivative = 6.01

$$\int \frac{(ax+bx^2)^{3/2}}{x^4(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a*x)^(3/2)/x^4/(d*x+c)^2,x)
```

output
```
( - 75*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*
x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*c*d**2*x**2 - 75*sqr
t(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(
x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*d**3*x**3 + 90*sqrt(c)*sqrt(a*
d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*s
qrt(b))/(sqrt(c)*sqrt(b)))*a*b*c**2*d*x**2 + 90*sqrt(c)*sqrt(a*d - b*c)*at
an((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sq
rt(c)*sqrt(b)))*a*b*c*d**2*x**3 - 24*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*
d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(
b)))*b**2*c**3*x**2 - 24*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - s
qrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**2*c*
*2*d*x**3 - 75*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqr
t(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*c*d**2*x**2
 - 75*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x)
 + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*d**3*x**3 + 90*sqrt(c)
*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*s
qrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b*c**2*d*x**2 + 90*sqrt(c)*sqrt(a*d -
 b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt
(b))/(sqrt(c)*sqrt(b)))*a*b*c*d**2*x**3 - 24*sqrt(c)*sqrt(a*d - b*c)*atan(
(sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sq...
```

3.107 $\int \frac{(ax+bx^2)^{3/2}}{x^5(c+dx)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 214

$$\int \frac{(ax+bx^2)^{3/2}}{x^5(c+dx)^2}\,dx = -\frac{(27bc-35ad)\sqrt{ax+bx^2}}{15c^3x^2} - \frac{(6b^2c^2-95abcd+105a^2d^2)\sqrt{ax+bx^2}}{15ac^4x} - \frac{2a\sqrt{ax+bx^2}}{5cx^3(c+dx)} + \frac{(5bc-7ad)\sqrt{ax+bx^2}}{5c^2x^2(c+dx)} - \frac{d(4bc-7ad)\sqrt{bc-ad}\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{9/2}}$$

output

```
-1/15*(-35*a*d+27*b*c)*(b*x^2+a*x)^(1/2)/c^3/x^2-1/15*(105*a^2*d^2-95*a*b*
c*d+6*b^2*c^2)*(b*x^2+a*x)^(1/2)/a/c^4/x-2/5*a*(b*x^2+a*x)^(1/2)/c/x^3/(d*
x+c)+1/5*(-7*a*d+5*b*c)*(b*x^2+a*x)^(1/2)/c^2/x^2/(d*x+c)-d*(-7*a*d+4*b*c)
*(-a*d+b*c)^(1/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^
(9/2)
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.98

$$\int \frac{(ax+bx^2)^{3/2}}{x^5(c+dx)^2}\,dx = \frac{(x(a+bx))^{3/2}\left(-\frac{\sqrt{c}(6b^2c^2x^2(c+dx)+abcx(12c^2-68cdx-95d^2x^2)+a^2(6c^3-14c^2dx+70cd^2x^2+105d^3x^3))}{a(a+bx)(c+dx)}\right.}{15c^{9/2}x^4}$$

input
```
Integrate[(a*x + b*x^2)^(3/2)/(x^5*(c + d*x)^2),x]
```

output
```
((x*(a + b*x))^(3/2)*(-((Sqrt[c]*(6*b^2*c^2*x^2*(c + d*x) + a*b*c*x*(12*c^
2 - 68*c*d*x - 95*d^2*x^2) + a^2*(6*c^3 - 14*c^2*d*x + 70*c*d^2*x^2 + 105*
d^3*x^3)))/(a*(a + b*x)*(c + d*x))) - (15*d*(4*b*c - 7*a*d)*Sqrt[-(b*c) +
a*d]*x^(5/2)*ArcTan[(-(d*Sqrt[x]*Sqrt[a + b*x]) + Sqrt[b]*(c + d*x))/(Sqrt
[c]*Sqrt[-(b*c) + a*d])])/(a + b*x)^(3/2)))/(15*c^(9/2)*x^4)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.28, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1261, 109, 27, 168, 27, 169, 27, 169, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}}{x^5(c+dx)^2}\,dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{(ax+bx^2)^{3/2}\int \frac{(a+bx)^{3/2}}{x^{7/2}(c+dx)^2}dx}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{109}$$

$$\frac{(ax+bx^2)^{3/2}\left(-\frac{2\int -\frac{a(6bc-7ad)+b(5bc-6ad)x}{2x^{5/2}\sqrt{a+bx}(c+dx)^2}dx}{5c}-\frac{2a\sqrt{a+bx}}{5cx^{5/2}(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow 27$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\int \frac{a(6bc-7ad)+b(5bc-6ad)x}{x^{5/2}\sqrt{a+bx}(c+dx)^2}dx}{5c}-\frac{2a\sqrt{a+bx}}{5cx^{5/2}(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow 168$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\frac{\sqrt{a+bx}(5bc-7ad)}{cx^{3/2}(c+dx)}-\frac{\int -\frac{a(27bc-35ad)(bc-ad)+4b(5bc-7ad)x(bc-ad)}{2x^{5/2}\sqrt{a+bx}(c+dx)}dx}{c(bc-ad)}}{5c}-\frac{2a\sqrt{a+bx}}{5cx^{5/2}(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow 27$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\frac{\int \frac{a(27bc-35ad)(bc-ad)+4b(5bc-7ad)x(bc-ad)}{x^{5/2}\sqrt{a+bx}(c+dx)}dx}{2c(bc-ad)}+\frac{\sqrt{a+bx}(5bc-7ad)}{cx^{3/2}(c+dx)}}{5c}-\frac{2a\sqrt{a+bx}}{5cx^{5/2}(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow 169$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\frac{-\frac{2\int -\frac{a(bc-ad)\left(6b^2c^2-95abdc+105a^2d^2-2bd(27bc-35ad)x\right)}{2x^{3/2}\sqrt{a+bx}(c+dx)}dx}{3ac}-\frac{2\sqrt{a+bx}(27bc-35ad)(bc-ad)}{3cx^{3/2}}}{2c(bc-ad)}+\frac{\sqrt{a+bx}(5bc-7ad)}{cx^{3/2}(c+dx)}}{5c}-\frac{2a\sqrt{a+bx}}{5cx^{5/2}(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow 27$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\frac{\frac{(bc-ad)\int \frac{6b^2c^2-95abdc+105a^2d^2-2bd(27bc-35ad)x}{x^{3/2}\sqrt{a+bx}(c+dx)}dx}{3c}-\frac{2\sqrt{a+bx}(27bc-35ad)(bc-ad)}{3cx^{3/2}}}{2c(bc-ad)}+\frac{\sqrt{a+bx}(5bc-7ad)}{cx^{3/2}(c+dx)}}{5c}-\frac{2a\sqrt{a+bx}}{5cx^{5/2}(c+dx)}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow 169$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\frac{\frac{(bc-ad)\left(-\frac{2\int \frac{15ad(4bc-7ad)(bc-ad)}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{ac}-\frac{2\sqrt{a+bx}\left(\frac{6b^2c}{a}+\frac{105ad^2}{c}-95bd\right)}{\sqrt{x}}\right)}{3c}-\frac{2\sqrt{a+bx}(27bc-35ad)(bc-ad)}{3cx^{3/2}}}{2c(bc-ad)}+\frac{\sqrt{a+bx}(5bc-7ad)}{cx^{3/2}(c+dx)}}{5c}-\cdots\right.}{x^{3/2}(a+bx)^{3/2}}$$

$\big\downarrow$ 27

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\frac{\frac{(bc-ad)\left(-\frac{15d(4bc-7ad)(bc-ad)\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{c}-\frac{2\sqrt{a+bx}\left(\frac{6b^2c}{a}+\frac{105ad^2}{c}-95bd\right)}{\sqrt{x}}\right)}{3c}-\frac{2\sqrt{a+bx}(27bc-35ad)(bc-ad)}{3cx^{3/2}}}{2c(bc-ad)}+\frac{\sqrt{a+bx}(\cdots}{cx^{3/2}\cdots}\cdots\right.}{x^{3/2}(a+bx)^{3/2}}$$

$\big\downarrow$ 104

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\frac{\frac{(bc-ad)\left(-\frac{30d(4bc-7ad)(bc-ad)\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}-\frac{2\sqrt{a+bx}\left(\frac{6b^2c}{a}+\frac{105ad^2}{c}-95bd\right)}{\sqrt{x}}\right)}{3c}-\frac{2\sqrt{a+bx}(27bc-35ad)(bc-ad)}{3cx^{3/2}}}{2c(bc-ad)}+\frac{\sqrt{a+b\cdots}}{cx^{3\cdots}}\cdots\right.}{x^{3/2}(a+bx)^{3/2}}$$

$\big\downarrow$ 221

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\frac{\frac{(bc-ad)\left(-\frac{30d(4bc-7ad)\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}}-\frac{2\sqrt{a+bx}\left(\frac{6b^2c}{a}+\frac{105ad^2}{c}-95bd\right)}{\sqrt{x}}\right)}{3c}-\frac{2\sqrt{a+bx}(27bc-35ad)(bc-ad)}{3cx^{3/2}}}{2c(bc-ad)}+\frac{\sqrt{a+\cdots}}{c\cdots}\cdots\right.}{x^{3/2}(a+bx)^{3/2}}$$

input
```
Int[(a*x + b*x^2)^(3/2)/(x^5*(c + d*x)^2),x]
```

output
```
((a*x + b*x^2)^(3/2)*((-2*a*Sqrt[a + b*x])/(5*c*x^(5/2)*(c + d*x)) + (((5*
b*c - 7*a*d)*Sqrt[a + b*x])/(c*x^(3/2)*(c + d*x)) + ((-2*(27*b*c - 35*a*d)
*(b*c - a*d)*Sqrt[a + b*x])/(3*c*x^(3/2)) + ((b*c - a*d)*((-2*((6*b^2*c)/a
 - 95*b*d + (105*a*d^2)/c)*Sqrt[a + b*x])/Sqrt[x] - (30*d*(4*b*c - 7*a*d)*
Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])]
)/c^(3/2)))/(3*c))/(2*c*(b*c - a*d)))/(5*c)))/(x^(3/2)*(a + b*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$-\frac{2\left((bx+a)^2c^3-\frac{7dx\left(-\frac{3}{7}b^2x^2+\frac{34}{7}abx+a^2\right)c^2}{3}+\frac{35d^2x^2\left(-\frac{19bx}{14}+a\right)ac}{3}+\frac{35a^2d^3x^3}{2}\right)\sqrt{c(ad-bc)}\,\sqrt{x(bx+a)}-35(dx+c)dx^3(a}{5\sqrt{c(ad-bc)}\,c^4x^3(dx+c)a}$
risch	$-\frac{2(bx+a)\left(45a^2d^2x^2-40abcd\,x^2+3b^2c^2x^2-10a^2cdx+6ab\,c^2x+3a^2c^2\right)}{15a\,c^4\sqrt{x(bx+a)}\,x^2}-\frac{d\left(-\frac{\left(3a^2d^2-4abcd+b^2c^2\right)\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ac}{}}{}\right)}{}\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a*x)^(3/2)/x^5/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/5/(c*(a*d-b*c))^(1/2)*(2*((b*x+a)^2*c^3-7/3*d*x*(-3/7*b^2*x^2+34/7*a*b*
x+a^2)*c^2+35/3*d^2*x^2*(-19/14*b*x+a)*a*c+35/2*a^2*d^3*x^3)*(c*(a*d-b*c))
^(1/2)*(x*(b*x+a))^(1/2)-35*(d*x+c)*d*x^3*(a*d-b*c)*(a*d-4/7*b*c)*a*arctan
((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2)))/c^4/x^3/(d*x+c)/a
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 468, normalized size of antiderivative = 2.19

$$\int \frac{(ax+bx^2)^{3/2}}{x^5(c+dx)^2}\,dx = \left[-\frac{15\left((4\,abcd^2-7\,a^2d^3)x^4+(4\,abc^2d-7\,a^2cd^2)x^3\right)\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)x+2\,\sqrt{bx^2+}}{dx+c}\right.}{}\right.$$

$$-\frac{15\left((4\,abcd^2-7\,a^2d^3)x^4+(4\,abc^2d-7\,a^2cd^2)x^3\right)\sqrt{-\frac{bc-ad}{c}}\arctan\left(-\frac{\sqrt{bx^2+ax}c\sqrt{-\frac{bc-ad}{c}}}{(bc-ad)x}\right)+(6\,a^2c^3+(6\,b^2}{15\,(ac^4dx^4-}$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^5/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[-1/30*(15*((4*a*b*c*d^2 - 7*a^2*d^3)*x^4 + (4*a*b*c^2*d - 7*a^2*c*d^2)*x^
3)*sqrt((b*c - a*d)/c)*log((a*c + (2*b*c - a*d)*x + 2*sqrt(b*x^2 + a*x)*c*
sqrt((b*c - a*d)/c))/(d*x + c)) + 2*(6*a^2*c^3 + (6*b^2*c^2*d - 95*a*b*c*d
^2 + 105*a^2*d^3)*x^3 + 2*(3*b^2*c^3 - 34*a*b*c^2*d + 35*a^2*c*d^2)*x^2 +
2*(6*a*b*c^3 - 7*a^2*c^2*d)*x)*sqrt(b*x^2 + a*x))/(a*c^4*d*x^4 + a*c^5*x^3
), -1/15*(15*((4*a*b*c*d^2 - 7*a^2*d^3)*x^4 + (4*a*b*c^2*d - 7*a^2*c*d^2)*
x^3)*sqrt(-(b*c - a*d)/c)*arctan(-sqrt(b*x^2 + a*x)*c*sqrt(-(b*c - a*d)/c)
/((b*c - a*d)*x)) + (6*a^2*c^3 + (6*b^2*c^2*d - 95*a*b*c*d^2 + 105*a^2*d^3
)*x^3 + 2*(3*b^2*c^3 - 34*a*b*c^2*d + 35*a^2*c*d^2)*x^2 + 2*(6*a*b*c^3 - 7
*a^2*c^2*d)*x)*sqrt(b*x^2 + a*x))/(a*c^4*d*x^4 + a*c^5*x^3)]
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x^5(c+dx)^2}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}}{x^5\,(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a*x)**(3/2)/x**5/(d*x+c)**2,x)
```

output
```
Integral((x*(a + b*x))**(3/2)/(x**5*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x^5(c+dx)^2}\,dx = \int \frac{(bx^2+ax)^{\frac{3}{2}}}{(dx+c)^2x^5}\,dx$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^5/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(3/2)/((d*x + c)^2*x^5), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ax+bx^2)^{3/2}}{x^5(c+dx)^2}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^5/(d*x+c)^2,x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: 2796
51166 icas_eval sage2279.664 NTL factor begin279.665 NTL factor end279.665
 NTL factor begin279.665 NTL factor endPsr 9.14286, Mod 191.109, Heu 11764
.9, Min9.14286GCD d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{x^5(c+dx)^2}\,dx = \int \frac{(b\,x^2+a\,x)^{3/2}}{x^5\,(c+d\,x)^2}\,dx$$

input
```
int((a*x + b*x^2)^(3/2)/(x^5*(c + d*x)^2),x)
```

output
```
int((a*x + b*x^2)^(3/2)/(x^5*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 749, normalized size of antiderivative = 3.50

$$\int \frac{(ax+bx^2)^{3/2}}{x^5(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a*x)^(3/2)/x^5/(d*x+c)^2,x)
```

output

```
(105*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x)
 - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*c*d**2*x**3 + 105*sqrt
(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x
)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*d**3*x**4 - 60*sqrt(c)*sqrt(a*d
 - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sq
rt(b))/(sqrt(c)*sqrt(b)))*a*b*c**2*d*x**3 - 60*sqrt(c)*sqrt(a*d - b*c)*ata
n((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqr
t(c)*sqrt(b)))*a*b*c*d**2*x**4 + 105*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*
d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(
b)))*a**2*c*d**2*x**3 + 105*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c)
+ sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2
*d**3*x**4 - 60*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sq
rt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b*c**2*d*x**3
- 60*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x)
 + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b*c*d**2*x**4 - 6*sqrt(x)
*sqrt(a + b*x)*a**2*c**4 + 14*sqrt(x)*sqrt(a + b*x)*a**2*c**3*d*x - 70*sqr
t(x)*sqrt(a + b*x)*a**2*c**2*d**2*x**2 - 105*sqrt(x)*sqrt(a + b*x)*a**2*c*
d**3*x**3 - 12*sqrt(x)*sqrt(a + b*x)*a*b*c**4*x + 68*sqrt(x)*sqrt(a + b*x)
*a*b*c**3*d*x**2 + 95*sqrt(x)*sqrt(a + b*x)*a*b*c**2*d**2*x**3 - 6*sqrt(x)
*sqrt(a + b*x)*b**2*c**4*x**2 - 6*sqrt(x)*sqrt(a + b*x)*b**2*c**3*d*x**...
```

3.108 $\int \frac{x^2\left(ax+bx^2\right)^{3/2}}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 332

$$\begin{aligned}\int \frac{x^2(ax+bx^2)^{3/2}}{(c+dx)^3}\,dx &= \frac{(80b^2c^2-52abcd+a^2d^2)\sqrt{ax+bx^2}}{8bd^5} \\ &- \frac{(30bc-17ad)x\sqrt{ax+bx^2}}{6d^4} + \frac{(40bc-21ad)x^2\sqrt{ax+bx^2}}{12cd^3} \\ &- \frac{(10bc-7ad)x^3\sqrt{ax+bx^2}}{4cd^2(c+dx)} - \frac{x^2(ax+bx^2)^{3/2}}{2d(c+dx)^2} \\ &- \frac{(160b^3c^3-144ab^2c^2d+18a^2bcd^2+a^3d^3)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{8b^{3/2}d^6} \\ &+ \frac{c^{3/2}(80b^2c^2-112abcd+35a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{4d^6\sqrt{bc-ad}}\end{aligned}$$

output

```
1/8*(a^2*d^2-52*a*b*c*d+80*b^2*c^2)*(b*x^2+a*x)^(1/2)/b/d^5-1/6*(-17*a*d+3
0*b*c)*x*(b*x^2+a*x)^(1/2)/d^4+1/12*(-21*a*d+40*b*c)*x^2*(b*x^2+a*x)^(1/2)
/c/d^3-1/4*(-7*a*d+10*b*c)*x^3*(b*x^2+a*x)^(1/2)/c/d^2/(d*x+c)-1/2*x^2*(b*
x^2+a*x)^(3/2)/d/(d*x+c)^2-1/8*(a^3*d^3+18*a^2*b*c*d^2-144*a*b^2*c^2*d+160
*b^3*c^3)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(3/2)/d^6+1/4*c^(3/2)*(35
*a^2*d^2-112*a*b*c*d+80*b^2*c^2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2
+a*x)^(1/2))/d^6/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 10.95 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.14

$$\int \frac{x^2(ax+bx^2)^{3/2}}{(c+dx)^3}\,dx =$$
$$-\frac{\sqrt{x}\left(3(-160b^4c^4+304ab^3c^3d-162a^2b^2c^2d^2+17a^3bcd^3+a^4d^4)(a+bx)(c+dx)^2\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)+\sqrt{a}\sqrt{}\right.}{}$$

input
```
Integrate[(x^2*(a*x + b*x^2)^(3/2))/(c + d*x)^3,x]
```

output
```
-1/24*(Sqrt[x]*(3*(-160*b^4*c^4 + 304*a*b^3*c^3*d - 162*a^2*b^2*c^2*d^2 +
17*a^3*b*c*d^3 + a^4*d^4)*(a + b*x)*(c + d*x)^2*ArcSinh[(Sqrt[b]*Sqrt[x])/
Sqrt[a]] + Sqrt[a]*Sqrt[b]*Sqrt[1 + (b*x)/a]*(d*(b*c - a*d)*Sqrt[x]*(a + b
*x)*(3*a^2*d^2*(c + d*x)^2 - 2*a*b*d*(78*c^3 + 122*c^2*d*x + 31*c*d^2*x^2
- 7*d^3*x^3) + 4*b^2*(60*c^4 + 90*c^3*d*x + 20*c^2*d^2*x^2 - 5*c*d^3*x^3 +
 2*d^4*x^4)) + 6*b*c^(3/2)*Sqrt[b*c - a*d]*(80*b^2*c^2 - 112*a*b*c*d + 35*
a^2*d^2)*Sqrt[a + b*x]*(c + d*x)^2*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt
[c]*Sqrt[a + b*x])])))/(Sqrt[a]*b^(3/2)*d^6*(-(b*c) + a*d)*Sqrt[x*(a + b*x
)]*Sqrt[1 + (b*x)/a]*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.29, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.708$, Rules used = {1261, 108, 27, 166, 27, 171, 27, 171, 27, 171, 27, 27, 175, 65, 104, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(ax+bx^2\right)^{3/2}}{(c+dx)^3}dx$$
$$\Big\downarrow \ 1261$$
$$\frac{\left(ax+bx^2\right)^{3/2}\int \frac{x^{7/2}(a+bx)^{3/2}}{(c+dx)^3}dx}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \ 108$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\int \frac{x^{5/2}\sqrt{a+bx}(7a+10bx)}{2(c+dx)^2}dx}{2d}-\frac{x^{7/2}(a+bx)^{3/2}}{2d(c+dx)^2}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\int \frac{x^{5/2}\sqrt{a+bx}(7a+10bx)}{(c+dx)^2}dx}{4d}-\frac{x^{7/2}(a+bx)^{3/2}}{2d(c+dx)^2}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \ 166$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{-\frac{\int -\frac{x^{3/2}\sqrt{a+bx}(5a(10bc-7ad)+2b(40bc-31ad)x)}{2(c+dx)}dx}{d(bc-ad)}-\frac{x^{5/2}(a+bx)^{3/2}(10bc-7ad)}{d(c+dx)(bc-ad)}}{4d}-\frac{x^{7/2}(a+bx)^{3/2}}{2d(c+dx)^2}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\frac{\int \frac{x^{3/2}\sqrt{a+bx}(5a(10bc-7ad)+2b(40bc-31ad)x)}{c+dx}dx}{2d(bc-ad)}-\frac{x^{5/2}(a+bx)^{3/2}(10bc-7ad)}{d(c+dx)(bc-ad)}}{4d}-\frac{x^{7/2}(a+bx)^{3/2}}{2d(c+dx)^2}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \ 171$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\frac{\frac{\int -\frac{3b\sqrt{x}\sqrt{a+bx}\left(ac(40bc-31ad)+4\left(20b^2c^2-18abdc+a^2d^2\right)x\right)}{c+dx}dx}{3bd}+\frac{2x^{3/2}(a+bx)^{3/2}(40bc-31ad)}{3d}}{2d(bc-ad)}-\frac{x^{5/2}(a+bx)^{3/2}(10bc-7ad)}{d(c+dx)(bc-ad)}}{4d}-\frac{x^{7/\cdots}}{2\cdots}\right.}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\frac{\frac{2x^{3/2}(a+bx)^{3/2}(40bc-31ad)}{3d}-\frac{\int \frac{\sqrt{x}\sqrt{a+bx}\left(ac(40bc-31ad)+4\left(20b^2c^2-18abdc+a^2d^2\right)x\right)}{c+dx}dx}{d}}{2d(bc-ad)}-\frac{x^{5/2}(a+bx)^{3/2}(10bc-7ad)}{d(c+dx)(bc-ad)}}{4d}-\frac{x^{7/2}(a\cdots}{2d(c\cdots}\right.}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \ 171$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\frac{\frac{2x^{3/2}(a+bx)^{3/2}(40bc-31ad)}{3d}-\frac{\frac{\int -\frac{2\sqrt{a+bx}\left(ac\left(20b^2c^2-18abdc+a^2d^2\right)+\left(80b^3c^3-92ab^2dc^2+17a^2bd^2c+a^3d^3\right)x\right)}{\sqrt{x}(c+dx)}dx}{2bd}-2\sqrt{x}(a+bx)^{3/2}\left(\cdots\right.}{d}}{2d(bc-ad)}}{4d}\cdots\right.}{x^{3/2}(a+bx)^{3/2}\cdots}$$

$\big\downarrow$ 27

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\frac{\frac{2x^{3/2}(a+bx)^{3/2}(40bc-31ad)}{3d}-\frac{-\frac{\int \frac{\sqrt{a+bx}\left(ac\left(20b^2c^2-18abdc+a^2d^2\right)+\left(80b^3c^3-92ab^2dc^2+17a^2bd^2c+a^3d^3\right)x\right)}{\sqrt{x}(c+dx)}dx}{bd}-2\sqrt{x}(a+bx)^{3/2}\left(\cdots\right.}{d}}{2d(bc-ad)}}{4d}\cdots\right.}{x^{3/2}(a+bx)^{3/2}\cdots}$$

$\big\downarrow$ 171

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\frac{\frac{2x^{3/2}(a+bx)^{3/2}(40bc-31ad)}{3d}-\frac{-\frac{\frac{\int -\frac{ac(bc-ad)\left(80b^2c^2-52abdc+a^2d^2\right)+(bc-ad)\left(160b^3c^3-144ab^2dc^2+18a^2bd^2c+a^3d^3\right)x}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}+\frac{\sqrt{x}\sqrt{\cdots}}{\cdots}}{bd}\cdots}{d}}{2d(bc-ad)}}{4d}\cdots\right.}{x^{3/2}(a+bx)^{3/2}\cdots}$$

$\big\downarrow$ 27

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\frac{\frac{2x^{3/2}(a+bx)^{3/2}(40bc-31ad)}{3d}-\frac{-\frac{\frac{\sqrt{x}\sqrt{a+bx}\left(a^3d^3+17a^2bcd^2-92ab^2c^2d+80b^3c^3\right)}{d}-\frac{\int \frac{(bc-ad)\left(ac\left(80b^2c^2-52abdc+a^2d^2\right)+\left(160b^3c^3-\cdots\right.\right.}{\sqrt{x}\sqrt{a+bx}(c+dx)}\cdots}{2d}}{bd}\cdots}{d}}{2d(bc-ad)}}{4d}\cdots\right.}{x^{3/2}(a+bx)^{3/2}\cdots}$$

$\big\downarrow$ 27

$$\left(ax+bx^2\right)^{3/2}\left(\frac{\frac{2x^{3/2}(a+bx)^{3/2}(40bc-31ad)}{3d}-\frac{-\frac{\frac{\sqrt{x}\sqrt{a+bx}\left(a^3d^3+17a^2bcd^2-92ab^2c^2d+80b^3c^3\right)}{d}-\frac{(bc-ad)\int\frac{ac\left(80b^2c^2-52abdc+a^2d^2\right)+\left(160b^3c^3-\cdots\right.}{\sqrt{x}\sqrt{a+bx}(c+\cdots}}{2d}}{bd}}{d}}{2d(bc-ad)}}{4d}\cdots\right)$$

$$\overline{\qquad x^{3/2}(a+bx)^{3/2}\cdots}$$

$\downarrow$ 175

$$\left(ax+bx^2\right)^{3/2}\left(\frac{\frac{2x^{3/2}(a+bx)^{3/2}(40bc-31ad)}{3d}-\frac{-\frac{\frac{\sqrt{x}\sqrt{a+bx}\left(a^3d^3+17a^2bcd^2-92ab^2c^2d+80b^3c^3\right)}{d}-\frac{(bc-ad)\left(\frac{\left(a^3d^3+18a^2bcd^2-144ab^2c^2d+160b^3c^3\right)\cdots}{d}\cdots\right.}{bd}}{\cdots}}{2d(bc-ad)}\cdots\right)$$

$$\overline{\qquad x^{3/\cdots}}$$

$\downarrow$ 65

$$\left(ax+bx^2\right)^{3/2}\left(\frac{\frac{2x^{3/2}(a+bx)^{3/2}(40bc-31ad)}{3d}-\frac{-\frac{\frac{\sqrt{x}\sqrt{a+bx}\left(a^3d^3+17a^2bcd^2-92ab^2c^2d+80b^3c^3\right)}{d}-\frac{(bc-ad)\left(\frac{2\left(a^3d^3+18a^2bcd^2-144ab^2c^2d+160b^3c^3\cdots\right.}{d}\cdots\right.}{bd}}{\cdots}}{2d(bc-ad)}\cdots\right)$$

$$\overline{\qquad x\cdots}$$

$\downarrow$ 104

$$\left(ax+bx^2\right)^{3/2}\left(\frac{\frac{2x^{3/2}(a+bx)^{3/2}(40bc-31ad)}{3d}-\frac{-\frac{\sqrt{x}\sqrt{a+bx}\left(a^3d^3+17a^2bcd^2-92ab^2c^2d+80b^3c^3\right)}{d}-\frac{(bc-ad)\left(\frac{2\left(a^3d^3+18a^2bcd^2-144ab^2c^2d+160b^3c^3\right.}{d}\right.}{bd}}{2d(bc-ad)}}{}\right.$$

$\downarrow$ 219

$$\left(ax+bx^2\right)^{3/2}\left(\frac{\frac{2x^{3/2}(a+bx)^{3/2}(40bc-31ad)}{3d}-\frac{-\frac{\sqrt{x}\sqrt{a+bx}\left(a^3d^3+17a^2bcd^2-92ab^2c^2d+80b^3c^3\right)}{d}-\frac{(bc-ad)\left(\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)\left(a^3d^3+18a^2bcd^2\right.}{\sqrt{b}d}\right.}{bd}}{2d(bc-ad}}{}\right.$$

$\downarrow$ 221

$$\left(ax+bx^2\right)^{3/2}\left(\frac{\frac{2x^{3/2}(a+bx)^{3/2}(40bc-31ad)}{3d}-\frac{-2\sqrt{x}(a+bx)^{3/2}\left(-\frac{a^2d}{b}+18ac-\frac{20bc^2}{d}\right)-\frac{\sqrt{x}\sqrt{a+bx}\left(a^3d^3+17a^2bcd^2-92ab^2c^2d+80b^3c^3\right)}{d}-\frac{(bc-ad)\left(\right.}{}}{2d(bc-}}{}\right.$$

input
```
Int[(x^2*(a*x + b*x^2)^(3/2))/(c + d*x)^3,x]
```

output
```
((a*x + b*x^2)^(3/2)*(-1/2*(x^(7/2)*(a + b*x)^(3/2))/(d*(c + d*x)^2) + (-(
((10*b*c - 7*a*d)*x^(5/2)*(a + b*x)^(3/2))/(d*(b*c - a*d)*(c + d*x))) + ((
2*(40*b*c - 31*a*d)*x^(3/2)*(a + b*x)^(3/2))/(3*d) - (-2*(18*a*c - (20*b*c
^2)/d - (a^2*d)/b)*Sqrt[x]*(a + b*x)^(3/2) - (((80*b^3*c^3 - 92*a*b^2*c^2*
d + 17*a^2*b*c*d^2 + a^3*d^3)*Sqrt[x]*Sqrt[a + b*x])/d - ((b*c - a*d)*((2*
(160*b^3*c^3 - 144*a*b^2*c^2*d + 18*a^2*b*c*d^2 + a^3*d^3)*ArcTanh[(Sqrt[b
]*Sqrt[x])/Sqrt[a + b*x]])/(Sqrt[b]*d) - (4*b*c^(3/2)*(80*b^2*c^2 - 112*a*
b*c*d + 35*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*
x])])/(d*Sqrt[b*c - a*d])))/(2*d))/(b*d))/d)/(2*d*(b*c - a*d)))/(4*d)))/(x
^(3/2)*(a + b*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.98

method	result
pseudoelliptic	$-\frac{4\left(5(dx+c)^2 x\, b^{\frac{5}{2}} a\left(b^2c^2-\frac{7}{5}abcd+\frac{7}{16}a^2d^2\right)c^2 \arctan\left(\frac{\sqrt{x(bx+a)}\, c}{x\sqrt{c(ad-bc)}}\right)+\sqrt{c(ad-bc)}\left(\frac{abx(dx+c)^2\left(a^3d^3+18a^2bc\,d^2-144a\,b^2c\right.}{32}\right.\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int(x^2*(b*x^2+a*x)^(3/2)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-4*(5*(d*x+c)^2*x*b^(5/2)*a*(b^2*c^2-7/5*a*b*c*d+7/16*a^2*d^2)*c^2*arctan(
(x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))+(c*(a*d-b*c))^(1/2)*(1/32*a*b*x
*(d*x+c)^2*(a^3*d^3+18*a^2*b*c*d^2-144*a*b^2*c^2*d+160*b^3*c^3)*arctanh((x
*(b*x+a))^(1/2)/x/b^(1/2))+d*b^(3/2)*((-b^2*c^4+11/16*a*b*c^3*d)*(x*(b*x+a
))^(3/2)-1/32*(x*(b*x+a))^(1/2)*x*((-32*b^3*x+48*a*b^2)*c^4-30*d*b*a*(-71/
15*b*x+a)*c^3+a*d^2*(80/3*b^2*x^2-244/3*a*b*x+a^2)*c^2+2*d^3*x*(-10/3*b^2*
x^2-31/3*a*b*x+a^2)*a*c+(4*b*x+a)*d^4*x^2*a*(2/3*b*x+a)))))/(c*(a*d-b*c))^
(1/2)/a/b^(5/2)/d^6/x/(d*x+c)^2
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 2129, normalized size of antiderivative = 6.41

$$\int \frac{x^2(ax+bx^2)^{3/2}}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x^2+a*x)^(3/2)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
[1/48*(3*(160*b^3*c^5 - 144*a*b^2*c^4*d + 18*a^2*b*c^3*d^2 + a^3*c^2*d^3 +
 (160*b^3*c^3*d^2 - 144*a*b^2*c^2*d^3 + 18*a^2*b*c*d^4 + a^3*d^5)*x^2 + 2*
(160*b^3*c^4*d - 144*a*b^2*c^3*d^2 + 18*a^2*b*c^2*d^3 + a^3*c*d^4)*x)*sqrt
(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 6*(80*b^4*c^5 - 112*a*b
^3*c^4*d + 35*a^2*b^2*c^3*d^2 + (80*b^4*c^3*d^2 - 112*a*b^3*c^2*d^3 + 35*a
^2*b^2*c*d^4)*x^2 + 2*(80*b^4*c^4*d - 112*a*b^3*c^3*d^2 + 35*a^2*b^2*c^2*d
^3)*x)*sqrt(c/(b*c - a*d))*log((a*c + (2*b*c - a*d)*x + 2*sqrt(b*x^2 + a*x
)*(b*c - a*d)*sqrt(c/(b*c - a*d)))/(d*x + c)) + 2*(8*b^3*d^5*x^4 + 240*b^3
*c^4*d - 156*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 - 2*(10*b^3*c*d^4 - 7*a*b^2*d
^5)*x^3 + (80*b^3*c^2*d^3 - 62*a*b^2*c*d^4 + 3*a^2*b*d^5)*x^2 + 2*(180*b^3
*c^3*d^2 - 122*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4)*x)*sqrt(b*x^2 + a*x))/(b^2*d
^8*x^2 + 2*b^2*c*d^7*x + b^2*c^2*d^6), 1/48*(12*(80*b^4*c^5 - 112*a*b^3*c^
4*d + 35*a^2*b^2*c^3*d^2 + (80*b^4*c^3*d^2 - 112*a*b^3*c^2*d^3 + 35*a^2*b^
2*c*d^4)*x^2 + 2*(80*b^4*c^4*d - 112*a*b^3*c^3*d^2 + 35*a^2*b^2*c^2*d^3)*x
)*sqrt(-c/(b*c - a*d))*arctan(-sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(-c/(b*c
- a*d))/(b*c*x + a*c)) + 3*(160*b^3*c^5 - 144*a*b^2*c^4*d + 18*a^2*b*c^3*d
^2 + a^3*c^2*d^3 + (160*b^3*c^3*d^2 - 144*a*b^2*c^2*d^3 + 18*a^2*b*c*d^4 +
 a^3*d^5)*x^2 + 2*(160*b^3*c^4*d - 144*a*b^2*c^3*d^2 + 18*a^2*b*c^2*d^3 +
a^3*c*d^4)*x)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(8*
b^3*d^5*x^4 + 240*b^3*c^4*d - 156*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 - 2*(...
```

Sympy [F]

$$\int \frac{x^2(ax+bx^2)^{3/2}}{(c+dx)^3}\,dx = \int \frac{x^2(x(a+bx))^{\frac{3}{2}}}{(c+dx)^3}\,dx$$

input
```
integrate(x**2*(b*x**2+a*x)**(3/2)/(d*x+c)**3,x)
```

output
```
Integral(x**2*(x*(a + b*x))**(3/2)/(c + d*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(ax+bx^2)^{3/2}}{(c+dx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(b*x^2+a*x)^(3/2)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(292) = 584$.

Time = 0.20 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.84

$$\int \frac{x^2(ax+bx^2)^{3/2}}{(c+dx)^3}\,dx = \frac{1}{24}\sqrt{bx^2+ax}\left(2x\left(\frac{4bx}{d^3} - \frac{18b^3cd^{14} - 7ab^2d^{15}}{b^2d^{18}}\right) + \frac{3(48b^3c^2d^{13} - 30ab^2cd^{14} + c}{b^2d^{18}}\right.$$
$$+ \frac{(80b^2c^4 - 112abc^3d + 35a^2c^2d^2)\arctan\left(-\frac{\left(\sqrt{b}x - \sqrt{bx^2+ax}\right)d + \sqrt{b}c}{\sqrt{-bc^2+acd}}\right)}{4\sqrt{-bc^2+acd}d^6}$$
$$+ \frac{(160b^3c^3 - 144ab^2c^2d + 18a^2bcd^2 + a^3d^3)\log\left(\left|2\left(\sqrt{b}x - \sqrt{bx^2+ax}\right)\sqrt{b} + a\right|\right)}{16b^{\frac{3}{2}}d^6}$$
$$+ \frac{40\left(\sqrt{b}x - \sqrt{bx^2+ax}\right)^3 b^2c^4d - 48\left(\sqrt{b}x - \sqrt{bx^2+ax}\right)^3 abc^3d^2 + 13\left(\sqrt{b}x - \sqrt{bx^2+ax}\right)^3 a^2c^2d^3 + 72\,(}{}$$

input
```
integrate(x^2*(b*x^2+a*x)^(3/2)/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/24*sqrt(b*x^2 + a*x)*(2*x*(4*b*x/d^3 - (18*b^3*c*d^14 - 7*a*b^2*d^15)/(b
^2*d^18)) + 3*(48*b^3*c^2*d^13 - 30*a*b^2*c*d^14 + a^2*b*d^15)/(b^2*d^18))
 + 1/4*(80*b^2*c^4 - 112*a*b*c^3*d + 35*a^2*c^2*d^2)*arctan(-((sqrt(b)*x -
 sqrt(b*x^2 + a*x))*d + sqrt(b)*c)/sqrt(-b*c^2 + a*c*d))/(sqrt(-b*c^2 + a*
c*d)*d^6) + 1/16*(160*b^3*c^3 - 144*a*b^2*c^2*d + 18*a^2*b*c*d^2 + a^3*d^3
)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/(b^(3/2)*d^6) +
1/4*(40*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*b^2*c^4*d - 48*(sqrt(b)*x - sqrt
(b*x^2 + a*x))^3*a*b*c^3*d^2 + 13*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^2*c^
2*d^3 + 72*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*b^(5/2)*c^5 - 64*(sqrt(b)*x -
 sqrt(b*x^2 + a*x))^2*a*b^(3/2)*c^4*d + 7*(sqrt(b)*x - sqrt(b*x^2 + a*x))^
2*a^2*sqrt(b)*c^3*d^2 + 72*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a*b^2*c^5 - 68*
(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^2*b*c^4*d + 11*(sqrt(b)*x - sqrt(b*x^2 +
 a*x))*a^3*c^3*d^2 + 18*a^2*b^(3/2)*c^5 - 13*a^3*sqrt(b)*c^4*d)/(((sqrt(b)
*x - sqrt(b*x^2 + a*x))^2*d + 2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b)*c
+ a*c)^2*d^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(ax+bx^2)^{3/2}}{(c+dx)^3}\,dx = \int \frac{x^2\,(b\,x^2+a\,x)^{3/2}}{(c+d\,x)^3}\,dx$$

input
```
int((x^2*(a*x + b*x^2)^(3/2))/(c + d*x)^3,x)
```

output
```
int((x^2*(a*x + b*x^2)^(3/2))/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{x^2(ax+bx^2)^{3/2}}{(c+dx)^3}\,dx = \int \frac{x^2(b\,x^2+ax)^{\frac{3}{2}}}{(dx+c)^3}dx$$

input
```
int(x^2*(b*x^2+a*x)^(3/2)/(d*x+c)^3,x)
```

output
```
int(x^2*(b*x^2+a*x)^(3/2)/(d*x+c)^3,x)
```

3.109 $\int \frac{x(ax+bx^2)^{3/2}}{(c+dx)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 266

$$\int \frac{x(ax+bx^2)^{3/2}}{(c+dx)^3} dx = -\frac{3(2bc-ad)\sqrt{ax+bx^2}}{d^4} + \frac{(12bc-5ad)x\sqrt{ax+bx^2}}{4cd^3} - \frac{(8bc-5ad)x^2\sqrt{ax+bx^2}}{4cd^2(c+dx)} - \frac{x(ax+bx^2)^{3/2}}{2d(c+dx)^2} + \frac{3(16b^2c^2-12abcd+a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4\sqrt{b}d^5} - \frac{3\sqrt{c}(16b^2c^2-20abcd+5a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{4d^5\sqrt{bc-ad}}$$

output
```
-3*(-a*d+2*b*c)*(b*x^2+a*x)^(1/2)/d^4+1/4*(-5*a*d+12*b*c)*x*(b*x^2+a*x)^(1
/2)/c/d^3-1/4*(-5*a*d+8*b*c)*x^2*(b*x^2+a*x)^(1/2)/c/d^2/(d*x+c)-1/2*x*(b*
x^2+a*x)^(3/2)/d/(d*x+c)^2+3/4*(a^2*d^2-12*a*b*c*d+16*b^2*c^2)*arctanh(b^(
1/2)*x/(b*x^2+a*x)^(1/2))/b^(1/2)/d^5-3/4*c^(1/2)*(5*a^2*d^2-20*a*b*c*d+16
*b^2*c^2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/d^5/(-a*d+
b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 10.94 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.03

$$\int \frac{x(ax+bx^2)^{3/2}}{(c+dx)^3}\,dx = \frac{\sqrt{x(a+bx)}\left(\frac{d(-bc+ad)\sqrt{x}\left(ad\left(12c^2+19cdx+5d^2x^2\right)-2b\left(12c^3+18c^2dx+4cd^2x^2-d^3x^3\right)\right)}{(c+dx)^2}+\frac{3(-16b^3c^3+\cdots}{\cdots}\right.}{4d^5(-bc\cdots}$$

input
```
Integrate[(x*(a*x + b*x^2)^(3/2))/(c + d*x)^3,x]
```

output
```
(Sqrt[x*(a + b*x)]*((d*(-(b*c) + a*d)*Sqrt[x]*(a*d*(12*c^2 + 19*c*d*x + 5*
d^2*x^2) - 2*b*(12*c^3 + 18*c^2*d*x + 4*c*d^2*x^2 - d^3*x^3)))/(c + d*x)^2
 + (3*(-16*b^3*c^3 + 28*a*b^2*c^2*d - 13*a^2*b*c*d^2 + a^3*d^3)*ArcSinh[(S
qrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[1 + (b*x)/a]) + (3*Sqrt[c]
*Sqrt[b*c - a*d]*(16*b^2*c^2 - 20*a*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[b*c -
 a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/Sqrt[a + b*x]))/(4*d^5*(-(b*c) +
a*d)*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1230, 27, 1230, 1269, 1091, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(ax+bx^2\right)^{3/2}}{(c+dx)^3}dx$$

$$\Big\downarrow \ 1230$$

$$\frac{\left(ax+bx^2\right)^{3/2}(2c+dx)}{2d^2(c+dx)^2}-\frac{3\int \frac{2(2ac+(4bc-ad)x)\sqrt{bx^2+ax}}{(c+dx)^2}dx}{8d^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(ax+bx^2\right)^{3/2}(2c+dx)}{2d^2(c+dx)^2}-\frac{3\int \frac{(2ac+(4bc-ad)x)\sqrt{bx^2+ax}}{(c+dx)^2}dx}{4d^2}$$

$$\Big\downarrow \; 1230$$

$$\frac{\left(ax+bx^2\right)^{3/2}(2c+dx)}{2d^2(c+dx)^2}-\frac{3\left(\frac{\sqrt{ax+bx^2}(dx(4bc-ad)+4c(2bc-ad))}{d^2(c+dx)}-\frac{\int \frac{4ac(2bc-ad)-\left(4abcd-(ad-4bc)^2\right)x}{(c+dx)\sqrt{bx^2+ax}}dx}{2d^2}\right)}{4d^2}$$

$$\Big\downarrow \; 1269$$

$$\frac{\left(ax+bx^2\right)^{3/2}(2c+dx)}{2d^2(c+dx)^2}-\frac{3\left(\frac{\sqrt{ax+bx^2}(dx(4bc-ad)+4c(2bc-ad))}{d^2(c+dx)}-\frac{-\frac{c\left(5a^2d^2-20abcd+16b^2c^2\right)\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}-\frac{\left(4abcd-(ad-4bc)^2\right)\int \frac{1}{\sqrt{bx^2+ax}}dx}{d}}{2d^2}\right)}{4d^2}$$

$$\Big\downarrow \; 1091$$

$$\frac{\left(ax+bx^2\right)^{3/2}(2c+dx)}{2d^2(c+dx)^2}-\frac{3\left(\frac{\sqrt{ax+bx^2}(dx(4bc-ad)+4c(2bc-ad))}{d^2(c+dx)}-\frac{-\frac{c\left(5a^2d^2-20abcd+16b^2c^2\right)\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}-\frac{2\left(4abcd-(ad-4bc)^2\right)\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{d}}{2d^2}\right)}{4d^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\left(ax+bx^2\right)^{3/2}(2c+dx)}{2d^2(c+dx)^2}-\frac{3\left(\frac{\sqrt{ax+bx^2}(dx(4bc-ad)+4c(2bc-ad))}{d^2(c+dx)}-\frac{-\frac{c\left(5a^2d^2-20abcd+16b^2c^2\right)\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}-\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)\left(4abcd-(ad-4bc)^2\right)}{\sqrt{b}d}}{2d^2}\right)}{4d^2}$$

$$\Big\downarrow \; 1154$$

$$\frac{\left(ax+bx^2\right)^{3/2}(2c+dx)}{2d^2(c+dx)^2}-\frac{3\left(\frac{\sqrt{ax+bx^2}(dx(4bc-ad)+4c(2bc-ad))}{d^2(c+dx)}-\frac{\frac{2c\left(5a^2d^2-20abcd+16b^2c^2\right)\int \frac{1}{4c(bc-ad)-\frac{(ac+(2bc-ad)x)^2}{bx^2+ax}}d\left(-\frac{ac+(2bc-ad)x}{\sqrt{bx^2+ax}}\right)}{d}-\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right.}{\sqrt{b}}}{2d^2}\right.}{4d^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\left(ax+bx^2\right)^{3/2}(2c+dx)}{2d^2(c+dx)^2}-\frac{3\left(\frac{\sqrt{ax+bx^2}(dx(4bc-ad)+4c(2bc-ad))}{d^2(c+dx)}-\frac{-\frac{\sqrt{c}\left(5a^2d^2-20abcd+16b^2c^2\right)\operatorname{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{d\sqrt{bc-ad}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)\left(4abcd-(ad\right.}{\sqrt{b}d}}{2d^2}\right.}{4d^2}$$

input
```
Int[(x*(a*x + b*x^2)^(3/2))/(c + d*x)^3,x]
```

output
```
((2*c + d*x)*(a*x + b*x^2)^(3/2))/(2*d^2*(c + d*x)^2) - (3*(((4*c*(2*b*c -
 a*d) + d*(4*b*c - a*d)*x)*Sqrt[a*x + b*x^2])/(d^2*(c + d*x)) - ((-2*(4*a*
b*c*d - (-4*b*c + a*d)^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(Sqrt[b]
*d) - (Sqrt[c]*(16*b^2*c^2 - 20*a*b*c*d + 5*a^2*d^2)*ArcTanh[(a*c + (2*b*c
 - a*d)*x)/(2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[a*x + b*x^2])])/(d*Sqrt[b*c - a
*d]))/(2*d^2)))/(4*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$\frac{12(dx+c)^2\left(b^2c^2-\frac{5}{4}abcd+\frac{5}{16}a^2d^2\right)x\sqrt{b}\,ac\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)+3\sqrt{c(ad-bc)}\left(\frac{ax(dx+c)^2\left(a^2d^2-12abcd+16b^2c^2\right)\operatorname{arctanh}\left(\cdots\right.}{4}\cdots\right.}{ax\sqrt{\cdots}}$
risch	Expression too large to display
default	Expression too large to display

input
```
int(x*(b*x^2+a*x)^(3/2)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
3*(4*(d*x+c)^2*(b^2*c^2-5/4*a*b*c*d+5/16*a^2*d^2)*x*b^(1/2)*a*c*arctan((x*
(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))+(c*(a*d-b*c))^(1/2)*(1/4*a*x*(d*x+
c)^2*(a^2*d^2-12*a*b*c*d+16*b^2*c^2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))+
d*((-b*c^3+7/12*a*c^2*d)*(x*(b*x+a))^(3/2)+5/12*(-12/5*(-b*x+a)*b*c^3+d*a*
(-43/5*b*x+a)*c^2+19/5*d^2*x*a*(-8/19*b*x+a)*c+a*d^3*x^2*(2/5*b*x+a))*x*(x
*(b*x+a))^(1/2))*b^(1/2)))/b^(1/2)/(c*(a*d-b*c))^(1/2)/a/d^5/x/(d*x+c)^2
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1635, normalized size of antiderivative = 6.15

$$\int \frac{x(ax+bx^2)^{3/2}}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(b*x^2+a*x)^(3/2)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
[1/8*(3*(16*b^2*c^4 - 12*a*b*c^3*d + a^2*c^2*d^2 + (16*b^2*c^2*d^2 - 12*a*
b*c*d^3 + a^2*d^4)*x^2 + 2*(16*b^2*c^3*d - 12*a*b*c^2*d^2 + a^2*c*d^3)*x)*
sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 3*(16*b^3*c^4 - 20*
a*b^2*c^3*d + 5*a^2*b*c^2*d^2 + (16*b^3*c^2*d^2 - 20*a*b^2*c*d^3 + 5*a^2*b
*d^4)*x^2 + 2*(16*b^3*c^3*d - 20*a*b^2*c^2*d^2 + 5*a^2*b*c*d^3)*x)*sqrt(c/
(b*c - a*d))*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)*(b*c - a*d)*
sqrt(c/(b*c - a*d)))/(d*x + c)) + 2*(2*b^2*d^4*x^3 - 24*b^2*c^3*d + 12*a*b
*c^2*d^2 - (8*b^2*c*d^3 - 5*a*b*d^4)*x^2 - (36*b^2*c^2*d^2 - 19*a*b*c*d^3)
*x)*sqrt(b*x^2 + a*x))/(b*d^7*x^2 + 2*b*c*d^6*x + b*c^2*d^5), -1/8*(6*(16*
b^3*c^4 - 20*a*b^2*c^3*d + 5*a^2*b*c^2*d^2 + (16*b^3*c^2*d^2 - 20*a*b^2*c*
d^3 + 5*a^2*b*d^4)*x^2 + 2*(16*b^3*c^3*d - 20*a*b^2*c^2*d^2 + 5*a^2*b*c*d^
3)*x)*sqrt(-c/(b*c - a*d))*arctan(-sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(-c/(
b*c - a*d))/(b*c*x + a*c)) - 3*(16*b^2*c^4 - 12*a*b*c^3*d + a^2*c^2*d^2 +
(16*b^2*c^2*d^2 - 12*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(16*b^2*c^3*d - 12*a*b*c
^2*d^2 + a^2*c*d^3)*x)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)
) - 2*(2*b^2*d^4*x^3 - 24*b^2*c^3*d + 12*a*b*c^2*d^2 - (8*b^2*c*d^3 - 5*a*
b*d^4)*x^2 - (36*b^2*c^2*d^2 - 19*a*b*c*d^3)*x)*sqrt(b*x^2 + a*x))/(b*d^7*
x^2 + 2*b*c*d^6*x + b*c^2*d^5), -1/8*(6*(16*b^2*c^4 - 12*a*b*c^3*d + a^2*c
^2*d^2 + (16*b^2*c^2*d^2 - 12*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(16*b^2*c^3*d -
 12*a*b*c^2*d^2 + a^2*c*d^3)*x)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(...
```

Sympy [F]

$$\int \frac{x(ax+bx^2)^{3/2}}{(c+dx)^3}\,dx = \int \frac{x(x(a+bx))^{\frac{3}{2}}}{(c+dx)^3}\,dx$$

input
```
integrate(x*(b*x**2+a*x)**(3/2)/(d*x+c)**3,x)
```

output
```
Integral(x*(x*(a + b*x))**(3/2)/(c + d*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(ax+bx^2)^{3/2}}{(c+dx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(b*x^2+a*x)^(3/2)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(232) = 464$.

Time = 0.19 (sec) , antiderivative size = 548, normalized size of antiderivative = 2.06

$$\int \frac{x(ax+bx^2)^{3/2}}{(c+dx)^3}\,dx = \frac{1}{4}\sqrt{bx^2+ax}\left(\frac{2\,bx}{d^3} - \frac{12\,b^2cd^8 - 5\,abd^9}{bd^{12}}\right)$$
$$- \frac{3\,(16\,b^2c^3 - 20\,abc^2d + 5\,a^2cd^2)\arctan\left(-\frac{\left(\sqrt{b}x - \sqrt{bx^2+ax}\right)d + \sqrt{b}c}{\sqrt{-bc^2+acd}}\right)}{4\,\sqrt{-bc^2+acd}d^5}$$
$$- \frac{3\,(16\,b^2c^2 - 12\,abcd + a^2d^2)\log\left(\left|2\left(\sqrt{b}x - \sqrt{bx^2+ax}\right)\sqrt{b} + a\right|\right)}{8\,\sqrt{b}d^5}$$
$$- \frac{32\left(\sqrt{b}x - \sqrt{bx^2+ax}\right)^3 b^2c^3d - 36\left(\sqrt{b}x - \sqrt{bx^2+ax}\right)^3 abc^2d^2 + 9\left(\sqrt{b}x - \sqrt{bx^2+ax}\right)^3 a^2cd^3 + 56\,(\sqrt{}}{}$$

input
```
integrate(x*(b*x^2+a*x)^(3/2)/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/4*sqrt(b*x^2 + a*x)*(2*b*x/d^3 - (12*b^2*c*d^8 - 5*a*b*d^9)/(b*d^12)) -
3/4*(16*b^2*c^3 - 20*a*b*c^2*d + 5*a^2*c*d^2)*arctan(-((sqrt(b)*x - sqrt(b
*x^2 + a*x))*d + sqrt(b)*c)/sqrt(-b*c^2 + a*c*d))/(sqrt(-b*c^2 + a*c*d)*d^
5) - 3/8*(16*b^2*c^2 - 12*a*b*c*d + a^2*d^2)*log(abs(2*(sqrt(b)*x - sqrt(b
*x^2 + a*x))*sqrt(b) + a))/(sqrt(b)*d^5) - 1/4*(32*(sqrt(b)*x - sqrt(b*x^2
 + a*x))^3*b^2*c^3*d - 36*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a*b*c^2*d^2 +
9*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^2*c*d^3 + 56*(sqrt(b)*x - sqrt(b*x^2
 + a*x))^2*b^(5/2)*c^4 - 44*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a*b^(3/2)*c^
3*d + 3*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2*sqrt(b)*c^2*d^2 + 56*(sqrt(b
)*x - sqrt(b*x^2 + a*x))*a*b^2*c^4 - 48*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^
2*b*c^3*d + 7*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^3*c^2*d^2 + 14*a^2*b^(3/2)
*c^4 - 9*a^3*sqrt(b)*c^3*d)/(((sqrt(b)*x - sqrt(b*x^2 + a*x))^2*d + 2*(sqr
t(b)*x - sqrt(b*x^2 + a*x))*sqrt(b)*c + a*c)^2*d^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(ax+bx^2)^{3/2}}{(c+dx)^3}\,dx = \int \frac{x\,(b\,x^2+a\,x)^{3/2}}{(c+d\,x)^3}\,dx$$

input
```
int((x*(a*x + b*x^2)^(3/2))/(c + d*x)^3,x)
```

output
```
int((x*(a*x + b*x^2)^(3/2))/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{x(ax+bx^2)^{3/2}}{(c+dx)^3}\,dx = \int \frac{x(b\,x^2+ax)^{\frac{3}{2}}}{(dx+c)^3}dx$$

input
```
int(x*(b*x^2+a*x)^(3/2)/(d*x+c)^3,x)
```

output
```
int(x*(b*x^2+a*x)^(3/2)/(d*x+c)^3,x)
```

3.110 $\int \frac{(ax+bx^2)^{3/2}}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 219

$$\int \frac{(ax+bx^2)^{3/2}}{(c+dx)^3}\,dx = \frac{3(4bc-ad)\sqrt{ax+bx^2}}{4cd^3} - \frac{3(2bc-ad)x\sqrt{ax+bx^2}}{4cd^2(c+dx)} - \frac{(ax+bx^2)^{3/2}}{2d(c+dx)^2} - \frac{3\sqrt{b}(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{d^4} + \frac{3(8b^2c^2-8abcd+a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{4\sqrt{c}d^4\sqrt{bc-ad}}$$

output
```
3/4*(-a*d+4*b*c)*(b*x^2+a*x)^(1/2)/c/d^3-3/4*(-a*d+2*b*c)*x*(b*x^2+a*x)^(1
/2)/c/d^2/(d*x+c)-1/2*(b*x^2+a*x)^(3/2)/d/(d*x+c)^2-3*b^(1/2)*(-a*d+2*b*c)
*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/d^4+3/4*(a^2*d^2-8*a*b*c*d+8*b^2*c^2
)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(1/2)/d^4/(-a*d+
b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 10.89 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.08

$$\int \frac{(ax+bx^2)^{3/2}}{(c+dx)^3}\,dx = \frac{\sqrt{x(a+bx)}\left(\frac{d(-bc+ad)\sqrt{x}(-ad(3c+5dx)+2b(6c^2+9cdx+2d^2x^2))}{(c+dx)^2}+\frac{12\sqrt{b}(2b^2c^2-3abcd+a^2d^2)\operatorname{arcsinh}}{\sqrt{a}\sqrt{1+\frac{bx}{a}}}\right.}{4d^4(-bc+ad)\sqrt{x}}$$

input
```
Integrate[(a*x + b*x^2)^(3/2)/(c + d*x)^3,x]
```

output
```
(Sqrt[x*(a + b*x)]*((d*(-(b*c) + a*d)*Sqrt[x]*(-(a*d*(3*c + 5*d*x)) + 2*b*
(6*c^2 + 9*c*d*x + 2*d^2*x^2)))/(c + d*x)^2 + (12*Sqrt[b]*(2*b^2*c^2 - 3*a
*b*c*d + a^2*d^2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[1 + (b
*x)/a]) - (3*Sqrt[b*c - a*d]*(8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*ArcTanh[(Sq
rt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(Sqrt[c]*Sqrt[a + b*x])))
/(4*d^4*(-(b*c) + a*d)*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1161, 1230, 1269, 1091, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}}{(c+dx)^3}\,dx$$

$$\Big\downarrow \text{ 1161}$$

$$\frac{3\int \frac{(a+2bx)\sqrt{bx^2+ax}}{(c+dx)^2}dx}{4d}-\frac{(ax+bx^2)^{3/2}}{2d(c+dx)^2}$$

$$\Big\downarrow \text{ 1230}$$

$$\frac{3\left(\frac{\sqrt{ax+bx^2}(-ad+4bc+2bdx)}{d^2(c+dx)}-\frac{\int \frac{a(4bc-ad)+4b(2bc-ad)x}{(c+dx)\sqrt{bx^2+ax}}dx}{2d^2}\right)}{4d}-\frac{(ax+bx^2)^{3/2}}{2d(c+dx)^2}$$

$$\Big\downarrow \; 1269$$

$$\frac{3\left(\frac{\sqrt{ax+bx^2}(-ad+4bc+2bdx)}{d^2(c+dx)}-\frac{\frac{4b(2bc-ad)\int\frac{1}{\sqrt{bx^2+ax}}dx}{d}-\frac{\left(a^2d^2-8abcd+8b^2c^2\right)\int\frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{2d^2}\right)}{4d}-\frac{\left(ax+bx^2\right)^{3/2}}{2d(c+dx)^2}$$

$$\Big\downarrow \; 1091$$

$$\frac{3\left(\frac{\sqrt{ax+bx^2}(-ad+4bc+2bdx)}{d^2(c+dx)}-\frac{\frac{8b(2bc-ad)\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{d}-\frac{\left(a^2d^2-8abcd+8b^2c^2\right)\int\frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{2d^2}\right)}{4d}-\frac{\left(ax+bx^2\right)^{3/2}}{2d(c+dx)^2}$$

$$\Big\downarrow \; 219$$

$$\frac{3\left(\frac{\sqrt{ax+bx^2}(-ad+4bc+2bdx)}{d^2(c+dx)}-\frac{\frac{8\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(2bc-ad)}{d}-\frac{\left(a^2d^2-8abcd+8b^2c^2\right)\int\frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{2d^2}\right)}{4d}-\frac{\left(ax+bx^2\right)^{3/2}}{2d(c+dx)^2}$$

$$\Big\downarrow \; 1154$$

$$\frac{3\left(\frac{\sqrt{ax+bx^2}(-ad+4bc+2bdx)}{d^2(c+dx)}-\frac{\frac{2\left(a^2d^2-8abcd+8b^2c^2\right)\int\frac{1}{4c(bc-ad)-\frac{(ac+(2bc-ad)x)^2}{bx^2+ax}}d\left(-\frac{ac+(2bc-ad)x}{\sqrt{bx^2+ax}}\right)}{d}+\frac{8\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(2bc-ad)}{d}}{2d^2}\right)}{4d}$$

$$\frac{\left(ax+bx^2\right)^{3/2}}{2d(c+dx)^2}$$

$$\Big\downarrow \; 219$$

$$\frac{3\left(\frac{\sqrt{ax+bx^2}(-ad+4bc+2bdx)}{d^2(c+dx)}-\frac{\frac{8\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(2bc-ad)}{d}-\frac{\left(a^2d^2-8abcd+8b^2c^2\right)\text{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{\sqrt{c}d\sqrt{bc-ad}}}{2d^2}\right)}{4d}-\frac{\left(ax+bx^2\right)^{3/2}}{2d(c+dx)^2}$$

input
```
Int[(a*x + b*x^2)^(3/2)/(c + d*x)^3,x]
```

output
```
-1/2*(a*x + b*x^2)^(3/2)/(d*(c + d*x)^2) + (3*(((4*b*c - a*d + 2*b*d*x)*Sq
rt[a*x + b*x^2])/(d^2*(c + d*x)) - ((8*Sqrt[b]*(2*b*c - a*d)*ArcTanh[(Sqrt
[b]*x)/Sqrt[a*x + b*x^2]])/d - ((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*ArcTanh[
(a*c + (2*b*c - a*d)*x)/(2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[a*x + b*x^2])])/(S
qrt[c]*d*Sqrt[b*c - a*d]))/(2*d^2)))/(4*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$-\frac{2\left(3(dx+c)^2\left(b^2c^2-abcd+\frac{1}{8}a^2d^2\right)\sqrt{b}\,xa\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)+\left(-\frac{3abx(dx+c)^2(ad-2bc)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{2}+d\sqrt{b}\left((-\cdots\right.\right.\right.}{\sqrt{b}\,\sqrt{c(ad-bc)}\,a\,d^4x(d\cdots}$
risch	$\frac{x(bx+a)b}{d^3\sqrt{x(bx+a)}}+\frac{-\frac{2\left(a^2d^2-6abcd+6b^2c^2\right)\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(\cdots}{}}}{x+\frac{c}{d}}\right.}{d^2\sqrt{-\frac{c(ad-bc)}{d^2}}}\cdots}{\cdots}$
default	Expression too large to display

input
```
int((b*x^2+a*x)^(3/2)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/b^(1/2)/(c*(a*d-b*c))^(1/2)*(3*(d*x+c)^2*(b^2*c^2-a*b*c*d+1/8*a^2*d^2)*
b^(1/2)*x*a*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))+(-3/2*a*b*x*
(d*x+c)^2*(a*d-2*b*c)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))+d*b^(1/2)*((-b*
c^2+3/8*a*c*d)*(x*(b*x+a))^(3/2)+5/8*x*(-4/5*b*(-2*b*x+a)*c^2-21/5*a*b*c*d
*x+a*d^2*x*(-4/5*b*x+a))*(x*(b*x+a))^(1/2)))*(c*(a*d-b*c))^(1/2))/a/d^4/x/
(d*x+c)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(189) = 378.

Time = 0.15 (sec) , antiderivative size = 1745, normalized size of antiderivative = 7.97

$$\int \frac{(ax+bx^2)^{3/2}}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a*x)^(3/2)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
[-1/8*(12*(2*b^2*c^5 - 3*a*b*c^4*d + a^2*c^3*d^2 + (2*b^2*c^3*d^2 - 3*a*b*
c^2*d^3 + a^2*c*d^4)*x^2 + 2*(2*b^2*c^4*d - 3*a*b*c^3*d^2 + a^2*c^2*d^3)*x
)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 3*(8*b^2*c^4 - 8*
a*b*c^3*d + a^2*c^2*d^2 + (8*b^2*c^2*d^2 - 8*a*b*c*d^3 + a^2*d^4)*x^2 + 2*
(8*b^2*c^3*d - 8*a*b*c^2*d^2 + a^2*c*d^3)*x)*sqrt(b*c^2 - a*c*d)*log((a*c
+ (2*b*c - a*d)*x + 2*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) -
2*(12*b^2*c^4*d - 15*a*b*c^3*d^2 + 3*a^2*c^2*d^3 + 4*(b^2*c^2*d^3 - a*b*c*
d^4)*x^2 + (18*b^2*c^3*d^2 - 23*a*b*c^2*d^3 + 5*a^2*c*d^4)*x)*sqrt(b*x^2 +
 a*x))/(b*c^4*d^4 - a*c^3*d^5 + (b*c^2*d^6 - a*c*d^7)*x^2 + 2*(b*c^3*d^5 -
 a*c^2*d^6)*x), -1/4*(3*(8*b^2*c^4 - 8*a*b*c^3*d + a^2*c^2*d^2 + (8*b^2*c^
2*d^2 - 8*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(8*b^2*c^3*d - 8*a*b*c^2*d^2 + a^2*
c*d^3)*x)*sqrt(-b*c^2 + a*c*d)*arctan(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a*
x)/(b*c*x + a*c)) + 6*(2*b^2*c^5 - 3*a*b*c^4*d + a^2*c^3*d^2 + (2*b^2*c^3*
d^2 - 3*a*b*c^2*d^3 + a^2*c*d^4)*x^2 + 2*(2*b^2*c^4*d - 3*a*b*c^3*d^2 + a^
2*c^2*d^3)*x)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - (12*b
^2*c^4*d - 15*a*b*c^3*d^2 + 3*a^2*c^2*d^3 + 4*(b^2*c^2*d^3 - a*b*c*d^4)*x^
2 + (18*b^2*c^3*d^2 - 23*a*b*c^2*d^3 + 5*a^2*c*d^4)*x)*sqrt(b*x^2 + a*x))/
(b*c^4*d^4 - a*c^3*d^5 + (b*c^2*d^6 - a*c*d^7)*x^2 + 2*(b*c^3*d^5 - a*c^2*
d^6)*x), 1/8*(24*(2*b^2*c^5 - 3*a*b*c^4*d + a^2*c^3*d^2 + (2*b^2*c^3*d^2 -
 3*a*b*c^2*d^3 + a^2*c*d^4)*x^2 + 2*(2*b^2*c^4*d - 3*a*b*c^3*d^2 + a^2*...
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/2}}{(c+dx)^3}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}}{(c+dx)^3}\,dx$$

input
```
integrate((b*x**2+a*x)**(3/2)/(d*x+c)**3,x)
```

output
```
Integral((x*(a + b*x))**(3/2)/(c + d*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ax+bx^2)^{3/2}}{(c+dx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a*x)^(3/2)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(189) = 378$.

Time = 0.18 (sec) , antiderivative size = 497, normalized size of antiderivative = 2.27

$$\int \frac{(ax+bx^2)^{3/2}}{(c+dx)^3}\,dx = \frac{\sqrt{bx^2+ax}\,b}{d^3} + \frac{3\,(8\,b^2c^2-8\,abcd+a^2d^2)\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)d+\sqrt{b}c}{\sqrt{-bc^2+acd}}\right)}{4\,\sqrt{-bc^2+acd}\,d^4} + \frac{3\,(2\,b^2c-abd)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{2\,\sqrt{b}d^4} + \frac{24\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3 b^2c^2d-24\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3 abcd^2+5\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3 a^2d^3+40\left(\sqrt{b}\right.}{}$$

input
```
integrate((b*x^2+a*x)^(3/2)/(d*x+c)^3,x, algorithm="giac")
```

output
```
sqrt(b*x^2 + a*x)*b/d^3 + 3/4*(8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*arctan(-((
sqrt(b)*x - sqrt(b*x^2 + a*x))*d + sqrt(b)*c)/sqrt(-b*c^2 + a*c*d))/(sqrt(
-b*c^2 + a*c*d)*d^4) + 3/2*(2*b^2*c - a*b*d)*log(abs(2*(sqrt(b)*x - sqrt(b
*x^2 + a*x))*sqrt(b) + a))/(sqrt(b)*d^4) + 1/4*(24*(sqrt(b)*x - sqrt(b*x^2
 + a*x))^3*b^2*c^2*d - 24*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a*b*c*d^2 + 5*
(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^2*d^3 + 40*(sqrt(b)*x - sqrt(b*x^2 + a
*x))^2*b^(5/2)*c^3 - 24*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a*b^(3/2)*c^2*d
- (sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2*sqrt(b)*c*d^2 + 40*(sqrt(b)*x - sq
rt(b*x^2 + a*x))*a*b^2*c^3 - 28*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^2*b*c^2*
d + 3*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^3*c*d^2 + 10*a^2*b^(3/2)*c^3 - 5*a
^3*sqrt(b)*c^2*d)/(((sqrt(b)*x - sqrt(b*x^2 + a*x))^2*d + 2*(sqrt(b)*x - s
qrt(b*x^2 + a*x))*sqrt(b)*c + a*c)^2*d^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{(c+dx)^3}\,dx=\int \frac{\left(b\,x^2+a\,x\right)^{3/2}}{\left(c+d\,x\right)^3}\,dx$$

input
```
int((a*x + b*x^2)^(3/2)/(c + d*x)^3,x)
```

output
```
int((a*x + b*x^2)^(3/2)/(c + d*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 2312, normalized size of antiderivative = 10.56

$$\int \frac{(ax+bx^2)^{3/2}}{(c+dx)^3}\,dx=\text{Too large to display}$$

input
```
int((b*x^2+a*x)^(3/2)/(d*x+c)^3,x)
```

output

```
( - 12*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*
x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*c**2*d**3 - 24*sqrt(
c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)
*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*c*d**4*x - 12*sqrt(c)*sqrt(a*d -
 b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt
(b))/(sqrt(c)*sqrt(b)))*a**3*d**5*x**2 + 120*sqrt(c)*sqrt(a*d - b*c)*atan(
(sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(
c)*sqrt(b)))*a**2*b*c**3*d**2 + 240*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d
 - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b
)))*a**2*b*c**2*d**3*x + 120*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c)
 - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**
2*b*c*d**4*x**2 - 288*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt
(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c**
4*d - 576*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a +
 b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c**3*d**2*x - 2
88*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) -
 sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c**2*d**3*x**2 + 192*s
qrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqr
t(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**3*c**5 + 384*sqrt(c)*sqrt(a*d
- b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*...
```

3.111 $\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 179

$$\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)^3}\,dx = -\frac{(4bc-ad)\sqrt{ax+bx^2}}{4cd^2(c+dx)} - \frac{(ax+bx^2)^{3/2}}{2dx(c+dx)^2} + \frac{2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{d^3} - \frac{(8b^2c^2-ad(4bc+ad))\,\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{4c^{3/2}d^3\sqrt{bc-ad}}$$

output
```
-1/4*(-a*d+4*b*c)*(b*x^2+a*x)^(1/2)/c/d^2/(d*x+c)-1/2*(b*x^2+a*x)^(3/2)/d/
x/(d*x+c)^2+2*b^(3/2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/d^3-1/4*(8*b^2*
c^2-a*d*(a*d+4*b*c))*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))
/c^(3/2)/d^3/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 10.79 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.06

$$\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)^3}\,dx = \frac{\sqrt{x(a+bx)}\left(\frac{d\sqrt{x}(ad(-c+dx)-2bc(2c+3dx))}{c(c+dx)^2} + \frac{8b^{3/2}\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1+\frac{bx}{a}}} - \frac{(8b^2c^2-4abcd-a^2d^2)\text{arctan}}{c^{3/2}\sqrt{bc-ad}\sqrt{a-}}\right.}{4d^3\sqrt{x}}$$

input
```
Integrate[(a*x + b*x^2)^(3/2)/(x*(c + d*x)^3),x]
```

output
```
(Sqrt[x*(a + b*x)]*((d*Sqrt[x]*(a*d*(-c + d*x) - 2*b*c*(2*c + 3*d*x)))/(c*
(c + d*x)^2) + (8*b^(3/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqr
t[1 + (b*x)/a]) - ((8*b^2*c^2 - 4*a*b*c*d - a^2*d^2)*ArcTanh[(Sqrt[b*c - a
*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(c^(3/2)*Sqrt[b*c - a*d]*Sqrt[a + b
*x])))/(4*d^3*Sqrt[x])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 681 vs. $2(179) = 358$.

Time = 1.88 (sec) , antiderivative size = 681, normalized size of antiderivative = 3.80, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1260, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)^3}\,dx$$

$$\Big\downarrow \text{ 1260}$$

$$\int \left(-\frac{d(ax+bx^2)^{3/2}}{c^3(c+dx)}+\frac{(ax+bx^2)^{3/2}}{c^3x}-\frac{d(ax+bx^2)^{3/2}}{c^2(c+dx)^2}-\frac{d(ax+bx^2)^{3/2}}{c(c+dx)^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^3\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{8b^{3/2}c^3}-\frac{3\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)\left(a^2d^2-8abcd+8b^2c^2\right)}{4\sqrt{b}c^2d^3}-\frac{3\left(a^2d^2-8abcd+8b^2c^2\right)\operatorname{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{8c^{3/2}d^3\sqrt{bc-ad}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(2bc-ad)\left(-a^2d^2-8abcd+8b^2c^2\right)}{8b^{3/2}c^3d^3}-\frac{\sqrt{ax+bx^2}\left(a^2d^2-2bdx(2bc-ad)-10abcd+8b^2c^2\right)}{8bc^3d^2}-\frac{(bc-ad)^{3/2}\operatorname{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{c^{3/2}d^3}+\frac{3\sqrt{bc-ad}(2bc-ad)\operatorname{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{2c^{3/2}d^3}+\frac{3\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(2bc-ad)}{cd^3}+\frac{a(a+2bx)\sqrt{ax+bx^2}}{8bc^3}+\frac{3\sqrt{ax+bx^2}(-3ad+4bc-2bdx)}{4c^2d^2}+\frac{\left(ax+bx^2\right)^{3/2}}{c^2(c+dx)}-\frac{3\sqrt{ax+bx^2}(-ad+4bc+2bdx)}{4cd^2(c+dx)}+\frac{\left(ax+bx^2\right)^{3/2}}{2c(c+dx)^2}$$

input
```
Int[(a*x + b*x^2)^(3/2)/(x*(c + d*x)^3),x]
```

output
```
(a*(a + 2*b*x)*Sqrt[a*x + b*x^2])/(8*b*c^3) + (3*(4*b*c - 3*a*d - 2*b*d*x)
*Sqrt[a*x + b*x^2])/(4*c^2*d^2) - (3*(4*b*c - a*d + 2*b*d*x)*Sqrt[a*x + b*
x^2])/(4*c*d^2*(c + d*x)) - ((8*b^2*c^2 - 10*a*b*c*d + a^2*d^2 - 2*b*d*(2*
b*c - a*d)*x)*Sqrt[a*x + b*x^2])/(8*b*c^3*d^2) + (a*x + b*x^2)^(3/2)/(2*c*
(c + d*x)^2) + (a*x + b*x^2)^(3/2)/(c^2*(c + d*x)) - (a^3*ArcTanh[(Sqrt[b]
*x)/Sqrt[a*x + b*x^2]])/(8*b^(3/2)*c^3) + (3*Sqrt[b]*(2*b*c - a*d)*ArcTanh
[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(c*d^3) + ((2*b*c - a*d)*(8*b^2*c^2 - 8*a
*b*c*d - a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(8*b^(3/2)*c^3*d
^3) - (3*(8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x +
b*x^2]])/(4*Sqrt[b]*c^2*d^3) - ((b*c - a*d)^(3/2)*ArcTanh[(a*c + (2*b*c -
a*d)*x)/(2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[a*x + b*x^2])])/(c^(3/2)*d^3) + (3
*Sqrt[b*c - a*d]*(2*b*c - a*d)*ArcTanh[(a*c + (2*b*c - a*d)*x)/(2*Sqrt[c]*
Sqrt[b*c - a*d]*Sqrt[a*x + b*x^2])])/(2*c^(3/2)*d^3) - (3*(8*b^2*c^2 - 8*a
*b*c*d + a^2*d^2)*ArcTanh[(a*c + (2*b*c - a*d)*x)/(2*Sqrt[c]*Sqrt[b*c - a*
d]*Sqrt[a*x + b*x^2])])/(8*c^(3/2)*d^3*Sqrt[b*c - a*d])
```

Defintions of rubi rules used

rule 1260
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x + c*x^2)^p
, (d + e*x)^m*(f + g*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ
[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + n + 2*p + 1, 0] && ILtQ[m, 0] && ILtQ
[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.09

method	result
pseudoelliptic	$\frac{-ax(dx+c)^2(a^2d^2+4abcd-8b^2c^2)\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)+\sqrt{c(ad-bc)}\left(8acx\,b^{\frac{3}{2}}(dx+c)^2\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)+d\left((-acd\right.\right.}{4\sqrt{c(ad-bc)}\,a\,d^3x(dx+c)^2c}$
default	Expression too large to display

input
```
int((b*x^2+a*x)^(3/2)/x/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/(c*(a*d-b*c))^(1/2)*(-a*x*(d*x+c)^2*(a^2*d^2+4*a*b*c*d-8*b^2*c^2)*arct
an((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))+(c*(a*d-b*c))^(1/2)*(8*a*c*x
*b^(3/2)*(d*x+c)^2*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))+d*((-a*c*d-4*b*c^2
)*(x*(b*x+a))^(3/2)+x^2*(x*(b*x+a))^(1/2)*(a*d-b*c)*(a*d-4*b*c))))/a/d^3/x
/(d*x+c)^2/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(153) = 306.

Time = 0.15 (sec) , antiderivative size = 1538, normalized size of antiderivative = 8.59

$$\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a*x)^(3/2)/x/(d*x+c)^3,x, algorithm="fricas")
```

output
```
[1/8*(8*(b^2*c^5 - a*b*c^4*d + (b^2*c^3*d^2 - a*b*c^2*d^3)*x^2 + 2*(b^2*c^
4*d - a*b*c^3*d^2)*x)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))
 - (8*b^2*c^4 - 4*a*b*c^3*d - a^2*c^2*d^2 + (8*b^2*c^2*d^2 - 4*a*b*c*d^3 -
 a^2*d^4)*x^2 + 2*(8*b^2*c^3*d - 4*a*b*c^2*d^2 - a^2*c*d^3)*x)*sqrt(b*c^2
- a*c*d)*log((a*c + (2*b*c - a*d)*x + 2*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a
*x))/(d*x + c)) - 2*(4*b^2*c^4*d - 3*a*b*c^3*d^2 - a^2*c^2*d^3 + (6*b^2*c^
3*d^2 - 7*a*b*c^2*d^3 + a^2*c*d^4)*x)*sqrt(b*x^2 + a*x))/(b*c^5*d^3 - a*c^
4*d^4 + (b*c^3*d^5 - a*c^2*d^6)*x^2 + 2*(b*c^4*d^4 - a*c^3*d^5)*x), 1/4*((
8*b^2*c^4 - 4*a*b*c^3*d - a^2*c^2*d^2 + (8*b^2*c^2*d^2 - 4*a*b*c*d^3 - a^2
*d^4)*x^2 + 2*(8*b^2*c^3*d - 4*a*b*c^2*d^2 - a^2*c*d^3)*x)*sqrt(-b*c^2 + a
*c*d)*arctan(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) + 4*(b^
2*c^5 - a*b*c^4*d + (b^2*c^3*d^2 - a*b*c^2*d^3)*x^2 + 2*(b^2*c^4*d - a*b*c
^3*d^2)*x)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - (4*b^2*c
^4*d - 3*a*b*c^3*d^2 - a^2*c^2*d^3 + (6*b^2*c^3*d^2 - 7*a*b*c^2*d^3 + a^2*
c*d^4)*x)*sqrt(b*x^2 + a*x))/(b*c^5*d^3 - a*c^4*d^4 + (b*c^3*d^5 - a*c^2*d
^6)*x^2 + 2*(b*c^4*d^4 - a*c^3*d^5)*x), -1/8*(16*(b^2*c^5 - a*b*c^4*d + (b
^2*c^3*d^2 - a*b*c^2*d^3)*x^2 + 2*(b^2*c^4*d - a*b*c^3*d^2)*x)*sqrt(-b)*ar
ctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (8*b^2*c^4 - 4*a*b*c^3*d - a^
2*c^2*d^2 + (8*b^2*c^2*d^2 - 4*a*b*c*d^3 - a^2*d^4)*x^2 + 2*(8*b^2*c^3*d -
 4*a*b*c^2*d^2 - a^2*c*d^3)*x)*sqrt(b*c^2 - a*c*d)*log((a*c + (2*b*c - ...
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)^3}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}}{x\,(c+dx)^3}\,dx$$

input
```
integrate((b*x**2+a*x)**(3/2)/x/(d*x+c)**3,x)
```

output
```
Integral((x*(a + b*x))**(3/2)/(x*(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)^3}\,dx = \int \frac{(bx^2+ax)^{\frac{3}{2}}}{(dx+c)^3x}\,dx$$

input
```
integrate((b*x^2+a*x)^(3/2)/x/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(3/2)/((d*x + c)^3*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. 2(153) = 306.

Time = 0.17 (sec) , antiderivative size = 475, normalized size of antiderivative = 2.65

$$\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)^3}\,dx = -\frac{b^{\frac{3}{2}}\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{d^3} - \frac{(8\,b^2c^2-4\,abcd-a^2d^2)\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)d+\sqrt{b}c}{\sqrt{-bc^2+acd}}\right)}{4\sqrt{-bc^2+acd}cd^3} - \frac{16\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3b^2c^2d-12\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3abcd^2+\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3a^2d^3+24\left(\sqrt{b}x\right.}{}$$

input
```
integrate((b*x^2+a*x)^(3/2)/x/(d*x+c)^3,x, algorithm="giac")
```

output
```
-b^(3/2)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/d^3 - 1/4
*(8*b^2*c^2 - 4*a*b*c*d - a^2*d^2)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a*x)
)*d + sqrt(b)*c)/sqrt(-b*c^2 + a*c*d))/(sqrt(-b*c^2 + a*c*d)*c*d^3) - 1/4*
(16*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*b^2*c^2*d - 12*(sqrt(b)*x - sqrt(b*x
^2 + a*x))^3*a*b*c*d^2 + (sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^2*d^3 + 24*(s
qrt(b)*x - sqrt(b*x^2 + a*x))^2*b^(5/2)*c^3 - 4*(sqrt(b)*x - sqrt(b*x^2 +
a*x))^2*a*b^(3/2)*c^2*d - 5*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2*sqrt(b)*
c*d^2 + 24*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a*b^2*c^3 - 8*(sqrt(b)*x - sqrt
(b*x^2 + a*x))*a^2*b*c^2*d - (sqrt(b)*x - sqrt(b*x^2 + a*x))*a^3*c*d^2 + 6
*a^2*b^(3/2)*c^3 - a^3*sqrt(b)*c^2*d)/(((sqrt(b)*x - sqrt(b*x^2 + a*x))^2*
d + 2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b)*c + a*c)^2*c*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)^3}\,dx=\int \frac{{\left(b\,x^2+a\,x\right)}^{3/2}}{x\,{\left(c+d\,x\right)}^3}\,dx$$

input
```
int((a*x + b*x^2)^(3/2)/(x*(c + d*x)^3),x)
```

output
```
int((a*x + b*x^2)^(3/2)/(x*(c + d*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 2151, normalized size of antiderivative = 12.02

$$\int \frac{(ax+bx^2)^{3/2}}{x(c+dx)^3}\,dx=\text{Too large to display}$$

input
```
int((b*x^2+a*x)^(3/2)/x/(d*x+c)^3,x)
```

output
```
( - 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x
) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*c**2*d**3 - 4*sqrt(c)
*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*s
qrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*c*d**4*x - 2*sqrt(c)*sqrt(a*d - b*
c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b)
)/(sqrt(c)*sqrt(b)))*a**3*d**5*x**2 - 4*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt
(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sq
rt(b)))*a**2*b*c**3*d**2 - 8*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c)
 - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**
2*b*c**2*d**3*x - 4*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d
)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*b*c*d**
4*x**2 + 32*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a
 + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c**4*d + 64*s
qrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqr
t(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c**3*d**2*x + 32*sqrt(c)*s
qrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqr
t(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c**2*d**3*x**2 - 32*sqrt(c)*sqrt(a
*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*
sqrt(b))/(sqrt(c)*sqrt(b)))*b**3*c**5 - 64*sqrt(c)*sqrt(a*d - b*c)*atan((s
qrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt...
```

3.112 $\int \frac{(ax+bx^2)^{3/2}}{x^2(c+dx)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int \frac{(ax+bx^2)^{3/2}}{x^2(c+dx)^3}\,dx=\frac{3a\sqrt{ax+bx^2}}{4c^2(c+dx)}+\frac{(ax+bx^2)^{3/2}}{2cx(c+dx)^2}+\frac{3a^2\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{4c^{5/2}\sqrt{bc-ad}}$$

output
```
3/4*a*(b*x^2+a*x)^(1/2)/c^2/(d*x+c)+1/2*(b*x^2+a*x)^(3/2)/c/x/(d*x+c)^2+3/
4*a^2*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(5/2)/(-a*d+
b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 10.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.09

$$\int \frac{(ax+bx^2)^{3/2}}{x^2(c+dx)^3}\,dx=\frac{(x(a+bx))^{3/2}\left(\frac{\sqrt{x}(5ac+2bcx+3adx)}{c(a+bx)(c+dx)^2}+\frac{3a^2\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\sqrt{x}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}\sqrt{bc-ad}(a+bx)^{3/2}}\right)}{4cx^{3/2}}$$

input
```
Integrate[(a*x + b*x^2)^(3/2)/(x^2*(c + d*x)^3),x]
```

output
```
((x*(a + b*x))^(3/2)*((Sqrt[x]*(5*a*c + 2*b*c*x + 3*a*d*x))/(c*(a + b*x)*(
c + d*x)^2) + (3*a^2*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b
*x])])/(c^(3/2)*Sqrt[b*c - a*d]*(a + b*x)^(3/2))))/(4*c*x^(3/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1261, 105, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}}{x^2(c+dx)^3}dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{(ax+bx^2)^{3/2}\int \frac{(a+bx)^{3/2}}{\sqrt{x}(c+dx)^3}dx}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{105}$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{3a\int \frac{\sqrt{a+bx}}{\sqrt{x}(c+dx)^2}dx}{4c}+\frac{\sqrt{x}(a+bx)^{3/2}}{2c(c+dx)^2}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{105}$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{3a\left(\frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{2c}+\frac{\sqrt{x}\sqrt{a+bx}}{c(c+dx)}\right)}{4c}+\frac{\sqrt{x}(a+bx)^{3/2}}{2c(c+dx)^2}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{104}$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{3a\left(\frac{a\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}+\frac{\sqrt{x}\sqrt{a+bx}}{c(c+dx)}\right)}{4c}+\frac{\sqrt{x}(a+bx)^{3/2}}{2c(c+dx)^2}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{3a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{x}\sqrt{a+bx}}{c(c+dx)}\right)}{4c}+\frac{\sqrt{x}(a+bx)^{3/2}}{2c(c+dx)^2}\right)}{x^{3/2}(a+bx)^{3/2}}$$

input
```
Int[(a*x + b*x^2)^(3/2)/(x^2*(c + d*x)^3),x]
```

output
```
((a*x + b*x^2)^(3/2)*((Sqrt[x]*(a + b*x)^(3/2))/(2*c*(c + d*x)^2) + (3*a*(
(Sqrt[x]*Sqrt[a + b*x])/(c*(c + d*x)) + (a*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x
])/(Sqrt[c]*Sqrt[a + b*x])])/(c^(3/2)*Sqrt[b*c - a*d])))/(4*c)))/(x^(3/2)*
(a + b*x)^(3/2))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$-\frac{a^{2}\left(-\frac{\sqrt{x(bx+a)}\,(3adx+2cbx+5ac)}{a^{2}(dx+c)^{2}}+\frac{3\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{\sqrt{c(ad-bc)}}\right)}{4c^{2}}$	87
default	Expression too large to display	3328

input
```
int((b*x^2+a*x)^(3/2)/x^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2/c^2*(-(x*(b*x+a))^(1/2)*(3*a*d*x+2*b*c*x+5*a*c)/a^2/(d*x+c)^2+3/(
c*(a*d-b*c))^(1/2)*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(96) = 192$.

Time = 0.10 (sec) , antiderivative size = 401, normalized size of antiderivative = 3.46

$$\int \frac{(ax+bx^2)^{3/2}}{x^2(c+dx)^3}\,dx = \left[\frac{3\,(a^2d^2x^2+2\,a^2cdx+a^2c^2)\sqrt{bc^2-acd}\log\left(\frac{ac+(2\,bc-ad)x+2\,\sqrt{bc^2-acd}\sqrt{bx^2+ax}}{dx+c}\right)+2\,(5\,a}{8\,(bc^6-ac^5d+(bc^4d^2-ac^3d^3)x^2+2\,(bc}\right.$$

$$-\frac{3\,(a^2d^2x^2+2\,a^2cdx+a^2c^2)\sqrt{-bc^2+acd}\arctan\left(\frac{\sqrt{-bc^2+acd}\sqrt{bx^2+ax}}{bcx+ac}\right)-(5\,abc^3-5\,a^2c^2d+(2\,b^2c^3+abc^2d}{4\,(bc^6-ac^5d+(bc^4d^2-ac^3d^3)x^2+2\,(bc^5d-ac^4d^2)x)}$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^2/(d*x+c)^3,x, algorithm="fricas")
```

output
```
[1/8*(3*(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2)*sqrt(b*c^2 - a*c*d)*log((a*c
 + (2*b*c - a*d)*x + 2*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) +
 2*(5*a*b*c^3 - 5*a^2*c^2*d + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x)*sqr
t(b*x^2 + a*x))/(b*c^6 - a*c^5*d + (b*c^4*d^2 - a*c^3*d^3)*x^2 + 2*(b*c^5*
d - a*c^4*d^2)*x), -1/4*(3*(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2)*sqrt(-b*c
^2 + a*c*d)*arctan(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) -
 (5*a*b*c^3 - 5*a^2*c^2*d + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(
b*x^2 + a*x))/(b*c^6 - a*c^5*d + (b*c^4*d^2 - a*c^3*d^3)*x^2 + 2*(b*c^5*d
- a*c^4*d^2)*x)]
```

Sympy [F]

$$\int \frac{(ax + bx^2)^{3/2}}{x^2(c + dx)^3}\,dx = \int \frac{(x(a + bx))^{\frac{3}{2}}}{x^2\,(c + dx)^3}\,dx$$

input
```
integrate((b*x**2+a*x)**(3/2)/x**2/(d*x+c)**3,x)
```

output
```
Integral((x*(a + b*x))**(3/2)/(x**2*(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{(ax + bx^2)^{3/2}}{x^2(c + dx)^3}\,dx = \int \frac{(bx^2 + ax)^{\frac{3}{2}}}{(dx + c)^3 x^2}\,dx$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^2/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(3/2)/((d*x + c)^3*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(96) = 192$.

Time = 0.15 (sec) , antiderivative size = 388, normalized size of antiderivative = 3.34

$$\int \frac{(ax+bx^2)^{3/2}}{x^2(c+dx)^3}\,dx = \frac{3\,a^2\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)d+\sqrt{b}c}{\sqrt{-bc^2+acd}}\right)}{4\sqrt{-bc^2+acd}c^2} + \frac{8\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3 b^2c^2d - 3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3 a^2d^3 + 8\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2 b^{\frac{5}{2}}c^3 + 16\left(\sqrt{b}x-\right.}{}$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^2/(d*x+c)^3,x, algorithm="giac")
```

output
```
3/4*a^2*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a*x))*d + sqrt(b)*c)/sqrt(-b*c^
2 + a*c*d))/(sqrt(-b*c^2 + a*c*d)*c^2) + 1/4*(8*(sqrt(b)*x - sqrt(b*x^2 +
a*x))^3*b^2*c^2*d - 3*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^2*d^3 + 8*(sqrt(
b)*x - sqrt(b*x^2 + a*x))^2*b^(5/2)*c^3 + 16*(sqrt(b)*x - sqrt(b*x^2 + a*x
))^2*a*b^(3/2)*c^2*d - 9*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2*sqrt(b)*c*d
^2 + 8*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a*b^2*c^3 + 12*(sqrt(b)*x - sqrt(b*
x^2 + a*x))*a^2*b*c^2*d - 5*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^3*c*d^2 + 2*
a^2*b^(3/2)*c^3 + 3*a^3*sqrt(b)*c^2*d)/(((sqrt(b)*x - sqrt(b*x^2 + a*x))^2
*d + 2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b)*c + a*c)^2*c^2*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{x^2(c+dx)^3}\,dx = \int \frac{\left(b\,x^2+a\,x\right)^{3/2}}{x^2\,\left(c+d\,x\right)^3}\,dx$$

input
```
int((a*x + b*x^2)^(3/2)/(x^2*(c + d*x)^3),x)
```

output
```
int((a*x + b*x^2)^(3/2)/(x^2*(c + d*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 1056, normalized size of antiderivative = 9.10

$$\int \frac{(ax+bx^2)^{3/2}}{x^2(c+dx)^3}\,dx = \text{Too large to display}$$

input

```
int((b*x^2+a*x)^(3/2)/x^2/(d*x+c)^3,x)
```

output

```
( - 6*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x
) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*c**2*d**2 - 12*sqrt(c
)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*
sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*c*d**3*x - 6*sqrt(c)*sqrt(a*d - b
*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b
))/(sqrt(c)*sqrt(b)))*a**3*d**4*x**2 + 12*sqrt(c)*sqrt(a*d - b*c)*atan((sq
rt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*
sqrt(b)))*a**2*b*c**3*d + 24*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c)
 - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**
2*b*c**2*d**2*x + 12*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(
d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*b*c*d*
*3*x**2 - 6*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a
 + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*c**2*d**2 - 12*
sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sq
rt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*c*d**3*x - 6*sqrt(c)*sqrt(a
*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*
sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*d**4*x**2 + 12*sqrt(c)*sqrt(a*d - b*c)*at
an((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sq
rt(c)*sqrt(b)))*a**2*b*c**3*d + 24*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d
- b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt...
```

3.113 $\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 165

$$\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)^3} \, dx = \frac{(bc-5ad)\sqrt{ax+bx^2}}{2c^2(c+dx)^2} - \frac{2a\sqrt{ax+bx^2}}{cx(c+dx)^2} + \frac{(2bc-15ad)\sqrt{ax+bx^2}}{4c^3(c+dx)} + \frac{3a(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{4c^{7/2}\sqrt{bc-ad}}$$

output
```
1/2*(-5*a*d+b*c)*(b*x^2+a*x)^(1/2)/c^2/(d*x+c)^2-2*a*(b*x^2+a*x)^(1/2)/c/x
/(d*x+c)^2+1/4*(-15*a*d+2*b*c)*(b*x^2+a*x)^(1/2)/c^3/(d*x+c)+3/4*a*(-5*a*d
+4*b*c)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(7/2)/(-a*
d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 10.21 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.05

$$\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)^3} \, dx = \frac{(x(a+bx))^{3/2}\left(-8(a+bx) + \frac{(4bc-5ad)\sqrt{x}\left(\sqrt{c}\sqrt{bc-ad}\sqrt{x}\sqrt{a+bx}(5ac+2bcx+3adx)+3a^2(c+dx)^2\text{ar}\right.}{c^{5/2}\sqrt{bc-ad}(a+bx)^{3/2}}}{4acx^2(c+dx)^2}$$

input
```
Integrate[(a*x + b*x^2)^(3/2)/(x^3*(c + d*x)^3),x]
```

output
```
((x*(a + b*x))^(3/2)*(-8*(a + b*x) + ((4*b*c - 5*a*d)*Sqrt[x]*(Sqrt[c]*Sqr
t[b*c - a*d]*Sqrt[x]*Sqrt[a + b*x]*(5*a*c + 2*b*c*x + 3*a*d*x) + 3*a^2*(c
+ d*x)^2*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])]))/(c^(
5/2)*Sqrt[b*c - a*d]*(a + b*x)^(3/2))))/(4*a*c*x^2*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1261, 107, 105, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{3/2}}{x^3(c+dx)^3}dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{\left(ax+bx^2\right)^{3/2}\int \frac{(a+bx)^{3/2}}{x^{3/2}(c+dx)^3}dx}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{107}$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{(4bc-5ad)\int \frac{(a+bx)^{3/2}}{\sqrt{x}(c+dx)^3}dx}{ac}-\frac{2(a+bx)^{5/2}}{ac\sqrt{x}(c+dx)^2}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{105}$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{(4bc-5ad)\left(\frac{3a\int \frac{\sqrt{a+bx}}{\sqrt{x}(c+dx)^2}dx}{4c}+\frac{\sqrt{x}(a+bx)^{3/2}}{2c(c+dx)^2}\right)}{ac}-\frac{2(a+bx)^{5/2}}{ac\sqrt{x}(c+dx)^2}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{105}$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{(4bc-5ad)\left(\frac{3a\left(\frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{2c}+\frac{\sqrt{x}\sqrt{a+bx}}{c(c+dx)}\right)}{4c}+\frac{\sqrt{x}(a+bx)^{3/2}}{2c(c+dx)^2}\right)}{ac}-\frac{2(a+bx)^{5/2}}{ac\sqrt{x}(c+dx)^2}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$\Big\downarrow$ 104

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{(4bc-5ad)\left(\frac{3a\left(\frac{a\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}+\frac{\sqrt{x}\sqrt{a+bx}}{c(c+dx)}\right)}{4c}+\frac{\sqrt{x}(a+bx)^{3/2}}{2c(c+dx)^2}\right)}{ac}-\frac{2(a+bx)^{5/2}}{ac\sqrt{x}(c+dx)^2}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$\Big\downarrow$ 221

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{(4bc-5ad)\left(\frac{3a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{x}\sqrt{a+bx}}{c(c+dx)}\right)}{4c}+\frac{\sqrt{x}(a+bx)^{3/2}}{2c(c+dx)^2}\right)}{ac}-\frac{2(a+bx)^{5/2}}{ac\sqrt{x}(c+dx)^2}\right)}{x^{3/2}(a+bx)^{3/2}}$$

input
```
Int[(a*x + b*x^2)^(3/2)/(x^3*(c + d*x)^3),x]
```

output
```
((a*x + b*x^2)^(3/2)*((-2*(a + b*x)^(5/2))/(a*c*Sqrt[x]*(c + d*x)^2) + ((4
*b*c - 5*a*d)*((Sqrt[x]*(a + b*x)^(3/2))/(2*c*(c + d*x)^2) + (3*a*((Sqrt[x
]*Sqrt[a + b*x])/(c*(c + d*x)) + (a*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqr
t[c]*Sqrt[a + b*x])])/(c^(3/2)*Sqrt[b*c - a*d])))/(4*c)))/(a*c)))/(x^(3/2)
*(a + b*x)^(3/2))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$-\frac{2\left(-\frac{15(dx+c)^2\left(ad-\frac{4bc}{5}\right)x\,a^2\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{8}+\sqrt{c(ad-bc)}\left(\left(\frac{9}{8}acd-\frac{1}{2}b\,c^2\right)(x(bx+a))^{\frac{3}{2}}+\sqrt{x(bx+a)}\left(\left(\frac{b^2x^2}{2}+a^2\right)c\right.\right.\right.}{\sqrt{c(ad-bc)}\,a\,c^3x(dx+c)^2}$
risch	$-\frac{2a(bx+a)}{c^3\sqrt{x(bx+a)}}-\frac{-\frac{a^2\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{x+\frac{c}{d}}\right)}{\sqrt{-\frac{c(ad-bc)}{d^2}}}+\frac{c\left(a^2d\right.}{}}{}$
default	Expression too large to display

input
```
int((b*x^2+a*x)^(3/2)/x^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/(c*(a*d-b*c))^(1/2)*(-15/8*(d*x+c)^2*(a*d-4/5*b*c)*x*a^2*arctan((x*(b*x
+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))+(c*(a*d-b*c))^(1/2)*((9/8*a*c*d-1/2*b*
c^2)*(x*(b*x+a))^(3/2)+(x*(b*x+a))^(1/2)*((1/2*b^2*x^2+a^2)*c^2+2*d*x*a*(-
11/16*b*x+a)*c+15/8*a^2*d^2*x^2)))/a/c^3/x/(d*x+c)^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 562, normalized size of antiderivative = 3.41

$$\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)^3}\,dx = \left[-\frac{3\left((4\,abcd^2-5\,a^2d^3)x^3+2\,(4\,abc^2d-5\,a^2cd^2)x^2+(4\,abc^3-5\,a^2c^2d)x\right)\sqrt{bc^2-a}}{}\right.$$

$$-\frac{3\left((4\,abcd^2-5\,a^2d^3)x^3+2\,(4\,abc^2d-5\,a^2cd^2)x^2+(4\,abc^3-5\,a^2c^2d)x\right)\sqrt{-bc^2+acd}\arctan\left(\frac{\sqrt{-bc^2+acd}\sqrt{b}}{bcx+ac}\right.}{4\left((bc^5d^2-ac^4d^3)x^3+2\,(b\right.}$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^3/(d*x+c)^3,x, algorithm="fricas")
```

output
```
[-1/8*(3*((4*a*b*c*d^2 - 5*a^2*d^3)*x^3 + 2*(4*a*b*c^2*d - 5*a^2*c*d^2)*x^
2 + (4*a*b*c^3 - 5*a^2*c^2*d)*x)*sqrt(b*c^2 - a*c*d)*log((a*c + (2*b*c - a
*d)*x - 2*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) + 2*(8*a*b*c^4
 - 8*a^2*c^3*d - (2*b^2*c^3*d - 17*a*b*c^2*d^2 + 15*a^2*c*d^3)*x^2 - (4*b^
2*c^4 - 29*a*b*c^3*d + 25*a^2*c^2*d^2)*x)*sqrt(b*x^2 + a*x))/((b*c^5*d^2 -
 a*c^4*d^3)*x^3 + 2*(b*c^6*d - a*c^5*d^2)*x^2 + (b*c^7 - a*c^6*d)*x), -1/4
*(3*((4*a*b*c*d^2 - 5*a^2*d^3)*x^3 + 2*(4*a*b*c^2*d - 5*a^2*c*d^2)*x^2 + (
4*a*b*c^3 - 5*a^2*c^2*d)*x)*sqrt(-b*c^2 + a*c*d)*arctan(sqrt(-b*c^2 + a*c*
d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) + (8*a*b*c^4 - 8*a^2*c^3*d - (2*b^2*c^
3*d - 17*a*b*c^2*d^2 + 15*a^2*c*d^3)*x^2 - (4*b^2*c^4 - 29*a*b*c^3*d + 25*
a^2*c^2*d^2)*x)*sqrt(b*x^2 + a*x))/((b*c^5*d^2 - a*c^4*d^3)*x^3 + 2*(b*c^6
*d - a*c^5*d^2)*x^2 + (b*c^7 - a*c^6*d)*x)]
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)^3}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}}{x^3\,(c+dx)^3}\,dx$$

input
```
integrate((b*x**2+a*x)**(3/2)/x**3/(d*x+c)**3,x)
```

output
```
Integral((x*(a + b*x))**(3/2)/(x**3*(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)^3}\,dx = \int \frac{(bx^2+ax)^{\frac{3}{2}}}{(dx+c)^3x^3}\,dx$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^3/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(3/2)/((d*x + c)^3*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^3/(d*x+c)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)^3}\,dx = \int \frac{(b\,x^2+a\,x)^{3/2}}{x^3\,(c+d\,x)^3}\,dx$$

input
```
int((a*x + b*x^2)^(3/2)/(x^3*(c + d*x)^3),x)
```

output
```
int((a*x + b*x^2)^(3/2)/(x^3*(c + d*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 99.58 (sec) , antiderivative size = 1613, normalized size of antiderivative = 9.78

$$\int \frac{(ax+bx^2)^{3/2}}{x^3(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a*x)^(3/2)/x^3/(d*x+c)^3,x)
```

output
```
(75*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x)
- sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*c**2*d**3*x + 150*sqrt(
c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)
*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*c*d**4*x**2 + 75*sqrt(c)*sqrt(a*
d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*s
qrt(b))/(sqrt(c)*sqrt(b)))*a**3*d**5*x**3 - 180*sqrt(c)*sqrt(a*d - b*c)*at
an((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sq
rt(c)*sqrt(b)))*a**2*b*c**3*d**2*x - 360*sqrt(c)*sqrt(a*d - b*c)*atan((sqr
t(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*s
qrt(b)))*a**2*b*c**2*d**3*x**2 - 180*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*
d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(
b)))*a**2*b*c*d**4*x**3 + 96*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c)
 - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b
**2*c**4*d*x + 192*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)
*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c**3*d
**2*x**2 + 96*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt
(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c**2*d**3*x
**3 + 75*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a +
b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*c**2*d**3*x + 150*
sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) +...
```

3.114 $\int \frac{(ax+bx^2)^{3/2}}{x^4(c+dx)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 219

$$\int \frac{(ax+bx^2)^{3/2}}{x^4(c+dx)^3}\,dx = -\frac{5(10bc-21ad)\sqrt{ax+bx^2}}{12c^4x} - \frac{2a\sqrt{ax+bx^2}}{3cx^2(c+dx)^2} + \frac{(3bc-7ad)\sqrt{ax+bx^2}}{6c^2x(c+dx)^2} + \frac{(12bc-35ad)\sqrt{ax+bx^2}}{12c^3x(c+dx)} + \frac{(8b^2c^2-40abcd+35a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{4c^{9/2}\sqrt{bc-ad}}$$

output

```
-5/12*(-21*a*d+10*b*c)*(b*x^2+a*x)^(1/2)/c^4/x-2/3*a*(b*x^2+a*x)^(1/2)/c/x
^2/(d*x+c)^2+1/6*(-7*a*d+3*b*c)*(b*x^2+a*x)^(1/2)/c^2/x/(d*x+c)^2+1/12*(-3
5*a*d+12*b*c)*(b*x^2+a*x)^(1/2)/c^3/x/(d*x+c)+1/4*(35*a^2*d^2-40*a*b*c*d+8
*b^2*c^2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(9/2)/(-
a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 10.51 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.98

$$\int \frac{(ax+bx^2)^{3/2}}{x^4(c+dx)^3}\,dx = \frac{(x(a+bx))^{3/2}\left(6d(a+bx)+\frac{(c+dx)\left(3c^{5/2}d(6bc-7ad)(a+bx)^{5/2}+(8b^2c^2-40abcd+35a^2d^2)(c+dx)\left(\sqrt{\right.}\right.}{c^{7/2}(bc-ad)}\right.}{12c(-bc+ad)x^3(c+dx)^2}$$

input
```
Integrate[(a*x + b*x^2)^(3/2)/(x^4*(c + d*x)^3),x]
```

output
```
((x*(a + b*x))^(3/2)*(6*d*(a + b*x) + ((c + d*x)*(3*c^(5/2)*d*(6*b*c - 7*a
*d)*(a + b*x)^(5/2) + (8*b^2*c^2 - 40*a*b*c*d + 35*a^2*d^2)*(c + d*x)*(Sqr
t[c]*Sqrt[a + b*x]*(4*b*c*x + a*(c - 3*d*x)) - 3*(b*c - a*d)^(3/2)*x^(3/2)
*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])))/(c^(7/2)*(b
*c - a*d)*(a + b*x)^(3/2))))/(12*c*(-(b*c) + a*d)*x^3*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.29, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1261, 109, 27, 168, 27, 168, 27, 169, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}}{x^4(c+dx)^3}\,dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{(ax+bx^2)^{3/2}\int \frac{(a+bx)^{3/2}}{x^{5/2}(c+dx)^3}dx}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{109}$$

$$\frac{(ax+bx^2)^{3/2}\left(-\frac{2\int -\frac{a(4bc-7ad)+3b(bc-2ad)x}{2x^{3/2}\sqrt{a+bx}(c+dx)^3}dx}{3c}-\frac{2a\sqrt{a+bx}}{3cx^{3/2}(c+dx)^2}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\int \frac{a(4bc-7ad)+3b(bc-2ad)x}{x^{3/2}\sqrt{a+bx}(c+dx)^3}dx}{3c}-\frac{2a\sqrt{a+bx}}{3cx^{3/2}(c+dx)^2}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \; 168$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\frac{\sqrt{a+bx}(3bc-7ad)}{2c\sqrt{x}(c+dx)^2}-\frac{\int -\frac{a(19bc-35ad)(bc-ad)+4b(3bc-7ad)x(bc-ad)}{2x^{3/2}\sqrt{a+bx}(c+dx)^2}dx}{2c(bc-ad)}}{3c}-\frac{2a\sqrt{a+bx}}{3cx^{3/2}(c+dx)^2}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\frac{\int \frac{a(19bc-35ad)(bc-ad)+4b(3bc-7ad)x(bc-ad)}{x^{3/2}\sqrt{a+bx}(c+dx)^2}dx}{4c(bc-ad)}+\frac{\sqrt{a+bx}(3bc-7ad)}{2c\sqrt{x}(c+dx)^2}}{3c}-\frac{2a\sqrt{a+bx}}{3cx^{3/2}(c+dx)^2}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \; 168$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\frac{\frac{\sqrt{a+bx}(12bc-35ad)(bc-ad)}{c\sqrt{x}(c+dx)}-\frac{\int -\frac{(bc-ad)^2(5a(10bc-21ad)+2b(12bc-35ad)x)}{2x^{3/2}\sqrt{a+bx}(c+dx)}dx}{c(bc-ad)}}{4c(bc-ad)}+\frac{\sqrt{a+bx}(3bc-7ad)}{2c\sqrt{x}(c+dx)^2}}{3c}-\frac{2a\sqrt{a+bx}}{3cx^{3/2}(c+dx)^2}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(ax+bx^2\right)^{3/2}\left(\frac{\frac{\frac{(bc-ad)\int \frac{5a(10bc-21ad)+2b(12bc-35ad)x}{x^{3/2}\sqrt{a+bx}(c+dx)}dx}{2c}+\frac{\sqrt{a+bx}(12bc-35ad)(bc-ad)}{c\sqrt{x}(c+dx)}}{4c(bc-ad)}+\frac{\sqrt{a+bx}(3bc-7ad)}{2c\sqrt{x}(c+dx)^2}}{3c}-\frac{2a\sqrt{a+bx}}{3cx^{3/2}(c+dx)^2}\right)}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \; 169$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\frac{\frac{(bc-ad)\left(-\frac{2\int -\frac{3a\left(8b^2c^2-40abdc+35a^2d^2\right)}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{ac}-\frac{10\sqrt{a+bx}(10bc-21ad)}{c\sqrt{x}}\right)}{2c}+\frac{\sqrt{a+bx}(12bc-35ad)(bc-ad)}{c\sqrt{x}(c+dx)}}{4c(bc-ad)}+\frac{\sqrt{a+bx}(3bc-7ad)}{2c\sqrt{x}(c+dx)^2}}{3c}-\frac{2c}{3cx^{3}}\right.}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\frac{\frac{(bc-ad)\left(\frac{3\left(35a^2d^2-40abcd+8b^2c^2\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{c}-\frac{10\sqrt{a+bx}(10bc-21ad)}{c\sqrt{x}}\right)}{2c}+\frac{\sqrt{a+bx}(12bc-35ad)(bc-ad)}{c\sqrt{x}(c+dx)}}{4c(bc-ad)}+\frac{\sqrt{a+bx}(3bc-7ad)}{2c\sqrt{x}(c+dx)^2}}{3c}\right.}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \; 104$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\frac{\frac{(bc-ad)\left(\frac{6\left(35a^2d^2-40abcd+8b^2c^2\right)\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}-\frac{10\sqrt{a+bx}(10bc-21ad)}{c\sqrt{x}}\right)}{2c}+\frac{\sqrt{a+bx}(12bc-35ad)(bc-ad)}{c\sqrt{x}(c+dx)}}{4c(bc-ad)}+\frac{\sqrt{a+bx}(3bc-7}{2c\sqrt{x}(c+dx)}\right.}{x^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{(ax+bx^2)^{3/2}\left(\frac{\frac{\frac{(bc-ad)\left(\frac{6\left(35a^2d^2-40abcd+8b^2c^2\right)\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}\sqrt{bc-ad}}-\frac{10\sqrt{a+bx}(10bc-21ad)}{c\sqrt{x}}\right)}{2c}+\frac{\sqrt{a+bx}(12bc-35ad)(bc-ad)}{c\sqrt{x}(c+dx)}}{4c(bc-ad)}+\frac{\sqrt{a+bx}(3bc-}{2c\sqrt{x}(c+d}\right.}{x^{3/2}(a+bx)^{3/2}}$$

input
```
Int[(a*x + b*x^2)^(3/2)/(x^4*(c + d*x)^3),x]
```

output
```
((a*x + b*x^2)^(3/2)*((-2*a*Sqrt[a + b*x])/(3*c*x^(3/2)*(c + d*x)^2) + (((
3*b*c - 7*a*d)*Sqrt[a + b*x])/(2*c*Sqrt[x]*(c + d*x)^2) + (((12*b*c - 35*a
*d)*(b*c - a*d)*Sqrt[a + b*x])/(c*Sqrt[x]*(c + d*x)) + ((b*c - a*d)*((-10*
(10*b*c - 21*a*d)*Sqrt[a + b*x])/(c*Sqrt[x]) + (6*(8*b^2*c^2 - 40*a*b*c*d
+ 35*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/
(c^(3/2)*Sqrt[b*c - a*d])))/(2*c))/(4*c*(b*c - a*d)))/(3*c)))/(x^(3/2)*(a
+ b*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$-\frac{2\left(\frac{105(dx+c)^2\left(a^2d^2-\frac{8}{7}abcd+\frac{8}{35}b^2c^2\right)x^2a\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{8}+\sqrt{c(ad-bc)}\left(3x\left(-\frac{13}{8}ad^2c+bc^2d\right)(x(bx+a))^{\frac{3}{2}}+\sqrt{x(bx\cdots}\right.\right.}{3\sqrt{c(ad-bc)}\,c^4x^2(dx+c)^2a}$
risch	$-\frac{2(bx+a)(-9adx+4cbx+ac)}{3c^4\sqrt{x(bx+a)}\,x}+\frac{c^2\left(a^2d^2-2abcd+b^2c^2\right)\left(\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{2c(ad-bc)\left(x+\frac{c}{d}\right)^2}+\frac{3(ad-2bc)d\left(\frac{d^2\sqrt{b\left(x\cdots}}{\cdots}\right.}{\cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int((b*x^2+a*x)^(3/2)/x^4/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3/(c*(a*d-b*c))^(1/2)*(105/8*(d*x+c)^2*(a^2*d^2-8/7*a*b*c*d+8/35*b^2*c^
2)*x^2*a*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))+(c*(a*d-b*c))^(
1/2)*(3*x*(-13/8*a*d^2*c+b*c^2*d)*(x*(b*x+a))^(3/2)+(x*(b*x+a))^(1/2)*(a*(
4*b*x+a)*c^3-7*d*(3/7*b^2*x^2-8/7*a*b*x+a^2)*x*c^2-17*d^2*x^2*(-89/136*b*x
+a)*a*c-105/8*a^2*d^3*x^3)))/c^4/x^2/(d*x+c)^2/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 710, normalized size of antiderivative = 3.24

$$\int \frac{(ax+bx^2)^{3/2}}{x^4(c+dx)^3}\,dx = \left[\frac{3\left((8b^2c^2d^2-40abcd^3+35a^2d^4)x^4+2(8b^2c^3d-40abc^2d^2+35a^2cd^3)x^3+(8b^2c\cdots\right.}{\cdots}\right.$$

$$-\frac{3\left((8b^2c^2d^2-40abcd^3+35a^2d^4)x^4+2(8b^2c^3d-40abc^2d^2+35a^2cd^3)x^3+(8b^2c^4-40abc^3d+35a^2c^2\cdots\right.}{\cdots}$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^4/(d*x+c)^3,x, algorithm="fricas")
```

output
```
[1/24*(3*((8*b^2*c^2*d^2 - 40*a*b*c*d^3 + 35*a^2*d^4)*x^4 + 2*(8*b^2*c^3*d
 - 40*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^3 + (8*b^2*c^4 - 40*a*b*c^3*d + 35*a^2
*c^2*d^2)*x^2)*sqrt(b*c^2 - a*c*d)*log((a*c + (2*b*c - a*d)*x + 2*sqrt(b*c
^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) - 2*(8*a*b*c^5 - 8*a^2*c^4*d + 5
*(10*b^2*c^3*d^2 - 31*a*b*c^2*d^3 + 21*a^2*c*d^4)*x^3 + (88*b^2*c^4*d - 26
3*a*b*c^3*d^2 + 175*a^2*c^2*d^3)*x^2 + 8*(4*b^2*c^5 - 11*a*b*c^4*d + 7*a^2
*c^3*d^2)*x)*sqrt(b*x^2 + a*x))/((b*c^6*d^2 - a*c^5*d^3)*x^4 + 2*(b*c^7*d
- a*c^6*d^2)*x^3 + (b*c^8 - a*c^7*d)*x^2), -1/12*(3*((8*b^2*c^2*d^2 - 40*a
*b*c*d^3 + 35*a^2*d^4)*x^4 + 2*(8*b^2*c^3*d - 40*a*b*c^2*d^2 + 35*a^2*c*d^
3)*x^3 + (8*b^2*c^4 - 40*a*b*c^3*d + 35*a^2*c^2*d^2)*x^2)*sqrt(-b*c^2 + a*
c*d)*arctan(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) + (8*a*b
*c^5 - 8*a^2*c^4*d + 5*(10*b^2*c^3*d^2 - 31*a*b*c^2*d^3 + 21*a^2*c*d^4)*x^
3 + (88*b^2*c^4*d - 263*a*b*c^3*d^2 + 175*a^2*c^2*d^3)*x^2 + 8*(4*b^2*c^5
- 11*a*b*c^4*d + 7*a^2*c^3*d^2)*x)*sqrt(b*x^2 + a*x))/((b*c^6*d^2 - a*c^5*
d^3)*x^4 + 2*(b*c^7*d - a*c^6*d^2)*x^3 + (b*c^8 - a*c^7*d)*x^2)]
```

Sympy [F]

$$\int \frac{(ax + bx^2)^{3/2}}{x^4(c + dx)^3} \, dx = \int \frac{(x(a + bx))^{\frac{3}{2}}}{x^4 (c + dx)^3} \, dx$$

input
```
integrate((b*x**2+a*x)**(3/2)/x**4/(d*x+c)**3,x)
```

output
```
Integral((x*(a + b*x))**(3/2)/(x**4*(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x^4(c+dx)^3}\,dx = \int \frac{(bx^2+ax)^{\frac{3}{2}}}{(dx+c)^3x^4}\,dx$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^4/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(3/2)/((d*x + c)^3*x^4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ax+bx^2)^{3/2}}{x^4(c+dx)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a*x)^(3/2)/x^4/(d*x+c)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{x^4(c+dx)^3}\,dx = \int \frac{(b\,x^2+a\,x)^{3/2}}{x^4\,(c+d\,x)^3}\,dx$$

input
```
int((a*x + b*x^2)^(3/2)/(x^4*(c + d*x)^3),x)
```

output
```
int((a*x + b*x^2)^(3/2)/(x^4*(c + d*x)^3), x)
```

Reduce [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x^4(c+dx)^3}\,dx = \int \frac{(b\,x^2+ax)^{\frac{3}{2}}}{x^4\,(dx+c)^3}dx$$

input
```
int((b*x^2+a*x)^(3/2)/x^4/(d*x+c)^3,x)
```

output
```
int((b*x^2+a*x)^(3/2)/x^4/(d*x+c)^3,x)
```

3.115 $\int \frac{x\left(ax+bx^2\right)^{5/2}}{c+dx}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 493

$$\begin{aligned}
\int \frac{x(ax+bx^2)^{5/2}}{c+dx}\,dx =
&-\frac{(512b^5c^5-1152ab^4c^4d+704a^2b^3c^3d^2-40a^3b^2c^2d^3-12a^4bcd^4-5a^5d^5)\sqrt{ax+bx^2}}{512b^3d^6} \\
&+\frac{(384b^4c^4-832ab^3c^3d+472a^2b^2c^2d^2-12a^3bcd^3-5a^4d^4)\,x\sqrt{ax+bx^2}}{768b^2d^5} \\
&-\frac{(320b^3c^3-680ab^2c^2d+372a^2bcd^2-5a^3d^3)\,x^2\sqrt{ax+bx^2}}{960bd^4} \\
&+\frac{(40b^2c^2-52abcd+5a^2d^2)\,x^3\sqrt{ax+bx^2}}{160d^3} \\
&-\frac{(12bc-5ad)x^2(ax+bx^2)^{3/2}}{60d^2}+\frac{x(ax+bx^2)^{5/2}}{6d} \\
&+\frac{(1024b^6c^6-2560ab^5c^5d+1920a^2b^4c^4d^2-320a^3b^3c^3d^3-40a^4b^2c^2d^4-12a^5bcd^5-5a^6d^6)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{512b^{7/2}d^7} \\
&-\frac{2c^{7/2}(bc-ad)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{d^7}
\end{aligned}$$

output
```
-1/512*(-5*a^5*d^5-12*a^4*b*c*d^4-40*a^3*b^2*c^2*d^3+704*a^2*b^3*c^3*d^2-1
152*a*b^4*c^4*d+512*b^5*c^5)*(b*x^2+a*x)^(1/2)/b^3/d^6+1/768*(-5*a^4*d^4-1
2*a^3*b*c*d^3+472*a^2*b^2*c^2*d^2-832*a*b^3*c^3*d+384*b^4*c^4)*x*(b*x^2+a*
x)^(1/2)/b^2/d^5-1/960*(-5*a^3*d^3+372*a^2*b*c*d^2-680*a*b^2*c^2*d+320*b^3
*c^3)*x^2*(b*x^2+a*x)^(1/2)/b/d^4+1/160*(5*a^2*d^2-52*a*b*c*d+40*b^2*c^2)*
x^3*(b*x^2+a*x)^(1/2)/d^3-1/60*(-5*a*d+12*b*c)*x^2*(b*x^2+a*x)^(3/2)/d^2+1
/6*x*(b*x^2+a*x)^(5/2)/d+1/512*(-5*a^6*d^6-12*a^5*b*c*d^5-40*a^4*b^2*c^2*d
^4-320*a^3*b^3*c^3*d^3+1920*a^2*b^4*c^4*d^2-2560*a*b^5*c^5*d+1024*b^6*c^6)
*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(7/2)/d^7-2*c^(7/2)*(-a*d+b*c)^(5/
2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/d^7
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.85 (sec) , antiderivative size = 702, normalized size of antiderivative = 1.42

$$\int \frac{x(ax+bx^2)^{5/2}}{c+dx}\,dx = \frac{(x(a+bx))^{5/2}\left(\sqrt{b}d\sqrt{x}\sqrt{a+bx}(75a^5d^5+10a^4bd^4(18c-5dx)+40a^3b^2d^3(15c^2-\right.}{}$$

input
```
Integrate[(x*(a*x + b*x^2)^(5/2))/(c + d*x),x]
```

output
```
((x*(a + b*x))^(5/2)*(Sqrt[b]*d*Sqrt[x]*Sqrt[a + b*x]*(75*a^5*d^5 + 10*a^4
*b*d^4*(18*c - 5*d*x) + 40*a^3*b^2*d^3*(15*c^2 - 3*c*d*x + d^2*x^2) + 16*a
^2*b^3*d^2*(-660*c^3 + 295*c^2*d*x - 186*c*d^2*x^2 + 135*d^3*x^3) + 64*a*b
^4*d*(270*c^4 - 130*c^3*d*x + 85*c^2*d^2*x^2 - 63*c*d^3*x^3 + 50*d^4*x^4)
- 128*b^5*(60*c^5 - 30*c^4*d*x + 20*c^3*d^2*x^2 - 15*c^2*d^3*x^3 + 12*c*d^
4*x^4 - 10*d^5*x^5)) + 15360*b^(5/2)*c^(5/2)*(b*c - a*d)^2*(b*c - a*d - I*
Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d])*Sqrt[-(b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[
d]*Sqrt[b*c - a*d]]*ArcTan[(Sqrt[-(b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sq
rt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*(-Sqrt[a] + Sqrt[a + b*x]))] + 15360*b^(5
/2)*c^(5/2)*(b*c - a*d)^2*(b*c - a*d + I*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d])*
Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*ArcTan[(Sqrt[
-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*
(-Sqrt[a] + Sqrt[a + b*x]))] + 30*(1024*b^6*c^6 - 2560*a*b^5*c^5*d + 1920*
a^2*b^4*c^4*d^2 - 320*a^3*b^3*c^3*d^3 - 40*a^4*b^2*c^2*d^4 - 12*a^5*b*c*d^
5 - 5*a^6*d^6)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(76
80*b^(7/2)*d^7*x^(5/2)*(a + b*x)^(5/2))
```

Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 488, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1231, 27, 1231, 27, 1231, 27, 1269, 1091, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(ax+bx^2\right)^{5/2}}{c+dx}dx$$

$$\Big\downarrow \text{1231}$$

$$-\frac{\int -\frac{(ac(12bc-5ad)+(24b^2c^2-12abdc-5a^2d^2)x)\left(bx^2+ax\right)^{3/2}}{2(c+dx)}dx}{12bd^2}-\frac{\left(ax+bx^2\right)^{5/2}(-5ad+12bc-10bdx)}{60bd^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(ac(12bc-5ad)+(24b^2c^2-12abdc-5a^2d^2)x)\left(bx^2+ax\right)^{3/2}}{c+dx}dx}{24bd^2}-\frac{\left(ax+bx^2\right)^{5/2}(-5ad+12bc-10bdx)}{60bd^2}$$

$$\Big\downarrow \text{1231}$$

$$\frac{-\frac{\int -\frac{3\left(ac\left(64b^3c^3-88ab^2dc^2+12a^2bd^2c+5a^3d^3\right)+\left(128b^4c^4-192ab^3dc^3+40a^2b^2d^2c^2+12a^3bd^3c+5a^4d^4\right)x\right)\sqrt{bx^2+ax}}{2(c+dx)}dx}{8bd^2}-\frac{(ax+bx^2)^{3/2}(5a^3d^3-2bdx(}{}}{24bd^2}$$

$$\frac{(ax+bx^2)^{5/2}(-5ad+12bc-10bdx)}{60bd^2}$$

$\downarrow$ 27

$$\frac{\frac{3\int \frac{\left(ac\left(64b^3c^3-88ab^2dc^2+12a^2bd^2c+5a^3d^3\right)+\left(128b^4c^4-192ab^3dc^3+40a^2b^2d^2c^2+12a^3bd^3c+5a^4d^4\right)x\right)\sqrt{bx^2+ax}}{c+dx}dx}{16bd^2}-\frac{(ax+bx^2)^{3/2}(5a^3d^3-2bdx(-5a}{}}{24bd^2}$$

$$\frac{(ax+bx^2)^{5/2}(-5ad+12bc-10bdx)}{60bd^2}$$

$\downarrow$ 1231

$$3\left(-\frac{\int -\frac{ac\left(512b^5c^5-1152ab^4dc^4+704a^2b^3d^2c^3-40a^3b^2d^3c^2-12a^4bd^4c-5a^5d^5\right)+\left(1024b^6c^6-2560ab^5dc^5+1920a^2b^4d^2c^4-320a^3b^3d^3c^3-40a^4b^2d^4c^2-12a^5\right.}{2(c+dx)\sqrt{bx^2+ax}}}{4bd^2}\right.$$

$$\frac{(ax+bx^2)^{5/2}(-5ad+12bc-10bdx)}{60bd^2}$$

$\downarrow$ 27

$$3\left(\frac{\int \frac{ac\left(512b^5c^5-1152ab^4dc^4+704a^2b^3d^2c^3-40a^3b^2d^3c^2-12a^4bd^4c-5a^5d^5\right)+\left(1024b^6c^6-2560ab^5dc^5+1920a^2b^4d^2c^4-320a^3b^3d^3c^3-40a^4b^2d^4c^2-12a^5bd^5\right.}{(c+dx)\sqrt{bx^2+ax}}}{8bd^2}\right.$$

1

$$\frac{(ax+bx^2)^{5/2}(-5ad+12bc-10bdx)}{60bd^2}$$

$\downarrow$ 1269

$$\frac{3\left(\frac{\frac{\left(-5a^6d^6-12a^5bcd^5-40a^4b^2c^2d^4-320a^3b^3c^3d^3+1920a^2b^4c^4d^2-2560ab^5c^5d+1024b^6c^6\right)\int\frac{1}{\sqrt{bx^2+ax}}dx}{d}-\frac{1024b^3c^4(bc-ad)^3\int\frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{8bd^2}-\frac{\sqrt{a}}{}\right.}{16bd^2}$$

$$\frac{(ax+bx^2)^{5/2}(-5ad+12bc-10bdx)}{60bd^2}$$

$\downarrow$ 1091

$$3\left(\frac{\frac{2\left(-5a^6d^6-12a^5bcd^5-40a^4b^2c^2d^4-320a^3b^3c^3d^3+1920a^2b^4c^4d^2-2560ab^5c^5d+1024b^6c^6\right)\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{d}}{8bd^2}-\frac{1024b^3c^4(bc-ad)^3\int \frac{1}{(c+dx)\sqrt{bx^2+\cdots}}}{d}\cdots\right.}{16bd\cdots}$$

$$\frac{\left(ax+bx^2\right)^{5/2}(-5ad+12bc-10bdx)}{60bd^2}$$

$$\Big\downarrow \; 219$$

$$3\left(\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)\left(-5a^6d^6-12a^5bcd^5-40a^4b^2c^2d^4-320a^3b^3c^3d^3+1920a^2b^4c^4d^2-2560ab^5c^5d+1024b^6c^6\right)}{\sqrt{b}d}}{8bd^2}-\frac{1024b^3c^4(bc-ad)^3\int \frac{1}{(c+dx)\sqrt{bx^2+a\cdots}}}{d}\cdots\right.}{16bd^2\cdots}$$

$$\frac{\left(ax+bx^2\right)^{5/2}(-5ad+12bc-10bdx)}{60bd^2}$$

$$\Big\downarrow \; 1154$$

$$3\left(\frac{\frac{2048b^3c^4(bc-ad)^3\int \frac{1}{4c(bc-ad)-\frac{(ac+(2bc-ad)x)^2}{bx^2+ax}}d\left(-\frac{ac+(2bc-ad)x}{\sqrt{bx^2+ax}}\right)}{d}+\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)\left(-5a^6d^6-12a^5bcd^5-40a^4b^2c^2d^4-320a^3b^3c^3d^3+192\cdots\right)}{\sqrt{b}d}}{8bd^2}\cdots\right.}{\cdots}$$

$$\frac{\left(ax+bx^2\right)^{5/2}(-5ad+12bc-10bdx)}{60bd^2}$$

$$\Big\downarrow \; 219$$

$$3\left(\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)\left(-5a^6d^6-12a^5bcd^5-40a^4b^2c^2d^4-320a^3b^3c^3d^3+1920a^2b^4c^4d^2-2560ab^5c^5d+1024b^6c^6\right)}{\sqrt{b}d}-\frac{1024b^3c^{7/2}(bc-ad)^{5/2}\text{arctanh}\left(\cdots\right)}{d}}{8bd^2}\cdots\right.}{\cdots}$$

$$\frac{\left(ax+bx^2\right)^{5/2}(-5ad+12bc-10bdx)}{60bd^2}$$

input
```
Int[(x*(a*x + b*x^2)^(5/2))/(c + d*x),x]
```

output
```
-1/60*((12*b*c - 5*a*d - 10*b*d*x)*(a*x + b*x^2)^(5/2))/(b*d^2) + (-1/8*((
64*b^3*c^3 - 88*a*b^2*c^2*d + 12*a^2*b*c*d^2 + 5*a^3*d^3 - 2*b*d*(24*b^2*c
^2 - 12*a*b*c*d - 5*a^2*d^2)*x)*(a*x + b*x^2)^(3/2))/(b*d^2) + (3*(-1/4*((
512*b^5*c^5 - 1152*a*b^4*c^4*d + 704*a^2*b^3*c^3*d^2 - 40*a^3*b^2*c^2*d^3
- 12*a^4*b*c*d^4 - 5*a^5*d^5 - 2*b*d*(128*b^4*c^4 - 192*a*b^3*c^3*d + 40*a
^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + 5*a^4*d^4)*x)*Sqrt[a*x + b*x^2])/(b*d^2)
 + ((2*(1024*b^6*c^6 - 2560*a*b^5*c^5*d + 1920*a^2*b^4*c^4*d^2 - 320*a^3*b
^3*c^3*d^3 - 40*a^4*b^2*c^2*d^4 - 12*a^5*b*c*d^5 - 5*a^6*d^6)*ArcTanh[(Sqr
t[b]*x)/Sqrt[a*x + b*x^2]])/(Sqrt[b]*d) - (1024*b^3*c^(7/2)*(b*c - a*d)^(5
/2)*ArcTanh[(a*c + (2*b*c - a*d)*x)/(2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[a*x +
b*x^2])])/d)/(8*b*d^2)))/(16*b*d^2))/(24*b*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.17

method	result
risch	$\frac{(1280b^5d^5x^5+3200a\,b^4d^5x^4-1536b^5c\,d^4x^4+2160a^2b^3d^5x^3-4032a\,b^4c\,d^4x^3+1920b^5c^2d^3x^3+40a^3b^2d^5x^2-2976a^2b^3c\,d^4x^2+5440a}{}$
default	Expression too large to display

input
```
int(x*(b*x^2+a*x)^(5/2)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/7680/b^3*(1280*b^5*d^5*x^5+3200*a*b^4*d^5*x^4-1536*b^5*c*d^4*x^4+2160*a^
2*b^3*d^5*x^3-4032*a*b^4*c*d^4*x^3+1920*b^5*c^2*d^3*x^3+40*a^3*b^2*d^5*x^2
-2976*a^2*b^3*c*d^4*x^2+5440*a*b^4*c^2*d^3*x^2-2560*b^5*c^3*d^2*x^2-50*a^4
*b*d^5*x-120*a^3*b^2*c*d^4*x+4720*a^2*b^3*c^2*d^3*x-8320*a*b^4*c^3*d^2*x+3
840*b^5*c^4*d*x+75*a^5*d^5+180*a^4*b*c*d^4+600*a^3*b^2*c^2*d^3-10560*a^2*b
^3*c^3*d^2+17280*a*b^4*c^4*d-7680*b^5*c^5)*x*(b*x+a)/d^6/(x*(b*x+a))^(1/2)
-1/1024/d^6/b^3*((5*a^6*d^6+12*a^5*b*c*d^5+40*a^4*b^2*c^2*d^4+320*a^3*b^3*
c^3*d^3-1920*a^2*b^4*c^4*d^2+2560*a*b^5*c^5*d-1024*b^6*c^6)/d*ln((1/2*a+b*
x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)+1024*c^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*
b^2*c^2*d-b^3*c^3)*b^3/d^2/(-c*(a*d-b*c)/d^2)^(1/2)*ln((-2*c*(a*d-b*c)/d^2
+(a*d-2*b*c)/d*(x+c/d)+2*(-c*(a*d-b*c)/d^2)^(1/2)*(b*(x+c/d)^2+(a*d-2*b*c)
/d*(x+c/d)-c*(a*d-b*c)/d^2)^(1/2))/(x+c/d)))
```

Fricas [A] (verification not implemented)

Time = 3.42 (sec) , antiderivative size = 1920, normalized size of antiderivative = 3.89

$$\int \frac{x(ax+bx^2)^{5/2}}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(x*(b*x^2+a*x)^(5/2)/(d*x+c),x, algorithm="fricas")
```

output
```
[-1/15360*(15*(1024*b^6*c^6 - 2560*a*b^5*c^5*d + 1920*a^2*b^4*c^4*d^2 - 32
0*a^3*b^3*c^3*d^3 - 40*a^4*b^2*c^2*d^4 - 12*a^5*b*c*d^5 - 5*a^6*d^6)*sqrt(
b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 15360*(b^6*c^5 - 2*a*b^5
*c^4*d + a^2*b^4*c^3*d^2)*sqrt(b*c^2 - a*c*d)*log((a*c + (2*b*c - a*d)*x -
 2*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) - 2*(1280*b^6*d^6*x^5
 - 7680*b^6*c^5*d + 17280*a*b^5*c^4*d^2 - 10560*a^2*b^4*c^3*d^3 + 600*a^3*
b^3*c^2*d^4 + 180*a^4*b^2*c*d^5 + 75*a^5*b*d^6 - 128*(12*b^6*c*d^5 - 25*a*
b^5*d^6)*x^4 + 48*(40*b^6*c^2*d^4 - 84*a*b^5*c*d^5 + 45*a^2*b^4*d^6)*x^3 -
 8*(320*b^6*c^3*d^3 - 680*a*b^5*c^2*d^4 + 372*a^2*b^4*c*d^5 - 5*a^3*b^3*d^
6)*x^2 + 10*(384*b^6*c^4*d^2 - 832*a*b^5*c^3*d^3 + 472*a^2*b^4*c^2*d^4 - 1
2*a^3*b^3*c*d^5 - 5*a^4*b^2*d^6)*x)*sqrt(b*x^2 + a*x))/(b^4*d^7), 1/15360*
(30720*(b^6*c^5 - 2*a*b^5*c^4*d + a^2*b^4*c^3*d^2)*sqrt(-b*c^2 + a*c*d)*ar
ctan(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) - 15*(1024*b^6*
c^6 - 2560*a*b^5*c^5*d + 1920*a^2*b^4*c^4*d^2 - 320*a^3*b^3*c^3*d^3 - 40*a
^4*b^2*c^2*d^4 - 12*a^5*b*c*d^5 - 5*a^6*d^6)*sqrt(b)*log(2*b*x + a - 2*sqr
t(b*x^2 + a*x)*sqrt(b)) + 2*(1280*b^6*d^6*x^5 - 7680*b^6*c^5*d + 17280*a*b
^5*c^4*d^2 - 10560*a^2*b^4*c^3*d^3 + 600*a^3*b^3*c^2*d^4 + 180*a^4*b^2*c*d
^5 + 75*a^5*b*d^6 - 128*(12*b^6*c*d^5 - 25*a*b^5*d^6)*x^4 + 48*(40*b^6*c^2
*d^4 - 84*a*b^5*c*d^5 + 45*a^2*b^4*d^6)*x^3 - 8*(320*b^6*c^3*d^3 - 680*a*b
^5*c^2*d^4 + 372*a^2*b^4*c*d^5 - 5*a^3*b^3*d^6)*x^2 + 10*(384*b^6*c^4*d...
```

Sympy [F]

$$\int \frac{x(ax+bx^2)^{5/2}}{c+dx}\,dx = \int \frac{x(x(a+bx))^{\frac{5}{2}}}{c+dx}\,dx$$

input
```
integrate(x*(b*x**2+a*x)**(5/2)/(d*x+c),x)
```

output
```
Integral(x*(x*(a + b*x))**(5/2)/(c + d*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(ax+bx^2)^{5/2}}{c+dx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(b*x^2+a*x)^(5/2)/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-2*b*c>0)', see `assume?` for
 more deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(ax+bx^2)^{5/2}}{c+dx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(b*x^2+a*x)^(5/2)/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(ax+bx^2)^{5/2}}{c+dx}\,dx = \int \frac{x\,\left(b\,x^2+a\,x\right)^{5/2}}{c+d\,x}\,dx$$

input
```
int((x*(a*x + b*x^2)^(5/2))/(c + d*x),x)
```

output
```
int((x*(a*x + b*x^2)^(5/2))/(c + d*x), x)
```

Reduce [F]

$$\int \frac{x(ax+bx^2)^{5/2}}{c+dx}\,dx = \int \frac{x(b\,x^2+ax)^{\frac{5}{2}}}{dx+c}dx$$

input
```
int(x*(b*x^2+a*x)^(5/2)/(d*x+c),x)
```

output
```
int(x*(b*x^2+a*x)^(5/2)/(d*x+c),x)
```

3.116 $\int \frac{(ax+bx^2)^{5/2}}{c+dx}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 380

$$\int \frac{(ax+bx^2)^{5/2}}{c+dx}\,dx = \frac{(128b^4c^4-288ab^3c^3d+176a^2b^2c^2d^2-10a^3bcd^3-3a^4d^4)\sqrt{ax+bx^2}}{128b^2d^5} - \frac{(96b^3c^3-208ab^2c^2d+118a^2bcd^2-3a^3d^3)\,x\sqrt{ax+bx^2}}{192bd^4} + \frac{(16b^2c^2-22abcd+3a^2d^2)\,x^2\sqrt{ax+bx^2}}{48d^3} - \frac{(2bc-ad)x(ax+bx^2)^{3/2}}{8d^2} + \frac{(ax+bx^2)^{5/2}}{5d} - \frac{(2bc-ad)\,(128b^4c^4-256ab^3c^3d+112a^2b^2c^2d^2+16a^3bcd^3+3a^4d^4)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{128b^{5/2}d^6} + \frac{2c^{5/2}(bc-ad)^{5/2}\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{d^6}$$

output
```
1/128*(-3*a^4*d^4-10*a^3*b*c*d^3+176*a^2*b^2*c^2*d^2-288*a*b^3*c^3*d+128*b
^4*c^4)*(b*x^2+a*x)^(1/2)/b^2/d^5-1/192*(-3*a^3*d^3+118*a^2*b*c*d^2-208*a*
b^2*c^2*d+96*b^3*c^3)*x*(b*x^2+a*x)^(1/2)/b/d^4+1/48*(3*a^2*d^2-22*a*b*c*d
+16*b^2*c^2)*x^2*(b*x^2+a*x)^(1/2)/d^3-1/8*(-a*d+2*b*c)*x*(b*x^2+a*x)^(3/2
)/d^2+1/5*(b*x^2+a*x)^(5/2)/d-1/128*(-a*d+2*b*c)*(3*a^4*d^4+16*a^3*b*c*d^3
+112*a^2*b^2*c^2*d^2-256*a*b^3*c^3*d+128*b^4*c^4)*arctanh(b^(1/2)*x/(b*x^2
+a*x)^(1/2))/b^(5/2)/d^6+2*c^(5/2)*(-a*d+b*c)^(5/2)*arctanh((-a*d+b*c)^(1/
2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/d^6
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.58 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.64

$$\int \frac{(ax+bx^2)^{5/2}}{c+dx}\,dx = \frac{(x(a+bx))^{5/2}\left(\sqrt{b}d\sqrt{x}\sqrt{a+bx}(-45a^4d^4+30a^3bd^3(-5c+dx)+4a^2b^2d^2(660c^2-\right.}{}$$

input
```
Integrate[(a*x + b*x^2)^(5/2)/(c + d*x),x]
```

output
```
((x*(a + b*x))^(5/2)*(Sqrt[b]*d*Sqrt[x]*Sqrt[a + b*x]*(-45*a^4*d^4 + 30*a^
3*b*d^3*(-5*c + d*x) + 4*a^2*b^2*d^2*(660*c^2 - 295*c*d*x + 186*d^2*x^2) +
 16*a*b^3*d*(-270*c^3 + 130*c^2*d*x - 85*c*d^2*x^2 + 63*d^3*x^3) + 32*b^4*
(60*c^4 - 30*c^3*d*x + 20*c^2*d^2*x^2 - 15*c*d^3*x^3 + 12*d^4*x^4)) - 3840
*b^(3/2)*c^(3/2)*(b*c - a*d)^2*(b*c - a*d - I*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a
*d])*Sqrt[-(b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*ArcTan[(
Sqrt[-(b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqr
t[c]*(-Sqrt[a] + Sqrt[a + b*x]))] - 3840*b^(3/2)*c^(3/2)*(b*c - a*d)^2*(b*
c - a*d + I*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d])*Sqrt[-(b*c) + 2*a*d + (2*I)*S
qrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*ArcTan[(Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a
]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*(-Sqrt[a] + Sqrt[a + b*x]))]
+ 30*(256*b^5*c^5 - 640*a*b^4*c^4*d + 480*a^2*b^3*c^3*d^2 - 80*a^3*b^2*c^2
*d^3 - 10*a^4*b*c*d^4 - 3*a^5*d^5)*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sq
rt[a + b*x])]))/(1920*b^(5/2)*d^6*x^(5/2)*(a + b*x)^(5/2))
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {1162, 1231, 27, 1231, 27, 1269, 1091, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{5/2}}{c+dx}\,dx$$

$\Big\downarrow$ 1162

$$\frac{(ax+bx^2)^{5/2}}{5d}-\frac{\int \frac{(ac+(2bc-ad)x)(bx^2+ax)^{3/2}}{c+dx}dx}{2d}$$

$\Big\downarrow$ 1231

$$\frac{(ax+bx^2)^{5/2}}{5d}-\frac{-\frac{\int -\frac{\left(ac\left(16b^2c^2-22abdc+3a^2d^2\right)+(2bc-ad)\left(16b^2c^2-16abdc-3a^2d^2\right)x\right)\sqrt{bx^2+ax}}{2(c+dx)}dx}{8bd^2}-\frac{(ax+bx^2)^{3/2}(3a^2d^2-6bdx(2bc-ad)-22abcd+16b^2c^2)}{24bd^2}}{2d}$$

$\Big\downarrow$ 27

$$\frac{(ax+bx^2)^{5/2}}{5d}-\frac{\frac{\int \frac{\left(ac\left(16b^2c^2-22abdc+3a^2d^2\right)+(2bc-ad)\left(16b^2c^2-16abdc-3a^2d^2\right)x\right)\sqrt{bx^2+ax}}{c+dx}dx}{16bd^2}-\frac{(ax+bx^2)^{3/2}(3a^2d^2-6bdx(2bc-ad)-22abcd+16b^2c^2)}{24bd^2}}{2d}$$

$\Big\downarrow$ 1231

$$\frac{(ax+bx^2)^{5/2}}{5d}-\frac{-\frac{\int -\frac{ac\left(128b^4c^4-288ab^3dc^3+176a^2b^2d^2c^2-10a^3bd^3c-3a^4d^4\right)+(2bc-ad)\left(128b^4c^4-256ab^3dc^3+112a^2b^2d^2c^2+16a^3bd^3c+3a^4d^4\right)x}{2(c+dx)\sqrt{bx^2+ax}}dx}{4bd^2}-\frac{\sqrt{ax+bx^2}\left(-3a^4d^4-\cdots\right.}{\cdots}}{16bd^2}\cdots}{2d}$$

$\Big\downarrow$ 27

$$\frac{(ax+bx^2)^{5/2}}{5d}-\frac{\frac{\int \frac{ac\left(128b^4c^4-288ab^3dc^3+176a^2b^2d^2c^2-10a^3bd^3c-3a^4d^4\right)+(2bc-ad)\left(128b^4c^4-256ab^3dc^3+112a^2b^2d^2c^2+16a^3bd^3c+3a^4d^4\right)x}{(c+dx)\sqrt{bx^2+ax}}dx}{8bd^2}-\frac{\sqrt{ax+bx^2}\left(-3a^4d^4-10a\cdots\right.}{\cdots}}{16bd^2}\cdots}{2d}$$

$\Big\downarrow$ 1269

$$\frac{(ax+bx^2)^{5/2}}{5d}-\frac{\frac{\frac{(2bc-ad)\left(3a^4d^4+16a^3bcd^3+112a^2b^2c^2d^2-256ab^3c^3d+128b^4c^4\right)\int \frac{1}{\sqrt{bx^2+ax}}dx}{d}-\frac{256b^2c^3(bc-ad)^3\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{8bd^2}-\frac{\sqrt{ax+bx^2}\left(-3a^4d^4-10a^3bcd^3-2\cdots\right.}{\cdots}}{16bd^2}\cdots}{2d}$$

$\Big\downarrow$ 1091

$$\frac{\left(ax+bx^2\right)^{5/2}}{5d}-\frac{\frac{\frac{\frac{2(2bc-ad)\left(3a^4d^4+16a^3bcd^3+112a^2b^2c^2d^2-256ab^3c^3d+128b^4c^4\right)\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{d}-\frac{256b^2c^3(bc-ad)^3\int\frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{8bd^2}-\frac{\sqrt{ax+bx^2}\left(-3a^4d^4-\cdots\right.}{\cdots}}{16bd^2}\cdots}{2d}$$

$\downarrow$ 219

$$\frac{\left(ax+bx^2\right)^{5/2}}{5d}-\frac{\frac{\frac{\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(2bc-ad)\left(3a^4d^4+16a^3bcd^3+112a^2b^2c^2d^2-256ab^3c^3d+128b^4c^4\right)}{\sqrt{b}d}-\frac{256b^2c^3(bc-ad)^3\int\frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}}{8bd^2}-\frac{\sqrt{ax+bx^2}\left(-3a^4d^4-1\cdots\right.}{\cdots}}{16bd^2}\cdots}{2d}$$

$\downarrow$ 1154

$$\frac{\left(ax+bx^2\right)^{5/2}}{5d}-\frac{\frac{\frac{\frac{512b^2c^3(bc-ad)^3\int\frac{1}{4c(bc-ad)-\frac{(ac+(2bc-ad)x)^2}{bx^2+ax}}d\left(-\frac{ac+(2bc-ad)x}{\sqrt{bx^2+ax}}\right)}{d}+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(2bc-ad)\left(3a^4d^4+16a^3bcd^3+112a^2b^2c^2d^2-256ab^3c^3d+12\cdots\right.}{\sqrt{b}d}}{8bd^2}\cdots}{16bd^2}\cdots}{\cdots}$$

$\downarrow$ 219

$$\frac{\left(ax+bx^2\right)^{5/2}}{5d}-\frac{\frac{\frac{\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(2bc-ad)\left(3a^4d^4+16a^3bcd^3+112a^2b^2c^2d^2-256ab^3c^3d+128b^4c^4\right)}{\sqrt{b}d}-\frac{256b^2c^{5/2}(bc-ad)^{5/2}\operatorname{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{d}}{8bd^2}-\frac{\sqrt{\cdots}}{\cdots}}{16bd^2}\cdots}{\cdots}$$

input
```
Int[(a*x + b*x^2)^(5/2)/(c + d*x),x]
```

output
```
(a*x + b*x^2)^(5/2)/(5*d) - (-1/24*((16*b^2*c^2 - 22*a*b*c*d + 3*a^2*d^2 -
 6*b*d*(2*b*c - a*d)*x)*(a*x + b*x^2)^(3/2))/(b*d^2) + (-1/4*((128*b^4*c^4
 - 288*a*b^3*c^3*d + 176*a^2*b^2*c^2*d^2 - 10*a^3*b*c*d^3 - 3*a^4*d^4 - 2*
b*d*(2*b*c - a*d)*(16*b^2*c^2 - 16*a*b*c*d - 3*a^2*d^2)*x)*Sqrt[a*x + b*x^
2])/(b*d^2) + ((2*(2*b*c - a*d)*(128*b^4*c^4 - 256*a*b^3*c^3*d + 112*a^2*b
^2*c^2*d^2 + 16*a^3*b*c*d^3 + 3*a^4*d^4)*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*
x^2]])/(Sqrt[b]*d) - (256*b^2*c^(5/2)*(b*c - a*d)^(5/2)*ArcTanh[(a*c + (2*
b*c - a*d)*x)/(2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[a*x + b*x^2])])/d)/(8*b*d^2)
)/(16*b*d^2))/(2*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.25

method	result
risch	$-\frac{\left(-384b^4d^4x^4-1008a\,b^3d^4x^3+480b^4c\,d^3x^3-744a^2b^2d^4x^2+1360a\,b^3c\,d^3x^2-640b^4c^2d^2x^2-30a^3b\,d^4x+1180a^2b^2c\,d^3x-2080a\,b^3c\right.}{1920b^2d^5\sqrt{x(bx+a)}}$
default	$\frac{\left(b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}\right)^{\frac{5}{2}}}{5}+\frac{(ad-2bc)\left(\frac{\left(2b\left(x+\frac{c}{d}\right)+\frac{ad-2bc}{d}\right)\left(b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}\right)^{\frac{3}{2}}}{8b}+\frac{3\left(-\frac{4bc(ad-}{d^2}\right.}{}\right.}{}$

input
```
int((b*x^2+a*x)^(5/2)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/1920/b^2*(-384*b^4*d^4*x^4-1008*a*b^3*d^4*x^3+480*b^4*c*d^3*x^3-744*a^2
*b^2*d^4*x^2+1360*a*b^3*c*d^3*x^2-640*b^4*c^2*d^2*x^2-30*a^3*b*d^4*x+1180*
a^2*b^2*c*d^3*x-2080*a*b^3*c^2*d^2*x+960*b^4*c^3*d*x+45*a^4*d^4+150*a^3*b*
c*d^3-2640*a^2*b^2*c^2*d^2+4320*a*b^3*c^3*d-1920*b^4*c^4)*x*(b*x+a)/d^5/(x
*(b*x+a))^(1/2)+1/256/d^5/b^2*((3*a^5*d^5+10*a^4*b*c*d^4+80*a^3*b^2*c^2*d^
3-480*a^2*b^3*c^3*d^2+640*a*b^4*c^4*d-256*b^5*c^5)/d*ln((1/2*a+b*x)/b^(1/2
)+(b*x^2+a*x)^(1/2))/b^(1/2)+256*c^3*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-
b^3*c^3)*b^2/d^2/(-c*(a*d-b*c)/d^2)^(1/2)*ln((-2*c*(a*d-b*c)/d^2+(a*d-2*b*
c)/d*(x+c/d)+2*(-c*(a*d-b*c)/d^2)^(1/2)*(b*(x+c/d)^2+(a*d-2*b*c)/d*(x+c/d)
-c*(a*d-b*c)/d^2)^(1/2))/(x+c/d)))
```

Fricas [A] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 1544, normalized size of antiderivative = 4.06

$$\int \frac{(ax+bx^2)^{5/2}}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a*x)^(5/2)/(d*x+c),x, algorithm="fricas")
```

output
```
[-1/3840*(15*(256*b^5*c^5 - 640*a*b^4*c^4*d + 480*a^2*b^3*c^3*d^2 - 80*a^3
*b^2*c^2*d^3 - 10*a^4*b*c*d^4 - 3*a^5*d^5)*sqrt(b)*log(2*b*x + a + 2*sqrt(
b*x^2 + a*x)*sqrt(b)) - 3840*(b^5*c^4 - 2*a*b^4*c^3*d + a^2*b^3*c^2*d^2)*s
qrt(b*c^2 - a*c*d)*log((a*c + (2*b*c - a*d)*x + 2*sqrt(b*c^2 - a*c*d)*sqrt
(b*x^2 + a*x))/(d*x + c)) - 2*(384*b^5*d^5*x^4 + 1920*b^5*c^4*d - 4320*a*b
^4*c^3*d^2 + 2640*a^2*b^3*c^2*d^3 - 150*a^3*b^2*c*d^4 - 45*a^4*b*d^5 - 48*
(10*b^5*c*d^4 - 21*a*b^4*d^5)*x^3 + 8*(80*b^5*c^2*d^3 - 170*a*b^4*c*d^4 +
93*a^2*b^3*d^5)*x^2 - 10*(96*b^5*c^3*d^2 - 208*a*b^4*c^2*d^3 + 118*a^2*b^3
*c*d^4 - 3*a^3*b^2*d^5)*x)*sqrt(b*x^2 + a*x))/(b^3*d^6), -1/3840*(7680*(b^
5*c^4 - 2*a*b^4*c^3*d + a^2*b^3*c^2*d^2)*sqrt(-b*c^2 + a*c*d)*arctan(sqrt(
-b*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) + 15*(256*b^5*c^5 - 640*a
*b^4*c^4*d + 480*a^2*b^3*c^3*d^2 - 80*a^3*b^2*c^2*d^3 - 10*a^4*b*c*d^4 - 3
*a^5*d^5)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*(384*b^
5*d^5*x^4 + 1920*b^5*c^4*d - 4320*a*b^4*c^3*d^2 + 2640*a^2*b^3*c^2*d^3 - 1
50*a^3*b^2*c*d^4 - 45*a^4*b*d^5 - 48*(10*b^5*c*d^4 - 21*a*b^4*d^5)*x^3 + 8
*(80*b^5*c^2*d^3 - 170*a*b^4*c*d^4 + 93*a^2*b^3*d^5)*x^2 - 10*(96*b^5*c^3*
d^2 - 208*a*b^4*c^2*d^3 + 118*a^2*b^3*c*d^4 - 3*a^3*b^2*d^5)*x)*sqrt(b*x^2
 + a*x))/(b^3*d^6), 1/1920*(15*(256*b^5*c^5 - 640*a*b^4*c^4*d + 480*a^2*b^
3*c^3*d^2 - 80*a^3*b^2*c^2*d^3 - 10*a^4*b*c*d^4 - 3*a^5*d^5)*sqrt(-b)*arct
an(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + 1920*(b^5*c^4 - 2*a*b^4*c^3*...
```

Sympy [F]

$$\int \frac{(ax + bx^2)^{5/2}}{c + dx}\,dx = \int \frac{(x(a + bx))^{\frac{5}{2}}}{c + dx}\,dx$$

input
```
integrate((b*x**2+a*x)**(5/2)/(d*x+c),x)
```

output
```
Integral((x*(a + b*x))**(5/2)/(c + d*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ax+bx^2)^{5/2}}{c+dx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a*x)^(5/2)/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ax+bx^2)^{5/2}}{c+dx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a*x)^(5/2)/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{5/2}}{c+dx}\,dx = \int \frac{(b\,x^2+a\,x)^{5/2}}{c+d\,x}\,dx$$

input
```
int((a*x + b*x^2)^(5/2)/(c + d*x),x)
```

output
```
int((a*x + b*x^2)^(5/2)/(c + d*x), x)
```

Reduce [F]

$$\int \frac{(ax + bx^2)^{5/2}}{c + dx}\, dx = \int \frac{(b\, x^2 + ax)^{\frac{5}{2}}}{dx + c} dx$$

input
```
int((b*x^2+a*x)^(5/2)/(d*x+c),x)
```

output
```
int((b*x^2+a*x)^(5/2)/(d*x+c),x)
```

3.117 $\int \frac{(ax+bx^2)^{5/2}}{x(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 291

$$\int \frac{(ax+bx^2)^{5/2}}{x(c+dx)}\,dx = -\frac{(64b^3c^3-144ab^2c^2d+88a^2bcd^2-5a^3d^3)\sqrt{ax+bx^2}}{64bd^4} + \frac{(4bc-5ad)(4bc-ad)x\sqrt{ax+bx^2}}{32d^3} - \frac{(8bc-5ad)\,(ax+bx^2)^{3/2}}{24d^2} + \frac{(ax+bx^2)^{5/2}}{4dx} + \frac{(128b^4c^4-320ab^3c^3d+240a^2b^2c^2d^2-40a^3bcd^3-5a^4d^4)\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{64b^{3/2}d^5} - \frac{2c^{3/2}(bc-ad)^{5/2}\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{d^5}$$

output
```
-1/64*(-5*a^3*d^3+88*a^2*b*c*d^2-144*a*b^2*c^2*d+64*b^3*c^3)*(b*x^2+a*x)^(
1/2)/b/d^4+1/32*(-5*a*d+4*b*c)*(-a*d+4*b*c)*x*(b*x^2+a*x)^(1/2)/d^3-1/24*(
-5*a*d+8*b*c)*(b*x^2+a*x)^(3/2)/d^2+1/4*(b*x^2+a*x)^(5/2)/d/x+1/64*(-5*a^4
*d^4-40*a^3*b*c*d^3+240*a^2*b^2*c^2*d^2-320*a*b^3*c^3*d+128*b^4*c^4)*arcta
nh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(3/2)/d^5-2*c^(3/2)*(-a*d+b*c)^(5/2)*arc
tanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/d^5
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.80 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.92

$$\int \frac{(ax+bx^2)^{5/2}}{x(c+dx)}\,dx = \frac{(x(a+bx))^{5/2}\left(\sqrt{b}d\sqrt{x}\sqrt{a+bx}(15a^3d^3+2a^2bd^2(-132c+59dx)+8ab^2d(54c^2-2\right.}{}$$

input
```
Integrate[(a*x + b*x^2)^(5/2)/(x*(c + d*x)),x]
```

output
```
((x*(a + b*x))^(5/2)*(Sqrt[b]*d*Sqrt[x]*Sqrt[a + b*x]*(15*a^3*d^3 + 2*a^2*
b*d^2*(-132*c + 59*d*x) + 8*a*b^2*d*(54*c^2 - 26*c*d*x + 17*d^2*x^2) - 16*
b^3*(12*c^3 - 6*c^2*d*x + 4*c*d^2*x^2 - 3*d^3*x^3)) + 384*Sqrt[b]*Sqrt[c]*
(b*c - a*d)^2*(b*c - a*d - I*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d])*Sqrt[-(b*c)
+ 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*ArcTan[(Sqrt[-(b*c) + 2*a
*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*(-Sqrt[a] +
Sqrt[a + b*x]))] + 384*Sqrt[b]*Sqrt[c]*(b*c - a*d)^2*(b*c - a*d + I*Sqrt[a
]*Sqrt[d]*Sqrt[b*c - a*d])*Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqr
t[b*c - a*d]]*ArcTan[(Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c
 - a*d]]*Sqrt[x])/(Sqrt[c]*(-Sqrt[a] + Sqrt[a + b*x]))] + 6*(128*b^4*c^4 -
 320*a*b^3*c^3*d + 240*a^2*b^2*c^2*d^2 - 40*a^3*b*c*d^3 - 5*a^4*d^4)*ArcTa
nh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(192*b^(3/2)*d^5*x^(5/2
)*(a + b*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.22, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {1261, 112, 27, 171, 27, 171, 27, 171, 27, 175, 65, 104, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{5/2}}{x(c+dx)}\,dx$$

$$\Big\downarrow \; 1261$$

$$\frac{\left(ax+bx^2\right)^{5/2}\int \frac{x^{3/2}(a+bx)^{5/2}}{c+dx}dx}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \; 112$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{x^{3/2}(a+bx)^{5/2}}{4d}-\frac{\int \frac{\sqrt{x}(a+bx)^{3/2}(3ac+(8bc-5ad)x)}{2(c+dx)}dx}{4d}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{x^{3/2}(a+bx)^{5/2}}{4d}-\frac{\int \frac{\sqrt{x}(a+bx)^{3/2}(3ac+(8bc-5ad)x)}{c+dx}dx}{8d}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \; 171$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{x^{3/2}(a+bx)^{5/2}}{4d}-\frac{\frac{\int -\frac{(a+bx)^{3/2}\left(ac(8bc-5ad)+\left(48b^2c^2-40abdc-5a^2d^2\right)x\right)}{2\sqrt{x}(c+dx)}dx}{3bd}+\frac{\sqrt{x}(a+bx)^{5/2}(8bc-5ad)}{3bd}}{8d}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{x^{3/2}(a+bx)^{5/2}}{4d}-\frac{\frac{\sqrt{x}(a+bx)^{5/2}(8bc-5ad)}{3bd}-\frac{\int \frac{(a+bx)^{3/2}\left(ac(8bc-5ad)+\left(48b^2c^2-40abdc-5a^2d^2\right)x\right)}{\sqrt{x}(c+dx)}dx}{6bd}}{8d}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \; 171$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{x^{3/2}(a+bx)^{5/2}}{4d}-\frac{\frac{\sqrt{x}(a+bx)^{5/2}(8bc-5ad)}{3bd}-\frac{\frac{\int -\frac{3\sqrt{a+bx}\left(ac(4bc-5ad)(4bc-ad)+\left(64b^3c^3-112ab^2dc^2+40a^2bd^2c+5a^3d^3\right)x\right)}{2\sqrt{x}(c+dx)}dx}{2d}+}{6bd}}{8d}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{x^{3/2}(a+bx)^{5/2}}{4d}-\frac{\frac{\sqrt{x}(a+bx)^{5/2}(8bc-5ad)}{3bd}-\frac{\frac{\sqrt{x}(a+bx)^{3/2}\left(-5a^2d^2-40abcd+48b^2c^2\right)}{2d}-\frac{3\int\frac{\frac{\sqrt{a+bx}\left(ac(4bc-5ad)(4bc-ad)+\left(64b^3c^3\right.\right.}{\sqrt{x}(c+}}{4d}}{6bd}}{8d}}{x^{5/2}(a+bx)^{5/2}}$$

$\downarrow$ 171

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{x^{3/2}(a+bx)^{5/2}}{4d}-\frac{\frac{\sqrt{x}(a+bx)^{5/2}(8bc-5ad)}{3bd}-\frac{\frac{\sqrt{x}(a+bx)^{3/2}\left(-5a^2d^2-40abcd+48b^2c^2\right)}{2d}-\frac{3\left(\int-\frac{ac\left(64b^3c^3-144ab^2dc^2+88a^2bd^2c-5\right.}{}\right.}{}}{}}{x^{5/2}(a+bx)^{5/2}}$$

$\downarrow$ 27

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{x^{3/2}(a+bx)^{5/2}}{4d}-\frac{\frac{\sqrt{x}(a+bx)^{5/2}(8bc-5ad)}{3bd}-\frac{\frac{\sqrt{x}(a+bx)^{3/2}\left(-5a^2d^2-40abcd+48b^2c^2\right)}{2d}-\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}\left(5a^3d^3+40a^2bcd^2-112ab^2c^2\right.}{d}\right.}{}}{}}{x^{5/2}(a+bx)^{5/2}}$$

$\downarrow$ 175

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{x^{3/2}(a+bx)^{5/2}}{4d}-\frac{\frac{\sqrt{x}(a+bx)^{5/2}(8bc-5ad)}{3bd}-\frac{\frac{\sqrt{x}(a+bx)^{3/2}\left(-5a^2d^2-40abcd+48b^2c^2\right)}{2d}-\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}\left(5a^3d^3+40a^2bcd^2-112ab^2c^2\right.}{d}\right.}{}}{}}{x^{5/2}(a+bx)^{5/2}}$$

$\downarrow$ 65

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{x^{3/2}(a+bx)^{5/2}}{4d}-\frac{\frac{\sqrt{x}(a+bx)^{5/2}(8bc-5ad)}{3bd}-\frac{\frac{\sqrt{x}(a+bx)^{3/2}\left(-5a^2d^2-40abcd+48b^2c^2\right)}{2d}-\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}\left(5a^3d^3+40a^2bcd^2-112ab^2c^2\cdots\right.}{d}\cdots\right.}{\cdots}}{\cdots}}{\cdots}\cdots\right.}{x^{5/2}(a+bx)^{5/\cdots}}$$

$\downarrow$ 104

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{x^{3/2}(a+bx)^{5/2}}{4d}-\frac{\frac{\sqrt{x}(a+bx)^{5/2}(8bc-5ad)}{3bd}-\frac{\frac{\sqrt{x}(a+bx)^{3/2}\left(-5a^2d^2-40abcd+48b^2c^2\right)}{2d}-\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}\left(5a^3d^3+40a^2bcd^2-112ab^2c^2\cdots\right.}{d}\cdots\right.}{\cdots}}{\cdots}}{\cdots}\cdots\right.}{x^{5/2}(a+bx)^{5\cdots}}$$

$\downarrow$ 219

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{x^{3/2}(a+bx)^{5/2}}{4d}-\frac{\frac{\sqrt{x}(a+bx)^{5/2}(8bc-5ad)}{3bd}-\frac{\frac{\sqrt{x}(a+bx)^{3/2}\left(-5a^2d^2-40abcd+48b^2c^2\right)}{2d}-\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}\left(5a^3d^3+40a^2bcd^2-112ab^2c^2\cdots\right.}{d}\cdots\right.}{\cdots}}{\cdots}}{\cdots}\cdots\right.}{x^{5/2}(a+bx)^{5\cdots}}$$

$\downarrow$ 221

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{x^{3/2}(a+bx)^{5/2}}{4d}-\frac{\frac{\sqrt{x}(a+bx)^{5/2}(8bc-5ad)}{3bd}-\frac{\frac{\sqrt{x}(a+bx)^{3/2}\left(-5a^2d^2-40abcd+48b^2c^2\right)}{2d}-\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}\left(5a^3d^3+40a^2bcd^2-112ab^2c^2\right.}{d}\right.}{}}{}}{}\right)}{x^{5/2}(a+b}$$

input
```
Int[(a*x + b*x^2)^(5/2)/(x*(c + d*x)),x]
```

output
```
((a*x + b*x^2)^(5/2)*((x^(3/2)*(a + b*x)^(5/2))/(4*d) - (((8*b*c - 5*a*d)*
Sqrt[x]*(a + b*x)^(5/2))/(3*b*d) - (((48*b^2*c^2 - 40*a*b*c*d - 5*a^2*d^2)
*Sqrt[x]*(a + b*x)^(3/2))/(2*d) - (3*(((64*b^3*c^3 - 112*a*b^2*c^2*d + 40*
a^2*b*c*d^2 + 5*a^3*d^3)*Sqrt[x]*Sqrt[a + b*x])/d - ((2*(128*b^4*c^4 - 320
*a*b^3*c^3*d + 240*a^2*b^2*c^2*d^2 - 40*a^3*b*c*d^3 - 5*a^4*d^4)*ArcTanh[(
Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(Sqrt[b]*d) - (256*b*c^(3/2)*(b*c - a*d)^
(5/2)*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/d)/(2*d)
))/(4*d))/(6*b*d))/(8*d)))/(x^(5/2)*(a + b*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 112
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*(m + n +
p + 1))), x] - Simp[1/(f*(m + n + p + 1))   Int[(a + b*x)^(m - 1)*(c + d*x)
^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*(b*e - a
*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] &&
GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m, 2*n, 2*p
] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$-\frac{5\left(\frac{128c^{2}\left(a^{3}b^{\frac{3}{2}}d^{3}-3a^{2}b^{\frac{5}{2}}c\,d^{2}+3a\,b^{\frac{7}{2}}c^{2}d-b^{\frac{9}{2}}c^{3}\right)\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{5}+\left(\left(a^{4}d^{4}+8a^{3}bc\,d^{3}-48a^{2}b^{2}c^{2}d^{2}+64a\,b^{3}c^{3}d-\frac{128}{5}c\right.\right.\right.}{}$
risch	$\frac{\left(48b^{3}d^{3}x^{3}+136a\,b^{2}d^{3}x^{2}-64b^{3}c\,d^{2}x^{2}+118a^{2}b\,d^{3}x-208a\,b^{2}c\,d^{2}x+96b^{3}c^{2}dx+15a^{3}d^{3}-264a^{2}bc\,d^{2}+432a\,b^{2}c^{2}d-192b^{3}c^{3}\right)x}{192b\,d^{4}\sqrt{x(bx+a)}}$
default	Expression too large to display

input
```
int((b*x^2+a*x)^(5/2)/x/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-5/64/b^(3/2)/(c*(a*d-b*c))^(1/2)*(128/5*c^2*(a^3*b^(3/2)*d^3-3*a^2*b^(5/2
)*c*d^2+3*a*b^(7/2)*c^2*d-b^(9/2)*c^3)*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*
d-b*c))^(1/2))+((a^4*d^4+8*a^3*b*c*d^3-48*a^2*b^2*c^2*d^2+64*a*b^3*c^3*d-1
28/5*c^4*b^4)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-d*(16/5*(d^3*x^3-4/3*c*
d^2*x^2+2*c^2*d*x-4*c^3)*b^(7/2)+(8/5*(17/3*d^2*x^2-26/3*c*d*x+18*c^2)*b^(
5/2)+d*a*(2/5*(59/3*d*x-44*c)*b^(3/2)+b^(1/2)*a*d))*d*a)*(x*(b*x+a))^(1/2)
)*(c*(a*d-b*c))^(1/2))/d^5
```

Fricas [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 1216, normalized size of antiderivative = 4.18

$$\int \frac{(ax+bx^2)^{5/2}}{x(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a*x)^(5/2)/x/(d*x+c),x, algorithm="fricas")
```

output
```
[-1/384*(3*(128*b^4*c^4 - 320*a*b^3*c^3*d + 240*a^2*b^2*c^2*d^2 - 40*a^3*b
*c*d^3 - 5*a^4*d^4)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) -
 384*(b^4*c^3 - 2*a*b^3*c^2*d + a^2*b^2*c*d^2)*sqrt(b*c^2 - a*c*d)*log((a*
c + (2*b*c - a*d)*x - 2*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c))
- 2*(48*b^4*d^4*x^3 - 192*b^4*c^3*d + 432*a*b^3*c^2*d^2 - 264*a^2*b^2*c*d^
3 + 15*a^3*b*d^4 - 8*(8*b^4*c*d^3 - 17*a*b^3*d^4)*x^2 + 2*(48*b^4*c^2*d^2
- 104*a*b^3*c*d^3 + 59*a^2*b^2*d^4)*x)*sqrt(b*x^2 + a*x))/(b^2*d^5), 1/384
*(768*(b^4*c^3 - 2*a*b^3*c^2*d + a^2*b^2*c*d^2)*sqrt(-b*c^2 + a*c*d)*arcta
n(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) - 3*(128*b^4*c^4 -
 320*a*b^3*c^3*d + 240*a^2*b^2*c^2*d^2 - 40*a^3*b*c*d^3 - 5*a^4*d^4)*sqrt(
b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(48*b^4*d^4*x^3 - 192*
b^4*c^3*d + 432*a*b^3*c^2*d^2 - 264*a^2*b^2*c*d^3 + 15*a^3*b*d^4 - 8*(8*b^
4*c*d^3 - 17*a*b^3*d^4)*x^2 + 2*(48*b^4*c^2*d^2 - 104*a*b^3*c*d^3 + 59*a^2
*b^2*d^4)*x)*sqrt(b*x^2 + a*x))/(b^2*d^5), -1/192*(3*(128*b^4*c^4 - 320*a*
b^3*c^3*d + 240*a^2*b^2*c^2*d^2 - 40*a^3*b*c*d^3 - 5*a^4*d^4)*sqrt(-b)*arc
tan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - 192*(b^4*c^3 - 2*a*b^3*c^2*d +
 a^2*b^2*c*d^2)*sqrt(b*c^2 - a*c*d)*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*
c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) - (48*b^4*d^4*x^3 - 192*b^4*c^3
*d + 432*a*b^3*c^2*d^2 - 264*a^2*b^2*c*d^3 + 15*a^3*b*d^4 - 8*(8*b^4*c*d^3
 - 17*a*b^3*d^4)*x^2 + 2*(48*b^4*c^2*d^2 - 104*a*b^3*c*d^3 + 59*a^2*b^2...
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{5/2}}{x(c+dx)}\,dx = \int \frac{(x(a+bx))^{\frac{5}{2}}}{x\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a*x)**(5/2)/x/(d*x+c),x)
```

output
```
Integral((x*(a + b*x))**(5/2)/(x*(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ax+bx^2)^{5/2}}{x(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a*x)^(5/2)/x/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ax+bx^2)^{5/2}}{x(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a*x)^(5/2)/x/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{5/2}}{x(c+dx)}\,dx = \int \frac{(b\,x^2+a\,x)^{5/2}}{x\,(c+d\,x)}\,dx$$

input
```
int((a*x + b*x^2)^(5/2)/(x*(c + d*x)),x)
```

output
```
int((a*x + b*x^2)^(5/2)/(x*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 10.78 (sec) , antiderivative size = 699, normalized size of antiderivative = 2.40

$$\int \frac{(ax+bx^2)^{5/2}}{x(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a*x)^(5/2)/x/(d*x+c),x)
```

output
```
( - 384*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b
*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*b**2*c*d**2 + 768*s
qrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqr
t(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**3*c**2*d - 384*sqrt(c)*sqrt(
a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)
*sqrt(b))/(sqrt(c)*sqrt(b)))*b**4*c**3 - 384*sqrt(c)*sqrt(a*d - b*c)*atan(
(sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(
c)*sqrt(b)))*a**2*b**2*c*d**2 + 768*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d
 - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b
)))*a*b**3*c**2*d - 384*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sq
rt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**4*c**
3 + 15*sqrt(x)*sqrt(a + b*x)*a**3*b*d**4 - 264*sqrt(x)*sqrt(a + b*x)*a**2*
b**2*c*d**3 + 118*sqrt(x)*sqrt(a + b*x)*a**2*b**2*d**4*x + 432*sqrt(x)*sqr
t(a + b*x)*a*b**3*c**2*d**2 - 208*sqrt(x)*sqrt(a + b*x)*a*b**3*c*d**3*x +
136*sqrt(x)*sqrt(a + b*x)*a*b**3*d**4*x**2 - 192*sqrt(x)*sqrt(a + b*x)*b**
4*c**3*d + 96*sqrt(x)*sqrt(a + b*x)*b**4*c**2*d**2*x - 64*sqrt(x)*sqrt(a +
 b*x)*b**4*c*d**3*x**2 + 48*sqrt(x)*sqrt(a + b*x)*b**4*d**4*x**3 - 15*sqrt
(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4*d**4 - 120*sqrt(b)
*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*b*c*d**3 + 720*sqrt(b
)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*b**2*c**2*d**2 - ...
```

3.118 $\int \frac{(ax+bx^2)^{5/2}}{x^2(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 219

$$\int \frac{(ax+bx^2)^{5/2}}{x^2(c+dx)}\,dx = \frac{(4bc-5ad)(2bc-ad)\sqrt{ax+bx^2}}{8d^3} - \frac{(6bc-5ad)\,(ax+bx^2)^{3/2}}{12d^2x} + \frac{(ax+bx^2)^{5/2}}{3dx^2} - \frac{(16b^3c^3-40ab^2c^2d+30a^2bcd^2-5a^3d^3)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{8\sqrt{b}d^4} + \frac{2\sqrt{c}(bc-ad)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{d^4}$$

output
```
1/8*(-5*a*d+4*b*c)*(-a*d+2*b*c)*(b*x^2+a*x)^(1/2)/d^3-1/12*(-5*a*d+6*b*c)*
(b*x^2+a*x)^(3/2)/d^2/x+1/3*(b*x^2+a*x)^(5/2)/d/x^2-1/8*(-5*a^3*d^3+30*a^2
*b*c*d^2-40*a*b^2*c^2*d+16*b^3*c^3)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b
^(1/2)/d^4+2*c^(1/2)*(-a*d+b*c)^(5/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(
b*x^2+a*x)^(1/2))/d^4
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.18 (sec) , antiderivative size = 499, normalized size of antiderivative = 2.28

$$\int \frac{(ax+bx^2)^{5/2}}{x^2(c+dx)}\,dx = \frac{(x(a+bx))^{5/2}\left(b\sqrt{c}d\sqrt{x}\sqrt{a+bx}(33a^2d^2+2abd(-27c+13dx)+4b^2(6c^2-3cdx+\right.}{}$$

input
```
Integrate[(a*x + b*x^2)^(5/2)/(x^2*(c + d*x)),x]
```

output
```
((x*(a + b*x))^(5/2)*(b*Sqrt[c]*d*Sqrt[x]*Sqrt[a + b*x]*(33*a^2*d^2 + 2*a*
b*d*(-27*c + 13*d*x) + 4*b^2*(6*c^2 - 3*c*d*x + 2*d^2*x^2)) - 48*(b*c - a*
d)^2*(b*c - a*d - I*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d])*Sqrt[-(b*c) + 2*a*d -
 (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*ArcTan[(Sqrt[-(b*c) + 2*a*d - (2*I
)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*(-Sqrt[a] + Sqrt[a +
b*x]))] - 48*(b*c - a*d)^2*(b*c - a*d + I*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d])
*Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*ArcTan[(Sqrt
[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]
*(-Sqrt[a] + Sqrt[a + b*x]))] + 6*Sqrt[b]*Sqrt[c]*(16*b^3*c^3 - 40*a*b^2*c
^2*d + 30*a^2*b*c*d^2 - 5*a^3*d^3)*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sq
rt[a + b*x])]))/(24*b*Sqrt[c]*d^4*x^(5/2)*(a + b*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.20, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1261, 112, 27, 171, 27, 171, 27, 175, 65, 104, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{5/2}}{x^2(c+dx)}\,dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{\left(ax+bx^2\right)^{5/2}\int \frac{\sqrt{x}(a+bx)^{5/2}}{c+dx}dx}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \text{ 112}$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{\sqrt{x}(a+bx)^{5/2}}{3d}-\frac{\int \frac{(a+bx)^{3/2}(ac+(6bc-5ad)x)}{2\sqrt{x}(c+dx)}dx}{3d}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{\sqrt{x}(a+bx)^{5/2}}{3d}-\frac{\int \frac{(a+bx)^{3/2}(ac+(6bc-5ad)x)}{\sqrt{x}(c+dx)}dx}{6d}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \text{ 171}$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{\sqrt{x}(a+bx)^{5/2}}{3d}-\frac{\frac{\int -\frac{3\sqrt{a+bx}(ac(2bc-3ad)+(4bc-5ad)(2bc-ad)x)}{2\sqrt{x}(c+dx)}dx}{2d}+\frac{\sqrt{x}(a+bx)^{3/2}(6bc-5ad)}{2d}}{6d}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{\sqrt{x}(a+bx)^{5/2}}{3d}-\frac{\frac{\sqrt{x}(a+bx)^{3/2}(6bc-5ad)}{2d}-\frac{3\int \frac{\sqrt{a+bx}(ac(2bc-3ad)+(4bc-5ad)(2bc-ad)x)}{\sqrt{x}(c+dx)}dx}{4d}}{6d}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \text{ 171}$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{\sqrt{x}(a+bx)^{5/2}}{3d}-\frac{\frac{\sqrt{x}(a+bx)^{3/2}(6bc-5ad)}{2d}-\frac{3\left(\frac{\int -\frac{ac\left(8b^2c^2-18abdc+11a^2d^2\right)+\left(16b^3c^3-40ab^2dc^2+30a^2bd^2c-5a^3d^3\right)x}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}+\sqrt{x}\sqrt{a \cdots}\right.}{4d}}{6d}\right.}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{\sqrt{x}(a+bx)^{5/2}}{3d}-\frac{\frac{\sqrt{x}(a+bx)^{3/2}(6bc-5ad)}{2d}-\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}(4bc-5ad)(2bc-ad)}{d}-\frac{\int \frac{ac\left(8b^2c^2-18abdc+11a^2d^2\right)+\left(16b^3c^3-40ab^2dc^2+3\cdots\right.}{\sqrt{x}\sqrt{a+bx}(c+dx)}\cdots}{2d}\cdots\right.}{4d}\cdots}{6d}\cdots\right.}{x^{5/2}(a+bx)^{5/2}}$$

$\downarrow$ 175

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{\sqrt{x}(a+bx)^{5/2}}{3d}-\frac{\frac{\sqrt{x}(a+bx)^{3/2}(6bc-5ad)}{2d}-\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}(4bc-5ad)(2bc-ad)}{d}-\frac{\frac{\left(-5a^3d^3+30a^2bcd^2-40ab^2c^2d+16b^3c^3\right)\int \frac{1}{\sqrt{x}\sqrt{a+b\cdots}}\cdots}{d}\cdots}{2d}\cdots\right.}{4d}\cdots}{6d}\cdots\right.}{x^{5/2}(a+bx)^{5/2}}$$

$\downarrow$ 65

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{\sqrt{x}(a+bx)^{5/2}}{3d}-\frac{\frac{\sqrt{x}(a+bx)^{3/2}(6bc-5ad)}{2d}-\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}(4bc-5ad)(2bc-ad)}{d}-\frac{\frac{2\left(-5a^3d^3+30a^2bcd^2-40ab^2c^2d+16b^3c^3\right)\int \frac{1}{1-\frac{bx}{a+bx}}\cdots}{d}\cdots}{2d}\cdots\right.}{4d}\cdots}{6d}\cdots\right.}{x^{5/2}(a+bx)^{5/2}}$$

$\downarrow$ 104

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{\sqrt{x}(a+bx)^{5/2}}{3d}-\frac{\frac{\sqrt{x}(a+bx)^{3/2}(6bc-5ad)}{2d}-\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}(4bc-5ad)(2bc-ad)}{d}-\frac{\frac{2\left(-5a^3d^3+30a^2bcd^2-40ab^2c^2d+16b^3c^3\right)\int \frac{1}{1-\frac{bx}{a+bx}}\cdots}{d}\cdots}{2d}\cdots\right.}{4d}\cdots}{6d}\cdots\right.}{x^{5/2}(a+bx)^{5/2}}$$

$\downarrow$ 219

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{\sqrt{x}(a+bx)^{5/2}}{3d}-\frac{\frac{\sqrt{x}(a+bx)^{3/2}(6bc-5ad)}{2d}-\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}(4bc-5ad)(2bc-ad)}{d}-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)\left(-5a^3d^3+30a^2bcd^2-40ab^2c^2\right.}{\sqrt{b}d}}{2d}\right)}{4d}}{6d}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$\Big\downarrow$ 221

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{\sqrt{x}(a+bx)^{5/2}}{3d}-\frac{\frac{\sqrt{x}(a+bx)^{3/2}(6bc-5ad)}{2d}-\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}(4bc-5ad)(2bc-ad)}{d}-\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)\left(-5a^3d^3+30a^2bcd^2-40ab^2c^2\right.}{\sqrt{b}d}\right)}{4d}}{6d}\right)}{x^{5/2}(a+bx)^{5/2}}$$

input
```
Int[(a*x + b*x^2)^(5/2)/(x^2*(c + d*x)),x]
```

output
```
((a*x + b*x^2)^(5/2)*((Sqrt[x]*(a + b*x)^(5/2))/(3*d) - (((6*b*c - 5*a*d)*
Sqrt[x]*(a + b*x)^(3/2))/(2*d) - (3*(((4*b*c - 5*a*d)*(2*b*c - a*d)*Sqrt[x
]*Sqrt[a + b*x])/d - ((2*(16*b^3*c^3 - 40*a*b^2*c^2*d + 30*a^2*b*c*d^2 - 5
*a^3*d^3)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(Sqrt[b]*d) - (32*Sqrt
[c]*(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a +
b*x])])/d)/(2*d)))/(4*d))/(6*d)))/(x^(5/2)*(a + b*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 112
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*(m + n +
p + 1))), x] - Simp[1/(f*(m + n + p + 1))   Int[(a + b*x)^(m - 1)*(c + d*x)
^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*(b*e - a
*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] &&
GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m, 2*n, 2*p
] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$-\frac{-\frac{d\sqrt{x(bx+a)}\left(8b^2d^2x^2+26ab\,d^2x-12b^2cxd+33a^2d^2-54abcd+24b^2c^2\right)}{24}-\frac{\left(5a^3d^3-30a^2bc\,d^2+40a\,b^2c^2d-16b^3c^3\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx\cdots}}{x\sqrt{b}}\cdots\right.}{8\sqrt{b}}\cdots}{d^4}$
risch	$\frac{(8b^2d^2x^2+26ab\,d^2x-12b^2cxd+33a^2d^2-54abcd+24b^2c^2)x(bx+a)}{24d^3\sqrt{x(bx+a)}}+\frac{\frac{\left(5a^3d^3-30a^2bc\,d^2+40a\,b^2c^2d-16b^3c^3\right)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b}\cdots\right.}{d\sqrt{b}}\cdots}{\cdots}$
default	Expression too large to display

input
```
int((b*x^2+a*x)^(5/2)/x^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/d^4*(-1/24*d*(x*(b*x+a))^(1/2)*(8*b^2*d^2*x^2+26*a*b*d^2*x-12*b^2*c*d*x
+33*a^2*d^2-54*a*b*c*d+24*b^2*c^2)-1/8*(5*a^3*d^3-30*a^2*b*c*d^2+40*a*b^2*
c^2*d-16*b^3*c^3)/b^(1/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-2*(a*d-b*c)
^3*c/(c*(a*d-b*c))^(1/2)*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))
)
```

Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 932, normalized size of antiderivative = 4.26

$$\int \frac{(ax+bx^2)^{5/2}}{x^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a*x)^(5/2)/x^2/(d*x+c),x, algorithm="fricas")
```

output
```
[-1/48*(3*(16*b^3*c^3 - 40*a*b^2*c^2*d + 30*a^2*b*c*d^2 - 5*a^3*d^3)*sqrt(
b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 48*(b^3*c^2 - 2*a*b^2*c*
d + a^2*b*d^2)*sqrt(b*c^2 - a*c*d)*log((a*c + (2*b*c - a*d)*x + 2*sqrt(b*c
^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) - 2*(8*b^3*d^3*x^2 + 24*b^3*c^2*
d - 54*a*b^2*c*d^2 + 33*a^2*b*d^3 - 2*(6*b^3*c*d^2 - 13*a*b^2*d^3)*x)*sqrt
(b*x^2 + a*x))/(b*d^4), -1/48*(96*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*sqrt
(-b*c^2 + a*c*d)*arctan(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*
c)) + 3*(16*b^3*c^3 - 40*a*b^2*c^2*d + 30*a^2*b*c*d^2 - 5*a^3*d^3)*sqrt(b)
*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*(8*b^3*d^3*x^2 + 24*b^3*
c^2*d - 54*a*b^2*c*d^2 + 33*a^2*b*d^3 - 2*(6*b^3*c*d^2 - 13*a*b^2*d^3)*x)*
sqrt(b*x^2 + a*x))/(b*d^4), 1/24*(3*(16*b^3*c^3 - 40*a*b^2*c^2*d + 30*a^2*
b*c*d^2 - 5*a^3*d^3)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a))
 + 24*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*sqrt(b*c^2 - a*c*d)*log((a*c + (
2*b*c - a*d)*x + 2*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) + (8*
b^3*d^3*x^2 + 24*b^3*c^2*d - 54*a*b^2*c*d^2 + 33*a^2*b*d^3 - 2*(6*b^3*c*d^
2 - 13*a*b^2*d^3)*x)*sqrt(b*x^2 + a*x))/(b*d^4), -1/24*(48*(b^3*c^2 - 2*a*
b^2*c*d + a^2*b*d^2)*sqrt(-b*c^2 + a*c*d)*arctan(sqrt(-b*c^2 + a*c*d)*sqrt
(b*x^2 + a*x)/(b*c*x + a*c)) - 3*(16*b^3*c^3 - 40*a*b^2*c^2*d + 30*a^2*b*c
*d^2 - 5*a^3*d^3)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) -
(8*b^3*d^3*x^2 + 24*b^3*c^2*d - 54*a*b^2*c*d^2 + 33*a^2*b*d^3 - 2*(6*b^...
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{5/2}}{x^2(c+dx)}\,dx = \int \frac{(x(a+bx))^{\frac{5}{2}}}{x^2\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a*x)**(5/2)/x**2/(d*x+c),x)
```

output
```
Integral((x*(a + b*x))**(5/2)/(x**2*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{5/2}}{x^2(c+dx)}\,dx = \int \frac{(bx^2+ax)^{\frac{5}{2}}}{(dx+c)x^2}\,dx$$

input
```
integrate((b*x^2+a*x)^(5/2)/x^2/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(5/2)/((d*x + c)*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ax+bx^2)^{5/2}}{x^2(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a*x)^(5/2)/x^2/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{5/2}}{x^2(c+dx)}\,dx = \int \frac{(b\,x^2+a\,x)^{5/2}}{x^2\,(c+d\,x)}\,dx$$

input
```
int((a*x + b*x^2)^(5/2)/(x^2*(c + d*x)),x)
```

output
```
int((a*x + b*x^2)^(5/2)/(x^2*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 570, normalized size of antiderivative = 2.60

$$\int \frac{(ax+bx^2)^{5/2}}{x^2(c+dx)}\,dx = \frac{48\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)a^2b\,d^2-96\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-b}}{}\right.}{}$$

input
```
int((b*x^2+a*x)^(5/2)/x^2/(d*x+c),x)
```

output
```
(48*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x)
- sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*b*d**2 - 96*sqrt(c)*sqr
t(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(
d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c*d + 48*sqrt(c)*sqrt(a*d - b*c)*ata
n((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqr
t(c)*sqrt(b)))*b**3*c**2 + 48*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c
) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*
*2*b*d**2 - 96*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqr
t(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c*d + 48*s
qrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqr
t(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**3*c**2 + 33*sqrt(x)*sqrt(a + b
*x)*a**2*b*d**3 - 54*sqrt(x)*sqrt(a + b*x)*a*b**2*c*d**2 + 26*sqrt(x)*sqrt
(a + b*x)*a*b**2*d**3*x + 24*sqrt(x)*sqrt(a + b*x)*b**3*c**2*d - 12*sqrt(x
)*sqrt(a + b*x)*b**3*c*d**2*x + 8*sqrt(x)*sqrt(a + b*x)*b**3*d**3*x**2 + 1
5*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*d**3 - 90*sq
rt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*b*c*d**2 + 120*s
qrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b**2*c**2*d - 48*s
qrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b**3*c**3)/(24*b*d**
4)
```

3.119 $\int \frac{(ax+bx^2)^{5/2}}{x^3(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 166

$$\int \frac{(ax+bx^2)^{5/2}}{x^3(c+dx)}\,dx = -\frac{b(4bc-7ad)\sqrt{ax+bx^2}}{4d^2} + \frac{b(ax+bx^2)^{3/2}}{2dx} + \frac{\sqrt{b}(8b^2c^2-20abcd+15a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4d^3} - \frac{2(bc-ad)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{\sqrt{c}d^3}$$

output
```
-1/4*b*(-7*a*d+4*b*c)*(b*x^2+a*x)^(1/2)/d^2+1/2*b*(b*x^2+a*x)^(3/2)/d/x+1/
4*b^(1/2)*(15*a^2*d^2-20*a*b*c*d+8*b^2*c^2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^
(1/2))/d^3-2*(-a*d+b*c)^(5/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*
x)^(1/2))/c^(1/2)/d^3
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.15

$$\int \frac{(ax+bx^2)^{5/2}}{x^3(c+dx)}\,dx = \frac{(x(a+bx))^{5/2}\left(\frac{bd\sqrt{x}(-4bc+9ad+2bdx)}{(a+bx)^2}-\frac{8(-bc+ad)^{5/2}\arctan\left(\frac{-d\sqrt{x}\sqrt{a+bx}+\sqrt{b}(c+dx)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{c}(a+bx)^{5/2}}-\frac{\sqrt{b}(8b^2c^2-}{}\right.}{4d^3x^{5/2}}$$

input
```
Integrate[(a*x + b*x^2)^(5/2)/(x^3*(c + d*x)),x]
```

output
```
((x*(a + b*x))^(5/2)*((b*d*Sqrt[x]*(-4*b*c + 9*a*d + 2*b*d*x))/(a + b*x)^2
 - (8*(-(b*c) + a*d)^(5/2)*ArcTan[(-(d*Sqrt[x]*Sqrt[a + b*x]) + Sqrt[b]*(c
 + d*x))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(Sqrt[c]*(a + b*x)^(5/2)) - (Sqrt[
b]*(8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a +
 b*x]])/(a + b*x)^(5/2)))/(4*d^3*x^(5/2))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.23, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1261, 113, 27, 171, 27, 175, 65, 104, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{5/2}}{x^3(c+dx)}\,dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{(ax+bx^2)^{5/2}\int \frac{(a+bx)^{5/2}}{\sqrt{x}(c+dx)}dx}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \text{113}$$

$$\frac{(ax+bx^2)^{5/2}\left(\frac{\int -\frac{\sqrt{a+bx}(a(bc-4ad)+b(4bc-7ad)x)}{2\sqrt{x}(c+dx)}dx}{2d}+\frac{b\sqrt{x}(a+bx)^{3/2}}{2d}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{b\sqrt{x}(a+bx)^{3/2}}{2d}-\frac{\int \frac{\sqrt{a+bx}(a(bc-4ad)+b(4bc-7ad)x)}{\sqrt{x}(c+dx)}dx}{4d}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \; 171$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{b\sqrt{x}(a+bx)^{3/2}}{2d}-\frac{\frac{\int -\frac{a\left(4b^2c^2-9abdc+8a^2d^2\right)+b\left(8b^2c^2-20abdc+15a^2d^2\right)x}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}+\frac{b\sqrt{x}\sqrt{a+bx}(4bc-7ad)}{d}}{4d}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{b\sqrt{x}(a+bx)^{3/2}}{2d}-\frac{\frac{b\sqrt{x}\sqrt{a+bx}(4bc-7ad)}{d}-\frac{\int \frac{a\left(4b^2c^2-9abdc+8a^2d^2\right)+b\left(8b^2c^2-20abdc+15a^2d^2\right)x}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{2d}}{4d}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \; 175$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{b\sqrt{x}(a+bx)^{3/2}}{2d}-\frac{\frac{b\sqrt{x}\sqrt{a+bx}(4bc-7ad)}{d}-\frac{\frac{b\left(15a^2d^2-20abcd+8b^2c^2\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{d}-\frac{8(bc-ad)^3\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}}{2d}}{4d}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \; 65$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{b\sqrt{x}(a+bx)^{3/2}}{2d}-\frac{\frac{b\sqrt{x}\sqrt{a+bx}(4bc-7ad)}{d}-\frac{\frac{2b\left(15a^2d^2-20abcd+8b^2c^2\right)\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}-\frac{8(bc-ad)^3\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}}{2d}}{4d}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \; 104$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{b\sqrt{x}(a+bx)^{3/2}}{2d}-\frac{\frac{b\sqrt{x}\sqrt{a+bx}(4bc-7ad)}{d}-\frac{\frac{2b\left(15a^2d^2-20abcd+8b^2c^2\right)\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}-\frac{16(bc-ad)^3\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}}{2d}}{4d}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(ax+bx^2)^{5/2}\left(\frac{b\sqrt{x}(a+bx)^{3/2}}{2d}-\frac{\frac{b\sqrt{x}\sqrt{a+bx}(4bc-7ad)}{d}-\frac{\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)(15a^2d^2-20abcd+8b^2c^2)}{d}-\frac{16(bc-ad)^3\int\frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}}{2d}}{4d}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{(ax+bx^2)^{5/2}\left(\frac{b\sqrt{x}(a+bx)^{3/2}}{2d}-\frac{\frac{b\sqrt{x}\sqrt{a+bx}(4bc-7ad)}{d}-\frac{\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)(15a^2d^2-20abcd+8b^2c^2)}{d}-\frac{16(bc-ad)^{5/2}\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{\sqrt{c}d}}{2d}}{4d}\right)}{x^{5/2}(a+bx)^{5/2}}$$

input
```
Int[(a*x + b*x^2)^(5/2)/(x^3*(c + d*x)),x]
```

output
```
((a*x + b*x^2)^(5/2)*((b*Sqrt[x]*(a + b*x)^(3/2))/(2*d) - ((b*(4*b*c - 7*a
*d)*Sqrt[x]*Sqrt[a + b*x])/d - ((2*Sqrt[b]*(8*b^2*c^2 - 20*a*b*c*d + 15*a^
2*d^2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/d - (16*(b*c - a*d)^(5/2)
*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(Sqrt[c]*d))/
(2*d))/(4*d)))/(x^(5/2)*(a + b*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 113
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$-\frac{\frac{b\left(-d\sqrt{x(bx+a)}\,(2bdx+9ad-4bc)-\frac{\left(15a^2d^2-20abcd+8b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{\sqrt{b}}\right)}{4}+\frac{2(ad-bc)^3\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{\sqrt{c(ad-bc)}}}{d^3}$
risch	$\frac{b(2bdx+9ad-4bc)x(bx+a)}{4d^2\sqrt{x(bx+a)}}+\frac{\frac{\sqrt{b}\left(15a^2d^2-20abcd+8b^2c^2\right)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{d}-\frac{8\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\ln\left(\frac{-2}{\ldots}\right.}{\ldots}}{8d^2}$
default	Expression too large to display

input
```
int((b*x^2+a*x)^(5/2)/x^3/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/d^3*(1/4*b*(-d*(x*(b*x+a))^(1/2)*(2*b*d*x+9*a*d-4*b*c)-(15*a^2*d^2-20*a
*b*c*d+8*b^2*c^2)/b^(1/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2)))+2*(a*d-b*c
)^3/(c*(a*d-b*c))^(1/2)*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 722, normalized size of antiderivative = 4.35

$$\int \frac{(ax+bx^2)^{5/2}}{x^3(c+dx)}\,dx = \left[\frac{(8\,b^2c^2-20\,abcd+15\,a^2d^2)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+8\,(b^2c^2-2\,abcd\right.$$

$$-\frac{16\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{-\frac{bc-ad}{c}}\arctan\left(-\frac{\sqrt{bx^2+ax}c\sqrt{-\frac{bc-ad}{c}}}{(bc-ad)x}\right)-(8\,b^2c^2-20\,abcd+15\,a^2d^2)\sqrt{b}\log\left(2\,b\right.}{8\,d^3}$$

$$-\frac{(8\,b^2c^2-20\,abcd+15\,a^2d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-4\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)}{}\right.}{4\,d^3}$$

$$-\frac{(8\,b^2c^2-20\,abcd+15\,a^2d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+8\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{-\frac{bc-ad}{c}}\arctan\left(-\frac{\sqrt{bx}}{}\right.}{4\,d^3}$$

input
```
integrate((b*x^2+a*x)^(5/2)/x^3/(d*x+c),x, algorithm="fricas")
```

output
```
[1/8*((8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*sqrt(b)*log(2*b*x + a + 2*sqrt
(b*x^2 + a*x)*sqrt(b)) + 8*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d
)/c)*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)*c*sqrt((b*c - a*d)/c
))/(d*x + c)) + 2*(2*b^2*d^2*x - 4*b^2*c*d + 9*a*b*d^2)*sqrt(b*x^2 + a*x))
/d^3, -1/8*(16*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/c)*arctan
(-sqrt(b*x^2 + a*x)*c*sqrt(-(b*c - a*d)/c)/((b*c - a*d)*x)) - (8*b^2*c^2 -
 20*a*b*c*d + 15*a^2*d^2)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt
(b)) - 2*(2*b^2*d^2*x - 4*b^2*c*d + 9*a*b*d^2)*sqrt(b*x^2 + a*x))/d^3, -1/
4*((8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)
*sqrt(-b)/(b*x + a)) - 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/
c)*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)*c*sqrt((b*c - a*d)/c))
/(d*x + c)) - (2*b^2*d^2*x - 4*b^2*c*d + 9*a*b*d^2)*sqrt(b*x^2 + a*x))/d^3
, -1/4*((8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*sqrt(-b)*arctan(sqrt(b*x^2 +
 a*x)*sqrt(-b)/(b*x + a)) + 8*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c -
 a*d)/c)*arctan(-sqrt(b*x^2 + a*x)*c*sqrt(-(b*c - a*d)/c)/((b*c - a*d)*x))
 - (2*b^2*d^2*x - 4*b^2*c*d + 9*a*b*d^2)*sqrt(b*x^2 + a*x))/d^3]
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{5/2}}{x^3(c+dx)}\,dx = \int \frac{(x(a+bx))^{\frac{5}{2}}}{x^3\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a*x)**(5/2)/x**3/(d*x+c),x)
```

output
```
Integral((x*(a + b*x))**(5/2)/(x**3*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{5/2}}{x^3(c+dx)}\,dx = \int \frac{(bx^2+ax)^{\frac{5}{2}}}{(dx+c)x^3}\,dx$$

input
```
integrate((b*x^2+a*x)^(5/2)/x^3/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(5/2)/((d*x + c)*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ax+bx^2)^{5/2}}{x^3(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a*x)^(5/2)/x^3/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{5/2}}{x^3(c+dx)}\,dx=\int \frac{(b\,x^2+a\,x)^{5/2}}{x^3\,(c+d\,x)}\,dx$$

input
```
int((a*x + b*x^2)^(5/2)/(x^3*(c + d*x)),x)
```

output
```
int((a*x + b*x^2)^(5/2)/(x^3*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.87

$$\int \frac{(ax+bx^2)^{5/2}}{x^3(c+dx)}\,dx=\frac{-8\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)a^2d^2+16\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}}{}\right.}{}$$

input
```
int((b*x^2+a*x)^(5/2)/x^3/(d*x+c),x)
```

output
```
( - 8*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x
) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*d**2 + 16*sqrt(c)*sqr
t(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(
d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b*c*d - 8*sqrt(c)*sqrt(a*d - b*c)*atan((s
qrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)
*sqrt(b)))*b**2*c**2 - 8*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + s
qrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*d*
*2 + 16*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b
*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b*c*d - 8*sqrt(c)*sqrt
(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d
)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**2*c**2 + 9*sqrt(x)*sqrt(a + b*x)*a*b*c*d*
*2 - 4*sqrt(x)*sqrt(a + b*x)*b**2*c**2*d + 2*sqrt(x)*sqrt(a + b*x)*b**2*c*
d**2*x + 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*c*
d**2 - 20*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b*c**2*
d + 8*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b**2*c**3)/(4
*c*d**3)
```

3.120 $\int \frac{\left(ax+bx^2\right)^{5/2}}{x^4(c+dx)} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 147

$$\int \frac{\left(ax+bx^2\right)^{5/2}}{x^4(c+dx)} \, dx = \frac{b(bc+2ad)\sqrt{ax+bx^2}}{cd} - \frac{2a\left(ax+bx^2\right)^{3/2}}{cx^2} - \frac{b^{3/2}(2bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{d^2} + \frac{2(bc-ad)^{5/2}\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{3/2}d^2}$$

output
```
b*(2*a*d+b*c)*(b*x^2+a*x)^(1/2)/c/d-2*a*(b*x^2+a*x)^(3/2)/c/x^2-b^(3/2)*(-
5*a*d+2*b*c)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/d^2+2*(-a*d+b*c)^(5/2)*a
rctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(3/2)/d^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.43 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.01

$$\int \frac{\left(ax+bx^2\right)^{5/2}}{x^4(c+dx)} \, dx = \frac{(x(a+bx))^{5/2} \left(bc^{3/2}d\sqrt{a+bx}(-2a^2d+b^2cx) - 2(bc-ad)^2 \left(bc-ad-i\sqrt{a}\sqrt{d}\sqrt{bc\right.\right.}{}$$

input
```
Integrate[(a*x + b*x^2)^(5/2)/(x^4*(c + d*x)),x]
```

output
```
((x*(a + b*x))^(5/2)*(b*c^(3/2)*d*Sqrt[a + b*x]*(-2*a^2*d + b^2*c*x) - 2*(
b*c - a*d)^2*(b*c - a*d - I*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d])*Sqrt[-(b*c) +
 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x]*ArcTan[(Sqrt[-(b*c
) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*(-Sqr
t[a] + Sqrt[a + b*x]))] - 2*(b*c - a*d)^2*(b*c - a*d + I*Sqrt[a]*Sqrt[d]*S
qrt[b*c - a*d])*Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d
]]*Sqrt[x]*ArcTan[(Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c -
a*d]]*Sqrt[x])/(Sqrt[c]*(-Sqrt[a] + Sqrt[a + b*x]))] + 2*b^(5/2)*c^(5/2)*(
2*b*c - 5*a*d)*Sqrt[x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])
]))/(b*c^(5/2)*d^2*x^3*(a + b*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.26, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1261, 109, 27, 171, 27, 175, 65, 104, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{5/2}}{x^4(c+dx)}dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{(ax+bx^2)^{5/2}\int \frac{(a+bx)^{5/2}}{x^{3/2}(c+dx)}dx}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \text{109}$$

$$\frac{(ax+bx^2)^{5/2}\left(-\frac{2\int -\frac{\sqrt{a+bx}(a(4bc-ad)+b(bc+2ad)x)}{2\sqrt{x}(c+dx)}dx}{c}-\frac{2a(a+bx)^{3/2}}{c\sqrt{x}}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{\int \frac{\sqrt{a+bx}(a(4bc-ad)+b(bc+2ad)x)}{\sqrt{x}(c+dx)}dx}{c}-\frac{2a(a+bx)^{3/2}}{c\sqrt{x}}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \; 171$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{\frac{\int -\frac{c(2bc-5ad)xb^2+a\left(b^2c^2-6abdc+2a^2d^2\right)}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}+\frac{b\sqrt{x}\sqrt{a+bx}(2ad+bc)}{d}}{c}-\frac{2a(a+bx)^{3/2}}{c\sqrt{x}}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{\frac{b\sqrt{x}\sqrt{a+bx}(2ad+bc)}{d}-\frac{\int \frac{c(2bc-5ad)xb^2+a\left(b^2c^2-6abdc+2a^2d^2\right)}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{2d}}{c}-\frac{2a(a+bx)^{3/2}}{c\sqrt{x}}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \; 175$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{\frac{b\sqrt{x}\sqrt{a+bx}(2ad+bc)}{d}-\frac{\frac{b^2c(2bc-5ad)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{d}-\frac{2(bc-ad)^3\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}}{2d}}{c}-\frac{2a(a+bx)^{3/2}}{c\sqrt{x}}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \; 65$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{\frac{b\sqrt{x}\sqrt{a+bx}(2ad+bc)}{d}-\frac{\frac{2b^2c(2bc-5ad)\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}-\frac{2(bc-ad)^3\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}}{2d}}{c}-\frac{2a(a+bx)^{3/2}}{c\sqrt{x}}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \; 104$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{\frac{b\sqrt{x}\sqrt{a+bx}(2ad+bc)}{d}-\frac{\frac{2b^2c(2bc-5ad)\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}-\frac{4(bc-ad)^3\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}}{2d}}{c}-\frac{2a(a+bx)^{3/2}}{c\sqrt{x}}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{\frac{b\sqrt{x}\sqrt{a+bx}(2ad+bc)}{d}-\frac{\frac{2b^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)(2bc-5ad)}{d}-\frac{4(bc-ad)^3\int\frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}}{2d}}{c}-\frac{2a(a+bx)^{3/2}}{c\sqrt{x}}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$\Big\downarrow$ 221

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{\frac{b\sqrt{x}\sqrt{a+bx}(2ad+bc)}{d}-\frac{\frac{2b^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)(2bc-5ad)}{d}-\frac{4(bc-ad)^{5/2}\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{\sqrt{c}d}}{2d}}{c}-\frac{2a(a+bx)^{3/2}}{c\sqrt{x}}\right)}{x^{5/2}(a+bx)^{5/2}}$$

input
```
Int[(a*x + b*x^2)^(5/2)/(x^4*(c + d*x)),x]
```

output
```
((a*x + b*x^2)^(5/2)*((-2*a*(a + b*x)^(3/2))/(c*Sqrt[x]) + ((b*(b*c + 2*a*
d)*Sqrt[x]*Sqrt[a + b*x])/d - ((2*b^(3/2)*c*(2*b*c - 5*a*d)*ArcTanh[(Sqrt[
b]*Sqrt[x])/Sqrt[a + b*x]])/d - (4*(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b*c - a
*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(Sqrt[c]*d))/(2*d))/c))/(x^(5/2)*(a
 + b*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] && !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$-\frac{2\left(x(-ad+bc)^3\sqrt{b}\,\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)-\frac{\sqrt{c(ad-bc)}\left(x\left(5cda\,b^2-2b^3c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)+d\left(b^2cx-2a^2d\right)\sqrt{b}\,\sqrt{x(bx+a}}{2}\right)}{\sqrt{b}\,\sqrt{c(ad-bc)}\,cx\,d^2}$
risch	$-\frac{(bx+a)(-b^2cx+2a^2d)}{c\sqrt{x(bx+a)}\,d}+\frac{\frac{2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^2}}\,\sqrt{b\left(x+\frac{c}{d}\right)^2}}{x+\frac{c}{d}}\right.}{d^2\sqrt{-\frac{c(ad-bc)}{d^2}}}}{2cd}$
default	Expression too large to display

input
```
int((b*x^2+a*x)^(5/2)/x^4/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2/b^(1/2)/(c*(a*d-b*c))^(1/2)*(x*(-a*d+b*c)^3*b^(1/2)*arctan((x*(b*x+a))^
(1/2)/x*c/(c*(a*d-b*c))^(1/2))-1/2*(c*(a*d-b*c))^(1/2)*(x*(5*a*b^2*c*d-2*b
^3*c^2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))+d*(b^2*c*x-2*a^2*d)*b^(1/2)*(
x*(b*x+a))^(1/2)))/c/x/d^2
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 685, normalized size of antiderivative = 4.66

$$\int \frac{(ax+bx^2)^{5/2}}{x^4(c+dx)}\,dx = \left[-\frac{(2\,b^2c^2-5\,abcd)\sqrt{b}x\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,(b^2c^2-2\,abcd+a^2d^2)}{2\,cd^2}\right.$$

input
```
integrate((b*x^2+a*x)^(5/2)/x^4/(d*x+c),x, algorithm="fricas")
```

output
```
[-1/2*((2*b^2*c^2 - 5*a*b*c*d)*sqrt(b)*x*log(2*b*x + a + 2*sqrt(b*x^2 + a*
x)*sqrt(b)) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*sqrt((b*c - a*d)/c)*log(
(a*c + (2*b*c - a*d)*x + 2*sqrt(b*x^2 + a*x)*c*sqrt((b*c - a*d)/c))/(d*x +
 c)) - 2*(b^2*c*d*x - 2*a^2*d^2)*sqrt(b*x^2 + a*x))/(c*d^2*x), 1/2*(4*(b^2
*c^2 - 2*a*b*c*d + a^2*d^2)*x*sqrt(-(b*c - a*d)/c)*arctan(-sqrt(b*x^2 + a*
x)*c*sqrt(-(b*c - a*d)/c)/((b*c - a*d)*x)) - (2*b^2*c^2 - 5*a*b*c*d)*sqrt(
b)*x*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(b^2*c*d*x - 2*a^2*d
^2)*sqrt(b*x^2 + a*x))/(c*d^2*x), ((2*b^2*c^2 - 5*a*b*c*d)*sqrt(-b)*x*arct
an(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*x*sqrt((b*c - a*d)/c)*log((a*c + (2*b*c - a*d)*x + 2*sqrt(b*x^2 + a*x)*c*
sqrt((b*c - a*d)/c))/(d*x + c)) + (b^2*c*d*x - 2*a^2*d^2)*sqrt(b*x^2 + a*x
))/(c*d^2*x), ((2*b^2*c^2 - 5*a*b*c*d)*sqrt(-b)*x*arctan(sqrt(b*x^2 + a*x)
*sqrt(-b)/(b*x + a)) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*sqrt(-(b*c - a*
d)/c)*arctan(-sqrt(b*x^2 + a*x)*c*sqrt(-(b*c - a*d)/c)/((b*c - a*d)*x)) +
(b^2*c*d*x - 2*a^2*d^2)*sqrt(b*x^2 + a*x))/(c*d^2*x)]
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{5/2}}{x^4(c+dx)}\,dx = \int \frac{(x(a+bx))^{\frac{5}{2}}}{x^4\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a*x)**(5/2)/x**4/(d*x+c),x)
```

output
```
Integral((x*(a + b*x))**(5/2)/(x**4*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{5/2}}{x^4(c+dx)}\,dx = \int \frac{(bx^2+ax)^{\frac{5}{2}}}{(dx+c)x^4}\,dx$$

input
```
integrate((b*x^2+a*x)^(5/2)/x^4/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(5/2)/((d*x + c)*x^4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ax+bx^2)^{5/2}}{x^4(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a*x)^(5/2)/x^4/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{5/2}}{x^4(c+dx)}\,dx = \int \frac{(b\,x^2+a\,x)^{5/2}}{x^4\,(c+d\,x)}\,dx$$

input
```
int((a*x + b*x^2)^(5/2)/(x^4*(c + d*x)),x)
```

output
```
int((a*x + b*x^2)^(5/2)/(x^4*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 466, normalized size of antiderivative = 3.17

$$\int \frac{(ax+bx^2)^{5/2}}{x^4(c+dx)}\,dx = \frac{8\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)a^2d^2x-16\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}}{}\right.}{}$$

input
```
int((b*x^2+a*x)^(5/2)/x^4/(d*x+c),x)
```

output
```
(8*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) -
 sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*d**2*x - 16*sqrt(c)*sqrt
(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d
)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b*c*d*x + 8*sqrt(c)*sqrt(a*d - b*c)*atan((
sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c
)*sqrt(b)))*b**2*c**2*x + 8*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c)
+ sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2
*d**2*x - 16*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(
a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b*c*d*x + 8*sqrt(
c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)
*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**2*c**2*x - 8*sqrt(x)*sqrt(a + b*x)
*a**2*c*d**2 + 4*sqrt(x)*sqrt(a + b*x)*b**2*c**2*d*x + 20*sqrt(b)*log((sqr
t(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b*c**2*d*x - 8*sqrt(b)*log((sqrt(
a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b**2*c**3*x - 8*sqrt(b)*a**2*c*d**2*x
 - sqrt(b)*a*b*c**2*d*x)/(4*c**2*d**2*x)
```

3.121 $\int \frac{(ax+bx^2)^{5/2}}{x^5(c+dx)} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 142

$$\int \frac{(ax+bx^2)^{5/2}}{x^5(c+dx)} \, dx = -\frac{2a(2bc-ad)\sqrt{ax+bx^2}}{c^2x} - \frac{2a(ax+bx^2)^{3/2}}{3cx^3} + \frac{2b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{d} - \frac{2(bc-ad)^{5/2}\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{5/2}d}$$

output
```
-2*a*(-a*d+2*b*c)*(b*x^2+a*x)^(1/2)/c^2/x-2/3*a*(b*x^2+a*x)^(3/2)/c/x^3+2*
b^(5/2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/d-2*(-a*d+b*c)^(5/2)*arctanh(
(-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(5/2)/d
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.23

$$\int \frac{(ax+bx^2)^{5/2}}{x^5(c+dx)} \, dx = -\frac{2\sqrt{x(a+bx)}\left(3(-bc+ad)^{5/2}x^{3/2}\arctan\left(\frac{-d\sqrt{x}\sqrt{a+bx}+\sqrt{b}(c+dx)}{\sqrt{c}\sqrt{-bc+ad}}\right)+\sqrt{c}\left(ad\sqrt{a+bx}(7bcx+a(c-3dx))+\right.\right.}{3c^{5/2}dx^2\sqrt{a+bx}}$$

input
```
Integrate[(a*x + b*x^2)^(5/2)/(x^5*(c + d*x)),x]
```

output
```
(-2*Sqrt[x*(a + b*x)]*(3*(-(b*c) + a*d)^(5/2)*x^(3/2)*ArcTan[(-(d*Sqrt[x]*
Sqrt[a + b*x]) + Sqrt[b]*(c + d*x))/(Sqrt[c]*Sqrt[-(b*c) + a*d])] + Sqrt[c
]*(a*d*Sqrt[a + b*x]*(7*b*c*x + a*(c - 3*d*x)) + 3*b^(5/2)*c^2*x^(3/2)*Log
[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])))/(3*c^(5/2)*d*x^2*Sqrt[a + b*x])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 904 vs. 2(142) = 284.

Time = 2.13 (sec) , antiderivative size = 904, normalized size of antiderivative = 6.37, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1260, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{5/2}}{x^5(c+dx)}dx$$

$$\Big\downarrow \text{1260}$$

$$\int \left(-\frac{d^5(ax+bx^2)^{5/2}}{c^5(c+dx)}+\frac{d^4(ax+bx^2)^{5/2}}{c^5x}-\frac{d^3(ax+bx^2)^{5/2}}{c^4x^2}+\frac{d^2(ax+bx^2)^{5/2}}{c^3x^3}-\frac{d(ax+bx^2)^{5/2}}{c^2x^4}+\frac{(ax+bx}{cx^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3d^4\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+ax}}\right)a^5}{128b^{5/2}c^5}+\frac{5d^3\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+ax}}\right)a^4}{64b^{3/2}c^4}+\frac{5d^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+ax}}\right)a^3}{8\sqrt{b}c^3}-\frac{3d^4(a+2bx)\sqrt{bx^2+ax}a^3}{128b^2c^5}-\frac{15\sqrt{b}d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+ax}}\right)a^2}{4c^2}-\frac{5d^3(a+2bx)\sqrt{bx^2+ax}a^2}{64bc^4}+\frac{2d\sqrt{bx^2+ax}a^2}{c^2x}-\frac{5d^3\left(bx^2+ax\right)^{3/2}a}{24c^4}+\frac{d^4(a+2bx)\left(bx^2+ax\right)^{3/2}a}{16bc^5}+\frac{5b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+ax}}\right)a}{c}-\frac{9bd\sqrt{bx^2+ax}a}{4c^2}-\frac{5d^2(a+2bx)\sqrt{bx^2+ax}a}{8c^3}-\frac{10b\sqrt{bx^2+ax}a}{3cx}-\frac{d^3\left(bx^2+ax\right)^{5/2}}{4c^4x}+\frac{2d^2\left(bx^2+ax\right)^{5/2}}{c^3x^2}-\frac{2\left(bx^2+ax\right)^{5/2}}{3cx^4}-\frac{5bd^2\left(bx^2+ax\right)^{3/2}}{3c^3}-\frac{d^2\left(16b^2c^2-22abdc+3a^2d^2-6bd(2bc-ad)x\right)\left(bx^2+ax\right)^{3/2}}{48bc^5}+\frac{(2bc-ad)\left(128b^4c^4-256ab^3dc^3+112a^2b^2d^2c^2+16a^3bd^3c+3a^4d^4\right)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+ax}}\right)}{128b^{5/2}c^5d}-\frac{(bc-ad)^{5/2}\operatorname{arctanh}\left(\frac{ac+(2bc-ad)x}{2\sqrt{c}\sqrt{bc-ad}\sqrt{bx^2+ax}}\right)}{c^{5/2}d}-\frac{b^2dx\sqrt{bx^2+ax}}{2c^2}-\frac{\left(128b^4c^4-288ab^3dc^3+176a^2b^2d^2c^2-10a^3bd^3c-3a^4d^4-2bd(2bc-ad)\left(16b^2c^2-16abdc-3a^2d^2\right)x\right)\sqrt{bx^2+}}{128b^2c^5}\frac{5b^2\sqrt{bx^2+ax}}{3c}$$

input
```
Int[(a*x + b*x^2)^(5/2)/(x^5*(c + d*x)),x]
```

output
```
(5*b^2*Sqrt[a*x + b*x^2])/(3*c) - (9*a*b*d*Sqrt[a*x + b*x^2])/(4*c^2) - (1
0*a*b*Sqrt[a*x + b*x^2])/(3*c*x) + (2*a^2*d*Sqrt[a*x + b*x^2])/(c^2*x) - (
b^2*d*x*Sqrt[a*x + b*x^2])/(2*c^2) - (5*a*d^2*(a + 2*b*x)*Sqrt[a*x + b*x^2
])/(8*c^3) - (5*a^2*d^3*(a + 2*b*x)*Sqrt[a*x + b*x^2])/(64*b*c^4) - (3*a^3
*d^4*(a + 2*b*x)*Sqrt[a*x + b*x^2])/(128*b^2*c^5) - ((128*b^4*c^4 - 288*a*
b^3*c^3*d + 176*a^2*b^2*c^2*d^2 - 10*a^3*b*c*d^3 - 3*a^4*d^4 - 2*b*d*(2*b*
c - a*d)*(16*b^2*c^2 - 16*a*b*c*d - 3*a^2*d^2)*x)*Sqrt[a*x + b*x^2])/(128*
b^2*c^5) - (5*b*d^2*(a*x + b*x^2)^(3/2))/(3*c^3) - (5*a*d^3*(a*x + b*x^2)^
(3/2))/(24*c^4) + (a*d^4*(a + 2*b*x)*(a*x + b*x^2)^(3/2))/(16*b*c^5) - (d^
2*(16*b^2*c^2 - 22*a*b*c*d + 3*a^2*d^2 - 6*b*d*(2*b*c - a*d)*x)*(a*x + b*x
^2)^(3/2))/(48*b*c^5) - (2*(a*x + b*x^2)^(5/2))/(3*c*x^4) + (2*d^2*(a*x +
b*x^2)^(5/2))/(c^3*x^2) - (d^3*(a*x + b*x^2)^(5/2))/(4*c^4*x) + (5*a*b^(3/
2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/c - (15*a^2*Sqrt[b]*d*ArcTanh[(
Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*c^2) + (5*a^3*d^2*ArcTanh[(Sqrt[b]*x)/Sq
rt[a*x + b*x^2]])/(8*Sqrt[b]*c^3) + (5*a^4*d^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a*
x + b*x^2]])/(64*b^(3/2)*c^4) + (3*a^5*d^4*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x +
b*x^2]])/(128*b^(5/2)*c^5) + ((2*b*c - a*d)*(128*b^4*c^4 - 256*a*b^3*c^3*d
 + 112*a^2*b^2*c^2*d^2 + 16*a^3*b*c*d^3 + 3*a^4*d^4)*ArcTanh[(Sqrt[b]*x)/S
qrt[a*x + b*x^2]])/(128*b^(5/2)*c^5*d) - ((b*c - a*d)^(5/2)*ArcTanh[(a*c +
 (2*b*c - a*d)*x)/(2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[a*x + b*x^2])])/(c^(5...
```

Defintions of rubi rules used

rule 1260
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x + c*x^2)^p
, (d + e*x)^m*(f + g*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ
[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + n + 2*p + 1, 0] && ILtQ[m, 0] && ILtQ
[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$-\frac{2\left(3x^2(ad-bc)^3\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)+\left(-3b^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)c^2x^2+d\sqrt{x(bx+a)}\,((7bx+a)c-3adx)a\right)\sqrt{c(ad-b}}{3\sqrt{c(ad-bc)}\,c^2x^2d}$
risch	$-\frac{2a(bx+a)(-3adx+7cbx+ac)}{3c^2\sqrt{x(bx+a)}\,x}+\frac{\frac{b^{\frac{5}{2}}c^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{d}-\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc}{}}{}\right)}{d^2}}{c^2}$
default	Expression too large to display

input
```
int((b*x^2+a*x)^(5/2)/x^5/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(3*x^2*(a*d-b*c)^3*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))+
(-3*b^(5/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*c^2*x^2+d*(x*(b*x+a))^(1/
2)*((7*b*x+a)*c-3*a*d*x)*a)*(c*(a*d-b*c))^(1/2))/(c*(a*d-b*c))^(1/2)/c^2/x
^2/d
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 698, normalized size of antiderivative = 4.92

$$\int \frac{(ax+bx^2)^{5/2}}{x^5(c+dx)}\,dx = \left[\frac{3\,b^{\frac{5}{2}}c^2x^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+3\,(b^2c^2-2\,abcd+a^2d^2)x^2\sqrt{\frac{bc-ad}{c}}\log\left(\right.}{3\,c^2dx^2}\right.$$

$$-\frac{6\,\sqrt{-b}b^2c^2x^2\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-3\,(b^2c^2-2\,abcd+a^2d^2)x^2\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)x-2\,\sqrt{bx^2+ax}c\sqrt{\frac{bc-ad}{c}}}{dx+c}\right.}{3\,c^2dx^2}$$

$$-\frac{2\left(3\,\sqrt{-b}b^2c^2x^2\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+3\,(b^2c^2-2\,abcd+a^2d^2)x^2\sqrt{-\frac{bc-ad}{c}}\arctan\left(-\frac{\sqrt{bx^2+ax}c\sqrt{-\frac{bc-ad}{c}}}{(bc-ad)x}\right)\right.}{3\,c^2dx^2}$$

input
```
integrate((b*x^2+a*x)^(5/2)/x^5/(d*x+c),x, algorithm="fricas")
```

output
```
[1/3*(3*b^(5/2)*c^2*x^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 3*(
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*sqrt((b*c - a*d)/c)*log((a*c + (2*b*c -
 a*d)*x - 2*sqrt(b*x^2 + a*x)*c*sqrt((b*c - a*d)/c))/(d*x + c)) - 2*(a^2*c
*d + (7*a*b*c*d - 3*a^2*d^2)*x)*sqrt(b*x^2 + a*x))/(c^2*d*x^2), 1/3*(3*b^(
5/2)*c^2*x^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 6*(b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*x^2*sqrt(-(b*c - a*d)/c)*arctan(-sqrt(b*x^2 + a*x)*c*s
qrt(-(b*c - a*d)/c)/((b*c - a*d)*x)) - 2*(a^2*c*d + (7*a*b*c*d - 3*a^2*d^2
)*x)*sqrt(b*x^2 + a*x))/(c^2*d*x^2), -1/3*(6*sqrt(-b)*b^2*c^2*x^2*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x
^2*sqrt((b*c - a*d)/c)*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)*c*
sqrt((b*c - a*d)/c))/(d*x + c)) + 2*(a^2*c*d + (7*a*b*c*d - 3*a^2*d^2)*x)*
sqrt(b*x^2 + a*x))/(c^2*d*x^2), -2/3*(3*sqrt(-b)*b^2*c^2*x^2*arctan(sqrt(b
*x^2 + a*x)*sqrt(-b)/(b*x + a)) + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*sq
rt(-(b*c - a*d)/c)*arctan(-sqrt(b*x^2 + a*x)*c*sqrt(-(b*c - a*d)/c)/((b*c
- a*d)*x)) + (a^2*c*d + (7*a*b*c*d - 3*a^2*d^2)*x)*sqrt(b*x^2 + a*x))/(c^2
*d*x^2)]
```

Sympy [F]

$$\int \frac{(ax + bx^2)^{5/2}}{x^5(c + dx)} \, dx = \int \frac{(x(a + bx))^{\frac{5}{2}}}{x^5 \, (c + dx)} \, dx$$

input
```
integrate((b*x**2+a*x)**(5/2)/x**5/(d*x+c),x)
```

output
```
Integral((x*(a + b*x))**(5/2)/(x**5*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(ax + bx^2)^{5/2}}{x^5(c + dx)} dx = \int \frac{(bx^2 + ax)^{\frac{5}{2}}}{(dx + c)x^5} dx$$

input
```
integrate((b*x^2+a*x)^(5/2)/x^5/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(5/2)/((d*x + c)*x^5), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ax + bx^2)^{5/2}}{x^5(c + dx)} dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a*x)^(5/2)/x^5/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax + bx^2)^{5/2}}{x^5(c + dx)} dx = \int \frac{(b\,x^2 + a\,x)^{5/2}}{x^5\,(c + d\,x)} dx$$

input
```
int((a*x + b*x^2)^(5/2)/(x^5*(c + d*x)),x)
```

output
```
int((a*x + b*x^2)^(5/2)/(x^5*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 471, normalized size of antiderivative = 3.32

$$\int \frac{(ax+bx^2)^{5/2}}{x^5(c+dx)}\,dx = \frac{-2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)a^2d^2x^2+4\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-}}{}\right.}{}$$

input

```
int((b*x^2+a*x)^(5/2)/x^5/(d*x+c),x)
```

output

```
(2*( - 3*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a +
b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*d**2*x**2 + 6*sqrt
(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x
)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b*c*d*x**2 - 3*sqrt(c)*sqrt(a*d -
b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(
b))/(sqrt(c)*sqrt(b)))*b**2*c**2*x**2 - 3*sqrt(c)*sqrt(a*d - b*c)*atan((sq
rt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*
sqrt(b)))*a**2*d**2*x**2 + 6*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c)
 + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b
*c*d*x**2 - 3*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt
(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**2*c**2*x**2 - s
qrt(x)*sqrt(a + b*x)*a**2*c**2*d + 3*sqrt(x)*sqrt(a + b*x)*a**2*c*d**2*x -
 7*sqrt(x)*sqrt(a + b*x)*a*b*c**2*d*x + 3*sqrt(b)*log((sqrt(a + b*x) + sqr
t(x)*sqrt(b))/sqrt(a))*b**2*c**3*x**2 - sqrt(b)*a**2*c*d**2*x**2 + sqrt(b)
*a*b*c**2*d*x**2))/(3*c**3*d*x**2)
```

3.122 $\int \frac{(ax+bx^2)^{5/2}}{x^6(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 138

$$\int \frac{(ax+bx^2)^{5/2}}{x^6(c+dx)}\,dx = -\frac{2(bc-ad)^2\sqrt{ax+bx^2}}{c^3x} - \frac{2(bc-ad)\,(ax+bx^2)^{3/2}}{3c^2x^3} - \frac{2(ax+bx^2)^{5/2}}{5cx^5} + \frac{2(bc-ad)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{7/2}}$$

output
```
-2*(-a*d+b*c)^2*(b*x^2+a*x)^(1/2)/c^3/x-2/3*(-a*d+b*c)*(b*x^2+a*x)^(3/2)/c
^2/x^3-2/5*(b*x^2+a*x)^(5/2)/c/x^5+2*(-a*d+b*c)^(5/2)*arctanh((-a*d+b*c)^(
1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(7/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.03 (sec) , antiderivative size = 421, normalized size of antiderivative = 3.05

$$\int \frac{(ax+bx^2)^{5/2}}{x^6(c+dx)}\,dx = \frac{2(x(a+bx))^{5/2}\left(-bc^{3/2}\sqrt{a+bx}(23b^2c^2x^2+abcx(11c-35dx)+a^2(3c^2-5cdx+1\right.}{}$$

input
```
Integrate[(a*x + b*x^2)^(5/2)/(x^6*(c + d*x)),x]
```

output
```
(2*(x*(a + b*x))^(5/2)*(-(b*c^(3/2)*Sqrt[a + b*x]*(23*b^2*c^2*x^2 + a*b*c*
x*(11*c - 35*d*x) + a^2*(3*c^2 - 5*c*d*x + 15*d^2*x^2))) - 15*(b*c - a*d)^
2*(b*c - a*d - I*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d])*Sqrt[-(b*c) + 2*a*d - (2
*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*x^(5/2)*ArcTan[(Sqrt[-(b*c) + 2*a*d -
 (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*(-Sqrt[a] + Sqrt
[a + b*x]))] - 15*(b*c - a*d)^2*(b*c - a*d + I*Sqrt[a]*Sqrt[d]*Sqrt[b*c -
a*d])*Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*x^(5/2)
*ArcTan[(Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt
[x])/(Sqrt[c]*(-Sqrt[a] + Sqrt[a + b*x]))]))/(15*b*c^(9/2)*x^5*(a + b*x)^(
5/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1261, 105, 105, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{5/2}}{x^6(c+dx)}dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{(ax+bx^2)^{5/2}\int \frac{(a+bx)^{5/2}}{x^{7/2}(c+dx)}dx}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \text{105}$$

$$\frac{(ax+bx^2)^{5/2}\left(\frac{(bc-ad)\int \frac{(a+bx)^{3/2}}{x^{5/2}(c+dx)}dx}{c}-\frac{2(a+bx)^{5/2}}{5cx^{5/2}}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \text{105}$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{(bc-ad)\left(\frac{(bc-ad)\int\frac{\sqrt{a+bx}}{x^{3/2}(c+dx)}dx}{c}-\frac{2(a+bx)^{3/2}}{3cx^{3/2}}\right)}{c}-\frac{2(a+bx)^{5/2}}{5cx^{5/2}}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$\Big\downarrow$ 105

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{(bc-ad)\left(\frac{(bc-ad)\left(\frac{(bc-ad)\int\frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{c}-\frac{2\sqrt{a+bx}}{c\sqrt{x}}\right)}{c}-\frac{2(a+bx)^{3/2}}{3cx^{3/2}}\right)}{c}-\frac{2(a+bx)^{5/2}}{5cx^{5/2}}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$\Big\downarrow$ 104

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{(bc-ad)\left(\frac{(bc-ad)\left(\frac{2(bc-ad)\int\frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}-\frac{2\sqrt{a+bx}}{c\sqrt{x}}\right)}{c}-\frac{2(a+bx)^{3/2}}{3cx^{3/2}}\right)}{c}-\frac{2(a+bx)^{5/2}}{5cx^{5/2}}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$\Big\downarrow$ 221

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{(bc-ad)\left(\frac{(bc-ad)\left(\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}}-\frac{2\sqrt{a+bx}}{c\sqrt{x}}\right)}{c}-\frac{2(a+bx)^{3/2}}{3cx^{3/2}}\right)}{c}-\frac{2(a+bx)^{5/2}}{5cx^{5/2}}\right)}{x^{5/2}(a+bx)^{5/2}}$$

input
```
Int[(a*x + b*x^2)^(5/2)/(x^6*(c + d*x)),x]
```

output
```
((a*x + b*x^2)^(5/2)*((-2*(a + b*x)^(5/2))/(5*c*x^(5/2)) + ((b*c - a*d)*((
-2*(a + b*x)^(3/2))/(3*c*x^(3/2)) + ((b*c - a*d)*((-2*Sqrt[a + b*x])/(c*Sq
rt[x]) + (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqr
t[a + b*x])])/c^(3/2)))/c))/c))/(x^(5/2)*(a + b*x)^(5/2))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{-\frac{2\sqrt{c(ad-bc)}\left(\left(\frac{23}{3}b^2x^2+\frac{11}{3}abx+a^2\right)c^2-\frac{5adx(7bx+a)c}{3}+5a^2d^2x^2\right)\sqrt{x(bx+a)}}{5}+2x^3(ad-bc)^3\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{c^3x^3\sqrt{c(ad-bc)}}$
risch	$-\frac{2(bx+a)(15a^2d^2x^2-35abcd\,x^2+23b^2c^2x^2-5a^2cdx+11ab\,c^2x+3a^2c^2)}{15c^3\sqrt{x(bx+a)}\,x^2}+\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\ln\left(-\frac{2c(a}{c}\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a*x)^(5/2)/x^6/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
2/5/(c*(a*d-b*c))^(1/2)*(-(c*(a*d-b*c))^(1/2)*((23/3*b^2*x^2+11/3*a*b*x+a^
2)*c^2-5/3*a*d*x*(7*b*x+a)*c+5*a^2*d^2*x^2)*(x*(b*x+a))^(1/2)+5*x^3*(a*d-b
*c)^3*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2)))/c^3/x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.37

$$\int \frac{(ax+bx^2)^{5/2}}{x^6(c+dx)}\,dx = \left[\frac{15\,(b^2c^2-2\,abcd+a^2d^2)x^3\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)x+2\,\sqrt{bx^2+ax}c\sqrt{\frac{bc-ad}{c}}}{dx+c}\right)-2\,(3\,a^2c^2}{15\,c^3x^3}\right.$$

input
```
integrate((b*x^2+a*x)^(5/2)/x^6/(d*x+c),x, algorithm="fricas")
```

output
```
[1/15*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^3*sqrt((b*c - a*d)/c)*log((a*c
 + (2*b*c - a*d)*x + 2*sqrt(b*x^2 + a*x)*c*sqrt((b*c - a*d)/c))/(d*x + c))
 - 2*(3*a^2*c^2 + (23*b^2*c^2 - 35*a*b*c*d + 15*a^2*d^2)*x^2 + (11*a*b*c^2
 - 5*a^2*c*d)*x)*sqrt(b*x^2 + a*x))/(c^3*x^3), 2/15*(15*(b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*x^3*sqrt(-(b*c - a*d)/c)*arctan(-sqrt(b*x^2 + a*x)*c*sqrt(-(
b*c - a*d)/c)/((b*c - a*d)*x)) - (3*a^2*c^2 + (23*b^2*c^2 - 35*a*b*c*d + 1
5*a^2*d^2)*x^2 + (11*a*b*c^2 - 5*a^2*c*d)*x)*sqrt(b*x^2 + a*x))/(c^3*x^3)]
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{5/2}}{x^6(c+dx)}\,dx = \int \frac{(x(a+bx))^{\frac{5}{2}}}{x^6\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a*x)**(5/2)/x**6/(d*x+c),x)
```

output
```
Integral((x*(a + b*x))**(5/2)/(x**6*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{5/2}}{x^6(c+dx)}\,dx = \int \frac{(bx^2+ax)^{\frac{5}{2}}}{(dx+c)x^6}\,dx$$

input
```
integrate((b*x^2+a*x)^(5/2)/x^6/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(5/2)/((d*x + c)*x^6), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(118) = 236$.

