Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.7-Improper-general-binomial/86-1.1.7.1

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3.267 $\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx$ 2286

3.268 $\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx$ . . . 2292
3.269 $\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx$ . . . 2298
3.270 $\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx$ . . . 2304
3.271 $\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx$ . . . 2310
3.272 $\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx$ . . . 2318
3.273 $\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx$ . . . 2325
3.274 $\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx$ . . . 2333
3.275 $\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx$ . . . 2340
3.276 $\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx$ . . . 2349
3.277 $\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx$ . . . 2354
3.278 $\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx$ . . . 2361
3.279 $\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx$ . . . 2369
3.280 $\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx$ . . . 2374
3.281 $\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx$ . . . 2380
3.282 $\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx$ . . . 2389
3.283 $\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx$ . . . 2394
3.284 $\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx$ . . . 2401
3.285 $\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx$ . . . 2411
3.286 $\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx$ . . . 2416
3.287 $\int \frac{x}{ax^3+bx^4}\,dx$ . . . 2423
3.288 $\int \frac{1}{ax^3+bx^4}\,dx$ . . . 2428
3.289 $\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx$ . . . 2433
3.290 $\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx$ . . . 2439
3.291 $\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx$ . . . 2445
3.292 $\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx$ . . . 2451
3.293 $\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx$ . . . 2456
3.294 $\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx$ . . . 2461
3.295 $\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx$ . . . 2466
3.296 $\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx$ . . . 2472
3.297 $\int \frac{1}{x^3+bx^5}\,dx$ . . . 2478
3.298 $\int \frac{1}{-x^3+bx^5}\,dx$ . . . 2483
3.299 $\int \frac{1}{ax+bx}\,dx$ . . . 2488
3.300 $\int \frac{1}{(ax+bx)^2}\,dx$ . . . 2493
3.301 $\int \frac{1}{(ax+bx)^3}\,dx$ . . . 2498
3.302 $\int \frac{1}{ax^2+bx^2}\,dx$ . . . 2503
3.303 $\int \frac{1}{ax^n+bx^n}\,dx$ . . . 2508

3.304 $\int \frac{1}{(ax^n+bx^n)^2}\,dx$ . . . 2513
3.305 $\int \frac{1}{(ax^n+bx^n)^3}\,dx$ . . . 2518
3.306 $\int (ax+bx^{14})^{12}\,dx$ . . . 2523
3.307 $\int x^{12}(ax+bx^{26})^{12}\,dx$ . . . 2530
3.308 $\int x^{24}(ax+bx^{38})^{12}\,dx$ . . . 2537
3.309 $\int x^{12(-1+m)}(ax+bx^{2+12m})^{12}\,dx$ . . . 2544
3.310 $\int (ax+bx^{14})^{12}\,dx$ . . . 2551
3.311 $\int (ax^2+bx^{27})^{12}\,dx$ . . . 2558
3.312 $\int (ax^3+bx^{40})^{12}\,dx$ . . . 2565
3.313 $\int (ax^m+bx^{1+13m})^{12}\,dx$ . . . 2572
3.314 $\int (ax^m+bx^{1+6m})^5\,dx$ . . . 2580
3.315 $\int \frac{1}{(bx^{1-2m}+ax^m)^3}\,dx$ . . . 2586
3.316 $\int \frac{1}{\frac{b}{x}+ax}\,dx$ . . . 2591
3.317 $\int \frac{1}{\frac{b}{x^2}+ax}\,dx$ . . . 2596
3.318 $\int \frac{1}{\frac{b}{x^3}+ax}\,dx$ . . . 2601
3.319 $\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx$ . . . 2606
3.320 $\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx$ . . . 2611
3.321 $\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx$ . . . 2616
3.322 $\int \left(\frac{a}{x}+bx\right)^2\,dx$ . . . 2621
3.323 $\int \left(\frac{a}{x}+bx\right)^3\,dx$ . . . 2626
3.324 $\int \left(\frac{a}{x}+bx\right)^4\,dx$ . . . 2631
3.325 $\int \frac{x}{\frac{1}{x}+x}\,dx$ . . . 2636
3.326 $\int \frac{1}{\frac{1}{x^2}+x^3}\,dx$ . . . 2641
3.327 $\int x^p(ax^n+bx^{1+13n+p})^{12}\,dx$ . . . 2651
3.328 $\int x^{12}(a+bx^{13})^{12}\,dx$ . . . 2659
3.329 $\int x^{12}(ax+bx^{26})^{12}\,dx$ . . . 2665
3.330 $\int x^{12}(ax^2+bx^{39})^{12}\,dx$ . . . 2672
3.331 $\int x^{24}(a+bx^{25})^{12}\,dx$ . . . 2679
3.332 $\int x^{24}(ax+bx^{38})^{12}\,dx$ . . . 2685
3.333 $\int x^{36}(a+bx^{37})^{12}\,dx$ . . . 2692
3.334 $\int \frac{1}{ax+bx^n}\,dx$ . . . 2698
3.335 $\int \frac{1}{ax+bx^{1+n}}\,dx$ . . . 2703
3.336 $\int \frac{1}{ax+bx^{1-n}}\,dx$ . . . 2709
3.337 $\int \frac{1}{2x+3x^{1+n}}\,dx$ . . . 2714
3.338 $\int \frac{1}{2x+3x^{1-n}}\,dx$ . . . 2720

3.339 $\int \frac{1}{-\sqrt{x}+x}\,dx$ . . . 2725
3.340 $\int \frac{1}{-x^{3/5}+x}\,dx$ . . . 2730
3.341 $\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx$ . . . 2735
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3.343 $\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx$ . . . 2746
3.344 $\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx$ . . . 2751
3.345 $\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx$ . . . 2757
3.346 $\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx$ . . . 2762
3.347 $\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx$ . . . 2767
3.348 $\int \frac{\sqrt{a+bx^n}}{cx}\,dx$ . . . 2772
3.349 $\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx$ . . . 2778
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3.351 $\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx$ . . . 2788
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3.354 $\int \frac{\left(ax^2+bx^n\right)^{3/2}}{c^4x^4}\,dx$ . . . 2805
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3.359 $\int (cx)^{7/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}\,dx$ . . . 2835
3.360 $\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx$ . . . 2841
3.361 $\int \sqrt{\frac{a+bx}{x^2}}\,dx$ . . . 2847
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3.363 $\int \sqrt{\frac{a+bx^3}{x^2}}\,dx$ . . . 2859
3.364 $\int \sqrt{\frac{a+bx^n}{x^2}}\,dx$ . . . 2865
3.365 $\int \sqrt{\frac{-a+bx}{x^2}}\,dx$ . . . 2871
3.366 $\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx$ . . . 2877
3.367 $\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx$ . . . 2883
3.368 $\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx$ . . . 2889
3.369 $\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx$ . . . 2895

3.370 $\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx$ . . . 2900
3.371 $\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx$ . . . 2905
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3.377 $\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx$ . . . 2935
3.378 $\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}}\,dx$ . . . 2941
3.379 $\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx$ . . . 2946
3.380 $\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx$ . . . 2951
3.381 $\int \frac{1}{cx(a+bx^n)^{3/2}}\,dx$ . . . 2956
3.382 $\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx$ . . . 2962
3.383 $\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx$ . . . 2967
3.384 $\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx$ . . . 2972
3.385 $\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx$ . . . 2977
3.386 $\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx$ . . . 2982
3.387 $\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx$ . . . 2988
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3.390 $\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx$ . . . 3004
3.391 $\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx$ . . . 3010
3.392 $\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx$ . . . 3015
3.393 $\int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}}\,dx$ . . . 3021
3.394 $\int \frac{1}{\sqrt{x^n(a+bx^{2-n})}}\,dx$ . . . 3026
3.395 $\int \frac{1}{\sqrt{x^2(b+ax^{-2+n})}}\,dx$ . . . 3031
3.396 $\int \frac{1}{\sqrt{x(bx+ax^{-1+n})}}\,dx$ . . . 3036
3.397 $\int \frac{1}{\sqrt{x^n(a-bx^{2-n})}}\,dx$ . . . 3041
3.398 $\int \frac{1}{\sqrt{x^2(-b+ax^{-2+n})}}\,dx$ . . . 3046

3.399 $\int \frac{1}{\sqrt{x(-bx+ax^{-1+n})}}\,dx$ 3051
3.400 $\int (cx)^m \left(ax^j+bx^n\right)^{3/2}\,dx$ 3056
3.401 $\int (cx)^m \sqrt{ax^j+bx^n}\,dx$ 3062
3.402 $\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx$ 3067
3.403 $\int \frac{(cx)^m}{(ax^j+bx^n)^{3/2}}\,dx$ 3072
3.404 $\int \frac{(cx)^m}{(ax^j+bx^n)^{5/2}}\,dx$ 3077
3.405 $\int \left(ax^j+bx^n\right)^{3/2}\,dx$ 3082
3.406 $\int \sqrt{ax^j+bx^n}\,dx$ 3088
3.407 $\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx$ 3093
3.408 $\int \frac{1}{(ax^j+bx^n)^{3/2}}\,dx$ 3098
3.409 $\int \frac{1}{(ax^j+bx^n)^{5/2}}\,dx$ 3103
3.410 $\int \sqrt{\frac{1+x}{x^5}}\,dx$ 3108
3.411 $\int \sqrt{x+x^{5/2}}\,dx$ 3113
3.412 $\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx$ 3118
3.413 $\int x\sqrt{x^2\left(a+bx^3\right)}\,dx$ 3123
3.414 $\int x\sqrt{ax^2+bx^5}\,dx$ 3128
3.415 $\int \sqrt{x^4\left(a+bx^3\right)}\,dx$ 3133
3.416 $\int (cx)^m \left(ax^q+bx^r\right)^3\,dx$ 3138
3.417 $\int (cx)^m \left(ax^q+bx^r\right)^2\,dx$ 3147
3.418 $\int (cx)^m \left(ax^q+bx^r\right)\,dx$ 3155
3.419 $\int \frac{(cx)^m}{ax^q+bx^r}\,dx$ 3161
3.420 $\int \frac{(cx)^m}{(ax^q+bx^r)^2}\,dx$ 3166
3.421 $\int \frac{(cx)^m}{(ax^q+bx^r)^3}\,dx$ 3171
3.422 $\int x^m(ax^j+bx^n)^p\,dx$ 3176
3.423 $\int x^{-1-pq}(bx^n+ax^q)^p\,dx$ 3181
3.424 $\int x^{-1-np}(bx^n+ax^q)^p\,dx$ 3186
3.425 $\int x^{-1-n-(-1+p)q}(bx^n+ax^q)^p\,dx$ 3191
3.426 $\int x^{-1-n(-1+p)-q}(bx^n+ax^q)^p\,dx$ 3196
3.427 $\int \frac{(-\frac{1}{x}+x)^p}{x}\,dx$ 3201
3.428 $\int \left(ax^m+bx^{1+m+mp}\right)^p\,dx$ 3206
3.429 $\int \left(x^m(a+bx^{1+mp})\right)^p\,dx$ 3210
3.430 $\int x^n(x^m(a+bx^{1+n+mp}))^p\,dx$ 3215
3.431 $\int x^n(ax^m+bx^{1+m+n+mp})^p\,dx$ 3220
3.432 $\int \sqrt{x^{2(-1+n)}\left(a+bx^n\right)}\,dx$ 3224
3.433 $\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^n\right)}\,dx$ 3229
3.434 $\int \sqrt[4]{x^{4(-1+n)}\left(a+bx^n\right)}\,dx$ 3234

3.435 $\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}}\,dx$ 3239
3.436 $\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p\,dx$ 3244
3.437 $\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx$ 3249
3.438 $\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx$ 3253

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CHAPTER 1

INTRODUCTION

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**438**]. This is test number [86].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (438)	0.00 (0)
Mathematica	100.00 (438)	0.00 (0)
Maple	84.02 (368)	15.98 (70)
Fricas	73.06 (320)	26.94 (118)
Reduce	62.56 (274)	37.44 (164)
Giac	57.08 (250)	42.92 (188)
Mupad	45.43 (199)	54.57 (239)
Maxima	37.44 (164)	62.56 (274)
Sympy	29.45 (129)	70.55 (309)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

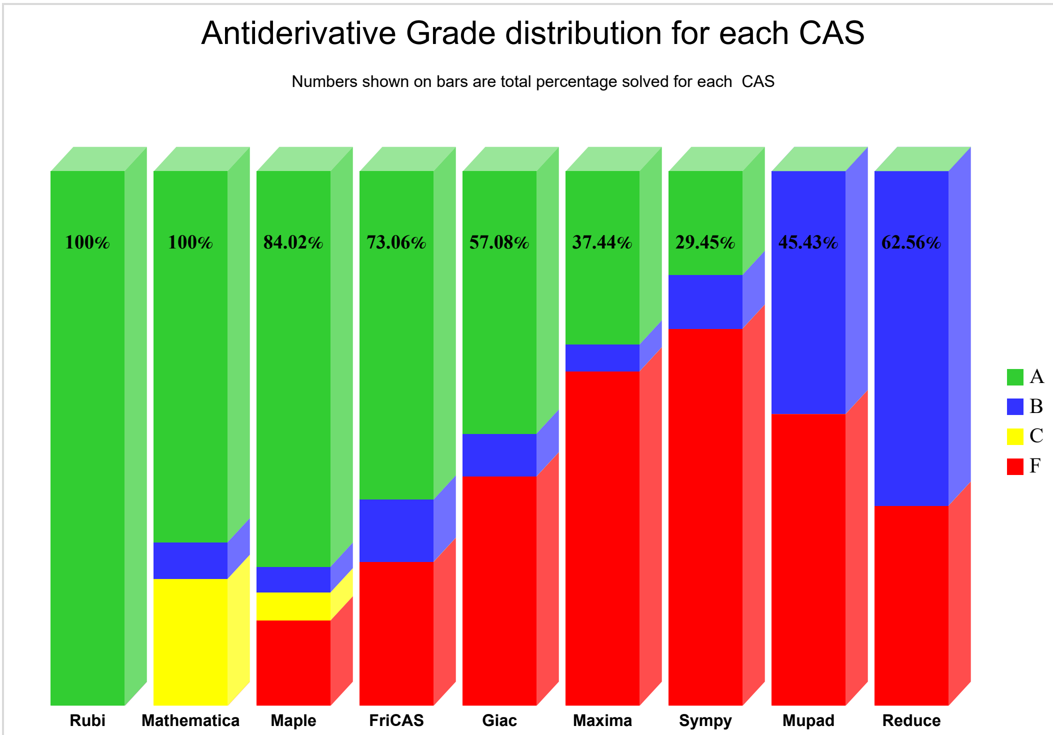
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

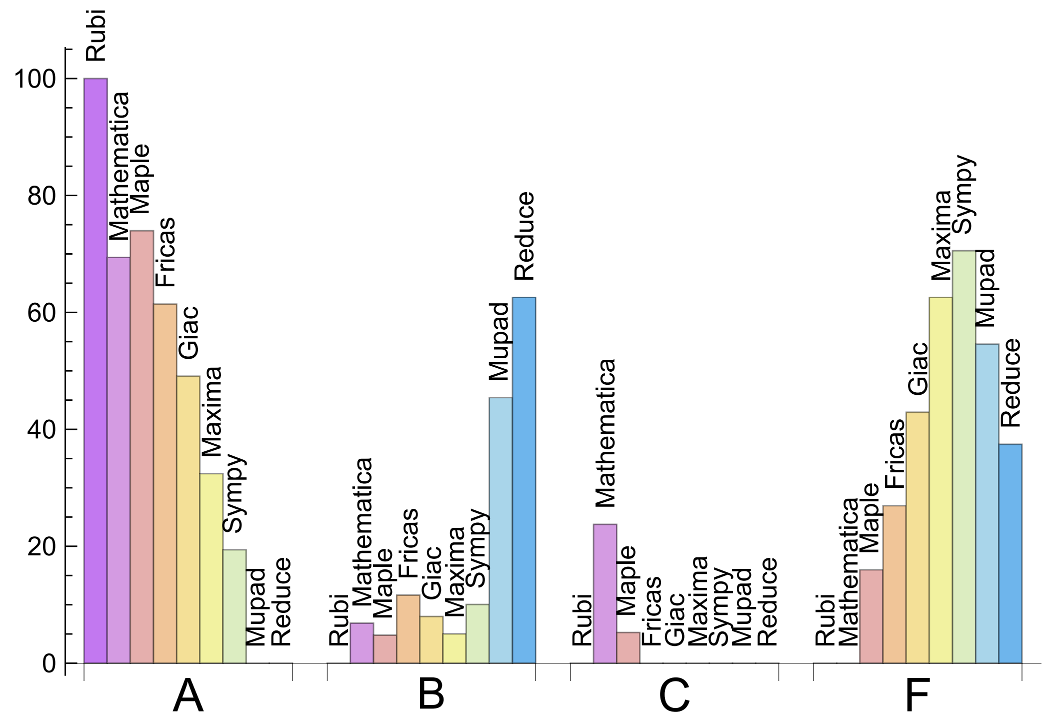
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Maple	73.973	4.795	5.251	15.982
Mathematica	69.406	6.849	23.744	0.000
Fricas	61.416	11.644	0.000	26.941
Giac	49.087	7.991	0.000	42.922
Maxima	32.420	5.023	0.000	62.557
Sympy	19.406	10.046	0.000	70.548
Mupad	0.000	45.434	0.000	54.566
Reduce	0.000	62.557	0.000	37.443

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	70	100.00	0.00	0.00
Fricas	118	46.61	15.25	38.14
Reduce	164	100.00	0.00	0.00
Giac	188	97.87	0.00	2.13
Mupad	239	0.00	100.00	0.00
Maxima	274	100.00	0.00	0.00
Sympy	309	91.91	8.09	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.04
Reduce	0.22
Giac	0.31
Rubi	0.45
Maple	0.99
Sympy	1.55
Mathematica	2.50
Mupad	5.85
Fricas	7.28

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	41.55	1.60	26.00	0.91
Mupad	45.86	1.44	34.00	0.88
Mathematica	65.39	1.16	58.00	0.95
Reduce	72.66	1.51	43.00	0.91
Rubi	122.16	1.04	68.50	1.00
Maple	125.05	1.27	55.50	0.88
Fricas	128.49	1.87	54.00	1.06
Sympy	220.79	4.22	26.00	0.89
Giac	549.39	17.86	48.00	0.97

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

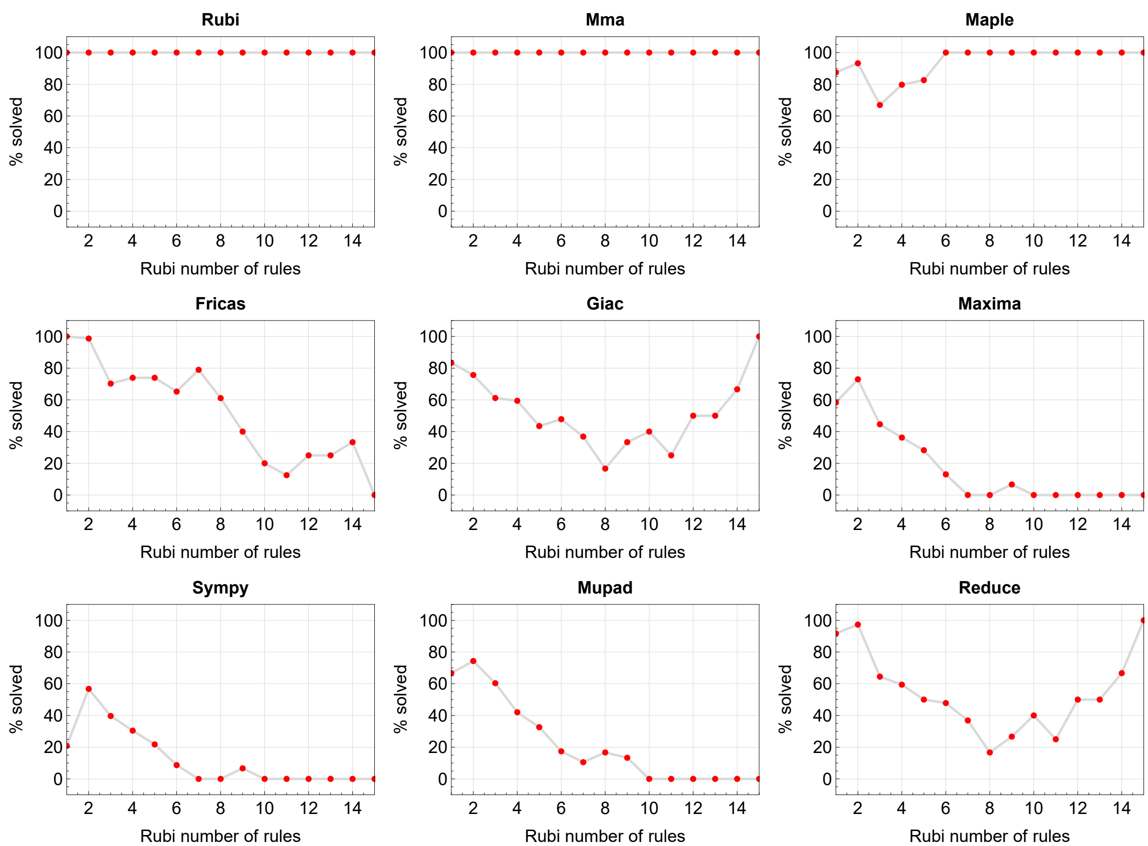


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

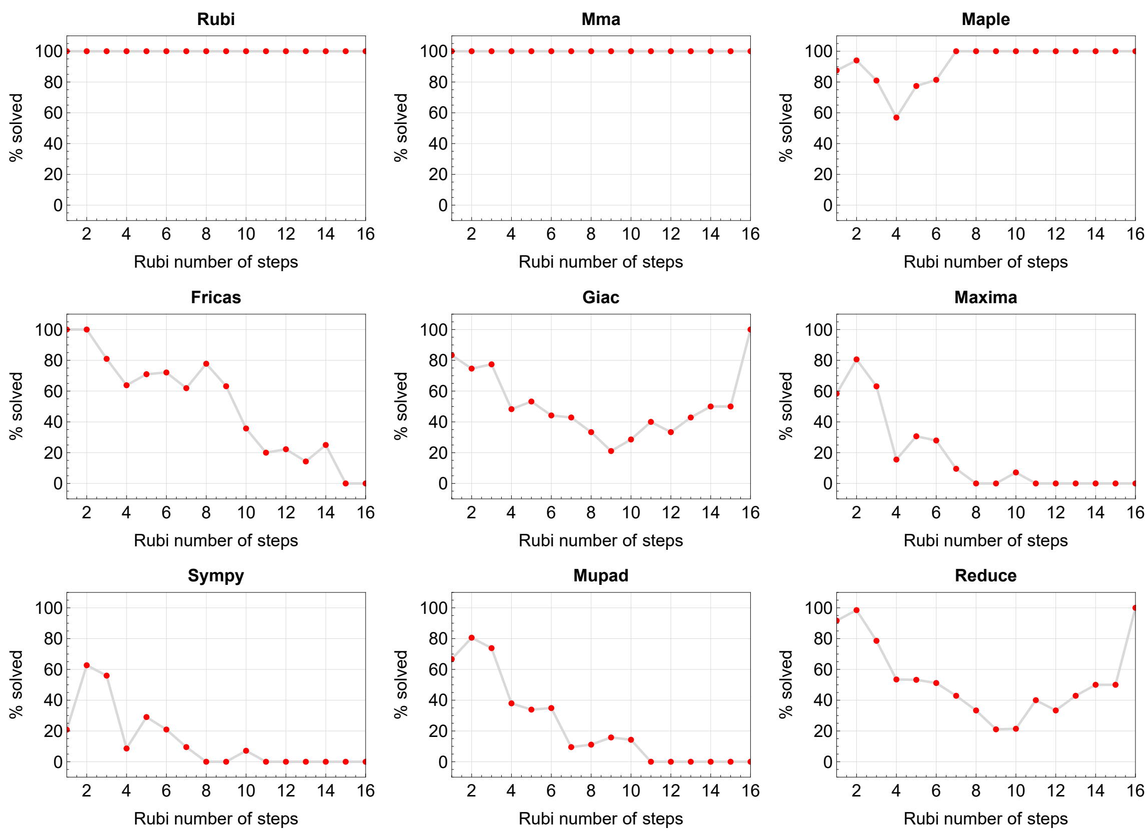


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

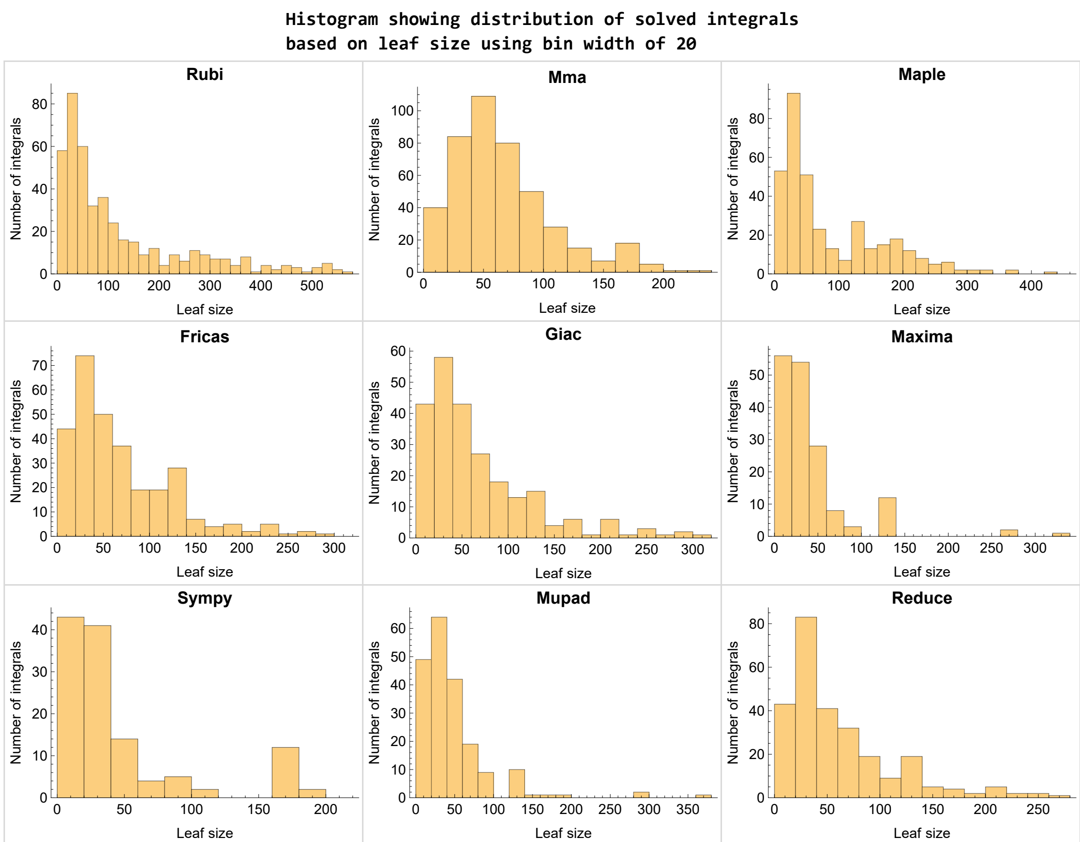


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

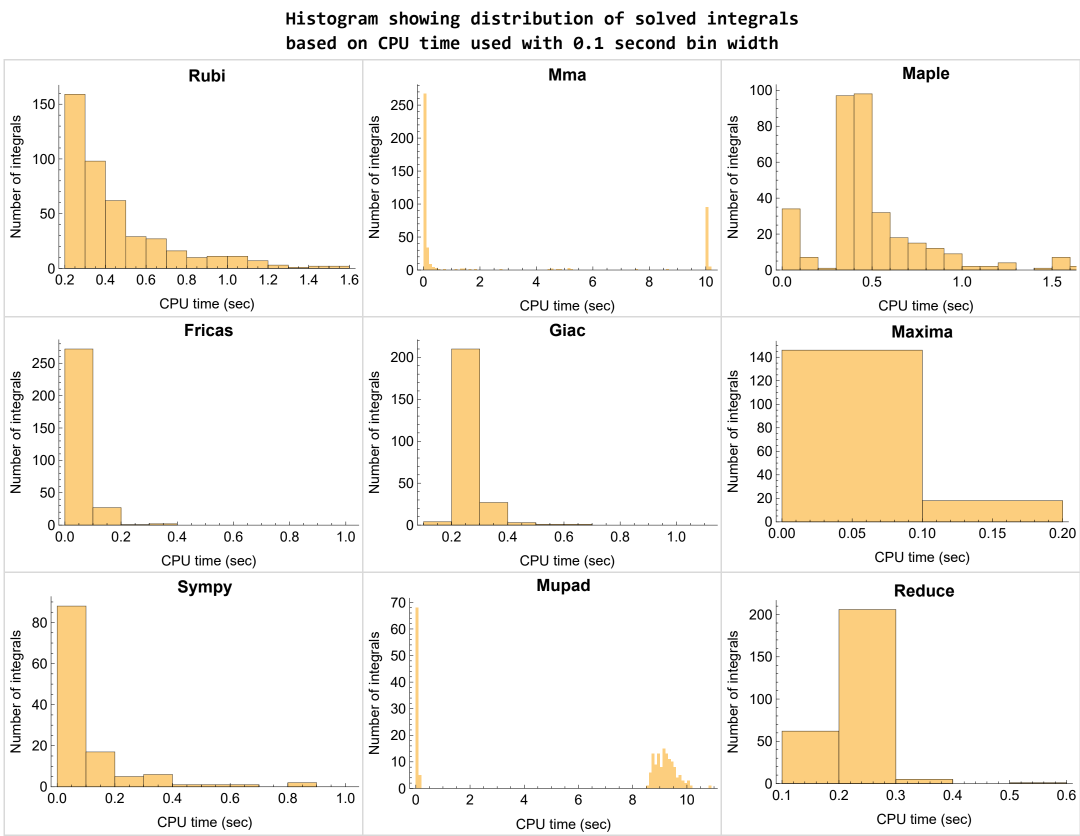


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

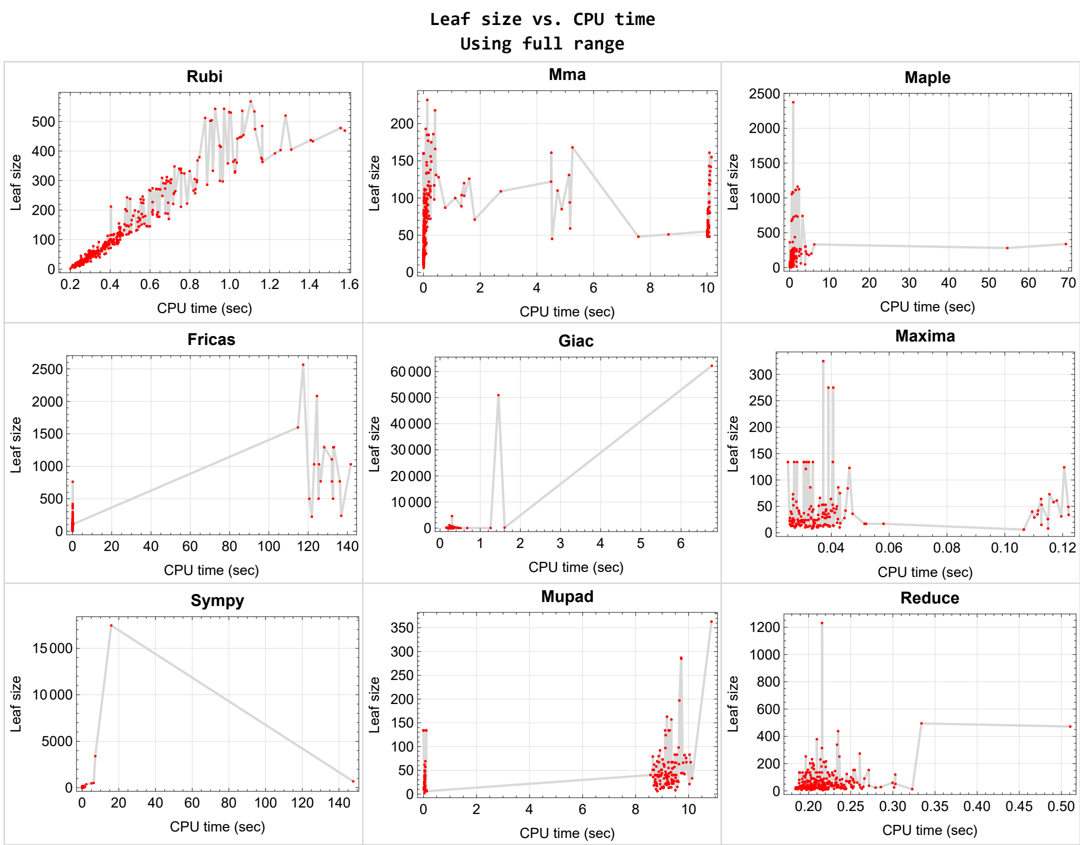


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 271, 272, 273}

Mathematica {}

Maple $\{4, 5, 6, 7, 8, 12, 13, 14, 15, 16, 17, 185, 186, 190, 191, 322, 323, 324, 416, 417\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

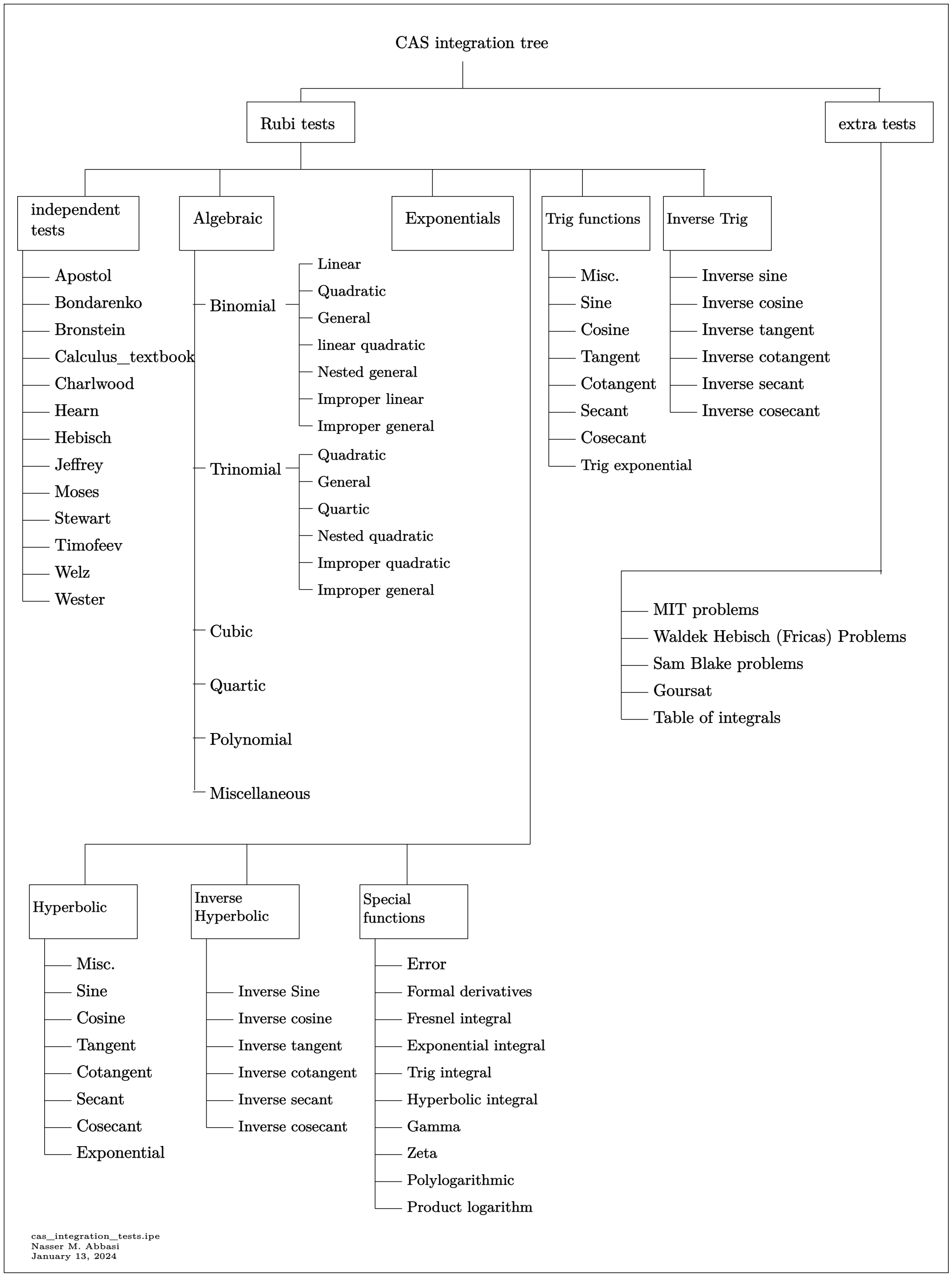
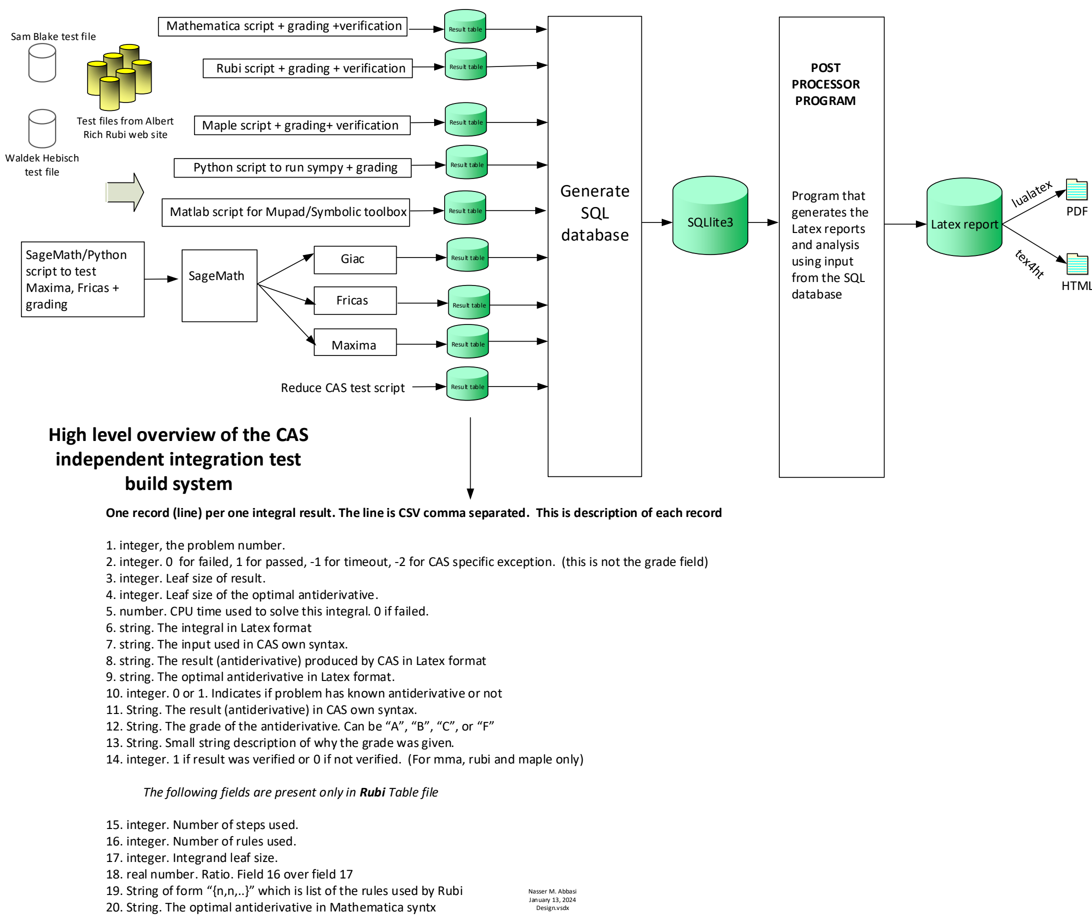


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS 37
2.2 Detailed conclusion table per each integral for all CAS systems 46
2.3 Detailed conclusion table specific for Rubi results 156

2.1 List of integrals sorted by grade for each CAS

Rubi 37
Mma 38
Maple 39
Fricas 40
Maxima 41
Giac 42
Mupad 43
Sympy 44
Reduce 45

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 36, 38, 40, 42, 43, 44, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 276, 279, 282, 285, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 309, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438 }

B grade { 33, 35, 37, 39, 41, 306, 307, 308, 310, 311, 312, 313, 314, 327, 328, 329, 330, 331, 332, 333, 371, 389, 393, 394, 395, 396, 397, 398, 399, 400 }

C grade { 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 160, 161, 162, 163, 179, 180, 181, 268, 269, 270, 271, 272, 273, 274, 275, 277, 278, 280, 281, 283, 284, 286, 390, 391, 392 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 36, 37, 38, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 276, 282, 285, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 328, 331, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 348, 356, 361, 362, 363, 364, 365, 367, 368, 373, 381, 387, 391, 410, 411, 412, 413, 414, 415, 432, 433, 434 }

B grade { 93, 251, 279, 306, 307, 308, 309, 310, 311, 312, 313, 314, 327, 329, 330, 332, 366, 386, 390, 418, 438 }

C grade { 33, 35, 39, 41, 104, 105, 106, 107, 108, 109, 222, 274, 275, 277, 278, 280, 281, 283, 284, 286, 326, 416, 417 }

F normal fail { 256, 257, 258, 343, 344, 345, 346, 347, 349, 350, 351, 352, 353, 354, 355, 357, 358, 359, 360, 369, 370, 371, 372, 374, 375, 376, 377, 378, 379, 380, 382, 383, 384, 385, 388, 389, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 435, 436, 437 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 38, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 109, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 259, 260, 261, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 276, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 302, 303, 304, 316, 317, 318, 322, 323, 324, 325, 326, 334, 335, 336, 337, 338, 339, 340, 341, 342, 348, 356, 361, 362, 363, 364, 365, 366, 367, 368, 373, 381, 387, 388, 389, 391, 392, 393, 395, 398, 410, 411, 412, 413, 414, 415, 418, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438 }

B grade { 35, 37, 39, 41, 86, 91, 146, 147, 148, 149, 150, 155, 156, 157, 158, 164, 165, 166, 167, 168, 173, 174, 175, 176, 262, 301, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 319, 320, 321, 327, 328, 329, 330, 331, 332, 333, 386, 390, 416, 417 }

C grade { }

F normal fail { 104, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 256, 257, 258, 274, 275, 277, 278, 419, 420, 421, 422, 423, 424, 425, 426, 427 }

F(-1) timedout fail { 151, 152, 153, 154, 159, 160, 161, 162, 163, 169, 170, 171, 172, 177, 178, 179, 180, 181 }

F(-2) exception fail { 343, 344, 345, 346, 347, 349, 350, 351, 352, 353, 354, 355, 357, 358, 359, 360, 369, 370, 371, 372, 374, 375, 376, 377, 378, 379, 380, 382, 383, 384, 385, 394, 396, 397, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 38, 40, 41, 42, 43, 44, 101, 102, 103, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 219, 220, 221, 222, 223, 224, 231, 232, 233, 234, 239, 240, 241, 242, 263, 264, 265, 282, 285, 287, 288, 297, 298, 299, 300, 301, 302, 303, 304, 316, 317, 318, 322, 323, 324, 325, 326, 328, 331, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 348, 356, 362, 366, 373, 381, 412, 413, 414, 415, 416, 417, 418, 432, 433, 434 }

B grade { 35, 37, 39, 262, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 319, 320, 321, 327, 329, 330, 332 }

C grade { }

F normal fail { 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 215, 216, 217, 218, 225, 226, 227, 228, 229, 230, 235, 236, 237, 238, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 283, 284, 286, 289, 290, 291, 292, 293, 294, 295, 296, 343, 344, 345, 346, 347, 349, 350, 351, 352, 353, 354, 355, 357, 358, 359, 360, 361, 363, 364, 365, 367, 368, 369, 370, 371, 372, 374, 375, 376, 377, 378, 379, 380, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 435, 436, 437, 438 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 36, 38, 40, 42, 43, 44, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 105, 146, 147, 148, 149, 150, 151, 152, 153, 154, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 215, 216, 217, 218, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 262, 263, 264, 265, 266, 267, 276, 279, 282, 285, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 328, 331, 333, 339, 340, 361, 363, 365, 366, 367, 387, 391, 411, 412, 413, 414, 415 }

B grade { 33, 35, 37, 39, 41, 155, 156, 157, 158, 214, 219, 220, 221, 222, 223, 224, 306, 307, 308, 309, 310, 311, 312, 313, 314, 327, 329, 330, 332, 341, 362, 410, 416, 417, 418 }

C grade { }

F normal fail { 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 256, 257, 258, 259, 260, 261, 268, 269, 270, 271, 272, 273, 274, 275, 277, 278, 280, 281, 283, 284, 286, 303, 304, 305, 315, 334, 335, 336, 337, 338, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 364, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 389, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438 }

F(-1) timedout fail { }

F(-2) exception fail { 386, 388, 390, 392 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 48, 55, 68, 79, 101, 102, 103, 105, 113, 121, 132, 141, 149, 157, 168, 177, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 215, 219, 220, 221, 222, 223, 224, 231, 232, 233, 234, 237, 239, 240, 241, 242, 245, 247, 259, 260, 261, 262, 263, 264, 265, 270, 287, 288, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 350, 361, 362, 363, 365, 366, 367, 371, 394, 395, 396, 397, 398, 399, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 428 }

C grade { }

F normal fail { }

F(-1) timedout fail { 45, 46, 47, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 104, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 214, 216, 217, 218, 225, 226, 227, 228, 229, 230, 235, 236, 238, 243, 244, 246, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 289, 290, 291, 292, 343, 344, 345, 346, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 364, 368, 369, 370, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 400, 401, 402, 403, 404, 419, 420, 421, 422, 423, 424, 425, 426, 427, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 21, 23, 25, 27, 29, 30, 31, 32, 34, 36, 38, 40, 41, 42, 43, 44, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 287, 288, 297, 298, 299, 300, 302, 316, 317, 318, 322, 323, 324, 325, 326, 339, 341, 342, 348, 356, 373, 412 }

B grade { 12, 20, 22, 24, 26, 28, 33, 35, 37, 39, 262, 301, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 319, 320, 321, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 340, 381, 416, 417, 418 }

C grade { }

F normal fail { 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 289, 290, 291, 292, 293, 294, 295, 296, 343, 344, 345, 346, 347, 349, 350, 351, 352, 354, 355, 357, 358, 360, 361, 362, 364, 365, 366, 368, 369, 370, 371, 372, 374, 375, 376, 377, 379, 380, 382, 383, 385, 386, 387, 389, 390, 391, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 414, 415, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 431, 435, 436, 437 }

F(-1) timedout fail { 82, 83, 84, 85, 86, 87, 88, 89, 127, 163, 315, 353, 359, 363, 367, 378, 384, 388, 392, 413, 430, 432, 433, 434, 438 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 259, 260, 261, 262, 263, 264, 265, 266, 267, 276, 279, 282, 285, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 361, 362, 363, 365, 366, 367, 386, 387, 388, 390, 391, 392, 410, 411, 412, 413, 414, 415, 416, 417, 418, 429, 430, 432, 433, 434, 435, 436, 437, 438 }

C grade { }

F normal fail { 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 256, 257, 258, 268, 269, 270, 271, 272, 273, 274, 275, 277, 278, 280, 281, 283, 284, 286, 326, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 364, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 389, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 431 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	15	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.88	0.76
time (sec)	N/A	0.240	0.002	0.164	0.033	0.060	0.016	0.256	0.196	0.025

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	15	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.88	0.76
time (sec)	N/A	0.241	0.002	0.085	0.036	0.064	0.028	0.255	0.203	0.023

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	14	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.82	0.76
time (sec)	N/A	0.231	0.000	0.083	0.041	0.066	0.018	0.282	0.184	0.020

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	12	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	1.00	0.83
time (sec)	N/A	0.229	0.000	0.036	0.034	0.074	0.017	0.243	0.219	0.018

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	14	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	1.08	0.85	0.85
time (sec)	N/A	0.243	0.002	0.038	0.033	0.067	0.031	0.262	0.203	0.025

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	13	5	10	13	10
N.S.	1	1.00	1.00	1.10	1.00	1.30	0.50	1.00	1.30	1.00
time (sec)	N/A	0.247	0.001	0.040	0.043	0.065	0.032	0.263	0.205	0.026

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	17	10	20	17	11
N.S.	1	1.00	1.00	0.92	0.85	1.31	0.77	1.54	1.31	0.85
time (sec)	N/A	0.242	0.002	0.035	0.032	0.068	0.045	0.247	0.190	0.043

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	14	13	15	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.93	0.87	1.00	0.87
time (sec)	N/A	0.251	0.003	0.036	0.036	0.062	0.049	0.231	0.202	0.025

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	26	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.87	0.80
time (sec)	N/A	0.269	0.001	0.370	0.030	0.074	0.017	0.240	0.186	9.246

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	34	30	25	24	24	24	24	26	24
N.S.	1	1.13	1.00	0.83	0.80	0.80	0.80	0.80	0.87	0.80
time (sec)	N/A	0.283	0.001	0.381	0.027	0.065	0.016	0.214	0.199	0.036

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	26	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.87	0.80
time (sec)	N/A	0.284	0.004	0.379	0.034	0.073	0.017	0.221	0.238	0.033

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	24	24	24	24	25	24
N.S.	1	1.00	1.00	0.94	1.50	1.50	1.50	1.50	1.56	1.50
time (sec)	N/A	0.227	0.003	0.368	0.034	0.062	0.021	0.254	0.301	0.036

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	24	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.96	0.84
time (sec)	N/A	0.260	0.002	0.342	0.029	0.066	0.018	0.276	0.279	0.032

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	23	30	23	22	21	21	20	24	21	21
N.S.	1	1.30	1.00	0.96	0.91	0.91	0.87	1.04	0.91	0.91
time (sec)	N/A	0.266	0.001	0.389	0.029	0.073	0.036	0.227	0.212	0.032

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	25	19	22	25	22
N.S.	1	1.00	1.00	0.96	0.92	1.04	0.79	0.92	1.04	0.92
time (sec)	N/A	0.267	0.001	0.328	0.026	0.063	0.036	0.285	0.196	0.037

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	27	28	27	24	23	27	24	32	27	23
N.S.	1	1.04	1.00	0.89	0.85	1.00	0.89	1.19	1.00	0.85
time (sec)	N/A	0.265	0.002	0.318	0.027	0.067	0.054	0.288	0.201	9.155

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	22	26	22	22	26	24
N.S.	1	1.00	1.00	0.96	0.96	1.13	0.96	0.96	1.13	1.04
time (sec)	N/A	0.260	0.001	0.322	0.026	0.065	0.061	0.268	0.194	0.030

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	37	36	36	42	36	37	36
N.S.	1	1.00	1.00	0.80	0.78	0.78	0.91	0.78	0.80	0.78
time (sec)	N/A	0.309	0.003	0.377	0.026	0.075	0.027	0.271	0.190	0.042

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	26	27	23	23	22	20	24	22	22
N.S.	1	0.96	1.00	0.85	0.85	0.81	0.74	0.89	0.81	0.81
time (sec)	N/A	0.281	0.005	0.418	0.026	0.064	0.062	0.248	0.198	9.378

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	26	82	56	26	26	23
N.S.	1	1.00	1.00	0.87	0.84	2.65	1.81	0.84	0.84	0.74
time (sec)	N/A	0.254	0.012	0.419	0.115	0.077	0.062	0.244	0.261	9.228

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87	0.87
time (sec)	N/A	0.234	0.004	0.394	0.026	0.066	0.043	0.257	0.187	9.249

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	23	16
N.S.	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.96	0.67
time (sec)	N/A	0.236	0.004	0.428	0.113	0.077	0.055	0.217	0.206	0.060

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	26	22	21	20	18	15	24	20	18
N.S.	1	1.18	1.00	0.95	0.91	0.82	0.68	1.09	0.91	0.82
time (sec)	N/A	0.279	0.005	0.385	0.027	0.079	0.088	0.233	0.199	0.065

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	30	29	82	65	29	30	26
N.S.	1	1.00	1.00	0.88	0.85	2.41	1.91	0.85	0.88	0.76
time (sec)	N/A	0.257	0.012	0.431	0.110	0.078	0.077	0.260	0.197	0.048

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	36	35	32	31	33	31	43	33	31
N.S.	1	1.03	1.00	0.91	0.89	0.94	0.89	1.23	0.94	0.89
time (sec)	N/A	0.302	0.006	0.404	0.030	0.074	0.130	0.245	0.214	0.064

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	51	43	39	40	106	87	40	43	37
N.S.	1	1.19	1.00	0.91	0.93	2.47	2.02	0.93	1.00	0.86
time (sec)	N/A	0.275	0.019	0.405	0.109	0.076	0.131	0.235	0.199	0.051

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	50	49	44	44	45	42	57	47	46
N.S.	1	1.02	1.00	0.90	0.90	0.92	0.86	1.16	0.96	0.94
time (sec)	N/A	0.323	0.007	0.408	0.038	0.073	0.206	0.289	0.208	9.334

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	35	120	78	35	61	33
N.S.	1	1.00	1.00	0.80	0.78	2.67	1.73	0.78	1.36	0.73
time (sec)	N/A	0.253	0.019	0.401	0.111	0.077	0.092	0.236	0.196	9.188

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	39	33	35	34	47	34	47	59	34
N.S.	1	1.03	0.87	0.92	0.89	1.24	0.89	1.24	1.55	0.89
time (sec)	N/A	0.299	0.015	0.402	0.035	0.072	0.130	0.246	0.248	0.049

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	61	54	45	49	136	92	47	72	44
N.S.	1	1.13	1.00	0.83	0.91	2.52	1.70	0.87	1.33	0.81
time (sec)	N/A	0.292	0.029	0.438	0.122	0.088	0.155	0.259	0.202	8.990

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	52	41	52	50	73	51	51	81	51
N.S.	1	1.06	0.84	1.06	1.02	1.49	1.04	1.04	1.65	1.04
time (sec)	N/A	0.327	0.034	0.418	0.033	0.075	0.164	0.238	0.189	8.961

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	78	67	55	64	172	114	59	88	58
N.S.	1	1.16	1.00	0.82	0.96	2.57	1.70	0.88	1.31	0.87
time (sec)	N/A	0.316	0.047	0.422	0.113	0.081	0.159	0.248	0.192	9.081

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	29	21	21	21	19	23	21	11
N.S.	1	1.00	2.23	1.62	1.62	1.62	1.46	1.77	1.62	0.85
time (sec)	N/A	0.251	0.006	0.352	0.029	0.067	0.038	0.243	0.200	8.933

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	15	18	14	14	15	18	14
N.S.	1	1.00	0.90	0.75	0.90	0.70	0.70	0.75	0.90	0.70
time (sec)	N/A	0.265	0.006	0.349	0.034	0.064	0.030	0.249	0.188	0.036

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	22	14	16	16	14	18	16	6
N.S.	1	1.00	3.67	2.33	2.67	2.67	2.33	3.00	2.67	1.00
time (sec)	N/A	0.225	0.004	0.359	0.038	0.066	0.034	0.251	0.199	0.056

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	13	8	8	15	13	8
N.S.	1	1.00	1.00	0.75	1.08	0.67	0.67	1.25	1.08	0.67
time (sec)	N/A	0.215	0.003	0.341	0.028	0.077	0.028	0.274	0.199	0.031

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	19	3	13	13	12	15	13	2
N.S.	1	1.00	9.50	1.50	6.50	6.50	6.00	7.50	6.50	1.00
time (sec)	N/A	0.201	0.003	0.336	0.029	0.068	0.035	0.250	0.187	0.035

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	19	15	12	15	11	10	16	15	11
N.S.	1	1.27	1.00	0.80	1.00	0.73	0.67	1.07	1.00	0.73
time (sec)	N/A	0.256	0.004	0.368	0.038	0.070	0.040	0.267	0.198	0.054

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	24	16	18	20	15	20	20	8
N.S.	1	1.00	3.00	2.00	2.25	2.50	1.88	2.50	2.50	1.00
time (sec)	N/A	0.222	0.004	0.345	0.027	0.065	0.043	0.282	0.201	0.034

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	24	22	17	20	24	17	26	32	16
N.S.	1	1.09	1.00	0.77	0.91	1.09	0.77	1.18	1.45	0.73
time (sec)	N/A	0.269	0.004	0.380	0.027	0.064	0.041	0.254	0.253	0.034

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	31	22	25	30	24	27	30	13
N.S.	1	1.00	2.07	1.47	1.67	2.00	1.60	1.80	2.00	0.87
time (sec)	N/A	0.226	0.004	0.351	0.028	0.073	0.053	0.285	0.209	0.039

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	31	29	23	27	30	22	33	37	23
N.S.	1	1.07	1.00	0.79	0.93	1.03	0.76	1.14	1.28	0.79
time (sec)	N/A	0.268	0.004	0.386	0.027	0.063	0.048	0.275	0.208	0.034

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	19	15	14	13	13	12	18	13	14
N.S.	1	1.27	1.00	0.93	0.87	0.87	0.80	1.20	0.87	0.93
time (sec)	N/A	0.250	0.004	0.372	0.034	0.072	0.057	0.297	0.202	8.983

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	20	18	16	15	15	12	18	25	16
N.S.	1	1.11	1.00	0.89	0.83	0.83	0.67	1.00	1.39	0.89
time (sec)	N/A	0.252	0.005	0.367	0.030	0.064	0.057	0.245	0.207	9.054

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	174	95	158	0	59	0	0	85	0
N.S.	1	1.07	0.58	0.97	0.00	0.36	0.00	0.00	0.52	0.00
time (sec)	N/A	0.542	10.080	0.558	0.000	0.097	0.000	0.000	0.224	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	300	80	197	0	57	0	0	63	0
N.S.	1	1.07	0.28	0.70	0.00	0.20	0.00	0.00	0.22	0.00
time (sec)	N/A	0.683	10.040	0.495	0.000	0.082	0.000	0.000	0.237	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	142	79	146	0	49	0	0	64	0
N.S.	1	1.04	0.58	1.07	0.00	0.36	0.00	0.00	0.47	0.00
time (sec)	N/A	0.447	10.026	0.497	0.000	0.083	0.000	0.000	0.236	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	268	51	175	0	43	0	0	39	40
N.S.	1	1.05	0.20	0.69	0.00	0.17	0.00	0.00	0.15	0.16
time (sec)	N/A	0.597	8.640	0.504	0.000	0.078	0.000	0.000	0.224	8.958

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	48	124	0	34	0	0	40	0
N.S.	1	1.00	0.42	1.10	0.00	0.30	0.00	0.00	0.35	0.00
time (sec)	N/A	0.388	7.578	0.484	0.000	0.082	0.000	0.000	0.272	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	266	51	177	0	40	0	0	45	0
N.S.	1	1.07	0.21	0.71	0.00	0.16	0.00	0.00	0.18	0.00
time (sec)	N/A	0.601	10.013	0.500	0.000	0.077	0.000	0.000	0.246	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	53	123	0	36	0	0	51	0
N.S.	1	1.00	0.46	1.06	0.00	0.31	0.00	0.00	0.44	0.00
time (sec)	N/A	0.406	10.015	0.500	0.000	0.078	0.000	0.000	0.246	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	298	53	195	0	54	0	0	54	0
N.S.	1	1.05	0.19	0.69	0.00	0.19	0.00	0.00	0.19	0.00
time (sec)	N/A	0.693	10.016	0.518	0.000	0.072	0.000	0.000	0.267	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	200	94	169	0	70	0	0	106	0
N.S.	1	1.08	0.51	0.91	0.00	0.38	0.00	0.00	0.57	0.00
time (sec)	N/A	0.614	10.063	0.492	0.000	0.101	0.000	0.000	0.241	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	326	84	210	0	68	0	0	84	0
N.S.	1	1.07	0.28	0.69	0.00	0.22	0.00	0.00	0.28	0.00
time (sec)	N/A	0.758	10.032	0.493	0.000	0.098	0.000	0.000	0.250	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	166	83	158	0	60	0	0	85	40
N.S.	1	1.05	0.53	1.00	0.00	0.38	0.00	0.00	0.54	0.25
time (sec)	N/A	0.508	10.029	0.464	0.000	0.102	0.000	0.000	0.269	8.744

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	291	52	195	0	58	0	0	58	0
N.S.	1	1.06	0.19	0.71	0.00	0.21	0.00	0.00	0.21	0.00
time (sec)	N/A	0.667	10.017	0.510	0.000	0.091	0.000	0.000	0.249	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	139	49	143	0	47	0	0	59	0
N.S.	1	1.04	0.37	1.07	0.00	0.35	0.00	0.00	0.44	0.00
time (sec)	N/A	0.446	10.017	0.476	0.000	0.112	0.000	0.000	0.302	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	290	52	188	0	52	0	0	64	0
N.S.	1	1.06	0.19	0.69	0.00	0.19	0.00	0.00	0.23	0.00
time (sec)	N/A	0.678	10.016	0.509	0.000	0.090	0.000	0.000	0.247	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	137	54	139	0	45	0	0	68	0
N.S.	1	1.02	0.40	1.04	0.00	0.34	0.00	0.00	0.51	0.00
time (sec)	N/A	0.450	10.017	0.491	0.000	0.086	0.000	0.000	0.251	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	292	54	189	0	51	0	0	70	0
N.S.	1	1.05	0.19	0.68	0.00	0.18	0.00	0.00	0.25	0.00
time (sec)	N/A	0.703	10.015	0.553	0.000	0.084	0.000	0.000	0.346	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	142	54	139	0	44	0	0	70	0
N.S.	1	1.04	0.39	1.01	0.00	0.32	0.00	0.00	0.51	0.00
time (sec)	N/A	0.452	10.018	0.549	0.000	0.086	0.000	0.000	0.347	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	324	54	210	0	67	0	0	71	0
N.S.	1	1.06	0.18	0.69	0.00	0.22	0.00	0.00	0.23	0.00
time (sec)	N/A	0.773	10.025	0.612	0.000	0.081	0.000	0.000	0.400	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	171	54	160	0	59	0	0	71	0
N.S.	1	1.05	0.33	0.98	0.00	0.36	0.00	0.00	0.44	0.00
time (sec)	N/A	0.551	10.021	0.700	0.000	0.080	0.000	0.000	0.445	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	148	80	147	0	48	0	0	64	0
N.S.	1	1.06	0.57	1.05	0.00	0.34	0.00	0.00	0.46	0.00
time (sec)	N/A	0.488	10.029	0.515	0.000	0.091	0.000	0.000	0.236	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	274	66	178	0	43	0	0	44	0
N.S.	1	1.06	0.26	0.69	0.00	0.17	0.00	0.00	0.17	0.00
time (sec)	N/A	0.613	10.030	0.505	0.000	0.083	0.000	0.000	0.224	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	64	127	0	34	0	0	45	0
N.S.	1	1.00	0.55	1.09	0.00	0.29	0.00	0.00	0.39	0.00
time (sec)	N/A	0.416	10.039	0.482	0.000	0.088	0.000	0.000	0.228	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	246	53	158	0	22	0	0	22	0
N.S.	1	1.07	0.23	0.69	0.00	0.10	0.00	0.00	0.10	0.00
time (sec)	N/A	0.563	10.018	0.431	0.000	0.084	0.000	0.000	0.207	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	49	108	0	14	0	0	24	40
N.S.	1	1.00	0.53	1.17	0.00	0.15	0.00	0.00	0.26	0.43
time (sec)	N/A	0.343	10.027	0.429	0.000	0.076	0.000	0.000	0.208	8.908

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	272	48	182	0	42	0	0	26	0
N.S.	1	1.08	0.19	0.72	0.00	0.17	0.00	0.00	0.10	0.00
time (sec)	N/A	0.631	10.013	0.503	0.000	0.093	0.000	0.000	0.198	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	53	129	0	36	0	0	26	0
N.S.	1	1.00	0.45	1.08	0.00	0.30	0.00	0.00	0.22	0.00
time (sec)	N/A	0.420	10.013	0.480	0.000	0.075	0.000	0.000	0.210	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	304	53	195	0	56	0	0	26	0
N.S.	1	1.06	0.19	0.68	0.00	0.20	0.00	0.00	0.09	0.00
time (sec)	N/A	0.707	10.017	0.510	0.000	0.081	0.000	0.000	0.216	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	177	80	172	0	80	0	0	149	0
N.S.	1	1.10	0.50	1.07	0.00	0.50	0.00	0.00	0.93	0.00
time (sec)	N/A	0.565	10.101	0.961	0.000	0.099	0.000	0.000	0.353	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	303	68	200	0	76	0	0	125	0
N.S.	1	1.09	0.24	0.72	0.00	0.27	0.00	0.00	0.45	0.00
time (sec)	N/A	0.705	10.024	0.940	0.000	0.090	0.000	0.000	0.271	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	145	67	147	0	68	0	0	128	0
N.S.	1	1.06	0.49	1.07	0.00	0.50	0.00	0.00	0.93	0.00
time (sec)	N/A	0.494	10.023	0.833	0.000	0.095	0.000	0.000	0.252	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	271	57	182	0	61	0	0	105	0
N.S.	1	1.07	0.23	0.72	0.00	0.24	0.00	0.00	0.42	0.00
time (sec)	N/A	0.632	10.022	0.467	0.000	0.087	0.000	0.000	0.280	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	54	130	0	51	0	0	106	0
N.S.	1	1.00	0.47	1.13	0.00	0.44	0.00	0.00	0.92	0.00
time (sec)	N/A	0.401	10.018	0.458	0.000	0.087	0.000	0.000	0.251	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	270	56	184	0	60	0	0	33	0
N.S.	1	1.06	0.22	0.72	0.00	0.24	0.00	0.00	0.13	0.00
time (sec)	N/A	0.651	10.017	0.444	0.000	0.087	0.000	0.000	0.210	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	54	132	0	51	0	0	35	0
N.S.	1	1.00	0.47	1.16	0.00	0.45	0.00	0.00	0.31	0.00
time (sec)	N/A	0.438	10.020	0.438	0.000	0.087	0.000	0.000	0.219	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	297	51	206	0	72	0	0	37	40
N.S.	1	1.09	0.19	0.75	0.00	0.26	0.00	0.00	0.14	0.15
time (sec)	N/A	0.687	10.017	0.814	0.000	0.081	0.000	0.000	0.283	9.529

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	147	56	150	0	68	0	0	37	0
N.S.	1	1.06	0.40	1.08	0.00	0.49	0.00	0.00	0.27	0.00
time (sec)	N/A	0.481	10.017	0.777	0.000	0.095	0.000	0.000	0.234	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	332	56	228	0	90	0	0	37	0
N.S.	1	1.08	0.18	0.75	0.00	0.29	0.00	0.00	0.12	0.00
time (sec)	N/A	0.799	10.016	0.885	0.000	0.087	0.000	0.000	0.241	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	189	128	212	0	382	0	91	339	0
N.S.	1	1.19	0.81	1.33	0.00	2.40	0.00	0.57	2.13	0.00
time (sec)	N/A	0.678	0.533	0.489	0.000	0.105	0.000	0.274	0.234	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	146	68	70	0	108	0	77	98	0
N.S.	1	1.16	0.54	0.56	0.00	0.86	0.00	0.61	0.78	0.00
time (sec)	N/A	0.582	0.060	0.460	0.000	0.085	0.000	0.255	0.196	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	145	117	198	0	354	0	78	314	0
N.S.	1	1.12	0.90	1.52	0.00	2.72	0.00	0.60	2.42	0.00
time (sec)	N/A	0.600	0.374	0.426	0.000	0.105	0.000	0.337	0.216	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	117	64	59	0	97	0	55	87	0
N.S.	1	1.16	0.63	0.58	0.00	0.96	0.00	0.54	0.86	0.00
time (sec)	N/A	0.482	0.050	0.430	0.000	0.085	0.000	0.299	0.207	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	27	0	61	0	17	116	0
N.S.	1	1.00	1.00	1.08	0.00	2.44	0.00	0.68	4.64	0.00
time (sec)	N/A	0.261	0.094	0.415	0.000	0.092	0.000	0.296	0.202	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	84	46	48	0	86	0	41	76	0
N.S.	1	1.11	0.61	0.63	0.00	1.13	0.00	0.54	1.00	0.00
time (sec)	N/A	0.397	0.042	0.434	0.000	0.084	0.000	0.262	0.253	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	38	37	0	76	0	29	136	0
N.S.	1	1.00	0.75	0.73	0.00	1.49	0.00	0.57	2.67	0.00
time (sec)	N/A	0.320	0.096	0.428	0.000	0.088	0.000	0.321	0.213	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	35	37	0	75	0	24	65	0
N.S.	1	1.00	0.69	0.73	0.00	1.47	0.00	0.47	1.27	0.00
time (sec)	N/A	0.317	0.037	0.425	0.000	0.139	0.000	0.303	0.193	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	84	56	48	0	87	0	43	155	0
N.S.	1	1.11	0.74	0.63	0.00	1.14	0.00	0.57	2.04	0.00
time (sec)	N/A	0.389	0.091	0.424	0.000	0.102	0.000	0.287	0.199	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	27	0	63	0	14	55	0
N.S.	1	1.00	1.00	1.08	0.00	2.52	0.00	0.56	2.20	0.00
time (sec)	N/A	0.256	0.013	0.421	0.000	0.101	0.000	0.283	0.203	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	117	58	59	0	95	0	55	170	0
N.S.	1	1.16	0.57	0.58	0.00	0.94	0.00	0.54	1.68	0.00
time (sec)	N/A	0.453	0.086	0.418	0.000	0.093	0.000	0.300	0.204	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	145	106	217	0	357	0	81	438	0
N.S.	1	1.12	0.82	1.67	0.00	2.75	0.00	0.62	3.37	0.00
time (sec)	N/A	0.589	0.120	0.426	0.000	0.113	0.000	0.313	0.235	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	145	68	70	0	110	0	90	186	0
N.S.	1	1.15	0.54	0.56	0.00	0.87	0.00	0.71	1.48	0.00
time (sec)	N/A	0.547	0.104	0.454	0.000	0.125	0.000	0.294	0.206	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	188	122	234	0	393	0	104	472	0
N.S.	1	1.18	0.77	1.47	0.00	2.47	0.00	0.65	2.97	0.00
time (sec)	N/A	0.689	0.187	0.478	0.000	0.103	0.000	0.288	0.511	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	177	79	81	0	121	0	147	210	0
N.S.	1	1.16	0.52	0.53	0.00	0.80	0.00	0.97	1.38	0.00
time (sec)	N/A	0.647	0.113	0.453	0.000	0.149	0.000	0.284	0.219	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	222	134	247	0	419	0	138	495	0
N.S.	1	1.17	0.71	1.31	0.00	2.22	0.00	0.73	2.62	0.00
time (sec)	N/A	0.786	0.186	0.491	0.000	0.110	0.000	0.292	0.334	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	211	90	92	0	132	0	202	231	0
N.S.	1	1.17	0.50	0.51	0.00	0.73	0.00	1.12	1.28	0.00
time (sec)	N/A	0.755	0.134	0.509	0.000	0.200	0.000	0.321	0.214	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	82	45	0	133	0	45	62	0
N.S.	1	1.00	1.49	0.82	0.00	2.42	0.00	0.82	1.13	0.00
time (sec)	N/A	0.337	0.023	3.882	0.000	0.161	0.000	0.302	0.225	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	62	25	0	94	0	23	43	0
N.S.	1	1.00	1.94	0.78	0.00	2.94	0.00	0.72	1.34	0.00
time (sec)	N/A	0.273	0.009	0.639	0.000	0.168	0.000	0.292	0.193	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	26	19	0	14	20	19
N.S.	1	1.00	1.00	0.87	1.13	0.83	0.00	0.61	0.87	0.83
time (sec)	N/A	0.257	0.010	0.626	0.042	0.074	0.000	0.261	0.215	9.173

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	31	28	38	29	0	30	30	27
N.S.	1	1.00	0.65	0.58	0.79	0.60	0.00	0.62	0.62	0.56
time (sec)	N/A	0.315	0.012	0.745	0.041	0.075	0.000	0.289	0.265	9.438

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	80	44	41	50	40	0	47	41	40
N.S.	1	1.08	0.59	0.55	0.68	0.54	0.00	0.64	0.55	0.54
time (sec)	N/A	0.388	0.012	0.960	0.042	0.087	0.000	0.291	0.229	9.374

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	64	688	0	0	0	0	45	0
N.S.	1	1.00	0.29	3.07	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.566	10.036	0.649	0.000	0.000	0.000	0.000	0.389	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	49	671	0	16	0	39	24	40
N.S.	1	1.00	0.25	3.41	0.00	0.08	0.00	0.20	0.12	0.20
time (sec)	N/A	0.483	10.012	0.456	0.000	0.088	0.000	0.324	0.266	9.438

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	53	696	0	43	0	0	26	0
N.S.	1	1.00	0.24	3.09	0.00	0.19	0.00	0.00	0.12	0.00
time (sec)	N/A	0.557	10.014	0.678	0.000	0.077	0.000	0.000	0.351	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	503	534	66	1079	0	0	0	0	47	0
N.S.	1	1.06	0.13	2.15	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.124	10.031	0.877	0.000	0.000	0.000	0.000	0.365	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	474	504	53	1054	0	0	0	0	23	0
N.S.	1	1.06	0.11	2.22	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.910	10.018	0.503	0.000	0.000	0.000	0.000	0.223	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	497	530	48	1083	0	24	0	0	26	0
N.S.	1	1.07	0.10	2.18	0.00	0.05	0.00	0.00	0.05	0.00
time (sec)	N/A	1.005	10.016	0.699	0.000	0.074	0.000	0.000	0.237	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	364	155	264	0	0	0	0	160	0
N.S.	1	1.21	0.51	0.88	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	1.018	10.155	1.275	0.000	0.000	0.000	0.000	0.278	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	448	136	273	0	0	0	0	124	0
N.S.	1	1.09	0.33	0.66	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	1.059	10.108	0.783	0.000	0.000	0.000	0.000	0.308	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	258	118	198	0	0	0	0	101	0
N.S.	1	1.21	0.55	0.93	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.720	10.098	0.602	0.000	0.000	0.000	0.000	0.259	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	340	94	207	0	0	0	0	65	40
N.S.	1	1.05	0.29	0.64	0.00	0.00	0.00	0.00	0.20	0.12
time (sec)	N/A	0.755	10.067	0.391	0.000	0.000	0.000	0.000	0.255	9.197

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	155	54	132	0	0	0	0	41	0
N.S.	1	1.26	0.44	1.07	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.444	10.044	0.430	0.000	0.000	0.000	0.000	0.250	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	340	59	213	0	0	0	0	48	0
N.S.	1	1.05	0.18	0.66	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.746	10.035	0.615	0.000	0.000	0.000	0.000	0.315	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	227	59	179	0	0	0	0	48	0
N.S.	1	1.21	0.31	0.95	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.613	10.044	0.744	0.000	0.000	0.000	0.000	0.454	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	446	59	281	0	0	0	0	48	0
N.S.	1	1.08	0.14	0.68	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.050	10.042	1.191	0.000	0.000	0.000	0.000	0.856	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	333	59	245	0	0	0	0	48	0
N.S.	1	1.21	0.21	0.89	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.915	10.047	1.543	0.000	0.000	0.000	0.000	1.443	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	358	142	197	0	0	0	0	160	0
N.S.	1	1.20	0.48	0.66	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.990	10.130	2.924	0.000	0.000	0.000	0.000	0.258	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	442	123	261	0	0	0	0	124	0
N.S.	1	1.08	0.30	0.64	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	1.038	10.110	1.744	0.000	0.000	0.000	0.000	0.311	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	248	106	163	0	0	0	0	101	40
N.S.	1	1.19	0.51	0.78	0.00	0.00	0.00	0.00	0.49	0.19
time (sec)	N/A	0.662	10.070	1.206	0.000	0.000	0.000	0.000	0.268	8.562

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	335	60	205	0	0	0	0	60	0
N.S.	1	1.05	0.19	0.64	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.753	10.056	0.724	0.000	0.000	0.000	0.000	0.235	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	181	60	130	0	0	0	0	62	0
N.S.	1	1.26	0.42	0.90	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.516	10.046	0.823	0.000	0.000	0.000	0.000	0.312	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	368	62	235	0	0	0	0	69	0
N.S.	1	1.05	0.18	0.67	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.838	10.044	1.576	0.000	0.000	0.000	0.000	0.425	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	255	62	168	0	0	0	0	71	0
N.S.	1	1.20	0.29	0.79	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.720	10.043	2.410	0.000	0.000	0.000	0.000	0.790	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	474	62	301	0	0	0	0	71	0
N.S.	1	1.08	0.14	0.69	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	1.127	10.043	3.878	0.000	0.000	0.000	0.000	1.261	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	361	62	201	0	0	0	0	71	0
N.S.	1	1.20	0.21	0.67	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	1.035	10.046	5.451	0.000	0.000	0.000	0.000	2.528	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	370	161	196	0	0	0	0	160	0
N.S.	1	1.22	0.53	0.64	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	1.027	10.085	4.289	0.000	0.000	0.000	0.000	0.485	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	454	143	261	0	0	0	0	124	0
N.S.	1	1.10	0.35	0.63	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	1.069	10.071	2.629	0.000	0.000	0.000	0.000	0.435	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	264	124	163	0	0	0	0	101	0
N.S.	1	1.22	0.57	0.75	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.727	10.083	1.599	0.000	0.000	0.000	0.000	0.283	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	348	106	210	0	0	0	0	65	0
N.S.	1	1.07	0.33	0.64	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.723	10.066	1.049	0.000	0.000	0.000	0.000	0.252	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	152	80	127	0	0	0	0	46	40
N.S.	1	1.21	0.63	1.01	0.00	0.00	0.00	0.00	0.37	0.32
time (sec)	N/A	0.473	10.038	0.526	0.000	0.000	0.000	0.000	0.267	8.962

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	312	54	195	0	0	0	0	26	0
N.S.	1	1.06	0.18	0.66	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.685	10.064	0.823	0.000	0.000	0.000	0.000	0.225	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	199	59	142	0	0	0	0	26	0
N.S.	1	1.22	0.36	0.87	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.524	10.041	1.516	0.000	0.000	0.000	0.000	0.240	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	418	59	262	0	0	0	0	26	0
N.S.	1	1.08	0.15	0.68	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.950	10.045	2.762	0.000	0.000	0.000	0.000	0.337	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	305	59	179	0	0	0	0	26	0
N.S.	1	1.22	0.24	0.71	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.824	10.041	4.809	0.000	0.000	0.000	0.000	0.441	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	485	131	303	0	0	0	0	210	0
N.S.	1	1.11	0.30	0.69	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	1.163	10.088	4.016	0.000	0.000	0.000	0.000	0.445	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	295	119	228	0	0	0	0	184	0
N.S.	1	1.23	0.50	0.95	0.00	0.00	0.00	0.00	0.77	0.00
time (sec)	N/A	0.835	10.077	2.404	0.000	0.000	0.000	0.000	0.372	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	379	94	237	0	0	0	0	151	0
N.S.	1	1.09	0.27	0.68	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.848	10.079	1.740	0.000	0.000	0.000	0.000	0.333	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	187	82	160	0	0	0	0	127	0
N.S.	1	1.26	0.55	1.07	0.00	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	0.559	10.070	1.260	0.000	0.000	0.000	0.000	0.337	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	309	62	197	0	0	0	0	33	40
N.S.	1	1.04	0.21	0.67	0.00	0.00	0.00	0.00	0.11	0.14
time (sec)	N/A	0.660	10.025	0.756	0.000	0.000	0.000	0.000	0.272	9.405

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	195	62	162	0	0	0	0	35	0
N.S.	1	1.23	0.39	1.03	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.542	10.061	1.593	0.000	0.000	0.000	0.000	0.271	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	414	64	267	0	0	0	0	35	0
N.S.	1	1.08	0.17	0.70	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.955	10.042	2.533	0.000	0.000	0.000	0.000	0.322	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	301	64	231	0	0	0	0	35	0
N.S.	1	1.22	0.26	0.94	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.836	10.048	4.035	0.000	0.000	0.000	0.000	0.462	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	471	520	64	333	0	0	0	0	35	0
N.S.	1	1.10	0.14	0.71	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.280	10.047	6.169	0.000	0.000	0.000	0.000	0.858	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	437	185	156	0	1293	0	396	153	0
N.S.	1	1.18	0.50	0.42	0.00	3.49	0.00	1.07	0.41	0.00
time (sec)	N/A	1.406	0.126	0.378	0.000	128.201	0.000	0.267	0.253	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	331	133	123	0	1031	0	312	120	0
N.S.	1	1.17	0.47	0.43	0.00	3.64	0.00	1.10	0.42	0.00
time (sec)	N/A	1.027	0.102	0.351	0.000	123.049	0.000	0.293	0.224	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	225	96	90	0	767	0	228	87	0
N.S.	1	1.15	0.49	0.46	0.00	3.93	0.00	1.17	0.45	0.00
time (sec)	N/A	0.691	0.074	0.366	0.000	132.140	0.000	0.292	0.240	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	121	74	57	0	501	0	143	54	40
N.S.	1	1.11	0.68	0.52	0.00	4.60	0.00	1.31	0.50	0.37
time (sec)	N/A	0.436	0.052	0.381	0.000	120.597	0.000	0.295	0.232	8.738

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	27	0	224	0	23	20	0
N.S.	1	1.00	1.00	1.17	0.00	9.74	0.00	1.00	0.87	0.00
time (sec)	N/A	0.249	0.044	0.374	0.000	121.877	0.000	0.268	0.252	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	89	76	79	0	0	0	72	82	0
N.S.	1	0.99	0.84	0.88	0.00	0.00	0.00	0.80	0.91	0.00
time (sec)	N/A	0.393	0.188	0.381	0.000	0.000	0.000	0.310	0.209	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	193	112	125	0	0	0	108	138	0
N.S.	1	1.08	0.63	0.70	0.00	0.00	0.00	0.61	0.78	0.00
time (sec)	N/A	0.640	0.230	0.385	0.000	0.000	0.000	0.299	0.198	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	299	149	167	0	0	0	177	195	0
N.S.	1	1.12	0.56	0.63	0.00	0.00	0.00	0.67	0.73	0.00
time (sec)	N/A	0.951	0.276	0.378	0.000	0.000	0.000	0.326	0.206	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	405	186	209	0	0	0	192	252	0
N.S.	1	1.14	0.53	0.59	0.00	0.00	0.00	0.54	0.71	0.00
time (sec)	N/A	1.309	0.348	0.385	0.000	0.000	0.000	0.345	0.237	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	403	168	145	0	1293	0	770	153	0
N.S.	1	1.17	0.49	0.42	0.00	3.77	0.00	2.24	0.45	0.00
time (sec)	N/A	1.255	5.255	0.387	0.000	132.793	0.000	0.312	0.271	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	297	131	112	0	1031	0	602	120	0
N.S.	1	1.16	0.51	0.44	0.00	4.04	0.00	2.36	0.47	0.00
time (sec)	N/A	0.965	5.133	0.384	0.000	141.716	0.000	0.274	0.197	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	193	94	79	0	768	0	434	87	40
N.S.	1	1.14	0.56	0.47	0.00	4.54	0.00	2.57	0.51	0.24
time (sec)	N/A	0.611	5.158	0.382	0.000	126.390	0.000	0.275	0.204	8.735

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	90	59	48	0	501	0	265	54	0
N.S.	1	1.07	0.70	0.57	0.00	5.96	0.00	3.15	0.64	0.00
time (sec)	N/A	0.382	5.173	0.380	0.000	132.687	0.000	0.278	0.218	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	80	88	67	0	0	0	83	62	0
N.S.	1	1.03	1.13	0.86	0.00	0.00	0.00	1.06	0.79	0.00
time (sec)	N/A	0.381	10.078	0.391	0.000	0.000	0.000	0.299	0.207	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	115	61	93	0	0	0	80	103	0
N.S.	1	1.02	0.54	0.82	0.00	0.00	0.00	0.71	0.91	0.00
time (sec)	N/A	0.455	10.061	0.343	0.000	0.000	0.000	0.343	0.213	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	221	61	139	0	0	0	143	160	0
N.S.	1	1.09	0.30	0.68	0.00	0.00	0.00	0.70	0.79	0.00
time (sec)	N/A	0.716	10.050	0.348	0.000	0.000	0.000	0.332	0.206	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	327	61	181	0	0	0	164	217	0
N.S.	1	1.12	0.21	0.62	0.00	0.00	0.00	0.56	0.75	0.00
time (sec)	N/A	1.025	10.060	0.357	0.000	0.000	0.000	0.345	0.221	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	433	61	223	0	0	0	245	274	0
N.S.	1	1.14	0.16	0.59	0.00	0.00	0.00	0.65	0.72	0.00
time (sec)	N/A	1.417	10.062	0.356	0.000	0.000	0.000	0.366	0.261	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	469	185	167	0	1294	0	206	153	0
N.S.	1	1.17	0.46	0.42	0.00	3.23	0.00	0.51	0.38	0.00
time (sec)	N/A	1.578	0.135	0.348	0.000	133.147	0.000	0.325	0.255	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	363	148	134	0	1031	0	164	120	0
N.S.	1	1.16	0.47	0.43	0.00	3.29	0.00	0.52	0.38	0.00
time (sec)	N/A	1.164	0.110	0.347	0.000	125.558	0.000	0.276	0.303	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	257	111	101	0	768	0	122	87	0
N.S.	1	1.14	0.49	0.45	0.00	3.41	0.00	0.54	0.39	0.00
time (sec)	N/A	0.826	0.086	0.348	0.000	136.129	0.000	0.284	0.252	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	151	74	68	0	502	0	80	54	0
N.S.	1	1.10	0.54	0.50	0.00	3.66	0.00	0.58	0.39	0.00
time (sec)	N/A	0.506	0.067	0.358	0.000	125.319	0.000	0.290	0.213	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	36	36	0	238	0	36	22	40
N.S.	1	1.00	0.77	0.77	0.00	5.06	0.00	0.77	0.47	0.85
time (sec)	N/A	0.287	0.038	0.349	0.000	136.850	0.000	0.284	0.197	9.110

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	61	0	0	0	48	61	0
N.S.	1	1.00	1.00	1.00	0.00	0.00	0.00	0.79	1.00	0.00
time (sec)	N/A	0.320	0.131	0.357	0.000	0.000	0.000	0.275	0.216	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	165	101	123	0	0	0	109	119	0
N.S.	1	1.08	0.66	0.80	0.00	0.00	0.00	0.71	0.78	0.00
time (sec)	N/A	0.549	0.252	0.352	0.000	0.000	0.000	0.295	0.228	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	271	138	183	0	0	0	136	176	0
N.S.	1	1.12	0.57	0.76	0.00	0.00	0.00	0.56	0.73	0.00
time (sec)	N/A	0.828	0.246	0.343	0.000	0.000	0.000	0.312	0.220	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	377	175	243	0	0	0	211	233	0
N.S.	1	1.15	0.53	0.74	0.00	0.00	0.00	0.64	0.71	0.00
time (sec)	N/A	1.159	0.275	0.329	0.000	0.000	0.000	0.325	0.204	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	392	161	143	0	2566	0	214	133	0
N.S.	1	1.17	0.48	0.43	0.00	7.64	0.00	0.64	0.40	0.00
time (sec)	N/A	1.227	4.509	0.354	0.000	117.473	0.000	0.441	0.197	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	286	122	110	0	2083	0	169	100	0
N.S.	1	1.15	0.49	0.44	0.00	8.40	0.00	0.68	0.40	0.00
time (sec)	N/A	0.886	4.499	0.315	0.000	124.488	0.000	1.608	0.250	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	180	85	77	0	1598	0	112	67	0
N.S.	1	1.12	0.53	0.48	0.00	9.99	0.00	0.70	0.42	0.00
time (sec)	N/A	0.576	4.873	0.319	0.000	114.813	0.000	0.412	0.208	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	45	45	0	1107	0	66	35	0
N.S.	1	1.09	0.66	0.66	0.00	16.28	0.00	0.97	0.51	0.00
time (sec)	N/A	0.337	4.534	0.328	0.000	132.052	0.000	0.277	0.191	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	71	56	0	0	0	71	68	40
N.S.	1	1.00	1.18	0.93	0.00	0.00	0.00	1.18	1.13	0.67
time (sec)	N/A	0.303	1.806	0.322	0.000	0.000	0.000	0.299	0.195	8.825

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	160	110	88	0	0	0	105	109	0
N.S.	1	1.10	0.75	0.60	0.00	0.00	0.00	0.72	0.75	0.00
time (sec)	N/A	0.538	4.728	0.332	0.000	0.000	0.000	0.287	0.196	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	266	48	126	0	0	0	156	146	0
N.S.	1	1.13	0.20	0.53	0.00	0.00	0.00	0.66	0.62	0.00
time (sec)	N/A	0.818	10.084	0.329	0.000	0.000	0.000	0.325	0.231	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	372	48	159	0	0	0	207	179	0
N.S.	1	1.15	0.15	0.49	0.00	0.00	0.00	0.64	0.55	0.00
time (sec)	N/A	1.162	10.068	0.329	0.000	0.000	0.000	0.351	0.215	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	478	48	192	0	0	0	258	212	0
N.S.	1	1.16	0.12	0.47	0.00	0.00	0.00	0.63	0.51	0.00
time (sec)	N/A	1.556	10.073	0.336	0.000	0.000	0.000	0.382	0.218	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.76
time (sec)	N/A	0.237	0.002	0.055	0.035	0.078	0.017	0.250	0.218	0.023

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.76
time (sec)	N/A	0.246	0.002	0.038	0.031	0.087	0.019	0.217	0.243	0.022

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.76
time (sec)	N/A	0.234	0.000	0.030	0.034	0.114	0.017	0.264	0.221	0.021

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.76
time (sec)	N/A	0.246	0.001	0.039	0.033	0.085	0.017	0.253	0.213	0.022

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	11	10	10	8	10	10	10
N.S.	1	1.00	0.86	0.79	0.71	0.71	0.57	0.71	0.71	0.71
time (sec)	N/A	0.213	0.000	0.036	0.031	0.082	0.017	0.261	0.220	0.018

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.270	0.002	0.305	0.035	0.081	0.017	0.299	0.242	0.043

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80	0.80
time (sec)	N/A	0.259	0.002	0.303	0.031	0.071	0.020	0.254	0.223	0.033

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.278	0.002	0.299	0.027	0.070	0.018	0.257	0.234	0.032

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80	0.80
time (sec)	N/A	0.261	0.003	0.306	0.030	0.073	0.019	0.220	0.243	0.035

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.253	0.002	0.305	0.032	0.084	0.023	0.343	0.237	0.034

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	52	52	52	49	53	52	51
N.S.	1	1.00	1.00	0.91	0.91	0.91	0.86	0.93	0.91	0.89
time (sec)	N/A	0.314	0.004	0.423	0.027	0.082	0.061	0.240	0.219	8.686

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	41	42	41	37	43	41	40
N.S.	1	1.00	1.00	0.93	0.95	0.93	0.84	0.98	0.93	0.91
time (sec)	N/A	0.291	0.005	0.385	0.029	0.068	0.060	0.247	0.218	0.040

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	29	29	26	30	29	29
N.S.	1	1.00	1.00	0.97	0.94	0.94	0.84	0.97	0.94	0.94
time (sec)	N/A	0.273	0.004	0.389	0.033	0.067	0.051	0.232	0.251	0.042

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	17	14	19	17	18
N.S.	1	1.00	1.00	1.06	1.00	0.94	0.78	1.06	0.94	1.00
time (sec)	N/A	0.253	0.002	0.378	0.034	0.069	0.042	0.238	0.220	8.690

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	11	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.10	1.00	1.00
time (sec)	N/A	0.216	0.001	0.385	0.027	0.069	0.020	0.205	0.204	0.021

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	18	16	10	20	15	15
N.S.	1	1.00	1.00	0.89	1.00	0.89	0.56	1.11	0.83	0.83
time (sec)	N/A	0.226	0.004	0.380	0.043	0.072	0.075	0.266	0.243	0.048

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	26	28	26	19	30	26	25
N.S.	1	1.00	1.00	0.93	1.00	0.93	0.68	1.07	0.93	0.89
time (sec)	N/A	0.290	0.008	0.375	0.038	0.079	0.083	0.234	0.243	8.763

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	41	40	41	31	45	43	38
N.S.	1	1.00	1.00	0.98	0.95	0.98	0.74	1.07	1.02	0.90
time (sec)	N/A	0.289	0.009	0.378	0.026	0.088	0.097	0.254	0.251	0.059

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	53	51	54	44	56	54	48
N.S.	1	1.00	1.00	0.95	0.91	0.96	0.79	1.00	0.96	0.86
time (sec)	N/A	0.297	0.005	0.385	0.036	0.076	0.109	0.249	0.227	0.061

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	54	57	59	73	54	62	71	62
N.S.	1	1.00	0.93	0.98	1.02	1.26	0.93	1.07	1.22	1.07
time (sec)	N/A	0.330	0.027	0.381	0.028	0.079	0.089	0.239	0.224	0.040

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	45	47	62	44	48	60	50
N.S.	1	1.00	0.93	0.98	1.02	1.35	0.96	1.04	1.30	1.09
time (sec)	N/A	0.304	0.012	0.384	0.028	0.075	0.121	0.239	0.232	0.046

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	34	36	47	31	34	46	36
N.S.	1	1.00	0.88	1.03	1.09	1.42	0.94	1.03	1.39	1.09
time (sec)	N/A	0.277	0.011	0.386	0.037	0.077	0.097	0.249	0.208	8.661

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	24	26	28	20	24	33	23
N.S.	1	1.00	0.87	1.04	1.13	1.22	0.87	1.04	1.43	1.00
time (sec)	N/A	0.271	0.007	0.362	0.027	0.075	0.076	1.258	0.228	0.038

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	13	13	10	12	12	12
N.S.	1	1.00	1.00	1.08	1.08	1.08	0.83	1.00	1.00	1.00
time (sec)	N/A	0.221	0.002	0.379	0.026	0.065	0.069	0.320	0.223	0.032

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	30	28	39	22	31	44	26
N.S.	1	1.00	0.83	1.03	0.97	1.34	0.76	1.07	1.52	0.90
time (sec)	N/A	0.275	0.013	0.385	0.038	0.080	0.101	0.257	0.242	8.728

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	35	43	45	63	37	45	70	41
N.S.	1	1.00	0.83	1.02	1.07	1.50	0.88	1.07	1.67	0.98
time (sec)	N/A	0.295	0.036	0.388	0.032	0.085	0.120	0.226	0.247	0.060

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	53	57	64	86	54	64	86	57
N.S.	1	1.00	0.91	0.98	1.10	1.48	0.93	1.10	1.48	0.98
time (sec)	N/A	0.326	0.043	0.389	0.027	0.090	0.141	0.254	0.215	0.068

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	68	73	95	66	73	97	69
N.S.	1	1.00	0.96	0.99	1.06	1.38	0.96	1.06	1.41	1.00
time (sec)	N/A	0.362	0.047	0.382	0.027	0.074	0.198	0.248	0.233	0.070

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	79	79	86	108	80	86	108	79
N.S.	1	1.00	0.94	0.94	1.02	1.29	0.95	1.02	1.29	0.94
time (sec)	N/A	0.369	0.043	0.382	0.033	0.078	0.170	0.256	0.225	8.650

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	117	53	32	53	62	0	131	52	62
N.S.	1	1.11	0.50	0.30	0.50	0.59	0.00	1.25	0.50	0.59
time (sec)	N/A	0.463	0.012	0.448	0.037	0.074	0.000	0.219	0.239	8.700

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	86	42	21	42	51	0	108	41	51
N.S.	1	1.08	0.52	0.26	0.52	0.64	0.00	1.35	0.51	0.64
time (sec)	N/A	0.375	0.009	0.440	0.045	0.076	0.000	0.219	0.215	8.648

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	41	13	30	39	0	81	29	39
N.S.	1	1.00	0.79	0.25	0.58	0.75	0.00	1.56	0.56	0.75
time (sec)	N/A	0.292	0.006	0.424	0.039	0.074	0.000	0.218	0.207	8.795

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	25	12	26	0	50	16	0
N.S.	1	1.00	0.92	1.00	0.48	1.04	0.00	2.00	0.64	0.00
time (sec)	N/A	0.244	0.008	0.432	0.040	0.073	0.000	0.214	0.263	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	53	36	0	116	0	67	38	73
N.S.	1	1.00	1.04	0.71	0.00	2.27	0.00	1.31	0.75	1.43
time (sec)	N/A	0.309	0.021	0.454	0.000	0.086	0.000	0.171	0.272	9.018

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	64	50	0	132	0	42	51	0
N.S.	1	1.00	1.23	0.96	0.00	2.54	0.00	0.81	0.98	0.00
time (sec)	N/A	0.307	0.015	0.467	0.000	0.082	0.000	0.213	0.303	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	82	81	61	0	154	0	72	72	0
N.S.	1	0.98	0.96	0.73	0.00	1.83	0.00	0.86	0.86	0.00
time (sec)	N/A	0.382	0.025	0.484	0.000	0.085	0.000	0.252	0.245	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	116	93	72	0	180	0	80	90	0
N.S.	1	1.04	0.83	0.64	0.00	1.61	0.00	0.71	0.80	0.00
time (sec)	N/A	0.449	0.025	0.503	0.000	0.085	0.000	0.245	0.222	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	185	80	32	86	95	0	282	85	80
N.S.	1	1.15	0.50	0.20	0.53	0.59	0.00	1.75	0.53	0.50
time (sec)	N/A	0.614	0.016	0.459	0.042	0.082	0.000	0.249	0.232	8.870

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	154	69	21	75	84	0	246	74	69
N.S.	1	1.13	0.51	0.15	0.55	0.62	0.00	1.81	0.54	0.51
time (sec)	N/A	0.572	0.013	0.457	0.043	0.075	0.000	0.243	0.208	8.810

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	120	58	13	64	73	0	210	63	58
N.S.	1	1.11	0.54	0.12	0.59	0.68	0.00	1.94	0.58	0.54
time (sec)	N/A	0.460	0.015	0.424	0.040	0.074	0.000	0.169	0.224	8.781

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	86	47	35	53	62	0	173	52	47
N.S.	1	1.08	0.59	0.44	0.66	0.78	0.00	2.16	0.65	0.59
time (sec)	N/A	0.394	0.014	0.438	0.040	0.079	0.000	0.155	0.232	8.878

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	31	35	41	50	0	136	40	36
N.S.	1	1.00	0.60	0.67	0.79	0.96	0.00	2.62	0.77	0.69
time (sec)	N/A	0.315	0.010	0.445	0.040	0.072	0.000	0.268	0.218	8.858

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	28	37	0	89	27	28
N.S.	1	1.00	0.92	1.08	1.12	1.48	0.00	3.56	1.08	1.12
time (sec)	N/A	0.259	0.007	0.462	0.045	0.077	0.000	0.202	0.221	9.206

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	76	68	61	0	135	0	85	51	0
N.S.	1	1.03	0.92	0.82	0.00	1.82	0.00	1.15	0.69	0.00
time (sec)	N/A	0.380	0.042	0.484	0.000	0.084	0.000	0.232	0.223	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	77	66	70	0	141	0	56	59	0
N.S.	1	1.05	0.90	0.96	0.00	1.93	0.00	0.77	0.81	0.00
time (sec)	N/A	0.382	0.034	0.509	0.000	0.087	0.000	0.267	0.218	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	80	82	67	0	159	0	70	73	0
N.S.	1	0.99	1.01	0.83	0.00	1.96	0.00	0.86	0.90	0.00
time (sec)	N/A	0.378	0.030	0.546	0.000	0.095	0.000	0.273	0.211	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	110	94	81	0	180	0	80	90	0
N.S.	1	1.01	0.86	0.74	0.00	1.65	0.00	0.73	0.83	0.00
time (sec)	N/A	0.450	0.031	0.560	0.000	0.085	0.000	0.284	0.218	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	144	104	92	0	202	0	109	107	0
N.S.	1	1.05	0.76	0.67	0.00	1.47	0.00	0.80	0.78	0.00
time (sec)	N/A	0.542	0.037	0.599	0.000	0.085	0.000	0.274	0.194	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	178	116	103	0	224	0	108	124	0
N.S.	1	1.08	0.70	0.62	0.00	1.36	0.00	0.65	0.75	0.00
time (sec)	N/A	0.638	0.181	0.637	0.000	0.085	0.000	0.307	0.245	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	115	53	52	53	51	0	64	41	51
N.S.	1	1.12	0.51	0.50	0.51	0.50	0.00	0.62	0.40	0.50
time (sec)	N/A	0.445	0.011	0.474	0.037	0.073	0.000	0.280	0.202	8.986

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	81	42	41	42	40	0	52	30	40
N.S.	1	1.08	0.56	0.55	0.56	0.53	0.00	0.69	0.40	0.53
time (sec)	N/A	0.367	0.009	0.439	0.040	0.070	0.000	0.240	0.194	8.806

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	30	30	30	28	0	38	18	31
N.S.	1	1.00	0.61	0.61	0.61	0.57	0.00	0.78	0.37	0.63
time (sec)	N/A	0.293	0.007	0.434	0.037	0.070	0.000	0.264	0.207	8.743

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	21	12	21	0	27	11	17
N.S.	1	1.00	0.91	0.91	0.52	0.91	0.00	1.17	0.48	0.74
time (sec)	N/A	0.240	0.007	0.451	0.037	0.076	0.000	0.239	0.205	8.804

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	46	13	0	79	0	45	31	0
N.S.	1	1.00	1.53	0.43	0.00	2.63	0.00	1.50	1.03	0.00
time (sec)	N/A	0.250	0.008	0.408	0.000	0.079	0.000	0.252	0.204	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	60	18	0	132	0	48	51	0
N.S.	1	1.00	1.11	0.33	0.00	2.44	0.00	0.89	0.94	0.00
time (sec)	N/A	0.309	0.015	0.432	0.000	0.079	0.000	0.240	0.217	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	88	83	36	0	158	0	73	73	44
N.S.	1	1.01	0.95	0.41	0.00	1.82	0.00	0.84	0.84	0.51
time (sec)	N/A	0.374	0.019	0.462	0.000	0.089	0.000	0.234	0.200	9.272

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	122	96	56	0	180	0	76	90	0
N.S.	1	1.06	0.83	0.49	0.00	1.57	0.00	0.66	0.78	0.00
time (sec)	N/A	0.455	0.019	0.464	0.000	0.090	0.000	0.252	0.233	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	110	50	56	41	60	0	79	42	57
N.S.	1	1.12	0.51	0.57	0.42	0.61	0.00	0.81	0.43	0.58
time (sec)	N/A	0.434	0.008	0.544	0.038	0.075	0.000	0.242	0.226	9.464

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	78	39	46	30	49	0	70	31	47
N.S.	1	1.08	0.54	0.64	0.42	0.68	0.00	0.97	0.43	0.65
time (sec)	N/A	0.364	0.011	0.483	0.039	0.072	0.000	0.232	0.196	9.126

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	26	34	19	38	0	48	20	35
N.S.	1	1.00	0.55	0.72	0.40	0.81	0.00	1.02	0.43	0.74
time (sec)	N/A	0.312	0.006	0.490	0.042	0.079	0.000	0.247	0.211	9.159

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	27	12	29	0	27	13	28
N.S.	1	1.00	0.90	1.29	0.57	1.38	0.00	1.29	0.62	1.33
time (sec)	N/A	0.247	0.004	0.473	0.039	0.070	0.000	0.243	0.207	8.919

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	54	31	0	161	0	77	57	0
N.S.	1	1.00	1.04	0.60	0.00	3.10	0.00	1.48	1.10	0.00
time (sec)	N/A	0.314	0.010	0.444	0.000	0.083	0.000	0.228	0.208	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	80	62	20	0	194	0	72	73	0
N.S.	1	1.07	0.83	0.27	0.00	2.59	0.00	0.96	0.97	0.00
time (sec)	N/A	0.368	0.014	0.441	0.000	0.093	0.000	0.276	0.211	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	117	84	13	0	224	0	92	92	42
N.S.	1	1.06	0.76	0.12	0.00	2.04	0.00	0.84	0.84	0.38
time (sec)	N/A	0.455	0.018	0.423	0.000	0.099	0.000	0.242	0.206	9.162

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	151	96	31	0	246	0	107	103	0
N.S.	1	1.09	0.70	0.22	0.00	1.78	0.00	0.78	0.75	0.00
time (sec)	N/A	0.558	0.020	0.455	0.000	0.100	0.000	0.249	0.210	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	185	106	50	0	268	0	122	114	44
N.S.	1	1.11	0.64	0.30	0.00	1.61	0.00	0.73	0.69	0.27
time (sec)	N/A	0.635	0.025	0.453	0.000	0.094	0.000	0.259	0.229	9.722

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	132	110	100	0	188	0	83	76	0
N.S.	1	1.06	0.88	0.80	0.00	1.50	0.00	0.66	0.61	0.00
time (sec)	N/A	0.470	0.040	0.411	0.000	0.088	0.000	0.226	0.198	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	96	99	89	0	167	0	71	57	0
N.S.	1	1.01	1.04	0.94	0.00	1.76	0.00	0.75	0.60	0.00
time (sec)	N/A	0.394	0.025	0.404	0.000	0.091	0.000	0.259	0.227	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	80	78	0	139	0	55	38	0
N.S.	1	1.00	1.33	1.30	0.00	2.32	0.00	0.92	0.63	0.00
time (sec)	N/A	0.321	0.022	0.405	0.000	0.103	0.000	0.235	0.213	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	53	58	0	85	0	37	25	0
N.S.	1	1.00	1.56	1.71	0.00	2.50	0.00	1.09	0.74	0.00
time (sec)	N/A	0.257	0.010	0.396	0.000	0.081	0.000	0.246	0.217	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	25	0	21	0	34	23	0
N.S.	1	1.00	0.92	1.00	0.00	0.84	0.00	1.36	0.92	0.00
time (sec)	N/A	0.242	0.008	0.397	0.000	0.074	0.000	0.246	0.221	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	31	31	0	29	0	52	40	0
N.S.	1	1.00	0.55	0.55	0.00	0.52	0.00	0.93	0.71	0.00
time (sec)	N/A	0.300	0.011	0.408	0.000	0.080	0.000	0.275	0.193	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	92	44	44	0	40	0	81	61	0
N.S.	1	1.07	0.51	0.51	0.00	0.47	0.00	0.94	0.71	0.00
time (sec)	N/A	0.375	0.012	0.409	0.000	0.079	0.000	0.260	0.217	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	128	55	55	0	51	0	107	80	0
N.S.	1	1.10	0.47	0.47	0.00	0.44	0.00	0.92	0.69	0.00
time (sec)	N/A	0.449	0.013	0.408	0.000	0.082	0.000	0.274	0.217	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	0	0	139	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	2.28	0.00
time (sec)	N/A	0.309	0.020	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	61	59	0	0	0	0	0	66	0
N.S.	1	1.27	1.23	0.00	0.00	0.00	0.00	0.00	1.38	0.00
time (sec)	N/A	0.313	0.014	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	52	0	0	0	0	0	26	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.301	0.010	0.000	0.000	0.000	0.000	0.000	0.214	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	30	36	0	38	0	0	38	54
N.S.	1	1.00	0.94	1.12	0.00	1.19	0.00	0.00	1.19	1.69
time (sec)	N/A	0.257	0.016	0.459	0.000	0.082	0.000	0.000	0.231	9.392

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	44	50	0	70	0	0	62	98
N.S.	1	1.00	0.63	0.71	0.00	1.00	0.00	0.00	0.89	1.40
time (sec)	N/A	0.355	0.026	0.497	0.000	0.091	0.000	0.000	0.222	9.622

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	114	72	84	0	111	0	0	98	157
N.S.	1	0.98	0.62	0.72	0.00	0.96	0.00	0.00	0.84	1.35
time (sec)	N/A	0.462	0.030	0.489	0.000	0.088	0.000	0.000	0.203	9.349

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	23	36	36	36	22	28	37
N.S.	1	1.00	1.26	1.21	1.89	1.89	1.89	1.16	1.47	1.95
time (sec)	N/A	0.232	0.010	0.375	0.029	0.070	0.169	0.255	0.195	9.165

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	86	46	43	46	42	0	65	34	42
N.S.	1	1.08	0.58	0.54	0.58	0.52	0.00	0.81	0.42	0.52
time (sec)	N/A	0.390	0.012	1.056	0.041	0.076	0.000	0.241	0.234	9.845

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	34	32	34	30	0	48	22	33
N.S.	1	1.00	0.65	0.62	0.65	0.58	0.00	0.92	0.42	0.63
time (sec)	N/A	0.314	0.010	0.836	0.038	0.074	0.000	0.251	0.208	10.129

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	14	21	0	29	13	21
N.S.	1	1.00	1.00	0.88	0.56	0.84	0.00	1.16	0.52	0.84
time (sec)	N/A	0.252	0.007	0.632	0.039	0.071	0.000	0.253	0.192	10.031

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	54	43	0	80	0	47	36	0
N.S.	1	1.00	1.69	1.34	0.00	2.50	0.00	1.47	1.12	0.00
time (sec)	N/A	0.259	0.011	0.530	0.000	0.084	0.000	0.250	0.199	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	76	66	0	132	0	52	61	0
N.S.	1	1.00	1.29	1.12	0.00	2.24	0.00	0.88	1.03	0.00
time (sec)	N/A	0.319	0.019	0.662	0.000	0.084	0.000	0.246	0.258	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	68	248	0	37	0	0	39	0
N.S.	1	1.00	0.29	1.04	0.00	0.16	0.00	0.00	0.16	0.00
time (sec)	N/A	0.500	10.032	0.898	0.000	0.075	0.000	0.000	0.275	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	52	231	0	14	0	0	20	0
N.S.	1	1.00	0.25	1.09	0.00	0.07	0.00	0.00	0.09	0.00
time (sec)	N/A	0.404	10.015	0.544	0.000	0.070	0.000	0.000	0.300	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	55	248	0	38	0	0	24	44
N.S.	1	1.00	0.23	1.02	0.00	0.16	0.00	0.00	0.10	0.18
time (sec)	N/A	0.485	10.014	0.644	0.000	0.072	0.000	0.000	0.226	9.778

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	514	543	68	25	0	45	0	0	42	0
N.S.	1	1.06	0.13	0.05	0.00	0.09	0.00	0.00	0.08	0.00
time (sec)	N/A	0.927	10.023	0.638	0.000	0.076	0.000	0.000	0.249	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	512	55	15	0	22	0	0	21	0
N.S.	1	1.06	0.11	0.03	0.00	0.05	0.00	0.00	0.04	0.00
time (sec)	N/A	0.877	10.023	0.501	0.000	0.079	0.000	0.000	0.222	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	510	543	50	20	0	46	0	0	24	0
N.S.	1	1.06	0.10	0.04	0.00	0.09	0.00	0.00	0.05	0.00
time (sec)	N/A	0.972	10.018	0.539	0.000	0.076	0.000	0.000	0.203	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	273	86	742	0	0	0	0	64	0
N.S.	1	1.03	0.32	2.80	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.676	10.038	1.566	0.000	0.000	0.000	0.000	0.356	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	525	536	70	1115	0	0	0	0	47	0
N.S.	1	1.02	0.13	2.12	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.063	10.031	1.543	0.000	0.000	0.000	0.000	0.362	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	82	79	0	148	0	52	62	0
N.S.	1	1.00	1.26	1.22	0.00	2.28	0.00	0.80	0.95	0.00
time (sec)	N/A	0.340	0.024	0.839	0.000	0.164	0.000	0.267	0.218	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	70	727	0	0	0	0	45	0
N.S.	1	1.00	0.30	3.07	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.553	10.026	0.983	0.000	0.000	0.000	0.000	0.341	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	492	502	57	2374	0	0	0	0	23	0
N.S.	1	1.02	0.12	4.83	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.903	10.020	0.904	0.000	0.000	0.000	0.000	0.256	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	60	59	0	101	0	40	43	0
N.S.	1	1.00	1.67	1.64	0.00	2.81	0.00	1.11	1.19	0.00
time (sec)	N/A	0.273	0.009	0.441	0.000	0.159	0.000	0.241	0.233	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	55	437	0	16	0	0	24	0
N.S.	1	1.00	0.27	2.15	0.00	0.08	0.00	0.00	0.12	0.00
time (sec)	N/A	0.477	10.012	1.298	0.000	0.081	0.000	0.000	0.228	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	519	532	55	1115	0	24	0	0	26	0
N.S.	1	1.03	0.11	2.15	0.00	0.05	0.00	0.00	0.05	0.00
time (sec)	N/A	0.997	10.014	1.467	0.000	0.075	0.000	0.000	0.257	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	29	26	21	0	28	20	0
N.S.	1	1.00	1.00	1.07	0.96	0.78	0.00	1.04	0.74	0.00
time (sec)	N/A	0.251	0.009	0.774	0.036	0.079	0.000	0.253	0.242	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	57	732	0	48	0	0	26	0
N.S.	1	1.00	0.24	3.11	0.00	0.20	0.00	0.00	0.11	0.00
time (sec)	N/A	0.568	10.014	1.960	0.000	0.078	0.000	0.000	0.273	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	555	568	57	1125	0	55	0	0	26	0
N.S.	1	1.02	0.10	2.03	0.00	0.10	0.00	0.00	0.05	0.00
time (sec)	N/A	1.105	10.013	2.352	0.000	0.078	0.000	0.000	0.315	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	35	37	38	31	0	43	30	0
N.S.	1	1.00	0.62	0.66	0.68	0.55	0.00	0.77	0.54	0.00
time (sec)	N/A	0.312	0.010	0.642	0.037	0.080	0.000	0.285	0.234	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	271	57	742	0	62	0	0	26	0
N.S.	1	1.02	0.22	2.80	0.00	0.23	0.00	0.00	0.10	0.00
time (sec)	N/A	0.642	10.015	3.218	0.000	0.089	0.000	0.000	0.293	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	26	28	26	19	30	26	25
N.S.	1	1.00	1.00	0.93	1.00	0.93	0.68	1.07	0.93	0.89
time (sec)	N/A	0.286	0.004	0.378	0.026	0.081	0.080	0.229	0.224	0.048

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	41	40	41	31	45	43	38
N.S.	1	1.00	1.00	0.98	0.95	0.98	0.74	1.07	1.02	0.90
time (sec)	N/A	0.326	0.003	0.376	0.029	0.073	0.092	0.236	0.206	0.052

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	119	112	91	0	176	0	104	76	0
N.S.	1	1.06	1.00	0.81	0.00	1.57	0.00	0.93	0.68	0.00
time (sec)	N/A	0.493	0.036	0.774	0.000	0.087	0.000	0.263	0.214	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	87	101	69	0	155	0	88	57	0
N.S.	1	1.01	1.17	0.80	0.00	1.80	0.00	1.02	0.66	0.00
time (sec)	N/A	0.415	0.019	0.717	0.000	0.094	0.000	0.236	0.189	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	82	47	0	127	0	69	38	0
N.S.	1	1.00	1.46	0.84	0.00	2.27	0.00	1.23	0.68	0.00
time (sec)	N/A	0.332	0.020	0.662	0.000	0.088	0.000	0.245	0.219	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	57	25	0	79	0	47	25	0
N.S.	1	1.00	1.78	0.78	0.00	2.47	0.00	1.47	0.78	0.00
time (sec)	N/A	0.270	0.009	0.617	0.000	0.085	0.000	0.240	0.205	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	20	0	21	0	27	23	21
N.S.	1	1.00	0.91	0.87	0.00	0.91	0.00	1.17	1.00	0.91
time (sec)	N/A	0.241	0.006	0.581	0.000	0.092	0.000	0.271	0.212	9.257

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	31	26	0	29	0	53	40	42
N.S.	1	1.00	0.60	0.50	0.00	0.56	0.00	1.02	0.77	0.81
time (sec)	N/A	0.299	0.008	0.602	0.000	0.077	0.000	0.260	0.200	9.165

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	86	42	39	0	40	0	82	61	40
N.S.	1	1.08	0.52	0.49	0.00	0.50	0.00	1.02	0.76	0.50
time (sec)	N/A	0.372	0.007	0.649	0.000	0.121	0.000	0.248	0.209	9.482

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	120	53	50	0	51	0	111	80	92
N.S.	1	1.11	0.49	0.46	0.00	0.47	0.00	1.03	0.74	0.85
time (sec)	N/A	0.452	0.011	0.704	0.000	0.082	0.000	0.258	0.237	8.921

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	27	26	23	22	28	22	32	28	22
N.S.	1	1.04	1.00	0.88	0.85	1.08	0.85	1.23	1.08	0.85
time (sec)	N/A	0.289	0.005	0.368	0.029	0.081	0.099	0.224	0.192	8.798

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	26	27	23	22	28	22	32	41	22
N.S.	1	0.96	1.00	0.85	0.81	1.04	0.81	1.19	1.52	0.81
time (sec)	N/A	0.303	0.005	0.361	0.034	0.083	0.102	0.240	0.235	8.751

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	14	9	14	8	5	15	8	8
N.S.	1	1.00	1.75	1.12	1.75	1.00	0.62	1.88	1.00	1.00
time (sec)	N/A	0.209	0.003	0.025	0.026	0.073	0.025	0.276	0.198	0.029

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	16	18	15	16	18	10
N.S.	1	1.00	1.00	1.10	1.60	1.80	1.50	1.60	1.80	1.00
time (sec)	N/A	0.213	0.003	0.028	0.029	0.071	0.027	0.228	0.239	8.769

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	16	26	27	16	26	26
N.S.	1	1.00	1.00	0.92	1.33	2.17	2.25	1.33	2.17	2.17
time (sec)	N/A	0.212	0.003	0.030	0.026	0.072	0.028	0.226	0.203	0.039

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	10	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.00	1.00	1.00
time (sec)	N/A	0.211	0.001	0.382	0.035	0.067	0.023	0.238	0.218	0.027

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	21	22	32	0	23	19
N.S.	1	1.00	1.00	0.95	1.05	1.10	1.60	0.00	1.15	0.95
time (sec)	N/A	0.229	0.004	0.043	0.030	0.082	0.264	0.000	0.204	8.792

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	40	36	82	0	44	21
N.S.	1	1.00	1.00	1.05	2.00	1.80	4.10	0.00	2.20	1.05
time (sec)	N/A	0.228	0.003	0.040	0.033	0.081	0.360	0.000	0.198	8.928

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	53	52	119	0	61	21
N.S.	1	1.00	1.00	1.05	2.65	2.60	5.95	0.00	3.05	1.05
time (sec)	N/A	0.234	0.003	0.050	0.038	0.081	0.418	0.000	0.225	9.122

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	135	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.44	8.38
time (sec)	N/A	0.242	0.003	0.496	0.041	0.082	0.044	0.251	0.216	9.246

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	135	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.44	8.38
time (sec)	N/A	0.228	0.004	0.933	0.032	0.066	0.045	0.264	0.201	9.288

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	135	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.44	8.38
time (sec)	N/A	0.228	0.004	1.756	0.034	0.064	0.048	0.242	0.209	0.105

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	338	275	231	520	285	202	287
N.S.	1	1.00	0.89	12.52	10.19	8.56	19.26	10.56	7.48	10.63
time (sec)	N/A	0.251	0.013	69.341	0.041	0.107	6.290	0.333	0.205	9.716

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	135	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.44	8.38
time (sec)	N/A	0.243	0.002	0.470	0.031	0.067	0.042	0.241	0.193	0.004

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	135	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.44	8.38
time (sec)	N/A	0.244	0.005	0.865	0.025	0.065	0.046	0.251	0.223	9.114

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	135	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.44	8.38
time (sec)	N/A	0.245	0.006	1.694	0.030	0.074	0.047	0.200	0.210	0.105

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	193	281	275	205	471	62225	202	285
N.S.	1	1.00	7.15	10.41	10.19	7.59	17.44	2304.63	7.48	10.56
time (sec)	N/A	0.259	0.079	54.632	0.039	0.093	5.090	6.766	0.202	9.724

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	88	120	121	93	197	93	90	124
N.S.	1	1.00	3.26	4.44	4.48	3.44	7.30	3.44	3.33	4.59
time (sec)	N/A	0.270	0.049	1.198	0.031	0.090	0.600	0.274	0.266	9.134

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	39	66	82	0	0	72	38
N.S.	1	1.00	1.00	1.44	2.44	3.04	0.00	0.00	2.67	1.41
time (sec)	N/A	0.271	0.056	0.502	0.036	0.096	0.000	0.000	0.249	8.941

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87	0.87
time (sec)	N/A	0.250	0.004	0.030	0.035	0.066	0.046	0.215	0.323	9.054

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	37	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	2.47	0.87
time (sec)	N/A	0.265	0.004	0.033	0.026	0.065	0.064	0.237	0.215	0.050

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	47	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	3.13	0.87
time (sec)	N/A	0.252	0.005	0.035	0.027	0.067	0.071	0.241	0.211	9.052

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	23	36	36	36	22	28	37
N.S.	1	1.00	1.26	1.21	1.89	1.89	1.89	1.16	1.47	1.95
time (sec)	N/A	0.246	0.011	0.046	0.038	0.066	0.135	0.237	0.189	0.041

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	23	36	36	36	22	28	37
N.S.	1	1.00	1.26	1.21	1.89	1.89	1.89	1.16	1.47	1.95
time (sec)	N/A	0.261	0.008	0.056	0.047	0.076	0.235	0.239	0.235	0.063

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	23	36	36	36	22	28	37
N.S.	1	1.00	1.26	1.21	1.89	1.89	1.89	1.16	1.47	1.95
time (sec)	N/A	0.253	0.010	0.075	0.039	0.073	0.336	0.227	0.286	9.250

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	25	19	22	25	22
N.S.	1	1.00	1.00	0.96	0.92	1.04	0.79	0.92	1.04	0.92
time (sec)	N/A	0.282	0.002	0.046	0.031	0.067	0.038	0.266	0.195	0.043

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	40	42	40	35	34	38	37	46	38	34
N.S.	1	1.05	1.00	0.88	0.85	0.95	0.92	1.15	0.95	0.85
time (sec)	N/A	0.314	0.007	0.096	0.032	0.069	0.066	0.242	0.212	0.039

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	44	48	49	45	48	47
N.S.	1	1.00	1.00	0.90	0.88	0.96	0.98	0.90	0.96	0.94
time (sec)	N/A	0.325	0.006	0.052	0.034	0.068	0.082	0.245	0.204	0.049

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	3	6	6	6
N.S.	1	1.00	1.00	1.17	1.00	1.00	0.50	1.00	1.00	1.00
time (sec)	N/A	0.216	0.005	0.092	0.107	0.072	0.040	0.284	0.202	8.822

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	170	144	34	124	122	36	112	13	197
N.S.	1	1.01	0.86	0.20	0.74	0.73	0.21	0.67	0.08	1.17
time (sec)	N/A	0.695	0.115	0.084	0.121	0.084	0.843	0.244	0.204	9.650

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	232	363	325	297	690	50971	253	363
N.S.	1	1.00	8.00	12.52	11.21	10.24	23.79	1757.62	8.72	12.52
time (sec)	N/A	0.257	0.135	0.036	0.037	0.101	147.772	1.455	0.197	10.860

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	15	14	134	160	14	135	14
N.S.	1	1.00	10.00	0.94	0.88	8.38	10.00	0.88	8.44	0.88
time (sec)	N/A	0.236	0.005	0.509	0.031	0.083	0.039	0.234	0.223	8.989

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	135	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.44	8.38
time (sec)	N/A	0.235	0.001	0.890	0.028	0.082	0.043	0.231	0.198	0.002

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	135	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.44	8.38
time (sec)	N/A	0.232	0.003	1.748	0.031	0.076	0.053	0.238	0.190	9.252

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	15	14	134	160	14	135	14
N.S.	1	1.00	10.00	0.94	0.88	8.38	10.00	0.88	8.44	0.88
time (sec)	N/A	0.228	0.003	0.907	0.034	0.084	0.041	0.222	0.189	9.348

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	135	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.44	8.38
time (sec)	N/A	0.230	0.002	1.721	0.027	0.080	0.047	0.218	0.228	0.002

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	15	14	134	160	14	135	14
N.S.	1	1.00	10.00	0.94	0.88	8.38	10.00	0.88	8.44	0.88
time (sec)	N/A	0.226	0.002	1.733	0.036	0.076	0.044	0.230	0.202	9.282

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	27	37	27	53	0	26	26
N.S.	1	1.00	1.00	1.17	1.61	1.17	2.30	0.00	1.13	1.13
time (sec)	N/A	0.269	0.021	0.428	0.031	0.096	0.300	0.000	0.185	9.368

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	26	25	29	27	28	49	0	22	31
N.S.	1	1.13	1.09	1.26	1.17	1.22	2.13	0.00	0.96	1.35
time (sec)	N/A	0.270	0.008	0.416	0.033	0.087	0.326	0.000	0.214	9.355

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	31	19	28	51	0	15	34
N.S.	1	1.00	1.00	2.07	1.27	1.87	3.40	0.00	1.00	2.27
time (sec)	N/A	0.256	0.003	0.426	0.031	0.088	0.363	0.000	0.244	9.506

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	25	24	25	16	26	31	0	20	26
N.S.	1	1.14	1.09	1.14	0.73	1.18	1.41	0.00	0.91	1.18
time (sec)	N/A	0.258	0.023	0.419	0.029	0.090	0.393	0.000	0.195	9.614

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	27	11	26	31	0	13	28
N.S.	1	1.00	1.00	1.80	0.73	1.73	2.07	0.00	0.87	1.87
time (sec)	N/A	0.250	0.017	0.436	0.035	0.079	0.286	0.000	0.214	9.580

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	10	9	8	8	8	9	7	8
N.S.	1	1.00	0.83	0.75	0.67	0.67	0.67	0.75	0.58	0.67
time (sec)	N/A	0.242	0.011	0.330	0.033	0.081	0.060	0.511	0.192	0.110

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	25	11	17	8	22	18	17	8
N.S.	1	1.00	1.79	0.79	1.21	0.57	1.57	1.29	1.21	0.57
time (sec)	N/A	0.246	0.013	0.549	0.028	0.070	0.103	0.457	0.215	9.500

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	10	32	30	8
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.83	2.67	2.50	0.67
time (sec)	N/A	0.241	0.013	0.329	0.115	0.077	0.086	0.322	0.203	0.100

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	33	24	28	31	24	32	0	30	26
N.S.	1	1.38	1.00	1.17	1.29	1.00	1.33	0.00	1.25	1.08
time (sec)	N/A	0.276	0.028	0.398	0.120	0.088	0.215	0.000	0.213	9.381

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	104	0	0	0	0	0	125	0
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	1.67	0.00
time (sec)	N/A	0.375	0.193	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	95	109	0	0	0	0	0	135	0
N.S.	1	0.96	1.10	0.00	0.00	0.00	0.00	0.00	1.36	0.00
time (sec)	N/A	0.434	0.242	0.000	0.000	0.000	0.000	0.000	0.217	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	103	0	0	0	0	0	30	0
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.418	1.435	0.000	0.000	0.000	0.000	0.000	0.475	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	69	99	0	0	0	0	0	22	0
N.S.	1	0.97	1.39	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.360	0.137	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	100	0	0	0	0	0	95	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	1.09	0.00
time (sec)	N/A	0.414	1.120	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	46	45	39	58	94	78	0	44	0
N.S.	1	0.90	0.88	0.76	1.14	1.84	1.53	0.00	0.86	0.00
time (sec)	N/A	0.290	0.013	0.779	0.117	0.082	0.835	0.000	0.224	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	0	0	0	0	0	80	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.95	0.00
time (sec)	N/A	0.422	0.090	0.000	0.000	0.000	0.000	0.000	0.204	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	77	0	0	0	0	0	17	97
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.28	1.59
time (sec)	N/A	0.354	0.088	0.000	0.000	0.000	0.000	0.000	0.212	9.123

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	84	0	0	0	0	0	20	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.436	0.083	0.000	0.000	0.000	0.000	0.000	0.274	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	131	131	0	0	0	0	0	167	0
N.S.	1	0.93	0.93	0.00	0.00	0.00	0.00	0.00	1.18	0.00
time (sec)	N/A	0.531	0.442	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	133	126	0	0	0	0	0	60	0
N.S.	1	1.04	0.98	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.537	1.614	0.000	0.000	0.000	0.000	0.000	0.855	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	101	117	0	0	0	0	0	99	0
N.S.	1	0.97	1.12	0.00	0.00	0.00	0.00	0.00	0.95	0.00
time (sec)	N/A	0.418	0.152	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	127	120	0	0	0	0	0	125	0
N.S.	1	1.04	0.98	0.00	0.00	0.00	0.00	0.00	1.02	0.00
time (sec)	N/A	0.498	1.433	0.000	0.000	0.000	0.000	0.000	0.244	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	64	58	51	73	117	88	0	63	0
N.S.	1	0.88	0.79	0.70	1.00	1.60	1.21	0.00	0.86	0.00
time (sec)	N/A	0.296	0.023	0.745	0.115	0.088	1.527	0.000	0.212	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	122	97	0	0	0	0	0	102	0
N.S.	1	1.04	0.83	0.00	0.00	0.00	0.00	0.00	0.87	0.00
time (sec)	N/A	0.495	0.103	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	95	94	0	0	0	0	0	44	0
N.S.	1	0.97	0.96	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.416	0.097	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	125	100	0	0	0	0	0	48	0
N.S.	1	1.02	0.82	0.00	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.506	0.105	0.000	0.000	0.000	0.000	0.000	0.282	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	97	96	0	0	0	0	0	46	0
N.S.	1	0.97	0.96	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.440	0.095	0.000	0.000	0.000	0.000	0.000	0.258	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	51	58	48	0	97	0	67	38	67
N.S.	1	1.13	1.29	1.07	0.00	2.16	0.00	1.49	0.84	1.49
time (sec)	N/A	0.372	0.026	0.154	0.000	0.084	0.000	0.286	0.240	8.883

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	62	61	53	102	0	69	60	55
N.S.	1	1.00	1.48	1.45	1.26	2.43	0.00	1.64	1.43	1.31
time (sec)	N/A	0.301	0.046	0.080	0.113	0.099	0.000	0.258	0.300	9.239

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	66	56	0	110	0	71	45	63
N.S.	1	1.00	1.29	1.10	0.00	2.16	0.00	1.39	0.88	1.24
time (sec)	N/A	0.347	0.025	0.089	0.000	0.086	0.000	0.176	0.260	9.051

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	69	74	0	118	0	0	41	0
N.S.	1	1.00	1.13	1.21	0.00	1.93	0.00	0.00	0.67	0.00
time (sec)	N/A	0.359	0.018	0.892	0.000	0.082	0.000	0.000	0.236	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	53	66	56	0	107	0	61	29	67
N.S.	1	1.08	1.35	1.14	0.00	2.18	0.00	1.24	0.59	1.37
time (sec)	N/A	0.379	0.038	0.177	0.000	0.078	0.000	0.265	0.202	10.076

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	68	81	34	108	0	63	36	54
N.S.	1	1.00	1.58	1.88	0.79	2.51	0.00	1.47	0.84	1.26
time (sec)	N/A	0.294	0.041	0.064	0.122	0.087	0.000	0.239	0.207	9.383

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	73	73	0	120	0	65	68	63
N.S.	1	1.00	1.38	1.38	0.00	2.26	0.00	1.23	1.28	1.19
time (sec)	N/A	0.345	0.047	0.131	0.000	0.089	0.000	0.200	0.216	9.634

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	77	105	0	128	0	0	47	0
N.S.	1	1.00	1.22	1.67	0.00	2.03	0.00	0.00	0.75	0.00
time (sec)	N/A	0.356	0.053	0.917	0.000	0.081	0.000	0.000	0.211	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	98	0	0	0	0	0	44	0
N.S.	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	0.344	0.379	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	89	0	0	0	0	0	22	0
N.S.	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.314	1.335	0.000	0.000	0.000	0.000	0.000	0.277	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	78	0	0	0	0	0	16	67
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	0.43	1.81
time (sec)	N/A	0.256	0.075	0.000	0.000	0.000	0.000	0.000	0.212	9.352

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	87	0	0	0	0	0	35	0
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.303	0.766	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	42	73	27	0	27	0
N.S.	1	1.00	1.00	0.84	1.35	2.35	0.87	0.00	0.87	0.00
time (sec)	N/A	0.254	0.006	0.790	0.111	0.080	0.646	0.000	0.227	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	68	0	0	0	0	0	32	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.318	0.077	0.000	0.000	0.000	0.000	0.000	0.209	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	66	0	0	0	0	0	23	0
N.S.	1	1.00	1.65	0.00	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.282	0.075	0.000	0.000	0.000	0.000	0.000	0.209	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	68	0	0	0	0	0	27	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.326	0.084	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	98	117	0	0	0	0	0	62	0
N.S.	1	0.92	1.09	0.00	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.441	0.383	0.000	0.000	0.000	0.000	0.000	0.214	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	109	0	0	0	0	0	51	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.437	2.726	0.000	0.000	0.000	0.000	0.000	0.791	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	70	91	0	0	0	0	0	47	0
N.S.	1	0.97	1.26	0.00	0.00	0.00	0.00	0.00	0.65	0.00
time (sec)	N/A	0.358	0.157	0.000	0.000	0.000	0.000	0.000	0.223	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	104	0	0	0	0	0	45	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.414	1.343	0.000	0.000	0.000	0.000	0.000	0.223	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	49	52	42	61	145	185	0	41	0
N.S.	1	0.91	0.96	0.78	1.13	2.69	3.43	0.00	0.76	0.00
time (sec)	N/A	0.287	0.023	0.431	0.118	0.088	1.171	0.000	0.207	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	78	0	0	0	0	0	48	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.432	0.095	0.000	0.000	0.000	0.000	0.000	0.204	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	70	74	0	0	0	0	0	48	0
N.S.	1	0.97	1.03	0.00	0.00	0.00	0.00	0.00	0.67	0.00
time (sec)	N/A	0.367	0.092	0.000	0.000	0.000	0.000	0.000	0.195	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	78	0	0	0	0	0	46	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.442	0.097	0.000	0.000	0.000	0.000	0.000	0.317	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	70	74	0	0	0	0	0	42	0
N.S.	1	0.97	1.03	0.00	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.365	0.091	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	64	56	0	111	0	0	43	0
N.S.	1	1.00	2.00	1.75	0.00	3.47	0.00	0.00	1.34	0.00
time (sec)	N/A	0.281	0.016	0.346	0.000	0.164	0.000	0.000	0.235	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	60	49	0	74	0	40	27	0
N.S.	1	1.00	1.88	1.53	0.00	2.31	0.00	1.25	0.84	0.00
time (sec)	N/A	0.281	0.017	0.109	0.000	0.080	0.000	0.206	0.219	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	64	0	0	102	0	0	47	0
N.S.	1	1.00	2.00	0.00	0.00	3.19	0.00	0.00	1.47	0.00
time (sec)	N/A	0.278	0.017	0.000	0.000	0.383	0.000	0.000	0.219	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	76	0	0	111	0	0	28	0
N.S.	1	1.00	2.05	0.00	0.00	3.00	0.00	0.00	0.76	0.00
time (sec)	N/A	0.299	0.032	0.000	0.000	0.086	0.000	0.000	0.206	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	72	60	0	121	0	0	62	0
N.S.	1	1.00	2.18	1.82	0.00	3.67	0.00	0.00	1.88	0.00
time (sec)	N/A	0.279	0.162	2.790	0.000	0.176	0.000	0.000	0.254	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	68	51	0	77	0	47	17	0
N.S.	1	1.00	2.06	1.55	0.00	2.33	0.00	1.42	0.52	0.00
time (sec)	N/A	0.278	0.016	0.126	0.000	0.085	0.000	0.234	0.220	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	72	0	0	111	0	0	62	0
N.S.	1	1.00	2.18	0.00	0.00	3.36	0.00	0.00	1.88	0.00
time (sec)	N/A	0.281	0.244	0.000	0.000	0.392	0.000	0.000	0.233	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	78	0	0	119	0	0	32	0
N.S.	1	1.00	2.05	0.00	0.00	3.13	0.00	0.00	0.84	0.00
time (sec)	N/A	0.299	0.066	0.000	0.000	0.088	0.000	0.000	0.189	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	78	0	0	0	0	0	16	67
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	0.43	1.81
time (sec)	N/A	0.295	0.070	0.000	0.000	0.000	0.000	0.000	0.197	9.915

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	78	0	0	117	0	0	16	67
N.S.	1	1.00	2.11	0.00	0.00	3.16	0.00	0.00	0.43	1.81
time (sec)	N/A	0.290	0.012	0.000	0.000	0.095	0.000	0.000	0.232	9.232

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	78	0	0	0	0	0	16	67
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	0.43	1.81
time (sec)	N/A	0.293	0.012	0.000	0.000	0.000	0.000	0.000	0.212	9.406

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	80	0	0	0	0	0	30	66
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	0.79	1.74
time (sec)	N/A	0.293	0.076	0.000	0.000	0.000	0.000	0.000	0.201	9.525

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	80	0	0	121	0	0	30	66
N.S.	1	1.00	2.11	0.00	0.00	3.18	0.00	0.00	0.79	1.74
time (sec)	N/A	0.289	0.012	0.000	0.000	0.088	0.000	0.000	0.208	9.498

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	80	0	0	0	0	0	30	66
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	0.79	1.74
time (sec)	N/A	0.283	0.012	0.000	0.000	0.000	0.000	0.000	0.226	9.791

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	218	0	0	0	0	0	0	0
N.S.	1	1.00	2.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.433	0.408	0.000	0.000	0.000	0.000	0.000	0.253	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	156	0	0	0	0	0	425	0
N.S.	1	1.00	1.56	0.00	0.00	0.00	0.00	0.00	4.25	0.00
time (sec)	N/A	0.433	0.253	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	120	106	0	0	0	0	0	35	0
N.S.	1	1.18	1.04	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.452	0.190	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	123	116	0	0	0	0	0	52	0
N.S.	1	1.11	1.05	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.451	0.248	0.000	0.000	0.000	0.000	0.000	0.215	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	123	166	0	0	0	0	0	69	0
N.S.	1	1.11	1.50	0.00	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	0.442	0.414	0.000	0.000	0.000	0.000	0.000	0.240	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	177	0	0	0	0	0	1088	82
N.S.	1	1.00	1.82	0.00	0.00	0.00	0.00	0.00	11.22	0.85
time (sec)	N/A	0.396	0.161	0.000	0.000	0.000	0.000	0.000	0.218	9.843

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	101	134	0	0	0	0	0	220	82
N.S.	1	1.16	1.54	0.00	0.00	0.00	0.00	0.00	2.53	0.94
time (sec)	N/A	0.400	0.122	0.000	0.000	0.000	0.000	0.000	0.227	9.847

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	88	0	0	0	0	0	28	83
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.30	0.89
time (sec)	N/A	0.392	0.066	0.000	0.000	0.000	0.000	0.000	0.274	9.475

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	104	0	0	0	0	0	45	83
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.45	0.82
time (sec)	N/A	0.400	0.155	0.000	0.000	0.000	0.000	0.000	0.262	9.557

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	185	0	0	0	0	0	62	83
N.S.	1	1.00	1.83	0.00	0.00	0.00	0.00	0.00	0.61	0.82
time (sec)	N/A	0.405	0.184	0.000	0.000	0.000	0.000	0.000	0.301	10.033

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	19	16	0	16	0	50	28	15
N.S.	1	1.00	1.06	0.89	0.00	0.89	0.00	2.78	1.56	0.83
time (sec)	N/A	0.254	0.016	0.082	0.000	0.075	0.000	0.226	0.213	9.573

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	18	0	19	0	11	15	27
N.S.	1	1.00	1.00	0.90	0.00	0.95	0.00	0.55	0.75	1.35
time (sec)	N/A	0.226	0.027	0.332	0.000	0.130	0.000	0.225	0.230	9.542

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	5	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.62	0.75
time (sec)	N/A	0.228	0.011	0.348	0.107	0.077	0.074	0.214	0.217	0.122

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	14	28	0	27	20	22
N.S.	1	1.00	1.00	1.16	0.56	1.12	0.00	1.08	0.80	0.88
time (sec)	N/A	0.252	0.010	0.589	0.035	0.081	0.000	0.235	0.193	9.423

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	14	28	0	27	20	29
N.S.	1	1.00	1.00	1.16	0.56	1.12	0.00	1.08	0.80	1.16
time (sec)	N/A	0.242	0.001	0.584	0.037	0.080	0.000	0.217	0.197	9.413

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	14	28	0	14	20	22
N.S.	1	1.00	1.00	1.16	0.56	1.12	0.00	0.56	0.80	0.88
time (sec)	N/A	0.264	0.008	0.924	0.041	0.083	0.000	0.677	0.218	8.757

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	105	100	86	1162	123	763	17454	4599	1231	163
N.S.	1	0.95	0.82	11.07	1.17	7.27	166.23	43.80	11.72	1.55
time (sec)	N/A	0.450	0.103	1.965	0.046	0.109	15.873	0.297	0.216	9.183

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	71	70	58	367	84	275	3398	1159	379	62
N.S.	1	0.99	0.82	5.17	1.18	3.87	47.86	16.32	5.34	0.87
time (sec)	N/A	0.392	0.076	0.777	0.046	0.089	7.174	0.232	0.210	9.076

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	40	30	85	45	69	367	164	64	33
N.S.	1	1.08	0.81	2.30	1.22	1.86	9.92	4.43	1.73	0.89
time (sec)	N/A	0.330	0.043	0.172	0.042	0.085	2.253	0.217	0.199	9.072

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	69	0	0	0	0	0	23	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.374	0.049	0.000	0.000	0.000	0.000	0.000	0.212	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	69	0	0	0	0	0	40	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.359	0.043	0.000	0.000	0.000	0.000	0.000	0.249	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	69	0	0	0	0	0	57	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.360	0.052	0.000	0.000	0.000	0.000	0.000	0.199	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	0	0	0	0	0	424	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	4.61	0.00
time (sec)	N/A	0.407	0.113	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	73	0	0	0	0	0	135	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	1.96	0.00
time (sec)	N/A	0.361	0.126	0.000	0.000	0.000	0.000	0.000	0.243	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	72	74	0	0	0	0	0	137	0
N.S.	1	0.97	1.00	0.00	0.00	0.00	0.00	0.00	1.85	0.00
time (sec)	N/A	0.403	0.102	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	82	0	0	0	0	0	143	0
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	2.07	0.00
time (sec)	N/A	0.361	0.155	0.000	0.000	0.000	0.000	0.000	0.231	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	95	83	0	0	0	0	0	142	0
N.S.	1	1.13	0.99	0.00	0.00	0.00	0.00	0.00	1.69	0.00
time (sec)	N/A	0.412	0.132	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	0	0	49	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.11	0.00
time (sec)	N/A	0.301	0.066	0.000	0.000	0.000	0.000	0.000	0.231	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	0	0	64	0	0	20	76
N.S.	1	1.00	0.98	0.00	0.00	1.45	0.00	0.00	0.45	1.73
time (sec)	N/A	0.292	0.051	0.000	0.000	0.090	0.000	0.000	0.227	9.906