Time = 0.14 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.70

$$\int \frac{(ax+bx^2)^{5/2}}{x^6(c+dx)}\,dx = \frac{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)d+\sqrt{b}c}{\sqrt{-bc^2+acd}}\right)}{\sqrt{-bc^2+acd}c^3} + \frac{2\left(45\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4ab^2c^2-45\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4a^2bcd+15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4a^3d^2+45\right.}{}$$

input
```
integrate((b*x^2+a*x)^(5/2)/x^6/(d*x+c),x, algorithm="giac")
```

output
```
2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(-((sqrt(b)*x
- sqrt(b*x^2 + a*x))*d + sqrt(b)*c)/sqrt(-b*c^2 + a*c*d))/(sqrt(-b*c^2 + a
*c*d)*c^3) + 2/15*(45*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a*b^2*c^2 - 45*(sq
rt(b)*x - sqrt(b*x^2 + a*x))^4*a^2*b*c*d + 15*(sqrt(b)*x - sqrt(b*x^2 + a*
x))^4*a^3*d^2 + 45*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^2*b^(3/2)*c^2 - 15*
(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^3*sqrt(b)*c*d + 35*(sqrt(b)*x - sqrt(b
*x^2 + a*x))^2*a^3*b*c^2 - 5*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^4*c*d + 1
5*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^4*sqrt(b)*c^2 + 3*a^5*c^2)/((sqrt(b)*x
 - sqrt(b*x^2 + a*x))^5*c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{5/2}}{x^6(c+dx)}\,dx = \int \frac{\left(b\,x^2+a\,x\right)^{5/2}}{x^6\,\left(c+d\,x\right)}\,dx$$

input
```
int((a*x + b*x^2)^(5/2)/(x^6*(c + d*x)),x)
```

output
```
int((a*x + b*x^2)^(5/2)/(x^6*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 507, normalized size of antiderivative = 3.67

$$\int \frac{(ax+bx^2)^{5/2}}{x^6(c+dx)}\,dx = \frac{2\sqrt{c}\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\sqrt{bx+a}-\sqrt{x}\sqrt{d}\sqrt{b}}{\sqrt{c}\sqrt{b}}\right)a^2d^2x^3-4\sqrt{c}\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}\cdots}{}\right.}{}$$

input
```
int((b*x^2+a*x)^(5/2)/x^6/(d*x+c),x)
```

output
```
(2*(15*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*
x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*d**2*x**3 - 30*sqrt(
c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)
*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b*c*d*x**3 + 15*sqrt(c)*sqrt(a*d -
b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(
b))/(sqrt(c)*sqrt(b)))*b**2*c**2*x**3 + 15*sqrt(c)*sqrt(a*d - b*c)*atan((s
qrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)
*sqrt(b)))*a**2*d**2*x**3 - 30*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*
c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a
*b*c*d*x**3 + 15*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*s
qrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**2*c**2*x**3
- 3*sqrt(x)*sqrt(a + b*x)*a**2*c**3 + 5*sqrt(x)*sqrt(a + b*x)*a**2*c**2*d*
x - 15*sqrt(x)*sqrt(a + b*x)*a**2*c*d**2*x**2 - 11*sqrt(x)*sqrt(a + b*x)*a
*b*c**3*x + 35*sqrt(x)*sqrt(a + b*x)*a*b*c**2*d*x**2 - 23*sqrt(x)*sqrt(a +
 b*x)*b**2*c**3*x**2 + 9*sqrt(b)*a**2*c*d**2*x**3 - 17*sqrt(b)*a*b*c**2*d*
x**3 + 5*sqrt(b)*b**2*c**3*x**3))/(15*c**4*x**3)
```

3.123 $\int \frac{(ax+bx^2)^{5/2}}{x^7(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 220

$$\int \frac{(ax+bx^2)^{5/2}}{x^7(c+dx)}\,dx = -\frac{2a(10bc-7ad)\sqrt{ax+bx^2}}{35c^2x^3} - \frac{2(45b^2c^2-77abcd+35a^2d^2)\sqrt{ax+bx^2}}{105c^3x^2} - \frac{2(15b^3c^3-161ab^2c^2d+245a^2bcd^2-105a^3d^3)\sqrt{ax+bx^2}}{105ac^4x} - \frac{2a(ax+bx^2)^{3/2}}{7cx^5} - \frac{2d(bc-ad)^{5/2}\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{9/2}}$$

output

```
-2/35*a*(-7*a*d+10*b*c)*(b*x^2+a*x)^(1/2)/c^2/x^3-2/105*(35*a^2*d^2-77*a*b
*c*d+45*b^2*c^2)*(b*x^2+a*x)^(1/2)/c^3/x^2-2/105*(-105*a^3*d^3+245*a^2*b*c
*d^2-161*a*b^2*c^2*d+15*b^3*c^3)*(b*x^2+a*x)^(1/2)/a/c^4/x-2/7*a*(b*x^2+a*
x)^(3/2)/c/x^5-2*d*(-a*d+b*c)^(5/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*
x^2+a*x)^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.95

$$\int \frac{(ax+bx^2)^{5/2}}{x^7(c+dx)}\,dx = \frac{2(x(a+bx))^{5/2}\left(-\frac{\sqrt{c}(15b^3c^3x^3+ab^2c^2x^2(45c-161dx)+a^2bcx(45c^2-77cdx+245d^2x^2)+a^3(15c^3-21c^2dx+}{a(a+bx)^2}\right.}{105c^{9/2}x^6}$$

input
```
Integrate[(a*x + b*x^2)^(5/2)/(x^7*(c + d*x)),x]
```

output
```
(2*(x*(a + b*x))^(5/2)*(-((Sqrt[c]*(15*b^3*c^3*x^3 + a*b^2*c^2*x^2*(45*c -
 161*d*x) + a^2*b*c*x*(45*c^2 - 77*c*d*x + 245*d^2*x^2) + a^3*(15*c^3 - 21
*c^2*d*x + 35*c*d^2*x^2 - 105*d^3*x^3)))/(a*(a + b*x)^2)) - (105*d*(-(b*c)
 + a*d)^(5/2)*x^(7/2)*ArcTan[(-(d*Sqrt[x]*Sqrt[a + b*x]) + Sqrt[b]*(c + d*
x))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(a + b*x)^(5/2)))/(105*c^(9/2)*x^6)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1261, 107, 105, 105, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{5/2}}{x^7(c+dx)}\,dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{(ax+bx^2)^{5/2}\int \frac{(a+bx)^{5/2}}{x^{9/2}(c+dx)}dx}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \text{107}$$

$$\frac{(ax+bx^2)^{5/2}\left(-\frac{d\int \frac{(a+bx)^{5/2}}{x^{7/2}(c+dx)}dx}{c}-\frac{2(a+bx)^{7/2}}{7acx^{7/2}}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \text{105}$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(-\frac{d\left(\frac{(bc-ad)\int \frac{(a+bx)^{3/2}}{x^{5/2}(c+dx)}dx}{c}-\frac{2(a+bx)^{5/2}}{5cx^{5/2}}\right)}{c}-\frac{2(a+bx)^{7/2}}{7acx^{7/2}}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \text{105}$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(-\frac{d\left(\frac{(bc-ad)\left(\frac{(bc-ad)\int \frac{\sqrt{a+bx}}{x^{3/2}(c+dx)}dx}{c}-\frac{2(a+bx)^{3/2}}{3cx^{3/2}}\right)}{c}-\frac{2(a+bx)^{5/2}}{5cx^{5/2}}\right)}{c}-\frac{2(a+bx)^{7/2}}{7acx^{7/2}}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \text{105}$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(-\frac{d\left(\frac{(bc-ad)\left(\frac{(bc-ad)\left(\frac{(bc-ad)\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{c}-\frac{2\sqrt{a+bx}}{c\sqrt{x}}\right)}{c}-\frac{2(a+bx)^{3/2}}{3cx^{3/2}}\right)}{c}-\frac{2(a+bx)^{5/2}}{5cx^{5/2}}\right)}{c}-\frac{2(a+bx)^{7/2}}{7acx^{7/2}}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \text{104}$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(-\frac{d\left(\frac{d\left(\frac{(bc-ad)\left(\frac{(bc-ad)\left(\frac{2(bc-ad)\int\frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}-\frac{2\sqrt{a+bx}}{c\sqrt{x}}\right)}{c}-\frac{2(a+bx)^{3/2}}{3cx^{3/2}}\right)}{c}-\frac{2(a+bx)^{5/2}}{5cx^{5/2}}\right)}{c}-\frac{2(a+bx)^{7/2}}{7acx^{7/2}}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(-\frac{d\left(\frac{d\left(\frac{(bc-ad)\left(\frac{(bc-ad)\left(\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}}-\frac{2\sqrt{a+bx}}{c\sqrt{x}}\right)}{c}-\frac{2(a+bx)^{3/2}}{3cx^{3/2}}\right)}{c}-\frac{2(a+bx)^{5/2}}{5cx^{5/2}}\right)}{c}-\frac{2(a+bx)^{7/2}}{7acx^{7/2}}\right)}{x^{5/2}(a+bx)^{5/2}}$$

input
```
Int[(a*x + b*x^2)^(5/2)/(x^7*(c + d*x)),x]
```

output
```
((a*x + b*x^2)^(5/2)*((-2*(a + b*x)^(7/2))/(7*a*c*x^(7/2)) - (d*((-2*(a +
b*x)^(5/2))/(5*c*x^(5/2)) + ((b*c - a*d)*((-2*(a + b*x)^(3/2))/(3*c*x^(3/2
)) + ((b*c - a*d)*((-2*Sqrt[a + b*x])/(c*Sqrt[x]) + (2*Sqrt[b*c - a*d]*Arc
Tanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/c^(3/2)))/c))/c))
/c))/(x^(5/2)*(a + b*x)^(5/2))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$-\frac{2\left(\left(\left(c^{3}(bx+a)^{3}-\frac{7dx\left(\frac{23}{3}b^{2}x^{2}+\frac{11}{3}abx+a^{2}\right)a\,c^{2}}{5}+\frac{7a^{2}d^{2}x^{2}(7bx+a)c}{3}-7a^{3}d^{3}x^{3}\right)\sqrt{c(ad-bc)}\,\sqrt{x(bx+a)}+7da\,x^{4}(ad-bc)^{3}\,\mathrm{ar}\right.\right.}{7\sqrt{c(ad-bc)}\,c^{4}x^{4}a}$
risch	$-\frac{2(bx+a)\left(-105a^{3}d^{3}x^{3}+245a^{2}bc\,d^{2}x^{3}-161a\,b^{2}c^{2}d\,x^{3}+15b^{3}c^{3}x^{3}+35a^{3}c\,d^{2}x^{2}-77a^{2}b\,c^{2}x^{2}d+45a\,b^{2}c^{3}x^{2}-21a^{3}c^{2}dx+45\right.}{105a\,c^{4}\sqrt{x(bx+a)}\,x^{3}}$
default	Expression too large to display

input
```
int((b*x^2+a*x)^(5/2)/x^7/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2/7*((c^3*(b*x+a)^3-7/5*d*x*(23/3*b^2*x^2+11/3*a*b*x+a^2)*a*c^2+7/3*a^2*d
^2*x^2*(7*b*x+a)*c-7*a^3*d^3*x^3)*(c*(a*d-b*c))^(1/2)*(x*(b*x+a))^(1/2)+7*
d*a*x^4*(a*d-b*c)^3*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2)))/(c*
(a*d-b*c))^(1/2)/c^4/x^4/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.04

$$\int \frac{(ax+bx^2)^{5/2}}{x^7(c+dx)}\,dx = \left[\frac{105\,(ab^2c^2d-2\,a^2bcd^2+a^3d^3)x^4\sqrt{\frac{bc-ad}{c}}\log\left(\frac{ac+(2\,bc-ad)x-2\,\sqrt{bx^2+axc}\sqrt{\frac{bc-ad}{c}}}{dx+c}\right)-2\,(1\right.$$

$$-\frac{2\left(105\,(ab^2c^2d-2\,a^2bcd^2+a^3d^3)x^4\sqrt{-\frac{bc-ad}{c}}\arctan\left(-\frac{\sqrt{bx^2+axc}\sqrt{-\frac{bc-ad}{c}}}{(bc-ad)x}\right)+(15\,a^3c^3+(15\,b^3c^3-161\,ab\right.}{10}$$

input
```
integrate((b*x^2+a*x)^(5/2)/x^7/(d*x+c),x, algorithm="fricas")
```

output
```
[1/105*(105*(a*b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3)*x^4*sqrt((b*c - a*d)/c
)*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)*c*sqrt((b*c - a*d)/c))/
(d*x + c)) - 2*(15*a^3*c^3 + (15*b^3*c^3 - 161*a*b^2*c^2*d + 245*a^2*b*c*d
^2 - 105*a^3*d^3)*x^3 + (45*a*b^2*c^3 - 77*a^2*b*c^2*d + 35*a^3*c*d^2)*x^2
 + 3*(15*a^2*b*c^3 - 7*a^3*c^2*d)*x)*sqrt(b*x^2 + a*x))/(a*c^4*x^4), -2/10
5*(105*(a*b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3)*x^4*sqrt(-(b*c - a*d)/c)*ar
ctan(-sqrt(b*x^2 + a*x)*c*sqrt(-(b*c - a*d)/c)/((b*c - a*d)*x)) + (15*a^3*
c^3 + (15*b^3*c^3 - 161*a*b^2*c^2*d + 245*a^2*b*c*d^2 - 105*a^3*d^3)*x^3 +
 (45*a*b^2*c^3 - 77*a^2*b*c^2*d + 35*a^3*c*d^2)*x^2 + 3*(15*a^2*b*c^3 - 7*
a^3*c^2*d)*x)*sqrt(b*x^2 + a*x))/(a*c^4*x^4)]
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{5/2}}{x^7(c+dx)}\,dx = \int \frac{(x(a+bx))^{\frac{5}{2}}}{x^7\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a*x)**(5/2)/x**7/(d*x+c),x)
```

output
```
Integral((x*(a + b*x))**(5/2)/(x**7*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{5/2}}{x^7(c+dx)}\,dx = \int \frac{(bx^2+ax)^{\frac{5}{2}}}{(dx+c)x^7}\,dx$$

input
```
integrate((b*x^2+a*x)^(5/2)/x^7/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(5/2)/((d*x + c)*x^7), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. 2(194) = 388.

Time = 0.14 (sec) , antiderivative size = 600, normalized size of antiderivative = 2.73

$$\int \frac{(ax+bx^2)^{5/2}}{x^7(c+dx)}\,dx =$$

$$-\frac{2\,(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3-a^3d^4)\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)d+\sqrt{b}c}{\sqrt{-bc^2+acd}}\right)}{\sqrt{-bc^2+acd}c^4}$$

$$+\frac{2\left(105\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6b^3c^3-315\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6ab^2c^2d+315\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6a^2bcd^2\ldots\right.}{}$$

input
```
integrate((b*x^2+a*x)^(5/2)/x^7/(d*x+c),x, algorithm="giac")
```

output
```
-2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*arctan(-((sqrt(
b)*x - sqrt(b*x^2 + a*x))*d + sqrt(b)*c)/sqrt(-b*c^2 + a*c*d))/(sqrt(-b*c^
2 + a*c*d)*c^4) + 2/105*(105*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*b^3*c^3 - 3
15*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a*b^2*c^2*d + 315*(sqrt(b)*x - sqrt(b
*x^2 + a*x))^6*a^2*b*c*d^2 - 105*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a^3*d^3
 + 315*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a*b^(5/2)*c^3 - 315*(sqrt(b)*x -
sqrt(b*x^2 + a*x))^5*a^2*b^(3/2)*c^2*d + 105*(sqrt(b)*x - sqrt(b*x^2 + a*x
))^5*a^3*sqrt(b)*c*d^2 + 525*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^2*b^2*c^3
 - 245*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^3*b*c^2*d + 35*(sqrt(b)*x - sqr
t(b*x^2 + a*x))^4*a^4*c*d^2 + 525*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^3*b^
(3/2)*c^3 - 105*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^4*sqrt(b)*c^2*d + 315*
(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^4*b*c^3 - 21*(sqrt(b)*x - sqrt(b*x^2 +
 a*x))^2*a^5*c^2*d + 105*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^5*sqrt(b)*c^3 +
 15*a^6*c^3)/((sqrt(b)*x - sqrt(b*x^2 + a*x))^7*c^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{5/2}}{x^7(c+dx)}\,dx = \int \frac{\left(b\,x^2+a\,x\right)^{5/2}}{x^7\,\left(c+d\,x\right)}\,dx$$

input
```
int((a*x + b*x^2)^(5/2)/(x^7*(c + d*x)),x)
```

output
```
int((a*x + b*x^2)^(5/2)/(x^7*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 631, normalized size of antiderivative = 2.87

$$\int \frac{(ax+bx^2)^{5/2}}{x^7(c+dx)}\,dx = \frac{-2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)a^3d^3x^4+4\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-}}{}\right.}{}$$

input
```
int((b*x^2+a*x)^(5/2)/x^7/(d*x+c),x)
```

output

```
(2*( - 105*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a
+ b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*d**3*x**4 + 210*
sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sq
rt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*b*c*d**2*x**4 - 105*sqrt(c)
*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*s
qrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c**2*d*x**4 - 105*sqrt(c)*sqrt(a
*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*
sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*d**3*x**4 + 210*sqrt(c)*sqrt(a*d - b*c)*a
tan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(s
qrt(c)*sqrt(b)))*a**2*b*c*d**2*x**4 - 105*sqrt(c)*sqrt(a*d - b*c)*atan((sq
rt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*
sqrt(b)))*a*b**2*c**2*d*x**4 - 15*sqrt(x)*sqrt(a + b*x)*a**3*c**4 + 21*sqr
t(x)*sqrt(a + b*x)*a**3*c**3*d*x - 35*sqrt(x)*sqrt(a + b*x)*a**3*c**2*d**2
*x**2 + 105*sqrt(x)*sqrt(a + b*x)*a**3*c*d**3*x**3 - 45*sqrt(x)*sqrt(a + b
*x)*a**2*b*c**4*x + 77*sqrt(x)*sqrt(a + b*x)*a**2*b*c**3*d*x**2 - 245*sqrt
(x)*sqrt(a + b*x)*a**2*b*c**2*d**2*x**3 - 45*sqrt(x)*sqrt(a + b*x)*a*b**2*
c**4*x**2 + 161*sqrt(x)*sqrt(a + b*x)*a*b**2*c**3*d*x**3 - 15*sqrt(x)*sqrt
(a + b*x)*b**3*c**4*x**3 - 75*sqrt(b)*a**3*c*d**3*x**4 + 155*sqrt(b)*a**2*
b*c**2*d**2*x**4 - 71*sqrt(b)*a*b**2*c**3*d*x**4 - 15*sqrt(b)*b**3*c**4*x*
*4))/(105*a*c**5*x**4)
```

3.124 $\int \frac{(ax+bx^2)^{5/2}}{x(c+dx)^6}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 259

$$\int \frac{(ax+bx^2)^{5/2}}{x(c+dx)^6}\,dx = \frac{3a^4\sqrt{ax+bx^2}}{128c^3(bc-ad)^2(c+dx)} + \frac{a^3(ax+bx^2)^{3/2}}{64c^2(bc-ad)^2x(c+dx)^2} + \frac{a^2(ax+bx^2)^{5/2}}{80c(bc-ad)^2x^2(c+dx)^3} + \frac{(ax+bx^2)^{7/2}}{5(bc-ad)x^2(c+dx)^5} - \frac{3a(ax+bx^2)^{7/2}}{40(bc-ad)^2x^3(c+dx)^4} + \frac{3a^5\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{128c^{7/2}(bc-ad)^{5/2}}$$

output
```
3/128*a^4*(b*x^2+a*x)^(1/2)/c^3/(-a*d+b*c)^2/(d*x+c)+1/64*a^3*(b*x^2+a*x)^
(3/2)/c^2/(-a*d+b*c)^2/x/(d*x+c)^2+1/80*a^2*(b*x^2+a*x)^(5/2)/c/(-a*d+b*c)
^2/x^2/(d*x+c)^3+1/5*(b*x^2+a*x)^(7/2)/(-a*d+b*c)/x^2/(d*x+c)^5-3/40*a*(b*
x^2+a*x)^(7/2)/(-a*d+b*c)^2/x^3/(d*x+c)^4+3/128*a^5*arctanh((-a*d+b*c)^(1/
2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(7/2)/(-a*d+b*c)^(5/2)
```

Mathematica [A] (verified)

Time = 10.59 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.92

$$\int \frac{(ax+bx^2)^{5/2}}{x(c+dx)^6}\,dx = \frac{(x(a+bx))^{5/2}\left(-\frac{2x^{3/2}(a+bx)}{(c+dx)^5}+\frac{3a\sqrt{x}(a+bx)}{4(bc-ad)(c+dx)^4}+\frac{3a^2\left(\frac{\sqrt{x}(a+bx)^{5/2}}{3c(c+dx)^3}+\frac{5a\sqrt{x}\sqrt{a+bx}(5ac+2bcx+3adx)}{24c^3(c+dx)^2}+\frac{5a^3\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\sqrt{x}}{\sqrt{c}\sqrt{a+bx}}\right)}{8c^{7/2}\sqrt{bc-ad}}\right)}{8(-bc+ad)(a+bx)^{5/2}}\right)}{10(-bc+ad)x^{5/2}}$$

input
```
Integrate[(a*x + b*x^2)^(5/2)/(x*(c + d*x)^6),x]
```

output
```
((x*(a + b*x))^(5/2)*((-2*x^(3/2)*(a + b*x))/(c + d*x)^5 + (3*a*Sqrt[x]*(a
 + b*x))/(4*(b*c - a*d)*(c + d*x)^4) + (3*a^2*((Sqrt[x]*(a + b*x)^(5/2))/(
3*c*(c + d*x)^3) + (5*a*Sqrt[x]*Sqrt[a + b*x]*(5*a*c + 2*b*c*x + 3*a*d*x))
/(24*c^3*(c + d*x)^2) + (5*a^3*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*
Sqrt[a + b*x])])/(8*c^(7/2)*Sqrt[b*c - a*d])))/(8*(-(b*c) + a*d)*(a + b*x)
^(5/2))))/(10*(-(b*c) + a*d)*x^(5/2))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1261, 105, 105, 105, 105, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{5/2}}{x(c+dx)^6}\,dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{(ax+bx^2)^{5/2}\int \frac{x^{3/2}(a+bx)^{5/2}}{(c+dx)^6}dx}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \text{105}$$

$$\frac{(ax+bx^2)^{5/2}\left(\frac{x^{3/2}(a+bx)^{7/2}}{5(c+dx)^5(bc-ad)}-\frac{3a\int\frac{\sqrt{x}(a+bx)^{5/2}}{(c+dx)^5}dx}{10(bc-ad)}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \text{ 105}$$

$$\frac{(ax+bx^2)^{5/2}\left(\frac{x^{3/2}(a+bx)^{7/2}}{5(c+dx)^5(bc-ad)}-\frac{3a\left(\frac{\sqrt{x}(a+bx)^{7/2}}{4(c+dx)^4(bc-ad)}-\frac{a\int\frac{(a+bx)^{5/2}}{\sqrt{x}(c+dx)^4}dx}{8(bc-ad)}\right)}{10(bc-ad)}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \text{ 105}$$

$$\frac{(ax+bx^2)^{5/2}\left(\frac{x^{3/2}(a+bx)^{7/2}}{5(c+dx)^5(bc-ad)}-\frac{3a\left(\frac{\sqrt{x}(a+bx)^{7/2}}{4(c+dx)^4(bc-ad)}-\frac{a\left(\frac{5a\int\frac{(a+bx)^{3/2}}{\sqrt{x}(c+dx)^3}dx}{6c}+\frac{\sqrt{x}(a+bx)^{5/2}}{3c(c+dx)^3}\right)}{8(bc-ad)}\right)}{10(bc-ad)}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \text{ 105}$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{x^{3/2}(a+bx)^{7/2}}{5(c+dx)^5(bc-ad)}-\frac{3a\left(\frac{\sqrt{x}(a+bx)^{7/2}}{4(c+dx)^4(bc-ad)}-\frac{a\left(\frac{5a\left(\frac{3a\int\frac{\sqrt{a+bx}}{\sqrt{x}(c+dx)^2}dx}{4c}+\frac{\sqrt{x}(a+bx)^{3/2}}{2c(c+dx)^2}\right)}{6c}+\frac{\sqrt{x}(a+bx)^{5/2}}{3c(c+dx)^3}\right)}{8(bc-ad)}\right)}{10(bc-ad)}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \; 105$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{x^{3/2}(a+bx)^{7/2}}{5(c+dx)^5(bc-ad)}-\frac{3a\left(\frac{\sqrt{x}(a+bx)^{7/2}}{4(c+dx)^4(bc-ad)}-\frac{a\left(\frac{5a\left(\frac{3a\left(\frac{a\int\frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{2c}+\frac{\sqrt{x}\sqrt{a+bx}}{c(c+dx)}\right)}{4c}+\frac{\sqrt{x}(a+bx)^{3/2}}{2c(c+dx)^2}\right)}{6c}+\frac{\sqrt{x}(a+bx)^{5/2}}{3c(c+dx)^3}}{8(bc-ad)}\right)}{10(bc-ad)}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$\downarrow$ 104

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{x^{3/2}(a+bx)^{7/2}}{5(c+dx)^5(bc-ad)}-\frac{3a\left(\frac{\sqrt{x}(a+bx)^{7/2}}{4(c+dx)^4(bc-ad)}-\frac{a\left(\frac{5a\left(\frac{3a\left(\frac{a\int\frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}+\frac{\sqrt{x}\sqrt{a+bx}}{c(c+dx)}\right)}{4c}+\frac{\sqrt{x}(a+bx)^{3/2}}{2c(c+dx)^2}\right)}{6c}+\frac{\sqrt{x}(a+bx)^{5/}}{3c(c+dx)^3}\right)}{8(bc-ad)}\right)}{10(bc-ad)}\right)}{x^{5/2}(a+bx)^{5/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{\left(ax+bx^2\right)^{5/2}\left(\frac{x^{3/2}(a+bx)^{7/2}}{5(c+dx)^5(bc-ad)}-\frac{3a\left(\frac{\sqrt{x}(a+bx)^{7/2}}{4(c+dx)^4(bc-ad)}-\frac{a\left(\frac{5a\left(\frac{3a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{x}\sqrt{a+bx}}{c(c+dx)}\right)}{4c}+\frac{\sqrt{x}(a+bx)^{3/2}}{2c(c+dx)^2}\right)}{6c}+\frac{\sqrt{x}(a+bx)}{3c(c+dx)}}{8(bc-ad)}}{10(bc-ad)}}{x^{5/2}(a+bx)^{5/2}}$$

input
```
Int[(a*x + b*x^2)^(5/2)/(x*(c + d*x)^6),x]
```

output
```
((a*x + b*x^2)^(5/2)*((x^(3/2)*(a + b*x)^(7/2))/(5*(b*c - a*d)*(c + d*x)^5
) - (3*a*((Sqrt[x]*(a + b*x)^(7/2))/(4*(b*c - a*d)*(c + d*x)^4) - (a*((Sqr
t[x]*(a + b*x)^(5/2))/(3*c*(c + d*x)^3) + (5*a*((Sqrt[x]*(a + b*x)^(3/2))/
(2*c*(c + d*x)^2) + (3*a*((Sqrt[x]*Sqrt[a + b*x])/(c*(c + d*x)) + (a*ArcTa
nh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(c^(3/2)*Sqrt[b*c -
 a*d])))/(4*c)))/(6*c)))/(8*(b*c - a*d))))/(10*(b*c - a*d))))/(x^(5/2)*(a
+ b*x)^(5/2))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.91

method	result
pseudoelliptic	$-\frac{3\left(\sqrt{c(ad-bc)}\left(\left(-d^4x^4-\frac{14}{3}c\,d^3x^3-\frac{128}{15}d^2c^2x^2+\frac{14}{3}c^3dx+c^4\right)a^4-\frac{2xb\left(d^3x^3+\frac{23}{5}c\,d^2x^2-\frac{233}{5}c^2dx+c^3\right)c\,a^3}{3}-\frac{248\left(\frac{1}{31}d^2x^2-\right.}{}\right.\right.}{128\sqrt{c(ad-bc)}\,(dx+c)^5}$
default	Expression too large to display

input
```
int((b*x^2+a*x)^(5/2)/x/(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
-3/128*((c*(a*d-b*c))^(1/2)*((-d^4*x^4-14/3*c*d^3*x^3-128/15*d^2*c^2*x^2+1
4/3*c^3*d*x+c^4)*a^4-2/3*x*b*(d^3*x^3+23/5*c*d^2*x^2-233/5*c^2*d*x+c^3)*c*
a^3-248/15*(1/31*d^2*x^2-64/31*c*d*x+c^2)*x^2*b^2*c^2*a^2-112/5*x^3*b^3*(-
11/21*d*x+c)*c^3*a-128/15*b^4*c^4*x^4)*(x*(b*x+a))^(1/2)+a^5*(d*x+c)^5*arc
tan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2)))/(c*(a*d-b*c))^(1/2)/(d*x+c
)^5/c^3/(a*d-b*c)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 670 vs. 2(227) = 454.

Time = 0.11 (sec) , antiderivative size = 1356, normalized size of antiderivative = 5.24

$$\int \frac{(ax+bx^2)^{5/2}}{x(c+dx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a*x)^(5/2)/x/(d*x+c)^6,x, algorithm="fricas")
```

output
```
[1/1280*(15*(a^5*d^5*x^5 + 5*a^5*c*d^4*x^4 + 10*a^5*c^2*d^3*x^3 + 10*a^5*c
^3*d^2*x^2 + 5*a^5*c^4*d*x + a^5*c^5)*sqrt(b*c^2 - a*c*d)*log((a*c + (2*b*
c - a*d)*x + 2*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) - 2*(15*a
^4*b*c^6 - 15*a^5*c^5*d - (128*b^5*c^6 - 304*a*b^4*c^5*d + 184*a^2*b^3*c^4
*d^2 + 2*a^3*b^2*c^3*d^3 + 5*a^4*b*c^2*d^4 - 15*a^5*c*d^5)*x^4 - 2*(168*a*
b^4*c^6 - 424*a^2*b^3*c^5*d + 279*a^3*b^2*c^4*d^2 + 12*a^4*b*c^3*d^3 - 35*
a^5*c^2*d^4)*x^3 - 2*(124*a^2*b^3*c^6 - 357*a^3*b^2*c^5*d + 297*a^4*b*c^4*
d^2 - 64*a^5*c^3*d^3)*x^2 - 10*(a^3*b^2*c^6 - 8*a^4*b*c^5*d + 7*a^5*c^4*d^
2)*x)*sqrt(b*x^2 + a*x))/(b^3*c^12 - 3*a*b^2*c^11*d + 3*a^2*b*c^10*d^2 - a
^3*c^9*d^3 + (b^3*c^7*d^5 - 3*a*b^2*c^6*d^6 + 3*a^2*b*c^5*d^7 - a^3*c^4*d^
8)*x^5 + 5*(b^3*c^8*d^4 - 3*a*b^2*c^7*d^5 + 3*a^2*b*c^6*d^6 - a^3*c^5*d^7)
*x^4 + 10*(b^3*c^9*d^3 - 3*a*b^2*c^8*d^4 + 3*a^2*b*c^7*d^5 - a^3*c^6*d^6)*
x^3 + 10*(b^3*c^10*d^2 - 3*a*b^2*c^9*d^3 + 3*a^2*b*c^8*d^4 - a^3*c^7*d^5)*
x^2 + 5*(b^3*c^11*d - 3*a*b^2*c^10*d^2 + 3*a^2*b*c^9*d^3 - a^3*c^8*d^4)*x)
, -1/640*(15*(a^5*d^5*x^5 + 5*a^5*c*d^4*x^4 + 10*a^5*c^2*d^3*x^3 + 10*a^5*
c^3*d^2*x^2 + 5*a^5*c^4*d*x + a^5*c^5)*sqrt(-b*c^2 + a*c*d)*arctan(sqrt(-b
*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) + (15*a^4*b*c^6 - 15*a^5*c^
5*d - (128*b^5*c^6 - 304*a*b^4*c^5*d + 184*a^2*b^3*c^4*d^2 + 2*a^3*b^2*c^3
*d^3 + 5*a^4*b*c^2*d^4 - 15*a^5*c*d^5)*x^4 - 2*(168*a*b^4*c^6 - 424*a^2*b^
3*c^5*d + 279*a^3*b^2*c^4*d^2 + 12*a^4*b*c^3*d^3 - 35*a^5*c^2*d^4)*x^3 ...
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{5/2}}{x(c+dx)^6}\,dx = \int \frac{(x(a+bx))^{\frac{5}{2}}}{x\,(c+dx)^6}\,dx$$

input
```
integrate((b*x**2+a*x)**(5/2)/x/(d*x+c)**6,x)
```

output
```
Integral((x*(a + b*x))**(5/2)/(x*(c + d*x)**6), x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{5/2}}{x(c+dx)^6}\,dx = \int \frac{(bx^2+ax)^{\frac{5}{2}}}{(dx+c)^6 x}\,dx$$

input
```
integrate((b*x^2+a*x)^(5/2)/x/(d*x+c)^6,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(5/2)/((d*x + c)^6*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2074 vs. $2(227) = 454$.

Time = 0.21 (sec) , antiderivative size = 2074, normalized size of antiderivative = 8.01

$$\int \frac{(ax+bx^2)^{5/2}}{x(c+dx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a*x)^(5/2)/x/(d*x+c)^6,x, algorithm="giac")
```

output
```
3/128*a^5*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a*x))*d + sqrt(b)*c)/sqrt(-b*
c^2 + a*c*d))/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*sqrt(-b*c^2 + a*c*d))
 + 1/640*(1280*(sqrt(b)*x - sqrt(b*x^2 + a*x))^9*b^5*c^5*d^4 - 2560*(sqrt(
b)*x - sqrt(b*x^2 + a*x))^9*a*b^4*c^4*d^5 + 1280*(sqrt(b)*x - sqrt(b*x^2 +
 a*x))^9*a^2*b^3*c^3*d^6 - 15*(sqrt(b)*x - sqrt(b*x^2 + a*x))^9*a^5*d^9 +
5120*(sqrt(b)*x - sqrt(b*x^2 + a*x))^8*b^(11/2)*c^6*d^3 - 6400*(sqrt(b)*x
- sqrt(b*x^2 + a*x))^8*a*b^(9/2)*c^5*d^4 - 2560*(sqrt(b)*x - sqrt(b*x^2 +
a*x))^8*a^2*b^(7/2)*c^4*d^5 + 3840*(sqrt(b)*x - sqrt(b*x^2 + a*x))^8*a^3*b
^(5/2)*c^3*d^6 - 135*(sqrt(b)*x - sqrt(b*x^2 + a*x))^8*a^5*sqrt(b)*c*d^8 +
 10240*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*b^6*c^7*d^2 - 8960*(sqrt(b)*x - s
qrt(b*x^2 + a*x))^7*a*b^5*c^6*d^3 - 6400*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7
*a^2*b^4*c^5*d^4 - 1280*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*a^3*b^3*c^4*d^5
+ 6400*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*a^4*b^2*c^3*d^6 - 470*(sqrt(b)*x
- sqrt(b*x^2 + a*x))^7*a^5*b*c^2*d^7 - 70*(sqrt(b)*x - sqrt(b*x^2 + a*x))^
7*a^6*c*d^8 + 10240*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*b^(13/2)*c^8*d + 128
0*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a*b^(11/2)*c^7*d^2 - 19200*(sqrt(b)*x
- sqrt(b*x^2 + a*x))^6*a^2*b^(9/2)*c^6*d^3 + 1280*(sqrt(b)*x - sqrt(b*x^2
+ a*x))^6*a^4*b^(5/2)*c^4*d^5 + 5630*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a^5
*b^(3/2)*c^3*d^6 - 490*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a^6*sqrt(b)*c^2*d
^7 + 4096*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*b^7*c^9 + 14848*(sqrt(b)*x ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{5/2}}{x(c+dx)^6}\,dx = \int \frac{\left(b\,x^2+a\,x\right)^{5/2}}{x\,\left(c+d\,x\right)^6}\,dx$$

input
```
int((a*x + b*x^2)^(5/2)/(x*(c + d*x)^6),x)
```

output
```
int((a*x + b*x^2)^(5/2)/(x*(c + d*x)^6), x)
```

Reduce [F]

$$\int \frac{(ax+bx^2)^{5/2}}{x(c+dx)^6}\,dx = \int \frac{(b\,x^2+ax)^{\frac{5}{2}}}{x\,(dx+c)^6}dx$$

input
```
int((b*x^2+a*x)^(5/2)/x/(d*x+c)^6,x)
```

output
```
int((b*x^2+a*x)^(5/2)/x/(d*x+c)^6,x)
```

3.125 $\int \frac{x^3(c+dx)}{\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 162

$$\int \frac{x^3(c+dx)}{\sqrt{ax+bx^2}}\,dx = \frac{5a^2(8bc-7ad)\sqrt{ax+bx^2}}{64b^4} - \frac{5a(8bc-7ad)x\sqrt{ax+bx^2}}{96b^3} + \frac{(8bc-7ad)x^2\sqrt{ax+bx^2}}{24b^2} + \frac{dx^3\sqrt{ax+bx^2}}{4b} - \frac{5a^3(8bc-7ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{64b^{9/2}}$$

output
```
5/64*a^2*(-7*a*d+8*b*c)*(b*x^2+a*x)^(1/2)/b^4-5/96*a*(-7*a*d+8*b*c)*x*(b*x
^2+a*x)^(1/2)/b^3+1/24*(-7*a*d+8*b*c)*x^2*(b*x^2+a*x)^(1/2)/b^2+1/4*d*x^3*
(b*x^2+a*x)^(1/2)/b-5/64*a^3*(-7*a*d+8*b*c)*arctanh(b^(1/2)*x/(b*x^2+a*x)^
(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.16

$$\int \frac{x^3(c+dx)}{\sqrt{ax+bx^2}}\,dx$$
$$=\frac{\sqrt{x}(a+bx)\left(120a^2bc\sqrt{x}-105a^3d\sqrt{x}-80ab^2cx^{3/2}+70a^2bdx^{3/2}+64b^3cx^{5/2}-56ab^2dx^{5/2}+48b^3dx^{7/2}\right)}{192b^4\sqrt{x(a+bx)}}$$
$$+\frac{5a^3(-8bc+7ad)\sqrt{x}\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{32b^{9/2}\sqrt{x(a+bx)}}$$

input
```
Integrate[(x^3*(c + d*x))/Sqrt[a*x + b*x^2],x]
```

output
```
(Sqrt[x]*(a + b*x)*(120*a^2*b*c*Sqrt[x] - 105*a^3*d*Sqrt[x] - 80*a*b^2*c*x
^(3/2) + 70*a^2*b*d*x^(3/2) + 64*b^3*c*x^(5/2) - 56*a*b^2*d*x^(5/2) + 48*b
^3*d*x^(7/2)))/(192*b^4*Sqrt[x*(a + b*x)]) + (5*a^3*(-8*b*c + 7*a*d)*Sqrt[
x]*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(3
2*b^(9/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1221, 1134, 1134, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx)}{\sqrt{ax+bx^2}}\,dx$$
$$\Big\downarrow \text{1221}$$
$$\frac{(8bc-7ad)\int \frac{x^3}{\sqrt{bx^2+ax}}dx}{8b}+\frac{dx^3\sqrt{ax+bx^2}}{4b}$$
$$\Big\downarrow \text{1134}$$

$$\frac{(8bc-7ad)\left(\frac{x^2\sqrt{ax+bx^2}}{3b}-\frac{5a\int\frac{x^2}{\sqrt{bx^2+ax}}dx}{6b}\right)}{8b}+\frac{dx^3\sqrt{ax+bx^2}}{4b}$$

$\big\downarrow$ 1134

$$\frac{(8bc-7ad)\left(\frac{x^2\sqrt{ax+bx^2}}{3b}-\frac{5a\left(\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\int\frac{x}{\sqrt{bx^2+ax}}dx}{4b}\right)}{6b}\right)}{8b}+\frac{dx^3\sqrt{ax+bx^2}}{4b}$$

$\big\downarrow$ 1160

$$\frac{(8bc-7ad)\left(\frac{x^2\sqrt{ax+bx^2}}{3b}-\frac{5a\left(\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\left(\frac{\sqrt{ax+bx^2}}{b}-\frac{a\int\frac{1}{\sqrt{bx^2+ax}}dx}{2b}\right)}{4b}\right)}{6b}\right)}{8b}+\frac{dx^3\sqrt{ax+bx^2}}{4b}$$

$\big\downarrow$ 1091

$$\frac{(8bc-7ad)\left(\frac{x^2\sqrt{ax+bx^2}}{3b}-\frac{5a\left(\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\left(\frac{\sqrt{ax+bx^2}}{b}-\frac{a\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{b}\right)}{4b}\right)}{6b}\right)}{8b}+\frac{dx^3\sqrt{ax+bx^2}}{4b}$$

$\big\downarrow$ 219

$$\frac{\left(\frac{x^2\sqrt{ax+bx^2}}{3b}-\frac{5a\left(\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\left(\frac{\sqrt{ax+bx^2}}{b}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{3/2}}\right)}{4b}\right)}{6b}\right)(8bc-7ad)}{8b}+\frac{dx^3\sqrt{ax+bx^2}}{4b}$$

input
```
Int[(x^3*(c + d*x))/Sqrt[a*x + b*x^2],x]
```

output
```
(d*x^3*Sqrt[a*x + b*x^2])/(4*b) + ((8*b*c - 7*a*d)*((x^2*Sqrt[a*x + b*x^2]
)/(3*b) - (5*a*((x*Sqrt[a*x + b*x^2])/(2*b) - (3*a*(Sqrt[a*x + b*x^2]/b -
(a*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/b^(3/2)))/(4*b)))/(6*b)))/(8*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.60

method	result
pseudoelliptic	$\frac{\frac{35a^3\left(ad-\frac{8bc}{7}\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{64}-\frac{35\left(-\frac{8\left(\frac{7dx}{12}+c\right)a^2b^{\frac{3}{2}}}{7}+\frac{16\left(\frac{7dx}{10}+c\right)xa\,b^{\frac{5}{2}}}{21}-\frac{64x^2\left(\frac{3dx}{4}+c\right)b^{\frac{7}{2}}}{105}+\sqrt{b}\,a^3d\right)\sqrt{x(bx+a)}}{64}}{b^{\frac{9}{2}}}$
risch	$-\frac{(-48b^3d\,x^3+56a\,b^2d\,x^2-64b^3c\,x^2-70a^2bdx+80a\,b^2cx+105a^3d-120c\,a^2b)x(bx+a)}{192b^4\sqrt{x(bx+a)}}+\frac{5a^3(7ad-8bc)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x\right.}{128b^{\frac{9}{2}}}$
default	$c\left(\frac{x^2\sqrt{b\,x^2+ax}}{3b}-\frac{5a\left(\frac{x\sqrt{b\,x^2+ax}}{2b}-\frac{3a\left(\frac{\sqrt{b\,x^2+ax}}{b}-\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)}{6b}\right)+d\left(\frac{x^3\sqrt{b\,x^2+ax}}{4b}-\frac{7a\left(x^2\sqrt{\right.}{}\right.$

input
```
int(x^3*(d*x+c)/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
35/64/b^(9/2)*(a^3*(a*d-8/7*b*c)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-(-8/
7*(7/12*d*x+c)*a^2*b^(3/2)+16/21*(7/10*d*x+c)*x*a*b^(5/2)-64/105*x^2*(3/4*
d*x+c)*b^(7/2)+b^(1/2)*a^3*d)*(x*(b*x+a))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.58

$$\int \frac{x^3(c+dx)}{\sqrt{ax+bx^2}}\,dx$$
$$= \left[-\frac{15\,(8\,a^3bc-7\,a^4d)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,(48\,b^4dx^3+120\,a^2b^2c-105\,a^3bd+8\,(8\,b^4}{384\,b^5}\right.$$

input
```
integrate(x^3*(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/384*(15*(8*a^3*b*c - 7*a^4*d)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a
*x)*sqrt(b)) - 2*(48*b^4*d*x^3 + 120*a^2*b^2*c - 105*a^3*b*d + 8*(8*b^4*c
- 7*a*b^3*d)*x^2 - 10*(8*a*b^3*c - 7*a^2*b^2*d)*x)*sqrt(b*x^2 + a*x))/b^5,
 1/192*(15*(8*a^3*b*c - 7*a^4*d)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b
)/(b*x + a)) + (48*b^4*d*x^3 + 120*a^2*b^2*c - 105*a^3*b*d + 8*(8*b^4*c -
7*a*b^3*d)*x^2 - 10*(8*a*b^3*c - 7*a^2*b^2*d)*x)*sqrt(b*x^2 + a*x))/b^5]
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.26

$$\int \frac{x^3(c+dx)}{\sqrt{ax+bx^2}}\,dx$$
$$= \begin{cases} -\frac{5a^3\left(-\frac{7ad}{8b}+c\right)\left(\begin{cases}\frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0\\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{16b^3}+\sqrt{ax+bx^2}\cdot\left(\frac{5a^2\left(-\frac{7ad}{8b}+c\right)}{8b^3}-\frac{5ax\left(-\frac{7ad}{8b}+c\right)}{12b^2}+\frac{dx^3}{4b}\right. \\ \frac{2\left(\frac{c(ax)^{\frac{7}{2}}}{7}+\frac{d(ax)^{\frac{9}{2}}}{9a}\right)}{a^4} \\ \tilde{\infty}\left(\frac{cx^4}{4}+\frac{dx^5}{5}\right) \end{cases}$$

input
```
integrate(x**3*(d*x+c)/(b*x**2+a*x)**(1/2),x)
```

output
```
Piecewise((-5*a**3*(-7*a*d/(8*b) + c)*Piecewise((log(a + 2*sqrt(b)*sqrt(a*
x + b*x**2) + 2*b*x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) +
 x)/sqrt(b*(a/(2*b) + x)**2), True))/(16*b**3) + sqrt(a*x + b*x**2)*(5*a**
2*(-7*a*d/(8*b) + c)/(8*b**3) - 5*a*x*(-7*a*d/(8*b) + c)/(12*b**2) + d*x**
3/(4*b) + x**2*(-7*a*d/(8*b) + c)/(3*b)), Ne(b, 0)), (2*(c*(a*x)**(7/2)/7
+ d*(a*x)**(9/2)/(9*a))/a**4, Ne(a, 0)), (zoo*(c*x**4/4 + d*x**5/5), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.27

$$\int \frac{x^3(c+dx)}{\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{bx^2+ax}dx^3}{4\,b} + \frac{\sqrt{bx^2+ax}cx^2}{3\,b} - \frac{7\,\sqrt{bx^2+ax}adx^2}{24\,b^2} - \frac{5\,\sqrt{bx^2+ax}acx}{12\,b^2} + \frac{35\,\sqrt{bx^2+ax}a^2dx}{96\,b^3} - \frac{5\,a^3c\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{7}{2}}} + \frac{35\,a^4d\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{9}{2}}} + \frac{5\,\sqrt{bx^2+ax}a^2c}{8\,b^3} - \frac{35\,\sqrt{bx^2+ax}a^3d}{64\,b^4}$$

input
```
integrate(x^3*(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(b*x^2 + a*x)*d*x^3/b + 1/3*sqrt(b*x^2 + a*x)*c*x^2/b - 7/24*sqrt(
b*x^2 + a*x)*a*d*x^2/b^2 - 5/12*sqrt(b*x^2 + a*x)*a*c*x/b^2 + 35/96*sqrt(b
*x^2 + a*x)*a^2*d*x/b^3 - 5/16*a^3*c*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*s
qrt(b))/b^(7/2) + 35/128*a^4*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)
)/b^(9/2) + 5/8*sqrt(b*x^2 + a*x)*a^2*c/b^3 - 35/64*sqrt(b*x^2 + a*x)*a^3*
d/b^4
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.83

$$\int \frac{x^3(c+dx)}{\sqrt{ax+bx^2}}\,dx$$
$$=\frac{1}{192}\sqrt{bx^2+ax}\left(2\left(4\left(\frac{6\,dx}{b}+\frac{8\,b^3c-7\,ab^2d}{b^4}\right)x-\frac{5\,(8\,ab^2c-7\,a^2bd)}{b^4}\right)x+\frac{15\,(8\,a^2bc-7\,a^3d)}{b^4}\right)$$
$$+\frac{5\,(8\,a^3bc-7\,a^4d)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{128\,b^{\frac{9}{2}}}$$

input
```
integrate(x^3*(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/192*sqrt(b*x^2 + a*x)*(2*(4*(6*d*x/b + (8*b^3*c - 7*a*b^2*d)/b^4)*x - 5*
(8*a*b^2*c - 7*a^2*b*d)/b^4)*x + 15*(8*a^2*b*c - 7*a^3*d)/b^4) + 5/128*(8*
a^3*b*c - 7*a^4*d)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))
/b^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx)}{\sqrt{ax+bx^2}}\,dx=\int \frac{x^3\,(c+d\,x)}{\sqrt{b\,x^2+a\,x}}\,dx$$

input
```
int((x^3*(c + d*x))/(a*x + b*x^2)^(1/2),x)
```

output
```
int((x^3*(c + d*x))/(a*x + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.09

$$\int \frac{x^3(c+dx)}{\sqrt{ax+bx^2}}\,dx$$
$$= \frac{-105\sqrt{x}\,\sqrt{bx+a}\,a^3bd+120\sqrt{x}\,\sqrt{bx+a}\,a^2b^2c+70\sqrt{x}\,\sqrt{bx+a}\,a^2b^2dx-80\sqrt{x}\,\sqrt{bx+a}\,a\,b^3cx-56\sqrt{x}}{}$$

input
```
int(x^3*(d*x+c)/(b*x^2+a*x)^(1/2),x)
```

output
```
( - 105*sqrt(x)*sqrt(a + b*x)*a**3*b*d + 120*sqrt(x)*sqrt(a + b*x)*a**2*b*
*2*c + 70*sqrt(x)*sqrt(a + b*x)*a**2*b**2*d*x - 80*sqrt(x)*sqrt(a + b*x)*a
*b**3*c*x - 56*sqrt(x)*sqrt(a + b*x)*a*b**3*d*x**2 + 64*sqrt(x)*sqrt(a + b
*x)*b**4*c*x**2 + 48*sqrt(x)*sqrt(a + b*x)*b**4*d*x**3 + 105*sqrt(b)*log((
sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4*d - 120*sqrt(b)*log((sqrt(a
 + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*b*c)/(192*b**5)
```

3.126 $\int \frac{x^2(c+dx)}{\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{x^2(c+dx)}{\sqrt{ax+bx^2}}\,dx = -\frac{a(6bc-5ad)\sqrt{ax+bx^2}}{8b^3}+\frac{(6bc-5ad)x\sqrt{ax+bx^2}}{12b^2} + \frac{dx^2\sqrt{ax+bx^2}}{3b}+\frac{a^2(6bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{8b^{7/2}}$$

output
```
-1/8*a*(-5*a*d+6*b*c)*(b*x^2+a*x)^(1/2)/b^3+1/12*(-5*a*d+6*b*c)*x*(b*x^2+a
*x)^(1/2)/b^2+1/3*d*x^2*(b*x^2+a*x)^(1/2)/b+1/8*a^2*(-5*a*d+6*b*c)*arctanh
(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.98

$$\int \frac{x^2(c+dx)}{\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{b}x(a+bx)\left(15a^2d+4b^2x(3c+2dx)-2ab(9c+5dx)\right)+6a^2(-6bc+5ad)\sqrt{x}\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a-}}\right.}{24b^{7/2}\sqrt{x(a+bx)}}$$

input
```
Integrate[(x^2*(c + d*x))/Sqrt[a*x + b*x^2],x]
```

output
```
(Sqrt[b]*x*(a + b*x)*(15*a^2*d + 4*b^2*x*(3*c + 2*d*x) - 2*a*b*(9*c + 5*d*
x)) + 6*a^2*(-6*b*c + 5*a*d)*Sqrt[x]*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x
])/(Sqrt[a] - Sqrt[a + b*x])])/(24*b^(7/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1221, 1134, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx)}{\sqrt{ax+bx^2}}dx$$

$$\Big\downarrow \text{ 1221}$$

$$\frac{(6bc-5ad)\int \frac{x^2}{\sqrt{bx^2+ax}}dx}{6b}+\frac{dx^2\sqrt{ax+bx^2}}{3b}$$

$$\Big\downarrow \text{ 1134}$$

$$\frac{(6bc-5ad)\left(\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\int \frac{x}{\sqrt{bx^2+ax}}dx}{4b}\right)}{6b}+\frac{dx^2\sqrt{ax+bx^2}}{3b}$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{(6bc-5ad)\left(\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\left(\frac{\sqrt{ax+bx^2}}{b}-\frac{a\int \frac{1}{\sqrt{bx^2+ax}}dx}{2b}\right)}{4b}\right)}{6b}+\frac{dx^2\sqrt{ax+bx^2}}{3b}$$

$$\Big\downarrow \text{ 1091}$$

$$\frac{(6bc-5ad)\left(\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\left(\frac{\sqrt{ax+bx^2}}{b}-\frac{a\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{b}\right)}{4b}\right)}{6b}+\frac{dx^2\sqrt{ax+bx^2}}{3b}$$

$\Big\downarrow$ 219

$$\frac{\left(\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\left(\frac{\sqrt{ax+bx^2}}{b}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{3/2}}\right)}{4b}\right)(6bc-5ad)}{6b}+\frac{dx^2\sqrt{ax+bx^2}}{3b}$$

input
```
Int[(x^2*(c + d*x))/Sqrt[a*x + b*x^2],x]
```

output
```
(d*x^2*Sqrt[a*x + b*x^2])/(3*b) + ((6*b*c - 5*a*d)*((x*Sqrt[a*x + b*x^2])/
(2*b) - (3*a*(Sqrt[a*x + b*x^2]/b - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^
2]])/b^(3/2)))/(4*b)))/(6*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$-\frac{5\left(a^{2}\left(a d-\frac{6 b c}{5}\right) \operatorname{arctanh}\left(\frac{\sqrt{x(b x+a)}}{x \sqrt{b}}\right)-\left(-\frac{6 a\left(\frac{5 d x}{9}+c\right) b^{\frac{3}{2}}}{5}+\frac{4 x\left(\frac{2 d x}{3}+c\right) b^{\frac{5}{2}}}{5}+\sqrt{b}\, a^{2} d\right) \sqrt{x(b x+a)}\right)}{8 b^{\frac{7}{2}}}$
risch	$\frac{\left(8 b^{2} d\, x^{2}-10 a b d x+12 b^{2} c x+15 a^{2} d-18 a b c\right) x(b x+a)}{24 b^{3} \sqrt{x(b x+a)}}-\frac{a^{2}(5 a d-6 b c) \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\, x^{2}+a x}\right)}{16 b^{\frac{7}{2}}}$
default	$c\left(\frac{x \sqrt{b\, x^{2}+a x}}{2 b}-\frac{3 a\left(\frac{\sqrt{b\, x^{2}+a x}}{b}-\frac{a \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\, x^{2}+a x}\right)}{2 b^{\frac{3}{2}}}\right)}{4 b}\right)+d\left(\frac{x^{2} \sqrt{b\, x^{2}+a x}}{3 b}-\frac{5 a\left(\frac{x \sqrt{b\, x^{2}+a x}}{2 b}-\frac{3 a\left(\frac{\sqrt{b\, x^{2}+\ldots}}{b}\ldots\right.}{\ldots}\right.}{\ldots}\right.$

input
```
int(x^2*(d*x+c)/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-5/8/b^(7/2)*(a^2*(a*d-6/5*b*c)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-(-6/5
*a*(5/9*d*x+c)*b^(3/2)+4/5*x*(2/3*d*x+c)*b^(5/2)+b^(1/2)*a^2*d)*(x*(b*x+a)
)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.65

$$\int \frac{x^2(c+dx)}{\sqrt{ax+bx^2}}\,dx$$

$$= \left[-\frac{3\,(6\,a^2bc-5\,a^3d)\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,(8\,b^3dx^2-18\,ab^2c+15\,a^2bd+2\,(6\,b^3c-5}{48\,b^4}\right.$$

$$-\frac{3\,(6\,a^2bc-5\,a^3d)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(8\,b^3dx^2-18\,ab^2c+15\,a^2bd+2\,(6\,b^3c-5\,ab^2d)x)\sqrt{bx}}{24\,b^4}$$

input
```
integrate(x^2*(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(6*a^2*b*c - 5*a^3*d)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x
)*sqrt(b)) - 2*(8*b^3*d*x^2 - 18*a*b^2*c + 15*a^2*b*d + 2*(6*b^3*c - 5*a*b
^2*d)*x)*sqrt(b*x^2 + a*x))/b^4, -1/24*(3*(6*a^2*b*c - 5*a^3*d)*sqrt(-b)*a
rctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (8*b^3*d*x^2 - 18*a*b^2*c +
15*a^2*b*d + 2*(6*b^3*c - 5*a*b^2*d)*x)*sqrt(b*x^2 + a*x))/b^4]
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.42

$$\int \frac{x^2(c+dx)}{\sqrt{ax+bx^2}}\,dx = \begin{cases} \frac{3a^2\left(-\frac{5ad}{6b}+c\right)\left(\begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{8b^2} + \sqrt{ax+bx^2}\left(-\frac{3a\left(-\frac{5ad}{6b}+c\right)}{4b^2}+\frac{dx^2}{3b}+\frac{x\left(-\frac{5ad}{6b}+c\right)}{2b}\right) & \text{fo} \\ \frac{2\left(\frac{c(ax)^{\frac{5}{2}}}{5}+\frac{d(ax)^{\frac{7}{2}}}{7a}\right)}{a^3} & \text{fo} \\ \tilde{\infty}\left(\frac{cx^3}{3}+\frac{dx^4}{4}\right) & \text{o} \end{cases}$$

input
```
integrate(x**2*(d*x+c)/(b*x**2+a*x)**(1/2),x)
```

output
```
Piecewise((3*a**2*(-5*a*d/(6*b) + c)*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x
 + b*x**2) + 2*b*x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) +
x)/sqrt(b*(a/(2*b) + x)**2), True))/(8*b**2) + sqrt(a*x + b*x**2)*(-3*a*(-
5*a*d/(6*b) + c)/(4*b**2) + d*x**2/(3*b) + x*(-5*a*d/(6*b) + c)/(2*b)), Ne
(b, 0)), (2*(c*(a*x)**(5/2)/5 + d*(a*x)**(7/2)/(7*a))/a**3, Ne(a, 0)), (zo
o*(c*x**3/3 + d*x**4/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.26

$$\begin{aligned}\int \frac{x^2(c+dx)}{\sqrt{ax+bx^2}}\,dx &= \frac{\sqrt{bx^2+ax}dx^2}{3\,b} + \frac{\sqrt{bx^2+ax}cx}{2\,b} - \frac{5\,\sqrt{bx^2+ax}adx}{12\,b^2} \\ &\quad + \frac{3\,a^2c\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{5}{2}}} \\ &\quad - \frac{5\,a^3d\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{7}{2}}} \\ &\quad - \frac{3\,\sqrt{bx^2+ax}ac}{4\,b^2} + \frac{5\,\sqrt{bx^2+ax}a^2d}{8\,b^3}\end{aligned}$$

input
```
integrate(x^2*(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(b*x^2 + a*x)*d*x^2/b + 1/2*sqrt(b*x^2 + a*x)*c*x/b - 5/12*sqrt(b*
x^2 + a*x)*a*d*x/b^2 + 3/8*a^2*c*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(
b))/b^(5/2) - 5/16*a^3*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(7
/2) - 3/4*sqrt(b*x^2 + a*x)*a*c/b^2 + 5/8*sqrt(b*x^2 + a*x)*a^2*d/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.83

$$\int \frac{x^2(c+dx)}{\sqrt{ax+bx^2}}\,dx = \frac{1}{24}\sqrt{bx^2+ax}\left(2\left(\frac{4\,dx}{b}+\frac{6\,b^2c-5\,abd}{b^3}\right)x-\frac{3\,(6\,abc-5\,a^2d)}{b^3}\right) - \frac{(6\,a^2bc-5\,a^3d)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{16\,b^{\frac{7}{2}}}$$

input
```
integrate(x^2*(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(b*x^2 + a*x)*(2*(4*d*x/b + (6*b^2*c - 5*a*b*d)/b^3)*x - 3*(6*a*b
*c - 5*a^2*d)/b^3) - 1/16*(6*a^2*b*c - 5*a^3*d)*log(abs(2*(sqrt(b)*x - sqr
t(b*x^2 + a*x))*sqrt(b) + a))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx)}{\sqrt{ax+bx^2}}\,dx = \int \frac{x^2\,(c+d\,x)}{\sqrt{b\,x^2+a\,x}}\,dx$$

input
```
int((x^2*(c + d*x))/(a*x + b*x^2)^(1/2),x)
```

output
```
int((x^2*(c + d*x))/(a*x + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.08

$$\int \frac{x^2(c+dx)}{\sqrt{ax+bx^2}}\,dx$$
$$= \frac{15\sqrt{x}\,\sqrt{bx+a}\,a^2bd-18\sqrt{x}\,\sqrt{bx+a}\,a\,b^2c-10\sqrt{x}\,\sqrt{bx+a}\,a\,b^2dx+12\sqrt{x}\,\sqrt{bx+a}\,b^3cx+8\sqrt{x}\,\sqrt{bx+}}{24b^4}$$

input
```
int(x^2*(d*x+c)/(b*x^2+a*x)^(1/2),x)
```

output
```
(15*sqrt(x)*sqrt(a + b*x)*a**2*b*d - 18*sqrt(x)*sqrt(a + b*x)*a*b**2*c - 1
0*sqrt(x)*sqrt(a + b*x)*a*b**2*d*x + 12*sqrt(x)*sqrt(a + b*x)*b**3*c*x + 8
*sqrt(x)*sqrt(a + b*x)*b**3*d*x**2 - 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(
x)*sqrt(b))/sqrt(a))*a**3*d + 18*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt
(b))/sqrt(a))*a**2*b*c)/(24*b**4)
```

3.127 $\int \frac{x(c+dx)}{\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 92

$$\int \frac{x(c+dx)}{\sqrt{ax+bx^2}}\,dx=\frac{(4bc-3ad)\sqrt{ax+bx^2}}{4b^2}+\frac{dx\sqrt{ax+bx^2}}{2b}-\frac{a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{5/2}}$$

output
```
1/4*(-3*a*d+4*b*c)*(b*x^2+a*x)^(1/2)/b^2+1/2*d*x*(b*x^2+a*x)^(1/2)/b-1/4*a
*(-3*a*d+4*b*c)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.17

$$\int \frac{x(c+dx)}{\sqrt{ax+bx^2}}\,dx=\frac{\sqrt{x}\left(\sqrt{b}\sqrt{x}(a+bx)(4bc-3ad+2bdx)+2a(-4bc+3ad)\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)\right)}{4b^{5/2}\sqrt{x(a+bx)}}$$

input
```
Integrate[(x*(c + d*x))/Sqrt[a*x + b*x^2],x]
```

output
```
(Sqrt[x]*(Sqrt[b]*Sqrt[x]*(a + b*x)*(4*b*c - 3*a*d + 2*b*d*x) + 2*a*(-4*b*
c + 3*a*d)*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*
x])]))/(4*b^(5/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1225, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx)}{\sqrt{ax+bx^2}}dx$$

$$\Big\downarrow \; 1225$$

$$\frac{\sqrt{ax+bx^2}(-3ad+4bc+2bdx)}{4b^2}-\frac{a(4bc-3ad)\int \frac{1}{\sqrt{bx^2+ax}}dx}{8b^2}$$

$$\Big\downarrow \; 1091$$

$$\frac{\sqrt{ax+bx^2}(-3ad+4bc+2bdx)}{4b^2}-\frac{a(4bc-3ad)\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{ax+bx^2}(-3ad+4bc+2bdx)}{4b^2}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(4bc-3ad)}{4b^{5/2}}$$

input
```
Int[(x*(c + d*x))/Sqrt[a*x + b*x^2],x]
```

output
```
((4*b*c - 3*a*d + 2*b*d*x)*Sqrt[a*x + b*x^2])/(4*b^2) - (a*(4*b*c - 3*a*d)
*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*b^(5/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{(-2bdx+3ad-4bc)x(bx+a)}{4b^2\sqrt{x(bx+a)}}+\frac{a(3ad-4bc)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8b^{\frac{5}{2}}}$
pseudoelliptic	$\frac{2b^{\frac{3}{2}}dx\sqrt{x(bx+a)}+3\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)a^2d-4\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)abc-3\sqrt{x(bx+a)}\,\sqrt{b}\,ad+4\sqrt{x(bx+a)}\,b^{\frac{3}{2}}c}{4b^{\frac{5}{2}}}$
default	$c\left(\frac{\sqrt{b\,x^2+ax}}{b}-\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2b^{\frac{3}{2}}}\right)+d\left(\frac{x\sqrt{b\,x^2+ax}}{2b}-\frac{3a\left(\frac{\sqrt{b\,x^2+ax}}{b}-\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)$

input
```
int(x*(d*x+c)/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*b*d*x+3*a*d-4*b*c)*x*(b*x+a)/b^2/(x*(b*x+a))^(1/2)+1/8*a*(3*a*d-4
*b*c)/b^(5/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.72

$$\int \frac{x(c+dx)}{\sqrt{ax+bx^2}}\,dx = \left[-\frac{(4\,abc-3\,a^2d)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,(2\,b^2dx+4\,b^2c-3\,abd)\sqrt{bx^2+ax}}{8\,b^3}, \frac{(4\,abc-3\right.$$

input
```
integrate(x*(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*((4*a*b*c - 3*a^2*d)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqr
t(b)) - 2*(2*b^2*d*x + 4*b^2*c - 3*a*b*d)*sqrt(b*x^2 + a*x))/b^3, 1/4*((4*
a*b*c - 3*a^2*d)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (
2*b^2*d*x + 4*b^2*c - 3*a*b*d)*sqrt(b*x^2 + a*x))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.64

$$\int \frac{x(c+dx)}{\sqrt{ax+bx^2}}\,dx = \begin{cases} -\frac{a\left(-\frac{3ad}{4b}+c\right)\left(\begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ \frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{2b}+\sqrt{ax+bx^2}\left(\frac{dx}{2b}+\frac{-\frac{3ad}{4b}+c}{b}\right) & \text{for}\ b\neq 0 \\ \frac{2\left(\frac{c(ax)^{\frac{3}{2}}}{3}+\frac{d(ax)^{\frac{5}{2}}}{5a}\right)}{a^2} & \text{for}\ a\neq 0 \\ \tilde{\infty}\left(\frac{cx^2}{2}+\frac{dx^3}{3}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x*(d*x+c)/(b*x**2+a*x)**(1/2),x)
```

output
```
Piecewise((-a*(-3*a*d/(4*b) + c)*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b
*x**2) + 2*b*x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/s
qrt(b*(a/(2*b) + x)**2), True))/(2*b) + sqrt(a*x + b*x**2)*(d*x/(2*b) + (-
3*a*d/(4*b) + c)/b), Ne(b, 0)), (2*(c*(a*x)**(3/2)/3 + d*(a*x)**(5/2)/(5*a
))/a**2, Ne(a, 0)), (zoo*(c*x**2/2 + d*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.25

$$\int \frac{x(c+dx)}{\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{bx^2+ax}dx}{2\,b} - \frac{ac\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{2\,b^{\frac{3}{2}}} + \frac{3\,a^2d\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{5}{2}}} + \frac{\sqrt{bx^2+ax}c}{b} - \frac{3\,\sqrt{bx^2+ax}ad}{4\,b^2}$$

input
```
integrate(x*(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a*x)*d*x/b - 1/2*a*c*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*
sqrt(b))/b^(3/2) + 3/8*a^2*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/
b^(5/2) + sqrt(b*x^2 + a*x)*c/b - 3/4*sqrt(b*x^2 + a*x)*a*d/b^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

$$\int \frac{x(c+dx)}{\sqrt{ax+bx^2}}\,dx = \frac{1}{4}\,\sqrt{bx^2+ax}\left(\frac{2\,dx}{b}+\frac{4\,bc-3\,ad}{b^2}\right) + \frac{(4\,abc-3\,a^2d)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,b^{\frac{5}{2}}}$$

input
```
integrate(x*(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(b*x^2 + a*x)*(2*d*x/b + (4*b*c - 3*a*d)/b^2) + 1/8*(4*a*b*c - 3*a
^2*d)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx)}{\sqrt{ax+bx^2}}\,dx = \int \frac{x\,(c+d\,x)}{\sqrt{b\,x^2+a\,x}}\,dx$$

input
```
int((x*(c + d*x))/(a*x + b*x^2)^(1/2),x)
```

output
```
int((x*(c + d*x))/(a*x + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08

$$\int \frac{x(c+dx)}{\sqrt{ax+bx^2}}\,dx = \frac{-3\sqrt{x}\,\sqrt{bx+a}\,abd+4\sqrt{x}\,\sqrt{bx+a}\,b^2c+2\sqrt{x}\,\sqrt{bx+a}\,b^2dx+3\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2d-4\sqrt{b}\log\left(\frac{\sqrt{b}}{}\right.}{4b^3}$$

input
```
int(x*(d*x+c)/(b*x^2+a*x)^(1/2),x)
```

output
```
( - 3*sqrt(x)*sqrt(a + b*x)*a*b*d + 4*sqrt(x)*sqrt(a + b*x)*b**2*c + 2*sqr
t(x)*sqrt(a + b*x)*b**2*d*x + 3*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(
b))/sqrt(a))*a**2*d - 4*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt
(a))*a*b*c)/(4*b**3)
```

3.128 $\int \frac{c+dx}{\sqrt{ax+bx^2}} dx$

Optimal result

Integrand size = 19, antiderivative size = 55

$$\int \frac{c+dx}{\sqrt{ax+bx^2}} dx = \frac{d\sqrt{ax+bx^2}}{b} + \frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{3/2}}$$

output
```
d*(b*x^2+a*x)^(1/2)/b+(-a*d+2*b*c)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^
(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.65

$$\int \frac{c+dx}{\sqrt{ax+bx^2}} dx = \frac{\sqrt{x}\left(\sqrt{b}d\sqrt{x}(a+bx)+2(2bc-ad)\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)\right)}{b^{3/2}\sqrt{x(a+bx)}}$$

input
```
Integrate[(c + d*x)/Sqrt[a*x + b*x^2],x]
```

output
```
(Sqrt[x]*(Sqrt[b]*d*Sqrt[x]*(a + b*x) + 2*(2*b*c - a*d)*Sqrt[a + b*x]*ArcT
anh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(b^(3/2)*Sqrt[x*(a + b
*x)])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{ax+bx^2}}dx$$
$$\Big\downarrow \text{1160}$$
$$\frac{(2bc-ad)\int \frac{1}{\sqrt{bx^2+ax}}dx}{2b}+\frac{d\sqrt{ax+bx^2}}{b}$$
$$\Big\downarrow \text{1091}$$
$$\frac{(2bc-ad)\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{b}+\frac{d\sqrt{ax+bx^2}}{b}$$
$$\Big\downarrow \text{219}$$
$$\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(2bc-ad)}{b^{3/2}}+\frac{d\sqrt{ax+bx^2}}{b}$$

input
```
Int[(c + d*x)/Sqrt[a*x + b*x^2],x]
```

output
```
(d*Sqrt[a*x + b*x^2])/b + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*
x^2]])/b^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{d\sqrt{x(bx+a)}}{b}-\frac{(ad-2bc)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{b^{\frac{3}{2}}}$	46
risch	$\frac{dx(bx+a)}{b\sqrt{x(bx+a)}}-\frac{(ad-2bc)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2b^{\frac{3}{2}}}$	59
default	$\frac{c\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{\sqrt{b}}+d\left(\frac{\sqrt{b\,x^{2}+ax}}{b}-\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2b^{\frac{3}{2}}}\right)$	79

input
```
int((d*x+c)/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d*(x*(b*x+a))^(1/2)/b-(a*d-2*b*c)/b^(3/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1
/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.16

$$\int \frac{c+dx}{\sqrt{ax+bx^2}}\,dx$$
$$=\left[\frac{2\,\sqrt{bx^2+ax}bd-(2\,bc-ad)\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{2\,b^2},\frac{\sqrt{bx^2+ax}bd-(2\,bc-ad)\sqrt{-b}\operatorname{arct}}{b^2}\right.$$

input
```
integrate((d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*sqrt(b*x^2 + a*x)*b*d - (2*b*c - a*d)*sqrt(b)*log(2*b*x + a - 2*sq
rt(b*x^2 + a*x)*sqrt(b)))/b^2, (sqrt(b*x^2 + a*x)*b*d - (2*b*c - a*d)*sqrt
(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(48) = 96$.

Time = 0.38 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.22

$$\int \frac{c+dx}{\sqrt{ax+bx^2}}\,dx = \begin{cases} \left(-\frac{ad}{2b}+c\right)\left(\begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ \frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise} \end{cases}\right)+\frac{d\sqrt{ax+bx^2}}{b} & \text{for}\ b\neq 0 \\ \frac{2c\sqrt{ax}+\frac{2d(ax)^{\frac{3}{2}}}{3a}}{a} & \text{for}\ a\neq 0 \\ \tilde{\infty}\left(cx+\frac{dx^2}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)/(b*x**2+a*x)**(1/2),x)
```

output
```
Piecewise(((-a*d/(2*b) + c)*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2
) + 2*b*x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b
*(a/(2*b) + x)**2), True)) + d*sqrt(a*x + b*x**2)/b, Ne(b, 0)), ((2*c*sqrt
(a*x) + 2*d*(a*x)**(3/2)/(3*a))/a, Ne(a, 0)), (zoo*(c*x + d*x**2/2), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.36

$$\int \frac{c+dx}{\sqrt{ax+bx^2}}\,dx = \frac{c\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}} - \frac{ad\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{2\,b^{\frac{3}{2}}} + \frac{\sqrt{bx^2+ax}d}{b}$$

input
```
integrate((d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
c*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b) - 1/2*a*d*log(2*b*x
 + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(3/2) + sqrt(b*x^2 + a*x)*d/b
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07

$$\int \frac{c+dx}{\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{bx^2+ax}d}{b} - \frac{(2\,bc-ad)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
sqrt(b*x^2 + a*x)*d/b - 1/2*(2*b*c - a*d)*log(abs(2*(sqrt(b)*x - sqrt(b*x^
2 + a*x))*sqrt(b) + a))/b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40

$$\int \frac{c+dx}{\sqrt{ax+bx^2}}\,dx = \frac{d\sqrt{b\,x^2+a\,x}}{b}+\frac{c\,\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}} - \frac{a\,d\,\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{2\,b^{3/2}}$$

input
```
int((c + d*x)/(a*x + b*x^2)^(1/2),x)
```

output
```
(d*(a*x + b*x^2)^(1/2))/b + (c*log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/
2)))/b^(1/2) - (a*d*log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2)))/(2*b^(
3/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16

$$\int \frac{c+dx}{\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{x}\,\sqrt{bx+a}\,bd-\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)ad+2\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)bc}{b^2}$$

input
```
int((d*x+c)/(b*x^2+a*x)^(1/2),x)
```

output
```
(sqrt(x)*sqrt(a + b*x)*b*d - sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))
/sqrt(a))*a*d + 2*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b
*c)/b**2
```

3.129 $\int \frac{c+dx}{x\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 52

$$\int \frac{c+dx}{x\sqrt{ax+bx^2}}\,dx=-\frac{2c\sqrt{ax+bx^2}}{ax}+\frac{2d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

output
```
-2*c*(b*x^2+a*x)^(1/2)/a/x+2*d*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(1/2
)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.42

$$\int \frac{c+dx}{x\sqrt{ax+bx^2}}\,dx=-\frac{2\left(\sqrt{b}c(a+bx)+ad\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)\right)}{a\sqrt{b}\sqrt{x(a+bx)}}$$

input
```
Integrate[(c + d*x)/(x*Sqrt[a*x + b*x^2]),x]
```

output
```
(-2*(Sqrt[b]*c*(a + b*x) + a*d*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]
) + Sqrt[a + b*x]]))/(a*Sqrt[b]*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1220, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{x\sqrt{ax+bx^2}}\,dx$$
$$\Big\downarrow \ 1220$$
$$d\int \frac{1}{\sqrt{bx^2+ax}}dx-\frac{2c\sqrt{ax+bx^2}}{ax}$$
$$\Big\downarrow \ 1091$$
$$2d\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}-\frac{2c\sqrt{ax+bx^2}}{ax}$$
$$\Big\downarrow \ 219$$
$$\frac{2d\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}-\frac{2c\sqrt{ax+bx^2}}{ax}$$

input
```
Int[(c + d*x)/(x*Sqrt[a*x + b*x^2]),x]
```

output
```
(-2*c*Sqrt[a*x + b*x^2])/(a*x) + (2*d*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2
]])/Sqrt[b]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

method	result	size
pseudoelliptic	$\frac{2ad\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)x-2c\sqrt{x(bx+a)}\,\sqrt{b}}{xa\sqrt{b}}$	49
default	$\frac{d\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}-\frac{2c\sqrt{b\,x^2+ax}}{ax}$	51
risch	$-\frac{2c(bx+a)}{a\sqrt{x(bx+a)}}+\frac{d\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$	51

input
```
int((d*x+c)/x/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a*d*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*x-c*(x*(b*x+a))^(1/2)*b^(1/2))
/x/b^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.25

$$\int \frac{c+dx}{x\sqrt{ax+bx^2}}\,dx = \left[\frac{a\sqrt{b}dx\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,\sqrt{bx^2+ax}bc}{abx}, -\frac{2\left(a\sqrt{-b}dx\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+\sqrt{bx^2+ax}bc\right)}{abx}\right]$$

input
```
integrate((d*x+c)/x/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[(a*sqrt(b)*d*x*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*sqrt(b*x^
2 + a*x)*b*c)/(a*b*x), -2*(a*sqrt(-b)*d*x*arctan(sqrt(b*x^2 + a*x)*sqrt(-b
)/(b*x + a)) + sqrt(b*x^2 + a*x)*b*c)/(a*b*x)]
```

Sympy [F]

$$\int \frac{c+dx}{x\sqrt{ax+bx^2}}\,dx = \int \frac{c+dx}{x\sqrt{x\,(a+bx)}}\,dx$$

input
```
integrate((d*x+c)/x/(b*x**2+a*x)**(1/2),x)
```

output
```
Integral((c + d*x)/(x*sqrt(x*(a + b*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{c+dx}{x\sqrt{ax+bx^2}}\,dx = \frac{d\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}} - \frac{2\,\sqrt{bx^2+ax}c}{ax}$$

input
```
integrate((d*x+c)/x/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b) - 2*sqrt(b*x^2 + a*
x)*c/(a*x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int \frac{c+dx}{x\sqrt{ax+bx^2}}\,dx = -\frac{d\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{\sqrt{b}}+\frac{2\,c}{\sqrt{b}x-\sqrt{bx^2+ax}}$$

input
```
integrate((d*x+c)/x/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
-d*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/sqrt(b) + 2*c/(
sqrt(b)*x - sqrt(b*x^2 + a*x))
```

Mupad [B] (verification not implemented)

Time = 9.59 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{c+dx}{x\sqrt{ax+bx^2}}\,dx = \frac{d\,\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}-\frac{2\,c\,\sqrt{b\,x^2+a\,x}}{a\,x}$$

input
```
int((c + d*x)/(x*(a*x + b*x^2)^(1/2)),x)
```

output
```
(d*log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2)))/b^(1/2) - (2*c*(a*x + b
*x^2)^(1/2))/(a*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06

$$\int \frac{c+dx}{x\sqrt{ax+bx^2}}\,dx=\frac{-2\sqrt{x}\,\sqrt{bx+a}\,bc+2\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)adx-2\sqrt{b}\,bcx}{abx}$$

input
```
int((d*x+c)/x/(b*x^2+a*x)^(1/2),x)
```

output
```
(2*( - sqrt(x)*sqrt(a + b*x)*b*c + sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sq
rt(b))/sqrt(a))*a*d*x - sqrt(b)*b*c*x))/(a*b*x)
```

3.130 $\int \frac{c+dx}{x^2\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 57

$$\int \frac{c+dx}{x^2\sqrt{ax+bx^2}}\,dx = -\frac{2c\sqrt{ax+bx^2}}{3ax^2}+\frac{2(2bc-3ad)\sqrt{ax+bx^2}}{3a^2x}$$

output
```
-2/3*c*(b*x^2+a*x)^(1/2)/a/x^2+2/3*(-3*a*d+2*b*c)*(b*x^2+a*x)^(1/2)/a^2/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.61

$$\int \frac{c+dx}{x^2\sqrt{ax+bx^2}}\,dx = -\frac{2\sqrt{x(a+bx)}(-2bcx+a(c+3dx))}{3a^2x^2}$$

input
```
Integrate[(c + d*x)/(x^2*Sqrt[a*x + b*x^2]),x]
```

output
```
(-2*Sqrt[x*(a + b*x)]*(-2*b*c*x + a*(c + 3*d*x)))/(3*a^2*x^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1220, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{x^2\sqrt{ax+bx^2}}dx$$

$$\Big\downarrow \text{1220}$$

$$-\frac{(2bc-3ad)\int \frac{1}{x\sqrt{bx^2+ax}}dx}{3a}-\frac{2c\sqrt{ax+bx^2}}{3ax^2}$$

$$\Big\downarrow \text{1123}$$

$$\frac{2\sqrt{ax+bx^2}(2bc-3ad)}{3a^2x}-\frac{2c\sqrt{ax+bx^2}}{3ax^2}$$

input
```
Int[(c + d*x)/(x^2*Sqrt[a*x + b*x^2]),x]
```

output
```
(-2*c*Sqrt[a*x + b*x^2])/(3*a*x^2) + (2*(2*b*c - 3*a*d)*Sqrt[a*x + b*x^2])
/(3*a^2*x)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.56

method	result	size
pseudoelliptic	$-\frac{2\sqrt{x(bx+a)}\,((3dx+c)a-2cbx)}{3a^2x^2}$	32
trager	$-\frac{2(3adx-2cbx+ac)\sqrt{b\,x^2+ax}}{3a^2x^2}$	34
risch	$-\frac{2(bx+a)(3adx-2cbx+ac)}{3a^2x\sqrt{x(bx+a)}}$	37
gosper	$-\frac{2(bx+a)(3adx-2cbx+ac)}{3x\,a^2\sqrt{b\,x^2+ax}}$	39
orering	$-\frac{2(bx+a)(3adx-2cbx+ac)}{3x\,a^2\sqrt{b\,x^2+ax}}$	39
default	$c\left(-\frac{2\sqrt{b\,x^2+ax}}{3a\,x^2}+\frac{4b\sqrt{b\,x^2+ax}}{3a^2x}\right)-\frac{2d\sqrt{b\,x^2+ax}}{ax}$	64

input
```
int((d*x+c)/x^2/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(x*(b*x+a))^(1/2)*((3*d*x+c)*a-2*c*b*x)/a^2/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.61

$$\int \frac{c+dx}{x^2\sqrt{ax+bx^2}}\,dx = -\frac{2\sqrt{bx^2+ax}(ac-(2\,bc-3\,ad)x)}{3\,a^2x^2}$$

input
```
integrate((d*x+c)/x^2/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(b*x^2 + a*x)*(a*c - (2*b*c - 3*a*d)*x)/(a^2*x^2)
```

Sympy [F]

$$\int \frac{c+dx}{x^2\sqrt{ax+bx^2}}\,dx = \int \frac{c+dx}{x^2\sqrt{x\,(a+bx)}}\,dx$$

input
```
integrate((d*x+c)/x**2/(b*x**2+a*x)**(1/2),x)
```

output
```
Integral((c + d*x)/(x**2*sqrt(x*(a + b*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

$$\int \frac{c+dx}{x^2\sqrt{ax+bx^2}}\,dx = \frac{4\sqrt{bx^2+ax}bc}{3\,a^2x} - \frac{2\sqrt{bx^2+ax}d}{ax} - \frac{2\sqrt{bx^2+ax}c}{3\,ax^2}$$

input
```
integrate((d*x+c)/x^2/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
4/3*sqrt(b*x^2 + a*x)*b*c/(a^2*x) - 2*sqrt(b*x^2 + a*x)*d/(a*x) - 2/3*sqrt
(b*x^2 + a*x)*c/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.33

$$\int \frac{c+dx}{x^2\sqrt{ax+bx^2}}\,dx=\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2d+3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}c+ac\right)}{3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3}$$

input
```
integrate((d*x+c)/x^2/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*d + 3*(sqrt(b)*x - sqrt(b*x^2 + a
*x))*sqrt(b)*c + a*c)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^3
```

Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.58

$$\int \frac{c+dx}{x^2\sqrt{ax+bx^2}}\,dx=-\frac{2\sqrt{b\,x^2+a\,x}\,\left(a\,c+3\,a\,d\,x-2\,b\,c\,x\right)}{3\,a^2\,x^2}$$

input
```
int((c + d*x)/(x^2*(a*x + b*x^2)^(1/2)),x)
```

output
```
-(2*(a*x + b*x^2)^(1/2)*(a*c + 3*a*d*x - 2*b*c*x))/(3*a^2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.12

$$\int \frac{c+dx}{x^2\sqrt{ax+bx^2}}\,dx=\frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,ac}{3}-2\sqrt{x}\,\sqrt{bx+a}\,adx+\frac{4\sqrt{x}\,\sqrt{bx+a}\,bcx}{3}+\frac{2\sqrt{b}\,ad\,x^2}{3}-\frac{4\sqrt{b}\,bc\,x^2}{3}}{a^2x^2}$$

input
```
int((d*x+c)/x^2/(b*x^2+a*x)^(1/2),x)
```

output

```
(2*( - sqrt(x)*sqrt(a + b*x)*a*c - 3*sqrt(x)*sqrt(a + b*x)*a*d*x + 2*sqrt(
x)*sqrt(a + b*x)*b*c*x + sqrt(b)*a*d*x**2 - 2*sqrt(b)*b*c*x**2))/(3*a**2*x
**2)
```

3.131 $\int \frac{c+dx}{x^3\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{c+dx}{x^3\sqrt{ax+bx^2}}\,dx = -\frac{2c\sqrt{ax+bx^2}}{5ax^3}+\frac{2(4bc-5ad)\sqrt{ax+bx^2}}{15a^2x^2}-\frac{4b(4bc-5ad)\sqrt{ax+bx^2}}{15a^3x}$$

output
```
-2/5*c*(b*x^2+a*x)^(1/2)/a/x^3+2/15*(-5*a*d+4*b*c)*(b*x^2+a*x)^(1/2)/a^2/x
^2-4/15*b*(-5*a*d+4*b*c)*(b*x^2+a*x)^(1/2)/a^3/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int \frac{c+dx}{x^3\sqrt{ax+bx^2}}\,dx = -\frac{2\sqrt{x(a+bx)}(8b^2cx^2-2abx(2c+5dx)+a^2(3c+5dx))}{15a^3x^3}$$

input
```
Integrate[(c + d*x)/(x^3*Sqrt[a*x + b*x^2]),x]
```

output
```
(-2*Sqrt[x*(a + b*x)]*(8*b^2*c*x^2 - 2*a*b*x*(2*c + 5*d*x) + a^2*(3*c + 5*
d*x)))/(15*a^3*x^3)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1220, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{x^3\sqrt{ax+bx^2}}\,dx$$

$$\Big\downarrow \textbf{1220}$$

$$-\frac{(4bc-5ad)\int \frac{1}{x^2\sqrt{bx^2+ax}}dx}{5a}-\frac{2c\sqrt{ax+bx^2}}{5ax^3}$$

$$\Big\downarrow \textbf{1129}$$

$$-\frac{(4bc-5ad)\left(-\frac{2b\int \frac{1}{x\sqrt{bx^2+ax}}dx}{3a}-\frac{2\sqrt{ax+bx^2}}{3ax^2}\right)}{5a}-\frac{2c\sqrt{ax+bx^2}}{5ax^3}$$

$$\Big\downarrow \textbf{1123}$$

$$-\frac{\left(\frac{4b\sqrt{ax+bx^2}}{3a^2x}-\frac{2\sqrt{ax+bx^2}}{3ax^2}\right)(4bc-5ad)}{5a}-\frac{2c\sqrt{ax+bx^2}}{5ax^3}$$

input
```
Int[(c + d*x)/(x^3*Sqrt[a*x + b*x^2]),x]
```

output
```
(-2*c*Sqrt[a*x + b*x^2])/(5*a*x^3) - ((4*b*c - 5*a*d)*((-2*Sqrt[a*x + b*x^
2])/(3*a*x^2) + (4*b*Sqrt[a*x + b*x^2])/(3*a^2*x)))/(5*a)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.54

method	result	size
pseudoelliptic	$-\frac{2\sqrt{x(bx+a)}\left(\left(\frac{5dx}{3}+c\right)a^2-\frac{4xb\left(\frac{5dx}{2}+c\right)a}{3}+\frac{8b^2c\,x^2}{3}\right)}{5a^3x^3}$	49
trager	$-\frac{2(-10abd\,x^2+8b^2c\,x^2+5a^2dx-4abcx+3a^2c)\sqrt{b\,x^2+ax}}{15a^3x^3}$	57
risch	$-\frac{2(bx+a)(-10abd\,x^2+8b^2c\,x^2+5a^2dx-4abcx+3a^2c)}{15a^3x^2\sqrt{x(bx+a)}}$	60
gosper	$-\frac{2(bx+a)(-10abd\,x^2+8b^2c\,x^2+5a^2dx-4abcx+3a^2c)}{15x^2a^3\sqrt{b\,x^2+ax}}$	62
orering	$-\frac{2(bx+a)(-10abd\,x^2+8b^2c\,x^2+5a^2dx-4abcx+3a^2c)}{15x^2a^3\sqrt{b\,x^2+ax}}$	62
default	$c\left(-\frac{2\sqrt{b\,x^2+ax}}{5a\,x^3}-\frac{4b\left(-\frac{2\sqrt{b\,x^2+ax}}{3a\,x^2}+\frac{4b\sqrt{b\,x^2+ax}}{3a^2x}\right)}{5a}\right)+d\left(-\frac{2\sqrt{b\,x^2+ax}}{3a\,x^2}+\frac{4b\sqrt{b\,x^2+ax}}{3a^2x}\right)$	112

input
```
int((d*x+c)/x^3/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(x*(b*x+a))^(1/2)*((5/3*d*x+c)*a^2-4/3*x*b*(5/2*d*x+c)*a+8/3*b^2*c*x^
2)/a^3/x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

$$\int \frac{c+dx}{x^3\sqrt{ax+bx^2}}\,dx = -\frac{2\left(3\,a^2c+2\,(4\,b^2c-5\,abd)x^2-(4\,abc-5\,a^2d)x\right)\sqrt{bx^2+ax}}{15\,a^3x^3}$$

input
```
integrate((d*x+c)/x^3/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/15*(3*a^2*c + 2*(4*b^2*c - 5*a*b*d)*x^2 - (4*a*b*c - 5*a^2*d)*x)*sqrt(b
*x^2 + a*x)/(a^3*x^3)
```

Sympy [F]

$$\int \frac{c+dx}{x^3\sqrt{ax+bx^2}}\,dx = \int \frac{c+dx}{x^3\sqrt{x\,(a+bx)}}\,dx$$

input
```
integrate((d*x+c)/x**3/(b*x**2+a*x)**(1/2),x)
```

output
```
Integral((c + d*x)/(x**3*sqrt(x*(a + b*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.18

$$\int \frac{c+dx}{x^3\sqrt{ax+bx^2}}\,dx = -\frac{16\,\sqrt{bx^2+ax}b^2c}{15\,a^3x} + \frac{4\,\sqrt{bx^2+ax}bd}{3\,a^2x} + \frac{8\,\sqrt{bx^2+ax}bc}{15\,a^2x^2} - \frac{2\,\sqrt{bx^2+ax}d}{3\,ax^2} - \frac{2\,\sqrt{bx^2+ax}c}{5\,ax^3}$$

input
```
integrate((d*x+c)/x^3/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
-16/15*sqrt(b*x^2 + a*x)*b^2*c/(a^3*x) + 4/3*sqrt(b*x^2 + a*x)*b*d/(a^2*x)
 + 8/15*sqrt(b*x^2 + a*x)*b*c/(a^2*x^2) - 2/3*sqrt(b*x^2 + a*x)*d/(a*x^2)
- 2/5*sqrt(b*x^2 + a*x)*c/(a*x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.48

$$\int \frac{c+dx}{x^3\sqrt{ax+bx^2}}\,dx = \frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3\sqrt{b}d + 20\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2bc + 5\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2ad + 15\left(\sqrt{b}x-\right.\right.}{15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5}$$

input
```
integrate((d*x+c)/x^3/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
2/15*(15*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*sqrt(b)*d + 20*(sqrt(b)*x - sqr
t(b*x^2 + a*x))^2*b*c + 5*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a*d + 15*(sqrt
(b)*x - sqrt(b*x^2 + a*x))*a*sqrt(b)*c + 3*a^2*c)/(sqrt(b)*x - sqrt(b*x^2
+ a*x))^5
```

Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int \frac{c+dx}{x^3\sqrt{ax+bx^2}}\,dx = -\frac{2\sqrt{b\,x^2+a\,x}\,\left(5\,d\,a^2\,x+3\,c\,a^2-10\,d\,a\,b\,x^2-4\,c\,a\,b\,x+8\,c\,b^2\,x^2\right)}{15\,a^3\,x^3}$$

input
```
int((c + d*x)/(x^3*(a*x + b*x^2)^(1/2)),x)
```

output
```
-(2*(a*x + b*x^2)^(1/2)*(3*a^2*c + 8*b^2*c*x^2 + 5*a^2*d*x - 10*a*b*d*x^2
- 4*a*b*c*x))/(15*a^3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.18

$$\int \frac{c+dx}{x^3\sqrt{ax+bx^2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2c}{5}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2dx}{3}+\frac{8\sqrt{x}\,\sqrt{bx+a}\,abcx}{15}+\frac{4\sqrt{x}\,\sqrt{bx+a}\,abd\,x^2}{3}-\frac{16\sqrt{x}\,\sqrt{bx+a}\,b^2c\,x^2}{15}-\frac{4\sqrt{b}\,abd\,x^3}{3}+\frac{16\sqrt{b}\,b^2c\,x^3}{15}}{a^3x^3}$$

input
```
int((d*x+c)/x^3/(b*x^2+a*x)^(1/2),x)
```

output
```
(2*( - 3*sqrt(x)*sqrt(a + b*x)*a**2*c - 5*sqrt(x)*sqrt(a + b*x)*a**2*d*x +
 4*sqrt(x)*sqrt(a + b*x)*a*b*c*x + 10*sqrt(x)*sqrt(a + b*x)*a*b*d*x**2 - 8
*sqrt(x)*sqrt(a + b*x)*b**2*c*x**2 - 10*sqrt(b)*a*b*d*x**3 + 8*sqrt(b)*b**
2*c*x**3))/(15*a**3*x**3)
```

3.132 $\int \frac{c+dx}{x^4\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 125

$$\int \frac{c+dx}{x^4\sqrt{ax+bx^2}}\,dx = -\frac{2c\sqrt{ax+bx^2}}{7ax^4}+\frac{2(6bc-7ad)\sqrt{ax+bx^2}}{35a^2x^3} - \frac{8b(6bc-7ad)\sqrt{ax+bx^2}}{105a^3x^2}+\frac{16b^2(6bc-7ad)\sqrt{ax+bx^2}}{105a^4x}$$

output
```
-2/7*c*(b*x^2+a*x)^(1/2)/a/x^4+2/35*(-7*a*d+6*b*c)*(b*x^2+a*x)^(1/2)/a^2/x
^3-8/105*b*(-7*a*d+6*b*c)*(b*x^2+a*x)^(1/2)/a^3/x^2+16/105*b^2*(-7*a*d+6*b
*c)*(b*x^2+a*x)^(1/2)/a^4/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.61

$$\int \frac{c+dx}{x^4\sqrt{ax+bx^2}}\,dx = -\frac{2\sqrt{x(a+bx)}(-48b^3cx^3+8ab^2x^2(3c+7dx)+3a^3(5c+7dx)-2a^2bx(9c+14dx))}{105a^4x^4}$$

input
```
Integrate[(c + d*x)/(x^4*Sqrt[a*x + b*x^2]),x]
```

output
```
(-2*Sqrt[x*(a + b*x)]*(-48*b^3*c*x^3 + 8*a*b^2*x^2*(3*c + 7*d*x) + 3*a^3*(
5*c + 7*d*x) - 2*a^2*b*x*(9*c + 14*d*x)))/(105*a^4*x^4)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1220, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{x^4\sqrt{ax+bx^2}}\,dx$$

$$\Big\downarrow \text{1220}$$

$$-\frac{(6bc-7ad)\int \frac{1}{x^3\sqrt{bx^2+ax}}dx}{7a}-\frac{2c\sqrt{ax+bx^2}}{7ax^4}$$

$$\Big\downarrow \text{1129}$$

$$-\frac{(6bc-7ad)\left(-\frac{4b\int \frac{1}{x^2\sqrt{bx^2+ax}}dx}{5a}-\frac{2\sqrt{ax+bx^2}}{5ax^3}\right)}{7a}-\frac{2c\sqrt{ax+bx^2}}{7ax^4}$$

$$\Big\downarrow \text{1129}$$

$$-\frac{(6bc-7ad)\left(-\frac{4b\left(-\frac{2b\int \frac{1}{x\sqrt{bx^2+ax}}dx}{3a}-\frac{2\sqrt{ax+bx^2}}{3ax^2}\right)}{5a}-\frac{2\sqrt{ax+bx^2}}{5ax^3}\right)}{7a}-\frac{2c\sqrt{ax+bx^2}}{7ax^4}$$

$$\Big\downarrow \text{1123}$$

$$-\frac{\left(-\frac{4b\left(\frac{4b\sqrt{ax+bx^2}}{3a^2x}-\frac{2\sqrt{ax+bx^2}}{3ax^2}\right)}{5a}-\frac{2\sqrt{ax+bx^2}}{5ax^3}\right)(6bc-7ad)}{7a}-\frac{2c\sqrt{ax+bx^2}}{7ax^4}$$

input
```
Int[(c + d*x)/(x^4*Sqrt[a*x + b*x^2]),x]
```

output
```
(-2*c*Sqrt[a*x + b*x^2])/(7*a*x^4) - ((6*b*c - 7*a*d)*((-2*Sqrt[a*x + b*x^
2])/(5*a*x^3) - (4*b*((-2*Sqrt[a*x + b*x^2])/(3*a*x^2) + (4*b*Sqrt[a*x + b
*x^2])/(3*a^2*x)))/(5*a)))/(7*a)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.53

method	result
pseudoelliptic	$-\frac{2\left(\left(\frac{7dx}{5}+c\right)a^{3}-\frac{6\left(\frac{14dx}{9}+c\right)xb\,a^{2}}{5}+\frac{8x^{2}b^{2}\left(\frac{7dx}{3}+c\right)a}{5}-\frac{16b^{3}c\,x^{3}}{5}\right)\sqrt{x(bx+a)}}{7a^{4}x^{4}}$
trager	$-\frac{2\left(56a\,b^{2}d\,x^{3}-48b^{3}c\,x^{3}-28a^{2}bd\,x^{2}+24a\,b^{2}c\,x^{2}+21a^{3}dx-18a^{2}bcx+15c\,a^{3}\right)\sqrt{b\,x^{2}+ax}}{105a^{4}x^{4}}$
risch	$-\frac{2(bx+a)\left(56a\,b^{2}d\,x^{3}-48b^{3}c\,x^{3}-28a^{2}bd\,x^{2}+24a\,b^{2}c\,x^{2}+21a^{3}dx-18a^{2}bcx+15c\,a^{3}\right)}{105a^{4}x^{3}\sqrt{x(bx+a)}}$
gosper	$-\frac{2(bx+a)\left(56a\,b^{2}d\,x^{3}-48b^{3}c\,x^{3}-28a^{2}bd\,x^{2}+24a\,b^{2}c\,x^{2}+21a^{3}dx-18a^{2}bcx+15c\,a^{3}\right)}{105x^{3}a^{4}\sqrt{b\,x^{2}+ax}}$
orering	$-\frac{2(bx+a)\left(56a\,b^{2}d\,x^{3}-48b^{3}c\,x^{3}-28a^{2}bd\,x^{2}+24a\,b^{2}c\,x^{2}+21a^{3}dx-18a^{2}bcx+15c\,a^{3}\right)}{105x^{3}a^{4}\sqrt{b\,x^{2}+ax}}$
default	$c\left(-\frac{2\sqrt{b\,x^{2}+ax}}{7a\,x^{4}}-\frac{6b\left(-\frac{2\sqrt{b\,x^{2}+ax}}{5a\,x^{3}}-\frac{4b\left(-\frac{2\sqrt{b\,x^{2}+ax}}{3a\,x^{2}}+\frac{4b\sqrt{b\,x^{2}+ax}}{3a^{2}x}\right)}{5a}\right)}{7a}\right)+d\left(-\frac{2\sqrt{b\,x^{2}+ax}}{5a\,x^{3}}-\frac{4b\left(-\frac{2\sqrt{b\,x^{2}+a}}{3a\,x^{2}}\right.}{5}\right.$

input
```
int((d*x+c)/x^4/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7*((7/5*d*x+c)*a^3-6/5*(14/9*d*x+c)*x*b*a^2+8/5*x^2*b^2*(7/3*d*x+c)*a-1
6/5*b^3*c*x^3)*(x*(b*x+a))^(1/2)/a^4/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.66

$$\int \frac{c+dx}{x^4\sqrt{ax+bx^2}}\,dx = -\frac{2\,(15\,a^3c-8\,(6\,b^3c-7\,ab^2d)x^3+4\,(6\,ab^2c-7\,a^2bd)x^2-3\,(6\,a^2bc-7\,a^3d)x)\sqrt{bx^2+ax}}{105\,a^4x^4}$$

input
```
integrate((d*x+c)/x^4/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/105*(15*a^3*c - 8*(6*b^3*c - 7*a*b^2*d)*x^3 + 4*(6*a*b^2*c - 7*a^2*b*d)
*x^2 - 3*(6*a^2*b*c - 7*a^3*d)*x)*sqrt(b*x^2 + a*x)/(a^4*x^4)
```

Sympy [F]

$$\int \frac{c+dx}{x^4\sqrt{ax+bx^2}}\,dx=\int \frac{c+dx}{x^4\sqrt{x\,(a+bx)}}\,dx$$

input
```
integrate((d*x+c)/x**4/(b*x**2+a*x)**(1/2),x)
```

output
```
Integral((c + d*x)/(x**4*sqrt(x*(a + b*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.22

$$\begin{aligned}\int \frac{c+dx}{x^4\sqrt{ax+bx^2}}\,dx=&\ \frac{32\,\sqrt{bx^2+ax}b^3c}{35\,a^4x}-\frac{16\,\sqrt{bx^2+ax}b^2d}{15\,a^3x}\\&-\frac{16\,\sqrt{bx^2+ax}b^2c}{35\,a^3x^2}+\frac{8\,\sqrt{bx^2+ax}bd}{15\,a^2x^2}\\&+\frac{12\,\sqrt{bx^2+ax}bc}{35\,a^2x^3}-\frac{2\,\sqrt{bx^2+ax}d}{5\,ax^3}-\frac{2\,\sqrt{bx^2+ax}c}{7\,ax^4}\end{aligned}$$

input
```
integrate((d*x+c)/x^4/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
32/35*sqrt(b*x^2 + a*x)*b^3*c/(a^4*x) - 16/15*sqrt(b*x^2 + a*x)*b^2*d/(a^3
*x) - 16/35*sqrt(b*x^2 + a*x)*b^2*c/(a^3*x^2) + 8/15*sqrt(b*x^2 + a*x)*b*d
/(a^2*x^2) + 12/35*sqrt(b*x^2 + a*x)*b*c/(a^2*x^3) - 2/5*sqrt(b*x^2 + a*x)
*d/(a*x^3) - 2/7*sqrt(b*x^2 + a*x)*c/(a*x^4)
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.53

$$\int \frac{c+dx}{x^4\sqrt{ax+bx^2}}\,dx = \frac{2\left(140\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4bd+210\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3b^{\frac{3}{2}}c+105\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3a\sqrt{b}d+252\right.}{105\left(\sqrt{b}x-\sqrt{\right.}}$$

input
```
integrate((d*x+c)/x^4/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
2/105*(140*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*b*d + 210*(sqrt(b)*x - sqrt(b
*x^2 + a*x))^3*b^(3/2)*c + 105*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a*sqrt(b)
*d + 252*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a*b*c + 21*(sqrt(b)*x - sqrt(b*
x^2 + a*x))^2*a^2*d + 105*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^2*sqrt(b)*c +
15*a^3*c)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^7
```

Mupad [B] (verification not implemented)

Time = 9.51 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90

$$\int \frac{c+dx}{x^4\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{b\,x^2+a\,x}\,\left(96\,b^3\,c-112\,a\,b^2\,d\right)}{105\,a^4\,x}-\frac{\sqrt{b\,x^2+a\,x}\,\left(14\,a\,d-12\,b\,c\right)}{35\,a^2\,x^3}-\frac{2\,c\,\sqrt{b\,x^2+a\,x}}{7\,a\,x^4}-\frac{\sqrt{b\,x^2+a\,x}\,\left(48\,b^2\,c-56\,a\,b\,d\right)}{105\,a^3\,x^2}$$

input
```
int((c + d*x)/(x^4*(a*x + b*x^2)^(1/2)),x)
```

output
```
((a*x + b*x^2)^(1/2)*(96*b^3*c - 112*a*b^2*d))/(105*a^4*x) - ((a*x + b*x^2
)^(1/2)*(14*a*d - 12*b*c))/(35*a^2*x^3) - (2*c*(a*x + b*x^2)^(1/2))/(7*a*x
^4) - ((a*x + b*x^2)^(1/2)*(48*b^2*c - 56*a*b*d))/(105*a^3*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.18

$$\int \frac{c+dx}{x^4\sqrt{ax+bx^2}}\,dx$$
$$= \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3c}{7}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3dx}{5}+\frac{12\sqrt{x}\,\sqrt{bx+a}\,a^2bcx}{35}+\frac{8\sqrt{x}\,\sqrt{bx+a}\,a^2bd\,x^2}{15}-\frac{16\sqrt{x}\,\sqrt{bx+a}\,a\,b^2c\,x^2}{35}-\frac{16\sqrt{x}\,\sqrt{bx+a}\,a\,b^2d\,x^3}{15}}{a^4x^4}$$

input
```
int((d*x+c)/x^4/(b*x^2+a*x)^(1/2),x)
```

output
```
(2*( - 15*sqrt(x)*sqrt(a + b*x)*a**3*c - 21*sqrt(x)*sqrt(a + b*x)*a**3*d*x
 + 18*sqrt(x)*sqrt(a + b*x)*a**2*b*c*x + 28*sqrt(x)*sqrt(a + b*x)*a**2*b*d
*x**2 - 24*sqrt(x)*sqrt(a + b*x)*a*b**2*c*x**2 - 56*sqrt(x)*sqrt(a + b*x)*
a*b**2*d*x**3 + 48*sqrt(x)*sqrt(a + b*x)*b**3*c*x**3 + 56*sqrt(b)*a*b**2*d
*x**4 - 48*sqrt(b)*b**3*c*x**4))/(105*a**4*x**4)
```

3.133 $\int \frac{c+dx}{x^5\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 160

$$\int \frac{c+dx}{x^5\sqrt{ax+bx^2}}\,dx = -\frac{2c\sqrt{ax+bx^2}}{9ax^5}+\frac{2(8bc-9ad)\sqrt{ax+bx^2}}{63a^2x^4} - \frac{4b(8bc-9ad)\sqrt{ax+bx^2}}{105a^3x^3}+\frac{16b^2(8bc-9ad)\sqrt{ax+bx^2}}{315a^4x^2} - \frac{32b^3(8bc-9ad)\sqrt{ax+bx^2}}{315a^5x}$$

output
```
-2/9*c*(b*x^2+a*x)^(1/2)/a/x^5+2/63*(-9*a*d+8*b*c)*(b*x^2+a*x)^(1/2)/a^2/x
^4-4/105*b*(-9*a*d+8*b*c)*(b*x^2+a*x)^(1/2)/a^3/x^3+16/315*b^2*(-9*a*d+8*b
*c)*(b*x^2+a*x)^(1/2)/a^4/x^2-32/315*b^3*(-9*a*d+8*b*c)*(b*x^2+a*x)^(1/2)/
a^5/x
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.59

$$\int \frac{c+dx}{x^5\sqrt{ax+bx^2}}\,dx = -\frac{2\sqrt{x(a+bx)}(128b^4cx^4+24a^2b^2x^2(2c+3dx)-16ab^3x^3(4c+9dx)+5a^4(7c+9dx)-2a^3bx(20c+2}{315a^5x^5}$$

input
```
Integrate[(c + d*x)/(x^5*Sqrt[a*x + b*x^2]),x]
```

output
```
(-2*Sqrt[x*(a + b*x)]*(128*b^4*c*x^4 + 24*a^2*b^2*x^2*(2*c + 3*d*x) - 16*a
*b^3*x^3*(4*c + 9*d*x) + 5*a^4*(7*c + 9*d*x) - 2*a^3*b*x*(20*c + 27*d*x)))
/(315*a^5*x^5)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1220, 1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{x^5\sqrt{ax+bx^2}}dx$$

$$\Big\downarrow \ 1220$$

$$-\frac{(8bc-9ad)\int \frac{1}{x^4\sqrt{bx^2+ax}}dx}{9a}-\frac{2c\sqrt{ax+bx^2}}{9ax^5}$$

$$\Big\downarrow \ 1129$$

$$-\frac{(8bc-9ad)\left(-\frac{6b\int \frac{1}{x^3\sqrt{bx^2+ax}}dx}{7a}-\frac{2\sqrt{ax+bx^2}}{7ax^4}\right)}{9a}-\frac{2c\sqrt{ax+bx^2}}{9ax^5}$$

$$\Big\downarrow \ 1129$$

$$-\frac{(8bc-9ad)\left(-\frac{6b\left(-\frac{4b\int \frac{1}{x^2\sqrt{bx^2+ax}}dx}{5a}-\frac{2\sqrt{ax+bx^2}}{5ax^3}\right)}{7a}-\frac{2\sqrt{ax+bx^2}}{7ax^4}\right)}{9a}-\frac{2c\sqrt{ax+bx^2}}{9ax^5}$$

$\downarrow$ 1129

$$-\frac{(8bc-9ad)\left(-\frac{6b\left(-\frac{4b\left(-\frac{2b\int \frac{1}{x\sqrt{bx^2+ax}}dx}{3a}-\frac{2\sqrt{ax+bx^2}}{3ax^2}\right)}{5a}-\frac{2\sqrt{ax+bx^2}}{5ax^3}\right)}{7a}-\frac{2\sqrt{ax+bx^2}}{7ax^4}\right)}{9a}-\frac{2c\sqrt{ax+bx^2}}{9ax^5}$$

$\downarrow$ 1123

$$-\frac{\left(-\frac{6b\left(-\frac{4b\left(\frac{4b\sqrt{ax+bx^2}}{3a^2x}-\frac{2\sqrt{ax+bx^2}}{3ax^2}\right)}{5a}-\frac{2\sqrt{ax+bx^2}}{5ax^3}\right)}{7a}-\frac{2\sqrt{ax+bx^2}}{7ax^4}\right)(8bc-9ad)}{9a}-\frac{2c\sqrt{ax+bx^2}}{9ax^5}$$

input
```
Int[(c + d*x)/(x^5*Sqrt[a*x + b*x^2]),x]
```

output
```
(-2*c*Sqrt[a*x + b*x^2])/(9*a*x^5) - ((8*b*c - 9*a*d)*((-2*Sqrt[a*x + b*x^
2])/(7*a*x^4) - (6*b*((-2*Sqrt[a*x + b*x^2])/(5*a*x^3) - (4*b*((-2*Sqrt[a*
x + b*x^2])/(3*a*x^2) + (4*b*Sqrt[a*x + b*x^2])/(3*a^2*x)))/(5*a)))/(7*a))
)/(9*a)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.52

method	result
pseudoelliptic	$-\frac{2\sqrt{x(bx+a)}\left(\left(\frac{9dx}{7}+c\right)a^4-\frac{8\left(\frac{27dx}{20}+c\right)xb\,a^3}{7}+\frac{48x^2b^2\left(\frac{3dx}{2}+c\right)a^2}{35}-\frac{64\left(\frac{9dx}{4}+c\right)x^3b^3a}{35}+\frac{128x^4b^4c}{35}\right)}{9a^5x^5}$
trager	$-\frac{2(-144x^4a\,b^3d+128x^4b^4c+72a^2b^2d\,x^3-64a\,b^3c\,x^3-54a^3bd\,x^2+48a^2b^2c\,x^2+45a^4dx-40a^3bcx+35c\,a^4)\sqrt{b\,x^2+ax}}{315a^5x^5}$
risch	$-\frac{2(bx+a)(-144x^4a\,b^3d+128x^4b^4c+72a^2b^2d\,x^3-64a\,b^3c\,x^3-54a^3bd\,x^2+48a^2b^2c\,x^2+45a^4dx-40a^3bcx+35c\,a^4)}{315a^5x^4\sqrt{x(bx+a)}}$
gosper	$-\frac{2(bx+a)(-144x^4a\,b^3d+128x^4b^4c+72a^2b^2d\,x^3-64a\,b^3c\,x^3-54a^3bd\,x^2+48a^2b^2c\,x^2+45a^4dx-40a^3bcx+35c\,a^4)}{315x^4a^5\sqrt{b\,x^2+ax}}$
orering	$-\frac{2(bx+a)(-144x^4a\,b^3d+128x^4b^4c+72a^2b^2d\,x^3-64a\,b^3c\,x^3-54a^3bd\,x^2+48a^2b^2c\,x^2+45a^4dx-40a^3bcx+35c\,a^4)}{315x^4a^5\sqrt{b\,x^2+ax}}$
default	$c\left(-\frac{2\sqrt{b\,x^2+ax}}{9a\,x^5}-\frac{8b\left(-\frac{2\sqrt{b\,x^2+ax}}{7a\,x^4}-\frac{6b\left(-\frac{2\sqrt{b\,x^2+ax}}{5a\,x^3}-\frac{4b\left(-\frac{2\sqrt{b\,x^2+ax}}{3a\,x^2}+\frac{4b\sqrt{b\,x^2+ax}}{3a^2x}\right)}{5a}\right)}{7a}\right)}{9a}\right)+d\left(-\frac{2\sqrt{b\,x^2+ax}}{7a\,x^4}\right.$

input
```
int((d*x+c)/x^5/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(x*(b*x+a))^(1/2)*((9/7*d*x+c)*a^4-8/7*(27/20*d*x+c)*x*b*a^3+48/35*x^
2*b^2*(3/2*d*x+c)*a^2-64/35*(9/4*d*x+c)*x^3*b^3*a+128/35*x^4*b^4*c)/a^5/x^
5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.66

$$\int \frac{c+dx}{x^5\sqrt{ax+bx^2}}\,dx = -\frac{2\,(35\,a^4c+16\,(8\,b^4c-9\,ab^3d)x^4-8\,(8\,ab^3c-9\,a^2b^2d)x^3+6\,(8\,a^2b^2c-9\,a^3bd)x^2-5\,(8\,a^3bc-9\,a^4d)}{315\,a^5x^5}$$

input
```
integrate((d*x+c)/x^5/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/315*(35*a^4*c + 16*(8*b^4*c - 9*a*b^3*d)*x^4 - 8*(8*a*b^3*c - 9*a^2*b^2
*d)*x^3 + 6*(8*a^2*b^2*c - 9*a^3*b*d)*x^2 - 5*(8*a^3*b*c - 9*a^4*d)*x)*sqr
t(b*x^2 + a*x)/(a^5*x^5)
```

Sympy [F]

$$\int \frac{c+dx}{x^5\sqrt{ax+bx^2}}\,dx = \int \frac{c+dx}{x^5\sqrt{x\,(a+bx)}}\,dx$$

input
```
integrate((d*x+c)/x**5/(b*x**2+a*x)**(1/2),x)
```

output
```
Integral((c + d*x)/(x**5*sqrt(x*(a + b*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.24

$$\int \frac{c+dx}{x^5\sqrt{ax+bx^2}}\,dx = -\frac{256\,\sqrt{bx^2+ax}b^4c}{315\,a^5x}+\frac{32\,\sqrt{bx^2+ax}b^3d}{35\,a^4x}+\frac{128\,\sqrt{bx^2+ax}b^3c}{315\,a^4x^2}-\frac{16\,\sqrt{bx^2+ax}b^2d}{35\,a^3x^2}-\frac{32\,\sqrt{bx^2+ax}b^2c}{105\,a^3x^3}+\frac{12\,\sqrt{bx^2+ax}bd}{35\,a^2x^3}+\frac{16\,\sqrt{bx^2+ax}bc}{63\,a^2x^4}-\frac{2\,\sqrt{bx^2+ax}d}{7\,ax^4}-\frac{2\,\sqrt{bx^2+ax}c}{9\,ax^5}$$

input
```
integrate((d*x+c)/x^5/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
-256/315*sqrt(b*x^2 + a*x)*b^4*c/(a^5*x) + 32/35*sqrt(b*x^2 + a*x)*b^3*d/(
a^4*x) + 128/315*sqrt(b*x^2 + a*x)*b^3*c/(a^4*x^2) - 16/35*sqrt(b*x^2 + a*
x)*b^2*d/(a^3*x^2) - 32/105*sqrt(b*x^2 + a*x)*b^2*c/(a^3*x^3) + 12/35*sqrt
(b*x^2 + a*x)*b*d/(a^2*x^3) + 16/63*sqrt(b*x^2 + a*x)*b*c/(a^2*x^4) - 2/7*
sqrt(b*x^2 + a*x)*d/(a*x^4) - 2/9*sqrt(b*x^2 + a*x)*c/(a*x^5)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.57

$$\int \frac{c+dx}{x^5\sqrt{ax+bx^2}}\,dx = \frac{2\left(630\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5 b^{\frac{3}{2}}d+1008\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4 b^2c+756\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4 abd+168\right.}{}$$

input
```
integrate((d*x+c)/x^5/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
2/315*(630*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*b^(3/2)*d + 1008*(sqrt(b)*x -
 sqrt(b*x^2 + a*x))^4*b^2*c + 756*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a*b*d
+ 1680*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a*b^(3/2)*c + 315*(sqrt(b)*x - sq
rt(b*x^2 + a*x))^3*a^2*sqrt(b)*d + 1080*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*
a^2*b*c + 45*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^3*d + 315*(sqrt(b)*x - sq
rt(b*x^2 + a*x))*a^3*sqrt(b)*c + 35*a^4*c)/(sqrt(b)*x - sqrt(b*x^2 + a*x))
^9
```

Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.91

$$\int \frac{c+dx}{x^5\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{b\,x^2+a\,x}\,\left(128\,b^3\,c-144\,a\,b^2\,d\right)}{315\,a^4\,x^2}-\frac{\sqrt{b\,x^2+a\,x}\,\left(18\,a\,d-16\,b\,c\right)}{63\,a^2\,x^4}-\frac{\sqrt{b\,x^2+a\,x}\,\left(256\,b^4\,c-288\,a\,b^3\,d\right)}{315\,a^5\,x}-\frac{2\,c\,\sqrt{b\,x^2+a\,x}}{9\,a\,x^5}-\frac{\sqrt{b\,x^2+a\,x}\,\left(32\,b^2\,c-36\,a\,b\,d\right)}{105\,a^3\,x^3}$$

input
```
int((c + d*x)/(x^5*(a*x + b*x^2)^(1/2)),x)
```

output
```
((a*x + b*x^2)^(1/2)*(128*b^3*c - 144*a*b^2*d))/(315*a^4*x^2) - ((a*x + b*
x^2)^(1/2)*(18*a*d - 16*b*c))/(63*a^2*x^4) - ((a*x + b*x^2)^(1/2)*(256*b^4
*c - 288*a*b^3*d))/(315*a^5*x) - (2*c*(a*x + b*x^2)^(1/2))/(9*a*x^5) - ((a
*x + b*x^2)^(1/2)*(32*b^2*c - 36*a*b*d))/(105*a^3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.18

$$\int \frac{c+dx}{x^5\sqrt{ax+bx^2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4c}{9}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4dx}{7}+\frac{16\sqrt{x}\,\sqrt{bx+a}\,a^3bcx}{63}+\frac{12\sqrt{x}\,\sqrt{bx+a}\,a^3bd\,x^2}{35}-\frac{32\sqrt{x}\,\sqrt{bx+a}\,a^2b^2c\,x^2}{105}-\frac{16\sqrt{x}\,\sqrt{bx+a}\,a^2b^2d}{35}}{a^5x^5}$$

input
```
int((d*x+c)/x^5/(b*x^2+a*x)^(1/2),x)
```

output
```
(2*( - 35*sqrt(x)*sqrt(a + b*x)*a**4*c - 45*sqrt(x)*sqrt(a + b*x)*a**4*d*x
 + 40*sqrt(x)*sqrt(a + b*x)*a**3*b*c*x + 54*sqrt(x)*sqrt(a + b*x)*a**3*b*d
*x**2 - 48*sqrt(x)*sqrt(a + b*x)*a**2*b**2*c*x**2 - 72*sqrt(x)*sqrt(a + b*
x)*a**2*b**2*d*x**3 + 64*sqrt(x)*sqrt(a + b*x)*a*b**3*c*x**3 + 144*sqrt(x)
*sqrt(a + b*x)*a*b**3*d*x**4 - 128*sqrt(x)*sqrt(a + b*x)*b**4*c*x**4 - 144
*sqrt(b)*a*b**3*d*x**5 + 128*sqrt(b)*b**4*c*x**5))/(315*a**5*x**5)
```

3.134 $\int \frac{x^3(c+dx)^2}{\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 249

$$\begin{aligned}\int \frac{x^3(c+dx)^2}{\sqrt{ax+bx^2}}\,dx &= \frac{a^2(80b^2c^2-7ad(20bc-9ad))\sqrt{ax+bx^2}}{128b^5} \\ &\quad - \frac{a(80b^2c^2-7ad(20bc-9ad))\,x\sqrt{ax+bx^2}}{192b^4} \\ &\quad + \frac{\left(80c^2-\frac{7ad(20bc-9ad)}{b^2}\right)x^2\sqrt{ax+bx^2}}{240b} \\ &\quad + \frac{d(20bc-9ad)x^3\sqrt{ax+bx^2}}{40b^2} + \frac{d^2x^4\sqrt{ax+bx^2}}{5b} \\ &\quad - \frac{a^3(80b^2c^2-7ad(20bc-9ad))\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{128b^{11/2}}\end{aligned}$$

output
```
1/128*a^2*(80*b^2*c^2-7*a*d*(-9*a*d+20*b*c))*(b*x^2+a*x)^(1/2)/b^5-1/192*a
*(80*b^2*c^2-7*a*d*(-9*a*d+20*b*c))*x*(b*x^2+a*x)^(1/2)/b^4+1/240*(80*c^2-
7*a*d*(-9*a*d+20*b*c)/b^2)*x^2*(b*x^2+a*x)^(1/2)/b+1/40*d*(-9*a*d+20*b*c)*
x^3*(b*x^2+a*x)^(1/2)/b^2+1/5*d^2*x^4*(b*x^2+a*x)^(1/2)/b-1/128*a^3*(80*b^
2*c^2-7*a*d*(-9*a*d+20*b*c))*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.98

$$\int \frac{x^3(c+dx)^2}{\sqrt{ax+bx^2}}\,dx$$
$$= \frac{x(a+bx)\left(1200a^2b^2c^2-2100a^3bcd+945a^4d^2-800ab^3c^2x+1400a^2b^2cdx-630a^3bd^2x+640b^4c^2x^2-1\right.}{1920b^5\sqrt{x(a+bx)}}$$
$$-\frac{a^3(80b^2c^2-140abcd+63a^2d^2)\sqrt{x}\sqrt{a+bx}\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{64b^{11/2}\sqrt{x(a+bx)}}$$

input
```
Integrate[(x^3*(c + d*x)^2)/Sqrt[a*x + b*x^2],x]
```

output
```
(x*(a + b*x)*(1200*a^2*b^2*c^2 - 2100*a^3*b*c*d + 945*a^4*d^2 - 800*a*b^3*
c^2*x + 1400*a^2*b^2*c*d*x - 630*a^3*b*d^2*x + 640*b^4*c^2*x^2 - 1120*a*b^
3*c*d*x^2 + 504*a^2*b^2*d^2*x^2 + 960*b^4*c*d*x^3 - 432*a*b^3*d^2*x^3 + 38
4*b^4*d^2*x^4))/(1920*b^5*Sqrt[x*(a + b*x)]) - (a^3*(80*b^2*c^2 - 140*a*b*
c*d + 63*a^2*d^2)*Sqrt[x]*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a
] + Sqrt[a + b*x])])/(64*b^(11/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.83, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1262, 27, 1221, 1134, 1134, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx)^2}{\sqrt{ax+bx^2}}\,dx$$
$$\Big\downarrow \text{1262}$$
$$\frac{\int \frac{x^3(10bc^2+d(20bc-9ad)x)}{2\sqrt{bx^2+ax}}dx}{5b}+\frac{d^2x^4\sqrt{ax+bx^2}}{5b}$$
$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x^3(10bc^2+d(20bc-9ad)x)}{\sqrt{bx^2+ax}}dx}{10b}+\frac{d^2x^4\sqrt{ax+bx^2}}{5b}$$

$\big\downarrow$ 1221

$$\frac{\frac{(63a^2d^2-140abcd+80b^2c^2)\int \frac{x^3}{\sqrt{bx^2+ax}}dx}{8b}+\frac{dx^3\sqrt{ax+bx^2}(20bc-9ad)}{4b}}{10b}+\frac{d^2x^4\sqrt{ax+bx^2}}{5b}$$

$\big\downarrow$ 1134

$$\frac{\frac{(63a^2d^2-140abcd+80b^2c^2)\left(\frac{x^2\sqrt{ax+bx^2}}{3b}-\frac{5a\int \frac{x^2}{\sqrt{bx^2+ax}}dx}{6b}\right)}{8b}+\frac{dx^3\sqrt{ax+bx^2}(20bc-9ad)}{4b}}{10b}+\frac{d^2x^4\sqrt{ax+bx^2}}{5b}$$

$\big\downarrow$ 1134

$$\frac{\frac{(63a^2d^2-140abcd+80b^2c^2)\left(\frac{x^2\sqrt{ax+bx^2}}{3b}-\frac{5a\left(\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\int \frac{x}{\sqrt{bx^2+ax}}dx}{4b}\right)}{6b}\right)}{8b}+\frac{dx^3\sqrt{ax+bx^2}(20bc-9ad)}{4b}}{10b}+\frac{d^2x^4\sqrt{ax+bx^2}}{5b}$$

$\big\downarrow$ 1160

$$\frac{\frac{(63a^2d^2-140abcd+80b^2c^2)\left(\frac{x^2\sqrt{ax+bx^2}}{3b}-\frac{5a\left(\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\left(\frac{\sqrt{ax+bx^2}}{b}-\frac{a\int \frac{1}{\sqrt{bx^2+ax}}dx}{2b}\right)}{4b}\right)}{6b}\right)}{8b}+\frac{dx^3\sqrt{ax+bx^2}(20bc-9ad)}{4b}}{10b}+\frac{d^2x^4\sqrt{ax+bx^2}}{5b}$$

$\big\downarrow$ 1091

$$\frac{\frac{(63a^2d^2-140abcd+80b^2c^2)\left(\frac{x^2\sqrt{ax+bx^2}}{3b}-\frac{5a\left(\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\left(\frac{\sqrt{ax+bx^2}}{b}-\frac{a\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{b}\right)}{4b}\right)}{6b}\right)}{8b}+\frac{dx^3\sqrt{ax+bx^2}(20bc-9ad)}{4b}}{10b}+\frac{d^2x^4\sqrt{ax+bx^2}}{5b}$$

$\Big\downarrow$ 219

$$\frac{\frac{\left(\frac{x^2\sqrt{ax+bx^2}}{3b}-\frac{5a\left(\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\left(\frac{\sqrt{ax+bx^2}}{b}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{3/2}}\right)}{4b}\right)}{6b}\right)(63a^2d^2-140abcd+80b^2c^2)}{8b}+\frac{dx^3\sqrt{ax+bx^2}(20bc-9ad)}{4b}}{10b}+\frac{d^2x^4\sqrt{ax+bx^2}}{5b}$$

input
```
Int[(x^3*(c + d*x)^2)/Sqrt[a*x + b*x^2],x]
```

output
```
(d^2*x^4*Sqrt[a*x + b*x^2])/(5*b) + ((d*(20*b*c - 9*a*d)*x^3*Sqrt[a*x + b*
x^2])/(4*b) + ((80*b^2*c^2 - 140*a*b*c*d + 63*a^2*d^2)*((x^2*Sqrt[a*x + b*
x^2])/(3*b) - (5*a*((x*Sqrt[a*x + b*x^2])/(2*b) - (3*a*(Sqrt[a*x + b*x^2]/
b - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/b^(3/2)))/(4*b)))/(6*b)))/(
8*b))/(10*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$-\frac{63\left(a^3\left(a^2d^2-\frac{20}{9}abcd+\frac{80}{63}b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)-\left(\frac{80\left(\frac{21}{50}d^2x^2+\frac{7}{6}cdx+c^2\right)a^2b^{\frac{5}{2}}}{63}-\frac{160xa\left(\frac{27}{50}d^2x^2+\frac{7}{5}cdx+c^2\right)b^{\frac{7}{2}}}{189}+\frac{12}{}\right.\right.}{128b^{\frac{11}{2}}}$
risch	$\frac{(384b^4d^2x^4-432a\,b^3d^2x^3+960b^4cd\,x^3+504a^2b^2d^2x^2-1120a\,b^3cd\,x^2+640c^2x^2b^4-630a^3b\,d^2x+1400a^2b^2cdx-800a\,b^3c^2x}{1920b^5\sqrt{x(bx+a)}}$
default	$c^2\left(\frac{x^2\sqrt{b\,x^2+ax}}{3b}-\frac{5a\left(\frac{x\sqrt{b\,x^2+ax}}{2b}-\frac{3a\left(\frac{\sqrt{b\,x^2+ax}}{b}-\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)}{6b}\right)+d^2\left(\frac{x^4\sqrt{b\,x^2+ax}}{5b}-\frac{9a\left(\frac{x}{}\right.}{}\right.$

input
```
int(x^3*(d*x+c)^2/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-63/128*(a^3*(a^2*d^2-20/9*a*b*c*d+80/63*b^2*c^2)*arctanh((x*(b*x+a))^(1/2
)/x/b^(1/2))-(80/63*(21/50*d^2*x^2+7/6*c*d*x+c^2)*a^2*b^(5/2)-160/189*x*a*
(27/50*d^2*x^2+7/5*c*d*x+c^2)*b^(7/2)+128/189*x^2*(3/5*d^2*x^2+3/2*c*d*x+c
^2)*b^(9/2)+d*((-2/3*d*x-20/9*c)*b^(3/2)+b^(1/2)*a*d)*a^3)*(x*(b*x+a))^(1/
2))/b^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.67

$$\int \frac{x^3(c+dx)^2}{\sqrt{ax+bx^2}}\,dx = \left[\frac{15\,(80\,a^3b^2c^2-140\,a^4bcd+63\,a^5d^2)\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(384\,b^5d^2x^4+1200\,a^2b^3c^2}{}\right.$$

input
```
integrate(x^3*(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/3840*(15*(80*a^3*b^2*c^2 - 140*a^4*b*c*d + 63*a^5*d^2)*sqrt(b)*log(2*b*
x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(384*b^5*d^2*x^4 + 1200*a^2*b^3*c
^2 - 2100*a^3*b^2*c*d + 945*a^4*b*d^2 + 48*(20*b^5*c*d - 9*a*b^4*d^2)*x^3
+ 8*(80*b^5*c^2 - 140*a*b^4*c*d + 63*a^2*b^3*d^2)*x^2 - 10*(80*a*b^4*c^2 -
 140*a^2*b^3*c*d + 63*a^3*b^2*d^2)*x)*sqrt(b*x^2 + a*x))/b^6, 1/1920*(15*(
80*a^3*b^2*c^2 - 140*a^4*b*c*d + 63*a^5*d^2)*sqrt(-b)*arctan(sqrt(b*x^2 +
a*x)*sqrt(-b)/(b*x + a)) + (384*b^5*d^2*x^4 + 1200*a^2*b^3*c^2 - 2100*a^3*
b^2*c*d + 945*a^4*b*d^2 + 48*(20*b^5*c*d - 9*a*b^4*d^2)*x^3 + 8*(80*b^5*c^
2 - 140*a*b^4*c*d + 63*a^2*b^3*d^2)*x^2 - 10*(80*a*b^4*c^2 - 140*a^2*b^3*c
*d + 63*a^3*b^2*d^2)*x)*sqrt(b*x^2 + a*x))/b^6]
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.30

$$\int \frac{x^3(c+dx)^2}{\sqrt{ax+bx^2}}\,dx$$

$$= \begin{cases} -\frac{5a^3\left(-\frac{7a\left(-\frac{9ad^2}{10b}+2cd\right)}{8b}+c^2\right)\left(\begin{cases}\frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0\\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{16b^3} + \sqrt{ax+bx^2}\cdot\left(\frac{5a^2\left(-\frac{7a\left(-\frac{9ad^2}{10b}+2cd\right)}{8b}+c^2\right)}{8b^3}\right. \\ \frac{2\left(\frac{c^2(ax)^{\frac{7}{2}}}{7}+\frac{2cd(ax)^{\frac{9}{2}}}{9a}+\frac{d^2(ax)^{\frac{11}{2}}}{11a^2}\right)}{a^4} \\ \tilde{\infty}\left(\frac{c^2x^4}{4}+\frac{2cdx^5}{5}+\frac{d^2x^6}{6}\right)\end{cases}$$

input
```
integrate(x**3*(d*x+c)**2/(b*x**2+a*x)**(1/2),x)
```

output
```
Piecewise((-5*a**3*(-7*a*(-9*a*d**2/(10*b) + 2*c*d)/(8*b) + c**2)*Piecewis
e((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(a**2/b, 0)),
((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2), True))/(16*b**3)
 + sqrt(a*x + b*x**2)*(5*a**2*(-7*a*(-9*a*d**2/(10*b) + 2*c*d)/(8*b) + c**
2)/(8*b**3) - 5*a*x*(-7*a*(-9*a*d**2/(10*b) + 2*c*d)/(8*b) + c**2)/(12*b**
2) + d**2*x**4/(5*b) + x**3*(-9*a*d**2/(10*b) + 2*c*d)/(4*b) + x**2*(-7*a*
(-9*a*d**2/(10*b) + 2*c*d)/(8*b) + c**2)/(3*b)), Ne(b, 0)), (2*(c**2*(a*x)
**(7/2)/7 + 2*c*d*(a*x)**(9/2)/(9*a) + d**2*(a*x)**(11/2)/(11*a**2))/a**4,
 Ne(a, 0)), (zoo*(c**2*x**4/4 + 2*c*d*x**5/5 + d**2*x**6/6), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.48

$$\int \frac{x^3(c+dx)^2}{\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{bx^2+ax}d^2x^4}{5\,b} + \frac{\sqrt{bx^2+ax}cdx^3}{2\,b} - \frac{9\,\sqrt{bx^2+ax}ad^2x^3}{40\,b^2} + \frac{\sqrt{bx^2+ax}c^2x^2}{3\,b} - \frac{7\,\sqrt{bx^2+ax}acdx^2}{12\,b^2} + \frac{21\,\sqrt{bx^2+ax}a^2d^2x^2}{80\,b^3} - \frac{5\,\sqrt{bx^2+ax}ac^2x}{12\,b^2} + \frac{35\,\sqrt{bx^2+ax}a^2cdx}{48\,b^3} - \frac{21\,\sqrt{bx^2+ax}a^3d^2x}{64\,b^4} - \frac{5\,a^3c^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{7}{2}}} + \frac{35\,a^4cd\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{64\,b^{\frac{9}{2}}} - \frac{63\,a^5d^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{256\,b^{\frac{11}{2}}} + \frac{5\,\sqrt{bx^2+ax}a^2c^2}{8\,b^3} - \frac{35\,\sqrt{bx^2+ax}a^3cd}{32\,b^4} + \frac{63\,\sqrt{bx^2+ax}a^4d^2}{128\,b^5}$$

input
```
integrate(x^3*(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
1/5*sqrt(b*x^2 + a*x)*d^2*x^4/b + 1/2*sqrt(b*x^2 + a*x)*c*d*x^3/b - 9/40*s
qrt(b*x^2 + a*x)*a*d^2*x^3/b^2 + 1/3*sqrt(b*x^2 + a*x)*c^2*x^2/b - 7/12*sq
rt(b*x^2 + a*x)*a*c*d*x^2/b^2 + 21/80*sqrt(b*x^2 + a*x)*a^2*d^2*x^2/b^3 -
5/12*sqrt(b*x^2 + a*x)*a*c^2*x/b^2 + 35/48*sqrt(b*x^2 + a*x)*a^2*c*d*x/b^3
 - 21/64*sqrt(b*x^2 + a*x)*a^3*d^2*x/b^4 - 5/16*a^3*c^2*log(2*b*x + a + 2*
sqrt(b*x^2 + a*x)*sqrt(b))/b^(7/2) + 35/64*a^4*c*d*log(2*b*x + a + 2*sqrt(
b*x^2 + a*x)*sqrt(b))/b^(9/2) - 63/256*a^5*d^2*log(2*b*x + a + 2*sqrt(b*x^
2 + a*x)*sqrt(b))/b^(11/2) + 5/8*sqrt(b*x^2 + a*x)*a^2*c^2/b^3 - 35/32*sqr
t(b*x^2 + a*x)*a^3*c*d/b^4 + 63/128*sqrt(b*x^2 + a*x)*a^4*d^2/b^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.87

$$\int \frac{x^3(c+dx)^2}{\sqrt{ax+bx^2}}\,dx$$
$$= \frac{1}{1920}\sqrt{bx^2+ax}\left(2\left(4\left(6\left(\frac{8\,d^2x}{b}+\frac{20\,b^4cd-9\,ab^3d^2}{b^5}\right)x+\frac{80\,b^4c^2-140\,ab^3cd+63\,a^2b^2d^2}{b^5}\right)x-\frac{5\,(80\,c\right.\right.$$
$$+\frac{(80\,a^3b^2c^2-140\,a^4bcd+63\,a^5d^2)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{256\,b^{\frac{11}{2}}}$$

input
```
integrate(x^3*(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/1920*sqrt(b*x^2 + a*x)*(2*(4*(6*(8*d^2*x/b + (20*b^4*c*d - 9*a*b^3*d^2)/
b^5)*x + (80*b^4*c^2 - 140*a*b^3*c*d + 63*a^2*b^2*d^2)/b^5)*x - 5*(80*a*b^
3*c^2 - 140*a^2*b^2*c*d + 63*a^3*b*d^2)/b^5)*x + 15*(80*a^2*b^2*c^2 - 140*
a^3*b*c*d + 63*a^4*d^2)/b^5) + 1/256*(80*a^3*b^2*c^2 - 140*a^4*b*c*d + 63*
a^5*d^2)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx)^2}{\sqrt{ax+bx^2}}\,dx=\int \frac{x^3\,{\left(c+d\,x\right)}^2}{\sqrt{b\,x^2+a\,x}}\,dx$$

input
```
int((x^3*(c + d*x)^2)/(a*x + b*x^2)^(1/2),x)
```

output
```
int((x^3*(c + d*x)^2)/(a*x + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.29

$$\int \frac{x^3(c+dx)^2}{\sqrt{ax+bx^2}}\,dx$$
$$= \frac{945\sqrt{x}\,\sqrt{bx+a}\,a^4b\,d^2-2100\sqrt{x}\,\sqrt{bx+a}\,a^3b^2cd-630\sqrt{x}\,\sqrt{bx+a}\,a^3b^2d^2x+1200\sqrt{x}\,\sqrt{bx+a}\,a^2b^3c^2+}{}$$

input
```
int(x^3*(d*x+c)^2/(b*x^2+a*x)^(1/2),x)
```

output
```
(945*sqrt(x)*sqrt(a + b*x)*a**4*b*d**2 - 2100*sqrt(x)*sqrt(a + b*x)*a**3*b
**2*c*d - 630*sqrt(x)*sqrt(a + b*x)*a**3*b**2*d**2*x + 1200*sqrt(x)*sqrt(a
 + b*x)*a**2*b**3*c**2 + 1400*sqrt(x)*sqrt(a + b*x)*a**2*b**3*c*d*x + 504*
sqrt(x)*sqrt(a + b*x)*a**2*b**3*d**2*x**2 - 800*sqrt(x)*sqrt(a + b*x)*a*b*
*4*c**2*x - 1120*sqrt(x)*sqrt(a + b*x)*a*b**4*c*d*x**2 - 432*sqrt(x)*sqrt(
a + b*x)*a*b**4*d**2*x**3 + 640*sqrt(x)*sqrt(a + b*x)*b**5*c**2*x**2 + 960
*sqrt(x)*sqrt(a + b*x)*b**5*c*d*x**3 + 384*sqrt(x)*sqrt(a + b*x)*b**5*d**2
*x**4 - 945*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**5*d*
*2 + 2100*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4*b*c*
d - 1200*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*b**2*
c**2)/(1920*b**6)
```

3.135 $\int \frac{x^2(c+dx)^2}{\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 202

$$\int \frac{x^2(c+dx)^2}{\sqrt{ax+bx^2}}\,dx = -\frac{a(48b^2c^2-5ad(16bc-7ad))\sqrt{ax+bx^2}}{64b^4} + \frac{(48b^2c^2-80abcd+35a^2d^2)\,x\sqrt{ax+bx^2}}{96b^3} + \frac{d(16bc-7ad)x^2\sqrt{ax+bx^2}}{24b^2} + \frac{d^2x^3\sqrt{ax+bx^2}}{4b} + \frac{a^2(48b^2c^2-5ad(16bc-7ad))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{64b^{9/2}}$$

output
```
-1/64*a*(48*b^2*c^2-5*a*d*(-7*a*d+16*b*c))*(b*x^2+a*x)^(1/2)/b^4+1/96*(35*
a^2*d^2-80*a*b*c*d+48*b^2*c^2)*x*(b*x^2+a*x)^(1/2)/b^3+1/24*d*(-7*a*d+16*b
*c)*x^2*(b*x^2+a*x)^(1/2)/b^2+1/4*d^2*x^3*(b*x^2+a*x)^(1/2)/b+1/64*a^2*(48
*b^2*c^2-5*a*d*(-7*a*d+16*b*c))*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(9/
2)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.91

$$\int \frac{x^2(c+dx)^2}{\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{x}\left(\sqrt{b}\sqrt{x}(a+bx)\left(-105a^3d^2+10a^2bd(24c+7dx)+16b^3x(6c^2+8cdx+3d^2x^2)-8ab^2(18c^2+20cdx\right.\right.}{192b^{9/2}\sqrt{x(a+bx)}}$$

input
```
Integrate[(x^2*(c + d*x)^2)/Sqrt[a*x + b*x^2],x]
```

output
```
(Sqrt[x]*(Sqrt[b]*Sqrt[x]*(a + b*x)*(-105*a^3*d^2 + 10*a^2*b*d*(24*c + 7*d
*x) + 16*b^3*x*(6*c^2 + 8*c*d*x + 3*d^2*x^2) - 8*a*b^2*(18*c^2 + 20*c*d*x
+ 7*d^2*x^2)) + 6*a^2*(48*b^2*c^2 - 80*a*b*c*d + 35*a^2*d^2)*Sqrt[a + b*x]
*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(192*b^(9/2)*Sqrt
[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1262, 27, 1221, 1134, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx)^2}{\sqrt{ax+bx^2}}\,dx$$

$$\Big\downarrow \; 1262$$

$$\frac{\int \frac{x^2(8bc^2+d(16bc-7ad)x)}{2\sqrt{bx^2+ax}}dx}{4b}+\frac{d^2x^3\sqrt{ax+bx^2}}{4b}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{x^2(8bc^2+d(16bc-7ad)x)}{\sqrt{bx^2+ax}}dx}{8b}+\frac{d^2x^3\sqrt{ax+bx^2}}{4b}$$

$$\Big\downarrow \ 1221$$

$$\frac{\frac{\left(35a^2d^2-80abcd+48b^2c^2\right)\int \frac{x^2}{\sqrt{bx^2+ax}}dx}{6b}+\frac{dx^2\sqrt{ax+bx^2}(16bc-7ad)}{3b}}{8b}+\frac{d^2x^3\sqrt{ax+bx^2}}{4b}$$

$$\Big\downarrow \ 1134$$

$$\frac{\frac{\left(35a^2d^2-80abcd+48b^2c^2\right)\left(\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\int \frac{x}{\sqrt{bx^2+ax}}dx}{4b}\right)}{6b}+\frac{dx^2\sqrt{ax+bx^2}(16bc-7ad)}{3b}}{8b}+\frac{d^2x^3\sqrt{ax+bx^2}}{4b}$$

$$\Big\downarrow \ 1160$$

$$\frac{\frac{\left(35a^2d^2-80abcd+48b^2c^2\right)\left(\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\left(\frac{\sqrt{ax+bx^2}}{b}-\frac{a\int \frac{1}{\sqrt{bx^2+ax}}dx}{2b}\right)}{4b}\right)}{6b}+\frac{dx^2\sqrt{ax+bx^2}(16bc-7ad)}{3b}}{8b}+\frac{d^2x^3\sqrt{ax+bx^2}}{4b}$$

$$\Big\downarrow \ 1091$$

$$\frac{\frac{\left(35a^2d^2-80abcd+48b^2c^2\right)\left(\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\left(\frac{\sqrt{ax+bx^2}}{b}-\frac{a\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{b}\right)}{4b}\right)}{6b}+\frac{dx^2\sqrt{ax+bx^2}(16bc-7ad)}{3b}}{8b}+\frac{d^2x^3\sqrt{ax+bx^2}}{4b}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\left(\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\left(\frac{\sqrt{ax+bx^2}}{b}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{3/2}}\right)}{4b}\right)\left(35a^2d^2-80abcd+48b^2c^2\right)}{6b}+\frac{dx^2\sqrt{ax+bx^2}(16bc-7ad)}{3b}}{8b}+\frac{d^2x^3\sqrt{ax+bx^2}}{4b}$$

input
```
Int[(x^2*(c + d*x)^2)/Sqrt[a*x + b*x^2],x]
```

output
```
(d^2*x^3*Sqrt[a*x + b*x^2])/(4*b) + ((d*(16*b*c - 7*a*d)*x^2*Sqrt[a*x + b*
x^2])/(3*b) + ((48*b^2*c^2 - 80*a*b*c*d + 35*a^2*d^2)*((x*Sqrt[a*x + b*x^2
])/(2*b) - (3*a*(Sqrt[a*x + b*x^2]/b - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b
*x^2]])/b^(3/2)))/(4*b)))/(6*b))/(8*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$\frac{\frac{35a^2\left(a^2d^2-\frac{16}{7}abcd+\frac{48}{35}b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{64}-\frac{35\left(\frac{48\left(\frac{7}{18}d^2x^2+\frac{10}{9}cdx+c^2\right)ab^{\frac{5}{2}}}{35}-\frac{32\left(\frac{1}{2}d^2x^2+\frac{4}{3}cdx+c^2\right)xb^{\frac{7}{2}}}{35}+da^2\left(\left(-\frac{2d\ldots}{3}\ldots\right.\right.\right.}{64}}{b^{\frac{9}{2}}}$
risch	$-\frac{(-48b^3d^2x^3+56ab^2d^2x^2-128b^3cdx^2-70a^2bd^2x+160ab^2cdx-96b^3c^2x+105a^3d^2-240a^2bcd+144ac^2b^2)x(bx+a)}{192b^4\sqrt{x(bx+a)}}+\ldots$
default	$c^2\left(\frac{x\sqrt{bx^2+ax}}{2b}-\frac{3a\left(\frac{\sqrt{bx^2+ax}}{b}-\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{bx^2+ax}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)+d^2\left(\frac{x^3\sqrt{bx^2+ax}}{4b}-\frac{7a\left(\frac{x^2\sqrt{bx^2+ax}}{3b}-\frac{5a\left(\frac{x\sqrt{\ldots}}{\ldots}\ldots\right.}{\ldots}\right.}{\ldots}\right.$

input
```
int(x^2*(d*x+c)^2/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
35/64*(a^2*(a^2*d^2-16/7*a*b*c*d+48/35*b^2*c^2)*arctanh((x*(b*x+a))^(1/2)/
x/b^(1/2))-(48/35*(7/18*d^2*x^2+10/9*c*d*x+c^2)*a*b^(5/2)-32/35*(1/2*d^2*x
^2+4/3*c*d*x+c^2)*x*b^(7/2)+d*a^2*((-2/3*d*x-16/7*c)*b^(3/2)+b^(1/2)*a*d))
*(x*(b*x+a))^(1/2))/b^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.69

$$\int \frac{x^2(c+dx)^2}{\sqrt{ax+bx^2}}\,dx$$
$$= \left[\frac{3\,(48\,a^2b^2c^2 - 80\,a^3bcd + 35\,a^4d^2)\sqrt{b}\log\left(2\,bx + a + 2\,\sqrt{bx^2+ax}\sqrt{b}\right) + 2\,(48\,b^4d^2x^3 - 144\,ab^3c^2 + 24\right.}{384\,b^5}$$
$$-\frac{3\,(48\,a^2b^2c^2 - 80\,a^3bcd + 35\,a^4d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right) - (48\,b^4d^2x^3 - 144\,ab^3c^2 + 240\,a^2b^2cd -}{192\,b^5}$$

input
```
integrate(x^2*(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/384*(3*(48*a^2*b^2*c^2 - 80*a^3*b*c*d + 35*a^4*d^2)*sqrt(b)*log(2*b*x +
 a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(48*b^4*d^2*x^3 - 144*a*b^3*c^2 + 24
0*a^2*b^2*c*d - 105*a^3*b*d^2 + 8*(16*b^4*c*d - 7*a*b^3*d^2)*x^2 + 2*(48*b
^4*c^2 - 80*a*b^3*c*d + 35*a^2*b^2*d^2)*x)*sqrt(b*x^2 + a*x))/b^5, -1/192*
(3*(48*a^2*b^2*c^2 - 80*a^3*b*c*d + 35*a^4*d^2)*sqrt(-b)*arctan(sqrt(b*x^2
 + a*x)*sqrt(-b)/(b*x + a)) - (48*b^4*d^2*x^3 - 144*a*b^3*c^2 + 240*a^2*b^
2*c*d - 105*a^3*b*d^2 + 8*(16*b^4*c*d - 7*a*b^3*d^2)*x^2 + 2*(48*b^4*c^2 -
 80*a*b^3*c*d + 35*a^2*b^2*d^2)*x)*sqrt(b*x^2 + a*x))/b^5]
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.40

$$\int \frac{x^2(c+dx)^2}{\sqrt{ax+bx^2}}\,dx$$

$$= \begin{cases} \frac{3a^2\left(-\frac{5a\left(-\frac{7ad^2}{8b}+2cd\right)}{6b}+c^2\right)\left(\begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{8b^2} + \sqrt{ax+bx^2}\left(-\frac{3a\left(-\frac{5a\left(-\frac{7ad^2}{8b}+2cd\right)}{6b}+c^2\right)}{4b^2} + \right. \\ \frac{2\left(\frac{c^2(ax)^{\frac{5}{2}}}{5}+\frac{2cd(ax)^{\frac{7}{2}}}{7a}+\frac{d^2(ax)^{\frac{9}{2}}}{9a^2}\right)}{a^3} \\ \tilde{\infty}\left(\frac{c^2x^3}{3}+\frac{cdx^4}{2}+\frac{d^2x^5}{5}\right) \end{cases}$$

input
```
integrate(x**2*(d*x+c)**2/(b*x**2+a*x)**(1/2),x)
```

output
```
Piecewise((3*a**2*(-5*a*(-7*a*d**2/(8*b) + 2*c*d)/(6*b) + c**2)*Piecewise(
(log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(a**2/b, 0)), ((
a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2), True))/(8*b**2) +
sqrt(a*x + b*x**2)*(-3*a*(-5*a*(-7*a*d**2/(8*b) + 2*c*d)/(6*b) + c**2)/(4*
b**2) + d**2*x**3/(4*b) + x**2*(-7*a*d**2/(8*b) + 2*c*d)/(3*b) + x*(-5*a*(
-7*a*d**2/(8*b) + 2*c*d)/(6*b) + c**2)/(2*b)), Ne(b, 0)), (2*(c**2*(a*x)**
(5/2)/5 + 2*c*d*(a*x)**(7/2)/(7*a) + d**2*(a*x)**(9/2)/(9*a**2))/a**3, Ne(
a, 0)), (zoo*(c**2*x**3/3 + c*d*x**4/2 + d**2*x**5/5), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.46

$$\int \frac{x^2(c+dx)^2}{\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{bx^2+ax}d^2x^3}{4\,b} + \frac{2\,\sqrt{bx^2+ax}cdx^2}{3\,b} - \frac{7\,\sqrt{bx^2+ax}ad^2x^2}{24\,b^2} + \frac{\sqrt{bx^2+ax}c^2x}{2\,b} - \frac{5\,\sqrt{bx^2+ax}acdx}{6\,b^2} + \frac{35\,\sqrt{bx^2+ax}a^2d^2x}{96\,b^3} + \frac{3\,a^2c^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{5}{2}}} - \frac{5\,a^3cd\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{7}{2}}} + \frac{35\,a^4d^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{9}{2}}} - \frac{3\,\sqrt{bx^2+ax}ac^2}{4\,b^2} + \frac{5\,\sqrt{bx^2+ax}a^2cd}{4\,b^3} - \frac{35\,\sqrt{bx^2+ax}a^3d^2}{64\,b^4}$$

input
```
integrate(x^2*(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(b*x^2 + a*x)*d^2*x^3/b + 2/3*sqrt(b*x^2 + a*x)*c*d*x^2/b - 7/24*s
qrt(b*x^2 + a*x)*a*d^2*x^2/b^2 + 1/2*sqrt(b*x^2 + a*x)*c^2*x/b - 5/6*sqrt(
b*x^2 + a*x)*a*c*d*x/b^2 + 35/96*sqrt(b*x^2 + a*x)*a^2*d^2*x/b^3 + 3/8*a^2
*c^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5/2) - 5/8*a^3*c*d*lo
g(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(7/2) + 35/128*a^4*d^2*log(2*
b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(9/2) - 3/4*sqrt(b*x^2 + a*x)*a*c
^2/b^2 + 5/4*sqrt(b*x^2 + a*x)*a^2*c*d/b^3 - 35/64*sqrt(b*x^2 + a*x)*a^3*d
^2/b^4
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.87

$$\int \frac{x^2(c+dx)^2}{\sqrt{ax+bx^2}}\,dx$$
$$= \frac{1}{192}\sqrt{bx^2+ax}\left(2\left(4\left(\frac{6\,d^2x}{b}+\frac{16\,b^3cd-7\,ab^2d^2}{b^4}\right)x+\frac{48\,b^3c^2-80\,ab^2cd+35\,a^2bd^2}{b^4}\right)x-\frac{3\,(48\,ab^2c^2-\right.$$
$$-\frac{(48\,a^2b^2c^2-80\,a^3bcd+35\,a^4d^2)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{128\,b^{\frac{9}{2}}}$$

input
```
integrate(x^2*(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/192*sqrt(b*x^2 + a*x)*(2*(4*(6*d^2*x/b + (16*b^3*c*d - 7*a*b^2*d^2)/b^4)
*x + (48*b^3*c^2 - 80*a*b^2*c*d + 35*a^2*b*d^2)/b^4)*x - 3*(48*a*b^2*c^2 -
 80*a^2*b*c*d + 35*a^3*d^2)/b^4) - 1/128*(48*a^2*b^2*c^2 - 80*a^3*b*c*d +
35*a^4*d^2)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(9/2
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx)^2}{\sqrt{ax+bx^2}}\,dx = \int \frac{x^2\,{\left(c+d\,x\right)}^2}{\sqrt{b\,x^2+a\,x}}\,dx$$

input
```
int((x^2*(c + d*x)^2)/(a*x + b*x^2)^(1/2),x)
```

output
```
int((x^2*(c + d*x)^2)/(a*x + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.27

$$\int \frac{x^2(c+dx)^2}{\sqrt{ax+bx^2}}\,dx$$
$$= \frac{-105\sqrt{x}\,\sqrt{bx+a}\,a^3b\,d^2+240\sqrt{x}\,\sqrt{bx+a}\,a^2b^2cd+70\sqrt{x}\,\sqrt{bx+a}\,a^2b^2d^2x-144\sqrt{x}\,\sqrt{bx+a}\,a\,b^3c^2-1}{}$$

input
```
int(x^2*(d*x+c)^2/(b*x^2+a*x)^(1/2),x)
```

output
```
( - 105*sqrt(x)*sqrt(a + b*x)*a**3*b*d**2 + 240*sqrt(x)*sqrt(a + b*x)*a**2
*b**2*c*d + 70*sqrt(x)*sqrt(a + b*x)*a**2*b**2*d**2*x - 144*sqrt(x)*sqrt(a
 + b*x)*a*b**3*c**2 - 160*sqrt(x)*sqrt(a + b*x)*a*b**3*c*d*x - 56*sqrt(x)*
sqrt(a + b*x)*a*b**3*d**2*x**2 + 96*sqrt(x)*sqrt(a + b*x)*b**4*c**2*x + 12
8*sqrt(x)*sqrt(a + b*x)*b**4*c*d*x**2 + 48*sqrt(x)*sqrt(a + b*x)*b**4*d**2
*x**3 + 105*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4*d*
*2 - 240*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*b*c*d
 + 144*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*b**2*c*
*2)/(192*b**5)
```

3.136 $\int \frac{x(c+dx)^2}{\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 155

$$\int \frac{x(c+dx)^2}{\sqrt{ax+bx^2}}\,dx = \frac{(8b^2c^2-12abcd+5a^2d^2)\sqrt{ax+bx^2}}{8b^3}+\frac{d(12bc-5ad)x\sqrt{ax+bx^2}}{12b^2}+\frac{d^2x^2\sqrt{ax+bx^2}}{3b}-\frac{a(8b^2c^2-12abcd+5a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{8b^{7/2}}$$

output
```
1/8*(5*a^2*d^2-12*a*b*c*d+8*b^2*c^2)*(b*x^2+a*x)^(1/2)/b^3+1/12*d*(-5*a*d+
12*b*c)*x*(b*x^2+a*x)^(1/2)/b^2+1/3*d^2*x^2*(b*x^2+a*x)^(1/2)/b-1/8*a*(5*a
^2*d^2-12*a*b*c*d+8*b^2*c^2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.99

$$\int \frac{x(c+dx)^2}{\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{x}\left(\sqrt{b}\sqrt{x}(a+bx)\left(15a^2d^2-2abd(18c+5dx)+8b^2(3c^2+3cdx+d^2x^2)\right)+6a(8b^2c^2-12abcd+5a^2d^2\right.}{24b^{7/2}\sqrt{x(a+bx)}}$$

input
```
Integrate[(x*(c + d*x)^2)/Sqrt[a*x + b*x^2],x]
```

output
```
(Sqrt[x]*(Sqrt[b]*Sqrt[x]*(a + b*x)*(15*a^2*d^2 - 2*a*b*d*(18*c + 5*d*x) +
 8*b^2*(3*c^2 + 3*c*d*x + d^2*x^2)) + 6*a*(8*b^2*c^2 - 12*a*b*c*d + 5*a^2*
d^2)*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])]))/
(24*b^(7/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1262, 27, 1225, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx)^2}{\sqrt{ax+bx^2}}dx$$

$$\Big\downarrow \text{1262}$$

$$\frac{\int \frac{x(6bc^2+d(12bc-5ad)x)}{2\sqrt{bx^2+ax}}dx}{3b}+\frac{d^2x^2\sqrt{ax+bx^2}}{3b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x(6bc^2+d(12bc-5ad)x)}{\sqrt{bx^2+ax}}dx}{6b}+\frac{d^2x^2\sqrt{ax+bx^2}}{3b}$$

$$\Big\downarrow \text{1225}$$

$$\frac{\frac{\sqrt{ax+bx^2}(3(8b^2c^2-ad(12bc-5ad))+2bdx(12bc-5ad))}{4b^2}-\frac{3a(8b^2c^2-ad(12bc-5ad))\int \frac{1}{\sqrt{bx^2+ax}}dx}{8b^2}}{6b}+\frac{d^2x^2\sqrt{ax+bx^2}}{3b}$$

$$\Big\downarrow \text{1091}$$

$$\frac{\frac{\sqrt{ax+bx^2}(3(8b^2c^2-ad(12bc-5ad))+2bdx(12bc-5ad))}{4b^2}-\frac{3a(8b^2c^2-ad(12bc-5ad))\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b^2}}{6b}+\frac{d^2x^2\sqrt{ax+bx^2}}{3b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\sqrt{ax+bx^2}(3(8b^2c^2-ad(12bc-5ad))+2bdx(12bc-5ad))}{4b^2}-\frac{3a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(8b^2c^2-ad(12bc-5ad))}{4b^{5/2}}}{6b}+\frac{d^2x^2\sqrt{ax+bx^2}}{3b}$$

input
```
Int[(x*(c + d*x)^2)/Sqrt[a*x + b*x^2],x]
```

output
```
(d^2*x^2*Sqrt[a*x + b*x^2])/(3*b) + (((3*(8*b^2*c^2 - a*d*(12*b*c - 5*a*d)
) + 2*b*d*(12*b*c - 5*a*d)*x)*Sqrt[a*x + b*x^2])/(4*b^2) - (3*a*(8*b^2*c^2
 - a*d*(12*b*c - 5*a*d))*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*b^(5/2
)))/(6*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$-\frac{5\left(a\left(a^2d^2-\frac{12}{5}abcd+\frac{8}{5}b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)-\left(\frac{8\left(\frac{1}{3}d^2x^2+cdx+c^2\right)b^{\frac{5}{2}}}{5}+d\left(2\left(-\frac{dx}{3}-\frac{6c}{5}\right)b^{\frac{3}{2}}+\sqrt{b}\,ad\right)a\right)\sqrt{x(bx+a)}}{8b^{\frac{7}{2}}}$
risch	$\frac{\left(8b^2d^2x^2-10ab\,d^2x+24b^2cxd+15a^2d^2-36abcd+24b^2c^2\right)x(bx+a)}{24b^3\sqrt{x(bx+a)}}-\frac{a\left(5a^2d^2-12abcd+8b^2c^2\right)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{16b^{\frac{7}{2}}}$
default	$c^2\left(\frac{\sqrt{b\,x^2+ax}}{b}-\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2b^{\frac{3}{2}}}\right)+d^2\left(\frac{x^2\sqrt{b\,x^2+ax}}{3b}-\frac{5a\left(\frac{x\sqrt{b\,x^2+ax}}{2b}-\frac{3a\left(\frac{\sqrt{b\,x^2+ax}}{b}-\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}\right.}{2b}\right.}{4b}\right.}{6b}\right.$

input
```
int(x*(d*x+c)^2/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-5/8/b^(7/2)*(a*(a^2*d^2-12/5*a*b*c*d+8/5*b^2*c^2)*arctanh((x*(b*x+a))^(1/
2)/x/b^(1/2))-(8/5*(1/3*d^2*x^2+c*d*x+c^2)*b^(5/2)+d*(2*(-1/3*d*x-6/5*c)*b
^(3/2)+b^(1/2)*a*d)*a)*(x*(b*x+a))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.70

$$\int \frac{x(c+dx)^2}{\sqrt{ax+bx^2}}\,dx = \left[\frac{3\left(8\,ab^2c^2-12\,a^2bcd+5\,a^3d^2\right)\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\left(8\,b^3d^2x^2+24\,b^3c^2-36\,ab^2cd+\right.}{48\,b^4}\right.$$

input
```
integrate(x*(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(8*a*b^2*c^2 - 12*a^2*b*c*d + 5*a^3*d^2)*sqrt(b)*log(2*b*x + a -
2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(8*b^3*d^2*x^2 + 24*b^3*c^2 - 36*a*b^2*c*
d + 15*a^2*b*d^2 + 2*(12*b^3*c*d - 5*a*b^2*d^2)*x)*sqrt(b*x^2 + a*x))/b^4,
 1/24*(3*(8*a*b^2*c^2 - 12*a^2*b*c*d + 5*a^3*d^2)*sqrt(-b)*arctan(sqrt(b*x
^2 + a*x)*sqrt(-b)/(b*x + a)) + (8*b^3*d^2*x^2 + 24*b^3*c^2 - 36*a*b^2*c*d
 + 15*a^2*b*d^2 + 2*(12*b^3*c*d - 5*a*b^2*d^2)*x)*sqrt(b*x^2 + a*x))/b^4]
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.55

$$\int \frac{x(c+dx)^2}{\sqrt{ax+bx^2}}\,dx$$

$$= \begin{cases} -\frac{a\left(-\frac{3a\left(-\frac{5ad^2}{6b}+2cd\right)}{4b}+c^2\right)\left(\begin{cases}\frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0\\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{2b}+\sqrt{ax+bx^2}\left(\frac{d^2x^2}{3b}+\frac{x\left(-\frac{5ad^2}{6b}+2cd\right)}{2b}+\frac{-\frac{3a}{}}{}\right. \\ \frac{2\left(\frac{c^2(ax)^{\frac{3}{2}}}{3}+\frac{2cd(ax)^{\frac{5}{2}}}{5a}+\frac{d^2(ax)^{\frac{7}{2}}}{7a^2}\right)}{a^2} \\ \tilde{\infty}\left(\frac{c^2x^2}{2}+\frac{2cdx^3}{3}+\frac{d^2x^4}{4}\right) \end{cases}$$

input
```
integrate(x*(d*x+c)**2/(b*x**2+a*x)**(1/2),x)
```

output
```
Piecewise((-a*(-3*a*(-5*a*d**2/(6*b) + 2*c*d)/(4*b) + c**2)*Piecewise((log
(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2
*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2), True))/(2*b) + sqrt(a*
x + b*x**2)*(d**2*x**2/(3*b) + x*(-5*a*d**2/(6*b) + 2*c*d)/(2*b) + (-3*a*(
-5*a*d**2/(6*b) + 2*c*d)/(4*b) + c**2)/b), Ne(b, 0)), (2*(c**2*(a*x)**(3/2
)/3 + 2*c*d*(a*x)**(5/2)/(5*a) + d**2*(a*x)**(7/2)/(7*a**2))/a**2, Ne(a, 0
)), (zoo*(c**2*x**2/2 + 2*c*d*x**3/3 + d**2*x**4/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.42

$$\int \frac{x(c+dx)^2}{\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{bx^2+ax}d^2x^2}{3\,b} + \frac{\sqrt{bx^2+ax}cdx}{b} - \frac{5\,\sqrt{bx^2+ax}ad^2x}{12\,b^2} - \frac{ac^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{2\,b^{\frac{3}{2}}} + \frac{3\,a^2cd\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{4\,b^{\frac{5}{2}}} - \frac{5\,a^3d^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{7}{2}}} + \frac{\sqrt{bx^2+ax}c^2}{b} - \frac{3\,\sqrt{bx^2+ax}acd}{2\,b^2} + \frac{5\,\sqrt{bx^2+ax}a^2d^2}{8\,b^3}$$

input
```
integrate(x*(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(b*x^2 + a*x)*d^2*x^2/b + sqrt(b*x^2 + a*x)*c*d*x/b - 5/12*sqrt(b*
x^2 + a*x)*a*d^2*x/b^2 - 1/2*a*c^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqr
t(b))/b^(3/2) + 3/4*a^2*c*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b
^(5/2) - 5/16*a^3*d^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(7/2)
 + sqrt(b*x^2 + a*x)*c^2/b - 3/2*sqrt(b*x^2 + a*x)*a*c*d/b^2 + 5/8*sqrt(b*
x^2 + a*x)*a^2*d^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.86

$$\int \frac{x(c+dx)^2}{\sqrt{ax+bx^2}}\,dx = \frac{1}{24}\,\sqrt{bx^2+ax}\left(2\left(\frac{4\,d^2x}{b} + \frac{12\,b^2cd-5\,abd^2}{b^3}\right)x + \frac{3\,(8\,b^2c^2-12\,abcd+5\,a^2d^2)}{b^3}\right) + \frac{(8\,ab^2c^2-12\,a^2bcd+5\,a^3d^2)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{16\,b^{\frac{7}{2}}}$$

input
```
integrate(x*(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(b*x^2 + a*x)*(2*(4*d^2*x/b + (12*b^2*c*d - 5*a*b*d^2)/b^3)*x + 3
*(8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)/b^3) + 1/16*(8*a*b^2*c^2 - 12*a^2*b*
c*d + 5*a^3*d^2)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b
^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx)^2}{\sqrt{ax+bx^2}}\,dx=\int \frac{x\,{\left(c+d\,x\right)}^2}{\sqrt{b\,x^2+a\,x}}\,dx$$

input
```
int((x*(c + d*x)^2)/(a*x + b*x^2)^(1/2),x)
```

output
```
int((x*(c + d*x)^2)/(a*x + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.25

$$\int \frac{x(c+dx)^2}{\sqrt{ax+bx^2}}\,dx$$
$$=\frac{15\sqrt{x}\,\sqrt{bx+a}\,a^2b\,d^2-36\sqrt{x}\,\sqrt{bx+a}\,a\,b^2cd-10\sqrt{x}\,\sqrt{bx+a}\,a\,b^2d^2x+24\sqrt{x}\,\sqrt{bx+a}\,b^3c^2+24\sqrt{x}\,\sqrt{b}}{}$$

input
```
int(x*(d*x+c)^2/(b*x^2+a*x)^(1/2),x)
```

output
```
(15*sqrt(x)*sqrt(a + b*x)*a**2*b*d**2 - 36*sqrt(x)*sqrt(a + b*x)*a*b**2*c*
d - 10*sqrt(x)*sqrt(a + b*x)*a*b**2*d**2*x + 24*sqrt(x)*sqrt(a + b*x)*b**3
*c**2 + 24*sqrt(x)*sqrt(a + b*x)*b**3*c*d*x + 8*sqrt(x)*sqrt(a + b*x)*b**3
*d**2*x**2 - 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**
3*d**2 + 36*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*b*
c*d - 24*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b**2*c**
2)/(24*b**4)
```

3.137 $\int \frac{(c+dx)^2}{\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{(c+dx)^2}{\sqrt{ax+bx^2}}\,dx = \frac{d(8bc-3ad)\sqrt{ax+bx^2}}{4b^2} + \frac{d^2x\sqrt{ax+bx^2}}{2b} + \frac{(8b^2c^2-8abcd+3a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{5/2}}$$

output
```
1/4*d*(-3*a*d+8*b*c)*(b*x^2+a*x)^(1/2)/b^2+1/2*d^2*x*(b*x^2+a*x)^(1/2)/b+1
/4*(3*a^2*d^2-8*a*b*c*d+8*b^2*c^2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^
(5/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx)^2}{\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{b}dx(a+bx)(8bc-3ad+2bdx)+(-8b^2c^2+8abcd-3a^2d^2)\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{4b^{5/2}\sqrt{x(a+bx)}}$$

input
```
Integrate[(c + d*x)^2/Sqrt[a*x + b*x^2],x]
```

output
```
(Sqrt[b]*d*x*(a + b*x)*(8*b*c - 3*a*d + 2*b*d*x) + (-8*b^2*c^2 + 8*a*b*c*d
 - 3*a^2*d^2)*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]
])/(4*b^(5/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1166, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{\sqrt{ax+bx^2}}dx$$

$$\Big\downarrow \text{ 1166}$$

$$\frac{\int \frac{c(4bc-ad)+3d(2bc-ad)x}{2\sqrt{bx^2+ax}}dx}{2b}+\frac{d\sqrt{ax+bx^2}(c+dx)}{2b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{c(4bc-ad)+3d(2bc-ad)x}{\sqrt{bx^2+ax}}dx}{4b}+\frac{d\sqrt{ax+bx^2}(c+dx)}{2b}$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{\frac{(3a^2d^2-8abcd+8b^2c^2)\int \frac{1}{\sqrt{bx^2+ax}}dx}{2b}+\frac{3d\sqrt{ax+bx^2}(2bc-ad)}{b}}{4b}+\frac{d\sqrt{ax+bx^2}(c+dx)}{2b}$$

$$\Big\downarrow \text{ 1091}$$

$$\frac{\frac{(3a^2d^2-8abcd+8b^2c^2)\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{b}+\frac{3d\sqrt{ax+bx^2}(2bc-ad)}{b}}{4b}+\frac{d\sqrt{ax+bx^2}(c+dx)}{2b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(3a^2d^2-8abcd+8b^2c^2)}{b^{3/2}}+\frac{3d\sqrt{ax+bx^2}(2bc-ad)}{b}}{4b}+\frac{d\sqrt{ax+bx^2}(c+dx)}{2b}$$

input
```
Int[(c + d*x)^2/Sqrt[a*x + b*x^2],x]
```

output
```
(d*(c + d*x)*Sqrt[a*x + b*x^2])/(2*b) + ((3*d*(2*b*c - a*d)*Sqrt[a*x + b*x
^2])/b + ((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x
 + b*x^2]])/b^(3/2))/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{\frac{3\left(a^{2} d^{2}-\frac{8}{3} a b c d+\frac{8}{3} b^{2} c^{2}\right) \operatorname{arctanh}\left(\frac{\sqrt{x(b x+a)}}{x \sqrt{b}}\right)}{4}-\frac{3 d\left(\frac{2(-d x-4 c) b^{\frac{3}{2}}}{3}+\sqrt{b}\, a d\right) \sqrt{x(b x+a)}}{4}}{b^{\frac{5}{2}}}$
risch	$-\frac{(-2 b d x+3 a d-8 b c) d x(b x+a)}{4 b^{2} \sqrt{x(b x+a)}}+\frac{\left(3 a^{2} d^{2}-8 a b c d+8 b^{2} c^{2}\right) \ln\left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\,x^{2}+a x}\right)}{8 b^{\frac{5}{2}}}$
default	$\frac{c^{2} \ln\left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\,x^{2}+a x}\right)}{\sqrt{b}}+d^{2}\left(\frac{x\sqrt{b\,x^{2}+a x}}{2 b}-\frac{3 a\left(\frac{\sqrt{b\,x^{2}+a x}}{b}-\frac{a \ln\left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\,x^{2}+a x}\right)}{2 b^{\frac{3}{2}}}\right)}{4 b}\right)+2 c d\left(\frac{\sqrt{b\,x^{2}+a x}}{b}\right.$

input
```
int((d*x+c)^2/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/4/b^(5/2)*((a^2*d^2-8/3*a*b*c*d+8/3*b^2*c^2)*arctanh((x*(b*x+a))^(1/2)/x
/b^(1/2))-d*(2/3*(-d*x-4*c)*b^(3/2)+b^(1/2)*a*d)*(x*(b*x+a))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.77

$$\int \frac{(c+dx)^2}{\sqrt{ax+bx^2}}\,dx = \left[\frac{(8\,b^2c^2-8\,abcd+3\,a^2d^2)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(2\,b^2d^2x+8\,b^2cd-3\,abd^2)\sqrt{bx^2+a}\right.}{8\,b^3}$$
$$\left. -\frac{(8\,b^2c^2-8\,abcd+3\,a^2d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(2\,b^2d^2x+8\,b^2cd-3\,abd^2)\sqrt{bx^2+ax}}{4\,b^3}\right]$$

input
```
integrate((d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*sqrt(b)*log(2*b*x + a + 2*sqrt(b
*x^2 + a*x)*sqrt(b)) + 2*(2*b^2*d^2*x + 8*b^2*c*d - 3*a*b*d^2)*sqrt(b*x^2
+ a*x))/b^3, -1/4*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*sqrt(-b)*arctan(sqr
t(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (2*b^2*d^2*x + 8*b^2*c*d - 3*a*b*d^2)
*sqrt(b*x^2 + a*x))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.71

$$\int \frac{(c+dx)^2}{\sqrt{ax+bx^2}}\,dx = \begin{cases} \sqrt{ax+bx^2}\left(\frac{d^2x}{2b}+\frac{-\frac{3ad^2}{4b}+2cd}{b}\right)+\left(-\frac{a\left(-\frac{3ad^2}{4b}+2cd\right)}{2b}+c^2\right)\left(\begin{cases}\frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0\\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise}\end{cases}\right) & \text{for } \\ \frac{2\left(c^2\sqrt{ax}+\frac{2cd(ax)^{\frac{3}{2}}}{3a}+\frac{d^2(ax)^{\frac{5}{2}}}{5a^2}\right)}{a} & \text{for } \\ \tilde{\infty}\left(\begin{cases}c^2x & \text{for } d=0\\ \frac{(c+dx)^3}{3d} & \text{otherwise}\end{cases}\right) & \text{othe} \end{cases}$$

input
```
integrate((d*x+c)**2/(b*x**2+a*x)**(1/2),x)
```

output
```
Piecewise((sqrt(a*x + b*x**2)*(d**2*x/(2*b) + (-3*a*d**2/(4*b) + 2*c*d)/b)
 + (-a*(-3*a*d**2/(4*b) + 2*c*d)/(2*b) + c**2)*Piecewise((log(a + 2*sqrt(b
)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(
a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2), True)), Ne(b, 0)), (2*(c**2*sqrt(a*
x) + 2*c*d*(a*x)**(3/2)/(3*a) + d**2*(a*x)**(5/2)/(5*a**2))/a, Ne(a, 0)),
(zoo*Piecewise((c**2*x, Eq(d, 0)), ((c + d*x)**3/(3*d), True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.43

$$\int \frac{(c+dx)^2}{\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{bx^2+ax}d^2x}{2\,b} + \frac{c^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}} - \frac{acd\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{b^{\frac{3}{2}}} + \frac{3\,a^2d^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{5}{2}}} + \frac{2\,\sqrt{bx^2+ax}cd}{b} - \frac{3\,\sqrt{bx^2+ax}ad^2}{4\,b^2}$$

input
```
integrate((d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a*x)*d^2*x/b + c^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sq
rt(b))/sqrt(b) - a*c*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(3/2
) + 3/8*a^2*d^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5/2) + 2*s
qrt(b*x^2 + a*x)*c*d/b - 3/4*sqrt(b*x^2 + a*x)*a*d^2/b^2
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx)^2}{\sqrt{ax+bx^2}}\,dx = \frac{1}{4}\sqrt{bx^2+ax}\left(\frac{2\,d^2x}{b}+\frac{8\,bcd-3\,ad^2}{b^2}\right) - \frac{(8\,b^2c^2-8\,abcd+3\,a^2d^2)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,b^{\frac{5}{2}}}$$

input
```
integrate((d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(b*x^2 + a*x)*(2*d^2*x/b + (8*b*c*d - 3*a*d^2)/b^2) - 1/8*(8*b^2*c
^2 - 8*a*b*c*d + 3*a^2*d^2)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt
(b) + a))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{\sqrt{ax+bx^2}}\,dx = \int \frac{(c+d\,x)^2}{\sqrt{b\,x^2+a\,x}}\,dx$$

input
```
int((c + d*x)^2/(a*x + b*x^2)^(1/2),x)
```

output
```
int((c + d*x)^2/(a*x + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.25

$$\int \frac{(c+dx)^2}{\sqrt{ax+bx^2}}\,dx = \frac{-3\sqrt{x}\sqrt{bx+a}\,ab\,d^2+8\sqrt{x}\sqrt{bx+a}\,b^2cd+2\sqrt{x}\sqrt{bx+a}\,b^2d^2x+3\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\sqrt{b}}{\sqrt{a}}\right)a^2d^2-8\sqrt{b}\log}{4b^3}$$

input
```
int((d*x+c)^2/(b*x^2+a*x)^(1/2),x)
```

output
```
( - 3*sqrt(x)*sqrt(a + b*x)*a*b*d**2 + 8*sqrt(x)*sqrt(a + b*x)*b**2*c*d +
2*sqrt(x)*sqrt(a + b*x)*b**2*d**2*x + 3*sqrt(b)*log((sqrt(a + b*x) + sqrt(
x)*sqrt(b))/sqrt(a))*a**2*d**2 - 8*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sq
rt(b))/sqrt(a))*a*b*c*d + 8*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/
sqrt(a))*b**2*c**2)/(4*b**3)
```

3.138 $\int \frac{(c+dx)^2}{x\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{(c+dx)^2}{x\sqrt{ax+bx^2}}\,dx = \frac{d^2\sqrt{ax+bx^2}}{b} - \frac{2c^2\sqrt{ax+bx^2}}{ax} + \frac{d(4bc-ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{3/2}}$$

output
```
d^2*(b*x^2+a*x)^(1/2)/b-2*c^2*(b*x^2+a*x)^(1/2)/a/x+d*(-a*d+4*b*c)*arctanh
(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.24

$$\int \frac{(c+dx)^2}{x\sqrt{ax+bx^2}}\,dx = \frac{\frac{\sqrt{b}(a+bx)(-2bc^2+ad^2x)}{a} + 2d(4bc-ad)\sqrt{x}\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{b^{3/2}\sqrt{x(a+bx)}}$$

input
```
Integrate[(c + d*x)^2/(x*Sqrt[a*x + b*x^2]),x]
```

output
```
((Sqrt[b]*(a + b*x)*(-2*b*c^2 + a*d^2*x))/a + 2*d*(4*b*c - a*d)*Sqrt[x]*Sq
rt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(b^(3/2
)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1262, 27, 1220, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{x\sqrt{ax+bx^2}}dx$$

$$\Big\downarrow \text{1262}$$

$$\frac{\int \frac{2bc^2+d(4bc-ad)x}{2x\sqrt{bx^2+ax}}dx}{b}+\frac{d^2\sqrt{ax+bx^2}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2bc^2+d(4bc-ad)x}{x\sqrt{bx^2+ax}}dx}{2b}+\frac{d^2\sqrt{ax+bx^2}}{b}$$

$$\Big\downarrow \text{1220}$$

$$\frac{d(4bc-ad)\int \frac{1}{\sqrt{bx^2+ax}}dx-\frac{4bc^2\sqrt{ax+bx^2}}{ax}}{2b}+\frac{d^2\sqrt{ax+bx^2}}{b}$$

$$\Big\downarrow \text{1091}$$

$$\frac{2d(4bc-ad)\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}-\frac{4bc^2\sqrt{ax+bx^2}}{ax}}{2b}+\frac{d^2\sqrt{ax+bx^2}}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{2d\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(4bc-ad)}{\sqrt{b}}-\frac{4bc^2\sqrt{ax+bx^2}}{ax}}{2b}+\frac{d^2\sqrt{ax+bx^2}}{b}$$

input
```
Int[(c + d*x)^2/(x*Sqrt[a*x + b*x^2]),x]
```

output
```
(d^2*Sqrt[a*x + b*x^2])/b + ((-4*b*c^2*Sqrt[a*x + b*x^2])/(a*x) + (2*d*(4*
b*c - a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b])/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.90

method	result	size
risch	$\frac{(bx+a)(a\,d^2x-2b\,c^2)}{a\sqrt{x(bx+a)}\,b}-\frac{(ad-4bc)d\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2b^{\frac{3}{2}}}$	74
pseudoelliptic	$\frac{-\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)a^2d^2x+4\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)abcdx+a\,d^2x\sqrt{x(bx+a)}\,\sqrt{b}-2c^2\sqrt{x(bx+a)}\,b^{\frac{3}{2}}}{b^{\frac{3}{2}}xa}$	97
default	$-\frac{2c^2\sqrt{b\,x^2+ax}}{ax}+d^2\left(\frac{\sqrt{b\,x^2+ax}}{b}-\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2b^{\frac{3}{2}}}\right)+\frac{2cd\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$	105

input
```
int((d*x+c)^2/x/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b*x+a)*(a*d^2*x-2*b*c^2)/a/(x*(b*x+a))^(1/2)/b-1/2*(a*d-4*b*c)*d/b^(3/2)*
ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.11

$$\int\frac{(c+dx)^2}{x\sqrt{ax+bx^2}}\,dx$$
$$=\left[-\frac{(4\,abcd-a^2d^2)\sqrt{b}x\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,(abd^2x-2\,b^2c^2)\sqrt{bx^2+ax}}{2\,ab^2x},\right.$$
$$\left.-\frac{(4\,abcd-a^2d^2)\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(abd^2x-2\,b^2c^2)\sqrt{bx^2+ax}}{ab^2x}\right]$$

input
```
integrate((d*x+c)^2/x/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*((4*a*b*c*d - a^2*d^2)*sqrt(b)*x*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)
*sqrt(b)) - 2*(a*b*d^2*x - 2*b^2*c^2)*sqrt(b*x^2 + a*x))/(a*b^2*x), -((4*a
*b*c*d - a^2*d^2)*sqrt(-b)*x*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a))
- (a*b*d^2*x - 2*b^2*c^2)*sqrt(b*x^2 + a*x))/(a*b^2*x)]
```

Sympy [F]

$$\int \frac{(c+dx)^2}{x\sqrt{ax+bx^2}}\,dx = \int \frac{(c+dx)^2}{x\sqrt{x\,(a+bx)}}\,dx$$

input
```
integrate((d*x+c)**2/x/(b*x**2+a*x)**(1/2),x)
```

output
```
Integral((c + d*x)**2/(x*sqrt(x*(a + b*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.26

$$\int \frac{(c+dx)^2}{x\sqrt{ax+bx^2}}\,dx = \frac{2\,cd\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}} - \frac{ad^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{2\,b^{\frac{3}{2}}} + \frac{\sqrt{bx^2+ax}d^2}{b} - \frac{2\,\sqrt{bx^2+ax}c^2}{ax}$$

input
```
integrate((d*x+c)^2/x/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
2*c*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b) - 1/2*a*d^2*log
(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(3/2) + sqrt(b*x^2 + a*x)*d^2/
b - 2*sqrt(b*x^2 + a*x)*c^2/(a*x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^2}{x\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{bx^2+ax}d^2}{b} + \frac{2\,c^2}{\sqrt{b}x-\sqrt{bx^2+ax}} - \frac{(4\,bcd-ad^2)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)^2/x/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
sqrt(b*x^2 + a*x)*d^2/b + 2*c^2/(sqrt(b)*x - sqrt(b*x^2 + a*x)) - 1/2*(4*b
*c*d - a*d^2)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(3
/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{x\sqrt{ax+bx^2}}\,dx = \int \frac{\left(c+d\,x\right)^2}{x\,\sqrt{b\,x^2+a\,x}}\,dx$$

input
```
int((c + d*x)^2/(x*(a*x + b*x^2)^(1/2)),x)
```

output
```
int((c + d*x)^2/(x*(a*x + b*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.49

$$\int \frac{(c+dx)^2}{x\sqrt{ax+bx^2}}\,dx = \frac{4\sqrt{x}\,\sqrt{bx+a}\,ab\,d^2x-8\sqrt{x}\,\sqrt{bx+a}\,b^2c^2-4\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2d^2x+16\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)abcd}{4a\,b^2x}$$

input
```
int((d*x+c)^2/x/(b*x^2+a*x)^(1/2),x)
```

output
```
(4*sqrt(x)*sqrt(a + b*x)*a*b*d**2*x - 8*sqrt(x)*sqrt(a + b*x)*b**2*c**2 -
4*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*d**2*x + 16*
sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b*c*d*x - sqrt(b)
*a**2*d**2*x - 8*sqrt(b)*b**2*c**2*x)/(4*a*b**2*x)
```

3.139 $\int \frac{(c+dx)^2}{x^2\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 90

$$\int \frac{(c+dx)^2}{x^2\sqrt{ax+bx^2}}\,dx=-\frac{2c^2\sqrt{ax+bx^2}}{3ax^2}+\frac{4c(bc-3ad)\sqrt{ax+bx^2}}{3a^2x}+\frac{2d^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

output
```
-2/3*c^2*(b*x^2+a*x)^(1/2)/a/x^2+4/3*c*(-3*a*d+b*c)*(b*x^2+a*x)^(1/2)/a^2/
x+2*d^2*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int \frac{(c+dx)^2}{x^2\sqrt{ax+bx^2}}\,dx=-\frac{2c(a+bx)(ac-2bcx+6adx)}{3a^2x\sqrt{x(a+bx)}}-\frac{2d^2\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

input
```
Integrate[(c + d*x)^2/(x^2*Sqrt[a*x + b*x^2]),x]
```

output
```
(-2*c*(a + b*x)*(a*c - 2*b*c*x + 6*a*d*x))/(3*a^2*x*Sqrt[x*(a + b*x)]) - (
2*d^2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt
[b]*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1290}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{x^2\sqrt{ax+bx^2}}\,dx$$

$$\Big\downarrow \text{ 1290}$$

Indeterminate

input
```
Int[(c + d*x)^2/(x^2*Sqrt[a*x + b*x^2]),x]
```

output
```
Indeterminate
```

Defintions of rubi rules used

rule 1290
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(f + g*x)^
n, d + e*x, x], R = PolynomialRemainder[(f + g*x)^n, d + e*x, x]}, Simp[(e*
R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*
e^2)), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1
)*(a + b*x + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R
*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 1] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.78

method	result	size
risch	$-\frac{2c(bx+a)(6adx-2cbx+ac)}{3a^2x\sqrt{x(bx+a)}}+\frac{d^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$	70
pseudoelliptic	$\frac{2a^2d^2\,\mathrm{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)x^2-\frac{2\left(-2b^{\frac{3}{2}}cx+a\sqrt{b}\,(6dx+c)\right)c\sqrt{x(bx+a)}}{3}}{a^2x^2\sqrt{b}}$	71
default	$\frac{d^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}+c^2\left(-\frac{2\sqrt{b\,x^2+ax}}{3a\,x^2}+\frac{4b\sqrt{b\,x^2+ax}}{3a^2x}\right)-\frac{4cd\sqrt{b\,x^2+ax}}{ax}$	98

input
```
int((d*x+c)^2/x^2/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*c*(b*x+a)*(6*a*d*x-2*b*c*x+a*c)/a^2/x/(x*(b*x+a))^(1/2)+d^2*ln((1/2*a
+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.96

$$\int \frac{(c+dx)^2}{x^2\sqrt{ax+bx^2}}\,dx$$
$$=\left[\frac{3\,a^2\sqrt{b}d^2x^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,(abc^2-2\,(b^2c^2-3\,abcd)x)\sqrt{bx^2+ax}}{3\,a^2bx^2},\right.$$
$$\left.-\frac{2\left(3\,a^2\sqrt{-b}d^2x^2\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+(abc^2-2\,(b^2c^2-3\,abcd)x)\sqrt{bx^2+ax}\right)}{3\,a^2bx^2}\right]$$

input
```
integrate((d*x+c)^2/x^2/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/3*(3*a^2*sqrt(b)*d^2*x^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) -
 2*(a*b*c^2 - 2*(b^2*c^2 - 3*a*b*c*d)*x)*sqrt(b*x^2 + a*x))/(a^2*b*x^2), -
2/3*(3*a^2*sqrt(-b)*d^2*x^2*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) +
 (a*b*c^2 - 2*(b^2*c^2 - 3*a*b*c*d)*x)*sqrt(b*x^2 + a*x))/(a^2*b*x^2)]
```

Sympy [F]

$$\int \frac{(c+dx)^2}{x^2\sqrt{ax+bx^2}}\,dx = \int \frac{(c+dx)^2}{x^2\sqrt{x\,(a+bx)}}\,dx$$

input
```
integrate((d*x+c)**2/x**2/(b*x**2+a*x)**(1/2),x)
```

output
```
Integral((c + d*x)**2/(x**2*sqrt(x*(a + b*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)^2}{x^2\sqrt{ax+bx^2}}\,dx = \frac{d^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}} + \frac{4\,\sqrt{bx^2+ax}bc^2}{3\,a^2x} - \frac{4\,\sqrt{bx^2+ax}cd}{ax} - \frac{2\,\sqrt{bx^2+ax}c^2}{3\,ax^2}$$

input
```
integrate((d*x+c)^2/x^2/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
d^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b) + 4/3*sqrt(b*x^2
+ a*x)*b*c^2/(a^2*x) - 4*sqrt(b*x^2 + a*x)*c*d/(a*x) - 2/3*sqrt(b*x^2 + a*
x)*c^2/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.31

$$\int \frac{(c+dx)^2}{x^2\sqrt{ax+bx^2}}\,dx = -\frac{d^2\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{\sqrt{b}} + \frac{2\left(6\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2cd+3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}c^2+ac^2\right)}{3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3}$$

input
```
integrate((d*x+c)^2/x^2/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
-d^2*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/sqrt(b) + 2/3
*(6*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*c*d + 3*(sqrt(b)*x - sqrt(b*x^2 + a*
x))*sqrt(b)*c^2 + a*c^2)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{x^2\sqrt{ax+bx^2}}\,dx = \int \frac{(c+d\,x)^2}{x^2\,\sqrt{b\,x^2+a\,x}}\,dx$$

input
```
int((c + d*x)^2/(x^2*(a*x + b*x^2)^(1/2)),x)
```

output
```
int((c + d*x)^2/(x^2*(a*x + b*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx)^2}{x^2\sqrt{ax+bx^2}}\,dx$$
$$= \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,ab\,c^2}{3} - 4\sqrt{x}\,\sqrt{bx+a}\,abcdx + \frac{4\sqrt{x}\,\sqrt{bx+a}\,b^2c^2x}{3} + 2\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2d^2x^2 + \frac{4\sqrt{b}\,abcd\,x^2}{3} - 4\sqrt{}}{a^2b\,x^2}$$

input
```
int((d*x+c)^2/x^2/(b*x^2+a*x)^(1/2),x)
```

output
```
(2*( - sqrt(x)*sqrt(a + b*x)*a*b*c**2 - 6*sqrt(x)*sqrt(a + b*x)*a*b*c*d*x
+ 2*sqrt(x)*sqrt(a + b*x)*b**2*c**2*x + 3*sqrt(b)*log((sqrt(a + b*x) + sqr
t(x)*sqrt(b))/sqrt(a))*a**2*d**2*x**2 + 2*sqrt(b)*a*b*c*d*x**2 - 2*sqrt(b)
*b**2*c**2*x**2))/(3*a**2*b*x**2)
```

3.140 $\int \frac{(c+dx)^2}{x^3\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 103

$$\int \frac{(c+dx)^2}{x^3\sqrt{ax+bx^2}}\,dx = \frac{8c(bc-ad)\sqrt{ax+bx^2}}{15a^2x^2} - \frac{8(2bc-3ad)(bc-ad)\sqrt{ax+bx^2}}{15a^3x} - \frac{2(c+dx)^2\sqrt{ax+bx^2}}{5ax^3}$$

output
```
8/15*c*(-a*d+b*c)*(b*x^2+a*x)^(1/2)/a^2/x^2-8/15*(-3*a*d+2*b*c)*(-a*d+b*c)
*(b*x^2+a*x)^(1/2)/a^3/x-2/5*(d*x+c)^2*(b*x^2+a*x)^(1/2)/a/x^3
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.66

$$\int \frac{(c+dx)^2}{x^3\sqrt{ax+bx^2}}\,dx = -\frac{2\sqrt{x(a+bx)}(8b^2c^2x^2-4abcx(c+5dx)+a^2(3c^2+10cdx+15d^2x^2))}{15a^3x^3}$$

input
```
Integrate[(c + d*x)^2/(x^3*Sqrt[a*x + b*x^2]),x]
```

output
```
(-2*Sqrt[x*(a + b*x)]*(8*b^2*c^2*x^2 - 4*a*b*c*x*(c + 5*d*x) + a^2*(3*c^2
+ 10*c*d*x + 15*d^2*x^2)))/(15*a^3*x^3)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.34, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1262, 27, 1220, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{x^3\sqrt{ax+bx^2}}dx$$

$$\Big\downarrow \text{ 1262}$$

$$-\frac{\int -\frac{2bc^2+d(4bc-3ad)x}{2x^3\sqrt{bx^2+ax}}dx}{b}-\frac{d^2\sqrt{ax+bx^2}}{bx^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{2bc^2+d(4bc-3ad)x}{x^3\sqrt{bx^2+ax}}dx}{2b}-\frac{d^2\sqrt{ax+bx^2}}{bx^2}$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{-\frac{(15a^2d^2-20abcd+8b^2c^2)\int \frac{1}{x^2\sqrt{bx^2+ax}}dx}{5a}-\frac{4bc^2\sqrt{ax+bx^2}}{5ax^3}}{2b}-\frac{d^2\sqrt{ax+bx^2}}{bx^2}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{-\frac{(15a^2d^2-20abcd+8b^2c^2)\left(-\frac{2b\int \frac{1}{x\sqrt{bx^2+ax}}dx}{3a}-\frac{2\sqrt{ax+bx^2}}{3ax^2}\right)}{5a}-\frac{4bc^2\sqrt{ax+bx^2}}{5ax^3}}{2b}-\frac{d^2\sqrt{ax+bx^2}}{bx^2}$$

$$\Big\downarrow \text{ 1123}$$

$$\frac{-\frac{\left(\frac{4b\sqrt{ax+bx^2}}{3a^2x}-\frac{2\sqrt{ax+bx^2}}{3ax^2}\right)(15a^2d^2-20abcd+8b^2c^2)}{5a}-\frac{4bc^2\sqrt{ax+bx^2}}{5ax^3}}{2b}-\frac{d^2\sqrt{ax+bx^2}}{bx^2}$$

input
```
Int[(c + d*x)^2/(x^3*Sqrt[a*x + b*x^2]),x]
```

output
```
-((d^2*Sqrt[a*x + b*x^2])/(b*x^2)) + ((-4*b*c^2*Sqrt[a*x + b*x^2])/(5*a*x^
3) - ((8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*((-2*Sqrt[a*x + b*x^2])/(3*a*x
^2) + (4*b*Sqrt[a*x + b*x^2])/(3*a^2*x)))/(5*a))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$-\frac{2\left(\left(5d^{2}x^{2}+\frac{10}{3}cdx+c^{2}\right)a^{2}-\frac{4bcx(5dx+c)a}{3}+\frac{8b^{2}c^{2}x^{2}}{3}\right)\sqrt{x(bx+a)}}{5a^{3}x^{3}}$
trager	$-\frac{2\left(15a^{2}d^{2}x^{2}-20abcd\,x^{2}+8b^{2}c^{2}x^{2}+10a^{2}cdx-4ab\,c^{2}x+3a^{2}c^{2}\right)\sqrt{b\,x^{2}+ax}}{15a^{3}x^{3}}$
risch	$-\frac{2(bx+a)\left(15a^{2}d^{2}x^{2}-20abcd\,x^{2}+8b^{2}c^{2}x^{2}+10a^{2}cdx-4ab\,c^{2}x+3a^{2}c^{2}\right)}{15a^{3}x^{2}\sqrt{x(bx+a)}}$
gosper	$-\frac{2(bx+a)\left(15a^{2}d^{2}x^{2}-20abcd\,x^{2}+8b^{2}c^{2}x^{2}+10a^{2}cdx-4ab\,c^{2}x+3a^{2}c^{2}\right)}{15x^{2}a^{3}\sqrt{b\,x^{2}+ax}}$
orering	$-\frac{2(bx+a)\left(15a^{2}d^{2}x^{2}-20abcd\,x^{2}+8b^{2}c^{2}x^{2}+10a^{2}cdx-4ab\,c^{2}x+3a^{2}c^{2}\right)}{15x^{2}a^{3}\sqrt{b\,x^{2}+ax}}$
default	$c^{2}\left(-\frac{2\sqrt{b\,x^{2}+ax}}{5a\,x^{3}}-\frac{4b\left(-\frac{2\sqrt{b\,x^{2}+ax}}{3a\,x^{2}}+\frac{4b\sqrt{b\,x^{2}+ax}}{3a^{2}x}\right)}{5a}\right)-\frac{2d^{2}\sqrt{b\,x^{2}+ax}}{ax}+2cd\left(-\frac{2\sqrt{b\,x^{2}+ax}}{3a\,x^{2}}+\frac{4b\sqrt{b\,x^{2}+ax}}{3a^{2}x}\right)$

input
```
int((d*x+c)^2/x^3/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*((5*d^2*x^2+10/3*c*d*x+c^2)*a^2-4/3*b*c*x*(5*d*x+c)*a+8/3*b^2*c^2*x^2
)*(x*(b*x+a))^(1/2)/a^3/x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.71

$$\int \frac{(c+dx)^2}{x^3\sqrt{ax+bx^2}}\,dx = -\frac{2\,(3\,a^2c^2+(8\,b^2c^2-20\,abcd+15\,a^2d^2)x^2-2\,(2\,abc^2-5\,a^2cd)x)\sqrt{bx^2+ax}}{15\,a^3x^3}$$

input
```
integrate((d*x+c)^2/x^3/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/15*(3*a^2*c^2 + (8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*x^2 - 2*(2*a*b*c^
2 - 5*a^2*c*d)*x)*sqrt(b*x^2 + a*x)/(a^3*x^3)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{x^3\sqrt{ax+bx^2}}\,dx = \int \frac{(c+dx)^2}{x^3\sqrt{x\,(a+bx)}}\,dx$$

input
```
integrate((d*x+c)**2/x**3/(b*x**2+a*x)**(1/2),x)
```

output
```
Integral((c + d*x)**2/(x**3*sqrt(x*(a + b*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.32

$$\int \frac{(c+dx)^2}{x^3\sqrt{ax+bx^2}}\,dx = -\frac{16\,\sqrt{bx^2+ax}b^2c^2}{15\,a^3x} + \frac{8\,\sqrt{bx^2+ax}bcd}{3\,a^2x} - \frac{2\,\sqrt{bx^2+ax}d^2}{ax} + \frac{8\,\sqrt{bx^2+ax}bc^2}{15\,a^2x^2} - \frac{4\,\sqrt{bx^2+ax}cd}{3\,ax^2} - \frac{2\,\sqrt{bx^2+ax}c^2}{5\,ax^3}$$

input
```
integrate((d*x+c)^2/x^3/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
-16/15*sqrt(b*x^2 + a*x)*b^2*c^2/(a^3*x) + 8/3*sqrt(b*x^2 + a*x)*b*c*d/(a^
2*x) - 2*sqrt(b*x^2 + a*x)*d^2/(a*x) + 8/15*sqrt(b*x^2 + a*x)*b*c^2/(a^2*x
^2) - 4/3*sqrt(b*x^2 + a*x)*c*d/(a*x^2) - 2/5*sqrt(b*x^2 + a*x)*c^2/(a*x^3
)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.62

$$\int \frac{(c+dx)^2}{x^3\sqrt{ax+bx^2}}\,dx$$
$$= \frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4 d^2+30\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3\sqrt{b}cd+20\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2 bc^2+10\left(\sqrt{b}\right.\right.}{15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5}$$

input
```
integrate((d*x+c)^2/x^3/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
2/15*(15*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*d^2 + 30*(sqrt(b)*x - sqrt(b*x^
2 + a*x))^3*sqrt(b)*c*d + 20*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*b*c^2 + 10*
(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a*c*d + 15*(sqrt(b)*x - sqrt(b*x^2 + a*x
))*a*sqrt(b)*c^2 + 3*a^2*c^2)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^5
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.73

$$\int \frac{(c+dx)^2}{x^3\sqrt{ax+bx^2}}\,dx = -\frac{2\sqrt{b\,x^2+a\,x}\,\left(3\,a^2\,c^2+10\,a^2\,c\,d\,x+15\,a^2\,d^2\,x^2-4\,a\,b\,c^2\,x-20\,a\,b\,c\,d\,x^2+8\,b^2\,c^2\,x^2\right)}{15\,a^3\,x^3}$$

input
```
int((c + d*x)^2/(x^3*(a*x + b*x^2)^(1/2)),x)
```

output
```
-(2*(a*x + b*x^2)^(1/2)*(3*a^2*c^2 + 15*a^2*d^2*x^2 + 8*b^2*c^2*x^2 - 4*a*
b*c^2*x + 10*a^2*c*d*x - 20*a*b*c*d*x^2))/(15*a^3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.45

$$\int \frac{(c+dx)^2}{x^3\sqrt{ax+bx^2}}\,dx$$
$$= \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2c^2}{5} - \frac{4\sqrt{x}\,\sqrt{bx+a}\,a^2cdx}{3} - 2\sqrt{x}\,\sqrt{bx+a}\,a^2d^2x^2 + \frac{8\sqrt{x}\,\sqrt{bx+a}\,ab\,c^2x}{15} + \frac{8\sqrt{x}\,\sqrt{bx+a}\,abcd\,x^2}{3} - \frac{16\sqrt{x}\,\sqrt{bx+a}\,b}{15}}{a^3x^3}$$

input
```
int((d*x+c)^2/x^3/(b*x^2+a*x)^(1/2),x)
```

output
```
(2*( - 3*sqrt(x)*sqrt(a + b*x)*a**2*c**2 - 10*sqrt(x)*sqrt(a + b*x)*a**2*c
*d*x - 15*sqrt(x)*sqrt(a + b*x)*a**2*d**2*x**2 + 4*sqrt(x)*sqrt(a + b*x)*a
*b*c**2*x + 20*sqrt(x)*sqrt(a + b*x)*a*b*c*d*x**2 - 8*sqrt(x)*sqrt(a + b*x
)*b**2*c**2*x**2 + 9*sqrt(b)*a**2*d**2*x**3 - 20*sqrt(b)*a*b*c*d*x**3 + 8*
sqrt(b)*b**2*c**2*x**3))/(15*a**3*x**3)
```

3.141 $\int \frac{(c+dx)^2}{x^4\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 165

$$\int \frac{(c+dx)^2}{x^4\sqrt{ax+bx^2}}\,dx = -\frac{8(bc-ad)(6bc+ad)\sqrt{ax+bx^2}}{105a^3x^2} + \frac{8(2bc-3ad)(bc-ad)(6bc+ad)\sqrt{ax+bx^2}}{105a^4cx} + \frac{2(6bc+ad)(c+dx)^2\sqrt{ax+bx^2}}{35a^2cx^3} - \frac{2(c+dx)^3\sqrt{ax+bx^2}}{7acx^4}$$

output
```
-8/105*(-a*d+b*c)*(a*d+6*b*c)*(b*x^2+a*x)^(1/2)/a^3/x^2+8/105*(-3*a*d+2*b*
c)*(-a*d+b*c)*(a*d+6*b*c)*(b*x^2+a*x)^(1/2)/a^4/c/x+2/35*(a*d+6*b*c)*(d*x+
c)^2*(b*x^2+a*x)^(1/2)/a^2/c/x^3-2/7*(d*x+c)^3*(b*x^2+a*x)^(1/2)/a/c/x^4
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.61

$$\int \frac{(c+dx)^2}{x^4\sqrt{ax+bx^2}}\,dx = -\frac{2\sqrt{x(a+bx)}(-48b^3c^2x^3+8ab^2cx^2(3c+14dx)-2a^2bx(9c^2+28cdx+35d^2x^2)+a^3(15c^2+42cdx+}{105a^4x^4}$$

input
```
Integrate[(c + d*x)^2/(x^4*Sqrt[a*x + b*x^2]),x]
```

output
```
(-2*Sqrt[x*(a + b*x)]*(-48*b^3*c^2*x^3 + 8*a*b^2*c*x^2*(3*c + 14*d*x) - 2*
a^2*b*x*(9*c^2 + 28*c*d*x + 35*d^2*x^2) + a^3*(15*c^2 + 42*c*d*x + 35*d^2*
x^2)))/(105*a^4*x^4)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1262, 27, 1220, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{x^4\sqrt{ax+bx^2}}dx$$

$$\Big\downarrow \text{1262}$$

$$-\frac{\int -\frac{4bc^2+d(8bc-5ad)x}{2x^4\sqrt{bx^2+ax}}dx}{2b}-\frac{d^2\sqrt{ax+bx^2}}{2bx^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4bc^2+d(8bc-5ad)x}{x^4\sqrt{bx^2+ax}}dx}{4b}-\frac{d^2\sqrt{ax+bx^2}}{2bx^3}$$

$$\Big\downarrow \text{1220}$$

$$\frac{-\frac{(35a^2d^2-56abcd+24b^2c^2)\int \frac{1}{x^3\sqrt{bx^2+ax}}dx}{7a}-\frac{8bc^2\sqrt{ax+bx^2}}{7ax^4}}{4b}-\frac{d^2\sqrt{ax+bx^2}}{2bx^3}$$

$$\Big\downarrow \text{1129}$$

$$\frac{-\frac{(35a^2d^2-56abcd+24b^2c^2)\left(-\frac{4b\int \frac{1}{x^2\sqrt{bx^2+ax}}dx}{5a}-\frac{2\sqrt{ax+bx^2}}{5ax^3}\right)}{7a}-\frac{8bc^2\sqrt{ax+bx^2}}{7ax^4}}{4b}-\frac{d^2\sqrt{ax+bx^2}}{2bx^3}$$

$$\Big\downarrow \text{1129}$$

$$-\frac{-\frac{(35a^2d^2-56abcd+24b^2c^2)\left(-\frac{4b\left(-\frac{2b\int\frac{1}{x\sqrt{bx^2+ax}}dx}{3a}-\frac{2\sqrt{ax+bx^2}}{3ax^2}\right)}{5a}-\frac{2\sqrt{ax+bx^2}}{5ax^3}\right)}{7a}-\frac{8bc^2\sqrt{ax+bx^2}}{7ax^4}}{4b}-\frac{d^2\sqrt{ax+bx^2}}{2bx^3}$$

$$\Big\downarrow \text{1123}$$

$$-\frac{-\frac{\left(-\frac{4b\left(\frac{4b\sqrt{ax+bx^2}}{3a^2x}-\frac{2\sqrt{ax+bx^2}}{3ax^2}\right)}{5a}-\frac{2\sqrt{ax+bx^2}}{5ax^3}\right)(35a^2d^2-56abcd+24b^2c^2)}{7a}-\frac{8bc^2\sqrt{ax+bx^2}}{7ax^4}}{4b}-\frac{d^2\sqrt{ax+bx^2}}{2bx^3}$$

input
```
Int[(c + d*x)^2/(x^4*Sqrt[a*x + b*x^2]),x]
```

output
```
-1/2*(d^2*Sqrt[a*x + b*x^2])/(b*x^3) + ((-8*b*c^2*Sqrt[a*x + b*x^2])/(7*a*
x^4) - ((24*b^2*c^2 - 56*a*b*c*d + 35*a^2*d^2)*((-2*Sqrt[a*x + b*x^2])/(5*
a*x^3) - (4*b*((-2*Sqrt[a*x + b*x^2])/(3*a*x^2) + (4*b*Sqrt[a*x + b*x^2])/
(3*a^2*x)))/(5*a)))/(7*a))/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.55

method	result
pseudoelliptic	$-\frac{2\sqrt{x(bx+a)}\left(\left(\frac{7}{3}d^2x^2+\frac{14}{5}cdx+c^2\right)a^3-\frac{6xb\left(\frac{35}{9}d^2x^2+\frac{28}{9}cdx+c^2\right)a^2}{5}+\frac{8x^2b^2\left(\frac{14dx}{3}+c\right)ca}{5}-\frac{16b^3c^2x^3}{5}\right)}{7a^4x^4}$
trager	$-\frac{2(-70d^2x^3a^2b+112a\,b^2cd\,x^3-48b^3c^2x^3+35a^3d^2x^2-56x^2a^2bcd+24a\,b^2c^2x^2+42a^3cdx-18a^2b\,c^2x+15c^2a^3)\sqrt{b\,x^2+ax}}{105a^4x^4}$
risch	$-\frac{2(bx+a)(-70d^2x^3a^2b+112a\,b^2cd\,x^3-48b^3c^2x^3+35a^3d^2x^2-56x^2a^2bcd+24a\,b^2c^2x^2+42a^3cdx-18a^2b\,c^2x+15c^2a^3)}{105a^4x^3\sqrt{x(bx+a)}}$
gosper	$-\frac{2(bx+a)(-70d^2x^3a^2b+112a\,b^2cd\,x^3-48b^3c^2x^3+35a^3d^2x^2-56x^2a^2bcd+24a\,b^2c^2x^2+42a^3cdx-18a^2b\,c^2x+15c^2a^3)}{105x^3a^4\sqrt{b\,x^2+ax}}$
orering	$-\frac{2(bx+a)(-70d^2x^3a^2b+112a\,b^2cd\,x^3-48b^3c^2x^3+35a^3d^2x^2-56x^2a^2bcd+24a\,b^2c^2x^2+42a^3cdx-18a^2b\,c^2x+15c^2a^3)}{105x^3a^4\sqrt{b\,x^2+ax}}$
default	$c^2\left(-\frac{2\sqrt{b\,x^2+ax}}{7a\,x^4}-\frac{6b\left(-\frac{2\sqrt{b\,x^2+ax}}{5a\,x^3}-\frac{4b\left(-\frac{2\sqrt{b\,x^2+ax}}{3a\,x^2}+\frac{4b\sqrt{b\,x^2+ax}}{3a^2x}\right)}{5a}\right)}{7a}\right)+d^2\left(-\frac{2\sqrt{b\,x^2+ax}}{3a\,x^2}+\frac{4b\sqrt{b\,x^2+ax}}{3a^2x}\right.$

input
```
int((d*x+c)^2/x^4/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7*(x*(b*x+a))^(1/2)*((7/3*d^2*x^2+14/5*c*d*x+c^2)*a^3-6/5*x*b*(35/9*d^2
*x^2+28/9*c*d*x+c^2)*a^2+8/5*x^2*b^2*(14/3*d*x+c)*c*a-16/5*b^3*c^2*x^3)/a^
4/x^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.66

$$\int \frac{(c+dx)^2}{x^4\sqrt{ax+bx^2}}\,dx = -\frac{2\left(15\,a^3c^2-2\left(24\,b^3c^2-56\,ab^2cd+35\,a^2bd^2\right)x^3+\left(24\,ab^2c^2-56\,a^2bcd+35\,a^3d^2\right)x^2-6\left(3\,a^2bc^2-7\,a\right.\right.}{105\,a^4x^4}$$

input
```
integrate((d*x+c)^2/x^4/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/105*(15*a^3*c^2 - 2*(24*b^3*c^2 - 56*a*b^2*c*d + 35*a^2*b*d^2)*x^3 + (2
4*a*b^2*c^2 - 56*a^2*b*c*d + 35*a^3*d^2)*x^2 - 6*(3*a^2*b*c^2 - 7*a^3*c*d)
*x)*sqrt(b*x^2 + a*x)/(a^4*x^4)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{x^4\sqrt{ax+bx^2}}\,dx = \int \frac{(c+dx)^2}{x^4\sqrt{x\,(a+bx)}}\,dx$$

input
```
integrate((d*x+c)**2/x**4/(b*x**2+a*x)**(1/2),x)
```

output
```
Integral((c + d*x)**2/(x**4*sqrt(x*(a + b*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.26

$$\int \frac{(c+dx)^2}{x^4\sqrt{ax+bx^2}}\,dx = \frac{32\,\sqrt{bx^2+ax}b^3c^2}{35\,a^4x} - \frac{32\,\sqrt{bx^2+ax}b^2cd}{15\,a^3x} + \frac{4\,\sqrt{bx^2+ax}bd^2}{3\,a^2x} - \frac{16\,\sqrt{bx^2+ax}b^2c^2}{35\,a^3x^2} + \frac{16\,\sqrt{bx^2+ax}bcd}{15\,a^2x^2} - \frac{2\,\sqrt{bx^2+ax}d^2}{3\,ax^2} + \frac{12\,\sqrt{bx^2+ax}bc^2}{35\,a^2x^3} - \frac{4\,\sqrt{bx^2+ax}cd}{5\,ax^3} - \frac{2\,\sqrt{bx^2+ax}c^2}{7\,ax^4}$$

input
```
integrate((d*x+c)^2/x^4/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
32/35*sqrt(b*x^2 + a*x)*b^3*c^2/(a^4*x) - 32/15*sqrt(b*x^2 + a*x)*b^2*c*d/
(a^3*x) + 4/3*sqrt(b*x^2 + a*x)*b*d^2/(a^2*x) - 16/35*sqrt(b*x^2 + a*x)*b^
2*c^2/(a^3*x^2) + 16/15*sqrt(b*x^2 + a*x)*b*c*d/(a^2*x^2) - 2/3*sqrt(b*x^2
 + a*x)*d^2/(a*x^2) + 12/35*sqrt(b*x^2 + a*x)*b*c^2/(a^2*x^3) - 4/5*sqrt(b
*x^2 + a*x)*c*d/(a*x^3) - 2/7*sqrt(b*x^2 + a*x)*c^2/(a*x^4)
```

Giac [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.56

$$\int \frac{(c+dx)^2}{x^4\sqrt{ax+bx^2}}\,dx$$
$$= \frac{2\left(105\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5\sqrt{b}d^2+280\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4bcd+35\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4ad^2+210\right.}{}$$

input
```
integrate((d*x+c)^2/x^4/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
2/105*(105*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*sqrt(b)*d^2 + 280*(sqrt(b)*x
- sqrt(b*x^2 + a*x))^4*b*c*d + 35*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a*d^2
+ 210*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*b^(3/2)*c^2 + 210*(sqrt(b)*x - sqr
t(b*x^2 + a*x))^3*a*sqrt(b)*c*d + 252*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a*
b*c^2 + 42*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2*c*d + 105*(sqrt(b)*x - sq
rt(b*x^2 + a*x))*a^2*sqrt(b)*c^2 + 15*a^3*c^2)/(sqrt(b)*x - sqrt(b*x^2 + a
*x))^7
```

Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx)^2}{x^4\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{b\,x^2+a\,x}\,\left(140\,a^2\,b\,d^2-224\,a\,b^2\,c\,d+96\,b^3\,c^2\right)}{105\,a^4\,x} - \frac{2\,c^2\,\sqrt{b\,x^2+a\,x}}{7\,a\,x^4} - \frac{\sqrt{b\,x^2+a\,x}\,\left(70\,a^2\,d^2-112\,a\,b\,c\,d+48\,b^2\,c^2\right)}{105\,a^3\,x^2} + \frac{\sqrt{b\,x^2+a\,x}\,\left(12\,b\,c^2-28\,a\,c\,d\right)}{35\,a^2\,x^3}$$

input
```
int((c + d*x)^2/(x^4*(a*x + b*x^2)^(1/2)),x)
```

output
```
((a*x + b*x^2)^(1/2)*(96*b^3*c^2 + 140*a^2*b*d^2 - 224*a*b^2*c*d))/(105*a^
4*x) - (2*c^2*(a*x + b*x^2)^(1/2))/(7*a*x^4) - ((a*x + b*x^2)^(1/2)*(70*a^
2*d^2 + 48*b^2*c^2 - 112*a*b*c*d))/(105*a^3*x^2) + ((a*x + b*x^2)^(1/2)*(1
2*b*c^2 - 28*a*c*d))/(35*a^2*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.30

$$\int \frac{(c+dx)^2}{x^4\sqrt{ax+bx^2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3c^2}{7}-\frac{4\sqrt{x}\,\sqrt{bx+a}\,a^3cdx}{5}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3d^2x^2}{3}+\frac{12\sqrt{x}\,\sqrt{bx+a}\,a^2b\,c^2x}{35}+\frac{16\sqrt{x}\,\sqrt{bx+a}\,a^2bcd\,x^2}{15}+\frac{4\sqrt{x}\,\sqrt{bx+a}\,a^2b\,a}{3}}{a^4x}$$

input
```
int((d*x+c)^2/x^4/(b*x^2+a*x)^(1/2),x)
```

output
```
(2*( - 15*sqrt(x)*sqrt(a + b*x)*a**3*c**2 - 42*sqrt(x)*sqrt(a + b*x)*a**3*
c*d*x - 35*sqrt(x)*sqrt(a + b*x)*a**3*d**2*x**2 + 18*sqrt(x)*sqrt(a + b*x)
*a**2*b*c**2*x + 56*sqrt(x)*sqrt(a + b*x)*a**2*b*c*d*x**2 + 70*sqrt(x)*sqr
t(a + b*x)*a**2*b*d**2*x**3 - 24*sqrt(x)*sqrt(a + b*x)*a*b**2*c**2*x**2 -
112*sqrt(x)*sqrt(a + b*x)*a*b**2*c*d*x**3 + 48*sqrt(x)*sqrt(a + b*x)*b**3*
c**2*x**3 - 70*sqrt(b)*a**2*b*d**2*x**4 + 112*sqrt(b)*a*b**2*c*d*x**4 - 48
*sqrt(b)*b**3*c**2*x**4))/(105*a**4*x**4)
```

3.142 $\int \frac{(c+dx)^2}{x^5\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 199

$$\begin{aligned}\int \frac{(c+dx)^2}{x^5\sqrt{ax+bx^2}}\,dx = & -\frac{2c^2\sqrt{ax+bx^2}}{9ax^5}+\frac{4c(4bc-9ad)\sqrt{ax+bx^2}}{63a^2x^4} \\ & -\frac{2\left(21d^2+\frac{4bc(4bc-9ad)}{a^2}\right)\sqrt{ax+bx^2}}{105ax^3} \\ & +\frac{8b(21a^2d^2+4bc(4bc-9ad))\sqrt{ax+bx^2}}{315a^4x^2} \\ & -\frac{16b^2(21a^2d^2+4bc(4bc-9ad))\sqrt{ax+bx^2}}{315a^5x}\end{aligned}$$

output
```
-2/9*c^2*(b*x^2+a*x)^(1/2)/a/x^5+4/63*c*(-9*a*d+4*b*c)*(b*x^2+a*x)^(1/2)/a
^2/x^4-2/105*(21*d^2+4*b*c*(-9*a*d+4*b*c)/a^2)*(b*x^2+a*x)^(1/2)/a/x^3+8/3
15*b*(21*a^2*d^2+4*b*c*(-9*a*d+4*b*c))*(b*x^2+a*x)^(1/2)/a^4/x^2-16/315*b^
2*(21*a^2*d^2+4*b*c*(-9*a*d+4*b*c))*(b*x^2+a*x)^(1/2)/a^5/x
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.65

$$\int \frac{(c+dx)^2}{x^5\sqrt{ax+bx^2}}\,dx = -\frac{2\sqrt{x(a+bx)}(128b^4c^2x^4-32ab^3cx^3(2c+9dx)+24a^2b^2x^2(2c^2+6cdx+7d^2x^2)-4a^3bx(10c^2+27cd}{315a^5x^5}$$

input
```
Integrate[(c + d*x)^2/(x^5*Sqrt[a*x + b*x^2]),x]
```

output
```
(-2*Sqrt[x*(a + b*x)]*(128*b^4*c^2*x^4 - 32*a*b^3*c*x^3*(2*c + 9*d*x) + 24
*a^2*b^2*x^2*(2*c^2 + 6*c*d*x + 7*d^2*x^2) - 4*a^3*b*x*(10*c^2 + 27*c*d*x
+ 21*d^2*x^2) + a^4*(35*c^2 + 90*c*d*x + 63*d^2*x^2)))/(315*a^5*x^5)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1262, 27, 1220, 1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{x^5\sqrt{ax+bx^2}}\,dx$$

$$\Big\downarrow \; 1262$$

$$-\frac{\int -\frac{6bc^2+d(12bc-7ad)x}{2x^5\sqrt{bx^2+ax}}dx}{3b}-\frac{d^2\sqrt{ax+bx^2}}{3bx^4}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{6bc^2+d(12bc-7ad)x}{x^5\sqrt{bx^2+ax}}dx}{6b}-\frac{d^2\sqrt{ax+bx^2}}{3bx^4}$$

$$\Big\downarrow \; 1220$$

$$\frac{-\frac{(21a^2d^2-36abcd+16b^2c^2)\int \frac{1}{x^4\sqrt{bx^2+ax}}dx}{3a}-\frac{4bc^2\sqrt{ax+bx^2}}{3ax^5}}{6b}-\frac{d^2\sqrt{ax+bx^2}}{3bx^4}$$

$$\Big\downarrow \text{1129}$$

$$\frac{-\frac{(21a^2d^2-36abcd+16b^2c^2)\left(-\frac{6b\int \frac{1}{x^3\sqrt{bx^2+ax}}dx}{7a}-\frac{2\sqrt{ax+bx^2}}{7ax^4}\right)}{3a}-\frac{4bc^2\sqrt{ax+bx^2}}{3ax^5}}{6b}-\frac{d^2\sqrt{ax+bx^2}}{3bx^4}$$

$$\Big\downarrow \text{1129}$$

$$\frac{-\frac{(21a^2d^2-36abcd+16b^2c^2)\left(-\frac{6b\left(-\frac{4b\int \frac{1}{x^2\sqrt{bx^2+ax}}dx}{5a}-\frac{2\sqrt{ax+bx^2}}{5ax^3}\right)}{7a}-\frac{2\sqrt{ax+bx^2}}{7ax^4}\right)}{3a}-\frac{4bc^2\sqrt{ax+bx^2}}{3ax^5}}{6b}-\frac{d^2\sqrt{ax+bx^2}}{3bx^4}$$

$$\Big\downarrow \text{1129}$$

$$\frac{-\frac{(21a^2d^2-36abcd+16b^2c^2)\left(-\frac{6b\left(-\frac{4b\left(-\frac{2b\int \frac{1}{x\sqrt{bx^2+ax}}dx}{3a}-\frac{2\sqrt{ax+bx^2}}{3ax^2}\right)}{5a}-\frac{2\sqrt{ax+bx^2}}{5ax^3}\right)}{7a}-\frac{2\sqrt{ax+bx^2}}{7ax^4}\right)}{3a}-\frac{4bc^2\sqrt{ax+bx^2}}{3ax^5}}{6b}-\frac{d^2\sqrt{ax+bx^2}}{3bx^4}$$

$$\Big\downarrow \text{1123}$$

$$\frac{-\frac{\left(-\frac{6b\left(-\frac{4b\left(\frac{4b\sqrt{ax+bx^2}}{3a^2x}-\frac{2\sqrt{ax+bx^2}}{3ax^2}\right)}{5a}-\frac{2\sqrt{ax+bx^2}}{5ax^3}\right)}{7a}-\frac{2\sqrt{ax+bx^2}}{7ax^4}\right)(21a^2d^2-36abcd+16b^2c^2)}{3a}-\frac{4bc^2\sqrt{ax+bx^2}}{3ax^5}}{6b}-\frac{d^2\sqrt{ax+bx^2}}{3bx^4}$$

input
```
Int[(c + d*x)^2/(x^5*Sqrt[a*x + b*x^2]),x]
```

output
```
-1/3*(d^2*Sqrt[a*x + b*x^2])/(b*x^4) + ((-4*b*c^2*Sqrt[a*x + b*x^2])/(3*a*
x^5) - ((16*b^2*c^2 - 36*a*b*c*d + 21*a^2*d^2)*((-2*Sqrt[a*x + b*x^2])/(7*
a*x^4) - (6*b*((-2*Sqrt[a*x + b*x^2])/(5*a*x^3) - (4*b*((-2*Sqrt[a*x + b*x
^2])/(3*a*x^2) + (4*b*Sqrt[a*x + b*x^2])/(3*a^2*x)))/(5*a)))/(7*a)))/(3*a)
)/(6*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.60

method	result
pseudoelliptic	$-\frac{2\sqrt{x(bx+a)}\left(\left(\frac{9}{5}d^2x^2+\frac{18}{7}cdx+c^2\right)a^4-\frac{8xb\left(\frac{21}{10}d^2x^2+\frac{27}{10}cdx+c^2\right)a^3}{7}+\frac{48x^2b^2\left(\frac{7}{2}d^2x^2+3cdx+c^2\right)a^2}{35}-\frac{64x^3b^3\left(\frac{9dx}{2}+c\right)ca}{35}+\frac{12\cdots}{}\right.}{9a^5x^5}$
trager	$-\frac{2(168a^2b^2d^2x^4-288ab^3cdx^4+128b^4c^2x^4-84a^3bd^2x^3+144a^2b^2cdx^3-64ab^3c^2x^3+63a^4d^2x^2-108a^3dcbx^2+48a^2b^2c^2\cdots}{315a^5x^5}$
risch	$-\frac{2(bx+a)(168a^2b^2d^2x^4-288ab^3cdx^4+128b^4c^2x^4-84a^3bd^2x^3+144a^2b^2cdx^3-64ab^3c^2x^3+63a^4d^2x^2-108a^3dcbx^2+48\cdots}{315a^5x^4\sqrt{x(bx+a)}}$
gosper	$-\frac{2(bx+a)(168a^2b^2d^2x^4-288ab^3cdx^4+128b^4c^2x^4-84a^3bd^2x^3+144a^2b^2cdx^3-64ab^3c^2x^3+63a^4d^2x^2-108a^3dcbx^2+48\cdots}{315x^4a^5\sqrt{bx^2+ax}}$
orering	$-\frac{2(bx+a)(168a^2b^2d^2x^4-288ab^3cdx^4+128b^4c^2x^4-84a^3bd^2x^3+144a^2b^2cdx^3-64ab^3c^2x^3+63a^4d^2x^2-108a^3dcbx^2+48\cdots}{315x^4a^5\sqrt{bx^2+ax}}$
default	$c^2\left(-\frac{2\sqrt{bx^2+ax}}{9ax^5}-\frac{8b\left(-\frac{2\sqrt{bx^2+ax}}{7ax^4}-\frac{6b\left(-\frac{2\sqrt{bx^2+ax}}{5ax^3}-\frac{4b\left(-\frac{2\sqrt{bx^2+ax}}{3ax^2}+\frac{4b\sqrt{bx^2+ax}}{3a^2x}\right)}{5a}\right)}{7a}\right)}{9a}\right)+d^2\left(-\frac{2\sqrt{bx^2+\cdots}}{5ax^3}\cdots\right.$

input
```
int((d*x+c)^2/x^5/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(x*(b*x+a))^(1/2)*((9/5*d^2*x^2+18/7*c*d*x+c^2)*a^4-8/7*x*b*(21/10*d^
2*x^2+27/10*c*d*x+c^2)*a^3+48/35*x^2*b^2*(7/2*d^2*x^2+3*c*d*x+c^2)*a^2-64/
35*x^3*b^3*(9/2*d*x+c)*c*a+128/35*b^4*c^2*x^4)/a^5/x^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.74

$$\int \frac{(c+dx)^2}{x^5\sqrt{ax+bx^2}}\,dx = -\frac{2\,(35\,a^4c^2+8\,(16\,b^4c^2-36\,ab^3cd+21\,a^2b^2d^2)x^4-4\,(16\,ab^3c^2-36\,a^2b^2cd+21\,a^3bd^2)x^3+3\,(16\,a^2b^2c}{315\,a^5x^5}$$

input
```
integrate((d*x+c)^2/x^5/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/315*(35*a^4*c^2 + 8*(16*b^4*c^2 - 36*a*b^3*c*d + 21*a^2*b^2*d^2)*x^4 -
4*(16*a*b^3*c^2 - 36*a^2*b^2*c*d + 21*a^3*b*d^2)*x^3 + 3*(16*a^2*b^2*c^2 -
 36*a^3*b*c*d + 21*a^4*d^2)*x^2 - 10*(4*a^3*b*c^2 - 9*a^4*c*d)*x)*sqrt(b*x
^2 + a*x)/(a^5*x^5)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{x^5\sqrt{ax+bx^2}}\,dx = \int \frac{(c+dx)^2}{x^5\sqrt{x\,(a+bx)}}\,dx$$

input
```
integrate((d*x+c)**2/x**5/(b*x**2+a*x)**(1/2),x)
```

output
```
Integral((c + d*x)**2/(x**5*sqrt(x*(a + b*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.42

$$\int \frac{(c+dx)^2}{x^5\sqrt{ax+bx^2}}\,dx = -\frac{256\sqrt{bx^2+ax}b^4c^2}{315\,a^5x} + \frac{64\sqrt{bx^2+ax}b^3cd}{35\,a^4x} - \frac{16\sqrt{bx^2+ax}b^2d^2}{15\,a^3x} + \frac{128\sqrt{bx^2+ax}b^3c^2}{315\,a^4x^2} - \frac{32\sqrt{bx^2+ax}b^2cd}{35\,a^3x^2} + \frac{8\sqrt{bx^2+ax}bd^2}{15\,a^2x^2} - \frac{32\sqrt{bx^2+ax}b^2c^2}{105\,a^3x^3} + \frac{24\sqrt{bx^2+ax}bcd}{35\,a^2x^3} - \frac{2\sqrt{bx^2+ax}d^2}{5\,ax^3} + \frac{16\sqrt{bx^2+ax}bc^2}{63\,a^2x^4} - \frac{4\sqrt{bx^2+ax}cd}{7\,ax^4} - \frac{2\sqrt{bx^2+ax}c^2}{9\,ax^5}$$

input
```
integrate((d*x+c)^2/x^5/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
-256/315*sqrt(b*x^2 + a*x)*b^4*c^2/(a^5*x) + 64/35*sqrt(b*x^2 + a*x)*b^3*c
*d/(a^4*x) - 16/15*sqrt(b*x^2 + a*x)*b^2*d^2/(a^3*x) + 128/315*sqrt(b*x^2
+ a*x)*b^3*c^2/(a^4*x^2) - 32/35*sqrt(b*x^2 + a*x)*b^2*c*d/(a^3*x^2) + 8/1
5*sqrt(b*x^2 + a*x)*b*d^2/(a^2*x^2) - 32/105*sqrt(b*x^2 + a*x)*b^2*c^2/(a^
3*x^3) + 24/35*sqrt(b*x^2 + a*x)*b*c*d/(a^2*x^3) - 2/5*sqrt(b*x^2 + a*x)*d
^2/(a*x^3) + 16/63*sqrt(b*x^2 + a*x)*b*c^2/(a^2*x^4) - 4/7*sqrt(b*x^2 + a*
x)*c*d/(a*x^4) - 2/9*sqrt(b*x^2 + a*x)*c^2/(a*x^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.76

$$\int \frac{(c+dx)^2}{x^5\sqrt{ax+bx^2}}\,dx = \frac{2\left(420\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6bd^2 + 1260\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5b^{\frac{3}{2}}cd + 315\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5a\sqrt{b}d^2 -\right.}{}$$

input
```
integrate((d*x+c)^2/x^5/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
2/315*(420*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*b*d^2 + 1260*(sqrt(b)*x - sqr
t(b*x^2 + a*x))^5*b^(3/2)*c*d + 315*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a*sq
rt(b)*d^2 + 1008*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*b^2*c^2 + 1512*(sqrt(b)
*x - sqrt(b*x^2 + a*x))^4*a*b*c*d + 63*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a
^2*d^2 + 1680*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a*b^(3/2)*c^2 + 630*(sqrt(
b)*x - sqrt(b*x^2 + a*x))^3*a^2*sqrt(b)*c*d + 1080*(sqrt(b)*x - sqrt(b*x^2
 + a*x))^2*a^2*b*c^2 + 90*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^3*c*d + 315*
(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^3*sqrt(b)*c^2 + 35*a^4*c^2)/(sqrt(b)*x -
 sqrt(b*x^2 + a*x))^9
```

Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx)^2}{x^5\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{b\,x^2+a\,x}\,(168\,a^2\,b\,d^2-288\,a\,b^2\,c\,d+128\,b^3\,c^2)}{315\,a^4\,x^2} - \frac{2\,c^2\,\sqrt{b\,x^2+a\,x}}{9\,a\,x^5} - \frac{\sqrt{b\,x^2+a\,x}\,(42\,a^2\,d^2-72\,a\,b\,c\,d+32\,b^2\,c^2)}{105\,a^3\,x^3} - \frac{\sqrt{b\,x^2+a\,x}\,(336\,a^2\,b^2\,d^2-576\,a\,b^3\,c\,d+256\,b^4\,c^2)}{315\,a^5\,x} + \frac{\sqrt{b\,x^2+a\,x}\,(16\,b\,c^2-36\,a\,c\,d)}{63\,a^2\,x^4}$$

input
```
int((c + d*x)^2/(x^5*(a*x + b*x^2)^(1/2)),x)
```

output
```
((a*x + b*x^2)^(1/2)*(128*b^3*c^2 + 168*a^2*b*d^2 - 288*a*b^2*c*d))/(315*a
^4*x^2) - (2*c^2*(a*x + b*x^2)^(1/2))/(9*a*x^5) - ((a*x + b*x^2)^(1/2)*(42
*a^2*d^2 + 32*b^2*c^2 - 72*a*b*c*d))/(105*a^3*x^3) - ((a*x + b*x^2)^(1/2)*
(256*b^4*c^2 + 336*a^2*b^2*d^2 - 576*a*b^3*c*d))/(315*a^5*x) + ((a*x + b*x
^2)^(1/2)*(16*b*c^2 - 36*a*c*d))/(63*a^2*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.42

$$\int \frac{(c+dx)^2}{x^5\sqrt{ax+bx^2}}\,dx$$
$$= \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4c^2}{9} - \frac{4\sqrt{x}\,\sqrt{bx+a}\,a^4cdx}{7} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4d^2x^2}{5} + \frac{16\sqrt{x}\,\sqrt{bx+a}\,a^3b\,c^2x}{63} + \frac{24\sqrt{x}\,\sqrt{bx+a}\,a^3bcd\,x^2}{35} + \frac{8\sqrt{x}\,\sqrt{bx+a}\,a^3b\,a}{15}}{}$$

input
```
int((d*x+c)^2/x^5/(b*x^2+a*x)^(1/2),x)
```

output
```
(2*( - 35*sqrt(x)*sqrt(a + b*x)*a**4*c**2 - 90*sqrt(x)*sqrt(a + b*x)*a**4*
c*d*x - 63*sqrt(x)*sqrt(a + b*x)*a**4*d**2*x**2 + 40*sqrt(x)*sqrt(a + b*x)
*a**3*b*c**2*x + 108*sqrt(x)*sqrt(a + b*x)*a**3*b*c*d*x**2 + 84*sqrt(x)*sq
rt(a + b*x)*a**3*b*d**2*x**3 - 48*sqrt(x)*sqrt(a + b*x)*a**2*b**2*c**2*x**
2 - 144*sqrt(x)*sqrt(a + b*x)*a**2*b**2*c*d*x**3 - 168*sqrt(x)*sqrt(a + b*
x)*a**2*b**2*d**2*x**4 + 64*sqrt(x)*sqrt(a + b*x)*a*b**3*c**2*x**3 + 288*s
qrt(x)*sqrt(a + b*x)*a*b**3*c*d*x**4 - 128*sqrt(x)*sqrt(a + b*x)*b**4*c**2
*x**4 + 168*sqrt(b)*a**2*b**2*d**2*x**5 - 288*sqrt(b)*a*b**3*c*d*x**5 + 12
8*sqrt(b)*b**4*c**2*x**5))/(315*a**5*x**5)
```

3.143 $\int \frac{x^3}{(c+dx)\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 168

$$\int \frac{x^3}{(c+dx)\sqrt{ax+bx^2}}\,dx = -\frac{(4bc+3ad)\sqrt{ax+bx^2}}{4b^2d^2}+\frac{x\sqrt{ax+bx^2}}{2bd}+\frac{(8b^2c^2+4abcd+3a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{5/2}d^3}-\frac{2c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{d^3\sqrt{bc-ad}}$$

output