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	0	0	61	0	0	39	0
N.S.	1	1.00	0.98	0.00	0.00	1.39	0.00	0.00	0.89	0.00
time (sec)	N/A	0.308	0.001	0.000	0.000	0.092	0.000	0.000	0.194	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	45	0	0	76	0	0	47	0
N.S.	1	1.00	0.98	0.00	0.00	1.65	0.00	0.00	1.02	0.00
time (sec)	N/A	0.339	0.062	0.000	0.000	0.089	0.000	0.000	0.195	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	45	0	0	79	0	0	25	0
N.S.	1	1.00	0.98	0.00	0.00	1.72	0.00	0.00	0.54	0.00
time (sec)	N/A	0.281	0.003	0.000	0.000	0.091	0.000	0.000	0.211	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	36	40	17	44	0	0	23	0
N.S.	1	1.00	0.82	0.91	0.39	1.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.304	0.045	0.418	0.052	0.085	0.000	0.000	0.194	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	36	40	17	44	0	0	17	0
N.S.	1	1.00	0.82	0.91	0.39	1.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.313	0.048	0.417	0.058	0.082	0.000	0.000	0.206	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	36	40	17	44	0	0	17	0
N.S.	1	1.00	0.82	0.91	0.39	1.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.301	0.045	0.412	0.051	0.099	0.000	0.000	0.204	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	47	0	0	47	0	0	31	0
N.S.	1	1.00	0.82	0.00	0.00	0.82	0.00	0.00	0.54	0.00
time (sec)	N/A	0.328	0.042	0.000	0.000	0.082	0.000	0.000	0.239	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	45	0	0	54	0	0	28	0
N.S.	1	1.00	0.74	0.00	0.00	0.89	0.00	0.00	0.46	0.00
time (sec)	N/A	0.333	0.022	0.000	0.000	0.080	0.000	0.000	0.201	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	40	0	0	76	0	0	51	0
N.S.	1	1.00	1.03	0.00	0.00	1.95	0.00	0.00	1.31	0.00
time (sec)	N/A	0.285	0.100	0.000	0.000	0.089	0.000	0.000	0.222	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	38	86	0	64	0	0	38	0
N.S.	1	1.00	0.95	2.15	0.00	1.60	0.00	0.00	0.95	0.00
time (sec)	N/A	0.348	0.085	1.654	0.000	0.087	0.000	0.000	0.206	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [326] had the largest ratio of [1]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.00	13	0.231
2	A	3	3	1.00	11	0.273
3	A	1	1	1.00	9	0.111
4	A	2	2	1.00	13	0.154
5	A	3	3	1.00	13	0.231
6	A	3	3	1.00	13	0.231
7	A	3	3	1.00	13	0.231
8	A	3	3	1.00	13	0.231
9	A	3	3	1.00	15	0.200
10	A	5	4	1.13	13	0.308
11	A	3	3	1.00	11	0.273
12	A	2	2	1.00	15	0.133
13	A	3	3	1.00	15	0.200
14	A	5	4	1.30	15	0.267
15	A	3	3	1.00	15	0.200
16	A	5	4	1.04	15	0.267
17	A	3	3	1.00	15	0.200
18	A	3	3	1.00	11	0.273
19	A	5	4	0.96	15	0.267
20	A	3	3	1.00	15	0.200
21	A	2	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
22	A	2	2	1.00	13	0.154
23	A	6	5	1.18	11	0.455
24	A	3	3	1.00	15	0.200
25	A	5	4	1.03	15	0.267
26	A	4	4	1.19	15	0.267
27	A	5	4	1.02	15	0.267
28	A	3	3	1.00	15	0.200
29	A	5	4	1.03	13	0.308
30	A	4	4	1.13	11	0.364
31	A	5	4	1.06	15	0.267
32	A	5	5	1.16	15	0.333
33	A	3	3	1.00	13	0.231
34	A	5	4	1.00	13	0.308
35	A	3	3	1.00	13	0.231
36	A	2	2	1.00	13	0.154
37	A	2	2	1.00	11	0.182
38	A	6	5	1.27	9	0.556
39	A	3	3	1.00	13	0.231
40	A	5	4	1.09	13	0.308
41	A	4	4	1.00	13	0.308
42	A	5	4	1.07	13	0.308
43	A	6	5	1.27	9	0.556
44	A	7	6	1.11	11	0.545
45	A	7	6	1.07	17	0.353
46	A	9	8	1.07	17	0.471
47	A	6	5	1.04	15	0.333
48	A	8	7	1.05	13	0.538
49	A	5	4	1.00	17	0.235
50	A	8	7	1.07	17	0.412
51	A	5	4	1.00	17	0.235
52	A	9	8	1.05	17	0.471
53	A	8	7	1.08	17	0.412

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	10	9	1.07	15	0.600
55	A	7	6	1.05	13	0.462
56	A	9	8	1.06	17	0.471
57	A	6	5	1.04	17	0.294
58	A	9	8	1.06	17	0.471
59	A	6	5	1.02	17	0.294
60	A	9	8	1.05	17	0.471
61	A	6	5	1.04	17	0.294
62	A	10	9	1.06	17	0.529
63	A	7	6	1.05	17	0.353
64	A	6	5	1.06	17	0.294
65	A	8	7	1.06	17	0.412
66	A	5	4	1.00	17	0.235
67	A	7	6	1.07	15	0.400
68	A	4	3	1.00	13	0.231
69	A	8	7	1.08	17	0.412
70	A	5	4	1.00	17	0.235
71	A	9	8	1.06	17	0.471
72	A	7	6	1.10	17	0.353
73	A	9	8	1.09	17	0.471
74	A	6	5	1.06	17	0.294
75	A	8	7	1.07	17	0.412
76	A	5	4	1.00	17	0.235
77	A	8	7	1.06	17	0.412
78	A	5	4	1.00	15	0.267
79	A	9	8	1.09	13	0.615
80	A	6	5	1.06	17	0.294
81	A	10	9	1.08	17	0.529
82	A	8	7	1.19	19	0.368
83	A	5	5	1.16	19	0.263
84	A	7	6	1.12	19	0.316
85	A	4	4	1.16	19	0.211

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
86	A	1	1	1.00	19	0.053
87	A	3	3	1.11	19	0.158
88	A	2	2	1.00	19	0.105
89	A	2	2	1.00	19	0.105
90	A	3	3	1.11	19	0.158
91	A	1	1	1.00	19	0.053
92	A	4	4	1.16	19	0.211
93	A	7	6	1.12	19	0.316
94	A	5	5	1.15	19	0.263
95	A	8	7	1.18	19	0.368
96	A	6	6	1.16	19	0.316
97	A	9	8	1.17	19	0.421
98	A	7	7	1.17	19	0.368
99	A	4	3	1.00	17	0.176
100	A	3	2	1.00	15	0.133
101	A	1	1	1.00	17	0.059
102	A	2	2	1.00	17	0.118
103	A	3	3	1.08	17	0.176
104	A	5	4	1.00	17	0.235
105	A	4	3	1.00	13	0.231
106	A	5	4	1.00	17	0.235
107	A	8	7	1.06	17	0.412
108	A	7	6	1.06	17	0.353
109	A	8	7	1.07	17	0.412
110	A	12	11	1.21	19	0.579
111	A	13	12	1.09	19	0.632
112	A	9	8	1.21	17	0.471
113	A	10	9	1.05	15	0.600
114	A	6	5	1.26	19	0.263
115	A	10	9	1.05	19	0.474
116	A	8	7	1.21	19	0.368
117	A	13	12	1.08	19	0.632

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	11	10	1.21	19	0.526
119	A	12	11	1.20	19	0.579
120	A	13	12	1.08	17	0.706
121	A	9	8	1.19	15	0.533
122	A	10	9	1.05	19	0.474
123	A	7	6	1.26	19	0.316
124	A	11	10	1.05	19	0.526
125	A	9	8	1.20	19	0.421
126	A	14	13	1.08	19	0.684
127	A	12	11	1.20	19	0.579
128	A	12	11	1.22	19	0.579
129	A	13	12	1.10	19	0.632
130	A	9	8	1.22	19	0.421
131	A	10	9	1.07	17	0.529
132	A	6	5	1.21	15	0.333
133	A	9	8	1.06	19	0.421
134	A	7	6	1.22	19	0.316
135	A	12	11	1.08	19	0.579
136	A	10	9	1.22	19	0.474
137	A	14	13	1.11	19	0.684
138	A	10	9	1.23	19	0.474
139	A	11	10	1.09	19	0.526
140	A	7	6	1.26	17	0.353
141	A	9	8	1.04	15	0.533
142	A	7	6	1.23	19	0.316
143	A	12	11	1.08	19	0.579
144	A	10	9	1.22	19	0.474
145	A	15	14	1.10	19	0.737
146	A	13	13	1.18	19	0.684
147	A	10	10	1.17	19	0.526
148	A	7	7	1.15	17	0.412
149	A	4	4	1.11	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
150	A	1	1	1.00	19	0.053
151	A	5	4	0.99	19	0.211
152	A	8	7	1.08	19	0.368
153	A	11	10	1.12	19	0.526
154	A	14	13	1.14	19	0.684
155	A	12	12	1.17	19	0.632
156	A	9	9	1.16	17	0.529
157	A	6	6	1.14	15	0.400
158	A	3	3	1.07	19	0.158
159	A	5	4	1.03	19	0.211
160	A	6	5	1.02	19	0.263
161	A	9	8	1.09	19	0.421
162	A	12	11	1.12	19	0.579
163	A	15	14	1.14	19	0.737
164	A	14	14	1.17	19	0.737
165	A	11	11	1.16	19	0.579
166	A	8	8	1.14	19	0.421
167	A	5	5	1.10	17	0.294
168	A	2	2	1.00	15	0.133
169	A	4	3	1.00	19	0.158
170	A	7	6	1.08	19	0.316
171	A	10	9	1.12	19	0.474
172	A	13	12	1.15	19	0.632
173	A	12	12	1.17	19	0.632
174	A	9	9	1.15	19	0.474
175	A	6	6	1.12	19	0.316
176	A	3	3	1.09	17	0.176
177	A	4	3	1.00	15	0.200
178	A	7	6	1.10	19	0.316
179	A	10	9	1.13	19	0.474
180	A	13	12	1.15	19	0.632
181	A	16	15	1.16	19	0.789

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	3	3	1.00	15	0.200
183	A	3	3	1.00	13	0.231
184	A	1	1	1.00	11	0.091
185	A	3	3	1.00	15	0.200
186	A	2	2	1.00	15	0.133
187	A	3	3	1.00	17	0.176
188	A	3	3	1.00	15	0.200
189	A	3	3	1.00	13	0.231
190	A	3	3	1.00	17	0.176
191	A	3	3	1.00	17	0.176
192	A	3	3	1.00	17	0.176
193	A	3	3	1.00	17	0.176
194	A	3	3	1.00	17	0.176
195	A	3	3	1.00	17	0.176
196	A	2	2	1.00	17	0.118
197	A	4	4	1.00	15	0.267
198	A	3	3	1.00	13	0.231
199	A	3	3	1.00	17	0.176
200	A	3	3	1.00	17	0.176
201	A	3	3	1.00	17	0.176
202	A	3	3	1.00	17	0.176
203	A	3	3	1.00	17	0.176
204	A	3	3	1.00	17	0.176
205	A	2	2	1.00	17	0.118
206	A	3	3	1.00	17	0.176
207	A	3	3	1.00	17	0.176
208	A	3	3	1.00	15	0.200
209	A	3	3	1.00	13	0.231
210	A	3	3	1.00	17	0.176
211	A	4	4	1.11	19	0.211
212	A	3	3	1.08	17	0.176
213	A	2	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	1	1	1.00	19	0.053
215	A	4	3	1.00	19	0.158
216	A	4	3	1.00	19	0.158
217	A	5	4	0.98	19	0.211
218	A	6	5	1.04	19	0.263
219	A	6	6	1.15	19	0.316
220	A	5	5	1.13	17	0.294
221	A	4	4	1.11	15	0.267
222	A	3	3	1.08	19	0.158
223	A	2	2	1.00	19	0.105
224	A	1	1	1.00	19	0.053
225	A	5	4	1.03	19	0.211
226	A	5	4	1.05	19	0.211
227	A	5	4	0.99	19	0.211
228	A	6	5	1.01	19	0.263
229	A	7	6	1.05	19	0.316
230	A	8	7	1.08	19	0.368
231	A	4	4	1.12	19	0.211
232	A	3	3	1.08	19	0.158
233	A	2	2	1.00	19	0.105
234	A	1	1	1.00	17	0.059
235	A	3	2	1.00	15	0.133
236	A	4	3	1.00	19	0.158
237	A	5	4	1.01	19	0.211
238	A	6	5	1.06	19	0.263
239	A	4	4	1.12	19	0.211
240	A	3	3	1.08	19	0.158
241	A	2	2	1.00	19	0.105
242	A	1	1	1.00	19	0.053
243	A	4	3	1.00	19	0.158
244	A	5	4	1.07	17	0.235
245	A	6	5	1.06	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
246	A	7	6	1.09	19	0.316
247	A	8	7	1.11	19	0.368
248	A	6	5	1.06	21	0.238
249	A	5	4	1.01	21	0.190
250	A	4	3	1.00	21	0.143
251	A	3	2	1.00	21	0.095
252	A	1	1	1.00	21	0.048
253	A	2	2	1.00	21	0.095
254	A	3	3	1.07	21	0.143
255	A	4	4	1.10	21	0.190
256	A	3	3	1.00	21	0.143
257	A	3	3	1.27	19	0.158
258	A	3	3	1.00	21	0.143
259	A	1	1	1.00	21	0.048
260	A	2	2	1.00	21	0.095
261	A	3	3	0.98	21	0.143
262	A	2	2	1.00	17	0.118
263	A	3	3	1.08	19	0.158
264	A	2	2	1.00	19	0.105
265	A	1	1	1.00	19	0.053
266	A	3	2	1.00	15	0.133
267	A	4	3	1.00	19	0.158
268	A	3	3	1.00	19	0.158
269	A	2	2	1.00	17	0.118
270	A	3	3	1.00	19	0.158
271	A	5	5	1.06	19	0.263
272	A	4	4	1.06	19	0.211
273	A	5	5	1.06	19	0.263
274	A	6	5	1.03	21	0.238
275	A	8	7	1.02	21	0.333
276	A	4	3	1.00	21	0.143
277	A	5	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	7	6	1.02	21	0.286
279	A	3	2	1.00	21	0.095
280	A	4	3	1.00	21	0.143
281	A	8	7	1.03	21	0.333
282	A	1	1	1.00	21	0.048
283	A	5	4	1.00	21	0.190
284	A	9	8	1.02	21	0.381
285	A	2	2	1.00	21	0.095
286	A	6	5	1.02	21	0.238
287	A	3	3	1.00	15	0.200
288	A	3	3	1.00	13	0.231
289	A	6	5	1.06	19	0.263
290	A	5	4	1.01	19	0.211
291	A	4	3	1.00	19	0.158
292	A	3	2	1.00	17	0.118
293	A	1	1	1.00	15	0.067
294	A	2	2	1.00	19	0.105
295	A	3	3	1.08	19	0.158
296	A	4	4	1.11	19	0.211
297	A	5	4	1.04	11	0.364
298	A	6	5	0.96	13	0.385
299	A	2	2	1.00	9	0.222
300	A	2	2	1.00	9	0.222
301	A	2	2	1.00	9	0.222
302	A	2	2	1.00	13	0.154
303	A	2	2	1.00	13	0.154
304	A	2	2	1.00	13	0.154
305	A	2	2	1.00	13	0.154
306	A	2	2	1.00	11	0.182
307	A	2	2	1.00	15	0.133
308	A	2	2	1.00	15	0.133
309	A	2	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
310	A	2	2	1.00	11	0.182
311	A	2	2	1.00	13	0.154
312	A	2	2	1.00	13	0.154
313	A	2	2	1.00	17	0.118
314	A	2	2	1.00	17	0.118
315	A	2	2	1.00	17	0.118
316	A	2	2	1.00	11	0.182
317	A	2	2	1.00	11	0.182
318	A	2	2	1.00	11	0.182
319	A	2	2	1.00	11	0.182
320	A	2	2	1.00	13	0.154
321	A	2	2	1.00	13	0.154
322	A	3	3	1.00	11	0.273
323	A	5	4	1.05	11	0.364
324	A	3	3	1.00	11	0.273
325	A	3	3	1.00	9	0.333
326	A	10	9	1.01	9	1.000
327	A	2	2	1.00	22	0.091
328	A	1	1	1.00	13	0.077
329	A	2	2	1.00	15	0.133
330	A	2	2	1.00	17	0.118
331	A	1	1	1.00	13	0.077
332	A	2	2	1.00	15	0.133
333	A	1	1	1.00	13	0.077
334	A	2	2	1.00	11	0.182
335	A	6	5	1.13	13	0.385
336	A	2	2	1.00	15	0.133
337	A	6	5	1.14	13	0.385
338	A	2	2	1.00	15	0.133
339	A	2	2	1.00	11	0.182
340	A	2	2	1.00	11	0.182
341	A	2	2	1.00	9	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	6	5	1.38	11	0.455
343	A	4	3	1.00	25	0.120
344	A	5	4	0.96	27	0.148
345	A	5	4	1.00	23	0.174
346	A	5	4	0.97	22	0.182
347	A	5	4	1.00	21	0.190
348	A	6	5	0.90	18	0.278
349	A	5	4	1.00	23	0.174
350	A	4	3	1.00	15	0.200
351	A	5	4	1.00	23	0.174
352	A	6	5	0.93	27	0.185
353	A	6	5	1.04	23	0.217
354	A	6	5	0.97	22	0.227
355	A	6	5	1.04	21	0.238
356	A	7	6	0.88	18	0.333
357	A	6	5	1.04	23	0.217
358	A	6	5	0.97	22	0.227
359	A	6	5	1.02	23	0.217
360	A	6	5	0.97	22	0.227
361	A	6	5	1.13	13	0.385
362	A	6	5	1.00	15	0.333
363	A	5	4	1.00	15	0.267
364	A	5	4	1.00	15	0.267
365	A	6	5	1.08	15	0.333
366	A	6	5	1.00	17	0.294
367	A	5	4	1.00	17	0.235
368	A	5	4	1.00	17	0.235
369	A	4	3	1.00	27	0.111
370	A	4	3	1.00	23	0.130
371	A	3	2	1.00	15	0.133
372	A	4	3	1.00	21	0.143
373	A	5	4	1.00	18	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
374	A	4	3	1.00	23	0.130
375	A	4	3	1.00	22	0.136
376	A	4	3	1.00	23	0.130
377	A	5	4	0.92	27	0.148
378	A	5	4	1.00	23	0.174
379	A	5	4	0.97	22	0.182
380	A	5	4	1.00	21	0.190
381	A	6	5	0.91	18	0.278
382	A	5	4	1.00	23	0.174
383	A	5	4	0.97	22	0.182
384	A	5	4	1.00	23	0.174
385	A	5	4	0.97	22	0.182
386	A	4	3	1.00	15	0.200
387	A	4	3	1.00	15	0.200
388	A	4	3	1.00	15	0.200
389	A	4	3	1.00	19	0.158
390	A	4	3	1.00	16	0.188
391	A	4	3	1.00	16	0.188
392	A	4	3	1.00	16	0.188
393	A	4	3	1.00	20	0.150
394	A	4	3	1.00	19	0.158
395	A	4	3	1.00	17	0.176
396	A	4	3	1.00	17	0.176
397	A	4	3	1.00	20	0.150
398	A	4	3	1.00	19	0.158
399	A	4	3	1.00	18	0.167
400	A	3	3	1.00	21	0.143
401	A	3	3	1.00	21	0.143
402	A	3	3	1.18	21	0.143
403	A	3	3	1.11	21	0.143
404	A	3	3	1.11	21	0.143
405	A	3	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
406	A	3	3	1.16	15	0.200
407	A	3	3	1.00	15	0.200
408	A	3	3	1.00	15	0.200
409	A	3	3	1.00	15	0.200
410	A	2	2	1.00	11	0.182
411	A	1	1	1.00	11	0.091
412	A	4	3	1.00	13	0.231
413	A	1	1	1.00	17	0.059
414	A	1	1	1.00	17	0.059
415	A	2	2	1.00	15	0.133
416	A	5	4	0.95	19	0.211
417	A	5	4	0.99	19	0.211
418	A	5	4	1.08	17	0.235
419	A	4	3	1.00	19	0.158
420	A	4	3	1.00	19	0.158
421	A	4	3	1.00	19	0.158
422	A	3	3	1.00	17	0.176
423	A	4	3	1.00	22	0.136
424	A	4	3	0.97	22	0.136
425	A	4	3	1.00	27	0.111
426	A	4	3	1.13	27	0.111
427	A	3	3	1.00	13	0.231
428	A	1	1	1.00	18	0.056
429	A	2	2	1.00	17	0.118
430	A	2	2	1.00	22	0.091
431	A	1	1	1.00	23	0.043
432	A	2	2	1.00	19	0.105
433	A	2	2	1.00	19	0.105
434	A	2	2	1.00	19	0.105
435	A	2	2	1.00	19	0.105
436	A	2	2	1.00	19	0.105
437	A	1	1	1.00	25	0.040

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
438	A	2	2	1.00	28	0.071

CHAPTER 3

Listing of integrals

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3.25 $\int \frac{1}{x^2(ax+bx^3)}\,dx$ . . . 306

3.26 $\int \frac{1}{x^3(ax+bx^3)}\,dx$ 311
3.27 $\int \frac{1}{x^4(ax+bx^3)}\,dx$ 317
3.28 $\int \frac{x^2}{(ax+bx^3)^2}\,dx$ 322
3.29 $\int \frac{x}{(ax+bx^3)^2}\,dx$ 327
3.30 $\int \frac{1}{(ax+bx^3)^2}\,dx$ 333
3.31 $\int \frac{1}{x(ax+bx^3)^2}\,dx$ 339
3.32 $\int \frac{1}{x^2(ax+bx^3)^2}\,dx$ 345
3.33 $\int \frac{x^5}{x-x^3}\,dx$ 351
3.34 $\int \frac{x^4}{x-x^3}\,dx$ 356
3.35 $\int \frac{x^3}{x-x^3}\,dx$ 361
3.36 $\int \frac{x^2}{x-x^3}\,dx$ 367
3.37 $\int \frac{x}{x-x^3}\,dx$ 372
3.38 $\int \frac{1}{x-x^3}\,dx$ 377
3.39 $\int \frac{1}{x(x-x^3)}\,dx$ 382
3.40 $\int \frac{1}{x^2(x-x^3)}\,dx$ 388
3.41 $\int \frac{1}{x^3(x-x^3)}\,dx$ 393
3.42 $\int \frac{1}{x^4(x-x^3)}\,dx$ 398
3.43 $\int \frac{1}{x+bx^3}\,dx$ 403
3.44 $\int \frac{1}{-x+bx^3}\,dx$ 408
3.45 $\int x^3\sqrt{ax+bx^3}\,dx$ 414
3.46 $\int x^2\sqrt{ax+bx^3}\,dx$ 421
3.47 $\int x\sqrt{ax+bx^3}\,dx$ 429
3.48 $\int \sqrt{ax+bx^3}\,dx$ 436
3.49 $\int \frac{\sqrt{ax+bx^3}}{x}\,dx$ 444
3.50 $\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx$ 450
3.51 $\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx$ 458
3.52 $\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx$ 464
3.53 $\int x^2(ax+bx^3)^{3/2}\,dx$ 472
3.54 $\int x(ax+bx^3)^{3/2}\,dx$ 479
3.55 $\int (ax+bx^3)^{3/2}\,dx$ 488
3.56 $\int \frac{(ax+bx^3)^{3/2}}{x}\,dx$ 495
3.57 $\int \frac{(ax+bx^3)^{3/2}}{x^2}\,dx$ 503
3.58 $\int \frac{(ax+bx^3)^{3/2}}{x^3}\,dx$ 509
3.59 $\int \frac{(ax+bx^3)^{3/2}}{x^4}\,dx$ 517
3.60 $\int \frac{(ax+bx^3)^{3/2}}{x^5}\,dx$ 523

3.61 $\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx$ 531

3.62 $\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx$ 537

3.63 $\int \frac{(ax+bx^3)^{3/2}}{x^8}\,dx$ 546

3.64 $\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx$ 552

3.65 $\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx$ 558

3.66 $\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx$ 565

3.67 $\int \frac{x}{\sqrt{ax+bx^3}}\,dx$ 571

3.68 $\int \frac{1}{\sqrt{ax+bx^3}}\,dx$ 578

3.69 $\int \frac{1}{x\sqrt{ax+bx^3}}\,dx$ 583

3.70 $\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx$ 591

3.71 $\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx$ 597

3.72 $\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx$ 605

3.73 $\int \frac{x^6}{(ax+bx^3)^{3/2}}\,dx$ 612

3.74 $\int \frac{x^5}{(ax+bx^3)^{3/2}}\,dx$ 620

3.75 $\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx$ 627

3.76 $\int \frac{x^3}{(ax+bx^3)^{3/2}}\,dx$ 635

3.77 $\int \frac{x^2}{(ax+bx^3)^{3/2}}\,dx$ 641

3.78 $\int \frac{x}{(ax+bx^3)^{3/2}}\,dx$ 649

3.79 $\int \frac{1}{(ax+bx^3)^{3/2}}\,dx$ 655

3.80 $\int \frac{1}{x(ax+bx^3)^{3/2}}\,dx$ 664

3.81 $\int \frac{1}{x^2(ax+bx^3)^{3/2}}\,dx$ 671

3.82 $\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}}\,dx$ 681

3.83 $\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}}\,dx$ 690

3.84 $\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}}\,dx$ 696

3.85 $\int \frac{x^{23/2}}{(ax+bx^3)^{9/2}}\,dx$ 703

3.86 $\int \frac{x^{21/2}}{(ax+bx^3)^{9/2}}\,dx$ 709

3.87 $\int \frac{x^{19/2}}{(ax+bx^3)^{9/2}}\,dx$ 714

3.88 $\int \frac{x^{17/2}}{(ax+bx^3)^{9/2}}\,dx$ 719

3.89 $\int \frac{x^{15/2}}{(ax+bx^3)^{9/2}}\,dx$ 724

3.90 $\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}}\,dx$ 729

3.91 $\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx$ 735

3.92 $\int \frac{x^{9/2}}{(ax+bx^3)^{9/2}}\,dx$ 740

3.93 $\int \frac{x^{7/2}}{(ax+bx^3)^{9/2}}\,dx$ . . . 746
3.94 $\int \frac{x^{5/2}}{(ax+bx^3)^{9/2}}\,dx$ . . . 753
3.95 $\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx$ . . . 759
3.96 $\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}}\,dx$ . . . 769
3.97 $\int \frac{1}{\sqrt{x}(ax+bx^3)^{9/2}}\,dx$ . . . 776
3.98 $\int \frac{1}{x^{3/2}(ax+bx^3)^{9/2}}\,dx$ . . . 787
3.99 $\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx$ . . . 796
3.100 $\int \frac{x}{\sqrt{ax+bx^4}}\,dx$ . . . 802
3.101 $\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx$ . . . 807
3.102 $\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx$ . . . 812
3.103 $\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx$ . . . 817
3.104 $\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx$ . . . 822
3.105 $\int \frac{1}{\sqrt{ax+bx^4}}\,dx$ . . . 830
3.106 $\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx$ . . . 836
3.107 $\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx$ . . . 844
3.108 $\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx$ . . . 853
3.109 $\int \frac{1}{x\sqrt{ax+bx^4}}\,dx$ . . . 861
3.110 $\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx$ . . . 870
3.111 $\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx$ . . . 888
3.112 $\int x\sqrt{b\sqrt[3]{x}+ax}\,dx$ . . . 908
3.113 $\int \sqrt{b\sqrt[3]{x}+ax}\,dx$ . . . 917
3.114 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx$ . . . 926
3.115 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx$ . . . 932
3.116 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx$ . . . 941
3.117 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx$ . . . 948
3.118 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx$ . . . 966
3.119 $\int x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$ . . . 982
3.120 $\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$ . . . 998
3.121 $\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$ . . . 1011
3.122 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx$ . . . 1019
3.123 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx$ . . . 1028

3.124 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx$ . . . 1035
3.125 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx$ . . . 1044
3.126 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx$ . . . 1052
3.127 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx$ . . . 1071
3.128 $\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$ . . . 1086
3.129 $\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$ . . . 1106
3.130 $\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$ . . . 1128
3.131 $\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$ . . . 1139
3.132 $\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$ . . . 1149
3.133 $\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx$ . . . 1155
3.134 $\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx$ . . . 1163
3.135 $\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx$ . . . 1170
3.136 $\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx$ . . . 1188
3.137 $\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$ . . . 1203
3.138 $\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$ . . . 1227
3.139 $\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$ . . . 1243
3.140 $\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$ . . . 1256
3.141 $\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$ . . . 1263
3.142 $\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$ . . . 1271
3.143 $\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$ . . . 1278
3.144 $\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$ . . . 1296
3.145 $\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$ . . . 1312
3.146 $\int x^3\sqrt{bx^{2/3}+ax}\,dx$ . . . 1338
3.147 $\int x^2\sqrt{bx^{2/3}+ax}\,dx$ . . . 1361
3.148 $\int x\sqrt{bx^{2/3}+ax}\,dx$ . . . 1377
3.149 $\int \sqrt{bx^{2/3}+ax}\,dx$ . . . 1387

3.150 $\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx$ 1394
3.151 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx$ 1399
3.152 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx$ 1405
3.153 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx$ 1414
3.154 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx$ 1430
3.155 $\int x^2\left(bx^{2/3}+ax\right)^{3/2}\,dx$ 1453
3.156 $\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx$ 1475
3.157 $\int \left(bx^{2/3}+ax\right)^{3/2}\,dx$ 1491
3.158 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx$ 1501
3.159 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx$ 1507
3.160 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx$ 1513
3.161 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx$ 1519
3.162 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx$ 1529
3.163 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx$ 1545
3.164 $\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx$ 1568
3.165 $\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx$ 1594
3.166 $\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx$ 1613
3.167 $\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx$ 1626
3.168 $\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx$ 1633
3.169 $\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx$ 1638
3.170 $\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx$ 1644
3.171 $\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx$ 1651
3.172 $\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx$ 1665
3.173 $\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$ 1686
3.174 $\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$ 1708
3.175 $\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$ 1724
3.176 $\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$ 1732
3.177 $\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$ 1738
3.178 $\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx$ 1744
3.179 $\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx$ 1752
3.180 $\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx$ 1766

3.181 $\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx$ . . . 1787
3.182 $\int x^2(ax^2+bx^3)\,dx$ . . . 1814
3.183 $\int x(ax^2+bx^3)\,dx$ . . . 1819
3.184 $\int (ax^2+bx^3)\,dx$ . . . 1824
3.185 $\int \frac{ax^2+bx^3}{x}\,dx$ . . . 1829
3.186 $\int \frac{ax^2+bx^3}{x^2}\,dx$ . . . 1834
3.187 $\int x^2(ax^2+bx^3)^2\,dx$ . . . 1839
3.188 $\int x(ax^2+bx^3)^2\,dx$ . . . 1844
3.189 $\int (ax^2+bx^3)^2\,dx$ . . . 1849
3.190 $\int \frac{(ax^2+bx^3)^2}{x}\,dx$ . . . 1854
3.191 $\int \frac{(ax^2+bx^3)^2}{x^2}\,dx$ . . . 1859
3.192 $\int \frac{x^6}{ax^2+bx^3}\,dx$ . . . 1864
3.193 $\int \frac{x^5}{ax^2+bx^3}\,dx$ . . . 1869
3.194 $\int \frac{x^4}{ax^2+bx^3}\,dx$ . . . 1874
3.195 $\int \frac{x^3}{ax^2+bx^3}\,dx$ . . . 1879
3.196 $\int \frac{x^2}{ax^2+bx^3}\,dx$ . . . 1884
3.197 $\int \frac{x}{ax^2+bx^3}\,dx$ . . . 1889
3.198 $\int \frac{1}{ax^2+bx^3}\,dx$ . . . 1894
3.199 $\int \frac{1}{x(ax^2+bx^3)}\,dx$ . . . 1899
3.200 $\int \frac{1}{x^2(ax^2+bx^3)}\,dx$ . . . 1904
3.201 $\int \frac{x^8}{(ax^2+bx^3)^2}\,dx$ . . . 1909
3.202 $\int \frac{x^7}{(ax^2+bx^3)^2}\,dx$ . . . 1915
3.203 $\int \frac{x^6}{(ax^2+bx^3)^2}\,dx$ . . . 1920
3.204 $\int \frac{x^5}{(ax^2+bx^3)^2}\,dx$ . . . 1925
3.205 $\int \frac{x^4}{(ax^2+bx^3)^2}\,dx$ . . . 1930
3.206 $\int \frac{x^3}{(ax^2+bx^3)^2}\,dx$ . . . 1935
3.207 $\int \frac{x^2}{(ax^2+bx^3)^2}\,dx$ . . . 1940
3.208 $\int \frac{x}{(ax^2+bx^3)^2}\,dx$ . . . 1945
3.209 $\int \frac{1}{(ax^2+bx^3)^2}\,dx$ . . . 1951
3.210 $\int \frac{1}{x(ax^2+bx^3)^2}\,dx$ . . . 1957
3.211 $\int x^2\sqrt{ax^2+bx^3}\,dx$ . . . 1963
3.212 $\int x\sqrt{ax^2+bx^3}\,dx$ . . . 1969
3.213 $\int \sqrt{ax^2+bx^3}\,dx$ . . . 1975
3.214 $\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx$ . . . 1980
3.215 $\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx$ . . . 1985

3.216 $\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx$ . . . 1990
3.217 $\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx$ . . . 1996
3.218 $\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx$ . . . 2002
3.219 $\int x^2(ax^2+bx^3)^{3/2}\,dx$ . . . 2008
3.220 $\int x(ax^2+bx^3)^{3/2}\,dx$ . . . 2016
3.221 $\int (ax^2+bx^3)^{3/2}\,dx$ . . . 2023
3.222 $\int \frac{(ax^2+bx^3)^{3/2}}{x}\,dx$ . . . 2029
3.223 $\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx$ . . . 2035
3.224 $\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx$ . . . 2040
3.225 $\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx$ . . . 2045
3.226 $\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx$ . . . 2051
3.227 $\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx$ . . . 2057
3.228 $\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx$ . . . 2062
3.229 $\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx$ . . . 2068
3.230 $\int \frac{(ax^2+bx^3)^{3/2}}{x^9}\,dx$ . . . 2075
3.231 $\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx$ . . . 2083
3.232 $\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx$ . . . 2089
3.233 $\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx$ . . . 2094
3.234 $\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx$ . . . 2099
3.235 $\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx$ . . . 2104
3.236 $\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx$ . . . 2109
3.237 $\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx$ . . . 2115
3.238 $\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx$ . . . 2121
3.239 $\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx$ . . . 2128
3.240 $\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx$ . . . 2134
3.241 $\int \frac{x^4}{(ax^2+bx^3)^{3/2}}\,dx$ . . . 2140
3.242 $\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx$ . . . 2145
3.243 $\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx$ . . . 2150
3.244 $\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx$ . . . 2156
3.245 $\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx$ . . . 2162
3.246 $\int \frac{1}{x(ax^2+bx^3)^{3/2}}\,dx$ . . . 2169
3.247 $\int \frac{1}{x^2(ax^2+bx^3)^{3/2}}\,dx$ . . . 2176
3.248 $\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx$ . . . 2185

3.249 $\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx$ . . . 2192

3.250 $\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx$ . . . 2198

3.251 $\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx$ . . . 2203

3.252 $\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx$ . . . 2208

3.253 $\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx$ . . . 2213

3.254 $\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx$ . . . 2218

3.255 $\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx$ . . . 2224

3.256 $\int x^{1-3p}(ax^2+bx^3)^p\,dx$ . . . 2230

3.257 $\int x^{-3p}(ax^2+bx^3)^p\,dx$ . . . 2235

3.258 $\int x^{-1-3p}(ax^2+bx^3)^p\,dx$ . . . 2240

3.259 $\int x^{-2-3p}(ax^2+bx^3)^p\,dx$ . . . 2245

3.260 $\int x^{-3-3p}(ax^2+bx^3)^p\,dx$ . . . 2250

3.261 $\int x^{-4-3p}(ax^2+bx^3)^p\,dx$ . . . 2255

3.262 $\int \frac{x^{11}}{(ax^2+bx^5)^3}\,dx$ . . . 2261

3.263 $\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx$ . . . 2266

3.264 $\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx$ . . . 2271

3.265 $\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx$ . . . 2276

3.266 $\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx$ . . . 2281

3.267 $\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx$ . . . 2286

3.268 $\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx$ . . . 2292

3.269 $\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx$ . . . 2298

3.270 $\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx$ . . . 2304

3.271 $\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx$ . . . 2310

3.272 $\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx$ . . . 2318

3.273 $\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx$ . . . 2325

3.274 $\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx$ . . . 2333

3.275 $\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx$ . . . 2340

3.276 $\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx$ . . . 2349

3.277 $\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx$ . . . 2354

3.278 $\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx$ . . . 2361

3.279 $\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx$ . . . 2369

3.280 $\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx$ . . . 2374

3.281 $\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx$ . . . 2380

3.282 $\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx$ . . . 2389

3.283 $\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx$ . . . 2394

3.284 $\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx$ 2401
3.285 $\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx$ 2411
3.286 $\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx$ 2416
3.287 $\int \frac{x}{ax^3+bx^4}\,dx$ 2423
3.288 $\int \frac{1}{ax^3+bx^4}\,dx$ 2428
3.289 $\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx$ 2433
3.290 $\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx$ 2439
3.291 $\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx$ 2445
3.292 $\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx$ 2451
3.293 $\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx$ 2456
3.294 $\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx$ 2461
3.295 $\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx$ 2466
3.296 $\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx$ 2472
3.297 $\int \frac{1}{x^3+bx^5}\,dx$ 2478
3.298 $\int \frac{1}{-x^3+bx^5}\,dx$ 2483
3.299 $\int \frac{1}{ax+bx}\,dx$ 2488
3.300 $\int \frac{1}{(ax+bx)^2}\,dx$ 2493
3.301 $\int \frac{1}{(ax+bx)^3}\,dx$ 2498
3.302 $\int \frac{1}{ax^2+bx^2}\,dx$ 2503
3.303 $\int \frac{1}{ax^n+bx^n}\,dx$ 2508
3.304 $\int \frac{1}{(ax^n+bx^n)^2}\,dx$ 2513
3.305 $\int \frac{1}{(ax^n+bx^n)^3}\,dx$ 2518
3.306 $\int \left(ax+bx^{14}\right)^{12}\,dx$ 2523
3.307 $\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx$ 2530
3.308 $\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx$ 2537
3.309 $\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx$ 2544
3.310 $\int \left(ax+bx^{14}\right)^{12}\,dx$ 2551
3.311 $\int \left(ax^2+bx^{27}\right)^{12}\,dx$ 2558
3.312 $\int \left(ax^3+bx^{40}\right)^{12}\,dx$ 2565
3.313 $\int \left(ax^m+bx^{1+13m}\right)^{12}\,dx$ 2572
3.314 $\int \left(ax^m+bx^{1+6m}\right)^5\,dx$ 2580
3.315 $\int \frac{1}{(bx^{1-2m}+ax^m)^3}\,dx$ 2586
3.316 $\int \frac{1}{\frac{b}{x}+ax}\,dx$ 2591
3.317 $\int \frac{1}{\frac{b}{x^2}+ax}\,dx$ 2596
3.318 $\int \frac{1}{\frac{b}{x^3}+ax}\,dx$ 2601
3.319 $\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx$ 2606

3.320 $\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx$ . . . 2611
3.321 $\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx$ . . . 2616
3.322 $\int \left(\frac{a}{x}+bx\right)^2\,dx$ . . . 2621
3.323 $\int \left(\frac{a}{x}+bx\right)^3\,dx$ . . . 2626
3.324 $\int \left(\frac{a}{x}+bx\right)^4\,dx$ . . . 2631
3.325 $\int \frac{x}{\frac{1}{x}+x}\,dx$ . . . 2636
3.326 $\int \frac{1}{\frac{1}{x^2}+x^3}\,dx$ . . . 2641
3.327 $\int x^p(ax^n+bx^{1+13n+p})^{12}\,dx$ . . . 2651
3.328 $\int x^{12}(a+bx^{13})^{12}\,dx$ . . . 2659
3.329 $\int x^{12}(ax+bx^{26})^{12}\,dx$ . . . 2665
3.330 $\int x^{12}(ax^2+bx^{39})^{12}\,dx$ . . . 2672
3.331 $\int x^{24}(a+bx^{25})^{12}\,dx$ . . . 2679
3.332 $\int x^{24}(ax+bx^{38})^{12}\,dx$ . . . 2685
3.333 $\int x^{36}(a+bx^{37})^{12}\,dx$ . . . 2692
3.334 $\int \frac{1}{ax+bx^n}\,dx$ . . . 2698
3.335 $\int \frac{1}{ax+bx^{1+n}}\,dx$ . . . 2703
3.336 $\int \frac{1}{ax+bx^{1-n}}\,dx$ . . . 2709
3.337 $\int \frac{1}{2x+3x^{1+n}}\,dx$ . . . 2714
3.338 $\int \frac{1}{2x+3x^{1-n}}\,dx$ . . . 2720
3.339 $\int \frac{1}{-\sqrt{x}+x}\,dx$ . . . 2725
3.340 $\int \frac{1}{-x^{3/5}+x}\,dx$ . . . 2730
3.341 $\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx$ . . . 2735
3.342 $\int \frac{1}{x+x^{\sqrt{2}}}\,dx$ . . . 2740
3.343 $\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx$ . . . 2746
3.344 $\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx$ . . . 2751
3.345 $\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx$ . . . 2757
3.346 $\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx$ . . . 2762
3.347 $\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx$ . . . 2767
3.348 $\int \frac{\sqrt{a+bx^n}}{cx}\,dx$ . . . 2772
3.349 $\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx$ . . . 2778
3.350 $\int \sqrt{\frac{a}{x^2}+bx^n}\,dx$ . . . 2783
3.351 $\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx$ . . . 2788
3.352 $\int (cx)^{-1-\frac{3j}{2}}\left(ax^j+bx^n\right)^{3/2}\,dx$ . . . 2793
3.353 $\int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}}\,dx$ . . . 2799

3.354 $\int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4}\,dx$ 2805
3.355 $\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}\,dx$ 2811
3.356 $\int \frac{(a+bx^n)^{3/2}}{cx}\,dx$ 2817
3.357 $\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}\,dx$ 2823
3.358 $\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}\,dx$ 2829
3.359 $\int (cx)^{7/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}\,dx$ 2835
3.360 $\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx$ 2841
3.361 $\int \sqrt{\frac{a+bx}{x^2}}\,dx$ 2847
3.362 $\int \sqrt{\frac{a+bx^2}{x^2}}\,dx$ 2853
3.363 $\int \sqrt{\frac{a+bx^3}{x^2}}\,dx$ 2859
3.364 $\int \sqrt{\frac{a+bx^n}{x^2}}\,dx$ 2865
3.365 $\int \sqrt{\frac{-a+bx}{x^2}}\,dx$ 2871
3.366 $\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx$ 2877
3.367 $\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx$ 2883
3.368 $\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx$ 2889
3.369 $\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx$ 2895
3.370 $\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx$ 2900
3.371 $\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx$ 2905
3.372 $\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx$ 2910
3.373 $\int \frac{1}{cx\sqrt{a+bx^n}}\,dx$ 2915
3.374 $\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx$ 2920
3.375 $\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx$ 2925
3.376 $\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx$ 2930
3.377 $\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx$ 2935
3.378 $\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}}\,dx$ 2941
3.379 $\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx$ 2946
3.380 $\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx$ 2951
3.381 $\int \frac{1}{cx(a+bx^n)^{3/2}}\,dx$ 2956
3.382 $\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx$ 2962

3.383 $\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx$ . . . 2967
3.384 $\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx$ . . . 2972
3.385 $\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx$ . . . 2977
3.386 $\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx$ . . . 2982
3.387 $\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx$ . . . 2988
3.388 $\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx$ . . . 2993
3.389 $\int \frac{1}{\sqrt{x^{2-n}(a+bx^n)}}\,dx$ . . . 2999
3.390 $\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx$ . . . 3004
3.391 $\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx$ . . . 3010
3.392 $\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx$ . . . 3015
3.393 $\int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}}\,dx$ . . . 3021
3.394 $\int \frac{1}{\sqrt{x^n(a+bx^{2-n})}}\,dx$ . . . 3026
3.395 $\int \frac{1}{\sqrt{x^2(b+ax^{-2+n})}}\,dx$ . . . 3031
3.396 $\int \frac{1}{\sqrt{x(bx+ax^{-1+n})}}\,dx$ . . . 3036
3.397 $\int \frac{1}{\sqrt{x^n(a-bx^{2-n})}}\,dx$ . . . 3041
3.398 $\int \frac{1}{\sqrt{x^2(-b+ax^{-2+n})}}\,dx$ . . . 3046
3.399 $\int \frac{1}{\sqrt{x(-bx+ax^{-1+n})}}\,dx$ . . . 3051
3.400 $\int (cx)^m\left(ax^j+bx^n\right)^{3/2}\,dx$ . . . 3056
3.401 $\int (cx)^m\sqrt{ax^j+bx^n}\,dx$ . . . 3062
3.402 $\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx$ . . . 3067
3.403 $\int \frac{(cx)^m}{(ax^j+bx^n)^{3/2}}\,dx$ . . . 3072
3.404 $\int \frac{(cx)^m}{(ax^j+bx^n)^{5/2}}\,dx$ . . . 3077
3.405 $\int \left(ax^j+bx^n\right)^{3/2}\,dx$ . . . 3082
3.406 $\int \sqrt{ax^j+bx^n}\,dx$ . . . 3088
3.407 $\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx$ . . . 3093
3.408 $\int \frac{1}{(ax^j+bx^n)^{3/2}}\,dx$ . . . 3098
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3.414 $\int x\sqrt{ax^2+bx^5}\,dx$ . . . 3128
3.415 $\int \sqrt{x^4\left(a+bx^3\right)}\,dx$ . . . 3133
3.416 $\int (cx)^m \left(ax^q+bx^r\right)^3\,dx$ . . . 3138
3.417 $\int (cx)^m \left(ax^q+bx^r\right)^2\,dx$ . . . 3147
3.418 $\int (cx)^m \left(ax^q+bx^r\right)\,dx$ . . . 3155
3.419 $\int \frac{(cx)^m}{ax^q+bx^r}\,dx$ . . . 3161
3.420 $\int \frac{(cx)^m}{(ax^q+bx^r)^2}\,dx$ . . . 3166
3.421 $\int \frac{(cx)^m}{(ax^q+bx^r)^3}\,dx$ . . . 3171
3.422 $\int x^m(ax^j+bx^n)^p\,dx$ . . . 3176
3.423 $\int x^{-1-pq}(bx^n+ax^q)^p\,dx$ . . . 3181
3.424 $\int x^{-1-np}(bx^n+ax^q)^p\,dx$ . . . 3186
3.425 $\int x^{-1-n-(-1+p)q}(bx^n+ax^q)^p\,dx$ . . . 3191
3.426 $\int x^{-1-n(-1+p)-q}(bx^n+ax^q)^p\,dx$ . . . 3196
3.427 $\int \frac{\left(-\frac{1}{x}+x\right)^p}{x}\,dx$ . . . 3201
3.428 $\int (ax^m+bx^{1+m+mp})^p\,dx$ . . . 3206
3.429 $\int (x^m(a+bx^{1+mp}))^p\,dx$ . . . 3210
3.430 $\int x^n(x^m(a+bx^{1+n+mp}))^p\,dx$ . . . 3215
3.431 $\int x^n(ax^m+bx^{1+m+n+mp})^p\,dx$ . . . 3220
3.432 $\int \sqrt{x^{2(-1+n)}\left(a+bx^n\right)}\,dx$ . . . 3224
3.433 $\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^n\right)}\,dx$ . . . 3229
3.434 $\int \sqrt[4]{x^{4(-1+n)}\left(a+bx^n\right)}\,dx$ . . . 3234
3.435 $\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}}\,dx$ . . . 3239
3.436 $\int \left(x^{\frac{-1+n}{p}}\left(a+bx^n\right)\right)^p\,dx$ . . . 3244
3.437 $\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx$ . . . 3249
3.438 $\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx$ . . . 3253

3.1 $\int x^2(ax+bx^3)\,dx$

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Optimal result

Integrand size = 13, antiderivative size = 17

$$\int x^2\left(ax+bx^3\right)\,dx=\frac{ax^4}{4}+\frac{bx^6}{6}$$

output
```
1/4*a*x^4+1/6*b*x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^2\left(ax+bx^3\right)\,dx=\frac{ax^4}{4}+\frac{bx^6}{6}$$

input
```
Integrate[x^2*(a*x + b*x^3),x]
```

output
```
(a*x^4)/4 + (b*x^6)/6
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(ax+bx^3\right) \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^3 \left(a+bx^2\right) dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(ax^3+bx^5\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ax^4}{4}+\frac{bx^6}{6}$$

input
```
Int[x^2*(a*x + b*x^3),x]
```

output
```
(a*x^4)/4 + (b*x^6)/6
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{1}{4}a\,x^{4}+\frac{1}{6}b\,x^{6}$	14
norman	$\frac{1}{4}a\,x^{4}+\frac{1}{6}b\,x^{6}$	14
risch	$\frac{1}{4}a\,x^{4}+\frac{1}{6}b\,x^{6}$	14
parallelrisch	$\frac{1}{4}a\,x^{4}+\frac{1}{6}b\,x^{6}$	14
gosper	$\frac{x^{4}\left(2b\,x^{2}+3a\right)}{12}$	16
orering	$\frac{x^{3}\left(2b\,x^{2}+3a\right)\left(b\,x^{3}+ax\right)}{12b\,x^{2}+12a}$	34

input
```
int(x^2*(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^4+1/6*b*x^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2\left(ax+bx^3\right)\,dx=\frac{1}{6}\,bx^6+\frac{1}{4}\,ax^4$$

input
```
integrate(x^2*(b*x^3+a*x),x, algorithm="fricas")
```

output
```
1/6*b*x^6 + 1/4*a*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x^2(ax + bx^3)\ dx = \frac{ax^4}{4} + \frac{bx^6}{6}$$

input
```
integrate(x**2*(b*x**3+a*x),x)
```

output
```
a*x**4/4 + b*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2(ax + bx^3)\ dx = \frac{1}{6}\,bx^6 + \frac{1}{4}\,ax^4$$

input
```
integrate(x^2*(b*x^3+a*x),x, algorithm="maxima")
```

output
```
1/6*b*x^6 + 1/4*a*x^4
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2(ax + bx^3)\ dx = \frac{1}{6}\,bx^6 + \frac{1}{4}\,ax^4$$

input
```
integrate(x^2*(b*x^3+a*x),x, algorithm="giac")
```

output
```
1/6*b*x^6 + 1/4*a*x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2(ax+bx^3)\,dx=\frac{b\,x^6}{6}+\frac{a\,x^4}{4}$$

input
```
int(x^2*(a*x + b*x^3),x)
```

output
```
(a*x^4)/4 + (b*x^6)/6
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2(ax+bx^3)\,dx=\frac{x^4(2b\,x^2+3a)}{12}$$

input
```
int(x^2*(b*x^3+a*x),x)
```

output
```
(x**4*(3*a + 2*b*x**2))/12
```

3.2 $\int x(ax+bx^3)\,dx$

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Reduce [B] (verification not implemented) 194

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int x(ax+bx^3)\,dx=\frac{ax^3}{3}+\frac{bx^5}{5}$$

output
```
1/3*a*x^3+1/5*b*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x(ax+bx^3)\,dx=\frac{ax^3}{3}+\frac{bx^5}{5}$$

input
```
Integrate[x*(a*x + b*x^3),x]
```

output
```
(a*x^3)/3 + (b*x^5)/5
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(ax+bx^3)\,dx$$
$$\Big\downarrow 9$$
$$\int x^2(a+bx^2)\,dx$$
$$\Big\downarrow 244$$
$$\int (ax^2+bx^4)\,dx$$
$$\Big\downarrow 2009$$
$$\frac{ax^3}{3}+\frac{bx^5}{5}$$

input
```
Int[x*(a*x + b*x^3),x]
```

output
```
(a*x^3)/3 + (b*x^5)/5
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}$	14
norman	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}$	14
risch	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}$	14
parallelrisch	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}$	14
gosper	$\frac{x^{3}\left(3b\,x^{2}+5a\right)}{15}$	16
orering	$\frac{x^{2}\left(3b\,x^{2}+5a\right)\left(b\,x^{3}+ax\right)}{15b\,x^{2}+15a}$	34

input
```
int(x*(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3+1/5*b*x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\left(ax+bx^3\right)\,dx=\frac{1}{5}\,bx^5+\frac{1}{3}\,ax^3$$

input
```
integrate(x*(b*x^3+a*x),x, algorithm="fricas")
```

output
```
1/5*b*x^5 + 1/3*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x(ax + bx^3)\,dx = \frac{ax^3}{3} + \frac{bx^5}{5}$$

input
```
integrate(x*(b*x**3+a*x),x)
```

output
```
a*x**3/3 + b*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(ax + bx^3)\,dx = \frac{1}{5}\,bx^5 + \frac{1}{3}\,ax^3$$

input
```
integrate(x*(b*x^3+a*x),x, algorithm="maxima")
```

output
```
1/5*b*x^5 + 1/3*a*x^3
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(ax + bx^3)\,dx = \frac{1}{5}\,bx^5 + \frac{1}{3}\,ax^3$$

input
```
integrate(x*(b*x^3+a*x),x, algorithm="giac")
```

output
```
1/5*b*x^5 + 1/3*a*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(ax+bx^3)\;dx=\frac{b\,x^5}{5}+\frac{a\,x^3}{3}$$

input
```
int(x*(a*x + b*x^3),x)
```

output
```
(a*x^3)/3 + (b*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x(ax+bx^3)\;dx=\frac{x^3(3b\,x^2+5a)}{15}$$

input
```
int(x*(b*x^3+a*x),x)
```

output
```
(x**3*(5*a + 3*b*x**2))/15
```

3.3 $\int (ax + bx^3) \, dx$

Optimal result 195
Mathematica [A] (verified) 195
Rubi [A] (verified) 196
Maple [A] (verified) 197
Fricas [A] (verification not implemented) 197
Sympy [A] (verification not implemented) 198
Maxima [A] (verification not implemented) 198
Giac [A] (verification not implemented) 198
Mupad [B] (verification not implemented) 199
Reduce [B] (verification not implemented) 199

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int (ax + bx^3) \, dx = \frac{ax^2}{2} + \frac{bx^4}{4}$$

output
```
1/2*a*x^2+1/4*b*x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int (ax + bx^3) \, dx = \frac{ax^2}{2} + \frac{bx^4}{4}$$

input
```
Integrate[a*x + b*x^3,x]
```

output
```
(a*x^2)/2 + (b*x^4)/4
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^3\right) \, dx$$

$$\Big\downarrow \; 2009$$

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

input
```
Int[a*x + b*x^3,x]
```

output
```
(a*x^2)/2 + (b*x^4)/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
norman	$\frac{1}{2}a\,x^{2}+\frac{1}{4}b\,x^{4}$	14
risch	$\frac{1}{2}a\,x^{2}+\frac{1}{4}b\,x^{4}$	14
parallelrisch	$\frac{1}{2}a\,x^{2}+\frac{1}{4}b\,x^{4}$	14
parts	$\frac{1}{2}a\,x^{2}+\frac{1}{4}b\,x^{4}$	14
gosper	$\frac{x^{2}(b\,x^{2}+2a)}{4}$	15
default	$\frac{(b\,x^{2}+a)^{2}}{4b}$	15
orering	$\frac{x(b\,x^{2}+2a)(b\,x^{3}+ax)}{4b\,x^{2}+4a}$	31

input
```
int(b*x^3+a*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+1/4*b*x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(ax+bx^3\right)\,dx=\frac{1}{4}\,bx^4+\frac{1}{2}\,ax^2$$

input
```
integrate(b*x^3+a*x,x, algorithm="fricas")
```

output
```
1/4*b*x^4 + 1/2*a*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int (ax + bx^3) \, dx = \frac{ax^2}{2} + \frac{bx^4}{4}$$

input
```
integrate(b*x**3+a*x,x)
```

output
```
a*x**2/2 + b*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (ax + bx^3) \, dx = \frac{1}{4} bx^4 + \frac{1}{2} ax^2$$

input
```
integrate(b*x^3+a*x,x, algorithm="maxima")
```

output
```
1/4*b*x^4 + 1/2*a*x^2
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (ax + bx^3) \, dx = \frac{1}{4} bx^4 + \frac{1}{2} ax^2$$

input
```
integrate(b*x^3+a*x,x, algorithm="giac")
```

output
```
1/4*b*x^4 + 1/2*a*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(ax+bx^3\right)\,dx=\frac{b\,x^4}{4}+\frac{a\,x^2}{2}$$

input
```
int(a*x + b*x^3,x)
```

output
```
(a*x^2)/2 + (b*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \left(ax+bx^3\right)\,dx=\frac{x^2(b\,x^2+2a)}{4}$$

input
```
int(b*x^3+a*x,x)
```

output
```
(x**2*(2*a + b*x**2))/4
```

3.4 $\int \frac{ax+bx^3}{x} \, dx$

Optimal result 200
Mathematica [A] (verified) 200
Rubi [A] (verified) 201
Maple [A] (warning: unable to verify) 202
Fricas [A] (verification not implemented) 202
Sympy [A] (verification not implemented) 203
Maxima [A] (verification not implemented) 203
Giac [A] (verification not implemented) 203
Mupad [B] (verification not implemented) 204
Reduce [B] (verification not implemented) 204

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{ax+bx^3}{x} \, dx = ax + \frac{bx^3}{3}$$

output
```
a*x+1/3*b*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{ax+bx^3}{x} \, dx = ax + \frac{bx^3}{3}$$

input
```
Integrate[(a*x + b*x^3)/x,x]
```

output
```
a*x + (b*x^3)/3
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {9, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax+bx^3}{x}\,dx$$

$$\Big\downarrow 9$$

$$\int \left(a+bx^2\right)dx$$

$$\Big\downarrow 2009$$

$$ax+\frac{bx^3}{3}$$

input
```
Int[(a*x + b*x^3)/x,x]
```

output
```
a*x + (b*x^3)/3
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$ax+\frac{1}{3}b\,x^{3}$	11
norman	$ax+\frac{1}{3}b\,x^{3}$	11
risch	$ax+\frac{1}{3}b\,x^{3}$	11
parallelrisch	$ax+\frac{1}{3}b\,x^{3}$	11
parts	$ax+\frac{1}{3}b\,x^{3}$	11
gosper	$\frac{x\left(b\,x^{2}+3a\right)}{3}$	13
orering	$\frac{\left(b\,x^{2}+3a\right)\left(b\,x^{3}+ax\right)}{3b\,x^{2}+3a}$	30

input
```
int((b*x^3+a*x)/x,x,method=_RETURNVERBOSE)
```

output
```
a*x+1/3*b*x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ax+bx^3}{x}\,dx=\frac{1}{3}\,bx^3+ax$$

input
```
integrate((b*x^3+a*x)/x,x, algorithm="fricas")
```

output
```
1/3*b*x^3 + a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{ax + bx^3}{x} \, dx = ax + \frac{bx^3}{3}$$

input
```
integrate((b*x**3+a*x)/x,x)
```

output
```
a*x + b*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ax + bx^3}{x} \, dx = \frac{1}{3} bx^3 + ax$$

input
```
integrate((b*x^3+a*x)/x,x, algorithm="maxima")
```

output
```
1/3*b*x^3 + a*x
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ax + bx^3}{x} \, dx = \frac{1}{3} bx^3 + ax$$

input
```
integrate((b*x^3+a*x)/x,x, algorithm="giac")
```

output
```
1/3*b*x^3 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ax + bx^3}{x}\,dx = \frac{b\,x^3}{3} + a\,x$$

input
```
int((a*x + b*x^3)/x,x)
```

output
```
a*x + (b*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{ax + bx^3}{x}\,dx = \frac{x(b\,x^2 + 3a)}{3}$$

input
```
int((b*x^3+a*x)/x,x)
```

output
```
(x*(3*a + b*x**2))/3
```

3.5 $\int \frac{ax+bx^3}{x^2}\,dx$

Optimal result 205
Mathematica [A] (verified) 205
Rubi [A] (verified) 206
Maple [A] (warning: unable to verify) 207
Fricas [A] (verification not implemented) 207
Sympy [A] (verification not implemented) 208
Maxima [A] (verification not implemented) 208
Giac [A] (verification not implemented) 208
Mupad [B] (verification not implemented) 209
Reduce [B] (verification not implemented) 209

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{ax+bx^3}{x^2}\,dx = \frac{bx^2}{2} + a\log(x)$$

output
```
1/2*b*x^2+a*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{ax+bx^3}{x^2}\,dx = \frac{bx^2}{2} + a\log(x)$$

input
```
Integrate[(a*x + b*x^3)/x^2,x]
```

output
```
(b*x^2)/2 + a*Log[x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax+bx^3}{x^2}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{a+bx^2}{x}dx$$

$$\Big\downarrow \; 244$$

$$\int \left(\frac{a}{x}+bx\right)dx$$

$$\Big\downarrow \; 2009$$

$$a\log(x)+\frac{bx^2}{2}$$

input
```
Int[(a*x + b*x^3)/x^2,x]
```

output
```
(b*x^2)/2 + a*Log[x]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{b\,x^{2}}{2}+a\ln\left(x\right)$	12
norman	$\frac{b\,x^{2}}{2}+a\ln\left(x\right)$	12
risch	$\frac{b\,x^{2}}{2}+a\ln\left(x\right)$	12
parallelrisch	$\frac{b\,x^{2}}{2}+a\ln\left(x\right)$	12

input
```
int((b*x^3+a*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*b*x^2+a*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{ax+bx^3}{x^2}\,dx = \frac{1}{2}\,bx^2 + a\log\left(x\right)$$

input
```
integrate((b*x^3+a*x)/x^2,x, algorithm="fricas")
```

output
```
1/2*b*x^2 + a*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{ax+bx^3}{x^2}\,dx = a\log{(x)} + \frac{bx^2}{2}$$

input
```
integrate((b*x**3+a*x)/x**2,x)
```

output
```
a*log(x) + b*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{ax+bx^3}{x^2}\,dx = \frac{1}{2}\,bx^2 + a\log\left(x\right)$$

input
```
integrate((b*x^3+a*x)/x^2,x, algorithm="maxima")
```

output
```
1/2*b*x^2 + a*log(x)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{ax+bx^3}{x^2}\,dx = \frac{1}{2}\,bx^2 + \frac{1}{2}\,a\log\left(x^2\right)$$

input
```
integrate((b*x^3+a*x)/x^2,x, algorithm="giac")
```

output
```
1/2*b*x^2 + 1/2*a*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{ax+bx^3}{x^2}\,dx=\frac{b\,x^2}{2}+a\,\ln\left(x\right)$$

input
```
int((a*x + b*x^3)/x^2,x)
```

output
```
(b*x^2)/2 + a*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{ax+bx^3}{x^2}\,dx=\log(x)\,a+\frac{b\,x^2}{2}$$

input
```
int((b*x^3+a*x)/x^2,x)
```

output
```
(2*log(x)*a + b*x**2)/2
```

3.6 $\int \frac{ax+bx^3}{x^3} \, dx$

Optimal result 210
Mathematica [A] (verified) 210
Rubi [A] (verified) 211
Maple [A] (warning: unable to verify) 212
Fricas [A] (verification not implemented) 212
Sympy [A] (verification not implemented) 213
Maxima [A] (verification not implemented) 213
Giac [A] (verification not implemented) 213
Mupad [B] (verification not implemented) 214
Reduce [B] (verification not implemented) 214

Optimal result

Integrand size = 13, antiderivative size = 10

$$\int \frac{ax+bx^3}{x^3} \, dx = -\frac{a}{x} + bx$$

output
```
-a/x+b*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{ax+bx^3}{x^3} \, dx = -\frac{a}{x} + bx$$

input
```
Integrate[(a*x + b*x^3)/x^3,x]
```

output
```
-(a/x) + b*x
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax+bx^3}{x^3}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{a+bx^2}{x^2}dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(\frac{a}{x^2}+b\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$bx-\frac{a}{x}$$

input
```
Int[(a*x + b*x^3)/x^3,x]
```

output
```
-(a/x) + b*x
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$-\frac{a}{x}+bx$	11
risch	$-\frac{a}{x}+bx$	11
gosper	$-\frac{-b\,x^{2}+a}{x}$	14
parallelrisch	$\frac{b\,x^{2}-a}{x}$	14
norman	$\frac{b\,x^{3}-ax}{x^{2}}$	15
orering	$-\frac{\left(-b\,x^{2}+a\right)\left(b\,x^{3}+ax\right)}{x^{2}\left(b\,x^{2}+a\right)}$	32

input
```
int((b*x^3+a*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-a/x+b*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

$$\int \frac{ax+bx^3}{x^3}\,dx=\frac{bx^2-a}{x}$$

input
```
integrate((b*x^3+a*x)/x^3,x, algorithm="fricas")
```

output
```
(b*x^2 - a)/x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.50

$$\int \frac{ax + bx^3}{x^3} \, dx = -\frac{a}{x} + bx$$

input
```
integrate((b*x**3+a*x)/x**3,x)
```

output
```
-a/x + b*x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{ax + bx^3}{x^3} \, dx = bx - \frac{a}{x}$$

input
```
integrate((b*x^3+a*x)/x^3,x, algorithm="maxima")
```

output
```
b*x - a/x
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{ax + bx^3}{x^3} \, dx = bx - \frac{a}{x}$$

input
```
integrate((b*x^3+a*x)/x^3,x, algorithm="giac")
```

output
```
b*x - a/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{ax + bx^3}{x^3} \, dx = b\,x - \frac{a}{x}$$

input
```
int((a*x + b*x^3)/x^3,x)
```

output
```
b*x - a/x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

$$\int \frac{ax + bx^3}{x^3} \, dx = \frac{b\,x^2 - a}{x}$$

input
```
int((b*x^3+a*x)/x^3,x)
```

output
```
( - a + b*x**2)/x
```

3.7 $\int \frac{ax+bx^3}{x^4} \, dx$

Optimal result 215
Mathematica [A] (verified) 215
Rubi [A] (verified) 216
Maple [A] (warning: unable to verify) 217
Fricas [A] (verification not implemented) 217
Sympy [A] (verification not implemented) 218
Maxima [A] (verification not implemented) 218
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Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{ax+bx^3}{x^4} \, dx = -\frac{a}{2x^2} + b\log(x)$$

output
```
-1/2*a/x^2+b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{ax+bx^3}{x^4} \, dx = -\frac{a}{2x^2} + b\log(x)$$

input
```
Integrate[(a*x + b*x^3)/x^4,x]
```

output
```
-1/2*a/x^2 + b*Log[x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax+bx^3}{x^4} \, dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{a+bx^2}{x^3}dx$$
$$\Big\downarrow \; 244$$
$$\int \left(\frac{a}{x^3}+\frac{b}{x}\right) dx$$
$$\Big\downarrow \; 2009$$
$$b\log(x)-\frac{a}{2x^2}$$

input
```
Int[(a*x + b*x^3)/x^4,x]
```

output
```
-1/2*a/x^2 + b*Log[x]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a}{2x^2}+b\ln\left(x\right)$	12
norman	$-\frac{a}{2x^2}+b\ln\left(x\right)$	12
risch	$-\frac{a}{2x^2}+b\ln\left(x\right)$	12
parallelrisch	$\frac{2b\ln(x)x^2-a}{2x^2}$	18

input
```
int((b*x^3+a*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/x^2+b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{ax+bx^3}{x^4}\,dx=\frac{2\,bx^2\log\left(x\right)-a}{2\,x^2}$$

input
```
integrate((b*x^3+a*x)/x^4,x, algorithm="fricas")
```

output
```
1/2*(2*b*x^2*log(x) - a)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{ax + bx^3}{x^4} \, dx = -\frac{a}{2x^2} + b \log (x)$$

input
```
integrate((b*x**3+a*x)/x**4,x)
```

output
```
-a/(2*x**2) + b*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{ax + bx^3}{x^4} \, dx = b \log (x) - \frac{a}{2 \, x^2}$$

input
```
integrate((b*x^3+a*x)/x^4,x, algorithm="maxima")
```

output
```
b*log(x) - 1/2*a/x^2
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \frac{ax + bx^3}{x^4} \, dx = \frac{1}{2} \, b \log \left(x^2\right) - \frac{bx^2 + a}{2 \, x^2}$$

input
```
integrate((b*x^3+a*x)/x^4,x, algorithm="giac")
```

output
```
1/2*b*log(x^2) - 1/2*(b*x^2 + a)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{ax + bx^3}{x^4} \, dx = b \ln(x) - \frac{a}{2 x^2}$$

input
```
int((a*x + b*x^3)/x^4,x)
```

output
```
b*log(x) - a/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{ax + bx^3}{x^4} \, dx = \frac{2 \log(x) b x^2 - a}{2x^2}$$

input
```
int((b*x^3+a*x)/x^4,x)
```

output
```
(2*log(x)*b*x**2 - a)/(2*x**2)
```

3.8 $\int \frac{ax+bx^3}{x^5}\,dx$

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Mupad [B] (verification not implemented) . . . 224
Reduce [B] (verification not implemented) . . . 224

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{ax+bx^3}{x^5}\,dx=-\frac{a}{3x^3}-\frac{b}{x}$$

output
```
-1/3*a/x^3-b/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{ax+bx^3}{x^5}\,dx=-\frac{a}{3x^3}-\frac{b}{x}$$

input
```
Integrate[(a*x + b*x^3)/x^5,x]
```

output
```
-1/3*a/x^3 - b/x
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax+bx^3}{x^5} \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{a+bx^2}{x^4}dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(\frac{a}{x^4}+\frac{b}{x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a}{3x^3}-\frac{b}{x}$$

input
```
Int[(a*x + b*x^3)/x^5,x]
```

output
```
-1/3*a/x^3 - b/x
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{3b\,x^{2}+a}{3x^{3}}$	14
default	$-\frac{a}{3x^{3}}-\frac{b}{x}$	14
risch	$\frac{-b\,x^{2}-\frac{a}{3}}{x^{3}}$	15
norman	$\frac{-\frac{1}{3}ax-b\,x^{3}}{x^{4}}$	16
parallelrisch	$\frac{-3b\,x^{2}-a}{3x^{3}}$	16
orering	$-\frac{\left(3b\,x^{2}+a\right)\left(b\,x^{3}+ax\right)}{3x^{4}\left(b\,x^{2}+a\right)}$	32

input
```
int((b*x^3+a*x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3*b*x^2+a)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{ax+bx^3}{x^5}\,dx=-\frac{3\,bx^2+a}{3\,x^3}$$

input
```
integrate((b*x^3+a*x)/x^5,x, algorithm="fricas")
```

output
```
-1/3*(3*b*x^2 + a)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{ax + bx^3}{x^5}\,dx = \frac{-a - 3bx^2}{3x^3}$$

input
```
integrate((b*x**3+a*x)/x**5,x)
```

output
```
(-a - 3*b*x**2)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{ax + bx^3}{x^5}\,dx = -\frac{3\,bx^2 + a}{3\,x^3}$$

input
```
integrate((b*x^3+a*x)/x^5,x, algorithm="maxima")
```

output
```
-1/3*(3*b*x^2 + a)/x^3
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{ax + bx^3}{x^5}\,dx = -\frac{3\,bx^2 + a}{3\,x^3}$$

input
```
integrate((b*x^3+a*x)/x^5,x, algorithm="giac")
```

output
```
-1/3*(3*b*x^2 + a)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{ax+bx^3}{x^5}\,dx = -\frac{3\,b\,x^2+a}{3\,x^3}$$

input
```
int((a*x + b*x^3)/x^5,x)
```

output
```
-(a + 3*b*x^2)/(3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{ax+bx^3}{x^5}\,dx = \frac{-3b\,x^2-a}{3x^3}$$

input
```
int((b*x^3+a*x)/x^5,x)
```

output
```
( - a - 3*b*x**2)/(3*x**3)
```

3.9 $\int x^2 (ax + bx^3)^2 \, dx$

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Rubi [A] (verified) 226
Maple [A] (verified) 227
Fricas [A] (verification not implemented) 227
Sympy [A] (verification not implemented) 228
Maxima [A] (verification not implemented) 228
Giac [A] (verification not implemented) 228
Mupad [B] (verification not implemented) 229
Reduce [B] (verification not implemented) 229

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int x^2 (ax + bx^3)^2 \, dx = \frac{a^2 x^5}{5} + \frac{2}{7} a b x^7 + \frac{b^2 x^9}{9}$$

output
```
1/5*a^2*x^5+2/7*a*b*x^7+1/9*b^2*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x^2 (ax + bx^3)^2 \, dx = \frac{a^2 x^5}{5} + \frac{2}{7} a b x^7 + \frac{b^2 x^9}{9}$$

input
```
Integrate[x^2*(a*x + b*x^3)^2,x]
```

output
```
(a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(ax+bx^3\right)^2 dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^4 \left(a+bx^2\right)^2 dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(a^2x^4+2abx^6+b^2x^8\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

input
```
Int[x^2*(a*x + b*x^3)^2,x]
```

output
```
(a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{1}{5}a^2x^5+\frac{2}{7}ab\,x^7+\frac{1}{9}b^2x^9$	25
norman	$\frac{1}{5}a^2x^5+\frac{2}{7}ab\,x^7+\frac{1}{9}b^2x^9$	25
risch	$\frac{1}{5}a^2x^5+\frac{2}{7}ab\,x^7+\frac{1}{9}b^2x^9$	25
parallelrisch	$\frac{1}{5}a^2x^5+\frac{2}{7}ab\,x^7+\frac{1}{9}b^2x^9$	25
gosper	$\frac{x^5(35b^2x^4+90ab\,x^2+63a^2)}{315}$	27
orering	$\frac{x^3(35b^2x^4+90ab\,x^2+63a^2)(b\,x^3+ax)^2}{315(b\,x^2+a)^2}$	47

input
```
int(x^2*(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^2*x^5+2/7*a*b*x^7+1/9*b^2*x^9
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2(ax+bx^3)^2\,dx=\frac{1}{9}\,b^2x^9+\frac{2}{7}\,abx^7+\frac{1}{5}\,a^2x^5$$

input
```
integrate(x^2*(b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^2 \left(ax+bx^3\right)^2 \, dx = \frac{a^2x^5}{5} + \frac{2abx^7}{7} + \frac{b^2x^9}{9}$$

input
```
integrate(x**2*(b*x**3+a*x)**2,x)
```

output
```
a**2*x**5/5 + 2*a*b*x**7/7 + b**2*x**9/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2 \left(ax+bx^3\right)^2 \, dx = \frac{1}{9} \, b^2x^9 + \frac{2}{7} \, abx^7 + \frac{1}{5} \, a^2x^5$$

input
```
integrate(x^2*(b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2 \left(ax+bx^3\right)^2 \, dx = \frac{1}{9} \, b^2x^9 + \frac{2}{7} \, abx^7 + \frac{1}{5} \, a^2x^5$$

input
```
integrate(x^2*(b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2 (ax + bx^3)^2 \, dx = \frac{a^2 \, x^5}{5} + \frac{2 \, a \, b \, x^7}{7} + \frac{b^2 \, x^9}{9}$$

input
```
int(x^2*(a*x + b*x^3)^2,x)
```

output
```
(a^2*x^5)/5 + (b^2*x^9)/9 + (2*a*b*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^2 (ax + bx^3)^2 \, dx = \frac{x^5 (35b^2 x^4 + 90ab \, x^2 + 63a^2)}{315}$$

input
```
int(x^2*(b*x^3+a*x)^2,x)
```

output
```
(x**5*(63*a**2 + 90*a*b*x**2 + 35*b**2*x**4))/315
```

3.10 $\int x(ax+bx^3)^2\,dx$

Optimal result 230
Mathematica [A] (verified) 230
Rubi [A] (verified) 231
Maple [A] (verified) 232
Fricas [A] (verification not implemented) 233
Sympy [A] (verification not implemented) 233
Maxima [A] (verification not implemented) 233
Giac [A] (verification not implemented) 234
Mupad [B] (verification not implemented) 234
Reduce [B] (verification not implemented) 234

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int x\left(ax+bx^3\right)^2\,dx=\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

output
```
1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x\left(ax+bx^3\right)^2\,dx=\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

input
```
Integrate[x*(a*x + b*x^3)^2,x]
```

output
```
(a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \left(a x+b x^3\right)^2 \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^3 \left(a+b x^2\right)^2 dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} \int x^2 \left(b x^2+a\right)^2 dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2} \int \left(b^2 x^6+2 a b x^4+a^2 x^2\right) dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(\frac{a^2 x^4}{2}+\frac{2}{3} a b x^6+\frac{b^2 x^8}{4}\right)$$

input
```
Int[x*(a*x + b*x^3)^2,x]
```

output
```
((a^2*x^4)/2 + (2*a*b*x^6)/3 + (b^2*x^8)/4)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{1}{4}x^4a^2+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
norman	$\frac{1}{4}x^4a^2+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
risch	$\frac{1}{4}x^4a^2+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
parallelrisch	$\frac{1}{4}x^4a^2+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
gosper	$\frac{x^4(3b^2x^4+8ab\,x^2+6a^2)}{24}$	27
orering	$\frac{x^2(3b^2x^4+8ab\,x^2+6a^2)(b\,x^3+ax)^2}{24(b\,x^2+a)^2}$	47

input
```
int(x*(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*a^2+1/3*a*b*x^6+1/8*b^2*x^8
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x\left(ax+bx^3\right)^2\,dx=\frac{1}{8}\,b^2x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

input
```
integrate(x*(b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x\left(ax+bx^3\right)^2\,dx=\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{b^2x^8}{8}$$

input
```
integrate(x*(b*x**3+a*x)**2,x)
```

output
```
a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x\left(ax+bx^3\right)^2\,dx=\frac{1}{8}\,b^2x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

input
```
integrate(x*(b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(ax+bx^3)^2\,dx=\frac{1}{8}\,b^2x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

input
```
integrate(x*(b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(ax+bx^3)^2\,dx=\frac{a^2\,x^4}{4}+\frac{a\,b\,x^6}{3}+\frac{b^2\,x^8}{8}$$

input
```
int(x*(a*x + b*x^3)^2,x)
```

output
```
(a^2*x^4)/4 + (b^2*x^8)/8 + (a*b*x^6)/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x(ax+bx^3)^2\,dx=\frac{x^4(3b^2x^4+8ab\,x^2+6a^2)}{24}$$

input
```
int(x*(b*x^3+a*x)^2,x)
```

output
```
(x**4*(6*a**2 + 8*a*b*x**2 + 3*b**2*x**4))/24
```

3.11 $\int (ax + bx^3)^2 \, dx$

Optimal result 235
Mathematica [A] (verified) 235
Rubi [A] (verified) 236
Maple [A] (verified) 237
Fricas [A] (verification not implemented) 237
Sympy [A] (verification not implemented) 238
Maxima [A] (verification not implemented) 238
Giac [A] (verification not implemented) 238
Mupad [B] (verification not implemented) 239
Reduce [B] (verification not implemented) 239

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int (ax + bx^3)^2 \, dx = \frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{b^2x^7}{7}$$

output
```
1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (ax + bx^3)^2 \, dx = \frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{b^2x^7}{7}$$

input
```
Integrate[(a*x + b*x^3)^2,x]
```

output
```
(a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2027, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^3\right)^2 dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int x^2\left(a+bx^2\right)^2 dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(a^2x^2+2abx^4+b^2x^6\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

input
```
Int[(a*x + b*x^3)^2,x]
```

output
```
(a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
norman	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
risch	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
parallelrisch	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
gosper	$\frac{x^3(15b^2x^4+42ab\,x^2+35a^2)}{105}$	27
orering	$\frac{x(15b^2x^4+42ab\,x^2+35a^2)(b\,x^3+ax)^2}{105(b\,x^2+a)^2}$	45

input
```
int((b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (ax+bx^3)^2\,dx=\frac{1}{7}\,b^2x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

input
```
integrate((b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \left(ax + bx^3\right)^2 \, dx = \frac{a^2x^3}{3} + \frac{2abx^5}{5} + \frac{b^2x^7}{7}$$

input
```
integrate((b*x**3+a*x)**2,x)
```

output
```
a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(ax + bx^3\right)^2 \, dx = \frac{1}{7}\,b^2x^7 + \frac{2}{5}\,abx^5 + \frac{1}{3}\,a^2x^3$$

input
```
integrate((b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(ax + bx^3\right)^2 \, dx = \frac{1}{7}\,b^2x^7 + \frac{2}{5}\,abx^5 + \frac{1}{3}\,a^2x^3$$

input
```
integrate((b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(ax+bx^3\right)^2\,dx=\frac{a^2\,x^3}{3}+\frac{2\,a\,b\,x^5}{5}+\frac{b^2\,x^7}{7}$$

input
```
int((a*x + b*x^3)^2,x)
```

output
```
(a^2*x^3)/3 + (b^2*x^7)/7 + (2*a*b*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \left(ax+bx^3\right)^2\,dx=\frac{x^3(15b^2x^4+42ab\,x^2+35a^2)}{105}$$

input
```
int((b*x^3+a*x)^2,x)
```

output
```
(x**3*(35*a**2 + 42*a*b*x**2 + 15*b**2*x**4))/105
```

3.12 $\int \frac{(ax+bx^3)^2}{x} dx$

Optimal result 240
Mathematica [A] (verified) 240
Rubi [A] (verified) 241
Maple [A] (warning: unable to verify) 242
Fricas [A] (verification not implemented) 242
Sympy [B] (verification not implemented) 243
Maxima [A] (verification not implemented) 243
Giac [A] (verification not implemented) 243
Mupad [B] (verification not implemented) 244
Reduce [B] (verification not implemented) 244

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int \frac{(ax+bx^3)^2}{x} dx = \frac{(a+bx^2)^3}{6b}$$

output
```
1/6*(b*x^2+a)^3/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(ax+bx^3)^2}{x} dx = \frac{(a+bx^2)^3}{6b}$$

input
```
Integrate[(a*x + b*x^3)^2/x,x]
```

output
```
(a + b*x^2)^3/(6*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {9, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^3)^2}{x} \, dx$$
$$\Big\downarrow \ 9$$
$$\int x(a+bx^2)^2 \, dx$$
$$\Big\downarrow \ 241$$
$$\frac{(a+bx^2)^3}{6b}$$

input
```
Int[(a*x + b*x^3)^2/x,x]
```

output
```
(a + b*x^2)^3/(6*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{(b\,x^2+a)^3}{6b}$	15
norman	$\frac{1}{6}b^2x^6+\frac{1}{2}ab\,x^4+\frac{1}{2}a^2x^2$	25
parallelrisch	$\frac{1}{6}b^2x^6+\frac{1}{2}ab\,x^4+\frac{1}{2}a^2x^2$	25
gosper	$\frac{x^2(b^2x^4+3ab\,x^2+3a^2)}{6}$	26
risch	$\frac{b^2x^6}{6}+\frac{ab\,x^4}{2}+\frac{a^2x^2}{2}+\frac{a^3}{6b}$	33
orering	$\frac{(b^2x^4+3ab\,x^2+3a^2)(b\,x^3+ax)^2}{6(b\,x^2+a)^2}$	43

input
```
int((b*x^3+a*x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/6*(b*x^2+a)^3/b
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{(ax+bx^3)^2}{x}\,dx = \frac{1}{6}\,b^2x^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

input
```
integrate((b*x^3+a*x)^2/x,x, algorithm="fricas")
```

output
```
1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{(ax+bx^3)^2}{x}\,dx = \frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}$$

input
```
integrate((b*x**3+a*x)**2/x,x)
```

output
```
a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{(ax+bx^3)^2}{x}\,dx = \frac{1}{6}\,b^2x^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

input
```
integrate((b*x^3+a*x)^2/x,x, algorithm="maxima")
```

output
```
1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{(ax+bx^3)^2}{x}\,dx = \frac{1}{6}\,b^2x^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

input
```
integrate((b*x^3+a*x)^2/x,x, algorithm="giac")
```

output
```
1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{(ax+bx^3)^2}{x}\,dx = \frac{a^2\,x^2}{2}+\frac{a\,b\,x^4}{2}+\frac{b^2\,x^6}{6}$$

input
```
int((a*x + b*x^3)^2/x,x)
```

output
```
(a^2*x^2)/2 + (b^2*x^6)/6 + (a*b*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int \frac{(ax+bx^3)^2}{x}\,dx = \frac{x^2(b^2x^4+3ab\,x^2+3a^2)}{6}$$

input
```
int((b*x^3+a*x)^2/x,x)
```

output
```
(x**2*(3*a**2 + 3*a*b*x**2 + b**2*x**4))/6
```

3.13 $\int \frac{(ax+bx^3)^2}{x^2} dx$

Optimal result 245
Mathematica [A] (verified) 245
Rubi [A] (verified) 246
Maple [A] (warning: unable to verify) 247
Fricas [A] (verification not implemented) 247
Sympy [A] (verification not implemented) 248
Maxima [A] (verification not implemented) 248
Giac [A] (verification not implemented) 248
Mupad [B] (verification not implemented) 249
Reduce [B] (verification not implemented) 249

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{(ax+bx^3)^2}{x^2} dx = a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

output
```
a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(ax+bx^3)^2}{x^2} dx = a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

input
```
Integrate[(a*x + b*x^3)^2/x^2,x]
```

output
```
a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {9, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^3)^2}{x^2} \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\int (a+bx^2)^2 \, dx$$

$$\Big\downarrow \text{ 210}$$

$$\int (a^2+2abx^2+b^2x^4) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

input
```
Int[(a*x + b*x^3)^2/x^2,x]
```

output
```
a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
default	$a^{2}x+\frac{2}{3}ab\,x^{3}+\frac{1}{5}b^{2}x^{5}$	22
risch	$a^{2}x+\frac{2}{3}ab\,x^{3}+\frac{1}{5}b^{2}x^{5}$	22
parallelrisch	$a^{2}x+\frac{2}{3}ab\,x^{3}+\frac{1}{5}b^{2}x^{5}$	22
gosper	$\frac{x\left(3b^{2}x^{4}+10ab\,x^{2}+15a^{2}\right)}{15}$	25
norman	$\frac{a^{2}x^{2}+\frac{1}{5}b^{2}x^{6}+\frac{2}{3}ab\,x^{4}}{x}$	28
orering	$\frac{\left(3b^{2}x^{4}+10ab\,x^{2}+15a^{2}\right)\left(b\,x^{3}+ax\right)^{2}}{15x\left(b\,x^{2}+a\right)^{2}}$	47

input
```
int((b*x^3+a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(ax+bx^3)^2}{x^2}\,dx=\frac{1}{5}\,b^2x^5+\frac{2}{3}\,abx^3+a^2x$$

input
```
integrate((b*x^3+a*x)^2/x^2,x, algorithm="fricas")
```

output
```
1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(ax+bx^3)^2}{x^2}\,dx = a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}$$

input
```
integrate((b*x**3+a*x)**2/x**2,x)
```

output
```
a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(ax+bx^3)^2}{x^2}\,dx = \frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

input
```
integrate((b*x^3+a*x)^2/x^2,x, algorithm="maxima")
```

output
```
1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(ax+bx^3)^2}{x^2}\,dx = \frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

input
```
integrate((b*x^3+a*x)^2/x^2,x, algorithm="giac")
```

output
```
1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(ax+bx^3)^2}{x^2}\,dx = a^2\,x + \frac{2\,a\,b\,x^3}{3} + \frac{b^2\,x^5}{5}$$

input
```
int((a*x + b*x^3)^2/x^2,x)
```

output
```
a^2*x + (b^2*x^5)/5 + (2*a*b*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(ax+bx^3)^2}{x^2}\,dx = \frac{x(3b^2x^4 + 10ab\,x^2 + 15a^2)}{15}$$

input
```
int((b*x^3+a*x)^2/x^2,x)
```

output
```
(x*(15*a**2 + 10*a*b*x**2 + 3*b**2*x**4))/15
```

3.14 $\int \frac{(ax+bx^3)^2}{x^3} dx$

Optimal result . . . 250
Mathematica [A] (verified) . . . 250
Rubi [A] (verified) . . . 251
Maple [A] (warning: unable to verify) . . . 252
Fricas [A] (verification not implemented) . . . 253
Sympy [A] (verification not implemented) . . . 253
Maxima [A] (verification not implemented) . . . 253
Giac [A] (verification not implemented) . . . 254
Mupad [B] (verification not implemented) . . . 254
Reduce [B] (verification not implemented) . . . 254

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \frac{(ax+bx^3)^2}{x^3} dx = abx^2 + \frac{b^2x^4}{4} + a^2\log(x)$$

output
```
a*b*x^2+1/4*b^2*x^4+a^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(ax+bx^3)^2}{x^3} dx = abx^2 + \frac{b^2x^4}{4} + a^2\log(x)$$

input
```
Integrate[(a*x + b*x^3)^2/x^3,x]
```

output
```
a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^3)^2}{x^3}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{(a+bx^2)^2}{x}dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^2}dx^2$$

$$\Big\downarrow \ 49$$

$$\frac{1}{2}\int \left(\frac{a^2}{x^2}+2ba+b^2x^2\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(a^2\log\left(x^2\right)+2abx^2+\frac{b^2x^4}{2}\right)$$

input
```
Int[(a*x + b*x^3)^2/x^3,x]
```

output
```
(2*a*b*x^2 + (b^2*x^4)/2 + a^2*Log[x^2])/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
default	$ab\,x^2+\frac{b^2x^4}{4}+a^2\ln\left(x\right)$	22
parallelrisch	$ab\,x^2+\frac{b^2x^4}{4}+a^2\ln\left(x\right)$	22
risch	$\frac{b^2x^4}{4}+ab\,x^2+a^2+a^2\ln\left(x\right)$	25
norman	$\frac{ab\,x^4+\frac{1}{4}b^2x^6}{x^2}+a^2\ln\left(x\right)$	27

input
```
int((b*x^3+a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*b*x^2+1/4*b^2*x^4+a^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(ax+bx^3)^2}{x^3}\,dx = \frac{1}{4}\,b^2x^4 + abx^2 + a^2\log{(x)}$$

input
```
integrate((b*x^3+a*x)^2/x^3,x, algorithm="fricas")
```

output
```
1/4*b^2*x^4 + a*b*x^2 + a^2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(ax+bx^3)^2}{x^3}\,dx = a^2\log{(x)} + abx^2 + \frac{b^2x^4}{4}$$

input
```
integrate((b*x**3+a*x)**2/x**3,x)
```

output
```
a**2*log(x) + a*b*x**2 + b**2*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(ax+bx^3)^2}{x^3}\,dx = \frac{1}{4}\,b^2x^4 + abx^2 + a^2\log{(x)}$$

input
```
integrate((b*x^3+a*x)^2/x^3,x, algorithm="maxima")
```

output
```
1/4*b^2*x^4 + a*b*x^2 + a^2*log(x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{(ax+bx^3)^2}{x^3} \, dx = \frac{1}{4}b^2x^4 + abx^2 + \frac{1}{2}a^2 \log(x^2)$$

input
```
integrate((b*x^3+a*x)^2/x^3,x, algorithm="giac")
```

output
```
1/4*b^2*x^4 + a*b*x^2 + 1/2*a^2*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(ax+bx^3)^2}{x^3} \, dx = a^2 \ln(x) + \frac{b^2 x^4}{4} + a\, b\, x^2$$

input
```
int((a*x + b*x^3)^2/x^3,x)
```

output
```
a^2*log(x) + (b^2*x^4)/4 + a*b*x^2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(ax+bx^3)^2}{x^3} \, dx = \log(x)\, a^2 + ab\, x^2 + \frac{b^2x^4}{4}$$

input
```
int((b*x^3+a*x)^2/x^3,x)
```

output
```
(4*log(x)*a**2 + 4*a*b*x**2 + b**2*x**4)/4
```

3.15 $\int \frac{(ax+bx^3)^2}{x^4}\,dx$

Optimal result 255
Mathematica [A] (verified) 255
Rubi [A] (verified) 256
Maple [A] (warning: unable to verify) 257
Fricas [A] (verification not implemented) 257
Sympy [A] (verification not implemented) 258
Maxima [A] (verification not implemented) 258
Giac [A] (verification not implemented) 258
Mupad [B] (verification not implemented) 259
Reduce [B] (verification not implemented) 259

Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{(ax+bx^3)^2}{x^4}\,dx = -\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

output
```
-a^2/x+2*a*b*x+1/3*b^2*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(ax+bx^3)^2}{x^4}\,dx = -\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

input
```
Integrate[(a*x + b*x^3)^2/x^4,x]
```

output
```
-(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^3)^2}{x^4} \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{(a+bx^2)^2}{x^2}dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(\frac{a^2}{x^2}+2ab+b^2x^2\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

input
```
Int[(a*x + b*x^3)^2/x^4,x]
```

output
```
-(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$	23
risch	$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$	23
parallelrisch	$\frac{b^2x^4+6ab\,x^2-3a^2}{3x}$	26
gosper	$-\frac{-b^2x^4-6ab\,x^2+3a^2}{3x}$	27
norman	$\frac{-a^2x^2+\frac{1}{3}b^2x^6+2ab\,x^4}{x^3}$	29
orering	$-\frac{\left(-b^2x^4-6ab\,x^2+3a^2\right)\left(b\,x^3+ax\right)^2}{3x^3\left(b\,x^2+a\right)^2}$	47

input
```
int((b*x^3+a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-a^2/x+2*a*b*x+1/3*b^2*x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{(ax+bx^3)^2}{x^4}\,dx=\frac{b^2x^4+6\,abx^2-3\,a^2}{3\,x}$$

input
```
integrate((b*x^3+a*x)^2/x^4,x, algorithm="fricas")
```

output
```
1/3*(b^2*x^4 + 6*a*b*x^2 - 3*a^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{(ax+bx^3)^2}{x^4}\,dx = -\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

input
```
integrate((b*x**3+a*x)**2/x**4,x)
```

output
```
-a**2/x + 2*a*b*x + b**2*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(ax+bx^3)^2}{x^4}\,dx = \frac{1}{3}\,b^2x^3 + 2\,abx - \frac{a^2}{x}$$

input
```
integrate((b*x^3+a*x)^2/x^4,x, algorithm="maxima")
```

output
```
1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(ax+bx^3)^2}{x^4}\,dx = \frac{1}{3}\,b^2x^3 + 2\,abx - \frac{a^2}{x}$$

input
```
integrate((b*x^3+a*x)^2/x^4,x, algorithm="giac")
```

output
```
1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(ax+bx^3)^2}{x^4}\,dx = \frac{b^2\,x^3}{3} - \frac{a^2}{x} + 2\,a\,b\,x$$

input
```
int((a*x + b*x^3)^2/x^4,x)
```

output
```
(b^2*x^3)/3 - a^2/x + 2*a*b*x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{(ax+bx^3)^2}{x^4}\,dx = \frac{b^2x^4 + 6ab\,x^2 - 3a^2}{3x}$$

input
```
int((b*x^3+a*x)^2/x^4,x)
```

output
```
( - 3*a**2 + 6*a*b*x**2 + b**2*x**4)/(3*x)
```

3.16 $\int \frac{(ax+bx^3)^2}{x^5} dx$

Optimal result 260
Mathematica [A] (verified) 260
Rubi [A] (verified) 261
Maple [A] (warning: unable to verify) 262
Fricas [A] (verification not implemented) 263
Sympy [A] (verification not implemented) 263
Maxima [A] (verification not implemented) 263
Giac [A] (verification not implemented) 264
Mupad [B] (verification not implemented) 264
Reduce [B] (verification not implemented) 264

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{(ax+bx^3)^2}{x^5} dx = -\frac{a^2}{2x^2} + \frac{b^2x^2}{2} + 2ab\log(x)$$

output
```
-1/2*a^2/x^2+1/2*b^2*x^2+2*a*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(ax+bx^3)^2}{x^5} dx = -\frac{a^2}{2x^2} + \frac{b^2x^2}{2} + 2ab\log(x)$$

input
```
Integrate[(a*x + b*x^3)^2/x^5,x]
```

output
```
-1/2*a^2/x^2 + (b^2*x^2)/2 + 2*a*b*Log[x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^3)^2}{x^5} dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{(a+bx^2)^2}{x^3}dx$$
$$\Big\downarrow \; 243$$
$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^4}dx^2$$
$$\Big\downarrow \; 49$$
$$\frac{1}{2}\int \left(\frac{a^2}{x^4}+\frac{2ba}{x^2}+b^2\right) dx^2$$
$$\Big\downarrow \; 2009$$
$$\frac{1}{2}\left(-\frac{a^2}{x^2}+2ab\log\left(x^2\right)+b^2x^2\right)$$

input
```
Int[(a*x + b*x^3)^2/x^5,x]
```

output
```
(-(a^2/x^2) + b^2*x^2 + 2*a*b*Log[x^2])/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a^2}{2x^2}+\frac{b^2x^2}{2}+2ab\ln(x)$	24
risch	$-\frac{a^2}{2x^2}+\frac{b^2x^2}{2}+2ab\ln(x)$	24
parallelrisch	$\frac{b^2x^4+4ab\ln(x)x^2-a^2}{2x^2}$	28
norman	$\frac{-\frac{1}{2}a^2x^2+\frac{1}{2}b^2x^6}{x^4}+2ab\ln(x)$	29

input
```
int((b*x^3+a*x)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/x^2+1/2*b^2*x^2+2*a*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(ax+bx^3)^2}{x^5}\,dx = \frac{b^2x^4 + 4\,abx^2\log(x) - a^2}{2\,x^2}$$

input
```
integrate((b*x^3+a*x)^2/x^5,x, algorithm="fricas")
```

output
```
1/2*(b^2*x^4 + 4*a*b*x^2*log(x) - a^2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(ax+bx^3)^2}{x^5}\,dx = -\frac{a^2}{2x^2} + 2ab\log(x) + \frac{b^2x^2}{2}$$

input
```
integrate((b*x**3+a*x)**2/x**5,x)
```

output
```
-a**2/(2*x**2) + 2*a*b*log(x) + b**2*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{(ax+bx^3)^2}{x^5}\,dx = \frac{1}{2}\,b^2x^2 + 2\,ab\log(x) - \frac{a^2}{2\,x^2}$$

input
```
integrate((b*x^3+a*x)^2/x^5,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2 + 2*a*b*log(x) - 1/2*a^2/x^2
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{(ax+bx^3)^2}{x^5}\,dx = \frac{1}{2}\,b^2x^2 + ab\log\left(x^2\right) - \frac{2\,abx^2 + a^2}{2\,x^2}$$

input
```
integrate((b*x^3+a*x)^2/x^5,x, algorithm="giac")
```

output
```
1/2*b^2*x^2 + a*b*log(x^2) - 1/2*(2*a*b*x^2 + a^2)/x^2
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{(ax+bx^3)^2}{x^5}\,dx = \frac{b^2\,x^2}{2} - \frac{a^2}{2\,x^2} + 2\,a\,b\,\ln\left(x\right)$$

input
```
int((a*x + b*x^3)^2/x^5,x)
```

output
```
(b^2*x^2)/2 - a^2/(2*x^2) + 2*a*b*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(ax+bx^3)^2}{x^5}\,dx = \frac{4\log(x)\,ab\,x^2 - a^2 + b^2x^4}{2x^2}$$

input
```
int((b*x^3+a*x)^2/x^5,x)
```

output
```
(4*log(x)*a*b*x**2 - a**2 + b**2*x**4)/(2*x**2)
```

3.17 $\int \frac{(ax+bx^3)^2}{x^6}\,dx$

Optimal result 265
Mathematica [A] (verified) 265
Rubi [A] (verified) 266
Maple [A] (warning: unable to verify) 267
Fricas [A] (verification not implemented) 267
Sympy [A] (verification not implemented) 268
Maxima [A] (verification not implemented) 268
Giac [A] (verification not implemented) 268
Mupad [B] (verification not implemented) 269
Reduce [B] (verification not implemented) 269

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \frac{(ax+bx^3)^2}{x^6}\,dx = -\frac{a^2}{3x^3} - \frac{2ab}{x} + b^2x$$

output
```
-1/3*a^2/x^3-2*a*b/x+b^2*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(ax+bx^3)^2}{x^6}\,dx = -\frac{a^2}{3x^3} - \frac{2ab}{x} + b^2x$$

input
```
Integrate[(a*x + b*x^3)^2/x^6,x]
```

output
```
-1/3*a^2/x^3 - (2*a*b)/x + b^2*x
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^3)^2}{x^6} \, dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{(a+bx^2)^2}{x^4}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^2}{x^4}+\frac{2ab}{x^2}+b^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$$

input
```
Int[(a*x + b*x^3)^2/x^6,x]
```

output
```
-1/3*a^2/x^3 - (2*a*b)/x + b^2*x
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$	22
risch	$b^2x+\frac{-2ab\,x^2-\frac{1}{3}a^2}{x^3}$	24
gosper	$-\frac{-3b^2x^4+6ab\,x^2+a^2}{3x^3}$	25
parallelrisch	$\frac{3b^2x^4-6ab\,x^2-a^2}{3x^3}$	27
norman	$\frac{b^2x^6-\frac{1}{3}a^2x^2-2ab\,x^4}{x^5}$	28
orering	$-\frac{\left(-3b^2x^4+6ab\,x^2+a^2\right)\left(b\,x^3+ax\right)^2}{3x^5\left(b\,x^2+a\right)^2}$	45

input
```
int((b*x^3+a*x)^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2/x^3-2*a*b/x+b^2*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{(ax+bx^3)^2}{x^6}\,dx=\frac{3\,b^2x^4-6\,abx^2-a^2}{3\,x^3}$$

input
```
integrate((b*x^3+a*x)^2/x^6,x, algorithm="fricas")
```

output
```
1/3*(3*b^2*x^4 - 6*a*b*x^2 - a^2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(ax+bx^3)^2}{x^6}\,dx = b^2x + \frac{-a^2-6abx^2}{3x^3}$$

input
```
integrate((b*x**3+a*x)**2/x**6,x)
```

output
```
b**2*x + (-a**2 - 6*a*b*x**2)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(ax+bx^3)^2}{x^6}\,dx = b^2x - \frac{6\,abx^2+a^2}{3\,x^3}$$

input
```
integrate((b*x^3+a*x)^2/x^6,x, algorithm="maxima")
```

output
```
b^2*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(ax+bx^3)^2}{x^6}\,dx = b^2x - \frac{6\,abx^2+a^2}{3\,x^3}$$

input
```
integrate((b*x^3+a*x)^2/x^6,x, algorithm="giac")
```

output
```
b^2*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{(ax+bx^3)^2}{x^6}\,dx = b^2\,x - \frac{\frac{a^2}{3} + 2\,b\,a\,x^2}{x^3}$$

input
```
int((a*x + b*x^3)^2/x^6,x)
```

output
```
b^2*x - (a^2/3 + 2*a*b*x^2)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{(ax+bx^3)^2}{x^6}\,dx = \frac{3b^2x^4 - 6ab\,x^2 - a^2}{3x^3}$$

input
```
int((b*x^3+a*x)^2/x^6,x)
```

output
```
( - a**2 - 6*a*b*x**2 + 3*b**2*x**4)/(3*x**3)
```

3.18 $\int (-4x+3x^3)^6\,dx$

Optimal result 270
Mathematica [A] (verified) 270
Rubi [A] (verified) 271
Maple [A] (verified) 272
Fricas [A] (verification not implemented) 273
Sympy [A] (verification not implemented) 273
Maxima [A] (verification not implemented) 274
Giac [A] (verification not implemented) 274
Mupad [B] (verification not implemented) 274
Reduce [B] (verification not implemented) 275

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int (-4x+3x^3)^6\,dx = \frac{4096x^7}{7} - 2048x^9 + \frac{34560x^{11}}{11} - \frac{34560x^{13}}{13} + 1296x^{15} - \frac{5832x^{17}}{17} + \frac{729x^{19}}{19}$$

output
```
4096/7*x^7-2048*x^9+34560/11*x^11-34560/13*x^13+1296*x^15-5832/17*x^17+729
/19*x^19
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (-4x+3x^3)^6\,dx = \frac{4096x^7}{7} - 2048x^9 + \frac{34560x^{11}}{11} - \frac{34560x^{13}}{13} + 1296x^{15} - \frac{5832x^{17}}{17} + \frac{729x^{19}}{19}$$

input
```
Integrate[(-4*x + 3*x^3)^6,x]
```

output
```
(4096*x^7)/7 - 2048*x^9 + (34560*x^11)/11 - (34560*x^13)/13 + 1296*x^15 -
(5832*x^17)/17 + (729*x^19)/19
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2027, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x^3-4x\right)^6 dx$$

$$\Big\downarrow \text{2027}$$

$$\int x^6 \left(3x^2-4\right)^6 dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(729x^{18}-5832x^{16}+19440x^{14}-34560x^{12}+34560x^{10}-18432x^8+4096x^6\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{729x^{19}}{19}-\frac{5832x^{17}}{17}+1296x^{15}-\frac{34560x^{13}}{13}+\frac{34560x^{11}}{11}-2048x^9+\frac{4096x^7}{7}$$

input
```
Int[(-4*x + 3*x^3)^6,x]
```

output
```
(4096*x^7)/7 - 2048*x^9 + (34560*x^11)/11 - (34560*x^13)/13 + 1296*x^15 -
(5832*x^17)/17 + (729*x^19)/19
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result	siz
default	$\frac{4096}{7}x^{7}-2048x^{9}+\frac{34560}{11}x^{11}-\frac{34560}{13}x^{13}+1296x^{15}-\frac{5832}{17}x^{17}+\frac{729}{19}x^{19}$	37
norman	$\frac{4096}{7}x^{7}-2048x^{9}+\frac{34560}{11}x^{11}-\frac{34560}{13}x^{13}+1296x^{15}-\frac{5832}{17}x^{17}+\frac{729}{19}x^{19}$	37
risch	$\frac{4096}{7}x^{7}-2048x^{9}+\frac{34560}{11}x^{11}-\frac{34560}{13}x^{13}+1296x^{15}-\frac{5832}{17}x^{17}+\frac{729}{19}x^{19}$	37
parallelrisch	$\frac{4096}{7}x^{7}-2048x^{9}+\frac{34560}{11}x^{11}-\frac{34560}{13}x^{13}+1296x^{15}-\frac{5832}{17}x^{17}+\frac{729}{19}x^{19}$	37
gosper	$\frac{x^{7}\left(12405393x^{12}-110918808x^{10}+419026608x^{8}-859541760x^{6}+1015822080x^{4}-662165504x^{2}+189190144\right)}{323323}$	38
orering	$\frac{x\left(12405393x^{12}-110918808x^{10}+419026608x^{8}-859541760x^{6}+1015822080x^{4}-662165504x^{2}+189190144\right)\left(3x^{3}-4x\right)^{6}}{323323\left(3x^{2}-4\right)^{6}}$	56

input
```
int((3*x^3-4*x)^6,x,method=_RETURNVERBOSE)
```

output
```
4096/7*x^7-2048*x^9+34560/11*x^11-34560/13*x^13+1296*x^15-5832/17*x^17+729
/19*x^19
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \left(-4x+3x^3\right)^6\,dx=\frac{729}{19}\,x^{19}-\frac{5832}{17}\,x^{17}+1296\,x^{15}-\frac{34560}{13}\,x^{13}+\frac{34560}{11}\,x^{11}-2048\,x^9+\frac{4096}{7}\,x^7$$

input
```
integrate((3*x^3-4*x)^6,x, algorithm="fricas")
```

output
```
729/19*x^19 - 5832/17*x^17 + 1296*x^15 - 34560/13*x^13 + 34560/11*x^11 - 2
048*x^9 + 4096/7*x^7
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \left(-4x+3x^3\right)^6\,dx=\frac{729x^{19}}{19}-\frac{5832x^{17}}{17}+1296x^{15}-\frac{34560x^{13}}{13}+\frac{34560x^{11}}{11}-2048x^9+\frac{4096x^7}{7}$$

input
```
integrate((3*x**3-4*x)**6,x)
```

output
```
729*x**19/19 - 5832*x**17/17 + 1296*x**15 - 34560*x**13/13 + 34560*x**11/1
1 - 2048*x**9 + 4096*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \left(-4x+3x^3\right)^6\,dx=\frac{729}{19}\,x^{19}-\frac{5832}{17}\,x^{17}+1296\,x^{15}-\frac{34560}{13}\,x^{13}+\frac{34560}{11}\,x^{11}-2048\,x^9+\frac{4096}{7}\,x^7$$

input
```
integrate((3*x^3-4*x)^6,x, algorithm="maxima")
```

output
```
729/19*x^19 - 5832/17*x^17 + 1296*x^15 - 34560/13*x^13 + 34560/11*x^11 - 2
048*x^9 + 4096/7*x^7
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \left(-4x+3x^3\right)^6\,dx=\frac{729}{19}\,x^{19}-\frac{5832}{17}\,x^{17}+1296\,x^{15}-\frac{34560}{13}\,x^{13}+\frac{34560}{11}\,x^{11}-2048\,x^9+\frac{4096}{7}\,x^7$$

input
```
integrate((3*x^3-4*x)^6,x, algorithm="giac")
```

output
```
729/19*x^19 - 5832/17*x^17 + 1296*x^15 - 34560/13*x^13 + 34560/11*x^11 - 2
048*x^9 + 4096/7*x^7
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \left(-4x+3x^3\right)^6\,dx=\frac{729\,x^{19}}{19}-\frac{5832\,x^{17}}{17}+1296\,x^{15}-\frac{34560\,x^{13}}{13}+\frac{34560\,x^{11}}{11}-2048\,x^9+\frac{4096\,x^7}{7}$$

input
```
int((4*x - 3*x^3)^6,x)
```

output
```
(4096*x^7)/7 - 2048*x^9 + (34560*x^11)/11 - (34560*x^13)/13 + 1296*x^15 -
(5832*x^17)/17 + (729*x^19)/19
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \left(-4x+3x^3\right)^6 \, dx$$
$$= \frac{x^7(12405393x^{12} - 110918808x^{10} + 419026608x^8 - 859541760x^6 + 1015822080x^4 - 662165504x^2 + 189}{323323}$$

input
```
int((3*x^3-4*x)^6,x)
```

output
```
(x**7*(12405393*x**12 - 110918808*x**10 + 419026608*x**8 - 859541760*x**6
+ 1015822080*x**4 - 662165504*x**2 + 189190144))/323323
```

3.19 $\int \frac{x^4}{ax+bx^3}\,dx$

Optimal result 276
Mathematica [A] (verified) 276
Rubi [A] (verified) 277
Maple [A] (verified) 278
Fricas [A] (verification not implemented) 279
Sympy [A] (verification not implemented) 279
Maxima [A] (verification not implemented) 279
Giac [A] (verification not implemented) 280
Mupad [B] (verification not implemented) 280
Reduce [B] (verification not implemented) 280

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{x^4}{ax+bx^3}\,dx=\frac{x^2}{2b}-\frac{a\log\left(a+bx^2\right)}{2b^2}$$

output
```
1/2*x^2/b-1/2*a*ln(b*x^2+a)/b^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{ax+bx^3}\,dx=\frac{x^2}{2b}-\frac{a\log\left(a+bx^2\right)}{2b^2}$$

input
```
Integrate[x^4/(a*x + b*x^3),x]
```

output
```
x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{ax+bx^3}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^3}{a+bx^2}dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{x^2}{bx^2+a}dx^2$$

$$\Big\downarrow \; 49$$

$$\frac{1}{2}\int \left(\frac{1}{b}-\frac{a}{b\left(bx^2+a\right)}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{x^2}{b}-\frac{a\log\left(a+bx^2\right)}{b^2}\right)$$

input
```
Int[x^4/(a*x + b*x^3),x]
```

output
```
(x^2/b - (a*Log[a + b*x^2])/b^2)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$-\frac{-b\,x^{2}+a\ln\left(b\,x^{2}+a\right)}{2b^{2}}$	23
default	$\frac{x^{2}}{2b}-\frac{a\ln\left(b\,x^{2}+a\right)}{2b^{2}}$	24
norman	$\frac{x^{2}}{2b}-\frac{a\ln\left(b\,x^{2}+a\right)}{2b^{2}}$	24
risch	$\frac{x^{2}}{2b}-\frac{a\ln\left(b\,x^{2}+a\right)}{2b^{2}}$	24

input
```
int(x^4/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-b*x^2+a*ln(b*x^2+a))/b^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^4}{ax + bx^3}\, dx = \frac{bx^2 - a\log\left(bx^2 + a\right)}{2\, b^2}$$

input
```
integrate(x^4/(b*x^3+a*x),x, algorithm="fricas")
```

output
```
1/2*(b*x^2 - a*log(b*x^2 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x^4}{ax + bx^3}\, dx = -\frac{a\log\left(a + bx^2\right)}{2b^2} + \frac{x^2}{2b}$$

input
```
integrate(x**4/(b*x**3+a*x),x)
```

output
```
-a*log(a + b*x**2)/(2*b**2) + x**2/(2*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{ax + bx^3}\, dx = \frac{x^2}{2\, b} - \frac{a\log\left(bx^2 + a\right)}{2\, b^2}$$

input
```
integrate(x^4/(b*x^3+a*x),x, algorithm="maxima")
```

output
```
1/2*x^2/b - 1/2*a*log(b*x^2 + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x^4}{ax+bx^3}\,dx = \frac{x^2}{2\,b} - \frac{a\log\left(|bx^2+a|\right)}{2\,b^2}$$

input
```
integrate(x^4/(b*x^3+a*x),x, algorithm="giac")
```

output
```
1/2*x^2/b - 1/2*a*log(abs(b*x^2 + a))/b^2
```

Mupad [B] (verification not implemented)

Time = 9.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^4}{ax+bx^3}\,dx = -\frac{a\,\ln\left(b\,x^2+a\right)-b\,x^2}{2\,b^2}$$

input
```
int(x^4/(a*x + b*x^3),x)
```

output
```
-(a*log(a + b*x^2) - b*x^2)/(2*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^4}{ax+bx^3}\,dx = \frac{-\log(b\,x^2+a)\,a+b\,x^2}{2b^2}$$

input
```
int(x^4/(b*x^3+a*x),x)
```

output
```
( - log(a + b*x**2)*a + b*x**2)/(2*b**2)
```

3.20 $\int \frac{x^3}{ax+bx^3}\,dx$

Optimal result 281
Mathematica [A] (verified) 281
Rubi [A] (verified) 282
Maple [A] (verified) 283
Fricas [A] (verification not implemented) 283
Sympy [B] (verification not implemented) 284
Maxima [A] (verification not implemented) 284
Giac [A] (verification not implemented) 285
Mupad [B] (verification not implemented) 285
Reduce [B] (verification not implemented) 285

Optimal result

Integrand size = 15, antiderivative size = 31

$$\int \frac{x^3}{ax+bx^3}\,dx=\frac{x}{b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

output
```
x/b-a^(1/2)*arctan(b^(1/2)*x/a^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{ax+bx^3}\,dx=\frac{x}{b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

input
```
Integrate[x^3/(a*x + b*x^3),x]
```

output
```
x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {9, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{ax+bx^3}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^2}{a+bx^2}dx$$

$$\Big\downarrow \; 262$$

$$\frac{x}{b}-\frac{a\int \frac{1}{bx^2+a}dx}{b}$$

$$\Big\downarrow \; 218$$

$$\frac{x}{b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

input
```
Int[x^3/(a*x + b*x^3),x]
```

output
```
x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{x}{b}-\frac{a \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	27
risch	$\frac{x}{b}+\frac{\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)}{2b^{2}}-\frac{\sqrt{-ab}\,\ln\left(\sqrt{-ab}\,x-a\right)}{2b^{2}}$	56

input
```
int(x^3/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
x/b-a/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.65

$$\int \frac{x^3}{ax+bx^3}\,dx=\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2\,x}{2\,b},-\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-x}{b}\right]$$

input
```
integrate(x^3/(b*x^3+a*x),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*x)/b,
 -(sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - x)/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(26) = 52$.

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

$$\int \frac{x^3}{ax+bx^3}\,dx = \frac{\sqrt{-\frac{a}{b^3}}\log\left(-b\sqrt{-\frac{a}{b^3}}+x\right)}{2} - \frac{\sqrt{-\frac{a}{b^3}}\log\left(b\sqrt{-\frac{a}{b^3}}+x\right)}{2} + \frac{x}{b}$$

input
```
integrate(x**3/(b*x**3+a*x),x)
```

output
```
sqrt(-a/b**3)*log(-b*sqrt(-a/b**3) + x)/2 - sqrt(-a/b**3)*log(b*sqrt(-a/b*
*3) + x)/2 + x/b
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{ax+bx^3}\,dx = -\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{x}{b}$$

input
```
integrate(x^3/(b*x^3+a*x),x, algorithm="maxima")
```

output
```
-a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{ax+bx^3}\,dx = -\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{x}{b}$$

input
```
integrate(x^3/(b*x^3+a*x),x, algorithm="giac")
```

output
```
-a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{ax+bx^3}\,dx = \frac{x}{b} - \frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

input
```
int(x^3/(a*x + b*x^3),x)
```

output
```
x/b - (a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/b^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{ax+bx^3}\,dx = \frac{-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)+bx}{b^2}$$

input
```
int(x^3/(b*x^3+a*x),x)
```

output
```
( - sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a))) + b*x)/b**2
```

3.21 $\int \frac{x^2}{ax+bx^3}\,dx$

Optimal result 286
Mathematica [A] (verified) 286
Rubi [A] (verified) 287
Maple [A] (verified) 288
Fricas [A] (verification not implemented) 288
Sympy [A] (verification not implemented) 288
Maxima [A] (verification not implemented) 289
Giac [A] (verification not implemented) 289
Mupad [B] (verification not implemented) 289
Reduce [B] (verification not implemented) 290

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{x^2}{ax+bx^3}\,dx=\frac{\log\left(a+bx^2\right)}{2b}$$

output
```
1/2*ln(b*x^2+a)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{ax+bx^3}\,dx=\frac{\log\left(a+bx^2\right)}{2b}$$

input
```
Integrate[x^2/(a*x + b*x^3),x]
```

output
```
Log[a + b*x^2]/(2*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {9, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{ax+bx^3}\,dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{x}{a+bx^2}dx$$
$$\Big\downarrow \; 240$$
$$\frac{\log\left(a+bx^2\right)}{2b}$$

input
```
Int[x^2/(a*x + b*x^3),x]
```

output
```
Log[a + b*x^2]/(2*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln(b\,x^2+a)}{2b}$	14
norman	$\frac{\ln(b\,x^2+a)}{2b}$	14
risch	$\frac{\ln(b\,x^2+a)}{2b}$	14
parallelrisch	$\frac{\ln(b\,x^2+a)}{2b}$	14

input
```
int(x^2/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(b*x^2+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{ax+bx^3}\,dx = \frac{\log(bx^2+a)}{2\,b}$$

input
```
integrate(x^2/(b*x^3+a*x),x, algorithm="fricas")
```

output
```
1/2*log(b*x^2 + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{ax+bx^3}\,dx = \frac{\log(a+bx^2)}{2b}$$

input
```
integrate(x**2/(b*x**3+a*x),x)
```

output
```
log(a + b*x**2)/(2*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{ax+bx^3}\,dx = \frac{\log(bx^2+a)}{2\,b}$$

input
```
integrate(x^2/(b*x^3+a*x),x, algorithm="maxima")
```

output
```
1/2*log(b*x^2 + a)/b
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{ax+bx^3}\,dx = \frac{\log(|bx^2+a|)}{2\,b}$$

input
```
integrate(x^2/(b*x^3+a*x),x, algorithm="giac")
```

output
```
1/2*log(abs(b*x^2 + a))/b
```

Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{ax+bx^3}\,dx = \frac{\ln(b\,x^2+a)}{2\,b}$$

input
```
int(x^2/(a*x + b*x^3),x)
```

output
```
log(a + b*x^2)/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{ax + bx^3} \, dx = \frac{\log(b\,x^2 + a)}{2b}$$

input
```
int(x^2/(b*x^3+a*x),x)
```

output
```
log(a + b*x**2)/(2*b)
```

3.22 $\int \frac{x}{ax+bx^3}\,dx$

Optimal result 291
Mathematica [A] (verified) 291
Rubi [A] (verified) 292
Maple [A] (verified) 293
Fricas [A] (verification not implemented) 293
Sympy [B] (verification not implemented) 293
Maxima [A] (verification not implemented) 294
Giac [A] (verification not implemented) 294
Mupad [B] (verification not implemented) 295
Reduce [B] (verification not implemented) 295

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{x}{ax+bx^3}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

output
```
arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x}{ax+bx^3}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Integrate[x/(a*x + b*x^3),x]
```

output
```
ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {9, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{ax+bx^3}dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{1}{a+bx^2}dx$$
$$\Big\downarrow \; 218$$
$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Int[x/(a*x + b*x^3),x]
```

output
```
ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$	16
risch	$-\frac{\ln\left(bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}$	41

input
```
int(x/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.79

$$\int \frac{x}{ax+bx^3}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab}\right]$$

input
```
integrate(x/(b*x^3+a*x),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt
(a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(22) = 44.

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.21

$$\int \frac{x}{ax+bx^3}\,dx = -\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

input
```
integrate(x/(b*x**3+a*x),x)
```

output
```
-sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-
1/(a*b)) + x)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{x}{ax + bx^3} \, dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(x/(b*x^3+a*x),x, algorithm="maxima")
```

output
```
arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{x}{ax + bx^3} \, dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(x/(b*x^3+a*x),x, algorithm="giac")
```

output
```
arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{x}{ax+bx^3}\,dx=\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

input
```
int(x/(a*x + b*x^3),x)
```

output
```
atan((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{x}{ax+bx^3}\,dx=\frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)}{ab}$$

input
```
int(x/(b*x^3+a*x),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a))))/(a*b)
```

3.23 $\int \frac{1}{ax+bx^3} dx$

Optimal result 296
Mathematica [A] (verified) 296
Rubi [A] (verified) 297
Maple [A] (verified) 298
Fricas [A] (verification not implemented) 299
Sympy [A] (verification not implemented) 299
Maxima [A] (verification not implemented) 299
Giac [A] (verification not implemented) 300
Mupad [B] (verification not implemented) 300
Reduce [B] (verification not implemented) 300

Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \frac{1}{ax+bx^3} dx = \frac{\log(x)}{a} - \frac{\log(a+bx^2)}{2a}$$

output
```
ln(x)/a-1/2*ln(b*x^2+a)/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax+bx^3} dx = \frac{\log(x)}{a} - \frac{\log(a+bx^2)}{2a}$$

input
```
Integrate[(a*x + b*x^3)^(-1),x]
```

output
```
Log[x]/a - Log[a + b*x^2]/(2*a)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {2026, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax+bx^3}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{1}{x\left(a+bx^2\right)}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2\left(bx^2+a\right)}dx^2$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{x^2}dx^2}{a}-\frac{b\int \frac{1}{bx^2+a}dx^2}{a}\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(\frac{\log\left(x^2\right)}{a}-\frac{b\int \frac{1}{bx^2+a}dx^2}{a}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{\log\left(x^2\right)}{a}-\frac{\log\left(a+bx^2\right)}{a}\right)$$

input
```
Int[(a*x + b*x^3)^(-1),x]
```

output
```
(Log[x^2]/a - Log[a + b*x^2]/a)/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\ln(x)}{a}-\frac{\ln(b\,x^{2}+a)}{2a}$	21
norman	$\frac{\ln(x)}{a}-\frac{\ln(b\,x^{2}+a)}{2a}$	21
risch	$\frac{\ln(x)}{a}-\frac{\ln(b\,x^{2}+a)}{2a}$	21
parallelrisch	$\frac{2\ln(x)-\ln(b\,x^{2}+a)}{2a}$	21

input
```
int(1/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a-1/2*ln(b*x^2+a)/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{ax+bx^3}\,dx = -\frac{\log\left(bx^2+a\right)-2\,\log\left(x\right)}{2\,a}$$

input
```
integrate(1/(b*x^3+a*x),x, algorithm="fricas")
```

output
```
-1/2*(log(b*x^2 + a) - 2*log(x))/a
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{1}{ax+bx^3}\,dx = \frac{\log\left(x\right)}{a} - \frac{\log\left(\frac{a}{b}+x^2\right)}{2a}$$

input
```
integrate(1/(b*x**3+a*x),x)
```

output
```
log(x)/a - log(a/b + x**2)/(2*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{ax+bx^3}\,dx = -\frac{\log\left(bx^2+a\right)}{2\,a}+\frac{\log\left(x\right)}{a}$$

input
```
integrate(1/(b*x^3+a*x),x, algorithm="maxima")
```

output
```
-1/2*log(b*x^2 + a)/a + log(x)/a
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{ax+bx^3}\,dx = \frac{\log(x^2)}{2\,a} - \frac{\log(|bx^2+a|)}{2\,a}$$

input
```
integrate(1/(b*x^3+a*x),x, algorithm="giac")
```

output
```
1/2*log(x^2)/a - 1/2*log(abs(b*x^2 + a))/a
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{ax+bx^3}\,dx = -\frac{\ln(b\,x^2+a) - 2\,\ln(x)}{2\,a}$$

input
```
int(1/(a*x + b*x^3),x)
```

output
```
-(log(a + b*x^2) - 2*log(x))/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{ax+bx^3}\,dx = \frac{-\log(b\,x^2+a) + 2\log(x)}{2a}$$

input
```
int(1/(b*x^3+a*x),x)
```

output
```
( - log(a + b*x**2) + 2*log(x))/(2*a)
```

3.24 $\int \frac{1}{x(ax+bx^3)} dx$

Optimal result 301
Mathematica [A] (verified) 301
Rubi [A] (verified) 302
Maple [A] (verified) 303
Fricas [A] (verification not implemented) 303
Sympy [B] (verification not implemented) 304
Maxima [A] (verification not implemented) 304
Giac [A] (verification not implemented) 305
Mupad [B] (verification not implemented) 305
Reduce [B] (verification not implemented) 305

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{1}{x(ax+bx^3)} dx = -\frac{1}{ax} - \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
-1/a/x-b^(1/2)*arctan(b^(1/2)*x/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(ax+bx^3)} dx = -\frac{1}{ax} - \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[1/(x*(a*x + b*x^3)),x]
```

output
```
-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {9, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax+bx^3\right)}dx$$
$$\Big\downarrow \ 9$$
$$\int \frac{1}{x^2\left(a+bx^2\right)}dx$$
$$\Big\downarrow \ 264$$
$$-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}$$
$$\Big\downarrow \ 218$$
$$-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

input
```
Int[1/(x*(a*x + b*x^3)),x]
```

output
```
-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{b \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{1}{ax}$	30
risch	$-\frac{1}{ax}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{3}\_Z^{2}+b\right)}\_R\ln\left(\left(3\_R^{2}a^{3}+2b\right)x+a^{2}\_R\right)\right)}{2}$	48

input
```
int(1/x/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
-b/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/a/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.41

$$\int \frac{1}{x\,(ax+bx^3)}\,dx=\left[\frac{x\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2}{2\,ax},-\frac{x\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+1}{ax}\right]$$

input
```
integrate(1/x/(b*x^3+a*x),x, algorithm="fricas")
```

output
```
[1/2*(x*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2)/(a
*x), -(x*sqrt(b/a)*arctan(x*sqrt(b/a)) + 1)/(a*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(29) = 58$.

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.91

$$\int \frac{1}{x\left(ax+bx^3\right)}\,dx=\frac{\sqrt{-\frac{b}{a^3}}\log\left(-\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^3}}\log\left(\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{1}{ax}$$

input
```
integrate(1/x/(b*x**3+a*x),x)
```

output
```
sqrt(-b/a**3)*log(-a**2*sqrt(-b/a**3)/b + x)/2 - sqrt(-b/a**3)*log(a**2*sq
rt(-b/a**3)/b + x)/2 - 1/(a*x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\left(ax+bx^3\right)}\,dx=-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a}-\frac{1}{ax}$$

input
```
integrate(1/x/(b*x^3+a*x),x, algorithm="maxima")
```

output
```
-b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\left(ax+bx^3\right)}\,dx=-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a}-\frac{1}{ax}$$

input
```
integrate(1/x/(b*x^3+a*x),x, algorithm="giac")
```

output
```
-b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\left(ax+bx^3\right)}\,dx=-\frac{1}{a\,x}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
int(1/(x*(a*x + b*x^3)),x)
```

output
```
- 1/(a*x) - (b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\left(ax+bx^3\right)}\,dx=\frac{-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)x-a}{a^2x}$$

input
```
int(1/x/(b*x^3+a*x),x)
```

output
```
( - (sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*x + a))/(a**2*x)
```

3.25 $\int \frac{1}{x^2(ax+bx^3)}\,dx$

Optimal result 306
Mathematica [A] (verified) 306
Rubi [A] (verified) 307
Maple [A] (verified) 308
Fricas [A] (verification not implemented) 309
Sympy [A] (verification not implemented) 309
Maxima [A] (verification not implemented) 309
Giac [A] (verification not implemented) 310
Mupad [B] (verification not implemented) 310
Reduce [B] (verification not implemented) 310

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{1}{x^2(ax+bx^3)}\,dx = -\frac{1}{2ax^2} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx^2)}{2a^2}$$

output
```
-1/2/a/x^2-b*ln(x)/a^2+1/2*b*ln(b*x^2+a)/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(ax+bx^3)}\,dx = -\frac{1}{2ax^2} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx^2)}{2a^2}$$

input
```
Integrate[1/(x^2*(a*x + b*x^3)),x]
```

output
```
-1/2*1/(a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {9, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (ax+bx^3)} \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x^3 (a+bx^2)}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} \int \frac{1}{x^4 (bx^2+a)}dx^2$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{2} \int \left(\frac{b^2}{a^2 (bx^2+a)}-\frac{b}{a^2x^2}+\frac{1}{ax^4}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(-\frac{b\log\left(x^2\right)}{a^2}+\frac{b\log\left(a+bx^2\right)}{a^2}-\frac{1}{ax^2}\right)$$

input
```
Int[1/(x^2*(a*x + b*x^3)),x]
```

output
```
(-(1/(a*x^2)) - (b*Log[x^2])/a^2 + (b*Log[a + b*x^2])/a^2)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{1}{2 a \,x^{2}}-\frac{b \ln \left(x \right)}{a^{2}}+\frac{b \ln \left(b \,x^{2}+a \right)}{2 a^{2}}$	32
norman	$-\frac{1}{2 a \,x^{2}}-\frac{b \ln \left(x \right)}{a^{2}}+\frac{b \ln \left(b \,x^{2}+a \right)}{2 a^{2}}$	32
parallelrisch	$-\frac{2 b \ln \left(x \right) x^{2}-b \ln \left(b \,x^{2}+a \right) x^{2}+a}{2 a^{2} x^{2}}$	33
risch	$-\frac{1}{2 a \,x^{2}}-\frac{b \ln \left(x \right)}{a^{2}}+\frac{b \ln \left(-b \,x^{2}-a \right)}{2 a^{2}}$	35

input
```
int(1/x^2/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/x^2-b*ln(x)/a^2+1/2*b*ln(b*x^2+a)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2 (ax + bx^3)} \, dx = \frac{bx^2 \log(bx^2 + a) - 2 bx^2 \log(x) - a}{2 a^2 x^2}$$

input
```
integrate(1/x^2/(b*x^3+a*x),x, algorithm="fricas")
```

output
```
1/2*(b*x^2*log(b*x^2 + a) - 2*b*x^2*log(x) - a)/(a^2*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2 (ax + bx^3)} \, dx = -\frac{1}{2ax^2} - \frac{b \log(x)}{a^2} + \frac{b \log\left(\frac{a}{b} + x^2\right)}{2a^2}$$

input
```
integrate(1/x**2/(b*x**3+a*x),x)
```

output
```
-1/(2*a*x**2) - b*log(x)/a**2 + b*log(a/b + x**2)/(2*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2 (ax + bx^3)} \, dx = \frac{b \log(bx^2 + a)}{2 a^2} - \frac{b \log(x)}{a^2} - \frac{1}{2 ax^2}$$

input
```
integrate(1/x^2/(b*x^3+a*x),x, algorithm="maxima")
```

output
```
1/2*b*log(b*x^2 + a)/a^2 - b*log(x)/a^2 - 1/2/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^2\left(ax+bx^3\right)}\,dx=-\frac{b\log\left(x^2\right)}{2\,a^2}+\frac{b\log\left(\left|bx^2+a\right|\right)}{2\,a^2}+\frac{bx^2-a}{2\,a^2x^2}$$

input
```
integrate(1/x^2/(b*x^3+a*x),x, algorithm="giac")
```

output
```
-1/2*b*log(x^2)/a^2 + 1/2*b*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 - a)/(a^2
*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2\left(ax+bx^3\right)}\,dx=\frac{b\,\ln\left(b\,x^2+a\right)}{2\,a^2}-\frac{1}{2\,a\,x^2}-\frac{b\,\ln\left(x\right)}{a^2}$$

input
```
int(1/(x^2*(a*x + b*x^3)),x)
```

output
```
(b*log(a + b*x^2))/(2*a^2) - 1/(2*a*x^2) - (b*log(x))/a^2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2\left(ax+bx^3\right)}\,dx=\frac{\log(b\,x^2+a)\,b\,x^2-2\log(x)\,b\,x^2-a}{2a^2x^2}$$

input
```
int(1/x^2/(b*x^3+a*x),x)
```

output
```
(log(a + b*x**2)*b*x**2 - 2*log(x)*b*x**2 - a)/(2*a**2*x**2)
```

3.26 $\int \frac{1}{x^3(ax+bx^3)}\,dx$

Optimal result 311
Mathematica [A] (verified) 311
Rubi [A] (verified) 312
Maple [A] (verified) 313
Fricas [A] (verification not implemented) 314
Sympy [B] (verification not implemented) 314
Maxima [A] (verification not implemented) 315
Giac [A] (verification not implemented) 315
Mupad [B] (verification not implemented) 315
Reduce [B] (verification not implemented) 316

Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{1}{x^3\left(ax+bx^3\right)}\,dx=-\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
-1/3/a/x^3+b/a^2/x+b^(3/2)*arctan(b^(1/2)*x/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\left(ax+bx^3\right)}\,dx=-\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[1/(x^3*(a*x + b*x^3)),x]
```

output
```
-1/3*1/(a*x^3) + b/(a^2*x) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {9, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(ax+bx^3\right)}dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{1}{x^4\left(a+bx^2\right)}dx$$
$$\Big\downarrow \text{ 264}$$
$$-\frac{b\int \frac{1}{x^2(bx^2+a)}dx}{a}-\frac{1}{3ax^3}$$
$$\Big\downarrow \text{ 264}$$
$$-\frac{b\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}$$
$$\Big\downarrow \text{ 218}$$
$$-\frac{b\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}$$

input
```
Int[1/(x^3*(a*x + b*x^3)),x]
```

output
```
-1/3*1/(a*x^3) - (b*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^
(3/2)))/a
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{1}{3a\,x^{3}}+\frac{b}{a^{2}x}+\frac{b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^{2}\sqrt{ab}}$	39
risch	$\frac{\frac{b\,x^{2}}{a^{2}}-\frac{1}{3a}}{x^{3}}+\frac{\sqrt{-ab}\,b\ln\left(-bx-\sqrt{-ab}\right)}{2a^{3}}-\frac{\sqrt{-ab}\,b\ln\left(-bx+\sqrt{-ab}\right)}{2a^{3}}$	70

input
```
int(1/x^3/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a/x^3+b/a^2/x+b^2/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.47

$$\int \frac{1}{x^3\left(ax+bx^3\right)}\,dx = \left[\frac{3\,bx^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+6\,bx^2-2\,a}{6\,a^2x^3}, \frac{3\,bx^3\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+3\,bx^2-a}{3\,a^2x^3}\right]$$

input
```
integrate(1/x^3/(b*x^3+a*x),x, algorithm="fricas")
```

output
```
[1/6*(3*b*x^3*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) +
 6*b*x^2 - 2*a)/(a^2*x^3), 1/3*(3*b*x^3*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*
b*x^2 - a)/(a^2*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(37) = 74.

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.02

$$\int \frac{1}{x^3\left(ax+bx^3\right)}\,dx = -\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2} + \frac{\sqrt{-\frac{b^3}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2} + \frac{-a+3bx^2}{3a^2x^3}$$

input
```
integrate(1/x**3/(b*x**3+a*x),x)
```

output
```
-sqrt(-b**3/a**5)*log(-a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + sqrt(-b**3/a**5
)*log(a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + (-a + 3*b*x**2)/(3*a**2*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^3 (ax + bx^3)} \, dx = \frac{b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{3\,bx^2 - a}{3\,a^2x^3}$$

input
```
integrate(1/x^3/(b*x^3+a*x),x, algorithm="maxima")
```

output
```
b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^3 (ax + bx^3)} \, dx = \frac{b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{3\,bx^2 - a}{3\,a^2x^3}$$

input
```
integrate(1/x^3/(b*x^3+a*x),x, algorithm="giac")
```

output
```
b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^3 (ax + bx^3)} \, dx = \frac{b^{3/2} \operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}} - \frac{\frac{1}{3\,a} - \frac{b\,x^2}{a^2}}{x^3}$$

input
```
int(1/(x^3*(a*x + b*x^3)),x)
```

output
```
(b^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(5/2) - (1/(3*a) - (b*x^2)/a^2)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\,(ax+bx^3)}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b\,x^3-a^2+3ab\,x^2}{3a^3x^3}$$

input
```
int(1/x^3/(b*x^3+a*x),x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*x**3 - a**2 + 3*a*b*x**
2)/(3*a**3*x**3)
```

3.27 $\int \frac{1}{x^4(ax+bx^3)}\,dx$

Optimal result 317
Mathematica [A] (verified) 317
Rubi [A] (verified) 318
Maple [A] (verified) 319
Fricas [A] (verification not implemented) 320
Sympy [A] (verification not implemented) 320
Maxima [A] (verification not implemented) 320
Giac [A] (verification not implemented) 321
Mupad [B] (verification not implemented) 321
Reduce [B] (verification not implemented) 321

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{1}{x^4(ax+bx^3)}\,dx=-\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx^2)}{2a^3}$$

output
```
-1/4/a/x^4+1/2*b/a^2/x^2+b^2*ln(x)/a^3-1/2*b^2*ln(b*x^2+a)/a^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4(ax+bx^3)}\,dx=-\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx^2)}{2a^3}$$

input
```
Integrate[1/(x^4*(a*x + b*x^3)),x]
```

output
```
-1/4*1/(a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(
2*a^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {9, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (ax + bx^3)} \, dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{1}{x^5 (a + bx^2)} dx$$
$$\Big\downarrow \; 243$$
$$\frac{1}{2} \int \frac{1}{x^6 (bx^2 + a)} dx^2$$
$$\Big\downarrow \; 54$$
$$\frac{1}{2} \int \left(-\frac{b^3}{a^3 (bx^2 + a)} + \frac{b^2}{a^3 x^2} - \frac{b}{a^2 x^4} + \frac{1}{a x^6} \right) dx^2$$
$$\Big\downarrow \; 2009$$
$$\frac{1}{2} \left(\frac{b^2 \log (x^2)}{a^3} - \frac{b^2 \log (a + bx^2)}{a^3} + \frac{b}{a^2 x^2} - \frac{1}{2 a x^4} \right)$$

input
```
Int[1/(x^4*(a*x + b*x^3)),x]
```

output
```
(-1/2*1/(a*x^4) + b/(a^2*x^2) + (b^2*Log[x^2])/a^3 - (b^2*Log[a + b*x^2])/
a^3)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{1}{4a\,x^{4}}+\frac{b}{2a^{2}x^{2}}+\frac{b^{2}\ln\left(x\right)}{a^{3}}-\frac{b^{2}\ln\left(b\,x^{2}+a\right)}{2a^{3}}$	44
norman	$\frac{-\frac{1}{4a}+\frac{b\,x^{2}}{2a^{2}}}{x^{4}}+\frac{b^{2}\ln\left(x\right)}{a^{3}}-\frac{b^{2}\ln\left(b\,x^{2}+a\right)}{2a^{3}}$	46
risch	$\frac{-\frac{1}{4a}+\frac{b\,x^{2}}{2a^{2}}}{x^{4}}+\frac{b^{2}\ln\left(x\right)}{a^{3}}-\frac{b^{2}\ln\left(b\,x^{2}+a\right)}{2a^{3}}$	46
parallelrisch	$\frac{4b^{2}\ln\left(x\right)x^{4}-2b^{2}\ln\left(b\,x^{2}+a\right)x^{4}+2ab\,x^{2}-a^{2}}{4a^{3}x^{4}}$	48

input
```
int(1/x^4/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a/x^4+1/2*b/a^2/x^2+b^2*ln(x)/a^3-1/2*b^2*ln(b*x^2+a)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4 (ax + bx^3)} \, dx = -\frac{2 b^2 x^4 \log (bx^2 + a) - 4 b^2 x^4 \log (x) - 2 abx^2 + a^2}{4 a^3 x^4}$$

input
```
integrate(1/x^4/(b*x^3+a*x),x, algorithm="fricas")
```

output
```
-1/4*(2*b^2*x^4*log(b*x^2 + a) - 4*b^2*x^4*log(x) - 2*a*b*x^2 + a^2)/(a^3*
x^4)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^4 (ax + bx^3)} \, dx = \frac{-a + 2bx^2}{4a^2x^4} + \frac{b^2 \log (x)}{a^3} - \frac{b^2 \log \left(\frac{a}{b} + x^2\right)}{2a^3}$$

input
```
integrate(1/x**4/(b*x**3+a*x),x)
```

output
```
(-a + 2*b*x**2)/(4*a**2*x**4) + b**2*log(x)/a**3 - b**2*log(a/b + x**2)/(2
*a**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^4 (ax + bx^3)} \, dx = -\frac{b^2 \log (bx^2 + a)}{2 a^3} + \frac{b^2 \log (x)}{a^3} + \frac{2 bx^2 - a}{4 a^2 x^4}$$

input
```
integrate(1/x^4/(b*x^3+a*x),x, algorithm="maxima")
```

output
```
-1/2*b^2*log(b*x^2 + a)/a^3 + b^2*log(x)/a^3 + 1/4*(2*b*x^2 - a)/(a^2*x^4)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^4 (ax + bx^3)} \, dx = \frac{b^2 \log (x^2)}{2 a^3} - \frac{b^2 \log (|bx^2 + a|)}{2 a^3} - \frac{3 b^2 x^4 - 2 abx^2 + a^2}{4 a^3 x^4}$$

input
```
integrate(1/x^4/(b*x^3+a*x),x, algorithm="giac")
```

output
```
1/2*b^2*log(x^2)/a^3 - 1/2*b^2*log(abs(b*x^2 + a))/a^3 - 1/4*(3*b^2*x^4 -
2*a*b*x^2 + a^2)/(a^3*x^4)
```

Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4 (ax + bx^3)} \, dx = \frac{b^2 \ln (x)}{a^3} - \frac{b^2 \ln (b x^2 + a)}{2 a^3} - \frac{\frac{1}{4 a} - \frac{b x^2}{2 a^2}}{x^4}$$

input
```
int(1/(x^4*(a*x + b*x^3)),x)
```

output
```
(b^2*log(x))/a^3 - (b^2*log(a + b*x^2))/(2*a^3) - (1/(4*a) - (b*x^2)/(2*a^
2))/x^4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^4 (ax + bx^3)} \, dx = \frac{-2 \log(b x^2 + a) b^2 x^4 + 4 \log(x) b^2 x^4 - a^2 + 2ab x^2}{4a^3 x^4}$$

input
```
int(1/x^4/(b*x^3+a*x),x)
```

output
```
( - 2*log(a + b*x**2)*b**2*x**4 + 4*log(x)*b**2*x**4 - a**2 + 2*a*b*x**2)/
(4*a**3*x**4)
```

3.28 $\int \frac{x^2}{(ax+bx^3)^2} \, dx$

Optimal result 322
Mathematica [A] (verified) 322
Rubi [A] (verified) 323
Maple [A] (verified) 324
Fricas [A] (verification not implemented) 324
Sympy [B] (verification not implemented) 325
Maxima [A] (verification not implemented) 325
Giac [A] (verification not implemented) 326
Mupad [B] (verification not implemented) 326
Reduce [B] (verification not implemented) 326

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \frac{x^2}{(ax+bx^3)^2} \, dx = \frac{x}{2a(a+bx^2)} + \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

output
```
1/2*x/a/(b*x^2+a)+1/2*arctan(b^(1/2)*x/a^(1/2))/a^(3/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(ax+bx^3)^2} \, dx = \frac{x}{2a(a+bx^2)} + \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

input
```
Integrate[x^2/(a*x + b*x^3)^2,x]
```

output
```
x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {9, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax+bx^3)^2}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{1}{(a+bx^2)^2}dx$$

$$\Big\downarrow \; 215$$

$$\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a(a+bx^2)}$$

$$\Big\downarrow \; 218$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a(a+bx^2)}$$

input
```
Int[x^2/(a*x + b*x^3)^2,x]
```

output
```
x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x}{2a(b\,x^2+a)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}$	36
risch	$\frac{x}{2a(b\,x^2+a)}-\frac{\ln\left(bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,a}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,a}$	62

input
```
int(x^2/(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.67

$$\int \frac{x^2}{(ax+bx^3)^2}\,dx = \left[\frac{2\,abx-(bx^2+a)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{4\,(a^2b^2x^2+a^3b)}, \frac{abx+(bx^2+a)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2\,(a^2b^2x^2+a^3b)}\right]$$

input
```
integrate(x^2/(b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*a*b*x - (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b
*x^2 + a)))/(a^2*b^2*x^2 + a^3*b), 1/2*(a*b*x + (b*x^2 + a)*sqrt(a*b)*arct
an(sqrt(a*b)*x/a))/(a^2*b^2*x^2 + a^3*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(36) = 72$.