```
-1/4*(3*a*d+4*b*c)*(b*x^2+a*x)^(1/2)/b^2/d^2+1/2*x*(b*x^2+a*x)^(1/2)/b/d+1
/4*(3*a^2*d^2+4*a*b*c*d+8*b^2*c^2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^
(5/2)/d^3-2*c^(5/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/
d^3/(-a*d+b*c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.64 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.79

$$\int \frac{x^3}{(c+dx)\sqrt{ax+bx^2}}\,dx =$$
$$-\frac{\sqrt{x}\left(\sqrt{b}\left(d\sqrt{bc-ad}\sqrt{x}(a+bx)(4bc+3ad-2bdx)+8bc^{3/2}\left(-i\sqrt{a}\sqrt{d}+\sqrt{bc-ad}\right)\sqrt{-bc+2ad-\cdots}\right.\right.}{\cdots}$$

input
```
Integrate[x^3/((c + d*x)*Sqrt[a*x + b*x^2]),x]
```

output
```
-1/4*(Sqrt[x]*(Sqrt[b]*(d*Sqrt[b*c - a*d]*Sqrt[x]*(a + b*x)*(4*b*c + 3*a*d
 - 2*b*d*x) + 8*b*c^(3/2)*((-I)*Sqrt[a]*Sqrt[d] + Sqrt[b*c - a*d])*Sqrt[-(
b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[a + b*x]*ArcTan
[(Sqrt[-(b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(S
qrt[c]*(Sqrt[a] - Sqrt[a + b*x]))] + 8*b*c^(3/2)*(I*Sqrt[a]*Sqrt[d] + Sqrt
[b*c - a*d])*Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*
Sqrt[a + b*x]*ArcTan[(Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c
 - a*d]]*Sqrt[x])/(Sqrt[c]*(Sqrt[a] - Sqrt[a + b*x]))]) + 2*Sqrt[b*c - a*d
]*(8*b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[
x])/(Sqrt[a] - Sqrt[a + b*x])]))/(b^(5/2)*d^3*Sqrt[b*c - a*d]*Sqrt[x*(a +
b*x)])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.29, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1261, 113, 27, 171, 27, 175, 65, 104, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{ax+bx^2}(c+dx)}\,dx$$

$$\Big\downarrow \text{ 1261}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\int \frac{x^{5/2}}{\sqrt{a+bx}(c+dx)}dx}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{ 113}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\int -\frac{\sqrt{x}(3ac+(4bc+3ad)x)}{2\sqrt{a+bx}(c+dx)}dx}{2bd}+\frac{x^{3/2}\sqrt{a+bx}}{2bd}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{x^{3/2}\sqrt{a+bx}}{2bd}-\frac{\int \frac{\sqrt{x}(3ac+(4bc+3ad)x)}{\sqrt{a+bx}(c+dx)}dx}{4bd}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{ 171}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{x^{3/2}\sqrt{a+bx}}{2bd}-\frac{\frac{\int -\frac{ac(4bc+3ad)+\left(8b^2c^2+4abdc+3a^2d^2\right)x}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{bd}+\frac{\sqrt{x}\sqrt{a+bx}(3ad+4bc)}{bd}}{4bd}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{x^{3/2}\sqrt{a+bx}}{2bd}-\frac{\frac{\sqrt{x}\sqrt{a+bx}(3ad+4bc)}{bd}-\frac{\int \frac{ac(4bc+3ad)+\left(8b^2c^2+4abdc+3a^2d^2\right)x}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{2bd}}{4bd}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{ 175}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{x^{3/2}\sqrt{a+bx}}{2bd}-\frac{\frac{\sqrt{x}\sqrt{a+bx}(3ad+4bc)}{bd}-\frac{\frac{\left(3a^2d^2+4abcd+8b^2c^2\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{d}-\frac{8b^2c^3\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}}{2bd}}{4bd}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{ 65}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{x^{3/2}\sqrt{a+bx}}{2bd}-\frac{\frac{\sqrt{x}\sqrt{a+bx}(3ad+4bc)}{bd}-\frac{\frac{2\left(3a^2d^2+4abcd+8b^2c^2\right)\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}-\frac{8b^2c^3\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}}{2bd}}{4bd}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{104}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{x^{3/2}\sqrt{a+bx}}{2bd}-\frac{\frac{\sqrt{x}\sqrt{a+bx}(3ad+4bc)}{bd}-\frac{\frac{2\left(3a^2d^2+4abcd+8b^2c^2\right)\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}-\frac{16b^2c^3\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}}{2bd}}{4bd}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{x^{3/2}\sqrt{a+bx}}{2bd}-\frac{\frac{\sqrt{x}\sqrt{a+bx}(3ad+4bc)}{bd}-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)\left(3a^2d^2+4abcd+8b^2c^2\right)}{\sqrt{b}d}-\frac{16b^2c^3\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}}{2bd}}{4bd}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{x^{3/2}\sqrt{a+bx}}{2bd}-\frac{\frac{\sqrt{x}\sqrt{a+bx}(3ad+4bc)}{bd}-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)\left(3a^2d^2+4abcd+8b^2c^2\right)}{\sqrt{b}d}-\frac{16b^2c^{5/2}\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{d\sqrt{bc-ad}}}{2bd}}{4bd}\right)}{\sqrt{ax+bx^2}}$$

input
```
Int[x^3/((c + d*x)*Sqrt[a*x + b*x^2]),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*((x^(3/2)*Sqrt[a + b*x])/(2*b*d) - (((4*b*c + 3*a*d
)*Sqrt[x]*Sqrt[a + b*x])/(b*d) - ((2*(8*b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*A
rcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(Sqrt[b]*d) - (16*b^2*c^(5/2)*Arc
Tanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(d*Sqrt[b*c - a*d
]))/(2*b*d))/(4*b*d)))/Sqrt[a*x + b*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 113
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$-\frac{\frac{d\sqrt{x(bx+a)}\,(-2bdx+3ad+4bc)}{4b^{2}}-\frac{\left(3a^{2}d^{2}+4abcd+8b^{2}c^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{4b^{\frac{5}{2}}}-\frac{2c^{3}\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{\sqrt{c(ad-bc)}}}{d^{3}}$
risch	$-\frac{(-2bdx+3ad+4bc)x(bx+a)}{4b^{2}d^{2}\sqrt{x(bx+a)}}+\frac{\frac{\left(3a^{2}d^{2}+4abcd+8b^{2}c^{2}\right)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{d\sqrt{b}}+\frac{8c^{3}b^{2}\ln\left(\frac{-\frac{2c(ad-bc)}{d^{2}}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2 \cdots}{\cdots}\right)}{d \cdots}}{8b^{2}d^{2}}$
default	$\frac{c^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{d^{3}\sqrt{b}}+\frac{\frac{x\sqrt{b\,x^{2}+ax}}{2b}-\frac{3a\left(\frac{\sqrt{b\,x^{2}+ax}}{b}-\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2b^{\frac{3}{2}}}\right)}{4b}}{d}-\frac{c\left(\frac{\sqrt{b\,x^{2}+ax}}{b}-\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x \cdots}\right)}{2b^{\frac{3}{2}}} \cdots\right)}{d^{2}}$

input
```
int(x^3/(d*x+c)/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d^3*(1/4*d*(x*(b*x+a))^(1/2)*(-2*b*d*x+3*a*d+4*b*c)/b^2-1/4*(3*a^2*d^2+
4*a*b*c*d+8*b^2*c^2)/b^(5/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-2*c^3/(c
*(a*d-b*c))^(1/2)*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 696, normalized size of antiderivative = 4.14

$$\int \frac{x^3}{(c+dx)\sqrt{ax+bx^2}}\,dx$$
$$= \left[\frac{8\,b^3c^2\sqrt{\frac{c}{bc-ad}}\log\left(\frac{ac+(2\,bc-ad)x-2\,\sqrt{bx^2+ax}(bc-ad)\sqrt{\frac{c}{bc-ad}}}{dx+c}\right)+(8\,b^2c^2+4\,abcd+3\,a^2d^2)\sqrt{b}\log\left(2\,bx+a+2\right.}{8\,b^3d^3}\right.$$
$$-\frac{16\,b^3c^2\sqrt{-\frac{c}{bc-ad}}\arctan\left(-\frac{\sqrt{bx^2+ax}(bc-ad)\sqrt{-\frac{c}{bc-ad}}}{bcx+ac}\right)-(8\,b^2c^2+4\,abcd+3\,a^2d^2)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{\right.}}{8\,b^3d^3}$$
$$-\frac{8\,b^3c^2\sqrt{-\frac{c}{bc-ad}}\arctan\left(-\frac{\sqrt{bx^2+ax}(bc-ad)\sqrt{-\frac{c}{bc-ad}}}{bcx+ac}\right)+(8\,b^2c^2+4\,abcd+3\,a^2d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-}}{bx+a}\right.}{4\,b^3d^3}$$

input
```
integrate(x^3/(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(8*b^3*c^2*sqrt(c/(b*c - a*d))*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*
x^2 + a*x)*(b*c - a*d)*sqrt(c/(b*c - a*d)))/(d*x + c)) + (8*b^2*c^2 + 4*a*
b*c*d + 3*a^2*d^2)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) +
2*(2*b^2*d^2*x - 4*b^2*c*d - 3*a*b*d^2)*sqrt(b*x^2 + a*x))/(b^3*d^3), -1/8
*(16*b^3*c^2*sqrt(-c/(b*c - a*d))*arctan(-sqrt(b*x^2 + a*x)*(b*c - a*d)*sq
rt(-c/(b*c - a*d))/(b*c*x + a*c)) - (8*b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*sq
rt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*(2*b^2*d^2*x - 4*b^
2*c*d - 3*a*b*d^2)*sqrt(b*x^2 + a*x))/(b^3*d^3), 1/4*(4*b^3*c^2*sqrt(c/(b*
c - a*d))*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)*(b*c - a*d)*sqr
t(c/(b*c - a*d)))/(d*x + c)) - (8*b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*sqrt(-b
)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (2*b^2*d^2*x - 4*b^2*c*d
- 3*a*b*d^2)*sqrt(b*x^2 + a*x))/(b^3*d^3), -1/4*(8*b^3*c^2*sqrt(-c/(b*c -
a*d))*arctan(-sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(-c/(b*c - a*d))/(b*c*x +
a*c)) + (8*b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*sqrt(-b)*arctan(sqrt(b*x^2 + a
*x)*sqrt(-b)/(b*x + a)) - (2*b^2*d^2*x - 4*b^2*c*d - 3*a*b*d^2)*sqrt(b*x^2
 + a*x))/(b^3*d^3)]
```

Sympy [F]

$$\int \frac{x^3}{(c+dx)\sqrt{ax+bx^2}}\,dx = \int \frac{x^3}{\sqrt{x(a+bx)}\,(c+dx)}\,dx$$

input
```
integrate(x**3/(d*x+c)/(b*x**2+a*x)**(1/2),x)
```

output
```
Integral(x**3/(sqrt(x*(a + b*x))*(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+dx)\sqrt{ax+bx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((a/d-(2*b*c)/d^2)^2>0)', see `as
sume?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+dx)\sqrt{ax+bx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+dx)\sqrt{ax+bx^2}}\,dx = \int \frac{x^3}{\sqrt{b\,x^2+a\,x}\,\left(c+d\,x\right)}\,dx$$

input
```
int(x^3/((a*x + b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int(x^3/((a*x + b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.99

$$\int \frac{x^3}{(c+dx)\sqrt{ax+bx^2}}\,dx = \frac{8\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)b^3c^2+8\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}+\sqrt{d}\,\sqrt{bx+a}+\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)b^3c}{}$$

input
```
int(x^3/(d*x+c)/(b*x^2+a*x)^(1/2),x)
```

output
```
(8*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) -
 sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**3*c**2 + 8*sqrt(c)*sqrt(a*
d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*s
qrt(b))/(sqrt(c)*sqrt(b)))*b**3*c**2 - 3*sqrt(x)*sqrt(a + b*x)*a**2*b*d**3
 - sqrt(x)*sqrt(a + b*x)*a*b**2*c*d**2 + 2*sqrt(x)*sqrt(a + b*x)*a*b**2*d*
*3*x + 4*sqrt(x)*sqrt(a + b*x)*b**3*c**2*d - 2*sqrt(x)*sqrt(a + b*x)*b**3*
c*d**2*x + 3*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*d
**3 + sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*b*c*d**2
 + 4*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b**2*c**2*d
- 8*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b**3*c**3)/(4*b
**3*d**3*(a*d - b*c))
```

3.144 $\int \frac{x^2}{(c+dx)\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{x^2}{(c+dx)\sqrt{ax+bx^2}}\,dx=\frac{\sqrt{ax+bx^2}}{bd}-\frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{3/2}d^2}+\frac{2c^{3/2}\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{d^2\sqrt{bc-ad}}$$

output
```
(b*x^2+a*x)^(1/2)/b/d-(a*d+2*b*c)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(
3/2)/d^2+2*c^(3/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/d
^2/(-a*d+b*c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.48 (sec) , antiderivative size = 431, normalized size of antiderivative = 3.75

$$\int \frac{x^2}{(c+dx)\sqrt{ax+bx^2}}\,dx$$
$$= \frac{\sqrt{x}\left(\sqrt{b}d\sqrt{x}(a+bx)+\frac{2\sqrt{b}\sqrt{c}\left(-i\sqrt{a}\sqrt{d}+\sqrt{bc-ad}\right)\sqrt{-bc+2ad-2i\sqrt{a}\sqrt{d}\sqrt{bc-ad}}\sqrt{a+bx}\arctan\left(\frac{\sqrt{-bc+2ad-2i\sqrt{a}\sqrt{d}\sqrt{bc-ad}}\sqrt{x}}{\sqrt{c}(\sqrt{a}-\sqrt{a+bx})}\right)}{\sqrt{bc-ad}} -$$

input
```
Integrate[x^2/((c + d*x)*Sqrt[a*x + b*x^2]),x]
```

output
```
(Sqrt[x]*(Sqrt[b]*d*Sqrt[x]*(a + b*x) + (2*Sqrt[b]*Sqrt[c]*((-I)*Sqrt[a]*S
qrt[d] + Sqrt[b*c - a*d])*Sqrt[-(b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt
[b*c - a*d]]*Sqrt[a + b*x]*ArcTan[(Sqrt[-(b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqr
t[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*(Sqrt[a] - Sqrt[a + b*x]))])/Sqrt[
b*c - a*d] + (2*Sqrt[b]*Sqrt[c]*(I*Sqrt[a]*Sqrt[d] + Sqrt[b*c - a*d])*Sqrt
[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[a + b*x]*Arc
Tan[(Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])
/(Sqrt[c]*(Sqrt[a] - Sqrt[a + b*x]))])/Sqrt[b*c - a*d] + 2*(2*b*c + a*d)*S
qrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])]))/(b^(3/
2)*d^2*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.36, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1261, 113, 27, 175, 65, 104, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{ax+bx^2}(c+dx)}\,dx$$
$$\Big\downarrow \text{1261}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\int \frac{x^{3/2}}{\sqrt{a+bx}(c+dx)}dx}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{113}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\int -\frac{ac+(2bc+ad)x}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{bd}+\frac{\sqrt{x}\sqrt{a+bx}}{bd}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\sqrt{x}\sqrt{a+bx}}{bd}-\frac{\int \frac{ac+(2bc+ad)x}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{2bd}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{175}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\sqrt{x}\sqrt{a+bx}}{bd}-\frac{\frac{(ad+2bc)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{d}-\frac{2bc^2\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}}{2bd}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{65}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\sqrt{x}\sqrt{a+bx}}{bd}-\frac{\frac{2(ad+2bc)\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}-\frac{2bc^2\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}}{2bd}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{104}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\sqrt{x}\sqrt{a+bx}}{bd}-\frac{\frac{2(ad+2bc)\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}-\frac{4bc^2\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}}{2bd}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\sqrt{x}\sqrt{a+bx}}{bd}-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)(ad+2bc)}{\sqrt{b}d}-\frac{4bc^2\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}}{2bd}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\sqrt{x}\sqrt{a+bx}}{bd}-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)(ad+2bc)}{\sqrt{bd}}-\frac{4bc^{3/2}\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{d\sqrt{bc-ad}}}{2bd}\right)}{\sqrt{ax+bx^2}}$$

input
```
Int[x^2/((c + d*x)*Sqrt[a*x + b*x^2]),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*((Sqrt[x]*Sqrt[a + b*x])/(b*d) - ((2*(2*b*c + a*d)*
ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(Sqrt[b]*d) - (4*b*c^(3/2)*ArcTa
nh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(d*Sqrt[b*c - a*d])
)/(2*b*d)))/Sqrt[a*x + b*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 113
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{-\frac{d\sqrt{x(bx+a)}}{b}+\frac{(ad+2bc)\,\mathrm{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{b^{\frac{3}{2}}}+\frac{2c^{2}\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{\sqrt{c(ad-bc)}}}{d^{2}}$
risch	$\frac{x(bx+a)}{bd\sqrt{x(bx+a)}}-\frac{\frac{(ad+2bc)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{d\sqrt{b}}+\frac{2c^{2}b\ln\left(\frac{-\frac{2c(ad-bc)}{d^{2}}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^{2}}}\sqrt{b\left(x+\frac{c}{d}\right)^{2}+\frac{(ad-2\cdots}{\cdots}}}{x+\frac{c}{d}}\right)}{d^{2}\sqrt{-\frac{c(ad-bc)}{d^{2}}}}}{2bd}$
default	$\frac{\frac{\sqrt{b\,x^{2}+ax}}{b}-\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2b^{\frac{3}{2}}}}{d}-\frac{c^{2}\ln\left(\frac{-\frac{2c(ad-bc)}{d^{2}}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^{2}}}\sqrt{b\left(x+\frac{c}{d}\right)^{2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\cdots}}{x+\frac{c}{d}}\right)}{d^{3}\sqrt{-\frac{c(ad-bc)}{d^{2}}}}$

input
```
int(x^2/(d*x+c)/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d^2*(-d*(x*(b*x+a))^(1/2)/b+(a*d+2*b*c)/b^(3/2)*arctanh((x*(b*x+a))^(1/
2)/x/b^(1/2))+2*c^2/(c*(a*d-b*c))^(1/2)*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a
*d-b*c))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 534, normalized size of antiderivative = 4.64

$$\int \frac{x^2}{(c+dx)\sqrt{ax+bx^2}}\,dx = \left[\frac{2\,b^2c\sqrt{\frac{c}{bc-ad}}\log\left(\frac{ac+(2\,bc-ad)x+2\,\sqrt{bx^2+ax}(bc-ad)\sqrt{\frac{c}{bc-ad}}}{dx+c}\right)+2\,\sqrt{bx^2+ax}bd+(2\,bc+ad)\sqrt{b}\log\left(2\,bx+a-\right.}{2\,b^2d^2}\right.$$

input
```
integrate(x^2/(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*b^2*c*sqrt(c/(b*c - a*d))*log((a*c + (2*b*c - a*d)*x + 2*sqrt(b*x^
2 + a*x)*(b*c - a*d)*sqrt(c/(b*c - a*d)))/(d*x + c)) + 2*sqrt(b*x^2 + a*x)
*b*d + (2*b*c + a*d)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)))
/(b^2*d^2), 1/2*(4*b^2*c*sqrt(-c/(b*c - a*d))*arctan(-sqrt(b*x^2 + a*x)*(b
*c - a*d)*sqrt(-c/(b*c - a*d))/(b*c*x + a*c)) + 2*sqrt(b*x^2 + a*x)*b*d +
(2*b*c + a*d)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)))/(b^2*d
^2), (b^2*c*sqrt(c/(b*c - a*d))*log((a*c + (2*b*c - a*d)*x + 2*sqrt(b*x^2
+ a*x)*(b*c - a*d)*sqrt(c/(b*c - a*d)))/(d*x + c)) + sqrt(b*x^2 + a*x)*b*d
 + (2*b*c + a*d)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)))/(b
^2*d^2), (2*b^2*c*sqrt(-c/(b*c - a*d))*arctan(-sqrt(b*x^2 + a*x)*(b*c - a*
d)*sqrt(-c/(b*c - a*d))/(b*c*x + a*c)) + sqrt(b*x^2 + a*x)*b*d + (2*b*c +
a*d)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)))/(b^2*d^2)]
```

Sympy [F]

$$\int \frac{x^2}{(c+dx)\sqrt{ax+bx^2}}\,dx = \int \frac{x^2}{\sqrt{x\,(a+bx)}\,(c+dx)}\,dx$$

input
```
integrate(x**2/(d*x+c)/(b*x**2+a*x)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(x*(a + b*x))*(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+dx)\sqrt{ax+bx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((a/d-(2*b*c)/d^2)^2>0)', see `as
sume?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+dx)\sqrt{ax+bx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+dx)\sqrt{ax+bx^2}}\,dx = \int \frac{x^2}{\sqrt{b\,x^2+a\,x}\,\left(c+d\,x\right)}\,dx$$

input
```
int(x^2/((a*x + b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int(x^2/((a*x + b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.06

$$\int \frac{x^2}{(c+dx)\sqrt{ax+bx^2}}\,dx = \frac{-2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)b^2c-2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}+\sqrt{d}\,\sqrt{bx+a}+\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)b^2$$

input
```
int(x^2/(d*x+c)/(b*x^2+a*x)^(1/2),x)
```

output
```
( - 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x
) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**2*c - 2*sqrt(c)*sqrt(a*
d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*s
qrt(b))/(sqrt(c)*sqrt(b)))*b**2*c + sqrt(x)*sqrt(a + b*x)*a*b*d**2 - sqrt(
x)*sqrt(a + b*x)*b**2*c*d - sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/
sqrt(a))*a**2*d**2 - sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)
)*a*b*c*d + 2*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b**2*
c**2)/(b**2*d**2*(a*d - b*c))
```

3.145 $\int \frac{x}{(c+dx)\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{x}{(c+dx)\sqrt{ax+bx^2}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}d} - \frac{2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{d\sqrt{bc-ad}}$$

output
```
2*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(1/2)/d-2*c^(1/2)*arctanh((-a*d+b
*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/d/(-a*d+b*c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.99 (sec) , antiderivative size = 380, normalized size of antiderivative = 4.37

$$\int \frac{x}{(c+dx)\sqrt{ax+bx^2}}\,dx = \frac{2\sqrt{x}\sqrt{a+bx}\left(\frac{\left(-i\sqrt{a}\sqrt{d}+\sqrt{bc-ad}\right)\sqrt{-bc+2ad-2i\sqrt{a}\sqrt{d}\sqrt{bc-ad}}\arctan\left(\frac{\sqrt{-bc+2ad-2i\sqrt{a}\sqrt{d}\sqrt{bc-ad}}\sqrt{x}}{\sqrt{c}\left(-\sqrt{a}+\sqrt{a+bx}\right)}\right)}{\sqrt{c}\sqrt{bc-ad}} + \frac{\left(i\sqrt{a}\sqrt{d}+\sqrt{bc-ad}\right)\sqrt{-}\cdots}{\cdots}\right.}{bd\sqrt{x(a+bx)}}$$

input
```
Integrate[x/((c + d*x)*Sqrt[a*x + b*x^2]),x]
```

output
```
(2*Sqrt[x]*Sqrt[a + b*x]*((((-I)*Sqrt[a]*Sqrt[d] + Sqrt[b*c - a*d])*Sqrt[-
(b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*ArcTan[(Sqrt[-(b*c)
 + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*(-Sqrt
[a] + Sqrt[a + b*x]))])/(Sqrt[c]*Sqrt[b*c - a*d]) + ((I*Sqrt[a]*Sqrt[d] +
Sqrt[b*c - a*d])*Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*
d]]*ArcTan[(Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*S
qrt[x])/(Sqrt[c]*(-Sqrt[a] + Sqrt[a + b*x]))])/(Sqrt[c]*Sqrt[b*c - a*d]) +
 2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(b*d*Sq
rt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1269, 1091, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{ax+bx^2}(c+dx)}\,dx$$

$$\Big\downarrow \text{1269}$$

$$\frac{\int \frac{1}{\sqrt{bx^2+ax}}dx}{d}-\frac{c\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}$$

$$\Big\downarrow \text{1091}$$

$$\frac{2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{d}-\frac{c\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}d}-\frac{c\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{d}$$

$$\Big\downarrow \text{1154}$$

$$\frac{2c\int \frac{1}{4c(bc-ad)-\frac{(ac+(2bc-ad)x)^2}{bx^2+ax}}d\left(-\frac{ac+(2bc-ad)x}{\sqrt{bx^2+ax}}\right)}{d}+\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}d}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}d}-\frac{\sqrt{c}\text{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{d\sqrt{bc-ad}}$$

input
```
Int[x/((c + d*x)*Sqrt[a*x + b*x^2]),x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(Sqrt[b]*d) - (Sqrt[c]*ArcTanh[
(a*c + (2*b*c - a*d)*x)/(2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[a*x + b*x^2])])/(d
*Sqrt[b*c - a*d])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{\sqrt{b}}+\frac{2c\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{\sqrt{c(ad-bc)}}}{d}$
default	$\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{d\sqrt{b}}+\frac{c\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{x+\frac{c}{d}}\right)}{d^2\sqrt{-\frac{c(ad-bc)}{d^2}}}$

input
```
int(x/(d*x+c)/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/b^(1/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))+c/(c*(a*d-b*c))^(1/2)*
arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 432, normalized size of antiderivative = 4.97

$$\int \frac{x}{(c+dx)\sqrt{ax+bx^2}}\,dx$$
$$= \left[\frac{b\sqrt{\frac{c}{bc-ad}}\log\left(\frac{ac+(2\,bc-ad)x-2\,\sqrt{bx^2+ax}(bc-ad)\sqrt{\frac{c}{bc-ad}}}{dx+c}\right)+\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{bd},\right.$$
$$-\frac{2\,b\sqrt{-\frac{c}{bc-ad}}\arctan\left(-\frac{\sqrt{bx^2+ax}(bc-ad)\sqrt{-\frac{c}{bc-ad}}}{bcx+ac}\right)-\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{bd},\ \frac{b\sqrt{\frac{c}{bc-ad}}\log\left(\right.}{}$$
$$\left.-\frac{2\left(b\sqrt{-\frac{c}{bc-ad}}\arctan\left(-\frac{\sqrt{bx^2+ax}(bc-ad)\sqrt{-\frac{c}{bc-ad}}}{bcx+ac}\right)+\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)\right)}{bd}\right]$$

input
```
integrate(x/(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[(b*sqrt(c/(b*c - a*d))*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)*(
b*c - a*d)*sqrt(c/(b*c - a*d)))/(d*x + c)) + sqrt(b)*log(2*b*x + a + 2*sqr
t(b*x^2 + a*x)*sqrt(b)))/(b*d), -(2*b*sqrt(-c/(b*c - a*d))*arctan(-sqrt(b*
x^2 + a*x)*(b*c - a*d)*sqrt(-c/(b*c - a*d))/(b*c*x + a*c)) - sqrt(b)*log(2
*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)))/(b*d), (b*sqrt(c/(b*c - a*d))*log
((a*c + (2*b*c - a*d)*x - 2*sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(c/(b*c - a*
d)))/(d*x + c)) - 2*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)))
/(b*d), -2*(b*sqrt(-c/(b*c - a*d))*arctan(-sqrt(b*x^2 + a*x)*(b*c - a*d)*s
qrt(-c/(b*c - a*d))/(b*c*x + a*c)) + sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqr
t(-b)/(b*x + a)))/(b*d)]
```

Sympy [F]

$$\int \frac{x}{(c+dx)\sqrt{ax+bx^2}}\,dx = \int \frac{x}{\sqrt{x\,(a+bx)}\,(c+dx)}\,dx$$

input
```
integrate(x/(d*x+c)/(b*x**2+a*x)**(1/2),x)
```

output
```
Integral(x/(sqrt(x*(a + b*x))*(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+dx)\sqrt{ax+bx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((a/d-(2*b*c)/d^2)^2>0)', see `as
sume?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(c+dx)\sqrt{ax+bx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+dx)\sqrt{ax+bx^2}}\,dx = \int \frac{x}{\sqrt{b\,x^2+a\,x}\,(c+d\,x)}\,dx$$

input
```
int(x/((a*x + b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int(x/((a*x + b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.91

$$\int \frac{x}{(c+dx)\sqrt{ax+bx^2}}\,dx$$
$$= \frac{2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)b+2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}+\sqrt{d}\,\sqrt{bx+a}+\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)b+2\sqrt{}}{bd\,(ad-bc)}$$

input
```
int(x/(d*x+c)/(b*x^2+a*x)^(1/2),x)
```

output
```
(2*(sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x)
- sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b + sqrt(c)*sqrt(a*d - b*c)*
atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(
sqrt(c)*sqrt(b)))*b + sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a
))*a*d - sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b*c))/(b*d
*(a*d - b*c))
```

3.146 $\int \frac{1}{(c+dx)\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 52

$$\int \frac{1}{(c+dx)\sqrt{ax+bx^2}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}}$$

output
```
2*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(1/2)/(-a*d+b*c)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.77

$$\int \frac{1}{(c+dx)\sqrt{ax+bx^2}}\,dx=-\frac{2\sqrt{x}\sqrt{a+bx}\arctan\left(\frac{-d\sqrt{x}\sqrt{a+bx}+\sqrt{b}(c+dx)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{c}\sqrt{-bc+ad}\sqrt{x(a+bx)}}$$

input
```
Integrate[1/((c + d*x)*Sqrt[a*x + b*x^2]),x]
```

output
```
(-2*Sqrt[x]*Sqrt[a + b*x]*ArcTan[(-(d*Sqrt[x]*Sqrt[a + b*x]) + Sqrt[b]*(c
+ d*x))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(Sqrt[c]*Sqrt[-(b*c) + a*d]*Sqrt[x*
(a + b*x)])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.31, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax+bx^2}(c+dx)} \, dx$$

$$\Big\downarrow \text{1154}$$

$$-2 \int \frac{1}{4c(bc-ad)-\frac{(ac+(2bc-ad)x)^2}{bx^2+ax}}d\left(-\frac{ac+(2bc-ad)x}{\sqrt{bx^2+ax}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{\sqrt{c}\sqrt{bc-ad}}$$

input
```
Int[1/((c + d*x)*Sqrt[a*x + b*x^2]),x]
```

output
```
ArcTanh[(a*c + (2*b*c - a*d)*x)/(2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[a*x + b*x^
2])]/(Sqrt[c]*Sqrt[b*c - a*d])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$-\frac{2\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{\sqrt{c(ad-bc)}}$	42
default	$-\frac{\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{x+\frac{c}{d}}\right)}{d\sqrt{-\frac{c(ad-bc)}{d^2}}}$	132

input
```
int(1/(d*x+c)/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(c*(a*d-b*c))^(1/2)*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.48

$$\int \frac{1}{(c+dx)\sqrt{ax+bx^2}}\,dx = \left[\frac{\log\left(\frac{ac+(2\,bc-ad)x+2\,\sqrt{bc^2-acd}\sqrt{bx^2+ax}}{dx+c}\right)}{\sqrt{bc^2-acd}},\ -\frac{2\,\sqrt{-bc^2+acd}\arctan\left(\frac{\sqrt{-bc^2+acd}\sqrt{bx^2+ax}}{bcx+ac}\right)}{bc^2-acd}\right]$$

input
```
integrate(1/(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[log((a*c + (2*b*c - a*d)*x + 2*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*
x + c))/sqrt(b*c^2 - a*c*d), -2*sqrt(-b*c^2 + a*c*d)*arctan(sqrt(-b*c^2 +
a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c))/(b*c^2 - a*c*d)]
```

Sympy [F]

$$\int \frac{1}{(c+dx)\sqrt{ax+bx^2}}\,dx = \int \frac{1}{\sqrt{x\,(a+bx)}\,(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(b*x**2+a*x)**(1/2),x)
```

output
```
Integral(1/(sqrt(x*(a + b*x))*(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+dx)\sqrt{ax+bx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

$$\int \frac{1}{(c+dx)\sqrt{ax+bx^2}}\,dx = \frac{2\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)d+\sqrt{b}c}{\sqrt{-bc^2+acd}}\right)}{\sqrt{-bc^2+acd}}$$

input
```
integrate(1/(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a*x))*d + sqrt(b)*c)/sqrt(-b*c^2 + a*
c*d))/sqrt(-b*c^2 + a*c*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt{ax+bx^2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a\,x}\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/((a*x + b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int(1/((a*x + b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.94

$$\int \frac{1}{(c+dx)\sqrt{ax+bx^2}}\,dx = -\frac{2\sqrt{c}\,\sqrt{ad-bc}\,\left(atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)+atan\left(\frac{\sqrt{ad-bc}+\sqrt{d}\,\sqrt{bx+a}+\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)\right)}{c\,(ad-bc)}$$

input
```
int(1/(d*x+c)/(b*x^2+a*x)^(1/2),x)
```

output
```
( - 2*sqrt(c)*sqrt(a*d - b*c)*(atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*
x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b))) + atan((sqrt(a*d - b*c) +
 sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))))/(c*
(a*d - b*c))
```

3.147 $\int \frac{1}{x(c+dx)\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 78

$$\int \frac{1}{x(c+dx)\sqrt{ax+bx^2}}\,dx=-\frac{2\sqrt{ax+bx^2}}{acx}-\frac{2d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{3/2}\sqrt{bc-ad}}$$

output
```
-2*(b*x^2+a*x)^(1/2)/a/c/x-2*d*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a
*x)^(1/2))/c^(3/2)/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.41

$$\int \frac{1}{x(c+dx)\sqrt{ax+bx^2}}\,dx=\frac{2\left(-\frac{\sqrt{c}(a+bx)}{a}+\frac{d\sqrt{x}\sqrt{a+bx}\arctan\left(\frac{-d\sqrt{x}\sqrt{a+bx}+\sqrt{b}(c+dx)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}\right)}{c^{3/2}\sqrt{x(a+bx)}}$$

input
```
Integrate[1/(x*(c + d*x)*Sqrt[a*x + b*x^2]),x]
```

output
```
(2*(-((Sqrt[c]*(a + b*x))/a) + (d*Sqrt[x]*Sqrt[a + b*x]*ArcTan[(-(d*Sqrt[x
]*Sqrt[a + b*x]) + Sqrt[b]*(c + d*x))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/Sqrt[
-(b*c) + a*d]))/(c^(3/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.33, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1261, 107, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{ax+bx^2}(c+dx)}dx$$

$$\Big\downarrow \ 1261$$

$$\frac{\sqrt{x}\sqrt{a+bx}\int \frac{1}{x^{3/2}\sqrt{a+bx}(c+dx)}dx}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 107$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{d\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{c}-\frac{2\sqrt{a+bx}}{ac\sqrt{x}}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 104$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{2d\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}-\frac{2\sqrt{a+bx}}{ac\sqrt{x}}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 221$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{2d\,\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{a+bx}}{ac\sqrt{x}}\right)}{\sqrt{ax+bx^2}}$$

input
```
Int[1/(x*(c + d*x)*Sqrt[a*x + b*x^2]),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*((-2*Sqrt[a + b*x])/(a*c*Sqrt[x]) - (2*d*ArcTanh[(S
qrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(c^(3/2)*Sqrt[b*c - a*d]
)))/Sqrt[a*x + b*x^2]
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

method	result	size
pseudoelliptic	$\frac{2\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)adx-2\sqrt{x(bx+a)}\,\sqrt{c(ad-bc)}}{acx\sqrt{c(ad-bc)}}$	79
default	$-\frac{2\sqrt{b\,x^2+ax}}{acx}+\frac{\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{x+\frac{c}{d}}\right)}{c\sqrt{-\frac{c(ad-bc)}{d^2}}}$	154
risch	$-\frac{2(bx+a)}{ac\sqrt{x(bx+a)}}+\frac{\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{x+\frac{c}{d}}\right)}{c\sqrt{-\frac{c(ad-bc)}{d^2}}}$	154

input
```
int(1/x/(d*x+c)/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))*a*d*x-(x*(b*x+a))^(1/
2)*(c*(a*d-b*c))^(1/2))/a/c/x/(c*(a*d-b*c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.77

$$\int \frac{1}{x(c+dx)\sqrt{ax+bx^2}}\,dx$$
$$=\left[\frac{\sqrt{bc^2-acd}adx\log\left(\frac{ac+(2\,bc-ad)x-2\,\sqrt{bc^2-acd}\sqrt{bx^2+ax}}{dx+c}\right)-2\,(bc^2-acd)\sqrt{bx^2+ax}}{(abc^3-a^2c^2d)x},\ \frac{2\left(\sqrt{-bc^2+acd}adx\,\mathrm{arc}\right.}{}\right.$$

input
```
integrate(1/x/(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[(sqrt(b*c^2 - a*c*d)*a*d*x*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*c^2 - a*
c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) - 2*(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/
((a*b*c^3 - a^2*c^2*d)*x), 2*(sqrt(-b*c^2 + a*c*d)*a*d*x*arctan(sqrt(-b*c^
2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) - (b*c^2 - a*c*d)*sqrt(b*x^2 +
 a*x))/((a*b*c^3 - a^2*c^2*d)*x)]
```

Sympy [F]

$$\int \frac{1}{x(c+dx)\sqrt{ax+bx^2}}\,dx = \int \frac{1}{x\sqrt{x\,(a+bx)}\,(c+dx)}\,dx$$

input
```
integrate(1/x/(d*x+c)/(b*x**2+a*x)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(x*(a + b*x))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{x(c+dx)\sqrt{ax+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+ax}(dx+c)x}\,dx$$

input
```
integrate(1/x/(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a*x)*(d*x + c)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(c+dx)\sqrt{ax+bx^2}}\,dx$$
$$= -\frac{2\,d\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)d+\sqrt{b}c}{\sqrt{-bc^2+acd}}\right)}{\sqrt{-bc^2+acd}c} + \frac{2}{\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)c}$$

input
```
integrate(1/x/(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
-2*d*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a*x))*d + sqrt(b)*c)/sqrt(-b*c^2 +
 a*c*d))/(sqrt(-b*c^2 + a*c*d)*c) + 2/((sqrt(b)*x - sqrt(b*x^2 + a*x))*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(c+dx)\sqrt{ax+bx^2}}\,dx = \int \frac{1}{x\,\sqrt{b\,x^2+a\,x}\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/(x*(a*x + b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int(1/(x*(a*x + b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.15

$$\int \frac{1}{x(c+dx)\sqrt{ax+bx^2}}\,dx$$
$$= \frac{2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)adx + 2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}+\sqrt{d}\,\sqrt{bx+a}+\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)adx}{a\,c^2x\,(ad-bc)}$$

input
```
int(1/x/(d*x+c)/(b*x^2+a*x)^(1/2),x)
```

output
```
(2*(sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x)
- sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*d*x + sqrt(c)*sqrt(a*d - b
*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b
))/(sqrt(c)*sqrt(b)))*a*d*x - sqrt(x)*sqrt(a + b*x)*a*c*d + sqrt(x)*sqrt(a
 + b*x)*b*c**2 - sqrt(b)*a*c*d*x + sqrt(b)*b*c**2*x))/(a*c**2*x*(a*d - b*c
))
```

3.148 $\int \frac{1}{x^2(c+dx)\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 117

$$\int \frac{1}{x^2(c+dx)\sqrt{ax+bx^2}}\,dx = -\frac{2\sqrt{ax+bx^2}}{3acx^2}+\frac{2(2bc+3ad)\sqrt{ax+bx^2}}{3a^2c^2x} + \frac{2d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{5/2}\sqrt{bc-ad}}$$

output
```
-2/3*(b*x^2+a*x)^(1/2)/a/c/x^2+2/3*(3*a*d+2*b*c)*(b*x^2+a*x)^(1/2)/a^2/c^2
/x+2*d^2*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(5/2)/(-a
*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^2(c+dx)\sqrt{ax+bx^2}}\,dx = \frac{2\left(\frac{\sqrt{c}(a+bx)(-ac+2bcx+3adx)}{a^2}-\frac{3d^2x^{3/2}\sqrt{a+bx}\arctan\left(\frac{-d\sqrt{x}\sqrt{a+bx}+\sqrt{b}(c+dx)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}\right)}{3c^{5/2}x\sqrt{x(a+bx)}}$$

input
```
Integrate[1/(x^2*(c + d*x)*Sqrt[a*x + b*x^2]),x]
```

output
```
(2*((Sqrt[c]*(a + b*x)*(-(a*c) + 2*b*c*x + 3*a*d*x))/a^2 - (3*d^2*x^(3/2)*
Sqrt[a + b*x]*ArcTan[(-(d*Sqrt[x]*Sqrt[a + b*x]) + Sqrt[b]*(c + d*x))/(Sqr
t[c]*Sqrt[-(b*c) + a*d])]/Sqrt[-(b*c) + a*d]))/(3*c^(5/2)*x*Sqrt[x*(a + b
*x)])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1261, 115, 27, 169, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{ax+bx^2}(c+dx)}dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\int \frac{1}{x^{5/2}\sqrt{a+bx}(c+dx)}dx}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{115}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{2\int \frac{2bc+3ad+2bdx}{2x^{3/2}\sqrt{a+bx}(c+dx)}dx}{3ac}-\frac{2\sqrt{a+bx}}{3acx^{3/2}}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{\int \frac{2bc+3ad+2bdx}{x^{3/2}\sqrt{a+bx}(c+dx)}dx}{3ac}-\frac{2\sqrt{a+bx}}{3acx^{3/2}}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{169}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{-\frac{2\int \frac{3a^2d^2}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{ac}-\frac{2\sqrt{a+bx}(3ad+2bc)}{ac\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}}{3acx^{3/2}}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{-\frac{3ad^2\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{c}-\frac{2\sqrt{a+bx}(3ad+2bc)}{ac\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}}{3acx^{3/2}}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 104$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{-\frac{6ad^2\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}-\frac{2\sqrt{a+bx}(3ad+2bc)}{ac\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}}{3acx^{3/2}}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 221$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{-\frac{6ad^2\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{a+bx}(3ad+2bc)}{ac\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}}{3acx^{3/2}}\right)}{\sqrt{ax+bx^2}}$$

input
```
Int[1/(x^2*(c + d*x)*Sqrt[a*x + b*x^2]),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*((-2*Sqrt[a + b*x])/(3*a*c*x^(3/2)) - ((-2*(2*b*c +
 3*a*d)*Sqrt[a + b*x])/(a*c*Sqrt[x]) - (6*a*d^2*ArcTanh[(Sqrt[b*c - a*d]*S
qrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(c^(3/2)*Sqrt[b*c - a*d]))/(3*a*c)))/Sqr
t[a*x + b*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 115
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ[2*m, 2
*n, 2*p]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$\frac{-2d^2 \arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)a^2x^3+2\left(-\frac{(x(bx+a))^{\frac{3}{2}}c}{3}+\sqrt{x(bx+a)}\,x^2(ad+bc)\right)\sqrt{c(ad-bc)}}{a^2c^2x^3\sqrt{c(ad-bc)}}$
risch	$-\frac{2(bx+a)(-3adx-2cbx+ac)}{3a^2c^2\sqrt{x(bx+a)}\,x}-\frac{d\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{x+\frac{c}{d}}\right)}{c^2\sqrt{-\frac{c(ad-bc)}{d^2}}}$
default	$\frac{-\frac{2\sqrt{b\,x^2+ax}}{3a\,x^2}+\frac{4b\sqrt{b\,x^2+ax}}{3a^2x}}{c}-\frac{d\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{x+\frac{c}{d}}\right)}{c^2\sqrt{-\frac{c(ad-bc)}{d^2}}}$

input
```
int(1/x^2/(d*x+c)/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-d^2*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))*a^2*x^3+(-1/3*(x
*(b*x+a))^(3/2)*c+(x*(b*x+a))^(1/2)*x^2*(a*d+b*c))*(c*(a*d-b*c))^(1/2))/(c
*(a*d-b*c))^(1/2)/a^2/c^2/x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.56

$$\int \frac{1}{x^2(c+dx)\sqrt{ax+bx^2}}\,dx$$
$$= \left[\frac{3\sqrt{bc^2-acd}a^2d^2x^2\log\left(\frac{ac+(2\,bc-ad)x+2\sqrt{bc^2-acd}\sqrt{bx^2+ax}}{dx+c}\right)-2\,(abc^3-a^2c^2d-(2\,b^2c^3+abc^2d-3\,a^2cd^2)x}{3\,(a^2bc^4-a^3c^3d)x^2}\right.$$
$$-\frac{2\left(3\sqrt{-bc^2+acd}a^2d^2x^2\arctan\left(\frac{\sqrt{-bc^2+acd}\sqrt{bx^2+ax}}{bcx+ac}\right)+(abc^3-a^2c^2d-(2\,b^2c^3+abc^2d-3\,a^2cd^2)x)\sqrt{}\right.}{3\,(a^2bc^4-a^3c^3d)x^2}$$

input
```
integrate(1/x^2/(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/3*(3*sqrt(b*c^2 - a*c*d)*a^2*d^2*x^2*log((a*c + (2*b*c - a*d)*x + 2*sqr
t(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) - 2*(a*b*c^3 - a^2*c^2*d -
(2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(b*x^2 + a*x))/((a^2*b*c^4 -
a^3*c^3*d)*x^2), -2/3*(3*sqrt(-b*c^2 + a*c*d)*a^2*d^2*x^2*arctan(sqrt(-b*c
^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) + (a*b*c^3 - a^2*c^2*d - (2*b
^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(b*x^2 + a*x))/((a^2*b*c^4 - a^3*
c^3*d)*x^2)]
```

Sympy [F]

$$\int \frac{1}{x^2(c+dx)\sqrt{ax+bx^2}}\,dx = \int \frac{1}{x^2\sqrt{x\,(a+bx)}\,(c+dx)}\,dx$$

input
```
integrate(1/x**2/(d*x+c)/(b*x**2+a*x)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(x*(a + b*x))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^2(c+dx)\sqrt{ax+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+ax}(dx+c)x^2}\,dx$$

input
```
integrate(1/x^2/(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a*x)*(d*x + c)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^2(c+dx)\sqrt{ax+bx^2}}\,dx = \frac{2\,d^2\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)d+\sqrt{b}c}{\sqrt{-bc^2+acd}}\right)}{\sqrt{-bc^2+acd}c^2} - \frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2d-3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}c-ac\right)}{3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3c^2}$$

input
```
integrate(1/x^2/(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
2*d^2*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a*x))*d + sqrt(b)*c)/sqrt(-b*c^2
+ a*c*d))/(sqrt(-b*c^2 + a*c*d)*c^2) - 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a*
x))^2*d - 3*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b)*c - a*c)/((sqrt(b)*x -
 sqrt(b*x^2 + a*x))^3*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(c+dx)\sqrt{ax+bx^2}}\,dx = \int \frac{1}{x^2\,\sqrt{b\,x^2+a\,x}\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/(x^2*(a*x + b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int(1/(x^2*(a*x + b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.25

$$\int \frac{1}{x^2(c+dx)\sqrt{ax+bx^2}}\,dx$$
$$= \frac{-2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)a^2d^2x^2 - 2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}+\sqrt{d}\,\sqrt{bx+a}+\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right.}{}$$

input
```
int(1/x^2/(d*x+c)/(b*x^2+a*x)^(1/2),x)
```

output
```
(2*( - 3*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a +
b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*d**2*x**2 - 3*sqrt
(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x
)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*d**2*x**2 - sqrt(x)*sqrt(a + b*
x)*a**2*c**2*d + 3*sqrt(x)*sqrt(a + b*x)*a**2*c*d**2*x + sqrt(x)*sqrt(a +
b*x)*a*b*c**3 - sqrt(x)*sqrt(a + b*x)*a*b*c**2*d*x - 2*sqrt(x)*sqrt(a + b*
x)*b**2*c**3*x - sqrt(b)*a**2*c*d**2*x**2 - sqrt(b)*a*b*c**2*d*x**2 + 2*sq
rt(b)*b**2*c**3*x**2))/(3*a**2*c**3*x**2*(a*d - b*c))
```

3.149 $\int \frac{1}{x^3(c+dx)\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 166

$$\int \frac{1}{x^3(c+dx)\sqrt{ax+bx^2}}\,dx = -\frac{2\sqrt{ax+bx^2}}{5acx^3}+\frac{2(4bc+5ad)\sqrt{ax+bx^2}}{15a^2c^2x^2} - \frac{2(8b^2c^2+10abcd+15a^2d^2)\sqrt{ax+bx^2}}{15a^3c^3x} - \frac{2d^3\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{7/2}\sqrt{bc-ad}}$$

output
```
-2/5*(b*x^2+a*x)^(1/2)/a/c/x^3+2/15*(5*a*d+4*b*c)*(b*x^2+a*x)^(1/2)/a^2/c^
2/x^2-2/15*(15*a^2*d^2+10*a*b*c*d+8*b^2*c^2)*(b*x^2+a*x)^(1/2)/a^3/c^3/x-2
*d^3*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(7/2)/(-a*d+b
*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^3(c+dx)\sqrt{ax+bx^2}}\,dx = \frac{2\left(-\frac{\sqrt{c}(a+bx)\left(8b^2c^2x^2+2abcx(-2c+5dx)+a^2\left(3c^2-5cdx+15d^2x^2\right)\right)}{a^3}+\frac{15d^3x^{5/2}\sqrt{a+bx}\arctan\left(\frac{-d\sqrt{x}\sqrt{a+bx}+\sqrt{b}(c+dx)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}\right)}{15c^{7/2}x^2\sqrt{x(a+bx)}}$$

input
```
Integrate[1/(x^3*(c + d*x)*Sqrt[a*x + b*x^2]),x]
```

output
```
(2*(-((Sqrt[c]*(a + b*x)*(8*b^2*c^2*x^2 + 2*a*b*c*x*(-2*c + 5*d*x) + a^2*(
3*c^2 - 5*c*d*x + 15*d^2*x^2)))/a^3) + (15*d^3*x^(5/2)*Sqrt[a + b*x]*ArcTa
n[(-(d*Sqrt[x]*Sqrt[a + b*x]) + Sqrt[b]*(c + d*x))/(Sqrt[c]*Sqrt[-(b*c) +
a*d])])/Sqrt[-(b*c) + a*d]))/(15*c^(7/2)*x^2*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1261, 115, 27, 169, 27, 169, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{ax+bx^2}(c+dx)}\,dx$$

$$\Big\downarrow \text{ 1261}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\int \frac{1}{x^{7/2}\sqrt{a+bx}(c+dx)}dx}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{ 115}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{2\int \frac{4bc+5ad+4bdx}{2x^{5/2}\sqrt{a+bx}(c+dx)}dx}{5ac}-\frac{2\sqrt{a+bx}}{5acx^{5/2}}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{\int \frac{4bc+5ad+4bdx}{x^{5/2}\sqrt{a+bx}(c+dx)}dx}{5ac}-\frac{2\sqrt{a+bx}}{5acx^{5/2}}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 169$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{-\frac{2\int \frac{8b^2c^2+10abdc+15a^2d^2+2bd(4bc+5ad)x}{2x^{3/2}\sqrt{a+bx}(c+dx)}dx}{3ac}-\frac{2\sqrt{a+bx}(5ad+4bc)}{3acx^{3/2}}}{5ac}-\frac{2\sqrt{a+bx}}{5acx^{5/2}}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{-\frac{\int \frac{8b^2c^2+10abdc+15a^2d^2+2bd(4bc+5ad)x}{x^{3/2}\sqrt{a+bx}(c+dx)}dx}{3ac}-\frac{2\sqrt{a+bx}(5ad+4bc)}{3acx^{3/2}}}{5ac}-\frac{2\sqrt{a+bx}}{5acx^{5/2}}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 169$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{-\frac{-\frac{2\int \frac{15a^3d^3}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{ac}-\frac{2\sqrt{a+bx}\left(\frac{8b^2c}{a}+\frac{15ad^2}{c}+10bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(5ad+4bc)}{3acx^{3/2}}}{5ac}-\frac{2\sqrt{a+bx}}{5acx^{5/2}}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{-\frac{-\frac{15a^2d^3\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{c}-\frac{2\sqrt{a+bx}\left(\frac{8b^2c}{a}+\frac{15ad^2}{c}+10bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(5ad+4bc)}{3acx^{3/2}}}{5ac}-\frac{2\sqrt{a+bx}}{5acx^{5/2}}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 104$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{-\frac{-\frac{30a^2d^3\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}-\frac{2\sqrt{a+bx}\left(\frac{8b^2c}{a}+\frac{15ad^2}{c}+10bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(5ad+4bc)}{3acx^{3/2}}}{5ac}-\frac{2\sqrt{a+bx}}{5acx^{5/2}}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 221$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{-\frac{-\frac{30a^2d^3\operatorname{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{a+bx}\left(\frac{8b^2c}{a}+\frac{15ad^2}{c}+10bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(5ad+4bc)}{3acx^{3/2}}}{5ac}-\frac{2\sqrt{a+bx}}{5acx^{5/2}}\right)}{\sqrt{ax+bx^2}}$$

input
```
Int[1/(x^3*(c + d*x)*Sqrt[a*x + b*x^2]),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*((-2*Sqrt[a + b*x])/(5*a*c*x^(5/2)) - ((-2*(4*b*c +
 5*a*d)*Sqrt[a + b*x])/(3*a*c*x^(3/2)) - ((-2*((8*b^2*c)/a + 10*b*d + (15*
a*d^2)/c)*Sqrt[a + b*x])/Sqrt[x] - (30*a^2*d^3*ArcTanh[(Sqrt[b*c - a*d]*Sq
rt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(c^(3/2)*Sqrt[b*c - a*d]))/(3*a*c))/(5*a*
c)))/Sqrt[a*x + b*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 115
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ[2*m, 2
*n, 2*p]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$-\frac{2\left(\sqrt{c(ad-bc)}\left(\left(\frac{8}{3}b^2x^2-\frac{4}{3}abx+a^2\right)c^2-\frac{5d(-2bx+a)acx}{3}+5a^2d^2x^2\right)\sqrt{x(bx+a)}-5\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)a^3d^3x^3\right)}{5\sqrt{c(ad-bc)}\,c^3a^3x^3}$
risch	$-\frac{2(bx+a)\left(15a^2d^2x^2+10abcd\,x^2+8b^2c^2x^2-5a^2cdx-4ab\,c^2x+3a^2c^2\right)}{15a^3c^3\sqrt{x(bx+a)}\,x^2}+\frac{d^2\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-}{d^2}}}{}\right)}{c^3\sqrt{-\frac{c(}{}}}$
default	$\frac{-\frac{2\sqrt{b\,x^2+ax}}{5a\,x^3}-\frac{4b\left(-\frac{2\sqrt{b\,x^2+ax}}{3a\,x^2}+\frac{4b\sqrt{b\,x^2+ax}}{3a^2x}\right)}{5a}}{c}-\frac{2d^2\sqrt{b\,x^2+ax}}{c^3ax}-\frac{d\left(-\frac{2\sqrt{b\,x^2+ax}}{3a\,x^2}+\frac{4b\sqrt{b\,x^2+ax}}{3a^2x}\right)}{c^2}+\frac{d^2\ln\left(\frac{-\frac{2c(ad-b}{d^2}}{}\right)}{}$

input
```
int(1/x^3/(d*x+c)/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*((c*(a*d-b*c))^(1/2)*((8/3*b^2*x^2-4/3*a*b*x+a^2)*c^2-5/3*d*(-2*b*x+a
)*a*c*x+5*a^2*d^2*x^2)*(x*(b*x+a))^(1/2)-5*arctan((x*(b*x+a))^(1/2)/x*c/(c
*(a*d-b*c))^(1/2))*a^3*d^3*x^3)/(c*(a*d-b*c))^(1/2)/c^3/a^3/x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.44

$$\int \frac{1}{x^3(c+dx)\sqrt{ax+bx^2}}\,dx$$
$$= \left[\frac{15\sqrt{bc^2-acd}a^3d^3x^3\log\left(\frac{ac+(2\,bc-ad)x-2\,\sqrt{bc^2-acd}\sqrt{bx^2+ax}}{dx+c}\right)-2\,(3\,a^2bc^4-3\,a^3c^3d+(8\,b^3c^4+2\,ab^2c^3d+5}{15\,(a^3bc^5-a^4c^4d)x^3}\right.$$

input
```
integrate(1/x^3/(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/15*(15*sqrt(b*c^2 - a*c*d)*a^3*d^3*x^3*log((a*c + (2*b*c - a*d)*x - 2*s
qrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) - 2*(3*a^2*b*c^4 - 3*a^3*
c^3*d + (8*b^3*c^4 + 2*a*b^2*c^3*d + 5*a^2*b*c^2*d^2 - 15*a^3*c*d^3)*x^2 -
 (4*a*b^2*c^4 + a^2*b*c^3*d - 5*a^3*c^2*d^2)*x)*sqrt(b*x^2 + a*x))/((a^3*b
*c^5 - a^4*c^4*d)*x^3), 2/15*(15*sqrt(-b*c^2 + a*c*d)*a^3*d^3*x^3*arctan(s
qrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) - (3*a^2*b*c^4 - 3*a^
3*c^3*d + (8*b^3*c^4 + 2*a*b^2*c^3*d + 5*a^2*b*c^2*d^2 - 15*a^3*c*d^3)*x^2
 - (4*a*b^2*c^4 + a^2*b*c^3*d - 5*a^3*c^2*d^2)*x)*sqrt(b*x^2 + a*x))/((a^3
*b*c^5 - a^4*c^4*d)*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^3(c+dx)\sqrt{ax+bx^2}}\,dx = \int \frac{1}{x^3\sqrt{x\,(a+bx)}\,(c+dx)}\,dx$$

input
```
integrate(1/x**3/(d*x+c)/(b*x**2+a*x)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(x*(a + b*x))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^3(c+dx)\sqrt{ax+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+ax}(dx+c)x^3}\,dx$$

input
```
integrate(1/x^3/(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a*x)*(d*x + c)*x^3), x)
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^3(c+dx)\sqrt{ax+bx^2}}\,dx = -\frac{2\,d^3\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)d+\sqrt{b}c}{\sqrt{-bc^2+acd}}\right)}{\sqrt{-bc^2+acd}c^3} + \frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4d^2-15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3\sqrt{b}cd+20\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2bc^2-5\left(\sqrt{\right.}\right.}{15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5c^3}$$

input
```
integrate(1/x^3/(d*x+c)/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
-2*d^3*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a*x))*d + sqrt(b)*c)/sqrt(-b*c^2
 + a*c*d))/(sqrt(-b*c^2 + a*c*d)*c^3) + 2/15*(15*(sqrt(b)*x - sqrt(b*x^2 +
 a*x))^4*d^2 - 15*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*sqrt(b)*c*d + 20*(sqrt
(b)*x - sqrt(b*x^2 + a*x))^2*b*c^2 - 5*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a
*c*d + 15*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a*sqrt(b)*c^2 + 3*a^2*c^2)/((sqr
t(b)*x - sqrt(b*x^2 + a*x))^5*c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(c+dx)\sqrt{ax+bx^2}}\,dx=\int \frac{1}{x^3\,\sqrt{b\,x^2+a\,x}\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/(x^3*(a*x + b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int(1/(x^3*(a*x + b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.24

$$\int \frac{1}{x^3(c+dx)\sqrt{ax+bx^2}}\,dx = \frac{2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)a^3d^3x^3+2\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}+\sqrt{d}\,\sqrt{bx+a}+\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)c}{}$$

input
```
int(1/x^3/(d*x+c)/(b*x^2+a*x)^(1/2),x)
```

output
```
(2*(15*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*
x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*d**3*x**3 + 15*sqrt(
c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)
*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*d**3*x**3 - 3*sqrt(x)*sqrt(a + b
*x)*a**3*c**3*d + 5*sqrt(x)*sqrt(a + b*x)*a**3*c**2*d**2*x - 15*sqrt(x)*sq
rt(a + b*x)*a**3*c*d**3*x**2 + 3*sqrt(x)*sqrt(a + b*x)*a**2*b*c**4 - sqrt(
x)*sqrt(a + b*x)*a**2*b*c**3*d*x + 5*sqrt(x)*sqrt(a + b*x)*a**2*b*c**2*d**
2*x**2 - 4*sqrt(x)*sqrt(a + b*x)*a*b**2*c**4*x + 2*sqrt(x)*sqrt(a + b*x)*a
*b**2*c**3*d*x**2 + 8*sqrt(x)*sqrt(a + b*x)*b**3*c**4*x**2 + 9*sqrt(b)*a**
3*c*d**3*x**3 + sqrt(b)*a**2*b*c**2*d**2*x**3 - 2*sqrt(b)*a*b**2*c**3*d*x*
*3 - 8*sqrt(b)*b**3*c**4*x**3))/(15*a**3*c**4*x**3*(a*d - b*c))
```

3.150 $\int \frac{x^3}{(c+dx)^2\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 179

$$\int \frac{x^3}{(c+dx)^2\sqrt{ax+bx^2}}\,dx = \frac{(2bc-ad)\sqrt{ax+bx^2}}{bd^2(bc-ad)} - \frac{cx\sqrt{ax+bx^2}}{d(bc-ad)(c+dx)} - \frac{(4bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{3/2}d^3} + \frac{c^{3/2}(4bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{d^3(bc-ad)^{3/2}}$$

output
```
(-a*d+2*b*c)*(b*x^2+a*x)^(1/2)/b/d^2/(-a*d+b*c)-c*x*(b*x^2+a*x)^(1/2)/d/(-
a*d+b*c)/(d*x+c)-(a*d+4*b*c)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(3/2)/
d^3+c^(3/2)*(-5*a*d+4*b*c)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^
(1/2))/d^3/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 10.64 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{(c+dx)^2\sqrt{ax+bx^2}}\,dx$$
$$= \frac{\sqrt{x}\left(-\frac{\sqrt{b}d\sqrt{x}(a+bx)(ad(c+dx)-bc(2c+dx))}{(bc-ad)(c+dx)}-\sqrt{a}(4bc+ad)\sqrt{1+\frac{bx}{a}}\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)+\frac{b^{3/2}c^{3/2}(4bc-5ad)\sqrt{a+bx}\text{arcta}}{(bc-ad)^{3/2}}\right.}{b^{3/2}d^3\sqrt{x(a+bx)}}$$

input
```
Integrate[x^3/((c + d*x)^2*Sqrt[a*x + b*x^2]),x]
```

output
```
(Sqrt[x]*(-((Sqrt[b]*d*Sqrt[x]*(a + b*x)*(a*d*(c + d*x) - b*c*(2*c + d*x))
)/((b*c - a*d)*(c + d*x))) - Sqrt[a]*(4*b*c + a*d)*Sqrt[1 + (b*x)/a]*ArcSi
nh[(Sqrt[b]*Sqrt[x])/Sqrt[a]] + (b^(3/2)*c^(3/2)*(4*b*c - 5*a*d)*Sqrt[a +
b*x]*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(b*c - a*
d)^(3/2)))/(b^(3/2)*d^3*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.32, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1261, 109, 27, 171, 25, 175, 65, 104, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{ax+bx^2}(c+dx)^2}\,dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\int \frac{x^{5/2}}{\sqrt{a+bx}(c+dx)^2}dx}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{109}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\int \frac{\sqrt{x}(3ac+2(2bc-ad)x)}{2\sqrt{a+bx}(c+dx)}dx}{d(bc-ad)}-\frac{cx^{3/2}\sqrt{a+bx}}{d(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\int \frac{\sqrt{x}(3ac+2(2bc-ad)x)}{\sqrt{a+bx}(c+dx)}dx}{2d(bc-ad)}-\frac{cx^{3/2}\sqrt{a+bx}}{d(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \; 171$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\frac{\int -\frac{ac(2bc-ad)+(bc-ad)(4bc+ad)x}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{bd}+\frac{2\sqrt{x}\sqrt{a+bx}(2bc-ad)}{bd}}{2d(bc-ad)}-\frac{cx^{3/2}\sqrt{a+bx}}{d(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\frac{2\sqrt{x}\sqrt{a+bx}(2bc-ad)}{bd}-\frac{\int \frac{ac(2bc-ad)+(bc-ad)(4bc+ad)x}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{bd}}{2d(bc-ad)}-\frac{cx^{3/2}\sqrt{a+bx}}{d(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \; 175$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\frac{2\sqrt{x}\sqrt{a+bx}(2bc-ad)}{bd}-\frac{\frac{(bc-ad)(ad+4bc)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{d}-\frac{bc^2(4bc-5ad)\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}}{bd}}{2d(bc-ad)}-\frac{cx^{3/2}\sqrt{a+bx}}{d(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \; 65$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\frac{2\sqrt{x}\sqrt{a+bx}(2bc-ad)}{bd}-\frac{\frac{2(bc-ad)(ad+4bc)\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}-\frac{bc^2(4bc-5ad)\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}}{bd}}{2d(bc-ad)}-\frac{cx^{3/2}\sqrt{a+bx}}{d(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \; 104$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\frac{2\sqrt{x}\sqrt{a+bx}(2bc-ad)}{bd}-\frac{\frac{2(bc-ad)(ad+4bc)\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}-\frac{2bc^2(4bc-5ad)\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}}{bd}}{2d(bc-ad)}-\frac{cx^{3/2}\sqrt{a+bx}}{d(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$\downarrow$ 219

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\frac{2\sqrt{x}\sqrt{a+bx}(2bc-ad)}{bd}-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)(bc-ad)(ad+4bc)}{\sqrt{b}d}-\frac{2bc^2(4bc-5ad)\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}}{bd}}{2d(bc-ad)}-\frac{cx^{3/2}\sqrt{a+bx}}{d(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$\downarrow$ 221

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\frac{2\sqrt{x}\sqrt{a+bx}(2bc-ad)}{bd}-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)(bc-ad)(ad+4bc)}{\sqrt{b}d}-\frac{2bc^{3/2}(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{d\sqrt{bc-ad}}}{bd}}{2d(bc-ad)}-\frac{cx^{3/2}\sqrt{a+bx}}{d(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

input
```
Int[x^3/((c + d*x)^2*Sqrt[a*x + b*x^2]),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(-((c*x^(3/2)*Sqrt[a + b*x])/(d*(b*c - a*d)*(c + d*
x))) + ((2*(2*b*c - a*d)*Sqrt[x]*Sqrt[a + b*x])/(b*d) - ((2*(b*c - a*d)*(4
*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(Sqrt[b]*d) - (2*b*c
^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a +
 b*x])])/(d*Sqrt[b*c - a*d]))/(b*d))/(2*d*(b*c - a*d))))/Sqrt[a*x + b*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{4(dx+c)b^{\frac{5}{2}}\left(bc-\frac{5ad}{4}\right)c^2\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)-2\left(\frac{b(ad+4bc)(ad-bc)(dx+c)\,\mathrm{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{2}+d\sqrt{x(bx+a)}\,b^{\frac{3}{2}}\left(b\,c^2-\frac{d(}{}\right.\right.}{b^{\frac{5}{2}}d^3(ad-bc)(dx+c)\sqrt{c(ad-bc)}}$
risch	$\frac{x(bx+a)}{d^2b\sqrt{x(bx+a)}}-\frac{\frac{(ad+4bc)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{d\sqrt{b}}+\frac{6c^2b\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-}{}}}{x+\frac{c}{d}}\right.}{d^2\sqrt{-\frac{c(ad-bc)}{d^2}}}}{}$
default	$\frac{\frac{\sqrt{b\,x^2+ax}}{b}-\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2b^{\frac{3}{2}}}}{d^2}-\frac{2c\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{d^3\sqrt{b}}-\frac{3c^2\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc}{d^2}}}{x+}\right.}{d^4\sqrt{-\frac{c(ad-}{d^2}}}$

input
```
int(x^3/(d*x+c)^2/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4*((d*x+c)*b^(5/2)*(b*c-5/4*a*d)*c^2*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-
b*c))^(1/2))-1/2*(1/2*b*(a*d+4*b*c)*(a*d-b*c)*(d*x+c)*arctanh((x*(b*x+a))^
(1/2)/x/b^(1/2))+d*(x*(b*x+a))^(1/2)*b^(3/2)*(b*c^2-1/2*d*(-b*x+a)*c-1/2*a
*d^2*x))*(c*(a*d-b*c))^(1/2))/(c*(a*d-b*c))^(1/2)/b^(5/2)/d^3/(a*d-b*c)/(d
*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1151, normalized size of antiderivative = 6.43

$$\int \frac{x^3}{(c+dx)^2\sqrt{ax+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*((4*b^2*c^3 - 3*a*b*c^2*d - a^2*c*d^2 + (4*b^2*c^2*d - 3*a*b*c*d^2 -
a^2*d^3)*x)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) + (4*b^3*
c^3 - 5*a*b^2*c^2*d + (4*b^3*c^2*d - 5*a*b^2*c*d^2)*x)*sqrt(c/(b*c - a*d))
*log((a*c + (2*b*c - a*d)*x + 2*sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(c/(b*c
- a*d)))/(d*x + c)) + 2*(2*b^2*c^2*d - a*b*c*d^2 + (b^2*c*d^2 - a*b*d^3)*x
)*sqrt(b*x^2 + a*x))/(b^3*c^2*d^3 - a*b^2*c*d^4 + (b^3*c*d^4 - a*b^2*d^5)*
x), 1/2*(2*(4*b^3*c^3 - 5*a*b^2*c^2*d + (4*b^3*c^2*d - 5*a*b^2*c*d^2)*x)*s
qrt(-c/(b*c - a*d))*arctan(-sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(-c/(b*c - a
*d))/(b*c*x + a*c)) + (4*b^2*c^3 - 3*a*b*c^2*d - a^2*c*d^2 + (4*b^2*c^2*d
- 3*a*b*c*d^2 - a^2*d^3)*x)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sq
rt(b)) + 2*(2*b^2*c^2*d - a*b*c*d^2 + (b^2*c*d^2 - a*b*d^3)*x)*sqrt(b*x^2
+ a*x))/(b^3*c^2*d^3 - a*b^2*c*d^4 + (b^3*c*d^4 - a*b^2*d^5)*x), 1/2*(2*(4
*b^2*c^3 - 3*a*b*c^2*d - a^2*c*d^2 + (4*b^2*c^2*d - 3*a*b*c*d^2 - a^2*d^3)
*x)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (4*b^3*c^3 - 5
*a*b^2*c^2*d + (4*b^3*c^2*d - 5*a*b^2*c*d^2)*x)*sqrt(c/(b*c - a*d))*log((a
*c + (2*b*c - a*d)*x + 2*sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(c/(b*c - a*d))
)/(d*x + c)) + 2*(2*b^2*c^2*d - a*b*c*d^2 + (b^2*c*d^2 - a*b*d^3)*x)*sqrt(
b*x^2 + a*x))/(b^3*c^2*d^3 - a*b^2*c*d^4 + (b^3*c*d^4 - a*b^2*d^5)*x), ((4
*b^3*c^3 - 5*a*b^2*c^2*d + (4*b^3*c^2*d - 5*a*b^2*c*d^2)*x)*sqrt(-c/(b*c -
 a*d))*arctan(-sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(-c/(b*c - a*d))/(b*c*...
```

Sympy [F]

$$\int \frac{x^3}{(c+dx)^2\sqrt{ax+bx^2}}\,dx = \int \frac{x^3}{\sqrt{x\,(a+bx)}\,(c+dx)^2}\,dx$$

input
```
integrate(x**3/(d*x+c)**2/(b*x**2+a*x)**(1/2),x)
```

output
```
Integral(x**3/(sqrt(x*(a + b*x))*(c + d*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+dx)^2\sqrt{ax+bx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((a/d-(2*b*c)/d^2)^2>0)', see `as
sume?` for
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+dx)^2\sqrt{ax+bx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+dx)^2\sqrt{ax+bx^2}}\,dx = \int \frac{x^3}{\sqrt{b\,x^2+a\,x}\,(c+d\,x)^2}\,dx$$

input
```
int(x^3/((a*x + b*x^2)^(1/2)*(c + d*x)^2),x)
```

output
```
int(x^3/((a*x + b*x^2)^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 873, normalized size of antiderivative = 4.88

$$\int \frac{x^3}{(c+dx)^2\sqrt{ax+bx^2}}\,dx = \text{Too large to display}$$

input
```
int(x^3/(d*x+c)^2/(b*x^2+a*x)^(1/2),x)
```

output
```
( - 5*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x
) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c**2*d - 5*sqrt(c)*
sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sq
rt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c*d**2*x + 4*sqrt(c)*sqrt(a*d - b
*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b
))/(sqrt(c)*sqrt(b)))*b**3*c**3 + 4*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d
 - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b
)))*b**3*c**2*d*x - 5*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt
(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c**
2*d - 5*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b
*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b**2*c*d**2*x + 4*sqrt
(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x
)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**3*c**3 + 4*sqrt(c)*sqrt(a*d - b*c
)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))
/(sqrt(c)*sqrt(b)))*b**3*c**2*d*x + sqrt(x)*sqrt(a + b*x)*a**2*b*c*d**3 +
sqrt(x)*sqrt(a + b*x)*a**2*b*d**4*x - 3*sqrt(x)*sqrt(a + b*x)*a*b**2*c**2*
d**2 - 2*sqrt(x)*sqrt(a + b*x)*a*b**2*c*d**3*x + 2*sqrt(x)*sqrt(a + b*x)*b
**3*c**3*d + sqrt(x)*sqrt(a + b*x)*b**3*c**2*d**2*x - sqrt(b)*log((sqrt(a
+ b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*c*d**3 - sqrt(b)*log((sqrt(a + b*x
) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*d**4*x - 2*sqrt(b)*log((sqrt(a + b*x...
```

3.151 $\int \frac{x^2}{(c+dx)^2\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \frac{x^2}{(c+dx)^2\sqrt{ax+bx^2}}\,dx = -\frac{c\sqrt{ax+bx^2}}{d(bc-ad)(c+dx)}+\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}d^2} - \frac{\sqrt{c}(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{d^2(bc-ad)^{3/2}}$$

output
```
-c*(b*x^2+a*x)^(1/2)/d/(-a*d+b*c)/(d*x+c)+2*arctanh(b^(1/2)*x/(b*x^2+a*x)^
(1/2))/b^(1/2)/d^2-c^(1/2)*(-3*a*d+2*b*c)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/
2)/(b*x^2+a*x)^(1/2))/d^2/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.33

$$\int \frac{x^2}{(c+dx)^2\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{x}\left(\frac{cd\sqrt{x}(a+bx)}{(-bc+ad)(c+dx)}-\frac{\sqrt{c}(2bc-3ad)\sqrt{a+bx}\arctan\left(\frac{-d\sqrt{x}\sqrt{a+bx}+\sqrt{b}(c+dx)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}-\frac{2\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}}\right)}{d^2\sqrt{x(a+bx)}}$$

input
```
Integrate[x^2/((c + d*x)^2*Sqrt[a*x + b*x^2]),x]
```

output
```
(Sqrt[x]*((c*d*Sqrt[x]*(a + b*x))/((-(b*c) + a*d)*(c + d*x)) - (Sqrt[c]*(2
*b*c - 3*a*d)*Sqrt[a + b*x]*ArcTan[(-(d*Sqrt[x]*Sqrt[a + b*x]) + Sqrt[b]*(
c + d*x))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(-(b*c) + a*d)^(3/2) - (2*Sqrt[a
+ b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/Sqrt[b]))/(d^2*Sqrt[x*(a +
 b*x)])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.42, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1261, 109, 27, 175, 65, 104, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{ax+bx^2}(c+dx)^2}\,dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\int \frac{x^{3/2}}{\sqrt{a+bx}(c+dx)^2}dx}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{109}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\int \frac{ac+2(bc-ad)x}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d(bc-ad)}-\frac{c\sqrt{x}\sqrt{a+bx}}{d(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\int \frac{ac+2(bc-ad)x}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{2d(bc-ad)}-\frac{c\sqrt{x}\sqrt{a+bx}}{d(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{175}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\frac{2(bc-ad)\int\frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{d}-\frac{c(2bc-3ad)\int\frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}}{2d(bc-ad)}-\frac{c\sqrt{x}\sqrt{a+bx}}{d(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{ 65}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\frac{4(bc-ad)\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}-\frac{c(2bc-3ad)\int\frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{d}}{2d(bc-ad)}-\frac{c\sqrt{x}\sqrt{a+bx}}{d(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{ 104}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\frac{4(bc-ad)\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}-\frac{2c(2bc-3ad)\int\frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}}{2d(bc-ad)}-\frac{c\sqrt{x}\sqrt{a+bx}}{d(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\frac{4\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)(bc-ad)}{\sqrt{b}d}-\frac{2c(2bc-3ad)\int\frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{d}}{2d(bc-ad)}-\frac{c\sqrt{x}\sqrt{a+bx}}{d(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\frac{4\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)(bc-ad)}{\sqrt{b}d}-\frac{2\sqrt{c}(2bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{d\sqrt{bc-ad}}}{2d(bc-ad)}-\frac{c\sqrt{x}\sqrt{a+bx}}{d(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

input

```
Int[x^2/((c + d*x)^2*Sqrt[a*x + b*x^2]),x]
```

output

```
(Sqrt[x]*Sqrt[a + b*x]*(-((c*Sqrt[x]*Sqrt[a + b*x])/(d*(b*c - a*d)*(c + d*
x))) + ((4*(b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(Sqrt[b]*
d) - (2*Sqrt[c]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]
*Sqrt[a + b*x])])/(d*Sqrt[b*c - a*d]))/(2*d*(b*c - a*d))))/Sqrt[a*x + b*x^
2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$-\frac{2\left((dx+c)\sqrt{b}\left(bc-\frac{3ad}{2}\right)c\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)-\frac{\left(2(dx+c)(ad-bc)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)+\sqrt{b}\,\sqrt{x(bx+a)}\,cd\right)\sqrt{c(ad-bc)}}{2}\right)}{\sqrt{b}\,\sqrt{c(ad-bc)}\,d^2(ad-bc)(dx+c)}$
default	$\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{d^2\sqrt{b}}+\frac{c^2\left(\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{c(ad-bc)\left(x+\frac{c}{d}\right)}-\frac{(ad-2bc)d\ln\left(-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\ldots\right.}{2c(ad-b\ldots}\right.}{d^4}$

input
```
int(x^2/(d*x+c)^2/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/b^(1/2)/(c*(a*d-b*c))^(1/2)*((d*x+c)*b^(1/2)*(b*c-3/2*a*d)*c*arctan((x*
(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))-1/2*(2*(d*x+c)*(a*d-b*c)*arctanh((
x*(b*x+a))^(1/2)/x/b^(1/2))+b^(1/2)*(x*(b*x+a))^(1/2)*c*d)*(c*(a*d-b*c))^(
1/2))/d^2/(a*d-b*c)/(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 851, normalized size of antiderivative = 6.45

$$\int \frac{x^2}{(c+dx)^2\sqrt{ax+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*(2*sqrt(b*x^2 + a*x)*b*c*d - 2*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)*s
qrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - (2*b^2*c^2 - 3*a*b*c
*d + (2*b^2*c*d - 3*a*b*d^2)*x)*sqrt(c/(b*c - a*d))*log((a*c + (2*b*c - a*
d)*x - 2*sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(c/(b*c - a*d)))/(d*x + c)))/(b
^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 - a*b*d^4)*x), -(sqrt(b*x^2 + a*x)*b*c
*d + (2*b^2*c^2 - 3*a*b*c*d + (2*b^2*c*d - 3*a*b*d^2)*x)*sqrt(-c/(b*c - a*
d))*arctan(-sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(-c/(b*c - a*d))/(b*c*x + a*
c)) - (b*c^2 - a*c*d + (b*c*d - a*d^2)*x)*sqrt(b)*log(2*b*x + a + 2*sqrt(b
*x^2 + a*x)*sqrt(b)))/(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 - a*b*d^4)*x),
 -1/2*(2*sqrt(b*x^2 + a*x)*b*c*d + 4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)*s
qrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (2*b^2*c^2 - 3*a*b*
c*d + (2*b^2*c*d - 3*a*b*d^2)*x)*sqrt(c/(b*c - a*d))*log((a*c + (2*b*c - a
*d)*x - 2*sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(c/(b*c - a*d)))/(d*x + c)))/(
b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 - a*b*d^4)*x), -(sqrt(b*x^2 + a*x)*b*
c*d + (2*b^2*c^2 - 3*a*b*c*d + (2*b^2*c*d - 3*a*b*d^2)*x)*sqrt(-c/(b*c - a
*d))*arctan(-sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(-c/(b*c - a*d))/(b*c*x + a
*c)) + 2*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)*sqrt(-b)*arctan(sqrt(b*x^2 +
a*x)*sqrt(-b)/(b*x + a)))/(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 - a*b*d^4)
*x)]
```

Sympy [F]

$$\int \frac{x^2}{(c+dx)^2\sqrt{ax+bx^2}}\,dx = \int \frac{x^2}{\sqrt{x\,(a+bx)}\,(c+dx)^2}\,dx$$

input
```
integrate(x**2/(d*x+c)**2/(b*x**2+a*x)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(x*(a + b*x))*(c + d*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+dx)^2\sqrt{ax+bx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((a/d-(2*b*c)/d^2)^2>0)', see `as
sume?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+dx)^2\sqrt{ax+bx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+dx)^2\sqrt{ax+bx^2}}\,dx = \int \frac{x^2}{\sqrt{b\,x^2+a\,x}\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(x^2/((a*x + b*x^2)^(1/2)*(c + d*x)^2),x)
```

output
```
int(x^2/((a*x + b*x^2)^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 709, normalized size of antiderivative = 5.37

$$\int \frac{x^2}{(c+dx)^2\sqrt{ax+bx^2}}\,dx = \text{Too large to display}$$

input
```
int(x^2/(d*x+c)^2/(b*x^2+a*x)^(1/2),x)
```

output

```
(3*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) -
 sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b*c*d + 3*sqrt(c)*sqrt(a*d
- b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqr
t(b))/(sqrt(c)*sqrt(b)))*a*b*d**2*x - 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt
(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sq
rt(b)))*b**2*c**2 - 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt
(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**2*c*d*x
 + 3*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x)
 + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b*c*d + 3*sqrt(c)*sqrt(a*
d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*s
qrt(b))/(sqrt(c)*sqrt(b)))*a*b*d**2*x - 2*sqrt(c)*sqrt(a*d - b*c)*atan((sq
rt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*
sqrt(b)))*b**2*c**2 - 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sq
rt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b**2*c*d
*x + sqrt(x)*sqrt(a + b*x)*a*b*c*d**2 - sqrt(x)*sqrt(a + b*x)*b**2*c**2*d
+ 2*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*c*d**2 + 2
*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*d**3*x - 4*sq
rt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b*c**2*d - 4*sqrt(b
)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b*c*d**2*x + 2*sqrt(b)*
log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b**2*c**3 + 2*sqrt(b)*lo...
```

3.152 $\int \frac{x}{(c+dx)^2\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{x}{(c+dx)^2\sqrt{ax+bx^2}}\,dx=\frac{\sqrt{ax+bx^2}}{(bc-ad)(c+dx)}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{\sqrt{c}(bc-ad)^{3/2}}$$

output
```
(b*x^2+a*x)^(1/2)/(-a*d+b*c)/(d*x+c)-a*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/
(b*x^2+a*x)^(1/2))/c^(1/2)/(-a*d+b*c)^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.57 (sec) , antiderivative size = 590, normalized size of antiderivative = 6.94

$$\int \frac{x}{(c+dx)^2\sqrt{ax+bx^2}}\,dx$$

$$=\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{a\sqrt{x}}{(bc-ad)(c+dx)(-\sqrt{a}+\sqrt{a+bx})}+\frac{bx^{3/2}}{(bc-ad)(c+dx)(-\sqrt{a}+\sqrt{a+bx})}+\frac{\sqrt{a}\sqrt{x}\sqrt{a+bx}}{(-bc+ad)(c+dx)(-\sqrt{a}+\sqrt{a+bx})}+\frac{a\left(i\sqrt{a}\sqrt{d}+\cdots\right.}{\sqrt{c}\cdots}\right.}{\sqrt{\cdots}}$$

input
```
Integrate[x/((c + d*x)^2*Sqrt[a*x + b*x^2]),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*((a*Sqrt[x])/((b*c - a*d)*(c + d*x)*(-Sqrt[a] + Sqr
t[a + b*x])) + (b*x^(3/2))/((b*c - a*d)*(c + d*x)*(-Sqrt[a] + Sqrt[a + b*x
])) + (Sqrt[a]*Sqrt[x]*Sqrt[a + b*x])/((-(b*c) + a*d)*(c + d*x)*(-Sqrt[a]
+ Sqrt[a + b*x])) + (a*(I*Sqrt[a]*Sqrt[d] + Sqrt[b*c - a*d])*ArcTan[(Sqrt[
-(b*c) + 2*a*d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*
(Sqrt[a] - Sqrt[a + b*x]))])/(Sqrt[c]*(b*c - a*d)^(3/2)*Sqrt[-(b*c) + 2*a*
d - (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]) + (a*ArcTan[(Sqrt[-(b*c) + 2*a
*d + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*(Sqrt[a] - S
qrt[a + b*x]))])/(Sqrt[c]*(b*c - a*d)*Sqrt[-(b*c) + 2*a*d + (2*I)*Sqrt[a]*
Sqrt[d]*Sqrt[b*c - a*d]]) + (I*a^(3/2)*Sqrt[d]*ArcTan[(Sqrt[-(b*c) + 2*a*d
 + (2*I)*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]]*Sqrt[x])/(Sqrt[c]*(-Sqrt[a] + Sq
rt[a + b*x]))])/(Sqrt[c]*(b*c - a*d)^(3/2)*Sqrt[-(b*c) + 2*a*d + (2*I)*Sqr
t[a]*Sqrt[d]*Sqrt[b*c - a*d]])))/Sqrt[x*(a + b*x)]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{ax+bx^2}(c+dx)^2}\,dx$$

$$\Big\downarrow \text{ 1228}$$

$$\frac{\sqrt{ax+bx^2}}{(c+dx)(bc-ad)}-\frac{a\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{2(bc-ad)}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{a\int \frac{1}{4c(bc-ad)-\frac{(ac+(2bc-ad)x)^2}{bx^2+ax}}d\left(-\frac{ac+(2bc-ad)x}{\sqrt{bx^2+ax}}\right)}{bc-ad}+\frac{\sqrt{ax+bx^2}}{(c+dx)(bc-ad)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{ax+bx^2}}{(c+dx)(bc-ad)}-\frac{a\text{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{2\sqrt{c}(bc-ad)^{3/2}}$$

input
```
Int[x/((c + d*x)^2*Sqrt[a*x + b*x^2]),x]
```

output
```
Sqrt[a*x + b*x^2]/((b*c - a*d)*(c + d*x)) - (a*ArcTanh[(a*c + (2*b*c - a*d
)*x)/(2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[a*x + b*x^2])])/(2*Sqrt[c]*(b*c - a*d
)^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{x(bx+a)}}{dx+c}-\frac{a\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{\sqrt{c(ad-bc)}}}{ad-bc}$
default	$-\frac{\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{x+\frac{c}{d}}\right)}{d^2\sqrt{-\frac{c(ad-bc)}{d^2}}}-\frac{c\left(\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2b}{}}}{c(ad-bc)(}\right.}{}$

input
```
int(x/(d*x+c)^2/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(a*d-b*c)*(-(x*(b*x+a))^(1/2)/(d*x+c)-a/(c*(a*d-b*c))^(1/2)*arctan((x*(b
*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.49

$$\int \frac{x}{(c+dx)^2\sqrt{ax+bx^2}}\,dx$$
$$=\left[-\frac{\sqrt{bc^2-acd}(adx+ac)\log\left(\frac{ac+(2\,bc-ad)x+2\,\sqrt{bc^2-acd}\sqrt{bx^2+ax}}{dx+c}\right)-2\,(bc^2-acd)\sqrt{bx^2+ax}}{2\,(b^2c^4-2\,abc^3d+a^2c^2d^2+(b^2c^3d-2\,abc^2d^2+a^2cd^3)x)},\frac{\sqrt{-bc^2+acd}(}{b^2c}\right.$$

input
```
integrate(x/(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(b*c^2 - a*c*d)*(a*d*x + a*c)*log((a*c + (2*b*c - a*d)*x + 2*sq
rt(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) - 2*(b*c^2 - a*c*d)*sqrt(b
*x^2 + a*x))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2
*d^2 + a^2*c*d^3)*x), (sqrt(-b*c^2 + a*c*d)*(a*d*x + a*c)*arctan(sqrt(-b*c
^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) + (b*c^2 - a*c*d)*sqrt(b*x^2
+ a*x))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2
+ a^2*c*d^3)*x)]
```

Sympy [F]

$$\int \frac{x}{(c+dx)^2\sqrt{ax+bx^2}}\,dx = \int \frac{x}{\sqrt{x\,(a+bx)}\,(c+dx)^2}\,dx$$

input
```
integrate(x/(d*x+c)**2/(b*x**2+a*x)**(1/2),x)
```

output
```
Integral(x/(sqrt(x*(a + b*x))*(c + d*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+dx)^2\sqrt{ax+bx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(73) = 146$.

Time = 0.29 (sec) , antiderivative size = 379, normalized size of antiderivative = 4.46

$$\int \frac{x}{(c+dx)^2\sqrt{ax+bx^2}}\,dx = \frac{\frac{2\sqrt{b-\frac{2\,bc}{dx+c}+\frac{bc^2}{(dx+c)^2}+\frac{ad}{dx+c}-\frac{acd}{(dx+c)^2}}\,d^2\mathrm{sgn}\left(\frac{1}{dx+c}\right)\mathrm{sgn}(d)}{bc\mathrm{sgn}\left(\frac{1}{dx+c}\right)^2\mathrm{sgn}(d)^2-ad\mathrm{sgn}\left(\frac{1}{dx+c}\right)^2\mathrm{sgn}(d)^2}-\frac{\left(ad^4\log\left(\left|2\,bcd-ad^2-2\,\sqrt{bc^2-acd}\sqrt{b}|d|\right|\right)+2\,\sqrt{bc^2-acd}\sqrt{b}d^2|d|\right)\mathrm{sgn}\left(\frac{1}{dx+c}\right)\mathrm{s}}{\sqrt{bc^2-acd}bc|d|-\sqrt{bc^2-acd}ad|d|}}{2\,d^3}$$

input
```
integrate(x/(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/2*(2*sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d/(d*x + c) - a*c*
d/(d*x + c)^2)*d^2*sgn(1/(d*x + c))*sgn(d)/(b*c*sgn(1/(d*x + c))^2*sgn(d)^
2 - a*d*sgn(1/(d*x + c))^2*sgn(d)^2) - (a*d^4*log(abs(2*b*c*d - a*d^2 - 2*
sqrt(b*c^2 - a*c*d)*sqrt(b)*abs(d))) + 2*sqrt(b*c^2 - a*c*d)*sqrt(b)*d^2*a
bs(d))*sgn(1/(d*x + c))*sgn(d)/(sqrt(b*c^2 - a*c*d)*b*c*abs(d) - sqrt(b*c^
2 - a*c*d)*a*d*abs(d)) + a*d^4*log(abs(2*b*c*d - a*d^2 - 2*sqrt(b*c^2 - a*
c*d)*(sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d/(d*x + c) - a*c*d
/(d*x + c)^2) + sqrt(b*c^2*d^2 - a*c*d^3)/((d*x + c)*d))*abs(d)))/(sqrt(b*
c^2 - a*c*d)*(b*c - a*d)*abs(d)*sgn(1/(d*x + c))*sgn(d)))/d^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+dx)^2\sqrt{ax+bx^2}}\,dx = \int \frac{x}{\sqrt{b\,x^2+a\,x}\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(x/((a*x + b*x^2)^(1/2)*(c + d*x)^2),x)
```

output
```
int(x/((a*x + b*x^2)^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.45

$$\int \frac{x}{(c+dx)^2\sqrt{ax+bx^2}}\,dx$$
$$= \frac{-\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)ac-\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)adx-}{c\left(a^2d^3x-\right.}$$

input
```
int(x/(d*x+c)^2/(b*x^2+a*x)^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x)
- sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*c - sqrt(c)*sqrt(a*d - b*c
)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))
/(sqrt(c)*sqrt(b)))*a*d*x - sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c)
+ sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*c
- sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) +
sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*d*x - sqrt(x)*sqrt(a + b*x)*
a*c*d + sqrt(x)*sqrt(a + b*x)*b*c**2)/(c*(a**2*c*d**2 + a**2*d**3*x - 2*a*
b*c**2*d - 2*a*b*c*d**2*x + b**2*c**3 + b**2*c**2*d*x))
```

3.153 $\int \frac{1}{(c+dx)^2\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \frac{1}{(c+dx)^2\sqrt{ax+bx^2}}\,dx = -\frac{d\sqrt{ax+bx^2}}{c(bc-ad)(c+dx)}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{3/2}(bc-ad)^{3/2}}$$

output
```
-d*(b*x^2+a*x)^(1/2)/c/(-a*d+b*c)/(d*x+c)+(-a*d+2*b*c)*arctanh((-a*d+b*c)^
(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(3/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.41

$$\int \frac{1}{(c+dx)^2\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{x}\left(-\frac{\sqrt{c}d\sqrt{x}(a+bx)}{(bc-ad)(c+dx)}+\frac{(2bc-ad)\sqrt{a+bx}\arctan\left(\frac{-d\sqrt{x}\sqrt{a+bx}+\sqrt{b}(c+dx)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}\right)}{c^{3/2}\sqrt{x(a+bx)}}$$

input
```
Integrate[1/((c + d*x)^2*Sqrt[a*x + b*x^2]),x]
```

output
```
(Sqrt[x]*(-((Sqrt[c]*d*Sqrt[x]*(a + b*x))/((b*c - a*d)*(c + d*x))) + ((2*b
*c - a*d)*Sqrt[a + b*x]*ArcTan[(-(d*Sqrt[x]*Sqrt[a + b*x]) + Sqrt[b]*(c +
d*x))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(-(b*c) + a*d)^(3/2)))/(c^(3/2)*Sqrt[
x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1157, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax+bx^2}(c+dx)^2}dx$$

$$\Big\downarrow \text{ 1157}$$

$$\frac{(2bc-ad)\int \frac{1}{(c+dx)\sqrt{bx^2+ax}}dx}{2c(bc-ad)}-\frac{d\sqrt{ax+bx^2}}{c(c+dx)(bc-ad)}$$

$$\Big\downarrow \text{ 1154}$$

$$-\frac{(2bc-ad)\int \frac{1}{4c(bc-ad)-\frac{(ac+(2bc-ad)x)^2}{bx^2+ax}}d\left(-\frac{ac+(2bc-ad)x}{\sqrt{bx^2+ax}}\right)}{c(bc-ad)}-\frac{d\sqrt{ax+bx^2}}{c(c+dx)(bc-ad)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{(2bc-ad)\text{arctanh}\left(\frac{x(2bc-ad)+ac}{2\sqrt{c}\sqrt{ax+bx^2}\sqrt{bc-ad}}\right)}{2c^{3/2}(bc-ad)^{3/2}}-\frac{d\sqrt{ax+bx^2}}{c(c+dx)(bc-ad)}$$

input
```
Int[1/((c + d*x)^2*Sqrt[a*x + b*x^2]),x]
```

output
```
-((d*Sqrt[a*x + b*x^2])/(c*(b*c - a*d)*(c + d*x))) + ((2*b*c - a*d)*ArcTan
h[(a*c + (2*b*c - a*d)*x)/(2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[a*x + b*x^2])])/
(2*c^(3/2)*(b*c - a*d)^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1157
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2))
  Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[m + 2*p + 3, 0]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$\frac{\frac{d\sqrt{x(bx+a)}}{dx+c}-\frac{(ad-2bc)\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)}{\sqrt{c(ad-bc)}}}{c(ad-bc)}$
default	$\frac{\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{c(ad-bc)\left(x+\frac{c}{d}\right)}-\frac{(ad-2bc)d\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)}{d}\cdots}}{x+\frac{c}{d}}\cdots\right)}{2c(ad-bc)\sqrt{-\frac{c(ad-bc)}{d^2}}}\cdots}{d^2}$

input
```
int(1/(d*x+c)^2/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/c/(a*d-b*c)*(d*(x*(b*x+a))^(1/2)/(d*x+c)-(a*d-2*b*c)/(c*(a*d-b*c))^(1/2)
*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 344, normalized size of antiderivative = 3.55

$$\int \frac{1}{(c+dx)^2\sqrt{ax+bx^2}}\,dx$$
$$= \left[\frac{(2\,bc^2-acd+(2\,bcd-ad^2)x)\sqrt{bc^2-acd}\log\left(\frac{ac+(2\,bc-ad)x+2\,\sqrt{bc^2-acd}\sqrt{bx^2+ax}}{dx+c}\right)-2\,(bc^2d-acd^2)\sqrt{bx^2+}}{2\,(b^2c^5-2\,abc^4d+a^2c^3d^2+(b^2c^4d-2\,abc^3d^2+a^2c^2d^3)x)}\right.$$
$$\left.-\frac{(2\,bc^2-acd+(2\,bcd-ad^2)x)\sqrt{-bc^2+acd}\arctan\left(\frac{\sqrt{-bc^2+acd}\sqrt{bx^2+ax}}{bcx+ac}\right)+(bc^2d-acd^2)\sqrt{bx^2+ax}}{b^2c^5-2\,abc^4d+a^2c^3d^2+(b^2c^4d-2\,abc^3d^2+a^2c^2d^3)x}\right]$$

input
```
integrate(1/(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*((2*b*c^2 - a*c*d + (2*b*c*d - a*d^2)*x)*sqrt(b*c^2 - a*c*d)*log((a*c
 + (2*b*c - a*d)*x + 2*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) -
 2*(b*c^2*d - a*c*d^2)*sqrt(b*x^2 + a*x))/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3
*d^2 + (b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x), -((2*b*c^2 - a*c*d +
(2*b*c*d - a*d^2)*x)*sqrt(-b*c^2 + a*c*d)*arctan(sqrt(-b*c^2 + a*c*d)*sqrt
(b*x^2 + a*x)/(b*c*x + a*c)) + (b*c^2*d - a*c*d^2)*sqrt(b*x^2 + a*x))/(b^2
*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^
3)*x)]
```

Sympy [F]

$$\int \frac{1}{(c+dx)^2\sqrt{ax+bx^2}}\,dx = \int \frac{1}{\sqrt{x\,(a+bx)}\,(c+dx)^2}\,dx$$

input
```
integrate(1/(d*x+c)**2/(b*x**2+a*x)**(1/2),x)
```

output
```
Integral(1/(sqrt(x*(a + b*x))*(c + d*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+dx)^2\sqrt{ax+bx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(c+dx)^2\sqrt{ax+bx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2\sqrt{ax+bx^2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a\,x}\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(1/((a*x + b*x^2)^(1/2)*(c + d*x)^2),x)
```

output
```
int(1/((a*x + b*x^2)^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 518, normalized size of antiderivative = 5.34

$$\int \frac{1}{(c+dx)^2\sqrt{ax+bx^2}}\,dx$$
$$= \frac{-\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)acd-\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{bx+a}-\sqrt{x}\,\sqrt{d}\,\sqrt{b}}{\sqrt{c}\,\sqrt{b}}\right)a\,d^2}{}$$

input
```
int(1/(d*x+c)^2/(b*x^2+a*x)^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x)
- sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*c*d - sqrt(c)*sqrt(a*d - b
*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b
))/(sqrt(c)*sqrt(b)))*a*d**2*x + 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d
- b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)
))*b*c**2 + 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt
(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b*c*d*x - sqrt(c)*
sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sq
rt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*c*d - sqrt(c)*sqrt(a*d - b*c)*atan((sq
rt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*
sqrt(b)))*a*d**2*x + 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqr
t(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b*c**2 +
2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) +
sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*b*c*d*x + sqrt(x)*sqrt(a + b*x
)*a*c*d**2 - sqrt(x)*sqrt(a + b*x)*b*c**2*d)/(c**2*(a**2*c*d**2 + a**2*d**
3*x - 2*a*b*c**2*d - 2*a*b*c*d**2*x + b**2*c**3 + b**2*c**2*d*x))
```

3.154 $\int \frac{1}{x(c+dx)^2\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{1}{x(c+dx)^2\sqrt{ax+bx^2}}\,dx = -\frac{(2bc-3ad)\sqrt{ax+bx^2}}{ac^2(bc-ad)x} - \frac{d\sqrt{ax+bx^2}}{c(bc-ad)x(c+dx)} - \frac{d(4bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{5/2}(bc-ad)^{3/2}}$$

output
```
-(-3*a*d+2*b*c)*(b*x^2+a*x)^(1/2)/a/c^2/(-a*d+b*c)/x-d*(b*x^2+a*x)^(1/2)/c
/(-a*d+b*c)/x/(d*x+c)-d*(-3*a*d+4*b*c)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/
(b*x^2+a*x)^(1/2))/c^(5/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.08

$$\int \frac{1}{x(c+dx)^2\sqrt{ax+bx^2}}\,dx = \frac{\frac{\sqrt{c}(a+bx)(2bc(c+dx)-ad(2c+3dx))}{a(-bc+ad)(c+dx)} - \frac{d(4bc-3ad)\sqrt{x}\sqrt{a+bx}\arctan\left(\frac{-d\sqrt{x}\sqrt{a+bx}+\sqrt{b}(c+dx)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{c^{5/2}\sqrt{x(a+bx)}}$$

input
```
Integrate[1/(x*(c + d*x)^2*Sqrt[a*x + b*x^2]),x]
```

output
```
((Sqrt[c]*(a + b*x)*(2*b*c*(c + d*x) - a*d*(2*c + 3*d*x)))/(a*(-(b*c) + a*
d)*(c + d*x)) - (d*(4*b*c - 3*a*d)*Sqrt[x]*Sqrt[a + b*x]*ArcTan[(-(d*Sqrt[
x]*Sqrt[a + b*x]) + Sqrt[b]*(c + d*x))/(Sqrt[c]*Sqrt[-(b*c) + a*d])]])/(-(b
*c) + a*d)^(3/2))/(c^(5/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1261, 114, 27, 169, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{ax+bx^2}(c+dx)^2}dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\int \frac{1}{x^{3/2}\sqrt{a+bx}(c+dx)^2}dx}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{114}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{\int -\frac{2bc-3ad-2bdx}{2x^{3/2}\sqrt{a+bx}(c+dx)}dx}{c(bc-ad)}-\frac{d\sqrt{a+bx}}{c\sqrt{x}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\int \frac{2bc-3ad-2bdx}{x^{3/2}\sqrt{a+bx}(c+dx)}dx}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{c\sqrt{x}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{169}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{2\int \frac{ad(4bc-3ad)}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{ac}-\frac{2\sqrt{a+bx}(2bc-3ad)}{ac\sqrt{x}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{c\sqrt{x}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{d(4bc-3ad)\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{c}-\frac{2\sqrt{a+bx}(2bc-3ad)}{ac\sqrt{x}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{c\sqrt{x}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{104}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{2d(4bc-3ad)\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}-\frac{2\sqrt{a+bx}(2bc-3ad)}{ac\sqrt{x}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{c\sqrt{x}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{2d(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{a+bx}(2bc-3ad)}{ac\sqrt{x}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{c\sqrt{x}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

input

```
Int[1/(x*(c + d*x)^2*Sqrt[a*x + b*x^2]),x]
```

output

```
(Sqrt[x]*Sqrt[a + b*x]*(-((d*Sqrt[a + b*x])/(c*(b*c - a*d)*Sqrt[x]*(c + d*
x))) + ((-2*(2*b*c - 3*a*d)*Sqrt[a + b*x])/(a*c*Sqrt[x]) - (2*d*(4*b*c - 3
*a*d)*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(c^(3/2)
*Sqrt[b*c - a*d]))/(2*c*(b*c - a*d))))/Sqrt[a*x + b*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.91

method	result
pseudoelliptic	$\frac{-2\sqrt{x(bx+a)}\left(-b\,c^2+d(-bx+a)c+\frac{3a\,d^2x}{2}\right)\sqrt{c(ad-bc)}+dxa\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)(dx+c)(3ad-4bc)}{\sqrt{c(ad-bc)}\,c^2x(ad-bc)(dx+c)a}$
default	$-\frac{2\sqrt{b\,x^2+ax}}{c^2ax}+\frac{\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{x+\frac{c}{d}}\right)}{c^2\sqrt{-\frac{c(ad-bc)}{d^2}}}-\frac{d^2\sqrt{b\left(x+\cdots\right.}}{\cdots}$
risch	$-\frac{2(bx+a)}{a\,c^2\sqrt{x(bx+a)}}-\frac{d\left(-\frac{\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{c(ad-bc)}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{x+\frac{c}{d}}\right)}{d\sqrt{-\frac{c(ad-bc)}{d^2}}}+\frac{c\,\cdots}{\cdots}\right.}{\cdots}$

input
```
int(1/x/(d*x+c)^2/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-2*(x*(b*x+a))^(1/2)*(-b*c^2+d*(-b*x+a)*c+3/2*a*d^2*x)*(c*(a*d-b*c))^(1/2
)+d*x*a*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))*(d*x+c)*(3*a*d-4
*b*c))/(c*(a*d-b*c))^(1/2)/c^2/x/(a*d-b*c)/(d*x+c)/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 488, normalized size of antiderivative = 3.37

$$\int \frac{1}{x(c+dx)^2\sqrt{ax+bx^2}}\,dx$$
$$=\left[\frac{\sqrt{bc^2-acd}\left(\left(4\,abcd^2-3\,a^2d^3\right)x^2+\left(4\,abc^2d-3\,a^2cd^2\right)x\right)\log\left(\frac{ac+(2\,bc-ad)x-2\,\sqrt{bc^2-acd}\sqrt{bx^2+ax}}{dx+c}\right)-2\,(2\,b^2c\cdots}{2\left(\left(ab^2c^5d-2\,a^2bc^4d^2+a^3c^3d^3\right)x^2+\left(ab^2c^6-2\,a^2b\cdots\right.\right.}\right.$$

input
```
integrate(1/x/(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b*c^2 - a*c*d)*((4*a*b*c*d^2 - 3*a^2*d^3)*x^2 + (4*a*b*c^2*d -
3*a^2*c*d^2)*x)*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*c^2 - a*c*d)*sqrt(b*
x^2 + a*x))/(d*x + c)) - 2*(2*b^2*c^4 - 4*a*b*c^3*d + 2*a^2*c^2*d^2 + (2*b
^2*c^3*d - 5*a*b*c^2*d^2 + 3*a^2*c*d^3)*x)*sqrt(b*x^2 + a*x))/((a*b^2*c^5*
d - 2*a^2*b*c^4*d^2 + a^3*c^3*d^3)*x^2 + (a*b^2*c^6 - 2*a^2*b*c^5*d + a^3*
c^4*d^2)*x), (sqrt(-b*c^2 + a*c*d)*((4*a*b*c*d^2 - 3*a^2*d^3)*x^2 + (4*a*b
*c^2*d - 3*a^2*c*d^2)*x)*arctan(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*
c*x + a*c)) - (2*b^2*c^4 - 4*a*b*c^3*d + 2*a^2*c^2*d^2 + (2*b^2*c^3*d - 5*
a*b*c^2*d^2 + 3*a^2*c*d^3)*x)*sqrt(b*x^2 + a*x))/((a*b^2*c^5*d - 2*a^2*b*c
^4*d^2 + a^3*c^3*d^3)*x^2 + (a*b^2*c^6 - 2*a^2*b*c^5*d + a^3*c^4*d^2)*x)]
```

Sympy [F]

$$\int \frac{1}{x(c+dx)^2\sqrt{ax+bx^2}}\,dx = \int \frac{1}{x\sqrt{x\,(a+bx)}\,(c+dx)^2}\,dx$$

input
```
integrate(1/x/(d*x+c)**2/(b*x**2+a*x)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(x*(a + b*x))*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{x(c+dx)^2\sqrt{ax+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+ax}(dx+c)^2x}\,dx$$

input
```
integrate(1/x/(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a*x)*(d*x + c)^2*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 739 vs. $2(131) = 262$.

Time = 0.48 (sec) , antiderivative size = 739, normalized size of antiderivative = 5.10

$$\int \frac{1}{x(c+dx)^2\sqrt{ax+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/6*(6*sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d/(d*x + c) - a*c*
d/(d*x + c)^2)*c^2*d^2*sgn(1/(d*x + c))*sgn(d)/(b*c^5*sgn(1/(d*x + c))^2*s
gn(d)^2 - a*c^4*d*sgn(1/(d*x + c))^2*sgn(d)^2) - (4*b*c*d^3*log(abs(8*b^2*
c^2*d - 8*a*b*c*d^2 + a^2*d^3 - 8*sqrt(b*c^2 - a*c*d)*b^(3/2)*c*abs(d) + 4
*sqrt(b*c^2 - a*c*d)*a*sqrt(b)*d*abs(d))) - 3*a*d^4*log(abs(8*b^2*c^2*d -
8*a*b*c*d^2 + a^2*d^3 - 8*sqrt(b*c^2 - a*c*d)*b^(3/2)*c*abs(d) + 4*sqrt(b*
c^2 - a*c*d)*a*sqrt(b)*d*abs(d))) + 6*sqrt(b*c^2 - a*c*d)*sqrt(b)*d^2*abs(
d))*sgn(1/(d*x + c))*sgn(d)/(sqrt(b*c^2 - a*c*d)*b*c^3*abs(d) - sqrt(b*c^2
 - a*c*d)*a*c^2*d*abs(d)) + (4*b*c*d^3 - 3*a*d^4)*log(abs(-2*sqrt(b*c^2 -
a*c*d)*c*(sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d/(d*x + c) - a
*c*d/(d*x + c)^2) + sqrt(b*c^2*d^2 - a*c*d^3)/((d*x + c)*d))^3*abs(d) + 2*
b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3 - 2*sqrt(b*c^2 - a*c*d)*(3*b*c - 2*a*d)*
(sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d/(d*x + c) - a*c*d/(d*x
 + c)^2) + sqrt(b*c^2*d^2 - a*c*d^3)/((d*x + c)*d))*abs(d) + (6*b*c^2*d -
5*a*c*d^2)*(sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d/(d*x + c) -
 a*c*d/(d*x + c)^2) + sqrt(b*c^2*d^2 - a*c*d^3)/((d*x + c)*d))^2))/((b*c^3
 - a*c^2*d)*sqrt(b*c^2 - a*c*d)*abs(d)*sgn(1/(d*x + c))*sgn(d)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(c+dx)^2\sqrt{ax+bx^2}}\,dx = \int \frac{1}{x\,\sqrt{b\,x^2+a\,x}\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(1/(x*(a*x + b*x^2)^(1/2)*(c + d*x)^2),x)
```

output
```
int(1/(x*(a*x + b*x^2)^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 712, normalized size of antiderivative = 4.91

$$\int \frac{1}{x(c+dx)^2\sqrt{ax+bx^2}}\,dx = \text{Too large to display}$$

input
```
int(1/x/(d*x+c)^2/(b*x^2+a*x)^(1/2),x)
```

output
```
(3*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) -
 sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*c*d**2*x + 3*sqrt(c)*sqr
t(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(
d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*d**3*x**2 - 4*sqrt(c)*sqrt(a*d - b*c)*
atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(
sqrt(c)*sqrt(b)))*a*b*c**2*d*x - 4*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d
- b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)
))*a*b*c*d**2*x**2 + 3*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqr
t(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*c*d*
*2*x + 3*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a +
b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**2*d**3*x**2 - 4*sqrt
(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x
)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a*b*c**2*d*x - 4*sqrt(c)*sqrt(a*d -
b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(
b))/(sqrt(c)*sqrt(b)))*a*b*c*d**2*x**2 - 2*sqrt(x)*sqrt(a + b*x)*a**2*c**2
*d**2 - 3*sqrt(x)*sqrt(a + b*x)*a**2*c*d**3*x + 4*sqrt(x)*sqrt(a + b*x)*a*
b*c**3*d + 5*sqrt(x)*sqrt(a + b*x)*a*b*c**2*d**2*x - 2*sqrt(x)*sqrt(a + b*
x)*b**2*c**4 - 2*sqrt(x)*sqrt(a + b*x)*b**2*c**3*d*x + sqrt(b)*a**2*c**2*d
**2*x + sqrt(b)*a**2*c*d**3*x**2 - 3*sqrt(b)*a*b*c**3*d*x - 3*sqrt(b)*a*b*
c**2*d**2*x**2 + 2*sqrt(b)*b**2*c**4*x + 2*sqrt(b)*b**2*c**3*d*x**2)/(a...
```

3.155 $\int \frac{1}{x^2(c+dx)^2\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 207

$$\int \frac{1}{x^2(c+dx)^2\sqrt{ax+bx^2}}\,dx = -\frac{(2bc-5ad)\sqrt{ax+bx^2}}{3ac^2(bc-ad)x^2} + \frac{(4b^2c^2+8abcd-15a^2d^2)\sqrt{ax+bx^2}}{3a^2c^3(bc-ad)x} - \frac{d\sqrt{ax+bx^2}}{c(bc-ad)x^2(c+dx)} + \frac{d^2(6bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{7/2}(bc-ad)^{3/2}}$$

output

```
-1/3*(-5*a*d+2*b*c)*(b*x^2+a*x)^(1/2)/a/c^2/(-a*d+b*c)/x^2+1/3*(-15*a^2*d^
2+8*a*b*c*d+4*b^2*c^2)*(b*x^2+a*x)^(1/2)/a^2/c^3/(-a*d+b*c)/x-d*(b*x^2+a*x
)^(1/2)/c/(-a*d+b*c)/x^2/(d*x+c)+d^2*(-5*a*d+6*b*c)*arctanh((-a*d+b*c)^(1/
2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(7/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^2(c+dx)^2\sqrt{ax+bx^2}}\,dx$$
$$= \frac{\frac{\sqrt{c}(a+bx)\left(-4b^2c^2x(c+dx)+2abc\left(c^2-3cdx-4d^2x^2\right)+a^2d\left(-2c^2+10cdx+15d^2x^2\right)\right)}{a^2(-bc+ad)(c+dx)}+\frac{3d^2(6bc-5ad)x^{3/2}\sqrt{a+bx}\arctan\left(\frac{-d\sqrt{x}\sqrt{a+bx}+\sqrt{b}(c+d}{\sqrt{c}\sqrt{-bc+ad}}\right.}{(-bc+ad)^{3/2}}}{3c^{7/2}x\sqrt{x(a+bx)}}$$

input
```
Integrate[1/(x^2*(c + d*x)^2*Sqrt[a*x + b*x^2]),x]
```

output
```
((Sqrt[c]*(a + b*x)*(-4*b^2*c^2*x*(c + d*x) + 2*a*b*c*(c^2 - 3*c*d*x - 4*d
^2*x^2) + a^2*d*(-2*c^2 + 10*c*d*x + 15*d^2*x^2)))/(a^2*(-(b*c) + a*d)*(c
+ d*x)) + (3*d^2*(6*b*c - 5*a*d)*x^(3/2)*Sqrt[a + b*x]*ArcTan[(-(d*Sqrt[x]
*Sqrt[a + b*x]) + Sqrt[b]*(c + d*x))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(-(b*c
) + a*d)^(3/2))/(3*c^(7/2)*x*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1261, 114, 27, 169, 27, 169, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{ax+bx^2}(c+dx)^2}\,dx$$
$$\Big\downarrow \text{1261}$$
$$\frac{\sqrt{x}\sqrt{a+bx}\int \frac{1}{x^{5/2}\sqrt{a+bx}(c+dx)^2}dx}{\sqrt{ax+bx^2}}$$
$$\Big\downarrow \text{114}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{\int -\frac{2bc-5ad-4bdx}{2x^{5/2}\sqrt{a+bx}(c+dx)}dx}{c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\int \frac{2bc-5ad-4bdx}{x^{5/2}\sqrt{a+bx}(c+dx)}dx}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{ 169}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{2\int \frac{4b^2c^2+8abdc-15a^2d^2+2bd(2bc-5ad)x}{2x^{3/2}\sqrt{a+bx}(c+dx)}dx}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{\int \frac{4b^2c^2+8abdc-15a^2d^2+2bd(2bc-5ad)x}{x^{3/2}\sqrt{a+bx}(c+dx)}dx}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{ 169}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{-\frac{2\int \frac{3a^2d^2(6bc-5ad)}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{ac}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{15ad^2}{c}+8bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{-\frac{3ad^2(6bc-5ad)\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{c}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{15ad^2}{c}+8bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{ 104}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{-\frac{-\frac{6ad^2(6bc-5ad)\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{15ad^2}{c}+8bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{-\frac{-\frac{6ad^2(6bc-5ad)\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{15ad^2}{c}+8bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

input
```
Int[1/(x^2*(c + d*x)^2*Sqrt[a*x + b*x^2]),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(-((d*Sqrt[a + b*x])/(c*(b*c - a*d)*x^(3/2)*(c + d*
x))) + ((-2*(2*b*c - 5*a*d)*Sqrt[a + b*x])/(3*a*c*x^(3/2)) - ((-2*((4*b^2*
c)/a + 8*b*d - (15*a*d^2)/c)*Sqrt[a + b*x])/Sqrt[x] - (6*a*d^2*(6*b*c - 5*
a*d)*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(c^(3/2)*
Sqrt[b*c - a*d]))/(3*a*c))/(2*c*(b*c - a*d))))/Sqrt[a*x + b*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{-2\left(\left(2b^2x-ab\right)c^3+d(2bx+a)(bx+a)c^2-5a\,d^2x\left(-\frac{4bx}{5}+a\right)c-\frac{15a^2d^3x^2}{2}\right)\sqrt{x(bx+a)}\,\sqrt{c(ad-bc)}-3d^2x^2a^2\arctan\left(\frac{\sqrt{x(bx+\cdots}}{x\sqrt{c(ad\cdots}}\right.\cdots}{3\sqrt{c(ad-bc)}\,c^3x^2(ad-bc)(dx+c)a^2}$
risch	$-\frac{2(bx+a)(-6adx-2cbx+ac)}{3a^2c^3\sqrt{x(bx+a)}\,x}+\frac{d^2\left(\frac{c\left(\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{c(ad-bc)\left(x+\frac{c}{d}\right)}-\frac{(ad-2bc)d\ln\left(-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(\cdots\right)}{d}\cdots\right)}{\cdots}\right)}{d^2}\cdots\right)}{\cdots}$
default	$\frac{-\frac{2\sqrt{b\,x^2+ax}}{3a\,x^2}+\frac{4b\sqrt{b\,x^2+ax}}{3a^2x}}{c^2}+\frac{\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{c(ad-bc)\left(x+\frac{c}{d}\right)}-\frac{(ad-2bc)d\ln\left(-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{\cdots}\right)}{2c(ad-bc)\cdots}}{c^2}$

input

```
int(1/x^2/(d*x+c)^2/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/3*(-2*((2*b^2*x-a*b)*c^3+d*(2*b*x+a)*(b*x+a)*c^2-5*a*d^2*x*(-4/5*b*x+a)*
c-15/2*a^2*d^3*x^2)*(x*(b*x+a))^(1/2)*(c*(a*d-b*c))^(1/2)-3*d^2*x^2*a^2*ar
ctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))*(d*x+c)*(5*a*d-6*b*c))/(c*
(a*d-b*c))^(1/2)/c^3/x^2/(a*d-b*c)/(d*x+c)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 646, normalized size of antiderivative = 3.12

$$\int \frac{1}{x^2(c+dx)^2\sqrt{ax+bx^2}}\,dx$$
$$= \left[\frac{3\left((6a^2bcd^3-5a^3d^4)x^3+(6a^2bc^2d^2-5a^3cd^3)x^2\right)\sqrt{bc^2-acd}\log\left(\frac{ac+(2bc-ad)x+2\sqrt{bc^2-acd}\sqrt{bx^2+ax}}{dx+c}\right)-2(\cdots}{6\left((a^2b^2c^6d-2a^3\cdots\right.}\right.$$
$$-\frac{3\left((6a^2bcd^3-5a^3d^4)x^3+(6a^2bc^2d^2-5a^3cd^3)x^2\right)\sqrt{-bc^2+acd}\arctan\left(\frac{\sqrt{-bc^2+acd}\sqrt{bx^2+ax}}{bcx+ac}\right)+(2ab^2c^5\cdots}{3\left((a^2b^2c^6d-2a^3bc^5\cdots\right.}$$

input
```
integrate(1/x^2/(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*((6*a^2*b*c*d^3 - 5*a^3*d^4)*x^3 + (6*a^2*b*c^2*d^2 - 5*a^3*c*d^3)
*x^2)*sqrt(b*c^2 - a*c*d)*log((a*c + (2*b*c - a*d)*x + 2*sqrt(b*c^2 - a*c*
d)*sqrt(b*x^2 + a*x))/(d*x + c)) - 2*(2*a*b^2*c^5 - 4*a^2*b*c^4*d + 2*a^3*
c^3*d^2 - (4*b^3*c^4*d + 4*a*b^2*c^3*d^2 - 23*a^2*b*c^2*d^3 + 15*a^3*c*d^4
)*x^2 - 2*(2*b^3*c^5 + a*b^2*c^4*d - 8*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x)*s
qrt(b*x^2 + a*x))/((a^2*b^2*c^6*d - 2*a^3*b*c^5*d^2 + a^4*c^4*d^3)*x^3 + (
a^2*b^2*c^7 - 2*a^3*b*c^6*d + a^4*c^5*d^2)*x^2), -1/3*(3*((6*a^2*b*c*d^3 -
 5*a^3*d^4)*x^3 + (6*a^2*b*c^2*d^2 - 5*a^3*c*d^3)*x^2)*sqrt(-b*c^2 + a*c*d
)*arctan(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) + (2*a*b^2*
c^5 - 4*a^2*b*c^4*d + 2*a^3*c^3*d^2 - (4*b^3*c^4*d + 4*a*b^2*c^3*d^2 - 23*
a^2*b*c^2*d^3 + 15*a^3*c*d^4)*x^2 - 2*(2*b^3*c^5 + a*b^2*c^4*d - 8*a^2*b*c
^3*d^2 + 5*a^3*c^2*d^3)*x)*sqrt(b*x^2 + a*x))/((a^2*b^2*c^6*d - 2*a^3*b*c^
5*d^2 + a^4*c^4*d^3)*x^3 + (a^2*b^2*c^7 - 2*a^3*b*c^6*d + a^4*c^5*d^2)*x^2
)]
```

Sympy [F]

$$\int \frac{1}{x^2(c+dx)^2\sqrt{ax+bx^2}}\,dx = \int \frac{1}{x^2\sqrt{x\,(a+bx)}\,(c+dx)^2}\,dx$$

input
```
integrate(1/x**2/(d*x+c)**2/(b*x**2+a*x)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(x*(a + b*x))*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{x^2(c+dx)^2\sqrt{ax+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+ax}(dx+c)^2x^2}\,dx$$

input
```
integrate(1/x^2/(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a*x)*(d*x + c)^2*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1082 vs. 2(187) = 374.

Time = 6.88 (sec) , antiderivative size = 1082, normalized size of antiderivative = 5.23

$$\int \frac{1}{x^2(c+dx)^2\sqrt{ax+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
-1/10*(10*sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d/(d*x + c) - a
*c*d/(d*x + c)^2)*c^3*d^4*sgn(1/(d*x + c))*sgn(d)/(b*c^7*sgn(1/(d*x + c))^
2*sgn(d)^2 - a*c^6*d*sgn(1/(d*x + c))^2*sgn(d)^2) - (6*b*c*d^5*log(abs(32*
b^3*c^3*d - 48*a*b^2*c^2*d^2 + 18*a^2*b*c*d^3 - a^3*d^4 - 32*sqrt(b*c^2 -
a*c*d)*b^(5/2)*c^2*abs(d) + 32*sqrt(b*c^2 - a*c*d)*a*b^(3/2)*c*d*abs(d) -
6*sqrt(b*c^2 - a*c*d)*a^2*sqrt(b)*d^2*abs(d))) - 5*a*d^6*log(abs(32*b^3*c^
3*d - 48*a*b^2*c^2*d^2 + 18*a^2*b*c*d^3 - a^3*d^4 - 32*sqrt(b*c^2 - a*c*d)
*b^(5/2)*c^2*abs(d) + 32*sqrt(b*c^2 - a*c*d)*a*b^(3/2)*c*d*abs(d) - 6*sqrt
(b*c^2 - a*c*d)*a^2*sqrt(b)*d^2*abs(d))) + 10*sqrt(b*c^2 - a*c*d)*sqrt(b)*
d^4*abs(d))*sgn(1/(d*x + c))*sgn(d)/(sqrt(b*c^2 - a*c*d)*b*c^4*abs(d) - sq
rt(b*c^2 - a*c*d)*a*c^3*d*abs(d)) + (6*b*c*d^5 - 5*a*d^6)*log(abs(-2*sqrt(
b*c^2 - a*c*d)*c^2*(sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d/(d*
x + c) - a*c*d/(d*x + c)^2) + sqrt(b*c^2*d^2 - a*c*d^3)/((d*x + c)*d))^5*a
bs(d) + 2*b^3*c^3*d - 5*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3 - a^3*d^4 - 4*(5*b*c
^2 - 4*a*c*d)*sqrt(b*c^2 - a*c*d)*(sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x +
 c)^2 + a*d/(d*x + c) - a*c*d/(d*x + c)^2) + sqrt(b*c^2*d^2 - a*c*d^3)/((d
*x + c)*d))^3*abs(d) + (10*b*c^3*d - 9*a*c^2*d^2)*(sqrt(b - 2*b*c/(d*x + c
) + b*c^2/(d*x + c)^2 + a*d/(d*x + c) - a*c*d/(d*x + c)^2) + sqrt(b*c^2*d^
2 - a*c*d^3)/((d*x + c)*d))^4 - 2*(5*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*sqrt
(b*c^2 - a*c*d)*(sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d/(d*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(c+dx)^2\sqrt{ax+bx^2}}\,dx = \int \frac{1}{x^2\,\sqrt{b\,x^2+a\,x}\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(1/(x^2*(a*x + b*x^2)^(1/2)*(c + d*x)^2),x)
```

output
```
int(1/(x^2*(a*x + b*x^2)^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 1291, normalized size of antiderivative = 6.24

$$\int \frac{1}{x^2(c+dx)^2\sqrt{ax+bx^2}}\,dx = \text{Too large to display}$$

input
```
int(1/x^2/(d*x+c)^2/(b*x^2+a*x)^(1/2),x)
```

output
```
( - 75*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*
x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**4*c*d**4*x**2 - 75*sqr
t(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(
x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**4*d**5*x**3 + 150*sqrt(c)*sqrt(a
*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*
sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*b*c**2*d**3*x**2 + 150*sqrt(c)*sqrt(a*d -
 b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt
(b))/(sqrt(c)*sqrt(b)))*a**3*b*c*d**4*x**3 - 72*sqrt(c)*sqrt(a*d - b*c)*at
an((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sq
rt(c)*sqrt(b)))*a**2*b**2*c**3*d**2*x**2 - 72*sqrt(c)*sqrt(a*d - b*c)*atan
((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt
(c)*sqrt(b)))*a**2*b**2*c**2*d**3*x**3 - 75*sqrt(c)*sqrt(a*d - b*c)*atan((
sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c
)*sqrt(b)))*a**4*c*d**4*x**2 - 75*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d -
 b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b))
)*a**4*d**5*x**3 + 150*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqr
t(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*b*c*
*2*d**3*x**2 + 150*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)
*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*b*c*d**4
*x**3 - 72*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt...
```

3.156 $\int \frac{1}{x^3(c+dx)^2\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 281

$$\int \frac{1}{x^3(c+dx)^2\sqrt{ax+bx^2}}\,dx = -\frac{(2bc-7ad)\sqrt{ax+bx^2}}{5ac^2(bc-ad)x^3} + \frac{(8b^2c^2+12abcd-35a^2d^2)\sqrt{ax+bx^2}}{15a^2c^3(bc-ad)x^2} - \frac{(16b^3c^3+24ab^2c^2d+50a^2bcd^2-105a^3d^3)\sqrt{ax+bx^2}}{15a^3c^4(bc-ad)x} - \frac{d\sqrt{ax+bx^2}}{c(bc-ad)x^3(c+dx)} - \frac{d^3(8bc-7ad)\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{9/2}(bc-ad)^{3/2}}$$

output
```
-1/5*(-7*a*d+2*b*c)*(b*x^2+a*x)^(1/2)/a/c^2/(-a*d+b*c)/x^3+1/15*(-35*a^2*d
^2+12*a*b*c*d+8*b^2*c^2)*(b*x^2+a*x)^(1/2)/a^2/c^3/(-a*d+b*c)/x^2-1/15*(-1
05*a^3*d^3+50*a^2*b*c*d^2+24*a*b^2*c^2*d+16*b^3*c^3)*(b*x^2+a*x)^(1/2)/a^3
/c^4/(-a*d+b*c)/x-d*(b*x^2+a*x)^(1/2)/c/(-a*d+b*c)/x^3/(d*x+c)-d^3*(-7*a*d
+8*b*c)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(9/2)/(-a*
d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^3(c+dx)^2\sqrt{ax+bx^2}}\,dx$$
$$=\frac{\frac{\sqrt{c}(a+bx)\left(16b^3c^3x^2(c+dx)-8ab^2c^2x\left(c^2-2cdx-3d^2x^2\right)+2a^2bc\left(3c^3-3c^2dx+19cd^2x^2+25d^3x^3\right)-a^3d\left(6c^3-14c^2dx+70cd^2x^2+105d^3x^3\right)\right)}{a^3(-bc+ad)(c+dx)}-}{15c^{9/2}x^2\sqrt{x(a+bx)}}$$

input
```
Integrate[1/(x^3*(c + d*x)^2*Sqrt[a*x + b*x^2]),x]
```

output
```
((Sqrt[c]*(a + b*x)*(16*b^3*c^3*x^2*(c + d*x) - 8*a*b^2*c^2*x*(c^2 - 2*c*d
*x - 3*d^2*x^2) + 2*a^2*b*c*(3*c^3 - 3*c^2*d*x + 19*c*d^2*x^2 + 25*d^3*x^3
) - a^3*d*(6*c^3 - 14*c^2*d*x + 70*c*d^2*x^2 + 105*d^3*x^3)))/(a^3*(-(b*c)
 + a*d)*(c + d*x)) - (15*d^3*(8*b*c - 7*a*d)*x^(5/2)*Sqrt[a + b*x]*ArcTan[
(-(d*Sqrt[x]*Sqrt[a + b*x]) + Sqrt[b]*(c + d*x))/(Sqrt[c]*Sqrt[-(b*c) + a*
d])])/(-(b*c) + a*d)^(3/2))/(15*c^(9/2)*x^2*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {1261, 114, 27, 169, 27, 169, 27, 169, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{ax+bx^2}(c+dx)^2}\,dx$$

$$\big\downarrow \ 1261$$

$$\frac{\sqrt{x}\sqrt{a+bx}\int \frac{1}{x^{7/2}\sqrt{a+bx}(c+dx)^2}dx}{\sqrt{ax+bx^2}}$$

$$\big\downarrow \ 114$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{\int -\frac{2bc-7ad-6bdx}{2x^{7/2}\sqrt{a+bx}(c+dx)}dx}{c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{5/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\int \frac{2bc-7ad-6bdx}{x^{7/2}\sqrt{a+bx}(c+dx)}dx}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{5/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \textbf{169}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{2\int \frac{8b^2c^2+12abdc-35a^2d^2+4bd(2bc-7ad)x}{2x^{5/2}\sqrt{a+bx}(c+dx)}dx}{5ac}-\frac{2\sqrt{a+bx}(2bc-7ad)}{5acx^{5/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{5/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{\int \frac{8b^2c^2+12abdc-35a^2d^2+4bd(2bc-7ad)x}{x^{5/2}\sqrt{a+bx}(c+dx)}dx}{5ac}-\frac{2\sqrt{a+bx}(2bc-7ad)}{5acx^{5/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{5/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \textbf{169}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{-\frac{2\int \frac{16b^3c^3+24ab^2dc^2+50a^2bd^2c-105a^3d^3+2bd\left(8b^2c^2+12abdc-35a^2d^2\right)x}{2x^{3/2}\sqrt{a+bx}(c+dx)}dx}{3ac}-\frac{2\sqrt{a+bx}\left(\frac{8b^2c}{a}-\frac{35ad^2}{c}+12bd\right)}{3x^{3/2}}}{5ac}-\frac{2\sqrt{a+bx}(2bc-7ad)}{5acx^{5/2}}}{2c(bc-ad)} -}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{-\frac{\int \frac{16b^3c^3+24ab^2dc^2+50a^2bd^2c-105a^3d^3+2bd\left(8b^2c^2+12abdc-35a^2d^2\right)x}{x^{3/2}\sqrt{a+bx}(c+dx)}dx}{3ac}-\frac{2\sqrt{a+bx}\left(\frac{8b^2c}{a}-\frac{35ad^2}{c}+12bd\right)}{3x^{3/2}}}{5ac}-\frac{2\sqrt{a+bx}(2bc-7ad)}{5acx^{5/2}}}{2c(bc-ad)} -}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \textbf{169}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{-\frac{-\frac{-\frac{2\int\frac{15a^3d^3(8bc-7ad)}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{ac}-\frac{2\sqrt{a+bx}\left(-105a^3d^3+50a^2bcd^2+24ab^2c^2d+16b^3c^3\right)}{ac\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}\left(\frac{8b^2c}{a}-\frac{35ad^2}{c}+12bd\right)}{3x^{3/2}}}{5ac}-\frac{2\sqrt{a+bx}(2bc-7a\cdots}{5acx^{5/2}}\cdots}{2c(bc-ad)}\cdots\right.}{\sqrt{ax+bx^2}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{-\frac{-\frac{-\frac{15a^2d^3(8bc-7ad)\int\frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{c}-\frac{2\sqrt{a+bx}\left(-105a^3d^3+50a^2bcd^2+24ab^2c^2d+16b^3c^3\right)}{ac\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}\left(\frac{8b^2c}{a}-\frac{35ad^2}{c}+12bd\right)}{3x^{3/2}}}{5ac}-\cdots}{2c(bc-ad)}\cdots\right.}{\sqrt{ax+bx^2}}$$

$\Big\downarrow$ 104

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{-\frac{-\frac{-\frac{30a^2d^3(8bc-7ad)\int\frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}-\frac{2\sqrt{a+bx}\left(-105a^3d^3+50a^2bcd^2+24ab^2c^2d+16b^3c^3\right)}{ac\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}\left(\frac{8b^2c}{a}-\frac{35ad^2}{c}+12bd\right)}{3x^{3/2}}}{5ac}-\cdots}{2c(bc-ad)}\cdots\right.}{\sqrt{ax+bx^2}}$$

$\Big\downarrow$ 221

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{-\frac{-\frac{-\frac{30a^2d^3(8bc-7ad)\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{a+bx}\left(-105a^3d^3+50a^2bcd^2+24ab^2c^2d+16b^3c^3\right)}{ac\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}\left(\frac{8b^2c}{a}-\frac{35ad^2}{c}+12bd\right)}{3x^{3/2}}}{5ac}\cdots}{2c(bc-ad)}\cdots\right.}{\sqrt{ax+bx^2}}$$

input

```
Int[1/(x^3*(c + d*x)^2*Sqrt[a*x + b*x^2]),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(-((d*Sqrt[a + b*x])/(c*(b*c - a*d)*x^(5/2)*(c + d*
x))) + ((-2*(2*b*c - 7*a*d)*Sqrt[a + b*x])/(5*a*c*x^(5/2)) - ((-2*((8*b^2*
c)/a + 12*b*d - (35*a*d^2)/c)*Sqrt[a + b*x])/(3*x^(3/2)) - ((-2*(16*b^3*c^
3 + 24*a*b^2*c^2*d + 50*a^2*b*c*d^2 - 105*a^3*d^3)*Sqrt[a + b*x])/(a*c*Sqr
t[x]) - (30*a^2*d^3*(8*b*c - 7*a*d)*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqr
t[c]*Sqrt[a + b*x])])/(c^(3/2)*Sqrt[b*c - a*d]))/(3*a*c))/(5*a*c))/(2*c*(b
*c - a*d))))/Sqrt[a*x + b*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{7(dx+c)d^3x^3a^3\left(ad-\frac{8bc}{7}\right)\arctan\left(\frac{\sqrt{x(bx+a)}\,c}{x\sqrt{c(ad-bc)}}\right)-\frac{2\sqrt{x(bx+a)}\,\sqrt{c(ad-bc)}\left(\left(-\frac{8}{3}b^3x^2+\frac{4}{3}xab^2-a^2b\right)c^4+d(bx+a)\left(-\frac{8b^2x^2}{3}+a^2\right)\right.}{5}}{c^4x^3(ad-bc)(dx+c)\sqrt{c(ad-bc)}\,a^3}$
risch	$-\frac{2(bx+a)(45a^2d^2x^2+20abcdx^2+8b^2c^2x^2-10a^2cdx-4abc^2x+3a^2c^2)}{15a^3c^4\sqrt{x(bx+a)}\,x^2}-\frac{d^3\left(c\left(\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-}{d^2}}}{c(ad-bc)\left(x+\frac{c}{d}\right)}\right.\right.}{}$
default	$\frac{-\frac{2\sqrt{bx^2+ax}}{5ax^3}-\frac{4b\left(-\frac{2\sqrt{bx^2+ax}}{3ax^2}+\frac{4b\sqrt{bx^2+ax}}{3a^2x}\right)}{5a}}{c^2}-\frac{6d^2\sqrt{bx^2+ax}}{c^4ax}-\frac{2d\left(-\frac{2\sqrt{bx^2+ax}}{3ax^2}+\frac{4b\sqrt{bx^2+ax}}{3a^2x}\right)}{c^3}+\frac{3d^2\ln\left(\frac{-\frac{2c(ad}{d}}{}\right.}{}$

input
```
int(1/x^3/(d*x+c)^2/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
7*((d*x+c)*d^3*x^3*a^3*(a*d-8/7*b*c)*arctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-
b*c))^(1/2))-2/35*(x*(b*x+a))^(1/2)*(c*(a*d-b*c))^(1/2)*((-8/3*b^3*x^2+4/3
*x*a*b^2-a^2*b)*c^4+d*(b*x+a)*(-8/3*b^2*x^2+a^2)*c^3-7/3*d^2*x*(b*x+a)*(12
/7*b*x+a)*a*c^2+35/3*d^3*(-5/7*b*x+a)*x^2*a^2*c+35/2*a^3*d^4*x^3))/(c*(a*d
-b*c))^(1/2)/c^4/x^3/(a*d-b*c)/(d*x+c)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 787, normalized size of antiderivative = 2.80

$$\int \frac{1}{x^3(c+dx)^2\sqrt{ax+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/30*(15*((8*a^3*b*c*d^4 - 7*a^4*d^5)*x^4 + (8*a^3*b*c^2*d^3 - 7*a^4*c*d^
4)*x^3)*sqrt(b*c^2 - a*c*d)*log((a*c + (2*b*c - a*d)*x - 2*sqrt(b*c^2 - a*
c*d)*sqrt(b*x^2 + a*x))/(d*x + c)) - 2*(6*a^2*b^2*c^6 - 12*a^3*b*c^5*d + 6
*a^4*c^4*d^2 + (16*b^4*c^5*d + 8*a*b^3*c^4*d^2 + 26*a^2*b^2*c^3*d^3 - 155*
a^3*b*c^2*d^4 + 105*a^4*c*d^5)*x^3 + 2*(8*b^4*c^6 + 11*a^2*b^2*c^4*d^2 - 5
4*a^3*b*c^3*d^3 + 35*a^4*c^2*d^4)*x^2 - 2*(4*a*b^3*c^6 - a^2*b^2*c^5*d - 1
0*a^3*b*c^4*d^2 + 7*a^4*c^3*d^3)*x)*sqrt(b*x^2 + a*x))/((a^3*b^2*c^7*d - 2
*a^4*b*c^6*d^2 + a^5*c^5*d^3)*x^4 + (a^3*b^2*c^8 - 2*a^4*b*c^7*d + a^5*c^6
*d^2)*x^3), 1/15*(15*((8*a^3*b*c*d^4 - 7*a^4*d^5)*x^4 + (8*a^3*b*c^2*d^3 -
 7*a^4*c*d^4)*x^3)*sqrt(-b*c^2 + a*c*d)*arctan(sqrt(-b*c^2 + a*c*d)*sqrt(b
*x^2 + a*x)/(b*c*x + a*c)) - (6*a^2*b^2*c^6 - 12*a^3*b*c^5*d + 6*a^4*c^4*d
^2 + (16*b^4*c^5*d + 8*a*b^3*c^4*d^2 + 26*a^2*b^2*c^3*d^3 - 155*a^3*b*c^2*
d^4 + 105*a^4*c*d^5)*x^3 + 2*(8*b^4*c^6 + 11*a^2*b^2*c^4*d^2 - 54*a^3*b*c^
3*d^3 + 35*a^4*c^2*d^4)*x^2 - 2*(4*a*b^3*c^6 - a^2*b^2*c^5*d - 10*a^3*b*c^
4*d^2 + 7*a^4*c^3*d^3)*x)*sqrt(b*x^2 + a*x))/((a^3*b^2*c^7*d - 2*a^4*b*c^6
*d^2 + a^5*c^5*d^3)*x^4 + (a^3*b^2*c^8 - 2*a^4*b*c^7*d + a^5*c^6*d^2)*x^3)
]
```

Sympy [F]

$$\int \frac{1}{x^3(c+dx)^2\sqrt{ax+bx^2}}\,dx = \int \frac{1}{x^3\sqrt{x\,(a+bx)}\,(c+dx)^2}\,dx$$

input
```
integrate(1/x**3/(d*x+c)**2/(b*x**2+a*x)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(x*(a + b*x))*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{x^3(c+dx)^2\sqrt{ax+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+ax}(dx+c)^2x^3}\,dx$$

input
```
integrate(1/x^3/(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a*x)*(d*x + c)^2*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^3(c+dx)^2\sqrt{ax+bx^2}}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(1/x^3/(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: 1603
70139 icas_eval sage2Psr 0, Mod 3.16228, Heu 6.4, MinOGCD dim 3 degree 1 p
srgcdop 0 heuop 6.4 modgcdop 8,125// Using PSR gcd 160.382 NTL factor begi
n160.382 NTL factor
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(c+dx)^2\sqrt{ax+bx^2}}\,dx = \int \frac{1}{x^3\,\sqrt{b\,x^2+a\,x}\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(1/(x^3*(a*x + b*x^2)^(1/2)*(c + d*x)^2),x)
```

output
```
int(1/(x^3*(a*x + b*x^2)^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 1494, normalized size of antiderivative = 5.32

$$\int \frac{1}{x^3(c+dx)^2\sqrt{ax+bx^2}}\,dx = \text{Too large to display}$$

input
```
int(1/x^3/(d*x+c)^2/(b*x^2+a*x)^(1/2),x)
```

output

```
(735*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x)
 - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**5*c*d**5*x**3 + 735*sqrt
(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x
)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**5*d**6*x**4 - 1260*sqrt(c)*sqrt(a
*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*
sqrt(b))/(sqrt(c)*sqrt(b)))*a**4*b*c**2*d**4*x**3 - 1260*sqrt(c)*sqrt(a*d
 - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqr
t(b))/(sqrt(c)*sqrt(b)))*a**4*b*c*d**5*x**4 + 480*sqrt(c)*sqrt(a*d - b*c)*
atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(
sqrt(c)*sqrt(b)))*a**3*b**2*c**3*d**3*x**3 + 480*sqrt(c)*sqrt(a*d - b*c)*a
tan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(s
qrt(c)*sqrt(b)))*a**3*b**2*c**2*d**4*x**4 + 735*sqrt(c)*sqrt(a*d - b*c)*at
an((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sq
rt(c)*sqrt(b)))*a**5*c*d**5*x**3 + 735*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(
a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqr
t(b)))*a**5*d**6*x**4 - 1260*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c)
 + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**
4*b*c**2*d**4*x**3 - 1260*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) +
sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**4*b
*c*d**5*x**4 + 480*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt...
```

3.157 $\int \frac{1}{x^2(c+dx)^2\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 207

$$\int \frac{1}{x^2(c+dx)^2\sqrt{ax+bx^2}}\,dx = -\frac{(2bc-5ad)\sqrt{ax+bx^2}}{3ac^2(bc-ad)x^2} + \frac{(4b^2c^2+8abcd-15a^2d^2)\sqrt{ax+bx^2}}{3a^2c^3(bc-ad)x} - \frac{d\sqrt{ax+bx^2}}{c(bc-ad)x^2(c+dx)} + \frac{d^2(6bc-5ad)\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{7/2}(bc-ad)^{3/2}}$$

output

```
-1/3*(-5*a*d+2*b*c)*(b*x^2+a*x)^(1/2)/a/c^2/(-a*d+b*c)/x^2+1/3*(-15*a^2*d^
2+8*a*b*c*d+4*b^2*c^2)*(b*x^2+a*x)^(1/2)/a^2/c^3/(-a*d+b*c)/x-d*(b*x^2+a*x
)^(1/2)/c/(-a*d+b*c)/x^2/(d*x+c)+d^2*(-5*a*d+6*b*c)*arctanh((-a*d+b*c)^(1/
2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(7/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^2(c+dx)^2\sqrt{ax+bx^2}}\,dx = \frac{\frac{\sqrt{c}(a+bx)\left(-4b^2c^2x(c+dx)+2abc\left(c^2-3cdx-4d^2x^2\right)+a^2d\left(-2c^2+10cdx+15d^2x^2\right)\right)}{a^2(-bc+ad)(c+dx)}+\frac{3d^2(6bc-5ad)x^{3/2}\sqrt{a+bx}\arctan\left(\frac{-d\sqrt{x}\sqrt{a+bx}+\sqrt{b}(c+d}{\sqrt{c}\sqrt{-bc+ad}}\right.}{(-bc+ad)^{3/2}}}{3c^{7/2}x\sqrt{x(a+bx)}}$$

input
```
Integrate[1/(x^2*(c + d*x)^2*Sqrt[a*x + b*x^2]),x]
```

output
```
((Sqrt[c]*(a + b*x)*(-4*b^2*c^2*x*(c + d*x) + 2*a*b*c*(c^2 - 3*c*d*x - 4*d
^2*x^2) + a^2*d*(-2*c^2 + 10*c*d*x + 15*d^2*x^2)))/(a^2*(-(b*c) + a*d)*(c
+ d*x)) + (3*d^2*(6*b*c - 5*a*d)*x^(3/2)*Sqrt[a + b*x]*ArcTan[(-(d*Sqrt[x]
*Sqrt[a + b*x]) + Sqrt[b]*(c + d*x))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(-(b*c
) + a*d)^(3/2))/(3*c^(7/2)*x*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1261, 114, 27, 169, 27, 169, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{ax+bx^2}(c+dx)^2}\,dx$$

$$\Big\downarrow \text{ 1261}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\int \frac{1}{x^{5/2}\sqrt{a+bx}(c+dx)^2}dx}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{ 114}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{\int -\frac{2bc-5ad-4bdx}{2x^{5/2}\sqrt{a+bx}(c+dx)}dx}{c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\int \frac{2bc-5ad-4bdx}{x^{5/2}\sqrt{a+bx}(c+dx)}dx}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{169}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{2\int \frac{4b^2c^2+8abdc-15a^2d^2+2bd(2bc-5ad)x}{2x^{3/2}\sqrt{a+bx}(c+dx)}dx}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{\int \frac{4b^2c^2+8abdc-15a^2d^2+2bd(2bc-5ad)x}{x^{3/2}\sqrt{a+bx}(c+dx)}dx}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{169}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{-\frac{2\int \frac{3a^2d^2(6bc-5ad)}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{ac}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{15ad^2}{c}+8bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{-\frac{3ad^2(6bc-5ad)\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{c}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{15ad^2}{c}+8bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{104}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{-\frac{-\frac{6ad^2(6bc-5ad)\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{15ad^2}{c}+8bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 221$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{-\frac{-\frac{6ad^2(6bc-5ad)\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{15ad^2}{c}+8bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

input
```
Int[1/(x^2*(c + d*x)^2*Sqrt[a*x + b*x^2]),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(-((d*Sqrt[a + b*x])/(c*(b*c - a*d)*x^(3/2)*(c + d*
x))) + ((-2*(2*b*c - 5*a*d)*Sqrt[a + b*x])/(3*a*c*x^(3/2)) - ((-2*((4*b^2*
c)/a + 8*b*d - (15*a*d^2)/c)*Sqrt[a + b*x])/Sqrt[x] - (6*a*d^2*(6*b*c - 5*
a*d)*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(c^(3/2)*
Sqrt[b*c - a*d]))/(3*a*c))/(2*c*(b*c - a*d))))/Sqrt[a*x + b*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{-2\left(\left(2b^2x-ab\right)c^3+d(2bx+a)(bx+a)c^2-5a\,d^2x\left(-\frac{4bx}{5}+a\right)c-\frac{15a^2d^3x^2}{2}\right)\sqrt{x(bx+a)}\,\sqrt{c(ad-bc)}-3d^2x^2a^2\arctan\left(\frac{\sqrt{x(bx+}}{x\sqrt{c(ad}}\right.}{3\sqrt{c(ad-bc)}\,c^3x^2(ad-bc)(dx+c)a^2}$
risch	$-\frac{2(bx+a)(-6adx-2cbx+ac)}{3a^2c^3\sqrt{x(bx+a)}\,x}+\frac{d^2\left(\frac{c\left(\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{c(ad-bc)\left(x+\frac{c}{d}\right)}-\frac{(ad-2bc)d\ln\left(-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(\right.}{d}\right.}{}\right)}{d^2}\right.}{}$
default	$\frac{-\frac{2\sqrt{b\,x^2+ax}}{3a\,x^2}+\frac{4b\sqrt{b\,x^2+ax}}{3a^2x}}{c^2}+\frac{\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{c(ad-bc)\left(x+\frac{c}{d}\right)}-\frac{(ad-2bc)d\ln\left(-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{\right.}{2c(ad-bc)}}{c^2}$

input

```
int(1/x^2/(d*x+c)^2/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/3*(-2*((2*b^2*x-a*b)*c^3+d*(2*b*x+a)*(b*x+a)*c^2-5*a*d^2*x*(-4/5*b*x+a)*
c-15/2*a^2*d^3*x^2)*(x*(b*x+a))^(1/2)*(c*(a*d-b*c))^(1/2)-3*d^2*x^2*a^2*ar
ctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))*(d*x+c)*(5*a*d-6*b*c))/(c*
(a*d-b*c))^(1/2)/c^3/x^2/(a*d-b*c)/(d*x+c)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 646, normalized size of antiderivative = 3.12

$$\int \frac{1}{x^2(c+dx)^2\sqrt{ax+bx^2}}\,dx$$
$$= \left[\frac{3\,((6\,a^2bcd^3-5\,a^3d^4)x^3+(6\,a^2bc^2d^2-5\,a^3cd^3)x^2)\sqrt{bc^2-acd}\log\left(\frac{ac+(2\,bc-ad)x+2\,\sqrt{bc^2-acd}\sqrt{bx^2+ax}}{dx+c}\right)-2\,(}{6\,((a^2b^2c^6d-2\,a^3}\right.$$
$$-\frac{3\,((6\,a^2bcd^3-5\,a^3d^4)x^3+(6\,a^2bc^2d^2-5\,a^3cd^3)x^2)\sqrt{-bc^2+acd}\arctan\left(\frac{\sqrt{-bc^2+acd}\sqrt{bx^2+ax}}{bcx+ac}\right)+(2\,ab^2c^5}{3\,((a^2b^2c^6d-2\,a^3bc^5}$$

input
```
integrate(1/x^2/(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*((6*a^2*b*c*d^3 - 5*a^3*d^4)*x^3 + (6*a^2*b*c^2*d^2 - 5*a^3*c*d^3)
*x^2)*sqrt(b*c^2 - a*c*d)*log((a*c + (2*b*c - a*d)*x + 2*sqrt(b*c^2 - a*c*
d)*sqrt(b*x^2 + a*x))/(d*x + c)) - 2*(2*a*b^2*c^5 - 4*a^2*b*c^4*d + 2*a^3*
c^3*d^2 - (4*b^3*c^4*d + 4*a*b^2*c^3*d^2 - 23*a^2*b*c^2*d^3 + 15*a^3*c*d^4
)*x^2 - 2*(2*b^3*c^5 + a*b^2*c^4*d - 8*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x)*s
qrt(b*x^2 + a*x))/((a^2*b^2*c^6*d - 2*a^3*b*c^5*d^2 + a^4*c^4*d^3)*x^3 + (
a^2*b^2*c^7 - 2*a^3*b*c^6*d + a^4*c^5*d^2)*x^2), -1/3*(3*((6*a^2*b*c*d^3 -
 5*a^3*d^4)*x^3 + (6*a^2*b*c^2*d^2 - 5*a^3*c*d^3)*x^2)*sqrt(-b*c^2 + a*c*d
)*arctan(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) + (2*a*b^2*
c^5 - 4*a^2*b*c^4*d + 2*a^3*c^3*d^2 - (4*b^3*c^4*d + 4*a*b^2*c^3*d^2 - 23*
a^2*b*c^2*d^3 + 15*a^3*c*d^4)*x^2 - 2*(2*b^3*c^5 + a*b^2*c^4*d - 8*a^2*b*c
^3*d^2 + 5*a^3*c^2*d^3)*x)*sqrt(b*x^2 + a*x))/((a^2*b^2*c^6*d - 2*a^3*b*c^
5*d^2 + a^4*c^4*d^3)*x^3 + (a^2*b^2*c^7 - 2*a^3*b*c^6*d + a^4*c^5*d^2)*x^2
)]
```

Sympy [F]

$$\int \frac{1}{x^2(c+dx)^2\sqrt{ax+bx^2}}\,dx = \int \frac{1}{x^2\sqrt{x\,(a+bx)}\,(c+dx)^2}\,dx$$

input
```
integrate(1/x**2/(d*x+c)**2/(b*x**2+a*x)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(x*(a + b*x))*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{x^2(c+dx)^2\sqrt{ax+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+ax}(dx+c)^2x^2}\,dx$$

input
```
integrate(1/x^2/(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a*x)*(d*x + c)^2*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1082 vs. $2(187) = 374$.

Time = 6.86 (sec) , antiderivative size = 1082, normalized size of antiderivative = 5.23

$$\int \frac{1}{x^2(c+dx)^2\sqrt{ax+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(d*x+c)^2/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
-1/10*(10*sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d/(d*x + c) - a
*c*d/(d*x + c)^2)*c^3*d^4*sgn(1/(d*x + c))*sgn(d)/(b*c^7*sgn(1/(d*x + c))^
2*sgn(d)^2 - a*c^6*d*sgn(1/(d*x + c))^2*sgn(d)^2) - (6*b*c*d^5*log(abs(32*
b^3*c^3*d - 48*a*b^2*c^2*d^2 + 18*a^2*b*c*d^3 - a^3*d^4 - 32*sqrt(b*c^2 -
a*c*d)*b^(5/2)*c^2*abs(d) + 32*sqrt(b*c^2 - a*c*d)*a*b^(3/2)*c*d*abs(d) -
6*sqrt(b*c^2 - a*c*d)*a^2*sqrt(b)*d^2*abs(d))) - 5*a*d^6*log(abs(32*b^3*c^
3*d - 48*a*b^2*c^2*d^2 + 18*a^2*b*c*d^3 - a^3*d^4 - 32*sqrt(b*c^2 - a*c*d)
*b^(5/2)*c^2*abs(d) + 32*sqrt(b*c^2 - a*c*d)*a*b^(3/2)*c*d*abs(d) - 6*sqrt
(b*c^2 - a*c*d)*a^2*sqrt(b)*d^2*abs(d))) + 10*sqrt(b*c^2 - a*c*d)*sqrt(b)*
d^4*abs(d))*sgn(1/(d*x + c))*sgn(d)/(sqrt(b*c^2 - a*c*d)*b*c^4*abs(d) - sq
rt(b*c^2 - a*c*d)*a*c^3*d*abs(d)) + (6*b*c*d^5 - 5*a*d^6)*log(abs(-2*sqrt(
b*c^2 - a*c*d)*c^2*(sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d/(d*
x + c) - a*c*d/(d*x + c)^2) + sqrt(b*c^2*d^2 - a*c*d^3)/((d*x + c)*d))^5*a
bs(d) + 2*b^3*c^3*d - 5*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3 - a^3*d^4 - 4*(5*b*c
^2 - 4*a*c*d)*sqrt(b*c^2 - a*c*d)*(sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x +
 c)^2 + a*d/(d*x + c) - a*c*d/(d*x + c)^2) + sqrt(b*c^2*d^2 - a*c*d^3)/((d
*x + c)*d))^3*abs(d) + (10*b*c^3*d - 9*a*c^2*d^2)*(sqrt(b - 2*b*c/(d*x + c
) + b*c^2/(d*x + c)^2 + a*d/(d*x + c) - a*c*d/(d*x + c)^2) + sqrt(b*c^2*d^
2 - a*c*d^3)/((d*x + c)*d))^4 - 2*(5*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*sqrt
(b*c^2 - a*c*d)*(sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d/(d*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(c+dx)^2\sqrt{ax+bx^2}}\,dx = \int \frac{1}{x^2\,\sqrt{b\,x^2+a\,x}\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(1/(x^2*(a*x + b*x^2)^(1/2)*(c + d*x)^2),x)
```

output
```
int(1/(x^2*(a*x + b*x^2)^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 1291, normalized size of antiderivative = 6.24

$$\int \frac{1}{x^2(c+dx)^2\sqrt{ax+bx^2}}\,dx = \text{Too large to display}$$

input
```
int(1/x^2/(d*x+c)^2/(b*x^2+a*x)^(1/2),x)
```

output
```
( - 75*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*
x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**4*c*d**4*x**2 - 75*sqr
t(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(
x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**4*d**5*x**3 + 150*sqrt(c)*sqrt(a
*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*
sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*b*c**2*d**3*x**2 + 150*sqrt(c)*sqrt(a*d -
 b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt
(b))/(sqrt(c)*sqrt(b)))*a**3*b*c*d**4*x**3 - 72*sqrt(c)*sqrt(a*d - b*c)*at
an((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sq
rt(c)*sqrt(b)))*a**2*b**2*c**3*d**2*x**2 - 72*sqrt(c)*sqrt(a*d - b*c)*atan
((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt
(c)*sqrt(b)))*a**2*b**2*c**2*d**3*x**3 - 75*sqrt(c)*sqrt(a*d - b*c)*atan((
sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c
)*sqrt(b)))*a**4*c*d**4*x**2 - 75*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d -
 b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b))
)*a**4*d**5*x**3 + 150*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqr
t(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*b*c*
*2*d**3*x**2 + 150*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)
*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*b*c*d**4
*x**3 - 72*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt...
```

3.158 $\int \frac{1}{x^2\sqrt{x(a+bx)}(c+dx)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 207

$$\int \frac{1}{x^2\sqrt{x(a+bx)}(c+dx)^2}\,dx = -\frac{(2bc-5ad)\sqrt{ax+bx^2}}{3ac^2(bc-ad)x^2} + \frac{(4b^2c^2+8abcd-15a^2d^2)\sqrt{ax+bx^2}}{3a^2c^3(bc-ad)x} - \frac{d\sqrt{ax+bx^2}}{c(bc-ad)x^2(c+dx)} + \frac{d^2(6bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{7/2}(bc-ad)^{3/2}}$$

output
```
-1/3*(-5*a*d+2*b*c)*(b*x^2+a*x)^(1/2)/a/c^2/(-a*d+b*c)/x^2+1/3*(-15*a^2*d^
2+8*a*b*c*d+4*b^2*c^2)*(b*x^2+a*x)^(1/2)/a^2/c^3/(-a*d+b*c)/x-d*(b*x^2+a*x
)^(1/2)/c/(-a*d+b*c)/x^2/(d*x+c)+d^2*(-5*a*d+6*b*c)*arctanh((-a*d+b*c)^(1/
2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(7/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^2\sqrt{x(a+bx)}(c+dx)^2}\,dx$$
$$= \frac{\frac{\sqrt{c}(a+bx)\left(-4b^2c^2x(c+dx)+2abc\left(c^2-3cdx-4d^2x^2\right)+a^2d\left(-2c^2+10cdx+15d^2x^2\right)\right)}{a^2(-bc+ad)(c+dx)}+\frac{3d^2(6bc-5ad)x^{3/2}\sqrt{a+bx}\arctan\left(\frac{-d\sqrt{x}\sqrt{a+bx}+\sqrt{b}(c+dx)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{3c^{7/2}x\sqrt{x(a+bx)}}$$

input
```
Integrate[1/(x^2*Sqrt[x*(a + b*x)]*(c + d*x)^2),x]
```

output
```
((Sqrt[c]*(a + b*x)*(-4*b^2*c^2*x*(c + d*x) + 2*a*b*c*(c^2 - 3*c*d*x - 4*d
^2*x^2) + a^2*d*(-2*c^2 + 10*c*d*x + 15*d^2*x^2)))/(a^2*(-(b*c) + a*d)*(c
+ d*x)) + (3*d^2*(6*b*c - 5*a*d)*x^(3/2)*Sqrt[a + b*x]*ArcTan[(-(d*Sqrt[x]
*Sqrt[a + b*x]) + Sqrt[b]*(c + d*x))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(-(b*c
) + a*d)^(3/2))/(3*c^(7/2)*x*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {2048, 1261, 114, 27, 169, 27, 169, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{x(a+bx)}(c+dx)^2}\,dx$$
$$\Big\downarrow \text{2048}$$
$$\int \frac{1}{x^2\sqrt{ax+bx^2}(c+dx)^2}dx$$
$$\Big\downarrow \text{1261}$$
$$\frac{\sqrt{x}\sqrt{a+bx}\int \frac{1}{x^{5/2}\sqrt{a+bx}(c+dx)^2}dx}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{114}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{\int -\frac{2bc-5ad-4bdx}{2x^{5/2}\sqrt{a+bx}(c+dx)}dx}{c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\int \frac{2bc-5ad-4bdx}{x^{5/2}\sqrt{a+bx}(c+dx)}dx}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{169}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{2\int \frac{4b^2c^2+8abdc-15a^2d^2+2bd(2bc-5ad)x}{2x^{3/2}\sqrt{a+bx}(c+dx)}dx}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{\int \frac{4b^2c^2+8abdc-15a^2d^2+2bd(2bc-5ad)x}{x^{3/2}\sqrt{a+bx}(c+dx)}dx}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{169}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{-\frac{2\int \frac{3a^2d^2(6bc-5ad)}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{ac}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{15ad^2}{c}+8bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{-\frac{3ad^2(6bc-5ad)\int \frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{c}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{15ad^2}{c}+8bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{104}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{-\frac{-\frac{6ad^2(6bc-5ad)\int \frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{15ad^2}{c}+8bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{-\frac{-\frac{6ad^2(6bc-5ad)\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{15ad^2}{c}+8bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

input
```
Int[1/(x^2*Sqrt[x*(a + b*x)]*(c + d*x)^2),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(-((d*Sqrt[a + b*x])/(c*(b*c - a*d)*x^(3/2)*(c + d*
x))) + ((-2*(2*b*c - 5*a*d)*Sqrt[a + b*x])/(3*a*c*x^(3/2)) - ((-2*((4*b^2*
c)/a + 8*b*d - (15*a*d^2)/c)*Sqrt[a + b*x])/Sqrt[x] - (6*a*d^2*(6*b*c - 5*
a*d)*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(c^(3/2)*
Sqrt[b*c - a*d]))/(3*a*c))/(2*c*(b*c - a*d))))/Sqrt[a*x + b*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{-2\left(\left(2b^2x-ab\right)c^3+d(2bx+a)(bx+a)c^2-5a\,d^2x\left(-\frac{4bx}{5}+a\right)c-\frac{15a^2d^3x^2}{2}\right)\sqrt{x(bx+a)}\,\sqrt{c(ad-bc)}-3d^2x^2a^2\arctan\left(\frac{\sqrt{x(bx+}}{x\sqrt{c(ad}}\right.}{3\sqrt{c(ad-bc)}\,c^3x^2(ad-bc)(dx+c)a^2}$
risch	$-\frac{2(bx+a)(-6adx-2cbx+ac)}{3a^2c^3\sqrt{x(bx+a)}\,x}+\frac{d^2\left(\frac{c\left(\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{c(ad-bc)\left(x+\frac{c}{d}\right)}-\frac{(ad-2bc)d\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)(}{d}}{}\right.}{}\right.}{d^2}\right.}{}$
default	$\frac{-\frac{2\sqrt{b\,x^2+ax}}{3a\,x^2}+\frac{4b\sqrt{b\,x^2+ax}}{3a^2x}}{c^2}+\frac{\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{c(ad-bc)\left(x+\frac{c}{d}\right)}-\frac{(ad-2bc)d\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{}}{}\right.}{2c(ad-bc}}{c^2}$

input
```
int(1/x^2/(x*(b*x+a))^(1/2)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-2*((2*b^2*x-a*b)*c^3+d*(2*b*x+a)*(b*x+a)*c^2-5*a*d^2*x*(-4/5*b*x+a)*
c-15/2*a^2*d^3*x^2)*(x*(b*x+a))^(1/2)*(c*(a*d-b*c))^(1/2)-3*d^2*x^2*a^2*ar
ctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))*(d*x+c)*(5*a*d-6*b*c))/(c*
(a*d-b*c))^(1/2)/c^3/x^2/(a*d-b*c)/(d*x+c)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 646, normalized size of antiderivative = 3.12

$$\int \frac{1}{x^2\sqrt{x(a+bx)}(c+dx)^2}\,dx$$

$$= \left[\frac{3\left((6\,a^2bcd^3-5\,a^3d^4)x^3+(6\,a^2bc^2d^2-5\,a^3cd^3)x^2\right)\sqrt{bc^2-acd}\log\left(\frac{ac+(2\,bc-ad)x+2\,\sqrt{bc^2-acd}\sqrt{bx^2+ax}}{dx+c}\right)-2\,(\ldots}{6\,\left((a^2b^2c^6d-2\,a^3\ldots\right.}\right.$$

$$-\frac{3\left((6\,a^2bcd^3-5\,a^3d^4)x^3+(6\,a^2bc^2d^2-5\,a^3cd^3)x^2\right)\sqrt{-bc^2+acd}\arctan\left(\frac{\sqrt{-bc^2+acd}\sqrt{bx^2+ax}}{bcx+ac}\right)+(2\,ab^2c^5\ldots}{3\,\left((a^2b^2c^6d-2\,a^3bc^5\ldots\right.}$$

input
```
integrate(1/x^2/(x*(b*x+a))^(1/2)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[1/6*(3*((6*a^2*b*c*d^3 - 5*a^3*d^4)*x^3 + (6*a^2*b*c^2*d^2 - 5*a^3*c*d^3)
*x^2)*sqrt(b*c^2 - a*c*d)*log((a*c + (2*b*c - a*d)*x + 2*sqrt(b*c^2 - a*c*
d)*sqrt(b*x^2 + a*x))/(d*x + c)) - 2*(2*a*b^2*c^5 - 4*a^2*b*c^4*d + 2*a^3*
c^3*d^2 - (4*b^3*c^4*d + 4*a*b^2*c^3*d^2 - 23*a^2*b*c^2*d^3 + 15*a^3*c*d^4
)*x^2 - 2*(2*b^3*c^5 + a*b^2*c^4*d - 8*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x)*s
qrt(b*x^2 + a*x))/((a^2*b^2*c^6*d - 2*a^3*b*c^5*d^2 + a^4*c^4*d^3)*x^3 + (
a^2*b^2*c^7 - 2*a^3*b*c^6*d + a^4*c^5*d^2)*x^2), -1/3*(3*((6*a^2*b*c*d^3 -
 5*a^3*d^4)*x^3 + (6*a^2*b*c^2*d^2 - 5*a^3*c*d^3)*x^2)*sqrt(-b*c^2 + a*c*d
)*arctan(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) + (2*a*b^2*
c^5 - 4*a^2*b*c^4*d + 2*a^3*c^3*d^2 - (4*b^3*c^4*d + 4*a*b^2*c^3*d^2 - 23*
a^2*b*c^2*d^3 + 15*a^3*c*d^4)*x^2 - 2*(2*b^3*c^5 + a*b^2*c^4*d - 8*a^2*b*c
^3*d^2 + 5*a^3*c^2*d^3)*x)*sqrt(b*x^2 + a*x))/((a^2*b^2*c^6*d - 2*a^3*b*c^
5*d^2 + a^4*c^4*d^3)*x^3 + (a^2*b^2*c^7 - 2*a^3*b*c^6*d + a^4*c^5*d^2)*x^2
)]
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{x(a+bx)}(c+dx)^2}\,dx = \int \frac{1}{x^2\sqrt{x\,(a+bx)}\,(c+dx)^2}\,dx$$

input
```
integrate(1/x**2/(x*(b*x+a))**(1/2)/(d*x+c)**2,x)
```

output
```
Integral(1/(x**2*sqrt(x*(a + b*x))*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{x(a+bx)}(c+dx)^2}\,dx = \int \frac{1}{\sqrt{(bx+a)x}(dx+c)^2x^2}\,dx$$

input
```
integrate(1/x^2/(x*(b*x+a))^(1/2)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt((b*x + a)*x)*(d*x + c)^2*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1082 vs. $2(187) = 374$.

Time = 8.53 (sec) , antiderivative size = 1082, normalized size of antiderivative = 5.23

$$\int \frac{1}{x^2\sqrt{x(a+bx)}(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(x*(b*x+a))^(1/2)/(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/10*(10*sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d/(d*x + c) - a
*c*d/(d*x + c)^2)*c^3*d^4*sgn(1/(d*x + c))*sgn(d)/(b*c^7*sgn(1/(d*x + c))^
2*sgn(d)^2 - a*c^6*d*sgn(1/(d*x + c))^2*sgn(d)^2) - (6*b*c*d^5*log(abs(32*
b^3*c^3*d - 48*a*b^2*c^2*d^2 + 18*a^2*b*c*d^3 - a^3*d^4 - 32*sqrt(b*c^2 -
a*c*d)*b^(5/2)*c^2*abs(d) + 32*sqrt(b*c^2 - a*c*d)*a*b^(3/2)*c*d*abs(d) -
6*sqrt(b*c^2 - a*c*d)*a^2*sqrt(b)*d^2*abs(d))) - 5*a*d^6*log(abs(32*b^3*c^
3*d - 48*a*b^2*c^2*d^2 + 18*a^2*b*c*d^3 - a^3*d^4 - 32*sqrt(b*c^2 - a*c*d)
*b^(5/2)*c^2*abs(d) + 32*sqrt(b*c^2 - a*c*d)*a*b^(3/2)*c*d*abs(d) - 6*sqrt
(b*c^2 - a*c*d)*a^2*sqrt(b)*d^2*abs(d))) + 10*sqrt(b*c^2 - a*c*d)*sqrt(b)*
d^4*abs(d))*sgn(1/(d*x + c))*sgn(d)/(sqrt(b*c^2 - a*c*d)*b*c^4*abs(d) - sq
rt(b*c^2 - a*c*d)*a*c^3*d*abs(d)) + (6*b*c*d^5 - 5*a*d^6)*log(abs(-2*sqrt(
b*c^2 - a*c*d)*c^2*(sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d/(d*
x + c) - a*c*d/(d*x + c)^2) + sqrt(b*c^2*d^2 - a*c*d^3)/((d*x + c)*d))^5*a
bs(d) + 2*b^3*c^3*d - 5*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3 - a^3*d^4 - 4*(5*b*c
^2 - 4*a*c*d)*sqrt(b*c^2 - a*c*d)*(sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x +
 c)^2 + a*d/(d*x + c) - a*c*d/(d*x + c)^2) + sqrt(b*c^2*d^2 - a*c*d^3)/((d
*x + c)*d))^3*abs(d) + (10*b*c^3*d - 9*a*c^2*d^2)*(sqrt(b - 2*b*c/(d*x + c
) + b*c^2/(d*x + c)^2 + a*d/(d*x + c) - a*c*d/(d*x + c)^2) + sqrt(b*c^2*d^
2 - a*c*d^3)/((d*x + c)*d))^4 - 2*(5*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*sqrt
(b*c^2 - a*c*d)*(sqrt(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d/(d*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{x(a+bx)}(c+dx)^2}\,dx = \int \frac{1}{x^2\,\sqrt{x\,\left(a+b\,x\right)}\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(1/(x^2*(x*(a + b*x))^(1/2)*(c + d*x)^2),x)
```

output
```
int(1/(x^2*(x*(a + b*x))^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 1291, normalized size of antiderivative = 6.24

$$\int \frac{1}{x^2\sqrt{x(a+bx)}(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int(1/x^2/(x*(b*x+a))^(1/2)/(d*x+c)^2,x)
```

output
```
( - 75*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*
x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**4*c*d**4*x**2 - 75*sqr
t(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(
x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**4*d**5*x**3 + 150*sqrt(c)*sqrt(a
*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*
sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*b*c**2*d**3*x**2 + 150*sqrt(c)*sqrt(a*d -
 b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt
(b))/(sqrt(c)*sqrt(b)))*a**3*b*c*d**4*x**3 - 72*sqrt(c)*sqrt(a*d - b*c)*at
an((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sq
rt(c)*sqrt(b)))*a**2*b**2*c**3*d**2*x**2 - 72*sqrt(c)*sqrt(a*d - b*c)*atan
((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt
(c)*sqrt(b)))*a**2*b**2*c**2*d**3*x**3 - 75*sqrt(c)*sqrt(a*d - b*c)*atan((
sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c
)*sqrt(b)))*a**4*c*d**4*x**2 - 75*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d -
 b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b))
)*a**4*d**5*x**3 + 150*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqr
t(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*b*c*
*2*d**3*x**2 + 150*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)
*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*b*c*d**4
*x**3 - 72*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt...
```

3.159 $\int \frac{1}{\sqrt{ax+bx^2}(c^2x^2+2cdx^3+d^2x^4)} dx$

Optimal result

Integrand size = 38, antiderivative size = 207

$$\int \frac{1}{\sqrt{ax+bx^2}\,(c^2x^2+2cdx^3+d^2x^4)}\,dx = -\frac{(2bc-5ad)\sqrt{ax+bx^2}}{3ac^2(bc-ad)x^2} + \frac{(4b^2c^2+8abcd-15a^2d^2)\,\sqrt{ax+bx^2}}{3a^2c^3(bc-ad)x} - \frac{d\sqrt{ax+bx^2}}{c(bc-ad)x^2(c+dx)} + \frac{d^2(6bc-5ad)\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{7/2}(bc-ad)^{3/2}}$$

output
```
-1/3*(-5*a*d+2*b*c)*(b*x^2+a*x)^(1/2)/a/c^2/(-a*d+b*c)/x^2+1/3*(-15*a^2*d^
2+8*a*b*c*d+4*b^2*c^2)*(b*x^2+a*x)^(1/2)/a^2/c^3/(-a*d+b*c)/x-d*(b*x^2+a*x
)^(1/2)/c/(-a*d+b*c)/x^2/(d*x+c)+d^2*(-5*a*d+6*b*c)*arctanh((-a*d+b*c)^(1/
2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(7/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt{ax+bx^2}\,(c^2x^2+2cdx^3+d^2x^4)}\,dx$$
$$= \frac{\frac{\sqrt{c}(a+bx)\left(-4b^2c^2x(c+dx)+2abc\left(c^2-3cdx-4d^2x^2\right)+a^2d\left(-2c^2+10cdx+15d^2x^2\right)\right)}{a^2(-bc+ad)(c+dx)}+\frac{3d^2(6bc-5ad)x^{3/2}\sqrt{a+bx}\arctan\left(\frac{-d\sqrt{x}\sqrt{a+bx}+\sqrt{b}(c+d}{\sqrt{c}\sqrt{-bc+ad}}\right.}{(-bc+ad)^{3/2}}}{3c^{7/2}x\sqrt{x(a+bx)}}$$

input
```
Integrate[1/(Sqrt[a*x + b*x^2]*(c^2*x^2 + 2*c*d*x^3 + d^2*x^4)),x]
```

output
```
((Sqrt[c]*(a + b*x)*(-4*b^2*c^2*x*(c + d*x) + 2*a*b*c*(c^2 - 3*c*d*x - 4*d
^2*x^2) + a^2*d*(-2*c^2 + 10*c*d*x + 15*d^2*x^2)))/(a^2*(-(b*c) + a*d)*(c
+ d*x)) + (3*d^2*(6*b*c - 5*a*d)*x^(3/2)*Sqrt[a + b*x]*ArcTan[(-(d*Sqrt[x]
*Sqrt[a + b*x]) + Sqrt[b]*(c + d*x))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(-(b*c
) + a*d)^(3/2))/(3*c^(7/2)*x*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.12, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2026, 1331, 27, 1261, 114, 27, 169, 27, 169, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax+bx^2}\,(c^2x^2+2cdx^3+d^2x^4)}\,dx$$
$$\Big\downarrow \; 2026$$
$$\int \frac{1}{x^2\sqrt{ax+bx^2}\,(c^2+2cdx+d^2x^2)}dx$$
$$\Big\downarrow \; 1331$$
$$d^2\int \frac{1}{d^2x^2(c+dx)^2\sqrt{bx^2+ax}}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{1}{x^2\sqrt{ax+bx^2}(c+dx)^2}dx$$

$$\Big\downarrow \; 1261$$

$$\frac{\sqrt{x}\sqrt{a+bx}\int \frac{1}{x^{5/2}\sqrt{a+bx}(c+dx)^2}dx}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \; 114$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{\int -\frac{2bc-5ad-4bdx}{2x^{5/2}\sqrt{a+bx}(c+dx)}dx}{c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\int \frac{2bc-5ad-4bdx}{x^{5/2}\sqrt{a+bx}(c+dx)}dx}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \; 169$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{2\int \frac{4b^2c^2+8abdc-15a^2d^2+2bd(2bc-5ad)x}{2x^{3/2}\sqrt{a+bx}(c+dx)}dx}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{\int \frac{4b^2c^2+8abdc-15a^2d^2+2bd(2bc-5ad)x}{x^{3/2}\sqrt{a+bx}(c+dx)}dx}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \; 169$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{-\frac{2\int \frac{3a^2d^2(6bc-5ad)}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{ac}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{15ad^2}{c}+8bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{-\frac{3ad^2(6bc-5ad)\int\frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{c}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{15ad^2}{c}+8bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{104}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{-\frac{6ad^2(6bc-5ad)\int\frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{15ad^2}{c}+8bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{-\frac{6ad^2(6bc-5ad)\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{15ad^2}{c}+8bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

input
```
Int[1/(Sqrt[a*x + b*x^2]*(c^2*x^2 + 2*c*d*x^3 + d^2*x^4)),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(-((d*Sqrt[a + b*x])/(c*(b*c - a*d)*x^(3/2)*(c + d*
x))) + ((-2*(2*b*c - 5*a*d)*Sqrt[a + b*x])/(3*a*c*x^(3/2)) - ((-2*((4*b^2*
c)/a + 8*b*d - (15*a*d^2)/c)*Sqrt[a + b*x])/Sqrt[x] - (6*a*d^2*(6*b*c - 5*
a*d)*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(c^(3/2)*
Sqrt[b*c - a*d]))/(3*a*c))/(2*c*(b*c - a*d))))/Sqrt[a*x + b*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

rule 1331
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((d
_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/c^p   Int[(g +
h*x)^m*(b/2 + c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, m, q}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{-2\left(\left(2b^2x-ab\right)c^3+d(2bx+a)(bx+a)c^2-5a\,d^2x\left(-\frac{4bx}{5}+a\right)c-\frac{15a^2d^3x^2}{2}\right)\sqrt{x(bx+a)}\,\sqrt{c(ad-bc)}-3d^2x^2a^2\arctan\left(\frac{\sqrt{x(bx+\ldots}}{x\sqrt{c(ad\ldots}}\right.}{3\sqrt{c(ad-bc)}\,c^3x^2(ad-bc)(dx+c)a^2}$
risch	$-\frac{2(bx+a)(-6adx-2cbx+ac)}{3a^2c^3\sqrt{x(bx+a)}\,x}+\frac{d^2\left(\frac{c\left(\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{c(ad-bc)\left(x+\frac{c}{d}\right)}-\frac{(ad-2bc)d\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)(\ldots}{d}\ldots}{\ldots}\right.}{\ldots}\right)}{d^2}\ldots\right.}{\ldots}$
default	$\frac{-\frac{2\sqrt{b\,x^2+ax}}{3a\,x^2}+\frac{4b\sqrt{b\,x^2+ax}}{3a^2x}}{c^2}+\frac{\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{c(ad-bc)\left(x+\frac{c}{d}\right)}-\frac{(ad-2bc)d\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{\ldots}}{\ldots}\right.}{2c(ad-bc\ldots}}{c^2}$

input
```
int(1/(b*x^2+a*x)^(1/2)/(d^2*x^4+2*c*d*x^3+c^2*x^2),x,method=_RETURNVERBOS
E)
```

output
```
1/3*(-2*((2*b^2*x-a*b)*c^3+d*(2*b*x+a)*(b*x+a)*c^2-5*a*d^2*x*(-4/5*b*x+a)*
c-15/2*a^2*d^3*x^2)*(x*(b*x+a))^(1/2)*(c*(a*d-b*c))^(1/2)-3*d^2*x^2*a^2*ar
ctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))*(d*x+c)*(5*a*d-6*b*c))/(c*
(a*d-b*c))^(1/2)/c^3/x^2/(a*d-b*c)/(d*x+c)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 646, normalized size of antiderivative = 3.12

$$\int \frac{1}{\sqrt{ax+bx^2}\,(c^2x^2+2cdx^3+d^2x^4)}\,dx$$

$$= \left[\frac{3\,((6\,a^2bcd^3-5\,a^3d^4)x^3+(6\,a^2bc^2d^2-5\,a^3cd^3)x^2)\sqrt{bc^2-acd}\log\left(\frac{ac+(2\,bc-ad)x+2\,\sqrt{bc^2-acd}\sqrt{bx^2+ax}}{dx+c}\right)-2\,(}{6\,((a^2b^2c^6d-2\,a^3 }\right.$$

$$-\frac{3\,((6\,a^2bcd^3-5\,a^3d^4)x^3+(6\,a^2bc^2d^2-5\,a^3cd^3)x^2)\sqrt{-bc^2+acd}\arctan\left(\frac{\sqrt{-bc^2+acd}\sqrt{bx^2+ax}}{bcx+ac}\right)+(2\,ab^2c^5}{3\,((a^2b^2c^6d-2\,a^3bc^5}$$

input
```
integrate(1/(b*x^2+a*x)^(1/2)/(d^2*x^4+2*c*d*x^3+c^2*x^2),x, algorithm="fr
icas")
```

output
```
[1/6*(3*((6*a^2*b*c*d^3 - 5*a^3*d^4)*x^3 + (6*a^2*b*c^2*d^2 - 5*a^3*c*d^3)
*x^2)*sqrt(b*c^2 - a*c*d)*log((a*c + (2*b*c - a*d)*x + 2*sqrt(b*c^2 - a*c*
d)*sqrt(b*x^2 + a*x))/(d*x + c)) - 2*(2*a*b^2*c^5 - 4*a^2*b*c^4*d + 2*a^3*
c^3*d^2 - (4*b^3*c^4*d + 4*a*b^2*c^3*d^2 - 23*a^2*b*c^2*d^3 + 15*a^3*c*d^4
)*x^2 - 2*(2*b^3*c^5 + a*b^2*c^4*d - 8*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x)*s
qrt(b*x^2 + a*x))/((a^2*b^2*c^6*d - 2*a^3*b*c^5*d^2 + a^4*c^4*d^3)*x^3 + (
a^2*b^2*c^7 - 2*a^3*b*c^6*d + a^4*c^5*d^2)*x^2), -1/3*(3*((6*a^2*b*c*d^3 -
 5*a^3*d^4)*x^3 + (6*a^2*b*c^2*d^2 - 5*a^3*c*d^3)*x^2)*sqrt(-b*c^2 + a*c*d
)*arctan(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) + (2*a*b^2*
c^5 - 4*a^2*b*c^4*d + 2*a^3*c^3*d^2 - (4*b^3*c^4*d + 4*a*b^2*c^3*d^2 - 23*
a^2*b*c^2*d^3 + 15*a^3*c*d^4)*x^2 - 2*(2*b^3*c^5 + a*b^2*c^4*d - 8*a^2*b*c
^3*d^2 + 5*a^3*c^2*d^3)*x)*sqrt(b*x^2 + a*x))/((a^2*b^2*c^6*d - 2*a^3*b*c^
5*d^2 + a^4*c^4*d^3)*x^3 + (a^2*b^2*c^7 - 2*a^3*b*c^6*d + a^4*c^5*d^2)*x^2
)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{ax+bx^2}\,(c^2x^2+2cdx^3+d^2x^4)}\,dx = \int \frac{1}{x^2\sqrt{x\,(a+bx)}\,(c+dx)^2}\,dx$$

input
```
integrate(1/(b*x**2+a*x)**(1/2)/(d**2*x**4+2*c*d*x**3+c**2*x**2),x)
```

output
```
Integral(1/(x**2*sqrt(x*(a + b*x))*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{ax+bx^2}\,(c^2x^2+2cdx^3+d^2x^4)}\,dx = \int \frac{1}{(d^2x^4+2\,cdx^3+c^2x^2)\sqrt{bx^2+ax}}\,dx$$

input
```
integrate(1/(b*x^2+a*x)^(1/2)/(d^2*x^4+2*c*d*x^3+c^2*x^2),x, algorithm="ma
xima")
```

output
```
integrate(1/((d^2*x^4 + 2*c*d*x^3 + c^2*x^2)*sqrt(b*x^2 + a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{ax+bx^2}\,(c^2x^2+2cdx^3+d^2x^4)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*x^2+a*x)^(1/2)/(d^2*x^4+2*c*d*x^3+c^2*x^2),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ax+bx^2}\,(c^2x^2+2cdx^3+d^2x^4)}\,dx=\int \frac{1}{\sqrt{b\,x^2+a\,x}\,(c^2\,x^2+2\,c\,d\,x^3+d^2\,x^4)}\,dx$$

input
```
int(1/((a*x + b*x^2)^(1/2)*(c^2*x^2 + d^2*x^4 + 2*c*d*x^3)),x)
```

output
```
int(1/((a*x + b*x^2)^(1/2)*(c^2*x^2 + d^2*x^4 + 2*c*d*x^3)), x)
```

Reduce [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 1291, normalized size of antiderivative = 6.24

$$\int \frac{1}{\sqrt{ax+bx^2}\,(c^2x^2+2cdx^3+d^2x^4)}\,dx=\text{Too large to display}$$

input
```
int(1/(b*x^2+a*x)^(1/2)/(d^2*x^4+2*c*d*x^3+c^2*x^2),x)
```

output
```
( - 75*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*
x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**4*c*d**4*x**2 - 75*sqr
t(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(
x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**4*d**5*x**3 + 150*sqrt(c)*sqrt(a
*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*
sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*b*c**2*d**3*x**2 + 150*sqrt(c)*sqrt(a*d -
 b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt
(b))/(sqrt(c)*sqrt(b)))*a**3*b*c*d**4*x**3 - 72*sqrt(c)*sqrt(a*d - b*c)*at
an((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sq
rt(c)*sqrt(b)))*a**2*b**2*c**3*d**2*x**2 - 72*sqrt(c)*sqrt(a*d - b*c)*atan
((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt
(c)*sqrt(b)))*a**2*b**2*c**2*d**3*x**3 - 75*sqrt(c)*sqrt(a*d - b*c)*atan((
sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c
)*sqrt(b)))*a**4*c*d**4*x**2 - 75*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d -
 b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b))
)*a**4*d**5*x**3 + 150*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqr
t(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*b*c*
*2*d**3*x**2 + 150*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)
*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*b*c*d**4
*x**3 - 72*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt...
```

3.160 $\int \frac{1}{\sqrt{x(a+bx)}\left(c^2x^2+2cdx^3+d^2x^4\right)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 207

$$\int \frac{1}{\sqrt{x(a+bx)}\left(c^2x^2+2cdx^3+d^2x^4\right)}\,dx = -\frac{(2bc-5ad)\sqrt{ax+bx^2}}{3ac^2(bc-ad)x^2} + \frac{\left(4b^2c^2+8abcd-15a^2d^2\right)\sqrt{ax+bx^2}}{3a^2c^3(bc-ad)x} - \frac{d\sqrt{ax+bx^2}}{c(bc-ad)x^2(c+dx)} + \frac{d^2(6bc-5ad)\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{ax+bx^2}}\right)}{c^{7/2}(bc-ad)^{3/2}}$$

output
```
-1/3*(-5*a*d+2*b*c)*(b*x^2+a*x)^(1/2)/a/c^2/(-a*d+b*c)/x^2+1/3*(-15*a^2*d^
2+8*a*b*c*d+4*b^2*c^2)*(b*x^2+a*x)^(1/2)/a^2/c^3/(-a*d+b*c)/x-d*(b*x^2+a*x
)^(1/2)/c/(-a*d+b*c)/x^2/(d*x+c)+d^2*(-5*a*d+6*b*c)*arctanh((-a*d+b*c)^(1/
2)*x/c^(1/2)/(b*x^2+a*x)^(1/2))/c^(7/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt{x(a+bx)}\,(c^2x^2+2cdx^3+d^2x^4)}\,dx$$
$$= \frac{\frac{\sqrt{c}(a+bx)\left(-4b^2c^2x(c+dx)+2abc(c^2-3cdx-4d^2x^2)+a^2d(-2c^2+10cdx+15d^2x^2)\right)}{a^2(-bc+ad)(c+dx)}+\frac{3d^2(6bc-5ad)x^{3/2}\sqrt{a+bx}\arctan\left(\frac{-d\sqrt{x}\sqrt{a+bx}+\sqrt{b}(c+d}{\sqrt{c}\sqrt{-bc+ad}}\right.}{(-bc+ad)^{3/2}}}{3c^{7/2}x\sqrt{x(a+bx)}}$$

input
```
Integrate[1/(Sqrt[x*(a + b*x)]*(c^2*x^2 + 2*c*d*x^3 + d^2*x^4)),x]
```

output
```
((Sqrt[c]*(a + b*x)*(-4*b^2*c^2*x*(c + d*x) + 2*a*b*c*(c^2 - 3*c*d*x - 4*d
^2*x^2) + a^2*d*(-2*c^2 + 10*c*d*x + 15*d^2*x^2)))/(a^2*(-(b*c) + a*d)*(c
+ d*x)) + (3*d^2*(6*b*c - 5*a*d)*x^(3/2)*Sqrt[a + b*x]*ArcTan[(-(d*Sqrt[x]
*Sqrt[a + b*x]) + Sqrt[b]*(c + d*x))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(-(b*c
) + a*d)^(3/2))/(3*c^(7/2)*x*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.12, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2026, 2007, 2048, 1261, 114, 27, 169, 27, 169, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x(a+bx)}\,(c^2x^2+2cdx^3+d^2x^4)}\,dx$$
$$\Big\downarrow \; 2026$$
$$\int \frac{1}{x^2\sqrt{x(a+bx)}\,(c^2+2cdx+d^2x^2)}dx$$
$$\Big\downarrow \; 2007$$
$$\int \frac{1}{x^2\sqrt{x(a+bx)}(c+dx)^2}dx$$

$$\Big\downarrow \ 2048$$

$$\int \frac{1}{x^2\sqrt{ax+bx^2}(c+dx)^2}dx$$

$$\Big\downarrow \ 1261$$

$$\frac{\sqrt{x}\sqrt{a+bx}\int \frac{1}{x^{5/2}\sqrt{a+bx}(c+dx)^2}dx}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 114$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(-\frac{\int -\frac{2bc-5ad-4bdx}{2x^{5/2}\sqrt{a+bx}(c+dx)}dx}{c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{\int \frac{2bc-5ad-4bdx}{x^{5/2}\sqrt{a+bx}(c+dx)}dx}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 169$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{2\int \frac{4b^2c^2+8abdc-15a^2d^2+2bd(2bc-5ad)x}{2x^{3/2}\sqrt{a+bx}(c+dx)}dx}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{\int \frac{4b^2c^2+8abdc-15a^2d^2+2bd(2bc-5ad)x}{x^{3/2}\sqrt{a+bx}(c+dx)}dx}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 169$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{-\frac{2\int \frac{3a^2d^2(6bc-5ad)}{2\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{ac}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{15ad^2}{c}+8bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{-\frac{3ad^2(6bc-5ad)\int\frac{1}{\sqrt{x}\sqrt{a+bx}(c+dx)}dx}{c}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{15ad^2}{c}+8bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \; 104$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{-\frac{6ad^2(6bc-5ad)\int\frac{1}{c-\frac{(bc-ad)x}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{c}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{15ad^2}{c}+8bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

$$\Big\downarrow \; 221$$

$$\frac{\sqrt{x}\sqrt{a+bx}\left(\frac{-\frac{-\frac{6ad^2(6bc-5ad)\text{arctanh}\left(\frac{\sqrt{x}\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx}}\right)}{c^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{a+bx}\left(\frac{4b^2c}{a}-\frac{15ad^2}{c}+8bd\right)}{\sqrt{x}}}{3ac}-\frac{2\sqrt{a+bx}(2bc-5ad)}{3acx^{3/2}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{cx^{3/2}(c+dx)(bc-ad)}\right)}{\sqrt{ax+bx^2}}$$

input

```
Int[1/(Sqrt[x*(a + b*x)]*(c^2*x^2 + 2*c*d*x^3 + d^2*x^4)),x]
```

output

```
(Sqrt[x]*Sqrt[a + b*x]*(-((d*Sqrt[a + b*x])/(c*(b*c - a*d)*x^(3/2)*(c + d*
x))) + ((-2*(2*b*c - 5*a*d)*Sqrt[a + b*x])/(3*a*c*x^(3/2)) - ((-2*((4*b^2*
c)/a + 8*b*d - (15*a*d^2)/c)*Sqrt[a + b*x])/Sqrt[x] - (6*a*d^2*(6*b*c - 5*
a*d)*ArcTanh[(Sqrt[b*c - a*d]*Sqrt[x])/(Sqrt[c]*Sqrt[a + b*x])])/(c^(3/2)*
Sqrt[b*c - a*d]))/(3*a*c))/(2*c*(b*c - a*d))))/Sqrt[a*x + b*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

rule 2007
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^(Ex
pon[Px, x]*p), x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; IntegerQ[p] && Pol
yQ[Px, x] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{-2\left(\left(2b^2x-ab\right)c^3+d(2bx+a)(bx+a)c^2-5a\,d^2x\left(-\frac{4bx}{5}+a\right)c-\frac{15a^2d^3x^2}{2}\right)\sqrt{x(bx+a)}\,\sqrt{c(ad-bc)}-3d^2x^2a^2\arctan\left(\frac{\sqrt{x(bx+}}{x\sqrt{c(ad}}\right.}{3\sqrt{c(ad-bc)}\,c^3x^2(ad-bc)(dx+c)a^2}$
risch	$-\frac{2(bx+a)(-6adx-2cbx+ac)}{3a^2c^3\sqrt{x(bx+a)}\,x}+\frac{d^2\left(\frac{c\left(\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{c(ad-bc)\left(x+\frac{c}{d}\right)}-\frac{(ad-2bc)d\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)(}{d}}{}\right.}{}\right.}{d^2}\right.}{}$
default	$\frac{-\frac{2\sqrt{b\,x^2+ax}}{3a\,x^2}+\frac{4b\sqrt{b\,x^2+ax}}{3a^2x}}{c^2}+\frac{\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}-\frac{c(ad-bc)}{d^2}}}{c(ad-bc)\left(x+\frac{c}{d}\right)}-\frac{(ad-2bc)d\ln\left(\frac{-\frac{2c(ad-bc)}{d^2}+\frac{(ad-2bc)\left(x+\frac{c}{d}\right)}{d}+2\sqrt{}}{}\right.}{2c(ad-bc}}{c^2}$

input
```
int(1/(x*(b*x+a))^(1/2)/(d^2*x^4+2*c*d*x^3+c^2*x^2),x,method=_RETURNVERBOS
E)
```

output
```
1/3*(-2*((2*b^2*x-a*b)*c^3+d*(2*b*x+a)*(b*x+a)*c^2-5*a*d^2*x*(-4/5*b*x+a)*
c-15/2*a^2*d^3*x^2)*(x*(b*x+a))^(1/2)*(c*(a*d-b*c))^(1/2)-3*d^2*x^2*a^2*ar
ctan((x*(b*x+a))^(1/2)/x*c/(c*(a*d-b*c))^(1/2))*(d*x+c)*(5*a*d-6*b*c))/(c*
(a*d-b*c))^(1/2)/c^3/x^2/(a*d-b*c)/(d*x+c)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 646, normalized size of antiderivative = 3.12

$$\int \frac{1}{\sqrt{x(a+bx)}\,(c^2x^2+2cdx^3+d^2x^4)}\,dx$$

$$= \left[\frac{3\left((6\,a^2bcd^3-5\,a^3d^4)x^3+(6\,a^2bc^2d^2-5\,a^3cd^3)x^2\right)\sqrt{bc^2-acd}\log\left(\frac{ac+(2\,bc-ad)x+2\,\sqrt{bc^2-acd}\sqrt{bx^2+ax}}{dx+c}\right)-2\,(\cdots}{6\left((a^2b^2c^6d-2\,a^3\cdots\right.}\right.$$

$$-\frac{3\left((6\,a^2bcd^3-5\,a^3d^4)x^3+(6\,a^2bc^2d^2-5\,a^3cd^3)x^2\right)\sqrt{-bc^2+acd}\arctan\left(\frac{\sqrt{-bc^2+acd}\sqrt{bx^2+ax}}{bcx+ac}\right)+(2\,ab^2c^5\cdots}{3\left((a^2b^2c^6d-2\,a^3bc^5\cdots\right.}$$

input
```
integrate(1/(x*(b*x+a))^(1/2)/(d^2*x^4+2*c*d*x^3+c^2*x^2),x, algorithm="fr
icas")
```

output
```
[1/6*(3*((6*a^2*b*c*d^3 - 5*a^3*d^4)*x^3 + (6*a^2*b*c^2*d^2 - 5*a^3*c*d^3)
*x^2)*sqrt(b*c^2 - a*c*d)*log((a*c + (2*b*c - a*d)*x + 2*sqrt(b*c^2 - a*c*
d)*sqrt(b*x^2 + a*x))/(d*x + c)) - 2*(2*a*b^2*c^5 - 4*a^2*b*c^4*d + 2*a^3*
c^3*d^2 - (4*b^3*c^4*d + 4*a*b^2*c^3*d^2 - 23*a^2*b*c^2*d^3 + 15*a^3*c*d^4
)*x^2 - 2*(2*b^3*c^5 + a*b^2*c^4*d - 8*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x)*s
qrt(b*x^2 + a*x))/((a^2*b^2*c^6*d - 2*a^3*b*c^5*d^2 + a^4*c^4*d^3)*x^3 + (
a^2*b^2*c^7 - 2*a^3*b*c^6*d + a^4*c^5*d^2)*x^2), -1/3*(3*((6*a^2*b*c*d^3 -
 5*a^3*d^4)*x^3 + (6*a^2*b*c^2*d^2 - 5*a^3*c*d^3)*x^2)*sqrt(-b*c^2 + a*c*d
)*arctan(sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a*x)/(b*c*x + a*c)) + (2*a*b^2*
c^5 - 4*a^2*b*c^4*d + 2*a^3*c^3*d^2 - (4*b^3*c^4*d + 4*a*b^2*c^3*d^2 - 23*
a^2*b*c^2*d^3 + 15*a^3*c*d^4)*x^2 - 2*(2*b^3*c^5 + a*b^2*c^4*d - 8*a^2*b*c
^3*d^2 + 5*a^3*c^2*d^3)*x)*sqrt(b*x^2 + a*x))/((a^2*b^2*c^6*d - 2*a^3*b*c^
5*d^2 + a^4*c^4*d^3)*x^3 + (a^2*b^2*c^7 - 2*a^3*b*c^6*d + a^4*c^5*d^2)*x^2
)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{x(a+bx)}\,(c^2x^2+2cdx^3+d^2x^4)}\,dx = \int \frac{1}{x^2\sqrt{x\,(a+bx)}\,(c+dx)^2}\,dx$$

input
```
integrate(1/(x*(b*x+a))**(1/2)/(d**2*x**4+2*c*d*x**3+c**2*x**2),x)
```

output
```
Integral(1/(x**2*sqrt(x*(a + b*x))*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x(a+bx)}\,(c^2x^2+2cdx^3+d^2x^4)}\,dx = \int \frac{1}{(d^2x^4+2\,cdx^3+c^2x^2)\sqrt{(bx+a)x}}\,dx$$

input
```
integrate(1/(x*(b*x+a))^(1/2)/(d^2*x^4+2*c*d*x^3+c^2*x^2),x, algorithm="ma
xima")
```

output
```
integrate(1/((d^2*x^4 + 2*c*d*x^3 + c^2*x^2)*sqrt((b*x + a)*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{x(a+bx)}\,(c^2x^2+2cdx^3+d^2x^4)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(x*(b*x+a))^(1/2)/(d^2*x^4+2*c*d*x^3+c^2*x^2),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x(a+bx)}\,(c^2x^2+2cdx^3+d^2x^4)}\,dx = \int \frac{1}{\sqrt{x\,(a+b\,x)}\,(c^2\,x^2+2\,c\,d\,x^3+d^2\,x^4)}\,dx$$

input
```
int(1/((x*(a + b*x))^(1/2)*(c^2*x^2 + d^2*x^4 + 2*c*d*x^3)),x)
```

output
```
int(1/((x*(a + b*x))^(1/2)*(c^2*x^2 + d^2*x^4 + 2*c*d*x^3)), x)
```

Reduce [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 1291, normalized size of antiderivative = 6.24

$$\int \frac{1}{\sqrt{x(a+bx)}\,(c^2x^2+2cdx^3+d^2x^4)}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(b*x+a))^(1/2)/(d^2*x^4+2*c*d*x^3+c^2*x^2),x)
```

output
```
( - 75*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*
x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**4*c*d**4*x**2 - 75*sqr
t(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(
x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**4*d**5*x**3 + 150*sqrt(c)*sqrt(a
*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*
sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*b*c**2*d**3*x**2 + 150*sqrt(c)*sqrt(a*d -
 b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt
(b))/(sqrt(c)*sqrt(b)))*a**3*b*c*d**4*x**3 - 72*sqrt(c)*sqrt(a*d - b*c)*at
an((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sq
rt(c)*sqrt(b)))*a**2*b**2*c**3*d**2*x**2 - 72*sqrt(c)*sqrt(a*d - b*c)*atan
((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x) - sqrt(x)*sqrt(d)*sqrt(b))/(sqrt
(c)*sqrt(b)))*a**2*b**2*c**2*d**3*x**3 - 75*sqrt(c)*sqrt(a*d - b*c)*atan((
sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c
)*sqrt(b)))*a**4*c*d**4*x**2 - 75*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d -
 b*c) + sqrt(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b))
)*a**4*d**5*x**3 + 150*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqr
t(d)*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*b*c*
*2*d**3*x**2 + 150*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)
*sqrt(a + b*x) + sqrt(x)*sqrt(d)*sqrt(b))/(sqrt(c)*sqrt(b)))*a**3*b*c*d**4
*x**3 - 72*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt...
```

3.161 $\int \frac{x^3(c+dx)}{(ax+bx^2)^{3/2}}\, dx$

Optimal result

Integrand size = 22, antiderivative size = 121

$$\int \frac{x^3(c+dx)}{(ax+bx^2)^{3/2}}\, dx = -\frac{2(bc-ad)x^2}{b^2\sqrt{ax+bx^2}}+\frac{3(4bc-5ad)\sqrt{ax+bx^2}}{4b^3} + \frac{dx\sqrt{ax+bx^2}}{2b^2}-\frac{3a(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{7/2}}$$

output
```
-2*(-a*d+b*c)*x^2/b^2/(b*x^2+a*x)^(1/2)+3/4*(-5*a*d+4*b*c)*(b*x^2+a*x)^(1/
2)/b^3+1/2*d*x*(b*x^2+a*x)^(1/2)/b^2-3/4*a*(-5*a*d+4*b*c)*arctanh(b^(1/2)*
x/(b*x^2+a*x)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.03

$$\int \frac{x^3(c+dx)}{(ax+bx^2)^{3/2}}\, dx = \frac{x^{3/2}\left(\sqrt{b}\sqrt{x}(a+bx)\left(-15a^2d+ab(12c-5dx)+2b^2x(2c+dx)\right)+6a(-4bc+5ad)(\right.}{4b^{7/2}(x(a+bx))^{3/2}}$$

input
```
Integrate[(x^3*(c + d*x))/(a*x + b*x^2)^(3/2),x]
```

output
```
(x^(3/2)*(Sqrt[b]*Sqrt[x]*(a + b*x)*(-15*a^2*d + a*b*(12*c - 5*d*x) + 2*b^
2*x*(2*c + d*x)) + 6*a*(-4*b*c + 5*a*d)*(a + b*x)^(3/2)*ArcTanh[(Sqrt[b]*S
qrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(4*b^(7/2)*(x*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1211, 25, 2192, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx)}{(ax+bx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 1211}$$

$$\frac{\int -\frac{-b^2dx^2-b(bc-ad)x+a(bc-ad)}{\sqrt{bx^2+ax}}dx}{b^3}+\frac{2ax(bc-ad)}{b^3\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2ax(bc-ad)}{b^3\sqrt{ax+bx^2}}-\frac{\int \frac{-b^2dx^2-b(bc-ad)x+a(bc-ad)}{\sqrt{bx^2+ax}}dx}{b^3}$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{2ax(bc-ad)}{b^3\sqrt{ax+bx^2}}-\frac{\frac{\int \frac{b(4a(bc-ad)-b(4bc-7ad)x)}{2\sqrt{bx^2+ax}}dx}{2b}-\frac{1}{2}bdx\sqrt{ax+bx^2}}{b^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2ax(bc-ad)}{b^3\sqrt{ax+bx^2}}-\frac{\frac{1}{4}\int \frac{4a(bc-ad)-b(4bc-7ad)x}{\sqrt{bx^2+ax}}dx-\frac{1}{2}bdx\sqrt{ax+bx^2}}{b^3}$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{2ax(bc-ad)}{b^3\sqrt{ax+bx^2}}-\frac{\frac{1}{4}\left(\frac{3}{2}a(4bc-5ad)\int \frac{1}{\sqrt{bx^2+ax}}dx-\sqrt{ax+bx^2}(4bc-7ad)\right)-\frac{1}{2}bdx\sqrt{ax+bx^2}}{b^3}$$

$$\Big\downarrow \text{ 1091}$$

$$\frac{\frac{2ax(bc-ad)}{b^3\sqrt{ax+bx^2}}-}{\frac{\frac{1}{4}\left(3a(4bc-5ad)\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}-\sqrt{ax+bx^2}(4bc-7ad)\right)-\frac{1}{2}bdx\sqrt{ax+bx^2}}{b^3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2ax(bc-ad)}{b^3\sqrt{ax+bx^2}}-\frac{\frac{1}{4}\left(\frac{3a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(4bc-5ad)}{\sqrt{b}}-\sqrt{ax+bx^2}(4bc-7ad)\right)-\frac{1}{2}bdx\sqrt{ax+bx^2}}{b^3}$$

input
```
Int[(x^3*(c + d*x))/(a*x + b*x^2)^(3/2),x]
```

output
```
(2*a*(b*c - a*d)*x)/(b^3*Sqrt[a*x + b*x^2]) - (-1/2*(b*d*x*Sqrt[a*x + b*x^
2]) + (-((4*b*c - 7*a*d)*Sqrt[a*x + b*x^2]) + (3*a*(4*b*c - 5*a*d)*ArcTanh
[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b])/4)/b^3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{2b^{\frac{5}{2}}dx^3+4x^2b^{\frac{5}{2}}c-5x^2b^{\frac{3}{2}}ad+12b^{\frac{3}{2}}acx+15\sqrt{x(bx+a)}\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)a^2d-12\sqrt{x(bx+a)}\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)abc-}{4b^{\frac{7}{2}}\sqrt{x(bx+a)}}$
risch	$-\frac{(-2bdx+7ad-4bc)x(bx+a)}{4b^3\sqrt{x(bx+a)}}+\frac{a\left(\frac{15ad\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}-12\sqrt{b}\,c\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)-\frac{16(ad-bc)\sqrt{\left(x+\frac{a}{b}\right)^2b-}}{b\left(x+\frac{a}{b}\right)}\right.}{8b^3}$
default	$c\left(\frac{x^2}{b\sqrt{b\,x^2+ax}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^2+ax}}-\frac{a\left(-\frac{1}{b\sqrt{b\,x^2+ax}}+\frac{2bx+a}{ab\sqrt{b\,x^2+ax}}\right)}{2b}+\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)+d\left(\frac{x^3}{2b\sqrt{b\,x^2+ax}}\right.$

input
```
int(x^3*(d*x+c)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*b^(5/2)*d*x^3+4*x^2*b^(5/2)*c-5*x^2*b^(3/2)*a*d+12*b^(3/2)*a*c*x+15
*(x*(b*x+a))^(1/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*a^2*d-12*(x*(b*x+a
))^(1/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*a*b*c-15*b^(1/2)*a^2*d*x)/b^
(7/2)/(x*(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.15

$$\int \frac{x^3(c+dx)}{(ax+bx^2)^{3/2}}\,dx=\left[-\frac{3\left(4\,a^2bc-5\,a^3d+\left(4\,ab^2c-5\,a^2bd\right)x\right)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,(2}{8\left(b^5x+ab^4\right)}\right.$$

input
```
integrate(x^3*(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(3*(4*a^2*b*c - 5*a^3*d + (4*a*b^2*c - 5*a^2*b*d)*x)*sqrt(b)*log(2*b
*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*(2*b^3*d*x^2 + 12*a*b^2*c - 15*a
^2*b*d + (4*b^3*c - 5*a*b^2*d)*x)*sqrt(b*x^2 + a*x))/(b^5*x + a*b^4), 1/4*
(3*(4*a^2*b*c - 5*a^3*d + (4*a*b^2*c - 5*a^2*b*d)*x)*sqrt(-b)*arctan(sqrt(
b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (2*b^3*d*x^2 + 12*a*b^2*c - 15*a^2*b*d
+ (4*b^3*c - 5*a*b^2*d)*x)*sqrt(b*x^2 + a*x))/(b^5*x + a*b^4)]
```

Sympy [F]

$$\int \frac{x^3(c+dx)}{(ax+bx^2)^{3/2}}\,dx = \int \frac{x^3(c+dx)}{(x\,(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(d*x+c)/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral(x**3*(c + d*x)/(x*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.35

$$\begin{aligned}\int \frac{x^3(c+dx)}{(ax+bx^2)^{3/2}}\,dx &= \frac{dx^3}{2\sqrt{bx^2+ax}b} + \frac{cx^2}{\sqrt{bx^2+ax}b} \\ &- \frac{5\,adx^2}{4\sqrt{bx^2+ax}b^2} + \frac{3\,acx}{\sqrt{bx^2+ax}b^2} - \frac{15\,a^2dx}{4\sqrt{bx^2+ax}b^3} \\ &- \frac{3\,ac\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{2\,b^{\frac{5}{2}}} + \frac{15\,a^2d\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{7}{2}}}\end{aligned}$$

input
```
integrate(x^3*(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
1/2*d*x^3/(sqrt(b*x^2 + a*x)*b) + c*x^2/(sqrt(b*x^2 + a*x)*b) - 5/4*a*d*x^
2/(sqrt(b*x^2 + a*x)*b^2) + 3*a*c*x/(sqrt(b*x^2 + a*x)*b^2) - 15/4*a^2*d*x
/(sqrt(b*x^2 + a*x)*b^3) - 3/2*a*c*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqr
t(b))/b^(5/2) + 15/8*a^2*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^
(7/2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.12

$$\int \frac{x^3(c+dx)}{(ax+bx^2)^{3/2}}\,dx = \frac{1}{4}\sqrt{bx^2+ax}\left(\frac{2\,dx}{b^2}+\frac{4\,b^6c-7\,ab^5d}{b^8}\right) + \frac{3\,(4\,abc-5\,a^2d)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,b^{\frac{7}{2}}} + \frac{2\left(a^2b^{\frac{3}{2}}c-a^3\sqrt{b}d\right)}{\left(\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right)b^4}$$

input
```
integrate(x^3*(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
1/4*sqrt(b*x^2 + a*x)*(2*d*x/b^2 + (4*b^6*c - 7*a*b^5*d)/b^8) + 3/8*(4*a*b
*c - 5*a^2*d)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(7
/2) + 2*(a^2*b^(3/2)*c - a^3*sqrt(b)*d)/(((sqrt(b)*x - sqrt(b*x^2 + a*x))*
sqrt(b) + a)*b^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx)}{(ax+bx^2)^{3/2}}\,dx = \int \frac{x^3\,(c+d\,x)}{{\left(b\,x^2+a\,x\right)}^{3/2}}\,dx$$

input
```
int((x^3*(c + d*x))/(a*x + b*x^2)^(3/2),x)
```

output
```
int((x^3*(c + d*x))/(a*x + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.26

$$\int \frac{x^3(c+dx)}{(ax+bx^2)^{3/2}}\,dx = \frac{15\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2d - 12\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)abc - 10\sqrt{b}}{}$$

input

```
int(x^3*(d*x+c)/(b*x^2+a*x)^(3/2),x)
```

output

```
(15*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a
**2*d - 12*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqr
t(a))*a*b*c - 10*sqrt(b)*sqrt(a + b*x)*a**2*d + 9*sqrt(b)*sqrt(a + b*x)*a*
b*c - 15*sqrt(x)*a**2*b*d + 12*sqrt(x)*a*b**2*c - 5*sqrt(x)*a*b**2*d*x + 4
*sqrt(x)*b**3*c*x + 2*sqrt(x)*b**3*d*x**2)/(4*sqrt(a + b*x)*b**4)
```

3.162 $\int \frac{x^2(c+dx)}{(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 82

$$\int \frac{x^2(c+dx)}{(ax+bx^2)^{3/2}}\,dx = -\frac{2(bc-ad)x}{b^2\sqrt{ax+bx^2}}+\frac{d\sqrt{ax+bx^2}}{b^2}+\frac{(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{5/2}}$$

output
```
-2*(-a*d+b*c)*x/b^2/(b*x^2+a*x)^(1/2)+d*(b*x^2+a*x)^(1/2)/b^2+(-3*a*d+2*b*
c)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.26

$$\int \frac{x^2(c+dx)}{(ax+bx^2)^{3/2}}\,dx = \frac{x^{3/2}\left(\sqrt{b}\sqrt{x}(a+bx)(-2bc+3ad+bdx)+2(2bc-3ad)(a+bx)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a}}\right.\right.}{b^{5/2}(x(a+bx))^{3/2}}$$

input
```
Integrate[(x^2*(c + d*x))/(a*x + b*x^2)^(3/2),x]
```

output
```
(x^(3/2)*(Sqrt[b]*Sqrt[x]*(a + b*x)*(-2*b*c + 3*a*d + b*d*x) + 2*(2*b*c -
3*a*d)*(a + b*x)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x]
)]))/(b^(5/2)*(x*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1211, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx)}{(ax+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 1211}$$

$$\frac{\int \frac{bc-ad+bdx}{\sqrt{bx^2+ax}}dx}{b^2}-\frac{2x(bc-ad)}{b^2\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{\frac{1}{2}(2bc-3ad)\int \frac{1}{\sqrt{bx^2+ax}}dx+d\sqrt{ax+bx^2}}{b^2}-\frac{2x(bc-ad)}{b^2\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{ 1091}$$

$$\frac{(2bc-3ad)\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}+d\sqrt{ax+bx^2}}{b^2}-\frac{2x(bc-ad)}{b^2\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(2bc-3ad)}{\sqrt{b}}+d\sqrt{ax+bx^2}}{b^2}-\frac{2x(bc-ad)}{b^2\sqrt{ax+bx^2}}$$

input
```
Int[(x^2*(c + d*x))/(a*x + b*x^2)^(3/2),x]
```

output
```
(-2*(b*c - a*d)*x)/(b^2*Sqrt[a*x + b*x^2]) + (d*Sqrt[a*x + b*x^2] + ((2*b*
c - 3*a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b])/b^2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{\frac{2(ad-bc)x}{\sqrt{x(bx+a)}}+d\sqrt{x(bx+a)}-\frac{(3ad-2bc)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{\sqrt{b}}}{b^2}$
risch	$\frac{dx(bx+a)}{b^2\sqrt{x(bx+a)}}-\frac{\frac{3ad\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}-2\sqrt{b}\,c\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)-\frac{4(ad-bc)\sqrt{\left(x+\frac{a}{b}\right)^2b-a\left(x+\frac{a}{b}\right)}}{b\left(x+\frac{a}{b}\right)}}{2b^2}$
default	$c\left(-\frac{x}{b\sqrt{b\,x^2+ax}}-\frac{a\left(-\frac{1}{b\sqrt{b\,x^2+ax}}+\frac{2bx+a}{ab\sqrt{b\,x^2+ax}}\right)}{2b}+\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{b^{\frac{3}{2}}}\right)+d\left(\frac{x^2}{b\sqrt{b\,x^2+ax}}-\frac{3a\left(-\frac{}{b\sqrt{b\,x}}\right.}{}\right.$

input
```
int(x^2*(d*x+c)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(2*(a*d-b*c)*x/(x*(b*x+a))^(1/2)+d*(x*(b*x+a))^(1/2)-(3*a*d-2*b*c)/b
^(1/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.50

$$\int \frac{x^2(c+dx)}{(ax+bx^2)^{3/2}}\,dx = \left[-\frac{(2\,abc-3\,a^2d+(2\,b^2c-3\,abd)x)\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,(b^2dx}{2\,(b^4x+ab^3)}\right.$$

$$\left.-\frac{(2\,abc-3\,a^2d+(2\,b^2c-3\,abd)x)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(b^2dx-2\,b^2c+3\,abd)\sqrt{bx^2+ax}}{b^4x+ab^3}\right]$$

input
```
integrate(x^2*(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*((2*a*b*c - 3*a^2*d + (2*b^2*c - 3*a*b*d)*x)*sqrt(b)*log(2*b*x + a -
 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*(b^2*d*x - 2*b^2*c + 3*a*b*d)*sqrt(b*x^2
 + a*x))/(b^4*x + a*b^3), -((2*a*b*c - 3*a^2*d + (2*b^2*c - 3*a*b*d)*x)*sq
rt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (b^2*d*x - 2*b^2*c +
 3*a*b*d)*sqrt(b*x^2 + a*x))/(b^4*x + a*b^3)]
```

Sympy [F]

$$\int \frac{x^2(c+dx)}{(ax+bx^2)^{3/2}}\,dx = \int \frac{x^2(c+dx)}{(x\,(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(d*x+c)/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral(x**2*(c + d*x)/(x*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.40

$$\int \frac{x^2(c+dx)}{(ax+bx^2)^{3/2}}\,dx = \frac{dx^2}{\sqrt{bx^2+ax}b} - \frac{2\,cx}{\sqrt{bx^2+ax}b} + \frac{3\,adx}{\sqrt{bx^2+ax}b^2} + \frac{c\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{b^{\frac{3}{2}}} - \frac{3\,ad\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{2\,b^{\frac{5}{2}}}$$

input
```
integrate(x^2*(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
d*x^2/(sqrt(b*x^2 + a*x)*b) - 2*c*x/(sqrt(b*x^2 + a*x)*b) + 3*a*d*x/(sqrt(
b*x^2 + a*x)*b^2) + c*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(3/2)
 - 3/2*a*d*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.30

$$\int \frac{x^2(c+dx)}{(ax+bx^2)^{3/2}}\,dx = \frac{\sqrt{bx^2+ax}d}{b^2} - \frac{(2\,bc-3\,ad)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{2\,b^{\frac{5}{2}}} - \frac{2\left(ab^{\frac{3}{2}}c-a^2\sqrt{b}d\right)}{\left(\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right)b^3}$$

input
```
integrate(x^2*(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
sqrt(b*x^2 + a*x)*d/b^2 - 1/2*(2*b*c - 3*a*d)*log(abs(2*(sqrt(b)*x - sqrt(
b*x^2 + a*x))*sqrt(b) + a))/b^(5/2) - 2*(a*b^(3/2)*c - a^2*sqrt(b)*d)/(((s
qrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)*b^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx)}{(ax+bx^2)^{3/2}}\,dx = \int \frac{x^2\,(c+d\,x)}{{(b\,x^2+a\,x)}^{3/2}}\,dx$$

input
```
int((x^2*(c + d*x))/(a*x + b*x^2)^(3/2),x)
```

output
```
int((x^2*(c + d*x))/(a*x + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.49

$$\int \frac{x^2(c+dx)}{(ax+bx^2)^{3/2}}\,dx = \frac{-12\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)ad + 8\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)bc + 9\sqrt{b}\,\surd}{4\sqrt{bx+a}\,b^3}$$

input
```
int(x^2*(d*x+c)/(b*x^2+a*x)^(3/2),x)
```

output
```
( - 12*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)
)*a*d + 8*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt
(a))*b*c + 9*sqrt(b)*sqrt(a + b*x)*a*d - 8*sqrt(b)*sqrt(a + b*x)*b*c + 12*
sqrt(x)*a*b*d - 8*sqrt(x)*b**2*c + 4*sqrt(x)*b**2*d*x)/(4*sqrt(a + b*x)*b*
*3)
```

3.163 $\int \frac{x(c+dx)}{(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \frac{x(c+dx)}{(ax+bx^2)^{3/2}}\,dx = \frac{2(bc-ad)x}{ab\sqrt{ax+bx^2}} + \frac{2d\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{3/2}}$$

output
```
2*(-a*d+b*c)*x/a/b/(b*x^2+a*x)^(1/2)+2*d*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/
2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.28

$$\int \frac{x(c+dx)}{(ax+bx^2)^{3/2}}\,dx = -\frac{2\left(\sqrt{b}(-bc+ad)x + ad\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)\right)}{ab^{3/2}\sqrt{x(a+bx)}}$$

input
```
Integrate[(x*(c + d*x))/(a*x + b*x^2)^(3/2),x]
```

output
```
(-2*(Sqrt[b]*(-(b*c) + a*d)*x + a*d*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sq
rt[x]) + Sqrt[a + b*x]]))/(a*b^(3/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1211, 27, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx)}{(ax+bx^2)^{3/2}}dx$$
$$\Big\downarrow \text{1211}$$
$$\frac{\int \frac{d}{\sqrt{bx^2+ax}}dx}{b}+\frac{2x(bc-ad)}{ab\sqrt{ax+bx^2}}$$
$$\Big\downarrow \text{27}$$
$$\frac{d\int \frac{1}{\sqrt{bx^2+ax}}dx}{b}+\frac{2x(bc-ad)}{ab\sqrt{ax+bx^2}}$$
$$\Big\downarrow \text{1091}$$
$$\frac{2d\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{b}+\frac{2x(bc-ad)}{ab\sqrt{ax+bx^2}}$$
$$\Big\downarrow \text{219}$$
$$\frac{2d\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{3/2}}+\frac{2x(bc-ad)}{ab\sqrt{ax+bx^2}}$$

input
```
Int[(x*(c + d*x))/(a*x + b*x^2)^(3/2),x]
```

output
```
(2*(b*c - a*d)*x)/(a*b*Sqrt[a*x + b*x^2]) + (2*d*ArcTanh[(Sqrt[b]*x)/Sqrt[
a*x + b*x^2]])/b^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$\frac{2 b^{\frac{3}{2}} c x -2 a d x \sqrt{b}+2 \,\operatorname{arctanh}\left(\frac{\sqrt{x \left(b x +a \right)}}{x \sqrt{b}}\right) a d \sqrt{x \left(b x +a \right)}}{b^{\frac{3}{2}} \sqrt{x \left(b x +a \right)}\, a}$
default	$c \left(-\frac{1}{b \sqrt{b \,x^{2}+a x}}+\frac{2 b x +a}{a b \sqrt{b \,x^{2}+a x}}\right)+d \left(-\frac{x}{b \sqrt{b \,x^{2}+a x}}-\frac{a \left(-\frac{1}{b \sqrt{b \,x^{2}+a x}}+\frac{2 b x +a}{a b \sqrt{b \,x^{2}+a x}}\right)}{2 b}+\frac{\ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b \,x^{2}+a x}\right.}{b^{\frac{3}{2}}}\right.$

input
```
int(x*(d*x+c)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(2*b^(3/2)*c*x-2*a*d*x*b^(1/2)+2*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*a*d*
(x*(b*x+a))^(1/2))/b^(3/2)/(x*(b*x+a))^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.77

$$\int \frac{x(c+dx)}{(ax+bx^2)^{3/2}}\,dx = \left[\frac{(abdx+a^2d)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(b^2c-abd)\sqrt{bx^2+ax}}{ab^3x+a^2b^2},\right.$$
$$\left.-\frac{2\left((abdx+a^2d)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(b^2c-abd)\sqrt{bx^2+ax}\right)}{ab^3x+a^2b^2}\right]$$

input
```
integrate(x*(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[((a*b*d*x + a^2*d)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) +
 2*(b^2*c - a*b*d)*sqrt(b*x^2 + a*x))/(a*b^3*x + a^2*b^2), -2*((a*b*d*x +
a^2*d)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (b^2*c - a*
b*d)*sqrt(b*x^2 + a*x))/(a*b^3*x + a^2*b^2)]
```

Sympy [F]

$$\int \frac{x(c+dx)}{(ax+bx^2)^{3/2}}\,dx = \int \frac{x(c+dx)}{(x\,(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(d*x+c)/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral(x*(c + d*x)/(x*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int \frac{x(c+dx)}{(ax+bx^2)^{3/2}}\,dx = \frac{2\,cx}{\sqrt{bx^2+ax}a} - \frac{2\,dx}{\sqrt{bx^2+ax}b} + \frac{d\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{b^{\frac{3}{2}}}$$

input
```
integrate(x*(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
2*c*x/(sqrt(b*x^2 + a*x)*a) - 2*d*x/(sqrt(b*x^2 + a*x)*b) + d*log(2*b*x +
a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.33

$$\int \frac{x(c+dx)}{(ax+bx^2)^{3/2}}\,dx = -\frac{d\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{b^{\frac{3}{2}}} + \frac{2\left(b^{\frac{3}{2}}c-a\sqrt{b}d\right)}{\left(\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right)b^2}$$

input
```
integrate(x*(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
-d*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(3/2) + 2*(b^
(3/2)*c - a*sqrt(b)*d)/(((sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)*b^2)
```

Mupad [B] (verification not implemented)

Time = 9.77 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07

$$\int \frac{x(c+dx)}{(ax+bx^2)^{3/2}}\,dx = \frac{d\,\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{b^{3/2}}+\frac{2\,c\,x}{a\,\sqrt{x\,\left(a+b\,x\right)}}-\frac{2\,d\,x}{b\,\sqrt{b\,x^2+a\,x}}$$

input
```
int((x*(c + d*x))/(a*x + b*x^2)^(3/2),x)
```

output
```
(d*log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2)))/b^(3/2) + (2*c*x)/(a*(x
*(a + b*x))^(1/2)) - (2*d*x)/(b*(a*x + b*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.38

$$\int \frac{x(c+dx)}{(ax+bx^2)^{3/2}}\,dx = \frac{2\sqrt{b}\,\sqrt{bx+a}\,\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)ad-2\sqrt{b}\,\sqrt{bx+a}\,ad+2\sqrt{b}\,\sqrt{bx+a}\,bc-2\sqrt{x}\,abd}{\sqrt{bx+a}\,a\,b^2}$$

input
```
int(x*(d*x+c)/(b*x^2+a*x)^(3/2),x)
```

output
```
(2*(sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a
*d - sqrt(b)*sqrt(a + b*x)*a*d + sqrt(b)*sqrt(a + b*x)*b*c - sqrt(x)*a*b*d
 + sqrt(x)*b**2*c))/(sqrt(a + b*x)*a*b**2)
```

3.164 $\int \frac{c+dx}{(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \frac{c+dx}{(ax+bx^2)^{3/2}}\,dx=-\frac{2c}{a\sqrt{ax+bx^2}}-\frac{2(2bc-ad)x}{a^2\sqrt{ax+bx^2}}$$

output
```
-2*c/a/(b*x^2+a*x)^(1/2)-2*(-a*d+2*b*c)*x/a^2/(b*x^2+a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.62

$$\int \frac{c+dx}{(ax+bx^2)^{3/2}}\,dx=\frac{2(-ac-2bcx+adx)}{a^2\sqrt{x(a+bx)}}$$

input
```
Integrate[(c + d*x)/(a*x + b*x^2)^(3/2),x]
```

output
```
(2*(-(a*c) - 2*b*c*x + a*d*x))/(a^2*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.69, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(ax+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1158}$$

$$-\frac{2(x(2bc-ad)+ac)}{a^2\sqrt{ax+bx^2}}$$

input
```
Int[(c + d*x)/(a*x + b*x^2)^(3/2),x]
```

output
```
(-2*(a*c + (2*b*c - a*d)*x))/(a^2*Sqrt[a*x + b*x^2])
```

Defintions of rubi rules used

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.62

method	result	size
pseudoelliptic	$\frac{(2dx-2c)a-4cbx}{\sqrt{x(bx+a)}\,a^2}$	30
gosper	$-\frac{2x(bx+a)(-adx+2cbx+ac)}{a^2(b\,x^2+ax)^{\frac{3}{2}}}$	37
orering	$-\frac{2x(bx+a)(-adx+2cbx+ac)}{a^2(b\,x^2+ax)^{\frac{3}{2}}}$	37
trager	$-\frac{2(-adx+2cbx+ac)\sqrt{b\,x^2+ax}}{(bx+a)a^2x}$	41
risch	$-\frac{2c(bx+a)}{a^2\sqrt{x(bx+a)}}+\frac{2x(ad-bc)}{\sqrt{x(bx+a)}\,a^2}$	45
default	$-\frac{2c(2bx+a)}{a^2\sqrt{b\,x^2+ax}}+d\left(-\frac{1}{b\sqrt{b\,x^2+ax}}+\frac{2bx+a}{ab\sqrt{b\,x^2+ax}}\right)$	68

input
```
int((d*x+c)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((2*d*x-2*c)*a-4*c*b*x)/(x*(b*x+a))^(1/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{c+dx}{(ax+bx^2)^{3/2}}\,dx = -\frac{2\,\sqrt{bx^2+ax}(ac+(2\,bc-ad)x)}{a^2bx^2+a^3x}$$

input
```
integrate((d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(b*x^2 + a*x)*(a*c + (2*b*c - a*d)*x)/(a^2*b*x^2 + a^3*x)
```

Sympy [F]

$$\int \frac{c+dx}{\left(ax+bx^2\right)^{3/2}}\,dx=\int \frac{c+dx}{\left(x\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral((c + d*x)/(x*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int \frac{c+dx}{\left(ax+bx^2\right)^{3/2}}\,dx=-\frac{4\,bcx}{\sqrt{bx^2+ax}a^2}+\frac{2\,dx}{\sqrt{bx^2+ax}a}-\frac{2\,c}{\sqrt{bx^2+ax}a}$$

input
```
integrate((d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
-4*b*c*x/(sqrt(b*x^2 + a*x)*a^2) + 2*d*x/(sqrt(b*x^2 + a*x)*a) - 2*c/(sqrt
(b*x^2 + a*x)*a)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.69

$$\int \frac{c+dx}{\left(ax+bx^2\right)^{3/2}}\,dx=-\frac{2\left(\frac{c}{a}+\frac{(2\,bc-ad)x}{a^2}\right)}{\sqrt{bx^2+ax}}$$

input
```
integrate((d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
-2*(c/a + (2*b*c - a*d)*x/a^2)/sqrt(b*x^2 + a*x)
```

Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.65

$$\int \frac{c+dx}{\left(ax+bx^2\right)^{3/2}}\,dx=-\frac{2\,a\,c-2\,a\,d\,x+4\,b\,c\,x}{a^2\,\sqrt{b\,x^2+a\,x}}$$

input
```
int((c + d*x)/(a*x + b*x^2)^(3/2),x)
```

output
```
-(2*a*c - 2*a*d*x + 4*b*c*x)/(a^2*(a*x + b*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.42

$$\int \frac{c+dx}{\left(ax+bx^2\right)^{3/2}}\,dx=\frac{2\sqrt{b}\,\sqrt{bx+a}\,adx-4\sqrt{b}\,\sqrt{bx+a}\,bcx-2\sqrt{x}\,abc+2\sqrt{x}\,abdx-4\sqrt{x}\,b^2cx}{\sqrt{bx+a}\,a^2bx}$$

input
```
int((d*x+c)/(b*x^2+a*x)^(3/2),x)
```

output
```
(2*(sqrt(b)*sqrt(a + b*x)*a*d*x - 2*sqrt(b)*sqrt(a + b*x)*b*c*x - sqrt(x)*
a*b*c + sqrt(x)*a*b*d*x - 2*sqrt(x)*b**2*c*x))/(sqrt(a + b*x)*a**2*b*x)
```

3.165 $\int \frac{c+dx}{x\left(ax+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{c+dx}{x\left(ax+bx^2\right)^{3/2}}\,dx=-\frac{2(4bc-3ad)}{3a^2\sqrt{ax+bx^2}}-\frac{2c}{3ax\sqrt{ax+bx^2}}+\frac{4(4bc-3ad)\sqrt{ax+bx^2}}{3a^3x}$$

output
```
1/3*(6*a*d-8*b*c)/a^2/(b*x^2+a*x)^(1/2)-2/3*c/a/x/(b*x^2+a*x)^(1/2)+4/3*(-
3*a*d+4*b*c)*(b*x^2+a*x)^(1/2)/a^3/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.63

$$\int \frac{c+dx}{x\left(ax+bx^2\right)^{3/2}}\,dx=-\frac{2(-8b^2cx^2+2abx(-2c+3dx)+a^2(c+3dx))}{3a^3x\sqrt{x(a+bx)}}$$

input
```
Integrate[(c + d*x)/(x*(a*x + b*x^2)^(3/2)),x]
```

output
```
(-2*(-8*b^2*c*x^2 + 2*a*b*x*(-2*c + 3*d*x) + a^2*(c + 3*d*x)))/(3*a^3*x*Sq
rt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.70, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1220, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{x\left(ax+bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1220}$$

$$-\frac{(4bc-3ad)\int \frac{1}{(bx^2+ax)^{3/2}}dx}{3a}-\frac{2c}{3ax\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{ 1088}$$

$$\frac{2(a+2bx)(4bc-3ad)}{3a^3\sqrt{ax+bx^2}}-\frac{2c}{3ax\sqrt{ax+bx^2}}$$

input
```
Int[(c + d*x)/(x*(a*x + b*x^2)^(3/2)),x]
```

output
```
(-2*c)/(3*a*x*Sqrt[a*x + b*x^2]) + (2*(4*b*c - 3*a*d)*(a + 2*b*x))/(3*a^3*
Sqrt[a*x + b*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.57

method	result	size
pseudoelliptic	$-\frac{2\left(\left(3dx+c\right)a^{2}-4xb\left(-\frac{3dx}{2}+c\right)a-8b^{2}c\,x^{2}\right)}{3\sqrt{x\left(bx+a\right)}\,a^{3}x}$	49
gosper	$-\frac{2\left(bx+a\right)\left(6abd\,x^{2}-8b^{2}c\,x^{2}+3a^{2}dx-4abcx+a^{2}c\right)}{3a^{3}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}$	58
orering	$-\frac{2\left(bx+a\right)\left(6abd\,x^{2}-8b^{2}c\,x^{2}+3a^{2}dx-4abcx+a^{2}c\right)}{3a^{3}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}$	58
risch	$-\frac{2\left(bx+a\right)\left(3adx-5cbx+ac\right)}{3a^{3}x\sqrt{x\left(bx+a\right)}}-\frac{2b\left(ad-bc\right)x}{\sqrt{x\left(bx+a\right)}\,a^{3}}$	62
trager	$-\frac{2\left(6abd\,x^{2}-8b^{2}c\,x^{2}+3a^{2}dx-4abcx+a^{2}c\right)\sqrt{b\,x^{2}+ax}}{3\left(bx+a\right)a^{3}x^{2}}$	63
default	$-\frac{2d\left(2bx+a\right)}{a^{2}\sqrt{b\,x^{2}+ax}}+c\left(-\frac{2}{3ax\sqrt{b\,x^{2}+ax}}+\frac{8b\left(2bx+a\right)}{3a^{3}\sqrt{b\,x^{2}+ax}}\right)$	70

input
```
int((d*x+c)/x/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(x*(b*x+a))^(1/2)*((3*d*x+c)*a^2-4*x*b*(-3/2*d*x+c)*a-8*b^2*c*x^2)/a^
3/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

$$\int \frac{c+dx}{x\left(ax+bx^2\right)^{3/2}}\,dx = -\frac{2\left(a^2c-2\left(4\,b^2c-3\,abd\right)x^2-\left(4\,abc-3\,a^2d\right)x\right)\sqrt{bx^2+ax}}{3\left(a^3bx^3+a^4x^2\right)}$$

input
```
integrate((d*x+c)/x/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(a^2*c - 2*(4*b^2*c - 3*a*b*d)*x^2 - (4*a*b*c - 3*a^2*d)*x)*sqrt(b*x^
2 + a*x)/(a^3*b*x^3 + a^4*x^2)
```

Sympy [F]

$$\int \frac{c+dx}{x\left(ax+bx^2\right)^{3/2}}\,dx = \int \frac{c+dx}{x\left(x\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)/x/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral((c + d*x)/(x*(x*(a + b*x))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.12

$$\int \frac{c+dx}{x\left(ax+bx^2\right)^{3/2}}\,dx = \frac{16\,b^2cx}{3\,\sqrt{bx^2+ax}a^3} - \frac{4\,bdx}{\sqrt{bx^2+ax}a^2} + \frac{8\,bc}{3\,\sqrt{bx^2+ax}a^2} - \frac{2\,d}{\sqrt{bx^2+ax}a} - \frac{2\,c}{3\,\sqrt{bx^2+ax}ax}$$

input
```
integrate((d*x+c)/x/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
16/3*b^2*c*x/(sqrt(b*x^2 + a*x)*a^3) - 4*b*d*x/(sqrt(b*x^2 + a*x)*a^2) + 8
/3*b*c/(sqrt(b*x^2 + a*x)*a^2) - 2*d/(sqrt(b*x^2 + a*x)*a) - 2/3*c/(sqrt(b
*x^2 + a*x)*a*x)
```

Giac [F]

$$\int \frac{c+dx}{x\left(ax+bx^2\right)^{3/2}}\,dx=\int \frac{dx+c}{\left(bx^2+ax\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((d*x+c)/x/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/((b*x^2 + a*x)^(3/2)*x), x)
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.72

$$\int \frac{c+dx}{x\left(ax+bx^2\right)^{3/2}}\,dx=-\frac{2\sqrt{b\,x^2+a\,x}\,\left(3\,d\,a^2\,x+c\,a^2+6\,d\,a\,b\,x^2-4\,c\,a\,b\,x-8\,c\,b^2\,x^2\right)}{3\,a^3\,x^2\,\left(a+b\,x\right)}$$

input
```
int((c + d*x)/(x*(a*x + b*x^2)^(3/2)),x)
```

output
```
-(2*(a*x + b*x^2)^(1/2)*(a^2*c - 8*b^2*c*x^2 + 3*a^2*d*x + 6*a*b*d*x^2 - 4
*a*b*c*x))/(3*a^3*x^2*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.08

$$\int \frac{c+dx}{x\left(ax+bx^2\right)^{3/2}}\,dx = \frac{4\sqrt{b}\,\sqrt{bx+a}\,ad\,x^2-\frac{16\sqrt{b}\,\sqrt{bx+a}\,bc\,x^2}{3}-\frac{2\sqrt{x}\,a^2c}{3}-2\sqrt{x}\,a^2dx+\frac{8\sqrt{x}\,abcx}{3}-4\sqrt{x}\,abd\,x^2}{\sqrt{bx+a}\,a^3x^2}$$

input
```
int((d*x+c)/x/(b*x^2+a*x)^(3/2),x)
```

output
```
(2*(6*sqrt(b)*sqrt(a + b*x)*a*d*x**2 - 8*sqrt(b)*sqrt(a + b*x)*b*c*x**2 -
sqrt(x)*a**2*c - 3*sqrt(x)*a**2*d*x + 4*sqrt(x)*a*b*c*x - 6*sqrt(x)*a*b*d*
x**2 + 8*sqrt(x)*b**2*c*x**2))/(3*sqrt(a + b*x)*a**3*x**2)
```

3.166 $\int \frac{c+dx}{x^2(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 122

$$\int \frac{c+dx}{x^2(ax+bx^2)^{3/2}}\,dx = -\frac{2c}{5ax^2\sqrt{ax+bx^2}} - \frac{2(6bc-5ad)}{5a^2x\sqrt{ax+bx^2}} + \frac{8(6bc-5ad)\sqrt{ax+bx^2}}{15a^3x^2} - \frac{16b(6bc-5ad)\sqrt{ax+bx^2}}{15a^4x}$$

output
```
-2/5*c/a/x^2/(b*x^2+a*x)^(1/2)-2/5*(-5*a*d+6*b*c)/a^2/x/(b*x^2+a*x)^(1/2)+
8/15*(-5*a*d+6*b*c)*(b*x^2+a*x)^(1/2)/a^3/x^2-16/15*b*(-5*a*d+6*b*c)*(b*x^
2+a*x)^(1/2)/a^4/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.61

$$\int \frac{c+dx}{x^2(ax+bx^2)^{3/2}}\,dx = -\frac{2(48b^3cx^3+8ab^2x^2(3c-5dx)+a^3(3c+5dx)-2a^2bx(3c+10dx))}{15a^4x^2\sqrt{x(a+bx)}}$$

input
```
Integrate[(c + d*x)/(x^2*(a*x + b*x^2)^(3/2)),x]
```

output
```
(-2*(48*b^3*c*x^3 + 8*a*b^2*x^2*(3*c - 5*d*x) + a^3*(3*c + 5*d*x) - 2*a^2*
b*x*(3*c + 10*d*x)))/(15*a^4*x^2*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.75, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1220, 1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{x^2 (ax+bx^2)^{3/2}} dx$$

$$\Big\downarrow \text{1220}$$

$$-\frac{(6bc-5ad)\int \frac{1}{x(bx^2+ax)^{3/2}}dx}{5a} - \frac{2c}{5ax^2\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1129}$$

$$-\frac{(6bc-5ad)\left(-\frac{4b\int \frac{1}{(bx^2+ax)^{3/2}}dx}{3a} - \frac{2}{3ax\sqrt{ax+bx^2}}\right)}{5a} - \frac{2c}{5ax^2\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1088}$$

$$-\frac{\left(\frac{8b(a+2bx)}{3a^3\sqrt{ax+bx^2}} - \frac{2}{3ax\sqrt{ax+bx^2}}\right)(6bc-5ad)}{5a} - \frac{2c}{5ax^2\sqrt{ax+bx^2}}$$

input
```
Int[(c + d*x)/(x^2*(a*x + b*x^2)^(3/2)),x]
```

output
```
(-2*c)/(5*a*x^2*Sqrt[a*x + b*x^2]) - ((6*b*c - 5*a*d)*(-2/(3*a*x*Sqrt[a*x
+ b*x^2]) + (8*b*(a + 2*b*x))/(3*a^3*Sqrt[a*x + b*x^2])))/(5*a)
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.54

method	result	size
pseudoelliptic	$-\frac{2\left(\left(\frac{5dx}{3}+c\right)a^3-2x\left(\frac{10dx}{3}+c\right)b\,a^2+8x^2b^2\left(-\frac{5dx}{3}+c\right)a+16b^3c\,x^3\right)}{5\sqrt{x(bx+a)}\,x^2a^4}$	66
gosper	$-\frac{2(bx+a)\left(-40a\,b^2d\,x^3+48b^3c\,x^3-20a^2bd\,x^2+24a\,b^2c\,x^2+5a^3dx-6a^2bcx+3c\,a^3\right)}{15x\,a^4\left(b\,x^2+ax\right)^{\frac{3}{2}}}$	86
orering	$-\frac{2(bx+a)\left(-40a\,b^2d\,x^3+48b^3c\,x^3-20a^2bd\,x^2+24a\,b^2c\,x^2+5a^3dx-6a^2bcx+3c\,a^3\right)}{15x\,a^4\left(b\,x^2+ax\right)^{\frac{3}{2}}}$	86
risch	$-\frac{2(bx+a)\left(-25abd\,x^2+33b^2c\,x^2+5a^2dx-9abcx+3a^2c\right)}{15a^4x^2\sqrt{x(bx+a)}}+\frac{2b^2(ad-bc)x}{\sqrt{x(bx+a)}\,a^4}$	87
trager	$-\frac{2\left(-40a\,b^2d\,x^3+48b^3c\,x^3-20a^2bd\,x^2+24a\,b^2c\,x^2+5a^3dx-6a^2bcx+3c\,a^3\right)\sqrt{b\,x^2+ax}}{15(bx+a)a^4x^3}$	88
default	$c\left(-\frac{2}{5a\,x^2\sqrt{b\,x^2+ax}}-\frac{6b\left(-\frac{2}{3ax\sqrt{b\,x^2+ax}}+\frac{8b(2bx+a)}{3a^3\sqrt{b\,x^2+ax}}\right)}{5a}\right)+d\left(-\frac{2}{3ax\sqrt{b\,x^2+ax}}+\frac{8b(2bx+a)}{3a^3\sqrt{b\,x^2+ax}}\right)$	118

input
```
int((d*x+c)/x^2/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/(x*(b*x+a))^(1/2)*((5/3*d*x+c)*a^3-2*x*(10/3*d*x+c)*b*a^2+8*x^2*b^2*(
-5/3*d*x+c)*a+16*b^3*c*x^3)/x^2/a^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.77

$$\int \frac{c+dx}{x^2\left(ax+bx^2\right)^{3/2}}\,dx = -\frac{2\left(3\,a^3c+8\left(6\,b^3c-5\,ab^2d\right)x^3+4\left(6\,ab^2c-5\,a^2bd\right)x^2-\left(6\,a^2bc-5\,a^3d\right)x\right)\sqrt{bx^2+ax}}{15\left(a^4bx^4+a^5x^3\right)}$$

input
```
integrate((d*x+c)/x^2/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/15*(3*a^3*c + 8*(6*b^3*c - 5*a*b^2*d)*x^3 + 4*(6*a*b^2*c - 5*a^2*b*d)*x
^2 - (6*a^2*b*c - 5*a^3*d)*x)*sqrt(b*x^2 + a*x)/(a^4*b*x^4 + a^5*x^3)
```

Sympy [F]

$$\int \frac{c+dx}{x^2 (ax+bx^2)^{3/2}}\,dx = \int \frac{c+dx}{x^2 (x(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)/x**2/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral((c + d*x)/(x**2*(x*(a + b*x))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.16

$$\int \frac{c+dx}{x^2 (ax+bx^2)^{3/2}}\,dx = -\frac{32\,b^3cx}{5\,\sqrt{bx^2+ax}a^4} + \frac{16\,b^2dx}{3\,\sqrt{bx^2+ax}a^3} - \frac{16\,b^2c}{5\,\sqrt{bx^2+ax}a^3} + \frac{8\,bd}{3\,\sqrt{bx^2+ax}a^2} + \frac{4\,bc}{5\,\sqrt{bx^2+ax}a^2x} - \frac{2\,d}{3\,\sqrt{bx^2+ax}ax} - \frac{2\,c}{5\,\sqrt{bx^2+ax}ax^2}$$

input
```
integrate((d*x+c)/x^2/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
-32/5*b^3*c*x/(sqrt(b*x^2 + a*x)*a^4) + 16/3*b^2*d*x/(sqrt(b*x^2 + a*x)*a^
3) - 16/5*b^2*c/(sqrt(b*x^2 + a*x)*a^3) + 8/3*b*d/(sqrt(b*x^2 + a*x)*a^2)
+ 4/5*b*c/(sqrt(b*x^2 + a*x)*a^2*x) - 2/3*d/(sqrt(b*x^2 + a*x)*a*x) - 2/5*
c/(sqrt(b*x^2 + a*x)*a*x^2)
```

Giac [F]

$$\int \frac{c+dx}{x^2 (ax+bx^2)^{3/2}}\,dx = \int \frac{dx+c}{(bx^2+ax)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((d*x+c)/x^2/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/((b*x^2 + a*x)^(3/2)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 9.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.71

$$\int \frac{c+dx}{x^2 (ax+bx^2)^{3/2}} \, dx = -\frac{2\sqrt{b\,x^2+a\,x}\,(5\,d\,a^3\,x+3\,c\,a^3-20\,d\,a^2\,b\,x^2-6\,c\,a^2\,b\,x-40\,d\,a\,b^2\,x^3+24\,c\,a\,b^2\,x^2+48\,c\,b^3\,x^3)}{15\,a^4\,x^3\,(a+b\,x)}$$

input
```
int((c + d*x)/(x^2*(a*x + b*x^2)^(3/2)),x)
```

output
```
-(2*(a*x + b*x^2)^(1/2)*(3*a^3*c + 48*b^3*c*x^3 + 5*a^3*d*x - 6*a^2*b*c*x
+ 24*a*b^2*c*x^2 - 20*a^2*b*d*x^2 - 40*a*b^2*d*x^3))/(15*a^4*x^3*(a + b*x)
)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \frac{c+dx}{x^2 (ax+bx^2)^{3/2}} \, dx = \frac{-\frac{16\sqrt{b}\,\sqrt{bx+a}\,abd\,x^3}{3}+\frac{32\sqrt{b}\,\sqrt{bx+a}\,b^2c\,x^3}{5}-\frac{2\sqrt{x}\,a^3c}{5}-\frac{2\sqrt{x}\,a^3dx}{3}+\frac{4\sqrt{x}\,a^2bcx}{5}+\frac{8\sqrt{x}\,a^2bd\,x^2}{3}-}{\sqrt{bx+a}\,a^4x^3}$$

input
```
int((d*x+c)/x^2/(b*x^2+a*x)^(3/2),x)
```

output
```
(2*( - 40*sqrt(b)*sqrt(a + b*x)*a*b*d*x**3 + 48*sqrt(b)*sqrt(a + b*x)*b**2
*c*x**3 - 3*sqrt(x)*a**3*c - 5*sqrt(x)*a**3*d*x + 6*sqrt(x)*a**2*b*c*x + 2
0*sqrt(x)*a**2*b*d*x**2 - 24*sqrt(x)*a*b**2*c*x**2 + 40*sqrt(x)*a*b**2*d*x
**3 - 48*sqrt(x)*b**3*c*x**3))/(15*sqrt(a + b*x)*a**4*x**3)
```

3.167 $\int \frac{c+dx}{x^3(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 157

$$\int \frac{c+dx}{x^3(ax+bx^2)^{3/2}}\,dx = -\frac{2c}{7ax^3\sqrt{ax+bx^2}} - \frac{2(8bc-7ad)}{7a^2x^2\sqrt{ax+bx^2}} + \frac{12(8bc-7ad)\sqrt{ax+bx^2}}{35a^3x^3} - \frac{16b(8bc-7ad)\sqrt{ax+bx^2}}{35a^4x^2} + \frac{32b^2(8bc-7ad)\sqrt{ax+bx^2}}{35a^5x}$$

output
```
-2/7*c/a/x^3/(b*x^2+a*x)^(1/2)-2/7*(-7*a*d+8*b*c)/a^2/x^2/(b*x^2+a*x)^(1/2
)+12/35*(-7*a*d+8*b*c)*(b*x^2+a*x)^(1/2)/a^3/x^3-16/35*b*(-7*a*d+8*b*c)*(b
*x^2+a*x)^(1/2)/a^4/x^2+32/35*b^2*(-7*a*d+8*b*c)*(b*x^2+a*x)^(1/2)/a^5/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.60

$$\int \frac{c+dx}{x^3(ax+bx^2)^{3/2}}\,dx = -\frac{2(-128b^4cx^4+16ab^3x^3(-4c+7dx)+8a^2b^2x^2(2c+7dx)-2a^3bx(4c+7dx)+a^4(5c+7dx))}{35a^5x^3\sqrt{x(a+bx)}}$$

input
```
Integrate[(c + d*x)/(x^3*(a*x + b*x^2)^(3/2)),x]
```

output
```
(-2*(-128*b^4*c*x^4 + 16*a*b^3*x^3*(-4*c + 7*d*x) + 8*a^2*b^2*x^2*(2*c + 7
*d*x) - 2*a^3*b*x*(4*c + 7*d*x) + a^4*(5*c + 7*d*x)))/(35*a^5*x^3*Sqrt[x*(
a + b*x)])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.79, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1220, 1129, 1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{x^3 (ax+bx^2)^{3/2}} dx$$

$$\Big\downarrow \text{1220}$$

$$-\frac{(8bc-7ad)\int \frac{1}{x^2(bx^2+ax)^{3/2}}dx}{7a}-\frac{2c}{7ax^3\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1129}$$

$$-\frac{(8bc-7ad)\left(-\frac{6b\int \frac{1}{x\left(bx^2+ax\right)^{3/2}}dx}{5a}-\frac{2}{5ax^2\sqrt{ax+bx^2}}\right)}{7a}-\frac{2c}{7ax^3\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1129}$$

$$-\frac{(8bc-7ad)\left(-\frac{6b\left(-\frac{4b\int \frac{1}{\left(bx^2+ax\right)^{3/2}}dx}{3a}-\frac{2}{3ax\sqrt{ax+bx^2}}\right)}{5a}-\frac{2}{5ax^2\sqrt{ax+bx^2}}\right)}{7a}-\frac{2c}{7ax^3\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1088}$$

$$-\frac{\left(-\frac{6b\left(\frac{8b(a+2bx)}{3a^3\sqrt{ax+bx^2}}-\frac{2}{3ax\sqrt{ax+bx^2}}\right)}{5a}-\frac{2}{5ax^2\sqrt{ax+bx^2}}\right)(8bc-7ad)}{7a}-\frac{2c}{7ax^3\sqrt{ax+bx^2}}$$

input
```
Int[(c + d*x)/(x^3*(a*x + b*x^2)^(3/2)),x]
```

output
```
(-2*c)/(7*a*x^3*Sqrt[a*x + b*x^2]) - ((8*b*c - 7*a*d)*(-2/(5*a*x^2*Sqrt[a*
x + b*x^2]) - (6*b*(-2/(3*a*x*Sqrt[a*x + b*x^2]) + (8*b*(a + 2*b*x))/(3*a^
3*Sqrt[a*x + b*x^2])))/(5*a)))/(7*a)
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.53

method	result
pseudoelliptic	$-\frac{2\left(\left(\frac{7dx}{5}+c\right)a^4-\frac{8xb\left(\frac{7dx}{4}+c\right)a^3}{5}+\frac{16x^2\left(\frac{7dx}{2}+c\right)b^2a^2}{5}-\frac{64x^3b^3\left(-\frac{7dx}{4}+c\right)a}{5}-\frac{128x^4b^4c}{5}\right)}{7\sqrt{x(bx+a)}\,x^3a^5}$
gosper	$-\frac{2(bx+a)\left(112x^4a\,b^3d-128x^4b^4c+56a^2b^2d\,x^3-64a\,b^3c\,x^3-14a^3bd\,x^2+16a^2b^2c\,x^2+7a^4dx-8a^3bcx+5c\,a^4\right)}{35x^2a^5\left(b\,x^2+ax\right)^{\frac{3}{2}}}$
orering	$-\frac{2(bx+a)\left(112x^4a\,b^3d-128x^4b^4c+56a^2b^2d\,x^3-64a\,b^3c\,x^3-14a^3bd\,x^2+16a^2b^2c\,x^2+7a^4dx-8a^3bcx+5c\,a^4\right)}{35x^2a^5\left(b\,x^2+ax\right)^{\frac{3}{2}}}$
risch	$-\frac{2(bx+a)\left(77a\,b^2d\,x^3-93b^3c\,x^3-21a^2bd\,x^2+29a\,b^2c\,x^2+7a^3dx-13a^2bcx+5c\,a^3\right)}{35a^5x^3\sqrt{x(bx+a)}}-\frac{2b^3(ad-bc)x}{\sqrt{x(bx+a)}\,a^5}$
trager	$-\frac{2\left(112x^4a\,b^3d-128x^4b^4c+56a^2b^2d\,x^3-64a\,b^3c\,x^3-14a^3bd\,x^2+16a^2b^2c\,x^2+7a^4dx-8a^3bcx+5c\,a^4\right)\sqrt{b\,x^2+ax}}{35(bx+a)a^5x^4}$
default	$c\left(-\frac{2}{7a\,x^3\sqrt{b\,x^2+ax}}-\frac{8b\left(-\frac{2}{5a\,x^2\sqrt{b\,x^2+ax}}-\frac{6b\left(-\frac{2}{3ax\sqrt{b\,x^2+ax}}+\frac{8b(2bx+a)}{3a^3\sqrt{b\,x^2+ax}}\right)}{5a}\right)}{7a}\right)+d\left(-\frac{2}{5a\,x^2\sqrt{b\,x^2+ax}}-\right.$

input
```
int((d*x+c)/x^3/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7/(x*(b*x+a))^(1/2)*((7/5*d*x+c)*a^4-8/5*x*b*(7/4*d*x+c)*a^3+16/5*x^2*(
7/2*d*x+c)*b^2*a^2-64/5*x^3*b^3*(-7/4*d*x+c)*a-128/5*x^4*b^4*c)/x^3/a^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.75

$$\int \frac{c+dx}{x^3\left(ax+bx^2\right)^{3/2}}\,dx = -\frac{2\left(5\,a^4c-16\left(8\,b^4c-7\,ab^3d\right)x^4-8\left(8\,ab^3c-7\,a^2b^2d\right)x^3+2\left(8\,a^2b^2c-7\,a^3bd\right)x^2-\left(8\,a^3bc-7\,a^4d\right)x\right)\sqrt{bx}}{35\left(a^5bx^5+a^6x^4\right)}$$

input
```
integrate((d*x+c)/x^3/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/35*(5*a^4*c - 16*(8*b^4*c - 7*a*b^3*d)*x^4 - 8*(8*a*b^3*c - 7*a^2*b^2*d
)*x^3 + 2*(8*a^2*b^2*c - 7*a^3*b*d)*x^2 - (8*a^3*b*c - 7*a^4*d)*x)*sqrt(b*
x^2 + a*x)/(a^5*b*x^5 + a^6*x^4)
```

Sympy [F]

$$\int \frac{c+dx}{x^3 (ax+bx^2)^{3/2}}\,dx = \int \frac{c+dx}{x^3 (x(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)/x**3/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral((c + d*x)/(x**3*(x*(a + b*x))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.20

$$\begin{aligned}\int \frac{c+dx}{x^3 (ax+bx^2)^{3/2}}\,dx &= \frac{256\,b^4cx}{35\,\sqrt{bx^2+ax}a^5} - \frac{32\,b^3dx}{5\,\sqrt{bx^2+ax}a^4} \\ &+ \frac{128\,b^3c}{35\,\sqrt{bx^2+ax}a^4} - \frac{16\,b^2d}{5\,\sqrt{bx^2+ax}a^3} - \frac{32\,b^2c}{35\,\sqrt{bx^2+ax}a^3x} + \frac{4\,bd}{5\,\sqrt{bx^2+ax}a^2x} \\ &+ \frac{16\,bc}{35\,\sqrt{bx^2+ax}a^2x^2} - \frac{2\,d}{5\,\sqrt{bx^2+ax}ax^2} - \frac{2\,c}{7\,\sqrt{bx^2+ax}ax^3}\end{aligned}$$

input
```
integrate((d*x+c)/x^3/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
256/35*b^4*c*x/(sqrt(b*x^2 + a*x)*a^5) - 32/5*b^3*d*x/(sqrt(b*x^2 + a*x)*a
^4) + 128/35*b^3*c/(sqrt(b*x^2 + a*x)*a^4) - 16/5*b^2*d/(sqrt(b*x^2 + a*x)
*a^3) - 32/35*b^2*c/(sqrt(b*x^2 + a*x)*a^3*x) + 4/5*b*d/(sqrt(b*x^2 + a*x)
*a^2*x) + 16/35*b*c/(sqrt(b*x^2 + a*x)*a^2*x^2) - 2/5*d/(sqrt(b*x^2 + a*x)
*a*x^2) - 2/7*c/(sqrt(b*x^2 + a*x)*a*x^3)
```

Giac [F]

$$\int \frac{c+dx}{x^3 (ax+bx^2)^{3/2}}\,dx = \int \frac{dx+c}{(bx^2+ax)^{\frac{3}{2}} x^3}\,dx$$

input
```
integrate((d*x+c)/x^3/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/((b*x^2 + a*x)^(3/2)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 9.71 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.03

$$\int \frac{c+dx}{x^3 (ax+bx^2)^{3/2}}\,dx = \frac{2\,b\,\sqrt{b\,x^2+a\,x}\,(21\,a\,d-29\,b\,c)}{35\,a^4\,x^2} - \frac{2\,c\,\sqrt{b\,x^2+a\,x}}{7\,a^2\,x^4} - \frac{\sqrt{b\,x^2+a\,x}\,(14\,a^2\,d-26\,a\,b\,c)}{35\,a^4\,x^3} - \frac{\sqrt{b\,x^2+a\,x}\left(x\left(\frac{116\,b^4\,c-84\,a\,b^3\,d}{35\,a^5}+\frac{4\,b^3\,(77\,a\,d-93\,b\,c)}{35\,a^5}\right)+\frac{2\,b^2\,(77\,a\,d-93\,b\,c)}{35\,a^4}\right)}{x\,(a+b\,x)}$$

input
```
int((c + d*x)/(x^3*(a*x + b*x^2)^(3/2)),x)
```

output
```
(2*b*(a*x + b*x^2)^(1/2)*(21*a*d - 29*b*c))/(35*a^4*x^2) - (2*c*(a*x + b*x
^2)^(1/2))/(7*a^2*x^4) - ((a*x + b*x^2)^(1/2)*(14*a^2*d - 26*a*b*c))/(35*a
^4*x^3) - ((a*x + b*x^2)^(1/2)*(x*((116*b^4*c - 84*a*b^3*d)/(35*a^5) + (4*
b^3*(77*a*d - 93*b*c))/(35*a^5)) + (2*b^2*(77*a*d - 93*b*c))/(35*a^4)))/(x
*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.98

$$\int \frac{c+dx}{x^3\left(ax+bx^2\right)^{3/2}}\,dx = \frac{\frac{32\sqrt{b}\,\sqrt{bx+a}\,a\,b^2 d\,x^4}{5}-\frac{256\sqrt{b}\,\sqrt{bx+a}\,b^3 c\,x^4}{35}-\frac{2\sqrt{x}\,a^4 c}{7}-\frac{2\sqrt{x}\,a^4 dx}{5}+\frac{16\sqrt{x}\,a^3 bcx}{35}+\frac{4\sqrt{x}\,a^3 bd\,x^2}{5}-}{\sqrt{bx+a}\,a^5 x^4}$$

input
```
int((d*x+c)/x^3/(b*x^2+a*x)^(3/2),x)
```

output
```
(2*(112*sqrt(b)*sqrt(a + b*x)*a*b**2*d*x**4 - 128*sqrt(b)*sqrt(a + b*x)*b*
*3*c*x**4 - 5*sqrt(x)*a**4*c - 7*sqrt(x)*a**4*d*x + 8*sqrt(x)*a**3*b*c*x +
 14*sqrt(x)*a**3*b*d*x**2 - 16*sqrt(x)*a**2*b**2*c*x**2 - 56*sqrt(x)*a**2*
b**2*d*x**3 + 64*sqrt(x)*a*b**3*c*x**3 - 112*sqrt(x)*a*b**3*d*x**4 + 128*s
qrt(x)*b**4*c*x**4))/(35*sqrt(a + b*x)*a**5*x**4)
```

3.168 $\int \frac{x^4(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 254

$$\int \frac{x^4(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = \frac{2(bc-ad)^2x^4}{ab^2\sqrt{ax+bx^2}} - \frac{5a(48b^2c^2-112abcd+63a^2d^2)\sqrt{ax+bx^2}}{64b^5} + \frac{5(48b^2c^2-112abcd+63a^2d^2)\,x\sqrt{ax+bx^2}}{96b^4} - \frac{(48b^2c^2-112abcd+63a^2d^2)\,x^2\sqrt{ax+bx^2}}{24ab^3} + \frac{d^2x^3\sqrt{ax+bx^2}}{4b^2} + \frac{5a^2(48b^2c^2-112abcd+63a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{64b^{11/2}}$$

output

```
2*(-a*d+b*c)^2*x^4/a/b^2/(b*x^2+a*x)^(1/2)-5/64*a*(63*a^2*d^2-112*a*b*c*d+
48*b^2*c^2)*(b*x^2+a*x)^(1/2)/b^5+5/96*(63*a^2*d^2-112*a*b*c*d+48*b^2*c^2)
*x*(b*x^2+a*x)^(1/2)/b^4-1/24*(63*a^2*d^2-112*a*b*c*d+48*b^2*c^2)*x^2*(b*x
^2+a*x)^(1/2)/a/b^3+1/4*d^2*x^3*(b*x^2+a*x)^(1/2)/b^2+5/64*a^2*(63*a^2*d^2
-112*a*b*c*d+48*b^2*c^2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.97

$$\int \frac{x^4(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = \frac{x^2(a+bx)\left(-720a^2b^2c^2+1680a^3bcd-945a^4d^2-240ab^3c^2x+560a^2b^2cdx-315a^3 \ldots\right.}{192b^5(x(a \ldots}$$
$$+\frac{5a^2(48b^2c^2-112abcd+63a^2d^2)\,x^{3/2}(a+bx)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{32b^{11/2}(x(a+bx))^{3/2}}$$

input
```
Integrate[(x^4*(c + d*x)^2)/(a*x + b*x^2)^(3/2),x]
```

output
```
(x^2*(a + b*x)*(-720*a^2*b^2*c^2 + 1680*a^3*b*c*d - 945*a^4*d^2 - 240*a*b^
3*c^2*x + 560*a^2*b^2*c*d*x - 315*a^3*b*d^2*x + 96*b^4*c^2*x^2 - 224*a*b^3
*c*d*x^2 + 126*a^2*b^2*d^2*x^2 + 128*b^4*c*d*x^3 - 72*a*b^3*d^2*x^3 + 48*b
^4*d^2*x^4))/(192*b^5*(x*(a + b*x))^(3/2)) + (5*a^2*(48*b^2*c^2 - 112*a*b*
c*d + 63*a^2*d^2)*x^(3/2)*(a + b*x)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt
[a] + Sqrt[a + b*x])])/(32*b^(11/2)*(x*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1211, 2192, 27, 2192, 27, 2192, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1211}$$

$$\frac{\int \frac{b^4d^2x^4+b^3d(2bc-ad)x^3+b^2(bc-ad)^2x^2-ab(bc-ad)^2x+a^2(bc-ad)^2}{\sqrt{bx^2+ax}}dx}{b^5}-\frac{2a^2x(bc-ad)^2}{b^5\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{\frac{\int \frac{d(16bc-15ad)x^3b^4+8(bc-ad)^2x^2b^3-8a(bc-ad)^2xb^2+8a^2(bc-ad)^2b}{2\sqrt{bx^2+ax}}dx}{4b}+\frac{1}{4}b^3d^2x^3\sqrt{ax+bx^2}}{b^5}-\frac{2a^2x(bc-ad)^2}{b^5\sqrt{ax+bx^2}}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{d(16bc-15ad)x^3b^4+8(bc-ad)^2x^2b^3-8a(bc-ad)^2xb^2+8a^2(bc-ad)^2b}{\sqrt{bx^2+ax}}dx}{8b}+\frac{1}{4}b^3d^2x^3\sqrt{ax+bx^2}}{b^5}-\frac{2a^2x(bc-ad)^2}{b^5\sqrt{ax+bx^2}}$$

$\downarrow$ 2192

$$\frac{\frac{\frac{\int \frac{\left(48b^2c^2-176abdc+123a^2d^2\right)x^2b^4-48a(bc-ad)^2xb^3+48a^2(bc-ad)^2b^2}{2\sqrt{bx^2+ax}}dx}{3b}+\frac{1}{3}b^3dx^2\sqrt{ax+bx^2}(16bc-15ad)}{8b}+\frac{1}{4}b^3d^2x^3\sqrt{ax+bx^2}}{b^5}-\frac{2a^2x(bc-ad)^2}{b^5\sqrt{ax+bx^2}}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{\left(48b^2c^2-176abdc+123a^2d^2\right)x^2b^4-48a(bc-ad)^2xb^3+48a^2(bc-ad)^2b^2}{\sqrt{bx^2+ax}}dx}{6b}+\frac{1}{3}b^3dx^2\sqrt{ax+bx^2}(16bc-15ad)}{8b}+\frac{1}{4}b^3d^2x^3\sqrt{ax+bx^2}}{b^5}-\frac{2a^2x(bc-ad)^2}{b^5\sqrt{ax+bx^2}}$$

$\downarrow$ 2192

$$\frac{\frac{\frac{\frac{\int \frac{3ab^3\left(64a(bc-ad)^2-b\left(112b^2c^2-304abdc+187a^2d^2\right)x\right)}{2\sqrt{bx^2+ax}}dx}{2b}+\frac{1}{2}b^3x\sqrt{ax+bx^2}\left(123a^2d^2-176abcd+48b^2c^2\right)}{6b}+\frac{1}{3}b^3dx^2\sqrt{ax+bx^2}(16bc-15ad)}{8b}+\frac{1}{4}b^3d^2x^3\sqrt{}}{b^5}$$

$$\frac{2a^2x(bc-ad)^2}{b^5\sqrt{ax+bx^2}}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\frac{3}{4}ab^2\int \frac{64a(bc-ad)^2-b\left(112b^2c^2-304abdc+187a^2d^2\right)x}{\sqrt{bx^2+ax}}dx+\frac{1}{2}b^3x\sqrt{ax+bx^2}\left(123a^2d^2-176abcd+48b^2c^2\right)}{6b}+\frac{1}{3}b^3dx^2\sqrt{ax+bx^2}(16bc-15ad)}{8b}+\frac{1}{4}b^3d^2x^3\sqrt{}}{b^5}$$

$$\frac{2a^2x(bc-ad)^2}{b^5\sqrt{ax+bx^2}}$$

$\downarrow$ 1160

$$\frac{\frac{\frac{\frac{3}{4}ab^2\left(\frac{5}{2}a\left(63a^2d^2-112abcd+48b^2c^2\right)\int\frac{1}{\sqrt{bx^2+ax}}dx-\sqrt{ax+bx^2}\left(187a^2d^2-304abcd+112b^2c^2\right)\right)+\frac{1}{2}b^3x\sqrt{ax+bx^2}\left(123a^2d^2-176abcd+48b^2c^2\right)}{6b}+\frac{1}{3}b^3dx^2\sqrt{ax+}}{8b}}{b^5}$$

$$\frac{2a^2x(bc-ad)^2}{b^5\sqrt{ax+bx^2}}$$

$\Big\downarrow$ 1091

$$\frac{\frac{\frac{\frac{3}{4}ab^2\left(5a\left(63a^2d^2-112abcd+48b^2c^2\right)\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}-\sqrt{ax+bx^2}\left(187a^2d^2-304abcd+112b^2c^2\right)\right)+\frac{1}{2}b^3x\sqrt{ax+bx^2}\left(123a^2d^2-176abcd+48b^2c^2\right)}{6b}+\frac{1}{3}b}{8b}}{b^5}$$

$$\frac{2a^2x(bc-ad)^2}{b^5\sqrt{ax+bx^2}}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{\frac{3}{4}ab^2\left(\frac{5a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)\left(63a^2d^2-112abcd+48b^2c^2\right)}{\sqrt{b}}-\sqrt{ax+bx^2}\left(187a^2d^2-304abcd+112b^2c^2\right)\right)+\frac{1}{2}b^3x\sqrt{ax+bx^2}\left(123a^2d^2-176abcd+48b^2c^2\right)}{6b}+\frac{1}{3}b^3}{8b}}{b^5}$$

$$\frac{2a^2x(bc-ad)^2}{b^5\sqrt{ax+bx^2}}$$

input
```
Int[(x^4*(c + d*x)^2)/(a*x + b*x^2)^(3/2),x]
```

output
```
(-2*a^2*(b*c - a*d)^2*x)/(b^5*Sqrt[a*x + b*x^2]) + ((b^3*d^2*x^3*Sqrt[a*x
+ b*x^2])/4 + ((b^3*d*(16*b*c - 15*a*d)*x^2*Sqrt[a*x + b*x^2])/3 + ((b^3*(
48*b^2*c^2 - 176*a*b*c*d + 123*a^2*d^2)*x*Sqrt[a*x + b*x^2])/2 + (3*a*b^2*
(-((112*b^2*c^2 - 304*a*b*c*d + 187*a^2*d^2)*Sqrt[a*x + b*x^2]) + (5*a*(48
*b^2*c^2 - 112*a*b*c*d + 63*a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]
])/Sqrt[b]))/4)/(6*b))/(8*b))/b^5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$\frac{\frac{315\sqrt{x(bx+a)}\,a^2\left(a^2d^2-\frac{16}{9}abcd+\frac{16}{21}b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{64}-315x\left(\frac{16a^2\left(-\frac{7}{40}d^2x^2-\frac{7}{9}cdx+c^2\right)b^{\frac{5}{2}}}{21}+\frac{16x\left(\frac{3}{10}d^2x^2+\frac{14}{15}cdx+c^2\right)}{63}\right.}{b^{\frac{11}{2}}\sqrt{x(bx+a)}}$
risch	$-\frac{\left(-48b^3d^2x^3+120a\,b^2d^2x^2-128b^3cd\,x^2-246a^2b\,d^2x+352a\,b^2cdx-96b^3c^2x+561a^3d^2-912a^2bcd+336a\,c^2b^2\right)x(bx+a)}{192b^5\sqrt{x(bx+a)}}$
default	$c^2\left(\frac{x^3}{2b\sqrt{b\,x^2+ax}}-\frac{5a\left(\frac{x^2}{b\sqrt{b\,x^2+ax}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^2+ax}}-\frac{a\left(-\frac{1}{b\sqrt{b\,x^2+ax}}+\frac{2bx+a}{ab\sqrt{b\,x^2+ax}}\right)}{2b}+\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)}{4b}\right)$

input
```
int(x^4*(d*x+c)^2/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
315/64*((x*(b*x+a))^(1/2)*a^2*(a^2*d^2-16/9*a*b*c*d+16/21*b^2*c^2)*arctanh
((x*(b*x+a))^(1/2)/x/b^(1/2))-x*(16/21*a^2*(-7/40*d^2*x^2-7/9*c*d*x+c^2)*b
^(5/2)+16/63*x*(3/10*d^2*x^2+14/15*c*d*x+c^2)*a*b^(7/2)-32/315*x^2*(1/2*d^
2*x^2+4/3*c*d*x+c^2)*b^(9/2)+d*a^3*((1/3*d*x-16/9*c)*b^(3/2)+b^(1/2)*a*d))
)/b^(11/2)/(x*(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.98

$$\int \frac{x^4(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = \left[\frac{15\,(48\,a^3b^2c^2 - 112\,a^4bcd + 63\,a^5d^2 + (48\,a^2b^3c^2 - 112\,a^3b^2cd + 63\,a^4bd^2)x)\sqrt{b}\log}{}\right.$$

$$-\frac{15\,(48\,a^3b^2c^2 - 112\,a^4bcd + 63\,a^5d^2 + (48\,a^2b^3c^2 - 112\,a^3b^2cd + 63\,a^4bd^2)x)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right) -}{}$$

input
```
integrate(x^4*(d*x+c)^2/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[1/384*(15*(48*a^3*b^2*c^2 - 112*a^4*b*c*d + 63*a^5*d^2 + (48*a^2*b^3*c^2
- 112*a^3*b^2*c*d + 63*a^4*b*d^2)*x)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2
+ a*x)*sqrt(b)) + 2*(48*b^5*d^2*x^4 - 720*a^2*b^3*c^2 + 1680*a^3*b^2*c*d -
 945*a^4*b*d^2 + 8*(16*b^5*c*d - 9*a*b^4*d^2)*x^3 + 2*(48*b^5*c^2 - 112*a*
b^4*c*d + 63*a^2*b^3*d^2)*x^2 - 5*(48*a*b^4*c^2 - 112*a^2*b^3*c*d + 63*a^3
*b^2*d^2)*x)*sqrt(b*x^2 + a*x))/(b^7*x + a*b^6), -1/192*(15*(48*a^3*b^2*c^
2 - 112*a^4*b*c*d + 63*a^5*d^2 + (48*a^2*b^3*c^2 - 112*a^3*b^2*c*d + 63*a^
4*b*d^2)*x)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (48*b^
5*d^2*x^4 - 720*a^2*b^3*c^2 + 1680*a^3*b^2*c*d - 945*a^4*b*d^2 + 8*(16*b^5
*c*d - 9*a*b^4*d^2)*x^3 + 2*(48*b^5*c^2 - 112*a*b^4*c*d + 63*a^2*b^3*d^2)*
x^2 - 5*(48*a*b^4*c^2 - 112*a^2*b^3*c*d + 63*a^3*b^2*d^2)*x)*sqrt(b*x^2 +
a*x))/(b^7*x + a*b^6)]
```

Sympy [F]

$$\int \frac{x^4(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = \int \frac{x^4(c+dx)^2}{(x\,(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(d*x+c)**2/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral(x**4*(c + d*x)**2/(x*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.48

$$\begin{aligned}\int \frac{x^4(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx &= \frac{d^2x^5}{4\,\sqrt{bx^2+ax}b} + \frac{2\,cdx^4}{3\,\sqrt{bx^2+ax}b} - \frac{3\,ad^2x^4}{8\,\sqrt{bx^2+ax}b^2} \\ &+ \frac{c^2x^3}{2\,\sqrt{bx^2+ax}b} - \frac{7\,acdx^3}{6\,\sqrt{bx^2+ax}b^2} + \frac{21\,a^2d^2x^3}{32\,\sqrt{bx^2+ax}b^3} - \frac{5\,ac^2x^2}{4\,\sqrt{bx^2+ax}b^2} \\ &+ \frac{35\,a^2cdx^2}{12\,\sqrt{bx^2+ax}b^3} - \frac{105\,a^3d^2x^2}{64\,\sqrt{bx^2+ax}b^4} - \frac{15\,a^2c^2x}{4\,\sqrt{bx^2+ax}b^3} + \frac{35\,a^3cdx}{4\,\sqrt{bx^2+ax}b^4} \\ &- \frac{315\,a^4d^2x}{64\,\sqrt{bx^2+ax}b^5} + \frac{15\,a^2c^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{7}{2}}} \\ &- \frac{35\,a^3cd\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{9}{2}}} \\ &+ \frac{315\,a^4d^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{11}{2}}}\end{aligned}$$

input
```
integrate(x^4*(d*x+c)^2/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
1/4*d^2*x^5/(sqrt(b*x^2 + a*x)*b) + 2/3*c*d*x^4/(sqrt(b*x^2 + a*x)*b) - 3/
8*a*d^2*x^4/(sqrt(b*x^2 + a*x)*b^2) + 1/2*c^2*x^3/(sqrt(b*x^2 + a*x)*b) -
7/6*a*c*d*x^3/(sqrt(b*x^2 + a*x)*b^2) + 21/32*a^2*d^2*x^3/(sqrt(b*x^2 + a*
x)*b^3) - 5/4*a*c^2*x^2/(sqrt(b*x^2 + a*x)*b^2) + 35/12*a^2*c*d*x^2/(sqrt(
b*x^2 + a*x)*b^3) - 105/64*a^3*d^2*x^2/(sqrt(b*x^2 + a*x)*b^4) - 15/4*a^2*
c^2*x/(sqrt(b*x^2 + a*x)*b^3) + 35/4*a^3*c*d*x/(sqrt(b*x^2 + a*x)*b^4) - 3
15/64*a^4*d^2*x/(sqrt(b*x^2 + a*x)*b^5) + 15/8*a^2*c^2*log(2*b*x + a + 2*s
qrt(b*x^2 + a*x)*sqrt(b))/b^(7/2) - 35/8*a^3*c*d*log(2*b*x + a + 2*sqrt(b*
x^2 + a*x)*sqrt(b))/b^(9/2) + 315/128*a^4*d^2*log(2*b*x + a + 2*sqrt(b*x^2
 + a*x)*sqrt(b))/b^(11/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.96

$$\int \frac{x^4(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = \frac{1}{192}\sqrt{bx^2+ax}\left(2\left(4\left(\frac{6\,d^2x}{b^2}+\frac{16\,b^{17}cd-15\,ab^{16}d^2}{b^{19}}\right)x+\frac{48\,b^{17}c^2-176\,ab^{16}cd+1\cdots}{b^{19}}\right.\right.$$
$$-\frac{5\,(48\,a^2b^2c^2-112\,a^3bcd+63\,a^4d^2)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{128\,b^{\frac{11}{2}}}$$
$$-\frac{2\left(a^3b^{\frac{5}{2}}c^2-2\,a^4b^{\frac{3}{2}}cd+a^5\sqrt{b}d^2\right)}{\left(\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right)b^6}$$

input
```
integrate(x^4*(d*x+c)^2/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
1/192*sqrt(b*x^2 + a*x)*(2*(4*(6*d^2*x/b^2 + (16*b^17*c*d - 15*a*b^16*d^2)
/b^19)*x + (48*b^17*c^2 - 176*a*b^16*c*d + 123*a^2*b^15*d^2)/b^19)*x - 3*(
112*a*b^16*c^2 - 304*a^2*b^15*c*d + 187*a^3*b^14*d^2)/b^19) - 5/128*(48*a^
2*b^2*c^2 - 112*a^3*b*c*d + 63*a^4*d^2)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2
+ a*x))*sqrt(b) + a))/b^(11/2) - 2*(a^3*b^(5/2)*c^2 - 2*a^4*b^(3/2)*c*d +
a^5*sqrt(b)*d^2)/(((sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)*b^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = \int \frac{x^4\,(c+d\,x)^2}{(b\,x^2+a\,x)^{3/2}}\,dx$$

input
```
int((x^4*(c + d*x)^2)/(a*x + b*x^2)^(3/2),x)
```

output
```
int((x^4*(c + d*x)^2)/(a*x + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.28

$$\int \frac{x^4(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = \frac{945\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^4d^2 - 1680\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^3bcd -}{}$$

input
```
int(x^4*(d*x+c)^2/(b*x^2+a*x)^(3/2),x)
```

output
```
(945*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*
a**4*d**2 - 1680*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b
))/sqrt(a))*a**3*b*c*d + 720*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sq
rt(x)*sqrt(b))/sqrt(a))*a**2*b**2*c**2 - 567*sqrt(b)*sqrt(a + b*x)*a**4*d*
*2 + 1050*sqrt(b)*sqrt(a + b*x)*a**3*b*c*d - 480*sqrt(b)*sqrt(a + b*x)*a**
2*b**2*c**2 - 945*sqrt(x)*a**4*b*d**2 + 1680*sqrt(x)*a**3*b**2*c*d - 315*s
qrt(x)*a**3*b**2*d**2*x - 720*sqrt(x)*a**2*b**3*c**2 + 560*sqrt(x)*a**2*b*
*3*c*d*x + 126*sqrt(x)*a**2*b**3*d**2*x**2 - 240*sqrt(x)*a*b**4*c**2*x - 2
24*sqrt(x)*a*b**4*c*d*x**2 - 72*sqrt(x)*a*b**4*d**2*x**3 + 96*sqrt(x)*b**5
*c**2*x**2 + 128*sqrt(x)*b**5*c*d*x**3 + 48*sqrt(x)*b**5*d**2*x**4)/(192*s
qrt(a + b*x)*b**6)
```

3.169 $\int \frac{x^3(c+dx)^2}{\left(ax+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 205

$$\int \frac{x^3(c+dx)^2}{\left(ax+bx^2\right)^{3/2}}\,dx = \frac{2(bc-ad)^2x^3}{ab^2\sqrt{ax+bx^2}} + \frac{\left(24b^2c^2-60abcd+35a^2d^2\right)\sqrt{ax+bx^2}}{8b^4} - \frac{\left(24b^2c^2-60abcd+35a^2d^2\right)x\sqrt{ax+bx^2}}{12ab^3} + \frac{d^2x^2\sqrt{ax+bx^2}}{3b^2} - \frac{a\left(24b^2c^2-60abcd+35a^2d^2\right)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{8b^{9/2}}$$

output
```
2*(-a*d+b*c)^2*x^3/a/b^2/(b*x^2+a*x)^(1/2)+1/8*(35*a^2*d^2-60*a*b*c*d+24*b
^2*c^2)*(b*x^2+a*x)^(1/2)/b^4-1/12*(35*a^2*d^2-60*a*b*c*d+24*b^2*c^2)*x*(b
*x^2+a*x)^(1/2)/a/b^3+1/3*d^2*x^2*(b*x^2+a*x)^(1/2)/b^2-1/8*a*(35*a^2*d^2-
60*a*b*c*d+24*b^2*c^2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.88

$$\int \frac{x^3(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = \frac{x^{3/2}\left(\sqrt{b}\sqrt{x}(a+bx)\left(105a^3d^2+5a^2bd(-36c+7dx)+2ab^2(36c^2-30cdx-7d^2x^2)\right.\right.}{24b^{9/}}$$

input
```
Integrate[(x^3*(c + d*x)^2)/(a*x + b*x^2)^(3/2),x]
```

output
```
(x^(3/2)*(Sqrt[b]*Sqrt[x]*(a + b*x)*(105*a^3*d^2 + 5*a^2*b*d*(-36*c + 7*d*
x) + 2*a*b^2*(36*c^2 - 30*c*d*x - 7*d^2*x^2) + 8*b^3*x*(3*c^2 + 3*c*d*x +
d^2*x^2)) - 6*a*(24*b^2*c^2 - 60*a*b*c*d + 35*a^2*d^2)*(a + b*x)^(3/2)*Arc
Tanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(24*b^(9/2)*(x*(a + b
*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1211, 25, 2192, 27, 2192, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1211}$$

$$\frac{\int -\frac{-b^3d^2x^3-b^2d(2bc-ad)x^2-b(bc-ad)^2x+a(bc-ad)^2}{\sqrt{bx^2+ax}}dx}{b^4}+\frac{2ax(bc-ad)^2}{b^4\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2ax(bc-ad)^2}{b^4\sqrt{ax+bx^2}}-\frac{\int \frac{-b^3d^2x^3-b^2d(2bc-ad)x^2-b(bc-ad)^2x+a(bc-ad)^2}{\sqrt{bx^2+ax}}dx}{b^4}$$

$$\Big\downarrow \text{2192}$$

$$\frac{2ax(bc-ad)^2}{b^4\sqrt{ax+bx^2}}-\frac{\frac{\int \frac{-d(12bc-11ad)x^2b^3-6(bc-ad)^2xb^2+6a(bc-ad)^2b}{2\sqrt{bx^2+ax}}dx}{3b}-\frac{1}{3}b^2d^2x^2\sqrt{ax+bx^2}}{b^4}$$

$\Big\downarrow$ 27

$$\frac{2ax(bc-ad)^2}{b^4\sqrt{ax+bx^2}}-\frac{\frac{\int \frac{-d(12bc-11ad)x^2b^3-6(bc-ad)^2xb^2+6a(bc-ad)^2b}{\sqrt{bx^2+ax}}dx}{6b}-\frac{1}{3}b^2d^2x^2\sqrt{ax+bx^2}}{b^4}$$

$\Big\downarrow$ 2192

$$\frac{2ax(bc-ad)^2}{b^4\sqrt{ax+bx^2}}-\frac{\frac{\frac{\int \frac{3b^2\left(8a(bc-ad)^2-b\left(8b^2c^2-28abdc+19a^2d^2\right)x\right)}{2\sqrt{bx^2+ax}}dx}{2b}-\frac{1}{2}b^2dx\sqrt{ax+bx^2}(12bc-11ad)}{6b}-\frac{1}{3}b^2d^2x^2\sqrt{ax+bx^2}}{b^4}$$

$\Big\downarrow$ 27

$$\frac{2ax(bc-ad)^2}{b^4\sqrt{ax+bx^2}}-\frac{\frac{\frac{3}{4}b\int \frac{8a(bc-ad)^2-b\left(8b^2c^2-28abdc+19a^2d^2\right)x}{\sqrt{bx^2+ax}}dx-\frac{1}{2}b^2dx\sqrt{ax+bx^2}(12bc-11ad)}{6b}-\frac{1}{3}b^2d^2x^2\sqrt{ax+bx^2}}{b^4}$$

$\Big\downarrow$ 1160

$$\frac{2ax(bc-ad)^2}{b^4\sqrt{ax+bx^2}}-\frac{\frac{\frac{3}{4}b\left(\frac{1}{2}a\left(35a^2d^2-60abcd+24b^2c^2\right)\int\frac{1}{\sqrt{bx^2+ax}}dx-\sqrt{ax+bx^2}\left(19a^2d^2-28abcd+8b^2c^2\right)\right)-\frac{1}{2}b^2dx\sqrt{ax+bx^2}(12bc-11ad)}{6b}-\frac{1}{3}b^2d^2x^2\sqrt{ax+bx^2}}{b^4}$$

$\Big\downarrow$ 1091

$$\frac{2ax(bc-ad)^2}{b^4\sqrt{ax+bx^2}}-\frac{\frac{\frac{3}{4}b\left(a\left(35a^2d^2-60abcd+24b^2c^2\right)\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}-\sqrt{ax+bx^2}\left(19a^2d^2-28abcd+8b^2c^2\right)\right)-\frac{1}{2}b^2dx\sqrt{ax+bx^2}(12bc-11ad)}{6b}-\frac{1}{3}b^2d^2x^2\sqrt{ax+bx^2}}{b^4}$$

$\Big\downarrow$ 219

$$\frac{2ax(bc-ad)^2}{b^4\sqrt{ax+bx^2}} - \frac{\frac{\frac{3}{4}b\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(35a^2d^2-60abcd+24b^2c^2)}{\sqrt{b}}-\sqrt{ax+bx^2}(19a^2d^2-28abcd+8b^2c^2)\right)-\frac{1}{2}b^2dx\sqrt{ax+bx^2}(12bc-11ad)}{6b} - \frac{1}{3}b^2d^2x^2\sqrt{ax}}{b^4}$$

input
```
Int[(x^3*(c + d*x)^2)/(a*x + b*x^2)^(3/2),x]
```

output
```
(2*a*(b*c - a*d)^2*x)/(b^4*Sqrt[a*x + b*x^2]) - (-1/3*(b^2*d^2*x^2*Sqrt[a*
x + b*x^2]) + (-1/2*(b^2*d*(12*b*c - 11*a*d)*x*Sqrt[a*x + b*x^2]) + (3*b*(
-((8*b^2*c^2 - 28*a*b*c*d + 19*a^2*d^2)*Sqrt[a*x + b*x^2]) + (a*(24*b^2*c^
2 - 60*a*b*c*d + 35*a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[
b]))/4)/(6*b))/b^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$-\frac{35\left(\sqrt{x(bx+a)}\,a\left(a^2d^2-\frac{12}{7}abcd+\frac{24}{35}b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)-x\left(\frac{24\left(-\frac{7}{36}d^2x^2-\frac{5}{6}cdx+c^2\right)a\,b^{\frac{5}{2}}}{35}+\frac{8x\left(\frac{1}{3}d^2x^2+cdx+c^2\right)b}{35}\right.\right.}{8\sqrt{x(bx+a)}\,b^{\frac{9}{2}}}$
risch	$\frac{(8b^2d^2x^2-22ab\,d^2x+24b^2cxd+57a^2d^2-84abcd+24b^2c^2)x(bx+a)}{24b^4\sqrt{x(bx+a)}}-\frac{a\left(\frac{35a^2d^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}+24b^{\frac{3}{2}}c^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}\right.\right.}{}$
default	$c^2\left(\frac{x^2}{b\sqrt{b\,x^2+ax}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^2+ax}}-\frac{a\left(-\frac{1}{b\sqrt{b\,x^2+ax}}+\frac{2bx+a}{ab\sqrt{b\,x^2+ax}}\right)}{2b}+\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)+d^2\left(\frac{x^4}{3b\sqrt{b\,x^2+a}}\right.$

input

```
int(x^3*(d*x+c)^2/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-35/8/(x*(b*x+a))^(1/2)/b^(9/2)*((x*(b*x+a))^(1/2)*a*(a^2*d^2-12/7*a*b*c*d
+24/35*b^2*c^2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-x*(24/35*(-7/36*d^2*x
^2-5/6*c*d*x+c^2)*a*b^(5/2)+8/35*x*(1/3*d^2*x^2+c*d*x+c^2)*b^(7/2)+d*((1/3
*d*x-12/7*c)*b^(3/2)+b^(1/2)*a*d)*a^2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.05

$$\int \frac{x^3(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = \left[\frac{3\left(24\,a^2b^2c^2-60\,a^3bcd+35\,a^4d^2+(24\,ab^3c^2-60\,a^2b^2cd+35\,a^3bd^2)x\right)\sqrt{b}\log\left(2\,b\right.}{}\right.$$

input
```
integrate(x^3*(d*x+c)^2/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(24*a^2*b^2*c^2 - 60*a^3*b*c*d + 35*a^4*d^2 + (24*a*b^3*c^2 - 60*
a^2*b^2*c*d + 35*a^3*b*d^2)*x)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)
*sqrt(b)) + 2*(8*b^4*d^2*x^3 + 72*a*b^3*c^2 - 180*a^2*b^2*c*d + 105*a^3*b*
d^2 + 2*(12*b^4*c*d - 7*a*b^3*d^2)*x^2 + (24*b^4*c^2 - 60*a*b^3*c*d + 35*a
^2*b^2*d^2)*x)*sqrt(b*x^2 + a*x))/(b^6*x + a*b^5), 1/24*(3*(24*a^2*b^2*c^2
 - 60*a^3*b*c*d + 35*a^4*d^2 + (24*a*b^3*c^2 - 60*a^2*b^2*c*d + 35*a^3*b*d
^2)*x)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (8*b^4*d^2*
x^3 + 72*a*b^3*c^2 - 180*a^2*b^2*c*d + 105*a^3*b*d^2 + 2*(12*b^4*c*d - 7*a
*b^3*d^2)*x^2 + (24*b^4*c^2 - 60*a*b^3*c*d + 35*a^2*b^2*d^2)*x)*sqrt(b*x^2
 + a*x))/(b^6*x + a*b^5)]
```

Sympy [F]

$$\int \frac{x^3(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = \int \frac{x^3(c+dx)^2}{(x\,(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(d*x+c)**2/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral(x**3*(c + d*x)**2/(x*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.46

$$\int \frac{x^3(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = \frac{d^2x^4}{3\sqrt{bx^2+ax}b} + \frac{cdx^3}{\sqrt{bx^2+ax}b} - \frac{7\,ad^2x^3}{12\sqrt{bx^2+ax}b^2} + \frac{c^2x^2}{\sqrt{bx^2+ax}b} - \frac{5\,acdx^2}{2\sqrt{bx^2+ax}b^2} + \frac{35\,a^2d^2x^2}{24\sqrt{bx^2+ax}b^3} + \frac{3\,ac^2x}{\sqrt{bx^2+ax}b^2} - \frac{15\,a^2cdx}{2\sqrt{bx^2+ax}b^3} + \frac{35\,a^3d^2x}{8\sqrt{bx^2+ax}b^4} - \frac{3\,ac^2\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{2\,b^{\frac{5}{2}}} + \frac{15\,a^2cd\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{4\,b^{\frac{7}{2}}} - \frac{35\,a^3d^2\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{9}{2}}}$$

input
```
integrate(x^3*(d*x+c)^2/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
1/3*d^2*x^4/(sqrt(b*x^2 + a*x)*b) + c*d*x^3/(sqrt(b*x^2 + a*x)*b) - 7/12*a
*d^2*x^3/(sqrt(b*x^2 + a*x)*b^2) + c^2*x^2/(sqrt(b*x^2 + a*x)*b) - 5/2*a*c
*d*x^2/(sqrt(b*x^2 + a*x)*b^2) + 35/24*a^2*d^2*x^2/(sqrt(b*x^2 + a*x)*b^3)
 + 3*a*c^2*x/(sqrt(b*x^2 + a*x)*b^2) - 15/2*a^2*c*d*x/(sqrt(b*x^2 + a*x)*b
^3) + 35/8*a^3*d^2*x/(sqrt(b*x^2 + a*x)*b^4) - 3/2*a*c^2*log(2*b*x + a + 2
*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5/2) + 15/4*a^2*c*d*log(2*b*x + a + 2*sqrt(
b*x^2 + a*x)*sqrt(b))/b^(7/2) - 35/16*a^3*d^2*log(2*b*x + a + 2*sqrt(b*x^2
 + a*x)*sqrt(b))/b^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00

$$\int \frac{x^3(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = \frac{1}{24}\sqrt{bx^2+ax}\left(2\left(\frac{4\,d^2x}{b^2} + \frac{12\,b^{11}cd-11\,ab^{10}d^2}{b^{13}}\right)x + \frac{3\,(8\,b^{11}c^2-28\,ab^{10}cd+19\,a^2b\right. + \frac{(24\,ab^2c^2-60\,a^2bcd+35\,a^3d^2)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{16\,b^{\frac{9}{2}}} + \frac{2\left(a^2b^{\frac{5}{2}}c^2-2\,a^3b^{\frac{3}{2}}cd+a^4\sqrt{b}d^2\right)}{\left(\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right)b^5}$$

input
```
integrate(x^3*(d*x+c)^2/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
1/24*sqrt(b*x^2 + a*x)*(2*(4*d^2*x/b^2 + (12*b^11*c*d - 11*a*b^10*d^2)/b^1
3)*x + 3*(8*b^11*c^2 - 28*a*b^10*c*d + 19*a^2*b^9*d^2)/b^13) + 1/16*(24*a*
b^2*c^2 - 60*a^2*b*c*d + 35*a^3*d^2)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a
*x))*sqrt(b) + a))/b^(9/2) + 2*(a^2*b^(5/2)*c^2 - 2*a^3*b^(3/2)*c*d + a^4*
sqrt(b)*d^2)/(((sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)*b^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx=\int \frac{x^3\,(c+d\,x)^2}{(b\,x^2+a\,x)^{3/2}}\,dx$$

input
```
int((x^3*(c + d*x)^2)/(a*x + b*x^2)^(3/2),x)
```

output
```
int((x^3*(c + d*x)^2)/(a*x + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.34

$$\int \frac{x^3(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx=\frac{-840\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^3d^2+1440\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2bcd}{}$$

input
```
int(x^3*(d*x+c)^2/(b*x^2+a*x)^(3/2),x)
```

output
```
( - 840*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a
))*a**3*d**2 + 1440*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqr
t(b))/sqrt(a))*a**2*b*c*d - 576*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) +
 sqrt(x)*sqrt(b))/sqrt(a))*a*b**2*c**2 + 525*sqrt(b)*sqrt(a + b*x)*a**3*d*
*2 - 960*sqrt(b)*sqrt(a + b*x)*a**2*b*c*d + 432*sqrt(b)*sqrt(a + b*x)*a*b*
*2*c**2 + 840*sqrt(x)*a**3*b*d**2 - 1440*sqrt(x)*a**2*b**2*c*d + 280*sqrt(
x)*a**2*b**2*d**2*x + 576*sqrt(x)*a*b**3*c**2 - 480*sqrt(x)*a*b**3*c*d*x -
 112*sqrt(x)*a*b**3*d**2*x**2 + 192*sqrt(x)*b**4*c**2*x + 192*sqrt(x)*b**4
*c*d*x**2 + 64*sqrt(x)*b**4*d**2*x**3)/(192*sqrt(a + b*x)*b**5)
```

3.170 $\int \frac{x^2(c+dx)^2}{(ax+bx^2)^{3/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 158

$$\int \frac{x^2(c+dx)^2}{(ax+bx^2)^{3/2}} \, dx = \frac{2(bc-ad)^2x^2}{ab^2\sqrt{ax+bx^2}} - \frac{(8b^2c^2-24abcd+15a^2d^2)\sqrt{ax+bx^2}}{4ab^3} + \frac{d^2x\sqrt{ax+bx^2}}{2b^2} + \frac{(8b^2c^2-24abcd+15a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{7/2}}$$

output
```
2*(-a*d+b*c)^2*x^2/a/b^2/(b*x^2+a*x)^(1/2)-1/4*(15*a^2*d^2-24*a*b*c*d+8*b^
2*c^2)*(b*x^2+a*x)^(1/2)/a/b^3+1/2*d^2*x*(b*x^2+a*x)^(1/2)/b^2+1/4*(15*a^2
*d^2-24*a*b*c*d+8*b^2*c^2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

$$\int \frac{x^2(c+dx)^2}{(ax+bx^2)^{3/2}} \, dx = \frac{x^{3/2}\left(\sqrt{b}\sqrt{x}(a+bx)\left(-15a^2d^2+abd(24c-5dx)+b^2(-8c^2+8cdx+2d^2x^2)\right)+2(8\right.}{4b^{7/2}(x(a+bx))^{3/2}}$$

input
```
Integrate[(x^2*(c + d*x)^2)/(a*x + b*x^2)^(3/2),x]
```

output
```
(x^(3/2)*(Sqrt[b]*Sqrt[x]*(a + b*x)*(-15*a^2*d^2 + a*b*d*(24*c - 5*d*x) +
b^2*(-8*c^2 + 8*c*d*x + 2*d^2*x^2)) + 2*(8*b^2*c^2 - 24*a*b*c*d + 15*a^2*d
^2)*(a + b*x)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])
)/(4*b^(7/2)*(x*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1211, 2192, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 1211$$

$$\frac{\int \frac{(bc-ad)^2+b^2d^2x^2+bd(2bc-ad)x}{\sqrt{bx^2+ax}}dx}{b^3}-\frac{2x(bc-ad)^2}{b^3\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 2192$$

$$\frac{\frac{\int \frac{b\left(4(bc-ad)^2+bd(8bc-7ad)x\right)}{2\sqrt{bx^2+ax}}dx}{2b}+\frac{1}{2}bd^2x\sqrt{ax+bx^2}}{b^3}-\frac{2x(bc-ad)^2}{b^3\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{4}\int \frac{4(bc-ad)^2+bd(8bc-7ad)x}{\sqrt{bx^2+ax}}dx+\frac{1}{2}bd^2x\sqrt{ax+bx^2}}{b^3}-\frac{2x(bc-ad)^2}{b^3\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 1160$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(15a^2d^2-24abcd+8b^2c^2\right)\int \frac{1}{\sqrt{bx^2+ax}}dx+d\sqrt{ax+bx^2}(8bc-7ad)\right)+\frac{1}{2}bd^2x\sqrt{ax+bx^2}}{b^3}-\frac{2x(bc-ad)^2}{b^3\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 1091$$

$$\frac{\frac{1}{4}\left(\left(15a^2d^2-24abcd+8b^2c^2\right)\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}+d\sqrt{ax+bx^2}(8bc-7ad)\right)+\frac{1}{2}bd^2x\sqrt{ax+bx^2}}{b^3}-\frac{2x(bc-ad)^2}{b^3\sqrt{ax+bx^2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{4}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)\left(15a^2d^2-24abcd+8b^2c^2\right)}{\sqrt{b}}+d\sqrt{ax+bx^2}(8bc-7ad)\right)+\frac{1}{2}bd^2x\sqrt{ax+bx^2}}{b^3}-\frac{2x(bc-ad)^2}{b^3\sqrt{ax+bx^2}}$$

input
```
Int[(x^2*(c + d*x)^2)/(a*x + b*x^2)^(3/2),x]
```

output
```
(-2*(b*c - a*d)^2*x)/(b^3*Sqrt[a*x + b*x^2]) + ((b*d^2*x*Sqrt[a*x + b*x^2]
)/2 + (d*(8*b*c - 7*a*d)*Sqrt[a*x + b*x^2] + ((8*b^2*c^2 - 24*a*b*c*d + 15
*a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b])/4)/b^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{\frac{15\sqrt{x(bx+a)}\left(a^2d^2-\frac{8}{5}abcd+\frac{8}{15}b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{4}-\frac{15x\left(\frac{2\left(-d^2x^2-4cdx+4c^2\right)b^{\frac{5}{2}}}{15}+d\left(\left(\frac{dx}{3}-\frac{8c}{5}\right)b^{\frac{3}{2}}+\sqrt{b}\,ad\right)a\right)}{4}}{b^{\frac{7}{2}}\sqrt{x(bx+a)}}$
risch	$-\frac{d(-2bdx+7ad-8bc)x(bx+a)}{4b^3\sqrt{x(bx+a)}}+\frac{\frac{15a^2d^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}+8b^{\frac{3}{2}}c^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)-\frac{16\left(a^2d^2-2abcd+b^2c^2\right)\cdots}{b\left(x+\cdots\right.}}{8b^3}$
default	$c^2\left(-\frac{x}{b\sqrt{b\,x^2+ax}}-\frac{a\left(-\frac{1}{b\sqrt{b\,x^2+ax}}+\frac{2bx+a}{ab\sqrt{b\,x^2+ax}}\right)}{2b}+\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{b^{\frac{3}{2}}}\right)+d^2\left(\frac{x^3}{2b\sqrt{b\,x^2+ax}}-\frac{5a\left(\frac{\cdots}{b\sqrt{\cdots}}\cdots\right.}{\cdots}\right.$

input
```
int(x^2*(d*x+c)^2/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
15/4/b^(7/2)/(x*(b*x+a))^(1/2)*((x*(b*x+a))^(1/2)*(a^2*d^2-8/5*a*b*c*d+8/1
5*b^2*c^2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-x*(2/15*(-d^2*x^2-4*c*d*x+
4*c^2)*b^(5/2)+d*((1/3*d*x-8/5*c)*b^(3/2)+b^(1/2)*a*d)*a))
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.13

$$\int \frac{x^2(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = \left[\frac{(8\,ab^2c^2-24\,a^2bcd+15\,a^3d^2+(8\,b^3c^2-24\,ab^2cd+15\,a^2bd^2)x)\sqrt{b}\log\left(2\,bx+a+\cdots\right.}{\cdots}\right.$$

$$-\frac{(8\,ab^2c^2-24\,a^2bcd+15\,a^3d^2+(8\,b^3c^2-24\,ab^2cd+15\,a^2bd^2)x)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(2\,b^3d^2x^2-\cdots}{4\,(b^5x+ab^4)}$$

input
```
integrate(x^2*(d*x+c)^2/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*((8*a*b^2*c^2 - 24*a^2*b*c*d + 15*a^3*d^2 + (8*b^3*c^2 - 24*a*b^2*c*d
 + 15*a^2*b*d^2)*x)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) +
 2*(2*b^3*d^2*x^2 - 8*b^3*c^2 + 24*a*b^2*c*d - 15*a^2*b*d^2 + (8*b^3*c*d -
 5*a*b^2*d^2)*x)*sqrt(b*x^2 + a*x))/(b^5*x + a*b^4), -1/4*((8*a*b^2*c^2 -
24*a^2*b*c*d + 15*a^3*d^2 + (8*b^3*c^2 - 24*a*b^2*c*d + 15*a^2*b*d^2)*x)*s
qrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (2*b^3*d^2*x^2 - 8*
b^3*c^2 + 24*a*b^2*c*d - 15*a^2*b*d^2 + (8*b^3*c*d - 5*a*b^2*d^2)*x)*sqrt(
b*x^2 + a*x))/(b^5*x + a*b^4)]
```

Sympy [F]

$$\int \frac{x^2(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = \int \frac{x^2(c+dx)^2}{(x\,(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(d*x+c)**2/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral(x**2*(c + d*x)**2/(x*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.42

$$\begin{aligned}\int \frac{x^2(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx &= \frac{d^2x^3}{2\sqrt{bx^2+ax}b} + \frac{2\,cdx^2}{\sqrt{bx^2+ax}b} \\ &- \frac{5\,ad^2x^2}{4\sqrt{bx^2+ax}b^2} - \frac{2\,c^2x}{\sqrt{bx^2+ax}b} + \frac{6\,acdx}{\sqrt{bx^2+ax}b^2} - \frac{15\,a^2d^2x}{4\sqrt{bx^2+ax}b^3} \\ &+ \frac{c^2\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{b^{\frac{3}{2}}} - \frac{3\,acd\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{b^{\frac{5}{2}}} \\ &+ \frac{15\,a^2d^2\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{7}{2}}}\end{aligned}$$

input
```
integrate(x^2*(d*x+c)^2/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
1/2*d^2*x^3/(sqrt(b*x^2 + a*x)*b) + 2*c*d*x^2/(sqrt(b*x^2 + a*x)*b) - 5/4*
a*d^2*x^2/(sqrt(b*x^2 + a*x)*b^2) - 2*c^2*x/(sqrt(b*x^2 + a*x)*b) + 6*a*c*
d*x/(sqrt(b*x^2 + a*x)*b^2) - 15/4*a^2*d^2*x/(sqrt(b*x^2 + a*x)*b^3) + c^2
*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(3/2) - 3*a*c*d*log(2*b*x
+ a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5/2) + 15/8*a^2*d^2*log(2*b*x + a +
2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.03

$$\int \frac{x^2(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = \frac{1}{4}\sqrt{bx^2+ax}\left(\frac{2\,d^2x}{b^2}+\frac{8\,b^6cd-7\,ab^5d^2}{b^8}\right) - \frac{(8\,b^2c^2-24\,abcd+15\,a^2d^2)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,b^{\frac{7}{2}}} - \frac{2\left(ab^{\frac{5}{2}}c^2-2\,a^2b^{\frac{3}{2}}cd+a^3\sqrt{b}d^2\right)}{\left(\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right)b^4}$$

input
```
integrate(x^2*(d*x+c)^2/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
1/4*sqrt(b*x^2 + a*x)*(2*d^2*x/b^2 + (8*b^6*c*d - 7*a*b^5*d^2)/b^8) - 1/8*
(8*b^2*c^2 - 24*a*b*c*d + 15*a^2*d^2)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 +
a*x))*sqrt(b) + a))/b^(7/2) - 2*(a*b^(5/2)*c^2 - 2*a^2*b^(3/2)*c*d + a^3*s
qrt(b)*d^2)/(((sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)*b^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = \int \frac{x^2\,{\left(c+d\,x\right)}^2}{{\left(b\,x^2+a\,x\right)}^{3/2}}\,dx$$

input
```
int((x^2*(c + d*x)^2)/(a*x + b*x^2)^(3/2),x)
```

output
```
int((x^2*(c + d*x)^2)/(a*x + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.43

$$\int \frac{x^2(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = \frac{15\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2d^2 - 24\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)abcd + 8\sqrt{}}{}$$

input
```
int(x^2*(d*x+c)^2/(b*x^2+a*x)^(3/2),x)
```

output
```
(15*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a
**2*d**2 - 24*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/
sqrt(a))*a*b*c*d + 8*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sq
rt(b))/sqrt(a))*b**2*c**2 - 10*sqrt(b)*sqrt(a + b*x)*a**2*d**2 + 18*sqrt(b
)*sqrt(a + b*x)*a*b*c*d - 8*sqrt(b)*sqrt(a + b*x)*b**2*c**2 - 15*sqrt(x)*a
**2*b*d**2 + 24*sqrt(x)*a*b**2*c*d - 5*sqrt(x)*a*b**2*d**2*x - 8*sqrt(x)*b
**3*c**2 + 8*sqrt(x)*b**3*c*d*x + 2*sqrt(x)*b**3*d**2*x**2)/(4*sqrt(a + b*
x)*b**4)
```

3.171 $\int \frac{x(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{x(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx=\frac{2(bc-ad)^2x}{ab^2\sqrt{ax+bx^2}}+\frac{d^2\sqrt{ax+bx^2}}{b^2}+\frac{d(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{5/2}}$$

output
```
2*(-a*d+b*c)^2*x/a/b^2/(b*x^2+a*x)^(1/2)+d^2*(b*x^2+a*x)^(1/2)/b^2+d*(-3*a
*d+4*b*c)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.17

$$\int \frac{x(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx=\frac{\sqrt{b}x(2b^2c^2+3a^2d^2+abd(-4c+dx))+ad(-4bc+3ad)\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\right.}{ab^{5/2}\sqrt{x(a+bx)}}$$

input
```
Integrate[(x*(c + d*x)^2)/(a*x + b*x^2)^(3/2),x]
```

output
```
(Sqrt[b]*x*(2*b^2*c^2 + 3*a^2*d^2 + a*b*d*(-4*c + d*x)) + a*d*(-4*b*c + 3*
a*d)*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(a*b^(
5/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1211, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1211}$$

$$\frac{\int \frac{d(2bc-ad+bdx)}{\sqrt{bx^2+ax}}dx}{b^2}+\frac{2x(bc-ad)^2}{ab^2\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int \frac{2bc-ad+bdx}{\sqrt{bx^2+ax}}dx}{b^2}+\frac{2x(bc-ad)^2}{ab^2\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{d\left(\frac{1}{2}(4bc-3ad)\int \frac{1}{\sqrt{bx^2+ax}}dx+d\sqrt{ax+bx^2}\right)}{b^2}+\frac{2x(bc-ad)^2}{ab^2\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1091}$$

$$\frac{d\left((4bc-3ad)\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}+d\sqrt{ax+bx^2}\right)}{b^2}+\frac{2x(bc-ad)^2}{ab^2\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)(4bc-3ad)}{\sqrt{b}}+d\sqrt{ax+bx^2}\right)}{b^2}+\frac{2x(bc-ad)^2}{ab^2\sqrt{ax+bx^2}}$$

input
```
Int[(x*(c + d*x)^2)/(a*x + b*x^2)^(3/2),x]
```

output
```
(2*(b*c - a*d)^2*x)/(a*b^2*Sqrt[a*x + b*x^2]) + (d*(d*Sqrt[a*x + b*x^2] +
((4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]))/b^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.39

method	result
pseudoelliptic	$\frac{2b^{\frac{5}{2}}c^2x-4b^{\frac{3}{2}}acdx+x^2b^{\frac{3}{2}}a\,d^2-3\sqrt{x(bx+a)}\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)a^2d^2+4\sqrt{x(bx+a)}\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)abcd+3\sqrt{b}\,a^2d^2}{b^{\frac{5}{2}}\sqrt{x(bx+a)}\,a}$
risch	$\frac{d^2x(bx+a)}{b^2\sqrt{x(bx+a)}}-\frac{\frac{2\left(-2a^2d^2+4abcd-2b^2c^2\right)\sqrt{\left(x+\frac{a}{b}\right)^2b-a\left(x+\frac{a}{b}\right)}}{ba\left(x+\frac{a}{b}\right)}+\frac{3a\,d^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}-4d\sqrt{b}\,c\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+a}\right.}{2b^2}$
default	$c^2\left(-\frac{1}{b\sqrt{b\,x^2+ax}}+\frac{2bx+a}{ab\sqrt{b\,x^2+ax}}\right)+d^2\left(\frac{x^2}{b\sqrt{b\,x^2+ax}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^2+ax}}-\frac{a\left(-\frac{1}{b\sqrt{b\,x^2+ax}}+\frac{2bx+a}{ab\sqrt{b\,x^2+ax}}\right)}{2b}+\frac{\ln\left(\frac{a}{2}\right.}{}\right.}{2b}\right.$

input
```
int(x*(d*x+c)^2/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(2*b^(5/2)*c^2*x-4*b^(3/2)*a*c*d*x+x^2*b^(3/2)*a*d^2-3*(x*(b*x+a))^(1/2)*a
rctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*a^2*d^2+4*(x*(b*x+a))^(1/2)*arctanh((x
*(b*x+a))^(1/2)/x/b^(1/2))*a*b*c*d+3*b^(1/2)*a^2*d^2*x)/b^(5/2)/(x*(b*x+a)
)^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.97

$$\int \frac{x(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = \left[-\frac{(4\,a^2bcd-3\,a^3d^2+(4\,ab^2cd-3\,a^2bd^2)x)\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)-}{2\,(ab^4x+a^2b^3)}\right.$$

$$-\frac{(4\,a^2bcd-3\,a^3d^2+(4\,ab^2cd-3\,a^2bd^2)x)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(ab^2d^2x+2\,b^3c^2-4\,ab^2cd+3\,a^2b}{ab^4x+a^2b^3}$$

input
```
integrate(x*(d*x+c)^2/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x)*sqrt(b)*l
og(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*(a*b^2*d^2*x + 2*b^3*c^2 -
 4*a*b^2*c*d + 3*a^2*b*d^2)*sqrt(b*x^2 + a*x))/(a*b^4*x + a^2*b^3), -((4*a
^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x)*sqrt(-b)*arctan(sqrt
(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (a*b^2*d^2*x + 2*b^3*c^2 - 4*a*b^2*c*d
 + 3*a^2*b*d^2)*sqrt(b*x^2 + a*x))/(a*b^4*x + a^2*b^3)]
```

Sympy [F]

$$\int \frac{x(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx=\int \frac{x(c+dx)^2}{(x\,(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(d*x+c)**2/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral(x*(c + d*x)**2/(x*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.60

$$\int \frac{x(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx=\frac{d^2x^2}{\sqrt{bx^2+ax}b}+\frac{2\,c^2x}{\sqrt{bx^2+ax}a}-\frac{4\,cdx}{\sqrt{bx^2+ax}b}+\frac{3\,ad^2x}{\sqrt{bx^2+ax}b^2}+\frac{2\,cd\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{b^{\frac{3}{2}}}-\frac{3\,ad^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{2\,b^{\frac{5}{2}}}$$

input
```
integrate(x*(d*x+c)^2/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
d^2*x^2/(sqrt(b*x^2 + a*x)*b) + 2*c^2*x/(sqrt(b*x^2 + a*x)*a) - 4*c*d*x/(s
qrt(b*x^2 + a*x)*b) + 3*a*d^2*x/(sqrt(b*x^2 + a*x)*b^2) + 2*c*d*log(2*b*x
+ a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(3/2) - 3/2*a*d^2*log(2*b*x + a + 2*s
qrt(b*x^2 + a*x)*sqrt(b))/b^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.36

$$\int \frac{x(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = \frac{\sqrt{bx^2+ax}\,d^2}{b^2} - \frac{(4\,bcd-3\,ad^2)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{2\,b^{\frac{5}{2}}} + \frac{2\left(b^{\frac{5}{2}}c^2-2\,ab^{\frac{3}{2}}cd+a^2\sqrt{b}d^2\right)}{\left(\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right)b^3}$$

input
```
integrate(x*(d*x+c)^2/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
sqrt(b*x^2 + a*x)*d^2/b^2 - 1/2*(4*b*c*d - 3*a*d^2)*log(abs(2*(sqrt(b)*x -
 sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(5/2) + 2*(b^(5/2)*c^2 - 2*a*b^(3/2)*c
*d + a^2*sqrt(b)*d^2)/(((sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)*b^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = \int \frac{x\,{(c+d\,x)}^2}{{(b\,x^2+a\,x)}^{3/2}}\,dx$$

input
```
int((x*(c + d*x)^2)/(a*x + b*x^2)^(3/2),x)
```

output
```
int((x*(c + d*x)^2)/(a*x + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.91

$$\int \frac{x(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = \frac{-12\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2d^2+16\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)abcd+9}{}$$

input
```
int(x*(d*x+c)^2/(b*x^2+a*x)^(3/2),x)
```

output
```
( - 12*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)
)*a**2*d**2 + 16*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b
))/sqrt(a))*a*b*c*d + 9*sqrt(b)*sqrt(a + b*x)*a**2*d**2 - 16*sqrt(b)*sqrt(
a + b*x)*a*b*c*d + 8*sqrt(b)*sqrt(a + b*x)*b**2*c**2 + 12*sqrt(x)*a**2*b*d
**2 - 16*sqrt(x)*a*b**2*c*d + 4*sqrt(x)*a*b**2*d**2*x + 8*sqrt(x)*b**3*c**
2)/(4*sqrt(a + b*x)*a*b**3)
```

3.172 $\int \frac{(c+dx)^2}{\left(ax+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 91

$$\int \frac{(c+dx)^2}{\left(ax+bx^2\right)^{3/2}}\,dx=-\frac{2c^2}{a\sqrt{ax+bx^2}}-\frac{2\left(\frac{d^2}{b}+\frac{2c(bc-ad)}{a^2}\right)x}{\sqrt{ax+bx^2}}+\frac{2d^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{3/2}}$$

output
```
-2*c^2/a/(b*x^2+a*x)^(1/2)-2*(d^2/b+2*c*(-a*d+b*c)/a^2)*x/(b*x^2+a*x)^(1/2
)+2*d^2*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^2}{\left(ax+bx^2\right)^{3/2}}\,dx = -\frac{2\left(\sqrt{b}(2b^2c^2x+a^2d^2x+abc(c-2dx))+a^2d^2\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)\right)}{a^2b^{3/2}\sqrt{x(a+bx)}}$$

input
```
Integrate[(c + d*x)^2/(a*x + b*x^2)^(3/2),x]
```

output
```
(-2*(Sqrt[b]*(2*b^2*c^2*x + a^2*d^2*x + a*b*c*(c - 2*d*x)) + a^2*d^2*Sqrt[
x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]))/(a^2*b^(3/2)*Sq
rt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1164, 25, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(ax+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{1164}$$

$$-\frac{2\int -\frac{d(ac+(2bc-ad)x)}{\sqrt{bx^2+ax}}dx}{a^2}-\frac{2(c+dx)(x(2bc-ad)+ac)}{a^2\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\int \frac{d(ac+(2bc-ad)x)}{\sqrt{bx^2+ax}}dx}{a^2}-\frac{2(c+dx)(x(2bc-ad)+ac)}{a^2\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2d\int \frac{ac+(2bc-ad)x}{\sqrt{bx^2+ax}}dx}{a^2}-\frac{2(c+dx)(x(2bc-ad)+ac)}{a^2\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{2d\left(\frac{a^2d\int \frac{1}{\sqrt{bx^2+ax}}dx}{2b}+\frac{\sqrt{ax+bx^2}(2bc-ad)}{b}\right)}{a^2}-\frac{2(c+dx)(x(2bc-ad)+ac)}{a^2\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1091}$$

$$\frac{2d\left(\frac{a^2d\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{b}+\frac{\sqrt{ax+bx^2}(2bc-ad)}{b}\right)}{a^2}-\frac{2(c+dx)(x(2bc-ad)+ac)}{a^2\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2d\left(\frac{a^2 d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{3/2}}+\frac{\sqrt{ax+bx^2}(2bc-ad)}{b}\right)}{a^2}-\frac{2(c+dx)(x(2bc-ad)+ac)}{a^2\sqrt{ax+bx^2}}$$

input

```
Int[(c + d*x)^2/(a*x + b*x^2)^(3/2),x]
```

output

```
(-2*(c + d*x)*(a*c + (2*b*c - a*d)*x))/(a^2*Sqrt[a*x + b*x^2]) + (2*d*(((2
*b*c - a*d)*Sqrt[a*x + b*x^2])/b + (a^2*d*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b
*x^2]])/b^(3/2)))/a^2
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$\frac{-2ac(-2dx+c)b^{\frac{3}{2}}-4b^{\frac{5}{2}}c^2x+2\left(\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)\sqrt{x(bx+a)}-\sqrt{b}\,x\right)d^2a^2}{b^{\frac{3}{2}}\sqrt{x(bx+a)}\,a^2}$
risch	$-\frac{2c^2(bx+a)}{a^2\sqrt{x(bx+a)}}+\frac{\frac{d^2a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{b^{\frac{3}{2}}}-\frac{2\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{\left(x+\frac{a}{b}\right)^2b-a\left(x+\frac{a}{b}\right)}}{b^2a\left(x+\frac{a}{b}\right)}}{a}$
default	$-\frac{2c^2(2bx+a)}{a^2\sqrt{b\,x^2+ax}}+d^2\left(-\frac{x}{b\sqrt{b\,x^2+ax}}-\frac{a\left(-\frac{1}{b\sqrt{b\,x^2+ax}}+\frac{2bx+a}{ab\sqrt{b\,x^2+ax}}\right)}{2b}+\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{b^{\frac{3}{2}}}\right)+2cd\left(-\frac{}{b}\right.$

input
```
int((d*x+c)^2/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^(3/2)/(x*(b*x+a))^(1/2)*(-a*c*(-2*d*x+c)*b^(3/2)-2*b^(5/2)*c^2*x+(arct
anh((x*(b*x+a))^(1/2)/x/b^(1/2))*(x*(b*x+a))^(1/2)-b^(1/2)*x)*d^2*a^2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.67

$$\int \frac{(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = \left[\frac{(a^2bd^2x^2+a^3d^2x)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,(ab^2c^2+(2\,b^3c^2-2\,ab^2}{a^2b^3x^2+a^3b^2x}\right.$$
$$\left.-\frac{2\left((a^2bd^2x^2+a^3d^2x)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+(ab^2c^2+(2\,b^3c^2-2\,ab^2cd+a^2bd^2)x)\sqrt{bx^2+ax}\right)}{a^2b^3x^2+a^3b^2x}\right]$$

input
```
integrate((d*x+c)^2/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[((a^2*b*d^2*x^2 + a^3*d^2*x)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*
sqrt(b)) - 2*(a*b^2*c^2 + (2*b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x)*sqrt(b*
x^2 + a*x))/(a^2*b^3*x^2 + a^3*b^2*x), -2*((a^2*b*d^2*x^2 + a^3*d^2*x)*sqr
t(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (a*b^2*c^2 + (2*b^3*c
^2 - 2*a*b^2*c*d + a^2*b*d^2)*x)*sqrt(b*x^2 + a*x))/(a^2*b^3*x^2 + a^3*b^2
*x)]
```

Sympy [F]

$$\int \frac{(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = \int \frac{(c+dx)^2}{(x(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**2/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral((c + d*x)**2/(x*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.21

$$\int \frac{(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = -\frac{4\,bc^2x}{\sqrt{bx^2+ax}a^2} + \frac{4\,cdx}{\sqrt{bx^2+ax}a} - \frac{2\,d^2x}{\sqrt{bx^2+ax}b} + \frac{d^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{b^{\frac{3}{2}}} - \frac{2\,c^2}{\sqrt{bx^2+ax}a}$$

input
```
integrate((d*x+c)^2/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
-4*b*c^2*x/(sqrt(b*x^2 + a*x)*a^2) + 4*c*d*x/(sqrt(b*x^2 + a*x)*a) - 2*d^2
*x/(sqrt(b*x^2 + a*x)*b) + d^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)
)/b^(3/2) - 2*c^2/(sqrt(b*x^2 + a*x)*a)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = -\frac{d^2\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{b^{\frac{3}{2}}} - \frac{2\left(\frac{c^2}{a}+\frac{(2\,b^2c^2-2\,abcd+a^2d^2)x}{a^2b}\right)}{\sqrt{bx^2+ax}}$$

input
```
integrate((d*x+c)^2/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
-d^2*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(3/2) - 2*(
c^2/a + (2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(a^2*b))/sqrt(b*x^2 + a*x)
```

Mupad [B] (verification not implemented)

Time = 9.83 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.05

$$\int \frac{(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = \frac{d^2\,\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{b^{3/2}} - \frac{c^2\,(2\,a+4\,b\,x)}{a^2\,\sqrt{b\,x^2+a\,x}} - \frac{2\,d^2\,x}{b\,\sqrt{b\,x^2+a\,x}} + \frac{4\,c\,d\,x}{a\,\sqrt{x\,(a+b\,x)}}$$

input
```
int((c + d*x)^2/(a*x + b*x^2)^(3/2),x)
```

output
```
(d^2*log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2)))/b^(3/2) - (c^2*(2*a +
 4*b*x))/(a^2*(a*x + b*x^2)^(1/2)) - (2*d^2*x)/(b*(a*x + b*x^2)^(1/2)) + (
4*c*d*x)/(a*(x*(a + b*x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.63

$$\int \frac{(c+dx)^2}{(ax+bx^2)^{3/2}}\,dx = \frac{2\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2d^2x-2\sqrt{b}\,\sqrt{bx+a}\,a^2d^2x+4\sqrt{b}\,\sqrt{bx+a}\,abcdx-}{\sqrt{bx+a}\,a^2}$$

input
```
int((d*x+c)^2/(b*x^2+a*x)^(3/2),x)
```

output
```
(2*(sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a
**2*d**2*x - sqrt(b)*sqrt(a + b*x)*a**2*d**2*x + 2*sqrt(b)*sqrt(a + b*x)*a
*b*c*d*x - 2*sqrt(b)*sqrt(a + b*x)*b**2*c**2*x - sqrt(x)*a**2*b*d**2*x - s
qrt(x)*a*b**2*c**2 + 2*sqrt(x)*a*b**2*c*d*x - 2*sqrt(x)*b**3*c**2*x))/(sqr
t(a + b*x)*a**2*b**2*x)
```

3.173 $\int \frac{(c+dx)^2}{x(ax+bx^2)^{3/2}} dx$

Optimal result

Integrand size = 24, antiderivative size = 98

$$\int \frac{(c+dx)^2}{x(ax+bx^2)^{3/2}} dx = \frac{8c(bc-ad)}{3a^2\sqrt{ax+bx^2}} + \frac{8(bc-ad)(2bc-ad)x}{3a^3\sqrt{ax+bx^2}} - \frac{2(c+dx)^2}{3ax\sqrt{ax+bx^2}}$$

output
```
8/3*c*(-a*d+b*c)/a^2/(b*x^2+a*x)^(1/2)+8/3*(-a*d+b*c)*(-a*d+2*b*c)*x/a^3/(
b*x^2+a*x)^(1/2)-2/3*(d*x+c)^2/a/x/(b*x^2+a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.68

$$\int \frac{(c+dx)^2}{x(ax+bx^2)^{3/2}} dx = \frac{16b^2c^2x^2 + 8abcx(c-3dx) - 2a^2(c^2+6cdx-3d^2x^2)}{3a^3x\sqrt{x(a+bx)}}$$

input
```
Integrate[(c + d*x)^2/(x*(a*x + b*x^2)^(3/2)),x]
```

output
```
(16*b^2*c^2*x^2 + 8*a*b*c*x*(c - 3*d*x) - 2*a^2*(c^2 + 6*c*d*x - 3*d^2*x^2
))/(3*a^3*x*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1262, 27, 1220, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{x\left(ax+bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1262}$$

$$-\frac{\int -\frac{2bc^2+d(4bc-ad)x}{2x(bx^2+ax)^{3/2}}dx}{b}-\frac{d^2}{b\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2bc^2+d(4bc-ad)x}{x(bx^2+ax)^{3/2}}dx}{2b}-\frac{d^2}{b\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1220}$$

$$\frac{-\frac{(8b^2c^2-3ad(4bc-ad))\int \frac{1}{(bx^2+ax)^{3/2}}dx}{3a}-\frac{4bc^2}{3ax\sqrt{ax+bx^2}}}{2b}-\frac{d^2}{b\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{\frac{2(a+2bx)(8b^2c^2-3ad(4bc-ad))}{3a^3\sqrt{ax+bx^2}}-\frac{4bc^2}{3ax\sqrt{ax+bx^2}}}{2b}-\frac{d^2}{b\sqrt{ax+bx^2}}$$

input
```
Int[(c + d*x)^2/(x*(a*x + b*x^2)^(3/2)),x]
```

output
```
-(d^2/(b*Sqrt[a*x + b*x^2])) + ((-4*b*c^2)/(3*a*x*Sqrt[a*x + b*x^2]) + (2*
(8*b^2*c^2 - 3*a*d*(4*b*c - a*d))*(a + 2*b*x))/(3*a^3*Sqrt[a*x + b*x^2]))/
(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$-\frac{2\left(\left(-3d^{2}x^{2}+6cdx+c^{2}\right)a^{2}-4bcx\left(-3dx+c\right)a-8b^{2}c^{2}x^{2}\right)}{3\sqrt{x\left(bx+a\right)}\,a^{3}x}$	63
risch	$-\frac{2c\left(bx+a\right)\left(6adx-5cbx+ac\right)}{3a^{3}x\sqrt{x\left(bx+a\right)}}+\frac{2\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)x}{\sqrt{x\left(bx+a\right)}\,a^{3}}$	75
gosper	$-\frac{2\left(bx+a\right)\left(-3a^{2}d^{2}x^{2}+12abcd\,x^{2}-8b^{2}c^{2}x^{2}+6a^{2}cdx-4ab\,c^{2}x+a^{2}c^{2}\right)}{3a^{3}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}$	77
orering	$-\frac{2\left(bx+a\right)\left(-3a^{2}d^{2}x^{2}+12abcd\,x^{2}-8b^{2}c^{2}x^{2}+6a^{2}cdx-4ab\,c^{2}x+a^{2}c^{2}\right)}{3a^{3}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}$	77
trager	$-\frac{2\left(-3a^{2}d^{2}x^{2}+12abcd\,x^{2}-8b^{2}c^{2}x^{2}+6a^{2}cdx-4ab\,c^{2}x+a^{2}c^{2}\right)\sqrt{b\,x^{2}+ax}}{3\left(bx+a\right)a^{3}x^{2}}$	82
default	$c^{2}\left(-\frac{2}{3ax\sqrt{b\,x^{2}+ax}}+\frac{8b\left(2bx+a\right)}{3a^{3}\sqrt{b\,x^{2}+ax}}\right)+d^{2}\left(-\frac{1}{b\sqrt{b\,x^{2}+ax}}+\frac{2bx+a}{ab\sqrt{b\,x^{2}+ax}}\right)-\frac{4cd\left(2bx+a\right)}{a^{2}\sqrt{b\,x^{2}+ax}}$	118

input
```
int((d*x+c)^2/x/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(x*(b*x+a))^(1/2)*((-3*d^2*x^2+6*c*d*x+c^2)*a^2-4*b*c*x*(-3*d*x+c)*a-
8*b^2*c^2*x^2)/a^3/x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx)^2}{x\left(ax+bx^2\right)^{3/2}}\,dx = -\frac{2\left(a^2c^2-\left(8\,b^2c^2-12\,abcd+3\,a^2d^2\right)x^2-2\left(2\,abc^2-3\,a^2cd\right)x\right)\sqrt{bx^2+ax}}{3\left(a^3bx^3+a^4x^2\right)}$$

input
```
integrate((d*x+c)^2/x/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(a^2*c^2 - (8*b^2*c^2 - 12*a*b*c*d + 3*a^2*d^2)*x^2 - 2*(2*a*b*c^2 -
3*a^2*c*d)*x)*sqrt(b*x^2 + a*x)/(a^3*b*x^3 + a^4*x^2)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{x\,(ax+bx^2)^{3/2}}\,dx = \int \frac{(c+dx)^2}{x\,(x\,(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**2/x/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral((c + d*x)**2/(x*(x*(a + b*x))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx)^2}{x\,(ax+bx^2)^{3/2}}\,dx = \frac{16\,b^2c^2x}{3\,\sqrt{bx^2+ax}a^3} - \frac{8\,bcdx}{\sqrt{bx^2+ax}a^2} + \frac{2\,d^2x}{\sqrt{bx^2+ax}a} + \frac{8\,bc^2}{3\,\sqrt{bx^2+ax}a^2} - \frac{4\,cd}{\sqrt{bx^2+ax}a} - \frac{2\,c^2}{3\,\sqrt{bx^2+ax}ax}$$

input
```
integrate((d*x+c)^2/x/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
16/3*b^2*c^2*x/(sqrt(b*x^2 + a*x)*a^3) - 8*b*c*d*x/(sqrt(b*x^2 + a*x)*a^2)
 + 2*d^2*x/(sqrt(b*x^2 + a*x)*a) + 8/3*b*c^2/(sqrt(b*x^2 + a*x)*a^2) - 4*c
*d/(sqrt(b*x^2 + a*x)*a) - 2/3*c^2/(sqrt(b*x^2 + a*x)*a*x)
```

Giac [F]

$$\int \frac{(c+dx)^2}{x\,(ax+bx^2)^{3/2}}\,dx = \int \frac{(dx+c)^2}{(bx^2+ax)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((d*x+c)^2/x/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/((b*x^2 + a*x)^(3/2)*x), x)
```

Mupad [B] (verification not implemented)

Time = 9.61 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx)^2}{x\left(ax+bx^2\right)^{3/2}}\,dx = -\frac{2\sqrt{b\,x^2+a\,x}\,\left(a^2\,c^2+6\,a^2\,c\,d\,x-3\,a^2\,d^2\,x^2-4\,a\,b\,c^2\,x+12\,a\,b\,c\,d\,x^2-8\,b^2\,c^2\,x^2\right)}{3\,a^3\,x^2\,\left(a+b\,x\right)}$$

input
```
int((c + d*x)^2/(x*(a*x + b*x^2)^(3/2)),x)
```

output
```
-(2*(a*x + b*x^2)^(1/2)*(a^2*c^2 - 3*a^2*d^2*x^2 - 8*b^2*c^2*x^2 - 4*a*b*c
^2*x + 6*a^2*c*d*x + 12*a*b*c*d*x^2))/(3*a^3*x^2*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.52

$$\int \frac{(c+dx)^2}{x\left(ax+bx^2\right)^{3/2}}\,dx = \frac{-4\sqrt{b}\,\sqrt{bx+a}\,a^2d^2x^2+8\sqrt{b}\,\sqrt{bx+a}\,abcd\,x^2-\frac{16\sqrt{b}\,\sqrt{bx+a}\,b^2c^2x^2}{3}-\frac{2\sqrt{x}\,a^2b\,c^2}{3}-4\sqrt{x}}{\sqrt{bx+a}\,a^3b\,x^2}$$

input
```
int((d*x+c)^2/x/(b*x^2+a*x)^(3/2),x)
```

output
```
(2*( - 6*sqrt(b)*sqrt(a + b*x)*a**2*d**2*x**2 + 12*sqrt(b)*sqrt(a + b*x)*a
*b*c*d*x**2 - 8*sqrt(b)*sqrt(a + b*x)*b**2*c**2*x**2 - sqrt(x)*a**2*b*c**2
 - 6*sqrt(x)*a**2*b*c*d*x + 3*sqrt(x)*a**2*b*d**2*x**2 + 4*sqrt(x)*a*b**2*
c**2*x - 12*sqrt(x)*a*b**2*c*d*x**2 + 8*sqrt(x)*b**3*c**2*x**2))/(3*sqrt(a
 + b*x)*a**3*b*x**2)
```

3.174 $\int \frac{(c+dx)^2}{x^2(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 163

$$\int \frac{(c+dx)^2}{x^2(ax+bx^2)^{3/2}}\,dx = -\frac{8(bc-ad)(6bc-ad)}{15a^3\sqrt{ax+bx^2}} - \frac{8(bc-ad)(2bc-ad)(6bc-ad)x}{15a^4c\sqrt{ax+bx^2}} + \frac{2(6bc-ad)(c+dx)^2}{15a^2cx\sqrt{ax+bx^2}} - \frac{2(c+dx)^3}{5acx^2\sqrt{ax+bx^2}}$$

output
```
-8/15*(-a*d+b*c)*(-a*d+6*b*c)/a^3/(b*x^2+a*x)^(1/2)-8/15*(-a*d+b*c)*(-a*d+
2*b*c)*(-a*d+6*b*c)*x/a^4/c/(b*x^2+a*x)^(1/2)+2/15*(-a*d+6*b*c)*(d*x+c)^2/
a^2/c/x/(b*x^2+a*x)^(1/2)-2/5*(d*x+c)^3/a/c/x^2/(b*x^2+a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.61

$$\int \frac{(c+dx)^2}{x^2(ax+bx^2)^{3/2}}\,dx = -\frac{2(48b^3c^2x^3+8ab^2cx^2(3c-10dx)+2a^2bx(-3c^2-20cdx+15d^2x^2)+a^3(3c^2+10cdx+15d^2x^2))}{15a^4x^2\sqrt{x(a+bx)}}$$

input
```
Integrate[(c + d*x)^2/(x^2*(a*x + b*x^2)^(3/2)),x]
```

output
```
(-2*(48*b^3*c^2*x^3 + 8*a*b^2*c*x^2*(3*c - 10*d*x) + 2*a^2*b*x*(-3*c^2 - 2
0*c*d*x + 15*d^2*x^2) + a^3*(3*c^2 + 10*c*d*x + 15*d^2*x^2)))/(15*a^4*x^2*
Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1262, 27, 1220, 1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{x^2\left(ax+bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1262}$$

$$-\frac{\int -\frac{4bc^2+d(8bc-3ad)x}{2x^2(bx^2+ax)^{3/2}}dx}{2b}-\frac{d^2}{2bx\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4bc^2+d(8bc-3ad)x}{x^2(bx^2+ax)^{3/2}}dx}{4b}-\frac{d^2}{2bx\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1220}$$

$$\frac{-\frac{(15a^2d^2-40abcd+24b^2c^2)\int \frac{1}{x\left(bx^2+ax\right)^{3/2}}dx}{5a}-\frac{8bc^2}{5ax^2\sqrt{ax+bx^2}}}{4b}-\frac{d^2}{2bx\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1129}$$

$$\frac{-\frac{(15a^2d^2-40abcd+24b^2c^2)\left(-\frac{4b\int \frac{1}{\left(bx^2+ax\right)^{3/2}}dx}{3a}-\frac{2}{3ax\sqrt{ax+bx^2}}\right)}{5a}-\frac{8bc^2}{5ax^2\sqrt{ax+bx^2}}}{4b}-\frac{d^2}{2bx\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1088}$$

$$-\frac{-\frac{\left(\frac{8b(a+2bx)}{3a^3\sqrt{ax+bx^2}}-\frac{2}{3ax\sqrt{ax+bx^2}}\right)(15a^2d^2-40abcd+24b^2c^2)}{5a}-\frac{8bc^2}{5ax^2\sqrt{ax+bx^2}}}{4b}-\frac{d^2}{2bx\sqrt{ax+bx^2}}$$

input
```
Int[(c + d*x)^2/(x^2*(a*x + b*x^2)^(3/2)),x]
```

output
```
-1/2*d^2/(b*x*Sqrt[a*x + b*x^2]) + ((-8*b*c^2)/(5*a*x^2*Sqrt[a*x + b*x^2])
 - ((24*b^2*c^2 - 40*a*b*c*d + 15*a^2*d^2)*(-2/(3*a*x*Sqrt[a*x + b*x^2]) +
 (8*b*(a + 2*b*x))/(3*a^3*Sqrt[a*x + b*x^2])))/(5*a))/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.56

method	result
pseudoelliptic	$-\frac{2\left(\left(5d^{2}x^{2}+\frac{10}{3}cdx+c^{2}\right)a^{3}-2xb\left(-5d^{2}x^{2}+\frac{20}{3}cdx+c^{2}\right)a^{2}+8x^{2}\left(-\frac{10dx}{3}+c\right)b^{2}ca+16b^{3}c^{2}x^{3}\right)}{5\sqrt{x(bx+a)}\,x^{2}a^{4}}$
risch	$-\frac{2(bx+a)\left(15a^{2}d^{2}x^{2}-50abcd\,x^{2}+33b^{2}c^{2}x^{2}+10a^{2}cdx-9ab\,c^{2}x+3a^{2}c^{2}\right)}{15a^{4}x^{2}\sqrt{x(bx+a)}}-\frac{2b\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)x}{\sqrt{x(bx+a)}\,a^{4}}$
gosper	$-\frac{2(bx+a)\left(30d^{2}x^{3}a^{2}b-80a\,b^{2}cd\,x^{3}+48b^{3}c^{2}x^{3}+15a^{3}d^{2}x^{2}-40x^{2}a^{2}bcd+24a\,b^{2}c^{2}x^{2}+10a^{3}cdx-6a^{2}b\,c^{2}x+3c^{2}a^{3}\right)}{15x\,a^{4}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}$
orering	$-\frac{2(bx+a)\left(30d^{2}x^{3}a^{2}b-80a\,b^{2}cd\,x^{3}+48b^{3}c^{2}x^{3}+15a^{3}d^{2}x^{2}-40x^{2}a^{2}bcd+24a\,b^{2}c^{2}x^{2}+10a^{3}cdx-6a^{2}b\,c^{2}x+3c^{2}a^{3}\right)}{15x\,a^{4}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}$
trager	$-\frac{2\left(30d^{2}x^{3}a^{2}b-80a\,b^{2}cd\,x^{3}+48b^{3}c^{2}x^{3}+15a^{3}d^{2}x^{2}-40x^{2}a^{2}bcd+24a\,b^{2}c^{2}x^{2}+10a^{3}cdx-6a^{2}b\,c^{2}x+3c^{2}a^{3}\right)\sqrt{b\,x^{2}+ax}}{15(bx+a)a^{4}x^{3}}$
default	$-\frac{2d^{2}(2bx+a)}{a^{2}\sqrt{b\,x^{2}+ax}}+c^{2}\left(-\frac{2}{5a\,x^{2}\sqrt{b\,x^{2}+ax}}-\frac{6b\left(-\frac{2}{3ax\sqrt{b\,x^{2}+ax}}+\frac{8b(2bx+a)}{3a^{3}\sqrt{b\,x^{2}+ax}}\right)}{5a}\right)+2cd\left(-\frac{2}{3ax\sqrt{b\,x^{2}+ax}}+\frac{8}{3a}\right.$

input
```
int((d*x+c)^2/x^2/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/(x*(b*x+a))^(1/2)*((5*d^2*x^2+10/3*c*d*x+c^2)*a^3-2*x*b*(-5*d^2*x^2+2
0/3*c*d*x+c^2)*a^2+8*x^2*(-10/3*d*x+c)*b^2*c*a+16*b^3*c^2*x^3)/x^2/a^4
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.74

$$\int \frac{(c+dx)^2}{x^2 (ax+bx^2)^{3/2}}\,dx =$$
$$-\frac{2\,(3\,a^3c^2+2\,(24\,b^3c^2-40\,ab^2cd+15\,a^2bd^2)x^3+(24\,ab^2c^2-40\,a^2bcd+15\,a^3d^2)x^2-2\,(3\,a^2bc^2-5\,a^3cd)}{15\,(a^4bx^4+a^5x^3)}$$

input
```
integrate((d*x+c)^2/x^2/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/15*(3*a^3*c^2 + 2*(24*b^3*c^2 - 40*a*b^2*c*d + 15*a^2*b*d^2)*x^3 + (24*
a*b^2*c^2 - 40*a^2*b*c*d + 15*a^3*d^2)*x^2 - 2*(3*a^2*b*c^2 - 5*a^3*c*d)*x
)*sqrt(b*x^2 + a*x)/(a^4*b*x^4 + a^5*x^3)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{x^2 (ax+bx^2)^{3/2}}\,dx = \int \frac{(c+dx)^2}{x^2 (x(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**2/x**2/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral((c + d*x)**2/(x**2*(x*(a + b*x))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.18

$$\int \frac{(c+dx)^2}{x^2 (ax+bx^2)^{3/2}}\,dx = -\frac{32\,b^3c^2x}{5\,\sqrt{bx^2+ax}a^4}+\frac{32\,b^2cdx}{3\,\sqrt{bx^2+ax}a^3} - \frac{4\,bd^2x}{\sqrt{bx^2+ax}a^2}-\frac{16\,b^2c^2}{5\,\sqrt{bx^2+ax}a^3}+\frac{16\,bcd}{3\,\sqrt{bx^2+ax}a^2}-\frac{2\,d^2}{\sqrt{bx^2+ax}a} + \frac{4\,bc^2}{5\,\sqrt{bx^2+ax}a^2x}-\frac{4\,cd}{3\,\sqrt{bx^2+ax}ax}-\frac{2\,c^2}{5\,\sqrt{bx^2+ax}ax^2}$$

input
```
integrate((d*x+c)^2/x^2/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
-32/5*b^3*c^2*x/(sqrt(b*x^2 + a*x)*a^4) + 32/3*b^2*c*d*x/(sqrt(b*x^2 + a*x
)*a^3) - 4*b*d^2*x/(sqrt(b*x^2 + a*x)*a^2) - 16/5*b^2*c^2/(sqrt(b*x^2 + a*
x)*a^3) + 16/3*b*c*d/(sqrt(b*x^2 + a*x)*a^2) - 2*d^2/(sqrt(b*x^2 + a*x)*a)
 + 4/5*b*c^2/(sqrt(b*x^2 + a*x)*a^2*x) - 4/3*c*d/(sqrt(b*x^2 + a*x)*a*x) -
 2/5*c^2/(sqrt(b*x^2 + a*x)*a*x^2)
```

Giac [F]

$$\int \frac{(c+dx)^2}{x^2\left(ax+bx^2\right)^{3/2}}\,dx = \int \frac{(dx+c)^2}{(bx^2+ax)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((d*x+c)^2/x^2/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/((b*x^2 + a*x)^(3/2)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 9.52 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.74

$$\int \frac{(c+dx)^2}{x^2\left(ax+bx^2\right)^{3/2}}\,dx = -\frac{2\sqrt{b\,x^2+a\,x}\,(3\,a^3\,c^2+10\,a^3\,c\,d\,x+15\,a^3\,d^2\,x^2-6\,a^2\,b\,c^2\,x-40\,a^2\,b\,c\,d\,x^2+30\,a^2\,b\,d^2\,x^3+24\,a\,b^2\,c^2\,x^2 \cdots}{15\,a^4\,x^3\,(a+b\,x)}$$

input
```
int((c + d*x)^2/(x^2*(a*x + b*x^2)^(3/2)),x)
```

output
```
-(2*(a*x + b*x^2)^(1/2)*(3*a^3*c^2 + 15*a^3*d^2*x^2 + 48*b^3*c^2*x^3 + 24*
a*b^2*c^2*x^2 + 30*a^2*b*d^2*x^3 + 10*a^3*c*d*x - 6*a^2*b*c^2*x - 40*a^2*b
*c*d*x^2 - 80*a*b^2*c*d*x^3))/(15*a^4*x^3*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx)^2}{x^2 (ax+bx^2)^{3/2}}\,dx = \frac{4\sqrt{b}\,\sqrt{bx+a}\,a^2d^2x^3 - \frac{32\sqrt{b}\,\sqrt{bx+a}\,abcd\,x^3}{3} + \frac{32\sqrt{b}\,\sqrt{bx+a}\,b^2c^2x^3}{5} - \frac{2\sqrt{x}\,a^3c^2}{5} - \frac{4\sqrt{x}\,a^3cdx}{3} - }{}$$

input
```
int((d*x+c)^2/x^2/(b*x^2+a*x)^(3/2),x)
```

output
```
(2*(30*sqrt(b)*sqrt(a + b*x)*a**2*d**2*x**3 - 80*sqrt(b)*sqrt(a + b*x)*a*b
*c*d*x**3 + 48*sqrt(b)*sqrt(a + b*x)*b**2*c**2*x**3 - 3*sqrt(x)*a**3*c**2
- 10*sqrt(x)*a**3*c*d*x - 15*sqrt(x)*a**3*d**2*x**2 + 6*sqrt(x)*a**2*b*c**
2*x + 40*sqrt(x)*a**2*b*c*d*x**2 - 30*sqrt(x)*a**2*b*d**2*x**3 - 24*sqrt(x
)*a*b**2*c**2*x**2 + 80*sqrt(x)*a*b**2*c*d*x**3 - 48*sqrt(x)*b**3*c**2*x**
3))/(15*sqrt(a + b*x)*a**4*x**3)
```

3.175 $\int \frac{(c+dx)^2}{x^3(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 196

$$\int \frac{(c+dx)^2}{x^3(ax+bx^2)^{3/2}}\,dx = -\frac{2c^2}{7ax^3\sqrt{ax+bx^2}} + \frac{4c(4bc-7ad)}{35a^2x^2\sqrt{ax+bx^2}} + \frac{2\left(35d^2+\frac{12bc(4bc-7ad)}{a^2}\right)}{35ax\sqrt{ax+bx^2}} - \frac{8(35a^2d^2+12bc(4bc-7ad))\sqrt{ax+bx^2}}{105a^4x^2} + \frac{16b(35a^2d^2+12bc(4bc-7ad))\sqrt{ax+bx^2}}{105a^5x}$$

output
```
-2/7*c^2/a/x^3/(b*x^2+a*x)^(1/2)+4/35*c*(-7*a*d+4*b*c)/a^2/x^2/(b*x^2+a*x)
^(1/2)+2/35*(35*d^2+12*b*c*(-7*a*d+4*b*c)/a^2)/a/x/(b*x^2+a*x)^(1/2)-8/105
*(35*a^2*d^2+12*b*c*(-7*a*d+4*b*c))*(b*x^2+a*x)^(1/2)/a^4/x^2+16/105*b*(35
*a^2*d^2+12*b*c*(-7*a*d+4*b*c))*(b*x^2+a*x)^(1/2)/a^5/x
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.67

$$\int \frac{(c+dx)^2}{x^3\left(ax+bx^2\right)^{3/2}}\,dx = \frac{2(384b^4c^2x^4+96ab^3cx^3(2c-7dx)+8a^2b^2x^2(-6c^2-42cdx+35d^2x^2)+4a^3bx(6}{105a^5x^3\sqrt{x(a+bx)}}$$

input
```
Integrate[(c + d*x)^2/(x^3*(a*x + b*x^2)^(3/2)),x]
```

output
```
(2*(384*b^4*c^2*x^4 + 96*a*b^3*c*x^3*(2*c - 7*d*x) + 8*a^2*b^2*x^2*(-6*c^2
 - 42*c*d*x + 35*d^2*x^2) + 4*a^3*b*x*(6*c^2 + 21*c*d*x + 35*d^2*x^2) - a^
4*(15*c^2 + 42*c*d*x + 35*d^2*x^2)))/(105*a^5*x^3*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1262, 27, 1220, 1129, 1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{x^3\left(ax+bx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1262}$$

$$-\frac{\int -\frac{6bc^2+d(12bc-5ad)x}{2x^3\left(bx^2+ax\right)^{3/2}}dx}{3b}-\frac{d^2}{3bx^2\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{6bc^2+d(12bc-5ad)x}{x^3\left(bx^2+ax\right)^{3/2}}dx}{6b}-\frac{d^2}{3bx^2\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1220}$$

$$\frac{-\frac{\left(35a^2d^2-84abcd+48b^2c^2\right)\int \frac{1}{x^2\left(bx^2+ax\right)^{3/2}}dx}{7a}-\frac{12bc^2}{7ax^3\sqrt{ax+bx^2}}}{6b}-\frac{d^2}{3bx^2\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 1129$$

$$-\frac{-\frac{\left(35a^2d^2-84abcd+48b^2c^2\right)\left(-\frac{6b\int \frac{1}{x\left(bx^2+ax\right)^{3/2}}dx}{5a}-\frac{2}{5ax^2\sqrt{ax+bx^2}}\right)}{7a}-\frac{12bc^2}{7ax^3\sqrt{ax+bx^2}}}{6b}-\frac{d^2}{3bx^2\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 1129$$

$$-\frac{-\frac{\left(35a^2d^2-84abcd+48b^2c^2\right)\left(-\frac{6b\left(-\frac{4b\int \frac{1}{\left(bx^2+ax\right)^{3/2}}dx}{3a}-\frac{2}{3ax\sqrt{ax+bx^2}}\right)}{5a}-\frac{2}{5ax^2\sqrt{ax+bx^2}}\right)}{7a}-\frac{12bc^2}{7ax^3\sqrt{ax+bx^2}}}{6b}-\frac{d^2}{3bx^2\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 1088$$

$$-\frac{-\frac{\left(-\frac{6b\left(\frac{8b(a+2bx)}{3a^3\sqrt{ax+bx^2}}-\frac{2}{3ax\sqrt{ax+bx^2}}\right)}{5a}-\frac{2}{5ax^2\sqrt{ax+bx^2}}\right)\left(35a^2d^2-84abcd+48b^2c^2\right)}{7a}-\frac{12bc^2}{7ax^3\sqrt{ax+bx^2}}}{6b}-\frac{d^2}{3bx^2\sqrt{ax+bx^2}}$$

input
```
Int[(c + d*x)^2/(x^3*(a*x + b*x^2)^(3/2)),x]
```

output
```
-1/3*d^2/(b*x^2*Sqrt[a*x + b*x^2]) + ((-12*b*c^2)/(7*a*x^3*Sqrt[a*x + b*x^
2]) - ((48*b^2*c^2 - 84*a*b*c*d + 35*a^2*d^2)*(-2/(5*a*x^2*Sqrt[a*x + b*x^
2]) - (6*b*(-2/(3*a*x*Sqrt[a*x + b*x^2]) + (8*b*(a + 2*b*x))/(3*a^3*Sqrt[a
*x + b*x^2])))/(5*a)))/(7*a))/(6*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$\frac{(-70d^2x^2-84cdx-30c^2)a^4+48x(\frac{35}{6}d^2x^2+\frac{7}{2}cdx+c^2)b a^3-96x^2(-\frac{35}{6}d^2x^2+7cdx+c^2)b^2a^2+384\left(-\frac{7dx}{2}+c\right)x^3b^3ca+768b}{105\sqrt{x(bx+a)}\,x^3a^5}$
risch	$-\frac{2(bx+a)(-175d^2x^3a^2b+462a\,b^2cd\,x^3-279b^3c^2x^3+35a^3d^2x^2-126x^2a^2bcd+87a\,b^2c^2x^2+42a^3cdx-39a^2b\,c^2x+15c^2a^3)}{105a^5x^3\sqrt{x(bx+a)}}$
gosper	$-\frac{2(bx+a)(-280a^2b^2d^2x^4+672a\,b^3cd\,x^4-384b^4c^2x^4-140a^3b\,d^2x^3+336a^2b^2cd\,x^3-192a\,b^3c^2x^3+35a^4d^2x^2-84a^3dcb\,x^2-}{105x^2a^5(b\,x^2+ax)^{\frac{3}{2}}}$
orering	$-\frac{2(bx+a)(-280a^2b^2d^2x^4+672a\,b^3cd\,x^4-384b^4c^2x^4-140a^3b\,d^2x^3+336a^2b^2cd\,x^3-192a\,b^3c^2x^3+35a^4d^2x^2-84a^3dcb\,x^2-}{105x^2a^5(b\,x^2+ax)^{\frac{3}{2}}}$
trager	$-\frac{2(-280a^2b^2d^2x^4+672a\,b^3cd\,x^4-384b^4c^2x^4-140a^3b\,d^2x^3+336a^2b^2cd\,x^3-192a\,b^3c^2x^3+35a^4d^2x^2-84a^3dcb\,x^2+48a^2b^2}{105(bx+a)a^5x^4}$
default	$c^2\left(-\frac{2}{7a\,x^3\sqrt{b\,x^2+ax}}-\frac{8b\left(-\frac{2}{5a\,x^2\sqrt{b\,x^2+ax}}-\frac{6b\left(-\frac{2}{3ax\sqrt{b\,x^2+ax}}+\frac{8b(2bx+a)}{3a^3\sqrt{b\,x^2+ax}}\right)}{5a}\right)}{7a}\right)+d^2\left(-\frac{2}{3ax\sqrt{b\,x^2+ax}}+\right.$

input
```
int((d*x+c)^2/x^3/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*((-70*d^2*x^2-84*c*d*x-30*c^2)*a^4+48*x*(35/6*d^2*x^2+7/2*c*d*x+c^2)
*b*a^3-96*x^2*(-35/6*d^2*x^2+7*c*d*x+c^2)*b^2*a^2+384*(-7/2*d*x+c)*x^3*b^3
*c*a+768*b^4*c^2*x^4)/(x*(b*x+a))^(1/2)/x^3/a^5
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.81

$$\int \frac{(c+dx)^2}{x^3\left(ax+bx^2\right)^{3/2}}\,dx =$$
$$-\frac{2\left(15\,a^4c^2-8\left(48\,b^4c^2-84\,ab^3cd+35\,a^2b^2d^2\right)x^4-4\left(48\,ab^3c^2-84\,a^2b^2cd+35\,a^3bd^2\right)x^3+\left(48\,a^2b^2c^2-8\right.\right.}{105\left(a^5bx^5+a^6x^4\right)}$$

input
```
integrate((d*x+c)^2/x^3/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/105*(15*a^4*c^2 - 8*(48*b^4*c^2 - 84*a*b^3*c*d + 35*a^2*b^2*d^2)*x^4 -
4*(48*a*b^3*c^2 - 84*a^2*b^2*c*d + 35*a^3*b*d^2)*x^3 + (48*a^2*b^2*c^2 - 8
4*a^3*b*c*d + 35*a^4*d^2)*x^2 - 6*(4*a^3*b*c^2 - 7*a^4*c*d)*x)*sqrt(b*x^2
+ a*x)/(a^5*b*x^5 + a^6*x^4)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{x^3\,(ax+bx^2)^{3/2}}\,dx = \int \frac{(c+dx)^2}{x^3\,(x\,(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**2/x**3/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral((c + d*x)**2/(x**3*(x*(a + b*x))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.36

$$\begin{aligned}\int \frac{(c+dx)^2}{x^3\,(ax+bx^2)^{3/2}}\,dx &= \frac{256\,b^4c^2x}{35\,\sqrt{bx^2+ax}a^5} - \frac{64\,b^3cdx}{5\,\sqrt{bx^2+ax}a^4} \\ &+ \frac{16\,b^2d^2x}{3\,\sqrt{bx^2+ax}a^3} + \frac{128\,b^3c^2}{35\,\sqrt{bx^2+ax}a^4} - \frac{32\,b^2cd}{5\,\sqrt{bx^2+ax}a^3} + \frac{8\,bd^2}{3\,\sqrt{bx^2+ax}a^2} \\ &- \frac{32\,b^2c^2}{35\,\sqrt{bx^2+ax}a^3x} + \frac{8\,bcd}{5\,\sqrt{bx^2+ax}a^2x} - \frac{2\,d^2}{3\,\sqrt{bx^2+ax}ax} \\ &+ \frac{16\,bc^2}{35\,\sqrt{bx^2+ax}a^2x^2} - \frac{4\,cd}{5\,\sqrt{bx^2+ax}ax^2} - \frac{2\,c^2}{7\,\sqrt{bx^2+ax}ax^3}\end{aligned}$$

input
```
integrate((d*x+c)^2/x^3/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
256/35*b^4*c^2*x/(sqrt(b*x^2 + a*x)*a^5) - 64/5*b^3*c*d*x/(sqrt(b*x^2 + a*
x)*a^4) + 16/3*b^2*d^2*x/(sqrt(b*x^2 + a*x)*a^3) + 128/35*b^3*c^2/(sqrt(b*
x^2 + a*x)*a^4) - 32/5*b^2*c*d/(sqrt(b*x^2 + a*x)*a^3) + 8/3*b*d^2/(sqrt(b
*x^2 + a*x)*a^2) - 32/35*b^2*c^2/(sqrt(b*x^2 + a*x)*a^3*x) + 8/5*b*c*d/(sq
rt(b*x^2 + a*x)*a^2*x) - 2/3*d^2/(sqrt(b*x^2 + a*x)*a*x) + 16/35*b*c^2/(sq
rt(b*x^2 + a*x)*a^2*x^2) - 4/5*c*d/(sqrt(b*x^2 + a*x)*a*x^2) - 2/7*c^2/(sq
rt(b*x^2 + a*x)*a*x^3)
```

Giac [F]

$$\int \frac{(c+dx)^2}{x^3\left(ax+bx^2\right)^{3/2}}\,dx=\int \frac{(dx+c)^2}{\left(bx^2+ax\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((d*x+c)^2/x^3/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/((b*x^2 + a*x)^(3/2)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 9.88 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.11

$$\int \frac{(c+dx)^2}{x^3\left(ax+bx^2\right)^{3/2}}\,dx=-\frac{2\,c^2\,\sqrt{b\,x^2+a\,x}}{7\,a^2\,x^4}$$
$$-\frac{\sqrt{b\,x^2+a\,x}\,\left(70\,a^3\,d^2-252\,a^2\,b\,c\,d+174\,a\,b^2\,c^2\right)}{105\,a^5\,x^2}$$
$$-\frac{\sqrt{b\,x^2+a\,x}\,\left(x\,\left(\frac{140\,a^2\,b^2\,d^2-504\,a\,b^3\,c\,d+348\,b^4\,c^2}{105\,a^5}-\frac{4\,b^2\,\left(175\,a^2\,d^2-462\,a\,b\,c\,d+279\,b^2\,c^2\right)}{105\,a^5}\right)-\frac{2\,b\,\left(175\,a^2\,d^2-462\,a\,b\,c\,d+279\,b^2\,c^2\right)}{105\,a^4}\right)}{x\,\left(a+b\,x\right)}$$
$$-\frac{2\,c\,\sqrt{b\,x^2+a\,x}\,\left(14\,a\,d-13\,b\,c\right)}{35\,a^3\,x^3}$$

input
```
int((c + d*x)^2/(x^3*(a*x + b*x^2)^(3/2)),x)
```

output
```
- (2*c^2*(a*x + b*x^2)^(1/2))/(7*a^2*x^4) - ((a*x + b*x^2)^(1/2)*(70*a^3*d
^2 + 174*a*b^2*c^2 - 252*a^2*b*c*d))/(105*a^5*x^2) - ((a*x + b*x^2)^(1/2)*
(x*((348*b^4*c^2 + 140*a^2*b^2*d^2 - 504*a*b^3*c*d)/(105*a^5) - (4*b^2*(17
5*a^2*d^2 + 279*b^2*c^2 - 462*a*b*c*d))/(105*a^5)) - (2*b*(175*a^2*d^2 + 2
79*b^2*c^2 - 462*a*b*c*d))/(105*a^4)))/(x*(a + b*x)) - (2*c*(a*x + b*x^2)^
(1/2)*(14*a*d - 13*b*c))/(35*a^3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx)^2}{x^3\,(ax+bx^2)^{3/2}}\,dx = \frac{-\frac{16\sqrt{b}\,\sqrt{bx+a}\,a^2 b\,d^2 x^4}{3} + \frac{64\sqrt{b}\,\sqrt{bx+a}\,a\,b^2 cd\,x^4}{5} - \frac{256\sqrt{b}\,\sqrt{bx+a}\,b^3 c^2 x^4}{35} - \frac{2\sqrt{x}\,a^4 c^2}{7} - \frac{4\sqrt{x}\,a^4 cdx}{5} - $$

input
```
int((d*x+c)^2/x^3/(b*x^2+a*x)^(3/2),x)
```

output
```
(2*( - 280*sqrt(b)*sqrt(a + b*x)*a**2*b*d**2*x**4 + 672*sqrt(b)*sqrt(a + b
*x)*a*b**2*c*d*x**4 - 384*sqrt(b)*sqrt(a + b*x)*b**3*c**2*x**4 - 15*sqrt(x
)*a**4*c**2 - 42*sqrt(x)*a**4*c*d*x - 35*sqrt(x)*a**4*d**2*x**2 + 24*sqrt(
x)*a**3*b*c**2*x + 84*sqrt(x)*a**3*b*c*d*x**2 + 140*sqrt(x)*a**3*b*d**2*x*
*3 - 48*sqrt(x)*a**2*b**2*c**2*x**2 - 336*sqrt(x)*a**2*b**2*c*d*x**3 + 280
*sqrt(x)*a**2*b**2*d**2*x**4 + 192*sqrt(x)*a*b**3*c**2*x**3 - 672*sqrt(x)*
a*b**3*c*d*x**4 + 384*sqrt(x)*b**4*c**2*x**4))/(105*sqrt(a + b*x)*a**5*x**
4)
```

3.176 $\int \frac{(c+dx)^2}{x^4(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 228

$$\begin{aligned}\int \frac{(c+dx)^2}{x^4(ax+bx^2)^{3/2}}\,dx &= -\frac{2c^2}{9ax^4\sqrt{ax+bx^2}}+\frac{4c(5bc-9ad)}{63a^2x^3\sqrt{ax+bx^2}}\\ &+\frac{2(20bc-21ad)(4bc-3ad)}{63a^3x^2\sqrt{ax+bx^2}}-\frac{4(20bc-21ad)(4bc-3ad)\sqrt{ax+bx^2}}{105a^4x^3}\\ &+\frac{16b(20bc-21ad)(4bc-3ad)\sqrt{ax+bx^2}}{315a^5x^2}\\ &-\frac{32b^2(20bc-21ad)(4bc-3ad)\sqrt{ax+bx^2}}{315a^6x}\end{aligned}$$

output
```
-2/9*c^2/a/x^4/(b*x^2+a*x)^(1/2)+4/63*c*(-9*a*d+5*b*c)/a^2/x^3/(b*x^2+a*x)
^(1/2)+2/63*(-21*a*d+20*b*c)*(-3*a*d+4*b*c)/a^3/x^2/(b*x^2+a*x)^(1/2)-4/10
5*(-21*a*d+20*b*c)*(-3*a*d+4*b*c)*(b*x^2+a*x)^(1/2)/a^4/x^3+16/315*b*(-21*
a*d+20*b*c)*(-3*a*d+4*b*c)*(b*x^2+a*x)^(1/2)/a^5/x^2-32/315*b^2*(-21*a*d+2
0*b*c)*(-3*a*d+4*b*c)*(b*x^2+a*x)^(1/2)/a^6/x
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.70

$$\int \frac{(c+dx)^2}{x^4(ax+bx^2)^{3/2}}\,dx = -\frac{2(1280b^5c^2x^5+128ab^4cx^4(5c-18dx)+16a^2b^3x^3(-10c^2-72cdx+63d^2x^2)+8a^3b^2x^2(10c^2+36cdx+}{315a^6x^4\sqrt{x(a+bx)}}$$

input
```
Integrate[(c + d*x)^2/(x^4*(a*x + b*x^2)^(3/2)),x]
```

output
```
(-2*(1280*b^5*c^2*x^5 + 128*a*b^4*c*x^4*(5*c - 18*d*x) + 16*a^2*b^3*x^3*(-
10*c^2 - 72*c*d*x + 63*d^2*x^2) + 8*a^3*b^2*x^2*(10*c^2 + 36*c*d*x + 63*d^
2*x^2) - 2*a^4*b*x*(25*c^2 + 72*c*d*x + 63*d^2*x^2) + a^5*(35*c^2 + 90*c*d
*x + 63*d^2*x^2)))/(315*a^6*x^4*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1262, 27, 1220, 1129, 1129, 1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{x^4(ax+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1262}$$

$$-\frac{\int -\frac{8bc^2+d(16bc-7ad)x}{2x^4(bx^2+ax)^{3/2}}dx}{4b}-\frac{d^2}{4bx^3\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{8bc^2+d(16bc-7ad)x}{x^4(bx^2+ax)^{3/2}}dx}{8b}-\frac{d^2}{4bx^3\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1220}$$

$$-\frac{-\frac{(20bc-21ad)(4bc-3ad)\int \frac{1}{x^3\left(bx^2+ax\right)^{3/2}}dx}{9a}-\frac{16bc^2}{9ax^4\sqrt{ax+bx^2}}}{8b}-\frac{d^2}{4bx^3\sqrt{ax+bx^2}}$$

$\downarrow$ 1129

$$-\frac{-\frac{(20bc-21ad)(4bc-3ad)\left(-\frac{8b\int \frac{1}{x^2\left(bx^2+ax\right)^{3/2}}dx}{7a}-\frac{2}{7ax^3\sqrt{ax+bx^2}}\right)}{9a}-\frac{16bc^2}{9ax^4\sqrt{ax+bx^2}}}{8b}-\frac{d^2}{4bx^3\sqrt{ax+bx^2}}$$

$\downarrow$ 1129

$$-\frac{-\frac{(20bc-21ad)(4bc-3ad)\left(-\frac{8b\left(-\frac{6b\int \frac{1}{x\left(bx^2+ax\right)^{3/2}}dx}{5a}-\frac{2}{5ax^2\sqrt{ax+bx^2}}\right)}{7a}-\frac{2}{7ax^3\sqrt{ax+bx^2}}\right)}{9a}-\frac{16bc^2}{9ax^4\sqrt{ax+bx^2}}}{8b}-\frac{d^2}{4bx^3\sqrt{ax+bx^2}}$$

$\downarrow$ 1129

$$-\frac{-\frac{(20bc-21ad)(4bc-3ad)\left(-\frac{8b\left(-\frac{6b\left(-\frac{4b\int \frac{1}{\left(bx^2+ax\right)^{3/2}}dx}{3a}-\frac{2}{3ax\sqrt{ax+bx^2}}\right)}{5a}-\frac{2}{5ax^2\sqrt{ax+bx^2}}\right)}{7a}-\frac{2}{7ax^3\sqrt{ax+bx^2}}\right)}{9a}-\frac{16bc^2}{9ax^4\sqrt{ax+bx^2}}}{8b}-\frac{d^2}{4bx^3\sqrt{ax+bx^2}}$$

$\downarrow$ 1088

$$\frac{-\frac{\left(-\frac{8b\left(-\frac{6b\left(\frac{8b(a+2bx)}{3a^3\sqrt{ax+bx^2}}-\frac{2}{3ax\sqrt{ax+bx^2}}\right)}{5a}-\frac{2}{5ax^2\sqrt{ax+bx^2}}\right)}{7a}-\frac{2}{7ax^3\sqrt{ax+bx^2}}\right)(20bc-21ad)(4bc-3ad)}{9a}-\frac{16bc^2}{9ax^4\sqrt{ax+bx^2}}}{8b}-\frac{d^2}{4bx^3\sqrt{ax+bx^2}}$$

input
```
Int[(c + d*x)^2/(x^4*(a*x + b*x^2)^(3/2)),x]
```

output
```
-1/4*d^2/(b*x^3*Sqrt[a*x + b*x^2]) + ((-16*b*c^2)/(9*a*x^4*Sqrt[a*x + b*x^
2]) - ((20*b*c - 21*a*d)*(4*b*c - 3*a*d)*(-2/(7*a*x^3*Sqrt[a*x + b*x^2]) -
 (8*b*(-2/(5*a*x^2*Sqrt[a*x + b*x^2]) - (6*b*(-2/(3*a*x*Sqrt[a*x + b*x^2])
 + (8*b*(a + 2*b*x))/(3*a^3*Sqrt[a*x + b*x^2])))/(5*a)))/(7*a)))/(9*a))/(8
*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$\frac{(-126d^2x^2-180cdx-70c^2)a^5+100(\frac{63}{25}d^2x^2+\frac{72}{25}cdx+c^2)xba^4-160x^2(\frac{63}{10}d^2x^2+\frac{18}{5}cdx+c^2)b^2a^3+320x^3b^3(-\frac{63}{10}d^2x^2+\frac{36}{5}}{315\sqrt{x(bx+a)}\,x^4a^6}$
risch	$-\frac{2(bx+a)(693a^2b^2d^2x^4-1674ab^3cdx^4+965b^4c^2x^4-189a^3bd^2x^3+522a^2b^2cdx^3-325ab^3c^2x^3+63a^4d^2x^2-234a^3dcbx^2}{315a^6x^4\sqrt{x(bx+a)}}$
gosper	$-\frac{2(bx+a)(1008a^2b^3d^2x^5-2304ab^4cdx^5+1280b^5c^2x^5+504a^3b^2d^2x^4-1152a^2b^3cdx^4+640ab^4c^2x^4-126a^4bd^2x^3+288a^3}{315x^3a^6(bx^2+ax)^{\frac{3}{2}}}$
orering	$-\frac{2(bx+a)(1008a^2b^3d^2x^5-2304ab^4cdx^5+1280b^5c^2x^5+504a^3b^2d^2x^4-1152a^2b^3cdx^4+640ab^4c^2x^4-126a^4bd^2x^3+288a^3}{315x^3a^6(bx^2+ax)^{\frac{3}{2}}}$
trager	$-\frac{2(1008a^2b^3d^2x^5-2304ab^4cdx^5+1280b^5c^2x^5+504a^3b^2d^2x^4-1152a^2b^3cdx^4+640ab^4c^2x^4-126a^4bd^2x^3+288a^3b^2cdx^3}{315(bx+a)a^6x^5}$
default	$c^2\left(-\frac{2}{9ax^4\sqrt{bx^2+ax}}-\frac{10b\left(-\frac{2}{7ax^3\sqrt{bx^2+ax}}-\frac{8b\left(-\frac{2}{5ax^2\sqrt{bx^2+ax}}-\frac{6b\left(-\frac{2}{3ax\sqrt{bx^2+ax}}+\frac{8b(2bx+a)}{3a^3\sqrt{bx^2+ax}}\right)}{5a}\right)}{7a}\right)}{9a}\right)+$

input
```
int((d*x+c)^2/x^4/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*((-126*d^2*x^2-180*c*d*x-70*c^2)*a^5+100*(63/25*d^2*x^2+72/25*c*d*x+
c^2)*x*b*a^4-160*x^2*(63/10*d^2*x^2+18/5*c*d*x+c^2)*b^2*a^3+320*x^3*b^3*(-
63/10*d^2*x^2+36/5*c*d*x+c^2)*a^2-1280*x^4*(-18/5*d*x+c)*b^4*c*a-2560*b^5*
c^2*x^5)/(x*(b*x+a))^(1/2)/x^4/a^6
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx)^2}{x^4(ax+bx^2)^{3/2}}\,dx =$$

$$-\frac{2\,(35\,a^5c^2+16\,(80\,b^5c^2-144\,ab^4cd+63\,a^2b^3d^2)x^5+8\,(80\,ab^4c^2-144\,a^2b^3cd+63\,a^3b^2d^2)x^4-2\,(80\,a^2b^3}{315\,(a^6ba}$$

input
```
integrate((d*x+c)^2/x^4/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/315*(35*a^5*c^2 + 16*(80*b^5*c^2 - 144*a*b^4*c*d + 63*a^2*b^3*d^2)*x^5
+ 8*(80*a*b^4*c^2 - 144*a^2*b^3*c*d + 63*a^3*b^2*d^2)*x^4 - 2*(80*a^2*b^3*
c^2 - 144*a^3*b^2*c*d + 63*a^4*b*d^2)*x^3 + (80*a^3*b^2*c^2 - 144*a^4*b*c*
d + 63*a^5*d^2)*x^2 - 10*(5*a^4*b*c^2 - 9*a^5*c*d)*x)*sqrt(b*x^2 + a*x)/(a
^6*b*x^6 + a^7*x^5)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{x^4 (ax+bx^2)^{3/2}}\,dx = \int \frac{(c+dx)^2}{x^4 (x(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**2/x**4/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral((c + d*x)**2/(x**4*(x*(a + b*x))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.50

$$\int \frac{(c+dx)^2}{x^4 (ax+bx^2)^{3/2}}\,dx = -\frac{512\,b^5c^2x}{63\,\sqrt{bx^2+ax}a^6} + \frac{512\,b^4cdx}{35\,\sqrt{bx^2+ax}a^5} - \frac{32\,b^3d^2x}{5\,\sqrt{bx^2+ax}a^4} - \frac{256\,b^4c^2}{63\,\sqrt{bx^2+ax}a^5} + \frac{256\,b^3cd}{35\,\sqrt{bx^2+ax}a^4} - \frac{16\,b^2d^2}{5\,\sqrt{bx^2+ax}a^3} + \frac{64\,b^3c^2}{63\,\sqrt{bx^2+ax}a^4x} - \frac{64\,b^2cd}{35\,\sqrt{bx^2+ax}a^3x} + \frac{4\,bd^2}{5\,\sqrt{bx^2+ax}a^2x} - \frac{32\,b^2c^2}{63\,\sqrt{bx^2+ax}a^3x^2} + \frac{32\,bcd}{35\,\sqrt{bx^2+ax}a^2x^2} - \frac{2\,d^2}{5\,\sqrt{bx^2+ax}ax^2} + \frac{20\,bc^2}{63\,\sqrt{bx^2+ax}a^2x^3} - \frac{4\,cd}{7\,\sqrt{bx^2+ax}ax^3} - \frac{2\,c^2}{9\,\sqrt{bx^2+ax}ax^4}$$

input
```
integrate((d*x+c)^2/x^4/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
-512/63*b^5*c^2*x/(sqrt(b*x^2 + a*x)*a^6) + 512/35*b^4*c*d*x/(sqrt(b*x^2 +
 a*x)*a^5) - 32/5*b^3*d^2*x/(sqrt(b*x^2 + a*x)*a^4) - 256/63*b^4*c^2/(sqrt
(b*x^2 + a*x)*a^5) + 256/35*b^3*c*d/(sqrt(b*x^2 + a*x)*a^4) - 16/5*b^2*d^2
/(sqrt(b*x^2 + a*x)*a^3) + 64/63*b^3*c^2/(sqrt(b*x^2 + a*x)*a^4*x) - 64/35
*b^2*c*d/(sqrt(b*x^2 + a*x)*a^3*x) + 4/5*b*d^2/(sqrt(b*x^2 + a*x)*a^2*x) -
 32/63*b^2*c^2/(sqrt(b*x^2 + a*x)*a^3*x^2) + 32/35*b*c*d/(sqrt(b*x^2 + a*x
)*a^2*x^2) - 2/5*d^2/(sqrt(b*x^2 + a*x)*a*x^2) + 20/63*b*c^2/(sqrt(b*x^2 +
 a*x)*a^2*x^3) - 4/7*c*d/(sqrt(b*x^2 + a*x)*a*x^3) - 2/9*c^2/(sqrt(b*x^2 +
 a*x)*a*x^4)
```

Giac [F]

$$\int \frac{(c+dx)^2}{x^4 (ax+bx^2)^{3/2}}\,dx = \int \frac{(dx+c)^2}{(bx^2+ax)^{\frac{3}{2}} x^4}\,dx$$

input
```
integrate((d*x+c)^2/x^4/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/((b*x^2 + a*x)^(3/2)*x^4), x)
```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.14

$$\int \frac{(c+dx)^2}{x^4 (ax+bx^2)^{3/2}}\,dx = \frac{\sqrt{b\,x^2+a\,x}\,\left(x\,\left(\frac{756\,a^2\,b^3\,d^2-2088\,a\,b^4\,c\,d+1300\,b^5\,c^2}{315\,a^6}-\frac{4\,b^3\,\left(693\,a^2\,d^2-1674\,a\,b\,c\,d+965\,b^2\,c^2\right)}{315\,a^6}\right)-\frac{2}{}\right.}{x\,\left(a+b\,x\right)} - \frac{\sqrt{b\,x^2+a\,x}\,\left(42\,a^3\,d^2-156\,a^2\,b\,c\,d+110\,a\,b^2\,c^2\right)}{105\,a^5\,x^3} - \frac{2\,c^2\,\sqrt{b\,x^2+a\,x}}{9\,a^2\,x^5} - \frac{2\,c\,\sqrt{b\,x^2+a\,x}\,\left(18\,a\,d-17\,b\,c\right)}{63\,a^3\,x^4} + \frac{2\,b\,\sqrt{b\,x^2+a\,x}\,\left(189\,a^2\,d^2-522\,a\,b\,c\,d+325\,b^2\,c^2\right)}{315\,a^5\,x^2}$$

input
```
int((c + d*x)^2/(x^4*(a*x + b*x^2)^(3/2)),x)
```

output
```
((a*x + b*x^2)^(1/2)*(x*((1300*b^5*c^2 + 756*a^2*b^3*d^2 - 2088*a*b^4*c*d)
/(315*a^6) - (4*b^3*(693*a^2*d^2 + 965*b^2*c^2 - 1674*a*b*c*d))/(315*a^6))
 - (2*b^2*(693*a^2*d^2 + 965*b^2*c^2 - 1674*a*b*c*d))/(315*a^5)))/(x*(a +
b*x)) - ((a*x + b*x^2)^(1/2)*(42*a^3*d^2 + 110*a*b^2*c^2 - 156*a^2*b*c*d))
/(105*a^5*x^3) - (2*c^2*(a*x + b*x^2)^(1/2))/(9*a^2*x^5) - (2*c*(a*x + b*x
^2)^(1/2)*(18*a*d - 17*b*c))/(63*a^3*x^4) + (2*b*(a*x + b*x^2)^(1/2)*(189*
a^2*d^2 + 325*b^2*c^2 - 522*a*b*c*d))/(315*a^5*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.24

$$\int \frac{(c+dx)^2}{x^4 (ax+bx^2)^{3/2}}\,dx = \frac{\frac{32\sqrt{b}\,\sqrt{bx+a}\,a^2b^2d^2x^5}{5} - \frac{512\sqrt{b}\,\sqrt{bx+a}\,a\,b^3cd\,x^5}{35} + \frac{512\sqrt{b}\,\sqrt{bx+a}\,b^4c^2x^5}{63} - \frac{2\sqrt{x}\,a^5c^2}{9} - \frac{4\sqrt{x}\,a^5cdx}{7} -$$

input
```
int((d*x+c)^2/x^4/(b*x^2+a*x)^(3/2),x)
```

output
```
(2*(1008*sqrt(b)*sqrt(a + b*x)*a**2*b**2*d**2*x**5 - 2304*sqrt(b)*sqrt(a +
 b*x)*a*b**3*c*d*x**5 + 1280*sqrt(b)*sqrt(a + b*x)*b**4*c**2*x**5 - 35*sqr
t(x)*a**5*c**2 - 90*sqrt(x)*a**5*c*d*x - 63*sqrt(x)*a**5*d**2*x**2 + 50*sq
rt(x)*a**4*b*c**2*x + 144*sqrt(x)*a**4*b*c*d*x**2 + 126*sqrt(x)*a**4*b*d**
2*x**3 - 80*sqrt(x)*a**3*b**2*c**2*x**2 - 288*sqrt(x)*a**3*b**2*c*d*x**3 -
 504*sqrt(x)*a**3*b**2*d**2*x**4 + 160*sqrt(x)*a**2*b**3*c**2*x**3 + 1152*
sqrt(x)*a**2*b**3*c*d*x**4 - 1008*sqrt(x)*a**2*b**3*d**2*x**5 - 640*sqrt(x
)*a*b**4*c**2*x**4 + 2304*sqrt(x)*a*b**4*c*d*x**5 - 1280*sqrt(x)*b**5*c**2
*x**5))/(315*sqrt(a + b*x)*a**6*x**5)
```

3.177 $\int \frac{x^4(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 202

$$\begin{aligned}\int \frac{x^4(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx &= \frac{2(bc-ad)^2x^4}{3ab^2(ax+bx^2)^{3/2}}+\frac{4(bc-4ad)(bc-ad)x^2}{3ab^3\sqrt{ax+bx^2}}\\ &\quad-\frac{(8b^2c^2-40abcd+35a^2d^2)\sqrt{ax+bx^2}}{4ab^4}+\frac{d^2x\sqrt{ax+bx^2}}{2b^3}\\ &\quad+\frac{(8b^2c^2-40abcd+35a^2d^2)\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{9/2}}\end{aligned}$$

output
```
2/3*(-a*d+b*c)^2*x^4/a/b^2/(b*x^2+a*x)^(3/2)+4/3*(-4*a*d+b*c)*(-a*d+b*c)*x
^2/a/b^3/(b*x^2+a*x)^(1/2)-1/4*(35*a^2*d^2-40*a*b*c*d+8*b^2*c^2)*(b*x^2+a*
x)^(1/2)/a/b^4+1/2*d^2*x*(b*x^2+a*x)^(1/2)/b^3+1/4*(35*a^2*d^2-40*a*b*c*d+
8*b^2*c^2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.89

$$\int \frac{x^4(c+dx)^2}{\left(ax+bx^2\right)^{5/2}}\,dx = \frac{x^{5/2}\left(\sqrt{b}\sqrt{x}(a+bx)\left(-105a^3d^2+20a^2bd(6c-7dx)+ab^2(-24c^2+160cdx-21d^2x\right.\right.}{12}$$

input
```
Integrate[(x^4*(c + d*x)^2)/(a*x + b*x^2)^(5/2),x]
```

output
```
(x^(5/2)*(Sqrt[b]*Sqrt[x]*(a + b*x)*(-105*a^3*d^2 + 20*a^2*b*d*(6*c - 7*d*
x) + a*b^2*(-24*c^2 + 160*c*d*x - 21*d^2*x^2) + 2*b^3*x*(-16*c^2 + 12*c*d*
x + 3*d^2*x^2)) + 6*(8*b^2*c^2 - 40*a*b*c*d + 35*a^2*d^2)*(a + b*x)^(5/2)*
ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(12*b^(9/2)*(x*(a
+ b*x))^(5/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(c+dx)^2}{\left(ax+bx^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1242}$$

$$\frac{2x^4\text{PolynomialRemainder}\left[(c+dx)^2,0,x\right]}{3a\left(ax+bx^2\right)^{3/2}}-\frac{2\int \frac{ax^3\text{PolynomialRemainder}\left[(c+dx)^2,0,x\right]}{\left(bx^2+ax\right)^{3/2}}dx}{3a^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2x^4\text{PolynomialRemainder}\left[(c+dx)^2,0,x\right]}{3a\left(ax+bx^2\right)^{3/2}}-\frac{2\int \frac{x^3\text{PolynomialRemainder}\left[(c+dx)^2,0,x\right]}{\left(bx^2+ax\right)^{3/2}}dx}{3a}$$

$$\Big\downarrow \text{ 2467}$$

$$\frac{2x^4\text{PolynomialRemainder}\left[(c+dx)^2,0,x\right]}{3a\left(ax+bx^2\right)^{3/2}}-\frac{2\sqrt{x}\sqrt{a+bx}\int\frac{x^{3/2}\text{PolynomialRemainder}\left[(c+dx)^2,0,x\right]}{(a+bx)^{3/2}}dx}{3a\sqrt{ax+bx^2}}$$

$\Big\downarrow$ 7284

$$\frac{2x^4\text{PolynomialRemainder}\left[(c+dx)^2,0,x\right]}{3a\left(ax+bx^2\right)^{3/2}}-\frac{4\sqrt{x}\sqrt{a+bx}\int\frac{x^2\text{PolynomialRemainder}\left[(c+dx)^2,0,x\right]}{(a+bx)^{3/2}}d\sqrt{x}}{3a\sqrt{ax+bx^2}}$$

$\Big\downarrow$ 7299

$$\frac{2x^4\text{PolynomialRemainder}\left[(c+dx)^2,0,x\right]}{3a\left(ax+bx^2\right)^{3/2}}-\frac{4\sqrt{x}\sqrt{a+bx}\int\frac{x^2\text{PolynomialRemainder}\left[(c+dx)^2,0,x\right]}{(a+bx)^{3/2}}d\sqrt{x}}{3a\sqrt{ax+bx^2}}$$

input
```
Int[(x^4*(c + d*x)^2)/(a*x + b*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1242
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(f + g*x)
^n, a*e + c*d*x, x], R = PolynomialRemainder[(f + g*x)^n, a*e + c*d*x, x]},
 Simp[R*(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p + 1)*(b^
2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(
a + b*x + c*x^2)^(p + 1)*ExpandToSum[d*e*(p + 1)*(b^2 - 4*a*c)*Q - R*(2*c*d
 - b*e)*(m + 2*p + 2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IG
tQ[n, 1] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

rule 7284
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{120dx\,a^{2}\left(-\frac{7dx}{6}+c\right)b^{\frac{3}{2}}-24x\left(\frac{7}{8}d^{2}x^{2}-\frac{20}{3}cdx+c^{2}\right)a\,b^{\frac{5}{2}}-32x^{2}\left(-\frac{3}{16}d^{2}x^{2}-\frac{3}{4}cdx+c^{2}\right)b^{\frac{7}{2}}-105\sqrt{b}\,a^{3}d^{2}x+105\left(a^{2}d^{2}-\frac{8}{7}abcd+\right.}{\sqrt{x(bx+a)}\,b^{\frac{9}{2}}\left(12bx+12a\right)}$
risch	$-\frac{d(-2bdx+11ad-8bc)x(bx+a)}{4b^{4}\sqrt{x(bx+a)}}+\frac{\frac{35a^{2}d^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{\sqrt{b}}+8b^{\frac{3}{2}}c^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)-\frac{32\left(2a^{2}d^{2}-3abcd+b^{2}c^{2}\right.}{b\left(x\right.}}{}$

input
```
int(x^4*(d*x+c)^2/(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
105/(x*(b*x+a))^(1/2)/b^(9/2)*(8/7*d*x*a^2*(-7/6*d*x+c)*b^(3/2)-8/35*x*(7/
8*d^2*x^2-20/3*c*d*x+c^2)*a*b^(5/2)-32/105*x^2*(-3/16*d^2*x^2-3/4*c*d*x+c^
2)*b^(7/2)-b^(1/2)*a^3*d^2*x+(a^2*d^2-8/7*a*b*c*d+8/35*b^2*c^2)*(b*x+a)*(x
*(b*x+a))^(1/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2)))/(12*b*x+12*a)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 511, normalized size of antiderivative = 2.53

$$\int \frac{x^4(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = \left[\frac{3\,(8\,a^2b^2c^2-40\,a^3bcd+35\,a^4d^2+(8\,b^4c^2-40\,ab^3cd+35\,a^2b^2d^2)x^2+2\,(8\,ab^3c^2-\right.$$

$$-\frac{3\,(8\,a^2b^2c^2-40\,a^3bcd+35\,a^4d^2+(8\,b^4c^2-40\,ab^3cd+35\,a^2b^2d^2)x^2+2\,(8\,ab^3c^2-40\,a^2b^2cd+35\,a^3bd^2)x}{}$$

input
```
integrate(x^4*(d*x+c)^2/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

output
```
[1/24*(3*(8*a^2*b^2*c^2 - 40*a^3*b*c*d + 35*a^4*d^2 + (8*b^4*c^2 - 40*a*b^
3*c*d + 35*a^2*b^2*d^2)*x^2 + 2*(8*a*b^3*c^2 - 40*a^2*b^2*c*d + 35*a^3*b*d
^2)*x)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(6*b^4*d^2
*x^3 - 24*a*b^3*c^2 + 120*a^2*b^2*c*d - 105*a^3*b*d^2 + 3*(8*b^4*c*d - 7*a
*b^3*d^2)*x^2 - 4*(8*b^4*c^2 - 40*a*b^3*c*d + 35*a^2*b^2*d^2)*x)*sqrt(b*x^
2 + a*x))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5), -1/12*(3*(8*a^2*b^2*c^2 - 40*a^
3*b*c*d + 35*a^4*d^2 + (8*b^4*c^2 - 40*a*b^3*c*d + 35*a^2*b^2*d^2)*x^2 + 2
*(8*a*b^3*c^2 - 40*a^2*b^2*c*d + 35*a^3*b*d^2)*x)*sqrt(-b)*arctan(sqrt(b*x
^2 + a*x)*sqrt(-b)/(b*x + a)) - (6*b^4*d^2*x^3 - 24*a*b^3*c^2 + 120*a^2*b^
2*c*d - 105*a^3*b*d^2 + 3*(8*b^4*c*d - 7*a*b^3*d^2)*x^2 - 4*(8*b^4*c^2 - 4
0*a*b^3*c*d + 35*a^2*b^2*d^2)*x)*sqrt(b*x^2 + a*x))/(b^7*x^2 + 2*a*b^6*x +
 a^2*b^5)]
```

Sympy [F]

$$\int \frac{x^4(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = \int \frac{x^4(c+dx)^2}{(x\,(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4*(d*x+c)**2/(b*x**2+a*x)**(5/2),x)
```

output
```
Integral(x**4*(c + d*x)**2/(x*(a + b*x))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. 2(178) = 356.

Time = 0.04 (sec) , antiderivative size = 531, normalized size of antiderivative = 2.63

$$\begin{aligned}\int \frac{x^4(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx &= \frac{d^2x^5}{2\,(bx^2+ax)^{\frac{3}{2}}b} \\ &- \frac{1}{3}\,c^2x\left(\frac{3\,x^2}{(bx^2+ax)^{\frac{3}{2}}b}+\frac{ax}{(bx^2+ax)^{\frac{3}{2}}b^2}-\frac{2\,x}{\sqrt{bx^2+ax}ab}-\frac{1}{\sqrt{bx^2+ax}b^2}\right) \\ &+ \frac{5\,acdx\left(\frac{3\,x^2}{(bx^2+ax)^{\frac{3}{2}}b}+\frac{ax}{(bx^2+ax)^{\frac{3}{2}}b^2}-\frac{2\,x}{\sqrt{bx^2+ax}ab}-\frac{1}{\sqrt{bx^2+ax}b^2}\right)}{3\,b} \\ &- \frac{35\,a^2d^2x\left(\frac{3\,x^2}{(bx^2+ax)^{\frac{3}{2}}b}+\frac{ax}{(bx^2+ax)^{\frac{3}{2}}b^2}-\frac{2\,x}{\sqrt{bx^2+ax}ab}-\frac{1}{\sqrt{bx^2+ax}b^2}\right)}{24\,b^2} \\ &+ \frac{2\,cdx^4}{(bx^2+ax)^{\frac{3}{2}}b}-\frac{7\,ad^2x^4}{4\,(bx^2+ax)^{\frac{3}{2}}b^2}-\frac{4\,c^2x}{3\,\sqrt{bx^2+ax}b^2}+\frac{20\,acdx}{3\,\sqrt{bx^2+ax}b^3} \\ &- \frac{35\,a^2d^2x}{6\,\sqrt{bx^2+ax}b^4}+\frac{c^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{b^{\frac{5}{2}}} \\ &- \frac{5\,acd\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{b^{\frac{7}{2}}} \\ &+ \frac{35\,a^2d^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{9}{2}}} \\ &- \frac{2\,\sqrt{bx^2+ax}c^2}{3\,ab^2}+\frac{10\,\sqrt{bx^2+ax}cd}{3\,b^3}-\frac{35\,\sqrt{bx^2+ax}ad^2}{12\,b^4}\end{aligned}$$

input
```
integrate(x^4*(d*x+c)^2/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

output
```
1/2*d^2*x^5/((b*x^2 + a*x)^(3/2)*b) - 1/3*c^2*x*(3*x^2/((b*x^2 + a*x)^(3/2
)*b) + a*x/((b*x^2 + a*x)^(3/2)*b^2) - 2*x/(sqrt(b*x^2 + a*x)*a*b) - 1/(sq
rt(b*x^2 + a*x)*b^2)) + 5/3*a*c*d*x*(3*x^2/((b*x^2 + a*x)^(3/2)*b) + a*x/(
(b*x^2 + a*x)^(3/2)*b^2) - 2*x/(sqrt(b*x^2 + a*x)*a*b) - 1/(sqrt(b*x^2 + a
*x)*b^2))/b - 35/24*a^2*d^2*x*(3*x^2/((b*x^2 + a*x)^(3/2)*b) + a*x/((b*x^2
 + a*x)^(3/2)*b^2) - 2*x/(sqrt(b*x^2 + a*x)*a*b) - 1/(sqrt(b*x^2 + a*x)*b^
2))/b^2 + 2*c*d*x^4/((b*x^2 + a*x)^(3/2)*b) - 7/4*a*d^2*x^4/((b*x^2 + a*x)
^(3/2)*b^2) - 4/3*c^2*x/(sqrt(b*x^2 + a*x)*b^2) + 20/3*a*c*d*x/(sqrt(b*x^2
 + a*x)*b^3) - 35/6*a^2*d^2*x/(sqrt(b*x^2 + a*x)*b^4) + c^2*log(2*b*x + a
+ 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5/2) - 5*a*c*d*log(2*b*x + a + 2*sqrt(b*
x^2 + a*x)*sqrt(b))/b^(7/2) + 35/8*a^2*d^2*log(2*b*x + a + 2*sqrt(b*x^2 +
a*x)*sqrt(b))/b^(9/2) - 2/3*sqrt(b*x^2 + a*x)*c^2/(a*b^2) + 10/3*sqrt(b*x^
2 + a*x)*c*d/b^3 - 35/12*sqrt(b*x^2 + a*x)*a*d^2/b^4
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.69

$$\int \frac{x^4(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = \frac{1}{4}\sqrt{bx^2+ax}\left(\frac{2\,d^2x}{b^3}+\frac{8\,b^8cd-11\,ab^7d^2}{b^{11}}\right)$$
$$-\frac{(8\,b^2c^2-40\,abcd+35\,a^2d^2)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,b^{\frac{9}{2}}}$$
$$-\frac{2\left(6\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2ab^3c^2-18\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2a^2b^2cd+12\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2a^3bd^2+9\right.}{3\left(\left(\sqrt{\right.\right.}}$$

input
```
integrate(x^4*(d*x+c)^2/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

output
```
1/4*sqrt(b*x^2 + a*x)*(2*d^2*x/b^3 + (8*b^8*c*d - 11*a*b^7*d^2)/b^11) - 1/
8*(8*b^2*c^2 - 40*a*b*c*d + 35*a^2*d^2)*log(abs(2*(sqrt(b)*x - sqrt(b*x^2
+ a*x))*sqrt(b) + a))/b^(9/2) - 2/3*(6*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a
*b^3*c^2 - 18*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2*b^2*c*d + 12*(sqrt(b)*
x - sqrt(b*x^2 + a*x))^2*a^3*b*d^2 + 9*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^2
*b^(5/2)*c^2 - 30*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^3*b^(3/2)*c*d + 21*(sq
rt(b)*x - sqrt(b*x^2 + a*x))*a^4*sqrt(b)*d^2 + 4*a^3*b^2*c^2 - 14*a^4*b*c*
d + 10*a^5*d^2)/(((sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)^3*b^(9/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = \int \frac{x^4\,(c+d\,x)^2}{(b\,x^2+a\,x)^{5/2}}\,dx$$

input
```
int((x^4*(c + d*x)^2)/(a*x + b*x^2)^(5/2),x)
```

output
```
int((x^4*(c + d*x)^2)/(a*x + b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.01

$$\int \frac{x^4(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = \frac{840\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^3d^2 - 960\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2bcd +}{}$$

input
```
int(x^4*(d*x+c)^2/(b*x^2+a*x)^(5/2),x)
```

output
```
(840*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*
a**3*d**2 - 960*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b)
)/sqrt(a))*a**2*b*c*d + 840*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqr
t(x)*sqrt(b))/sqrt(a))*a**2*b*d**2*x + 192*sqrt(b)*sqrt(a + b*x)*log((sqrt
(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b**2*c**2 - 960*sqrt(b)*sqrt(a + b
*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b**2*c*d*x + 192*sqrt
(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b**3*c**2
*x + 175*sqrt(b)*sqrt(a + b*x)*a**3*d**2 - 160*sqrt(b)*sqrt(a + b*x)*a**2*
b*c*d + 175*sqrt(b)*sqrt(a + b*x)*a**2*b*d**2*x - 160*sqrt(b)*sqrt(a + b*x
)*a*b**2*c*d*x - 840*sqrt(x)*a**3*b*d**2 + 960*sqrt(x)*a**2*b**2*c*d - 112
0*sqrt(x)*a**2*b**2*d**2*x - 192*sqrt(x)*a*b**3*c**2 + 1280*sqrt(x)*a*b**3
*c*d*x - 168*sqrt(x)*a*b**3*d**2*x**2 - 256*sqrt(x)*b**4*c**2*x + 192*sqrt
(x)*b**4*c*d*x**2 + 48*sqrt(x)*b**4*d**2*x**3)/(96*sqrt(a + b*x)*b**5*(a +
 b*x))
```

3.178 $\int \frac{x^3(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 122

$$\int \frac{x^3(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = \frac{2(bc-ad)^2x^3}{3ab^2(ax+bx^2)^{3/2}} - \frac{4d(bc-ad)x}{b^3\sqrt{ax+bx^2}} + \frac{d^2\sqrt{ax+bx^2}}{b^3} + \frac{d(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{7/2}}$$

output
```
2/3*(-a*d+b*c)^2*x^3/a/b^2/(b*x^2+a*x)^(3/2)-4*d*(-a*d+b*c)*x/b^3/(b*x^2+a
*x)^(1/2)+d^2*(b*x^2+a*x)^(1/2)/b^3+d*(-5*a*d+4*b*c)*arctanh(b^(1/2)*x/(b*
x^2+a*x)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.19

$$\int \frac{x^3(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = \frac{x^{5/2}\left(\frac{\sqrt{b}\sqrt{x}(a+bx)\left(15a^3d^2+2b^3c^2x+ab^2dx(-16c+3dx)+4a^2bd(-3c+5dx)\right)}{a} + 6d(4bc-5ad)(a+bx)^{5}\right.}{3b^{7/2}(x(a+bx))^{5/2}}$$

input
```
Integrate[(x^3*(c + d*x)^2)/(a*x + b*x^2)^(5/2),x]
```

output
```
(x^(5/2)*((Sqrt[b]*Sqrt[x]*(a + b*x)*(15*a^3*d^2 + 2*b^3*c^2*x + a*b^2*d*x
*(-16*c + 3*d*x) + 4*a^2*b*d*(-3*c + 5*d*x)))/a + 6*d*(4*b*c - 5*a*d)*(a +
 b*x)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(3*b^(
7/2)*(x*(a + b*x))^(5/2))
```

Rubi [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.27, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1242, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx)^2}{(ax+bx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 1242}$$

$$\frac{2x^3\text{PolynomialRemainder}\left[(c+dx)^2,0,x\right]}{3a\left(ax+bx^2\right)^{3/2}}-\frac{2\int 0dx}{3a^2}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{2x^3\text{PolynomialRemainder}\left[(c+dx)^2,0,x\right]}{3a\left(ax+bx^2\right)^{3/2}}$$

input
```
Int[(x^3*(c + d*x)^2)/(a*x + b*x^2)^(5/2),x]
```

output
```
(2*x^3*PolynomialRemainder[(c + d*x)^2, 0, x])/(3*a*(a*x + b*x^2)^(3/2))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 1242
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(f + g*x)
^n, a*e + c*d*x, x], R = PolynomialRemainder[(f + g*x)^n, a*e + c*d*x, x]},
 Simp[R*(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p + 1)*(b^
2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(
a + b*x + c*x^2)^(p + 1)*ExpandToSum[d*e*(p + 1)*(b^2 - 4*a*c)*Q - R*(2*c*d
 - b*e)*(m + 2*p + 2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IG
tQ[n, 1] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98

method	result
pseudoelliptic	$-\frac{5\left(d\left(ad-\frac{4bc}{5}\right)(bx+a)a\sqrt{x(bx+a)}\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)-\left(\left(-\frac{4d\left(-\frac{5dx}{3}+c\right)a^2b^{\frac{3}{2}}}{5}-\frac{16d\left(-\frac{3dx}{16}+c\right)xa\,b^{\frac{5}{2}}}{15}+\sqrt{b}\,a^3d^2+\frac{2b^{\frac{7}{2}}c}{15}\right.\right.\right.}{\sqrt{x(bx+a)}\,b^{\frac{7}{2}}(bx+a)a}$
risch	$\frac{d^2x(bx+a)}{b^3\sqrt{x(bx+a)}}-\frac{\frac{2\left(-6a^2d^2+8abcd-2b^2c^2\right)\sqrt{\left(x+\frac{a}{b}\right)^2b-a\left(x+\frac{a}{b}\right)}}{ba\left(x+\frac{a}{b}\right)}+\frac{5a\,d^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}-4d\sqrt{b}\,c\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+a\right.}{2b^3}$
default	$c^2\left(-\frac{x^2}{b(b\,x^2+ax)^{\frac{3}{2}}}+\frac{a\left(-\frac{x}{2b\left(b\,x^2+ax\right)^{\frac{3}{2}}}-\frac{a\left(-\frac{1}{3b\left(b\,x^2+ax\right)^{\frac{3}{2}}}-\frac{a\left(-\frac{2(2bx+a)}{3a^2\left(b\,x^2+ax\right)^{\frac{3}{2}}}+\frac{16b(2bx+a)}{3a^4\sqrt{b\,x^2+ax}}\right)}{2b}\right)}{4b}\right)}{2b}\right)+d^2$

input
```
int(x^3*(d*x+c)^2/(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-5/(x*(b*x+a))^(1/2)/b^(7/2)*(d*(a*d-4/5*b*c)*(b*x+a)*a*(x*(b*x+a))^(1/2)*
arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-(-4/5*d*(-5/3*d*x+c)*a^2*b^(3/2)-16/1
5*d*(-3/16*d*x+c)*x*a*b^(5/2)+b^(1/2)*a^3*d^2+2/15*b^(7/2)*c^2*x)*x)/(b*x+
a)/a
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 399, normalized size of antiderivative = 3.27

$$\int \frac{x^3(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = \left[-\frac{3\left(4\,a^3bcd-5\,a^4d^2+(4\,ab^3cd-5\,a^2b^2d^2)x^2+2\,(4\,a^2b^2cd-5\,a^3bd^2)x\right)\sqrt{b}\log\left(2\,b\right.}{}\right.$$

$$-\frac{3\left(4\,a^3bcd-5\,a^4d^2+(4\,ab^3cd-5\,a^2b^2d^2)x^2+2\,(4\,a^2b^2cd-5\,a^3bd^2)x\right)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(3\,ab}{3\,(ab^6x^2+2\,a^2b^5x+a^3b^4)}$$

input
```
integrate(x^3*(d*x+c)^2/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*(3*(4*a^3*b*c*d - 5*a^4*d^2 + (4*a*b^3*c*d - 5*a^2*b^2*d^2)*x^2 + 2*
(4*a^2*b^2*c*d - 5*a^3*b*d^2)*x)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*
x)*sqrt(b)) - 2*(3*a*b^3*d^2*x^2 - 12*a^2*b^2*c*d + 15*a^3*b*d^2 + 2*(b^4*
c^2 - 8*a*b^3*c*d + 10*a^2*b^2*d^2)*x)*sqrt(b*x^2 + a*x))/(a*b^6*x^2 + 2*a
^2*b^5*x + a^3*b^4), -1/3*(3*(4*a^3*b*c*d - 5*a^4*d^2 + (4*a*b^3*c*d - 5*a
^2*b^2*d^2)*x^2 + 2*(4*a^2*b^2*c*d - 5*a^3*b*d^2)*x)*sqrt(-b)*arctan(sqrt(
b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (3*a*b^3*d^2*x^2 - 12*a^2*b^2*c*d + 15*
a^3*b*d^2 + 2*(b^4*c^2 - 8*a*b^3*c*d + 10*a^2*b^2*d^2)*x)*sqrt(b*x^2 + a*x
))/(a*b^6*x^2 + 2*a^2*b^5*x + a^3*b^4)]
```

Sympy [F]

$$\int \frac{x^3(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = \int \frac{x^3(c+dx)^2}{(x\,(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*(d*x+c)**2/(b*x**2+a*x)**(5/2),x)
```

output
```
Integral(x**3*(c + d*x)**2/(x*(a + b*x))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(108) = 216$.

Time = 0.05 (sec) , antiderivative size = 410, normalized size of antiderivative = 3.36

$$\begin{aligned}\int \frac{x^3(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx =& -\frac{2}{3}\,cdx\left(\frac{3\,x^2}{(bx^2+ax)^{\frac{3}{2}}b}+\frac{ax}{(bx^2+ax)^{\frac{3}{2}}b^2}-\frac{2\,x}{\sqrt{bx^2+ax}ab}-\frac{1}{\sqrt{bx^2+ax}b^2}\right)\\
&+\frac{5\,ad^2x\left(\frac{3\,x^2}{(bx^2+ax)^{\frac{3}{2}}b}+\frac{ax}{(bx^2+ax)^{\frac{3}{2}}b^2}-\frac{2\,x}{\sqrt{bx^2+ax}ab}-\frac{1}{\sqrt{bx^2+ax}b^2}\right)}{6\,b}\\
&+\frac{d^2x^4}{(bx^2+ax)^{\frac{3}{2}}b}-\frac{c^2x^2}{(bx^2+ax)^{\frac{3}{2}}b}-\frac{ac^2x}{3\,(bx^2+ax)^{\frac{3}{2}}b^2}+\frac{2\,c^2x}{3\,\sqrt{bx^2+ax}ab}\\
&-\frac{8\,cdx}{3\,\sqrt{bx^2+ax}b^2}+\frac{10\,ad^2x}{3\,\sqrt{bx^2+ax}b^3}+\frac{2\,cd\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{b^{\frac{5}{2}}}\\
&-\frac{5\,ad^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{2\,b^{\frac{7}{2}}}\\
&+\frac{c^2}{3\,\sqrt{bx^2+ax}b^2}-\frac{4\,\sqrt{bx^2+ax}cd}{3\,ab^2}+\frac{5\,\sqrt{bx^2+ax}d^2}{3\,b^3}\end{aligned}$$

input
```
integrate(x^3*(d*x+c)^2/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

output
```
-2/3*c*d*x*(3*x^2/((b*x^2 + a*x)^(3/2)*b) + a*x/((b*x^2 + a*x)^(3/2)*b^2)
- 2*x/(sqrt(b*x^2 + a*x)*a*b) - 1/(sqrt(b*x^2 + a*x)*b^2)) + 5/6*a*d^2*x*(
3*x^2/((b*x^2 + a*x)^(3/2)*b) + a*x/((b*x^2 + a*x)^(3/2)*b^2) - 2*x/(sqrt(
b*x^2 + a*x)*a*b) - 1/(sqrt(b*x^2 + a*x)*b^2))/b + d^2*x^4/((b*x^2 + a*x)^
(3/2)*b) - c^2*x^2/((b*x^2 + a*x)^(3/2)*b) - 1/3*a*c^2*x/((b*x^2 + a*x)^(3
/2)*b^2) + 2/3*c^2*x/(sqrt(b*x^2 + a*x)*a*b) - 8/3*c*d*x/(sqrt(b*x^2 + a*x
)*b^2) + 10/3*a*d^2*x/(sqrt(b*x^2 + a*x)*b^3) + 2*c*d*log(2*b*x + a + 2*sq
rt(b*x^2 + a*x)*sqrt(b))/b^(5/2) - 5/2*a*d^2*log(2*b*x + a + 2*sqrt(b*x^2
+ a*x)*sqrt(b))/b^(7/2) + 1/3*c^2/(sqrt(b*x^2 + a*x)*b^2) - 4/3*sqrt(b*x^2
 + a*x)*c*d/(a*b^2) + 5/3*sqrt(b*x^2 + a*x)*d^2/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(108) = 216$.

Time = 0.21 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.44

$$\int \frac{x^3(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = \frac{\sqrt{bx^2+ax}d^2}{b^3} - \frac{(4\,bcd-5\,ad^2)\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{2\,b^{\frac{7}{2}}} + \frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2b^3c^2-12\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2ab^2cd+9\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2a^2bd^2+3\left(\sqrt{b}\right.\right.}{3\left(\left(\sqrt{b}x-\right.\right.}$$

input
```
integrate(x^3*(d*x+c)^2/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

output
```
sqrt(b*x^2 + a*x)*d^2/b^3 - 1/2*(4*b*c*d - 5*a*d^2)*log(abs(2*(sqrt(b)*x -
 sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(7/2) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2
 + a*x))^2*b^3*c^2 - 12*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a*b^2*c*d + 9*(s
qrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2*b*d^2 + 3*(sqrt(b)*x - sqrt(b*x^2 + a*
x))*a*b^(5/2)*c^2 - 18*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^2*b^(3/2)*c*d + 1
5*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^3*sqrt(b)*d^2 + a^2*b^2*c^2 - 8*a^3*b*
c*d + 7*a^4*d^2)/(((sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)^3*b^(7/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = \int \frac{x^3\,(c+d\,x)^2}{(b\,x^2+a\,x)^{5/2}}\,dx$$

input
```
int((x^3*(c + d*x)^2)/(a*x + b*x^2)^(5/2),x)
```

output
```
int((x^3*(c + d*x)^2)/(a*x + b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.49

$$\int \frac{x^3(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = \frac{-30\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^3d^2+24\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2bcd-}{}$$

input
```
int(x^3*(d*x+c)^2/(b*x^2+a*x)^(5/2),x)
```

output
```
( - 30*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)
)*a**3*d**2 + 24*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b
))/sqrt(a))*a**2*b*c*d - 30*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqr
t(x)*sqrt(b))/sqrt(a))*a**2*b*d**2*x + 24*sqrt(b)*sqrt(a + b*x)*log((sqrt(
a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b**2*c*d*x - 5*sqrt(b)*sqrt(a + b*x
)*a**3*d**2 - 5*sqrt(b)*sqrt(a + b*x)*a**2*b*d**2*x + 4*sqrt(b)*sqrt(a + b
*x)*a*b**2*c**2 + 4*sqrt(b)*sqrt(a + b*x)*b**3*c**2*x + 30*sqrt(x)*a**3*b*
d**2 - 24*sqrt(x)*a**2*b**2*c*d + 40*sqrt(x)*a**2*b**2*d**2*x - 32*sqrt(x)
*a*b**3*c*d*x + 6*sqrt(x)*a*b**3*d**2*x**2 + 4*sqrt(x)*b**4*c**2*x)/(6*sqr
t(a + b*x)*a*b**4*(a + b*x))
```

3.179 $\int \frac{x^2(c+dx)^2}{\left(ax+bx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 108

$$\int \frac{x^2(c+dx)^2}{\left(ax+bx^2\right)^{5/2}}\,dx=\frac{2(bc-ad)^2x^2}{3ab^2\left(ax+bx^2\right)^{3/2}}+\frac{4(bc-ad)(bc+2ad)x}{3a^2b^2\sqrt{ax+bx^2}}+\frac{2d^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{5/2}}$$

output
```
2/3*(-a*d+b*c)^2*x^2/a/b^2/(b*x^2+a*x)^(3/2)+4/3*(-a*d+b*c)*(2*a*d+b*c)*x/
a^2/b^2/(b*x^2+a*x)^(1/2)+2*d^2*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(5/
2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.03

$$\int \frac{x^2(c+dx)^2}{\left(ax+bx^2\right)^{5/2}}\,dx=\frac{x\left(2\sqrt{b}(bc-ad)x(3a^2d+2b^2cx+ab(3c+4dx))-6a^2d^2\sqrt{x}(a+bx)^{3/2}\log\left(-\sqrt{b}\sqrt{\right.\right.}{3a^2b^{5/2}(x(a+bx))^{3/2}}$$

input
```
Integrate[(x^2*(c + d*x)^2)/(a*x + b*x^2)^(5/2),x]
```

output
```
(x*(2*Sqrt[b]*(b*c - a*d)*x*(3*a^2*d + 2*b^2*c*x + a*b*(3*c + 4*d*x)) - 6*
a^2*d^2*Sqrt[x]*(a + b*x)^(3/2)*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]))/
(3*a^2*b^(5/2)*(x*(a + b*x))^(3/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1242}$$

$$\frac{2x^2\text{PolynomialRemainder}\left[(c+dx)^2,0,x\right]}{3a\,(ax+bx^2)^{3/2}}-\frac{2\int -\frac{ax\text{PolynomialRemainder}\left[(c+dx)^2,0,x\right]}{(bx^2+ax)^{3/2}}dx}{3a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\int \frac{ax\text{PolynomialRemainder}\left[(c+dx)^2,0,x\right]}{(bx^2+ax)^{3/2}}dx}{3a^2}+\frac{2x^2\text{PolynomialRemainder}\left[(c+dx)^2,0,x\right]}{3a\,(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{x\text{PolynomialRemainder}\left[(c+dx)^2,0,x\right]}{(bx^2+ax)^{3/2}}dx}{3a}+\frac{2x^2\text{PolynomialRemainder}\left[(c+dx)^2,0,x\right]}{3a\,(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{2467}$$

$$\frac{2\sqrt{x}\sqrt{a+bx}\int \frac{\text{PolynomialRemainder}\left[(c+dx)^2,0,x\right]}{\sqrt{x}(a+bx)^{3/2}}dx}{3a\sqrt{ax+bx^2}}+\frac{2x^2\text{PolynomialRemainder}\left[(c+dx)^2,0,x\right]}{3a\,(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{7284}$$

$$\frac{4\sqrt{x}\sqrt{a+bx}\int \frac{\text{PolynomialRemainder}\left[(c+dx)^2,0,x\right]}{(a+bx)^{3/2}}d\sqrt{x}}{3a\sqrt{ax+bx^2}}+\frac{2x^2\text{PolynomialRemainder}\left[(c+dx)^2,0,x\right]}{3a\,(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{7299}$$

$$\frac{4\sqrt{x}\sqrt{a+bx}\int \frac{\text{PolynomialRemainder}[(c+dx)^2,0,x]}{(a+bx)^{3/2}}d\sqrt{x}}{3a\sqrt{ax+bx^2}}+\frac{2x^2\text{PolynomialRemainder}\left[(c+dx)^2,0,x\right]}{3a\left(ax+bx^2\right)^{3/2}}$$

input
```
Int[(x^2*(c + d*x)^2)/(a*x + b*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1242
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(f + g*x)
^n, a*e + c*d*x, x], R = PolynomialRemainder[(f + g*x)^n, a*e + c*d*x, x]},
 Simp[R*(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p + 1)*(b^
2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(
a + b*x + c*x^2)^(p + 1)*ExpandToSum[d*e*(p + 1)*(b^2 - 4*a*c)*Q - R*(2*c*d
 - b*e)*(m + 2*p + 2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IG
tQ[n, 1] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

rule 7284
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$\frac{2a^2\sqrt{x(bx+a)}\,d^2(bx+a)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)-2x\left(-\left(\frac{2dx}{3}+c\right)ac\,b^{\frac{5}{2}}+\sqrt{b}\,a^3d^2+\frac{4a^2d^2x\,b^{\frac{3}{2}}}{3}-\frac{2b^{\frac{7}{2}}c^2x}{3}\right)}{b^{\frac{5}{2}}(bx+a)\sqrt{x(bx+a)}\,a^2}$
default	$c^2\left(-\frac{x}{2b(b\,x^2+ax)^{\frac{3}{2}}}-\frac{a\left(-\frac{1}{3b\left(b\,x^2+ax\right)^{\frac{3}{2}}}-\frac{a\left(-\frac{2(2bx+a)}{3a^2\left(b\,x^2+ax\right)^{\frac{3}{2}}}+\frac{16b(2bx+a)}{3a^4\sqrt{b\,x^2+ax}}\right)}{2b}\right)}{4b}\right)+d^2\left(-\frac{x^3}{3b(b\,x^2+ax)^{\frac{3}{2}}}-\right.$

input
```
int(x^2*(d*x+c)^2/(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/(x*(b*x+a))^(1/2)/b^(5/2)*(a^2*(x*(b*x+a))^(1/2)*d^2*(b*x+a)*arctanh((x*
(b*x+a))^(1/2)/x/b^(1/2))-x*(-(2/3*d*x+c)*a*c*b^(5/2)+b^(1/2)*a^3*d^2+4/3*
a^2*d^2*x*b^(3/2)-2/3*b^(7/2)*c^2*x))/(b*x+a)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.86

$$\int \frac{x^2(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = \left[\frac{3\,(a^2b^2d^2x^2+2\,a^3bd^2x+a^4d^2)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(3\,ab^3c^2-3\right.}{3\,(a^2b^5x^2+2\,a^3b^4x+a^4b^3)}$$

$$-\frac{2\left(3\,(a^2b^2d^2x^2+2\,a^3bd^2x+a^4d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(3\,ab^3c^2-3\,a^3bd^2+2\,(b^4c^2+ab^3cd-2\,a\right.}{3\,(a^2b^5x^2+2\,a^3b^4x+a^4b^3)}$$

input
```
integrate(x^2*(d*x+c)^2/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(a^2*b^2*d^2*x^2 + 2*a^3*b*d^2*x + a^4*d^2)*sqrt(b)*log(2*b*x + a
+ 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(3*a*b^3*c^2 - 3*a^3*b*d^2 + 2*(b^4*c^2
 + a*b^3*c*d - 2*a^2*b^2*d^2)*x)*sqrt(b*x^2 + a*x))/(a^2*b^5*x^2 + 2*a^3*b
^4*x + a^4*b^3), -2/3*(3*(a^2*b^2*d^2*x^2 + 2*a^3*b*d^2*x + a^4*d^2)*sqrt(
-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (3*a*b^3*c^2 - 3*a^3*b*
d^2 + 2*(b^4*c^2 + a*b^3*c*d - 2*a^2*b^2*d^2)*x)*sqrt(b*x^2 + a*x))/(a^2*b
^5*x^2 + 2*a^3*b^4*x + a^4*b^3)]
```

Sympy [F]

$$\int \frac{x^2(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = \int \frac{x^2(c+dx)^2}{(x\,(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*(d*x+c)**2/(b*x**2+a*x)**(5/2),x)
```

output
```
Integral(x**2*(c + d*x)**2/(x*(a + b*x))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(94) = 188$.

Time = 0.04 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.73

$$\int \frac{x^2(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = -\frac{1}{3}\,d^2x\left(\frac{3\,x^2}{(bx^2+ax)^{\frac{3}{2}}b}+\frac{ax}{(bx^2+ax)^{\frac{3}{2}}b^2}-\frac{2\,x}{\sqrt{bx^2+ax}ab}-\frac{1}{\sqrt{bx^2+ax}b^2}\right) - \frac{2\,cdx^2}{(bx^2+ax)^{\frac{3}{2}}b}+\frac{4\,c^2x}{3\,\sqrt{bx^2+ax}a^2}-\frac{2\,c^2x}{3\,(bx^2+ax)^{\frac{3}{2}}b}-\frac{2\,acdx}{3\,(bx^2+ax)^{\frac{3}{2}}b^2} + \frac{4\,cdx}{3\,\sqrt{bx^2+ax}ab}-\frac{4\,d^2x}{3\,\sqrt{bx^2+ax}b^2}+\frac{d^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{b^{\frac{5}{2}}} + \frac{2\,c^2}{3\,\sqrt{bx^2+ax}ab}+\frac{2\,cd}{3\,\sqrt{bx^2+ax}b^2}-\frac{2\,\sqrt{bx^2+ax}d^2}{3\,ab^2}$$

input
```
integrate(x^2*(d*x+c)^2/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*d^2*x*(3*x^2/((b*x^2 + a*x)^(3/2)*b) + a*x/((b*x^2 + a*x)^(3/2)*b^2)
- 2*x/(sqrt(b*x^2 + a*x)*a*b) - 1/(sqrt(b*x^2 + a*x)*b^2)) - 2*c*d*x^2/((b
*x^2 + a*x)^(3/2)*b) + 4/3*c^2*x/(sqrt(b*x^2 + a*x)*a^2) - 2/3*c^2*x/((b*x
^2 + a*x)^(3/2)*b) - 2/3*a*c*d*x/((b*x^2 + a*x)^(3/2)*b^2) + 4/3*c*d*x/(sq
rt(b*x^2 + a*x)*a*b) - 4/3*d^2*x/(sqrt(b*x^2 + a*x)*b^2) + d^2*log(2*b*x +
 a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5/2) + 2/3*c^2/(sqrt(b*x^2 + a*x)*a*b
) + 2/3*c*d/(sqrt(b*x^2 + a*x)*b^2) - 2/3*sqrt(b*x^2 + a*x)*d^2/(a*b^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(94) = 188$.

Time = 0.26 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.18

$$\int \frac{x^2(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = -\frac{d^2\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{b^{\frac{5}{2}}} + \frac{2\left(6\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2b^2cd-6\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2abd^2+3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)b^{\frac{5}{2}}c^2+6\left(\sqrt{b}x-\right.\right.}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+\right.}$$

input
```
integrate(x^2*(d*x+c)^2/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

output
```
-d^2*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(5/2) + 2/3
*(6*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*b^2*c*d - 6*(sqrt(b)*x - sqrt(b*x^2
+ a*x))^2*a*b*d^2 + 3*(sqrt(b)*x - sqrt(b*x^2 + a*x))*b^(5/2)*c^2 + 6*(sqr
t(b)*x - sqrt(b*x^2 + a*x))*a*b^(3/2)*c*d - 9*(sqrt(b)*x - sqrt(b*x^2 + a*
x))*a^2*sqrt(b)*d^2 + 2*a*b^2*c^2 + 2*a^2*b*c*d - 4*a^3*d^2)/(((sqrt(b)*x
- sqrt(b*x^2 + a*x))*sqrt(b) + a)^3*b^(5/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = \int \frac{x^2\,(c+d\,x)^2}{(b\,x^2+a\,x)^{5/2}}\,dx$$

input
```
int((x^2*(c + d*x)^2)/(a*x + b*x^2)^(5/2),x)
```

output
```
int((x^2*(c + d*x)^2)/(a*x + b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.03

$$\int \frac{x^2(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = \frac{2\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^3d^2+2\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2b\,d^2x+\frac{4\sqrt{}}{}}{}$$

input
```
int(x^2*(d*x+c)^2/(b*x^2+a*x)^(5/2),x)
```

output
```
(2*(3*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))
*a**3*d**2 + 3*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))
/sqrt(a))*a**2*b*d**2*x + 2*sqrt(b)*sqrt(a + b*x)*a**2*b*c*d - 2*sqrt(b)*s
qrt(a + b*x)*a*b**2*c**2 + 2*sqrt(b)*sqrt(a + b*x)*a*b**2*c*d*x - 2*sqrt(b
)*sqrt(a + b*x)*b**3*c**2*x - 3*sqrt(x)*a**3*b*d**2 - 4*sqrt(x)*a**2*b**2*
d**2*x + 3*sqrt(x)*a*b**3*c**2 + 2*sqrt(x)*a*b**3*c*d*x + 2*sqrt(x)*b**4*c
**2*x))/(3*sqrt(a + b*x)*a**2*b**3*(a + b*x))
```

3.180 $\int \frac{x(c+dx)^2}{(ax+bx^2)^{5/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 65

$$\int \frac{x(c+dx)^2}{(ax+bx^2)^{5/2}} \, dx = \frac{2x(c+dx)^2}{3a(ax+bx^2)^{3/2}} - \frac{8c(ac+(2bc-ad)x)}{3a^3\sqrt{ax+bx^2}}$$

output
```
2/3*x*(d*x+c)^2/a/(b*x^2+a*x)^(3/2)-8/3*c*(a*c+(-a*d+2*b*c)*x)/a^3/(b*x^2+
a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int \frac{x(c+dx)^2}{(ax+bx^2)^{5/2}} \, dx = \frac{2x(-8b^2c^2x^2 + 4abcx(-3c+dx) + a^2(-3c^2+6cdx+d^2x^2))}{3a^3(x(a+bx))^{3/2}}$$

input
```
Integrate[(x*(c + d*x)^2)/(a*x + b*x^2)^(5/2),x]
```

output
```
(2*x*(-8*b^2*c^2*x^2 + 4*a*b*c*x*(-3*c + d*x) + a^2*(-3*c^2 + 6*c*d*x + d^
2*x^2)))/(3*a^3*(x*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1227, 1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1227}$$

$$\frac{4c\int \frac{c+dx}{(bx^2+ax)^{3/2}}dx}{3a}+\frac{2x(c+dx)^2}{3a\,(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 1158}$$

$$\frac{2x(c+dx)^2}{3a\,(ax+bx^2)^{3/2}}-\frac{8c(x(2bc-ad)+ac)}{3a^3\sqrt{ax+bx^2}}$$

input
```
Int[(x*(c + d*x)^2)/(a*x + b*x^2)^(5/2),x]
```

output
```
(2*x*(c + d*x)^2)/(3*a*(a*x + b*x^2)^(3/2)) - (8*c*(a*c + (2*b*c - a*d)*x)
)/(3*a^3*Sqrt[a*x + b*x^2])
```

Defintions of rubi rules used

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1227
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(
(b*f - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[m*((b*(
e*f + d*g) - 2*(c*d*f + a*e*g))/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m
 - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
&& EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

method	result
risch	$-\frac{2c^2(bx+a)}{a^3\sqrt{x(bx+a)}}+\frac{2x(adx+5cbx+6ac)(ad-bc)}{3\sqrt{x(bx+a)}\,(bx+a)a^3}$
pseudoelliptic	$\frac{(2d^2x^2+12cdx-6c^2)a^2-24\left(-\frac{dx}{3}+c\right)xbca-16b^2c^2x^2}{3\sqrt{x(bx+a)}\,(bx+a)a^3}$
gosper	$-\frac{2x^2(bx+a)(-a^2d^2x^2-4abcd\,x^2+8b^2c^2x^2-6a^2cdx+12ab\,c^2x+3a^2c^2)}{3a^3(b\,x^2+ax)^{\frac{5}{2}}}$
orering	$-\frac{2x^2(bx+a)(-a^2d^2x^2-4abcd\,x^2+8b^2c^2x^2-6a^2cdx+12ab\,c^2x+3a^2c^2)}{3a^3(b\,x^2+ax)^{\frac{5}{2}}}$
trager	$-\frac{2(-a^2d^2x^2-4abcd\,x^2+8b^2c^2x^2-6a^2cdx+12ab\,c^2x+3a^2c^2)\sqrt{b\,x^2+ax}}{3a^3(bx+a)^2x}$
default	$c^2\left(-\frac{1}{3b(b\,x^2+ax)^{\frac{3}{2}}}-\frac{a\left(-\frac{2(2bx+a)}{3a^2\left(b\,x^2+ax\right)^{\frac{3}{2}}}+\frac{16b(2bx+a)}{3a^4\sqrt{b\,x^2+ax}}\right)}{2b}\right)+d^2\left(-\frac{x^2}{b(b\,x^2+ax)^{\frac{3}{2}}}+\frac{a\left(-\frac{x}{2b\left(b\,x^2+ax\right)^{\frac{3}{2}}}-\cdots\right.}{\cdots}\right.$

input
```
int(x*(d*x+c)^2/(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/a^3*c^2*(b*x+a)/(x*(b*x+a))^(1/2)+2/3*x*(a*d*x+5*b*c*x+6*a*c)*(a*d-b*c)
/(x*(b*x+a))^(1/2)/(b*x+a)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.45

$$\int \frac{x(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = -\frac{2\left(3\,a^2c^2+(8\,b^2c^2-4\,abcd-a^2d^2)x^2+6\,(2\,abc^2-a^2cd)x\right)\sqrt{bx^2+ax}}{3\,(a^3b^2x^3+2\,a^4bx^2+a^5x)}$$

input
```
integrate(x*(d*x+c)^2/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(3*a^2*c^2 + (8*b^2*c^2 - 4*a*b*c*d - a^2*d^2)*x^2 + 6*(2*a*b*c^2 - a
^2*c*d)*x)*sqrt(b*x^2 + a*x)/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x)
```

Sympy [F]

$$\int \frac{x(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = \int \frac{x(c+dx)^2}{(x\,(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(d*x+c)**2/(b*x**2+a*x)**(5/2),x)
```

output
```
Integral(x*(c + d*x)**2/(x*(a + b*x))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(57) = 114.

Time = 0.04 (sec) , antiderivative size = 205, normalized size of antiderivative = 3.15

$$\int \frac{x(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = -\frac{d^2x^2}{(bx^2+ax)^{\frac{3}{2}}b}+\frac{2\,c^2x}{3\,(bx^2+ax)^{\frac{3}{2}}a} - \frac{16\,bc^2x}{3\,\sqrt{bx^2+ax}a^3}+\frac{8\,cdx}{3\,\sqrt{bx^2+ax}a^2}-\frac{4\,cdx}{3\,(bx^2+ax)^{\frac{3}{2}}b}-\frac{ad^2x}{3\,(bx^2+ax)^{\frac{3}{2}}b^2} + \frac{2\,d^2x}{3\,\sqrt{bx^2+ax}ab}-\frac{8\,c^2}{3\,\sqrt{bx^2+ax}a^2}+\frac{4\,cd}{3\,\sqrt{bx^2+ax}ab}+\frac{d^2}{3\,\sqrt{bx^2+ax}b^2}$$

input
```
integrate(x*(d*x+c)^2/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

output
```
-d^2*x^2/((b*x^2 + a*x)^(3/2)*b) + 2/3*c^2*x/((b*x^2 + a*x)^(3/2)*a) - 16/
3*b*c^2*x/(sqrt(b*x^2 + a*x)*a^3) + 8/3*c*d*x/(sqrt(b*x^2 + a*x)*a^2) - 4/
3*c*d*x/((b*x^2 + a*x)^(3/2)*b) - 1/3*a*d^2*x/((b*x^2 + a*x)^(3/2)*b^2) +
2/3*d^2*x/(sqrt(b*x^2 + a*x)*a*b) - 8/3*c^2/(sqrt(b*x^2 + a*x)*a^2) + 4/3*
c*d/(sqrt(b*x^2 + a*x)*a*b) + 1/3*d^2/(sqrt(b*x^2 + a*x)*b^2)
```

Giac [F]

$$\int \frac{x(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = \int \frac{(dx+c)^2x}{(bx^2+ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(d*x+c)^2/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*x/(b*x^2 + a*x)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 9.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26

$$\int \frac{x(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = -\frac{2\sqrt{b\,x^2+a\,x}\,(3\,a^2\,c^2-6\,a^2\,c\,d\,x-a^2\,d^2\,x^2+12\,a\,b\,c^2\,x-4\,a\,b\,c\,d\,x^2+8\,b^2\,c^2\,x^2)}{3\,a^3\,x\,(a+b\,x)^2}$$

input
```
int((x*(c + d*x)^2)/(a*x + b*x^2)^(5/2),x)
```

output
```
-(2*(a*x + b*x^2)^(1/2)*(3*a^2*c^2 - a^2*d^2*x^2 + 8*b^2*c^2*x^2 + 12*a*b*
c^2*x - 6*a^2*c*d*x - 4*a*b*c*d*x^2))/(3*a^3*x*(a + b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.32

$$\int \frac{x(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = \frac{\frac{4\sqrt{b}\,\sqrt{bx+a}\,a^3d^2x}{3} - \frac{8\sqrt{b}\,\sqrt{bx+a}\,a^2bcdx}{3} + \frac{4\sqrt{b}\,\sqrt{bx+a}\,a^2b\,d^2x^2}{3} + \frac{16\sqrt{b}\,\sqrt{bx+a}\,a\,b^2c^2x}{3} - \frac{8\sqrt{b}\,\sqrt{bx+a}\,a\,b^2c}{3}}{}$$

input
```
int(x*(d*x+c)^2/(b*x^2+a*x)^(5/2),x)
```

output
```
(2*(2*sqrt(b)*sqrt(a + b*x)*a**3*d**2*x - 4*sqrt(b)*sqrt(a + b*x)*a**2*b*c
*d*x + 2*sqrt(b)*sqrt(a + b*x)*a**2*b*d**2*x**2 + 8*sqrt(b)*sqrt(a + b*x)*
a*b**2*c**2*x - 4*sqrt(b)*sqrt(a + b*x)*a*b**2*c*d*x**2 + 8*sqrt(b)*sqrt(a
 + b*x)*b**3*c**2*x**2 - 3*sqrt(x)*a**2*b**2*c**2 + 6*sqrt(x)*a**2*b**2*c*
d*x + sqrt(x)*a**2*b**2*d**2*x**2 - 12*sqrt(x)*a*b**3*c**2*x + 4*sqrt(x)*a
*b**3*c*d*x**2 - 8*sqrt(x)*b**4*c**2*x**2))/(3*sqrt(a + b*x)*a**3*b**2*x*(
a + b*x))
```

3.181 $\int \frac{(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 169

$$\int \frac{(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = \frac{8(bc-ad)^2(2bc-ad)x^2}{3a^3bc\,(ax+bx^2)^{3/2}} + \frac{2(2bc-ad)x(c+dx)^2}{a^2c\,(ax+bx^2)^{3/2}} - \frac{2(c+dx)^3}{3ac\,(ax+bx^2)^{3/2}} + \frac{8(bc-ad)(2bc-ad)(2bc+ad)x}{3a^4bc\sqrt{ax+bx^2}}$$

output
```
8/3*(-a*d+b*c)^2*(-a*d+2*b*c)*x^2/a^3/b/c/(b*x^2+a*x)^(3/2)+2*(-a*d+2*b*c)
*x*(d*x+c)^2/a^2/c/(b*x^2+a*x)^(3/2)-2/3*(d*x+c)^3/a/c/(b*x^2+a*x)^(3/2)+8
/3*(-a*d+b*c)*(-a*d+2*b*c)*(a*d+2*b*c)*x/a^4/b/c/(b*x^2+a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.56

$$\int \frac{(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = \frac{32b^3c^2x^3+16ab^2cx^2(3c-2dx)-2a^3(c^2+6cdx-3d^2x^2)+4a^2bx(3c^2-12cdx+d^2}{3a^4(x(a+bx))^{3/2}}$$

input
```
Integrate[(c + d*x)^2/(a*x + b*x^2)^(5/2),x]
```

output
```
(32*b^3*c^2*x^3 + 16*a*b^2*c*x^2*(3*c - 2*d*x) - 2*a^3*(c^2 + 6*c*d*x - 3*
d^2*x^2) + 4*a^2*b*x*(3*c^2 - 12*c*d*x + d^2*x^2))/(3*a^4*(x*(a + b*x))^(3
/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.46, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1156, 1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(ax+bx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 1156}$$

$$-\frac{4(2bc-ad)\int \frac{c+dx}{(bx^2+ax)^{3/2}}dx}{3a^2}-\frac{2(a+2bx)(c+dx)^2}{3a^2\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1158}$$

$$\frac{8(2bc-ad)(x(2bc-ad)+ac)}{3a^4\sqrt{ax+bx^2}}-\frac{2(a+2bx)(c+dx)^2}{3a^2\left(ax+bx^2\right)^{3/2}}$$

input
```
Int[(c + d*x)^2/(a*x + b*x^2)^(5/2),x]
```

output
```
(-2*(a + 2*b*x)*(c + d*x)^2)/(3*a^2*(a*x + b*x^2)^(3/2)) + (8*(2*b*c - a*d
)*(a*c + (2*b*c - a*d)*x))/(3*a^4*Sqrt[a*x + b*x^2])
```

Defintions of rubi rules used

rule 1156
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] + Simp[m*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*c)))   Int[
(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.56

method	result
risch	$-\frac{2c(bx+a)(6adx-8cbx+ac)}{3a^4x\sqrt{x(bx+a)}}+\frac{2x\left(2abdx-8b^2cx+3a^2d-9abc\right)(ad-bc)}{3\sqrt{x(bx+a)}\,(bx+a)a^4}$
pseudoelliptic	$-\frac{2\left(\left(-3d^2x^2+6cdx+c^2\right)a^3-6x\left(\frac{1}{3}d^2x^2-4cdx+c^2\right)b\,a^2-24x^2\left(-\frac{2dx}{3}+c\right)b^2ca-16b^3c^2x^3\right)}{3\sqrt{x(bx+a)}\,x(bx+a)a^4}$
gosper	$-\frac{2x(bx+a)\left(-2d^2x^3a^2b+16a\,b^2cd\,x^3-16b^3c^2x^3-3a^3d^2x^2+24x^2a^2bcd-24a\,b^2c^2x^2+6a^3cdx-6a^2b\,c^2x+c^2a^3\right)}{3a^4\left(b\,x^2+ax\right)^{\frac{5}{2}}}$
orering	$-\frac{2x(bx+a)\left(-2d^2x^3a^2b+16a\,b^2cd\,x^3-16b^3c^2x^3-3a^3d^2x^2+24x^2a^2bcd-24a\,b^2c^2x^2+6a^3cdx-6a^2b\,c^2x+c^2a^3\right)}{3a^4\left(b\,x^2+ax\right)^{\frac{5}{2}}}$
trager	$-\frac{2\left(-2d^2x^3a^2b+16a\,b^2cd\,x^3-16b^3c^2x^3-3a^3d^2x^2+24x^2a^2bcd-24a\,b^2c^2x^2+6a^3cdx-6a^2b\,c^2x+c^2a^3\right)\sqrt{b\,x^2+ax}}{3a^4x^2(bx+a)^2}$
default	$c^2\left(-\frac{2(2bx+a)}{3a^2\left(b\,x^2+ax\right)^{\frac{3}{2}}}+\frac{16b(2bx+a)}{3a^4\sqrt{b\,x^2+ax}}\right)+d^2\left(-\frac{x}{2b\left(b\,x^2+ax\right)^{\frac{3}{2}}}-\frac{a\left(-\frac{1}{3b\left(b\,x^2+ax\right)^{\frac{3}{2}}}-\frac{a\left(-\frac{2(2bx+a)}{3a^2\left(b\,x^2+ax\right)^{\frac{3}{2}}}+\frac{1}{3a\cdots}\right.}{2b}\right.}{4b}\right.$

input
```
int((d*x+c)^2/(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*c*(b*x+a)*(6*a*d*x-8*b*c*x+a*c)/a^4/x/(x*(b*x+a))^(1/2)+2/3*x*(2*a*b*
d*x-8*b^2*c*x+3*a^2*d-9*a*b*c)*(a*d-b*c)/(x*(b*x+a))^(1/2)/(b*x+a)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.76

$$\int \frac{(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = -\frac{2\,(a^3c^2-2\,(8\,b^3c^2-8\,ab^2cd+a^2bd^2)x^3-3\,(8\,ab^2c^2-8\,a^2bcd+a^3d^2)x^2-6\,(a^2bc^2-a^3cd)x)\sqrt{bx^2+ax}}{3\,(a^4b^2x^4+2\,a^5bx^3+a^6x^2)}$$

input
```
integrate((d*x+c)^2/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(a^3*c^2 - 2*(8*b^3*c^2 - 8*a*b^2*c*d + a^2*b*d^2)*x^3 - 3*(8*a*b^2*c
^2 - 8*a^2*b*c*d + a^3*d^2)*x^2 - 6*(a^2*b*c^2 - a^3*c*d)*x)*sqrt(b*x^2 +
a*x)/(a^4*b^2*x^4 + 2*a^5*b*x^3 + a^6*x^2)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = \int \frac{(c+dx)^2}{(x\,(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)**2/(b*x**2+a*x)**(5/2),x)
```

output
```
Integral((c + d*x)**2/(x*(a + b*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.20

$$\int \frac{(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = -\frac{4\,bc^2x}{3\,(bx^2+ax)^{\frac{3}{2}}a^2} + \frac{32\,b^2c^2x}{3\,\sqrt{bx^2+ax}a^4} + \frac{4\,cdx}{3\,(bx^2+ax)^{\frac{3}{2}}a} - \frac{32\,bcdx}{3\,\sqrt{bx^2+ax}a^3} + \frac{4\,d^2x}{3\,\sqrt{bx^2+ax}a^2} - \frac{2\,d^2x}{3\,(bx^2+ax)^{\frac{3}{2}}b} - \frac{2\,c^2}{3\,(bx^2+ax)^{\frac{3}{2}}a} + \frac{16\,bc^2}{3\,\sqrt{bx^2+ax}a^3} - \frac{16\,cd}{3\,\sqrt{bx^2+ax}a^2} + \frac{2\,d^2}{3\,\sqrt{bx^2+ax}ab}$$

input
```
integrate((d*x+c)^2/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

output
```
-4/3*b*c^2*x/((b*x^2 + a*x)^(3/2)*a^2) + 32/3*b^2*c^2*x/(sqrt(b*x^2 + a*x)
*a^4) + 4/3*c*d*x/((b*x^2 + a*x)^(3/2)*a) - 32/3*b*c*d*x/(sqrt(b*x^2 + a*x
)*a^3) + 4/3*d^2*x/(sqrt(b*x^2 + a*x)*a^2) - 2/3*d^2*x/((b*x^2 + a*x)^(3/2
)*b) - 2/3*c^2/((b*x^2 + a*x)^(3/2)*a) + 16/3*b*c^2/(sqrt(b*x^2 + a*x)*a^3
) - 16/3*c*d/(sqrt(b*x^2 + a*x)*a^2) + 2/3*d^2/(sqrt(b*x^2 + a*x)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.65

$$\int \frac{(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = \frac{2\left(\left(x\left(\frac{2\,(8\,b^3c^2-8\,ab^2cd+a^2bd^2)x}{a^4}+\frac{3\,(8\,ab^2c^2-8\,a^2bcd+a^3d^2)}{a^4}\right)+\frac{6\,(a^2bc^2-a^3cd)}{a^4}\right)x-\frac{c^2}{a}\right)}{3\,(bx^2+ax)^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)^2/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

output
```
2/3*((x*(2*(8*b^3*c^2 - 8*a*b^2*c*d + a^2*b*d^2)*x/a^4 + 3*(8*a*b^2*c^2 -
8*a^2*b*c*d + a^3*d^2)/a^4) + 6*(a^2*b*c^2 - a^3*c*d)/a^4)*x - c^2/a)/(b*x
^2 + a*x)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 9.60 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.66

$$\int \frac{(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = \frac{2\left(-a^3\,c^2-6\,a^3\,c\,d\,x+3\,a^3\,d^2\,x^2+6\,a^2\,b\,c^2\,x-24\,a^2\,b\,c\,d\,x^2+2\,a^2\,b\,d^2\,x^3+24\,a\,b^2\,c\right.}{3\,a^4\,{\left(b\,x^2+a\,x\right)}^{3/2}}$$

input
```
int((c + d*x)^2/(a*x + b*x^2)^(5/2),x)
```

output
```
(2*(3*a^3*d^2*x^2 - a^3*c^2 + 16*b^3*c^2*x^3 + 24*a*b^2*c^2*x^2 + 2*a^2*b*
d^2*x^3 - 6*a^3*c*d*x + 6*a^2*b*c^2*x - 24*a^2*b*c*d*x^2 - 16*a*b^2*c*d*x^
3))/(3*a^4*(a*x + b*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.56

$$\int \frac{(c+dx)^2}{(ax+bx^2)^{5/2}}\,dx = \frac{-4\sqrt{b}\,\sqrt{bx+a}\,a^3d^2x^2+\frac{32\sqrt{b}\,\sqrt{bx+a}\,a^2bcd\,x^2}{3}-4\sqrt{b}\,\sqrt{bx+a}\,a^2b\,d^2x^3-\frac{32\sqrt{b}\,\sqrt{bx+a}\,a\,b^2c^2}{3}}{}$$

input
```
int((d*x+c)^2/(b*x^2+a*x)^(5/2),x)
```

output
```
(2*( - 6*sqrt(b)*sqrt(a + b*x)*a**3*d**2*x**2 + 16*sqrt(b)*sqrt(a + b*x)*a
**2*b*c*d*x**2 - 6*sqrt(b)*sqrt(a + b*x)*a**2*b*d**2*x**3 - 16*sqrt(b)*sqr
t(a + b*x)*a*b**2*c**2*x**2 + 16*sqrt(b)*sqrt(a + b*x)*a*b**2*c*d*x**3 - 1
6*sqrt(b)*sqrt(a + b*x)*b**3*c**2*x**3 - sqrt(x)*a**3*b*c**2 - 6*sqrt(x)*a
**3*b*c*d*x + 3*sqrt(x)*a**3*b*d**2*x**2 + 6*sqrt(x)*a**2*b**2*c**2*x - 24
*sqrt(x)*a**2*b**2*c*d*x**2 + 2*sqrt(x)*a**2*b**2*d**2*x**3 + 24*sqrt(x)*a
*b**3*c**2*x**2 - 16*sqrt(x)*a*b**3*c*d*x**3 + 16*sqrt(x)*b**4*c**2*x**3))
/(3*sqrt(a + b*x)*a**4*b*x**2*(a + b*x))
```

3.182 $\int \frac{(c+dx)^2}{x(ax+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 187

$$\int \frac{(c+dx)^2}{x(ax+bx^2)^{5/2}}\,dx = \frac{4c(4bc-5ad)}{15a^2(ax+bx^2)^{3/2}} - \frac{2c^2}{5ax(ax+bx^2)^{3/2}} + \frac{2\left(5d^2+\frac{4bc(4bc-5ad)}{a^2}\right)x}{15a(ax+bx^2)^{3/2}} + \frac{8(5a^2d^2+4bc(4bc-5ad))}{15a^4\sqrt{ax+bx^2}} - \frac{16(5a^2d^2+4bc(4bc-5ad))\sqrt{ax+bx^2}}{15a^5x}$$

output

```
4/15*c*(-5*a*d+4*b*c)/a^2/(b*x^2+a*x)^(3/2)-2/5*c^2/a/x/(b*x^2+a*x)^(3/2)+
2/15*(5*d^2+4*b*c*(-5*a*d+4*b*c)/a^2)*x/a/(b*x^2+a*x)^(3/2)+8/15*(5*a^2*d^
2+4*b*c*(-5*a*d+4*b*c))/a^4/(b*x^2+a*x)^(1/2)-16/15*(5*a^2*d^2+4*b*c*(-5*a
*d+4*b*c))*(b*x^2+a*x)^(1/2)/a^5/x
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.70

$$\int \frac{(c+dx)^2}{x\,(ax+bx^2)^{5/2}}\,dx = -\frac{2(128b^4c^2x^4+32ab^3cx^3(6c-5dx)+8a^2b^2x^2(6c^2-30cdx+5d^2x^2)+4a^3bx(-2c^2-15cdx+15d^2x^2)+}{15a^5x(x(a+bx))^{3/2}}$$

input
```
Integrate[(c + d*x)^2/(x*(a*x + b*x^2)^(5/2)),x]
```

output
```
(-2*(128*b^4*c^2*x^4 + 32*a*b^3*c*x^3*(6*c - 5*d*x) + 8*a^2*b^2*x^2*(6*c^2
 - 30*c*d*x + 5*d^2*x^2) + 4*a^3*b*x*(-2*c^2 - 15*c*d*x + 15*d^2*x^2) + a^
4*(3*c^2 + 10*c*d*x + 15*d^2*x^2)))/(15*a^5*x*(x*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.76, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1262, 27, 1220, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{x\,(ax+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1262}$$

$$-\frac{\int -\frac{3(2bc^2+d(4bc-ad)x)}{2x(bx^2+ax)^{5/2}}dx}{3b}-\frac{d^2}{3b\,(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2bc^2+d(4bc-ad)x}{x(bx^2+ax)^{5/2}}dx}{2b}-\frac{d^2}{3b\,(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{1220}$$

$$\frac{-\frac{(5a^2d^2-20abcd+16b^2c^2)\int \frac{1}{(bx^2+ax)^{5/2}}dx}{5a}-\frac{4bc^2}{5ax(ax+bx^2)^{3/2}}}{2b}-\frac{d^2}{3b\,(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{1089}$$

$$\frac{-\frac{(5a^2d^2-20abcd+16b^2c^2)\left(-\frac{8b\int \frac{1}{(bx^2+ax)^{3/2}}dx}{3a^2}-\frac{2(a+2bx)}{3a^2(ax+bx^2)^{3/2}}\right)}{5a}-\frac{4bc^2}{5ax(ax+bx^2)^{3/2}}}{2b}-\frac{d^2}{3b\,(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{-\frac{\left(\frac{16b(a+2bx)}{3a^4\sqrt{ax+bx^2}}-\frac{2(a+2bx)}{3a^2(ax+bx^2)^{3/2}}\right)(5a^2d^2-20abcd+16b^2c^2)}{5a}-\frac{4bc^2}{5ax(ax+bx^2)^{3/2}}}{2b}-\frac{d^2}{3b\,(ax+bx^2)^{3/2}}$$

input
```
Int[(c + d*x)^2/(x*(a*x + b*x^2)^(5/2)),x]
```

output
```
-1/3*d^2/(b*(a*x + b*x^2)^(3/2)) + ((-4*b*c^2)/(5*a*x*(a*x + b*x^2)^(3/2))
 - ((16*b^2*c^2 - 20*a*b*c*d + 5*a^2*d^2)*((-2*(a + 2*b*x))/(3*a^2*(a*x +
b*x^2)^(3/2)) + (16*b*(a + 2*b*x))/(3*a^4*Sqrt[a*x + b*x^2])))/(5*a))/(2*b
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$\frac{(-30d^2x^2-20cdx-6c^2)a^4+16xb(-\frac{15}{2}d^2x^2+\frac{15}{2}cdx+c^2)a^3-96(\frac{5}{6}d^2x^2-5cdx+c^2)x^2b^2a^2-384x^3b^3c\left(-\frac{5dx}{6}+c\right)a-256b^4c}{15\sqrt{x(bx+a)}\,x^2(bx+a)a^5}$
risch	$-\frac{2(bx+a)(15a^2d^2x^2-80abcd\,x^2+73b^2c^2x^2+10a^2cdx-14ab\,c^2x+3a^2c^2)}{15a^5x^2\sqrt{x(bx+a)}}-\frac{2x(5abdx-11b^2cx+6a^2d-12abc)(ad-bc)b}{3\sqrt{x(bx+a)}\,(bx+a)a^5}$
gosper	$-\frac{2(bx+a)(40a^2b^2d^2x^4-160a\,b^3cd\,x^4+128b^4c^2x^4+60a^3b\,d^2x^3-240a^2b^2cd\,x^3+192a\,b^3c^2x^3+15a^4d^2x^2-60a^3dcb\,x^2+48a}{15a^5(b\,x^2+ax)^{\frac{5}{2}}}$
orering	$-\frac{2(bx+a)(40a^2b^2d^2x^4-160a\,b^3cd\,x^4+128b^4c^2x^4+60a^3b\,d^2x^3-240a^2b^2cd\,x^3+192a\,b^3c^2x^3+15a^4d^2x^2-60a^3dcb\,x^2+48a}{15a^5(b\,x^2+ax)^{\frac{5}{2}}}$
trager	$-\frac{2(40a^2b^2d^2x^4-160a\,b^3cd\,x^4+128b^4c^2x^4+60a^3b\,d^2x^3-240a^2b^2cd\,x^3+192a\,b^3c^2x^3+15a^4d^2x^2-60a^3dcb\,x^2+48a^2b^2c^2x}{15a^5x^3(bx+a)^2}$
default	$c^2\left(-\frac{2}{5ax(b\,x^2+ax)^{\frac{3}{2}}}-\frac{8b\left(-\frac{2(2bx+a)}{3a^2(b\,x^2+ax)^{\frac{3}{2}}}+\frac{16b(2bx+a)}{3a^4\sqrt{b\,x^2+ax}}\right)}{5a}\right)+d^2\left(-\frac{1}{3b(b\,x^2+ax)^{\frac{3}{2}}}-\frac{a\left(-\frac{2(2bx+a)}{3a^2(b\,x^2+ax)}\right.}{2}\right.$

input
```
int((d*x+c)^2/x/(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*((-30*d^2*x^2-20*c*d*x-6*c^2)*a^4+16*x*b*(-15/2*d^2*x^2+15/2*c*d*x+c^
2)*a^3-96*(5/6*d^2*x^2-5*c*d*x+c^2)*x^2*b^2*a^2-384*x^3*b^3*c*(-5/6*d*x+c)
*a-256*b^4*c^2*x^4)/(x*(b*x+a))^(1/2)/x^2/(b*x+a)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx)^2}{x\,(ax+bx^2)^{5/2}}\,dx =$$

$$-\frac{2\,(3\,a^4c^2+8\,(16\,b^4c^2-20\,ab^3cd+5\,a^2b^2d^2)x^4+12\,(16\,ab^3c^2-20\,a^2b^2cd+5\,a^3bd^2)x^3+3\,(16\,a^2b^2c^2-20}{15\,(a^5b^2x^5+2\,a^6bx^4+a^7x^3)}$$

input
```
integrate((d*x+c)^2/x/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/15*(3*a^4*c^2 + 8*(16*b^4*c^2 - 20*a*b^3*c*d + 5*a^2*b^2*d^2)*x^4 + 12*
(16*a*b^3*c^2 - 20*a^2*b^2*c*d + 5*a^3*b*d^2)*x^3 + 3*(16*a^2*b^2*c^2 - 20
*a^3*b*c*d + 5*a^4*d^2)*x^2 - 2*(4*a^3*b*c^2 - 5*a^4*c*d)*x)*sqrt(b*x^2 +
a*x)/(a^5*b^2*x^5 + 2*a^6*b*x^4 + a^7*x^3)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{x\,(ax+bx^2)^{5/2}}\,dx = \int \frac{(c+dx)^2}{x\,(x\,(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)**2/x/(b*x**2+a*x)**(5/2),x)
```

output
```
Integral((c + d*x)**2/(x*(x*(a + b*x))**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.34

$$\begin{aligned}\int \frac{(c+dx)^2}{x\,(ax+bx^2)^{5/2}}\,dx &= \frac{32\,b^2c^2x}{15\,(bx^2+ax)^{\frac{3}{2}}a^3} - \frac{256\,b^3c^2x}{15\,\sqrt{bx^2+ax}a^5} \\ &\quad - \frac{8\,bcdx}{3\,(bx^2+ax)^{\frac{3}{2}}a^2} + \frac{64\,b^2cdx}{3\,\sqrt{bx^2+ax}a^4} + \frac{2\,d^2x}{3\,(bx^2+ax)^{\frac{3}{2}}a} \\ &\quad - \frac{16\,bd^2x}{3\,\sqrt{bx^2+ax}a^3} + \frac{16\,bc^2}{15\,(bx^2+ax)^{\frac{3}{2}}a^2} - \frac{128\,b^2c^2}{15\,\sqrt{bx^2+ax}a^4} \\ &\quad - \frac{4\,cd}{3\,(bx^2+ax)^{\frac{3}{2}}a} + \frac{32\,bcd}{3\,\sqrt{bx^2+ax}a^3} - \frac{8\,d^2}{3\,\sqrt{bx^2+ax}a^2} - \frac{2\,c^2}{5\,(bx^2+ax)^{\frac{3}{2}}ax}\end{aligned}$$

input
```
integrate((d*x+c)^2/x/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

output
```
32/15*b^2*c^2*x/((b*x^2 + a*x)^(3/2)*a^3) - 256/15*b^3*c^2*x/(sqrt(b*x^2 +
 a*x)*a^5) - 8/3*b*c*d*x/((b*x^2 + a*x)^(3/2)*a^2) + 64/3*b^2*c*d*x/(sqrt(
b*x^2 + a*x)*a^4) + 2/3*d^2*x/((b*x^2 + a*x)^(3/2)*a) - 16/3*b*d^2*x/(sqrt
(b*x^2 + a*x)*a^3) + 16/15*b*c^2/((b*x^2 + a*x)^(3/2)*a^2) - 128/15*b^2*c^
2/(sqrt(b*x^2 + a*x)*a^4) - 4/3*c*d/((b*x^2 + a*x)^(3/2)*a) + 32/3*b*c*d/(
sqrt(b*x^2 + a*x)*a^3) - 8/3*d^2/(sqrt(b*x^2 + a*x)*a^2) - 2/5*c^2/((b*x^2
 + a*x)^(3/2)*a*x)
```

Giac [F]

$$\int \frac{(c+dx)^2}{x\left(ax+bx^2\right)^{5/2}}\,dx = \int \frac{(dx+c)^2}{\left(bx^2+ax\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((d*x+c)^2/x/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/((b*x^2 + a*x)^(5/2)*x), x)
```

Mupad [B] (verification not implemented)

Time = 9.75 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx)^2}{x\left(ax+bx^2\right)^{5/2}}\,dx = -\frac{2\sqrt{b\,x^2+a\,x}\,\left(3\,a^4\,c^2+10\,a^4\,c\,d\,x+15\,a^4\,d^2\,x^2-8\,a^3\,b\,c^2\,x-60\,a^3\,b\,c\,d\,x^2+60\,a^3\,b\,d^2\,x^3+48\,a^2\,b^2\,c^2\,x^2\right.}{15\,a^5\,x^3\,{\left(a+b\,x\right)}^2}$$

input
```
int((c + d*x)^2/(x*(a*x + b*x^2)^(5/2)),x)
```

output
```
-(2*(a*x + b*x^2)^(1/2)*(3*a^4*c^2 + 15*a^4*d^2*x^2 + 128*b^4*c^2*x^4 + 19
2*a*b^3*c^2*x^3 + 60*a^3*b*d^2*x^3 + 10*a^4*c*d*x + 48*a^2*b^2*c^2*x^2 + 4
0*a^2*b^2*d^2*x^4 - 8*a^3*b*c^2*x - 60*a^3*b*c*d*x^2 - 160*a*b^3*c*d*x^4 -
 240*a^2*b^2*c*d*x^3))/(15*a^5*x^3*(a + b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.60

$$\int \frac{(c+dx)^2}{x\,(ax+bx^2)^{5/2}}\,dx = \frac{\frac{16\sqrt{b}\,\sqrt{bx+a}\,a^3d^2x^3}{3} - \frac{64\sqrt{b}\,\sqrt{bx+a}\,a^2bcd\,x^3}{3} + \frac{16\sqrt{b}\,\sqrt{bx+a}\,a^2b\,d^2x^4}{3} + \frac{256\sqrt{b}\,\sqrt{bx+a}\,a\,b^2c^2x^3}{15} - 64\sqrt{b}$$

input
```
int((d*x+c)^2/x/(b*x^2+a*x)^(5/2),x)
```

output
```
(2*(40*sqrt(b)*sqrt(a + b*x)*a**3*d**2*x**3 - 160*sqrt(b)*sqrt(a + b*x)*a*
*2*b*c*d*x**3 + 40*sqrt(b)*sqrt(a + b*x)*a**2*b*d**2*x**4 + 128*sqrt(b)*sq
rt(a + b*x)*a*b**2*c**2*x**3 - 160*sqrt(b)*sqrt(a + b*x)*a*b**2*c*d*x**4 +
 128*sqrt(b)*sqrt(a + b*x)*b**3*c**2*x**4 - 3*sqrt(x)*a**4*c**2 - 10*sqrt(
x)*a**4*c*d*x - 15*sqrt(x)*a**4*d**2*x**2 + 8*sqrt(x)*a**3*b*c**2*x + 60*s
qrt(x)*a**3*b*c*d*x**2 - 60*sqrt(x)*a**3*b*d**2*x**3 - 48*sqrt(x)*a**2*b**
2*c**2*x**2 + 240*sqrt(x)*a**2*b**2*c*d*x**3 - 40*sqrt(x)*a**2*b**2*d**2*x
**4 - 192*sqrt(x)*a*b**3*c**2*x**3 + 160*sqrt(x)*a*b**3*c*d*x**4 - 128*sqr
t(x)*b**4*c**2*x**4))/(15*sqrt(a + b*x)*a**5*x**3*(a + b*x))
```

3.183 $\int \frac{(c+dx)^2}{x^2\left(ax+bx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 238

$$\begin{aligned}\int \frac{(c+dx)^2}{x^2\left(ax+bx^2\right)^{5/2}}\,dx =& \frac{2\left(35d^2+\frac{16bc(5bc-7ad)}{a^2}\right)}{105a\left(ax+bx^2\right)^{3/2}}-\frac{2c^2}{7ax^2\left(ax+bx^2\right)^{3/2}}\\&+\frac{4c(5bc-7ad)}{35a^2x\left(ax+bx^2\right)^{3/2}}+\frac{4(35a^2d^2+16bc(5bc-7ad))}{35a^4x\sqrt{ax+bx^2}}\\&-\frac{16(35a^2d^2+16bc(5bc-7ad))\sqrt{ax+bx^2}}{105a^5x^2}\\&+\frac{32b(35a^2d^2+16bc(5bc-7ad))\sqrt{ax+bx^2}}{105a^6x}\end{aligned}$$

output
```
2/105*(35*d^2+16*b*c*(-7*a*d+5*b*c)/a^2)/a/(b*x^2+a*x)^(3/2)-2/7*c^2/a/x^2
/(b*x^2+a*x)^(3/2)+4/35*c*(-7*a*d+5*b*c)/a^2/x/(b*x^2+a*x)^(3/2)+4/35*(35*
a^2*d^2+16*b*c*(-7*a*d+5*b*c))/a^4/x/(b*x^2+a*x)^(1/2)-16/105*(35*a^2*d^2+
16*b*c*(-7*a*d+5*b*c))*(b*x^2+a*x)^(1/2)/a^5/x^2+32/105*b*(35*a^2*d^2+16*b
*c*(-7*a*d+5*b*c))*(b*x^2+a*x)^(1/2)/a^6/x
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.68

$$\int \frac{(c+dx)^2}{x^2\left(ax+bx^2\right)^{5/2}}\,dx=\frac{2560b^5c^2x^5+256ab^4cx^4(15c-14dx)+32a^2b^3x^3(30c^2-168cdx+35d^2x^2)-2a^5}{10}$$

input
```
Integrate[(c + d*x)^2/(x^2*(a*x + b*x^2)^(5/2)),x]
```

output
```
(2560*b^5*c^2*x^5 + 256*a*b^4*c*x^4*(15*c - 14*d*x) + 32*a^2*b^3*x^3*(30*c
^2 - 168*c*d*x + 35*d^2*x^2) - 2*a^5*(15*c^2 + 42*c*d*x + 35*d^2*x^2) + 16
*a^3*b^2*x^2*(-10*c^2 - 84*c*d*x + 105*d^2*x^2) + 4*a^4*b*x*(15*c^2 + 56*c
*d*x + 105*d^2*x^2))/(105*a^6*x^2*(x*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1262, 27, 1220, 1129, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{x^2\left(ax+bx^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{1262}$$

$$-\frac{\int -\frac{8bc^2+d(16bc-5ad)x}{2x^2\left(bx^2+ax\right)^{5/2}}dx}{4b}-\frac{d^2}{4bx\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{8bc^2+d(16bc-5ad)x}{x^2\left(bx^2+ax\right)^{5/2}}dx}{8b}-\frac{d^2}{4bx\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1220}$$

$$\frac{-\frac{(35a^2d^2-112abcd+80b^2c^2)\int \frac{1}{x\left(bx^2+ax\right)^{5/2}}dx}{7a}-\frac{16bc^2}{7ax^2(ax+bx^2)^{3/2}}}{8b}-\frac{d^2}{4bx\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1129}$$

$$\frac{-\frac{(35a^2d^2-112abcd+80b^2c^2)\left(-\frac{8b\int \frac{1}{\left(bx^2+ax\right)^{5/2}}dx}{5a}-\frac{2}{5ax\left(ax+bx^2\right)^{3/2}}\right)}{7a}-\frac{16bc^2}{7ax^2(ax+bx^2)^{3/2}}}{8b}-\frac{d^2}{4bx\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1089}$$

$$\frac{-\frac{(35a^2d^2-112abcd+80b^2c^2)\left(-\frac{8b\left(-\frac{8b\int \frac{1}{\left(bx^2+ax\right)^{3/2}}dx}{3a^2}-\frac{2(a+2bx)}{3a^2\left(ax+bx^2\right)^{3/2}}\right)}{5a}-\frac{2}{5ax\left(ax+bx^2\right)^{3/2}}\right)}{7a}-\frac{16bc^2}{7ax^2(ax+bx^2)^{3/2}}}{8b}-\frac{d^2}{4bx\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{-\frac{\left(-\frac{8b\left(\frac{16b(a+2bx)}{3a^4\sqrt{ax+bx^2}}-\frac{2(a+2bx)}{3a^2\left(ax+bx^2\right)^{3/2}}\right)}{5a}-\frac{2}{5ax\left(ax+bx^2\right)^{3/2}}\right)(35a^2d^2-112abcd+80b^2c^2)}{7a}-\frac{16bc^2}{7ax^2(ax+bx^2)^{3/2}}}{8b}-\frac{d^2}{4bx\left(ax+bx^2\right)^{3/2}}$$

input

```
Int[(c + d*x)^2/(x^2*(a*x + b*x^2)^(5/2)),x]
```

output

```
-1/4*d^2/(b*x*(a*x + b*x^2)^(3/2)) + ((-16*b*c^2)/(7*a*x^2*(a*x + b*x^2)^(
3/2)) - ((80*b^2*c^2 - 112*a*b*c*d + 35*a^2*d^2)*(-2/(5*a*x*(a*x + b*x^2)^
(3/2)) - (8*b*((-2*(a + 2*b*x))/(3*a^2*(a*x + b*x^2)^(3/2)) + (16*b*(a + 2
*b*x))/(3*a^4*Sqrt[a*x + b*x^2])))/(5*a)))/(7*a))/(8*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{\left(-70 d^{2} x^{2}-84 c d x-30 c^{2}\right) a^{5}+60\left(7 d^{2} x^{2}+\frac{56}{15} c d x+c^{2}\right) x b\, a^{4}-160 x^{2}\left(-\frac{21}{2} d^{2} x^{2}+\frac{42}{5} c d x+c^{2}\right) b^{2} a^{3}+960 x^{3}\left(\frac{7}{6} d^{2} x^{2}-\frac{28}{5} c d x+c^{2}\right)}{105 \sqrt{x\left(b x+a\right)}\, x^{3}\left(b x+a\right) a^{6}}$
risch	$-\frac{2\left(b x+a\right)\left(-280 d^{2} x^{3} a^{2} b+1022 a\, b^{2} c d\, x^{3}-790 b^{3} c^{2} x^{3}+35 a^{3} d^{2} x^{2}-196 x^{2} a^{2} b c d+185 a\, b^{2} c^{2} x^{2}+42 a^{3} c d x-60 a^{2} b\, c^{2} x+15 c^{2} a\right.}{105 a^{6} x^{3} \sqrt{x\left(b x+a\right)}}$
gosper	$-\frac{2\left(b x+a\right)\left(-560 a^{2} b^{3} d^{2} x^{5}+1792 a\, b^{4} c d\, x^{5}-1280 b^{5} c^{2} x^{5}-840 a^{3} b^{2} d^{2} x^{4}+2688 a^{2} b^{3} c d\, x^{4}-1920 a\, b^{4} c^{2} x^{4}-210 a^{4} b\, d^{2} x^{3}+672\right.}{105 x\, a^{6}\left(b\, x^{2}+a x\right)^{\frac{5}{2}}}$
orering	$-\frac{2\left(b x+a\right)\left(-560 a^{2} b^{3} d^{2} x^{5}+1792 a\, b^{4} c d\, x^{5}-1280 b^{5} c^{2} x^{5}-840 a^{3} b^{2} d^{2} x^{4}+2688 a^{2} b^{3} c d\, x^{4}-1920 a\, b^{4} c^{2} x^{4}-210 a^{4} b\, d^{2} x^{3}+672\right.}{105 x\, a^{6}\left(b\, x^{2}+a x\right)^{\frac{5}{2}}}$
trager	$-\frac{2\left(-560 a^{2} b^{3} d^{2} x^{5}+1792 a\, b^{4} c d\, x^{5}-1280 b^{5} c^{2} x^{5}-840 a^{3} b^{2} d^{2} x^{4}+2688 a^{2} b^{3} c d\, x^{4}-1920 a\, b^{4} c^{2} x^{4}-210 a^{4} b\, d^{2} x^{3}+672 a^{3} b^{2} c d\right.}{105 a^{6} x^{4}\left(b x+a\right)^{2}}$
default	$d^{2}\left(-\frac{2\left(2 b x+a\right)}{3 a^{2}\left(b\, x^{2}+a x\right)^{\frac{3}{2}}}+\frac{16 b\left(2 b x+a\right)}{3 a^{4} \sqrt{b\, x^{2}+a x}}\right)+c^{2}\left(-\frac{2}{7 a\, x^{2}\left(b\, x^{2}+a x\right)^{\frac{3}{2}}}-\frac{10 b\left(-\frac{2}{5 a x\left(b\, x^{2}+a x\right)^{\frac{3}{2}}}-\frac{8 b\left(-\frac{2\left(2 b x+a\right)}{3 a^{2}\left(b\, x^{2}+a x\right.}\right.}{}\right)}{7 a}\right.$

input
```
int((d*x+c)^2/x^2/(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*((-70*d^2*x^2-84*c*d*x-30*c^2)*a^5+60*(7*d^2*x^2+56/15*c*d*x+c^2)*x*
b*a^4-160*x^2*(-21/2*d^2*x^2+42/5*c*d*x+c^2)*b^2*a^3+960*x^3*(7/6*d^2*x^2-
28/5*c*d*x+c^2)*b^3*a^2+3840*(-14/15*d*x+c)*x^4*b^4*c*a+2560*b^5*c^2*x^5)/
(x*(b*x+a))^(1/2)/x^3/(b*x+a)/a^6
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx)^2}{x^2 (ax+bx^2)^{5/2}}\,dx =$$
$$-\frac{2\,(15\,a^5c^2 - 16\,(80\,b^5c^2 - 112\,ab^4cd + 35\,a^2b^3d^2)x^5 - 24\,(80\,ab^4c^2 - 112\,a^2b^3cd + 35\,a^3b^2d^2)x^4 - 6\,(80\,a^2}{105\,(a^6b^2x^6 + }$$

input
```
integrate((d*x+c)^2/x^2/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/105*(15*a^5*c^2 - 16*(80*b^5*c^2 - 112*a*b^4*c*d + 35*a^2*b^3*d^2)*x^5
- 24*(80*a*b^4*c^2 - 112*a^2*b^3*c*d + 35*a^3*b^2*d^2)*x^4 - 6*(80*a^2*b^3
*c^2 - 112*a^3*b^2*c*d + 35*a^4*b*d^2)*x^3 + (80*a^3*b^2*c^2 - 112*a^4*b*c
*d + 35*a^5*d^2)*x^2 - 6*(5*a^4*b*c^2 - 7*a^5*c*d)*x)*sqrt(b*x^2 + a*x)/(a
^6*b^2*x^6 + 2*a^7*b*x^5 + a^8*x^4)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{x^2 (ax+bx^2)^{5/2}}\,dx = \int \frac{(c+dx)^2}{x^2 (x (a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)**2/x**2/(b*x**2+a*x)**(5/2),x)
```

output
```
Integral((c + d*x)**2/(x**2*(x*(a + b*x))**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.36

$$\int \frac{(c+dx)^2}{x^2(ax+bx^2)^{5/2}}\,dx = -\frac{64\,b^3c^2x}{21\,(bx^2+ax)^{\frac{3}{2}}a^4} + \frac{512\,b^4c^2x}{21\,\sqrt{bx^2+ax}a^6} + \frac{64\,b^2cdx}{15\,(bx^2+ax)^{\frac{3}{2}}a^3} - \frac{512\,b^3cdx}{15\,\sqrt{bx^2+ax}a^5} - \frac{4\,bd^2x}{3\,(bx^2+ax)^{\frac{3}{2}}a^2} + \frac{32\,b^2d^2x}{3\,\sqrt{bx^2+ax}a^4} - \frac{32\,b^2c^2}{21\,(bx^2+ax)^{\frac{3}{2}}a^3} + \frac{256\,b^3c^2}{21\,\sqrt{bx^2+ax}a^5} + \frac{32\,bcd}{15\,(bx^2+ax)^{\frac{3}{2}}a^2} - \frac{256\,b^2cd}{15\,\sqrt{bx^2+ax}a^4} - \frac{2\,d^2}{3\,(bx^2+ax)^{\frac{3}{2}}a} + \frac{16\,bd^2}{3\,\sqrt{bx^2+ax}a^3} + \frac{4\,bc^2}{7\,(bx^2+ax)^{\frac{3}{2}}a^2x} - \frac{4\,cd}{5\,(bx^2+ax)^{\frac{3}{2}}ax} - \frac{2\,c^2}{7\,(bx^2+ax)^{\frac{3}{2}}ax^2}$$

input
```
integrate((d*x+c)^2/x^2/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

output
```
-64/21*b^3*c^2*x/((b*x^2 + a*x)^(3/2)*a^4) + 512/21*b^4*c^2*x/(sqrt(b*x^2
+ a*x)*a^6) + 64/15*b^2*c*d*x/((b*x^2 + a*x)^(3/2)*a^3) - 512/15*b^3*c*d*x
/(sqrt(b*x^2 + a*x)*a^5) - 4/3*b*d^2*x/((b*x^2 + a*x)^(3/2)*a^2) + 32/3*b^
2*d^2*x/(sqrt(b*x^2 + a*x)*a^4) - 32/21*b^2*c^2/((b*x^2 + a*x)^(3/2)*a^3)
+ 256/21*b^3*c^2/(sqrt(b*x^2 + a*x)*a^5) + 32/15*b*c*d/((b*x^2 + a*x)^(3/2
)*a^2) - 256/15*b^2*c*d/(sqrt(b*x^2 + a*x)*a^4) - 2/3*d^2/((b*x^2 + a*x)^(
3/2)*a) + 16/3*b*d^2/(sqrt(b*x^2 + a*x)*a^3) + 4/7*b*c^2/((b*x^2 + a*x)^(3
/2)*a^2*x) - 4/5*c*d/((b*x^2 + a*x)^(3/2)*a*x) - 2/7*c^2/((b*x^2 + a*x)^(3
/2)*a*x^2)
```

Giac [F]

$$\int \frac{(c+dx)^2}{x^2(ax+bx^2)^{5/2}}\,dx = \int \frac{(dx+c)^2}{(bx^2+ax)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((d*x+c)^2/x^2/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/((b*x^2 + a*x)^(5/2)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 9.98 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.24

$$\int \frac{(c+dx)^2}{x^2\left(ax+bx^2\right)^{5/2}}\,dx = \frac{\left(\frac{560\,a^2\,b\,d^2-1792\,a\,b^2\,c\,d+1280\,b^3\,c^2}{105\,a^5}+\frac{2\,b\,x\,\left(560\,a^2\,b\,d^2-1792\,a\,b^2\,c\,d+1280\,b^3\,c^2\right)}{105\,a^6}\right)\,\sqrt{b\,x^2+a\,x}}{x\,\left(a+b\,x\right)}$$
$$-\frac{2\,c^2\,\sqrt{b\,x^2+a\,x}}{7\,a^3\,x^4}$$
$$-\frac{\sqrt{b\,x^2+a\,x}\,\left(\frac{70\,a^4\,d^2-392\,a^3\,b\,c\,d+370\,a^2\,b^2\,c^2}{105\,a^5}-x\,\left(\frac{a\,\left(\frac{16\,b^3\,c\,\left(7\,a\,d-10\,b\,c\right)}{105\,a^5}-\frac{8\,b^3\,c\,\left(91\,a\,d-115\,b\,c\right)}{105\,a^5}\right)}{b}-\frac{2\,b\,\left(70\,a^4\,d^2-392\,a^3\,b\,c\,d+370\cdots\right)}{105\,a^6}\cdots\right)\right)}{x^2\,{\left(a+b\,x\right)}^2}$$
$$-\frac{4\,c\,\sqrt{b\,x^2+a\,x}\,\left(7\,a\,d-10\,b\,c\right)}{35\,a^4\,x^3}$$

input
```
int((c + d*x)^2/(x^2*(a*x + b*x^2)^(5/2)),x)
```

output
```
(((1280*b^3*c^2 + 560*a^2*b*d^2 - 1792*a*b^2*c*d)/(105*a^5) + (2*b*x*(1280
*b^3*c^2 + 560*a^2*b*d^2 - 1792*a*b^2*c*d))/(105*a^6))*(a*x + b*x^2)^(1/2)
)/(x*(a + b*x)) - (2*c^2*(a*x + b*x^2)^(1/2))/(7*a^3*x^4) - ((a*x + b*x^2)
^(1/2)*((70*a^4*d^2 + 370*a^2*b^2*c^2 - 392*a^3*b*c*d)/(105*a^5) - x*((a*(
(16*b^3*c*(7*a*d - 10*b*c))/(105*a^5) - (8*b^3*c*(91*a*d - 115*b*c))/(105*
a^5)))/b - (2*b*(70*a^4*d^2 + 370*a^2*b^2*c^2 - 392*a^3*b*c*d))/(105*a^6)
+ (4*b^2*c*(91*a*d - 115*b*c))/(105*a^4))))/(x^2*(a + b*x)^2) - (4*c*(a*x
+ b*x^2)^(1/2)*(7*a*d - 10*b*c))/(35*a^4*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.47

$$\int \frac{(c+dx)^2}{x^2\,(ax+bx^2)^{5/2}}\,dx = \frac{-\frac{32\sqrt{b}\,\sqrt{bx+a}\,a^3b\,d^2x^4}{3}+\frac{512\sqrt{b}\,\sqrt{bx+a}\,a^2b^2cd\,x^4}{15}-\frac{32\sqrt{b}\,\sqrt{bx+a}\,a^2b^2d^2x^5}{3}-\frac{512\sqrt{b}\,\sqrt{bx+a}\,a\,b^3c^2x^4}{21}-}{}$$

input
```
int((d*x+c)^2/x^2/(b*x^2+a*x)^(5/2),x)
```

output
```
(2*( - 560*sqrt(b)*sqrt(a + b*x)*a**3*b*d**2*x**4 + 1792*sqrt(b)*sqrt(a +
b*x)*a**2*b**2*c*d*x**4 - 560*sqrt(b)*sqrt(a + b*x)*a**2*b**2*d**2*x**5 -
1280*sqrt(b)*sqrt(a + b*x)*a*b**3*c**2*x**4 + 1792*sqrt(b)*sqrt(a + b*x)*a
*b**3*c*d*x**5 - 1280*sqrt(b)*sqrt(a + b*x)*b**4*c**2*x**5 - 15*sqrt(x)*a*
*5*c**2 - 42*sqrt(x)*a**5*c*d*x - 35*sqrt(x)*a**5*d**2*x**2 + 30*sqrt(x)*a
**4*b*c**2*x + 112*sqrt(x)*a**4*b*c*d*x**2 + 210*sqrt(x)*a**4*b*d**2*x**3
- 80*sqrt(x)*a**3*b**2*c**2*x**2 - 672*sqrt(x)*a**3*b**2*c*d*x**3 + 840*sq
rt(x)*a**3*b**2*d**2*x**4 + 480*sqrt(x)*a**2*b**3*c**2*x**3 - 2688*sqrt(x)
*a**2*b**3*c*d*x**4 + 560*sqrt(x)*a**2*b**3*d**2*x**5 + 1920*sqrt(x)*a*b**
4*c**2*x**4 - 1792*sqrt(x)*a*b**4*c*d*x**5 + 1280*sqrt(x)*b**5*c**2*x**5))
/(105*sqrt(a + b*x)*a**6*x**4*(a + b*x))
```

3.184 $\int \frac{(c+dx)^2}{x^3(ax+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 289

$$\int \frac{(c+dx)^2}{x^3(ax+bx^2)^{5/2}}\,dx = -\frac{2c^2}{9ax^3(ax+bx^2)^{3/2}}+\frac{4c(2bc-3ad)}{21a^2x^2(ax+bx^2)^{3/2}} + \frac{2\left(21d^2+\frac{20bc(2bc-3ad)}{a^2}\right)}{63ax(ax+bx^2)^{3/2}}+\frac{16(21a^2d^2+20bc(2bc-3ad))}{63a^4x^2\sqrt{ax+bx^2}} - \frac{32(21a^2d^2+20bc(2bc-3ad))\sqrt{ax+bx^2}}{105a^5x^3} + \frac{128b(21a^2d^2+20bc(2bc-3ad))\sqrt{ax+bx^2}}{315a^6x^2} - \frac{256b^2(21a^2d^2+20bc(2bc-3ad))\sqrt{ax+bx^2}}{315a^7x}$$

output
```
-2/9*c^2/a/x^3/(b*x^2+a*x)^(3/2)+4/21*c*(-3*a*d+2*b*c)/a^2/x^2/(b*x^2+a*x)
^(3/2)+2/63*(21*d^2+20*b*c*(-3*a*d+2*b*c)/a^2)/a/x/(b*x^2+a*x)^(3/2)+16/63
*(21*a^2*d^2+20*b*c*(-3*a*d+2*b*c))/a^4/x^2/(b*x^2+a*x)^(1/2)-32/105*(21*a
^2*d^2+20*b*c*(-3*a*d+2*b*c))*(b*x^2+a*x)^(1/2)/a^5/x^3+128/315*b*(21*a^2*
d^2+20*b*c*(-3*a*d+2*b*c))*(b*x^2+a*x)^(1/2)/a^6/x^2-256/315*b^2*(21*a^2*d
^2+20*b*c*(-3*a*d+2*b*c))*(b*x^2+a*x)^(1/2)/a^7/x
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.65

$$\int \frac{(c+dx)^2}{x^3\left(ax+bx^2\right)^{5/2}}\,dx = -\frac{2(5120b^6c^2x^6+7680ab^5cx^5(c-dx)+384a^2b^4x^4(5c^2-30cdx+7d^2x^2)-12a^5bx(5c^2+15cdx+14d^2x^2)}{315a^7x^3(x(}$$

input
```
Integrate[(c + d*x)^2/(x^3*(a*x + b*x^2)^(5/2)),x]
```

output
```
(-2*(5120*b^6*c^2*x^6 + 7680*a*b^5*c*x^5*(c - d*x) + 384*a^2*b^4*x^4*(5*c^
2 - 30*c*d*x + 7*d^2*x^2) - 12*a^5*b*x*(5*c^2 + 15*c*d*x + 14*d^2*x^2) + 2
4*a^4*b^2*x^2*(5*c^2 + 20*c*d*x + 42*d^2*x^2) + 64*a^3*b^3*x^3*(-5*c^2 - 4
5*c*d*x + 63*d^2*x^2) + a^6*(35*c^2 + 90*c*d*x + 63*d^2*x^2)))/(315*a^7*x^
3*(x*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.73, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1262, 27, 1220, 1129, 1129, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{x^3\left(ax+bx^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \ 1262$$

$$-\frac{\int -\frac{10bc^2+d(20bc-7ad)x}{2x^3(bx^2+ax)^{5/2}}dx}{5b}-\frac{d^2}{5bx^2\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{10bc^2+d(20bc-7ad)x}{x^3(bx^2+ax)^{5/2}}dx}{10b}-\frac{d^2}{5bx^2\left(ax+bx^2\right)^{3/2}}$$

$\downarrow$ 1220

$$\frac{-\frac{(21a^2d^2-60abcd+40b^2c^2)\int \frac{1}{x^2\left(bx^2+ax\right)^{5/2}}dx}{3a}-\frac{20bc^2}{9ax^3(ax+bx^2)^{3/2}}}{10b}-\frac{d^2}{5bx^2\left(ax+bx^2\right)^{3/2}}$$

$\downarrow$ 1129

$$\frac{-\frac{(21a^2d^2-60abcd+40b^2c^2)\left(-\frac{10b\int \frac{1}{x\left(bx^2+ax\right)^{5/2}}dx}{7a}-\frac{2}{7ax^2\left(ax+bx^2\right)^{3/2}}\right)}{3a}-\frac{20bc^2}{9ax^3(ax+bx^2)^{3/2}}}{10b}-\frac{d^2}{5bx^2\left(ax+bx^2\right)^{3/2}}$$

$\downarrow$ 1129

$$\frac{-\frac{(21a^2d^2-60abcd+40b^2c^2)\left(-\frac{10b\left(-\frac{8b\int \frac{1}{\left(bx^2+ax\right)^{5/2}}dx}{5a}-\frac{2}{5ax\left(ax+bx^2\right)^{3/2}}\right)}{7a}-\frac{2}{7ax^2\left(ax+bx^2\right)^{3/2}}\right)}{3a}-\frac{20bc^2}{9ax^3(ax+bx^2)^{3/2}}}{10b}-\frac{d^2}{5bx^2\left(ax+bx^2\right)^{3/2}}$$