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.73

$$\int \frac{x^2}{(ax+bx^3)^2}\,dx = \frac{x}{2a^2+2abx^2} - \frac{\sqrt{-\frac{1}{a^3b}}\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b}}\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4}$$

input
```
integrate(x**2/(b*x**3+a*x)**2,x)
```

output
```
x/(2*a**2 + 2*a*b*x**2) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a**3*b)) +
x)/4 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) + x)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{(ax+bx^3)^2}\,dx = \frac{x}{2(abx^2+a^2)} + \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a}$$

input
```
integrate(x^2/(b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
1/2*x/(a*b*x^2 + a^2) + 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{(ax+bx^3)^2}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a} + \frac{x}{2\,(bx^2+a)a}$$

input
```
integrate(x^2/(b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*x/((b*x^2 + a)*a)
```

Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{(ax+bx^3)^2}\,dx = \frac{x}{2\,a\,\left(b\,x^2+a\right)} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

input
```
int(x^2/(a*x + b*x^3)^2,x)
```

output
```
x/(2*a*(a + b*x^2)) + atan((b^(1/2)*x)/a^(1/2))/(2*a^(3/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int \frac{x^2}{(ax+bx^3)^2}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b\,x^2+abx}{2a^2b\left(b\,x^2+a\right)}$$

input
```
int(x^2/(b*x^3+a*x)^2,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a + sqrt(b)*sqrt(a)*atan((b
*x)/(sqrt(b)*sqrt(a)))*b*x**2 + a*b*x)/(2*a**2*b*(a + b*x**2))
```

3.29 $\int \frac{x}{(ax+bx^3)^2}\,dx$

Optimal result 327
Mathematica [A] (verified) 327
Rubi [A] (verified) 328
Maple [A] (verified) 329
Fricas [A] (verification not implemented) 330
Sympy [A] (verification not implemented) 330
Maxima [A] (verification not implemented) 330
Giac [A] (verification not implemented) 331
Mupad [B] (verification not implemented) 331
Reduce [B] (verification not implemented) 331

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{x}{(ax+bx^3)^2}\,dx = \frac{1}{2a(a+bx^2)} + \frac{\log(x)}{a^2} - \frac{\log(a+bx^2)}{2a^2}$$

output
```
1/2/a/(b*x^2+a)+ln(x)/a^2-1/2*ln(b*x^2+a)/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{x}{(ax+bx^3)^2}\,dx = \frac{\frac{a}{a+bx^2} + 2\log(x) - \log(a+bx^2)}{2a^2}$$

input
```
Integrate[x/(a*x + b*x^3)^2,x]
```

output
```
(a/(a + b*x^2) + 2*Log[x] - Log[a + b*x^2])/(2*a^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {9, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(ax+bx^3)^2}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{1}{x(a+bx^2)^2}dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{1}{x^2(bx^2+a)^2}dx^2$$

$$\Big\downarrow \; 54$$

$$\frac{1}{2}\int \left(-\frac{b}{a^2(bx^2+a)}-\frac{b}{a(bx^2+a)^2}+\frac{1}{a^2x^2}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{\log(a+bx^2)}{a^2}+\frac{\log(x^2)}{a^2}+\frac{1}{a(a+bx^2)}\right)$$

input
```
Int[x/(a*x + b*x^3)^2,x]
```

output
```
(1/(a*(a + b*x^2)) + Log[x^2]/a^2 - Log[a + b*x^2]/a^2)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{1}{2a(b\,x^2+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(b\,x^2+a)}{2a^2}$	35
norman	$-\frac{b\,x^2}{2a^2(b\,x^2+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(b\,x^2+a)}{2a^2}$	39
default	$\frac{\ln(x)}{a^2}-\frac{b\left(-\frac{a}{b\left(b\,x^2+a\right)}+\frac{\ln\left(b\,x^2+a\right)}{b}\right)}{2a^2}$	42
parallelrisch	$\frac{2b\ln(x)x^2-b\ln\left(b\,x^2+a\right)x^2-b\,x^2+2a\ln(x)-a\ln\left(b\,x^2+a\right)}{2a^2(b\,x^2+a)}$	60

input
```
int(x/(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/a/(b*x^2+a)+ln(x)/a^2-1/2*ln(b*x^2+a)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{x}{(ax+bx^3)^2}\,dx = -\frac{(bx^2+a)\log(bx^2+a) - 2(bx^2+a)\log(x) - a}{2(a^2bx^2+a^3)}$$

input
```
integrate(x/(b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
-1/2*((b*x^2 + a)*log(b*x^2 + a) - 2*(b*x^2 + a)*log(x) - a)/(a^2*b*x^2 +
a^3)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{x}{(ax+bx^3)^2}\,dx = \frac{1}{2a^2+2abx^2} + \frac{\log(x)}{a^2} - \frac{\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

input
```
integrate(x/(b*x**3+a*x)**2,x)
```

output
```
1/(2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(a/b + x**2)/(2*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{x}{(ax+bx^3)^2}\,dx = \frac{1}{2(abx^2+a^2)} - \frac{\log(bx^2+a)}{2a^2} + \frac{\log(x)}{a^2}$$

input
```
integrate(x/(b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
1/2/(a*b*x^2 + a^2) - 1/2*log(b*x^2 + a)/a^2 + log(x)/a^2
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{x}{(ax+bx^3)^2}\,dx = \frac{\log(x^2)}{2\,a^2} - \frac{\log(|bx^2+a|)}{2\,a^2} + \frac{bx^2+2\,a}{2\,(bx^2+a)a^2}$$

input
```
integrate(x/(b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
1/2*log(x^2)/a^2 - 1/2*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 + 2*a)/((b*x^2
 + a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{x}{(ax+bx^3)^2}\,dx = \frac{\ln(x)}{a^2} + \frac{1}{2\,a\,\left(b\,x^2+a\right)} - \frac{\ln\left(b\,x^2+a\right)}{2\,a^2}$$

input
```
int(x/(a*x + b*x^3)^2,x)
```

output
```
log(x)/a^2 + 1/(2*a*(a + b*x^2)) - log(a + b*x^2)/(2*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.55

$$\int \frac{x}{(ax+bx^3)^2}\,dx = \frac{-\log(b\,x^2+a)\,a - \log(b\,x^2+a)\,b\,x^2 + 2\log(x)\,a + 2\log(x)\,b\,x^2 - b\,x^2}{2a^2\left(b\,x^2+a\right)}$$

input
```
int(x/(b*x^3+a*x)^2,x)
```

output
```
( - log(a + b*x**2)*a - log(a + b*x**2)*b*x**2 + 2*log(x)*a + 2*log(x)*b*x
**2 - b*x**2)/(2*a**2*(a + b*x**2))
```

3.30 $\int \frac{1}{(ax+bx^3)^2}\,dx$

Optimal result 333
Mathematica [A] (verified) 333
Rubi [A] (verified) 334
Maple [A] (verified) 335
Fricas [A] (verification not implemented) 336
Sympy [A] (verification not implemented) 336
Maxima [A] (verification not implemented) 337
Giac [A] (verification not implemented) 337
Mupad [B] (verification not implemented) 337
Reduce [B] (verification not implemented) 338

Optimal result

Integrand size = 11, antiderivative size = 54

$$\int \frac{1}{(ax+bx^3)^2}\,dx=-\frac{1}{a^2x}-\frac{bx}{2a^2(a+bx^2)}-\frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}$$

output
```
-1/a^2/x-1/2*b*x/a^2/(b*x^2+a)-3/2*b^(1/2)*arctan(b^(1/2)*x/a^(1/2))/a^(5/
2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ax+bx^3)^2}\,dx=-\frac{1}{a^2x}-\frac{bx}{2a^2(a+bx^2)}-\frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}$$

input
```
Integrate[(a*x + b*x^3)^(-2),x]
```

output
```
-(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(5/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2026, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax+bx^3)^2}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{1}{x^2(a+bx^2)^2}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{3\int \frac{1}{x^2(bx^2+a)}dx}{2a}+\frac{1}{2ax(a+bx^2)}$$

$$\Big\downarrow \text{264}$$

$$\frac{3\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{2a}+\frac{1}{2ax(a+bx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{3\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{2a}+\frac{1}{2ax(a+bx^2)}$$

input
```
Int[(a*x + b*x^3)^(-2),x]
```

output
```
1/(2*a*x*(a + b*x^2)) + (3*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/a^(3/2)))/(2*a)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{1}{a^{2} x}-\frac{b \left(\frac{x}{2 b \,x^{2}+2 a}+\frac{3 \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}}\right)}{a^{2}}$	45
risch	$\frac{-\frac{3 b \,x^{2}}{2 a^{2}}-\frac{1}{a}}{x \left(b \,x^{2}+a \right)}+\frac{3 \left(\sum_{\_R =\operatorname{RootOf}\left(a^{5} \_Z^{2}+b \right)}\_R \ln \left(\left(3 \_R^{2} a^{5}+2 b \right) x +a^{3} \_R \right)\right)}{4}$	68

input
```
int(1/(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2/x-b/a^2*(1/2*x/(b*x^2+a)+3/2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.52

$$\int \frac{1}{(ax+bx^3)^2}\,dx = \left[-\frac{6\,bx^2-3\,(bx^3+ax)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+4\,a}{4\,(a^2bx^3+a^3x)}, -\frac{3\,bx^2+3\,(bx^3+ax)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+2\,a}{2\,(a^2bx^3+a^3x)}\right]$$

input
```
integrate(1/(b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
[-1/4*(6*b*x^2 - 3*(b*x^3 + a*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a)
- a)/(b*x^2 + a)) + 4*a)/(a^2*b*x^3 + a^3*x), -1/2*(3*b*x^2 + 3*(b*x^3 + a
*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 2*a)/(a^2*b*x^3 + a^3*x)]
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.70

$$\int \frac{1}{(ax+bx^3)^2}\,dx = \frac{3\sqrt{-\frac{b}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4} - \frac{3\sqrt{-\frac{b}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4} + \frac{-2a-3bx^2}{2a^3x+2a^2bx^3}$$

input
```
integrate(1/(b*x**3+a*x)**2,x)
```

output
```
3*sqrt(-b/a**5)*log(-a**3*sqrt(-b/a**5)/b + x)/4 - 3*sqrt(-b/a**5)*log(a**
3*sqrt(-b/a**5)/b + x)/4 + (-2*a - 3*b*x**2)/(2*a**3*x + 2*a**2*b*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{1}{(ax+bx^3)^2}\,dx = -\frac{3\,bx^2+2\,a}{2\,(a^2bx^3+a^3x)} - \frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2}$$

input
```
integrate(1/(b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
-1/2*(3*b*x^2 + 2*a)/(a^2*b*x^3 + a^3*x) - 3/2*b*arctan(b*x/sqrt(a*b))/(sq
rt(a*b)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{1}{(ax+bx^3)^2}\,dx = -\frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2} - \frac{3\,bx^2+2\,a}{2\,(bx^3+ax)a^2}$$

input
```
integrate(1/(b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
-3/2*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*x^2 + 2*a)/((b*x^3
 + a*x)*a^2)
```

Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{1}{(ax+bx^3)^2}\,dx = -\frac{\frac{1}{a}+\frac{3\,b\,x^2}{2\,a^2}}{b\,x^3+a\,x} - \frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

input
```
int(1/(a*x + b*x^3)^2,x)
```

output
```
- (1/a + (3*b*x^2)/(2*a^2))/(a*x + b*x^3) - (3*b^(1/2)*atan((b^(1/2)*x)/a^
(1/2)))/(2*a^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

$$\int \frac{1}{(ax+bx^3)^2}\,dx = \frac{-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ax - 3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b\,x^3 - 2a^2 - 3ab\,x^2}{2a^3x\,(b\,x^2+a)}$$

input
```
int(1/(b*x^3+a*x)^2,x)
```

output
```
( - 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*x - 3*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*x**3 - 2*a**2 - 3*a*b*x**2)/(2*a**3*x*(a
 + b*x**2))
```

3.31 $\int \frac{1}{x\left(ax+bx^3\right)^2} \, dx$

Optimal result 339
Mathematica [A] (verified) 339
Rubi [A] (verified) 340
Maple [A] (verified) 341
Fricas [A] (verification not implemented) 342
Sympy [A] (verification not implemented) 342
Maxima [A] (verification not implemented) 342
Giac [A] (verification not implemented) 343
Mupad [B] (verification not implemented) 343
Reduce [B] (verification not implemented) 344

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{1}{x\left(ax+bx^3\right)^2} \, dx = -\frac{1}{2a^2x^2} - \frac{b}{2a^2\left(a+bx^2\right)} - \frac{2b\log(x)}{a^3} + \frac{b\log\left(a+bx^2\right)}{a^3}$$

output
```
-1/2/a^2/x^2-1/2*b/a^2/(b*x^2+a)-2*b*ln(x)/a^3+b*ln(b*x^2+a)/a^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{1}{x\left(ax+bx^3\right)^2} \, dx = -\frac{a\left(\frac{1}{x^2}+\frac{b}{a+bx^2}\right)+4b\log(x)-2b\log\left(a+bx^2\right)}{2a^3}$$

input
```
Integrate[1/(x*(a*x + b*x^3)^2),x]
```

output
```
-1/2*(a*(x^(-2) + b/(a + b*x^2)) + 4*b*Log[x] - 2*b*Log[a + b*x^2])/a^3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {9, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax+bx^3\right)^2}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x^3\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{1}{x^4\left(bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{2}\int \left(\frac{2b^2}{a^3\left(bx^2+a\right)}+\frac{b^2}{a^2\left(bx^2+a\right)^2}-\frac{2b}{a^3x^2}+\frac{1}{a^2x^4}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{2b\log\left(x^2\right)}{a^3}+\frac{2b\log\left(a+bx^2\right)}{a^3}-\frac{b}{a^2\left(a+bx^2\right)}-\frac{1}{a^2x^2}\right)$$

input
```
Int[1/(x*(a*x + b*x^3)^2),x]
```

output
```
(-(1/(a^2*x^2)) - b/(a^2*(a + b*x^2)) - (2*b*Log[x^2])/a^3 + (2*b*Log[a +
b*x^2])/a^3)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

method	result	size
norman	$\frac{\frac{b^2x^4}{a^3}-\frac{1}{2a}}{(b\,x^2+a)x^2}+\frac{b\ln\left(b\,x^2+a\right)}{a^3}-\frac{2b\ln\left(x\right)}{a^3}$	52
risch	$\frac{-\frac{b\,x^2}{a^2}-\frac{1}{2a}}{(b\,x^2+a)x^2}-\frac{2b\ln\left(x\right)}{a^3}+\frac{b\ln\left(-b\,x^2-a\right)}{a^3}$	54
default	$-\frac{1}{2a^2x^2}-\frac{2b\ln\left(x\right)}{a^3}+\frac{b^2\left(-\frac{a}{b\left(b\,x^2+a\right)}+\frac{2\ln\left(b\,x^2+a\right)}{b}\right)}{2a^3}$	55
parallelrisch	$-\frac{4b^2\ln\left(x\right)x^4-2b^2\ln\left(b\,x^2+a\right)x^4-2b^2x^4+4ab\ln\left(x\right)x^2-2\ln\left(b\,x^2+a\right)x^2ab+a^2}{2a^3x^2\left(b\,x^2+a\right)}$	80

input
```
int(1/x/(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
(b^2/a^3*x^4-1/2/a)/(b*x^2+a)/x^2+b*ln(b*x^2+a)/a^3-2*b*ln(x)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.49

$$\int \frac{1}{x\left(ax+bx^3\right)^2}\,dx = -\frac{2\,abx^2+a^2-2\left(b^2x^4+abx^2\right)\log\left(bx^2+a\right)+4\left(b^2x^4+abx^2\right)\log\left(x\right)}{2\left(a^3bx^4+a^4x^2\right)}$$

input
```
integrate(1/x/(b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*b*x^2 + a^2 - 2*(b^2*x^4 + a*b*x^2)*log(b*x^2 + a) + 4*(b^2*x^4
+ a*b*x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(ax+bx^3\right)^2}\,dx = \frac{-a-2bx^2}{2a^3x^2+2a^2bx^4} - \frac{2b\log\left(x\right)}{a^3} + \frac{b\log\left(\frac{a}{b}+x^2\right)}{a^3}$$

input
```
integrate(1/x/(b*x**3+a*x)**2,x)
```

output
```
(-a - 2*b*x**2)/(2*a**3*x**2 + 2*a**2*b*x**4) - 2*b*log(x)/a**3 + b*log(a/
b + x**2)/a**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\left(ax+bx^3\right)^2}\,dx = -\frac{2\,bx^2+a}{2\left(a^2bx^4+a^3x^2\right)} + \frac{b\log\left(bx^2+a\right)}{a^3} - \frac{2\,b\log\left(x\right)}{a^3}$$

input
```
integrate(1/x/(b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*b*x^2 + a)/(a^2*b*x^4 + a^3*x^2) + b*log(b*x^2 + a)/a^3 - 2*b*log(
x)/a^3
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(ax+bx^3\right)^2}\,dx = -\frac{b\log\left(x^2\right)}{a^3} + \frac{b\log\left(\left|bx^2+a\right|\right)}{a^3} - \frac{2\,bx^2+a}{2\,(bx^4+ax^2)a^2}$$

input
```
integrate(1/x/(b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
-b*log(x^2)/a^3 + b*log(abs(b*x^2 + a))/a^3 - 1/2*(2*b*x^2 + a)/((b*x^4 +
a*x^2)*a^2)
```

Mupad [B] (verification not implemented)

Time = 8.96 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(ax+bx^3\right)^2}\,dx = \frac{b\,\ln\left(b\,x^2+a\right)}{a^3} - \frac{\frac{1}{2\,a}+\frac{b\,x^2}{a^2}}{b\,x^4+a\,x^2} - \frac{2\,b\,\ln\left(x\right)}{a^3}$$

input
```
int(1/(x*(a*x + b*x^3)^2),x)
```

output
```
(b*log(a + b*x^2))/a^3 - (1/(2*a) + (b*x^2)/a^2)/(a*x^2 + b*x^4) - (2*b*lo
g(x))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.65

$$\int \frac{1}{x\left(ax+bx^3\right)^2}\,dx$$
$$= \frac{2\log(b\,x^2+a)\,ab\,x^2+2\log(b\,x^2+a)\,b^2x^4-4\log(x)\,ab\,x^2-4\log(x)\,b^2x^4-a^2+2b^2x^4}{2a^3x^2\,(b\,x^2+a)}$$

input
```
int(1/x/(b*x^3+a*x)^2,x)
```

output
```
(2*log(a + b*x**2)*a*b*x**2 + 2*log(a + b*x**2)*b**2*x**4 - 4*log(x)*a*b*x
**2 - 4*log(x)*b**2*x**4 - a**2 + 2*b**2*x**4)/(2*a**3*x**2*(a + b*x**2))
```

3.32 $\int \frac{1}{x^2(ax+bx^3)^2}\,dx$

Optimal result 345
Mathematica [A] (verified) 345
Rubi [A] (verified) 346
Maple [A] (verified) 347
Fricas [A] (verification not implemented) 348
Sympy [A] (verification not implemented) 348
Maxima [A] (verification not implemented) 349
Giac [A] (verification not implemented) 349
Mupad [B] (verification not implemented) 350
Reduce [B] (verification not implemented) 350

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \frac{1}{x^2\left(ax+bx^3\right)^2}\,dx=-\frac{1}{3a^2x^3}+\frac{2b}{a^3x}+\frac{b^2x}{2a^3\left(a+bx^2\right)}+\frac{5b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}$$

output
```
-1/3/a^2/x^3+2*b/a^3/x+1/2*b^2*x/a^3/(b*x^2+a)+5/2*b^(3/2)*arctan(b^(1/2)*
x/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(ax+bx^3\right)^2}\,dx=-\frac{1}{3a^2x^3}+\frac{2b}{a^3x}+\frac{b^2x}{2a^3\left(a+bx^2\right)}+\frac{5b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}$$

input
```
Integrate[1/(x^2*(a*x + b*x^3)^2),x]
```

output
```
-1/3*1/(a^2*x^3) + (2*b)/(a^3*x) + (b^2*x)/(2*a^3*(a + b*x^2)) + (5*b^(3/2
)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {9, 253, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (ax+bx^3)^2} \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x^4 (a+bx^2)^2} dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{5\int \frac{1}{x^4(bx^2+a)}dx}{2a} + \frac{1}{2ax^3 (a+bx^2)}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{5\left(-\frac{b\int \frac{1}{x^2\left(bx^2+a\right)}dx}{a} - \frac{1}{3ax^3}\right)}{2a} + \frac{1}{2ax^3 (a+bx^2)}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{5\left(-\frac{b\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a} - \frac{1}{ax}\right)}{a} - \frac{1}{3ax^3}\right)}{2a} + \frac{1}{2ax^3 (a+bx^2)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{5\left(-\frac{b\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{ax}\right)}{a} - \frac{1}{3ax^3}\right)}{2a} + \frac{1}{2ax^3 (a+bx^2)}$$

input
```
Int[1/(x^2*(a*x + b*x^3)^2),x]
```

output
```
1/(2*a*x^3*(a + b*x^2)) + (5*(-1/3*1/(a*x^3) - (b*(-(1/(a*x)) - (Sqrt[b]*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))/a))/(2*a)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{1}{3a^{2}x^{3}}+\frac{2b}{a^{3}x}+\frac{b^{2}\left(\frac{x}{2b\,x^{2}+2a}+\frac{5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{3}}$	55
risch	$\frac{\frac{5b^{2}x^{4}}{2a^{3}}+\frac{5b\,x^{2}}{3a^{2}}-\frac{1}{3a}}{x^{3}\left(b\,x^{2}+a\right)}+\frac{5\sqrt{-ab}\,b\ln\left(-bx-\sqrt{-ab}\right)}{4a^{4}}-\frac{5\sqrt{-ab}\,b\ln\left(-bx+\sqrt{-ab}\right)}{4a^{4}}$	91

input
```
int(1/x^2/(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^2/x^3+2*b/a^3/x+b^2/a^3*(1/2*x/(b*x^2+a)+5/2/(a*b)^(1/2)*arctan(b*x
/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.57

$$\int \frac{1}{x^2 (ax+bx^3)^2} dx = \left[\frac{30 b^2x^4 + 20 abx^2 + 15 (b^2x^5 + abx^3)\sqrt{-\frac{b}{a}} \log \left(\frac{bx^2+2 ax\sqrt{-\frac{b}{a}}-a}{bx^2+a} \right) - 4 a^2}{12 (a^3bx^5 + a^4x^3)}, \frac{15 b^2x^4 + 10 abx^2 + 15 (b^2x^5 +}{6 (a^3bx^}\right.$$

input
```
integrate(1/x^2/(b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
[1/12*(30*b^2*x^4 + 20*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(-b/a)*log((b*
x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 4*a^2)/(a^3*b*x^5 + a^4*x^3), 1
/6*(15*b^2*x^4 + 10*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(b/a)*arctan(x*sq
rt(b/a)) - 2*a^2)/(a^3*b*x^5 + a^4*x^3)]
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.70

$$\int \frac{1}{x^2 (ax+bx^3)^2} dx = -\frac{5\sqrt{-\frac{b^3}{a^7}} \log \left(-\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2} + x \right)}{4} + \frac{5\sqrt{-\frac{b^3}{a^7}} \log \left(\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2} + x \right)}{4} + \frac{-2a^2 + 10abx^2 + 15b^2x^4}{6a^4x^3 + 6a^3bx^5}$$

input
```
integrate(1/x**2/(b*x**3+a*x)**2,x)
```

output
```
-5*sqrt(-b**3/a**7)*log(-a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + 5*sqrt(-b**3/
a**7)*log(a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + (-2*a**2 + 10*a*b*x**2 + 15*
b**2*x**4)/(6*a**4*x**3 + 6*a**3*b*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^2\left(ax+bx^3\right)^2}\,dx = \frac{15\,b^2x^4+10\,abx^2-2\,a^2}{6\,(a^3bx^5+a^4x^3)}+\frac{5\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3}$$

input
```
integrate(1/x^2/(b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
1/6*(15*b^2*x^4 + 10*a*b*x^2 - 2*a^2)/(a^3*b*x^5 + a^4*x^3) + 5/2*b^2*arct
an(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\left(ax+bx^3\right)^2}\,dx = \frac{5\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3}+\frac{b^2x}{2\,(bx^2+a)a^3}+\frac{6\,bx^2-a}{3\,a^3x^3}$$

input
```
integrate(1/x^2/(b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
5/2*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/2*b^2*x/((b*x^2 + a)*a^3
) + 1/3*(6*b*x^2 - a)/(a^3*x^3)
```

Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^2 (ax+bx^3)^2} \, dx = \frac{\frac{5\,b\,x^2}{3\,a^2} - \frac{1}{3\,a} + \frac{5\,b^2\,x^4}{2\,a^3}}{b\,x^5 + a\,x^3} + \frac{5\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{7/2}}$$

input
```
int(1/(x^2*(a*x + b*x^3)^2),x)
```

output
```
((5*b*x^2)/(3*a^2) - 1/(3*a) + (5*b^2*x^4)/(2*a^3))/(a*x^3 + b*x^5) + (5*b
^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \frac{1}{x^2 (ax+bx^3)^2} \, dx = \frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) ab\,x^3 + 15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) b^2x^5 - 2a^3 + 10a^2b\,x^2 + 15a\,b^2x^4}{6a^4x^3\,(b\,x^2 + a)}$$

input
```
int(1/x^2/(b*x^3+a*x)^2,x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*x**3 + 15*sqrt(b)*sq
rt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*x**5 - 2*a**3 + 10*a**2*b*x**2 +
15*a*b**2*x**4)/(6*a**4*x**3*(a + b*x**2))
```

3.33 $\int \frac{x^5}{x-x^3} dx$

Optimal result 351
Mathematica [B] (verified) 351
Rubi [A] (verified) 352
Maple [C] (verified) 353
Fricas [A] (verification not implemented) 353
Sympy [B] (verification not implemented) 354
Maxima [A] (verification not implemented) 354
Giac [B] (verification not implemented) 354
Mupad [B] (verification not implemented) 355
Reduce [B] (verification not implemented) 355

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x^5}{x-x^3} dx = -x - \frac{x^3}{3} + \operatorname{arctanh}(x)$$

output
```
-x-1/3*x^3+arctanh(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 29 vs. 2(13) = 26.

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.23

$$\int \frac{x^5}{x-x^3} dx = -x - \frac{x^3}{3} - \frac{1}{2}\log(1-x) + \frac{1}{2}\log(1+x)$$

input
```
Integrate[x^5/(x - x^3),x]
```

output
```
-x - x^3/3 - Log[1 - x]/2 + Log[1 + x]/2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {9, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{x-x^3} \, dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{x^4}{1-x^2}dx$$
$$\Big\downarrow \; 254$$
$$\int \left(-x^2+\frac{1}{1-x^2}-1\right)dx$$
$$\Big\downarrow \; 2009$$
$$\text{arctanh}(x)-\frac{x^3}{3}-x$$

input
```
Int[x^5/(x - x^3),x]
```

output
```
-x - x^3/3 + ArcTanh[x]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

method	result	size
meijerg	$-\frac{i\left(-\frac{2ix\left(5x^2+15\right)}{15}+2i\,\mathrm{arctanh}\left(x\right)\right)}{2}$	21
default	$-\frac{x^3}{3}-x-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	22
norman	$-\frac{x^3}{3}-x-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	22
risch	$-\frac{x^3}{3}-x-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	22
parallelrisch	$-\frac{x^3}{3}-x-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	22

input
```
int(x^5/(-x^3+x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*(-2/15*I*x*(5*x^2+15)+2*I*arctanh(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{x^5}{x-x^3}\,dx = -\frac{1}{3}\,x^3-x+\frac{1}{2}\,\log\left(x+1\right)-\frac{1}{2}\,\log\left(x-1\right)$$

input
```
integrate(x^5/(-x^3+x),x, algorithm="fricas")
```

output
```
-1/3*x^3 - x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{x^5}{x - x^3} \, dx = -\frac{x^3}{3} - x - \frac{\log(x-1)}{2} + \frac{\log(x+1)}{2}$$

input
```
integrate(x**5/(-x**3+x),x)
```

output
```
-x**3/3 - x - log(x - 1)/2 + log(x + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{x^5}{x - x^3} \, dx = -\frac{1}{3}\,x^3 - x + \frac{1}{2}\,\log(x+1) - \frac{1}{2}\,\log(x-1)$$

input
```
integrate(x^5/(-x^3+x),x, algorithm="maxima")
```

output
```
-1/3*x^3 - x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{x^5}{x - x^3} \, dx = -\frac{1}{3}\,x^3 - x + \frac{1}{2}\,\log(|x+1|) - \frac{1}{2}\,\log(|x-1|)$$

input
```
integrate(x^5/(-x^3+x),x, algorithm="giac")
```

output
```
-1/3*x^3 - x + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{x-x^3}\,dx = \operatorname{atanh}(x) - x - \frac{x^3}{3}$$

input
```
int(x^5/(x - x^3),x)
```

output
```
atanh(x) - x - x^3/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{x^5}{x-x^3}\,dx = -\frac{\log(x-1)}{2} + \frac{\log(x+1)}{2} - \frac{x^3}{3} - x$$

input
```
int(x^5/(-x^3+x),x)
```

output
```
( - 3*log(x - 1) + 3*log(x + 1) - 2*x**3 - 6*x)/6
```

3.34 $\int \frac{x^4}{x-x^3} dx$

Optimal result 356
Mathematica [A] (verified) 356
Rubi [A] (verified) 357
Maple [A] (verified) 358
Fricas [A] (verification not implemented) 359
Sympy [A] (verification not implemented) 359
Maxima [A] (verification not implemented) 359
Giac [A] (verification not implemented) 360
Mupad [B] (verification not implemented) 360
Reduce [B] (verification not implemented) 360

Optimal result

Integrand size = 13, antiderivative size = 20

$$\int \frac{x^4}{x-x^3} dx = -\frac{x^2}{2} - \frac{1}{2}\log\left(1-x^2\right)$$

output
```
-1/2*x^2-1/2*ln(-x^2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x^4}{x-x^3} dx = -\frac{x^2}{2} - \frac{1}{2}\log\left(-1+x^2\right)$$

input
```
Integrate[x^4/(x - x^3),x]
```

output
```
-1/2*x^2 - Log[-1 + x^2]/2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x-x^3} \, dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{x^3}{1-x^2}dx$$
$$\Big\downarrow \; 243$$
$$\frac{1}{2}\int \frac{x^2}{1-x^2}dx^2$$
$$\Big\downarrow \; 49$$
$$\frac{1}{2}\int \left(\frac{1}{1-x^2}-1\right)dx^2$$
$$\Big\downarrow \; 2009$$
$$\frac{1}{2}\left(-x^2-\log\left(1-x^2\right)\right)$$

input
```
Int[x^4/(x - x^3),x]
```

output
```
(-x^2 - Log[1 - x^2])/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{x^2}{2}-\frac{\ln(x^2-1)}{2}$	15
meijerg	$-\frac{x^2}{2}-\frac{\ln(-x^2+1)}{2}$	17
default	$-\frac{x^2}{2}-\frac{\ln(x-1)}{2}-\frac{\ln(x+1)}{2}$	19
norman	$-\frac{x^2}{2}-\frac{\ln(x-1)}{2}-\frac{\ln(x+1)}{2}$	19
parallelrisch	$-\frac{x^2}{2}-\frac{\ln(x-1)}{2}-\frac{\ln(x+1)}{2}$	19

input
```
int(x^4/(-x^3+x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2-1/2*ln(x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{x^4}{x-x^3}\,dx = -\frac{1}{2}\,x^2 - \frac{1}{2}\,\log\left(x^2-1\right)$$

input
```
integrate(x^4/(-x^3+x),x, algorithm="fricas")
```

output
```
-1/2*x^2 - 1/2*log(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{x^4}{x-x^3}\,dx = -\frac{x^2}{2} - \frac{\log\left(x^2-1\right)}{2}$$

input
```
integrate(x**4/(-x**3+x),x)
```

output
```
-x**2/2 - log(x**2 - 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x^4}{x-x^3}\,dx = -\frac{1}{2}\,x^2 - \frac{1}{2}\,\log\left(x+1\right) - \frac{1}{2}\,\log\left(x-1\right)$$

input
```
integrate(x^4/(-x^3+x),x, algorithm="maxima")
```

output
```
-1/2*x^2 - 1/2*log(x + 1) - 1/2*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{x^4}{x-x^3}\,dx = -\frac{1}{2}\,x^2 - \frac{1}{2}\,\log\left(\left|x^2-1\right|\right)$$

input
```
integrate(x^4/(-x^3+x),x, algorithm="giac")
```

output
```
-1/2*x^2 - 1/2*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{x^4}{x-x^3}\,dx = -\frac{\ln\left(x^2-1\right)}{2} - \frac{x^2}{2}$$

input
```
int(x^4/(x - x^3),x)
```

output
```
- log(x^2 - 1)/2 - x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x^4}{x-x^3}\,dx = -\frac{\log(x-1)}{2} - \frac{\log(x+1)}{2} - \frac{x^2}{2}$$

input
```
int(x^4/(-x^3+x),x)
```

output
```
( - (log(x - 1) + log(x + 1) + x**2))/2
```

3.35 $\int \frac{x^3}{x-x^3} dx$

Optimal result . . . 361
Mathematica [B] (verified) . . . 361
Rubi [A] (verified) . . . 362
Maple [C] (verified) . . . 363
Fricas [B] (verification not implemented) . . . 364
Sympy [B] (verification not implemented) . . . 364
Maxima [B] (verification not implemented) . . . 364
Giac [B] (verification not implemented) . . . 365
Mupad [B] (verification not implemented) . . . 365
Reduce [B] (verification not implemented) . . . 365

Optimal result

Integrand size = 13, antiderivative size = 6

$$\int \frac{x^3}{x-x^3} dx = -x + \operatorname{arctanh}(x)$$

output
```
-x+arctanh(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 22 vs. 2(6) = 12.

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 3.67

$$\int \frac{x^3}{x-x^3} dx = -x - \frac{1}{2}\log(1-x) + \frac{1}{2}\log(1+x)$$

input
```
Integrate[x^3/(x - x^3),x]
```

output
```
-x - Log[1 - x]/2 + Log[1 + x]/2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {9, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x-x^3} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^2}{1-x^2}dx$$

$$\Big\downarrow \; 262$$

$$\int \frac{1}{1-x^2}dx - x$$

$$\Big\downarrow \; 219$$

$$\text{arctanh}(x) - x$$

input
```
Int[x^3/(x - x^3),x]
```

output
```
-x + ArcTanh[x]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.33

method	result	size
meijerg	$\frac{i(2ix-2i\,\text{arctanh}(x))}{2}$	14
default	$-x-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	17
norman	$-x-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	17
risch	$-x-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	17
parallelrisch	$-x-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	17

input
```
int(x^3/(-x^3+x),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*(2*I*x-2*I*arctanh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.67

$$\int \frac{x^3}{x - x^3}\,dx = -x + \frac{1}{2}\,\log\left(x + 1\right) - \frac{1}{2}\,\log\left(x - 1\right)$$

input
```
integrate(x^3/(-x^3+x),x, algorithm="fricas")
```

output
```
-x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(3) = 6.

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.33

$$\int \frac{x^3}{x - x^3}\,dx = -x - \frac{\log\left(x - 1\right)}{2} + \frac{\log\left(x + 1\right)}{2}$$

input
```
integrate(x**3/(-x**3+x),x)
```

output
```
-x - log(x - 1)/2 + log(x + 1)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.67

$$\int \frac{x^3}{x - x^3}\,dx = -x + \frac{1}{2}\,\log\left(x + 1\right) - \frac{1}{2}\,\log\left(x - 1\right)$$

input
```
integrate(x^3/(-x^3+x),x, algorithm="maxima")
```

output
```
-x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(6) = 12$.

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 3.00

$$\int \frac{x^3}{x - x^3} \, dx = -x + \frac{1}{2} \log(|x + 1|) - \frac{1}{2} \log(|x - 1|)$$

input
```
integrate(x^3/(-x^3+x),x, algorithm="giac")
```

output
```
-x + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{x - x^3} \, dx = \operatorname{atanh}(x) - x$$

input
```
int(x^3/(x - x^3),x)
```

output
```
atanh(x) - x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.67

$$\int \frac{x^3}{x - x^3} \, dx = -\frac{\log(x - 1)}{2} + \frac{\log(x + 1)}{2} - x$$

input
```
int(x^3/(-x^3+x),x)
```

output
```
( - log(x - 1) + log(x + 1) - 2*x)/2
```

3.36 $\int \frac{x^2}{x-x^3} \, dx$

Optimal result 367
Mathematica [A] (verified) 367
Rubi [A] (verified) 368
Maple [A] (verified) 369
Fricas [A] (verification not implemented) 369
Sympy [A] (verification not implemented) 370
Maxima [A] (verification not implemented) 370
Giac [A] (verification not implemented) 370
Mupad [B] (verification not implemented) 371
Reduce [B] (verification not implemented) 371

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{x^2}{x-x^3} \, dx = -\frac{1}{2}\log \left(1-x^2\right)$$

output
```
-1/2*ln(-x^2+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{x-x^3} \, dx = -\frac{1}{2}\log \left(1-x^2\right)$$

input
```
Integrate[x^2/(x - x^3),x]
```

output
```
-1/2*Log[1 - x^2]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {9, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x-x^3} \, dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{x}{1-x^2}dx$$
$$\Big\downarrow \; 240$$
$$-\frac{1}{2}\log\left(1-x^2\right)$$

input
```
Int[x^2/(x - x^3),x]
```

output
```
-1/2*Log[1 - x^2]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{\ln(x^2-1)}{2}$	9
meijerg	$-\frac{\ln(-x^2+1)}{2}$	11
default	$-\frac{\ln(x-1)}{2}-\frac{\ln(x+1)}{2}$	14
norman	$-\frac{\ln(x-1)}{2}-\frac{\ln(x+1)}{2}$	14
parallelrisch	$-\frac{\ln(x-1)}{2}-\frac{\ln(x+1)}{2}$	14

input
```
int(x^2/(-x^3+x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{x-x^3}\,dx=-\frac{1}{2}\,\log\left(x^2-1\right)$$

input
```
integrate(x^2/(-x^3+x),x, algorithm="fricas")
```

output
```
-1/2*log(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{x-x^3}\,dx = -\frac{\log{(x^2-1)}}{2}$$

input
```
integrate(x**2/(-x**3+x),x)
```

output
```
-log(x**2 - 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{x-x^3}\,dx = -\frac{1}{2}\,\log\left(x+1\right) - \frac{1}{2}\,\log\left(x-1\right)$$

input
```
integrate(x^2/(-x^3+x),x, algorithm="maxima")
```

output
```
-1/2*log(x + 1) - 1/2*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{x^2}{x-x^3}\,dx = -\frac{1}{2}\,\log\left(|x+1|\right) - \frac{1}{2}\,\log\left(|x-1|\right)$$

input
```
integrate(x^2/(-x^3+x),x, algorithm="giac")
```

output
```
-1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{x-x^3}\,dx=-\frac{\ln\left(x^2-1\right)}{2}$$

input
```
int(x^2/(x - x^3),x)
```

output
```
-log(x^2 - 1)/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{x-x^3}\,dx=-\frac{\log(x-1)}{2}-\frac{\log(x+1)}{2}$$

input
```
int(x^2/(-x^3+x),x)
```

output
```
( - (log(x - 1) + log(x + 1)))/2
```

3.37 $\int \frac{x}{x-x^3} dx$

Optimal result 372
Mathematica [B] (verified) 372
Rubi [A] (verified) 373
Maple [A] (verified) 374
Fricas [B] (verification not implemented) 374
Sympy [B] (verification not implemented) 375
Maxima [B] (verification not implemented) 375
Giac [B] (verification not implemented) 375
Mupad [B] (verification not implemented) 376
Reduce [B] (verification not implemented) 376

Optimal result

Integrand size = 11, antiderivative size = 2

$$\int \frac{x}{x-x^3} dx = \operatorname{arctanh}(x)$$

output
```
arctanh(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 19 vs. 2(2) = 4.

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 9.50

$$\int \frac{x}{x-x^3} dx = -\frac{1}{2}\log(1-x) + \frac{1}{2}\log(1+x)$$

input
```
Integrate[x/(x - x^3),x]
```

output
```
-1/2*Log[1 - x] + Log[1 + x]/2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {9, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x-x^3} \, dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{1}{1-x^2}dx$$
$$\Big\downarrow \; 219$$
$$\text{arctanh}(x)$$

input
```
Int[x/(x - x^3),x]
```

output
```
ArcTanh[x]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\operatorname{arctanh}(x)$	3
meijerg	$\operatorname{arctanh}(x)$	3
norman	$-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	14
risch	$-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	14
parallelrisch	$-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	14

input
```
int(x/(-x^3+x),x,method=_RETURNVERBOSE)
```

output
```
arctanh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. 2(2) = 4.

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 6.50

$$\int \frac{x}{x-x^3}\,dx=\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x-1)$$

input
```
integrate(x/(-x^3+x),x, algorithm="fricas")
```

output
```
1/2*log(x + 1) - 1/2*log(x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(2) = 4$.

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 6.00

$$\int \frac{x}{x - x^3} \, dx = -\frac{\log (x - 1)}{2} + \frac{\log (x + 1)}{2}$$

input
```
integrate(x/(-x**3+x),x)
```

output
```
-log(x - 1)/2 + log(x + 1)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(2) = 4$.

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 6.50

$$\int \frac{x}{x - x^3} \, dx = \frac{1}{2} \log (x + 1) - \frac{1}{2} \log (x - 1)$$

input
```
integrate(x/(-x^3+x),x, algorithm="maxima")
```

output
```
1/2*log(x + 1) - 1/2*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(2) = 4$.

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 7.50

$$\int \frac{x}{x - x^3} \, dx = \frac{1}{2} \log (|x + 1|) - \frac{1}{2} \log (|x - 1|)$$

input
```
integrate(x/(-x^3+x),x, algorithm="giac")
```

output
```
1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{x}{x - x^3} \, dx = \operatorname{atanh}(x)$$

input
```
int(x/(x - x^3),x)
```

output
```
atanh(x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 6.50

$$\int \frac{x}{x - x^3} \, dx = -\frac{\log(x - 1)}{2} + \frac{\log(x + 1)}{2}$$

input
```
int(x/(-x^3+x),x)
```

output
```
( - log(x - 1) + log(x + 1))/2
```

3.38 $\int \frac{1}{x-x^3} \, dx$

Optimal result 377
Mathematica [A] (verified) 377
Rubi [A] (verified) 378
Maple [A] (verified) 379
Fricas [A] (verification not implemented) 380
Sympy [A] (verification not implemented) 380
Maxima [A] (verification not implemented) 380
Giac [A] (verification not implemented) 381
Mupad [B] (verification not implemented) 381
Reduce [B] (verification not implemented) 381

Optimal result

Integrand size = 9, antiderivative size = 15

$$\int \frac{1}{x-x^3} \, dx = \log(x) - \frac{1}{2} \log \left(1-x^2\right)$$

output
```
ln(x)-1/2*ln(-x^2+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{x-x^3} \, dx = \log(x) - \frac{1}{2} \log \left(1-x^2\right)$$

input
```
Integrate[(x - x^3)^(-1),x]
```

output
```
Log[x] - Log[1 - x^2]/2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {2026, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x-x^3} \, dx$$
$$\Big\downarrow \text{ 2026}$$
$$\int \frac{1}{x\left(1-x^2\right)}dx$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2}\int \frac{1}{x^2\left(1-x^2\right)}dx^2$$
$$\Big\downarrow \text{ 47}$$
$$\frac{1}{2}\left(\int \frac{1}{x^2}dx^2+\int \frac{1}{1-x^2}dx^2\right)$$
$$\Big\downarrow \text{ 14}$$
$$\frac{1}{2}\left(\int \frac{1}{1-x^2}dx^2+\log\left(x^2\right)\right)$$
$$\Big\downarrow \text{ 16}$$
$$\frac{1}{2}\left(\log\left(x^2\right)-\log\left(1-x^2\right)\right)$$

input
```
Int[(x - x^3)^(-1),x]
```

output
```
(Log[x^2] - Log[1 - x^2])/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
risch	$\ln\left(x\right)-\frac{\ln\left(x^{2}-1\right)}{2}$	12
default	$\ln\left(x\right)-\frac{\ln\left(x+1\right)}{2}-\frac{\ln\left(x-1\right)}{2}$	16
norman	$\ln\left(x\right)-\frac{\ln\left(x+1\right)}{2}-\frac{\ln\left(x-1\right)}{2}$	16
parallelrisch	$\ln\left(x\right)-\frac{\ln\left(x+1\right)}{2}-\frac{\ln\left(x-1\right)}{2}$	16
meijerg	$-\frac{\ln\left(-x^{2}+1\right)}{2}+\ln\left(x\right)+\frac{i\pi}{2}$	18

input
```
int(1/(-x^3+x),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/2*ln(x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1}{x-x^3}\,dx = -\frac{1}{2}\log\left(x^2-1\right)+\log\left(x\right)$$

input
```
integrate(1/(-x^3+x),x, algorithm="fricas")
```

output
```
-1/2*log(x^2 - 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1}{x-x^3}\,dx = \log\left(x\right)-\frac{\log\left(x^2-1\right)}{2}$$

input
```
integrate(1/(-x**3+x),x)
```

output
```
log(x) - log(x**2 - 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{x-x^3}\,dx = -\frac{1}{2}\log\left(x+1\right)-\frac{1}{2}\log\left(x-1\right)+\log\left(x\right)$$

input
```
integrate(1/(-x^3+x),x, algorithm="maxima")
```

output
```
-1/2*log(x + 1) - 1/2*log(x - 1) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{1}{x-x^3}\,dx = \frac{1}{2}\log\left(x^2\right) - \frac{1}{2}\log\left(\left|x^2-1\right|\right)$$

input
```
integrate(1/(-x^3+x),x, algorithm="giac")
```

output
```
1/2*log(x^2) - 1/2*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1}{x-x^3}\,dx = \ln(x) - \frac{\ln(x^2-1)}{2}$$

input
```
int(1/(x - x^3),x)
```

output
```
log(x) - log(x^2 - 1)/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{x-x^3}\,dx = -\frac{\log(x-1)}{2} - \frac{\log(x+1)}{2} + \log(x)$$

input
```
int(1/(-x^3+x),x)
```

output
```
( - log(x - 1) - log(x + 1) + 2*log(x))/2
```

3.39 $\int \frac{1}{x(x-x^3)}\,dx$

Optimal result 382
Mathematica [B] (verified) 382
Rubi [A] (verified) 383
Maple [C] (verified) 384
Fricas [B] (verification not implemented) 385
Sympy [B] (verification not implemented) 385
Maxima [B] (verification not implemented) 385
Giac [B] (verification not implemented) 386
Mupad [B] (verification not implemented) 386
Reduce [B] (verification not implemented) 386

Optimal result

Integrand size = 13, antiderivative size = 8

$$\int \frac{1}{x(x-x^3)}\,dx = -\frac{1}{x} + \operatorname{arctanh}(x)$$

output
```
-1/x+arctanh(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 24 vs. 2(8) = 16.

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 3.00

$$\int \frac{1}{x(x-x^3)}\,dx = -\frac{1}{x} - \frac{1}{2}\log(1-x) + \frac{1}{2}\log(1+x)$$

input
```
Integrate[1/(x*(x - x^3)),x]
```

output
```
-x^(-1) - Log[1 - x]/2 + Log[1 + x]/2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {9, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(x-x^3)}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x^2(1-x^2)}dx$$

$$\Big\downarrow \text{ 264}$$

$$\int \frac{1}{1-x^2}dx-\frac{1}{x}$$

$$\Big\downarrow \text{ 219}$$

$$\text{arctanh}(x)-\frac{1}{x}$$

input
```
Int[1/(x*(x - x^3)),x]
```

output
```
-x^(-1) + ArcTanh[x]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

method	result	size
meijerg	$\frac{i\left(\frac{2i}{x}-2i \operatorname{arctanh}\left(x\right)\right)}{2}$	16
default	$\frac{\ln \left(x+1\right)}{2}-\frac{\ln \left(x-1\right)}{2}-\frac{1}{x}$	19
norman	$\frac{\ln \left(x+1\right)}{2}-\frac{\ln \left(x-1\right)}{2}-\frac{1}{x}$	19
risch	$\frac{\ln \left(x+1\right)}{2}-\frac{\ln \left(x-1\right)}{2}-\frac{1}{x}$	19
parallelrisch	$-\frac{\ln \left(x-1\right) x-\ln \left(x+1\right) x+2}{2x}$	21

input
```
int(1/x/(-x^3+x),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*(2*I/x-2*I*arctanh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(8) = 16$.

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.50

$$\int \frac{1}{x(x-x^3)}\,dx = \frac{x\log(x+1) - x\log(x-1) - 2}{2\,x}$$

input
```
integrate(1/x/(-x^3+x),x, algorithm="fricas")
```

output
```
1/2*(x*log(x + 1) - x*log(x - 1) - 2)/x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{1}{x(x-x^3)}\,dx = -\frac{\log(x-1)}{2} + \frac{\log(x+1)}{2} - \frac{1}{x}$$

input
```
integrate(1/x/(-x**3+x),x)
```

output
```
-log(x - 1)/2 + log(x + 1)/2 - 1/x
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(8) = 16$.

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{1}{x(x-x^3)}\,dx = -\frac{1}{x} + \frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

input
```
integrate(1/x/(-x^3+x),x, algorithm="maxima")
```

output
```
-1/x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(8) = 16.

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.50

$$\int \frac{1}{x(x-x^3)}\,dx = -\frac{1}{x} + \frac{1}{2}\,\log\left(|x+1|\right) - \frac{1}{2}\,\log\left(|x-1|\right)$$

input
```
integrate(1/x/(-x^3+x),x, algorithm="giac")
```

output
```
-1/x + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(x-x^3)}\,dx = \operatorname{atanh}(x) - \frac{1}{x}$$

input
```
int(1/(x*(x - x^3)),x)
```

output
```
atanh(x) - 1/x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.50

$$\int \frac{1}{x(x-x^3)}\,dx = \frac{-\log(x-1)\,x + \log(x+1)\,x - 2}{2x}$$

input
```
int(1/x/(-x^3+x),x)
```

output
```
( - log(x - 1)*x + log(x + 1)*x - 2)/(2*x)
```

3.40 $\int \frac{1}{x^2(x-x^3)}\,dx$

Optimal result 388
Mathematica [A] (verified) 388
Rubi [A] (verified) 389
Maple [A] (verified) 390
Fricas [A] (verification not implemented) 391
Sympy [A] (verification not implemented) 391
Maxima [A] (verification not implemented) 391
Giac [A] (verification not implemented) 392
Mupad [B] (verification not implemented) 392
Reduce [B] (verification not implemented) 392

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1}{x^2(x-x^3)}\,dx = -\frac{1}{2x^2} + \log(x) - \frac{1}{2}\log\left(1-x^2\right)$$

output
```
-1/2/x^2+ln(x)-1/2*ln(-x^2+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(x-x^3)}\,dx = -\frac{1}{2x^2} + \log(x) - \frac{1}{2}\log\left(1-x^2\right)$$

input
```
Integrate[1/(x^2*(x - x^3)),x]
```

output
```
-1/2*1/x^2 + Log[x] - Log[1 - x^2]/2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {9, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (x-x^3)} \, dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{1}{x^3 (1-x^2)}dx$$
$$\Big\downarrow \; 243$$
$$\frac{1}{2} \int \frac{1}{x^4 (1-x^2)}dx^2$$
$$\Big\downarrow \; 54$$
$$\frac{1}{2} \int \left(\frac{1}{x^2}+\frac{1}{x^4}+\frac{1}{1-x^2}\right)dx^2$$
$$\Big\downarrow \; 2009$$
$$\frac{1}{2} \left(-\frac{1}{x^2}+\log \left(x^2\right)-\log \left(1-x^2\right)\right)$$

input
```
Int[1/(x^2*(x - x^3)),x]
```

output
```
(-x^(-2) + Log[x^2] - Log[1 - x^2])/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{1}{2x^2}+\ln(x)-\frac{\ln(x^2-1)}{2}$	17
default	$-\frac{1}{2x^2}+\ln(x)-\frac{\ln(x+1)}{2}-\frac{\ln(x-1)}{2}$	21
norman	$-\frac{1}{2x^2}+\ln(x)-\frac{\ln(x+1)}{2}-\frac{\ln(x-1)}{2}$	21
meijerg	$-\frac{\ln(-x^2+1)}{2}+\ln(x)+\frac{i\pi}{2}-\frac{1}{2x^2}$	23
parallelrisch	$\frac{2\ln(x)x^2-\ln(x-1)x^2-\ln(x+1)x^2-1}{2x^2}$	33

input
```
int(1/x^2/(-x^3+x),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2+ln(x)-1/2*ln(x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (x - x^3)} \, dx = -\frac{x^2 \log(x^2 - 1) - 2\, x^2 \log(x) + 1}{2\, x^2}$$

input
```
integrate(1/x^2/(-x^3+x),x, algorithm="fricas")
```

output
```
-1/2*(x^2*log(x^2 - 1) - 2*x^2*log(x) + 1)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^2 (x - x^3)} \, dx = \log(x) - \frac{\log(x^2 - 1)}{2} - \frac{1}{2x^2}$$

input
```
integrate(1/x**2/(-x**3+x),x)
```

output
```
log(x) - log(x**2 - 1)/2 - 1/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2 (x - x^3)} \, dx = -\frac{1}{2\, x^2} - \frac{1}{2} \log(x + 1) - \frac{1}{2} \log(x - 1) + \log(x)$$

input
```
integrate(1/x^2/(-x^3+x),x, algorithm="maxima")
```

output
```
-1/2/x^2 - 1/2*log(x + 1) - 1/2*log(x - 1) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^2 (x - x^3)} \, dx = -\frac{x^2 + 1}{2 x^2} + \frac{1}{2} \log(x^2) - \frac{1}{2} \log(|x^2 - 1|)$$

input
```
integrate(1/x^2/(-x^3+x),x, algorithm="giac")
```

output
```
-1/2*(x^2 + 1)/x^2 + 1/2*log(x^2) - 1/2*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^2 (x - x^3)} \, dx = \ln(x) - \frac{\ln(x^2 - 1)}{2} - \frac{1}{2 x^2}$$

input
```
int(1/(x^2*(x - x^3)),x)
```

output
```
log(x) - log(x^2 - 1)/2 - 1/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^2 (x - x^3)} \, dx = \frac{-\log(x-1) x^2 - \log(x+1) x^2 + 2\log(x) x^2 - 1}{2x^2}$$

input
```
int(1/x^2/(-x^3+x),x)
```

output
```
( - log(x - 1)*x**2 - log(x + 1)*x**2 + 2*log(x)*x**2 - 1)/(2*x**2)
```

3.41 $\int \frac{1}{x^3(x-x^3)}\,dx$

Optimal result . . . 393
Mathematica [B] (verified) . . . 393
Rubi [A] (verified) . . . 394
Maple [C] (verified) . . . 395
Fricas [B] (verification not implemented) . . . 396
Sympy [A] (verification not implemented) . . . 396
Maxima [A] (verification not implemented) . . . 396
Giac [B] (verification not implemented) . . . 397
Mupad [B] (verification not implemented) . . . 397
Reduce [B] (verification not implemented) . . . 397

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{1}{x^3(x-x^3)}\,dx = -\frac{1}{3x^3}-\frac{1}{x}+\operatorname{arctanh}(x)$$

output
```
-1/3/x^3-1/x+arctanh(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 31 vs. 2(15) = 30.

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \frac{1}{x^3(x-x^3)}\,dx = -\frac{1}{3x^3}-\frac{1}{x}-\frac{1}{2}\log(1-x)+\frac{1}{2}\log(1+x)$$

input
```
Integrate[1/(x^3*(x - x^3)),x]
```

output
```
-1/3*1/x^3 - x^(-1) - Log[1 - x]/2 + Log[1 + x]/2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {9, 264, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (x-x^3)} \, dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{1}{x^4 (1-x^2)}dx$$
$$\Big\downarrow \; 264$$
$$\int \frac{1}{x^2 (1-x^2)}dx-\frac{1}{3 x^3}$$
$$\Big\downarrow \; 264$$
$$\int \frac{1}{1-x^2}dx-\frac{1}{3 x^3}-\frac{1}{x}$$
$$\Big\downarrow \; 219$$
$$\text{arctanh}(x)-\frac{1}{3 x^3}-\frac{1}{x}$$

input
```
Int[1/(x^3*(x - x^3)),x]
```

output
```
-1/3*1/x^3 - x^(-1) + ArcTanh[x]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

method	result	size
meijerg	$-\frac{i\left(-\frac{2i}{x}-\frac{2i}{3x^3}+2i\,\mathrm{arctanh}(x)\right)}{2}$	22
default	$-\frac{1}{3x^3}-\frac{1}{x}+\frac{\ln(x+1)}{2}-\frac{\ln(x-1)}{2}$	24
norman	$\frac{-\frac{1}{3}-x^2}{x^3}-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	25
risch	$\frac{-\frac{1}{3}-x^2}{x^3}-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	25
parallelrisch	$-\frac{3\ln(x-1)x^3-3\ln(x+1)x^3+2+6x^2}{6x^3}$	31

input
```
int(1/x^3/(-x^3+x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*(-2*I/x-2/3*I/x^3+2*I*arctanh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(13) = 26$.

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.00

$$\int \frac{1}{x^3 (x - x^3)} \, dx = \frac{3 \, x^3 \log \left(x + 1\right) - 3 \, x^3 \log \left(x - 1\right) - 6 \, x^2 - 2}{6 \, x^3}$$

input
```
integrate(1/x^3/(-x^3+x),x, algorithm="fricas")
```

output
```
1/6*(3*x^3*log(x + 1) - 3*x^3*log(x - 1) - 6*x^2 - 2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \frac{1}{x^3 (x - x^3)} \, dx = -\frac{\log \left(x - 1\right)}{2} + \frac{\log \left(x + 1\right)}{2} - \frac{3x^2 + 1}{3x^3}$$

input
```
integrate(1/x**3/(-x**3+x),x)
```

output
```
-log(x - 1)/2 + log(x + 1)/2 - (3*x**2 + 1)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \frac{1}{x^3 (x - x^3)} \, dx = -\frac{3 \, x^2 + 1}{3 \, x^3} + \frac{1}{2} \, \log \left(x + 1\right) - \frac{1}{2} \, \log \left(x - 1\right)$$

input
```
integrate(1/x^3/(-x^3+x),x, algorithm="maxima")
```

output
```
-1/3*(3*x^2 + 1)/x^3 + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \frac{1}{x^3(x-x^3)}\,dx = -\frac{3\,x^2+1}{3\,x^3} + \frac{1}{2}\,\log\left(|x+1|\right) - \frac{1}{2}\,\log\left(|x-1|\right)$$

input
```
integrate(1/x^3/(-x^3+x),x, algorithm="giac")
```

output
```
-1/3*(3*x^2 + 1)/x^3 + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3(x-x^3)}\,dx = \mathrm{atanh}(x) - \frac{x^2+\frac{1}{3}}{x^3}$$

input
```
int(1/(x^3*(x - x^3)),x)
```

output
```
atanh(x) - (x^2 + 1/3)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.00

$$\int \frac{1}{x^3(x-x^3)}\,dx = \frac{-3\log(x-1)\,x^3 + 3\log(x+1)\,x^3 - 6x^2 - 2}{6x^3}$$

input
```
int(1/x^3/(-x^3+x),x)
```

output
```
( - 3*log(x - 1)*x**3 + 3*log(x + 1)*x**3 - 6*x**2 - 2)/(6*x**3)
```

3.42 $\int \frac{1}{x^4(x-x^3)}\,dx$

Optimal result 398
Mathematica [A] (verified) 398
Rubi [A] (verified) 399
Maple [A] (verified) 400
Fricas [A] (verification not implemented) 401
Sympy [A] (verification not implemented) 401
Maxima [A] (verification not implemented) 401
Giac [A] (verification not implemented) 402
Mupad [B] (verification not implemented) 402
Reduce [B] (verification not implemented) 402

Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \frac{1}{x^4(x-x^3)}\,dx=-\frac{1}{4x^4}-\frac{1}{2x^2}+\log(x)-\frac{1}{2}\log\left(1-x^2\right)$$

output
```
-1/4/x^4-1/2/x^2+ln(x)-1/2*ln(-x^2+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4(x-x^3)}\,dx=-\frac{1}{4x^4}-\frac{1}{2x^2}+\log(x)-\frac{1}{2}\log\left(1-x^2\right)$$

input
```
Integrate[1/(x^4*(x - x^3)),x]
```

output
```
-1/4*1/x^4 - 1/(2*x^2) + Log[x] - Log[1 - x^2]/2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {9, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (x-x^3)} \, dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{1}{x^5 (1-x^2)}dx$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2} \int \frac{1}{x^6 (1-x^2)}dx^2$$
$$\Big\downarrow \text{ 54}$$
$$\frac{1}{2} \int \left(\frac{1}{x^2}+\frac{1}{x^4}+\frac{1}{x^6}+\frac{1}{1-x^2}\right)dx^2$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2} \left(-\frac{1}{2 x^4}-\frac{1}{x^2}+\log \left(x^2\right)-\log \left(1-x^2\right)\right)$$

input
```
Int[1/(x^4*(x - x^3)),x]
```

output
```
(-1/2*1/x^4 - x^(-2) + Log[x^2] - Log[1 - x^2])/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{-\frac{1}{4}-\frac{x^2}{2}}{x^4}+\ln\left(x\right)-\frac{\ln\left(x^2-1\right)}{2}$	23
default	$-\frac{1}{4x^4}-\frac{1}{2x^2}+\ln\left(x\right)-\frac{\ln(x+1)}{2}-\frac{\ln(x-1)}{2}$	26
norman	$\frac{-\frac{1}{4}-\frac{x^2}{2}}{x^4}+\ln\left(x\right)-\frac{\ln(x+1)}{2}-\frac{\ln(x-1)}{2}$	27
meijerg	$-\frac{\ln(-x^2+1)}{2}+\ln\left(x\right)+\frac{i\pi}{2}-\frac{1}{4x^4}-\frac{1}{2x^2}$	28
parallelrisch	$\frac{4\ln(x)x^4-2\ln(x-1)x^4-2\ln(x+1)x^4-1-2x^2}{4x^4}$	38

input
```
int(1/x^4/(-x^3+x),x,method=_RETURNVERBOSE)
```

output
```
(-1/4-1/2*x^2)/x^4+ln(x)-1/2*ln(x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^4 (x - x^3)} dx = -\frac{2 x^4 \log(x^2 - 1) - 4 x^4 \log(x) + 2 x^2 + 1}{4 x^4}$$

input
```
integrate(1/x^4/(-x^3+x),x, algorithm="fricas")
```

output
```
-1/4*(2*x^4*log(x^2 - 1) - 4*x^4*log(x) + 2*x^2 + 1)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^4 (x - x^3)} dx = \log(x) - \frac{\log(x^2 - 1)}{2} - \frac{2x^2 + 1}{4x^4}$$

input
```
integrate(1/x**4/(-x**3+x),x)
```

output
```
log(x) - log(x**2 - 1)/2 - (2*x**2 + 1)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^4 (x - x^3)} dx = -\frac{2 x^2 + 1}{4 x^4} - \frac{1}{2} \log(x + 1) - \frac{1}{2} \log(x - 1) + \log(x)$$

input
```
integrate(1/x^4/(-x^3+x),x, algorithm="maxima")
```

output
```
-1/4*(2*x^2 + 1)/x^4 - 1/2*log(x + 1) - 1/2*log(x - 1) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^4(x-x^3)}\,dx = -\frac{3\,x^4+2\,x^2+1}{4\,x^4}+\frac{1}{2}\,\log\left(x^2\right)-\frac{1}{2}\,\log\left(\left|x^2-1\right|\right)$$

input
```
integrate(1/x^4/(-x^3+x),x, algorithm="giac")
```

output
```
-1/4*(3*x^4 + 2*x^2 + 1)/x^4 + 1/2*log(x^2) - 1/2*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^4(x-x^3)}\,dx = \ln(x)-\frac{\ln(x^2-1)}{2}-\frac{\frac{x^2}{2}+\frac{1}{4}}{x^4}$$

input
```
int(1/(x^4*(x - x^3)),x)
```

output
```
log(x) - log(x^2 - 1)/2 - (x^2/2 + 1/4)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^4(x-x^3)}\,dx = \frac{-2\log(x-1)\,x^4-2\log(x+1)\,x^4+4\log(x)\,x^4-2x^2-1}{4x^4}$$

input
```
int(1/x^4/(-x^3+x),x)
```

output
```
( - 2*log(x - 1)*x**4 - 2*log(x + 1)*x**4 + 4*log(x)*x**4 - 2*x**2 - 1)/(4
*x**4)
```

3.43 $\int \frac{1}{x+bx^3} dx$

Optimal result 403
Mathematica [A] (verified) 403
Rubi [A] (verified) 404
Maple [A] (verified) 405
Fricas [A] (verification not implemented) 406
Sympy [A] (verification not implemented) 406
Maxima [A] (verification not implemented) 406
Giac [A] (verification not implemented) 407
Mupad [B] (verification not implemented) 407
Reduce [B] (verification not implemented) 407

Optimal result

Integrand size = 9, antiderivative size = 15

$$\int \frac{1}{x+bx^3} dx = \log(x) - \frac{1}{2}\log\left(1+bx^2\right)$$

output
```
ln(x)-1/2*ln(b*x^2+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{x+bx^3} dx = \log(x) - \frac{1}{2}\log\left(1+bx^2\right)$$

input
```
Integrate[(x + b*x^3)^(-1),x]
```

output
```
Log[x] - Log[1 + b*x^2]/2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {2026, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{bx^3+x}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{1}{x(bx^2+1)}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2(bx^2+1)}dx^2$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(\int \frac{1}{x^2}dx^2-b\int \frac{1}{bx^2+1}dx^2\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(\log \left(x^2\right)-b\int \frac{1}{bx^2+1}dx^2\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\log \left(x^2\right)-\log \left(bx^2+1\right)\right)$$

input
```
Int[(x + b*x^3)^(-1),x]
```

output
```
(Log[x^2] - Log[1 + b*x^2])/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\ln\left(x\right)-\frac{\ln\left(b\,x^{2}+1\right)}{2}$	14
norman	$\ln\left(x\right)-\frac{\ln\left(b\,x^{2}+1\right)}{2}$	14
risch	$\ln\left(x\right)-\frac{\ln\left(b\,x^{2}+1\right)}{2}$	14
parallelrisch	$\ln\left(x\right)-\frac{\ln\left(b\,x^{2}+1\right)}{2}$	14
meijerg	$-\frac{\ln\left(b\,x^{2}+1\right)}{2}+\ln\left(x\right)+\frac{\ln\left(b\right)}{2}$	18

input
```
int(1/(b*x^3+x),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/2*ln(b*x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{x+bx^3}\,dx=-\frac{1}{2}\log\left(bx^2+1\right)+\log\left(x\right)$$

input
```
integrate(1/(b*x^3+x),x, algorithm="fricas")
```

output
```
-1/2*log(b*x^2 + 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{1}{x+bx^3}\,dx=\log\left(x\right)-\frac{\log\left(x^2+\frac{1}{b}\right)}{2}$$

input
```
integrate(1/(b*x**3+x),x)
```

output
```
log(x) - log(x**2 + 1/b)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{x+bx^3}\,dx=-\frac{1}{2}\log\left(bx^2+1\right)+\log\left(x\right)$$

input
```
integrate(1/(b*x^3+x),x, algorithm="maxima")
```

output
```
-1/2*log(b*x^2 + 1) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{1}{x+bx^3}\,dx = \frac{1}{2}\log\left(x^2\right) - \frac{1}{2}\log\left(\left|bx^2+1\right|\right)$$

input
```
integrate(1/(b*x^3+x),x, algorithm="giac")
```

output
```
1/2*log(x^2) - 1/2*log(abs(b*x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{1}{x+bx^3}\,dx = \ln(x) - \frac{\ln\left(\frac{3\,b\,x^2}{2}+\frac{3}{2}\right)}{2}$$

input
```
int(1/(x + b*x^3),x)
```

output
```
log(x) - log((3*b*x^2)/2 + 3/2)/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{x+bx^3}\,dx = -\frac{\log(b\,x^2+1)}{2} + \log(x)$$

input
```
int(1/(b*x^3+x),x)
```

output
```
( - log(b*x**2 + 1) + 2*log(x))/2
```

3.44 $\int \frac{1}{-x+bx^3} \, dx$

Optimal result 408
Mathematica [A] (verified) 408
Rubi [A] (verified) 409
Maple [A] (verified) 410
Fricas [A] (verification not implemented) 411
Sympy [A] (verification not implemented) 411
Maxima [A] (verification not implemented) 412
Giac [A] (verification not implemented) 412
Mupad [B] (verification not implemented) 412
Reduce [B] (verification not implemented) 413

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{1}{-x+bx^3} \, dx = -\log(x) + \frac{1}{2}\log\left(1-bx^2\right)$$

output
```
-ln(x)+1/2*ln(-b*x^2+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{-x+bx^3} \, dx = -\log(x) + \frac{1}{2}\log\left(1-bx^2\right)$$

input
```
Integrate[(-x + b*x^3)^(-1),x]
```

output
```
-Log[x] + Log[1 - b*x^2]/2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {2026, 243, 25, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{bx^3 - x} \, dx$$
$$\Big\downarrow \text{2026}$$
$$\int \frac{1}{x (bx^2 - 1)} dx$$
$$\Big\downarrow \text{243}$$
$$\frac{1}{2} \int -\frac{1}{x^2 (1 - bx^2)} dx^2$$
$$\Big\downarrow \text{25}$$
$$-\frac{1}{2} \int \frac{1}{x^2 (1 - bx^2)} dx^2$$
$$\Big\downarrow \text{47}$$
$$\frac{1}{2} \left(-b \int \frac{1}{1 - bx^2} dx^2 - \int \frac{1}{x^2} dx^2 \right)$$
$$\Big\downarrow \text{14}$$
$$\frac{1}{2} \left(-b \int \frac{1}{1 - bx^2} dx^2 - \log (x^2) \right)$$
$$\Big\downarrow \text{16}$$
$$\frac{1}{2} (\log (1 - bx^2) - \log (x^2))$$

input
```
Int[(-x + b*x^3)^(-1),x]
```

output
```
(-Log[x^2] + Log[1 - b*x^2])/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\ln\left(b\,x^{2}-1\right)}{2}-\ln\left(x\right)$	16
norman	$\frac{\ln\left(b\,x^{2}-1\right)}{2}-\ln\left(x\right)$	16
parallelrisch	$\frac{\ln\left(b\,x^{2}-1\right)}{2}-\ln\left(x\right)$	16
risch	$-\ln\left(x\right)+\frac{\ln\left(-b\,x^{2}+1\right)}{2}$	17
meijerg	$\frac{\ln\left(-b\,x^{2}+1\right)}{2}-\ln\left(x\right)-\frac{\ln\left(-b\right)}{2}$	23

input
```
int(1/(b*x^3-x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(b*x^2-1)-ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{-x+bx^3}\,dx = \frac{1}{2}\log\left(bx^2-1\right) - \log(x)$$

input
```
integrate(1/(b*x^3-x),x, algorithm="fricas")
```

output
```
1/2*log(b*x^2 - 1) - log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{1}{-x+bx^3}\,dx = -\log(x) + \frac{\log\left(x^2-\frac{1}{b}\right)}{2}$$

input
```
integrate(1/(b*x**3-x),x)
```

output
```
-log(x) + log(x**2 - 1/b)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{-x+bx^3}\,dx = \frac{1}{2}\log\left(bx^2-1\right) - \log\left(x\right)$$

input
```
integrate(1/(b*x^3-x),x, algorithm="maxima")
```

output
```
1/2*log(b*x^2 - 1) - log(x)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{-x+bx^3}\,dx = -\frac{1}{2}\log\left(x^2\right) + \frac{1}{2}\log\left(\left|bx^2-1\right|\right)$$

input
```
integrate(1/(b*x^3-x),x, algorithm="giac")
```

output
```
-1/2*log(x^2) + 1/2*log(abs(b*x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{-x+bx^3}\,dx = \frac{\ln\left(\frac{3}{2}-\frac{3\,b\,x^2}{2}\right)}{2} - \ln\left(x\right)$$

input
```
int(-1/(x - b*x^3),x)
```

output
```
log(3/2 - (3*b*x^2)/2)/2 - log(x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{1}{-x+bx^3}\,dx = \frac{\log\left(-\sqrt{b}+bx\right)}{2} + \frac{\log\left(\sqrt{b}+bx\right)}{2} - \log(x)$$

input
```
int(1/(b*x^3-x),x)
```

output
```
(log( - sqrt(b) + b*x) + log(sqrt(b) + b*x) - 2*log(x))/2
```

3.45 $\int x^3\sqrt{ax+bx^3}\,dx$

Optimal result 414
Mathematica [C] (verified) 415
Rubi [A] (verified) 415
Maple [A] (verified) 418
Fricas [A] (verification not implemented) 418
Sympy [F] 419
Maxima [F] 419
Giac [F] 419
Mupad [F(-1)] 420
Reduce [F] 420

Optimal result

Integrand size = 17, antiderivative size = 163

$$\int x^3\sqrt{ax+bx^3}\,dx = -\frac{20a^2\sqrt{ax+bx^3}}{231b^2}+\frac{4ax^2\sqrt{ax+bx^3}}{77b}+\frac{2}{11}x^4\sqrt{ax+bx^3} + \frac{10a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{ax+bx^3}}$$

output
```
-20/231*a^2*(b*x^3+a*x)^(1/2)/b^2+4/77*a*x^2*(b*x^3+a*x)^(1/2)/b+2/11*x^4*
(b*x^3+a*x)^(1/2)+10/231*a^(11/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(
a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x^(1/2)/a^(1/
4)),1/2*2^(1/2))/b^(9/4)/(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.58

$$\int x^3\sqrt{ax+bx^3}\,dx = \frac{2\sqrt{x(a+bx^2)}\left(\sqrt{1+\frac{bx^2}{a}}(-5a^2+2abx^2+7b^2x^4)+5a^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{77b^2\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[x^3*Sqrt[a*x + b*x^3],x]
```

output
```
(2*Sqrt[x*(a + b*x^2)]*(Sqrt[1 + (b*x^2)/a]*(-5*a^2 + 2*a*b*x^2 + 7*b^2*x^
4) + 5*a^2*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)]))/(77*b^2*Sqrt[
1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1927, 1930, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{ax+bx^3}\,dx$$

$$\Big\downarrow \text{1927}$$

$$\frac{2}{11}a\int \frac{x^4}{\sqrt{bx^3+ax}}dx+\frac{2}{11}x^4\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{1930}$$

$$\frac{2}{11}a\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\int \frac{x^2}{\sqrt{bx^3+ax}}dx}{7b}\right)+\frac{2}{11}x^4\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{1930}$$

$$\frac{2}{11}a\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\int\frac{1}{\sqrt{bx^3+ax}}dx}{3b}\right)}{7b}\right)+\frac{2}{11}x^4\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{1917}$$

$$\frac{2}{11}a\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\sqrt{x}\sqrt{a+bx^2}\int\frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3b\sqrt{ax+bx^3}}\right)}{7b}\right)+\frac{2}{11}x^4\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{11}a\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{2a\sqrt{x}\sqrt{a+bx^2}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3b\sqrt{ax+bx^3}}\right)}{7b}\right)+\frac{2}{11}x^4\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{11}a\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}\right)}{7b}\right)+\frac{2}{11}x^4\sqrt{ax+bx^3}$$

input
```
Int[x^3*Sqrt[a*x + b*x^3],x]
```

output
```
(2*x^4*Sqrt[a*x + b*x^3])/11 + (2*a*((2*x^2*Sqrt[a*x + b*x^3])/(7*b) - (5*
a*((2*Sqrt[a*x + b*x^3])/(3*b) - (a^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sq
rt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x
])/a^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[a*x + b*x^3])))/(7*b)))/11
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{2(-21b^2x^4-6ab\,x^2+10a^2)x(b\,x^2+a)}{231b^2\sqrt{x(b\,x^2+a)}}+\frac{10a^3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\right)}{231b^3\sqrt{b\,x^3+ax}}$
default	$\frac{2x^4\sqrt{b\,x^3+ax}}{11}+\frac{4a\,x^2\sqrt{b\,x^3+ax}}{77b}-\frac{20a^2\sqrt{b\,x^3+ax}}{231b^2}+\frac{10a^3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\right)}{231b^3\sqrt{b\,x^3+ax}}$
elliptic	$\frac{2x^4\sqrt{b\,x^3+ax}}{11}+\frac{4a\,x^2\sqrt{b\,x^3+ax}}{77b}-\frac{20a^2\sqrt{b\,x^3+ax}}{231b^2}+\frac{10a^3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\right)}{231b^3\sqrt{b\,x^3+ax}}$

input
```
int(x^3*(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/231*(-21*b^2*x^4-6*a*b*x^2+10*a^2)/b^2*x*(b*x^2+a)/(x*(b*x^2+a))^(1/2)+
10/231*a^3/b^3*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*
(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^
3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.36

$$\int x^3\sqrt{ax+bx^3}\,dx = \frac{2\left(10\,a^3\sqrt{b}\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+(21\,b^3x^4+6\,ab^2x^2-10\,a^2b)\sqrt{bx^3+ax}\right)}{231\,b^3}$$

input
```
integrate(x^3*(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
2/231*(10*a^3*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + (21*b^3*x^4 + 6*
a*b^2*x^2 - 10*a^2*b)*sqrt(b*x^3 + a*x))/b^3
```

Sympy [F]

$$\int x^3\sqrt{ax+bx^3}\,dx = \int x^3\sqrt{x\,(a+bx^2)}\,dx$$

input
```
integrate(x**3*(b*x**3+a*x)**(1/2),x)
```

output
```
Integral(x**3*sqrt(x*(a + b*x**2)), x)
```

Maxima [F]

$$\int x^3\sqrt{ax+bx^3}\,dx = \int \sqrt{bx^3+ax}x^3\,dx$$

input
```
integrate(x^3*(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x)*x^3, x)
```

Giac [F]

$$\int x^3\sqrt{ax+bx^3}\,dx = \int \sqrt{bx^3+ax}x^3\,dx$$

input
```
integrate(x^3*(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a*x)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{ax+bx^3}\,dx=\int x^3\,\sqrt{b\,x^3+a\,x}\,dx$$

input
```
int(x^3*(a*x + b*x^3)^(1/2),x)
```

output
```
int(x^3*(a*x + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int x^3\sqrt{ax+bx^3}\,dx$$
$$=\frac{-\frac{20\sqrt{x}\,\sqrt{b\,x^2+a}\,a^2}{231}+\frac{4\sqrt{x}\,\sqrt{b\,x^2+a}\,ab\,x^2}{77}+\frac{2\sqrt{x}\,\sqrt{b\,x^2+a}\,b^2x^4}{11}+\frac{10\left(\int\frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^3+ax}dx\right)a^3}{231}}{b^2}$$

input
```
int(x^3*(b*x^3+a*x)^(1/2),x)
```

output
```
(2*( - 10*sqrt(x)*sqrt(a + b*x**2)*a**2 + 6*sqrt(x)*sqrt(a + b*x**2)*a*b*x
**2 + 21*sqrt(x)*sqrt(a + b*x**2)*b**2*x**4 + 5*int((sqrt(x)*sqrt(a + b*x*
*2))/(a*x + b*x**3),x)*a**3))/(231*b**2)
```

3.46 $\int x^2\sqrt{ax+bx^3}\,dx$

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Optimal result

Integrand size = 17, antiderivative size = 281

$$\begin{aligned}\int x^2\sqrt{ax+bx^3}\,dx &= -\frac{4a^2x(a+bx^2)}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{4ax\sqrt{ax+bx^3}}{45b}+\frac{2}{9}x^3\sqrt{ax+bx^3}\\ &+\frac{4a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{ax+bx^3}}\\ &-\frac{2a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15b^{7/4}\sqrt{ax+bx^3}}\end{aligned}$$

output
```
-4/15*a^2*x*(b*x^2+a)/b^(3/2)/(a^(1/2)+b^(1/2)*x)/(b*x^3+a*x)^(1/2)+4/45*a
*x*(b*x^3+a*x)^(1/2)/b+2/9*x^3*(b*x^3+a*x)^(1/2)+4/15*a^(9/4)*x^(1/2)*(a^(
1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*ar
ctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))/b^(7/4)/(b*x^3+a*x)^(1/2)-2/15
*a^(9/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/
2)*InverseJacobiAM(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)),1/2*2^(1/2))/b^(7/4)/
(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.28

$$\int x^2\sqrt{ax+bx^3}\,dx = \frac{2x\sqrt{x(a+bx^2)}\left((a+bx^2)\sqrt{1+\frac{bx^2}{a}}-a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{9b\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[x^2*Sqrt[a*x + b*x^3],x]
```

output
```
(2*x*Sqrt[x*(a + b*x^2)]*((a + b*x^2)*Sqrt[1 + (b*x^2)/a] - a*Hypergeometr
ic2F1[-1/2, 3/4, 7/4, -((b*x^2)/a)]))/(9*b*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1927, 1930, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{ax+bx^3}\,dx$$

$$\Big\downarrow \text{1927}$$

$$\frac{2}{9}a\int \frac{x^3}{\sqrt{bx^3+ax}}dx+\frac{2}{9}x^3\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{1930}$$

$$\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{3a\int \frac{x}{\sqrt{bx^3+ax}}dx}{5b}\right)+\frac{2}{9}x^3\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{1938}$$

$$\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{3a\sqrt{x}\sqrt{a+bx^2}\int\frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{5b\sqrt{ax+bx^3}}\right)+\frac{2}{9}x^3\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\int\frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{5b\sqrt{ax+bx^3}}\right)+\frac{2}{9}x^3\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5b\sqrt{ax+bx^3}}\right)+\frac{2}{9}x^3\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5b\sqrt{ax+bx^3}}\right)+\frac{2}{9}x^3\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5b\sqrt{ax+bx^3}}\right)+\frac{2}{9}x^3\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{ 1510}$$

$$\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+\dots}{\left(\sqrt{a}+\dots\right.}}}{\sqrt[4]{\dots}}\dots\right.}{5b\sqrt{ax+bx^3}}\right.$$

$$\frac{2}{9}x^3\sqrt{ax+bx^3}$$

input
```
Int[x^2*Sqrt[a*x + b*x^3],x]
```

output
```
(2*x^3*Sqrt[a*x + b*x^3])/9 + (2*a*((2*x*Sqrt[a*x + b*x^3])/(5*b) - (6*a*S
qrt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*
x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x
)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a
+ b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqr
t[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/
(2*b^(3/4)*Sqrt[a + b*x^2])))/(5*b*Sqrt[a*x + b*x^3])))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.70

method	result
default	$\frac{2x^3\sqrt{bx^3+ax}}{9}+\frac{4ax\sqrt{bx^3+ax}}{45b}-\frac{2a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\right.}{b}\right.}{15b^2\sqrt{bx^3+ax}}$
elliptic	$\frac{2x^3\sqrt{bx^3+ax}}{9}+\frac{4ax\sqrt{bx^3+ax}}{45b}-\frac{2a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\right.}{b}\right.}{15b^2\sqrt{bx^3+ax}}$
risch	$\frac{2x^2(5bx^2+2a)(bx^2+a)}{45b\sqrt{x(bx^2+a)}}-\frac{2a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{15b^2\sqrt{bx^3+ax}}$

input

```
int(x^2*(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2/9*x^3*(b*x^3+a*x)^(1/2)+4/45*a*x*(b*x^3+a*x)^(1/2)/b-2/15*a^2/b^2*(-a*b)
^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-
a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*
b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)
)+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2
^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.20

$$\int x^2\sqrt{ax+bx^3}\,dx = \frac{2\left(6\,a^2\sqrt{b}\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\left(5\,b^2x^3+2\,abx\right)\sqrt{bx^3+ax}\right)}{45\,b^2}$$

input
```
integrate(x^2*(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
2/45*(6*a^2*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b,
 0, x)) + (5*b^2*x^3 + 2*a*b*x)*sqrt(b*x^3 + a*x))/b^2
```

Sympy [F]

$$\int x^2\sqrt{ax+bx^3}\,dx = \int x^2\sqrt{x\left(a+bx^2\right)}\,dx$$

input
```
integrate(x**2*(b*x**3+a*x)**(1/2),x)
```

output
```
Integral(x**2*sqrt(x*(a + b*x**2)), x)
```

Maxima [F]

$$\int x^2\sqrt{ax+bx^3}\,dx = \int \sqrt{bx^3+ax}x^2\,dx$$

input
```
integrate(x^2*(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x)*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{ax+bx^3}\,dx=\int \sqrt{bx^3+ax}x^2\,dx$$

input
```
integrate(x^2*(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a*x)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{ax+bx^3}\,dx=\int x^2\,\sqrt{b\,x^3+a\,x}\,dx$$

input
```
int(x^2*(a*x + b*x^3)^(1/2),x)
```

output
```
int(x^2*(a*x + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{ax+bx^3}\,dx=\frac{\frac{4\sqrt{x}\,\sqrt{b\,x^2+a}\,a x}{45}+\frac{2\sqrt{x}\,\sqrt{b\,x^2+a}\,b\,x^3}{9}-\frac{2\left(\int\frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^2+a}dx\right)a^2}{15}}{b}$$

input
```
int(x^2*(b*x^3+a*x)^(1/2),x)
```

output
```
(2*(2*sqrt(x)*sqrt(a + b*x**2)*a*x + 5*sqrt(x)*sqrt(a + b*x**2)*b*x**3 - 3
*int((sqrt(x)*sqrt(a + b*x**2))/(a + b*x**2),x)*a**2))/(45*b)
```

3.47 $\int x\sqrt{ax+bx^3}\,dx$

Optimal result 429
Mathematica [C] (verified) 430
Rubi [A] (verified) 430
Maple [A] (verified) 432
Fricas [A] (verification not implemented) 433
Sympy [F] 433
Maxima [F] 434
Giac [F] 434
Mupad [F(-1)] 434
Reduce [F] 435

Optimal result

Integrand size = 15, antiderivative size = 137

$$\int x\sqrt{ax+bx^3}\,dx = \frac{4a\sqrt{ax+bx^3}}{21b}+\frac{2}{7}x^2\sqrt{ax+bx^3} - \frac{2a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{21b^{5/4}\sqrt{ax+bx^3}}$$

output
```
4/21*a*(b*x^3+a*x)^(1/2)/b+2/7*x^2*(b*x^3+a*x)^(1/2)-2/21*a^(7/4)*x^(1/2)*
(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiA
M(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)),1/2*2^(1/2))/b^(5/4)/(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.58

$$\int x\sqrt{ax+bx^3}\,dx = \frac{2\sqrt{x(a+bx^2)}\left((a+bx^2)\sqrt{1+\frac{bx^2}{a}}-a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{7b\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[x*Sqrt[a*x + b*x^3],x]
```

output
```
(2*Sqrt[x*(a + b*x^2)]*((a + b*x^2)*Sqrt[1 + (b*x^2)/a] - a*Hypergeometric
2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)]))/(7*b*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1927, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{ax+bx^3}\,dx$$

$$\Big\downarrow \text{1927}$$

$$\frac{2}{7}a\int \frac{x^2}{\sqrt{bx^3+ax}}dx+\frac{2}{7}x^2\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{1930}$$

$$\frac{2}{7}a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\int \frac{1}{\sqrt{bx^3+ax}}dx}{3b}\right)+\frac{2}{7}x^2\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{1917}$$

$$\frac{2}{7}a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\sqrt{x}\sqrt{a+bx^2}\int\frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3b\sqrt{ax+bx^3}}\right)+\frac{2}{7}x^2\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{7}a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{2a\sqrt{x}\sqrt{a+bx^2}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3b\sqrt{ax+bx^3}}\right)+\frac{2}{7}x^2\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{7}a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}\right)+\frac{2}{7}x^2\sqrt{ax+bx^3}$$

input
```
Int[x*Sqrt[a*x + b*x^3],x]
```

output
```
(2*x^2*Sqrt[a*x + b*x^3])/7 + (2*a*((2*Sqrt[a*x + b*x^3])/(3*b) - (a^(3/4)
*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*E
llipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[a*x +
b*x^3])))/7
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.07

method	result	si
default	$\frac{2x^{2}\sqrt{b\,x^{3}+ax}}{7}+\frac{4a\sqrt{b\,x^{3}+ax}}{21b}-\frac{2a^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{21b^{2}\sqrt{b\,x^{3}+ax}}$	14
elliptic	$\frac{2x^{2}\sqrt{b\,x^{3}+ax}}{7}+\frac{4a\sqrt{b\,x^{3}+ax}}{21b}-\frac{2a^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{21b^{2}\sqrt{b\,x^{3}+ax}}$	14
risch	$\frac{2\left(3b\,x^{2}+2a\right)x\left(b\,x^{2}+a\right)}{21b\sqrt{x\left(b\,x^{2}+a\right)}}-\frac{2a^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{21b^{2}\sqrt{b\,x^{3}+ax}}$	14

input
```
int(x*(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*x^2*(b*x^3+a*x)^(1/2)+4/21*a*(b*x^3+a*x)^(1/2)/b-2/21*a^2/b^2*(-a*b)^(
1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*
b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(
((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.36

$$\int x\sqrt{ax+bx^3}\,dx = -\frac{2\left(2a^2\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)-(3b^2x^2+2ab)\sqrt{bx^3+ax}\right)}{21b^2}$$

input
```
integrate(x*(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/21*(2*a^2*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) - (3*b^2*x^2 + 2*a*
b)*sqrt(b*x^3 + a*x))/b^2
```

Sympy [F]

$$\int x\sqrt{ax+bx^3}\,dx = \int x\sqrt{x(a+bx^2)}\,dx$$

input
```
integrate(x*(b*x**3+a*x)**(1/2),x)
```

output
```
Integral(x*sqrt(x*(a + b*x**2)), x)
```

Maxima [F]

$$\int x\sqrt{ax+bx^3}\,dx = \int \sqrt{bx^3+ax}x\,dx$$

input
```
integrate(x*(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x)*x, x)
```

Giac [F]

$$\int x\sqrt{ax+bx^3}\,dx = \int \sqrt{bx^3+ax}x\,dx$$

input
```
integrate(x*(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a*x)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{ax+bx^3}\,dx = \int x\,\sqrt{b\,x^3+a\,x}\,dx$$

input
```
int(x*(a*x + b*x^3)^(1/2),x)
```

output
```
int(x*(a*x + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{ax+bx^3}\,dx = \frac{\frac{4\sqrt{x}\,\sqrt{b\,x^2+a}\,a}{21}+\frac{2\sqrt{x}\,\sqrt{b\,x^2+a}\,b\,x^2}{7}-\frac{2\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^3+ax}dx\right)a^2}{21}}{b}$$

input
```
int(x*(b*x^3+a*x)^(1/2),x)
```

output
```
(2*(2*sqrt(x)*sqrt(a + b*x**2)*a + 3*sqrt(x)*sqrt(a + b*x**2)*b*x**2 - int
((sqrt(x)*sqrt(a + b*x**2))/(a*x + b*x**3),x)*a**2))/(21*b)
```

3.48 $\int \sqrt{ax+bx^3}\,dx$

Optimal result 436
Mathematica [C] (verified) 437
Rubi [A] (verified) 437
Maple [A] (verified) 440
Fricas [A] (verification not implemented) 441
Sympy [F] 441
Maxima [F] 442
Giac [F] 442
Mupad [B] (verification not implemented) 442
Reduce [F] 443

Optimal result

Integrand size = 13, antiderivative size = 255

$$\int \sqrt{ax+bx^3}\,dx = \frac{4ax(a+bx^2)}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3} - \frac{4a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+bx^3}} + \frac{2a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+bx^3}}$$

output
```
4/5*a*x*(b*x^2+a)/b^(1/2)/(a^(1/2)+b^(1/2)*x)/(b*x^3+a*x)^(1/2)+2/5*x*(b*x
^3+a*x)^(1/2)-4/5*a^(5/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+
b^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2
^(1/2))/b^(3/4)/(b*x^3+a*x)^(1/2)+2/5*a^(5/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*
((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x
^(1/2)/a^(1/4)),1/2*2^(1/2))/b^(3/4)/(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.64 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.20

$$\int \sqrt{ax+bx^3}\,dx=\frac{2x\sqrt{x(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)}{3\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[Sqrt[a*x + b*x^3],x]
```

output
```
(2*x*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^2)/a)])/
(3*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {1910, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ax+bx^3}\,dx$$

$$\Big\downarrow \text{1910}$$

$$\frac{2}{5}a\int \frac{x}{\sqrt{bx^3+ax}}dx+\frac{2}{5}x\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{1938}$$

$$\frac{2a\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{266}$$

$$\frac{4a\sqrt{x}\sqrt{a+bx^2}\int \frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}$$

$\downarrow$ 834

$$\frac{4a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}$$

$\downarrow$ 27

$$\frac{4a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}$$

$\downarrow$ 761

$$\frac{4a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}$$

$\downarrow$ 1510

$$\frac{4a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right)}{5\sqrt{ax+bx^3}}$$

$$\frac{2}{5}x\sqrt{ax+bx^3}$$

input
```
Int[Sqrt[a*x + b*x^3],x]
```

output
```
(2*x*Sqrt[a*x + b*x^3])/5 + (4*a*Sqrt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x]*Sq
rt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqr
t[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x]
)/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a]
+ Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[
(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(5*Sqrt[a*
x + b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.69

method	result
default	$\frac{2x\sqrt{b\,x^3+ax}}{5}+\frac{2a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\ \mathrm{EllipticF}\cdots}{\cdots}\right.}{5b\sqrt{b\,x^3+ax}}$
elliptic	$\frac{2x\sqrt{b\,x^3+ax}}{5}+\frac{2a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\ \mathrm{EllipticF}\cdots}{\cdots}\right.}{5b\sqrt{b\,x^3+ax}}$
risch	$\frac{2x^2\left(b\,x^2+a\right)}{5\sqrt{x\left(b\,x^2+a\right)}}+\frac{2a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\ \mathrm{EllipticF}\cdots}{\cdots}\right.}{5b\sqrt{b\,x^3+ax}}$

input
```
int((b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*x*(b*x^3+a*x)^(1/2)+2/5*a*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1
/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)
*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2
)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b
)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.17

$$\int \sqrt{ax+bx^3}\,dx = \frac{2\left(\sqrt{bx^3+ax}bx-2\,a\sqrt{b}\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)\right)}{5\,b}$$

input
```
integrate((b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(sqrt(b*x^3 + a*x)*b*x - 2*a*sqrt(b)*weierstrassZeta(-4*a/b, 0, weiers
trassPInverse(-4*a/b, 0, x)))/b
```

Sympy [F]

$$\int \sqrt{ax+bx^3}\,dx = \int \sqrt{ax+bx^3}\,dx$$

input
```
integrate((b*x**3+a*x)**(1/2),x)
```

output
```
Integral(sqrt(a*x + b*x**3), x)
```

Maxima [F]

$$\int \sqrt{ax+bx^3}\,dx=\int \sqrt{bx^3+ax}\,dx$$

input
```
integrate((b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x), x)
```

Giac [F]

$$\int \sqrt{ax+bx^3}\,dx=\int \sqrt{bx^3+ax}\,dx$$

input
```
integrate((b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a*x), x)
```

Mupad [B] (verification not implemented)

Time = 8.96 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.16

$$\int \sqrt{ax+bx^3}\,dx=\frac{2\,x\,\sqrt{b\,x^3+a\,x}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{3}{4};\ \frac{7}{4};\ -\frac{b\,x^2}{a}\right)}{3\,\sqrt{\frac{b\,x^2}{a}+1}}$$

input
```
int((a*x + b*x^3)^(1/2),x)
```

output
```
(2*x*(a*x + b*x^3)^(1/2)*hypergeom([-1/2, 3/4], 7/4, -(b*x^2)/a))/(3*((b*x
^2)/a + 1)^(1/2))
```

Reduce [F]

$$\int \sqrt{ax+bx^3}\,dx = \frac{2\sqrt{x}\,\sqrt{b\,x^2+a}\,x}{5} + \frac{2\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^2+a}dx\right)a}{5}$$

input
```
int((b*x^3+a*x)^(1/2),x)
```

output
```
(2*(sqrt(x)*sqrt(a + b*x**2)*x + int((sqrt(x)*sqrt(a + b*x**2))/(a + b*x**
2),x)*a))/5
```

3.49 $\int \frac{\sqrt{ax+bx^3}}{x}\,dx$

Optimal result 444
Mathematica [C] (verified) 444
Rubi [A] (verified) 445
Maple [A] (verified) 447
Fricas [A] (verification not implemented) 447
Sympy [F] 448
Maxima [F] 448
Giac [F] 448
Mupad [F(-1)] 449
Reduce [F] 449

Optimal result

Integrand size = 17, antiderivative size = 113

$$\int \frac{\sqrt{ax+bx^3}}{x}\,dx = \frac{2}{3}\sqrt{ax+bx^3}+\frac{2a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{ax+bx^3}}$$

output
```
2/3*(b*x^3+a*x)^(1/2)+2/3*a^(3/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(
a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x^(1/2)/a^(1/
4)),1/2*2^(1/2))/b^(1/4)/(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.58 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{ax+bx^3}}{x}\,dx = \frac{2\sqrt{x(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^2}{a}\right)}{\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[Sqrt[a*x + b*x^3]/x,x]
```

output
```
(2*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)])/Sq
rt[1 + (b*x^2)/a]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1927, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^3}}{x}\,dx$$

$$\Big\downarrow \text{1927}$$

$$\frac{2}{3}a\int \frac{1}{\sqrt{bx^3+ax}}dx+\frac{2}{3}\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{1917}$$

$$\frac{2a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3\sqrt{ax+bx^3}}+\frac{2}{3}\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{266}$$

$$\frac{4a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3\sqrt{ax+bx^3}}+\frac{2}{3}\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{761}$$

$$\frac{2a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{ax+bx^3}}+\frac{2}{3}\sqrt{ax+bx^3}$$

input
```
Int[Sqrt[a*x + b*x^3]/x,x]
```

output
```
(2*Sqrt[a*x + b*x^3])/3 + (2*a^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^
(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a*x + b*x^3])
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{2\sqrt{b\,x^{3}+a x}}{3}+\frac{2 a \sqrt{-a b}\, \sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\, \sqrt{-\frac{2 \left(x-\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\, \sqrt{-\frac{b x}{\sqrt{-a b}}}\, \operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}, \frac{\sqrt{2}}{2}\right)}{3 b \sqrt{b\,x^{3}+a x}}$	124
elliptic	$\frac{2\sqrt{b\,x^{3}+a x}}{3}+\frac{2 a \sqrt{-a b}\, \sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\, \sqrt{-\frac{2 \left(x-\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\, \sqrt{-\frac{b x}{\sqrt{-a b}}}\, \operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}, \frac{\sqrt{2}}{2}\right)}{3 b \sqrt{b\,x^{3}+a x}}$	124
risch	$\frac{2 x \left(b\,x^{2}+a \right)}{3 \sqrt{x \left(b\,x^{2}+a \right)}}+\frac{2 a \sqrt{-a b}\, \sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\, \sqrt{-\frac{2 \left(x-\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\, \sqrt{-\frac{b x}{\sqrt{-a b}}}\, \operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}, \frac{\sqrt{2}}{2}\right)}{3 b \sqrt{b\,x^{3}+a x}}$	132

input
```
int((b*x^3+a*x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/3*(b*x^3+a*x)^(1/2)+2/3*a*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2
)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b
*x)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^
(1/2),1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt{ax+bx^3}}{x}\,dx = \frac{2\left(2\,a\sqrt{b}\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\sqrt{bx^3+ax}b\right)}{3\,b}$$

input
```
integrate((b*x^3+a*x)^(1/2)/x,x, algorithm="fricas")
```

output
```
2/3*(2*a*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b*x^3 + a*x)*b)/
b
```

Sympy [F]

$$\int \frac{\sqrt{ax + bx^3}}{x}\, dx = \int \frac{\sqrt{x\,(a + bx^2)}}{x}\, dx$$

input
```
integrate((b*x**3+a*x)**(1/2)/x,x)
```

output
```
Integral(sqrt(x*(a + b*x**2))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax + bx^3}}{x}\, dx = \int \frac{\sqrt{bx^3 + ax}}{x}\, dx$$

input
```
integrate((b*x^3+a*x)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x)/x, x)
```

Giac [F]

$$\int \frac{\sqrt{ax + bx^3}}{x}\, dx = \int \frac{\sqrt{bx^3 + ax}}{x}\, dx$$

input
```
integrate((b*x^3+a*x)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a*x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^3}}{x}\,dx=\int \frac{\sqrt{b\,x^3+a\,x}}{x}\,dx$$

input
```
int((a*x + b*x^3)^(1/2)/x,x)
```

output
```
int((a*x + b*x^3)^(1/2)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{ax+bx^3}}{x}\,dx=\frac{2\sqrt{x}\,\sqrt{b\,x^2+a}}{3}+\frac{2\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^3+ax}dx\right)a}{3}$$

input
```
int((b*x^3+a*x)^(1/2)/x,x)
```

output
```
(2*(sqrt(x)*sqrt(a + b*x**2) + int((sqrt(x)*sqrt(a + b*x**2))/(a*x + b*x**
3),x)*a))/3
```

3.50 $\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx$

Optimal result 450
Mathematica [C] (verified) 451
Rubi [A] (verified) 451
Maple [A] (verified) 454
Fricas [A] (verification not implemented) 455
Sympy [F] 455
Maxima [F] 456
Giac [F] 456
Mupad [F(-1)] 456
Reduce [F] 457

Optimal result

Integrand size = 17, antiderivative size = 248

$$\begin{aligned}
\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx
&= \frac{4\sqrt{b}x(a+bx^2)}{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{x} \\
&\quad - \frac{4\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{ax+bx^3}} \\
&\quad + \frac{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt{ax+bx^3}}
\end{aligned}$$

output

```
4*b^(1/2)*x*(b*x^2+a)/(a^(1/2)+b^(1/2)*x)/(b*x^3+a*x)^(1/2)-2*(b*x^3+a*x)^
(1/2)/x-4*a^(1/4)*b^(1/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+
b^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2
^(1/2))/(b*x^3+a*x)^(1/2)+2*a^(1/4)*b^(1/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((
b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x^(
1/2)/a^(1/4)),1/2*2^(1/2))/(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx=-\frac{2\sqrt{x(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{3}{4},-\frac{bx^2}{a}\right)}{x\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[Sqrt[a*x + b*x^3]/x^2,x]
```

output
```
(-2*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-1/2, -1/4, 3/4, -((b*x^2)/a)])/
(x*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {1926, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^3}}{x^2}dx$$

$$\Big\downarrow \text{1926}$$

$$2b\int \frac{x}{\sqrt{bx^3+ax}}dx-\frac{2\sqrt{ax+bx^3}}{x}$$

$$\Big\downarrow \text{1938}$$

$$\frac{2b\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}$$

$$\Big\downarrow \text{266}$$

$$\frac{4b\sqrt{x}\sqrt{a+bx^2}\int \frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}$$

$$\Big\downarrow \text{834}$$

$$\frac{4b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{4b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}$$

$$\Big\downarrow \text{761}$$

$$\frac{4b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}$$

$$\Big\downarrow \text{1510}$$

$$\frac{4b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{\sqrt{ax+bx^3}}$$

$$\frac{2\sqrt{ax+bx^3}}{x}$$

input
```
Int[Sqrt[a*x + b*x^3]/x^2,x]
```

output
```
(-2*Sqrt[a*x + b*x^3])/x + (4*b*Sqrt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x]*Sqr
t[a + b*x^2])/(Sqrt[a] + Sqrt[b]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt
[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])
/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] +
 Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(
b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/Sqrt[a*x +
b*x^3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.71

method	result
default	$-\frac{2(bx^2+a)}{\sqrt{x(bx^2+a)}}+\frac{2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\ \mathrm{EllipticF}}{}\right.}{\sqrt{bx^3+ax}}$
risch	$-\frac{2(bx^2+a)}{\sqrt{x(bx^2+a)}}+\frac{2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\ \mathrm{EllipticF}}{}\right.}{\sqrt{bx^3+ax}}$
elliptic	$-\frac{2(bx^2+a)}{\sqrt{x(bx^2+a)}}+\frac{2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\ \mathrm{EllipticF}}{}\right.}{\sqrt{bx^3+ax}}$

input
```
int((b*x^3+a*x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(b*x^2+a)/(x*(b*x^2+a))^(1/2)+2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)
^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1
/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(
1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-
a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.16

$$\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx = -\frac{2\left(2\sqrt{b}x\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\sqrt{bx^3+ax}\right)}{x}$$

input
```
integrate((b*x^3+a*x)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-2*(2*sqrt(b)*x*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0,
x)) + sqrt(b*x^3 + a*x))/x
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx = \int \frac{\sqrt{x\left(a+bx^2\right)}}{x^2}\,dx$$

input
```
integrate((b*x**3+a*x)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(x*(a + b*x**2))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx = \int \frac{\sqrt{bx^3+ax}}{x^2}\,dx$$

input
```
integrate((b*x^3+a*x)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx = \int \frac{\sqrt{bx^3+ax}}{x^2}\,dx$$

input
```
integrate((b*x^3+a*x)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a*x)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx = \int \frac{\sqrt{b\,x^3+a\,x}}{x^2}\,dx$$

input
```
int((a*x + b*x^3)^(1/2)/x^2,x)
```

output
```
int((a*x + b*x^3)^(1/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx = \frac{2\sqrt{b\,x^2+a}+2\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^4+a\,x^2}dx\right)a}{\sqrt{x}}$$

input
```
int((b*x^3+a*x)^(1/2)/x^2,x)
```

output
```
(2*(sqrt(a + b*x**2) + sqrt(x)*int((sqrt(x)*sqrt(a + b*x**2))/(a*x**2 + b*
x**4),x)*a))/sqrt(x)
```

3.51 $\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx$

Optimal result 458
Mathematica [C] (verified) 459
Rubi [A] (verified) 459
Maple [A] (verified) 461
Fricas [A] (verification not implemented) 461
Sympy [F] 462
Maxima [F] 462
Giac [F] 462
Mupad [F(-1)] 463
Reduce [F] 463

Optimal result

Integrand size = 17, antiderivative size = 116

$$\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx = -\frac{2\sqrt{ax+bx^3}}{3x^2} + \frac{2b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{ax+bx^3}}$$

output
```
-2/3*(b*x^3+a*x)^(1/2)/x^2+2/3*b^(3/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2
+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x^(1/2)/
a^(1/4)),1/2*2^(1/2))/a^(1/4)/(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx = -\frac{2\sqrt{x(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{4},-\frac{bx^2}{a}\right)}{3x^2\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[Sqrt[a*x + b*x^3]/x^3,x]
```

output
```
(-2*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/4, -1/2, 1/4, -((b*x^2)/a)])/
(3*x^2*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1926, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^3}}{x^3}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{2}{3}b\int \frac{1}{\sqrt{bx^3+ax}}dx-\frac{2\sqrt{ax+bx^3}}{3x^2}$$

$$\Big\downarrow \text{1917}$$

$$\frac{2b\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3x^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{4b\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3x^2}$$

$$\Big\downarrow \text{761}$$

$$\frac{2b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3x^2}$$

input
```
Int[Sqrt[a*x + b*x^3]/x^3,x]
```

output
```
(-2*Sqrt[a*x + b*x^3])/(3*x^2) + (2*b^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*
Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt
[x])/a^(1/4)], 1/2])/(3*a^(1/4)*Sqrt[a*x + b*x^3])
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{2\sqrt{b\,x^{3}+ax}}{3x^{2}}+\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3\sqrt{b\,x^{3}+ax}}$	123
elliptic	$-\frac{2\sqrt{b\,x^{3}+ax}}{3x^{2}}+\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3\sqrt{b\,x^{3}+ax}}$	123
risch	$-\frac{2\left(b\,x^{2}+a\right)}{3x\sqrt{x\left(b\,x^{2}+a\right)}}+\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3\sqrt{b\,x^{3}+ax}}$	130

input
```
int((b*x^3+a*x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x^3+a*x)^(1/2)/x^2+2/3*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/
2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*
b*x)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)
^(1/2),1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.31

$$\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx=\frac{2\left(2\sqrt{b}x^2\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)-\sqrt{bx^3+ax}\right)}{3\,x^2}$$

input
```
integrate((b*x^3+a*x)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
2/3*(2*sqrt(b)*x^2*weierstrassPInverse(-4*a/b, 0, x) - sqrt(b*x^3 + a*x))/
x^2
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx=\int \frac{\sqrt{x\,(a+bx^2)}}{x^3}\,dx$$

input
```
integrate((b*x**3+a*x)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(x*(a + b*x**2))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx=\int \frac{\sqrt{bx^3+ax}}{x^3}\,dx$$

input
```
integrate((b*x^3+a*x)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x)/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx=\int \frac{\sqrt{bx^3+ax}}{x^3}\,dx$$

input
```
integrate((b*x^3+a*x)^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a*x)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx=\int \frac{\sqrt{b\,x^3+a\,x}}{x^3}\,dx$$

input
```
int((a*x + b*x^3)^(1/2)/x^3,x)
```

output
```
int((a*x + b*x^3)^(1/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx=\frac{-2\sqrt{b\,x^2+a}-2\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^5+a\,x^3}dx\right)ax}{\sqrt{x}\,x}$$

input
```
int((b*x^3+a*x)^(1/2)/x^3,x)
```

output
```
( - 2*(sqrt(a + b*x**2) + sqrt(x)*int((sqrt(x)*sqrt(a + b*x**2))/(a*x**3 +
 b*x**5),x)*a*x))/(sqrt(x)*x)
```

3.52 $\int \frac{\sqrt{ax+bx^3}}{x^4} dx$

Optimal result 464
Mathematica [C] (verified) 465
Rubi [A] (verified) 465
Maple [A] (verified) 469
Fricas [A] (verification not implemented) 470
Sympy [F] 470
Maxima [F] 470
Giac [F] 471
Mupad [F(-1)] 471
Reduce [F] 471

Optimal result

Integrand size = 17, antiderivative size = 283

$$\begin{aligned}\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx &= \frac{4b^{3/2}x(a+bx^2)}{5a\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{5x^3}-\frac{4b\sqrt{ax+bx^3}}{5ax}\\ &-\frac{4b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+bx^3}}\\ &+\frac{2b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+bx^3}}\end{aligned}$$

output
```
4/5*b^(3/2)*x*(b*x^2+a)/a/(a^(1/2)+b^(1/2)*x)/(b*x^3+a*x)^(1/2)-2/5*(b*x^3
+a*x)^(1/2)/x^3-4/5*b*(b*x^3+a*x)^(1/2)/a/x-4/5*b^(5/4)*x^(1/2)*(a^(1/2)+b
^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(b
^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))/a^(3/4)/(b*x^3+a*x)^(1/2)+2/5*b^(5/4
)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*Inve
rseJacobiAM(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)),1/2*2^(1/2))/a^(3/4)/(b*x^3+
a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx = -\frac{2\sqrt{x\left(a+bx^2\right)}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},-\frac{bx^2}{a}\right)}{5x^3\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[Sqrt[a*x + b*x^3]/x^4,x]
```

output
```
(-2*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-5/4, -1/2, -1/4, -((b*x^2)/a)])
/(5*x^3*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1926, 1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^3}}{x^4}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{2}{5}b\int \frac{1}{x\sqrt{bx^3+ax}}dx-\frac{2\sqrt{ax+bx^3}}{5x^3}$$

$$\Big\downarrow \text{1931}$$

$$\frac{2}{5}b\left(\frac{b\int \frac{x}{\sqrt{bx^3+ax}}dx}{a}-\frac{2\sqrt{ax+bx^3}}{ax}\right)-\frac{2\sqrt{ax+bx^3}}{5x^3}$$

$$\Big\downarrow \text{1938}$$

$$\frac{2}{5}b\left(\frac{b\sqrt{x}\sqrt{a+bx^2}\int\frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)-\frac{2\sqrt{ax+bx^3}}{5x^3}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2}{5}b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\int\frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)-\frac{2\sqrt{ax+bx^3}}{5x^3}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{2}{5}b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)-\frac{2\sqrt{ax+bx^3}}{5x^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{5}b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)-\frac{2\sqrt{ax+bx^3}}{5x^3}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{2}{5}b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)-\frac{2\sqrt{ax+bx^3}}{5x^3}$$

$$\Big\downarrow \text{ 1510}$$

$$\frac{2}{5}b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{}}\right.\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{a\sqrt{ax+bx^3}}\right.$$

$$\frac{2\sqrt{ax+bx^3}}{5x^3}$$

input

```
Int[Sqrt[a*x + b*x^3]/x^4,x]
```

output

```
(-2*Sqrt[a*x + b*x^3])/(5*x^3) + (2*b*((-2*Sqrt[a*x + b*x^3])/(a*x) + (2*b
*Sqrt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt[b
]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]
*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[
a + b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(S
qrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2]
)/(2*b^(3/4)*Sqrt[a + b*x^2])))/(a*Sqrt[a*x + b*x^3])))/5
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{2(bx^2+a)(2bx^2+a)}{5x^2\sqrt{x(bx^2+a)}\,a}+\frac{2b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-a}}{}\right.}{5a\sqrt{bx^3+ax}}$
default	$-\frac{2\sqrt{bx^3+ax}}{5x^3}-\frac{4(bx^2+a)b}{5a\sqrt{x(bx^2+a)}}+\frac{2b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\right.}{b}\right.}{5a\sqrt{bx^3+ax}}$
elliptic	$-\frac{2\sqrt{bx^3+ax}}{5x^3}-\frac{4(bx^2+a)b}{5a\sqrt{x(bx^2+a)}}+\frac{2b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\right.}{b}\right.}{5a\sqrt{bx^3+ax}}$

input
```
int((b*x^3+a*x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/5*(b*x^2+a)*(2*b*x^2+a)/x^2/(x*(b*x^2+a))^(1/2)/a+2/5/a*b*(-a*b)^(1/2)*
((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1
/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2
)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b
)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx = -\frac{2\left(2\,b^{\frac{3}{2}}x^3\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\sqrt{bx^3+ax}(2\,bx^2+a)\right)}{5\,ax^3}$$

input
```
integrate((b*x^3+a*x)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-2/5*(2*b^(3/2)*x^3*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b,
 0, x)) + sqrt(b*x^3 + a*x)*(2*b*x^2 + a))/(a*x^3)
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx = \int \frac{\sqrt{x\,(a+bx^2)}}{x^4}\,dx$$

input
```
integrate((b*x**3+a*x)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(x*(a + b*x**2))/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx = \int \frac{\sqrt{bx^3+ax}}{x^4}\,dx$$

input
```
integrate((b*x^3+a*x)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x)/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx = \int \frac{\sqrt{bx^3+ax}}{x^4}\,dx$$

input
```
integrate((b*x^3+a*x)^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a*x)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx = \int \frac{\sqrt{b\,x^3+a\,x}}{x^4}\,dx$$

input
```
int((a*x + b*x^3)^(1/2)/x^4,x)
```

output
```
int((a*x + b*x^3)^(1/2)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx = \frac{-2\sqrt{b\,x^2+a}-2\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^6+a\,x^4}dx\right)a\,x^2}{3\sqrt{x}\,x^2}$$

input
```
int((b*x^3+a*x)^(1/2)/x^4,x)
```

output
```
( - 2*(sqrt(a + b*x**2) + sqrt(x)*int((sqrt(x)*sqrt(a + b*x**2))/(a*x**4 +
 b*x**6),x)*a*x**2))/(3*sqrt(x)*x**2)
```

3.53 $\int x^2(ax+bx^3)^{3/2}\,dx$

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Optimal result

Integrand size = 17, antiderivative size = 186

$$\int x^2(ax+bx^3)^{3/2}\,dx = -\frac{8a^3\sqrt{ax+bx^3}}{231b^2}+\frac{8a^2x^2\sqrt{ax+bx^3}}{385b}+\frac{4}{55}ax^4\sqrt{ax+bx^3}+\frac{2}{15}x^3(ax+bx^3)^{3/2}+\frac{4a^{15/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{ax+bx^3}}$$

output
```
-8/231*a^3*(b*x^3+a*x)^(1/2)/b^2+8/385*a^2*x^2*(b*x^3+a*x)^(1/2)/b+4/55*a*
x^4*(b*x^3+a*x)^(1/2)+2/15*x^3*(b*x^3+a*x)^(3/2)+4/231*a^(15/4)*x^(1/2)*(a
^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(
2*arctan(b^(1/4)*x^(1/2)/a^(1/4)),1/2*2^(1/2))/b^(9/4)/(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.51

$$\int x^2(ax+bx^3)^{3/2}\,dx = \frac{2\sqrt{x(a+bx^2)}\left(-\left((5a-11bx^2)(a+bx^2)^2\sqrt{1+\frac{bx^2}{a}}\right)+5a^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4}\right.\right.}{165b^2\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[x^2*(a*x + b*x^3)^(3/2),x]
```

output
```
(2*Sqrt[x*(a + b*x^2)]*(-((5*a - 11*b*x^2)*(a + b*x^2)^2*Sqrt[1 + (b*x^2)/
a]) + 5*a^3*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)]))/(165*b^2*Sqr
t[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {1927, 1927, 1930, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(ax+bx^3)^{3/2}\,dx$$

$$\Big\downarrow \text{1927}$$

$$\frac{2}{5}a\int x^3\sqrt{bx^3+ax}dx+\frac{2}{15}x^3(ax+bx^3)^{3/2}$$

$$\Big\downarrow \text{1927}$$

$$\frac{2}{5}a\left(\frac{2}{11}a\int \frac{x^4}{\sqrt{bx^3+ax}}dx+\frac{2}{11}x^4\sqrt{ax+bx^3}\right)+\frac{2}{15}x^3(ax+bx^3)^{3/2}$$

$$\Big\downarrow \text{1930}$$

$$\frac{2}{5}a\left(\frac{2}{11}a\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\int\frac{x^2}{\sqrt{bx^3+ax}}dx}{7b}\right)+\frac{2}{11}x^4\sqrt{ax+bx^3}\right)+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{1930}$$

$$\frac{2}{5}a\left(\frac{2}{11}a\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\int\frac{1}{\sqrt{bx^3+ax}}dx}{3b}\right)}{7b}\right)+\frac{2}{11}x^4\sqrt{ax+bx^3}\right)+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{1917}$$

$$\frac{2}{5}a\left(\frac{2}{11}a\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\sqrt{x}\sqrt{a+bx^2}\int\frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3b\sqrt{ax+bx^3}}\right)}{7b}\right)+\frac{2}{11}x^4\sqrt{ax+bx^3}\right)+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{5}a\left(\frac{2}{11}a\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{2a\sqrt{x}\sqrt{a+bx^2}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3b\sqrt{ax+bx^3}}\right)}{7b}\right)+\frac{2}{11}x^4\sqrt{ax+bx^3}\right)+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{5}a\left(\frac{2}{11}a\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}\right)}{7b}\right)+\frac{2}{11}x^4\right.$$

$$\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}$$

input
```
Int[x^2*(a*x + b*x^3)^(3/2),x]
```

output
```
(2*x^3*(a*x + b*x^3)^(3/2))/15 + (2*a*((2*x^4*Sqrt[a*x + b*x^3])/11 + (2*a
*((2*x^2*Sqrt[a*x + b*x^3])/(7*b) - (5*a*((2*Sqrt[a*x + b*x^3])/(3*b) - (a
^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x
)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[
a*x + b*x^3])))/(7*b)))/11))/5
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{2(-77b^3x^6-119a\,b^2x^4-12a^2b\,x^2+20a^3)x(b\,x^2+a)}{1155b^2\sqrt{x(b\,x^2+a)}}+\frac{4a^4\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\left(\right.}\right.}{231b^3\sqrt{b\,x^3+ax}}$
default	$\frac{2b\,x^6\sqrt{b\,x^3+ax}}{15}+\frac{34a\,x^4\sqrt{b\,x^3+ax}}{165}+\frac{8a^2x^2\sqrt{b\,x^3+ax}}{385b}-\frac{8a^3\sqrt{b\,x^3+ax}}{231b^2}+\frac{4a^4\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-}}{231b^3\sqrt{b\,x^3+}}$
elliptic	$\frac{2b\,x^6\sqrt{b\,x^3+ax}}{15}+\frac{34a\,x^4\sqrt{b\,x^3+ax}}{165}+\frac{8a^2x^2\sqrt{b\,x^3+ax}}{385b}-\frac{8a^3\sqrt{b\,x^3+ax}}{231b^2}+\frac{4a^4\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-}}{231b^3\sqrt{b\,x^3+}}$

input
```
int(x^2*(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/1155*(-77*b^3*x^6-119*a*b^2*x^4-12*a^2*b*x^2+20*a^3)/b^2*x*(b*x^2+a)/(x
*(b*x^2+a))^(1/2)+4/231*a^4/b^3*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1
/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)
*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b
)^(1/2),1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.38

$$\int x^2 (ax + bx^3)^{3/2} \, dx = \frac{2 \left(20 a^4 \sqrt{b} \text{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) + (77 b^4 x^6 + 119 a b^3 x^4 + 12 a^2 b^2 x^2 - 20 a^3 b) \sqrt{bx}\right.}{1155 b^3}$$

input
```
integrate(x^2*(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
2/1155*(20*a^4*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + (77*b^4*x^6 + 1
19*a*b^3*x^4 + 12*a^2*b^2*x^2 - 20*a^3*b)*sqrt(b*x^3 + a*x))/b^3
```

Sympy [F]

$$\int x^2 (ax + bx^3)^{3/2} \, dx = \int x^2 \left(x \left(a + bx^2\right)\right)^{\frac{3}{2}} \, dx$$

input
```
integrate(x**2*(b*x**3+a*x)**(3/2),x)
```

output
```
Integral(x**2*(x*(a + b*x**2))**(3/2), x)
```

Maxima [F]

$$\int x^2 (ax + bx^3)^{3/2} \, dx = \int \left(bx^3 + ax\right)^{\frac{3}{2}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x)^(3/2)*x^2, x)
```

Giac [F]

$$\int x^2 \left(ax+bx^3\right)^{3/2}\,dx=\int \left(bx^3+ax\right)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x)^(3/2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(ax+bx^3\right)^{3/2}\,dx=\int x^2\,\left(b\,x^3+a\,x\right)^{3/2}\,dx$$

input
```
int(x^2*(a*x + b*x^3)^(3/2),x)
```

output
```
int(x^2*(a*x + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int x^2 \left(ax+bx^3\right)^{3/2}\,dx=\frac{-\frac{8\sqrt{x}\,\sqrt{b\,x^2+a}\,a^3}{231}+\frac{8\sqrt{x}\,\sqrt{b\,x^2+a}\,a^2b\,x^2}{385}+\frac{34\sqrt{x}\,\sqrt{b\,x^2+a}\,a\,b^2x^4}{165}+\frac{2\sqrt{x}\,\sqrt{b\,x^2+a}\,b^3x^6}{15}+\frac{4\left(\int\frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^3+ax}dx\right)a^4}{231}}{b^2}$$

input
```
int(x^2*(b*x^3+a*x)^(3/2),x)
```

output
```
(2*( - 20*sqrt(x)*sqrt(a + b*x**2)*a**3 + 12*sqrt(x)*sqrt(a + b*x**2)*a**2
*b*x**2 + 119*sqrt(x)*sqrt(a + b*x**2)*a*b**2*x**4 + 77*sqrt(x)*sqrt(a + b
*x**2)*b**3*x**6 + 10*int((sqrt(x)*sqrt(a + b*x**2))/(a*x + b*x**3),x)*a**
4))/(1155*b**2)
```

3.54 $\int x(ax+bx^3)^{3/2}\,dx$

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Optimal result

Integrand size = 15, antiderivative size = 304

$$\int x\left(ax+bx^3\right)^{3/2}\,dx = -\frac{8a^3x(a+bx^2)}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} + \frac{8a^2x\sqrt{ax+bx^3}}{195b} + \frac{4}{39}ax^3\sqrt{ax+bx^3} + \frac{2}{13}x^2\left(ax+bx^3\right)^{3/2} + \frac{8a^{13/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{ax+bx^3}} - \frac{4a^{13/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{65b^{7/4}\sqrt{ax+bx^3}}$$

output
```
-8/65*a^3*x*(b*x^2+a)/b^(3/2)/(a^(1/2)+b^(1/2)*x)/(b*x^3+a*x)^(1/2)+8/195*
a^2*x*(b*x^3+a*x)^(1/2)/b+4/39*a*x^3*(b*x^3+a*x)^(1/2)+2/13*x^2*(b*x^3+a*x
)^(3/2)+8/65*a^(13/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1
/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/
2))/b^(7/4)/(b*x^3+a*x)^(1/2)-4/65*a^(13/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((
b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x^(
1/2)/a^(1/4)),1/2*2^(1/2))/b^(7/4)/(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.28

$$\int x\left(ax+bx^3\right)^{3/2}\,dx=\frac{2x\sqrt{x(a+bx^2)}\left((a+bx^2)^2\sqrt{1+\frac{bx^2}{a}}-a^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{13b\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[x*(a*x + b*x^3)^(3/2),x]
```

output
```
(2*x*Sqrt[x*(a + b*x^2)]*((a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] - a^2*Hypergeo
metric2F1[-3/2, 3/4, 7/4, -((b*x^2)/a)]))/(13*b*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {1927, 1927, 1930, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax+bx^3\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1927}$$

$$\frac{6}{13}a\int x^2\sqrt{bx^3+ax}dx+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{1927}$$

$$\frac{6}{13}a\left(\frac{2}{9}a\int\frac{x^3}{\sqrt{bx^3+ax}}dx+\frac{2}{9}x^3\sqrt{ax+bx^3}\right)+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{1930}$$

$$\frac{6}{13}a\left(\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{3a\int \frac{x}{\sqrt{bx^3+ax}}dx}{5b}\right)+\frac{2}{9}x^3\sqrt{ax+bx^3}\right)+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{ 1938}$$

$$\frac{6}{13}a\left(\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{3a\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{5b\sqrt{ax+bx^3}}\right)+\frac{2}{9}x^3\sqrt{ax+bx^3}\right)+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{6}{13}a\left(\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\int \frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{5b\sqrt{ax+bx^3}}\right)+\frac{2}{9}x^3\sqrt{ax+bx^3}\right)+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{6}{13}a\left(\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5b\sqrt{ax+bx^3}}\right)+\frac{2}{9}x^3\sqrt{ax+bx^3}\right)+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{6}{13}a\left(\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5b\sqrt{ax+bx^3}}\right)+\frac{2}{9}x^3\sqrt{ax+bx^3}\right)+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{6}{13}a\left(\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right.}{5b\sqrt{ax+bx^3}}\right.\right.$$

$$\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}$$

$\downarrow$ 1510

$$\frac{6}{13}a\left(\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)}{}\right.}{5b\sqrt{ax+bx^3}}\right.\right.$$

$$\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}$$

input
```
Int[x*(a*x + b*x^3)^(3/2),x]
```

output
```
(2*x^2*(a*x + b*x^3)^(3/2))/13 + (6*a*((2*x^3*Sqrt[a*x + b*x^3]])/9 + (2*a*
((2*x*Sqrt[a*x + b*x^3]])/(5*b) - (6*a*Sqrt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[
x]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x
)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sq
rt[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqr
t[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*Ar
cTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(5*b*
Sqrt[a*x + b*x^3])))/9))/13
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.69

method	result
risch	$\frac{2x^2(15b^2x^4+25abx^2+4a^2)(bx^2+a)}{195b\sqrt{x(bx^2+a)}} - \frac{4a^3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\right.}{b}\right.}{65b^2\sqrt{bx^3+ax}}$
default	$\frac{2bx^5\sqrt{bx^3+ax}}{13} + \frac{10ax^3\sqrt{bx^3+ax}}{39} + \frac{8a^2x\sqrt{bx^3+ax}}{195b} - \frac{4a^3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \text{Ell}}{}\right.}{65b^2\sqrt{b}}$
elliptic	$\frac{2bx^5\sqrt{bx^3+ax}}{13} + \frac{10ax^3\sqrt{bx^3+ax}}{39} + \frac{8a^2x\sqrt{bx^3+ax}}{195b} - \frac{4a^3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \text{Ell}}{}\right.}{65b^2\sqrt{b}}$

input
```
int(x*(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/195*x^2*(15*b^2*x^4+25*a*b*x^2+4*a^2)/b*(b*x^2+a)/(x*(b*x^2+a))^(1/2)-4/
65*a^3/b^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(
-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*
x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^
(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/
2)*b)^(1/2),1/2*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.22

$$\int x\left(ax+bx^3\right)^{3/2}\,dx=\frac{2\left(12\,a^3\sqrt{b}\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\left(15\,b^3x^5+25\,ab^2x^3+\right)\right)}{195\,b^2}$$

input
```
integrate(x*(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
2/195*(12*a^3*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/
b, 0, x)) + (15*b^3*x^5 + 25*a*b^2*x^3 + 4*a^2*b*x)*sqrt(b*x^3 + a*x))/b^2
```

Sympy [F]

$$\int x\left(ax+bx^3\right)^{3/2}\,dx=\int x\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(b*x**3+a*x)**(3/2),x)
```

output
```
Integral(x*(x*(a + b*x**2))**(3/2), x)
```

Maxima [F]

$$\int x(ax+bx^3)^{3/2}\,dx=\int (bx^3+ax)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x)^(3/2)*x, x)
```

Giac [F]

$$\int x(ax+bx^3)^{3/2}\,dx=\int (bx^3+ax)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x)^(3/2)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(ax+bx^3)^{3/2}\,dx=\int x\,(b\,x^3+a\,x)^{3/2}\,dx$$

input
```
int(x*(a*x + b*x^3)^(3/2),x)
```

output
```
int(x*(a*x + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int x(ax+bx^3)^{3/2}\,dx = \frac{\frac{8\sqrt{x}\,\sqrt{b\,x^2+a}\,a^2x}{195}+\frac{10\sqrt{x}\,\sqrt{b\,x^2+a}\,ab\,x^3}{39}+\frac{2\sqrt{x}\,\sqrt{b\,x^2+a}\,b^2x^5}{13}-\frac{4\left(\int\frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^2+a}dx\right)a^3}{65}}{b}$$

input
```
int(x*(b*x^3+a*x)^(3/2),x)
```

output
```
(2*(4*sqrt(x)*sqrt(a + b*x**2)*a**2*x + 25*sqrt(x)*sqrt(a + b*x**2)*a*b*x*
*3 + 15*sqrt(x)*sqrt(a + b*x**2)*b**2*x**5 - 6*int((sqrt(x)*sqrt(a + b*x**
2))/(a + b*x**2),x)*a**3))/(195*b)
```

3.55 $\int (ax+bx^3)^{3/2}\,dx$

Optimal result . . . 488
Mathematica [C] (verified) . . . 488
Rubi [A] (verified) . . . 489
Maple [A] (verified) . . . 492
Fricas [A] (verification not implemented) . . . 492
Sympy [F] . . . 493
Maxima [F] . . . 493
Giac [F] . . . 493
Mupad [B] (verification not implemented) . . . 494
Reduce [F] . . . 494

Optimal result

Integrand size = 13, antiderivative size = 158

$$\int (ax+bx^3)^{3/2}\,dx = \frac{8a^2\sqrt{ax+bx^3}}{77b} + \frac{12}{77}ax^2\sqrt{ax+bx^3} + \frac{2}{11}x(ax+bx^3)^{3/2} - \frac{4a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{77b^{5/4}\sqrt{ax+bx^3}}$$

output
```
8/77*a^2*(b*x^3+a*x)^(1/2)/b+12/77*a*x^2*(b*x^3+a*x)^(1/2)+2/11*x*(b*x^3+a
*x)^(3/2)-4/77*a^(11/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^
(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)),1/2*2^
(1/2))/b^(5/4)/(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.53

$$\int (ax+bx^3)^{3/2}\,dx=\frac{2\sqrt{x(a+bx^2)}\left((a+bx^2)^2\sqrt{1+\frac{bx^2}{a}}-a^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{11b\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a*x + b*x^3)^(3/2),x]
```

output
```
(2*Sqrt[x*(a + b*x^2)]*((a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] - a^2*Hypergeome
tric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)]))/(11*b*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {1910, 1927, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax+bx^3)^{3/2}\,dx$$

$$\Big\downarrow \text{1910}$$

$$\frac{6}{11}a\int x\sqrt{bx^3+ax}dx+\frac{2}{11}x(ax+bx^3)^{3/2}$$

$$\Big\downarrow \text{1927}$$

$$\frac{6}{11}a\left(\frac{2}{7}a\int \frac{x^2}{\sqrt{bx^3+ax}}dx+\frac{2}{7}x^2\sqrt{ax+bx^3}\right)+\frac{2}{11}x(ax+bx^3)^{3/2}$$

$$\Big\downarrow \text{1930}$$

$$\frac{6}{11}a\left(\frac{2}{7}a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\int \frac{1}{\sqrt{bx^3+ax}}dx}{3b}\right)+\frac{2}{7}x^2\sqrt{ax+bx^3}\right)+\frac{2}{11}x(ax+bx^3)^{3/2}$$

$$\Big\downarrow \text{1917}$$

$$\frac{6}{11}a\left(\frac{2}{7}a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\sqrt{x}\sqrt{a+bx^2}\int\frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3b\sqrt{ax+bx^3}}\right)+\frac{2}{7}x^2\sqrt{ax+bx^3}\right)+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{266}$$

$$\frac{6}{11}a\left(\frac{2}{7}a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{2a\sqrt{x}\sqrt{a+bx^2}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3b\sqrt{ax+bx^3}}\right)+\frac{2}{7}x^2\sqrt{ax+bx^3}\right)+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{761}$$

$$\frac{6}{11}a\left(\frac{2}{7}a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}\right)+\frac{2}{7}x^2\sqrt{ax+\right.$$

$$\frac{2}{11}x\left(ax+bx^3\right)^{3/2}$$

input
```
Int[(a*x + b*x^3)^(3/2),x]
```

output
```
(2*x*(a*x + b*x^3)^(3/2))/11 + (6*a*((2*x^2*Sqrt[a*x + b*x^3])/7 + (2*a*((
2*Sqrt[a*x + b*x^3])/(3*b) - (a^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(
a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a
^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[a*x + b*x^3])))/7))/11
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00

method	result
risch	$\frac{2(7b^2x^4+13abx^2+4a^2)x(bx^2+a)}{77b\sqrt{x(bx^2+a)}}-\frac{4a^3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{77b^2\sqrt{bx^3+ax}}$
default	$\frac{2bx^4\sqrt{bx^3+ax}}{11}+\frac{26ax^2\sqrt{bx^3+ax}}{77}+\frac{8a^2\sqrt{bx^3+ax}}{77b}-\frac{4a^3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{(\right.}{77b^2\sqrt{bx^3+ax}}$
elliptic	$\frac{2bx^4\sqrt{bx^3+ax}}{11}+\frac{26ax^2\sqrt{bx^3+ax}}{77}+\frac{8a^2\sqrt{bx^3+ax}}{77b}-\frac{4a^3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{(\right.}{77b^2\sqrt{bx^3+ax}}$

input
```
int((b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/77*(7*b^2*x^4+13*a*b*x^2+4*a^2)/b*x*(b*x^2+a)/(x*(b*x^2+a))^(1/2)-4/77*a
^3/b^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b
)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(
1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.38

$$\int \left(ax+bx^3\right)^{3/2}\,dx = -\frac{2\left(4\,a^3\sqrt{b}\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-(7\,b^3x^4+13\,ab^2x^2+4\,a^2b)\sqrt{bx^3+ax}\right)}{77\,b^2}$$

input
```
integrate((b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/77*(4*a^3*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) - (7*b^3*x^4 + 13*a
*b^2*x^2 + 4*a^2*b)*sqrt(b*x^3 + a*x))/b^2
```

Sympy [F]

$$\int \left(ax+bx^3\right)^{3/2} dx = \int \left(ax+bx^3\right)^{\frac{3}{2}} dx$$

input
```
integrate((b*x**3+a*x)**(3/2),x)
```

output
```
Integral((a*x + b*x**3)**(3/2), x)
```

Maxima [F]

$$\int \left(ax+bx^3\right)^{3/2} dx = \int \left(bx^3+ax\right)^{\frac{3}{2}} dx$$

input
```
integrate((b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x)^(3/2), x)
```

Giac [F]

$$\int \left(ax+bx^3\right)^{3/2} dx = \int \left(bx^3+ax\right)^{\frac{3}{2}} dx$$

input
```
integrate((b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 8.74 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.25

$$\int (ax+bx^3)^{3/2}\,dx=\frac{2\,x\,{\left(b\,x^3+a\,x\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{5}{4};\ \frac{9}{4};\ -\frac{b\,x^2}{a}\right)}{5\,{\left(\frac{b\,x^2}{a}+1\right)}^{3/2}}$$

input
```
int((a*x + b*x^3)^(3/2),x)
```

output
```
(2*x*(a*x + b*x^3)^(3/2)*hypergeom([-3/2, 5/4], 9/4, -(b*x^2)/a))/(5*((b*x
^2)/a + 1)^(3/2))
```

Reduce [F]

$$\int (ax+bx^3)^{3/2}\,dx=\frac{\frac{8\sqrt{x}\,\sqrt{b\,x^2+a}\,a^2}{77}+\frac{26\sqrt{x}\,\sqrt{b\,x^2+a}\,ab\,x^2}{77}+\frac{2\sqrt{x}\,\sqrt{b\,x^2+a}\,b^2x^4}{11}-\frac{4\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^3+ax}dx\right)a^3}{77}}{b}$$

input
```
int((b*x^3+a*x)^(3/2),x)
```

output
```
(2*(4*sqrt(x)*sqrt(a + b*x**2)*a**2 + 13*sqrt(x)*sqrt(a + b*x**2)*a*b*x**2
 + 7*sqrt(x)*sqrt(a + b*x**2)*b**2*x**4 - 2*int((sqrt(x)*sqrt(a + b*x**2))
/(a*x + b*x**3),x)*a**3))/(77*b)
```

3.56 $\int \frac{(ax+bx^3)^{3/2}}{x} \, dx$

Optimal result . . . 495
Mathematica [C] (verified) . . . 496
Rubi [A] (verified) . . . 496
Maple [A] (verified) . . . 500
Fricas [A] (verification not implemented) . . . 501
Sympy [F] . . . 501
Maxima [F] . . . 501
Giac [F] . . . 502
Mupad [F(-1)] . . . 502
Reduce [F] . . . 502

Optimal result

Integrand size = 17, antiderivative size = 275

$$\int \frac{(ax+bx^3)^{3/2}}{x} \, dx = \frac{8a^2x(a+bx^2)}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} + \frac{4}{15}ax\sqrt{ax+bx^3} + \frac{2}{9}\left(ax+bx^3\right)^{3/2} - \frac{8a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{ax+bx^3}} + \frac{4a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{ax+bx^3}}$$

output
```
8/15*a^2*x*(b*x^2+a)/b^(1/2)/(a^(1/2)+b^(1/2)*x)/(b*x^3+a*x)^(1/2)+4/15*a*
x*(b*x^3+a*x)^(1/2)+2/9*(b*x^3+a*x)^(3/2)-8/15*a^(9/4)*x^(1/2)*(a^(1/2)+b^
(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(b^
(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))/b^(3/4)/(b*x^3+a*x)^(1/2)+4/15*a^(9/4
)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*Inve
rseJacobiAM(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)),1/2*2^(1/2))/b^(3/4)/(b*x^3+
a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.19

$$\int \frac{(ax+bx^3)^{3/2}}{x}\,dx = \frac{2ax\sqrt{x(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)}{3\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a*x + b*x^3)^(3/2)/x,x]
```

output
```
(2*a*x*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^2)/a)]
)/(3*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1927, 1910, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^3)^{3/2}}{x}\,dx$$

$$\Big\downarrow \text{1927}$$

$$\frac{2}{3}a\int \sqrt{bx^3+ax}dx+\frac{2}{9}(ax+bx^3)^{3/2}$$

$$\Big\downarrow \text{1910}$$

$$\frac{2}{3}a\left(\frac{2}{5}a\int \frac{x}{\sqrt{bx^3+ax}}dx+\frac{2}{5}x\sqrt{ax+bx^3}\right)+\frac{2}{9}(ax+bx^3)^{3/2}$$

$$\Big\downarrow \text{1938}$$

$$\frac{2}{3}a\left(\frac{2a\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}\right)+\frac{2}{9}(ax+bx^3)^{3/2}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{3}a\left(\frac{4a\sqrt{x}\sqrt{a+bx^2}\int \frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}\right)+\frac{2}{9}\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{834}$$

$$\frac{2}{3}a\left(\frac{4a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}\right)+\frac{2}{9}\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}a\left(\frac{4a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}\right)+\frac{2}{9}\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{3}a\left(\frac{4a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}\right)+\frac{2}{9}\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{1510}$$

$$\frac{2}{3}a\left(\frac{4a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{5\sqrt{ax+bx^3}}\right.$$

$$\frac{2}{9}\left(ax+bx^3\right)^{3/2}$$

input
```
Int[(a*x + b*x^3)^(3/2)/x,x]
```

output
```
(2*(a*x + b*x^3)^(3/2))/9 + (2*a*((2*x*Sqrt[a*x + b*x^3])/5 + (4*a*Sqrt[x]
*Sqrt[a + b*x^2]*(-((-((Sqrt[x]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*x)) +
(a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*E
llipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^
2]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] +
 Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(
3/4)*Sqrt[a + b*x^2])))/(5*Sqrt[a*x + b*x^3])))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.71

method	result
default	$\frac{2b\,x^3\sqrt{b\,x^3+ax}}{9}+\frac{22ax\sqrt{b\,x^3+ax}}{45}+\frac{4a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\operatorname{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\right.}{b}\right.}{15b\sqrt{b\,x^3+ax}}$
elliptic	$\frac{2b\,x^3\sqrt{b\,x^3+ax}}{9}+\frac{22ax\sqrt{b\,x^3+ax}}{45}+\frac{4a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\operatorname{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\right.}{b}\right.}{15b\sqrt{b\,x^3+ax}}$
risch	$\frac{2x^2\left(5b\,x^2+11a\right)\left(b\,x^2+a\right)}{45\sqrt{x\left(b\,x^2+a\right)}}+\frac{4a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\operatorname{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{15b\sqrt{b\,x^3+ax}}$

input
```
int((b*x^3+a*x)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/9*b*x^3*(b*x^3+a*x)^(1/2)+22/45*a*x*(b*x^3+a*x)^(1/2)+4/15*a^2*(-a*b)^(1
/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a
*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b
)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^
(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.21

$$\int \frac{(ax+bx^3)^{3/2}}{x}\,dx = -\frac{2\left(12\,a^2\sqrt{b}\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)-(5\,b^2x^3+11\,abx)\sqrt{bx^3+ax}\right)}{45\,b}$$

input
```
integrate((b*x^3+a*x)^(3/2)/x,x, algorithm="fricas")
```

output
```
-2/45*(12*a^2*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/
b, 0, x)) - (5*b^2*x^3 + 11*a*b*x)*sqrt(b*x^3 + a*x))/b
```

Sympy [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x}\,dx = \int \frac{(x(a+bx^2))^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((b*x**3+a*x)**(3/2)/x,x)
```

output
```
Integral((x*(a + b*x**2))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x, x)
```

Giac [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x}\,dx=\int \frac{(bx^3+ax)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^3)^{3/2}}{x}\,dx=\int \frac{(b\,x^3+a\,x)^{3/2}}{x}\,dx$$

input
```
int((a*x + b*x^3)^(3/2)/x,x)
```

output
```
int((a*x + b*x^3)^(3/2)/x, x)
```

Reduce [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x}\,dx=\frac{22\sqrt{x}\,\sqrt{b\,x^2+a}\,ax}{45}+\frac{2\sqrt{x}\,\sqrt{b\,x^2+a}\,b\,x^3}{9}+\frac{4\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^2+a}dx\right)a^2}{15}$$

input
```
int((b*x^3+a*x)^(3/2)/x,x)
```

output
```
(2*(11*sqrt(x)*sqrt(a + b*x**2)*a*x + 5*sqrt(x)*sqrt(a + b*x**2)*b*x**3 +
6*int((sqrt(x)*sqrt(a + b*x**2))/(a + b*x**2),x)*a**2))/45
```

3.57 $\int \frac{(ax+bx^3)^{3/2}}{x^2} dx$

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Optimal result

Integrand size = 17, antiderivative size = 134

$$\int \frac{(ax+bx^3)^{3/2}}{x^2} dx = \frac{4}{7}a\sqrt{ax+bx^3} + \frac{2(ax+bx^3)^{3/2}}{7x} + \frac{4a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{ax+bx^3}}$$

output
```
4/7*a*(b*x^3+a*x)^(1/2)+2/7*(b*x^3+a*x)^(3/2)/x+4/7*a^(7/4)*x^(1/2)*(a^(1/
2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*ar
ctan(b^(1/4)*x^(1/2)/a^(1/4)),1/2*2^(1/2))/b^(1/4)/(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.37

$$\int \frac{(ax+bx^3)^{3/2}}{x^2} dx = \frac{2a\sqrt{x(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^2}{a}\right)}{\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a*x + b*x^3)^(3/2)/x^2,x]
```

output
```
(2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)])/
Sqrt[1 + (b*x^2)/a]
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1927, 1927, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^3)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{1927}$$

$$\frac{6}{7}a\int \frac{\sqrt{bx^3+ax}}{x}dx+\frac{2(ax+bx^3)^{3/2}}{7x}$$

$$\Big\downarrow \text{1927}$$

$$\frac{6}{7}a\left(\frac{2}{3}a\int \frac{1}{\sqrt{bx^3+ax}}dx+\frac{2}{3}\sqrt{ax+bx^3}\right)+\frac{2(ax+bx^3)^{3/2}}{7x}$$

$$\Big\downarrow \text{1917}$$

$$\frac{6}{7}a\left(\frac{2a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3\sqrt{ax+bx^3}}+\frac{2}{3}\sqrt{ax+bx^3}\right)+\frac{2(ax+bx^3)^{3/2}}{7x}$$

$$\Big\downarrow \text{266}$$

$$\frac{6}{7}a\left(\frac{4a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3\sqrt{ax+bx^3}}+\frac{2}{3}\sqrt{ax+bx^3}\right)+\frac{2(ax+bx^3)^{3/2}}{7x}$$

$$\Big\downarrow \text{761}$$

$$\frac{6}{7}a\left(\frac{2a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{ax+bx^3}}+\frac{2}{3}\sqrt{ax+bx^3}\right)+\frac{2\left(ax+bx^3\right)^{3/2}}{7x}$$

input
```
Int[(a*x + b*x^3)^(3/2)/x^2,x]
```

output
```
(2*(a*x + b*x^3)^(3/2))/(7*x) + (6*a*((2*Sqrt[a*x + b*x^3])/3 + (2*a^(3/4)
*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*E
llipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a*x +
b*x^3])))/7
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.07

method	result	s
risch	$\frac{2(b\,x^2+3a)x(b\,x^2+a)}{7\sqrt{x(b\,x^2+a)}}+\frac{4a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{7b\sqrt{b\,x^3+ax}}$	1
default	$\frac{2b\,x^2\sqrt{b\,x^3+ax}}{7}+\frac{6a\sqrt{b\,x^3+ax}}{7}+\frac{4a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{7b\sqrt{b\,x^3+ax}}$	1
elliptic	$\frac{2b\,x^2\sqrt{b\,x^3+ax}}{7}+\frac{6a\sqrt{b\,x^3+ax}}{7}+\frac{4a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{7b\sqrt{b\,x^3+ax}}$	1

input
```
int((b*x^3+a*x)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2/7*(b*x^2+3*a)*x*(b*x^2+a)/(x*(b*x^2+a))^(1/2)+4/7*a^2*(-a*b)^(1/2)/b*((x
+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)
*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*
b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.35

$$\int \frac{(ax+bx^3)^{3/2}}{x^2}\,dx = \frac{2\left(4\,a^2\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+(b^2x^2+3\,ab)\sqrt{bx^3+ax}\right)}{7\,b}$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
2/7*(4*a^2*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + (b^2*x^2 + 3*a*b)*s
qrt(b*x^3 + a*x))/b
```

Sympy [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^2}\,dx = \int \frac{(x(a+bx^2))^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x**3+a*x)**(3/2)/x**2,x)
```

output
```
Integral((x*(a + b*x**2))**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^2}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^2}\,dx=\int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^3)^{3/2}}{x^2}\,dx=\int \frac{(b\,x^3+a\,x)^{3/2}}{x^2}\,dx$$

input
```
int((a*x + b*x^3)^(3/2)/x^2,x)
```

output
```
int((a*x + b*x^3)^(3/2)/x^2, x)
```

Reduce [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^2}\,dx=\frac{6\sqrt{x}\,\sqrt{b\,x^2+a}\,a}{7}+\frac{2\sqrt{x}\,\sqrt{b\,x^2+a}\,b\,x^2}{7}+\frac{4\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^3+ax}dx\right)a^2}{7}$$

input
```
int((b*x^3+a*x)^(3/2)/x^2,x)
```

output
```
(2*(3*sqrt(x)*sqrt(a + b*x**2)*a + sqrt(x)*sqrt(a + b*x**2)*b*x**2 + 2*int
((sqrt(x)*sqrt(a + b*x**2))/(a*x + b*x**3),x)*a**2))/7
```

3.58 $\int \frac{(ax+bx^3)^{3/2}}{x^3}\,dx$

Optimal result . . . 509
Mathematica [C] (verified) . . . 510
Rubi [A] (verified) . . . 510
Maple [A] (verified) . . . 514
Fricas [A] (verification not implemented) . . . 515
Sympy [F] . . . 515
Maxima [F] . . . 515
Giac [F] . . . 516
Mupad [F(-1)] . . . 516
Reduce [F] . . . 516

Optimal result

Integrand size = 17, antiderivative size = 274

$$\int \frac{(ax+bx^3)^{3/2}}{x^3}\,dx = \frac{24a\sqrt{b}x(a+bx^2)}{5\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} + \frac{12}{5}bx\sqrt{ax+bx^3} - \frac{2(ax+bx^3)^{3/2}}{x^2} - \frac{24a^{5/4}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{ax+bx^3}} + \frac{12a^{5/4}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5\sqrt{ax+bx^3}}$$

output
```
24/5*a*b^(1/2)*x*(b*x^2+a)/(a^(1/2)+b^(1/2)*x)/(b*x^3+a*x)^(1/2)+12/5*b*x*
(b*x^3+a*x)^(1/2)-2*(b*x^3+a*x)^(3/2)/x^2-24/5*a^(5/4)*b^(1/4)*x^(1/2)*(a^
(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*a
rctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))/(b*x^3+a*x)^(1/2)+12/5*a^(5/4
)*b^(1/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1
/2)*InverseJacobiAM(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)),1/2*2^(1/2))/(b*x^3+
a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.19

$$\int \frac{(ax+bx^3)^{3/2}}{x^3}\,dx = -\frac{2a\sqrt{x(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{4},\frac{3}{4},-\frac{bx^2}{a}\right)}{x\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a*x + b*x^3)^(3/2)/x^3,x]
```

output
```
(-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, -1/4, 3/4, -((b*x^2)/a)]
)/(x*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1926, 1910, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^3)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{1926}$$

$$6b\int \sqrt{bx^3+ax}dx-\frac{2(ax+bx^3)^{3/2}}{x^2}$$

$$\Big\downarrow \text{1910}$$

$$6b\left(\frac{2}{5}a\int \frac{x}{\sqrt{bx^3+ax}}dx+\frac{2}{5}x\sqrt{ax+bx^3}\right)-\frac{2(ax+bx^3)^{3/2}}{x^2}$$

$$\Big\downarrow \text{1938}$$

$$6b\left(\frac{2a\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}$$

$\Big\downarrow$ 266

$$6b\left(\frac{4a\sqrt{x}\sqrt{a+bx^2}\int \frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}$$

$\Big\downarrow$ 834

$$6b\left(\frac{4a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}$$

$\Big\downarrow$ 27

$$6b\left(\frac{4a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}$$

$\Big\downarrow$ 761

$$6b\left(\frac{4a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}$$

$\Big\downarrow$ 1510

$$6b\left(\frac{4a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{}}\right.\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{5\sqrt{ax+bx^3}}\right.$$

$$\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}$$

input
```
Int[(a*x + b*x^3)^(3/2)/x^3,x]
```

output
```
(-2*(a*x + b*x^3)^(3/2))/x^2 + 6*b*((2*x*Sqrt[a*x + b*x^3])/5 + (4*a*Sqrt[
x]*Sqrt[a + b*x^2]*(-((-(-((Sqrt[x]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*x))
 + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]
*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*
x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
 + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b
^(3/4)*Sqrt[a + b*x^2])))/(5*Sqrt[a*x + b*x^3]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{2(bx^2+a)(-bx^2+5a)}{5\sqrt{x(bx^2+a)}}+\frac{12a\sqrt{-ab}\sqrt{\frac{(x+\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}}\sqrt{-\frac{2(x-\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\Big(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\Big(\sqrt{\frac{(x+\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\Big)}{b}+}{5\sqrt{bx^3+ax}}$
default	$-\frac{2(bx^2+a)a}{\sqrt{x(bx^2+a)}}+\frac{2bx\sqrt{bx^3+ax}}{5}+\frac{12a\sqrt{-ab}\sqrt{\frac{(x+\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}}\sqrt{-\frac{2(x-\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\Big(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\Big(\sqrt{\frac{(x+\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}},}{b}}{5\sqrt{bx^3+ax}}$
elliptic	$-\frac{2(bx^2+a)a}{\sqrt{x(bx^2+a)}}+\frac{2bx\sqrt{bx^3+ax}}{5}+\frac{12a\sqrt{-ab}\sqrt{\frac{(x+\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}}\sqrt{-\frac{2(x-\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\Big(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\Big(\sqrt{\frac{(x+\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}},}{b}}{5\sqrt{bx^3+ax}}$

input
```
int((b*x^3+a*x)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-2/5*(b*x^2+a)*(-b*x^2+5*a)/(x*(b*x^2+a))^(1/2)+12/5*a*(-a*b)^(1/2)*((x+(-
a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)
^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*El
lipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2
)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.19

$$\int \frac{(ax+bx^3)^{3/2}}{x^3}\,dx = -\frac{2\left(12\,a\sqrt{b}x\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)-\sqrt{bx^3+ax}(bx^2-5\,a)\right)}{5\,x}$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
-2/5*(12*a*sqrt(b)*x*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b
, 0, x)) - sqrt(b*x^3 + a*x)*(b*x^2 - 5*a))/x
```

Sympy [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^3}\,dx = \int \frac{(x(a+bx^2))^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b*x**3+a*x)**(3/2)/x**3,x)
```

output
```
Integral((x*(a + b*x**2))**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^3}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^3, x)
```

Giac [F]

$$\int \frac{(ax + bx^3)^{3/2}}{x^3}\,dx = \int \frac{(bx^3 + ax)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^3,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax + bx^3)^{3/2}}{x^3}\,dx = \int \frac{(b\,x^3 + a\,x)^{3/2}}{x^3}\,dx$$

input
```
int((a*x + b*x^3)^(3/2)/x^3,x)
```

output
```
int((a*x + b*x^3)^(3/2)/x^3, x)
```

Reduce [F]

$$\int \frac{(ax + bx^3)^{3/2}}{x^3}\,dx = \frac{\frac{14\sqrt{b\,x^2+a}\,a}{5} + \frac{2\sqrt{b\,x^2+a}\,b\,x^2}{5} + \frac{12\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^4+a\,x^2}dx\right)a^2}{5}}{\sqrt{x}}$$

input
```
int((b*x^3+a*x)^(3/2)/x^3,x)
```

output
```
(2*(7*sqrt(a + b*x**2)*a + sqrt(a + b*x**2)*b*x**2 + 6*sqrt(x)*int((sqrt(x
)*sqrt(a + b*x**2))/(a*x**2 + b*x**4),x)*a**2))/(5*sqrt(x))
```

3.59 $\int \frac{(ax+bx^3)^{3/2}}{x^4}\,dx$

Optimal result 517
Mathematica [C] (verified) 517
Rubi [A] (verified) 518
Maple [A] (verified) 520
Fricas [A] (verification not implemented) 521
Sympy [F] 521
Maxima [F] 521
Giac [F] 522
Mupad [F(-1)] 522
Reduce [F] 522

Optimal result

Integrand size = 17, antiderivative size = 134

$$\int \frac{(ax+bx^3)^{3/2}}{x^4}\,dx = \frac{4}{3}b\sqrt{ax+bx^3} - \frac{2(ax+bx^3)^{3/2}}{3x^3} + \frac{4a^{3/4}b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt{ax+bx^3}}$$

output
```
4/3*b*(b*x^3+a*x)^(1/2)-2/3*(b*x^3+a*x)^(3/2)/x^3+4/3*a^(3/4)*b^(3/4)*x^(1
/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJac
obiAM(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)),1/2*2^(1/2))/(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.40

$$\int \frac{(ax+bx^3)^{3/2}}{x^4}\,dx = -\frac{2a\sqrt{x\,(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},\frac{1}{4},-\frac{bx^2}{a}\right)}{3x^2\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a*x + b*x^3)^(3/2)/x^4,x]
```

output
```
(-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^2)/a)]
)/(3*x^2*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1926, 1927, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^3\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{ 1926}$$

$$2b\int \frac{\sqrt{bx^3+ax}}{x}dx-\frac{2\left(ax+bx^3\right)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{ 1927}$$

$$2b\left(\frac{2}{3}a\int \frac{1}{\sqrt{bx^3+ax}}dx+\frac{2}{3}\sqrt{ax+bx^3}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{ 1917}$$

$$2b\left(\frac{2a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3\sqrt{ax+bx^3}}+\frac{2}{3}\sqrt{ax+bx^3}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{ 266}$$

$$2b\left(\frac{4a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3\sqrt{ax+bx^3}}+\frac{2}{3}\sqrt{ax+bx^3}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{ 761}$$

$$2b\left(\frac{2a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{ax+bx^3}}+\frac{2}{3}\sqrt{ax+bx^3}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{3x^3}$$

input
```
Int[(a*x + b*x^3)^(3/2)/x^4,x]
```

output
```
(-2*(a*x + b*x^3)^(3/2))/(3*x^3) + 2*b*((2*Sqrt[a*x + b*x^3])/3 + (2*a^(3/
4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]
*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a*x
+ b*x^3]))
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.04

method	result	si
default	$-\frac{2a\sqrt{b\,x^3+ax}}{3x^2}+\frac{2b\sqrt{b\,x^3+ax}}{3}+\frac{4a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3\sqrt{b\,x^3+ax}}$	13
risch	$-\frac{2(b\,x^2+a)(-b\,x^2+a)}{3x\sqrt{x(b\,x^2+a)}}+\frac{4a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3\sqrt{b\,x^3+ax}}$	13
elliptic	$-\frac{2a\sqrt{b\,x^3+ax}}{3x^2}+\frac{2b\sqrt{b\,x^3+ax}}{3}+\frac{4a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3\sqrt{b\,x^3+ax}}$	13

input
```
int((b*x^3+a*x)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/3*a*(b*x^3+a*x)^(1/2)/x^2+2/3*b*(b*x^3+a*x)^(1/2)+4/3*a*(-a*b)^(1/2)*((
x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2
)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a
*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.34

$$\int \frac{(ax+bx^3)^{3/2}}{x^4}\,dx = \frac{2\left(4\,a\sqrt{b}x^2\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\sqrt{bx^3+ax}(bx^2-a)\right)}{3\,x^2}$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
2/3*(4*a*sqrt(b)*x^2*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b*x^3 + a*x)
*(b*x^2 - a))/x^2
```

Sympy [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^4}\,dx = \int \frac{(x(a+bx^2))^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x**3+a*x)**(3/2)/x**4,x)
```

output
```
Integral((x*(a + b*x**2))**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^4}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^4, x)
```

Giac [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^4}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^4,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^3)^{3/2}}{x^4}\,dx = \int \frac{(b\,x^3+a\,x)^{3/2}}{x^4}\,dx$$

input
```
int((a*x + b*x^3)^(3/2)/x^4,x)
```

output
```
int((a*x + b*x^3)^(3/2)/x^4, x)
```

Reduce [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^4}\,dx = \frac{-\frac{10\sqrt{b\,x^2+a}\,a}{3}+\frac{2\sqrt{b\,x^2+a}\,b\,x^2}{3}-4\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^5+a\,x^3}dx\right)a^2x}{\sqrt{x}\,x}$$

input
```
int((b*x^3+a*x)^(3/2)/x^4,x)
```

output
```
(2*( - 5*sqrt(a + b*x**2)*a + sqrt(a + b*x**2)*b*x**2 - 6*sqrt(x)*int((sqr
t(x)*sqrt(a + b*x**2))/(a*x**3 + b*x**5),x)*a**2*x))/(3*sqrt(x)*x)
```

3.60 $\int \frac{(ax+bx^3)^{3/2}}{x^5}\,dx$

Optimal result . . . 523
Mathematica [C] (verified) . . . 524
Rubi [A] (verified) . . . 524
Maple [A] (verified) . . . 528
Fricas [A] (verification not implemented) . . . 529
Sympy [F] . . . 529
Maxima [F] . . . 529
Giac [F] . . . 530
Mupad [F(-1)] . . . 530
Reduce [F] . . . 530

Optimal result

Integrand size = 17, antiderivative size = 277

$$\int \frac{(ax+bx^3)^{3/2}}{x^5}\,dx = \frac{24b^{3/2}x(a+bx^2)}{5\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{12b\sqrt{ax+bx^3}}{5x} - \frac{2(ax+bx^3)^{3/2}}{5x^4} - \frac{24\sqrt[4]{a}b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{ax+bx^3}} + \frac{12\sqrt[4]{a}b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5\sqrt{ax+bx^3}}$$

output
```
24/5*b^(3/2)*x*(b*x^2+a)/(a^(1/2)+b^(1/2)*x)/(b*x^3+a*x)^(1/2)-12/5*b*(b*x
^3+a*x)^(1/2)/x-2/5*(b*x^3+a*x)^(3/2)/x^4-24/5*a^(1/4)*b^(5/4)*x^(1/2)*(a^
(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*a
rctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))/(b*x^3+a*x)^(1/2)+12/5*a^(1/4
)*b^(5/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1
/2)*InverseJacobiAM(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)),1/2*2^(1/2))/(b*x^3+
a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.19

$$\int \frac{(ax+bx^3)^{3/2}}{x^5}\,dx = -\frac{2a\sqrt{x\,(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{5}{4},-\frac{1}{4},-\frac{bx^2}{a}\right)}{5x^3\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a*x + b*x^3)^(3/2)/x^5,x]
```

output
```
(-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^2)/a)
])/(5*x^3*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1926, 1926, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^3)^{3/2}}{x^5}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{6}{5}b\int \frac{\sqrt{bx^3+ax}}{x^2}dx-\frac{2(ax+bx^3)^{3/2}}{5x^4}$$

$$\Big\downarrow \text{1926}$$

$$\frac{6}{5}b\left(2b\int \frac{x}{\sqrt{bx^3+ax}}dx-\frac{2\sqrt{ax+bx^3}}{x}\right)-\frac{2(ax+bx^3)^{3/2}}{5x^4}$$

$$\Big\downarrow \text{1938}$$

$$\frac{6}{5}b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}$$

$\Big\downarrow$ 266

$$\frac{6}{5}b\left(\frac{4b\sqrt{x}\sqrt{a+bx^2}\int \frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}$$

$\Big\downarrow$ 834

$$\frac{6}{5}b\left(\frac{4b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}$$

$\Big\downarrow$ 27

$$\frac{6}{5}b\left(\frac{4b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}$$

$\Big\downarrow$ 761

$$\frac{6}{5}b\left(\frac{4b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}$$

$\Big\downarrow$ 1510

$$\frac{6}{5}b\left(\frac{4b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\ldots}{\sqrt[4]{\ldots}}\right.\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\ldots\right.}{\sqrt{ax+bx^3}}\ldots\right.$$

$$\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}$$

input
```
Int[(a*x + b*x^3)^(3/2)/x^5,x]
```

output
```
(-2*(a*x + b*x^3)^(3/2))/(5*x^4) + (6*b*((-2*Sqrt[a*x + b*x^3])/x + (4*b*S
qrt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*
x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x
)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a
+ b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqr
t[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/
(2*b^(3/4)*Sqrt[a + b*x^2])))/Sqrt[a*x + b*x^3]))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{2(b\,x^2+a)(7b\,x^2+a)}{5x^2\sqrt{x(b\,x^2+a)}}+\frac{12b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-}}{}$
default	$-\frac{2a\sqrt{b\,x^3+ax}}{5x^3}-\frac{14(b\,x^2+a)b}{5\sqrt{x(b\,x^2+a)}}+\frac{12b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\right.}{b}\right.}{5\sqrt{b\,x^3+ax}}$
elliptic	$-\frac{2a\sqrt{b\,x^3+ax}}{5x^3}-\frac{14(b\,x^2+a)b}{5\sqrt{x(b\,x^2+a)}}+\frac{12b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\right.}{b}\right.}{5\sqrt{b\,x^3+ax}}$

input
```
int((b*x^3+a*x)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-2/5*(b*x^2+a)*(7*b*x^2+a)/x^2/(x*(b*x^2+a))^(1/2)+12/5*b*(-a*b)^(1/2)*((x
+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)
*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b
*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(
1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.18

$$\int \frac{(ax+bx^3)^{3/2}}{x^5}\,dx = -\frac{2\left(12\,b^{\frac{3}{2}}x^3\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\sqrt{bx^3+ax}(7\,bx^2+a)\right)}{5\,x^3}$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
-2/5*(12*b^(3/2)*x^3*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b
, 0, x)) + sqrt(b*x^3 + a*x)*(7*b*x^2 + a))/x^3
```

Sympy [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^5}\,dx = \int \frac{(x(a+bx^2))^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x**3+a*x)**(3/2)/x**5,x)
```

output
```
Integral((x*(a + b*x**2))**(3/2)/x**5, x)
```

Maxima [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^5}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^5, x)
```

Giac [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^5}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^5,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^3)^{3/2}}{x^5}\,dx = \int \frac{(b\,x^3+a\,x)^{3/2}}{x^5}\,dx$$

input
```
int((a*x + b*x^3)^(3/2)/x^5,x)
```

output
```
int((a*x + b*x^3)^(3/2)/x^5, x)
```

Reduce [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^5}\,dx = \frac{-2\sqrt{b\,x^2+a}\,a+2\sqrt{b\,x^2+a}\,b\,x^2-4\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^6+a\,x^4}dx\right)a^2x^2}{\sqrt{x}\,x^2}$$

input
```
int((b*x^3+a*x)^(3/2)/x^5,x)
```

output
```
(2*( - sqrt(a + b*x**2)*a + sqrt(a + b*x**2)*b*x**2 - 2*sqrt(x)*int((sqrt(
x)*sqrt(a + b*x**2))/(a*x**4 + b*x**6),x)*a**2*x**2))/(sqrt(x)*x**2)
```

3.61 $\int \frac{(ax+bx^3)^{3/2}}{x^6} dx$

Optimal result . . . 531
Mathematica [C] (verified) . . . 531
Rubi [A] (verified) . . . 532
Maple [A] (verified) . . . 534
Fricas [A] (verification not implemented) . . . 535
Sympy [F] . . . 535
Maxima [F] . . . 535
Giac [F] . . . 536
Mupad [F(-1)] . . . 536
Reduce [F] . . . 536

Optimal result

Integrand size = 17, antiderivative size = 137

$$\int \frac{(ax+bx^3)^{3/2}}{x^6} dx = -\frac{4b\sqrt{ax+bx^3}}{7x^2} - \frac{2(ax+bx^3)^{3/2}}{7x^5} + \frac{4b^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{7\sqrt[4]{a}\sqrt{ax+bx^3}}$$

output
```
-4/7*b*(b*x^3+a*x)^(1/2)/x^2-2/7*(b*x^3+a*x)^(3/2)/x^5+4/7*b^(7/4)*x^(1/2)
*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobi
AM(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)),1/2*2^(1/2))/a^(1/4)/(b*x^3+a*x)^(1/2
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.39

$$\int \frac{(ax+bx^3)^{3/2}}{x^6} dx = -\frac{2a\sqrt{x(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{3}{2},-\frac{3}{4},-\frac{bx^2}{a}\right)}{7x^4\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a*x + b*x^3)^(3/2)/x^6,x]
```

output
```
(-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-7/4, -3/2, -3/4, -((b*x^2)/a)
])/(7*x^4*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1926, 1926, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^3\right)^{3/2}}{x^6}dx$$

$$\Big\downarrow \text{ 1926}$$

$$\frac{6}{7}b\int \frac{\sqrt{bx^3+ax}}{x^3}dx-\frac{2\left(ax+bx^3\right)^{3/2}}{7x^5}$$

$$\Big\downarrow \text{ 1926}$$

$$\frac{6}{7}b\left(\frac{2}{3}b\int \frac{1}{\sqrt{bx^3+ax}}dx-\frac{2\sqrt{ax+bx^3}}{3x^2}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{7x^5}$$

$$\Big\downarrow \text{ 1917}$$

$$\frac{6}{7}b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3x^2}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{7x^5}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{6}{7}b\left(\frac{4b\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3x^2}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{7x^5}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{6}{7}b\left(\frac{2b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3x^2}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{7x^5}$$

input
```
Int[(a*x + b*x^3)^(3/2)/x^6,x]
```

output
```
(-2*(a*x + b*x^3)^(3/2))/(7*x^5) + (6*b*((-2*Sqrt[a*x + b*x^3])/(3*x^2) +
(2*b^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[
b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*a^(1/4)*S
qrt[a*x + b*x^3])))/7
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01

method	result	si
risch	$-\frac{2(b x^2+a)(3b x^2+a)}{7x^3\sqrt{x(b x^2+a)}}+\frac{4b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{7\sqrt{b x^3+ax}}$	13
default	$-\frac{2a\sqrt{b x^3+ax}}{7x^4}-\frac{6b\sqrt{b x^3+ax}}{7x^2}+\frac{4b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{7\sqrt{b x^3+ax}}$	14
elliptic	$-\frac{2a\sqrt{b x^3+ax}}{7x^4}-\frac{6b\sqrt{b x^3+ax}}{7x^2}+\frac{4b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{7\sqrt{b x^3+ax}}$	14

input
```
int((b*x^3+a*x)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-2/7*(b*x^2+a)*(3*b*x^2+a)/x^3/(x*(b*x^2+a))^(1/2)+4/7*b*(-a*b)^(1/2)*((x+
(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*
b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b
)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.32

$$\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx = \frac{2\left(4\,b^{\frac{3}{2}}x^4\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-\sqrt{bx^3+ax}(3\,bx^2+a)\right)}{7\,x^4}$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
2/7*(4*b^(3/2)*x^4*weierstrassPInverse(-4*a/b, 0, x) - sqrt(b*x^3 + a*x)*(
3*b*x^2 + a))/x^4
```

Sympy [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx = \int \frac{(x(a+bx^2))^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x**3+a*x)**(3/2)/x**6,x)
```

output
```
Integral((x*(a + b*x**2))**(3/2)/x**6, x)
```

Maxima [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^6, x)
```

Giac [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^6,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx = \int \frac{(b\,x^3+a\,x)^{3/2}}{x^6}\,dx$$

input
```
int((a*x + b*x^3)^(3/2)/x^6,x)
```

output
```
int((a*x + b*x^3)^(3/2)/x^6, x)
```

Reduce [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx = \frac{\frac{2\sqrt{b\,x^2+a}\,a}{5} - 2\sqrt{b\,x^2+a}\,b\,x^2 + \frac{12\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^7+a\,x^5}dx\right)a^2x^3}{5}}{\sqrt{x}\,x^3}$$

input
```
int((b*x^3+a*x)^(3/2)/x^6,x)
```

output
```
(2*(sqrt(a + b*x**2)*a - 5*sqrt(a + b*x**2)*b*x**2 + 6*sqrt(x)*int((sqrt(x
)*sqrt(a + b*x**2))/(a*x**5 + b*x**7),x)*a**2*x**3))/(5*sqrt(x)*x**3)
```

3.62 $\int \frac{(ax+bx^3)^{3/2}}{x^7} dx$

Optimal result 537
Mathematica [C] (verified) 538
Rubi [A] (verified) 538
Maple [A] (verified) 542
Fricas [A] (verification not implemented) 543
Sympy [F] 543
Maxima [F] 544
Giac [F] 544
Mupad [F(-1)] 544
Reduce [F] 545

Optimal result

Integrand size = 17, antiderivative size = 306

$$\int \frac{(ax+bx^3)^{3/2}}{x^7} dx = \frac{8b^{5/2}x(a+bx^2)}{15a\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{4b\sqrt{ax+bx^3}}{15x^3} - \frac{8b^2\sqrt{ax+bx^3}}{15ax} - \frac{2(ax+bx^3)^{3/2}}{9x^6} - \frac{8b^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{ax+bx^3}} + \frac{4b^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15a^{3/4}\sqrt{ax+bx^3}}$$

output

```
8/15*b^(5/2)*x*(b*x^2+a)/a/(a^(1/2)+b^(1/2)*x)/(b*x^3+a*x)^(1/2)-4/15*b*(b
*x^3+a*x)^(1/2)/x^3-8/15*b^2*(b*x^3+a*x)^(1/2)/a/x-2/9*(b*x^3+a*x)^(3/2)/x
^6-8/15*b^(9/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)
^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))/a^
(3/4)/(b*x^3+a*x)^(1/2)+4/15*b^(9/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a
)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x^(1/2)/a^
(1/4)),1/2*2^(1/2))/a^(3/4)/(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.18

$$\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx = -\frac{2a\sqrt{x\,(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},-\frac{3}{2},-\frac{5}{4},-\frac{bx^2}{a}\right)}{9x^5\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a*x + b*x^3)^(3/2)/x^7,x]
```

output
```
(-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-9/4, -3/2, -5/4, -((b*x^2)/a)
])/(9*x^5*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1926, 1926, 1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^3)^{3/2}}{x^7}dx$$

$$\Big\downarrow \text{ 1926}$$

$$\frac{2}{3}b\int \frac{\sqrt{bx^3+ax}}{x^4}dx-\frac{2(ax+bx^3)^{3/2}}{9x^6}$$

$$\Big\downarrow \text{ 1926}$$

$$\frac{2}{3}b\left(\frac{2}{5}b\int \frac{1}{x\sqrt{bx^3+ax}}dx-\frac{2\sqrt{ax+bx^3}}{5x^3}\right)-\frac{2(ax+bx^3)^{3/2}}{9x^6}$$

$$\Big\downarrow \text{ 1931}$$

$$\frac{2}{3}b\left(\frac{2}{5}b\left(\frac{b\int \frac{x}{\sqrt{bx^3+ax}}dx}{a}-\frac{2\sqrt{ax+bx^3}}{ax}\right)-\frac{2\sqrt{ax+bx^3}}{5x^3}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{9x^6}$$

$$\Big\downarrow \text{ 1938}$$

$$\frac{2}{3}b\left(\frac{2}{5}b\left(\frac{b\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)-\frac{2\sqrt{ax+bx^3}}{5x^3}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{9x^6}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2}{3}b\left(\frac{2}{5}b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\int \frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)-\frac{2\sqrt{ax+bx^3}}{5x^3}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{9x^6}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{2}{3}b\left(\frac{2}{5}b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)-\frac{2\sqrt{ax+bx^3}}{5x^3}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{9x^6}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{3}b\left(\frac{2}{5}b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)-\frac{2\sqrt{ax+bx^3}}{5x^3}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{9x^6}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{2}{3}b\left(\frac{2}{5}b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right.\right.$$

$$\frac{2\left(ax+bx^3\right)^{3/2}}{9x^6}$$

$\big\downarrow$ 1510

$$\frac{2}{3}b\left(\frac{2}{5}b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{a\sqrt{ax+bx^3}}\right.\right.$$

$$\frac{2\left(ax+bx^3\right)^{3/2}}{9x^6}$$

input
```
Int[(a*x + b*x^3)^(3/2)/x^7,x]
```

output
```
(-2*(a*x + b*x^3)^(3/2))/(9*x^6) + (2*b*((-2*Sqrt[a*x + b*x^3])/(5*x^3) +
(2*b*((-2*Sqrt[a*x + b*x^3])/(a*x) + (2*b*Sqrt[x]*Sqrt[a + b*x^2]*(-((-((S
qrt[x]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[
b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4
)*Sqrt[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*
(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[
2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(
a*Sqrt[a*x + b*x^3]))/5))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{2(b x^2+a)(12b^2x^4+11ab x^2+5a^2)}{45x^4\sqrt{x(b x^2+a)}\,a}+\frac{4b^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{-ab}}}\right.}{b}\right.}{15a\sqrt{b x^3+ax}}$
default	$-\frac{2a\sqrt{b x^3+ax}}{9x^5}-\frac{22b\sqrt{b x^3+ax}}{45x^3}-\frac{8(b x^2+a)b^2}{15a\sqrt{x(b x^2+a)}}+\frac{4b^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \text{Ellipt}}{}\right.}{15a\sqrt{b x^3}}$
elliptic	$-\frac{2a\sqrt{b x^3+ax}}{9x^5}-\frac{22b\sqrt{b x^3+ax}}{45x^3}-\frac{8(b x^2+a)b^2}{15a\sqrt{x(b x^2+a)}}+\frac{4b^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \text{Ellipt}}{}\right.}{15a\sqrt{b x^3}}$

input
```
int((b*x^3+a*x)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-2/45*(b*x^2+a)*(12*b^2*x^4+11*a*b*x^2+5*a^2)/x^4/(x*(b*x^2+a))^(1/2)/a+4/
15/a*b^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a
*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)
^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1
/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)
*b)^(1/2),1/2*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.22

$$\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx = -\frac{2\left(12\,b^{\frac{5}{2}}x^5\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\left(12\,b^2x^4+11\,abx^2+5\,a^2\right)\sqrt{bx^3+}\right.}{45\,ax^5}$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
-2/45*(12*b^(5/2)*x^5*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/
b, 0, x)) + (12*b^2*x^4 + 11*a*b*x^2 + 5*a^2)*sqrt(b*x^3 + a*x))/(a*x^5)
```

Sympy [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx = \int \frac{(x(a+bx^2))^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((b*x**3+a*x)**(3/2)/x**7,x)
```

output
```
Integral((x*(a + b*x**2))**(3/2)/x**7, x)
```

Maxima [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^7, x)
```

Giac [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^7,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx = \int \frac{(b\,x^3+a\,x)^{3/2}}{x^7}\,dx$$

input
```
int((a*x + b*x^3)^(3/2)/x^7,x)
```

output
```
int((a*x + b*x^3)^(3/2)/x^7, x)
```

Reduce [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx = \frac{-\frac{2\sqrt{b\,x^2+a}\,a}{21}-\frac{2\sqrt{b\,x^2+a}\,b\,x^2}{3}+\frac{4\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^8+a\,x^6}dx\right)a^2x^4}{7}}{\sqrt{x}\,x^4}$$

input
```
int((b*x^3+a*x)^(3/2)/x^7,x)
```

output
```
(2*( - sqrt(a + b*x**2)*a - 7*sqrt(a + b*x**2)*b*x**2 + 6*sqrt(x)*int((sqr
t(x)*sqrt(a + b*x**2))/(a*x**6 + b*x**8),x)*a**2*x**4))/(21*sqrt(x)*x**4)
```

3.63 $\int \frac{(ax+bx^3)^{3/2}}{x^8} dx$

Optimal result . . . 546
Mathematica [C] (verified) . . . 546
Rubi [A] (verified) . . . 547
Maple [A] (verified) . . . 549
Fricas [A] (verification not implemented) . . . 550
Sympy [F] . . . 550
Maxima [F] . . . 550
Giac [F] . . . 551
Mupad [F(-1)] . . . 551
Reduce [F] . . . 551

Optimal result

Integrand size = 17, antiderivative size = 163

$$\int \frac{(ax+bx^3)^{3/2}}{x^8} dx = -\frac{12b\sqrt{ax+bx^3}}{77x^4} - \frac{8b^2\sqrt{ax+bx^3}}{77ax^2} - \frac{2(ax+bx^3)^{3/2}}{11x^7} - \frac{4b^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{77a^{5/4}\sqrt{ax+bx^3}}$$

output
```
-12/77*b*(b*x^3+a*x)^(1/2)/x^4-8/77*b^2*(b*x^3+a*x)^(1/2)/a/x^2-2/11*(b*x^
3+a*x)^(3/2)/x^7-4/77*b^(11/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(
1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))
,1/2*2^(1/2))/a^(5/4)/(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.33

$$\int \frac{(ax+bx^3)^{3/2}}{x^8} dx = -\frac{2a\sqrt{x(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\frac{3}{2},-\frac{7}{4},-\frac{bx^2}{a}\right)}{11x^6\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a*x + b*x^3)^(3/2)/x^8,x]
```

output
```
(-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-11/4, -3/2, -7/4, -((b*x^2)/a
)])/(11*x^6*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1926, 1926, 1931, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^3\right)^{3/2}}{x^8}\,dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{6}{11}b\int \frac{\sqrt{bx^3+ax}}{x^5}dx-\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}$$

$$\Big\downarrow \text{1926}$$

$$\frac{6}{11}b\left(\frac{2}{7}b\int \frac{1}{x^2\sqrt{bx^3+ax}}dx-\frac{2\sqrt{ax+bx^3}}{7x^4}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}$$

$$\Big\downarrow \text{1931}$$

$$\frac{6}{11}b\left(\frac{2}{7}b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax}}dx}{3a}-\frac{2\sqrt{ax+bx^3}}{3ax^2}\right)-\frac{2\sqrt{ax+bx^3}}{7x^4}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}$$

$$\Big\downarrow \text{1917}$$

$$\frac{6}{11}b\left(\frac{2}{7}b\left(-\frac{b\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3ax^2}\right)-\frac{2\sqrt{ax+bx^3}}{7x^4}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}$$

$$\Big\downarrow \text{266}$$

$$\frac{6}{11}b\left(\frac{2}{7}b\left(-\frac{2b\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3ax^2}\right)-\frac{2\sqrt{ax+bx^3}}{7x^4}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}$$

$$\Big\downarrow \text{761}$$

$$\frac{6}{11}b\left(\frac{2}{7}b\left(-\frac{b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3a^{5/4}\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3ax^2}\right)-\frac{2\sqrt{ax+bx^3}}{7x^4}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}$$

input
```
Int[(a*x + b*x^3)^(3/2)/x^8,x]
```

output
```
(-2*(a*x + b*x^3)^(3/2))/(11*x^7) + (6*b*((-2*Sqrt[a*x + b*x^3])/(7*x^4) +
 (2*b*((-2*Sqrt[a*x + b*x^3])/(3*a*x^2) - (b^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt
[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/
4)*Sqrt[x])/a^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[a*x + b*x^3])))/7))/11
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{2(b\,x^2+a)(4b^2x^4+13ab\,x^2+7a^2)}{77x^5\sqrt{x(b\,x^2+a)}\,a}-\frac{4b^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{77a\sqrt{b\,x^3+ax}}$
default	$-\frac{2a\sqrt{b\,x^3+ax}}{11x^6}-\frac{26b\sqrt{b\,x^3+ax}}{77x^4}-\frac{8b^2\sqrt{b\,x^3+ax}}{77a\,x^2}-\frac{4b^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{(x+\cdots}{\cdots}}\right.}{77a\sqrt{b\,x^3+ax}}$
elliptic	$-\frac{2a\sqrt{b\,x^3+ax}}{11x^6}-\frac{26b\sqrt{b\,x^3+ax}}{77x^4}-\frac{8b^2\sqrt{b\,x^3+ax}}{77a\,x^2}-\frac{4b^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{(x+\cdots}{\cdots}}\right.}{77a\sqrt{b\,x^3+ax}}$

input
```
int((b*x^3+a*x)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-2/77*(b*x^2+a)*(4*b^2*x^4+13*a*b*x^2+7*a^2)/x^5/(x*(b*x^2+a))^(1/2)/a-4/7
7/a*b^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*
b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^
(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.36

$$\int \frac{(ax+bx^3)^{3/2}}{x^8}\,dx = -\frac{2\left(4\,b^{\frac{5}{2}}x^6\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\left(4\,b^2x^4+13\,abx^2+7\,a^2\right)\sqrt{bx^3+ax}\right)}{77\,ax^6}$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
-2/77*(4*b^(5/2)*x^6*weierstrassPInverse(-4*a/b, 0, x) + (4*b^2*x^4 + 13*a
*b*x^2 + 7*a^2)*sqrt(b*x^3 + a*x))/(a*x^6)
```

Sympy [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^8}\,dx = \int \frac{(x(a+bx^2))^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((b*x**3+a*x)**(3/2)/x**8,x)
```

output
```
Integral((x*(a + b*x**2))**(3/2)/x**8, x)
```

Maxima [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^8}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^8, x)
```

Giac [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^8}\,dx=\int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^8,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^3)^{3/2}}{x^8}\,dx=\int \frac{(b\,x^3+a\,x)^{3/2}}{x^8}\,dx$$

input
```
int((a*x + b*x^3)^(3/2)/x^8,x)
```

output
```
int((a*x + b*x^3)^(3/2)/x^8, x)
```

Reduce [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^8}\,dx=\frac{-\frac{2\sqrt{b\,x^2+a}\,a}{15}-\frac{2\sqrt{b\,x^2+a}\,b\,x^2}{5}+\frac{4\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^9+a\,x^7}dx\right)a^2x^5}{15}}{\sqrt{x}\,x^5}$$

input
```
int((b*x^3+a*x)^(3/2)/x^8,x)
```

output
```
(2*( - sqrt(a + b*x**2)*a - 3*sqrt(a + b*x**2)*b*x**2 + 2*sqrt(x)*int((sqr
t(x)*sqrt(a + b*x**2))/(a*x**7 + b*x**9),x)*a**2*x**5))/(15*sqrt(x)*x**5)
```

3.64 $\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx$

Optimal result 552
Mathematica [C] (verified) 553
Rubi [A] (verified) 553
Maple [A] (verified) 555
Fricas [A] (verification not implemented) 556
Sympy [F] 556
Maxima [F] 556
Giac [F] 557
Mupad [F(-1)] 557
Reduce [F] 557

Optimal result

Integrand size = 17, antiderivative size = 140

$$\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx = -\frac{10a\sqrt{ax+bx^3}}{21b^2}+\frac{2x^2\sqrt{ax+bx^3}}{7b} + \frac{5a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{21b^{9/4}\sqrt{ax+bx^3}}$$

output
```
-10/21*a*(b*x^3+a*x)^(1/2)/b^2+2/7*x^2*(b*x^3+a*x)^(1/2)/b+5/21*a^(7/4)*x^
(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJ
acobiAM(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)),1/2*2^(1/2))/b^(9/4)/(b*x^3+a*x)
^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.57

$$\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx = \frac{2x\left(-5a^2-2abx^2+3b^2x^4+5a^2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{21b^2\sqrt{x\,(a+bx^2)}}$$

input
```
Integrate[x^4/Sqrt[a*x + b*x^3],x]
```

output
```
(2*x*(-5*a^2 - 2*a*b*x^2 + 3*b^2*x^4 + 5*a^2*Sqrt[1 + (b*x^2)/a]*Hypergeom
etric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(21*b^2*Sqrt[x*(a + b*x^2)])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1930, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\int \frac{x^2}{\sqrt{bx^3+ax}}dx}{7b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\int \frac{1}{\sqrt{bx^3+ax}}dx}{3b}\right)}{7b}$$

$$\Big\downarrow \text{1917}$$

$$\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\sqrt{x}\sqrt{a+bx^2}\int\frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3b\sqrt{ax+bx^3}}\right)}{7b}$$

$$\Big\downarrow \text{266}$$

$$\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{2a\sqrt{x}\sqrt{a+bx^2}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3b\sqrt{ax+bx^3}}\right)}{7b}$$

$$\Big\downarrow \text{761}$$

$$\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}\right)}{7b}$$

input
```
Int[x^4/Sqrt[a*x + b*x^3],x]
```

output
```
(2*x^2*Sqrt[a*x + b*x^3])/(7*b) - (5*a*((2*Sqrt[a*x + b*x^3])/(3*b) - (a^(
3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^
2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[a*
x + b*x^3])))/(7*b)
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.05

method	result	s
risch	$-\frac{2(-3b\,x^2+5a)x(b\,x^2+a)}{21b^2\sqrt{x(b\,x^2+a)}}+\frac{5a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{21b^3\sqrt{b\,x^3+ax}}$	1
default	$\frac{2x^2\sqrt{b\,x^3+ax}}{7b}-\frac{10a\sqrt{b\,x^3+ax}}{21b^2}+\frac{5a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{21b^3\sqrt{b\,x^3+ax}}$	1
elliptic	$\frac{2x^2\sqrt{b\,x^3+ax}}{7b}-\frac{10a\sqrt{b\,x^3+ax}}{21b^2}+\frac{5a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{21b^3\sqrt{b\,x^3+ax}}$	1

input
```
int(x^4/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*(-3*b*x^2+5*a)/b^2*x*(b*x^2+a)/(x*(b*x^2+a))^(1/2)+5/21*a^2/b^3*(-a*
b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/
(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*Ellipt
icF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.34

$$\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx = \frac{2\left(5\,a^2\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\left(3\,b^2x^2-5\,ab\right)\sqrt{bx^3+ax}\right)}{21\,b^3}$$

input
```
integrate(x^4/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
2/21*(5*a^2*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + (3*b^2*x^2 - 5*a*b
)*sqrt(b*x^3 + a*x))/b^3
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^4}{\sqrt{x\left(a+bx^2\right)}}\,dx$$

input
```
integrate(x**4/(b*x**3+a*x)**(1/2),x)
```

output
```
Integral(x**4/sqrt(x*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^4}{\sqrt{bx^3+ax}}\,dx$$

input
```
integrate(x^4/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(b*x^3 + a*x), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^4}{\sqrt{bx^3+ax}}\,dx$$

input
```
integrate(x^4/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(b*x^3 + a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^4}{\sqrt{b\,x^3+a\,x}}\,dx$$

input
```
int(x^4/(a*x + b*x^3)^(1/2),x)
```

output
```
int(x^4/(a*x + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx = \frac{-10\sqrt{x}\,\sqrt{b\,x^2+a}\,a+6\sqrt{x}\,\sqrt{b\,x^2+a}\,b\,x^2+5\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^3+ax}dx\right)a^2}{21b^2}$$

input
```
int(x^4/(b*x^3+a*x)^(1/2),x)
```

output
```
( - 10*sqrt(x)*sqrt(a + b*x**2)*a + 6*sqrt(x)*sqrt(a + b*x**2)*b*x**2 + 5*
int((sqrt(x)*sqrt(a + b*x**2))/(a*x + b*x**3),x)*a**2)/(21*b**2)
```

3.65 $\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx$

Optimal result 558
Mathematica [C] (verified) 559
Rubi [A] (verified) 559
Maple [A] (verified) 562
Fricas [A] (verification not implemented) 563
Sympy [F] 563
Maxima [F] 563
Giac [F] 564
Mupad [F(-1)] 564
Reduce [F] 564

Optimal result

Integrand size = 17, antiderivative size = 258

$$\begin{aligned}\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx &= -\frac{6ax(a+bx^2)}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{2x\sqrt{ax+bx^3}}{5b}\\
&+\frac{6a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{ax+bx^3}}\\
&-\frac{3a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5b^{7/4}\sqrt{ax+bx^3}}\end{aligned}$$

output
```
-6/5*a*x*(b*x^2+a)/b^(3/2)/(a^(1/2)+b^(1/2)*x)/(b*x^3+a*x)^(1/2)+2/5*x*(b*
x^3+a*x)^(1/2)/b+6/5*a^(5/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/
2)+b^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/
2*2^(1/2))/b^(7/4)/(b*x^3+a*x)^(1/2)-3/5*a^(5/4)*x^(1/2)*(a^(1/2)+b^(1/2)*
x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4
)*x^(1/2)/a^(1/4)),1/2*2^(1/2))/b^(7/4)/(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.26

$$\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx=\frac{2x^2\left(a+bx^2-a\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{5b\sqrt{x\left(a+bx^2\right)}}$$

input
```
Integrate[x^3/Sqrt[a*x + b*x^3],x]
```

output
```
(2*x^2*(a + b*x^2 - a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 3/4, 7/4,
 -((b*x^2)/a)]))/(5*b*Sqrt[x*(a + b*x^2)])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {1930, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{ax+bx^3}}dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{3a\int \frac{x}{\sqrt{bx^3+ax}}dx}{5b}$$

$$\Big\downarrow \text{1938}$$

$$\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{3a\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{5b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\int \frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{5b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{834}$$

$$\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{761}$$

$$\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1510}$$

$$\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{5b\sqrt{ax+bx^3}}$$

input
```
Int[x^3/Sqrt[a*x + b*x^3],x]
```

output
```
(2*x*Sqrt[a*x + b*x^3])/(5*b) - (6*a*Sqrt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x
]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)
*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqr
t[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt
[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*Arc
Tan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(5*b*S
qrt[a*x + b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.69

method	result
default	$\frac{2x\sqrt{b\,x^3+ax}}{5b}-\frac{3a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,\text{EllipticF}}{}\right.}{5b^2\sqrt{b\,x^3+ax}}$
elliptic	$\frac{2x\sqrt{b\,x^3+ax}}{5b}-\frac{3a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,\text{EllipticF}}{}\right.}{5b^2\sqrt{b\,x^3+ax}}$
risch	$\frac{2x^2\left(b\,x^2+a\right)}{5b\sqrt{x\left(b\,x^2+a\right)}}-\frac{3a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,\text{Elliptic}}{}\right.}{5b^2\sqrt{b\,x^3+ax}}$

input
```
int(x^3/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*x*(b*x^3+a*x)^(1/2)/b-3/5*a/b^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b
)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(
1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^
(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(
-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.17

$$\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx = \frac{2\left(\sqrt{bx^3+ax}bx+3\,a\sqrt{b}\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)\right)}{5\,b^2}$$

input
```
integrate(x^3/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(sqrt(b*x^3 + a*x)*b*x + 3*a*sqrt(b)*weierstrassZeta(-4*a/b, 0, weiers
trassPInverse(-4*a/b, 0, x)))/b^2
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^3}{\sqrt{x\,(a+bx^2)}}\,dx$$

input
```
integrate(x**3/(b*x**3+a*x)**(1/2),x)
```

output
```
Integral(x**3/sqrt(x*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^3}{\sqrt{bx^3+ax}}\,dx$$

input
```
integrate(x^3/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(b*x^3 + a*x), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx=\int \frac{x^3}{\sqrt{bx^3+ax}}\,dx$$

input
```
integrate(x^3/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(b*x^3 + a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx=\int \frac{x^3}{\sqrt{b\,x^3+a\,x}}\,dx$$

input
```
int(x^3/(a*x + b*x^3)^(1/2),x)
```

output
```
int(x^3/(a*x + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx=\frac{2\sqrt{x}\,\sqrt{b\,x^2+a}\,x-3\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^2+a}dx\right)a}{5b}$$

input
```
int(x^3/(b*x^3+a*x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(a + b*x**2)*x - 3*int((sqrt(x)*sqrt(a + b*x**2))/(a + b*x*
*2),x)*a)/(5*b)
```

3.66 $\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx$

Optimal result . . . 565
Mathematica [C] (verified) . . . 565
Rubi [A] (verified) . . . 566
Maple [A] (verified) . . . 568
Fricas [A] (verification not implemented) . . . 568
Sympy [F] . . . 569
Maxima [F] . . . 569
Giac [F] . . . 569
Mupad [F(-1)] . . . 570
Reduce [F] . . . 570

Optimal result

Integrand size = 17, antiderivative size = 116

$$\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx = \frac{2\sqrt{ax+bx^3}}{3b} - \frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}$$

output
```
2/3*(b*x^3+a*x)^(1/2)/b-1/3*a^(3/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)
/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x^(1/2)/a^(
1/4)),1/2*2^(1/2))/b^(5/4)/(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.55

$$\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx = \frac{2x\left(a+bx^2-a\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{3b\sqrt{x\left(a+bx^2\right)}}$$

input
```
Integrate[x^2/Sqrt[a*x + b*x^3],x]
```

output
```
(2*x*(a + b*x^2 - a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -
((b*x^2)/a)]))/(3*b*Sqrt[x*(a + b*x^2)])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{ax+bx^3}}dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\int \frac{1}{\sqrt{bx^3+ax}}dx}{3b}$$

$$\Big\downarrow \text{1917}$$

$$\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2\sqrt{ax+bx^3}}{3b}-\frac{2a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{761}$$

$$\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}$$

input
```
Int[x^2/Sqrt[a*x + b*x^3],x]
```

output
```
(2*Sqrt[a*x + b*x^3])/(3*b) - (a^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[
(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/
a^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[a*x + b*x^3])
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{2\sqrt{b\,x^3+ax}}{3b}-\frac{a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3b^2\sqrt{b\,x^3+ax}}$	127
elliptic	$\frac{2\sqrt{b\,x^3+ax}}{3b}-\frac{a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3b^2\sqrt{b\,x^3+ax}}$	127
risch	$\frac{2x\left(b\,x^2+a\right)}{3b\sqrt{x\left(b\,x^2+a\right)}}-\frac{a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3b^2\sqrt{b\,x^3+ax}}$	135

input
```
int(x^2/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(b*x^3+a*x)^(1/2)/b-1/3*a/b^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^
(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/
2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)
*b)^(1/2),1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.29

$$\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx = -\frac{2\left(a\sqrt{b}\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)-\sqrt{bx^3+ax}b\right)}{3\,b^2}$$

input
```
integrate(x^2/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(a*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) - sqrt(b*x^3 + a*x)*b)/b
^2
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^2}{\sqrt{x\,(a+bx^2)}}\,dx$$

input
```
integrate(x**2/(b*x**3+a*x)**(1/2),x)
```

output
```
Integral(x**2/sqrt(x*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^2}{\sqrt{bx^3+ax}}\,dx$$

input
```
integrate(x^2/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(b*x^3 + a*x), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^2}{\sqrt{bx^3+ax}}\,dx$$

input
```
integrate(x^2/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(b*x^3 + a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx=\int \frac{x^2}{\sqrt{b\,x^3+a\,x}}\,dx$$

input
```
int(x^2/(a*x + b*x^3)^(1/2),x)
```

output
```
int(x^2/(a*x + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx=\frac{2\sqrt{x}\,\sqrt{b\,x^2+a}-\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^3+ax}dx\right)a}{3b}$$

input
```
int(x^2/(b*x^3+a*x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(a + b*x**2) - int((sqrt(x)*sqrt(a + b*x**2))/(a*x + b*x**3
),x)*a)/(3*b)
```

3.67 $\int \frac{x}{\sqrt{ax+bx^3}}\,dx$

Optimal result 571
Mathematica [C] (verified) 572
Rubi [A] (verified) 572
Maple [A] (verified) 575
Fricas [A] (verification not implemented) 575
Sympy [F] 576
Maxima [F] 576
Giac [F] 576
Mupad [F(-1)] 577
Reduce [F] 577

Optimal result

Integrand size = 15, antiderivative size = 229

$$\int \frac{x}{\sqrt{ax+bx^3}}\,dx = \frac{2x(a+bx^2)}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{2\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+bx^3}} + \frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+bx^3}}$$

output
```
2*x*(b*x^2+a)/b^(1/2)/(a^(1/2)+b^(1/2)*x)/(b*x^3+a*x)^(1/2)-2*a^(1/4)*x^(1
/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*EllipticE(
sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))/b^(3/4)/(b*x^3+a*x)^(1
/2)+a^(1/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^
(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)),1/2*2^(1/2))/b^(3/
4)/(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.23

$$\int \frac{x}{\sqrt{ax+bx^3}}\,dx = \frac{2x^2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)}{3\sqrt{x\left(a+bx^2\right)}}$$

input

```
Integrate[x/Sqrt[a*x + b*x^3],x]
```

output

```
(2*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^2)/a)])
/(3*Sqrt[x*(a + b*x^2)])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{ax+bx^3}}dx$$

$$\Big\downarrow \text{ 1938}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2\sqrt{x}\sqrt{a+bx^2}\int \frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{2\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{761}$$

$$\frac{2\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1510}$$

$$\frac{2\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{\sqrt{ax+bx^3}}$$

input
```
Int[x/Sqrt[a*x + b*x^3],x]
```

output
```
(2*Sqrt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt
[b]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[
b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqr
t[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/
(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/
2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/Sqrt[a*x + b*x^3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.69

method	result
default	$\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}\right)}{b\sqrt{b\,x^{3}+ax}}$
elliptic	$\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}\right)}{b\sqrt{b\,x^{3}+ax}}$

input
```
int(x/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/
2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*
(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2
*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/
2),1/2*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.10

$$\int \frac{x}{\sqrt{ax+bx^3}}\,dx = -\frac{2\,\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)}{\sqrt{b}}$$

input
```
integrate(x/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x))/sqrt(b)
```

Sympy [F]

$$\int \frac{x}{\sqrt{ax+bx^3}}\,dx = \int \frac{x}{\sqrt{x\,(a+bx^2)}}\,dx$$

input
```
integrate(x/(b*x**3+a*x)**(1/2),x)
```

output
```
Integral(x/sqrt(x*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{ax+bx^3}}\,dx = \int \frac{x}{\sqrt{bx^3+ax}}\,dx$$

input
```
integrate(x/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(b*x^3 + a*x), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{ax+bx^3}}\,dx = \int \frac{x}{\sqrt{bx^3+ax}}\,dx$$

input
```
integrate(x/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(b*x^3 + a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{ax+bx^3}}\,dx=\int \frac{x}{\sqrt{b\,x^3+a\,x}}\,dx$$

input
```
int(x/(a*x + b*x^3)^(1/2),x)
```

output
```
int(x/(a*x + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{ax+bx^3}}\,dx=\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^2+a}dx$$

input
```
int(x/(b*x^3+a*x)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(a + b*x**2))/(a + b*x**2),x)
```

3.68 $\int \frac{1}{\sqrt{ax+bx^3}}\,dx$

Optimal result 578
Mathematica [C] (verified) 578
Rubi [A] (verified) 579
Maple [A] (verified) 580
Fricas [A] (verification not implemented) 581
Sympy [F] 581
Maxima [F] 581
Giac [F] 582
Mupad [B] (verification not implemented) 582
Reduce [F] 582

Optimal result

Integrand size = 13, antiderivative size = 92

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx=\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax+bx^3}}$$

output
```
x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*Invers
eJacobiAM(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)),1/2*2^(1/2))/a^(1/4)/b^(1/4)/(
b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.53

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx=\frac{2x\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)}{\sqrt{x\left(a+bx^2\right)}}$$

input
```
Integrate[1/Sqrt[a*x + b*x^3],x]
```

output
```
(2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])/S
qrt[x*(a + b*x^2)]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx$$

$$\Big\downarrow \text{1917}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{761}$$

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax+bx^3}}$$

input
```
Int[1/Sqrt[a*x + b*x^3],x]
```

output
```
(Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*E
llipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(a^(1/4)*b^(1/4)*Sqrt[
a*x + b*x^3])
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{b\,x^{3}+ax}}$	108
elliptic	$\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{b\,x^{3}+ax}}$	108

input
```
int(1/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/
2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*
EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.15

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx = \frac{2\,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)}{\sqrt{b}}$$

input
```
integrate(1/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
2*weierstrassPInverse(-4*a/b, 0, x)/sqrt(b)
```

Sympy [F]

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx = \int \frac{1}{\sqrt{ax+bx^3}}\,dx$$

input
```
integrate(1/(b*x**3+a*x)**(1/2),x)
```

output
```
Integral(1/sqrt(a*x + b*x**3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax}}\,dx$$

input
```
integrate(1/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*x^3 + a*x), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax}}\,dx$$

input
```
integrate(1/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*x^3 + a*x), x)
```

Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx = \frac{2\,x\,\sqrt{\frac{b\,x^2}{a}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{b\,x^2}{a}\right)}{\sqrt{b\,x^3+a\,x}}$$

input
```
int(1/(a*x + b*x^3)^(1/2),x)
```

output
```
(2*x*((b*x^2)/a + 1)^(1/2)*hypergeom([1/4, 1/2], 5/4, -(b*x^2)/a))/(a*x +
b*x^3)^(1/2)
```

Reduce [F]

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx = \int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^3+ax}dx$$

input
```
int(1/(b*x^3+a*x)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(a + b*x**2))/(a*x + b*x**3),x)
```

3.69 $\int \frac{1}{x\sqrt{ax+bx^3}}\,dx$

Optimal result 583
Mathematica [C] (verified) 584
Rubi [A] (verified) 584
Maple [A] (verified) 587
Fricas [A] (verification not implemented) 588
Sympy [F] 588
Maxima [F] 589
Giac [F] 589
Mupad [F(-1)] 589
Reduce [F] 590

Optimal result

Integrand size = 17, antiderivative size = 253

$$\begin{aligned}\int \frac{1}{x\sqrt{ax+bx^3}}\,dx &= \frac{2\sqrt{b}x(a+bx^2)}{a\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\\ &\quad-\frac{2\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{ax+bx^3}}\\ &\quad+\frac{\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{a^{3/4}\sqrt{ax+bx^3}}\end{aligned}$$

output
```
2*b^(1/2)*x*(b*x^2+a)/a/(a^(1/2)+b^(1/2)*x)/(b*x^3+a*x)^(1/2)-2*(b*x^3+a*x
)^(1/2)/a/x-2*b^(1/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1
/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/
2))/a^(3/4)/(b*x^3+a*x)^(1/2)+b^(1/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+
a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x^(1/2)/a
^(1/4)),1/2*2^(1/2))/a^(3/4)/(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.19

$$\int \frac{1}{x\sqrt{ax+bx^3}}\,dx = -\frac{2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{bx^2}{a}\right)}{\sqrt{x(a+bx^2)}}$$

input
```
Integrate[1/(x*Sqrt[a*x + b*x^3]),x]
```

output
```
(-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((b*x^2)/a)])/S
qrt[x*(a + b*x^2)]
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{ax+bx^3}}dx$$

$$\Big\downarrow \text{ 1931}$$

$$\frac{b\int \frac{x}{\sqrt{bx^3+ax}}dx}{a}-\frac{2\sqrt{ax+bx^3}}{ax}$$

$$\Big\downarrow \text{ 1938}$$

$$\frac{b\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2b\sqrt{x}\sqrt{a+bx^2}\int \frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}$$

$$\Big\downarrow \text{834}$$

$$\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}$$

$$\Big\downarrow \text{761}$$

$$\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}$$

$$\Big\downarrow \text{1510}$$

$$\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{a\sqrt{ax+bx^3}}$$

$$\frac{2\sqrt{ax+bx^3}}{ax}$$

input
```
Int[1/(x*Sqrt[a*x + b*x^3]),x]
```

output
```
(-2*Sqrt[a*x + b*x^3])/(a*x) + (2*b*Sqrt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x]
*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*
Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt
[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcT
an[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(a*Sqrt
[a*x + b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.72

method	result
default	$-\frac{2(bx^2+a)}{a\sqrt{x(bx^2+a)}}+\frac{\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,\mathrm{EllipticF}\ldots}{\ldots}\right.}{a\sqrt{bx^3+ax}}$ …
risch	$-\frac{2(bx^2+a)}{a\sqrt{x(bx^2+a)}}+\frac{\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,\mathrm{EllipticF}\ldots}{\ldots}\right.}{a\sqrt{bx^3+ax}}$ …
elliptic	$-\frac{2(bx^2+a)}{a\sqrt{x(bx^2+a)}}+\frac{\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,\mathrm{EllipticF}\ldots}{\ldots}\right.}{a\sqrt{bx^3+ax}}$ …

input
```
int(1/x/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(b*x^2+a)/a/(x*(b*x^2+a))^(1/2)+1/a*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-
a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b
)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*
b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((
x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.17

$$\int \frac{1}{x\sqrt{ax+bx^3}}\,dx = -\frac{2\left(\sqrt{b}x\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\sqrt{bx^3+ax}\right)}{ax}$$

input
```
integrate(1/x/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(b)*x*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x)
) + sqrt(b*x^3 + a*x))/(a*x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{ax+bx^3}}\,dx = \int \frac{1}{x\sqrt{x(a+bx^2)}}\,dx$$

input
```
integrate(1/x/(b*x**3+a*x)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(x*(a + b*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{ax+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax}x}\,dx$$

input
```
integrate(1/x/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{ax+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax}x}\,dx$$

input
```
integrate(1/x/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{ax+bx^3}}\,dx = \int \frac{1}{x\,\sqrt{b\,x^3+a\,x}}\,dx$$

input
```
int(1/(x*(a*x + b*x^3)^(1/2)),x)
```

output
```
int(1/(x*(a*x + b*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{ax+bx^3}}\,dx = \int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^4+a\,x^2}dx$$

input
```
int(1/x/(b*x^3+a*x)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(a + b*x**2))/(a*x**2 + b*x**4),x)
```

3.70 $\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx$

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Optimal result

Integrand size = 17, antiderivative size = 119

$$\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx = -\frac{2\sqrt{ax+bx^3}}{3ax^2} - \frac{b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3a^{5/4}\sqrt{ax+bx^3}}$$

output
```
-2/3*(b*x^3+a*x)^(1/2)/a/x^2-1/3*b^(3/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x
^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x^(1/2
)/a^(1/4)),1/2*2^(1/2))/a^(5/4)/(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx = -\frac{2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-\frac{bx^2}{a}\right)}{3x\sqrt{x(a+bx^2)}}$$

input
```
Integrate[1/(x^2*Sqrt[a*x + b*x^3]),x]
```

output
```
(-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/4, 1/2, 1/4, -((b*x^2)/a)])/(
3*x*Sqrt[x*(a + b*x^2)])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1931, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{ax+bx^3}}dx$$

$$\Big\downarrow \text{1931}$$

$$-\frac{b\int \frac{1}{\sqrt{bx^3+ax}}dx}{3a}-\frac{2\sqrt{ax+bx^3}}{3ax^2}$$

$$\Big\downarrow \text{1917}$$

$$-\frac{b\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3ax^2}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2b\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3ax^2}$$

$$\Big\downarrow \text{761}$$

$$-\frac{b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3a^{5/4}\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3ax^2}$$

input
```
Int[1/(x^2*Sqrt[a*x + b*x^3]),x]
```

output
```
(-2*Sqrt[a*x + b*x^3])/(3*a*x^2) - (b^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*
Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt
[x])/a^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[a*x + b*x^3])
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{2\sqrt{b\,x^{3}+ax}}{3a\,x^{2}}-\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3a\sqrt{b\,x^{3}+ax}}$	129
elliptic	$-\frac{2\sqrt{b\,x^{3}+ax}}{3a\,x^{2}}-\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3a\sqrt{b\,x^{3}+ax}}$	129
risch	$-\frac{2\left(b\,x^{2}+a\right)}{3ax\sqrt{x\left(b\,x^{2}+a\right)}}-\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3a\sqrt{b\,x^{3}+ax}}$	136

input
```
int(1/x^2/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x^3+a*x)^(1/2)/a/x^2-1/3/a*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)
^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1
/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2
)*b)^(1/2),1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.30

$$\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx = -\frac{2\left(\sqrt{b}x^2\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\sqrt{bx^3+ax}\right)}{3\,ax^2}$$

input
```
integrate(1/x^2/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(b)*x^2*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b*x^3 + a*x))/(
a*x^2)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx = \int \frac{1}{x^2\sqrt{x\,(a+bx^2)}}\,dx$$

input
```
integrate(1/x**2/(b*x**3+a*x)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(x*(a + b*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx=\int \frac{1}{x^2\,\sqrt{b\,x^3+a\,x}}\,dx$$

input
```
int(1/(x^2*(a*x + b*x^3)^(1/2)),x)
```

output
```
int(1/(x^2*(a*x + b*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx=\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^5+a\,x^3}dx$$

input
```
int(1/x^2/(b*x^3+a*x)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(a + b*x**2))/(a*x**3 + b*x**5),x)
```

3.71 $\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx$

Optimal result 597
Mathematica [C] (verified) 598
Rubi [A] (verified) 598
Maple [A] (verified) 602
Fricas [A] (verification not implemented) 603
Sympy [F] 603
Maxima [F] 603
Giac [F] 604
Mupad [F(-1)] 604
Reduce [F] 604

Optimal result

Integrand size = 17, antiderivative size = 286

$$\begin{aligned}\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx &= -\frac{6b^{3/2}x(a+bx^2)}{5a^2\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{5ax^3}+\frac{6b\sqrt{ax+bx^3}}{5a^2x}\\ &+\frac{6b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{ax+bx^3}}\\ &-\frac{3b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5a^{7/4}\sqrt{ax+bx^3}}\end{aligned}$$

output
```
-6/5*b^(3/2)*x*(b*x^2+a)/a^2/(a^(1/2)+b^(1/2)*x)/(b*x^3+a*x)^(1/2)-2/5*(b*
x^3+a*x)^(1/2)/a/x^3+6/5*b*(b*x^3+a*x)^(1/2)/a^2/x+6/5*b^(5/4)*x^(1/2)*(a^
(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*a
rctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))/a^(7/4)/(b*x^3+a*x)^(1/2)-3/5
*b^(5/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/
2)*InverseJacobiAM(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)),1/2*2^(1/2))/a^(7/4)/
(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.19

$$\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx=-\frac{2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},-\frac{1}{4},-\frac{bx^2}{a}\right)}{5x^2\sqrt{x(a+bx^2)}}$$

input
```
Integrate[1/(x^3*Sqrt[a*x + b*x^3]),x]
```

output
```
(-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-5/4, 1/2, -1/4, -((b*x^2)/a)])/
(5*x^2*Sqrt[x*(a + b*x^2)])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1931, 1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{ax+bx^3}}dx$$

$$\Big\downarrow \text{1931}$$

$$-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax}}dx}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{3b\left(\frac{b\int \frac{x}{\sqrt{bx^3+ax}}dx}{a}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}$$

$$\Big\downarrow \text{1938}$$

$$-\frac{3b\left(\frac{b\sqrt{x}\sqrt{a+bx^2}\int\frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}$$

$$\Big\downarrow \text{ 266}$$

$$-\frac{3b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\int\frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}$$

$$\Big\downarrow \text{ 834}$$

$$-\frac{3b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}$$

$$\Big\downarrow \text{ 761}$$

$$-\frac{3b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}$$

$$\Big\downarrow \text{ 1510}$$

$$-\frac{3b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{a\sqrt{ax+bx^3}}}{5a}$$

$$\frac{2\sqrt{ax+bx^3}}{5ax^3}$$

input
```
Int[1/(x^3*Sqrt[a*x + b*x^3]),x]
```

output
```
(-2*Sqrt[a*x + b*x^3])/(5*a*x^3) - (3*b*((-2*Sqrt[a*x + b*x^3])/(a*x) + (2
*b*Sqrt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt
[b]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[
b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqr
t[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/
(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/
2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(a*Sqrt[a*x + b*x^3])))/(5*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{2(bx^2+a)(-3bx^2+a)}{5a^2x^2\sqrt{x(bx^2+a)}}-\frac{3b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{5a^2\sqrt{bx^3+ax}}$
default	$-\frac{2\sqrt{bx^3+ax}}{5ax^3}+\frac{6(bx^2+a)b}{5a^2\sqrt{x(bx^2+a)}}-\frac{3b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\right.\right.}{b}\right.}{5a^2\sqrt{bx^3+ax}}$
elliptic	$-\frac{2\sqrt{bx^3+ax}}{5ax^3}+\frac{6(bx^2+a)b}{5a^2\sqrt{x(bx^2+a)}}-\frac{3b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\right.\right.}{b}\right.}{5a^2\sqrt{bx^3+ax}}$

input
```
int(1/x^3/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(b*x^2+a)*(-3*b*x^2+a)/a^2/x^2/(x*(b*x^2+a))^(1/2)-3/5*b/a^2*(-a*b)^(
1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*
b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)
^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+
(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(
1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.20

$$\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx = \frac{2\left(3\,b^{\frac{3}{2}}x^3\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\sqrt{bx^3+ax}(3\,bx^2-a)\right)}{5\,a^2x^3}$$

input
```
integrate(1/x^3/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(3*b^(3/2)*x^3*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b,
0, x)) + sqrt(b*x^3 + a*x)*(3*b*x^2 - a))/(a^2*x^3)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx = \int \frac{1}{x^3\sqrt{x\,(a+bx^2)}}\,dx$$

input
```
integrate(1/x**3/(b*x**3+a*x)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(x*(a + b*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx = \int \frac{1}{x^3\,\sqrt{b\,x^3+a\,x}}\,dx$$

input
```
int(1/(x^3*(a*x + b*x^3)^(1/2)),x)
```

output
```
int(1/(x^3*(a*x + b*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx = \int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^6+a\,x^4}dx$$

input
```
int(1/x^3/(b*x^3+a*x)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(a + b*x**2))/(a*x**4 + b*x**6),x)
```

3.72 $\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx$

Optimal result 605
Mathematica [C] (verified) 606
Rubi [A] (verified) 606
Maple [A] (verified) 609
Fricas [A] (verification not implemented) 609
Sympy [F] 610
Maxima [F] 610
Giac [F] 610
Mupad [F(-1)] 611
Reduce [F] 611

Optimal result

Integrand size = 17, antiderivative size = 161

$$\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx = -\frac{x^5}{b\sqrt{ax+bx^3}} - \frac{15a\sqrt{ax+bx^3}}{7b^3} + \frac{9x^2\sqrt{ax+bx^3}}{7b^2} + \frac{15a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{14b^{13/4}\sqrt{ax+bx^3}}$$

output
```
-x^5/b/(b*x^3+a*x)^(1/2)-15/7*a*(b*x^3+a*x)^(1/2)/b^3+9/7*x^2*(b*x^3+a*x)^
(1/2)/b^2+15/14*a^(7/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^
(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)),1/2*2^
(1/2))/b^(13/4)/(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.50

$$\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx = \frac{x\left(-15a^2-6abx^2+2b^2x^4+15a^2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{7b^3\sqrt{x(a+bx^2)}}$$

input
```
Integrate[x^7/(a*x + b*x^3)^(3/2),x]
```

output
```
(x*(-15*a^2 - 6*a*b*x^2 + 2*b^2*x^4 + 15*a^2*Sqrt[1 + (b*x^2)/a]*Hypergeom
etric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(7*b^3*Sqrt[x*(a + b*x^2)])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1928, 1930, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1928}$$

$$\frac{9\int \frac{x^4}{\sqrt{bx^3+ax}}dx}{2b}-\frac{x^5}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1930}$$

$$\frac{9\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\int \frac{x^2}{\sqrt{bx^3+ax}}dx}{7b}\right)}{2b}-\frac{x^5}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1930}$$

$$\frac{9\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\int\frac{1}{\sqrt{bx^3+ax}}dx}{3b}\right)}{7b}\right)}{2b}-\frac{x^5}{b\sqrt{ax+bx^3}}$$

$\Big\downarrow$ 1917

$$\frac{9\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\sqrt{x}\sqrt{a+bx^2}\int\frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3b\sqrt{ax+bx^3}}\right)}{7b}\right)}{2b}-\frac{x^5}{b\sqrt{ax+bx^3}}$$

$\Big\downarrow$ 266

$$\frac{9\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{2a\sqrt{x}\sqrt{a+bx^2}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3b\sqrt{ax+bx^3}}\right)}{7b}\right)}{2b}-\frac{x^5}{b\sqrt{ax+bx^3}}$$

$\Big\downarrow$ 761

$$\frac{9\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}\right)}{7b}\right)}{2b}-\frac{x^5}{b\sqrt{ax+bx^3}}$$

input
```
Int[x^7/(a*x + b*x^3)^(3/2),x]
```

output
```
-(x^5/(b*Sqrt[a*x + b*x^3])) + (9*((2*x^2*Sqrt[a*x + b*x^3])/(7*b) - (5*a*
((2*Sqrt[a*x + b*x^3])/(3*b) - (a^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt
[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])
/a^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[a*x + b*x^3])))/(7*b)))/(2*b)
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1928
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.07

method	result
default	$-\frac{x a^2}{b^3\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{2x^2\sqrt{b x^3+ax}}{7b^2}-\frac{8a\sqrt{b x^3+ax}}{7b^3}+\frac{15a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\cdots}\right.}{14b^4\sqrt{b x^3+ax}}$
elliptic	$-\frac{x a^2}{b^3\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{2x^2\sqrt{b x^3+ax}}{7b^2}-\frac{8a\sqrt{b x^3+ax}}{7b^3}+\frac{15a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\cdots}\right.}{14b^4\sqrt{b x^3+ax}}$
risch	$-\frac{2\left(-b x^2+4a\right)\left(b x^2+a\right)x}{7b^3\sqrt{x\left(b x^2+a\right)}}+\frac{a^2\left(\frac{11\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{b x^3+ax}}-7a\left(\frac{\cdots}{a\sqrt{\left(x^2\cdots\right.}}\right.\right.}{7b^3\cdots}$

input
```
int(x^7/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b^3*x*a^2/((x^2+a/b)*b*x)^(1/2)+2/7*x^2*(b*x^3+a*x)^(1/2)/b^2-8/7*a*(b*
x^3+a*x)^(1/2)/b^3+15/14/b^4*a^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(
1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2
)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*
b)^(1/2),1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.50

$$\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx=\frac{15\left(a^2bx^2+a^3\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\left(2b^3x^4-6ab^2x^2-15a^2b\right)\sqrt{bx^3\cdots}}{7\left(b^5x^2+ab^4\right)}$$

input
```
integrate(x^7/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
1/7*(15*(a^2*b*x^2 + a^3)*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + (2*b
^3*x^4 - 6*a*b^2*x^2 - 15*a^2*b)*sqrt(b*x^3 + a*x))/(b^5*x^2 + a*b^4)
```

Sympy [F]

$$\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^7}{(x\,(a+bx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**7/(b*x**3+a*x)**(3/2),x)
```

output
```
Integral(x**7/(x*(a + b*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^7}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^7/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^7/(b*x^3 + a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^7}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^7/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^7/(b*x^3 + a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^7}{(b\,x^3+a\,x)^{3/2}}\,dx$$

input
```
int(x^7/(a*x + b*x^3)^(3/2),x)
```

output
```
int(x^7/(a*x + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx = \frac{-30\sqrt{x}\,\sqrt{b\,x^2+a}\,a^2-6\sqrt{x}\,\sqrt{b\,x^2+a}\,ab\,x^2+2\sqrt{x}\,\sqrt{b\,x^2+a}\,b^2x^4+15\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2}}{b^2x^5+2ab\,x^3}\right.}{7b^3\,(b\,x^2+a)}$$

input
```
int(x^7/(b*x^3+a*x)^(3/2),x)
```

output
```
( - 30*sqrt(x)*sqrt(a + b*x**2)*a**2 - 6*sqrt(x)*sqrt(a + b*x**2)*a*b*x**2
 + 2*sqrt(x)*sqrt(a + b*x**2)*b**2*x**4 + 15*int((sqrt(x)*sqrt(a + b*x**2)
)/(a**2*x + 2*a*b*x**3 + b**2*x**5),x)*a**4 + 15*int((sqrt(x)*sqrt(a + b*x
**2))/(a**2*x + 2*a*b*x**3 + b**2*x**5),x)*a**3*b*x**2)/(7*b**3*(a + b*x**
2))
```

3.73 $\int \frac{x^6}{(ax+bx^3)^{3/2}}\,dx$

Optimal result 612
Mathematica [C] (verified) 613
Rubi [A] (verified) 613
Maple [A] (verified) 617
Fricas [A] (verification not implemented) 618
Sympy [F] 618
Maxima [F] 618
Giac [F] 619
Mupad [F(-1)] 619
Reduce [F] 619

Optimal result

Integrand size = 17, antiderivative size = 279

$$\int \frac{x^6}{(ax+bx^3)^{3/2}}\,dx = -\frac{x^4}{b\sqrt{ax+bx^3}} - \frac{21ax(a+bx^2)}{5b^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} + \frac{7x\sqrt{ax+bx^3}}{5b^2} + \frac{21a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{11/4}\sqrt{ax+bx^3}} - \frac{21a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{10b^{11/4}\sqrt{ax+bx^3}}$$

output

```
-x^4/b/(b*x^3+a*x)^(1/2)-21/5*a*x*(b*x^2+a)/b^(5/2)/(a^(1/2)+b^(1/2)*x)/(b
*x^3+a*x)^(1/2)+7/5*x*(b*x^3+a*x)^(1/2)/b^2+21/5*a^(5/4)*x^(1/2)*(a^(1/2)+
b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(
b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))/b^(11/4)/(b*x^3+a*x)^(1/2)-21/10*a^
(5/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*
InverseJacobiAM(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)),1/2*2^(1/2))/b^(11/4)/(b
*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.24

$$\int \frac{x^6}{(ax+bx^3)^{3/2}}\,dx=\frac{2x^2\left(-7a+bx^2+7a\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{5b^2\sqrt{x\left(a+bx^2\right)}}$$

input
```
Integrate[x^6/(a*x + b*x^3)^(3/2),x]
```

output
```
(2*x^2*(-7*a + b*x^2 + 7*a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2,
 7/4, -((b*x^2)/a)]))/(5*b^2*Sqrt[x*(a + b*x^2)])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1928, 1930, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(ax+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{1928}$$

$$\frac{7\int \frac{x^3}{\sqrt{bx^3+ax}}dx}{2b}-\frac{x^4}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1930}$$

$$\frac{7\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{3a\int \frac{x}{\sqrt{bx^3+ax}}dx}{5b}\right)}{2b}-\frac{x^4}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1938}$$

$$\frac{7\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{3a\sqrt{x}\sqrt{a+bx^2}\int\frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{5b\sqrt{ax+bx^3}}\right)}{2b}-\frac{x^4}{b\sqrt{ax+bx^3}}$$

$\big\downarrow$ 266

$$\frac{7\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\int\frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{5b\sqrt{ax+bx^3}}\right)}{2b}-\frac{x^4}{b\sqrt{ax+bx^3}}$$

$\big\downarrow$ 834

$$\frac{7\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5b\sqrt{ax+bx^3}}\right)}{2b}-\frac{x^4}{b\sqrt{ax+bx^3}}$$

$\big\downarrow$ 27

$$\frac{7\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5b\sqrt{ax+bx^3}}\right)}{2b}-\frac{x^4}{b\sqrt{ax+bx^3}}$$

$\big\downarrow$ 761

$$\frac{7\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5b\sqrt{ax+bx^3}}\right)}{2b}-\frac{x^4}{b\sqrt{ax+bx^3}}$$

$\big\downarrow$ 1510

$$\frac{7\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\right.\right.}{\frac{\sqrt[4]{b}\sqrt{a+bx^2}}{\sqrt{b}}}\right.}{5b\sqrt{ax+bx^3}}\right.}{2b}$$

$$\frac{x^4}{b\sqrt{ax+bx^3}}$$

input
```
Int[x^6/(a*x + b*x^3)^(3/2),x]
```

output
```
-(x^4/(b*Sqrt[a*x + b*x^3])) + (7*((2*x*Sqrt[a*x + b*x^3])/(5*b) - (6*a*Sq
rt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*x
)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)
^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a +
 b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt
[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(
2*b^(3/4)*Sqrt[a + b*x^2])))/(5*b*Sqrt[a*x + b*x^3])))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1928
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.72

method	result
default	$\frac{x^2 a}{b^2\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{2x\sqrt{b\,x^3+ax}}{5b^2}-\frac{21a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\ldots\right)}{b}\ldots\right)}{10b^3\sqrt{b\,x^3+ax}}\ldots$
elliptic	$\frac{x^2 a}{b^2\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{2x\sqrt{b\,x^3+ax}}{5b^2}-\frac{21a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\ldots\right)}{b}\ldots\right)}{10b^3\sqrt{b\,x^3+ax}}\ldots$
risch	$\frac{2x^2\left(b\,x^2+a\right)}{5b^2\sqrt{x\left(b\,x^2+a\right)}}-\frac{a\left(\frac{8\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,\mathrm{Elliptic}\ldots}{\ldots}\right)}{b\sqrt{b\,x^3+ax}}\ldots\right)}{\ldots}$

input
```
int(x^6/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*x^2*a/((x^2+a/b)*b*x)^(1/2)+2/5*x*(b*x^3+a*x)^(1/2)/b^2-21/10*a/b^3*
(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)
/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-
2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2
^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)
,1/2*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.27

$$\int \frac{x^6}{(ax+bx^3)^{3/2}}\,dx = \frac{21\,(abx^2+a^2)\sqrt{b}\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+(2\,b^2x^3}{5\,(b^4x^2+ab^3)}$$

input
```
integrate(x^6/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
1/5*(21*(a*b*x^2 + a^2)*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierstrassPInv
erse(-4*a/b, 0, x)) + (2*b^2*x^3 + 7*a*b*x)*sqrt(b*x^3 + a*x))/(b^4*x^2 +
a*b^3)
```

Sympy [F]

$$\int \frac{x^6}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^6}{(x\,(a+bx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**6/(b*x**3+a*x)**(3/2),x)
```

output
```
Integral(x**6/(x*(a + b*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^6}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^6}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^6/(b*x^3 + a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x^6}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^6}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^3 + a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^6}{(b\,x^3+a\,x)^{3/2}}\,dx$$

input
```
int(x^6/(a*x + b*x^3)^(3/2),x)
```

output
```
int(x^6/(a*x + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^6}{(ax+bx^3)^{3/2}}\,dx = \frac{-14\sqrt{x}\,\sqrt{b\,x^2+a}\,ax+2\sqrt{x}\,\sqrt{b\,x^2+a}\,b\,x^3+21\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^2x^4+2ab\,x^2+a^2}dx\right)a^3+21\left(\int \frac{\sqrt{x}}{b^2x^4+}\right.}{5b^2\,(b\,x^2+a)}$$

input
```
int(x^6/(b*x^3+a*x)^(3/2),x)
```

output
```
( - 14*sqrt(x)*sqrt(a + b*x**2)*a*x + 2*sqrt(x)*sqrt(a + b*x**2)*b*x**3 +
21*int((sqrt(x)*sqrt(a + b*x**2))/(a**2 + 2*a*b*x**2 + b**2*x**4),x)*a**3
+ 21*int((sqrt(x)*sqrt(a + b*x**2))/(a**2 + 2*a*b*x**2 + b**2*x**4),x)*a**
2*b*x**2)/(5*b**2*(a + b*x**2))
```

3.74 $\int \frac{x^5}{(ax+bx^3)^{3/2}}\,dx$

Optimal result . . . 620
Mathematica [C] (verified) . . . 620
Rubi [A] (verified) . . . 621
Maple [A] (verified) . . . 623
Fricas [A] (verification not implemented) . . . 624
Sympy [F] . . . 624
Maxima [F] . . . 625
Giac [F] . . . 625
Mupad [F(-1)] . . . 625
Reduce [F] . . . 626

Optimal result

Integrand size = 17, antiderivative size = 137

$$\int \frac{x^5}{(ax+bx^3)^{3/2}}\,dx = -\frac{x^3}{b\sqrt{ax+bx^3}}+\frac{5\sqrt{ax+bx^3}}{3b^2} - \frac{5a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{9/4}\sqrt{ax+bx^3}}$$

output
```
-x^3/b/(b*x^3+a*x)^(1/2)+5/3*(b*x^3+a*x)^(1/2)/b^2-5/6*a^(3/4)*x^(1/2)*(a^
(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2
*arctan(b^(1/4)*x^(1/2)/a^(1/4)),1/2*2^(1/2))/b^(9/4)/(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.49

$$\int \frac{x^5}{(ax+bx^3)^{3/2}}\,dx = \frac{x\left(5a+2bx^2-5a\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{3b^2\sqrt{x\left(a+bx^2\right)}}$$

input
```
Integrate[x^5/(a*x + b*x^3)^(3/2),x]
```

output
```
(x*(5*a + 2*b*x^2 - 5*a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/
4, -((b*x^2)/a)]))/(3*b^2*Sqrt[x*(a + b*x^2)])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1928, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(ax+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{1928}$$

$$\frac{5\int \frac{x^2}{\sqrt{bx^3+ax}}dx}{2b}-\frac{x^3}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1930}$$

$$\frac{5\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\int \frac{1}{\sqrt{bx^3+ax}}dx}{3b}\right)}{2b}-\frac{x^3}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1917}$$

$$\frac{5\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3b\sqrt{ax+bx^3}}\right)}{2b}-\frac{x^3}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{266}$$

$$\frac{5\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{2a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3b\sqrt{ax+bx^3}}\right)}{2b}-\frac{x^3}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{761}$$

$$\frac{5\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}\right)}{2b}-\frac{x^3}{b\sqrt{ax+bx^3}}$$

input
```
Int[x^5/(a*x + b*x^3)^(3/2),x]
```

output
```
-(x^3/(b*Sqrt[a*x + b*x^3])) + (5*((2*Sqrt[a*x + b*x^3])/(3*b) - (a^(3/4)*
Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[a*x + b
*x^3])))/(2*b)
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1928
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.07

method	result
default	$\frac{xa}{b^2\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{2\sqrt{b\,x^3+ax}}{3b^2}-\frac{5a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{6b^3\sqrt{b\,x^3+ax}}$
elliptic	$\frac{xa}{b^2\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{2\sqrt{b\,x^3+ax}}{3b^2}-\frac{5a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{6b^3\sqrt{b\,x^3+ax}}$
risch	$\frac{2(b\,x^2+a)x}{3b^2\sqrt{x(b\,x^2+a)}}-\frac{a\left(\frac{4\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{b\,x^3+ax}}-3a\left(\frac{x}{a\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\cdots\right.\right.}{3b^2}$

input
```
int(x^5/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*x*a/((x^2+a/b)*b*x)^(1/2)+2/3*(b*x^3+a*x)^(1/2)/b^2-5/6*a/b^3*(-a*b)
^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-
a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*Elliptic
F(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.50

$$\int \frac{x^5}{(ax+bx^3)^{3/2}}\,dx = -\frac{5\,(abx^2+a^2)\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-(2\,b^2x^2+5\,ab)\sqrt{bx^3+ax}}{3\,(b^4x^2+ab^3)}$$

input
```
integrate(x^5/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(5*(a*b*x^2 + a^2)*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) - (2*b^2
*x^2 + 5*a*b)*sqrt(b*x^3 + a*x))/(b^4*x^2 + a*b^3)
```

Sympy [F]

$$\int \frac{x^5}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^5}{(x\,(a+bx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5/(b*x**3+a*x)**(3/2),x)
```

output
```
Integral(x**5/(x*(a + b*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^5}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^5}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^5/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^5/(b*x^3 + a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x^5}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^5}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^5/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^5/(b*x^3 + a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^5}{(b\,x^3+a\,x)^{3/2}}\,dx$$

input
```
int(x^5/(a*x + b*x^3)^(3/2),x)
```

output
```
int(x^5/(a*x + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^5}{(ax+bx^3)^{3/2}}\,dx = \frac{10\sqrt{x}\,\sqrt{b\,x^2+a}\,a+2\sqrt{x}\,\sqrt{b\,x^2+a}\,b\,x^2-5\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^2x^5+2ab\,x^3+a^2x}dx\right)a^3-5\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^{}}}{b^2x^5+2ab\,x}\right.}{3b^2\left(b\,x^2+a\right)}$$

input
```
int(x^5/(b*x^3+a*x)^(3/2),x)
```

output
```
(10*sqrt(x)*sqrt(a + b*x**2)*a + 2*sqrt(x)*sqrt(a + b*x**2)*b*x**2 - 5*int
((sqrt(x)*sqrt(a + b*x**2))/(a**2*x + 2*a*b*x**3 + b**2*x**5),x)*a**3 - 5*
int((sqrt(x)*sqrt(a + b*x**2))/(a**2*x + 2*a*b*x**3 + b**2*x**5),x)*a**2*b
*x**2)/(3*b**2*(a + b*x**2))
```

3.75 $\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx$

Optimal result . . . 627
Mathematica [C] (verified) . . . 628
Rubi [A] (verified) . . . 628
Maple [A] (verified) . . . 631
Fricas [A] (verification not implemented) . . . 632
Sympy [F] . . . 632
Maxima [F] . . . 633
Giac [F] . . . 633
Mupad [F(-1)] . . . 633
Reduce [F] . . . 634

Optimal result

Integrand size = 17, antiderivative size = 253

$$\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx = -\frac{x^2}{b\sqrt{ax+bx^3}}+\frac{3x(a+bx^2)}{b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{3\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{7/4}\sqrt{ax+bx^3}} + \frac{3\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{7/4}\sqrt{ax+bx^3}}$$

output
```
-x^2/b/(b*x^3+a*x)^(1/2)+3*x*(b*x^2+a)/b^(3/2)/(a^(1/2)+b^(1/2)*x)/(b*x^3+
a*x)^(1/2)-3*a^(1/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/
2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2
))/b^(7/4)/(b*x^3+a*x)^(1/2)+3/2*a^(1/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x
^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x^(1/2
)/a^(1/4)),1/2*2^(1/2))/b^(7/4)/(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.23

$$\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx = -\frac{2x^2\left(-1+\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{b\sqrt{x(a+bx^2)}}$$

input
```
Integrate[x^4/(a*x + b*x^3)^(3/2),x]
```

output
```
(-2*x^2*(-1 + Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^
2)/a)]))/(b*Sqrt[x*(a + b*x^2)])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {1928, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1928}$$

$$\frac{3\int \frac{x}{\sqrt{bx^3+ax}}dx}{2b}-\frac{x^2}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{ 1938}$$

$$\frac{3\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{2b\sqrt{ax+bx^3}}-\frac{x^2}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{3\sqrt{x}\sqrt{a+bx^2}\int\frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{b\sqrt{ax+bx^3}}-\frac{x^2}{b\sqrt{ax+bx^3}}$$

$\Big\downarrow$ 834

$$\frac{3\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{b\sqrt{ax+bx^3}}-\frac{x^2}{b\sqrt{ax+bx^3}}$$

$\Big\downarrow$ 27

$$\frac{3\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{b\sqrt{ax+bx^3}}-\frac{x^2}{b\sqrt{ax+bx^3}}$$

$\Big\downarrow$ 761

$$\frac{3\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{b\sqrt{ax+bx^3}}-\frac{x^2}{b\sqrt{ax+bx^3}}$$

$\Big\downarrow$ 1510

$$\frac{3\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{b\sqrt{ax+bx^3}}$$

$$\frac{x^2}{b\sqrt{ax+bx^3}}$$

input
```
Int[x^4/(a*x + b*x^3)^(3/2),x]
```

output
```
-(x^2/(b*Sqrt[a*x + b*x^3])) + (3*Sqrt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x]*S
qrt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sq
rt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x
])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a]
 + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan
[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(b*Sqrt[a
*x + b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1928
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.72

method	result
default	$-\frac{x^{2}}{b\sqrt{\left(x^{2}+\frac{a}{b}\right)bx}}+\frac{3\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,\mathrm{Elliptic}}{}\right.}{2b^{2}\sqrt{b\,x^{3}+ax}}$
elliptic	$-\frac{x^{2}}{b\sqrt{\left(x^{2}+\frac{a}{b}\right)bx}}+\frac{3\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,\mathrm{Elliptic}}{}\right.}{2b^{2}\sqrt{b\,x^{3}+ax}}$

input
```
int(x^4/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b*x^2/((x^2+a/b)*b*x)^(1/2)+3/2/b^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-
a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b
)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*
b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((
x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx = -\frac{\sqrt{bx^3+ax}bx + 3\left(bx^2+a\right)\sqrt{b}\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)}{b^3x^2+ab^2}$$

input
```
integrate(x^4/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-(sqrt(b*x^3 + a*x)*b*x + 3*(b*x^2 + a)*sqrt(b)*weierstrassZeta(-4*a/b, 0,
 weierstrassPInverse(-4*a/b, 0, x)))/(b^3*x^2 + a*b^2)
```

Sympy [F]

$$\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^4}{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4/(b*x**3+a*x)**(3/2),x)
```

output
```
Integral(x**4/(x*(a + b*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^4}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^3 + a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^4}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^3 + a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^4}{(b\,x^3+a\,x)^{3/2}}\,dx$$

input
```
int(x^4/(a*x + b*x^3)^(3/2),x)
```

output
```
int(x^4/(a*x + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{b\,x^2+a}\,x-3\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^2x^4+2ab\,x^2+a^2}dx\right)a^2-3\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^2x^4+2ab\,x^2+a^2}dx\right)ab\,x^2}{b\,(b\,x^2+a)}$$

input
```
int(x^4/(b*x^3+a*x)^(3/2),x)
```

output
```
(2*sqrt(x)*sqrt(a + b*x**2)*x - 3*int((sqrt(x)*sqrt(a + b*x**2))/(a**2 + 2
*a*b*x**2 + b**2*x**4),x)*a**2 - 3*int((sqrt(x)*sqrt(a + b*x**2))/(a**2 +
2*a*b*x**2 + b**2*x**4),x)*a*b*x**2)/(b*(a + b*x**2))
```

3.76 $\int \frac{x^3}{(ax+bx^3)^{3/2}}\,dx$

Optimal result 635
Mathematica [C] (verified) 635
Rubi [A] (verified) 636
Maple [A] (verified) 638
Fricas [A] (verification not implemented) 638
Sympy [F] 639
Maxima [F] 639
Giac [F] 639
Mupad [F(-1)] 640
Reduce [F] 640

Optimal result

Integrand size = 17, antiderivative size = 115

$$\int \frac{x^3}{(ax+bx^3)^{3/2}}\,dx = -\frac{x}{b\sqrt{ax+bx^3}} + \frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}b^{5/4}\sqrt{ax+bx^3}}$$

output
```
-x/b/(b*x^3+a*x)^(1/2)+1/2*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)
+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)),1/2
*2^(1/2))/a^(1/4)/b^(5/4)/(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.47

$$\int \frac{x^3}{(ax+bx^3)^{3/2}}\,dx = \frac{x\left(-1+\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{b\sqrt{x\,(a+bx^2)}}$$

input
```
Integrate[x^3/(a*x + b*x^3)^(3/2),x]
```

output
```
(x*(-1 + Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)
]))/(b*Sqrt[x*(a + b*x^2)])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1928, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(ax+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{1928}$$

$$\frac{\int \frac{1}{\sqrt{bx^3+ax}}dx}{2b}-\frac{x}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1917}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{2b\sqrt{ax+bx^3}}-\frac{x}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{b\sqrt{ax+bx^3}}-\frac{x}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{761}$$

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}b^{5/4}\sqrt{ax+bx^3}}-\frac{x}{b\sqrt{ax+bx^3}}$$

input
```
Int[x^3/(a*x + b*x^3)^(3/2),x]
```

output
```
-(x/(b*Sqrt[a*x + b*x^3])) + (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^
2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)],
 1/2])/(2*a^(1/4)*b^(5/4)*Sqrt[a*x + b*x^3])
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1928
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.13

method	result	size
default	$-\frac{x}{b\sqrt{\left(x^{2}+\frac{a}{b}\right)bx}}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{2b^{2}\sqrt{b\,x^{3}+ax}}$	130
elliptic	$-\frac{x}{b\sqrt{\left(x^{2}+\frac{a}{b}\right)bx}}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{2b^{2}\sqrt{b\,x^{3}+ax}}$	130

input
```
int(x^3/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b*x/((x^2+a/b)*b*x)^(1/2)+1/2/b^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*
b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^
(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1
/2)*b)^(1/2),1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.44

$$\int \frac{x^3}{(ax+bx^3)^{3/2}}\,dx=\frac{(bx^2+a)\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)-\sqrt{bx^3+ax}b}{b^3x^2+ab^2}$$

input
```
integrate(x^3/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
((b*x^2 + a)*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) - sqrt(b*x^3 + a*x)
*b)/(b^3*x^2 + a*b^2)
```

Sympy [F]

$$\int \frac{x^3}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^3}{(x\,(a+bx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(b*x**3+a*x)**(3/2),x)
```

output
```
Integral(x**3/(x*(a + b*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^3}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(b*x^3 + a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x^3}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^3}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^3/(b*x^3 + a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(ax+bx^3)^{3/2}}\,dx=\int \frac{x^3}{\left(b\,x^3+a\,x\right)^{3/2}}\,dx$$

input
```
int(x^3/(a*x + b*x^3)^(3/2),x)
```

output
```
int(x^3/(a*x + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3}{(ax+bx^3)^{3/2}}\,dx=\frac{-2\sqrt{x}\,\sqrt{b\,x^2+a}+\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^2x^5+2ab\,x^3+a^2x}dx\right)a^2+\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^2x^5+2ab\,x^3+a^2x}dx\right)ab\,x^2}{b\left(b\,x^2+a\right)}$$

input
```
int(x^3/(b*x^3+a*x)^(3/2),x)
```

output
```
( - 2*sqrt(x)*sqrt(a + b*x**2) + int((sqrt(x)*sqrt(a + b*x**2))/(a**2*x +
2*a*b*x**3 + b**2*x**5),x)*a**2 + int((sqrt(x)*sqrt(a + b*x**2))/(a**2*x +
 2*a*b*x**3 + b**2*x**5),x)*a*b*x**2)/(b*(a + b*x**2))
```

3.77 $\int \frac{x^2}{(ax+bx^3)^{3/2}}\,dx$

Optimal result . . . 641
Mathematica [C] (verified) . . . 642
Rubi [A] (verified) . . . 642
Maple [A] (verified) . . . 645
Fricas [A] (verification not implemented) . . . 646
Sympy [F] . . . 646
Maxima [F] . . . 647
Giac [F] . . . 647
Mupad [F(-1)] . . . 647
Reduce [F] . . . 648

Optimal result

Integrand size = 17, antiderivative size = 254

$$\int \frac{x^2}{(ax+bx^3)^{3/2}}\,dx=\frac{x^2}{a\sqrt{ax+bx^3}}-\frac{x(a+bx^2)}{a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{ax+bx^3}}-\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{ax+bx^3}}$$

output

```
x^2/a/(b*x^3+a*x)^(1/2)-x*(b*x^2+a)/a/b^(1/2)/(a^(1/2)+b^(1/2)*x)/(b*x^3+a
*x)^(1/2)+x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1
/2)*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))/a^(3/4)/
b^(3/4)/(b*x^3+a*x)^(1/2)-1/2*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1
/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)),
1/2*2^(1/2))/a^(3/4)/b^(3/4)/(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.22

$$\int \frac{x^2}{(ax+bx^3)^{3/2}}\,dx=\frac{2x^2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{bx^2}{a}\right)}{3a\sqrt{x(a+bx^2)}}$$

input
```
Integrate[x^2/(a*x + b*x^3)^(3/2),x]
```

output
```
(2*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^2)/a)])
/(3*a*Sqrt[x*(a + b*x^2)])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {1929, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1929}$$

$$\frac{x^2}{a\sqrt{ax+bx^3}}-\frac{\int \frac{x}{\sqrt{bx^3+ax}}dx}{2a}$$

$$\Big\downarrow \text{ 1938}$$

$$\frac{x^2}{a\sqrt{ax+bx^3}}-\frac{\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{2a\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{x^2}{a\sqrt{ax+bx^3}}-\frac{\sqrt{x}\sqrt{a+bx^2}\int \frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{a\sqrt{ax+bx^3}}$$

$$\Big\downarrow \textcolor{blue}{834}$$

$$\frac{x^2}{a\sqrt{ax+bx^3}}-\frac{\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{x^2}{a\sqrt{ax+bx^3}}-\frac{\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}$$

$$\Big\downarrow \textcolor{blue}{761}$$

$$\frac{x^2}{a\sqrt{ax+bx^3}}-\frac{\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}$$

$$\Big\downarrow \textcolor{blue}{1510}$$

$$\frac{x^2}{a\sqrt{ax+bx^3}}-\frac{\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{a\sqrt{ax+bx^3}}$$

input
```
Int[x^2/(a*x + b*x^3)^(3/2),x]
```

output
```
x^2/(a*Sqrt[a*x + b*x^3]) - (Sqrt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x]*Sqrt[a
 + b*x^2])/(Sqrt[a] + Sqrt[b]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^
(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sq
rt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(
1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(a*Sqrt[a*x +
b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.72

method	result
default	$\frac{x^2}{a\sqrt{(x^2+\frac{a}{b})bx}}-\frac{\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,\text{EllipticF}\left(\right.}{}\right.}{2ab\sqrt{b\,x^3+ax}}$
elliptic	$\frac{x^2}{a\sqrt{(x^2+\frac{a}{b})bx}}-\frac{\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,\text{EllipticF}\left(\right.}{}\right.}{2ab\sqrt{b\,x^3+ax}}$

input
```
int(x^2/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x^2/a/((x^2+a/b)*b*x)^(1/2)-1/2/a*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b
)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(
1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^
(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(
-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.24

$$\int \frac{x^2}{(ax+bx^3)^{3/2}}\,dx = \frac{\sqrt{bx^3+ax}bx + (bx^2+a)\sqrt{b}\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)}{ab^2x^2+a^2b}$$

input
```
integrate(x^2/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
(sqrt(b*x^3 + a*x)*b*x + (b*x^2 + a)*sqrt(b)*weierstrassZeta(-4*a/b, 0, we
ierstrassPInverse(-4*a/b, 0, x)))/(a*b^2*x^2 + a^2*b)
```

Sympy [F]

$$\int \frac{x^2}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^2}{(x\,(a+bx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(b*x**3+a*x)**(3/2),x)
```

output
```
Integral(x**2/(x*(a + b*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^2}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^3 + a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^2}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^3 + a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^2}{(b\,x^3+a\,x)^{3/2}}\,dx$$

input
```
int(x^2/(a*x + b*x^3)^(3/2),x)
```

output
```
int(x^2/(a*x + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2}{(ax+bx^3)^{3/2}}\,dx = \int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^2x^4+2ab\,x^2+a^2}dx$$

input
```
int(x^2/(b*x^3+a*x)^(3/2),x)
```

output
```
int((sqrt(x)*sqrt(a + b*x**2))/(a**2 + 2*a*b*x**2 + b**2*x**4),x)
```

3.78 $\int \frac{x}{(ax+bx^3)^{3/2}}\,dx$

Optimal result 649
Mathematica [C] (verified) 649
Rubi [A] (verified) 650
Maple [A] (verified) 652
Fricas [A] (verification not implemented) 652
Sympy [F] 653
Maxima [F] 653
Giac [F] 653
Mupad [F(-1)] 654
Reduce [F] 654

Optimal result

Integrand size = 15, antiderivative size = 114

$$\int \frac{x}{(ax+bx^3)^{3/2}}\,dx = \frac{x}{a\sqrt{ax+bx^3}} + \frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\sqrt{ax+bx^3}}$$

output
```
x/a/(b*x^3+a*x)^(1/2)+1/2*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+
b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)),1/2*
2^(1/2))/a^(5/4)/b^(1/4)/(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.47

$$\int \frac{x}{(ax+bx^3)^{3/2}}\,dx = \frac{x + x\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)}{a\sqrt{x(a+bx^2)}}$$

input
```
Integrate[x/(a*x + b*x^3)^(3/2),x]
```

output
```
(x + x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])
/(a*Sqrt[x*(a + b*x^2)])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1929, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(ax+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{ 1929}$$

$$\frac{\int \frac{1}{\sqrt{bx^3+ax}}dx}{2a}+\frac{x}{a\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{ 1917}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{2a\sqrt{ax+bx^3}}+\frac{x}{a\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{a\sqrt{ax+bx^3}}+\frac{x}{a\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\sqrt{ax+bx^3}}+\frac{x}{a\sqrt{ax+bx^3}}$$

input
```
Int[x/(a*x + b*x^3)^(3/2),x]
```

output
```
x/(a*Sqrt[a*x + b*x^3]) + (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/
(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/
2])/(2*a^(5/4)*b^(1/4)*Sqrt[a*x + b*x^3])
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.16

method	result	size
default	$\frac{x}{a\sqrt{\left(x^{2}+\frac{a}{b}\right) b x}}+\frac{\sqrt{-a b}\, \sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\, \sqrt{-\frac{2\left(x-\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\, \sqrt{-\frac{b x}{\sqrt{-a b}}}\, \operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}, \frac{\sqrt{2}}{2}\right)}{2 a b \sqrt{b\,x^{3}+a x}}$	132
elliptic	$\frac{x}{a\sqrt{\left(x^{2}+\frac{a}{b}\right) b x}}+\frac{\sqrt{-a b}\, \sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\, \sqrt{-\frac{2\left(x-\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\, \sqrt{-\frac{b x}{\sqrt{-a b}}}\, \operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}, \frac{\sqrt{2}}{2}\right)}{2 a b \sqrt{b\,x^{3}+a x}}$	132

input
```
int(x/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x/a/((x^2+a/b)*b*x)^(1/2)+1/2/a*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^
(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/
2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)
*b)^(1/2),1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.45

$$\int \frac{x}{(ax+bx^3)^{3/2}}\,dx = \frac{(bx^2+a)\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\sqrt{bx^3+ax}b}{ab^2x^2+a^2b}$$

input
```
integrate(x/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
((b*x^2 + a)*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b*x^3 + a*x)
*b)/(a*b^2*x^2 + a^2*b)
```

Sympy [F]

$$\int \frac{x}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x}{(x(a+bx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x**3+a*x)**(3/2),x)
```

output
```
Integral(x/(x*(a + b*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(b*x^3 + a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/(b*x^3 + a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x}{(b\,x^3+a\,x)^{3/2}}\,dx$$

input
```
int(x/(a*x + b*x^3)^(3/2),x)
```

output
```
int(x/(a*x + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{x}{(ax+bx^3)^{3/2}}\,dx = \int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^2x^5+2ab\,x^3+a^2x}dx$$

input
```
int(x/(b*x^3+a*x)^(3/2),x)
```

output
```
int((sqrt(x)*sqrt(a + b*x**2))/(a**2*x + 2*a*b*x**3 + b**2*x**5),x)
```

3.79 $\int \frac{1}{(ax+bx^3)^{3/2}}\,dx$

Optimal result 655
Mathematica [C] (verified) 656
Rubi [A] (verified) 656
Maple [A] (verified) 660
Fricas [A] (verification not implemented) 661
Sympy [F] 661
Maxima [F] 662
Giac [F] 662
Mupad [B] (verification not implemented) 662
Reduce [F] 663

Optimal result

Integrand size = 13, antiderivative size = 273

$$\begin{aligned}\int \frac{1}{(ax+bx^3)^{3/2}}\,dx &= \frac{1}{a\sqrt{ax+bx^3}}+\frac{3\sqrt{b}x(a+bx^2)}{a^2\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} \\ &- \frac{3\sqrt{ax+bx^3}}{a^2x}-\frac{3\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{7/4}\sqrt{ax+bx^3}} \\ &+ \frac{3\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{7/4}\sqrt{ax+bx^3}}\end{aligned}$$

output
```
1/a/(b*x^3+a*x)^(1/2)+3*b^(1/2)*x*(b*x^2+a)/a^2/(a^(1/2)+b^(1/2)*x)/(b*x^3
+a*x)^(1/2)-3*(b*x^3+a*x)^(1/2)/a^2/x-3*b^(1/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x
)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x
^(1/2)/a^(1/4))),1/2*2^(1/2))/a^(7/4)/(b*x^3+a*x)^(1/2)+3/2*b^(1/4)*x^(1/2
)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacob
iAM(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)),1/2*2^(1/2))/a^(7/4)/(b*x^3+a*x)^(1/
2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.19

$$\int \frac{1}{(ax+bx^3)^{3/2}}\,dx = -\frac{2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{4},-\frac{bx^2}{a}\right)}{a\sqrt{x\,(a+bx^2)}}$$

input
```
Integrate[(a*x + b*x^3)^(-3/2),x]
```

output
```
(-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/4, 3/2, 3/4, -((b*x^2)/a)])/(
a*Sqrt[x*(a + b*x^2)])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {1912, 1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{1912}$$

$$\frac{3\int \frac{1}{x\sqrt{bx^3+ax}}dx}{2a}+\frac{1}{a\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{3\left(\frac{b\int \frac{x}{\sqrt{bx^3+ax}}dx}{a}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{2a}+\frac{1}{a\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1938}$$

$$\frac{3\left(\frac{b\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{2a}+\frac{1}{a\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{3\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\int \frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{2a}+\frac{1}{a\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{3\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{2a}+\frac{1}{a\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{2a}+\frac{1}{a\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{3\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{2a}+\frac{1}{a\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{ 1510}$$

$$\frac{3\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}-\cdots}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}\cdots\right)}{2a}$$

$$\frac{1}{a\sqrt{ax+bx^3}}$$

input
```
Int[(a*x + b*x^3)^(-3/2),x]
```

output
```
1/(a*Sqrt[a*x + b*x^3]) + (3*((-2*Sqrt[a*x + b*x^3])/(a*x) + (2*b*Sqrt[x]*
Sqrt[a + b*x^2]*(-((-((Sqrt[x]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*x)) + (
a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^2
]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] +
Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(3
/4)*Sqrt[a + b*x^2])))/(a*Sqrt[a*x + b*x^3])))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1912
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.75

method	result
default	$-\frac{2(b\,x^{2}+a)}{a^{2}\sqrt{x(b\,x^{2}+a)}}-\frac{b\,x^{2}}{a^{2}\sqrt{(x^{2}+\frac{a}{b})bx}}+\frac{3\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\operatorname{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\cdots\right.}{b}\cdots\right.}{2a^{2}\sqrt{b\,x^{3}+ax}\cdots}$
elliptic	$-\frac{2(b\,x^{2}+a)}{a^{2}\sqrt{x(b\,x^{2}+a)}}-\frac{b\,x^{2}}{a^{2}\sqrt{(x^{2}+\frac{a}{b})bx}}+\frac{3\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\operatorname{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\cdots\right.}{b}\cdots\right.}{2a^{2}\sqrt{b\,x^{3}+ax}\cdots}$
risch	$-\frac{2(b\,x^{2}+a)}{a^{2}\sqrt{x(b\,x^{2}+a)}}+\frac{b^{2}\left(\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\operatorname{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,\operatorname{Ellipt}\cdots}{\cdots}\right.}{b^{2}\sqrt{b\,x^{3}+ax}\cdots}\right.}{\cdots}$

input
```
int(1/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(b*x^2+a)/a^2/(x*(b*x^2+a))^(1/2)-b*x^2/a^2/((x^2+a/b)*b*x)^(1/2)+3/2/a
^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1
/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)
*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/
2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1
/2),1/2*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.26

$$\int \frac{1}{(ax+bx^3)^{3/2}}\,dx = -\frac{3\,(bx^3+ax)\sqrt{b}\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\sqrt{bx^3+ax}(3\,bx^2+2\,a)}{a^2bx^3+a^3x}$$

input
```
integrate(1/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-(3*(b*x^3 + a*x)*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-
4*a/b, 0, x)) + sqrt(b*x^3 + a*x)*(3*b*x^2 + 2*a))/(a^2*b*x^3 + a^3*x)
```

Sympy [F]

$$\int \frac{1}{(ax+bx^3)^{3/2}}\,dx = \int \frac{1}{(ax+bx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**3+a*x)**(3/2),x)
```

output
```
Integral((a*x + b*x**3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(ax+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(ax+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.15

$$\int \frac{1}{(ax+bx^3)^{3/2}}\,dx = -\frac{2\,x\,\left(\frac{b\,x^2}{a}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{3}{2};\ \frac{3}{4};\ -\frac{b\,x^2}{a}\right)}{(b\,x^3+a\,x)^{3/2}}$$

input
```
int(1/(a*x + b*x^3)^(3/2),x)
```

output
```
-(2*x*((b*x^2)/a + 1)^(3/2)*hypergeom([-1/4, 3/2], 3/4, -(b*x^2)/a))/(a*x
+ b*x^3)^(3/2)
```

Reduce [F]

$$\int \frac{1}{(ax + bx^3)^{3/2}} \, dx = \int \frac{\sqrt{x}\,\sqrt{b\,x^2 + a}}{b^2x^6 + 2ab\,x^4 + a^2x^2} dx$$

input
```
int(1/(b*x^3+a*x)^(3/2),x)
```

output
```
int((sqrt(x)*sqrt(a + b*x**2))/(a**2*x**2 + 2*a*b*x**4 + b**2*x**6),x)
```

3.80 $\int \frac{1}{x(ax+bx^3)^{3/2}} dx$

Optimal result 664
Mathematica [C] (verified) 664
Rubi [A] (verified) 665
Maple [A] (verified) 667
Fricas [A] (verification not implemented) 668
Sympy [F] 668
Maxima [F] 669
Giac [F] 669
Mupad [F(-1)] 669
Reduce [F] 670

Optimal result

Integrand size = 17, antiderivative size = 139

$$\int \frac{1}{x(ax+bx^3)^{3/2}} dx = \frac{1}{ax\sqrt{ax+bx^3}} - \frac{5\sqrt{ax+bx^3}}{3a^2x^2} - \frac{5b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{9/4}\sqrt{ax+bx^3}}$$

output
```
1/a/x/(b*x^3+a*x)^(1/2)-5/3*(b*x^3+a*x)^(1/2)/a^2/x^2-5/6*b^(3/4)*x^(1/2)*
(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiA
M(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)),1/2*2^(1/2))/a^(9/4)/(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.40

$$\int \frac{1}{x(ax+bx^3)^{3/2}} dx = -\frac{2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{2},\frac{1}{4},-\frac{bx^2}{a}\right)}{3ax\sqrt{x(a+bx^2)}}$$

input
```
Integrate[1/(x*(a*x + b*x^3)^(3/2)),x]
```

output
```
(-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((b*x^2)/a)])/(
3*a*x*Sqrt[x*(a + b*x^2)])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1929, 1931, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{5\int \frac{1}{x^2\sqrt{bx^3+ax}}dx}{2a}+\frac{1}{ax\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{5\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax}}dx}{3a}-\frac{2\sqrt{ax+bx^3}}{3ax^2}\right)}{2a}+\frac{1}{ax\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1917}$$

$$\frac{5\left(-\frac{b\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3ax^2}\right)}{2a}+\frac{1}{ax\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{266}$$

$$\frac{5\left(-\frac{2b\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3ax^2}\right)}{2a}+\frac{1}{ax\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{761}$$

$$\frac{5\left(-\frac{b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3a^{5/4}\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3ax^2}\right)}{2a}+\frac{1}{ax\sqrt{ax+bx^3}}$$

input
```
Int[1/(x*(a*x + b*x^3)^(3/2)),x]
```

output
```
1/(a*x*Sqrt[a*x + b*x^3]) + (5*((-2*Sqrt[a*x + b*x^3])/(3*a*x^2) - (b^(3/4
)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*
EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[a*x +
 b*x^3])))/(2*a)
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.08

method	result
default	$-\frac{bx}{a^{2}\sqrt{\left(x^{2}+\frac{a}{b}\right)bx}}-\frac{2\sqrt{b\,x^{3}+ax}}{3a^{2}x^{2}}-\frac{5\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{6a^{2}\sqrt{b\,x^{3}+ax}}$
elliptic	$-\frac{bx}{a^{2}\sqrt{\left(x^{2}+\frac{a}{b}\right)bx}}-\frac{2\sqrt{b\,x^{3}+ax}}{3a^{2}x^{2}}-\frac{5\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{6a^{2}\sqrt{b\,x^{3}+ax}}$
risch	$-\frac{2\left(b\,x^{2}+a\right)}{3a^{2}x\sqrt{x\left(b\,x^{2}+a\right)}}-\frac{b\left(\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{b\,x^{3}+ax}}+3a\left(\frac{x}{a\sqrt{\left(x^{2}+\frac{a}{b}\right)bx}}+\right.\right.}{3a^{2}}$

input
```
int(1/x/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-b*x/a^2/((x^2+a/b)*b*x)^(1/2)-2/3*(b*x^3+a*x)^(1/2)/a^2/x^2-5/6/a^2*(-a*b
)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(
-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*Ellipti
cF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.49

$$\int \frac{1}{x\left(ax+bx^3\right)^{3/2}}\,dx = -\frac{5\left(bx^4+ax^2\right)\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\sqrt{bx^3+ax}\left(5\,bx^2+2\,a\right)}{3\left(a^2bx^4+a^3x^2\right)}$$

input
```
integrate(1/x/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(5*(b*x^4 + a*x^2)*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b
*x^3 + a*x)*(5*b*x^2 + 2*a))/(a^2*b*x^4 + a^3*x^2)
```

Sympy [F]

$$\int \frac{1}{x\left(ax+bx^3\right)^{3/2}}\,dx = \int \frac{1}{x\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(b*x**3+a*x)**(3/2),x)
```

output
```
Integral(1/(x*(x*(a + b*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(ax+bx^3\right)^{3/2}}\,dx = \int \frac{1}{\left(bx^3+ax\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a*x)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{1}{x\left(ax+bx^3\right)^{3/2}}\,dx = \int \frac{1}{\left(bx^3+ax\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a*x)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(ax+bx^3\right)^{3/2}}\,dx = \int \frac{1}{x\left(b\,x^3+a\,x\right)^{3/2}}\,dx$$

input
```
int(1/(x*(a*x + b*x^3)^(3/2)),x)
```

output
```
int(1/(x*(a*x + b*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\left(ax+bx^3\right)^{3/2}}\,dx=\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^2x^7+2ab\,x^5+a^2x^3}dx$$

input
```
int(1/x/(b*x^3+a*x)^(3/2),x)
```

output
```
int((sqrt(x)*sqrt(a + b*x**2))/(a**2*x**3 + 2*a*b*x**5 + b**2*x**7),x)
```

3.81 $\int \frac{1}{x^2(ax+bx^3)^{3/2}}\,dx$

Optimal result . . . 671
Mathematica [C] (verified) . . . 672
Rubi [A] (verified) . . . 672
Maple [A] (verified) . . . 677
Fricas [A] (verification not implemented) . . . 679
Sympy [F] . . . 679
Maxima [F] . . . 679
Giac [F] . . . 680
Mupad [F(-1)] . . . 680
Reduce [F] . . . 680

Optimal result

Integrand size = 17, antiderivative size = 306

$$
\begin{aligned}
\int \frac{1}{x^2\left(ax+bx^3\right)^{3/2}}\,dx &= \frac{1}{ax^2\sqrt{ax+bx^3}} \\
&- \frac{21b^{3/2}x(a+bx^2)}{5a^3\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} + \frac{21b\sqrt{ax+bx^3}}{5a^3x} \\
&+ \frac{21b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{11/4}\sqrt{ax+bx^3}} \\
&- \frac{21b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{10a^{11/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

output
```
1/a/x^2/(b*x^3+a*x)^(1/2)-21/5*b^(3/2)*x*(b*x^2+a)/a^3/(a^(1/2)+b^(1/2)*x)
/(b*x^3+a*x)^(1/2)-7/5*(b*x^3+a*x)^(1/2)/a^2/x^3+21/5*b*(b*x^3+a*x)^(1/2)/
a^3/x+21/5*b^(5/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)
*x)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))
/a^(11/4)/(b*x^3+a*x)^(1/2)-21/10*b^(5/4)*x^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*
x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x^(1/
2)/a^(1/4)),1/2*2^(1/2))/a^(11/4)/(b*x^3+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^2 (ax+bx^3)^{3/2}}\,dx = -\frac{2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{3}{2},-\frac{1}{4},-\frac{bx^2}{a}\right)}{5ax^2\sqrt{x(a+bx^2)}}$$

input
```
Integrate[1/(x^2*(a*x + b*x^3)^(3/2)),x]
```

output
```
(-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-5/4, 3/2, -1/4, -((b*x^2)/a)])/
(5*a*x^2*Sqrt[x*(a + b*x^2)])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1929, 1931, 1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (ax+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{7\int \frac{1}{x^3\sqrt{bx^3+ax}}dx}{2a}+\frac{1}{ax^2\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{7\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax}}dx}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}\right)}{2a}+\frac{1}{ax^2\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{7\left(-\frac{3b\left(\frac{b\int \frac{x}{\sqrt{bx^3+ax}}dx}{a}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}\right)}{2a}+\frac{1}{ax^2\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1938}$$

$$\frac{7\left(-\frac{3b\left(\frac{b\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}\right)}{2a}+\frac{1}{ax^2\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{266}$$

$$\frac{7\left(-\frac{3b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\int \frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}\right)}{2a}+\frac{1}{ax^2\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{834}$$

$$\frac{7\left(-\frac{3b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}\right)}{2a}+\frac{1}{ax^2\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{7\left(-\frac{3b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}\right)}{2a}+\frac{1}{ax^2\sqrt{ax+bx^3}}$$

$\Big\downarrow$ 761

$$\frac{7\left(-\frac{3b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}\right)}{2a}+\frac{1}{ax^2\sqrt{ax+bx^3}}$$

$\Big\downarrow$ 1510

$$\frac{7\left(-\frac{3b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}}{a\sqrt{ax+bx^3}}}{5a}}{2a}$$

$$\frac{1}{ax^2\sqrt{ax+bx^3}}$$

input
```
Int[1/(x^2*(a*x + b*x^3)^(3/2)),x]
```

output
```
1/(a*x^2*Sqrt[a*x + b*x^3]) + (7*((-2*Sqrt[a*x + b*x^3])/(5*a*x^3) - (3*b*
((-2*Sqrt[a*x + b*x^3])/(a*x) + (2*b*Sqrt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x
]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)
*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqr
t[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt
[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*Arc
Tan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(a*Sqr
t[a*x + b*x^3])))/(5*a)))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.75

method	result
default	$-\frac{2\sqrt{b\,x^3+ax}}{5a^2x^3}+\frac{16(b\,x^2+a)b}{5a^3\sqrt{x(b\,x^2+a)}}+\frac{b^2x^2}{a^3\sqrt{(x^2+\frac{a}{b})bx}}-\frac{21b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{Ellip}}{}\right.}{10a^3\sqrt{b\,x}}$
elliptic	$-\frac{2\sqrt{b\,x^3+ax}}{5a^2x^3}+\frac{16(b\,x^2+a)b}{5a^3\sqrt{x(b\,x^2+a)}}+\frac{b^2x^2}{a^3\sqrt{(x^2+\frac{a}{b})bx}}-\frac{21b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{Ellip}}{}\right.}{10a^3\sqrt{b\,x}}$
risch	$-\frac{2(b\,x^2+a)(-8b\,x^2+a)}{5a^3x^2\sqrt{x(b\,x^2+a)}}-\frac{b^2\left(\frac{8\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-}}{}\right.}{b\sqrt{b\,x^3+ax}}\right.}{}$

input
```
int(1/x^2/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(b*x^3+a*x)^(1/2)/a^2/x^3+16/5*(b*x^2+a)*b/a^3/(x*(b*x^2+a))^(1/2)+b^
2*x^2/a^3/((x^2+a/b)*b*x)^(1/2)-21/10*b/a^3*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/
b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/
(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x
+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*Ellipti
cF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.29

$$\int \frac{1}{x^2 (ax+bx^3)^{3/2}} dx = \frac{21 (b^2x^5 + abx^3)\sqrt{b}\text{weierstrassZeta}\left(-\frac{4a}{b}, 0, \text{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + (2}{5 (a^3bx^5 + a^4x^3)}$$

input
```
integrate(1/x^2/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
1/5*(21*(b^2*x^5 + a*b*x^3)*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierstrass
PInverse(-4*a/b, 0, x)) + (21*b^2*x^4 + 14*a*b*x^2 - 2*a^2)*sqrt(b*x^3 + a
*x))/(a^3*b*x^5 + a^4*x^3)
```

Sympy [F]

$$\int \frac{1}{x^2 (ax+bx^3)^{3/2}} dx = \int \frac{1}{x^2 (x (a+bx^2))^{\frac{3}{2}}} dx$$

input
```
integrate(1/x**2/(b*x**3+a*x)**(3/2),x)
```

output
```
Integral(1/(x**2*(x*(a + b*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (ax+bx^3)^{3/2}} dx = \int \frac{1}{(bx^3+ax)^{\frac{3}{2}} x^2} dx$$

input
```
integrate(1/x^2/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a*x)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (ax+bx^3)^{3/2}} \, dx = \int \frac{1}{(bx^3+ax)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a*x)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (ax+bx^3)^{3/2}} \, dx = \int \frac{1}{x^2 (b\,x^3+a\,x)^{3/2}} \, dx$$

input
```
int(1/(x^2*(a*x + b*x^3)^(3/2)),x)
```

output
```
int(1/(x^2*(a*x + b*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (ax+bx^3)^{3/2}} \, dx = \int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^2x^8+2ab\,x^6+a^2x^4} dx$$

input
```
int(1/x^2/(b*x^3+a*x)^(3/2),x)
```

output
```
int((sqrt(x)*sqrt(a + b*x**2))/(a**2*x**4 + 2*a*b*x**6 + b**2*x**8),x)
```

3.82 $\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}}\,dx$

Optimal result 681
Mathematica [A] (verified) 681
Rubi [A] (verified) 682
Maple [A] (verified) 686
Fricas [A] (verification not implemented) 687
Sympy [F(-1)] 688
Maxima [F] 688
Giac [A] (verification not implemented) 688
Mupad [F(-1)] 689
Reduce [B] (verification not implemented) 689

Optimal result

Integrand size = 19, antiderivative size = 159

$$\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{x^{25/2}}{7b(ax+bx^3)^{7/2}} - \frac{9x^{19/2}}{35b^2(ax+bx^3)^{5/2}} - \frac{3x^{13/2}}{5b^3(ax+bx^3)^{3/2}} - \frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}} + \frac{9\sqrt{x}\sqrt{ax+bx^3}}{2b^5} - \frac{9a\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax+bx^3}}\right)}{2b^{11/2}}$$

output
```
-1/7*x^(25/2)/b/(b*x^3+a*x)^(7/2)-9/35*x^(19/2)/b^2/(b*x^3+a*x)^(5/2)-3/5*
x^(13/2)/b^3/(b*x^3+a*x)^(3/2)-3*x^(7/2)/b^4/(b*x^3+a*x)^(1/2)+9/2*x^(1/2)
*(b*x^3+a*x)^(1/2)/b^5-9/2*a*arctanh(b^(1/2)*x^(3/2)/(b*x^3+a*x)^(1/2))/b^
(11/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.81

$$\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{9/2}\left(\sqrt{b}(a+bx^2)\left(315a^4x+1050a^3bx^3+1218a^2b^2x^5+528ab^3x^7+35b^4x^9\right)-630a\right.}{70b^{11/2}\left(x\left(a+bx^2\right)\right)^{9/2}}$$

input
```
Integrate[x^(29/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
(x^(9/2)*(Sqrt[b]*(a + b*x^2)*(315*a^4*x + 1050*a^3*b*x^3 + 1218*a^2*b^2*x
^5 + 528*a*b^3*x^7 + 35*b^4*x^9) - 630*a*(a + b*x^2)^(9/2)*ArcTanh[(Sqrt[b
]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])]))/(70*b^(11/2)*(x*(a + b*x^2))^(9/2))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1928, 1928, 1928, 1928, 1930, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}}dx$$

$$\Big\downarrow \text{1928}$$

$$\frac{9\int \frac{x^{23/2}}{(bx^3+ax)^{7/2}}dx}{7b}-\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1928}$$

$$\frac{9\left(\frac{7\int \frac{x^{17/2}}{(bx^3+ax)^{5/2}}dx}{5b}-\frac{x^{19/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1928}$$

$$\frac{9\left(\frac{7\left(\frac{5\int \frac{x^{11/2}}{(bx^3+ax)^{3/2}}dx}{3b}-\frac{x^{13/2}}{3b\left(ax+bx^3\right)^{3/2}}\right)}{5b}-\frac{x^{19/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1928}$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\int \frac{x^{5/2}}{\sqrt{bx^3+ax}}dx}{b}-\frac{x^{7/2}}{b\sqrt{ax+bx^3}}\right)}{3b}-\frac{x^{13/2}}{3b\left(ax+bx^3\right)^{3/2}}\right)}{5b}-\frac{x^{19/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

$\Big\downarrow$ 1930

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\sqrt{x}\sqrt{ax+bx^3}}{2b}-\frac{a\int \frac{\sqrt{x}}{\sqrt{bx^3+ax}}dx}{2b}\right)}{b}-\frac{x^{7/2}}{b\sqrt{ax+bx^3}}\right)}{3b}-\frac{x^{13/2}}{3b\left(ax+bx^3\right)^{3/2}}\right)}{5b}-\frac{x^{19/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

$\Big\downarrow$ 1935

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\sqrt{x}\sqrt{ax+bx^3}}{2b}-\frac{a\int\frac{1}{1-\frac{bx^3}{bx^3+ax}}d\frac{x^{3/2}}{\sqrt{bx^3+ax}}}{2b}\right)}{b}-\frac{x^{7/2}}{b\sqrt{ax+bx^3}}\right)}{3b}-\frac{x^{13/2}}{3b\left(ax+bx^3\right)^{3/2}}\right)}{5b}-\frac{x^{19/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

$\big\downarrow$ 219

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\sqrt{x}\sqrt{ax+bx^3}}{2b}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax+bx^3}}\right)}{2b^{3/2}}\right)}{b}-\frac{x^{7/2}}{b\sqrt{ax+bx^3}}\right)}{3b}-\frac{x^{13/2}}{3b\left(ax+bx^3\right)^{3/2}}\right)}{5b}-\frac{x^{19/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

input
```
Int[x^(29/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
-1/7*x^(25/2)/(b*(a*x + b*x^3)^(7/2)) + (9*(-1/5*x^(19/2)/(b*(a*x + b*x^3)
^(5/2)) + (7*(-1/3*x^(13/2)/(b*(a*x + b*x^3)^(3/2)) + (5*(-(x^(7/2)/(b*Sqr
t[a*x + b*x^3])) + (3*((Sqrt[x]*Sqrt[a*x + b*x^3])/(2*b) - (a*ArcTanh[(Sqr
t[b]*x^(3/2))/Sqrt[a*x + b*x^3]])/(2*b^(3/2))))/b))/(3*b)))/(5*b)))/(7*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1928
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.33

method	result
default	$-\frac{\sqrt{x(b\,x^2+a)}\,\left(-35x^9 b^{\frac{9}{2}}+315\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)a\,b^3x^6\sqrt{b\,x^2+a}-528b^{\frac{7}{2}}a\,x^7+945\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)a^2b^2x^4\sqrt{b\,x^2+a}-1218b^{\cdots}\right.}{70b^{\frac{11}{2}}\sqrt{x}\,(b\,x^2+\cdots}$
risch	$\frac{x^{\frac{3}{2}}\,(b\,x^2+a)}{2b^5\sqrt{x(b\,x^2+a)}}+\frac{\left(-\frac{53a^2\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2b+2\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)}}{560b^7\left(x-\frac{\sqrt{-ab}}{b}\right)^3}-\frac{571a^2\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2b+2\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)}}{1120b^6\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)^2}+\frac{97a\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2b+\cdots}}{35b^6\left(x-\cdots\right.}\right.}{\cdots}$

input
```
int(x^(29/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/70*(x*(b*x^2+a))^(1/2)/b^(11/2)*(-35*x^9*b^(9/2)+315*ln(b^(1/2)*x+(b*x^
2+a)^(1/2))*a*b^3*x^6*(b*x^2+a)^(1/2)-528*b^(7/2)*a*x^7+945*ln(b^(1/2)*x+(
b*x^2+a)^(1/2))*a^2*b^2*x^4*(b*x^2+a)^(1/2)-1218*b^(5/2)*a^2*x^5+945*ln(b^
(1/2)*x+(b*x^2+a)^(1/2))*a^3*b*x^2*(b*x^2+a)^(1/2)-1050*b^(3/2)*a^3*x^3+31
5*ln(b^(1/2)*x+(b*x^2+a)^(1/2))*a^4*(b*x^2+a)^(1/2)-315*b^(1/2)*a^4*x)/x^(
1/2)/(b*x^2+a)^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.40

$$\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}}\,dx = \left[\frac{315\,(ab^4x^8+4\,a^2b^3x^6+6\,a^3b^2x^4+4\,a^4bx^2+a^5)\sqrt{b}\log\left(2\,bx^2-2\,\sqrt{bx^3+ax}\sqrt{b}\sqrt{\cdot}\right.}{140\,(b^{10}x^8+4\,ab^9x^6+\cdots}\right.$$

input
```
integrate(x^(29/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
[1/140*(315*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*b*x^2 + a^5
)*sqrt(b)*log(2*b*x^2 - 2*sqrt(b*x^3 + a*x)*sqrt(b)*sqrt(x) + a) + 2*(35*b
^5*x^8 + 528*a*b^4*x^6 + 1218*a^2*b^3*x^4 + 1050*a^3*b^2*x^2 + 315*a^4*b)*
sqrt(b*x^3 + a*x)*sqrt(x))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3
*b^7*x^2 + a^4*b^6), 1/70*(315*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4
+ 4*a^4*b*x^2 + a^5)*sqrt(-b)*arctan(sqrt(b*x^3 + a*x)*sqrt(-b)*sqrt(x)/(b
*x^2 + a)) + (35*b^5*x^8 + 528*a*b^4*x^6 + 1218*a^2*b^3*x^4 + 1050*a^3*b^2
*x^2 + 315*a^4*b)*sqrt(b*x^3 + a*x)*sqrt(x))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a
^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(29/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{29}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(29/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(29/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.57

$$\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{\left(\left(\left(x^2\left(\frac{35\,x^2}{b}+\frac{528\,a}{b^2}\right)+\frac{1218\,a^2}{b^3}\right)x^2+\frac{1050\,a^3}{b^4}\right)x^2+\frac{315\,a^4}{b^5}\right)x}{70\,(bx^2+a)^{\frac{7}{2}}} + \frac{9\,a\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{11}{2}}}$$

input
```
integrate(x^(29/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
1/70*(((x^2*(35*x^2/b + 528*a/b^2) + 1218*a^2/b^3)*x^2 + 1050*a^3/b^4)*x^2
 + 315*a^4/b^5)*x/(b*x^2 + a)^(7/2) + 9/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^
2 + a)))/b^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{29/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\int \frac{x^{29/2}}{{\left(b\,x^3+a\,x\right)}^{9/2}}\,dx$$

input
```
int(x^(29/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(29/2)/(a*x + b*x^3)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.13

$$\int \frac{x^{29/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{315\sqrt{b\,x^2+a}\,a^4bx+1050\sqrt{b\,x^2+a}\,a^3b^2x^3+1218\sqrt{b\,x^2+a}\,a^2b^3x^5+528\sqrt{b\,x^2+a}}{}$$

input
```
int(x^(29/2)/(b*x^3+a*x)^(9/2),x)
```

output
```
(315*sqrt(a + b*x**2)*a**4*b*x + 1050*sqrt(a + b*x**2)*a**3*b**2*x**3 + 12
18*sqrt(a + b*x**2)*a**2*b**3*x**5 + 528*sqrt(a + b*x**2)*a*b**4*x**7 + 35
*sqrt(a + b*x**2)*b**5*x**9 - 315*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*
x)/sqrt(a))*a**5 - 1260*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a)
)*a**4*b*x**2 - 1890*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a
**3*b**2*x**4 - 1260*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a
**2*b**3*x**6 - 315*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*
b**4*x**8 - 213*sqrt(b)*a**5 - 852*sqrt(b)*a**4*b*x**2 - 1278*sqrt(b)*a**3
*b**2*x**4 - 852*sqrt(b)*a**2*b**3*x**6 - 213*sqrt(b)*a*b**4*x**8)/(70*b**
6*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.83 $\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}}\,dx$

Optimal result 690
Mathematica [A] (verified) 690
Rubi [A] (verified) 691
Maple [A] (verified) 693
Fricas [A] (verification not implemented) 693
Sympy [F(-1)] 694
Maxima [F] 694
Giac [A] (verification not implemented) 694
Mupad [F(-1)] 695
Reduce [B] (verification not implemented) 695

Optimal result

Integrand size = 19, antiderivative size = 126

$$\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{x^{23/2}}{7b(ax+bx^3)^{7/2}} - \frac{8x^{17/2}}{35b^2(ax+bx^3)^{5/2}} - \frac{16x^{11/2}}{35b^3(ax+bx^3)^{3/2}} - \frac{64x^{5/2}}{35b^4\sqrt{ax+bx^3}} + \frac{128\sqrt{ax+bx^3}}{35b^5\sqrt{x}}$$

output
```
-1/7*x^(23/2)/b/(b*x^3+a*x)^(7/2)-8/35*x^(17/2)/b^2/(b*x^3+a*x)^(5/2)-16/3
5*x^(11/2)/b^3/(b*x^3+a*x)^(3/2)-64/35*x^(5/2)/b^4/(b*x^3+a*x)^(1/2)+128/3
5*(b*x^3+a*x)^(1/2)/b^5/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.54

$$\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{7/2}(128a^4+448a^3bx^2+560a^2b^2x^4+280ab^3x^6+35b^4x^8)}{35b^5(x(a+bx^2))^{7/2}}$$

input
```
Integrate[x^(27/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
(x^(7/2)*(128*a^4 + 448*a^3*b*x^2 + 560*a^2*b^2*x^4 + 280*a*b^3*x^6 + 35*b
^4*x^8))/(35*b^5*(x*(a + b*x^2))^(7/2))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1921, 1921, 1921, 1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{8\int \frac{x^{21/2}}{(bx^3+ax)^{7/2}}dx}{7b}-\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{8\left(\frac{6\int \frac{x^{15/2}}{\left(bx^3+ax\right)^{5/2}}dx}{5b}-\frac{x^{17/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{8\left(\frac{6\left(\frac{4\int \frac{x^{9/2}}{\left(bx^3+ax\right)^{3/2}}dx}{3b}-\frac{x^{11/2}}{3b\left(ax+bx^3\right)^{3/2}}\right)}{5b}-\frac{x^{17/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{8\left(\frac{6\left(\frac{4\left(\frac{2\int\frac{x^{3/2}}{\sqrt{bx^3+ax}}dx}{b}-\frac{x^{5/2}}{b\sqrt{ax+bx^3}}\right)}{3b}-\frac{x^{11/2}}{3b\left(ax+bx^3\right)^{3/2}}\right)}{5b}-\frac{x^{17/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{8\left(\frac{6\left(\frac{4\left(\frac{2\sqrt{ax+bx^3}}{b^2\sqrt{x}}-\frac{x^{5/2}}{b\sqrt{ax+bx^3}}\right)}{3b}-\frac{x^{11/2}}{3b\left(ax+bx^3\right)^{3/2}}\right)}{5b}-\frac{x^{17/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

input
```
Int[x^(27/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
-1/7*x^(23/2)/(b*(a*x + b*x^3)^(7/2)) + (8*(-1/5*x^(17/2)/(b*(a*x + b*x^3)
^(5/2)) + (6*(-1/3*x^(11/2)/(b*(a*x + b*x^3)^(3/2)) + (4*(-(x^(5/2)/(b*Sqr
t[a*x + b*x^3])) + (2*Sqrt[a*x + b*x^3])/(b^2*Sqrt[x])))/(3*b)))/(5*b)))/(
7*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.56

method	result	size
gosper	$\frac{\left(b\,x^{2}+a\right)\left(35 b^{4} x^{8}+280 a\,b^{3} x^{6}+560 a^{2} b^{2} x^{4}+448 a^{3} b\,x^{2}+128 a^{4}\right) x^{\frac{9}{2}}}{35 b^{5} \left(b\,x^{3}+a x \right)^{\frac{9}{2}}}$	70
orering	$\frac{\left(b\,x^{2}+a\right)\left(35 b^{4} x^{8}+280 a\,b^{3} x^{6}+560 a^{2} b^{2} x^{4}+448 a^{3} b\,x^{2}+128 a^{4}\right) x^{\frac{9}{2}}}{35 b^{5} \left(b\,x^{3}+a x \right)^{\frac{9}{2}}}$	70
default	$\frac{\sqrt{x \left(b\,x^{2}+a \right)}\,\left(35 b^{4} x^{8}+280 a\,b^{3} x^{6}+560 a^{2} b^{2} x^{4}+448 a^{3} b\,x^{2}+128 a^{4}\right)}{35 \sqrt{x}\,\left(b\,x^{2}+a \right)^{4} b^{5}}$	72
risch	$\frac{\left(b\,x^{2}+a \right) \sqrt{x}}{b^{5} \sqrt{x \left(b\,x^{2}+a \right)}}+\frac{\left(b\,x^{2}+a \right) \left(140 b^{3} x^{6}+350 a\,b^{2} x^{4}+308 a^{2} b\,x^{2}+93 a^{3}\right) a \sqrt{x}}{35 b^{5} \left(b^{4} x^{8}+4 a\,b^{3} x^{6}+6 a^{2} b^{2} x^{4}+4 a^{3} b\,x^{2}+a^{4}\right) \sqrt{x \left(b\,x^{2}+a \right)}}$	128

input
```
int(x^(27/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*(b*x^2+a)*(35*b^4*x^8+280*a*b^3*x^6+560*a^2*b^2*x^4+448*a^3*b*x^2+128
*a^4)*x^(9/2)/b^5/(b*x^3+a*x)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

$$\int \frac{x^{27/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{\left(35\,b^{4}x^{8}+280\,ab^{3}x^{6}+560\,a^{2}b^{2}x^{4}+448\,a^{3}bx^{2}+128\,a^{4}\right)\sqrt{bx^{3}+ax}\sqrt{x}}{35\,\left(b^{9}x^{9}+4\,ab^{8}x^{7}+6\,a^{2}b^{7}x^{5}+4\,a^{3}b^{6}x^{3}+a^{4}b^{5}x\right)}$$

input
```
integrate(x^(27/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
1/35*(35*b^4*x^8 + 280*a*b^3*x^6 + 560*a^2*b^2*x^4 + 448*a^3*b*x^2 + 128*a
^4)*sqrt(b*x^3 + a*x)*sqrt(x)/(b^9*x^9 + 4*a*b^8*x^7 + 6*a^2*b^7*x^5 + 4*a
^3*b^6*x^3 + a^4*b^5*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(27/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{27}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(27/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(27/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.61

$$\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{\frac{35\sqrt{bx^2+a}}{b} + \frac{140\,(bx^2+a)^3a-70\,(bx^2+a)^2a^2+28\,(bx^2+a)a^3-5\,a^4}{(bx^2+a)^{\frac{7}{2}}b}}{35\,b^4}$$

input
```
integrate(x^(27/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
1/35*(35*sqrt(b*x^2 + a)/b + (140*(b*x^2 + a)^3*a - 70*(b*x^2 + a)^2*a^2 +
 28*(b*x^2 + a)*a^3 - 5*a^4)/((b*x^2 + a)^(7/2)*b))/b^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{27/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(x^(27/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(27/2)/(a*x + b*x^3)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.78

$$\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{\sqrt{b\,x^2+a}\,(35b^4x^8+280a\,b^3x^6+560a^2b^2x^4+448a^3b\,x^2+128a^4)}{35b^5\,(b^4x^8+4a\,b^3x^6+6a^2b^2x^4+4a^3b\,x^2+a^4)}$$

input
```
int(x^(27/2)/(b*x^3+a*x)^(9/2),x)
```

output
```
(sqrt(a + b*x**2)*(128*a**4 + 448*a**3*b*x**2 + 560*a**2*b**2*x**4 + 280*a
*b**3*x**6 + 35*b**4*x**8))/(35*b**5*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x
**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.84 $\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}}\,dx$

Optimal result 696
Mathematica [A] (verified) 696
Rubi [A] (verified) 697
Maple [A] (verified) 699
Fricas [A] (verification not implemented) 699
Sympy [F(-1)] 700
Maxima [F] 700
Giac [A] (verification not implemented) 701
Mupad [F(-1)] 701
Reduce [B] (verification not implemented) 701