$\downarrow$ 1089

$$\frac{-\frac{(21a^2d^2-60abcd+40b^2c^2)\left(-\frac{10b\left(-\frac{8b\left(-\frac{8b\int \frac{1}{\left(bx^2+ax\right)^{3/2}}dx}{3a^2}-\frac{2(a+2bx)}{3a^2\left(ax+bx^2\right)^{3/2}}\right)}{5a}-\frac{2}{5ax\left(ax+bx^2\right)^{3/2}}\right)}{7a}-\frac{2}{7ax^2\left(ax+bx^2\right)^{3/2}}\right)}{3a}-\frac{20bc^2}{9ax^3(ax+bx}{10b}$$

$$\frac{d^2}{5bx^2\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1088}$$

$$-\frac{-\frac{\left(-\frac{10b\left(-\frac{8b\left(\frac{16b(a+2bx)}{3a^4\sqrt{ax+bx^2}}-\frac{2(a+2bx)}{3a^2\left(ax+bx^2\right)^{3/2}}\right)}{5a}-\frac{2}{5ax\left(ax+bx^2\right)^{3/2}}\right)}{7a}-\frac{2}{7ax^2\left(ax+bx^2\right)^{3/2}}\right)(21a^2d^2-60abcd+40b^2c^2)}{3a}-\frac{20bc^2}{9ax^3(ax+bx^2)^{3/2}}}{10b}-\frac{d^2}{5bx^2\left(ax+bx^2\right)^{3/2}}$$

input
```
Int[(c + d*x)^2/(x^3*(a*x + b*x^2)^(5/2)),x]
```

output
```
-1/5*d^2/(b*x^2*(a*x + b*x^2)^(3/2)) + ((-20*b*c^2)/(9*a*x^3*(a*x + b*x^2)
^(3/2)) - ((40*b^2*c^2 - 60*a*b*c*d + 21*a^2*d^2)*(-2/(7*a*x^2*(a*x + b*x^
2)^(3/2)) - (10*b*(-2/(5*a*x*(a*x + b*x^2)^(3/2)) - (8*b*((-2*(a + 2*b*x))
/(3*a^2*(a*x + b*x^2)^(3/2)) + (16*b*(a + 2*b*x))/(3*a^4*Sqrt[a*x + b*x^2]
)))/(5*a)))/(7*a)))/(3*a))/(10*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{\left(-126d^2x^2-180cdx-70c^2\right)a^6+120xb\left(\frac{14}{5}d^2x^2+3cdx+c^2\right)a^5-240x^2\left(\frac{42}{5}d^2x^2+4cdx+c^2\right)b^2a^4+640\left(-\frac{63}{5}d^2x^2+9cdx+c^2\right)x}{315\sqrt{x(bx+a)}\,x^4(bx+a)a^7}$
risch	$-\frac{2(bx+a)\left(1533a^2b^2d^2x^4-4740a\,b^3cd\,x^4+3335b^4c^2x^4-294a^3b\,d^2x^3+1110a^2b^2cd\,x^3-880a\,b^3c^2x^3+63a^4d^2x^2-360a^3dcb\right.}{315a^7x^4\sqrt{x(bx+a)}}$
gosper	$-\frac{2(bx+a)\left(2688a^2b^4d^2x^6-7680a\,b^5cd\,x^6+5120b^6c^2x^6+4032a^3b^3d^2x^5-11520a^2b^4cd\,x^5+7680a\,b^5c^2x^5+1008a^4b^2d^2x^4-\right.}{}$
orering	$-\frac{2(bx+a)\left(2688a^2b^4d^2x^6-7680a\,b^5cd\,x^6+5120b^6c^2x^6+4032a^3b^3d^2x^5-11520a^2b^4cd\,x^5+7680a\,b^5c^2x^5+1008a^4b^2d^2x^4-\right.}{}$
trager	$-\frac{2\left(2688a^2b^4d^2x^6-7680a\,b^5cd\,x^6+5120b^6c^2x^6+4032a^3b^3d^2x^5-11520a^2b^4cd\,x^5+7680a\,b^5c^2x^5+1008a^4b^2d^2x^4-2880a^3\right.}{}$
default	$c^2\left(-\frac{2}{9a\,x^3(b\,x^2+ax)^{\frac{3}{2}}}-\frac{4b\left(-\frac{2}{7a\,x^2\left(b\,x^2+ax\right)^{\frac{3}{2}}}-\frac{10b\left(-\frac{2}{5ax\left(b\,x^2+ax\right)^{\frac{3}{2}}}-\frac{8b\left(-\frac{2(2bx+a)}{3a^2\left(b\,x^2+ax\right)^{\frac{3}{2}}}+\frac{16b(2bx+a)}{3a^4\sqrt{b\,x^2+ax}}\right)}{5a}\right)}{7a}\right)}{3a}\right.$

input
```
int((d*x+c)^2/x^3/(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*((-126*d^2*x^2-180*c*d*x-70*c^2)*a^6+120*x*b*(14/5*d^2*x^2+3*c*d*x+c
^2)*a^5-240*x^2*(42/5*d^2*x^2+4*c*d*x+c^2)*b^2*a^4+640*(-63/5*d^2*x^2+9*c*
d*x+c^2)*x^3*b^3*a^3-3840*x^4*b^4*(7/5*d^2*x^2-6*c*d*x+c^2)*a^2-15360*b^5*
c*x^5*(-d*x+c)*a-10240*b^6*c^2*x^6)/(x*(b*x+a))^(1/2)/x^4/(b*x+a)/a^7
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx)^2}{x^3 (ax+bx^2)^{5/2}}\,dx =$$
$$-\frac{2\,(35\,a^6c^2+128\,(40\,b^6c^2-60\,ab^5cd+21\,a^2b^4d^2)x^6+192\,(40\,ab^5c^2-60\,a^2b^4cd+21\,a^3b^3d^2)x^5+48\,(40\,a$$

input
```
integrate((d*x+c)^2/x^3/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/315*(35*a^6*c^2 + 128*(40*b^6*c^2 - 60*a*b^5*c*d + 21*a^2*b^4*d^2)*x^6
+ 192*(40*a*b^5*c^2 - 60*a^2*b^4*c*d + 21*a^3*b^3*d^2)*x^5 + 48*(40*a^2*b^
4*c^2 - 60*a^3*b^3*c*d + 21*a^4*b^2*d^2)*x^4 - 8*(40*a^3*b^3*c^2 - 60*a^4*
b^2*c*d + 21*a^5*b*d^2)*x^3 + 3*(40*a^4*b^2*c^2 - 60*a^5*b*c*d + 21*a^6*d^
2)*x^2 - 30*(2*a^5*b*c^2 - 3*a^6*c*d)*x)*sqrt(b*x^2 + a*x)/(a^7*b^2*x^7 +
2*a^8*b*x^6 + a^9*x^5)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{x^3 (ax+bx^2)^{5/2}}\,dx = \int \frac{(c+dx)^2}{x^3 (x\,(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)**2/x**3/(b*x**2+a*x)**(5/2),x)
```

output
```
Integral((c + d*x)**2/(x**3*(x*(a + b*x))**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.38

$$
\begin{aligned}
\int \frac{(c+dx)^2}{x^3 (ax+bx^2)^{5/2}}\,dx &= \frac{256\,b^4c^2x}{63\,(bx^2+ax)^{\frac{3}{2}}a^5} - \frac{2048\,b^5c^2x}{63\,\sqrt{bx^2+ax}a^7} \\
&- \frac{128\,b^3cdx}{21\,(bx^2+ax)^{\frac{3}{2}}a^4} + \frac{1024\,b^4cdx}{21\,\sqrt{bx^2+ax}a^6} + \frac{32\,b^2d^2x}{15\,(bx^2+ax)^{\frac{3}{2}}a^3} \\
&- \frac{256\,b^3d^2x}{15\,\sqrt{bx^2+ax}a^5} + \frac{128\,b^3c^2}{63\,(bx^2+ax)^{\frac{3}{2}}a^4} - \frac{1024\,b^4c^2}{63\,\sqrt{bx^2+ax}a^6} \\
&- \frac{64\,b^2cd}{21\,(bx^2+ax)^{\frac{3}{2}}a^3} + \frac{512\,b^3cd}{21\,\sqrt{bx^2+ax}a^5} + \frac{16\,bd^2}{15\,(bx^2+ax)^{\frac{3}{2}}a^2} - \frac{128\,b^2d^2}{15\,\sqrt{bx^2+ax}a^4} \\
&- \frac{16\,b^2c^2}{21\,(bx^2+ax)^{\frac{3}{2}}a^3x} + \frac{8\,bcd}{7\,(bx^2+ax)^{\frac{3}{2}}a^2x} - \frac{2\,d^2}{5\,(bx^2+ax)^{\frac{3}{2}}ax} \\
&+ \frac{8\,bc^2}{21\,(bx^2+ax)^{\frac{3}{2}}a^2x^2} - \frac{4\,cd}{7\,(bx^2+ax)^{\frac{3}{2}}ax^2} - \frac{2\,c^2}{9\,(bx^2+ax)^{\frac{3}{2}}ax^3}
\end{aligned}
$$

input
```
integrate((d*x+c)^2/x^3/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

output
```
256/63*b^4*c^2*x/((b*x^2 + a*x)^(3/2)*a^5) - 2048/63*b^5*c^2*x/(sqrt(b*x^2
 + a*x)*a^7) - 128/21*b^3*c*d*x/((b*x^2 + a*x)^(3/2)*a^4) + 1024/21*b^4*c*
d*x/(sqrt(b*x^2 + a*x)*a^6) + 32/15*b^2*d^2*x/((b*x^2 + a*x)^(3/2)*a^3) -
256/15*b^3*d^2*x/(sqrt(b*x^2 + a*x)*a^5) + 128/63*b^3*c^2/((b*x^2 + a*x)^(
3/2)*a^4) - 1024/63*b^4*c^2/(sqrt(b*x^2 + a*x)*a^6) - 64/21*b^2*c*d/((b*x^
2 + a*x)^(3/2)*a^3) + 512/21*b^3*c*d/(sqrt(b*x^2 + a*x)*a^5) + 16/15*b*d^2
/((b*x^2 + a*x)^(3/2)*a^2) - 128/15*b^2*d^2/(sqrt(b*x^2 + a*x)*a^4) - 16/2
1*b^2*c^2/((b*x^2 + a*x)^(3/2)*a^3*x) + 8/7*b*c*d/((b*x^2 + a*x)^(3/2)*a^2
*x) - 2/5*d^2/((b*x^2 + a*x)^(3/2)*a*x) + 8/21*b*c^2/((b*x^2 + a*x)^(3/2)*
a^2*x^2) - 4/7*c*d/((b*x^2 + a*x)^(3/2)*a*x^2) - 2/9*c^2/((b*x^2 + a*x)^(3
/2)*a*x^3)
```

Giac [F]

$$\int \frac{(c+dx)^2}{x^3(ax+bx^2)^{5/2}}\,dx=\int \frac{(dx+c)^2}{(bx^2+ax)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((d*x+c)^2/x^3/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/((b*x^2 + a*x)^(5/2)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 9.79 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.31

$$\int \frac{(c+dx)^2}{x^3(ax+bx^2)^{5/2}}\,dx=\frac{\left(x\left(\frac{a\left(\frac{168\,a^2\,b^3\,d^2-960\,a\,b^4\,c\,d+920\,b^5\,c^2}{315\,a^6}-\frac{4\,b^3\,\left(273\,a^2\,d^2-1380\,a\,b\,c\,d+1235\,b^2\,c^2\right)}{315\,a^6}\right)}{b}+\frac{8\,b^2\,\left(147\,a^2\,d^2-555\,a\,b\right.}{315\,a^5}\right.\right.}{x^2}$$
$$-\frac{2\,c^2\,\sqrt{b\,x^2+a\,x}}{9\,a^3\,x^5}-\frac{\sqrt{b\,x^2+a\,x}\,\left(42\,a^4\,d^2-240\,a^3\,b\,c\,d+230\,a^2\,b^2\,c^2\right)}{105\,a^7\,x^3}$$
$$-\frac{\sqrt{b\,x^2+a\,x}\,\left(\frac{2688\,a^2\,b^2\,d^2-7680\,a\,b^3\,c\,d+5120\,b^4\,c^2}{315\,a^6}+\frac{2\,b\,x\,\left(2688\,a^2\,b^2\,d^2-7680\,a\,b^3\,c\,d+5120\,b^4\,c^2\right)}{315\,a^7}\right)}{x\,\left(a+b\,x\right)}$$
$$-\frac{4\,c\,\sqrt{b\,x^2+a\,x}\,\left(9\,a\,d-13\,b\,c\right)}{63\,a^4\,x^4}$$

input
```
int((c + d*x)^2/(x^3*(a*x + b*x^2)^(5/2)),x)
```

output
```
((x*((a*((920*b^5*c^2 + 168*a^2*b^3*d^2 - 960*a*b^4*c*d)/(315*a^6) - (4*b^
3*(273*a^2*d^2 + 1235*b^2*c^2 - 1380*a*b*c*d))/(315*a^6)))/b + (8*b^2*(147
*a^2*d^2 + 440*b^2*c^2 - 555*a*b*c*d))/(315*a^5) + (2*b^2*(273*a^2*d^2 + 1
235*b^2*c^2 - 1380*a*b*c*d))/(315*a^5)) + (4*b*(147*a^2*d^2 + 440*b^2*c^2
- 555*a*b*c*d))/(315*a^4))*(a*x + b*x^2)^(1/2))/(x^2*(a + b*x)^2) - (2*c^2
*(a*x + b*x^2)^(1/2))/(9*a^3*x^5) - ((a*x + b*x^2)^(1/2)*(42*a^4*d^2 + 230
*a^2*b^2*c^2 - 240*a^3*b*c*d))/(105*a^7*x^3) - ((a*x + b*x^2)^(1/2)*((5120
*b^4*c^2 + 2688*a^2*b^2*d^2 - 7680*a*b^3*c*d)/(315*a^6) + (2*b*x*(5120*b^4
*c^2 + 2688*a^2*b^2*d^2 - 7680*a*b^3*c*d))/(315*a^7)))/(x*(a + b*x)) - (4*
c*(a*x + b*x^2)^(1/2)*(9*a*d - 13*b*c))/(63*a^4*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.38

$$\int \frac{(c+dx)^2}{x^3 \left(ax+bx^2\right)^{5/2}}\,dx = \frac{\frac{256\sqrt{b}\,\sqrt{bx+a}\,a^3b^2d^2x^5}{15} - \frac{1024\sqrt{b}\,\sqrt{bx+a}\,a^2b^3cd\,x^5}{21} + \frac{256\sqrt{b}\,\sqrt{bx+a}\,a^2b^3d^2x^6}{15} + \frac{2048\sqrt{b}\,\sqrt{bx+a}\,a\,b^4c^2x^5}{63}}{}$$

input
```
int((d*x+c)^2/x^3/(b*x^2+a*x)^(5/2),x)
```

output
```
(2*(2688*sqrt(b)*sqrt(a + b*x)*a**3*b**2*d**2*x**5 - 7680*sqrt(b)*sqrt(a +
 b*x)*a**2*b**3*c*d*x**5 + 2688*sqrt(b)*sqrt(a + b*x)*a**2*b**3*d**2*x**6
+ 5120*sqrt(b)*sqrt(a + b*x)*a*b**4*c**2*x**5 - 7680*sqrt(b)*sqrt(a + b*x)
*a*b**4*c*d*x**6 + 5120*sqrt(b)*sqrt(a + b*x)*b**5*c**2*x**6 - 35*sqrt(x)*
a**6*c**2 - 90*sqrt(x)*a**6*c*d*x - 63*sqrt(x)*a**6*d**2*x**2 + 60*sqrt(x)
*a**5*b*c**2*x + 180*sqrt(x)*a**5*b*c*d*x**2 + 168*sqrt(x)*a**5*b*d**2*x**
3 - 120*sqrt(x)*a**4*b**2*c**2*x**2 - 480*sqrt(x)*a**4*b**2*c*d*x**3 - 100
8*sqrt(x)*a**4*b**2*d**2*x**4 + 320*sqrt(x)*a**3*b**3*c**2*x**3 + 2880*sqr
t(x)*a**3*b**3*c*d*x**4 - 4032*sqrt(x)*a**3*b**3*d**2*x**5 - 1920*sqrt(x)*
a**2*b**4*c**2*x**4 + 11520*sqrt(x)*a**2*b**4*c*d*x**5 - 2688*sqrt(x)*a**2
*b**4*d**2*x**6 - 7680*sqrt(x)*a*b**5*c**2*x**5 + 7680*sqrt(x)*a*b**5*c*d*
x**6 - 5120*sqrt(x)*b**6*c**2*x**6))/(315*sqrt(a + b*x)*a**7*x**5*(a + b*x
))
```

3.185 $\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{(ex)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 199

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{(ex)^{3/2}}\,dx = \frac{2c^3\sqrt{ax+bx^2}}{e\sqrt{ex}} + \frac{2d(3b^2c^2-3abcd+a^2d^2)\left(ax+bx^2\right)^{3/2}}{3b^3(ex)^{3/2}} + \frac{2d^2(3bc-2ad)e\left(ax+bx^2\right)^{5/2}}{5b^3(ex)^{5/2}} + \frac{2d^3e^2\left(ax+bx^2\right)^{7/2}}{7b^3(ex)^{7/2}} - \frac{2\sqrt{a}c^3\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{ex}}\right)}{e^{3/2}}$$

output

```
2*c^3*(b*x^2+a*x)^(1/2)/e/(e*x)^(1/2)+2/3*d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*
(b*x^2+a*x)^(3/2)/b^3/(e*x)^(3/2)+2/5*d^2*(-2*a*d+3*b*c)*e*(b*x^2+a*x)^(5/
2)/b^3/(e*x)^(5/2)+2/7*d^3*e^2*(b*x^2+a*x)^(7/2)/b^3/(e*x)^(7/2)-2*a^(1/2)
*c^3*arctanh(e^(1/2)*(b*x^2+a*x)^(1/2)/a^(1/2)/(e*x)^(1/2))/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.80

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{(ex)^{3/2}}\,dx = \frac{2(x(a+bx))^{3/2}\left(\sqrt{a+bx}(8a^3d^3-2a^2bd^2(21c+2dx)+3ab^2d(35c^2+7cdx+\right.}{105b^3(ex)}$$

input
```
Integrate[((c + d*x)^3*Sqrt[a*x + b*x^2])/(e*x)^(3/2),x]
```

output
```
(2*(x*(a + b*x))^(3/2)*(Sqrt[a + b*x]*(8*a^3*d^3 - 2*a^2*b*d^2*(21*c + 2*d
*x) + 3*a*b^2*d*(35*c^2 + 7*c*d*x + d^2*x^2) + 3*b^3*(35*c^3 + 35*c^2*d*x
+ 21*c*d^2*x^2 + 5*d^3*x^3)) - 105*Sqrt[a]*b^3*c^3*ArcTanh[Sqrt[a + b*x]/S
qrt[a]]))/(105*b^3*(e*x)^(3/2)*(a + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1262, 27, 2169, 27, 1221, 1131, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}(c+dx)^3}{(ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{1262}$$

$$\frac{2\int \frac{\sqrt{bx^2+ax}(7bc^3e^3+d^2(21bc-4ad)x^2e^3+21bc^2dxe^3)}{2(ex)^{3/2}}dx}{7be^3}+\frac{2d^3\sqrt{ex}\left(ax+bx^2\right)^{3/2}}{7be^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{bx^2+ax}(7bc^3e^3+d^2(21bc-4ad)x^2e^3+21bc^2dxe^3)}{(ex)^{3/2}}dx}{7be^3}+\frac{2d^3\sqrt{ex}\left(ax+bx^2\right)^{3/2}}{7be^2}$$

$$\Big\downarrow \text{2169}$$

$$\frac{\frac{2\int \frac{e^5\left(35b^2c^3+d\left(105b^2c^2-42abdc+8a^2d^2\right)x\right)\sqrt{bx^2+ax}}{2(ex)^{3/2}}dx}{5be^2}+\frac{2d^2e^2\left(ax+bx^2\right)^{3/2}(21bc-4ad)}{5b\sqrt{ex}}}{7be^3}+\frac{2d^3\sqrt{ex}\left(ax+bx^2\right)^{3/2}}{7be^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{e^3\int \frac{\left(35b^2c^3+d\left(105b^2c^2-42abdc+8a^2d^2\right)x\right)\sqrt{bx^2+ax}}{(ex)^{3/2}}dx}{5b}+\frac{2d^2e^2\left(ax+bx^2\right)^{3/2}(21bc-4ad)}{5b\sqrt{ex}}}{7be^3}+\frac{2d^3\sqrt{ex}\left(ax+bx^2\right)^{3/2}}{7be^2}$$

$$\Big\downarrow \; 1221$$

$$\frac{\frac{e^3\left(35b^2c^3\int \frac{\sqrt{bx^2+ax}}{(ex)^{3/2}}dx+\frac{2d\left(ax+bx^2\right)^{3/2}\left(8a^2d^2-42abcd+105b^2c^2\right)}{3b(ex)^{3/2}}\right)}{5b}+\frac{2d^2e^2\left(ax+bx^2\right)^{3/2}(21bc-4ad)}{5b\sqrt{ex}}}{7be^3}+\frac{2d^3\sqrt{ex}\left(ax+bx^2\right)^{3/2}}{7be^2}$$

$$\Big\downarrow \; 1131$$

$$\frac{\frac{e^3\left(35b^2c^3\left(\frac{a\int \frac{1}{\sqrt{ex}\sqrt{bx^2+ax}}dx}{e}+\frac{2\sqrt{ax+bx^2}}{e\sqrt{ex}}\right)+\frac{2d\left(ax+bx^2\right)^{3/2}\left(8a^2d^2-42abcd+105b^2c^2\right)}{3b(ex)^{3/2}}\right)}{5b}+\frac{2d^2e^2\left(ax+bx^2\right)^{3/2}(21bc-4ad)}{5b\sqrt{ex}}}{7be^3}+\frac{2d^3\sqrt{ex}\left(ax+bx^2\right)^{3/2}}{7be^2}$$

$$\Big\downarrow \; 1136$$

$$\frac{\frac{e^3\left(35b^2c^3\left(2a\int \frac{1}{\frac{e\left(bx^2+ax\right)}{x}-ae}d\frac{\sqrt{bx^2+ax}}{\sqrt{ex}}+\frac{2\sqrt{ax+bx^2}}{e\sqrt{ex}}\right)+\frac{2d\left(ax+bx^2\right)^{3/2}\left(8a^2d^2-42abcd+105b^2c^2\right)}{3b(ex)^{3/2}}\right)}{5b}+\frac{2d^2e^2\left(ax+bx^2\right)^{3/2}(21bc-4ad)}{5b\sqrt{ex}}}{7be^3}+\frac{2d^3\sqrt{ex}\left(ax+bx^2\right)^{3/2}}{7be^2}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{e^3\left(\frac{2d\left(ax+bx^2\right)^{3/2}\left(8a^2d^2-42abcd+105b^2c^2\right)}{3b(ex)^{3/2}}+35b^2c^3\left(\frac{2\sqrt{ax+bx^2}}{e\sqrt{ex}}-\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{ex}}\right)}{e^{3/2}}\right)\right)}{5b}+\frac{2d^2e^2\left(ax+bx^2\right)^{3/2}(21bc-4ad)}{5b\sqrt{ex}}}{7be^3}+\frac{2d^3\sqrt{ex}\left(ax+bx^2\right)^{3/2}}{7be^2}$$

input
```
Int[((c + d*x)^3*Sqrt[a*x + b*x^2])/(e*x)^(3/2),x]
```

output
```
(2*d^3*Sqrt[e*x]*(a*x + b*x^2)^(3/2))/(7*b*e^2) + ((2*d^2*(21*b*c - 4*a*d)
*e^2*(a*x + b*x^2)^(3/2))/(5*b*Sqrt[e*x]) + (e^3*((2*d*(105*b^2*c^2 - 42*a
*b*c*d + 8*a^2*d^2)*(a*x + b*x^2)^(3/2))/(3*b*(e*x)^(3/2)) + 35*b^2*c^3*((
2*Sqrt[a*x + b*x^2])/(e*Sqrt[e*x]) - (2*Sqrt[a]*ArcTanh[(Sqrt[e]*Sqrt[a*x
+ b*x^2])/(Sqrt[a]*Sqrt[e*x])])/e^(3/2))))/(5*b))/(7*b*e^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

rule 2169
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d -
b*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2
, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.54

method	result
default	$-\frac{2\sqrt{x(bx+a)}\left(-15b^3d^3x^3\sqrt{(bx+a)e}\,\sqrt{ae}-3a\,b^2d^3x^2\sqrt{(bx+a)e}\,\sqrt{ae}-63b^3c\,d^2x^2\sqrt{(bx+a)e}\,\sqrt{ae}+105a\,b^3c^3e\ \operatorname{arctanh}\left(\frac{\sqrt{(bx+a)}}{\sqrt{ae}}\right.\right.}{}$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(1/2)/(e*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*(x*(b*x+a))^(1/2)/e*(-15*b^3*d^3*x^3*((b*x+a)*e)^(1/2)*(a*e)^(1/2)-
3*a*b^2*d^3*x^2*((b*x+a)*e)^(1/2)*(a*e)^(1/2)-63*b^3*c*d^2*x^2*((b*x+a)*e)
^(1/2)*(a*e)^(1/2)+105*a*b^3*c^3*e*arctanh(((b*x+a)*e)^(1/2)/(a*e)^(1/2))+
4*a^2*b*d^3*x*((b*x+a)*e)^(1/2)*(a*e)^(1/2)-21*a*b^2*c*d^2*x*((b*x+a)*e)^(
1/2)*(a*e)^(1/2)-105*b^3*c^2*d*x*((b*x+a)*e)^(1/2)*(a*e)^(1/2)-8*a^3*d^3*(
(b*x+a)*e)^(1/2)*(a*e)^(1/2)+42*a^2*b*c*d^2*((b*x+a)*e)^(1/2)*(a*e)^(1/2)-
105*a*b^2*c^2*d*((b*x+a)*e)^(1/2)*(a*e)^(1/2)-105*b^3*c^3*((b*x+a)*e)^(1/2
)*(a*e)^(1/2))/(e*x)^(1/2)/((b*x+a)*e)^(1/2)/b^3/(a*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.87

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{(ex)^{3/2}}\,dx = \left[\frac{105\,b^3c^3ex\sqrt{\frac{a}{e}}\log\left(-\frac{bx^2+2\,ax-2\,\sqrt{bx^2+ax}\sqrt{ex}\sqrt{\frac{a}{e}}}{x^2}\right)+2\,(15\,b^3d^3x^3+105\,b^3c^3+\cdots}{\cdots}\right.$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/(e*x)^(3/2),x, algorithm="fricas")
```

output
```
[1/105*(105*b^3*c^3*e*x*sqrt(a/e)*log(-(b*x^2 + 2*a*x - 2*sqrt(b*x^2 + a*x
)*sqrt(e*x)*sqrt(a/e))/x^2) + 2*(15*b^3*d^3*x^3 + 105*b^3*c^3 + 105*a*b^2*
c^2*d - 42*a^2*b*c*d^2 + 8*a^3*d^3 + 3*(21*b^3*c*d^2 + a*b^2*d^3)*x^2 + (1
05*b^3*c^2*d + 21*a*b^2*c*d^2 - 4*a^2*b*d^3)*x)*sqrt(b*x^2 + a*x)*sqrt(e*x
))/(b^3*e^2*x), 2/105*(105*b^3*c^3*e*x*sqrt(-a/e)*arctan(sqrt(b*x^2 + a*x)
*sqrt(e*x)*sqrt(-a/e)/(a*x)) + (15*b^3*d^3*x^3 + 105*b^3*c^3 + 105*a*b^2*c
^2*d - 42*a^2*b*c*d^2 + 8*a^3*d^3 + 3*(21*b^3*c*d^2 + a*b^2*d^3)*x^2 + (10
5*b^3*c^2*d + 21*a*b^2*c*d^2 - 4*a^2*b*d^3)*x)*sqrt(b*x^2 + a*x)*sqrt(e*x)
)/(b^3*e^2*x)]
```

Sympy [F]

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{(ex)^{3/2}}\,dx = \int \frac{\sqrt{x\,(a+bx)}(c+dx)^3}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**3*(b*x**2+a*x)**(1/2)/(e*x)**(3/2),x)
```

output
```
Integral(sqrt(x*(a + b*x))*(c + d*x)**3/(e*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{(ex)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+ax}(dx+c)^3}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/(e*x)^(3/2),x, algorithm="maxima")
```

output
```
c^3*integrate(sqrt(b*x + a)/x, x)/e^(3/2) + 2/105*(105*b^3*c^2*d*x^3 + 105
*a*b^2*c^2*d*x^2 + (15*b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 - 4*a^2*b*d^3*x + 8*a
^3*d^3)*x^2 + 21*(3*b^3*c*d^2*x^3 + a*b^2*c*d^2*x^2 - 2*a^2*b*c*d^2*x)*x)*
sqrt(b*x + a)/(b^3*e^(3/2)*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(165) = 330.

Time = 0.22 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.71

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{(ex)^{3/2}}\,dx = \frac{2\left(\frac{\frac{105\,ac^3e^4|e|\arctan\left(\frac{\sqrt{bex+ae}}{\sqrt{-ae}}\right)}{\sqrt{-ae}}+\frac{105\,\sqrt{bex+ae}b^{21}c^3e^3|e|+105\,(bex+ae)^{\frac{3}{2}}b^{20}c^2de^2|e|-105\,(bex+ae)^{\frac{3}{2}}ab^{19}cd^2 \cdots}{\cdots}}{e \cdots}\right.}{\cdots}$$

input
```
integrate((d*x+c)^3*(b*x^2+a*x)^(1/2)/(e*x)^(3/2),x, algorithm="giac")
```

output
```
2/105*((105*a*c^3*e^4*abs(e)*arctan(sqrt(b*e*x + a*e)/sqrt(-a*e))/sqrt(-a*
e) + (105*sqrt(b*e*x + a*e)*b^21*c^3*e^3*abs(e) + 105*(b*e*x + a*e)^(3/2)*
b^20*c^2*d*e^2*abs(e) - 105*(b*e*x + a*e)^(3/2)*a*b^19*c*d^2*e^2*abs(e) +
35*(b*e*x + a*e)^(3/2)*a^2*b^18*d^3*e^2*abs(e) + 63*(b*e*x + a*e)^(5/2)*b^
19*c*d^2*e*abs(e) - 42*(b*e*x + a*e)^(5/2)*a*b^18*d^3*e*abs(e) + 15*(b*e*x
 + a*e)^(7/2)*b^18*d^3*abs(e))/b^21)/e^5 - (105*a*b^3*c^3*e*abs(e)*arctan(
sqrt(a*e)/sqrt(-a*e)) + 105*sqrt(a*e)*sqrt(-a*e)*b^3*c^3*abs(e) + 105*sqrt
(a*e)*sqrt(-a*e)*a*b^2*c^2*d*abs(e) - 42*sqrt(a*e)*sqrt(-a*e)*a^2*b*c*d^2*
abs(e) + 8*sqrt(a*e)*sqrt(-a*e)*a^3*d^3*abs(e))/(sqrt(-a*e)*b^3*e^2))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{(ex)^{3/2}}\,dx=\int \frac{\sqrt{b\,x^2+a\,x}\,(c+d\,x)^3}{(e\,x)^{3/2}}\,dx$$

input
```
int(((a*x + b*x^2)^(1/2)*(c + d*x)^3)/(e*x)^(3/2),x)
```

output
```
int(((a*x + b*x^2)^(1/2)*(c + d*x)^3)/(e*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)^3\sqrt{ax+bx^2}}{(ex)^{3/2}}\,dx=\frac{\sqrt{e}\left(16\sqrt{bx+a}\,a^3d^3-84\sqrt{bx+a}\,a^2bc\,d^2-8\sqrt{bx+a}\,a^2b\,d^3x+210\sqrt{bx+a}\right.}{}$$

input
```
int((d*x+c)^3*(b*x^2+a*x)^(1/2)/(e*x)^(3/2),x)
```

output
```
(sqrt(e)*(16*sqrt(a + b*x)*a**3*d**3 - 84*sqrt(a + b*x)*a**2*b*c*d**2 - 8*
sqrt(a + b*x)*a**2*b*d**3*x + 210*sqrt(a + b*x)*a*b**2*c**2*d + 42*sqrt(a
+ b*x)*a*b**2*c*d**2*x + 6*sqrt(a + b*x)*a*b**2*d**3*x**2 + 210*sqrt(a + b
*x)*b**3*c**3 + 210*sqrt(a + b*x)*b**3*c**2*d*x + 126*sqrt(a + b*x)*b**3*c
*d**2*x**2 + 30*sqrt(a + b*x)*b**3*d**3*x**3 + 105*sqrt(a)*log(sqrt(a + b*
x) - sqrt(a))*b**3*c**3 - 105*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**3*c*
*3))/(105*b**3*e**2)
```

3.186 $\int \frac{(ex)^{9/2}}{(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 185

$$\int \frac{(ex)^{9/2}}{(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx = \frac{2a^3e^4\sqrt{ex}}{b^3(bc-ad)\sqrt{ax+bx^2}} - \frac{2(bc+2ad)e^5\sqrt{ax+bx^2}}{b^3d^2\sqrt{ex}} + \frac{2e^6\left(ax+bx^2\right)^{3/2}}{3b^3d(ex)^{3/2}} + \frac{2c^3e^{9/2}\arctan\left(\frac{\sqrt{d}\sqrt{e}\sqrt{ax+bx^2}}{\sqrt{bc-ad}\sqrt{ex}}\right)}{d^{5/2}(bc-ad)^{3/2}}$$

output
```
2*a^3*e^4*(e*x)^(1/2)/b^3/(-a*d+b*c)/(b*x^2+a*x)^(1/2)-2*(2*a*d+b*c)*e^5*(
b*x^2+a*x)^(1/2)/b^3/d^2/(e*x)^(1/2)+2/3*e^6*(b*x^2+a*x)^(3/2)/b^3/d/(e*x)
^(3/2)+2*c^3*e^(9/2)*arctan(d^(1/2)*e^(1/2)*(b*x^2+a*x)^(1/2)/(-a*d+b*c)^(
1/2)/(e*x)^(1/2))/d^(5/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.91

$$\int \frac{(ex)^{9/2}}{(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx = \frac{2e^4\sqrt{ex}\left(-\sqrt{d}\sqrt{bc-ad}\left(-8a^3d^2+2a^2bd(c-2dx)+b^3cx(3c-dx)+ab^2(3\right.\right.}{3b^3d^{5/2}(bc-ad)^{3/2}\sqrt{x(a}}$$

input
```
Integrate[(e*x)^(9/2)/((c + d*x)*(a*x + b*x^2)^(3/2)),x]
```

output
```
(2*e^4*Sqrt[e*x]*(-(Sqrt[d]*Sqrt[b*c - a*d]*(-8*a^3*d^2 + 2*a^2*b*d*(c - 2
*d*x) + b^3*c*x*(3*c - d*x) + a*b^2*(3*c^2 + c*d*x + d^2*x^2))) + 3*b^3*c^
3*Sqrt[a + b*x]*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]]))/(3*b^3*d
^(5/2)*(b*c - a*d)^(3/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1261, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{9/2}}{(ax+bx^2)^{3/2}(c+dx)}dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{(ex)^{9/2}(a+bx)^{3/2}\int \frac{x^3}{(a+bx)^{3/2}(c+dx)}dx}{x^3(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{98}$$

$$\frac{(ex)^{9/2}(a+bx)^{3/2}\int \left(-\frac{a^3}{b^2(bc-ad)(a+bx)^{3/2}}+\frac{-bc-ad}{b^2d^2\sqrt{a+bx}}+\frac{x}{bd\sqrt{a+bx}}-\frac{c^3}{d^2(ad-bc)\sqrt{a+bx}(c+dx)}\right)dx}{x^3(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ex)^{9/2}(a+bx)^{3/2}\left(\frac{2a^3}{b^3\sqrt{a+bx}(bc-ad)}+\frac{2c^3\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{5/2}(bc-ad)^{3/2}}-\frac{2\sqrt{a+bx}(ad+bc)}{b^3d^2}-\frac{2a\sqrt{a+bx}}{b^3d}+\frac{2(a+bx)^{3/2}}{3b^3d}\right)}{x^3(ax+bx^2)^{3/2}}$$

input
```
Int[(e*x)^(9/2)/((c + d*x)*(a*x + b*x^2)^(3/2)),x]
```

output
```
((e*x)^(9/2)*(a + b*x)^(3/2)*((2*a^3)/(b^3*(b*c - a*d)*Sqrt[a + b*x]) - (2
*a*Sqrt[a + b*x])/(b^3*d) - (2*(b*c + a*d)*Sqrt[a + b*x])/(b^3*d^2) + (2*(
a + b*x)^(3/2))/(3*b^3*d) + (2*c^3*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c
 - a*d]])/(d^(5/2)*(b*c - a*d)^(3/2))))/(x^3*(a*x + b*x^2)^(3/2))
```

Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{2(-bdx+5ad+3bc)(bx+a)e^{5}x}{3b^{3}d^{2}\sqrt{ex}\,\sqrt{x(bx+a)}}+\frac{2\left(-\frac{a^{3}d^{2}}{(ad-bc)\sqrt{bex+ae}}+\frac{b^{3}c^{3}\,\mathrm{arctanh}\left(\frac{d\sqrt{bex+ae}}{\sqrt{e(ad-bc)d}}\right)}{(ad-bc)\sqrt{e(ad-bc)d}}\right)e^{5}\sqrt{(bx+a)e}\,x}{b^{3}d^{2}\sqrt{ex}\,\sqrt{x(bx+a)}}$
default	$\frac{2e^{4}\sqrt{ex}\,\sqrt{x(bx+a)}\left(3\sqrt{(bx+a)e}\,\mathrm{arctanh}\left(\frac{d\sqrt{(bx+a)e}}{\sqrt{e(ad-bc)d}}\right)b^{3}c^{3}+\sqrt{e(ad-bc)d}\,a\,b^{2}d^{2}x^{2}-\sqrt{e(ad-bc)d}\,b^{3}cd\,x^{2}-4\sqrt{e(ad-bc)d}\,a^{2}b\,d^{2}}{3x(bx+a)b^{3}d^{2}(ad-bc)\sqrt{e(}}$

input
```
int((e*x)^(9/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-b*d*x+5*a*d+3*b*c)*(b*x+a)/b^3/d^2*e^5/(e*x)^(1/2)/(x*(b*x+a))^(1/2
)*x+2/b^3/d^2*(-a^3*d^2/(a*d-b*c)/(b*e*x+a*e)^(1/2)+b^3*c^3/(a*d-b*c)/(e*(
a*d-b*c)*d)^(1/2)*arctanh(d*(b*e*x+a*e)^(1/2)/(e*(a*d-b*c)*d)^(1/2)))*e^5*
((b*x+a)*e)^(1/2)/(e*x)^(1/2)/(x*(b*x+a))^(1/2)*x
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 539, normalized size of antiderivative = 2.91

$$\int \frac{(ex)^{9/2}}{(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx = \left[-\frac{3\left(b^4c^3e^4x^2+ab^3c^3e^4x\right)\sqrt{-\frac{e}{bcd-ad^2}}\log\left(-\frac{bdex^2-(bc-2\,ad)ex-2\left(bcd-ad^2\right)\sqrt{bx^2+}}{dx^2+cx}\right.}{}\right.$$

input
```
integrate((e*x)^(9/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/3*(3*(b^4*c^3*e^4*x^2 + a*b^3*c^3*e^4*x)*sqrt(-e/(b*c*d - a*d^2))*log(
-(b*d*e*x^2 - (b*c - 2*a*d)*e*x - 2*(b*c*d - a*d^2)*sqrt(b*x^2 + a*x)*sqrt
(e*x)*sqrt(-e/(b*c*d - a*d^2)))/(d*x^2 + c*x)) - 2*((b^3*c*d - a*b^2*d^2)*
e^4*x^2 - (3*b^3*c^2 + a*b^2*c*d - 4*a^2*b*d^2)*e^4*x - (3*a*b^2*c^2 + 2*a
^2*b*c*d - 8*a^3*d^2)*e^4)*sqrt(b*x^2 + a*x)*sqrt(e*x))/((b^5*c*d^2 - a*b^
4*d^3)*x^2 + (a*b^4*c*d^2 - a^2*b^3*d^3)*x), 2/3*(3*(b^4*c^3*e^4*x^2 + a*b
^3*c^3*e^4*x)*sqrt(e/(b*c*d - a*d^2))*arctan(-sqrt(b*x^2 + a*x)*(b*c - a*d
)*sqrt(e*x)*sqrt(e/(b*c*d - a*d^2))/(b*e*x^2 + a*e*x)) + ((b^3*c*d - a*b^2
*d^2)*e^4*x^2 - (3*b^3*c^2 + a*b^2*c*d - 4*a^2*b*d^2)*e^4*x - (3*a*b^2*c^2
 + 2*a^2*b*c*d - 8*a^3*d^2)*e^4)*sqrt(b*x^2 + a*x)*sqrt(e*x))/((b^5*c*d^2
- a*b^4*d^3)*x^2 + (a*b^4*c*d^2 - a^2*b^3*d^3)*x)]
```

Sympy [F]

$$\int \frac{(ex)^{9/2}}{(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx = \int \frac{(ex)^{\frac{9}{2}}}{\left(x\left(a+bx\right)\right)^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate((e*x)**(9/2)/(d*x+c)/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral((e*x)**(9/2)/((x*(a + b*x))**(3/2)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(ex)^{9/2}}{(c+dx)\,(ax+bx^2)^{3/2}}\,dx=\int \frac{(ex)^{\frac{9}{2}}}{(bx^2+ax)^{\frac{3}{2}}(dx+c)}\,dx$$

input
```
integrate((e*x)^(9/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
2/3*((b^3*d*e^(9/2)*x^2 - a*b^2*d*e^(9/2)*x - 2*a^2*b*d*e^(9/2))*x^3 - 2*(
a^2*b*c*e^(9/2) + a^3*d*e^(9/2) + (b^3*c*e^(9/2) + a*b^2*d*e^(9/2))*x^2 +
2*(a*b^2*c*e^(9/2) + a^2*b*d*e^(9/2))*x)*x^2)/((b^4*d^2*x^3 + a*b^3*c*d*x
+ (b^4*c*d + a*b^3*d^2)*x^2)*sqrt(b*x + a)) + integrate(2/3*((4*a^3*b*c*d*
e^(9/2) + (3*b^4*c^2*e^(9/2) + 4*a*b^3*c*d*e^(9/2) + 3*a^2*b^2*d^2*e^(9/2)
)*x^2 + (3*a*b^3*c^2*e^(9/2) + 8*a^2*b^2*c*d*e^(9/2) + 3*a^3*b*d^2*e^(9/2)
)*x)*x^3 + (2*a^3*b*c^2*e^(9/2) + 2*a^4*c*d*e^(9/2) + 5*(a*b^3*c^2*e^(9/2)
 + a^2*b^2*c*d*e^(9/2))*x^2 + 7*(a^2*b^2*c^2*e^(9/2) + a^3*b*c*d*e^(9/2))*
x)*x^2)/((b^5*d^3*x^6 + a^2*b^3*c^2*d*x^2 + 2*(b^5*c*d^2 + a*b^4*d^3)*x^5
+ (b^5*c^2*d + 4*a*b^4*c*d^2 + a^2*b^3*d^3)*x^4 + 2*(a*b^4*c^2*d + a^2*b^3
*c*d^2)*x^3)*sqrt(b*x + a)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(155) = 310$.

Time = 0.25 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.04

$$\int \frac{(ex)^{9/2}}{(c+dx)\,(ax+bx^2)^{3/2}}\,dx=\frac{2}{3}\left(\frac{3\,c^3e^2\arctan\left(\frac{\sqrt{bex+aed}}{\sqrt{bcde-ad^2e}}\right)}{(bcd^2|e|-ad^3|e|)\sqrt{bcde-ad^2e}}+\frac{3\,a^3e^2}{(b^4c|e|-ab^3d|e|)\sqrt{bex+ae}}-\frac{3\,\sqrt{be}}{}\right.$$
$$-\frac{2\left(3\,\sqrt{ae}b^3c^3e^6\arctan\left(\frac{\sqrt{aed}}{\sqrt{bcde-ad^2e}}\right)-3\,\sqrt{bcde-ad^2e}eab^2c^2e^6-2\,\sqrt{bcde-ad^2e}ea^2bcde^6+8\,\sqrt{bcde-ad^2}\right.}{3\left(\sqrt{bcde-ad^2e}\sqrt{ae}b^4cd^2|e|-\sqrt{bcde-ad^2e}\sqrt{ae}ab^3d^3|e|\right)}$$

input
```
integrate((e*x)^(9/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
2/3*(3*c^3*e^2*arctan(sqrt(b*e*x + a*e)*d/sqrt(b*c*d*e - a*d^2*e))/((b*c*d
^2*abs(e) - a*d^3*abs(e))*sqrt(b*c*d*e - a*d^2*e)) + 3*a^3*e^2/((b^4*c*abs
(e) - a*b^3*d*abs(e))*sqrt(b*e*x + a*e)) - (3*sqrt(b*e*x + a*e)*b^7*c*d*e^
3 + 6*sqrt(b*e*x + a*e)*a*b^6*d^2*e^3 - (b*e*x + a*e)^(3/2)*b^6*d^2*e^2)/(
b^9*d^3*e^2*abs(e)))*e^4 - 2/3*(3*sqrt(a*e)*b^3*c^3*e^6*arctan(sqrt(a*e)*d
/sqrt(b*c*d*e - a*d^2*e)) - 3*sqrt(b*c*d*e - a*d^2*e)*a*b^2*c^2*e^6 - 2*sq
rt(b*c*d*e - a*d^2*e)*a^2*b*c*d*e^6 + 8*sqrt(b*c*d*e - a*d^2*e)*a^3*d^2*e^
6)/(sqrt(b*c*d*e - a*d^2*e)*sqrt(a*e)*b^4*c*d^2*abs(e) - sqrt(b*c*d*e - a*
d^2*e)*sqrt(a*e)*a*b^3*d^3*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{9/2}}{(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx=\int \frac{{\left(e\,x\right)}^{9/2}}{{\left(b\,x^2+a\,x\right)}^{3/2}\,\left(c+d\,x\right)}\,dx$$

input
```
int((e*x)^(9/2)/((a*x + b*x^2)^(3/2)*(c + d*x)),x)
```

output
```
int((e*x)^(9/2)/((a*x + b*x^2)^(3/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.19

$$\int \frac{(ex)^{9/2}}{(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx=\frac{2\sqrt{e}\,e^4\left(3\sqrt{d}\,\sqrt{bx+a}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)b^3c^3-8a^4d^4+10a^3bc\ldots}{3\ldots}$$

input
```
int((e*x)^(9/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x)
```

output
```
(2*sqrt(e)*e**4*(3*sqrt(d)*sqrt(a + b*x)*sqrt( - a*d + b*c)*atan((sqrt(a +
 b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*b**3*c**3 - 8*a**4*d**4 + 10*a**3*b
*c*d**3 - 4*a**3*b*d**4*x + a**2*b**2*c**2*d**2 + 5*a**2*b**2*c*d**3*x + a
**2*b**2*d**4*x**2 - 3*a*b**3*c**3*d + 2*a*b**3*c**2*d**2*x - 2*a*b**3*c*d
**3*x**2 - 3*b**4*c**3*d*x + b**4*c**2*d**2*x**2))/(3*sqrt(a + b*x)*b**3*d
**3*(a**2*d**2 - 2*a*b*c*d + b**2*c**2))
```

3.187 $\int \frac{(ex)^{7/2}}{(c+dx)(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 144

$$\int \frac{(ex)^{7/2}}{(c+dx)(ax+bx^2)^{3/2}}\,dx = -\frac{2a^2e^3\sqrt{ex}}{b^2(bc-ad)\sqrt{ax+bx^2}} + \frac{2e^4\sqrt{ax+bx^2}}{b^2d\sqrt{ex}} - \frac{2c^2e^{7/2}\arctan\left(\frac{\sqrt{d}\sqrt{e}\sqrt{ax+bx^2}}{\sqrt{bc-ad}\sqrt{ex}}\right)}{d^{3/2}(bc-ad)^{3/2}}$$

output
```
-2*a^2*e^3*(e*x)^(1/2)/b^2/(-a*d+b*c)/(b*x^2+a*x)^(1/2)+2*e^4*(b*x^2+a*x)^
(1/2)/b^2/d/(e*x)^(1/2)-2*c^2*e^(7/2)*arctan(d^(1/2)*e^(1/2)*(b*x^2+a*x)^(
1/2)/(-a*d+b*c)^(1/2)/(e*x)^(1/2))/d^(3/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90

$$\int \frac{(ex)^{7/2}}{(c+dx)(ax+bx^2)^{3/2}}\,dx = \frac{2e^3\sqrt{ex}\left(\sqrt{d}\sqrt{bc-ad}(-2a^2d+b^2cx+ab(c-dx)) - b^2c^2\sqrt{a+bx}\arctan\left(\right.\right.}{b^2d^{3/2}(bc-ad)^{3/2}\sqrt{x(a+bx)}}$$

input
```
Integrate[(e*x)^(7/2)/((c + d*x)*(a*x + b*x^2)^(3/2)),x]
```

output
```
(2*e^3*Sqrt[e*x]*(Sqrt[d]*Sqrt[b*c - a*d]*(-2*a^2*d + b^2*c*x + a*b*(c - d
*x)) - b^2*c^2*Sqrt[a + b*x]*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d
]]))/(b^2*d^(3/2)*(b*c - a*d)^(3/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1261, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{7/2}}{(ax+bx^2)^{3/2}(c+dx)}dx$$

$$\Big\downarrow \text{ 1261}$$

$$\frac{(ex)^{7/2}(a+bx)^{3/2}\int \frac{x^2}{(a+bx)^{3/2}(c+dx)}dx}{x^2(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 98}$$

$$\frac{(ex)^{7/2}(a+bx)^{3/2}\int \left(\frac{a^2}{b(bc-ad)(a+bx)^{3/2}}+\frac{1}{bd\sqrt{a+bx}}+\frac{c^2}{d(ad-bc)\sqrt{a+bx}(c+dx)}\right)dx}{x^2(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(ex)^{7/2}(a+bx)^{3/2}\left(-\frac{2a^2}{b^2\sqrt{a+bx}(bc-ad)}-\frac{2c^2\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{3/2}(bc-ad)^{3/2}}+\frac{2\sqrt{a+bx}}{b^2d}\right)}{x^2(ax+bx^2)^{3/2}}$$

input
```
Int[(e*x)^(7/2)/((c + d*x)*(a*x + b*x^2)^(3/2)),x]
```

output
```
((e*x)^(7/2)*(a + b*x)^(3/2)*((-2*a^2)/(b^2*(b*c - a*d)*Sqrt[a + b*x]) + (
2*Sqrt[a + b*x])/(b^2*d) - (2*c^2*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c
- a*d]])/(d^(3/2)*(b*c - a*d)^(3/2))))/(x^2*(a*x + b*x^2)^(3/2))
```

Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.05

method	result
risch	$\frac{2(bx+a)e^4x}{d\,b^2\sqrt{ex}\,\sqrt{x(bx+a)}}-\frac{2\left(-\frac{a^2d}{(ad-bc)\sqrt{bex+ae}}+\frac{b^2c^2\,\mathrm{arctanh}\left(\frac{d\sqrt{bex+ae}}{\sqrt{e(ad-bc)d}}\right)}{(ad-bc)\sqrt{e(ad-bc)d}}\right)e^4\sqrt{(bx+a)e}\,x}{b^2d\sqrt{ex}\,\sqrt{x(bx+a)}}$
default	$-\frac{2e^3\sqrt{ex}\,\sqrt{x(bx+a)}\left(\sqrt{(bx+a)e}\,\mathrm{arctanh}\left(\frac{d\sqrt{(bx+a)e}}{\sqrt{e(ad-bc)d}}\right)b^2c^2-\sqrt{e(ad-bc)d}\,abdx+\sqrt{e(ad-bc)d}\,b^2cx-2\sqrt{e(ad-bc)d}\,a^2d+\sqrt{e(a}\right.}{x(bx+a)b^2d(ad-bc)\sqrt{e(ad-bc)d}}$

input
```
int((e*x)^(7/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/b^2*(b*x+a)*e^4/(e*x)^(1/2)/(x*(b*x+a))^(1/2)*x-2/b^2/d*(-a^2*d/(a*d-b
*c)/(b*e*x+a*e)^(1/2)+b^2*c^2/(a*d-b*c)/(e*(a*d-b*c)*d)^(1/2)*arctanh(d*(b
*e*x+a*e)^(1/2)/(e*(a*d-b*c)*d)^(1/2)))*e^4*((b*x+a)*e)^(1/2)/(e*x)^(1/2)/
(x*(b*x+a))^(1/2)*x
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.93

$$\int \frac{(ex)^{7/2}}{(c+dx)\,(ax+bx^2)^{3/2}}\,dx = \left[-\frac{(b^3c^2e^3x^2+ab^2c^2e^3x)\sqrt{-\frac{e}{bcd-ad^2}}\log\left(-\frac{bdex^2-(bc-2\,ad)ex+2\,(bcd-ad^2)\sqrt{bx^2+a}}{dx^2+cx}\right.}{(b^4cd-ab^3d^2)x^2+}\right.$$

input
```
integrate((e*x)^(7/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[-((b^3*c^2*e^3*x^2 + a*b^2*c^2*e^3*x)*sqrt(-e/(b*c*d - a*d^2))*log(-(b*d*
e*x^2 - (b*c - 2*a*d)*e*x + 2*(b*c*d - a*d^2)*sqrt(b*x^2 + a*x)*sqrt(e*x)*
sqrt(-e/(b*c*d - a*d^2)))/(d*x^2 + c*x)) - 2*((b^2*c - a*b*d)*e^3*x + (a*b
*c - 2*a^2*d)*e^3)*sqrt(b*x^2 + a*x)*sqrt(e*x))/((b^4*c*d - a*b^3*d^2)*x^2
 + (a*b^3*c*d - a^2*b^2*d^2)*x), -2*((b^3*c^2*e^3*x^2 + a*b^2*c^2*e^3*x)*s
qrt(e/(b*c*d - a*d^2))*arctan(-sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(e*x)*sqr
t(e/(b*c*d - a*d^2))/(b*e*x^2 + a*e*x)) - ((b^2*c - a*b*d)*e^3*x + (a*b*c
- 2*a^2*d)*e^3)*sqrt(b*x^2 + a*x)*sqrt(e*x))/((b^4*c*d - a*b^3*d^2)*x^2 +
(a*b^3*c*d - a^2*b^2*d^2)*x)]
```

Sympy [F]

$$\int \frac{(ex)^{7/2}}{(c+dx)\,(ax+bx^2)^{3/2}}\,dx = \int \frac{(ex)^{\frac{7}{2}}}{(x\,(a+bx))^{\frac{3}{2}}\,(c+dx)}\,dx$$

input
```
integrate((e*x)**(7/2)/(d*x+c)/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral((e*x)**(7/2)/((x*(a + b*x))**(3/2)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(ex)^{7/2}}{(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx = \int \frac{(ex)^{\frac{7}{2}}}{\left(bx^2+ax\right)^{\frac{3}{2}}(dx+c)}\,dx$$

input
```
integrate((e*x)^(7/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
2*(b*e^(7/2)*x + a*e^(7/2))*x^2/((b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)*s
qrt(b*x + a)) - integrate(2*((a*b*c*e^(7/2) + a^2*d*e^(7/2) + (b^2*c*e^(7/
2) + a*b*d*e^(7/2))*x)*x^3 + 2*(a*b*c*e^(7/2)*x + a^2*c*e^(7/2))*x^2)/((b^
3*d^2*x^5 + a^2*b*c^2*x + 2*(b^3*c*d + a*b^2*d^2)*x^4 + (b^3*c^2 + 4*a*b^2
*c*d + a^2*b*d^2)*x^3 + 2*(a*b^2*c^2 + a^2*b*c*d)*x^2)*sqrt(b*x + a)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(120) = 240$.

Time = 0.18 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.93

$$\int \frac{(ex)^{7/2}}{(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx = 2\,e^3\left(\frac{\sqrt{ae}b^2c^2e^2\arctan\left(\frac{\sqrt{ae}d}{\sqrt{bcde-ad^2e}}\right) - \sqrt{bcde-ad^2e}eabce^2 + 2\sqrt{bcde-ad^2}}{\sqrt{bcde-ad^2e}\sqrt{ae}b^3cd|e| - \sqrt{bcde-ad^2e}\sqrt{ae}ab^2d^2|e|}\right.$$

input
```
integrate((e*x)^(7/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
2*e^3*((sqrt(a*e)*b^2*c^2*e^2*arctan(sqrt(a*e)*d/sqrt(b*c*d*e - a*d^2*e))
- sqrt(b*c*d*e - a*d^2*e)*a*b*c*e^2 + 2*sqrt(b*c*d*e - a*d^2*e)*a^2*d*e^2)
/(sqrt(b*c*d*e - a*d^2*e)*sqrt(a*e)*b^3*c*d*abs(e) - sqrt(b*c*d*e - a*d^2*
e)*sqrt(a*e)*a*b^2*d^2*abs(e)) - (b^2*c^2*e*arctan(sqrt(b*e*x + a*e)*d/sqr
t(b*c*d*e - a*d^2*e))/(sqrt(b*c*d*e - a*d^2*e)*(b*c*d*abs(e) - a*d^2*abs(e
))) + a^2*e/(sqrt(b*e*x + a*e)*(b*c*abs(e) - a*d*abs(e))) - sqrt(b*e*x + a
*e)/(d*abs(e)))*e/b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}}{(c+dx)\,(ax+bx^2)^{3/2}}\,dx = \int \frac{(e\,x)^{7/2}}{(b\,x^2+a\,x)^{3/2}\,(c+d\,x)}\,dx$$

input
```
int((e*x)^(7/2)/((a*x + b*x^2)^(3/2)*(c + d*x)),x)
```

output
```
int((e*x)^(7/2)/((a*x + b*x^2)^(3/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.04

$$\int \frac{(ex)^{7/2}}{(c+dx)\,(ax+bx^2)^{3/2}}\,dx = \frac{2\sqrt{e}\,e^3\left(-\sqrt{d}\,\sqrt{bx+a}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)b^2c^2+2a^3d^3-3a^2bc\,d}{\sqrt{bx+a}\,b^2d^2\,(a^2d^2-2abcd+b^2c}$$

input
```
int((e*x)^(7/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x)
```

output
```
(2*sqrt(e)*e**3*( - sqrt(d)*sqrt(a + b*x)*sqrt( - a*d + b*c)*atan((sqrt(a
+ b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*b**2*c**2 + 2*a**3*d**3 - 3*a**2*b
*c*d**2 + a**2*b*d**3*x + a*b**2*c**2*d - 2*a*b**2*c*d**2*x + b**3*c**2*d*
x))/(sqrt(a + b*x)*b**2*d**2*(a**2*d**2 - 2*a*b*c*d + b**2*c**2))
```

3.188 $\int \frac{(ex)^{5/2}}{(c+dx)(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 109

$$\int \frac{(ex)^{5/2}}{(c+dx)(ax+bx^2)^{3/2}}\,dx=\frac{2ae^2\sqrt{ex}}{b(bc-ad)\sqrt{ax+bx^2}}+\frac{2ce^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{e}\sqrt{ax+bx^2}}{\sqrt{bc-ad}\sqrt{ex}}\right)}{\sqrt{d}(bc-ad)^{3/2}}$$

output
```
2*a*e^2*(e*x)^(1/2)/b/(-a*d+b*c)/(b*x^2+a*x)^(1/2)+2*c*e^(5/2)*arctan(d^(1
/2)*e^(1/2)*(b*x^2+a*x)^(1/2)/(-a*d+b*c)^(1/2)/(e*x)^(1/2))/d^(1/2)/(-a*d+
b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \frac{(ex)^{5/2}}{(c+dx)(ax+bx^2)^{3/2}}\,dx=\frac{2e^2\sqrt{ex}\left(a\sqrt{d}\sqrt{bc-ad}+bc\sqrt{a+bx}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)\right)}{b\sqrt{d}(bc-ad)^{3/2}\sqrt{x(a+bx)}}$$

input
```
Integrate[(e*x)^(5/2)/((c + d*x)*(a*x + b*x^2)^(3/2)),x]
```

output
```
(2*e^2*Sqrt[e*x]*(a*Sqrt[d]*Sqrt[b*c - a*d] + b*c*Sqrt[a + b*x]*ArcTan[(Sq
rt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]]))/(b*Sqrt[d]*(b*c - a*d)^(3/2)*Sqrt[
x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1261, 87, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2}}{(ax+bx^2)^{3/2}(c+dx)}dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{(ex)^{5/2}(a+bx)^{3/2}\int \frac{x}{(a+bx)^{3/2}(c+dx)}dx}{x(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{87}$$

$$\frac{(ex)^{5/2}(a+bx)^{3/2}\left(\frac{c\int \frac{1}{\sqrt{a+bx}(c+dx)}dx}{bc-ad}+\frac{2a}{b\sqrt{a+bx}(bc-ad)}\right)}{x(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{(ex)^{5/2}(a+bx)^{3/2}\left(\frac{2c\int \frac{1}{c-\frac{ad}{b}+\frac{d(a+bx)}{b}}d\sqrt{a+bx}}{b(bc-ad)}+\frac{2a}{b\sqrt{a+bx}(bc-ad)}\right)}{x(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{(ex)^{5/2}(a+bx)^{3/2}\left(\frac{2c\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{\sqrt{d}(bc-ad)^{3/2}}+\frac{2a}{b\sqrt{a+bx}(bc-ad)}\right)}{x(ax+bx^2)^{3/2}}$$

input
```
Int[(e*x)^(5/2)/((c + d*x)*(a*x + b*x^2)^(3/2)),x]
```

output
```
((e*x)^(5/2)*(a + b*x)^(3/2)*((2*a)/(b*(b*c - a*d)*Sqrt[a + b*x]) + (2*c*A
rcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(Sqrt[d]*(b*c - a*d)^(3/2)
)))/(x*(a*x + b*x^2)^(3/2))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{2e^2\sqrt{ex}\,\sqrt{x(bx+a)}\left(bc\,\operatorname{arctanh}\left(\frac{d\sqrt{(bx+a)e}}{\sqrt{e(ad-bc)d}}\right)\sqrt{(bx+a)e}-a\sqrt{e(ad-bc)d}\right)}{x(bx+a)b(ad-bc)\sqrt{e(ad-bc)d}}$	110

input
```
int((e*x)^(5/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*e^2/x*(e*x)^(1/2)*(x*(b*x+a))^(1/2)*(b*c*arctanh(d*((b*x+a)*e)^(1/2)/(e*
(a*d-b*c)*d)^(1/2))*((b*x+a)*e)^(1/2)-a*(e*(a*d-b*c)*d)^(1/2))/(b*x+a)/b/(
a*d-b*c)/(e*(a*d-b*c)*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 337, normalized size of antiderivative = 3.09

$$\int \frac{(ex)^{5/2}}{(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx = \left[\frac{2\sqrt{bx^2+ax}\sqrt{ex}ae^2-(b^2ce^2x^2+abce^2x)\sqrt{-\frac{e}{bcd-ad^2}}\log\left(-\frac{bdex^2-(bc-2\,ad)}{}\right.}{(b^3c-ab^2d)x^2+(ab^2c-a^2bd)x}\right.$$

input
```
integrate((e*x)^(5/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[(2*sqrt(b*x^2 + a*x)*sqrt(e*x)*a*e^2 - (b^2*c*e^2*x^2 + a*b*c*e^2*x)*sqrt
(-e/(b*c*d - a*d^2))*log(-(b*d*e*x^2 - (b*c - 2*a*d)*e*x - 2*(b*c*d - a*d^
2)*sqrt(b*x^2 + a*x)*sqrt(e*x)*sqrt(-e/(b*c*d - a*d^2)))/(d*x^2 + c*x)))/(
(b^3*c - a*b^2*d)*x^2 + (a*b^2*c - a^2*b*d)*x), 2*(sqrt(b*x^2 + a*x)*sqrt(
e*x)*a*e^2 + (b^2*c*e^2*x^2 + a*b*c*e^2*x)*sqrt(e/(b*c*d - a*d^2))*arctan(
-sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(e*x)*sqrt(e/(b*c*d - a*d^2))/(b*e*x^2
+ a*e*x)))/((b^3*c - a*b^2*d)*x^2 + (a*b^2*c - a^2*b*d)*x)]
```

Sympy [F]

$$\int \frac{(ex)^{5/2}}{(c+dx)(ax+bx^2)^{3/2}}\,dx = \int \frac{(ex)^{\frac{5}{2}}}{(x(a+bx))^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((e*x)**(5/2)/(d*x+c)/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral((e*x)**(5/2)/((x*(a + b*x))**(3/2)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(ex)^{5/2}}{(c+dx)(ax+bx^2)^{3/2}}\,dx = \int \frac{(ex)^{\frac{5}{2}}}{(bx^2+ax)^{\frac{3}{2}}(dx+c)}\,dx$$

input
```
integrate((e*x)^(5/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^(5/2)/((b*x^2 + a*x)^(3/2)*(d*x + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(89) = 178.

Time = 0.16 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.93

$$\int \frac{(ex)^{5/2}}{(c+dx)(ax+bx^2)^{3/2}}\,dx = \frac{2\left(\frac{bc\arctan\left(\frac{\sqrt{bex+aed}}{\sqrt{bcde-ad^2e}}\right)}{\sqrt{bcde-ad^2e}(bc|e|-ad|e|)} + \frac{a}{\sqrt{bex+ae}(bc|e|-ad|e|)}\right)e^4}{b} - \frac{2\left(\sqrt{ae}bce^4\arctan\left(\frac{\sqrt{aed}}{\sqrt{bcde-ad^2e}}\right) + \sqrt{bcde-ad^2e}ae^4\right)}{\sqrt{bcde-ad^2e}\sqrt{ae}b^2c|e| - \sqrt{bcde-ad^2e}\sqrt{ae}abd|e|}$$

input
```
integrate((e*x)^(5/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
2*(b*c*arctan(sqrt(b*e*x + a*e)*d/sqrt(b*c*d*e - a*d^2*e))/(sqrt(b*c*d*e -
 a*d^2*e)*(b*c*abs(e) - a*d*abs(e))) + a/(sqrt(b*e*x + a*e)*(b*c*abs(e) -
a*d*abs(e))))*e^4/b - 2*(sqrt(a*e)*b*c*e^4*arctan(sqrt(a*e)*d/sqrt(b*c*d*e
 - a*d^2*e)) + sqrt(b*c*d*e - a*d^2*e)*a*e^4)/(sqrt(b*c*d*e - a*d^2*e)*sqr
t(a*e)*b^2*c*abs(e) - sqrt(b*c*d*e - a*d^2*e)*sqrt(a*e)*a*b*d*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}}{(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx=\int \frac{{\left(e\,x\right)}^{5/2}}{{\left(b\,x^2+a\,x\right)}^{3/2}\,\left(c+d\,x\right)}\,dx$$

input
```
int((e*x)^(5/2)/((a*x + b*x^2)^(3/2)*(c + d*x)),x)
```

output
```
int((e*x)^(5/2)/((a*x + b*x^2)^(3/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \frac{(ex)^{5/2}}{(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx=\frac{2\sqrt{e}\,e^2\left(\sqrt{d}\,\sqrt{bx+a}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)bc-a^2d^2+abcd\right)}{\sqrt{bx+a}\,bd\left(a^2d^2-2abcd+b^2c^2\right)}$$

input
```
int((e*x)^(5/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x)
```

output
```
(2*sqrt(e)*e**2*(sqrt(d)*sqrt(a + b*x)*sqrt( - a*d + b*c)*atan((sqrt(a + b
*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*b*c - a**2*d**2 + a*b*c*d))/(sqrt(a +
 b*x)*b*d*(a**2*d**2 - 2*a*b*c*d + b**2*c**2))
```

3.189 $\int \frac{(ex)^{3/2}}{(c+dx)(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 102

$$\int \frac{(ex)^{3/2}}{(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx=-\frac{2e\sqrt{ex}}{(bc-ad)\sqrt{ax+bx^2}}-\frac{2\sqrt{d}e^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{e}\sqrt{ax+bx^2}}{\sqrt{bc-ad}\sqrt{ex}}\right)}{(bc-ad)^{3/2}}$$

output
```
-2*e*(e*x)^(1/2)/(-a*d+b*c)/(b*x^2+a*x)^(1/2)-2*d^(1/2)*e^(3/2)*arctan(d^(
1/2)*e^(1/2)*(b*x^2+a*x)^(1/2)/(-a*d+b*c)^(1/2)/(e*x)^(1/2))/(-a*d+b*c)^(3
/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.87

$$\int \frac{(ex)^{3/2}}{(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx=-\frac{2e\sqrt{ex}\left(\sqrt{bc-ad}+\sqrt{d}\sqrt{a+bx}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)\right)}{(bc-ad)^{3/2}\sqrt{x(a+bx)}}$$

input
```
Integrate[(e*x)^(3/2)/((c + d*x)*(a*x + b*x^2)^(3/2)),x]
```

output
```
(-2*e*Sqrt[e*x]*(Sqrt[b*c - a*d] + Sqrt[d]*Sqrt[a + b*x]*ArcTan[(Sqrt[d]*S
qrt[a + b*x])/Sqrt[b*c - a*d]]))/((b*c - a*d)^(3/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1261, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}}{(ax+bx^2)^{3/2}(c+dx)}dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{(ex)^{3/2}(a+bx)^{3/2}\int \frac{1}{(a+bx)^{3/2}(c+dx)}dx}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{(ex)^{3/2}(a+bx)^{3/2}\left(-\frac{d\int \frac{1}{\sqrt{a+bx}(c+dx)}dx}{bc-ad}-\frac{2}{\sqrt{a+bx}(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{(ex)^{3/2}(a+bx)^{3/2}\left(-\frac{2d\int \frac{1}{c-\frac{ad}{b}+\frac{d(a+bx)}{b}}d\sqrt{a+bx}}{b(bc-ad)}-\frac{2}{\sqrt{a+bx}(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{(ex)^{3/2}(a+bx)^{3/2}\left(-\frac{2\sqrt{d}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}-\frac{2}{\sqrt{a+bx}(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

input
```
Int[(e*x)^(3/2)/((c + d*x)*(a*x + b*x^2)^(3/2)),x]
```

output
```
((e*x)^(3/2)*(a + b*x)^(3/2)*(-2/((b*c - a*d)*Sqrt[a + b*x]) - (2*Sqrt[d]*
ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2)))/(a*x
+ b*x^2)^(3/2)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.01

method	result	size
default	$-\frac{2e\sqrt{ex}\,\sqrt{x(bx+a)}\left(d\,\operatorname{arctanh}\left(\frac{d\sqrt{(bx+a)e}}{\sqrt{e(ad-bc)d}}\right)\sqrt{(bx+a)e}-\sqrt{e(ad-bc)d}\right)}{x(bx+a)(ad-bc)\sqrt{e(ad-bc)d}}$	103

input
```
int((e*x)^(3/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*e/x*(e*x)^(1/2)*(x*(b*x+a))^(1/2)*(d*arctanh(d*((b*x+a)*e)^(1/2)/(e*(a*
d-b*c)*d)^(1/2))*((b*x+a)*e)^(1/2)-(e*(a*d-b*c)*d)^(1/2))/(b*x+a)/(a*d-b*c
)/(e*(a*d-b*c)*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.88

$$\int \frac{(ex)^{3/2}}{(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx = \left[-\frac{\left(bex^2+aex\right)\sqrt{-\frac{de}{bc-ad}}\log\left(-\frac{bdex^2-(bc-2\,ad)ex+2\,\sqrt{bx^2+ax}(bc-ad)\sqrt{ex}\sqrt{-\frac{de}{bc-ad}}}{dx^2+cx}\right.}{(b^2c-abd)x^2+(abc-a^2d)x}\right.$$

input
```
integrate((e*x)^(3/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[-((b*e*x^2 + a*e*x)*sqrt(-d*e/(b*c - a*d))*log(-(b*d*e*x^2 - (b*c - 2*a*d
)*e*x + 2*sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(e*x)*sqrt(-d*e/(b*c - a*d)))/
(d*x^2 + c*x)) + 2*sqrt(b*x^2 + a*x)*sqrt(e*x)*e)/((b^2*c - a*b*d)*x^2 + (
a*b*c - a^2*d)*x), -2*((b*e*x^2 + a*e*x)*sqrt(d*e/(b*c - a*d))*arctan(-sqr
t(b*x^2 + a*x)*(b*c - a*d)*sqrt(e*x)*sqrt(d*e/(b*c - a*d))/(b*d*e*x^2 + a*
d*e*x)) + sqrt(b*x^2 + a*x)*sqrt(e*x)*e)/((b^2*c - a*b*d)*x^2 + (a*b*c - a
^2*d)*x)]
```

Sympy [F]

$$\int \frac{(ex)^{3/2}}{(c+dx)\,(ax+bx^2)^{3/2}}\,dx = \int \frac{(ex)^{\frac{3}{2}}}{(x\,(a+bx))^{\frac{3}{2}}\,(c+dx)}\,dx$$

input
```
integrate((e*x)**(3/2)/(d*x+c)/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral((e*x)**(3/2)/((x*(a + b*x))**(3/2)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(ex)^{3/2}}{(c+dx)\,(ax+bx^2)^{3/2}}\,dx = \int \frac{(ex)^{\frac{3}{2}}}{(bx^2+ax)^{\frac{3}{2}}(dx+c)}\,dx$$

input
```
integrate((e*x)^(3/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^(3/2)/((b*x^2 + a*x)^(3/2)*(d*x + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(82) = 164$.

Time = 0.14 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.00

$$\int \frac{(ex)^{3/2}}{(c+dx)\,(ax+bx^2)^{3/2}}\,dx = -2\,e^4\left(\frac{d\arctan\left(\frac{\sqrt{bex+aed}}{\sqrt{bcde-ad^2e}}\right)}{\sqrt{bcde-ad^2e}(bce|e|-ade|e|)} + \frac{1}{(bce|e|-ade|e|)\sqrt{bex+ae}}\right) + \frac{2\left(\sqrt{ae}de^3\arctan\left(\frac{\sqrt{ae}d}{\sqrt{bcde-ad^2e}}\right)+\sqrt{bcde-ad^2e}e^3\right)}{\sqrt{bcde-ad^2e}\sqrt{ae}bc|e|-\sqrt{bcde-ad^2e}\sqrt{ae}ad|e|}$$

input
```
integrate((e*x)^(3/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
-2*e^4*(d*arctan(sqrt(b*e*x + a*e)*d/sqrt(b*c*d*e - a*d^2*e))/(sqrt(b*c*d*
e - a*d^2*e)*(b*c*e*abs(e) - a*d*e*abs(e))) + 1/((b*c*e*abs(e) - a*d*e*abs
(e))*sqrt(b*e*x + a*e))) + 2*(sqrt(a*e)*d*e^3*arctan(sqrt(a*e)*d/sqrt(b*c*
d*e - a*d^2*e)) + sqrt(b*c*d*e - a*d^2*e)*e^3)/(sqrt(b*c*d*e - a*d^2*e)*sq
rt(a*e)*b*c*abs(e) - sqrt(b*c*d*e - a*d^2*e)*sqrt(a*e)*a*d*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}}{(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx = \int \frac{(e\,x)^{3/2}}{\left(b\,x^2+a\,x\right)^{3/2}\,(c+d\,x)}\,dx$$

input
```
int((e*x)^(3/2)/((a*x + b*x^2)^(3/2)*(c + d*x)),x)
```

output
```
int((e*x)^(3/2)/((a*x + b*x^2)^(3/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.85

$$\int \frac{(ex)^{3/2}}{(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx = \frac{2\sqrt{e}\,e\left(-\sqrt{d}\,\sqrt{bx+a}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)+ad-bc\right)}{\sqrt{bx+a}\,\left(a^2d^2-2abcd+b^2c^2\right)}$$

input
```
int((e*x)^(3/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x)
```

output
```
(2*sqrt(e)*e*( - sqrt(d)*sqrt(a + b*x)*sqrt( - a*d + b*c)*atan((sqrt(a + b
*x)*d)/(sqrt(d)*sqrt( - a*d + b*c))) + a*d - b*c))/(sqrt(a + b*x)*(a**2*d*
*2 - 2*a*b*c*d + b**2*c**2))
```

3.190 $\int \frac{\sqrt{ex}}{(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 155

$$\int \frac{\sqrt{ex}}{(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx = \frac{2b\sqrt{ex}}{a(bc-ad)\sqrt{ax+bx^2}} + \frac{2d^{3/2}\sqrt{e}\arctan\left(\frac{\sqrt{d}\sqrt{e}\sqrt{ax+bx^2}}{\sqrt{bc-ad}\sqrt{ex}}\right)}{c(bc-ad)^{3/2}} - \frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{ex}}\right)}{a^{3/2}c}$$

output
```
2*b*(e*x)^(1/2)/a/(-a*d+b*c)/(b*x^2+a*x)^(1/2)+2*d^(3/2)*e^(1/2)*arctan(d^
(1/2)*e^(1/2)*(b*x^2+a*x)^(1/2)/(-a*d+b*c)^(1/2)/(e*x)^(1/2))/c/(-a*d+b*c)
^(3/2)-2*e^(1/2)*arctanh(e^(1/2)*(b*x^2+a*x)^(1/2)/a^(1/2)/(e*x)^(1/2))/a^
(3/2)/c
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{ex}}{(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx = \frac{2\sqrt{ex}\left(\sqrt{a}\left(bc\sqrt{bc-ad}+ad^{3/2}\sqrt{a+bx}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)\right)-(bc-ad)^{3/2}\cdots\right.}{a^{3/2}c(bc-ad)^{3/2}\sqrt{x(a+bx)}}$$

input
```
Integrate[Sqrt[e*x]/((c + d*x)*(a*x + b*x^2)^(3/2)),x]
```

output
```
(2*Sqrt[e*x]*(Sqrt[a]*(b*c*Sqrt[b*c - a*d] + a*d^(3/2)*Sqrt[a + b*x]*ArcTa
n[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]]) - (b*c - a*d)^(3/2)*Sqrt[a + b
*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(a^(3/2)*c*(b*c - a*d)^(3/2)*Sqrt[x*(
a + b*x)])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1261, 96, 25, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}}{(ax+bx^2)^{3/2}(c+dx)}dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{x\sqrt{ex}(a+bx)^{3/2}\int \frac{1}{x(a+bx)^{3/2}(c+dx)}dx}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{96}$$

$$\frac{x\sqrt{ex}(a+bx)^{3/2}\left(\frac{2b}{a\sqrt{a+bx}(bc-ad)}-\frac{\int -\frac{bc-ad+bdx}{x\sqrt{a+bx}(c+dx)}dx}{a(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{x\sqrt{ex}(a+bx)^{3/2}\left(\frac{\int \frac{bc-ad+bdx}{x\sqrt{a+bx}(c+dx)}dx}{a(bc-ad)}+\frac{2b}{a\sqrt{a+bx}(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{174}$$

$$\frac{x\sqrt{ex}(a+bx)^{3/2}\left(\frac{\frac{ad^2\int \frac{1}{\sqrt{a+bx}(c+dx)}dx}{c}+\frac{(bc-ad)\int \frac{1}{x\sqrt{a+bx}}dx}{c}}{a(bc-ad)}+\frac{2b}{a\sqrt{a+bx}(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{x\sqrt{ex}(a+bx)^{3/2}\left(\frac{\frac{2ad^2\int\frac{1}{c-\frac{ad}{b}+\frac{d(a+bx)}{b}}d\sqrt{a+bx}}{bc}+\frac{2(bc-ad)\int\frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}}{a(bc-ad)}+\frac{2b}{a\sqrt{a+bx}(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \textbf{218}$$

$$\frac{x\sqrt{ex}(a+bx)^{3/2}\left(\frac{\frac{2(bc-ad)\int\frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}+\frac{2ad^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{a(bc-ad)}+\frac{2b}{a\sqrt{a+bx}(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{x\sqrt{ex}(a+bx)^{3/2}\left(\frac{\frac{2ad^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)(bc-ad)}{\sqrt{a}c}}{a(bc-ad)}+\frac{2b}{a\sqrt{a+bx}(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

input
```
Int[Sqrt[e*x]/((c + d*x)*(a*x + b*x^2)^(3/2)),x]
```

output
```
(x*Sqrt[e*x]*(a + b*x)^(3/2)*((2*b)/(a*(b*c - a*d)*Sqrt[a + b*x]) + ((2*a*
d^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(c*Sqrt[b*c - a*d
]) - (2*(b*c - a*d)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*c))/(a*(b*c -
 a*d))))/(a*x + b*x^2)^(3/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] && !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.36

method	result
default	$\frac{2\sqrt{ex}\,\sqrt{x(bx+a)}\left(d^2\operatorname{arctanh}\left(\frac{d\sqrt{(bx+a)e}}{\sqrt{e(ad-bc)d}}\right)a\sqrt{ae}\,\sqrt{(bx+a)e}-\operatorname{arctanh}\left(\frac{\sqrt{(bx+a)e}}{\sqrt{ae}}\right)\sqrt{e(ad-bc)d}\,\sqrt{(bx+a)e}\,ad+\operatorname{arctanh}\left(\frac{\sqrt{(bx+a}}{\sqrt{ae}}\right.}{x(bx+a)ac\sqrt{ae}\,(ad-bc)\sqrt{e(ad-bc)d}}$

input
```
int((e*x)^(1/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(e*x)^(1/2)*(x*(b*x+a))^(1/2)*(d^2*arctanh(d*((b*x+a)*e)^(1/2)/(e*(a*d-b
*c)*d)^(1/2))*a*(a*e)^(1/2)*((b*x+a)*e)^(1/2)-arctanh(((b*x+a)*e)^(1/2)/(a
*e)^(1/2))*(e*(a*d-b*c)*d)^(1/2)*((b*x+a)*e)^(1/2)*a*d+arctanh(((b*x+a)*e)
^(1/2)/(a*e)^(1/2))*(e*(a*d-b*c)*d)^(1/2)*((b*x+a)*e)^(1/2)*b*c-b*c*(a*e)^
(1/2)*(e*(a*d-b*c)*d)^(1/2))/x/(b*x+a)/a/c/(a*e)^(1/2)/(a*d-b*c)/(e*(a*d-b
*c)*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 967, normalized size of antiderivative = 6.24

$$\int \frac{\sqrt{ex}}{(c+dx)\,(ax+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x)^(1/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[(2*sqrt(b*x^2 + a*x)*sqrt(e*x)*b*c - (a*b*d*x^2 + a^2*d*x)*sqrt(-d*e/(b*c
 - a*d))*log(-(b*d*e*x^2 - (b*c - 2*a*d)*e*x - 2*sqrt(b*x^2 + a*x)*(b*c -
a*d)*sqrt(e*x)*sqrt(-d*e/(b*c - a*d)))/(d*x^2 + c*x)) + ((b^2*c - a*b*d)*x
^2 + (a*b*c - a^2*d)*x)*sqrt(e/a)*log(-(b*e*x^2 + 2*a*e*x - 2*sqrt(b*x^2 +
 a*x)*sqrt(e*x)*a*sqrt(e/a))/x^2))/((a*b^2*c^2 - a^2*b*c*d)*x^2 + (a^2*b*c
^2 - a^3*c*d)*x), (2*sqrt(b*x^2 + a*x)*sqrt(e*x)*b*c + 2*((b^2*c - a*b*d)*
x^2 + (a*b*c - a^2*d)*x)*sqrt(-e/a)*arctan(sqrt(b*x^2 + a*x)*sqrt(e*x)*a*s
qrt(-e/a)/(b*e*x^2 + a*e*x)) - (a*b*d*x^2 + a^2*d*x)*sqrt(-d*e/(b*c - a*d)
)*log(-(b*d*e*x^2 - (b*c - 2*a*d)*e*x - 2*sqrt(b*x^2 + a*x)*(b*c - a*d)*sq
rt(e*x)*sqrt(-d*e/(b*c - a*d)))/(d*x^2 + c*x)))/((a*b^2*c^2 - a^2*b*c*d)*x
^2 + (a^2*b*c^2 - a^3*c*d)*x), (2*sqrt(b*x^2 + a*x)*sqrt(e*x)*b*c + 2*(a*b
*d*x^2 + a^2*d*x)*sqrt(d*e/(b*c - a*d))*arctan(-sqrt(b*x^2 + a*x)*(b*c - a
*d)*sqrt(e*x)*sqrt(d*e/(b*c - a*d))/(b*d*e*x^2 + a*d*e*x)) + ((b^2*c - a*b
*d)*x^2 + (a*b*c - a^2*d)*x)*sqrt(e/a)*log(-(b*e*x^2 + 2*a*e*x - 2*sqrt(b*
x^2 + a*x)*sqrt(e*x)*a*sqrt(e/a))/x^2))/((a*b^2*c^2 - a^2*b*c*d)*x^2 + (a^
2*b*c^2 - a^3*c*d)*x), 2*(sqrt(b*x^2 + a*x)*sqrt(e*x)*b*c + (a*b*d*x^2 + a
^2*d*x)*sqrt(d*e/(b*c - a*d))*arctan(-sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(e
*x)*sqrt(d*e/(b*c - a*d))/(b*d*e*x^2 + a*d*e*x)) + ((b^2*c - a*b*d)*x^2 +
(a*b*c - a^2*d)*x)*sqrt(-e/a)*arctan(sqrt(b*x^2 + a*x)*sqrt(e*x)*a*sqrt(-e
/a)/(b*e*x^2 + a*e*x)))/((a*b^2*c^2 - a^2*b*c*d)*x^2 + (a^2*b*c^2 - a^3...
```

Sympy [F]

$$\int \frac{\sqrt{ex}}{(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{ex}}{\left(x\left(a+bx\right)\right)^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate((e*x)**(1/2)/(d*x+c)/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral(sqrt(e*x)/((x*(a + b*x))**(3/2)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ex}}{(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{ex}}{\left(bx^2+ax\right)^{\frac{3}{2}}\left(dx+c\right)}\,dx$$

input
```
integrate((e*x)^(1/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x)/((b*x^2 + a*x)^(3/2)*(d*x + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(123) = 246.

Time = 0.32 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.41

$$\int \frac{\sqrt{ex}}{(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx=2\left(\frac{d^2\arctan\left(\frac{\sqrt{bex+aed}}{\sqrt{bcde-ad^2e}}\right)}{\left(bc^2e^2|e|-acde^2|e|\right)\sqrt{bcde-ad^2e}}+\frac{b}{\left(abce^2|e|-a^2de^2|e|\right)\sqrt{bex+ae}}+\right.$$
$$-\frac{2\left(\sqrt{ae}\sqrt{-ae}ad^2e^2\arctan\left(\frac{\sqrt{ae}d}{\sqrt{bcde-ad^2e}}\right)+\sqrt{bcde-ad^2e}\sqrt{ae}bce^2\arctan\left(\frac{\sqrt{ae}}{\sqrt{-ae}}\right)-\sqrt{bcde-ad^2e}\sqrt{ae}ade\right.}{\sqrt{bcde-ad^2e}\sqrt{ae}\sqrt{-ae}abc^2|e|-\sqrt{bcde-ad^2e}\sqrt{ae}\sqrt{-ae}a^2cd}$$

input
```
integrate((e*x)^(1/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
2*(d^2*arctan(sqrt(b*e*x + a*e)*d/sqrt(b*c*d*e - a*d^2*e))/((b*c^2*e^2*abs
(e) - a*c*d*e^2*abs(e))*sqrt(b*c*d*e - a*d^2*e)) + b/((a*b*c*e^2*abs(e) -
a^2*d*e^2*abs(e))*sqrt(b*e*x + a*e)) + arctan(sqrt(b*e*x + a*e)/sqrt(-a*e)
)/(sqrt(-a*e)*a*c*e^2*abs(e)))*e^4 - 2*(sqrt(a*e)*sqrt(-a*e)*a*d^2*e^2*arc
tan(sqrt(a*e)*d/sqrt(b*c*d*e - a*d^2*e)) + sqrt(b*c*d*e - a*d^2*e)*sqrt(a*
e)*b*c*e^2*arctan(sqrt(a*e)/sqrt(-a*e)) - sqrt(b*c*d*e - a*d^2*e)*sqrt(a*e
)*a*d*e^2*arctan(sqrt(a*e)/sqrt(-a*e)) + sqrt(b*c*d*e - a*d^2*e)*sqrt(-a*e
)*b*c*e^2)/(sqrt(b*c*d*e - a*d^2*e)*sqrt(a*e)*sqrt(-a*e)*a*b*c^2*abs(e) -
sqrt(b*c*d*e - a*d^2*e)*sqrt(a*e)*sqrt(-a*e)*a^2*c*d*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}}{(c+dx)\,(ax+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{e\,x}}{(b\,x^2+a\,x)^{3/2}\,(c+d\,x)}\,dx$$

input
```
int((e*x)^(1/2)/((a*x + b*x^2)^(3/2)*(c + d*x)),x)
```

output
```
int((e*x)^(1/2)/((a*x + b*x^2)^(3/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{ex}}{(c+dx)\,(ax+bx^2)^{3/2}}\,dx = \frac{\sqrt{e}\left(2\sqrt{d}\,\sqrt{bx+a}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)a^2d+\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{b\right.$$

input
```
int((e*x)^(1/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x)
```

output
```
(sqrt(e)*(2*sqrt(d)*sqrt(a + b*x)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d
)/(sqrt(d)*sqrt( - a*d + b*c)))*a**2*d + sqrt(a)*sqrt(a + b*x)*log(sqrt(a
+ b*x) - sqrt(a))*a**2*d**2 - 2*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) -
sqrt(a))*a*b*c*d + sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**2
*c**2 - sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a**2*d**2 + 2*s
qrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a*b*c*d - sqrt(a)*sqrt(a
 + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**2*c**2 - 2*a**2*b*c*d + 2*a*b**2*c
**2))/(sqrt(a + b*x)*a**2*c*(a**2*d**2 - 2*a*b*c*d + b**2*c**2))
```

3.191 $\int \frac{1}{\sqrt{ex}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 206

$$\int \frac{1}{\sqrt{ex}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx = -\frac{1}{ac\sqrt{ex}\sqrt{ax+bx^2}}-\frac{b(3bc-ad)\sqrt{ex}}{a^2c(bc-ad)e\sqrt{ax+bx^2}} -\frac{2d^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{e}\sqrt{ax+bx^2}}{\sqrt{bc-ad}\sqrt{ex}}\right)}{c^2(bc-ad)^{3/2}\sqrt{e}}+\frac{(3bc+2ad)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{ex}}\right)}{a^{5/2}c^2\sqrt{e}}$$

output

```
-1/a/c/(e*x)^(1/2)/(b*x^2+a*x)^(1/2)-b*(-a*d+3*b*c)*(e*x)^(1/2)/a^2/c/(-a*
d+b*c)/e/(b*x^2+a*x)^(1/2)-2*d^(5/2)*arctan(d^(1/2)*e^(1/2)*(b*x^2+a*x)^(1
/2)/(-a*d+b*c)^(1/2)/(e*x)^(1/2))/c^2/(-a*d+b*c)^(3/2)/e^(1/2)+(2*a*d+3*b*
c)*arctanh(e^(1/2)*(b*x^2+a*x)^(1/2)/a^(1/2)/(e*x)^(1/2))/a^(5/2)/c^2/e^(1
/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{ex}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx = \frac{\sqrt{a}\Big(c\sqrt{bc-ad}(-abc+a^2d-3b^2cx+abdx)-2a^2d^{5/2}x\sqrt{a+bx}\,\mathrm{arcta}}{a^{5/2}c^2(bc-ad}$$

input
```
Integrate[1/(Sqrt[e*x]*(c + d*x)*(a*x + b*x^2)^(3/2)),x]
```

output
```
(Sqrt[a]*(c*Sqrt[b*c - a*d]*(-(a*b*c) + a^2*d - 3*b^2*c*x + a*b*d*x) - 2*a
^2*d^(5/2)*x*Sqrt[a + b*x]*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]]
) + Sqrt[b*c - a*d]*(3*b^2*c^2 - a*b*c*d - 2*a^2*d^2)*x*Sqrt[a + b*x]*ArcT
anh[Sqrt[a + b*x]/Sqrt[a]])/(a^(5/2)*c^2*(b*c - a*d)^(3/2)*Sqrt[e*x]*Sqrt[
x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1261, 114, 27, 169, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ex}\left(ax+bx^2\right)^{3/2}(c+dx)}dx$$

$$\Big\downarrow \; 1261$$

$$\frac{x^2(a+bx)^{3/2}\int \frac{1}{x^2(a+bx)^{3/2}(c+dx)}dx}{\sqrt{ex}\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 114$$

$$\frac{x^2(a+bx)^{3/2}\left(-\frac{\int \frac{3bc+2ad+3bdx}{2x(a+bx)^{3/2}(c+dx)}dx}{ac}-\frac{1}{acx\sqrt{a+bx}}\right)}{\sqrt{ex}\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{x^2(a+bx)^{3/2}\left(-\frac{\int \frac{3bc+2ad+3bdx}{x(a+bx)^{3/2}(c+dx)}dx}{2ac}-\frac{1}{acx\sqrt{a+bx}}\right)}{\sqrt{ex}\left(ax+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 169

$$\frac{x^2(a+bx)^{3/2}\left(-\frac{\frac{2\int \frac{(bc-ad)(3bc+2ad)+bd(3bc-ad)x}{2x\sqrt{a+bx}(c+dx)}dx}{a(bc-ad)}+\frac{2b(3bc-ad)}{a\sqrt{a+bx}(bc-ad)}}{2ac}-\frac{1}{acx\sqrt{a+bx}}\right)}{\sqrt{ex}\left(ax+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{x^2(a+bx)^{3/2}\left(-\frac{\frac{\int \frac{(bc-ad)(3bc+2ad)+bd(3bc-ad)x}{x\sqrt{a+bx}(c+dx)}dx}{a(bc-ad)}+\frac{2b(3bc-ad)}{a\sqrt{a+bx}(bc-ad)}}{2ac}-\frac{1}{acx\sqrt{a+bx}}\right)}{\sqrt{ex}\left(ax+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 174

$$\frac{x^2(a+bx)^{3/2}\left(-\frac{\frac{\frac{2a^2d^3\int \frac{1}{\sqrt{a+bx}(c+dx)}dx}{c}+\frac{(bc-ad)(2ad+3bc)\int \frac{1}{x\sqrt{a+bx}}dx}{c}}{a(bc-ad)}+\frac{2b(3bc-ad)}{a\sqrt{a+bx}(bc-ad)}}{2ac}-\frac{1}{acx\sqrt{a+bx}}\right)}{\sqrt{ex}\left(ax+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 73

$$\frac{x^2(a+bx)^{3/2}\left(-\frac{\frac{\frac{4a^2d^3\int \frac{1}{c-\frac{ad}{b}+\frac{d(a+bx)}{b}}d\sqrt{a+bx}}{bc}+\frac{2(bc-ad)(2ad+3bc)\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}}{a(bc-ad)}+\frac{2b(3bc-ad)}{a\sqrt{a+bx}(bc-ad)}}{2ac}-\frac{1}{acx\sqrt{a+bx}}\right)}{\sqrt{ex}\left(ax+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 218

$$\frac{x^2(a+bx)^{3/2}\left(-\frac{\frac{\frac{2(bc-ad)(2ad+3bc)\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}+\frac{4a^2d^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{a(bc-ad)}+\frac{2b(3bc-ad)}{a\sqrt{a+bx}(bc-ad)}}{2ac}-\frac{1}{acx\sqrt{a+bx}}\right)}{\sqrt{ex}\left(ax+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 221

$$\frac{x^2(a+bx)^{3/2}\left(-\frac{\frac{\frac{4a^2d^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)(bc-ad)(2ad+3bc)}{\sqrt{a}c}}{a(bc-ad)}+\frac{2b(3bc-ad)}{a\sqrt{a+bx}(bc-ad)}}{2ac}-\frac{1}{acx\sqrt{a+bx}}\right)}{\sqrt{ex}\,(ax+bx^2)^{3/2}}$$

input
```
Int[1/(Sqrt[e*x]*(c + d*x)*(a*x + b*x^2)^(3/2)),x]
```

output
```
(x^2*(a + b*x)^(3/2)*(-(1/(a*c*x*Sqrt[a + b*x])) - ((2*b*(3*b*c - a*d))/(a
*(b*c - a*d)*Sqrt[a + b*x]) + ((4*a^2*d^(5/2)*ArcTan[(Sqrt[d]*Sqrt[a + b*x
])/Sqrt[b*c - a*d]])/(c*Sqrt[b*c - a*d]) - (2*(b*c - a*d)*(3*b*c + 2*a*d)*
ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*c))/(a*(b*c - a*d)))/(2*a*c)))/(S
qrt[e*x]*(a*x + b*x^2)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.92

method	result
risch	$-\frac{bx+a}{a^{2}c\sqrt{ex}\,\sqrt{x(bx+a)}}-\frac{b\left(-\frac{(2ad+3bc)\operatorname{arctanh}\left(\frac{\sqrt{bex+ae}}{\sqrt{ae}}\right)}{bc\sqrt{ae}}-\frac{2bc}{(ad-bc)\sqrt{bex+ae}}+\frac{2a^{2}d^{3}\operatorname{arctanh}\left(\frac{d\sqrt{bex+ae}}{\sqrt{e(ad-bc)d}}\right)}{(ad-bc)bc\sqrt{e(ad-bc)d}}\right)\sqrt{(bx+a)e}\,x}{a^{2}c\sqrt{ex}\,\sqrt{x(bx+a)}}$
default	$\frac{\left(-2\sqrt{(bx+a)e}\,\sqrt{ae}\,\operatorname{arctanh}\left(\frac{d\sqrt{(bx+a)e}}{\sqrt{e(ad-bc)d}}\right)a^{2}d^{3}x+2\sqrt{(bx+a)e}\,\operatorname{arctanh}\left(\frac{\sqrt{(bx+a)e}}{\sqrt{ae}}\right)\sqrt{e(ad-bc)d}\,a^{2}d^{2}x+\sqrt{(bx+a)e}\,\operatorname{arctanh}\left(\right.\right.}{}$

input
```
int(1/(e*x)^(1/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^2/c*(b*x+a)/(e*x)^(1/2)/(x*(b*x+a))^(1/2)-1/a^2*b/c*(-(2*a*d+3*b*c)/b
/c/(a*e)^(1/2)*arctanh((b*e*x+a*e)^(1/2)/(a*e)^(1/2))-2*b*c/(a*d-b*c)/(b*e
*x+a*e)^(1/2)+2/(a*d-b*c)*a^2*d^3/b/c/(e*(a*d-b*c)*d)^(1/2)*arctanh(d*(b*e
*x+a*e)^(1/2)/(e*(a*d-b*c)*d)^(1/2)))*((b*x+a)*e)^(1/2)/(e*x)^(1/2)/(x*(b*
x+a))^(1/2)*x
```

Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1294, normalized size of antiderivative = 6.28

$$\int \frac{1}{\sqrt{ex}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x)^(1/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(2*(a^3*b*d^2*e*x^3 + a^4*d^2*e*x^2)*sqrt(-d/((b*c - a*d)*e))*log(-(
b*d*x^2 + 2*sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(e*x)*sqrt(-d/((b*c - a*d)*e
)) - (b*c - 2*a*d)*x)/(d*x^2 + c*x)) - ((3*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d
^2)*x^3 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^2)*sqrt(a*e)*log(-(b*e*x
^2 + 2*a*e*x + 2*sqrt(b*x^2 + a*x)*sqrt(a*e)*sqrt(e*x))/x^2) + 2*(a^2*b*c^
2 - a^3*c*d + (3*a*b^2*c^2 - a^2*b*c*d)*x)*sqrt(b*x^2 + a*x)*sqrt(e*x))/((
a^3*b^2*c^3 - a^4*b*c^2*d)*e*x^3 + (a^4*b*c^3 - a^5*c^2*d)*e*x^2), -(((3*b
^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2)*x^3 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d
^2)*x^2)*sqrt(-a*e)*arctan(sqrt(b*x^2 + a*x)*sqrt(-a*e)*sqrt(e*x)/(a*e*x))
 + (a^3*b*d^2*e*x^3 + a^4*d^2*e*x^2)*sqrt(-d/((b*c - a*d)*e))*log(-(b*d*x^
2 + 2*sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(e*x)*sqrt(-d/((b*c - a*d)*e)) - (
b*c - 2*a*d)*x)/(d*x^2 + c*x)) + (a^2*b*c^2 - a^3*c*d + (3*a*b^2*c^2 - a^2
*b*c*d)*x)*sqrt(b*x^2 + a*x)*sqrt(e*x))/((a^3*b^2*c^3 - a^4*b*c^2*d)*e*x^3
 + (a^4*b*c^3 - a^5*c^2*d)*e*x^2), -1/2*(4*(a^3*b*d^2*e*x^3 + a^4*d^2*e*x^
2)*sqrt(d/((b*c - a*d)*e))*arctan(-sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(e*x)
*sqrt(d/((b*c - a*d)*e))/(b*d*x^2 + a*d*x)) - ((3*b^3*c^2 - a*b^2*c*d - 2*
a^2*b*d^2)*x^3 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^2)*sqrt(a*e)*log(
-(b*e*x^2 + 2*a*e*x + 2*sqrt(b*x^2 + a*x)*sqrt(a*e)*sqrt(e*x))/x^2) + 2*(a
^2*b*c^2 - a^3*c*d + (3*a*b^2*c^2 - a^2*b*c*d)*x)*sqrt(b*x^2 + a*x)*sqrt(e
*x))/((a^3*b^2*c^3 - a^4*b*c^2*d)*e*x^3 + (a^4*b*c^3 - a^5*c^2*d)*e*x^2...
```

Sympy [F]

$$\int \frac{1}{\sqrt{ex}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx = \int \frac{1}{\sqrt{ex}\,(x\,(a+bx))^{\frac{3}{2}}\,(c+dx)}\,dx$$

input
```
integrate(1/(e*x)**(1/2)/(d*x+c)/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral(1/(sqrt(e*x)*(x*(a + b*x))**(3/2)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{ex}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx = \int \frac{1}{\left(bx^2+ax\right)^{\frac{3}{2}}(dx+c)\sqrt{ex}}\,dx$$

input
```
integrate(1/(e*x)^(1/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a*x)^(3/2)*(d*x + c)*sqrt(e*x)), x)
```

Giac [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{ex}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx = -\left(\frac{2\,d^3\arctan\left(\frac{\sqrt{bex+aed}}{\sqrt{bcde-ad^2e}}\right)}{\left(bc^3e^3|e|-ac^2de^3|e|\right)\sqrt{bcde-ad^2e}} + \frac{2\,ab^2ce-3\,(bex+ae)b^2c+(bex+ae)abd}{\left(a^2bc^2e^3|e|-a^3cde^3|e|\right)\left(\sqrt{bex+ae}ae-(bex+ae)^{\frac{3}{2}}\right)} + \frac{(3\,bc+$$

input
```
integrate(1/(e*x)^(1/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
-(2*d^3*arctan(sqrt(b*e*x + a*e)*d/sqrt(b*c*d*e - a*d^2*e))/((b*c^3*e^3*ab
s(e) - a*c^2*d*e^3*abs(e))*sqrt(b*c*d*e - a*d^2*e)) + (2*a*b^2*c*e - 3*(b*
e*x + a*e)*b^2*c + (b*e*x + a*e)*a*b*d)/((a^2*b*c^2*e^3*abs(e) - a^3*c*d*e
^3*abs(e))*(sqrt(b*e*x + a*e)*a*e - (b*e*x + a*e)^(3/2))) + (3*b*c + 2*a*d
)*arctan(sqrt(b*e*x + a*e)/sqrt(-a*e))/(sqrt(-a*e)*a^2*c^2*e^3*abs(e)))*e^
4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ex}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(b\,x^2+a\,x\right)^{3/2}\,\sqrt{e\,x}\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/((a*x + b*x^2)^(3/2)*(e*x)^(1/2)*(c + d*x)),x)
```

output
```
int(1/((a*x + b*x^2)^(3/2)*(e*x)^(1/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.91

$$\int \frac{1}{\sqrt{ex}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx=\frac{\sqrt{e}\left(-4\sqrt{d}\,\sqrt{bx+a}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)a^3d^2x-2\sqrt{a}\,\sqrt{bx+}\right.}{}$$

input
```
int(1/(e*x)^(1/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x)
```

output
```
(sqrt(e)*( - 4*sqrt(d)*sqrt(a + b*x)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x
)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*a**3*d**2*x - 2*sqrt(a)*sqrt(a + b*x)*l
og(sqrt(a + b*x) - sqrt(a))*a**3*d**3*x + sqrt(a)*sqrt(a + b*x)*log(sqrt(a
 + b*x) - sqrt(a))*a**2*b*c*d**2*x + 4*sqrt(a)*sqrt(a + b*x)*log(sqrt(a +
b*x) - sqrt(a))*a*b**2*c**2*d*x - 3*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x
) - sqrt(a))*b**3*c**3*x + 2*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqr
t(a))*a**3*d**3*x - sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a**
2*b*c*d**2*x - 4*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a*b**2
*c**2*d*x + 3*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**3*c**3
*x - 2*a**4*c*d**2 + 4*a**3*b*c**2*d - 2*a**3*b*c*d**2*x - 2*a**2*b**2*c**
3 + 8*a**2*b**2*c**2*d*x - 6*a*b**3*c**3*x))/(2*sqrt(a + b*x)*a**3*c**2*e*
x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2))
```

3.192 $\int \frac{1}{(ex)^{3/2}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 283

$$\int \frac{1}{(ex)^{3/2}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx = -\frac{1}{2ac(ex)^{3/2}\sqrt{ax+bx^2}} + \frac{5bc+4ad}{4a^2c^2e\sqrt{ex}\sqrt{ax+bx^2}} + \frac{b(15b^2c^2-3abcd-4a^2d^2)\sqrt{ex}}{4a^3c^2(bc-ad)e^2\sqrt{ax+bx^2}} + \frac{2d^{7/2}\arctan\left(\frac{\sqrt{d}\sqrt{e}\sqrt{ax+bx^2}}{\sqrt{bc-ad}\sqrt{ex}}\right)}{c^3(bc-ad)^{3/2}e^{3/2}} - \frac{(15b^2c^2+12abcd+8a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{ex}}\right)}{4a^{7/2}c^3e^{3/2}}$$

output

```
-1/2/a/c/(e*x)^(3/2)/(b*x^2+a*x)^(1/2)+1/4*(4*a*d+5*b*c)/a^2/c^2/e/(e*x)^(
1/2)/(b*x^2+a*x)^(1/2)+1/4*b*(-4*a^2*d^2-3*a*b*c*d+15*b^2*c^2)*(e*x)^(1/2)
/a^3/c^2/(-a*d+b*c)/e^2/(b*x^2+a*x)^(1/2)+2*d^(7/2)*arctan(d^(1/2)*e^(1/2)
*(b*x^2+a*x)^(1/2)/(-a*d+b*c)^(1/2)/(e*x)^(1/2))/c^3/(-a*d+b*c)^(3/2)/e^(3
/2)-1/4*(8*a^2*d^2+12*a*b*c*d+15*b^2*c^2)*arctanh(e^(1/2)*(b*x^2+a*x)^(1/2
)/a^(1/2)/(e*x)^(1/2))/a^(7/2)/c^3/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.90

$$\int \frac{1}{(ex)^{3/2}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx =$$
$$-\frac{\sqrt{a}\left(c\sqrt{bc-ad}(-15b^3c^2x^2-2a^3d(c-2dx)+ab^2cx(-5c+3dx)+a^2b(2c^2+cdx+4d^2x^2))-8a^3d^{7/2}x^2\right.}{4a^{7/2}c^3(bc-ad)^{3/}}$$

input
```
Integrate[1/((e*x)^(3/2)*(c + d*x)*(a*x + b*x^2)^(3/2)),x]
```

output
```
-1/4*(Sqrt[a]*(c*Sqrt[b*c - a*d]*(-15*b^3*c^2*x^2 - 2*a^3*d*(c - 2*d*x) +
a*b^2*c*x*(-5*c + 3*d*x) + a^2*b*(2*c^2 + c*d*x + 4*d^2*x^2)) - 8*a^3*d^(7
/2)*x^2*Sqrt[a + b*x]*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]]) + S
qrt[b*c - a*d]*(15*b^3*c^3 - 3*a*b^2*c^2*d - 4*a^2*b*c*d^2 - 8*a^3*d^3)*x^
2*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(a^(7/2)*c^3*(b*c - a*d)^(
3/2)*(e*x)^(3/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {1261, 114, 27, 168, 27, 169, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ex)^{3/2}\left(ax+bx^2\right)^{3/2}(c+dx)}dx$$
$$\Big\downarrow \text{ 1261}$$
$$\frac{x^3(a+bx)^{3/2}\int \frac{1}{x^3(a+bx)^{3/2}(c+dx)}dx}{(ex)^{3/2}\left(ax+bx^2\right)^{3/2}}$$
$$\Big\downarrow \text{ 114}$$

$$\frac{x^3(a+bx)^{3/2}\left(-\frac{\int \frac{5bc+4ad+5bdx}{2x^2(a+bx)^{3/2}(c+dx)}dx}{2ac}-\frac{1}{2acx^2\sqrt{a+bx}}\right)}{(ex)^{3/2}\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{x^3(a+bx)^{3/2}\left(-\frac{\int \frac{5bc+4ad+5bdx}{x^2(a+bx)^{3/2}(c+dx)}dx}{4ac}-\frac{1}{2acx^2\sqrt{a+bx}}\right)}{(ex)^{3/2}\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 168$$

$$\frac{x^3(a+bx)^{3/2}\left(-\frac{-\frac{\int \frac{15b^2c^2+12abdc+8a^2d^2+3bd(5bc+4ad)x}{2x(a+bx)^{3/2}(c+dx)}dx}{ac}-\frac{4ad+5bc}{acx\sqrt{a+bx}}}{4ac}-\frac{1}{2acx^2\sqrt{a+bx}}\right)}{(ex)^{3/2}\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{x^3(a+bx)^{3/2}\left(-\frac{-\frac{\int \frac{15b^2c^2+12abdc+8a^2d^2+3bd(5bc+4ad)x}{x(a+bx)^{3/2}(c+dx)}dx}{2ac}-\frac{4ad+5bc}{acx\sqrt{a+bx}}}{4ac}-\frac{1}{2acx^2\sqrt{a+bx}}\right)}{(ex)^{3/2}\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 169$$

$$\frac{x^3(a+bx)^{3/2}\left(-\frac{-\frac{\frac{2\int \frac{(bc-ad)\left(15b^2c^2+12abdc+8a^2d^2\right)+bd\left(15b^2c^2-3abdc-4a^2d^2\right)x}{2x\sqrt{a+bx}(c+dx)}dx}{a(bc-ad)}+\frac{2b\left(-4a^2d^2-3abcd+15b^2c^2\right)}{a\sqrt{a+bx}(bc-ad)}}{2ac}-\frac{4ad+5bc}{acx\sqrt{a+bx}}}{4ac}-\frac{1}{2acx^2\sqrt{a+bx}}\right)}{(ex)^{3/2}\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{x^3(a+bx)^{3/2}\left(-\frac{-\frac{\frac{\int \frac{(bc-ad)\left(15b^2c^2+12abdc+8a^2d^2\right)+bd\left(15b^2c^2-3abdc-4a^2d^2\right)x}{x\sqrt{a+bx}(c+dx)}dx}{a(bc-ad)}+\frac{2b\left(-4a^2d^2-3abcd+15b^2c^2\right)}{a\sqrt{a+bx}(bc-ad)}}{2ac}-\frac{4ad+5bc}{acx\sqrt{a+bx}}}{4ac}-\frac{1}{2acx^2\sqrt{a+bx}}\right)}{(ex)^{3/2}\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 174$$

$$\frac{x^3(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{\frac{8a^3d^4\int\frac{1}{\sqrt{a+bx}(c+dx)}dx}{c}+\frac{(bc-ad)\left(8a^2d^2+12abcd+15b^2c^2\right)\int\frac{1}{x\sqrt{a+bx}}dx}{c}}{a(bc-ad)}+\frac{2b\left(-4a^2d^2-3abcd+15b^2c^2\right)}{a\sqrt{a+bx}(bc-ad)}}{2ac}-\frac{4ad+5bc}{acx\sqrt{a+bx}}}{4ac}-\frac{\cdots}{2acx\cdots}\cdots\right)}{(ex)^{3/2}\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 73$$

$$\frac{x^3(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{\frac{16a^3d^4\int\frac{1}{c-\frac{ad}{b}+\frac{d(a+bx)}{b}}d\sqrt{a+bx}}{bc}+\frac{2(bc-ad)\left(8a^2d^2+12abcd+15b^2c^2\right)\int\frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}}{a(bc-ad)}+\frac{2b\left(-4a^2d^2-3abcd+15b^2c^2\right)}{a\sqrt{a+bx}(bc-ad)}}{2ac}-\frac{4ad\cdots}{acx\cdots}}{4ac}\cdots\right)}{(ex)^{3/2}\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 218$$

$$\frac{x^3(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{\frac{2(bc-ad)\left(8a^2d^2+12abcd+15b^2c^2\right)\int\frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}+\frac{16a^3d^{7/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{a(bc-ad)}+\frac{2b\left(-4a^2d^2-3abcd+15b^2c^2\right)}{a\sqrt{a+bx}(bc-ad)}}{2ac}-\frac{4ad+5\cdots}{acx\sqrt{a+\cdots}}}{4ac}\cdots\right)}{(ex)^{3/2}\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{x^3(a+bx)^{3/2}\left(-\frac{-\frac{\frac{2b\left(-4a^2d^2-3abcd+15b^2c^2\right)}{a\sqrt{a+bx}(bc-ad)}+\frac{\frac{16a^3d^{7/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)(bc-ad)\left(8a^2d^2+12abcd+15b^2c^2\right)}{\sqrt{a}c}}{a(bc-ad)}}{2ac}-\frac{4ad+\cdots}{acx\sqrt{a\cdots}}}{4ac}\cdots\right)}{(ex)^{3/2}\left(ax+bx^2\right)^{3/2}}$$

input
```
Int[1/((e*x)^(3/2)*(c + d*x)*(a*x + b*x^2)^(3/2)),x]
```

output
```
(x^3*(a + b*x)^(3/2)*(-1/2*1/(a*c*x^2*Sqrt[a + b*x]) - (-((5*b*c + 4*a*d)/
(a*c*x*Sqrt[a + b*x])) - ((2*b*(15*b^2*c^2 - 3*a*b*c*d - 4*a^2*d^2))/(a*(b
*c - a*d)*Sqrt[a + b*x]) + ((16*a^3*d^(7/2)*ArcTan[(Sqrt[d]*Sqrt[a + b*x])
/Sqrt[b*c - a*d]])/(c*Sqrt[b*c - a*d]) - (2*(b*c - a*d)*(15*b^2*c^2 + 12*a
*b*c*d + 8*a^2*d^2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*c))/(a*(b*c -
 a*d)))/(2*a*c))/(4*a*c)))/((e*x)^(3/2)*(a*x + b*x^2)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{(bx+a)(-4adx-7cbx+2ac)}{4a^3c^2xe\sqrt{ex}\,\sqrt{x(bx+a)}}+\frac{b\left(-\frac{\left(8a^2d^2+12abcd+15b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{bex+ae}}{\sqrt{ae}}\right)}{bc\sqrt{ae}}-\frac{8b^2c^2}{(ad-bc)\sqrt{bex+ae}}+\frac{8a^3d^4\operatorname{arctanh}\left(\frac{d\sqrt{bex+ae}}{\sqrt{e(ad-bc)d}}\right)}{(ad-bc)bc\sqrt{e(ad-bc)d}}\right.}{4a^3c^2e\sqrt{ex}\,\sqrt{x(bx+a)}}$
default	$\frac{\sqrt{x(bx+a)}\left(8\sqrt{(bx+a)e}\sqrt{ae}\operatorname{arctanh}\left(\frac{d\sqrt{(bx+a)e}}{\sqrt{e(ad-bc)d}}\right)a^3d^4x^2-8\sqrt{(bx+a)e}\operatorname{arctanh}\left(\frac{\sqrt{(bx+a)e}}{\sqrt{ae}}\right)\sqrt{e(ad-bc)d}\,a^3d^3x^2-4\sqrt{(bx+a)e}\right.}{}$

input
```
int(1/(e*x)^(3/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b*x+a)*(-4*a*d*x-7*b*c*x+2*a*c)/a^3/c^2/x/e/(e*x)^(1/2)/(x*(b*x+a))^
(1/2)+1/4/a^3*b/c^2*(-(8*a^2*d^2+12*a*b*c*d+15*b^2*c^2)/b/c/(a*e)^(1/2)*ar
ctanh((b*e*x+a*e)^(1/2)/(a*e)^(1/2))-8*b^2*c^2/(a*d-b*c)/(b*e*x+a*e)^(1/2)
+8/(a*d-b*c)*a^3*d^4/b/c/(e*(a*d-b*c)*d)^(1/2)*arctanh(d*(b*e*x+a*e)^(1/2)
/(e*(a*d-b*c)*d)^(1/2)))/e*((b*x+a)*e)^(1/2)/(e*x)^(1/2)/(x*(b*x+a))^(1/2)
*x
```

Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 1640, normalized size of antiderivative = 5.80

$$\int \frac{1}{(ex)^{3/2}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x)^(3/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(8*(a^4*b*d^3*e*x^4 + a^5*d^3*e*x^3)*sqrt(-d/((b*c - a*d)*e))*log(-(
b*d*x^2 - 2*sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(e*x)*sqrt(-d/((b*c - a*d)*e
)) - (b*c - 2*a*d)*x)/(d*x^2 + c*x)) - ((15*b^4*c^3 - 3*a*b^3*c^2*d - 4*a^
2*b^2*c*d^2 - 8*a^3*b*d^3)*x^4 + (15*a*b^3*c^3 - 3*a^2*b^2*c^2*d - 4*a^3*b
*c*d^2 - 8*a^4*d^3)*x^3)*sqrt(a*e)*log(-(b*e*x^2 + 2*a*e*x - 2*sqrt(b*x^2
+ a*x)*sqrt(a*e)*sqrt(e*x))/x^2) + 2*(2*a^3*b*c^3 - 2*a^4*c^2*d - (15*a*b^
3*c^3 - 3*a^2*b^2*c^2*d - 4*a^3*b*c*d^2)*x^2 - (5*a^2*b^2*c^3 - a^3*b*c^2*
d - 4*a^4*c*d^2)*x)*sqrt(b*x^2 + a*x)*sqrt(e*x))/((a^4*b^2*c^4 - a^5*b*c^3
*d)*e^2*x^4 + (a^5*b*c^4 - a^6*c^3*d)*e^2*x^3), 1/4*(((15*b^4*c^3 - 3*a*b^
3*c^2*d - 4*a^2*b^2*c*d^2 - 8*a^3*b*d^3)*x^4 + (15*a*b^3*c^3 - 3*a^2*b^2*c
^2*d - 4*a^3*b*c*d^2 - 8*a^4*d^3)*x^3)*sqrt(-a*e)*arctan(sqrt(b*x^2 + a*x)
*sqrt(-a*e)*sqrt(e*x)/(a*e*x)) - 4*(a^4*b*d^3*e*x^4 + a^5*d^3*e*x^3)*sqrt(
-d/((b*c - a*d)*e))*log(-(b*d*x^2 - 2*sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(e
*x)*sqrt(-d/((b*c - a*d)*e)) - (b*c - 2*a*d)*x)/(d*x^2 + c*x)) - (2*a^3*b*
c^3 - 2*a^4*c^2*d - (15*a*b^3*c^3 - 3*a^2*b^2*c^2*d - 4*a^3*b*c*d^2)*x^2 -
 (5*a^2*b^2*c^3 - a^3*b*c^2*d - 4*a^4*c*d^2)*x)*sqrt(b*x^2 + a*x)*sqrt(e*x
))/((a^4*b^2*c^4 - a^5*b*c^3*d)*e^2*x^4 + (a^5*b*c^4 - a^6*c^3*d)*e^2*x^3)
, 1/8*(16*(a^4*b*d^3*e*x^4 + a^5*d^3*e*x^3)*sqrt(d/((b*c - a*d)*e))*arctan
(-sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(e*x)*sqrt(d/((b*c - a*d)*e))/(b*d*x^2
 + a*d*x)) + ((15*b^4*c^3 - 3*a*b^3*c^2*d - 4*a^2*b^2*c*d^2 - 8*a^3*b*d...
```

Sympy [F]

$$\int \frac{1}{(ex)^{3/2}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx=\int \frac{1}{(ex)^{\frac{3}{2}}\left(x\left(a+bx\right)\right)^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/(e*x)**(3/2)/(d*x+c)/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral(1/((e*x)**(3/2)*(x*(a + b*x))**(3/2)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{(ex)^{3/2}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx=\int \frac{1}{(bx^2+ax)^{\frac{3}{2}}(dx+c)\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x)^(3/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a*x)^(3/2)*(d*x + c)*(e*x)^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.95

$$\int \frac{1}{(ex)^{3/2}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx=\frac{1}{4}\left(\frac{8\,d^4\arctan\left(\frac{\sqrt{bex+ae}d}{\sqrt{bcde-ad^2e}}\right)}{(bc^4e^4|e|-ac^3de^4|e|)\sqrt{bcde-ad^2e}}+\frac{8\,b^3}{(a^3bce^4|e|-a^4de^4|e|)\sqrt{be}}\right.$$

input
```
integrate(1/(e*x)^(3/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
1/4*(8*d^4*arctan(sqrt(b*e*x + a*e)*d/sqrt(b*c*d*e - a*d^2*e))/((b*c^4*e^4
*abs(e) - a*c^3*d*e^4*abs(e))*sqrt(b*c*d*e - a*d^2*e)) + 8*b^3/((a^3*b*c*e
^4*abs(e) - a^4*d*e^4*abs(e))*sqrt(b*e*x + a*e)) + (15*b^2*c^2 + 12*a*b*c*
d + 8*a^2*d^2)*arctan(sqrt(b*e*x + a*e)/sqrt(-a*e))/(sqrt(-a*e)*a^3*c^3*e^
4*abs(e)) - (9*sqrt(b*e*x + a*e)*a*b^2*c*e + 4*sqrt(b*e*x + a*e)*a^2*b*d*e
 - 7*(b*e*x + a*e)^(3/2)*b^2*c - 4*(b*e*x + a*e)^(3/2)*a*b*d)/(a^3*b^2*c^2
*e^6*x^2*abs(e)))*e^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{3/2}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx = \int \frac{1}{\left(b\,x^2+a\,x\right)^{3/2}\,(e\,x)^{3/2}\,(c+d\,x)}\,dx$$

input
```
int(1/((a*x + b*x^2)^(3/2)*(e*x)^(3/2)*(c + d*x)),x)
```

output
```
int(1/((a*x + b*x^2)^(3/2)*(e*x)^(3/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.96

$$\int \frac{1}{(ex)^{3/2}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx = \frac{\sqrt{e}\left(16\sqrt{d}\,\sqrt{bx+a}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)a^4d^3x^2+8\sqrt{a}\,\sqrt{bx}\right.}{}$$

input
```
int(1/(e*x)^(3/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x)
```

output
```
(sqrt(e)*(16*sqrt(d)*sqrt(a + b*x)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*
d)/(sqrt(d)*sqrt( - a*d + b*c)))*a**4*d**3*x**2 + 8*sqrt(a)*sqrt(a + b*x)*
log(sqrt(a + b*x) - sqrt(a))*a**4*d**4*x**2 - 4*sqrt(a)*sqrt(a + b*x)*log(
sqrt(a + b*x) - sqrt(a))*a**3*b*c*d**3*x**2 - sqrt(a)*sqrt(a + b*x)*log(sq
rt(a + b*x) - sqrt(a))*a**2*b**2*c**2*d**2*x**2 - 18*sqrt(a)*sqrt(a + b*x)
*log(sqrt(a + b*x) - sqrt(a))*a*b**3*c**3*d*x**2 + 15*sqrt(a)*sqrt(a + b*x
)*log(sqrt(a + b*x) - sqrt(a))*b**4*c**4*x**2 - 8*sqrt(a)*sqrt(a + b*x)*lo
g(sqrt(a + b*x) + sqrt(a))*a**4*d**4*x**2 + 4*sqrt(a)*sqrt(a + b*x)*log(sq
rt(a + b*x) + sqrt(a))*a**3*b*c*d**3*x**2 + sqrt(a)*sqrt(a + b*x)*log(sqrt
(a + b*x) + sqrt(a))*a**2*b**2*c**2*d**2*x**2 + 18*sqrt(a)*sqrt(a + b*x)*l
og(sqrt(a + b*x) + sqrt(a))*a*b**3*c**3*d*x**2 - 15*sqrt(a)*sqrt(a + b*x)*
log(sqrt(a + b*x) + sqrt(a))*b**4*c**4*x**2 - 4*a**5*c**2*d**2 + 8*a**5*c*
d**3*x + 8*a**4*b*c**3*d - 6*a**4*b*c**2*d**2*x + 8*a**4*b*c*d**3*x**2 - 4
*a**3*b**2*c**4 - 12*a**3*b**2*c**3*d*x - 2*a**3*b**2*c**2*d**2*x**2 + 10*
a**2*b**3*c**4*x - 36*a**2*b**3*c**3*d*x**2 + 30*a*b**4*c**4*x**2))/(8*sqr
t(a + b*x)*a**4*c**3*e**2*x**2*(a**2*d**2 - 2*a*b*c*d + b**2*c**2))
```

3.193 $\int \frac{1}{(ex)^{5/2}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 367

$$\int \frac{1}{(ex)^{5/2}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx=-\frac{1}{3ac(ex)^{5/2}\sqrt{ax+bx^2}}+\frac{7bc+6ad}{12a^2c^2e(ex)^{3/2}\sqrt{ax+bx^2}}-\frac{35b^2c^2+30abcd+24a^2d^2}{24a^3c^3e^2\sqrt{ex}\sqrt{ax+bx^2}}-\frac{b(35b^3c^3-5ab^2c^2d-6a^2bcd^2-8a^3d^3)\sqrt{ex}}{8a^4c^3(bc-ad)e^3\sqrt{ax+bx^2}}-\frac{2d^{9/2}\arctan\left(\frac{\sqrt{d}\sqrt{e}\sqrt{ax+bx^2}}{\sqrt{bc-ad}\sqrt{ex}}\right)}{c^4(bc-ad)^{3/2}e^{5/2}}+\frac{(35b^3c^3+30ab^2c^2d+24a^2bcd^2+16a^3d^3)\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{ex}}\right)}{8a^{9/2}c^4e^{5/2}}$$

output
```
-1/3/a/c/(e*x)^(5/2)/(b*x^2+a*x)^(1/2)+1/12*(6*a*d+7*b*c)/a^2/c^2/e/(e*x)^
(3/2)/(b*x^2+a*x)^(1/2)-1/24*(24*a^2*d^2+30*a*b*c*d+35*b^2*c^2)/a^3/c^3/e^
2/(e*x)^(1/2)/(b*x^2+a*x)^(1/2)-1/8*b*(-8*a^3*d^3-6*a^2*b*c*d^2-5*a*b^2*c^
2*d+35*b^3*c^3)*(e*x)^(1/2)/a^4/c^3/(-a*d+b*c)/e^3/(b*x^2+a*x)^(1/2)-2*d^(
9/2)*arctan(d^(1/2)*e^(1/2)*(b*x^2+a*x)^(1/2)/(-a*d+b*c)^(1/2)/(e*x)^(1/2)
)/c^4/(-a*d+b*c)^(3/2)/e^(5/2)+1/8*(16*a^3*d^3+24*a^2*b*c*d^2+30*a*b^2*c^2
*d+35*b^3*c^3)*arctanh(e^(1/2)*(b*x^2+a*x)^(1/2)/a^(1/2)/(e*x)^(1/2))/a^(9
/2)/c^4/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.89

$$\int \frac{1}{(ex)^{5/2}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx =$$
$$-\frac{\sqrt{a}\left(c\sqrt{bc-ad}(105b^4c^3x^3+5ab^3c^2x^2(7c-3dx)-4a^4d(2c^2-3cdx+6d^2x^2)-a^2b^2cx(14c^2+5cdx+18\right.}{}$$

input
```
Integrate[1/((e*x)^(5/2)*(c + d*x)*(a*x + b*x^2)^(3/2)),x]
```

output
```
-1/24*(Sqrt[a]*(c*Sqrt[b*c - a*d]*(105*b^4*c^3*x^3 + 5*a*b^3*c^2*x^2*(7*c
 - 3*d*x) - 4*a^4*d*(2*c^2 - 3*c*d*x + 6*d^2*x^2) - a^2*b^2*c*x*(14*c^2 + 5
*c*d*x + 18*d^2*x^2) + 2*a^3*b*(4*c^3 + c^2*d*x - 3*c*d^2*x^2 - 12*d^3*x^3
)) + 48*a^4*d^(9/2)*x^3*Sqrt[a + b*x]*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[
b*c - a*d]]) - 3*Sqrt[b*c - a*d]*(35*b^4*c^4 - 5*a*b^3*c^3*d - 6*a^2*b^2*c
^2*d^2 - 8*a^3*b*c*d^3 - 16*a^4*d^4)*x^3*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*
x]/Sqrt[a]])/(a^(9/2)*c^4*(b*c - a*d)^(3/2)*(e*x)^(5/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {1261, 114, 27, 168, 27, 168, 27, 169, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ex)^{5/2}\left(ax+bx^2\right)^{3/2}(c+dx)}dx$$
$$\Big\downarrow \text{ 1261}$$
$$\frac{x^4(a+bx)^{3/2}\int \frac{1}{x^4(a+bx)^{3/2}(c+dx)}dx}{(ex)^{5/2}\left(ax+bx^2\right)^{3/2}}$$
$$\Big\downarrow \text{ 114}$$

$$\frac{x^4(a+bx)^{3/2}\left(-\frac{\int \frac{7bc+6ad+7bdx}{2x^3(a+bx)^{3/2}(c+dx)}dx}{3ac}-\frac{1}{3acx^3\sqrt{a+bx}}\right)}{(ex)^{5/2}\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{x^4(a+bx)^{3/2}\left(-\frac{\int \frac{7bc+6ad+7bdx}{x^3(a+bx)^{3/2}(c+dx)}dx}{6ac}-\frac{1}{3acx^3\sqrt{a+bx}}\right)}{(ex)^{5/2}\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 168$$

$$\frac{x^4(a+bx)^{3/2}\left(-\frac{-\frac{\int \frac{35b^2c^2+30abdc+24a^2d^2+5bd(7bc+6ad)x}{2x^2(a+bx)^{3/2}(c+dx)}dx}{2ac}-\frac{6ad+7bc}{2acx^2\sqrt{a+bx}}}{6ac}-\frac{1}{3acx^3\sqrt{a+bx}}\right)}{(ex)^{5/2}\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{x^4(a+bx)^{3/2}\left(-\frac{-\frac{\int \frac{35b^2c^2+30abdc+24a^2d^2+5bd(7bc+6ad)x}{x^2(a+bx)^{3/2}(c+dx)}dx}{4ac}-\frac{6ad+7bc}{2acx^2\sqrt{a+bx}}}{6ac}-\frac{1}{3acx^3\sqrt{a+bx}}\right)}{(ex)^{5/2}\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 168$$

$$\frac{x^4(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{\int \frac{3\left(35b^3c^3+30ab^2dc^2+24a^2bd^2c+16a^3d^3+bd\left(35b^2c^2+30abdc+24a^2d^2\right)x\right)}{2x(a+bx)^{3/2}(c+dx)}dx}{ac}-\frac{\frac{35b^2c}{a}+\frac{24ad^2}{c}+30bd}{x\sqrt{a+bx}}}{4ac}-\frac{6ad+7bc}{2acx^2\sqrt{a+bx}}}{6ac}-\frac{1}{3acx^3\sqrt{a+bx}}\right)}{(ex)^{5/2}\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{x^4(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{3\int \frac{35b^3c^3+30ab^2dc^2+24a^2bd^2c+16a^3d^3+bd\left(35b^2c^2+30abdc+24a^2d^2\right)x}{x(a+bx)^{3/2}(c+dx)}dx}{2ac}-\frac{\frac{35b^2c}{a}+\frac{24ad^2}{c}+30bd}{x\sqrt{a+bx}}}{4ac}-\frac{6ad+7bc}{2acx^2\sqrt{a+bx}}}{6ac}-\frac{1}{3acx^3\sqrt{a+bx}}\right)}{(ex)^{5/2}\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 169$$

$$\frac{x^4(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{3\left(\frac{2\int \frac{(bc-ad)\left(35b^3c^3+30ab^2dc^2+24a^2bd^2c+16a^3d^3\right)+bd\left(35b^3c^3-5ab^2dc^2-6a^2bd^2c-8a^3d^3\right)x}{2x\sqrt{a+bx}(c+dx)}dx}{a(bc-ad)}+\frac{2b\left(-8a^3d^3-6a^2bcd^2-5ab\cdots\right.}{a\sqrt{a+bx}(bc-a\cdots}\right.}{2ac}\cdots}{4ac}\cdots}{6ac}\cdots\right.}{(ex)^{5/2}\left(ax+bx^2\right)^{3/2}}$$

$\downarrow$ 27

$$\frac{x^4(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{3\left(\frac{\int \frac{(bc-ad)\left(35b^3c^3+30ab^2dc^2+24a^2bd^2c+16a^3d^3\right)+bd\left(35b^3c^3-5ab^2dc^2-6a^2bd^2c-8a^3d^3\right)x}{x\sqrt{a+bx}(c+dx)}dx}{a(bc-ad)}+\frac{2b\left(-8a^3d^3-6a^2bcd^2-5ab^2\cdots\right.}{a\sqrt{a+bx}(bc-ad\cdots}\right.}{2ac}\cdots}{4ac}\cdots}{6ac}\cdots\right.}{(ex)^{5/2}\left(ax+bx^2\right)^{3/2}}$$

$\downarrow$ 174

$$\frac{x^4(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{3\left(\frac{\frac{16a^4d^5\int \frac{1}{\sqrt{a+bx}(c+dx)}dx}{c}+\frac{(bc-ad)\left(16a^3d^3+24a^2bcd^2+30ab^2c^2d+35b^3c^3\right)\int \frac{1}{x\sqrt{a+bx}}dx}{c}}{a(bc-ad)}+\frac{2b\left(-8a^3d^3-6a^2bcd^2-5ab^2c^2d\cdots\right.}{a\sqrt{a+bx}(bc-ad)}\cdots\right.}{2ac}\cdots}{4ac}\cdots}{6ac}\cdots\right.}{(ex)^{5/2}\left(ax+bx^2\right)^{3/2}}$$

$\downarrow$ 73

$$\frac{x^4(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{3\left(\frac{\frac{32a^4d^5\int\frac{1}{c-\frac{ad}{b}+\frac{d(a+bx)}{b}}d\sqrt{a+bx}}{bc}+\frac{2(bc-ad)\left(16a^3d^3+24a^2bcd^2+30ab^2c^2d+35b^3c^3\right)\int\frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}}{a(bc-ad)}+\frac{2b\left(-8a^3d^3-6a\cdots\right.}{a\sqrt{\cdots}}\cdots\right.}{2ac}\cdots}{4ac}\cdots}{6ac}\cdots\right.}{(ex)^{5/2}\left(ax+bx^2\right)^{3/2}\cdots}$$

$\downarrow$ 218

$$\frac{x^4(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{3\left(\frac{\frac{2(bc-ad)\left(16a^3d^3+24a^2bcd^2+30ab^2c^2d+35b^3c^3\right)\int\frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}+\frac{32a^4d^{9/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{a(bc-ad)}+\frac{2b\left(-8a^3d^3-6a^2bcd\cdots\right.}{a\sqrt{a+b\cdots}}\cdots\right.}{2ac}\cdots}{4ac}\cdots}{6ac}\cdots\right.}{(ex)^{5/2}\left(ax+bx^2\right)^{3/2}\cdots}$$

$\downarrow$ 221

$$\frac{x^4(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{3\left(\frac{2b\left(-8a^3d^3-6a^2bcd^2-5ab^2c^2d+35b^3c^3\right)}{a\sqrt{a+bx}(bc-ad)}+\frac{\frac{32a^4d^{9/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)(bc-ad)\left(16a^3d^3+24a^2bcd\cdots\right.}{\sqrt{a}c}}{a(bc-ad)}\cdots\right.}{2ac}\cdots}{4ac}\cdots}{6ac}\cdots\right.}{(ex)^{5/2}\left(ax+bx^2\right)^{3/2}\cdots}$$

input

```
Int[1/((e*x)^(5/2)*(c + d*x)*(a*x + b*x^2)^(3/2)),x]
```

output
```
(x^4*(a + b*x)^(3/2)*(-1/3*1/(a*c*x^3*Sqrt[a + b*x]) - (-1/2*(7*b*c + 6*a*
d)/(a*c*x^2*Sqrt[a + b*x]) - (-(((35*b^2*c)/a + 30*b*d + (24*a*d^2)/c)/(x*
Sqrt[a + b*x])) - (3*((2*b*(35*b^3*c^3 - 5*a*b^2*c^2*d - 6*a^2*b*c*d^2 - 8
*a^3*d^3))/(a*(b*c - a*d)*Sqrt[a + b*x]) + ((32*a^4*d^(9/2)*ArcTan[(Sqrt[d
]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(c*Sqrt[b*c - a*d]) - (2*(b*c - a*d)*(3
5*b^3*c^3 + 30*a*b^2*c^2*d + 24*a^2*b*c*d^2 + 16*a^3*d^3)*ArcTanh[Sqrt[a +
 b*x]/Sqrt[a]])/(Sqrt[a]*c))/(a*(b*c - a*d))))/(2*a*c))/(4*a*c))/(6*a*c)))
/((e*x)^(5/2)*(a*x + b*x^2)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{(bx+a)\left(24a^2d^2x^2+42abcd\,x^2+57b^2c^2x^2-12a^2cdx-22ab\,c^2x+8a^2c^2\right)}{24a^4c^3x^2e^2\sqrt{ex}\,\sqrt{x(bx+a)}}-\frac{b\left(-\frac{\left(16a^3d^3+24a^2bc\,d^2+30a\,b^2c^2d+35b^3c^3\right)\operatorname{arctanh}\left(\sqrt{}\right.}{bc\sqrt{ae}}\right.}{8c}$
default	$-\frac{\sqrt{x(bx+a)}\left(48\sqrt{(bx+a)e}\,\sqrt{ae}\,\operatorname{arctanh}\left(\frac{d\sqrt{(bx+a)e}}{\sqrt{e(ad-bc)d}}\right)a^4d^5x^3-48\sqrt{(bx+a)e}\,\operatorname{arctanh}\left(\frac{\sqrt{(bx+a)e}}{\sqrt{ae}}\right)\sqrt{e(ad-bc)d}\,a^4d^4x^3-24\sqrt{(b\right.}{}$

input
```
int(1/(e*x)^(5/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(b*x+a)*(24*a^2*d^2*x^2+42*a*b*c*d*x^2+57*b^2*c^2*x^2-12*a^2*c*d*x-2
2*a*b*c^2*x+8*a^2*c^2)/a^4/c^3/x^2/e^2/(e*x)^(1/2)/(x*(b*x+a))^(1/2)-1/8/c
^3/a^4*b*(-(16*a^3*d^3+24*a^2*b*c*d^2+30*a*b^2*c^2*d+35*b^3*c^3)/b/c/(a*e)
^(1/2)*arctanh((b*e*x+a*e)^(1/2)/(a*e)^(1/2))-16*b^3*c^3/(a*d-b*c)/(b*e*x+
a*e)^(1/2)+16/(a*d-b*c)*a^4*d^5/b/c/(e*(a*d-b*c)*d)^(1/2)*arctanh(d*(b*e*x
+a*e)^(1/2)/(e*(a*d-b*c)*d)^(1/2)))/e^2*((b*x+a)*e)^(1/2)/(e*x)^(1/2)/(x*(
b*x+a))^(1/2)*x
```

Fricas [A] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 2013, normalized size of antiderivative = 5.49

$$\int \frac{1}{(ex)^{5/2}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x)^(5/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/48*(48*(a^5*b*d^4*e*x^5 + a^6*d^4*e*x^4)*sqrt(-d/((b*c - a*d)*e))*log(
-(b*d*x^2 + 2*sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(e*x)*sqrt(-d/((b*c - a*d)
*e)) - (b*c - 2*a*d)*x)/(d*x^2 + c*x)) - 3*((35*b^5*c^4 - 5*a*b^4*c^3*d -
6*a^2*b^3*c^2*d^2 - 8*a^3*b^2*c*d^3 - 16*a^4*b*d^4)*x^5 + (35*a*b^4*c^4 -
5*a^2*b^3*c^3*d - 6*a^3*b^2*c^2*d^2 - 8*a^4*b*c*d^3 - 16*a^5*d^4)*x^4)*sqr
t(a*e)*log(-(b*e*x^2 + 2*a*e*x + 2*sqrt(b*x^2 + a*x)*sqrt(a*e)*sqrt(e*x))/
x^2) + 2*(8*a^4*b*c^4 - 8*a^5*c^3*d + 3*(35*a*b^4*c^4 - 5*a^2*b^3*c^3*d -
6*a^3*b^2*c^2*d^2 - 8*a^4*b*c*d^3)*x^3 + (35*a^2*b^3*c^4 - 5*a^3*b^2*c^3*d
 - 6*a^4*b*c^2*d^2 - 24*a^5*c*d^3)*x^2 - 2*(7*a^3*b^2*c^4 - a^4*b*c^3*d -
6*a^5*c^2*d^2)*x)*sqrt(b*x^2 + a*x)*sqrt(e*x))/((a^5*b^2*c^5 - a^6*b*c^4*d
)*e^3*x^5 + (a^6*b*c^5 - a^7*c^4*d)*e^3*x^4), -1/24*(3*((35*b^5*c^4 - 5*a*
b^4*c^3*d - 6*a^2*b^3*c^2*d^2 - 8*a^3*b^2*c*d^3 - 16*a^4*b*d^4)*x^5 + (35*
a*b^4*c^4 - 5*a^2*b^3*c^3*d - 6*a^3*b^2*c^2*d^2 - 8*a^4*b*c*d^3 - 16*a^5*d
^4)*x^4)*sqrt(-a*e)*arctan(sqrt(b*x^2 + a*x)*sqrt(-a*e)*sqrt(e*x)/(a*e*x))
 + 24*(a^5*b*d^4*e*x^5 + a^6*d^4*e*x^4)*sqrt(-d/((b*c - a*d)*e))*log(-(b*d
*x^2 + 2*sqrt(b*x^2 + a*x)*(b*c - a*d)*sqrt(e*x)*sqrt(-d/((b*c - a*d)*e))
- (b*c - 2*a*d)*x)/(d*x^2 + c*x)) + (8*a^4*b*c^4 - 8*a^5*c^3*d + 3*(35*a*b
^4*c^4 - 5*a^2*b^3*c^3*d - 6*a^3*b^2*c^2*d^2 - 8*a^4*b*c*d^3)*x^3 + (35*a^
2*b^3*c^4 - 5*a^3*b^2*c^3*d - 6*a^4*b*c^2*d^2 - 24*a^5*c*d^3)*x^2 - 2*(7*a
^3*b^2*c^4 - a^4*b*c^3*d - 6*a^5*c^2*d^2)*x)*sqrt(b*x^2 + a*x)*sqrt(e*x...
```

Sympy [F]

$$\int \frac{1}{(ex)^{5/2}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx = \int \frac{1}{(ex)^{\frac{5}{2}}\left(x\left(a+bx\right)\right)^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/(e*x)**(5/2)/(d*x+c)/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral(1/((e*x)**(5/2)*(x*(a + b*x))**(3/2)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{(ex)^{5/2}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^2+ax\right)^{\frac{3}{2}}(dx+c)\,(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x)^(5/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a*x)^(3/2)*(d*x + c)*(e*x)^(5/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ex)^{5/2}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx=$$
$$-\frac{1}{24}\left(\frac{48\,d^5\arctan\left(\frac{\sqrt{bex+aed}}{\sqrt{bcde-ad^2e}}\right)}{(bc^5e^5|e|-ac^4de^5|e|)\sqrt{bcde-ad^2e}}+\frac{48\,b^4}{(a^4bce^5|e|-a^5de^5|e|)\sqrt{bex+ae}}+\frac{3\,(35\,b^3c^3+30\,ab^2c^2d+2}{\sqrt{}}\right.$$

input
```
integrate(1/(e*x)^(5/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
-1/24*(48*d^5*arctan(sqrt(b*e*x + a*e)*d/sqrt(b*c*d*e - a*d^2*e))/((b*c^5*
e^5*abs(e) - a*c^4*d*e^5*abs(e))*sqrt(b*c*d*e - a*d^2*e)) + 48*b^4/((a^4*b
*c*e^5*abs(e) - a^5*d*e^5*abs(e))*sqrt(b*e*x + a*e)) + 3*(35*b^3*c^3 + 30*
a*b^2*c^2*d + 24*a^2*b*c*d^2 + 16*a^3*d^3)*arctan(sqrt(b*e*x + a*e)/sqrt(-
a*e))/(sqrt(-a*e)*a^4*c^4*e^5*abs(e)) + (87*sqrt(b*e*x + a*e)*a^2*b^3*c^2*
e^2 + 54*sqrt(b*e*x + a*e)*a^3*b^2*c*d*e^2 + 24*sqrt(b*e*x + a*e)*a^4*b*d^
2*e^2 - 136*(b*e*x + a*e)^(3/2)*a*b^3*c^2*e - 96*(b*e*x + a*e)^(3/2)*a^2*b
^2*c*d*e - 48*(b*e*x + a*e)^(3/2)*a^3*b*d^2*e + 57*(b*e*x + a*e)^(5/2)*b^3
*c^2 + 42*(b*e*x + a*e)^(5/2)*a*b^2*c*d + 24*(b*e*x + a*e)^(5/2)*a^2*b*d^2
)/(a^4*b^3*c^3*e^8*x^3*abs(e)))*e^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{5/2}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(b\,x^2+a\,x\right)^{3/2}\,\left(e\,x\right)^{5/2}\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/((a*x + b*x^2)^(3/2)*(e*x)^(5/2)*(c + d*x)),x)
```

output
```
int(1/((a*x + b*x^2)^(3/2)*(e*x)^(5/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 721, normalized size of antiderivative = 1.96

$$\int \frac{1}{(ex)^{5/2}(c+dx)\left(ax+bx^2\right)^{3/2}}\,dx=\frac{\sqrt{e}\left(-48\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)a^5d^5x^3-105\sqrt{a}\,\sqrt{bx+a}\right.}{}$$

input
```
int(1/(e*x)^(5/2)/(d*x+c)/(b*x^2+a*x)^(3/2),x)
```

output

```
(sqrt(e)*( - 96*sqrt(d)*sqrt(a + b*x)*sqrt( - a*d + b*c)*atan((sqrt(a + b*
x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*a**5*d**4*x**3 - 48*sqrt(a)*sqrt(a + b
*x)*log(sqrt(a + b*x) - sqrt(a))*a**5*d**5*x**3 + 24*sqrt(a)*sqrt(a + b*x)
*log(sqrt(a + b*x) - sqrt(a))*a**4*b*c*d**4*x**3 + 6*sqrt(a)*sqrt(a + b*x)
*log(sqrt(a + b*x) - sqrt(a))*a**3*b**2*c**2*d**3*x**3 + 3*sqrt(a)*sqrt(a
+ b*x)*log(sqrt(a + b*x) - sqrt(a))*a**2*b**3*c**3*d**2*x**3 + 120*sqrt(a)
*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b**4*c**4*d*x**3 - 105*sqrt(
a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**5*c**5*x**3 + 48*sqrt(a)*
sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a**5*d**5*x**3 - 24*sqrt(a)*sqr
t(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a**4*b*c*d**4*x**3 - 6*sqrt(a)*sqr
t(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a**3*b**2*c**2*d**3*x**3 - 3*sqrt(
a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a**2*b**3*c**3*d**2*x**3 - 1
20*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a*b**4*c**4*d*x**3 +
 105*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**5*c**5*x**3 - 1
6*a**6*c**3*d**2 + 24*a**6*c**2*d**3*x - 48*a**6*c*d**4*x**2 + 32*a**5*b*c
**4*d - 20*a**5*b*c**3*d**2*x + 36*a**5*b*c**2*d**3*x**2 - 48*a**5*b*c*d**
4*x**3 - 16*a**4*b**2*c**5 - 32*a**4*b**2*c**4*d*x + 2*a**4*b**2*c**3*d**2
*x**2 + 12*a**4*b**2*c**2*d**3*x**3 + 28*a**3*b**3*c**5*x + 80*a**3*b**3*c
**4*d*x**2 + 6*a**3*b**3*c**3*d**2*x**3 - 70*a**2*b**4*c**5*x**2 + 240*a**
2*b**4*c**4*d*x**3 - 210*a*b**5*c**5*x**3))/(48*sqrt(a + b*x)*a**5*c**4...
```

3.194 $\int \frac{x^{7/2}}{(1-x)^2(2x-x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 41

$$\int \frac{x^{7/2}}{(1-x)^2(2x-x^2)^{3/2}}\,dx = \frac{8}{\sqrt{2-x}}+\frac{\sqrt{2-x}}{1-x}-7\text{arctanh}\left(\sqrt{2-x}\right)$$

output
```
8/(2-x)^(1/2)+(2-x)^(1/2)/(1-x)-7*arctanh((2-x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{x^{7/2}}{(1-x)^2(2x-x^2)^{3/2}}\,dx = \frac{\sqrt{x}\left(-10+9x+7\sqrt{-2+x}(-1+x)\arctan\left(\sqrt{-2+x}\right)\right)}{(-1+x)\sqrt{-((-2+x)x)}}$$

input
```
Integrate[x^(7/2)/((1 - x)^2*(2*x - x^2)^(3/2)),x]
```

output
```
(Sqrt[x]*(-10 + 9*x + 7*Sqrt[-2 + x]*(-1 + x)*ArcTan[Sqrt[-2 + x]]))/((-1
+ x)*Sqrt[-((-2 + x)*x)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.80, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1261, 100, 27, 87, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{(1-x)^2 (2x-x^2)^{3/2}} \, dx$$

$$\Big\downarrow \ 1261$$

$$\frac{(2-x)^{3/2} x^{3/2} \int \frac{x^2}{(1-x)^2 (2-x)^{3/2}} dx}{(2x-x^2)^{3/2}}$$

$$\Big\downarrow \ 100$$

$$\frac{(2-x)^{3/2} x^{3/2} \left(\frac{1}{(1-x)\sqrt{2-x}} - \int \frac{2x+5}{2(1-x)(2-x)^{3/2}} dx\right)}{(2x-x^2)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{(2-x)^{3/2} x^{3/2} \left(\frac{1}{(1-x)\sqrt{2-x}} - \frac{1}{2}\int \frac{2x+5}{(1-x)(2-x)^{3/2}} dx\right)}{(2x-x^2)^{3/2}}$$

$$\Big\downarrow \ 87$$

$$\frac{(2-x)^{3/2} x^{3/2} \left(\frac{1}{2}\left(\frac{18}{\sqrt{2-x}} - 7\int \frac{1}{(1-x)\sqrt{2-x}} dx\right) + \frac{1}{(1-x)\sqrt{2-x}}\right)}{(2x-x^2)^{3/2}}$$

$$\Big\downarrow \ 73$$

$$\frac{(2-x)^{3/2} x^{3/2} \left(\frac{1}{2}\left(14\int \frac{1}{1-x} d\sqrt{2-x} + \frac{18}{\sqrt{2-x}}\right) + \frac{1}{(1-x)\sqrt{2-x}}\right)}{(2x-x^2)^{3/2}}$$

$$\Big\downarrow \ 220$$

$$\frac{(2-x)^{3/2} x^{3/2} \left(\frac{1}{2}\left(\frac{18}{\sqrt{2-x}} - 14\text{arctanh}\left(\sqrt{2-x}\right)\right) + \frac{1}{(1-x)\sqrt{2-x}}\right)}{(2x-x^2)^{3/2}}$$

input
```
Int[x^(7/2)/((1 - x)^2*(2*x - x^2)^(3/2)),x]
```

output
```
((2 - x)^(3/2)*x^(3/2)*(1/((1 - x)*Sqrt[2 - x]) + (18/Sqrt[2 - x] - 14*Arc
Tanh[Sqrt[2 - x]])/2))/(2*x - x^2)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.27

method	result	size
risch	$\frac{(9x-10)\sqrt{x}}{(x-1)\sqrt{-x(x-2)}}-\frac{7\,\operatorname{arctanh}\left(\sqrt{2-x}\right)\sqrt{2-x}\,\sqrt{x}}{\sqrt{-x(x-2)}}$	52
default	$\frac{\sqrt{-x(x-2)}\,\left(7\ln\left(\sqrt{2-x}-1\right)\sqrt{2-x}\,x-7\ln\left(\sqrt{2-x}+1\right)\sqrt{2-x}\,x-7\ln\left(\sqrt{2-x}-1\right)\sqrt{2-x}+7\ln\left(\sqrt{2-x}+1\right)\sqrt{2-x}+18x-20\right)}{2\sqrt{x}\,(x-2)\left(\sqrt{2-x}-1\right)\left(\sqrt{2-x}+1\right)}$	124

input
```
int(x^(7/2)/(1-x)^2/(-x^2+2*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(9*x-10)/(x-1)*x^(1/2)/(-x*(x-2))^(1/2)-7*arctanh((2-x)^(1/2))*(2-x)^(1/2)
*x^(1/2)/(-x*(x-2))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(34) = 68.

Time = 0.13 (sec) , antiderivative size = 134, normalized size of antiderivative = 3.27

$$\int \frac{x^{7/2}}{(1-x)^2\,(2x-x^2)^{3/2}}\,dx = -\frac{2\sqrt{-x^2+2\,x}(9\,x-10)\sqrt{x}-7\,(x^3-3\,x^2+2\,x)\log\left(\frac{x^2+\sqrt{-x^2+2\,x}\sqrt{x}-2\,x}{x^2-2\,x}\right)+7\,(x^3-3\,x^2+2\,x)\log\left(-\frac{x^2-\cdots}{\cdots}\right.}{2\,(x^3-3\,x^2+2\,x)}$$

input
```
integrate(x^(7/2)/(1-x)^2/(-x^2+2*x)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(-x^2 + 2*x)*(9*x - 10)*sqrt(x) - 7*(x^3 - 3*x^2 + 2*x)*log((x
^2 + sqrt(-x^2 + 2*x)*sqrt(x) - 2*x)/(x^2 - 2*x)) + 7*(x^3 - 3*x^2 + 2*x)*
log(-(x^2 - sqrt(-x^2 + 2*x)*sqrt(x) - 2*x)/(x^2 - 2*x)))/(x^3 - 3*x^2 + 2
*x)
```

Sympy [F]

$$\int \frac{x^{7/2}}{(1-x)^2 (2x-x^2)^{3/2}}\,dx = \int \frac{x^{\frac{7}{2}}}{(-x(x-2))^{\frac{3}{2}}(x-1)^2}\,dx$$

input
```
integrate(x**(7/2)/(1-x)**2/(-x**2+2*x)**(3/2),x)
```

output
```
Integral(x**(7/2)/((-x*(x - 2))**(3/2)*(x - 1)**2), x)
```

Maxima [F]

$$\int \frac{x^{7/2}}{(1-x)^2 (2x-x^2)^{3/2}}\,dx = \int \frac{x^{\frac{7}{2}}}{(-x^2+2x)^{\frac{3}{2}}(x-1)^2}\,dx$$

input
```
integrate(x^(7/2)/(1-x)^2/(-x^2+2*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^(7/2)/((-x^2 + 2*x)^(3/2)*(x - 1)^2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.27

$$\int \frac{x^{7/2}}{(1-x)^2\,(2x-x^2)^{3/2}}\,dx = -\frac{9\,x-10}{(-x+2)^{\frac{3}{2}}-\sqrt{-x+2}} - \frac{7}{2}\,\log\left(\sqrt{-x+2}+1\right) + \frac{7}{2}\,\log\left(\left|\sqrt{-x+2}-1\right|\right)$$

input
```
integrate(x^(7/2)/(1-x)^2/(-x^2+2*x)^(3/2),x, algorithm="giac")
```

output
```
-(9*x - 10)/((-x + 2)^(3/2) - sqrt(-x + 2)) - 7/2*log(sqrt(-x + 2) + 1) +
7/2*log(abs(sqrt(-x + 2) - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{(1-x)^2\,(2x-x^2)^{3/2}}\,dx = \int \frac{x^{7/2}}{(2\,x-x^2)^{3/2}\,(x-1)^2}\,dx$$

input
```
int(x^(7/2)/((2*x - x^2)^(3/2)*(x - 1)^2),x)
```

output
```
int(x^(7/2)/((2*x - x^2)^(3/2)*(x - 1)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.20

$$\int \frac{x^{7/2}}{(1-x)^2\,(2x-x^2)^{3/2}}\,dx = \frac{7\sqrt{-x+2}\log\left(\sqrt{-x+2}-1\right)x-7\sqrt{-x+2}\log\left(\sqrt{-x+2}-1\right)-7\sqrt{-x+}}{2\sqrt{-x+2}\,(x}$$

input
```
int(x^(7/2)/(1-x)^2/(-x^2+2*x)^(3/2),x)
```

output
```
(7*sqrt( - x + 2)*log(sqrt( - x + 2) - 1)*x - 7*sqrt( - x + 2)*log(sqrt( -
 x + 2) - 1) - 7*sqrt( - x + 2)*log(sqrt( - x + 2) + 1)*x + 7*sqrt( - x +
2)*log(sqrt( - x + 2) + 1) + 18*x - 20)/(2*sqrt( - x + 2)*(x - 1))
```

3.195 $\int \frac{x^4\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 376

$$\begin{aligned}\int \frac{x^4\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = &-\frac{4(b^2c^2+4abcd-24a^2d^2)\,x\sqrt{c+dx}}{15b^3d^2\sqrt{ax+bx^2}}-\frac{2x^3\sqrt{c+dx}}{b\sqrt{ax+bx^2}}\\ &+\frac{2(bc-24ad)\sqrt{c+dx}\sqrt{ax+bx^2}}{15b^3d}+\frac{12x\sqrt{c+dx}\sqrt{ax+bx^2}}{5b^2}\\ &+\frac{4\sqrt{a}(b^2c^2+4abcd-24a^2d^2)\,\sqrt{x}\sqrt{c+dx}E\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15b^{7/2}d^2\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}}\\ &-\frac{2a^{3/2}(bc-24ad)\sqrt{x}\sqrt{c+dx}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15b^{7/2}d\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}}\end{aligned}$$

output
```
-4/15*(-24*a^2*d^2+4*a*b*c*d+b^2*c^2)*x*(d*x+c)^(1/2)/b^3/d^2/(b*x^2+a*x)^
(1/2)-2*x^3*(d*x+c)^(1/2)/b/(b*x^2+a*x)^(1/2)+2/15*(-24*a*d+b*c)*(d*x+c)^(
1/2)*(b*x^2+a*x)^(1/2)/b^3/d+12/5*x*(d*x+c)^(1/2)*(b*x^2+a*x)^(1/2)/b^2+4/
15*a^(1/2)*(-24*a^2*d^2+4*a*b*c*d+b^2*c^2)*x^(1/2)*(d*x+c)^(1/2)*EllipticE
(b^(1/2)*x^(1/2)/a^(1/2)/(1+b*x/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(7/2)/d^2/(a
*(d*x+c)/c/(b*x+a))^(1/2)/(b*x^2+a*x)^(1/2)-2/15*a^(3/2)*(-24*a*d+b*c)*x^(
1/2)*(d*x+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x^(1/2)/a^(1/2)),(1-a*d/
b/c)^(1/2))/b^(7/2)/d/(a*(d*x+c)/c/(b*x+a))^(1/2)/(b*x^2+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 16.31 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.82

$$\int \frac{x^4\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \frac{2(c+dx)\left(48a^3d^2-8a^2bd(c-3dx)+b^3x(-2c^2+cdx+3d^2x^2)-ab^2(2c^2+7cdx+\right.}{}$$

input
```
Integrate[(x^4*Sqrt[c + d*x])/(a*x + b*x^2)^(3/2),x]
```

output
```
(2*(c + d*x)*(48*a^3*d^2 - 8*a^2*b*d*(c - 3*d*x) + b^3*x*(-2*c^2 + c*d*x +
 3*d^2*x^2) - a*b^2*(2*c^2 + 7*c*d*x + 6*d^2*x^2)) + (4*I)*Sqrt[a/b]*b*d*(
-(b^2*c^2) - 4*a*b*c*d + 24*a^2*d^2)*Sqrt[1 + a/(b*x)]*Sqrt[1 + c/(d*x)]*x
^(3/2)*EllipticE[I*ArcSinh[Sqrt[a/b]/Sqrt[x]], (b*c)/(a*d)] - (2*I)*Sqrt[a
/b]*b*d*(-(b^2*c^2) - 32*a*b*c*d + 48*a^2*d^2)*Sqrt[1 + a/(b*x)]*Sqrt[1 +
c/(d*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[a/b]/Sqrt[x]], (b*c)/(a*d)])/(15
*b^4*d^2*Sqrt[x*(a + b*x)]*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {1261, 108, 27, 171, 27, 171, 27, 176, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{x^{3/2}(a+bx)^{3/2}\int \frac{x^{5/2}\sqrt{c+dx}}{(a+bx)^{3/2}}dx}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{108}$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2\int \frac{x^{3/2}(5c+6dx)}{2\sqrt{a+bx}\sqrt{c+dx}}dx}{b}-\frac{2x^{5/2}\sqrt{c+dx}}{b\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{\int \frac{x^{3/2}(5c+6dx)}{\sqrt{a+bx}\sqrt{c+dx}}dx}{b}-\frac{2x^{5/2}\sqrt{c+dx}}{b\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 171$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{\frac{2\int -\frac{d\sqrt{x}(18ac-(bc-24ad)x)}{2\sqrt{a+bx}\sqrt{c+dx}}dx}{5bd}+\frac{12x^{3/2}\sqrt{a+bx}\sqrt{c+dx}}{5b}}{b}-\frac{2x^{5/2}\sqrt{c+dx}}{b\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{\frac{12x^{3/2}\sqrt{a+bx}\sqrt{c+dx}}{5b}-\frac{\int \frac{\sqrt{x}(18ac-(bc-24ad)x)}{\sqrt{a+bx}\sqrt{c+dx}}dx}{5b}}{b}-\frac{2x^{5/2}\sqrt{c+dx}}{b\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 171$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{\frac{12x^{3/2}\sqrt{a+bx}\sqrt{c+dx}}{5b}-\frac{\frac{2\int \frac{ac(bc-24ad)+2\left(b^2c^2+4abdc-24a^2d^2\right)x}{2\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{3bd}-\frac{2\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}(bc-24ad)}{3bd}}{5b}}{b}-\frac{2x^{5/2}\sqrt{c+dx}}{b\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{\frac{12x^{3/2}\sqrt{a+bx}\sqrt{c+dx}}{5b}-\frac{\frac{\int \frac{ac(bc-24ad)+2\left(b^2c^2+4abdc-24a^2d^2\right)x}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{3bd}-\frac{2\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}(bc-24ad)}{3bd}}{5b}}{b}-\frac{2x^{5/2}\sqrt{c+dx}}{b\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 176$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{\frac{12x^{3/2}\sqrt{a+bx}\sqrt{c+dx}}{5b}-\frac{\frac{\frac{2\left(-24a^2d^2+4abcd+b^2c^2\right)\int\frac{\sqrt{c+dx}}{\sqrt{x}\sqrt{a+bx}}dx}{d}-\frac{c\left(-24a^2d^2+7abcd+2b^2c^2\right)\int\frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{d}}{3bd}-2\sqrt{x}\sqrt{a+bx}\cdots}{5b}}{b}\right.}{(ax+bx^2)^{3/2}}$$

$\downarrow$ 122

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{\frac{12x^{3/2}\sqrt{a+bx}\sqrt{c+dx}}{5b}-\frac{\frac{\frac{2\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\left(-24a^2d^2+4abcd+b^2c^2\right)\int\frac{\sqrt{\frac{dx}{c}+1}}{\sqrt{x}\sqrt{\frac{bx}{a}+1}}dx}{d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-\frac{c\left(-24a^2d^2+7abcd+2b^2c^2\right)\int\frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}d\cdots}{d}}{3bd}\cdots}{5b}}{b}\right.}{(ax+bx^2)^{3/2}}$$

$\downarrow$ 120

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{\frac{12x^{3/2}\sqrt{a+bx}\sqrt{c+dx}}{5b}-\frac{\frac{\frac{4\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\left(-24a^2d^2+4abcd+b^2c^2\right)E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-\frac{c\left(-24a^2d^2+7abcd+2b^2c^2\right)\int\frac{\cdots}{\sqrt{x}\sqrt{a}\cdots}}{d}}{3bd}\cdots}{5b}}{b}\right.}{(ax+bx^2)^{3/2}}$$

$\downarrow$ 127

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{\frac{12x^{3/2}\sqrt{a+bx}\sqrt{c+dx}}{5b}-\frac{\frac{\frac{4\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\left(-24a^2d^2+4abcd+b^2c^2\right)E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-\frac{c\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}\left(-24a^2d^2+7abcd+\cdots\right.}{d\sqrt{a+bx}\cdots}}{3bd}\cdots}{5b}}{b}\right.}{(ax+bx^2)^{3/2}}$$

$\downarrow$ 126

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{\frac{12x^{3/2}\sqrt{a+bx}\sqrt{c+dx}}{5b}-\frac{\frac{\frac{4\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\left(-24a^2d^2+4abcd+b^2c^2\right)E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-\frac{2\sqrt{-a}c\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}\left(-24a^2d^2+7\ldots\right.}{\sqrt{b}d\ldots}}{3bd}}{5b}}{b}\right.}{\left(ax+bx^2\right)^{3/2}}$$

input
```
Int[(x^4*Sqrt[c + d*x])/(a*x + b*x^2)^(3/2),x]
```

output
```
(x^(3/2)*(a + b*x)^(3/2)*((-2*x^(5/2)*Sqrt[c + d*x])/(b*Sqrt[a + b*x]) + (
(12*x^(3/2)*Sqrt[a + b*x]*Sqrt[c + d*x])/(5*b) - ((-2*(b*c - 24*a*d)*Sqrt[
x]*Sqrt[a + b*x]*Sqrt[c + d*x])/(3*b*d) + ((4*Sqrt[-a]*(b^2*c^2 + 4*a*b*c*
d - 24*a^2*d^2)*Sqrt[1 + (b*x)/a]*Sqrt[c + d*x]*EllipticE[ArcSin[(Sqrt[b]*
Sqrt[x])/Sqrt[-a]], (a*d)/(b*c)])/(Sqrt[b]*d*Sqrt[a + b*x]*Sqrt[1 + (d*x)/
c]) - (2*Sqrt[-a]*c*(2*b^2*c^2 + 7*a*b*c*d - 24*a^2*d^2)*Sqrt[1 + (b*x)/a]
*Sqrt[1 + (d*x)/c]*EllipticF[ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[-a]], (a*d)/(b*
c)])/(Sqrt[b]*d*Sqrt[a + b*x]*Sqrt[c + d*x]))/(3*b*d))/(5*b))/b))/(a*x + b
*x^2)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] && !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.55 (sec), antiderivative size = 571, normalized size of antiderivative = 1.52

method	result
elliptic	$\frac{\sqrt{x(bx+a)(dx+c)}\left(-\frac{2\left(bd\,x^2+cbx\right)a^2}{b^4\sqrt{\left(x+\frac{a}{b}\right)\left(bd\,x^2+cbx\right)}}+\frac{2x\sqrt{bd\,x^3+ad\,x^2+bc\,x^2+acx}}{5b^2}+\frac{2\left(-\frac{ad-bc}{b^2}-\frac{2(2ad+2bc)}{5b^2}\right)\sqrt{bd\,x^3+ad\,x^2+bc\,x^2+acx}}{3bd}+\frac{2\left(\right.}{}$
default	$-\frac{2\sqrt{dx+c}\,\sqrt{x(bx+a)}\left(48\sqrt{\frac{dx+c}{c}}\,\sqrt{\frac{d(bx+a)}{ad-bc}}\,\sqrt{-\frac{xd}{c}}\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+c}{c}},\sqrt{-\frac{bc}{ad-bc}}\right)a^3cd^3-32\sqrt{\frac{dx+c}{c}}\,\sqrt{\frac{d(bx+a)}{ad-bc}}\,\sqrt{-\frac{xd}{c}}\,\mathrm{Elliptic}}{}$

input
```
int(x^4*(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(x*(b*x+a)*(d*x+c))^(1/2)/(x*(b*x+a))^(1/2)/(d*x+c)^(1/2)*(-2*(b*d*x^2+b*c
*x)*a^2/b^4/((x+a/b)*(b*d*x^2+b*c*x))^(1/2)+2/5/b^2*x*(b*d*x^3+a*d*x^2+b*c
*x^2+a*c*x)^(1/2)+2/3*(-1/b^2*(a*d-b*c)-2/5/b^2*(2*a*d+2*b*c))/b/d*(b*d*x^
3+a*d*x^2+b*c*x^2+a*c*x)^(1/2)+2*(1/b^3*c*a^2-1/3*(-1/b^2*(a*d-b*c)-2/5/b^
2*(2*a*d+2*b*c))/b/d*a*c)*c/d*((x+c/d)/c*d)^(1/2)*((x+a/b)/(-c/d+a/b))^(1/
2)*(-1/c*x*d)^(1/2)/(b*d*x^3+a*d*x^2+b*c*x^2+a*c*x)^(1/2)*EllipticF(((x+c/
d)/c*d)^(1/2),(-c/d/(-c/d+a/b))^(1/2))+2*(a*(a*d-b*c)/b^3+a^2*d/b^3-3/5*a*
c/b^2-2/3*(-1/b^2*(a*d-b*c)-2/5/b^2*(2*a*d+2*b*c))/b/d*(a*d+b*c))*c/d*((x+
c/d)/c*d)^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)*(-1/c*x*d)^(1/2)/(b*d*x^3+a*d*x
^2+b*c*x^2+a*c*x)^(1/2)*((-c/d+a/b)*EllipticE(((x+c/d)/c*d)^(1/2),(-c/d/(-
c/d+a/b))^(1/2))-a/b*EllipticF(((x+c/d)/c*d)^(1/2),(-c/d/(-c/d+a/b))^(1/2)
)))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.38

$$\int \frac{x^4\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \frac{2\left((2\,ab^3c^3+7\,a^2b^2c^2d+32\,a^3bcd^2-48\,a^4d^3+(2\,b^4c^3+7\,ab^3c^2d+32\,a^2b^2cd^2-48\right.}{}$$

input
```
integrate(x^4*(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
2/45*((2*a*b^3*c^3 + 7*a^2*b^2*c^2*d + 32*a^3*b*c*d^2 - 48*a^4*d^3 + (2*b^
4*c^3 + 7*a*b^3*c^2*d + 32*a^2*b^2*c*d^2 - 48*a^3*b*d^3)*x)*sqrt(b*d)*weie
rstrassPInverse(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*
c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), 1/3*(3*b*d*x +
 b*c + a*d)/(b*d)) + 6*(a*b^3*c^2*d + 4*a^2*b^2*c*d^2 - 24*a^3*b*d^3 + (b^
4*c^2*d + 4*a*b^3*c*d^2 - 24*a^2*b^2*d^3)*x)*sqrt(b*d)*weierstrassZeta(4/3
*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d
 - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), weierstrassPInverse(4/3*(b^2*c^2
- a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b
*c*d^2 + 2*a^3*d^3)/(b^3*d^3), 1/3*(3*b*d*x + b*c + a*d)/(b*d))) + 3*(3*b^
4*d^3*x^2 + a*b^3*c*d^2 - 24*a^2*b^2*d^3 + (b^4*c*d^2 - 6*a*b^3*d^3)*x)*sq
rt(b*x^2 + a*x)*sqrt(d*x + c))/(b^6*d^3*x + a*b^5*d^3)
```

Sympy [F]

$$\int \frac{x^4\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{x^4\sqrt{c+dx}}{(x\,(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(d*x+c)**(1/2)/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral(x**4*sqrt(c + d*x)/(x*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx+c}x^4}{(bx^2+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)*x^4/(b*x^2 + a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx+c}x^4}{(bx^2+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)*x^4/(b*x^2 + a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{x^4\,\sqrt{c+d\,x}}{{\left(b\,x^2+a\,x\right)}^{3/2}}\,dx$$

input
```
int((x^4*(c + d*x)^(1/2))/(a*x + b*x^2)^(3/2),x)
```

output
```
int((x^4*(c + d*x)^(1/2))/(a*x + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^4*(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x)
```

output
```
(36*sqrt(x)*sqrt(c + d*x)*sqrt(a + b*x)*a*c*d - 24*sqrt(x)*sqrt(c + d*x)*s
qrt(a + b*x)*a*d**2*x - 6*sqrt(x)*sqrt(c + d*x)*sqrt(a + b*x)*b*c**2 + 4*s
qrt(x)*sqrt(c + d*x)*sqrt(a + b*x)*b*c*d*x + 12*sqrt(x)*sqrt(c + d*x)*sqrt
(a + b*x)*b*d**2*x**2 - 18*int((sqrt(c + d*x)*sqrt(a + b*x))/(sqrt(x)*a**2
*c + sqrt(x)*a**2*d*x + 2*sqrt(x)*a*b*c*x + 2*sqrt(x)*a*b*d*x**2 + sqrt(x)
*b**2*c*x**2 + sqrt(x)*b**2*d*x**3),x)*a**3*c**2*d + 3*int((sqrt(c + d*x)*
sqrt(a + b*x))/(sqrt(x)*a**2*c + sqrt(x)*a**2*d*x + 2*sqrt(x)*a*b*c*x + 2*
sqrt(x)*a*b*d*x**2 + sqrt(x)*b**2*c*x**2 + sqrt(x)*b**2*d*x**3),x)*a**2*b*
c**3 - 18*int((sqrt(c + d*x)*sqrt(a + b*x))/(sqrt(x)*a**2*c + sqrt(x)*a**2
*d*x + 2*sqrt(x)*a*b*c*x + 2*sqrt(x)*a*b*d*x**2 + sqrt(x)*b**2*c*x**2 + sq
rt(x)*b**2*d*x**3),x)*a**2*b*c**2*d*x + 3*int((sqrt(c + d*x)*sqrt(a + b*x)
)/(sqrt(x)*a**2*c + sqrt(x)*a**2*d*x + 2*sqrt(x)*a*b*c*x + 2*sqrt(x)*a*b*d
*x**2 + sqrt(x)*b**2*c*x**2 + sqrt(x)*b**2*d*x**3),x)*a*b**2*c**3*x + 48*i
nt((sqrt(x)*sqrt(c + d*x)*sqrt(a + b*x)*x)/(a**2*c + a**2*d*x + 2*a*b*c*x
 + 2*a*b*d*x**2 + b**2*c*x**2 + b**2*d*x**3),x)*a**3*d**3 - 32*int((sqrt(x)
*sqrt(c + d*x)*sqrt(a + b*x)*x)/(a**2*c + a**2*d*x + 2*a*b*c*x + 2*a*b*d*x
**2 + b**2*c*x**2 + b**2*d*x**3),x)*a**2*b*c*d**2 + 48*int((sqrt(x)*sqrt(c
 + d*x)*sqrt(a + b*x)*x)/(a**2*c + a**2*d*x + 2*a*b*c*x + 2*a*b*d*x**2 + b
**2*c*x**2 + b**2*d*x**3),x)*a**2*b*d**3*x - int((sqrt(x)*sqrt(c + d*x)*sq
rt(a + b*x)*x)/(a**2*c + a**2*d*x + 2*a*b*c*x + 2*a*b*d*x**2 + b**2*c*x...
```

3.196 $\int \frac{x^3\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 296

$$\int \frac{x^3\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \frac{2(bc-8ad)x\sqrt{c+dx}}{3b^2d\sqrt{ax+bx^2}} - \frac{2x^2\sqrt{c+dx}}{b\sqrt{ax+bx^2}} + \frac{8\sqrt{c+dx}\sqrt{ax+bx^2}}{3b^2} - \frac{2\sqrt{a}(bc-8ad)\sqrt{x}\sqrt{c+dx}E\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3b^{5/2}d\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}} - \frac{8a^{3/2}\sqrt{x}\sqrt{c+dx}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3b^{5/2}\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}}$$

output
```
2/3*(-8*a*d+b*c)*x*(d*x+c)^(1/2)/b^2/d/(b*x^2+a*x)^(1/2)-2*x^2*(d*x+c)^(1/
2)/b/(b*x^2+a*x)^(1/2)+8/3*(d*x+c)^(1/2)*(b*x^2+a*x)^(1/2)/b^2-2/3*a^(1/2)
*(-8*a*d+b*c)*x^(1/2)*(d*x+c)^(1/2)*EllipticE(b^(1/2)*x^(1/2)/a^(1/2)/(1+b
*x/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(5/2)/d/(a*(d*x+c)/c/(b*x+a))^(1/2)/(b*x^
2+a*x)^(1/2)-8/3*a^(3/2)*x^(1/2)*(d*x+c)^(1/2)*InverseJacobiAM(arctan(b^(1
/2)*x^(1/2)/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(5/2)/(a*(d*x+c)/c/(b*x+a))^(1/2
)/(b*x^2+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.12 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.80

$$\int \frac{x^3\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \frac{2(c+dx)\left(-8a^2d+ab(c-4dx)+b^2x(c+dx)\right)-2i\sqrt{\frac{a}{b}}bd(-bc+8ad)\sqrt{1+\frac{a}{bx}}\sqrt{1}}{}$$

input
```
Integrate[(x^3*Sqrt[c + d*x])/(a*x + b*x^2)^(3/2),x]
```

output
```
(2*(c + d*x)*(-8*a^2*d + a*b*(c - 4*d*x) + b^2*x*(c + d*x)) - (2*I)*Sqrt[a
/b]*b*d*(-(b*c) + 8*a*d)*Sqrt[1 + a/(b*x)]*Sqrt[1 + c/(d*x)]*x^(3/2)*Ellip
ticE[I*ArcSinh[Sqrt[a/b]/Sqrt[x]], (b*c)/(a*d)] + (2*I)*Sqrt[a/b]*b*d*(-5*
b*c + 8*a*d)*Sqrt[1 + a/(b*x)]*Sqrt[1 + c/(d*x)]*x^(3/2)*EllipticF[I*ArcSi
nh[Sqrt[a/b]/Sqrt[x]], (b*c)/(a*d)])/(3*b^3*d*Sqrt[x*(a + b*x)]*Sqrt[c + d
*x])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1261, 108, 27, 171, 27, 176, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{c+dx}}{(ax+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 1261}$$

$$\frac{x^{3/2}(a+bx)^{3/2}\int \frac{x^{3/2}\sqrt{c+dx}}{(a+bx)^{3/2}}dx}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 108}$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2\int \frac{\sqrt{x}(3c+4dx)}{2\sqrt{a+bx}\sqrt{c+dx}}dx}{b}-\frac{2x^{3/2}\sqrt{c+dx}}{b\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{\int \frac{\sqrt{x}(3c+4dx)}{\sqrt{a+bx}\sqrt{c+dx}}dx}{b}-\frac{2x^{3/2}\sqrt{c+dx}}{b\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 171}$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{\frac{2\int -\frac{d(4ac-(bc-8ad)x)}{2\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{3bd}+\frac{8\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}{3b}}{b}-\frac{2x^{3/2}\sqrt{c+dx}}{b\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{\frac{8\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}{3b}-\frac{\int \frac{4ac-(bc-8ad)x}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{3b}}{b}-\frac{2x^{3/2}\sqrt{c+dx}}{b\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 176}$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{\frac{8\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}{3b}-\frac{\frac{c(bc-4ad)\int \frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{d}-\frac{(bc-8ad)\int \frac{\sqrt{c+dx}}{\sqrt{x}\sqrt{a+bx}}dx}{d}}{3b}}{b}-\frac{2x^{3/2}\sqrt{c+dx}}{b\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 122}$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{\frac{8\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}{3b}-\frac{\frac{c(bc-4ad)\int \frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{d}-\frac{\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}(bc-8ad)\int \frac{\sqrt{\frac{dx}{c}+1}}{\sqrt{x}\sqrt{\frac{bx}{a}+1}}dx}{d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}}{3b}}{b}-\frac{2x^{3/2}\sqrt{c+dx}}{b\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 120}$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{\frac{8\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}{3b}-\frac{\frac{c(bc-4ad)\int\frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{d}-\frac{2\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}(bc-8ad)E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}}{3b}}{b}-\frac{2x^{3/2}\sqrt{c+dx}}{b\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{127}$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{\frac{8\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}{3b}-\frac{\frac{c\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}(bc-4ad)\int\frac{1}{\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}}dx}{d\sqrt{a+bx}\sqrt{c+dx}}-\frac{2\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}(bc-8ad)E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}}{3b}}{b}-\right.}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{126}$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{\frac{8\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}{3b}-\frac{\frac{2\sqrt{-a}c\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}(bc-4ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a+bx}\sqrt{c+dx}}-\frac{2\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}(bc-8ad)E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right.\right.}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}}{3b}}{b}\right.}{(ax+bx^2)^{3/2}}$$

input
```
Int[(x^3*Sqrt[c + d*x])/(a*x + b*x^2)^(3/2),x]
```

output
```
(x^(3/2)*(a + b*x)^(3/2)*((-2*x^(3/2)*Sqrt[c + d*x])/(b*Sqrt[a + b*x]) + (
(8*Sqrt[x]*Sqrt[a + b*x]*Sqrt[c + d*x])/(3*b) - ((-2*Sqrt[-a]*(b*c - 8*a*d
)*Sqrt[1 + (b*x)/a]*Sqrt[c + d*x]*EllipticE[ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[
-a]], (a*d)/(b*c)])/(Sqrt[b]*d*Sqrt[a + b*x]*Sqrt[1 + (d*x)/c]) + (2*Sqrt[
-a]*c*(b*c - 4*a*d)*Sqrt[1 + (b*x)/a]*Sqrt[1 + (d*x)/c]*EllipticF[ArcSin[(
Sqrt[b]*Sqrt[x])/Sqrt[-a]], (a*d)/(b*c)])/(Sqrt[b]*d*Sqrt[a + b*x]*Sqrt[c
+ d*x]))/(3*b))/b))/(a*x + b*x^2)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.44

method	result
elliptic	$\frac{\sqrt{x(bx+a)(dx+c)}\left(\frac{2\left(bd\,x^2+cbx\right)a}{b^3\sqrt{\left(x+\frac{a}{b}\right)\left(bd\,x^2+cbx\right)}}+\frac{2\sqrt{bd\,x^3+ad\,x^2+bc\,x^2+acx}}{3b^2}-\frac{8a\,c^2\sqrt{\frac{\left(x+\frac{c}{d}\right)d}{c}}\sqrt{\frac{x+\frac{a}{b}}{-\frac{c}{d}+\frac{a}{b}}}\sqrt{-\frac{xd}{c}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{c}{d}\right)d}{c}},\sqrt{-\ldots}\right.}{3b^2d\sqrt{bd\,x^3+ad\,x^2+bc\,x^2+acx}}\right.}{}$
default	$\frac{2\sqrt{dx+c}\,\sqrt{x(bx+a)}\left(8\sqrt{\frac{dx+c}{c}}\,\sqrt{\frac{d(bx+a)}{ad-bc}}\,\sqrt{-\frac{xd}{c}}\,\text{EllipticF}\left(\sqrt{\frac{dx+c}{c}},\sqrt{-\frac{bc}{ad-bc}}\right)a^2c\,d^2-5a\,c^2\sqrt{\frac{dx+c}{c}}\,\sqrt{\frac{d(bx+a)}{ad-bc}}\,\sqrt{-\frac{xd}{c}}\,\text{EllipticF}\right.}{}$

input
```
int(x^3*(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(x*(b*x+a)*(d*x+c))^(1/2)/(x*(b*x+a))^(1/2)/(d*x+c)^(1/2)*(2*(b*d*x^2+b*c*
x)*a/b^3/((x+a/b)*(b*d*x^2+b*c*x))^(1/2)+2/3/b^2*(b*d*x^3+a*d*x^2+b*c*x^2+
a*c*x)^(1/2)-8/3*a*c^2/b^2/d*((x+c/d)/c*d)^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2
)*(-1/c*x*d)^(1/2)/(b*d*x^3+a*d*x^2+b*c*x^2+a*c*x)^(1/2)*EllipticF(((x+c/d
)/c*d)^(1/2),(-c/d/(-c/d+a/b))^(1/2))+2*(-1/b^2*(a*d-b*c)-1/b^2*d*a-2/3/b^
2*(a*d+b*c))*c/d*((x+c/d)/c*d)^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)*(-1/c*x*d)
^(1/2)/(b*d*x^3+a*d*x^2+b*c*x^2+a*c*x)^(1/2)*((-c/d+a/b)*EllipticE(((x+c/d
)/c*d)^(1/2),(-c/d/(-c/d+a/b))^(1/2))-a/b*EllipticF(((x+c/d)/c*d)^(1/2),(-
c/d/(-c/d+a/b))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.44

$$\int \frac{x^3\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx =$$
$$-\frac{2\left(\left(ab^2c^2+5\,a^2bcd-8\,a^3d^2+(b^3c^2+5\,ab^2cd-8\,a^2bd^2)x\right)\sqrt{bd}\text{weierstrassPInverse}\left(\frac{4\,(b^2c^2-abcd+a^2d^2)}{3\,b^2d^2},-\frac{4}{}\right.}{}$$

input
```
integrate(x^3*(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/9*((a*b^2*c^2 + 5*a^2*b*c*d - 8*a^3*d^2 + (b^3*c^2 + 5*a*b^2*c*d - 8*a^
2*b*d^2)*x)*sqrt(b*d)*weierstrassPInverse(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2
)/(b^2*d^2), -4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)
/(b^3*d^3), 1/3*(3*b*d*x + b*c + a*d)/(b*d)) + 3*(a*b^2*c*d - 8*a^2*b*d^2
+ (b^3*c*d - 8*a*b^2*d^2)*x)*sqrt(b*d)*weierstrassZeta(4/3*(b^2*c^2 - a*b*
c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
 + 2*a^3*d^3)/(b^3*d^3), weierstrassPInverse(4/3*(b^2*c^2 - a*b*c*d + a^2*
d^2)/(b^2*d^2), -4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d
^3)/(b^3*d^3), 1/3*(3*b*d*x + b*c + a*d)/(b*d))) - 3*(b^3*d^2*x + 4*a*b^2*
d^2)*sqrt(b*x^2 + a*x)*sqrt(d*x + c))/(b^5*d^2*x + a*b^4*d^2)
```

Sympy [F]

$$\int \frac{x^3\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{x^3\sqrt{c+dx}}{(x(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(d*x+c)**(1/2)/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral(x**3*sqrt(c + d*x)/(x*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx+c}x^3}{(bx^2+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)*x^3/(b*x^2 + a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x^3\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx+c}x^3}{(bx^2+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)*x^3/(b*x^2 + a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx=\int \frac{x^3\,\sqrt{c+d\,x}}{{\left(b\,x^2+a\,x\right)}^{3/2}} \,d x$$

input
```
int((x^3*(c + d*x)^(1/2))/(a*x + b*x^2)^(3/2),x)
```

output
```
int((x^3*(c + d*x)^(1/2))/(a*x + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx=\frac{-6\sqrt{x}\,\sqrt{dx+c}\,\sqrt{bx+a}\,c+4\sqrt{x}\,\sqrt{dx+c}\,\sqrt{bx+a}\,dx+3\left(\int \frac{\sqrt{dx-}}{\sqrt{x}\,a^2c+\sqrt{x}\,a^2dx+2\sqrt{x}\,abcx+}\right.}{}$$

input
```
int(x^3*(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x)
```

output
```
( - 6*sqrt(x)*sqrt(c + d*x)*sqrt(a + b*x)*c + 4*sqrt(x)*sqrt(c + d*x)*sqrt
(a + b*x)*d*x + 3*int((sqrt(c + d*x)*sqrt(a + b*x))/(sqrt(x)*a**2*c + sqrt
(x)*a**2*d*x + 2*sqrt(x)*a*b*c*x + 2*sqrt(x)*a*b*d*x**2 + sqrt(x)*b**2*c*x
**2 + sqrt(x)*b**2*d*x**3),x)*a**2*c**2 + 3*int((sqrt(c + d*x)*sqrt(a + b*
x))/(sqrt(x)*a**2*c + sqrt(x)*a**2*d*x + 2*sqrt(x)*a*b*c*x + 2*sqrt(x)*a*b
*d*x**2 + sqrt(x)*b**2*c*x**2 + sqrt(x)*b**2*d*x**3),x)*a*b*c**2*x - 8*int
((sqrt(x)*sqrt(c + d*x)*sqrt(a + b*x)*x)/(a**2*c + a**2*d*x + 2*a*b*c*x +
2*a*b*d*x**2 + b**2*c*x**2 + b**2*d*x**3),x)*a**2*d**2 + 5*int((sqrt(x)*sq
rt(c + d*x)*sqrt(a + b*x)*x)/(a**2*c + a**2*d*x + 2*a*b*c*x + 2*a*b*d*x**2
 + b**2*c*x**2 + b**2*d*x**3),x)*a*b*c*d - 8*int((sqrt(x)*sqrt(c + d*x)*sq
rt(a + b*x)*x)/(a**2*c + a**2*d*x + 2*a*b*c*x + 2*a*b*d*x**2 + b**2*c*x**2
 + b**2*d*x**3),x)*a*b*d**2*x + 5*int((sqrt(x)*sqrt(c + d*x)*sqrt(a + b*x)
*x)/(a**2*c + a**2*d*x + 2*a*b*c*x + 2*a*b*d*x**2 + b**2*c*x**2 + b**2*d*x
**3),x)*b**2*c*d*x)/(6*b*d*(a + b*x))
```

3.197 $\int \frac{x^2\sqrt{c+dx}}{\left(ax+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 209

$$\int \frac{x^2\sqrt{c+dx}}{\left(ax+bx^2\right)^{3/2}}\,dx=\frac{2x\sqrt{c+dx}}{b\sqrt{ax+bx^2}}-\frac{4\sqrt{a}\sqrt{x}\sqrt{c+dx}E\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{b^{3/2}\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}}+\frac{2\sqrt{a}\sqrt{x}\sqrt{c+dx}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{b^{3/2}\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}}$$

output
```
2*x*(d*x+c)^(1/2)/b/(b*x^2+a*x)^(1/2)-4*a^(1/2)*x^(1/2)*(d*x+c)^(1/2)*Elli
pticE(b^(1/2)*x^(1/2)/a^(1/2)/(1+b*x/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(3/2)/(
a*(d*x+c)/c/(b*x+a))^(1/2)/(b*x^2+a*x)^(1/2)+2*a^(1/2)*x^(1/2)*(d*x+c)^(1/
2)*InverseJacobiAM(arctan(b^(1/2)*x^(1/2)/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(3
/2)/(a*(d*x+c)/c/(b*x+a))^(1/2)/(b*x^2+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.09 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00

$$\int \frac{x^2\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \frac{2a(2a+bx)(c+dx)+4ia\sqrt{\frac{a}{b}}bd\sqrt{1+\frac{a}{bx}}\sqrt{1+\frac{c}{dx}}x^{3/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{a}{b}}}{\sqrt{x}}\right)|\frac{bc}{ad}\right)-2i\sqrt{}}{ab^2\sqrt{x(a+bx)}\sqrt{c}}$$

input
```
Integrate[(x^2*Sqrt[c + d*x])/(a*x + b*x^2)^(3/2),x]
```

output
```
(2*a*(2*a + b*x)*(c + d*x) + (4*I)*a*Sqrt[a/b]*b*d*Sqrt[1 + a/(b*x)]*Sqrt[
1 + c/(d*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[a/b]/Sqrt[x]], (b*c)/(a*d)]
- (2*I)*Sqrt[a/b]*b*(-(b*c) + 2*a*d)*Sqrt[1 + a/(b*x)]*Sqrt[1 + c/(d*x)]*x
^(3/2)*EllipticF[I*ArcSinh[Sqrt[a/b]/Sqrt[x]], (b*c)/(a*d)])/(a*b^2*Sqrt[x
*(a + b*x)]*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1261, 108, 27, 176, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1261}$$

$$\frac{x^{3/2}(a+bx)^{3/2}\int \frac{\sqrt{x}\sqrt{c+dx}}{(a+bx)^{3/2}}dx}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 108}$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2\int \frac{c+2dx}{2\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{b}-\frac{2\sqrt{x}\sqrt{c+dx}}{b\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{\int \frac{c+2dx}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{b}-\frac{2\sqrt{x}\sqrt{c+dx}}{b\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 176$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2\int \frac{\sqrt{c+dx}}{\sqrt{x}\sqrt{a+bx}}dx-c\int \frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{b}-\frac{2\sqrt{x}\sqrt{c+dx}}{b\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 122$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{\frac{2\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\int \frac{\sqrt{\frac{dx}{c}+1}}{\sqrt{x}\sqrt{\frac{bx}{a}+1}}dx}{\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-c\int \frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{b}-\frac{2\sqrt{x}\sqrt{c+dx}}{b\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 120$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{\frac{4\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-c\int \frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{b}-\frac{2\sqrt{x}\sqrt{c+dx}}{b\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 127$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{\frac{4\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-\frac{c\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}\int \frac{1}{\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}}dx}{\sqrt{a+bx}\sqrt{c+dx}}}{b}-\frac{2\sqrt{x}\sqrt{c+dx}}{b\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 126$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{\frac{4\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-\frac{2\sqrt{-a}c\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a+bx}\sqrt{c+dx}}}{b}-\frac{2\sqrt{x}\sqrt{c+dx}}{b\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

input
```
Int[(x^2*Sqrt[c + d*x])/(a*x + b*x^2)^(3/2),x]
```

output
```
(x^(3/2)*(a + b*x)^(3/2)*((-2*Sqrt[x]*Sqrt[c + d*x])/(b*Sqrt[a + b*x]) + (
(4*Sqrt[-a]*Sqrt[1 + (b*x)/a]*Sqrt[c + d*x]*EllipticE[ArcSin[(Sqrt[b]*Sqrt
[x])/Sqrt[-a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[a + b*x]*Sqrt[1 + (d*x)/c]) -
(2*Sqrt[-a]*c*Sqrt[1 + (b*x)/a]*Sqrt[1 + (d*x)/c]*EllipticF[ArcSin[(Sqrt[b
]*Sqrt[x])/Sqrt[-a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[a + b*x]*Sqrt[c + d*x]))
/b))/(a*x + b*x^2)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.68

method	result
default	$-\frac{2\sqrt{dx+c}\,\sqrt{x(bx+a)}\left(2\sqrt{\frac{dx+c}{c}}\,\sqrt{\frac{d(bx+a)}{ad-bc}}\,\sqrt{-\frac{xd}{c}}\,\text{EllipticF}\left(\sqrt{\frac{dx+c}{c}},\sqrt{-\frac{bc}{ad-bc}}\right)acd-\sqrt{\frac{dx+c}{c}}\,\sqrt{\frac{d(bx+a)}{ad-bc}}\,\sqrt{-\frac{xd}{c}}\,\text{EllipticF}\left(\sqrt{\right.\right.}{}}$
elliptic	$\frac{\sqrt{x(bx+a)(dx+c)}\left(-\frac{2(bd\,x^2+cbx)}{b^2\sqrt{\left(x+\frac{a}{b}\right)(bd\,x^2+cbx)}}+\frac{2c^2\sqrt{\frac{\left(x+\frac{c}{d}\right)d}{c}}\,\sqrt{\frac{x+\frac{a}{b}}{-\frac{c}{d}+\frac{a}{b}}}\,\sqrt{-\frac{xd}{c}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{c}{d}\right)d}{c}},\sqrt{-\frac{c}{d\left(-\frac{c}{d}+\frac{a}{b}\right)}}\right)}{bd\sqrt{bd\,x^3+ad\,x^2+bc\,x^2+acx}}+\frac{4c\sqrt{\frac{\left(x+\frac{c}{d}\right)d}{c}}\cdots}{\cdots}\right.}{\sqrt{x(bx+a)}\,\sqrt{dx+c}\cdots}$

input
```
int(x^2*(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(d*x+c)^(1/2)*(x*(b*x+a))^(1/2)*(2*((d*x+c)/c)^(1/2)*(d*(b*x+a)/(a*d-b*
c))^(1/2)*(-1/c*x*d)^(1/2)*EllipticF(((d*x+c)/c)^(1/2),(-b*c/(a*d-b*c))^(1
/2))*a*c*d-((d*x+c)/c)^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-1/c*x*d)^(1/2)*
EllipticF(((d*x+c)/c)^(1/2),(-b*c/(a*d-b*c))^(1/2))*b*c^2-2*((d*x+c)/c)^(1
/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-1/c*x*d)^(1/2)*EllipticE(((d*x+c)/c)^(1/
2),(-b*c/(a*d-b*c))^(1/2))*a*c*d+2*((d*x+c)/c)^(1/2)*(d*(b*x+a)/(a*d-b*c))
^(1/2)*(-1/c*x*d)^(1/2)*EllipticE(((d*x+c)/c)^(1/2),(-b*c/(a*d-b*c))^(1/2)
)*b*c^2+b*x^2*d^2+b*c*d*x)/x/(b*d*x^2+a*d*x+b*c*x+a*c)/b^2/d
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.70

$$\int \frac{x^2\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx =$$
$$-\frac{2\left(3\sqrt{bx^2+ax}\sqrt{dx+c}b^2d-(abc-2a^2d+(b^2c-2abd)x)\sqrt{bd}\text{weierstrassPInverse}\left(\frac{4(b^2c^2-abcd+a^2d^2)}{3b^2d^2},-\cdots\right.\right.}{\cdots}$$

input
```
integrate(x^2*(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(3*sqrt(b*x^2 + a*x)*sqrt(d*x + c)*b^2*d - (a*b*c - 2*a^2*d + (b^2*c
- 2*a*b*d)*x)*sqrt(b*d)*weierstrassPInverse(4/3*(b^2*c^2 - a*b*c*d + a^2*d
^2)/(b^2*d^2), -4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^
3)/(b^3*d^3), 1/3*(3*b*d*x + b*c + a*d)/(b*d)) + 6*(b^2*d*x + a*b*d)*sqrt(
b*d)*weierstrassZeta(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2
*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), weierstra
ssPInverse(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3 -
 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), 1/3*(3*b*d*x + b*c
+ a*d)/(b*d))))/(b^4*d*x + a*b^3*d)
```

Sympy [F]

$$\int \frac{x^2\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{x^2\sqrt{c+dx}}{(x\,(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(d*x+c)**(1/2)/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral(x**2*sqrt(c + d*x)/(x*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx+c}x^2}{(bx^2+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)*x^2/(b*x^2 + a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{c+dx}}{\left(ax+bx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{dx+c}x^2}{\left(bx^2+ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)*x^2/(b*x^2 + a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{c+dx}}{\left(ax+bx^2\right)^{3/2}}\,dx=\int \frac{x^2\,\sqrt{c+d\,x}}{{\left(b\,x^2+a\,x\right)}^{3/2}}\,dx$$

input
```
int((x^2*(c + d*x)^(1/2))/(a*x + b*x^2)^(3/2),x)
```

output
```
int((x^2*(c + d*x)^(1/2))/(a*x + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{c+dx}}{\left(ax+bx^2\right)^{3/2}}\,dx=\frac{2\sqrt{x}\,\sqrt{dx+c}\,\sqrt{bx+a}\,c-\left(\int \frac{\sqrt{dx+c}\,\sqrt{bx+a}}{\sqrt{x}\,a^2c+\sqrt{x}\,a^2dx+2\sqrt{x}\,abcx+2\sqrt{x}\,abd\,x^2+\sqrt{x}\,b^2c\,x^2+\sqrt{x}\,b^2d\,x^3}dx\right)a}{}$$

input
```
int(x^2*(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x)
```

output

```
(2*sqrt(x)*sqrt(c + d*x)*sqrt(a + b*x)*c - int((sqrt(c + d*x)*sqrt(a + b*x
))/(sqrt(x)*a**2*c + sqrt(x)*a**2*d*x + 2*sqrt(x)*a*b*c*x + 2*sqrt(x)*a*b*
d*x**2 + sqrt(x)*b**2*c*x**2 + sqrt(x)*b**2*d*x**3),x)*a**2*c**2 - int((sq
rt(c + d*x)*sqrt(a + b*x))/(sqrt(x)*a**2*c + sqrt(x)*a**2*d*x + 2*sqrt(x)*
a*b*c*x + 2*sqrt(x)*a*b*d*x**2 + sqrt(x)*b**2*c*x**2 + sqrt(x)*b**2*d*x**3
),x)*a*b*c**2*x + 2*int((sqrt(x)*sqrt(c + d*x)*sqrt(a + b*x)*x)/(a**2*c +
a**2*d*x + 2*a*b*c*x + 2*a*b*d*x**2 + b**2*c*x**2 + b**2*d*x**3),x)*a**2*d
**2 - int((sqrt(x)*sqrt(c + d*x)*sqrt(a + b*x)*x)/(a**2*c + a**2*d*x + 2*a
*b*c*x + 2*a*b*d*x**2 + b**2*c*x**2 + b**2*d*x**3),x)*a*b*c*d + 2*int((sqr
t(x)*sqrt(c + d*x)*sqrt(a + b*x)*x)/(a**2*c + a**2*d*x + 2*a*b*c*x + 2*a*b
*d*x**2 + b**2*c*x**2 + b**2*d*x**3),x)*a*b*d**2*x - int((sqrt(x)*sqrt(c +
 d*x)*sqrt(a + b*x)*x)/(a**2*c + a**2*d*x + 2*a*b*c*x + 2*a*b*d*x**2 + b**
2*c*x**2 + b**2*d*x**3),x)*b**2*c*d*x)/(2*a*d*(a + b*x))
```

3.198 $\int \frac{x\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 90

$$\int \frac{x\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx=\frac{2\sqrt{x}\sqrt{c+dx}\,E\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}}$$

output
```
2*x^(1/2)*(d*x+c)^(1/2)*EllipticE(b^(1/2)*x^(1/2)/a^(1/2)/(1+b*x/a)^(1/2),
(1-a*d/b/c)^(1/2))/a^(1/2)/b^(1/2)/(a*(d*x+c)/c/(b*x+a))^(1/2)/(b*x^2+a*x)
^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.78

$$\int \frac{x\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx=\frac{2\sqrt{\frac{bx}{a}}\left(\sqrt{\frac{bx}{a}}(c+dx)+ic\sqrt{1+\frac{bx}{a}}\sqrt{1+\frac{dx}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{bx}{a}}\right)|\frac{ad}{bc}\right)-ic\sqrt{1+\frac{bx}{a}}\sqrt{1}\right.}{b\sqrt{x(a+bx)}\sqrt{c+dx}}$$

input
```
Integrate[(x*Sqrt[c + d*x])/(a*x + b*x^2)^(3/2),x]
```

output
```
(2*Sqrt[(b*x)/a]*(Sqrt[(b*x)/a]*(c + d*x) + I*c*Sqrt[1 + (b*x)/a]*Sqrt[1 +
 (d*x)/c]*EllipticE[I*ArcSinh[Sqrt[(b*x)/a]], (a*d)/(b*c)] - I*c*Sqrt[1 +
(b*x)/a]*Sqrt[1 + (d*x)/c]*EllipticF[I*ArcSinh[Sqrt[(b*x)/a]], (a*d)/(b*c)
]))/(b*Sqrt[x*(a + b*x)]*Sqrt[c + d*x])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 231 vs. $2(90) = 180$.

Time = 0.62 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.57, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1234, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{c+dx}}{(ax+bx^2)^{3/2}} dx$$

$$\Big\downarrow \text{1234}$$

$$\frac{2x\sqrt{c+dx}}{a\sqrt{ax+bx^2}} - \frac{2\int \frac{adx}{2\sqrt{c+dx}\sqrt{bx^2+ax}}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2x\sqrt{c+dx}}{a\sqrt{ax+bx^2}} - \frac{d\int \frac{x}{\sqrt{c+dx}\sqrt{bx^2+ax}}dx}{a}$$

$$\Big\downarrow \text{1269}$$

$$\frac{2x\sqrt{c+dx}}{a\sqrt{ax+bx^2}} - \frac{d\left(\frac{\int \frac{\sqrt{c+dx}}{\sqrt{bx^2+ax}}dx}{d} - \frac{c\int \frac{1}{\sqrt{c+dx}\sqrt{bx^2+ax}}dx}{d}\right)}{a}$$

$$\Big\downarrow \text{1169}$$

$$\frac{2x\sqrt{c+dx}}{a\sqrt{ax+bx^2}} - \frac{d\left(\frac{\sqrt{x}\sqrt{a+bx}\int \frac{\sqrt{c+dx}}{\sqrt{x}\sqrt{a+bx}}dx}{d\sqrt{ax+bx^2}} - \frac{c\sqrt{x}\sqrt{a+bx}\int \frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{d\sqrt{ax+bx^2}}\right)}{a}$$

$$\Big\downarrow \text{122}$$

$$\frac{2x\sqrt{c+dx}}{a\sqrt{ax+bx^2}}-\frac{d\left(\frac{\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\int\frac{\sqrt{\frac{dx}{c}+1}}{\sqrt{x}\sqrt{\frac{bx}{a}+1}}dx}{d\sqrt{ax+bx^2}\sqrt{\frac{dx}{c}+1}}-\frac{c\sqrt{x}\sqrt{a+bx}\int\frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{d\sqrt{ax+bx^2}}\right)}{a}$$

$$\Big\downarrow \textbf{120}$$

$$\frac{2x\sqrt{c+dx}}{a\sqrt{ax+bx^2}}-\frac{d\left(\frac{2\sqrt{-a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{ax+bx^2}\sqrt{\frac{dx}{c}+1}}-\frac{c\sqrt{x}\sqrt{a+bx}\int\frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{d\sqrt{ax+bx^2}}\right)}{a}$$

$$\Big\downarrow \textbf{127}$$

$$\frac{2x\sqrt{c+dx}}{a\sqrt{ax+bx^2}}-\frac{d\left(\frac{2\sqrt{-a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{ax+bx^2}\sqrt{\frac{dx}{c}+1}}-\frac{c\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}\int\frac{1}{\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}}dx}{d\sqrt{ax+bx^2}\sqrt{c+dx}}\right)}{a}$$

$$\Big\downarrow \textbf{126}$$

$$\frac{2x\sqrt{c+dx}}{a\sqrt{ax+bx^2}}-\frac{d\left(\frac{2\sqrt{-a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{ax+bx^2}\sqrt{\frac{dx}{c}+1}}-\frac{2\sqrt{-a}c\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{ax+bx^2}\sqrt{c+dx}}\right)}{a}$$

input

```
Int[(x*Sqrt[c + d*x])/(a*x + b*x^2)^(3/2),x]
```

output

```
(2*x*Sqrt[c + d*x])/(a*Sqrt[a*x + b*x^2]) - (d*((2*Sqrt[-a]*Sqrt[x]*Sqrt[1
 + (b*x)/a]*Sqrt[c + d*x]*EllipticE[ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[-a]], (a
*d)/(b*c)])/(Sqrt[b]*d*Sqrt[1 + (d*x)/c]*Sqrt[a*x + b*x^2]) - (2*Sqrt[-a]*
c*Sqrt[x]*Sqrt[1 + (b*x)/a]*Sqrt[1 + (d*x)/c]*EllipticF[ArcSin[(Sqrt[b]*Sq
rt[x])/Sqrt[-a]], (a*d)/(b*c)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a*x + b*x^2]
)))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1234
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(
(f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p +
 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*Simp[g
*(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*
(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1
] && GtQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(83) = 166$.

Time = 1.49 (sec) , antiderivative size = 280, normalized size of antiderivative = 3.11

method	result
default	$\frac{2\sqrt{dx+c}\,\sqrt{x(bx+a)}\left(\sqrt{\frac{dx+c}{c}}\,\sqrt{\frac{d(bx+a)}{ad-bc}}\,\sqrt{-\frac{xd}{c}}\,\operatorname{EllipticF}\left(\sqrt{\frac{dx+c}{c}},\sqrt{-\frac{bc}{ad-bc}}\right)acd-\sqrt{\frac{dx+c}{c}}\,\sqrt{\frac{d(bx+a)}{ad-bc}}\,\sqrt{-\frac{xd}{c}}\,\operatorname{EllipticE}\left(\sqrt{\frac{dx+c}{c}} \cdots\right.\right.}{badx(bd\,x^2+adx+cbx+ac) \cdots}$
elliptic	$\frac{\sqrt{x(bx+a)(dx+c)}\left(\frac{2bd\,x^2+2cbx}{ba\sqrt{\left(x+\frac{a}{b}\right)\left(bd\,x^2+cbx\right)}}+\frac{2\left(\frac{d}{b}-\frac{ad-bc}{ab}-\frac{c}{a}\right)c\sqrt{\frac{\left(x+\frac{c}{d}\right)d}{c}}\,\sqrt{\frac{x+\frac{a}{b}}{-\frac{c}{d}+\frac{a}{b}}}\,\sqrt{-\frac{xd}{c}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{c}{d}\right)d}{c}},\sqrt{-\frac{c}{d\left(-\frac{c}{d}+\frac{a}{b}\right)}}\right)}{d\sqrt{bd\,x^3+ad\,x^2+bc\,x^2+acx}}-\cdots\right.}{\sqrt{x(bx+a)}\,\sqrt{d \cdots}}$

input
```
int(x*(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(d*x+c)^(1/2)*(x*(b*x+a))^(1/2)*(((d*x+c)/c)^(1/2)*(d*(b*x+a)/(a*d-b*c))
^(1/2)*(-1/c*x*d)^(1/2)*EllipticF(((d*x+c)/c)^(1/2),(-b*c/(a*d-b*c))^(1/2)
)*a*c*d-((d*x+c)/c)^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-1/c*x*d)^(1/2)*Ell
ipticE(((d*x+c)/c)^(1/2),(-b*c/(a*d-b*c))^(1/2))*a*c*d+((d*x+c)/c)^(1/2)*(
d*(b*x+a)/(a*d-b*c))^(1/2)*(-1/c*x*d)^(1/2)*EllipticE(((d*x+c)/c)^(1/2),(-
b*c/(a*d-b*c))^(1/2))*b*c^2+b*x^2*d^2+b*c*d*x)/b/a/d/x/(b*d*x^2+a*d*x+b*c*
x+a*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(83) = 166$.

Time = 0.08 (sec) , antiderivative size = 356, normalized size of antiderivative = 3.96

$$\int \frac{x\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \frac{2\left(3\sqrt{bx^2+ax}\sqrt{dx+c}b^2d+(abc+a^2d+(b^2c+abd)x)\sqrt{bd}\text{weierstrassPInverse}\left(\frac{4}{\cdots}\right.\right.}{\cdots}$$

input
```
integrate(x*(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(3*sqrt(b*x^2 + a*x)*sqrt(d*x + c)*b^2*d + (a*b*c + a^2*d + (b^2*c + a
*b*d)*x)*sqrt(b*d)*weierstrassPInverse(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(
b^2*d^2), -4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b
^3*d^3), 1/3*(3*b*d*x + b*c + a*d)/(b*d)) + 3*(b^2*d*x + a*b*d)*sqrt(b*d)*
weierstrassZeta(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*
c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), weierstrassPIn
verse(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3 - 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), 1/3*(3*b*d*x + b*c + a*d
)/(b*d))))/(a*b^3*d*x + a^2*b^2*d)
```

Sympy [F]

$$\int \frac{x\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{x\sqrt{c+dx}}{(x(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(d*x+c)**(1/2)/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral(x*sqrt(c + d*x)/(x*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx+c}x}{(bx^2+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)*x/(b*x^2 + a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx+c}x}{(bx^2+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)*x/(b*x^2 + a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{x\,\sqrt{c+d\,x}}{\left(b\,x^2+a\,x\right)^{3/2}}\,dx$$

input
```
int((x*(c + d*x)^(1/2))/(a*x + b*x^2)^(3/2),x)
```

output
```
int((x*(c + d*x)^(1/2))/(a*x + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx+c}\,\sqrt{bx+a}}{\sqrt{x}\,a^2+2\sqrt{x}\,abx+\sqrt{x}\,b^2x^2}dx$$

input
```
int(x*(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x)
```

output
```
int((sqrt(c + d*x)*sqrt(a + b*x))/(sqrt(x)*a**2 + 2*sqrt(x)*a*b*x + sqrt(x
)*b**2*x**2),x)
```

3.199 $\int \frac{\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 212

$$\int \frac{\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = -\frac{2\sqrt{c+dx}}{a\sqrt{ax+bx^2}} - \frac{4\sqrt{b}\sqrt{x}\sqrt{c+dx}\,E\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{a^{3/2}\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}} + \frac{2d\sqrt{x}\sqrt{c+dx}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}c\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}}$$

output
```
-2*(d*x+c)^(1/2)/a/(b*x^2+a*x)^(1/2)-4*b^(1/2)*x^(1/2)*(d*x+c)^(1/2)*Ellip
ticE(b^(1/2)*x^(1/2)/a^(1/2)/(1+b*x/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(3/2)/(a
*(d*x+c)/c/(b*x+a))^(1/2)/(b*x^2+a*x)^(1/2)+2*d*x^(1/2)*(d*x+c)^(1/2)*Inve
rseJacobiAM(arctan(b^(1/2)*x^(1/2)/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(1/2)/b^(
1/2)/c/(a*(d*x+c)/c/(b*x+a))^(1/2)/(b*x^2+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.37 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \frac{2\sqrt{\frac{a}{b}}(c+dx)+4id\sqrt{1+\frac{a}{bx}}\sqrt{1+\frac{c}{dx}}x^{3/2}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{\frac{a}{b}}}{\sqrt{x}}\right)\mid\frac{bc}{ad}\right)-2id\sqrt{1+\frac{a}{bx}}\sqrt{1}}{a\sqrt{\frac{a}{b}}\sqrt{x(a+bx)}\sqrt{c+dx}}$$

input
```
Integrate[Sqrt[c + d*x]/(a*x + b*x^2)^(3/2),x]
```

output
```
(2*Sqrt[a/b]*(c + d*x) + (4*I)*d*Sqrt[1 + a/(b*x)]*Sqrt[1 + c/(d*x)]*x^(3/
2)*EllipticE[I*ArcSinh[Sqrt[a/b]/Sqrt[x]], (b*c)/(a*d)] - (2*I)*d*Sqrt[1 +
 a/(b*x)]*Sqrt[1 + c/(d*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[a/b]/Sqrt[x]]
, (b*c)/(a*d)])/(a*Sqrt[a/b]*Sqrt[x*(a + b*x)]*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {1163, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 1163$$

$$\frac{2\int \frac{d(a+2bx)}{2\sqrt{c+dx}\sqrt{bx^2+ax}}dx}{a^2}-\frac{2(a+2bx)\sqrt{c+dx}}{a^2\sqrt{ax+bx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{d\int \frac{a+2bx}{\sqrt{c+dx}\sqrt{bx^2+ax}}dx}{a^2}-\frac{2(a+2bx)\sqrt{c+dx}}{a^2\sqrt{ax+bx^2}}$$

$$\Big\downarrow \; 1269$$

$$\frac{d\left(\frac{2b\int \frac{\sqrt{c+dx}}{\sqrt{bx^2+ax}}dx}{d}-\frac{(2bc-ad)\int \frac{1}{\sqrt{c+dx}\sqrt{bx^2+ax}}dx}{d}\right)}{a^2}-\frac{2(a+2bx)\sqrt{c+dx}}{a^2\sqrt{ax+bx^2}}$$

$\Big\downarrow$ 1169

$$\frac{d\left(\frac{2b\sqrt{x}\sqrt{a+bx}\int \frac{\sqrt{c+dx}}{\sqrt{x}\sqrt{a+bx}}dx}{d\sqrt{ax+bx^2}}-\frac{\sqrt{x}\sqrt{a+bx}(2bc-ad)\int \frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{d\sqrt{ax+bx^2}}\right)}{a^2}-\frac{2(a+2bx)\sqrt{c+dx}}{a^2\sqrt{ax+bx^2}}$$

$\Big\downarrow$ 122

$$\frac{d\left(\frac{2b\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\int \frac{\sqrt{\frac{dx}{c}+1}}{\sqrt{x}\sqrt{\frac{bx}{a}+1}}dx}{d\sqrt{ax+bx^2}\sqrt{\frac{dx}{c}+1}}-\frac{\sqrt{x}\sqrt{a+bx}(2bc-ad)\int \frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{d\sqrt{ax+bx^2}}\right)}{a^2}-\frac{2(a+2bx)\sqrt{c+dx}}{a^2\sqrt{ax+bx^2}}$$

$\Big\downarrow$ 120

$$\frac{d\left(\frac{4\sqrt{-a}\sqrt{b}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{d\sqrt{ax+bx^2}\sqrt{\frac{dx}{c}+1}}-\frac{\sqrt{x}\sqrt{a+bx}(2bc-ad)\int \frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{d\sqrt{ax+bx^2}}\right)}{a^2}-\frac{2(a+2bx)\sqrt{c+dx}}{a^2\sqrt{ax+bx^2}}$$

$\Big\downarrow$ 127

$$\frac{d\left(\frac{4\sqrt{-a}\sqrt{b}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{d\sqrt{ax+bx^2}\sqrt{\frac{dx}{c}+1}}-\frac{\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}(2bc-ad)\int \frac{1}{\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}}dx}{d\sqrt{ax+bx^2}\sqrt{c+dx}}\right)}{a^2}-\frac{2(a+2bx)\sqrt{c+dx}}{a^2\sqrt{ax+bx^2}}$$

$\Big\downarrow$ 126

$$\frac{d\left(\frac{4\sqrt{-a}\sqrt{b}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{d\sqrt{ax+bx^2}\sqrt{\frac{dx}{c}+1}}-\frac{2\sqrt{-a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}(2bc-ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{ax+bx^2}\sqrt{c+dx}}\right)}{a^2}-\frac{2(a+2bx)\sqrt{c+dx}}{a^2\sqrt{ax+bx^2}}$$

input
```
Int[Sqrt[c + d*x]/(a*x + b*x^2)^(3/2),x]
```

output
```
(-2*(a + 2*b*x)*Sqrt[c + d*x])/(a^2*Sqrt[a*x + b*x^2]) + (d*((4*Sqrt[-a]*S
qrt[b]*Sqrt[x]*Sqrt[1 + (b*x)/a]*Sqrt[c + d*x]*EllipticE[ArcSin[(Sqrt[b]*S
qrt[x])/Sqrt[-a]], (a*d)/(b*c)])/(d*Sqrt[1 + (d*x)/c]*Sqrt[a*x + b*x^2]) -
 (2*Sqrt[-a]*(2*b*c - a*d)*Sqrt[x]*Sqrt[1 + (b*x)/a]*Sqrt[1 + (d*x)/c]*Ell
ipticF[ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[-a]], (a*d)/(b*c)])/(Sqrt[b]*d*Sqrt[c
 + d*x]*Sqrt[a*x + b*x^2])))/a^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1163
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] - Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1
)*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p +
1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 0] && (LtQ[
m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c, d,
e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.31

method	result
default	$-\frac{2\left(\sqrt{\frac{dx+c}{c}}\,\sqrt{\frac{d(bx+a)}{ad-bc}}\,\sqrt{-\frac{xd}{c}}\ \text{EllipticF}\left(\sqrt{\frac{dx+c}{c}},\sqrt{-\frac{bc}{ad-bc}}\right)acd-2\sqrt{\frac{dx+c}{c}}\,\sqrt{\frac{d(bx+a)}{ad-bc}}\,\sqrt{-\frac{xd}{c}}\ \text{EllipticE}\left(\sqrt{\frac{dx+c}{c}},\sqrt{-\frac{bc}{ad-bc}}\right)a}{x(bx+a)d\,a^2\sqrt{dx+}}$
elliptic	$\frac{\sqrt{x(bx+a)(dx+c)}\left(-\frac{2\left(bd\,x^2+cbx\right)}{a^2\sqrt{\left(x+\frac{a}{b}\right)\left(bd\,x^2+cbx\right)}}-\frac{2\left(bd\,x^2+adx+cbx+ac\right)}{a^2\sqrt{x\left(bd\,x^2+adx+cbx+ac\right)}}+\frac{2\left(\frac{ad-bc}{a^2}+\frac{bc}{a^2}\right)c\sqrt{\frac{\left(x+\frac{c}{d}\right)d}{c}}\,\sqrt{\frac{x+\frac{a}{b}}{-\frac{c}{d}+\frac{a}{b}}}\,\sqrt{-\frac{xd}{c}}\ \text{EllipticF}\left(\sqrt{}\right.}{d\sqrt{bd\,x^3+ad\,x^2+bc\,x^2+acx}}\right.}{}$

input
```
int((d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(((d*x+c)/c)^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-1/c*x*d)^(1/2)*Ellipti
cF(((d*x+c)/c)^(1/2),(-b*c/(a*d-b*c))^(1/2))*a*c*d-2*((d*x+c)/c)^(1/2)*(d*
(b*x+a)/(a*d-b*c))^(1/2)*(-1/c*x*d)^(1/2)*EllipticE(((d*x+c)/c)^(1/2),(-b*
c/(a*d-b*c))^(1/2))*a*c*d+2*((d*x+c)/c)^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)*
(-1/c*x*d)^(1/2)*EllipticE(((d*x+c)/c)^(1/2),(-b*c/(a*d-b*c))^(1/2))*b*c^2
+2*b*x^2*d^2+a*d^2*x+2*b*c*d*x+a*c*d)/x*(x*(b*x+a))^(1/2)/(b*x+a)/d/a^2/(d
*x+c)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(185) = 370$.