Optimal result

Integrand size = 19, antiderivative size = 130

$$\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{x^{21/2}}{7b\,(ax+bx^3)^{7/2}} - \frac{x^{15/2}}{5b^2\,(ax+bx^3)^{5/2}} - \frac{x^{9/2}}{3b^3\,(ax+bx^3)^{3/2}} - \frac{x^{3/2}}{b^4\sqrt{ax+bx^3}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax+bx^3}}\right)}{b^{9/2}}$$

output
```
-1/7*x^(21/2)/b/(b*x^3+a*x)^(7/2)-1/5*x^(15/2)/b^2/(b*x^3+a*x)^(5/2)-1/3*x
^(9/2)/b^3/(b*x^3+a*x)^(3/2)-x^(3/2)/b^4/(b*x^3+a*x)^(1/2)+arctanh(b^(1/2)
*x^(3/2)/(b*x^3+a*x)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.90

$$\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{9/2}\left(-\sqrt{b}x(a+bx^2)\left(105a^3+350a^2bx^2+406ab^2x^4+176b^3x^6\right)+210(a+bx^2)^{9/2}\text{ a}\right.}{105b^{9/2}\left(x\left(a+bx^2\right)\right)^{9/2}}$$

input
```
Integrate[x^(25/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
(x^(9/2)*(-(Sqrt[b]*x*(a + b*x^2)*(105*a^3 + 350*a^2*b*x^2 + 406*a*b^2*x^4
 + 176*b^3*x^6)) + 210*(a + b*x^2)^(9/2)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + S
qrt[a + b*x^2])]))/(105*b^(9/2)*(x*(a + b*x^2))^(9/2))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1928, 1928, 1928, 1928, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}}dx$$

$$\Big\downarrow \text{1928}$$

$$\frac{\int \frac{x^{19/2}}{(bx^3+ax)^{7/2}}dx}{b}-\frac{x^{21/2}}{7b(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1928}$$

$$\frac{\frac{\int \frac{x^{13/2}}{\left(bx^3+ax\right)^{5/2}}dx}{b}-\frac{x^{15/2}}{5b(ax+bx^3)^{5/2}}}{b}-\frac{x^{21/2}}{7b(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1928}$$

$$\frac{\frac{\frac{\int \frac{x^{7/2}}{\left(bx^3+ax\right)^{3/2}}dx}{b}-\frac{x^{9/2}}{3b\left(ax+bx^3\right)^{3/2}}}{b}-\frac{x^{15/2}}{5b(ax+bx^3)^{5/2}}}{b}-\frac{x^{21/2}}{7b(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1928}$$

$$\frac{\frac{\frac{\frac{\int \frac{\sqrt{x}}{\sqrt{bx^3+ax}}dx}{b}-\frac{x^{3/2}}{b\sqrt{ax+bx^3}}}{b}-\frac{x^{9/2}}{3b\left(ax+bx^3\right)^{3/2}}}{b}-\frac{x^{15/2}}{5b(ax+bx^3)^{5/2}}}{b}-\frac{x^{21/2}}{7b(ax+bx^3)^{7/2}}$$

$$\downarrow \text{1935}$$

$$\frac{\frac{\frac{\frac{\int \frac{1}{1-\frac{bx^3}{bx^3+ax}}d\frac{x^{3/2}}{\sqrt{bx^3+ax}}}{b}-\frac{x^{3/2}}{b\sqrt{ax+bx^3}}}{b}-\frac{x^{9/2}}{3b\left(ax+bx^3\right)^{3/2}}}{b}-\frac{x^{15/2}}{5b(ax+bx^3)^{5/2}}}{b}-\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

$$\downarrow \text{219}$$

$$\frac{\frac{\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax+bx^3}}\right)}{b^{3/2}}-\frac{x^{3/2}}{b\sqrt{ax+bx^3}}}{b}-\frac{x^{9/2}}{3b\left(ax+bx^3\right)^{3/2}}}{b}-\frac{x^{15/2}}{5b(ax+bx^3)^{5/2}}}{b}-\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

input
```
Int[x^(25/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
-1/7*x^(21/2)/(b*(a*x + b*x^3)^(7/2)) + (-1/5*x^(15/2)/(b*(a*x + b*x^3)^(5
/2)) + (-1/3*x^(9/2)/(b*(a*x + b*x^3)^(3/2)) + (-(x^(3/2)/(b*Sqrt[a*x + b*
x^3])) + ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x + b*x^3]]/b^(3/2))/b)/b)/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1928
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.52

method	result
default	$\frac{\sqrt{x(b\,x^2+a)}\left(105\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)b^3x^6\sqrt{b\,x^2+a}-176x^7b^{\frac{7}{2}}+315\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)a\,b^2x^4\sqrt{b\,x^2+a}-406b^{\frac{5}{2}}\,a\,x^5+315\ln\left(\sqrt{b}\,x\right.\right.}{105b^{\frac{9}{2}}\sqrt{x}\,(b\,x^2+a)^4}$

input
```
int(x^(25/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*(x*(b*x^2+a))^(1/2)/b^(9/2)*(105*ln(b^(1/2)*x+(b*x^2+a)^(1/2))*b^3*x
^6*(b*x^2+a)^(1/2)-176*x^7*b^(7/2)+315*ln(b^(1/2)*x+(b*x^2+a)^(1/2))*a*b^2
*x^4*(b*x^2+a)^(1/2)-406*b^(5/2)*a*x^5+315*ln(b^(1/2)*x+(b*x^2+a)^(1/2))*a
^2*b*x^2*(b*x^2+a)^(1/2)-350*b^(3/2)*a^2*x^3+105*ln(b^(1/2)*x+(b*x^2+a)^(1
/2))*a^3*(b*x^2+a)^(1/2)-105*x*a^3*b^(1/2))/x^(1/2)/(b*x^2+a)^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.72

$$\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}}\,dx = \left[\frac{105\,(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4)\sqrt{b}\log\left(2\,bx^2+2\,\sqrt{bx^3+ax}\sqrt{b}\sqrt{x}\right.}{210\,(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4\right.}$$

input
```
integrate(x^(25/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
[1/210*(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sq
rt(b)*log(2*b*x^2 + 2*sqrt(b*x^3 + a*x)*sqrt(b)*sqrt(x) + a) - 2*(176*b^4*
x^6 + 406*a*b^3*x^4 + 350*a^2*b^2*x^2 + 105*a^3*b)*sqrt(b*x^3 + a*x)*sqrt(
x))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5), -1/
105*(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(
-b)*arctan(sqrt(b*x^3 + a*x)*sqrt(-b)*sqrt(x)/(b*x^2 + a)) + (176*b^4*x^6
+ 406*a*b^3*x^4 + 350*a^2*b^2*x^2 + 105*a^3*b)*sqrt(b*x^3 + a*x)*sqrt(x))/
(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(25/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{25}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(25/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(25/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.60

$$\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{\left(2\left(x^2\left(\frac{88\,x^2}{b}+\frac{203\,a}{b^2}\right)+\frac{175\,a^2}{b^3}\right)x^2+\frac{105\,a^3}{b^4}\right)x}{105\,(bx^2+a)^{\frac{7}{2}}} - \frac{\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}}$$

input
```
integrate(x^(25/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
-1/105*(2*(x^2*(88*x^2/b + 203*a/b^2) + 175*a^2/b^3)*x^2 + 105*a^3/b^4)*x/
(b*x^2 + a)^(7/2) - log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{25/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(x^(25/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(25/2)/(a*x + b*x^3)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.42

$$\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{-105\sqrt{b\,x^2+a}\,a^3bx-350\sqrt{b\,x^2+a}\,a^2b^2x^3-406\sqrt{b\,x^2+a}\,a\,b^3x^5-176\sqrt{b\,x^2+a}\,b}{}$$

input
```
int(x^(25/2)/(b*x^3+a*x)^(9/2),x)
```

output
```
( - 105*sqrt(a + b*x**2)*a**3*b*x - 350*sqrt(a + b*x**2)*a**2*b**2*x**3 -
406*sqrt(a + b*x**2)*a*b**3*x**5 - 176*sqrt(a + b*x**2)*b**4*x**7 + 105*sq
rt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4 + 420*sqrt(b)*log((
sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*x**2 + 630*sqrt(b)*log((sqrt
(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**2*x**4 + 420*sqrt(b)*log((sqrt(
a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**3*x**6 + 105*sqrt(b)*log((sqrt(a +
b*x**2) + sqrt(b)*x)/sqrt(a))*b**4*x**8 + 56*sqrt(b)*a**4 + 224*sqrt(b)*a*
*3*b*x**2 + 336*sqrt(b)*a**2*b**2*x**4 + 224*sqrt(b)*a*b**3*x**6 + 56*sqrt
(b)*b**4*x**8)/(105*b**5*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b*
*3*x**6 + b**4*x**8))
```

3.85 $\int \frac{x^{23/2}}{(ax+bx^3)^{9/2}}\,dx$

Optimal result 703
Mathematica [A] (verified) 703
Rubi [A] (verified) 704
Maple [A] (verified) 706
Fricas [A] (verification not implemented) 706
Sympy [F(-1)] 707
Maxima [F] 707
Giac [A] (verification not implemented) 707
Mupad [F(-1)] 708
Reduce [B] (verification not implemented) 708

Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \frac{x^{23/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{x^{19/2}}{7b(ax+bx^3)^{7/2}} - \frac{6x^{13/2}}{35b^2(ax+bx^3)^{5/2}} - \frac{8x^{7/2}}{35b^3(ax+bx^3)^{3/2}} - \frac{16\sqrt{x}}{35b^4\sqrt{ax+bx^3}}$$

output
```
-1/7*x^(19/2)/b/(b*x^3+a*x)^(7/2)-6/35*x^(13/2)/b^2/(b*x^3+a*x)^(5/2)-8/35
*x^(7/2)/b^3/(b*x^3+a*x)^(3/2)-16/35*x^(1/2)/b^4/(b*x^3+a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

$$\int \frac{x^{23/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{9/2}(a+bx^2)(-16a^3-56a^2bx^2-70ab^2x^4-35b^3x^6)}{35b^4(x(a+bx^2))^{9/2}}$$

input
```
Integrate[x^(23/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
(x^(9/2)*(a + b*x^2)*(-16*a^3 - 56*a^2*b*x^2 - 70*a*b^2*x^4 - 35*b^3*x^6))
/(35*b^4*(x*(a + b*x^2))^(9/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1921, 1921, 1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{23/2}}{(ax+bx^3)^{9/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{6\int \frac{x^{17/2}}{(bx^3+ax)^{7/2}}dx}{7b}-\frac{x^{19/2}}{7b(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{6\left(\frac{4\int \frac{x^{11/2}}{(bx^3+ax)^{5/2}}dx}{5b}-\frac{x^{13/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{19/2}}{7b(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{6\left(\frac{4\left(\frac{2\int \frac{x^{5/2}}{(bx^3+ax)^{3/2}}dx}{3b}-\frac{x^{7/2}}{3b(ax+bx^3)^{3/2}}\right)}{5b}-\frac{x^{13/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{19/2}}{7b(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{6\left(\frac{4\left(-\frac{2\sqrt{x}}{3b^2\sqrt{ax+bx^3}}-\frac{x^{7/2}}{3b\left(ax+bx^3\right)^{3/2}}\right)}{5b}-\frac{x^{13/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

input
```
Int[x^(23/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
-1/7*x^(19/2)/(b*(a*x + b*x^3)^(7/2)) + (6*(-1/5*x^(13/2)/(b*(a*x + b*x^3)
^(5/2)) + (4*(-1/3*x^(7/2)/(b*(a*x + b*x^3)^(3/2)) - (2*Sqrt[x])/(3*b^2*Sq
rt[a*x + b*x^3])))/(5*b)))/(7*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.58

method	result	size
gosper	$-\frac{(b x^2+a)(35b^3x^6+70a b^2x^4+56a^2b x^2+16a^3)x^{\frac{9}{2}}}{35b^4(b x^3+ax)^{\frac{9}{2}}}$	59
orering	$-\frac{(b x^2+a)(35b^3x^6+70a b^2x^4+56a^2b x^2+16a^3)x^{\frac{9}{2}}}{35b^4(b x^3+ax)^{\frac{9}{2}}}$	59
default	$-\frac{\sqrt{x(b x^2+a)}\,(35b^3x^6+70a b^2x^4+56a^2b x^2+16a^3)}{35\sqrt{x}\,(b x^2+a)^4 b^4}$	61

input
```
int(x^(23/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35*(b*x^2+a)*(35*b^3*x^6+70*a*b^2*x^4+56*a^2*b*x^2+16*a^3)*x^(9/2)/b^4/
(b*x^3+a*x)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int \frac{x^{23/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{(35\,b^3x^6+70\,ab^2x^4+56\,a^2bx^2+16\,a^3)\sqrt{bx^3+ax}\sqrt{x}}{35\,(b^8x^9+4\,ab^7x^7+6\,a^2b^6x^5+4\,a^3b^5x^3+a^4b^4x)}$$

input
```
integrate(x^(23/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
-1/35*(35*b^3*x^6 + 70*a*b^2*x^4 + 56*a^2*b*x^2 + 16*a^3)*sqrt(b*x^3 + a*x
)*sqrt(x)/(b^8*x^9 + 4*a*b^7*x^7 + 6*a^2*b^6*x^5 + 4*a^3*b^5*x^3 + a^4*b^4
*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{23/2}}{(ax+bx^3)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(23/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{23/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{23}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(23/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(23/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.54

$$\int \frac{x^{23/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{35\,(bx^2+a)^3-35\,(bx^2+a)^2a+21\,(bx^2+a)a^2-5\,a^3}{35\,(bx^2+a)^{\frac{7}{2}}b^4}$$

input
```
integrate(x^(23/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
-1/35*(35*(b*x^2 + a)^3 - 35*(b*x^2 + a)^2*a + 21*(b*x^2 + a)*a^2 - 5*a^3)
/((b*x^2 + a)^(7/2)*b^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{23/2}}{(ax+bx^3)^{9/2}}\,dx=\int \frac{x^{23/2}}{{\left(b\,x^3+a\,x\right)}^{9/2}} \,d x$$

input
```
int(x^(23/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(23/2)/(a*x + b*x^3)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

$$\int \frac{x^{23/2}}{(ax+bx^3)^{9/2}}\,dx=\frac{\sqrt{b\,x^2+a}\,\left(-35b^3x^6-70a\,b^2x^4-56a^2b\,x^2-16a^3\right)}{35b^4\left(b^4x^8+4a\,b^3x^6+6a^2b^2x^4+4a^3b\,x^2+a^4\right)}$$

input
```
int(x^(23/2)/(b*x^3+a*x)^(9/2),x)
```

output
```
(sqrt(a + b*x**2)*( - 16*a**3 - 56*a**2*b*x**2 - 70*a*b**2*x**4 - 35*b**3*
x**6))/(35*b**4*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 +
 b**4*x**8))
```

3.86 $\int \frac{x^{21/2}}{(ax+bx^3)^{9/2}}\,dx$

Optimal result 709
Mathematica [A] (verified) 709
Rubi [A] (verified) 710
Maple [A] (verified) 710
Fricas [B] (verification not implemented) 711
Sympy [F(-1)] 711
Maxima [F] 712
Giac [A] (verification not implemented) 712
Mupad [F(-1)] 712
Reduce [B] (verification not implemented) 713

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{x^{21/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{21/2}}{7a\,(ax+bx^3)^{7/2}}$$

output
```
1/7*x^(21/2)/a/(b*x^3+a*x)^(7/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^{21/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{21/2}}{7a\,(x\,(a+bx^2))^{7/2}}$$

input
```
Integrate[x^(21/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
x^(21/2)/(7*a*(x*(a + b*x^2))^(7/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{21/2}}{(ax+bx^3)^{9/2}}\,dx$$

$$\Big\downarrow \; 1920$$

$$\frac{x^{21/2}}{7a\,(ax+bx^3)^{7/2}}$$

input
```
Int[x^(21/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
x^(21/2)/(7*a*(a*x + b*x^3)^(7/2))
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

method	result	size
gosper	$\frac{(b\,x^2+a)x^{\frac{23}{2}}}{7a(b\,x^3+ax)^{\frac{9}{2}}}$	27
orering	$\frac{(b\,x^2+a)x^{\frac{23}{2}}}{7a(b\,x^3+ax)^{\frac{9}{2}}}$	27
default	$\frac{x^{\frac{13}{2}}\sqrt{x(b\,x^2+a)}}{7a(b\,x^2+a)^4}$	29

input
```
int(x^(21/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*(b*x^2+a)*x^(23/2)/a/(b*x^3+a*x)^(9/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(19) = 38$.

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.44

$$\int \frac{x^{21/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{\sqrt{bx^3+ax}x^{\frac{13}{2}}}{7\,(ab^4x^8+4\,a^2b^3x^6+6\,a^3b^2x^4+4\,a^4bx^2+a^5)}$$

input
```
integrate(x^(21/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
1/7*sqrt(b*x^3 + a*x)*x^(13/2)/(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4
+ 4*a^4*b*x^2 + a^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{21/2}}{(ax+bx^3)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(21/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{21/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{21}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(21/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(21/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{x^{21/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^7}{7\,(bx^2+a)^{\frac{7}{2}}a}$$

input
```
integrate(x^(21/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
1/7*x^7/((b*x^2 + a)^(7/2)*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{21/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{21/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(x^(21/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(21/2)/(a*x + b*x^3)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 4.64

$$\int \frac{x^{21/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{\sqrt{b\,x^2+a}\,b^4x^7+\sqrt{b}\,a^4+4\sqrt{b}\,a^3b\,x^2+6\sqrt{b}\,a^2b^2x^4+4\sqrt{b}\,a\,b^3x^6+\sqrt{b}\,b^4x^8}{7a\,b^4\left(b^4x^8+4a\,b^3x^6+6a^2b^2x^4+4a^3b\,x^2+a^4\right)}$$

input
```
int(x^(21/2)/(b*x^3+a*x)^(9/2),x)
```

output
```
(sqrt(a + b*x**2)*b**4*x**7 + sqrt(b)*a**4 + 4*sqrt(b)*a**3*b*x**2 + 6*sqr
t(b)*a**2*b**2*x**4 + 4*sqrt(b)*a*b**3*x**6 + sqrt(b)*b**4*x**8)/(7*a*b**4
*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.87 $\int \frac{x^{19/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal result 714
Mathematica [A] (verified) 714
Rubi [A] (verified) 715
Maple [A] (verified) 716
Fricas [A] (verification not implemented) 717
Sympy [F(-1)] 717
Maxima [F] 717
Giac [A] (verification not implemented) 718
Mupad [F(-1)] 718
Reduce [B] (verification not implemented) 718

Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \frac{x^{19/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = -\frac{x^{15/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{4x^{9/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{8x^{3/2}}{105b^3\left(ax+bx^3\right)^{3/2}}$$

output
```
-1/7*x^(15/2)/b/(b*x^3+a*x)^(7/2)-4/35*x^(9/2)/b^2/(b*x^3+a*x)^(5/2)-8/105
*x^(3/2)/b^3/(b*x^3+a*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61

$$\int \frac{x^{19/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = \frac{x^{7/2}(-8a^2-28abx^2-35b^2x^4)}{105b^3\left(x\left(a+bx^2\right)\right)^{7/2}}$$

input
```
Integrate[x^(19/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
(x^(7/2)*(-8*a^2 - 28*a*b*x^2 - 35*b^2*x^4))/(105*b^3*(x*(a + b*x^2))^(7/2
))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1921, 1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{19/2}}{(ax+bx^3)^{9/2}}\,dx$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{4\int \frac{x^{13/2}}{(bx^3+ax)^{7/2}}dx}{7b}-\frac{x^{15/2}}{7b\,(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{4\left(\frac{2\int \frac{x^{7/2}}{\left(bx^3+ax\right)^{5/2}}dx}{5b}-\frac{x^{9/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{15/2}}{7b\,(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{4\left(-\frac{2x^{3/2}}{15b^2(ax+bx^3)^{3/2}}-\frac{x^{9/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{15/2}}{7b\,(ax+bx^3)^{7/2}}$$

input
```
Int[x^(19/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
-1/7*x^(15/2)/(b*(a*x + b*x^3)^(7/2)) + (4*(-1/5*x^(9/2)/(b*(a*x + b*x^3)^
(5/2)) - (2*x^(3/2))/(15*b^2*(a*x + b*x^3)^(3/2))))/(7*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.63

method	result	size
gosper	$-\frac{(b\,x^2+a)\,(35b^2x^4+28ab\,x^2+8a^2)x^{\frac{9}{2}}}{105b^3(b\,x^3+ax)^{\frac{9}{2}}}$	48
orering	$-\frac{(b\,x^2+a)\,(35b^2x^4+28ab\,x^2+8a^2)x^{\frac{9}{2}}}{105b^3(b\,x^3+ax)^{\frac{9}{2}}}$	48
default	$-\frac{\sqrt{x(b\,x^2+a)}\,(35b^2x^4+28ab\,x^2+8a^2)}{105\sqrt{x}\,(b\,x^2+a)^4b^3}$	50

input
```
int(x^(19/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/105*(b*x^2+a)*(35*b^2*x^4+28*a*b*x^2+8*a^2)*x^(9/2)/b^3/(b*x^3+a*x)^(9/
2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13

$$\int \frac{x^{19/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{(35\,b^2x^4+28\,abx^2+8\,a^2)\sqrt{bx^3+ax}\sqrt{x}}{105\,(b^7x^9+4\,ab^6x^7+6\,a^2b^5x^5+4\,a^3b^4x^3+a^4b^3x)}$$

input
```
integrate(x^(19/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
-1/105*(35*b^2*x^4 + 28*a*b*x^2 + 8*a^2)*sqrt(b*x^3 + a*x)*sqrt(x)/(b^7*x^
9 + 4*a*b^6*x^7 + 6*a^2*b^5*x^5 + 4*a^3*b^4*x^3 + a^4*b^3*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{19/2}}{(ax+bx^3)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(19/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{19/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{19}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(19/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(19/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.54

$$\int \frac{x^{19/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{35\,(bx^2+a)^2 - 42\,(bx^2+a)a + 15\,a^2}{105\,(bx^2+a)^{\frac{7}{2}}b^3}$$

input
```
integrate(x^(19/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
-1/105*(35*(b*x^2 + a)^2 - 42*(b*x^2 + a)*a + 15*a^2)/((b*x^2 + a)^(7/2)*b
^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{19/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{19/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(x^(19/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(19/2)/(a*x + b*x^3)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \frac{x^{19/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{\sqrt{b\,x^2+a}\,(-35b^2x^4 - 28ab\,x^2 - 8a^2)}{105b^3\,(b^4x^8 + 4a\,b^3x^6 + 6a^2b^2x^4 + 4a^3b\,x^2 + a^4)}$$

input
```
int(x^(19/2)/(b*x^3+a*x)^(9/2),x)
```

output
```
(sqrt(a + b*x**2)*( - 8*a**2 - 28*a*b*x**2 - 35*b**2*x**4))/(105*b**3*(a**
4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.88 $\int \frac{x^{17/2}}{(ax+bx^3)^{9/2}} \, dx$

Optimal result 719
Mathematica [A] (verified) 719
Rubi [A] (verified) 720
Maple [A] (verified) 721
Fricas [A] (verification not implemented) 721
Sympy [F(-1)] 722
Maxima [F] 722
Giac [A] (verification not implemented) 722
Mupad [F(-1)] 723
Reduce [B] (verification not implemented) 723

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int \frac{x^{17/2}}{(ax+bx^3)^{9/2}} \, dx = \frac{x^{17/2}}{7a\,(ax+bx^3)^{7/2}} + \frac{2x^{15/2}}{35a^2\,(ax+bx^3)^{5/2}}$$

output
```
1/7*x^(17/2)/a/(b*x^3+a*x)^(7/2)+2/35*x^(15/2)/a^2/(b*x^3+a*x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.75

$$\int \frac{x^{17/2}}{(ax+bx^3)^{9/2}} \, dx = \frac{x^{7/2}(7ax^5+2bx^7)}{35a^2\,(x\,(a+bx^2))^{7/2}}$$

input
```
Integrate[x^(17/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
(x^(7/2)*(7*a*x^5 + 2*b*x^7))/(35*a^2*(x*(a + b*x^2))^(7/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{17/2}}{(ax+bx^3)^{9/2}}\,dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{2\int \frac{x^{15/2}}{(bx^3+ax)^{7/2}}dx}{7a}+\frac{x^{17/2}}{7a\,(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2x^{15/2}}{35a^2\,(ax+bx^3)^{5/2}}+\frac{x^{17/2}}{7a\,(ax+bx^3)^{7/2}}$$

input
```
Int[x^(17/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
x^(17/2)/(7*a*(a*x + b*x^3)^(7/2)) + (2*x^(15/2))/(35*a^2*(a*x + b*x^3)^(5
/2))
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

method	result	size
gosper	$\frac{(b\,x^2+a)\,x^{\frac{19}{2}}\,(2b\,x^2+7a)}{35a^2(b\,x^3+ax)^{\frac{9}{2}}}$	37
orering	$\frac{(b\,x^2+a)\,x^{\frac{19}{2}}\,(2b\,x^2+7a)}{35a^2(b\,x^3+ax)^{\frac{9}{2}}}$	37
default	$\frac{x^{\frac{9}{2}}\,\sqrt{x(b\,x^2+a)}\,(2b\,x^2+7a)}{35a^2(b\,x^2+a)^4}$	39

input
```
int(x^(17/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*(b*x^2+a)*x^(19/2)*(2*b*x^2+7*a)/a^2/(b*x^3+a*x)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.49

$$\int \frac{x^{17/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{(2\,bx^6+7\,ax^4)\sqrt{bx^3+ax}\sqrt{x}}{35\,(a^2b^4x^8+4\,a^3b^3x^6+6\,a^4b^2x^4+4\,a^5bx^2+a^6)}$$

input
```
integrate(x^(17/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
1/35*(2*b*x^6 + 7*a*x^4)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^2*b^4*x^8 + 4*a^3*b^
3*x^6 + 6*a^4*b^2*x^4 + 4*a^5*b*x^2 + a^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{17/2}}{(ax+bx^3)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(17/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{17/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{17}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(17/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(17/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

$$\int \frac{x^{17/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^5\left(\frac{2\,bx^2}{a^2}+\frac{7}{a}\right)}{35\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate(x^(17/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
1/35*x^5*(2*b*x^2/a^2 + 7/a)/(b*x^2 + a)^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{17/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{17/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(x^(17/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(17/2)/(a*x + b*x^3)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.67

$$\int \frac{x^{17/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{7\sqrt{b\,x^2+a}\,a\,b^3x^5+2\sqrt{b\,x^2+a}\,b^4x^7-2\sqrt{b}\,a^4-8\sqrt{b}\,a^3b\,x^2-12\sqrt{b}\,a^2b^2x^4-8\sqrt{b}\,a}{35a^2b^3\left(b^4x^8+4a\,b^3x^6+6a^2b^2x^4+4a^3b\,x^2+a^4\right)}$$

input
```
int(x^(17/2)/(b*x^3+a*x)^(9/2),x)
```

output
```
(7*sqrt(a + b*x**2)*a*b**3*x**5 + 2*sqrt(a + b*x**2)*b**4*x**7 - 2*sqrt(b)
*a**4 - 8*sqrt(b)*a**3*b*x**2 - 12*sqrt(b)*a**2*b**2*x**4 - 8*sqrt(b)*a*b*
*3*x**6 - 2*sqrt(b)*b**4*x**8)/(35*a**2*b**3*(a**4 + 4*a**3*b*x**2 + 6*a**
2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.89 $\int \frac{x^{15/2}}{(ax+bx^3)^{9/2}} dx$

Optimal result . . . 724
Mathematica [A] (verified) . . . 724
Rubi [A] (verified) . . . 725
Maple [A] (verified) . . . 726
Fricas [A] (verification not implemented) . . . 726
Sympy [F(-1)] . . . 727
Maxima [F] . . . 727
Giac [A] (verification not implemented) . . . 727
Mupad [F(-1)] . . . 728
Reduce [B] (verification not implemented) . . . 728

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int \frac{x^{15/2}}{(ax+bx^3)^{9/2}} dx = -\frac{x^{11/2}}{7b(ax+bx^3)^{7/2}} - \frac{2x^{5/2}}{35b^2(ax+bx^3)^{5/2}}$$

output
```
-1/7*x^(11/2)/b/(b*x^3+a*x)^(7/2)-2/35*x^(5/2)/b^2/(b*x^3+a*x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{x^{15/2}}{(ax+bx^3)^{9/2}} dx = \frac{x^{7/2}(-2a-7bx^2)}{35b^2(x(a+bx^2))^{7/2}}$$

input
```
Integrate[x^(15/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
(x^(7/2)*(-2*a - 7*b*x^2))/(35*b^2*(x*(a + b*x^2))^(7/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15/2}}{(ax+bx^3)^{9/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{2\int \frac{x^{9/2}}{(bx^3+ax)^{7/2}}dx}{7b}-\frac{x^{11/2}}{7b\,(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1920}$$

$$-\frac{2x^{5/2}}{35b^2\,(ax+bx^3)^{5/2}}-\frac{x^{11/2}}{7b\,(ax+bx^3)^{7/2}}$$

input
```
Int[x^(15/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
-1/7*x^(11/2)/(b*(a*x + b*x^3)^(7/2)) - (2*x^(5/2))/(35*b^2*(a*x + b*x^3)^
(5/2))
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)\left(7 b\,x^{2}+2 a\right) x^{\frac{9}{2}}}{35 b^{2} \left(b\,x^{3}+a x\right)^{\frac{9}{2}}}$	37
orering	$-\frac{\left(b\,x^{2}+a\right)\left(7 b\,x^{2}+2 a\right) x^{\frac{9}{2}}}{35 b^{2} \left(b\,x^{3}+a x\right)^{\frac{9}{2}}}$	37
default	$-\frac{\sqrt{x \left(b\,x^{2}+a\right)}\,\left(7 b\,x^{2}+2 a\right)}{35 \sqrt{x}\,\left(b\,x^{2}+a\right)^{4} b^{2}}$	39

input
```
int(x^(15/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35*(b*x^2+a)*(7*b*x^2+2*a)*x^(9/2)/b^2/(b*x^3+a*x)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.47

$$\int \frac{x^{15/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{\sqrt{bx^3+ax}(7\,bx^2+2\,a)\sqrt{x}}{35\,(b^6x^9+4\,ab^5x^7+6\,a^2b^4x^5+4\,a^3b^3x^3+a^4b^2x)}$$

input
```
integrate(x^(15/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
-1/35*sqrt(b*x^3 + a*x)*(7*b*x^2 + 2*a)*sqrt(x)/(b^6*x^9 + 4*a*b^5*x^7 + 6
*a^2*b^4*x^5 + 4*a^3*b^3*x^3 + a^4*b^2*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{15/2}}{(ax+bx^3)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(15/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{15/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{15}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(15/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(15/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.47

$$\int \frac{x^{15/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{7\,bx^2+2\,a}{35\,(bx^2+a)^{\frac{7}{2}}b^2}$$

input
```
integrate(x^(15/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
-1/35*(7*b*x^2 + 2*a)/((b*x^2 + a)^(7/2)*b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{15/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{15/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(x^(15/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(15/2)/(a*x + b*x^3)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.27

$$\int \frac{x^{15/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{\sqrt{b\,x^2+a}\,(-7b\,x^2-2a)}{35b^2\,(b^4x^8+4a\,b^3x^6+6a^2b^2x^4+4a^3b\,x^2+a^4)}$$

input
```
int(x^(15/2)/(b*x^3+a*x)^(9/2),x)
```

output
```
(sqrt(a + b*x**2)*( - 2*a - 7*b*x**2))/(35*b**2*(a**4 + 4*a**3*b*x**2 + 6*
a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.90 $\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}} dx$

Optimal result 729
Mathematica [A] (verified) 729
Rubi [A] (verified) 730
Maple [A] (verified) 731
Fricas [A] (verification not implemented) 732
Sympy [F] 732
Maxima [F] 732
Giac [A] (verification not implemented) 733
Mupad [F(-1)] 733
Reduce [B] (verification not implemented) 733

Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}} dx = \frac{x^{13/2}}{7a(ax+bx^3)^{7/2}} + \frac{4x^{11/2}}{35a^2(ax+bx^3)^{5/2}} + \frac{8x^{9/2}}{105a^3(ax+bx^3)^{3/2}}$$

output
```
1/7*x^(13/2)/a/(b*x^3+a*x)^(7/2)+4/35*x^(11/2)/a^2/(b*x^3+a*x)^(5/2)+8/105
*x^(9/2)/a^3/(b*x^3+a*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

$$\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}} dx = \frac{x^{9/2}(a+bx^2)(35a^2x^3+28abx^5+8b^2x^7)}{105a^3(x(a+bx^2))^{9/2}}$$

input
```
Integrate[x^(13/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
(x^(9/2)*(a + b*x^2)*(35*a^2*x^3 + 28*a*b*x^5 + 8*b^2*x^7))/(105*a^3*(x*(a
 + b*x^2))^(9/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1921, 1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{4\int \frac{x^{11/2}}{(bx^3+ax)^{7/2}}dx}{7a}+\frac{x^{13/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{4\left(\frac{2\int \frac{x^{9/2}}{\left(bx^3+ax\right)^{5/2}}dx}{5a}+\frac{x^{11/2}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{13/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{4\left(\frac{2x^{9/2}}{15a^2(ax+bx^3)^{3/2}}+\frac{x^{11/2}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{13/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

input
```
Int[x^(13/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
x^(13/2)/(7*a*(a*x + b*x^3)^(7/2)) + (4*(x^(11/2)/(5*a*(a*x + b*x^3)^(5/2)
) + (2*x^(9/2))/(15*a^2*(a*x + b*x^3)^(3/2))))/(7*a)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.63

method	result	size
gosper	$\frac{(b\,x^2+a)x^{\frac{15}{2}}(8b^2x^4+28ab\,x^2+35a^2)}{105a^3(b\,x^3+ax)^{\frac{9}{2}}}$	48
orering	$\frac{(b\,x^2+a)x^{\frac{15}{2}}(8b^2x^4+28ab\,x^2+35a^2)}{105a^3(b\,x^3+ax)^{\frac{9}{2}}}$	48
default	$\frac{x^{\frac{5}{2}}\sqrt{x(b\,x^2+a)}\,(8b^2x^4+28ab\,x^2+35a^2)}{105a^3(b\,x^2+a)^4}$	50

input
```
int(x^(13/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*(b*x^2+a)*x^(15/2)*(8*b^2*x^4+28*a*b*x^2+35*a^2)/a^3/(b*x^3+a*x)^(9/
2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.14

$$\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{(8\,b^2x^6+28\,abx^4+35\,a^2x^2)\sqrt{bx^3+ax}\sqrt{x}}{105\,(a^3b^4x^8+4\,a^4b^3x^6+6\,a^5b^2x^4+4\,a^6bx^2+a^7)}$$

input
```
integrate(x^(13/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(8*b^2*x^6 + 28*a*b*x^4 + 35*a^2*x^2)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^3
*b^4*x^8 + 4*a^4*b^3*x^6 + 6*a^5*b^2*x^4 + 4*a^6*b*x^2 + a^7)
```

Sympy [F]

$$\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{13}{2}}}{(x\,(a+bx^2))^{\frac{9}{2}}}\,dx$$

input
```
integrate(x**(13/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Integral(x**(13/2)/(x*(a + b*x**2))**(9/2), x)
```

Maxima [F]

$$\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{13}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(13/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(13/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.57

$$\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{\left(4\,x^2\left(\frac{2\,b^2x^2}{a^3}+\frac{7\,b}{a^2}\right)+\frac{35}{a}\right)x^3}{105\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate(x^(13/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
1/105*(4*x^2*(2*b^2*x^2/a^3 + 7*b/a^2) + 35/a)*x^3/(b*x^2 + a)^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{13/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(x^(13/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(13/2)/(a*x + b*x^3)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.04

$$\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{35\sqrt{b\,x^2+a}\,a^2b^2x^3+28\sqrt{b\,x^2+a}\,a\,b^3x^5+8\sqrt{b\,x^2+a}\,b^4x^7-8\sqrt{b}\,a^4-32\sqrt{b}\,a^3b\,x^2}{105a^3b^2\left(b^4x^8+4a\,b^3x^6+6a^2b^2x^4+4a^3b\,x^2+\right.}$$

input
```
int(x^(13/2)/(b*x^3+a*x)^(9/2),x)
```

output
```
(35*sqrt(a + b*x**2)*a**2*b**2*x**3 + 28*sqrt(a + b*x**2)*a*b**3*x**5 + 8*
sqrt(a + b*x**2)*b**4*x**7 - 8*sqrt(b)*a**4 - 32*sqrt(b)*a**3*b*x**2 - 48*
sqrt(b)*a**2*b**2*x**4 - 32*sqrt(b)*a*b**3*x**6 - 8*sqrt(b)*b**4*x**8)/(10
5*a**3*b**2*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**
4*x**8))
```

3.91 $\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx$

Optimal result 735
Mathematica [A] (verified) 735
Rubi [A] (verified) 736
Maple [A] (verified) 736
Fricas [B] (verification not implemented) 737
Sympy [F] 737
Maxima [F] 738
Giac [A] (verification not implemented) 738
Mupad [F(-1)] 738
Reduce [B] (verification not implemented) 739

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{x^{7/2}}{7b\,(ax+bx^3)^{7/2}}$$

output
```
-1/7*x^(7/2)/b/(b*x^3+a*x)^(7/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{x^{7/2}}{7b\,(x\,(a+bx^2))^{7/2}}$$

input
```
Integrate[x^(11/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
-1/7*x^(7/2)/(b*(x*(a + b*x^2))^(7/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}} \, dx$$
$$\Big\downarrow \; 1920$$
$$-\frac{x^{7/2}}{7b\,(ax+bx^3)^{7/2}}$$

input
```
Int[x^(11/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
-1/7*x^(7/2)/(b*(a*x + b*x^3)^(7/2))
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

method	result	size
gosper	$-\frac{(b\,x^{2}+a)\,x^{\frac{9}{2}}}{7b(b\,x^{3}+ax)^{\frac{9}{2}}}$	27
orering	$-\frac{(b\,x^{2}+a)\,x^{\frac{9}{2}}}{7b(b\,x^{3}+ax)^{\frac{9}{2}}}$	27
default	$-\frac{\sqrt{x(b\,x^{2}+a)}}{7\sqrt{x}\,(b\,x^{2}+a)^{4}b}$	29

input
```
int(x^(11/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7*(b*x^2+a)/b*x^(9/2)/(b*x^3+a*x)^(9/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(19) = 38$.

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{\sqrt{bx^3+ax}\sqrt{x}}{7\,(b^5x^9+4\,ab^4x^7+6\,a^2b^3x^5+4\,a^3b^2x^3+a^4bx)}$$

input
```
integrate(x^(11/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
-1/7*sqrt(b*x^3 + a*x)*sqrt(x)/(b^5*x^9 + 4*a*b^4*x^7 + 6*a^2*b^3*x^5 + 4*
a^3*b^2*x^3 + a^4*b*x)
```

Sympy [F]

$$\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{11}{2}}}{(x\,(a+bx^2))^{\frac{9}{2}}}\,dx$$

input
```
integrate(x**(11/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Integral(x**(11/2)/(x*(a + b*x**2))**(9/2), x)
```

Maxima [F]

$$\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{11}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(11/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(11/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{1}{7\,(bx^2+a)^{\frac{7}{2}}b}$$

input
```
integrate(x^(11/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
-1/7/((b*x^2 + a)^(7/2)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{11/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(x^(11/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(11/2)/(a*x + b*x^3)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.20

$$\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{\sqrt{b\,x^2+a}}{7b\,(b^4x^8+4a\,b^3x^6+6a^2b^2x^4+4a^3b\,x^2+a^4)}$$

input
```
int(x^(11/2)/(b*x^3+a*x)^(9/2),x)
```

output
```
( - sqrt(a + b*x**2))/(7*b*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*
b**3*x**6 + b**4*x**8))
```

3.92 $\int \frac{x^{9/2}}{(ax+bx^3)^{9/2}} dx$

Optimal result . . . 740
Mathematica [A] (verified) . . . 740
Rubi [A] (verified) . . . 741
Maple [A] (verified) . . . 743
Fricas [A] (verification not implemented) . . . 743
Sympy [F] . . . 744
Maxima [F] . . . 744
Giac [A] (verification not implemented) . . . 744
Mupad [F(-1)] . . . 745
Reduce [B] (verification not implemented) . . . 745

Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \frac{x^{9/2}}{(ax+bx^3)^{9/2}} dx = \frac{x^{9/2}}{7a(ax+bx^3)^{7/2}} + \frac{6x^{7/2}}{35a^2(ax+bx^3)^{5/2}} + \frac{8x^{5/2}}{35a^3(ax+bx^3)^{3/2}} + \frac{16x^{3/2}}{35a^4\sqrt{ax+bx^3}}$$

output
```
1/7*x^(9/2)/a/(b*x^3+a*x)^(7/2)+6/35*x^(7/2)/a^2/(b*x^3+a*x)^(5/2)+8/35*x^
(5/2)/a^3/(b*x^3+a*x)^(3/2)+16/35*x^(3/2)/a^4/(b*x^3+a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.57

$$\int \frac{x^{9/2}}{(ax+bx^3)^{9/2}} dx = \frac{x^{7/2}(35a^3x + 70a^2bx^3 + 56ab^2x^5 + 16b^3x^7)}{35a^4(x(a+bx^2))^{7/2}}$$

input
```
Integrate[x^(9/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
(x^(7/2)*(35*a^3*x + 70*a^2*b*x^3 + 56*a*b^2*x^5 + 16*b^3*x^7))/(35*a^4*(x
*(a + b*x^2))^(7/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1921, 1921, 1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}}{(ax+bx^3)^{9/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{6\int \frac{x^{7/2}}{(bx^3+ax)^{7/2}}dx}{7a}+\frac{x^{9/2}}{7a(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{6\left(\frac{4\int \frac{x^{5/2}}{\left(bx^3+ax\right)^{5/2}}dx}{5a}+\frac{x^{7/2}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{9/2}}{7a(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{6\left(\frac{4\left(\frac{2\int \frac{x^{3/2}}{\left(bx^3+ax\right)^{3/2}}dx}{3a}+\frac{x^{5/2}}{3a\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{x^{7/2}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{9/2}}{7a(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{6\left(\frac{4\left(\frac{2x^{3/2}}{3a^2\sqrt{ax+bx^3}}+\frac{x^{5/2}}{3a\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{x^{7/2}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

input
```
Int[x^(9/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
x^(9/2)/(7*a*(a*x + b*x^3)^(7/2)) + (6*(x^(7/2)/(5*a*(a*x + b*x^3)^(5/2))
+ (4*(x^(5/2)/(3*a*(a*x + b*x^3)^(3/2)) + (2*x^(3/2))/(3*a^2*Sqrt[a*x + b*
x^3])))/(5*a)))/(7*a)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.58

method	result	size
gosper	$\frac{(b\,x^2+a)\,x^{\frac{11}{2}}\,(16b^3x^6+56a\,b^2x^4+70a^2b\,x^2+35a^3)}{35a^4(b\,x^3+ax)^{\frac{9}{2}}}$	59
orering	$\frac{(b\,x^2+a)\,x^{\frac{11}{2}}\,(16b^3x^6+56a\,b^2x^4+70a^2b\,x^2+35a^3)}{35a^4(b\,x^3+ax)^{\frac{9}{2}}}$	59
default	$\frac{\sqrt{x}\,\sqrt{x(b\,x^2+a)}\,(16b^3x^6+56a\,b^2x^4+70a^2b\,x^2+35a^3)}{35(b\,x^2+a)^4a^4}$	61

input
```
int(x^(9/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*(b*x^2+a)*x^(11/2)*(16*b^3*x^6+56*a*b^2*x^4+70*a^2*b*x^2+35*a^3)/a^4/
(b*x^3+a*x)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

$$\int \frac{x^{9/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{(16\,b^3x^6+56\,ab^2x^4+70\,a^2bx^2+35\,a^3)\sqrt{bx^3+ax}\sqrt{x}}{35\,(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8)}$$

input
```
integrate(x^(9/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
1/35*(16*b^3*x^6 + 56*a*b^2*x^4 + 70*a^2*b*x^2 + 35*a^3)*sqrt(b*x^3 + a*x)
*sqrt(x)/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)
```

Sympy [F]

$$\int \frac{x^{9/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{9}{2}}}{(x(a+bx^2))^{\frac{9}{2}}}\,dx$$

input
```
integrate(x**(9/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Integral(x**(9/2)/(x*(a + b*x**2))**(9/2), x)
```

Maxima [F]

$$\int \frac{x^{9/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{9}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(9/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(9/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.54

$$\int \frac{x^{9/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{\left(2\left(4x^2\left(\frac{2b^3x^2}{a^4}+\frac{7b^2}{a^3}\right)+\frac{35b}{a^2}\right)x^2+\frac{35}{a}\right)x}{35(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate(x^(9/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
1/35*(2*(4*x^2*(2*b^3*x^2/a^4 + 7*b^2/a^3) + 35*b/a^2)*x^2 + 35/a)*x/(b*x^
2 + a)^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{9/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{9/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(x^(9/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(9/2)/(a*x + b*x^3)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.68

$$\int \frac{x^{9/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{35\sqrt{b\,x^2+a}\,a^3bx+70\sqrt{b\,x^2+a}\,a^2b^2x^3+56\sqrt{b\,x^2+a}\,a\,b^3x^5+16\sqrt{b\,x^2+a}\,b^4x^7-}{35a^4b\,(b^4x^8+4a\,b^3x^6+6a^2b^2x^4+}$$

input
```
int(x^(9/2)/(b*x^3+a*x)^(9/2),x)
```

output
```
(35*sqrt(a + b*x**2)*a**3*b*x + 70*sqrt(a + b*x**2)*a**2*b**2*x**3 + 56*sq
rt(a + b*x**2)*a*b**3*x**5 + 16*sqrt(a + b*x**2)*b**4*x**7 - 16*sqrt(b)*a*
*4 - 64*sqrt(b)*a**3*b*x**2 - 96*sqrt(b)*a**2*b**2*x**4 - 64*sqrt(b)*a*b**
3*x**6 - 16*sqrt(b)*b**4*x**8)/(35*a**4*b*(a**4 + 4*a**3*b*x**2 + 6*a**2*b
**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.93 $\int \frac{x^{7/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal result 746
Mathematica [A] (verified) 746
Rubi [A] (verified) 747
Maple [B] (verified) 749
Fricas [A] (verification not implemented) 749
Sympy [F] 750
Maxima [F] 750
Giac [A] (verification not implemented) 750
Mupad [F(-1)] 751
Reduce [B] (verification not implemented) 751

Optimal result

Integrand size = 19, antiderivative size = 130

$$\int \frac{x^{7/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}}+\frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{a^{9/2}}$$

output
```
1/7*x^(7/2)/a/(b*x^3+a*x)^(7/2)+1/5*x^(5/2)/a^2/(b*x^3+a*x)^(5/2)+1/3*x^(3
/2)/a^3/(b*x^3+a*x)^(3/2)+x^(1/2)/a^4/(b*x^3+a*x)^(1/2)-arctanh(a^(1/2)*x^
(1/2)/(b*x^3+a*x)^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.82

$$\int \frac{x^{7/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{\sqrt{x}\left(\sqrt{a}(176a^3+406a^2bx^2+350ab^2x^4+105b^3x^6)-105(a+bx^2)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right.\right.}{105a^{9/2}\left(a+bx^2\right)^3\sqrt{x\left(a+bx^2\right)}}$$

input
```
Integrate[x^(7/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
(Sqrt[x]*(Sqrt[a]*(176*a^3 + 406*a^2*b*x^2 + 350*a*b^2*x^4 + 105*b^3*x^6)
- 105*(a + b*x^2)^(7/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/(105*a^(9/2)*(a
 + b*x^2)^3*Sqrt[x*(a + b*x^2)])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1929, 1929, 1929, 1929, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{(ax+bx^3)^{9/2}}dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{\int \frac{x^{5/2}}{(bx^3+ax)^{7/2}}dx}{a}+\frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1929}$$

$$\frac{\frac{\int \frac{x^{3/2}}{\left(bx^3+ax\right)^{5/2}}dx}{a}+\frac{x^{5/2}}{5a(ax+bx^3)^{5/2}}}{a}+\frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1929}$$

$$\frac{\frac{\frac{\int \frac{\sqrt{x}}{\left(bx^3+ax\right)^{3/2}}dx}{a}+\frac{x^{3/2}}{3a\left(ax+bx^3\right)^{3/2}}}{a}+\frac{x^{5/2}}{5a(ax+bx^3)^{5/2}}}{a}+\frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1929}$$

$$\frac{\frac{\frac{\frac{\int \frac{1}{\sqrt{x}\sqrt{bx^3+ax}}dx}{a}+\frac{\sqrt{x}}{a\sqrt{ax+bx^3}}}{a}+\frac{x^{3/2}}{3a\left(ax+bx^3\right)^{3/2}}}{a}+\frac{x^{5/2}}{5a(ax+bx^3)^{5/2}}}{a}+\frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{\frac{\frac{\frac{\sqrt{x}}{a\sqrt{ax+bx^3}}-\frac{\int \frac{1}{1-\frac{ax}{bx^3+ax}}d\frac{\sqrt{x}}{\sqrt{bx^3+ax}}}{a}}{a}+\frac{x^{3/2}}{3a\left(ax+bx^3\right)^{3/2}}}{a}+\frac{x^{5/2}}{5a(ax+bx^3)^{5/2}}}{a}+\frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{\frac{\sqrt{x}}{a\sqrt{ax+bx^3}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{a^{3/2}}}{a}+\frac{x^{3/2}}{3a\left(ax+bx^3\right)^{3/2}}}{a}+\frac{x^{5/2}}{5a(ax+bx^3)^{5/2}}}{a}+\frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

input
```
Int[x^(7/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
x^(7/2)/(7*a*(a*x + b*x^3)^(7/2)) + (x^(5/2)/(5*a*(a*x + b*x^3)^(5/2)) + (
x^(3/2)/(3*a*(a*x + b*x^3)^(3/2)) + (Sqrt[x]/(a*Sqrt[a*x + b*x^3]) - ArcTa
nh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^3]]/a^(3/2))/a)/a)/a
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(100) = 200$.

Time = 0.43 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.67

method	result
default	$-\frac{\sqrt{x(b\,x^2+a)}\left(105\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)b^3x^6\sqrt{b\,x^2+a}-105\sqrt{a}\,b^3x^6+315\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)a\,b^2x^4\sqrt{b\,x^2+a}-350a^{\frac{3}{2}}b^2x^4+3}{105a^{\frac{9}{2}}\sqrt{x}\,(b\,x^2+a)^4}$

input
```
int(x^(7/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/105*(x*(b*x^2+a))^(1/2)/a^(9/2)*(105*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x
)*b^3*x^6*(b*x^2+a)^(1/2)-105*a^(1/2)*b^3*x^6+315*ln(2*(a^(1/2)*(b*x^2+a)^
(1/2)+a)/x)*a*b^2*x^4*(b*x^2+a)^(1/2)-350*a^(3/2)*b^2*x^4+315*ln(2*(a^(1/2
)*(b*x^2+a)^(1/2)+a)/x)*a^2*b*x^2*(b*x^2+a)^(1/2)-406*a^(5/2)*b*x^2+105*ln
(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*a^3*(b*x^2+a)^(1/2)-176*a^(7/2))/x^(1/2)
/(b*x^2+a)^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.75

$$\int \frac{x^{7/2}}{(ax+bx^3)^{9/2}}\,dx = \left[\frac{105\,(b^4x^9+4\,ab^3x^7+6\,a^2b^2x^5+4\,a^3bx^3+a^4x)\sqrt{a}\log\left(\frac{bx^3+2\,ax-2\,\sqrt{bx^3+ax}\sqrt{a}\sqrt{x}}{x^3}\right)+}{210\,(a^5b^4x^9+4\,a^6b^3x^7+6\,a^7b^2x^5+}\right.$$

input
```
integrate(x^(7/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
[1/210*(105*(b^4*x^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*x)*
sqrt(a)*log((b*x^3 + 2*a*x - 2*sqrt(b*x^3 + a*x)*sqrt(a)*sqrt(x))/x^3) + 2
*(105*a*b^3*x^6 + 350*a^2*b^2*x^4 + 406*a^3*b*x^2 + 176*a^4)*sqrt(b*x^3 +
a*x)*sqrt(x))/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 +
 a^9*x), 1/105*(105*(b^4*x^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 +
 a^4*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt(x)/sqrt(b*x^3 + a*x)) + (105*a*b^3*x
^6 + 350*a^2*b^2*x^4 + 406*a^3*b*x^2 + 176*a^4)*sqrt(b*x^3 + a*x)*sqrt(x))
/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)]
```

Sympy [F]

$$\int \frac{x^{7/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{7}{2}}}{(x(a+bx^2))^{\frac{9}{2}}}\,dx$$

input
```
integrate(x**(7/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Integral(x**(7/2)/(x*(a + b*x**2))**(9/2), x)
```

Maxima [F]

$$\int \frac{x^{7/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{7}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(7/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(7/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.62

$$\int \frac{x^{7/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4} + \frac{105\,(bx^2+a)^3+35\,(bx^2+a)^2a+21\,(bx^2+a)a^2+15\,a^3}{105\,(bx^2+a)^{\frac{7}{2}}a^4}$$

input
```
integrate(x^(7/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4) + 1/105*(105*(b*x^2 + a)^3
 + 35*(b*x^2 + a)^2*a + 21*(b*x^2 + a)*a^2 + 15*a^3)/((b*x^2 + a)^(7/2)*a^
4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{7/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(x^(7/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(7/2)/(a*x + b*x^3)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 438, normalized size of antiderivative = 3.37

$$\int \frac{x^{7/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{176\sqrt{b\,x^2+a}\,a^4+406\sqrt{b\,x^2+a}\,a^3b\,x^2+350\sqrt{b\,x^2+a}\,a^2b^2x^4+105\sqrt{b\,x^2+a}\,a\,b^3x^{6}}{}$$

input
```
int(x^(7/2)/(b*x^3+a*x)^(9/2),x)
```

output
```
(176*sqrt(a + b*x**2)*a**4 + 406*sqrt(a + b*x**2)*a**3*b*x**2 + 350*sqrt(a
 + b*x**2)*a**2*b**2*x**4 + 105*sqrt(a + b*x**2)*a*b**3*x**6 + 105*sqrt(a)
*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**4 + 420*sqrt(a)*
log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**3*b*x**2 + 630*sq
rt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b**2*x**4
 + 420*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**
3*x**6 + 105*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))
*b**4*x**8 - 105*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt
(a))*a**4 - 420*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(
a))*a**3*b*x**2 - 630*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)
/sqrt(a))*a**2*b**2*x**4 - 420*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + s
qrt(b)*x)/sqrt(a))*a*b**3*x**6 - 105*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(
a) + sqrt(b)*x)/sqrt(a))*b**4*x**8)/(105*a**5*(a**4 + 4*a**3*b*x**2 + 6*a*
*2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.94 $\int \frac{x^{5/2}}{(ax+bx^3)^{9/2}} dx$

Optimal result 753
Mathematica [A] (verified) 753
Rubi [A] (verified) 754
Maple [A] (verified) 756
Fricas [A] (verification not implemented) 756
Sympy [F] 757
Maxima [F] 757
Giac [A] (verification not implemented) 757
Mupad [F(-1)] 758
Reduce [B] (verification not implemented) 758

Optimal result

Integrand size = 19, antiderivative size = 126

$$\int \frac{x^{5/2}}{(ax+bx^3)^{9/2}} dx = \frac{x^{5/2}}{7a(ax+bx^3)^{7/2}} + \frac{8x^{3/2}}{35a^2(ax+bx^3)^{5/2}} + \frac{16\sqrt{x}}{35a^3(ax+bx^3)^{3/2}} + \frac{64}{35a^4\sqrt{x}\sqrt{ax+bx^3}} - \frac{128\sqrt{ax+bx^3}}{35a^5x^{3/2}}$$

output
```
1/7*x^(5/2)/a/(b*x^3+a*x)^(7/2)+8/35*x^(3/2)/a^2/(b*x^3+a*x)^(5/2)+16/35*x
^(1/2)/a^3/(b*x^3+a*x)^(3/2)+64/35/a^4/x^(1/2)/(b*x^3+a*x)^(1/2)-128/35*(b
*x^3+a*x)^(1/2)/a^5/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.54

$$\int \frac{x^{5/2}}{(ax+bx^3)^{9/2}} dx = \frac{x^{5/2}(-35a^4 - 280a^3bx^2 - 560a^2b^2x^4 - 448ab^3x^6 - 128b^4x^8)}{35a^5(x(a+bx^2))^{7/2}}$$

input
```
Integrate[x^(5/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
(x^(5/2)*(-35*a^4 - 280*a^3*b*x^2 - 560*a^2*b^2*x^4 - 448*a*b^3*x^6 - 128*
b^4*x^8))/(35*a^5*(x*(a + b*x^2))^(7/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1921, 1921, 1921, 1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(ax+bx^3)^{9/2}}dx$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{8\int \frac{x^{3/2}}{(bx^3+ax)^{7/2}}dx}{7a}+\frac{x^{5/2}}{7a(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{8\left(\frac{6\int \frac{\sqrt{x}}{(bx^3+ax)^{5/2}}dx}{5a}+\frac{x^{3/2}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{5/2}}{7a(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{8\left(\frac{6\left(\frac{4\int \frac{1}{\sqrt{x}(bx^3+ax)^{3/2}}dx}{3a}+\frac{\sqrt{x}}{3a(ax+bx^3)^{3/2}}\right)}{5a}+\frac{x^{3/2}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{5/2}}{7a(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{8\left(\frac{6\left(\frac{4\left(\frac{2\int \frac{1}{x^{3/2}\sqrt{bx^3+ax}}dx}{a}+\frac{1}{a\sqrt{x}\sqrt{ax+bx^3}}\right)}{3a}+\frac{\sqrt{x}}{3a\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{x^{3/2}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{8\left(\frac{6\left(\frac{4\left(\frac{1}{a\sqrt{x}\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{a^2x^{3/2}}\right)}{3a}+\frac{\sqrt{x}}{3a\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{x^{3/2}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

input
```
Int[x^(5/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
x^(5/2)/(7*a*(a*x + b*x^3)^(7/2)) + (8*(x^(3/2)/(5*a*(a*x + b*x^3)^(5/2))
+ (6*(Sqrt[x]/(3*a*(a*x + b*x^3)^(3/2)) + (4*(1/(a*Sqrt[x]*Sqrt[a*x + b*x^
3]) - (2*Sqrt[a*x + b*x^3])/(a^2*x^(3/2))))/(3*a)))/(5*a)))/(7*a)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.56

method	result	size
gosper	$-\frac{x^{\frac{7}{2}} \left(b\,x^{2}+a\right) \left(128 b^{4} x^{8}+448 a\,b^{3} x^{6}+560 a^{2} b^{2} x^{4}+280 a^{3} b\,x^{2}+35 a^{4}\right)}{35 a^{5} \left(b\,x^{3}+a x\right)^{\frac{9}{2}}}$	70
orering	$-\frac{x^{\frac{7}{2}} \left(b\,x^{2}+a\right) \left(128 b^{4} x^{8}+448 a\,b^{3} x^{6}+560 a^{2} b^{2} x^{4}+280 a^{3} b\,x^{2}+35 a^{4}\right)}{35 a^{5} \left(b\,x^{3}+a x\right)^{\frac{9}{2}}}$	70
default	$-\frac{\sqrt{x \left(b\,x^{2}+a\right)}\, \left(128 b^{4} x^{8}+448 a\,b^{3} x^{6}+560 a^{2} b^{2} x^{4}+280 a^{3} b\,x^{2}+35 a^{4}\right)}{35 x^{\frac{3}{2}} \left(b\,x^{2}+a\right)^{4} a^{5}}$	72
risch	$-\frac{b\,x^{2}+a}{a^{5} \sqrt{x}\, \sqrt{x \left(b\,x^{2}+a\right)}}-\frac{\left(b\,x^{2}+a\right) x^{\frac{3}{2}} \left(93 b^{3} x^{6}+308 a\,b^{2} x^{4}+350 a^{2} b\,x^{2}+140 a^{3}\right) b}{35 \left(b^{4} x^{8}+4 a\,b^{3} x^{6}+6 a^{2} b^{2} x^{4}+4 a^{3} b\,x^{2}+a^{4}\right) a^{5} \sqrt{x \left(b\,x^{2}+a\right)}}$	129

input
```
int(x^(5/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35*x^(7/2)*(b*x^2+a)*(128*b^4*x^8+448*a*b^3*x^6+560*a^2*b^2*x^4+280*a^3
*b*x^2+35*a^4)/a^5/(b*x^3+a*x)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87

$$\int \frac{x^{5/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = -\frac{\left(128\,b^4x^8+448\,ab^3x^6+560\,a^2b^2x^4+280\,a^3bx^2+35\,a^4\right)\sqrt{bx^3+ax}\sqrt{x}}{35\,\left(a^5b^4x^{10}+4\,a^6b^3x^8+6\,a^7b^2x^6+4\,a^8bx^4+a^9x^2\right)}$$

input
```
integrate(x^(5/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
-1/35*(128*b^4*x^8 + 448*a*b^3*x^6 + 560*a^2*b^2*x^4 + 280*a^3*b*x^2 + 35*
a^4)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^5*b^4*x^10 + 4*a^6*b^3*x^8 + 6*a^7*b^2*x
^6 + 4*a^8*b*x^4 + a^9*x^2)
```

Sympy [F]

$$\int \frac{x^{5/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{5}{2}}}{\left(x\left(a+bx^2\right)\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x**(5/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Integral(x**(5/2)/(x*(a + b*x**2))**(9/2), x)
```

Maxima [F]

$$\int \frac{x^{5/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{5}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(5/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(5/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.71

$$\int \frac{x^{5/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{\left(\left(x^2\left(\frac{93\,b^4x^2}{a^5}+\frac{308\,b^3}{a^4}\right)+\frac{350\,b^2}{a^3}\right)x^2+\frac{140\,b}{a^2}\right)x}{35\left(bx^2+a\right)^{\frac{7}{2}}} + \frac{2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a^4}$$

input
```
integrate(x^(5/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
-1/35*((x^2*(93*b^4*x^2/a^5 + 308*b^3/a^4) + 350*b^2/a^3)*x^2 + 140*b/a^2)
*x/(b*x^2 + a)^(7/2) + 2*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^
4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{5/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(x^(5/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(5/2)/(a*x + b*x^3)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.48

$$\int \frac{x^{5/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{-35\sqrt{b\,x^2+a}\,a^4-280\sqrt{b\,x^2+a}\,a^3b\,x^2-560\sqrt{b\,x^2+a}\,a^2b^2x^4-448\sqrt{b\,x^2+a}\,a\,b^3x}{35a^5x\,(b^4x^8+4a}$$

input
```
int(x^(5/2)/(b*x^3+a*x)^(9/2),x)
```

output
```
( - 35*sqrt(a + b*x**2)*a**4 - 280*sqrt(a + b*x**2)*a**3*b*x**2 - 560*sqrt
(a + b*x**2)*a**2*b**2*x**4 - 448*sqrt(a + b*x**2)*a*b**3*x**6 - 128*sqrt(
a + b*x**2)*b**4*x**8 + 128*sqrt(b)*a**4*x + 512*sqrt(b)*a**3*b*x**3 + 768
*sqrt(b)*a**2*b**2*x**5 + 512*sqrt(b)*a*b**3*x**7 + 128*sqrt(b)*b**4*x**9)
/(35*a**5*x*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**
4*x**8))
```

3.95 $\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx$

Optimal result 759
Mathematica [A] (verified) 759
Rubi [A] (verified) 760
Maple [A] (verified) 764
Fricas [A] (verification not implemented) 765
Sympy [F] 766
Maxima [F] 766
Giac [A] (verification not implemented) 766
Mupad [F(-1)] 767
Reduce [B] (verification not implemented) 767

Optimal result

Integrand size = 19, antiderivative size = 159

$$\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{3/2}}{7a\,(ax+bx^3)^{7/2}}+\frac{9\sqrt{x}}{35a^2\,(ax+bx^3)^{5/2}}+\frac{3}{5a^3\sqrt{x}\,(ax+bx^3)^{3/2}}+\frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}}-\frac{9\sqrt{ax+bx^3}}{2a^5x^{5/2}}+\frac{9b\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{2a^{11/2}}$$

output
```
1/7*x^(3/2)/a/(b*x^3+a*x)^(7/2)+9/35*x^(1/2)/a^2/(b*x^3+a*x)^(5/2)+3/5/a^3
/x^(1/2)/(b*x^3+a*x)^(3/2)+3/a^4/x^(3/2)/(b*x^3+a*x)^(1/2)-9/2*(b*x^3+a*x)
^(1/2)/a^5/x^(5/2)+9/2*b*arctanh(a^(1/2)*x^(1/2)/(b*x^3+a*x)^(1/2))/a^(11/
2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.77

$$\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{\sqrt{x\,(a+bx^2)}\Big(-\sqrt{a}(35a^4+528a^3bx^2+1218a^2b^2x^4+1050ab^3x^6+315b^4x^8)+315b}{70a^{11/2}x^{5/2}\,(a+bx^2)^4}$$

input
```
Integrate[x^(3/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
(Sqrt[x*(a + b*x^2)]*(-(Sqrt[a]*(35*a^4 + 528*a^3*b*x^2 + 1218*a^2*b^2*x^4
 + 1050*a*b^3*x^6 + 315*b^4*x^8)) + 315*b*x^2*(a + b*x^2)^(7/2)*ArcTanh[Sq
rt[a + b*x^2]/Sqrt[a]]))/(70*a^(11/2)*x^(5/2)*(a + b*x^2)^4)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1929, 1929, 1929, 1929, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{9\int \frac{\sqrt{x}}{(bx^3+ax)^{7/2}}dx}{7a}+\frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1929}$$

$$\frac{9\left(\frac{7\int \frac{1}{\sqrt{x}\left(bx^3+ax\right)^{5/2}}dx}{5a}+\frac{\sqrt{x}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1929}$$

$$\frac{9\left(\frac{7\left(\frac{5\int \frac{1}{x^{3/2}\left(bx^3+ax\right)^{3/2}}dx}{3a}+\frac{1}{3a\sqrt{x}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{\sqrt{x}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1929}$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\int\frac{1}{x^{5/2}\sqrt{bx^3+ax}}dx}{a}+\frac{1}{ax^{3/2}\sqrt{ax+bx^3}}\right)}{3a}+\frac{1}{3a\sqrt{x}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{\sqrt{x}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{ 1931}$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(-\frac{b\int\frac{1}{\sqrt{x}\sqrt{bx^3+ax}}dx}{2a}-\frac{\sqrt{ax+bx^3}}{2ax^{5/2}}\right)}{a}+\frac{1}{ax^{3/2}\sqrt{ax+bx^3}}\right)}{3a}+\frac{1}{3a\sqrt{x}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{\sqrt{x}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{ 1935}$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{b\int \frac{1}{1-\frac{ax}{bx^3+ax}}d\frac{\sqrt{x}}{\sqrt{bx^3+ax}}}{2a}-\frac{\sqrt{ax+bx^3}}{2ax^{5/2}}\right)}{a}+\frac{1}{ax^{3/2}\sqrt{ax+bx^3}}\right)}{3a}+\frac{1}{3a\sqrt{x}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{\sqrt{x}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{2a^{3/2}}-\frac{\sqrt{ax+bx^3}}{2ax^{5/2}}\right)}{a}+\frac{1}{ax^{3/2}\sqrt{ax+bx^3}}\right)}{3a}+\frac{1}{3a\sqrt{x}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{\sqrt{x}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

input
```
Int[x^(3/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
x^(3/2)/(7*a*(a*x + b*x^3)^(7/2)) + (9*(Sqrt[x]/(5*a*(a*x + b*x^3)^(5/2))
+ (7*(1/(3*a*Sqrt[x]*(a*x + b*x^3)^(3/2)) + (5*(1/(a*x^(3/2)*Sqrt[a*x + b*
x^3]) + (3*(-1/2*Sqrt[a*x + b*x^3]/(a*x^(5/2)) + (b*ArcTanh[(Sqrt[a]*Sqrt[
x])/Sqrt[a*x + b*x^3]])/(2*a^(3/2))))/a))/(3*a)))/(5*a)))/(7*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.47

method	result
default	$\frac{\sqrt{x(b\,x^2+a)}\left(315\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)b^4x^8\sqrt{b\,x^2+a}-315\sqrt{a}\,b^4x^8+945\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)a\,b^3x^6\sqrt{b\,x^2+a}-1050a^{\frac{3}{2}}b^3x^6+945\cdots\right.}{70a^{\frac{11}{2}}x^{\frac{5}{2}}(b\,x^2+\cdots}$
risch	$-\frac{b\,x^2+a}{2a^5x^{\frac{3}{2}}\sqrt{x(b\,x^2+a)}}+\frac{\left(\frac{9b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{2a^{\frac{11}{2}}}-\frac{2629b\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2b+2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}}{1120a^5\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}+\frac{2629b\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2b-2\sqrt{-ab}\left(x+\cdots\right.}}{1120a^5\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}\cdots\right.}{\cdots}$

input
```
int(x^(3/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/70*(x*(b*x^2+a))^(1/2)/a^(11/2)*(315*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)
*b^4*x^8*(b*x^2+a)^(1/2)-315*a^(1/2)*b^4*x^8+945*ln(2*(a^(1/2)*(b*x^2+a)^(
1/2)+a)/x)*a*b^3*x^6*(b*x^2+a)^(1/2)-1050*a^(3/2)*b^3*x^6+945*ln(2*(a^(1/2
)*(b*x^2+a)^(1/2)+a)/x)*a^2*b^2*x^4*(b*x^2+a)^(1/2)-1218*a^(5/2)*b^2*x^4+3
15*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*a^3*b*x^2*(b*x^2+a)^(1/2)-528*a^(7/
2)*b*x^2-35*a^(9/2))/x^(5/2)/(b*x^2+a)^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.47

$$\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx = \left[\frac{315\,(b^5x^{11}+4\,ab^4x^9+6\,a^2b^3x^7+4\,a^3b^2x^5+a^4bx^3)\sqrt{a}\log\left(\frac{bx^3+2\,ax+2\,\sqrt{bx^3+ax}\sqrt{a}\sqrt{x}}{x^3}\right)}{140\,(a^6b^4x^{11}+4\,a^7b^3x^9+6\right.$$

input
```
integrate(x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
[1/140*(315*(b^5*x^11 + 4*a*b^4*x^9 + 6*a^2*b^3*x^7 + 4*a^3*b^2*x^5 + a^4*
b*x^3)*sqrt(a)*log((b*x^3 + 2*a*x + 2*sqrt(b*x^3 + a*x)*sqrt(a)*sqrt(x))/x
^3) - 2*(315*a*b^4*x^8 + 1050*a^2*b^3*x^6 + 1218*a^3*b^2*x^4 + 528*a^4*b*x
^2 + 35*a^5)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*
a^8*b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3), -1/70*(315*(b^5*x^11 + 4*a*b^4*x^9
+ 6*a^2*b^3*x^7 + 4*a^3*b^2*x^5 + a^4*b*x^3)*sqrt(-a)*arctan(sqrt(-a)*sqrt
(x)/sqrt(b*x^3 + a*x)) + (315*a*b^4*x^8 + 1050*a^2*b^3*x^6 + 1218*a^3*b^2*
x^4 + 528*a^4*b*x^2 + 35*a^5)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^6*b^4*x^11 + 4
*a^7*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3)]
```

Sympy [F]

$$\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{3}{2}}}{(x(a+bx^2))^{\frac{9}{2}}}\,dx$$

input
```
integrate(x**(3/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Integral(x**(3/2)/(x*(a + b*x**2))**(9/2), x)
```

Maxima [F]

$$\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{3}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(3/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.65

$$\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{9\,b\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}a^5} - \frac{\sqrt{bx^2+a}}{2\,a^5x^2} - \frac{140\,(bx^2+a)^3b + 35\,(bx^2+a)^2ab + 14\,(bx^2+a)a^2b + 5\,a^3b}{35\,(bx^2+a)^{\frac{7}{2}}a^5}$$

input
```
integrate(x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
-9/2*b*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^5) - 1/2*sqrt(b*x^2 +
a)/(a^5*x^2) - 1/35*(140*(b*x^2 + a)^3*b + 35*(b*x^2 + a)^2*a*b + 14*(b*x^
2 + a)*a^2*b + 5*a^3*b)/((b*x^2 + a)^(7/2)*a^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{3/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(x^(3/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(3/2)/(a*x + b*x^3)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 472, normalized size of antiderivative = 2.97

$$\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{-35\sqrt{b\,x^2+a}\,a^5-528\sqrt{b\,x^2+a}\,a^4b\,x^2-1218\sqrt{b\,x^2+a}\,a^3b^2x^4-1050\sqrt{b\,x^2+a}\,a^2}{}$$

input
```
int(x^(3/2)/(b*x^3+a*x)^(9/2),x)
```

output
```
( - 35*sqrt(a + b*x**2)*a**5 - 528*sqrt(a + b*x**2)*a**4*b*x**2 - 1218*sqr
t(a + b*x**2)*a**3*b**2*x**4 - 1050*sqrt(a + b*x**2)*a**2*b**3*x**6 - 315*
sqrt(a + b*x**2)*a*b**4*x**8 - 315*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a)
 + sqrt(b)*x)/sqrt(a))*a**4*b*x**2 - 1260*sqrt(a)*log((sqrt(a + b*x**2) -
sqrt(a) + sqrt(b)*x)/sqrt(a))*a**3*b**2*x**4 - 1890*sqrt(a)*log((sqrt(a +
b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b**3*x**6 - 1260*sqrt(a)*log(
(sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**4*x**8 - 315*sqrt(a
)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**5*x**10 + 315*s
qrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**4*b*x**2 +
 1260*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**3*b
**2*x**4 + 1890*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(
a))*a**2*b**3*x**6 + 1260*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b
)*x)/sqrt(a))*a*b**4*x**8 + 315*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) +
sqrt(b)*x)/sqrt(a))*b**5*x**10)/(70*a**6*x**2*(a**4 + 4*a**3*b*x**2 + 6*a*
*2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.96 $\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}} dx$

Optimal result 769
Mathematica [A] (verified) 769
Rubi [A] (verified) 770
Maple [A] (verified) 773
Fricas [A] (verification not implemented) 773
Sympy [F] 774
Maxima [F] 774
Giac [A] (verification not implemented) 774
Mupad [F(-1)] 775
Reduce [B] (verification not implemented) 775

Optimal result

Integrand size = 19, antiderivative size = 152

$$\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}} dx = \frac{\sqrt{x}}{7a(ax+bx^3)^{7/2}} + \frac{2}{7a^2\sqrt{x}(ax+bx^3)^{5/2}} + \frac{16}{21a^3x^{3/2}(ax+bx^3)^{3/2}} + \frac{32}{7a^4x^{5/2}\sqrt{ax+bx^3}} - \frac{128\sqrt{ax+bx^3}}{21a^5x^{7/2}} + \frac{256b\sqrt{ax+bx^3}}{21a^6x^{3/2}}$$

output
```
1/7*x^(1/2)/a/(b*x^3+a*x)^(7/2)+2/7/a^2/x^(1/2)/(b*x^3+a*x)^(5/2)+16/21/a^
3/x^(3/2)/(b*x^3+a*x)^(3/2)+32/7/a^4/x^(5/2)/(b*x^3+a*x)^(1/2)-128/21*(b*x
^3+a*x)^(1/2)/a^5/x^(7/2)+256/21*b*(b*x^3+a*x)^(1/2)/a^6/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}} dx = \frac{\sqrt{x}(-7a^5+70a^4bx^2+560a^3b^2x^4+1120a^2b^3x^6+896ab^4x^8+256b^5x^{10})}{21a^6(x(a+bx^2))^{7/2}}$$

input
```
Integrate[Sqrt[x]/(a*x + b*x^3)^(9/2),x]
```

output
```
(Sqrt[x]*(-7*a^5 + 70*a^4*b*x^2 + 560*a^3*b^2*x^4 + 1120*a^2*b^3*x^6 + 896
*a*b^4*x^8 + 256*b^5*x^10))/(21*a^6*(x*(a + b*x^2))^(7/2))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1921, 1921, 1921, 1921, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}}\,dx$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{10\int \frac{1}{\sqrt{x}(bx^3+ax)^{7/2}}dx}{7a}+\frac{\sqrt{x}}{7a\,(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{10\left(\frac{8\int \frac{1}{x^{3/2}\left(bx^3+ax\right)^{5/2}}dx}{5a}+\frac{1}{5a\sqrt{x}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{\sqrt{x}}{7a\,(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{10\left(\frac{8\left(\frac{2\int \frac{1}{x^{5/2}\left(bx^3+ax\right)^{3/2}}dx}{a}+\frac{1}{3ax^{3/2}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{1}{5a\sqrt{x}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{\sqrt{x}}{7a\,(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{10\left(\frac{8\left(\frac{2\left(\frac{4\int\frac{1}{x^{7/2}\sqrt{bx^3+ax}}dx}{a}+\frac{1}{ax^{5/2}\sqrt{ax+bx^3}}\right)}{a}+\frac{1}{3ax^{3/2}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{1}{5a\sqrt{x}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{10\left(\frac{8\left(\frac{2\left(\frac{4\left(-\frac{2b\int\frac{1}{x^{3/2}\sqrt{bx^3+ax}}dx}{3a}-\frac{\sqrt{ax+bx^3}}{3ax^{7/2}}\right)}{a}+\frac{1}{ax^{5/2}\sqrt{ax+bx^3}}\right)}{a}+\frac{1}{3ax^{3/2}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{1}{5a\sqrt{x}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{10\left(\frac{8\left(\frac{2\left(\frac{4\left(\frac{2b\sqrt{ax+bx^3}}{3a^2x^{3/2}}-\frac{\sqrt{ax+bx^3}}{3ax^{7/2}}\right)}{a}+\frac{1}{ax^{5/2}\sqrt{ax+bx^3}}\right)}{a}+\frac{1}{3ax^{3/2}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{1}{5a\sqrt{x}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}}$$

input
```
Int[Sqrt[x]/(a*x + b*x^3)^(9/2),x]
```

output
```
Sqrt[x]/(7*a*(a*x + b*x^3)^(7/2)) + (10*(1/(5*a*Sqrt[x]*(a*x + b*x^3)^(5/2
)) + (8*(1/(3*a*x^(3/2)*(a*x + b*x^3)^(3/2)) + (2*(1/(a*x^(5/2)*Sqrt[a*x +
 b*x^3]) + (4*(-1/3*Sqrt[a*x + b*x^3]/(a*x^(7/2)) + (2*b*Sqrt[a*x + b*x^3]
)/(3*a^2*x^(3/2))))/a))/a))/(5*a)))/(7*a)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.53

method	result	size
gosper	$-\frac{x^{\frac{3}{2}}(b\,x^{2}+a)(-256b^{5}x^{10}-896a\,b^{4}x^{8}-1120a^{2}x^{6}b^{3}-560a^{3}x^{4}b^{2}-70a^{4}x^{2}b+7a^{5})}{21a^{6}(b\,x^{3}+ax)^{\frac{9}{2}}}$	81
orering	$-\frac{x^{\frac{3}{2}}(b\,x^{2}+a)(-256b^{5}x^{10}-896a\,b^{4}x^{8}-1120a^{2}x^{6}b^{3}-560a^{3}x^{4}b^{2}-70a^{4}x^{2}b+7a^{5})}{21a^{6}(b\,x^{3}+ax)^{\frac{9}{2}}}$	81
default	$-\frac{\sqrt{x(b\,x^{2}+a)}\,(-256b^{5}x^{10}-896a\,b^{4}x^{8}-1120a^{2}x^{6}b^{3}-560a^{3}x^{4}b^{2}-70a^{4}x^{2}b+7a^{5})}{21x^{\frac{7}{2}}(b\,x^{2}+a)^{4}a^{6}}$	83
risch	$-\frac{(b\,x^{2}+a)(-14b\,x^{2}+a)}{3a^{6}x^{\frac{5}{2}}\sqrt{x(b\,x^{2}+a)}}+\frac{(b\,x^{2}+a)x^{\frac{3}{2}}(158b^{3}x^{6}+511a\,b^{2}x^{4}+560a^{2}b\,x^{2}+210a^{3})b^{2}}{21a^{6}(b^{4}x^{8}+4a\,b^{3}x^{6}+6a^{2}b^{2}x^{4}+4a^{3}b\,x^{2}+a^{4})\sqrt{x(b\,x^{2}+a)}}$	139

input
```
int(x^(1/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/21*x^(3/2)*(b*x^2+a)*(-256*b^5*x^10-896*a*b^4*x^8-1120*a^2*b^3*x^6-560*
a^3*b^2*x^4-70*a^4*b*x^2+7*a^5)/a^6/(b*x^3+a*x)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}}\,dx = \frac{(256\,b^5x^{10}+896\,ab^4x^8+1120\,a^2b^3x^6+560\,a^3b^2x^4+70\,a^4bx^2-7\,a^5)\sqrt{bx^3+ax}\sqrt{x}}{21\,(a^6b^4x^{12}+4\,a^7b^3x^{10}+6\,a^8b^2x^8+4\,a^9bx^6+a^{10}x^4)}$$

input
```
integrate(x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
1/21*(256*b^5*x^10 + 896*a*b^4*x^8 + 1120*a^2*b^3*x^6 + 560*a^3*b^2*x^4 +
70*a^4*b*x^2 - 7*a^5)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^6*b^4*x^12 + 4*a^7*b^3*
x^10 + 6*a^8*b^2*x^8 + 4*a^9*b*x^6 + a^10*x^4)
```

Sympy [F]

$$\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{\sqrt{x}}{(x(a+bx^2))^{\frac{9}{2}}}\,dx$$

input
```
integrate(x**(1/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Integral(sqrt(x)/(x*(a + b*x**2))**(9/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{\sqrt{x}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{158\,b^5x^2}{a^6}+\frac{511\,b^4}{a^5}\right)+\frac{560\,b^3}{a^4}\right)x^2+\frac{210\,b^2}{a^3}\right)x}{21\,(bx^2+a)^{\frac{7}{2}}} - \frac{4\left(6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4b^{\frac{3}{2}}-15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2ab^{\frac{3}{2}}+7\,a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3a^5}$$

input
```
integrate(x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
1/21*((x^2*(158*b^5*x^2/a^6 + 511*b^4/a^5) + 560*b^3/a^4)*x^2 + 210*b^2/a^
3)*x/(b*x^2 + a)^(7/2) - 4/3*(6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(3/2) -
15*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3/2) + 7*a^2*b^(3/2))/(((sqrt(b)*x
 - sqrt(b*x^2 + a))^2 - a)^3*a^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{\sqrt{x}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(x^(1/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(1/2)/(a*x + b*x^3)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}}\,dx = \frac{-7\sqrt{b\,x^2+a}\,a^5+70\sqrt{b\,x^2+a}\,a^4b\,x^2+560\sqrt{b\,x^2+a}\,a^3b^2x^4+1120\sqrt{b\,x^2+a}\,a^2b^3x}{21}$$

input
```
int(x^(1/2)/(b*x^3+a*x)^(9/2),x)
```

output
```
( - 7*sqrt(a + b*x**2)*a**5 + 70*sqrt(a + b*x**2)*a**4*b*x**2 + 560*sqrt(a
 + b*x**2)*a**3*b**2*x**4 + 1120*sqrt(a + b*x**2)*a**2*b**3*x**6 + 896*sqr
t(a + b*x**2)*a*b**4*x**8 + 256*sqrt(a + b*x**2)*b**5*x**10 - 256*sqrt(b)*
a**4*b*x**3 - 1024*sqrt(b)*a**3*b**2*x**5 - 1536*sqrt(b)*a**2*b**3*x**7 -
1024*sqrt(b)*a*b**4*x**9 - 256*sqrt(b)*b**5*x**11)/(21*a**6*x**3*(a**4 + 4
*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.97 $\int \frac{1}{\sqrt{x}\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal result . . . 776
Mathematica [A] (verified) . . . 777
Rubi [A] (verified) . . . 777
Maple [A] (verified) . . . 783
Fricas [A] (verification not implemented) . . . 783
Sympy [F] . . . 784
Maxima [F] . . . 784
Giac [A] (verification not implemented) . . . 785
Mupad [F(-1)] . . . 785
Reduce [B] (verification not implemented) . . . 786

Optimal result

Integrand size = 19, antiderivative size = 189

$$\int \frac{1}{\sqrt{x}\left(ax+bx^3\right)^{9/2}}\,dx = \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}}+\frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}}+\frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}}+\frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}}-\frac{33\sqrt{ax+bx^3}}{4a^5x^{9/2}}+\frac{99b\sqrt{ax+bx^3}}{8a^6x^{5/2}}-\frac{99b^2\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{8a^{13/2}}$$

output
```
1/7/a/x^(1/2)/(b*x^3+a*x)^(7/2)+11/35/a^2/x^(3/2)/(b*x^3+a*x)^(5/2)+33/35/
a^3/x^(5/2)/(b*x^3+a*x)^(3/2)+33/5/a^4/x^(7/2)/(b*x^3+a*x)^(1/2)-33/4*(b*x
^3+a*x)^(1/2)/a^5/x^(9/2)+99/8*b*(b*x^3+a*x)^(1/2)/a^6/x^(5/2)-99/8*b^2*ar
ctanh(a^(1/2)*x^(1/2)/(b*x^3+a*x)^(1/2))/a^(13/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3)^{9/2}}\,dx = \frac{\sqrt{x\,(a+bx^2)}\Big(\sqrt{a}(-70a^5+385a^4bx^2+5808a^3b^2x^4+13398a^2b^3x^6+11550ab^4x}{280a^{13/2}x^{9/2}\,(a+bx^2)^4}$$

input
```
Integrate[1/(Sqrt[x]*(a*x + b*x^3)^(9/2)),x]
```

output
```
(Sqrt[x*(a + b*x^2)]*(Sqrt[a]*(-70*a^5 + 385*a^4*b*x^2 + 5808*a^3*b^2*x^4
+ 13398*a^2*b^3*x^6 + 11550*a*b^4*x^8 + 3465*b^5*x^10) - 3465*b^2*x^4*(a +
 b*x^2)^(7/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/(280*a^(13/2)*x^(9/2)*(a
+ b*x^2)^4)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1929, 1929, 1929, 1929, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3)^{9/2}}\,dx$$

$$\Big\downarrow \; 1929$$

$$\frac{11\int \frac{1}{x^{3/2}(bx^3+ax)^{7/2}}dx}{7a}+\frac{1}{7a\sqrt{x}\,(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \; 1929$$

$$\frac{11\left(\frac{9\int \frac{1}{x^{5/2}\left(bx^3+ax\right)^{5/2}}dx}{5a}+\frac{1}{5ax^{3/2}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{1}{7a\sqrt{x}\,(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \; 1929$$

$$\frac{11\left(\frac{9\left(\frac{7\int \frac{1}{x^{7/2}\left(bx^3+ax\right)^{3/2}}dx}{3a}+\frac{1}{3ax^{5/2}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{1}{5ax^{3/2}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}}$$

$\downarrow$ 1929

$$\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\int \frac{1}{x^{9/2}\sqrt{bx^3+ax}}dx}{a}+\frac{1}{ax^{7/2}\sqrt{ax+bx^3}}\right)}{3a}+\frac{1}{3ax^{5/2}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{1}{5ax^{3/2}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}}$$

$\downarrow$ 1931

$$\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(-\frac{3b\int \frac{1}{x^{5/2}\sqrt{bx^3+ax}}dx}{4a}-\frac{\sqrt{ax+bx^3}}{4ax^{9/2}}\right)}{a}+\frac{1}{ax^{7/2}\sqrt{ax+bx^3}}\right)}{3a}+\frac{1}{3ax^{5/2}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{1}{5ax^{3/2}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}}$$

$\downarrow$ 1931

$$\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(-\frac{3b\left(-\frac{b\int\frac{1}{\sqrt{x}\sqrt{bx^3+ax}}dx}{2a}-\frac{\sqrt{ax+bx^3}}{2ax^{5/2}}\right)}{4a}-\frac{\sqrt{ax+bx^3}}{4ax^{9/2}}\right)}{a}+\frac{1}{ax^{7/2}\sqrt{ax+bx^3}}\right)}{3a}+\frac{1}{3ax^{5/2}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{1}{5ax^{3/2}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}}$$

$\downarrow$ 1935

$$\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(-\frac{3b\left(\frac{b\int\frac{1}{1-\frac{ax}{bx^3+ax}}d\frac{\sqrt{x}}{\sqrt{bx^3+ax}}}{2a}-\frac{\sqrt{ax+bx^3}}{2ax^{5/2}}\right)}{4a}-\frac{\sqrt{ax+bx^3}}{4ax^{9/2}}\right)}{a}+\frac{1}{ax^{7/2}\sqrt{ax+bx^3}}\right)}{3a}+\frac{1}{3ax^{5/2}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{1}{5ax^{3/2}(ax+bx^3)^{5/2}}\right)}{7a}+$$

$$\frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}}$$

$\downarrow$ 219

$$\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{2a^{3/2}}-\frac{\sqrt{ax+bx^3}}{2ax^{5/2}}\right)}{4a}-\frac{\sqrt{ax+bx^3}}{4ax^{9/2}}\right)}{a}+\frac{1}{ax^{7/2}\sqrt{ax+bx^3}}\right)}{3a}+\frac{1}{3ax^{5/2}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{1}{5ax^{3/2}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}}$$

input
```
Int[1/(Sqrt[x]*(a*x + b*x^3)^(9/2)),x]
```

output
```
1/(7*a*Sqrt[x]*(a*x + b*x^3)^(7/2)) + (11*(1/(5*a*x^(3/2)*(a*x + b*x^3)^(5
/2)) + (9*(1/(3*a*x^(5/2)*(a*x + b*x^3)^(3/2)) + (7*(1/(a*x^(7/2)*Sqrt[a*x
 + b*x^3]) + (5*(-1/4*Sqrt[a*x + b*x^3]/(a*x^(9/2)) - (3*b*(-1/2*Sqrt[a*x
+ b*x^3]/(a*x^(5/2)) + (b*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^3]])/(2
*a^(3/2))))/(4*a)))/a))/(3*a)))/(5*a)))/(7*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.31

method	result
default	$-\frac{\sqrt{x(bx^2+a)}\left(3465\ln\left(\frac{2a+2\sqrt{a}\sqrt{bx^2+a}}{x}\right)b^5x^{10}\sqrt{bx^2+a}-3465\sqrt{a}\,b^5x^{10}+10395\ln\left(\frac{2a+2\sqrt{a}\sqrt{bx^2+a}}{x}\right)a\,b^4x^8\sqrt{bx^2+a}-11550a^{\frac{3}{2}}\cdots\right.}{\cdots}$
risch	$-\frac{(bx^2+a)(-19bx^2+2a)}{8a^6x^{\frac{7}{2}}\sqrt{x(bx^2+a)}}+\frac{\left(-\frac{99b^2\ln\left(\frac{2a+2\sqrt{a}\sqrt{bx^2+a}}{x}\right)}{8a^{\frac{13}{2}}}+\frac{6311b^2\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2b+2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}}{1120a^6\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}-\frac{6311b^2\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2b-\cdots}}{1120a^6\sqrt{-ab}\left(\cdots\right.}\cdots\right.}{\cdots}$

input
```
int(1/x^(1/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/280*(x*(b*x^2+a))^(1/2)/a^(13/2)*(3465*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)
/x)*b^5*x^10*(b*x^2+a)^(1/2)-3465*a^(1/2)*b^5*x^10+10395*ln(2*(a^(1/2)*(b*
x^2+a)^(1/2)+a)/x)*a*b^4*x^8*(b*x^2+a)^(1/2)-11550*a^(3/2)*b^4*x^8+10395*l
n(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*a^2*b^3*x^6*(b*x^2+a)^(1/2)-13398*a^(5/
2)*b^3*x^6+3465*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*a^3*b^2*x^4*(b*x^2+a)^
(1/2)-5808*a^(7/2)*b^2*x^4-385*a^(9/2)*b*x^2+70*a^(11/2))/x^(9/2)/(b*x^2+a
)^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.22

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3)^{9/2}}\,dx = \left[\frac{3465\,(b^6x^{13}+4\,ab^5x^{11}+6\,a^2b^4x^9+4\,a^3b^3x^7+a^4b^2x^5)\sqrt{a}\log\left(\frac{bx^3+2\,ax-2\,\sqrt{bx^3+a\cdots}}{x^3}\right.\cdots}{560\,(a^7b^4x^{13}+\cdots}\right.$$

input
```
integrate(1/x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
[1/560*(3465*(b^6*x^13 + 4*a*b^5*x^11 + 6*a^2*b^4*x^9 + 4*a^3*b^3*x^7 + a^
4*b^2*x^5)*sqrt(a)*log((b*x^3 + 2*a*x - 2*sqrt(b*x^3 + a*x)*sqrt(a)*sqrt(x
))/x^3) + 2*(3465*a*b^5*x^10 + 11550*a^2*b^4*x^8 + 13398*a^3*b^3*x^6 + 580
8*a^4*b^2*x^4 + 385*a^5*b*x^2 - 70*a^6)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^7*b^
4*x^13 + 4*a^8*b^3*x^11 + 6*a^9*b^2*x^9 + 4*a^10*b*x^7 + a^11*x^5), 1/280*
(3465*(b^6*x^13 + 4*a*b^5*x^11 + 6*a^2*b^4*x^9 + 4*a^3*b^3*x^7 + a^4*b^2*x
^5)*sqrt(-a)*arctan(sqrt(-a)*sqrt(x)/sqrt(b*x^3 + a*x)) + (3465*a*b^5*x^10
 + 11550*a^2*b^4*x^8 + 13398*a^3*b^3*x^6 + 5808*a^4*b^2*x^4 + 385*a^5*b*x^
2 - 70*a^6)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^7*b^4*x^13 + 4*a^8*b^3*x^11 + 6*
a^9*b^2*x^9 + 4*a^10*b*x^7 + a^11*x^5)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3)^{9/2}}\,dx = \int \frac{1}{\sqrt{x}\,(x\,(a+bx^2))^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/x**(1/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Integral(1/(sqrt(x)*(x*(a + b*x**2))**(9/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3)^{9/2}}\,dx = \int \frac{1}{(bx^3+ax)^{\frac{9}{2}}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a*x)^(9/2)*sqrt(x)), x)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3)^{9/2}}\,dx = \frac{99\,b^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^6} + \frac{350\,(bx^2+a)^3b^2+70\,(bx^2+a)^2ab^2+21\,(bx^2+a)a^2b^2+5\,a^3b^2}{35\,(bx^2+a)^{\frac{7}{2}}a^6} + \frac{19\,(bx^2+a)^{\frac{3}{2}}b^2-21\,\sqrt{bx^2+a}ab^2}{8\,a^6b^2x^4}$$

input
```
integrate(1/x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
99/8*b^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^6) + 1/35*(350*(b*x^
2 + a)^3*b^2 + 70*(b*x^2 + a)^2*a*b^2 + 21*(b*x^2 + a)*a^2*b^2 + 5*a^3*b^2
)/((b*x^2 + a)^(7/2)*a^6) + 1/8*(19*(b*x^2 + a)^(3/2)*b^2 - 21*sqrt(b*x^2
+ a)*a*b^2)/(a^6*b^2*x^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3)^{9/2}}\,dx = \int \frac{1}{\sqrt{x}\,(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(1/(x^(1/2)*(a*x + b*x^3)^(9/2)),x)
```

output
```
int(1/(x^(1/2)*(a*x + b*x^3)^(9/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.62

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3)^{9/2}}\,dx = \frac{-70\sqrt{b\,x^2+a}\,a^6+385\sqrt{b\,x^2+a}\,a^5b\,x^2+5808\sqrt{b\,x^2+a}\,a^4b^2x^4+13398\sqrt{b\,x^2+}$$

input
```
int(1/x^(1/2)/(b*x^3+a*x)^(9/2),x)
```

output
```
( - 70*sqrt(a + b*x**2)*a**6 + 385*sqrt(a + b*x**2)*a**5*b*x**2 + 5808*sqr
t(a + b*x**2)*a**4*b**2*x**4 + 13398*sqrt(a + b*x**2)*a**3*b**3*x**6 + 115
50*sqrt(a + b*x**2)*a**2*b**4*x**8 + 3465*sqrt(a + b*x**2)*a*b**5*x**10 +
3465*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**4*b*
*2*x**4 + 13860*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(
a))*a**3*b**3*x**6 + 20790*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(
b)*x)/sqrt(a))*a**2*b**4*x**8 + 13860*sqrt(a)*log((sqrt(a + b*x**2) - sqrt
(a) + sqrt(b)*x)/sqrt(a))*a*b**5*x**10 + 3465*sqrt(a)*log((sqrt(a + b*x**2
) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**6*x**12 - 3465*sqrt(a)*log((sqrt(a +
b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**4*b**2*x**4 - 13860*sqrt(a)*log
((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**3*b**3*x**6 - 20790*
sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b**4*x*
*8 - 13860*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a
*b**5*x**10 - 3465*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sq
rt(a))*b**6*x**12)/(280*a**7*x**4*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4
 + 4*a*b**3*x**6 + b**4*x**8))
```

3.98 $\int \frac{1}{x^{3/2}(ax+bx^3)^{9/2}}\,dx$

Optimal result 787
Mathematica [A] (verified) 788
Rubi [A] (verified) 788
Maple [A] (verified) 792
Fricas [A] (verification not implemented) 793
Sympy [F] 793
Maxima [F] 793
Giac [A] (verification not implemented) 794
Mupad [F(-1)] 794
Reduce [B] (verification not implemented) 795

Optimal result

Integrand size = 19, antiderivative size = 180

$$\int \frac{1}{x^{3/2}(ax+bx^3)^{9/2}}\,dx = \frac{1}{7ax^{3/2}(ax+bx^3)^{7/2}} + \frac{12}{35a^2x^{5/2}(ax+bx^3)^{5/2}} + \frac{8}{7a^3x^{7/2}(ax+bx^3)^{3/2}} + \frac{64}{7a^4x^{9/2}\sqrt{ax+bx^3}} - \frac{384\sqrt{ax+bx^3}}{35a^5x^{11/2}} + \frac{512b\sqrt{ax+bx^3}}{35a^6x^{7/2}} - \frac{1024b^2\sqrt{ax+bx^3}}{35a^7x^{3/2}}$$

output
```
1/7/a/x^(3/2)/(b*x^3+a*x)^(7/2)+12/35/a^2/x^(5/2)/(b*x^3+a*x)^(5/2)+8/7/a^
3/x^(7/2)/(b*x^3+a*x)^(3/2)+64/7/a^4/x^(9/2)/(b*x^3+a*x)^(1/2)-384/35*(b*x
^3+a*x)^(1/2)/a^5/x^(11/2)+512/35*b*(b*x^3+a*x)^(1/2)/a^6/x^(7/2)-1024/35*
b^2*(b*x^3+a*x)^(1/2)/a^7/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{9/2}}\,dx = \frac{-7a^6+28a^5bx^2-280a^4b^2x^4-2240a^3b^3x^6-4480a^2b^4x^8-3584ab^5x^{10}-1024b}{35a^7x^{3/2}\left(x\left(a+bx^2\right)\right)^{7/2}}$$

input
```
Integrate[1/(x^(3/2)*(a*x + b*x^3)^(9/2)),x]
```

output
```
(-7*a^6 + 28*a^5*b*x^2 - 280*a^4*b^2*x^4 - 2240*a^3*b^3*x^6 - 4480*a^2*b^4
*x^8 - 3584*a*b^5*x^10 - 1024*b^6*x^12)/(35*a^7*x^(3/2)*(x*(a + b*x^2))^(7
/2))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1921, 1921, 1921, 1921, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{9/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{12\int \frac{1}{x^{5/2}\left(bx^3+ax\right)^{7/2}}dx}{7a}+\frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{12\left(\frac{2\int \frac{1}{x^{7/2}\left(bx^3+ax\right)^{5/2}}dx}{a}+\frac{1}{5ax^{5/2}\left(ax+bx^3\right)^{5/2}}\right)}{7a}+\frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{12\left(\frac{2\left(\frac{8\int \frac{1}{x^{9/2}\left(bx^3+ax\right)^{3/2}}dx}{3a}+\frac{1}{3ax^{7/2}\left(ax+bx^3\right)^{3/2}}\right)}{a}+\frac{1}{5ax^{5/2}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}}$$

$\Big\downarrow$ 1921

$$\frac{12\left(\frac{2\left(\frac{8\left(\frac{6\int \frac{1}{x^{11/2}\sqrt{bx^3+ax}}dx}{a}+\frac{1}{ax^{9/2}\sqrt{ax+bx^3}}\right)}{3a}+\frac{1}{3ax^{7/2}\left(ax+bx^3\right)^{3/2}}\right)}{a}+\frac{1}{5ax^{5/2}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}}$$

$\Big\downarrow$ 1922

$$\frac{12\left(\frac{2\left(\frac{8\left(\frac{6\left(-\frac{4b\int \frac{1}{x^{7/2}\sqrt{bx^3+ax}}dx}{5a}-\frac{\sqrt{ax+bx^3}}{5ax^{11/2}}\right)}{a}+\frac{1}{ax^{9/2}\sqrt{ax+bx^3}}\right)}{3a}+\frac{1}{3ax^{7/2}\left(ax+bx^3\right)^{3/2}}\right)}{a}+\frac{1}{5ax^{5/2}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}}$$

$\Big\downarrow$ 1922

$$\frac{12\left(\frac{2\left(\frac{8\left(\frac{6\left(-\frac{4b\left(-\frac{2b\int\frac{1}{x^{3/2}\sqrt{bx^3+ax}}dx}{3a}-\frac{\sqrt{ax+bx^3}}{3ax^{7/2}}\right)}{5a}-\frac{\sqrt{ax+bx^3}}{5ax^{11/2}}\right)}{a}+\frac{1}{ax^{9/2}\sqrt{ax+bx^3}}\right)}{3a}+\frac{1}{3ax^{7/2}\left(ax+bx^3\right)^{3/2}}\right)}{a}+\frac{1}{5ax^{5/2}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}}$$

$$\downarrow \ 1920$$

$$\frac{12\left(\frac{2\left(\frac{8\left(\frac{6\left(-\frac{4b\left(\frac{2b\sqrt{ax+bx^3}}{3a^2x^{3/2}}-\frac{\sqrt{ax+bx^3}}{3ax^{7/2}}\right)}{5a}-\frac{\sqrt{ax+bx^3}}{5ax^{11/2}}\right)}{a}+\frac{1}{ax^{9/2}\sqrt{ax+bx^3}}\right)}{3a}+\frac{1}{3ax^{7/2}\left(ax+bx^3\right)^{3/2}}\right)}{a}+\frac{1}{5ax^{5/2}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}}$$

input
```
Int[1/(x^(3/2)*(a*x + b*x^3)^(9/2)),x]
```

output
```
1/(7*a*x^(3/2)*(a*x + b*x^3)^(7/2)) + (12*(1/(5*a*x^(5/2)*(a*x + b*x^3)^(5
/2)) + (2*(1/(3*a*x^(7/2)*(a*x + b*x^3)^(3/2)) + (8*(1/(a*x^(9/2)*Sqrt[a*x
 + b*x^3]) + (6*(-1/5*Sqrt[a*x + b*x^3]/(a*x^(11/2)) - (4*b*(-1/3*Sqrt[a*x
 + b*x^3]/(a*x^(7/2)) + (2*b*Sqrt[a*x + b*x^3])/(3*a^2*x^(3/2))))/(5*a))/
a))/(3*a)))/a))/(7*a)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)\left(1024 b^{6} x^{12}+3584 b^{5} x^{10} a+4480 b^{4} x^{8} a^{2}+2240 b^{3} x^{6} a^{3}+280 b^{2} x^{4} a^{4}-28 b\,x^{2} a^{5}+7 a^{6}\right)}{35 \sqrt{x}\, a^{7} \left(b\,x^{3}+a x\right)^{\frac{9}{2}}}$	92
orering	$-\frac{\left(b\,x^{2}+a\right)\left(1024 b^{6} x^{12}+3584 b^{5} x^{10} a+4480 b^{4} x^{8} a^{2}+2240 b^{3} x^{6} a^{3}+280 b^{2} x^{4} a^{4}-28 b\,x^{2} a^{5}+7 a^{6}\right)}{35 \sqrt{x}\, a^{7} \left(b\,x^{3}+a x\right)^{\frac{9}{2}}}$	92
default	$-\frac{\sqrt{x \left(b\,x^{2}+a\right)}\,\left(1024 b^{6} x^{12}+3584 b^{5} x^{10} a+4480 b^{4} x^{8} a^{2}+2240 b^{3} x^{6} a^{3}+280 b^{2} x^{4} a^{4}-28 b\,x^{2} a^{5}+7 a^{6}\right)}{35 x^{\frac{11}{2}} \left(b\,x^{2}+a\right)^{4} a^{7}}$	94
risch	$-\frac{\left(b\,x^{2}+a\right)\left(66 b^{2} x^{4}-8 a b\,x^{2}+a^{2}\right)}{5 a^{7} x^{\frac{9}{2}} \sqrt{x \left(b\,x^{2}+a\right)}}-\frac{\left(b\,x^{2}+a\right) x^{\frac{3}{2}} \left(562 b^{3} x^{6}+1792 a\,b^{2} x^{4}+1925 a^{2} b\,x^{2}+700 a^{3}\right) b^{3}}{35 a^{7} \left(b^{4} x^{8}+4 a\,b^{3} x^{6}+6 a^{2} b^{2} x^{4}+4 a^{3} b\,x^{2}+a^{4}\right) \sqrt{x \left(b\,x^{2}+a\right)}}$	150

input
```
int(1/x^(3/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35*(b*x^2+a)*(1024*b^6*x^12+3584*a*b^5*x^10+4480*a^2*b^4*x^8+2240*a^3*b
^3*x^6+280*a^4*b^2*x^4-28*a^5*b*x^2+7*a^6)/x^(1/2)/a^7/(b*x^3+a*x)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{9/2}}\,dx = -\frac{\left(1024\,b^6x^{12}+3584\,ab^5x^{10}+4480\,a^2b^4x^8+2240\,a^3b^3x^6+280\,a^4b^2x^4-28\,a^5bx^2+7\,a^6\right)\sqrt{bx^3+ax}\sqrt{x}}{35\left(a^7b^4x^{14}+4\,a^8b^3x^{12}+6\,a^9b^2x^{10}+4\,a^{10}bx^8+a^{11}x^6\right)}$$

input
```
integrate(1/x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
-1/35*(1024*b^6*x^12 + 3584*a*b^5*x^10 + 4480*a^2*b^4*x^8 + 2240*a^3*b^3*x
^6 + 280*a^4*b^2*x^4 - 28*a^5*b*x^2 + 7*a^6)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^
7*b^4*x^14 + 4*a^8*b^3*x^12 + 6*a^9*b^2*x^10 + 4*a^10*b*x^8 + a^11*x^6)
```

Sympy [F]

$$\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{9/2}}\,dx = \int \frac{1}{x^{\frac{3}{2}}\left(x\left(a+bx^2\right)\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/x**(3/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Integral(1/(x**(3/2)*(x*(a + b*x**2))**(9/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{9/2}}\,dx = \int \frac{1}{\left(bx^3+ax\right)^{\frac{9}{2}}x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a*x)^(9/2)*x^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{9/2}}\,dx = -\frac{\left(\left(2\,x^2\left(\frac{281\,b^6x^2}{a^7}+\frac{896\,b^5}{a^6}\right)+\frac{1925\,b^4}{a^5}\right)x^2+\frac{700\,b^3}{a^4}\right)x}{35\,(bx^2+a)^{\frac{7}{2}}} + \frac{4\left(25\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8b^{\frac{5}{2}}-120\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6ab^{\frac{5}{2}}+210\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4a^2b^{\frac{5}{2}}-140\left(\sqrt{b}x\right.\right.}{5\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^5a^6}$$

input
```
integrate(1/x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
-1/35*((2*x^2*(281*b^6*x^2/a^7 + 896*b^5/a^6) + 1925*b^4/a^5)*x^2 + 700*b^
3/a^4)*x/(b*x^2 + a)^(7/2) + 4/5*(25*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(5/
2) - 120*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(5/2) + 210*(sqrt(b)*x - sqrt
(b*x^2 + a))^4*a^2*b^(5/2) - 140*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(5/
2) + 33*a^4*b^(5/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5*a^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{9/2}}\,dx = \int \frac{1}{x^{3/2}\,\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

input
```
int(1/(x^(3/2)*(a*x + b*x^3)^(9/2)),x)
```

output
```
int(1/(x^(3/2)*(a*x + b*x^3)^(9/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^{3/2} (ax + bx^3)^{9/2}} \, dx = \frac{-7\sqrt{b\,x^2+a}\,a^6 + 28\sqrt{b\,x^2+a}\,a^5 b\,x^2 - 280\sqrt{b\,x^2+a}\,a^4 b^2 x^4 - 2240\sqrt{b\,x^2+a}\,a}{}$$

input
```
int(1/x^(3/2)/(b*x^3+a*x)^(9/2),x)
```

output
```
( - 7*sqrt(a + b*x**2)*a**6 + 28*sqrt(a + b*x**2)*a**5*b*x**2 - 280*sqrt(a
 + b*x**2)*a**4*b**2*x**4 - 2240*sqrt(a + b*x**2)*a**3*b**3*x**6 - 4480*sq
rt(a + b*x**2)*a**2*b**4*x**8 - 3584*sqrt(a + b*x**2)*a*b**5*x**10 - 1024*
sqrt(a + b*x**2)*b**6*x**12 + 1024*sqrt(b)*a**4*b**2*x**5 + 4096*sqrt(b)*a
**3*b**3*x**7 + 6144*sqrt(b)*a**2*b**4*x**9 + 4096*sqrt(b)*a*b**5*x**11 +
1024*sqrt(b)*b**6*x**13)/(35*a**7*x**5*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2
*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.99 $\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx$

Optimal result 796
Mathematica [A] (verified) 796
Rubi [A] (verified) 797
Maple [A] (verified) 798
Fricas [A] (verification not implemented) 799
Sympy [F] 799
Maxima [F] 800
Giac [A] (verification not implemented) 800
Mupad [F(-1)] 800
Reduce [B] (verification not implemented) 801

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx=\frac{x\sqrt{ax+bx^4}}{3b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax+bx^4}}\right)}{3b^{3/2}}$$

output
```
1/3*x*(b*x^4+a*x)^(1/2)/b-1/3*a*arctanh(b^(1/2)*x^2/(b*x^4+a*x)^(1/2))/b^(
3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.49

$$\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx=\frac{\sqrt{b}x^2(a+bx^3)-a\sqrt{x}\sqrt{a+bx^3}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{3b^{3/2}\sqrt{x(a+bx^3)}}$$

input
```
Integrate[x^4/Sqrt[a*x + b*x^4],x]
```

output
```
(Sqrt[b]*x^2*(a + b*x^3) - a*Sqrt[x]*Sqrt[a + b*x^3]*Log[Sqrt[b]*x^(3/2) +
 Sqrt[a + b*x^3]])/(3*b^(3/2)*Sqrt[x*(a + b*x^3)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1930, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{x\sqrt{ax+bx^4}}{3b}-\frac{a\int \frac{x}{\sqrt{bx^4+ax}}dx}{2b}$$

$$\Big\downarrow \text{1935}$$

$$\frac{x\sqrt{ax+bx^4}}{3b}-\frac{a\int \frac{1}{1-\frac{bx^4}{bx^4+ax}}d\frac{x^2}{\sqrt{bx^4+ax}}}{3b}$$

$$\Big\downarrow \text{219}$$

$$\frac{x\sqrt{ax+bx^4}}{3b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax+bx^4}}\right)}{3b^{3/2}}$$

input
```
Int[x^4/Sqrt[a*x + b*x^4],x]
```

output
```
(x*Sqrt[a*x + b*x^4])/(3*b) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x + b*x^4]])
/(3*b^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 3.88 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{-x\sqrt{x(b\,x^3+a)}\,\sqrt{b}+\operatorname{arctanh}\left(\frac{\sqrt{x\left(b\,x^3+a\right)}}{x^2\sqrt{b}}\right)a}{3b^{\frac{3}{2}}}$	45
pseudoelliptic	$-\frac{-x\sqrt{x(b\,x^3+a)}\,\sqrt{b}+\operatorname{arctanh}\left(\frac{\sqrt{x\left(b\,x^3+a\right)}}{x^2\sqrt{b}}\right)a}{3b^{\frac{3}{2}}}$	45
risch	$\frac{x^2\left(b\,x^3+a\right)}{3b\sqrt{x(b\,x^3+a)}}-\frac{a\,\operatorname{arctanh}\left(\frac{\sqrt{x\left(b\,x^3+a\right)}}{x^2\sqrt{b}}\right)}{3b^{\frac{3}{2}}}$	53
elliptic	Expression too large to display	997

input
```
int(x^4/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/b^(3/2)*(-x*(x*(b*x^3+a))^(1/2)*b^(1/2)+arctanh((x*(b*x^3+a))^(1/2)/x
^2/b^(1/2))*a)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.42

$$\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx$$
$$= \left[\frac{4\sqrt{bx^4+ax}bx + a\sqrt{b}\log\left(-8\,b^2x^6 - 8\,abx^3 - a^2 + 4\,(2\,bx^4+ax)\sqrt{bx^4+ax}\sqrt{b}\right)}{12\,b^2}, \frac{2\sqrt{bx^4+ax}bx + a\cdots}{}\right.$$

input
```
integrate(x^4/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(4*sqrt(b*x^4 + a*x)*b*x + a*sqrt(b)*log(-8*b^2*x^6 - 8*a*b*x^3 - a^
2 + 4*(2*b*x^4 + a*x)*sqrt(b*x^4 + a*x)*sqrt(b)))/b^2, 1/6*(2*sqrt(b*x^4 +
 a*x)*b*x + a*sqrt(-b)*arctan(2*sqrt(b*x^4 + a*x)*sqrt(-b)*x/(2*b*x^3 + a)
))/b^2]
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^4}{\sqrt{x\,(a+bx^3)}}\,dx$$

input
```
integrate(x**4/(b*x**4+a*x)**(1/2),x)
```

output
```
Integral(x**4/sqrt(x*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^4}{\sqrt{bx^4+ax}}\,dx$$

input
```
integrate(x^4/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(b*x^4 + a*x), x)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx = \frac{\sqrt{bx^4+ax}\,x}{3\,b} + \frac{a\arctan\left(\frac{\sqrt{b+\frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\,\sqrt{-b}b}$$

input
```
integrate(x^4/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(b*x^4 + a*x)*x/b + 1/3*a*arctan(sqrt(b + a/x^3)/sqrt(-b))/(sqrt(-
b)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^4}{\sqrt{b\,x^4+a\,x}}\,dx$$

input
```
int(x^4/(a*x + b*x^4)^(1/2),x)
```

output
```
int(x^4/(a*x + b*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13

$$\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx$$
$$= \frac{2\sqrt{x}\,\sqrt{b\,x^3+a}\,bx+\sqrt{b}\log\left(\sqrt{b\,x^3+a}-\sqrt{x}\,\sqrt{b}\,x\right)a-\sqrt{b}\log\left(\sqrt{b\,x^3+a}+\sqrt{x}\,\sqrt{b}\,x\right)a}{6b^2}$$

input
```
int(x^4/(b*x^4+a*x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(a + b*x**3)*b*x + sqrt(b)*log(sqrt(a + b*x**3) - sqrt(x)*s
qrt(b)*x)*a - sqrt(b)*log(sqrt(a + b*x**3) + sqrt(x)*sqrt(b)*x)*a)/(6*b**2
)
```

3.100 $\int \frac{x}{\sqrt{ax+bx^4}}\,dx$

Optimal result 802
Mathematica [A] (verified) 802
Rubi [A] (verified) 803
Maple [A] (verified) 804
Fricas [A] (verification not implemented) 804
Sympy [F] 805
Maxima [F] 805
Giac [A] (verification not implemented) 805
Mupad [F(-1)] 806
Reduce [B] (verification not implemented) 806

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{x}{\sqrt{ax+bx^4}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax+bx^4}}\right)}{3\sqrt{b}}$$

output
```
2/3*arctanh(b^(1/2)*x^2/(b*x^4+a*x)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.94

$$\int \frac{x}{\sqrt{ax+bx^4}}\,dx = \frac{2\sqrt{x}\sqrt{a+bx^3}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{3\sqrt{b}\sqrt{x(a+bx^3)}}$$

input
```
Integrate[x/Sqrt[a*x + b*x^4],x]
```

output
```
(2*Sqrt[x]*Sqrt[a + b*x^3]*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]])/(3*Sqrt
[b]*Sqrt[x*(a + b*x^3)])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{ax+bx^4}}\,dx$$

$$\Big\downarrow \text{1935}$$

$$\frac{2}{3}\int \frac{1}{1-\frac{bx^4}{bx^4+ax}}d\frac{x^2}{\sqrt{bx^4+ax}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax+bx^4}}\right)}{3\sqrt{b}}$$

input
```
Int[x/Sqrt[a*x + b*x^4],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x + b*x^4]])/(3*Sqrt[b])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{x\left(b\,x^{3}+a\right)}}{x^{2}\sqrt{b}}\right)}{3\sqrt{b}}$	25
pseudoelliptic	$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{x\left(b\,x^{3}+a\right)}}{x^{2}\sqrt{b}}\right)}{3\sqrt{b}}$	25
elliptic	Expression too large to display	979

input
```
int(x/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/b^(1/2)*arctanh((x*(b*x^3+a))^(1/2)/x^2/b^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.94

$$\int \frac{x}{\sqrt{ax+bx^4}}\,dx = \left[\frac{\log\left(-8\,b^2x^6-8\,abx^3-a^2-4\,(2\,bx^4+ax)\sqrt{bx^4+ax}\sqrt{b}\right)}{6\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(\frac{2\,\sqrt{bx^4+ax}\sqrt{-b}x}{2\,bx^3+a}\right)}{3\,b}\right]$$

input
```
integrate(x/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*log(-8*b^2*x^6 - 8*a*b*x^3 - a^2 - 4*(2*b*x^4 + a*x)*sqrt(b*x^4 + a*x
)*sqrt(b))/sqrt(b), -1/3*sqrt(-b)*arctan(2*sqrt(b*x^4 + a*x)*sqrt(-b)*x/(2
*b*x^3 + a))/b]
```

Sympy [F]

$$\int \frac{x}{\sqrt{ax+bx^4}}\,dx = \int \frac{x}{\sqrt{x\left(a+bx^3\right)}}\,dx$$

input
```
integrate(x/(b*x**4+a*x)**(1/2),x)
```

output
```
Integral(x/sqrt(x*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{ax+bx^4}}\,dx = \int \frac{x}{\sqrt{bx^4+ax}}\,dx$$

input
```
integrate(x/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(b*x^4 + a*x), x)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{x}{\sqrt{ax+bx^4}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{b+\frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\sqrt{-b}}$$

input
```
integrate(x/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

output
```
-2/3*arctan(sqrt(b + a/x^3)/sqrt(-b))/sqrt(-b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{ax+bx^4}}\,dx=\int \frac{x}{\sqrt{b\,x^4+a\,x}}\,dx$$

input
```
int(x/(a*x + b*x^4)^(1/2),x)
```

output
```
int(x/(a*x + b*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.34

$$\int \frac{x}{\sqrt{ax+bx^4}}\,dx=\frac{\sqrt{b}\left(-\log\left(\sqrt{b\,x^3+a}-\sqrt{x}\,\sqrt{b}\,x\right)+\log\left(\sqrt{b\,x^3+a}+\sqrt{x}\,\sqrt{b}\,x\right)\right)}{3b}$$

input
```
int(x/(b*x^4+a*x)^(1/2),x)
```

output
```
(sqrt(b)*( - log(sqrt(a + b*x**3) - sqrt(x)*sqrt(b)*x) + log(sqrt(a + b*x*
*3) + sqrt(x)*sqrt(b)*x)))/(3*b)
```

3.101 $\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx$

Optimal result 807
Mathematica [A] (verified) 807
Rubi [A] (verified) 808
Maple [A] (verified) 808
Fricas [A] (verification not implemented) 809
Sympy [F] 810
Maxima [A] (verification not implemented) 810
Giac [A] (verification not implemented) 810
Mupad [B] (verification not implemented) 811
Reduce [B] (verification not implemented) 811

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{ax+bx^4}}{3ax^2}$$

output
```
-2/3*(b*x^4+a*x)^(1/2)/a/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{x(a+bx^3)}}{3ax^2}$$

input
```
Integrate[1/(x^2*Sqrt[a*x + b*x^4]),x]
```

output
```
(-2*Sqrt[x*(a + b*x^3)])/(3*a*x^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx$$
$$\Big\downarrow \; 1920$$
$$-\frac{2\sqrt{ax+bx^4}}{3ax^2}$$

input
```
Int[1/(x^2*Sqrt[a*x + b*x^4]),x]
```

output
```
(-2*Sqrt[a*x + b*x^4])/(3*a*x^2)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{2\sqrt{b\,x^{4}+a x}}{3 a\,x^{2}}$	20
trager	$-\frac{2\sqrt{b\,x^{4}+a x}}{3 a\,x^{2}}$	20
elliptic	$-\frac{2\sqrt{b\,x^{4}+a x}}{3 a\,x^{2}}$	20
pseudoelliptic	$-\frac{2\sqrt{x\left(b\,x^{3}+a\right)}}{3 a\,x^{2}}$	20
gosper	$-\frac{2\left(b\,x^{3}+a\right)}{3 x a \sqrt{b\,x^{4}+a x}}$	27
risch	$-\frac{2\left(b\,x^{3}+a\right)}{3 a x \sqrt{x\left(b\,x^{3}+a\right)}}$	27
orering	$-\frac{2\left(b\,x^{3}+a\right)}{3 x a \sqrt{b\,x^{4}+a x}}$	27

input
```
int(1/x^2/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x^4+a*x)^(1/2)/a/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx=-\frac{2\,\sqrt{bx^4+ax}}{3\,ax^2}$$

input
```
integrate(1/x^2/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(b*x^4 + a*x)/(a*x^2)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx = \int \frac{1}{x^2\sqrt{x\,(a+bx^3)}}\,dx$$

input
```
integrate(1/x**2/(b*x**4+a*x)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(x*(a + b*x**3))), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx = -\frac{2\,(bx^4+ax)}{3\,\sqrt{bx^3+a}ax^{\frac{5}{2}}}$$

input
```
integrate(1/x^2/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

output
```
-2/3*(b*x^4 + a*x)/(sqrt(b*x^3 + a)*a*x^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx = -\frac{2\,\sqrt{b+\frac{a}{x^3}}}{3\,a}$$

input
```
integrate(1/x^2/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(b + a/x^3)/a
```

Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{b\,x^4+a\,x}}{3\,a\,x^2}$$

input
```
int(1/(x^2*(a*x + b*x^4)^(1/2)),x)
```

output
```
-(2*(a*x + b*x^4)^(1/2))/(3*a*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{b\,x^3+a}}{3\sqrt{x}\,ax}$$

input
```
int(1/x^2/(b*x^4+a*x)^(1/2),x)
```

output
```
( - 2*sqrt(a + b*x**3))/(3*sqrt(x)*a*x)
```

3.102 $\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx$

Optimal result 812
Mathematica [A] (verified) 812
Rubi [A] (verified) 813
Maple [A] (verified) 814
Fricas [A] (verification not implemented) 814
Sympy [F] 815
Maxima [A] (verification not implemented) 815
Giac [A] (verification not implemented) 815
Mupad [B] (verification not implemented) 816
Reduce [B] (verification not implemented) 816

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{ax+bx^4}}{9ax^5}+\frac{4b\sqrt{ax+bx^4}}{9a^2x^2}$$

output
```
-2/9*(b*x^4+a*x)^(1/2)/a/x^5+4/9*b*(b*x^4+a*x)^(1/2)/a^2/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx = -\frac{2(a-2bx^3)\sqrt{x(a+bx^3)}}{9a^2x^5}$$

input
```
Integrate[1/(x^5*Sqrt[a*x + b*x^4]),x]
```

output
```
(-2*(a - 2*b*x^3)*Sqrt[x*(a + b*x^3)])/(9*a^2*x^5)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{ax+bx^4}}dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{2b\int \frac{1}{x^2\sqrt{bx^4+ax}}dx}{3a}-\frac{2\sqrt{ax+bx^4}}{9ax^5}$$

$$\Big\downarrow \text{1920}$$

$$\frac{4b\sqrt{ax+bx^4}}{9a^2x^2}-\frac{2\sqrt{ax+bx^4}}{9ax^5}$$

input
```
Int[1/(x^5*Sqrt[a*x + b*x^4]),x]
```

output
```
(-2*Sqrt[a*x + b*x^4])/(9*a*x^5) + (4*b*Sqrt[a*x + b*x^4])/(9*a^2*x^2)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.58

method	result	size
trager	$-\frac{2\left(-2b\,x^{3}+a\right)\sqrt{b\,x^{4}+a x}}{9a^{2}x^{5}}$	28
pseudoelliptic	$-\frac{2\left(-2b\,x^{3}+a\right)\sqrt{x\left(b\,x^{3}+a\right)}}{9a^{2}x^{5}}$	28
gosper	$-\frac{2\left(b\,x^{3}+a\right)\left(-2b\,x^{3}+a\right)}{9x^{4}a^{2}\sqrt{b\,x^{4}+a x}}$	35
risch	$-\frac{2\left(b\,x^{3}+a\right)\left(-2b\,x^{3}+a\right)}{9a^{2}x^{4}\sqrt{x\left(b\,x^{3}+a\right)}}$	35
orering	$-\frac{2\left(b\,x^{3}+a\right)\left(-2b\,x^{3}+a\right)}{9x^{4}a^{2}\sqrt{b\,x^{4}+a x}}$	35
default	$-\frac{2\sqrt{b\,x^{4}+a x}}{9a\,x^{5}}+\frac{4b\sqrt{b\,x^{4}+a x}}{9a^{2}x^{2}}$	41
elliptic	$-\frac{2\sqrt{b\,x^{4}+a x}}{9a\,x^{5}}+\frac{4b\sqrt{b\,x^{4}+a x}}{9a^{2}x^{2}}$	41

input
```
int(1/x^5/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(-2*b*x^3+a)/a^2/x^5*(b*x^4+a*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx = \frac{2\sqrt{bx^4+ax}(2\,bx^3-a)}{9\,a^2x^5}$$

input
```
integrate(1/x^5/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

output
```
2/9*sqrt(b*x^4 + a*x)*(2*b*x^3 - a)/(a^2*x^5)
```

Sympy [F]

$$\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx = \int \frac{1}{x^5\sqrt{x\left(a+bx^3\right)}}\,dx$$

input
```
integrate(1/x**5/(b*x**4+a*x)**(1/2),x)
```

output
```
Integral(1/(x**5*sqrt(x*(a + b*x**3))), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx = \frac{2\left(2\,b^2x^7+abx^4-a^2x\right)}{9\,\sqrt{bx^3+a}a^2x^{\frac{11}{2}}}$$

input
```
integrate(1/x^5/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

output
```
2/9*(2*b^2*x^7 + a*b*x^4 - a^2*x)/(sqrt(b*x^3 + a)*a^2*x^(11/2))
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx = -\frac{2\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}}{9\,a^2}+\frac{2\,\sqrt{b+\frac{a}{x^3}}b}{3\,a^2}$$

input
```
integrate(1/x^5/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

output
```
-2/9*(b + a/x^3)^(3/2)/a^2 + 2/3*sqrt(b + a/x^3)*b/a^2
```

Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx = -\frac{2\,\sqrt{b\,x^4+a\,x}\,\left(a-2\,b\,x^3\right)}{9\,a^2\,x^5}$$

input
```
int(1/(x^5*(a*x + b*x^4)^(1/2)),x)
```

output
```
-(2*(a*x + b*x^4)^(1/2)*(a - 2*b*x^3))/(9*a^2*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx = \frac{2\sqrt{b\,x^3+a}\,(2b\,x^3-a)}{9\sqrt{x}\,a^2x^4}$$

input
```
int(1/x^5/(b*x^4+a*x)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3)*( - a + 2*b*x**3))/(9*sqrt(x)*a**2*x**4)
```

3.103 $\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx$

Optimal result 817
Mathematica [A] (verified) 817
Rubi [A] (verified) 818
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Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{ax+bx^4}}{15ax^8} + \frac{8b\sqrt{ax+bx^4}}{45a^2x^5} - \frac{16b^2\sqrt{ax+bx^4}}{45a^3x^2}$$

output
```
-2/15*(b*x^4+a*x)^(1/2)/a/x^8+8/45*b*(b*x^4+a*x)^(1/2)/a^2/x^5-16/45*b^2*(
b*x^4+a*x)^(1/2)/a^3/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{x(a+bx^3)}(3a^2-4abx^3+8b^2x^6)}{45a^3x^8}$$

input
```
Integrate[1/(x^8*Sqrt[a*x + b*x^4]),x]
```

output
```
(-2*Sqrt[x*(a + b*x^3)]*(3*a^2 - 4*a*b*x^3 + 8*b^2*x^6))/(45*a^3*x^8)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx$$

$$\Big\downarrow \text{ 1922}$$

$$-\frac{4b\int \frac{1}{x^5\sqrt{bx^4+ax}}dx}{5a}-\frac{2\sqrt{ax+bx^4}}{15ax^8}$$

$$\Big\downarrow \text{ 1922}$$

$$-\frac{4b\left(-\frac{2b\int \frac{1}{x^2\sqrt{bx^4+ax}}dx}{3a}-\frac{2\sqrt{ax+bx^4}}{9ax^5}\right)}{5a}-\frac{2\sqrt{ax+bx^4}}{15ax^8}$$

$$\Big\downarrow \text{ 1920}$$

$$-\frac{4b\left(\frac{4b\sqrt{ax+bx^4}}{9a^2x^2}-\frac{2\sqrt{ax+bx^4}}{9ax^5}\right)}{5a}-\frac{2\sqrt{ax+bx^4}}{15ax^8}$$

input
```
Int[1/(x^8*Sqrt[a*x + b*x^4]),x]
```

output
```
(-2*Sqrt[a*x + b*x^4])/(15*a*x^8) - (4*b*((-2*Sqrt[a*x + b*x^4])/(9*a*x^5)
 + (4*b*Sqrt[a*x + b*x^4])/(9*a^2*x^2)))/(5*a)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

method	result	size
trager	$-\frac{2\left(8b^{2}x^{6}-4ab\,x^{3}+3a^{2}\right)\sqrt{b\,x^{4}+ax}}{45x^{8}a^{3}}$	41
pseudoelliptic	$-\frac{2\left(8b^{2}x^{6}-4ab\,x^{3}+3a^{2}\right)\sqrt{x\left(b\,x^{3}+a\right)}}{45x^{8}a^{3}}$	41
gosper	$-\frac{2\left(b\,x^{3}+a\right)\left(8b^{2}x^{6}-4ab\,x^{3}+3a^{2}\right)}{45x^{7}a^{3}\sqrt{b\,x^{4}+ax}}$	48
risch	$-\frac{2\left(b\,x^{3}+a\right)\left(8b^{2}x^{6}-4ab\,x^{3}+3a^{2}\right)}{45a^{3}x^{7}\sqrt{x\left(b\,x^{3}+a\right)}}$	48
orering	$-\frac{2\left(b\,x^{3}+a\right)\left(8b^{2}x^{6}-4ab\,x^{3}+3a^{2}\right)}{45x^{7}a^{3}\sqrt{b\,x^{4}+ax}}$	48
default	$-\frac{2\sqrt{b\,x^{4}+ax}}{15a\,x^{8}}+\frac{8b\sqrt{b\,x^{4}+ax}}{45a^{2}x^{5}}-\frac{16b^{2}\sqrt{b\,x^{4}+ax}}{45a^{3}x^{2}}$	63
elliptic	$-\frac{2\sqrt{b\,x^{4}+ax}}{15a\,x^{8}}+\frac{8b\sqrt{b\,x^{4}+ax}}{45a^{2}x^{5}}-\frac{16b^{2}\sqrt{b\,x^{4}+ax}}{45a^{3}x^{2}}$	63

input
```
int(1/x^8/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*(8*b^2*x^6-4*a*b*x^3+3*a^2)/x^8/a^3*(b*x^4+a*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx = -\frac{2\,(8\,b^2x^6-4\,abx^3+3\,a^2)\sqrt{bx^4+ax}}{45\,a^3x^8}$$

input
```
integrate(1/x^8/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/45*(8*b^2*x^6 - 4*a*b*x^3 + 3*a^2)*sqrt(b*x^4 + a*x)/(a^3*x^8)
```

Sympy [F]

$$\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx = \int \frac{1}{x^8\sqrt{x\,(a+bx^3)}}\,dx$$

input
```
integrate(1/x**8/(b*x**4+a*x)**(1/2),x)
```

output
```
Integral(1/(x**8*sqrt(x*(a + b*x**3))), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx = -\frac{2\,(8\,b^3x^{10}+4\,ab^2x^7-a^2bx^4+3\,a^3x)}{45\,\sqrt{bx^3+a}a^3x^{\frac{17}{2}}}$$

input
```
integrate(1/x^8/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

output
```
-2/45*(8*b^3*x^10 + 4*a*b^2*x^7 - a^2*b*x^4 + 3*a^3*x)/(sqrt(b*x^3 + a)*a^
3*x^(17/2))
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{b+\frac{a}{x^3}}b^2}{3\,a^3} - \frac{2\left(3\left(b+\frac{a}{x^3}\right)^{\frac{5}{2}} - 10\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}b\right)}{45\,a^3}$$

input
```
integrate(1/x^8/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(b + a/x^3)*b^2/a^3 - 2/45*(3*(b + a/x^3)^(5/2) - 10*(b + a/x^3)^
(3/2)*b)/a^3
```

Mupad [B] (verification not implemented)

Time = 9.37 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{b\,x^4+a\,x}\,(3\,a^2-4\,a\,b\,x^3+8\,b^2\,x^6)}{45\,a^3\,x^8}$$

input
```
int(1/(x^8*(a*x + b*x^4)^(1/2)),x)
```

output
```
-(2*(a*x + b*x^4)^(1/2)*(3*a^2 + 8*b^2*x^6 - 4*a*b*x^3))/(45*a^3*x^8)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx = \frac{2\sqrt{b\,x^3+a}\,(-8b^2x^6+4ab\,x^3-3a^2)}{45\sqrt{x}\,a^3x^7}$$

input
```
int(1/x^8/(b*x^4+a*x)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3)*( - 3*a**2 + 4*a*b*x**3 - 8*b**2*x**6))/(45*sqrt(x)*a*
*3*x**7)
```

3.104 $\int \frac{x^3}{\sqrt{ax+bx^4}} dx$

Optimal result 822
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Rubi [A] (verified) 823
Maple [C] (verified) 825
Fricas [F] 827
Sympy [F] 827
Maxima [F] 828
Giac [F] 828
Mupad [F(-1)] 828
Reduce [F] 829

Optimal result

Integrand size = 17, antiderivative size = 224

$$\int \frac{x^3}{\sqrt{ax+bx^4}} dx = \frac{\sqrt{ax+bx^4}}{2b} - \frac{a^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

output
```
1/2*(b*x^4+a*x)^(1/2)/b-1/12*a^(2/3)*x*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*Inverse
JacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/
3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/b/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(
a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^4+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.29

$$\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx=\frac{x\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)\right)}{2b\sqrt{x\left(a+bx^3\right)}}$$

input
```
Integrate[x^3/Sqrt[a*x + b*x^4],x]
```

output
```
(x*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((
b*x^3)/a)]))/(2*b*Sqrt[x*(a + b*x^3)])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1930, 1917, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{ax+bx^4}}dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{\sqrt{ax+bx^4}}{2b}-\frac{a\int \frac{1}{\sqrt{bx^4+ax}}dx}{4b}$$

$$\Big\downarrow \text{1917}$$

$$\frac{\sqrt{ax+bx^4}}{2b}-\frac{a\sqrt{x}\sqrt{a+bx^3}\int \frac{1}{\sqrt{x}\sqrt{bx^3+a}}dx}{4b\sqrt{ax+bx^4}}$$

$$\Big\downarrow \text{851}$$

$$\frac{\sqrt{ax+bx^4}}{2b}-\frac{a\sqrt{x}\sqrt{a+bx^3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b\sqrt{ax+bx^4}}$$

$$\Big\downarrow \text{766}$$

$$\frac{\sqrt{ax+bx^4}}{2b}-\frac{a^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

input
```
Int[x^3/Sqrt[a*x + b*x^4],x]
```

output
```
Sqrt[a*x + b*x^4]/(2*b) - (a^(2/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*E
llipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/3)*x*(a^(1/3) +
b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 688, normalized size of antiderivative = 3.07

method	result
default	$\frac{\sqrt{b x^4+ax}}{2b} - \frac{a\left(\frac{(-a b^2)^{\frac{1}{3}}}{2b} - \frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-a b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{(-a b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{(-a b^2)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{(-a b^2)^{\frac{1}{3}}}{b}\right)^2\sqrt{\frac{(-a b^2)^{\frac{1}{3}}\cdots}{b\left(-\frac{(-a b^2)^{\frac{1}{3}}}{2b}\cdots\right.}}\cdots}{2\left(-\frac{3(-a b^2)^{\frac{1}{3}}}{2b}+\cdots\right.}$
elliptic	$\frac{\sqrt{b x^4+ax}}{2b} - \frac{a\left(\frac{(-a b^2)^{\frac{1}{3}}}{2b} - \frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-a b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{(-a b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{(-a b^2)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{(-a b^2)^{\frac{1}{3}}}{b}\right)^2\sqrt{\frac{(-a b^2)^{\frac{1}{3}}\cdots}{b\left(-\frac{(-a b^2)^{\frac{1}{3}}}{2b}\cdots\right.}}\cdots}{2\left(-\frac{3(-a b^2)^{\frac{1}{3}}}{2b}+\cdots\right.}$
risch	$\frac{x(b x^3+a)}{2b\sqrt{x(b x^3+a)}} - \frac{a\left(\frac{(-a b^2)^{\frac{1}{3}}}{2b} - \frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-a b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{(-a b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{(-a b^2)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{(-a b^2)^{\frac{1}{3}}}{b}\right)^2\sqrt{\frac{(-a b\cdots}{b\left(-\frac{(-a\cdots}{2\cdots}\right.}}\cdots}{2\left(-\frac{3(-a b^2)\cdots}{2b}\cdots\right.}$

input
```
int(x^3/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*x^4+a*x)^(1/2)/b-1/2*a*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(
1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*x*(x-1/b*(-a*b^2)^(
1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx=\int \frac{x^3}{\sqrt{bx^4+ax}}\,dx$$

input
```
integrate(x^3/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 + a*x)*x^2/(b*x^3 + a), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx=\int \frac{x^3}{\sqrt{x\,(a+bx^3)}}\,dx$$

input
```
integrate(x**3/(b*x**4+a*x)**(1/2),x)
```

output
```
Integral(x**3/sqrt(x*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^3}{\sqrt{bx^4+ax}}\,dx$$

input
```
integrate(x^3/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(b*x^4 + a*x), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^3}{\sqrt{bx^4+ax}}\,dx$$

input
```
integrate(x^3/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(b*x^4 + a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^3}{\sqrt{b\,x^4+a\,x}}\,dx$$

input
```
int(x^3/(a*x + b*x^4)^(1/2),x)
```

output
```
int(x^3/(a*x + b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{ax + bx^4}} \, dx = \frac{2\sqrt{x}\,\sqrt{b\,x^3 + a} - \left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^4+ax} dx\right) a}{4b}$$

input
```
int(x^3/(b*x^4+a*x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(a + b*x**3) - int((sqrt(x)*sqrt(a + b*x**3))/(a*x + b*x**4
),x)*a)/(4*b)
```

3.105 $\int \frac{1}{\sqrt{ax+bx^4}}\,dx$

Optimal result 830
Mathematica [C] (verified) 831
Rubi [A] (verified) 831
Maple [C] (verified) 833
Fricas [A] (verification not implemented) 834
Sympy [F] 834
Maxima [F] 834
Giac [A] (verification not implemented) 835
Mupad [B] (verification not implemented) 835
Reduce [F] 835

Optimal result

Integrand size = 13, antiderivative size = 197

$$\int \frac{1}{\sqrt{ax+bx^4}}\,dx = \frac{x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

output
```
1/3*x*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3
)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/
2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3
^(3/4)/a^(1/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)
*x)^2)^(1/2)/(b*x^4+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.25

$$\int \frac{1}{\sqrt{ax+bx^4}}\,dx=\frac{2x\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)}{\sqrt{x\left(a+bx^3\right)}}$$

input
```
Integrate[1/Sqrt[a*x + b*x^4],x]
```

output
```
(2*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)])/S
qrt[x*(a + b*x^3)]
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1917, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax+bx^4}}dx$$

$$\Big\downarrow \text{1917}$$

$$\frac{\sqrt{x}\sqrt{a+bx^3}\int \frac{1}{\sqrt{x}\sqrt{bx^3+a}}dx}{\sqrt{ax+bx^4}}$$

$$\Big\downarrow \text{851}$$

$$\frac{2\sqrt{x}\sqrt{a+bx^3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{\sqrt{ax+bx^4}}$$

$$\Big\downarrow \text{766}$$

$$\frac{x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

input
```
Int[1/Sqrt[a*x + b*x^4],x]
```

output
```
(x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sq
rt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/
(3^(1/4)*a^(1/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sq
rt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 671, normalized size of antiderivative = 3.41

method	result
default	$\frac{2\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\,\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2}\sqrt{\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\right.}{b\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right.}}}{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)}$
elliptic	$\frac{2\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\,\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2}\sqrt{\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\right.}{b\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right.}}}{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)}$

input

```
int(1/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^
2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)
^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x
-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*b/(-a*b^2)^(1/3)/(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/
(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.08

$$\int \frac{1}{\sqrt{ax+bx^4}}\,dx = -\frac{2\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)}{\sqrt{a}}$$

input
```
integrate(1/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2*weierstrassPInverse(0, -4*b/a, 1/x)/sqrt(a)
```

Sympy [F]

$$\int \frac{1}{\sqrt{ax+bx^4}}\,dx = \int \frac{1}{\sqrt{ax+bx^4}}\,dx$$

input
```
integrate(1/(b*x**4+a*x)**(1/2),x)
```

output
```
Integral(1/sqrt(a*x + b*x**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{ax+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+ax}}\,dx$$

input
```
integrate(1/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*x^4 + a*x), x)
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.20

$$\int \frac{1}{\sqrt{ax+bx^4}}\,dx = \frac{1}{3}\sqrt{bx^4+ax}\,x - \frac{a\arctan\left(\frac{\sqrt{b+\frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\sqrt{-b}}$$

input
```
integrate(1/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(b*x^4 + a*x)*x - 1/3*a*arctan(sqrt(b + a/x^3)/sqrt(-b))/sqrt(-b)
```

Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.20

$$\int \frac{1}{\sqrt{ax+bx^4}}\,dx = \frac{2\,x\,\sqrt{\frac{b\,x^3}{a}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{6},\frac{1}{2};\ \frac{7}{6};\ -\frac{b\,x^3}{a}\right)}{\sqrt{b\,x^4+a\,x}}$$

input
```
int(1/(a*x + b*x^4)^(1/2),x)
```

output
```
(2*x*((b*x^3)/a + 1)^(1/2)*hypergeom([1/6, 1/2], 7/6, -(b*x^3)/a))/(a*x +
b*x^4)^(1/2)
```

Reduce [F]

$$\int \frac{1}{\sqrt{ax+bx^4}}\,dx = \int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^4+ax}dx$$

input
```
int(1/(b*x^4+a*x)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(a + b*x**3))/(a*x + b*x**4),x)
```

3.106 $\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx$

Optimal result . . . 836
Mathematica [C] (verified) . . . 837
Rubi [A] (verified) . . . 837
Maple [C] (verified) . . . 839
Fricas [A] (verification not implemented) . . . 841
Sympy [F] . . . 842
Maxima [F] . . . 842
Giac [F] . . . 842
Mupad [F(-1)] . . . 843
Reduce [F] . . . 843