Time = 0.10 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.79

$$\int \frac{\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx =$$
$$-\frac{2\left(((2\,b^2c-abd)x^2+(2\,abc-a^2d)x)\sqrt{bd}\text{weierstrassPInverse}\left(\frac{4\,(b^2c^2-abcd+a^2d^2)}{3\,b^2d^2},-\frac{4\,(2\,b^3c^3-3\,ab^2c^2d-3\,a^2bcd^2+}{27\,b^3d^3}\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(((2*b^2*c - a*b*d)*x^2 + (2*a*b*c - a^2*d)*x)*sqrt(b*d)*weierstrassP
Inverse(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3 - 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), 1/3*(3*b*d*x + b*c + a
*d)/(b*d)) + 6*(b^2*d*x^2 + a*b*d*x)*sqrt(b*d)*weierstrassZeta(4/3*(b^2*c^
2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), weierstrassPInverse(4/3*(b^2*c^2 - a*b*c*
d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 +
 2*a^3*d^3)/(b^3*d^3), 1/3*(3*b*d*x + b*c + a*d)/(b*d))) + 3*(2*b^2*d*x +
a*b*d)*sqrt(b*x^2 + a*x)*sqrt(d*x + c))/(a^2*b^2*d*x^2 + a^3*b*d*x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{c+dx}}{(x\,(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**(1/2)/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral(sqrt(c + d*x)/(x*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx+c}}{(bx^2+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)/(b*x^2 + a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx+c}}{(bx^2+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)/(b*x^2 + a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{c+d\,x}}{{\left(b\,x^2+a\,x\right)}^{3/2}}\,dx$$

input
```
int((c + d*x)^(1/2)/(a*x + b*x^2)^(3/2),x)
```

output
```
int((c + d*x)^(1/2)/(a*x + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx+c}\,\sqrt{bx+a}}{\sqrt{x}\,a^2x+2\sqrt{x}\,ab\,x^2+\sqrt{x}\,b^2x^3}dx$$

input
```
int((d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x)
```

output
```
int((sqrt(c + d*x)*sqrt(a + b*x))/(sqrt(x)*a**2*x + 2*sqrt(x)*a*b*x**2 + s
qrt(x)*b**2*x**3),x)
```

3.200 $\int \frac{\sqrt{c+dx}}{x\left(ax+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 304

$$\int \frac{\sqrt{c+dx}}{x\left(ax+bx^2\right)^{3/2}}\,dx=\frac{2(8bc-ad)\sqrt{c+dx}}{3a^2c\sqrt{ax+bx^2}}+\frac{2\sqrt{c+dx}}{ax\sqrt{ax+bx^2}}-\frac{8\sqrt{c+dx}\sqrt{ax+bx^2}}{3a^2x^2}+\frac{2\sqrt{b}(8bc-ad)\sqrt{x}\sqrt{c+dx}E\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{5/2}c\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}}-\frac{8\sqrt{b}d\sqrt{x}\sqrt{c+dx}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3a^{3/2}c\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}}$$

output
```
2/3*(-a*d+8*b*c)*(d*x+c)^(1/2)/a^2/c/(b*x^2+a*x)^(1/2)+2*(d*x+c)^(1/2)/a/x
/(b*x^2+a*x)^(1/2)-8/3*(d*x+c)^(1/2)*(b*x^2+a*x)^(1/2)/a^2/x^2+2/3*b^(1/2)
*(-a*d+8*b*c)*x^(1/2)*(d*x+c)^(1/2)*EllipticE(b^(1/2)*x^(1/2)/a^(1/2)/(1+b
*x/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(5/2)/c/(a*(d*x+c)/c/(b*x+a))^(1/2)/(b*x^
2+a*x)^(1/2)-8/3*b^(1/2)*d*x^(1/2)*(d*x+c)^(1/2)*InverseJacobiAM(arctan(b^
(1/2)*x^(1/2)/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(3/2)/c/(a*(d*x+c)/c/(b*x+a))^
(1/2)/(b*x^2+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.61 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{c+dx}}{x\left(ax+bx^2\right)^{3/2}}\,dx = \frac{-2ac(a+4bx)(c+dx)+2i\sqrt{\frac{a}{b}}bd(-8bc+ad)\sqrt{1+\frac{a}{bx}}\sqrt{1+\frac{c}{dx}}x^{5/2}E\Big(i\text{arcsinh}\Big(}{3a^3cx\sqrt{x(}}$$

input
```
Integrate[Sqrt[c + d*x]/(x*(a*x + b*x^2)^(3/2)),x]
```

output
```
(-2*a*c*(a + 4*b*x)*(c + d*x) + (2*I)*Sqrt[a/b]*b*d*(-8*b*c + a*d)*Sqrt[1
+ a/(b*x)]*Sqrt[1 + c/(d*x)]*x^(5/2)*EllipticE[I*ArcSinh[Sqrt[a/b]/Sqrt[x]
], (b*c)/(a*d)] - (2*I)*Sqrt[a/b]*b*d*(-4*b*c + a*d)*Sqrt[1 + a/(b*x)]*Sqr
t[1 + c/(d*x)]*x^(5/2)*EllipticF[I*ArcSinh[Sqrt[a/b]/Sqrt[x]], (b*c)/(a*d)
])/(3*a^3*c*x*Sqrt[x*(a + b*x)]*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.17, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {1261, 110, 27, 169, 27, 169, 27, 176, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}}{x\left(ax+bx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 1261$$

$$\frac{x^{3/2}(a+bx)^{3/2}\int \frac{\sqrt{c+dx}}{x^{5/2}(a+bx)^{3/2}}dx}{\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 110$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2\int -\frac{4bc-ad+3bdx}{2x^{3/2}(a+bx)^{3/2}\sqrt{c+dx}}dx}{3a}-\frac{2\sqrt{c+dx}}{3ax^{3/2}\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{\int \frac{4bc-ad+3bdx}{x^{3/2}(a+bx)^{3/2}\sqrt{c+dx}}dx}{3a}-\frac{2\sqrt{c+dx}}{3ax^{3/2}\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 169$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{2\int \frac{b(c(8bc-5ad)+d(4bc-ad)x)}{2\sqrt{x}(a+bx)^{3/2}\sqrt{c+dx}}dx}{ac}-\frac{2\sqrt{c+dx}(4bc-ad)}{ac\sqrt{x}\sqrt{a+bx}}}{3a}-\frac{2\sqrt{c+dx}}{3ax^{3/2}\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{b\int \frac{c(8bc-5ad)+d(4bc-ad)x}{\sqrt{x}(a+bx)^{3/2}\sqrt{c+dx}}dx}{ac}-\frac{2\sqrt{c+dx}(4bc-ad)}{ac\sqrt{x}\sqrt{a+bx}}}{3a}-\frac{2\sqrt{c+dx}}{3ax^{3/2}\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 169$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{b\left(\frac{2\int -\frac{d(bc-ad)(4ac+(8bc-ad)x)}{2\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{a(bc-ad)}+\frac{2\sqrt{x}\sqrt{c+dx}(8bc-ad)}{a\sqrt{a+bx}}\right)}{ac}-\frac{2\sqrt{c+dx}(4bc-ad)}{ac\sqrt{x}\sqrt{a+bx}}}{3a}-\frac{2\sqrt{c+dx}}{3ax^{3/2}\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{b\left(\frac{2\sqrt{x}\sqrt{c+dx}(8bc-ad)}{a\sqrt{a+bx}}-\frac{d\int \frac{4ac+(8bc-ad)x}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{a}\right)}{ac}-\frac{2\sqrt{c+dx}(4bc-ad)}{ac\sqrt{x}\sqrt{a+bx}}}{3a}-\frac{2\sqrt{c+dx}}{3ax^{3/2}\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 176$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{b\left(\frac{2\sqrt{x}\sqrt{c+dx}(8bc-ad)}{a\sqrt{a+bx}}-\frac{d\left(\frac{(8bc-ad)\int\frac{\sqrt{c+dx}}{\sqrt{x}\sqrt{a+bx}}dx}{d}-\frac{c(8bc-5ad)\int\frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{d}\right)}{a}\right)}{ac}-\frac{2\sqrt{c+dx}(4bc-ad)}{ac\sqrt{x}\sqrt{a+bx}}}{3a}-\frac{2\sqrt{c-}}{3ax^{3/2}}\cdots\right)}{(ax+bx^2)^{3/2}}$$

$\Big\downarrow$ 122

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{b\left(\frac{2\sqrt{x}\sqrt{c+dx}(8bc-ad)}{a\sqrt{a+bx}}-\frac{d\left(\frac{\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}(8bc-ad)\int\frac{\sqrt{\frac{dx}{c}+1}}{\sqrt{x}\sqrt{\frac{bx}{a}+1}}dx}{d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-\frac{c(8bc-5ad)\int\frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{d}\right)}{a}\right)}{ac}-\frac{2\sqrt{c+dx}(4bc-a\cdots}{ac\sqrt{x}\sqrt{a+bx}}}{3a}\cdots\right)}{(ax+bx^2)^{3/2}}$$

$\Big\downarrow$ 120

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{b\left(\frac{2\sqrt{x}\sqrt{c+dx}(8bc-ad)}{a\sqrt{a+bx}}-\frac{d\left(\frac{2\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}(8bc-ad)E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-\frac{c(8bc-5ad)\int\frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{d}\right)}{a}\right)}{ac}-2\cdots}{3a}\cdots\right)}{(ax+bx^2)^{3/2}}$$

$\downarrow$ 127

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{b\left(\frac{2\sqrt{x}\sqrt{c+dx}(8bc-ad)}{a\sqrt{a+bx}}-\frac{d\left(\frac{2\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}(8bc-ad)E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-\frac{c\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}(8bc-5ad)\int\frac{1}{\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{}}}{d\sqrt{a+bx}\sqrt{c+dx}}\right)}{a}\right)}{ac}}{3a}\right)}{(ax+bx^2)^{3/2}}$$

$\downarrow$ 126

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{b\left(\frac{2\sqrt{x}\sqrt{c+dx}(8bc-ad)}{a\sqrt{a+bx}}-\frac{d\left(\frac{2\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}(8bc-ad)E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-\frac{2\sqrt{-a}c\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}(8bc-5ad)\,\text{EllipticF}}{\sqrt{b}d\sqrt{a+bx}\sqrt{c+dx}}\right)}{a}\right)}{ac}}{3a}\right)}{(ax+bx^2)^{3/2}}$$

input
```
Int[Sqrt[c + d*x]/(x*(a*x + b*x^2)^(3/2)),x]
```

output
```
(x^(3/2)*(a + b*x)^(3/2)*((-2*Sqrt[c + d*x])/(3*a*x^(3/2)*Sqrt[a + b*x]) -
 ((-2*(4*b*c - a*d)*Sqrt[c + d*x])/(a*c*Sqrt[x]*Sqrt[a + b*x]) - (b*((2*(8
*b*c - a*d)*Sqrt[x]*Sqrt[c + d*x])/(a*Sqrt[a + b*x]) - (d*((2*Sqrt[-a]*(8*
b*c - a*d)*Sqrt[1 + (b*x)/a]*Sqrt[c + d*x]*EllipticE[ArcSin[(Sqrt[b]*Sqrt[
x])/Sqrt[-a]], (a*d)/(b*c)])/(Sqrt[b]*d*Sqrt[a + b*x]*Sqrt[1 + (d*x)/c]) -
 (2*Sqrt[-a]*c*(8*b*c - 5*a*d)*Sqrt[1 + (b*x)/a]*Sqrt[1 + (d*x)/c]*Ellipti
cF[ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[-a]], (a*d)/(b*c)])/(Sqrt[b]*d*Sqrt[a + b
*x]*Sqrt[c + d*x])))/a))/(a*c))/(3*a)))/(a*x + b*x^2)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.65

method	result
default	$-\frac{2\left(x\sqrt{\frac{dx+c}{c}}\,\sqrt{\frac{d(bx+a)}{ad-bc}}\,\sqrt{-\frac{xd}{c}}\ \text{EllipticF}\left(\sqrt{\frac{dx+c}{c}},\sqrt{-\frac{bc}{ad-bc}}\right)a^2cd^2-4x\sqrt{\frac{dx+c}{c}}\,\sqrt{\frac{d(bx+a)}{ad-bc}}\,\sqrt{-\frac{xd}{c}}\ \text{EllipticF}\left(\sqrt{\frac{dx+c}{c}},\sqrt{-\frac{bc}{ad-}}\right.\right.}{}$
elliptic	$\frac{\sqrt{x(bx+a)(dx+c)}\left(\frac{2\left(bd\,x^2+cbx\right)b}{a^3\sqrt{\left(x+\frac{a}{b}\right)\left(bd\,x^2+cbx\right)}}-\frac{2\sqrt{bd\,x^3+ad\,x^2+bc\,x^2+acx}}{3a^2x^2}-\frac{2\left(bd\,x^2+adx+cbx+ac\right)(ad-5bc)}{3a^3c\sqrt{x\left(bd\,x^2+adx+cbx+ac\right)}}+\frac{2\left(-\frac{b(ad-bc)}{a^3}-\frac{b^2c}{a^3}-\frac{db}{3a^2}\right)}{}\right.}{}$

input
```
int((d*x+c)^(1/2)/x/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(x*((d*x+c)/c)^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-1/c*x*d)^(1/2)*Ell
ipticF(((d*x+c)/c)^(1/2),(-b*c/(a*d-b*c))^(1/2))*a^2*c*d^2-4*x*((d*x+c)/c)
^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-1/c*x*d)^(1/2)*EllipticF(((d*x+c)/c)^
(1/2),(-b*c/(a*d-b*c))^(1/2))*a*b*c^2*d-x*((d*x+c)/c)^(1/2)*(d*(b*x+a)/(a*
d-b*c))^(1/2)*(-1/c*x*d)^(1/2)*EllipticE(((d*x+c)/c)^(1/2),(-b*c/(a*d-b*c)
)^(1/2))*a^2*c*d^2+9*x*((d*x+c)/c)^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-1/c
*x*d)^(1/2)*EllipticE(((d*x+c)/c)^(1/2),(-b*c/(a*d-b*c))^(1/2))*a*b*c^2*d-
8*x*((d*x+c)/c)^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-1/c*x*d)^(1/2)*Ellipti
cE(((d*x+c)/c)^(1/2),(-b*c/(a*d-b*c))^(1/2))*b^2*c^3+a*b*d^3*x^3-8*b^2*c*x
^3*d^2+a^2*d^3*x^2-3*a*b*c*d^2*x^2-8*b^2*c^2*d*x^2+2*a^2*c*d^2*x-4*a*b*c^2
*d*x+a^2*c^2*d)/x^2*(x*(b*x+a))^(1/2)/(b*x+a)/c/d/a^3/(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.59

$$\int \frac{\sqrt{c+dx}}{x\left(ax+bx^2\right)^{3/2}}\,dx = \frac{2\left(\left((8\,b^3c^2-5\,ab^2cd-a^2bd^2)x^3+(8\,ab^2c^2-5\,a^2bcd-a^3d^2)x^2\right)\sqrt{bd}\text{weierstrassPI}\right.}{}$$

input
```
integrate((d*x+c)^(1/2)/x/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
2/9*(((8*b^3*c^2 - 5*a*b^2*c*d - a^2*b*d^2)*x^3 + (8*a*b^2*c^2 - 5*a^2*b*c
*d - a^3*d^2)*x^2)*sqrt(b*d)*weierstrassPInverse(4/3*(b^2*c^2 - a*b*c*d +
a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a
^3*d^3)/(b^3*d^3), 1/3*(3*b*d*x + b*c + a*d)/(b*d)) + 3*((8*b^3*c*d - a*b^
2*d^2)*x^3 + (8*a*b^2*c*d - a^2*b*d^2)*x^2)*sqrt(b*d)*weierstrassZeta(4/3*
(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d
- 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), weierstrassPInverse(4/3*(b^2*c^2 -
 a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*
c*d^2 + 2*a^3*d^3)/(b^3*d^3), 1/3*(3*b*d*x + b*c + a*d)/(b*d))) - 3*(a^2*b
*c*d - (8*b^3*c*d - a*b^2*d^2)*x^2 - (4*a*b^2*c*d - a^2*b*d^2)*x)*sqrt(b*x
^2 + a*x)*sqrt(d*x + c))/(a^3*b^2*c*d*x^3 + a^4*b*c*d*x^2)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}}{x\left(ax+bx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{c+dx}}{x\left(x\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**(1/2)/x/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral(sqrt(c + d*x)/(x*(x*(a + b*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}}{x\left(ax+bx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{dx+c}}{\left(bx^2+ax\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((d*x+c)^(1/2)/x/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)/((b*x^2 + a*x)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}}{x\left(ax+bx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{dx+c}}{\left(bx^2+ax\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((d*x+c)^(1/2)/x/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)/((b*x^2 + a*x)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}}{x\left(ax+bx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{c+d\,x}}{x\,\left(b\,x^2+a\,x\right)^{3/2}}\,dx$$

input
```
int((c + d*x)^(1/2)/(x*(a*x + b*x^2)^(3/2)),x)
```

output
```
int((c + d*x)^(1/2)/(x*(a*x + b*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}}{x\left(ax+bx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{dx+c}\,\sqrt{bx+a}}{\sqrt{x}\,a^2x^2+2\sqrt{x}\,ab\,x^3+\sqrt{x}\,b^2x^4}dx$$

input
```
int((d*x+c)^(1/2)/x/(b*x^2+a*x)^(3/2),x)
```

output
```
int((sqrt(c + d*x)*sqrt(a + b*x))/(sqrt(x)*a**2*x**2 + 2*sqrt(x)*a*b*x**3
+ sqrt(x)*b**2*x**4),x)
```

3.201 $\int \frac{\sqrt{c+dx}}{x^2(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 385

$$\begin{aligned}\int \frac{\sqrt{c+dx}}{x^2\left(ax+bx^2\right)^{3/2}}\,dx = &-\frac{4(24b^2c^2-4abcd-a^2d^2)\sqrt{c+dx}}{15a^3c^2\sqrt{ax+bx^2}}\\ &+\frac{2\sqrt{c+dx}}{ax^2\sqrt{ax+bx^2}}-\frac{12\sqrt{c+dx}\sqrt{ax+bx^2}}{5a^2x^3}+\frac{2(24bc-ad)\sqrt{c+dx}\sqrt{ax+bx^2}}{15a^3cx^2}\\ &-\frac{4\sqrt{b}(24b^2c^2-4abcd-a^2d^2)\sqrt{x}\sqrt{c+dx}E\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{7/2}c^2\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}}\\ &+\frac{2\sqrt{b}d(24bc-ad)\sqrt{x}\sqrt{c+dx}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15a^{5/2}c^2\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}}\end{aligned}$$

output

```
-4/15*(-a^2*d^2-4*a*b*c*d+24*b^2*c^2)*(d*x+c)^(1/2)/a^3/c^2/(b*x^2+a*x)^(1
/2)+2*(d*x+c)^(1/2)/a/x^2/(b*x^2+a*x)^(1/2)-12/5*(d*x+c)^(1/2)*(b*x^2+a*x)
^(1/2)/a^2/x^3+2/15*(-a*d+24*b*c)*(d*x+c)^(1/2)*(b*x^2+a*x)^(1/2)/a^3/c/x^
2-4/15*b^(1/2)*(-a^2*d^2-4*a*b*c*d+24*b^2*c^2)*x^(1/2)*(d*x+c)^(1/2)*Ellip
ticE(b^(1/2)*x^(1/2)/a^(1/2)/(1+b*x/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(7/2)/c^
2/(a*(d*x+c)/c/(b*x+a))^(1/2)/(b*x^2+a*x)^(1/2)+2/15*b^(1/2)*d*(-a*d+24*b*
c)*x^(1/2)*(d*x+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x^(1/2)/a^(1/2)),(
1-a*d/b/c)^(1/2))/a^(5/2)/c^2/(a*(d*x+c)/c/(b*x+a))^(1/2)/(b*x^2+a*x)^(1/2
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 15.08 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{c+dx}}{x^2\left(ax+bx^2\right)^{3/2}}\,dx = \frac{-2ac(c+dx)\left(-24b^2cx^2+abx(-6c+dx)+a^2(3c+dx)\right)-4i\sqrt{\frac{a}{b}}bd(-24b^2c^2+\cdots}{\cdots}$$

input
```
Integrate[Sqrt[c + d*x]/(x^2*(a*x + b*x^2)^(3/2)),x]
```

output
```
(-2*a*c*(c + d*x)*(-24*b^2*c*x^2 + a*b*x*(-6*c + d*x) + a^2*(3*c + d*x)) -
 (4*I)*Sqrt[a/b]*b*d*(-24*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*Sqrt[1 + a/(b*x)]
*Sqrt[1 + c/(d*x)]*x^(7/2)*EllipticE[I*ArcSinh[Sqrt[a/b]/Sqrt[x]], (b*c)/(
a*d)] + (2*I)*Sqrt[a/b]*b*d*(-24*b^2*c^2 + 7*a*b*c*d + 2*a^2*d^2)*Sqrt[1 +
 a/(b*x)]*Sqrt[1 + c/(d*x)]*x^(7/2)*EllipticF[I*ArcSinh[Sqrt[a/b]/Sqrt[x]]
, (b*c)/(a*d)])/(15*a^4*c^2*x^2*Sqrt[x*(a + b*x)]*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.19, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {1261, 110, 27, 169, 27, 169, 27, 169, 27, 176, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}}{x^2\left(ax+bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 1261$$

$$\frac{x^{3/2}(a+bx)^{3/2}\int \frac{\sqrt{c+dx}}{x^{7/2}(a+bx)^{3/2}}dx}{\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 110$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2\int -\frac{6bc-ad+5bdx}{2x^{5/2}(a+bx)^{3/2}\sqrt{c+dx}}dx}{5a}-\frac{2\sqrt{c+dx}}{5ax^{5/2}\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{\int \frac{6bc-ad+5bdx}{x^{5/2}(a+bx)^{3/2}\sqrt{c+dx}}dx}{5a}-\frac{2\sqrt{c+dx}}{5ax^{5/2}\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \; 169$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{2\int \frac{24b^2c^2-7abdc-2a^2d^2+3bd(6bc-ad)x}{2x^{3/2}(a+bx)^{3/2}\sqrt{c+dx}}dx}{3ac}-\frac{2\sqrt{c+dx}(6bc-ad)}{3acx^{3/2}\sqrt{a+bx}}}{5a}-\frac{2\sqrt{c+dx}}{5ax^{5/2}\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{\int \frac{24b^2c^2-7abdc-2a^2d^2+3bd(6bc-ad)x}{x^{3/2}(a+bx)^{3/2}\sqrt{c+dx}}dx}{3ac}-\frac{2\sqrt{c+dx}(6bc-ad)}{3acx^{3/2}\sqrt{a+bx}}}{5a}-\frac{2\sqrt{c+dx}}{5ax^{5/2}\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \; 169$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{2\int \frac{b\left(c\left(48b^2c^2-32abdc-a^2d^2\right)+d\left(24b^2c^2-7abdc-2a^2d^2\right)x\right)}{2\sqrt{x}(a+bx)^{3/2}\sqrt{c+dx}}dx}{ac}-\frac{2\sqrt{c+dx}\left(\frac{24b^2c}{a}-\frac{2ad^2}{c}-7bd\right)}{\sqrt{x}\sqrt{a+bx}}}{3ac}-\frac{2\sqrt{c+dx}(6bc-ad)}{3acx^{3/2}\sqrt{a+bx}}}{5a}-\frac{2\sqrt{c+dx}}{5ax^{5/2}\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{b\int \frac{c\left(48b^2c^2-32abdc-a^2d^2\right)+d\left(24b^2c^2-7abdc-2a^2d^2\right)x}{\sqrt{x}(a+bx)^{3/2}\sqrt{c+dx}}dx}{ac}-\frac{2\sqrt{c+dx}\left(\frac{24b^2c}{a}-\frac{2ad^2}{c}-7bd\right)}{\sqrt{x}\sqrt{a+bx}}}{3ac}-\frac{2\sqrt{c+dx}(6bc-ad)}{3acx^{3/2}\sqrt{a+bx}}}{5a}-\frac{2\sqrt{c+dx}}{5ax^{5/2}\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$\Big\downarrow$ 169

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{b\left(\frac{2\int -\frac{d(bc-ad)\left(ac(24bc-ad)+2\left(24b^2c^2-4abdc-a^2d^2\right)x\right)}{2\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{a(bc-ad)}+\frac{4\sqrt{x}\sqrt{c+dx}\left(-a^2d^2-4abcd+24b^2c^2\right)}{a\sqrt{a+bx}}\right)}{ac}-\frac{2\sqrt{c+dx}\left(\frac{24b^2c}{a}-\cdots\right.}{\sqrt{x}\sqrt{a+bx}\cdots}\cdots}{3ac}\cdots}{5a}\cdots\right.}{(ax+bx^2)^{3/2}\cdots}$$

$\Big\downarrow$ 27

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{b\left(\frac{4\sqrt{x}\sqrt{c+dx}\left(-a^2d^2-4abcd+24b^2c^2\right)}{a\sqrt{a+bx}}-\frac{d\int \frac{ac(24bc-ad)+2\left(24b^2c^2-4abdc-a^2d^2\right)x}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{a}\right)}{ac}-\frac{2\sqrt{c+dx}\left(\frac{24b^2c}{a}-\frac{2ad^2}{c}-7bd\right)}{\sqrt{x}\sqrt{a+bx}}-\cdots}{3ac}\cdots}{5a}\cdots\right.}{(ax+bx^2)^{3/2}\cdots}$$

$\Big\downarrow$ 176

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{b\left(\frac{4\sqrt{x}\sqrt{c+dx}\left(-a^2d^2-4abcd+24b^2c^2\right)}{a\sqrt{a+bx}}-\frac{d\left(\frac{2\left(-a^2d^2-4abcd+24b^2c^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{x}\sqrt{a+bx}}dx}{d}-\frac{c\left(-a^2d^2-32abcd+48b^2c^2\right)\int \frac{\cdots}{\sqrt{x}\sqrt{a+\cdots}}}{d}\cdots\right.}{a}\cdots\right.}{ac}\cdots}{3ac}\cdots}{5a}\cdots\right.}{(ax+bx^2)^{3/2}\cdots}$$

$\Big\downarrow$ 122

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{b\left(\frac{4\sqrt{x}\sqrt{c+dx}\left(-a^2d^2-4abcd+24b^2c^2\right)}{a\sqrt{a+bx}}-\frac{d\left(\frac{2\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\left(-a^2d^2-4abcd+24b^2c^2\right)\int\frac{\sqrt{\frac{dx}{c}+1}}{\sqrt{x}\sqrt{\frac{bx}{a}+1}}dx}{d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-\frac{c\left(-a^2d^2-32abcd+4\cdots\right.}{\cdots}\right.}{a}\cdots\right.}{ac}\cdots}{3ac}\cdots}{5a}\cdots\right.}{\left(ax+bx^2\right)^{3/2}\cdots}$$

$\downarrow$ 120

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{b\left(\frac{4\sqrt{x}\sqrt{c+dx}\left(-a^2d^2-4abcd+24b^2c^2\right)}{a\sqrt{a+bx}}-\frac{d\left(\frac{4\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\left(-a^2d^2-4abcd+24b^2c^2\right)E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-\frac{c\left(-a^2d^{\cdots}\right.}{\cdots}\right.}{a}\cdots\right.}{ac}\cdots}{3ac}\cdots}{5a}\cdots\right.}{\left(ax+bx^2\right)^{3/2}\cdots}$$

$\downarrow$ 127

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{b\left(\frac{4\sqrt{x}\sqrt{c+dx}\left(-a^2d^2-4abcd+24b^2c^2\right)}{a\sqrt{a+bx}}-\frac{d\left(\frac{4\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\left(-a^2d^2-4abcd+24b^2c^2\right)E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-\frac{c\sqrt{\frac{bx}{a}+\cdots}}{\cdots}\cdots\right)}{a}\cdots\right)}{ac}\cdots}{3ac}\cdots}{5a}\cdots\right)}{(ax+bx^2)^{3/2}\cdots}$$

$$\Big\downarrow \; 126$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{b\left(\frac{4\sqrt{x}\sqrt{c+dx}\left(-a^2d^2-4abcd+24b^2c^2\right)}{a\sqrt{a+bx}}-\frac{d\left(\frac{4\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\left(-a^2d^2-4abcd+24b^2c^2\right)E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-\frac{2\sqrt{-a}c\cdots}{\cdots}\cdots\right)}{a}\cdots\right)}{ac}\cdots}{3ac}\cdots}{5\cdots}\cdots\right)}{(ax+bx\cdots}$$

input
```
Int[Sqrt[c + d*x]/(x^2*(a*x + b*x^2)^(3/2)),x]
```

output
```
(x^(3/2)*(a + b*x)^(3/2)*((-2*Sqrt[c + d*x])/(5*a*x^(5/2)*Sqrt[a + b*x]) -
 ((-2*(6*b*c - a*d)*Sqrt[c + d*x])/(3*a*c*x^(3/2)*Sqrt[a + b*x]) - ((-2*((
24*b^2*c)/a - 7*b*d - (2*a*d^2)/c)*Sqrt[c + d*x])/(Sqrt[x]*Sqrt[a + b*x])
- (b*((4*(24*b^2*c^2 - 4*a*b*c*d - a^2*d^2)*Sqrt[x]*Sqrt[c + d*x])/(a*Sqrt
[a + b*x]) - (d*((4*Sqrt[-a]*(24*b^2*c^2 - 4*a*b*c*d - a^2*d^2)*Sqrt[1 + (
b*x)/a]*Sqrt[c + d*x]*EllipticE[ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[-a]], (a*d)/
(b*c)])/(Sqrt[b]*d*Sqrt[a + b*x]*Sqrt[1 + (d*x)/c]) - (2*Sqrt[-a]*c*(48*b^
2*c^2 - 32*a*b*c*d - a^2*d^2)*Sqrt[1 + (b*x)/a]*Sqrt[1 + (d*x)/c]*Elliptic
F[ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[-a]], (a*d)/(b*c)])/(Sqrt[b]*d*Sqrt[a + b*
x]*Sqrt[c + d*x])))/a))/(a*c))/(3*a*c))/(5*a)))/(a*x + b*x^2)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 3.08 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.54

method	result
elliptic	$\frac{\sqrt{x(bx+a)(dx+c)}\left(-\frac{2\sqrt{bd\,x^3+ad\,x^2+bc\,x^2+acx}}{5a^2x^3}-\frac{2(ad-9bc)\sqrt{bd\,x^3+ad\,x^2+bc\,x^2+acx}}{15a^3c\,x^2}+\frac{2\left(bd\,x^2+adx+cbx+ac\right)\left(2a^2d^2+8abcd-33b^2c^2\right)}{15a^4c^2\sqrt{x\left(bd\,x^2+adx+cbx+ac\right)}}-\cdots\right.}{\cdots}$
default	$\frac{2\left(2x^2\sqrt{\frac{dx+c}{c}}\,\sqrt{\frac{d(bx+a)}{ad-bc}}\,\sqrt{-\frac{xd}{c}}\,\text{EllipticF}\left(\sqrt{\frac{dx+c}{c}},\sqrt{-\frac{bc}{ad-bc}}\right)a^3c\,d^3+7x^2\sqrt{\frac{dx+c}{c}}\,\sqrt{\frac{d(bx+a)}{ad-bc}}\,\sqrt{-\frac{xd}{c}}\,\text{EllipticF}\left(\sqrt{\frac{dx+c}{c}},\sqrt{-\frac{b}{ad-}}\right.\cdots}{\cdots}$

input

```
int((d*x+c)^(1/2)/x^2/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
(x*(b*x+a)*(d*x+c))^(1/2)/(x*(b*x+a))^(1/2)/(d*x+c)^(1/2)*(-2/5/a^2/x^3*(b
*d*x^3+a*d*x^2+b*c*x^2+a*c*x)^(1/2)-2/15/a^3/c*(a*d-9*b*c)*(b*d*x^3+a*d*x^
2+b*c*x^2+a*c*x)^(1/2)/x^2+2/15*(b*d*x^2+a*d*x+b*c*x+a*c)/a^4/c^2*(2*a^2*d
^2+8*a*b*c*d-33*b^2*c^2)/(x*(b*d*x^2+a*d*x+b*c*x+a*c))^(1/2)-2*(b*d*x^2+b*
c*x)*b^2/a^4/((x+a/b)*(b*d*x^2+b*c*x))^(1/2)+2*(-1/15*b*d*(a*d-9*b*c)/c/a^
3+b^2*(a*d-b*c)/a^4+1/a^4*b^3*c)*c/d*((x+c/d)/c*d)^(1/2)*((x+a/b)/(-c/d+a/
b))^(1/2)*(-1/c*x*d)^(1/2)/(b*d*x^3+a*d*x^2+b*c*x^2+a*c*x)^(1/2)*EllipticF
(((x+c/d)/c*d)^(1/2),(-c/d/(-c/d+a/b))^(1/2))+2*(-1/15*b*d*(2*a^2*d^2+8*a*
b*c*d-33*b^2*c^2)/a^4/c^2+1/a^4*b^3*d)*c/d*((x+c/d)/c*d)^(1/2)*((x+a/b)/(-
c/d+a/b))^(1/2)*(-1/c*x*d)^(1/2)/(b*d*x^3+a*d*x^2+b*c*x^2+a*c*x)^(1/2)*((-
c/d+a/b)*EllipticE(((x+c/d)/c*d)^(1/2),(-c/d/(-c/d+a/b))^(1/2))-a/b*Ellipt
icF(((x+c/d)/c*d)^(1/2),(-c/d/(-c/d+a/b))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 600, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{c+dx}}{x^2\left(ax+bx^2\right)^{3/2}}\,dx =$$
$$-\frac{2\left(\left((48\,b^4c^3-32\,ab^3c^2d-7\,a^2b^2cd^2-2\,a^3bd^3)x^4+(48\,ab^3c^3-32\,a^2b^2c^2d-7\,a^3bcd^2-2\,a^4d^3)x^3\right)\sqrt{bd}\text{we}\right.}{}$$

input
```
integrate((d*x+c)^(1/2)/x^2/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/45*(((48*b^4*c^3 - 32*a*b^3*c^2*d - 7*a^2*b^2*c*d^2 - 2*a^3*b*d^3)*x^4
+ (48*a*b^3*c^3 - 32*a^2*b^2*c^2*d - 7*a^3*b*c*d^2 - 2*a^4*d^3)*x^3)*sqrt(
b*d)*weierstrassPInverse(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/2
7*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), 1/3*(
3*b*d*x + b*c + a*d)/(b*d)) + 6*((24*b^4*c^2*d - 4*a*b^3*c*d^2 - a^2*b^2*d
^3)*x^4 + (24*a*b^3*c^2*d - 4*a^2*b^2*c*d^2 - a^3*b*d^3)*x^3)*sqrt(b*d)*we
ierstrassZeta(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^
3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), weierstrassPInve
rse(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3 - 3*a*b^
2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), 1/3*(3*b*d*x + b*c + a*d)/
(b*d))) + 3*(3*a^3*b*c^2*d + 2*(24*b^4*c^2*d - 4*a*b^3*c*d^2 - a^2*b^2*d^3
)*x^3 + (24*a*b^3*c^2*d - 7*a^2*b^2*c*d^2 - 2*a^3*b*d^3)*x^2 - (6*a^2*b^2*
c^2*d - a^3*b*c*d^2)*x)*sqrt(b*x^2 + a*x)*sqrt(d*x + c))/(a^4*b^2*c^2*d*x^
4 + a^5*b*c^2*d*x^3)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}}{x^2\left(ax+bx^2\right)^{3/2}}\,dx = \int \frac{\sqrt{c+dx}}{x^2\left(x\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**(1/2)/x**2/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral(sqrt(c + d*x)/(x**2*(x*(a + b*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}}{x^2 (ax+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx+c}}{(bx^2+ax)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((d*x+c)^(1/2)/x^2/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)/((b*x^2 + a*x)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}}{x^2 (ax+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx+c}}{(bx^2+ax)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((d*x+c)^(1/2)/x^2/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)/((b*x^2 + a*x)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}}{x^2 (ax+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{c+d\,x}}{x^2\,(b\,x^2+a\,x)^{3/2}}\,dx$$

input
```
int((c + d*x)^(1/2)/(x^2*(a*x + b*x^2)^(3/2)),x)
```

output
```
int((c + d*x)^(1/2)/(x^2*(a*x + b*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}}{x^2 \left(ax+bx^2\right)^{3/2}}\,dx = \int \frac{\sqrt{dx+c}\,\sqrt{bx+a}}{\sqrt{x}\,a^2x^3+2\sqrt{x}\,ab\,x^4+\sqrt{x}\,b^2x^5}dx$$

input
```
int((d*x+c)^(1/2)/x^2/(b*x^2+a*x)^(3/2),x)
```

output
```
int((sqrt(c + d*x)*sqrt(a + b*x))/(sqrt(x)*a**2*x**3 + 2*sqrt(x)*a*b*x**4
+ sqrt(x)*b**2*x**5),x)
```

3.202 $\int \frac{x^5}{\sqrt{c+dx}(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 406

$$\begin{aligned}\int \frac{x^5}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx &= \frac{2(8b^2c^2+13abcd+24a^2d^2)\,x\sqrt{c+dx}}{15b^3d^3\sqrt{ax+bx^2}} \\ &- \frac{4(2bc+3ad)x^2\sqrt{c+dx}}{15b^2d^2\sqrt{ax+bx^2}} + \frac{2x^3\sqrt{c+dx}}{5bd\sqrt{ax+bx^2}} \\ &- \frac{2\sqrt{a}(8b^3c^3+9ab^2c^2d+16a^2bcd^2-48a^3d^3)\,\sqrt{x}\sqrt{c+dx}\,E\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15b^{7/2}d^3(bc-ad)\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}} \\ &+ \frac{2a^{3/2}(4b^2c^2+5abcd-24a^2d^2)\,\sqrt{x}\sqrt{c+dx}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15b^{7/2}d^2(bc-ad)\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}}\end{aligned}$$

output

```
2/15*(24*a^2*d^2+13*a*b*c*d+8*b^2*c^2)*x*(d*x+c)^(1/2)/b^3/d^3/(b*x^2+a*x)
^(1/2)-4/15*(3*a*d+2*b*c)*x^2*(d*x+c)^(1/2)/b^2/d^2/(b*x^2+a*x)^(1/2)+2/5*
x^3*(d*x+c)^(1/2)/b/d/(b*x^2+a*x)^(1/2)-2/15*a^(1/2)*(-48*a^3*d^3+16*a^2*b
*c*d^2+9*a*b^2*c^2*d+8*b^3*c^3)*x^(1/2)*(d*x+c)^(1/2)*EllipticE(b^(1/2)*x^
(1/2)/a^(1/2)/(1+b*x/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(7/2)/d^3/(-a*d+b*c)/(a
*(d*x+c)/c/(b*x+a))^(1/2)/(b*x^2+a*x)^(1/2)+2/15*a^(3/2)*(-24*a^2*d^2+5*a*
b*c*d+4*b^2*c^2)*x^(1/2)*(d*x+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x^(1
/2)/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(7/2)/d^2/(-a*d+b*c)/(a*(d*x+c)/c/(b*x+a
))^(1/2)/(b*x^2+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.49 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \frac{2\Big(adx(c+dx)\,(15a^3d^2-(bc-ad)(4bc+9ad)(a+bx)+3bd(bc-ad)x(a}{}$$

input
```
Integrate[x^5/(Sqrt[c + d*x]*(a*x + b*x^2)^(3/2)),x]
```

output
```
(2*(a*d*x*(c + d*x)*(15*a^3*d^2 - (b*c - a*d)*(4*b*c + 9*a*d)*(a + b*x) +
3*b*d*(b*c - a*d)*x*(a + b*x)) + Sqrt[a/b]*(Sqrt[a/b]*(8*b^3*c^3 + 9*a*b^2
*c^2*d + 16*a^2*b*c*d^2 - 48*a^3*d^3)*(a + b*x)*(c + d*x) + I*a*d*(8*b^3*c
^3 + 9*a*b^2*c^2*d + 16*a^2*b*c*d^2 - 48*a^3*d^3)*Sqrt[1 + a/(b*x)]*Sqrt[1
 + c/(d*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[a/b]/Sqrt[x]], (b*c)/(a*d)] -
 (4*I)*a*d*(b^3*c^3 + a*b^2*c^2*d + 10*a^2*b*c*d^2 - 12*a^3*d^3)*Sqrt[1 +
a/(b*x)]*Sqrt[1 + c/(d*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[a/b]/Sqrt[x]],
 (b*c)/(a*d)])))/(15*a*b^3*d^3*(b*c - a*d)*Sqrt[x*(a + b*x)]*Sqrt[c + d*x]
)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {1261, 109, 27, 171, 27, 171, 27, 176, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(ax+bx^2)^{3/2}\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \; 1261$$

$$\frac{x^{3/2}(a+bx)^{3/2}\int \frac{x^{7/2}}{(a+bx)^{3/2}\sqrt{c+dx}}dx}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 109$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2ax^{5/2}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{2\int \frac{x^{3/2}(5ac-(bc-6ad)x)}{2\sqrt{a+bx}\sqrt{c+dx}}dx}{b(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2ax^{5/2}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{\int \frac{x^{3/2}(5ac-(bc-6ad)x)}{\sqrt{a+bx}\sqrt{c+dx}}dx}{b(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 171$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2ax^{5/2}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{\frac{2\int \frac{\sqrt{x}\left(3ac(bc-6ad)+\left(4b^2c^2+5abdc-24a^2d^2\right)x\right)}{2\sqrt{a+bx}\sqrt{c+dx}}dx}{5bd}-\frac{2x^{3/2}\sqrt{a+bx}\sqrt{c+dx}(bc-6ad)}{5bd}}{b(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2ax^{5/2}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{\frac{\int \frac{\sqrt{x}\left(3ac(bc-6ad)+\left(4b^2c^2+5abdc-24a^2d^2\right)x\right)}{\sqrt{a+bx}\sqrt{c+dx}}dx}{5bd}-\frac{2x^{3/2}\sqrt{a+bx}\sqrt{c+dx}(bc-6ad)}{5bd}}{b(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 171$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2ax^{5/2}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{\frac{\frac{2\int -\frac{ac\left(4b^2c^2+5abdc-24a^2d^2\right)+\left(8b^3c^3+9ab^2dc^2+16a^2bd^2c-48a^3d^3\right)x}{2\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{3bd}+\frac{2}{3}\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}\left(-\frac{24a^2d}{b}+5\right.}{5bd}}{b(bc-ad)}\right.}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{x^{3/2}(a+bx)^{3/2}\Bigg(\frac{2ax^{5/2}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{\frac{\frac{2}{3}\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}\left(-\frac{24a^2d}{b}+5ac+\frac{4bc^2}{d}\right)-\frac{\int\frac{ac\left(4b^2c^2+5abdc-24a^2d^2\right)+\left(8b^3c^3+9ab^2dc^2+16a^2bd^2c-48a^3\ldots\right)}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}\ldots}{3bd}}{5bd}\ldots}{b(bc-ad)}\ldots}{(ax+bx^2)^{3/2}}$$

$\downarrow$ 176

$$\frac{x^{3/2}(a+bx)^{3/2}\Bigg(\frac{2ax^{5/2}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{\frac{\frac{2}{3}\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}\left(-\frac{24a^2d}{b}+5ac+\frac{4bc^2}{d}\right)-\frac{\frac{\left(-48a^3d^3+16a^2bcd^2+9ab^2c^2d+8b^3c^3\right)\int\frac{\sqrt{c+dx}}{\sqrt{x}\sqrt{a+bx}}dx}{d}-\frac{c(bc-a\ldots}{\ldots}}{3bd}}{5bd}\ldots}{b(bc-ad)}\ldots}{(ax+bx^2)^{3/2}}$$

$\downarrow$ 122

$$\frac{x^{3/2}(a+bx)^{3/2}\Bigg(\frac{2ax^{5/2}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{\frac{\frac{2}{3}\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}\left(-\frac{24a^2d}{b}+5ac+\frac{4bc^2}{d}\right)-\frac{\frac{\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\left(-48a^3d^3+16a^2bcd^2+9ab^2c^2d+8b^3c^3\right)\int\frac{\sqrt{\frac{dx}{c}+\ldots}}{\sqrt{x}\sqrt{\frac{bx}{a}\ldots}}\ldots}{d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}\ldots}{3\ldots}}{5bd}\ldots}{b(bc-ad)}\ldots}{(ax+bx^2)^{3/2}}$$

$\downarrow$ 120

$$\frac{x^{3/2}(a+bx)^{3/2}\Bigg(\frac{2ax^{5/2}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{\frac{\frac{2}{3}\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}\left(-\frac{24a^2d}{b}+5ac+\frac{4bc^2}{d}\right)-\frac{2\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\left(-48a^3d^3+16a^2bcd^2+9ab^2c^2d+8b^3c^3\right)E\left(\ldots\right)}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}\ldots}{5bd}\ldots}{b(b\ldots}\ldots}{(ax+bx^2)^{3/2}}$$

$\downarrow$ 127

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2ax^{5/2}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{\frac{2}{3}\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}\left(-\frac{24a^2d}{b}+5ac+\frac{4bc^2}{d}\right)-\frac{\frac{2\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\left(-48a^3d^3+16a^2bcd^2+9ab^2c^2d+8b^3c^3\right)E\left(\text{a}\right.}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}}{5bd}}{}\right.}{(ax+bx^2)^{3/2}}$$

$\Big\downarrow$ 126

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2ax^{5/2}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{\frac{2}{3}\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}\left(-\frac{24a^2d}{b}+5ac+\frac{4bc^2}{d}\right)-\frac{2\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\left(-48a^3d^3+16a^2bcd^2+9ab^2c^2d+8b^3c^3\right)E\left(\text{a}\right.}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}}{5}\right.}{(ax+bx^2)^{3}}$$

input

```
Int[x^5/(Sqrt[c + d*x]*(a*x + b*x^2)^(3/2)),x]
```

output

```
(x^(3/2)*(a + b*x)^(3/2)*((2*a*x^(5/2)*Sqrt[c + d*x])/(b*(b*c - a*d)*Sqrt[
a + b*x]) - ((-2*(b*c - 6*a*d)*x^(3/2)*Sqrt[a + b*x]*Sqrt[c + d*x])/(5*b*d
) + ((2*(5*a*c + (4*b*c^2)/d - (24*a^2*d)/b)*Sqrt[x]*Sqrt[a + b*x]*Sqrt[c
+ d*x])/3 - ((2*Sqrt[-a]*(8*b^3*c^3 + 9*a*b^2*c^2*d + 16*a^2*b*c*d^2 - 48*
a^3*d^3)*Sqrt[1 + (b*x)/a]*Sqrt[c + d*x]*EllipticE[ArcSin[(Sqrt[b]*Sqrt[x]
)/Sqrt[-a]], (a*d)/(b*c)])/(Sqrt[b]*d*Sqrt[a + b*x]*Sqrt[1 + (d*x)/c]) - (
2*Sqrt[-a]*c*(b*c - a*d)*(8*b^2*c^2 + 13*a*b*c*d + 24*a^2*d^2)*Sqrt[1 + (b
*x)/a]*Sqrt[1 + (d*x)/c]*EllipticF[ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[-a]], (a*
d)/(b*c)])/(Sqrt[b]*d*Sqrt[a + b*x]*Sqrt[c + d*x]))/(3*b*d))/(5*b*d))/(b*(
b*c - a*d))))/(a*x + b*x^2)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.45

method	result
elliptic	$\frac{\sqrt{x(bx+a)(dx+c)}\left(-\frac{2\left(bd\,x^{2}+cbx\right)a^{3}}{(ad-bc)b^{4}\sqrt{\left(x+\frac{a}{b}\right)\left(bd\,x^{2}+cbx\right)}}+\frac{2x\sqrt{bd\,x^{3}+ad\,x^{2}+bc\,x^{2}+acx}}{5b^{2}d}+\frac{2\left(-\frac{a}{b^{2}}-\frac{2(2ad+2bc)}{5b^{2}d}\right)\sqrt{bd\,x^{3}+ad\,x^{2}+bc\,x^{2}+acx}}{3bd}+\right.}{}$
default	$-\frac{2\left(48\sqrt{\frac{dx+c}{c}}\,\sqrt{\frac{d(bx+a)}{ad-bc}}\,\sqrt{-\frac{xd}{c}}\,\operatorname{EllipticF}\left(\sqrt{\frac{dx+c}{c}},\sqrt{-\frac{bc}{ad-bc}}\right)a^{4}c\,d^{4}-40\sqrt{\frac{dx+c}{c}}\,\sqrt{\frac{d(bx+a)}{ad-bc}}\,\sqrt{-\frac{xd}{c}}\,\operatorname{EllipticF}\left(\sqrt{\frac{dx+c}{c}},\sqrt{-\frac{b}{ad-}}\right.\right.}{}$

input
```
int(x^5/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(x*(b*x+a)*(d*x+c))^(1/2)/(x*(b*x+a))^(1/2)/(d*x+c)^(1/2)*(-2*(b*d*x^2+b*c
*x)/(a*d-b*c)/b^4*a^3/((x+a/b)*(b*d*x^2+b*c*x))^(1/2)+2/5/b^2/d*x*(b*d*x^3
+a*d*x^2+b*c*x^2+a*c*x)^(1/2)+2/3*(-a/b^2-2/5/b^2/d*(2*a*d+2*b*c))/b/d*(b*
d*x^3+a*d*x^2+b*c*x^2+a*c*x)^(1/2)+2*(1/b^3*c/(a*d-b*c)*a^3-1/3*(-a/b^2-2/
5/b^2/d*(2*a*d+2*b*c))/b/d*a*c)*c/d*((x+c/d)/c*d)^(1/2)*((x+a/b)/(-c/d+a/b
))^(1/2)*(-1/c*x*d)^(1/2)/(b*d*x^3+a*d*x^2+b*c*x^2+a*c*x)^(1/2)*EllipticF(
((x+c/d)/c*d)^(1/2),(-c/d/(-c/d+a/b))^(1/2))+2*(1/b^3*a^2+d/b^3*a^3/(a*d-b
*c)-3/5/b^2/d*a*c-2/3*(-a/b^2-2/5/b^2/d*(2*a*d+2*b*c))/b/d*(a*d+b*c))*c/d*
((x+c/d)/c*d)^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)*(-1/c*x*d)^(1/2)/(b*d*x^3+a
*d*x^2+b*c*x^2+a*c*x)^(1/2)*((-c/d+a/b)*EllipticE(((x+c/d)/c*d)^(1/2),(-c/
d/(-c/d+a/b))^(1/2))-a/b*EllipticF(((x+c/d)/c*d)^(1/2),(-c/d/(-c/d+a/b))^(
1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.58

$$\int \frac{x^5}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx =$$

$$-\frac{2\left(\left(8\,ab^4c^4+5\,a^2b^3c^3d+10\,a^3b^2c^2d^2+40\,a^4bcd^3-48\,a^5d^4+\left(8\,b^5c^4+5\,ab^4c^3d+10\,a^2b^3c^2d^2+40\,a^3b^2c\right.\right.\right.}{}$$

input
```
integrate(x^5/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/45*((8*a*b^4*c^4 + 5*a^2*b^3*c^3*d + 10*a^3*b^2*c^2*d^2 + 40*a^4*b*c*d^
3 - 48*a^5*d^4 + (8*b^5*c^4 + 5*a*b^4*c^3*d + 10*a^2*b^3*c^2*d^2 + 40*a^3*
b^2*c*d^3 - 48*a^4*b*d^4)*x)*sqrt(b*d)*weierstrassPInverse(4/3*(b^2*c^2 -
a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c
*d^2 + 2*a^3*d^3)/(b^3*d^3), 1/3*(3*b*d*x + b*c + a*d)/(b*d)) + 3*(8*a*b^4
*c^3*d + 9*a^2*b^3*c^2*d^2 + 16*a^3*b^2*c*d^3 - 48*a^4*b*d^4 + (8*b^5*c^3*
d + 9*a*b^4*c^2*d^2 + 16*a^2*b^3*c*d^3 - 48*a^3*b^2*d^4)*x)*sqrt(b*d)*weie
rstrassZeta(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3
- 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), weierstrassPInvers
e(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3 - 3*a*b^2*
c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), 1/3*(3*b*d*x + b*c + a*d)/(b
*d))) + 3*(4*a*b^4*c^2*d^2 + 5*a^2*b^3*c*d^3 - 24*a^3*b^2*d^4 - 3*(b^5*c*d
^3 - a*b^4*d^4)*x^2 + 2*(2*b^5*c^2*d^2 + a*b^4*c*d^3 - 3*a^2*b^3*d^4)*x)*s
qrt(b*x^2 + a*x)*sqrt(d*x + c))/(a*b^6*c*d^4 - a^2*b^5*d^5 + (b^7*c*d^4 -
a*b^6*d^5)*x)
```

Sympy [F]

$$\int \frac{x^5}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{x^5}{(x\,(a+bx))^{\frac{3}{2}}\,\sqrt{c+dx}}\,dx$$

input
```
integrate(x**5/(d*x+c)**(1/2)/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral(x**5/((x*(a + b*x))**(3/2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x^5}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{x^5}{(bx^2+ax)^{\frac{3}{2}}\,\sqrt{dx+c}}\,dx$$

input
```
integrate(x^5/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^5/((b*x^2 + a*x)^(3/2)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{x^5}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{x^5}{(bx^2+ax)^{\frac{3}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^5/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^5/((b*x^2 + a*x)^(3/2)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{x^5}{(b\,x^2+a\,x)^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(x^5/((a*x + b*x^2)^(3/2)*(c + d*x)^(1/2)),x)
```

output
```
int(x^5/((a*x + b*x^2)^(3/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^5}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^5/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x)
```

output

```
(18*sqrt(x)*sqrt(c + d*x)*sqrt(a + b*x)*a*c*d - 12*sqrt(x)*sqrt(c + d*x)*s
qrt(a + b*x)*a*d**2*x + 12*sqrt(x)*sqrt(c + d*x)*sqrt(a + b*x)*b*c**2 - 8*
sqrt(x)*sqrt(c + d*x)*sqrt(a + b*x)*b*c*d*x + 6*sqrt(x)*sqrt(c + d*x)*sqrt
(a + b*x)*b*d**2*x**2 - 9*int((sqrt(c + d*x)*sqrt(a + b*x))/(sqrt(x)*a**2*
c + sqrt(x)*a**2*d*x + 2*sqrt(x)*a*b*c*x + 2*sqrt(x)*a*b*d*x**2 + sqrt(x)*
b**2*c*x**2 + sqrt(x)*b**2*d*x**3),x)*a**3*c**2*d - 6*int((sqrt(c + d*x)*s
qrt(a + b*x))/(sqrt(x)*a**2*c + sqrt(x)*a**2*d*x + 2*sqrt(x)*a*b*c*x + 2*s
qrt(x)*a*b*d*x**2 + sqrt(x)*b**2*c*x**2 + sqrt(x)*b**2*d*x**3),x)*a**2*b*c
**3 - 9*int((sqrt(c + d*x)*sqrt(a + b*x))/(sqrt(x)*a**2*c + sqrt(x)*a**2*d
*x + 2*sqrt(x)*a*b*c*x + 2*sqrt(x)*a*b*d*x**2 + sqrt(x)*b**2*c*x**2 + sqrt
(x)*b**2*d*x**3),x)*a**2*b*c**2*d*x - 6*int((sqrt(c + d*x)*sqrt(a + b*x))/
(sqrt(x)*a**2*c + sqrt(x)*a**2*d*x + 2*sqrt(x)*a*b*c*x + 2*sqrt(x)*a*b*d*x
**2 + sqrt(x)*b**2*c*x**2 + sqrt(x)*b**2*d*x**3),x)*a*b**2*c**3*x + 24*int
((sqrt(x)*sqrt(c + d*x)*sqrt(a + b*x)*x)/(a**2*c + a**2*d*x + 2*a*b*c*x +
2*a*b*d*x**2 + b**2*c*x**2 + b**2*d*x**3),x)*a**3*d**3 + 4*int((sqrt(x)*sq
rt(c + d*x)*sqrt(a + b*x)*x)/(a**2*c + a**2*d*x + 2*a*b*c*x + 2*a*b*d*x**2
 + b**2*c*x**2 + b**2*d*x**3),x)*a**2*b*c*d**2 + 24*int((sqrt(x)*sqrt(c +
d*x)*sqrt(a + b*x)*x)/(a**2*c + a**2*d*x + 2*a*b*c*x + 2*a*b*d*x**2 + b**2
*c*x**2 + b**2*d*x**3),x)*a**2*b*d**3*x + 2*int((sqrt(x)*sqrt(c + d*x)*sqr
t(a + b*x)*x)/(a**2*c + a**2*d*x + 2*a*b*c*x + 2*a*b*d*x**2 + b**2*c*x*...
```

3.203 $\int \frac{x^4}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 318

$$\int \frac{x^4}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = -\frac{4(bc+2ad)x\sqrt{c+dx}}{3b^2d^2\sqrt{ax+bx^2}} + \frac{2x^2\sqrt{c+dx}}{3bd\sqrt{ax+bx^2}} + \frac{2\sqrt{a}(2b^2c^2+3abcd-8a^2d^2)\sqrt{x}\sqrt{c+dx}\,E\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3b^{5/2}d^2(bc-ad)\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}} - \frac{2a^{3/2}(bc-4ad)\sqrt{x}\sqrt{c+dx}\,\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3b^{5/2}d(bc-ad)\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}}$$

output

```
-4/3*(2*a*d+b*c)*x*(d*x+c)^(1/2)/b^2/d^2/(b*x^2+a*x)^(1/2)+2/3*x^2*(d*x+c)
^(1/2)/b/d/(b*x^2+a*x)^(1/2)+2/3*a^(1/2)*(-8*a^2*d^2+3*a*b*c*d+2*b^2*c^2)*
x^(1/2)*(d*x+c)^(1/2)*EllipticE(b^(1/2)*x^(1/2)/a^(1/2)/(1+b*x/a)^(1/2),(1
-a*d/b/c)^(1/2))/b^(5/2)/d^2/(-a*d+b*c)/(a*(d*x+c)/c/(b*x+a))^(1/2)/(b*x^2
+a*x)^(1/2)-2/3*a^(3/2)*(-4*a*d+b*c)*x^(1/2)*(d*x+c)^(1/2)*InverseJacobiAM
(arctan(b^(1/2)*x^(1/2)/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(5/2)/d/(-a*d+b*c)/(
a*(d*x+c)/c/(b*x+a))^(1/2)/(b*x^2+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.78 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.97

$$\int \frac{x^4}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \frac{2(c+dx)\left(-8a^3d^2+a^2bd(3c-4dx)+b^3cx(2c-dx)+ab^2(2c^2+2cdx+a\right.}{}$$

input
```
Integrate[x^4/(Sqrt[c + d*x]*(a*x + b*x^2)^(3/2)),x]
```

output
```
(2*(c + d*x)*(-8*a^3*d^2 + a^2*b*d*(3*c - 4*d*x) + b^3*c*x*(2*c - d*x) + a
*b^2*(2*c^2 + 2*c*d*x + d^2*x^2)) - (2*I)*Sqrt[a/b]*b*d*(-2*b^2*c^2 - 3*a*
b*c*d + 8*a^2*d^2)*Sqrt[1 + a/(b*x)]*Sqrt[1 + c/(d*x)]*x^(3/2)*EllipticE[I
*ArcSinh[Sqrt[a/b]/Sqrt[x]], (b*c)/(a*d)] + (2*I)*Sqrt[a/b]*b*d*(-(b^2*c^2
) - 7*a*b*c*d + 8*a^2*d^2)*Sqrt[1 + a/(b*x)]*Sqrt[1 + c/(d*x)]*x^(3/2)*Ell
ipticF[I*ArcSinh[Sqrt[a/b]/Sqrt[x]], (b*c)/(a*d)])/(3*b^3*d^2*(-(b*c) + a*
d)*Sqrt[x*(a + b*x)]*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1261, 109, 27, 171, 27, 176, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(ax+bx^2)^{3/2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 1261}$$

$$\frac{x^{3/2}(a+bx)^{3/2}\int \frac{x^{5/2}}{(a+bx)^{3/2}\sqrt{c+dx}}dx}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 109}$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2ax^{3/2}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{2\int \frac{\sqrt{x}(3ac-(bc-4ad)x)}{2\sqrt{a+bx}\sqrt{c+dx}}dx}{b(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2ax^{3/2}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{\int \frac{\sqrt{x}(3ac-(bc-4ad)x)}{\sqrt{a+bx}\sqrt{c+dx}}dx}{b(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \; 171$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2ax^{3/2}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{\frac{2\int \frac{ac(bc-4ad)+\left(2b^2c^2+3abdc-8a^2d^2\right)x}{2\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{3bd}-\frac{2\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}(bc-4ad)}{3bd}}{b(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2ax^{3/2}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{\frac{\int \frac{ac(bc-4ad)+\left(2b^2c^2+3abdc-8a^2d^2\right)x}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{3bd}-\frac{2\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}(bc-4ad)}{3bd}}{b(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \; 176$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2ax^{3/2}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{\frac{\frac{\left(-8a^2d^2+3abcd+2b^2c^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{x}\sqrt{a+bx}}dx}{d}-\frac{2c(bc-ad)(2ad+bc)\int \frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{d}}{3bd}-\frac{2\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}(bc\cdots}{3bd}}{b(bc-ad)}\cdots\right.}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \; 122$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2ax^{3/2}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{\frac{\frac{\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\left(-8a^2d^2+3abcd+2b^2c^2\right)\int \frac{\sqrt{\frac{dx}{c}+1}}{\sqrt{x}\sqrt{\frac{bx}{a}+1}}dx}{d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-\frac{2c(bc-ad)(2ad+bc)\int \frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{d}}{3bd}-2\sqrt{x}\cdots}{b(bc-ad)}\cdots\right.}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \; 120$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2ax^{3/2}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{\frac{\frac{2\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\left(-8a^2d^2+3abcd+2b^2c^2\right)E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-\frac{2c(bc-ad)(2ad+bc)\int\frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+\cdots}}}{d}\cdots}{3bd}\cdots}{b(bc-ad)}\cdots\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \; 127$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2ax^{3/2}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{\frac{\frac{2\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\left(-8a^2d^2+3abcd+2b^2c^2\right)E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-\frac{2c\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}(bc-ad)(2ad+bc)\int\cdots}{d\sqrt{a+bx}\sqrt{c+dx}\cdots}\cdots}{3bd}\cdots}{b(bc-ad)}\cdots\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \; 126$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2ax^{3/2}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{\frac{\frac{2\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\left(-8a^2d^2+3abcd+2b^2c^2\right)E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-\frac{4\sqrt{-a}c\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}(bc-ad)(2ad+\cdots}{\sqrt{b}d\sqrt{a+b\cdots}}\cdots}{3bd}\cdots}{b(bc-ad)}\cdots\right)}{(ax+bx^2)^{3/2}}$$

input

```
Int[x^4/(Sqrt[c + d*x]*(a*x + b*x^2)^(3/2)),x]
```

output

```
(x^(3/2)*(a + b*x)^(3/2)*((2*a*x^(3/2)*Sqrt[c + d*x])/(b*(b*c - a*d)*Sqrt[
a + b*x]) - ((-2*(b*c - 4*a*d)*Sqrt[x]*Sqrt[a + b*x]*Sqrt[c + d*x])/(3*b*d
) + ((2*Sqrt[-a]*(2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^2)*Sqrt[1 + (b*x)/a]*Sqr
t[c + d*x]*EllipticE[ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[-a]], (a*d)/(b*c)])/(Sq
rt[b]*d*Sqrt[a + b*x]*Sqrt[1 + (d*x)/c]) - (4*Sqrt[-a]*c*(b*c - a*d)*(b*c
+ 2*a*d)*Sqrt[1 + (b*x)/a]*Sqrt[1 + (d*x)/c]*EllipticF[ArcSin[(Sqrt[b]*Sqr
t[x])/Sqrt[-a]], (a*d)/(b*c)])/(Sqrt[b]*d*Sqrt[a + b*x]*Sqrt[c + d*x]))/(3
*b*d))/(b*(b*c - a*d))))/(a*x + b*x^2)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.49

method	result
elliptic	$\sqrt{x(bx+a)(dx+c)}\left(\frac{2(bd\,x^2+cbx)a^2}{(ad-bc)b^3\sqrt{\left(x+\frac{a}{b}\right)(bd\,x^2+cbx)}}+\frac{2\sqrt{bd\,x^3+ad\,x^2+bc\,x^2+acx}}{3b^2 d}+\frac{2\left(-\frac{c\,a^2}{b^2(ad-bc)}-\frac{ac}{3b^2 d}\right)c\sqrt{\frac{\left(x+\frac{c}{d}\right)d}{c}}\sqrt{\frac{x+\frac{a}{b}}{-\frac{c}{d}+\frac{a}{b}}}\sqrt{-\frac{xd}{c}}\cdots}{d\sqrt{bd\,x^3+ad\,x^2+bc\,x\cdots}}\cdots\right.$
default	$2\left(8\sqrt{\frac{dx+c}{c}}\sqrt{\frac{d(bx+a)}{ad-bc}}\sqrt{-\frac{xd}{c}}\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+c}{c}},\sqrt{-\frac{bc}{ad-bc}}\right)a^3 c\,d^3-7\sqrt{\frac{dx+c}{c}}\sqrt{\frac{d(bx+a)}{ad-bc}}\sqrt{-\frac{xd}{c}}\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+c}{c}},\sqrt{-\frac{bc}{ad-bc}}\right)\cdots\right.$

input
```
int(x^4/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(x*(b*x+a)*(d*x+c))^(1/2)/(x*(b*x+a))^(1/2)/(d*x+c)^(1/2)*(2*(b*d*x^2+b*c*
x)/(a*d-b*c)/b^3*a^2/((x+a/b)*(b*d*x^2+b*c*x))^(1/2)+2/3/b^2/d*(b*d*x^3+a*
d*x^2+b*c*x^2+a*c*x)^(1/2)+2*(-1/b^2*c/(a*d-b*c)*a^2-1/3/b^2/d*a*c)*c/d*((
x+c/d)/c*d)^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)*(-1/c*x*d)^(1/2)/(b*d*x^3+a*d
*x^2+b*c*x^2+a*c*x)^(1/2)*EllipticF(((x+c/d)/c*d)^(1/2),(-c/d/(-c/d+a/b))^
(1/2))+2*(-a/b^2-d/b^2*a^2/(a*d-b*c)-2/3/b^2/d*(a*d+b*c))*c/d*((x+c/d)/c*d
)^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)*(-1/c*x*d)^(1/2)/(b*d*x^3+a*d*x^2+b*c*x
^2+a*c*x)^(1/2)*((-c/d+a/b)*EllipticE(((x+c/d)/c*d)^(1/2),(-c/d/(-c/d+a/b)
)^(1/2))-a/b*EllipticF(((x+c/d)/c*d)^(1/2),(-c/d/(-c/d+a/b))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.67

$$\int \frac{x^4}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \frac{2\left((2\,ab^3c^3+2\,a^2b^2c^2d+7\,a^3bcd^2-8\,a^4d^3+(2\,b^4c^3+2\,ab^3c^2d+7\,a^2b^2cd^2\right.}{}$$

input
```
integrate(x^4/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
2/9*((2*a*b^3*c^3 + 2*a^2*b^2*c^2*d + 7*a^3*b*c*d^2 - 8*a^4*d^3 + (2*b^4*c
^3 + 2*a*b^3*c^2*d + 7*a^2*b^2*c*d^2 - 8*a^3*b*d^3)*x)*sqrt(b*d)*weierstra
ssPInverse(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3 -
 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), 1/3*(3*b*d*x + b*c
+ a*d)/(b*d)) + 3*(2*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - 8*a^3*b*d^3 + (2*b^4*
c^2*d + 3*a*b^3*c*d^2 - 8*a^2*b^2*d^3)*x)*sqrt(b*d)*weierstrassZeta(4/3*(b
^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d -
3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), weierstrassPInverse(4/3*(b^2*c^2 - a
*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*
d^2 + 2*a^3*d^3)/(b^3*d^3), 1/3*(3*b*d*x + b*c + a*d)/(b*d))) + 3*(a*b^3*c
*d^2 - 4*a^2*b^2*d^3 + (b^4*c*d^2 - a*b^3*d^3)*x)*sqrt(b*x^2 + a*x)*sqrt(d
*x + c))/(a*b^5*c*d^3 - a^2*b^4*d^4 + (b^6*c*d^3 - a*b^5*d^4)*x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{x^4}{(x\,(a+bx))^{\frac{3}{2}}\,\sqrt{c+dx}}\,dx$$

input
```
integrate(x**4/(d*x+c)**(1/2)/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral(x**4/((x*(a + b*x))**(3/2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{x^4}{(bx^2+ax)^{\frac{3}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^4/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^2 + a*x)^(3/2)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{x^4}{(bx^2+ax)^{\frac{3}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^4/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^4/((b*x^2 + a*x)^(3/2)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{x^4}{{\left(b\,x^2+a\,x\right)}^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(x^4/((a*x + b*x^2)^(3/2)*(c + d*x)^(1/2)),x)
```

output
```
int(x^4/((a*x + b*x^2)^(3/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \frac{-6\sqrt{x}\,\sqrt{dx+c}\,\sqrt{bx+a}\,c+4\sqrt{x}\,\sqrt{dx+c}\,\sqrt{bx+a}\,dx+3\Big(\int \frac{}{\sqrt{x}\,a^2c+\sqrt{x}\,a^2dx}}{}$$

input
```
int(x^4/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x)
```

output
```
( - 6*sqrt(x)*sqrt(c + d*x)*sqrt(a + b*x)*c + 4*sqrt(x)*sqrt(c + d*x)*sqrt
(a + b*x)*d*x + 3*int((sqrt(c + d*x)*sqrt(a + b*x))/(sqrt(x)*a**2*c + sqrt
(x)*a**2*d*x + 2*sqrt(x)*a*b*c*x + 2*sqrt(x)*a*b*d*x**2 + sqrt(x)*b**2*c*x
**2 + sqrt(x)*b**2*d*x**3),x)*a**2*c**2 + 3*int((sqrt(c + d*x)*sqrt(a + b*
x))/(sqrt(x)*a**2*c + sqrt(x)*a**2*d*x + 2*sqrt(x)*a*b*c*x + 2*sqrt(x)*a*b
*d*x**2 + sqrt(x)*b**2*c*x**2 + sqrt(x)*b**2*d*x**3),x)*a*b*c**2*x - 8*int
((sqrt(x)*sqrt(c + d*x)*sqrt(a + b*x)*x)/(a**2*c + a**2*d*x + 2*a*b*c*x +
2*a*b*d*x**2 + b**2*c*x**2 + b**2*d*x**3),x)*a**2*d**2 - int((sqrt(x)*sqrt
(c + d*x)*sqrt(a + b*x)*x)/(a**2*c + a**2*d*x + 2*a*b*c*x + 2*a*b*d*x**2 +
 b**2*c*x**2 + b**2*d*x**3),x)*a*b*c*d - 8*int((sqrt(x)*sqrt(c + d*x)*sqrt
(a + b*x)*x)/(a**2*c + a**2*d*x + 2*a*b*c*x + 2*a*b*d*x**2 + b**2*c*x**2 +
 b**2*d*x**3),x)*a*b*d**2*x - int((sqrt(x)*sqrt(c + d*x)*sqrt(a + b*x)*x)/
(a**2*c + a**2*d*x + 2*a*b*c*x + 2*a*b*d*x**2 + b**2*c*x**2 + b**2*d*x**3)
,x)*b**2*c*d*x)/(6*b*d**2*(a + b*x))
```

3.204 $\int \frac{x^3}{\sqrt{c+dx}(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 243

$$\int \frac{x^3}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx=\frac{2x\sqrt{c+dx}}{bd\sqrt{ax+bx^2}}-\frac{2\sqrt{a}(bc-2ad)\sqrt{x}\sqrt{c+dx}E\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{b^{3/2}d(bc-ad)\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}}-\frac{2a^{3/2}\sqrt{x}\sqrt{c+dx}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{b^{3/2}(bc-ad)\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}}$$

output
```
2*x*(d*x+c)^(1/2)/b/d/(b*x^2+a*x)^(1/2)-2*a^(1/2)*(-2*a*d+b*c)*x^(1/2)*(d*
x+c)^(1/2)*EllipticE(b^(1/2)*x^(1/2)/a^(1/2)/(1+b*x/a)^(1/2),(1-a*d/b/c)^(
1/2))/b^(3/2)/d/(-a*d+b*c)/(a*(d*x+c)/c/(b*x+a))^(1/2)/(b*x^2+a*x)^(1/2)-2
*a^(3/2)*x^(1/2)*(d*x+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x^(1/2)/a^(1
/2)),(1-a*d/b/c)^(1/2))/b^(3/2)/(-a*d+b*c)/(a*(d*x+c)/c/(b*x+a))^(1/2)/(b*
x^2+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.25 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \frac{2(c+dx)\left(-2a^2d+b^2cx+ab(c-dx)\right)-2i\sqrt{\frac{a}{b}}bd(-bc+2ad)\sqrt{1+\frac{a}{bx}}\sqrt{1}}{}$$

input
```
Integrate[x^3/(Sqrt[c + d*x]*(a*x + b*x^2)^(3/2)),x]
```

output
```
(2*(c + d*x)*(-2*a^2*d + b^2*c*x + a*b*(c - d*x)) - (2*I)*Sqrt[a/b]*b*d*(-
(b*c) + 2*a*d)*Sqrt[1 + a/(b*x)]*Sqrt[1 + c/(d*x)]*x^(3/2)*EllipticE[I*Arc
Sinh[Sqrt[a/b]/Sqrt[x]], (b*c)/(a*d)] + (4*I)*Sqrt[a/b]*b*d*(-(b*c) + a*d)
*Sqrt[1 + a/(b*x)]*Sqrt[1 + c/(d*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[a/b]
/Sqrt[x]], (b*c)/(a*d)])/(b^2*d*(b*c - a*d)*Sqrt[x*(a + b*x)]*Sqrt[c + d*x
])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1261, 109, 27, 176, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(ax+bx^2)^{3/2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \; 1261$$

$$\frac{x^{3/2}(a+bx)^{3/2}\int \frac{x^{3/2}}{(a+bx)^{3/2}\sqrt{c+dx}}dx}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \; 109$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2a\sqrt{x}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{2\int\frac{ac-(bc-2ad)x}{2\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{b(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2a\sqrt{x}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{\int\frac{ac-(bc-2ad)x}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{b(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \; 176$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2a\sqrt{x}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{\frac{c(bc-ad)\int\frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{d}-\frac{(bc-2ad)\int\frac{\sqrt{c+dx}}{\sqrt{x}\sqrt{a+bx}}dx}{d}}{b(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \; 122$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2a\sqrt{x}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{\frac{c(bc-ad)\int\frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{d}-\frac{\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}(bc-2ad)\int\frac{\sqrt{\frac{dx}{c}+1}}{\sqrt{x}\sqrt{\frac{bx}{a}+1}}dx}{d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}}{b(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \; 120$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2a\sqrt{x}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{\frac{c(bc-ad)\int\frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{d}-\frac{2\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}(bc-2ad)E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}}{b(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \; 127$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2a\sqrt{x}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{\frac{c\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}(bc-ad)\int\frac{1}{\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}}dx}{d\sqrt{a+bx}\sqrt{c+dx}}-\frac{2\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}(bc-2ad)E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}}{b(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \; 126$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2a\sqrt{x}\sqrt{c+dx}}{b\sqrt{a+bx}(bc-ad)}-\frac{\frac{2\sqrt{-a}c\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}(bc-ad)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a+bx}\sqrt{c+dx}}-\frac{2\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}(bc-2ad)E\left(\arcsin\left(\frac{\sqrt{b}\cdots}{\sqrt{-}\cdots}\right.\right.}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}\cdots}{b(bc-ad)}\right.}{(ax+bx^2)^{3/2}}$$

input
```
Int[x^3/(Sqrt[c + d*x]*(a*x + b*x^2)^(3/2)),x]
```

output
```
(x^(3/2)*(a + b*x)^(3/2)*((2*a*Sqrt[x]*Sqrt[c + d*x])/(b*(b*c - a*d)*Sqrt[
a + b*x]) - ((-2*Sqrt[-a]*(b*c - 2*a*d)*Sqrt[1 + (b*x)/a]*Sqrt[c + d*x]*El
lipticE[ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[-a]], (a*d)/(b*c)])/(Sqrt[b]*d*Sqrt[
a + b*x]*Sqrt[1 + (d*x)/c]) + (2*Sqrt[-a]*c*(b*c - a*d)*Sqrt[1 + (b*x)/a]*
Sqrt[1 + (d*x)/c]*EllipticF[ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[-a]], (a*d)/(b*c
)])/(Sqrt[b]*d*Sqrt[a + b*x]*Sqrt[c + d*x]))/(b*(b*c - a*d))))/(a*x + b*x^
2)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.66

method	result
elliptic	$\frac{\sqrt{x(bx+a)(dx+c)}\left(-\frac{2\left(bd\,x^2+cbx\right)a}{(ad-bc)b^2\sqrt{\left(x+\frac{a}{b}\right)\left(bd\,x^2+cbx\right)}}+\frac{2c^2a\sqrt{\frac{\left(x+\frac{c}{d}\right)d}{c}}\sqrt{\frac{x+\frac{a}{b}}{-\frac{c}{d}+\frac{a}{b}}}\sqrt{-\frac{xd}{c}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{c}{d}\right)d}{c}},\sqrt{-\frac{c}{d\left(-\frac{c}{d}+\frac{a}{b}\right)}}\right)}{b(ad-bc)d\sqrt{bd\,x^3+ad\,x^2+bc\,x^2+acx}}+\frac{2\left(\frac{1}{b}-\cdots\right.}{\cdots}\right.}{\sqrt{x(bx+\cdots}}$
default	$-\frac{2\left(2\sqrt{\frac{dx+c}{c}}\sqrt{\frac{d(bx+a)}{ad-bc}}\sqrt{-\frac{xd}{c}}\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+c}{c}},\sqrt{-\frac{bc}{ad-bc}}\right)a^2c\,d^2-2a\,c^2\sqrt{\frac{dx+c}{c}}\sqrt{\frac{d(bx+a)}{ad-bc}}\sqrt{-\frac{xd}{c}}\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+c}{c}},\sqrt{-\cdots}\right.\cdots\right.}{\cdots}$

input
```
int(x^3/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(x*(b*x+a)*(d*x+c))^(1/2)/(x*(b*x+a))^(1/2)/(d*x+c)^(1/2)*(-2*(b*d*x^2+b*c
*x)/(a*d-b*c)/b^2*a/((x+a/b)*(b*d*x^2+b*c*x))^(1/2)+2/b*c^2/(a*d-b*c)*a/d*
((x+c/d)/c*d)^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)*(-1/c*x*d)^(1/2)/(b*d*x^3+a
*d*x^2+b*c*x^2+a*c*x)^(1/2)*EllipticF(((x+c/d)/c*d)^(1/2),(-c/d/(-c/d+a/b)
)^(1/2))+2*(1/b+a/b*d/(a*d-b*c))*c/d*((x+c/d)/c*d)^(1/2)*((x+a/b)/(-c/d+a/
b))^(1/2)*(-1/c*x*d)^(1/2)/(b*d*x^3+a*d*x^2+b*c*x^2+a*c*x)^(1/2)*((-c/d+a/
b)*EllipticE(((x+c/d)/c*d)^(1/2),(-c/d/(-c/d+a/b))^(1/2))-a/b*EllipticF(((
x+c/d)/c*d)^(1/2),(-c/d/(-c/d+a/b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(216) = 432$.

Time = 0.12 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.81

$$\int \frac{x^3}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \frac{2\left(3\sqrt{bx^2+ax}\sqrt{dx+c}\,ab^2d^2-\left(ab^2c^2+2\,a^2bcd-2\,a^3d^2+\left(b^3c^2+2\,ab^2cd\cdots\right.\right.\right.}{\cdots}$$

input
```
integrate(x^3/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(3*sqrt(b*x^2 + a*x)*sqrt(d*x + c)*a*b^2*d^2 - (a*b^2*c^2 + 2*a^2*b*c*
d - 2*a^3*d^2 + (b^3*c^2 + 2*a*b^2*c*d - 2*a^2*b*d^2)*x)*sqrt(b*d)*weierst
rassPInverse(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3
 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), 1/3*(3*b*d*x + b*
c + a*d)/(b*d)) - 3*(a*b^2*c*d - 2*a^2*b*d^2 + (b^3*c*d - 2*a*b^2*d^2)*x)*
sqrt(b*d)*weierstrassZeta(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/
27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), weie
rstrassPInverse(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*
c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), 1/3*(3*b*d*x +
 b*c + a*d)/(b*d))))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*
x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{x^3}{(x\,(a+bx))^{\frac{3}{2}}\sqrt{c+dx}}\,dx$$

input
```
integrate(x**3/(d*x+c)**(1/2)/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral(x**3/((x*(a + b*x))**(3/2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{x^3}{(bx^2+ax)^{\frac{3}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^3/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/((b*x^2 + a*x)^(3/2)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{x^3}{(bx^2+ax)^{\frac{3}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^3/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^3/((b*x^2 + a*x)^(3/2)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{x^3}{(b\,x^2+a\,x)^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(x^3/((a*x + b*x^2)^(3/2)*(c + d*x)^(1/2)),x)
```

output
```
int(x^3/((a*x + b*x^2)^(3/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{x}\,\sqrt{dx+c}\,\sqrt{bx+a}\,x}{b^2d\,x^3+2abd\,x^2+b^2c\,x^2+a^2dx+2abcx+a^2c}dx$$

input
```
int(x^3/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x)
```

output
```
int((sqrt(x)*sqrt(c + d*x)*sqrt(a + b*x)*x)/(a**2*c + a**2*d*x + 2*a*b*c*x
 + 2*a*b*d*x**2 + b**2*c*x**2 + b**2*d*x**3),x)
```

3.205 $\int \frac{x^2}{\sqrt{c+dx}\left(ax+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 201

$$\int \frac{x^2}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = -\frac{2\sqrt{a}\sqrt{x}\sqrt{c+dx}\,E\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{b}(bc-ad)\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}} + \frac{2\sqrt{a}\sqrt{x}\sqrt{c+dx}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}(bc-ad)\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}}$$

output
```
-2*a^(1/2)*x^(1/2)*(d*x+c)^(1/2)*EllipticE(b^(1/2)*x^(1/2)/a^(1/2)/(1+b*x/
a)^(1/2),(1-a*d/b/c)^(1/2))/b^(1/2)/(-a*d+b*c)/(a*(d*x+c)/c/(b*x+a))^(1/2)
/(b*x^2+a*x)^(1/2)+2*a^(1/2)*x^(1/2)*(d*x+c)^(1/2)*InverseJacobiAM(arctan(
b^(1/2)*x^(1/2)/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/(-a*d+b*c)/(a*(d*x+c)/
c/(b*x+a))^(1/2)/(b*x^2+a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 6.64 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \frac{2a\sqrt{-\frac{c}{d}}\sqrt{1+\frac{a}{bx}}(c+dx)-2c\sqrt{1+\frac{c}{dx}}\sqrt{x}(a+bx)E\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{\sqrt{x}}\right)|\frac{ad}{bc}\right)}{b\sqrt{-\frac{c}{d}}(bc-ad)\sqrt{1+\frac{a}{bx}}\sqrt{x(a+bx)}\sqrt{c+dx}}$$

input
```
Integrate[x^2/(Sqrt[c + d*x]*(a*x + b*x^2)^(3/2)),x]
```

output
```
(2*a*Sqrt[-(c/d)]*Sqrt[1 + a/(b*x)]*(c + d*x) - 2*c*Sqrt[1 + c/(d*x)]*Sqrt
[x]*(a + b*x)*EllipticE[ArcSin[Sqrt[-(c/d)]/Sqrt[x]], (a*d)/(b*c)])/(b*Sqr
t[-(c/d)]*(b*c - a*d)*Sqrt[1 + a/(b*x)]*Sqrt[x*(a + b*x)]*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.79, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1261, 110, 27, 122, 120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax+bx^2)^{3/2}\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{x^{3/2}(a+bx)^{3/2}\int \frac{\sqrt{x}}{(a+bx)^{3/2}\sqrt{c+dx}}dx}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{110}$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2\int \frac{\sqrt{c+dx}}{2\sqrt{x}\sqrt{a+bx}}dx}{bc-ad}-\frac{2\sqrt{x}\sqrt{c+dx}}{\sqrt{a+bx}(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{\int \frac{\sqrt{c+dx}}{\sqrt{x}\sqrt{a+bx}}dx}{bc-ad}-\frac{2\sqrt{x}\sqrt{c+dx}}{\sqrt{a+bx}(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \; 122$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\int \frac{\sqrt{\frac{dx}{c}+1}}{\sqrt{x}\sqrt{\frac{bx}{a}+1}}dx}{\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}(bc-ad)}-\frac{2\sqrt{x}\sqrt{c+dx}}{\sqrt{a+bx}(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \; 120$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(\frac{2\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}(bc-ad)}-\frac{2\sqrt{x}\sqrt{c+dx}}{\sqrt{a+bx}(bc-ad)}\right)}{(ax+bx^2)^{3/2}}$$

input
```
Int[x^2/(Sqrt[c + d*x]*(a*x + b*x^2)^(3/2)),x]
```

output
```
(x^(3/2)*(a + b*x)^(3/2)*((-2*Sqrt[x]*Sqrt[c + d*x])/((b*c - a*d)*Sqrt[a +
 b*x]) + (2*Sqrt[-a]*Sqrt[1 + (b*x)/a]*Sqrt[c + d*x]*EllipticE[ArcSin[(Sqr
t[b]*Sqrt[x])/Sqrt[-a]], (a*d)/(b*c)])/(Sqrt[b]*(b*c - a*d)*Sqrt[a + b*x]*
Sqrt[1 + (d*x)/c]))/(a*x + b*x^2)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(178) = 356$.

Time = 1.57 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.79

method	result
default	$\frac{2\left(\sqrt{\frac{dx+c}{c}}\,\sqrt{\frac{d(bx+a)}{ad-bc}}\,\sqrt{-\frac{xd}{c}}\,\operatorname{EllipticF}\left(\sqrt{\frac{dx+c}{c}},\sqrt{-\frac{bc}{ad-bc}}\right)acd-\sqrt{\frac{dx+c}{c}}\,\sqrt{\frac{d(bx+a)}{ad-bc}}\,\sqrt{-\frac{xd}{c}}\,\operatorname{EllipticF}\left(\sqrt{\frac{dx+c}{c}},\sqrt{-\frac{bc}{ad-bc}}\right)b\,c^2-\right.}{d}$
elliptic	$\frac{\sqrt{x(bx+a)(dx+c)}\left(\frac{2bd\,x^2+2cbx}{(ad-bc)b\sqrt{\left(x+\frac{a}{b}\right)\left(bd\,x^2+cbx\right)}}-\frac{2c^2\sqrt{\frac{\left(x+\frac{c}{d}\right)d}{c}}\,\sqrt{\frac{x+\frac{a}{b}}{-\frac{c}{d}+\frac{a}{b}}}\,\sqrt{-\frac{xd}{c}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{c}{d}\right)d}{c}},\sqrt{-\frac{c}{d\left(-\frac{c}{d}+\frac{a}{b}\right)}}\right)}{(ad-bc)d\sqrt{bd\,x^3+ad\,x^2+bc\,x^2+acx}}-\frac{2c\sqrt{\frac{\left(x+\frac{c}{d}\right.}{c}}}{}\right.}{\sqrt{x(bx+a)}\,\sqrt{dx+c}}$

input
```
int(x^2/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(((d*x+c)/c)^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-1/c*x*d)^(1/2)*Elliptic
F(((d*x+c)/c)^(1/2),(-b*c/(a*d-b*c))^(1/2))*a*c*d-((d*x+c)/c)^(1/2)*(d*(b*
x+a)/(a*d-b*c))^(1/2)*(-1/c*x*d)^(1/2)*EllipticF(((d*x+c)/c)^(1/2),(-b*c/(
a*d-b*c))^(1/2))*b*c^2-((d*x+c)/c)^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-1/c
*x*d)^(1/2)*EllipticE(((d*x+c)/c)^(1/2),(-b*c/(a*d-b*c))^(1/2))*a*c*d+((d*
x+c)/c)^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-1/c*x*d)^(1/2)*EllipticE(((d*x
+c)/c)^(1/2),(-b*c/(a*d-b*c))^(1/2))*b*c^2+b*x^2*d^2+b*c*d*x)*(x*(b*x+a))^
(1/2)*(d*x+c)^(1/2)/d/b/(a*d-b*c)/x/(b*d*x^2+a*d*x+b*c*x+a*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(178) = 356$.

Time = 0.09 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.90

$$\int \frac{x^2}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = -\frac{2\left(3\sqrt{bx^2+ax}\sqrt{dx+c}b^2d-(2\,abc-a^2d+(2\,b^2c-abd)x)\sqrt{bd}\text{weierstrassPInverse}\left(\frac{4\,(b^2c^2-abcd+a^2d^2)}{3\,b^2d^2},-\right.\right.}{}$$

input
```
integrate(x^2/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(3*sqrt(b*x^2 + a*x)*sqrt(d*x + c)*b^2*d - (2*a*b*c - a^2*d + (2*b^2*
c - a*b*d)*x)*sqrt(b*d)*weierstrassPInverse(4/3*(b^2*c^2 - a*b*c*d + a^2*d
^2)/(b^2*d^2), -4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^
3)/(b^3*d^3), 1/3*(3*b*d*x + b*c + a*d)/(b*d)) + 3*(b^2*d*x + a*b*d)*sqrt(
b*d)*weierstrassZeta(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2
*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), weierstra
ssPInverse(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3 -
 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), 1/3*(3*b*d*x + b*c
+ a*d)/(b*d))))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{x^2}{(x\,(a+bx))^{\frac{3}{2}}\sqrt{c+dx}}\,dx$$

input
```
integrate(x**2/(d*x+c)**(1/2)/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral(x**2/((x*(a + b*x))**(3/2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{x^2}{(bx^2+ax)^{\frac{3}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^2/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^2 + a*x)^(3/2)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{x^2}{(bx^2+ax)^{\frac{3}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^2/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/((b*x^2 + a*x)^(3/2)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{x^2}{(b\,x^2+a\,x)^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(x^2/((a*x + b*x^2)^(3/2)*(c + d*x)^(1/2)),x)
```

output
```
int(x^2/((a*x + b*x^2)^(3/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{x}\,\sqrt{dx+c}\,\sqrt{bx+a}}{b^2d\,x^3+2abd\,x^2+b^2c\,x^2+a^2dx+2abcx+a^2c}dx$$

input
```
int(x^2/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x)
```

output
```
int((sqrt(x)*sqrt(c + d*x)*sqrt(a + b*x))/(a**2*c + a**2*d*x + 2*a*b*c*x +
 2*a*b*d*x**2 + b**2*c*x**2 + b**2*d*x**3),x)
```

3.206 $\int \frac{x}{\sqrt{c+dx}\left(ax+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 205

$$\int \frac{x}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \frac{2\sqrt{b}\sqrt{x}\sqrt{c+dx}\,E\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}} - \frac{2\sqrt{a}d\sqrt{x}\sqrt{c+dx}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}c(bc-ad)\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}}$$

output
```
2*b^(1/2)*x^(1/2)*(d*x+c)^(1/2)*EllipticE(b^(1/2)*x^(1/2)/a^(1/2)/(1+b*x/a
)^(1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/(-a*d+b*c)/(a*(d*x+c)/c/(b*x+a))^(1/2)/
(b*x^2+a*x)^(1/2)-2*a^(1/2)*d*x^(1/2)*(d*x+c)^(1/2)*InverseJacobiAM(arctan
(b^(1/2)*x^(1/2)/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/(-a*d+b*c)/(a*(d*x+
c)/c/(b*x+a))^(1/2)/(b*x^2+a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 6.56 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.63

$$\int \frac{x}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \frac{2\sqrt{c+dx}\left(\sqrt{-\frac{a}{b}}\sqrt{1+\frac{c}{dx}}-\sqrt{1+\frac{a}{bx}}\sqrt{x}E\left(\arcsin\left(\frac{\sqrt{-\frac{a}{b}}}{\sqrt{x}}\right)|\frac{bc}{ad}\right)\right)}{\sqrt{-\frac{a}{b}}(-bc+ad)\sqrt{1+\frac{c}{dx}}\sqrt{x(a+bx)}}$$

input
```
Integrate[x/(Sqrt[c + d*x]*(a*x + b*x^2)^(3/2)),x]
```

output
```
(2*Sqrt[c + d*x]*(Sqrt[-(a/b)]*Sqrt[1 + c/(d*x)] - Sqrt[1 + a/(b*x)]*Sqrt[
x]*EllipticE[ArcSin[Sqrt[-(a/b)]/Sqrt[x]], (b*c)/(a*d)]))/(Sqrt[-(a/b)]*(-
(b*c) + a*d)*Sqrt[1 + c/(d*x)]*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.74, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1235, 27, 1268, 122, 120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(ax+bx^2)^{3/2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{1235}$$

$$\frac{2bx\sqrt{c+dx}}{a\sqrt{ax+bx^2}(bc-ad)}-\frac{2\int \frac{acd(a+bx)}{2\sqrt{c+dx}\sqrt{bx^2+ax}}dx}{a^2c(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2bx\sqrt{c+dx}}{a\sqrt{ax+bx^2}(bc-ad)}-\frac{d\int \frac{a+bx}{\sqrt{c+dx}\sqrt{bx^2+ax}}dx}{a(bc-ad)}$$

$$\Big\downarrow \text{1268}$$

$$\frac{2bx\sqrt{c+dx}}{a\sqrt{ax+bx^2}(bc-ad)}-\frac{d\sqrt{x}\sqrt{a+bx}\int \frac{\sqrt{a+bx}}{\sqrt{x}\sqrt{c+dx}}dx}{a\sqrt{ax+bx^2}(bc-ad)}$$

$$\Big\downarrow \text{ 122}$$

$$\frac{2bx\sqrt{c+dx}}{a\sqrt{ax+bx^2}(bc-ad)}-\frac{d\sqrt{x}(a+bx)\sqrt{\frac{dx}{c}+1}\int \frac{\sqrt{\frac{bx}{a}+1}}{\sqrt{x}\sqrt{\frac{dx}{c}+1}}dx}{a\sqrt{\frac{bx}{a}+1}\sqrt{ax+bx^2}\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \text{ 120}$$

$$\frac{2bx\sqrt{c+dx}}{a\sqrt{ax+bx^2}(bc-ad)}-\frac{2\sqrt{-c}\sqrt{d}\sqrt{x}(a+bx)\sqrt{\frac{dx}{c}+1}E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{x}}{\sqrt{-c}}\right)|\frac{bc}{ad}\right)}{a\sqrt{\frac{bx}{a}+1}\sqrt{ax+bx^2}\sqrt{c+dx}(bc-ad)}$$

input
```
Int[x/(Sqrt[c + d*x]*(a*x + b*x^2)^(3/2)),x]
```

output
```
(2*b*x*Sqrt[c + d*x])/(a*(b*c - a*d)*Sqrt[a*x + b*x^2]) - (2*Sqrt[-c]*Sqrt
[d]*Sqrt[x]*(a + b*x)*Sqrt[1 + (d*x)/c]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[x])
/Sqrt[-c]], (b*c)/(a*d)])/(a*(b*c - a*d)*Sqrt[1 + (b*x)/a]*Sqrt[c + d*x]*S
qrt[a*x + b*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.06

method	result
default	$\frac{2\left(\sqrt{\frac{dx+c}{c}}\sqrt{\frac{d(bx+a)}{ad-bc}}\sqrt{-\frac{xd}{c}}\,\text{EllipticE}\left(\sqrt{\frac{dx+c}{c}},\sqrt{-\frac{bc}{ad-bc}}\right)acd-\sqrt{\frac{dx+c}{c}}\sqrt{\frac{d(bx+a)}{ad-bc}}\sqrt{-\frac{xd}{c}}\,\text{EllipticE}\left(\sqrt{\frac{dx+c}{c}},\sqrt{-\frac{bc}{ad-bc}}\right)bc^2-}{ad(ad-bc)x(bdx^2+adx+cbx+ac)}$
elliptic	$\frac{\sqrt{x(bx+a)(dx+c)}\left(-\frac{2\left(bdx^2+cbx\right)}{a(ad-bc)\sqrt{\left(x+\frac{a}{b}\right)\left(bdx^2+cbx\right)}}+\frac{2\left(\frac{1}{a}+\frac{bc}{a(ad-bc)}\right)c\sqrt{\frac{\left(x+\frac{c}{d}\right)d}{c}}\sqrt{\frac{x+\frac{a}{b}}{-\frac{c}{d}+\frac{a}{b}}}\sqrt{-\frac{xd}{c}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{c}{d}\right)d}{c}},\sqrt{-\frac{c}{d\left(-\frac{c}{d}+\right.}}\right.}{d\sqrt{bdx^3+adx^2+bcx^2+acx}}\right.}{\sqrt{x(bx+a}}$

input
```
int(x/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(((d*x+c)/c)^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-1/c*x*d)^(1/2)*Elliptic
E(((d*x+c)/c)^(1/2),(-b*c/(a*d-b*c))^(1/2))*a*c*d-((d*x+c)/c)^(1/2)*(d*(b*
x+a)/(a*d-b*c))^(1/2)*(-1/c*x*d)^(1/2)*EllipticE(((d*x+c)/c)^(1/2),(-b*c/(
a*d-b*c))^(1/2))*b*c^2-b*x^2*d^2-b*c*d*x)*(x*(b*x+a))^(1/2)*(d*x+c)^(1/2)/
a/d/(a*d-b*c)/x/(b*d*x^2+a*d*x+b*c*x+a*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(182) = 364$.

Time = 0.09 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.86

$$\int \frac{x}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \frac{2\left(3\sqrt{bx^2+ax}\sqrt{dx+c}b^2d+(abc-2a^2d+(b^2c-2abd)x)\sqrt{bd}\text{weierstras}\right.}{}$$

input
```
integrate(x/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(3*sqrt(b*x^2 + a*x)*sqrt(d*x + c)*b^2*d + (a*b*c - 2*a^2*d + (b^2*c -
 2*a*b*d)*x)*sqrt(b*d)*weierstrassPInverse(4/3*(b^2*c^2 - a*b*c*d + a^2*d^
2)/(b^2*d^2), -4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3
)/(b^3*d^3), 1/3*(3*b*d*x + b*c + a*d)/(b*d)) + 3*(b^2*d*x + a*b*d)*sqrt(b
*d)*weierstrassZeta(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*
b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), weierstras
sPInverse(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3 -
3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), 1/3*(3*b*d*x + b*c +
 a*d)/(b*d))))/(a^2*b^2*c*d - a^3*b*d^2 + (a*b^3*c*d - a^2*b^2*d^2)*x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{x}{(x(a+bx))^{\frac{3}{2}}\sqrt{c+dx}}\,dx$$

input
```
integrate(x/(d*x+c)**(1/2)/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral(x/((x*(a + b*x))**(3/2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{x}{(bx^2+ax)^{\frac{3}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^2 + a*x)^(3/2)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{x}{(bx^2+ax)^{\frac{3}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/((b*x^2 + a*x)^(3/2)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{x}{(b\,x^2+a\,x)^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(x/((a*x + b*x^2)^(3/2)*(c + d*x)^(1/2)),x)
```

output
```
int(x/((a*x + b*x^2)^(3/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx+c}\,\sqrt{bx+a}}{\sqrt{x}\,a^2c+\sqrt{x}\,a^2dx+2\sqrt{x}\,abcx+2\sqrt{x}\,abd\,x^2+\sqrt{x}\,b^2c\,x^2+\sqrt{x}\,b^2d\,x^3}dx$$

input

```
int(x/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x)
```

output

```
int((sqrt(c + d*x)*sqrt(a + b*x))/(sqrt(x)*a**2*c + sqrt(x)*a**2*d*x + 2*s
qrt(x)*a*b*c*x + 2*sqrt(x)*a*b*d*x**2 + sqrt(x)*b**2*c*x**2 + sqrt(x)*b**2
*d*x**3),x)
```

3.207 $\int \frac{1}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 247

$$\int \frac{1}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = -\frac{2\sqrt{c+dx}}{ac\sqrt{ax+bx^2}} - \frac{2\sqrt{b}(2bc-ad)\sqrt{x}\sqrt{c+dx}\,E\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{a^{3/2}c(bc-ad)\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}} + \frac{2\sqrt{b}d\sqrt{x}\sqrt{c+dx}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{a}c(bc-ad)\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}}$$

output
```
-2*(d*x+c)^(1/2)/a/c/(b*x^2+a*x)^(1/2)-2*b^(1/2)*(-a*d+2*b*c)*x^(1/2)*(d*x
+c)^(1/2)*EllipticE(b^(1/2)*x^(1/2)/a^(1/2)/(1+b*x/a)^(1/2),(1-a*d/b/c)^(1
/2))/a^(3/2)/c/(-a*d+b*c)/(a*(d*x+c)/c/(b*x+a))^(1/2)/(b*x^2+a*x)^(1/2)+2*
b^(1/2)*d*x^(1/2)*(d*x+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x^(1/2)/a^(
1/2)),(1-a*d/b/c)^(1/2))/a^(1/2)/c/(-a*d+b*c)/(a*(d*x+c)/c/(b*x+a))^(1/2)/
(b*x^2+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.95 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \frac{-2abc(c+dx)+2i\sqrt{\frac{a}{b}}bd(-2bc+ad)\sqrt{1+\frac{a}{bx}}\sqrt{1+\frac{c}{dx}}x^{3/2}E\left(i\text{arcsinh}\left(\cdots\right.\right.}{a^2c(-bc+a\cdots}$$

input
```
Integrate[1/(Sqrt[c + d*x]*(a*x + b*x^2)^(3/2)),x]
```

output
```
(-2*a*b*c*(c + d*x) + (2*I)*Sqrt[a/b]*b*d*(-2*b*c + a*d)*Sqrt[1 + a/(b*x)]
*Sqrt[1 + c/(d*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[a/b]/Sqrt[x]], (b*c)/(
a*d)] - (2*I)*Sqrt[a/b]*b*d*(-(b*c) + a*d)*Sqrt[1 + a/(b*x)]*Sqrt[1 + c/(d
*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[a/b]/Sqrt[x]], (b*c)/(a*d)])/(a^2*c*
(-(b*c) + a*d)*Sqrt[x*(a + b*x)]*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {1165, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax+bx^2)^{3/2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \; 1165$$

$$-\frac{2\int -\frac{bd(ac+(2bc-ad)x)}{2\sqrt{c+dx}\sqrt{bx^2+ax}}dx}{a^2c(bc-ad)}-\frac{2\sqrt{c+dx}(bx(2bc-ad)+a(bc-ad))}{a^2c\sqrt{ax+bx^2}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{bd\int \frac{ac+(2bc-ad)x}{\sqrt{c+dx}\sqrt{bx^2+ax}}dx}{a^2c(bc-ad)}-\frac{2\sqrt{c+dx}(bx(2bc-ad)+a(bc-ad))}{a^2c\sqrt{ax+bx^2}(bc-ad)}$$

$$\Big\downarrow \text{1269}$$

$$\frac{bd\left(\frac{(2bc-ad)\int \frac{\sqrt{c+dx}}{\sqrt{bx^2+ax}}dx}{d}-\frac{2c(bc-ad)\int \frac{1}{\sqrt{c+dx}\sqrt{bx^2+ax}}dx}{d}\right)}{a^2c(bc-ad)}-\frac{2\sqrt{c+dx}(bx(2bc-ad)+a(bc-ad))}{a^2c\sqrt{ax+bx^2}(bc-ad)}$$

$$\Big\downarrow \text{1169}$$

$$\frac{bd\left(\frac{\sqrt{x}\sqrt{a+bx}(2bc-ad)\int \frac{\sqrt{c+dx}}{\sqrt{x}\sqrt{a+bx}}dx}{d\sqrt{ax+bx^2}}-\frac{2c\sqrt{x}\sqrt{a+bx}(bc-ad)\int \frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{d\sqrt{ax+bx^2}}\right)}{a^2c(bc-ad)}-\frac{2\sqrt{c+dx}(bx(2bc-ad)+a(bc-ad))}{a^2c\sqrt{ax+bx^2}(bc-ad)}$$

$$\Big\downarrow \text{122}$$

$$\frac{bd\left(\frac{\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}(2bc-ad)\int \frac{\sqrt{\frac{dx}{c}+1}}{\sqrt{x}\sqrt{\frac{bx}{a}+1}}dx}{d\sqrt{ax+bx^2}\sqrt{\frac{dx}{c}+1}}-\frac{2c\sqrt{x}\sqrt{a+bx}(bc-ad)\int \frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{d\sqrt{ax+bx^2}}\right)}{a^2c(bc-ad)}-\frac{2\sqrt{c+dx}(bx(2bc-ad)+a(bc-ad))}{a^2c\sqrt{ax+bx^2}(bc-ad)}$$

$$\Big\downarrow \text{120}$$

$$\frac{bd\left(\frac{2\sqrt{-a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}(2bc-ad)E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{ax+bx^2}\sqrt{\frac{dx}{c}+1}}-\frac{2c\sqrt{x}\sqrt{a+bx}(bc-ad)\int \frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{d\sqrt{ax+bx^2}}\right)}{a^2c(bc-ad)}-\frac{2\sqrt{c+dx}(bx(2bc-ad)+a(bc-ad))}{a^2c\sqrt{ax+bx^2}(bc-ad)}$$

$$\Big\downarrow \text{127}$$

$$\frac{bd\left(\frac{2\sqrt{-a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}(2bc-ad)E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{ax+bx^2}\sqrt{\frac{dx}{c}+1}}-\frac{2c\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}(bc-ad)\int \frac{1}{\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}}dx}{d\sqrt{ax+bx^2}\sqrt{c+dx}}\right)}{a^2c(bc-ad)}-\frac{2\sqrt{c+dx}(bx(2bc-ad)+a(bc-ad))}{a^2c\sqrt{ax+bx^2}(bc-ad)}$$

$$\Big\downarrow \text{126}$$

$$\frac{bd\left(\frac{2\sqrt{-a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}(2bc-ad)E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{ax+bx^2}\sqrt{\frac{dx}{c}+1}}-\frac{4\sqrt{-a}c\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}(bc-ad)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{ax+bx^2}\sqrt{c+dx}}\right)}{a^2c(bc-ad)}-\frac{2\sqrt{c+dx}(bx(2bc-ad)+a(bc-ad))}{a^2c\sqrt{ax+bx^2}(bc-ad)}$$

input
```
Int[1/(Sqrt[c + d*x]*(a*x + b*x^2)^(3/2)),x]
```

output
```
(-2*Sqrt[c + d*x]*(a*(b*c - a*d) + b*(2*b*c - a*d)*x))/(a^2*c*(b*c - a*d)*
Sqrt[a*x + b*x^2]) + (b*d*((2*Sqrt[-a]*(2*b*c - a*d)*Sqrt[x]*Sqrt[1 + (b*x
)/a]*Sqrt[c + d*x]*EllipticE[ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[-a]], (a*d)/(b*
c)])/(Sqrt[b]*d*Sqrt[1 + (d*x)/c]*Sqrt[a*x + b*x^2]) - (4*Sqrt[-a]*c*(b*c
- a*d)*Sqrt[x]*Sqrt[1 + (b*x)/a]*Sqrt[1 + (d*x)/c]*EllipticF[ArcSin[(Sqrt[
b]*Sqrt[x])/Sqrt[-a]], (a*d)/(b*c)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a*x + b
*x^2])))/(a^2*c*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(220) = 440$.

Time = 2.69 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.91

method	result
elliptic	$\frac{\sqrt{x(bx+a)(dx+c)}\left(\frac{2\left(bd\,x^2+cbx\right)b}{(ad-bc)a^2\sqrt{\left(x+\frac{a}{b}\right)\left(bd\,x^2+cbx\right)}}-\frac{2\left(bd\,x^2+adx+cbx+ac\right)}{a^2c\sqrt{x\left(bd\,x^2+adx+cbx+ac\right)}}+\frac{2\left(-\frac{b}{a^2}-\frac{b^2c}{(ad-bc)a^2}\right)c\sqrt{\frac{\left(x+\frac{c}{d}\right)d}{c}}\sqrt{\frac{x+\frac{a}{b}}{-\frac{c}{d}+\frac{a}{b}}}\sqrt{-\frac{xd}{c}}}{d\sqrt{bd\,x^3+ad\,x^2+bc}}\right.}{}$
default	$-\frac{2\left(\sqrt{\frac{dx+c}{c}}\sqrt{\frac{d(bx+a)}{ad-bc}}\sqrt{-\frac{xd}{c}}\,\text{EllipticF}\left(\sqrt{\frac{dx+c}{c}},\sqrt{-\frac{bc}{ad-bc}}\right)a^2c\,d^2-a\,c^2\sqrt{\frac{dx+c}{c}}\sqrt{\frac{d(bx+a)}{ad-bc}}\sqrt{-\frac{xd}{c}}\,\text{EllipticF}\left(\sqrt{\frac{dx+c}{c}},\sqrt{-\frac{bc}{ad-}}\right.\right.}{}$

input

```
int(1/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
(x*(b*x+a)*(d*x+c))^(1/2)/(x*(b*x+a))^(1/2)/(d*x+c)^(1/2)*(2*(b*d*x^2+b*c*
x)/(a*d-b*c)*b/a^2/((x+a/b)*(b*d*x^2+b*c*x))^(1/2)-2*(b*d*x^2+a*d*x+b*c*x+
a*c)/a^2/c/(x*(b*d*x^2+a*d*x+b*c*x+a*c))^(1/2)+2*(-b/a^2-b^2*c/(a*d-b*c)/a
^2)*c/d*((x+c/d)/c*d)^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)*(-1/c*x*d)^(1/2)/(b
*d*x^3+a*d*x^2+b*c*x^2+a*c*x)^(1/2)*EllipticF(((x+c/d)/c*d)^(1/2),(-c/d/(-
c/d+a/b))^(1/2))+2*(-1/a^2*d*b^2/(a*d-b*c)+1/a^2*d*b/c)*c/d*((x+c/d)/c*d)^
(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)*(-1/c*x*d)^(1/2)/(b*d*x^3+a*d*x^2+b*c*x^2
+a*c*x)^(1/2)*((-c/d+a/b)*EllipticE(((x+c/d)/c*d)^(1/2),(-c/d/(-c/d+a/b))^
(1/2))-a/b*EllipticF(((x+c/d)/c*d)^(1/2),(-c/d/(-c/d+a/b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. 2(220) = 440.

Time = 0.08 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx =$$
$$-\frac{2\left(\left((2\,b^3c^2-2\,ab^2cd-a^2bd^2)x^2+(2\,ab^2c^2-2\,a^2bcd-a^3d^2)x\right)\sqrt{bd}\text{weierstrassPInverse}\left(\frac{4\,(b^2c^2-abcd+a^2d^2)}{3\,b^2d^2}\right.\right.}{}$$

input
```
integrate(1/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(((2*b^3*c^2 - 2*a*b^2*c*d - a^2*b*d^2)*x^2 + (2*a*b^2*c^2 - 2*a^2*b*
c*d - a^3*d^2)*x)*sqrt(b*d)*weierstrassPInverse(4/3*(b^2*c^2 - a*b*c*d + a
^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^
3*d^3)/(b^3*d^3), 1/3*(3*b*d*x + b*c + a*d)/(b*d)) + 3*((2*b^3*c*d - a*b^2
*d^2)*x^2 + (2*a*b^2*c*d - a^2*b*d^2)*x)*sqrt(b*d)*weierstrassZeta(4/3*(b^
2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), weierstrassPInverse(4/3*(b^2*c^2 - a*
b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d
^2 + 2*a^3*d^3)/(b^3*d^3), 1/3*(3*b*d*x + b*c + a*d)/(b*d))) + 3*(a*b^2*c*
d - a^2*b*d^2 + (2*b^3*c*d - a*b^2*d^2)*x)*sqrt(b*x^2 + a*x)*sqrt(d*x + c)
)/((a^2*b^3*c^2*d - a^3*b^2*c*d^2)*x^2 + (a^3*b^2*c^2*d - a^4*b*c*d^2)*x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{1}{(x\,(a+bx))^{\frac{3}{2}}\,\sqrt{c+dx}}\,dx$$

input
```
integrate(1/(d*x+c)**(1/2)/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral(1/((x*(a + b*x))**(3/2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{3}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(1/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a*x)^(3/2)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{3}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(1/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a*x)^(3/2)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{1}{(b\,x^2+a\,x)^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(1/((a*x + b*x^2)^(3/2)*(c + d*x)^(1/2)),x)
```

output
```
int(1/((a*x + b*x^2)^(3/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \frac{-2\sqrt{dx+c}\,\sqrt{bx+a}-2\sqrt{x}\left(\int \frac{\sqrt{dx+c}\,\sqrt{bx+a}}{\sqrt{x}\,a^2c+\sqrt{x}\,a^2dx+2\sqrt{x}\,abcx+2\sqrt{x}\,abd\,x^2+\sqrt{x}\,b^2c\,x^2+\sqrt{x}}\right.}{}$$

input
```
int(1/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x)
```

output
```
( - 2*sqrt(c + d*x)*sqrt(a + b*x) - 2*sqrt(x)*int((sqrt(c + d*x)*sqrt(a +
b*x))/(sqrt(x)*a**2*c + sqrt(x)*a**2*d*x + 2*sqrt(x)*a*b*c*x + 2*sqrt(x)*a
*b*d*x**2 + sqrt(x)*b**2*c*x**2 + sqrt(x)*b**2*d*x**3),x)*a*b*c - 2*sqrt(x
)*int((sqrt(c + d*x)*sqrt(a + b*x))/(sqrt(x)*a**2*c + sqrt(x)*a**2*d*x + 2
*sqrt(x)*a*b*c*x + 2*sqrt(x)*a*b*d*x**2 + sqrt(x)*b**2*c*x**2 + sqrt(x)*b*
*2*d*x**3),x)*b**2*c*x - sqrt(x)*int((sqrt(x)*sqrt(c + d*x)*sqrt(a + b*x))
/(a**2*c + a**2*d*x + 2*a*b*c*x + 2*a*b*d*x**2 + b**2*c*x**2 + b**2*d*x**3
),x)*a*b*d - sqrt(x)*int((sqrt(x)*sqrt(c + d*x)*sqrt(a + b*x))/(a**2*c + a
**2*d*x + 2*a*b*c*x + 2*a*b*d*x**2 + b**2*c*x**2 + b**2*d*x**3),x)*b**2*d*
x)/(sqrt(x)*a*c*(a + b*x))
```

3.208 $\int \frac{1}{x\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 319

$$\int \frac{1}{x\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \frac{4(2bc+ad)\sqrt{c+dx}}{3a^2c^2\sqrt{ax+bx^2}}-\frac{2\sqrt{c+dx}}{3acx\sqrt{ax+bx^2}}+\frac{2\sqrt{b}(8b^2c^2-3abcd-2a^2d^2)\sqrt{x}\sqrt{c+dx}\,E\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{5/2}c^2(bc-ad)\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}}-\frac{2\sqrt{b}d(4bc-ad)\sqrt{x}\sqrt{c+dx}\,\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3a^{3/2}c^2(bc-ad)\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}}$$

output

```
4/3*(a*d+2*b*c)*(d*x+c)^(1/2)/a^2/c^2/(b*x^2+a*x)^(1/2)-2/3*(d*x+c)^(1/2)/
a/c/x/(b*x^2+a*x)^(1/2)+2/3*b^(1/2)*(-2*a^2*d^2-3*a*b*c*d+8*b^2*c^2)*x^(1/
2)*(d*x+c)^(1/2)*EllipticE(b^(1/2)*x^(1/2)/a^(1/2)/(1+b*x/a)^(1/2),(1-a*d/
b/c)^(1/2))/a^(5/2)/c^2/(-a*d+b*c)/(a*(d*x+c)/c/(b*x+a))^(1/2)/(b*x^2+a*x)
^(1/2)-2/3*b^(1/2)*d*(-a*d+4*b*c)*x^(1/2)*(d*x+c)^(1/2)*InverseJacobiAM(ar
ctan(b^(1/2)*x^(1/2)/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(3/2)/c^2/(-a*d+b*c)/(a
*(d*x+c)/c/(b*x+a))^(1/2)/(b*x^2+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.84 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \frac{-2ac(c+dx)\,(a^2d-4b^2cx+ab(-c+dx))-2i\sqrt{\frac{a}{b}}bd(-8b^2c^2+3abcd+$$

input
```
Integrate[1/(x*Sqrt[c + d*x]*(a*x + b*x^2)^(3/2)),x]
```

output
```
(-2*a*c*(c + d*x)*(a^2*d - 4*b^2*c*x + a*b*(-c + d*x)) - (2*I)*Sqrt[a/b]*b
*d*(-8*b^2*c^2 + 3*a*b*c*d + 2*a^2*d^2)*Sqrt[1 + a/(b*x)]*Sqrt[1 + c/(d*x)
]*x^(5/2)*EllipticE[I*ArcSinh[Sqrt[a/b]/Sqrt[x]], (b*c)/(a*d)] + (4*I)*Sqr
t[a/b]*b*d*(-2*b^2*c^2 + a*b*c*d + a^2*d^2)*Sqrt[1 + a/(b*x)]*Sqrt[1 + c/(
d*x)]*x^(5/2)*EllipticF[I*ArcSinh[Sqrt[a/b]/Sqrt[x]], (b*c)/(a*d)])/(3*a^3
*c^2*(-(b*c) + a*d)*x*Sqrt[x*(a + b*x)]*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.30, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {1261, 115, 27, 169, 27, 169, 27, 176, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\,(ax+bx^2)^{3/2}\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{1261}$$

$$\frac{x^{3/2}(a+bx)^{3/2}\int \frac{1}{x^{5/2}(a+bx)^{3/2}\sqrt{c+dx}}dx}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{115}$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{2\int \frac{4bc+2ad+3bdx}{2x^{3/2}(a+bx)^{3/2}\sqrt{c+dx}}dx}{3ac}-\frac{2\sqrt{c+dx}}{3acx^{3/2}\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{\int \frac{2(2bc+ad)+3bdx}{x^{3/2}(a+bx)^{3/2}\sqrt{c+dx}}dx}{3ac}-\frac{2\sqrt{c+dx}}{3acx^{3/2}\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{169}$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{2\int \frac{b(c(8bc+ad)+2d(2bc+ad)x)}{2\sqrt{x}(a+bx)^{3/2}\sqrt{c+dx}}dx}{ac}-\frac{4\sqrt{c+dx}(ad+2bc)}{ac\sqrt{x}\sqrt{a+bx}}}{3ac}-\frac{2\sqrt{c+dx}}{3acx^{3/2}\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{b\int \frac{c(8bc+ad)+2d(2bc+ad)x}{\sqrt{x}(a+bx)^{3/2}\sqrt{c+dx}}dx}{ac}-\frac{4\sqrt{c+dx}(ad+2bc)}{ac\sqrt{x}\sqrt{a+bx}}}{3ac}-\frac{2\sqrt{c+dx}}{3acx^{3/2}\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{169}$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{b\left(\frac{2\int -\frac{d\left(ac(4bc-ad)+\left(8b^2c^2-3abdc-2a^2d^2\right)x\right)}{2\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{a(bc-ad)}+\frac{2\sqrt{x}\sqrt{c+dx}\left(-2a^2d^2-3abcd+8b^2c^2\right)}{a\sqrt{a+bx}(bc-ad)}\right)}{ac}-\frac{4\sqrt{c+dx}(ad+2bc)}{ac\sqrt{x}\sqrt{a+bx}}}{3ac}-\frac{2\sqrt{c-}}{3acx^{3/2}}\right.}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{b\left(\frac{2\sqrt{x}\sqrt{c+dx}\left(-2a^2d^2-3abcd+8b^2c^2\right)}{a\sqrt{a+bx}(bc-ad)}-\frac{d\int \frac{ac(4bc-ad)+\left(8b^2c^2-3abdc-2a^2d^2\right)x}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{a(bc-ad)}\right)}{ac}-\frac{4\sqrt{c+dx}(ad+2bc)}{ac\sqrt{x}\sqrt{a+bx}}}{3ac}-\frac{2\sqrt{c+dx}}{3acx^{3/2}\sqrt{a+}}\right.}{(ax+bx^2)^{3/2}}$$

$\Big\downarrow$ 176

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{b\left(\frac{2\sqrt{x}\sqrt{c+dx}\left(-2a^2d^2-3abcd+8b^2c^2\right)}{a\sqrt{a+bx}(bc-ad)}-\frac{d\left(\frac{\left(-2a^2d^2-3abcd+8b^2c^2\right)\int\frac{\sqrt{c+dx}}{\sqrt{x}\sqrt{a+bx}}dx}{d}-\frac{c(bc-ad)(ad+8bc)\int\frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{d}\right.}{a(bc-ad)}}{ac}}{3ac}\right.}{\left(ax+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 122

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{b\left(\frac{2\sqrt{x}\sqrt{c+dx}\left(-2a^2d^2-3abcd+8b^2c^2\right)}{a\sqrt{a+bx}(bc-ad)}-\frac{d\left(\frac{\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\left(-2a^2d^2-3abcd+8b^2c^2\right)\int\frac{\sqrt{\frac{dx}{c}+1}}{\sqrt{x}\sqrt{\frac{bx}{a}+1}}dx}{d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-\frac{c(bc-ad)(ad+8bc)\int\frac{}{\sqrt{x}}}{d}\right.}{a(bc-ad)}}{ac}}{3ac}\right.}{\left(ax+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 120

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{b\left(\frac{2\sqrt{x}\sqrt{c+dx}\left(-2a^2d^2-3abcd+8b^2c^2\right)}{a\sqrt{a+bx}(bc-ad)}-\frac{d\left(\frac{2\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\left(-2a^2d^2-3abcd+8b^2c^2\right)E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-\frac{c(bc-ad)(\cdots}{\cdots}\right)}{a(bc-ad)}\cdots\right)}{ac}\cdots}{3ac}\cdots\right)}{\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 127$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{b\left(\frac{2\sqrt{x}\sqrt{c+dx}\left(-2a^2d^2-3abcd+8b^2c^2\right)}{a\sqrt{a+bx}(bc-ad)}-\frac{d\left(\frac{2\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\left(-2a^2d^2-3abcd+8b^2c^2\right)E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-\frac{c\sqrt{\frac{bx}{a}+1}\cdots}{\cdots}\right)}{a(bc-ad)}\cdots\right)}{ac}\cdots}{3ac}\cdots\right)}{\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 126$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{b\left(\frac{2\sqrt{x}\sqrt{c+dx}\left(-2a^2d^2-3abcd+8b^2c^2\right)}{a\sqrt{a+bx}(bc-ad)}-\frac{d\left(\frac{2\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\left(-2a^2d^2-3abcd+8b^2c^2\right)E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-\frac{2\sqrt{-a}c\sqrt{\cdots}}{\cdots}\right)}{a(bc-ad)}\cdots\right)}{ac}\cdots}{3ac}\cdots\right)}{\left(ax+bx^2\right)^{3/2}}$$

input
```
Int[1/(x*Sqrt[c + d*x]*(a*x + b*x^2)^(3/2)),x]
```

output
```
(x^(3/2)*(a + b*x)^(3/2)*((-2*Sqrt[c + d*x])/(3*a*c*x^(3/2)*Sqrt[a + b*x])
 - ((-4*(2*b*c + a*d)*Sqrt[c + d*x])/(a*c*Sqrt[x]*Sqrt[a + b*x]) - (b*((2*
(8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*Sqrt[x]*Sqrt[c + d*x])/(a*(b*c - a*d)*
Sqrt[a + b*x]) - (d*((2*Sqrt[-a]*(8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*Sqrt[
1 + (b*x)/a]*Sqrt[c + d*x]*EllipticE[ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[-a]], (
a*d)/(b*c)])/(Sqrt[b]*d*Sqrt[a + b*x]*Sqrt[1 + (d*x)/c]) - (2*Sqrt[-a]*c*(
b*c - a*d)*(8*b*c + a*d)*Sqrt[1 + (b*x)/a]*Sqrt[1 + (d*x)/c]*EllipticF[Arc
Sin[(Sqrt[b]*Sqrt[x])/Sqrt[-a]], (a*d)/(b*c)])/(Sqrt[b]*d*Sqrt[a + b*x]*Sq
rt[c + d*x])))/(a*(b*c - a*d))))/(a*c))/(3*a*c)))/(a*x + b*x^2)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 115
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ[2*m, 2
*n, 2*p]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 3.34 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.68

method	result
elliptic	$\frac{\sqrt{x(bx+a)(dx+c)}\left(-\frac{2\left(bd\,x^2+cbx\right)b^2}{(ad-bc)a^3\sqrt{\left(x+\frac{a}{b}\right)\left(bd\,x^2+cbx\right)}}-\frac{2\sqrt{bd\,x^3+ad\,x^2+bc\,x^2+acx}}{3a^2c\,x^2}+\frac{2\left(bd\,x^2+adx+cbx+ac\right)(2ad+5bc)}{3a^3c^2\sqrt{x\left(bd\,x^2+adx+cbx+ac\right)}}+\frac{2\left(\frac{b^2}{a^3}+\frac{b^3c}{(ad-bc)a}\right.}{}\right.}{}$
default	$\frac{2\left(2x\sqrt{\frac{dx+c}{c}}\,\sqrt{\frac{d(bx+a)}{ad-bc}}\,\sqrt{-\frac{xd}{c}}\ \text{EllipticF}\left(\sqrt{\frac{dx+c}{c}},\sqrt{-\frac{bc}{ad-bc}}\right)a^3c\,d^3+2x\sqrt{\frac{dx+c}{c}}\,\sqrt{\frac{d(bx+a)}{ad-bc}}\,\sqrt{-\frac{xd}{c}}\ \text{EllipticF}\left(\sqrt{\frac{dx+c}{c}},\sqrt{-\frac{bc}{ad-bc}}\right.\right.}{}$

input

```
int(1/x/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
(x*(b*x+a)*(d*x+c))^(1/2)/(x*(b*x+a))^(1/2)/(d*x+c)^(1/2)*(-2*(b*d*x^2+b*c
*x)/(a*d-b*c)*b^2/a^3/((x+a/b)*(b*d*x^2+b*c*x))^(1/2)-2/3/a^2/c/x^2*(b*d*x
^3+a*d*x^2+b*c*x^2+a*c*x)^(1/2)+2/3*(b*d*x^2+a*d*x+b*c*x+a*c)/a^3/c^2*(2*a
*d+5*b*c)/(x*(b*d*x^2+a*d*x+b*c*x+a*c))^(1/2)+2*(b^2/a^3+b^3*c/(a*d-b*c)/a
^3-1/3/a^2*d*b/c)*c/d*((x+c/d)/c*d)^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)*(-1/c
*x*d)^(1/2)/(b*d*x^3+a*d*x^2+b*c*x^2+a*c*x)^(1/2)*EllipticF(((x+c/d)/c*d)^
(1/2),(-c/d/(-c/d+a/b))^(1/2))+2*(b^3/a^3*d/(a*d-b*c)-1/3*b*d*(2*a*d+5*b*c
)/c^2/a^3)*c/d*((x+c/d)/c*d)^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)*(-1/c*x*d)^(
1/2)/(b*d*x^3+a*d*x^2+b*c*x^2+a*c*x)^(1/2)*((-c/d+a/b)*EllipticE(((x+c/d)/
c*d)^(1/2),(-c/d/(-c/d+a/b))^(1/2))-a/b*EllipticF(((x+c/d)/c*d)^(1/2),(-c/
d/(-c/d+a/b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 616 vs. 2(280) = 560.

Time = 0.10 (sec) , antiderivative size = 616, normalized size of antiderivative = 1.93

$$\int \frac{1}{x\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \frac{2\left(((8\,b^4c^3-7\,ab^3c^2d-2\,a^2b^2cd^2-2\,a^3bd^3)x^3+(8\,ab^3c^3-7\,a^2b^2c^2d-2\right.}{}$$

input
```
integrate(1/x/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
2/9*(((8*b^4*c^3 - 7*a*b^3*c^2*d - 2*a^2*b^2*c*d^2 - 2*a^3*b*d^3)*x^3 + (8
*a*b^3*c^3 - 7*a^2*b^2*c^2*d - 2*a^3*b*c*d^2 - 2*a^4*d^3)*x^2)*sqrt(b*d)*w
eierstrassPInverse(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b
^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), 1/3*(3*b*d*
x + b*c + a*d)/(b*d)) + 3*((8*b^4*c^2*d - 3*a*b^3*c*d^2 - 2*a^2*b^2*d^3)*x
^3 + (8*a*b^3*c^2*d - 3*a^2*b^2*c*d^2 - 2*a^3*b*d^3)*x^2)*sqrt(b*d)*weiers
trassZeta(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3 -
3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), weierstrassPInverse(
4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3 - 3*a*b^2*c^
2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), 1/3*(3*b*d*x + b*c + a*d)/(b*d
))) - 3*(a^2*b^2*c^2*d - a^3*b*c*d^2 - (8*b^4*c^2*d - 3*a*b^3*c*d^2 - 2*a^
2*b^2*d^3)*x^2 - 2*(2*a*b^3*c^2*d - a^2*b^2*c*d^2 - a^3*b*d^3)*x)*sqrt(b*x
^2 + a*x)*sqrt(d*x + c))/((a^3*b^3*c^3*d - a^4*b^2*c^2*d^2)*x^3 + (a^4*b^2
*c^3*d - a^5*b*c^2*d^2)*x^2)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{1}{x\,(x\,(a+bx))^{\frac{3}{2}}\,\sqrt{c+dx}}\,dx$$

input
```
integrate(1/x/(d*x+c)**(1/2)/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral(1/(x*(x*(a + b*x))**(3/2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{3}{2}}\sqrt{dx+c}x}\,dx$$

input
```
integrate(1/x/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a*x)^(3/2)*sqrt(d*x + c)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{3}{2}}\sqrt{dx+cx}}\,dx$$

input
```
integrate(1/x/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a*x)^(3/2)*sqrt(d*x + c)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{1}{x\,(b\,x^2+a\,x)^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(1/(x*(a*x + b*x^2)^(3/2)*(c + d*x)^(1/2)),x)
```

output
```
int(1/(x*(a*x + b*x^2)^(3/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/x/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x)
```

output

```
( - 2*sqrt(c + d*x)*sqrt(a + b*x)*a*c + 4*sqrt(c + d*x)*sqrt(a + b*x)*a*d*
x + 8*sqrt(c + d*x)*sqrt(a + b*x)*b*c*x + 2*sqrt(x)*int((sqrt(c + d*x)*sqr
t(a + b*x)*x)/(sqrt(x)*a**2*c + sqrt(x)*a**2*d*x + 2*sqrt(x)*a*b*c*x + 2*s
qrt(x)*a*b*d*x**2 + sqrt(x)*b**2*c*x**2 + sqrt(x)*b**2*d*x**3),x)*a**2*b*d
**2*x + 4*sqrt(x)*int((sqrt(c + d*x)*sqrt(a + b*x)*x)/(sqrt(x)*a**2*c + sq
rt(x)*a**2*d*x + 2*sqrt(x)*a*b*c*x + 2*sqrt(x)*a*b*d*x**2 + sqrt(x)*b**2*c
*x**2 + sqrt(x)*b**2*d*x**3),x)*a*b**2*c*d*x + 2*sqrt(x)*int((sqrt(c + d*x
)*sqrt(a + b*x)*x)/(sqrt(x)*a**2*c + sqrt(x)*a**2*d*x + 2*sqrt(x)*a*b*c*x
 + 2*sqrt(x)*a*b*d*x**2 + sqrt(x)*b**2*c*x**2 + sqrt(x)*b**2*d*x**3),x)*a*b
**2*d**2*x**2 + 4*sqrt(x)*int((sqrt(c + d*x)*sqrt(a + b*x)*x)/(sqrt(x)*a**
2*c + sqrt(x)*a**2*d*x + 2*sqrt(x)*a*b*c*x + 2*sqrt(x)*a*b*d*x**2 + sqrt(x
)*b**2*c*x**2 + sqrt(x)*b**2*d*x**3),x)*b**3*c*d*x**2 + sqrt(x)*int((sqrt(
c + d*x)*sqrt(a + b*x))/(sqrt(x)*a**2*c + sqrt(x)*a**2*d*x + 2*sqrt(x)*a*b
*c*x + 2*sqrt(x)*a*b*d*x**2 + sqrt(x)*b**2*c*x**2 + sqrt(x)*b**2*d*x**3),x
)*a**2*b*c*d*x + 8*sqrt(x)*int((sqrt(c + d*x)*sqrt(a + b*x))/(sqrt(x)*a**2
*c + sqrt(x)*a**2*d*x + 2*sqrt(x)*a*b*c*x + 2*sqrt(x)*a*b*d*x**2 + sqrt(x)
*b**2*c*x**2 + sqrt(x)*b**2*d*x**3),x)*a*b**2*c**2*x + sqrt(x)*int((sqrt(c
 + d*x)*sqrt(a + b*x))/(sqrt(x)*a**2*c + sqrt(x)*a**2*d*x + 2*sqrt(x)*a*b*
c*x + 2*sqrt(x)*a*b*d*x**2 + sqrt(x)*b**2*c*x**2 + sqrt(x)*b**2*d*x**3),x)
*a*b**2*c*d*x**2 + 8*sqrt(x)*int((sqrt(c + d*x)*sqrt(a + b*x))/(sqrt(x)...
```

3.209 $\int \frac{1}{x^2\sqrt{c+dx}\left(ax+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 406

$$\begin{aligned}\int \frac{1}{x^2\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = & -\frac{2(24b^2c^2+13abcd+8a^2d^2)\sqrt{c+dx}}{15a^3c^3\sqrt{ax+bx^2}} \\ & -\frac{2\sqrt{c+dx}}{5acx^2\sqrt{ax+bx^2}}+\frac{4(3bc+2ad)\sqrt{c+dx}}{15a^2c^2x\sqrt{ax+bx^2}} \\ & -\frac{2\sqrt{b}(48b^3c^3-16ab^2c^2d-9a^2bcd^2-8a^3d^3)\sqrt{x}\sqrt{c+dx}E\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{7/2}c^3(bc-ad)\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}} \\ & +\frac{2\sqrt{b}d(24b^2c^2-5abcd-4a^2d^2)\sqrt{x}\sqrt{c+dx}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15a^{5/2}c^3(bc-ad)\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax+bx^2}}\end{aligned}$$

output

```
-2/15*(8*a^2*d^2+13*a*b*c*d+24*b^2*c^2)*(d*x+c)^(1/2)/a^3/c^3/(b*x^2+a*x)^
(1/2)-2/5*(d*x+c)^(1/2)/a/c/x^2/(b*x^2+a*x)^(1/2)+4/15*(2*a*d+3*b*c)*(d*x+
c)^(1/2)/a^2/c^2/x/(b*x^2+a*x)^(1/2)-2/15*b^(1/2)*(-8*a^3*d^3-9*a^2*b*c*d^
2-16*a*b^2*c^2*d+48*b^3*c^3)*x^(1/2)*(d*x+c)^(1/2)*EllipticE(b^(1/2)*x^(1/
2)/a^(1/2)/(1+b*x/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(7/2)/c^3/(-a*d+b*c)/(a*(d
*x+c)/c/(b*x+a))^(1/2)/(b*x^2+a*x)^(1/2)+2/15*b^(1/2)*d*(-4*a^2*d^2-5*a*b*
c*d+24*b^2*c^2)*x^(1/2)*(d*x+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x^(1/
2)/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(5/2)/c^3/(-a*d+b*c)/(a*(d*x+c)/c/(b*x+a)
)^(1/2)/(b*x^2+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.99 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^2\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \frac{-2ac(c+dx)\,(24b^3c^2x^2+ab^2cx(6c-5dx)+a^3d(3c-4dx)-a^2b(3c^2+$$

input
```
Integrate[1/(x^2*Sqrt[c + d*x]*(a*x + b*x^2)^(3/2)),x]
```

output
```
(-2*a*c*(c + d*x)*(24*b^3*c^2*x^2 + a*b^2*c*x*(6*c - 5*d*x) + a^3*d*(3*c -
 4*d*x) - a^2*b*(3*c^2 + 2*c*d*x + 4*d^2*x^2)) + (2*I)*Sqrt[a/b]*b*d*(-48*
b^3*c^3 + 16*a*b^2*c^2*d + 9*a^2*b*c*d^2 + 8*a^3*d^3)*Sqrt[1 + a/(b*x)]*Sq
rt[1 + c/(d*x)]*x^(7/2)*EllipticE[I*ArcSinh[Sqrt[a/b]/Sqrt[x]], (b*c)/(a*d
)] - (2*I)*Sqrt[a/b]*b*d*(-24*b^3*c^3 + 11*a*b^2*c^2*d + 5*a^2*b*c*d^2 + 8
*a^3*d^3)*Sqrt[1 + a/(b*x)]*Sqrt[1 + c/(d*x)]*x^(7/2)*EllipticF[I*ArcSinh[
Sqrt[a/b]/Sqrt[x]], (b*c)/(a*d)])/(15*a^4*c^3*(-(b*c) + a*d)*x^2*Sqrt[x*(a
 + b*x)]*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.28, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {1261, 115, 27, 169, 27, 169, 27, 169, 27, 176, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\,(ax+bx^2)^{3/2}\,\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \; 1261$$

$$\frac{x^{3/2}(a+bx)^{3/2}\int \frac{1}{x^{7/2}(a+bx)^{3/2}\sqrt{c+dx}}dx}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \; 115$$

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{2\int \frac{6bc+4ad+5bdx}{2x^{5/2}(a+bx)^{3/2}\sqrt{c+dx}}dx}{5ac}-\frac{2\sqrt{c+dx}}{5acx^{5/2}\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$\downarrow$ 27

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{\int \frac{2(3bc+2ad)+5bdx}{x^{5/2}(a+bx)^{3/2}\sqrt{c+dx}}dx}{5ac}-\frac{2\sqrt{c+dx}}{5acx^{5/2}\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$\downarrow$ 169

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{2\int \frac{24b^2c^2+13abdc+8a^2d^2+6bd(3bc+2ad)x}{2x^{3/2}(a+bx)^{3/2}\sqrt{c+dx}}dx}{3ac}-\frac{4\sqrt{c+dx}(2ad+3bc)}{3acx^{3/2}\sqrt{a+bx}}}{5ac}-\frac{2\sqrt{c+dx}}{5acx^{5/2}\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$\downarrow$ 27

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{\int \frac{24b^2c^2+13abdc+8a^2d^2+6bd(3bc+2ad)x}{x^{3/2}(a+bx)^{3/2}\sqrt{c+dx}}dx}{3ac}-\frac{4\sqrt{c+dx}(2ad+3bc)}{3acx^{3/2}\sqrt{a+bx}}}{5ac}-\frac{2\sqrt{c+dx}}{5acx^{5/2}\sqrt{a+bx}}\right)}{(ax+bx^2)^{3/2}}$$

$\downarrow$ 169

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{2\int \frac{b\left(4c\left(12b^2c^2+2abdc+a^2d^2\right)+d\left(24b^2c^2+13abdc+8a^2d^2\right)x\right)}{2\sqrt{x}(a+bx)^{3/2}\sqrt{c+dx}}dx}{ac}-\frac{2\sqrt{c+dx}\left(\frac{24b^2c}{a}+\frac{8ad^2}{c}+13bd\right)}{\sqrt{x}\sqrt{a+bx}}}{3ac}-\frac{4\sqrt{c+dx}(2ad+3bc)}{3acx^{3/2}\sqrt{a+bx}}}{5ac}-\ldots\right)}{(ax+bx^2)^{3/2}}$$

$\downarrow$ 27

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{b\int \frac{4c\left(12b^2c^2+2abdc+a^2d^2\right)+d\left(24b^2c^2+13abdc+8a^2d^2\right)x}{\sqrt{x}(a+bx)^{3/2}\sqrt{c+dx}}dx}{ac}-\frac{2\sqrt{c+dx}\left(\frac{24b^2c}{a}+\frac{8ad^2}{c}+13bd\right)}{\sqrt{x}\sqrt{a+bx}}}{3ac}-\frac{4\sqrt{c+dx}(2ad+3bc)}{3acx^{3/2}\sqrt{a+bx}}}{5ac}-\ldots\right)}{(ax+bx^2)^{3/2}}$$

$\downarrow$ 169

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{b\left(\frac{2\int -\frac{d\left(ac\left(24b^2c^2-5abdc-4a^2d^2\right)+\left(48b^3c^3-16ab^2dc^2-9a^2bd^2c-8a^3d^3\right)x\right)}{2\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{a(bc-ad)}+\frac{2\sqrt{x}\sqrt{c+dx}\left(-8a^3d^3-9a^2bcd^2-16ab^2c^2d\right.}{a\sqrt{a+bx}(bc-ad)}}{ac}}{3ac}}{5ac}\right.}{(ax+bx^2)^{3/2}}$$

$\downarrow$ 27

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{b\left(\frac{2\sqrt{x}\sqrt{c+dx}\left(-8a^3d^3-9a^2bcd^2-16ab^2c^2d+48b^3c^3\right)}{a\sqrt{a+bx}(bc-ad)}-\frac{d\int \frac{ac\left(24b^2c^2-5abdc-4a^2d^2\right)+\left(48b^3c^3-16ab^2dc^2-9a^2bd^2c-8a^3d^3\right.}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}}{a(bc-ad)}\right.}{ac}}{3ac}}{5ac}\right.}{(ax+bx^2)^{3/2}}$$

$\downarrow$ 176

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{b\left(\frac{2\sqrt{x}\sqrt{c+dx}\left(-8a^3d^3-9a^2bcd^2-16ab^2c^2d+48b^3c^3\right)}{a\sqrt{a+bx}(bc-ad)}-\frac{d\left(\frac{\left(-8a^3d^3-9a^2bcd^2-16ab^2c^2d+48b^3c^3\right)\int \frac{\sqrt{c+dx}}{\sqrt{x}\sqrt{a+bx}}dx}{d}-\frac{4c(bc-a}{}\right.}{a(bc-ad)}\right.}{ac}}{3ac}}{5ac}\right.}{(ax+bx^2)^{3/2}}$$

$\downarrow$ 122

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{b\left(\frac{2\sqrt{x}\sqrt{c+dx}\left(-8a^3d^3-9a^2bcd^2-16ab^2c^2d+48b^3c^3\right)}{a\sqrt{a+bx}(bc-ad)}-\frac{d\left(\frac{\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\left(-8a^3d^3-9a^2bcd^2-16ab^2c^2d+48b^3c^3\right)\int\frac{\sqrt{\frac{dx}{c}+1}}{\sqrt{x}\sqrt{\frac{bx}{a}+}}}{d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}\right.}{a(bc-}\right.}{ac}}{3ac}}{5ac}\right.}{(ax+bx^2)}$$

$\downarrow$ 120

$$\frac{x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{b\left(\frac{2\sqrt{x}\sqrt{c+dx}\left(-8a^3d^3-9a^2bcd^2-16ab^2c^2d+48b^3c^3\right)}{a\sqrt{a+bx}(bc-ad)}-\frac{d\left(\frac{2\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\left(-8a^3d^3-9a^2bcd^2-16ab^2c^2d+48b^3c^3\right)E\left(\text{ar}\right.}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}\right.}{}\right.}{ac}}{3ac}}{}\right.}{(ax+}$$

$\downarrow$ 127

$$x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{b\left(\frac{2\sqrt{x}\sqrt{c+dx}\left(-8a^3d^3-9a^2bcd^2-16ab^2c^2d+48b^3c^3\right)}{a\sqrt{a+bx}(bc-ad)}-\frac{d\left(\frac{2\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\left(-8a^3d^3-9a^2bcd^2-16ab^2c^2d+48b^3c^3\right)E\left(\text{ar}\right.}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}\right.}{}\right)}{ac}}{}}{}\right.$$

$$\Big\downarrow \; 126$$

$$x^{3/2}(a+bx)^{3/2}\left(-\frac{-\frac{-\frac{b\left(\frac{2\sqrt{x}\sqrt{c+dx}\left(-8a^3d^3-9a^2bcd^2-16ab^2c^2d+48b^3c^3\right)}{a\sqrt{a+bx}(bc-ad)}-\frac{d\left(\frac{2\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\left(-8a^3d^3-9a^2bcd^2-16ab^2c^2d+48b^3c^3\right)E\left(\text{ar}\right.}{\sqrt{b}d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}\right.}{}\right)}{ac}}{}}{}\right.$$

input
```
Int[1/(x^2*Sqrt[c + d*x]*(a*x + b*x^2)^(3/2)),x]
```

output

```
(x^(3/2)*(a + b*x)^(3/2)*((-2*Sqrt[c + d*x])/(5*a*c*x^(5/2)*Sqrt[a + b*x])
 - ((-4*(3*b*c + 2*a*d)*Sqrt[c + d*x])/(3*a*c*x^(3/2)*Sqrt[a + b*x]) - ((-
2*((24*b^2*c)/a + 13*b*d + (8*a*d^2)/c)*Sqrt[c + d*x])/(Sqrt[x]*Sqrt[a + b
*x]) - (b*((2*(48*b^3*c^3 - 16*a*b^2*c^2*d - 9*a^2*b*c*d^2 - 8*a^3*d^3)*Sq
rt[x]*Sqrt[c + d*x])/(a*(b*c - a*d)*Sqrt[a + b*x]) - (d*((2*Sqrt[-a]*(48*b
^3*c^3 - 16*a*b^2*c^2*d - 9*a^2*b*c*d^2 - 8*a^3*d^3)*Sqrt[1 + (b*x)/a]*Sqr
t[c + d*x]*EllipticE[ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[-a]], (a*d)/(b*c)])/(Sq
rt[b]*d*Sqrt[a + b*x]*Sqrt[1 + (d*x)/c]) - (8*Sqrt[-a]*c*(b*c - a*d)*(12*b
^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[1 + (b*x)/a]*Sqrt[1 + (d*x)/c]*Elliptic
F[ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[-a]], (a*d)/(b*c)])/(Sqrt[b]*d*Sqrt[a + b*
x]*Sqrt[c + d*x])))/(a*(b*c - a*d))))/(a*c))/(3*a*c))/(5*a*c)))/(a*x + b*x
^2)^(3/2)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 115

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ[2*m, 2
*n, 2*p]
```

rule 120

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 3.97 (sec) , antiderivative size = 622, normalized size of antiderivative = 1.53

method	result
elliptic	$\frac{\sqrt{x(bx+a)(dx+c)}\left(-\frac{2\sqrt{bd\,x^3+ad\,x^2+bc\,x^2+acx}}{5a^2c\,x^3}+\frac{2(4ad+9bc)\sqrt{bd\,x^3+ad\,x^2+bc\,x^2+acx}}{15a^3c^2x^2}-\frac{2\left(bd\,x^2+adx+cbx+ac\right)\left(8a^2d^2+17abcd+33b^2c^2\right)}{15a^4c^3\sqrt{x\left(bd\,x^2+adx+cbx+ac\right)}}\right.}{}$
default	Expression too large to display

input
```
int(1/x^2/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(x*(b*x+a)*(d*x+c))^(1/2)/(x*(b*x+a))^(1/2)/(d*x+c)^(1/2)*(-2/5/a^2/c/x^3*
(b*d*x^3+a*d*x^2+b*c*x^2+a*c*x)^(1/2)+2/15/a^3/c^2*(4*a*d+9*b*c)*(b*d*x^3+
a*d*x^2+b*c*x^2+a*c*x)^(1/2)/x^2-2/15*(b*d*x^2+a*d*x+b*c*x+a*c)/a^4/c^3*(8
*a^2*d^2+17*a*b*c*d+33*b^2*c^2)/(x*(b*d*x^2+a*d*x+b*c*x+a*c))^(1/2)+2*(b*d
*x^2+b*c*x)/(a*d-b*c)*b^3/a^4/((x+a/b)*(b*d*x^2+b*c*x))^(1/2)+2*(1/15*b*d*
(4*a*d+9*b*c)/c^2/a^3-b^3/a^4-b^4*c/(a*d-b*c)/a^4)*c/d*((x+c/d)/c*d)^(1/2)
*((x+a/b)/(-c/d+a/b))^(1/2)*(-1/c*x*d)^(1/2)/(b*d*x^3+a*d*x^2+b*c*x^2+a*c*
x)^(1/2)*EllipticF(((x+c/d)/c*d)^(1/2),(-c/d/(-c/d+a/b))^(1/2))+2*(1/15*b*
d*(8*a^2*d^2+17*a*b*c*d+33*b^2*c^2)/a^4/c^3-b^4/a^4*d/(a*d-b*c))*c/d*((x+c
/d)/c*d)^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)*(-1/c*x*d)^(1/2)/(b*d*x^3+a*d*x^
2+b*c*x^2+a*c*x)^(1/2)*((-c/d+a/b)*EllipticE(((x+c/d)/c*d)^(1/2),(-c/d/(-c
/d+a/b))^(1/2))-a/b*EllipticF(((x+c/d)/c*d)^(1/2),(-c/d/(-c/d+a/b))^(1/2))
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 743 vs. $2(361) = 722$.

Time = 0.13 (sec) , antiderivative size = 743, normalized size of antiderivative = 1.83

$$\int \frac{1}{x^2\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/45*(((48*b^5*c^4 - 40*a*b^4*c^3*d - 10*a^2*b^3*c^2*d^2 - 5*a^3*b^2*c*d^
3 - 8*a^4*b*d^4)*x^4 + (48*a*b^4*c^4 - 40*a^2*b^3*c^3*d - 10*a^3*b^2*c^2*d
^2 - 5*a^4*b*c*d^3 - 8*a^5*d^4)*x^3)*sqrt(b*d)*weierstrassPInverse(4/3*(b^
2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), 1/3*(3*b*d*x + b*c + a*d)/(b*d)) + 3*
((48*b^5*c^3*d - 16*a*b^4*c^2*d^2 - 9*a^2*b^3*c*d^3 - 8*a^3*b^2*d^4)*x^4 +
 (48*a*b^4*c^3*d - 16*a^2*b^3*c^2*d^2 - 9*a^3*b^2*c*d^3 - 8*a^4*b*d^4)*x^3
)*sqrt(b*d)*weierstrassZeta(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -
4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), we
ierstrassPInverse(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(2*b^
3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), 1/3*(3*b*d*x
 + b*c + a*d)/(b*d))) + 3*(3*a^3*b^2*c^3*d - 3*a^4*b*c^2*d^2 + (48*b^5*c^3
*d - 16*a*b^4*c^2*d^2 - 9*a^2*b^3*c*d^3 - 8*a^3*b^2*d^4)*x^3 + (24*a*b^4*c
^3*d - 11*a^2*b^3*c^2*d^2 - 5*a^3*b^2*c*d^3 - 8*a^4*b*d^4)*x^2 - 2*(3*a^2*
b^3*c^3*d - a^3*b^2*c^2*d^2 - 2*a^4*b*c*d^3)*x)*sqrt(b*x^2 + a*x)*sqrt(d*x
 + c))/((a^4*b^3*c^4*d - a^5*b^2*c^3*d^2)*x^4 + (a^5*b^2*c^4*d - a^6*b*c^3
*d^2)*x^3)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{1}{x^2\,(x\,(a+bx))^{\frac{3}{2}}\,\sqrt{c+dx}}\,dx$$

input
```
integrate(1/x**2/(d*x+c)**(1/2)/(b*x**2+a*x)**(3/2),x)
```

output
```
Integral(1/(x**2*(x*(a + b*x))**(3/2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{3}{2}}\sqrt{dx+c}x^2}\,dx$$

input
```
integrate(1/x^2/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a*x)^(3/2)*sqrt(d*x + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{3}{2}}\sqrt{dx+c}x^2}\,dx$$

input
```
integrate(1/x^2/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a*x)^(3/2)*sqrt(d*x + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{1}{x^2\,(b\,x^2+a\,x)^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(1/(x^2*(a*x + b*x^2)^(3/2)*(c + d*x)^(1/2)),x)
```

output
```
int(1/(x^2*(a*x + b*x^2)^(3/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{c+dx}\,(ax+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input

```
int(1/x^2/(d*x+c)^(1/2)/(b*x^2+a*x)^(3/2),x)
```

output

```
( - 6*sqrt(c + d*x)*sqrt(a + b*x)*a*c + 8*sqrt(c + d*x)*sqrt(a + b*x)*a*d*
x + 12*sqrt(c + d*x)*sqrt(a + b*x)*b*c*x + 30*sqrt(c + d*x)*sqrt(a + b*x)*
b*d*x**2 + 15*sqrt(x)*int((sqrt(c + d*x)*sqrt(a + b*x)*x)/(sqrt(x)*a**2*c
 + sqrt(x)*a**2*d*x + 2*sqrt(x)*a*b*c*x + 2*sqrt(x)*a*b*d*x**2 + sqrt(x)*b*
*2*c*x**2 + sqrt(x)*b**2*d*x**3),x)*a*b**2*d**2*x**2 + 15*sqrt(x)*int((sqr
t(c + d*x)*sqrt(a + b*x)*x)/(sqrt(x)*a**2*c + sqrt(x)*a**2*d*x + 2*sqrt(x)
*a*b*c*x + 2*sqrt(x)*a*b*d*x**2 + sqrt(x)*b**2*c*x**2 + sqrt(x)*b**2*d*x**
3),x)*b**3*d**2*x**3 + 8*sqrt(x)*int((sqrt(c + d*x)*sqrt(a + b*x))/(sqrt(x
)*a**2*c*x + sqrt(x)*a**2*d*x**2 + 2*sqrt(x)*a*b*c*x**2 + 2*sqrt(x)*a*b*d*
x**3 + sqrt(x)*b**2*c*x**3 + sqrt(x)*b**2*d*x**4),x)*a**3*d**2*x**2 + 28*s
qrt(x)*int((sqrt(c + d*x)*sqrt(a + b*x))/(sqrt(x)*a**2*c*x + sqrt(x)*a**2*
d*x**2 + 2*sqrt(x)*a*b*c*x**2 + 2*sqrt(x)*a*b*d*x**3 + sqrt(x)*b**2*c*x**3
 + sqrt(x)*b**2*d*x**4),x)*a**2*b*c*d*x**2 + 8*sqrt(x)*int((sqrt(c + d*x)*
sqrt(a + b*x))/(sqrt(x)*a**2*c*x + sqrt(x)*a**2*d*x**2 + 2*sqrt(x)*a*b*c*x
**2 + 2*sqrt(x)*a*b*d*x**3 + sqrt(x)*b**2*c*x**3 + sqrt(x)*b**2*d*x**4),x)
*a**2*b*d**2*x**3 + 24*sqrt(x)*int((sqrt(c + d*x)*sqrt(a + b*x))/(sqrt(x)*
a**2*c*x + sqrt(x)*a**2*d*x**2 + 2*sqrt(x)*a*b*c*x**2 + 2*sqrt(x)*a*b*d*x*
*3 + sqrt(x)*b**2*c*x**3 + sqrt(x)*b**2*d*x**4),x)*a*b**2*c**2*x**2 + 28*s
qrt(x)*int((sqrt(c + d*x)*sqrt(a + b*x))/(sqrt(x)*a**2*c*x + sqrt(x)*a**2*
d*x**2 + 2*sqrt(x)*a*b*c*x**2 + 2*sqrt(x)*a*b*d*x**3 + sqrt(x)*b**2*c*x...
```

3.210 $\int \frac{\sqrt{x}}{\sqrt{-2+3x-x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 21

$$\int \frac{\sqrt{x}}{\sqrt{-2+3x-x^2}}\,dx = -2\sqrt{2}E\left(\arcsin\left(\sqrt{2-x}\right)|\frac{1}{2}\right)$$

output
```
-2*2^(1/2)*EllipticE((2-x)^(1/2),1/2*2^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 57 vs. 2(21) = 42.

Time = 20.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.71

$$\int \frac{\sqrt{x}}{\sqrt{-2+3x-x^2}}\,dx = \frac{2\sqrt{2-2x}\sqrt{2-x}\left(-E\left(\arcsin\left(\sqrt{x}\right)|\frac{1}{2}\right)+\text{EllipticF}\left(\arcsin\left(\sqrt{x}\right),\frac{1}{2}\right)\right)}{\sqrt{-2+3x-x^2}}$$

input
```
Integrate[Sqrt[x]/Sqrt[-2 + 3*x - x^2],x]
```

output
```
(2*Sqrt[2 - 2*x]*Sqrt[2 - x]*(-EllipticE[ArcSin[Sqrt[x]], 1/2] + EllipticF
[ArcSin[Sqrt[x]], 1/2]))/Sqrt[-2 + 3*x - x^2]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 52 vs. $2(21) = 42$.

Time = 0.37 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.48, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1170, 1452, 27, 389, 322, 331, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{-x^2+3x-2}}dx$$
$$\Big\downarrow \text{ 1170}$$
$$2\int \frac{x}{\sqrt{-x^2+3x-2}}d\sqrt{x}$$
$$\Big\downarrow \text{ 1452}$$
$$4\int \frac{x}{2\sqrt{2-x}\sqrt{x-1}}d\sqrt{x}$$
$$\Big\downarrow \text{ 27}$$
$$2\int \frac{x}{\sqrt{2-x}\sqrt{x-1}}d\sqrt{x}$$
$$\Big\downarrow \text{ 389}$$
$$2\left(2\int \frac{1}{\sqrt{2-x}\sqrt{x-1}}d\sqrt{x}-\int \frac{\sqrt{2-x}}{\sqrt{x-1}}d\sqrt{x}\right)$$
$$\Big\downarrow \text{ 322}$$
$$2\left(-\int \frac{\sqrt{2-x}}{\sqrt{x-1}}d\sqrt{x}-2\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt{x}}{\sqrt{2}}\right),2\right)\right)$$
$$\Big\downarrow \text{ 331}$$
$$2\left(-\frac{\sqrt{1-x}\int \frac{\sqrt{2-x}}{\sqrt{1-x}}d\sqrt{x}}{\sqrt{x-1}}-2\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt{x}}{\sqrt{2}}\right),2\right)\right)$$

$$\Big\downarrow \text{327}$$

$$2\left(-2\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt{x}}{\sqrt{2}}\right),2\right)-\frac{\sqrt{2}\sqrt{1-x}E\left(\arcsin\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x-1}}\right)$$

input
```
Int[Sqrt[x]/Sqrt[-2 + 3*x - x^2],x]
```

output
```
2*(-((Sqrt[2]*Sqrt[1 - x]*EllipticE[ArcSin[Sqrt[x]], 1/2])/Sqrt[-1 + x]) -
 2*EllipticF[ArcCos[Sqrt[x]/Sqrt[2]], 2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 322
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

rule 1170
```
Int[(x_)^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2
 Subst[Int[x^(2*m + 1)/Sqrt[a + b*x^2 + c*x^4], x], x, Sqrt[x]], x] /; Free
Q[{a, b, c}, x] && EqQ[m^2, 1/4]
```

rule 1452
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[x^2/(Sqrt[b + q + 2*c*x^2]*Sqrt
[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
&& LtQ[c, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.48

method	result	size
default	$\frac{2\sqrt{1-x}\,\sqrt{2-x}\,\left(\operatorname{EllipticE}\left(\sqrt{1-x},i\right)-2\operatorname{EllipticF}\left(\sqrt{1-x},i\right)\right)}{\sqrt{-x^{2}+3x-2}}$	52
elliptic	$-\frac{2\sqrt{-x\left(x^{2}-3x+2\right)}\,\sqrt{1-x}\,\sqrt{2-x}\,\left(-\operatorname{EllipticE}\left(\sqrt{1-x},i\right)+2\operatorname{EllipticF}\left(\sqrt{1-x},i\right)\right)}{\sqrt{-x^{2}+3x-2}\,\sqrt{-x^{3}+3x^{2}-2x}}$	83

input
```
int(x^(1/2)/(-x^2+3*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(1-x)^(1/2)*(2-x)^(1/2)*(EllipticE((1-x)^(1/2),I)-2*EllipticF((1-x)^(1/2
),I))/(-x^2+3*x-2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{x}}{\sqrt{-2+3x-x^2}}\,dx = -2i\,\text{weierstrassPInverse}(4,0,x-1) + 2i\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,x-1))$$

input
```
integrate(x^(1/2)/(-x^2+3*x-2)^(1/2),x, algorithm="fricas")
```

output
```
-2*I*weierstrassPInverse(4, 0, x - 1) + 2*I*weierstrassZeta(4, 0, weierstr
assPInverse(4, 0, x - 1))
```

Sympy [F]

$$\int \frac{\sqrt{x}}{\sqrt{-2+3x-x^2}}\,dx = \int \frac{\sqrt{x}}{\sqrt{-\left(x-2\right)\left(x-1\right)}}\,dx$$

input
```
integrate(x**(1/2)/(-x**2+3*x-2)**(1/2),x)
```

output
```
Integral(sqrt(x)/sqrt(-(x - 2)*(x - 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{-2+3x-x^2}}\,dx = \int \frac{\sqrt{x}}{\sqrt{-x^2+3\,x-2}}\,dx$$

input
```
integrate(x^(1/2)/(-x^2+3*x-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/sqrt(-x^2 + 3*x - 2), x)
```

Giac [F]

$$\int \frac{\sqrt{x}}{\sqrt{-2+3x-x^2}}\,dx = \int \frac{\sqrt{x}}{\sqrt{-x^2+3\,x-2}}\,dx$$

input
```
integrate(x^(1/2)/(-x^2+3*x-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x)/sqrt(-x^2 + 3*x - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{-2+3x-x^2}}\,dx = \int \frac{\sqrt{x}}{\sqrt{-x^2+3\,x-2}}\,dx$$

input
```
int(x^(1/2)/(3*x - x^2 - 2)^(1/2),x)
```

output
```
int(x^(1/2)/(3*x - x^2 - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{x}}{\sqrt{-2+3x-x^2}}\,dx = -\left(\int \frac{\sqrt{x}\,\sqrt{-x^2+3x-2}}{x^2-3x+2}dx\right)$$

input
```
int(x^(1/2)/(-x^2+3*x-2)^(1/2),x)
```

output
```
- int((sqrt(x)*sqrt( - x**2 + 3*x - 2))/(x**2 - 3*x + 2),x)
```

3.211 $\int \frac{x}{\sqrt{-1+x}\sqrt{2x-x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 21

$$\int \frac{x}{\sqrt{-1+x}\sqrt{2x-x^2}}\,dx = -2\sqrt{2}E\left(\arcsin\left(\sqrt{2-x}\right)|\frac{1}{2}\right)$$

output
```
-2*2^(1/2)*EllipticE((2-x)^(1/2),1/2*2^(1/2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.99 (sec) , antiderivative size = 115, normalized size of antiderivative = 5.48

$$\int \frac{x}{\sqrt{-1+x}\sqrt{2x-x^2}}\,dx = \frac{2\left(\sqrt{-2+x}(-1+x)\sqrt{\frac{x}{-2+x}}+i\sqrt{2}(-2+x)\sqrt{\frac{-1+x}{-2+x}}E\left(i\text{arcsinh}\left(\frac{\sqrt{2}}{\sqrt{-2+x}}\right)|\frac{1}{2}\right)\right)}{\sqrt{\frac{-2+x}{x}}\sqrt{\frac{-1+x}{x}}\sqrt{\frac{x}{-2+x}}\sqrt{-((-2+x)x)}}$$

input
```
Integrate[x/(Sqrt[-1 + x]*Sqrt[2*x - x^2]),x]
```

output
```
(2*(Sqrt[-2 + x]*(-1 + x)*Sqrt[x/(-2 + x)] + I*Sqrt[2]*(-2 + x)*Sqrt[(-1 +
 x)/(-2 + x)]*EllipticE[I*ArcSinh[Sqrt[2]/Sqrt[-2 + x]], 1/2]))/(Sqrt[(-2
+ x)/x]*Sqrt[(-1 + x)/x]*Sqrt[x/(-2 + x)]*Sqrt[-((-2 + x)*x)])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.81, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1269, 1113, 762, 1114, 836, 762, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x-1}\sqrt{2x-x^2}}dx$$

$$\Big\downarrow \text{1269}$$

$$\int \frac{1}{\sqrt{x-1}\sqrt{2x-x^2}}dx+\int \frac{\sqrt{x-1}}{\sqrt{2x-x^2}}dx$$

$$\Big\downarrow \text{1113}$$

$$\int \frac{\sqrt{x-1}}{\sqrt{2x-x^2}}dx+2\int \frac{1}{\sqrt{1-(x-1)^2}}d\sqrt{x-1}$$

$$\Big\downarrow \text{762}$$

$$\int \frac{\sqrt{x-1}}{\sqrt{2x-x^2}}dx+2\operatorname{EllipticF}\left(\arcsin\left(\sqrt{x-1}\right),-1\right)$$

$$\Big\downarrow \text{1114}$$

$$2\int \frac{x-1}{\sqrt{1-(x-1)^2}}d\sqrt{x-1}+2\operatorname{EllipticF}\left(\arcsin\left(\sqrt{x-1}\right),-1\right)$$

$$\Big\downarrow \text{836}$$

$$2\left(\int \frac{x}{\sqrt{1-(x-1)^2}}d\sqrt{x-1}-\int \frac{1}{\sqrt{1-(x-1)^2}}d\sqrt{x-1}\right)+2\operatorname{EllipticF}\left(\arcsin\left(\sqrt{x-1}\right),-1\right)$$

$$\Big\downarrow \text{762}$$

$$2\left(\int \frac{x}{\sqrt{1-(x-1)^2}}d\sqrt{x-1}-\text{EllipticF}\left(\arcsin\left(\sqrt{x-1}\right),-1\right)\right)+$$
$$2\,\text{EllipticF}\left(\arcsin\left(\sqrt{x-1}\right),-1\right)$$

$$\Big\downarrow \text{ 1388}$$

$$2\left(\int \frac{\sqrt{x}}{\sqrt{2-x}}d\sqrt{x-1}-\text{EllipticF}\left(\arcsin\left(\sqrt{x-1}\right),-1\right)\right)+2\,\text{EllipticF}\left(\arcsin\left(\sqrt{x-1}\right),-1\right)$$

$$\Big\downarrow \text{ 327}$$

$$2\,\text{EllipticF}\left(\arcsin\left(\sqrt{x-1}\right),-1\right)+$$
$$2\left(E\left(\arcsin\left(\sqrt{x-1}\right)\middle|-1\right)-\text{EllipticF}\left(\arcsin\left(\sqrt{x-1}\right),-1\right)\right)$$

input
```
Int[x/(Sqrt[-1 + x]*Sqrt[2*x - x^2]),x]
```

output
```
2*(EllipticE[ArcSin[Sqrt[-1 + x]], -1] - EllipticF[ArcSin[Sqrt[-1 + x]], -
1]) + 2*EllipticF[ArcSin[Sqrt[-1 + x]], -1]
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1113
```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Simp[(4/e)*Sqrt[-c/(b^2 - 4*a*c)]   Subst[Int[1/Sqrt[Simp[1 - b^
2*(x^4/(d^2*(b^2 - 4*a*c))), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 - 4*a*c), 0]
```

rule 1114
```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Simp[(4/e)*Sqrt[-c/(b^2 - 4*a*c)]   Subst[Int[x^2/Sqrt[Simp[1 - b^2*
(x^4/(d^2*(b^2 - 4*a*c))), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 - 4*a*c), 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.14

method	result	size
default	$-\frac{2\sqrt{x-1}\,\sqrt{-x(x-2)}\,\sqrt{1-x}\,\sqrt{2-x}\,\left(\operatorname{EllipticE}\left(\sqrt{1-x},i\right)-2\operatorname{EllipticF}\left(\sqrt{1-x},i\right)\right)}{\sqrt{x}\,\left(x^{2}-3x+2\right)}$	66
elliptic	$-\frac{2\sqrt{-(x-1)(x-2)x}\,\sqrt{1-x}\,\sqrt{2-x}\,\sqrt{x}\,\left(-\operatorname{EllipticE}\left(\sqrt{1-x},i\right)+2\operatorname{EllipticF}\left(\sqrt{1-x},i\right)\right)}{\sqrt{x-1}\,\sqrt{-x(x-2)}\,\sqrt{-x^{3}+3x^{2}-2x}}$	85

input
```
int(x/(x-1)^(1/2)/(-x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(x-1)^(1/2)*(-x*(x-2))^(1/2)*(1-x)^(1/2)*(2-x)^(1/2)/x^(1/2)*(EllipticE
((1-x)^(1/2),I)-2*EllipticF((1-x)^(1/2),I))/(x^2-3*x+2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{x}{\sqrt{-1+x}\sqrt{2x-x^2}}\,dx = -2i\,\text{weierstrassPInverse}(4,0,x-1) + 2i\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,x-1))$$

input
```
integrate(x/(x-1)^(1/2)/(-x^2+2*x)^(1/2),x, algorithm="fricas")
```

output
```
-2*I*weierstrassPInverse(4, 0, x - 1) + 2*I*weierstrassZeta(4, 0, weierstr
assPInverse(4, 0, x - 1))
```

Sympy [F]

$$\int \frac{x}{\sqrt{-1+x}\sqrt{2x-x^2}}\,dx = \int \frac{x}{\sqrt{-x\,(x-2)}\sqrt{x-1}}\,dx$$

input
```
integrate(x/(x-1)**(1/2)/(-x**2+2*x)**(1/2),x)
```

output
```
Integral(x/(sqrt(-x*(x - 2))*sqrt(x - 1)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{-1+x}\sqrt{2x-x^2}}\,dx = \int \frac{x}{\sqrt{-x^2+2\,x}\sqrt{x-1}}\,dx$$

input
```
integrate(x/(x-1)^(1/2)/(-x^2+2*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(-x^2 + 2*x)*sqrt(x - 1)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{-1+x}\sqrt{2x-x^2}}\,dx = \int \frac{x}{\sqrt{-x^2+2\,x}\sqrt{x-1}}\,dx$$

input
```
integrate(x/(x-1)^(1/2)/(-x^2+2*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(-x^2 + 2*x)*sqrt(x - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{-1+x}\sqrt{2x-x^2}}\,dx = \int \frac{x}{\sqrt{2\,x-x^2}\,\sqrt{x-1}}\,dx$$

input
```
int(x/((2*x - x^2)^(1/2)*(x - 1)^(1/2)),x)
```

output
```
int(x/((2*x - x^2)^(1/2)*(x - 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{-1+x}\sqrt{2x-x^2}}\,dx = -\left(\int \frac{\sqrt{x}\,\sqrt{x-1}\,\sqrt{-x+2}}{x^2-3x+2}dx\right)$$

input
```
int(x/(x-1)^(1/2)/(-x^2+2*x)^(1/2),x)
```

output
```
- int((sqrt(x)*sqrt(x - 1)*sqrt( - x + 2))/(x**2 - 3*x + 2),x)
```

3.212 $\int x^2(c+dx)^q \left(ax+bx^2\right)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 78

$$\int x^2(c+dx)^q \left(ax+bx^2\right)^p \, dx = \frac{x^3\left(1+\frac{bx}{a}\right)^{-p}(c+dx)^q\left(1+\frac{dx}{c}\right)^{-q}\left(ax+bx^2\right)^p \text{AppellF1}\left(3+p,-p,-q,4+p,-\frac{bx}{a},-\frac{dx}{c}\right)}{3+p}$$

output
```
x^3*(d*x+c)^q*(b*x^2+a*x)^p*AppellF1(3+p,-p,-q,4+p,-b*x/a,-d*x/c)/(3+p)/((
1+b*x/a)^p)/((1+d*x/c)^q)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int x^2(c+dx)^q \left(ax+bx^2\right)^p \, dx = \frac{x^3\left(\frac{a+bx}{a}\right)^{-p}(x(a+bx))^p(c+dx)^q\left(\frac{c+dx}{c}\right)^{-q} \text{AppellF1}\left(3+p,-p,-q,4+p,-\frac{bx}{a},-\frac{dx}{c}\right)}{3+p}$$

input
```
Integrate[x^2*(c + d*x)^q*(a*x + b*x^2)^p,x]
```

output
```
(x^3*(x*(a + b*x))^p*(c + d*x)^q*AppellF1[3 + p, -p, -q, 4 + p, -((b*x)/a)
, -((d*x)/c)])/((3 + p)*((a + b*x)/a)^p*((c + d*x)/c)^q)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1261, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(ax+bx^2\right)^p (c+dx)^q \, dx$$

$$\Big\downarrow \text{ 1261}$$

$$x^{-p}(a+bx)^{-p} \left(ax+bx^2\right)^p \int x^{p+2}(a+bx)^p(c+dx)^q dx$$

$$\Big\downarrow \text{ 152}$$

$$x^{-p}\left(\frac{bx}{a}+1\right)^{-p} \left(ax+bx^2\right)^p \int x^{p+2}\left(\frac{bx}{a}+1\right)^p (c+dx)^q dx$$

$$\Big\downarrow \text{ 152}$$

$$x^{-p}\left(\frac{bx}{a}+1\right)^{-p} \left(ax+bx^2\right)^p (c+dx)^q \left(\frac{dx}{c}+1\right)^{-q} \int x^{p+2}\left(\frac{bx}{a}+1\right)^p \left(\frac{dx}{c}+1\right)^q dx$$

$$\Big\downarrow \text{ 150}$$

$$\frac{x^3\left(\frac{bx}{a}+1\right)^{-p} \left(ax+bx^2\right)^p (c+dx)^q \left(\frac{dx}{c}+1\right)^{-q} \operatorname{AppellF1}\left(p+3,-p,-q,p+4,-\frac{bx}{a},-\frac{dx}{c}\right)}{p+3}$$

input
```
Int[x^2*(c + d*x)^q*(a*x + b*x^2)^p,x]
```

output
```
(x^3*(c + d*x)^q*(a*x + b*x^2)^p*AppellF1[3 + p, -p, -q, 4 + p, -((b*x)/a)
, -((d*x)/c)])/((3 + p)*(1 + (b*x)/a)^p*(1 + (d*x)/c)^q)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [F]

$$\int x^2(dx+c)^q\left(b\,x^2+ax\right)^p dx$$

input
```
int(x^2*(d*x+c)^q*(b*x^2+a*x)^p,x)
```

output
```
int(x^2*(d*x+c)^q*(b*x^2+a*x)^p,x)
```

Fricas [F]

$$\int x^2(c+dx)^q\left(ax+bx^2\right)^p\,dx=\int\left(bx^2+ax\right)^p(dx+c)^qx^2\,dx$$

input
```
integrate(x^2*(d*x+c)^q*(b*x^2+a*x)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a*x)^p*(d*x + c)^q*x^2, x)
```

Sympy [F]

$$\int x^2(c+dx)^q\left(ax+bx^2\right)^p\,dx=\int x^2(x(a+bx))^p\,(c+dx)^q\,dx$$

input
```
integrate(x**2*(d*x+c)**q*(b*x**2+a*x)**p,x)
```

output
```
Integral(x**2*(x*(a + b*x))**p*(c + d*x)**q, x)
```

Maxima [F]

$$\int x^2(c+dx)^q\left(ax+bx^2\right)^p\,dx=\int\left(bx^2+ax\right)^p(dx+c)^qx^2\,dx$$

input
```
integrate(x^2*(d*x+c)^q*(b*x^2+a*x)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^p*(d*x + c)^q*x^2, x)
```

Giac [F]

$$\int x^2(c+dx)^q\left(ax+bx^2\right)^p\,dx=\int\left(bx^2+ax\right)^p(dx+c)^qx^2\,dx$$

input
```
integrate(x^2*(d*x+c)^q*(b*x^2+a*x)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a*x)^p*(d*x + c)^q*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(c+dx)^q \left(ax+bx^2\right)^p \, dx = \int x^2 \left(b\,x^2+a\,x\right)^p \left(c+d\,x\right)^q \,dx$$

input
```
int(x^2*(a*x + b*x^2)^p*(c + d*x)^q,x)
```

output
```
int(x^2*(a*x + b*x^2)^p*(c + d*x)^q, x)
```

Reduce [F]

$$\int x^2(c+dx)^q \left(ax+bx^2\right)^p \, dx = \text{too large to display}$$

input
```
int(x^2*(d*x+c)^q*(b*x^2+a*x)^p,x)
```

output

```
((c + d*x)**q*(a*x + b*x**2)**p*a**3*c*d**2*p**3 + (c + d*x)**q*(a*x + b*x
**2)**p*a**3*c*d**2*p**2*q + 3*(c + d*x)**q*(a*x + b*x**2)**p*a**3*c*d**2*
p**2 + (c + d*x)**q*(a*x + b*x**2)**p*a**3*c*d**2*p*q + 2*(c + d*x)**q*(a*
x + b*x**2)**p*a**3*c*d**2*p - (c + d*x)**q*(a*x + b*x**2)**p*a**3*d**3*p*
*3*x - 2*(c + d*x)**q*(a*x + b*x**2)**p*a**3*d**3*p**2*q*x - 2*(c + d*x)**
q*(a*x + b*x**2)**p*a**3*d**3*p**2*x - (c + d*x)**q*(a*x + b*x**2)**p*a**3
*d**3*p*q**2*x - 2*(c + d*x)**q*(a*x + b*x**2)**p*a**3*d**3*p*q*x - 2*(c +
 d*x)**q*(a*x + b*x**2)**p*a**2*b*c**2*d*p**2*q - 2*(c + d*x)**q*(a*x + b*
x**2)**p*a**2*b*c**2*d*p*q - 2*(c + d*x)**q*(a*x + b*x**2)**p*a**2*b*c*d**
2*p**3*x - 4*(c + d*x)**q*(a*x + b*x**2)**p*a**2*b*c*d**2*p**2*x + 2*(c +
d*x)**q*(a*x + b*x**2)**p*a**2*b*c*d**2*p*q**2*x + 2*(c + d*x)**q*(a*x + b
*x**2)**p*a**2*b*d**3*p**3*x**2 + 3*(c + d*x)**q*(a*x + b*x**2)**p*a**2*b*
d**3*p**2*q*x**2 + (c + d*x)**q*(a*x + b*x**2)**p*a**2*b*d**3*p**2*x**2 +
(c + d*x)**q*(a*x + b*x**2)**p*a**2*b*d**3*p*q**2*x**2 + (c + d*x)**q*(a*x
 + b*x**2)**p*a**2*b*d**3*p*q*x**2 + 2*(c + d*x)**q*(a*x + b*x**2)**p*a*b*
*2*c**3*p**2*q + 4*(c + d*x)**q*(a*x + b*x**2)**p*a*b**2*c**3*p*q + 2*(c +
 d*x)**q*(a*x + b*x**2)**p*a*b**2*c**3*q + 2*(c + d*x)**q*(a*x + b*x**2)**
p*a*b**2*c**2*d*p**2*q*x - 2*(c + d*x)**q*(a*x + b*x**2)**p*a*b**2*c**2*d*
p*q**2*x - 2*(c + d*x)**q*(a*x + b*x**2)**p*a*b**2*c**2*d*p*q*x - 2*(c + d
*x)**q*(a*x + b*x**2)**p*a*b**2*c**2*d*q**2*x + 4*(c + d*x)**q*(a*x + b...
```

3.213 $\int x(c+dx)^q \left(ax+bx^2\right)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 78

$$\int x(c+dx)^q \left(ax+bx^2\right)^p \, dx = \frac{x^2\left(1+\frac{bx}{a}\right)^{-p}(c+dx)^q\left(1+\frac{dx}{c}\right)^{-q}\left(ax+bx^2\right)^p \text{AppellF1}\left(2+p,-p,-q,3+p,-\frac{bx}{a},-\frac{dx}{c}\right)}{2+p}$$

output
```
x^2*(d*x+c)^q*(b*x^2+a*x)^p*AppellF1(2+p,-p,-q,3+p,-b*x/a,-d*x/c)/(2+p)/((
1+b*x/a)^p)/((1+d*x/c)^q)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int x(c+dx)^q \left(ax+bx^2\right)^p \, dx = \frac{x^2\left(\frac{a+bx}{a}\right)^{-p}(x(a+bx))^p(c+dx)^q\left(\frac{c+dx}{c}\right)^{-q} \text{AppellF1}\left(2+p,-p,-q,3+p,-\frac{bx}{a},-\frac{dx}{c}\right)}{2+p}$$

input
```
Integrate[x*(c + d*x)^q*(a*x + b*x^2)^p,x]
```

output
```
(x^2*(x*(a + b*x))^p*(c + d*x)^q*AppellF1[2 + p, -p, -q, 3 + p, -((b*x)/a)
, -((d*x)/c)])/((2 + p)*((a + b*x)/a)^p*((c + d*x)/c)^q)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 209 vs. $2(78) = 156$.

Time = 0.58 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.68, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax+bx^2\right)^p (c+dx)^q\,dx$$

$$\Big\downarrow \text{1269}$$

$$\frac{\int (c+dx)^{q+1}\left(bx^2+ax\right)^p dx}{d}-\frac{c\int (c+dx)^q\left(bx^2+ax\right)^p dx}{d}$$

$$\Big\downarrow \text{1179}$$

$$\frac{\left(ax+bx^2\right)^p\left(-\frac{dx}{c}\right)^{-p}\left(1-\frac{b(c+dx)}{bc-ad}\right)^{-p}\int (c+dx)^{q+1}\left(1-\frac{c+dx}{c}\right)^p\left(1-\frac{b(c+dx)}{bc-ad}\right)^p d(c+dx)}{d^2}-\frac{c\left(ax+bx^2\right)^p\left(-\frac{dx}{c}\right)^{-p}\left(1-\frac{b(c+dx)}{bc-ad}\right)^{-p}\int (c+dx)^q\left(1-\frac{c+dx}{c}\right)^p\left(1-\frac{b(c+dx)}{bc-ad}\right)^p d(c+dx)}{d^2}$$

$$\Big\downarrow \text{150}$$

$$\frac{\left(ax+bx^2\right)^p\left(-\frac{dx}{c}\right)^{-p}(c+dx)^{q+2}\left(1-\frac{b(c+dx)}{bc-ad}\right)^{-p}\text{AppellF1}\left(q+2,-p,-p,q+3,\frac{c+dx}{c},\frac{b(c+dx)}{bc-ad}\right)}{d^2(q+2)}-\frac{c\left(ax+bx^2\right)^p\left(-\frac{dx}{c}\right)^{-p}(c+dx)^{q+1}\left(1-\frac{b(c+dx)}{bc-ad}\right)^{-p}\text{AppellF1}\left(q+1,-p,-p,q+2,\frac{c+dx}{c},\frac{b(c+dx)}{bc-ad}\right)}{d^2(q+1)}$$

input
```
Int[x*(c + d*x)^q*(a*x + b*x^2)^p,x]
```

output
```
-((c*(c + d*x)^(1 + q)*(a*x + b*x^2)^p*AppellF1[1 + q, -p, -p, 2 + q, (c +
 d*x)/c, (b*(c + d*x))/(b*c - a*d)])/(d^2*(1 + q)*(-((d*x)/c))^p*(1 - (b*(
c + d*x))/(b*c - a*d))^p)) + ((c + d*x)^(2 + q)*(a*x + b*x^2)^p*AppellF1[2
 + q, -p, -p, 3 + q, (c + d*x)/c, (b*(c + d*x))/(b*c - a*d)])/(d^2*(2 + q)
*(-((d*x)/c))^p*(1 - (b*(c + d*x))/(b*c - a*d))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [F]

$$\int x(dx+c)^q \left(b\,x^2+ax\right)^p dx$$

input
```
int(x*(d*x+c)^q*(b*x^2+a*x)^p,x)
```

output
```
int(x*(d*x+c)^q*(b*x^2+a*x)^p,x)
```

Fricas [F]

$$\int x(c+dx)^q \left(ax+bx^2\right)^p \, dx = \int \left(bx^2+ax\right)^p (dx+c)^q x \, dx$$

input
```
integrate(x*(d*x+c)^q*(b*x^2+a*x)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a*x)^p*(d*x + c)^q*x, x)
```

Sympy [F]

$$\int x(c+dx)^q \left(ax+bx^2\right)^p \, dx = \int x(x(a+bx))^p \, (c+dx)^q \, dx$$

input
```
integrate(x*(d*x+c)**q*(b*x**2+a*x)**p,x)
```

output
```
Integral(x*(x*(a + b*x))**p*(c + d*x)**q, x)
```

Maxima [F]

$$\int x(c+dx)^q \left(ax+bx^2\right)^p \, dx = \int \left(bx^2+ax\right)^p (dx+c)^q x \, dx$$

input
```
integrate(x*(d*x+c)^q*(b*x^2+a*x)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^p*(d*x + c)^q*x, x)
```

Giac [F]

$$\int x(c+dx)^q \left(ax+bx^2\right)^p \, dx = \int \left(bx^2+ax\right)^p (dx+c)^q x \, dx$$

input
```
integrate(x*(d*x+c)^q*(b*x^2+a*x)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a*x)^p*(d*x + c)^q*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(c+dx)^q \left(ax+bx^2\right)^p \, dx = \int x \left(b\,x^2+a\,x\right)^p (c+d\,x)^q \, dx$$

input
```
int(x*(a*x + b*x^2)^p*(c + d*x)^q,x)
```

output
```
int(x*(a*x + b*x^2)^p*(c + d*x)^q, x)
```

Reduce [F]

$$\int x(c+dx)^q \left(ax+bx^2\right)^p \, dx = \text{too large to display}$$

input
```
int(x*(d*x+c)^q*(b*x^2+a*x)^p,x)
```

output
```
( - (c + d*x)**q*(a*x + b*x**2)**p*a**2*c*d*p**2 - (c + d*x)**q*(a*x + b*x
**2)**p*a**2*c*d*p + (c + d*x)**q*(a*x + b*x**2)**p*a**2*d**2*p**2*x + (c
+ d*x)**q*(a*x + b*x**2)**p*a**2*d**2*p*q*x - (c + d*x)**q*(a*x + b*x**2)*
*p*a*b*c**2*p*q - (c + d*x)**q*(a*x + b*x**2)**p*a*b*c**2*q + 2*(c + d*x)*
*q*(a*x + b*x**2)**p*a*b*c*d*p**2*x + (c + d*x)**q*(a*x + b*x**2)**p*a*b*c
*d*p*q*x + (c + d*x)**q*(a*x + b*x**2)**p*a*b*c*d*q**2*x + 2*(c + d*x)**q*
(a*x + b*x**2)**p*a*b*d**2*p**2*x**2 + 3*(c + d*x)**q*(a*x + b*x**2)**p*a*
b*d**2*p*q*x**2 + (c + d*x)**q*(a*x + b*x**2)**p*a*b*d**2*p*x**2 + (c + d*
x)**q*(a*x + b*x**2)**p*a*b*d**2*q**2*x**2 + (c + d*x)**q*(a*x + b*x**2)**
p*a*b*d**2*q*x**2 + 2*(c + d*x)**q*(a*x + b*x**2)**p*b**2*c**2*p*q*x + 4*(
c + d*x)**q*(a*x + b*x**2)**p*b**2*c*d*p**2*x**2 + 2*(c + d*x)**q*(a*x + b
*x**2)**p*b**2*c*d*p*q*x**2 + 2*(c + d*x)**q*(a*x + b*x**2)**p*b**2*c*d*p*
x**2 - 4*int(((c + d*x)**q*(a*x + b*x**2)**p*x)/(4*a**2*c*d*p**3 + 8*a**2*
c*d*p**2*q + 6*a**2*c*d*p**2 + 5*a**2*c*d*p*q**2 + 9*a**2*c*d*p*q + 2*a**2
*c*d*p + a**2*c*d*q**3 + 3*a**2*c*d*q**2 + 2*a**2*c*d*q + 4*a**2*d**2*p**3
*x + 8*a**2*d**2*p**2*q*x + 6*a**2*d**2*p**2*x + 5*a**2*d**2*p*q**2*x + 9*
a**2*d**2*p*q*x + 2*a**2*d**2*p*x + a**2*d**2*q**3*x + 3*a**2*d**2*q**2*x
+ 2*a**2*d**2*q*x + 8*a*b*c**2*p**3 + 8*a*b*c**2*p**2*q + 12*a*b*c**2*p**2
 + 2*a*b*c**2*p*q**2 + 6*a*b*c**2*p*q + 4*a*b*c**2*p + 12*a*b*c*d*p**3*x +
 16*a*b*c*d*p**2*q*x + 18*a*b*c*d*p**2*x + 7*a*b*c*d*p*q**2*x + 15*a*b*...
```

3.214 $\int (c+dx)^q \left(ax+bx^2\right)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 76

$$\int (c+dx)^q \left(ax+bx^2\right)^p \, dx = \frac{x\left(1+\frac{bx}{a}\right)^{-p}(c+dx)^q\left(1+\frac{dx}{c}\right)^{-q}\left(ax+bx^2\right)^p \operatorname{AppellF1}\left(1+p,-p,-q,2+p,-\frac{bx}{a},-\frac{dx}{c}\right)}{1+p}$$

output
```
x*(d*x+c)^q*(b*x^2+a*x)^p*AppellF1(p+1,-p,-q,2+p,-b*x/a,-d*x/c)/(p+1)/((1+
b*x/a)^p)/((1+d*x/c)^q)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int (c+dx)^q \left(ax+bx^2\right)^p \, dx = \frac{x\left(\frac{a+bx}{a}\right)^{-p}(x(a+bx))^p(c+dx)^q\left(\frac{c+dx}{c}\right)^{-q} \operatorname{AppellF1}\left(1+p,-p,-q,2+p,-\frac{bx}{a},-\frac{dx}{c}\right)}{1+p}$$

input
```
Integrate[(c + d*x)^q*(a*x + b*x^2)^p,x]
```

output
```
(x*(x*(a + b*x))^p*(c + d*x)^q*AppellF1[1 + p, -p, -q, 2 + p, -((b*x)/a),
-((d*x)/c)])/((1 + p)*((a + b*x)/a)^p*((c + d*x)/c)^q)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.36, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^2\right)^p (c+dx)^q \, dx$$

$$\Big\downarrow \text{1179}$$

$$\frac{\left(ax+bx^2\right)^p \left(-\frac{dx}{c}\right)^{-p} \left(1-\frac{b(c+dx)}{bc-ad}\right)^{-p} \int (c+dx)^q \left(1-\frac{c+dx}{c}\right)^p \left(1-\frac{b(c+dx)}{bc-ad}\right)^p d(c+dx)}{d}$$

$$\Big\downarrow \text{150}$$

$$\frac{\left(ax+bx^2\right)^p \left(-\frac{dx}{c}\right)^{-p} (c+dx)^{q+1} \left(1-\frac{b(c+dx)}{bc-ad}\right)^{-p} \text{AppellF1}\left(q+1,-p,-p,q+2,\frac{c+dx}{c},\frac{b(c+dx)}{bc-ad}\right)}{d(q+1)}$$

input
```
Int[(c + d*x)^q*(a*x + b*x^2)^p,x]
```

output
```
((c + d*x)^(1 + q)*(a*x + b*x^2)^p*AppellF1[1 + q, -p, -p, 2 + q, (c + d*x
)/c, (b*(c + d*x))/(b*c - a*d)])/(d*(1 + q)*(-((d*x)/c))^p*(1 - (b*(c + d*
x))/(b*c - a*d))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

Maple [F]

$$\int (dx+c)^{q} \left(b\,x^{2}+ax\right)^{p} dx$$

input
```
int((d*x+c)^q*(b*x^2+a*x)^p,x)
```

output
```
int((d*x+c)^q*(b*x^2+a*x)^p,x)
```

Fricas [F]

$$\int (c+dx)^q \left(ax+bx^2\right)^p \,dx = \int \left(bx^2+ax\right)^p (dx+c)^q \,dx$$

input
```
integrate((d*x+c)^q*(b*x^2+a*x)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a*x)^p*(d*x + c)^q, x)
```

Sympy [F]

$$\int (c+dx)^q \left(ax+bx^2\right)^p \, dx = \int (x(a+bx))^p (c+dx)^q \, dx$$

input
```
integrate((d*x+c)**q*(b*x**2+a*x)**p,x)
```

output
```
Integral((x*(a + b*x))**p*(c + d*x)**q, x)
```

Maxima [F]

$$\int (c+dx)^q \left(ax+bx^2\right)^p \, dx = \int \left(bx^2+ax\right)^p (dx+c)^q \, dx$$

input
```
integrate((d*x+c)^q*(b*x^2+a*x)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^p*(d*x + c)^q, x)
```

Giac [F]

$$\int (c+dx)^q \left(ax+bx^2\right)^p \, dx = \int \left(bx^2+ax\right)^p (dx+c)^q \, dx$$

input
```
integrate((d*x+c)^q*(b*x^2+a*x)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a*x)^p*(d*x + c)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^q \left(ax+bx^2\right)^p \, dx = \int \left(b\,x^2+a\,x\right)^p\,{\left(c+d\,x\right)}^q \,dx$$

input
```
int((a*x + b*x^2)^p*(c + d*x)^q,x)
```

output
```
int((a*x + b*x^2)^p*(c + d*x)^q, x)
```

Reduce [F]

$$\int (c+dx)^q \left(ax+bx^2\right)^p \, dx = \text{too large to display}$$

input
```
int((d*x+c)^q*(b*x^2+a*x)^p,x)
```

output

```
((c + d*x)**q*(a*x + b*x**2)**p*a*c*p + (c + d*x)**q*(a*x + b*x**2)**p*a*c
*q + (c + d*x)**q*(a*x + b*x**2)**p*a*d*p*x + (c + d*x)**q*(a*x + b*x**2)*
*p*a*d*q*x + 2*(c + d*x)**q*(a*x + b*x**2)**p*b*c*p*x + 2*int(((c + d*x)**
q*(a*x + b*x**2)**p*x)/(2*a**2*c*d*p**2 + 3*a**2*c*d*p*q + a**2*c*d*p + a*
*2*c*d*q**2 + a**2*c*d*q + 2*a**2*d**2*p**2*x + 3*a**2*d**2*p*q*x + a**2*d
**2*p*x + a**2*d**2*q**2*x + a**2*d**2*q*x + 4*a*b*c**2*p**2 + 2*a*b*c**2*
p*q + 2*a*b*c**2*p + 6*a*b*c*d*p**2*x + 5*a*b*c*d*p*q*x + 3*a*b*c*d*p*x +
a*b*c*d*q**2*x + a*b*c*d*q*x + 2*a*b*d**2*p**2*x**2 + 3*a*b*d**2*p*q*x**2
+ a*b*d**2*p*x**2 + a*b*d**2*q**2*x**2 + a*b*d**2*q*x**2 + 4*b**2*c**2*p**
2*x + 2*b**2*c**2*p*q*x + 2*b**2*c**2*p*x + 4*b**2*c*d*p**2*x**2 + 2*b**2*
c*d*p*q*x**2 + 2*b**2*c*d*p*x**2),x)*a**3*d**3*p**4 + 5*int(((c + d*x)**q*
(a*x + b*x**2)**p*x)/(2*a**2*c*d*p**2 + 3*a**2*c*d*p*q + a**2*c*d*p + a**2
*c*d*q**2 + a**2*c*d*q + 2*a**2*d**2*p**2*x + 3*a**2*d**2*p*q*x + a**2*d**
2*p*x + a**2*d**2*q**2*x + a**2*d**2*q*x + 4*a*b*c**2*p**2 + 2*a*b*c**2*p*
q + 2*a*b*c**2*p + 6*a*b*c*d*p**2*x + 5*a*b*c*d*p*q*x + 3*a*b*c*d*p*x + a*
b*c*d*q**2*x + a*b*c*d*q*x + 2*a*b*d**2*p**2*x**2 + 3*a*b*d**2*p*q*x**2 +
a*b*d**2*p*x**2 + a*b*d**2*q**2*x**2 + a*b*d**2*q*x**2 + 4*b**2*c**2*p**2*
x + 2*b**2*c**2*p*q*x + 2*b**2*c**2*p*x + 4*b**2*c*d*p**2*x**2 + 2*b**2*c*
d*p*q*x**2 + 2*b**2*c*d*p*x**2),x)*a**3*d**3*p**3*q + int(((c + d*x)**q*(a
*x + b*x**2)**p*x)/(2*a**2*c*d*p**2 + 3*a**2*c*d*p*q + a**2*c*d*p + a**...
```

3.215 $\int \frac{(c+dx)^q (ax+bx^2)^p}{x} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{(c+dx)^q (ax+bx^2)^p}{x} \, dx = \frac{\left(1+\frac{bx}{a}\right)^{-p} (c+dx)^q \left(1+\frac{dx}{c}\right)^{-q} (ax+bx^2)^p \operatorname{AppellF1}\left(p,-p,-q,1+p,-\frac{bx}{a},-\frac{dx}{c}\right)}{p}$$

output
```
(d*x+c)^q*(b*x^2+a*x)^p*AppellF1(p,-p,-q,p+1,-b*x/a,-d*x/c)/p/((1+b*x/a)^p
)/((1+d*x/c)^q)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^q (ax+bx^2)^p}{x} \, dx = \frac{\left(\frac{a+bx}{a}\right)^{-p} (x(a+bx))^p (c+dx)^q \left(\frac{c+dx}{c}\right)^{-q} \operatorname{AppellF1}\left(p,-p,-q,1+p,-\frac{bx}{a},-\frac{dx}{c}\right)}{p}$$

input
```
Integrate[((c + d*x)^q*(a*x + b*x^2)^p)/x,x]
```

output
```
((x*(a + b*x))^p*(c + d*x)^q*AppellF1[p, -p, -q, 1 + p, -((b*x)/a), -((d*x
)/c)])/(p*((a + b*x)/a)^p*((c + d*x)/c)^q)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1261, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^p (c+dx)^q}{x} dx$$

$$\Big\downarrow \text{ 1261}$$

$$x^{-p}(a+bx)^{-p} \left(ax+bx^2\right)^p \int x^{p-1}(a+bx)^p(c+dx)^q dx$$

$$\Big\downarrow \text{ 152}$$

$$x^{-p}\left(\frac{bx}{a}+1\right)^{-p} \left(ax+bx^2\right)^p \int x^{p-1}\left(\frac{bx}{a}+1\right)^p (c+dx)^q dx$$

$$\Big\downarrow \text{ 152}$$

$$x^{-p}\left(\frac{bx}{a}+1\right)^{-p} \left(ax+bx^2\right)^p (c+dx)^q \left(\frac{dx}{c}+1\right)^{-q} \int x^{p-1}\left(\frac{bx}{a}+1\right)^p \left(\frac{dx}{c}+1\right)^q dx$$

$$\Big\downarrow \text{ 150}$$

$$\frac{\left(\frac{bx}{a}+1\right)^{-p} \left(ax+bx^2\right)^p (c+dx)^q \left(\frac{dx}{c}+1\right)^{-q} \text{AppellF1}\left(p,-p,-q,p+1,-\frac{bx}{a},-\frac{dx}{c}\right)}{p}$$

input
```
Int[((c + d*x)^q*(a*x + b*x^2)^p)/x,x]
```

output
```
((c + d*x)^q*(a*x + b*x^2)^p*AppellF1[p, -p, -q, 1 + p, -((b*x)/a), -((d*x
)/c)])/(p*(1 + (b*x)/a)^p*(1 + (d*x)/c)^q)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [F]

$$\int \frac{(dx+c)^q (b\,x^2+ax)^p}{x}dx$$

input
```
int((d*x+c)^q*(b*x^2+a*x)^p/x,x)
```

output
```
int((d*x+c)^q*(b*x^2+a*x)^p/x,x)
```

Fricas [F]

$$\int \frac{(c+dx)^q (ax+bx^2)^p}{x}\,dx = \int \frac{(bx^2+ax)^p (dx+c)^q}{x}\,dx$$

input
```
integrate((d*x+c)^q*(b*x^2+a*x)^p/x,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a*x)^p*(d*x + c)^q/x, x)
```

Sympy [F]

$$\int \frac{(c+dx)^q\,(ax+bx^2)^p}{x}\,dx = \int \frac{(x(a+bx))^p\,(c+dx)^q}{x}\,dx$$

input
```
integrate((d*x+c)**q*(b*x**2+a*x)**p/x,x)
```

output
```
Integral((x*(a + b*x))**p*(c + d*x)**q/x, x)
```

Maxima [F]

$$\int \frac{(c+dx)^q\,(ax+bx^2)^p}{x}\,dx = \int \frac{(bx^2+ax)^p(dx+c)^q}{x}\,dx$$

input
```
integrate((d*x+c)^q*(b*x^2+a*x)^p/x,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^p*(d*x + c)^q/x, x)
```

Giac [F]

$$\int \frac{(c+dx)^q\,(ax+bx^2)^p}{x}\,dx = \int \frac{(bx^2+ax)^p(dx+c)^q}{x}\,dx$$

input
```
integrate((d*x+c)^q*(b*x^2+a*x)^p/x,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a*x)^p*(d*x + c)^q/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^q\left(ax+bx^2\right)^p}{x}\,dx=\int \frac{{\left(b\,x^2+a\,x\right)}^p\,{\left(c+d\,x\right)}^q}{x}\,dx$$

input
```
int(((a*x + b*x^2)^p*(c + d*x)^q)/x,x)
```

output
```
int(((a*x + b*x^2)^p*(c + d*x)^q)/x, x)
```

Reduce [F]

$$\int \frac{(c+dx)^q\left(ax+bx^2\right)^p}{x}\,dx=\text{Too large to display}$$

input
```
int((d*x+c)^q*(b*x^2+a*x)^p/x,x)
```

output

```
((c + d*x)**q*(a*x + b*x**2)**p*a*d + (c + d*x)**q*(a*x + b*x**2)**p*b*c -
 int(((c + d*x)**q*(a*x + b*x**2)**p*x)/(a**2*c*d*p + a**2*c*d*q + a**2*d*
*2*p*x + a**2*d**2*q*x + 2*a*b*c**2*p + 3*a*b*c*d*p*x + a*b*c*d*q*x + a*b*
d**2*p*x**2 + a*b*d**2*q*x**2 + 2*b**2*c**2*p*x + 2*b**2*c*d*p*x**2),x)*a*
*2*b*d**3*p**2 - int(((c + d*x)**q*(a*x + b*x**2)**p*x)/(a**2*c*d*p + a**2
*c*d*q + a**2*d**2*p*x + a**2*d**2*q*x + 2*a*b*c**2*p + 3*a*b*c*d*p*x + a*
b*c*d*q*x + a*b*d**2*p*x**2 + a*b*d**2*q*x**2 + 2*b**2*c**2*p*x + 2*b**2*c
*d*p*x**2),x)*a**2*b*d**3*p*q - 2*int(((c + d*x)**q*(a*x + b*x**2)**p*x)/(
a**2*c*d*p + a**2*c*d*q + a**2*d**2*p*x + a**2*d**2*q*x + 2*a*b*c**2*p + 3
*a*b*c*d*p*x + a*b*c*d*q*x + a*b*d**2*p*x**2 + a*b*d**2*q*x**2 + 2*b**2*c*
*2*p*x + 2*b**2*c*d*p*x**2),x)*a*b**2*c*d**2*p**2 - int(((c + d*x)**q*(a*x
 + b*x**2)**p*x)/(a**2*c*d*p + a**2*c*d*q + a**2*d**2*p*x + a**2*d**2*q*x
+ 2*a*b*c**2*p + 3*a*b*c*d*p*x + a*b*c*d*q*x + a*b*d**2*p*x**2 + a*b*d**2*
q*x**2 + 2*b**2*c**2*p*x + 2*b**2*c*d*p*x**2),x)*a*b**2*c*d**2*p*q - int((
(c + d*x)**q*(a*x + b*x**2)**p*x)/(a**2*c*d*p + a**2*c*d*q + a**2*d**2*p*x
 + a**2*d**2*q*x + 2*a*b*c**2*p + 3*a*b*c*d*p*x + a*b*c*d*q*x + a*b*d**2*p
*x**2 + a*b*d**2*q*x**2 + 2*b**2*c**2*p*x + 2*b**2*c*d*p*x**2),x)*a*b**2*c
*d**2*q**2 - 2*int(((c + d*x)**q*(a*x + b*x**2)**p*x)/(a**2*c*d*p + a**2*c
*d*q + a**2*d**2*p*x + a**2*d**2*q*x + 2*a*b*c**2*p + 3*a*b*c*d*p*x + a*b*
c*d*q*x + a*b*d**2*p*x**2 + a*b*d**2*q*x**2 + 2*b**2*c**2*p*x + 2*b**2*...
```

3.216 $\int \frac{(c+dx)^q (ax+bx^2)^p}{x^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 79

$$\int \frac{(c+dx)^q (ax+bx^2)^p}{x^2} dx = -\frac{\left(1+\frac{bx}{a}\right)^{-p} (c+dx)^q \left(1+\frac{dx}{c}\right)^{-q} (ax+bx^2)^p \operatorname{AppellF1}\left(-1+p,-p,-q,p,-\frac{bx}{a},-\frac{dx}{c}\right)}{(1-p)x}$$

output
```
-(d*x+c)^q*(b*x^2+a*x)^p*AppellF1(-1+p,-p,-q,p,-b*x/a,-d*x/c)/(1-p)/x/((1+
b*x/a)^p)/((1+d*x/c)^q)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96

$$\int \frac{(c+dx)^q (ax+bx^2)^p}{x^2} dx = \frac{\left(\frac{a+bx}{a}\right)^{-p} (x(a+bx))^p (c+dx)^q \left(\frac{c+dx}{c}\right)^{-q} \operatorname{AppellF1}\left(-1+p,-p,-q,p,-\frac{bx}{a},-\frac{dx}{c}\right)}{(-1+p)x}$$

input
```
Integrate[((c + d*x)^q*(a*x + b*x^2)^p)/x^2,x]
```

output
```
((x*(a + b*x))^p*(c + d*x)^q*AppellF1[-1 + p, -p, -q, p, -((b*x)/a), -((d*
x)/c)])/((-1 + p)*x*((a + b*x)/a)^p*((c + d*x)/c)^q)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1261, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^p (c+dx)^q}{x^2} \, dx$$

$$\Big\downarrow \text{1261}$$

$$x^{-p}(a+bx)^{-p} (ax+bx^2)^p \int x^{p-2}(a+bx)^p(c+dx)^q dx$$

$$\Big\downarrow \text{152}$$

$$x^{-p}\left(\frac{bx}{a}+1\right)^{-p} (ax+bx^2)^p \int x^{p-2}\left(\frac{bx}{a}+1\right)^p (c+dx)^q dx$$

$$\Big\downarrow \text{152}$$

$$x^{-p}\left(\frac{bx}{a}+1\right)^{-p} (ax+bx^2)^p (c+dx)^q \left(\frac{dx}{c}+1\right)^{-q} \int x^{p-2}\left(\frac{bx}{a}+1\right)^p \left(\frac{dx}{c}+1\right)^q dx$$

$$\Big\downarrow \text{150}$$

$$-\frac{\left(\frac{bx}{a}+1\right)^{-p} (ax+bx^2)^p (c+dx)^q \left(\frac{dx}{c}+1\right)^{-q} \text{AppellF1}\left(p-1,-p,-q,p,-\frac{bx}{a},-\frac{dx}{c}\right)}{(1-p)x}$$

input
```
Int[((c + d*x)^q*(a*x + b*x^2)^p)/x^2,x]
```

output
```
-(((c + d*x)^q*(a*x + b*x^2)^p*AppellF1[-1 + p, -p, -q, p, -((b*x)/a), -((
d*x)/c)])/((1 - p)*x*(1 + (b*x)/a)^p*(1 + (d*x)/c)^q))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [F]

$$\int \frac{(dx+c)^q\,(b\,x^2+ax)^p}{x^2}dx$$

input
```
int((d*x+c)^q*(b*x^2+a*x)^p/x^2,x)
```

output
```
int((d*x+c)^q*(b*x^2+a*x)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(c+dx)^q\,(ax+bx^2)^p}{x^2}\,dx=\int \frac{(bx^2+ax)^p(dx+c)^q}{x^2}\,dx$$

input
```
integrate((d*x+c)^q*(b*x^2+a*x)^p/x^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a*x)^p*(d*x + c)^q/x^2, x)
```

Sympy [F]

$$\int \frac{(c+dx)^q (ax+bx^2)^p}{x^2} dx = \int \frac{(x(a+bx))^p (c+dx)^q}{x^2} dx$$

input
```
integrate((d*x+c)**q*(b*x**2+a*x)**p/x**2,x)
```

output
```
Integral((x*(a + b*x))**p*(c + d*x)**q/x**2, x)
```

Maxima [F]

$$\int \frac{(c+dx)^q (ax+bx^2)^p}{x^2} dx = \int \frac{(bx^2+ax)^p (dx+c)^q}{x^2} dx$$

input
```
integrate((d*x+c)^q*(b*x^2+a*x)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^p*(d*x + c)^q/x^2, x)
```

Giac [F]

$$\int \frac{(c+dx)^q (ax+bx^2)^p}{x^2} dx = \int \frac{(bx^2+ax)^p (dx+c)^q}{x^2} dx$$

input
```
integrate((d*x+c)^q*(b*x^2+a*x)^p/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a*x)^p*(d*x + c)^q/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^q (ax+bx^2)^p}{x^2} dx = \int \frac{(b x^2 + a x)^p (c + d x)^q}{x^2} dx$$

input
```
int(((a*x + b*x^2)^p*(c + d*x)^q)/x^2,x)
```

output
```
int(((a*x + b*x^2)^p*(c + d*x)^q)/x^2, x)
```

Reduce [F]

$$\int \frac{(c+dx)^q (ax+bx^2)^p}{x^2} dx = \text{too large to display}$$

input
```
int((d*x+c)^q*(b*x^2+a*x)^p/x^2,x)
```

output

```
((c + d*x)**q*(a*x + b*x**2)**p - int(((c + d*x)**q*(a*x + b*x**2)**p)/(a*
*2*c*d*p**2*x + a**2*c*d*p*q*x - 2*a**2*c*d*p*x - a**2*c*d*q*x + a**2*c*d*
x + a**2*d**2*p**2*x**2 + a**2*d**2*p*q*x**2 - 2*a**2*d**2*p*x**2 - a**2*d
**2*q*x**2 + a**2*d**2*x**2 + 2*a*b*c**2*p**2*x - 3*a*b*c**2*p*x + a*b*c**
2*x + 3*a*b*c*d*p**2*x**2 + a*b*c*d*p*q*x**2 - 5*a*b*c*d*p*x**2 - a*b*c*d*
q*x**2 + 2*a*b*c*d*x**2 + a*b*d**2*p**2*x**3 + a*b*d**2*p*q*x**3 - 2*a*b*d
**2*p*x**3 - a*b*d**2*q*x**3 + a*b*d**2*x**3 + 2*b**2*c**2*p**2*x**2 - 3*b
**2*c**2*p*x**2 + b**2*c**2*x**2 + 2*b**2*c*d*p**2*x**3 - 3*b**2*c*d*p*x**
3 + b**2*c*d*x**3),x)*a**2*d**2*p**2*q*x - int(((c + d*x)**q*(a*x + b*x**2
)**p)/(a**2*c*d*p**2*x + a**2*c*d*p*q*x - 2*a**2*c*d*p*x - a**2*c*d*q*x +
a**2*c*d*x + a**2*d**2*p**2*x**2 + a**2*d**2*p*q*x**2 - 2*a**2*d**2*p*x**2
 - a**2*d**2*q*x**2 + a**2*d**2*x**2 + 2*a*b*c**2*p**2*x - 3*a*b*c**2*p*x
+ a*b*c**2*x + 3*a*b*c*d*p**2*x**2 + a*b*c*d*p*q*x**2 - 5*a*b*c*d*p*x**2 -
 a*b*c*d*q*x**2 + 2*a*b*c*d*x**2 + a*b*d**2*p**2*x**3 + a*b*d**2*p*q*x**3
- 2*a*b*d**2*p*x**3 - a*b*d**2*q*x**3 + a*b*d**2*x**3 + 2*b**2*c**2*p**2*x
**2 - 3*b**2*c**2*p*x**2 + b**2*c**2*x**2 + 2*b**2*c*d*p**2*x**3 - 3*b**2*
c*d*p*x**3 + b**2*c*d*x**3),x)*a**2*d**2*p*q**2*x + 2*int(((c + d*x)**q*(a
*x + b*x**2)**p)/(a**2*c*d*p**2*x + a**2*c*d*p*q*x - 2*a**2*c*d*p*x - a**2
*c*d*q*x + a**2*c*d*x + a**2*d**2*p**2*x**2 + a**2*d**2*p*q*x**2 - 2*a**2*
d**2*p*x**2 - a**2*d**2*q*x**2 + a**2*d**2*x**2 + 2*a*b*c**2*p**2*x - 3...
```

3.217 $\int x^3(a+2bx)^q \left(ax+bx^2\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 232

$$\int x^3(a+2bx)^q \left(ax+bx^2\right)^p \, dx = \frac{x(a+2bx)^{1+q}\left(ax+bx^2\right)^{1+p}}{b^2(1+p)} - \frac{a^3(3+p+2q)(a+2bx)^{1+q}\left(ax+bx^2\right)^p\left(1-\frac{(a+2bx)^2}{a^2}\right)^{-p}\text{Hypergeometric2F1}\left(-1-p,\frac{1+q}{2},\frac{3+q}{2},\frac{(a+2bx)^2}{a^2}\right)}{16b^4(1+p)(1+q)} + \frac{a^2(7+3p+2q)(a+2bx)^{2+q}\left(ax+bx^2\right)^p\left(1-\frac{(a+2bx)^2}{a^2}\right)^{-p}\text{Hypergeometric2F1}\left(-1-p,\frac{2+q}{2},\frac{4+q}{2},\frac{(a+2bx)^2}{a^2}\right)}{16b^4(1+p)(2+q)}$$

output
```
x*(2*b*x+a)^(1+q)*(b*x^2+a*x)^(p+1)/b^2/(p+1)-1/16*a^3*(3+p+2*q)*(2*b*x+a)
^(1+q)*(b*x^2+a*x)^p*hypergeom([-1-p, 1/2+1/2*q],[3/2+1/2*q],(2*b*x+a)^2/a
^2)/b^4/(p+1)/(1+q)/((1-(2*b*x+a)^2/a^2)^p)+1/16*a^2*(7+3*p+2*q)*(2*b*x+a)
^(2+q)*(b*x^2+a*x)^p*hypergeom([-1-p, 1+1/2*q],[2+1/2*q],(2*b*x+a)^2/a^2)/
b^4/(p+1)/(2+q)/((1-(2*b*x+a)^2/a^2)^p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.34

$$\int x^3(a+2bx)^q\left(ax+bx^2\right)^p\,dx$$
$$=\frac{x^4\left(\frac{a+bx}{a}\right)^{-p}(x(a+bx))^p(a+2bx)^q\left(\frac{a+2bx}{a}\right)^{-q}\text{AppellF1}\left(4+p,-p,-q,5+p,-\frac{bx}{a},-\frac{2bx}{a}\right)}{4+p}$$

input
```
Integrate[x^3*(a + 2*b*x)^q*(a*x + b*x^2)^p,x]
```

output
```
(x^4*(x*(a + b*x))^p*(a + 2*b*x)^q*AppellF1[4 + p, -p, -q, 5 + p, -((b*x)/
a), (-2*b*x)/a])/((4 + p)*((a + b*x)/a)^p*((a + 2*b*x)/a)^q)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.34, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1261, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(ax+bx^2\right)^p(a+2bx)^q\,dx$$

$$\Big\downarrow \ 1261$$

$$x^{-p}(a+bx)^{-p}\left(ax+bx^2\right)^p\int x^{p+3}(a+bx)^p(a+2bx)^qdx$$

$$\Big\downarrow \ 152$$

$$x^{-p}\left(\frac{bx}{a}+1\right)^{-p}\left(ax+bx^2\right)^p\int x^{p+3}(a+2bx)^q\left(\frac{bx}{a}+1\right)^pdx$$

$$\Big\downarrow \ 152$$

$$x^{-p}\left(\frac{bx}{a}+1\right)^{-p}\left(ax+bx^2\right)^p(a+2bx)^q\left(\frac{2bx}{a}+1\right)^{-q}\int x^{p+3}\left(\frac{bx}{a}+1\right)^p\left(\frac{2bx}{a}+1\right)^q dx$$

$$\Big\downarrow \text{ 150}$$

$$\frac{x^4\left(\frac{bx}{a}+1\right)^{-p}\left(ax+bx^2\right)^p(a+2bx)^q\left(\frac{2bx}{a}+1\right)^{-q}\text{AppellF1}\left(p+4,-p,-q,p+5,-\frac{bx}{a},-\frac{2bx}{a}\right)}{p+4}$$

input
```
Int[x^3*(a + 2*b*x)^q*(a*x + b*x^2)^p,x]
```

output
```
(x^4*(a + 2*b*x)^q*(a*x + b*x^2)^p*AppellF1[4 + p, -p, -q, 5 + p, -((b*x)/
a), (-2*b*x)/a])/((4 + p)*(1 + (b*x)/a)^p*(1 + (2*b*x)/a)^q)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [F]

$$\int x^3(2bx+a)^q\,(b\,x^2+ax)^p\,dx$$

input
```
int(x^3*(2*b*x+a)^q*(b*x^2+a*x)^p,x)
```

output
```
int(x^3*(2*b*x+a)^q*(b*x^2+a*x)^p,x)
```

Fricas [F]

$$\int x^3(a+2bx)^q\,(ax+bx^2)^p\,dx=\int (bx^2+ax)^p(2\,bx+a)^q x^3\,dx$$

input
```
integrate(x^3*(2*b*x+a)^q*(b*x^2+a*x)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a*x)^p*(2*b*x + a)^q*x^3, x)
```

Sympy [F]

$$\int x^3(a+2bx)^q\,(ax+bx^2)^p\,dx=\int x^3(x(a+bx))^p\,(a+2bx)^q\,dx$$

input
```
integrate(x**3*(2*b*x+a)**q*(b*x**2+a*x)**p,x)
```

output
```
Integral(x**3*(x*(a + b*x))**p*(a + 2*b*x)**q, x)
```

Maxima [F]

$$\int x^3(a+2bx)^q\left(ax+bx^2\right)^p\,dx=\int\left(bx^2+ax\right)^p(2\,bx+a)^qx^3\,dx$$

input
```
integrate(x^3*(2*b*x+a)^q*(b*x^2+a*x)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^p*(2*b*x + a)^q*x^3, x)
```

Giac [F]

$$\int x^3(a+2bx)^q\left(ax+bx^2\right)^p\,dx=\int\left(bx^2+ax\right)^p(2\,bx+a)^qx^3\,dx$$

input
```
integrate(x^3*(2*b*x+a)^q*(b*x^2+a*x)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a*x)^p*(2*b*x + a)^q*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a+2bx)^q\left(ax+bx^2\right)^p\,dx=\int x^3\,\left(b\,x^2+a\,x\right)^p\,(a+2\,b\,x)^q\,dx$$

input
```
int(x^3*(a*x + b*x^2)^p*(a + 2*b*x)^q,x)
```

output
```
int(x^3*(a*x + b*x^2)^p*(a + 2*b*x)^q, x)
```

Reduce [F]

$$\int x^3 (a+2bx)^q \left(ax+bx^2\right)^p \, dx = \text{too large to display}$$

input
```
int(x^3*(2*b*x+a)^q*(b*x^2+a*x)^p,x)
```

output
```
( - 4*(a + 2*b*x)**q*(a*x + b*x**2)**p*a**4*p**4 - 4*(a + 2*b*x)**q*(a*x +
 b*x**2)**p*a**4*p**3*q - 24*(a + 2*b*x)**q*(a*x + b*x**2)**p*a**4*p**3 -
(a + 2*b*x)**q*(a*x + b*x**2)**p*a**4*p**2*q**2 - 14*(a + 2*b*x)**q*(a*x +
 b*x**2)**p*a**4*p**2*q - 44*(a + 2*b*x)**q*(a*x + b*x**2)**p*a**4*p**2 -
(a + 2*b*x)**q*(a*x + b*x**2)**p*a**4*p*q**2 - 13*(a + 2*b*x)**q*(a*x + b*
x**2)**p*a**4*p*q - 24*(a + 2*b*x)**q*(a*x + b*x**2)**p*a**4*p - 3*(a + 2*
b*x)**q*(a*x + b*x**2)**p*a**4*q + 16*(a + 2*b*x)**q*(a*x + b*x**2)**p*a**
3*b*p**4*x + 24*(a + 2*b*x)**q*(a*x + b*x**2)**p*a**3*b*p**3*q*x + 80*(a +
 2*b*x)**q*(a*x + b*x**2)**p*a**3*b*p**3*x + 12*(a + 2*b*x)**q*(a*x + b*x*
*2)**p*a**3*b*p**2*q**2*x + 80*(a + 2*b*x)**q*(a*x + b*x**2)**p*a**3*b*p**
2*q*x + 96*(a + 2*b*x)**q*(a*x + b*x**2)**p*a**3*b*p**2*x + 2*(a + 2*b*x)*
*q*(a*x + b*x**2)**p*a**3*b*p*q**3*x + 20*(a + 2*b*x)**q*(a*x + b*x**2)**p
*a**3*b*p*q**2*x + 60*(a + 2*b*x)**q*(a*x + b*x**2)**p*a**3*b*p*q*x + 6*(a
 + 2*b*x)**q*(a*x + b*x**2)**p*a**3*b*q**2*x - 32*(a + 2*b*x)**q*(a*x + b*
x**2)**p*a**2*b**2*p**4*x**2 - 48*(a + 2*b*x)**q*(a*x + b*x**2)**p*a**2*b*
*2*p**3*q*x**2 - 112*(a + 2*b*x)**q*(a*x + b*x**2)**p*a**2*b**2*p**3*x**2
- 24*(a + 2*b*x)**q*(a*x + b*x**2)**p*a**2*b**2*p**2*q**2*x**2 - 136*(a +
2*b*x)**q*(a*x + b*x**2)**p*a**2*b**2*p**2*q*x**2 - 48*(a + 2*b*x)**q*(a*x
 + b*x**2)**p*a**2*b**2*p**2*x**2 - 4*(a + 2*b*x)**q*(a*x + b*x**2)**p*a**
2*b**2*p*q**3*x**2 - 52*(a + 2*b*x)**q*(a*x + b*x**2)**p*a**2*b**2*p*q*...
```

3.218 $\int x^2(a+2bx)^q\left(ax+bx^2\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 219

$$\int x^2(a+2bx)^q\left(ax+bx^2\right)^p\,dx=\frac{(a+2bx)^{1+q}\left(ax+bx^2\right)^{1+p}}{2b^2(3+2p+q)}+\frac{a^2(2+p+q)(a+2bx)^{1+q}\left(ax+bx^2\right)^p\left(1-\frac{(a+2bx)^2}{a^2}\right)^{-p}\text{Hypergeometric2F1}\left(-p,\frac{1+q}{2},\frac{3+q}{2},\frac{(a+2bx)^2}{a^2}\right)}{4b^3(1+q)(3+2p+q)}-\frac{a(a+2bx)^{2+q}\left(ax+bx^2\right)^p\left(1-\frac{(a+2bx)^2}{a^2}\right)^{-p}\text{Hypergeometric2F1}\left(-p,\frac{2+q}{2},\frac{4+q}{2},\frac{(a+2bx)^2}{a^2}\right)}{4b^3(2+q)}$$

output
```
1/2*(2*b*x+a)^(1+q)*(b*x^2+a*x)^(p+1)/b^2/(3+2*p+q)+1/4*a^2*(2+p+q)*(2*b*x
+a)^(1+q)*(b*x^2+a*x)^p*hypergeom([-p, 1/2+1/2*q],[3/2+1/2*q],(2*b*x+a)^2/
a^2)/b^3/(1+q)/(3+2*p+q)/((1-(2*b*x+a)^2/a^2)^p)-1/4*a*(2*b*x+a)^(2+q)*(b*
x^2+a*x)^p*hypergeom([-p, 1+1/2*q],[2+1/2*q],(2*b*x+a)^2/a^2)/b^3/(2+q)/((
1-(2*b*x+a)^2/a^2)^p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.37

$$\int x^2(a+2bx)^q\left(ax+bx^2\right)^p\,dx$$
$$=\frac{x^3\left(\frac{a+bx}{a}\right)^{-p}(x(a+bx))^p(a+2bx)^q\left(\frac{a+2bx}{a}\right)^{-q}\text{AppellF1}\left(3+p,-p,-q,4+p,-\frac{bx}{a},-\frac{2bx}{a}\right)}{3+p}$$

input
```
Integrate[x^2*(a + 2*b*x)^q*(a*x + b*x^2)^p,x]
```

output
```
(x^3*(x*(a + b*x))^p*(a + 2*b*x)^q*AppellF1[3 + p, -p, -q, 4 + p, -((b*x)/
a), (-2*b*x)/a])/((3 + p)*((a + b*x)/a)^p*((a + 2*b*x)/a)^q)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.37, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1261, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(ax+bx^2\right)^p(a+2bx)^q\,dx$$
$$\Big\downarrow \text{1261}$$
$$x^{-p}(a+bx)^{-p}\left(ax+bx^2\right)^p\int x^{p+2}(a+bx)^p(a+2bx)^qdx$$
$$\Big\downarrow \text{152}$$
$$x^{-p}\left(\frac{bx}{a}+1\right)^{-p}\left(ax+bx^2\right)^p\int x^{p+2}(a+2bx)^q\left(\frac{bx}{a}+1\right)^pdx$$
$$\Big\downarrow \text{152}$$

$$x^{-p}\left(\frac{bx}{a}+1\right)^{-p}\left(ax+bx^2\right)^p(a+2bx)^q\left(\frac{2bx}{a}+1\right)^{-q}\int x^{p+2}\left(\frac{bx}{a}+1\right)^p\left(\frac{2bx}{a}+1\right)^q dx$$

$$\Big\downarrow 150$$

$$\frac{x^3\left(\frac{bx}{a}+1\right)^{-p}\left(ax+bx^2\right)^p(a+2bx)^q\left(\frac{2bx}{a}+1\right)^{-q}\operatorname{AppellF1}\left(p+3,-p,-q,p+4,-\frac{bx}{a},-\frac{2bx}{a}\right)}{p+3}$$

input
```
Int[x^2*(a + 2*b*x)^q*(a*x + b*x^2)^p,x]
```

output
```
(x^3*(a + 2*b*x)^q*(a*x + b*x^2)^p*AppellF1[3 + p, -p, -q, 4 + p, -((b*x)/
a), (-2*b*x)/a])/((3 + p)*(1 + (b*x)/a)^p*(1 + (2*b*x)/a)^q)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [F]

$$\int x^2(2bx+a)^q\left(b\,x^2+ax\right)^p dx$$

input
```
int(x^2*(2*b*x+a)^q*(b*x^2+a*x)^p,x)
```

output
```
int(x^2*(2*b*x+a)^q*(b*x^2+a*x)^p,x)
```

Fricas [F]

$$\int x^2(a+2bx)^q\left(ax+bx^2\right)^p\,dx=\int\left(bx^2+ax\right)^p(2\,bx+a)^q x^2\,dx$$

input
```
integrate(x^2*(2*b*x+a)^q*(b*x^2+a*x)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a*x)^p*(2*b*x + a)^q*x^2, x)
```

Sympy [F]

$$\int x^2(a+2bx)^q\left(ax+bx^2\right)^p\,dx=\int x^2(x(a+bx))^p\,(a+2bx)^q\,dx$$

input
```
integrate(x**2*(2*b*x+a)**q*(b*x**2+a*x)**p,x)
```

output
```
Integral(x**2*(x*(a + b*x))**p*(a + 2*b*x)**q, x)
```

Maxima [F]

$$\int x^2(a+2bx)^q\left(ax+bx^2\right)^p\,dx=\int \left(bx^2+ax\right)^p(2\,bx+a)^q x^2\,dx$$

input
```
integrate(x^2*(2*b*x+a)^q*(b*x^2+a*x)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^p*(2*b*x + a)^q*x^2, x)
```

Giac [F]

$$\int x^2(a+2bx)^q\left(ax+bx^2\right)^p\,dx=\int \left(bx^2+ax\right)^p(2\,bx+a)^q x^2\,dx$$

input
```
integrate(x^2*(2*b*x+a)^q*(b*x^2+a*x)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a*x)^p*(2*b*x + a)^q*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+2bx)^q\left(ax+bx^2\right)^p\,dx=\int x^2\left(b\,x^2+a\,x\right)^p\,(a+2\,b\,x)^q\,dx$$

input
```
int(x^2*(a*x + b*x^2)^p*(a + 2*b*x)^q,x)
```

output
```
int(x^2*(a*x + b*x^2)^p*(a + 2*b*x)^q, x)
```

Reduce [F]

$$\int x^2(a+2bx)^q\left(ax+bx^2\right)^p\,dx=\text{too large to display}$$

input
```
int(x^2*(2*b*x+a)^q*(b*x^2+a*x)^p,x)
```

output
```
(2*(a + 2*b*x)**q*(a*x + b*x**2)**p*a**3*p**3 + (a + 2*b*x)**q*(a*x + b*x*
*2)**p*a**3*p**2*q + 6*(a + 2*b*x)**q*(a*x + b*x**2)**p*a**3*p**2 + 2*(a +
 2*b*x)**q*(a*x + b*x**2)**p*a**3*p*q + 4*(a + 2*b*x)**q*(a*x + b*x**2)**p
*a**3*p + (a + 2*b*x)**q*(a*x + b*x**2)**p*a**3*q - 8*(a + 2*b*x)**q*(a*x
+ b*x**2)**p*a**2*b*p**3*x - 8*(a + 2*b*x)**q*(a*x + b*x**2)**p*a**2*b*p**
2*q*x - 16*(a + 2*b*x)**q*(a*x + b*x**2)**p*a**2*b*p**2*x - 2*(a + 2*b*x)*
*q*(a*x + b*x**2)**p*a**2*b*p*q**2*x - 12*(a + 2*b*x)**q*(a*x + b*x**2)**p
*a**2*b*p*q*x - 2*(a + 2*b*x)**q*(a*x + b*x**2)**p*a**2*b*q**2*x + 16*(a +
 2*b*x)**q*(a*x + b*x**2)**p*a*b**2*p**3*x**2 + 24*(a + 2*b*x)**q*(a*x + b
*x**2)**p*a*b**2*p**2*q*x**2 + 8*(a + 2*b*x)**q*(a*x + b*x**2)**p*a*b**2*p
**2*x**2 + 12*(a + 2*b*x)**q*(a*x + b*x**2)**p*a*b**2*p*q**2*x**2 + 8*(a +
 2*b*x)**q*(a*x + b*x**2)**p*a*b**2*p*q*x**2 + 2*(a + 2*b*x)**q*(a*x + b*x
**2)**p*a*b**2*q**3*x**2 + 2*(a + 2*b*x)**q*(a*x + b*x**2)**p*a*b**2*q**2*
x**2 + 32*(a + 2*b*x)**q*(a*x + b*x**2)**p*b**3*p**3*x**3 + 48*(a + 2*b*x)
**q*(a*x + b*x**2)**p*b**3*p**2*q*x**3 + 48*(a + 2*b*x)**q*(a*x + b*x**2)*
*p*b**3*p**2*x**3 + 24*(a + 2*b*x)**q*(a*x + b*x**2)**p*b**3*p*q**2*x**3 +
 48*(a + 2*b*x)**q*(a*x + b*x**2)**p*b**3*p*q*x**3 + 16*(a + 2*b*x)**q*(a*
x + b*x**2)**p*b**3*p*x**3 + 4*(a + 2*b*x)**q*(a*x + b*x**2)**p*b**3*q**3*
x**3 + 12*(a + 2*b*x)**q*(a*x + b*x**2)**p*b**3*q**2*x**3 + 8*(a + 2*b*x)*
*q*(a*x + b*x**2)**p*b**3*q*x**3 + 128*int(((a + 2*b*x)**q*(a*x + b*x**...
```

3.219 $\int x(a+2bx)^q \left(ax+bx^2\right)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 166

$$\int x(a+2bx)^q \left(ax+bx^2\right)^p \, dx = -\frac{a(a+2bx)^{1+q}\left(ax+bx^2\right)^p\left(1-\frac{(a+2bx)^2}{a^2}\right)^{-p}\text{Hypergeometric2F1}\left(-p,\frac{1+q}{2},\frac{3+q}{2},\frac{(a+2bx)^2}{a^2}\right)}{4b^2(1+q)} + \frac{(a+2bx)^{2+q}\left(ax+bx^2\right)^p\left(1-\frac{(a+2bx)^2}{a^2}\right)^{-p}\text{Hypergeometric2F1}\left(-p,\frac{2+q}{2},\frac{4+q}{2},\frac{(a+2bx)^2}{a^2}\right)}{4b^2(2+q)}$$

output

```
-1/4*a*(2*b*x+a)^(1+q)*(b*x^2+a*x)^p*hypergeom([-p, 1/2+1/2*q],[3/2+1/2*q]
,(2*b*x+a)^2/a^2)/b^2/(1+q)/((1-(2*b*x+a)^2/a^2)^p)+1/4*(2*b*x+a)^(2+q)*(b
*x^2+a*x)^p*hypergeom([-p, 1+1/2*q],[2+1/2*q],(2*b*x+a)^2/a^2)/b^2/(2+q)/(
(1-(2*b*x+a)^2/a^2)^p)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.80

$$\int x(a+2bx)^q\left(ax+bx^2\right)^p\,dx$$
$$=\frac{(x(a+bx))^p(a+2bx)^{1+q}\left(1-\frac{(a+2bx)^2}{a^2}\right)^{-p}\left(-a(2+q)\,\text{Hypergeometric2F1}\left(-p,\frac{1+q}{2},\frac{3+q}{2},\frac{(a+2bx)^2}{a^2}\right)+(1\right.}{4b^2(1+q)(2+q)}$$

input
```
Integrate[x*(a + 2*b*x)^q*(a*x + b*x^2)^p,x]
```

output
```
((x*(a + b*x))^p*(a + 2*b*x)^(1 + q)*(-(a*(2 + q)*Hypergeometric2F1[-p, (1
 + q)/2, (3 + q)/2, (a + 2*b*x)^2/a^2]) + (1 + q)*(a + 2*b*x)*Hypergeometr
ic2F1[-p, (2 + q)/2, (4 + q)/2, (a + 2*b*x)^2/a^2]))/(4*b^2*(1 + q)*(2 + q
)*(1 - (a + 2*b*x)^2/a^2)^p)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1269, 1118, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax+bx^2\right)^p(a+2bx)^q\,dx$$
$$\Big\downarrow \text{1269}$$
$$\frac{\int (a+2bx)^{q+1}\left(bx^2+ax\right)^p dx}{2b}-\frac{a\int (a+2bx)^q\left(bx^2+ax\right)^p dx}{2b}$$
$$\Big\downarrow \text{1118}$$
$$\frac{\int (a+2bx)^{q+1}\left(\frac{(a+2bx)^2}{4b}-\frac{a^2}{4b}\right)^p d(a+2bx)}{4b^2}-\frac{a\int (a+2bx)^q\left(\frac{(a+2bx)^2}{4b}-\frac{a^2}{4b}\right)^p d(a+2bx)}{4b^2}$$
$$\Big\downarrow \text{279}$$

$$\frac{\left(1-\frac{(a+2bx)^2}{a^2}\right)^{-p}\left(\frac{(a+2bx)^2}{4b}-\frac{a^2}{4b}\right)^p\int (a+2bx)^{q+1}\left(1-\frac{(a+2bx)^2}{a^2}\right)^p d(a+2bx)}{4b^2}-\frac{a\left(1-\frac{(a+2bx)^2}{a^2}\right)^{-p}\left(\frac{(a+2bx)^2}{4b}-\frac{a^2}{4b}\right)^p\int (a+2bx)^{q}\left(1-\frac{(a+2bx)^2}{a^2}\right)^p d(a+2bx)}{4b^2}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{\left(1-\frac{(a+2bx)^2}{a^2}\right)^{-p}\left(\frac{(a+2bx)^2}{4b}-\frac{a^2}{4b}\right)^p(a+2bx)^{q+2}\,\text{Hypergeometric2F1}\left(-p,\frac{q+2}{2},\frac{q+4}{2},\frac{(a+2bx)^2}{a^2}\right)}{4b^2(q+2)}-\frac{a\left(1-\frac{(a+2bx)^2}{a^2}\right)^{-p}\left(\frac{(a+2bx)^2}{4b}-\frac{a^2}{4b}\right)^p(a+2bx)^{q+1}\,\text{Hypergeometric2F1}\left(-p,\frac{q+1}{2},\frac{q+3}{2},\frac{(a+2bx)^2}{a^2}\right)}{4b^2(q+1)}$$

input
```
Int[x*(a + 2*b*x)^q*(a*x + b*x^2)^p,x]
```

output
```
-1/4*(a*(a + 2*b*x)^(1 + q)*(-1/4*a^2/b + (a + 2*b*x)^2/(4*b))^p*Hypergeom
etric2F1[-p, (1 + q)/2, (3 + q)/2, (a + 2*b*x)^2/a^2])/(b^2*(1 + q)*(1 - (
a + 2*b*x)^2/a^2)^p) + ((a + 2*b*x)^(2 + q)*(-1/4*a^2/b + (a + 2*b*x)^2/(4
*b))^p*Hypergeometric2F1[-p, (2 + q)/2, (4 + q)/2, (a + 2*b*x)^2/a^2])/(4*
b^2*(2 + q)*(1 - (a + 2*b*x)^2/a^2)^p)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1118
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[1/e   Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x,
d + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [F]

$$\int x(2bx+a)^{q} \left(b\,x^{2}+ax\right)^{p}dx$$

input
```
int(x*(2*b*x+a)^q*(b*x^2+a*x)^p,x)
```

output
```
int(x*(2*b*x+a)^q*(b*x^2+a*x)^p,x)
```

Fricas [F]

$$\int x(a+2bx)^q \left(ax+bx^2\right)^p \, dx = \int (bx^{2}+ax)^{p}(2\,bx+a)^{q}x\,dx$$

input
```
integrate(x*(2*b*x+a)^q*(b*x^2+a*x)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a*x)^p*(2*b*x + a)^q*x, x)
```

Sympy [F]

$$\int x(a+2bx)^q \left(ax+bx^2\right)^p \, dx = \int x(x(a+bx))^p (a+2bx)^q \, dx$$

input
```
integrate(x*(2*b*x+a)**q*(b*x**2+a*x)**p,x)
```

output
```
Integral(x*(x*(a + b*x))**p*(a + 2*b*x)**q, x)
```

Maxima [F]

$$\int x(a+2bx)^q \left(ax+bx^2\right)^p \, dx = \int (bx^2+ax)^p (2\,bx+a)^q x \, dx$$

input
```
integrate(x*(2*b*x+a)^q*(b*x^2+a*x)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^p*(2*b*x + a)^q*x, x)
```

Giac [F]

$$\int x(a+2bx)^q \left(ax+bx^2\right)^p \, dx = \int (bx^2+ax)^p (2\,bx+a)^q x \, dx$$

input
```
integrate(x*(2*b*x+a)^q*(b*x^2+a*x)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a*x)^p*(2*b*x + a)^q*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+2bx)^q \left(ax+bx^2\right)^p \, dx = \int x \left(b\,x^2+a\,x\right)^p \left(a+2\,b\,x\right)^q \,dx$$

input
```
int(x*(a*x + b*x^2)^p*(a + 2*b*x)^q,x)
```

output
```
int(x*(a*x + b*x^2)^p*(a + 2*b*x)^q, x)
```

Reduce [F]

$$\int x(a+2bx)^q \left(ax+bx^2\right)^p \, dx = \text{too large to display}$$

input
```
int(x*(2*b*x+a)^q*(b*x^2+a*x)^p,x)
```

output
```
( - (a + 2*b*x)**q*(a*x + b*x**2)**p*a**2*p - (a + 2*b*x)**q*(a*x + b*x**2
)**p*a**2 + 4*(a + 2*b*x)**q*(a*x + b*x**2)**p*a*b*p*x + 2*(a + 2*b*x)**q*
(a*x + b*x**2)**p*a*b*q*x + 8*(a + 2*b*x)**q*(a*x + b*x**2)**p*b**2*p*x**2
 + 4*(a + 2*b*x)**q*(a*x + b*x**2)**p*b**2*q*x**2 + 4*(a + 2*b*x)**q*(a*x
+ b*x**2)**p*b**2*x**2 - 16*int(((a + 2*b*x)**q*(a*x + b*x**2)**p*x)/(4*a*
*2*p**2 + 4*a**2*p*q + 6*a**2*p + a**2*q**2 + 3*a**2*q + 2*a**2 + 12*a*b*p
**2*x + 12*a*b*p*q*x + 18*a*b*p*x + 3*a*b*q**2*x + 9*a*b*q*x + 6*a*b*x + 8
*b**2*p**2*x**2 + 8*b**2*p*q*x**2 + 12*b**2*p*x**2 + 2*b**2*q**2*x**2 + 6*
b**2*q*x**2 + 4*b**2*x**2),x)*a**2*b**2*p**4 - 24*int(((a + 2*b*x)**q*(a*x
 + b*x**2)**p*x)/(4*a**2*p**2 + 4*a**2*p*q + 6*a**2*p + a**2*q**2 + 3*a**2
*q + 2*a**2 + 12*a*b*p**2*x + 12*a*b*p*q*x + 18*a*b*p*x + 3*a*b*q**2*x + 9
*a*b*q*x + 6*a*b*x + 8*b**2*p**2*x**2 + 8*b**2*p*q*x**2 + 12*b**2*p*x**2 +
 2*b**2*q**2*x**2 + 6*b**2*q*x**2 + 4*b**2*x**2),x)*a**2*b**2*p**3*q - 40*
int(((a + 2*b*x)**q*(a*x + b*x**2)**p*x)/(4*a**2*p**2 + 4*a**2*p*q + 6*a**
2*p + a**2*q**2 + 3*a**2*q + 2*a**2 + 12*a*b*p**2*x + 12*a*b*p*q*x + 18*a*
b*p*x + 3*a*b*q**2*x + 9*a*b*q*x + 6*a*b*x + 8*b**2*p**2*x**2 + 8*b**2*p*q
*x**2 + 12*b**2*p*x**2 + 2*b**2*q**2*x**2 + 6*b**2*q*x**2 + 4*b**2*x**2),x
)*a**2*b**2*p**3 - 12*int(((a + 2*b*x)**q*(a*x + b*x**2)**p*x)/(4*a**2*p**
2 + 4*a**2*p*q + 6*a**2*p + a**2*q**2 + 3*a**2*q + 2*a**2 + 12*a*b*p**2*x
+ 12*a*b*p*q*x + 18*a*b*p*x + 3*a*b*q**2*x + 9*a*b*q*x + 6*a*b*x + 8*b*...
```

3.220 $\int (a+2bx)^q \left(ax+bx^2\right)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 82

$$\int (a+2bx)^q \left(ax+bx^2\right)^p \, dx = \frac{(a+2bx)^{1+q} \left(ax+bx^2\right)^p \left(1-\frac{(a+2bx)^2}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(-p,\frac{1+q}{2},\frac{3+q}{2},\frac{(a+2bx)^2}{a^2}\right)}{2b(1+q)}$$

output
```
1/2*(2*b*x+a)^(1+q)*(b*x^2+a*x)^p*hypergeom([-p, 1/2+1/2*q],[3/2+1/2*q],(2
*b*x+a)^2/a^2)/b/(1+q)/((1-(2*b*x+a)^2/a^2)^p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int (a+2bx)^q \left(ax+bx^2\right)^p \, dx = \frac{x\left(\frac{a+bx}{a}\right)^{-p} (x(a+bx))^p (a+2bx)^q \left(\frac{a+2bx}{a}\right)^{-q} \text{AppellF1}\left(1+p,-p,-q,2+p,-\frac{bx}{a},-\frac{2bx}{a}\right)}{1+p}$$

input
```
Integrate[(a + 2*b*x)^q*(a*x + b*x^2)^p,x]
```

output
```
(x*(x*(a + b*x))^p*(a + 2*b*x)^q*AppellF1[1 + p, -p, -q, 2 + p, -((b*x)/a)
, (-2*b*x)/a])/((1 + p)*((a + b*x)/a)^p*((a + 2*b*x)/a)^q)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1118, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^2\right)^p (a+2bx)^q \, dx$$

$$\Big\downarrow \text{ 1118}$$

$$\frac{\int (a+2bx)^q \left(\frac{(a+2bx)^2}{4b}-\frac{a^2}{4b}\right)^p d(a+2bx)}{2b}$$

$$\Big\downarrow \text{ 279}$$

$$\frac{\left(1-\frac{(a+2bx)^2}{a^2}\right)^{-p} \left(\frac{(a+2bx)^2}{4b}-\frac{a^2}{4b}\right)^p \int (a+2bx)^q \left(1-\frac{(a+2bx)^2}{a^2}\right)^p d(a+2bx)}{2b}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{\left(1-\frac{(a+2bx)^2}{a^2}\right)^{-p} \left(\frac{(a+2bx)^2}{4b}-\frac{a^2}{4b}\right)^p (a+2bx)^{q+1} \text{Hypergeometric2F1}\left(-p,\frac{q+1}{2},\frac{q+3}{2},\frac{(a+2bx)^2}{a^2}\right)}{2b(q+1)}$$

input
```
Int[(a + 2*b*x)^q*(a*x + b*x^2)^p,x]
```

output
```
((a + 2*b*x)^(1 + q)*(-1/4*a^2/b + (a + 2*b*x)^2/(4*b))^p*Hypergeometric2F
1[-p, (1 + q)/2, (3 + q)/2, (a + 2*b*x)^2/a^2])/(2*b*(1 + q)*(1 - (a + 2*b
*x)^2/a^2)^p)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1118
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[1/e   Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x,
d + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[2*c*d - b*e, 0]
```

Maple [F]

$$\int (2bx+a)^q \left(b\,x^2+ax\right)^p dx$$

input
```
int((2*b*x+a)^q*(b*x^2+a*x)^p,x)
```

output
```
int((2*b*x+a)^q*(b*x^2+a*x)^p,x)
```

Fricas [F]

$$\int (a+2bx)^q \left(ax+bx^2\right)^p \, dx = \int \left(bx^2+ax\right)^p (2\,bx+a)^q \, dx$$

input
```
integrate((2*b*x+a)^q*(b*x^2+a*x)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a*x)^p*(2*b*x + a)^q, x)
```

Sympy [F]

$$\int (a+2bx)^q\,(ax+bx^2)^p\,dx = \int (x(a+bx))^p\,(a+2bx)^q\,dx$$

input
```
integrate((2*b*x+a)**q*(b*x**2+a*x)**p,x)
```

output
```
Integral((x*(a + b*x))**p*(a + 2*b*x)**q, x)
```

Maxima [F]

$$\int (a+2bx)^q\,(ax+bx^2)^p\,dx = \int (bx^2+ax)^p(2\,bx+a)^q\,dx$$

input
```
integrate((2*b*x+a)^q*(b*x^2+a*x)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^p*(2*b*x + a)^q, x)
```

Giac [F]

$$\int (a+2bx)^q\,(ax+bx^2)^p\,dx = \int (bx^2+ax)^p(2\,bx+a)^q\,dx$$

input
```
integrate((2*b*x+a)^q*(b*x^2+a*x)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a*x)^p*(2*b*x + a)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+2bx)^q \left(ax+bx^2\right)^p \, dx = \int \left(b\,x^2+a\,x\right)^p \,(a+2\,b\,x)^q \, dx$$

input
```
int((a*x + b*x^2)^p*(a + 2*b*x)^q,x)
```

output
```
int((a*x + b*x^2)^p*(a + 2*b*x)^q, x)
```

Reduce [F]

$$\int (a+2bx)^q \left(ax+bx^2\right)^p \, dx = \text{too large to display}$$

input
```
int((2*b*x+a)^q*(b*x^2+a*x)^p,x)
```

output

```
((a + 2*b*x)**q*(a*x + b*x**2)**p*a*p + (a + 2*b*x)**q*(a*x + b*x**2)**p*a
*q + 4*(a + 2*b*x)**q*(a*x + b*x**2)**p*b*p*x + 2*(a + 2*b*x)**q*(a*x + b*
x**2)**p*b*q*x + 16*int(((a + 2*b*x)**q*(a*x + b*x**2)**p*x)/(4*a**2*p**2
+ 4*a**2*p*q + 2*a**2*p + a**2*q**2 + a**2*q + 12*a*b*p**2*x + 12*a*b*p*q*
x + 6*a*b*p*x + 3*a*b*q**2*x + 3*a*b*q*x + 8*b**2*p**2*x**2 + 8*b**2*p*q*x
**2 + 4*b**2*p*x**2 + 2*b**2*q**2*x**2 + 2*b**2*q*x**2),x)*a*b**2*p**4 + 2
4*int(((a + 2*b*x)**q*(a*x + b*x**2)**p*x)/(4*a**2*p**2 + 4*a**2*p*q + 2*a
**2*p + a**2*q**2 + a**2*q + 12*a*b*p**2*x + 12*a*b*p*q*x + 6*a*b*p*x + 3*
a*b*q**2*x + 3*a*b*q*x + 8*b**2*p**2*x**2 + 8*b**2*p*q*x**2 + 4*b**2*p*x**
2 + 2*b**2*q**2*x**2 + 2*b**2*q*x**2),x)*a*b**2*p**3*q + 8*int(((a + 2*b*x
)**q*(a*x + b*x**2)**p*x)/(4*a**2*p**2 + 4*a**2*p*q + 2*a**2*p + a**2*q**2
 + a**2*q + 12*a*b*p**2*x + 12*a*b*p*q*x + 6*a*b*p*x + 3*a*b*q**2*x + 3*a*
b*q*x + 8*b**2*p**2*x**2 + 8*b**2*p*q*x**2 + 4*b**2*p*x**2 + 2*b**2*q**2*x
**2 + 2*b**2*q*x**2),x)*a*b**2*p**3 + 12*int(((a + 2*b*x)**q*(a*x + b*x**2
)**p*x)/(4*a**2*p**2 + 4*a**2*p*q + 2*a**2*p + a**2*q**2 + a**2*q + 12*a*b
*p**2*x + 12*a*b*p*q*x + 6*a*b*p*x + 3*a*b*q**2*x + 3*a*b*q*x + 8*b**2*p**
2*x**2 + 8*b**2*p*q*x**2 + 4*b**2*p*x**2 + 2*b**2*q**2*x**2 + 2*b**2*q*x**
2),x)*a*b**2*p**2*q**2 + 8*int(((a + 2*b*x)**q*(a*x + b*x**2)**p*x)/(4*a**
2*p**2 + 4*a**2*p*q + 2*a**2*p + a**2*q**2 + a**2*q + 12*a*b*p**2*x + 12*a
*b*p*q*x + 6*a*b*p*x + 3*a*b*q**2*x + 3*a*b*q*x + 8*b**2*p**2*x**2 + 8*...
```

3.221 $\int \frac{(a+2bx)^q(ax+bx^2)^p}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 165

$$\int \frac{(a+2bx)^q\,(ax+bx^2)^p}{x}\,dx = -\frac{(a+2bx)^{1+q}\,(ax+bx^2)^p\left(1-\frac{(a+2bx)^2}{a^2}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,\frac{1+q}{2},\frac{3+q}{2},\frac{(a+2bx)^2}{a^2}\right)}{a(1+q)} - \frac{(a+2bx)^{2+q}\,(ax+bx^2)^p\left(1-\frac{(a+2bx)^2}{a^2}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,\frac{2+q}{2},\frac{4+q}{2},\frac{(a+2bx)^2}{a^2}\right)}{a^2(2+q)}$$

output
```
-(2*b*x+a)^(1+q)*(b*x^2+a*x)^p*hypergeom([1-p, 1/2+1/2*q],[3/2+1/2*q],(2*b
*x+a)^2/a^2)/a/(1+q)/((1-(2*b*x+a)^2/a^2)^p)-(2*b*x+a)^(2+q)*(b*x^2+a*x)^p
*hypergeom([1-p, 1+1/2*q],[2+1/2*q],(2*b*x+a)^2/a^2)/a^2/(2+q)/((1-(2*b*x+
a)^2/a^2)^p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.44

$$\int \frac{(a+2bx)^q (ax+bx^2)^p}{x} \, dx = \frac{\left(\frac{a+bx}{a}\right)^{-p} (x(a+bx))^p (a+2bx)^q \left(\frac{a+2bx}{a}\right)^{-q} \text{AppellF1}\left(p,-p,-q,1+p,-\frac{bx}{a},-\frac{2bx}{a}\right)}{p}$$

input
```
Integrate[((a + 2*b*x)^q*(a*x + b*x^2)^p)/x,x]
```

output
```
((x*(a + b*x))^p*(a + 2*b*x)^q*AppellF1[p, -p, -q, 1 + p, -((b*x)/a), (-2*
b*x)/a])/(p*((a + b*x)/a)^p*((a + 2*b*x)/a)^q)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1261, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^p (a+2bx)^q}{x} \, dx$$

$$\Big\downarrow \; 1261$$

$$x^{-p}(a+bx)^{-p} (ax+bx^2)^p \int x^{p-1}(a+bx)^p(a+2bx)^q dx$$

$$\Big\downarrow \; 152$$

$$x^{-p}\left(\frac{bx}{a}+1\right)^{-p} (ax+bx^2)^p \int x^{p-1}(a+2bx)^q \left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \; 152$$

$$x^{-p}\left(\frac{bx}{a}+1\right)^{-p}\left(ax+bx^2\right)^p(a+2bx)^q\left(\frac{2bx}{a}+1\right)^{-q}\int x^{p-1}\left(\frac{bx}{a}+1\right)^p\left(\frac{2bx}{a}+1\right)^q dx$$

$$\Big\downarrow \, 150$$

$$\frac{\left(\frac{bx}{a}+1\right)^{-p}\left(ax+bx^2\right)^p(a+2bx)^q\left(\frac{2bx}{a}+1\right)^{-q}\text{AppellF1}\left(p,-p,-q,p+1,-\frac{bx}{a},-\frac{2bx}{a}\right)}{p}$$

input
```
Int[((a + 2*b*x)^q*(a*x + b*x^2)^p)/x,x]
```

output
```
((a + 2*b*x)^q*(a*x + b*x^2)^p*AppellF1[p, -p, -q, 1 + p, -((b*x)/a), (-2*
b*x)/a])/(p*(1 + (b*x)/a)^p*(1 + (2*b*x)/a)^q)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [F]

$$\int \frac{(2bx+a)^q\,(b\,x^2+ax)^p}{x}dx$$

input
```
int((2*b*x+a)^q*(b*x^2+a*x)^p/x,x)
```

output
```
int((2*b*x+a)^q*(b*x^2+a*x)^p/x,x)
```

Fricas [F]

$$\int \frac{(a+2bx)^q\,(ax+bx^2)^p}{x}\,dx = \int \frac{(bx^2+ax)^p(2\,bx+a)^q}{x}\,dx$$

input
```
integrate((2*b*x+a)^q*(b*x^2+a*x)^p/x,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a*x)^p*(2*b*x + a)^q/x, x)
```

Sympy [F]

$$\int \frac{(a+2bx)^q\,(ax+bx^2)^p}{x}\,dx = \int \frac{(x(a+bx))^p\,(a+2bx)^q}{x}\,dx$$

input
```
integrate((2*b*x+a)**q*(b*x**2+a*x)**p/x,x)
```

output
```
Integral((x*(a + b*x))**p*(a + 2*b*x)**q/x, x)
```

Maxima [F]

$$\int \frac{(a+2bx)^q (ax+bx^2)^p}{x}\,dx = \int \frac{(bx^2+ax)^p (2\,bx+a)^q}{x}\,dx$$

input
```
integrate((2*b*x+a)^q*(b*x^2+a*x)^p/x,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^p*(2*b*x + a)^q/x, x)
```

Giac [F]

$$\int \frac{(a+2bx)^q (ax+bx^2)^p}{x}\,dx = \int \frac{(bx^2+ax)^p (2\,bx+a)^q}{x}\,dx$$

input
```
integrate((2*b*x+a)^q*(b*x^2+a*x)^p/x,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a*x)^p*(2*b*x + a)^q/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+2bx)^q (ax+bx^2)^p}{x}\,dx = \int \frac{(b\,x^2+a\,x)^p\,(a+2\,b\,x)^q}{x}\,dx$$

input
```
int(((a*x + b*x^2)^p*(a + 2*b*x)^q)/x,x)
```

output
```
int(((a*x + b*x^2)^p*(a + 2*b*x)^q)/x, x)
```

Reduce [F]

$$\int \frac{(a+2bx)^q (ax+bx^2)^p}{x}\,dx$$
$$= \frac{3(2bx+a)^q (b\,x^2+ax)^p - 8\left(\int \frac{(2bx+a)^q (b\,x^2+ax)^p x}{4b^2p\,x^2+2b^2q\,x^2+6abpx+3abqx+2a^2p+a^2q}dx\right) b^2p^2 - 8\left(\int \frac{(2bx+a)^q (b\,x^2+ax)^p}{4b^2p\,x^2+2b^2q\,x^2+6abpx+3abqx}\right.}{}$$

input
```
int((2*b*x+a)^q*(b*x^2+a*x)^p/x,x)
```

output
```
(3*(a + 2*b*x)**q*(a*x + b*x**2)**p - 8*int(((a + 2*b*x)**q*(a*x + b*x**2)
**p*x)/(2*a**2*p + a**2*q + 6*a*b*p*x + 3*a*b*q*x + 4*b**2*p*x**2 + 2*b**2
*q*x**2),x)*b**2*p**2 - 8*int(((a + 2*b*x)**q*(a*x + b*x**2)**p*x)/(2*a**2
*p + a**2*q + 6*a*b*p*x + 3*a*b*q*x + 4*b**2*p*x**2 + 2*b**2*q*x**2),x)*b*
*2*p*q - 2*int(((a + 2*b*x)**q*(a*x + b*x**2)**p*x)/(2*a**2*p + a**2*q + 6
*a*b*p*x + 3*a*b*q*x + 4*b**2*p*x**2 + 2*b**2*q*x**2),x)*b**2*q**2 + 2*int
(((a + 2*b*x)**q*(a*x + b*x**2)**p)/(2*a**2*p*x + a**2*q*x + 6*a*b*p*x**2
+ 3*a*b*q*x**2 + 4*b**2*p*x**3 + 2*b**2*q*x**3),x)*a**2*p**2 + 5*int(((a +
 2*b*x)**q*(a*x + b*x**2)**p)/(2*a**2*p*x + a**2*q*x + 6*a*b*p*x**2 + 3*a*
b*q*x**2 + 4*b**2*p*x**3 + 2*b**2*q*x**3),x)*a**2*p*q + 2*int(((a + 2*b*x)
**q*(a*x + b*x**2)**p)/(2*a**2*p*x + a**2*q*x + 6*a*b*p*x**2 + 3*a*b*q*x**
2 + 4*b**2*p*x**3 + 2*b**2*q*x**3),x)*a**2*q**2)/(2*(2*p + q))
```

3.222 $\int \frac{(a+2bx)^q (ax+bx^2)^p}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 217

$$\int \frac{(a+2bx)^q (ax+bx^2)^p}{x^2}\,dx = -\frac{(a+2bx)^{1+q}(ax+bx^2)^{-1+p}}{2(1-2p-q)} - \frac{4b(p+q)(a+2bx)^{1+q}(ax+bx^2)^p\left(1-\frac{(a+2bx)^2}{a^2}\right)^{-p}\text{Hypergeometric2F1}\left(2-p,\frac{1+q}{2},\frac{3+q}{2},\frac{(a+2bx)^2}{a^2}\right)}{a^2(1-2p-q)(1+q)} + \frac{4b(a+2bx)^{2+q}(ax+bx^2)^p\left(1-\frac{(a+2bx)^2}{a^2}\right)^{-p}\text{Hypergeometric2F1}\left(2-p,\frac{2+q}{2},\frac{4+q}{2},\frac{(a+2bx)^2}{a^2}\right)}{a^3(2+q)}$$

output

```
-1/2*(2*b*x+a)^(1+q)*(b*x^2+a*x)^(-1+p)/(1-2*p-q)-4*b*(p+q)*(2*b*x+a)^(1+q
)*(b*x^2+a*x)^p*hypergeom([2-p, 1/2+1/2*q],[3/2+1/2*q],(2*b*x+a)^2/a^2)/a^
2/(1-2*p-q)/(1+q)/((1-(2*b*x+a)^2/a^2)^p)+4*b*(2*b*x+a)^(2+q)*(b*x^2+a*x)^
p*hypergeom([2-p, 1+1/2*q],[2+1/2*q],(2*b*x+a)^2/a^2)/a^3/(2+q)/((1-(2*b*x
+a)^2/a^2)^p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.36

$$\int \frac{(a+2bx)^q (ax+bx^2)^p}{x^2} dx$$
$$= \frac{\left(\frac{a+bx}{a}\right)^{-p} (x(a+bx))^p (a+2bx)^q \left(\frac{a+2bx}{a}\right)^{-q} \text{AppellF1}\left(-1+p,-p,-q,p,-\frac{bx}{a},-\frac{2bx}{a}\right)}{(-1+p)x}$$

input
```
Integrate[((a + 2*b*x)^q*(a*x + b*x^2)^p)/x^2,x]
```

output
```
((x*(a + b*x))^p*(a + 2*b*x)^q*AppellF1[-1 + p, -p, -q, p, -((b*x)/a), (-2
*b*x)/a])/((-1 + p)*x*((a + b*x)/a)^p*((a + 2*b*x)/a)^q)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.40 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.37, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1261, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^p (a+2bx)^q}{x^2} dx$$
$$\Big\downarrow \text{1261}$$
$$x^{-p}(a+bx)^{-p} (ax+bx^2)^p \int x^{p-2}(a+bx)^p(a+2bx)^q dx$$
$$\Big\downarrow \text{152}$$
$$x^{-p}\left(\frac{bx}{a}+1\right)^{-p} (ax+bx^2)^p \int x^{p-2}(a+2bx)^q \left(\frac{bx}{a}+1\right)^p dx$$
$$\Big\downarrow \text{152}$$

$$x^{-p}\left(\frac{bx}{a}+1\right)^{-p}\left(ax+bx^2\right)^p(a+2bx)^q\left(\frac{2bx}{a}+1\right)^{-q}\int x^{p-2}\left(\frac{bx}{a}+1\right)^p\left(\frac{2bx}{a}+1\right)^q dx$$

$$\Big\downarrow \text{150}$$

$$-\frac{\left(\frac{bx}{a}+1\right)^{-p}\left(ax+bx^2\right)^p(a+2bx)^q\left(\frac{2bx}{a}+1\right)^{-q}\text{AppellF1}\left(p-1,-p,-q,p,-\frac{bx}{a},-\frac{2bx}{a}\right)}{(1-p)x}$$

input
```
Int[((a + 2*b*x)^q*(a*x + b*x^2)^p)/x^2,x]
```

output
```
-(((a + 2*b*x)^q*(a*x + b*x^2)^p*AppellF1[-1 + p, -p, -q, p, -((b*x)/a), (
-2*b*x)/a])/((1 - p)*x*(1 + (b*x)/a)^p*(1 + (2*b*x)/a)^q))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 1261
```
Int[((e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((b_.)*(x_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Simp[(e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))
  Int[x^(m + p)*(f + g*x)^n*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, f, g, m,
 n}, x] &&  !IGtQ[n, 0]
```