Optimal result

Integrand size = 17, antiderivative size = 225

$$\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx=-\frac{2\sqrt{ax+bx^4}}{5ax^3}-\frac{2bx\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

output
```
-2/5*(b*x^4+a*x)^(1/2)/a/x^3-2/15*b*x*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJ
acobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3
)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(4/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)
*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^4+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.24

$$\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},-\frac{bx^3}{a}\right)}{5x^2\sqrt{x(a+bx^3)}}$$

input
```
Integrate[1/(x^3*Sqrt[a*x + b*x^4]),x]
```

output
```
(-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-5/6, 1/2, 1/6, -((b*x^3)/a)])/(
5*x^2*Sqrt[x*(a + b*x^3)])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1931, 1917, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{ax+bx^4}}dx$$

$$\Big\downarrow \text{ 1931}$$

$$-\frac{2b\int \frac{1}{\sqrt{bx^4+ax}}dx}{5a}-\frac{2\sqrt{ax+bx^4}}{5ax^3}$$

$$\Big\downarrow \text{ 1917}$$

$$-\frac{2b\sqrt{x}\sqrt{a+bx^3}\int \frac{1}{\sqrt{x}\sqrt{bx^3+a}}dx}{5a\sqrt{ax+bx^4}}-\frac{2\sqrt{ax+bx^4}}{5ax^3}$$

$$\Big\downarrow \text{ 851}$$

$$-\frac{4b\sqrt{x}\sqrt{a+bx^3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{5a\sqrt{ax+bx^4}}-\frac{2\sqrt{ax+bx^4}}{5ax^3}$$

$$\downarrow \text{766}$$

$$-\frac{2bx\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}-\frac{2\sqrt{ax+bx^4}}{5ax^3}$$

input
```
Int[1/(x^3*Sqrt[a*x + b*x^4]),x]
```

output
```
(-2*Sqrt[a*x + b*x^4])/(5*a*x^3) - (2*b*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + S
qrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(5*3^(1/4)*a^(4/3)*Sqrt[(b^(1/3)*x*
(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b
*x^4])
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 696, normalized size of antiderivative = 3.09

method	result
default	$-\frac{2\sqrt{b x^{4}+a x}}{5 a x^{3}}-\frac{4 b^{2}\left(\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\,\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}\right)\sqrt{\frac{\left(-\frac{3\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\,\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}\right) x}{\left(-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\,\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}\right)\left(x-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\,\left(x-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2}\sqrt{\frac{\left(-a \cdots\right.}{b\left(-\frac{\left(-\cdots\right.}{}\right.}}\cdots}{5 a\left(-\frac{3\left(-a b^{2}\cdots\right.}{2 b}\cdots\right.}$
elliptic	$-\frac{2\sqrt{b x^{4}+a x}}{5 a x^{3}}-\frac{4 b^{2}\left(\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\,\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}\right)\sqrt{\frac{\left(-\frac{3\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\,\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}\right) x}{\left(-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\,\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}\right)\left(x-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\,\left(x-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2}\sqrt{\frac{\left(-a \cdots\right.}{b\left(-\frac{\left(-\cdots\right.}{}\right.}}\cdots}{5 a\left(-\frac{3\left(-a b^{2}\cdots\right.}{2 b}\cdots\right.}$
risch	$-\frac{2\left(b x^{3}+a\right)}{5 a x^{2}\sqrt{x\left(b x^{3}+a\right)}}-\frac{4 b^{2}\left(\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\,\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}\right)\sqrt{\frac{\left(-\frac{3\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\,\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}\right) x}{\left(-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\,\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}\right)\left(x-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\,\left(x-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2}\sqrt{\frac{-\cdots}{b \cdots}}\cdots}{5 a\left(-\frac{3\cdots}{}\cdots\right.}$

input
```
int(1/x^3/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(b*x^4+a*x)^(1/2)/a/x^3-4/5*b^2/a*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^
(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*x*(x-1/b*(
-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.19

$$\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx = \frac{2\left(2\sqrt{a}bx^3\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)-\sqrt{bx^4+ax}a\right)}{5a^2x^3}$$

input
```
integrate(1/x^3/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(2*sqrt(a)*b*x^3*weierstrassPInverse(0, -4*b/a, 1/x) - sqrt(b*x^4 + a*
x)*a)/(a^2*x^3)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx=\int \frac{1}{x^3\sqrt{x\,(a+bx^3)}}\,dx$$

input
```
integrate(1/x**3/(b*x**4+a*x)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(x*(a + b*x**3))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx=\int \frac{1}{\sqrt{bx^4+ax}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^4 + a*x)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx=\int \frac{1}{\sqrt{bx^4+ax}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^4 + a*x)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx=\int \frac{1}{x^3\,\sqrt{b\,x^4+a\,x}}\,dx$$

input
```
int(1/(x^3*(a*x + b*x^4)^(1/2)),x)
```

output
```
int(1/(x^3*(a*x + b*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx=\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^7+a\,x^4}dx$$

input
```
int(1/x^3/(b*x^4+a*x)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(a + b*x**3))/(a*x**4 + b*x**7),x)
```

3.107 $\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx$

Optimal result 844
Mathematica [C] (verified) 845
Rubi [A] (verified) 845
Maple [C] (verified) 849
Fricas [F] 850
Sympy [F] 851
Maxima [F] 851
Giac [F] 851
Mupad [F(-1)] 852
Reduce [F] 852

Optimal result

Integrand size = 17, antiderivative size = 503

$$\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx = -\frac{5\left(1+\sqrt{3}\right)ax(a+bx^3)}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax+bx^4}}+\frac{x^2\sqrt{ax+bx^4}}{4b}$$
$$+\frac{5\sqrt[4]{3}a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{8b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$
$$+\frac{5\left(1-\sqrt{3}\right)a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{16\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

output
```
-5/8*(1+3^(1/2))*a*x*(b*x^3+a)/b^(5/3)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)/(b*
x^4+a*x)^(1/2)+1/4*x^2*(b*x^4+a*x)^(1/2)/b+5/8*3^(1/4)*a^(4/3)*x*(a^(1/3)+
b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b
^(1/3)*x)^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)*x)^2/(a^(1/3)
+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/b^(5/3)/(b^(1/3)
*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^4+a*x
)^(1/2)+5/48*(1-3^(1/2))*a^(4/3)*x*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJaco
biAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x
)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/b^(5/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)
/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^4+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.13

$$\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx = \frac{x^3\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)\right)}{4b\sqrt{x\left(a+bx^3\right)}}$$

input
```
Integrate[x^5/Sqrt[a*x + b*x^4],x]
```

output
```
(x^3*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/6,
-((b*x^3)/a)]))/(4*b*Sqrt[x*(a + b*x^3)])
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {1930, 1938, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx$$

$\big\downarrow$ 1930

$$\frac{x^2\sqrt{ax+bx^4}}{4b}-\frac{5a\int \frac{x^2}{\sqrt{bx^4+ax}}dx}{8b}$$

$\big\downarrow$ 1938

$$\frac{x^2\sqrt{ax+bx^4}}{4b}-\frac{5a\sqrt{x}\sqrt{a+bx^3}\int \frac{x^{3/2}}{\sqrt{bx^3+a}}dx}{8b\sqrt{ax+bx^4}}$$

$\big\downarrow$ 851

$$\frac{x^2\sqrt{ax+bx^4}}{4b}-\frac{5a\sqrt{x}\sqrt{a+bx^3}\int \frac{x^2}{\sqrt{bx^3+a}}d\sqrt{x}}{4b\sqrt{ax+bx^4}}$$

$\big\downarrow$ 837

$$\frac{x^2\sqrt{ax+bx^4}}{4b}-\frac{5a\sqrt{x}\sqrt{a+bx^3}\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{4b\sqrt{ax+bx^4}}$$

$\big\downarrow$ 25

$$\frac{x^2\sqrt{ax+bx^4}}{4b}-\frac{5a\sqrt{x}\sqrt{a+bx^3}\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{4b\sqrt{ax+bx^4}}$$

$\big\downarrow$ 766

$$\frac{x^2\sqrt{ax+bx^4}}{4b}-$$

$$\frac{5a\sqrt{x}\sqrt{a+bx^3}\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{4b\sqrt{ax+bx^4}}$$

$\big\downarrow$ 2420

$$\frac{x^2\sqrt{ax+bx^4}}{4b} - \frac{5a\sqrt{x}\sqrt{a+bx^3}\left(\frac{\frac{\left(1+\sqrt{3}\right)\sqrt{x}\sqrt{a+bx^3}}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x} - \frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{2b^{2/3}} - \cdots\right)}{4b\sqrt{ax+bx^4}}$$

input
```
Int[x^5/Sqrt[a*x + b*x^4],x]
```

output
```
(x^2*Sqrt[a*x + b*x^4])/(4*b) - (5*a*Sqrt[x]*Sqrt[a + b*x^3]*((((1 + Sqrt[
3])*Sqrt[x]*Sqrt[a + b*x^3])/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x) - (3^(1/4
)*a^(1/3)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^
(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2
+ Sqrt[3])/4])/(Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqr
t[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)
*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (
1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3]
)/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) +
 (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(4*b*Sqrt[a*x + b*x^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 1079, normalized size of antiderivative = 2.15

method	result	size
default	Expression too large to display	1079
elliptic	Expression too large to display	1079
risch	Expression too large to display	1086

input
```
int(x^5/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2*(b*x^4+a*x)^(1/2)/b-5/8*a/b*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))
^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2
)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(
x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*...
```

Fricas [F]

$$\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^5}{\sqrt{bx^4+ax}}\,dx$$

input
```
integrate(x^5/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 + a*x)*x^4/(b*x^3 + a), x)
```

Sympy [F]

$$\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^5}{\sqrt{x\,(a+bx^3)}}\,dx$$

input
```
integrate(x**5/(b*x**4+a*x)**(1/2),x)
```

output
```
Integral(x**5/sqrt(x*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^5}{\sqrt{bx^4+ax}}\,dx$$

input
```
integrate(x^5/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^5/sqrt(b*x^4 + a*x), x)
```

Giac [F]

$$\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^5}{\sqrt{bx^4+ax}}\,dx$$

input
```
integrate(x^5/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^5/sqrt(b*x^4 + a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx=\int \frac{x^5}{\sqrt{b\,x^4+a\,x}}\,dx$$

input
```
int(x^5/(a*x + b*x^4)^(1/2),x)
```

output
```
int(x^5/(a*x + b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx=\frac{2\sqrt{x}\,\sqrt{b\,x^3+a}\,x^2-5\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)a}{8b}$$

input
```
int(x^5/(b*x^4+a*x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(a + b*x**3)*x**2 - 5*int((sqrt(x)*sqrt(a + b*x**3)*x)/(a +
 b*x**3),x)*a)/(8*b)
```

3.108 $\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx$

Optimal result . . . 853
Mathematica [C] (verified) . . . 854
Rubi [A] (verified) . . . 854
Maple [C] (verified) . . . 858
Fricas [F] . . . 859
Sympy [F] . . . 859
Maxima [F] . . . 859
Giac [F] . . . 860
Mupad [F(-1)] . . . 860
Reduce [F] . . . 860

Optimal result

Integrand size = 17, antiderivative size = 474

$$\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx=\frac{\left(1+\sqrt{3}\right)x(a+bx^3)}{b^{2/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax+bx^4}}$$

$$-\frac{\sqrt[4]{3}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

output
```
(1+3^(1/2))*x*(b*x^3+a)/b^(2/3)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)/(b*x^4+a*x
)^(1/2)-3^(1/4)*a^(1/3)*x*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*EllipticE((1-(a^(1/3
)+(1-3^(1/2))*b^(1/3)*x)^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2),1/4*6^
(1/2)+1/4*2^(1/2))/b^(2/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1
/2))*b^(1/3)*x)^2)^(1/2)/(b*x^4+a*x)^(1/2)-1/6*(1-3^(1/2))*a^(1/3)*x*(a^(1
/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2
))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)
*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/b^(2
/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2
)/(b*x^4+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.11

$$\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx=\frac{2x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)}{5\sqrt{x\left(a+bx^3\right)}}$$

input
```
Integrate[x^2/Sqrt[a*x + b*x^4],x]
```

output
```
(2*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/6, -((b*x^3)/a)]
)/(5*Sqrt[x*(a + b*x^3)])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.353$, Rules used = {1938, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx$$

$$\Big\downarrow \text{ 1938}$$

$$\frac{\sqrt{x}\sqrt{a+bx^3}\int \frac{x^{3/2}}{\sqrt{bx^3+a}}dx}{\sqrt{ax+bx^4}}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{2\sqrt{x}\sqrt{a+bx^3}\int \frac{x^2}{\sqrt{bx^3+a}}d\sqrt{x}}{\sqrt{ax+bx^4}}$$

$$\Big\downarrow \text{ 837}$$

$$\frac{2\sqrt{x}\sqrt{a+bx^3}\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{\sqrt{ax+bx^4}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2\sqrt{x}\sqrt{a+bx^3}\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{\sqrt{ax+bx^4}}$$

$$\Big\downarrow \text{ 766}$$

$$\frac{2\sqrt{x}\sqrt{a+bx^3}\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\cdots}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\cdots}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{\sqrt{ax+bx^4}}$$

$$\Big\downarrow \text{ 2420}$$

$$\frac{2\sqrt{x}\sqrt{a+bx^3}\left(\frac{\frac{(1+\sqrt{3})\sqrt{x}\sqrt{a+bx^3}}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}-\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{(1-\sqrt{3})\sqrt[3]{b}x+\sqrt[3]{a}}{(1+\sqrt{3})\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{2b^{2/3}} - \cdots\right)}{\sqrt{ax+bx^4}}$$

input

```
Int[x^2/Sqrt[a*x + b*x^4],x]
```

output

```
(2*Sqrt[x]*Sqrt[a + b*x^3]*((((1 + Sqrt[3])*Sqrt[x]*Sqrt[a + b*x^3])/(a^(1
/3) + (1 + Sqrt[3])*b^(1/3)*x) - (3^(1/4)*a^(1/3)*Sqrt[x]*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqr
t[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(
a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*x*(a
^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3
]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(
1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) +
 (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(
1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[
a + b*x^3])))/Sqrt[a*x + b*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 1054, normalized size of antiderivative = 2.22

method	result	size
default	Expression too large to display	1054
elliptic	Expression too large to display	1054

input
```
int(x^2/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2
)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(
-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-
a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b
^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^
(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a...
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^2}{\sqrt{bx^4+ax}}\,dx$$

input
```
integrate(x^2/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 + a*x)*x/(b*x^3 + a), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^2}{\sqrt{x\,(a+bx^3)}}\,dx$$

input
```
integrate(x**2/(b*x**4+a*x)**(1/2),x)
```

output
```
Integral(x**2/sqrt(x*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^2}{\sqrt{bx^4+ax}}\,dx$$

input
```
integrate(x^2/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(b*x^4 + a*x), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^2}{\sqrt{bx^4+ax}}\,dx$$

input
```
integrate(x^2/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(b*x^4 + a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^2}{\sqrt{b\,x^4+a\,x}}\,dx$$

input
```
int(x^2/(a*x + b*x^4)^(1/2),x)
```

output
```
int(x^2/(a*x + b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx = \int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx$$

input
```
int(x^2/(b*x^4+a*x)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(a + b*x**3)*x)/(a + b*x**3),x)
```

3.109 $\int \frac{1}{x\sqrt{ax+bx^4}}\,dx$

Optimal result 861
Mathematica [C] (verified) 862
Rubi [A] (verified) 862
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Sympy [F] 868
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Reduce [F] 869

Optimal result

Integrand size = 17, antiderivative size = 497

$$\int \frac{1}{x\sqrt{ax+bx^4}}\,dx = \frac{2\left(1+\sqrt{3}\right)\sqrt[3]{b}x(a+bx^3)}{a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax+bx^4}} - \frac{2\sqrt{ax+bx^4}}{ax}$$
$$- \frac{2\sqrt[4]{3}\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$
$$- \frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

output
```
2*(1+3^(1/2))*b^(1/3)*x*(b*x^3+a)/a/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)/(b*x^4
+a*x)^(1/2)-2*(b*x^4+a*x)^(1/2)/a/x-2*3^(1/4)*b^(1/3)*x*(a^(1/3)+b^(1/3)*x
)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)
^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)*x)^2/(a^(1/3)+(1+3^(1/
2))*b^(1/3)*x)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/a^(2/3)/(b^(1/3)*x*(a^(1/
3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^4+a*x)^(1/2)-1
/3*(1-3^(1/2))*b^(1/3)*x*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b
^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arcco
s((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(
1/2)+1/4*2^(1/2))*3^(3/4)/a^(2/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+
(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^4+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.10

$$\int \frac{1}{x\sqrt{ax+bx^4}}\,dx=-\frac{2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},-\frac{bx^3}{a}\right)}{\sqrt{x\left(a+bx^3\right)}}$$

input
```
Integrate[1/(x*Sqrt[a*x + b*x^4]),x]
```

output
```
(-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/6, 1/2, 5/6, -((b*x^3)/a)])/S
qrt[x*(a + b*x^3)]
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {1931, 1938, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{ax+bx^4}}dx$$

$\big\downarrow$ 1931

$$\frac{2b\int \frac{x^2}{\sqrt{bx^4+ax}}dx}{a}-\frac{2\sqrt{ax+bx^4}}{ax}$$

$\big\downarrow$ 1938

$$\frac{2b\sqrt{x}\sqrt{a+bx^3}\int \frac{x^{3/2}}{\sqrt{bx^3+a}}dx}{a\sqrt{ax+bx^4}}-\frac{2\sqrt{ax+bx^4}}{ax}$$

$\big\downarrow$ 851

$$\frac{4b\sqrt{x}\sqrt{a+bx^3}\int \frac{x^2}{\sqrt{bx^3+a}}d\sqrt{x}}{a\sqrt{ax+bx^4}}-\frac{2\sqrt{ax+bx^4}}{ax}$$

$\big\downarrow$ 837

$$\frac{4b\sqrt{x}\sqrt{a+bx^3}\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{a\sqrt{ax+bx^4}}-\frac{2\sqrt{ax+bx^4}}{ax}$$

$\big\downarrow$ 25

$$\frac{4b\sqrt{x}\sqrt{a+bx^3}\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{a\sqrt{ax+bx^4}}-\frac{2\sqrt{ax+bx^4}}{ax}$$

$\big\downarrow$ 766

$$\frac{4b\sqrt{x}\sqrt{a+bx^3}\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}{\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}}\cdots\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\cdots\right)}{a\sqrt{ax+bx^4}}\cdots$$

$$\frac{2\sqrt{ax+bx^4}}{ax}$$

$\big\downarrow$ 2420

$$\frac{4b\sqrt{x}\sqrt{a+bx^3}\left(\frac{\frac{(1+\sqrt{3})\sqrt{x}\sqrt{a+bx^3}}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}-\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{(1-\sqrt{3})\sqrt[3]{b}x+\sqrt[3]{a}}{(1+\sqrt{3})\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{2b^{2/3}}-\right.}{a\sqrt{ax+bx^4}}$$

$$\frac{2\sqrt{ax+bx^4}}{ax}$$

input
```
Int[1/(x*Sqrt[a*x + b*x^4]),x]
```

output
```
(-2*Sqrt[a*x + b*x^4])/(a*x) + (4*b*Sqrt[x]*Sqrt[a + b*x^3]*((((1 + Sqrt[3
])*Sqrt[x]*Sqrt[a + b*x^3])/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x) - (3^(1/4)
*a^(1/3)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(
1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 +
 Sqrt[3])/4])/(Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt
[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*
Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1
 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])
/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) +
(1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(a*Sqrt[a*x + b*x^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 1083, normalized size of antiderivative = 2.18

method	result	size
default	Expression too large to display	1083
risch	Expression too large to display	1083
elliptic	Expression too large to display	1083

input
```
int(1/x/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(b*x^3+a)/a/(x*(b*x^3+a))^(1/2)+2*b/a*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(
1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-
a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/
(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1...
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.05

$$\int \frac{1}{x\sqrt{ax+bx^4}}\,dx = \frac{2\,\text{weierstrassZeta}\left(0,-\frac{4\,b}{a},\text{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)\right)}{\sqrt{a}}$$

input
```
integrate(1/x/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

output
```
2*weierstrassZeta(0, -4*b/a, weierstrassPInverse(0, -4*b/a, 1/x))/sqrt(a)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{ax+bx^4}}\,dx = \int \frac{1}{x\sqrt{x\left(a+bx^3\right)}}\,dx$$

input
```
integrate(1/x/(b*x**4+a*x)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(x*(a + b*x**3))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{ax+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+ax}x}\,dx$$

input
```
integrate(1/x/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^4 + a*x)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{ax+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+ax}x}\,dx$$

input
```
integrate(1/x/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^4 + a*x)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{ax+bx^4}}\,dx=\int \frac{1}{x\,\sqrt{b\,x^4+a\,x}}\,dx$$

input
```
int(1/(x*(a*x + b*x^4)^(1/2)),x)
```

output
```
int(1/(x*(a*x + b*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{ax+bx^4}}\,dx=\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^5+a\,x^2}dx$$

input
```
int(1/x/(b*x^4+a*x)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(a + b*x**3))/(a*x**2 + b*x**5),x)
```

3.110 $\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx$

Optimal result 870
Mathematica [C] (verified) 871
Rubi [A] (warning: unable to verify) 871
Maple [A] (verified) 884
Fricas [F] 885
Sympy [F] 885
Maxima [F] 886
Giac [F] 886
Mupad [F(-1)] 886
Reduce [F] 887

Optimal result

Integrand size = 19, antiderivative size = 301

$$\begin{aligned}\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx &= -\frac{884b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^6}+\frac{884b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{24035a^5} \\ &\quad -\frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4}+\frac{476b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^3} \\ &\quad -\frac{28b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^2}+\frac{4bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a}+\frac{2}{9}x^4\sqrt{b\sqrt[3]{x}+ax} \\ &\quad +\frac{442b^{27/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{14421a^{25/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

output
```
-884/14421*b^6*(b*x^(1/3)+a*x)^(1/2)/a^6+884/24035*b^5*x^(2/3)*(b*x^(1/3)+
a*x)^(1/2)/a^5-6188/216315*b^4*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a^4+476/19665
*b^3*x^2*(b*x^(1/3)+a*x)^(1/2)/a^3-28/1311*b^2*x^(8/3)*(b*x^(1/3)+a*x)^(1/
2)/a^2+4/207*b*x^(10/3)*(b*x^(1/3)+a*x)^(1/2)/a+2/9*x^4*(b*x^(1/3)+a*x)^(1
/2)+442/14421*b^(27/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a
^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*InverseJacobiAM(2*arctan(a^(1/4)*x^(1/6)/
b^(1/4)),1/2*2^(1/2))/a^(25/4)/(b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.51

$$\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx$$
$$=\frac{2\sqrt{b\sqrt[3]{x}+ax}\left(\sqrt{1+\frac{ax^{2/3}}{b}}\left(-9945b^6+3978ab^5x^{2/3}-3094a^2b^4x^{4/3}+2618a^3b^3x^2-2310a^4b^2x^{8/3}+2090\right.\right.}{216315a^6\sqrt{1+\frac{ax^{2/3}}{b}}}$$

input
```
Integrate[x^3*Sqrt[b*x^(1/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(1/3) + a*x]*(Sqrt[1 + (a*x^(2/3))/b]*(-9945*b^6 + 3978*a*b^5*
x^(2/3) - 3094*a^2*b^4*x^(4/3) + 2618*a^3*b^3*x^2 - 2310*a^4*b^2*x^(8/3) +
 2090*a^5*b*x^(10/3) + 24035*a^6*x^4) + 9945*b^6*Hypergeometric2F1[-1/2, 1
/4, 5/4, -((a*x^(2/3))/b)]))/(216315*a^6*Sqrt[1 + (a*x^(2/3))/b])
```

Rubi [A] (warning: unable to verify)

Time = 1.02 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {1924, 1927, 1930, 1930, 1930, 1930, 1930, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{ax+b\sqrt[3]{x}}\,dx$$

$$\Big\downarrow \text{1924}$$

$$3\int x^{11/3}\sqrt{\sqrt[3]{x}b+ax}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1927}$$

$$3\left(\frac{2}{27}b\int \frac{x^4}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}+\frac{2}{27}x^4\sqrt{ax+b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2}{27}b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{21b\int\frac{x^{10/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{23a}\right)+\frac{2}{27}x^4\sqrt{ax+b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2}{27}b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\int\frac{x^{8/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{19a}\right)}{23a}\right)+\frac{2}{27}x^4\sqrt{ax+b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2}{27}b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\int\frac{x^2}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{15a}\right)}{19a}\right)}{23a}\right)+\frac{2}{27}x^4\sqrt{ax+\right.$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2}{27}b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^{2}\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\int\frac{x^{4/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{11a}\right)}{15a}\right)}{19a}\right)}{23a}\right.\right.$$

↓ 1930

$$3\left(\frac{2}{27}b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^{2}\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\int \cdots}{\cdots}\right.}{11a}\right.}{15a}\right.}{19a}\right.}{23a}\right.\right.$$

↓ 1930

$$5b \left(2 \right.$$

$$9b \left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a} - \right.$$

$$13b \left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a} - \right.$$

$$17b \left(\frac{2x^{2}\sqrt{ax+b\sqrt[3]{x}}}{15a} - \frac{\quad}{15a} \right.$$

$$21b \left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a} - \frac{\quad}{19a} \right.$$

↓ 1917

$$5b\left(\frac{2}{-}\right.$$

$$9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\right.$$

$$13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\right.$$

$$17b\left(\frac{2x^{2}\sqrt{ax+b\sqrt[3]{x}}}{15a}-\right.$$

$$21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{\quad}{19a}\right.$$

$$3\left(\frac{2}{27}b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{\quad}{23a}\right.\right.$$

↓ 266

$$3\left(\frac{2}{27}b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^{2}\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{19a}\right.}{23a}\right.\right.$$

↓ 761

$$5b\left(\frac{2}{}\right.$$

$$9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\right.$$

$$13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\right.$$

$$17b\left(\frac{2x^{2}\sqrt{ax+b\sqrt[3]{x}}}{15a}-\right.$$

$$21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\right.$$

input
```
Int[x^3*Sqrt[b*x^(1/3) + a*x],x]
```

output
```
3*((2*x^4*Sqrt[b*x^(1/3) + a*x])/27 + (2*b*((2*x^(10/3)*Sqrt[b*x^(1/3) + a
*x])/(23*a) - (21*b*((2*x^(8/3)*Sqrt[b*x^(1/3) + a*x])/(19*a) - (17*b*((2*
x^2*Sqrt[b*x^(1/3) + a*x])/(15*a) - (13*b*((2*x^(4/3)*Sqrt[b*x^(1/3) + a*x
])/(11*a) - (9*b*((2*x^(2/3)*Sqrt[b*x^(1/3) + a*x])/(7*a) - (5*b*((2*Sqrt[
b*x^(1/3) + a*x])/(3*a) - (b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b + a*
x^(2/3)]*x^(1/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*Ellip
ticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[b*x^(1/3)
+ a*x]*Sqrt[b + a*x^(4/3)])))/(7*a)))/(11*a)))/(15*a)))/(19*a)))/(23*a)))/
27)
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{2x^4\sqrt{b\,x^{\frac{1}{3}}+ax}}{9}+\frac{4b\,x^{\frac{10}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{207a}-\frac{28b^2x^{\frac{8}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1311a^2}+\frac{476b^3x^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{19665a^3}-\frac{6188b^4x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{216315a^4}+$
default	$\frac{2x^4\sqrt{b\,x^{\frac{1}{3}}+ax}}{9}+\frac{4b\,x^{\frac{10}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{207a}-\frac{28b^2x^{\frac{8}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1311a^2}+\frac{476b^3x^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{19665a^3}-\frac{6188b^4x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{216315a^4}+$

input
```
int(x^3*(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*x^4*(b*x^(1/3)+a*x)^(1/2)+4/207*b*x^(10/3)*(b*x^(1/3)+a*x)^(1/2)/a-28/
1311*b^2*x^(8/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+476/19665*b^3*x^2*(b*x^(1/3)+a*
x)^(1/2)/a^3-6188/216315*b^4*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a^4+884/24035*b
^5*x^(2/3)*(b*x^(1/3)+a*x)^(1/2)/a^5-884/14421*b^6*(b*x^(1/3)+a*x)^(1/2)/a
^6+442/14421*b^7/a^7*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/
2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(
-a*b)^(1/2)*a)^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^
(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Fricas [F]

$$\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx=\int\sqrt{ax+bx^{\frac{1}{3}}}x^3\,dx$$

input
```
integrate(x^3*(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*x + b*x^(1/3))*x^3, x)
```

Sympy [F]

$$\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx=\int x^3\sqrt{ax+b\sqrt[3]{x}}\,dx$$

input
```
integrate(x**3*(b*x**(1/3)+a*x)**(1/2),x)
```

output
```
Integral(x**3*sqrt(a*x + b*x**(1/3)), x)
```

Maxima [F]

$$\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx=\int \sqrt{ax+bx^{\frac{1}{3}}}x^3\,dx$$

input
```
integrate(x^3*(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))*x^3, x)
```

Giac [F]

$$\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx=\int \sqrt{ax+bx^{\frac{1}{3}}}x^3\,dx$$

input
```
integrate(x^3*(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx=\int x^3\,\sqrt{a\,x+b\,x^{1/3}}\,dx$$

input
```
int(x^3*(a*x + b*x^(1/3))^(1/2),x)
```

output
```
int(x^3*(a*x + b*x^(1/3))^(1/2), x)
```

Reduce [F]

$$\int x^3 \sqrt{b\sqrt[3]{x} + ax}\, dx$$

$$= \frac{-\frac{28x^{\frac{17}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a^4b^2}{1311} + \frac{884x^{\frac{5}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a\,b^5}{24035} + \frac{4\sqrt{x}\,\sqrt{x^{\frac{2}{3}}a+b}\,a^5b\,x^3}{207} - \frac{6188\sqrt{x}\,\sqrt{x^{\frac{2}{3}}a+b}\,a^2b^4x}{216315} + \frac{2x^{\frac{25}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a^6}{9} + \frac{476x^{\frac{13}{6}}\sqrt{x}}{196}}{a^6}$$

input
```
int(x^3*(b*x^(1/3)+a*x)^(1/2),x)
```

output
```
(2*( - 2310*x**(5/6)*sqrt(x**(2/3)*a + b)*a**4*b**2*x**2 + 3978*x**(5/6)*s
qrt(x**(2/3)*a + b)*a*b**5 + 2090*sqrt(x)*sqrt(x**(2/3)*a + b)*a**5*b*x**3
 - 3094*sqrt(x)*sqrt(x**(2/3)*a + b)*a**2*b**4*x + 24035*x**(1/6)*sqrt(x**
(2/3)*a + b)*a**6*x**4 + 2618*x**(1/6)*sqrt(x**(2/3)*a + b)*a**3*b**3*x**2
 - 6630*x**(1/6)*sqrt(x**(2/3)*a + b)*b**6 + 1105*int(sqrt(x**(2/3)*a + b)
/(x**(5/6)*b + sqrt(x)*a*x),x)*b**7))/(216315*a**6)
```

3.111 $\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx$

Optimal result 888
Mathematica [C] (verified) 889
Rubi [A] (warning: unable to verify) 890
Maple [A] (verified) 904
Fricas [F] 905
Sympy [F] 905
Maxima [F] 905
Giac [F] 906
Mupad [F(-1)] 906
Reduce [F] 906

Optimal result

Integrand size = 19, antiderivative size = 411

$$\begin{aligned}
&\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx\\
&=\frac{44b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221a^{9/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{44b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^4}+\frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3}\\
&-\frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2}+\frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a}+\frac{2}{7}x^3\sqrt{b\sqrt[3]{x}+ax}\\
&-\frac{44b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221a^{19/4}\sqrt{b\sqrt[3]{x}+ax}}\\
&+\frac{22b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{221a^{19/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}$$

output
```
44/221*b^5*(b+a*x^(2/3))*x^(1/3)/a^(9/2)/(b^(1/2)+a^(1/2)*x^(1/3))/(b*x^(1
/3)+a*x)^(1/2)-44/663*b^4*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^4+220/4641*b^3*x
*(b*x^(1/3)+a*x)^(1/2)/a^3-60/1547*b^2*x^(5/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+4
/119*b*x^(7/3)*(b*x^(1/3)+a*x)^(1/2)/a+2/7*x^3*(b*x^(1/3)+a*x)^(1/2)-44/22
1*b^(21/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/
3))^2)^(1/2)*x^(1/6)*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*
2^(1/2))/a^(19/4)/(b*x^(1/3)+a*x)^(1/2)+22/221*b^(21/4)*(b^(1/2)+a^(1/2)*x
^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*InverseJ
acobiAM(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)),1/2*2^(1/2))/a^(19/4)/(b*x^(1/3)
+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.33

$$\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx$$
$$=\frac{2\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}\left(\sqrt{1+\frac{ax^{2/3}}{b}}\left(-385b^4+110ab^3x^{2/3}-90a^2b^2x^{4/3}+78a^3bx^2+663a^4x^{8/3}\right)+385b^4\,\text{Hype}\right.}{4641a^4\sqrt{1+\frac{ax^{2/3}}{b}}}$$

input
```
Integrate[x^2*Sqrt[b*x^(1/3) + a*x],x]
```

output
```
(2*x^(1/3)*Sqrt[b*x^(1/3) + a*x]*(Sqrt[1 + (a*x^(2/3))/b]*(-385*b^4 + 110*
a*b^3*x^(2/3) - 90*a^2*b^2*x^(4/3) + 78*a^3*b*x^2 + 663*a^4*x^(8/3)) + 385
*b^4*Hypergeometric2F1[-1/2, 3/4, 7/4, -((a*x^(2/3))/b)]))/(4641*a^4*Sqrt[
1 + (a*x^(2/3))/b])
```

Rubi [A] (warning: unable to verify)

Time = 1.06 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {1924, 1927, 1930, 1930, 1930, 1930, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{ax+b\sqrt[3]{x}}dx$$

$$\Big\downarrow \text{1924}$$

$$3\int x^{8/3}\sqrt{\sqrt[3]{x}b+ax}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1927}$$

$$3\left(\frac{2}{21}b\int \frac{x^3}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}+\frac{2}{21}x^3\sqrt{ax+b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2}{21}b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\int \frac{x^{7/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{17a}\right)+\frac{2}{21}x^3\sqrt{ax+b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2}{21}b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\int \frac{x^{5/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{13a}\right)}{17a}\right)+\frac{2}{21}x^3\sqrt{ax+b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2}{21}b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\int\frac{x}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{9a}\right)}{13a}\right)}{17a}\right)+\frac{2}{21}x^3\sqrt{ax+b\sqrt[3]{}}\right.$$

$\downarrow$ 1930

$$3\left(\frac{2}{21}b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\int\frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{5a}\right)}{9a}\right)}{13a}\right)}{17a}\right.\right.$$

↓ 1938

$$3\left(\frac{2}{21}b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d}{5a\sqrt{ax+b\sqrt[3]{x}}}\right.}{9a}\right.}{13a}\right.}{17a}\right.\right.$$

$\downarrow$ 266

$$3\Bigg(\frac{2}{21}b\Bigg(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\Bigg(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\Bigg(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\Bigg(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d}{5a\sqrt{ax+b\sqrt[3]{x}}}}{9a}}{13a}}{17a}$$

$\downarrow$ 834

$$3\left(\frac{2}{21}b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}}}}{\sqrt{a}}\right.}{5a\sqrt{}}\right.}{9a}\right.}{13a}\right.}{17a}\right.\right.$$

$\downarrow$ 27

$$3\left(\frac{2}{21}b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}\cdots}}\cdots}{\sqrt{a}}\cdots\right.}{5a\sqrt{ax\cdots}}\right.}{9a}\cdots\right.}{13a}\cdots\right.}{17a}\cdots\right.\right.$$

↓ 761

$$\sqrt[4]{b}\left(\sqrt{a}x^{2/3}\right.$$

$$6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\Bigg($$

$$7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\right.$$

$$11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\right.$$

$$15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\right.$$

$$3\left(\frac{2}{21}b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{\qquad}{17a}\right.\right.$$

↓ 1510

$$6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}\right.}{}\right.$$

$$7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{}{}\right.$$

$$11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{}{}\right.$$

$$15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{}{}\right.$$

$$3\left(\frac{2}{21}b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{}{}\right.\right.$$

input
```
Int[x^2*Sqrt[b*x^(1/3) + a*x],x]
```

output
```
3*((2*x^3*Sqrt[b*x^(1/3) + a*x])/21 + (2*b*((2*x^(7/3)*Sqrt[b*x^(1/3) + a*
x])/(17*a) - (15*b*((2*x^(5/3)*Sqrt[b*x^(1/3) + a*x])/(13*a) - (11*b*((2*x
*Sqrt[b*x^(1/3) + a*x])/(9*a) - (7*b*((2*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(5
*a) - (6*b*Sqrt[b + a*x^(2/3)]*x^(1/6)*(-((-((x^(1/6)*Sqrt[b + a*x^(4/3)])
/(Sqrt[b] + Sqrt[a]*x^(2/3))) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[
(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticE[2*ArcTan[(a^(1/4)
*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b + a*x^(4/3)]))/Sqrt[a]) + (b^(1/
4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(
2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(3/4)*S
qrt[b + a*x^(4/3)])))/(5*a*Sqrt[b*x^(1/3) + a*x])))/(9*a)))/(13*a)))/(17*a
)))/21)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{2x^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{7}+\frac{4b\,x^{\frac{7}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{119a}-\frac{60b^2x^{\frac{5}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1547a^2}+\frac{220b^3x\sqrt{b\,x^{\frac{1}{3}}+ax}}{4641a^3}-\frac{44b^4x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{663a^4}+\frac{22b^5}{}$
default	$\frac{2x^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{7}+\frac{4b\,x^{\frac{7}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{119a}-\frac{60b^2x^{\frac{5}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1547a^2}+\frac{220b^3x\sqrt{b\,x^{\frac{1}{3}}+ax}}{4641a^3}-\frac{44b^4x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{663a^4}+\frac{22b^5}{}$

input

```
int(x^2*(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2/7*x^3*(b*x^(1/3)+a*x)^(1/2)+4/119*b*x^(7/3)*(b*x^(1/3)+a*x)^(1/2)/a-60/1
547*b^2*x^(5/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+220/4641*b^3*x*(b*x^(1/3)+a*x)^(
1/2)/a^3-44/663*b^4*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^4+22/221*b^5/a^5*(-a*b
)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a
*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2)*a)^(1/2)/(b*x^
(1/3)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*
a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/
a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx=\int \sqrt{ax+bx^{\frac{1}{3}}}x^2\,dx$$

input
```
integrate(x^2*(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*x + b*x^(1/3))*x^2, x)
```

Sympy [F]

$$\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx=\int x^2\sqrt{ax+b\sqrt[3]{x}}\,dx$$

input
```
integrate(x**2*(b*x**(1/3)+a*x)**(1/2),x)
```

output
```
Integral(x**2*sqrt(a*x + b*x**(1/3)), x)
```

Maxima [F]

$$\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx=\int \sqrt{ax+bx^{\frac{1}{3}}}x^2\,dx$$

input
```
integrate(x^2*(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx=\int \sqrt{ax+bx^{\frac{1}{3}}}x^2\,dx$$

input
```
integrate(x^2*(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx=\int x^2\,\sqrt{a\,x+b\,x^{1/3}}\,dx$$

input
```
int(x^2*(a*x + b*x^(1/3))^(1/2),x)
```

output
```
int(x^2*(a*x + b*x^(1/3))^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx$$
$$=\frac{-\frac{60x^{\frac{11}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a^2b^2}{1547}+\frac{4\sqrt{x}\,\sqrt{x^{\frac{2}{3}}a+b}\,a^3b\,x^2}{119}-\frac{44\sqrt{x}\,\sqrt{x^{\frac{2}{3}}a+b}\,b^4}{663}+\frac{2x^{\frac{19}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a^4}{7}+\frac{220x^{\frac{7}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a\,b^3}{4641}+\frac{22\left(\int\frac{x^{\frac{1}{6}}\sqrt{x^{\frac{2}{3}}a+b}}{x^{\frac{2}{3}}b+x^{\frac{4}{3}}a}\right.}{663}}{a^4}$$

input
```
int(x^2*(b*x^(1/3)+a*x)^(1/2),x)
```

output
```
(2*( - 90*x**(5/6)*sqrt(x**(2/3)*a + b)*a**2*b**2*x + 78*sqrt(x)*sqrt(x**(
2/3)*a + b)*a**3*b*x**2 - 154*sqrt(x)*sqrt(x**(2/3)*a + b)*b**4 + 663*x**(
1/6)*sqrt(x**(2/3)*a + b)*a**4*x**3 + 110*x**(1/6)*sqrt(x**(2/3)*a + b)*a*
b**3*x + 77*int((x**(1/6)*sqrt(x**(2/3)*a + b))/(x**(2/3)*b + x**(1/3)*a*x
),x)*b**5))/(4641*a**4)
```

3.112 $\int x\sqrt{b\sqrt[3]{x}+ax}\,dx$

Optimal result 908
Mathematica [C] (verified) 909
Rubi [A] (warning: unable to verify) 909
Maple [A] (verified) 913
Fricas [F] 914
Sympy [F] 914
Maxima [F] 914
Giac [F] 915
Mupad [F(-1)] 915
Reduce [F] 915

Optimal result

Integrand size = 17, antiderivative size = 213

$$\int x\sqrt{b\sqrt[3]{x}+ax}\,dx = \frac{12b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^3} - \frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2} + \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a} + \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} - \frac{6b^{15/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77a^{13/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output

```
12/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^3-36/385*b^2*x^(2/3)*(b*x^(1/3)+a*x)^(1/
2)/a^2+4/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a+2/5*x^2*(b*x^(1/3)+a*x)^(1/2
)-6/77*b^(15/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*
x^(1/3))^2)^(1/2)*x^(1/6)*InverseJacobiAM(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)
),1/2*2^(1/2))/a^(13/4)/(b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.55

$$\int x\sqrt{b\sqrt[3]{x}+ax}\,dx = \frac{2\sqrt{b\sqrt[3]{x}+ax}\left(\sqrt{1+\frac{ax^{2/3}}{b}}\left(45b^3-18ab^2x^{2/3}+14a^2bx^{4/3}+77a^3x^2\right)-45b^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\right.\right.}{385a^3\sqrt{1+\frac{ax^{2/3}}{b}}}$$

input
```
Integrate[x*Sqrt[b*x^(1/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(1/3) + a*x]*(Sqrt[1 + (a*x^(2/3))/b]*(45*b^3 - 18*a*b^2*x^(2/
3) + 14*a^2*b*x^(4/3) + 77*a^3*x^2) - 45*b^3*Hypergeometric2F1[-1/2, 1/4,
5/4, -((a*x^(2/3))/b)]))/(385*a^3*Sqrt[1 + (a*x^(2/3))/b])
```

Rubi [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1924, 1927, 1930, 1930, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{ax+b\sqrt[3]{x}}\,dx$$

$$\Big\downarrow \text{1924}$$

$$3\int x^{5/3}\sqrt{\sqrt[3]{x}b+ax}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1927}$$

$$3\left(\frac{2}{15}b\int \frac{x^2}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}+\frac{2}{15}x^2\sqrt{ax+b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2}{15}b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\int\frac{x^{4/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{11a}\right)+\frac{2}{15}x^2\sqrt{ax+b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2}{15}b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\int\frac{x^{2/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{7a}\right)}{11a}\right)+\frac{2}{15}x^2\sqrt{ax+b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2}{15}b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b\int\frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{3a}\right)}{7a}\right)}{11a}\right)+\frac{2}{15}x^2\sqrt{ax+b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{1917}$$

$$3\left(\frac{2}{15}b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{3a\sqrt{ax+b\sqrt[3]{x}}}\right)}{7a}\right)}{11a}\right)+\frac{2}{15}x^{2}\cdots$$

$\big\downarrow$ 266

$$3\left(\frac{2}{15}b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{2b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3a\sqrt{ax+b\sqrt[3]{x}}}\right)}{7a}\right)}{11a}\right)+\frac{2}{15}x^{2}\sqrt{a\cdots}$$

$\big\downarrow$ 761

$$3\left(\frac{2}{15}b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^{2}}}\,\text{EllipticF}\cdots}{3a^{5/4}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}\cdots\right)}{7a}\cdots\right)}{11a}\cdots\right.\right.$$

input
```
Int[x*Sqrt[b*x^(1/3) + a*x],x]
```

output
```
3*((2*x^2*Sqrt[b*x^(1/3) + a*x])/15 + (2*b*((2*x^(4/3)*Sqrt[b*x^(1/3) + a*
x])/(11*a) - (9*b*((2*x^(2/3)*Sqrt[b*x^(1/3) + a*x])/(7*a) - (5*b*((2*Sqrt
[b*x^(1/3) + a*x])/(3*a) - (b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b + a
*x^(2/3)]*x^(1/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*Elli
pticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[b*x^(1/3)
 + a*x]*Sqrt[b + a*x^(4/3)])))/(7*a)))/(11*a)))/15)
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{2x^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{5}+\frac{4b\,x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{55a}-\frac{36b^2x^{\frac{2}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{385a^2}+\frac{12b^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{77a^3}-\frac{6b^4\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-}}{}$
default	$\frac{2x^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{5}+\frac{4b\,x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{55a}-\frac{36b^2x^{\frac{2}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{385a^2}+\frac{12b^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{77a^3}-\frac{6b^4\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-}}{}$

input
```
int(x*(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*x^2*(b*x^(1/3)+a*x)^(1/2)+4/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a-36/38
5*b^2*x^(2/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+12/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^
3-6/77*b^4/a^4*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1
/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^
(1/2)*a)^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))
*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Fricas [F]

$$\int x\sqrt{b\sqrt[3]{x}+ax}\,dx = \int \sqrt{ax+bx^{\frac{1}{3}}}x\,dx$$

input
```
integrate(x*(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*x + b*x^(1/3))*x, x)
```

Sympy [F]

$$\int x\sqrt{b\sqrt[3]{x}+ax}\,dx = \int x\sqrt{ax+b\sqrt[3]{x}}\,dx$$

input
```
integrate(x*(b*x**(1/3)+a*x)**(1/2),x)
```

output
```
Integral(x*sqrt(a*x + b*x**(1/3)), x)
```

Maxima [F]

$$\int x\sqrt{b\sqrt[3]{x}+ax}\,dx = \int \sqrt{ax+bx^{\frac{1}{3}}}x\,dx$$

input
```
integrate(x*(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))*x, x)
```

Giac [F]

$$\int x\sqrt{b\sqrt[3]{x}+ax}\,dx=\int \sqrt{ax+bx^{\frac{1}{3}}}x\,dx$$

input
```
integrate(x*(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{b\sqrt[3]{x}+ax}\,dx=\int x\,\sqrt{a\,x+b\,x^{1/3}}\,dx$$

input
```
int(x*(a*x + b*x^(1/3))^(1/2),x)
```

output
```
int(x*(a*x + b*x^(1/3))^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{b\sqrt[3]{x}+ax}\,dx$$

$$=\frac{-\frac{36x^{\frac{5}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a\,b^{2}}{385}+\frac{4\sqrt{x}\,\sqrt{x^{\frac{2}{3}}a+b}\,a^{2}bx}{55}+\frac{2x^{\frac{13}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a^{3}}{5}+\frac{12x^{\frac{1}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,b^{3}}{77}-\frac{2\left(\int\frac{\sqrt{x^{\frac{2}{3}}a+b}}{x^{\frac{5}{6}}b+\sqrt{x}\,ax}dx\right)b^{4}}{77}}{a^{3}}$$

input
```
int(x*(b*x^(1/3)+a*x)^(1/2),x)
```

output
```
(2*( - 18*x**(5/6)*sqrt(x**(2/3)*a + b)*a*b**2 + 14*sqrt(x)*sqrt(x**(2/3)*
a + b)*a**2*b*x + 77*x**(1/6)*sqrt(x**(2/3)*a + b)*a**3*x**2 + 30*x**(1/6)
*sqrt(x**(2/3)*a + b)*b**3 - 5*int(sqrt(x**(2/3)*a + b)/(x**(5/6)*b + sqrt
(x)*a*x),x)*b**4))/(385*a**3)
```

3.113 $\int \sqrt{b\sqrt[3]{x}+ax}\,dx$

Optimal result 917
Mathematica [C] (verified) 918
Rubi [A] (warning: unable to verify) 918
Maple [A] (verified) 922
Fricas [F] 923
Sympy [F] 923
Maxima [F] 924
Giac [F] 924
Mupad [B] (verification not implemented) 924
Reduce [F] 925

Optimal result

Integrand size = 15, antiderivative size = 323

$$\int \sqrt{b\sqrt[3]{x}+ax}\,dx = -\frac{4b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5a^{3/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a}+\frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}$$
$$+\frac{4b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{b\sqrt[3]{x}+ax}}$$
$$-\frac{2b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5a^{7/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
-4/5*b^2*(b+a*x^(2/3))*x^(1/3)/a^(3/2)/(b^(1/2)+a^(1/2)*x^(1/3))/(b*x^(1/3
)+a*x)^(1/2)+4/15*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a+2/3*x*(b*x^(1/3)+a*x)^
(1/2)+4/5*b^(9/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2
)*x^(1/3))^2)^(1/2)*x^(1/6)*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)
)),1/2*2^(1/2))/a^(7/4)/(b*x^(1/3)+a*x)^(1/2)-2/5*b^(9/4)*(b^(1/2)+a^(1/2)
*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*Invers
eJacobiAM(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)),1/2*2^(1/2))/a^(7/4)/(b*x^(1/3
)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.29

$$\int \sqrt{b\sqrt[3]{x}+ax}\,dx = \frac{2\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}\left(\left(b+ax^{2/3}\right)\sqrt{1+\frac{ax^{2/3}}{b}}-b\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{ax^{2/3}}{b}\right)\right)}{3a\sqrt{1+\frac{ax^{2/3}}{b}}}$$

input
```
Integrate[Sqrt[b*x^(1/3) + a*x],x]
```

output
```
(2*x^(1/3)*Sqrt[b*x^(1/3) + a*x]*((b + a*x^(2/3))*Sqrt[1 + (a*x^(2/3))/b]
- b*Hypergeometric2F1[-1/2, 3/4, 7/4, -((a*x^(2/3))/b)]))/(3*a*Sqrt[1 + (a
*x^(2/3))/b])
```

Rubi [A] (warning: unable to verify)

Time = 0.76 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {1910, 1924, 1930, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ax+b\sqrt[3]{x}}\,dx$$

$$\Big\downarrow \text{1910}$$

$$\frac{2}{9}b\int \frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}dx+\frac{2}{3}x\sqrt{ax+b\sqrt[3]{x}}$$

$$\Big\downarrow \text{1924}$$

$$\frac{2}{3}b\int \frac{x}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}+\frac{2}{3}x\sqrt{ax+b\sqrt[3]{x}}$$

$$\Big\downarrow \text{1930}$$

$$\frac{2}{3}b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\int\frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{5a}\right)+\frac{2}{3}x\sqrt{ax+b\sqrt[3]{x}}$$

$$\Big\downarrow \text{1938}$$

$$\frac{2}{3}b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)+\frac{2}{3}x\sqrt{ax+b\sqrt[3]{x}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{3}b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)+\frac{2}{3}x\sqrt{ax+b\sqrt[3]{x}}$$

$$\Big\downarrow \text{834}$$

$$\frac{2}{3}b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{b}\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)+\frac{2}{3}x\sqrt{ax+b\sqrt[3]{x}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)+\frac{2}{3}x\sqrt{ax+b\sqrt[3]{x}}$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{3}b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\int\frac{\sqrt{b}-\ldots}{\sqrt{ax^{4}\ldots}}}{\ldots}\right)}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\frac{2}{3}x\sqrt{ax+b\sqrt[3]{x}}$$

$$\Big\downarrow \text{1510}$$

$$\frac{2}{3}b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\sqrt[4]{b}(\sqrt{\ldots}}{\ldots}\right)}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\frac{2}{3}x\sqrt{ax+b\sqrt[3]{x}}$$

input
```
Int[Sqrt[b*x^(1/3) + a*x],x]
```

output
```
(2*x*Sqrt[b*x^(1/3) + a*x])/3 + (2*b*((2*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(5
*a) - (6*b*Sqrt[b + a*x^(2/3)]*x^(1/6)*(-((-((x^(1/6)*Sqrt[b + a*x^(4/3)])
/(Sqrt[b] + Sqrt[a]*x^(2/3))) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[
(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticE[2*ArcTan[(a^(1/4)
*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b + a*x^(4/3)]))/Sqrt[a]) + (b^(1/
4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(
2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(3/4)*S
qrt[b + a*x^(4/3)])))/(5*a*Sqrt[b*x^(1/3) + a*x])))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{2x\sqrt{b\,x^{\frac{1}{3}}+ax}}{3}+\frac{4b\,x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{15a}-\frac{2b^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{Ellipti}}{}\right.}{5a^{2}\sqrt{b\,x^{\frac{1}{3}}}}$
default	$\frac{2x\sqrt{b\,x^{\frac{1}{3}}+ax}}{3}+\frac{4b\,x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{15a}-\frac{2b^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{Ellipti}}{}\right.}{5a^{2}\sqrt{b\,x^{\frac{1}{3}}}}$

input
```
int((b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x*(b*x^(1/3)+a*x)^(1/2)+4/15*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a-2/5/a^2
*b^2*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x
^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2)*a)^(
1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*
b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((
x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \sqrt{b\sqrt[3]{x}+ax}\,dx=\int \sqrt{ax+bx^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*x + b*x^(1/3)), x)
```

Sympy [F]

$$\int \sqrt{b\sqrt[3]{x}+ax}\,dx=\int \sqrt{ax+b\sqrt[3]{x}}\,dx$$

input
```
integrate((b*x**(1/3)+a*x)**(1/2),x)
```

output
```
Integral(sqrt(a*x + b*x**(1/3)), x)
```

Maxima [F]

$$\int \sqrt{b\sqrt[3]{x}+ax}\,dx=\int \sqrt{ax+bx^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(1/3)), x)
```

Giac [F]

$$\int \sqrt{b\sqrt[3]{x}+ax}\,dx=\int \sqrt{ax+bx^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x + b*x^(1/3)), x)
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.12

$$\int \sqrt{b\sqrt[3]{x}+ax}\,dx=\frac{6\,x\,\sqrt{a\,x+b\,x^{1/3}}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ -\frac{a\,x^{2/3}}{b}\right)}{7\,\sqrt{\frac{a\,x^{2/3}}{b}+1}}$$

input
```
int((a*x + b*x^(1/3))^(1/2),x)
```

output
```
(6*x*(a*x + b*x^(1/3))^(1/2)*hypergeom([-1/2, 7/4], 11/4, -(a*x^(2/3))/b))
/(7*((a*x^(2/3))/b + 1)^(1/2))
```

Reduce [F]

$$\int \sqrt{b\sqrt[3]{x}+ax}\,dx = \frac{\frac{4\sqrt{x}\,\sqrt{x^{\frac{2}{3}}a+b}\,b}{15}+\frac{2x^{\frac{7}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a}{3}-\frac{2\left(\int \frac{x^{\frac{1}{6}}\sqrt{x^{\frac{2}{3}}a+b}}{x^{\frac{2}{3}}b+x^{\frac{4}{3}}a}dx\right)b^{2}}{15}}{a}$$

input
```
int((b*x^(1/3)+a*x)^(1/2),x)
```

output
```
(2*(2*sqrt(x)*sqrt(x**(2/3)*a + b)*b + 5*x**(1/6)*sqrt(x**(2/3)*a + b)*a*x
 - int((x**(1/6)*sqrt(x**(2/3)*a + b))/(x**(2/3)*b + x**(1/3)*a*x),x)*b**2
))/(15*a)
```

3.114 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx$

Optimal result 926
Mathematica [C] (verified) 927
Rubi [A] (warning: unable to verify) 927
Maple [A] (verified) 929
Fricas [F] 930
Sympy [F] 930
Maxima [F] 930
Giac [F] 931
Mupad [F(-1)] 931
Reduce [F] 931

Optimal result

Integrand size = 19, antiderivative size = 123

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx = 2\sqrt{b\sqrt[3]{x}+ax} + \frac{2b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
2*(b*x^(1/3)+a*x)^(1/2)+2*b^(3/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))
/(b^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*InverseJacobiAM(2*arctan(a^(1/
4)*x^(1/6)/b^(1/4)),1/2*2^(1/2))/a^(1/4)/(b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx=\frac{6\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{ax^{2/3}}{b}\right)}{\sqrt{1+\frac{ax^{2/3}}{b}}}$$

input
```
Integrate[Sqrt[b*x^(1/3) + a*x]/x,x]
```

output
```
(6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-1/2, 1/4, 5/4, -((a*x^(2/3))/b
)])/Sqrt[1 + (a*x^(2/3))/b]
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}=0.263$, Rules used = {1924, 1927, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x}dx$$

$$\Big\downarrow \text{ 1924}$$

$$3\int \frac{\sqrt{\sqrt[3]{x}b+ax}}{\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 1927}$$

$$3\left(\frac{2}{3}b\int \frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}+\frac{2}{3}\sqrt{ax+b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{ 1917}$$

$$3\left(\frac{2b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{3\sqrt{ax+b\sqrt[3]{x}}}+\frac{2}{3}\sqrt{ax+b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{266}$$

$$3\left(\frac{4b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3\sqrt{ax+b\sqrt[3]{x}}}+\frac{2}{3}\sqrt{ax+b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{761}$$

$$3\left(\frac{2b^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}+\frac{2}{3}\sqrt{ax+b\sqrt[3]{x}}\right)$$

input
```
Int[Sqrt[b*x^(1/3) + a*x]/x,x]
```

output
```
3*((2*Sqrt[b*x^(1/3) + a*x])/3 + (2*b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sq
rt[b + a*x^(2/3)]*x^(1/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))
^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(3*a^(1/4)*Sqrt[b
*x^(1/3) + a*x]*Sqrt[b + a*x^(4/3)]))
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.07

method	result
derivativedivides	$2\sqrt{b\,x^{\frac{1}{3}}+ax}+\frac{2b\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\;\mathrm{EllipticF}\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{b\,x^{\frac{1}{3}}+ax}}$
default	$2\sqrt{b\,x^{\frac{1}{3}}+ax}+\frac{2b\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\;\mathrm{EllipticF}\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{b\,x^{\frac{1}{3}}+ax}}$

input
```
int((b*x^(1/3)+a*x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*(b*x^(1/3)+a*x)^(1/2)+2*b/a*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(
-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-
x^(1/3)/(-a*b)^(1/2)*a)^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/
a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx = \int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x,x, algorithm="fricas")
```

output
```
integral(sqrt(a*x + b*x^(1/3))/x, x)
```

Sympy [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx = \int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x}\,dx$$

input
```
integrate((b*x**(1/3)+a*x)**(1/2)/x,x)
```

output
```
Integral(sqrt(a*x + b*x**(1/3))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx = \int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))/x, x)
```

Giac [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx=\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx=\int \frac{\sqrt{a\,x+b\,x^{1/3}}}{x}\,dx$$

input
```
int((a*x + b*x^(1/3))^(1/2)/x,x)
```

output
```
int((a*x + b*x^(1/3))^(1/2)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx=2x^{\frac{1}{6}}\sqrt{x^{\frac{2}{3}}a+b}+\frac{2\left(\int \frac{\sqrt{x^{\frac{2}{3}}a+b}}{x^{\frac{5}{6}}b+\sqrt{x}\,ax}dx\right)b}{3}$$

input
```
int((b*x^(1/3)+a*x)^(1/2)/x,x)
```

output
```
(2*(3*x**(1/6)*sqrt(x**(2/3)*a + b) + int(sqrt(x**(2/3)*a + b)/(x**(5/6)*b
 + sqrt(x)*a*x),x)*b))/3
```

3.115 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx$

Optimal result 932
Mathematica [C] (verified) 933
Rubi [A] (warning: unable to verify) 933
Maple [A] (verified) 937
Fricas [F] 938
Sympy [F] 939
Maxima [F] 939
Giac [F] 939
Mupad [F(-1)] 940
Reduce [F] 940

Optimal result

Integrand size = 19, antiderivative size = 325

$$\begin{aligned}\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx &= \frac{12a^{3/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}}\\
&\quad-\frac{12a^{5/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}\\
&\quad+\frac{6a^{5/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

output
```
12/5*a^(3/2)*(b+a*x^(2/3))*x^(1/3)/b/(b^(1/2)+a^(1/2)*x^(1/3))/(b*x^(1/3)+
a*x)^(1/2)-6/5*(b*x^(1/3)+a*x)^(1/2)/x-12/5*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(1
/3)-12/5*a^(5/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)
*x^(1/3))^2)^(1/2)*x^(1/6)*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))
),1/2*2^(1/2))/b^(3/4)/(b*x^(1/3)+a*x)^(1/2)+6/5*a^(5/4)*(b^(1/2)+a^(1/2)*
x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*Inverse
JacobiAM(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)),1/2*2^(1/2))/b^(3/4)/(b*x^(1/3)
+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx=-\frac{6\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},-\frac{ax^{2/3}}{b}\right)}{5\sqrt{1+\frac{ax^{2/3}}{b}}\,x}$$

input
```
Integrate[Sqrt[b*x^(1/3) + a*x]/x^2,x]
```

output
```
(-6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-5/4, -1/2, -1/4, -((a*x^(2/3)
)/b)])/(5*Sqrt[1 + (a*x^(2/3))/b]*x)
```

Rubi [A] (warning: unable to verify)

Time = 0.75 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1924, 1926, 1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x^2}\,dx$$

$$\Big\downarrow \; 1924$$

$$3\int \frac{\sqrt{\sqrt[3]{x}b+ax}}{x^{4/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1926}$$

$$3\left(\frac{2}{5}a\int \frac{1}{\sqrt[3]{x}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5x}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{2}{5}a\left(\frac{a\int \frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5x}\right)$$

$$\Big\downarrow \text{1938}$$

$$3\left(\frac{2}{5}a\left(\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5x}\right)$$

$$\Big\downarrow \text{266}$$

$$3\left(\frac{2}{5}a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5x}\right)$$

$$\Big\downarrow \text{834}$$

$$3\left(\frac{2}{5}a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{b}\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5x}\right)$$

$$\Big\downarrow \text{27}$$

$$3\left(\frac{2}{5}a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int \frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5x}\right)$$

$$\Big\downarrow \text{761}$$

$$3\left(\frac{2}{5}a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2}{\cdots}\right.\right.$$

$$\Big\downarrow \ 1510$$

$$3\left(\frac{2}{5}a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}\cdots}{\left(\sqrt{a}x^{2/3}\cdots\right.}}}{\sqrt[4]{a}\cdots}\cdots\right.}{b\sqrt{ax+b\sqrt[3]{x}}}\right.\right.$$

input
```
Int[Sqrt[b*x^(1/3) + a*x]/x^2,x]
```

output
```
3*((-2*Sqrt[b*x^(1/3) + a*x])/(5*x) + (2*a*((-2*Sqrt[b*x^(1/3) + a*x])/(b*
x^(1/3)) + (2*a*Sqrt[b + a*x^(2/3)]*x^(1/6)*(-((-((x^(1/6)*Sqrt[b + a*x^(4
/3)])/(Sqrt[b] + Sqrt[a]*x^(2/3))) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*
Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticE[2*ArcTan[(a^
(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b + a*x^(4/3)]))/Sqrt[a]) + (
b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a
]*x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(3
/4)*Sqrt[b + a*x^(4/3)])))/(b*Sqrt[b*x^(1/3) + a*x])))/5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1926

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.66

method	result
derivativedivides	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{5x}-\frac{12\left(b+a\,x^{\frac{2}{3}}\right)a}{5b\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}+\frac{6a\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{Ellip}\cdots}{\cdots}\right.}{5b\sqrt{b\,x^{\frac{1}{3}}\cdots}}$
default	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{5x}-\frac{12\left(b+a\,x^{\frac{2}{3}}\right)a}{5b\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}+\frac{6a\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{Ellip}\cdots}{\cdots}\right.}{5b\sqrt{b\,x^{\frac{1}{3}}\cdots}}$

input
```
int((b*x^(1/3)+a*x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-6/5*(b*x^(1/3)+a*x)^(1/2)/x-12/5*(b+a*x^(2/3))*a/b/(x^(1/3)*(b+a*x^(2/3))
)^(1/2)+6/5*a/b*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(
1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)
^(1/2)*a)^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/
3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*E
llipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx=\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^2}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a*x + b*x^(1/3))/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx = \int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x^2}\,dx$$

input
```
integrate((b*x**(1/3)+a*x)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(a*x + b*x**(1/3))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx = \int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^2}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx = \int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^2}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx=\int \frac{\sqrt{a\,x+b\,x^{1/3}}}{x^2}\,dx$$

input
```
int((a*x + b*x^(1/3))^(1/2)/x^2,x)
```

output
```
int((a*x + b*x^(1/3))^(1/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx=\frac{-2\sqrt{x^{\frac{2}{3}}a+b}-\frac{2x^{\frac{5}{6}}\left(\int \frac{\sqrt{x^{\frac{2}{3}}a+b}}{x^{\frac{11}{6}}b+\sqrt{x}\,a\,x^2}dx\right)b}{3}}{x^{\frac{5}{6}}}$$

input
```
int((b*x^(1/3)+a*x)^(1/2)/x^2,x)
```

output
```
(2*( - 3*sqrt(x**(2/3)*a + b) - x**(5/6)*int(sqrt(x**(2/3)*a + b)/(x**(5/6
)*b*x + sqrt(x)*a*x**2),x)*b))/(3*x**(5/6))
```

3.116 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx$

Optimal result 941
Mathematica [C] (verified) 942
Rubi [A] (warning: unable to verify) 942
Maple [A] (verified) 945
Fricas [F] 946
Sympy [F] 946
Maxima [F] 946
Giac [F] 947
Mupad [F(-1)] 947
Reduce [F] 947

Optimal result

Integrand size = 19, antiderivative size = 188

$$\begin{aligned}\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx &= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{77bx^{4/3}}+\frac{20a^2\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}}\\ &\quad+\frac{10a^{11/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77b^{9/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

output
```
-6/11*(b*x^(1/3)+a*x)^(1/2)/x^2-12/77*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(4/3)+20
/77*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2/3)+10/77*a^(11/4)*(b^(1/2)+a^(1/2)*
x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*Inverse
JacobiAM(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)),1/2*2^(1/2))/b^(9/4)/(b*x^(1/3)
+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.31

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx = -\frac{6\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\frac{1}{2},-\frac{7}{4},-\frac{ax^{2/3}}{b}\right)}{11\sqrt{1+\frac{ax^{2/3}}{b}}\,x^2}$$

input
```
Integrate[Sqrt[b*x^(1/3) + a*x]/x^3,x]
```

output
```
(-6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-11/4, -1/2, -7/4, -((a*x^(2/3
))/b)])/(11*Sqrt[1 + (a*x^(2/3))/b]*x^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1924, 1926, 1931, 1931, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x^3}\,dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{\sqrt{\sqrt[3]{x}b+ax}}{x^{7/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1926}$$

$$3\left(\frac{2}{11}a\int \frac{1}{x^{4/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11x^2}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{2}{11}a\left(-\frac{5a\int\frac{1}{x^{2/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11x^2}\right)$$

$\big\downarrow$ 1931

$$3\left(\frac{2}{11}a\left(-\frac{5a\left(-\frac{a\int\frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{3b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11x^2}\right)$$

$\big\downarrow$ 1917

$$3\left(\frac{2}{11}a\left(-\frac{5a\left(-\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11x^2}\right)$$

$\big\downarrow$ 266

$$3\left(\frac{2}{11}a\left(-\frac{5a\left(-\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11x^2}\right)$$

$\big\downarrow$ 761

$$3\left(\frac{2}{11}a\left(-\frac{5a\left(-\frac{a^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{a}}{7}\right.\right.$$

input
```
Int[Sqrt[b*x^(1/3) + a*x]/x^3,x]
```

output
```
3*((-2*Sqrt[b*x^(1/3) + a*x])/(11*x^2) + (2*a*((-2*Sqrt[b*x^(1/3) + a*x])/
(7*b*x^(4/3)) - (5*a*((-2*Sqrt[b*x^(1/3) + a*x])/(3*b*x^(2/3)) - (a^(3/4)*
(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b + a*x^(2/3)]*x^(1/6)*Sqrt[(b + a*x^(4/3
))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(
1/4)], 1/2])/(3*b^(5/4)*Sqrt[b*x^(1/3) + a*x]*Sqrt[b + a*x^(4/3)])))/(7*b)
))/11)
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.95

method	result
derivativedivides	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{11x^{2}}-\frac{12a\sqrt{b\,x^{\frac{1}{3}}+ax}}{77b\,x^{\frac{4}{3}}}+\frac{20a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{77b^{2}x^{\frac{2}{3}}}+\frac{10a^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x}{\sqrt{}}}}{77b^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}$
default	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{11x^{2}}-\frac{12a\sqrt{b\,x^{\frac{1}{3}}+ax}}{77b\,x^{\frac{4}{3}}}+\frac{20a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{77b^{2}x^{\frac{2}{3}}}+\frac{10a^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x}{\sqrt{}}}}{77b^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}$

input
```
int((b*x^(1/3)+a*x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-6/11*(b*x^(1/3)+a*x)^(1/2)/x^2-12/77*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(4/3)+20
/77*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2/3)+10/77*a^2/b^2*(-a*b)^(1/2)*((x^(
1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2)
)*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2)*a)^(1/2)/(b*x^(1/3)+a*x)^(1
/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2
))
```

Fricas [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx=\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^3}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a*x + b*x^(1/3))/x^3, x)
```

Sympy [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx=\int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x^3}\,dx$$

input
```
integrate((b*x**(1/3)+a*x)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(a*x + b*x**(1/3))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx=\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^3}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx = \int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^3}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx = \int \frac{\sqrt{a\,x+b\,x^{1/3}}}{x^3}\,dx$$

input
```
int((a*x + b*x^(1/3))^(1/2)/x^3,x)
```

output
```
int((a*x + b*x^(1/3))^(1/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx = \frac{-\frac{2\sqrt{x^{\frac{2}{3}}a+b}}{3}-\frac{2x^{\frac{11}{6}}\left(\int \frac{\sqrt{x^{\frac{2}{3}}a+b}}{x^{\frac{17}{6}}b+\sqrt{x}\,a\,x^3}dx\right)b}{9}}{x^{\frac{11}{6}}}$$

input
```
int((b*x^(1/3)+a*x)^(1/2)/x^3,x)
```

output
```
(2*( - 3*sqrt(x**(2/3)*a + b) - x**(5/6)*int(sqrt(x**(2/3)*a + b)/(x**(5/6
)*b*x**2 + sqrt(x)*a*x**3),x)*b*x))/(9*x**(5/6)*x)
```

3.117 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx$

Optimal result 948
Mathematica [C] (verified) 949
Rubi [A] (warning: unable to verify) 949
Maple [A] (verified) 963
Fricas [F] 964
Sympy [F] 964
Maxima [F] 964
Giac [F] 965
Mupad [F(-1)] 965
Reduce [F] 965

Optimal result

Integrand size = 19, antiderivative size = 413

$$\begin{aligned}
&\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx \\
&= -\frac{308a^{9/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{1105b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} \\
&\quad + \frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{308a^4\sqrt{b\sqrt[3]{x}+ax}}{1105b^4\sqrt[3]{x}} \\
&\quad + \frac{308a^{17/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105b^{15/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad - \frac{154a^{17/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{1105b^{15/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}$$

output
```
-308/1105*a^(9/2)*(b+a*x^(2/3))*x^(1/3)/b^4/(b^(1/2)+a^(1/2)*x^(1/3))/(b*x
^(1/3)+a*x)^(1/2)-6/17*(b*x^(1/3)+a*x)^(1/2)/x^3-12/221*a*(b*x^(1/3)+a*x)^
(1/2)/b/x^(7/3)+44/663*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(5/3)-308/3315*a^3*
(b*x^(1/3)+a*x)^(1/2)/b^3/x+308/1105*a^4*(b*x^(1/3)+a*x)^(1/2)/b^4/x^(1/3)
+308/1105*a^(17/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/
2)*x^(1/3))^2)^(1/2)*x^(1/6)*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4
))),1/2*2^(1/2))/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)-154/1105*a^(17/4)*(b^(1/2)
+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6
)*InverseJacobiAM(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)),1/2*2^(1/2))/b^(15/4)/
(b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.14

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx = -\frac{6\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{17}{4},-\frac{1}{2},-\frac{13}{4},-\frac{ax^{2/3}}{b}\right)}{17\sqrt{1+\frac{ax^{2/3}}{b}}x^3}$$

input
```
Integrate[Sqrt[b*x^(1/3) + a*x]/x^4,x]
```

output
```
(-6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-17/4, -1/2, -13/4, -((a*x^(2/
3))/b)])/(17*Sqrt[1 + (a*x^(2/3))/b]*x^3)
```

Rubi [A] (warning: unable to verify)

Time = 1.05 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {1924, 1926, 1931, 1931, 1931, 1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x^4}\,dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{\sqrt{\sqrt[3]{x}b+ax}}{x^{10/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1926}$$

$$3\left(\frac{2}{17}a\int \frac{1}{x^{7/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{17x^3}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{2}{17}a\left(-\frac{11a\int \frac{1}{x^{5/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{17x^3}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\int \frac{1}{x\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{17x^3}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\int \frac{1}{\sqrt[3]{x}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{17x^3}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{a\int\frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)-2\sqrt{\cdots}\right.$$

$\downarrow$ 1938

$$3\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{\ldots}}}{13bx^{7/3}}\right.\right.$$

↓ 266

$$3\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right.\right.$$

$\downarrow$ 834

$$3\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{b}\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{\cdots}}{9\cdots}\cdots}{13b}\cdots$$

↓ 27

$$3\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+}}{9bx^{5}}\right.}{13b}\right.\right.$$

↓ 761

$$3\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}\,d\sqrt[6]{x}}{\sqrt{a}}\cdots}{b\sqrt{ax+b\sqrt[3]{x}}}\cdots}{5b}\cdots}{9b}\cdots}{13b}\cdots$$

↓ 1510

$$3\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)}{}\right.}{b\sqrt{ax+b\sqrt[3]{x}}}}{5b}}{9b}}{13b}\right.\right.$$

input
```
Int[Sqrt[b*x^(1/3) + a*x]/x^4,x]
```

output
```
3*((-2*Sqrt[b*x^(1/3) + a*x])/(17*x^3) + (2*a*((-2*Sqrt[b*x^(1/3) + a*x])/
(13*b*x^(7/3)) - (11*a*((-2*Sqrt[b*x^(1/3) + a*x])/(9*b*x^(5/3)) - (7*a*((
-2*Sqrt[b*x^(1/3) + a*x])/(5*b*x) - (3*a*((-2*Sqrt[b*x^(1/3) + a*x])/(b*x^
(1/3)) + (2*a*Sqrt[b + a*x^(2/3)]*x^(1/6)*(-((-((x^(1/6)*Sqrt[b + a*x^(4/3
)])/(Sqrt[b] + Sqrt[a]*x^(2/3))) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sq
rt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticE[2*ArcTan[(a^(1
/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b + a*x^(4/3)]))/Sqrt[a]) + (b^
(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*
x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(3/4
)*Sqrt[b + a*x^(4/3)])))/(b*Sqrt[b*x^(1/3) + a*x])))/(5*b)))/(9*b)))/(13*b
)))/17)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.68

method	result
derivativedivides	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{17x^{3}}-\frac{12a\sqrt{b\,x^{\frac{1}{3}}+ax}}{221b\,x^{\frac{7}{3}}}+\frac{44a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{663b^{2}x^{\frac{5}{3}}}-\frac{308a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{3315b^{3}x}+\frac{308\left(b+a\,x^{\frac{2}{3}}\right)a^{4}}{1105b^{4}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}-\frac{154a^{4}\sqrt{}}{}$
default	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{17x^{3}}-\frac{12a\sqrt{b\,x^{\frac{1}{3}}+ax}}{221b\,x^{\frac{7}{3}}}+\frac{44a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{663b^{2}x^{\frac{5}{3}}}-\frac{308a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{3315b^{3}x}+\frac{308\left(b+a\,x^{\frac{2}{3}}\right)a^{4}}{1105b^{4}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}-\frac{154a^{4}\sqrt{}}{}$

input
```
int((b*x^(1/3)+a*x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-6/17*(b*x^(1/3)+a*x)^(1/2)/x^3-12/221*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(7/3)+4
4/663*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(5/3)-308/3315*a^3*(b*x^(1/3)+a*x)^(
1/2)/b^3/x+308/1105*(b+a*x^(2/3))*a^4/b^4/(x^(1/3)*(b+a*x^(2/3)))^(1/2)-15
4/1105*a^4/b^4*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1
/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^
(1/2)*a)^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3
)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*El
lipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx=\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^4}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
integral(sqrt(a*x + b*x^(1/3))/x^4, x)
```

Sympy [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx=\int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x^4}\,dx$$

input
```
integrate((b*x**(1/3)+a*x)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(a*x + b*x**(1/3))/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx=\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^4}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx = \int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^4}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx = \int \frac{\sqrt{a\,x+b\,x^{1/3}}}{x^4}\,dx$$

input
```
int((a*x + b*x^(1/3))^(1/2)/x^4,x)
```

output
```
int((a*x + b*x^(1/3))^(1/2)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx = \frac{-\frac{2\sqrt{x^{\frac{2}{3}}a+b}}{5}-\frac{2x^{\frac{17}{6}}\left(\int \frac{\sqrt{x^{\frac{2}{3}}a+b}}{x^{\frac{23}{6}}b+\sqrt{x}\,a\,x^4}dx\right)b}{15}}{x^{\frac{17}{6}}}$$

input
```
int((b*x^(1/3)+a*x)^(1/2)/x^4,x)
```

output
```
(2*( - 3*sqrt(x**(2/3)*a + b) - x**(5/6)*int(sqrt(x**(2/3)*a + b)/(x**(5/6
)*b*x**3 + sqrt(x)*a*x**4),x)*b*x**2))/(15*x**(5/6)*x**2)
```

3.118 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx$

Optimal result . . . 966
Mathematica [C] (verified) . . . 967
Rubi [A] (warning: unable to verify) . . . 967
Maple [A] (verified) . . . 978
Fricas [F] . . . 979
Sympy [F] . . . 979
Maxima [F] . . . 980
Giac [F] . . . 980
Mupad [F(-1)] . . . 980
Reduce [F] . . . 981

Optimal result

Integrand size = 19, antiderivative size = 276

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx = -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}}+\frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}}-\frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2}+\frac{7956a^4\sqrt{b\sqrt[3]{x}+ax}}{168245b^4x^{4/3}}-\frac{2652a^5\sqrt{b\sqrt[3]{x}+ax}}{33649b^5x^{2/3}}-\frac{1326a^{23/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{33649b^{21/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
-6/23*(b*x^(1/3)+a*x)^(1/2)/x^4-12/437*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(10/3)+
68/2185*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(8/3)-884/24035*a^3*(b*x^(1/3)+a*x
)^(1/2)/b^3/x^2+7956/168245*a^4*(b*x^(1/3)+a*x)^(1/2)/b^4/x^(4/3)-2652/336
49*a^5*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(2/3)-1326/33649*a^(23/4)*(b^(1/2)+a^(1
/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*Inv
erseJacobiAM(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)),1/2*2^(1/2))/b^(21/4)/(b*x^
(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx=-\frac{6\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{23}{4},-\frac{1}{2},-\frac{19}{4},-\frac{ax^{2/3}}{b}\right)}{23\sqrt{1+\frac{ax^{2/3}}{b}}x^4}$$

input
```
Integrate[Sqrt[b*x^(1/3) + a*x]/x^5,x]
```

output
```
(-6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-23/4, -1/2, -19/4, -((a*x^(2/
3))/b)])/(23*Sqrt[1 + (a*x^(2/3))/b]*x^4)
```

Rubi [A] (warning: unable to verify)

Time = 0.92 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.21, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1924, 1926, 1931, 1931, 1931, 1931, 1931, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x^5}dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{\sqrt{\sqrt[3]{x}b+ax}}{x^{13/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1926}$$

$$3\left(\frac{2}{23}a\int \frac{1}{x^{10/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{23x^4}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{2}{23}a\left(-\frac{17a\int \frac{1}{x^{8/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{19b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{19bx^{10/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{23x^4}\right)$$

$\Big\downarrow$ 1931

$$3\left(\frac{2}{23}a\left(-\frac{17a\left(-\frac{13a\int \frac{1}{x^2\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{19bx^{10/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{23x^4}\right)$$

$\Big\downarrow$ 1931

$$3\left(\frac{2}{23}a\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\int \frac{1}{x^{4/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^2}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{19bx^{10/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{23x^4}\right)$$

$\Big\downarrow$ 1931

$$3\left(\frac{2}{23}a\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\int\frac{1}{x^{2/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^2}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{19bx^{10/3}}\right)\right)$$

$$\Big\downarrow \; 1931$$

$$3\Bigg(\frac{2}{23}a\Bigg(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{a\int\frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{3b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^{2}}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}-\frac{2\sqrt{\ }}{1}$$

↓ 1917

$$3\left(\frac{2}{23}a\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^{2}}\right)}{15b}-\frac{2\sqrt{ax+}}{15bx}\right.}{19b}\right.\right.$$

↓ 266

$$3\left(\frac{2}{23}a\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^{2}}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{}}}{15bx^{8/3}}}{19b}\right.\right.$$

↓ 761

$$3\left(\frac{2}{23}a\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{a^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}}{11b}}{15b}}{19b}$$

input
```
Int[Sqrt[b*x^(1/3) + a*x]/x^5,x]
```

output
```
3*((-2*Sqrt[b*x^(1/3) + a*x])/(23*x^4) + (2*a*((-2*Sqrt[b*x^(1/3) + a*x])/
(19*b*x^(10/3)) - (17*a*((-2*Sqrt[b*x^(1/3) + a*x])/(15*b*x^(8/3)) - (13*a
*((-2*Sqrt[b*x^(1/3) + a*x])/(11*b*x^2) - (9*a*((-2*Sqrt[b*x^(1/3) + a*x])
/(7*b*x^(4/3)) - (5*a*((-2*Sqrt[b*x^(1/3) + a*x])/(3*b*x^(2/3)) - (a^(3/4)
*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b + a*x^(2/3)]*x^(1/6)*Sqrt[(b + a*x^(4/
3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^
(1/4)], 1/2])/(3*b^(5/4)*Sqrt[b*x^(1/3) + a*x]*Sqrt[b + a*x^(4/3)])))/(7*b
)))/(11*b)))/(15*b)))/(19*b)))/23)
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.89

method	result
derivativedivides	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{23x^{4}}-\frac{12a\sqrt{b\,x^{\frac{1}{3}}+ax}}{437b\,x^{\frac{10}{3}}}+\frac{68a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{2185b^{2}x^{\frac{8}{3}}}-\frac{884a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{24035b^{3}x^{2}}+\frac{7956a^{4}\sqrt{b\,x^{\frac{1}{3}}+ax}}{168245b^{4}x^{\frac{4}{3}}}-\frac{2652a^{5}\sqrt{b\,x}}{33649b^{5}}$
default	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{23x^{4}}-\frac{12a\sqrt{b\,x^{\frac{1}{3}}+ax}}{437b\,x^{\frac{10}{3}}}+\frac{68a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{2185b^{2}x^{\frac{8}{3}}}-\frac{884a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{24035b^{3}x^{2}}+\frac{7956a^{4}\sqrt{b\,x^{\frac{1}{3}}+ax}}{168245b^{4}x^{\frac{4}{3}}}-\frac{2652a^{5}\sqrt{b\,x}}{33649b^{5}}$

input
```
int((b*x^(1/3)+a*x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-6/23*(b*x^(1/3)+a*x)^(1/2)/x^4-12/437*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(10/3)+
68/2185*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(8/3)-884/24035*a^3*(b*x^(1/3)+a*x
)^(1/2)/b^3/x^2+7956/168245*a^4*(b*x^(1/3)+a*x)^(1/2)/b^4/x^(4/3)-2652/336
49*a^5*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(2/3)-1326/33649*a^5/b^5*(-a*b)^(1/2)*(
(x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(
1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2)*a)^(1/2)/(b*x^(1/3)+a*x
)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^
(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx = \int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^5}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
integral(sqrt(a*x + b*x^(1/3))/x^5, x)
```

Sympy [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx = \int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x^5}\,dx$$

input
```
integrate((b*x**(1/3)+a*x)**(1/2)/x**5,x)
```

output
```
Integral(sqrt(a*x + b*x**(1/3))/x**5, x)
```

Maxima [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx = \int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^5}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))/x^5, x)
```

Giac [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx = \int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^5}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x^5,x, algorithm="giac")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx = \int \frac{\sqrt{a\,x+b\,x^{1/3}}}{x^5}\,dx$$

input
```
int((a*x + b*x^(1/3))^(1/2)/x^5,x)
```

output
```
int((a*x + b*x^(1/3))^(1/2)/x^5, x)
```

Reduce [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx = \frac{-\frac{2\sqrt{x^{\frac{2}{3}}a+b}}{7}-\frac{2x^{\frac{23}{6}}\left(\int \frac{\sqrt{x^{\frac{2}{3}}a+b}}{x^{\frac{29}{6}}b+\sqrt{x}\,a\,x^5}dx\right)b}{21}}{x^{\frac{23}{6}}}$$

input

```
int((b*x^(1/3)+a*x)^(1/2)/x^5,x)
```

output

```
(2*( - 3*sqrt(x**(2/3)*a + b) - x**(5/6)*int(sqrt(x**(2/3)*a + b)/(x**(5/6
)*b*x**4 + sqrt(x)*a*x**5),x)*b*x**3))/(21*x**(5/6)*x**3)
```

3.119 $\int x^2 \left(b\sqrt[3]{x}+ax\right)^{3/2} \, dx$

Optimal result 982
Mathematica [C] (verified) 983
Rubi [A] (warning: unable to verify) 983
Maple [A] (verified) 994
Fricas [F] 995
Sympy [F] 995
Maxima [F] 995
Giac [F] 996
Mupad [F(-1)] 996
Reduce [F] 996

Optimal result

Integrand size = 19, antiderivative size = 298

$$\int x^2 \left(b\sqrt[3]{x}+ax\right)^{3/2} \, dx = \frac{1768b^6\sqrt{b\sqrt[3]{x}+ax}}{100947a^5} - \frac{1768b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{168245a^4} + \frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3} - \frac{136b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^2} + \frac{8b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a} + \frac{4}{69}bx^{10/3}\sqrt{b\sqrt[3]{x}+ax} + \frac{2}{9}x^3\left(b\sqrt[3]{x}+ax\right)^{3/2} - \frac{884b^{27/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{100947a^{21/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
1768/100947*b^6*(b*x^(1/3)+a*x)^(1/2)/a^5-1768/168245*b^5*x^(2/3)*(b*x^(1/
3)+a*x)^(1/2)/a^4+1768/216315*b^4*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a^3-136/19
665*b^3*x^2*(b*x^(1/3)+a*x)^(1/2)/a^2+8/1311*b^2*x^(8/3)*(b*x^(1/3)+a*x)^(
1/2)/a+4/69*b*x^(10/3)*(b*x^(1/3)+a*x)^(1/2)+2/9*x^3*(b*x^(1/3)+a*x)^(3/2)
-884/100947*b^(27/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(
1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*InverseJacobiAM(2*arctan(a^(1/4)*x^(1/6)/b^
(1/4)),1/2*2^(1/2))/a^(21/4)/(b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.48

$$\int x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\frac{2\sqrt{b\sqrt[3]{x}+ax}\left(\left(b+ax^{2/3}\right)^2\sqrt{1+\frac{ax^{2/3}}{b}}\left(3315b^4-7293ab^3x^{2/3}+12155a^2b^2x^{4/3}-17765a^3bx\right.\right.}{216315a^5\sqrt{1+\frac{ax^{2/3}}{b}}}$$

input
```
Integrate[x^2*(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
(2*Sqrt[b*x^(1/3) + a*x]*((b + a*x^(2/3))^2*Sqrt[1 + (a*x^(2/3))/b]*(3315*
b^4 - 7293*a*b^3*x^(2/3) + 12155*a^2*b^2*x^(4/3) - 17765*a^3*b*x^2 + 24035
*a^4*x^(8/3)) - 3315*b^6*Hypergeometric2F1[-3/2, 1/4, 5/4, -((a*x^(2/3))/b
)]))/(216315*a^5*Sqrt[1 + (a*x^(2/3))/b])
```

Rubi [A] (warning: unable to verify)

Time = 0.99 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {1924, 1927, 1927, 1930, 1930, 1930, 1930, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(ax+b\sqrt[3]{x}\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1924}$$

$$3\int x^{8/3}\left(\sqrt[3]{x}b+ax\right)^{3/2}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1927}$$

$$3\left(\frac{2}{9}b\int x^3\sqrt{\sqrt[3]{x}b+ax}d\sqrt[3]{x}+\frac{2}{27}x^3\left(ax+b\sqrt[3]{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{1927}$$

$$3\left(\frac{2}{9}b\left(\frac{2}{23}b\int \frac{x^{10/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}+\frac{2}{23}x^{10/3}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{27}x^3\left(ax+b\sqrt[3]{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2}{9}b\left(\frac{2}{23}b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\int \frac{x^{8/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{19a}\right)+\frac{2}{23}x^{10/3}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{27}x^3\left(ax+b\sqrt[3]{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2}{9}b\left(\frac{2}{23}b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\int \frac{x^2}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{15a}\right)}{19a}\right)+\frac{2}{23}x^{10/3}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{27}x^3\right.$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2}{9}b\left(\frac{2}{23}b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\int \frac{x^{4/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{11a}\right)}{15a}\right)}{19a}\right)+\frac{2}{23}x^{10/3}\sqrt{\right.\right.\right.$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2}{9}b\left(\frac{2}{23}b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\int\frac{x^{2/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{7a}\right)}{11a}\right.}{15a}\right.}{19a}\right.\right.\right.$$

$\downarrow$ 1930

$$3\left(\frac{2}{9}b\left(\frac{2}{23}b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b\int\cdots}{\cdots}\right)}{7a}\cdots\right)}{11a}\cdots\right)}{15a}\cdots\right)}{19a}\cdots\right.\right.\right.$$

$\downarrow$ 1917

$$3\left(\frac{2}{9}b\left(\frac{2}{23}b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b\sqrt[6]{x}}{\cdots}\right.}{\cdots}\right.}{11a}\right.}{15a}\right.}{19a}\right.\right.\right.$$

↓ 266

$$3\left(\frac{2}{9}b\left(\frac{2}{23}b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{2b\sqrt[6]{\ldots}}{\ldots}\right.}{\ldots}\right.}{11a}\right.}{15a}\right.}{19a}\right.\right.\right.$$

↓ 761

$$3\left(\frac{2}{9}b\left(\frac{2}{23}b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b^{3/4}\,\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{19a}\right.\right.\right.$$

input
```
Int[x^2*(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
3*((2*x^3*(b*x^(1/3) + a*x)^(3/2))/27 + (2*b*((2*x^(10/3)*Sqrt[b*x^(1/3) +
 a*x])/23 + (2*b*((2*x^(8/3)*Sqrt[b*x^(1/3) + a*x])/(19*a) - (17*b*((2*x^2
*Sqrt[b*x^(1/3) + a*x])/(15*a) - (13*b*((2*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/
(11*a) - (9*b*((2*x^(2/3)*Sqrt[b*x^(1/3) + a*x])/(7*a) - (5*b*((2*Sqrt[b*x
^(1/3) + a*x])/(3*a) - (b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b + a*x^(
2/3)]*x^(1/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*Elliptic
F[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[b*x^(1/3) + a
*x]*Sqrt[b + a*x^(4/3)])))/(7*a)))/(11*a)))/(15*a)))/(19*a)))/23))/9)
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.66

method	result
default	$-\frac{2\left(-216755x^{\frac{11}{3}}a^{6}b^{2}-380380x^{\frac{13}{3}}a^{7}b+616a^{5}b^{3}x^{3}+6630b^{7}\sqrt{-ab}\,\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\ \mathrm{Ellip}\right.}{1514205a^{6}\sqrt{x^{\frac{1}{3}}\left(b+a\right.}}$
derivativedivides	$\frac{2ax^{4}\sqrt{b\,x^{\frac{1}{3}}+ax}}{9}+\frac{58b\,x^{\frac{10}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{207}+\frac{8b^{2}x^{\frac{8}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1311a}-\frac{136b^{3}x^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{19665a^{2}}+\frac{1768b^{4}x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{216315a^{3}}-$

input
```
int(x^2*(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/1514205*(-216755*x^(11/3)*a^6*b^2-380380*x^(13/3)*a^7*b+616*a^5*b^3*x^3
+6630*b^7*(-a*b)^(1/2)*((x^(1/3)*a+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/
2)*((-x^(1/3)*a+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2)*a
)^(1/2)*EllipticF(((x^(1/3)*a+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2
))+1768*x^(5/3)*a^3*b^5-952*x^(7/3)*a^4*b^4-168245*a^8*x^5-5304*a^2*b^6*x-
13260*x^(1/3)*a*b^7)/a^6/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

Fricas [F]

$$\int x^2 \left(b\sqrt[3]{x}+ax\right)^{3/2} dx = \int \left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a*x^3 + b*x^(7/3))*sqrt(a*x + b*x^(1/3)), x)
```

Sympy [F]

$$\int x^2 \left(b\sqrt[3]{x}+ax\right)^{3/2} dx = \int x^2 \left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}} dx$$

input
```
integrate(x**2*(b*x**(1/3)+a*x)**(3/2),x)
```

output
```
Integral(x**2*(a*x + b*x**(1/3))**(3/2), x)
```

Maxima [F]

$$\int x^2 \left(b\sqrt[3]{x}+ax\right)^{3/2} dx = \int \left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)*x^2, x)
```

Giac [F]

$$\int x^2 \left(b\sqrt[3]{x}+ax\right)^{3/2} \, dx=\int \left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(b\sqrt[3]{x}+ax\right)^{3/2} \, dx=\int x^2\,{\left(a\,x+b\,x^{1/3}\right)}^{3/2} \,d x$$

input
```
int(x^2*(a*x + b*x^(1/3))^(3/2),x)
```

output
```
int(x^2*(a*x + b*x^(1/3))^(3/2), x)
```

Reduce [F]

$$\int x^2 \left(b\sqrt[3]{x}+ax\right)^{3/2} \, dx=\frac{\frac{8x^{\frac{17}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a^4b^2}{1311}-\frac{1768x^{\frac{5}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a\,b^5}{168245}+\frac{58\sqrt{x}\,\sqrt{x^{\frac{2}{3}}a+b}\,a^5b\,x^3}{207}+\frac{1768\sqrt{x}\,\sqrt{x^{\frac{2}{3}}a+b}\,a^2b^4x}{216315}+\frac{2x^{\frac{25}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a^6}{9}}{a^5}$$

input
```
int(x^2*(b*x^(1/3)+a*x)^(3/2),x)
```

output
```
(2*(4620*x**(5/6)*sqrt(x**(2/3)*a + b)*a**4*b**2*x**2 - 7956*x**(5/6)*sqrt
(x**(2/3)*a + b)*a*b**5 + 212135*sqrt(x)*sqrt(x**(2/3)*a + b)*a**5*b*x**3
+ 6188*sqrt(x)*sqrt(x**(2/3)*a + b)*a**2*b**4*x + 168245*x**(1/6)*sqrt(x**
(2/3)*a + b)*a**6*x**4 - 5236*x**(1/6)*sqrt(x**(2/3)*a + b)*a**3*b**3*x**2
 + 13260*x**(1/6)*sqrt(x**(2/3)*a + b)*b**6 - 2210*int(sqrt(x**(2/3)*a + b
)/(x**(5/6)*b + sqrt(x)*a*x),x)*b**7))/(1514205*a**5)
```

3.120 $\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

Optimal result 998
Mathematica [C] (verified) 999
Rubi [A] (warning: unable to verify) 999
Maple [A] (verified) 1008
Fricas [F] 1008
Sympy [F] 1009
Maxima [F] 1009
Giac [F] 1009
Mupad [F(-1)] 1010
Reduce [F] 1010

Optimal result

Integrand size = 17, antiderivative size = 408

$$\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=-\frac{88b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{1105a^{7/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{88b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{3315a^3}-\frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2}+\frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a}+\frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{7}x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{88b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105a^{15/4}\sqrt{b\sqrt[3]{x}+ax}}-\frac{44b^{21/4}\left(\sqrt{b}\right.}{}$$

output
```
-88/1105*b^5*(b+a*x^(2/3))*x^(1/3)/a^(7/2)/(b^(1/2)+a^(1/2)*x^(1/3))/(b*x^
(1/3)+a*x)^(1/2)+88/3315*b^4*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^3-88/4641*b^3
*x*(b*x^(1/3)+a*x)^(1/2)/a^2+24/1547*b^2*x^(5/3)*(b*x^(1/3)+a*x)^(1/2)/a+1
2/119*b*x^(7/3)*(b*x^(1/3)+a*x)^(1/2)+2/7*x^2*(b*x^(1/3)+a*x)^(3/2)+88/110
5*b^(21/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/
3))^2)^(1/2)*x^(1/6)*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*
2^(1/2))/a^(15/4)/(b*x^(1/3)+a*x)^(1/2)-44/1105*b^(21/4)*(b^(1/2)+a^(1/2)*
x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*Inverse
JacobiAM(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)),1/2*2^(1/2))/a^(15/4)/(b*x^(1/3
)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.30

$$\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\frac{2\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}\left(\left(b+ax^{2/3}\right)^2\sqrt{1+\frac{ax^{2/3}}{b}}\left(77b^2-143abx^{2/3}+221a^2x^{4/3}\right)-77b^4\,\text{Hypergeo}\right.}{1547a^3\sqrt{1+\frac{ax^{2/3}}{b}}}$$

input
```
Integrate[x*(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
(2*x^(1/3)*Sqrt[b*x^(1/3) + a*x]*((b + a*x^(2/3))^2*Sqrt[1 + (a*x^(2/3))/b
]*(77*b^2 - 143*a*b*x^(2/3) + 221*a^2*x^(4/3)) - 77*b^4*Hypergeometric2F1[
-3/2, 3/4, 7/4, -((a*x^(2/3))/b)]))/(1547*a^3*Sqrt[1 + (a*x^(2/3))/b])
```

Rubi [A] (warning: unable to verify)

Time = 1.04 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {1924, 1927, 1927, 1930, 1930, 1930, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax+b\sqrt[3]{x}\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1924}$$

$$3\int x^{5/3}\left(\sqrt[3]{x}b+ax\right)^{3/2}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1927}$$

$$3\left(\frac{2}{7}b\int x^2\sqrt{\sqrt[3]{x}b+ax}d\sqrt[3]{x}+\frac{2}{21}x^2\left(ax+b\sqrt[3]{x}\right)^{3/2}\right)$$

$$\Big\downarrow \; 1927$$

$$3\left(\frac{2}{7}b\left(\frac{2}{17}b\int \frac{x^{7/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}+\frac{2}{17}x^{7/3}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{21}x^2\left(ax+b\sqrt[3]{x}\right)^{3/2}\right)$$

$$\Big\downarrow \; 1930$$

$$3\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\int \frac{x^{5/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{13a}\right)+\frac{2}{17}x^{7/3}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{21}x^2\left(ax+b\sqrt[3]{x}\right)^{3/2}\right)$$

$$\Big\downarrow \; 1930$$

$$3\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\int \frac{x}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{9a}\right)}{13a}\right)+\frac{2}{17}x^{7/3}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{21}x^2(a$$

$$\Big\downarrow \; 1930$$

$$3\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\int \frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{5a}\right)}{9a}\right)}{13a}\right)+\frac{2}{17}x^{7/3}\sqrt{ax$$

$$\Big\downarrow \; 1938$$

$$3\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)}{9a}\right)}{13a}\right)+\cdots\right.\right.$$

$$\Big\downarrow \text{ 266}$$

$$3\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)}{9a}\right)}{13a}\right)+\cdots\right.\right.$$

$$\Big\downarrow \text{ 834}$$

$$3\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}\,d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int\cdots}{\cdots}\cdots\right.}{5a\sqrt{ax+b\sqrt[3]{x}}}\cdots\right.}{9a}\cdots\right.}{13a}\cdots\right.\right.\right.$$

$\downarrow$ 27

$$3\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int\frac{\sqrt{b}-\cdots}{\sqrt{ax\cdots}}\cdots}{\cdots}\cdots\right.}{5a\sqrt{ax+b\sqrt[3]{x}}}\cdots\right.}{9a}\cdots\right.}{13a}\cdots\right.\right.\right.$$

$\downarrow$ 761

$$3\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+}{\left(\sqrt{a}x^{2/3}+\right.}}}{2a^3}\right.}{}\right.}{9a}\right.}{13a}\right.\right.\right.$$

↓ 1510

$$3\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+}{\left(\sqrt{a}x^{2/3}+\right.}}}{2a^3}\right.}{}\right.}{}\right.}{}\right.\right.\right.$$

input
```
Int[x*(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
3*((2*x^2*(b*x^(1/3) + a*x)^(3/2))/21 + (2*b*((2*x^(7/3)*Sqrt[b*x^(1/3) +
a*x])/17 + (2*b*((2*x^(5/3)*Sqrt[b*x^(1/3) + a*x])/(13*a) - (11*b*((2*x*Sq
rt[b*x^(1/3) + a*x])/(9*a) - (7*b*((2*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(5*a)
 - (6*b*Sqrt[b + a*x^(2/3)]*x^(1/6)*(-((-((x^(1/6)*Sqrt[b + a*x^(4/3)])/(S
qrt[b] + Sqrt[a]*x^(2/3))) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b
+ a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticE[2*ArcTan[(a^(1/4)*x^
(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b + a*x^(4/3)]))/Sqrt[a]) + (b^(1/4)*
(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3
))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(3/4)*Sqrt
[b + a*x^(4/3)])))/(5*a*Sqrt[b*x^(1/3) + a*x])))/(9*a)))/(13*a)))/17))/7)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.64

method	result
default	$\frac{\frac{622 x^{\frac{8}{3}} a^{4} b^{2}}{1547}+\frac{80 x^{\frac{10}{3}} a^{5} b}{119}-\frac{16 a^{3} b^{3} x^{2}}{4641}-\frac{88 b^{6} \sqrt{\frac{x^{\frac{1}{3}} a+\sqrt{-a b}}{\sqrt{-a b}}}\, \sqrt{-\frac{2\left(x^{\frac{1}{3}} a-\sqrt{-a b}\right)}{\sqrt{-a b}}}\, \sqrt{-\frac{x^{\frac{1}{3}} a}{\sqrt{-a b}}}\, \operatorname{EllipticE}\left(\sqrt{\frac{x^{\frac{1}{3}} a+\sqrt{-a b}}{\sqrt{-a b}}}, \frac{\sqrt{2}}{2}\right)}{1105}+ \cdots 44}{a^{4} \sqrt{x^{\frac{1}{3}}\left(b+a \cdots\right.}}$ 44b
derivativedivides	$\frac{2 a x^{3} \sqrt{b x^{\frac{1}{3}}+a x}}{7}+\frac{46 b x^{\frac{7}{3}} \sqrt{b x^{\frac{1}{3}}+a x}}{119}+\frac{24 b^{2} x^{\frac{5}{3}} \sqrt{b x^{\frac{1}{3}}+a x}}{1547 a}-\frac{88 b^{3} x \sqrt{b x^{\frac{1}{3}}+a x}}{4641 a^{2}}+\frac{88 b^{4} x^{\frac{1}{3}} \sqrt{b x^{\frac{1}{3}}+a x}}{3315 a^{3}}-\cdots$

input
```
int(x*(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/23205/a^4*(4665*x^(8/3)*a^4*b^2+7800*x^(10/3)*a^5*b-40*a^3*b^3*x^2-924*b
^6*((x^(1/3)*a+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)*a-(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2)*a)^(1/2)*EllipticE(((x^(1/3
)*a+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+462*b^6*((x^(1/3)*a+(-a
*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)*a-(-a*b)^(1/2))/(-a*b)^(1/2))^
(1/2)*(-x^(1/3)/(-a*b)^(1/2)*a)^(1/2)*EllipticF(((x^(1/3)*a+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+3315*a^6*x^4+308*x^(2/3)*a*b^5+88*x^(4/3)
*a^2*b^4)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

Fricas [F]

$$\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a*x^2 + b*x^(4/3))*sqrt(a*x + b*x^(1/3)), x)
```

Sympy [F]

$$\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int x\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(b*x**(1/3)+a*x)**(3/2),x)
```

output
```
Integral(x*(a*x + b*x**(1/3))**(3/2), x)
```

Maxima [F]

$$\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)*x, x)
```

Giac [F]

$$\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int x\,\left(a\,x+b\,x^{1/3}\right)^{3/2}\,dx$$

input
```
int(x*(a*x + b*x^(1/3))^(3/2),x)
```

output
```
int(x*(a*x + b*x^(1/3))^(3/2), x)
```

Reduce [F]

$$\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\frac{\frac{24x^{\frac{11}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a^2b^2}{1547}+\frac{46\sqrt{x}\,\sqrt{x^{\frac{2}{3}}a+b}\,a^3b\,x^2}{119}+\frac{88\sqrt{x}\,\sqrt{x^{\frac{2}{3}}a+b}\,b^4}{3315}+\frac{2x^{\frac{19}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a^4}{7}-\frac{88x^{\frac{7}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a\,b^3}{4641}-\frac{44\left(\right.}{}}{a^3}$$

input
```
int(x*(b*x^(1/3)+a*x)^(3/2),x)
```

output
```
(2*(180*x**(5/6)*sqrt(x**(2/3)*a + b)*a**2*b**2*x + 4485*sqrt(x)*sqrt(x**(
2/3)*a + b)*a**3*b*x**2 + 308*sqrt(x)*sqrt(x**(2/3)*a + b)*b**4 + 3315*x**
(1/6)*sqrt(x**(2/3)*a + b)*a**4*x**3 - 220*x**(1/6)*sqrt(x**(2/3)*a + b)*a
*b**3*x - 154*int((x**(1/6)*sqrt(x**(2/3)*a + b))/(x**(2/3)*b + x**(1/3)*a
*x),x)*b**5))/(23205*a**3)
```

3.121 $\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

Optimal result 1011
Mathematica [C] (verified) 1012
Rubi [A] (warning: unable to verify) 1012
Maple [A] (verified) 1015
Fricas [F] 1016
Sympy [F] 1017
Maxima [F] 1017
Giac [F] 1017
Mupad [B] (verification not implemented) 1018
Reduce [F] 1018

Optimal result

Integrand size = 15, antiderivative size = 208

$$\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=-\frac{8b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^2}+\frac{24b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a}+\frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{4b^{15/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77a^{9/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
-8/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^2+24/385*b^2*x^(2/3)*(b*x^(1/3)+a*x)^(1/
2)/a+12/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)+2/5*x*(b*x^(1/3)+a*x)^(3/2)+4/7
7*b^(15/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/
3))^2)^(1/2)*x^(1/6)*InverseJacobiAM(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)),1/2
*2^(1/2))/a^(9/4)/(b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.51

$$\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\frac{2\sqrt{b\sqrt[3]{x}+ax}\left(-\left(\left(5b-11ax^{2/3}\right)\left(b+ax^{2/3}\right)^2\sqrt{1+\frac{ax^{2/3}}{b}}\right)+5b^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\ldots\right)\right)}{55a^2\sqrt{1+\frac{ax^{2/3}}{b}}}$$

input
```
Integrate[(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
(2*Sqrt[b*x^(1/3) + a*x]*(-((5*b - 11*a*x^(2/3))*(b + a*x^(2/3))^2*Sqrt[1
+ (a*x^(2/3))/b]) + 5*b^3*Hypergeometric2F1[-3/2, 1/4, 5/4, -((a*x^(2/3))/
b)]))/(55*a^2*Sqrt[1 + (a*x^(2/3))/b])
```

Rubi [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {1910, 1924, 1927, 1930, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+b\sqrt[3]{x}\right)^{3/2}dx$$

$$\Big\downarrow \text{1910}$$

$$\frac{2}{5}b\int \sqrt[3]{x}\sqrt{\sqrt[3]{x}b+ax}dx+\frac{2}{5}x\left(ax+b\sqrt[3]{x}\right)^{3/2}$$

$$\Big\downarrow \text{1924}$$

$$\frac{6}{5}b\int x\sqrt{\sqrt[3]{x}b+ax}d\sqrt[3]{x}+\frac{2}{5}x\left(ax+b\sqrt[3]{x}\right)^{3/2}$$

$$\Big\downarrow \text{1927}$$

$$\frac{6}{5}b\left(\frac{2}{11}b\int \frac{x^{4/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}+\frac{2}{11}x^{4/3}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{5}x\left(ax+b\sqrt[3]{x}\right)^{3/2}$$

$$\Big\downarrow \text{1930}$$

$$\frac{6}{5}b\left(\frac{2}{11}b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\int \frac{x^{2/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{7a}\right)+\frac{2}{11}x^{4/3}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{5}x\left(ax+b\sqrt[3]{x}\right)^{3/2}$$

$$\Big\downarrow \text{1930}$$

$$\frac{6}{5}b\left(\frac{2}{11}b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b\int \frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{3a}\right)}{7a}\right)+\frac{2}{11}x^{4/3}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{5}x\left(ax+b\sqrt[3]{x}\right)^{3/2}$$

$$\Big\downarrow \text{1917}$$

$$\frac{6}{5}b\left(\frac{2}{11}b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{3a\sqrt{ax+b\sqrt[3]{x}}}\right)}{7a}\right)+\frac{2}{11}x^{4/3}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{5}x\left(ax+b\sqrt[3]{x}\right)^{3/2}$$

$$\Big\downarrow \text{266}$$

$$\frac{6}{5}b\left(\frac{2}{11}b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{2b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3a\sqrt{ax+b\sqrt[3]{x}}}\right)}{7a}\right)+\frac{2}{11}x^{4/3}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{5}x\left(ax+b\sqrt[3]{x}\right)^{3/2}$$

$$\Big\downarrow \text{761}$$

$$\frac{6}{5}b\left(\frac{2}{11}b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}}{\sqrt[4]{}}\right.\right.}{3a^{5/4}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}\right.}{7a}\right.\right.$$

$$\frac{2}{5}x\left(ax+b\sqrt[3]{x}\right)^{3/2}$$

input
```
Int[(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
(2*x*(b*x^(1/3) + a*x)^(3/2))/5 + (6*b*((2*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/
11 + (2*b*((2*x^(2/3)*Sqrt[b*x^(1/3) + a*x])/(7*a) - (5*b*((2*Sqrt[b*x^(1/
3) + a*x])/(3*a) - (b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b + a*x^(2/3)
]*x^(1/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*
ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[b*x^(1/3) + a*x]*
Sqrt[b + a*x^(4/3)])))/(7*a)))/11))/5
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.78

method	result
default	$\frac{\frac{262 x^{\frac{5}{3}} a^{3} b^{2}}{385}+\frac{56 x^{\frac{7}{3}} a^{4} b}{55}+\frac{4 b^{4} \sqrt{-a b}\, \sqrt{\frac{x^{\frac{1}{3}} a+\sqrt{-a b}}{\sqrt{-a b}}}\, \sqrt{-\frac{2 \left(x^{\frac{1}{3}} a-\sqrt{-a b}\right)}{\sqrt{-a b}}}\, \sqrt{-\frac{x^{\frac{1}{3}} a}{\sqrt{-a b}}}\, \operatorname{EllipticF}\left(\sqrt{\frac{x^{\frac{1}{3}} a+\sqrt{-a b}}{\sqrt{-a b}}}, \frac{\sqrt{2}}{2}\right)}{77}-\frac{16 a^{2} b^{3} x}{385}+}{a^{3} \sqrt{x^{\frac{1}{3}} \left(b+a\,x^{\frac{2}{3}}\right)}}$
derivativedivides	$\frac{2 a\,x^{2} \sqrt{b\,x^{\frac{1}{3}}+a x}}{5}+\frac{34 b\,x^{\frac{4}{3}} \sqrt{b\,x^{\frac{1}{3}}+a x}}{55}+\frac{24 b^{2} x^{\frac{2}{3}} \sqrt{b\,x^{\frac{1}{3}}+a x}}{385 a}-\frac{8 b^{3} \sqrt{b\,x^{\frac{1}{3}}+a x}}{77 a^{2}}+\frac{4 b^{4} \sqrt{-a b}\, \sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-a b}}{a}\right) a}{\sqrt{-a b}}}}{}$

input
```
int((b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/385*(131*x^(5/3)*a^3*b^2+196*x^(7/3)*a^4*b+10*b^4*(-a*b)^(1/2)*((x^(1/3)
*a+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)*a-(-a*b)^(1/2))/(-a*b)^(
1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2)*a)^(1/2)*EllipticF(((x^(1/3)*a+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-8*a^2*b^3*x+77*a^5*x^3-20*x^(1/3)*a
*b^4)/a^3/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

Fricas [F]

$$\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int \left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a*x + b*x^(1/3))^(3/2), x)
```

Sympy [F]

$$\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int \left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x**(1/3)+a*x)**(3/2),x)
```

output
```
Integral((a*x + b*x**(1/3))**(3/2), x)
```

Maxima [F]

$$\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int \left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2), x)
```

Giac [F]

$$\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int \left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 8.56 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.19

$$\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\frac{2\,x\,{\left(a\,x+b\,x^{1/3}\right)}^{3/2}\,{{}}_2{\mathrm{F}}_1\left(-\frac{3}{2},\frac{9}{4};\ \frac{13}{4};\ -\frac{a\,x^{2/3}}{b}\right)}{3\,{\left(\frac{a\,x^{2/3}}{b}+1\right)}^{3/2}}$$

input
```
int((a*x + b*x^(1/3))^(3/2),x)
```

output
```
(2*x*(a*x + b*x^(1/3))^(3/2)*hypergeom([-3/2, 9/4], 13/4, -(a*x^(2/3))/b))
/(3*((a*x^(2/3))/b + 1)^(3/2))
```

Reduce [F]

$$\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\frac{\frac{24x^{\frac{5}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a\,b^{2}}{385}+\frac{34\sqrt{x}\,\sqrt{x^{\frac{2}{3}}a+b}\,a^{2}bx}{55}+\frac{2x^{\frac{13}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a^{3}}{5}-\frac{8x^{\frac{1}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,b^{3}}{77}+\frac{4\left(\int \frac{\sqrt{x^{\frac{2}{3}}a+b}}{x^{\frac{5}{6}}b+\sqrt{x}\,ax}dx\right)b^{4}}{231}}{a^{2}}$$

input
```
int((b*x^(1/3)+a*x)^(3/2),x)
```

output
```
(2*(36*x**(5/6)*sqrt(x**(2/3)*a + b)*a*b**2 + 357*sqrt(x)*sqrt(x**(2/3)*a
+ b)*a**2*b*x + 231*x**(1/6)*sqrt(x**(2/3)*a + b)*a**3*x**2 - 60*x**(1/6)*
sqrt(x**(2/3)*a + b)*b**3 + 10*int(sqrt(x**(2/3)*a + b)/(x**(5/6)*b + sqrt
(x)*a*x),x)*b**4))/(1155*a**2)
```

3.122 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx$

Optimal result 1019
Mathematica [C] (verified) 1020
Rubi [A] (warning: unable to verify) 1020
Maple [A] (verified) 1024
Fricas [F] 1025
Sympy [F] 1025
Maxima [F] 1026
Giac [F] 1026
Mupad [F(-1)] 1026
Reduce [F] 1027

Optimal result

Integrand size = 19, antiderivative size = 319

$$\begin{aligned}\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx &= \frac{8b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5\sqrt{a}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} \\ &+ \frac{4}{5}b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2} \\ &- \frac{8b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{b\sqrt[3]{x}+ax}} \\ &+ \frac{4b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5a^{3/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

output
```
8/5*b^2*(b+a*x^(2/3))*x^(1/3)/a^(1/2)/(b^(1/2)+a^(1/2)*x^(1/3))/(b*x^(1/3)
+a*x)^(1/2)+4/5*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)+2/3*(b*x^(1/3)+a*x)^(3/2)-
8/5*b^(9/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1
/3))^2)^(1/2)*x^(1/6)*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2
*2^(1/2))/a^(3/4)/(b*x^(1/3)+a*x)^(1/2)+4/5*b^(9/4)*(b^(1/2)+a^(1/2)*x^(1/
3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*InverseJacob
iAM(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)),1/2*2^(1/2))/a^(3/4)/(b*x^(1/3)+a*x)
^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.19

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx=\frac{2b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-\frac{ax^{2/3}}{b}\right)}{\sqrt{1+\frac{ax^{2/3}}{b}}}$$

input
```
Integrate[(b*x^(1/3) + a*x)^(3/2)/x,x]
```

output
```
(2*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-3/2, 3/4, 7/4, -((a*
x^(2/3))/b)])/Sqrt[1 + (a*x^(2/3))/b]
```

Rubi [A] (warning: unable to verify)

Time = 0.75 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1924, 1927, 1910, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+b\sqrt[3]{x}\right)^{3/2}}{x}\,dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{\left(\sqrt[3]{x}b+ax\right)^{3/2}}{\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1927}$$

$$3\left(\frac{2}{3}b\int \sqrt{\sqrt[3]{x}b+ax}d\sqrt[3]{x}+\frac{2}{9}\left(ax+b\sqrt[3]{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{1910}$$

$$3\left(\frac{2}{3}b\left(\frac{2}{5}b\int \frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}+\frac{2}{5}\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{9}\left(ax+b\sqrt[3]{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{1938}$$

$$3\left(\frac{2}{3}b\left(\frac{2b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{5\sqrt{ax+b\sqrt[3]{x}}}+\frac{2}{5}\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{9}\left(ax+b\sqrt[3]{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{266}$$

$$3\left(\frac{2}{3}b\left(\frac{4b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{5\sqrt{ax+b\sqrt[3]{x}}}+\frac{2}{5}\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{9}\left(ax+b\sqrt[3]{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{834}$$

$$3\left(\frac{2}{3}b\left(\frac{4b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{b}\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{5\sqrt{ax+b\sqrt[3]{x}}}+\frac{2}{5}\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{9}\left(ax+b\sqrt[3]{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{27}$$

$$3\left(\frac{2}{3}b\left(\frac{4b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int \frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{5\sqrt{ax+b\sqrt[3]{x}}}+\frac{2}{5}\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{9}\left(ax+b\sqrt[3]{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{761}$$

$$3\left(\frac{2}{3}b\left(\frac{4b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{5\sqrt{ax+b\sqrt[3]{x}}}+\frac{2}{5}\right.\right.$$

$\downarrow$ 1510

$$3\left(\frac{2}{3}b\left(\frac{4b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}}{\left(\sqrt{a}x^{2/3}+\right.}}}{\sqrt[4]{a}}\right.}{5\sqrt{ax+b\sqrt[3]{x}}}\right.\right.$$

input
```
Int[(b*x^(1/3) + a*x)^(3/2)/x,x]
```

output
```
3*((2*(b*x^(1/3) + a*x)^(3/2))/9 + (2*b*((2*x^(1/3)*Sqrt[b*x^(1/3) + a*x])
/5 + (4*b*Sqrt[b + a*x^(2/3)]*x^(1/6)*(-((-((x^(1/6)*Sqrt[b + a*x^(4/3)])/
(Sqrt[b] + Sqrt[a]*x^(2/3))) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(
b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticE[2*ArcTan[(a^(1/4)*
x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b + a*x^(4/3)]))/Sqrt[a]) + (b^(1/4
)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2
/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(3/4)*Sq
rt[b + a*x^(4/3)])))/(5*Sqrt[b*x^(1/3) + a*x])))/3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{2ax\sqrt{b\,x^{\frac{1}{3}}+ax}}{3}+\frac{22b\,x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{15}+\frac{4b^{2}\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{Elli}}{}\right.}{5a\sqrt{b\,x}}$
default	$\frac{\frac{8b^{3}\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}a-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\ \mathrm{EllipticE}\left(\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{5}-\frac{4b^{3}\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}a-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{}}{5}}{a\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}$

input
```
int((b*x^(1/3)+a*x)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/3*a*x*(b*x^(1/3)+a*x)^(1/2)+22/15*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)+4/5*b^
2/a*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^
(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2)*a)^(1
/2)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b
)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x
^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x,x, algorithm="fricas")
```

output
```
integral((a*x + b*x^(1/3))^(3/2)/x, x)
```

Sympy [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx=\int \frac{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((b*x**(1/3)+a*x)**(3/2)/x,x)
```

output
```
Integral((a*x + b*x**(1/3))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)/x, x)
```

Giac [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x,x, algorithm="giac")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx=\int \frac{\left(a\,x+b\,x^{1/3}\right)^{3/2}}{x}\,dx$$

input
```
int((a*x + b*x^(1/3))^(3/2)/x,x)
```

output
```
int((a*x + b*x^(1/3))^(3/2)/x, x)
```

Reduce [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx = \frac{22\sqrt{x}\,\sqrt{x^{\frac{2}{3}}a+b}\,b}{15}+\frac{2x^{\frac{7}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a}{3}+\frac{4\left(\int \frac{x^{\frac{1}{6}}\sqrt{x^{\frac{2}{3}}a+b}}{x^{\frac{2}{3}}b+x^{\frac{4}{3}}a}dx\right)b^2}{15}$$

input
```
int((b*x^(1/3)+a*x)^(3/2)/x,x)
```

output
```
(2*(11*sqrt(x)*sqrt(x**(2/3)*a + b)*b + 5*x**(1/6)*sqrt(x**(2/3)*a + b)*a*
x + 2*int((x**(1/6)*sqrt(x**(2/3)*a + b))/(x**(2/3)*b + x**(1/3)*a*x),x)*b
**2))/15
```

3.123 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx$

Optimal result . . . 1028
Mathematica [C] (verified) . . . 1029
Rubi [A] (warning: unable to verify) . . . 1029
Maple [A] (verified) . . . 1032
Fricas [F] . . . 1032
Sympy [F] . . . 1033
Maxima [F] . . . 1033
Giac [F] . . . 1033
Mupad [F(-1)] . . . 1034
Reduce [F] . . . 1034

Optimal result

Integrand size = 19, antiderivative size = 144

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx = 4a\sqrt{b\sqrt[3]{x}+ax}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x} + \frac{4a^{3/4}b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
4*a*(b*x^(1/3)+a*x)^(1/2)-2*(b*x^(1/3)+a*x)^(3/2)/x+4*a^(3/4)*b^(3/4)*(b^(
1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)*x^
(1/6)*InverseJacobiAM(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)),1/2*2^(1/2))/(b*x^
(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.42

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx = -\frac{2b\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},\frac{1}{4},-\frac{ax^{2/3}}{b}\right)}{\sqrt{1+\frac{ax^{2/3}}{b}}\,x^{2/3}}$$

input
```
Integrate[(b*x^(1/3) + a*x)^(3/2)/x^2,x]
```

output
```
(-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((a*x^(2/3
))/b)])/(Sqrt[1 + (a*x^(2/3))/b]*x^(2/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1924, 1926, 1927, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+b\sqrt[3]{x}\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{\left(\sqrt[3]{x}b+ax\right)^{3/2}}{x^{4/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1926}$$

$$3\left(2a\int \frac{\sqrt{\sqrt[3]{x}b+ax}}{\sqrt[3]{x}}d\sqrt[3]{x}-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{3x}\right)$$

$$\Big\downarrow \text{1927}$$

$$3\left(2a\left(\frac{2}{3}b\int\frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}+\frac{2}{3}\sqrt{ax+b\sqrt[3]{x}}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{3x}\right)$$

$$\Big\downarrow \text{1917}$$

$$3\left(2a\left(\frac{2b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{3\sqrt{ax+b\sqrt[3]{x}}}+\frac{2}{3}\sqrt{ax+b\sqrt[3]{x}}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{3x}\right)$$

$$\Big\downarrow \text{266}$$

$$3\left(2a\left(\frac{4b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3\sqrt{ax+b\sqrt[3]{x}}}+\frac{2}{3}\sqrt{ax+b\sqrt[3]{x}}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{3x}\right)$$

$$\Big\downarrow \text{761}$$

$$3\left(2a\left(\frac{2b^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}+\frac{2}{3}\sqrt{ax+b\sqrt[3]{x}}\right.\right.$$

input
```
Int[(b*x^(1/3) + a*x)^(3/2)/x^2,x]
```

output
```
3*((-2*(b*x^(1/3) + a*x)^(3/2))/(3*x) + 2*a*((2*Sqrt[b*x^(1/3) + a*x])/3 +
 (2*b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b + a*x^(2/3)]*x^(1/6)*Sqrt[(
b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*
x^(1/6))/b^(1/4)], 1/2])/(3*a^(1/4)*Sqrt[b*x^(1/3) + a*x]*Sqrt[b + a*x^(4/
3)])))
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.90

method	result
default	$\frac{4x^{\frac{1}{3}}\sqrt{-ab}\,\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}a-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)b+2x^{\frac{4}{3}}a^{2}-2b^{2}}{x^{\frac{1}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}$
derivativedivides	$-\frac{2b\sqrt{b\,x^{\frac{1}{3}}+ax}}{x^{\frac{2}{3}}}+2a\sqrt{b\,x^{\frac{1}{3}}+ax}+\frac{4b\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\right.}\right.}{\sqrt{b\,x^{\frac{1}{3}}+ax}}$

input
```
int((b*x^(1/3)+a*x)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2/x^(1/3)*(2*x^(1/3)*(-a*b)^(1/2)*((x^(1/3)*a+(-a*b)^(1/2))/(-a*b)^(1/2))^
(1/2)*(-2*(x^(1/3)*a-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1
/2)*a)^(1/2)*EllipticF(((x^(1/3)*a+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2
^(1/2))*b+x^(4/3)*a^2-b^2)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

Fricas [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
integral((a*x + b*x^(1/3))^(3/2)/x^2, x)
```

Sympy [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx=\int \frac{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x**(1/3)+a*x)**(3/2)/x**2,x)
```

output
```
Integral((a*x + b*x**(1/3))**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx=\int \frac{\left(a\,x+b\,x^{1/3}\right)^{3/2}}{x^2}\,dx$$

input
```
int((a*x + b*x^(1/3))^(3/2)/x^2,x)
```

output
```
int((a*x + b*x^(1/3))^(3/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx=\frac{2x^{\frac{2}{3}}\sqrt{x^{\frac{2}{3}}a+b}\,a-10\sqrt{x^{\frac{2}{3}}a+b}\,b-4\sqrt{x}\left(\int \frac{\sqrt{x^{\frac{2}{3}}a+b}}{\sqrt{x}\,bx+x^{\frac{13}{6}}a}dx\right)b^2}{\sqrt{x}}$$

input
```
int((b*x^(1/3)+a*x)^(3/2)/x^2,x)
```

output
```
(2*(x**(2/3)*sqrt(x**(2/3)*a + b)*a - 5*sqrt(x**(2/3)*a + b)*b - 2*sqrt(x)
*int(sqrt(x**(2/3)*a + b)/(sqrt(x)*b*x + x**(1/6)*a*x**2),x)*b**2))/sqrt(x
)
```

3.124 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx$

Optimal result 1035
Mathematica [C] (verified) 1036
Rubi [A] (warning: unable to verify) 1036
Maple [A] (verified) 1040
Fricas [F] 1041
Sympy [F] 1042
Maxima [F] 1042
Giac [F] 1042
Mupad [F(-1)] 1043
Reduce [F] 1043

Optimal result

Integrand size = 19, antiderivative size = 350

$$\begin{aligned}\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx &= \frac{8a^{5/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} \\ &- \frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2} \\ &- \frac{8a^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{b\sqrt[3]{x}+ax}} \\ &+ \frac{4a^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

output
```
8/5*a^(5/2)*(b+a*x^(2/3))*x^(1/3)/b/(b^(1/2)+a^(1/2)*x^(1/3))/(b*x^(1/3)+a
*x)^(1/2)-4/5*a*(b*x^(1/3)+a*x)^(1/2)/x-8/5*a^2*(b*x^(1/3)+a*x)^(1/2)/b/x^
(1/3)-2/3*(b*x^(1/3)+a*x)^(3/2)/x^2-8/5*a^(9/4)*(b^(1/2)+a^(1/2)*x^(1/3))*
((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*EllipticE(sin(2*
arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))/b^(3/4)/(b*x^(1/3)+a*x)^(1/2
)+4/5*a^(9/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^
(1/3))^2)^(1/2)*x^(1/6)*InverseJacobiAM(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)),
1/2*2^(1/2))/b^(3/4)/(b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.18

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx=-\frac{2b\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},-\frac{3}{2},-\frac{5}{4},-\frac{ax^{2/3}}{b}\right)}{3\sqrt{1+\frac{ax^{2/3}}{b}}x^{5/3}}$$

input
```
Integrate[(b*x^(1/3) + a*x)^(3/2)/x^3,x]
```

output
```
(-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-9/4, -3/2, -5/4, -((a*x^(2/
3))/b)])/(3*Sqrt[1 + (a*x^(2/3))/b]*x^(5/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.84 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1924, 1926, 1926, 1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+b\sqrt[3]{x}\right)^{3/2}}{x^3}\,dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{\left(\sqrt[3]{x}b+ax\right)^{3/2}}{x^{7/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1926}$$

$$3\left(\frac{2}{3}a\int \frac{\sqrt{\sqrt[3]{x}b+ax}}{x^{4/3}}d\sqrt[3]{x}-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{9x^2}\right)$$

$$\Big\downarrow \text{1926}$$

$$3\left(\frac{2}{3}a\left(\frac{2}{5}a\int \frac{1}{\sqrt[3]{x}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5x}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{9x^2}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{a\int \frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5x}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{9x^2}\right)$$

$$\Big\downarrow \text{1938}$$

$$3\left(\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5x}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{9x^2}\right)$$

$$\Big\downarrow \text{266}$$

$$3\left(\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5x}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{9x^2}\right)$$

$$\Big\downarrow \text{834}$$

$$3\left(\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{b}\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5x}\right)\right)$$

$$\Big\downarrow \text{27}$$

$$3\left(\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int \frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5x}\right)-\right.$$

$\downarrow$ 761

$$3\left(\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\int \frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}\right.\right.\right.$$

$\downarrow$ 1510

$$3\left(\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{\cdots}{\left(\sqrt{\cdots}\right.}}}{\cdots}\right.}{b\sqrt{ax+b\sqrt[3]{x}}}\right.\right.\right.$$

input

```
Int[(b*x^(1/3) + a*x)^(3/2)/x^3,x]
```

output
```
3*((-2*(b*x^(1/3) + a*x)^(3/2))/(9*x^2) + (2*a*((-2*Sqrt[b*x^(1/3) + a*x])
/(5*x) + (2*a*((-2*Sqrt[b*x^(1/3) + a*x])/(b*x^(1/3)) + (2*a*Sqrt[b + a*x^
(2/3)]*x^(1/6)*(-((-((x^(1/6)*Sqrt[b + a*x^(4/3)])/(Sqrt[b] + Sqrt[a]*x^(2
/3))) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b]
 + Sqrt[a]*x^(2/3))^2]*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2]
)/(a^(1/4)*Sqrt[b + a*x^(4/3)]))/Sqrt[a]) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^
(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*Arc
Tan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(3/4)*Sqrt[b + a*x^(4/3)])))/(b
*Sqrt[b*x^(1/3) + a*x])))/5))/3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.67

method	result
derivativedivides	$-\frac{2b\sqrt{b\,x^{\frac{1}{3}}+ax}}{3x^{\frac{5}{3}}}-\frac{22a\sqrt{b\,x^{\frac{1}{3}}+ax}}{15x}-\frac{8\left(b+a\,x^{\frac{2}{3}}\right)a^{2}}{5b\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}+\frac{4a^{2}\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-}}{}$
default	$-\frac{2\left(-12a^{2}b\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}a-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,x^{\frac{8}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\operatorname{EllipticE}\left(\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+6a^{2}b\right.}{}$

input
```
int((b*x^(1/3)+a*x)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3*b*(b*x^(1/3)+a*x)^(1/2)/x^(5/3)-22/15*a*(b*x^(1/3)+a*x)^(1/2)/x-8/5*(
b+a*x^(2/3))*a^2/b/(x^(1/3)*(b+a*x^(2/3)))^(1/2)+4/5*a^2/b*(-a*b)^(1/2)*((
x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1
/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2)*a)^(1/2)/(b*x^(1/3)+a*x)
^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(
1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(
1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^{3}}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
integral((a*x + b*x^(1/3))^(3/2)/x^3, x)
```

Sympy [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx=\int \frac{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b*x**(1/3)+a*x)**(3/2)/x**3,x)
```

output
```
Integral((a*x + b*x**(1/3))**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)/x^3, x)
```

Giac [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^3,x, algorithm="giac")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx=\int \frac{\left(a\,x+b\,x^{1/3}\right)^{3/2}}{x^3}\,dx$$

input
```
int((a*x + b*x^(1/3))^(3/2)/x^3,x)
```

output
```
int((a*x + b*x^(1/3))^(3/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx=\frac{-2x^{\frac{2}{3}}\sqrt{x^{\frac{2}{3}}a+b}\,a-\frac{2\sqrt{x^{\frac{2}{3}}a+b}\,b}{7}+\frac{4\sqrt{x}\left(\int \frac{\sqrt{x^{\frac{2}{3}}a+b}}{\sqrt{x}\,b\,x^{2}+x^{\frac{19}{6}}a}dx\right)b^{2}x}{7}}{\sqrt{x}\,x}$$

input
```
int((b*x^(1/3)+a*x)^(3/2)/x^3,x)
```

output
```
(2*( - 7*x**(2/3)*sqrt(x**(2/3)*a + b)*a - sqrt(x**(2/3)*a + b)*b + 2*sqrt
(x)*int(sqrt(x**(2/3)*a + b)/(sqrt(x)*b*x**2 + x**(1/6)*a*x**3),x)*b**2*x)
)/(7*sqrt(x)*x)
```

3.125 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx$

Optimal result 1044
Mathematica [C] (verified) 1045
Rubi [A] (warning: unable to verify) 1045
Maple [A] (verified) 1048
Fricas [F] 1049
Sympy [F] 1049
Maxima [F] 1050
Giac [F] 1050
Mupad [F(-1)] 1050
Reduce [F] 1051

Optimal result

Integrand size = 19, antiderivative size = 213

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx = -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{385bx^{4/3}} + \frac{8a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3} + \frac{4a^{15/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77b^{9/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
-12/55*a*(b*x^(1/3)+a*x)^(1/2)/x^2-24/385*a^2*(b*x^(1/3)+a*x)^(1/2)/b/x^(4
/3)+8/77*a^3*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2/3)-2/5*(b*x^(1/3)+a*x)^(3/2)/x
^3+4/77*a^(15/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)
*x^(1/3))^2)^(1/2)*x^(1/6)*InverseJacobiAM(2*arctan(a^(1/4)*x^(1/6)/b^(1/4
)),1/2*2^(1/2))/b^(9/4)/(b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.29

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx=-\frac{2b\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{15}{4},-\frac{3}{2},-\frac{11}{4},-\frac{ax^{2/3}}{b}\right)}{5\sqrt{1+\frac{ax^{2/3}}{b}}\,x^{8/3}}$$

input
```
Integrate[(b*x^(1/3) + a*x)^(3/2)/x^4,x]
```

output
```
(-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-15/4, -3/2, -11/4, -((a*x^(
2/3))/b)])/(5*Sqrt[1 + (a*x^(2/3))/b]*x^(8/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1924, 1926, 1926, 1931, 1931, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+b\sqrt[3]{x}\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{\left(\sqrt[3]{x}b+ax\right)^{3/2}}{x^{10/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1926}$$

$$3\left(\frac{2}{5}a\int \frac{\sqrt{\sqrt[3]{x}b+ax}}{x^{7/3}}d\sqrt[3]{x}-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{15x^3}\right)$$

$$\Big\downarrow \text{1926}$$

$$3\left(\frac{2}{5}a\left(\frac{2}{11}a\int\frac{1}{x^{4/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11x^2}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{15x^3}\right)$$

$\Big\downarrow$ 1931

$$3\left(\frac{2}{5}a\left(\frac{2}{11}a\left(-\frac{5a\int\frac{1}{x^{2/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11x^2}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{15x^3}\right)$$

$\Big\downarrow$ 1931

$$3\left(\frac{2}{5}a\left(\frac{2}{11}a\left(-\frac{5a\left(-\frac{a\int\frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{3b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11x^2}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/}}{15x^3}\right.$$

$\Big\downarrow$ 1917

$$3\left(\frac{2}{5}a\left(\frac{2}{11}a\left(-\frac{5a\left(-\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11x^2}\right)-\right.$$

$\Big\downarrow$ 266

$$3\left(\frac{2}{5}a\left(\frac{2}{11}a\left(-\frac{5a\left(-\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11x^2}\right)-\frac{2(a}{}\right.$$

$\Big\downarrow$ 761

$$3\left(\frac{2}{5}a\left(\frac{2}{11}a\left(-\frac{5a\left(-\frac{a^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\right.\right.\right.$$

input
```
Int[(b*x^(1/3) + a*x)^(3/2)/x^4,x]
```

output
```
3*((-2*(b*x^(1/3) + a*x)^(3/2))/(15*x^3) + (2*a*((-2*Sqrt[b*x^(1/3) + a*x]
)/(11*x^2) + (2*a*((-2*Sqrt[b*x^(1/3) + a*x])/(7*b*x^(4/3)) - (5*a*((-2*Sq
rt[b*x^(1/3) + a*x])/(3*b*x^(2/3)) - (a^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*
Sqrt[b + a*x^(2/3)]*x^(1/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3
))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(3*b^(5/4)*Sqrt
[b*x^(1/3) + a*x]*Sqrt[b + a*x^(4/3)])))/(7*b)))/11))/5)
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.79

method	result
default	$\frac{\frac{4a^3\sqrt{-ab}\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}a-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)x^{\frac{14}{3}}}{77}-\frac{262x^{\frac{11}{3}}a^2b^2}{385}+\frac{16x^{\frac{13}{3}}a^3b}{385}-\frac{56a}{}}{b^2\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,x^{\frac{14}{3}}}$
derivativedivides	$-\frac{2b\sqrt{b\,x^{\frac{1}{3}}+ax}}{5x^{\frac{8}{3}}}-\frac{34a\sqrt{b\,x^{\frac{1}{3}}+ax}}{55x^2}-\frac{24a^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{385b\,x^{\frac{4}{3}}}+\frac{8a^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{77b^2x^{\frac{2}{3}}}+\frac{4a^3\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2(}{}}}{}$

input
```
int((b*x^(1/3)+a*x)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
2/385*(10*a^3*(-a*b)^(1/2)*((x^(1/3)*a+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(
-2*(x^(1/3)*a-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2)*a)^
(1/2)*EllipticF(((x^(1/3)*a+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
*x^(14/3)-131*x^(11/3)*a^2*b^2+8*x^(13/3)*a^3*b-196*a*b^3*x^3+20*a^4*x^5-7
7*x^(7/3)*b^4)/b^2/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/x^(14/3)
```

Fricas [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
integral((a*x + b*x^(1/3))^(3/2)/x^4, x)
```

Sympy [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx=\int \frac{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x**(1/3)+a*x)**(3/2)/x**4,x)
```

output
```
Integral((a*x + b*x**(1/3))**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)/x^4, x)
```

Giac [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^4,x, algorithm="giac")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx=\int \frac{\left(a\,x+b\,x^{1/3}\right)^{3/2}}{x^4}\,dx$$

input
```
int((a*x + b*x^(1/3))^(3/2)/x^4,x)
```

output
```
int((a*x + b*x^(1/3))^(3/2)/x^4, x)
```

Reduce [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx = \frac{-\frac{2x^{\frac{2}{3}}\sqrt{x^{\frac{2}{3}}a+b}\,a}{3}-\frac{14\sqrt{x^{\frac{2}{3}}a+b}\,b}{39}+\frac{4\sqrt{x}\left(\int \frac{\sqrt{x^{\frac{2}{3}}a+b}}{\sqrt{x}\,b\,x^3+x^{\frac{25}{6}}a}dx\right)b^2x^2}{39}}{\sqrt{x}\,x^2}$$

input
```
int((b*x^(1/3)+a*x)^(3/2)/x^4,x)
```

output
```
(2*( - 13*x**(2/3)*sqrt(x**(2/3)*a + b)*a - 7*sqrt(x**(2/3)*a + b)*b + 2*s
qrt(x)*int(sqrt(x**(2/3)*a + b)/(sqrt(x)*b*x**3 + x**(1/6)*a*x**4),x)*b**2
*x**2))/(39*sqrt(x)*x**2)
```

3.126 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx$

Optimal result 1052
Mathematica [C] (verified) 1053
Rubi [A] (warning: unable to verify) 1053
Maple [A] (verified) 1068
Fricas [F] 1068
Sympy [F] 1069
Maxima [F] 1069
Giac [F] 1069
Mupad [F(-1)] 1070
Reduce [F] 1070

Optimal result

Integrand size = 19, antiderivative size = 438

$$
\begin{aligned}
\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx &= -\frac{88a^{11/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{1105b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}} \\
&\quad - \frac{88a^4\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{88a^5\sqrt{b\sqrt[3]{x}+ax}}{1105b^4\sqrt[3]{x}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{7x^4} \\
&\quad + \frac{88a^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105b^{15/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad - \frac{44a^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{1105b^{15/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

output
```
-88/1105*a^(11/2)*(b+a*x^(2/3))*x^(1/3)/b^4/(b^(1/2)+a^(1/2)*x^(1/3))/(b*x
^(1/3)+a*x)^(1/2)-12/119*a*(b*x^(1/3)+a*x)^(1/2)/x^3-24/1547*a^2*(b*x^(1/3
)+a*x)^(1/2)/b/x^(7/3)+88/4641*a^3*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(5/3)-88/33
15*a^4*(b*x^(1/3)+a*x)^(1/2)/b^3/x+88/1105*a^5*(b*x^(1/3)+a*x)^(1/2)/b^4/x
^(1/3)-2/7*(b*x^(1/3)+a*x)^(3/2)/x^4+88/1105*a^(21/4)*(b^(1/2)+a^(1/2)*x^(
1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*EllipticE(
sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))/b^(15/4)/(b*x^(1/3)+a*
x)^(1/2)-44/1105*a^(21/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2
)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*InverseJacobiAM(2*arctan(a^(1/4)*x^(1/
6)/b^(1/4)),1/2*2^(1/2))/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.14

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx=-\frac{2b\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{21}{4},-\frac{3}{2},-\frac{17}{4},-\frac{ax^{2/3}}{b}\right)}{7\sqrt{1+\frac{ax^{2/3}}{b}}x^{11/3}}$$

input
```
Integrate[(b*x^(1/3) + a*x)^(3/2)/x^5,x]
```

output
```
(-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-21/4, -3/2, -17/4, -((a*x^(
2/3))/b)])/(7*Sqrt[1 + (a*x^(2/3))/b]*x^(11/3))
```

Rubi [A] (warning: unable to verify)

Time = 1.13 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {1924, 1926, 1926, 1931, 1931, 1931, 1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+b\sqrt[3]{x}\right)^{3/2}}{x^5}\,dx$$

$$\Big\downarrow \ 1924$$

$$3\int \frac{\left(\sqrt[3]{x}b+ax\right)^{3/2}}{x^{13/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \ 1926$$

$$3\left(\frac{2}{7}a\int \frac{\sqrt{\sqrt[3]{x}b+ax}}{x^{10/3}}d\sqrt[3]{x}-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{21x^4}\right)$$

$$\Big\downarrow \ 1926$$

$$3\left(\frac{2}{7}a\left(\frac{2}{17}a\int \frac{1}{x^{7/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{17x^3}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{21x^4}\right)$$

$$\Big\downarrow \ 1931$$

$$3\left(\frac{2}{7}a\left(\frac{2}{17}a\left(-\frac{11a\int \frac{1}{x^{5/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{17x^3}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{21x^4}\right)$$

$$\Big\downarrow \ 1931$$

$$3\left(\frac{2}{7}a\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\int \frac{1}{x\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{17x^3}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right.}{21x^4}\right.$$

$$\Big\downarrow \ 1931$$

$$3\left(\frac{2}{7}a\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\int\frac{1}{\sqrt[3]{x}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)-\frac{2\sqrt{ax+b}}{17x^3}\right.\right.\right.$$

$\downarrow$ 1931

$$3\left(\frac{2}{7}a\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{a\int\frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right.\right.\right. -$$

↓ 1938

$$3\left(\frac{2}{7}a\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax-}}{13bx}\right.\right.\right.$$

$\downarrow$ 266

$$3\left(\frac{2}{7}a\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax}}{13b}\cdots$$

↓ 834

$$3\left(\frac{2}{7}a\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{b}\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\cdots}{13b}\right.\right.\right.$$

$\downarrow$ 27

$$3\left(\frac{2}{7}a\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-2\cdots}{13b}\right.\right.\right.$$

↓ 761

$$3\left(\frac{2}{7}a\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/}}{\sqrt{ax^{4/3}+b}}}{\sqrt{a}}\right.}{b\sqrt{ax+b\sqrt[3]{x}}}\right.}{5b}\right.}{9b}\right.}{13b}\right.\right.\right.$$

↓ 1510

$$3\left(\frac{2}{7}a\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}\right.}{}\right.}{b\sqrt{ax+b\sqrt[3]{x}}}\right.}{5b}\right.}{9b}\right.}{1}\right.\right.\right.$$

input
```
Int[(b*x^(1/3) + a*x)^(3/2)/x^5,x]
```

output
```
3*((-2*(b*x^(1/3) + a*x)^(3/2))/(21*x^4) + (2*a*((-2*Sqrt[b*x^(1/3) + a*x]
)/(17*x^3) + (2*a*((-2*Sqrt[b*x^(1/3) + a*x])/(13*b*x^(7/3)) - (11*a*((-2*
Sqrt[b*x^(1/3) + a*x])/(9*b*x^(5/3)) - (7*a*((-2*Sqrt[b*x^(1/3) + a*x])/(5
*b*x) - (3*a*((-2*Sqrt[b*x^(1/3) + a*x])/(b*x^(1/3)) + (2*a*Sqrt[b + a*x^(
2/3)]*x^(1/6)*(-((-((x^(1/6)*Sqrt[b + a*x^(4/3)])/(Sqrt[b] + Sqrt[a]*x^(2/
3))) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b]
+ Sqrt[a]*x^(2/3))^2]*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])
/(a^(1/4)*Sqrt[b + a*x^(4/3)]))/Sqrt[a]) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(
2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcT
an[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(3/4)*Sqrt[b + a*x^(4/3)])))/(b*
Sqrt[b*x^(1/3) + a*x])))/(5*b)))/(9*b)))/(13*b)))/17))/7)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 3.88 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.69

method	result
derivativedivides	$-\frac{2b\sqrt{b\,x^{\frac{1}{3}}+ax}}{7x^{\frac{11}{3}}}-\frac{46a\sqrt{b\,x^{\frac{1}{3}}+ax}}{119x^{3}}-\frac{24a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1547b\,x^{\frac{7}{3}}}+\frac{88a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{4641b^{2}x^{\frac{5}{3}}}-\frac{88a^{4}\sqrt{b\,x^{\frac{1}{3}}+ax}}{3315b^{3}x}+\frac{88\left(b+a\,x^{\frac{2}{3}}\right)\cdots}{1105b^{4}\sqrt{x^{\frac{1}{3}}\left(b\cdots\right.}}$
default	$-\frac{2\left(924a^{5}b\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}a-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,x^{\frac{20}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,\mathrm{EllipticE}\left(\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)-462a^{5}\cdots\right.}{\cdots}$

input

```
int((b*x^(1/3)+a*x)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output

```
-2/7*b*(b*x^(1/3)+a*x)^(1/2)/x^(11/3)-46/119*a*(b*x^(1/3)+a*x)^(1/2)/x^3-2
4/1547*a^2*(b*x^(1/3)+a*x)^(1/2)/b/x^(7/3)+88/4641*a^3*(b*x^(1/3)+a*x)^(1/
2)/b^2/x^(5/3)-88/3315*a^4*(b*x^(1/3)+a*x)^(1/2)/b^3/x+88/1105*(b+a*x^(2/3
))*a^5/b^4/(x^(1/3)*(b+a*x^(2/3)))^(1/2)-44/1105*a^5/b^4*(-a*b)^(1/2)*((x^
(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2
))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2)*a)^(1/2)/(b*x^(1/3)+a*x)^(
1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/
2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/
2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input

```
integrate((b*x^(1/3)+a*x)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
integral((a*x + b*x^(1/3))^(3/2)/x^5, x)
```

Sympy [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx=\int \frac{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x**(1/3)+a*x)**(3/2)/x**5,x)
```

output
```
Integral((a*x + b*x**(1/3))**(3/2)/x**5, x)
```

Maxima [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)/x^5, x)
```

Giac [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^5,x, algorithm="giac")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx=\int \frac{{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}{x^5}\,dx$$

input
```
int((a*x + b*x^(1/3))^(3/2)/x^5,x)
```

output
```
int((a*x + b*x^(1/3))^(3/2)/x^5, x)
```

Reduce [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx=\frac{-\frac{2x^{\frac{2}{3}}\sqrt{x^{\frac{2}{3}}a+b}\,a}{5}-\frac{26\sqrt{x^{\frac{2}{3}}a+b}\,b}{95}+\frac{4\sqrt{x}\left(\int \frac{\sqrt{x^{\frac{2}{3}}a+b}}{\sqrt{x}\,b\,x^4+x^{\frac{31}{6}}a}dx\right)b^2x^3}{95}}{\sqrt{x}\,x^3}$$

input
```
int((b*x^(1/3)+a*x)^(3/2)/x^5,x)
```

output
```
(2*( - 19*x**(2/3)*sqrt(x**(2/3)*a + b)*a - 13*sqrt(x**(2/3)*a + b)*b + 2*
sqrt(x)*int(sqrt(x**(2/3)*a + b)/(sqrt(x)*b*x**4 + x**(1/6)*a*x**5),x)*b**
2*x**3))/(95*sqrt(x)*x**3)
```

3.127 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx$

Optimal result . . . 1071
Mathematica [C] (verified) . . . 1072
Rubi [A] (warning: unable to verify) . . . 1072
Maple [A] (verified) . . . 1083
Fricas [F] . . . 1084
Sympy [F(-1)] . . . 1084
Maxima [F] . . . 1084
Giac [F] . . . 1085
Mupad [F(-1)] . . . 1085
Reduce [F] . . . 1085

Optimal result

Integrand size = 19, antiderivative size = 301

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx = -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x}+ax}}{216315b^3x^2} + \frac{1768a^5\sqrt{b\sqrt[3]{x}+ax}}{168245b^4x^{4/3}} - \frac{1768a^6\sqrt{b\sqrt[3]{x}+ax}}{100947b^5x^{2/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{9x^5} - \frac{884a^{27/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{100947b^{21/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
-4/69*a*(b*x^(1/3)+a*x)^(1/2)/x^4-8/1311*a^2*(b*x^(1/3)+a*x)^(1/2)/b/x^(10
/3)+136/19665*a^3*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(8/3)-1768/216315*a^4*(b*x^(
1/3)+a*x)^(1/2)/b^3/x^2+1768/168245*a^5*(b*x^(1/3)+a*x)^(1/2)/b^4/x^(4/3)-
1768/100947*a^6*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(2/3)-2/9*(b*x^(1/3)+a*x)^(3/2
)/x^5-884/100947*a^(27/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2
)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*InverseJacobiAM(2*arctan(a^(1/4)*x^(1/
6)/b^(1/4)),1/2*2^(1/2))/b^(21/4)/(b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.21

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx=-\frac{2b\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{27}{4},-\frac{3}{2},-\frac{23}{4},-\frac{ax^{2/3}}{b}\right)}{9\sqrt{1+\frac{ax^{2/3}}{b}}x^{14/3}}$$

input
```
Integrate[(b*x^(1/3) + a*x)^(3/2)/x^6,x]
```

output
```
(-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-27/4, -3/2, -23/4, -((a*x^(
2/3))/b)])/(9*Sqrt[1 + (a*x^(2/3))/b]*x^(14/3))
```

Rubi [A] (warning: unable to verify)

Time = 1.04 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {1924, 1926, 1926, 1931, 1931, 1931, 1931, 1931, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+b\sqrt[3]{x}\right)^{3/2}}{x^6}dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{\left(\sqrt[3]{x}b+ax\right)^{3/2}}{x^{16/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1926}$$

$$3\left(\frac{2}{9}a\int \frac{\sqrt{\sqrt[3]{x}b+ax}}{x^{13/3}}d\sqrt[3]{x}-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{27x^5}\right)$$

$$\Big\downarrow \text{1926}$$

$$3\left(\frac{2}{9}a\left(\frac{2}{23}a\int \frac{1}{x^{10/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{23x^4}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{27x^5}\right)$$

$$\Big\downarrow \text{ 1931}$$

$$3\left(\frac{2}{9}a\left(\frac{2}{23}a\left(-\frac{17a\int \frac{1}{x^{8/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{19b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{19bx^{10/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{23x^4}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{27x^5}\right)$$

$$\Big\downarrow \text{ 1931}$$

$$3\left(\frac{2}{9}a\left(\frac{2}{23}a\left(-\frac{17a\left(-\frac{13a\int \frac{1}{x^{2}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{19bx^{10/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{23x^4}\right)-\frac{2\left(ax+b\sqrt[3]{\cdots}\right.}{27x^5}\cdots\right.$$

$$\Big\downarrow \text{ 1931}$$

$$3\left(\frac{2}{9}a\left(\frac{2}{23}a\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\int \frac{1}{x^{4/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^{2}}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{19bx^{10/3}}\right)-\frac{2\sqrt{ax+\cdots}}{23x^4}\cdots\right.\right.$$

$$\Big\downarrow \text{ 1931}$$

$$3\left(\frac{2}{9}a\left(\frac{2}{23}a\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\int\frac{1}{x^{2/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^2}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}-\frac{2\sqrt{ax+b\sqrt[3]{}}}{19bx^{10/3}}\right.\right.\right.$$

$\downarrow$ 1931

$$3\left(\frac{2}{9}a\left(\frac{2}{23}a\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{a\int\frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{3b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^{2}}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}-\right.\right.\right.$$

↓ 1917

$$3\left(\frac{2}{9}a\left(\frac{2}{23}a\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{x^{2/3}a+b}\,\sqrt[6]{x}}d\sqrt[3]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^{2}}\right)}{15b}-2\ldots\right)}{19b}\right.\right.\right.$$

↓ 266

$$3\left(\frac{2}{9}a\left(\frac{2}{23}a\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{ax^{4/3}+b}}\,d\sqrt[6]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^{2}}\right)}{15b}-\frac{2\sqrt{a}}{15}\right.}{19b}\right.\right.\right.$$

↓ 761

$$3\left(\frac{2}{9}a\left(\frac{2}{23}a\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{a^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^{2}}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}-\frac{2\sqrt{ax+b\cdots}}{3bx^{2/\cdots}}\right.}{7b}\right.}{11b}\right.}{15b}\right.}{19b}\right.\right.\right.$$

input
```
Int[(b*x^(1/3) + a*x)^(3/2)/x^6,x]
```

output
```
3*((-2*(b*x^(1/3) + a*x)^(3/2))/(27*x^5) + (2*a*((-2*Sqrt[b*x^(1/3) + a*x]
)/(23*x^4) + (2*a*((-2*Sqrt[b*x^(1/3) + a*x])/(19*b*x^(10/3)) - (17*a*((-2
*Sqrt[b*x^(1/3) + a*x])/(15*b*x^(8/3)) - (13*a*((-2*Sqrt[b*x^(1/3) + a*x])
/(11*b*x^2) - (9*a*((-2*Sqrt[b*x^(1/3) + a*x])/(7*b*x^(4/3)) - (5*a*((-2*S
qrt[b*x^(1/3) + a*x])/(3*b*x^(2/3)) - (a^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))
*Sqrt[b + a*x^(2/3)]*x^(1/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/
3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(3*b^(5/4)*Sqr
t[b*x^(1/3) + a*x]*Sqrt[b + a*x^(4/3)]))))/(7*b)))/(11*b)))/(15*b)))/(19*b)
))/23))/9)
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 5.45 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.67

method	result
default	$-\frac{2\left(6630a^{6}\sqrt{-ab}\,\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}a-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)x^{\frac{26}{3}}-1768x^{\frac{23}{3}}a^{5}b^{2}+\cdots\right.}{1514205b^{5}\sqrt{x^{\frac{1}{3}}\left(b+a\cdots\right.}}$
derivativedivides	$-\frac{2b\sqrt{b\,x^{\frac{1}{3}}+ax}}{9x^{\frac{14}{3}}}-\frac{58a\sqrt{b\,x^{\frac{1}{3}}+ax}}{207x^{4}}-\frac{8a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1311b\,x^{\frac{10}{3}}}+\frac{136a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{19665b^{2}x^{\frac{8}{3}}}-\frac{1768a^{4}\sqrt{b\,x^{\frac{1}{3}}+ax}}{216315b^{3}x^{2}}+\frac{1768a^{5}\sqrt{b\,x\cdots}}{168245b^{4}\cdots}$

input
```
int((b*x^(1/3)+a*x)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-2/1514205*(6630*a^6*(-a*b)^(1/2)*((x^(1/3)*a+(-a*b)^(1/2))/(-a*b)^(1/2))^
(1/2)*(-2*(x^(1/3)*a-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1
/2)*a)^(1/2)*EllipticF(((x^(1/3)*a+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2
^(1/2))*x^(26/3)-1768*x^(23/3)*a^5*b^2+5304*x^(25/3)*a^6*b+952*a^4*b^3*x^7
+216755*x^(17/3)*a^2*b^5-616*x^(19/3)*a^3*b^4+380380*a*b^6*x^5+13260*a^7*x
^9+168245*x^(13/3)*b^7)/b^5/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/x^(26/3)
```

Fricas [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
integral((a*x + b*x^(1/3))^(3/2)/x^6, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx=\text{Timed out}$$

input
```
integrate((b*x**(1/3)+a*x)**(3/2)/x**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)/x^6, x)
```

Giac [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^6,x, algorithm="giac")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx=\int \frac{\left(a\,x+b\,x^{1/3}\right)^{3/2}}{x^6}\,dx$$

input
```
int((a*x + b*x^(1/3))^(3/2)/x^6,x)
```

output
```
int((a*x + b*x^(1/3))^(3/2)/x^6, x)
```

Reduce [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx=\frac{-\frac{2x^{\frac{2}{3}}\sqrt{x^{\frac{2}{3}}a+b}\,a}{7}-\frac{38\sqrt{x^{\frac{2}{3}}a+b}\,b}{175}+\frac{4\sqrt{x}\left(\int \frac{\sqrt{x^{\frac{2}{3}}a+b}}{\sqrt{x}\,b\,x^5+x^{\frac{37}{6}}a}dx\right)b^2x^4}{175}}{\sqrt{x}\,x^4}$$

input
```
int((b*x^(1/3)+a*x)^(3/2)/x^6,x)
```

output
```
(2*( - 25*x**(2/3)*sqrt(x**(2/3)*a + b)*a - 19*sqrt(x**(2/3)*a + b)*b + 2*
sqrt(x)*int(sqrt(x**(2/3)*a + b)/(sqrt(x)*b*x**5 + x**(1/6)*a*x**6),x)*b**
2*x**4))/(175*sqrt(x)*x**4)
```

3.128 $\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal result . . . 1086
Mathematica [C] (verified) . . . 1087
Rubi [A] (warning: unable to verify) . . . 1087
Maple [A] (verified) . . . 1102
Fricas [F] . . . 1103
Sympy [F] . . . 1103
Maxima [F] . . . 1103
Giac [F] . . . 1104
Mupad [F(-1)] . . . 1104
Reduce [F] . . . 1104

Optimal result

Integrand size = 19, antiderivative size = 304

$$\begin{aligned}\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx &= \frac{11050b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^7}-\frac{2210b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{4807a^6}\\ &\quad+\frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5}-\frac{1190b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{3933a^4}\\ &\quad+\frac{350b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^3}-\frac{50bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a^2}+\frac{2x^4\sqrt{b\sqrt[3]{x}+ax}}{9a}\\ &\quad-\frac{5525b^{27/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{14421a^{29/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

output
```
11050/14421*b^6*(b*x^(1/3)+a*x)^(1/2)/a^7-2210/4807*b^5*x^(2/3)*(b*x^(1/3)
+a*x)^(1/2)/a^6+15470/43263*b^4*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a^5-1190/393
3*b^3*x^2*(b*x^(1/3)+a*x)^(1/2)/a^4+350/1311*b^2*x^(8/3)*(b*x^(1/3)+a*x)^(
1/2)/a^3-50/207*b*x^(10/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+2/9*x^4*(b*x^(1/3)+a*
x)^(1/2)/a-5525/14421*b^(27/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b
^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*InverseJacobiAM(2*arctan(a^(1/4)*
x^(1/6)/b^(1/4)),1/2*2^(1/2))/a^(29/4)/(b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.53

$$\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \frac{2\sqrt{b\sqrt[3]{x}+ax}\Big(16575b^7+6630ab^6x^{2/3}-2210a^2b^5x^{4/3}+1190a^3b^4x^2-770a^4b^3x^{8/3}+550a^5b^2x^{10/3}-41}{43263a^7\left(b+ax^{2/3}\right)}$$

input
```
Integrate[x^4/Sqrt[b*x^(1/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(1/3) + a*x]*(16575*b^7 + 6630*a*b^6*x^(2/3) - 2210*a^2*b^5*x^
(4/3) + 1190*a^3*b^4*x^2 - 770*a^4*b^3*x^(8/3) + 550*a^5*b^2*x^(10/3) - 41
8*a^6*b*x^4 + 4807*a^7*x^(14/3) - 16575*b^7*Sqrt[1 + (a*x^(2/3))/b]*Hyperg
eometric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(43263*a^7*(b + a*x^(2/3)))
```

Rubi [A] (warning: unable to verify)

Time = 1.03 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.22, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {1924, 1930, 1930, 1930, 1930, 1930, 1930, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

$$\Big\downarrow \text{ 1924}$$

$$3\int \frac{x^{14/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 1930}$$

$$3\left(\frac{2x^4\sqrt{ax+b\sqrt[3]{x}}}{27a}-\frac{25b\int\frac{x^4}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{27a}\right)$$

$\Big\downarrow$ 1930

$$3\left(\frac{2x^4\sqrt{ax+b\sqrt[3]{x}}}{27a}-\frac{25b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{21b\int\frac{x^{10/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{23a}\right)}{27a}\right)$$

$\Big\downarrow$ 1930

$$3\left(\frac{2x^4\sqrt{ax+b\sqrt[3]{x}}}{27a}-\frac{25b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\int\frac{x^{8/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{19a}\right)}{23a}\right)}{27a}\right)$$

$\Big\downarrow$ 1930

$$3\left(\frac{2x^4\sqrt{ax+b\sqrt[3]{x}}}{27a}-\frac{25b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\int\frac{x^2}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{15a}\right)}{19a}\right)}{23a}\right)}{27a}\right)$$

$\downarrow$ 1930

$$3\left(\frac{2x^4\sqrt{ax+b\sqrt[3]{x}}}{27a}-\frac{25b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\int\frac{x^{4/3}}{\sqrt{\sqrt[3]{x}b+a}}}{11a}\right.}{15a}\right.}{19a}\right.}{23a}\right.}{27a}\right.$$

↓ 1930

$$9b\left(\frac{2x^{2/3}\sqrt{ax+\cdots}}{7a}\right.$$

$$13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{\cdots}{\cdots}\right.$$

$$17b\left(\frac{2x^{2}\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{\cdots}{15a}\right.$$

$$21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{\cdots}{19a}\right.$$

$$25b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{\cdots}{23a}\right.$$

↓ 1930

$$9b\left(\frac{2x^{2/3}\sqrt{ax+\cdots}}{7a}\right.$$

$$13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{\cdots}{\cdots}\right.$$

$$17b\left(\frac{2x^{2}\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{\cdots}{\cdots}\right.$$

$$21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{\cdots}{19a}\right.$$

↓ 1917

$9b\left(\frac{2x^{2/3}\sqrt{ax+\cdots}}{7a}\right.$

$13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\right.$

$17b\left(\frac{2x^{2}\sqrt{ax+b\sqrt[3]{x}}}{15a}-\right.$

$21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\right.$

$25b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{\cdots}{23a}\right.$

↓ 266

$$9b\left(\frac{2x^{2/3}\sqrt{ax+\cdots}}{7a}\right.$$

$$13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\right.$$

$$17b\left(\frac{2x^{2}\sqrt{ax+b\sqrt[3]{x}}}{15a}-\right.$$

$$21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\right.$$

$$25b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{\cdots}{23a}\right.$$

↓ 761

$$9b\left(\frac{2x^{2/3}\sqrt{ax+\cdots}}{7a}\right.$$

$$13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\right.$$

$$17b\left(\frac{2x^{2}\sqrt{ax+b\sqrt[3]{x}}}{15a}-\right.$$

$$21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\right.$$

input
```
Int[x^4/Sqrt[b*x^(1/3) + a*x],x]
```

output
```
3*((2*x^4*Sqrt[b*x^(1/3) + a*x])/(27*a) - (25*b*((2*x^(10/3)*Sqrt[b*x^(1/3
) + a*x])/(23*a) - (21*b*((2*x^(8/3)*Sqrt[b*x^(1/3) + a*x])/(19*a) - (17*b
*((2*x^2*Sqrt[b*x^(1/3) + a*x])/(15*a) - (13*b*((2*x^(4/3)*Sqrt[b*x^(1/3)
+ a*x])/(11*a) - (9*b*((2*x^(2/3)*Sqrt[b*x^(1/3) + a*x])/(7*a) - (5*b*((2*
Sqrt[b*x^(1/3) + a*x])/(3*a) - (b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b
 + a*x^(2/3)]*x^(1/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*
EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[b*x^(
1/3) + a*x]*Sqrt[b + a*x^(4/3)])))/(7*a)))/(11*a)))/(15*a)))/(19*a)))/(23*
a)))/(27*a))
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 4.29 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.64

method	result
default	$-\frac{-1100x^{\frac{11}{3}}a^{6}b^{2}+836x^{\frac{13}{3}}a^{7}b+1540a^{5}b^{3}x^{3}+4420x^{\frac{5}{3}}a^{3}b^{5}-2380x^{\frac{7}{3}}a^{4}b^{4}-9614a^{8}x^{5}+16575b^{7}\sqrt{-ab}\,\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}}{43263\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^{8}}$
derivativedivides	$\frac{2x^{4}\sqrt{b\,x^{\frac{1}{3}}+ax}}{9a}-\frac{50b\,x^{\frac{10}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{207a^{2}}+\frac{350b^{2}x^{\frac{8}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1311a^{3}}-\frac{1190b^{3}x^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{3933a^{4}}+\frac{15470b^{4}x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{43263a^{5}}$

input
```
int(x^4/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/43263*(-1100*x^(11/3)*a^6*b^2+836*x^(13/3)*a^7*b+1540*a^5*b^3*x^3+4420*
x^(5/3)*a^3*b^5-2380*x^(7/3)*a^4*b^4-9614*a^8*x^5+16575*b^7*(-a*b)^(1/2)*(
(x^(1/3)*a+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)*a-(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2)*a)^(1/2)*EllipticF(((x^(1/3)*a+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-13260*a^2*b^6*x-33150*x^(1/
3)*a*b^7)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/a^8
```

Fricas [F]

$$\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x^4}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^2*x^5 - a*b*x^(13/3) + b^2*x^(11/3))*sqrt(a*x + b*x^(1/3))/(a^
3*x^2 + b^3), x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x^4}{\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

input
```
integrate(x**4/(b*x**(1/3)+a*x)**(1/2),x)
```

output
```
Integral(x**4/sqrt(a*x + b*x**(1/3)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x^4}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(a*x + b*x^(1/3)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x^4}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(a*x + b*x^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x^4}{\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

input
```
int(x^4/(a*x + b*x^(1/3))^(1/2),x)
```

output
```
int(x^4/(a*x + b*x^(1/3))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$$

$$= \frac{11550x^{\frac{17}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a^4b^2 - 19890x^{\frac{5}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a\,b^5 - 10450\sqrt{x}\,\sqrt{x^{\frac{2}{3}}a+b}\,a^5b\,x^3 + 15470\sqrt{x}\,\sqrt{x^{\frac{2}{3}}a+b}\,a}{43}$$

input
```
int(x^4/(b*x^(1/3)+a*x)^(1/2),x)
```

output
```
(11550*x**(5/6)*sqrt(x**(2/3)*a + b)*a**4*b**2*x**2 - 19890*x**(5/6)*sqrt(
x**(2/3)*a + b)*a*b**5 - 10450*sqrt(x)*sqrt(x**(2/3)*a + b)*a**5*b*x**3 +
15470*sqrt(x)*sqrt(x**(2/3)*a + b)*a**2*b**4*x + 9614*x**(1/6)*sqrt(x**(2/
3)*a + b)*a**6*x**4 - 13090*x**(1/6)*sqrt(x**(2/3)*a + b)*a**3*b**3*x**2 +
 33150*x**(1/6)*sqrt(x**(2/3)*a + b)*b**6 - 5525*int(sqrt(x**(2/3)*a + b)/
(x**(5/6)*b + sqrt(x)*a*x),x)*b**7)/(43263*a**7)
```

3.129 $\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal result 1106
Mathematica [C] (verified) 1107
Rubi [A] (warning: unable to verify) 1108
Maple [A] (verified) 1124
Fricas [F] 1125
Sympy [F] 1126
Maxima [F] 1126
Giac [F] 1126
Mupad [F(-1)] 1127
Reduce [F] 1127

Optimal result

Integrand size = 19, antiderivative size = 414

$$\begin{aligned}
&\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx \\
&= -\frac{418b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221a^{11/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{418b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^5} \\
&\quad -\frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4}+\frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3} \\
&\quad -\frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^2}+\frac{2x^3\sqrt{b\sqrt[3]{x}+ax}}{7a} \\
&\quad +\frac{418b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221a^{23/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad -\frac{209b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{221a^{23/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}$$

output
```
-418/221*b^5*(b+a*x^(2/3))*x^(1/3)/a^(11/2)/(b^(1/2)+a^(1/2)*x^(1/3))/(b*x
^(1/3)+a*x)^(1/2)+418/663*b^4*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^5-2090/4641*
b^3*x*(b*x^(1/3)+a*x)^(1/2)/a^4+570/1547*b^2*x^(5/3)*(b*x^(1/3)+a*x)^(1/2)
/a^3-38/119*b*x^(7/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+2/7*x^3*(b*x^(1/3)+a*x)^(1
/2)/a+418/221*b^(21/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a
^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^
(1/4))),1/2*2^(1/2))/a^(23/4)/(b*x^(1/3)+a*x)^(1/2)-209/221*b^(21/4)*(b^(1
/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(
1/6)*InverseJacobiAM(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)),1/2*2^(1/2))/a^(23/
4)/(b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.35

$$\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \frac{2\sqrt{b\sqrt[3]{x}+ax}\left(1463b^5\sqrt[3]{x}+418ab^4x-190a^2b^3x^{5/3}+114a^3b^2x^{7/3}-78a^4bx^3+663a^5x^{11/3}-1463b^5\sqrt{1\right.}{4641a^5\left(b+ax^{2/3}\right)}$$

input
```
Integrate[x^3/Sqrt[b*x^(1/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(1/3) + a*x]*(1463*b^5*x^(1/3) + 418*a*b^4*x - 190*a^2*b^3*x^(
5/3) + 114*a^3*b^2*x^(7/3) - 78*a^4*b*x^3 + 663*a^5*x^(11/3) - 1463*b^5*Sq
rt[1 + (a*x^(2/3))/b]*x^(1/3)*Hypergeometric2F1[1/2, 3/4, 7/4, -((a*x^(2/3
))/b)]))/(4641*a^5*(b + a*x^(2/3)))
```

Rubi [A] (warning: unable to verify)

Time = 1.07 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {1924, 1930, 1930, 1930, 1930, 1930, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{ax+b\sqrt[3]{x}}}dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{x^{11/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\int \frac{x^3}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{21a}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\int \frac{x^{7/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{17a}\right)}{21a}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\int\frac{x^{5/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{13a}\right)}{17a}\right)}{21a}\right)$$

$\downarrow$ 1930

$$3\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\int\frac{x}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{9a}\right)}{13a}\right)}{17a}\right)}{21a}\right)$$

$\downarrow$ 1930

$$3\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\int\frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d}{5a}\right.}{9a}\right.}{13a}\right.}{17a}\right.}{21a}\right.$$

↓ 1938

$$3\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\sqrt[6]{x}\sqrt{ax^{2/3}+b \cdots}}{5a\sqrt{ax \cdots}} \cdots\right.}{9a}\cdots\right.}{13a}\cdots\right.}{17a}\cdots\right.}{21a}\cdots\right.$$

↓ 266

$$3\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b\cdots}}{5a\sqrt{ax\cdots}}\right.}{9a}\right.}{13a}\right.}{17a}\right.}{21a}\right.$$

↓ 834

$$6b\sqrt[6]{x}\sqrt{ax^{2/3}+b$$

$$7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\right.$$

$$11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\right.$$

$$15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\right.\quad 13a$$

$$19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\right.\quad 17a$$

$$3\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\right.\quad 21a$$

$\downarrow$ 27

$$3\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}}{\cdots}\right.}{\cdots}\right.}{13a}\right.}{17a}\right.}{21a}\right.$$

↓ 761

$$6b\sqrt[6]{x}\sqrt{ax^{2/3}+b$$

$$7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\right.$$

$$11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\right.$$

$$15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\right.$$

$$19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\right.$$

↓ 1510

$$6b\sqrt[6]{x}\sqrt{ax^{2/3}+b$$

$$7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\right.$$

$$11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\right.$$

$$15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\right.$$

$$19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\right.$$

input
```
Int[x^3/Sqrt[b*x^(1/3) + a*x],x]
```

output
```
3*((2*x^3*Sqrt[b*x^(1/3) + a*x])/(21*a) - (19*b*((2*x^(7/3)*Sqrt[b*x^(1/3)
 + a*x])/(17*a) - (15*b*((2*x^(5/3)*Sqrt[b*x^(1/3) + a*x])/(13*a) - (11*b*
((2*x*Sqrt[b*x^(1/3) + a*x])/(9*a) - (7*b*((2*x^(1/3)*Sqrt[b*x^(1/3) + a*x
])/(5*a) - (6*b*Sqrt[b + a*x^(2/3)]*x^(1/6)*(-((-((x^(1/6)*Sqrt[b + a*x^(4
/3)])/(Sqrt[b] + Sqrt[a]*x^(2/3))) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*
Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticE[2*ArcTan[(a^
(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b + a*x^(4/3)]))/Sqrt[a]) + (
b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a
]*x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(3
/4)*Sqrt[b + a*x^(4/3)])))/(5*a*Sqrt[b*x^(1/3) + a*x])))/(9*a)))/(13*a)))/
(17*a)))/(21*a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1930

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 2.63 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.63

method	result
default	$-\frac{-228x^{\frac{8}{3}}a^4b^2+156x^{\frac{10}{3}}a^5b+380a^3b^3x^2+8778b^6\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}a-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-}}{\sqrt{-ab}}}\right.}{46}$
derivativedivides	$\frac{2x^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{7a}-\frac{38b\,x^{\frac{7}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{119a^2}+\frac{570b^2x^{\frac{5}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1547a^3}-\frac{2090b^3x\sqrt{b\,x^{\frac{1}{3}}+ax}}{4641a^4}+\frac{418b^4x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{663a^5}-$

input
```
int(x^3/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4641/a^6*(-228*x^(8/3)*a^4*b^2+156*x^(10/3)*a^5*b+380*a^3*b^3*x^2+8778*
b^6*((x^(1/3)*a+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)*a-(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2)*a)^(1/2)*EllipticE(((x^(1/
3)*a+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-4389*b^6*((x^(1/3)*a+(
-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)*a-(-a*b)^(1/2))/(-a*b)^(1/2)
)^(1/2)*(-x^(1/3)/(-a*b)^(1/2)*a)^(1/2)*EllipticF(((x^(1/3)*a+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-1326*a^6*x^4-2926*x^(2/3)*a*b^5-836*x^(
4/3)*a^2*b^4)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

Fricas [F]

$$\int\frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int\frac{x^3}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^2*x^4 - a*b*x^(10/3) + b^2*x^(8/3))*sqrt(a*x + b*x^(1/3))/(a^3
*x^2 + b^3), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{x^3}{\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

input
```
integrate(x**3/(b*x**(1/3)+a*x)**(1/2),x)
```

output
```
Integral(x**3/sqrt(a*x + b*x**(1/3)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{x^3}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(a*x + b*x^(1/3)), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{x^3}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(a*x + b*x^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{x^3}{\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

input
```
int(x^3/(a*x + b*x^(1/3))^(1/2),x)
```

output
```
int(x^3/(a*x + b*x^(1/3))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$$
$$=\frac{1710x^{\frac{11}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a^2b^2-1482\sqrt{x}\,\sqrt{x^{\frac{2}{3}}a+b}\,a^3b\,x^2+2926\sqrt{x}\,\sqrt{x^{\frac{2}{3}}a+b}\,b^4+1326x^{\frac{19}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a^4-20}{4641a^5}$$

input
```
int(x^3/(b*x^(1/3)+a*x)^(1/2),x)
```

output
```
(1710*x**(5/6)*sqrt(x**(2/3)*a + b)*a**2*b**2*x - 1482*sqrt(x)*sqrt(x**(2/
3)*a + b)*a**3*b*x**2 + 2926*sqrt(x)*sqrt(x**(2/3)*a + b)*b**4 + 1326*x**(
1/6)*sqrt(x**(2/3)*a + b)*a**4*x**3 - 2090*x**(1/6)*sqrt(x**(2/3)*a + b)*a
*b**3*x - 1463*int((x**(1/6)*sqrt(x**(2/3)*a + b))/(x**(2/3)*b + x**(1/3)*
a*x),x)*b**5)/(4641*a**5)
```

3.130 $\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal result 1128
Mathematica [C] (verified) 1129
Rubi [A] (warning: unable to verify) 1129
Maple [A] (verified) 1136
Fricas [F] 1136
Sympy [F] 1137
Maxima [F] 1137
Giac [F] 1137
Mupad [F(-1)] 1138
Reduce [F] 1138

Optimal result

Integrand size = 19, antiderivative size = 216

$$\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = -\frac{78b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^4}+\frac{234b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^3}-\frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2}+\frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a} + \frac{39b^{15/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77a^{17/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
-78/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^4+234/385*b^2*x^(2/3)*(b*x^(1/3)+a*x)^(
1/2)/a^3-26/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+2/5*x^2*(b*x^(1/3)+a*x)
^(1/2)/a+39/77*b^(15/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+
a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*InverseJacobiAM(2*arctan(a^(1/4)*x^(1/6)
/b^(1/4)),1/2*2^(1/2))/a^(17/4)/(b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.57

$$\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \frac{2\sqrt{b\sqrt[3]{x}+ax}\left(-195b^4-78ab^3x^{2/3}+26a^2b^2x^{4/3}-14a^3bx^2+77a^4x^{8/3}+195b^4\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeome}\right.}{385a^4\left(b+ax^{2/3}\right)}$$

input
```
Integrate[x^2/Sqrt[b*x^(1/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(1/3) + a*x]*(-195*b^4 - 78*a*b^3*x^(2/3) + 26*a^2*b^2*x^(4/3)
 - 14*a^3*b*x^2 + 77*a^4*x^(8/3) + 195*b^4*Sqrt[1 + (a*x^(2/3))/b]*Hyperge
ometric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(385*a^4*(b + a*x^(2/3)))
```

Rubi [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1924, 1930, 1930, 1930, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{ax+b\sqrt[3]{x}}}dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{x^{8/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\int\frac{x^2}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{15a}\right)$$

$$\Big\downarrow \text{ 1930}$$

$$3\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\int\frac{x^{4/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{11a}\right)}{15a}\right)$$

$$\Big\downarrow \text{ 1930}$$

$$3\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\int\frac{x^{2/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{7a}\right)}{11a}\right)}{15a}\right)$$

$$\Big\downarrow \text{ 1930}$$

$$3\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b\int\frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{3a}\right)}{7a}\right)}{11a}\right)}{15a}\right)$$

$\downarrow$ 1917

$$3\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{3a\sqrt{ax+b\sqrt[3]{x}}}\right)}{7a}\right)}{11a}\right)}{15a}\right)$$

$\downarrow$ 266

$$3\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{2b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3a\sqrt{ax+b\sqrt[3]{x}}}\right)}{7a}\right)}{11a}\right)}{15a}\right)$$

$\downarrow$ 761

$$3\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}}{\left(\sqrt{a}x^{2/3}\right.}}}{3a^{5/4}\sqrt{ax+b\sqrt[3]{}}}\right)}{7a}\right)}{11a}\right)}{15a}\right)$$

input

```
Int[x^2/Sqrt[b*x^(1/3) + a*x],x]
```

output

```
3*((2*x^2*Sqrt[b*x^(1/3) + a*x])/(15*a) - (13*b*((2*x^(4/3)*Sqrt[b*x^(1/3)
 + a*x])/(11*a) - (9*b*((2*x^(2/3)*Sqrt[b*x^(1/3) + a*x])/(7*a) - (5*b*((2
*Sqrt[b*x^(1/3) + a*x])/(3*a) - (b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[
b + a*x^(2/3)]*x^(1/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]
*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[b*x^
(1/3) + a*x]*Sqrt[b + a*x^(4/3)])))/(7*a)))/(11*a)))/(15*a))
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.75

method	result
default	$\frac{52x^{\frac{5}{3}}a^3b^2-28x^{\frac{7}{3}}a^4b+195b^4\sqrt{-ab}\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}a-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)-156a}{385\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^5}$
derivativedivides	$\frac{2x^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{5a}-\frac{26b\,x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{55a^2}+\frac{234b^2x^{\frac{2}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{385a^3}-\frac{78b^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{77a^4}+\frac{39b^4\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}}{}$

input
```
int(x^2/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/385*(52*x^(5/3)*a^3*b^2-28*x^(7/3)*a^4*b+195*b^4*(-a*b)^(1/2)*((x^(1/3)*
a+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)*a-(-a*b)^(1/2))/(-a*b)^(1
/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2)*a)^(1/2)*EllipticF(((x^(1/3)*a+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-156*a^2*b^3*x+154*a^5*x^3-390*x^(1/3
)*a*b^4)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/a^5
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{x^2}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^2*x^3 - a*b*x^(7/3) + b^2*x^(5/3))*sqrt(a*x + b*x^(1/3))/(a^3*
x^2 + b^3), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x^2}{\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

input
```
integrate(x**2/(b*x**(1/3)+a*x)**(1/2),x)
```

output
```
Integral(x**2/sqrt(a*x + b*x**(1/3)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x^2}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(a*x + b*x^(1/3)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x^2}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(a*x + b*x^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x^2}{\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

input
```
int(x^2/(a*x + b*x^(1/3))^(1/2),x)
```

output
```
int(x^2/(a*x + b*x^(1/3))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \frac{234x^{\frac{5}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a\,b^2-182\sqrt{x}\,\sqrt{x^{\frac{2}{3}}a+b}\,a^2bx+154x^{\frac{13}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a^3-390x^{\frac{1}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,b^3+65\left(\int \frac{\sqrt{x^{\frac{2}{3}}a+b}}{x^{\frac{5}{6}}b+\cdots}\right.}{385a^4}$$

input
```
int(x^2/(b*x^(1/3)+a*x)^(1/2),x)
```

output
```
(234*x**(5/6)*sqrt(x**(2/3)*a + b)*a*b**2 - 182*sqrt(x)*sqrt(x**(2/3)*a +
b)*a**2*b*x + 154*x**(1/6)*sqrt(x**(2/3)*a + b)*a**3*x**2 - 390*x**(1/6)*s
qrt(x**(2/3)*a + b)*b**3 + 65*int(sqrt(x**(2/3)*a + b)/(x**(5/6)*b + sqrt(
x)*a*x),x)*b**4)/(385*a**4)
```

3.131 $\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal result 1139
Mathematica [C] (verified) 1140
Rubi [A] (warning: unable to verify) 1140
Maple [A] (verified) 1145
Fricas [F] 1146
Sympy [F] 1146
Maxima [F] 1147
Giac [F] 1147
Mupad [F(-1)] 1147
Reduce [F] 1148

Optimal result

Integrand size = 17, antiderivative size = 326

$$\begin{aligned}\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx &= \frac{14b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5a^{5/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^2}+\frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a}\\ &-\frac{14b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{11/4}\sqrt{b\sqrt[3]{x}+ax}}\\ &+\frac{7b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5a^{11/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

output
```
14/5*b^2*(b+a*x^(2/3))*x^(1/3)/a^(5/2)/(b^(1/2)+a^(1/2)*x^(1/3))/(b*x^(1/3
)+a*x)^(1/2)-14/15*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+2/3*x*(b*x^(1/3)+a*
x)^(1/2)/a-14/5*b^(9/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+
a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b
^(1/4))),1/2*2^(1/2))/a^(11/4)/(b*x^(1/3)+a*x)^(1/2)+7/5*b^(9/4)*(b^(1/2)+
a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)
*InverseJacobiAM(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)),1/2*2^(1/2))/a^(11/4)/(
b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.33

$$\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \frac{2\sqrt{b\sqrt[3]{x}+ax}\left(-7b^2\sqrt[3]{x}-2abx+5a^2x^{5/3}+7b^2\sqrt{1+\frac{ax^{2/3}}{b}}\sqrt[3]{x}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{ax^{2/3}}{b}\right)\right)}{15a^2\left(b+ax^{2/3}\right)}$$

input
```
Integrate[x/Sqrt[b*x^(1/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(1/3) + a*x]*(-7*b^2*x^(1/3) - 2*a*b*x + 5*a^2*x^(5/3) + 7*b^2
*Sqrt[1 + (a*x^(2/3))/b]*x^(1/3)*Hypergeometric2F1[1/2, 3/4, 7/4, -((a*x^(
2/3))/b)]))/(15*a^2*(b + a*x^(2/3)))
```

Rubi [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1924, 1930, 1930, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{x^{5/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\int \frac{x}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{9a}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\int \frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{5a}\right)}{9a}\right)$$

$$\Big\downarrow \text{1938}$$

$$3\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)}{9a}\right)$$

$$\Big\downarrow \text{266}$$

$$3\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)}{9a}\right)$$

$$\Big\downarrow \text{834}$$

$$3\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{b}\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)}{9a}\right)$$

$$\Big\downarrow \text{27}$$

$$3\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)}{9a}\right)$$

$$\Big\downarrow \text{761}$$

$$3\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}\right.}{5a\sqrt{ax+b\sqrt[3]{x}}}\right.}{9a}\right.$$

$\downarrow$ 1510

$$3\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}\right.}{5a\sqrt{ax+}}\right.}{9a}\right.$$

input
```
Int[x/Sqrt[b*x^(1/3) + a*x],x]
```

output
```
3*((2*x*Sqrt[b*x^(1/3) + a*x])/(9*a) - (7*b*((2*x^(1/3)*Sqrt[b*x^(1/3) + a
*x])/(5*a) - (6*b*Sqrt[b + a*x^(2/3)]*x^(1/6)*(-((-((x^(1/6)*Sqrt[b + a*x^
(4/3)])/(Sqrt[b] + Sqrt[a]*x^(2/3))) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3)
)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticE[2*ArcTan[(
a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b + a*x^(4/3)]))/Sqrt[a]) +
 (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt
[a]*x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^
(3/4)*Sqrt[b + a*x^(4/3)])))/(5*a*Sqrt[b*x^(1/3) + a*x])))/(9*a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{2x\sqrt{b\,x^{\frac{1}{3}}+ax}}{3a}-\frac{14b\,x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{15a^{2}}+\frac{7b^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{Ellipt}}{}\right.}{5a^{3}\sqrt{b\,x^{\frac{1}{3}}}}$
default	$-\frac{-42b^{3}\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}a-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\ \mathrm{EllipticE}\left(\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+21b^{3}\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}a\right.}{\sqrt{}}}}{15a^{3}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}$

input
```
int(x/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x*(b*x^(1/3)+a*x)^(1/2)/a-14/15*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+7/
5*b^2/a^3*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(
-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2)
*a)^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a
*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*Ellipti
cF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{x}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*
x^2 + b^3), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{x}{\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

input
```
integrate(x/(b*x**(1/3)+a*x)**(1/2),x)
```

output
```
Integral(x/sqrt(a*x + b*x**(1/3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(a*x + b*x^(1/3)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(a*x + b*x^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x}{\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

input
```
int(x/(a*x + b*x^(1/3))^(1/2),x)
```

output
```
int(x/(a*x + b*x^(1/3))^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \frac{-14\sqrt{x}\,\sqrt{x^{\frac{2}{3}}a+b}\,b+10x^{\frac{7}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a+7\left(\int \frac{x^{\frac{1}{6}}\sqrt{x^{\frac{2}{3}}a+b}}{x^{\frac{2}{3}}b+x^{\frac{4}{3}}a}dx\right)b^2}{15a^2}$$

input

```
int(x/(b*x^(1/3)+a*x)^(1/2),x)
```

output

```
( - 14*sqrt(x)*sqrt(x**(2/3)*a + b)*b + 10*x**(1/6)*sqrt(x**(2/3)*a + b)*a
*x + 7*int((x**(1/6)*sqrt(x**(2/3)*a + b))/(x**(2/3)*b + x**(1/3)*a*x),x)*
b**2)/(15*a**2)
```

3.132 $\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal result 1149
Mathematica [C] (verified) 1150
Rubi [A] (warning: unable to verify) 1150
Maple [A] (verified) 1152
Fricas [F] 1153
Sympy [F] 1153
Maxima [F] 1153
Giac [F] 1154
Mupad [B] (verification not implemented) 1154
Reduce [F] 1154

Optimal result

Integrand size = 15, antiderivative size = 126

$$\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \frac{2\sqrt{b\sqrt[3]{x}+ax}}{a} - \frac{b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{a^{5/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
2*(b*x^(1/3)+a*x)^(1/2)/a-b^(3/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))
/(b^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*InverseJacobiAM(2*arctan(a^(1/
4)*x^(1/6)/b^(1/4)),1/2*2^(1/2))/a^(5/4)/(b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \frac{2\sqrt{b\sqrt[3]{x}+ax}\left(b+ax^{2/3}-b\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{ax^{2/3}}{b}\right)\right)}{a\left(b+ax^{2/3}\right)}$$

input
```
Integrate[1/Sqrt[b*x^(1/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(1/3) + a*x]*(b + a*x^(2/3) - b*Sqrt[1 + (a*x^(2/3))/b]*Hyperg
eometric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(a*(b + a*x^(2/3)))
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1916, 1919, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

$$\Big\downarrow \text{1916}$$

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}}{a}-\frac{b\int \frac{1}{x^{2/3}\sqrt{\sqrt[3]{x}b+ax}}dx}{3a}$$

$$\Big\downarrow \text{1919}$$

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}}{a}-\frac{b\int \frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{a}$$

$$\Big\downarrow \text{1917}$$

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}}{a}-\frac{b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{a\sqrt{ax+b\sqrt[3]{x}}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}}{a}-\frac{2b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{a\sqrt{ax+b\sqrt[3]{x}}}$$

$$\Big\downarrow \text{761}$$

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}}{a}-\frac{b^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{a^{5/4}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}$$

input

```
Int[1/Sqrt[b*x^(1/3) + a*x],x]
```

output

```
(2*Sqrt[b*x^(1/3) + a*x])/a - (b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b
+ a*x^(2/3)]*x^(1/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*E
llipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(5/4)*Sqrt[b*x^(1/3
) + a*x]*Sqrt[b + a*x^(4/3)])
```

Defintions of rubi rules used

rule 266

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1916
```
Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[-2*(Sqrt
[a*x^j + b*x^n]/(b*(n - 2)*x^(n - 1))), x] - Simp[a*((2*n - j - 2)/(b*(n -
2)))   Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] &&
 LtQ[2*(n - 1), j, n]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1919
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
 && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.01

method	result	siz
default	$-\frac{b\sqrt{-ab}\,\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}a-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)-2x^{\frac{1}{3}}ab-2a^{2}x}{\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^{2}}$	12
derivativedivides	$\frac{2\sqrt{b\,x^{\frac{1}{3}}+ax}}{a}-\frac{b\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}$	13

input
```
int(1/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*(-a*b)^(1/2)*((x^(1/3)*a+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3
)*a-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2)*a)^(1/2)*Elli
pticF(((x^(1/3)*a+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-2*x^(1/3)
*a*b-2*a^2*x)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/a^2
```

Fricas [F]

$$\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(1/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*
x^3 + b^3*x), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

input
```
integrate(1/(b*x**(1/3)+a*x)**(1/2),x)
```

output
```
Integral(1/sqrt(a*x + b*x**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(1/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a*x + b*x^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(1/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(a*x + b*x^(1/3)), x)
```

Mupad [B] (verification not implemented)

Time = 8.96 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.32

$$\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\frac{2\,x\,\sqrt{\frac{b}{a\,x^{2/3}}+1}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ -\frac{b}{a\,x^{2/3}}\right)}{\sqrt{a\,x+b\,x^{1/3}}}$$

input
```
int(1/(a*x + b*x^(1/3))^(1/2),x)
```

output
```
(2*x*(b/(a*x^(2/3)) + 1)^(1/2)*hypergeom([-3/4, 1/2], 1/4, -b/(a*x^(2/3)))
)/(a*x + b*x^(1/3))^(1/2)
```

Reduce [F]

$$\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\frac{6x^{\frac{1}{6}}\sqrt{x^{\frac{2}{3}}a+b}-\left(\int \frac{\sqrt{x^{\frac{2}{3}}a+b}}{x^{\frac{5}{6}}b+\sqrt{x}\,ax}dx\right)b}{3a}$$

input
```
int(1/(b*x^(1/3)+a*x)^(1/2),x)
```

output
```
(6*x**(1/6)*sqrt(x**(2/3)*a + b) - int(sqrt(x**(2/3)*a + b)/(x**(5/6)*b +
sqrt(x)*a*x),x)*b)/(3*a)
```

3.133 $\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal result 1155
Mathematica [C] (verified) 1156
Rubi [A] (warning: unable to verify) 1156
Maple [A] (verified) 1160
Fricas [F] 1160
Sympy [F] 1161
Maxima [F] 1161
Giac [F] 1161
Mupad [F(-1)] 1162
Reduce [F] 1162

Optimal result

Integrand size = 19, antiderivative size = 294

$$\begin{aligned}\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx &= \frac{6\sqrt{a}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{6\sqrt{b\sqrt[3]{x}+ax}}{b\sqrt[3]{x}}\\ &\quad-\frac{6\sqrt[4]{a}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}\\ &\quad+\frac{3\sqrt[4]{a}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

output

```
6*a^(1/2)*(b+a*x^(2/3))*x^(1/3)/b/(b^(1/2)+a^(1/2)*x^(1/3))/(b*x^(1/3)+a*x
)^(1/2)-6*(b*x^(1/3)+a*x)^(1/2)/b/x^(1/3)-6*a^(1/4)*(b^(1/2)+a^(1/2)*x^(1/
3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*EllipticE(si
n(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))/b^(3/4)/(b*x^(1/3)+a*x)^
(1/2)+3*a^(1/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*
x^(1/3))^2)^(1/2)*x^(1/6)*InverseJacobiAM(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)
),1/2*2^(1/2))/b^(3/4)/(b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.18

$$\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx = -\frac{6\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{ax^{2/3}}{b}\right)}{\sqrt{b\sqrt[3]{x}+ax}}$$

input
```
Integrate[1/(x*Sqrt[b*x^(1/3) + a*x]),x]
```

output
```
(-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((a*x^(2/3)
)/b)])/Sqrt[b*x^(1/3) + a*x]
```

Rubi [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1924, 1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

$$\Big\downarrow \ 1924$$

$$3\int \frac{1}{\sqrt[3]{x}\sqrt{\sqrt[3]{x}b+ax}}\,d\sqrt[3]{x}$$

$$\Big\downarrow \ 1931$$

$$3\left(\frac{a\int \frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}\,d\sqrt[3]{x}}{b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \ 1938$$

$$3\left(\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)$$

$\downarrow$ 266

$$3\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)$$

$\downarrow$ 834

$$3\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{b}\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)$$

$\downarrow$ 27

$$3\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int \frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)$$

$\downarrow$ 761

$$3\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\int \frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax\cdots}}{b\sqrt[3]{\cdots}}\right.$$

$\downarrow$ 1510

$$3\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}}{\sqrt[4]{a}\sqrt{ax^{4/3}}}\right.}{b\sqrt{ax+b\sqrt[3]{x}}}\right.$$

input
```
Int[1/(x*Sqrt[b*x^(1/3) + a*x]),x]
```

output
```
3*((-2*Sqrt[b*x^(1/3) + a*x])/(b*x^(1/3)) + (2*a*Sqrt[b + a*x^(2/3)]*x^(1/
6)*(-((-((x^(1/6)*Sqrt[b + a*x^(4/3)])/(Sqrt[b] + Sqrt[a]*x^(2/3))) + (b^(
1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x
^(2/3))^2]*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*S
qrt[b + a*x^(4/3)]))/Sqrt[a]) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[
(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)
*x^(1/6))/b^(1/4)], 1/2])/(2*a^(3/4)*Sqrt[b + a*x^(4/3)])))/(b*Sqrt[b*x^(1
/3) + a*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.66

method	result
derivativedivides	$-\frac{6\left(b+a\,x^{\frac{2}{3}}\right)}{b\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}+\frac{3\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right) \ldots}{\sqrt{-ab}}} \ldots\right)}{a} \ldots\right.}{b\sqrt{b\,x^{\frac{1}{3}}+ax} \ldots}$
default	$-\frac{3\left(-2\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}a-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)b+\sqrt{x^{\frac{1}{3}}\left(b+a\,x \ldots\right.}\right.}{x^{\frac{1}{3}}\left(b \ldots\right.}$

input
```
int(1/x/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-6*(b+a*x^(2/3))/b/(x^(1/3)*(b+a*x^(2/3)))^(1/2)+3/b*(-a*b)^(1/2)*((x^(1/3
)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a
/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2)*a)^(1/2)/(b*x^(1/3)+a*x)^(1/2)
*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^
(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*
a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}\,x}\,dx$$

input
```
integrate(1/x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*
x^4 + b^3*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{x\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

input
```
integrate(1/x/(b*x**(1/3)+a*x)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a*x + b*x**(1/3))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}\,x}\,dx$$

input
```
integrate(1/x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*x^(1/3))*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}\,x}\,dx$$

input
```
integrate(1/x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*x + b*x^(1/3))*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{x\,\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

input
```
int(1/(x*(a*x + b*x^(1/3))^(1/2)),x)
```

output
```
int(1/(x*(a*x + b*x^(1/3))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{\sqrt{x^{\frac{1}{3}}b+ax}}{x^{\frac{4}{3}}b+a\,x^2}dx$$

input
```
int(1/x/(b*x^(1/3)+a*x)^(1/2),x)
```

output
```
int(sqrt(x**(1/3)*b + a*x)/(x**(1/3)*b*x + a*x**2),x)
```

3.134 $\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal result 1163
Mathematica [C] (verified) 1164
Rubi [A] (warning: unable to verify) 1164
Maple [A] (verified) 1167
Fricas [F] 1167
Sympy [F] 1168
Maxima [F] 1168
Giac [F] 1168
Mupad [F(-1)] 1169
Reduce [F] 1169

Optimal result

Integrand size = 19, antiderivative size = 163

$$\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx = -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{7bx^{4/3}}+\frac{10a\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{2/3}} + \frac{5a^{7/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{7b^{9/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
-6/7*(b*x^(1/3)+a*x)^(1/2)/b/x^(4/3)+10/7*a*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2
/3)+5/7*a^(7/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*
x^(1/3))^2)^(1/2)*x^(1/6)*InverseJacobiAM(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)
),1/2*2^(1/2))/b^(9/4)/(b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.36

$$\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx = -\frac{6\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{1}{2},-\frac{3}{4},-\frac{ax^{2/3}}{b}\right)}{7x\sqrt{b\sqrt[3]{x}+ax}}$$

input
```
Integrate[1/(x^2*Sqrt[b*x^(1/3) + a*x]),x]
```

output
```
(-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-7/4, 1/2, -3/4, -((a*x^(2/3
))/b)])/(7*x*Sqrt[b*x^(1/3) + a*x])
```

Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1924, 1931, 1931, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{ax+b\sqrt[3]{x}}}dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{1}{x^{4/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1931}$$

$$3\left(-\frac{5a\int \frac{1}{x^{2/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(-\frac{5a\left(-\frac{a\int \frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{3b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)$$

$$\Big\downarrow \text{1917}$$

$$3\left(-\frac{5a\left(-\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)$$

$$\Big\downarrow \text{266}$$

$$3\left(-\frac{5a\left(-\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)$$

$$\Big\downarrow \text{761}$$

$$3\left(-\frac{5a\left(-\frac{a^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)$$

input
```
Int[1/(x^2*Sqrt[b*x^(1/3) + a*x]),x]
```

output
```
3*((-2*Sqrt[b*x^(1/3) + a*x])/(7*b*x^(4/3)) - (5*a*((-2*Sqrt[b*x^(1/3) + a
*x])/(3*b*x^(2/3)) - (a^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b + a*x^(2/
3)]*x^(1/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[
2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[b*x^(1/3) + a*x
]*Sqrt[b + a*x^(4/3)])))/(7*b))
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.87

method	result
default	$\frac{5a\sqrt{-ab}\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}a-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)x^{\frac{4}{3}}+4abx+10x^{\frac{5}{3}}a^{2}-6x^{\frac{1}{3}}b^{2}}{7b^{2}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,x^{\frac{4}{3}}}$
derivativedivides	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{7b\,x^{\frac{4}{3}}}+\frac{10a\sqrt{b\,x^{\frac{1}{3}}+ax}}{7b^{2}x^{\frac{2}{3}}}+\frac{5a\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\left(x^{\frac{1}{3}}\right.}\right.}{7b^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}$

input
```
int(1/x^2/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*(5*a*(-a*b)^(1/2)*((x^(1/3)*a+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(x
^(1/3)*a-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2)*a)^(1/2)
*EllipticF(((x^(1/3)*a+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^(4
/3)+4*a*b*x+10*x^(5/3)*a^2-6*x^(1/3)*b^2)/b^2/(x^(1/3)*(b+a*x^(2/3)))^(1/2
)/x^(4/3)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}\,x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*
x^5 + b^3*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{1}{x^2\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

input
```
integrate(1/x**2/(b*x**(1/3)+a*x)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a*x + b*x**(1/3))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*x^(1/3))*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*x + b*x^(1/3))*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{x^2\,\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

input
```
int(1/(x^2*(a*x + b*x^(1/3))^(1/2)),x)
```

output
```
int(1/(x^2*(a*x + b*x^(1/3))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{\sqrt{x^{\frac{1}{3}}b+ax}}{x^{\frac{7}{3}}b+a\,x^3}dx$$

input
```
int(1/x^2/(b*x^(1/3)+a*x)^(1/2),x)
```

output
```
int(sqrt(x**(1/3)*b + a*x)/(x**(1/3)*b*x**2 + a*x**3),x)
```

3.135 $\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal result . . . 1170
Mathematica [C] (verified) . . . 1171
Rubi [A] (warning: unable to verify) . . . 1171
Maple [A] (verified) . . . 1184
Fricas [F] . . . 1185
Sympy [F] . . . 1186
Maxima [F] . . . 1186
Giac [F] . . . 1186
Mupad [F(-1)] . . . 1187
Reduce [F] . . . 1187

Optimal result

Integrand size = 19, antiderivative size = 388

$$\begin{aligned}\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx &= -\frac{154a^{7/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{65b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}}\\ &\quad+\frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}}-\frac{154a^2\sqrt{b\sqrt[3]{x}+ax}}{195b^3x}+\frac{154a^3\sqrt{b\sqrt[3]{x}+ax}}{65b^4\sqrt[3]{x}}\\ &\quad+\frac{154a^{13/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65b^{15/4}\sqrt{b\sqrt[3]{x}+ax}}\\ &\quad-\frac{77a^{13/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{65b^{15/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

output
```
-154/65*a^(7/2)*(b+a*x^(2/3))*x^(1/3)/b^4/(b^(1/2)+a^(1/2)*x^(1/3))/(b*x^(
1/3)+a*x)^(1/2)-6/13*(b*x^(1/3)+a*x)^(1/2)/b/x^(7/3)+22/39*a*(b*x^(1/3)+a*
x)^(1/2)/b^2/x^(5/3)-154/195*a^2*(b*x^(1/3)+a*x)^(1/2)/b^3/x+154/65*a^3*(b
*x^(1/3)+a*x)^(1/2)/b^4/x^(1/3)+154/65*a^(13/4)*(b^(1/2)+a^(1/2)*x^(1/3))*
((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*EllipticE(sin(2*
arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))/b^(15/4)/(b*x^(1/3)+a*x)^(1/
2)-77/65*a^(13/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2
)*x^(1/3))^2)^(1/2)*x^(1/6)*InverseJacobiAM(2*arctan(a^(1/4)*x^(1/6)/b^(1/
4)),1/2*2^(1/2))/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.15

$$\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx=-\frac{6\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(-\frac{13}{4},\frac{1}{2},-\frac{9}{4},-\frac{ax^{2/3}}{b}\right)}{13x^2\sqrt{b\sqrt[3]{x}+ax}}$$

input
```
Integrate[1/(x^3*Sqrt[b*x^(1/3) + a*x]),x]
```

output
```
(-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-13/4, 1/2, -9/4, -((a*x^(2/
3))/b)])/(13*x^2*Sqrt[b*x^(1/3) + a*x])
```

Rubi [A] (warning: unable to verify)

Time = 0.95 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {1924, 1931, 1931, 1931, 1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

$$\Big\downarrow \text{ 1924}$$

$$3\int \frac{1}{x^{7/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 1931}$$

$$3\left(-\frac{11a\int \frac{1}{x^{5/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)$$

$$\Big\downarrow \text{ 1931}$$

$$3\left(-\frac{11a\left(-\frac{7a\int \frac{1}{x\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)$$

$$\Big\downarrow \text{ 1931}$$

$$3\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\int \frac{1}{\sqrt[3]{x}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)$$

$$\Big\downarrow \text{ 1931}$$

$$3\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{a\int\frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)$$

$\downarrow$ 1938

$$3\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)$$

$$\Big\downarrow \; 266$$

$$3\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)$$

$\Big\downarrow$ 834

$$3\Bigg(-\frac{11a\Bigg(-\frac{7a\Bigg(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{b}\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\Bigg)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{\cdots}}}{9bx^{5/3}}\cdots}{13b}\cdots$$

$\downarrow$ 27

$$3\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}\right.$$

↓ 761

$$3\left(11a\left(7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b}\sqrt[3]{x}}-\frac{2\sqrt{\cdots}}{\cdots}\right)}{5b}\right.\right.\right.$$

$$-\frac{\cdots}{9b}\qquad -\frac{\cdots}{13b}$$

↓ 1510

$$3\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4\cdots}}{\left(\sqrt{a}x^{2\cdots}\right.}}\cdots}{\sqrt[4]{\cdots}}\cdots\right)}{b\sqrt{ax+b\sqrt[3]{x}}}\cdots\right)}{5b}\cdots\right)}{9b}\cdots\right)}{13b}\cdots\right.$$

input
```
Int[1/(x^3*Sqrt[b*x^(1/3) + a*x]),x]
```

output
```
3*((-2*Sqrt[b*x^(1/3) + a*x])/(13*b*x^(7/3)) - (11*a*((-2*Sqrt[b*x^(1/3) +
 a*x])/(9*b*x^(5/3)) - (7*a*((-2*Sqrt[b*x^(1/3) + a*x])/(5*b*x) - (3*a*((-
2*Sqrt[b*x^(1/3) + a*x])/(b*x^(1/3)) + (2*a*Sqrt[b + a*x^(2/3)]*x^(1/6)*(-
((-(x^(1/6)*Sqrt[b + a*x^(4/3)])/(Sqrt[b] + Sqrt[a]*x^(2/3))) + (b^(1/4)*
(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3
))^2]*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b
 + a*x^(4/3)]))/Sqrt[a]) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b +
a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1
/6))/b^(1/4)], 1/2])/(2*a^(3/4)*Sqrt[b + a*x^(4/3)])))/(b*Sqrt[b*x^(1/3) +
 a*x])))/(5*b)))/(9*b)))/(13*b))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.68

method	result
derivativedivides	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{13b\,x^{\frac{7}{3}}}+\frac{22a\sqrt{b\,x^{\frac{1}{3}}+ax}}{39b^{2}x^{\frac{5}{3}}}-\frac{154a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{195b^{3}x}+\frac{154\left(b+a\,x^{\frac{2}{3}}\right)a^{3}}{65b^{4}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}-\frac{77a^{3}\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{\cdots}}{\cdots}$
default	$\frac{-462a^{3}b\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}a-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,x^{\frac{10}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,\operatorname{EllipticE}\left(\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+231a^{3}b\sqrt{\cdots}}{\cdots}$

input
```
int(1/x^3/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-6/13*(b*x^(1/3)+a*x)^(1/2)/b/x^(7/3)+22/39*a*(b*x^(1/3)+a*x)^(1/2)/b^2/x^
(5/3)-154/195*a^2*(b*x^(1/3)+a*x)^(1/2)/b^3/x+154/65*(b+a*x^(2/3))*a^3/b^4
/(x^(1/3)*(b+a*x^(2/3)))^(1/2)-77/65*a^3/b^4*(-a*b)^(1/2)*((x^(1/3)+1/a*(-
a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^
(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2)*a)^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(
-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/
2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)
^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}\,x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*
x^6 + b^3*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{x^3\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

input
```
integrate(1/x**3/(b*x**(1/3)+a*x)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(a*x + b*x**(1/3))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*x^(1/3))*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*x + b*x^(1/3))*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{x^3\,\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

input
```
int(1/(x^3*(a*x + b*x^(1/3))^(1/2)),x)
```

output
```
int(1/(x^3*(a*x + b*x^(1/3))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{\sqrt{x^{\frac{1}{3}}b+ax}}{x^{\frac{10}{3}}b+a\,x^4}dx$$

input
```
int(1/x^3/(b*x^(1/3)+a*x)^(1/2),x)
```

output
```
int(sqrt(x**(1/3)*b + a*x)/(x**(1/3)*b*x**3 + a*x**4),x)
```

3.136 $\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal result 1188
Mathematica [C] (verified) 1189
Rubi [A] (warning: unable to verify) 1189
Maple [A] (verified) 1200
Fricas [F] 1201
Sympy [F] 1201
Maxima [F] 1201
Giac [F] 1202
Mupad [F(-1)] 1202
Reduce [F] 1202

Optimal result

Integrand size = 19, antiderivative size = 251

$$\begin{aligned}\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx &= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}}+\frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}}-\frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2}\\ &\quad+\frac{3978a^3\sqrt{b\sqrt[3]{x}+ax}}{7315b^4x^{4/3}}-\frac{1326a^4\sqrt{b\sqrt[3]{x}+ax}}{1463b^5x^{2/3}}\\ &\quad-\frac{663a^{19/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{1463b^{21/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

output
```
-6/19*(b*x^(1/3)+a*x)^(1/2)/b/x^(10/3)+34/95*a*(b*x^(1/3)+a*x)^(1/2)/b^2/x
^(8/3)-442/1045*a^2*(b*x^(1/3)+a*x)^(1/2)/b^3/x^2+3978/7315*a^3*(b*x^(1/3)
+a*x)^(1/2)/b^4/x^(4/3)-1326/1463*a^4*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(2/3)-66
3/1463*a^(19/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*
x^(1/3))^2)^(1/2)*x^(1/6)*InverseJacobiAM(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)
),1/2*2^(1/2))/b^(21/4)/(b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx = -\frac{6\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(-\frac{19}{4},\frac{1}{2},-\frac{15}{4},-\frac{ax^{2/3}}{b}\right)}{19x^3\sqrt{b\sqrt[3]{x}+ax}}$$

input
```
Integrate[1/(x^4*Sqrt[b*x^(1/3) + a*x]),x]
```

output
```
(-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-19/4, 1/2, -15/4, -((a*x^(2
/3))/b)])/(19*x^3*Sqrt[b*x^(1/3) + a*x])
```

Rubi [A] (warning: unable to verify)

Time = 0.82 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.22, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1924, 1931, 1931, 1931, 1931, 1931, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{ax+b\sqrt[3]{x}}}dx$$

$$\Big\downarrow \ 1924$$

$$3\int \frac{1}{x^{10/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}$$

$$\Big\downarrow \ 1931$$

$$3\left(-\frac{17a\int \frac{1}{x^{8/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{19b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{19bx^{10/3}}\right)$$

$$\Big\downarrow \ 1931$$

$$3\left(-\frac{17a\left(-\frac{13a\int \frac{1}{x^2\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{19bx^{10/3}}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\int \frac{1}{x^{4/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^2}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{19bx^{10/3}}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\int\frac{1}{x^{2/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^2}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{19bx^{10/3}}\right)$$

$$\downarrow \text{1931}$$

$$3\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{a\int\frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{3b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^{2}}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}-\frac{2\sqrt{ax+}}{19bx^{10}}\right.$$

↓ 1917

$$3\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{x^{2/3}a+b}\,\sqrt[6]{x}}d\sqrt[3]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^{2}}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}\right.$$

↓ 266

$$3\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^{2}}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}-$$

↓ 761

$$3\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{a^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax\cdots}}{7bx\cdots}}{11b}\cdots}{15b}\cdots}{19b}\cdots$$

input
```
Int[1/(x^4*Sqrt[b*x^(1/3) + a*x]),x]
```

output
```
3*((-2*Sqrt[b*x^(1/3) + a*x])/(19*b*x^(10/3)) - (17*a*((-2*Sqrt[b*x^(1/3)
+ a*x])/(15*b*x^(8/3)) - (13*a*((-2*Sqrt[b*x^(1/3) + a*x])/(11*b*x^2) - (9
*a*((-2*Sqrt[b*x^(1/3) + a*x])/(7*b*x^(4/3)) - (5*a*((-2*Sqrt[b*x^(1/3) +
a*x])/(3*b*x^(2/3)) - (a^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b + a*x^(2
/3)]*x^(1/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF
[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[b*x^(1/3) + a*
x]*Sqrt[b + a*x^(4/3)])))/(7*b)))/(11*b)))/(15*b)))/(19*b))
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 4.81 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.71

method	result
default	$-\frac{3315a^4\sqrt{-ab}\,\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}a-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)x^{\frac{16}{3}}+2652a^4b\,x^5+6630a}{7315b^5\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,x^{\frac{16}{3}}}$
derivativedivides	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{19b\,x^{\frac{10}{3}}}+\frac{34a\sqrt{b\,x^{\frac{1}{3}}+ax}}{95b^2x^{\frac{8}{3}}}-\frac{442a^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{1045b^3x^2}+\frac{3978a^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{7315b^4x^{\frac{4}{3}}}-\frac{1326a^4\sqrt{b\,x^{\frac{1}{3}}+ax}}{1463b^5x^{\frac{2}{3}}}-\frac{663a^4\sqrt{-}}{}$

input
```
int(1/x^4/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7315*(3315*a^4*(-a*b)^(1/2)*((x^(1/3)*a+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2)*(-2*(x^(1/3)*a-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2)
*a)^(1/2)*EllipticF(((x^(1/3)*a+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1
/2))*x^(16/3)+2652*a^4*b*x^5+6630*x^(17/3)*a^5+476*x^(11/3)*a^2*b^3-884*x^
(13/3)*a^3*b^2-308*a*b^4*x^3+2310*x^(7/3)*b^5)/b^5/(x^(1/3)*(b+a*x^(2/3)))
^(1/2)/x^(16/3)
```

Fricas [F]

$$\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*
x^7 + b^3*x^5), x)
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{x^4\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

input
```
integrate(1/x**4/(b*x**(1/3)+a*x)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt(a*x + b*x**(1/3))), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*x^(1/3))*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*x + b*x^(1/3))*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{x^4\,\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

input
```
int(1/(x^4*(a*x + b*x^(1/3))^(1/2)),x)
```

output
```
int(1/(x^4*(a*x + b*x^(1/3))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{\sqrt{x^{\frac{1}{3}}b+ax}}{x^{\frac{13}{3}}b+a\,x^5}dx$$

input
```
int(1/x^4/(b*x^(1/3)+a*x)^(1/2),x)
```

output
```
int(sqrt(x**(1/3)*b + a*x)/(x**(1/3)*b*x**4 + a*x**5),x)
```

3.137 $\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal result 1203
Mathematica [C] (verified) 1204
Rubi [A] (warning: unable to verify) 1204
Maple [A] (verified) 1224
Fricas [F] 1224
Sympy [F] 1225
Maxima [F] 1225
Giac [F] 1225
Mupad [F(-1)] 1226
Reduce [F] 1226

Optimal result

Integrand size = 19, antiderivative size = 437

$$\begin{aligned}\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = &-\frac{4807b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221a^{13/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} \\ &-\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{4807b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^6}-\frac{24035b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^5} \\ &+\frac{6555b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^4}-\frac{437bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^3}+\frac{23x^3\sqrt{b\sqrt[3]{x}+ax}}{7a^2} \\ &+\frac{4807b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221a^{27/4}\sqrt{b\sqrt[3]{x}+ax}} \\ &-\frac{4807b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{442a^{27/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

output
```
-4807/221*b^5*(b+a*x^(2/3))*x^(1/3)/a^(13/2)/(b^(1/2)+a^(1/2)*x^(1/3))/(b*
x^(1/3)+a*x)^(1/2)-3*x^4/a/(b*x^(1/3)+a*x)^(1/2)+4807/663*b^4*x^(1/3)*(b*x
^(1/3)+a*x)^(1/2)/a^6-24035/4641*b^3*x*(b*x^(1/3)+a*x)^(1/2)/a^5+6555/1547
*b^2*x^(5/3)*(b*x^(1/3)+a*x)^(1/2)/a^4-437/119*b*x^(7/3)*(b*x^(1/3)+a*x)^(
1/2)/a^3+23/7*x^3*(b*x^(1/3)+a*x)^(1/2)/a^2+4807/221*b^(21/4)*(b^(1/2)+a^(
1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*El
lipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))/a^(27/4)/(b*x^
(1/3)+a*x)^(1/2)-4807/442*b^(21/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3)
)/(b^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*InverseJacobiAM(2*arctan(a^(1
/4)*x^(1/6)/b^(1/4)),1/2*2^(1/2))/a^(27/4)/(b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.30

$$\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \frac{2x^{2/3}\left(-33649b^5+4807ab^4x^{2/3}-2185a^2b^3x^{4/3}+1311a^3b^2x^2-897a^4bx^{8/3}+663a\right.}{4641a^6\sqrt{b\sqrt[3]{x}+}$$

input
```
Integrate[x^4/(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
(2*x^(2/3)*(-33649*b^5 + 4807*a*b^4*x^(2/3) - 2185*a^2*b^3*x^(4/3) + 1311*
a^3*b^2*x^2 - 897*a^4*b*x^(8/3) + 663*a^5*x^(10/3) + 33649*b^5*Sqrt[1 + (a
*x^(2/3))/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -((a*x^(2/3))/b)]))/(4641*a^
6*Sqrt[b*x^(1/3) + a*x])
```

Rubi [A] (warning: unable to verify)

Time = 1.16 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {1924, 1928, 1930, 1930, 1930, 1930, 1930, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(ax+b\sqrt[3]{x}\right)^{3/2}}\,dx$$

$\big\downarrow$ 1924

$$3\int \frac{x^{14/3}}{\left(\sqrt[3]{x}b+ax\right)^{3/2}}d\sqrt[3]{x}$$

$\big\downarrow$ 1928

$$3\left(\frac{23\int \frac{x^{11/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{2a}-\frac{x^4}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\big\downarrow$ 1930

$$3\left(\frac{23\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\int \frac{x^3}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{21a}\right)}{2a}-\frac{x^4}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\big\downarrow$ 1930

$$3\left(\frac{23\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\int \frac{x^{7/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{17a}\right)}{21a}\right)}{2a}-\frac{x^4}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\big\downarrow$ 1930

$$3\left(\frac{23\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\int\frac{x^{5/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{13a}\right)}{17a}\right)}{21a}\right)}{2a}-\frac{x^4}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\downarrow$ 1930

$$3\left(\frac{23\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\int\frac{x}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{9a}\right)}{13a}\right)}{17a}\right)}{21a}\right)}{2a}-\frac{x}{a\sqrt{ax}}\right.$$

↓ 1930

$$\Bigg(23 \left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a} - \frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a} - \frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a} - \frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a} - \frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a} - \frac{3b\int\frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}\,d\sqrt[3]{x}}{5a}\right.}{9a}\right.}{13a}\right.}{17a}\right.}{21a}\right.$$

↓ 1938

$$23\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int}{5a\sqrt{ax+b\sqrt[3]{\ }}}\right.}{9a}\right.}{13a}\right.}{17a}\right.}{21a}\right.$$

↓ 266

$$3\,\frac{23\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\cdots}{5a\sqrt{ax+b\sqrt[3]{\cdots}}}\cdots\right)}{9a}\cdots\right)}{13a}\cdots\right)}{17a}\cdots\right)}{21a}\cdots\right)\cdots}{2a}\cdots$$

↓ 834

23 $\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a} - \frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a} - \frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a} - \frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a} - \frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a} - \frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{13a}\right.}{17a}\right.}{21a}$

↓ 27

$$6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\sqrt{b}\right.$$

$$7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\right.$$

$$11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\right. \qquad 9$$

$$15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\right. \qquad 13a$$

$$19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\right. \qquad 17a$$

23 $$\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}- \qquad 21a$$

↓ 761

$$6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\sqrt[4]{\ldots}\right.$$

$$7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{\ldots}{\ldots}\right.$$

$$11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{\ldots}{\ldots}\right.$$

$$15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{\ldots}{\ldots}\right.$$

$$19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{\ldots}{\ldots}\right.$$

↓ 1510

$$6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\sqrt[4]{\cdots}\right.$$

$$7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{\cdots}{\cdots}\right.$$

$$11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{\cdots}{\cdots}\right.$$

$$15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{\cdots}{\cdots}\right.$$

$$19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{\cdots}{\cdots}\right.$$

input
```
Int[x^4/(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
3*(-(x^4/(a*Sqrt[b*x^(1/3) + a*x])) + (23*((2*x^3*Sqrt[b*x^(1/3) + a*x])/(
21*a) - (19*b*((2*x^(7/3)*Sqrt[b*x^(1/3) + a*x])/(17*a) - (15*b*((2*x^(5/3
)*Sqrt[b*x^(1/3) + a*x])/(13*a) - (11*b*((2*x*Sqrt[b*x^(1/3) + a*x])/(9*a)
 - (7*b*((2*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(5*a) - (6*b*Sqrt[b + a*x^(2/3)
]*x^(1/6)*(-((-(-((x^(1/6)*Sqrt[b + a*x^(4/3)])/(Sqrt[b] + Sqrt[a]*x^(2/3)))
 + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sq
rt[a]*x^(2/3))^2]*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^
(1/4)*Sqrt[b + a*x^(4/3)]))/Sqrt[a]) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3)
)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTan[(
a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(3/4)*Sqrt[b + a*x^(4/3)])))/(5*a*Sq
rt[b*x^(1/3) + a*x])))/(9*a)))/(13*a)))/(17*a)))/(21*a)))/(2*a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1928
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 4.02 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{3x^{\frac{2}{3}}b^5}{a^6\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}+\frac{2x^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{7a^2}-\frac{80b\,x^{\frac{7}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{119a^3}+\frac{1914b^2x^{\frac{5}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1547a^4}-\frac{10112b^3x\sqrt{b\,x^{\frac{1}{3}}+ax}}{4641a^5}+$
default	$\frac{5244x^{\frac{8}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^4b^2-3588x^{\frac{10}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^5b-8740x^2\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^3b^3-201894b^6\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{}}{}$

input

```
int(x^4/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
3*x^(2/3)/a^6*b^5/((x^(2/3)+b/a)*x^(1/3)*a)^(1/2)+2/7*x^3*(b*x^(1/3)+a*x)^
(1/2)/a^2-80/119*b*x^(7/3)*(b*x^(1/3)+a*x)^(1/2)/a^3+1914/1547*b^2*x^(5/3)
*(b*x^(1/3)+a*x)^(1/2)/a^4-10112/4641*b^3*x*(b*x^(1/3)+a*x)^(1/2)/a^5+2818
/663*b^4*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^6-4807/442*b^5/a^7*(-a*b)^(1/2)*(
(x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(
1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2)*a)^(1/2)/(b*x^(1/3)+a*x
)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^
(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^
(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x^4}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate(x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^4*x^6 + 3*a^2*b^2*x^(14/3) - 2*a*b^3*x^4 - (2*a^3*b*x^5 - b^4*
x^3)*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)
```

Sympy [F]

$$\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{x^4}{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4/(b*x**(1/3)+a*x)**(3/2),x)
```

output
```
Integral(x**4/(a*x + b*x**(1/3))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{x^4}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(a*x + b*x^(1/3))^(3/2), x)
```

Giac [F]

$$\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{x^4}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^4/(a*x + b*x^(1/3))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x^4}{{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}\,dx$$

input
```
int(x^4/(a*x + b*x^(1/3))^(3/2),x)
```

output
```
int(x^4/(a*x + b*x^(1/3))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\frac{1326x^{\frac{23}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a^5-4370x^{\frac{11}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a^2b^3+2622\sqrt{x}\,\sqrt{x^{\frac{2}{3}}a+b}\,a^3b^2x^2-6729}{}$$

input
```
int(x^4/(b*x^(1/3)+a*x)^(3/2),x)
```

output
```
(1326*x**(5/6)*sqrt(x**(2/3)*a + b)*a**5*x**3 - 4370*x**(5/6)*sqrt(x**(2/3
)*a + b)*a**2*b**3*x + 2622*sqrt(x)*sqrt(x**(2/3)*a + b)*a**3*b**2*x**2 -
67298*sqrt(x)*sqrt(x**(2/3)*a + b)*b**5 - 1794*x**(1/6)*sqrt(x**(2/3)*a +
b)*a**4*b*x**3 + 9614*x**(1/6)*sqrt(x**(2/3)*a + b)*a*b**4*x + 33649*x**(2
/3)*int((x**(1/6)*sqrt(x**(2/3)*a + b))/(x**(2/3)*b**2 + 2*x**(1/3)*a*b*x
+ a**2*x**2),x)*a*b**6 + 33649*int((x**(1/6)*sqrt(x**(2/3)*a + b))/(x**(2/
3)*b**2 + 2*x**(1/3)*a*b*x + a**2*x**2),x)*b**7)/(4641*a**6*(x**(2/3)*a +
b))
```

3.138 $\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal result . . . 1227
Mathematica [C] (verified) . . . 1228
Rubi [A] (warning: unable to verify) . . . 1228
Maple [A] (verified) . . . 1239
Fricas [F] . . . 1240
Sympy [F] . . . 1240
Maxima [F] . . . 1241
Giac [F] . . . 1241
Mupad [F(-1)] . . . 1241
Reduce [F] . . . 1242

Optimal result

Integrand size = 19, antiderivative size = 239

$$\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=-\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}}-\frac{663b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^5}+\frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^4}-\frac{221bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^3}+\frac{17x^2\sqrt{b\sqrt[3]{x}+ax}}{5a^2}+\frac{663b^{15/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{154a^{21/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output

```
-3*x^3/a/(b*x^(1/3)+a*x)^(1/2)-663/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^5+1989/3
85*b^2*x^(2/3)*(b*x^(1/3)+a*x)^(1/2)/a^4-221/55*b*x^(4/3)*(b*x^(1/3)+a*x)^
(1/2)/a^3+17/5*x^2*(b*x^(1/3)+a*x)^(1/2)/a^2+663/154*b^(15/4)*(b^(1/2)+a^(
1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*In
verseJacobiAM(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)),1/2*2^(1/2))/a^(21/4)/(b*x
^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.50

$$\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \frac{-3315b^4\sqrt[3]{x}-1326ab^3x+442a^2b^2x^{5/3}-238a^3bx^{7/3}+154a^4x^3+3315b^4\sqrt{1+\frac{ax^2}{b}}}{385a^5\sqrt{b\sqrt[3]{x}+ax}}$$

input
```
Integrate[x^3/(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
(-3315*b^4*x^(1/3) - 1326*a*b^3*x + 442*a^2*b^2*x^(5/3) - 238*a^3*b*x^(7/3
) + 154*a^4*x^3 + 3315*b^4*Sqrt[1 + (a*x^(2/3))/b]*x^(1/3)*Hypergeometric2
F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)])/(385*a^5*Sqrt[b*x^(1/3) + a*x])
```

Rubi [A] (warning: unable to verify)

Time = 0.83 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1924, 1928, 1930, 1930, 1930, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(ax+b\sqrt[3]{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{x^{11/3}}{\left(\sqrt[3]{x}b+ax\right)^{3/2}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1928}$$

$$3\left(\frac{17\int \frac{x^{8/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{2a}-\frac{x^3}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\downarrow \text{1930}$$

$$3\left(\frac{17\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\int \frac{x^2}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{15a}\right)}{2a}-\frac{x^3}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\downarrow \text{1930}$$

$$3\left(\frac{17\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\int \frac{x^{4/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{11a}\right)}{15a}\right)}{2a}-\frac{x^3}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\downarrow \text{1930}$$

$$3\left(\frac{17\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\int\frac{x^{2/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{7a}\right)}{11a}\right)}{15a}\right)}{2a}-\frac{x^3}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

↓ 1930

$$3\left(\frac{17\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b\int\frac{1}{\sqrt{\sqrt[3]{x}b+ax}}\,d\sqrt[3]{x}}{3a}\right)}{7a}\right)}{11a}\right)}{15a}\right)}{2a}-\frac{x^3}{a\sqrt{ax+b}}\right.$$

↓ 1917

$$3\left(\frac{17\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{3a\sqrt{ax+b\sqrt[3]{x}}}\right)}{7a}\right)}{11a}\right)}{15a}\right)}{2a}\right.$$

↓ 266

$$3\left(\frac{17\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{2b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3a\sqrt{ax+b\sqrt[3]{x}}}\right)}{7a}\right)}{11a}\right)}{15a}\right)}{2a}-\right.$$

↓ 761

$$3\,\frac{17\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right.}}}{3a^{5/4}\sqrt{ax+b\sqrt[3]{x}}\sqrt{}}\right.}{7a}\right.}{11a}\right.}{15a}\right.}{2a}$$

input
```
Int[x^3/(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
3*(-(x^3/(a*Sqrt[b*x^(1/3) + a*x])) + (17*((2*x^2*Sqrt[b*x^(1/3) + a*x])/(
15*a) - (13*b*((2*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(11*a) - (9*b*((2*x^(2/3)
*Sqrt[b*x^(1/3) + a*x])/(7*a) - (5*b*((2*Sqrt[b*x^(1/3) + a*x])/(3*a) - (b
^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b + a*x^(2/3)]*x^(1/6)*Sqrt[(b + a
*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/
6))/b^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[b*x^(1/3) + a*x]*Sqrt[b + a*x^(4/3)]))
)/(7*a)))/(11*a)))/(15*a)))/(2*a))
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1928
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.95

method	result
derivativedivides	$-\frac{3x^{\frac{1}{3}}b^{4}}{a^{5}\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}+\frac{2x^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{5a^{2}}-\frac{56b\,x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{55a^{3}}+\frac{834b^{2}x^{\frac{2}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{385a^{4}}-\frac{432b^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{77a^{5}}+$ 66
default	$-\frac{-884x^{\frac{5}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^{3}b^{2}+476x^{\frac{7}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^{4}b-3315\sqrt{-ab}\,\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}a-\right.}{\sqrt{-}}}}{}$

input
```
int(x^3/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-3*x^(1/3)/a^5*b^4/((x^(2/3)+b/a)*x^(1/3)*a)^(1/2)+2/5*x^2*(b*x^(1/3)+a*x)
^(1/2)/a^2-56/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a^3+834/385*b^2*x^(2/3)*(
b*x^(1/3)+a*x)^(1/2)/a^4-432/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^5+663/154*b^4/
a^6*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^
(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2)*a)^(1
/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(
1/2))^(1/2),1/2*2^(1/2))
```

Fricas [F]

$$\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x^3}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^4*x^5 + 3*a^2*b^2*x^(11/3) - 2*a*b^3*x^3 - (2*a^3*b*x^4 - b^4*
x^2)*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)
```

Sympy [F]

$$\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x^3}{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(b*x**(1/3)+a*x)**(3/2),x)
```

output
```
Integral(x**3/(a*x + b*x**(1/3))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x^3}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(a*x + b*x^(1/3))^(3/2), x)
```

Giac [F]

$$\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x^3}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^3/(a*x + b*x^(1/3))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x^3}{\left(a\,x+b\,x^{1/3}\right)^{3/2}}\,dx$$

input
```
int(x^3/(a*x + b*x^(1/3))^(3/2),x)
```

output
```
int(x^3/(a*x + b*x^(1/3))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \frac{154x^{\frac{17}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a^4-1326x^{\frac{5}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,ab^3+442\sqrt{x}\,\sqrt{x^{\frac{2}{3}}a+b}\,a^2b^2x-238x^{\frac{13}{6}}\sqrt{}}{}$$

input
```
int(x^3/(b*x^(1/3)+a*x)^(3/2),x)
```

output
```
(154*x**(5/6)*sqrt(x**(2/3)*a + b)*a**4*x**2 - 1326*x**(5/6)*sqrt(x**(2/3)
*a + b)*a*b**3 + 442*sqrt(x)*sqrt(x**(2/3)*a + b)*a**2*b**2*x - 238*x**(1/
6)*sqrt(x**(2/3)*a + b)*a**3*b*x**2 - 6630*x**(1/6)*sqrt(x**(2/3)*a + b)*b
**4 + 1105*x**(2/3)*int(sqrt(x**(2/3)*a + b)/(x**(5/6)*b**2 + 2*sqrt(x)*a*
b*x + x**(1/6)*a**2*x**2),x)*a*b**5 + 1105*int(sqrt(x**(2/3)*a + b)/(x**(5
/6)*b**2 + 2*sqrt(x)*a*b*x + x**(1/6)*a**2*x**2),x)*b**6)/(385*a**5*(x**(2
/3)*a + b))
```

3.139 $\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal result 1243
Mathematica [C] (verified) 1244
Rubi [A] (warning: unable to verify) 1244
Maple [A] (verified) 1253
Fricas [F] 1253
Sympy [F] 1254
Maxima [F] 1254
Giac [F] 1254
Mupad [F(-1)] 1255
Reduce [F] 1255

Optimal result

Integrand size = 19, antiderivative size = 349

$$\begin{aligned}
\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx &= \frac{77b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5a^{7/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} \\
&- \frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^3} + \frac{11x\sqrt{b\sqrt[3]{x}+ax}}{3a^2} \\
&- \frac{77b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{15/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&+ \frac{77b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{10a^{15/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}$$

output
```
77/5*b^2*(b+a*x^(2/3))*x^(1/3)/a^(7/2)/(b^(1/2)+a^(1/2)*x^(1/3))/(b*x^(1/3
)+a*x)^(1/2)-3*x^2/a/(b*x^(1/3)+a*x)^(1/2)-77/15*b*x^(1/3)*(b*x^(1/3)+a*x)
^(1/2)/a^3+11/3*x*(b*x^(1/3)+a*x)^(1/2)/a^2-77/5*b^(9/4)*(b^(1/2)+a^(1/2)*
x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*Ellipti
cE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))/a^(15/4)/(b*x^(1/3)
+a*x)^(1/2)+77/10*b^(9/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2
)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*InverseJacobiAM(2*arctan(a^(1/4)*x^(1/
6)/b^(1/4)),1/2*2^(1/2))/a^(15/4)/(b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.27

$$\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \frac{2x^{2/3}\left(77b^2-11abx^{2/3}+5a^2x^{4/3}-77b^2\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\right.\right.}{15a^3\sqrt{b\sqrt[3]{x}+ax}}$$

input
```
Integrate[x^2/(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
(2*x^(2/3)*(77*b^2 - 11*a*b*x^(2/3) + 5*a^2*x^(4/3) - 77*b^2*Sqrt[1 + (a*x
^(2/3))/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -((a*x^(2/3))/b)]))/(15*a^3*Sq
rt[b*x^(1/3) + a*x])
```

Rubi [A] (warning: unable to verify)

Time = 0.85 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1924, 1928, 1930, 1930, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(ax+b\sqrt[3]{x}\right)^{3/2}}\,dx$$

$\downarrow$ 1924

$$3\int \frac{x^{8/3}}{\left(\sqrt[3]{x}b+ax\right)^{3/2}}d\sqrt[3]{x}$$

$\downarrow$ 1928

$$3\left(\frac{11\int \frac{x^{5/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{2a}-\frac{x^2}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\downarrow$ 1930

$$3\left(\frac{11\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\int \frac{x}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{9a}\right)}{2a}-\frac{x^2}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\downarrow$ 1930

$$3\left(\frac{11\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\int \frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{5a}\right)}{9a}\right)}{2a}-\frac{x^2}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\downarrow$ 1938

$$3\left(\frac{11\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)}{9a}\right)}{2a}-\frac{x^2}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\downarrow$ 266

$$3\left(\frac{11\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)}{9a}\right)}{2a}-\frac{x^2}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\downarrow$ 834

$$3\left(\frac{11\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{b}\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)}{9a}\right)}{2a}-\frac{x^2}{a\sqrt{ax+b\sqrt[3]{x}}}\right.$$

$$\Big\downarrow \text{27}$$

$$3\left(\frac{11\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)}{9a}\right)}{2a}-\frac{x^2}{a\sqrt{ax+b\sqrt[3]{x}}}\right.$$

↓ 761

$$3\left(\frac{11\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\cdots\right.}{5a\sqrt{ax+b\sqrt[3]{x}}}\right.}{9a}\right.}{2a}\right.$$

$\downarrow$ 1510

$$3\left(\frac{11\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^{2}}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\cdots\right)}{5a\sqrt{ax+b\sqrt[3]{\cdots}}}\cdots\right)}{9a}\cdots\right)}{2a}\cdots\right.$$

input
```
Int[x^2/(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
3*(-(x^2/(a*Sqrt[b*x^(1/3) + a*x])) + (11*((2*x*Sqrt[b*x^(1/3) + a*x])/(9*
a) - (7*b*((2*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(5*a) - (6*b*Sqrt[b + a*x^(2/
3)]*x^(1/6)*(-((-((x^(1/6)*Sqrt[b + a*x^(4/3)])/(Sqrt[b] + Sqrt[a]*x^(2/3)
)) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] +
Sqrt[a]*x^(2/3))^2]*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(
a^(1/4)*Sqrt[b + a*x^(4/3)]))/Sqrt[a]) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/
3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTan
[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(3/4)*Sqrt[b + a*x^(4/3)])))/(5*a*
Sqrt[b*x^(1/3) + a*x])))/(9*a)))/(2*a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1928
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.68

method	result
derivativedivides	$-\frac{3x^{\frac{2}{3}}b^2}{a^3\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}+\frac{2x\sqrt{b\,x^{\frac{1}{3}}+ax}}{3a^2}-\frac{32b\,x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{15a^3}+\frac{77b^2\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}}{}$
default	$-\frac{-462b^3\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}a-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\operatorname{EllipticE}\left(\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+231b^3\sqrt{\frac{x^{\frac{1}{3}}a}{}}}{}$

input
```
int(x^2/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-3*x^(2/3)/a^3*b^2/((x^(2/3)+b/a)*x^(1/3)*a)^(1/2)+2/3*x*(b*x^(1/3)+a*x)^(
1/2)/a^2-32/15*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^3+77/10*b^2/a^4*(-a*b)^(1
/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a
*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2)*a)^(1/2)/(b*x^(1/3
)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-
a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-
a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x^2}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^4*x^4 + 3*a^2*b^2*x^(8/3) - 2*a*b^3*x^2 - (2*a^3*b*x^3 - b^4*x
)*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)
```

Sympy [F]

$$\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x^2}{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(b*x**(1/3)+a*x)**(3/2),x)
```

output
```
Integral(x**2/(a*x + b*x**(1/3))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x^2}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(a*x + b*x^(1/3))^(3/2), x)
```

Giac [F]

$$\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x^2}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/(a*x + b*x^(1/3))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x^2}{{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}\,dx$$

input
```
int(x^2/(a*x + b*x^(1/3))^(3/2),x)
```

output
```
int(x^2/(a*x + b*x^(1/3))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\frac{10x^{\frac{11}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a^2+154\sqrt{x}\,\sqrt{x^{\frac{2}{3}}a+b}\,b^2-22x^{\frac{7}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,ab-77x^{\frac{2}{3}}\left(\int \frac{x^{\frac{1}{6}}\sqrt{x}}{x^{\frac{2}{3}}b^2+2x^{\frac{4}{3}}}\right.}{15a^3\left(x^{\frac{2}{3}}a+b\right)}$$

input
```
int(x^2/(b*x^(1/3)+a*x)^(3/2),x)
```

output
```
(10*x**(5/6)*sqrt(x**(2/3)*a + b)*a**2*x + 154*sqrt(x)*sqrt(x**(2/3)*a + b
)*b**2 - 22*x**(1/6)*sqrt(x**(2/3)*a + b)*a*b*x - 77*x**(2/3)*int((x**(1/6
)*sqrt(x**(2/3)*a + b))/(x**(2/3)*b**2 + 2*x**(1/3)*a*b*x + a**2*x**2),x)*
a*b**3 - 77*int((x**(1/6)*sqrt(x**(2/3)*a + b))/(x**(2/3)*b**2 + 2*x**(1/3
)*a*b*x + a**2*x**2),x)*b**4)/(15*a**3*(x**(2/3)*a + b))
```

3.140 $\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal result 1256
Mathematica [C] (verified) 1257
Rubi [A] (warning: unable to verify) 1257
Maple [A] (verified) 1260
Fricas [F] 1261
Sympy [F] 1261
Maxima [F] 1261
Giac [F] 1262
Mupad [F(-1)] 1262
Reduce [F] 1262

Optimal result

Integrand size = 17, antiderivative size = 149

$$\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = -\frac{3x}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{5\sqrt{b\sqrt[3]{x}+ax}}{a^2} - \frac{5b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{9/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
-3*x/a/(b*x^(1/3)+a*x)^(1/2)+5*(b*x^(1/3)+a*x)^(1/2)/a^2-5/2*b^(3/4)*(b^(1
/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(
1/6)*InverseJacobiAM(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)),1/2*2^(1/2))/a^(9/4
)/(b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.55

$$\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\frac{\sqrt{b\sqrt[3]{x}+ax}\left(5b+2ax^{2/3}-5b\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{ax^{2/3}}{b}\right)\right)}{a^2\left(b+ax^{2/3}\right)}$$

input
```
Integrate[x/(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
(Sqrt[b*x^(1/3) + a*x]*(5*b + 2*a*x^(2/3) - 5*b*Sqrt[1 + (a*x^(2/3))/b]*Hy
pergeometric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(a^2*(b + a*x^(2/3)))
```

Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.353$, Rules used = {1924, 1928, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(ax+b\sqrt[3]{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{x^{5/3}}{\left(\sqrt[3]{x}b+ax\right)^{3/2}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1928}$$

$$3\left(\frac{5\int \frac{x^{2/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{2a}-\frac{x}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{5\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b\int \frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{3a}\right)}{2a}-\frac{x}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\Big\downarrow$ 1917

$$3\left(\frac{5\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{3a\sqrt{ax+b\sqrt[3]{x}}}\right)}{2a}-\frac{x}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\Big\downarrow$ 266

$$3\left(\frac{5\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{2b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3a\sqrt{ax+b\sqrt[3]{x}}}\right)}{2a}-\frac{x}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\Big\downarrow$ 761

$$3\left(\frac{5\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3a^{5/4}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}\right)}{2a}-\frac{x}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

input
```
Int[x/(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
3*(-(x/(a*Sqrt[b*x^(1/3) + a*x])) + (5*((2*Sqrt[b*x^(1/3) + a*x])/(3*a) -
(b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b + a*x^(2/3)]*x^(1/6)*Sqrt[(b +
 a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(
1/6))/b^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[b*x^(1/3) + a*x]*Sqrt[b + a*x^(4/3)]
)))/(2*a))
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1928
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{3x^{\frac{1}{3}} b}{a^{2} \sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right) x^{\frac{1}{3}} a}}+\frac{2\sqrt{b\,x^{\frac{1}{3}}+a x}}{a^{2}}-\frac{5 b \sqrt{-a b}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-a b}}{a}\right) a}{\sqrt{-a b}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-a b}}{a}\right) a}{\sqrt{-a b}}}\,\sqrt{-\frac{x^{\frac{1}{3}} a}{\sqrt{-a b}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x^{\frac{1}{3}}\right.}{}}\right.}{2 a^{3} \sqrt{b\,x^{\frac{1}{3}}+a x}}$
default	$-\frac{5 \sqrt{-a b}\,\sqrt{x^{\frac{1}{3}} \left(b+a\,x^{\frac{2}{3}}\right)}\,\sqrt{\frac{x^{\frac{1}{3}} a+\sqrt{-a b}}{\sqrt{-a b}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}} a-\sqrt{-a b}\right)}{\sqrt{-a b}}}\,\sqrt{-\frac{x^{\frac{1}{3}} a}{\sqrt{-a b}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{x^{\frac{1}{3}} a+\sqrt{-a b}}{\sqrt{-a b}}}, \frac{\sqrt{2}}{2}\right) b-6\sqrt{b\,x^{\frac{1}{3}}+a}}{2 x^{\frac{1}{3}} \left(b+a\,x^{\frac{2}{3}}\right) a^{3}}$

input
```
int(x/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3*x^(1/3)/a^2*b/((x^(2/3)+b/a)*x^(1/3)*a)^(1/2)+2*(b*x^(1/3)+a*x)^(1/2)/a^
2-5/2*b/a^3*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)
*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/
2)*a)^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/
(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Fricas [F]

$$\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^
(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)
```

Sympy [F]

$$\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x}{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x**(1/3)+a*x)**(3/2),x)
```

output
```
Integral(x/(a*x + b*x**(1/3))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(a*x + b*x^(1/3))^(3/2), x)
```

Giac [F]

$$\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/(a*x + b*x^(1/3))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x}{\left(a\,x+b\,x^{1/3}\right)^{3/2}}\,dx$$

input
```
int(x/(a*x + b*x^(1/3))^(3/2),x)
```

output
```
int(x/(a*x + b*x^(1/3))^(3/2), x)
```

Reduce [F]

$$\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\frac{6x^{\frac{5}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,a+30x^{\frac{1}{6}}\sqrt{x^{\frac{2}{3}}a+b}\,b-5x^{\frac{2}{3}}\left(\int \frac{\sqrt{x^{\frac{2}{3}}a+b}}{x^{\frac{5}{6}}b^{2}+2\sqrt{x}\,abx+x^{\frac{13}{6}}a^{2}}dx\right)a\,b^{2}-5\left(\int \frac{}{x^{\frac{5}{6}}}\right.}{3a^{2}\left(x^{\frac{2}{3}}a+b\right)}$$

input
```
int(x/(b*x^(1/3)+a*x)^(3/2),x)
```

output
```
(6*x**(5/6)*sqrt(x**(2/3)*a + b)*a + 30*x**(1/6)*sqrt(x**(2/3)*a + b)*b -
5*x**(2/3)*int(sqrt(x**(2/3)*a + b)/(x**(5/6)*b**2 + 2*sqrt(x)*a*b*x + x**
(1/6)*a**2*x**2),x)*a*b**2 - 5*int(sqrt(x**(2/3)*a + b)/(x**(5/6)*b**2 + 2
*sqrt(x)*a*b*x + x**(1/6)*a**2*x**2),x)*b**3)/(3*a**2*(x**(2/3)*a + b))
```

3.141 $\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal result 1263
Mathematica [C] (verified) 1264
Rubi [A] (warning: unable to verify) 1264
Maple [A] (verified) 1267
Fricas [F] 1268
Sympy [F] 1269
Maxima [F] 1269
Giac [F] 1269
Mupad [B] (verification not implemented) 1270
Reduce [F] 1270

Optimal result

Integrand size = 15, antiderivative size = 296

$$\int \frac{1}{(b\sqrt[3]{x}+ax)^{3/2}}\,dx = -\frac{3\left(b+ax^{2/3}\right)\sqrt[3]{x}}{\sqrt{a}b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}}$$
$$+\frac{3\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}$$
$$-\frac{3\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
-3*(b+a*x^(2/3))*x^(1/3)/a^(1/2)/b/(b^(1/2)+a^(1/2)*x^(1/3))/(b*x^(1/3)+a*
x)^(1/2)+3*x^(2/3)/b/(b*x^(1/3)+a*x)^(1/2)+3*(b^(1/2)+a^(1/2)*x^(1/3))*((b
+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*EllipticE(sin(2*arc
tan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))/a^(3/4)/b^(3/4)/(b*x^(1/3)+a*x)
^(1/2)-3/2*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/
3))^2)^(1/2)*x^(1/6)*InverseJacobiAM(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)),1/2
*2^(1/2))/a^(3/4)/b^(3/4)/(b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.21

$$\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\frac{2\sqrt{1+\frac{ax^{2/3}}{b}}x^{2/3}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{ax^{2/3}}{b}\right)}{b\sqrt{b\sqrt[3]{x}+ax}}$$

input
```
Integrate[(b*x^(1/3) + a*x)^(-3/2),x]
```

output
```
(2*Sqrt[1 + (a*x^(2/3))/b]*x^(2/3)*Hypergeometric2F1[3/4, 3/2, 7/4, -((a*x
^(2/3))/b)])/(b*Sqrt[b*x^(1/3) + a*x])
```

Rubi [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {1912, 1924, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(ax+b\sqrt[3]{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1912}$$

$$\frac{3x^{2/3}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{\int \frac{1}{\sqrt[3]{x}\sqrt{\sqrt[3]{x}b+ax}}dx}{2b}$$

$$\Big\downarrow \text{1924}$$

$$\frac{3x^{2/3}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{3\int \frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{2b}$$

$$\Big\downarrow \text{1938}$$

$$\frac{3x^{2/3}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{3\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{2b\sqrt{ax+b\sqrt[3]{x}}}$$

$\downarrow$ 266

$$\frac{3x^{2/3}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{3\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}$$

$\downarrow$ 834

$$\frac{3x^{2/3}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{3\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{b}\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}$$

$\downarrow$ 27

$$\frac{3x^{2/3}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{3\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int \frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}$$

$\downarrow$ 761

$$\frac{3x^{2/3}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{3\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\int \frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}$$

$\downarrow$ 1510

$$\frac{3x^{2/3}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{3\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}E\left(2\,a\ldots\right.}{\sqrt[4]{a}\sqrt{ax^{4/3}+b}\ldots}}{\sqrt{a}\ldots}\right.}{b\sqrt{ax+b\sqrt[3]{x}}}$$

input
```
Int[(b*x^(1/3) + a*x)^(-3/2),x]
```

output
```
(3*x^(2/3))/(b*Sqrt[b*x^(1/3) + a*x]) - (3*Sqrt[b + a*x^(2/3)]*x^(1/6)*(-(
(-((x^(1/6)*Sqrt[b + a*x^(4/3)])/(Sqrt[b] + Sqrt[a]*x^(2/3))) + (b^(1/4)*(
Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3)
)^2]*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b
+ a*x^(4/3)]))/Sqrt[a]) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a
*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/
6))/b^(1/4)], 1/2])/(2*a^(3/4)*Sqrt[b + a*x^(4/3)])))/(b*Sqrt[b*x^(1/3) +
a*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1912
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{3x^{\frac{2}{3}}}{b\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}-\frac{3\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{EllipticE}\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\right.}{a}\right.}{2ba\sqrt{b\,x^{\frac{1}{3}}+ax}}$
default	$-\frac{3\left(2\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}a-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\ \mathrm{EllipticE}\left(\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)b-\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right.}\right.}{2a\,x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right.}$

input
```
int(1/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3*x^(2/3)/b/((x^(2/3)+b/a)*x^(1/3)*a)^(1/2)-3/2/b/a*(-a*b)^(1/2)*((x^(1/3)
+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/
(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2)*a)^(1/2)/(b*x^(1/3)+a*x)^(1/2)*
(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(
1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a
/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^
(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^5 + 2*a^3*b^3*x^3 + b^6*x), x)
```

Sympy [F]

$$\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**(1/3)+a*x)**(3/2),x)
```

output
```
Integral((a*x + b*x**(1/3))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(1/3))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*x + b*x^(1/3))^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 9.41 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.14

$$\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\frac{2\,x\,{\left(\frac{a\,x^{2/3}}{b}+1\right)}^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{4},\frac{3}{2};\ \frac{7}{4};\ -\frac{a\,x^{2/3}}{b}\right)}{{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}$$

input
```
int(1/(a*x + b*x^(1/3))^(3/2),x)
```

output
```
(2*x*((a*x^(2/3))/b + 1)^(3/2)*hypergeom([3/4, 3/2], 7/4, -(a*x^(2/3))/b))
/(a*x + b*x^(1/3))^(3/2)
```

Reduce [F]

$$\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^{\frac{1}{3}}\sqrt{x^{\frac{1}{3}}b+ax}\,b+\sqrt{x^{\frac{1}{3}}b+ax}\,ax}dx$$

input
```
int(1/(b*x^(1/3)+a*x)^(3/2),x)
```

output
```
int(1/(x**(1/3)*sqrt(x**(1/3)*b + a*x)*b + sqrt(x**(1/3)*b + a*x)*a*x),x)
```

3.142 $\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal result 1271
Mathematica [C] (verified) 1272
Rubi [A] (warning: unable to verify) 1272
Maple [A] (verified) 1275
Fricas [F] 1276
Sympy [F] 1276
Maxima [F] 1276
Giac [F] 1277
Mupad [F(-1)] 1277
Reduce [F] 1277

Optimal result

Integrand size = 19, antiderivative size = 158

$$\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}} - \frac{5\sqrt{b\sqrt[3]{x}+ax}}{b^2x^{2/3}} - \frac{5a^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{9/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
3/b/x^(1/3)/(b*x^(1/3)+a*x)^(1/2)-5*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2/3)-5/2*
a^(3/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/3))
^2)^(1/2)*x^(1/6)*InverseJacobiAM(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)),1/2*2^
(1/2))/b^(9/4)/(b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.39

$$\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=-\frac{2\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{2},\frac{1}{4},-\frac{ax^{2/3}}{b}\right)}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}$$

input
```
Integrate[1/(x*(b*x^(1/3) + a*x)^(3/2)),x]
```

output
```
(-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((a*x^(2/3)
)/b)])/(b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])
```

Rubi [A] (warning: unable to verify)

Time = 0.54 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1924, 1929, 1931, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax+b\sqrt[3]{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{1}{\sqrt[3]{x}\left(\sqrt[3]{x}b+ax\right)^{3/2}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1929}$$

$$3\left(\frac{5\int \frac{1}{x^{2/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{2b}+\frac{1}{b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{5\left(-\frac{a\int \frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{3b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{2b}+\frac{1}{b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\Big\downarrow$ 1917

$$3\left(\frac{5\left(-\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{2b}+\frac{1}{b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\Big\downarrow$ 266

$$3\left(\frac{5\left(-\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{2b}+\frac{1}{b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\Big\downarrow$ 761

$$3\left(\frac{5\left(-\frac{a^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{2b}+\frac{1}{b\sqrt[3]{x}\sqrt{ax+b}}\right.$$

input
```
Int[1/(x*(b*x^(1/3) + a*x)^(3/2)),x]
```

output
```
3*(1/(b*x^(1/3)*Sqrt[b*x^(1/3) + a*x]) + (5*((-2*Sqrt[b*x^(1/3) + a*x])/(3
*b*x^(2/3)) - (a^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b + a*x^(2/3)]*x^(
1/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTa
n[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[b*x^(1/3) + a*x]*Sqrt[
b + a*x^(4/3)])))/(2*b))
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.03

method	result
derivativedivides	$-\frac{2\sqrt{b\,x^{\frac{1}{3}}+ax}}{b^{2}x^{\frac{2}{3}}}-\frac{3x^{\frac{1}{3}}a}{b^{2}\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}-\frac{5\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\left(\sqrt{\cdots}\right.\right.}{2b^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}$ …
default	$-\frac{5\sqrt{-ab}\,\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,x^{\frac{2}{3}}\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}a-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+4b\,x^{\frac{1}{3}}\sqrt{\cdots}}{2b^{2}x\left(b+a\,x^{\frac{2}{3}}\right)}$ …

input
```
int(1/x/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2/3)-3*x^(1/3)*a/b^2/((x^(2/3)+b/a)*x^(1/3
)*a)^(1/2)-5/2/b^2*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2)
)^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a
*b)^(1/2)*a)^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1
/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Fricas [F]

$$\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^
(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^6 + 2*a^3*b^3*x^4 + b^6*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(b*x**(1/3)+a*x)**(3/2),x)
```

output
```
Integral(1/(x*(a*x + b*x**(1/3))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*x^(1/3))^(3/2)*x), x)
```

Giac [F]

$$\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*x + b*x^(1/3))^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x\,\left(a\,x+b\,x^{1/3}\right)^{3/2}}\,dx$$

input
```
int(1/(x*(a*x + b*x^(1/3))^(3/2)),x)
```

output
```
int(1/(x*(a*x + b*x^(1/3))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^{\frac{4}{3}}\sqrt{x^{\frac{1}{3}}b+ax}\,b+\sqrt{x^{\frac{1}{3}}b+ax}\,a\,x^{2}}dx$$

input
```
int(1/x/(b*x^(1/3)+a*x)^(3/2),x)
```

output
```
int(1/(x**(1/3)*sqrt(x**(1/3)*b + a*x)*b*x + sqrt(x**(1/3)*b + a*x)*a*x**2
),x)
```

3.143 $\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal result 1278
Mathematica [C] (verified) 1279
Rubi [A] (warning: unable to verify) 1279
Maple [A] (verified) 1293
Fricas [F] 1293
Sympy [F] 1294
Maxima [F] 1294
Giac [F] 1294
Mupad [F(-1)] 1295
Reduce [F] 1295

Optimal result

Integrand size = 19, antiderivative size = 383

$$
\begin{aligned}
\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx &= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}+\frac{77a^{5/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} \\
&-\frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}}+\frac{77a\sqrt{b\sqrt[3]{x}+ax}}{15b^3x}-\frac{77a^2\sqrt{b\sqrt[3]{x}+ax}}{5b^4\sqrt[3]{x}} \\
&-\frac{77a^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{15/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&+\frac{77a^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{10b^{15/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

output
```
3/b/x^(4/3)/(b*x^(1/3)+a*x)^(1/2)+77/5*a^(5/2)*(b+a*x^(2/3))*x^(1/3)/b^4/(
b^(1/2)+a^(1/2)*x^(1/3))/(b*x^(1/3)+a*x)^(1/2)-11/3*(b*x^(1/3)+a*x)^(1/2)/
b^2/x^(5/3)+77/15*a*(b*x^(1/3)+a*x)^(1/2)/b^3/x-77/5*a^2*(b*x^(1/3)+a*x)^(
1/2)/b^4/x^(1/3)-77/5*a^(9/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^
(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*EllipticE(sin(2*arctan(a^(1/4)*x^(
1/6)/b^(1/4))),1/2*2^(1/2))/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)+77/10*a^(9/4)*(
b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)
*x^(1/6)*InverseJacobiAM(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)),1/2*2^(1/2))/b^
(15/4)/(b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.17

$$\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=-\frac{2\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},\frac{3}{2},-\frac{5}{4},-\frac{ax^{2/3}}{b}\right)}{3bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}$$

input
```
Integrate[1/(x^2*(b*x^(1/3) + a*x)^(3/2)),x]
```

output
```
(-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-9/4, 3/2, -5/4, -((a*x^(2/3
))/b)])/(3*b*x^(4/3)*Sqrt[b*x^(1/3) + a*x])
```

Rubi [A] (warning: unable to verify)

Time = 0.96 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.579$, Rules used = {1924, 1929, 1931, 1931, 1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(ax+b\sqrt[3]{x}\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{1}{x^{4/3}\left(\sqrt[3]{x}b+ax\right)^{3/2}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1929}$$

$$3\left(\frac{11\int \frac{1}{x^{5/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{2b}+\frac{1}{bx^{4/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{11\left(-\frac{7a\int \frac{1}{x\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{2b}+\frac{1}{bx^{4/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{11\left(-\frac{7a\left(-\frac{3a\int \frac{1}{\sqrt[3]{x}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{2b}+\frac{1}{bx^{4/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{11\left(-\frac{7a\left(-\frac{3a\left(\frac{a\int\frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{2b}+\frac{1}{bx^{4/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\downarrow$ 1938

$$3\left(\frac{11\left(-\frac{7a\left(-\frac{3a\left(\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{2b}+\frac{1}{bx^{4/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \; 266$$

$$3\left(\frac{11\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{2b}+\frac{1}{bx^{4/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\downarrow 834$$

$$3\left(\frac{11\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{b}\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{2b}\right.$$

$$\Big\downarrow 27$$

$$3\left(\frac{11\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{2b}+\right.$$

↓ 761

$$3\left(11\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+\cdots}}{b\sqrt[3]{\cdots}}\cdots\right)}{5b}\cdots\right)}{9b}\cdots\right)}{2b}\cdots\right)$$

↓ 1510

$$3\left(\frac{11\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+\cdots}{\left(\sqrt{a}x^{2/3}+\cdots\right.}}\cdots}{\sqrt[4]{a}\sqrt{\cdots}}\cdots\right.}{b\sqrt{ax+b\sqrt[3]{x}}}\cdots\right)}{5b}\cdots\right)}{9b}\cdots\right)}{2b}\cdots\right.$$

input
```
Int[1/(x^2*(b*x^(1/3) + a*x)^(3/2)),x]
```

output
```
3*(1/(b*x^(4/3)*Sqrt[b*x^(1/3) + a*x]) + (11*((-2*Sqrt[b*x^(1/3) + a*x])/(
9*b*x^(5/3)) - (7*a*((-2*Sqrt[b*x^(1/3) + a*x])/(5*b*x) - (3*a*((-2*Sqrt[b
*x^(1/3) + a*x])/(b*x^(1/3)) + (2*a*Sqrt[b + a*x^(2/3)]*x^(1/6)*(-((-((x^(
1/6)*Sqrt[b + a*x^(4/3)])/(Sqrt[b] + Sqrt[a]*x^(2/3))) + (b^(1/4)*(Sqrt[b]
 + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*El
lipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b + a*x^(
4/3)]))/Sqrt[a]) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3
))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(
1/4)], 1/2])/(2*a^(3/4)*Sqrt[b + a*x^(4/3)])))/(b*Sqrt[b*x^(1/3) + a*x])))
/(5*b)))/(9*b)))/(2*b))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 2.53 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{2\sqrt{b\,x^{\frac{1}{3}}+ax}}{3b^{2}x^{\frac{5}{3}}}+\frac{32a\sqrt{b\,x^{\frac{1}{3}}+ax}}{15b^{3}x}-\frac{62\left(b+a\,x^{\frac{2}{3}}\right)a^{2}}{5b^{4}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}-\frac{3x^{\frac{2}{3}}a^{3}}{b^{4}\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}+\frac{77a^{2}\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\ \sqrt{}}{}$
default	$-\frac{-462a^{2}b\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}a-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,x^{\frac{8}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,\mathrm{EllipticE}\left(\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+231a^{2}b\sqrt{}}{}$

input
```
int(1/x^2/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(5/3)+32/15*a*(b*x^(1/3)+a*x)^(1/2)/b^3/x
-62/5*(b+a*x^(2/3))*a^2/b^4/(x^(1/3)*(b+a*x^(2/3)))^(1/2)-3*x^(2/3)*a^3/b^
4/((x^(2/3)+b/a)*x^(1/3)*a)^(1/2)+77/10*a^2/b^4*(-a*b)^(1/2)*((x^(1/3)+1/a
*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*
b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2)*a)^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2/
a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)
,1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a
*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \frac{1}{x^{2}\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^{2}}\,dx$$

input
```
integrate(1/x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^
(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^7 + 2*a^3*b^3*x^5 + b^6*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x^2 \left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^2 \left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(b*x**(1/3)+a*x)**(3/2),x)
```

output
```
Integral(1/(x**2*(a*x + b*x**(1/3))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 \left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*x^(1/3))^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 \left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*x + b*x^(1/3))^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^2\,{\left(a\,x+b\,x^{1/3}\right)}^{3/2}} \,d x$$

input
```
int(1/(x^2*(a*x + b*x^(1/3))^(3/2)),x)
```

output
```
int(1/(x^2*(a*x + b*x^(1/3))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^{\frac{7}{3}}\sqrt{x^{\frac{1}{3}} b+a x}\, b+\sqrt{x^{\frac{1}{3}} b+a x}\, a\,x^{3}}d x$$

input
```
int(1/x^2/(b*x^(1/3)+a*x)^(3/2),x)
```

output
```
int(1/(x**(1/3)*sqrt(x**(1/3)*b + a*x)*b*x**2 + sqrt(x**(1/3)*b + a*x)*a*x
**3),x)
```

3.144 $\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal result . . . 1296
Mathematica [C] (verified) . . . 1297
Rubi [A] (warning: unable to verify) . . . 1297
Maple [A] (verified) . . . 1308
Fricas [F] . . . 1309
Sympy [F] . . . 1309
Maxima [F] . . . 1310
Giac [F] . . . 1310
Mupad [F(-1)] . . . 1310
Reduce [F] . . . 1311

Optimal result

Integrand size = 19, antiderivative size = 246

$$\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}-\frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}}+\frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2}-\frac{1989a^2\sqrt{b\sqrt[3]{x}+ax}}{385b^4x^{4/3}}+\frac{663a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^5x^{2/3}}+\frac{663a^{15/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{154b^{21/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
3/b/x^(7/3)/(b*x^(1/3)+a*x)^(1/2)-17/5*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(8/3)+2
21/55*a*(b*x^(1/3)+a*x)^(1/2)/b^3/x^2-1989/385*a^2*(b*x^(1/3)+a*x)^(1/2)/b
^4/x^(4/3)+663/77*a^3*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(2/3)+663/154*a^(15/4)*(
b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)
*x^(1/6)*InverseJacobiAM(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)),1/2*2^(1/2))/b^
(21/4)/(b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^3 \left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = -\frac{2\sqrt{1+\frac{ax^{2/3}}{b}}\ \text{Hypergeometric2F1}\left(-\frac{15}{4},\frac{3}{2},-\frac{11}{4},-\frac{ax^{2/3}}{b}\right)}{5bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}$$

input
```
Integrate[1/(x^3*(b*x^(1/3) + a*x)^(3/2)),x]
```

output
```
(-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-15/4, 3/2, -11/4, -((a*x^(2
/3))/b)])/(5*b*x^(7/3)*Sqrt[b*x^(1/3) + a*x])
```

Rubi [A] (warning: unable to verify)

Time = 0.84 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.22, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1924, 1929, 1931, 1931, 1931, 1931, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(ax+b\sqrt[3]{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1924}$$

$$3\int \frac{1}{x^{7/3}\left(\sqrt[3]{x}b+ax\right)^{3/2}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 1929}$$

$$3\left(\frac{17\int \frac{1}{x^{8/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{2b}+\frac{1}{bx^{7/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \text{ 1931}$$

$$3\left(\frac{17\left(-\frac{13a\int \frac{1}{x^2\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{2b}+\frac{1}{bx^{7/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{17\left(-\frac{13a\left(-\frac{9a\int \frac{1}{x^{4/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^2}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{2b}+\frac{1}{bx^{7/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{17\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\int\frac{1}{x^{2/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^2}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{2b}+\frac{1}{bx^{7/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\downarrow$ 1931

$$3\left(\frac{17\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{a\int\frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{3b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^{2}}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{2b}+\frac{1}{bx^{7/3}\sqrt{ax+}}\right.$$

↓ 1917

$$3\left(\frac{17\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^{2}}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{2b}+\right.$$

↓ 266

$$3\left(\frac{17\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^{2}}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{2b}+\frac{\cdots}{b\cdots}\right.$$

↓ 761

$$3\frac{\left(17\frac{-\left(13a\frac{-\left(9a\frac{-\left(5a\left(-\frac{a^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^{2}}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{\cdots}}}{7bx^{4/3}}\cdots}{11b}\cdots}{15b}\cdots}{2b}\cdots$$

input
```
Int[1/(x^3*(b*x^(1/3) + a*x)^(3/2)),x]
```

output
```
3*(1/(b*x^(7/3)*Sqrt[b*x^(1/3) + a*x]) + (17*((-2*Sqrt[b*x^(1/3) + a*x])/(
15*b*x^(8/3)) - (13*a*((-2*Sqrt[b*x^(1/3) + a*x])/(11*b*x^2) - (9*a*((-2*S
qrt[b*x^(1/3) + a*x])/(7*b*x^(4/3)) - (5*a*((-2*Sqrt[b*x^(1/3) + a*x])/(3*
b*x^(2/3)) - (a^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b + a*x^(2/3)]*x^(1
/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTan
[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[b*x^(1/3) + a*x]*Sqrt[b
 + a*x^(4/3)])))/(7*b)))/(11*b)))/(15*b)))/(2*b))
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 4.04 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{3x^{\frac{1}{3}}a^4}{b^5\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}-\frac{2\sqrt{b\,x^{\frac{1}{3}}+ax}}{5b^2x^{\frac{8}{3}}}+\frac{56a\sqrt{b\,x^{\frac{1}{3}}+ax}}{55b^3x^2}-\frac{834a^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{385b^4x^{\frac{4}{3}}}+\frac{432a^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{77b^5x^{\frac{2}{3}}}+\frac{663a^3\sqrt{-ab}}{}$
default	$\frac{3315x^{\frac{14}{3}}\sqrt{-ab}\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}a-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^3-884x}{}$

input
```
int(1/x^3/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3*x^(1/3)*a^4/b^5/((x^(2/3)+b/a)*x^(1/3)*a)^(1/2)-2/5*(b*x^(1/3)+a*x)^(1/2
)/b^2/x^(8/3)+56/55*a*(b*x^(1/3)+a*x)^(1/2)/b^3/x^2-834/385*a^2*(b*x^(1/3)
+a*x)^(1/2)/b^4/x^(4/3)+432/77*a^3*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(2/3)+663/1
54*a^3/b^5*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*
(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2
)*a)^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(
-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Fricas [F]

$$\int \frac{1}{x^3 \left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^
(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^8 + 2*a^3*b^3*x^6 + b^6*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^3 \left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^3\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**3/(b*x**(1/3)+a*x)**(3/2),x)
```

output
```
Integral(1/(x**3*(a*x + b*x**(1/3))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*x^(1/3))^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*x + b*x^(1/3))^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^3\,\left(a\,x+b\,x^{1/3}\right)^{3/2}}\,dx$$

input
```
int(1/(x^3*(a*x + b*x^(1/3))^(3/2)),x)
```

output
```
int(1/(x^3*(a*x + b*x^(1/3))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 \left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^{\frac{10}{3}}\sqrt{x^{\frac{1}{3}}b+ax}\,b+\sqrt{x^{\frac{1}{3}}b+ax}\,a\,x^4}dx$$

input
```
int(1/x^3/(b*x^(1/3)+a*x)^(3/2),x)
```

output
```
int(1/(x**(1/3)*sqrt(x**(1/3)*b + a*x)*b*x**3 + sqrt(x**(1/3)*b + a*x)*a*x
**4),x)
```

3.145 $\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal result . . . 1312
Mathematica [C] (verified) . . . 1313
Rubi [A] (warning: unable to verify) . . . 1313
Maple [A] (verified) . . . 1335
Fricas [F] . . . 1335
Sympy [F] . . . 1336
Maxima [F] . . . 1336
Giac [F] . . . 1336
Mupad [F(-1)] . . . 1337
Reduce [F] . . . 1337

Optimal result

Integrand size = 19, antiderivative size = 471

$$\begin{aligned}\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx &= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}-\frac{4807a^{11/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221b^7\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}\\
&-\frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}}+\frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3}-\frac{6555a^2\sqrt{b\sqrt[3]{x}+ax}}{1547b^4x^{7/3}}\\
&+\frac{24035a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^5x^{5/3}}-\frac{4807a^4\sqrt{b\sqrt[3]{x}+ax}}{663b^6x}+\frac{4807a^5\sqrt{b\sqrt[3]{x}+ax}}{221b^7\sqrt[3]{x}}\\
&+\frac{4807a^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221b^{27/4}\sqrt{b\sqrt[3]{x}+ax}}\\
&-\frac{4807a^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{442b^{27/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

output
```
3/b/x^(10/3)/(b*x^(1/3)+a*x)^(1/2)-4807/221*a^(11/2)*(b+a*x^(2/3))*x^(1/3)
/b^7/(b^(1/2)+a^(1/2)*x^(1/3))/(b*x^(1/3)+a*x)^(1/2)-23/7*(b*x^(1/3)+a*x)^
(1/2)/b^2/x^(11/3)+437/119*a*(b*x^(1/3)+a*x)^(1/2)/b^3/x^3-6555/1547*a^2*(
b*x^(1/3)+a*x)^(1/2)/b^4/x^(7/3)+24035/4641*a^3*(b*x^(1/3)+a*x)^(1/2)/b^5/
x^(5/3)-4807/663*a^4*(b*x^(1/3)+a*x)^(1/2)/b^6/x+4807/221*a^5*(b*x^(1/3)+a
*x)^(1/2)/b^7/x^(1/3)+4807/221*a^(21/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^
(2/3))/(b^(1/2)+a^(1/2)*x^(1/3))^2)^(1/2)*x^(1/6)*EllipticE(sin(2*arctan(a
^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))/b^(27/4)/(b*x^(1/3)+a*x)^(1/2)-4807/
442*a^(21/4)*(b^(1/2)+a^(1/2)*x^(1/3))*((b+a*x^(2/3))/(b^(1/2)+a^(1/2)*x^(
1/3))^2)^(1/2)*x^(1/6)*InverseJacobiAM(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)),1
/2*2^(1/2))/b^(27/4)/(b*x^(1/3)+a*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^4 \left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = -\frac{2\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(-\frac{21}{4},\frac{3}{2},-\frac{17}{4},-\frac{ax^{2/3}}{b}\right)}{7bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}$$

input
```
Integrate[1/(x^4*(b*x^(1/3) + a*x)^(3/2)),x]
```

output
```
(-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-21/4, 3/2, -17/4, -((a*x^(2
/3))/b)])/(7*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])
```

Rubi [A] (warning: unable to verify)

Time = 1.28 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.10, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {1924, 1929, 1931, 1931, 1931, 1931, 1931, 1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(ax+b\sqrt[3]{x}\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1924}$$

$$3\int \frac{1}{x^{10/3}\left(\sqrt[3]{x}b+ax\right)^{3/2}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 1929}$$

$$3\left(\frac{23\int \frac{1}{x^{11/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{2b}+\frac{1}{bx^{10/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \text{ 1931}$$

$$3\left(\frac{23\left(-\frac{19a\int \frac{1}{x^3\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{21b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{21bx^{11/3}}\right)}{2b}+\frac{1}{bx^{10/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \text{ 1931}$$

$$3\left(\frac{23\left(-\frac{19a\left(-\frac{15a\int \frac{1}{x^{7/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{17b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{17bx^3}\right)}{21b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{21bx^{11/3}}\right)}{2b}+\frac{1}{bx^{10/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \text{ 1931}$$

$$3\left(\frac{23\left(-\frac{19a\left(-\frac{15a\left(-\frac{11a\int\frac{1}{x^{5/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)}{17b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{17bx^3}\right)}{21b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{21bx^{11/3}}\right)}{2b}+\frac{1}{bx^{10/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\downarrow 1931$$

$$3\left(\frac{23\left(-\frac{19a\left(-\frac{15a\left(-\frac{11a\left(-\frac{7a\int\frac{1}{x\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)}{17b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{17bx^{3}}\right)}{21b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{21bx^{11/3}}\right)}{2b}+\frac{1}{bx^{10/3}\sqrt{a}}\right.$$

↓ 1931

$$\Bigg(23 \Bigg(-\frac{19a \left(-\frac{15a \left(-\frac{11a \left(-\frac{7a \left(-\frac{3a \int \frac{1}{\sqrt[3]{x}\sqrt{\sqrt[3]{x}b+ax}}\, d\sqrt[3]{x}}{5b} - \frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx} \right)}{9b} - \frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}} \right)}{13b} - \frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}} \right)}{17b} - \frac{2\sqrt{ax+b\sqrt[3]{x}}}{17bx^3} \right)}{21b} - \frac{2\sqrt{ax\cdots}}{21bx\cdots}$$

↓ 1931

$$\left(\left(\left(19a\left(-\frac{15a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{a\int\frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}\,d\sqrt[3]{x}}{b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)}{17b}-\frac{2\sqrt{ax+b\sqrt[3]{}}}{17bx^{3}}\right.\right.\right.$$

$\downarrow$ 1938

$$\left(\left(19a\left(-\frac{15a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)}{17b}\right.\right.\right.$$

↓ 266

$$\left(23\left(-\frac{19a\left(-\frac{15a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)}{17b}}{21b}\right.\right.$$

↓ 834

$$\Bigg(\Bigg(\Bigg(19a\left(-\frac{15a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{b}\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}}{13b}}{17b}$$

↓ 27

$$\left(\left(\left(19a\left(-\frac{15a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{\cdots}}{\cdots}\right)}{13b}\cdots\right)}{17b}\cdots\right.\right.\right.\right.$$

↓ 761

$$2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x\cdots}{\sqrt{ax^{4/3}\cdots}}}{\sqrt{\cdots}}\cdots\right.$$

$$3a \quad \frac{\cdots}{b\sqrt{ax+b\sqrt[3]{x}}}$$

$$7a - \frac{\cdots}{5b}$$

$$11a - \frac{\cdots}{9b}$$

$$15a - \frac{\cdots}{13b}$$

↓ 1510

$$2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}} - \frac{\sqrt[4]{b}\left(\sqrt{a}x^{2}\right.}{}\right.$$

$$3a \quad \frac{}{b\sqrt{ax+b\sqrt[3]{x}}}$$

$$7a \quad - \quad \frac{}{5b}$$

$$11a \quad - \quad \frac{}{9b}$$

$$15a \quad - \quad \frac{}{}$$

input
```
Int[1/(x^4*(b*x^(1/3) + a*x)^(3/2)),x]
```

output
```
3*(1/(b*x^(10/3)*Sqrt[b*x^(1/3) + a*x]) + (23*((-2*Sqrt[b*x^(1/3) + a*x])/
(21*b*x^(11/3)) - (19*a*((-2*Sqrt[b*x^(1/3) + a*x])/(17*b*x^3) - (15*a*((-
2*Sqrt[b*x^(1/3) + a*x])/(13*b*x^(7/3)) - (11*a*((-2*Sqrt[b*x^(1/3) + a*x]
)/(9*b*x^(5/3)) - (7*a*((-2*Sqrt[b*x^(1/3) + a*x])/(5*b*x) - (3*a*((-2*Sqr
t[b*x^(1/3) + a*x])/(b*x^(1/3)) + (2*a*Sqrt[b + a*x^(2/3)]*x^(1/6)*(-((-((
x^(1/6)*Sqrt[b + a*x^(4/3)])/(Sqrt[b] + Sqrt[a]*x^(2/3))) + (b^(1/4)*(Sqrt
[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]
*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b + a*
x^(4/3)]))/Sqrt[a]) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(
4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/
b^(1/4)], 1/2])/(2*a^(3/4)*Sqrt[b + a*x^(4/3)])))/(b*Sqrt[b*x^(1/3) + a*x]
)))/(5*b)))/(9*b)))/(13*b)))/(17*b)))/(21*b)))/(2*b))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 6.17 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.71

method	result
derivativedivides	$-\frac{2\sqrt{b\,x^{\frac{1}{3}}+ax}}{7b^{2}x^{\frac{11}{3}}}+\frac{80a\sqrt{b\,x^{\frac{1}{3}}+ax}}{119b^{3}x^{3}}-\frac{1914a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1547b^{4}x^{\frac{7}{3}}}+\frac{10112a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{4641b^{5}x^{\frac{5}{3}}}-\frac{2818a^{4}\sqrt{b\,x^{\frac{1}{3}}+ax}}{663b^{6}x}+\frac{4144}{221b^{7}\sqrt{}}$
default	$-201894a^{5}b\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}a-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,x^{\frac{20}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,\mathrm{EllipticE}\left(\sqrt{\frac{x^{\frac{1}{3}}a+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+10094$

input
```
int(1/x^4/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(11/3)+80/119*a*(b*x^(1/3)+a*x)^(1/2)/b^3
/x^3-1914/1547*a^2*(b*x^(1/3)+a*x)^(1/2)/b^4/x^(7/3)+10112/4641*a^3*(b*x^(
1/3)+a*x)^(1/2)/b^5/x^(5/3)-2818/663*a^4*(b*x^(1/3)+a*x)^(1/2)/b^6/x+4144/
221*(b+a*x^(2/3))*a^5/b^7/(x^(1/3)*(b+a*x^(2/3)))^(1/2)+3*x^(2/3)*a^6/b^7/
((x^(2/3)+b/a)*x^(1/3)*a)^(1/2)-4807/442*a^5/b^7*(-a*b)^(1/2)*((x^(1/3)+1/
a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a
*b)^(1/2))^(1/2)*(-x^(1/3)/(-a*b)^(1/2)*a)^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2
/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2
),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-
a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^
(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^9 + 2*a^3*b^3*x^7 + b^6*x^5), x)
```

Sympy [F]

$$\int \frac{1}{x^4 \left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^4 \left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**4/(b*x**(1/3)+a*x)**(3/2),x)
```

output
```
Integral(1/(x**4*(a*x + b*x**(1/3))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 \left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*x^(1/3))^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 \left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*x + b*x^(1/3))^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^4\,{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}\,dx$$

input
```
int(1/(x^4*(a*x + b*x^(1/3))^(3/2)),x)
```

output
```
int(1/(x^4*(a*x + b*x^(1/3))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 \left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^{\frac{13}{3}}\sqrt{x^{\frac{1}{3}}b+ax}\,b+\sqrt{x^{\frac{1}{3}}b+ax}\,a\,x^5}dx$$

input
```
int(1/x^4/(b*x^(1/3)+a*x)^(3/2),x)
```

output
```
int(1/(x**(1/3)*sqrt(x**(1/3)*b + a*x)*b*x**4 + sqrt(x**(1/3)*b + a*x)*a*x
**5),x)
```

3.146 $\int x^3\sqrt{bx^{2/3}+ax}\,dx$

Optimal result 1338
Mathematica [A] (verified) 1339
Rubi [A] (verified) 1339
Maple [A] (verified) 1357
Fricas [B] (verification not implemented) 1357
Sympy [F] 1358
Maxima [F] 1359
Giac [A] (verification not implemented) 1359
Mupad [F(-1)] 1360
Reduce [B] (verification not implemented) 1360

Optimal result

Integrand size = 19, antiderivative size = 371

$$\begin{aligned}\int x^3\sqrt{bx^{2/3}+ax}\,dx = &-\frac{524288b^9\left(bx^{2/3}+ax\right)^{3/2}}{4345965a^{10}} \\ &+\frac{8388608b^{12}\left(bx^{2/3}+ax\right)^{3/2}}{152108775a^{13}x}-\frac{4194304b^{11}\left(bx^{2/3}+ax\right)^{3/2}}{50702925a^{12}x^{2/3}} \\ &+\frac{1048576b^{10}\left(bx^{2/3}+ax\right)^{3/2}}{10140585a^{11}\sqrt[3]{x}}+\frac{65536b^8\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{482885a^9} \\ &-\frac{360448b^7x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{2414425a^8}+\frac{90112b^6x\left(bx^{2/3}+ax\right)^{3/2}}{557175a^7} \\ &-\frac{45056b^5x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{260015a^6}+\frac{2816b^4x^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{15295a^5} \\ &-\frac{1408b^3x^2\left(bx^{2/3}+ax\right)^{3/2}}{7245a^4}+\frac{352b^2x^{7/3}\left(bx^{2/3}+ax\right)^{3/2}}{1725a^3} \\ &-\frac{16bx^{8/3}\left(bx^{2/3}+ax\right)^{3/2}}{75a^2}+\frac{2x^3\left(bx^{2/3}+ax\right)^{3/2}}{9a}\end{aligned}$$

output
```
-524288/4345965*b^9*(b*x^(2/3)+a*x)^(3/2)/a^10+8388608/152108775*b^12*(b*x
^(2/3)+a*x)^(3/2)/a^13/x-4194304/50702925*b^11*(b*x^(2/3)+a*x)^(3/2)/a^12/
x^(2/3)+1048576/10140585*b^10*(b*x^(2/3)+a*x)^(3/2)/a^11/x^(1/3)+65536/482
885*b^8*x^(1/3)*(b*x^(2/3)+a*x)^(3/2)/a^9-360448/2414425*b^7*x^(2/3)*(b*x^
(2/3)+a*x)^(3/2)/a^8+90112/557175*b^6*x*(b*x^(2/3)+a*x)^(3/2)/a^7-45056/26
0015*b^5*x^(4/3)*(b*x^(2/3)+a*x)^(3/2)/a^6+2816/15295*b^4*x^(5/3)*(b*x^(2/
3)+a*x)^(3/2)/a^5-1408/7245*b^3*x^2*(b*x^(2/3)+a*x)^(3/2)/a^4+352/1725*b^2
*x^(7/3)*(b*x^(2/3)+a*x)^(3/2)/a^3-16/75*b*x^(8/3)*(b*x^(2/3)+a*x)^(3/2)/a
^2+2/9*x^3*(b*x^(2/3)+a*x)^(3/2)/a
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.50

$$\int x^3\sqrt{bx^{2/3}+ax}\,dx=\frac{2\sqrt{bx^{2/3}+ax}\left(4194304b^{13}-2097152ab^{12}\sqrt[3]{x}+1572864a^2b^{11}x^{2/3}-1310720a^3b^{10}x\right.}{}$$

input
```
Integrate[x^3*Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(2/3) + a*x]*(4194304*b^13 - 2097152*a*b^12*x^(1/3) + 1572864*
a^2*b^11*x^(2/3) - 1310720*a^3*b^10*x + 1146880*a^4*b^9*x^(4/3) - 1032192*
a^5*b^8*x^(5/3) + 946176*a^6*b^7*x^2 - 878592*a^7*b^6*x^(7/3) + 823680*a^8
*b^5*x^(8/3) - 777920*a^9*b^4*x^3 + 739024*a^10*b^3*x^(10/3) - 705432*a^11
*b^2*x^(11/3) + 676039*a^12*b*x^4 + 16900975*a^13*x^(13/3)))/(152108775*a^
13*x^(1/3))
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.684$, Rules used = {1922, 1922, 1922, 1922, 1922, 1922, 1922, 1922, 1922, 1908, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sqrt{ax+bx^{2/3}}\,dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a}-\frac{8b\int x^{8/3}\sqrt{x^{2/3}b+ax}dx}{9a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a}-\frac{8b\left(\frac{6x^{8/3}\left(ax+bx^{2/3}\right)^{3/2}}{25a}-\frac{22b\int x^{7/3}\sqrt{x^{2/3}b+ax}dx}{25a}\right)}{9a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a}-\frac{8b\left(\frac{6x^{8/3}\left(ax+bx^{2/3}\right)^{3/2}}{25a}-\frac{22b\left(\frac{6x^{7/3}\left(ax+bx^{2/3}\right)^{3/2}}{23a}-\frac{20b\int x^2\sqrt{x^{2/3}b+ax}dx}{23a}\right)}{25a}\right)}{9a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a}-\frac{8b\left(\frac{6x^{8/3}\left(ax+bx^{2/3}\right)^{3/2}}{25a}-\frac{22b\left(\frac{6x^{7/3}\left(ax+bx^{2/3}\right)^{3/2}}{23a}-\frac{20b\left(\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a}-\frac{6b\int x^{5/3}\sqrt{x^{2/3}b+ax}dx}{7a}\right)}{23a}\right)}{25a}\right)}{9a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a}-\frac{8b\left(\frac{6x^{8/3}\left(ax+bx^{2/3}\right)^{3/2}}{25a}-\frac{22b\left(\frac{6x^{7/3}\left(ax+bx^{2/3}\right)^{3/2}}{23a}-\frac{20b\left(\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a}-\frac{6b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{3/2}}{19a}-\frac{16b\int x^{4/3}\sqrt{x^{2/3}b+ax}dx}{19a}\right)}{7a}\right)}{23a}\right)}{25a}\right)}{9a}$$

$\downarrow$ 1922

$$\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a} - \frac{8b\left(\frac{6x^{8/3}\left(ax+bx^{2/3}\right)^{3/2}}{25a} - \frac{22b\left(\frac{6x^{7/3}\left(ax+bx^{2/3}\right)^{3/2}}{23a} - \frac{20b\left(\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a} - \frac{6b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{3/2}}{19a} - \frac{16b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{17a} - \frac{14b\int x \cdots}{\cdots}\right)}{19a}\right)}{7a}\right)}{23a}\right)}{25a}\right)}{9a}$$

↓ 1922

$$\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a} -$$

$$8b\left(\frac{6x^{8/3}\left(ax+bx^{2/3}\right)^{3/2}}{25a} - \frac{22b\left(\frac{6x^{7/3}\left(ax+bx^{2/3}\right)^{3/2}}{23a} - \frac{20b\left(\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a} - \frac{6b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{3/2}}{19a} - \frac{16b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{17a} - \frac{14b\left(2 \cdots\right.}{\cdots}\right.}{\cdots}\right.}{7a}\right.}{23a}\right.}{25a}\right.$$

↓ 1922

$$\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a} -$$

$$14b\left(\frac{2}{}\right.$$

$$16b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{17a} - \right.$$

$$6b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{3/2}}{19a} - \right.$$

$$20b\left(\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a} - \right.$$

$$22b\left(\frac{6x^{7/3}\left(ax+bx^{2/3}\right)^{3/2}}{23a} - \frac{}{23a}\right.$$

↓ 1922

$$\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a} -$$

$$14b \Big(2$$

$$16b \left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{17a} - \right.$$

$$6b \left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{3/2}}{19a} - \right.$$

$$20b \left(\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a} - \right.$$

↓ 1908

$$\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a} -$$

$$14b \; \Bigg(\; 2$$

$$16b \left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{17a} - \right.$$

↓ 1922

$$\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a} -$$

$$14b$$

$$16b \left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{17a} - \right.$$

↓ 1922

$$\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a} -$$

$14b$

↓ 1920

$$\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a}-$$

$$\left(\left(\left(\left(\left(\left(14b\,\frac{2}{-}\right.\right.\right.\right.\right.\right.$$

input
```
Int[x^3*Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*x^3*(b*x^(2/3) + a*x)^(3/2))/(9*a) - (8*b*((6*x^(8/3)*(b*x^(2/3) + a*x)
^(3/2))/(25*a) - (22*b*((6*x^(7/3)*(b*x^(2/3) + a*x)^(3/2))/(23*a) - (20*b
*((2*x^2*(b*x^(2/3) + a*x)^(3/2))/(7*a) - (6*b*((6*x^(5/3)*(b*x^(2/3) + a*
x)^(3/2))/(19*a) - (16*b*((6*x^(4/3)*(b*x^(2/3) + a*x)^(3/2))/(17*a) - (14
*b*((2*x*(b*x^(2/3) + a*x)^(3/2))/(5*a) - (4*b*((6*x^(2/3)*(b*x^(2/3) + a*
x)^(3/2))/(13*a) - (10*b*((6*x^(1/3)*(b*x^(2/3) + a*x)^(3/2))/(11*a) - (8*
b*((2*(b*x^(2/3) + a*x)^(3/2))/(3*a) - (2*b*((6*(b*x^(2/3) + a*x)^(3/2))/(
7*a*x^(1/3)) - (4*b*((-4*b*(b*x^(2/3) + a*x)^(3/2))/(5*a^2*x) + (6*(b*x^(2
/3) + a*x)^(3/2))/(5*a*x^(2/3))))/(7*a)))/(3*a)))/(11*a)))/(13*a)))/(5*a))
)/(17*a)))/(19*a)))/(7*a)))/(23*a)))/(25*a)))/(9*a)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.42

method	result
derivativedivides	$\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(x^{\frac{1}{3}}a+b\right)\left(16900975a^{12}x^{4}-16224936a^{11}b\,x^{\frac{11}{3}}+15519504a^{10}b^{2}x^{\frac{10}{3}}-14780480a^{9}b^{3}x^{3}+14002560a^{8}b^{4}x^{\frac{8}{3}}\right.}{}$
default	$-\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(x^{\frac{1}{3}}a+b\right)\left(16224936a^{11}b\,x^{\frac{11}{3}}-15519504a^{10}b^{2}x^{\frac{10}{3}}-14002560a^{8}b^{4}x^{\frac{8}{3}}+13178880a^{7}b^{5}x^{\frac{7}{3}}+11354112a\right.}{}$

input
```
int(x^3*(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/152108775*(b*x^(2/3)+a*x)^(1/2)*(x^(1/3)*a+b)*(16900975*a^12*x^4-1622493
6*a^11*b*x^(11/3)+15519504*a^10*b^2*x^(10/3)-14780480*a^9*b^3*x^3+14002560
*a^8*b^4*x^(8/3)-13178880*a^7*b^5*x^(7/3)+12300288*a^6*b^6*x^2-11354112*a^
5*b^7*x^(5/3)+10321920*a^4*b^8*x^(4/3)-9175040*a^3*b^9*x+7864320*a^2*b^10*
x^(2/3)-6291456*a*b^11*x^(1/3)+4194304*b^12)/x^(1/3)/a^13
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1293 vs. 2(277) = 554.