Maple [F]

$$\int \frac{(2bx+a)^q\,(b\,x^2+ax)^p}{x^2}dx$$

input
```
int((2*b*x+a)^q*(b*x^2+a*x)^p/x^2,x)
```

output
```
int((2*b*x+a)^q*(b*x^2+a*x)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(a+2bx)^q\,(ax+bx^2)^p}{x^2}\,dx = \int \frac{(bx^2+ax)^p(2\,bx+a)^q}{x^2}\,dx$$

input
```
integrate((2*b*x+a)^q*(b*x^2+a*x)^p/x^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a*x)^p*(2*b*x + a)^q/x^2, x)
```

Sympy [F]

$$\int \frac{(a+2bx)^q\,(ax+bx^2)^p}{x^2}\,dx = \int \frac{(x(a+bx))^p\,(a+2bx)^q}{x^2}\,dx$$

input
```
integrate((2*b*x+a)**q*(b*x**2+a*x)**p/x**2,x)
```

output
```
Integral((x*(a + b*x))**p*(a + 2*b*x)**q/x**2, x)
```

Maxima [F]

$$\int \frac{(a+2bx)^q (ax+bx^2)^p}{x^2} dx = \int \frac{(bx^2+ax)^p (2bx+a)^q}{x^2} dx$$

input
```
integrate((2*b*x+a)^q*(b*x^2+a*x)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^p*(2*b*x + a)^q/x^2, x)
```

Giac [F]

$$\int \frac{(a+2bx)^q (ax+bx^2)^p}{x^2} dx = \int \frac{(bx^2+ax)^p (2bx+a)^q}{x^2} dx$$

input
```
integrate((2*b*x+a)^q*(b*x^2+a*x)^p/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a*x)^p*(2*b*x + a)^q/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+2bx)^q (ax+bx^2)^p}{x^2} dx = \int \frac{(bx^2+ax)^p (a+2bx)^q}{x^2} dx$$

input
```
int(((a*x + b*x^2)^p*(a + 2*b*x)^q)/x^2,x)
```

output
```
int(((a*x + b*x^2)^p*(a + 2*b*x)^q)/x^2, x)
```

Reduce [F]

$$\int \frac{(a+2bx)^q\,(ax+bx^2)^p}{x^2}\,dx = \text{too large to display}$$

input
```
int((2*b*x+a)^q*(b*x^2+a*x)^p/x^2,x)
```

output
```
((a + 2*b*x)**q*(a*x + b*x**2)**p - 4*int(((a + 2*b*x)**q*(a*x + b*x**2)**
p)/(4*a**2*p**2*x + 2*a**2*p*q*x - 7*a**2*p*x - 2*a**2*q*x + 3*a**2*x + 12
*a*b*p**2*x**2 + 6*a*b*p*q*x**2 - 21*a*b*p*x**2 - 6*a*b*q*x**2 + 9*a*b*x**
2 + 8*b**2*p**2*x**3 + 4*b**2*p*q*x**3 - 14*b**2*p*x**3 - 4*b**2*q*x**3 +
6*b**2*x**3),x)*a*b*p**3*x - 10*int(((a + 2*b*x)**q*(a*x + b*x**2)**p)/(4*
a**2*p**2*x + 2*a**2*p*q*x - 7*a**2*p*x - 2*a**2*q*x + 3*a**2*x + 12*a*b*p
**2*x**2 + 6*a*b*p*q*x**2 - 21*a*b*p*x**2 - 6*a*b*q*x**2 + 9*a*b*x**2 + 8*
b**2*p**2*x**3 + 4*b**2*p*q*x**3 - 14*b**2*p*x**3 - 4*b**2*q*x**3 + 6*b**2
*x**3),x)*a*b*p**2*q*x + 7*int(((a + 2*b*x)**q*(a*x + b*x**2)**p)/(4*a**2*
p**2*x + 2*a**2*p*q*x - 7*a**2*p*x - 2*a**2*q*x + 3*a**2*x + 12*a*b*p**2*x
**2 + 6*a*b*p*q*x**2 - 21*a*b*p*x**2 - 6*a*b*q*x**2 + 9*a*b*x**2 + 8*b**2*
p**2*x**3 + 4*b**2*p*q*x**3 - 14*b**2*p*x**3 - 4*b**2*q*x**3 + 6*b**2*x**3
),x)*a*b*p**2*x - 4*int(((a + 2*b*x)**q*(a*x + b*x**2)**p)/(4*a**2*p**2*x
+ 2*a**2*p*q*x - 7*a**2*p*x - 2*a**2*q*x + 3*a**2*x + 12*a*b*p**2*x**2 + 6
*a*b*p*q*x**2 - 21*a*b*p*x**2 - 6*a*b*q*x**2 + 9*a*b*x**2 + 8*b**2*p**2*x*
*3 + 4*b**2*p*q*x**3 - 14*b**2*p*x**3 - 4*b**2*q*x**3 + 6*b**2*x**3),x)*a*
b*p*q**2*x + 16*int(((a + 2*b*x)**q*(a*x + b*x**2)**p)/(4*a**2*p**2*x + 2*
a**2*p*q*x - 7*a**2*p*x - 2*a**2*q*x + 3*a**2*x + 12*a*b*p**2*x**2 + 6*a*b
*p*q*x**2 - 21*a*b*p*x**2 - 6*a*b*q*x**2 + 9*a*b*x**2 + 8*b**2*p**2*x**3 +
 4*b**2*p*q*x**3 - 14*b**2*p*x**3 - 4*b**2*q*x**3 + 6*b**2*x**3),x)*a*b...
```

3.223 $\int (3-x)^q x (6x-x^2)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 100

$$\int (3-x)^q x \left(6x-x^2\right)^p \, dx = -\frac{3^{1+2p}(3-x)^{1+q} \text{Hypergeometric2F1}\left(-p,\frac{1+q}{2},\frac{3+q}{2},\frac{1}{9}(3-x)^2\right)}{1+q} + \frac{9^p(3-x)^{2+q} \text{Hypergeometric2F1}\left(-p,\frac{2+q}{2},\frac{4+q}{2},\frac{1}{9}(3-x)^2\right)}{2+q}$$

output
```
-3^(1+2*p)*(3-x)^(1+q)*hypergeom([-p, 1/2+1/2*q],[3/2+1/2*q],1/9*(3-x)^2)/
(1+q)+9^p*(3-x)^(2+q)*hypergeom([-p, 1+1/2*q],[2+1/2*q],1/9*(3-x)^2)/(2+q)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.57

$$\int (3-x)^q x \left(6x-x^2\right)^p \, dx = \frac{2^p 3^{p+q}(6-x)^{-p} x^2 (-((-6+x)x))^p \text{AppellF1}\left(2+p,-q,-p,3+p,\frac{x}{3},\frac{x}{6}\right)}{2+p}$$

input
```
Integrate[(3 - x)^q*x*(6*x - x^2)^p,x]
```

output
```
(2^p*3^(p + q)*x^2*(-((-6 + x)*x))^p*AppellF1[2 + p, -q, -p, 3 + p, x/3, x
/6])/((2 + p)*(6 - x)^p)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1269, 1118, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(6x-x^2)^p (3-x)^q \, dx$$

$$\Big\downarrow \text{1269}$$

$$3\int (3-x)^q (6x-x^2)^p \, dx - \int (3-x)^{q+1} (6x-x^2)^p \, dx$$

$$\Big\downarrow \text{1118}$$

$$\int (9-(3-x)^2)^p (3-x)^{q+1} d(3-x) - 3\int (9-(3-x)^2)^p (3-x)^q d(3-x)$$

$$\Big\downarrow \text{278}$$

$$\frac{9^p(3-x)^{q+2} \text{Hypergeometric2F1}\left(-p,\frac{q+2}{2},\frac{q+4}{2},\frac{1}{9}(3-x)^2\right)}{q+2} - \frac{3^{2p+1}(3-x)^{q+1} \text{Hypergeometric2F1}\left(-p,\frac{q+1}{2},\frac{q+3}{2},\frac{1}{9}(3-x)^2\right)}{q+1}$$

input
```
Int[(3 - x)^q*x*(6*x - x^2)^p,x]
```

output
```
-((3^(1 + 2*p)*(3 - x)^(1 + q)*Hypergeometric2F1[-p, (1 + q)/2, (3 + q)/2,
 (3 - x)^2/9])/(1 + q)) + (9^p*(3 - x)^(2 + q)*Hypergeometric2F1[-p, (2 +
q)/2, (4 + q)/2, (3 - x)^2/9])/(2 + q)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 1118
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[1/e   Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x,
d + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [F]

$$\int (3-x)^q x(-x^2+6x)^p\,dx$$

input
```
int((3-x)^q*x*(-x^2+6*x)^p,x)
```

output
```
int((3-x)^q*x*(-x^2+6*x)^p,x)
```

Fricas [F]

$$\int (3-x)^q x(6x-x^2)^p\,dx = \int (-x^2+6\,x)^p x(-x+3)^q\,dx$$

input
```
integrate((3-x)^q*x*(-x^2+6*x)^p,x, algorithm="fricas")
```

output
```
integral((-x^2 + 6*x)^p*x*(-x + 3)^q, x)
```

Sympy [F]

$$\int (3-x)^q x (6x-x^2)^p \, dx = \int x(-x(x-6))^p (3-x)^q \, dx$$

input
```
integrate((3-x)**q*x*(-x**2+6*x)**p,x)
```

output
```
Integral(x*(-x*(x - 6))**p*(3 - x)**q, x)
```

Maxima [F]

$$\int (3-x)^q x (6x-x^2)^p \, dx = \int (-x^2+6x)^p x(-x+3)^q \, dx$$

input
```
integrate((3-x)^q*x*(-x^2+6*x)^p,x, algorithm="maxima")
```

output
```
integrate((-x^2 + 6*x)^p*x*(-x + 3)^q, x)
```

Giac [F]

$$\int (3-x)^q x (6x-x^2)^p \, dx = \int (-x^2+6x)^p x(-x+3)^q \, dx$$

input
```
integrate((3-x)^q*x*(-x^2+6*x)^p,x, algorithm="giac")
```

output
```
integrate((-x^2 + 6*x)^p*x*(-x + 3)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int (3-x)^q x(6x-x^2)^p \, dx = \int x\,(6\,x-x^2)^p\,(3-x)^q \, dx$$

input
```
int(x*(6*x - x^2)^p*(3 - x)^q,x)
```

output
```
int(x*(6*x - x^2)^p*(3 - x)^q, x)
```

Reduce [F]

$$\int (3-x)^q x(6x-x^2)^p \, dx = \text{too large to display}$$

input
```
int((3-x)^q*x*(-x^2+6*x)^p,x)
```

output

```
(2*( - x + 3)**q*( - x**2 + 6*x)**p*p*x**2 - 6*( - x + 3)**q*( - x**2 + 6*
x)**p*p*x - 9*( - x + 3)**q*( - x**2 + 6*x)**p*p + ( - x + 3)**q*( - x**2
+ 6*x)**p*q*x**2 - 3*( - x + 3)**q*( - x**2 + 6*x)**p*q*x + ( - x + 3)**q*
( - x**2 + 6*x)**p*x**2 - 9*( - x + 3)**q*( - x**2 + 6*x)**p - 72*int((( -
 x + 3)**q*( - x**2 + 6*x)**p*x)/(4*p**2*x**2 - 36*p**2*x + 72*p**2 + 4*p*
q*x**2 - 36*p*q*x + 72*p*q + 6*p*x**2 - 54*p*x + 108*p + q**2*x**2 - 9*q**
2*x + 18*q**2 + 3*q*x**2 - 27*q*x + 54*q + 2*x**2 - 18*x + 36),x)*p**4 - 1
08*int((( - x + 3)**q*( - x**2 + 6*x)**p*x)/(4*p**2*x**2 - 36*p**2*x + 72*
p**2 + 4*p*q*x**2 - 36*p*q*x + 72*p*q + 6*p*x**2 - 54*p*x + 108*p + q**2*x
**2 - 9*q**2*x + 18*q**2 + 3*q*x**2 - 27*q*x + 54*q + 2*x**2 - 18*x + 36),
x)*p**3*q - 180*int((( - x + 3)**q*( - x**2 + 6*x)**p*x)/(4*p**2*x**2 - 36
*p**2*x + 72*p**2 + 4*p*q*x**2 - 36*p*q*x + 72*p*q + 6*p*x**2 - 54*p*x + 1
08*p + q**2*x**2 - 9*q**2*x + 18*q**2 + 3*q*x**2 - 27*q*x + 54*q + 2*x**2
- 18*x + 36),x)*p**3 - 54*int((( - x + 3)**q*( - x**2 + 6*x)**p*x)/(4*p**2
*x**2 - 36*p**2*x + 72*p**2 + 4*p*q*x**2 - 36*p*q*x + 72*p*q + 6*p*x**2 -
54*p*x + 108*p + q**2*x**2 - 9*q**2*x + 18*q**2 + 3*q*x**2 - 27*q*x + 54*q
 + 2*x**2 - 18*x + 36),x)*p**2*q**2 - 180*int((( - x + 3)**q*( - x**2 + 6*
x)**p*x)/(4*p**2*x**2 - 36*p**2*x + 72*p**2 + 4*p*q*x**2 - 36*p*q*x + 72*p
*q + 6*p*x**2 - 54*p*x + 108*p + q**2*x**2 - 9*q**2*x + 18*q**2 + 3*q*x**2
 - 27*q*x + 54*q + 2*x**2 - 18*x + 36),x)*p**2*q - 144*int((( - x + 3)*...
```

3.224 $\int \frac{x^3(c+dx)}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 129

$$\int \frac{x^3(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2a^2(bc-ad)\sqrt{ax^2+bx^3}}{b^4x} - \frac{2a(2bc-3ad)\left(ax^2+bx^3\right)^{3/2}}{3b^4x^3} + \frac{2(bc-3ad)\left(ax^2+bx^3\right)^{5/2}}{5b^4x^5} + \frac{2d\left(ax^2+bx^3\right)^{7/2}}{7b^4x^7}$$

output
```
2*a^2*(-a*d+b*c)*(b*x^3+a*x^2)^(1/2)/b^4/x-2/3*a*(-3*a*d+2*b*c)*(b*x^3+a*x
^2)^(3/2)/b^4/x^3+2/5*(-3*a*d+b*c)*(b*x^3+a*x^2)^(5/2)/b^4/x^5+2/7*d*(b*x^
3+a*x^2)^(7/2)/b^4/x^7
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.58

$$\int \frac{x^3(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{x^2(a+bx)}\left(-48a^3d+8a^2b(7c+3dx)+3b^3x^2(7c+5dx)-2ab^2x(14c+9dx)\right)}{105b^4x}$$

input
```
Integrate[(x^3*(c + d*x))/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*Sqrt[x^2*(a + b*x)]*(-48*a^3*d + 8*a^2*b*(7*c + 3*d*x) + 3*b^3*x^2*(7*c
 + 5*d*x) - 2*a*b^2*x*(14*c + 9*d*x)))/(105*b^4*x)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1945, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx)}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(7bc-6ad)\int \frac{x^3}{\sqrt{bx^3+ax^2}}dx}{7b}+\frac{2dx^2\sqrt{ax^2+bx^3}}{7b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{(7bc-6ad)\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\int \frac{x^2}{\sqrt{bx^3+ax^2}}dx}{5b}\right)}{7b}+\frac{2dx^2\sqrt{ax^2+bx^3}}{7b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{(7bc-6ad)\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{2a\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}\right)}{5b}\right)}{7b}+\frac{2dx^2\sqrt{ax^2+bx^3}}{7b}$$

$$\Big\downarrow \text{1920}$$

$$\frac{\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\right)}{5b}\right)(7bc-6ad)}{7b}+\frac{2dx^2\sqrt{ax^2+bx^3}}{7b}$$

input
```
Int[(x^3*(c + d*x))/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*d*x^2*Sqrt[a*x^2 + b*x^3])/(7*b) + ((7*b*c - 6*a*d)*((2*x*Sqrt[a*x^2 +
b*x^3])/(5*b) - (4*a*((2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*a*Sqrt[a*x^2 + b*
x^3])/(3*b^2*x)))/(5*b)))/(7*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.58

method	result	size
pseudoelliptic	$\frac{256\sqrt{bx+a}\left(\frac{45x^3\left(c+\frac{7dx}{9}\right)b^4}{128}-\frac{27x^2a\left(\frac{20dx}{27}+c\right)b^3}{64}+\frac{9x\left(\frac{2dx}{3}+c\right)a^2b^2}{16}-\frac{9a^3\left(\frac{4dx}{9}+c\right)b}{8}+a^4d\right)}{315b^5}$	75
trager	$-\frac{2\left(-15b^3d\,x^3+18a\,b^2d\,x^2-21b^3c\,x^2-24a^2bdx+28a\,b^2cx+48a^3d-56c\,a^2b\right)\sqrt{b\,x^3+a\,x^2}}{105b^4x}$	80
risch	$-\frac{2(bx+a)x\left(-15b^3d\,x^3+18a\,b^2d\,x^2-21b^3c\,x^2-24a^2bdx+28a\,b^2cx+48a^3d-56c\,a^2b\right)}{105\sqrt{x^2(bx+a)}\,b^4}$	81
gosper	$-\frac{2(bx+a)\left(-15b^3d\,x^3+18a\,b^2d\,x^2-21b^3c\,x^2-24a^2bdx+28a\,b^2cx+48a^3d-56c\,a^2b\right)x}{105b^4\sqrt{b\,x^3+a\,x^2}}$	83
default	$-\frac{2(bx+a)\left(-15b^3d\,x^3+18a\,b^2d\,x^2-21b^3c\,x^2-24a^2bdx+28a\,b^2cx+48a^3d-56c\,a^2b\right)x}{105b^4\sqrt{b\,x^3+a\,x^2}}$	83
orering	$-\frac{2(bx+a)\left(-15b^3d\,x^3+18a\,b^2d\,x^2-21b^3c\,x^2-24a^2bdx+28a\,b^2cx+48a^3d-56c\,a^2b\right)x}{105b^4\sqrt{b\,x^3+a\,x^2}}$	83

input
```
int(x^3*(d*x+c)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
256/315*(b*x+a)^(1/2)*(45/128*x^3*(c+7/9*d*x)*b^4-27/64*x^2*a*(20/27*d*x+c
)*b^3+9/16*x*(2/3*d*x+c)*a^2*b^2-9/8*a^3*(4/9*d*x+c)*b+a^4*d)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.63

$$\int \frac{x^3(c+dx)}{\sqrt{ax^2+bx^3}}\,dx$$
$$=\frac{2\left(15\,b^3dx^3+56\,a^2bc-48\,a^3d+3\,(7\,b^3c-6\,ab^2d)x^2-4\,(7\,ab^2c-6\,a^2bd)x\right)\sqrt{bx^3+ax^2}}{105\,b^4x}$$

input
```
integrate(x^3*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*b^3*d*x^3 + 56*a^2*b*c - 48*a^3*d + 3*(7*b^3*c - 6*a*b^2*d)*x^2
- 4*(7*a*b^2*c - 6*a^2*b*d)*x)*sqrt(b*x^3 + a*x^2)/(b^4*x)
```

Sympy [F]

$$\int \frac{x^3(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^3(c+dx)}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(x**3*(d*x+c)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**3*(c + d*x)/sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.76

$$\int \frac{x^3(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,(3\,b^3x^3-ab^2x^2+4\,a^2bx+8\,a^3)c}{15\,\sqrt{bx+a}b^3} + \frac{2\,(5\,b^4x^4-ab^3x^3+2\,a^2b^2x^2-8\,a^3bx-16\,a^4)d}{35\,\sqrt{bx+a}b^4}$$

input
```
integrate(x^3*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(3*b^3*x^3 - a*b^2*x^2 + 4*a^2*b*x + 8*a^3)*c/(sqrt(b*x + a)*b^3) + 2
/35*(5*b^4*x^4 - a*b^3*x^3 + 2*a^2*b^2*x^2 - 8*a^3*b*x - 16*a^4)*d/(sqrt(b
*x + a)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.93

$$\int \frac{x^3(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\left(\frac{7\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)c}{b^2}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)d}{b^3}\right)}{105\,b\mathrm{sgn}\,(x)} - \frac{16\left(7\,a^{\frac{5}{2}}bc-6\,a^{\frac{7}{2}}d\right)\mathrm{sgn}(x)}{105\,b^4}$$

input
```
integrate(x^3*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
2/105*(7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)
*c/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*
a^2 - 35*sqrt(b*x + a)*a^3)*d/b^3)/(b*sgn(x)) - 16/105*(7*a^(5/2)*b*c - 6*
a^(7/2)*d)*sgn(x)/b^4
```

Mupad [B] (verification not implemented)

Time = 9.47 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.65

$$\int \frac{x^3(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,d\,x^3}{7\,b}-\frac{96\,a^3\,d-112\,a^2\,b\,c}{105\,b^4}+\frac{x^2\,\left(42\,b^3\,c-36\,a\,b^2\,d\right)}{105\,b^4}+\frac{8\,a\,x\,\left(6\,a\,d-7\,b\,c\right)}{105\,b^3}\right)}{x}$$

input
```
int((x^3*(c + d*x))/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
((a*x^2 + b*x^3)^(1/2)*((2*d*x^3)/(7*b) - (96*a^3*d - 112*a^2*b*c)/(105*b^
4) + (x^2*(42*b^3*c - 36*a*b^2*d))/(105*b^4) + (8*a*x*(6*a*d - 7*b*c))/(10
5*b^3)))/x
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.53

$$\int \frac{x^3(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{bx+a}\,(15b^3d\,x^3-18a\,b^2d\,x^2+21b^3c\,x^2+24a^2bdx-28a\,b^2cx-48a^3d+56a^2bc)}{105b^4}$$

input
```
int(x^3*(d*x+c)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 48*a**3*d + 56*a**2*b*c + 24*a**2*b*d*x - 28*a*b**2*c
*x - 18*a*b**2*d*x**2 + 21*b**3*c*x**2 + 15*b**3*d*x**3))/(105*b**4)
```

3.225 $\int \frac{x^2(c+dx)}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 92

$$\int \frac{x^2(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = -\frac{2a(bc-ad)\sqrt{ax^2+bx^3}}{b^3x} + \frac{2(bc-2ad)\left(ax^2+bx^3\right)^{3/2}}{3b^3x^3} + \frac{2d\left(ax^2+bx^3\right)^{5/2}}{5b^3x^5}$$

output
```
-2*a*(-a*d+b*c)*(b*x^3+a*x^2)^(1/2)/b^3/x+2/3*(-2*a*d+b*c)*(b*x^3+a*x^2)^(
3/2)/b^3/x^3+2/5*d*(b*x^3+a*x^2)^(5/2)/b^3/x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.60

$$\int \frac{x^2(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{x^2(a+bx)}(8a^2d-2ab(5c+2dx)+b^2x(5c+3dx))}{15b^3x}$$

input
```
Integrate[(x^2*(c + d*x))/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*Sqrt[x^2*(a + b*x)]*(8*a^2*d - 2*a*b*(5*c + 2*d*x) + b^2*x*(5*c + 3*d*x
)))/(15*b^3*x)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1945, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx)}{\sqrt{ax^2+bx^3}}\,dx$$

$$\Big\downarrow \text{ 1945}$$

$$\frac{(5bc-4ad)\int \frac{x^2}{\sqrt{bx^3+ax^2}}dx}{5b}+\frac{2dx\sqrt{ax^2+bx^3}}{5b}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{(5bc-4ad)\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{2a\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}\right)}{5b}+\frac{2dx\sqrt{ax^2+bx^3}}{5b}$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\right)(5bc-4ad)}{5b}+\frac{2dx\sqrt{ax^2+bx^3}}{5b}$$

input
```
Int[(x^2*(c + d*x))/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*d*x*Sqrt[a*x^2 + b*x^3])/(5*b) + ((5*b*c - 4*a*d)*((2*Sqrt[a*x^2 + b*x^
3])/(3*b) - (4*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)))/(5*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.61

method	result	size
trager	$\frac{2(3b^2d\,x^2-4abdx+5b^2cx+8a^2d-10abc)\sqrt{b\,x^3+a\,x^2}}{15b^3x}$	56
risch	$\frac{2(bx+a)x(3b^2d\,x^2-4abdx+5b^2cx+8a^2d-10abc)}{15\sqrt{x^2(bx+a)}\,b^3}$	57
pseudoelliptic	$-\frac{32\sqrt{bx+a}\left(-\frac{7\left(\frac{5dx}{7}+c\right)x^2b^3}{16}+\frac{7\left(\frac{9dx}{14}+c\right)xa\,b^2}{12}-\frac{7\left(\frac{3dx}{7}+c\right)a^2b}{6}+a^3d\right)}{35b^4}$	58
gosper	$\frac{2(bx+a)(3b^2d\,x^2-4abdx+5b^2cx+8a^2d-10abc)x}{15b^3\sqrt{b\,x^3+a\,x^2}}$	59
default	$\frac{2(bx+a)(3b^2d\,x^2-4abdx+5b^2cx+8a^2d-10abc)x}{15b^3\sqrt{b\,x^3+a\,x^2}}$	59
orering	$\frac{2(bx+a)(3b^2d\,x^2-4abdx+5b^2cx+8a^2d-10abc)x}{15b^3\sqrt{b\,x^3+a\,x^2}}$	59

input
```
int(x^2*(d*x+c)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(3*b^2*d*x^2-4*a*b*d*x+5*b^2*c*x+8*a^2*d-10*a*b*c)/b^3/x*(b*x^3+a*x^2
)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.61

$$\int \frac{x^2(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,(3\,b^2dx^2-10\,abc+8\,a^2d+(5\,b^2c-4\,abd)x)\sqrt{bx^3+ax^2}}{15\,b^3x}$$

input
```
integrate(x^2*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*b^2*d*x^2 - 10*a*b*c + 8*a^2*d + (5*b^2*c - 4*a*b*d)*x)*sqrt(b*x^3
 + a*x^2)/(b^3*x)
```

Sympy [F]

$$\int \frac{x^2(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^2(c+dx)}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(x**2*(d*x+c)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**2*(c + d*x)/sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int \frac{x^2(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,(b^2x^2-abx-2\,a^2)c}{3\,\sqrt{bx+a}b^2} + \frac{2\,(3\,b^3x^3-ab^2x^2+4\,a^2bx+8\,a^3)d}{15\,\sqrt{bx+a}b^3}$$

input
```
integrate(x^2*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(b^2*x^2 - a*b*x - 2*a^2)*c/(sqrt(b*x + a)*b^2) + 2/15*(3*b^3*x^3 - a*
b^2*x^2 + 4*a^2*b*x + 8*a^3)*d/(sqrt(b*x + a)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01

$$\int \frac{x^2(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\left(\frac{5\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}a\right)c}{b}+\frac{\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)d}{b^2}\right)}{15\,b\mathrm{sgn}\left(x\right)} + \frac{4\left(5\,a^{\frac{3}{2}}bc-4\,a^{\frac{5}{2}}d\right)\mathrm{sgn}(x)}{15\,b^3}$$

input
```
integrate(x^2*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
2/15*(5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*c/b + (3*(b*x + a)^(5/2) - 1
0*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*d/b^2)/(b*sgn(x)) + 4/15*(5*a^
(3/2)*b*c - 4*a^(5/2)*d)*sgn(x)/b^3
```

Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.67

$$\int \frac{x^2(c+dx)}{\sqrt{ax^2+bx^3}}\,dx=\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{16\,a^2\,d-20\,a\,b\,c}{15\,b^3}+\frac{x\,\left(10\,b^2\,c-8\,a\,b\,d\right)}{15\,b^3}+\frac{2\,d\,x^2}{5\,b}\right)}{x}$$

input
```
int((x^2*(c + d*x))/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
((a*x^2 + b*x^3)^(1/2)*((16*a^2*d - 20*a*b*c)/(15*b^3) + (x*(10*b^2*c - 8*
a*b*d))/(15*b^3) + (2*d*x^2)/(5*b)))/x
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.49

$$\int \frac{x^2(c+dx)}{\sqrt{ax^2+bx^3}}\,dx=\frac{2\sqrt{bx+a}\,(3b^2d\,x^2-4abdx+5b^2cx+8a^2d-10abc)}{15b^3}$$

input
```
int(x^2*(d*x+c)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*(8*a**2*d - 10*a*b*c - 4*a*b*d*x + 5*b**2*c*x + 3*b**2*d*
x**2))/(15*b**3)
```

3.226 $\int \frac{x(c+dx)}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 58

$$\int \frac{x(c+dx)}{\sqrt{ax^2+bx^3}}\,dx=\frac{2(bc-ad)\sqrt{ax^2+bx^3}}{b^2x}+\frac{2d(ax^2+bx^3)^{3/2}}{3b^2x^3}$$

output
```
2*(-a*d+b*c)*(b*x^3+a*x^2)^(1/2)/b^2/x+2/3*d*(b*x^3+a*x^2)^(3/2)/b^2/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.62

$$\int \frac{x(c+dx)}{\sqrt{ax^2+bx^3}}\,dx=\frac{2\sqrt{x^2(a+bx)}(3bc-2ad+bdx)}{3b^2x}$$

input
```
Integrate[(x*(c + d*x))/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*Sqrt[x^2*(a + b*x)]*(3*b*c - 2*a*d + b*d*x))/(3*b^2*x)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1945, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx)}{\sqrt{ax^2+bx^3}}\,dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(3bc-2ad)\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}+\frac{2d\sqrt{ax^2+bx^3}}{3b}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2\sqrt{ax^2+bx^3}(3bc-2ad)}{3b^2x}+\frac{2d\sqrt{ax^2+bx^3}}{3b}$$

input
```
Int[(x*(c + d*x))/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*d*Sqrt[a*x^2 + b*x^3])/(3*b) + (2*(3*b*c - 2*a*d)*Sqrt[a*x^2 + b*x^3])/
(3*b^2*x)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.62

method	result	size
trager	$-\frac{2(-bdx+2ad-3bc)\sqrt{b\,x^{3}+a\,x^{2}}}{3b^{2}x}$	36
risch	$-\frac{2(bx+a)x(-bdx+2ad-3bc)}{3\sqrt{x^{2}(bx+a)}\,b^{2}}$	37
gosper	$-\frac{2(bx+a)(-bdx+2ad-3bc)x}{3b^{2}\sqrt{b\,x^{3}+a\,x^{2}}}$	39
default	$-\frac{2(bx+a)(-bdx+2ad-3bc)x}{3b^{2}\sqrt{b\,x^{3}+a\,x^{2}}}$	39
orering	$-\frac{2(bx+a)(-bdx+2ad-3bc)x}{3b^{2}\sqrt{b\,x^{3}+a\,x^{2}}}$	39
pseudoelliptic	$\frac{16\sqrt{bx+a}\left(\frac{5x\left(\frac{3dx}{5}+c\right)b^{2}}{8}-\frac{5\left(\frac{2dx}{5}+c\right)ab}{4}+a^{2}d\right)}{15b^{3}}$	41

input
```
int(x*(d*x+c)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-b*d*x+2*a*d-3*b*c)/b^2/x*(b*x^3+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.59

$$\int \frac{x(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{bx^3+ax^2}(bdx+3\,bc-2\,ad)}{3\,b^2x}$$

input
```
integrate(x*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(b*x^3 + a*x^2)*(b*d*x + 3*b*c - 2*a*d)/(b^2*x)
```

Sympy [F]

$$\int \frac{x(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x(c+dx)}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(x*(d*x+c)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x*(c + d*x)/sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

$$\int \frac{x(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{bx+a}c}{b} + \frac{2\,(b^2x^2-abx-2\,a^2)d}{3\sqrt{bx+a}b^2}$$

input
```
integrate(x*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(b*x + a)*c/b + 2/3*(b^2*x^2 - a*b*x - 2*a^2)*d/(sqrt(b*x + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12

$$\int \frac{x(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = -\frac{2\left(3\sqrt{a}bc-2a^{\frac{3}{2}}d\right)\mathrm{sgn}(x)}{3b^2}+\frac{2\left(3\sqrt{bx+a}c+\frac{\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)d}{b}\right)}{3b\mathrm{sgn}(x)}$$

input
```
integrate(x*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-2/3*(3*sqrt(a)*b*c - 2*a^(3/2)*d)*sgn(x)/b^2 + 2/3*(3*sqrt(b*x + a)*c + (
(b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*d/b)/(b*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

$$\int \frac{x(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = -\frac{\left(\frac{4\,a\,d-6\,b\,c}{3\,b^2}-\frac{2\,d\,x}{3\,b}\right)\,\sqrt{b\,x^3+a\,x^2}}{x}$$

input
```
int((x*(c + d*x))/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
-((((4*a*d - 6*b*c)/(3*b^2) - (2*d*x)/(3*b))*(a*x^2 + b*x^3)^(1/2))/x
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.41

$$\int \frac{x(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{bx+a}\,(bdx-2ad+3bc)}{3b^2}$$

input
```
int(x*(d*x+c)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 2*a*d + 3*b*c + b*d*x))/(3*b**2)
```

3.227 $\int \frac{c+dx}{\sqrt{ax^2+bx^3}} dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{c+dx}{\sqrt{ax^2+bx^3}} dx = \frac{2d\sqrt{ax^2+bx^3}}{bx} - \frac{2c\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{\sqrt{a}}$$

output
```
2*d*(b*x^3+a*x^2)^(1/2)/b/x-2*c*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(
1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int \frac{c+dx}{\sqrt{ax^2+bx^3}} dx = \frac{2x\left(\sqrt{a}d(a+bx) - bc\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{a}b\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(c + d*x)/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*x*(Sqrt[a]*d*(a + b*x) - b*c*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a
]]))/(Sqrt[a]*b*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2450, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{2450}$$

$$\int \left(\frac{c}{\sqrt{ax^2+bx^3}}+\frac{dx}{\sqrt{ax^2+bx^3}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2d\sqrt{ax^2+bx^3}}{bx}-\frac{2c\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

input
```
Int[(c + d*x)/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*d*Sqrt[a*x^2 + b*x^3])/(b*x) - (2*c*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*
x^3]])/Sqrt[a]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2450
```
Int[(Pq_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[Expan
dIntegrand[Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && (Po
lyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IntegerQ[p] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.47

method	result	size
pseudoelliptic	$-\frac{2\sqrt{bx+a}\,(-bdx+2ad-3bc)}{3b^2}$	27
default	$\frac{2x\sqrt{bx+a}\,\left(\sqrt{bx+a}\,d\sqrt{a}-bc\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{\sqrt{b\,x^3+a\,x^2}\,b\sqrt{a}}$	59

input
```
int((d*x+c)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x+a)^(1/2)*(-b*d*x+2*a*d-3*b*c)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.36

$$\int \frac{c+dx}{\sqrt{ax^2+bx^3}}\,dx = \left[\frac{\sqrt{a}bcx\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}ad}{abx}, \frac{2\left(\sqrt{-a}bcx\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+\sqrt{bx^3+ax^2}\right.}{abx}\right.$$

input
```
integrate((d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[(sqrt(a)*b*c*x*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) +
 2*sqrt(b*x^3 + a*x^2)*a*d)/(a*b*x), 2*(sqrt(-a)*b*c*x*arctan(sqrt(b*x^3 +
 a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + sqrt(b*x^3 + a*x^2)*a*d)/(a*b*x)]
```

Sympy [F]

$$\int \frac{c+dx}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{c+dx}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate((d*x+c)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral((c + d*x)/sqrt(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{dx+c}{\sqrt{bx^3+ax^2}}\,dx$$

input
```
integrate((d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.33

$$\int \frac{c+dx}{\sqrt{ax^2+bx^3}}\,dx = -\frac{2\left(bc\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}d\right)\operatorname{sgn}(x)}{\sqrt{-a}b} + \frac{2\left(\frac{c\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{\sqrt{bx+a}d}{b}\right)}{\operatorname{sgn}(x)}$$

input
```
integrate((d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-2*(b*c*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a)*d)*sgn(x)/(sqrt(-a)*b)
 + 2*(c*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + sqrt(b*x + a)*d/b)/sgn(x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{c+d\,x}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int((c + d*x)/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
int((c + d*x)/(a*x^2 + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \frac{c+dx}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{bx+a}\,ad+\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)bc-\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)bc}{ab}$$

input
```
int((d*x+c)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*a*d + sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b*c - sqrt(a)*
log(sqrt(a + b*x) + sqrt(a))*b*c)/(a*b)
```

3.228 $\int \frac{c+dx}{x\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{c+dx}{x\sqrt{ax^2+bx^3}}\,dx=-\frac{c\sqrt{ax^2+bx^3}}{ax^2}+\frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{a^{3/2}}$$

output
```
-c*(b*x^3+a*x^2)^(1/2)/a/x^2+(-2*a*d+b*c)*arctanh((b*x^3+a*x^2)^(1/2)/a^(1
/2)/x)/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06

$$\int \frac{c+dx}{x\sqrt{ax^2+bx^3}}\,dx=\frac{-\sqrt{a}c(a+bx)+(bc-2ad)x\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(c + d*x)/(x*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-(Sqrt[a]*c*(a + b*x)) + (b*c - 2*a*d)*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b
*x]/Sqrt[a]])/(a^(3/2)*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1944, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{x\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \ 1944$$

$$-\frac{(bc-2ad)\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{c\sqrt{ax^2+bx^3}}{ax^2}$$

$$\Big\downarrow \ 1914$$

$$\frac{(bc-2ad)\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{c\sqrt{ax^2+bx^3}}{ax^2}$$

$$\Big\downarrow \ 219$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)(bc-2ad)}{a^{3/2}}-\frac{c\sqrt{ax^2+bx^3}}{ax^2}$$

input
```
Int[(c + d*x)/(x*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
-((c*Sqrt[a*x^2 + b*x^3])/(a*x^2)) + ((b*c - 2*a*d)*ArcTanh[(Sqrt[a]*x)/Sq
rt[a*x^2 + b*x^3]])/a^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.59

method	result	size
pseudoelliptic	$\frac{2\sqrt{bx+a}\, d\sqrt{a}-2bc\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b\sqrt{a}}$	38
risch	$-\frac{c(bx+a)}{a\sqrt{x^{2}(bx+a)}}-\frac{(2ad-bc)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{bx+a}\, x}{a^{\frac{3}{2}}\sqrt{x^{2}(bx+a)}}$	69
default	$-\frac{\sqrt{bx+a}\left(a^{\frac{3}{2}}c\sqrt{bx+a}+2\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^{2}dx-\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)abcx\right)}{\sqrt{b\,x^{3}+a\,x^{2}}\, a^{\frac{5}{2}}}$	76

input
```
int((d*x+c)/x/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((b*x+a)^(1/2)*d*a^(1/2)-b*c*arctanh((b*x+a)^(1/2)/a^(1/2)))/b/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.31

$$\int \frac{c+dx}{x\sqrt{ax^2+bx^3}}\,dx = \left[-\frac{(bc-2\,ad)\sqrt{a}x^2\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}ac}{2\,a^2x^2}, -\frac{(bc-2\,ad)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+\sqrt{bx^3+ax^2}ac}{a^2x^2}\right]$$

input
```
integrate((d*x+c)/x/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*((b*c - 2*a*d)*sqrt(a)*x^2*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2
)*sqrt(a))/x^2) + 2*sqrt(b*x^3 + a*x^2)*a*c)/(a^2*x^2), -((b*c - 2*a*d)*sq
rt(-a)*x^2*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + sqrt(b*x^3
 + a*x^2)*a*c)/(a^2*x^2)]
```

Sympy [F]

$$\int \frac{c+dx}{x\sqrt{ax^2+bx^3}}\,dx = \int \frac{c+dx}{x\sqrt{x^2\left(a+bx\right)}}\,dx$$

input
```
integrate((d*x+c)/x/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral((c + d*x)/(x*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{c+dx}{x\sqrt{ax^2+bx^3}}\,dx = \int \frac{dx+c}{\sqrt{bx^3+ax^2}x}\,dx$$

input
```
integrate((d*x+c)/x/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/(sqrt(b*x^3 + a*x^2)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int \frac{c+dx}{x\sqrt{ax^2+bx^3}}\,dx = -\frac{b\left(\frac{(bc-2\,ad)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}ab}+\frac{\sqrt{bx+a}c}{abx}\right)}{\operatorname{sgn}\left(x\right)}$$

input
```
integrate((d*x+c)/x/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-b*((b*c - 2*a*d)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a*b) + sqrt(b*x
 + a)*c/(a*b*x))/sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{x\sqrt{ax^2+bx^3}}\,dx = \int \frac{c+d\,x}{x\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int((c + d*x)/(x*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
int((c + d*x)/(x*(a*x^2 + b*x^3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.41

$$\int \frac{c+dx}{x\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{-2\sqrt{bx+a}\,ac+2\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)adx-\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)bcx-2\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)}{2a^2x}$$

input
```
int((d*x+c)/x/(b*x^3+a*x^2)^(1/2),x)
```

output
```
( - 2*sqrt(a + b*x)*a*c + 2*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*d*x - s
qrt(a)*log(sqrt(a + b*x) - sqrt(a))*b*c*x - 2*sqrt(a)*log(sqrt(a + b*x) +
sqrt(a))*a*d*x + sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b*c*x)/(2*a**2*x)
```

3.229 $\int \frac{c+dx}{x^2\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 105

$$\int \frac{c+dx}{x^2\sqrt{ax^2+bx^3}}\,dx=-\frac{c\sqrt{ax^2+bx^3}}{2ax^3}+\frac{(3bc-4ad)\sqrt{ax^2+bx^3}}{4a^2x^2}-\frac{b(3bc-4ad)\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{4a^{5/2}}$$

output
```
-1/2*c*(b*x^3+a*x^2)^(1/2)/a/x^3+1/4*(-4*a*d+3*b*c)*(b*x^3+a*x^2)^(1/2)/a^
2/x^2-1/4*b*(-4*a*d+3*b*c)*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{c+dx}{x^2\sqrt{ax^2+bx^3}}\,dx=\frac{-\sqrt{a}(a+bx)(-3bcx+2a(c+2dx))-b(3bc-4ad)x^2\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}x\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(c + d*x)/(x^2*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-(Sqrt[a]*(a + b*x)*(-3*b*c*x + 2*a*(c + 2*d*x))) - b*(3*b*c - 4*a*d)*x^2
*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(5/2)*x*Sqrt[x^2*(a +
b*x)])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1944, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{x^2\sqrt{ax^2+bx^3}}\,dx$$

$$\Big\downarrow \text{1944}$$

$$-\frac{(3bc-4ad)\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{c\sqrt{ax^2+bx^3}}{2ax^3}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{(3bc-4ad)\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{c\sqrt{ax^2+bx^3}}{2ax^3}$$

$$\Big\downarrow \text{1914}$$

$$-\frac{(3bc-4ad)\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{c\sqrt{ax^2+bx^3}}{2ax^3}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)(3bc-4ad)}{4a}-\frac{c\sqrt{ax^2+bx^3}}{2ax^3}$$

input
```
Int[(c + d*x)/(x^2*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
-1/2*(c*Sqrt[a*x^2 + b*x^3])/(a*x^3) - ((3*b*c - 4*a*d)*(-(Sqrt[a*x^2 + b*
x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/(4*
a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.41

method	result
pseudoelliptic	$-\frac{c\sqrt{bx+a}}{ax}-\frac{2\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\left(-\frac{bc}{2}+ad\right)}{a^{\frac{3}{2}}}$
risch	$-\frac{(bx+a)(4adx-3cbx+2ac)}{4a^{2}x\sqrt{x^{2}(bx+a)}}+\frac{(4ad-3bc)b\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{bx+a}\,x}{4a^{\frac{5}{2}}\sqrt{x^{2}(bx+a)}}$
default	$-\frac{\sqrt{bx+a}\left(4(bx+a)^{\frac{3}{2}}a^{\frac{7}{2}}d-3(bx+a)^{\frac{3}{2}}a^{\frac{5}{2}}bc-4\sqrt{bx+a}\,a^{\frac{9}{2}}d+5\sqrt{bx+a}\,a^{\frac{7}{2}}bc-4\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^{3}b^{2}d\,x^{2}+3\,\text{arctanh}\left(\frac{\sqrt{bx}}{\sqrt{}}\right.\right.}{4xb\sqrt{b\,x^{3}+a\,x^{2}}\,a^{\frac{9}{2}}}$

input
```
int((d*x+c)/x^2/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-c/a*(b*x+a)^(1/2)/x-2*arctanh((b*x+a)^(1/2)/a^(1/2))*(-1/2*b*c+a*d)/a^(3/
2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.88

$$\int \frac{c+dx}{x^2\sqrt{ax^2+bx^3}}\,dx$$
$$=\left[-\frac{(3\,b^2c-4\,abd)\sqrt{a}x^3\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}(2\,a^2c-(3\,abc-4\,a^2d)x)}{8\,a^3x^3},\ \frac{(3\,b^2c-4}{}\right.$$

input
```
integrate((d*x+c)/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*((3*b^2*c - 4*a*b*d)*sqrt(a)*x^3*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 +
 a*x^2)*sqrt(a))/x^2) + 2*sqrt(b*x^3 + a*x^2)*(2*a^2*c - (3*a*b*c - 4*a^2*
d)*x))/(a^3*x^3), 1/4*((3*b^2*c - 4*a*b*d)*sqrt(-a)*x^3*arctan(sqrt(b*x^3
+ a*x^2)*sqrt(-a)/(b*x^2 + a*x)) - sqrt(b*x^3 + a*x^2)*(2*a^2*c - (3*a*b*c
 - 4*a^2*d)*x))/(a^3*x^3)]
```

Sympy [F]

$$\int \frac{c+dx}{x^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{c+dx}{x^2\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate((d*x+c)/x**2/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral((c + d*x)/(x**2*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{c+dx}{x^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{dx+c}{\sqrt{bx^3+ax^2}x^2}\,dx$$

input
```
integrate((d*x+c)/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/(sqrt(b*x^3 + a*x^2)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10

$$\int \frac{c+dx}{x^2\sqrt{ax^2+bx^3}}\,dx = \frac{\frac{(3\,b^3c-4\,ab^2d)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{3\,(bx+a)^{\frac{3}{2}}b^3c-5\,\sqrt{bx+a}ab^3c-4\,(bx+a)^{\frac{3}{2}}ab^2d+4\,\sqrt{bx+a}a^2b^2d}{a^2b^2x^2}}{4\,b\mathrm{sgn}\left(x\right)}$$

input
```
integrate((d*x+c)/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*((3*b^3*c - 4*a*b^2*d)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) +
 (3*(b*x + a)^(3/2)*b^3*c - 5*sqrt(b*x + a)*a*b^3*c - 4*(b*x + a)^(3/2)*a*
b^2*d + 4*sqrt(b*x + a)*a^2*b^2*d)/(a^2*b^2*x^2))/(b*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{x^2\sqrt{ax^2+bx^3}}\,dx=\int \frac{c+d\,x}{x^2\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int((c + d*x)/(x^2*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
int((c + d*x)/(x^2*(a*x^2 + b*x^3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.26

$$\int \frac{c+dx}{x^2\sqrt{ax^2+bx^3}}\,dx = \frac{-4\sqrt{bx+a}\,a^2c-8\sqrt{bx+a}\,a^2dx+6\sqrt{bx+a}\,abcx-4\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)abd\,x^2+3\sqrt{a}\log\left(\sqrt{bx+}\right.}{8a^3x^2}$$

input
```
int((d*x+c)/x^2/(b*x^3+a*x^2)^(1/2),x)
```

output
```
( - 4*sqrt(a + b*x)*a**2*c - 8*sqrt(a + b*x)*a**2*d*x + 6*sqrt(a + b*x)*a*
b*c*x - 4*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b*d*x**2 + 3*sqrt(a)*log(
sqrt(a + b*x) - sqrt(a))*b**2*c*x**2 + 4*sqrt(a)*log(sqrt(a + b*x) + sqrt(
a))*a*b*d*x**2 - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**2*c*x**2)/(8*a*
*3*x**2)
```

3.230 $\int \frac{c+dx}{x^3\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 142

$$\int \frac{c+dx}{x^3\sqrt{ax^2+bx^3}}\,dx = -\frac{c\sqrt{ax^2+bx^3}}{3ax^4}+\frac{(5bc-6ad)\sqrt{ax^2+bx^3}}{12a^2x^3}-\frac{b(5bc-6ad)\sqrt{ax^2+bx^3}}{8a^3x^2}+\frac{b^2(5bc-6ad)\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{8a^{7/2}}$$

output
```
-1/3*c*(b*x^3+a*x^2)^(1/2)/a/x^4+1/12*(-6*a*d+5*b*c)*(b*x^3+a*x^2)^(1/2)/a
^2/x^3-1/8*b*(-6*a*d+5*b*c)*(b*x^3+a*x^2)^(1/2)/a^3/x^2+1/8*b^2*(-6*a*d+5*
b*c)*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.82

$$\int \frac{c+dx}{x^3\sqrt{ax^2+bx^3}}\,dx = \frac{-\sqrt{a}(a+bx)\left(15b^2cx^2+4a^2(2c+3dx)-2abx(5c+9dx)\right)+3b^2(5bc-6ad)x^3\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{24a^{7/2}x^2\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(c + d*x)/(x^3*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-(Sqrt[a]*(a + b*x)*(15*b^2*c*x^2 + 4*a^2*(2*c + 3*d*x) - 2*a*b*x*(5*c +
9*d*x))) + 3*b^2*(5*b*c - 6*a*d)*x^3*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/S
qrt[a]])/(24*a^(7/2)*x^2*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1944, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{x^3\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1944}$$

$$-\frac{(5bc-6ad)\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{6a}-\frac{c\sqrt{ax^2+bx^3}}{3ax^4}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{(5bc-6ad)\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{c\sqrt{ax^2+bx^3}}{3ax^4}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{(5bc-6ad)\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{c\sqrt{ax^2+bx^3}}{3ax^4}$$

$$\Big\downarrow \text{1914}$$

$$-\frac{(5bc-6ad)\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{c\sqrt{ax^2+bx^3}}{3ax^4}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\left(-\frac{3b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)(5bc-6ad)}{6a}-\frac{c\sqrt{ax^2+bx^3}}{3ax^4}$$

input
```
Int[(c + d*x)/(x^3*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
-1/3*(c*Sqrt[a*x^2 + b*x^3])/(a*x^4) - ((5*b*c - 6*a*d)*(-1/2*Sqrt[a*x^2 +
 b*x^3]/(a*x^3) - (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[
a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/(4*a)))/(6*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.45

method	result
pseudoelliptic	$\frac{b\,x^{2}\left(ad-\frac{3bc}{4}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\frac{3\sqrt{bx+a}\,\left(\frac{2(-2dx-c)a^{\frac{3}{2}}}{3}+\sqrt{a}\,bcx\right)}{4}}{a^{\frac{5}{2}}x^{2}}$
risch	$-\frac{(bx+a)\left(-18abd\,x^{2}+15b^{2}c\,x^{2}+12a^{2}dx-10abcx+8a^{2}c\right)}{24a^{3}x^{2}\sqrt{x^{2}(bx+a)}}-\frac{(6ad-5bc)b^{2}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{bx+a}\,x}{8a^{\frac{7}{2}}\sqrt{x^{2}(bx+a)}}$
default	$\frac{\sqrt{bx+a}\,\left(18(bx+a)^{\frac{5}{2}}a^{\frac{9}{2}}d-15(bx+a)^{\frac{5}{2}}a^{\frac{7}{2}}bc-48(bx+a)^{\frac{3}{2}}a^{\frac{11}{2}}d+40(bx+a)^{\frac{3}{2}}a^{\frac{9}{2}}bc+30\sqrt{bx+a}\,a^{\frac{13}{2}}d-33\sqrt{bx+a}\,a^{\frac{11}{2}}bc-18\ a \cdots\right.}{24x^{2}b\sqrt{b\,x^{3}+a\,x^{2}}\,a^{\frac{13}{2}}}$

input
```
int((d*x+c)/x^3/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^(5/2)*(b*x^2*(a*d-3/4*b*c)*arctanh((b*x+a)^(1/2)/a^(1/2))+3/4*(b*x+a)^
(1/2)*(2/3*(-2*d*x-c)*a^(3/2)+a^(1/2)*b*c*x))/x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.73

$$\int \frac{c+dx}{x^3\sqrt{ax^2+bx^3}}\,dx$$
$$= \left[-\frac{3\,(5\,b^3c-6\,ab^2d)\sqrt{a}x^4\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(8\,a^3c+3\,(5\,ab^2c-6\,a^2bd)x^2-2\,(5\,a^2bc-6\,a}{48\,a^4x^4}\right.$$
$$-\frac{3\,(5\,b^3c-6\,ab^2d)\sqrt{-a}x^4\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+(8\,a^3c+3\,(5\,ab^2c-6\,a^2bd)x^2-2\,(5\,a^2bc-6\,a^3d)x)}{24\,a^4x^4}$$

input
```
integrate((d*x+c)/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(5*b^3*c - 6*a*b^2*d)*sqrt(a)*x^4*log((b*x^2 + 2*a*x - 2*sqrt(b*
x^3 + a*x^2)*sqrt(a))/x^2) + 2*(8*a^3*c + 3*(5*a*b^2*c - 6*a^2*b*d)*x^2 -
2*(5*a^2*b*c - 6*a^3*d)*x)*sqrt(b*x^3 + a*x^2))/(a^4*x^4), -1/24*(3*(5*b^3
*c - 6*a*b^2*d)*sqrt(-a)*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 +
a*x)) + (8*a^3*c + 3*(5*a*b^2*c - 6*a^2*b*d)*x^2 - 2*(5*a^2*b*c - 6*a^3*d)
*x)*sqrt(b*x^3 + a*x^2))/(a^4*x^4)]
```

Sympy [F]

$$\int \frac{c+dx}{x^3\sqrt{ax^2+bx^3}}\,dx = \int \frac{c+dx}{x^3\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate((d*x+c)/x**3/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral((c + d*x)/(x**3*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{c+dx}{x^3\sqrt{ax^2+bx^3}}\,dx = \int \frac{dx+c}{\sqrt{bx^3+ax^2}x^3}\,dx$$

input
```
integrate((d*x+c)/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/(sqrt(b*x^3 + a*x^2)*x^3), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.92

$$\int \frac{c+dx}{x^3\sqrt{ax^2+bx^3}}\,dx = -\frac{b^3\left(\frac{3\,(5\,bc-6\,ad)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3b}+\frac{15\,(bx+a)^{\frac{5}{2}}bc-40\,(bx+a)^{\frac{3}{2}}abc+33\,\sqrt{bx+a}a^2bc-18\,(bx+a)^{\frac{5}{2}}ad+48\,(bx+a)^{\frac{3}{2}}a^2d-30\,\sqrt{bx+a}a^3d}{a^3b^4x^3}\right)}{24\,\mathrm{sgn}\,(x)}$$

input
```
integrate((d*x+c)/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/24*b^3*(3*(5*b*c - 6*a*d)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3*
b) + (15*(b*x + a)^(5/2)*b*c - 40*(b*x + a)^(3/2)*a*b*c + 33*sqrt(b*x + a)
*a^2*b*c - 18*(b*x + a)^(5/2)*a*d + 48*(b*x + a)^(3/2)*a^2*d - 30*sqrt(b*x
 + a)*a^3*d)/(a^3*b^4*x^3))/sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{x^3\sqrt{ax^2+bx^3}}\,dx = \int \frac{c+d\,x}{x^3\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int((c + d*x)/(x^3*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
int((c + d*x)/(x^3*(a*x^2 + b*x^3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.20

$$\int \frac{c+dx}{x^3\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{-16\sqrt{bx+a}\,a^3c-24\sqrt{bx+a}\,a^3dx+20\sqrt{bx+a}\,a^2bcx+36\sqrt{bx+a}\,a^2bd\,x^2-30\sqrt{bx+a}\,a\,b^2c\,x^2+18}{}$$

input
```
int((d*x+c)/x^3/(b*x^3+a*x^2)^(1/2),x)
```

output
```
( - 16*sqrt(a + b*x)*a**3*c - 24*sqrt(a + b*x)*a**3*d*x + 20*sqrt(a + b*x)
*a**2*b*c*x + 36*sqrt(a + b*x)*a**2*b*d*x**2 - 30*sqrt(a + b*x)*a*b**2*c*x
**2 + 18*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b**2*d*x**3 - 15*sqrt(a)*l
og(sqrt(a + b*x) - sqrt(a))*b**3*c*x**3 - 18*sqrt(a)*log(sqrt(a + b*x) + s
qrt(a))*a*b**2*d*x**3 + 15*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**3*c*x**
3)/(48*a**4*x**3)
```

3.231 $\int \frac{c+dx}{x^4\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 179

$$\int \frac{c+dx}{x^4\sqrt{ax^2+bx^3}}\,dx = -\frac{c\sqrt{ax^2+bx^3}}{4ax^5}+\frac{(7bc-8ad)\sqrt{ax^2+bx^3}}{24a^2x^4} - \frac{5b(7bc-8ad)\sqrt{ax^2+bx^3}}{96a^3x^3}+\frac{5b^2(7bc-8ad)\sqrt{ax^2+bx^3}}{64a^4x^2} - \frac{5b^3(7bc-8ad)\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{64a^{9/2}}$$

output

```
-1/4*c*(b*x^3+a*x^2)^(1/2)/a/x^5+1/24*(-8*a*d+7*b*c)*(b*x^3+a*x^2)^(1/2)/a
^2/x^4-5/96*b*(-8*a*d+7*b*c)*(b*x^3+a*x^2)^(1/2)/a^3/x^3+5/64*b^2*(-8*a*d+
7*b*c)*(b*x^3+a*x^2)^(1/2)/a^4/x^2-5/64*b^3*(-8*a*d+7*b*c)*arctanh((b*x^3+
a*x^2)^(1/2)/a^(1/2)/x)/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.75

$$\int \frac{c+dx}{x^4\sqrt{ax^2+bx^3}}\,dx = \frac{-\sqrt{a}(a+bx)\left(-105b^3cx^3+16a^3(3c+4dx)-8a^2bx(7c+10dx)+10ab^2x^2(7c+12dx)\right)-15b^3(7bc-8a\cdots}{192a^{9/2}x^3\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(c + d*x)/(x^4*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-(Sqrt[a]*(a + b*x)*(-105*b^3*c*x^3 + 16*a^3*(3*c + 4*d*x) - 8*a^2*b*x*(7
*c + 10*d*x) + 10*a*b^2*x^2*(7*c + 12*d*x))) - 15*b^3*(7*b*c - 8*a*d)*x^4*
Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(192*a^(9/2)*x^3*Sqrt[x^2*(a
 + b*x)])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1944, 1931, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{x^4\sqrt{ax^2+bx^3}}\,dx$$

$$\Big\downarrow \text{ 1944}$$

$$-\frac{(7bc-8ad)\int \frac{1}{x^3\sqrt{bx^3+ax^2}}dx}{8a}-\frac{c\sqrt{ax^2+bx^3}}{4ax^5}$$

$$\Big\downarrow \text{ 1931}$$

$$-\frac{(7bc-8ad)\left(-\frac{5b\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{c\sqrt{ax^2+bx^3}}{4ax^5}$$

$$\Big\downarrow \text{ 1931}$$

$$-\frac{(7bc-8ad)\left(-\frac{5b\left(-\frac{3b\int\frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{c\sqrt{ax^2+bx^3}}{4ax^5}$$

$\downarrow$ 1931

$$-\frac{(7bc-8ad)\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int\frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{c\sqrt{ax^2+bx^3}}{4ax^5}$$

$\downarrow$ 1914

$$-\frac{(7bc-8ad)\left(-\frac{5b\left(-\frac{3b\left(\frac{b\int\frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{c\sqrt{ax^2+bx^3}}{4ax^5}$$

$\downarrow$ 219

$$-\frac{\left(-\frac{5b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)(7bc-8ad)}{8a}-\frac{c\sqrt{ax^2+bx^3}}{4ax^5}$$

input
```
Int[(c + d*x)/(x^4*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
-1/4*(c*Sqrt[a*x^2 + b*x^3])/(a*x^5) - ((7*b*c - 8*a*d)*(-1/3*Sqrt[a*x^2 +
 b*x^3]/(a*x^4) - (5*b*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (3*b*(-(Sqrt[a*
x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/
2)))/(4*a)))/(6*a)))/(8*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.46

method	result
pseudoelliptic	$-\frac{3\left(b^{2}x^{3}\left(ad-\frac{5bc}{6}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\frac{4\left(-\frac{5xb\left(\frac{9dx}{5}+c\right)a^{\frac{3}{2}}}{4}+\left(\frac{3dx}{2}+c\right)a^{\frac{5}{2}}+\frac{15\sqrt{a}\,b^{2}c\,x^{2}}{8}\right)\sqrt{bx+a}}{9}\right)}{4a^{\frac{7}{2}}x^{3}}$
risch	$-\frac{(bx+a)\left(120a\,b^{2}d\,x^{3}-105b^{3}c\,x^{3}-80a^{2}bd\,x^{2}+70a\,b^{2}c\,x^{2}+64a^{3}dx-56a^{2}bcx+48c\,a^{3}\right)}{192a^{4}x^{3}\sqrt{x^{2}(bx+a)}}+\frac{5(8ad-7bc)b^{3}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right.}{64a^{\frac{9}{2}}\sqrt{x^{2}(bx+a)}}$
default	$\frac{\sqrt{bx+a}\left(264a^{\frac{17}{2}}\sqrt{bx+a}\,d-584a^{\frac{15}{2}}(bx+a)^{\frac{3}{2}}d+440a^{\frac{13}{2}}(bx+a)^{\frac{5}{2}}d-120a^{\frac{11}{2}}(bx+a)^{\frac{7}{2}}d-279a^{\frac{15}{2}}\sqrt{bx+a}\,bc+511a^{\frac{13}{2}}(bx+a)\right.}{192x^{3}b\sqrt{b\,x^{3}+a\,x}}$

input
```
int((d*x+c)/x^4/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-3/4*(b^2*x^3*(a*d-5/6*b*c)*arctanh((b*x+a)^(1/2)/a^(1/2))+4/9*(-5/4*x*b*(
9/5*d*x+c)*a^(3/2)+(3/2*d*x+c)*a^(5/2)+15/8*a^(1/2)*b^2*c*x^2)*(b*x+a)^(1/
2))/a^(7/2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.65

$$\int \frac{c+dx}{x^4\sqrt{ax^2+bx^3}}\,dx$$
$$=\left[-\frac{15\,(7\,b^4c-8\,ab^3d)\sqrt{a}x^5\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(48\,a^4c-15\,(7\,ab^3c-8\,a^2b^2d)x^3+10\,(7\,a^2b^2c}{384\,a^5x^5}\right.$$

input
```
integrate((d*x+c)/x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/384*(15*(7*b^4*c - 8*a*b^3*d)*sqrt(a)*x^5*log((b*x^2 + 2*a*x + 2*sqrt(
b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*(48*a^4*c - 15*(7*a*b^3*c - 8*a^2*b^2*d)*
x^3 + 10*(7*a^2*b^2*c - 8*a^3*b*d)*x^2 - 8*(7*a^3*b*c - 8*a^4*d)*x)*sqrt(b
*x^3 + a*x^2))/(a^5*x^5), 1/192*(15*(7*b^4*c - 8*a*b^3*d)*sqrt(-a)*x^5*arc
tan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) - (48*a^4*c - 15*(7*a*b^3*
c - 8*a^2*b^2*d)*x^3 + 10*(7*a^2*b^2*c - 8*a^3*b*d)*x^2 - 8*(7*a^3*b*c - 8
*a^4*d)*x)*sqrt(b*x^3 + a*x^2))/(a^5*x^5)]
```

Sympy [F]

$$\int \frac{c+dx}{x^4\sqrt{ax^2+bx^3}}\,dx=\int \frac{c+dx}{x^4\sqrt{x^2\left(a+bx\right)}}\,dx$$

input
```
integrate((d*x+c)/x**4/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral((c + d*x)/(x**4*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{c+dx}{x^4\sqrt{ax^2+bx^3}}\,dx=\int \frac{dx+c}{\sqrt{bx^3+ax^2}x^4}\,dx$$

input
```
integrate((d*x+c)/x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/(sqrt(b*x^3 + a*x^2)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.01

$$\int \frac{c+dx}{x^4\sqrt{ax^2+bx^3}}\,dx = \frac{\frac{15\,(7\,b^5c-8\,ab^4d)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4}+\frac{105\,(bx+a)^{\frac{7}{2}}b^5c-385\,(bx+a)^{\frac{5}{2}}ab^5c+511\,(bx+a)^{\frac{3}{2}}a^2b^5c-279\,\sqrt{bx+a}a^3b^5c-120\,(bx+a)^{\frac{7}{2}}ab^4d+440\,(bx}{a^4b^4x^4}}{192\,b\text{sgn}\left(x\right)}$$

input
```
integrate((d*x+c)/x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/192*(15*(7*b^5*c - 8*a*b^4*d)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a
^4) + (105*(b*x + a)^(7/2)*b^5*c - 385*(b*x + a)^(5/2)*a*b^5*c + 511*(b*x
+ a)^(3/2)*a^2*b^5*c - 279*sqrt(b*x + a)*a^3*b^5*c - 120*(b*x + a)^(7/2)*a
*b^4*d + 440*(b*x + a)^(5/2)*a^2*b^4*d - 584*(b*x + a)^(3/2)*a^3*b^4*d + 2
64*sqrt(b*x + a)*a^4*b^4*d)/(a^4*b^4*x^4))/(b*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{x^4\sqrt{ax^2+bx^3}}\,dx = \int \frac{c+d\,x}{x^4\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int((c + d*x)/(x^4*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
int((c + d*x)/(x^4*(a*x^2 + b*x^3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.15

$$\int \frac{c+dx}{x^4\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{-96\sqrt{bx+a}\,a^4c-128\sqrt{bx+a}\,a^4dx+112\sqrt{bx+a}\,a^3bcx+160\sqrt{bx+a}\,a^3bd\,x^2-140\sqrt{bx+a}\,a^2b^2c\,x^2}{}$$

input
```
int((d*x+c)/x^4/(b*x^3+a*x^2)^(1/2),x)
```

output
```
( - 96*sqrt(a + b*x)*a**4*c - 128*sqrt(a + b*x)*a**4*d*x + 112*sqrt(a + b*
x)*a**3*b*c*x + 160*sqrt(a + b*x)*a**3*b*d*x**2 - 140*sqrt(a + b*x)*a**2*b
**2*c*x**2 - 240*sqrt(a + b*x)*a**2*b**2*d*x**3 + 210*sqrt(a + b*x)*a*b**3
*c*x**3 - 120*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b**3*d*x**4 + 105*sqr
t(a)*log(sqrt(a + b*x) - sqrt(a))*b**4*c*x**4 + 120*sqrt(a)*log(sqrt(a + b
*x) + sqrt(a))*a*b**3*d*x**4 - 105*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b*
*4*c*x**4)/(384*a**5*x**4)
```

3.232 $\int \frac{x^3(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 188

$$\int \frac{x^3(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{2a^2(bc-ad)^2\sqrt{ax^2+bx^3}}{b^5x} - \frac{4a(bc-2ad)(bc-ad)\left(ax^2+bx^3\right)^{3/2}}{3b^5x^3} + \frac{2(b^2c^2-6abcd+6a^2d^2)\left(ax^2+bx^3\right)^{5/2}}{5b^5x^5} + \frac{4d(bc-2ad)\left(ax^2+bx^3\right)^{7/2}}{7b^5x^7} + \frac{2d^2\left(ax^2+bx^3\right)^{9/2}}{9b^5x^9}$$

output
```
2*a^2*(-a*d+b*c)^2*(b*x^3+a*x^2)^(1/2)/b^5/x-4/3*a*(-2*a*d+b*c)*(-a*d+b*c)
*(b*x^3+a*x^2)^(3/2)/b^5/x^3+2/5*(6*a^2*d^2-6*a*b*c*d+b^2*c^2)*(b*x^3+a*x^
2)^(5/2)/b^5/x^5+4/7*d*(-2*a*d+b*c)*(b*x^3+a*x^2)^(7/2)/b^5/x^7+2/9*d^2*(b
*x^3+a*x^2)^(9/2)/b^5/x^9
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.67

$$\int \frac{x^3(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{2\sqrt{x^2(a+bx)}(128a^4d^2-32a^3bd(9c+2dx)+24a^2b^2(7c^2+6cdx+2d^2x^2)-4ab^3x(21c^2+27cdx+10d}{315b^5x}$$

input
```
Integrate[(x^3*(c + d*x)^2)/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*Sqrt[x^2*(a + b*x)]*(128*a^4*d^2 - 32*a^3*b*d*(9*c + 2*d*x) + 24*a^2*b^
2*(7*c^2 + 6*c*d*x + 2*d^2*x^2) - 4*a*b^3*x*(21*c^2 + 27*c*d*x + 10*d^2*x^
2) + b^4*x^2*(63*c^2 + 90*c*d*x + 35*d^2*x^2)))/(315*b^5*x)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.90, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx$$
$$\Big\downarrow \text{1948}$$
$$\frac{x\sqrt{a+bx}\int \frac{x^2(c+dx)^2}{\sqrt{a+bx}}dx}{\sqrt{ax^2+bx^3}}$$
$$\Big\downarrow \text{99}$$
$$\frac{x\sqrt{a+bx}\int \left(\frac{d^2(a+bx)^{7/2}}{b^4}+\frac{2d(bc-2ad)(a+bx)^{5/2}}{b^4}+\frac{(b^2c^2-6abdc+6a^2d^2)(a+bx)^{3/2}}{b^4}+\frac{2a(bc-2ad)(ad-bc)\sqrt{a+bx}}{b^4}+\frac{a^2(ad-bc)^2}{b^4\sqrt{a+bx}}\right)}{\sqrt{ax^2+bx^3}}$$
$$\Big\downarrow \text{2009}$$

$$\frac{x\sqrt{a+bx}\left(\frac{2a^2\sqrt{a+bx}(bc-ad)^2}{b^5}+\frac{2(a+bx)^{5/2}(6a^2d^2-6abcd+b^2c^2)}{5b^5}+\frac{4d(a+bx)^{7/2}(bc-2ad)}{7b^5}-\frac{4a(a+bx)^{3/2}(bc-2ad)(bc-ad)}{3b^5}+\frac{2d^2(a}{9}\right.}{\sqrt{ax^2+bx^3}}$$

input
```
Int[(x^3*(c + d*x)^2)/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(x*Sqrt[a + b*x]*((2*a^2*(b*c - a*d)^2*Sqrt[a + b*x])/b^5 - (4*a*(b*c - 2*
a*d)*(b*c - a*d)*(a + b*x)^(3/2))/(3*b^5) + (2*(b^2*c^2 - 6*a*b*c*d + 6*a^
2*d^2)*(a + b*x)^(5/2))/(5*b^5) + (4*d*(b*c - 2*a*d)*(a + b*x)^(7/2))/(7*b
^5) + (2*d^2*(a + b*x)^(9/2))/(9*b^5)))/Sqrt[a*x^2 + b*x^3]
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$-\frac{512\sqrt{bx+a}\left(-\frac{99x^3\left(\frac{7}{11}d^2x^2+\frac{14}{9}cdx+c^2\right)b^5}{256}+\frac{297x^2\left(\frac{175}{297}d^2x^2+\frac{40}{27}cdx+c^2\right)a\,b^4}{640}-\frac{99x\,a^2\left(\frac{50}{99}d^2x^2+\frac{4}{3}cdx+c^2\right)b^3}{160}+\frac{99a^3\left(\frac{10}{33}d^2 \cdots\right)}{\cdots}\right)}{693b^6}$
trager	$\frac{2\left(35d^2x^4b^4-40a\,b^3d^2x^3+90b^4cd\,x^3+48a^2b^2d^2x^2-108a\,b^3cd\,x^2+63c^2x^2b^4-64a^3b\,d^2x+144a^2b^2cdx-84a\,b^3c^2x+128a^4d^2 \cdots\right)}{315b^5x}$
risch	$\frac{2(bx+a)x\left(35d^2x^4b^4-40a\,b^3d^2x^3+90b^4cd\,x^3+48a^2b^2d^2x^2-108a\,b^3cd\,x^2+63c^2x^2b^4-64a^3b\,d^2x+144a^2b^2cdx-84a\,b^3c^2x+ \cdots\right)}{315\sqrt{x^2(bx+a)}\,b^5}$
gosper	$\frac{2(bx+a)\left(35d^2x^4b^4-40a\,b^3d^2x^3+90b^4cd\,x^3+48a^2b^2d^2x^2-108a\,b^3cd\,x^2+63c^2x^2b^4-64a^3b\,d^2x+144a^2b^2cdx-84a\,b^3c^2x+1 \cdots\right)}{315b^5\sqrt{b\,x^3+a\,x^2}}$
default	$\frac{2(bx+a)\left(35d^2x^4b^4-40a\,b^3d^2x^3+90b^4cd\,x^3+48a^2b^2d^2x^2-108a\,b^3cd\,x^2+63c^2x^2b^4-64a^3b\,d^2x+144a^2b^2cdx-84a\,b^3c^2x+1 \cdots\right)}{315b^5\sqrt{b\,x^3+a\,x^2}}$
orering	$\frac{2(bx+a)\left(35d^2x^4b^4-40a\,b^3d^2x^3+90b^4cd\,x^3+48a^2b^2d^2x^2-108a\,b^3cd\,x^2+63c^2x^2b^4-64a^3b\,d^2x+144a^2b^2cdx-84a\,b^3c^2x+1 \cdots\right)}{315b^5\sqrt{b\,x^3+a\,x^2}}$

input
```
int(x^3*(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-512/693*(b*x+a)^(1/2)*(-99/256*x^3*(7/11*d^2*x^2+14/9*c*d*x+c^2)*b^5+297/
640*x^2*(175/297*d^2*x^2+40/27*c*d*x+c^2)*a*b^4-99/160*x*a^2*(50/99*d^2*x^
2+4/3*c*d*x+c^2)*b^3+99/80*a^3*(10/33*d^2*x^2+8/9*c*d*x+c^2)*b^2-11/5*d*a^
4*(5/22*d*x+c)*b+d^2*a^5)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.78

$$\int \frac{x^3(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{2\left(35\,b^4d^2x^4+168\,a^2b^2c^2-288\,a^3bcd+128\,a^4d^2+10\,(9\,b^4cd-4\,ab^3d^2)x^3+3\,(21\,b^4c^2-36\,ab^3cd+16\,a \cdots\right)}{315\,b^5x}$$

input
```
integrate(x^3*(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*b^4*d^2*x^4 + 168*a^2*b^2*c^2 - 288*a^3*b*c*d + 128*a^4*d^2 + 10
*(9*b^4*c*d - 4*a*b^3*d^2)*x^3 + 3*(21*b^4*c^2 - 36*a*b^3*c*d + 16*a^2*b^2
*d^2)*x^2 - 4*(21*a*b^3*c^2 - 36*a^2*b^2*c*d + 16*a^3*b*d^2)*x)*sqrt(b*x^3
 + a*x^2)/(b^5*x)
```

Sympy [F]

$$\int \frac{x^3(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^3(c+dx)^2}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(x**3*(d*x+c)**2/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**3*(c + d*x)**2/sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.89

$$\int \frac{x^3(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,(3\,b^3x^3-ab^2x^2+4\,a^2bx+8\,a^3)c^2}{15\,\sqrt{bx+a}b^3} + \frac{4\,(5\,b^4x^4-ab^3x^3+2\,a^2b^2x^2-8\,a^3bx-16\,a^4)cd}{35\,\sqrt{bx+a}b^4} + \frac{2\,(35\,b^5x^5-5\,ab^4x^4+8\,a^2b^3x^3-16\,a^3b^2x^2+64\,a^4bx+128\,a^5)d^2}{315\,\sqrt{bx+a}b^5}$$

input
```
integrate(x^3*(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(3*b^3*x^3 - a*b^2*x^2 + 4*a^2*b*x + 8*a^3)*c^2/(sqrt(b*x + a)*b^3) +
 4/35*(5*b^4*x^4 - a*b^3*x^3 + 2*a^2*b^2*x^2 - 8*a^3*b*x - 16*a^4)*c*d/(sq
rt(b*x + a)*b^4) + 2/315*(35*b^5*x^5 - 5*a*b^4*x^4 + 8*a^2*b^3*x^3 - 16*a^
3*b^2*x^2 + 64*a^4*b*x + 128*a^5)*d^2/(sqrt(b*x + a)*b^5)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.06

$$\int \frac{x^3(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{2\left(\frac{21\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)c^2}{b^2}+\frac{18\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)cd}{b^3}+\frac{\left(35\,(bx+a)^{\frac{9}{2}}-18\right.}{}\right.}{315\,b\mathrm{sgn}\,(x)}$$
$$-\frac{16\left(21\,a^{\frac{5}{2}}b^2c^2-36\,a^{\frac{7}{2}}bcd+16\,a^{\frac{9}{2}}d^2\right)\mathrm{sgn}(x)}{315\,b^5}$$

input
```
integrate(x^3*(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
2/315*(21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2
)*c^2/b^2 + 18*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3
/2)*a^2 - 35*sqrt(b*x + a)*a^3)*c*d/b^3 + (35*(b*x + a)^(9/2) - 180*(b*x +
 a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt
(b*x + a)*a^4)*d^2/b^4)/(b*sgn(x)) - 16/315*(21*a^(5/2)*b^2*c^2 - 36*a^(7/
2)*b*c*d + 16*a^(9/2)*d^2)*sgn(x)/b^5
```

Mupad [B] (verification not implemented)

Time = 9.63 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.81

$$\int \frac{x^3(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{2\,\sqrt{b\,x^3+a\,x^2}\,(-64\,a^3\,d^2+144\,a^2\,b\,c\,d+48\,a^2\,b\,d^2\,x-84\,a\,b^2\,c^2-108\,a\,b^2\,c\,d\,x-40\,a\,b^2\,d^2\,x^2+63\,b^3\,c^2}{315\,b^4}$$
$$+\frac{16\,a^2\,\sqrt{b\,x^3+a\,x^2}\,(16\,a^2\,d^2-36\,a\,b\,c\,d+21\,b^2\,c^2)}{315\,b^5\,x}$$

input
```
int((x^3*(c + d*x)^2)/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(63*b^3*c^2*x - 84*a*b^2*c^2 - 64*a^3*d^2 + 35*b^
3*d^2*x^3 - 40*a*b^2*d^2*x^2 + 144*a^2*b*c*d + 48*a^2*b*d^2*x + 90*b^3*c*d
*x^2 - 108*a*b^2*c*d*x))/(315*b^4) + (16*a^2*(a*x^2 + b*x^3)^(1/2)*(16*a^2
*d^2 + 21*b^2*c^2 - 36*a*b*c*d))/(315*b^5*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.74

$$\int \frac{x^3(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{2\sqrt{bx+a}\,(35b^4d^2x^4 - 40a\,b^3d^2x^3 + 90b^4cd\,x^3 + 48a^2b^2d^2x^2 - 108a\,b^3cd\,x^2 + 63b^4c^2x^2 - 64a^3b\,d^2x + 14}{315b^5}$$

input
```
int(x^3*(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*(128*a**4*d**2 - 288*a**3*b*c*d - 64*a**3*b*d**2*x + 168*
a**2*b**2*c**2 + 144*a**2*b**2*c*d*x + 48*a**2*b**2*d**2*x**2 - 84*a*b**3*
c**2*x - 108*a*b**3*c*d*x**2 - 40*a*b**3*d**2*x**3 + 63*b**4*c**2*x**2 + 9
0*b**4*c*d*x**3 + 35*b**4*d**2*x**4))/(315*b**5)
```

3.233 $\int \frac{x^2(c+dx)^2}{\sqrt{ax^2+bx^3}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 139

$$\int \frac{x^2(c+dx)^2}{\sqrt{ax^2+bx^3}} \, dx = -\frac{2a(bc-ad)^2\sqrt{ax^2+bx^3}}{b^4x} + \frac{2(bc-3ad)(bc-ad)\left(ax^2+bx^3\right)^{3/2}}{3b^4x^3} + \frac{2d(2bc-3ad)\left(ax^2+bx^3\right)^{5/2}}{5b^4x^5} + \frac{2d^2\left(ax^2+bx^3\right)^{7/2}}{7b^4x^7}$$

output
```
-2*a*(-a*d+b*c)^2*(b*x^3+a*x^2)^(1/2)/b^4/x+2/3*(-3*a*d+b*c)*(-a*d+b*c)*(b
*x^3+a*x^2)^(3/2)/b^4/x^3+2/5*d*(-3*a*d+2*b*c)*(b*x^3+a*x^2)^(5/2)/b^4/x^5
+2/7*d^2*(b*x^3+a*x^2)^(7/2)/b^4/x^7
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.69

$$\int \frac{x^2(c+dx)^2}{\sqrt{ax^2+bx^3}} \, dx = \frac{2\sqrt{x^2(a+bx)}(-48a^3d^2+8a^2bd(14c+3dx)-2ab^2(35c^2+28cdx+9d^2x^2)+b^3x(35c^2+42cdx+15d^2x}{105b^4x}$$

input
```
Integrate[(x^2*(c + d*x)^2)/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*Sqrt[x^2*(a + b*x)]*(-48*a^3*d^2 + 8*a^2*b*d*(14*c + 3*d*x) - 2*a*b^2*(
35*c^2 + 28*c*d*x + 9*d^2*x^2) + b^3*x*(35*c^2 + 42*c*d*x + 15*d^2*x^2)))/
(105*b^4*x)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx)^2}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1948}$$

$$\frac{x\sqrt{a+bx}\int \frac{x(c+dx)^2}{\sqrt{a+bx}}dx}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{86}$$

$$\frac{x\sqrt{a+bx}\int \left(\frac{d^2(a+bx)^{5/2}}{b^3}+\frac{d(2bc-3ad)(a+bx)^{3/2}}{b^3}+\frac{(bc-3ad)(bc-ad)\sqrt{a+bx}}{b^3}-\frac{a(ad-bc)^2}{b^3\sqrt{a+bx}}\right)dx}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\sqrt{a+bx}\left(\frac{2d(a+bx)^{5/2}(2bc-3ad)}{5b^4}+\frac{2(a+bx)^{3/2}(bc-3ad)(bc-ad)}{3b^4}-\frac{2a\sqrt{a+bx}(bc-ad)^2}{b^4}+\frac{2d^2(a+bx)^{7/2}}{7b^4}\right)}{\sqrt{ax^2+bx^3}}$$

input
```
Int[(x^2*(c + d*x)^2)/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(x*Sqrt[a + b*x]*((-2*a*(b*c - a*d)^2*Sqrt[a + b*x])/b^4 + (2*(b*c - 3*a*d
)*(b*c - a*d)*(a + b*x)^(3/2))/(3*b^4) + (2*d*(2*b*c - 3*a*d)*(a + b*x)^(5
/2))/(5*b^4) + (2*d^2*(a + b*x)^(7/2))/(7*b^4)))/Sqrt[a*x^2 + b*x^3]
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{256\left(\frac{63x^{2}\left(\frac{5}{9}d^{2}x^{2}+\frac{10}{7}cdx+c^{2}\right)b^{4}}{128}-\frac{21\left(\frac{10}{21}d^{2}x^{2}+\frac{9}{7}cdx+c^{2}\right)xa\,b^{3}}{32}+\frac{21a^{2}\left(\frac{2}{7}d^{2}x^{2}+\frac{6}{7}cdx+c^{2}\right)b^{2}}{16}-\frac{9d\left(\frac{2dx}{9}+c\right)a^{3}b}{4}+a^{4}d^{2}\right)\sqrt{bx+a}}{315b^{5}}$
trager	$-\frac{2\left(-15d^{2}x^{3}b^{3}+18a\,b^{2}d^{2}x^{2}-42b^{3}cd\,x^{2}-24a^{2}b\,d^{2}x+56a\,b^{2}cdx-35b^{3}c^{2}x+48a^{3}d^{2}-112a^{2}bcd+70a\,c^{2}b^{2}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{105b^{4}x}$
risch	$-\frac{2(bx+a)x\left(-15d^{2}x^{3}b^{3}+18a\,b^{2}d^{2}x^{2}-42b^{3}cd\,x^{2}-24a^{2}b\,d^{2}x+56a\,b^{2}cdx-35b^{3}c^{2}x+48a^{3}d^{2}-112a^{2}bcd+70a\,c^{2}b^{2}\right)}{105\sqrt{x^{2}(bx+a)}\,b^{4}}$
gosper	$-\frac{2(bx+a)\left(-15d^{2}x^{3}b^{3}+18a\,b^{2}d^{2}x^{2}-42b^{3}cd\,x^{2}-24a^{2}b\,d^{2}x+56a\,b^{2}cdx-35b^{3}c^{2}x+48a^{3}d^{2}-112a^{2}bcd+70a\,c^{2}b^{2}\right)x}{105b^{4}\sqrt{b\,x^{3}+a\,x^{2}}}$
default	$-\frac{2(bx+a)\left(-15d^{2}x^{3}b^{3}+18a\,b^{2}d^{2}x^{2}-42b^{3}cd\,x^{2}-24a^{2}b\,d^{2}x+56a\,b^{2}cdx-35b^{3}c^{2}x+48a^{3}d^{2}-112a^{2}bcd+70a\,c^{2}b^{2}\right)x}{105b^{4}\sqrt{b\,x^{3}+a\,x^{2}}}$
orering	$-\frac{2(bx+a)\left(-15d^{2}x^{3}b^{3}+18a\,b^{2}d^{2}x^{2}-42b^{3}cd\,x^{2}-24a^{2}b\,d^{2}x+56a\,b^{2}cdx-35b^{3}c^{2}x+48a^{3}d^{2}-112a^{2}bcd+70a\,c^{2}b^{2}\right)x}{105b^{4}\sqrt{b\,x^{3}+a\,x^{2}}}$

input
```
int(x^2*(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
256/315*(63/128*x^2*(5/9*d^2*x^2+10/7*c*d*x+c^2)*b^4-21/32*(10/21*d^2*x^2+
9/7*c*d*x+c^2)*x*a*b^3+21/16*a^2*(2/7*d^2*x^2+6/7*c*d*x+c^2)*b^2-9/4*d*(2/
9*d*x+c)*a^3*b+a^4*d^2)*(b*x+a)^(1/2)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.78

$$\int \frac{x^2(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,(15\,b^3d^2x^3 - 70\,ab^2c^2 + 112\,a^2bcd - 48\,a^3d^2 + 6\,(7\,b^3cd - 3\,ab^2d^2)x^2 + (35\,b^3c^2 - 56\,ab^2cd + 24\,a^2bd^2)x}{105\,b^4x}$$

input
```
integrate(x^2*(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*b^3*d^2*x^3 - 70*a*b^2*c^2 + 112*a^2*b*c*d - 48*a^3*d^2 + 6*(7*b
^3*c*d - 3*a*b^2*d^2)*x^2 + (35*b^3*c^2 - 56*a*b^2*c*d + 24*a^2*b*d^2)*x)*
sqrt(b*x^3 + a*x^2)/(b^4*x)
```

Sympy [F]

$$\int \frac{x^2(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^2(c+dx)^2}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(x**2*(d*x+c)**2/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**2*(c + d*x)**2/sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.96

$$\int \frac{x^2(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,(b^2x^2-abx-2\,a^2)c^2}{3\,\sqrt{bx+a}b^2} + \frac{4\,(3\,b^3x^3-ab^2x^2+4\,a^2bx+8\,a^3)cd}{15\,\sqrt{bx+a}b^3} + \frac{2\,(5\,b^4x^4-ab^3x^3+2\,a^2b^2x^2-8\,a^3bx-16\,a^4)d^2}{35\,\sqrt{bx+a}b^4}$$

input
```
integrate(x^2*(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(b^2*x^2 - a*b*x - 2*a^2)*c^2/(sqrt(b*x + a)*b^2) + 4/15*(3*b^3*x^3 -
a*b^2*x^2 + 4*a^2*b*x + 8*a^3)*c*d/(sqrt(b*x + a)*b^3) + 2/35*(5*b^4*x^4 -
 a*b^3*x^3 + 2*a^2*b^2*x^2 - 8*a^3*b*x - 16*a^4)*d^2/(sqrt(b*x + a)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.17

$$\int \frac{x^2(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx$$

$$= \frac{2\left(\frac{35\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)c^2}{b} + \frac{14\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)cd}{b^2} + \frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\sqrt{b}\right.}{b^3}}{105\,b\mathrm{sgn}\,(x)} + \frac{4\left(35\,a^{\frac{3}{2}}b^2c^2-56\,a^{\frac{5}{2}}bcd+24\,a^{\frac{7}{2}}d^2\right)\mathrm{sgn}(x)}{105\,b^4}$$

input
```
integrate(x^2*(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
2/105*(35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*c^2/b + 14*(3*(b*x + a)^(5
/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*c*d/b^2 + 3*(5*(b*x + a
)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)
*a^3)*d^2/b^3)/(b*sgn(x)) + 4/105*(35*a^(3/2)*b^2*c^2 - 56*a^(5/2)*b*c*d +
 24*a^(7/2)*d^2)*sgn(x)/b^4
```

Mupad [B] (verification not implemented)

Time = 9.74 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.82

$$\int \frac{x^2(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{b\,x^3+a\,x^2}\,\left(24\,a^2\,d^2-56\,a\,b\,c\,d-18\,a\,b\,d^2\,x+35\,b^2\,c^2+42\,b^2\,c\,d\,x+15\,b^2\,d^2\,x^2\right)}{105\,b^3} - \frac{4\,a\,\sqrt{b\,x^3+a\,x^2}\,\left(24\,a^2\,d^2-56\,a\,b\,c\,d+35\,b^2\,c^2\right)}{105\,b^4\,x}$$

input
```
int((x^2*(c + d*x)^2)/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(24*a^2*d^2 + 35*b^2*c^2 + 15*b^2*d^2*x^2 - 18*a*
b*d^2*x + 42*b^2*c*d*x - 56*a*b*c*d))/(105*b^3) - (4*a*(a*x^2 + b*x^3)^(1/
2)*(24*a^2*d^2 + 35*b^2*c^2 - 56*a*b*c*d))/(105*b^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.71

$$\int \frac{x^2(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{bx+a}\,(15b^3d^2x^3-18a\,b^2d^2x^2+42b^3cd\,x^2+24a^2b\,d^2x-56a\,b^2cdx+35b^3c^2x-48a^3d^2+112a^2bcd}{105b^4}$$

input
```
int(x^2*(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 48*a**3*d**2 + 112*a**2*b*c*d + 24*a**2*b*d**2*x - 70
*a*b**2*c**2 - 56*a*b**2*c*d*x - 18*a*b**2*d**2*x**2 + 35*b**3*c**2*x + 42
*b**3*c*d*x**2 + 15*b**3*d**2*x**3))/(105*b**4)
```

3.234 $\int \frac{x(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 96

$$\int \frac{x(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{2(bc-ad)^2\sqrt{ax^2+bx^3}}{b^3x} + \frac{4d(bc-ad)\left(ax^2+bx^3\right)^{3/2}}{3b^3x^3} + \frac{2d^2\left(ax^2+bx^3\right)^{5/2}}{5b^3x^5}$$

output
```
2*(-a*d+b*c)^2*(b*x^3+a*x^2)^(1/2)/b^3/x+4/3*d*(-a*d+b*c)*(b*x^3+a*x^2)^(3
/2)/b^3/x^3+2/5*d^2*(b*x^3+a*x^2)^(5/2)/b^3/x^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.70

$$\int \frac{x(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{x^2(a+bx)}(8a^2d^2-4abd(5c+dx)+b^2(15c^2+10cdx+3d^2x^2))}{15b^3x}$$

input
```
Integrate[(x*(c + d*x)^2)/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*Sqrt[x^2*(a + b*x)]*(8*a^2*d^2 - 4*a*b*d*(5*c + d*x) + b^2*(15*c^2 + 10
*c*d*x + 3*d^2*x^2)))/(15*b^3*x)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1948, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx)^2}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \ 1948$$

$$\frac{x\sqrt{a+bx}\int \frac{(c+dx)^2}{\sqrt{a+bx}}dx}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \ 53$$

$$\frac{x\sqrt{a+bx}\int \left(\frac{(a+bx)^{3/2}d^2}{b^2}+\frac{2(bc-ad)\sqrt{a+bx}d}{b^2}+\frac{(bc-ad)^2}{b^2\sqrt{a+bx}}\right)dx}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \ 2009$$

$$\frac{x\sqrt{a+bx}\left(\frac{4d(a+bx)^{3/2}(bc-ad)}{3b^3}+\frac{2\sqrt{a+bx}(bc-ad)^2}{b^3}+\frac{2d^2(a+bx)^{5/2}}{5b^3}\right)}{\sqrt{ax^2+bx^3}}$$

input
```
Int[(x*(c + d*x)^2)/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(x*Sqrt[a + b*x]*((2*(b*c - a*d)^2*Sqrt[a + b*x])/b^3 + (4*d*(b*c - a*d)*(
a + b*x)^(3/2))/(3*b^3) + (2*d^2*(a + b*x)^(5/2))/(5*b^3)))/Sqrt[a*x^2 + b
*x^3]
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.75

method	result	size
trager	$\frac{2\left(3b^{2}d^{2}x^{2}-4ab\,d^{2}x+10b^{2}cxd+8a^{2}d^{2}-20abcd+15b^{2}c^{2}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{15b^{3}x}$	72
risch	$\frac{2(bx+a)x\left(3b^{2}d^{2}x^{2}-4ab\,d^{2}x+10b^{2}cxd+8a^{2}d^{2}-20abcd+15b^{2}c^{2}\right)}{15\sqrt{x^{2}(bx+a)}\,b^{3}}$	73
gosper	$\frac{2(bx+a)\left(3b^{2}d^{2}x^{2}-4ab\,d^{2}x+10b^{2}cxd+8a^{2}d^{2}-20abcd+15b^{2}c^{2}\right)x}{15b^{3}\sqrt{b\,x^{3}+a\,x^{2}}}$	75
default	$\frac{2(bx+a)\left(3b^{2}d^{2}x^{2}-4ab\,d^{2}x+10b^{2}cxd+8a^{2}d^{2}-20abcd+15b^{2}c^{2}\right)x}{15b^{3}\sqrt{b\,x^{3}+a\,x^{2}}}$	75
orering	$\frac{2(bx+a)\left(3b^{2}d^{2}x^{2}-4ab\,d^{2}x+10b^{2}cxd+8a^{2}d^{2}-20abcd+15b^{2}c^{2}\right)x}{15b^{3}\sqrt{b\,x^{3}+a\,x^{2}}}$	75
pseudoelliptic	$-\frac{2\sqrt{bx+a}\,\left(-15d^{2}x^{3}b^{3}+18a\,b^{2}d^{2}x^{2}-42b^{3}cd\,x^{2}-24a^{2}b\,d^{2}x+56a\,b^{2}cdx-35b^{3}c^{2}x+48a^{3}d^{2}-112a^{2}bcd+70a\,c^{2}b^{2}\right)}{105b^{4}}$	100

input
```
int(x*(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(3*b^2*d^2*x^2-4*a*b*d^2*x+10*b^2*c*d*x+8*a^2*d^2-20*a*b*c*d+15*b^2*c
^2)/b^3/x*(b*x^3+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.76

$$\int \frac{x(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\left(3\,b^2d^2x^2+15\,b^2c^2-20\,abcd+8\,a^2d^2+2\,(5\,b^2cd-2\,abd^2)x\right)\sqrt{bx^3+ax^2}}{15\,b^3x}$$

input
```
integrate(x*(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*b^2*d^2*x^2 + 15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2 + 2*(5*b^2*c*d -
 2*a*b*d^2)*x)*sqrt(b*x^3 + a*x^2)/(b^3*x)
```

Sympy [F]

$$\int \frac{x(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x(c+dx)^2}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(x*(d*x+c)**2/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x*(c + d*x)**2/sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.97

$$\int \frac{x(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{bx+a}c^2}{b} + \frac{4\,(b^2x^2-abx-2\,a^2)cd}{3\sqrt{bx+a}b^2} + \frac{2\,(3\,b^3x^3-ab^2x^2+4\,a^2bx+8\,a^3)d^2}{15\sqrt{bx+a}b^3}$$

input
```
integrate(x*(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(b*x + a)*c^2/b + 4/3*(b^2*x^2 - a*b*x - 2*a^2)*c*d/(sqrt(b*x + a)*b
^2) + 2/15*(3*b^3*x^3 - a*b^2*x^2 + 4*a^2*b*x + 8*a^3)*d^2/(sqrt(b*x + a)*
b^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.27

$$\int \frac{x(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\left(15\sqrt{bx+a}c^2 + \frac{10\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)cd}{b} + \frac{\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)d^2}{b^2}\right)}{15\,b\mathrm{sgn}\left(x\right)} - \frac{2\left(15\sqrt{a}b^2c^2-20\,a^{\frac{3}{2}}bcd+8\,a^{\frac{5}{2}}d^2\right)\mathrm{sgn}(x)}{15\,b^3}$$

input
```
integrate(x*(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
2/15*(15*sqrt(b*x + a)*c^2 + 10*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*c*d/
b + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*d^2/
b^2)/(b*sgn(x)) - 2/15*(15*sqrt(a)*b^2*c^2 - 20*a^(3/2)*b*c*d + 8*a^(5/2)*
d^2)*sgn(x)/b^3
```

Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.76

$$\int \frac{x(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{16\,a^2\,d^2-40\,a\,b\,c\,d+30\,b^2\,c^2}{15\,b^3}+\frac{2\,d^2\,x^2}{5\,b}-\frac{4\,d\,x\,(2\,a\,d-5\,b\,c)}{15\,b^2}\right)}{x}$$

input
```
int((x*(c + d*x)^2)/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
((a*x^2 + b*x^3)^(1/2)*((16*a^2*d^2 + 30*b^2*c^2 - 40*a*b*c*d)/(15*b^3) +
(2*d^2*x^2)/(5*b) - (4*d*x*(2*a*d - 5*b*c))/(15*b^2)))/x
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.64

$$\int \frac{x(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{bx+a}\,(3b^2d^2x^2-4ab\,d^2x+10b^2cdx+8a^2d^2-20abcd+15b^2c^2)}{15b^3}$$

input
```
int(x*(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*(8*a**2*d**2 - 20*a*b*c*d - 4*a*b*d**2*x + 15*b**2*c**2 +
 10*b**2*c*d*x + 3*b**2*d**2*x**2))/(15*b**3)
```

3.235 $\int \frac{(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 97

$$\int \frac{(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{2d(2bc-ad)\sqrt{ax^2+bx^3}}{b^2x}+\frac{2d^2(ax^2+bx^3)^{3/2}}{3b^2x^3}-\frac{2c^2\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{\sqrt{a}}$$

output
```
2*d*(-a*d+2*b*c)*(b*x^3+a*x^2)^(1/2)/b^2/x+2/3*d^2*(b*x^3+a*x^2)^(3/2)/b^2
/x^3-2*c^2*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{a}dx(a+bx)(6bc-2ad+bdx)-6b^2c^2x\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{3\sqrt{a}b^2\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(c + d*x)^2/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*Sqrt[a]*d*x*(a + b*x)*(6*b*c - 2*a*d + b*d*x) - 6*b^2*c^2*x*Sqrt[a + b*
x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(3*Sqrt[a]*b^2*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2450, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx$$
$$\Big\downarrow \text{2450}$$
$$\int \left(\frac{c^2}{\sqrt{ax^2+bx^3}}+\frac{2cdx}{\sqrt{ax^2+bx^3}}+\frac{d^2x^2}{\sqrt{ax^2+bx^3}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{2c^2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}-\frac{4ad^2\sqrt{ax^2+bx^3}}{3b^2x}+\frac{4cd\sqrt{ax^2+bx^3}}{bx}+\frac{2d^2\sqrt{ax^2+bx^3}}{3b}$$

input
```
Int[(c + d*x)^2/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*d^2*Sqrt[a*x^2 + b*x^3])/(3*b) + (4*c*d*Sqrt[a*x^2 + b*x^3])/(b*x) - (4
*a*d^2*Sqrt[a*x^2 + b*x^3])/(3*b^2*x) - (2*c^2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*
x^2 + b*x^3]])/Sqrt[a]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2450
```
Int[(Pq_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[Expan
dIntegrand[Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && (Po
lyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IntegerQ[p] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.65

method	result	size
pseudoelliptic	$\frac{2\sqrt{bx+a}\,(3b^2d^2x^2-4ab\,d^2x+10b^2cxd+8a^2d^2-20abcd+15b^2c^2)}{15b^3}$	63
default	$\frac{2x\sqrt{bx+a}\left(d^2(bx+a)^{\frac{3}{2}}\sqrt{a}-3a^{\frac{3}{2}}d^2\sqrt{bx+a}+6bcd\sqrt{bx+a}\,\sqrt{a}-3b^2c^2\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{3\sqrt{b\,x^3+a\,x^2}\,b^2\sqrt{a}}$	95

input
```
int((d*x+c)^2/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(b*x+a)^(1/2)*(3*b^2*d^2*x^2-4*a*b*d^2*x+10*b^2*c*d*x+8*a^2*d^2-20*a*
b*c*d+15*b^2*c^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.94

$$\int \frac{(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx$$
$$=\left[\frac{3\sqrt{a}b^2c^2x\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(abd^2x+6\,abcd-2\,a^2d^2)\sqrt{bx^3+ax^2}}{3\,ab^2x}, \frac{2\left(3\sqrt{-a}b^2c^2x\operatorname{arctan}\right.}{}\right.$$

input
```
integrate((d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/3*(3*sqrt(a)*b^2*c^2*x*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(
a))/x^2) + 2*(a*b*d^2*x + 6*a*b*c*d - 2*a^2*d^2)*sqrt(b*x^3 + a*x^2))/(a*b
^2*x), 2/3*(3*sqrt(-a)*b^2*c^2*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^
2 + a*x)) + (a*b*d^2*x + 6*a*b*c*d - 2*a^2*d^2)*sqrt(b*x^3 + a*x^2))/(a*b^
2*x)]
```

Sympy [F]

$$\int \frac{(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{(c+dx)^2}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate((d*x+c)**2/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral((c + d*x)**2/sqrt(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{(dx+c)^2}{\sqrt{bx^3+ax^2}}\,dx$$

input
```
integrate((d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2/sqrt(b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.42

$$\int \frac{(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\left(\frac{3\,c^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{6\,\sqrt{bx+a}b^5cd+(bx+a)^{\frac{3}{2}}b^4d^2-3\,\sqrt{bx+a}ab^4d^2}{b^6}\right)}{3\,\mathrm{sgn}\,(x)} - \frac{2\left(3\,b^2c^2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+6\,\sqrt{-a}\sqrt{a}bcd-2\,\sqrt{-a}a^{\frac{3}{2}}d^2\right)\mathrm{sgn}(x)}{3\,\sqrt{-a}b^2}$$

input
```
integrate((d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
2/3*(3*c^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + (6*sqrt(b*x + a)*b^5*
c*d + (b*x + a)^(3/2)*b^4*d^2 - 3*sqrt(b*x + a)*a*b^4*d^2)/b^6)/sgn(x) - 2
/3*(3*b^2*c^2*arctan(sqrt(a)/sqrt(-a)) + 6*sqrt(-a)*sqrt(a)*b*c*d - 2*sqrt
(-a)*a^(3/2)*d^2)*sgn(x)/(sqrt(-a)*b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{(c+d\,x)^2}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int((c + d*x)^2/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
int((c + d*x)^2/(a*x^2 + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx)^2}{\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{-4\sqrt{bx+a}\,a^2d^2+12\sqrt{bx+a}\,abcd+2\sqrt{bx+a}\,ab\,d^2x+3\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2c^2-3\sqrt{a}\log\left(\sqrt{bx+a}\right.}{3a\,b^2}$$

input
```
int((d*x+c)^2/(b*x^3+a*x^2)^(1/2),x)
```

output
```
( - 4*sqrt(a + b*x)*a**2*d**2 + 12*sqrt(a + b*x)*a*b*c*d + 2*sqrt(a + b*x)
*a*b*d**2*x + 3*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**2*c**2 - 3*sqrt(a)
*log(sqrt(a + b*x) + sqrt(a))*b**2*c**2)/(3*a*b**2)
```

3.236 $\int \frac{(c+dx)^2}{x\sqrt{ax^2+bx^3}} dx$

Optimal result

Integrand size = 26, antiderivative size = 93

$$\int \frac{(c+dx)^2}{x\sqrt{ax^2+bx^3}} dx = -\frac{c^2\sqrt{ax^2+bx^3}}{ax^2}+\frac{2d^2\sqrt{ax^2+bx^3}}{bx}+\frac{c(bc-4ad)\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{a^{3/2}}$$

output
```
-c^2*(b*x^3+a*x^2)^(1/2)/a/x^2+2*d^2*(b*x^3+a*x^2)^(1/2)/b/x+c*(-4*a*d+b*c
)*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx)^2}{x\sqrt{ax^2+bx^3}} dx = \frac{\sqrt{a}(a+bx)\left(-bc^2+2ad^2x\right)+bc(bc-4ad)x\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}b\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(c + d*x)^2/(x*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(Sqrt[a]*(a + b*x)*(-(b*c^2) + 2*a*d^2*x) + b*c*(b*c - 4*a*d)*x*Sqrt[a + b
*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(a^(3/2)*b*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1948, 100, 27, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{x\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \; 1948$$

$$\frac{x\sqrt{a+bx}\int \frac{(c+dx)^2}{x^2\sqrt{a+bx}}dx}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 100$$

$$\frac{x\sqrt{a+bx}\left(\frac{\int -\frac{c(bc-4ad)-2ad^2x}{2x\sqrt{a+bx}}dx}{a}-\frac{c^2\sqrt{a+bx}}{ax}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 27$$

$$\frac{x\sqrt{a+bx}\left(-\frac{\int \frac{c(bc-4ad)-2ad^2x}{x\sqrt{a+bx}}dx}{2a}-\frac{c^2\sqrt{a+bx}}{ax}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 90$$

$$\frac{x\sqrt{a+bx}\left(-\frac{c(bc-4ad)\int \frac{1}{x\sqrt{a+bx}}dx-\frac{4ad^2\sqrt{a+bx}}{b}}{2a}-\frac{c^2\sqrt{a+bx}}{ax}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 73$$

$$\frac{x\sqrt{a+bx}\left(-\frac{\frac{2c(bc-4ad)\int\frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}-\frac{4ad^2\sqrt{a+bx}}{b}}{2a}-\frac{c^2\sqrt{a+bx}}{ax}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{2c\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)(bc-4ad)}{\sqrt{a}}-\frac{4ad^2\sqrt{a+bx}}{b}}{2a}-\frac{c^2\sqrt{a+bx}}{ax}\right)}{\sqrt{ax^2+bx^3}}$$

input
```
Int[(c + d*x)^2/(x*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(x*Sqrt[a + b*x]*(-((c^2*Sqrt[a + b*x])/(a*x)) - ((-4*a*d^2*Sqrt[a + b*x])
/b - (2*c*(b*c - 4*a*d)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a])/(2*a)))/S
qrt[a*x^2 + b*x^3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61

method	result	size
pseudoelliptic	$-\frac{2\left(b^{2} c^{2} \operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right)-2 d \left(-\frac{a^{\frac{3}{2}} d}{3}+b \sqrt{a}\, \left(\frac{d x}{6}+c \right)\right) \sqrt{b x+a}\right)}{\sqrt{a}\, b^{2}}$	57
risch	$-\frac{c^{2} \left(b x+a \right)}{a \sqrt{x^{2} \left(b x+a \right)}}+\frac{\left(2 a \,d^{2} \sqrt{b x+a}-\frac{b c \left(4 a d-b c \right) \operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right) \sqrt{b x+a}\, x}{a b \sqrt{x^{2} \left(b x+a \right)}}$	94
default	$\frac{\sqrt{b x+a}\, \left(2 d^{2} \sqrt{b x+a}\, a^{\frac{5}{2}} x-a^{\frac{3}{2}} \sqrt{b x+a}\, b \,c^{2}-4 \,\operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right) a^{2} b c d x+\operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right) a \,b^{2} c^{2} x \right)}{\sqrt{b \,x^{3}+a \,x^{2}}\, b \,a^{\frac{5}{2}}}$	103

input
```
int((d*x+c)^2/x/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/a^(1/2)*(b^2*c^2*arctanh((b*x+a)^(1/2)/a^(1/2))-2*d*(-1/3*a^(3/2)*d+b*a
^(1/2)*(1/6*d*x+c))*(b*x+a)^(1/2))/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.12

$$\int \frac{(c+dx)^2}{x\sqrt{ax^2+bx^3}}\,dx = \left[-\frac{(b^2c^2-4\,abcd)\sqrt{a}x^2\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,(2\,a^2d^2x-abc^2)\sqrt{bx^3+ax^2}}{2\,a^2bx^2}, -\frac{(b^2c^2-4\,abcd)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)-(2\,a^2d^2x-abc^2)\sqrt{bx^3+ax^2}}{a^2bx^2}\right]$$

input
```
integrate((d*x+c)^2/x/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*((b^2*c^2 - 4*a*b*c*d)*sqrt(a)*x^2*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3
 + a*x^2)*sqrt(a))/x^2) - 2*(2*a^2*d^2*x - a*b*c^2)*sqrt(b*x^3 + a*x^2))/(
a^2*b*x^2), -((b^2*c^2 - 4*a*b*c*d)*sqrt(-a)*x^2*arctan(sqrt(b*x^3 + a*x^2
)*sqrt(-a)/(b*x^2 + a*x)) - (2*a^2*d^2*x - a*b*c^2)*sqrt(b*x^3 + a*x^2))/(
a^2*b*x^2)]
```

Sympy [F]

$$\int \frac{(c+dx)^2}{x\sqrt{ax^2+bx^3}}\,dx = \int \frac{(c+dx)^2}{x\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate((d*x+c)**2/x/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral((c + d*x)**2/(x*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{(c+dx)^2}{x\sqrt{ax^2+bx^3}}\,dx = \int \frac{(dx+c)^2}{\sqrt{bx^3+ax^2}x}\,dx$$

input
```
integrate((d*x+c)^2/x/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2/(sqrt(b*x^3 + a*x^2)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx)^2}{x\sqrt{ax^2+bx^3}}\,dx = \frac{b\left(\frac{2\sqrt{bx+a}d^2}{b^2} - \frac{\sqrt{bx+a}c^2}{abx} - \frac{(bc^2-4\,acd)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}ab}\right)}{\operatorname{sgn}(x)}$$

input
```
integrate((d*x+c)^2/x/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
b*(2*sqrt(b*x + a)*d^2/b^2 - sqrt(b*x + a)*c^2/(a*b*x) - (b*c^2 - 4*a*c*d)
*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a*b))/sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{x\sqrt{ax^2+bx^3}}\,dx = \int \frac{(c+d\,x)^2}{x\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int((c + d*x)^2/(x*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
int((c + d*x)^2/(x*(a*x^2 + b*x^3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.32

$$\int \frac{(c+dx)^2}{x\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{4\sqrt{bx+a}\,a^2d^2x - 2\sqrt{bx+a}\,ab\,c^2 + 4\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)abcdx - \sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2c^2x - 4}{2a^2bx}$$

input
```
int((d*x+c)^2/x/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(4*sqrt(a + b*x)*a**2*d**2*x - 2*sqrt(a + b*x)*a*b*c**2 + 4*sqrt(a)*log(sq
rt(a + b*x) - sqrt(a))*a*b*c*d*x - sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b*
*2*c**2*x - 4*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*a*b*c*d*x + sqrt(a)*log
(sqrt(a + b*x) + sqrt(a))*b**2*c**2*x)/(2*a**2*b*x)
```

3.237 $\int \frac{(c+dx)^2}{x^2\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 121

$$\int \frac{(c+dx)^2}{x^2\sqrt{ax^2+bx^3}}\,dx = -\frac{c^2\sqrt{ax^2+bx^3}}{2ax^3}+\frac{c(3bc-8ad)\sqrt{ax^2+bx^3}}{4a^2x^2} - \frac{(3b^2c^2-8abcd+8a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{4a^{5/2}}$$

output
```
-1/2*c^2*(b*x^3+a*x^2)^(1/2)/a/x^3+1/4*c*(-8*a*d+3*b*c)*(b*x^3+a*x^2)^(1/2
)/a^2/x^2-1/4*(8*a^2*d^2-8*a*b*c*d+3*b^2*c^2)*arctanh((b*x^3+a*x^2)^(1/2)/
a^(1/2)/x)/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx)^2}{x^2\sqrt{ax^2+bx^3}}\,dx = \frac{-\sqrt{a}c(a+bx)(-3bcx+2a(c+4dx))-(3b^2c^2-8abcd+8a^2d^2)\,x^2\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}x\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(c + d*x)^2/(x^2*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-(Sqrt[a]*c*(a + b*x)*(-3*b*c*x + 2*a*(c + 4*d*x))) - (3*b^2*c^2 - 8*a*b*
c*d + 8*a^2*d^2)*x^2*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(5
/2)*x*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1948, 100, 27, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{x^2\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1948}$$

$$\frac{x\sqrt{a+bx}\int \frac{(c+dx)^2}{x^3\sqrt{a+bx}}dx}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{100}$$

$$\frac{x\sqrt{a+bx}\left(\frac{\int -\frac{c(3bc-8ad)-4ad^2x}{2x^2\sqrt{a+bx}}dx}{2a}-\frac{c^2\sqrt{a+bx}}{2ax^2}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\sqrt{a+bx}\left(-\frac{\int \frac{c(3bc-8ad)-4ad^2x}{x^2\sqrt{a+bx}}dx}{4a}-\frac{c^2\sqrt{a+bx}}{2ax^2}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{87}$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{(8a^2d^2-8abcd+3b^2c^2)\int \frac{1}{x\sqrt{a+bx}}dx}{2a}-\frac{c\sqrt{a+bx}(3bc-8ad)}{ax}}{4a}-\frac{c^2\sqrt{a+bx}}{2ax^2}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow 73$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{\left(8a^2d^2-8abcd+3b^2c^2\right)\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{ab}-\frac{c\sqrt{a+bx}(3bc-8ad)}{ax}}{4a}-\frac{c^2\sqrt{a+bx}}{2ax^2}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow 221$$

$$\frac{x\sqrt{a+bx}\left(-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\left(8a^2d^2-8abcd+3b^2c^2\right)}{a^{3/2}}-\frac{c\sqrt{a+bx}(3bc-8ad)}{ax}}{4a}-\frac{c^2\sqrt{a+bx}}{2ax^2}\right)}{\sqrt{ax^2+bx^3}}$$

input
```
Int[(c + d*x)^2/(x^2*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(x*Sqrt[a + b*x]*(-1/2*(c^2*Sqrt[a + b*x])/(a*x^2) - (-(((c*(3*b*c - 8*a*d)
*Sqrt[a + b*x])/(a*x)) + ((3*b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTanh[Sqrt
[a + b*x]/Sqrt[a]])/a^(3/2))/(4*a)))/Sqrt[a*x^2 + b*x^3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.53

method	result
pseudoelliptic	$\frac{bc\left(-\frac{c\sqrt{bx+a}}{ax}-\frac{(4ad-bc)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right)+2d^2\sqrt{bx+a}}{b}$
risch	$-\frac{c(bx+a)(8adx-3cbx+2ac)}{4a^2x\sqrt{x^2(bx+a)}}-\frac{(8a^2d^2-8abcd+3b^2c^2)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{bx+a}\,x}{4a^{\frac{5}{2}}\sqrt{x^2(bx+a)}}$
default	$\frac{\sqrt{bx+a}\left(8a^{\frac{9}{2}}\sqrt{bx+a}\,cd-8a^{\frac{7}{2}}(bx+a)^{\frac{3}{2}}cd-5a^{\frac{7}{2}}\sqrt{bx+a}\,b\,c^2+3a^{\frac{5}{2}}(bx+a)^{\frac{3}{2}}b\,c^2-8\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^4bd^2x^2+8\operatorname{arctanh}\left(\right.}{4xb\sqrt{b\,x^3+a\,x^2}\,a^{\frac{9}{2}}}$

input
```
int((d*x+c)^2/x^2/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(b*c*(-c/a*(b*x+a)^(1/2)/x-(4*a*d-b*c)/a^(3/2)*arctanh((b*x+a)^(1/2)/a
^(1/2)))+2*d^2*(b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.89

$$\int \frac{(c+dx)^2}{x^2\sqrt{ax^2+bx^3}}\,dx$$
$$= \left[\frac{(3\,b^2c^2-8\,abcd+8\,a^2d^2)\sqrt{a}x^3\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,(2\,a^2c^2-(3\,abc^2-8\,a^2cd)x)\sqrt{bx^3+ax^2}}{8\,a^3x^3}\right.$$

input
```
integrate((d*x+c)^2/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*((3*b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*sqrt(a)*x^3*log((b*x^2 + 2*a*x -
 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) - 2*(2*a^2*c^2 - (3*a*b*c^2 - 8*a^2*c
*d)*x)*sqrt(b*x^3 + a*x^2))/(a^3*x^3), 1/4*((3*b^2*c^2 - 8*a*b*c*d + 8*a^2
*d^2)*sqrt(-a)*x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) - (2
*a^2*c^2 - (3*a*b*c^2 - 8*a^2*c*d)*x)*sqrt(b*x^3 + a*x^2))/(a^3*x^3)]
```

Sympy [F]

$$\int \frac{(c+dx)^2}{x^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{(c+dx)^2}{x^2\sqrt{x^2\left(a+bx\right)}}\,dx$$

input
```
integrate((d*x+c)**2/x**2/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral((c + d*x)**2/(x**2*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{(c+dx)^2}{x^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{(dx+c)^2}{\sqrt{bx^3+ax^2}x^2}\,dx$$

input
```
integrate((d*x+c)^2/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2/(sqrt(b*x^3 + a*x^2)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^2}{x^2\sqrt{ax^2+bx^3}}\,dx = \frac{\frac{(3\,b^3c^2-8\,ab^2cd+8\,a^2bd^2)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{3\,(bx+a)^{\frac{3}{2}}b^3c^2-5\,\sqrt{bx+a}ab^3c^2-8\,(bx+a)^{\frac{3}{2}}ab^2cd+8\,\sqrt{bx+a}a^2b^2cd}{a^2b^2x^2}}{4\,b\mathrm{sgn}\left(x\right)}$$

input
```
integrate((d*x+c)^2/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*((3*b^3*c^2 - 8*a*b^2*c*d + 8*a^2*b*d^2)*arctan(sqrt(b*x + a)/sqrt(-a)
)/(sqrt(-a)*a^2) + (3*(b*x + a)^(3/2)*b^3*c^2 - 5*sqrt(b*x + a)*a*b^3*c^2
- 8*(b*x + a)^(3/2)*a*b^2*c*d + 8*sqrt(b*x + a)*a^2*b^2*c*d)/(a^2*b^2*x^2)
)/(b*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{x^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{{\left(c+d\,x\right)}^2}{x^2\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int((c + d*x)^2/(x^2*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
int((c + d*x)^2/(x^2*(a*x^2 + b*x^3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.58

$$\int \frac{(c+dx)^2}{x^2\sqrt{ax^2+bx^3}}\,dx = \frac{-4\sqrt{bx+a}\,a^2c^2-16\sqrt{bx+a}\,a^2cdx+6\sqrt{bx+a}\,ab\,c^2x+8\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)a^2d^2x^2-8\sqrt{a}\log\left(\sqrt{\right.}\right.}{}$$

input
```
int((d*x+c)^2/x^2/(b*x^3+a*x^2)^(1/2),x)
```

output
```
( - 4*sqrt(a + b*x)*a**2*c**2 - 16*sqrt(a + b*x)*a**2*c*d*x + 6*sqrt(a + b
*x)*a*b*c**2*x + 8*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a**2*d**2*x**2 - 8
*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b*c*d*x**2 + 3*sqrt(a)*log(sqrt(a
+ b*x) - sqrt(a))*b**2*c**2*x**2 - 8*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*
a**2*d**2*x**2 + 8*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*a*b*c*d*x**2 - 3*s
qrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**2*c**2*x**2)/(8*a**3*x**2)
```

3.238 $\int \frac{(c+dx)^2}{x^3\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 170

$$\int \frac{(c+dx)^2}{x^3\sqrt{ax^2+bx^3}}\,dx = -\frac{c^2\sqrt{ax^2+bx^3}}{3ax^4}+\frac{c(5bc-12ad)\sqrt{ax^2+bx^3}}{12a^2x^3} - \frac{(5b^2c^2-12abcd+8a^2d^2)\sqrt{ax^2+bx^3}}{8a^3x^2} + \frac{b(5b^2c^2-12abcd+8a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{8a^{7/2}}$$

output

```
-1/3*c^2*(b*x^3+a*x^2)^(1/2)/a/x^4+1/12*c*(-12*a*d+5*b*c)*(b*x^3+a*x^2)^(1
/2)/a^2/x^3-1/8*(8*a^2*d^2-12*a*b*c*d+5*b^2*c^2)*(b*x^3+a*x^2)^(1/2)/a^3/x
^2+1/8*b*(8*a^2*d^2-12*a*b*c*d+5*b^2*c^2)*arctanh((b*x^3+a*x^2)^(1/2)/a^(1
/2)/x)/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.82

$$\int \frac{(c+dx)^2}{x^3\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{-\sqrt{a}(a+bx)\left(15b^2c^2x^2-2abcx(5c+18dx)+8a^2\left(c^2+3cdx+3d^2x^2\right)\right)+3b\left(5b^2c^2-12abcd+8a^2d^2\right)x}{24a^{7/2}x^2\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(c + d*x)^2/(x^3*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-(Sqrt[a]*(a + b*x)*(15*b^2*c^2*x^2 - 2*a*b*c*x*(5*c + 18*d*x) + 8*a^2*(c
^2 + 3*c*d*x + 3*d^2*x^2))) + 3*b*(5*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*x^3
*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(24*a^(7/2)*x^2*Sqrt[x^2*(a
 + b*x)])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1948, 100, 27, 87, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{x^3\sqrt{ax^2+bx^3}}\,dx$$
$$\Big\downarrow \; 1948$$
$$\frac{x\sqrt{a+bx}\int \frac{(c+dx)^2}{x^4\sqrt{a+bx}}dx}{\sqrt{ax^2+bx^3}}$$
$$\Big\downarrow \; 100$$
$$\frac{x\sqrt{a+bx}\left(\frac{\int -\frac{c(5bc-12ad)-6ad^2x}{2x^3\sqrt{a+bx}}dx}{3a}-\frac{c^2\sqrt{a+bx}}{3ax^3}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\sqrt{a+bx}\left(-\frac{\int \frac{c(5bc-12ad)-6ad^2x}{x^3\sqrt{a+bx}}dx}{6a}-\frac{c^2\sqrt{a+bx}}{3ax^3}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{87}$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{3\left(8a^2d^2+bc(5bc-12ad)\right)\int \frac{1}{x^2\sqrt{a+bx}}dx}{4a}-\frac{c\sqrt{a+bx}(5bc-12ad)}{2ax^2}}{6a}-\frac{c^2\sqrt{a+bx}}{3ax^3}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{52}$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{3\left(8a^2d^2+bc(5bc-12ad)\right)\left(-\frac{b\int \frac{1}{x\sqrt{a+bx}}dx}{2a}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{c\sqrt{a+bx}(5bc-12ad)}{2ax^2}}{6a}-\frac{c^2\sqrt{a+bx}}{3ax^3}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{73}$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{3\left(8a^2d^2+bc(5bc-12ad)\right)\left(-\frac{\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{a}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{c\sqrt{a+bx}(5bc-12ad)}{2ax^2}}{6a}-\frac{c^2\sqrt{a+bx}}{3ax^3}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{221}$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{3\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}\right)\left(8a^2d^2+bc(5bc-12ad)\right)}{4a}-\frac{c\sqrt{a+bx}(5bc-12ad)}{2ax^2}}{6a}-\frac{c^2\sqrt{a+bx}}{3ax^3}\right)}{\sqrt{ax^2+bx^3}}$$

input
```
Int[(c + d*x)^2/(x^3*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(x*Sqrt[a + b*x]*(-1/3*(c^2*Sqrt[a + b*x])/(a*x^3) - (-1/2*(c*(5*b*c - 12*
a*d)*Sqrt[a + b*x])/(a*x^2) - (3*(8*a^2*d^2 + b*c*(5*b*c - 12*a*d))*(-(Sqr
t[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)))/(4*a))/(6
*a)))/Sqrt[a*x^2 + b*x^3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.46

method	result
pseudoelliptic	$-\frac{2\left(x^{2}\left(a^{2}d^{2}-abcd+\frac{3}{8}b^{2}c^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-\frac{3c\sqrt{bx+a}\left(\frac{2(-4dx-c)a^{\frac{3}{2}}}{3}+\sqrt{a}\,bcx\right)}{8}\right)}{a^{\frac{5}{2}}x^{2}}$
risch	$-\frac{(bx+a)\left(24a^{2}d^{2}x^{2}-36abcd\,x^{2}+15b^{2}c^{2}x^{2}+24a^{2}cdx-10ab\,c^{2}x+8a^{2}c^{2}\right)}{24a^{3}x^{2}\sqrt{x^{2}(bx+a)}}+\frac{\left(8a^{2}d^{2}-12abcd+5b^{2}c^{2}\right)b\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\cdots}{8a^{\frac{7}{2}}\sqrt{x^{2}(bx+a)}}$
default	$-\sqrt{bx+a}\,\Big(24a^{\frac{11}{2}}(bx+a)^{\frac{5}{2}}d^{2}-36a^{\frac{9}{2}}(bx+a)^{\frac{5}{2}}bcd+15a^{\frac{7}{2}}(bx+a)^{\frac{5}{2}}b^{2}c^{2}-48a^{\frac{13}{2}}(bx+a)^{\frac{3}{2}}d^{2}+96a^{\frac{11}{2}}(bx+a)^{\frac{3}{2}}bcd-40a^{\frac{9}{2}}(bx\cdots$

input
```
int((d*x+c)^2/x^3/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/a^(5/2)*(x^2*(a^2*d^2-a*b*c*d+3/8*b^2*c^2)*arctanh((b*x+a)^(1/2)/a^(1/2
))-3/8*c*(b*x+a)^(1/2)*(2/3*(-4*d*x-c)*a^(3/2)+a^(1/2)*b*c*x))/x^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.78

$$\int \frac{(c+dx)^2}{x^3\sqrt{ax^2+bx^3}}\,dx = \left[\frac{3\,(5\,b^3c^2-12\,ab^2cd+8\,a^2bd^2)\sqrt{a}x^4\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,(8\,a^3c^2+3\,(5\,ab^2c^2-12\,a^2bcd+8\,a\right.}{48\,a^4x^4}$$

$$-\frac{3\,(5\,b^3c^2-12\,ab^2cd+8\,a^2bd^2)\sqrt{-a}x^4\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+(8\,a^3c^2+3\,(5\,ab^2c^2-12\,a^2bcd+8\,a^3d^2}{24\,a^4x^4}$$

input
```
integrate((d*x+c)^2/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(5*b^3*c^2 - 12*a*b^2*c*d + 8*a^2*b*d^2)*sqrt(a)*x^4*log((b*x^2 +
 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) - 2*(8*a^3*c^2 + 3*(5*a*b^2*c
^2 - 12*a^2*b*c*d + 8*a^3*d^2)*x^2 - 2*(5*a^2*b*c^2 - 12*a^3*c*d)*x)*sqrt(
b*x^3 + a*x^2))/(a^4*x^4), -1/24*(3*(5*b^3*c^2 - 12*a*b^2*c*d + 8*a^2*b*d^
2)*sqrt(-a)*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + (8*a^
3*c^2 + 3*(5*a*b^2*c^2 - 12*a^2*b*c*d + 8*a^3*d^2)*x^2 - 2*(5*a^2*b*c^2 -
12*a^3*c*d)*x)*sqrt(b*x^3 + a*x^2))/(a^4*x^4)]
```

Sympy [F]

$$\int \frac{(c+dx)^2}{x^3\sqrt{ax^2+bx^3}}\,dx = \int \frac{(c+dx)^2}{x^3\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate((d*x+c)**2/x**3/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral((c + d*x)**2/(x**3*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{(c+dx)^2}{x^3\sqrt{ax^2+bx^3}}\,dx = \int \frac{(dx+c)^2}{\sqrt{bx^3+ax^2}x^3}\,dx$$

input
```
integrate((d*x+c)^2/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2/(sqrt(b*x^3 + a*x^2)*x^3), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.22

$$\int \frac{(c+dx)^2}{x^3\sqrt{ax^2+bx^3}}\,dx = -\frac{b^3\left(\frac{3\,(5\,b^2c^2-12\,abcd+8\,a^2d^2)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3b^2}+\frac{15\,(bx+a)^{\frac{5}{2}}b^2c^2-40\,(bx+a)^{\frac{3}{2}}ab^2c^2+33\,\sqrt{bx+a}a^2b^2c^2-36\,(bx+a)^{\frac{5}{2}}abcd+96\,(bx+a)^{\frac{3}{2}}}{a^3b^5x^3}\right)}{24\,\mathrm{sgn}\left(x\right)}$$

input
```
integrate((d*x+c)^2/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/24*b^3*(3*(5*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*arctan(sqrt(b*x + a)/sqr
t(-a))/(sqrt(-a)*a^3*b^2) + (15*(b*x + a)^(5/2)*b^2*c^2 - 40*(b*x + a)^(3/
2)*a*b^2*c^2 + 33*sqrt(b*x + a)*a^2*b^2*c^2 - 36*(b*x + a)^(5/2)*a*b*c*d +
 96*(b*x + a)^(3/2)*a^2*b*c*d - 60*sqrt(b*x + a)*a^3*b*c*d + 24*(b*x + a)^
(5/2)*a^2*d^2 - 48*(b*x + a)^(3/2)*a^3*d^2 + 24*sqrt(b*x + a)*a^4*d^2)/(a^
3*b^5*x^3))/sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{x^3\sqrt{ax^2+bx^3}}\,dx = \int \frac{(c+d\,x)^2}{x^3\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int((c + d*x)^2/(x^3*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
int((c + d*x)^2/(x^3*(a*x^2 + b*x^3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.48

$$\int \frac{(c+dx)^2}{x^3\sqrt{ax^2+bx^3}}\,dx = \frac{-16\sqrt{bx+a}\,a^3c^2-48\sqrt{bx+a}\,a^3cdx-48\sqrt{bx+a}\,a^3d^2x^2+20\sqrt{bx+a}\,a^2b\,c^2x+72\sqrt{bx+a}\,a^2bcd\,x^2-}{}$$

input
```
int((d*x+c)^2/x^3/(b*x^3+a*x^2)^(1/2),x)
```

output
```
( - 16*sqrt(a + b*x)*a**3*c**2 - 48*sqrt(a + b*x)*a**3*c*d*x - 48*sqrt(a +
 b*x)*a**3*d**2*x**2 + 20*sqrt(a + b*x)*a**2*b*c**2*x + 72*sqrt(a + b*x)*a
**2*b*c*d*x**2 - 30*sqrt(a + b*x)*a*b**2*c**2*x**2 - 24*sqrt(a)*log(sqrt(a
 + b*x) - sqrt(a))*a**2*b*d**2*x**3 + 36*sqrt(a)*log(sqrt(a + b*x) - sqrt(
a))*a*b**2*c*d*x**3 - 15*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**3*c**2*x*
*3 + 24*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*a**2*b*d**2*x**3 - 36*sqrt(a)
*log(sqrt(a + b*x) + sqrt(a))*a*b**2*c*d*x**3 + 15*sqrt(a)*log(sqrt(a + b*
x) + sqrt(a))*b**3*c**2*x**3)/(48*a**4*x**3)
```

3.239 $\int \frac{(c+dx)^2}{x^4\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 221

$$\int \frac{(c+dx)^2}{x^4\sqrt{ax^2+bx^3}}\,dx = -\frac{c^2\sqrt{ax^2+bx^3}}{4ax^5}+\frac{c(7bc-16ad)\sqrt{ax^2+bx^3}}{24a^2x^4} - \frac{(35b^2c^2-80abcd+48a^2d^2)\sqrt{ax^2+bx^3}}{96a^3x^3} + \frac{b(35b^2c^2-80abcd+48a^2d^2)\sqrt{ax^2+bx^3}}{64a^4x^2} - \frac{b^2(35b^2c^2-80abcd+48a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{64a^{9/2}}$$

output

```
-1/4*c^2*(b*x^3+a*x^2)^(1/2)/a/x^5+1/24*c*(-16*a*d+7*b*c)*(b*x^3+a*x^2)^(1
/2)/a^2/x^4-1/96*(48*a^2*d^2-80*a*b*c*d+35*b^2*c^2)*(b*x^3+a*x^2)^(1/2)/a^
3/x^3+1/64*b*(48*a^2*d^2-80*a*b*c*d+35*b^2*c^2)*(b*x^3+a*x^2)^(1/2)/a^4/x^
2-1/64*b^2*(48*a^2*d^2-80*a*b*c*d+35*b^2*c^2)*arctanh((b*x^3+a*x^2)^(1/2)/
a^(1/2)/x)/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.79

$$\int \frac{(c+dx)^2}{x^4\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{-\sqrt{a}(a+bx)\left(-105b^3c^2x^3+10ab^2cx^2(7c+24dx)+16a^3(3c^2+8cdx+6d^2x^2)-8a^2bx(7c^2+20cdx+\right.}{192a^{9/2}x^3\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(c + d*x)^2/(x^4*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-(Sqrt[a]*(a + b*x)*(-105*b^3*c^2*x^3 + 10*a*b^2*c*x^2*(7*c + 24*d*x) + 1
6*a^3*(3*c^2 + 8*c*d*x + 6*d^2*x^2) - 8*a^2*b*x*(7*c^2 + 20*c*d*x + 18*d^2
*x^2))) - 3*b^2*(35*b^2*c^2 - 80*a*b*c*d + 48*a^2*d^2)*x^4*Sqrt[a + b*x]*A
rcTanh[Sqrt[a + b*x]/Sqrt[a]])/(192*a^(9/2)*x^3*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.83, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1948, 100, 27, 87, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{x^4\sqrt{ax^2+bx^3}}\,dx$$
$$\Big\downarrow \text{1948}$$
$$\frac{x\sqrt{a+bx}\int \frac{(c+dx)^2}{x^5\sqrt{a+bx}}dx}{\sqrt{ax^2+bx^3}}$$
$$\Big\downarrow \text{100}$$
$$\frac{x\sqrt{a+bx}\left(\frac{\int -\frac{c(7bc-16ad)-8ad^2x}{2x^4\sqrt{a+bx}}dx}{4a}-\frac{c^2\sqrt{a+bx}}{4ax^4}\right)}{\sqrt{ax^2+bx^3}}$$

$$\big\downarrow \ 27$$

$$\frac{x\sqrt{a+bx}\left(-\frac{\int \frac{c(7bc-16ad)-8ad^2x}{x^4\sqrt{a+bx}}dx}{8a}-\frac{c^2\sqrt{a+bx}}{4ax^4}\right)}{\sqrt{ax^2+bx^3}}$$

$$\big\downarrow \ 87$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{\left(48a^2d^2+5bc(7bc-16ad)\right)\int \frac{1}{x^3\sqrt{a+bx}}dx}{6a}-\frac{c\sqrt{a+bx}(7bc-16ad)}{3ax^3}}{8a}-\frac{c^2\sqrt{a+bx}}{4ax^4}\right)}{\sqrt{ax^2+bx^3}}$$

$$\big\downarrow \ 52$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{\left(48a^2d^2+5bc(7bc-16ad)\right)\left(-\frac{3b\int \frac{1}{x^2\sqrt{a+bx}}dx}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)}{6a}-\frac{c\sqrt{a+bx}(7bc-16ad)}{3ax^3}}{8a}-\frac{c^2\sqrt{a+bx}}{4ax^4}\right)}{\sqrt{ax^2+bx^3}}$$

$$\big\downarrow \ 52$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{\left(48a^2d^2+5bc(7bc-16ad)\right)\left(-\frac{3b\left(-\frac{b\int \frac{1}{x\sqrt{a+bx}}dx}{2a}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)}{6a}-\frac{c\sqrt{a+bx}(7bc-16ad)}{3ax^3}}{8a}-\frac{c^2\sqrt{a+bx}}{4ax^4}\right)}{\sqrt{ax^2+bx^3}}$$

$$\big\downarrow \ 73$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{\left(48a^2d^2+5bc(7bc-16ad)\right)\left(-\frac{3b\left(-\frac{\int\frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{a}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)}{6a}-\frac{c\sqrt{a+bx}(7bc-16ad)}{3ax^3}}{8a}-\frac{c^2\sqrt{a+bx}}{4ax^4}\right)}{\sqrt{ax^2+bx^3}}$$

$\Big\downarrow$ 221

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)\left(48a^2d^2+5bc(7bc-16ad)\right)}{6a}-\frac{c\sqrt{a+bx}(7bc-16ad)}{3ax^3}}{8a}-\frac{c^2\sqrt{a+bx}}{4ax^4}\right)}{\sqrt{ax^2+bx^3}}$$

input
```
Int[(c + d*x)^2/(x^4*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(x*Sqrt[a + b*x]*(-1/4*(c^2*Sqrt[a + b*x])/(a*x^4) - (-1/3*(c*(7*b*c - 16*
a*d)*Sqrt[a + b*x])/(a*x^3) - ((48*a^2*d^2 + 5*b*c*(7*b*c - 16*a*d))*(-1/2
*Sqrt[a + b*x]/(a*x^2) - (3*b*(-(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a
+ b*x]/Sqrt[a]])/a^(3/2)))/(4*a)))/(6*a))/(8*a)))/Sqrt[a*x^2 + b*x^3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.47

method	result
pseudoelliptic	$\frac{b\,x^3\left(a^2d^2-\frac{3}{2}abcd+\frac{5}{8}b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-\frac{\left(\left(3d^2x^2+3cdx+c^2\right)a^{\frac{5}{2}}+\frac{15\left(\left(-\frac{12dx}{5}-\frac{2c}{3}\right)a^{\frac{3}{2}}+\sqrt{a}\,bcx\right)xbc}{8}\right)\sqrt{bx+a}}{3}}{a^{\frac{7}{2}}x^3}$
risch	$-\frac{(bx+a)\left(-144d^2x^3a^2b+240a\,b^2cd\,x^3-105b^3c^2x^3+96a^3d^2x^2-160x^2a^2bcd+70a\,b^2c^2x^2+128a^3cdx-56a^2b\,c^2x+48c^2a^3\right)}{192a^4x^3\sqrt{x^2(bx+a)}}$
default	$-\frac{\sqrt{bx+a}\,\left(240a^{\frac{19}{2}}\sqrt{bx+a}\,d^2-624a^{\frac{17}{2}}(bx+a)^{\frac{3}{2}}d^2+528a^{\frac{15}{2}}(bx+a)^{\frac{5}{2}}d^2-144a^{\frac{13}{2}}(bx+a)^{\frac{7}{2}}d^2-528a^{\frac{17}{2}}\sqrt{bx+a}\,bcd+1168a\right.}{}$

input
```
int((d*x+c)^2/x^4/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^(7/2)*(b*x^3*(a^2*d^2-3/2*a*b*c*d+5/8*b^2*c^2)*arctanh((b*x+a)^(1/2)/a
^(1/2))-1/3*((3*d^2*x^2+3*c*d*x+c^2)*a^(5/2)+15/8*((-12/5*d*x-2/3*c)*a^(3/
2)+a^(1/2)*b*c*x)*x*b*c)*(b*x+a)^(1/2))/x^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.71

$$\int\frac{(c+dx)^2}{x^4\sqrt{ax^2+bx^3}}\,dx$$
$$=\left[\frac{3\left(35\,b^4c^2-80\,ab^3cd+48\,a^2b^2d^2\right)\sqrt{a}x^5\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\left(48\,a^4c^2-3\left(35\,ab^3c^2-80\,a^2b^2c\right.\right.}{384\,a^5x^5}\right.$$

input
```
integrate((d*x+c)^2/x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/384*(3*(35*b^4*c^2 - 80*a*b^3*c*d + 48*a^2*b^2*d^2)*sqrt(a)*x^5*log((b*
x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) - 2*(48*a^4*c^2 - 3*(35*
a*b^3*c^2 - 80*a^2*b^2*c*d + 48*a^3*b*d^2)*x^3 + 2*(35*a^2*b^2*c^2 - 80*a^
3*b*c*d + 48*a^4*d^2)*x^2 - 8*(7*a^3*b*c^2 - 16*a^4*c*d)*x)*sqrt(b*x^3 + a
*x^2))/(a^5*x^5), 1/192*(3*(35*b^4*c^2 - 80*a*b^3*c*d + 48*a^2*b^2*d^2)*sq
rt(-a)*x^5*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) - (48*a^4*c^
2 - 3*(35*a*b^3*c^2 - 80*a^2*b^2*c*d + 48*a^3*b*d^2)*x^3 + 2*(35*a^2*b^2*c
^2 - 80*a^3*b*c*d + 48*a^4*d^2)*x^2 - 8*(7*a^3*b*c^2 - 16*a^4*c*d)*x)*sqrt
(b*x^3 + a*x^2))/(a^5*x^5)]
```

Sympy [F]

$$\int \frac{(c+dx)^2}{x^4\sqrt{ax^2+bx^3}}\,dx = \int \frac{(c+dx)^2}{x^4\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate((d*x+c)**2/x**4/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral((c + d*x)**2/(x**4*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{(c+dx)^2}{x^4\sqrt{ax^2+bx^3}}\,dx = \int \frac{(dx+c)^2}{\sqrt{bx^3+ax^2}x^4}\,dx$$

input
```
integrate((d*x+c)^2/x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2/(sqrt(b*x^3 + a*x^2)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.26

$$\int \frac{(c+dx)^2}{x^4\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{\frac{3\,(35\,b^5c^2-80\,ab^4cd+48\,a^2b^3d^2)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4}+\frac{105\,(bx+a)^{\frac{7}{2}}b^5c^2-385\,(bx+a)^{\frac{5}{2}}ab^5c^2+511\,(bx+a)^{\frac{3}{2}}a^2b^5c^2-279\,\sqrt{bx+a}a^3b^5c^2-240\,(bx\ldots}{}}{}$$

input
```
integrate((d*x+c)^2/x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/192*(3*(35*b^5*c^2 - 80*a*b^4*c*d + 48*a^2*b^3*d^2)*arctan(sqrt(b*x + a)
/sqrt(-a))/(sqrt(-a)*a^4) + (105*(b*x + a)^(7/2)*b^5*c^2 - 385*(b*x + a)^(
5/2)*a*b^5*c^2 + 511*(b*x + a)^(3/2)*a^2*b^5*c^2 - 279*sqrt(b*x + a)*a^3*b
^5*c^2 - 240*(b*x + a)^(7/2)*a*b^4*c*d + 880*(b*x + a)^(5/2)*a^2*b^4*c*d -
 1168*(b*x + a)^(3/2)*a^3*b^4*c*d + 528*sqrt(b*x + a)*a^4*b^4*c*d + 144*(b
*x + a)^(7/2)*a^2*b^3*d^2 - 528*(b*x + a)^(5/2)*a^3*b^3*d^2 + 624*(b*x + a
)^(3/2)*a^4*b^3*d^2 - 240*sqrt(b*x + a)*a^5*b^3*d^2)/(a^4*b^4*x^4))/(b*sgn
(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{x^4\sqrt{ax^2+bx^3}}\,dx=\int \frac{{\left(c+d\,x\right)}^2}{x^4\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int((c + d*x)^2/(x^4*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
int((c + d*x)^2/(x^4*(a*x^2 + b*x^3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.41

$$\int \frac{(c+dx)^2}{x^4\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{-96\sqrt{bx+a}\,a^4c^2-256\sqrt{bx+a}\,a^4cdx-192\sqrt{bx+a}\,a^4d^2x^2+112\sqrt{bx+a}\,a^3b\,c^2x+320\sqrt{bx+a}\,a^3bcd}{}$$

input
```
int((d*x+c)^2/x^4/(b*x^3+a*x^2)^(1/2),x)
```

output
```
( - 96*sqrt(a + b*x)*a**4*c**2 - 256*sqrt(a + b*x)*a**4*c*d*x - 192*sqrt(a
 + b*x)*a**4*d**2*x**2 + 112*sqrt(a + b*x)*a**3*b*c**2*x + 320*sqrt(a + b*
x)*a**3*b*c*d*x**2 + 288*sqrt(a + b*x)*a**3*b*d**2*x**3 - 140*sqrt(a + b*x
)*a**2*b**2*c**2*x**2 - 480*sqrt(a + b*x)*a**2*b**2*c*d*x**3 + 210*sqrt(a
+ b*x)*a*b**3*c**2*x**3 + 144*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a**2*b*
*2*d**2*x**4 - 240*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b**3*c*d*x**4 +
105*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**4*c**2*x**4 - 144*sqrt(a)*log(
sqrt(a + b*x) + sqrt(a))*a**2*b**2*d**2*x**4 + 240*sqrt(a)*log(sqrt(a + b*
x) + sqrt(a))*a*b**3*c*d*x**4 - 105*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b
**4*c**2*x**4)/(384*a**5*x**4)
```

3.240 $\int \frac{x^4}{(c+dx)\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 170

$$\int \frac{x^4}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \frac{2(b^2c^2+abcd+a^2d^2)\sqrt{ax^2+bx^3}}{b^3d^3x} - \frac{2(bc+2ad)\left(ax^2+bx^3\right)^{3/2}}{3b^3d^2x^3} + \frac{2\left(ax^2+bx^3\right)^{5/2}}{5b^3dx^5} - \frac{2c^3\arctan\left(\frac{\sqrt{d}\sqrt{ax^2+bx^3}}{\sqrt{bc-adx}}\right)}{d^{7/2}\sqrt{bc-ad}}$$

output
```
2*(a^2*d^2+a*b*c*d+b^2*c^2)*(b*x^3+a*x^2)^(1/2)/b^3/d^3/x-2/3*(2*a*d+b*c)*
(b*x^3+a*x^2)^(3/2)/b^3/d^2/x^3+2/5*(b*x^3+a*x^2)^(5/2)/b^3/d/x^5-2*c^3*ar
ctan(d^(1/2)*(b*x^3+a*x^2)^(1/2)/(-a*d+b*c)^(1/2)/x)/d^(7/2)/(-a*d+b*c)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.81

$$\int \frac{x^4}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \frac{2x\left(\frac{\sqrt{d}(a+bx)\left(8a^2d^2+2abd(5c-2dx)+b^2\left(15c^2-5cdx+3d^2x^2\right)\right)}{b^3}-\frac{15c^3\sqrt{a+bx}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}\right)}{15d^{7/2}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x^4/((c + d*x)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(2*x*((Sqrt[d]*(a + b*x)*(8*a^2*d^2 + 2*a*b*d*(5*c - 2*d*x) + b^2*(15*c^2
- 5*c*d*x + 3*d^2*x^2)))/b^3 - (15*c^3*Sqrt[a + b*x]*ArcTan[(Sqrt[d]*Sqrt[
a + b*x])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d]))/(15*d^(7/2)*Sqrt[x^2*(a + b*
x)])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}(c+dx)}\,dx$$

$$\Big\downarrow \text{1948}$$

$$\frac{x\sqrt{a+bx}\int \frac{x^3}{\sqrt{a+bx}(c+dx)}dx}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{99}$$

$$\frac{x\sqrt{a+bx}\int \left(-\frac{c^3}{d^3\sqrt{a+bx}(c+dx)}+\frac{(a+bx)^{3/2}}{b^2d}+\frac{(-bc-2ad)\sqrt{a+bx}}{b^2d^2}+\frac{b^2c^2+abdc+a^2d^2}{b^2d^3\sqrt{a+bx}}\right)dx}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\sqrt{a+bx}\left(\frac{2\sqrt{a+bx}(a^2d^2+abcd+b^2c^2)}{b^3d^3}-\frac{2c^3\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{7/2}\sqrt{bc-ad}}-\frac{2(a+bx)^{3/2}(2ad+bc)}{3b^3d^2}+\frac{2(a+bx)^{5/2}}{5b^3d}\right)}{\sqrt{ax^2+bx^3}}$$

input
```
Int[x^4/((c + d*x)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(x*Sqrt[a + b*x]*((2*(b^2*c^2 + a*b*c*d + a^2*d^2)*Sqrt[a + b*x])/(b^3*d^3
) - (2*(b*c + 2*a*d)*(a + b*x)^(3/2))/(3*b^3*d^2) + (2*(a + b*x)^(5/2))/(5
*b^3*d) - (2*c^3*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(d^(7/2)
*Sqrt[b*c - a*d])))/Sqrt[a*x^2 + b*x^3]
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.81

method	result
risch	$\frac{2(3b^2d^2x^2-4ab\,d^2x-5b^2cxd+8a^2d^2+10abcd+15b^2c^2)(bx+a)x}{15b^3d^3\sqrt{x^2(bx+a)}}+\frac{2c^3\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)\sqrt{bx+a}\,x}{d^3\sqrt{d(ad-bc)}\,\sqrt{x^2(bx+a)}}$
pseudoelliptic	$-\frac{32\left(\frac{35c^4b^4\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)}{16}+\sqrt{bx+a}\,\sqrt{d(ad-bc)}\left(\frac{\left(-5d^3x^3+7c\,d^2x^2-\frac{35}{3}c^2dx+35c^3\right)b^3}{16}+\frac{35d\left(\frac{9}{35}d^2x^2-\frac{2}{5}cdx+c^2\right)a\,b^2}{24}\right.\right.}{35\sqrt{d(ad-bc)}\,d^4b^4}$
default	$\frac{2x\sqrt{bx+a}\left(3\sqrt{d(ad-bc)}\,(bx+a)^{\frac{5}{2}}d^2-10\sqrt{d(ad-bc)}\,(bx+a)^{\frac{3}{2}}a\,d^2-5\sqrt{d(ad-bc)}\,(bx+a)^{\frac{3}{2}}bcd+15b^3c^3\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right.\right.}{15\sqrt{b\,x^3+a\,x^2}\,b^3d^3\sqrt{d(ad-bc)}}$

input
```
int(x^4/(d*x+c)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(3*b^2*d^2*x^2-4*a*b*d^2*x-5*b^2*c*d*x+8*a^2*d^2+10*a*b*c*d+15*b^2*c^
2)*(b*x+a)/b^3/d^3/(x^2*(b*x+a))^(1/2)*x+2/d^3*c^3/(d*(a*d-b*c))^(1/2)*arc
tanh(d*(b*x+a)^(1/2)/(d*(a*d-b*c))^(1/2))/(x^2*(b*x+a))^(1/2)*(b*x+a)^(1/2
)*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.47

$$\int \frac{x^4}{(c+dx)\sqrt{ax^2+bx^3}}\,dx$$
$$=\left[-\frac{15\sqrt{-bcd+ad^2}b^3c^3x\log\left(\frac{bdx^2-(bc-2\,ad)x+2\sqrt{bx^3+ax^2}\sqrt{-bcd+ad^2}}{dx^2+cx}\right)-2\left(15\,b^3c^3d-5\,ab^2c^2d^2-2\,a^2bcd^3-\right.}{15\left(b^4cd^4-ab^3d^5\right)x}\right.$$

input
```
integrate(x^4/(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/15*(15*sqrt(-b*c*d + a*d^2)*b^3*c^3*x*log((b*d*x^2 - (b*c - 2*a*d)*x +
 2*sqrt(b*x^3 + a*x^2)*sqrt(-b*c*d + a*d^2))/(d*x^2 + c*x)) - 2*(15*b^3*c^
3*d - 5*a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 - 8*a^3*d^4 + 3*(b^3*c*d^3 - a*b^2*d
^4)*x^2 - (5*b^3*c^2*d^2 - a*b^2*c*d^3 - 4*a^2*b*d^4)*x)*sqrt(b*x^3 + a*x^
2))/((b^4*c*d^4 - a*b^3*d^5)*x), 2/15*(15*sqrt(b*c*d - a*d^2)*b^3*c^3*x*ar
ctan(sqrt(b*x^3 + a*x^2)*sqrt(b*c*d - a*d^2)/(b*d*x^2 + a*d*x)) + (15*b^3*
c^3*d - 5*a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 - 8*a^3*d^4 + 3*(b^3*c*d^3 - a*b^2
*d^4)*x^2 - (5*b^3*c^2*d^2 - a*b^2*c*d^3 - 4*a^2*b*d^4)*x)*sqrt(b*x^3 + a*
x^2))/((b^4*c*d^4 - a*b^3*d^5)*x)]
```

Sympy [F]

$$\int \frac{x^4}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^4}{\sqrt{x^2\,(a+bx)}\,(c+dx)}\,dx$$

input
```
integrate(x**4/(d*x+c)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**4/(sqrt(x**2*(a + b*x))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x^4}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^4}{\sqrt{bx^3+ax^2}(dx+c)}\,dx$$

input
```
integrate(x^4/(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(sqrt(b*x^3 + a*x^2)*(d*x + c)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^4}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(x^4/(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^4}{\sqrt{b\,x^3+a\,x^2}\,\left(c+d\,x\right)}\,dx$$

input
```
int(x^4/((a*x^2 + b*x^3)^(1/2)*(c + d*x)),x)
```

output
```
int(x^4/((a*x^2 + b*x^3)^(1/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.25

$$\int \frac{x^4}{(c+dx)\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{2\sqrt{d}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)b^3c^3 + \frac{16\sqrt{bx+a}\,a^3d^4}{15} + \frac{4\sqrt{bx+a}\,a^2bc\,d^3}{15} - \frac{8\sqrt{bx+a}\,a^2b\,d^4x}{15} + \frac{2\sqrt{bx+a}\,a\,b^2c^2d^2}{3} - 2}{b^3d^4\,(ad-bc)}$$

input
```
int(x^4/(d*x+c)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*(15*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( -
a*d + b*c)))*b**3*c**3 + 8*sqrt(a + b*x)*a**3*d**4 + 2*sqrt(a + b*x)*a**2*
b*c*d**3 - 4*sqrt(a + b*x)*a**2*b*d**4*x + 5*sqrt(a + b*x)*a*b**2*c**2*d**
2 - sqrt(a + b*x)*a*b**2*c*d**3*x + 3*sqrt(a + b*x)*a*b**2*d**4*x**2 - 15*
sqrt(a + b*x)*b**3*c**3*d + 5*sqrt(a + b*x)*b**3*c**2*d**2*x - 3*sqrt(a +
b*x)*b**3*c*d**3*x**2))/(15*b**3*d**4*(a*d - b*c))
```

3.241 $\int \frac{x^3}{(c+dx)\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 121

$$\int \frac{x^3}{(c+dx)\sqrt{ax^2+bx^3}}\,dx=-\frac{2(bc+ad)\sqrt{ax^2+bx^3}}{b^2d^2x}+\frac{2(ax^2+bx^3)^{3/2}}{3b^2dx^3}+\frac{2c^2\arctan\left(\frac{\sqrt{d}\sqrt{ax^2+bx^3}}{\sqrt{bc-ad}x}\right)}{d^{5/2}\sqrt{bc-ad}}$$

output
```
-2*(a*d+b*c)*(b*x^3+a*x^2)^(1/2)/b^2/d^2/x+2/3*(b*x^3+a*x^2)^(3/2)/b^2/d/x
^3+2*c^2*arctan(d^(1/2)*(b*x^3+a*x^2)^(1/2)/(-a*d+b*c)^(1/2)/x)/d^(5/2)/(-
a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{(c+dx)\sqrt{ax^2+bx^3}}\,dx=\frac{2x\left(\frac{\sqrt{d}(a+bx)(-3bc-2ad+bdx)}{b^2}+\frac{3c^2\sqrt{a+bx}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}\right)}{3d^{5/2}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x^3/((c + d*x)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(2*x*((Sqrt[d]*(a + b*x)*(-3*b*c - 2*a*d + b*d*x))/b^2 + (3*c^2*Sqrt[a + b
*x]*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d]))/(3*
d^(5/2)*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}(c+dx)}dx$$
$$\Big\downarrow \text{1948}$$
$$\frac{x\sqrt{a+bx}\int \frac{x^2}{\sqrt{a+bx}(c+dx)}dx}{\sqrt{ax^2+bx^3}}$$
$$\Big\downarrow \text{99}$$
$$\frac{x\sqrt{a+bx}\int \left(\frac{c^2}{d^2\sqrt{a+bx}(c+dx)}+\frac{\sqrt{a+bx}}{bd}+\frac{-bc-ad}{bd^2\sqrt{a+bx}}\right)dx}{\sqrt{ax^2+bx^3}}$$
$$\Big\downarrow \text{2009}$$
$$\frac{x\sqrt{a+bx}\left(\frac{2c^2\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{5/2}\sqrt{bc-ad}}-\frac{2\sqrt{a+bx}(ad+bc)}{b^2d^2}+\frac{2(a+bx)^{3/2}}{3b^2d}\right)}{\sqrt{ax^2+bx^3}}$$

input
```
Int[x^3/((c + d*x)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(x*Sqrt[a + b*x]*((-2*(b*c + a*d)*Sqrt[a + b*x])/(b^2*d^2) + (2*(a + b*x)^
(3/2))/(3*b^2*d) + (2*c^2*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])
/(d^(5/2)*Sqrt[b*c - a*d])))/Sqrt[a*x^2 + b*x^3]
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

method	result	s
risch	$-\frac{2(-bdx+2ad+3bc)(bx+a)x}{3b^2d^2\sqrt{x^2(bx+a)}}-\frac{2c^2\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)\sqrt{bx+a}\,x}{d^2\sqrt{d(ad-bc)}\,\sqrt{x^2(bx+a)}}$	1
pseudoelliptic	$\frac{2b^3c^3\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)+\frac{16\sqrt{bx+a}\,\sqrt{d(ad-bc)}\left(\frac{\left(3d^2x^2-5cdx+15c^2\right)b^2}{8}+\frac{5d\left(-\frac{2dx}{5}+c\right)ab}{4}+a^2d^2\right)}{15}}{d^3\sqrt{d(ad-bc)}\,b^3}$	1
default	$\frac{2x\sqrt{bx+a}\left(\sqrt{d(ad-bc)}\,(bx+a)^{\frac{3}{2}}d-3b^2c^2\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)-3\sqrt{d(ad-bc)}\,\sqrt{bx+a}\,ad-3\sqrt{d(ad-bc)}\,\sqrt{bx+a}\,bc\right)}{3\sqrt{b\,x^3+a\,x^2}\,b^2d^2\sqrt{d(ad-bc)}}$	1

input
```
int(x^3/(d*x+c)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-b*d*x+2*a*d+3*b*c)*(b*x+a)/b^2/d^2/(x^2*(b*x+a))^(1/2)*x-2*c^2/d^2/
(d*(a*d-b*c))^(1/2)*arctanh(d*(b*x+a)^(1/2)/(d*(a*d-b*c))^(1/2))/(x^2*(b*x
+a))^(1/2)*(b*x+a)^(1/2)*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.61

$$\int \frac{x^3}{(c+dx)\sqrt{ax^2+bx^3}}\,dx$$
$$= \left[-\frac{3\sqrt{-bcd+ad^2}b^2c^2x\log\left(\frac{bdx^2-(bc-2\,ad)x-2\,\sqrt{bx^3+ax^2}\sqrt{-bcd+ad^2}}{dx^2+cx}\right)+2\,(3\,b^2c^2d-abcd^2-2\,a^2d^3-(b^2cd^2-}{3\,(b^3cd^3-ab^2d^4)x}\right.$$
$$-\frac{2\left(3\sqrt{bcd-ad^2}b^2c^2x\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{bcd-ad^2}}{bdx^2+adx}\right)+(3\,b^2c^2d-abcd^2-2\,a^2d^3-(b^2cd^2-abd^3)x)\sqrt{bx^3}\right.}{3\,(b^3cd^3-ab^2d^4)x}$$

input
```
integrate(x^3/(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/3*(3*sqrt(-b*c*d + a*d^2)*b^2*c^2*x*log((b*d*x^2 - (b*c - 2*a*d)*x - 2
*sqrt(b*x^3 + a*x^2)*sqrt(-b*c*d + a*d^2))/(d*x^2 + c*x)) + 2*(3*b^2*c^2*d
 - a*b*c*d^2 - 2*a^2*d^3 - (b^2*c*d^2 - a*b*d^3)*x)*sqrt(b*x^3 + a*x^2))/(
(b^3*c*d^3 - a*b^2*d^4)*x), -2/3*(3*sqrt(b*c*d - a*d^2)*b^2*c^2*x*arctan(s
qrt(b*x^3 + a*x^2)*sqrt(b*c*d - a*d^2)/(b*d*x^2 + a*d*x)) + (3*b^2*c^2*d -
 a*b*c*d^2 - 2*a^2*d^3 - (b^2*c*d^2 - a*b*d^3)*x)*sqrt(b*x^3 + a*x^2))/((b
^3*c*d^3 - a*b^2*d^4)*x)]
```

Sympy [F]

$$\int \frac{x^3}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^3}{\sqrt{x^2\,(a+bx)}\,(c+dx)}\,dx$$

input
```
integrate(x**3/(d*x+c)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**3/(sqrt(x**2*(a + b*x))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x^3}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^3}{\sqrt{bx^3+ax^2}(dx+c)}\,dx$$

input
```
integrate(x^3/(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(sqrt(b*x^3 + a*x^2)*(d*x + c)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^3}{\sqrt{b\,x^3+a\,x^2}\,\left(c+d\,x\right)}\,dx$$

input
```
int(x^3/((a*x^2 + b*x^3)^(1/2)*(c + d*x)),x)
```

output
```
int(x^3/((a*x^2 + b*x^3)^(1/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{(c+dx)\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{-2\sqrt{d}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)b^2c^2 - \frac{4\sqrt{bx+a}\,a^2d^3}{3} - \frac{2\sqrt{bx+a}\,abc\,d^2}{3} + \frac{2\sqrt{bx+a}\,ab\,d^3x}{3} + 2\sqrt{bx+a}\,b^2c^2d - }{b^2d^3\,(ad-bc)}$$

input
```
int(x^3/(d*x+c)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*( - 3*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt(
- a*d + b*c)))*b**2*c**2 - 2*sqrt(a + b*x)*a**2*d**3 - sqrt(a + b*x)*a*b*c
*d**2 + sqrt(a + b*x)*a*b*d**3*x + 3*sqrt(a + b*x)*b**2*c**2*d - sqrt(a +
b*x)*b**2*c*d**2*x))/(3*b**2*d**3*(a*d - b*c))
```

3.242 $\int \frac{x^2}{(c+dx)\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 84

$$\int \frac{x^2}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{ax^2+bx^3}}{bdx} - \frac{2c\arctan\left(\frac{\sqrt{d}\sqrt{ax^2+bx^3}}{\sqrt{bc-adx}}\right)}{d^{3/2}\sqrt{bc-ad}}$$

output
```
2*(b*x^3+a*x^2)^(1/2)/b/d/x-2*c*arctan(d^(1/2)*(b*x^3+a*x^2)^(1/2)/(-a*d+b
*c)^(1/2)/x)/d^(3/2)/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \frac{2x\left(\frac{\sqrt{d}(a+bx)}{b} - \frac{c\sqrt{a+bx}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}\right)}{d^{3/2}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x^2/((c + d*x)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(2*x*((Sqrt[d]*(a + b*x))/b - (c*Sqrt[a + b*x]*ArcTan[(Sqrt[d]*Sqrt[a + b*
x])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d]))/(d^(3/2)*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1948, 90, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}(c+dx)}dx$$

$$\Big\downarrow \text{1948}$$

$$\frac{x\sqrt{a+bx}\int \frac{x}{\sqrt{a+bx}(c+dx)}dx}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{90}$$

$$\frac{x\sqrt{a+bx}\left(\frac{2\sqrt{a+bx}}{bd}-\frac{c\int \frac{1}{\sqrt{a+bx}(c+dx)}dx}{d}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{73}$$

$$\frac{x\sqrt{a+bx}\left(\frac{2\sqrt{a+bx}}{bd}-\frac{2c\int \frac{1}{c-\frac{ad}{b}+\frac{d(a+bx)}{b}}d\sqrt{a+bx}}{bd}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{218}$$

$$\frac{x\sqrt{a+bx}\left(\frac{2\sqrt{a+bx}}{bd}-\frac{2c\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{3/2}\sqrt{bc-ad}}\right)}{\sqrt{ax^2+bx^3}}$$

input
```
Int[x^2/((c + d*x)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(x*Sqrt[a + b*x]*((2*Sqrt[a + b*x])/(b*d) - (2*c*ArcTan[(Sqrt[d]*Sqrt[a +
b*x])/Sqrt[b*c - a*d]])/(d^(3/2)*Sqrt[b*c - a*d])))/Sqrt[a*x^2 + b*x^3]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

method	result	size
pseudoelliptic	$\frac{-\frac{2\sqrt{bx+a}\,(-bdx+2ad+3bc)}{3}-\frac{2b^2c^2\,\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)}{\sqrt{d(ad-bc)}}}{b^2d^2}$	75
risch	$\frac{2(bx+a)x}{bd\sqrt{x^2(bx+a)}}+\frac{2c\,\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)\sqrt{bx+a}\,x}{d\sqrt{d(ad-bc)}\,\sqrt{x^2(bx+a)}}$	86
default	$\frac{2x\sqrt{bx+a}\left(bc\,\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)+\sqrt{bx+a}\,\sqrt{d(ad-bc)}\right)}{\sqrt{b\,x^3+a\,x^2}\,bd\sqrt{d(ad-bc)}}$	88

input
```
int(x^2/(d*x+c)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d^2*(-1/3*(b*x+a)^(1/2)*(-b*d*x+2*a*d+3*b*c)-b^2*c^2/(d*(a*d-b*c))^(1/2)
*arctanh(d*(b*x+a)^(1/2)/(d*(a*d-b*c))^(1/2)))/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.79

$$\int \frac{x^2}{(c+dx)\sqrt{ax^2+bx^3}}\,dx$$
$$= \left[-\frac{\sqrt{-bcd+ad^2}bcx\log\left(\frac{bdx^2-(bc-2\,ad)x+2\,\sqrt{bx^3+ax^2}\sqrt{-bcd+ad^2}}{dx^2+cx}\right)-2\,\sqrt{bx^3+ax^2}(bcd-ad^2)}{(b^2cd^2-abd^3)x}, \frac{2\left(\sqrt{bcd-ad^2}\right.}{}\right.$$

input
```
integrate(x^2/(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-(sqrt(-b*c*d + a*d^2)*b*c*x*log((b*d*x^2 - (b*c - 2*a*d)*x + 2*sqrt(b*x^
3 + a*x^2)*sqrt(-b*c*d + a*d^2))/(d*x^2 + c*x)) - 2*sqrt(b*x^3 + a*x^2)*(b
*c*d - a*d^2))/((b^2*c*d^2 - a*b*d^3)*x), 2*(sqrt(b*c*d - a*d^2)*b*c*x*arc
tan(sqrt(b*x^3 + a*x^2)*sqrt(b*c*d - a*d^2)/(b*d*x^2 + a*d*x)) + sqrt(b*x^
3 + a*x^2)*(b*c*d - a*d^2))/((b^2*c*d^2 - a*b*d^3)*x)]
```

Sympy [F]

$$\int \frac{x^2}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^2}{\sqrt{x^2\,(a+bx)}\,(c+dx)}\,dx$$

input
```
integrate(x**2/(d*x+c)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(x**2*(a + b*x))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x^2}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^2}{\sqrt{bx^3+ax^2}(dx+c)}\,dx$$

input
```
integrate(x^2/(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(b*x^3 + a*x^2)*(d*x + c)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^2}{\sqrt{b\,x^3+a\,x^2}\,(c+d\,x)}\,dx$$

input
```
int(x^2/((a*x^2 + b*x^3)^(1/2)*(c + d*x)),x)
```

output
```
int(x^2/((a*x^2 + b*x^3)^(1/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{d}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)bc+2\sqrt{bx+a}\,a\,d^2-2\sqrt{bx+a}\,bcd}{b\,d^2\,(ad-bc)}$$

input
```
int(x^2/(d*x+c)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*(sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d
 + b*c)))*b*c + sqrt(a + b*x)*a*d**2 - sqrt(a + b*x)*b*c*d))/(b*d**2*(a*d
- b*c))
```

3.243 $\int \frac{x}{(c+dx)\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 56

$$\int \frac{x}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \frac{2\arctan\left(\frac{\sqrt{d}\sqrt{ax^2+bx^3}}{\sqrt{bc-adx}}\right)}{\sqrt{d}\sqrt{bc-ad}}$$

output
```
2*arctan(d^(1/2)*(b*x^3+a*x^2)^(1/2)/(-a*d+b*c)^(1/2)/x)/d^(1/2)/(-a*d+b*c
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.25

$$\int \frac{x}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \frac{2x\sqrt{a+bx}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\sqrt{bc-ad}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x/((c + d*x)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(2*x*Sqrt[a + b*x]*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(Sqrt[
d]*Sqrt[b*c - a*d]*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.29, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1948, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{ax^2+bx^3}(c+dx)}\,dx$$

$$\Big\downarrow \text{ 1948}$$

$$\frac{x\sqrt{a+bx}\int \frac{1}{\sqrt{a+bx}(c+dx)}dx}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2x\sqrt{a+bx}\int \frac{1}{c-\frac{ad}{b}+\frac{d(a+bx)}{b}}d\sqrt{a+bx}}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{2x\sqrt{a+bx}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\sqrt{ax^2+bx^3}\sqrt{bc-ad}}$$

input
```
Int[x/((c + d*x)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(2*x*Sqrt[a + b*x]*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(Sqrt[
d]*Sqrt[b*c - a*d]*Sqrt[a*x^2 + b*x^3])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

method	result	size
pseudoelliptic	$\frac{2\sqrt{bx+a}+\frac{2bc\,\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)}{\sqrt{d(ad-bc)}}}{bd}$	54
default	$-\frac{2x\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)}{\sqrt{b\,x^{3}+a\,x^{2}}\,\sqrt{d(ad-bc)}}$	58

input
```
int(x/(d*x+c)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/d*((b*x+a)^(1/2)+b*c/(d*(a*d-b*c))^(1/2)*arctanh(d*(b*x+a)^(1/2)/(d*(a
*d-b*c))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.57

$$\int \frac{x}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \left[-\frac{\sqrt{-bcd+ad^2}\log\left(\frac{bdx^2-(bc-2\,ad)x-2\,\sqrt{bx^3+ax^2}\sqrt{-bcd+ad^2}}{dx^2+cx}\right)}{bcd-ad^2}, -\frac{2\,\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{bcd-ad^2}}{bdx^2+adx}\right)}{\sqrt{bcd-ad^2}}\right]$$

input
```
integrate(x/(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-sqrt(-b*c*d + a*d^2)*log((b*d*x^2 - (b*c - 2*a*d)*x - 2*sqrt(b*x^3 + a*x
^2)*sqrt(-b*c*d + a*d^2))/(d*x^2 + c*x))/(b*c*d - a*d^2), -2*arctan(sqrt(b
*x^3 + a*x^2)*sqrt(b*c*d - a*d^2)/(b*d*x^2 + a*d*x))/sqrt(b*c*d - a*d^2)]
```

Sympy [F]

$$\int \frac{x}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \int \frac{x}{\sqrt{x^2\,(a+bx)}\,(c+dx)}\,dx$$

input
```
integrate(x/(d*x+c)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x/(sqrt(x**2*(a + b*x))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \int \frac{x}{\sqrt{bx^3+ax^2}(dx+c)}\,dx$$

input
```
integrate(x/(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(b*x^3 + a*x^2)*(d*x + c)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.41

$$\int \frac{x}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)\operatorname{sgn}(x)}{\sqrt{bcd-ad^2}} + \frac{2\arctan\left(\frac{\sqrt{bx+a}d}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2}\operatorname{sgn}(x)}$$

input
```
integrate(x/(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2))*sgn(x)/sqrt(b*c*d - a*d^2) + 2*ar
ctan(sqrt(b*x + a)*d/sqrt(b*c*d - a*d^2))/(sqrt(b*c*d - a*d^2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \int \frac{x}{\sqrt{b\,x^3+a\,x^2}\,(c+d\,x)}\,dx$$

input
```
int(x/((a*x^2 + b*x^3)^(1/2)*(c + d*x)),x)
```

output
```
int(x/((a*x^2 + b*x^3)^(1/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \frac{x}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = -\frac{2\sqrt{d}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)}{d\,(ad-bc)}$$

input
```
int(x/(d*x+c)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
( - 2*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a
*d + b*c))))/(d*(a*d - b*c))
```

3.244 $\int \frac{1}{(c+dx)\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 95

$$\int \frac{1}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = -\frac{2\sqrt{d}\arctan\left(\frac{\sqrt{d}\sqrt{ax^2+bx^3}}{\sqrt{bc-ad}x}\right)}{c\sqrt{bc-ad}} - \frac{2\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{\sqrt{a}c}$$

output
```
-2*d^(1/2)*arctan(d^(1/2)*(b*x^3+a*x^2)^(1/2)/(-a*d+b*c)^(1/2)/x)/c/(-a*d+
b*c)^(1/2)-2*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(1/2)/c
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.20

$$\int \frac{1}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = -\frac{2x\sqrt{a+bx}\left(\sqrt{a}\sqrt{d}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)+\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{a}c\sqrt{bc-ad}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[1/((c + d*x)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-2*x*Sqrt[a + b*x]*(Sqrt[a]*Sqrt[d]*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b
*c - a*d]] + Sqrt[b*c - a*d]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(Sqrt[a]*c*S
qrt[b*c - a*d]*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2467, 97, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax^2+bx^3}(c+dx)}dx$$

$$\Big\downarrow \text{ 2467}$$

$$\frac{x\sqrt{a+bx}\int \frac{1}{x\sqrt{a+bx}(c+dx)}dx}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 97}$$

$$\frac{x\sqrt{a+bx}\left(\frac{\int \frac{1}{x\sqrt{a+bx}}dx}{c}-\frac{d\int \frac{1}{\sqrt{a+bx}(c+dx)}dx}{c}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{x\sqrt{a+bx}\left(\frac{2\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}-\frac{2d\int \frac{1}{c-\frac{ad}{b}+\frac{d(a+bx)}{b}}d\sqrt{a+bx}}{bc}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{x\sqrt{a+bx}\left(\frac{2\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}-\frac{2\sqrt{d}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{x\sqrt{a+bx}\left(-\frac{2\sqrt{d}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}c}\right)}{\sqrt{ax^2+bx^3}}$$

input
```
Int[1/((c + d*x)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(x*Sqrt[a + b*x]*((-2*Sqrt[d]*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*
d]])/(c*Sqrt[b*c - a*d]) - (2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*c))
)/Sqrt[a*x^2 + b*x^3]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.39

method	result	size
pseudoelliptic	$-\frac{2\,\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)}{\sqrt{d(ad-bc)}}$	37
default	$\frac{2x\sqrt{bx+a}\left(d\,\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)\sqrt{a}-\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{d(ad-bc)}\right)}{\sqrt{b\,x^{3}+a\,x^{2}}\,c\sqrt{a}\,\sqrt{d(ad-bc)}}$	96

input
```
int(1/(d*x+c)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(d*(a*d-b*c))^(1/2)*arctanh(d*(b*x+a)^(1/2)/(d*(a*d-b*c))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 455, normalized size of antiderivative = 4.79

$$\int \frac{1}{(c+dx)\sqrt{ax^2+bx^3}}\,dx$$
$$= \left[\frac{a\sqrt{-\frac{d}{bc-ad}}\log\left(\frac{bdx^2-(bc-2\,ad)x-2\,\sqrt{bx^3+ax^2}(bc-ad)\sqrt{-\frac{d}{bc-ad}}}{dx^2+cx}\right)+\sqrt{a}\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)}{ac}, \frac{a\sqrt{-\frac{d}{bc-ad}}\,\mathrm{lo}\cdots}{\cdots}\right.$$
$$-\frac{2\,a\sqrt{\frac{d}{bc-ad}}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{\frac{d}{bc-ad}}}{x}\right)-\sqrt{a}\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)}{ac},$$
$$\left.-\frac{2\left(a\sqrt{\frac{d}{bc-ad}}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{\frac{d}{bc-ad}}}{x}\right)-\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)\right)}{ac}\right]$$

input
```
integrate(1/(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[(a*sqrt(-d/(b*c - a*d))*log((b*d*x^2 - (b*c - 2*a*d)*x - 2*sqrt(b*x^3 + a
*x^2)*(b*c - a*d)*sqrt(-d/(b*c - a*d)))/(d*x^2 + c*x)) + sqrt(a)*log((b*x^
2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2))/(a*c), (a*sqrt(-d/(b*c -
a*d))*log((b*d*x^2 - (b*c - 2*a*d)*x - 2*sqrt(b*x^3 + a*x^2)*(b*c - a*d)*s
qrt(-d/(b*c - a*d)))/(d*x^2 + c*x)) + 2*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2
)*sqrt(-a)/(b*x^2 + a*x)))/(a*c), -(2*a*sqrt(d/(b*c - a*d))*arctan(sqrt(b*
x^3 + a*x^2)*sqrt(d/(b*c - a*d))/x) - sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(
b*x^3 + a*x^2)*sqrt(a))/x^2))/(a*c), -2*(a*sqrt(d/(b*c - a*d))*arctan(sqrt
(b*x^3 + a*x^2)*sqrt(d/(b*c - a*d))/x) - sqrt(-a)*arctan(sqrt(b*x^3 + a*x^
2)*sqrt(-a)/(b*x^2 + a*x)))/(a*c)]
```

Sympy [F]

$$\int \frac{1}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{x^2\,(a+bx)}\,(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(x**2*(a + b*x))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x^2)*(d*x + c)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.60

$$\int \frac{1}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \frac{2\left(\sqrt{-a}d\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)-\sqrt{bcd-ad^2}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\right)\mathrm{sgn}(x)}{\sqrt{bcd-ad^2}\sqrt{-a}c} - \frac{2\,d\arctan\left(\frac{\sqrt{bx+a}d}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2}c\,\mathrm{sgn}(x)} + \frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}c\,\mathrm{sgn}(x)}$$

input
```
integrate(1/(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
2*(sqrt(-a)*d*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - sqrt(b*c*d - a*d^2)*
arctan(sqrt(a)/sqrt(-a)))*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(-a)*c) - 2*d*ar
ctan(sqrt(b*x + a)*d/sqrt(b*c*d - a*d^2))/(sqrt(b*c*d - a*d^2)*c*sgn(x)) +
 2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*c*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{b\,x^3+a\,x^2}\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/((a*x^2 + b*x^3)^(1/2)*(c + d*x)),x)
```

output
```
int(1/((a*x^2 + b*x^3)^(1/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.28

$$\int \frac{1}{(c+dx)\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{2\sqrt{d}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)a+\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)ad-\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)bc-\sqrt{a}\log}{ac\,(ad-bc)}$$

input
```
int(1/(d*x+c)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d
+ b*c)))*a + sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*d - sqrt(a)*log(sqrt(a
 + b*x) - sqrt(a))*b*c - sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*a*d + sqrt(a
)*log(sqrt(a + b*x) + sqrt(a))*b*c)/(a*c*(a*d - b*c))
```

3.245 $\int \frac{1}{x(c+dx)\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 128

$$\int \frac{1}{x(c+dx)\sqrt{ax^2+bx^3}}\,dx=-\frac{\sqrt{ax^2+bx^3}}{acx^2}+\frac{2d^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{ax^2+bx^3}}{\sqrt{bc-adx}}\right)}{c^2\sqrt{bc-ad}}+\frac{(bc+2ad)\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{a^{3/2}c^2}$$

output
```
-(b*x^3+a*x^2)^(1/2)/a/c/x^2+2*d^(3/2)*arctan(d^(1/2)*(b*x^3+a*x^2)^(1/2)/
(-a*d+b*c)^(1/2)/x)/c^2/(-a*d+b*c)^(1/2)+(2*a*d+b*c)*arctanh((b*x^3+a*x^2)
^(1/2)/a^(1/2)/x)/a^(3/2)/c^2
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.22

$$\int \frac{1}{x(c+dx)\sqrt{ax^2+bx^3}}\,dx=-\frac{a+bx}{ac\sqrt{x^2(a+bx)}}+\frac{2d^{3/2}x\sqrt{a+bx}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{bc-ad}\sqrt{x^2(a+bx)}}+\frac{(bc+2ad)x\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}c^2\sqrt{x^2(a+bx)}}$$

input
```
Integrate[1/(x*(c + d*x)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
-((a + b*x)/(a*c*Sqrt[x^2*(a + b*x)])) + (2*d^(3/2)*x*Sqrt[a + b*x]*ArcTan
[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(c^2*Sqrt[b*c - a*d]*Sqrt[x^2*(
a + b*x)]) + ((b*c + 2*a*d)*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
)/(a^(3/2)*c^2*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1948, 114, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{ax^2+bx^3}(c+dx)}dx$$

$$\Big\downarrow \text{1948}$$

$$\frac{x\sqrt{a+bx}\int \frac{1}{x^2\sqrt{a+bx}(c+dx)}dx}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{114}$$

$$\frac{x\sqrt{a+bx}\left(-\frac{\int \frac{bc+2ad+bdx}{2x\sqrt{a+bx}(c+dx)}dx}{ac}-\frac{\sqrt{a+bx}}{acx}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\sqrt{a+bx}\left(-\frac{\int \frac{bc+2ad+bdx}{x\sqrt{a+bx}(c+dx)}dx}{2ac}-\frac{\sqrt{a+bx}}{acx}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{174}$$

$$\frac{x\sqrt{a+bx}\left(-\frac{\frac{(2ad+bc)\int \frac{1}{x\sqrt{a+bx}}dx}{c}-\frac{2ad^2\int \frac{1}{\sqrt{a+bx}(c+dx)}dx}{c}}{2ac}-\frac{\sqrt{a+bx}}{acx}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow 73$$

$$\frac{x\sqrt{a+bx}\left(-\frac{\frac{2(2ad+bc)\int\frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}-\frac{4ad^2\int\frac{1}{c-\frac{ad}{b}+\frac{d(a+bx)}{b}}d\sqrt{a+bx}}{bc}}{2ac}-\frac{\sqrt{a+bx}}{acx}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow 218$$

$$\frac{x\sqrt{a+bx}\left(-\frac{\frac{2(2ad+bc)\int\frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}-\frac{4ad^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{2ac}-\frac{\sqrt{a+bx}}{acx}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow 221$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{4ad^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)(2ad+bc)}{\sqrt{a}c}}{2ac}-\frac{\sqrt{a+bx}}{acx}\right)}{\sqrt{ax^2+bx^3}}$$

input
```
Int[1/(x*(c + d*x)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(x*Sqrt[a + b*x]*(-(Sqrt[a + b*x]/(a*c*x)) - ((-4*a*d^(3/2)*ArcTan[(Sqrt[d
]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(c*Sqrt[b*c - a*d]) - (2*(b*c + 2*a*d)*
ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*c))/(2*a*c)))/Sqrt[a*x^2 + b*x^3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.47

method	result
pseudoelliptic	$\frac{-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{2d\,\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)}{\sqrt{d(ad-bc)}}}{c}$
risch	$-\frac{bx+a}{ac\sqrt{x^2(bx+a)}}-\frac{b\left(-\frac{(2ad+bc)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{bc\sqrt{a}}+\frac{2a\,d^2\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)}{bc\sqrt{d(ad-bc)}}\right)\sqrt{bx+a}\,x}{ac\sqrt{x^2(bx+a)}}$
default	$-\frac{\sqrt{bx+a}\left(2d^2\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)a^{\frac{5}{2}}x+\sqrt{d(ad-bc)}\,a^{\frac{3}{2}}\sqrt{bx+a}\,c-2\sqrt{d(ad-bc)}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^2dx-\sqrt{d(ad-bc)}\,a\right.}{\sqrt{b\,x^3+a\,x^2}\,c^2\sqrt{d(ad-bc)}\,a^{\frac{5}{2}}}$

input
```
int(1/x/(d*x+c)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/c*(-1/a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+d/(d*(a*d-b*c))^(1/2)*arcta
nh(d*(b*x+a)^(1/2)/(d*(a*d-b*c))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 604, normalized size of antiderivative = 4.72

$$\int \frac{1}{x(c+dx)\sqrt{ax^2+bx^3}}\,dx$$
$$=\left[\frac{2\,a^2dx^2\sqrt{-\frac{d}{bc-ad}}\log\left(\frac{bdx^2-(bc-2\,ad)x+2\,\sqrt{bx^3+ax^2}(bc-ad)\sqrt{-\frac{d}{bc-ad}}}{dx^2+cx}\right)+(bc+2\,ad)\sqrt{a}x^2\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}}{x^2}\right.}{2\,a^2c^2x^2}\right.$$

input
```
integrate(1/x/(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a^2*d*x^2*sqrt(-d/(b*c - a*d))*log((b*d*x^2 - (b*c - 2*a*d)*x + 2*
sqrt(b*x^3 + a*x^2)*(b*c - a*d)*sqrt(-d/(b*c - a*d)))/(d*x^2 + c*x)) + (b*
c + 2*a*d)*sqrt(a)*x^2*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a))
/x^2) - 2*sqrt(b*x^3 + a*x^2)*a*c)/(a^2*c^2*x^2), (a^2*d*x^2*sqrt(-d/(b*c
- a*d))*log((b*d*x^2 - (b*c - 2*a*d)*x + 2*sqrt(b*x^3 + a*x^2)*(b*c - a*d)
*sqrt(-d/(b*c - a*d)))/(d*x^2 + c*x)) - (b*c + 2*a*d)*sqrt(-a)*x^2*arctan(
sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) - sqrt(b*x^3 + a*x^2)*a*c)/(a^
2*c^2*x^2), 1/2*(4*a^2*d*x^2*sqrt(d/(b*c - a*d))*arctan(sqrt(b*x^3 + a*x^2
)*sqrt(d/(b*c - a*d))/x) + (b*c + 2*a*d)*sqrt(a)*x^2*log((b*x^2 + 2*a*x +
2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) - 2*sqrt(b*x^3 + a*x^2)*a*c)/(a^2*c^2*
x^2), (2*a^2*d*x^2*sqrt(d/(b*c - a*d))*arctan(sqrt(b*x^3 + a*x^2)*sqrt(d/(
b*c - a*d))/x) - (b*c + 2*a*d)*sqrt(-a)*x^2*arctan(sqrt(b*x^3 + a*x^2)*sqr
t(-a)/(b*x^2 + a*x)) - sqrt(b*x^3 + a*x^2)*a*c)/(a^2*c^2*x^2)]
```

Sympy [F]

$$\int \frac{1}{x(c+dx)\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x\sqrt{x^2\,(a+bx)}\,(c+dx)}\,dx$$

input
```
integrate(1/x/(d*x+c)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(x**2*(a + b*x))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{x(c+dx)\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}(dx+c)x}\,dx$$

input
```
integrate(1/x/(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x^2)*(d*x + c)*x), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x(c+dx)\sqrt{ax^2+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(c+dx)\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x\,\sqrt{b\,x^3+a\,x^2}\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/(x*(a*x^2 + b*x^3)^(1/2)*(c + d*x)),x)
```

output
```
int(1/(x*(a*x^2 + b*x^3)^(1/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.69

$$\int \frac{1}{x(c+dx)\sqrt{ax^2+bx^3}}\,dx = \frac{-4\sqrt{d}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)a^2dx-2\sqrt{bx+a}\,a^2cd+2\sqrt{bx+a}\,ab\,c^2-2\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)}{}$$

input
```
int(1/x/(d*x+c)/(b*x^3+a*x^2)^(1/2),x)
```

output

```
( - 4*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a
*d + b*c)))*a**2*d*x - 2*sqrt(a + b*x)*a**2*c*d + 2*sqrt(a + b*x)*a*b*c**2
 - 2*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a**2*d**2*x + sqrt(a)*log(sqrt(a
 + b*x) - sqrt(a))*a*b*c*d*x + sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**2*c
**2*x + 2*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*a**2*d**2*x - sqrt(a)*log(s
qrt(a + b*x) + sqrt(a))*a*b*c*d*x - sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b
**2*c**2*x)/(2*a**2*c**2*x*(a*d - b*c))
```

3.246 $\int \frac{1}{x^2(c+dx)\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 185

$$\int \frac{1}{x^2(c+dx)\sqrt{ax^2+bx^3}}\,dx = -\frac{\sqrt{ax^2+bx^3}}{2acx^3}+\frac{(3bc+4ad)\sqrt{ax^2+bx^3}}{4a^2c^2x^2} - \frac{2d^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{ax^2+bx^3}}{\sqrt{bc-ad}x}\right)}{c^3\sqrt{bc-ad}} - \frac{(3b^2c^2+4abcd+8a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{4a^{5/2}c^3}$$

output
```
-1/2*(b*x^3+a*x^2)^(1/2)/a/c/x^3+1/4*(4*a*d+3*b*c)*(b*x^3+a*x^2)^(1/2)/a^2
/c^2/x^2-2*d^(5/2)*arctan(d^(1/2)*(b*x^3+a*x^2)^(1/2)/(-a*d+b*c)^(1/2)/x)/
c^3/(-a*d+b*c)^(1/2)-1/4*(8*a^2*d^2+4*a*b*c*d+3*b^2*c^2)*arctanh((b*x^3+a*
x^2)^(1/2)/a^(1/2)/x)/a^(5/2)/c^3
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2(c+dx)\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{\sqrt{a}\left(c\sqrt{bc-ad}(a+bx)(-2ac+3bcx+4adx)-8a^2d^{5/2}x^2\sqrt{a+bx}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)\right)-\sqrt{bc-ad}(3b^2c}{4a^{5/2}c^3\sqrt{bc-ad}x\sqrt{x^2(a+bx)}}$$

input
```
Integrate[1/(x^2*(c + d*x)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(Sqrt[a]*(c*Sqrt[b*c - a*d]*(a + b*x)*(-2*a*c + 3*b*c*x + 4*a*d*x) - 8*a^2
*d^(5/2)*x^2*Sqrt[a + b*x]*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]]
) - Sqrt[b*c - a*d]*(3*b^2*c^2 + 4*a*b*c*d + 8*a^2*d^2)*x^2*Sqrt[a + b*x]*
ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(5/2)*c^3*Sqrt[b*c - a*d]*x*Sqrt[x^2*
(a + b*x)])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1948, 114, 27, 168, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}(c+dx)}\,dx$$
$$\Big\downarrow \ 1948$$
$$\frac{x\sqrt{a+bx}\int \frac{1}{x^3\sqrt{a+bx}(c+dx)}dx}{\sqrt{ax^2+bx^3}}$$
$$\Big\downarrow \ 114$$
$$\frac{x\sqrt{a+bx}\left(-\frac{\int \frac{3bc+4ad+3bdx}{2x^2\sqrt{a+bx}(c+dx)}dx}{2ac}-\frac{\sqrt{a+bx}}{2acx^2}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 27$$

$$\frac{x\sqrt{a+bx}\left(-\frac{\int \frac{3bc+4ad+3bdx}{x^2\sqrt{a+bx}(c+dx)}dx}{4ac}-\frac{\sqrt{a+bx}}{2acx^2}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 168$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{\int \frac{3b^2c^2+4abdc+8a^2d^2+bd(3bc+4ad)x}{2x\sqrt{a+bx}(c+dx)}dx}{ac}-\frac{\sqrt{a+bx}(4ad+3bc)}{acx}}{4ac}-\frac{\sqrt{a+bx}}{2acx^2}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 27$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{\int \frac{3b^2c^2+4abdc+8a^2d^2+bd(3bc+4ad)x}{x\sqrt{a+bx}(c+dx)}dx}{2ac}-\frac{\sqrt{a+bx}(4ad+3bc)}{acx}}{4ac}-\frac{\sqrt{a+bx}}{2acx^2}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 174$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{\frac{\left(8a^2d^2+4abcd+3b^2c^2\right)\int \frac{1}{x\sqrt{a+bx}}dx}{c}-\frac{8a^2d^3\int \frac{1}{\sqrt{a+bx}(c+dx)}dx}{c}}{2ac}-\frac{\sqrt{a+bx}(4ad+3bc)}{acx}}{4ac}-\frac{\sqrt{a+bx}}{2acx^2}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 73$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{\frac{2\left(8a^2d^2+4abcd+3b^2c^2\right)\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}-\frac{16a^2d^3\int \frac{1}{c-\frac{ad}{b}+\frac{d(a+bx)}{b}}d\sqrt{a+bx}}{bc}}{2ac}-\frac{\sqrt{a+bx}(4ad+3bc)}{acx}}{4ac}-\frac{\sqrt{a+bx}}{2acx^2}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 218$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{\frac{2\left(8a^2d^2+4abcd+3b^2c^2\right)\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}-\frac{16a^2d^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{2ac}-\frac{\sqrt{a+bx}(4ad+3bc)}{acx}}{4ac}-\frac{\sqrt{a+bx}}{2acx^2}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 221$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{-\frac{16a^2d^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\left(8a^2d^2+4abcd+3b^2c^2\right)}{\sqrt{a}c}}{2ac}-\frac{\sqrt{a+bx}(4ad+3bc)}{acx}}{4ac}-\frac{\sqrt{a+bx}}{2acx^2}\right)}{\sqrt{ax^2+bx^3}}$$

input
```
Int[1/(x^2*(c + d*x)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(x*Sqrt[a + b*x]*(-1/2*Sqrt[a + b*x]/(a*c*x^2) - (-(((3*b*c + 4*a*d)*Sqrt[
a + b*x])/(a*c*x)) - ((-16*a^2*d^(5/2)*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt
[b*c - a*d]])/(c*Sqrt[b*c - a*d]) - (2*(3*b^2*c^2 + 4*a*b*c*d + 8*a^2*d^2)
*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*c))/(2*a*c))/(4*a*c)))/Sqrt[a*x^
2 + b*x^3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.46

method	result
pseudoelliptic	$\frac{-\frac{c\sqrt{bx+a}}{ax}+\frac{(2ad+bc)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{2d^2\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)}{\sqrt{d(ad-bc)}}}{c^2}$
risch	$-\frac{(bx+a)(-4adx-3cbx+2ac)}{4a^2c^2x\sqrt{x^2(bx+a)}}+\frac{b\left(-\frac{(8a^2d^2+4abcd+3b^2c^2)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{bc\sqrt{a}}+\frac{8a^2d^3\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)}{bc\sqrt{d(ad-bc)}}\right)\sqrt{bx+a}\,x}{4a^2c^2\sqrt{x^2(bx+a)}}$
default	$-\frac{\sqrt{bx+a}\left(-8d^3\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)b\,a^{\frac{9}{2}}x^2+4\sqrt{d(ad-bc)}\,a^{\frac{9}{2}}\sqrt{bx+a}\,cd-4\sqrt{d(ad-bc)}\,a^{\frac{7}{2}}(bx+a)^{\frac{3}{2}}cd+5\sqrt{d(ad-bc)}\,a\right.}{}$

input
```
int(1/x^2/(d*x+c)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(-c/a*(b*x+a)^(1/2)/x+(2*a*d+b*c)/a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1
/2))-2*d^2/(d*(a*d-b*c))^(1/2)*arctanh(d*(b*x+a)^(1/2)/(d*(a*d-b*c))^(1/2)
))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 774, normalized size of antiderivative = 4.18

$$\int \frac{1}{x^2(c+dx)\sqrt{ax^2+bx^3}}\,dx$$

$$= \left[\frac{8\,a^3d^2x^3\sqrt{-\frac{d}{bc-ad}}\log\left(\frac{bdx^2-(bc-2\,ad)x-2\,\sqrt{bx^3+ax^2}(bc-ad)\sqrt{-\frac{d}{bc-ad}}}{dx^2+cx}\right)+(3\,b^2c^2+4\,abcd+8\,a^2d^2)\sqrt{a}x^3\log\left(\frac{b}{}\right.}{8\,a^3c^3x^3}\right.$$

$$-\frac{16\,a^3d^2x^3\sqrt{\frac{d}{bc-ad}}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{\frac{d}{bc-ad}}}{x}\right)-(3\,b^2c^2+4\,abcd+8\,a^2d^2)\sqrt{a}x^3\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{}}{x^2}\right.}{8\,a^3c^3x^3}$$

$$-\frac{8\,a^3d^2x^3\sqrt{\frac{d}{bc-ad}}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{\frac{d}{bc-ad}}}{x}\right)-(3\,b^2c^2+4\,abcd+8\,a^2d^2)\sqrt{-a}x^3\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+}{4\,a^3c^3x^3}$$

input
```
integrate(1/x^2/(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(8*a^3*d^2*x^3*sqrt(-d/(b*c - a*d))*log((b*d*x^2 - (b*c - 2*a*d)*x -
2*sqrt(b*x^3 + a*x^2)*(b*c - a*d)*sqrt(-d/(b*c - a*d)))/(d*x^2 + c*x)) + (
3*b^2*c^2 + 4*a*b*c*d + 8*a^2*d^2)*sqrt(a)*x^3*log((b*x^2 + 2*a*x - 2*sqrt
(b*x^3 + a*x^2)*sqrt(a))/x^2) - 2*(2*a^2*c^2 - (3*a*b*c^2 + 4*a^2*c*d)*x)*
sqrt(b*x^3 + a*x^2))/(a^3*c^3*x^3), 1/4*(4*a^3*d^2*x^3*sqrt(-d/(b*c - a*d)
)*log((b*d*x^2 - (b*c - 2*a*d)*x - 2*sqrt(b*x^3 + a*x^2)*(b*c - a*d)*sqrt(
-d/(b*c - a*d)))/(d*x^2 + c*x)) + (3*b^2*c^2 + 4*a*b*c*d + 8*a^2*d^2)*sqrt
(-a)*x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) - (2*a^2*c^2 -
 (3*a*b*c^2 + 4*a^2*c*d)*x)*sqrt(b*x^3 + a*x^2))/(a^3*c^3*x^3), -1/8*(16*a
^3*d^2*x^3*sqrt(d/(b*c - a*d))*arctan(sqrt(b*x^3 + a*x^2)*sqrt(d/(b*c - a*
d))/x) - (3*b^2*c^2 + 4*a*b*c*d + 8*a^2*d^2)*sqrt(a)*x^3*log((b*x^2 + 2*a*
x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*(2*a^2*c^2 - (3*a*b*c^2 + 4*a^
2*c*d)*x)*sqrt(b*x^3 + a*x^2))/(a^3*c^3*x^3), -1/4*(8*a^3*d^2*x^3*sqrt(d/(
b*c - a*d))*arctan(sqrt(b*x^3 + a*x^2)*sqrt(d/(b*c - a*d))/x) - (3*b^2*c^2
 + 4*a*b*c*d + 8*a^2*d^2)*sqrt(-a)*x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)
/(b*x^2 + a*x)) + (2*a^2*c^2 - (3*a*b*c^2 + 4*a^2*c*d)*x)*sqrt(b*x^3 + a*x
^2))/(a^3*c^3*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^2(c+dx)\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^2\sqrt{x^2\left(a+bx\right)}\left(c+dx\right)}\,dx$$

input
```
integrate(1/x**2/(d*x+c)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(x**2*(a + b*x))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^2(c+dx)\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}(dx+c)x^2}\,dx$$

input
```
integrate(1/x^2/(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x^2)*(d*x + c)*x^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^2(c+dx)\sqrt{ax^2+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(c+dx)\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^2\,\sqrt{b\,x^3+a\,x^2}\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/(x^2*(a*x^2 + b*x^3)^(1/2)*(c + d*x)),x)
```

output
```
int(1/(x^2*(a*x^2 + b*x^3)^(1/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.87

$$\int \frac{1}{x^2(c+dx)\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{16\sqrt{d}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)a^3d^2x^2-4\sqrt{bx+a}\,a^3c^2d+8\sqrt{bx+a}\,a^3c\,d^2x+4\sqrt{bx+a}\,a^2b\,c^3-2}{}$$

input
```
int(1/x^2/(d*x+c)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(16*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d
 + b*c)))*a**3*d**2*x**2 - 4*sqrt(a + b*x)*a**3*c**2*d + 8*sqrt(a + b*x)*a
**3*c*d**2*x + 4*sqrt(a + b*x)*a**2*b*c**3 - 2*sqrt(a + b*x)*a**2*b*c**2*d
*x - 6*sqrt(a + b*x)*a*b**2*c**3*x + 8*sqrt(a)*log(sqrt(a + b*x) - sqrt(a)
)*a**3*d**3*x**2 - 4*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a**2*b*c*d**2*x*
*2 - sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b**2*c**2*d*x**2 - 3*sqrt(a)*l
og(sqrt(a + b*x) - sqrt(a))*b**3*c**3*x**2 - 8*sqrt(a)*log(sqrt(a + b*x) +
 sqrt(a))*a**3*d**3*x**2 + 4*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*a**2*b*c
*d**2*x**2 + sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*a*b**2*c**2*d*x**2 + 3*s
qrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**3*c**3*x**2)/(8*a**3*c**3*x**2*(a*d
 - b*c))
```

3.247 $\int \frac{1}{x^3(c+dx)\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 250

$$\int \frac{1}{x^3(c+dx)\sqrt{ax^2+bx^3}}\,dx = -\frac{\sqrt{ax^2+bx^3}}{3acx^4}+\frac{(5bc+6ad)\sqrt{ax^2+bx^3}}{12a^2c^2x^3} - \frac{(5b^2c^2+6abcd+8a^2d^2)\sqrt{ax^2+bx^3}}{8a^3c^3x^2}+\frac{2d^{7/2}\arctan\left(\frac{\sqrt{d}\sqrt{ax^2+bx^3}}{\sqrt{bc-ad}x}\right)}{c^4\sqrt{bc-ad}} + \frac{(5b^3c^3+6ab^2c^2d+8a^2bcd^2+16a^3d^3)\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{8a^{7/2}c^4}$$

output
```
-1/3*(b*x^3+a*x^2)^(1/2)/a/c/x^4+1/12*(6*a*d+5*b*c)*(b*x^3+a*x^2)^(1/2)/a^
2/c^2/x^3-1/8*(8*a^2*d^2+6*a*b*c*d+5*b^2*c^2)*(b*x^3+a*x^2)^(1/2)/a^3/c^3/
x^2+2*d^(7/2)*arctan(d^(1/2)*(b*x^3+a*x^2)^(1/2)/(-a*d+b*c)^(1/2)/x)/c^4/(
-a*d+b*c)^(1/2)+1/8*(16*a^3*d^3+8*a^2*b*c*d^2+6*a*b^2*c^2*d+5*b^3*c^3)*arc
tanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(7/2)/c^4
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^3(c+dx)\sqrt{ax^2+bx^3}}\,dx$$
$$=\frac{\sqrt{a}\left(-c\sqrt{bc-ad}(a+bx)\left(15b^2c^2x^2+2abcx(-5c+9dx)+4a^2(2c^2-3cdx+6d^2x^2)\right)+48a^3d^{7/2}x^3\sqrt{a+\right.}{24a^{7/2}c^4\sqrt{bc-ac}}$$

input
```
Integrate[1/(x^3*(c + d*x)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(Sqrt[a]*(-(c*Sqrt[b*c - a*d]*(a + b*x)*(15*b^2*c^2*x^2 + 2*a*b*c*x*(-5*c
+ 9*d*x) + 4*a^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2))) + 48*a^3*d^(7/2)*x^3*Sqrt
[a + b*x]*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]]) + 3*Sqrt[b*c -
a*d]*(5*b^3*c^3 + 6*a*b^2*c^2*d + 8*a^2*b*c*d^2 + 16*a^3*d^3)*x^3*Sqrt[a +
 b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(24*a^(7/2)*c^4*Sqrt[b*c - a*d]*x^2*
Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {1948, 114, 27, 168, 27, 168, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}(c+dx)}\,dx$$
$$\Big\downarrow \text{1948}$$
$$\frac{x\sqrt{a+bx}\int \frac{1}{x^4\sqrt{a+bx}(c+dx)}dx}{\sqrt{ax^2+bx^3}}$$
$$\Big\downarrow \text{114}$$

$$\frac{x\sqrt{a+bx}\left(-\frac{\int \frac{5bc+6ad+5bdx}{2x^3\sqrt{a+bx}(c+dx)}dx}{3ac}-\frac{\sqrt{a+bx}}{3acx^3}\right)}{\sqrt{ax^2+bx^3}}$$

$\downarrow$ 27

$$\frac{x\sqrt{a+bx}\left(-\frac{\int \frac{5bc+6ad+5bdx}{x^3\sqrt{a+bx}(c+dx)}dx}{6ac}-\frac{\sqrt{a+bx}}{3acx^3}\right)}{\sqrt{ax^2+bx^3}}$$

$\downarrow$ 168

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{\int \frac{3\left(5b^2c^2+6abdc+8a^2d^2+bd(5bc+6ad)x\right)}{2x^2\sqrt{a+bx}(c+dx)}dx}{2ac}-\frac{\sqrt{a+bx}(6ad+5bc)}{2acx^2}}{6ac}-\frac{\sqrt{a+bx}}{3acx^3}\right)}{\sqrt{ax^2+bx^3}}$$

$\downarrow$ 27

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{3\int \frac{5b^2c^2+6abdc+8a^2d^2+bd(5bc+6ad)x}{x^2\sqrt{a+bx}(c+dx)}dx}{4ac}-\frac{\sqrt{a+bx}(6ad+5bc)}{2acx^2}}{6ac}-\frac{\sqrt{a+bx}}{3acx^3}\right)}{\sqrt{ax^2+bx^3}}$$

$\downarrow$ 168

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{3\left(-\frac{\int \frac{5b^3c^3+6ab^2dc^2+8a^2bd^2c+16a^3d^3+bd\left(5b^2c^2+6abdc+8a^2d^2\right)x}{2x\sqrt{a+bx}(c+dx)}dx}{ac}-\frac{\sqrt{a+bx}\left(\frac{5b^2c}{a}+\frac{8ad^2}{c}+6bd\right)}{x}\right)}{4ac}-\frac{\sqrt{a+bx}(6ad+5bc)}{2acx^2}}{6ac}-\frac{\sqrt{a+bx}}{3acx^3}\right)}{\sqrt{ax^2+bx^3}}$$

$\downarrow$ 27

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{3\left(-\frac{\int \frac{5b^3c^3+6ab^2dc^2+8a^2bd^2c+16a^3d^3+bd\left(5b^2c^2+6abdc+8a^2d^2\right)x}{x\sqrt{a+bx}(c+dx)}dx}{2ac}-\frac{\sqrt{a+bx}\left(\frac{5b^2c}{a}+\frac{8ad^2}{c}+6bd\right)}{x}\right)}{4ac}-\frac{\sqrt{a+bx}(6ad+5bc)}{2acx^2}}{6ac}-\frac{\sqrt{a+bx}}{3acx^3}\right)}{\sqrt{ax^2+bx^3}}$$

$\downarrow$ 174

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{3\left(-\frac{\frac{\left(16a^3d^3+8a^2bcd^2+6ab^2c^2d+5b^3c^3\right)\int\frac{1}{x\sqrt{a+bx}}dx}{c}-\frac{16a^3d^4\int\frac{1}{\sqrt{a+bx}(c+dx)}dx}{c}}{2ac}-\frac{\sqrt{a+bx}\left(\frac{5b^2c}{a}+\frac{8ad^2}{c}+6bd\right)}{x}\right)}{4ac}-\frac{\sqrt{a+bx}(6ad+5bc\cdots}{2acx^2}}{6ac}\cdots\right.}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 73$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{3\left(-\frac{\frac{2\left(16a^3d^3+8a^2bcd^2+6ab^2c^2d+5b^3c^3\right)\int\frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}-\frac{32a^3d^4\int\frac{1}{c-\frac{ad}{b}+\frac{d(a+bx)}{b}}d\sqrt{a+bx}}{bc}}{2ac}-\frac{\sqrt{a+bx}\left(\frac{5b^2c}{a}+\frac{8ad^2}{c}+6bd\right)}{x}\right)}{4ac}-\cdots}{6ac}\cdots\right.}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 218$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{3\left(-\frac{\frac{2\left(16a^3d^3+8a^2bcd^2+6ab^2c^2d+5b^3c^3\right)\int\frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}-\frac{32a^3d^{7/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{2ac}-\frac{\sqrt{a+bx}\left(\frac{5b^2c}{a}+\frac{8ad^2}{c}+6bd\right)}{x}\right)}{4ac}-\frac{\sqrt{a+\cdots}}{\cdots}}{6ac}\cdots\right.}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 221$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{3\left(-\frac{-\frac{32a^3d^{7/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\left(16a^3d^3+8a^2bcd^2+6ab^2c^2d+5b^3c^3\right)}{\sqrt{ac}}}{2ac}-\frac{\sqrt{a+bx}\left(\frac{5b^2c}{a}+\frac{8ad^2}{c}+6bd\right)}{x}\right)}{4ac}-\frac{\sqrt{\cdots}}{\cdots}}{6ac}\cdots\right.}{\sqrt{ax^2+bx^3}}$$

input
```
Int[1/(x^3*(c + d*x)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(x*Sqrt[a + b*x]*(-1/3*Sqrt[a + b*x]/(a*c*x^3) - (-1/2*((5*b*c + 6*a*d)*Sq
rt[a + b*x])/(a*c*x^2) - (3*(-((((5*b^2*c)/a + 6*b*d + (8*a*d^2)/c)*Sqrt[a
 + b*x])/x) - ((-32*a^3*d^(7/2)*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c -
a*d]])/(c*Sqrt[b*c - a*d]) - (2*(5*b^3*c^3 + 6*a*b^2*c^2*d + 8*a^2*b*c*d^2
 + 16*a^3*d^3)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*c))/(2*a*c)))/(4*a
*c))/(6*a*c)))/Sqrt[a*x^2 + b*x^3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.46

method	result
pseudoelliptic	$\frac{-\frac{c\sqrt{bx+a}\,(-4adx-3cbx+2ac)}{4a^2x^2}-\frac{(8a^2d^2+4abcd+3b^2c^2)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{4a^{\frac{5}{2}}}+\frac{2d^3\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)}{\sqrt{d(ad-bc)}}}{c^3}$
risch	$-\frac{(bx+a)(24a^2d^2x^2+18abcd\,x^2+15b^2c^2x^2-12a^2cdx-10ab\,c^2x+8a^2c^2)}{24a^3c^3x^2\sqrt{x^2(bx+a)}}-\frac{b\left(-\frac{(16a^3d^3+8a^2bc\,d^2+6a\,b^2c^2d+5b^3c^3)\,\text{arcta}}{bc\sqrt{a}}\right.}{8a^3c}$
default	$-\frac{\sqrt{bx+a}\,\left(48d^4\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)b^2a^{\frac{13}{2}}x^3+24(bx+a)^{\frac{5}{2}}a^{\frac{11}{2}}\sqrt{d(ad-bc)}\,c\,d^2+18(bx+a)^{\frac{5}{2}}a^{\frac{9}{2}}\sqrt{d(ad-bc)}\,b\,c^2d+15(b\right.}{}$

input
```
int(1/x^3/(d*x+c)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(-1/4*c*(b*x+a)^(1/2)*(-4*a*d*x-3*b*c*x+2*a*c)/a^2/x^2-1/4*(8*a^2*d^
2+4*a*b*c*d+3*b^2*c^2)/a^(5/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2*d^3/(d*(a*
d-b*c))^(1/2)*arctanh(d*(b*x+a)^(1/2)/(d*(a*d-b*c))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 985, normalized size of antiderivative = 3.94

$$\int \frac{1}{x^3(c+dx)\sqrt{ax^2+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(48*a^4*d^3*x^4*sqrt(-d/(b*c - a*d))*log((b*d*x^2 - (b*c - 2*a*d)*x
+ 2*sqrt(b*x^3 + a*x^2)*(b*c - a*d)*sqrt(-d/(b*c - a*d)))/(d*x^2 + c*x)) +
 3*(5*b^3*c^3 + 6*a*b^2*c^2*d + 8*a^2*b*c*d^2 + 16*a^3*d^3)*sqrt(a)*x^4*lo
g((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) - 2*(8*a^3*c^3 + 3*
(5*a*b^2*c^3 + 6*a^2*b*c^2*d + 8*a^3*c*d^2)*x^2 - 2*(5*a^2*b*c^3 + 6*a^3*c
^2*d)*x)*sqrt(b*x^3 + a*x^2))/(a^4*c^4*x^4), 1/24*(24*a^4*d^3*x^4*sqrt(-d/
(b*c - a*d))*log((b*d*x^2 - (b*c - 2*a*d)*x + 2*sqrt(b*x^3 + a*x^2)*(b*c -
 a*d)*sqrt(-d/(b*c - a*d)))/(d*x^2 + c*x)) - 3*(5*b^3*c^3 + 6*a*b^2*c^2*d
+ 8*a^2*b*c*d^2 + 16*a^3*d^3)*sqrt(-a)*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt
(-a)/(b*x^2 + a*x)) - (8*a^3*c^3 + 3*(5*a*b^2*c^3 + 6*a^2*b*c^2*d + 8*a^3*
c*d^2)*x^2 - 2*(5*a^2*b*c^3 + 6*a^3*c^2*d)*x)*sqrt(b*x^3 + a*x^2))/(a^4*c^
4*x^4), 1/48*(96*a^4*d^3*x^4*sqrt(d/(b*c - a*d))*arctan(sqrt(b*x^3 + a*x^2
)*sqrt(d/(b*c - a*d))/x) + 3*(5*b^3*c^3 + 6*a*b^2*c^2*d + 8*a^2*b*c*d^2 +
16*a^3*d^3)*sqrt(a)*x^4*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a)
)/x^2) - 2*(8*a^3*c^3 + 3*(5*a*b^2*c^3 + 6*a^2*b*c^2*d + 8*a^3*c*d^2)*x^2
- 2*(5*a^2*b*c^3 + 6*a^3*c^2*d)*x)*sqrt(b*x^3 + a*x^2))/(a^4*c^4*x^4), 1/2
4*(48*a^4*d^3*x^4*sqrt(d/(b*c - a*d))*arctan(sqrt(b*x^3 + a*x^2)*sqrt(d/(b
*c - a*d))/x) - 3*(5*b^3*c^3 + 6*a*b^2*c^2*d + 8*a^2*b*c*d^2 + 16*a^3*d^3)
*sqrt(-a)*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) - (8*a^3*
c^3 + 3*(5*a*b^2*c^3 + 6*a^2*b*c^2*d + 8*a^3*c*d^2)*x^2 - 2*(5*a^2*b*c^...
```

Sympy [F]

$$\int \frac{1}{x^3(c+dx)\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^3\sqrt{x^2\,(a+bx)}\,(c+dx)}\,dx$$

input
```
integrate(1/x**3/(d*x+c)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(x**2*(a + b*x))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^3(c+dx)\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}(dx+c)x^3}\,dx$$

input
```
integrate(1/x^3/(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x^2)*(d*x + c)*x^3), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^3(c+dx)\sqrt{ax^2+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^3/(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(c+dx)\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^3\,\sqrt{b\,x^3+a\,x^2}\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/(x^3*(a*x^2 + b*x^3)^(1/2)*(c + d*x)),x)
```

output
```
int(1/(x^3*(a*x^2 + b*x^3)^(1/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.96

$$\int \frac{1}{x^3(c+dx)\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{-96\sqrt{d}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)a^4d^3x^3-16\sqrt{bx+a}\,a^4c^3d+24\sqrt{bx+a}\,a^4c^2d^2x-48\sqrt{bx+a}\,a^4c}{}$$

input
```
int(1/x^3/(d*x+c)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
( - 96*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( -
a*d + b*c)))*a**4*d**3*x**3 - 16*sqrt(a + b*x)*a**4*c**3*d + 24*sqrt(a + b
*x)*a**4*c**2*d**2*x - 48*sqrt(a + b*x)*a**4*c*d**3*x**2 + 16*sqrt(a + b*x
)*a**3*b*c**4 - 4*sqrt(a + b*x)*a**3*b*c**3*d*x + 12*sqrt(a + b*x)*a**3*b*
c**2*d**2*x**2 - 20*sqrt(a + b*x)*a**2*b**2*c**4*x + 6*sqrt(a + b*x)*a**2*
b**2*c**3*d*x**2 + 30*sqrt(a + b*x)*a*b**3*c**4*x**2 - 48*sqrt(a)*log(sqrt
(a + b*x) - sqrt(a))*a**4*d**4*x**3 + 24*sqrt(a)*log(sqrt(a + b*x) - sqrt(
a))*a**3*b*c*d**3*x**3 + 6*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a**2*b**2*
c**2*d**2*x**3 + 3*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b**3*c**3*d*x**3
 + 15*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**4*c**4*x**3 + 48*sqrt(a)*log
(sqrt(a + b*x) + sqrt(a))*a**4*d**4*x**3 - 24*sqrt(a)*log(sqrt(a + b*x) +
sqrt(a))*a**3*b*c*d**3*x**3 - 6*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*a**2*
b**2*c**2*d**2*x**3 - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*a*b**3*c**3*d
*x**3 - 15*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**4*c**4*x**3)/(48*a**4*c
**4*x**3*(a*d - b*c))
```

3.248 $\int \frac{x^3}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 135

$$\int \frac{x^3}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{ax^2+bx^3}}{bd^2x}+\frac{c^2\sqrt{ax^2+bx^3}}{d^2(bc-ad)x(c+dx)} - \frac{c(3bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{ax^2+bx^3}}{\sqrt{bc-ad}x}\right)}{d^{5/2}(bc-ad)^{3/2}}$$

output
```
2*(b*x^3+a*x^2)^(1/2)/b/d^2/x+c^2*(b*x^3+a*x^2)^(1/2)/d^2/(-a*d+b*c)/x/(d*
x+c)-c*(-4*a*d+3*b*c)*arctan(d^(1/2)*(b*x^3+a*x^2)^(1/2)/(-a*d+b*c)^(1/2)/
x)/d^(5/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \frac{x\left(\frac{\sqrt{d}(a+bx)(-2ad(c+dx)+bc(3c+2dx))}{b(bc-ad)(c+dx)}-\frac{c(3bc-4ad)\sqrt{a+bx}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}\right)}{d^{5/2}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x^3/((c + d*x)^2*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(x*((Sqrt[d]*(a + b*x)*(-2*a*d*(c + d*x) + b*c*(3*c + 2*d*x)))/(b*(b*c - a
*d)*(c + d*x)) - (c*(3*b*c - 4*a*d)*Sqrt[a + b*x]*ArcTan[(Sqrt[d]*Sqrt[a +
 b*x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2)))/(d^(5/2)*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1948, 100, 27, 90, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}(c+dx)^2}dx$$

$$\Big\downarrow \text{1948}$$

$$\frac{x\sqrt{a+bx}\int \frac{x^2}{\sqrt{a+bx}(c+dx)^2}dx}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{100}$$

$$\frac{x\sqrt{a+bx}\left(\frac{c^2\sqrt{a+bx}}{d^2(c+dx)(bc-ad)}-\frac{\int \frac{c(bc-2ad)-2d(bc-ad)x}{2\sqrt{a+bx}(c+dx)}dx}{d^2(bc-ad)}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\sqrt{a+bx}\left(\frac{c^2\sqrt{a+bx}}{d^2(c+dx)(bc-ad)}-\frac{\int \frac{c(bc-2ad)-2d(bc-ad)x}{\sqrt{a+bx}(c+dx)}dx}{2d^2(bc-ad)}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{90}$$

$$\frac{x\sqrt{a+bx}\left(\frac{c^2\sqrt{a+bx}}{d^2(c+dx)(bc-ad)}-\frac{c(3bc-4ad)\int \frac{1}{\sqrt{a+bx}(c+dx)}dx-\frac{4\sqrt{a+bx}(bc-ad)}{b}}{2d^2(bc-ad)}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{73}$$

$$\frac{x\sqrt{a+bx}\left(\frac{c^2\sqrt{a+bx}}{d^2(c+dx)(bc-ad)}-\frac{\frac{2c(3bc-4ad)\int\frac{1}{c-\frac{ad}{b}+\frac{d(a+bx)}{b}}d\sqrt{a+bx}}{b}-\frac{4\sqrt{a+bx}(bc-ad)}{b}}{2d^2(bc-ad)}\right)}{\sqrt{ax^2+bx^3}}$$

$\Big\downarrow$ 218

$$\frac{x\sqrt{a+bx}\left(\frac{c^2\sqrt{a+bx}}{d^2(c+dx)(bc-ad)}-\frac{\frac{2c(3bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\sqrt{bc-ad}}-\frac{4\sqrt{a+bx}(bc-ad)}{b}}{2d^2(bc-ad)}\right)}{\sqrt{ax^2+bx^3}}$$

input
```
Int[x^3/((c + d*x)^2*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(x*Sqrt[a + b*x]*((c^2*Sqrt[a + b*x])/(d^2*(b*c - a*d)*(c + d*x)) - ((-4*(
b*c - a*d)*Sqrt[a + b*x])/b + (2*c*(3*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a
+ b*x])/Sqrt[b*c - a*d]])/(Sqrt[d]*Sqrt[b*c - a*d]))/(2*d^2*(b*c - a*d))))
/Sqrt[a*x^2 + b*x^3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$\frac{-\frac{2\sqrt{bx+a}\,(-bdx+2ad+6bc)}{3}+\frac{b^{2}c^{2}\left(\frac{c\sqrt{bx+a}}{dx+c}-\frac{(6ad-5bc)\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)}{\sqrt{d(ad-bc)}}\right)}{ad-bc}}{d^{3}b^{2}}$
risch	$\frac{2(bx+a)x}{d^{2}b\sqrt{x^{2}(bx+a)}}-\frac{c\left(\frac{bc\sqrt{bx+a}}{(ad-bc)(d(bx+a)-ad+bc)}-\frac{(4ad-3bc)\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)}{(ad-bc)\sqrt{d(ad-bc)}}\right)\sqrt{bx+a}\,x}{d^{2}\sqrt{x^{2}(bx+a)}}$
default	$\frac{x\sqrt{bx+a}\left(4\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)abc\,d^{2}x-3\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)b^{2}c^{2}dx+2\sqrt{bx+a}\,\sqrt{d(ad-bc)}\,a\,d^{2}x-2\sqrt{bx+a}\,\sqrt{d(ad}\right.}{b\sqrt{b\,x^{3}+a\,x^{2}}\,d^{2}(ad}$

input
```
int(x^3/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(-2/3*(b*x+a)^(1/2)*(-b*d*x+2*a*d+6*b*c)+b^2*c^2/(a*d-b*c)*(c*(b*x+a
)^(1/2)/(d*x+c)-(6*a*d-5*b*c)/(d*(a*d-b*c))^(1/2)*arctanh(d*(b*x+a)^(1/2)/
(d*(a*d-b*c))^(1/2))))/b^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 503, normalized size of antiderivative = 3.73

$$\int \frac{x^3}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx$$
$$= \left[-\frac{\sqrt{-bcd+ad^2}((3\,b^2c^2d-4\,abcd^2)x^2+(3\,b^2c^3-4\,abc^2d)x)\log\left(\frac{bdx^2-(bc-2\,ad)x+2\,\sqrt{bx^3+ax^2}\sqrt{-bcd+ad^2}}{dx^2+cx}\right)-}{2\,((b^3c^2d^4-2\,ab^2cd^5+a^2bd^6)x^2+(b^3c^3d^3-2\right.$$

input
```
integrate(x^3/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(-b*c*d + a*d^2)*((3*b^2*c^2*d - 4*a*b*c*d^2)*x^2 + (3*b^2*c^3
- 4*a*b*c^2*d)*x)*log((b*d*x^2 - (b*c - 2*a*d)*x + 2*sqrt(b*x^3 + a*x^2)*s
qrt(-b*c*d + a*d^2))/(d*x^2 + c*x)) - 2*(3*b^2*c^3*d - 5*a*b*c^2*d^2 + 2*a
^2*c*d^3 + 2*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x)*sqrt(b*x^3 + a*x^2))
/((b^3*c^2*d^4 - 2*a*b^2*c*d^5 + a^2*b*d^6)*x^2 + (b^3*c^3*d^3 - 2*a*b^2*c
^2*d^4 + a^2*b*c*d^5)*x), (sqrt(b*c*d - a*d^2)*((3*b^2*c^2*d - 4*a*b*c*d^2
)*x^2 + (3*b^2*c^3 - 4*a*b*c^2*d)*x)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(b*c*d
 - a*d^2)/(b*d*x^2 + a*d*x)) + (3*b^2*c^3*d - 5*a*b*c^2*d^2 + 2*a^2*c*d^3
+ 2*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x)*sqrt(b*x^3 + a*x^2))/((b^3*c^
2*d^4 - 2*a*b^2*c*d^5 + a^2*b*d^6)*x^2 + (b^3*c^3*d^3 - 2*a*b^2*c^2*d^4 +
a^2*b*c*d^5)*x)]
```

Sympy [F]

$$\int \frac{x^3}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^3}{\sqrt{x^2\,(a+bx)}\,(c+dx)^2}\,dx$$

input
```
integrate(x**3/(d*x+c)**2/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**3/(sqrt(x**2*(a + b*x))*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{x^3}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^3}{\sqrt{bx^3+ax^2}(dx+c)^2}\,dx$$

input
```
integrate(x^3/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(sqrt(b*x^3 + a*x^2)*(d*x + c)^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^3}{\sqrt{b\,x^3+a\,x^2}\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(x^3/((a*x^2 + b*x^3)^(1/2)*(c + d*x)^2),x)
```

output
```
int(x^3/((a*x^2 + b*x^3)^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.44

$$\int \frac{x^3}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{4\sqrt{d}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)ab\,c^2d+4\sqrt{d}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)abc\,d^2x-3\sqrt{d}\,\sqrt{-ad+b}}{}$$

input
```
int(x^3/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(4*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d
+ b*c)))*a*b*c**2*d + 4*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/
(sqrt(d)*sqrt( - a*d + b*c)))*a*b*c*d**2*x - 3*sqrt(d)*sqrt( - a*d + b*c)*
atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*b**2*c**3 - 3*sqrt(d)
*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*b
**2*c**2*d*x + 2*sqrt(a + b*x)*a**2*c*d**3 + 2*sqrt(a + b*x)*a**2*d**4*x -
 5*sqrt(a + b*x)*a*b*c**2*d**2 - 4*sqrt(a + b*x)*a*b*c*d**3*x + 3*sqrt(a +
 b*x)*b**2*c**3*d + 2*sqrt(a + b*x)*b**2*c**2*d**2*x)/(b*d**3*(a**2*c*d**2
 + a**2*d**3*x - 2*a*b*c**2*d - 2*a*b*c*d**2*x + b**2*c**3 + b**2*c**2*d*x
))
```

3.249 $\int \frac{x^2}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 105

$$\int \frac{x^2}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = -\frac{c\sqrt{ax^2+bx^3}}{d(bc-ad)x(c+dx)}+\frac{(bc-2ad)\arctan\left(\frac{\sqrt{d}\sqrt{ax^2+bx^3}}{\sqrt{bc-ad}x}\right)}{d^{3/2}(bc-ad)^{3/2}}$$

output
```
-c*(b*x^3+a*x^2)^(1/2)/d/(-a*d+b*c)/x/(d*x+c)+(-2*a*d+b*c)*arctan(d^(1/2)*
(b*x^3+a*x^2)^(1/2)/(-a*d+b*c)^(1/2)/x)/d^(3/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \frac{-c\sqrt{d}\sqrt{bc-ad}x(a+bx)+(bc-2ad)x\sqrt{a+bx}(c+dx)\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{3/2}(bc-ad)^{3/2}\sqrt{x^2(a+bx)}(c+dx)}$$

input
```
Integrate[x^2/((c + d*x)^2*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-(c*Sqrt[d]*Sqrt[b*c - a*d]*x*(a + b*x)) + (b*c - 2*a*d)*x*Sqrt[a + b*x]*
(c + d*x)*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(d^(3/2)*(b*c -
 a*d)^(3/2)*Sqrt[x^2*(a + b*x)]*(c + d*x))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1948, 87, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}(c+dx)^2}dx$$

$$\Big\downarrow \ 1948$$

$$\frac{x\sqrt{a+bx}\int \frac{x}{\sqrt{a+bx}(c+dx)^2}dx}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \ 87$$

$$\frac{x\sqrt{a+bx}\left(\frac{(bc-2ad)\int \frac{1}{\sqrt{a+bx}(c+dx)}dx}{2d(bc-ad)}-\frac{c\sqrt{a+bx}}{d(c+dx)(bc-ad)}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \ 73$$

$$\frac{x\sqrt{a+bx}\left(\frac{(bc-2ad)\int \frac{1}{c-\frac{ad}{b}+\frac{d(a+bx)}{b}}d\sqrt{a+bx}}{bd(bc-ad)}-\frac{c\sqrt{a+bx}}{d(c+dx)(bc-ad)}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \ 218$$

$$\frac{x\sqrt{a+bx}\left(\frac{(bc-2ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{3/2}(bc-ad)^{3/2}}-\frac{c\sqrt{a+bx}}{d(c+dx)(bc-ad)}\right)}{\sqrt{ax^2+bx^3}}$$

input
```
Int[x^2/((c + d*x)^2*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(x*Sqrt[a + b*x]*(-((c*Sqrt[a + b*x])/(d*(b*c - a*d)*(c + d*x))) + ((b*c -
 2*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(d^(3/2)*(b*c - a
*d)^(3/2))))/Sqrt[a*x^2 + b*x^3]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$\frac{2\sqrt{bx+a}-\frac{bc\left(\frac{c\sqrt{bx+a}}{dx+c}-\frac{(4ad-3bc)\,\mathrm{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)}{\sqrt{d(ad-bc)}}\right)}{ad-bc}}{d^2b}$
default	$\frac{x\sqrt{bx+a}\left(-2\,\mathrm{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)a\,d^2x+\mathrm{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)bcdx-2\,\mathrm{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)acd+\mathrm{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc}}\right.\right.}{\sqrt{b\,x^3+a\,x^2}\,d(ad-bc)(dx+c)\sqrt{d(ad-bc)}}$

input
```
int(x^2/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^2*(2*(b*x+a)^(1/2)-b*c/(a*d-b*c)*(c*(b*x+a)^(1/2)/(d*x+c)-(4*a*d-3*b*c
)/(d*(a*d-b*c))^(1/2)*arctanh(d*(b*x+a)^(1/2)/(d*(a*d-b*c))^(1/2))))/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 378, normalized size of antiderivative = 3.60

$$\int \frac{x^2}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx$$
$$= \left[-\frac{\sqrt{-bcd+ad^2}((bcd-2\,ad^2)x^2+(bc^2-2\,acd)x)\log\left(\frac{bdx^2-(bc-2\,ad)x-2\,\sqrt{bx^3+ax^2}\sqrt{-bcd+ad^2}}{dx^2+cx}\right)+2\,(bc^2d-}{2\,((b^2c^2d^3-2\,abcd^4+a^2d^5)x^2+(b^2c^3d^2-2\,abc^2d^3+a^2cd^4)x)}\right.$$
$$-\frac{\sqrt{bcd-ad^2}((bcd-2\,ad^2)x^2+(bc^2-2\,acd)x)\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{bcd-ad^2}}{bdx^2+adx}\right)+(bc^2d-acd^2)\sqrt{bx^3+ax^2}}{(b^2c^2d^3-2\,abcd^4+a^2d^5)x^2+(b^2c^3d^2-2\,abc^2d^3+a^2cd^4)x}$$

input
```
integrate(x^2/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(-b*c*d + a*d^2)*((b*c*d - 2*a*d^2)*x^2 + (b*c^2 - 2*a*c*d)*x)*
log((b*d*x^2 - (b*c - 2*a*d)*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(-b*c*d + a*d^2
))/(d*x^2 + c*x)) + 2*(b*c^2*d - a*c*d^2)*sqrt(b*x^3 + a*x^2))/((b^2*c^2*d
^3 - 2*a*b*c*d^4 + a^2*d^5)*x^2 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4
)*x), -(sqrt(b*c*d - a*d^2)*((b*c*d - 2*a*d^2)*x^2 + (b*c^2 - 2*a*c*d)*x)*
arctan(sqrt(b*x^3 + a*x^2)*sqrt(b*c*d - a*d^2)/(b*d*x^2 + a*d*x)) + (b*c^2
*d - a*c*d^2)*sqrt(b*x^3 + a*x^2))/((b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*
x^2 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x)]
```

Sympy [F]

$$\int \frac{x^2}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^2}{\sqrt{x^2\,(a+bx)}\,(c+dx)^2}\,dx$$

input
```
integrate(x**2/(d*x+c)**2/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(x**2*(a + b*x))*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{x^2}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^2}{\sqrt{bx^3+ax^2}(dx+c)^2}\,dx$$

input
```
integrate(x^2/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(b*x^3 + a*x^2)*(d*x + c)^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^2}{\sqrt{b\,x^3+a\,x^2}\,(c+d\,x)^2}\,dx$$

input
```
int(x^2/((a*x^2 + b*x^3)^(1/2)*(c + d*x)^2),x)
```

output
```
int(x^2/((a*x^2 + b*x^3)^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.32

$$\int \frac{x^2}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{-2\sqrt{d}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)acd-2\sqrt{d}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)a\,d^2x+\sqrt{d}\,\sqrt{-ad+bc}\,at}{d^2\left(a^2d^3x-2abc\,d^2x+b^2c^2dx+\right.}$$

input
```
int(x^2/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x)
```

output
```
( - 2*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a
*d + b*c)))*a*c*d - 2*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(s
qrt(d)*sqrt( - a*d + b*c)))*a*d**2*x + sqrt(d)*sqrt( - a*d + b*c)*atan((sq
rt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*b*c**2 + sqrt(d)*sqrt( - a*d
+ b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*b*c*d*x + sqrt
(a + b*x)*a*c*d**2 - sqrt(a + b*x)*b*c**2*d)/(d**2*(a**2*c*d**2 + a**2*d**
3*x - 2*a*b*c**2*d - 2*a*b*c*d**2*x + b**2*c**3 + b**2*c**2*d*x))
```

3.250 $\int \frac{x}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{x}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{ax^2+bx^3}}{(bc-ad)x(c+dx)} + \frac{b\arctan\left(\frac{\sqrt{d}\sqrt{ax^2+bx^3}}{\sqrt{bc-adx}}\right)}{\sqrt{d}(bc-ad)^{3/2}}$$

output
```
(b*x^3+a*x^2)^(1/2)/(-a*d+b*c)/x/(d*x+c)+b*arctan(d^(1/2)*(b*x^3+a*x^2)^(1
/2)/(-a*d+b*c)^(1/2)/x)/d^(1/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.15

$$\int \frac{x}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \frac{x\left(\sqrt{d}\sqrt{bc-ad}(a+bx)+b\sqrt{a+bx}(c+dx)\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)\right)}{\sqrt{d}(bc-ad)^{3/2}\sqrt{x^2(a+bx)}(c+dx)}$$

input
```
Integrate[x/((c + d*x)^2*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(x*(Sqrt[d]*Sqrt[b*c - a*d]*(a + b*x) + b*Sqrt[a + b*x]*(c + d*x)*ArcTan[(
Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]]))/(Sqrt[d]*(b*c - a*d)^(3/2)*Sqrt[
x^2*(a + b*x)]*(c + d*x))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1948, 52, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{ax^2+bx^3}(c+dx)^2}dx$$

$$\Big\downarrow \text{1948}$$

$$\frac{x\sqrt{a+bx}\int \frac{1}{\sqrt{a+bx}(c+dx)^2}dx}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{52}$$

$$\frac{x\sqrt{a+bx}\left(\frac{b\int \frac{1}{\sqrt{a+bx}(c+dx)}dx}{2(bc-ad)}+\frac{\sqrt{a+bx}}{(c+dx)(bc-ad)}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{73}$$

$$\frac{x\sqrt{a+bx}\left(\frac{\int \frac{1}{c-\frac{ad}{b}+\frac{d(a+bx)}{b}}d\sqrt{a+bx}}{bc-ad}+\frac{\sqrt{a+bx}}{(c+dx)(bc-ad)}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{218}$$

$$\frac{x\sqrt{a+bx}\left(\frac{b\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{\sqrt{d}(bc-ad)^{3/2}}+\frac{\sqrt{a+bx}}{(c+dx)(bc-ad)}\right)}{\sqrt{ax^2+bx^3}}$$

input
```
Int[x/((c + d*x)^2*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(x*Sqrt[a + b*x]*(Sqrt[a + b*x]/((b*c - a*d)*(c + d*x)) + (b*ArcTan[(Sqrt[
d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(Sqrt[d]*(b*c - a*d)^(3/2))))/Sqrt[a*x
^2 + b*x^3]
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.83

method	result	size
pseudoelliptic	$\frac{\frac{c\sqrt{bx+a}}{dx+c}-\frac{(2ad-bc)\,\mathrm{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)}{\sqrt{d(ad-bc)}}}{d(ad-bc)}$	77
default	$-\frac{x\sqrt{bx+a}\left(-\mathrm{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)bdx-bc\,\mathrm{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)+\sqrt{bx+a}\,\sqrt{d(ad-bc)}\right)}{\sqrt{b\,x^{3}+a\,x^{2}}\,(ad-bc)(dx+c)\sqrt{d(ad-bc)}}$	127

input
```
int(x/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/(a*d-b*c)*(c*(b*x+a)^(1/2)/(d*x+c)-(2*a*d-b*c)/(d*(a*d-b*c))^(1/2)*arc
tanh(d*(b*x+a)^(1/2)/(d*(a*d-b*c))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 333, normalized size of antiderivative = 3.58

$$\int \frac{x}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx$$
$$=\left[\frac{(bdx^2+bcx)\sqrt{-bcd+ad^2}\log\left(\frac{bdx^2-(bc-2\,ad)x+2\,\sqrt{bx^3+ax^2}\sqrt{-bcd+ad^2}}{dx^2+cx}\right)+2\,\sqrt{bx^3+ax^2}(bcd-ad^2)}{2\left((b^2c^2d^2-2\,abcd^3+a^2d^4)x^2+(b^2c^3d-2\,abc^2d^2+a^2cd^3)x\right)},\right.$$
$$\left.-\frac{(bdx^2+bcx)\sqrt{bcd-ad^2}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{bcd-ad^2}}{bdx^2+adx}\right)-\sqrt{bx^3+ax^2}(bcd-ad^2)}{(b^2c^2d^2-2\,abcd^3+a^2d^4)x^2+(b^2c^3d-2\,abc^2d^2+a^2cd^3)x}\right]$$

input
```
integrate(x/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*((b*d*x^2 + b*c*x)*sqrt(-b*c*d + a*d^2)*log((b*d*x^2 - (b*c - 2*a*d)*
x + 2*sqrt(b*x^3 + a*x^2)*sqrt(-b*c*d + a*d^2))/(d*x^2 + c*x)) + 2*sqrt(b*
x^3 + a*x^2)*(b*c*d - a*d^2))/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 +
 (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x), -((b*d*x^2 + b*c*x)*sqrt(b*c*
d - a*d^2)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(b*c*d - a*d^2)/(b*d*x^2 + a*d*x
)) - sqrt(b*x^3 + a*x^2)*(b*c*d - a*d^2))/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^
2*d^4)*x^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)]
```

Sympy [F]

$$\int \frac{x}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{x}{\sqrt{x^2\,(a+bx)}\,(c+dx)^2}\,dx$$

input
```
integrate(x/(d*x+c)**2/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x/(sqrt(x**2*(a + b*x))*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{x}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{x}{\sqrt{bx^3+ax^2}(dx+c)^2}\,dx$$

input
```
integrate(x/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(b*x^3 + a*x^2)*(d*x + c)^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx=\int \frac{x}{\sqrt{b\,x^3+a\,x^2}\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(x/((a*x^2 + b*x^3)^(1/2)*(c + d*x)^2),x)
```

output
```
int(x/((a*x^2 + b*x^3)^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.68

$$\int \frac{x}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx$$
$$=\frac{\sqrt{d}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)bc+\sqrt{d}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)bdx-\sqrt{bx+a}\,a\,d^2+\sqrt{bx+a}\,}{d\left(a^2d^3x-2abc\,d^2x+b^2c^2dx+a^2c\,d^2-2ab\,c^2d+b^2c^3\right)}$$

input
```
int(x/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d +
b*c)))*b*c + sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sq
rt( - a*d + b*c)))*b*d*x - sqrt(a + b*x)*a*d**2 + sqrt(a + b*x)*b*c*d)/(d*
(a**2*c*d**2 + a**2*d**3*x - 2*a*b*c**2*d - 2*a*b*c*d**2*x + b**2*c**3 + b
**2*c**2*d*x))
```

3.251 $\int \frac{1}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 145

$$\int \frac{1}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = -\frac{d\sqrt{ax^2+bx^3}}{c(bc-ad)x(c+dx)} - \frac{\sqrt{d}(3bc-2ad)\arctan\left(\frac{\sqrt{d}\sqrt{ax^2+bx^3}}{\sqrt{bc-ad}x}\right)}{c^2(bc-ad)^{3/2}} - \frac{2\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{\sqrt{a}c^2}$$

output
```
-d*(b*x^3+a*x^2)^(1/2)/c/(-a*d+b*c)/x/(d*x+c)-d^(1/2)*(-2*a*d+3*b*c)*arcta
n(d^(1/2)*(b*x^3+a*x^2)^(1/2)/(-a*d+b*c)^(1/2)/x)/c^2/(-a*d+b*c)^(3/2)-2*a
rctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(1/2)/c^2
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.98

$$\int \frac{1}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \frac{x\left(-\frac{cd(a+bx)}{(bc-ad)(c+dx)}-\frac{\sqrt{d}(3bc-2ad)\sqrt{a+bx}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}-\frac{2\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{c^2\sqrt{x^2(a+bx)}}$$

input
```
Integrate[1/((c + d*x)^2*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(x*(-((c*d*(a + b*x))/((b*c - a*d)*(c + d*x))) - (Sqrt[d]*(3*b*c - 2*a*d)*
Sqrt[a + b*x]*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(b*c - a*d)
^(3/2) - (2*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]))/(c^2*S
qrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2467, 114, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax^2+bx^3}(c+dx)^2}\,dx$$

$$\Big\downarrow \ 2467$$

$$\frac{x\sqrt{a+bx}\int \frac{1}{x\sqrt{a+bx}(c+dx)^2}dx}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \ 114$$

$$\frac{x\sqrt{a+bx}\left(-\frac{\int -\frac{2bc-2ad-bdx}{2x\sqrt{a+bx}(c+dx)}dx}{c(bc-ad)}-\frac{d\sqrt{a+bx}}{c(c+dx)(bc-ad)}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 27$$

$$\frac{x\sqrt{a+bx}\left(\frac{\int \frac{2(bc-ad)-bdx}{x\sqrt{a+bx}(c+dx)}dx}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{c(c+dx)(bc-ad)}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 174$$

$$\frac{x\sqrt{a+bx}\left(\frac{\frac{2(bc-ad)\int \frac{1}{x\sqrt{a+bx}}dx}{c}-\frac{d(3bc-2ad)\int \frac{1}{\sqrt{a+bx}(c+dx)}dx}{c}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{c(c+dx)(bc-ad)}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 73$$

$$\frac{x\sqrt{a+bx}\left(\frac{\frac{4(bc-ad)\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}-\frac{2d(3bc-2ad)\int \frac{1}{c-\frac{ad}{b}+\frac{d(a+bx)}{b}}d\sqrt{a+bx}}{bc}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{c(c+dx)(bc-ad)}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 218$$

$$\frac{x\sqrt{a+bx}\left(\frac{\frac{4(bc-ad)\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}-\frac{2\sqrt{d}(3bc-2ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{c(c+dx)(bc-ad)}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 221$$

$$\frac{x\sqrt{a+bx}\left(\frac{-\frac{2\sqrt{d}(3bc-2ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{4\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)(bc-ad)}{\sqrt{a}c}}{2c(bc-ad)}-\frac{d\sqrt{a+bx}}{c(c+dx)(bc-ad)}\right)}{\sqrt{ax^2+bx^3}}$$

input

```
Int[1/((c + d*x)^2*Sqrt[a*x^2 + b*x^3]),x]
```

output

```
(x*Sqrt[a + b*x]*(-((d*Sqrt[a + b*x])/(c*(b*c - a*d)*(c + d*x))) + ((-2*Sq
rt[d]*(3*b*c - 2*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(c*
Sqrt[b*c - a*d]) - (4*(b*c - a*d)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]
*c))/(2*c*(b*c - a*d))))/Sqrt[a*x^2 + b*x^3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.47

method	result
pseudoelliptic	$\frac{b\left(-\frac{\sqrt{bx+a}}{b(dx+c)}+\frac{\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)}{\sqrt{d(ad-bc)}}\right)}{ad-bc}$
default	$\frac{x\sqrt{bx+a}\left(2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)d^3x-3\sqrt{a}\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)bc\,d^2x+2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)c\,d^2-3\sqrt{a}\operatorname{arcta}\right.}{}$

input
```
int(1/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
b/(a*d-b*c)*(-(b*x+a)^(1/2)/b/(d*x+c)+1/(d*(a*d-b*c))^(1/2)*arctanh(d*(b*x
+a)^(1/2)/(d*(a*d-b*c))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 941, normalized size of antiderivative = 6.49

$$\int \frac{1}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*(2*sqrt(b*x^3 + a*x^2)*a*c*d - ((3*a*b*c*d - 2*a^2*d^2)*x^2 + (3*a*b
*c^2 - 2*a^2*c*d)*x)*sqrt(-d/(b*c - a*d))*log((b*d*x^2 - (b*c - 2*a*d)*x -
 2*sqrt(b*x^3 + a*x^2)*(b*c - a*d)*sqrt(-d/(b*c - a*d)))/(d*x^2 + c*x)) -
2*((b*c*d - a*d^2)*x^2 + (b*c^2 - a*c*d)*x)*sqrt(a)*log((b*x^2 + 2*a*x - 2
*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2))/((a*b*c^3*d - a^2*c^2*d^2)*x^2 + (a*b*
c^4 - a^2*c^3*d)*x), -(sqrt(b*x^3 + a*x^2)*a*c*d + ((3*a*b*c*d - 2*a^2*d^2
)*x^2 + (3*a*b*c^2 - 2*a^2*c*d)*x)*sqrt(d/(b*c - a*d))*arctan(sqrt(b*x^3 +
 a*x^2)*sqrt(d/(b*c - a*d))/x) - ((b*c*d - a*d^2)*x^2 + (b*c^2 - a*c*d)*x)
*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2))/((a*b*c
^3*d - a^2*c^2*d^2)*x^2 + (a*b*c^4 - a^2*c^3*d)*x), -1/2*(2*sqrt(b*x^3 + a
*x^2)*a*c*d - 4*((b*c*d - a*d^2)*x^2 + (b*c^2 - a*c*d)*x)*sqrt(-a)*arctan(
sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) - ((3*a*b*c*d - 2*a^2*d^2)*x^2
 + (3*a*b*c^2 - 2*a^2*c*d)*x)*sqrt(-d/(b*c - a*d))*log((b*d*x^2 - (b*c - 2
*a*d)*x - 2*sqrt(b*x^3 + a*x^2)*(b*c - a*d)*sqrt(-d/(b*c - a*d)))/(d*x^2 +
 c*x)))/((a*b*c^3*d - a^2*c^2*d^2)*x^2 + (a*b*c^4 - a^2*c^3*d)*x), -(sqrt(
b*x^3 + a*x^2)*a*c*d - 2*((b*c*d - a*d^2)*x^2 + (b*c^2 - a*c*d)*x)*sqrt(-a
)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + ((3*a*b*c*d - 2*a^2
*d^2)*x^2 + (3*a*b*c^2 - 2*a^2*c*d)*x)*sqrt(d/(b*c - a*d))*arctan(sqrt(b*x
^3 + a*x^2)*sqrt(d/(b*c - a*d))/x))/((a*b*c^3*d - a^2*c^2*d^2)*x^2 + (a*b*
c^4 - a^2*c^3*d)*x)]
```

Sympy [F]

$$\int \frac{1}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx=\int \frac{1}{\sqrt{x^2\left(a+bx\right)}\left(c+dx\right)^2}\,dx$$

input
```
integrate(1/(d*x+c)**2/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(x**2*(a + b*x))*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx=\int \frac{1}{\sqrt{bx^3+ax^2}(dx+c)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x^2)*(d*x + c)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(127) = 254.

Time = 0.42 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.30

$$\int \frac{1}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx$$
$$=\frac{\left(3\sqrt{-a}bcd\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)-2\sqrt{-a}ad^2\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)-2\sqrt{bcd-ad^2}bc\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+2\sqrt{bcd}\right.}{\sqrt{bcd-ad^2}\sqrt{-a}bc^3-\sqrt{bcd-ad^2}\sqrt{-a}ac^2d}$$
$$-\frac{\sqrt{bx+a}bd}{(bc^2\mathrm{sgn}(x)-acd\mathrm{sgn}(x))(bc+(bx+a)d-ad)}$$
$$-\frac{(3bcd-2ad^2)\arctan\left(\frac{\sqrt{bx+a}d}{\sqrt{bcd-ad^2}}\right)}{(bc^3\mathrm{sgn}(x)-ac^2d\mathrm{sgn}(x))\sqrt{bcd-ad^2}}+\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}c^2\mathrm{sgn}(x)}$$

input
```
integrate(1/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
(3*sqrt(-a)*b*c*d*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 2*sqrt(-a)*a*d^2
*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 2*sqrt(b*c*d - a*d^2)*b*c*arctan(
sqrt(a)/sqrt(-a)) + 2*sqrt(b*c*d - a*d^2)*a*d*arctan(sqrt(a)/sqrt(-a)) + s
qrt(b*c*d - a*d^2)*sqrt(-a)*sqrt(a)*d)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(-a
)*b*c^3 - sqrt(b*c*d - a*d^2)*sqrt(-a)*a*c^2*d) - sqrt(b*x + a)*b*d/((b*c^
2*sgn(x) - a*c*d*sgn(x))*(b*c + (b*x + a)*d - a*d)) - (3*b*c*d - 2*a*d^2)*
arctan(sqrt(b*x + a)*d/sqrt(b*c*d - a*d^2))/((b*c^3*sgn(x) - a*c^2*d*sgn(x
))*sqrt(b*c*d - a*d^2)) + 2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*c^2*s
gn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{b\,x^3+a\,x^2}\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(1/((a*x^2 + b*x^3)^(1/2)*(c + d*x)^2),x)
```

output
```
int(1/((a*x^2 + b*x^3)^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 516, normalized size of antiderivative = 3.56

$$\int \frac{1}{(c+dx)^2\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{2\sqrt{d}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)a^2cd+2\sqrt{d}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)a^2d^2x-3\sqrt{d}\,\sqrt{-ad+bc}\,a}{}$$

input
```
int(1/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d
+ b*c)))*a**2*c*d + 2*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(s
qrt(d)*sqrt( - a*d + b*c)))*a**2*d**2*x - 3*sqrt(d)*sqrt( - a*d + b*c)*ata
n((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*a*b*c**2 - 3*sqrt(d)*sqr
t( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*a*b*c
*d*x + sqrt(a + b*x)*a**2*c*d**2 - sqrt(a + b*x)*a*b*c**2*d + sqrt(a)*log(
sqrt(a + b*x) - sqrt(a))*a**2*c*d**2 + sqrt(a)*log(sqrt(a + b*x) - sqrt(a)
)*a**2*d**3*x - 2*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b*c**2*d - 2*sqrt
(a)*log(sqrt(a + b*x) - sqrt(a))*a*b*c*d**2*x + sqrt(a)*log(sqrt(a + b*x)
- sqrt(a))*b**2*c**3 + sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**2*c**2*d*x
- sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*a**2*c*d**2 - sqrt(a)*log(sqrt(a +
b*x) + sqrt(a))*a**2*d**3*x + 2*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*a*b*c
**2*d + 2*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*a*b*c*d**2*x - sqrt(a)*log(
sqrt(a + b*x) + sqrt(a))*b**2*c**3 - sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*
b**2*c**2*d*x)/(a*c**2*(a**2*c*d**2 + a**2*d**3*x - 2*a*b*c**2*d - 2*a*b*c
*d**2*x + b**2*c**3 + b**2*c**2*d*x))
```

3.252 $\int \frac{1}{x(c+dx)^2\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 195

$$\int \frac{1}{x(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = -\frac{\sqrt{ax^2+bx^3}}{acx^2(c+dx)} - \frac{d(bc-2ad)\sqrt{ax^2+bx^3}}{ac^2(bc-ad)x(c+dx)} + \frac{d^{3/2}(5bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{ax^2+bx^3}}{\sqrt{bc-ad}x}\right)}{c^3(bc-ad)^{3/2}} + \frac{(bc+4ad)\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{a^{3/2}c^3}$$

output

```
-(b*x^3+a*x^2)^(1/2)/a/c/x^2/(d*x+c)-d*(-2*a*d+b*c)*(b*x^3+a*x^2)^(1/2)/a/
c^2/(-a*d+b*c)/x/(d*x+c)+d^(3/2)*(-4*a*d+5*b*c)*arctan(d^(1/2)*(b*x^3+a*x^
2)^(1/2)/(-a*d+b*c)^(1/2)/x)/c^3/(-a*d+b*c)^(3/2)+(4*a*d+b*c)*arctanh((b*x
^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(3/2)/c^3
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.87

$$\int \frac{1}{x(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \frac{\frac{c(a+bx)(bc(c+dx)-ad(c+2dx))}{a(-bc+ad)(c+dx)}+\frac{d^{3/2}(5bc-4ad)x\sqrt{a+bx}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}+\frac{(bc+4ad)x\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}}{c^3\sqrt{x^2(a+bx)}}$$

input
```
Integrate[1/(x*(c + d*x)^2*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
((c*(a + b*x)*(b*c*(c + d*x) - a*d*(c + 2*d*x)))/(a*(-(b*c) + a*d)*(c + d*
x)) + (d^(3/2)*(5*b*c - 4*a*d)*x*Sqrt[a + b*x]*ArcTan[(Sqrt[d]*Sqrt[a + b*
x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2) + ((b*c + 4*a*d)*x*Sqrt[a + b*x]*A
rcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2))/(c^3*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1948, 114, 27, 168, 25, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{ax^2+bx^3}(c+dx)^2}\,dx$$

$$\Big\downarrow \text{1948}$$

$$\frac{x\sqrt{a+bx}\int \frac{1}{x^2\sqrt{a+bx}(c+dx)^2}dx}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{114}$$

$$\frac{x\sqrt{a+bx}\left(-\frac{\int \frac{bc+4ad+3bdx}{2x\sqrt{a+bx}(c+dx)^2}dx}{ac}-\frac{\sqrt{a+bx}}{acx(c+dx)}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 27$$

$$\frac{x\sqrt{a+bx}\left(-\frac{\int \frac{bc+4ad+3bdx}{x\sqrt{a+bx}(c+dx)^2}dx}{2ac}-\frac{\sqrt{a+bx}}{acx(c+dx)}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 168$$

$$\frac{x\sqrt{a+bx}\left(-\frac{\frac{2d\sqrt{a+bx}(bc-2ad)}{c(c+dx)(bc-ad)}-\frac{\int -\frac{(bc-ad)(bc+4ad)+bd(bc-2ad)x}{x\sqrt{a+bx}(c+dx)}dx}{c(bc-ad)}}{2ac}-\frac{\sqrt{a+bx}}{acx(c+dx)}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 25$$

$$\frac{x\sqrt{a+bx}\left(-\frac{\frac{\int \frac{(bc-ad)(bc+4ad)+bd(bc-2ad)x}{x\sqrt{a+bx}(c+dx)}dx}{c(bc-ad)}+\frac{2d\sqrt{a+bx}(bc-2ad)}{c(c+dx)(bc-ad)}}{2ac}-\frac{\sqrt{a+bx}}{acx(c+dx)}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 174$$

$$\frac{x\sqrt{a+bx}\left(-\frac{\frac{\frac{(bc-ad)(4ad+bc)\int \frac{1}{x\sqrt{a+bx}}dx}{c}-\frac{ad^2(5bc-4ad)\int \frac{1}{\sqrt{a+bx}(c+dx)}dx}{c}}{c(bc-ad)}+\frac{2d\sqrt{a+bx}(bc-2ad)}{c(c+dx)(bc-ad)}}{2ac}-\frac{\sqrt{a+bx}}{acx(c+dx)}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 73$$

$$\frac{x\sqrt{a+bx}\left(-\frac{\frac{\frac{2(bc-ad)(4ad+bc)\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}-\frac{2ad^2(5bc-4ad)\int \frac{1}{c-\frac{ad}{b}+\frac{d(a+bx)}{b}}d\sqrt{a+bx}}{bc}}{c(bc-ad)}+\frac{2d\sqrt{a+bx}(bc-2ad)}{c(c+dx)(bc-ad)}}{2ac}-\frac{\sqrt{a+bx}}{acx(c+dx)}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 218$$

$$\frac{x\sqrt{a+bx}\left(-\frac{\frac{\frac{2(bc-ad)(4ad+bc)\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}-\frac{2ad^{3/2}(5bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{c(bc-ad)}+\frac{2d\sqrt{a+bx}(bc-2ad)}{c(c+dx)(bc-ad)}}{2ac}-\frac{\sqrt{a+bx}}{acx(c+dx)}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 221$$

$$\frac{x\sqrt{a+bx}\left(-\frac{\frac{-\frac{2ad^{3/2}(5bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)(bc-ad)(4ad+bc)}{\sqrt{a}c}}{c(bc-ad)}+\frac{2d\sqrt{a+bx}(bc-2ad)}{c(c+dx)(bc-ad)}}{2ac}-\frac{\sqrt{a+bx}}{acx(c+dx)}\right)}{\sqrt{ax^2+bx^3}}$$

input
```
Int[1/(x*(c + d*x)^2*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(x*Sqrt[a + b*x]*(-(Sqrt[a + b*x]/(a*c*x*(c + d*x))) - ((2*d*(b*c - 2*a*d)
*Sqrt[a + b*x])/(c*(b*c - a*d)*(c + d*x)) + ((-2*a*d^(3/2)*(5*b*c - 4*a*d)
*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(c*Sqrt[b*c - a*d]) - (2
*(b*c - a*d)*(b*c + 4*a*d)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*c))/(c
*(b*c - a*d)))/(2*a*c)))/Sqrt[a*x^2 + b*x^3]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.51

method	result
pseudoelliptic	$\frac{-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{d\left(-\frac{c\sqrt{bx+a}}{dx+c}-\frac{(2ad-3bc)\,\mathrm{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)}{\sqrt{d(ad-bc)}}\right)}{ad-bc}}{c^2}$
risch	$-\frac{bx+a}{a\,c^2\sqrt{x^2(bx+a)}}-\frac{b\left(-\frac{(4ad+bc)\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{bc\sqrt{a}}-\frac{2a\,d^2\left(-\frac{bc\sqrt{bx+a}}{2(ad-bc)(d(bx+a)-ad+bc)}-\frac{(4ad-5bc)\,\mathrm{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)}{2(ad-bc)\sqrt{d(ad-bc)}}\cdots\right)}{bc}\right)}{c^2a\sqrt{x^2(bx+a)}}\cdots$
default	$-\frac{\sqrt{bx+a}\left(4a^{\frac{7}{2}}\,\mathrm{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)d^4x^2-5a^{\frac{5}{2}}\,\mathrm{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)bc\,d^3x^2+4a^{\frac{7}{2}}\,\mathrm{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)c\,d^3x-5a^{\frac{5}{2}}\,\cdots\right)}{\cdots}$

input
```
int(1/x/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(-2/a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))-d/(a*d-b*c)*(-c*(b*x+a)^(
1/2)/(d*x+c)-(2*a*d-3*b*c)/(d*(a*d-b*c))^(1/2)*arctanh(d*(b*x+a)^(1/2)/(d*
(a*d-b*c))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1256, normalized size of antiderivative = 6.44

$$\int \frac{1}{x(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(((5*a^2*b*c*d^2 - 4*a^3*d^3)*x^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^2)*x^2
)*sqrt(-d/(b*c - a*d))*log((b*d*x^2 - (b*c - 2*a*d)*x + 2*sqrt(b*x^3 + a*x
^2)*(b*c - a*d)*sqrt(-d/(b*c - a*d)))/(d*x^2 + c*x)) + ((b^2*c^2*d + 3*a*b
*c*d^2 - 4*a^2*d^3)*x^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x^2)*sqrt(
a)*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) - 2*(a*b*c^3 -
 a^2*c^2*d + (a*b*c^2*d - 2*a^2*c*d^2)*x)*sqrt(b*x^3 + a*x^2))/((a^2*b*c^4
*d - a^3*c^3*d^2)*x^3 + (a^2*b*c^5 - a^3*c^4*d)*x^2), 1/2*(2*((5*a^2*b*c*d
^2 - 4*a^3*d^3)*x^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^2)*x^2)*sqrt(d/(b*c - a*d
))*arctan(sqrt(b*x^3 + a*x^2)*sqrt(d/(b*c - a*d))/x) + ((b^2*c^2*d + 3*a*b
*c*d^2 - 4*a^2*d^3)*x^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x^2)*sqrt(
a)*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) - 2*(a*b*c^3 -
 a^2*c^2*d + (a*b*c^2*d - 2*a^2*c*d^2)*x)*sqrt(b*x^3 + a*x^2))/((a^2*b*c^4
*d - a^3*c^3*d^2)*x^3 + (a^2*b*c^5 - a^3*c^4*d)*x^2), -1/2*(2*((b^2*c^2*d
+ 3*a*b*c*d^2 - 4*a^2*d^3)*x^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x^2
)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) - ((5*a^2*b*
c*d^2 - 4*a^3*d^3)*x^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^2)*x^2)*sqrt(-d/(b*c -
 a*d))*log((b*d*x^2 - (b*c - 2*a*d)*x + 2*sqrt(b*x^3 + a*x^2)*(b*c - a*d)*
sqrt(-d/(b*c - a*d)))/(d*x^2 + c*x)) + 2*(a*b*c^3 - a^2*c^2*d + (a*b*c^2*d
 - 2*a^2*c*d^2)*x)*sqrt(b*x^3 + a*x^2))/((a^2*b*c^4*d - a^3*c^3*d^2)*x^3 +
 (a^2*b*c^5 - a^3*c^4*d)*x^2), -(((b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3)...
```

Sympy [F]

$$\int \frac{1}{x(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x\sqrt{x^2\,(a+bx)}\,(c+dx)^2}\,dx$$

input
```
integrate(1/x/(d*x+c)**2/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(x**2*(a + b*x))*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{x(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}(dx+c)^2x}\,dx$$

input
```
integrate(1/x/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x^2)*(d*x + c)^2*x), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x\,\sqrt{b\,x^3+a\,x^2}\,(c+d\,x)^2}\,dx$$

input
```
int(1/(x*(a*x^2 + b*x^3)^(1/2)*(c + d*x)^2),x)
```

output
```
int(1/(x*(a*x^2 + b*x^3)^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 743, normalized size of antiderivative = 3.81

$$\int \frac{1}{x(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \text{Too large to display}$$

input
```
int(1/x/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x)
```

output
```
( - 8*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a
*d + b*c)))*a**3*c*d**2*x - 8*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*
x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*a**3*d**3*x**2 + 10*sqrt(d)*sqrt( - a*
d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*a**2*b*c**2*
d*x + 10*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt(
- a*d + b*c)))*a**2*b*c*d**2*x**2 - 2*sqrt(a + b*x)*a**3*c**2*d**2 - 4*sqr
t(a + b*x)*a**3*c*d**3*x + 4*sqrt(a + b*x)*a**2*b*c**3*d + 6*sqrt(a + b*x)
*a**2*b*c**2*d**2*x - 2*sqrt(a + b*x)*a*b**2*c**4 - 2*sqrt(a + b*x)*a*b**2
*c**3*d*x - 4*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a**3*c*d**3*x - 4*sqrt(
a)*log(sqrt(a + b*x) - sqrt(a))*a**3*d**4*x**2 + 7*sqrt(a)*log(sqrt(a + b*
x) - sqrt(a))*a**2*b*c**2*d**2*x + 7*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*
a**2*b*c*d**3*x**2 - 2*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b**2*c**3*d*
x - 2*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b**2*c**2*d**2*x**2 - sqrt(a)
*log(sqrt(a + b*x) - sqrt(a))*b**3*c**4*x - sqrt(a)*log(sqrt(a + b*x) - sq
rt(a))*b**3*c**3*d*x**2 + 4*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*a**3*c*d*
*3*x + 4*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*a**3*d**4*x**2 - 7*sqrt(a)*l
og(sqrt(a + b*x) + sqrt(a))*a**2*b*c**2*d**2*x - 7*sqrt(a)*log(sqrt(a + b*
x) + sqrt(a))*a**2*b*c*d**3*x**2 + 2*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*
a*b**2*c**3*d*x + 2*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*a*b**2*c**2*d**2*
x**2 + sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**3*c**4*x + sqrt(a)*log(s...
```

3.253 $\int \frac{1}{x^2(c+dx)^2\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 270

$$\begin{aligned}\int \frac{1}{x^2(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = &-\frac{\sqrt{ax^2+bx^3}}{2acx^3(c+dx)}+\frac{3(bc+2ad)\sqrt{ax^2+bx^3}}{4a^2c^2x^2(c+dx)}\\ &+\frac{d(3bc-4ad)(bc+3ad)\sqrt{ax^2+bx^3}}{4a^2c^3(bc-ad)x(c+dx)}\\ &-\frac{d^{5/2}(7bc-6ad)\arctan\left(\frac{\sqrt{d}\sqrt{ax^2+bx^3}}{\sqrt{bc-ad}x}\right)}{c^4(bc-ad)^{3/2}}\\ &-\frac{(3b^2c^2+8abcd+24a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{4a^{5/2}c^4}\end{aligned}$$

output

```
-1/2*(b*x^3+a*x^2)^(1/2)/a/c/x^3/(d*x+c)+3/4*(2*a*d+b*c)*(b*x^3+a*x^2)^(1/
2)/a^2/c^2/x^2/(d*x+c)+1/4*d*(-4*a*d+3*b*c)*(3*a*d+b*c)*(b*x^3+a*x^2)^(1/2
)/a^2/c^3/(-a*d+b*c)/x/(d*x+c)-d^(5/2)*(-6*a*d+7*b*c)*arctan(d^(1/2)*(b*x^
3+a*x^2)^(1/2)/(-a*d+b*c)^(1/2)/x)/c^4/(-a*d+b*c)^(3/2)-1/4*(24*a^2*d^2+8*
a*b*c*d+3*b^2*c^2)*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(5/2)/c^4
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \frac{\frac{c(a+bx)(-3b^2c^2x(c+dx)+abc(2c^2-3cdx-5d^2x^2)+2a^2d(-c^2+3cdx+6d^2x^2))}{a^2(-bc+ad)(c+dx)} - \frac{4d^{5/2}(7bc-6ad)x^2\sqrt{a+bx}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} - \left(3b^2c^2+\cdots\right.}{4c^4x\sqrt{x^2(a+bx)}}$$

input
```
Integrate[1/(x^2*(c + d*x)^2*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
((c*(a + b*x)*(-3*b^2*c^2*x*(c + d*x) + a*b*c*(2*c^2 - 3*c*d*x - 5*d^2*x^2
) + 2*a^2*d*(-c^2 + 3*c*d*x + 6*d^2*x^2)))/(a^2*(-(b*c) + a*d)*(c + d*x))
- (4*d^(5/2)*(7*b*c - 6*a*d)*x^2*Sqrt[a + b*x]*ArcTan[(Sqrt[d]*Sqrt[a + b*
x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2) - ((3*b^2*c^2 + 8*a*b*c*d + 24*a^2
*d^2)*x^2*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(5/2))/(4*c^4*x*
Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1948, 114, 27, 168, 27, 168, 25, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}(c+dx)^2}dx$$

$$\Big\downarrow \text{ 1948}$$

$$\frac{x\sqrt{a+bx}\int \frac{1}{x^3\sqrt{a+bx}(c+dx)^2}dx}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 114}$$

$$\frac{x\sqrt{a+bx}\left(-\frac{\int \frac{3(bc+2ad)+5bdx}{2x^2\sqrt{a+bx}(c+dx)^2}dx}{2ac}-\frac{\sqrt{a+bx}}{2acx^2(c+dx)}\right)}{\sqrt{ax^2+bx^3}}$$

$\Big\downarrow$ 27

$$\frac{x\sqrt{a+bx}\left(-\frac{\int \frac{3(bc+2ad)+5bdx}{x^2\sqrt{a+bx}(c+dx)^2}dx}{4ac}-\frac{\sqrt{a+bx}}{2acx^2(c+dx)}\right)}{\sqrt{ax^2+bx^3}}$$

$\Big\downarrow$ 168

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{\int \frac{3b^2c^2+8abdc+24a^2d^2+9bd(bc+2ad)x}{2x\sqrt{a+bx}(c+dx)^2}dx}{ac}-\frac{3\sqrt{a+bx}(2ad+bc)}{acx(c+dx)}}{4ac}-\frac{\sqrt{a+bx}}{2acx^2(c+dx)}\right)}{\sqrt{ax^2+bx^3}}$$

$\Big\downarrow$ 27

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{\int \frac{3b^2c^2+8abdc+24a^2d^2+9bd(bc+2ad)x}{x\sqrt{a+bx}(c+dx)^2}dx}{2ac}-\frac{3\sqrt{a+bx}(2ad+bc)}{acx(c+dx)}}{4ac}-\frac{\sqrt{a+bx}}{2acx^2(c+dx)}\right)}{\sqrt{ax^2+bx^3}}$$

$\Big\downarrow$ 168

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{\frac{2d\sqrt{a+bx}(3bc-4ad)(3ad+bc)}{c(c+dx)(bc-ad)}-\frac{\int -\frac{(bc-ad)\left(3b^2c^2+8abdc+24a^2d^2\right)+bd(3bc-4ad)(bc+3ad)x}{x\sqrt{a+bx}(c+dx)}dx}{c(bc-ad)}}{2ac}-\frac{3\sqrt{a+bx}(2ad+bc)}{acx(c+dx)}}{4ac}-\frac{\sqrt{a+bx}}{2acx^2(c+dx)}\right)}{\sqrt{ax^2+bx^3}}$$

$\Big\downarrow$ 25

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{\frac{\int \frac{(bc-ad)\left(3b^2c^2+8abdc+24a^2d^2\right)+bd(3bc-4ad)(bc+3ad)x}{x\sqrt{a+bx}(c+dx)}dx}{c(bc-ad)}+\frac{2d\sqrt{a+bx}(3bc-4ad)(3ad+bc)}{c(c+dx)(bc-ad)}}{2ac}-\frac{3\sqrt{a+bx}(2ad+bc)}{acx(c+dx)}}{4ac}-\frac{\sqrt{a+bx}}{2acx^2(c+dx)}\right)}{\sqrt{ax^2+bx^3}}$$

$\Big\downarrow$ 174

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{\frac{(bc-ad)\left(24a^2d^2+8abcd+3b^2c^2\right)\int\frac{1}{x\sqrt{a+bx}}dx}{c}-\frac{4a^2d^3(7bc-6ad)\int\frac{1}{\sqrt{a+bx}(c+dx)}dx}{c}}{c(bc-ad)}+\frac{2d\sqrt{a+bx}(3bc-4ad)(3ad+bc)}{c(c+dx)(bc-ad)}}{2ac}-\frac{3\sqrt{a+bx}(2ad+bc)}{acx(c+dx)}}{4ac}\right.}{\sqrt{ax^2+bx^3}}$$

$\big\downarrow$ 73

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{\frac{2(bc-ad)\left(24a^2d^2+8abcd+3b^2c^2\right)\int\frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}-\frac{8a^2d^3(7bc-6ad)\int\frac{1}{c-\frac{ad}{b}+\frac{d(a+bx)}{b}}d\sqrt{a+bx}}{bc}}{c(bc-ad)}+\frac{2d\sqrt{a+bx}(3bc-4ad)(3ad+bc)}{c(c+dx)(bc-ad)}}{2ac}-\right.}{\sqrt{ax^2+bx^3}}$$

$\big\downarrow$ 218

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{\frac{2(bc-ad)\left(24a^2d^2+8abcd+3b^2c^2\right)\int\frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}-\frac{8a^2d^{5/2}(7bc-6ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{c(bc-ad)}+\frac{2d\sqrt{a+bx}(3bc-4ad)(3ad+bc)}{c(c+dx)(bc-ad)}}{2ac}-\right.}{\sqrt{ax^2+bx^3}}$$

$\big\downarrow$ 221

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{-\frac{8a^2d^{5/2}(7bc-6ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)(bc-ad)\left(24a^2d^2+8abcd+3b^2c^2\right)}{\sqrt{a}c}}{c(bc-ad)}+\frac{2d\sqrt{a+bx}(3bc-4ad)(3ad+bc)}{c(c+dx)(bc-ad)}}{2ac}-\right.}{\sqrt{ax^2+bx^3}}$$

input
```
Int[1/(x^2*(c + d*x)^2*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(x*Sqrt[a + b*x]*(-1/2*Sqrt[a + b*x]/(a*c*x^2*(c + d*x)) - ((-3*(b*c + 2*a
*d)*Sqrt[a + b*x])/(a*c*x*(c + d*x)) - ((2*d*(3*b*c - 4*a*d)*(b*c + 3*a*d)
*Sqrt[a + b*x])/(c*(b*c - a*d)*(c + d*x)) + ((-8*a^2*d^(5/2)*(7*b*c - 6*a*
d)*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(c*Sqrt[b*c - a*d]) -
(2*(b*c - a*d)*(3*b^2*c^2 + 8*a*b*c*d + 24*a^2*d^2)*ArcTanh[Sqrt[a + b*x]/
Sqrt[a]])/(Sqrt[a]*c))/(c*(b*c - a*d)))/(2*a*c))/(4*a*c)))/Sqrt[a*x^2 + b*
x^3]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.46

method	result
pseudoelliptic	$\frac{-\frac{c\sqrt{bx+a}}{ax}+\frac{(4ad+bc)\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{d^{2}\left(-\frac{c\sqrt{bx+a}}{dx+c}-\frac{(4ad-5bc)\,\mathrm{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)}{\sqrt{d(ad-bc)}}\right)}{ad-bc}}{c^{3}}$
risch	$-\frac{(bx+a)(-8adx-3cbx+2ac)}{4a^{2}c^{3}x\sqrt{x^{2}(bx+a)}}+\frac{b\left(-\frac{\left(24a^{2}d^{2}+8abcd+3b^{2}c^{2}\right)\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{bc\sqrt{a}}-\frac{8a^{2}d^{3}\left(-\frac{bc\sqrt{bx+a}}{2(ad-bc)(d(bx+a)-ad+bc)}-\frac{(6ad}{}\right.}{bc}\right.}{4a^{2}c^{3}\sqrt{x^{2}(bx+a)}}$
default	$-\frac{\sqrt{bx+a}\left(-5\sqrt{d(ad-bc)}\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^{3}b^{3}c^{2}d^{2}x^{3}-3\sqrt{d(ad-bc)}\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^{2}b^{4}c^{3}d\,x^{3}-24\,\mathrm{arctanh}\left(\frac{d\sqrt{b}}{\sqrt{d(a}}\right.\right.}{}$

input
```
int(1/x^2/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(-c/a*(b*x+a)^(1/2)/x+(4*a*d+b*c)/a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1
/2))+d^2/(a*d-b*c)*(-c*(b*x+a)^(1/2)/(d*x+c)-(4*a*d-5*b*c)/(d*(a*d-b*c))^(
1/2)*arctanh(d*(b*x+a)^(1/2)/(d*(a*d-b*c))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1598, normalized size of antiderivative = 5.92

$$\int \frac{1}{x^2(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(4*((7*a^3*b*c*d^3 - 6*a^4*d^4)*x^4 + (7*a^3*b*c^2*d^2 - 6*a^4*c*d^3)
*x^3)*sqrt(-d/(b*c - a*d))*log((b*d*x^2 - (b*c - 2*a*d)*x - 2*sqrt(b*x^3 +
 a*x^2)*(b*c - a*d)*sqrt(-d/(b*c - a*d)))/(d*x^2 + c*x)) + ((3*b^3*c^3*d +
 5*a*b^2*c^2*d^2 + 16*a^2*b*c*d^3 - 24*a^3*d^4)*x^4 + (3*b^3*c^4 + 5*a*b^2
*c^3*d + 16*a^2*b*c^2*d^2 - 24*a^3*c*d^3)*x^3)*sqrt(a)*log((b*x^2 + 2*a*x
- 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) - 2*(2*a^2*b*c^4 - 2*a^3*c^3*d - (3*
a*b^2*c^3*d + 5*a^2*b*c^2*d^2 - 12*a^3*c*d^3)*x^2 - 3*(a*b^2*c^4 + a^2*b*c
^3*d - 2*a^3*c^2*d^2)*x)*sqrt(b*x^3 + a*x^2))/((a^3*b*c^5*d - a^4*c^4*d^2)
*x^4 + (a^3*b*c^6 - a^4*c^5*d)*x^3), -1/8*(8*((7*a^3*b*c*d^3 - 6*a^4*d^4)*
x^4 + (7*a^3*b*c^2*d^2 - 6*a^4*c*d^3)*x^3)*sqrt(d/(b*c - a*d))*arctan(sqrt
(b*x^3 + a*x^2)*sqrt(d/(b*c - a*d))/x) - ((3*b^3*c^3*d + 5*a*b^2*c^2*d^2 +
 16*a^2*b*c*d^3 - 24*a^3*d^4)*x^4 + (3*b^3*c^4 + 5*a*b^2*c^3*d + 16*a^2*b*
c^2*d^2 - 24*a^3*c*d^3)*x^3)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a
*x^2)*sqrt(a))/x^2) + 2*(2*a^2*b*c^4 - 2*a^3*c^3*d - (3*a*b^2*c^3*d + 5*a^
2*b*c^2*d^2 - 12*a^3*c*d^3)*x^2 - 3*(a*b^2*c^4 + a^2*b*c^3*d - 2*a^3*c^2*d
^2)*x)*sqrt(b*x^3 + a*x^2))/((a^3*b*c^5*d - a^4*c^4*d^2)*x^4 + (a^3*b*c^6
- a^4*c^5*d)*x^3), 1/4*(((3*b^3*c^3*d + 5*a*b^2*c^2*d^2 + 16*a^2*b*c*d^3 -
 24*a^3*d^4)*x^4 + (3*b^3*c^4 + 5*a*b^2*c^3*d + 16*a^2*b*c^2*d^2 - 24*a^3*
c*d^3)*x^3)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) +
2*((7*a^3*b*c*d^3 - 6*a^4*d^4)*x^4 + (7*a^3*b*c^2*d^2 - 6*a^4*c*d^3)*x^...
```

Sympy [F]

$$\int \frac{1}{x^2(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^2\sqrt{x^2\,(a+bx)}\,(c+dx)^2}\,dx$$

input
```
integrate(1/x**2/(d*x+c)**2/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(x**2*(a + b*x))*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{x^2(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}(dx+c)^2x^2}\,dx$$

input
```
integrate(1/x^2/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x^2)*(d*x + c)^2*x^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^2(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^2\,\sqrt{b\,x^3+a\,x^2}\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(1/(x^2*(a*x^2 + b*x^3)^(1/2)*(c + d*x)^2),x)
```

output
```
int(1/(x^2*(a*x^2 + b*x^3)^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 971, normalized size of antiderivative = 3.60

$$\int \frac{1}{x^2(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \text{Too large to display}$$

input
```
int(1/x^2/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(48*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d
 + b*c)))*a**4*c*d**3*x**2 + 48*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a +
b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*a**4*d**4*x**3 - 56*sqrt(d)*sqrt( -
a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*a**3*b*c**
2*d**2*x**2 - 56*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d
)*sqrt( - a*d + b*c)))*a**3*b*c*d**3*x**3 - 4*sqrt(a + b*x)*a**4*c**3*d**2
 + 12*sqrt(a + b*x)*a**4*c**2*d**3*x + 24*sqrt(a + b*x)*a**4*c*d**4*x**2 +
 8*sqrt(a + b*x)*a**3*b*c**4*d - 18*sqrt(a + b*x)*a**3*b*c**3*d**2*x - 34*
sqrt(a + b*x)*a**3*b*c**2*d**3*x**2 - 4*sqrt(a + b*x)*a**2*b**2*c**5 + 4*s
qrt(a + b*x)*a**2*b**2*c**3*d**2*x**2 + 6*sqrt(a + b*x)*a*b**3*c**5*x + 6*
sqrt(a + b*x)*a*b**3*c**4*d*x**2 + 24*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))
*a**4*c*d**4*x**2 + 24*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a**4*d**5*x**3
 - 40*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a**3*b*c**2*d**3*x**2 - 40*sqrt
(a)*log(sqrt(a + b*x) - sqrt(a))*a**3*b*c*d**4*x**3 + 11*sqrt(a)*log(sqrt(
a + b*x) - sqrt(a))*a**2*b**2*c**3*d**2*x**2 + 11*sqrt(a)*log(sqrt(a + b*x
) - sqrt(a))*a**2*b**2*c**2*d**3*x**3 + 2*sqrt(a)*log(sqrt(a + b*x) - sqrt
(a))*a*b**3*c**4*d*x**2 + 2*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b**3*c*
*3*d**2*x**3 + 3*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**4*c**5*x**2 + 3*s
qrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**4*c**4*d*x**3 - 24*sqrt(a)*log(sqrt
(a + b*x) + sqrt(a))*a**4*c*d**4*x**2 - 24*sqrt(a)*log(sqrt(a + b*x) + ...
```

3.254 $\int \frac{1}{x^3(c+dx)^2\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 362

$$\int \frac{1}{x^3(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = -\frac{\sqrt{ax^2+bx^3}}{3acx^4(c+dx)}+\frac{(5bc+8ad)\sqrt{ax^2+bx^3}}{12a^2c^2x^3(c+dx)}-\frac{(15b^2c^2+26abcd+48a^2d^2)\sqrt{ax^2+bx^3}}{24a^3c^3x^2(c+dx)}-\frac{d(5b^3c^3+7ab^2c^2d+12a^2bcd^2-32a^3d^3)\sqrt{ax^2+bx^3}}{8a^3c^4(bc-ad)x(c+dx)}+\frac{d^{7/2}(9bc-8ad)\arctan\left(\frac{\sqrt{d}\sqrt{ax^2+bx^3}}{\sqrt{bc-ad}\,x}\right)}{c^5(bc-ad)^{3/2}}+\frac{(5b^3c^3+12ab^2c^2d+24a^2bcd^2+64a^3d^3)\,\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}\,x}\right)}{8a^{7/2}c^5}$$

output
```
-1/3*(b*x^3+a*x^2)^(1/2)/a/c/x^4/(d*x+c)+1/12*(8*a*d+5*b*c)*(b*x^3+a*x^2)^
(1/2)/a^2/c^2/x^3/(d*x+c)-1/24*(48*a^2*d^2+26*a*b*c*d+15*b^2*c^2)*(b*x^3+a
*x^2)^(1/2)/a^3/c^3/x^2/(d*x+c)-1/8*d*(-32*a^3*d^3+12*a^2*b*c*d^2+7*a*b^2*
c^2*d+5*b^3*c^3)*(b*x^3+a*x^2)^(1/2)/a^3/c^4/(-a*d+b*c)/x/(d*x+c)+d^(7/2)*
(-8*a*d+9*b*c)*arctan(d^(1/2)*(b*x^3+a*x^2)^(1/2)/(-a*d+b*c)^(1/2)/x)/c^5/
(-a*d+b*c)^(3/2)+1/8*(64*a^3*d^3+24*a^2*b*c*d^2+12*a*b^2*c^2*d+5*b^3*c^3)*
arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(7/2)/c^5
```

Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3(c+dx)^2\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{\frac{c(a+bx)(15b^3c^3x^2(c+dx)+ab^2c^2x(-10c^2+11cdx+21d^2x^2)-8a^3d(c^3-2c^2dx+6cd^2x^2+12d^3x^3)+2a^2bc(4c^3-3c^2dx+11cd^2x^2+18d^3x^3))}{a^3(-bc+ad)(c+dx)}+\cdots}{24c^5x^2\sqrt{x^2(a+b\cdots}}$$

input
```
Integrate[1/(x^3*(c + d*x)^2*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
((c*(a + b*x)*(15*b^3*c^3*x^2*(c + d*x) + a*b^2*c^2*x*(-10*c^2 + 11*c*d*x
 + 21*d^2*x^2) - 8*a^3*d*(c^3 - 2*c^2*d*x + 6*c*d^2*x^2 + 12*d^3*x^3) + 2*a
^2*b*c*(4*c^3 - 3*c^2*d*x + 11*c*d^2*x^2 + 18*d^3*x^3)))/(a^3*(-(b*c) + a*
d)*(c + d*x)) + (24*d^(7/2)*(9*b*c - 8*a*d)*x^3*Sqrt[a + b*x]*ArcTan[(Sqrt
[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2) + (3*(5*b^3*c^3 + 1
2*a*b^2*c^2*d + 24*a^2*b*c*d^2 + 64*a^3*d^3)*x^3*Sqrt[a + b*x]*ArcTanh[Sqr
t[a + b*x]/Sqrt[a]])/a^(7/2))/(24*c^5*x^2*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1948, 114, 27, 168, 27, 168, 27, 168, 25, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}(c+dx)^2}\,dx$$
$$\Big\downarrow \text{1948}$$
$$\frac{x\sqrt{a+bx}\int \frac{1}{x^4\sqrt{a+bx}(c+dx)^2}dx}{\sqrt{ax^2+bx^3}}$$
$$\Big\downarrow \text{114}$$

$$\frac{x\sqrt{a+bx}\left(-\frac{\int \frac{5bc+8ad+7bdx}{2x^3\sqrt{a+bx}(c+dx)^2}dx}{3ac}-\frac{\sqrt{a+bx}}{3acx^3(c+dx)}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \ 27$$

$$\frac{x\sqrt{a+bx}\left(-\frac{\int \frac{5bc+8ad+7bdx}{x^3\sqrt{a+bx}(c+dx)^2}dx}{6ac}-\frac{\sqrt{a+bx}}{3acx^3(c+dx)}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \ 168$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{\int \frac{15b^2c^2+26abdc+48a^2d^2+5bd(5bc+8ad)x}{2x^2\sqrt{a+bx}(c+dx)^2}dx}{2ac}-\frac{\sqrt{a+bx}(8ad+5bc)}{2acx^2(c+dx)}}{6ac}-\frac{\sqrt{a+bx}}{3acx^3(c+dx)}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \ 27$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{\int \frac{15b^2c^2+26abdc+48a^2d^2+5bd(5bc+8ad)x}{x^2\sqrt{a+bx}(c+dx)^2}dx}{4ac}-\frac{\sqrt{a+bx}(8ad+5bc)}{2acx^2(c+dx)}}{6ac}-\frac{\sqrt{a+bx}}{3acx^3(c+dx)}\right)}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \ 168$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{-\frac{\int \frac{3\left(5b^3c^3+12ab^2dc^2+24a^2bd^2c+64a^3d^3+bd\left(15b^2c^2+26abdc+48a^2d^2\right)x\right)}{2x\sqrt{a+bx}(c+dx)^2}dx}{ac}-\frac{\sqrt{a+bx}\left(\frac{15b^2c}{a}+\frac{48ad^2}{c}+26bd\right)}{x(c+dx)}}{4ac}-\frac{\sqrt{a+bx}(8ad+5bc)}{2acx^2(c+dx)}}{6ac}-\right.}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \ 27$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{-\frac{3\int \frac{5b^3c^3+12ab^2dc^2+24a^2bd^2c+64a^3d^3+bd\left(15b^2c^2+26abdc+48a^2d^2\right)x}{x\sqrt{a+bx}(c+dx)^2}dx}{2ac}-\frac{\sqrt{a+bx}\left(\frac{15b^2c}{a}+\frac{48ad^2}{c}+26bd\right)}{x(c+dx)}}{4ac}-\frac{\sqrt{a+bx}(8ad+5bc)}{2acx^2(c+dx)}}{6ac}-\right.}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \ 168$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{-\frac{3\left(\frac{2d\sqrt{a+bx}\left(-32a^3d^3+12a^2bcd^2+7ab^2c^2d+5b^3c^3\right)}{c(c+dx)(bc-ad)}-\frac{\int -\frac{(bc-ad)\left(5b^3c^3+12ab^2dc^2+24a^2bd^2c+64a^3d^3\right)+bd\left(5b^3c^3+7ab^2dc^2+12a^2bc\cdots}{x\sqrt{a+bx}(c+dx)}}{c(bc-ad)}\cdots}{2ac}}{4ac}}{6ac}\cdots}{\sqrt{ax^2+bx^3}}$$

$\downarrow$ 25

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{-\frac{3\left(\frac{\int \frac{(bc-ad)\left(5b^3c^3+12ab^2dc^2+24a^2bd^2c+64a^3d^3\right)+bd\left(5b^3c^3+7ab^2dc^2+12a^2bd^2c-32a^3d^3\right)x}{x\sqrt{a+bx}(c+dx)}dx}{c(bc-ad)}+\frac{2d\sqrt{a+bx}\left(-32a^3d^3+12a^2bcd^2+7\cdots\right.}{c(c+dx)(bc-ad)}\cdots}{2ac}}{4ac}}{6ac}\cdots}{\sqrt{ax^2+bx^3}}$$

$\downarrow$ 174

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{-\frac{3\left(\frac{\frac{(bc-ad)\left(64a^3d^3+24a^2bcd^2+12ab^2c^2d+5b^3c^3\right)\int \frac{1}{x\sqrt{a+bx}}dx}{c}-\frac{8a^3d^4(9bc-8ad)\int \frac{1}{\sqrt{a+bx}(c+dx)}dx}{c}}{c(bc-ad)}+\frac{2d\sqrt{a+bx}\left(-32a^3d^3+12a^2bcd\cdots\right.}{c(c+dx)(bc-\cdots}\cdots}{2ac}}{4ac}}{6ac}\cdots}{\sqrt{ax^2+bx^3}}$$

$\downarrow$ 73

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{-\frac{3\left(\frac{\frac{2(bc-ad)\left(64a^3d^3+24a^2bcd^2+12ab^2c^2d+5b^3c^3\right)\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}-\frac{16a^3d^4(9bc-8ad)\int \frac{1}{c-\frac{ad}{b}+\frac{d(a+bx)}{b}}d\sqrt{a+bx}}{bc}}{c(bc-ad)}+\frac{2d\sqrt{a+bx}\left(-\cdots\right.}{\cdots}\cdots}{2ac}}{4ac}}{6ac}\cdots}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 218$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{-\frac{3\left(\frac{\frac{2(bc-ad)\left(64a^3d^3+24a^2bcd^2+12ab^2c^2d+5b^3c^3\right)\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}-\frac{16a^3d^{7/2}(9bc-8ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{c(bc-ad)}+\frac{2d\sqrt{a+bx}\left(-32a^3 \ldots\right.}{\ldots}\right.}{2ac}\ldots}{4ac}\ldots}{6ac}\ldots\right.}{\sqrt{ax^2+bx^3}\ldots}$$

$$\Big\downarrow \; 221$$

$$\frac{x\sqrt{a+bx}\left(-\frac{-\frac{-\frac{3\left(\frac{-\frac{16a^3d^{7/2}(9bc-8ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)(bc-ad)\left(64a^3d^3+24a^2bcd^2+12ab^2c^2d+5b^3c^3\right)}{\sqrt{a}c}}{c(bc-ad)}+\frac{2d\sqrt{a+bx}\left(-3 \ldots\right.}{\ldots}\right.}{2ac}\ldots}{4ac}\ldots}{6ac}\ldots\right.}{\sqrt{ax^2+bx^3}\ldots}$$

input

```
Int[1/(x^3*(c + d*x)^2*Sqrt[a*x^2 + b*x^3]),x]
```

output

```
(x*Sqrt[a + b*x]*(-1/3*Sqrt[a + b*x]/(a*c*x^3*(c + d*x)) - (-1/2*((5*b*c +
 8*a*d)*Sqrt[a + b*x])/(a*c*x^2*(c + d*x)) - (-((((15*b^2*c)/a + 26*b*d +
(48*a*d^2)/c)*Sqrt[a + b*x])/(x*(c + d*x))) - (3*((2*d*(5*b^3*c^3 + 7*a*b^
2*c^2*d + 12*a^2*b*c*d^2 - 32*a^3*d^3)*Sqrt[a + b*x])/(c*(b*c - a*d)*(c +
d*x)) + ((-16*a^3*d^(7/2)*(9*b*c - 8*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/S
qrt[b*c - a*d]])/(c*Sqrt[b*c - a*d]) - (2*(b*c - a*d)*(5*b^3*c^3 + 12*a*b^
2*c^2*d + 24*a^2*b*c*d^2 + 64*a^3*d^3)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sq
rt[a]*c))/(c*(b*c - a*d))))/(2*a*c))/(4*a*c))/(6*a*c)))/Sqrt[a*x^2 + b*x^3
]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.43

method	result
pseudoelliptic	$\frac{-\frac{c\sqrt{bx+a}\,(-8adx-3cbx+2ac)}{4a^{2}x^{2}}-\frac{\left(24a^{2}d^{2}+8abcd+3b^{2}c^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{4a^{\frac{5}{2}}}-\frac{d^{3}\left(-\frac{c\sqrt{bx+a}}{dx+c}-\frac{(6ad-7bc)\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{d(ad-bc)}}\right)}{\sqrt{d(ad-bc)}}\right)}{ad-bc}}{c^{4}}$
risch	$-\frac{(bx+a)\left(72a^{2}d^{2}x^{2}+36abcd\,x^{2}+15b^{2}c^{2}x^{2}-24a^{2}cdx-10ab\,c^{2}x+8a^{2}c^{2}\right)}{24a^{3}c^{4}x^{2}\sqrt{x^{2}(bx+a)}}-\frac{b\left(-\frac{\left(64a^{3}d^{3}+24a^{2}bc\,d^{2}+12a\,b^{2}c^{2}d+5b^{3}c^{3}\right)\text{arc}}{bc\sqrt{a}}\right.}{}$
default	Expression too large to display

input
```
int(1/x^3/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(-1/4*c*(b*x+a)^(1/2)*(-8*a*d*x-3*b*c*x+2*a*c)/a^2/x^2-1/4*(24*a^2*d
^2+8*a*b*c*d+3*b^2*c^2)/a^(5/2)*arctanh((b*x+a)^(1/2)/a^(1/2))-d^3/(a*d-b*
c)*(-c*(b*x+a)^(1/2)/(d*x+c)-(6*a*d-7*b*c)/(d*(a*d-b*c))^(1/2)*arctanh(d*(
b*x+a)^(1/2)/(d*(a*d-b*c))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 1979, normalized size of antiderivative = 5.47

$$\int \frac{1}{x^3(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(24*((9*a^4*b*c*d^4 - 8*a^5*d^5)*x^5 + (9*a^4*b*c^2*d^3 - 8*a^5*c*d^
4)*x^4)*sqrt(-d/(b*c - a*d))*log((b*d*x^2 - (b*c - 2*a*d)*x + 2*sqrt(b*x^3
 + a*x^2)*(b*c - a*d)*sqrt(-d/(b*c - a*d)))/(d*x^2 + c*x)) + 3*((5*b^4*c^4
*d + 7*a*b^3*c^3*d^2 + 12*a^2*b^2*c^2*d^3 + 40*a^3*b*c*d^4 - 64*a^4*d^5)*x
^5 + (5*b^4*c^5 + 7*a*b^3*c^4*d + 12*a^2*b^2*c^3*d^2 + 40*a^3*b*c^2*d^3 -
64*a^4*c*d^4)*x^4)*sqrt(a)*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt
(a))/x^2) - 2*(8*a^3*b*c^5 - 8*a^4*c^4*d + 3*(5*a*b^3*c^4*d + 7*a^2*b^2*c^
3*d^2 + 12*a^3*b*c^2*d^3 - 32*a^4*c*d^4)*x^3 + (15*a*b^3*c^5 + 11*a^2*b^2*
c^4*d + 22*a^3*b*c^3*d^2 - 48*a^4*c^2*d^3)*x^2 - 2*(5*a^2*b^2*c^5 + 3*a^3*
b*c^4*d - 8*a^4*c^3*d^2)*x)*sqrt(b*x^3 + a*x^2))/((a^4*b*c^6*d - a^5*c^5*d
^2)*x^5 + (a^4*b*c^7 - a^5*c^6*d)*x^4), 1/48*(48*((9*a^4*b*c*d^4 - 8*a^5*d
^5)*x^5 + (9*a^4*b*c^2*d^3 - 8*a^5*c*d^4)*x^4)*sqrt(d/(b*c - a*d))*arctan(
sqrt(b*x^3 + a*x^2)*sqrt(d/(b*c - a*d))/x) + 3*((5*b^4*c^4*d + 7*a*b^3*c^3
*d^2 + 12*a^2*b^2*c^2*d^3 + 40*a^3*b*c*d^4 - 64*a^4*d^5)*x^5 + (5*b^4*c^5
+ 7*a*b^3*c^4*d + 12*a^2*b^2*c^3*d^2 + 40*a^3*b*c^2*d^3 - 64*a^4*c*d^4)*x^
4)*sqrt(a)*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) - 2*(8
*a^3*b*c^5 - 8*a^4*c^4*d + 3*(5*a*b^3*c^4*d + 7*a^2*b^2*c^3*d^2 + 12*a^3*b
*c^2*d^3 - 32*a^4*c*d^4)*x^3 + (15*a*b^3*c^5 + 11*a^2*b^2*c^4*d + 22*a^3*b
*c^3*d^2 - 48*a^4*c^2*d^3)*x^2 - 2*(5*a^2*b^2*c^5 + 3*a^3*b*c^4*d - 8*a^4*
c^3*d^2)*x)*sqrt(b*x^3 + a*x^2))/((a^4*b*c^6*d - a^5*c^5*d^2)*x^5 + (a^...
```

Sympy [F]

$$\int \frac{1}{x^3(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^3\sqrt{x^2\,(a+bx)}\,(c+dx)^2}\,dx$$

input
```
integrate(1/x**3/(d*x+c)**2/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(x**2*(a + b*x))*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{x^3(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}(dx+c)^2x^3}\,dx$$

input
```
integrate(1/x^3/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x^2)*(d*x + c)^2*x^3), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^3(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^3/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^3\,\sqrt{b\,x^3+a\,x^2}\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(1/(x^3*(a*x^2 + b*x^3)^(1/2)*(c + d*x)^2),x)
```

output
```
int(1/(x^3*(a*x^2 + b*x^3)^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1238, normalized size of antiderivative = 3.42

$$\int \frac{1}{x^3(c+dx)^2\sqrt{ax^2+bx^3}}\,dx = \text{Too large to display}$$

input
```
int(1/x^3/(d*x+c)^2/(b*x^3+a*x^2)^(1/2),x)
```

output
```
( - 384*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( -
 a*d + b*c)))*a**5*c*d**4*x**3 - 384*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt
(a + b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*a**5*d**5*x**4 + 432*sqrt(d)*sq
rt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*a**4
*b*c**2*d**3*x**3 + 432*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/
(sqrt(d)*sqrt( - a*d + b*c)))*a**4*b*c*d**4*x**4 - 16*sqrt(a + b*x)*a**5*c
**4*d**2 + 32*sqrt(a + b*x)*a**5*c**3*d**3*x - 96*sqrt(a + b*x)*a**5*c**2*
d**4*x**2 - 192*sqrt(a + b*x)*a**5*c*d**5*x**3 + 32*sqrt(a + b*x)*a**4*b*c
**5*d - 44*sqrt(a + b*x)*a**4*b*c**4*d**2*x + 140*sqrt(a + b*x)*a**4*b*c**
3*d**3*x**2 + 264*sqrt(a + b*x)*a**4*b*c**2*d**4*x**3 - 16*sqrt(a + b*x)*a
**3*b**2*c**6 - 8*sqrt(a + b*x)*a**3*b**2*c**5*d*x - 22*sqrt(a + b*x)*a**3
*b**2*c**4*d**2*x**2 - 30*sqrt(a + b*x)*a**3*b**2*c**3*d**3*x**3 + 20*sqrt
(a + b*x)*a**2*b**3*c**6*x + 8*sqrt(a + b*x)*a**2*b**3*c**5*d*x**2 - 12*sq
rt(a + b*x)*a**2*b**3*c**4*d**2*x**3 - 30*sqrt(a + b*x)*a*b**4*c**6*x**2 -
 30*sqrt(a + b*x)*a*b**4*c**5*d*x**3 - 192*sqrt(a)*log(sqrt(a + b*x) - sqr
t(a))*a**5*c*d**5*x**3 - 192*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a**5*d**
6*x**4 + 312*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a**4*b*c**2*d**4*x**3 +
312*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a**4*b*c*d**5*x**4 - 84*sqrt(a)*l
og(sqrt(a + b*x) - sqrt(a))*a**3*b**2*c**3*d**3*x**3 - 84*sqrt(a)*log(sqrt
(a + b*x) - sqrt(a))*a**3*b**2*c**2*d**4*x**4 - 15*sqrt(a)*log(sqrt(a +...
```

3.255 $\int \frac{(ex)^{-3+\frac{3n}{2}}(c+dx)^3}{(ax^n+bx^{1+n})^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 246

$$\int \frac{(ex)^{-3+\frac{3n}{2}}(c+dx)^3}{(ax^n+bx^{1+n})^{3/2}}\,dx = \frac{2(bc-ad)^3x^{-n}(ex)^{3n/2}}{a^3be^3\sqrt{ax^n+bx^{1+n}}} + \frac{c^2(7bc-12ad)x^{-1-2n}(ex)^{3n/2}\sqrt{ax^n+bx^{1+n}}}{4a^3e^3} - \frac{c^3x^{-2(1+n)}(ex)^{3n/2}\sqrt{ax^n+bx^{1+n}}}{2a^2e^3} - \frac{3c(5b^2c^2-12abcd+8a^2d^2)\,x^{-n}(ex)^{3n/2}\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}e^3\sqrt{ax^n+bx^{1+n}}}$$

output

```
2*(-a*d+b*c)^3*(e*x)^(3/2*n)/a^3/b/e^3/(x^n)/(a*x^n+b*x^(1+n))^(1/2)+1/4*c
^2*(-12*a*d+7*b*c)*x^(-1-2*n)*(e*x)^(3/2*n)*(a*x^n+b*x^(1+n))^(1/2)/a^3/e^
3-1/2*c^3*(e*x)^(3/2*n)*(a*x^n+b*x^(1+n))^(1/2)/a^2/e^3/(x^(2+2*n))-3/4*c*
(8*a^2*d^2-12*a*b*c*d+5*b^2*c^2)*(e*x)^(3/2*n)*(b*x+a)^(1/2)*arctanh((b*x+
a)^(1/2)/a^(1/2))/a^(7/2)/e^3/(x^n)/(a*x^n+b*x^(1+n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.69

$$\int \frac{(ex)^{-3+\frac{3n}{2}}(c+dx)^3}{(ax^n+bx^{1+n})^{3/2}}\,dx =$$
$$-\frac{x^{-2-n}(ex)^{3n/2}\left(\sqrt{a}(-15b^3c^3x^2+8a^3d^3x^2+ab^2c^2x(-5c+36dx)+2a^2bc(c^2+6cdx-12d^2x^2))+3bc(5b\right.}{4a^{7/2}be^3\sqrt{x^n(a+bx)}}$$

input
```
Integrate[((e*x)^(-3 + (3*n)/2)*(c + d*x)^3)/(a*x^n + b*x^(1 + n))^(3/2),x
]
```

output
```
-1/4*(x^(-2 - n)*(e*x)^((3*n)/2)*(Sqrt[a]*(-15*b^3*c^3*x^2 + 8*a^3*d^3*x^2
 + a*b^2*c^2*x*(-5*c + 36*d*x) + 2*a^2*b*c*(c^2 + 6*c*d*x - 12*d^2*x^2)) +
 3*b*c*(5*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*x^2*Sqrt[a + b*x]*ArcTanh[Sqrt
[a + b*x]/Sqrt[a]]))/(a^(7/2)*b*e^3*Sqrt[x^n*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.82, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1948, 109, 27, 161, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3(ex)^{\frac{3n}{2}-3}}{(ax^n+bx^{n+1})^{3/2}}\,dx$$

$$\Big\downarrow \ 1948$$

$$\frac{x^{-n}\sqrt{a+bx}(ex)^{3n/2}\int \frac{(c+dx)^3}{x^3(a+bx)^{3/2}}dx}{e^3\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \ 109$$

$$\frac{x^{-n}\sqrt{a+bx}(ex)^{3n/2}\left(-\frac{\int \frac{(c+dx)(c(5bc-8ad)+d(bc-4ad)x)}{2x^2(a+bx)^{3/2}}dx}{2a}-\frac{c(c+dx)^2}{2ax^2\sqrt{a+bx}}\right)}{e^3\sqrt{ax^n+bx^{n+1}}}$$

$\downarrow$ 27

$$\frac{x^{-n}\sqrt{a+bx}(ex)^{3n/2}\left(-\frac{\int \frac{(c+dx)(c(5bc-8ad)+d(bc-4ad)x)}{x^2(a+bx)^{3/2}}dx}{4a}-\frac{c(c+dx)^2}{2ax^2\sqrt{a+bx}}\right)}{e^3\sqrt{ax^n+bx^{n+1}}}$$

$\downarrow$ 161

$$\frac{x^{-n}\sqrt{a+bx}(ex)^{3n/2}\left(-\frac{-\frac{3c\left(8a^2d^2-12abcd+5b^2c^2\right)\int \frac{1}{x\sqrt{a+bx}}dx}{2a^2}-\frac{x\left(-8a^3d^3+26a^2bcd^2-36ab^2c^2d+15b^3c^3\right)+abc^2(5bc-8ad)}{a^2bx\sqrt{a+bx}}}{4a}-\frac{c(c+dx)^2}{2ax^2\sqrt{a+bx}}\right)}{e^3\sqrt{ax^n+bx^{n+1}}}$$

$\downarrow$ 73

$$\frac{x^{-n}\sqrt{a+bx}(ex)^{3n/2}\left(-\frac{-\frac{3c\left(8a^2d^2-12abcd+5b^2c^2\right)\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{a^2b}-\frac{x\left(-8a^3d^3+26a^2bcd^2-36ab^2c^2d+15b^3c^3\right)+abc^2(5bc-8ad)}{a^2bx\sqrt{a+bx}}}{4a}-\frac{c(c+dx)^2}{2ax^2\sqrt{a+bx}}\right)}{e^3\sqrt{ax^n+bx^{n+1}}}$$

$\downarrow$ 221

$$\frac{x^{-n}\sqrt{a+bx}(ex)^{3n/2}\left(-\frac{\frac{3c\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\left(8a^2d^2-12abcd+5b^2c^2\right)}{a^{5/2}}-\frac{x\left(-8a^3d^3+26a^2bcd^2-36ab^2c^2d+15b^3c^3\right)+abc^2(5bc-8ad)}{a^2bx\sqrt{a+bx}}}{4a}-\frac{c(c+dx)^2}{2ax^2\sqrt{a+bx}}\right)}{e^3\sqrt{ax^n+bx^{n+1}}}$$

input
```
Int[((e*x)^(-3 + (3*n)/2)*(c + d*x)^3)/(a*x^n + b*x^(1 + n))^(3/2),x]
```

output
```
((e*x)^((3*n)/2)*Sqrt[a + b*x]*(-1/2*(c*(c + d*x)^2)/(a*x^2*Sqrt[a + b*x])
 - (-((a*b*c^2*(5*b*c - 8*a*d) + (15*b^3*c^3 - 36*a*b^2*c^2*d + 26*a^2*b*c
*d^2 - 8*a^3*d^3)*x)/(a^2*b*x*Sqrt[a + b*x])) + (3*c*(5*b^2*c^2 - 12*a*b*c
*d + 8*a^2*d^2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(5/2))/(4*a)))/(e^3*x^n*
Sqrt[a*x^n + b*x^(1 + n)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 161
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))
*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^2*c*d*e*g*(n + 1) + a^2*c*d*f*h*(n +
 1) + a*b*(d^2*e*g*(m + 1) + c^2*f*h*(m + 1) - c*d*(f*g + e*h)*(m + n + 2))
 + (a^2*d^2*f*h*(n + 1) - a*b*d^2*(f*g + e*h)*(n + 1) + b^2*(c^2*f*h*(m + 1
) - c*d*(f*g + e*h)*(m + 1) + d^2*e*g*(m + n + 2)))*x)/(b*d*(b*c - a*d)^2*(
m + 1)*(n + 1)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] - Simp[(a^2*d^2*f*
h*(2 + 3*n + n^2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
 3)) + b^2*(c^2*f*h*(2 + 3*m + m^2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
 d^2*e*g*(6 + m^2 + 5*n + n^2 + m*(2*n + 5))))/(b*d*(b*c - a*d)^2*(m + 1)*(
n + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e, f, g, h}, x] && LtQ[m, -1] && LtQ[n, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [F]

$$\int \frac{(ex)^{-3+\frac{3n}{2}} (dx+c)^3}{(a\,x^n+b\,x^{1+n})^{\frac{3}{2}}}dx$$

input
```
int((e*x)^(-3+3/2*n)*(d*x+c)^3/(a*x^n+b*x^(1+n))^(3/2),x)
```

output
```
int((e*x)^(-3+3/2*n)*(d*x+c)^3/(a*x^n+b*x^(1+n))^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 563, normalized size of antiderivative = 2.29

$$\int \frac{(ex)^{-3+\frac{3n}{2}}(c+dx)^3}{(ax^n+bx^{1+n})^{3/2}}\,dx = \left[\frac{3\left((5\,b^4c^3-12\,ab^3c^2d+8\,a^2b^2cd^2)x^3+(5\,ab^3c^3-12\,a^2b^2c^2d+8\,a^3bcd^2)x^2\right)\sqrt{}}{}\right.$$

input
```
integrate((e*x)^(-3+3/2*n)*(d*x+c)^3/(a*x^n+b*x^(1+n))^(3/2),x, algorithm=
"fricas")
```

output
```
[1/8*(3*((5*b^4*c^3 - 12*a*b^3*c^2*d + 8*a^2*b^2*c*d^2)*x^3 + (5*a*b^3*c^3
 - 12*a^2*b^2*c^2*d + 8*a^3*b*c*d^2)*x^2)*sqrt(a)*e^(3/2*n - 3)*x^(1/2*n +
 1/2)*log(((b*x + 2*a)*x^(1/2*n + 1/2) - 2*sqrt(a)*sqrt(x)*sqrt((b*x + a)*
x^(n + 1)/x))/(x*x^(1/2*n + 1/2))) - 2*(2*a^3*b*c^3 - (15*a*b^3*c^3 - 36*a
^2*b^2*c^2*d + 24*a^3*b*c*d^2 - 8*a^4*d^3)*x^2 - (5*a^2*b^2*c^3 - 12*a^3*b
*c^2*d)*x)*e^(3/2*n - 3)*sqrt(x)*sqrt((b*x + a)*x^(n + 1)/x))/((a^4*b^2*x^
3 + a^5*b*x^2)*x^(1/2*n + 1/2)), 1/4*(3*((5*b^4*c^3 - 12*a*b^3*c^2*d + 8*a
^2*b^2*c*d^2)*x^3 + (5*a*b^3*c^3 - 12*a^2*b^2*c^2*d + 8*a^3*b*c*d^2)*x^2)*
sqrt(-a)*e^(3/2*n - 3)*x^(1/2*n + 1/2)*arctan(sqrt(-a)*sqrt(x)*sqrt((b*x +
 a)*x^(n + 1)/x)/((b*x + a)*x^(1/2*n + 1/2))) - (2*a^3*b*c^3 - (15*a*b^3*c
^3 - 36*a^2*b^2*c^2*d + 24*a^3*b*c*d^2 - 8*a^4*d^3)*x^2 - (5*a^2*b^2*c^3 -
 12*a^3*b*c^2*d)*x)*e^(3/2*n - 3)*sqrt(x)*sqrt((b*x + a)*x^(n + 1)/x))/((a
^4*b^2*x^3 + a^5*b*x^2)*x^(1/2*n + 1/2))]
```

Sympy [F]

$$\int \frac{(ex)^{-3+\frac{3n}{2}}(c+dx)^3}{(ax^n+bx^{1+n})^{3/2}}\,dx = \int \frac{(ex)^{\frac{3n}{2}-3}(c+dx)^3}{(ax^n+bx^{n+1})^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)**(-3+3/2*n)*(d*x+c)**3/(a*x**n+b*x**(1+n))**(3/2),x)
```

output
```
Integral((e*x)**(3*n/2 - 3)*(c + d*x)**3/(a*x**n + b*x**(n + 1))**(3/2), x
)
```

Maxima [F]

$$\int \frac{(ex)^{-3+\frac{3n}{2}}(c+dx)^3}{(ax^n+bx^{1+n})^{3/2}}\,dx = \int \frac{(dx+c)^3(ex)^{\frac{3}{2}n-3}}{(bx^{n+1}+ax^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(-3+3/2*n)*(d*x+c)^3/(a*x^n+b*x^(1+n))^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((d*x + c)^3*(e*x)^(3/2*n - 3)/(b*x^(n + 1) + a*x^n)^(3/2), x)
```

Giac [F]

$$\int \frac{(ex)^{-3+\frac{3n}{2}}(c+dx)^3}{(ax^n+bx^{1+n})^{3/2}}\,dx=\int \frac{(dx+c)^3(ex)^{\frac{3}{2}n-3}}{(bx^{n+1}+ax^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(-3+3/2*n)*(d*x+c)^3/(a*x^n+b*x^(1+n))^(3/2),x, algorithm=
"giac")
```

output
```
integrate((d*x + c)^3*(e*x)^(3/2*n - 3)/(b*x^(n + 1) + a*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-3+\frac{3n}{2}}(c+dx)^3}{(ax^n+bx^{1+n})^{3/2}}\,dx=\int \frac{(e\,x)^{\frac{3\,n}{2}-3}\,(c+d\,x)^3}{(a\,x^n+b\,x^{n+1})^{3/2}}\,dx$$

input
```
int(((e*x)^((3*n)/2 - 3)*(c + d*x)^3)/(a*x^n + b*x^(n + 1))^(3/2),x)
```

output
```
int(((e*x)^((3*n)/2 - 3)*(c + d*x)^3)/(a*x^n + b*x^(n + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.22

$$\int \frac{(ex)^{-3+\frac{3n}{2}}(c+dx)^3}{(ax^n+bx^{1+n})^{3/2}}\,dx=\frac{e^{\frac{3n}{2}}\left(24\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)a^2bc\,d^2x^2-36\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+\right.\right.}{}$$

input
```
int((e*x)^(-3+3/2*n)*(d*x+c)^3/(a*x^n+b*x^(1+n))^(3/2),x)
```

output
```
(e**((3*n)/2)*(24*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a**2*
b*c*d**2*x**2 - 36*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b*
*2*c**2*d*x**2 + 15*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**
3*c**3*x**2 - 24*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a**2*b
*c*d**2*x**2 + 36*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a*b**
2*c**2*d*x**2 - 15*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**3
*c**3*x**2 - 16*a**4*d**3*x**2 - 4*a**3*b*c**3 - 24*a**3*b*c**2*d*x + 48*a
**3*b*c*d**2*x**2 + 10*a**2*b**2*c**3*x - 72*a**2*b**2*c**2*d*x**2 + 30*a*
b**3*c**3*x**2))/(8*sqrt(a + b*x)*a**4*b*e**3*x**2)
```

3.256 $\int \frac{(ex)^{-2+\frac{3n}{2}}(c+dx)^2}{\left(ax^n+bx^{1+n}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 172

$$\int \frac{(ex)^{-2+\frac{3n}{2}}(c+dx)^2}{\left(ax^n+bx^{1+n}\right)^{3/2}}\,dx=-\frac{2(bc-ad)^2x^{-n}(ex)^{3n/2}}{a^2be^2\sqrt{ax^n+bx^{1+n}}}-\frac{c^2x^{-1-2n}(ex)^{3n/2}\sqrt{ax^n+bx^{1+n}}}{a^2e^2}+\frac{c(3bc-4ad)x^{-n}(ex)^{3n/2}\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}e^2\sqrt{ax^n+bx^{1+n}}}$$

output
```
-2*(-a*d+b*c)^2*(e*x)^(3/2*n)/a^2/b/e^2/(x^n)/(a*x^n+b*x^(1+n))^(1/2)-c^2*
x^(-1-2*n)*(e*x)^(3/2*n)*(a*x^n+b*x^(1+n))^(1/2)/a^2/e^2+c*(-4*a*d+3*b*c)*
(e*x)^(3/2*n)*(b*x+a)^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2)/e^2/(x^
n)/(a*x^n+b*x^(1+n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.67

$$\int \frac{(ex)^{-2+\frac{3n}{2}}(c+dx)^2}{(ax^n+bx^{1+n})^{3/2}}\,dx = \frac{x^{-1-n}(ex)^{3n/2}\left(-\sqrt{a}(3b^2c^2x+2a^2d^2x+abc(c-4dx))+bc(3bc-4ad)x\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{a^{5/2}be^2\sqrt{x^n(a+bx)}}$$

input

```
Integrate[((e*x)^(-2 + (3*n)/2)*(c + d*x)^2)/(a*x^n + b*x^(1 + n))^(3/2),x
]
```

output

```
(x^(-1 - n)*(e*x)^((3*n)/2)*(-(Sqrt[a]*(3*b^2*c^2*x + 2*a^2*d^2*x + a*b*c*
(c - 4*d*x))) + b*c*(3*b*c - 4*a*d)*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/
Sqrt[a]]))/(a^(5/2)*b*e^2*Sqrt[x^n*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.82, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1948, 100, 27, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2(ex)^{\frac{3n}{2}-2}}{(ax^n+bx^{n+1})^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1948}$$

$$\frac{x^{-n}\sqrt{a+bx}(ex)^{3n/2}\int \frac{(c+dx)^2}{x^2(a+bx)^{3/2}}dx}{e^2\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{ 100}$$

$$\frac{x^{-n}\sqrt{a+bx}(ex)^{3n/2}\left(\frac{\int -\frac{c(3bc-4ad)-2ad^2x}{2x(a+bx)^{3/2}}dx}{a}-\frac{c^2}{ax\sqrt{a+bx}}\right)}{e^2\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^{-n}\sqrt{a+bx}(ex)^{3n/2}\left(-\frac{\int \frac{c(3bc-4ad)-2ad^2x}{x(a+bx)^{3/2}}dx}{2a}-\frac{c^2}{ax\sqrt{a+bx}}\right)}{e^2\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{x^{-n}\sqrt{a+bx}(ex)^{3n/2}\left(-\frac{\frac{c(3bc-4ad)\int \frac{1}{x\sqrt{a+bx}}dx}{a}+\frac{2\left(\frac{c(3bc-4ad)}{a}+\frac{2ad^2}{b}\right)}{\sqrt{a+bx}}}{2a}-\frac{c^2}{ax\sqrt{a+bx}}\right)}{e^2\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{x^{-n}\sqrt{a+bx}(ex)^{3n/2}\left(-\frac{\frac{2c(3bc-4ad)\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{ab}+\frac{2\left(\frac{c(3bc-4ad)}{a}+\frac{2ad^2}{b}\right)}{\sqrt{a+bx}}}{2a}-\frac{c^2}{ax\sqrt{a+bx}}\right)}{e^2\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{x^{-n}\sqrt{a+bx}(ex)^{3n/2}\left(-\frac{\frac{2\left(\frac{c(3bc-4ad)}{a}+\frac{2ad^2}{b}\right)}{\sqrt{a+bx}}-\frac{2c\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)(3bc-4ad)}{a^{3/2}}}{2a}-\frac{c^2}{ax\sqrt{a+bx}}\right)}{e^2\sqrt{ax^n+bx^{n+1}}}$$

input
```
Int[((e*x)^(-2 + (3*n)/2)*(c + d*x)^2)/(a*x^n + b*x^(1 + n))^(3/2),x]
```

output
```
((e*x)^((3*n)/2)*Sqrt[a + b*x]*(-(c^2/(a*x*Sqrt[a + b*x])) - ((2*((2*a*d^2
)/b + (c*(3*b*c - 4*a*d))/a))/Sqrt[a + b*x] - (2*c*(3*b*c - 4*a*d)*ArcTanh
[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2))/(2*a)))/(e^2*x^n*Sqrt[a*x^n + b*x^(1 + n
)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [F]

$$\int \frac{(ex)^{-2+\frac{3n}{2}}(dx+c)^2}{(a\,x^n+b\,x^{1+n})^{\frac{3}{2}}}dx$$

input
```
int((e*x)^(-2+3/2*n)*(d*x+c)^2/(a*x^n+b*x^(1+n))^(3/2),x)
```

output
```
int((e*x)^(-2+3/2*n)*(d*x+c)^2/(a*x^n+b*x^(1+n))^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.38

$$\int \frac{(ex)^{-2+\frac{3n}{2}}(c+dx)^2}{(ax^n+bx^{1+n})^{3/2}}\,dx = \left[-\frac{((3\,b^3c^2-4\,ab^2cd)x^2+(3\,ab^2c^2-4\,a^2bcd)x)\sqrt{a}e^{\frac{3}{2}\,n-2}x^{\frac{1}{2}\,n+\frac{1}{2}}\log\left(\frac{(bx+2\,a)x^{\frac{1}{2}}}{\cdots}\right.}{2\,(a^3\cdots}\right.$$

$$-\frac{((3\,b^3c^2-4\,ab^2cd)x^2+(3\,ab^2c^2-4\,a^2bcd)x)\sqrt{-a}e^{\frac{3}{2}\,n-2}x^{\frac{1}{2}\,n+\frac{1}{2}}\arctan\left(\frac{\sqrt{-a}\sqrt{x}\sqrt{\frac{(bx+a)x^{n+1}}{x}}}{(bx+a)x^{\frac{1}{2}\,n+\frac{1}{2}}}\right)+(a^2bc^2+(3\cdots}{(a^3b^2x^2+a^4bx)x^{\frac{1}{2}\,n+\frac{1}{2}}}$$

input
```
integrate((e*x)^(-2+3/2*n)*(d*x+c)^2/(a*x^n+b*x^(1+n))^(3/2),x, algorithm=
"fricas")
```

output
```
[-1/2*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^2 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x)*sqr
t(a)*e^(3/2*n - 2)*x^(1/2*n + 1/2)*log(((b*x + 2*a)*x^(1/2*n + 1/2) - 2*sq
rt(a)*sqrt(x)*sqrt((b*x + a)*x^(n + 1)/x))/(x*x^(1/2*n + 1/2))) + 2*(a^2*b
*c^2 + (3*a*b^2*c^2 - 4*a^2*b*c*d + 2*a^3*d^2)*x)*e^(3/2*n - 2)*sqrt(x)*sq
rt((b*x + a)*x^(n + 1)/x))/((a^3*b^2*x^2 + a^4*b*x)*x^(1/2*n + 1/2)), -(((
3*b^3*c^2 - 4*a*b^2*c*d)*x^2 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x)*sqrt(-a)*e^(
3/2*n - 2)*x^(1/2*n + 1/2)*arctan(sqrt(-a)*sqrt(x)*sqrt((b*x + a)*x^(n + 1
)/x)/((b*x + a)*x^(1/2*n + 1/2))) + (a^2*b*c^2 + (3*a*b^2*c^2 - 4*a^2*b*c*
d + 2*a^3*d^2)*x)*e^(3/2*n - 2)*sqrt(x)*sqrt((b*x + a)*x^(n + 1)/x))/((a^3
*b^2*x^2 + a^4*b*x)*x^(1/2*n + 1/2))]
```

Sympy [F]

$$\int \frac{(ex)^{-2+\frac{3n}{2}}(c+dx)^2}{(ax^n+bx^{1+n})^{3/2}}\,dx = \int \frac{(ex)^{\frac{3n}{2}-2}(c+dx)^2}{(ax^n+bx^{n+1})^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)**(-2+3/2*n)*(d*x+c)**2/(a*x**n+b*x**(1+n))**(3/2),x)
```

output
```
Integral((e*x)**(3*n/2 - 2)*(c + d*x)**2/(a*x**n + b*x**(n + 1))**(3/2), x
)
```

Maxima [F]

$$\int \frac{(ex)^{-2+\frac{3n}{2}}(c+dx)^2}{(ax^n+bx^{1+n})^{3/2}}\,dx = \int \frac{(dx+c)^2(ex)^{\frac{3}{2}n-2}}{(bx^{n+1}+ax^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(-2+3/2*n)*(d*x+c)^2/(a*x^n+b*x^(1+n))^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((d*x + c)^2*(e*x)^(3/2*n - 2)/(b*x^(n + 1) + a*x^n)^(3/2), x)
```

Giac [F]

$$\int \frac{(ex)^{-2+\frac{3n}{2}}(c+dx)^2}{(ax^n+bx^{1+n})^{3/2}}\,dx = \int \frac{(dx+c)^2(ex)^{\frac{3}{2}n-2}}{(bx^{n+1}+ax^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(-2+3/2*n)*(d*x+c)^2/(a*x^n+b*x^(1+n))^(3/2),x, algorithm=
"giac")
```

output
```
integrate((d*x + c)^2*(e*x)^(3/2*n - 2)/(b*x^(n + 1) + a*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-2+\frac{3n}{2}}(c+dx)^2}{(ax^n+bx^{1+n})^{3/2}}\,dx=\int \frac{(e\,x)^{\frac{3\,n}{2}-2}\,(c+d\,x)^2}{(a\,x^n+b\,x^{n+1})^{3/2}}\,dx$$

input
```
int(((e*x)^((3*n)/2 - 2)*(c + d*x)^2)/(a*x^n + b*x^(n + 1))^(3/2),x)
```

output
```
int(((e*x)^((3*n)/2 - 2)*(c + d*x)^2)/(a*x^n + b*x^(n + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.01

$$\int \frac{(ex)^{-2+\frac{3n}{2}}(c+dx)^2}{(ax^n+bx^{1+n})^{3/2}}\,dx=\frac{e^{\frac{3n}{2}}\left(4\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)abcdx-3\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\right.\right.}{}$$

input
```
int((e*x)^(-2+3/2*n)*(d*x+c)^2/(a*x^n+b*x^(1+n))^(3/2),x)
```

output
```
(e**((3*n)/2)*(4*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b*c*
d*x - 3*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**2*c**2*x - 4
*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a*b*c*d*x + 3*sqrt(a)*
sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**2*c**2*x - 4*a**3*d**2*x - 2
*a**2*b*c**2 + 8*a**2*b*c*d*x - 6*a*b**2*c**2*x))/(2*sqrt(a + b*x)*a**3*b*
e**2*x)
```

3.257 $\int \frac{(ex)^{-1+\frac{3n}{2}}(c+dx)}{(ax^n+bx^{1+n})^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 118

$$\int \frac{(ex)^{-1+\frac{3n}{2}}(c+dx)}{(ax^n+bx^{1+n})^{3/2}}\,dx=\frac{2(bc-ad)x^{-n}(ex)^{3n/2}}{abe\sqrt{ax^n+bx^{1+n}}}-\frac{2cx^{-n}(ex)^{3n/2}\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}e\sqrt{ax^n+bx^{1+n}}}$$

output
```
2*(-a*d+b*c)*(e*x)^(3/2*n)/a/b/e/(x^n)/(a*x^n+b*x^(1+n))^(1/2)-2*c*(e*x)^(
3/2*n)*(b*x+a)^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)/e/(x^n)/(a*x^n
+b*x^(1+n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.70

$$\int \frac{(ex)^{-1+\frac{3n}{2}}(c+dx)}{(ax^n+bx^{1+n})^{3/2}}\,dx = -\frac{2(ex)^{3n/2}(a+bx)\left(\sqrt{a}(-bc+ad)+bc\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{a^{3/2}be\,(x^n(a+bx))^{3/2}}$$

input
```
Integrate[((e*x)^(-1 + (3*n)/2)*(c + d*x))/(a*x^n + b*x^(1 + n))^(3/2),x]
```

output
```
(-2*(e*x)^((3*n)/2)*(a + b*x)*(Sqrt[a]*(-(b*c) + a*d) + b*c*Sqrt[a + b*x]*
ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(a^(3/2)*b*e*(x^n*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1948, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(ex)^{\frac{3n}{2}-1}}{(ax^n+bx^{n+1})^{3/2}}\,dx$$

$$\Big\downarrow \text{1948}$$

$$\frac{x^{-n}\sqrt{a+bx}(ex)^{3n/2}\int \frac{c+dx}{x(a+bx)^{3/2}}dx}{e\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{87}$$

$$\frac{x^{-n}\sqrt{a+bx}(ex)^{3n/2}\left(\frac{c\int \frac{1}{x\sqrt{a+bx}}dx}{a}+\frac{2(bc-ad)}{ab\sqrt{a+bx}}\right)}{e\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{73}$$

$$\frac{x^{-n}\sqrt{a+bx}(ex)^{3n/2}\left(\frac{2c\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{ab}+\frac{2(bc-ad)}{ab\sqrt{a+bx}}\right)}{e\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{221}$$

$$\frac{x^{-n}\sqrt{a+bx}(ex)^{3n/2}\left(\frac{2(bc-ad)}{ab\sqrt{a+bx}}-\frac{2c\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{e\sqrt{ax^n+bx^{n+1}}}$$

input
```
Int[((e*x)^(-1 + (3*n)/2)*(c + d*x))/(a*x^n + b*x^(1 + n))^(3/2),x]
```

output
```
((e*x)^((3*n)/2)*Sqrt[a + b*x]*((2*(b*c - a*d))/(a*b*Sqrt[a + b*x]) - (2*c
*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)))/(e*x^n*Sqrt[a*x^n + b*x^(1 + n)
])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [F]

$$\int \frac{(ex)^{-1+\frac{3n}{2}}(dx+c)}{(a\,x^n+b\,x^{1+n})^{\frac{3}{2}}}dx$$

input
```
int((e*x)^(-1+3/2*n)*(d*x+c)/(a*x^n+b*x^(1+n))^(3/2),x)
```

output
```
int((e*x)^(-1+3/2*n)*(d*x+c)/(a*x^n+b*x^(1+n))^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.45

$$\int \frac{(ex)^{-1+\frac{3n}{2}}(c+dx)}{(ax^n+bx^{1+n})^{3/2}}\,dx = \left[\frac{(b^2cx+abc)\sqrt{a}e^{\frac{3}{2}n-1}x^{\frac{1}{2}n+\frac{1}{2}}\log\left(\frac{(bx+2a)x^{\frac{1}{2}n+\frac{1}{2}}-2\sqrt{a}\sqrt{x}\sqrt{\frac{(bx+a)x^{n+1}}{x}}}{xx^{\frac{1}{2}n+\frac{1}{2}}}\right)+2(abc-}{(a^2b^2x+a^3b)x^{\frac{1}{2}n+\frac{1}{2}}}\right.$$

input
```
integrate((e*x)^(-1+3/2*n)*(d*x+c)/(a*x^n+b*x^(1+n))^(3/2),x, algorithm="f
ricas")
```

output
```
[((b^2*c*x + a*b*c)*sqrt(a)*e^(3/2*n - 1)*x^(1/2*n + 1/2)*log(((b*x + 2*a)
*x^(1/2*n + 1/2) - 2*sqrt(a)*sqrt(x)*sqrt((b*x + a)*x^(n + 1)/x))/(x*x^(1/
2*n + 1/2))) + 2*(a*b*c - a^2*d)*e^(3/2*n - 1)*sqrt(x)*sqrt((b*x + a)*x^(n
 + 1)/x))/((a^2*b^2*x + a^3*b)*x^(1/2*n + 1/2)), 2*((b^2*c*x + a*b*c)*sqrt
(-a)*e^(3/2*n - 1)*x^(1/2*n + 1/2)*arctan(sqrt(-a)*sqrt(x)*sqrt((b*x + a)*
x^(n + 1)/x)/((b*x + a)*x^(1/2*n + 1/2))) + (a*b*c - a^2*d)*e^(3/2*n - 1)*
sqrt(x)*sqrt((b*x + a)*x^(n + 1)/x))/((a^2*b^2*x + a^3*b)*x^(1/2*n + 1/2))
]
```

Sympy [F]

$$\int \frac{(ex)^{-1+\frac{3n}{2}}(c+dx)}{(ax^n+bx^{1+n})^{3/2}}\,dx = \int \frac{(ex)^{\frac{3n}{2}-1}(c+dx)}{(ax^n+bx^{n+1})^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)**(-1+3/2*n)*(d*x+c)/(a*x**n+b*x**(1+n))**(3/2),x)
```

output
```
Integral((e*x)**(3*n/2 - 1)*(c + d*x)/(a*x**n + b*x**(n + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+\frac{3n}{2}}(c+dx)}{(ax^n+bx^{1+n})^{3/2}}\,dx = \int \frac{(dx+c)(ex)^{\frac{3}{2}n-1}}{(bx^{n+1}+ax^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(-1+3/2*n)*(d*x+c)/(a*x^n+b*x^(1+n))^(3/2),x, algorithm="m
axima")
```

output
```
integrate((d*x + c)*(e*x)^(3/2*n - 1)/(b*x^(n + 1) + a*x^n)^(3/2), x)
```

Giac [F]

$$\int \frac{(ex)^{-1+\frac{3n}{2}}(c+dx)}{(ax^n+bx^{1+n})^{3/2}}\,dx = \int \frac{(dx+c)(ex)^{\frac{3}{2}n-1}}{(bx^{n+1}+ax^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(-1+3/2*n)*(d*x+c)/(a*x^n+b*x^(1+n))^(3/2),x, algorithm="g
iac")
```

output
```
integrate((d*x + c)*(e*x)^(3/2*n - 1)/(b*x^(n + 1) + a*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+\frac{3n}{2}}(c+dx)}{(ax^n+bx^{1+n})^{3/2}}\,dx=\int \frac{(e\,x)^{\frac{3\,n}{2}-1}\,(c+d\,x)}{(a\,x^n+b\,x^{n+1})^{3/2}}\,dx$$

input
```
int(((e*x)^((3*n)/2 - 1)*(c + d*x))/(a*x^n + b*x^(n + 1))^(3/2),x)
```

output
```
int(((e*x)^((3*n)/2 - 1)*(c + d*x))/(a*x^n + b*x^(n + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.68

$$\int \frac{(ex)^{-1+\frac{3n}{2}}(c+dx)}{(ax^n+bx^{1+n})^{3/2}}\,dx=\frac{e^{\frac{3n}{2}}\left(\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}-\sqrt{a}\right)bc-\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}+\sqrt{a}\right)bc-\right.}{\sqrt{bx+a}\,a^2be}$$

input
```
int((e*x)^(-1+3/2*n)*(d*x+c)/(a*x^n+b*x^(1+n))^(3/2),x)
```

output
```
(e**((3*n)/2)*(sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b*c - sq
rt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b*c - 2*a**2*d + 2*a*b*c)
)/(sqrt(a + b*x)*a**2*b*e)
```

3.258 $\int \frac{(ex)^{3n/2}}{\left(ax^n+bx^{1+n}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 36

$$\int \frac{(ex)^{3n/2}}{\left(ax^n+bx^{1+n}\right)^{3/2}}\,dx=-\frac{2x^{-n}(ex)^{3n/2}}{b\sqrt{ax^n+bx^{1+n}}}$$

output
```
-2*(e*x)^(3/2*n)/b/(x^n)/(a*x^n+b*x^(1+n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{(ex)^{3n/2}}{\left(ax^n+bx^{1+n}\right)^{3/2}}\,dx=-\frac{2(ex)^{3n/2}(a+bx)}{b\left(x^n(a+bx)\right)^{3/2}}$$

input
```
Integrate[(e*x)^((3*n)/2)/(a*x^n + b*x^(1 + n))^(3/2),x]
```

output
```
(-2*(e*x)^((3*n)/2)*(a + b*x))/(b*(x^n*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1938, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3n/2}}{(ax^n+bx^{n+1})^{3/2}}\,dx$$

$$\Big\downarrow \text{1938}$$

$$\frac{x^{-n}\sqrt{a+bx}(ex)^{3n/2}\int \frac{1}{(a+bx)^{3/2}}dx}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{17}$$

$$-\frac{2x^{-n}(ex)^{3n/2}}{b\sqrt{ax^n+bx^{n+1}}}$$

input
```
Int[(e*x)^((3*n)/2)/(a*x^n + b*x^(1 + n))^(3/2),x]
```

output
```
(-2*(e*x)^((3*n)/2))/(b*x^n*Sqrt[a*x^n + b*x^(1 + n)])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

method	result	size
orering	$-\frac{2(bx+a)(ex)^{\frac{3n}{2}}}{b(a\,x^{n}+b\,x^{1+n})^{\frac{3}{2}}}$	33

input
```
int((e*x)^(3/2*n)/(a*x^n+b*x^(1+n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(b*x+a)/b*(e*x)^(3/2*n)/(a*x^n+b*x^(1+n))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28

$$\int \frac{(ex)^{3n/2}}{(ax^n+bx^{1+n})^{3/2}}\,dx = -\frac{2\,e^{\frac{3}{2}\,n}\sqrt{x}\sqrt{\frac{(bx+a)x^{n+1}}{x}}}{(b^2x+ab)x^{\frac{1}{2}\,n+\frac{1}{2}}}$$

input
```
integrate((e*x)^(3/2*n)/(a*x^n+b*x^(1+n))^(3/2),x, algorithm="fricas")
```

output
```
-2*e^(3/2*n)*sqrt(x)*sqrt((b*x + a)*x^(n + 1)/x)/((b^2*x + a*b)*x^(1/2*n +
 1/2))
```

Sympy [F]

$$\int \frac{(ex)^{3n/2}}{(ax^n+bx^{1+n})^{3/2}}\,dx = \int \frac{(ex)^{\frac{3n}{2}}}{(ax^n+bx^{n+1})^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)**(3/2*n)/(a*x**n+b*x**(1+n))**(3/2),x)
```

output
```
Integral((e*x)**(3*n/2)/(a*x**n + b*x**(n + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \frac{(ex)^{3n/2}}{(ax^n + bx^{1+n})^{3/2}}\,dx = -\frac{2\,e^{\frac{3}{2}\,n}x^{\frac{3}{2}\,n}}{\sqrt{bx + a}b(x^n)^{\frac{3}{2}}}$$

input
```
integrate((e*x)^(3/2*n)/(a*x^n+b*x^(1+n))^(3/2),x, algorithm="maxima")
```

output
```
-2*e^(3/2*n)*x^(3/2*n)/(sqrt(b*x + a)*b*(x^n)^(3/2))
```

Giac [F]

$$\int \frac{(ex)^{3n/2}}{(ax^n + bx^{1+n})^{3/2}}\,dx = \int \frac{(ex)^{\frac{3}{2}\,n}}{(bx^{n+1} + ax^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2*n)/(a*x^n+b*x^(1+n))^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x)^(3/2*n)/(b*x^(n + 1) + a*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3n/2}}{(ax^n + bx^{1+n})^{3/2}}\,dx = \int \frac{(e\,x)^{\frac{3\,n}{2}}}{(a\,x^n + b\,x^{n+1})^{3/2}}\,dx$$

input
```
int((e*x)^((3*n)/2)/(a*x^n + b*x^(n + 1))^(3/2),x)
```

output
```
int((e*x)^((3*n)/2)/(a*x^n + b*x^(n + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.50

$$\int \frac{(ex)^{3n/2}}{\left(ax^n + bx^{1+n}\right)^{3/2}}\,dx = -\frac{2e^{\frac{3n}{2}}}{\sqrt{bx+a}\,b}$$

input
```
int((e*x)^(3/2*n)/(a*x^n+b*x^(1+n))^(3/2),x)
```

output
```
( - 2*e**((3*n)/2))/(sqrt(a + b*x)*b)
```

3.259 $\int \frac{(ex)^{1+\frac{3n}{2}}}{(c+dx)\left(ax^n+bx^{1+n}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 138

$$\int \frac{(ex)^{1+\frac{3n}{2}}}{(c+dx)\left(ax^n+bx^{1+n}\right)^{3/2}}\,dx = \frac{2aex^{-n}(ex)^{3n/2}}{b(bc-ad)\sqrt{ax^n+bx^{1+n}}} + \frac{2cex^{-n}(ex)^{3n/2}\sqrt{a+bx}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{\sqrt{d}(bc-ad)^{3/2}\sqrt{ax^n+bx^{1+n}}}$$

output
```
2*a*e*(e*x)^(3/2*n)/b/(-a*d+b*c)/(x^n)/(a*x^n+b*x^(1+n))^(1/2)+2*c*e*(e*x)
^(3/2*n)*(b*x+a)^(1/2)*arctan(d^(1/2)*(b*x+a)^(1/2)/(-a*d+b*c)^(1/2))/d^(1
/2)/(-a*d+b*c)^(3/2)/(x^n)/(a*x^n+b*x^(1+n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.80

$$\int \frac{(ex)^{1+\frac{3n}{2}}}{(c+dx)\left(ax^n+bx^{1+n}\right)^{3/2}}\,dx = \frac{2e(ex)^{3n/2}(a+bx)\left(a\sqrt{d}\sqrt{bc-ad}+bc\sqrt{a+bx}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)\right)}{b\sqrt{d}(bc-ad)^{3/2}\left(x^n(a+bx)\right)^{3/2}}$$

input
```
Integrate[(e*x)^(1 + (3*n)/2)/((c + d*x)*(a*x^n + b*x^(1 + n))^(3/2)),x]
```

output
```
(2*e*(e*x)^((3*n)/2)*(a + b*x)*(a*Sqrt[d]*Sqrt[b*c - a*d] + b*c*Sqrt[a + b
*x]*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]]))/(b*Sqrt[d]*(b*c - a*
d)^(3/2)*(x^n*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1948, 87, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{\frac{3n}{2}+1}}{(c+dx)\left(ax^n+bx^{n+1}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1948}$$

$$\frac{ex^{-n}\sqrt{a+bx}(ex)^{3n/2}\int \frac{x}{(a+bx)^{3/2}(c+dx)}dx}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{87}$$

$$\frac{ex^{-n}\sqrt{a+bx}(ex)^{3n/2}\left(\frac{c\int \frac{1}{\sqrt{a+bx}(c+dx)}dx}{bc-ad}+\frac{2a}{b\sqrt{a+bx}(bc-ad)}\right)}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{73}$$

$$\frac{ex^{-n}\sqrt{a+bx}(ex)^{3n/2}\left(\frac{2c\int \frac{1}{c-\frac{ad}{b}+\frac{d(a+bx)}{b}}d\sqrt{a+bx}}{b(bc-ad)}+\frac{2a}{b\sqrt{a+bx}(bc-ad)}\right)}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{218}$$

$$\frac{ex^{-n}\sqrt{a+bx}(ex)^{3n/2}\left(\frac{2c\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{\sqrt{d}(bc-ad)^{3/2}}+\frac{2a}{b\sqrt{a+bx}(bc-ad)}\right)}{\sqrt{ax^n+bx^{n+1}}}$$

input
```
Int[(e*x)^(1 + (3*n)/2)/((c + d*x)*(a*x^n + b*x^(1 + n))^(3/2)),x]
```

output
```
(e*(e*x)^((3*n)/2)*Sqrt[a + b*x]*((2*a)/(b*(b*c - a*d)*Sqrt[a + b*x]) + (2
*c*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(Sqrt[d]*(b*c - a*d)^(
3/2))))/(x^n*Sqrt[a*x^n + b*x^(1 + n)])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [F]

$$\int \frac{(ex)^{1+\frac{3n}{2}}}{(dx+c)\,(a\,x^{n}+b\,x^{1+n})^{\frac{3}{2}}}dx$$

input
```
int((e*x)^(1+3/2*n)/(d*x+c)/(a*x^n+b*x^(1+n))^(3/2),x)
```

output
```
int((e*x)^(1+3/2*n)/(d*x+c)/(a*x^n+b*x^(1+n))^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 439, normalized size of antiderivative = 3.18

$$\int \frac{(ex)^{1+\frac{3n}{2}}}{(c+dx)\,(ax^{n}+bx^{1+n})^{3/2}}\,dx = \left[\frac{(b^2cx+abc)\sqrt{-bcd+ad^2}e^{\frac{3}{2}n+1}x^{\frac{1}{2}n+\frac{1}{2}}\log\left(\frac{(bdx-bc+2\,ad)x^{\frac{1}{2}n+\frac{1}{2}}+2\sqrt{-bcd+}}{(dx+c)x^{\frac{1}{2}n+\frac{1}{2}}}\right.}{(ab^3c^2d-2\,a^2b^2cd^2+a^3bd^3+(b^4c^2d-}\right.$$

$$-\frac{2\left((b^2cx+abc)\sqrt{bcd-ad^2}e^{\frac{3}{2}n+1}x^{\frac{1}{2}n+\frac{1}{2}}\arctan\left(\frac{\sqrt{bcd-ad^2}\sqrt{x}\sqrt{\frac{(bx+a)x^{n+1}}{x}}}{(bdx+ad)x^{\frac{1}{2}n+\frac{1}{2}}}\right)-(abcd-a^2d^2)e^{\frac{3}{2}n+1}\sqrt{x}\sqrt{\frac{(bx+}{}}\right.}{(ab^3c^2d-2\,a^2b^2cd^2+a^3bd^3+(b^4c^2d-2\,ab^3cd^2+a^2b^2d^3)x)x^{\frac{1}{2}n+\frac{1}{2}}}$$

input
```
integrate((e*x)^(1+3/2*n)/(d*x+c)/(a*x^n+b*x^(1+n))^(3/2),x, algorithm="fr
icas")
```

output
```
[((b^2*c*x + a*b*c)*sqrt(-b*c*d + a*d^2)*e^(3/2*n + 1)*x^(1/2*n + 1/2)*log
(((b*d*x - b*c + 2*a*d)*x^(1/2*n + 1/2) + 2*sqrt(-b*c*d + a*d^2)*sqrt(x)*s
qrt((b*x + a)*x^(n + 1)/x))/((d*x + c)*x^(1/2*n + 1/2))) + 2*(a*b*c*d - a^
2*d^2)*e^(3/2*n + 1)*sqrt(x)*sqrt((b*x + a)*x^(n + 1)/x))/((a*b^3*c^2*d -
2*a^2*b^2*c*d^2 + a^3*b*d^3 + (b^4*c^2*d - 2*a*b^3*c*d^2 + a^2*b^2*d^3)*x)
*x^(1/2*n + 1/2)), -2*((b^2*c*x + a*b*c)*sqrt(b*c*d - a*d^2)*e^(3/2*n + 1)
*x^(1/2*n + 1/2)*arctan(sqrt(b*c*d - a*d^2)*sqrt(x)*sqrt((b*x + a)*x^(n +
1)/x)/((b*d*x + a*d)*x^(1/2*n + 1/2))) - (a*b*c*d - a^2*d^2)*e^(3/2*n + 1)
*sqrt(x)*sqrt((b*x + a)*x^(n + 1)/x))/((a*b^3*c^2*d - 2*a^2*b^2*c*d^2 + a^
3*b*d^3 + (b^4*c^2*d - 2*a*b^3*c*d^2 + a^2*b^2*d^3)*x)*x^(1/2*n + 1/2))]
```

Sympy [F]

$$\int \frac{(ex)^{1+\frac{3n}{2}}}{(c+dx)\left(ax^n+bx^{1+n}\right)^{3/2}}\,dx = \int \frac{(ex)^{\frac{3n}{2}+1}}{(c+dx)\left(ax^n+bx^{n+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)**(1+3/2*n)/(d*x+c)/(a*x**n+b*x**(1+n))**(3/2),x)
```

output
```
Integral((e*x)**(3*n/2 + 1)/((c + d*x)*(a*x**n + b*x**(n + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{(ex)^{1+\frac{3n}{2}}}{(c+dx)\left(ax^n+bx^{1+n}\right)^{3/2}}\,dx = \int \frac{(ex)^{\frac{3}{2}n+1}}{\left(bx^{n+1}+ax^n\right)^{\frac{3}{2}}(dx+c)}\,dx$$

input
```
integrate((e*x)^(1+3/2*n)/(d*x+c)/(a*x^n+b*x^(1+n))^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((e*x)^(3/2*n + 1)/((b*x^(n + 1) + a*x^n)^(3/2)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{(ex)^{1+\frac{3n}{2}}}{(c+dx)\left(ax^n+bx^{1+n}\right)^{3/2}}\,dx = \int \frac{(ex)^{\frac{3}{2}n+1}}{\left(bx^{n+1}+ax^n\right)^{\frac{3}{2}}(dx+c)}\,dx$$

input
```
integrate((e*x)^(1+3/2*n)/(d*x+c)/(a*x^n+b*x^(1+n))^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((e*x)^(3/2*n + 1)/((b*x^(n + 1) + a*x^n)^(3/2)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{1+\frac{3n}{2}}}{(c+dx)\left(ax^n+bx^{1+n}\right)^{3/2}}\,dx=\int \frac{(e\,x)^{\frac{3\,n}{2}+1}}{\left(a\,x^n+b\,x^{n+1}\right)^{3/2}\,(c+d\,x)}\,dx$$

input
```
int((e*x)^((3*n)/2 + 1)/((a*x^n + b*x^(n + 1))^(3/2)*(c + d*x)),x)
```

output
```
int((e*x)^((3*n)/2 + 1)/((a*x^n + b*x^(n + 1))^(3/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.75

$$\int \frac{(ex)^{1+\frac{3n}{2}}}{(c+dx)\left(ax^n+bx^{1+n}\right)^{3/2}}\,dx=\frac{2e^{\frac{3n}{2}}e\left(\sqrt{d}\,\sqrt{bx+a}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)bc-a^2d^2+abcd\right)}{\sqrt{bx+a}\,bd\left(a^2d^2-2abcd+b^2c^2\right)}$$

input
```
int((e*x)^(1+3/2*n)/(d*x+c)/(a*x^n+b*x^(1+n))^(3/2),x)
```

output
```
(2*e**((3*n)/2)*e*(sqrt(d)*sqrt(a + b*x)*sqrt( - a*d + b*c)*atan((sqrt(a +
 b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*b*c - a**2*d**2 + a*b*c*d))/(sqrt(a
 + b*x)*b*d*(a**2*d**2 - 2*a*b*c*d + b**2*c**2))
```

3.260 $\int \frac{(ex)^{2+\frac{3n}{2}}}{(c+dx)^2(ax^n+bx^{1+n})^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 210

$$\begin{aligned}\int \frac{(ex)^{2+\frac{3n}{2}}}{(c+dx)^2(ax^n+bx^{1+n})^{3/2}}\,dx &= -\frac{2a^2e^2x^{-n}(ex)^{3n/2}}{b(bc-ad)^2\sqrt{ax^n+bx^{1+n}}} \\ &\quad -\frac{c^2e^2x^{-2n}(ex)^{3n/2}\sqrt{ax^n+bx^{1+n}}}{d(bc-ad)^2(c+dx)} \\ &\quad +\frac{c(bc-4ad)e^2x^{-n}(ex)^{3n/2}\sqrt{a+bx}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{3/2}(bc-ad)^{5/2}\sqrt{ax^n+bx^{1+n}}}\end{aligned}$$

output
```
-2*a^2*e^2*(e*x)^(3/2*n)/b/(-a*d+b*c)^2/(x^n)/(a*x^n+b*x^(1+n))^(1/2)-c^2*
e^2*(e*x)^(3/2*n)*(a*x^n+b*x^(1+n))^(1/2)/d/(-a*d+b*c)^2/(x^(2*n))/(d*x+c)
+c*(-4*a*d+b*c)*e^2*(e*x)^(3/2*n)*(b*x+a)^(1/2)*arctan(d^(1/2)*(b*x+a)^(1/
2)/(-a*d+b*c)^(1/2))/d^(3/2)/(-a*d+b*c)^(5/2)/(x^n)/(a*x^n+b*x^(1+n))^(1/2
)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.75

$$\int \frac{(ex)^{2+\frac{3n}{2}}}{(c+dx)^2\left(ax^n+bx^{1+n}\right)^{3/2}}\,dx =$$
$$-\frac{e^2(ex)^{3n/2}(a+bx)\left(\sqrt{d}\sqrt{bc-ad}(abc^2+b^2c^2x+2a^2d(c+dx))-bc(bc-4ad)\sqrt{a+bx}(c+dx)\arctan\left(\right.\right.}{bd^{3/2}(bc-ad)^{5/2}\left(x^n(a+bx)\right)^{3/2}(c+dx)}$$

input
```
Integrate[(e*x)^(2 + (3*n)/2)/((c + d*x)^2*(a*x^n + b*x^(1 + n))^(3/2)),x]
```

output
```
-((e^2*(e*x)^((3*n)/2)*(a + b*x)*(Sqrt[d]*Sqrt[b*c - a*d]*(a*b*c^2 + b^2*c
^2*x + 2*a^2*d*(c + d*x)) - b*c*(b*c - 4*a*d)*Sqrt[a + b*x]*(c + d*x)*ArcT
an[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]]))/(b*d^(3/2)*(b*c - a*d)^(5/2)
*(x^n*(a + b*x))^(3/2)*(c + d*x)))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1948, 100, 27, 87, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{\frac{3n}{2}+2}}{(c+dx)^2\left(ax^n+bx^{n+1}\right)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 1948}$$
$$\frac{e^2x^{-n}\sqrt{a+bx}(ex)^{3n/2}\int \frac{x^2}{(a+bx)^{3/2}(c+dx)^2}dx}{\sqrt{ax^n+bx^{n+1}}}$$
$$\Big\downarrow \text{ 100}$$
$$\frac{e^2x^{-n}\sqrt{a+bx}(ex)^{3n/2}\left(\frac{2\int -\frac{a(bc+2ad)-b(bc-ad)x}{2\sqrt{a+bx}(c+dx)^2}dx}{b^2(bc-ad)}-\frac{2a^2}{b^2\sqrt{a+bx}(c+dx)(bc-ad)}\right)}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \; 27$$

$$\frac{e^2x^{-n}\sqrt{a+bx}(ex)^{3n/2}\left(-\frac{\int \frac{a(bc+2ad)-b(bc-ad)x}{\sqrt{a+bx}(c+dx)^2}dx}{b^2(bc-ad)}-\frac{2a^2}{b^2\sqrt{a+bx}(c+dx)(bc-ad)}\right)}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \; 87$$

$$\frac{e^2x^{-n}\sqrt{a+bx}(ex)^{3n/2}\left(-\frac{\frac{\sqrt{a+bx}(2a^2d^2+b^2c^2)}{d(c+dx)(bc-ad)}-\frac{b^2c(bc-4ad)\int \frac{1}{\sqrt{a+bx}(c+dx)}dx}{2d(bc-ad)}}{b^2(bc-ad)}-\frac{2a^2}{b^2\sqrt{a+bx}(c+dx)(bc-ad)}\right)}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \; 73$$

$$\frac{e^2x^{-n}\sqrt{a+bx}(ex)^{3n/2}\left(-\frac{\frac{\sqrt{a+bx}(2a^2d^2+b^2c^2)}{d(c+dx)(bc-ad)}-\frac{bc(bc-4ad)\int \frac{1}{c-\frac{ad}{b}+\frac{d(a+bx)}{b}}d\sqrt{a+bx}}{d(bc-ad)}}{b^2(bc-ad)}-\frac{2a^2}{b^2\sqrt{a+bx}(c+dx)(bc-ad)}\right)}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \; 218$$

$$\frac{e^2x^{-n}\sqrt{a+bx}(ex)^{3n/2}\left(-\frac{\frac{\sqrt{a+bx}(2a^2d^2+b^2c^2)}{d(c+dx)(bc-ad)}-\frac{b^2c(bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{3/2}(bc-ad)^{3/2}}}{b^2(bc-ad)}-\frac{2a^2}{b^2\sqrt{a+bx}(c+dx)(bc-ad)}\right)}{\sqrt{ax^n+bx^{n+1}}}$$

input
```
Int[(e*x)^(2 + (3*n)/2)/((c + d*x)^2*(a*x^n + b*x^(1 + n))^(3/2)),x]
```

output
```
(e^2*(e*x)^((3*n)/2)*Sqrt[a + b*x]*((-2*a^2)/(b^2*(b*c - a*d)*Sqrt[a + b*x
]*(c + d*x)) - (((b^2*c^2 + 2*a^2*d^2)*Sqrt[a + b*x])/(d*(b*c - a*d)*(c +
d*x)) - (b^2*c*(b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d
]])/(d^(3/2)*(b*c - a*d)^(3/2)))/(b^2*(b*c - a*d))))/(x^n*Sqrt[a*x^n + b*x
^(1 + n)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [F]

$$\int \frac{(ex)^{2+\frac{3n}{2}}}{(dx+c)^2 \left(a\,x^n+b\,x^{1+n}\right)^{\frac{3}{2}}}dx$$

input
```
int((e*x)^(2+3/2*n)/(d*x+c)^2/(a*x^n+b*x^(1+n))^(3/2),x)
```

output
```
int((e*x)^(2+3/2*n)/(d*x+c)^2/(a*x^n+b*x^(1+n))^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(188) = 376$.

Time = 0.12 (sec) , antiderivative size = 845, normalized size of antiderivative = 4.02

$$\int \frac{(ex)^{2+\frac{3n}{2}}}{(c+dx)^2 (ax^n+bx^{1+n})^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x)^(2+3/2*n)/(d*x+c)^2/(a*x^n+b*x^(1+n))^(3/2),x, algorithm="
fricas")
```

output
```
[1/2*((a*b^2*c^3 - 4*a^2*b*c^2*d + (b^3*c^2*d - 4*a*b^2*c*d^2)*x^2 + (b^3*
c^3 - 3*a*b^2*c^2*d - 4*a^2*b*c*d^2)*x)*sqrt(-b*c*d + a*d^2)*e^(3/2*n + 2)
*x^(1/2*n + 1/2)*log(((b*d*x - b*c + 2*a*d)*x^(1/2*n + 1/2) + 2*sqrt(-b*c*
d + a*d^2)*sqrt(x)*sqrt((b*x + a)*x^(n + 1)/x))/((d*x + c)*x^(1/2*n + 1/2)
)) - 2*(a*b^2*c^3*d + a^2*b*c^2*d^2 - 2*a^3*c*d^3 + (b^3*c^3*d - a*b^2*c^2
*d^2 + 2*a^2*b*c*d^3 - 2*a^3*d^4)*x)*e^(3/2*n + 2)*sqrt(x)*sqrt((b*x + a)*
x^(n + 1)/x))/((a*b^4*c^4*d^2 - 3*a^2*b^3*c^3*d^3 + 3*a^3*b^2*c^2*d^4 - a^
4*b*c*d^5 + (b^5*c^3*d^3 - 3*a*b^4*c^2*d^4 + 3*a^2*b^3*c*d^5 - a^3*b^2*d^6
)*x^2 + (b^5*c^4*d^2 - 2*a*b^4*c^3*d^3 + 2*a^3*b^2*c*d^5 - a^4*b*d^6)*x)*x
^(1/2*n + 1/2)), -((a*b^2*c^3 - 4*a^2*b*c^2*d + (b^3*c^2*d - 4*a*b^2*c*d^2
)*x^2 + (b^3*c^3 - 3*a*b^2*c^2*d - 4*a^2*b*c*d^2)*x)*sqrt(b*c*d - a*d^2)*e
^(3/2*n + 2)*x^(1/2*n + 1/2)*arctan(sqrt(b*c*d - a*d^2)*sqrt(x)*sqrt((b*x
+ a)*x^(n + 1)/x)/((b*d*x + a*d)*x^(1/2*n + 1/2))) + (a*b^2*c^3*d + a^2*b*
c^2*d^2 - 2*a^3*c*d^3 + (b^3*c^3*d - a*b^2*c^2*d^2 + 2*a^2*b*c*d^3 - 2*a^3
*d^4)*x)*e^(3/2*n + 2)*sqrt(x)*sqrt((b*x + a)*x^(n + 1)/x))/((a*b^4*c^4*d^
2 - 3*a^2*b^3*c^3*d^3 + 3*a^3*b^2*c^2*d^4 - a^4*b*c*d^5 + (b^5*c^3*d^3 - 3
*a*b^4*c^2*d^4 + 3*a^2*b^3*c*d^5 - a^3*b^2*d^6)*x^2 + (b^5*c^4*d^2 - 2*a*b
^4*c^3*d^3 + 2*a^3*b^2*c*d^5 - a^4*b*d^6)*x)*x^(1/2*n + 1/2))]
```

Sympy [F]

$$\int \frac{(ex)^{2+\frac{3n}{2}}}{(c+dx)^2\,(ax^n+bx^{1+n})^{3/2}}\,dx = \int \frac{(ex)^{\frac{3n}{2}+2}}{(c+dx)^2\,(ax^n+bx^{n+1})^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)**(2+3/2*n)/(d*x+c)**2/(a*x**n+b*x**(1+n))**(3/2),x)
```

output
```
Integral((e*x)**(3*n/2 + 2)/((c + d*x)**2*(a*x**n + b*x**(n + 1))**(3/2)),
 x)
```

Maxima [F]

$$\int \frac{(ex)^{2+\frac{3n}{2}}}{(c+dx)^2\left(ax^n+bx^{1+n}\right)^{3/2}}\,dx=\int \frac{(ex)^{\frac{3}{2}\,n+2}}{\left(bx^{n+1}+ax^n\right)^{\frac{3}{2}}(dx+c)^2}\,dx$$

input
```
integrate((e*x)^(2+3/2*n)/(d*x+c)^2/(a*x^n+b*x^(1+n))^(3/2),x, algorithm="
maxima")
```

output
```
integrate((e*x)^(3/2*n + 2)/((b*x^(n + 1) + a*x^n)^(3/2)*(d*x + c)^2), x)
```

Giac [F]

$$\int \frac{(ex)^{2+\frac{3n}{2}}}{(c+dx)^2\left(ax^n+bx^{1+n}\right)^{3/2}}\,dx=\int \frac{(ex)^{\frac{3}{2}\,n+2}}{\left(bx^{n+1}+ax^n\right)^{\frac{3}{2}}(dx+c)^2}\,dx$$

input
```
integrate((e*x)^(2+3/2*n)/(d*x+c)^2/(a*x^n+b*x^(1+n))^(3/2),x, algorithm="
giac")
```

output
```
integrate((e*x)^(3/2*n + 2)/((b*x^(n + 1) + a*x^n)^(3/2)*(d*x + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{2+\frac{3n}{2}}}{(c+dx)^2\left(ax^n+bx^{1+n}\right)^{3/2}}\,dx=\int \frac{(e\,x)^{\frac{3\,n}{2}+2}}{\left(a\,x^n+b\,x^{n+1}\right)^{3/2}\,(c+d\,x)^2}\,dx$$

input
```
int((e*x)^((3*n)/2 + 2)/((a*x^n + b*x^(n + 1))^(3/2)*(c + d*x)^2),x)
```

output
```
int((e*x)^((3*n)/2 + 2)/((a*x^n + b*x^(n + 1))^(3/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.80

$$\int \frac{(ex)^{2+\frac{3n}{2}}}{(c+dx)^2 (ax^n+bx^{1+n})^{3/2}}\,dx = \frac{e^{\frac{3n}{2}} e^2 \left(4\sqrt{d}\,\sqrt{bx+a}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right) ab\,c^2 d + 4\sqrt{d}\,\sqrt{bx+}\right.}{}$$

input
```
int((e*x)^(2+3/2*n)/(d*x+c)^2/(a*x^n+b*x^(1+n))^(3/2),x)
```

output
```
(e**((3*n)/2)*e**2*(4*sqrt(d)*sqrt(a + b*x)*sqrt( - a*d + b*c)*atan((sqrt(
a + b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*a*b*c**2*d + 4*sqrt(d)*sqrt(a +
b*x)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)
))*a*b*c*d**2*x - sqrt(d)*sqrt(a + b*x)*sqrt( - a*d + b*c)*atan((sqrt(a +
b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*b**2*c**3 - sqrt(d)*sqrt(a + b*x)*sq
rt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*b**2
*c**2*d*x - 2*a**3*c*d**3 - 2*a**3*d**4*x + a**2*b*c**2*d**2 + 2*a**2*b*c*
d**3*x + a*b**2*c**3*d - a*b**2*c**2*d**2*x + b**3*c**3*d*x))/(sqrt(a + b*
x)*b*d**2*(a**3*c*d**3 + a**3*d**4*x - 3*a**2*b*c**2*d**2 - 3*a**2*b*c*d**
3*x + 3*a*b**2*c**3*d + 3*a*b**2*c**2*d**2*x - b**3*c**4 - b**3*c**3*d*x))
```

3.261 $\int \frac{(ex)^{3+\frac{3n}{2}}}{(c+dx)^3 (ax^n+bx^{1+n})^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 299

$$\begin{aligned}\int \frac{(ex)^{3+\frac{3n}{2}}}{(c+dx)^3 (ax^n+bx^{1+n})^{3/2}}\,dx &= \frac{2a^3e^3x^{-n}(ex)^{3n/2}}{b(bc-ad)^3\sqrt{ax^n+bx^{1+n}}} \\ &+ \frac{c^3e^3x^{-2n}(ex)^{3n/2}\sqrt{ax^n+bx^{1+n}}}{2d^2(bc-ad)^2(c+dx)^2} - \frac{c^2(5bc-12ad)e^3x^{-2n}(ex)^{3n/2}\sqrt{ax^n+bx^{1+n}}}{4d^2(bc-ad)^3(c+dx)} \\ &+ \frac{3c(b^2c^2-4abcd+8a^2d^2)\,e^3x^{-n}(ex)^{3n/2}\sqrt{a+bx}\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{4d^{5/2}(bc-ad)^{7/2}\sqrt{ax^n+bx^{1+n}}}\end{aligned}$$

output

```
2*a^3*e^3*(e*x)^(3/2*n)/b/(-a*d+b*c)^3/(x^n)/(a*x^n+b*x^(1+n))^(1/2)+1/2*c
^3*e^3*(e*x)^(3/2*n)*(a*x^n+b*x^(1+n))^(1/2)/d^2/(-a*d+b*c)^2/(x^(2*n))/(d
*x+c)^2-1/4*c^2*(-12*a*d+5*b*c)*e^3*(e*x)^(3/2*n)*(a*x^n+b*x^(1+n))^(1/2)/
d^2/(-a*d+b*c)^3/(x^(2*n))/(d*x+c)+3/4*c*(8*a^2*d^2-4*a*b*c*d+b^2*c^2)*e^3
*(e*x)^(3/2*n)*(b*x+a)^(1/2)*arctan(d^(1/2)*(b*x+a)^(1/2)/(-a*d+b*c)^(1/2)
)/d^(5/2)/(-a*d+b*c)^(7/2)/(x^n)/(a*x^n+b*x^(1+n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.73

$$\int \frac{(ex)^{3+\frac{3n}{2}}}{(c+dx)^3\left(ax^n+bx^{1+n}\right)^{3/2}}\,dx = \frac{e^3(ex)^{3n/2}\left(\frac{\sqrt{d}(a+bx)\left(8a^3d^2(c+dx)^2-b^3c^3x(3c+5dx)+2a^2bc^2d(5c+6dx)+ab^2c^2\left(-3c^2+5c\right.\right.}{b(bc-ad)^3(c+dx)^2}\right.}{4d^{5/2}\left(x^n(a+bx)\right)}$$

input
```
Integrate[(e*x)^(3 + (3*n)/2)/((c + d*x)^3*(a*x^n + b*x^(1 + n))^(3/2)),x]
```

output
```
(e^3*(e*x)^((3*n)/2)*((Sqrt[d]*(a + b*x)*(8*a^3*d^2*(c + d*x)^2 - b^3*c^3*
x*(3*c + 5*d*x) + 2*a^2*b*c^2*d*(5*c + 6*d*x) + a*b^2*c^2*(-3*c^2 + 5*c*d*
x + 12*d^2*x^2)))/(b*(b*c - a*d)^3*(c + d*x)^2) + (3*c*(b^2*c^2 - 4*a*b*c*
d + 8*a^2*d^2)*(a + b*x)^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a
*d]])/(b*c - a*d)^(7/2)))/(4*d^(5/2)*(x^n*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.83, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1948, 109, 27, 162, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{\frac{3n}{2}+3}}{(c+dx)^3\left(ax^n+bx^{n+1}\right)^{3/2}}\,dx$$

$$\Big\downarrow \ 1948$$

$$\frac{e^3x^{-n}\sqrt{a+bx}(ex)^{3n/2}\int \frac{x^3}{(a+bx)^{3/2}(c+dx)^3}dx}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \ 109$$

$$\frac{e^3x^{-n}\sqrt{a+bx}(ex)^{3n/2}\left(\frac{2ax^2}{b\sqrt{a+bx}(c+dx)^2(bc-ad)}-\frac{2\int \frac{cx(4a-bx)}{2\sqrt{a+bx}(c+dx)^3}dx}{b(bc-ad)}\right)}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^3x^{-n}\sqrt{a+bx}(ex)^{3n/2}\left(\frac{2ax^2}{b\sqrt{a+bx}(c+dx)^2(bc-ad)}-\frac{c\int \frac{x(4a-bx)}{\sqrt{a+bx}(c+dx)^3}dx}{b(bc-ad)}\right)}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{162}$$

$$\frac{e^3x^{-n}\sqrt{a+bx}(ex)^{3n/2}\left(\frac{2ax^2}{b\sqrt{a+bx}(c+dx)^2(bc-ad)}-\frac{c\left(\frac{\sqrt{a+bx}\left(dx\left(-16a^2d^2-4abcd+5b^2c^2\right)+c(bc-4ad)(2ad+3bc)\right)}{4d^2(c+dx)^2(bc-ad)^2}-\frac{3b\left(8a^2d^2-4abcd+b^2c^2\right)}{8d^2(bc-\cdots}\right.}{b(bc-ad)}\right.}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{73}$$

$$\frac{e^3x^{-n}\sqrt{a+bx}(ex)^{3n/2}\left(\frac{2ax^2}{b\sqrt{a+bx}(c+dx)^2(bc-ad)}-\frac{c\left(\frac{\sqrt{a+bx}\left(dx\left(-16a^2d^2-4abcd+5b^2c^2\right)+c(bc-4ad)(2ad+3bc)\right)}{4d^2(c+dx)^2(bc-ad)^2}-\frac{3\left(8a^2d^2-4abcd+b^2c^2\right)}{4d^2(\cdots}\right.}{b(bc-ad)}\right.}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{218}$$

$$\frac{e^3x^{-n}\sqrt{a+bx}(ex)^{3n/2}\left(\frac{2ax^2}{b\sqrt{a+bx}(c+dx)^2(bc-ad)}-\frac{c\left(\frac{\sqrt{a+bx}\left(dx\left(-16a^2d^2-4abcd+5b^2c^2\right)+c(bc-4ad)(2ad+3bc)\right)}{4d^2(c+dx)^2(bc-ad)^2}-\frac{3b\left(8a^2d^2-4abcd+b^2c^2\right)}{4d^{5/2}(bc-\cdots}\right.}{b(bc-ad)}\right.}{\sqrt{ax^n+bx^{n+1}}}$$

input
```
Int[(e*x)^(3 + (3*n)/2)/((c + d*x)^3*(a*x^n + b*x^(1 + n))^(3/2)),x]
```

output
```
(e^3*(e*x)^((3*n)/2)*Sqrt[a + b*x]*((2*a*x^2)/(b*(b*c - a*d)*Sqrt[a + b*x]
*(c + d*x)^2) - (c*((Sqrt[a + b*x]*(c*(b*c - 4*a*d)*(3*b*c + 2*a*d) + d*(5
*b^2*c^2 - 4*a*b*c*d - 16*a^2*d^2)*x))/(4*d^2*(b*c - a*d)^2*(c + d*x)^2) -
 (3*b*(b^2*c^2 - 4*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqr
t[b*c - a*d]])/(4*d^(5/2)*(b*c - a*d)^(5/2))))/(b*(b*c - a*d))))/(x^n*Sqrt
[a*x^n + b*x^(1 + n)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [F]

$$\int \frac{(ex)^{3+\frac{3n}{2}}}{(dx+c)^3\,(a\,x^n+b\,x^{1+n})^{\frac{3}{2}}}dx$$

input
```
int((e*x)^(3+3/2*n)/(d*x+c)^3/(a*x^n+b*x^(1+n))^(3/2),x)
```

output
```
int((e*x)^(3+3/2*n)/(d*x+c)^3/(a*x^n+b*x^(1+n))^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 725 vs. 2(269) = 538.

Time = 0.21 (sec) , antiderivative size = 1473, normalized size of antiderivative = 4.93

$$\int \frac{(ex)^{3+\frac{3n}{2}}}{(c+dx)^3\,(ax^n+bx^{1+n})^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x)^(3+3/2*n)/(d*x+c)^3/(a*x^n+b*x^(1+n))^(3/2),x, algorithm="
fricas")
```

output
```
[1/8*(3*(a*b^3*c^5 - 4*a^2*b^2*c^4*d + 8*a^3*b*c^3*d^2 + (b^4*c^3*d^2 - 4*
a*b^3*c^2*d^3 + 8*a^2*b^2*c*d^4)*x^3 + (2*b^4*c^4*d - 7*a*b^3*c^3*d^2 + 12
*a^2*b^2*c^2*d^3 + 8*a^3*b*c*d^4)*x^2 + (b^4*c^5 - 2*a*b^3*c^4*d + 16*a^3*
b*c^2*d^3)*x)*sqrt(-b*c*d + a*d^2)*e^(3/2*n + 3)*x^(1/2*n + 1/2)*log(((b*d
*x - b*c + 2*a*d)*x^(1/2*n + 1/2) + 2*sqrt(-b*c*d + a*d^2)*sqrt(x)*sqrt((b
*x + a)*x^(n + 1)/x))/((d*x + c)*x^(1/2*n + 1/2))) - 2*(3*a*b^3*c^5*d - 13
*a^2*b^2*c^4*d^2 + 2*a^3*b*c^3*d^3 + 8*a^4*c^2*d^4 + (5*b^4*c^4*d^2 - 17*a
*b^3*c^3*d^3 + 12*a^2*b^2*c^2*d^4 - 8*a^3*b*c*d^5 + 8*a^4*d^6)*x^2 + (3*b^
4*c^5*d - 8*a*b^3*c^4*d^2 - 7*a^2*b^2*c^3*d^3 - 4*a^3*b*c^2*d^4 + 16*a^4*c
*d^5)*x)*e^(3/2*n + 3)*sqrt(x)*sqrt((b*x + a)*x^(n + 1)/x))/((a*b^5*c^6*d^
3 - 4*a^2*b^4*c^5*d^4 + 6*a^3*b^3*c^4*d^5 - 4*a^4*b^2*c^3*d^6 + a^5*b*c^2*
d^7 + (b^6*c^4*d^5 - 4*a*b^5*c^3*d^6 + 6*a^2*b^4*c^2*d^7 - 4*a^3*b^3*c*d^8
 + a^4*b^2*d^9)*x^3 + (2*b^6*c^5*d^4 - 7*a*b^5*c^4*d^5 + 8*a^2*b^4*c^3*d^6
 - 2*a^3*b^3*c^2*d^7 - 2*a^4*b^2*c*d^8 + a^5*b*d^9)*x^2 + (b^6*c^6*d^3 - 2
*a*b^5*c^5*d^4 - 2*a^2*b^4*c^4*d^5 + 8*a^3*b^3*c^3*d^6 - 7*a^4*b^2*c^2*d^7
 + 2*a^5*b*c*d^8)*x)*x^(1/2*n + 1/2)), -1/4*(3*(a*b^3*c^5 - 4*a^2*b^2*c^4*
d + 8*a^3*b*c^3*d^2 + (b^4*c^3*d^2 - 4*a*b^3*c^2*d^3 + 8*a^2*b^2*c*d^4)*x^
3 + (2*b^4*c^4*d - 7*a*b^3*c^3*d^2 + 12*a^2*b^2*c^2*d^3 + 8*a^3*b*c*d^4)*x
^2 + (b^4*c^5 - 2*a*b^3*c^4*d + 16*a^3*b*c^2*d^3)*x)*sqrt(b*c*d - a*d^2)*e
^(3/2*n + 3)*x^(1/2*n + 1/2)*arctan(sqrt(b*c*d - a*d^2)*sqrt(x)*sqrt((b...
```

Sympy [F]

$$\int \frac{(ex)^{3+\frac{3n}{2}}}{(c+dx)^3 (ax^n+bx^{1+n})^{3/2}}\,dx = \int \frac{(ex)^{\frac{3n}{2}+3}}{(c+dx)^3 (ax^n+bx^{n+1})^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)**(3+3/2*n)/(d*x+c)**3/(a*x**n+b*x**(1+n))**(3/2),x)
```

output
```
Integral((e*x)**(3*n/2 + 3)/((c + d*x)**3*(a*x**n + b*x**(n + 1))**(3/2)),
 x)
```

Maxima [F]

$$\int \frac{(ex)^{3+\frac{3n}{2}}}{(c+dx)^3 (ax^n+bx^{1+n})^{3/2}}\,dx = \int \frac{(ex)^{\frac{3}{2}n+3}}{(bx^{n+1}+ax^n)^{\frac{3}{2}}(dx+c)^3}\,dx$$

input
```
integrate((e*x)^(3+3/2*n)/(d*x+c)^3/(a*x^n+b*x^(1+n))^(3/2),x, algorithm="
maxima")
```

output
```
integrate((e*x)^(3/2*n + 3)/((b*x^(n + 1) + a*x^n)^(3/2)*(d*x + c)^3), x)
```

Giac [F]

$$\int \frac{(ex)^{3+\frac{3n}{2}}}{(c+dx)^3 (ax^n+bx^{1+n})^{3/2}}\,dx = \int \frac{(ex)^{\frac{3}{2}n+3}}{(bx^{n+1}+ax^n)^{\frac{3}{2}}(dx+c)^3}\,dx$$

input
```
integrate((e*x)^(3+3/2*n)/(d*x+c)^3/(a*x^n+b*x^(1+n))^(3/2),x, algorithm="
giac")
```

output
```
integrate((e*x)^(3/2*n + 3)/((b*x^(n + 1) + a*x^n)^(3/2)*(d*x + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3+\frac{3n}{2}}}{(c+dx)^3 (ax^n+bx^{1+n})^{3/2}}\,dx = \int \frac{(e\,x)^{\frac{3\,n}{2}+3}}{(a\,x^n+b\,x^{n+1})^{3/2}\,(c+d\,x)^3}\,dx$$

input
```
int((e*x)^((3*n)/2 + 3)/((a*x^n + b*x^(n + 1))^(3/2)*(c + d*x)^3),x)
```

output
```
int((e*x)^((3*n)/2 + 3)/((a*x^n + b*x^(n + 1))^(3/2)*(c + d*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 865, normalized size of antiderivative = 2.89

$$\int \frac{(ex)^{3+\frac{3n}{2}}}{(c+dx)^3 (ax^n + bx^{1+n})^{3/2}} \, dx = \text{Too large to display}$$

input
```
int((e*x)^(3+3/2*n)/(d*x+c)^3/(a*x^n+b*x^(1+n))^(3/2),x)
```

output
```
(e**((3*n)/2)*e**3*(24*sqrt(d)*sqrt(a + b*x)*sqrt( - a*d + b*c)*atan((sqrt
(a + b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*a**2*b*c**3*d**2 + 48*sqrt(d)*s
qrt(a + b*x)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*
d + b*c)))*a**2*b*c**2*d**3*x + 24*sqrt(d)*sqrt(a + b*x)*sqrt( - a*d + b*c
)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*a**2*b*c*d**4*x**2
 - 12*sqrt(d)*sqrt(a + b*x)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt
(d)*sqrt( - a*d + b*c)))*a*b**2*c**4*d - 24*sqrt(d)*sqrt(a + b*x)*sqrt( -
a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*a*b**2*c**
3*d**2*x - 12*sqrt(d)*sqrt(a + b*x)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)
*d)/(sqrt(d)*sqrt( - a*d + b*c)))*a*b**2*c**2*d**3*x**2 + 3*sqrt(d)*sqrt(a
 + b*x)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d + b
*c)))*b**3*c**5 + 6*sqrt(d)*sqrt(a + b*x)*sqrt( - a*d + b*c)*atan((sqrt(a
+ b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*b**3*c**4*d*x + 3*sqrt(d)*sqrt(a +
 b*x)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c
)))*b**3*c**3*d**2*x**2 - 8*a**4*c**2*d**4 - 16*a**4*c*d**5*x - 8*a**4*d**
6*x**2 - 2*a**3*b*c**3*d**3 + 4*a**3*b*c**2*d**4*x + 8*a**3*b*c*d**5*x**2
+ 13*a**2*b**2*c**4*d**2 + 7*a**2*b**2*c**3*d**3*x - 12*a**2*b**2*c**2*d**
4*x**2 - 3*a*b**3*c**5*d + 8*a*b**3*c**4*d**2*x + 17*a*b**3*c**3*d**3*x**2
 - 3*b**4*c**5*d*x - 5*b**4*c**4*d**2*x**2))/(4*sqrt(a + b*x)*b*d**3*(a**4
*c**2*d**4 + 2*a**4*c*d**5*x + a**4*d**6*x**2 - 4*a**3*b*c**3*d**3 - 8*...
```

3.262 $\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c+dx}\sqrt{ax^n+bx^{1+n}}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 99

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c+dx}\sqrt{ax^n+bx^{1+n}}}\,dx=\frac{2\sqrt{a}x^{n/2}\sqrt{c+dx}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}c\sqrt{\frac{a(c+dx)}{c(a+bx)}}\sqrt{ax^n+bx^{1+n}}}$$

output
```
2*a^(1/2)*x^(1/2*n)*(d*x+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x^(1/2)/a
^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/(a*(d*x+c)/c/(b*x+a))^(1/2)/(a*x^n+b*
x^(1+n))^(1/2)
```

Mathematica [A] (verified)

Time = 4.92 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.12

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c+dx}\sqrt{ax^n+bx^{1+n}}}\,dx = -\frac{2x^{\frac{1+n}{2}}\sqrt{a+bx}\sqrt{\frac{b(c+dx)}{d(a+bx)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),1-\frac{bc}{ad}\right)}{\sqrt{a}\sqrt{\frac{bx}{a+bx}}\sqrt{x^n(a+bx)}\sqrt{c+dx}}$$

input
```
Integrate[x^((-1 + n)/2)/(Sqrt[c + d*x]*Sqrt[a*x^n + b*x^(1 + n)]),x]
```

output
```
(-2*x^((1 + n)/2)*Sqrt[a + b*x]*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Elliptic
F[ArcSin[Sqrt[a]/Sqrt[a + b*x]], 1 - (b*c)/(a*d)])/(Sqrt[a]*Sqrt[(b*x)/(a
+ b*x)]*Sqrt[x^n*(a + b*x)]*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1948, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n-1}{2}}}{\sqrt{c+dx}\sqrt{ax^n+bx^{n+1}}}dx$$

$$\Big\downarrow \text{1948}$$

$$\frac{x^{n/2}\sqrt{a+bx}\int \frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c+dx}}dx}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{127}$$

$$\frac{x^{n/2}\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}\int \frac{1}{\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}}dx}{\sqrt{c+dx}\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{126}$$

$$\frac{2\sqrt{-a}x^{n/2}\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{c+dx}\sqrt{ax^n+bx^{n+1}}}$$

input
```
Int[x^((-1 + n)/2)/(Sqrt[c + d*x]*Sqrt[a*x^n + b*x^(1 + n)]),x]
```

output
```
(2*Sqrt[-a]*x^(n/2)*Sqrt[1 + (b*x)/a]*Sqrt[1 + (d*x)/c]*EllipticF[ArcSin[(
Sqrt[b]*Sqrt[x])/Sqrt[-a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[c + d*x]*Sqrt[a*x^
n + b*x^(1 + n)])
```

Defintions of rubi rules used

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [F]

$$\int \frac{x^{-\frac{1}{2}+\frac{n}{2}}}{\sqrt{dx+c}\,\sqrt{a\,x^{n}+b\,x^{1+n}}}dx$$

input
```
int(x^(-1/2+1/2*n)/(d*x+c)^(1/2)/(a*x^n+b*x^(1+n))^(1/2),x)
```

output
```
int(x^(-1/2+1/2*n)/(d*x+c)^(1/2)/(a*x^n+b*x^(1+n))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.09

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c+dx}\sqrt{ax^n+bx^{1+n}}}\,dx = \frac{2\sqrt{bd}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(b^2c^2-abcd+a^2d^2)}{3\,b^2d^2}, -\frac{4\,(2\,b^3c^3-3\,ab^2c^2d-3\,a^2bcd^2+2\,a^3d^3)}{27\,b^3d^3}, \frac{3\,bdx+bc+ad}{3\,bd}\right)}{bd}$$

input
```
integrate(x^(-1/2+1/2*n)/(d*x+c)^(1/2)/(a*x^n+b*x^(1+n))^(1/2),x, algorith
m="fricas")
```

output
```
2*sqrt(b*d)*weierstrassPInverse(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2
), -4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3)
, 1/3*(3*b*d*x + b*c + a*d)/(b*d))/(b*d)
```

Sympy [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c+dx}\sqrt{ax^n+bx^{1+n}}}\,dx = \int \frac{x^{\frac{n}{2}-\frac{1}{2}}}{\sqrt{c+dx}\sqrt{ax^n+bx^{n+1}}}\,dx$$

input
```
integrate(x**(-1/2+1/2*n)/(d*x+c)**(1/2)/(a*x**n+b*x**(1+n))**(1/2),x)
```

output
```
Integral(x**(n/2 - 1/2)/(sqrt(c + d*x)*sqrt(a*x**n + b*x**(n + 1))), x)
```

Maxima [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c+dx}\sqrt{ax^n+bx^{1+n}}}\,dx = \int \frac{x^{\frac{1}{2}\,n-\frac{1}{2}}}{\sqrt{bx^{n+1}+ax^n}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^(-1/2+1/2*n)/(d*x+c)^(1/2)/(a*x^n+b*x^(1+n))^(1/2),x, algorith
m="maxima")
```

output
```
integrate(x^(1/2*n - 1/2)/(sqrt(b*x^(n + 1) + a*x^n)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c+dx}\sqrt{ax^n+bx^{1+n}}}\,dx=\int \frac{x^{\frac{1}{2}\,n-\frac{1}{2}}}{\sqrt{bx^{n+1}+ax^n}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^(-1/2+1/2*n)/(d*x+c)^(1/2)/(a*x^n+b*x^(1+n))^(1/2),x, algorith
m="giac")
```

output
```
integrate(x^(1/2*n - 1/2)/(sqrt(b*x^(n + 1) + a*x^n)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c+dx}\sqrt{ax^n+bx^{1+n}}}\,dx=\int \frac{x^{\frac{n}{2}-\frac{1}{2}}}{\sqrt{a\,x^n+b\,x^{n+1}}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(x^(n/2 - 1/2)/((a*x^n + b*x^(n + 1))^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int(x^(n/2 - 1/2)/((a*x^n + b*x^(n + 1))^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c+dx}\sqrt{ax^n+bx^{1+n}}}\,dx=\int \frac{\sqrt{x}\,\sqrt{dx+c}\,\sqrt{bx+a}}{bd\,x^3+ad\,x^2+bc\,x^2+acx}dx$$

input
```
int(x^(-1/2+1/2*n)/(d*x+c)^(1/2)/(a*x^n+b*x^(1+n))^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(c + d*x)*sqrt(a + b*x))/(a*c*x + a*d*x**2 + b*c*x**2 + b
*d*x**3),x)
```

3.263 $\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c-dx}\sqrt{ax^n+bx^{1+n}}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 99

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c-dx}\sqrt{ax^n+bx^{1+n}}}\,dx = \frac{2\sqrt{c}x^{n/2}\sqrt{1+\frac{bx}{a}}\sqrt{1-\frac{dx}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}\sqrt{x}}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c-dx}\sqrt{ax^n+bx^{1+n}}}$$

output

```
2*c^(1/2)*x^(1/2*n)*(1+b*x/a)^(1/2)*(1-d*x/c)^(1/2)*EllipticF(d^(1/2)*x^(1
/2)/c^(1/2),(-b*c/a/d)^(1/2))/d^(1/2)/(-d*x+c)^(1/2)/(a*x^n+b*x^(1+n))^(1/
2)
```

Mathematica [A] (verified)

Time = 4.94 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.14

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c-dx}\sqrt{ax^n+bx^{1+n}}}\,dx = -\frac{2x^{\frac{1+n}{2}}\sqrt{a+bx}\sqrt{\frac{b(-c+dx)}{d(a+bx)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),1+\frac{bc}{ad}\right)}{\sqrt{a}\sqrt{\frac{bx}{a+bx}}\sqrt{x^n(a+bx)}\sqrt{c-dx}}$$

input
```
Integrate[x^((-1 + n)/2)/(Sqrt[c - d*x]*Sqrt[a*x^n + b*x^(1 + n)]),x]
```

output
```
(-2*x^((1 + n)/2)*Sqrt[a + b*x]*Sqrt[(b*(-c + d*x))/(d*(a + b*x))]*Ellipti
cF[ArcSin[Sqrt[a]/Sqrt[a + b*x]], 1 + (b*c)/(a*d)])/(Sqrt[a]*Sqrt[(b*x)/(a
 + b*x)]*Sqrt[x^n*(a + b*x)]*Sqrt[c - d*x])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {1948, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n-1}{2}}}{\sqrt{c-dx}\sqrt{ax^n+bx^{n+1}}}dx$$

$$\Big\downarrow \text{ 1948}$$

$$\frac{x^{n/2}\sqrt{a+bx}\int \frac{1}{\sqrt{x}\sqrt{a+bx}\sqrt{c-dx}}dx}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{ 127}$$

$$\frac{x^{n/2}\sqrt{\frac{bx}{a}+1}\sqrt{1-\frac{dx}{c}}\int \frac{1}{\sqrt{x}\sqrt{\frac{bx}{a}+1}\sqrt{1-\frac{dx}{c}}}dx}{\sqrt{c-dx}\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{ 126}$$

$$\frac{2\sqrt{c}x^{n/2}\sqrt{\frac{bx}{a}+1}\sqrt{1-\frac{dx}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}\sqrt{x}}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c-dx}\sqrt{ax^n+bx^{n+1}}}$$

input
```
Int[x^((-1 + n)/2)/(Sqrt[c - d*x]*Sqrt[a*x^n + b*x^(1 + n)]),x]
```

output
```
(2*Sqrt[c]*x^(n/2)*Sqrt[1 + (b*x)/a]*Sqrt[1 - (d*x)/c]*EllipticF[ArcSin[(S
qrt[d]*Sqrt[x])/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[c - d*x]*Sqrt[a*x
^n + b*x^(1 + n)])
```

Defintions of rubi rules used

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [F]

$$\int \frac{x^{-\frac{1}{2}+\frac{n}{2}}}{\sqrt{-dx+c}\,\sqrt{a\,x^{n}+b\,x^{1+n}}}dx$$

input
```
int(x^(-1/2+1/2*n)/(-d*x+c)^(1/2)/(a*x^n+b*x^(1+n))^(1/2),x)
```

output
```
int(x^(-1/2+1/2*n)/(-d*x+c)^(1/2)/(a*x^n+b*x^(1+n))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.10

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c-dx}\sqrt{ax^n+bx^{1+n}}}\,dx = -\frac{2\sqrt{-bd}\text{weierstrassPInverse}\left(\frac{4\,(b^2c^2+abcd+a^2d^2)}{3\,b^2d^2},\frac{4\,(2\,b^3c^3+3\,ab^2c^2d-3\,a^2bcd^2-2\,a^3d^3)}{27\,b^3d^3},\frac{3\,bdx-bc+ad}{3\,bd}\right)}{bd}$$

input
```
integrate(x^(-1/2+1/2*n)/(-d*x+c)^(1/2)/(a*x^n+b*x^(1+n))^(1/2),x, algorit
hm="fricas")
```

output
```
-2*sqrt(-b*d)*weierstrassPInverse(4/3*(b^2*c^2 + a*b*c*d + a^2*d^2)/(b^2*d
^2), 4/27*(2*b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 2*a^3*d^3)/(b^3*d^3
), 1/3*(3*b*d*x - b*c + a*d)/(b*d))/(b*d)
```

Sympy [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c-dx}\sqrt{ax^n+bx^{1+n}}}\,dx = \int \frac{x^{\frac{n}{2}-\frac{1}{2}}}{\sqrt{c-dx}\sqrt{ax^n+bx^{n+1}}}\,dx$$

input
```
integrate(x**(-1/2+1/2*n)/(-d*x+c)**(1/2)/(a*x**n+b*x**(1+n))**(1/2),x)
```

output
```
Integral(x**(n/2 - 1/2)/(sqrt(c - d*x)*sqrt(a*x**n + b*x**(n + 1))), x)
```

Maxima [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c-dx}\sqrt{ax^n+bx^{1+n}}}\,dx = \int \frac{x^{\frac{1}{2}\,n-\frac{1}{2}}}{\sqrt{bx^{n+1}+ax^n}\sqrt{-dx+c}}\,dx$$

input
```
integrate(x^(-1/2+1/2*n)/(-d*x+c)^(1/2)/(a*x^n+b*x^(1+n))^(1/2),x, algorit
hm="maxima")
```

output
```
integrate(x^(1/2*n - 1/2)/(sqrt(b*x^(n + 1) + a*x^n)*sqrt(-d*x + c)), x)
```

Giac [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c-dx}\sqrt{ax^n+bx^{1+n}}}\,dx=\int \frac{x^{\frac{1}{2}\,n-\frac{1}{2}}}{\sqrt{bx^{n+1}+ax^n}\sqrt{-dx+c}}\,dx$$

input
```
integrate(x^(-1/2+1/2*n)/(-d*x+c)^(1/2)/(a*x^n+b*x^(1+n))^(1/2),x, algorit
hm="giac")
```

output
```
integrate(x^(1/2*n - 1/2)/(sqrt(b*x^(n + 1) + a*x^n)*sqrt(-d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c-dx}\sqrt{ax^n+bx^{1+n}}}\,dx=\int \frac{x^{\frac{n}{2}-\frac{1}{2}}}{\sqrt{a\,x^n+b\,x^{n+1}}\,\sqrt{c-d\,x}}\,dx$$

input
```
int(x^(n/2 - 1/2)/((a*x^n + b*x^(n + 1))^(1/2)*(c - d*x)^(1/2)),x)
```

output
```
int(x^(n/2 - 1/2)/((a*x^n + b*x^(n + 1))^(1/2)*(c - d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c-dx}\sqrt{ax^n+bx^{1+n}}}\,dx=\int \frac{\sqrt{x}\,\sqrt{-dx+c}\,\sqrt{bx+a}}{-bd\,x^3-ad\,x^2+bc\,x^2+acx}dx$$

input
```
int(x^(-1/2+1/2*n)/(-d*x+c)^(1/2)/(a*x^n+b*x^(1+n))^(1/2),x)
```

output

```
int((sqrt(x)*sqrt(c - d*x)*sqrt(a + b*x))/(a*c*x - a*d*x**2 + b*c*x**2 - b
*d*x**3),x)
```

3.264 $\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c+dx}\sqrt{ax^n-bx^{1+n}}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 99

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c+dx}\sqrt{ax^n-bx^{1+n}}}\,dx = \frac{2\sqrt{a}x^{n/2}\sqrt{1-\frac{bx}{a}}\sqrt{1+\frac{dx}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{c+dx}\sqrt{ax^n-bx^{1+n}}}$$

output
```
2*a^(1/2)*x^(1/2*n)*(1-b*x/a)^(1/2)*(1+d*x/c)^(1/2)*EllipticF(b^(1/2)*x^(1
/2)/a^(1/2),(-a*d/b/c)^(1/2))/b^(1/2)/(d*x+c)^(1/2)/(a*x^n-b*x^(1+n))^(1/2
)
```

Mathematica [A] (verified)

Time = 4.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.21

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c+dx}\sqrt{ax^n-bx^{1+n}}}\,dx = \frac{2x^{\frac{1}{2}(-1+n)}(a-bx)^{3/2}\sqrt{\frac{bx}{-a+bx}}\sqrt{\frac{b(c+dx)}{d(-a+bx)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a}}{\sqrt{a-bx}}\right),1+\frac{bc}{ad}\right)}{\sqrt{a}b\sqrt{x^n(a-bx)}\sqrt{c+dx}}$$

input
```
Integrate[x^((-1 + n)/2)/(Sqrt[c + d*x]*Sqrt[a*x^n - b*x^(1 + n)]),x]
```

output
```
(2*x^((-1 + n)/2)*(a - b*x)^(3/2)*Sqrt[(b*x)/(-a + b*x)]*Sqrt[(b*(c + d*x)
)/(d*(-a + b*x))]*EllipticF[ArcSin[Sqrt[a]/Sqrt[a - b*x]], 1 + (b*c)/(a*d)
])/(Sqrt[a]*b*Sqrt[x^n*(a - b*x)]*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {1948, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n-1}{2}}}{\sqrt{c+dx}\sqrt{ax^n-bx^{n+1}}}dx$$

$$\Big\downarrow \text{ 1948}$$

$$\frac{x^{n/2}\sqrt{a-bx}\int \frac{1}{\sqrt{x}\sqrt{a-bx}\sqrt{c+dx}}dx}{\sqrt{ax^n-bx^{n+1}}}$$

$$\Big\downarrow \text{ 127}$$

$$\frac{x^{n/2}\sqrt{1-\frac{bx}{a}}\sqrt{\frac{dx}{c}+1}\int \frac{1}{\sqrt{x}\sqrt{1-\frac{bx}{a}}\sqrt{\frac{dx}{c}+1}}dx}{\sqrt{c+dx}\sqrt{ax^n-bx^{n+1}}}$$

$$\Big\downarrow \text{ 126}$$

$$\frac{2\sqrt{a}x^{n/2}\sqrt{1-\frac{bx}{a}}\sqrt{\frac{dx}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{c+dx}\sqrt{ax^n-bx^{n+1}}}$$

input
```
Int[x^((-1 + n)/2)/(Sqrt[c + d*x]*Sqrt[a*x^n - b*x^(1 + n)]),x]
```

output
```
(2*Sqrt[a]*x^(n/2)*Sqrt[1 - (b*x)/a]*Sqrt[1 + (d*x)/c]*EllipticF[ArcSin[(S
qrt[b]*Sqrt[x])/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[c + d*x]*Sqrt[a*x
^n - b*x^(1 + n)])
```

Defintions of rubi rules used

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [F]

$$\int \frac{x^{-\frac{1}{2}+\frac{n}{2}}}{\sqrt{dx+c}\,\sqrt{a\,x^{n}-b\,x^{1+n}}}dx$$

input
```
int(x^(-1/2+1/2*n)/(d*x+c)^(1/2)/(a*x^n-b*x^(1+n))^(1/2),x)
```

output
```
int(x^(-1/2+1/2*n)/(d*x+c)^(1/2)/(a*x^n-b*x^(1+n))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.10

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c+dx}\sqrt{ax^n-bx^{1+n}}}\,dx = -\frac{2\sqrt{-bd}\,\text{weierstrassPInverse}\left(\frac{4\,(b^2c^2+abcd+a^2d^2)}{3\,b^2d^2}, -\frac{4\,(2\,b^3c^3+3\,ab^2c^2d-3\,a^2bcd^2-2\,a^3d^3)}{27\,b^3d^3}, \frac{3\,bdx+bc-ad}{3\,bd}\right)}{bd}$$

input
```
integrate(x^(-1/2+1/2*n)/(d*x+c)^(1/2)/(a*x^n-b*x^(1+n))^(1/2),x, algorith
m="fricas")
```

output
```
-2*sqrt(-b*d)*weierstrassPInverse(4/3*(b^2*c^2 + a*b*c*d + a^2*d^2)/(b^2*d
^2), -4/27*(2*b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 2*a^3*d^3)/(b^3*d^
3), 1/3*(3*b*d*x + b*c - a*d)/(b*d))/(b*d)
```

Sympy [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c+dx}\sqrt{ax^n-bx^{1+n}}}\,dx = \int \frac{x^{\frac{n}{2}-\frac{1}{2}}}{\sqrt{c+dx}\sqrt{ax^n-bx^{n+1}}}\,dx$$

input
```
integrate(x**(-1/2+1/2*n)/(d*x+c)**(1/2)/(a*x**n-b*x**(1+n))**(1/2),x)
```

output
```
Integral(x**(n/2 - 1/2)/(sqrt(c + d*x)*sqrt(a*x**n - b*x**(n + 1))), x)
```

Maxima [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c+dx}\sqrt{ax^n-bx^{1+n}}}\,dx = \int \frac{x^{\frac{1}{2}\,n-\frac{1}{2}}}{\sqrt{-bx^{n+1}+ax^n}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^(-1/2+1/2*n)/(d*x+c)^(1/2)/(a*x^n-b*x^(1+n))^(1/2),x, algorith
m="maxima")
```

output
```
integrate(x^(1/2*n - 1/2)/(sqrt(-b*x^(n + 1) + a*x^n)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c+dx}\sqrt{ax^n-bx^{1+n}}}\,dx=\int \frac{x^{\frac{1}{2}\,n-\frac{1}{2}}}{\sqrt{-bx^{n+1}+ax^n}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^(-1/2+1/2*n)/(d*x+c)^(1/2)/(a*x^n-b*x^(1+n))^(1/2),x, algorith
m="giac")
```

output
```
integrate(x^(1/2*n - 1/2)/(sqrt(-b*x^(n + 1) + a*x^n)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c+dx}\sqrt{ax^n-bx^{1+n}}}\,dx=\int \frac{x^{\frac{n}{2}-\frac{1}{2}}}{\sqrt{a\,x^n-b\,x^{n+1}}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(x^(n/2 - 1/2)/((a*x^n - b*x^(n + 1))^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int(x^(n/2 - 1/2)/((a*x^n - b*x^(n + 1))^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c+dx}\sqrt{ax^n-bx^{1+n}}}\,dx=\int \frac{\sqrt{x}\,\sqrt{dx+c}\,\sqrt{-bx+a}}{-bd\,x^3+ad\,x^2-bc\,x^2+acx}dx$$

input
```
int(x^(-1/2+1/2*n)/(d*x+c)^(1/2)/(a*x^n-b*x^(1+n))^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(c + d*x)*sqrt(a - b*x))/(a*c*x + a*d*x**2 - b*c*x**2 - b
*d*x**3),x)
```

3.265 $\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c-dx}\sqrt{ax^n-bx^{1+n}}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 100

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c-dx}\sqrt{ax^n-bx^{1+n}}}\,dx=\frac{2\sqrt{a}x^{n/2}\sqrt{1-\frac{bx}{a}}\sqrt{1-\frac{dx}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{c-dx}\sqrt{ax^n-bx^{1+n}}}$$

output
```
2*a^(1/2)*x^(1/2*n)*(1-b*x/a)^(1/2)*(1-d*x/c)^(1/2)*EllipticF(b^(1/2)*x^(1
/2)/a^(1/2),(a*d/b/c)^(1/2))/b^(1/2)/(-d*x+c)^(1/2)/(a*x^n-b*x^(1+n))^(1/2
)
```

Mathematica [A] (verified)

Time = 4.00 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.22

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c-dx}\sqrt{ax^n-bx^{1+n}}}\,dx$$
$$=\frac{2x^{\frac{1}{2}(-1+n)}(a-bx)^{3/2}\sqrt{\frac{bx}{-a+bx}}\sqrt{\frac{b(c-dx)}{d(a-bx)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a}}{\sqrt{a-bx}}\right),1-\frac{bc}{ad}\right)}{\sqrt{a}b\sqrt{x^n(a-bx)}\sqrt{c-dx}}$$

input
```
Integrate[x^((-1 + n)/2)/(Sqrt[c - d*x]*Sqrt[a*x^n - b*x^(1 + n)]),x]
```

output
```
(2*x^((-1 + n)/2)*(a - b*x)^(3/2)*Sqrt[(b*x)/(-a + b*x)]*Sqrt[(b*(c - d*x)
)/(d*(a - b*x))]*EllipticF[ArcSin[Sqrt[a]/Sqrt[a - b*x]], 1 - (b*c)/(a*d)]
)/(Sqrt[a]*b*Sqrt[x^n*(a - b*x)]*Sqrt[c - d*x])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {1948, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n-1}{2}}}{\sqrt{c-dx}\sqrt{ax^n-bx^{n+1}}}dx$$

$$\Big\downarrow \text{1948}$$

$$\frac{x^{n/2}\sqrt{a-bx}\int \frac{1}{\sqrt{x}\sqrt{a-bx}\sqrt{c-dx}}dx}{\sqrt{ax^n-bx^{n+1}}}$$

$$\Big\downarrow \text{127}$$

$$\frac{x^{n/2}\sqrt{1-\frac{bx}{a}}\sqrt{1-\frac{dx}{c}}\int \frac{1}{\sqrt{x}\sqrt{1-\frac{bx}{a}}\sqrt{1-\frac{dx}{c}}}dx}{\sqrt{c-dx}\sqrt{ax^n-bx^{n+1}}}$$

$$\Big\downarrow \text{126}$$

$$\frac{2\sqrt{a}x^{n/2}\sqrt{1-\frac{bx}{a}}\sqrt{1-\frac{dx}{c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{c-dx}\sqrt{ax^n-bx^{n+1}}}$$

input
```
Int[x^((-1 + n)/2)/(Sqrt[c - d*x]*Sqrt[a*x^n - b*x^(1 + n)]),x]
```

output
```
(2*Sqrt[a]*x^(n/2)*Sqrt[1 - (b*x)/a]*Sqrt[1 - (d*x)/c]*EllipticF[ArcSin[(S
qrt[b]*Sqrt[x])/Sqrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[c - d*x]*Sqrt[a*x^n
- b*x^(1 + n)])
```

Defintions of rubi rules used

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [F]

$$\int \frac{x^{-\frac{1}{2}+\frac{n}{2}}}{\sqrt{-dx+c}\,\sqrt{a\,x^{n}-b\,x^{1+n}}}dx$$

input
```
int(x^(-1/2+1/2*n)/(-d*x+c)^(1/2)/(a*x^n-b*x^(1+n))^(1/2),x)
```

output
```
int(x^(-1/2+1/2*n)/(-d*x+c)^(1/2)/(a*x^n-b*x^(1+n))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.10

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c-dx}\sqrt{ax^n-bx^{1+n}}}\,dx = \frac{2\sqrt{bd}\text{weierstrassPInverse}\left(\frac{4\left(b^2c^2-abcd+a^2d^2\right)}{3\,b^2d^2},\frac{4\left(2\,b^3c^3-3\,ab^2c^2d-3\,a^2bcd^2+2\,a^3d^3\right)}{27\,b^3d^3},\frac{3\,bdx-bc-ad}{3\,bd}\right)}{bd}$$

input
```
integrate(x^(-1/2+1/2*n)/(-d*x+c)^(1/2)/(a*x^n-b*x^(1+n))^(1/2),x, algorit
hm="fricas")
```

output
```
2*sqrt(b*d)*weierstrassPInverse(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2
), 4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3),
 1/3*(3*b*d*x - b*c - a*d)/(b*d))/(b*d)
```

Sympy [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c-dx}\sqrt{ax^n-bx^{1+n}}}\,dx = \int \frac{x^{\frac{n}{2}-\frac{1}{2}}}{\sqrt{c-dx}\sqrt{ax^n-bx^{n+1}}}\,dx$$

input
```
integrate(x**(-1/2+1/2*n)/(-d*x+c)**(1/2)/(a*x**n-b*x**(1+n))**(1/2),x)
```

output
```
Integral(x**(n/2 - 1/2)/(sqrt(c - d*x)*sqrt(a*x**n - b*x**(n + 1))), x)
```

Maxima [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c-dx}\sqrt{ax^n-bx^{1+n}}}\,dx = \int \frac{x^{\frac{1}{2}\,n-\frac{1}{2}}}{\sqrt{-bx^{n+1}+ax^n}\sqrt{-dx+c}}\,dx$$

input
```
integrate(x^(-1/2+1/2*n)/(-d*x+c)^(1/2)/(a*x^n-b*x^(1+n))^(1/2),x, algorit
hm="maxima")
```

output
```
integrate(x^(1/2*n - 1/2)/(sqrt(-b*x^(n + 1) + a*x^n)*sqrt(-d*x + c)), x)
```

Giac [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c-dx}\sqrt{ax^n-bx^{1+n}}}\,dx=\int \frac{x^{\frac{1}{2}\,n-\frac{1}{2}}}{\sqrt{-bx^{n+1}+ax^n}\sqrt{-dx+c}}\,dx$$

input
```
integrate(x^(-1/2+1/2*n)/(-d*x+c)^(1/2)/(a*x^n-b*x^(1+n))^(1/2),x, algorit
hm="giac")
```

output
```
integrate(x^(1/2*n - 1/2)/(sqrt(-b*x^(n + 1) + a*x^n)*sqrt(-d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c-dx}\sqrt{ax^n-bx^{1+n}}}\,dx=\int \frac{x^{\frac{n}{2}-\frac{1}{2}}}{\sqrt{a\,x^n-b\,x^{n+1}}\,\sqrt{c-d\,x}}\,dx$$

input
```
int(x^(n/2 - 1/2)/((a*x^n - b*x^(n + 1))^(1/2)*(c - d*x)^(1/2)),x)
```

output
```
int(x^(n/2 - 1/2)/((a*x^n - b*x^(n + 1))^(1/2)*(c - d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{c-dx}\sqrt{ax^n-bx^{1+n}}}\,dx=\int \frac{\sqrt{x}\,\sqrt{-dx+c}\,\sqrt{-bx+a}}{bd\,x^3-ad\,x^2-bc\,x^2+acx}dx$$

input
```
int(x^(-1/2+1/2*n)/(-d*x+c)^(1/2)/(a*x^n-b*x^(1+n))^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(c - d*x)*sqrt(a - b*x))/(a*c*x - a*d*x**2 - b*c*x**2 + b
*d*x**3),x)
```

3.266 $\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1+5x}\sqrt{2x^n+3x^{1+n}}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 58

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1+5x}\sqrt{2x^n+3x^{1+n}}}\,dx = \frac{2x^{n/2}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{3}{2}}\sqrt{x}\right),-\frac{7}{3}\right)}{\sqrt{3}\sqrt{2x^n+3x^{1+n}}}$$

output
```
2/3*x^(1/2*n)*(2+3*x)^(1/2)*InverseJacobiAM(arctan(1/2*6^(1/2)*x^(1/2)),1/
3*I*21^(1/2))*3^(1/2)/(2*x^n+3*x^(1+n))^(1/2)
```

Mathematica [A] (verified)

Time = 2.82 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.55

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1+5x}\sqrt{2x^n+3x^{1+n}}}\,dx = -\frac{x^{\frac{1+n}{2}}\sqrt{\frac{2}{5}+2x}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{\sqrt{1+\frac{3x}{2}}}\right),\frac{7}{10}\right)}{\sqrt{\frac{x}{2+3x}}\sqrt{2+3x}\sqrt{x^n(2+3x)}\sqrt{\frac{1+5x}{2+3x}}}$$

input
```
Integrate[x^((-1 + n)/2)/(Sqrt[1 + 5*x]*Sqrt[2*x^n + 3*x^(1 + n)]),x]
```

output
```
-((x^((1 + n)/2)*Sqrt[2/5 + 2*x]*EllipticF[ArcSin[1/Sqrt[1 + (3*x)/2]], 7/
10])/(Sqrt[x/(2 + 3*x)]*Sqrt[2 + 3*x]*Sqrt[x^n*(2 + 3*x)]*Sqrt[(1 + 5*x)/(
2 + 3*x)]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{1948, 125\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n-1}{2}}}{\sqrt{5x+1}\sqrt{3x^{n+1}+2x^n}}dx$$

$$\Big\downarrow \text{ 1948}$$

$$\frac{\sqrt{3x+2}x^{n/2}\int \frac{1}{\sqrt{x}\sqrt{3x+2}\sqrt{5x+1}}dx}{\sqrt{3x^{n+1}+2x^n}}$$

$$\Big\downarrow \text{ 125}$$

$$-\frac{2i\sqrt{3x+2}x^{n/2}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}\sqrt{x}\right),\frac{10}{3}\right)}{\sqrt{3}\sqrt{3x^{n+1}+2x^n}}$$

input
```
Int[x^((-1 + n)/2)/(Sqrt[1 + 5*x]*Sqrt[2*x^n + 3*x^(1 + n)]),x]
```

output
```
((-2*I)*x^(n/2)*Sqrt[2 + 3*x]*EllipticF[I*ArcSinh[Sqrt[3/2]*Sqrt[x]], 10/3
])/(Sqrt[3]*Sqrt[2*x^n + 3*x^(1 + n)])
```

Defintions of rubi rules used

rule 125
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (GtQ[-b/d, 0] || LtQ[-b/f, 0])
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [F]

$$\int \frac{x^{-\frac{1}{2}+\frac{n}{2}}}{\sqrt{1+5x}\,\sqrt{2x^n+3x^{1+n}}}dx$$

input
```
int(x^(-1/2+1/2*n)/(1+5*x)^(1/2)/(2*x^n+3*x^(1+n))^(1/2),x)
```

output
```
int(x^(-1/2+1/2*n)/(1+5*x)^(1/2)/(2*x^n+3*x^(1+n))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.19

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1+5x}\sqrt{2x^n+3x^{1+n}}}\,dx = \frac{2}{15}\sqrt{15}\mathrm{weierstrassPInverse}\left(\frac{316}{675},-\frac{3536}{91125},x+\frac{13}{45}\right)$$

input
```
integrate(x^(-1/2+1/2*n)/(1+5*x)^(1/2)/(2*x^n+3*x^(1+n))^(1/2),x, algorith
m="fricas")
```

output
```
2/15*sqrt(15)*weierstrassPInverse(316/675, -3536/91125, x + 13/45)
```

Sympy [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1+5x}\sqrt{2x^n+3x^{1+n}}}\,dx = \int \frac{x^{\frac{n}{2}-\frac{1}{2}}}{\sqrt{5x+1}\sqrt{2x^n+3x^{n+1}}}\,dx$$

input
```
integrate(x**(-1/2+1/2*n)/(1+5*x)**(1/2)/(2*x**n+3*x**(1+n))**(1/2),x)
```

output
```
Integral(x**(n/2 - 1/2)/(sqrt(5*x + 1)*sqrt(2*x**n + 3*x**(n + 1))), x)
```

Maxima [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1+5x}\sqrt{2x^n+3x^{1+n}}}\,dx = \int \frac{x^{\frac{1}{2}\,n-\frac{1}{2}}}{\sqrt{5\,x+1}\sqrt{3\,x^{n+1}+2\,x^n}}\,dx$$

input
```
integrate(x^(-1/2+1/2*n)/(1+5*x)^(1/2)/(2*x^n+3*x^(1+n))^(1/2),x, algorith
m="maxima")
```

output
```
integrate(x^(1/2*n - 1/2)/(sqrt(5*x + 1)*sqrt(3*x^(n + 1) + 2*x^n)), x)
```

Giac [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1+5x}\sqrt{2x^n+3x^{1+n}}}\,dx = \int \frac{x^{\frac{1}{2}\,n-\frac{1}{2}}}{\sqrt{5\,x+1}\sqrt{3\,x^{n+1}+2\,x^n}}\,dx$$

input
```
integrate(x^(-1/2+1/2*n)/(1+5*x)^(1/2)/(2*x^n+3*x^(1+n))^(1/2),x, algorith
m="giac")
```

output
```
integrate(x^(1/2*n - 1/2)/(sqrt(5*x + 1)*sqrt(3*x^(n + 1) + 2*x^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1+5x}\sqrt{2x^n+3x^{1+n}}}\,dx=\int \frac{x^{\frac{n}{2}-\frac{1}{2}}}{\sqrt{2\,x^n+3\,x^{n+1}}\,\sqrt{5\,x+1}}\,dx$$

input
```
int(x^(n/2 - 1/2)/((2*x^n + 3*x^(n + 1))^(1/2)*(5*x + 1)^(1/2)),x)
```

output
```
int(x^(n/2 - 1/2)/((2*x^n + 3*x^(n + 1))^(1/2)*(5*x + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1+5x}\sqrt{2x^n+3x^{1+n}}}\,dx=\int \frac{\sqrt{x}\,\sqrt{3x+2}\,\sqrt{5x+1}}{15x^3+13x^2+2x}dx$$

input
```
int(x^(-1/2+1/2*n)/(1+5*x)^(1/2)/(2*x^n+3*x^(1+n))^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(3*x + 2)*sqrt(5*x + 1))/(15*x**3 + 13*x**2 + 2*x),x)
```

3.267 $\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1-5x}\sqrt{2x^n+3x^{1+n}}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 57

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1-5x}\sqrt{2x^n+3x^{1+n}}}\,dx = \frac{\sqrt{\frac{2}{5}}x^{n/2}\sqrt{2+3x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{5}\sqrt{x}\right),-\frac{3}{10}\right)}{\sqrt{2x^n+3x^{1+n}}}$$

output
```
1/5*10^(1/2)*x^(1/2*n)*(2+3*x)^(1/2)*EllipticF(x^(1/2)*5^(1/2),1/10*I*30^(
1/2))/(2*x^n+3*x^(1+n))^(1/2)
```

Mathematica [A] (verified)

Time = 2.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.28

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1-5x}\sqrt{2x^n+3x^{1+n}}}\,dx = -\frac{2x^{\frac{1+n}{2}}\sqrt{\frac{2+3x}{-1+5x}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{\sqrt{1-5x}}\right),\frac{13}{3}\right)}{\sqrt{3}\sqrt{x^n(2+3x)}\sqrt{\frac{x}{-1+5x}}}$$

input
```
Integrate[x^((-1 + n)/2)/(Sqrt[1 - 5*x]*Sqrt[2*x^n + 3*x^(1 + n)]),x]
```

output
```
(-2*x^((1 + n)/2)*Sqrt[(2 + 3*x)/(-1 + 5*x)]*EllipticF[ArcSin[1/Sqrt[1 - 5
*x]], 13/3])/(Sqrt[3]*Sqrt[x^n*(2 + 3*x)]*Sqrt[x/(-1 + 5*x)])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1948, 125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n-1}{2}}}{\sqrt{1-5x}\sqrt{3x^{n+1}+2x^n}}dx$$

$$\Big\downarrow \text{ 1948}$$

$$\frac{\sqrt{3x+2}x^{n/2}\int \frac{1}{\sqrt{1-5x}\sqrt{x}\sqrt{3x+2}}dx}{\sqrt{3x^{n+1}+2x^n}}$$

$$\Big\downarrow \text{ 125}$$

$$\frac{\sqrt{\frac{2}{5}}\sqrt{3x+2}x^{n/2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{5}\sqrt{x}\right),-\frac{3}{10}\right)}{\sqrt{3x^{n+1}+2x^n}}$$

input
```
Int[x^((-1 + n)/2)/(Sqrt[1 - 5*x]*Sqrt[2*x^n + 3*x^(1 + n)]),x]
```

output
```
(Sqrt[2/5]*x^(n/2)*Sqrt[2 + 3*x]*EllipticF[ArcSin[Sqrt[5]*Sqrt[x]], -3/10]
)/Sqrt[2*x^n + 3*x^(1 + n)]
```

Defintions of rubi rules used

rule 125
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (GtQ[-b/d, 0] || LtQ[-b/f, 0])
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [F]

$$\int \frac{x^{-\frac{1}{2}+\frac{n}{2}}}{\sqrt{1-5x}\,\sqrt{2x^n+3x^{1+n}}}dx$$

input
```
int(x^(-1/2+1/2*n)/(1-5*x)^(1/2)/(2*x^n+3*x^(1+n))^(1/2),x)
```

output
```
int(x^(-1/2+1/2*n)/(1-5*x)^(1/2)/(2*x^n+3*x^(1+n))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.19

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1-5x}\sqrt{2x^n+3x^{1+n}}}\,dx = -\frac{2}{15}\,\sqrt{-15}\text{weierstrassPInverse}\left(\frac{556}{675},-\frac{10304}{91125},x+\frac{7}{45}\right)$$

input
```
integrate(x^(-1/2+1/2*n)/(1-5*x)^(1/2)/(2*x^n+3*x^(1+n))^(1/2),x, algorith
m="fricas")
```

output
```
-2/15*sqrt(-15)*weierstrassPInverse(556/675, -10304/91125, x + 7/45)
```

Sympy [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1-5x}\sqrt{2x^n+3x^{1+n}}}\,dx = \int \frac{x^{\frac{n}{2}-\frac{1}{2}}}{\sqrt{1-5x}\sqrt{2x^n+3x^{n+1}}}\,dx$$

input
```
integrate(x**(-1/2+1/2*n)/(1-5*x)**(1/2)/(2*x**n+3*x**(1+n))**(1/2),x)
```

output
```
Integral(x**(n/2 - 1/2)/(sqrt(1 - 5*x)*sqrt(2*x**n + 3*x**(n + 1))), x)
```

Maxima [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1-5x}\sqrt{2x^n+3x^{1+n}}}\,dx = \int \frac{x^{\frac{1}{2}\,n-\frac{1}{2}}}{\sqrt{3\,x^{n+1}+2\,x^n}\sqrt{-5\,x+1}}\,dx$$

input
```
integrate(x^(-1/2+1/2*n)/(1-5*x)^(1/2)/(2*x^n+3*x^(1+n))^(1/2),x, algorith
m="maxima")
```

output
```
integrate(x^(1/2*n - 1/2)/(sqrt(3*x^(n + 1) + 2*x^n)*sqrt(-5*x + 1)), x)
```

Giac [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1-5x}\sqrt{2x^n+3x^{1+n}}}\,dx = \int \frac{x^{\frac{1}{2}\,n-\frac{1}{2}}}{\sqrt{3\,x^{n+1}+2\,x^n}\sqrt{-5\,x+1}}\,dx$$

input
```
integrate(x^(-1/2+1/2*n)/(1-5*x)^(1/2)/(2*x^n+3*x^(1+n))^(1/2),x, algorith
m="giac")
```

output
```
integrate(x^(1/2*n - 1/2)/(sqrt(3*x^(n + 1) + 2*x^n)*sqrt(-5*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1-5x}\sqrt{2x^n+3x^{1+n}}}\,dx=\int \frac{x^{\frac{n}{2}-\frac{1}{2}}}{\sqrt{2\,x^n+3\,x^{n+1}}\,\sqrt{1-5\,x}}\,dx$$

input
```
int(x^(n/2 - 1/2)/((2*x^n + 3*x^(n + 1))^(1/2)*(1 - 5*x)^(1/2)),x)
```

output
```
int(x^(n/2 - 1/2)/((2*x^n + 3*x^(n + 1))^(1/2)*(1 - 5*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1-5x}\sqrt{2x^n+3x^{1+n}}}\,dx=-\left(\int \frac{\sqrt{x}\,\sqrt{3x+2}\,\sqrt{-5x+1}}{15x^3+7x^2-2x}dx\right)$$

input
```
int(x^(-1/2+1/2*n)/(1-5*x)^(1/2)/(2*x^n+3*x^(1+n))^(1/2),x)
```

output
```
 - int((sqrt(x)*sqrt(3*x + 2)*sqrt( - 5*x + 1))/(15*x**3 + 7*x**2 - 2*x),x
)
```

3.268 $\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1+5x}\sqrt{2x^n-3x^{1+n}}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 58

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1+5x}\sqrt{2x^n-3x^{1+n}}}\,dx = \frac{2\sqrt{2-3x}x^{n/2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}\sqrt{x}\right),-\frac{10}{3}\right)}{\sqrt{3}\sqrt{2x^n-3x^{1+n}}}$$

output
```
2/3*(2-3*x)^(1/2)*x^(1/2*n)*EllipticF(1/2*6^(1/2)*x^(1/2),1/3*I*30^(1/2))*
3^(1/2)/(2*x^n-3*x^(1+n))^(1/2)
```

Mathematica [A] (verified)

Time = 2.41 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.62

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1+5x}\sqrt{2x^n-3x^{1+n}}}\,dx = \frac{\sqrt{\frac{2}{5}}(2-3x)^{3/2}x^{\frac{1}{2}(-1+n)}\sqrt{\frac{x}{-2+3x}}\sqrt{\frac{1+5x}{-2+3x}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{\sqrt{1-\frac{3x}{2}}}\right),\frac{13}{10}\right)}{\sqrt{(2-3x)x^n}\sqrt{1+5x}}$$

input
```
Integrate[x^((-1 + n)/2)/(Sqrt[1 + 5*x]*Sqrt[2*x^n - 3*x^(1 + n)]),x]
```

output
```
(Sqrt[2/5]*(2 - 3*x)^(3/2)*x^((-1 + n)/2)*Sqrt[x/(-2 + 3*x)]*Sqrt[(1 + 5*x
)/(-2 + 3*x)]*EllipticF[ArcSin[1/Sqrt[1 - (3*x)/2]], 13/10])/(Sqrt[(2 - 3*
x)*x^n]*Sqrt[1 + 5*x])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1948, 125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n-1}{2}}}{\sqrt{5x+1}\sqrt{2x^n-3x^{n+1}}}dx$$

$$\Big\downarrow \text{ 1948}$$

$$\frac{\sqrt{2-3x}x^{n/2}\int \frac{1}{\sqrt{2-3x}\sqrt{x}\sqrt{5x+1}}dx}{\sqrt{2x^n-3x^{n+1}}}$$

$$\Big\downarrow \text{ 125}$$

$$\frac{2\sqrt{2-3x}x^{n/2}\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}\sqrt{x}\right),-\frac{10}{3}\right)}{\sqrt{3}\sqrt{2x^n-3x^{n+1}}}$$

input
```
Int[x^((-1 + n)/2)/(Sqrt[1 + 5*x]*Sqrt[2*x^n - 3*x^(1 + n)]),x]
```

output
```
(2*Sqrt[2 - 3*x]*x^(n/2)*EllipticF[ArcSin[Sqrt[3/2]*Sqrt[x]], -10/3])/(Sqr
t[3]*Sqrt[2*x^n - 3*x^(1 + n)])
```

Defintions of rubi rules used

rule 125
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (GtQ[-b/d, 0] || LtQ[-b/f, 0])
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [F]

$$\int \frac{x^{-\frac{1}{2}+\frac{n}{2}}}{\sqrt{1+5x}\,\sqrt{2x^n-3x^{1+n}}}dx$$

input
```
int(x^(-1/2+1/2*n)/(1+5*x)^(1/2)/(2*x^n-3*x^(1+n))^(1/2),x)
```

output
```
int(x^(-1/2+1/2*n)/(1+5*x)^(1/2)/(2*x^n-3*x^(1+n))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.19

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1+5x}\sqrt{2x^n-3x^{1+n}}}\,dx = -\frac{2}{15}\sqrt{-15}\text{weierstrassPInverse}\left(\frac{556}{675},\frac{10304}{91125},x-\frac{7}{45}\right)$$

input
```
integrate(x^(-1/2+1/2*n)/(1+5*x)^(1/2)/(2*x^n-3*x^(1+n))^(1/2),x, algorith
m="fricas")
```

output
```
-2/15*sqrt(-15)*weierstrassPInverse(556/675, 10304/91125, x - 7/45)
```

Sympy [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1+5x}\sqrt{2x^n-3x^{1+n}}}\,dx = \int \frac{x^{\frac{n}{2}-\frac{1}{2}}}{\sqrt{5x+1}\sqrt{2x^n-3x^{n+1}}}\,dx$$

input
```
integrate(x**(-1/2+1/2*n)/(1+5*x)**(1/2)/(2*x**n-3*x**(1+n))**(1/2),x)
```

output
```
Integral(x**(n/2 - 1/2)/(sqrt(5*x + 1)*sqrt(2*x**n - 3*x**(n + 1))), x)
```

Maxima [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1+5x}\sqrt{2x^n-3x^{1+n}}}\,dx = \int \frac{x^{\frac{1}{2}\,n-\frac{1}{2}}}{\sqrt{5\,x+1}\sqrt{-3\,x^{n+1}+2\,x^n}}\,dx$$

input
```
integrate(x^(-1/2+1/2*n)/(1+5*x)^(1/2)/(2*x^n-3*x^(1+n))^(1/2),x, algorith
m="maxima")
```

output
```
integrate(x^(1/2*n - 1/2)/(sqrt(5*x + 1)*sqrt(-3*x^(n + 1) + 2*x^n)), x)
```

Giac [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1+5x}\sqrt{2x^n-3x^{1+n}}}\,dx = \int \frac{x^{\frac{1}{2}\,n-\frac{1}{2}}}{\sqrt{5\,x+1}\sqrt{-3\,x^{n+1}+2\,x^n}}\,dx$$

input
```
integrate(x^(-1/2+1/2*n)/(1+5*x)^(1/2)/(2*x^n-3*x^(1+n))^(1/2),x, algorith
m="giac")
```

output
```
integrate(x^(1/2*n - 1/2)/(sqrt(5*x + 1)*sqrt(-3*x^(n + 1) + 2*x^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1+5x}\sqrt{2x^n-3x^{1+n}}}\,dx=\int \frac{x^{\frac{n}{2}-\frac{1}{2}}}{\sqrt{2\,x^n-3\,x^{n+1}}\,\sqrt{5\,x+1}}\,dx$$

input
```
int(x^(n/2 - 1/2)/((2*x^n - 3*x^(n + 1))^(1/2)*(5*x + 1)^(1/2)),x)
```

output
```
int(x^(n/2 - 1/2)/((2*x^n - 3*x^(n + 1))^(1/2)*(5*x + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1+5x}\sqrt{2x^n-3x^{1+n}}}\,dx=-\left(\int \frac{\sqrt{x}\,\sqrt{5x+1}\,\sqrt{-3x+2}}{15x^3-7x^2-2x}dx\right)$$

input
```
int(x^(-1/2+1/2*n)/(1+5*x)^(1/2)/(2*x^n-3*x^(1+n))^(1/2),x)
```

output
```
- int((sqrt(x)*sqrt(5*x + 1)*sqrt( - 3*x + 2))/(15*x**3 - 7*x**2 - 2*x),x
)
```

3.269 $\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1-5x}\sqrt{2x^n-3x^{1+n}}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 57

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1-5x}\sqrt{2x^n-3x^{1+n}}}\,dx=\frac{\sqrt{\frac{2}{5}}\sqrt{2-3x}x^{n/2}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{5}\sqrt{x}\right),\frac{3}{10}\right)}{\sqrt{2x^n-3x^{1+n}}}$$

output
```
1/5*10^(1/2)*(2-3*x)^(1/2)*x^(1/2*n)*EllipticF(x^(1/2)*5^(1/2),1/10*30^(1/
2))/(2*x^n-3*x^(1+n))^(1/2)
```

Mathematica [A] (verified)

Time = 2.46 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1-5x}\sqrt{2x^n-3x^{1+n}}}\,dx=-\frac{2\sqrt{\frac{2-3x}{3-15x}}x^{\frac{1+n}{2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{\sqrt{1-5x}}\right),-\frac{7}{3}\right)}{\sqrt{(2-3x)x^n}\sqrt{\frac{x}{-1+5x}}}$$

input
```
Integrate[x^((-1 + n)/2)/(Sqrt[1 - 5*x]*Sqrt[2*x^n - 3*x^(1 + n)]),x]
```

output
```
(-2*Sqrt[(2 - 3*x)/(3 - 15*x)]*x^((1 + n)/2)*EllipticF[ArcSin[1/Sqrt[1 - 5
*x]], -7/3])/(Sqrt[(2 - 3*x)*x^n]*Sqrt[x/(-1 + 5*x)])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1948, 125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n-1}{2}}}{\sqrt{1-5x}\sqrt{2x^n-3x^{n+1}}} \, dx$$

$$\Big\downarrow \text{1948}$$

$$\frac{\sqrt{2-3x}x^{n/2}\int \frac{1}{\sqrt{1-5x}\sqrt{2-3x}\sqrt{x}}dx}{\sqrt{2x^n-3x^{n+1}}}$$

$$\Big\downarrow \text{125}$$

$$\frac{\sqrt{\frac{2}{5}}\sqrt{2-3x}x^{n/2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{5}\sqrt{x}\right),\frac{3}{10}\right)}{\sqrt{2x^n-3x^{n+1}}}$$

input
```
Int[x^((-1 + n)/2)/(Sqrt[1 - 5*x]*Sqrt[2*x^n - 3*x^(1 + n)]),x]
```

output
```
(Sqrt[2/5]*Sqrt[2 - 3*x]*x^(n/2)*EllipticF[ArcSin[Sqrt[5]*Sqrt[x]], 3/10])
/Sqrt[2*x^n - 3*x^(1 + n)]
```

Defintions of rubi rules used

rule 125
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (GtQ[-b/d, 0] || LtQ[-b/f, 0])
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [F]

$$\int \frac{x^{-\frac{1}{2}+\frac{n}{2}}}{\sqrt{1-5x}\,\sqrt{2x^n-3x^{1+n}}}dx$$

input
```
int(x^(-1/2+1/2*n)/(1-5*x)^(1/2)/(2*x^n-3*x^(1+n))^(1/2),x)
```

output
```
int(x^(-1/2+1/2*n)/(1-5*x)^(1/2)/(2*x^n-3*x^(1+n))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.19

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1-5x}\sqrt{2x^n-3x^{1+n}}}\,dx = \frac{2}{15}\,\sqrt{15}\text{weierstrassPInverse}\left(\frac{316}{675},\frac{3536}{91125},x-\frac{13}{45}\right)$$

input
```
integrate(x^(-1/2+1/2*n)/(1-5*x)^(1/2)/(2*x^n-3*x^(1+n))^(1/2),x, algorith
m="fricas")
```

output
```
2/15*sqrt(15)*weierstrassPInverse(316/675, 3536/91125, x - 13/45)
```

Sympy [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1-5x}\sqrt{2x^n-3x^{1+n}}}\,dx=\int \frac{x^{\frac{n}{2}-\frac{1}{2}}}{\sqrt{1-5x}\sqrt{2x^n-3x^{n+1}}}\,dx$$

input
```
integrate(x**(-1/2+1/2*n)/(1-5*x)**(1/2)/(2*x**n-3*x**(1+n))**(1/2),x)
```

output
```
Integral(x**(n/2 - 1/2)/(sqrt(1 - 5*x)*sqrt(2*x**n - 3*x**(n + 1))), x)
```

Maxima [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1-5x}\sqrt{2x^n-3x^{1+n}}}\,dx=\int \frac{x^{\frac{1}{2}\,n-\frac{1}{2}}}{\sqrt{-3\,x^{n+1}+2\,x^n}\sqrt{-5\,x+1}}\,dx$$

input
```
integrate(x^(-1/2+1/2*n)/(1-5*x)^(1/2)/(2*x^n-3*x^(1+n))^(1/2),x, algorith
m="maxima")
```

output
```
integrate(x^(1/2*n - 1/2)/(sqrt(-3*x^(n + 1) + 2*x^n)*sqrt(-5*x + 1)), x)
```

Giac [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1-5x}\sqrt{2x^n-3x^{1+n}}}\,dx=\int \frac{x^{\frac{1}{2}\,n-\frac{1}{2}}}{\sqrt{-3\,x^{n+1}+2\,x^n}\sqrt{-5\,x+1}}\,dx$$

input
```
integrate(x^(-1/2+1/2*n)/(1-5*x)^(1/2)/(2*x^n-3*x^(1+n))^(1/2),x, algorith
m="giac")
```

output
```
integrate(x^(1/2*n - 1/2)/(sqrt(-3*x^(n + 1) + 2*x^n)*sqrt(-5*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1-5x}\sqrt{2x^n-3x^{1+n}}}\,dx=\int \frac{x^{\frac{n}{2}-\frac{1}{2}}}{\sqrt{2\,x^n-3\,x^{n+1}}\,\sqrt{1-5\,x}}\,dx$$

input
```
int(x^(n/2 - 1/2)/((2*x^n - 3*x^(n + 1))^(1/2)*(1 - 5*x)^(1/2)),x)
```

output
```
int(x^(n/2 - 1/2)/((2*x^n - 3*x^(n + 1))^(1/2)*(1 - 5*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{\frac{1}{2}(-1+n)}}{\sqrt{1-5x}\sqrt{2x^n-3x^{1+n}}}\,dx=\int \frac{\sqrt{x}\,\sqrt{-3x+2}\,\sqrt{-5x+1}}{15x^3-13x^2+2x}dx$$

input
```
int(x^(-1/2+1/2*n)/(1-5*x)^(1/2)/(2*x^n-3*x^(1+n))^(1/2),x)
```

output
```
int((sqrt(x)*sqrt( - 3*x + 2)*sqrt( - 5*x + 1))/(15*x**3 - 13*x**2 + 2*x),
x)
```

3.270 $\int (ex)^m(c+dx)^q \left(ax^n+bx^{1+n}\right)^p \, dx$

Optimal result

Integrand size = 28, antiderivative size = 94

$$\int (ex)^m(c+dx)^q \left(ax^n+bx^{1+n}\right)^p \, dx = \frac{x(ex)^m \left(1+\frac{bx}{a}\right)^{-p} (c+dx)^q \left(1+\frac{dx}{c}\right)^{-q} \left(ax^n+bx^{1+n}\right)^p \text{AppellF1}\left(1+m+np,-p,-q,2+m+np,-\frac{bx}{a}\right.}{1+m+np}$$

output
```
x*(e*x)^m*(d*x+c)^q*(a*x^n+b*x^(1+n))^p*AppellF1(n*p+m+1,-p,-q,n*p+m+2,-b*
x/a,-d*x/c)/(n*p+m+1)/((1+b*x/a)^p)/((1+d*x/c)^q)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98

$$\int (ex)^m(c+dx)^q \left(ax^n+bx^{1+n}\right)^p \, dx = \frac{x(ex)^m \left(\frac{a+bx}{a}\right)^{-p} (x^n(a+bx))^p (c+dx)^q \left(\frac{c+dx}{c}\right)^{-q} \text{AppellF1}\left(1+m+np,-p,-q,2+m+np,-\frac{bx}{a},-\frac{dx}{c}\right.}{1+m+np}$$

input
```
Integrate[(e*x)^m*(c + d*x)^q*(a*x^n + b*x^(1 + n))^p,x]
```

output
```
(x*(e*x)^m*(x^n*(a + b*x))^p*(c + d*x)^q*AppellF1[1 + m + n*p, -p, -q, 2 +
 m + n*p, -((b*x)/a), -((d*x)/c)])/((1 + m + n*p)*((a + b*x)/a)^p*((c + d*
x)/c)^q)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1948, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m(c+dx)^q\left(ax^n+bx^{n+1}\right)^p\,dx$$

$$\Big\downarrow \text{1948}$$

$$(ex)^m(a+bx)^{-p}x^{-m-np}\left(ax^n+bx^{n+1}\right)^p\int x^{m+np}(a+bx)^p(c+dx)^qdx$$

$$\Big\downarrow \text{152}$$

$$(ex)^m\left(\frac{bx}{a}+1\right)^{-p}x^{-m-np}\left(ax^n+bx^{n+1}\right)^p\int x^{m+np}\left(\frac{bx}{a}+1\right)^p(c+dx)^qdx$$

$$\Big\downarrow \text{152}$$

$$(ex)^m\left(\frac{bx}{a}+1\right)^{-p}(c+dx)^q\left(\frac{dx}{c}+1\right)^{-q}x^{-m-np}\left(ax^n+bx^{n+1}\right)^p\int x^{m+np}\left(\frac{bx}{a}+1\right)^p\left(\frac{dx}{c}+1\right)^qdx$$

$$\Big\downarrow \text{150}$$

$$\frac{x(ex)^m\left(\frac{bx}{a}+1\right)^{-p}(c+dx)^q\left(\frac{dx}{c}+1\right)^{-q}\left(ax^n+bx^{n+1}\right)^p\operatorname{AppellF1}\left(m+np+1,-p,-q,m+np+2,-\frac{bx}{a},-\frac{dx}{c}\right)}{m+np+1}$$

input
```
Int[(e*x)^m*(c + d*x)^q*(a*x^n + b*x^(1 + n))^p,x]
```

output
```
(x*(e*x)^m*(c + d*x)^q*(a*x^n + b*x^(1 + n))^p*AppellF1[1 + m + n*p, -p, -
q, 2 + m + n*p, -((b*x)/a), -((d*x)/c)])/((1 + m + n*p)*(1 + (b*x)/a)^p*(1
 + (d*x)/c)^q)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [F]

$$\int (ex)^m (dx+c)^q \left(a\,x^n+b\,x^{1+n}\right)^p dx$$

input
```
int((e*x)^m*(d*x+c)^q*(a*x^n+b*x^(1+n))^p,x)
```

output
```
int((e*x)^m*(d*x+c)^q*(a*x^n+b*x^(1+n))^p,x)
```

Fricas [F]

$$\int (ex)^m(c+dx)^q\left(ax^n+bx^{1+n}\right)^p\,dx=\int \left(bx^{n+1}+ax^n\right)^p(dx+c)^q(ex)^m\,dx$$

input
```
integrate((e*x)^m*(d*x+c)^q*(a*x^n+b*x^(1+n))^p,x, algorithm="fricas")
```

output
```
integral((b*x^(n + 1) + a*x^n)^p*(d*x + c)^q*(e*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m(c+dx)^q\left(ax^n+bx^{1+n}\right)^p\,dx=\text{Timed out}$$

input
```
integrate((e*x)**m*(d*x+c)**q*(a*x**n+b*x**(1+n))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m(c+dx)^q\left(ax^n+bx^{1+n}\right)^p\,dx=\int \left(bx^{n+1}+ax^n\right)^p(dx+c)^q(ex)^m\,dx$$

input
```
integrate((e*x)^m*(d*x+c)^q*(a*x^n+b*x^(1+n))^p,x, algorithm="maxima")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^p*(d*x + c)^q*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m (c+dx)^q \left(ax^n+bx^{1+n}\right)^p \, dx = \int (bx^{n+1}+ax^n)^p (dx+c)^q (ex)^m \, dx$$

input
```
integrate((e*x)^m*(d*x+c)^q*(a*x^n+b*x^(1+n))^p,x, algorithm="giac")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^p*(d*x + c)^q*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m (c+dx)^q \left(ax^n+bx^{1+n}\right)^p \, dx = \int (e\,x)^m \left(a\,x^n+b\,x^{n+1}\right)^p (c+d\,x)^q \, dx$$

input
```
int((e*x)^m*(a*x^n + b*x^(n + 1))^p*(c + d*x)^q,x)
```

output
```
int((e*x)^m*(a*x^n + b*x^(n + 1))^p*(c + d*x)^q, x)
```

Reduce [F]

$$\int (ex)^m (c+dx)^q \left(ax^n+bx^{1+n}\right)^p \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(d*x+c)^q*(a*x^n+b*x^(1+n))^p,x)
```

output
```
(e**m*(x**m*(c + d*x)**q*(x**n*a + x**n*b*x)**p*a*c*p + x**m*(c + d*x)**q*
(x**n*a + x**n*b*x)**p*a*c*q + x**m*(c + d*x)**q*(x**n*a + x**n*b*x)**p*a*
d*m*x + x**m*(c + d*x)**q*(x**n*a + x**n*b*x)**p*a*d*n*p*x + x**m*(c + d*x
)**q*(x**n*a + x**n*b*x)**p*a*d*q*x + x**m*(c + d*x)**q*(x**n*a + x**n*b*x
)**p*b*c*m*x + x**m*(c + d*x)**q*(x**n*a + x**n*b*x)**p*b*c*n*p*x + x**m*(
c + d*x)**q*(x**n*a + x**n*b*x)**p*b*c*p*x + int((x**m*(c + d*x)**q*(x**n*
a + x**n*b*x)**p*x)/(a**2*c*d*m**2 + 2*a**2*c*d*m*n*p + a**2*c*d*m*p + 2*a
**2*c*d*m*q + a**2*c*d*m + a**2*c*d*n**2*p**2 + a**2*c*d*n*p**2 + 2*a**2*c
*d*n*p*q + a**2*c*d*n*p + a**2*c*d*p*q + a**2*c*d*q**2 + a**2*c*d*q + a**2
*d**2*m**2*x + 2*a**2*d**2*m*n*p*x + a**2*d**2*m*p*x + 2*a**2*d**2*m*q*x +
 a**2*d**2*m*x + a**2*d**2*n**2*p**2*x + a**2*d**2*n*p**2*x + 2*a**2*d**2*
n*p*q*x + a**2*d**2*n*p*x + a**2*d**2*p*q*x + a**2*d**2*q**2*x + a**2*d**2
*q*x + a*b*c**2*m**2 + 2*a*b*c**2*m*n*p + 2*a*b*c**2*m*p + a*b*c**2*m*q +
a*b*c**2*m + a*b*c**2*n**2*p**2 + 2*a*b*c**2*n*p**2 + a*b*c**2*n*p*q + a*b
*c**2*n*p + a*b*c**2*p**2 + a*b*c**2*p*q + a*b*c**2*p + 2*a*b*c*d*m**2*x +
 4*a*b*c*d*m*n*p*x + 3*a*b*c*d*m*p*x + 3*a*b*c*d*m*q*x + 2*a*b*c*d*m*x + 2
*a*b*c*d*n**2*p**2*x + 3*a*b*c*d*n*p**2*x + 3*a*b*c*d*n*p*q*x + 2*a*b*c*d*
n*p*x + a*b*c*d*p**2*x + 2*a*b*c*d*p*q*x + a*b*c*d*p*x + a*b*c*d*q**2*x +
a*b*c*d*q*x + a*b*d**2*m**2*x**2 + 2*a*b*d**2*m*n*p*x**2 + a*b*d**2*m*p*x*
*2 + 2*a*b*d**2*m*q*x**2 + a*b*d**2*m*x**2 + a*b*d**2*n**2*p**2*x**2 + ...
```

3.271 $\int x^2(c+dx)^q \left(ax^n+bx^{1+n}\right)^p \, dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int x^2(c+dx)^q \left(ax^n+bx^{1+n}\right)^p \, dx = \frac{x^3\left(1+\frac{bx}{a}\right)^{-p}(c+dx)^q\left(1+\frac{dx}{c}\right)^{-q}\left(ax^n+bx^{1+n}\right)^p \operatorname{AppellF1}\left(3+np,-p,-q,4+np,-\frac{bx}{a},-\frac{dx}{c}\right)}{3+np}$$

output
```
x^3*(d*x+c)^q*(a*x^n+b*x^(1+n))^p*AppellF1(n*p+3,-p,-q,n*p+4,-b*x/a,-d*x/c
)/(n*p+3)/((1+b*x/a)^p)/((1+d*x/c)^q)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.98

$$\int x^2(c+dx)^q \left(ax^n+bx^{1+n}\right)^p \, dx = \frac{x^3\left(\frac{a+bx}{a}\right)^{-p}(x^n(a+bx))^p(c+dx)^q\left(\frac{c+dx}{c}\right)^{-q} \operatorname{AppellF1}\left(3+np,-p,-q,4+np,-\frac{bx}{a},-\frac{dx}{c}\right)}{3+np}$$

input
```
Integrate[x^2*(c + d*x)^q*(a*x^n + b*x^(1 + n))^p,x]
```

output
```
(x^3*(x^n*(a + b*x))^p*(c + d*x)^q*AppellF1[3 + n*p, -p, -q, 4 + n*p, -((b
*x)/a), -((d*x)/c)])/((3 + n*p)*((a + b*x)/a)^p*((c + d*x)/c)^q)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1948, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(c+dx)^q \left(ax^n+bx^{n+1}\right)^p \, dx$$

$$\Big\downarrow \text{ 1948}$$

$$x^{-np}(a+bx)^{-p} \left(ax^n+bx^{n+1}\right)^p \int x^{np+2}(a+bx)^p(c+dx)^q dx$$

$$\Big\downarrow \text{ 152}$$

$$x^{-np}\left(\frac{bx}{a}+1\right)^{-p} \left(ax^n+bx^{n+1}\right)^p \int x^{np+2}\left(\frac{bx}{a}+1\right)^p (c+dx)^q dx$$

$$\Big\downarrow \text{ 152}$$

$$x^{-np}\left(\frac{bx}{a}+1\right)^{-p} (c+dx)^q \left(\frac{dx}{c}+1\right)^{-q} \left(ax^n+bx^{n+1}\right)^p \int x^{np+2}\left(\frac{bx}{a}+1\right)^p \left(\frac{dx}{c}+1\right)^q dx$$

$$\Big\downarrow \text{ 150}$$

$$\frac{x^3\left(\frac{bx}{a}+1\right)^{-p} (c+dx)^q \left(\frac{dx}{c}+1\right)^{-q} \left(ax^n+bx^{n+1}\right)^p \operatorname{AppellF1}\left(np+3,-p,-q,np+4,-\frac{bx}{a},-\frac{dx}{c}\right)}{np+3}$$

input
```
Int[x^2*(c + d*x)^q*(a*x^n + b*x^(1 + n))^p,x]
```

output
```
(x^3*(c + d*x)^q*(a*x^n + b*x^(1 + n))^p*AppellF1[3 + n*p, -p, -q, 4 + n*p
, -((b*x)/a), -((d*x)/c)])/((3 + n*p)*(1 + (b*x)/a)^p*(1 + (d*x)/c)^q)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [F]

$$\int x^2(dx+c)^q\left(a\,x^n+b\,x^{1+n}\right)^p dx$$

input
```
int(x^2*(d*x+c)^q*(a*x^n+b*x^(1+n))^p,x)
```

output
```
int(x^2*(d*x+c)^q*(a*x^n+b*x^(1+n))^p,x)
```

Fricas [F]

$$\int x^2(c+dx)^q \left(ax^n+bx^{1+n}\right)^p \, dx = \int (bx^{n+1}+ax^n)^p (dx+c)^q x^2 \, dx$$

input
```
integrate(x^2*(d*x+c)^q*(a*x^n+b*x^(1+n))^p,x, algorithm="fricas")
```

output
```
integral((b*x^(n + 1) + a*x^n)^p*(d*x + c)^q*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2(c+dx)^q \left(ax^n+bx^{1+n}\right)^p \, dx = \text{Timed out}$$

input
```
integrate(x**2*(d*x+c)**q*(a*x**n+b*x**(1+n))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2(c+dx)^q \left(ax^n+bx^{1+n}\right)^p \, dx = \int (bx^{n+1}+ax^n)^p (dx+c)^q x^2 \, dx$$

input
```
integrate(x^2*(d*x+c)^q*(a*x^n+b*x^(1+n))^p,x, algorithm="maxima")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^p*(d*x + c)^q*x^2, x)
```

Giac [F]

$$\int x^2(c+dx)^q\left(ax^n+bx^{1+n}\right)^p\,dx=\int \left(bx^{n+1}+ax^n\right)^p(dx+c)^qx^2\,dx$$

input
```
integrate(x^2*(d*x+c)^q*(a*x^n+b*x^(1+n))^p,x, algorithm="giac")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^p*(d*x + c)^q*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(c+dx)^q\left(ax^n+bx^{1+n}\right)^p\,dx=\int x^2\left(a\,x^n+b\,x^{n+1}\right)^p(c+d\,x)^q\,dx$$

input
```
int(x^2*(a*x^n + b*x^(n + 1))^p*(c + d*x)^q,x)
```

output
```
int(x^2*(a*x^n + b*x^(n + 1))^p*(c + d*x)^q, x)
```

Reduce [F]

$$\int x^2(c+dx)^q\left(ax^n+bx^{1+n}\right)^p\,dx=\int x^2(dx+c)^q\left(x^na+b\,x^{1+n}\right)^p\,dx$$

input
```
int(x^2*(d*x+c)^q*(a*x^n+b*x^(1+n))^p,x)
```

output
```
int(x^2*(d*x+c)^q*(a*x^n+b*x^(1+n))^p,x)
```

3.272 $\int x(c+dx)^q \left(ax^n+bx^{1+n}\right)^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int x(c+dx)^q \left(ax^n+bx^{1+n}\right)^p \, dx = \frac{x^2\left(1+\frac{bx}{a}\right)^{-p}(c+dx)^q\left(1+\frac{dx}{c}\right)^{-q}\left(ax^n+bx^{1+n}\right)^p \operatorname{AppellF1}\left(2+np,-p,-q,3+np,-\frac{bx}{a},-\frac{dx}{c}\right)}{2+np}$$

output
```
x^2*(d*x+c)^q*(a*x^n+b*x^(1+n))^p*AppellF1(n*p+2,-p,-q,n*p+3,-b*x/a,-d*x/c
)/(n*p+2)/((1+b*x/a)^p)/((1+d*x/c)^q)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.98

$$\int x(c+dx)^q \left(ax^n+bx^{1+n}\right)^p \, dx = \frac{x^2\left(\frac{a+bx}{a}\right)^{-p}\left(x^n(a+bx)\right)^p(c+dx)^q\left(\frac{c+dx}{c}\right)^{-q} \operatorname{AppellF1}\left(2+np,-p,-q,3+np,-\frac{bx}{a},-\frac{dx}{c}\right)}{2+np}$$

input
```
Integrate[x*(c + d*x)^q*(a*x^n + b*x^(1 + n))^p,x]
```

output
```
(x^2*(x^n*(a + b*x))^p*(c + d*x)^q*AppellF1[2 + n*p, -p, -q, 3 + n*p, -((b
*x)/a), -((d*x)/c)])/((2 + n*p)*((a + b*x)/a)^p*((c + d*x)/c)^q)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1948, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(c+dx)^q \left(ax^n+bx^{n+1}\right)^p dx$$

$$\Big\downarrow \text{1948}$$

$$x^{-np}(a+bx)^{-p}\left(ax^n+bx^{n+1}\right)^p\int x^{np+1}(a+bx)^p(c+dx)^q dx$$

$$\Big\downarrow \text{152}$$

$$x^{-np}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^n+bx^{n+1}\right)^p\int x^{np+1}\left(\frac{bx}{a}+1\right)^p(c+dx)^q dx$$

$$\Big\downarrow \text{152}$$

$$x^{-np}\left(\frac{bx}{a}+1\right)^{-p}(c+dx)^q\left(\frac{dx}{c}+1\right)^{-q}\left(ax^n+bx^{n+1}\right)^p\int x^{np+1}\left(\frac{bx}{a}+1\right)^p\left(\frac{dx}{c}+1\right)^q dx$$

$$\Big\downarrow \text{150}$$

$$\frac{x^2\left(\frac{bx}{a}+1\right)^{-p}(c+dx)^q\left(\frac{dx}{c}+1\right)^{-q}\left(ax^n+bx^{n+1}\right)^p \text{AppellF1}\left(np+2,-p,-q,np+3,-\frac{bx}{a},-\frac{dx}{c}\right)}{np+2}$$

input
```
Int[x*(c + d*x)^q*(a*x^n + b*x^(1 + n))^p,x]
```

output
```
(x^2*(c + d*x)^q*(a*x^n + b*x^(1 + n))^p*AppellF1[2 + n*p, -p, -q, 3 + n*p
, -((b*x)/a), -((d*x)/c)])/((2 + n*p)*(1 + (b*x)/a)^p*(1 + (d*x)/c)^q)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] && !IntegerQ[p]
 && NeQ[b*c - a*d, 0] && !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [F]

$$\int x(dx+c)^q\left(a\,x^n+b\,x^{1+n}\right)^p dx$$

input
```
int(x*(d*x+c)^q*(a*x^n+b*x^(1+n))^p,x)
```

output
```
int(x*(d*x+c)^q*(a*x^n+b*x^(1+n))^p,x)
```

Fricas [F]

$$\int x(c+dx)^q \left(ax^n+bx^{1+n}\right)^p \, dx=\int \left(bx^{n+1}+ax^n\right)^p(dx+c)^q x \, dx$$

input
```
integrate(x*(d*x+c)^q*(a*x^n+b*x^(1+n))^p,x, algorithm="fricas")
```

output
```
integral((b*x^(n + 1) + a*x^n)^p*(d*x + c)^q*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x(c+dx)^q \left(ax^n+bx^{1+n}\right)^p \, dx=\text{Timed out}$$

input
```
integrate(x*(d*x+c)**q*(a*x**n+b*x**(1+n))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x(c+dx)^q \left(ax^n+bx^{1+n}\right)^p \, dx=\int \left(bx^{n+1}+ax^n\right)^p(dx+c)^q x \, dx$$

input
```
integrate(x*(d*x+c)^q*(a*x^n+b*x^(1+n))^p,x, algorithm="maxima")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^p*(d*x + c)^q*x, x)
```

Giac [F]

$$\int x(c+dx)^q \left(ax^n+bx^{1+n}\right)^p \,dx=\int \left(bx^{n+1}+ax^n\right)^p (dx+c)^q x \,dx$$

input
```
integrate(x*(d*x+c)^q*(a*x^n+b*x^(1+n))^p,x, algorithm="giac")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^p*(d*x + c)^q*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(c+dx)^q \left(ax^n+bx^{1+n}\right)^p \,dx=\int x\left(a\,x^n+b\,x^{n+1}\right)^p (c+d\,x)^q \,dx$$

input
```
int(x*(a*x^n + b*x^(n + 1))^p*(c + d*x)^q,x)
```

output
```
int(x*(a*x^n + b*x^(n + 1))^p*(c + d*x)^q, x)
```

Reduce [F]

$$\int x(c+dx)^q \left(ax^n+bx^{1+n}\right)^p \,dx=\text{too large to display}$$

input
```
int(x*(d*x+c)^q*(a*x^n+b*x^(1+n))^p,x)
```

output

```
( - (c + d*x)**q*(x**n*a + x**n*b*x)**p*a**2*c*d*n*p**2 - (c + d*x)**q*(x*
*n*a + x**n*b*x)**p*a**2*c*d*p + (c + d*x)**q*(x**n*a + x**n*b*x)**p*a**2*
d**2*n*p**2*x + (c + d*x)**q*(x**n*a + x**n*b*x)**p*a**2*d**2*p*q*x - (c +
 d*x)**q*(x**n*a + x**n*b*x)**p*a*b*c**2*n*p*q - (c + d*x)**q*(x**n*a + x*
*n*b*x)**p*a*b*c**2*q + (c + d*x)**q*(x**n*a + x**n*b*x)**p*a*b*c*d*n*p**2
*x + (c + d*x)**q*(x**n*a + x**n*b*x)**p*a*b*c*d*n*p*q*x + (c + d*x)**q*(x
**n*a + x**n*b*x)**p*a*b*c*d*p**2*x + (c + d*x)**q*(x**n*a + x**n*b*x)**p*
a*b*c*d*q**2*x + (c + d*x)**q*(x**n*a + x**n*b*x)**p*a*b*d**2*n**2*p**2*x*
*2 + (c + d*x)**q*(x**n*a + x**n*b*x)**p*a*b*d**2*n*p**2*x**2 + 2*(c + d*x
)**q*(x**n*a + x**n*b*x)**p*a*b*d**2*n*p*q*x**2 + (c + d*x)**q*(x**n*a + x
**n*b*x)**p*a*b*d**2*n*p*x**2 + (c + d*x)**q*(x**n*a + x**n*b*x)**p*a*b*d*
*2*p*q*x**2 + (c + d*x)**q*(x**n*a + x**n*b*x)**p*a*b*d**2*q**2*x**2 + (c
+ d*x)**q*(x**n*a + x**n*b*x)**p*a*b*d**2*q*x**2 + (c + d*x)**q*(x**n*a +
x**n*b*x)**p*b**2*c**2*n*p*q*x + (c + d*x)**q*(x**n*a + x**n*b*x)**p*b**2*
c**2*p*q*x + (c + d*x)**q*(x**n*a + x**n*b*x)**p*b**2*c*d*n**2*p**2*x**2 +
 2*(c + d*x)**q*(x**n*a + x**n*b*x)**p*b**2*c*d*n*p**2*x**2 + (c + d*x)**q
*(x**n*a + x**n*b*x)**p*b**2*c*d*n*p*q*x**2 + (c + d*x)**q*(x**n*a + x**n*
b*x)**p*b**2*c*d*n*p*x**2 + (c + d*x)**q*(x**n*a + x**n*b*x)**p*b**2*c*d*p
**2*x**2 + (c + d*x)**q*(x**n*a + x**n*b*x)**p*b**2*c*d*p*q*x**2 + (c + d*
x)**q*(x**n*a + x**n*b*x)**p*b**2*c*d*p*x**2 - int(((c + d*x)**q*(x**n*...
```

3.273 $\int (c+dx)^q \left(ax^n+bx^{1+n}\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int (c+dx)^q \left(ax^n+bx^{1+n}\right)^p \, dx = \frac{x\left(1+\frac{bx}{a}\right)^{-p}(c+dx)^q\left(1+\frac{dx}{c}\right)^{-q}\left(ax^n+bx^{1+n}\right)^p \operatorname{AppellF1}\left(1+np,-p,-q,2+np,-\frac{bx}{a},-\frac{dx}{c}\right)}{1+np}$$

output
```
x*(d*x+c)^q*(a*x^n+b*x^(1+n))^p*AppellF1(n*p+1,-p,-q,n*p+2,-b*x/a,-d*x/c)/
(n*p+1)/((1+b*x/a)^p)/((1+d*x/c)^q)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.98

$$\int (c+dx)^q \left(ax^n+bx^{1+n}\right)^p \, dx = \frac{x\left(\frac{a+bx}{a}\right)^{-p}(x^n(a+bx))^p(c+dx)^q\left(\frac{c+dx}{c}\right)^{-q} \operatorname{AppellF1}\left(1+np,-p,-q,2+np,-\frac{bx}{a},-\frac{dx}{c}\right)}{1+np}$$

input
```
Integrate[(c + d*x)^q*(a*x^n + b*x^(1 + n))^p,x]
```

output
```
(x*(x^n*(a + b*x))^p*(c + d*x)^q*AppellF1[1 + n*p, -p, -q, 2 + n*p, -((b*x
)/a), -((d*x)/c)])/((1 + n*p)*((a + b*x)/a)^p*((c + d*x)/c)^q)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2468, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^q \left(ax^n+bx^{n+1}\right)^p \, dx$$

$$\Big\downarrow \text{ 2468}$$

$$x^{-np}(a+bx)^{-p}\left(ax^n+bx^{n+1}\right)^p \int x^{np}(a+bx)^p(c+dx)^q dx$$

$$\Big\downarrow \text{ 152}$$

$$x^{-np}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^n+bx^{n+1}\right)^p \int x^{np}\left(\frac{bx}{a}+1\right)^p (c+dx)^q dx$$

$$\Big\downarrow \text{ 152}$$

$$x^{-np}\left(\frac{bx}{a}+1\right)^{-p}(c+dx)^q\left(\frac{dx}{c}+1\right)^{-q}\left(ax^n+bx^{n+1}\right)^p \int x^{np}\left(\frac{bx}{a}+1\right)^p\left(\frac{dx}{c}+1\right)^q dx$$

$$\Big\downarrow \text{ 150}$$

$$\frac{x\left(\frac{bx}{a}+1\right)^{-p}(c+dx)^q\left(\frac{dx}{c}+1\right)^{-q}\left(ax^n+bx^{n+1}\right)^p \text{AppellF1}\left(np+1,-p,-q,np+2,-\frac{bx}{a},-\frac{dx}{c}\right)}{np+1}$$

input
```
Int[(c + d*x)^q*(a*x^n + b*x^(1 + n))^p,x]
```

output
```
(x*(c + d*x)^q*(a*x^n + b*x^(1 + n))^p*AppellF1[1 + n*p, -p, -q, 2 + n*p,
-((b*x)/a), -((d*x)/c)])/((1 + n*p)*(1 + (b*x)/a)^p*(1 + (d*x)/c)^q)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 2468
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_), x_Symbol] :> Simp[(a
*x^r + b*x^s)^p/(x^(p*r)*(a + b*x^(s - r))^p)   Int[x^(p*r)*(a + b*x^(s - r
))^p*Fx, x], x] /; FreeQ[{a, b, p, r, s}, x] &&  !IntegerQ[p] && PosQ[s - r
] &&  !(EqQ[p, 1] && EqQ[Fx, 1])
```

Maple [F]

$$\int (dx+c)^q \left(a\,x^n+b\,x^{1+n}\right)^p dx$$

input
```
int((d*x+c)^q*(a*x^n+b*x^(1+n))^p,x)
```

output
```
int((d*x+c)^q*(a*x^n+b*x^(1+n))^p,x)
```

Fricas [F]

$$\int (c+dx)^q \left(ax^n+bx^{1+n}\right)^p \, dx = \int \left(bx^{n+1}+ax^n\right)^p (dx+c)^q \, dx$$

input
```
integrate((d*x+c)^q*(a*x^n+b*x^(1+n))^p,x, algorithm="fricas")
```

output
```
integral((b*x^(n + 1) + a*x^n)^p*(d*x + c)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^q \left(ax^n+bx^{1+n}\right)^p \,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**q*(a*x**n+b*x**(1+n))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)^q \left(ax^n+bx^{1+n}\right)^p \,dx = \int (bx^{n+1}+ax^n)^p(dx+c)^q \,dx$$

input
```
integrate((d*x+c)^q*(a*x^n+b*x^(1+n))^p,x, algorithm="maxima")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^p*(d*x + c)^q, x)
```

Giac [F]

$$\int (c+dx)^q \left(ax^n+bx^{1+n}\right)^p \,dx = \int (bx^{n+1}+ax^n)^p(dx+c)^q \,dx$$

input
```
integrate((d*x+c)^q*(a*x^n+b*x^(1+n))^p,x, algorithm="giac")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^p*(d*x + c)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^q \left(ax^n+bx^{1+n}\right)^p \, dx = \int \left(a\,x^n+b\,x^{n+1}\right)^p\,(c+d\,x)^q \,dx$$

input
```
int((a*x^n + b*x^(n + 1))^p*(c + d*x)^q,x)
```

output
```
int((a*x^n + b*x^(n + 1))^p*(c + d*x)^q, x)
```

Reduce [F]

$$\int (c+dx)^q \left(ax^n+bx^{1+n}\right)^p \, dx = \text{too large to display}$$

input
```
int((d*x+c)^q*(a*x^n+b*x^(1+n))^p,x)
```

output
```
((c + d*x)**q*(x***n*a + x***n*b*x)**p*a*c*p + (c + d*x)**q*(x***n*a + x***n*b
*x)**p*a*c*q + (c + d*x)**q*(x***n*a + x***n*b*x)**p*a*d*n*p*x + (c + d*x)**
q*(x***n*a + x***n*b*x)**p*a*d*q*x + (c + d*x)**q*(x***n*a + x***n*b*x)**p*b*c
*n*p*x + (c + d*x)**q*(x***n*a + x***n*b*x)**p*b*c*p*x + int(((c + d*x)**q*(
x***n*a + x***n*b*x)**p*x)/(a**2*c*d*n**2*p**2 + a**2*c*d*n*p**2 + 2*a**2*c*
d*n*p*q + a**2*c*d*n*p + a**2*c*d*p*q + a**2*c*d*q**2 + a**2*c*d*q + a**2*
d**2*n**2*p**2*x + a**2*d**2*n*p**2*x + 2*a**2*d**2*n*p*q*x + a**2*d**2*n*
p*x + a**2*d**2*p*q*x + a**2*d**2*q**2*x + a**2*d**2*q*x + a*b*c**2*n**2*p
**2 + 2*a*b*c**2*n*p**2 + a*b*c**2*n*p*q + a*b*c**2*n*p + a*b*c**2*p**2 +
a*b*c**2*p*q + a*b*c**2*p + 2*a*b*c*d*n**2*p**2*x + 3*a*b*c*d*n*p**2*x + 3
*a*b*c*d*n*p*q*x + 2*a*b*c*d*n*p*x + a*b*c*d*p**2*x + 2*a*b*c*d*p*q*x + a*
b*c*d*p*x + a*b*c*d*q**2*x + a*b*c*d*q*x + a*b*d**2*n**2*p**2*x**2 + a*b*d
**2*n*p**2*x**2 + 2*a*b*d**2*n*p*q*x**2 + a*b*d**2*n*p*x**2 + a*b*d**2*p*q
*x**2 + a*b*d**2*q**2*x**2 + a*b*d**2*q*x**2 + b**2*c**2*n**2*p**2*x + 2*b
**2*c**2*n*p**2*x + b**2*c**2*n*p*q*x + b**2*c**2*n*p*x + b**2*c**2*p**2*x
 + b**2*c**2*p*q*x + b**2*c**2*p*x + b**2*c*d*n**2*p**2*x**2 + 2*b**2*c*d*
n*p**2*x**2 + b**2*c*d*n*p*q*x**2 + b**2*c*d*n*p*x**2 + b**2*c*d*p**2*x**2
 + b**2*c*d*p*q*x**2 + b**2*c*d*p*x**2),x)*a**3*d**3*n**3*p**4 + int(((c +
 d*x)**q*(x***n*a + x***n*b*x)**p*x)/(a**2*c*d*n**2*p**2 + a**2*c*d*n*p**2 +
 2*a**2*c*d*n*p*q + a**2*c*d*n*p + a**2*c*d*p*q + a**2*c*d*q**2 + a**2*...
```

3.274 $\int \frac{(c+dx)^q\left(ax^n+bx^{1+n}\right)^p}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 82

$$\int \frac{(c+dx)^q\left(ax^n+bx^{1+n}\right)^p}{x}\,dx = \frac{\left(1+\frac{bx}{a}\right)^{-p}(c+dx)^q\left(1+\frac{dx}{c}\right)^{-q}\left(ax^n+bx^{1+n}\right)^p \operatorname{AppellF1}\left(np,-p,-q,1+np,-\frac{bx}{a},-\frac{dx}{c}\right)}{np}$$

output
```
(d*x+c)^q*(a*x^n+b*x^(1+n))^p*AppellF1(n*p,-p,-q,n*p+1,-b*x/a,-d*x/c)/n/p/
((1+b*x/a)^p)/((1+d*x/c)^q)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx)^q\left(ax^n+bx^{1+n}\right)^p}{x}\,dx = \frac{\left(\frac{a+bx}{a}\right)^{-p}(x^n(a+bx))^p(c+dx)^q\left(\frac{c+dx}{c}\right)^{-q}\operatorname{AppellF1}\left(np,-p,-q,1+np,-\frac{bx}{a},-\frac{dx}{c}\right)}{np}$$

input
```
Integrate[((c + d*x)^q*(a*x^n + b*x^(1 + n))^p)/x,x]
```

output
```
((x^n*(a + b*x))^p*(c + d*x)^q*AppellF1[n*p, -p, -q, 1 + n*p, -((b*x)/a),
-((d*x)/c)])/(n*p*((a + b*x)/a)^p*((c + d*x)/c)^q)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1948, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^q \left(ax^n+bx^{n+1}\right)^p}{x}dx$$

$$\Big\downarrow \text{1948}$$

$$x^{-np}(a+bx)^{-p}\left(ax^n+bx^{n+1}\right)^p\int x^{np-1}(a+bx)^p(c+dx)^qdx$$

$$\Big\downarrow \text{152}$$

$$x^{-np}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^n+bx^{n+1}\right)^p\int x^{np-1}\left(\frac{bx}{a}+1\right)^p(c+dx)^qdx$$

$$\Big\downarrow \text{152}$$

$$x^{-np}\left(\frac{bx}{a}+1\right)^{-p}(c+dx)^q\left(\frac{dx}{c}+1\right)^{-q}\left(ax^n+bx^{n+1}\right)^p\int x^{np-1}\left(\frac{bx}{a}+1\right)^p\left(\frac{dx}{c}+1\right)^qdx$$

$$\Big\downarrow \text{150}$$

$$\frac{\left(\frac{bx}{a}+1\right)^{-p}(c+dx)^q\left(\frac{dx}{c}+1\right)^{-q}\left(ax^n+bx^{n+1}\right)^p\text{AppellF1}\left(np,-p,-q,np+1,-\frac{bx}{a},-\frac{dx}{c}\right)}{np}$$

input
```
Int[((c + d*x)^q*(a*x^n + b*x^(1 + n))^p)/x,x]
```

output
```
((c + d*x)^q*(a*x^n + b*x^(1 + n))^p*AppellF1[n*p, -p, -q, 1 + n*p, -((b*x
)/a), -((d*x)/c)])/(n*p*(1 + (b*x)/a)^p*(1 + (d*x)/c)^q)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [F]

$$\int \frac{(dx+c)^q\,(a\,x^n+b\,x^{1+n})^p}{x}dx$$

input
```
int((d*x+c)^q*(a*x^n+b*x^(1+n))^p/x,x)
```

output
```
int((d*x+c)^q*(a*x^n+b*x^(1+n))^p/x,x)
```

Fricas [F]

$$\int \frac{(c+dx)^q\,(ax^n+bx^{1+n})^p}{x}\,dx = \int \frac{(bx^{n+1}+ax^n)^p(dx+c)^q}{x}\,dx$$

input
```
integrate((d*x+c)^q*(a*x^n+b*x^(1+n))^p/x,x, algorithm="fricas")
```

output
```
integral((b*x^(n + 1) + a*x^n)^p*(d*x + c)^q/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^q\,(ax^n+bx^{1+n})^p}{x}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**q*(a*x**n+b*x**(1+n))**p/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)^q\,(ax^n+bx^{1+n})^p}{x}\,dx = \int \frac{(bx^{n+1}+ax^n)^p(dx+c)^q}{x}\,dx$$

input
```
integrate((d*x+c)^q*(a*x^n+b*x^(1+n))^p/x,x, algorithm="maxima")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^p*(d*x + c)^q/x, x)
```

Giac [F]

$$\int \frac{(c+dx)^q (ax^n+bx^{1+n})^p}{x}\,dx=\int \frac{(bx^{n+1}+ax^n)^p(dx+c)^q}{x}\,dx$$

input
```
integrate((d*x+c)^q*(a*x^n+b*x^(1+n))^p/x,x, algorithm="giac")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^p*(d*x + c)^q/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^q (ax^n+bx^{1+n})^p}{x}\,dx=\int \frac{(a\,x^n+b\,x^{n+1})^p\,(c+d\,x)^q}{x}\,dx$$

input
```
int(((a*x^n + b*x^(n + 1))^p*(c + d*x)^q)/x,x)
```

output
```
int(((a*x^n + b*x^(n + 1))^p*(c + d*x)^q)/x, x)
```

Reduce [F]

$$\int \frac{(c+dx)^q (ax^n+bx^{1+n})^p}{x}\,dx=\text{too large to display}$$

input
```
int((d*x+c)^q*(a*x^n+b*x^(1+n))^p/x,x)
```

output

```
((c + d*x)**q*(x**n*a + x**n*b*x)**p*a*d + (c + d*x)**q*(x**n*a + x**n*b*x
)**p*b*c - int(((c + d*x)**q*(x**n*a + x**n*b*x)**p*x)/(a**2*c*d*n*p + a**
2*c*d*q + a**2*d**2*n*p*x + a**2*d**2*q*x + a*b*c**2*n*p + a*b*c**2*p + 2*
a*b*c*d*n*p*x + a*b*c*d*p*x + a*b*c*d*q*x + a*b*d**2*n*p*x**2 + a*b*d**2*q
*x**2 + b**2*c**2*n*p*x + b**2*c**2*p*x + b**2*c*d*n*p*x**2 + b**2*c*d*p*x
**2),x)*a**2*b*d**3*n*p**2 - int(((c + d*x)**q*(x**n*a + x**n*b*x)**p*x)/(
a**2*c*d*n*p + a**2*c*d*q + a**2*d**2*n*p*x + a**2*d**2*q*x + a*b*c**2*n*p
 + a*b*c**2*p + 2*a*b*c*d*n*p*x + a*b*c*d*p*x + a*b*c*d*q*x + a*b*d**2*n*p
*x**2 + a*b*d**2*q*x**2 + b**2*c**2*n*p*x + b**2*c**2*p*x + b**2*c*d*n*p*x
**2 + b**2*c*d*p*x**2),x)*a**2*b*d**3*p*q - int(((c + d*x)**q*(x**n*a + x*
*n*b*x)**p*x)/(a**2*c*d*n*p + a**2*c*d*q + a**2*d**2*n*p*x + a**2*d**2*q*x
 + a*b*c**2*n*p + a*b*c**2*p + 2*a*b*c*d*n*p*x + a*b*c*d*p*x + a*b*c*d*q*x
 + a*b*d**2*n*p*x**2 + a*b*d**2*q*x**2 + b**2*c**2*n*p*x + b**2*c**2*p*x +
 b**2*c*d*n*p*x**2 + b**2*c*d*p*x**2),x)*a*b**2*c*d**2*n*p**2 - int(((c +
d*x)**q*(x**n*a + x**n*b*x)**p*x)/(a**2*c*d*n*p + a**2*c*d*q + a**2*d**2*n
*p*x + a**2*d**2*q*x + a*b*c**2*n*p + a*b*c**2*p + 2*a*b*c*d*n*p*x + a*b*c
*d*p*x + a*b*c*d*q*x + a*b*d**2*n*p*x**2 + a*b*d**2*q*x**2 + b**2*c**2*n*p
*x + b**2*c**2*p*x + b**2*c*d*n*p*x**2 + b**2*c*d*p*x**2),x)*a*b**2*c*d**2
*n*p*q - int(((c + d*x)**q*(x**n*a + x**n*b*x)**p*x)/(a**2*c*d*n*p + a**2*
c*d*q + a**2*d**2*n*p*x + a**2*d**2*q*x + a*b*c**2*n*p + a*b*c**2*p + 2...
```

3.275 $\int \frac{(c+dx)^q (ax^n+bx^{1+n})^p}{x^2} dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{(c+dx)^q (ax^n+bx^{1+n})^p}{x^2} dx = -\frac{\left(1+\frac{bx}{a}\right)^{-p} (c+dx)^q \left(1+\frac{dx}{c}\right)^{-q} (ax^n+bx^{1+n})^p \operatorname{AppellF1}\left(-1+np,-p,-q,np,-\frac{bx}{a},-\frac{dx}{c}\right)}{(1-np)x}$$

output
```
-(d*x+c)^q*(a*x^n+b*x^(1+n))^p*AppellF1(n*p-1,-p,-q,n*p,-b*x/a,-d*x/c)/(-n
*p+1)/x/((1+b*x/a)^p)/((1+d*x/c)^q)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx)^q (ax^n+bx^{1+n})^p}{x^2} dx = \frac{\left(\frac{a+bx}{a}\right)^{-p} (x^n(a+bx))^p (c+dx)^q \left(\frac{c+dx}{c}\right)^{-q} \operatorname{AppellF1}\left(-1+np,-p,-q,np,-\frac{bx}{a},-\frac{dx}{c}\right)}{(-1+np)x}$$

input
```
Integrate[((c + d*x)^q*(a*x^n + b*x^(1 + n))^p)/x^2,x]
```

output
```
((x^n*(a + b*x))^p*(c + d*x)^q*AppellF1[-1 + n*p, -p, -q, n*p, -((b*x)/a),
 -((d*x)/c)])/((-1 + n*p)*x*((a + b*x)/a)^p*((c + d*x)/c)^q)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1948, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^q \left(ax^n+bx^{n+1}\right)^p}{x^2} \, dx$$

$$\Big\downarrow \text{ 1948}$$

$$x^{-np}(a+bx)^{-p} \left(ax^n+bx^{n+1}\right)^p \int x^{np-2}(a+bx)^p(c+dx)^q dx$$

$$\Big\downarrow \text{ 152}$$

$$x^{-np}\left(\frac{bx}{a}+1\right)^{-p} \left(ax^n+bx^{n+1}\right)^p \int x^{np-2}\left(\frac{bx}{a}+1\right)^p(c+dx)^q dx$$

$$\Big\downarrow \text{ 152}$$

$$x^{-np}\left(\frac{bx}{a}+1\right)^{-p}(c+dx)^q\left(\frac{dx}{c}+1\right)^{-q} \left(ax^n+bx^{n+1}\right)^p \int x^{np-2}\left(\frac{bx}{a}+1\right)^p\left(\frac{dx}{c}+1\right)^q dx$$

$$\Big\downarrow \text{ 150}$$

$$-\frac{\left(\frac{bx}{a}+1\right)^{-p}(c+dx)^q\left(\frac{dx}{c}+1\right)^{-q}\left(ax^n+bx^{n+1}\right)^p \operatorname{AppellF1}\left(np-1,-p,-q,np,-\frac{bx}{a},-\frac{dx}{c}\right)}{x(1-np)}$$

input
```
Int[((c + d*x)^q*(a*x^n + b*x^(1 + n))^p)/x^2,x]
```

output
```
-(((c + d*x)^q*(a*x^n + b*x^(1 + n))^p*AppellF1[-1 + n*p, -p, -q, n*p, -((
b*x)/a), -((d*x)/c)])/((1 - n*p)*x*(1 + (b*x)/a)^p*(1 + (d*x)/c)^q))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [F]

$$\int \frac{(dx+c)^q \left(a\,x^n+b\,x^{1+n}\right)^p}{x^2}dx$$

input
```
int((d*x+c)^q*(a*x^n+b*x^(1+n))^p/x^2,x)
```

output
```
int((d*x+c)^q*(a*x^n+b*x^(1+n))^p/x^2,x)
```

Fricas [F]

$$\int \frac{(c+dx)^q \left(ax^n+bx^{1+n}\right)^p}{x^2}\,dx=\int \frac{(bx^{n+1}+ax^n)^p(dx+c)^q}{x^2}\,dx$$

input
```
integrate((d*x+c)^q*(a*x^n+b*x^(1+n))^p/x^2,x, algorithm="fricas")
```

output
```
integral((b*x^(n + 1) + a*x^n)^p*(d*x + c)^q/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^q \left(ax^n+bx^{1+n}\right)^p}{x^2}\,dx=\text{Timed out}$$

input
```
integrate((d*x+c)**q*(a*x**n+b*x**(1+n))**p/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)^q \left(ax^n+bx^{1+n}\right)^p}{x^2}\,dx=\int \frac{(bx^{n+1}+ax^n)^p(dx+c)^q}{x^2}\,dx$$

input
```
integrate((d*x+c)^q*(a*x^n+b*x^(1+n))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^p*(d*x + c)^q/x^2, x)
```

Giac [F]

$$\int \frac{(c+dx)^q (ax^n+bx^{1+n})^p}{x^2}\,dx = \int \frac{(bx^{n+1}+ax^n)^p (dx+c)^q}{x^2}\,dx$$

input
```
integrate((d*x+c)^q*(a*x^n+b*x^(1+n))^p/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^p*(d*x + c)^q/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^q (ax^n+bx^{1+n})^p}{x^2}\,dx = \int \frac{(a\,x^n+b\,x^{n+1})^p\,(c+d\,x)^q}{x^2}\,dx$$

input
```
int(((a*x^n + b*x^(n + 1))^p*(c + d*x)^q)/x^2,x)
```

output
```
int(((a*x^n + b*x^(n + 1))^p*(c + d*x)^q)/x^2, x)
```

Reduce [F]

$$\int \frac{(c+dx)^q (ax^n+bx^{1+n})^p}{x^2}\,dx = \text{too large to display}$$

input
```
int((d*x+c)^q*(a*x^n+b*x^(1+n))^p/x^2,x)
```

output

```
((c + d*x)**q*(x**n*a + x**n*b*x)**p - int(((c + d*x)**q*(x**n*a + x**n*b*
x)**p)/(a**2*c*d*n**2*p**2*x + a**2*c*d*n*p*q*x - 2*a**2*c*d*n*p*x - a**2*
c*d*q*x + a**2*c*d*x + a**2*d**2*n**2*p**2*x**2 + a**2*d**2*n*p*q*x**2 - 2
*a**2*d**2*n*p*x**2 - a**2*d**2*q*x**2 + a**2*d**2*x**2 + a*b*c**2*n**2*p*
*2*x + a*b*c**2*n*p**2*x - 2*a*b*c**2*n*p*x - a*b*c**2*p*x + a*b*c**2*x +
2*a*b*c*d*n**2*p**2*x**2 + a*b*c*d*n*p**2*x**2 + a*b*c*d*n*p*q*x**2 - 4*a*
b*c*d*n*p*x**2 - a*b*c*d*p*x**2 - a*b*c*d*q*x**2 + 2*a*b*c*d*x**2 + a*b*d*
*2*n**2*p**2*x**3 + a*b*d**2*n*p*q*x**3 - 2*a*b*d**2*n*p*x**3 - a*b*d**2*q
*x**3 + a*b*d**2*x**3 + b**2*c**2*n**2*p**2*x**2 + b**2*c**2*n*p**2*x**2 -
 2*b**2*c**2*n*p*x**2 - b**2*c**2*p*x**2 + b**2*c**2*x**2 + b**2*c*d*n**2*
p**2*x**3 + b**2*c*d*n*p**2*x**3 - 2*b**2*c*d*n*p*x**3 - b**2*c*d*p*x**3 +
 b**2*c*d*x**3),x)*a**2*d**2*n**2*p**2*q*x - int(((c + d*x)**q*(x**n*a + x
**n*b*x)**p)/(a**2*c*d*n**2*p**2*x + a**2*c*d*n*p*q*x - 2*a**2*c*d*n*p*x -
 a**2*c*d*q*x + a**2*c*d*x + a**2*d**2*n**2*p**2*x**2 + a**2*d**2*n*p*q*x*
*2 - 2*a**2*d**2*n*p*x**2 - a**2*d**2*q*x**2 + a**2*d**2*x**2 + a*b*c**2*n
**2*p**2*x + a*b*c**2*n*p**2*x - 2*a*b*c**2*n*p*x - a*b*c**2*p*x + a*b*c**
2*x + 2*a*b*c*d*n**2*p**2*x**2 + a*b*c*d*n*p**2*x**2 + a*b*c*d*n*p*q*x**2
- 4*a*b*c*d*n*p*x**2 - a*b*c*d*p*x**2 - a*b*c*d*q*x**2 + 2*a*b*c*d*x**2 +
a*b*d**2*n**2*p**2*x**3 + a*b*d**2*n*p*q*x**3 - 2*a*b*d**2*n*p*x**3 - a*b*
d**2*q*x**3 + a*b*d**2*x**3 + b**2*c**2*n**2*p**2*x**2 + b**2*c**2*n*p*...
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
           return 1
        else:
           return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
           return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
           if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
              return 1
           else:
              return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
           return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
     elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
           return 1
        else:
           return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
     else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
     m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
     m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
     return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
     return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
     m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
     return max(4,m1)   #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
     m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
     return max(5,m1)   #max(5,m1)
  elif is_appell_function(expn.operator()):
     m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
     return max(6,m1)   #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
           #is checked before calling the grading function that is passed.
           #but kept it here.
     m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
     return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
     return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
     print ("Enter grade_antiderivative for sagemath")
     print("Enter grade_antiderivative, result=",result)
     print("Enter grade_antiderivative, optimal=",optimal)
     print("type(anti)=",type(result))
     print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file