Time = 128.20 (sec) , antiderivative size = 1293, normalized size of antiderivative = 3.49

$$\int x^3\sqrt{bx^{2/3}+ax}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-1/304217550*((211106232532992*b^19 + 43980465111040*b^18 + 206158430208*(
64*a^3 - 3)*b^16 - 4123168604160*b^17 - 1073741824*(11264*a^3 - 53)*b^15 -
 393725113600*a^15 - 402653184*(5504*a^3 + 1)*b^14 + 12582912*(3194880*a^6
 - 114688*a^3 - 3)*b^13 + 469762048*(18816*a^6 + 103*a^3)*b^12 - 50331648*
(48816*a^6 + 23*a^3)*b^11 - 786432*(45731840*a^9 - 495872*a^6 - 15*a^3)*b^
10 - 7340032*(1349120*a^9 + 3439*a^6)*b^9 + 250478592*(5600*a^9 + 3*a^6)*b
^8 + 12288*(2616979456*a^12 - 21542400*a^9 - 693*a^6)*b^7 + 212992*(437436
16*a^12 + 89111*a^9)*b^6 - 638976*(1652476*a^12 + 935*a^9)*b^5 + 42432*(72
17086464*a^15 + 4969216*a^12 + 165*a^9)*b^4 + 7524608*(20570112*a^15 - 210
1*a^12)*b^3 + 2821728*(7815168*a^15 + 181*a^12)*b^2 + 2028117*(2072576*a^1
5 - 3*a^12)*b)*x - 4*(16900975*(16777216*a^13*b^6 + 6291456*a^13*b^5 + 196
608*a^13*b^4 - 262144*a^16 - 114688*a^13*b^3 - 2304*a^13*b^2 + 864*a^13*b
- 27*a^13)*x^5 + 739024*(16777216*a^10*b^9 + 6291456*a^10*b^8 + 196608*a^1
0*b^7 - 114688*a^10*b^6 - 2304*a^10*b^5 + 864*a^10*b^4 - (262144*a^13 + 27
*a^10)*b^3)*x^4 - 878592*(16777216*a^7*b^12 + 6291456*a^7*b^11 + 196608*a^
7*b^10 - 114688*a^7*b^9 - 2304*a^7*b^8 + 864*a^7*b^7 - (262144*a^10 + 27*a
^7)*b^6)*x^3 + 1146880*(16777216*a^4*b^15 + 6291456*a^4*b^14 + 196608*a^4*
b^13 - 114688*a^4*b^12 - 2304*a^4*b^11 + 864*a^4*b^10 - (262144*a^7 + 27*a
^4)*b^9)*x^2 - 2097152*(16777216*a*b^18 + 6291456*a*b^17 + 196608*a*b^16 -
 114688*a*b^15 - 2304*a*b^14 + 864*a*b^13 - (262144*a^4 + 27*a)*b^12)*x...
```

Sympy [F]

$$\int x^3 \sqrt{bx^{2/3} + ax}\, dx = \int x^3 \sqrt{ax + bx^{\frac{2}{3}}}\, dx$$

input
```
integrate(x**3*(b*x**(2/3)+a*x)**(1/2),x)
```

output
```
Integral(x**3*sqrt(a*x + b*x**(2/3)), x)
```

Maxima [F]

$$\int x^3\sqrt{bx^{2/3}+ax}\,dx=\int\sqrt{ax+bx^{\frac{2}{3}}}x^3\,dx$$

input
```
integrate(x^3*(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(2/3))*x^3, x)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.07

$$\int x^3\sqrt{bx^{2/3}+ax}\,dx=\text{Too large to display}$$

input
```
integrate(x^3*(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
-8388608/152108775*b^(27/2)/a^13 + 2/152108775*(27*(676039*(a*x^(1/3) + b)
^(25/2) - 8817900*(a*x^(1/3) + b)^(23/2)*b + 53117350*(a*x^(1/3) + b)^(21/
2)*b^2 - 195695500*(a*x^(1/3) + b)^(19/2)*b^3 + 492116625*(a*x^(1/3) + b)^
(17/2)*b^4 - 892371480*(a*x^(1/3) + b)^(15/2)*b^5 + 1201269300*(a*x^(1/3)
+ b)^(13/2)*b^6 - 1216870200*(a*x^(1/3) + b)^(11/2)*b^7 + 929553625*(a*x^(
1/3) + b)^(9/2)*b^8 - 531173500*(a*x^(1/3) + b)^(7/2)*b^9 + 223092870*(a*x
^(1/3) + b)^(5/2)*b^10 - 67603900*(a*x^(1/3) + b)^(3/2)*b^11 + 16900975*sq
rt(a*x^(1/3) + b)*b^12)*b/a^12 + 13*(1300075*(a*x^(1/3) + b)^(27/2) - 1825
3053*(a*x^(1/3) + b)^(25/2)*b + 119041650*(a*x^(1/3) + b)^(23/2)*b^2 - 478
056150*(a*x^(1/3) + b)^(21/2)*b^3 + 1320944625*(a*x^(1/3) + b)^(19/2)*b^4
- 2657429775*(a*x^(1/3) + b)^(17/2)*b^5 + 4015671660*(a*x^(1/3) + b)^(15/2
)*b^6 - 4633467300*(a*x^(1/3) + b)^(13/2)*b^7 + 4106936925*(a*x^(1/3) + b)
^(11/2)*b^8 - 2788660875*(a*x^(1/3) + b)^(9/2)*b^9 + 1434168450*(a*x^(1/3)
 + b)^(7/2)*b^10 - 547591590*(a*x^(1/3) + b)^(5/2)*b^11 + 152108775*(a*x^(
1/3) + b)^(3/2)*b^12 - 35102025*sqrt(a*x^(1/3) + b)*b^13)/a^12)/a
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{bx^{2/3}+ax}\,dx=\int x^3\,\sqrt{a\,x+b\,x^{2/3}}\,dx$$

input
```
int(x^3*(a*x + b*x^(2/3))^(1/2),x)
```

output
```
int(x^3*(a*x + b*x^(2/3))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.41

$$\int x^3\sqrt{bx^{2/3}+ax}\,dx=\frac{2\sqrt{x^{\frac{1}{3}}a+b}\left(-705432x^{\frac{11}{3}}a^{11}b^2+823680x^{\frac{8}{3}}a^8b^5-1032192x^{\frac{5}{3}}a^5b^8+1572864x^{\frac{2}{3}}a^2b\right.}{}$$

input
```
int(x^3*(b*x^(2/3)+a*x)^(1/2),x)
```

output
```
(2*sqrt(x**(1/3)*a + b)*( - 705432*x**(2/3)*a**11*b**2*x**3 + 823680*x**(2
/3)*a**8*b**5*x**2 - 1032192*x**(2/3)*a**5*b**8*x + 1572864*x**(2/3)*a**2*
b**11 + 16900975*x**(1/3)*a**13*x**4 + 739024*x**(1/3)*a**10*b**3*x**3 - 8
78592*x**(1/3)*a**7*b**6*x**2 + 1146880*x**(1/3)*a**4*b**9*x - 2097152*x**
(1/3)*a*b**12 + 676039*a**12*b*x**4 - 777920*a**9*b**4*x**3 + 946176*a**6*
b**7*x**2 - 1310720*a**3*b**10*x + 4194304*b**13))/(152108775*a**13)
```

3.147 $\int x^2\sqrt{bx^{2/3}+ax}\,dx$

Optimal result 1361
Mathematica [A] (verified) 1362
Rubi [A] (verified) 1362
Maple [A] (verified) 1373
Fricas [B] (verification not implemented) 1373
Sympy [F] 1374
Maxima [F] 1375
Giac [A] (verification not implemented) 1375
Mupad [F(-1)] 1376
Reduce [B] (verification not implemented) 1376

Optimal result

Integrand size = 19, antiderivative size = 283

$$\int x^2\sqrt{bx^{2/3}+ax}\,dx=\frac{8192b^6\left(bx^{2/3}+ax\right)^{3/2}}{46189a^7}-\frac{131072b^9\left(bx^{2/3}+ax\right)^{3/2}}{1616615a^{10}x}+\frac{196608b^8\left(bx^{2/3}+ax\right)^{3/2}}{1616615a^9x^{2/3}}-\frac{49152b^7\left(bx^{2/3}+ax\right)^{3/2}}{323323a^8\sqrt[3]{x}}-\frac{9216b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{46189a^6}+\frac{4608b^4x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{20995a^5}-\frac{384b^3x\left(bx^{2/3}+ax\right)^{3/2}}{1615a^4}+\frac{576b^2x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{2261a^3}-\frac{36bx^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{133a^2}+\frac{2x^2\left(bx^{2/3}+ax\right)^{3/2}}{7a}$$

output
```
8192/46189*b^6*(b*x^(2/3)+a*x)^(3/2)/a^7-131072/1616615*b^9*(b*x^(2/3)+a*x
)^(3/2)/a^10/x+196608/1616615*b^8*(b*x^(2/3)+a*x)^(3/2)/a^9/x^(2/3)-49152/
323323*b^7*(b*x^(2/3)+a*x)^(3/2)/a^8/x^(1/3)-9216/46189*b^5*x^(1/3)*(b*x^(
2/3)+a*x)^(3/2)/a^6+4608/20995*b^4*x^(2/3)*(b*x^(2/3)+a*x)^(3/2)/a^5-384/1
615*b^3*x*(b*x^(2/3)+a*x)^(3/2)/a^4+576/2261*b^2*x^(4/3)*(b*x^(2/3)+a*x)^(
3/2)/a^3-36/133*b*x^(5/3)*(b*x^(2/3)+a*x)^(3/2)/a^2+2/7*x^2*(b*x^(2/3)+a*x
)^(3/2)/a
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.47

$$\int x^2\sqrt{bx^{2/3}+ax}\,dx = \frac{2\left(bx^{2/3}+ax\right)^{3/2}\left(-65536b^9+98304ab^8\sqrt[3]{x}-122880a^2b^7x^{2/3}+143360a^3b^6x-16\right.}{}$$

input
```
Integrate[x^2*Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*(b*x^(2/3) + a*x)^(3/2)*(-65536*b^9 + 98304*a*b^8*x^(1/3) - 122880*a^2*
b^7*x^(2/3) + 143360*a^3*b^6*x - 161280*a^4*b^5*x^(4/3) + 177408*a^5*b^4*x
^(5/3) - 192192*a^6*b^3*x^2 + 205920*a^7*b^2*x^(7/3) - 218790*a^8*b*x^(8/3
) + 230945*a^9*x^3))/(1616615*a^10*x)
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1922, 1922, 1922, 1922, 1922, 1922, 1908, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{ax+bx^{2/3}}\,dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a}-\frac{6b\int x^{5/3}\sqrt{x^{2/3}b+ax}dx}{7a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a}-\frac{6b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{3/2}}{19a}-\frac{16b\int x^{4/3}\sqrt{x^{2/3}b+ax}dx}{19a}\right)}{7a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a}-\frac{6b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{3/2}}{19a}-\frac{16b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{17a}-\frac{14b\int x\sqrt{x^{2/3}b+ax}dx}{17a}\right)}{19a}\right)}{7a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a}-\frac{6b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{3/2}}{19a}-\frac{16b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{17a}-\frac{14b\left(\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a}-\frac{4b\int x^{2/3}\sqrt{x^{2/3}b+ax}dx}{5a}\right)}{17a}\right)}{19a}\right)}{7a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a}-\frac{6b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{3/2}}{19a}-\frac{16b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{17a}-\frac{14b\left(\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a}-\frac{4b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{13a}-\frac{10b\int \sqrt[3]{x}\sqrt{x^{2/3}b+ax}dx}{13a}\right)}{5a}\right)}{17a}\right)}{19a}\right)}{7a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a}-\frac{6b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{3/2}}{19a}-\frac{16b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{17a}-\frac{14b\left(\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a}-\frac{4b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{13a}-\frac{10b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{11a}-\frac{8b\int\sqrt{x}}{}\right.}{13a}\right.}{5a}\right.}{17a}\right.}{19a}\right.}{7a}$$

$\downarrow$ 1908

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a} -$$

$$8b\left(\frac{2\left(\cdots\right.}{\cdots}\right.$$

$$10b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{11a} - \cdots\right.$$

$$4b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{13a} - \cdots\right.$$

$$14b\left(\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a} - \frac{\cdots}{5a}\right.$$

$$16b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{17a} - \frac{\cdots}{17a}\right.$$

↓ 1922

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a} -$$

$$16b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{17a} - \frac{14b\left(\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a} - \frac{4b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{13a} - \frac{10b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{11a} - \frac{8b\left(\frac{2(a}{}\right.}{}\right.}{}\right.}{}\right.}{17a}$$

↓ 1922

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a} -$$

$$8b \quad \frac{2(a\ldots}{\ldots}$$

$$10b \quad \frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{11a} -$$

$$4b \quad \frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{13a} -$$

$$14b \quad \frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a} -$$

↓ 1920

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a}-$$

$$8b\ \Bigg(\frac{2\left(a\right.}{}$$

$$10b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{11a}-\right.$$

$$4b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{13a}-\right.$$

$$14b\left(\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a}-\right.$$

input
```
Int[x^2*Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*x^2*(b*x^(2/3) + a*x)^(3/2))/(7*a) - (6*b*((6*x^(5/3)*(b*x^(2/3) + a*x)
^(3/2))/(19*a) - (16*b*((6*x^(4/3)*(b*x^(2/3) + a*x)^(3/2))/(17*a) - (14*b
*((2*x*(b*x^(2/3) + a*x)^(3/2))/(5*a) - (4*b*((6*x^(2/3)*(b*x^(2/3) + a*x)
^(3/2))/(13*a) - (10*b*((6*x^(1/3)*(b*x^(2/3) + a*x)^(3/2))/(11*a) - (8*b*
((2*(b*x^(2/3) + a*x)^(3/2))/(3*a) - (2*b*((6*(b*x^(2/3) + a*x)^(3/2))/(7*
a*x^(1/3)) - (4*b*((-4*b*(b*x^(2/3) + a*x)^(3/2))/(5*a^2*x) + (6*(b*x^(2/3
) + a*x)^(3/2))/(5*a*x^(2/3))))/(7*a)))/(3*a)))/(11*a)))/(13*a)))/(5*a)))/
(17*a)))/(19*a)))/(7*a)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.43

method	result
derivativedivides	$\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(x^{\frac{1}{3}}a+b\right)(230945a^{9}x^{3}-218790a^{8}b\,x^{\frac{8}{3}}+205920a^{7}b^{2}x^{\frac{7}{3}}-192192a^{6}b^{3}x^{2}+177408a^{5}b^{4}x^{\frac{5}{3}}-161280a^{4}b^{5}x}{1616615x^{\frac{1}{3}}a^{10}}$
default	$-\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(x^{\frac{1}{3}}a+b\right)(218790a^{8}b\,x^{\frac{8}{3}}-205920a^{7}b^{2}x^{\frac{7}{3}}-177408a^{5}b^{4}x^{\frac{5}{3}}+161280a^{4}b^{5}x^{\frac{4}{3}}-230945a^{9}x^{3}+122880a^{2}b}{1616615x^{\frac{1}{3}}a^{10}}$

input

```
int(x^2*(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2/1616615*(b*x^(2/3)+a*x)^(1/2)*(x^(1/3)*a+b)*(230945*a^9*x^3-218790*a^8*b
*x^(8/3)+205920*a^7*b^2*x^(7/3)-192192*a^6*b^3*x^2+177408*a^5*b^4*x^(5/3)-
161280*a^4*b^5*x^(4/3)+143360*a^3*b^6*x-122880*a^2*b^7*x^(2/3)+98304*a*b^8
*x^(1/3)-65536*b^9)/x^(1/3)/a^10
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. 2(211) = 422.

Time = 123.05 (sec) , antiderivative size = 1031, normalized size of antiderivative = 3.64

$$\int x^2\sqrt{bx^{2/3}+ax}\,dx = \text{Too large to display}$$

input

```
integrate(x^2*(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
1/3233230*((3298534883328*b^16 + 687194767360*b^15 + 3221225472*(64*a^3 -
3)*b^13 - 64424509440*b^14 - 16777216*(11264*a^3 - 53)*b^12 + 5380094720*a
^12 - 6291456*(5504*a^3 + 1)*b^11 + 196608*(3194880*a^6 - 114688*a^3 - 3)*
b^10 + 7340032*(18816*a^6 + 103*a^3)*b^9 - 786432*(48816*a^6 + 23*a^3)*b^8
 - 12288*(45731840*a^9 - 495872*a^6 - 15*a^3)*b^7 - 114688*(1349120*a^9 +
3439*a^6)*b^6 + 3913728*(5600*a^9 + 3*a^6)*b^5 - 2112*(2027683840*a^12 + 1
958400*a^9 + 63*a^6)*b^4 - 36608*(59351040*a^12 - 8101*a^9)*b^3 - 549120*(
566272*a^12 + 17*a^9)*b^2 - 109395*(516096*a^12 - a^9)*b)*x + 4*(230945*(1
6777216*a^10*b^6 + 6291456*a^10*b^5 + 196608*a^10*b^4 - 262144*a^13 - 1146
88*a^10*b^3 - 2304*a^10*b^2 + 864*a^10*b - 27*a^10)*x^4 + 13728*(16777216*
a^7*b^9 + 6291456*a^7*b^8 + 196608*a^7*b^7 - 114688*a^7*b^6 - 2304*a^7*b^5
 + 864*a^7*b^4 - (262144*a^10 + 27*a^7)*b^3)*x^3 - 17920*(16777216*a^4*b^1
2 + 6291456*a^4*b^11 + 196608*a^4*b^10 - 114688*a^4*b^9 - 2304*a^4*b^8 + 8
64*a^4*b^7 - (262144*a^7 + 27*a^4)*b^6)*x^2 + 32768*(16777216*a*b^15 + 629
1456*a*b^14 + 196608*a*b^13 - 114688*a*b^12 - 2304*a*b^11 + 864*a*b^10 - (
262144*a^4 + 27*a)*b^9)*x - (1099511627776*b^16 + 412316860416*b^15 + 1288
4901888*b^14 - 7516192768*b^13 - 150994944*b^12 - 65536*(262144*a^3 + 27)*
b^10 + 56623104*b^11 - 12155*(16777216*a^9*b^7 + 6291456*a^9*b^6 + 196608*
a^9*b^5 - 114688*a^9*b^4 - 2304*a^9*b^3 + 864*a^9*b^2 - (262144*a^12 + 27*
a^9)*b)*x^3 + 14784*(16777216*a^6*b^10 + 6291456*a^6*b^9 + 196608*a^6*b...
```

Sympy [F]

$$\int x^2\sqrt{bx^{2/3}+ax}\,dx=\int x^2\sqrt{ax+bx^{\frac{2}{3}}}\,dx$$

input
```
integrate(x**2*(b*x**(2/3)+a*x)**(1/2),x)
```

output
```
Integral(x**2*sqrt(a*x + b*x**(2/3)), x)
```

Maxima [F]

$$\int x^2 \sqrt{bx^{2/3} + ax}\, dx = \int \sqrt{ax + bx^{\frac{2}{3}}}x^2\, dx$$

input
```
integrate(x^2*(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(2/3))*x^2, x)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.10

$$\int x^2 \sqrt{bx^{2/3} + ax}\, dx = \frac{131072\, b^{\frac{21}{2}}}{1616615\, a^{10}} + \frac{2\left(\frac{21\left(12155\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}-122265\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b+554268\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^2-1492260\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b^3+2645370\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^4-3233230\left(ax^{\frac{1}{3}}\right.\right.}{a^9}\right.}{}$$

input
```
integrate(x^2*(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
131072/1616615*b^(21/2)/a^10 + 2/1616615*(21*(12155*(a*x^(1/3) + b)^(19/2)
 - 122265*(a*x^(1/3) + b)^(17/2)*b + 554268*(a*x^(1/3) + b)^(15/2)*b^2 - 1
492260*(a*x^(1/3) + b)^(13/2)*b^3 + 2645370*(a*x^(1/3) + b)^(11/2)*b^4 - 3
233230*(a*x^(1/3) + b)^(9/2)*b^5 + 2771340*(a*x^(1/3) + b)^(7/2)*b^6 - 166
2804*(a*x^(1/3) + b)^(5/2)*b^7 + 692835*(a*x^(1/3) + b)^(3/2)*b^8 - 230945
*sqrt(a*x^(1/3) + b)*b^9)*b/a^9 + 5*(46189*(a*x^(1/3) + b)^(21/2) - 510510
*(a*x^(1/3) + b)^(19/2)*b + 2567565*(a*x^(1/3) + b)^(17/2)*b^2 - 7759752*(
a*x^(1/3) + b)^(15/2)*b^3 + 15668730*(a*x^(1/3) + b)^(13/2)*b^4 - 22221108
*(a*x^(1/3) + b)^(11/2)*b^5 + 22632610*(a*x^(1/3) + b)^(9/2)*b^6 - 1662804
0*(a*x^(1/3) + b)^(7/2)*b^7 + 8729721*(a*x^(1/3) + b)^(5/2)*b^8 - 3233230*
(a*x^(1/3) + b)^(3/2)*b^9 + 969969*sqrt(a*x^(1/3) + b)*b^10)/a^9)/a
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{bx^{2/3}+ax}\,dx = \int x^2\,\sqrt{a\,x+b\,x^{2/3}}\,dx$$

input
```
int(x^2*(a*x + b*x^(2/3))^(1/2),x)
```

output
```
int(x^2*(a*x + b*x^(2/3))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.42

$$\int x^2\sqrt{bx^{2/3}+ax}\,dx = \frac{2\sqrt{x^{\frac{1}{3}}a+b}\,\left(-12870x^{\frac{8}{3}}a^8b^2+16128x^{\frac{5}{3}}a^5b^5-24576x^{\frac{2}{3}}a^2b^8+230945x^{\frac{10}{3}}a^{10}+137$$

input
```
int(x^2*(b*x^(2/3)+a*x)^(1/2),x)
```

output
```
(2*sqrt(x**(1/3)*a + b)*( - 12870*x**(2/3)*a**8*b**2*x**2 + 16128*x**(2/3)
*a**5*b**5*x - 24576*x**(2/3)*a**2*b**8 + 230945*x**(1/3)*a**10*x**3 + 137
28*x**(1/3)*a**7*b**3*x**2 - 17920*x**(1/3)*a**4*b**6*x + 32768*x**(1/3)*a
*b**9 + 12155*a**9*b*x**3 - 14784*a**6*b**4*x**2 + 20480*a**3*b**7*x - 655
36*b**10))/(1616615*a**10)
```

3.148 $\int x\sqrt{bx^{2/3}+ax}\,dx$

Optimal result . . . 1377
Mathematica [A] (verified) . . . 1378
Rubi [A] (verified) . . . 1378
Maple [A] (verified) . . . 1383
Fricas [B] (verification not implemented) . . . 1384
Sympy [F] . . . 1385
Maxima [F] . . . 1385
Giac [A] (verification not implemented) . . . 1385
Mupad [F(-1)] . . . 1386
Reduce [B] (verification not implemented) . . . 1386

Optimal result

Integrand size = 17, antiderivative size = 195

$$\int x\sqrt{bx^{2/3}+ax}\,dx = -\frac{128b^3\left(bx^{2/3}+ax\right)^{3/2}}{429a^4}+\frac{2048b^6\left(bx^{2/3}+ax\right)^{3/2}}{15015a^7x} -\frac{1024b^5\left(bx^{2/3}+ax\right)^{3/2}}{5005a^6x^{2/3}}+\frac{256b^4\left(bx^{2/3}+ax\right)^{3/2}}{1001a^5\sqrt[3]{x}} +\frac{48b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{143a^3}-\frac{24bx^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{65a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{3/2}}{5a}$$

output

```
-128/429*b^3*(b*x^(2/3)+a*x)^(3/2)/a^4+2048/15015*b^6*(b*x^(2/3)+a*x)^(3/2
)/a^7/x-1024/5005*b^5*(b*x^(2/3)+a*x)^(3/2)/a^6/x^(2/3)+256/1001*b^4*(b*x^
(2/3)+a*x)^(3/2)/a^5/x^(1/3)+48/143*b^2*x^(1/3)*(b*x^(2/3)+a*x)^(3/2)/a^3-
24/65*b*x^(2/3)*(b*x^(2/3)+a*x)^(3/2)/a^2+2/5*x*(b*x^(2/3)+a*x)^(3/2)/a
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.49

$$\int x\sqrt{bx^{2/3}+ax}\,dx = \frac{2\left(bx^{2/3}+ax\right)^{3/2}\left(1024b^6-1536ab^5\sqrt[3]{x}+1920a^2b^4x^{2/3}-2240a^3b^3x+2520a^4b^2x^{4/3}\right.}{15015a^7x}$$

input
```
Integrate[x*Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*(b*x^(2/3) + a*x)^(3/2)*(1024*b^6 - 1536*a*b^5*x^(1/3) + 1920*a^2*b^4*x
^(2/3) - 2240*a^3*b^3*x + 2520*a^4*b^2*x^(4/3) - 2772*a^5*b*x^(5/3) + 3003
*a^6*x^2))/(15015*a^7*x)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {1922, 1922, 1922, 1908, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{ax+bx^{2/3}}\,dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a}-\frac{4b\int x^{2/3}\sqrt{x^{2/3}b+ax}dx}{5a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a}-\frac{4b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{13a}-\frac{10b\int \sqrt[3]{x}\sqrt{x^{2/3}b+ax}dx}{13a}\right)}{5a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a}-\frac{4b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{13a}-\frac{10b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{11a}-\frac{8b\int\sqrt{x^{2/3}b+ax}dx}{11a}\right)}{13a}\right)}{5a}$$

$\downarrow$ 1908

$$\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a}-\frac{4b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{13a}-\frac{10b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{11a}-\frac{8b\left(\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{3a}-\frac{2b\int\frac{\sqrt{x^{2/3}b+ax}}{\sqrt[3]{x}}dx}{3a}\right)}{11a}\right)}{13a}\right)}{5a}$$

$\downarrow$ 1922

$$\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a}-\frac{4b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{13a}-\frac{10b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{11a}-\frac{8b\left(\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{3a}-\frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{3/2}}{7a\sqrt[3]{x}}-\frac{4b\int\frac{\sqrt{x^{2/3}b+ax}}{x^{2/3}}dx}{7a}\right)}{3a}\right)}{11a}\right)}{13a}\right)}{5a}$$

$\downarrow$ 1922

$$\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a}-\frac{4b\Bigg(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{13a}-\frac{10b\Bigg(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{11a}-\frac{8b\Bigg(\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{3a}-\frac{2b\Bigg(\frac{6\left(ax+bx^{2/3}\right)^{3/2}}{7a\sqrt[3]{x}}-\frac{4b\Bigg(\frac{6\left(ax+bx^{2/3}\right)^{3/2}}{5ax^{2/3}}-\frac{2b\int\frac{\sqrt{x^{2/3}b+ax}}{x}dx}{5a}}{7a}}{3a}}{11a}}{13a}}{5a}$$

$\downarrow$ 1920

$$\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a}-\frac{4b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{13a}-\frac{10b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{11a}-\frac{8b\left(\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{3a}-\frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{3/2}}{7a\sqrt[3]{x}}-\frac{4b\left(\frac{6\left(ax+bx^{2/3}\right)^{3/2}}{5ax^{2/3}}-\frac{4b\left(ax+bx^{2/3}\right)^{3/2}}{5a^2x}\right)}{7a}\right)}{3a}\right)}{11a}\right)}{13a}\right)}{5a}$$

input
```
Int[x*Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*x*(b*x^(2/3) + a*x)^(3/2))/(5*a) - (4*b*((6*x^(2/3)*(b*x^(2/3) + a*x)^(
3/2))/(13*a) - (10*b*((6*x^(1/3)*(b*x^(2/3) + a*x)^(3/2))/(11*a) - (8*b*((
2*(b*x^(2/3) + a*x)^(3/2))/(3*a) - (2*b*((6*(b*x^(2/3) + a*x)^(3/2))/(7*a*
x^(1/3)) - (4*b*((-4*b*(b*x^(2/3) + a*x)^(3/2))/(5*a^2*x) + (6*(b*x^(2/3)
+ a*x)^(3/2))/(5*a*x^(2/3))))/(7*a)))/(3*a)))/(11*a)))/(13*a)))/(5*a)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.46

method	result
derivativedivides	$\frac{2\sqrt{b\,x^{\frac{2}{3}}+a x}\,\left(x^{\frac{1}{3}} a+b\right)\left(3003 a^{6} x^{2}-2772 a^{5} b\,x^{\frac{5}{3}}+2520 a^{4} b^{2} x^{\frac{4}{3}}-2240 b^{3} x\,a^{3}+1920 a^{2} b^{4} x^{\frac{2}{3}}-1536 a\,b^{5} x^{\frac{1}{3}}+1024 b^{6}\right)}{15015 x^{\frac{1}{3}} a^{7}}$
default	$-\frac{2\sqrt{b\,x^{\frac{2}{3}}+a x}\,\left(x^{\frac{1}{3}} a+b\right)\left(2772 a^{5} b\,x^{\frac{5}{3}}-2520 a^{4} b^{2} x^{\frac{4}{3}}-1920 a^{2} b^{4} x^{\frac{2}{3}}-3003 a^{6} x^{2}+1536 a\,b^{5} x^{\frac{1}{3}}+2240 b^{3} x\,a^{3}-1024 b^{6}\right)}{15015 x^{\frac{1}{3}} a^{7}}$

input
```
int(x*(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15015*(b*x^(2/3)+a*x)^(1/2)*(x^(1/3)*a+b)*(3003*a^6*x^2-2772*a^5*b*x^(5/
3)+2520*a^4*b^2*x^(4/3)-2240*b^3*x*a^3+1920*a^2*b^4*x^(2/3)-1536*a*b^5*x^(
1/3)+1024*b^6)/x^(1/3)/a^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 767 vs. $2(145) = 290$.

Time = 132.14 (sec) , antiderivative size = 767, normalized size of antiderivative = 3.93

$$\int x\sqrt{bx^{2/3} + ax}\,dx = \text{Too large to display}$$

input
```
integrate(x*(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-1/30030*((51539607552*b^13 + 10737418240*b^12 + 50331648*(64*a^3 - 3)*b^1
0 - 1006632960*b^11 - 262144*(11264*a^3 - 53)*b^9 - 69957888*a^9 - 98304*(
5504*a^3 + 1)*b^8 + 3072*(3194880*a^6 - 114688*a^3 - 3)*b^7 + 114688*(1881
6*a^6 + 103*a^3)*b^6 - 12288*(48816*a^6 + 23*a^3)*b^5 + 192*(302776320*a^9
 + 495872*a^6 + 15*a^3)*b^4 + 1792*(16588800*a^9 - 3439*a^6)*b^3 + 26208*(
163840*a^9 + 7*a^6)*b^2 + 693*(1024000*a^9 - 3*a^6)*b)*x - 4*(3003*(167772
16*a^7*b^6 + 6291456*a^7*b^5 + 196608*a^7*b^4 - 262144*a^10 - 114688*a^7*b
^3 - 2304*a^7*b^2 + 864*a^7*b - 27*a^7)*x^3 + 280*(16777216*a^4*b^9 + 6291
456*a^4*b^8 + 196608*a^4*b^7 - 114688*a^4*b^6 - 2304*a^4*b^5 + 864*a^4*b^4
 - (262144*a^7 + 27*a^4)*b^3)*x^2 - 512*(16777216*a*b^12 + 6291456*a*b^11
+ 196608*a*b^10 - 114688*a*b^9 - 2304*a*b^8 + 864*a*b^7 - (262144*a^4 + 27
*a)*b^6)*x + (17179869184*b^13 + 6442450944*b^12 + 201326592*b^11 - 117440
512*b^10 - 2359296*b^9 - 1024*(262144*a^3 + 27)*b^7 + 884736*b^8 + 231*(16
777216*a^6*b^7 + 6291456*a^6*b^6 + 196608*a^6*b^5 - 114688*a^6*b^4 - 2304*
a^6*b^3 + 864*a^6*b^2 - (262144*a^9 + 27*a^6)*b)*x^2 - 320*(16777216*a^3*b
^10 + 6291456*a^3*b^9 + 196608*a^3*b^8 - 114688*a^3*b^7 - 2304*a^3*b^6 + 8
64*a^3*b^5 - (262144*a^6 + 27*a^3)*b^4)*x)*x^(2/3) - 12*(21*(16777216*a^5*
b^8 + 6291456*a^5*b^7 + 196608*a^5*b^6 - 114688*a^5*b^5 - 2304*a^5*b^4 + 8
64*a^5*b^3 - (262144*a^8 + 27*a^5)*b^2)*x^2 - 32*(16777216*a^2*b^11 + 6291
456*a^2*b^10 + 196608*a^2*b^9 - 114688*a^2*b^8 - 2304*a^2*b^7 + 864*a^2...
```

Sympy [F]

$$\int x\sqrt{bx^{2/3}+ax}\,dx=\int x\sqrt{ax+bx^{\frac{2}{3}}}\,dx$$

input
```
integrate(x*(b*x**(2/3)+a*x)**(1/2),x)
```

output
```
Integral(x*sqrt(a*x + b*x**(2/3)), x)
```

Maxima [F]

$$\int x\sqrt{bx^{2/3}+ax}\,dx=\int \sqrt{ax+bx^{\frac{2}{3}}}x\,dx$$

input
```
integrate(x*(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(2/3))*x, x)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.17

$$\int x\sqrt{bx^{2/3}+ax}\,dx=-\frac{2048\,b^{\frac{15}{2}}}{15015\,a^{7}}+\frac{2\left(\frac{15\left(231\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}-1638\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b+5005\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^{2}-8580\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^{3}+9009\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{4}-6006\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{5}+3003\sqrt{ax}}{a^{6}}\right.}{}$$

input
```
integrate(x*(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
-2048/15015*b^(15/2)/a^7 + 2/15015*(15*(231*(a*x^(1/3) + b)^(13/2) - 1638*
(a*x^(1/3) + b)^(11/2)*b + 5005*(a*x^(1/3) + b)^(9/2)*b^2 - 8580*(a*x^(1/3
) + b)^(7/2)*b^3 + 9009*(a*x^(1/3) + b)^(5/2)*b^4 - 6006*(a*x^(1/3) + b)^(
3/2)*b^5 + 3003*sqrt(a*x^(1/3) + b)*b^6)*b/a^6 + 7*(429*(a*x^(1/3) + b)^(1
5/2) - 3465*(a*x^(1/3) + b)^(13/2)*b + 12285*(a*x^(1/3) + b)^(11/2)*b^2 -
25025*(a*x^(1/3) + b)^(9/2)*b^3 + 32175*(a*x^(1/3) + b)^(7/2)*b^4 - 27027*
(a*x^(1/3) + b)^(5/2)*b^5 + 15015*(a*x^(1/3) + b)^(3/2)*b^6 - 6435*sqrt(a*
x^(1/3) + b)*b^7)/a^6)/a
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{bx^{2/3}+ax}\,dx=\int x\,\sqrt{a\,x+b\,x^{2/3}}\,dx$$

input
```
int(x*(a*x + b*x^(2/3))^(1/2),x)
```

output
```
int(x*(a*x + b*x^(2/3))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.45

$$\int x\sqrt{bx^{2/3}+ax}\,dx=\frac{2\sqrt{x^{\frac{1}{3}}a+b}\,\left(-252x^{\frac{5}{3}}a^5b^2+384x^{\frac{2}{3}}a^2b^5+3003x^{\frac{7}{3}}a^7+280x^{\frac{4}{3}}a^4b^3-512x^{\frac{1}{3}}a\,b^6+231\right.}{15015a^7}$$

input
```
int(x*(b*x^(2/3)+a*x)^(1/2),x)
```

output
```
(2*sqrt(x**(1/3)*a + b)*( - 252*x**(2/3)*a**5*b**2*x + 384*x**(2/3)*a**2*b
**5 + 3003*x**(1/3)*a**7*x**2 + 280*x**(1/3)*a**4*b**3*x - 512*x**(1/3)*a*
b**6 + 231*a**6*b*x**2 - 320*a**3*b**4*x + 1024*b**7))/(15015*a**7)
```

3.149 $\int \sqrt{bx^{2/3}+ax}\,dx$

Optimal result . . . 1387
Mathematica [A] (verified) . . . 1387
Rubi [A] (verified) . . . 1388
Maple [A] (verified) . . . 1390
Fricas [B] (verification not implemented) . . . 1390
Sympy [F] . . . 1391
Maxima [F] . . . 1391
Giac [A] (verification not implemented) . . . 1392
Mupad [B] (verification not implemented) . . . 1392
Reduce [B] (verification not implemented) . . . 1393

Optimal result

Integrand size = 15, antiderivative size = 109

$$\int \sqrt{bx^{2/3}+ax}\,dx=\frac{2\left(bx^{2/3}+ax\right)^{3/2}}{3a}-\frac{32b^3\left(bx^{2/3}+ax\right)^{3/2}}{105a^4x}+\frac{16b^2\left(bx^{2/3}+ax\right)^{3/2}}{35a^3x^{2/3}}-\frac{4b\left(bx^{2/3}+ax\right)^{3/2}}{7a^2\sqrt[3]{x}}$$

output
```
2/3*(b*x^(2/3)+a*x)^(3/2)/a-32/105*b^3*(b*x^(2/3)+a*x)^(3/2)/a^4/x+16/35*b
^2*(b*x^(2/3)+a*x)^(3/2)/a^3/x^(2/3)-4/7*b*(b*x^(2/3)+a*x)^(3/2)/a^2/x^(1/
3)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.68

$$\int \sqrt{bx^{2/3}+ax}\,dx=\frac{2\sqrt{bx^{2/3}+ax}\left(-16b^4+8ab^3\sqrt[3]{x}-6a^2b^2x^{2/3}+5a^3bx+35a^4x^{4/3}\right)}{105a^4\sqrt[3]{x}}$$

input
```
Integrate[Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(2/3) + a*x]*(-16*b^4 + 8*a*b^3*x^(1/3) - 6*a^2*b^2*x^(2/3) +
5*a^3*b*x + 35*a^4*x^(4/3)))/(105*a^4*x^(1/3))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1908, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ax+bx^{2/3}}dx$$

$$\Big\downarrow \text{1908}$$

$$\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{3a}-\frac{2b\int \frac{\sqrt{x^{2/3}b+ax}}{\sqrt[3]{x}}dx}{3a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{3a}-\frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{3/2}}{7a\sqrt[3]{x}}-\frac{4b\int \frac{\sqrt{x^{2/3}b+ax}}{x^{2/3}}dx}{7a}\right)}{3a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{3a}-\frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{3/2}}{7a\sqrt[3]{x}}-\frac{4b\left(\frac{6\left(ax+bx^{2/3}\right)^{3/2}}{5ax^{2/3}}-\frac{2b\int \frac{\sqrt{x^{2/3}b+ax}}{x}dx}{5a}\right)}{7a}\right)}{3a}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{3a}-\frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{3/2}}{7a\sqrt[3]{x}}-\frac{4b\left(\frac{6\left(ax+bx^{2/3}\right)^{3/2}}{5ax^{2/3}}-\frac{4b\left(ax+bx^{2/3}\right)^{3/2}}{5a^2x}\right)}{7a}\right)}{3a}$$

input
```
Int[Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*(b*x^(2/3) + a*x)^(3/2))/(3*a) - (2*b*((6*(b*x^(2/3) + a*x)^(3/2))/(7*a
*x^(1/3)) - (4*b*((-4*b*(b*x^(2/3) + a*x)^(3/2))/(5*a^2*x) + (6*(b*x^(2/3)
 + a*x)^(3/2))/(5*a*x^(2/3))))/(7*a)))/(3*a)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.52

method	result	size
derivativedivides	$\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(x^{\frac{1}{3}}a+b\right)\left(35a^{3}x-30a^{2}b\,x^{\frac{2}{3}}+24a\,b^{2}x^{\frac{1}{3}}-16b^{3}\right)}{105x^{\frac{1}{3}}a^{4}}$	57
default	$-\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(x^{\frac{1}{3}}a+b\right)\left(30a^{2}b\,x^{\frac{2}{3}}-24a\,b^{2}x^{\frac{1}{3}}-35a^{3}x+16b^{3}\right)}{105x^{\frac{1}{3}}a^{4}}$	57

input
```
int((b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/105*(b*x^(2/3)+a*x)^(1/2)*(x^(1/3)*a+b)*(35*a^3*x-30*a^2*b*x^(2/3)+24*a*
b^2*x^(1/3)-16*b^3)/x^(1/3)/a^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. 2(81) = 162.

Time = 120.60 (sec) , antiderivative size = 501, normalized size of antiderivative = 4.60

$$\int \sqrt{bx^{2/3}+ax}\,dx = \frac{(805306368\,b^{10}+167772160\,b^{9}+786432\,(64\,a^{3}-3)b^{7}-15728640\,b^{8}-4096\,(11264\,a}{}$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
1/210*((805306368*b^10 + 167772160*b^9 + 786432*(64*a^3 - 3)*b^7 - 1572864
0*b^8 - 4096*(11264*a^3 - 53)*b^6 + 815360*a^6 - 1536*(5504*a^3 + 1)*b^5 -
 48*(15728640*a^6 + 114688*a^3 + 3)*b^4 - 1792*(221184*a^6 - 103*a^3)*b^3
- 192*(307200*a^6 + 23*a^3)*b^2 - 15*(499712*a^6 - 3*a^3)*b)*x + 4*(35*(16
777216*a^4*b^6 + 6291456*a^4*b^5 + 196608*a^4*b^4 - 262144*a^7 - 114688*a^
4*b^3 - 2304*a^4*b^2 + 864*a^4*b - 27*a^4)*x^2 - 6*(16777216*a^2*b^8 + 629
1456*a^2*b^7 + 196608*a^2*b^6 - 114688*a^2*b^5 - 2304*a^2*b^4 + 864*a^2*b^
3 - (262144*a^5 + 27*a^2)*b^2)*x^(4/3) + 8*(16777216*a*b^9 + 6291456*a*b^8
 + 196608*a*b^7 - 114688*a*b^6 - 2304*a*b^5 + 864*a*b^4 - (262144*a^4 + 27
*a)*b^3)*x - (268435456*b^10 + 100663296*b^9 + 3145728*b^8 - 1835008*b^7 -
 36864*b^6 - 16*(262144*a^3 + 27)*b^4 + 13824*b^5 - 5*(16777216*a^3*b^7 +
6291456*a^3*b^6 + 196608*a^3*b^5 - 114688*a^3*b^4 - 2304*a^3*b^3 + 864*a^3
*b^2 - (262144*a^6 + 27*a^3)*b)*x)*x^(2/3))*sqrt(a*x + b*x^(2/3)))/((16777
216*a^4*b^6 + 6291456*a^4*b^5 + 196608*a^4*b^4 - 262144*a^7 - 114688*a^4*b
^3 - 2304*a^4*b^2 + 864*a^4*b - 27*a^4)*x)
```

Sympy [F]

$$\int \sqrt{bx^{2/3} + ax}\, dx = \int \sqrt{ax + bx^{\frac{2}{3}}}\, dx$$

input
```
integrate((b*x**(2/3)+a*x)**(1/2),x)
```

output
```
Integral(sqrt(a*x + b*x**(2/3)), x)
```

Maxima [F]

$$\int \sqrt{bx^{2/3} + ax}\, dx = \int \sqrt{ax + bx^{\frac{2}{3}}}\, dx$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(2/3)), x)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.31

$$\int \sqrt{bx^{2/3}+ax}\,dx = \frac{32\,b^{\frac{9}{2}}}{105\,a^4} + \frac{2\left(\frac{9\left(5\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}-21\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b+35\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^2-35\sqrt{ax^{\frac{1}{3}}+b}\,b^3\right)b}{a^3}+\frac{35\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}-180\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b+378\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^2-420\left(a\right.}{a^3}\right.}{105\,a}$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
32/105*b^(9/2)/a^4 + 2/105*(9*(5*(a*x^(1/3) + b)^(7/2) - 21*(a*x^(1/3) + b
)^(5/2)*b + 35*(a*x^(1/3) + b)^(3/2)*b^2 - 35*sqrt(a*x^(1/3) + b)*b^3)*b/a
^3 + (35*(a*x^(1/3) + b)^(9/2) - 180*(a*x^(1/3) + b)^(7/2)*b + 378*(a*x^(1
/3) + b)^(5/2)*b^2 - 420*(a*x^(1/3) + b)^(3/2)*b^3 + 315*sqrt(a*x^(1/3) +
b)*b^4)/a^3)/a
```

Mupad [B] (verification not implemented)

Time = 8.74 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.37

$$\int \sqrt{bx^{2/3}+ax}\,dx = \frac{3\,x\,\sqrt{a\,x+b\,x^{2/3}}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},4;\;5;\;-\frac{a\,x^{1/3}}{b}\right)}{4\,\sqrt{\frac{a\,x^{1/3}}{b}+1}}$$

input
```
int((a*x + b*x^(2/3))^(1/2),x)
```

output
```
(3*x*(a*x + b*x^(2/3))^(1/2)*hypergeom([-1/2, 4], 5, -(a*x^(1/3))/b))/(4*(
(a*x^(1/3))/b + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.50

$$\int \sqrt{bx^{2/3}+ax}\,dx = \frac{2\sqrt{x^{\frac{1}{3}}a+b}\left(-6x^{\frac{2}{3}}a^2b^2+35x^{\frac{4}{3}}a^4+8x^{\frac{1}{3}}a\,b^3+5a^3bx-16b^4\right)}{105a^4}$$

input
```
int((b*x^(2/3)+a*x)^(1/2),x)
```

output
```
(2*sqrt(x**(1/3)*a + b)*( - 6*x**(2/3)*a**2*b**2 + 35*x**(1/3)*a**4*x + 8*
x**(1/3)*a*b**3 + 5*a**3*b*x - 16*b**4))/(105*a**4)
```

3.150 $\int \frac{\sqrt{bx^{2/3}+ax}}{x} dx$

Optimal result 1394
Mathematica [A] (verified) 1394
Rubi [A] (verified) 1395
Maple [A] (verified) 1395
Fricas [B] (verification not implemented) 1396
Sympy [F] 1397
Maxima [F] 1397
Giac [A] (verification not implemented) 1397
Mupad [F(-1)] 1398
Reduce [B] (verification not implemented) 1398

Optimal result

Integrand size = 19, antiderivative size = 23

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x} dx = \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{ax}$$

output
```
2*(b*x^(2/3)+a*x)^(3/2)/a/x
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x} dx = \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{ax}$$

input
```
Integrate[Sqrt[b*x^(2/3) + a*x]/x,x]
```

output
```
(2*(b*x^(2/3) + a*x)^(3/2))/(a*x)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^{2/3}}}{x} \, dx$$

$$\Big\downarrow \text{1920}$$

$$\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{ax}$$

input
```
Int[Sqrt[b*x^(2/3) + a*x]/x,x]
```

output
```
(2*(b*x^(2/3) + a*x)^(3/2))/(a*x)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(x^{\frac{1}{3}}a+b\right)}{x^{\frac{1}{3}}a}$	27
default	$\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(x^{\frac{1}{3}}a+b\right)}{x^{\frac{1}{3}}a}$	27

input
```
int((b*x^(2/3)+a*x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*(b*x^(2/3)+a*x)^(1/2)/x^(1/3)*(x^(1/3)*a+b)/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(19) = 38.

Time = 121.88 (sec) , antiderivative size = 224, normalized size of antiderivative = 9.74

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx =$$

$$-\frac{(50331648\,b^7 + 10485760\,b^6 + 49152\,(1024\,a^3 - 3)b^4 - 983040\,b^5 + 256\,(73728\,a^3 + 53)b^3 - 23296\,a^3 + 9}{}$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x,x, algorithm="fricas")
```

output
```
-1/2*((50331648*b^7 + 10485760*b^6 + 49152*(1024*a^3 - 3)*b^4 - 983040*b^5
 + 256*(73728*a^3 + 53)*b^3 - 23296*a^3 + 96*(16384*a^3 - 1)*b^2 - 9*(8192
*a^3 + 1)*b)*x - 4*((16777216*a*b^6 + 6291456*a*b^5 + 196608*a*b^4 - 26214
4*a^4 - 114688*a*b^3 - 2304*a*b^2 + 864*a*b - 27*a)*x + (16777216*b^7 + 62
91456*b^6 + 196608*b^5 - 114688*b^4 - 2304*b^3 - (262144*a^3 + 27)*b + 864
*b^2)*x^(2/3))*sqrt(a*x + b*x^(2/3)))/((16777216*a*b^6 + 6291456*a*b^5 + 1
96608*a*b^4 - 262144*a^4 - 114688*a*b^3 - 2304*a*b^2 + 864*a*b - 27*a)*x)
```

Sympy [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx = \int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x}\,dx$$

input
```
integrate((b*x**(2/3)+a*x)**(1/2)/x,x)
```

output
```
Integral(sqrt(a*x + b*x**(2/3))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx = \int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x}\,dx$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(2/3))/x, x)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx = \frac{2\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}}{a} - \frac{2\,b^{\frac{3}{2}}}{a}$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x,x, algorithm="giac")
```

output
```
2*(a*x^(1/3) + b)^(3/2)/a - 2*b^(3/2)/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx=\int \frac{\sqrt{a\,x+b\,x^{2/3}}}{x}\,dx$$

input
```
int((a*x + b*x^(2/3))^(1/2)/x,x)
```

output
```
int((a*x + b*x^(2/3))^(1/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx=\frac{2\sqrt{x^{\frac{1}{3}}a+b}\,\left(x^{\frac{1}{3}}a+b\right)}{a}$$

input
```
int((b*x^(2/3)+a*x)^(1/2)/x,x)
```

output
```
(2*sqrt(x**(1/3)*a + b)*(x**(1/3)*a + b))/a
```

3.151 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^2} dx$

Optimal result 1399
Mathematica [A] (verified) 1399
Rubi [A] (verified) 1400
Maple [A] (verified) 1401
Fricas [F(-1)] 1402
Sympy [F] 1402
Maxima [F] 1403
Giac [A] (verification not implemented) 1403
Mupad [F(-1)] 1403
Reduce [B] (verification not implemented) 1404

Optimal result

Integrand size = 19, antiderivative size = 90

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx=-\frac{3\sqrt{bx^{2/3}+ax}}{2x}-\frac{3a\sqrt{bx^{2/3}+ax}}{4bx^{2/3}}+\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{4b^{3/2}}$$

output
```
-3/2*(b*x^(2/3)+a*x)^(1/2)/x-3/4*a*(b*x^(2/3)+a*x)^(1/2)/b/x^(2/3)+3/4*a^2
*arctanh(b^(1/2)*x^(1/3)/(b*x^(2/3)+a*x)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx=-\frac{3\left(2b+a\sqrt[3]{x}\right)\sqrt{bx^{2/3}+ax}}{4bx}+\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{4b^{3/2}}$$

input
```
Integrate[Sqrt[b*x^(2/3) + a*x]/x^2,x]
```

output
```
(-3*(2*b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x])/(4*b*x) + (3*a^2*ArcTanh[(Sqr
t[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(4*b^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1926, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^{2/3}}}{x^2}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{4}a\int \frac{1}{x\sqrt{x^{2/3}b+ax}}dx-\frac{3\sqrt{ax+bx^{2/3}}}{2x}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{4}a\left(-\frac{a\int \frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{2x}$$

$$\Big\downarrow \text{1935}$$

$$\frac{1}{4}a\left(\frac{3a\int \frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{2x}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}a\left(\frac{3a\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{2x}$$

input
```
Int[Sqrt[b*x^(2/3) + a*x]/x^2,x]
```

output
```
(-3*Sqrt[b*x^(2/3) + a*x])/(2*x) + (a*((-3*Sqrt[b*x^(2/3) + a*x])/(b*x^(2/
3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/b^(3/2)))/4
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{3\sqrt{b\,x^{\frac{2}{3}}+a x}\,\left(\left(x^{\frac{1}{3}} a+b\right)^{\frac{3}{2}} b^{\frac{3}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{x^{\frac{1}{3}} a+b}}{\sqrt{b}}\right) b\,a^{2} x^{\frac{2}{3}}+\sqrt{x^{\frac{1}{3}} a+b}\,b^{\frac{5}{2}}\right)}{4 x \sqrt{x^{\frac{1}{3}} a+b}\,b^{\frac{5}{2}}}$	79
derivativedivides	$\frac{3\sqrt{b\,x^{\frac{2}{3}}+a x}\,\left(\operatorname{arctanh}\left(\frac{\sqrt{x^{\frac{1}{3}} a+b}}{\sqrt{b}}\right) b\,a^{2} x^{\frac{2}{3}}-\left(x^{\frac{1}{3}} a+b\right)^{\frac{3}{2}} b^{\frac{3}{2}}-\sqrt{x^{\frac{1}{3}} a+b}\,b^{\frac{5}{2}}\right)}{4 x \sqrt{x^{\frac{1}{3}} a+b}\,b^{\frac{5}{2}}}$	80

input
```
int((b*x^(2/3)+a*x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-3/4*(b*x^(2/3)+a*x)^(1/2)*((x^(1/3)*a+b)^(3/2)*b^(3/2)-arctanh((x^(1/3)*a
+b)^(1/2)/b^(1/2))*b*a^2*x^(2/3)+(x^(1/3)*a+b)^(1/2)*b^(5/2))/x/(x^(1/3)*a
+b)^(1/2)/b^(5/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx = \int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^2}\,dx$$

input
```
integrate((b*x**(2/3)+a*x)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(a*x + b*x**(2/3))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx = \int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^2}\,dx$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(2/3))/x^2, x)
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx = -\frac{3\left(\frac{a^3\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b}+\frac{\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^3+\sqrt{ax^{\frac{1}{3}}+b}\,a^3b}{a^2bx^{\frac{2}{3}}}\right)}{4\,a}$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x^2,x, algorithm="giac")
```

output
```
-3/4*(a^3*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) + ((a*x^(1/3)
+ b)^(3/2)*a^3 + sqrt(a*x^(1/3) + b)*a^3*b)/(a^2*b*x^(2/3)))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx = \int \frac{\sqrt{a\,x+b\,x^{2/3}}}{x^2}\,dx$$

input
```
int((a*x + b*x^(2/3))^(1/2)/x^2,x)
```

output
```
int((a*x + b*x^(2/3))^(1/2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx = \frac{-\frac{3x^{\frac{1}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,ab}{4}-\frac{3\sqrt{x^{\frac{1}{3}}a+b}\,b^2}{2}-\frac{3x^{\frac{2}{3}}\sqrt{b}\log\left(\sqrt{x^{\frac{1}{3}}a+b}-\sqrt{b}\right)a^2}{8}+\frac{3x^{\frac{2}{3}}\sqrt{b}\log\left(\sqrt{x^{\frac{1}{3}}a+b}+\sqrt{b}\right)a^2}{8}}{x^{\frac{2}{3}}b^2}$$

input

```
int((b*x^(2/3)+a*x)^(1/2)/x^2,x)
```

output

```
(3*( - 2*x**(1/3)*sqrt(x**(1/3)*a + b)*a*b - 4*sqrt(x**(1/3)*a + b)*b**2 -
 x**(2/3)*sqrt(b)*log(sqrt(x**(1/3)*a + b) - sqrt(b))*a**2 + x**(2/3)*sqrt
(b)*log(sqrt(x**(1/3)*a + b) + sqrt(b))*a**2))/(8*x**(2/3)*b**2)
```

3.152 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^3} dx$

Optimal result 1405
Mathematica [A] (verified) 1405
Rubi [A] (verified) 1406
Maple [A] (verified) 1410
Fricas [F(-1)] 1411
Sympy [F] 1411
Maxima [F] 1412
Giac [A] (verification not implemented) 1412
Mupad [F(-1)] 1412
Reduce [B] (verification not implemented) 1413

Optimal result

Integrand size = 19, antiderivative size = 178

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^3} dx = -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}} + \frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}} - \frac{7a^3\sqrt{bx^{2/3}+ax}}{64b^3x} + \frac{21a^4\sqrt{bx^{2/3}+ax}}{128b^4x^{2/3}} - \frac{21a^5\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{128b^{9/2}}$$

output
```
-3/5*(b*x^(2/3)+a*x)^(1/2)/x^2-3/40*a*(b*x^(2/3)+a*x)^(1/2)/b/x^(5/3)+7/80
*a^2*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(4/3)-7/64*a^3*(b*x^(2/3)+a*x)^(1/2)/b^3/
x+21/128*a^4*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(2/3)-21/128*a^5*arctanh(b^(1/2)*
x^(1/3)/(b*x^(2/3)+a*x)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^3} dx = \frac{\sqrt{bx^{2/3}+ax}\left(-384b^4 - 48ab^3\sqrt[3]{x} + 56a^2b^2x^{2/3} - 70a^3bx + 105a^4x^{4/3}\right)}{640b^4x^2} - \frac{21a^5\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{128b^{9/2}}$$

input
```
Integrate[Sqrt[b*x^(2/3) + a*x]/x^3,x]
```

output
```
(Sqrt[b*x^(2/3) + a*x]*(-384*b^4 - 48*a*b^3*x^(1/3) + 56*a^2*b^2*x^(2/3) -
 70*a^3*b*x + 105*a^4*x^(4/3)))/(640*b^4*x^2) - (21*a^5*ArcTanh[(Sqrt[b]*x
^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(128*b^(9/2))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1926, 1931, 1931, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^{2/3}}}{x^3}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{10}a\int \frac{1}{x^2\sqrt{x^{2/3}b+ax}}dx-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{10}a\left(-\frac{7a\int \frac{1}{x^{5/3}\sqrt{x^{2/3}b+ax}}dx}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{10}a\left(-\frac{7a\left(-\frac{5a\int \frac{1}{x^{4/3}\sqrt{x^{2/3}b+ax}}dx}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{10}a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\int\frac{1}{x\sqrt{x^{2/3}b+ax}}dx}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}$$

$\Big\downarrow$ 1931

$$\frac{1}{10}a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(-\frac{a\int\frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}$$

$\Big\downarrow$ 1935

$$\frac{1}{10}a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\int \frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{10}a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}$$

input
```
Int[Sqrt[b*x^(2/3) + a*x]/x^3,x]
```

output
```
(-3*Sqrt[b*x^(2/3) + a*x])/(5*x^2) + (a*((-3*Sqrt[b*x^(2/3) + a*x])/(4*b*x
^(5/3)) - (7*a*(-(Sqrt[b*x^(2/3) + a*x]/(b*x^(4/3))) - (5*a*((-3*Sqrt[b*x^
(2/3) + a*x])/(2*b*x) - (3*a*((-3*Sqrt[b*x^(2/3) + a*x])/(b*x^(2/3)) + (3*
a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/b^(3/2)))/(4*b)))/(6*b
)))/(8*b)))/10
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{\sqrt{b\,x^{\frac{2}{3}}+a x}\,\left(105 b^{\frac{17}{2}} \sqrt{x^{\frac{1}{3}} a+b}+790 b^{\frac{15}{2}} \left(x^{\frac{1}{3}} a+b\right)^{\frac{3}{2}}-896 b^{\frac{13}{2}} \left(x^{\frac{1}{3}} a+b\right)^{\frac{5}{2}}+490 b^{\frac{11}{2}} \left(x^{\frac{1}{3}} a+b\right)^{\frac{7}{2}}-105 b^{\frac{9}{2}} \left(x^{\frac{1}{3}} a+b\right)^{\frac{9}{2}}\right.}{640 x^{2} \sqrt{x^{\frac{1}{3}} a+b}\, b^{\frac{17}{2}}}$
default	$-\frac{\sqrt{b\,x^{\frac{2}{3}}+a x}\,\left(105 b^{\frac{17}{2}} \sqrt{x^{\frac{1}{3}} a+b}+790 b^{\frac{15}{2}} \left(x^{\frac{1}{3}} a+b\right)^{\frac{3}{2}}-896 b^{\frac{13}{2}} \left(x^{\frac{1}{3}} a+b\right)^{\frac{5}{2}}+490 b^{\frac{11}{2}} \left(x^{\frac{1}{3}} a+b\right)^{\frac{7}{2}}-105 b^{\frac{9}{2}} \left(x^{\frac{1}{3}} a+b\right)^{\frac{9}{2}}\right.}{640 x^{2} \sqrt{x^{\frac{1}{3}} a+b}\, b^{\frac{17}{2}}}$

input
```
int((b*x^(2/3)+a*x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/640*(b*x^(2/3)+a*x)^(1/2)*(105*b^(17/2)*(x^(1/3)*a+b)^(1/2)+790*b^(15/2
)*(x^(1/3)*a+b)^(3/2)-896*b^(13/2)*(x^(1/3)*a+b)^(5/2)+490*b^(11/2)*(x^(1/
3)*a+b)^(7/2)-105*b^(9/2)*(x^(1/3)*a+b)^(9/2)+105*arctanh((x^(1/3)*a+b)^(1
/2)/b^(1/2))*b^4*a^5*x^(5/3))/x^2/(x^(1/3)*a+b)^(1/2)/b^(17/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx = \int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^3}\,dx$$

input
```
integrate((b*x**(2/3)+a*x)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(a*x + b*x**(2/3))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx = \int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^3}\,dx$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(2/3))/x^3, x)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx = \frac{1}{640}a^5\left(\frac{105\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^4} + \frac{105\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}} - 490\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b + 896\left(ax^{\frac{1}{3}}+\right.}{a^5b^4x}\right.$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x^3,x, algorithm="giac")
```

output
```
1/640*a^5*(105*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) + (105*
(a*x^(1/3) + b)^(9/2) - 490*(a*x^(1/3) + b)^(7/2)*b + 896*(a*x^(1/3) + b)^
(5/2)*b^2 - 790*(a*x^(1/3) + b)^(3/2)*b^3 - 105*sqrt(a*x^(1/3) + b)*b^4)/(
a^5*b^4*x^(5/3)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx = \int \frac{\sqrt{a\,x+b\,x^{2/3}}}{x^3}\,dx$$

input
```
int((a*x + b*x^(2/3))^(1/2)/x^3,x)
```

output
```
int((a*x + b*x^(2/3))^(1/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx = \frac{112x^{\frac{2}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,a^2b^3+210x^{\frac{4}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,a^4b-96x^{\frac{1}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,a\,b^4-140\sqrt{x^{\frac{1}{3}}a+b}\,a}{}$$

input
```
int((b*x^(2/3)+a*x)^(1/2)/x^3,x)
```

output
```
(112*x**(2/3)*sqrt(x**(1/3)*a + b)*a**2*b**3 + 210*x**(1/3)*sqrt(x**(1/3)*
a + b)*a**4*b*x - 96*x**(1/3)*sqrt(x**(1/3)*a + b)*a*b**4 - 140*sqrt(x**(1
/3)*a + b)*a**3*b**2*x - 768*sqrt(x**(1/3)*a + b)*b**5 + 105*x**(2/3)*sqrt
(b)*log(sqrt(x**(1/3)*a + b) - sqrt(b))*a**5*x - 105*x**(2/3)*sqrt(b)*log(
sqrt(x**(1/3)*a + b) + sqrt(b))*a**5*x)/(1280*x**(2/3)*b**5*x)
```

3.153 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^4} dx$

Optimal result 1414
Mathematica [A] (verified) 1415
Rubi [A] (verified) 1415
Maple [A] (verified) 1427
Fricas [F(-1)] 1427
Sympy [F] 1428
Maxima [F] 1428
Giac [A] (verification not implemented) 1428
Mupad [F(-1)] 1429
Reduce [B] (verification not implemented) 1429

Optimal result

Integrand size = 19, antiderivative size = 266

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^4} dx = -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2} + \frac{1287a^4\sqrt{bx^{2/3}+ax}}{35840b^4x^{5/3}} - \frac{429a^5\sqrt{bx^{2/3}+ax}}{10240b^5x^{4/3}} + \frac{429a^6\sqrt{bx^{2/3}+ax}}{8192b^6x} - \frac{1287a^7\sqrt{bx^{2/3}+ax}}{16384b^7x^{2/3}} + \frac{1287a^8\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{16384b^{15/2}}$$

output
```
-3/8*(b*x^(2/3)+a*x)^(1/2)/x^3-3/112*a*(b*x^(2/3)+a*x)^(1/2)/b/x^(8/3)+13/
448*a^2*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(7/3)-143/4480*a^3*(b*x^(2/3)+a*x)^(1/
2)/b^3/x^2+1287/35840*a^4*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(5/3)-429/10240*a^5*
(b*x^(2/3)+a*x)^(1/2)/b^5/x^(4/3)+429/8192*a^6*(b*x^(2/3)+a*x)^(1/2)/b^6/x
-1287/16384*a^7*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(2/3)+1287/16384*a^8*arctanh(b
^(1/2)*x^(1/3)/(b*x^(2/3)+a*x)^(1/2))/b^(15/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx = \frac{\sqrt{bx^{2/3}+ax}\left(-215040b^7-15360ab^6\sqrt[3]{x}+16640a^2b^5x^{2/3}-18304a^3b^4x+20592a^4b^3\right.}{573440b^7x^3} + \frac{1287a^8\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{16384b^{15/2}}$$

input
```
Integrate[Sqrt[b*x^(2/3) + a*x]/x^4,x]
```

output
```
(Sqrt[b*x^(2/3) + a*x]*(-215040*b^7 - 15360*a*b^6*x^(1/3) + 16640*a^2*b^5*
x^(2/3) - 18304*a^3*b^4*x + 20592*a^4*b^3*x^(4/3) - 24024*a^5*b^2*x^(5/3)
+ 30030*a^6*b*x^2 - 45045*a^7*x^(7/3)))/(573440*b^7*x^3) + (1287*a^8*ArcTa
nh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(16384*b^(15/2))
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1926, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^{2/3}}}{x^4}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{16}a\int \frac{1}{x^3\sqrt{x^{2/3}b+ax}}dx-\frac{3\sqrt{ax+bx^{2/3}}}{8x^3}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{16}a\left(-\frac{13a\int \frac{1}{x^{8/3}\sqrt{x^{2/3}b+ax}}dx}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{8x^3}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{16}a\left(-\frac{13a\left(-\frac{11a\int \frac{1}{x^{7/3}\sqrt{x^{2/3}b+ax}}dx}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{8x^3}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{16}a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\int \frac{1}{x^2\sqrt{x^{2/3}b+ax}}dx}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{8x^3}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{16}a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\int\frac{1}{x^{5/3}\sqrt{x^{2/3}b+ax}}dx}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{8x^3}$$

$\Big\downarrow$ 1931

$$\frac{1}{16}a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\int \frac{1}{x^{4/3}\sqrt{x^{2/3}b+ax}}dx}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax-}}{7bx}\right.$$

$$\frac{3\sqrt{ax+bx^{2/3}}}{8x^{3}}$$

$\downarrow$ 1931

$$\frac{1}{16}a \Bigg(-\frac{13a\Bigg(-\frac{11a\Bigg(-\frac{9a\Bigg(-\frac{7a\Bigg(-\frac{5a\left(-\frac{3a\int\frac{1}{x\sqrt{x^{2/3}b+ax}}dx}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\Bigg)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\Bigg)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\Bigg)}{12b}-\frac{\sqrt{ax+bx^{2/}}}{2bx^{7/3}}}{14b}$$

↓ 1931

$$13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(-\frac{a\int\frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax\cdots}}{5\cdots}\cdots}{12b}\cdots\right.$$

↓ 1935

$$\left(\left(11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\int\frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\right.\right.\right.$$

↓ 219

$$\left(\left(\left(11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-3\right.\right.\right.\right.$$

input
```
Int[Sqrt[b*x^(2/3) + a*x]/x^4,x]
```

output
```
(-3*Sqrt[b*x^(2/3) + a*x])/(8*x^3) + (a*((-3*Sqrt[b*x^(2/3) + a*x])/(7*b*x
^(8/3)) - (13*a*(-1/2*Sqrt[b*x^(2/3) + a*x]/(b*x^(7/3)) - (11*a*((-3*Sqrt[
b*x^(2/3) + a*x])/(5*b*x^2) - (9*a*((-3*Sqrt[b*x^(2/3) + a*x])/(4*b*x^(5/3
)) - (7*a*(-(Sqrt[b*x^(2/3) + a*x]/(b*x^(4/3))) - (5*a*((-3*Sqrt[b*x^(2/3)
 + a*x])/(2*b*x) - (3*a*((-3*Sqrt[b*x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*Arc
Tanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/b^(3/2)))/(4*b)))/(6*b)))/(
8*b)))/(10*b)))/(12*b)))/(14*b)))/16
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.63

method	result
derivativedivides	$-\frac{\sqrt{b x^{\frac{2}{3}}+a x}\,\left(45045\left(x^{\frac{1}{3}} a+b\right)^{\frac{15}{2}} b^{\frac{15}{2}}-345345\left(x^{\frac{1}{3}} a+b\right)^{\frac{13}{2}} b^{\frac{17}{2}}+1150149\left(x^{\frac{1}{3}} a+b\right)^{\frac{11}{2}} b^{\frac{19}{2}}-2167737\left(x^{\frac{1}{3}} a+b\right)^{\frac{9}{2}} b^{\frac{21}{2}}}{}$
default	$-\frac{\sqrt{b x^{\frac{2}{3}}+a x}\,\left(45045\left(x^{\frac{1}{3}} a+b\right)^{\frac{15}{2}} b^{\frac{15}{2}}-345345\left(x^{\frac{1}{3}} a+b\right)^{\frac{13}{2}} b^{\frac{17}{2}}+1150149\left(x^{\frac{1}{3}} a+b\right)^{\frac{11}{2}} b^{\frac{19}{2}}-2167737\left(x^{\frac{1}{3}} a+b\right)^{\frac{9}{2}} b^{\frac{21}{2}}}{}$

input
```
int((b*x^(2/3)+a*x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/573440*(b*x^(2/3)+a*x)^(1/2)*(45045*(x^(1/3)*a+b)^(15/2)*b^(15/2)-34534
5*(x^(1/3)*a+b)^(13/2)*b^(17/2)+1150149*(x^(1/3)*a+b)^(11/2)*b^(19/2)-2167
737*(x^(1/3)*a+b)^(9/2)*b^(21/2)+2518087*(x^(1/3)*a+b)^(7/2)*b^(23/2)-1831
739*(x^(1/3)*a+b)^(5/2)*b^(25/2)+801535*(x^(1/3)*a+b)^(3/2)*b^(27/2)-45045
*arctanh((x^(1/3)*a+b)^(1/2)/b^(1/2))*b^7*a^8*x^(8/3)+45045*(x^(1/3)*a+b)^
(1/2)*b^(29/2))/x^3/(x^(1/3)*a+b)^(1/2)/b^(29/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx = \int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^4}\,dx$$

input
```
integrate((b*x**(2/3)+a*x)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(a*x + b*x**(2/3))/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx = \int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^4}\,dx$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(2/3))/x^4, x)
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx = -\frac{\frac{45045\,a^9\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^7} + \frac{45045\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^9 - 345345\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^9 b + 1150149\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^9 b^2 - 2167737\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^9 b^3 + 25180}{a^8 b^7 x^{\frac{8}{3}}}}{573440\,a}$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x^4,x, algorithm="giac")
```

output
```
-1/573440*(45045*a^9*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7) +
 (45045*(a*x^(1/3) + b)^(15/2)*a^9 - 345345*(a*x^(1/3) + b)^(13/2)*a^9*b +
 1150149*(a*x^(1/3) + b)^(11/2)*a^9*b^2 - 2167737*(a*x^(1/3) + b)^(9/2)*a^
9*b^3 + 2518087*(a*x^(1/3) + b)^(7/2)*a^9*b^4 - 1831739*(a*x^(1/3) + b)^(5
/2)*a^9*b^5 + 801535*(a*x^(1/3) + b)^(3/2)*a^9*b^6 + 45045*sqrt(a*x^(1/3)
+ b)*a^9*b^7)/(a^8*b^7*x^(8/3)))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx=\int \frac{\sqrt{a\,x+b\,x^{2/3}}}{x^4}\,dx$$

input
```
int((a*x + b*x^(2/3))^(1/2)/x^4,x)
```

output
```
int((a*x + b*x^(2/3))^(1/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx=\frac{-48048x^{\frac{5}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,a^5b^3+33280x^{\frac{2}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,a^2b^6-90090x^{\frac{7}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,a^7b+4118}{}$$

input
```
int((b*x^(2/3)+a*x)^(1/2)/x^4,x)
```

output
```
( - 48048*x**(2/3)*sqrt(x**(1/3)*a + b)*a**5*b**3*x + 33280*x**(2/3)*sqrt(
x**(1/3)*a + b)*a**2*b**6 - 90090*x**(1/3)*sqrt(x**(1/3)*a + b)*a**7*b*x**
2 + 41184*x**(1/3)*sqrt(x**(1/3)*a + b)*a**4*b**4*x - 30720*x**(1/3)*sqrt(
x**(1/3)*a + b)*a*b**7 + 60060*sqrt(x**(1/3)*a + b)*a**6*b**2*x**2 - 36608
*sqrt(x**(1/3)*a + b)*a**3*b**5*x - 430080*sqrt(x**(1/3)*a + b)*b**8 - 450
45*x**(2/3)*sqrt(b)*log(sqrt(x**(1/3)*a + b) - sqrt(b))*a**8*x**2 + 45045*
x**(2/3)*sqrt(b)*log(sqrt(x**(1/3)*a + b) + sqrt(b))*a**8*x**2)/(1146880*x
**(2/3)*b**8*x**2)
```

3.154 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx$

Optimal result 1430
Mathematica [A] (verified) 1431
Rubi [A] (verified) 1431
Maple [A] (verified) 1449
Fricas [F(-1)] 1449
Sympy [F] 1450
Maxima [F] 1450
Giac [A] (verification not implemented) 1450
Mupad [F(-1)] 1451
Reduce [B] (verification not implemented) 1451

Optimal result

Integrand size = 19, antiderivative size = 354

$$\begin{aligned}\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx = &-\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}\\&-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx^{2/3}+ax}}{19712b^4x^{8/3}}-\frac{4199a^5\sqrt{bx^{2/3}+ax}}{236544b^5x^{7/3}}\\&+\frac{4199a^6\sqrt{bx^{2/3}+ax}}{215040b^6x^2}-\frac{12597a^7\sqrt{bx^{2/3}+ax}}{573440b^7x^{5/3}}+\frac{4199a^8\sqrt{bx^{2/3}+ax}}{163840b^8x^{4/3}}\\&-\frac{4199a^9\sqrt{bx^{2/3}+ax}}{131072b^9x}+\frac{12597a^{10}\sqrt{bx^{2/3}+ax}}{262144b^{10}x^{2/3}}-\frac{12597a^{11}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{262144b^{21/2}}\end{aligned}$$

output
```
-3/11*(b*x^(2/3)+a*x)^(1/2)/x^4-3/220*a*(b*x^(2/3)+a*x)^(1/2)/b/x^(11/3)+1
9/1320*a^2*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(10/3)-323/21120*a^3*(b*x^(2/3)+a*x
)^(1/2)/b^3/x^3+323/19712*a^4*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(8/3)-4199/23654
4*a^5*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(7/3)+4199/215040*a^6*(b*x^(2/3)+a*x)^(1
/2)/b^6/x^2-12597/573440*a^7*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(5/3)+4199/163840
*a^8*(b*x^(2/3)+a*x)^(1/2)/b^8/x^(4/3)-4199/131072*a^9*(b*x^(2/3)+a*x)^(1/
2)/b^9/x+12597/262144*a^10*(b*x^(2/3)+a*x)^(1/2)/b^10/x^(2/3)-12597/262144
*a^11*arctanh(b^(1/2)*x^(1/3)/(b*x^(2/3)+a*x)^(1/2))/b^(21/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx = \frac{\sqrt{bx^{2/3}+ax}\left(-82575360b^{10}-4128768ab^9\sqrt[3]{x}+4358144a^2b^8x^{2/3}-4630528a^3b^7x+\right.}{}$$
$$-\frac{12597a^{11}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{262144b^{21/2}}$$

input
```
Integrate[Sqrt[b*x^(2/3) + a*x]/x^5,x]
```

output
```
(Sqrt[b*x^(2/3) + a*x]*(-82575360*b^10 - 4128768*a*b^9*x^(1/3) + 4358144*a
^2*b^8*x^(2/3) - 4630528*a^3*b^7*x + 4961280*a^4*b^6*x^(4/3) - 5374720*a^5
*b^5*x^(5/3) + 5912192*a^6*b^4*x^2 - 6651216*a^7*b^3*x^(7/3) + 7759752*a^8
*b^2*x^(8/3) - 9699690*a^9*b*x^3 + 14549535*a^10*x^(10/3)))/(302776320*b^1
0*x^4) - (12597*a^11*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(26
2144*b^(21/2))
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.14, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {1926, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^{2/3}}}{x^5}\,dx$$

$$\Big\downarrow \text{ 1926}$$

$$\frac{1}{22}a\int \frac{1}{x^4\sqrt{x^{2/3}b+ax}}dx-\frac{3\sqrt{ax+bx^{2/3}}}{11x^4}$$

$$\Big\downarrow \text{ 1931}$$

$$\frac{1}{22}a\left(-\frac{19a\int\frac{1}{x^{11/3}\sqrt{x^{2/3}b+ax}}dx}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{11x^4}$$

$\downarrow$ 1931

$$\frac{1}{22}a\left(-\frac{19a\left(-\frac{17a\int\frac{1}{x^{10/3}\sqrt{x^{2/3}b+ax}}dx}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{11x^4}$$

$\downarrow$ 1931

$$\frac{1}{22}a\left(-\frac{19a\left(-\frac{17a\left(-\frac{15a\int\frac{1}{x^3\sqrt{x^{2/3}b+ax}}dx}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{11x^4}$$

$\downarrow$ 1931

$$\frac{1}{22} a \left(-\frac{19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\int \frac{1}{x^{8/3}\sqrt{x^{2/3}b+ax}}dx}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b} - \frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}\right) - \frac{3\sqrt{ax+bx^{2/3}}}{11x^4}$$

$$\Big\downarrow \; 1931$$

$$\frac{1}{22}a\left(-\frac{19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\int\frac{1}{x^{7/3}\sqrt{x^{2/3}b+ax}}dx}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^{3}}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{a}}{10}\right.$$

$$\frac{3\sqrt{ax+bx^{2/3}}}{11x^4}$$

$\downarrow$ 1931

$$\frac{1}{22}a\left(-\frac{19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\int\frac{1}{x^2\sqrt{x^{2/3}b+ax}}dx}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+\cdots}}{3bx^{1\cdots}}\cdots\right)}{20b}\cdots\right.$$

↓ 1931

$$19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\int\frac{1}{x^{5/3}\sqrt{x^{2/3}b+ax}}\,dx}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\cdots}{\cdots}\right)}{18b}\cdots\right.$$

↓ 1931

$$\left(\left(\left(17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\int\frac{1}{x^{4/3}\sqrt{x^{2/3}b+ax}}dx}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\cdots\right)}{14b}\cdots\right)}{16b}\cdots\right.\right.\right.\right.$$

↓ 1931

$$\left(\left(\left(\left(\left(15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\int\frac{1}{x\sqrt{x^{2/3}b+ax}}dx}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2}}}{5bx^{2}}\right.}{12b}\right.}{14b}\right.\right.\right.\right.\right.\right.$$

↓ 1931

$$\Bigg(\Bigg(\Bigg(\Bigg(\Bigg(13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(-\frac{a\int\frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}\,dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-3\cdots}{10b}\cdots}{12b}\cdots$$

↓ 1935

$$5a\left(-\frac{3a\left(\frac{3a\int\frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)$$

$$7a\left(-\frac{\cdots}{6b}-\frac{\sqrt{ax+bx^{2/}}}{bx^{4/3}}\right.$$

$$9a\left(-\frac{\cdots}{8b}\right.$$

$$11a\left(-\frac{\cdots}{10b}\right.$$

↓ 219

$$\Bigg(\Bigg(\Bigg(\Bigg(\Bigg(11a\Bigg(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}\quad\cdots}{10b}\quad\cdots$$

input
```
Int[Sqrt[b*x^(2/3) + a*x]/x^5,x]
```

output
```
(-3*Sqrt[b*x^(2/3) + a*x])/(11*x^4) + (a*((-3*Sqrt[b*x^(2/3) + a*x])/(10*b
*x^(11/3)) - (19*a*(-1/3*Sqrt[b*x^(2/3) + a*x]/(b*x^(10/3)) - (17*a*((-3*S
qrt[b*x^(2/3) + a*x])/(8*b*x^3) - (15*a*((-3*Sqrt[b*x^(2/3) + a*x])/(7*b*x
^(8/3)) - (13*a*(-1/2*Sqrt[b*x^(2/3) + a*x]/(b*x^(7/3)) - (11*a*((-3*Sqrt[
b*x^(2/3) + a*x])/(5*b*x^2) - (9*a*((-3*Sqrt[b*x^(2/3) + a*x])/(4*b*x^(5/3
)) - (7*a*(-(Sqrt[b*x^(2/3) + a*x]/(b*x^(4/3))) - (5*a*((-3*Sqrt[b*x^(2/3)
 + a*x])/(2*b*x) - (3*a*((-3*Sqrt[b*x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*Arc
Tanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/b^(3/2)))/(4*b)))/(6*b)))/(
8*b)))/(10*b)))/(12*b)))/(14*b)))/(16*b)))/(18*b)))/(20*b)))/22
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.59

method	result
derivativedivides	$-\frac{\sqrt{b\,x^{\frac{2}{3}}+ax}\left(-14549535b^{\frac{21}{2}}\left(x^{\frac{1}{3}}a+b\right)^{\frac{21}{2}}+155195040b^{\frac{23}{2}}\left(x^{\frac{1}{3}}a+b\right)^{\frac{19}{2}}-749786037b^{\frac{25}{2}}\left(x^{\frac{1}{3}}a+b\right)^{\frac{17}{2}}+2163862272b\right.}{}$
default	$-\frac{\sqrt{b\,x^{\frac{2}{3}}+ax}\left(-14549535b^{\frac{21}{2}}\left(x^{\frac{1}{3}}a+b\right)^{\frac{21}{2}}+155195040b^{\frac{23}{2}}\left(x^{\frac{1}{3}}a+b\right)^{\frac{19}{2}}-749786037b^{\frac{25}{2}}\left(x^{\frac{1}{3}}a+b\right)^{\frac{17}{2}}+2163862272b\right.}{}$

input
```
int((b*x^(2/3)+a*x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/302776320*(b*x^(2/3)+a*x)^(1/2)*(-14549535*b^(21/2)*(x^(1/3)*a+b)^(21/2
)+155195040*b^(23/2)*(x^(1/3)*a+b)^(19/2)-749786037*b^(25/2)*(x^(1/3)*a+b)
^(17/2)+2163862272*b^(27/2)*(x^(1/3)*a+b)^(15/2)-4139920070*b^(29/2)*(x^(1
/3)*a+b)^(13/2)+5503713280*b^(31/2)*(x^(1/3)*a+b)^(11/2)-5174056250*b^(33/
2)*(x^(1/3)*a+b)^(9/2)+3424523520*b^(35/2)*(x^(1/3)*a+b)^(7/2)-1551313995*
b^(37/2)*(x^(1/3)*a+b)^(5/2)+14549535*arctanh((x^(1/3)*a+b)^(1/2)/b^(1/2))
*b^10*a^11*x^(11/3)+450357600*b^(39/2)*(x^(1/3)*a+b)^(3/2)+14549535*b^(41/
2)*(x^(1/3)*a+b)^(1/2))/x^4/(x^(1/3)*a+b)^(1/2)/b^(41/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx = \int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^5}\,dx$$

input
```
integrate((b*x**(2/3)+a*x)**(1/2)/x**5,x)
```

output
```
Integral(sqrt(a*x + b*x**(2/3))/x**5, x)
```

Maxima [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx = \int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^5}\,dx$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(2/3))/x^5, x)
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx = \frac{1}{302776320}\,a^{11}\left(\frac{14549535\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^{10}} + \frac{14549535\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}} - 155195040\right.}{}\right.$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x^5,x, algorithm="giac")
```

output
```
1/302776320*a^11*(14549535*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*
b^10) + (14549535*(a*x^(1/3) + b)^(21/2) - 155195040*(a*x^(1/3) + b)^(19/2
)*b + 749786037*(a*x^(1/3) + b)^(17/2)*b^2 - 2163862272*(a*x^(1/3) + b)^(1
5/2)*b^3 + 4139920070*(a*x^(1/3) + b)^(13/2)*b^4 - 5503713280*(a*x^(1/3) +
 b)^(11/2)*b^5 + 5174056250*(a*x^(1/3) + b)^(9/2)*b^6 - 3424523520*(a*x^(1
/3) + b)^(7/2)*b^7 + 1551313995*(a*x^(1/3) + b)^(5/2)*b^8 - 450357600*(a*x
^(1/3) + b)^(3/2)*b^9 - 14549535*sqrt(a*x^(1/3) + b)*b^10)/(a^11*b^10*x^(1
1/3)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx = \int \frac{\sqrt{a\,x+b\,x^{2/3}}}{x^5}\,dx$$

input
```
int((a*x + b*x^(2/3))^(1/2)/x^5,x)
```

output
```
int((a*x + b*x^(2/3))^(1/2)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx = \frac{15519504x^{\frac{8}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,a^8b^3 - 10749440x^{\frac{5}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,a^5b^6 + 8716288x^{\frac{2}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,a^2b}{}$$

input
```
int((b*x^(2/3)+a*x)^(1/2)/x^5,x)
```

output

```
(15519504*x**(2/3)*sqrt(x**(1/3)*a + b)*a**8*b**3*x**2 - 10749440*x**(2/3)
*sqrt(x**(1/3)*a + b)*a**5*b**6*x + 8716288*x**(2/3)*sqrt(x**(1/3)*a + b)*
a**2*b**9 + 29099070*x**(1/3)*sqrt(x**(1/3)*a + b)*a**10*b*x**3 - 13302432
*x**(1/3)*sqrt(x**(1/3)*a + b)*a**7*b**4*x**2 + 9922560*x**(1/3)*sqrt(x**(
1/3)*a + b)*a**4*b**7*x - 8257536*x**(1/3)*sqrt(x**(1/3)*a + b)*a*b**10 -
19399380*sqrt(x**(1/3)*a + b)*a**9*b**2*x**3 + 11824384*sqrt(x**(1/3)*a +
b)*a**6*b**5*x**2 - 9261056*sqrt(x**(1/3)*a + b)*a**3*b**8*x - 165150720*s
qrt(x**(1/3)*a + b)*b**11 + 14549535*x**(2/3)*sqrt(b)*log(sqrt(x**(1/3)*a
+ b) - sqrt(b))*a**11*x**3 - 14549535*x**(2/3)*sqrt(b)*log(sqrt(x**(1/3)*a
 + b) + sqrt(b))*a**11*x**3)/(605552640*x**(2/3)*b**11*x**3)
```

3.155 $\int x^2 \left(bx^{2/3}+ax\right)^{3/2} dx$

Optimal result . . . 1453
Mathematica [A] (verified) . . . 1454
Rubi [A] (verified) . . . 1454
Maple [A] (verified) . . . 1470
Fricas [B] (verification not implemented) . . . 1470
Sympy [F] . . . 1471
Maxima [F] . . . 1472
Giac [B] (verification not implemented) . . . 1472
Mupad [F(-1)] . . . 1473
Reduce [B] (verification not implemented) . . . 1474

Optimal result

Integrand size = 19, antiderivative size = 343

$$\begin{aligned}\int x^2 \left(bx^{2/3}+ax\right)^{3/2} dx &= \frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7} \\ &- \frac{1048576b^{11}\left(bx^{2/3}+ax\right)^{5/2}}{152108775a^{12}x^{5/3}} + \frac{524288b^{10}\left(bx^{2/3}+ax\right)^{5/2}}{30421755a^{11}x^{4/3}} \\ &- \frac{131072b^9\left(bx^{2/3}+ax\right)^{5/2}}{4345965a^{10}x} + \frac{65536b^8\left(bx^{2/3}+ax\right)^{5/2}}{1448655a^9x^{2/3}} \\ &- \frac{90112b^7\left(bx^{2/3}+ax\right)^{5/2}}{1448655a^8\sqrt[3]{x}} - \frac{11264b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{111435a^6} \\ &+ \frac{5632b^4x^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{45885a^5} - \frac{352b^3x\left(bx^{2/3}+ax\right)^{5/2}}{2415a^4} \\ &+ \frac{176b^2x^{4/3}\left(bx^{2/3}+ax\right)^{5/2}}{1035a^3} - \frac{44bx^{5/3}\left(bx^{2/3}+ax\right)^{5/2}}{225a^2} + \frac{2x^2\left(bx^{2/3}+ax\right)^{5/2}}{9a}\end{aligned}$$

output
```
45056/557175*b^6*(b*x^(2/3)+a*x)^(5/2)/a^7-1048576/152108775*b^11*(b*x^(2/
3)+a*x)^(5/2)/a^12/x^(5/3)+524288/30421755*b^10*(b*x^(2/3)+a*x)^(5/2)/a^11
/x^(4/3)-131072/4345965*b^9*(b*x^(2/3)+a*x)^(5/2)/a^10/x+65536/1448655*b^8
*(b*x^(2/3)+a*x)^(5/2)/a^9/x^(2/3)-90112/1448655*b^7*(b*x^(2/3)+a*x)^(5/2)
/a^8/x^(1/3)-11264/111435*b^5*x^(1/3)*(b*x^(2/3)+a*x)^(5/2)/a^6+5632/45885
*b^4*x^(2/3)*(b*x^(2/3)+a*x)^(5/2)/a^5-352/2415*b^3*x*(b*x^(2/3)+a*x)^(5/2
)/a^4+176/1035*b^2*x^(4/3)*(b*x^(2/3)+a*x)^(5/2)/a^3-44/225*b*x^(5/3)*(b*x
^(2/3)+a*x)^(5/2)/a^2+2/9*x^2*(b*x^(2/3)+a*x)^(5/2)/a
```

Mathematica [A] (verified)

Time = 5.25 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.49

$$\int x^2 \left(bx^{2/3}+ax\right)^{3/2} dx = \frac{2\left(b+a\sqrt[3]{x}\right)\left(bx^{2/3}+ax\right)^{3/2}\left(-524288b^{11}+1310720ab^{10}\sqrt[3]{x}-2293760a^2b^9x^{2/3}+3440640a^3 \cdots\right.}{\cdots}$$

input
```
Integrate[x^2*(b*x^(2/3) + a*x)^(3/2),x]
```

output
```
(2*(b + a*x^(1/3))*(b*x^(2/3) + a*x)^(3/2)*(-524288*b^11 + 1310720*a*b^10*
x^(1/3) - 2293760*a^2*b^9*x^(2/3) + 3440640*a^3*b^8*x - 4730880*a^4*b^7*x^
(4/3) + 6150144*a^5*b^6*x^(5/3) - 7687680*a^6*b^5*x^2 + 9335040*a^7*b^4*x^
(7/3) - 11085360*a^8*b^3*x^(8/3) + 12932920*a^9*b^2*x^3 - 14872858*a^10*b*
x^(10/3) + 16900975*a^11*x^(11/3)))/(152108775*a^12*x)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.17, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {1922, 1922, 1922, 1922, 1922, 1922, 1908, 1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(ax+bx^{2/3}\right)^{3/2} dx$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{5/2}}{9a}-\frac{22b\int x^{5/3}\left(x^{2/3}b+ax\right)^{3/2}dx}{27a}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{5/2}}{9a}-\frac{22b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{5/2}}{25a}-\frac{4b\int x^{4/3}\left(x^{2/3}b+ax\right)^{3/2}dx}{5a}\right)}{27a}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{5/2}}{9a}-\frac{22b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{5/2}}{25a}-\frac{4b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{5/2}}{23a}-\frac{18b\int x\left(x^{2/3}b+ax\right)^{3/2}dx}{23a}\right)}{5a}\right)}{27a}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{5/2}}{9a}-\frac{22b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{5/2}}{25a}-\frac{4b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{5/2}}{23a}-\frac{18b\left(\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}-\frac{16b\int x^{2/3}\left(x^{2/3}b+ax\right)^{3/2}dx}{21a}\right)}{23a}\right)}{5a}\right)}{27a}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{5/2}}{9a}-\frac{22b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{5/2}}{25a}-\frac{4b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{5/2}}{23a}-\frac{18b\left(\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}-\frac{16b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{19a}-\frac{14b\int \sqrt[3]{x}\left(x^{2/3}b+ax\right)^{3/2}dx}{19a}\right)}{21a}\right)}{23a}\right)}{5a}\right)}{27a}$$

$\downarrow$ 1922

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{5/2}}{9a}-$$

$$\frac{22b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{5/2}}{25a}-\frac{4b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{5/2}}{23a}-\frac{18b\left(\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}-\frac{16b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{19a}-\frac{14b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a}-\frac{12b\int\left(\right.}{}\right.}{19a}\right.}{21a}\right.}{23a}\right.}{5a}\right.}{27a}$$

↓ 1908

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{5/2}}{9a} -$$

$$4b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{5/2}}{23a} - \frac{18b\left(\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a} - \frac{16b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{19a} - \frac{14b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a} - \frac{12b\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{21a}\right.}{23a}\right.$$

↓ 1922

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{5/2}}{9a}-$$

$$12b\left(\right.$$

$$14b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a}-\right.$$

$$16b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{19a}-\right.$$

$$18b\left(\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}-\right.$$

$$4b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{5/2}}{23a}-\frac{\cdots}{23a}\right.$$

↓ 1922

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{5/2}}{9a} -$$

$$12b$$

$$14b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a} - \right.$$

$$16b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{19a} - \right.$$

$$18b\left(\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a} - \right.$$

↓ 1922

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{5/2}}{9a} -$$

$$12b$$

$$14b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a} -\right.$$

$$16b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{19a} -\right.$$

↓ 1922

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{5/2}}{9a} - \Bigg(\Bigg(\Bigg(\Bigg(\Bigg(\Bigg($$

$$12b \Bigg|$$

$$14b \left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a} - \right.$$

$\downarrow$ 1920

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{5/2}}{9a} -$$

$12b$

$14b \quad \frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a} -$

input
```
Int[x^2*(b*x^(2/3) + a*x)^(3/2),x]
```

output
```
(2*x^2*(b*x^(2/3) + a*x)^(5/2))/(9*a) - (22*b*((6*x^(5/3)*(b*x^(2/3) + a*x
)^(5/2))/(25*a) - (4*b*((6*x^(4/3)*(b*x^(2/3) + a*x)^(5/2))/(23*a) - (18*b
*((2*x*(b*x^(2/3) + a*x)^(5/2))/(7*a) - (16*b*((6*x^(2/3)*(b*x^(2/3) + a*x
)^(5/2))/(19*a) - (14*b*((6*x^(1/3)*(b*x^(2/3) + a*x)^(5/2))/(17*a) - (12*
b*((2*(b*x^(2/3) + a*x)^(5/2))/(5*a) - (2*b*((6*(b*x^(2/3) + a*x)^(5/2))/(
13*a*x^(1/3)) - (8*b*((6*(b*x^(2/3) + a*x)^(5/2))/(11*a*x^(2/3)) - (6*b*((
2*(b*x^(2/3) + a*x)^(5/2))/(3*a*x) - (4*b*((-12*b*(b*x^(2/3) + a*x)^(5/2))
/(35*a^2*x^(5/3)) + (6*(b*x^(2/3) + a*x)^(5/2))/(7*a*x^(4/3))))/(9*a)))/(1
1*a)))/(13*a)))/(3*a)))/(17*a)))/(19*a)))/(21*a)))/(23*a)))/(5*a)))/(27*a)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.42

method	result
derivativedivides	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(x^{\frac{1}{3}}a+b\right)\left(16900975a^{11}x^{\frac{11}{3}}-14872858a^{10}b\,x^{\frac{10}{3}}+12932920a^{9}b^{2}x^{3}-11085360a^{8}b^{3}x^{\frac{8}{3}}+9335040a^{7}b^{4}x\right.}{152108}$
default	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(x^{\frac{1}{3}}a+b\right)\left(16900975a^{11}x^{\frac{11}{3}}-14872858a^{10}b\,x^{\frac{10}{3}}+12932920a^{9}b^{2}x^{3}-11085360a^{8}b^{3}x^{\frac{8}{3}}+9335040a^{7}b^{4}x\right.}{152108}$

input
```
int(x^2*(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/152108775*(b*x^(2/3)+a*x)^(3/2)*(x^(1/3)*a+b)*(16900975*a^11*x^(11/3)-14
872858*a^10*b*x^(10/3)+12932920*a^9*b^2*x^3-11085360*a^8*b^3*x^(8/3)+93350
40*a^7*b^4*x^(7/3)-7687680*a^6*b^5*x^2+6150144*a^5*b^6*x^(5/3)-4730880*a^4
*b^7*x^(4/3)+3440640*a^3*b^8*x-2293760*a^2*b^9*x^(2/3)+1310720*a*b^10*x^(1
/3)-524288*b^11)/x/a^12
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1293 vs. 2(255) = 510.

Time = 132.79 (sec) , antiderivative size = 1293, normalized size of antiderivative = 3.77

$$\int x^2\left(bx^{2/3}+ax\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(x^2*(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
2/152108775*((6597069766656*b^19 + 1374389534720*b^18 + 6442450944*(64*a^3
 - 3)*b^16 - 128849018880*b^17 - 33554432*(11264*a^3 - 53)*b^15 + 98431278
400*a^15 - 12582912*(5504*a^3 + 1)*b^14 + 393216*(3194880*a^6 - 114688*a^3
 - 3)*b^13 + 14680064*(18816*a^6 + 103*a^3)*b^12 - 1572864*(48816*a^6 + 23
*a^3)*b^11 - 24576*(45731840*a^9 - 495872*a^6 - 15*a^3)*b^10 - 229376*(134
9120*a^9 + 3439*a^6)*b^9 + 7827456*(5600*a^9 + 3*a^6)*b^8 - 384*(620420562
944*a^12 + 21542400*a^9 + 693*a^6)*b^7 - 6656*(7444688384*a^12 - 89111*a^9
)*b^6 + 19968*(232361024*a^12 - 935*a^9)*b^5 - 1326*(173210075136*a^15 - 5
33564416*a^12 - 165*a^9)*b^4 - 1881152*(45121536*a^15 + 34547*a^12)*b^3 -
352716*(19243008*a^15 - 1339*a^12)*b^2 + 2028117*(237568*a^15 + 21*a^12)*b
)*x + (16900975*(16777216*a^13*b^6 + 6291456*a^13*b^5 + 196608*a^13*b^4 -
262144*a^16 - 114688*a^13*b^3 - 2304*a^13*b^2 + 864*a^13*b - 27*a^13)*x^5
- 92378*(16777216*a^10*b^9 + 6291456*a^10*b^8 + 196608*a^10*b^7 - 114688*a
^10*b^6 - 2304*a^10*b^5 + 864*a^10*b^4 - (262144*a^13 + 27*a^10)*b^3)*x^4
+ 109824*(16777216*a^7*b^12 + 6291456*a^7*b^11 + 196608*a^7*b^10 - 114688*
a^7*b^9 - 2304*a^7*b^8 + 864*a^7*b^7 - (262144*a^10 + 27*a^7)*b^6)*x^3 - 1
43360*(16777216*a^4*b^15 + 6291456*a^4*b^14 + 196608*a^4*b^13 - 114688*a^4
*b^12 - 2304*a^4*b^11 + 864*a^4*b^10 - (262144*a^7 + 27*a^4)*b^9)*x^2 + 26
2144*(16777216*a*b^18 + 6291456*a*b^17 + 196608*a*b^16 - 114688*a*b^15 - 2
304*a*b^14 + 864*a*b^13 - (262144*a^4 + 27*a)*b^12)*x - 4*(219902325555...
```

Sympy [F]

$$\int x^2 \left(bx^{2/3}+ax\right)^{3/2} dx = \int x^2 \left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}} dx$$

input
```
integrate(x**2*(b*x**(2/3)+a*x)**(3/2),x)
```

output
```
Integral(x**2*(a*x + b*x**(2/3))**(3/2), x)
```

Maxima [F]

$$\int x^2 \left(bx^{2/3} + ax\right)^{3/2} \, dx = \int \left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(2/3))^(3/2)*x^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(255) = 510$.

Time = 0.31 (sec) , antiderivative size = 770, normalized size of antiderivative = 2.24

$$\int x^2 \left(bx^{2/3} + ax\right)^{3/2} \, dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
2/16900975*b*(524288*b^(25/2)/a^12 + (25*(88179*(a*x^(1/3) + b)^(23/2) - 1
062347*(a*x^(1/3) + b)^(21/2)*b + 5870865*(a*x^(1/3) + b)^(19/2)*b^2 - 196
84665*(a*x^(1/3) + b)^(17/2)*b^3 + 44618574*(a*x^(1/3) + b)^(15/2)*b^4 - 7
2076158*(a*x^(1/3) + b)^(13/2)*b^5 + 85180914*(a*x^(1/3) + b)^(11/2)*b^6 -
 74364290*(a*x^(1/3) + b)^(9/2)*b^7 + 47805615*(a*x^(1/3) + b)^(7/2)*b^8 -
 22309287*(a*x^(1/3) + b)^(5/2)*b^9 + 7436429*(a*x^(1/3) + b)^(3/2)*b^10 -
 2028117*sqrt(a*x^(1/3) + b)*b^11)*b/a^11 + 3*(676039*(a*x^(1/3) + b)^(25/
2) - 8817900*(a*x^(1/3) + b)^(23/2)*b + 53117350*(a*x^(1/3) + b)^(21/2)*b^
2 - 195695500*(a*x^(1/3) + b)^(19/2)*b^3 + 492116625*(a*x^(1/3) + b)^(17/2
)*b^4 - 892371480*(a*x^(1/3) + b)^(15/2)*b^5 + 1201269300*(a*x^(1/3) + b)^
(13/2)*b^6 - 1216870200*(a*x^(1/3) + b)^(11/2)*b^7 + 929553625*(a*x^(1/3)
+ b)^(9/2)*b^8 - 531173500*(a*x^(1/3) + b)^(7/2)*b^9 + 223092870*(a*x^(1/3
) + b)^(5/2)*b^10 - 67603900*(a*x^(1/3) + b)^(3/2)*b^11 + 16900975*sqrt(a*
x^(1/3) + b)*b^12)/a^11)/a) - 2/152108775*a*(4194304*b^(27/2)/a^13 - (27*(
676039*(a*x^(1/3) + b)^(25/2) - 8817900*(a*x^(1/3) + b)^(23/2)*b + 5311735
0*(a*x^(1/3) + b)^(21/2)*b^2 - 195695500*(a*x^(1/3) + b)^(19/2)*b^3 + 4921
16625*(a*x^(1/3) + b)^(17/2)*b^4 - 892371480*(a*x^(1/3) + b)^(15/2)*b^5 +
1201269300*(a*x^(1/3) + b)^(13/2)*b^6 - 1216870200*(a*x^(1/3) + b)^(11/2)*
b^7 + 929553625*(a*x^(1/3) + b)^(9/2)*b^8 - 531173500*(a*x^(1/3) + b)^(7/2
)*b^9 + 223092870*(a*x^(1/3) + b)^(5/2)*b^10 - 67603900*(a*x^(1/3) + b)...
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(bx^{2/3}+ax\right)^{3/2} dx = \int x^2 \left(a\,x+b\,x^{2/3}\right)^{3/2} dx$$

input
```
int(x^2*(a*x + b*x^(2/3))^(3/2),x)
```

output
```
int(x^2*(a*x + b*x^(2/3))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.45

$$\int x^2 \left(bx^{2/3} + ax\right)^{3/2} dx = \frac{2\sqrt{x^{\frac{1}{3}}a + b}\left(88179x^{\frac{11}{3}}a^{11}b^2 - 102960x^{\frac{8}{3}}a^8b^5 + 129024x^{\frac{5}{3}}a^5b^8 - 196608x^{\frac{2}{3}}a^2b^{11} + 16900975x\right.}{}$$

input
```
int(x^2*(b*x^(2/3)+a*x)^(3/2),x)
```

output
```
(2*sqrt(x**(1/3)*a + b)*(88179*x**(2/3)*a**11*b**2*x**3 - 102960*x**(2/3)*
a**8*b**5*x**2 + 129024*x**(2/3)*a**5*b**8*x - 196608*x**(2/3)*a**2*b**11
+ 16900975*x**(1/3)*a**13*x**4 - 92378*x**(1/3)*a**10*b**3*x**3 + 109824*x
**(1/3)*a**7*b**6*x**2 - 143360*x**(1/3)*a**4*b**9*x + 262144*x**(1/3)*a*b
**12 + 18929092*a**12*b*x**4 + 97240*a**9*b**4*x**3 - 118272*a**6*b**7*x**
2 + 163840*a**3*b**10*x - 524288*b**13))/(152108775*a**12)
```

3.156 $\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx$

Optimal result 1475
Mathematica [A] (verified) 1476
Rubi [A] (verified) 1476
Maple [A] (verified) 1486
Fricas [B] (verification not implemented) 1486
Sympy [F] 1487
Maxima [F] 1488
Giac [B] (verification not implemented) 1488
Mupad [F(-1)] 1489
Reduce [B] (verification not implemented) 1490

Optimal result

Integrand size = 17, antiderivative size = 255

$$\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx=-\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}+\frac{65536b^8\left(bx^{2/3}+ax\right)^{5/2}}{4849845a^9x^{5/3}}-\frac{32768b^7\left(bx^{2/3}+ax\right)^{5/2}}{969969a^8x^{4/3}}+\frac{8192b^6\left(bx^{2/3}+ax\right)^{5/2}}{138567a^7x}-\frac{4096b^5\left(bx^{2/3}+ax\right)^{5/2}}{46189a^6x^{2/3}}+\frac{512b^4\left(bx^{2/3}+ax\right)^{5/2}}{4199a^5\sqrt[3]{x}}+\frac{64b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{323a^3}-\frac{32bx^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{133a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{5/2}}{7a}$$

output

```
-256/1615*b^3*(b*x^(2/3)+a*x)^(5/2)/a^4+65536/4849845*b^8*(b*x^(2/3)+a*x)^
(5/2)/a^9/x^(5/3)-32768/969969*b^7*(b*x^(2/3)+a*x)^(5/2)/a^8/x^(4/3)+8192/
138567*b^6*(b*x^(2/3)+a*x)^(5/2)/a^7/x-4096/46189*b^5*(b*x^(2/3)+a*x)^(5/2
)/a^6/x^(2/3)+512/4199*b^4*(b*x^(2/3)+a*x)^(5/2)/a^5/x^(1/3)+64/323*b^2*x^
(1/3)*(b*x^(2/3)+a*x)^(5/2)/a^3-32/133*b*x^(2/3)*(b*x^(2/3)+a*x)^(5/2)/a^2
+2/7*x*(b*x^(2/3)+a*x)^(5/2)/a
```

Mathematica [A] (verified)

Time = 5.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.51

$$\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx=\frac{2\left(b+a\sqrt[3]{x}\right)\left(bx^{2/3}+ax\right)^{3/2}\left(32768b^8-81920ab^7\sqrt[3]{x}+143360a^2b^6x^{2/3}-215040a^3b^5x+295\cdots\right.}{4849845a^9x}$$

input
```
Integrate[x*(b*x^(2/3) + a*x)^(3/2),x]
```

output
```
(2*(b + a*x^(1/3))*(b*x^(2/3) + a*x)^(3/2)*(32768*b^8 - 81920*a*b^7*x^(1/3
) + 143360*a^2*b^6*x^(2/3) - 215040*a^3*b^5*x + 295680*a^4*b^4*x^(4/3) - 3
84384*a^5*b^3*x^(5/3) + 480480*a^6*b^2*x^2 - 583440*a^7*b*x^(7/3) + 692835
*a^8*x^(8/3)))/(4849845*a^9*x)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1922, 1922, 1922, 1908, 1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax+bx^{2/3}\right)^{3/2}dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}-\frac{16b\int x^{2/3}\left(x^{2/3}b+ax\right)^{3/2}dx}{21a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}-\frac{16b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{19a}-\frac{14b\int\sqrt[3]{x}\left(x^{2/3}b+ax\right)^{3/2}dx}{19a}\right)}{21a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}-\frac{16b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{19a}-\frac{14b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a}-\frac{12b\int\left(x^{2/3}b+ax\right)^{3/2}dx}{17a}\right)}{19a}\right)}{21a}$$

$\downarrow$ 1908

$$\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}-\frac{16b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{19a}-\frac{14b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a}-\frac{12b\left(\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a}-\frac{2b\int\frac{\left(x^{2/3}b+ax\right)^{3/2}}{\sqrt[3]{x}}dx}{3a}\right)}{17a}\right)}{19a}\right)}{21a}$$

$\downarrow$ 1922

$$\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}-\frac{16b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{19a}-\frac{14b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a}-\frac{12b\left(\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a}-\frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{13a\sqrt[3]{x}}-\frac{8b\int\frac{\left(x^{2/3}b+ax\right)^{3/2}}{x^{2/3}}dx}{13a}\right)}{3a}\right)}{17a}\right)}{19a}\right)}{21a}$$

$\downarrow$ 1922

$$\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}-\frac{16b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{19a}-\frac{14b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a}-\frac{12b\left(\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a}-\frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{13a\sqrt[3]{x}}-\frac{8b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{11ax^{2/3}}-\frac{6b\int\frac{\left(x^{2/3}b+ax\right)^{3}}{x}}{11a}\right.}{13a}\right.}{3a}\right.}{17a}\right.}{19a}\right.}{21a}$$

↓ 1922

$$\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}-$$

$$16b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{19a}-\frac{14b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a}-\frac{12b\left(\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a}-\frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{13a\sqrt[3]{x}}-\frac{8b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{11ax^{2/3}}-\frac{6b\left(\frac{2\left(ax+bx^{2/3}\right)}{3ax}\right.}{}\right.}{13a}\right.}{3a}\right.}{17a}\right.}{19a}\right.$$

↓ 1922

$$\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}-$$

$$6b\left(\frac{2\left(ax+bx^{2/3}\right)}{3ax}\right.$$

$$8b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{11ax^{2/3}}-\right.$$

$$2b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{13a\sqrt[3]{x}}-\right.$$

$$12b\left(\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a}-\frac{\quad}{3a}\right.$$

$$14b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a}-\frac{\quad}{17a}\right.$$

↓ 1920

$$\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a} -$$

$$14b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a} - \frac{12b\left(\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a} - \frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{13a\sqrt[3]{x}} - \frac{8b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{11ax^{2/3}} - \frac{6b\left(\frac{2\left(ax+bx^{2/3}\right)}{3ax}\right.}{}\right.}{}\right.}{3a}\right.}{17a}\right.$$

input
```
Int[x*(b*x^(2/3) + a*x)^(3/2),x]
```

output
```
(2*x*(b*x^(2/3) + a*x)^(5/2))/(7*a) - (16*b*((6*x^(2/3)*(b*x^(2/3) + a*x)^
(5/2))/(19*a) - (14*b*((6*x^(1/3)*(b*x^(2/3) + a*x)^(5/2))/(17*a) - (12*b*
((2*(b*x^(2/3) + a*x)^(5/2))/(5*a) - (2*b*((6*(b*x^(2/3) + a*x)^(5/2))/(13
*a*x^(1/3)) - (8*b*((6*(b*x^(2/3) + a*x)^(5/2))/(11*a*x^(2/3)) - (6*b*((2*
(b*x^(2/3) + a*x)^(5/2))/(3*a*x) - (4*b*((-12*b*(b*x^(2/3) + a*x)^(5/2))/(
35*a^2*x^(5/3)) + (6*(b*x^(2/3) + a*x)^(5/2))/(7*a*x^(4/3))))/(9*a)))/(11*
a)))/(13*a)))/(3*a)))/(17*a)))/(19*a)))/(21*a)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.44

method	result
derivativedivides	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(x^{\frac{1}{3}}a+b\right)\left(692835a^{8}x^{\frac{8}{3}}-583440a^{7}b\,x^{\frac{7}{3}}+480480a^{6}b^{2}x^{2}-384384a^{5}b^{3}x^{\frac{5}{3}}+295680x^{\frac{4}{3}}a^{4}b^{4}-215040a^{3}b^{5}\right.}{4849845x\,a^{9}}$
default	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(x^{\frac{1}{3}}a+b\right)\left(692835a^{8}x^{\frac{8}{3}}-583440a^{7}b\,x^{\frac{7}{3}}+480480a^{6}b^{2}x^{2}-384384a^{5}b^{3}x^{\frac{5}{3}}+295680x^{\frac{4}{3}}a^{4}b^{4}-215040a^{3}b^{5}\right.}{4849845x\,a^{9}}$

input
```
int(x*(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/4849845*(b*x^(2/3)+a*x)^(3/2)*(x^(1/3)*a+b)*(692835*a^8*x^(8/3)-583440*a
^7*b*x^(7/3)+480480*a^6*b^2*x^2-384384*a^5*b^3*x^(5/3)+295680*x^(4/3)*a^4*
b^4-215040*a^3*b^5*x+143360*a^2*b^6*x^(2/3)-81920*x^(1/3)*a*b^7+32768*b^8)
/x/a^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. 2(189) = 378.

Time = 141.72 (sec) , antiderivative size = 1031, normalized size of antiderivative = 4.04

$$\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(x*(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-1/4849845*((824633720832*b^16 + 171798691840*b^15 + 805306368*(64*a^3 - 3
)*b^13 - 16106127360*b^14 - 4194304*(11264*a^3 - 53)*b^12 - 8070142080*a^1
2 - 1572864*(5504*a^3 + 1)*b^11 + 49152*(3194880*a^6 - 114688*a^3 - 3)*b^1
0 + 1835008*(18816*a^6 + 103*a^3)*b^9 - 196608*(48816*a^6 + 23*a^3)*b^8 +
3072*(6575923200*a^9 + 495872*a^6 + 15*a^3)*b^7 + 28672*(146455680*a^9 - 3
439*a^6)*b^6 - 419328*(934400*a^9 - 7*a^6)*b^5 + 1584*(12166103040*a^12 -
38275840*a^9 - 21*a^6)*b^4 + 164736*(43008000*a^12 + 33737*a^9)*b^3 + 5148
0*(10838016*a^12 - 799*a^9)*b^2 - 109395*(401408*a^12 + 33*a^9)*b)*x - 2*(
692835*(16777216*a^10*b^6 + 6291456*a^10*b^5 + 196608*a^10*b^4 - 262144*a^
13 - 114688*a^10*b^3 - 2304*a^10*b^2 + 864*a^10*b - 27*a^10)*x^4 - 6864*(1
6777216*a^7*b^9 + 6291456*a^7*b^8 + 196608*a^7*b^7 - 114688*a^7*b^6 - 2304
*a^7*b^5 + 864*a^7*b^4 - (262144*a^10 + 27*a^7)*b^3)*x^3 + 8960*(16777216*
a^4*b^12 + 6291456*a^4*b^11 + 196608*a^4*b^10 - 114688*a^4*b^9 - 2304*a^4*
b^8 + 864*a^4*b^7 - (262144*a^7 + 27*a^4)*b^6)*x^2 - 16384*(16777216*a*b^1
5 + 6291456*a*b^14 + 196608*a*b^13 - 114688*a*b^12 - 2304*a*b^11 + 864*a*b
^10 - (262144*a^4 + 27*a)*b^9)*x + 2*(274877906944*b^16 + 103079215104*b^1
5 + 3221225472*b^14 - 1879048192*b^13 - 37748736*b^12 - 16384*(262144*a^3
+ 27)*b^10 + 14155776*b^11 + 401115*(16777216*a^9*b^7 + 6291456*a^9*b^6 +
196608*a^9*b^5 - 114688*a^9*b^4 - 2304*a^9*b^3 + 864*a^9*b^2 - (262144*a^1
2 + 27*a^9)*b)*x^3 + 3696*(16777216*a^6*b^10 + 6291456*a^6*b^9 + 196608...
```

Sympy [F]

$$\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx=\int x\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(b*x**(2/3)+a*x)**(3/2),x)
```

output
```
Integral(x*(a*x + b*x**(2/3))**(3/2), x)
```

Maxima [F]

$$\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx=\int \left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(2/3))^(3/2)*x, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(189) = 378$.

Time = 0.27 (sec) , antiderivative size = 602, normalized size of antiderivative = 2.36

$$\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(x*(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
-2/692835*b*(32768*b^(19/2)/a^9 - (19*(6435*(a*x^(1/3) + b)^(17/2) - 58344
*(a*x^(1/3) + b)^(15/2)*b + 235620*(a*x^(1/3) + b)^(13/2)*b^2 - 556920*(a*
x^(1/3) + b)^(11/2)*b^3 + 850850*(a*x^(1/3) + b)^(9/2)*b^4 - 875160*(a*x^(
1/3) + b)^(7/2)*b^5 + 612612*(a*x^(1/3) + b)^(5/2)*b^6 - 291720*(a*x^(1/3)
 + b)^(3/2)*b^7 + 109395*sqrt(a*x^(1/3) + b)*b^8)*b/a^8 + 9*(12155*(a*x^(1
/3) + b)^(19/2) - 122265*(a*x^(1/3) + b)^(17/2)*b + 554268*(a*x^(1/3) + b)
^(15/2)*b^2 - 1492260*(a*x^(1/3) + b)^(13/2)*b^3 + 2645370*(a*x^(1/3) + b)
^(11/2)*b^4 - 3233230*(a*x^(1/3) + b)^(9/2)*b^5 + 2771340*(a*x^(1/3) + b)^
(7/2)*b^6 - 1662804*(a*x^(1/3) + b)^(5/2)*b^7 + 692835*(a*x^(1/3) + b)^(3/
2)*b^8 - 230945*sqrt(a*x^(1/3) + b)*b^9)/a^8)/a) + 2/1616615*a*(65536*b^(2
1/2)/a^10 + (21*(12155*(a*x^(1/3) + b)^(19/2) - 122265*(a*x^(1/3) + b)^(17
/2)*b + 554268*(a*x^(1/3) + b)^(15/2)*b^2 - 1492260*(a*x^(1/3) + b)^(13/2)
*b^3 + 2645370*(a*x^(1/3) + b)^(11/2)*b^4 - 3233230*(a*x^(1/3) + b)^(9/2)*
b^5 + 2771340*(a*x^(1/3) + b)^(7/2)*b^6 - 1662804*(a*x^(1/3) + b)^(5/2)*b^
7 + 692835*(a*x^(1/3) + b)^(3/2)*b^8 - 230945*sqrt(a*x^(1/3) + b)*b^9)*b/a
^9 + 5*(46189*(a*x^(1/3) + b)^(21/2) - 510510*(a*x^(1/3) + b)^(19/2)*b + 2
567565*(a*x^(1/3) + b)^(17/2)*b^2 - 7759752*(a*x^(1/3) + b)^(15/2)*b^3 + 1
5668730*(a*x^(1/3) + b)^(13/2)*b^4 - 22221108*(a*x^(1/3) + b)^(11/2)*b^5 +
 22632610*(a*x^(1/3) + b)^(9/2)*b^6 - 16628040*(a*x^(1/3) + b)^(7/2)*b^7 +
 8729721*(a*x^(1/3) + b)^(5/2)*b^8 - 3233230*(a*x^(1/3) + b)^(3/2)*b^9 ...
```

Mupad [F(-1)]

Timed out.

$$\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx=\int x\,\left(a\,x+b\,x^{2/3}\right)^{3/2}\,dx$$

input
```
int(x*(a*x + b*x^(2/3))^(3/2),x)
```

output
```
int(x*(a*x + b*x^(2/3))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.47

$$\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx=\frac{2\sqrt{x^{\frac{1}{3}}a+b}\,\left(6435x^{\frac{8}{3}}a^8b^2-8064x^{\frac{5}{3}}a^5b^5+12288x^{\frac{2}{3}}a^2b^8+692835x^{\frac{10}{3}}a^{10}-6864x^{\frac{7}{3}}a^7b^3+896\right.}{4849845a^9}$$

input
```
int(x*(b*x^(2/3)+a*x)^(3/2),x)
```

output
```
(2*sqrt(x**(1/3)*a + b)*(6435*x**(2/3)*a**8*b**2*x**2 - 8064*x**(2/3)*a**5
*b**5*x + 12288*x**(2/3)*a**2*b**8 + 692835*x**(1/3)*a**10*x**3 - 6864*x**
(1/3)*a**7*b**3*x**2 + 8960*x**(1/3)*a**4*b**6*x - 16384*x**(1/3)*a*b**9 +
 802230*a**9*b*x**3 + 7392*a**6*b**4*x**2 - 10240*a**3*b**7*x + 32768*b**1
0))/(4849845*a**9)
```

3.157 $\int \left(bx^{2/3}+ax\right)^{3/2}\,dx$

Optimal result 1491
Mathematica [A] (verified) 1492
Rubi [A] (verified) 1492
Maple [A] (verified) 1496
Fricas [B] (verification not implemented) 1497
Sympy [F] 1498
Maxima [F] 1498
Giac [B] (verification not implemented) 1498
Mupad [B] (verification not implemented) 1499
Reduce [B] (verification not implemented) 1499

Optimal result

Integrand size = 15, antiderivative size = 169

$$\int \left(bx^{2/3}+ax\right)^{3/2}\,dx=\frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a}-\frac{512b^5\left(bx^{2/3}+ax\right)^{5/2}}{15015a^6x^{5/3}}+\frac{256b^4\left(bx^{2/3}+ax\right)^{5/2}}{3003a^5x^{4/3}}-\frac{64b^3\left(bx^{2/3}+ax\right)^{5/2}}{429a^4x}+\frac{32b^2\left(bx^{2/3}+ax\right)^{5/2}}{143a^3x^{2/3}}-\frac{4b\left(bx^{2/3}+ax\right)^{5/2}}{13a^2\sqrt[3]{x}}$$

output
```
2/5*(b*x^(2/3)+a*x)^(5/2)/a-512/15015*b^5*(b*x^(2/3)+a*x)^(5/2)/a^6/x^(5/3
)+256/3003*b^4*(b*x^(2/3)+a*x)^(5/2)/a^5/x^(4/3)-64/429*b^3*(b*x^(2/3)+a*x
)^(5/2)/a^4/x+32/143*b^2*(b*x^(2/3)+a*x)^(5/2)/a^3/x^(2/3)-4/13*b*(b*x^(2/
3)+a*x)^(5/2)/a^2/x^(1/3)
```

Mathematica [A] (verified)

Time = 5.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.56

$$\int \left(bx^{2/3}+ax\right)^{3/2}\,dx=\frac{2\left(b+a\sqrt[3]{x}\right)\left(bx^{2/3}+ax\right)^{3/2}\left(-256b^5+640ab^4\sqrt[3]{x}-1120a^2b^3x^{2/3}+1680a^3b^2x-2310a^4bx^{4/}\right.}{15015a^6x}$$

input
```
Integrate[(b*x^(2/3) + a*x)^(3/2),x]
```

output
```
(2*(b + a*x^(1/3))*(b*x^(2/3) + a*x)^(3/2)*(-256*b^5 + 640*a*b^4*x^(1/3) -
 1120*a^2*b^3*x^(2/3) + 1680*a^3*b^2*x - 2310*a^4*b*x^(4/3) + 3003*a^5*x^(
5/3)))/(15015*a^6*x)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1908, 1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^{2/3}\right)^{3/2}\,dx$$

$$\Big\downarrow \text{ 1908}$$

$$\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a}-\frac{2b\int \frac{\left(x^{2/3}b+ax\right)^{3/2}}{\sqrt[3]{x}}dx}{3a}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a}-\frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{13a\sqrt[3]{x}}-\frac{8b\int \frac{\left(x^{2/3}b+ax\right)^{3/2}}{x^{2/3}}dx}{13a}\right)}{3a}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a}-\frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{13a\sqrt[3]{x}}-\frac{8b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{11ax^{2/3}}-\frac{6b\int\frac{\left(x^{2/3}b+ax\right)^{3/2}}{x}dx}{11a}\right)}{13a}\right)}{3a}$$

$\downarrow$ 1922

$$\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a}-\frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{13a\sqrt[3]{x}}-\frac{8b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{11ax^{2/3}}-\frac{6b\left(\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{3ax}-\frac{4b\int\frac{\left(x^{2/3}b+ax\right)^{3/2}}{x^{4/3}}dx}{9a}\right)}{11a}\right)}{13a}\right)}{3a}$$

$\downarrow$ 1922

$$\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a}-\frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{13a\sqrt[3]{x}}-\frac{8b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{11ax^{2/3}}-\frac{6b\left(\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{3ax}-\frac{4b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{7ax^{4/3}}-\frac{2b\int\frac{\left(x^{2/3}b+ax\right)^{3/2}}{x^{5/3}}dx}{7a}\right)}{9a}\right)}{11a}\right)}{13a}\right)}{3a}$$

$\downarrow$ 1920

$$\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a}-\frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{13a\sqrt[3]{x}}-\frac{8b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{11ax^{2/3}}-\frac{6b\left(\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{3ax}-\frac{4b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{7ax^{4/3}}-\frac{12b\left(ax+bx^{2/3}\right)^{5/2}}{35a^2x^{5/3}}\right)}{9a}\right)}{11a}\right)}{13a}\right)}{3a}$$

input

```
Int[(b*x^(2/3) + a*x)^(3/2),x]
```

output

```
(2*(b*x^(2/3) + a*x)^(5/2))/(5*a) - (2*b*((6*(b*x^(2/3) + a*x)^(5/2))/(13*
a*x^(1/3)) - (8*b*((6*(b*x^(2/3) + a*x)^(5/2))/(11*a*x^(2/3)) - (6*b*((2*(
b*x^(2/3) + a*x)^(5/2))/(3*a*x) - (4*b*((-12*b*(b*x^(2/3) + a*x)^(5/2))/(3
5*a^2*x^(5/3)) + (6*(b*x^(2/3) + a*x)^(5/2))/(7*a*x^(4/3))))/(9*a)))/(11*a
)))/(13*a)))/(3*a)
```

Defintions of rubi rules used

rule 1908

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.47

method	result	size
derivativedivides	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(x^{\frac{1}{3}}a+b\right)\left(3003a^{5}x^{\frac{5}{3}}-2310x^{\frac{4}{3}}a^{4}b+1680a^{3}b^{2}x-1120a^{2}b^{3}x^{\frac{2}{3}}+640x^{\frac{1}{3}}a\,b^{4}-256b^{5}\right)}{15015x\,a^{6}}$	79
default	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(x^{\frac{1}{3}}a+b\right)\left(3003a^{5}x^{\frac{5}{3}}-2310x^{\frac{4}{3}}a^{4}b+1680a^{3}b^{2}x-1120a^{2}b^{3}x^{\frac{2}{3}}+640x^{\frac{1}{3}}a\,b^{4}-256b^{5}\right)}{15015x\,a^{6}}$	79

input
```
int((b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/15015*(b*x^(2/3)+a*x)^(3/2)*(x^(1/3)*a+b)*(3003*a^5*x^(5/3)-2310*x^(4/3)
*a^4*b+1680*a^3*b^2*x-1120*a^2*b^3*x^(2/3)+640*x^(1/3)*a*b^4-256*b^5)/x/a^
6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 768 vs. $2(125) = 250$.

Time = 126.39 (sec) , antiderivative size = 768, normalized size of antiderivative = 4.54

$$\int \left(bx^{2/3} + ax\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
2/15015*(4*(805306368*b^13 + 167772160*b^12 + 786432*(64*a^3 - 3)*b^10 - 1
5728640*b^11 - 4096*(11264*a^3 - 53)*b^9 + 4372368*a^9 - 1536*(5504*a^3 +
1)*b^8 - 48*(242810880*a^6 + 114688*a^3 + 3)*b^7 - 1792*(1353984*a^6 - 103
*a^3)*b^6 + 192*(1152384*a^6 - 23*a^3)*b^5 - 3*(3633315840*a^9 - 12027392*
a^6 - 15*a^3)*b^4 - 112*(35389440*a^9 + 29281*a^6)*b^3 - 819*(368640*a^9 -
 31*a^6)*b^2 + 693*(40960*a^9 + 3*a^6)*b)*x + (3003*(16777216*a^7*b^6 + 62
91456*a^7*b^5 + 196608*a^7*b^4 - 262144*a^10 - 114688*a^7*b^3 - 2304*a^7*b
^2 + 864*a^7*b - 27*a^7)*x^3 - 70*(16777216*a^4*b^9 + 6291456*a^4*b^8 + 19
6608*a^4*b^7 - 114688*a^4*b^6 - 2304*a^4*b^5 + 864*a^4*b^4 - (262144*a^7 +
 27*a^4)*b^3)*x^2 + 128*(16777216*a*b^12 + 6291456*a*b^11 + 196608*a*b^10
- 114688*a*b^9 - 2304*a*b^8 + 864*a*b^7 - (262144*a^4 + 27*a)*b^6)*x - 16*
(268435456*b^13 + 100663296*b^12 + 3145728*b^11 - 1835008*b^10 - 36864*b^9
 - 16*(262144*a^3 + 27)*b^7 + 13824*b^8 - 231*(16777216*a^6*b^7 + 6291456*
a^6*b^6 + 196608*a^6*b^5 - 114688*a^6*b^4 - 2304*a^6*b^3 + 864*a^6*b^2 - (
262144*a^9 + 27*a^6)*b)*x^2 - 5*(16777216*a^3*b^10 + 6291456*a^3*b^9 + 196
608*a^3*b^8 - 114688*a^3*b^7 - 2304*a^3*b^6 + 864*a^3*b^5 - (262144*a^6 +
27*a^3)*b^4)*x)*x^(2/3) + 3*(21*(16777216*a^5*b^8 + 6291456*a^5*b^7 + 1966
08*a^5*b^6 - 114688*a^5*b^5 - 2304*a^5*b^4 + 864*a^5*b^3 - (262144*a^8 + 2
7*a^5)*b^2)*x^2 - 32*(16777216*a^2*b^11 + 6291456*a^2*b^10 + 196608*a^2*b^
9 - 114688*a^2*b^8 - 2304*a^2*b^7 + 864*a^2*b^6 - (262144*a^5 + 27*a^2)...
```

Sympy [F]

$$\int \left(bx^{2/3}+ax\right)^{3/2}\,dx=\int \left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x**(2/3)+a*x)**(3/2),x)
```

output
```
Integral((a*x + b*x**(2/3))**(3/2), x)
```

Maxima [F]

$$\int \left(bx^{2/3}+ax\right)^{3/2}\,dx=\int \left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(2/3))^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(125) = 250$.

Time = 0.28 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.57

$$\int \left(bx^{2/3}+ax\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
2/3003*b*(256*b^(13/2)/a^6 + (13*(63*(a*x^(1/3) + b)^(11/2) - 385*(a*x^(1/
3) + b)^(9/2)*b + 990*(a*x^(1/3) + b)^(7/2)*b^2 - 1386*(a*x^(1/3) + b)^(5/
2)*b^3 + 1155*(a*x^(1/3) + b)^(3/2)*b^4 - 693*sqrt(a*x^(1/3) + b)*b^5)*b/a
^5 + 3*(231*(a*x^(1/3) + b)^(13/2) - 1638*(a*x^(1/3) + b)^(11/2)*b + 5005*
(a*x^(1/3) + b)^(9/2)*b^2 - 8580*(a*x^(1/3) + b)^(7/2)*b^3 + 9009*(a*x^(1/
3) + b)^(5/2)*b^4 - 6006*(a*x^(1/3) + b)^(3/2)*b^5 + 3003*sqrt(a*x^(1/3) +
 b)*b^6)/a^5)/a) - 2/15015*a*(1024*b^(15/2)/a^7 - (15*(231*(a*x^(1/3) + b)
^(13/2) - 1638*(a*x^(1/3) + b)^(11/2)*b + 5005*(a*x^(1/3) + b)^(9/2)*b^2 -
 8580*(a*x^(1/3) + b)^(7/2)*b^3 + 9009*(a*x^(1/3) + b)^(5/2)*b^4 - 6006*(a
*x^(1/3) + b)^(3/2)*b^5 + 3003*sqrt(a*x^(1/3) + b)*b^6)*b/a^6 + 7*(429*(a*
x^(1/3) + b)^(15/2) - 3465*(a*x^(1/3) + b)^(13/2)*b + 12285*(a*x^(1/3) + b
)^(11/2)*b^2 - 25025*(a*x^(1/3) + b)^(9/2)*b^3 + 32175*(a*x^(1/3) + b)^(7/
2)*b^4 - 27027*(a*x^(1/3) + b)^(5/2)*b^5 + 15015*(a*x^(1/3) + b)^(3/2)*b^6
 - 6435*sqrt(a*x^(1/3) + b)*b^7)/a^6)/a)
```

Mupad [B] (verification not implemented)

Time = 8.74 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.24

$$\int \left(bx^{2/3}+ax\right)^{3/2}\,dx=\frac{x\,\left(a\,x+b\,x^{2/3}\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},6;\ 7;\ -\frac{a\,x^{1/3}}{b}\right)}{2\,\left(\frac{a\,x^{1/3}}{b}+1\right)^{3/2}}$$

input
```
int((a*x + b*x^(2/3))^(3/2),x)
```

output
```
(x*(a*x + b*x^(2/3))^(3/2)*hypergeom([-3/2, 6], 7, -(a*x^(1/3))/b))/(2*((a
*x^(1/3))/b + 1)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.51

$$\int \left(bx^{2/3}+ax\right)^{3/2}\,dx=\frac{2\sqrt{x^{\frac{1}{3}}a+b}\,\left(63x^{\frac{5}{3}}a^5b^2-96x^{\frac{2}{3}}a^2b^5+3003x^{\frac{7}{3}}a^7-70x^{\frac{4}{3}}a^4b^3+128x^{\frac{1}{3}}a\,b^6+3696a^6b\,x^2+80a\right.}{15015a^6}$$

input
```
int((b*x^(2/3)+a*x)^(3/2),x)
```

output
```
(2*sqrt(x**(1/3)*a + b)*(63*x**(2/3)*a**5*b**2*x - 96*x**(2/3)*a**2*b**5 +
 3003*x**(1/3)*a**7*x**2 - 70*x**(1/3)*a**4*b**3*x + 128*x**(1/3)*a*b**6 +
 3696*a**6*b*x**2 + 80*a**3*b**4*x - 256*b**7))/(15015*a**6)
```

3.158 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x} dx$

Optimal result 1501
Mathematica [A] (verified) 1501
Rubi [A] (verified) 1502
Maple [A] (verified) 1503
Fricas [B] (verification not implemented) 1504
Sympy [F] 1504
Maxima [F] 1505
Giac [B] (verification not implemented) 1505
Mupad [F(-1)] 1506
Reduce [B] (verification not implemented) 1506

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x} dx = \frac{16b^2\left(bx^{2/3}+ax\right)^{5/2}}{105a^3x^{5/3}} - \frac{8b\left(bx^{2/3}+ax\right)^{5/2}}{21a^2x^{4/3}} + \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{3ax}$$

output
```
16/105*b^2*(b*x^(2/3)+a*x)^(5/2)/a^3/x^(5/3)-8/21*b*(b*x^(2/3)+a*x)^(5/2)/
a^2/x^(4/3)+2/3*(b*x^(2/3)+a*x)^(5/2)/a/x
```

Mathematica [A] (verified)

Time = 5.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.70

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x} dx = \frac{2\left(b+a\sqrt[3]{x}\right)\left(8b^2-20ab\sqrt[3]{x}+35a^2x^{2/3}\right)\left(bx^{2/3}+ax\right)^{3/2}}{105a^3x}$$

input
```
Integrate[(b*x^(2/3) + a*x)^(3/2)/x,x]
```

output
```
(2*(b + a*x^(1/3))*(8*b^2 - 20*a*b*x^(1/3) + 35*a^2*x^(2/3))*(b*x^(2/3) +
a*x)^(3/2))/(105*a^3*x)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^{2/3}\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{3ax}-\frac{4b\int \frac{\left(x^{2/3}b+ax\right)^{3/2}}{x^{4/3}}dx}{9a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{3ax}-\frac{4b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{7ax^{4/3}}-\frac{2b\int \frac{\left(x^{2/3}b+ax\right)^{3/2}}{x^{5/3}}dx}{7a}\right)}{9a}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{3ax}-\frac{4b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{7ax^{4/3}}-\frac{12b\left(ax+bx^{2/3}\right)^{5/2}}{35a^2x^{5/3}}\right)}{9a}$$

input
```
Int[(b*x^(2/3) + a*x)^(3/2)/x,x]
```

output
```
(2*(b*x^(2/3) + a*x)^(5/2))/(3*a*x) - (4*b*((-12*b*(b*x^(2/3) + a*x)^(5/2)
)/(35*a^2*x^(5/3)) + (6*(b*x^(2/3) + a*x)^(5/2))/(7*a*x^(4/3))))/(9*a)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.57

method	result	size
derivativedivides	$\frac{2\left(b\,x^{\frac{2}{3}}+a x \right)^{\frac{3}{2}} \left(x^{\frac{1}{3}} a +b \right) \left(35 x^{\frac{2}{3}} a^{2}-20 x^{\frac{1}{3}} a b +8 b^{2}\right)}{105 x \,a^{3}}$	48
default	$\frac{2\left(b\,x^{\frac{2}{3}}+a x \right)^{\frac{3}{2}} \left(x^{\frac{1}{3}} a +b \right) \left(35 x^{\frac{2}{3}} a^{2}-20 x^{\frac{1}{3}} a b +8 b^{2}\right)}{105 x \,a^{3}}$	48

input
```
int((b*x^(2/3)+a*x)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/105*(b*x^(2/3)+a*x)^(3/2)*(x^(1/3)*a+b)*(35*x^(2/3)*a^2-20*x^(1/3)*a*b+8
*b^2)/x/a^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(62) = 124$.

Time = 132.69 (sec) , antiderivative size = 501, normalized size of antiderivative = 5.96

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx =$$

$$-\frac{(201326592\,b^{10}+41943040\,b^9+196608\,(6784\,a^3-3)b^7-3932160\,b^8+1024\,(257536\,a^3+53)b^6-40768}{}$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x,x, algorithm="fricas")
```

output
```
-1/105*((201326592*b^10 + 41943040*b^9 + 196608*(6784*a^3 - 3)*b^7 - 39321
60*b^8 + 1024*(257536*a^3 + 53)*b^6 - 407680*a^6 - 384*(72704*a^3 + 1)*b^5
 + 12*(94371840*a^6 - 437248*a^3 - 3)*b^4 + 896*(442368*a^6 + 449*a^3)*b^3
 + 24*(1105920*a^6 - 151*a^3)*b^2 - 15*(253952*a^6 + 15*a^3)*b)*x - 2*(35*
(16777216*a^4*b^6 + 6291456*a^4*b^5 + 196608*a^4*b^4 - 262144*a^7 - 114688
*a^4*b^3 - 2304*a^4*b^2 + 864*a^4*b - 27*a^4)*x^2 + 3*(16777216*a^2*b^8 +
6291456*a^2*b^7 + 196608*a^2*b^6 - 114688*a^2*b^5 - 2304*a^2*b^4 + 864*a^2
*b^3 - (262144*a^5 + 27*a^2)*b^2)*x^(4/3) - 4*(16777216*a*b^9 + 6291456*a*
b^8 + 196608*a*b^7 - 114688*a*b^6 - 2304*a*b^5 + 864*a*b^4 - (262144*a^4 +
 27*a)*b^3)*x + 2*(67108864*b^10 + 25165824*b^9 + 786432*b^8 - 458752*b^7
- 9216*b^6 - 4*(262144*a^3 + 27)*b^4 + 3456*b^5 + 25*(16777216*a^3*b^7 + 6
291456*a^3*b^6 + 196608*a^3*b^5 - 114688*a^3*b^4 - 2304*a^3*b^3 + 864*a^3*
b^2 - (262144*a^6 + 27*a^3)*b)*x)*x^(2/3))*sqrt(a*x + b*x^(2/3)))/((167772
16*a^3*b^6 + 6291456*a^3*b^5 + 196608*a^3*b^4 - 262144*a^6 - 114688*a^3*b^
3 - 2304*a^3*b^2 + 864*a^3*b - 27*a^3)*x)
```

Sympy [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx = \int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((b*x**(2/3)+a*x)**(3/2)/x,x)
```

output
```
Integral((a*x + b*x**(2/3))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx=\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(2/3))^(3/2)/x, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(62) = 124$.

Time = 0.28 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.15

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx =$$

$$-\frac{2}{35}\,b\left(\frac{8\,b^{\frac{7}{2}}}{a^3}-\frac{\frac{7\left(3\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}-10\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b+15\sqrt{ax^{\frac{1}{3}}+b}b^2\right)b}{a^2}+\frac{3\left(5\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}-21\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b+35\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^2-35\sqrt{ax^{\frac{1}{3}}+}\right.}{a^2}}{a}\right.$$

$$+\frac{2}{105}\,a\left(\frac{16\,b^{\frac{9}{2}}}{a^4}+\frac{\frac{9\left(5\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}-21\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b+35\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^2-35\sqrt{ax^{\frac{1}{3}}+b}b^3\right)b}{a^3}+\frac{35\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}-180\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b+378\left(ax^{\frac{1}{3}}\right.}{}}{a}\right.$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x,x, algorithm="giac")
```

output
```
-2/35*b*(8*b^(7/2)/a^3 - (7*(3*(a*x^(1/3) + b)^(5/2) - 10*(a*x^(1/3) + b)^
(3/2)*b + 15*sqrt(a*x^(1/3) + b)*b^2)*b/a^2 + 3*(5*(a*x^(1/3) + b)^(7/2) -
 21*(a*x^(1/3) + b)^(5/2)*b + 35*(a*x^(1/3) + b)^(3/2)*b^2 - 35*sqrt(a*x^(
1/3) + b)*b^3)/a^2)/a) + 2/105*a*(16*b^(9/2)/a^4 + (9*(5*(a*x^(1/3) + b)^(
7/2) - 21*(a*x^(1/3) + b)^(5/2)*b + 35*(a*x^(1/3) + b)^(3/2)*b^2 - 35*sqrt
(a*x^(1/3) + b)*b^3)*b/a^3 + (35*(a*x^(1/3) + b)^(9/2) - 180*(a*x^(1/3) +
b)^(7/2)*b + 378*(a*x^(1/3) + b)^(5/2)*b^2 - 420*(a*x^(1/3) + b)^(3/2)*b^3
 + 315*sqrt(a*x^(1/3) + b)*b^4)/a^3)/a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx=\int \frac{\left(a\,x+b\,x^{2/3}\right)^{3/2}}{x}\,dx$$

input
```
int((a*x + b*x^(2/3))^(3/2)/x,x)
```

output
```
int((a*x + b*x^(2/3))^(3/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx=\frac{2\sqrt{x^{\frac{1}{3}}a+b}\,\left(3x^{\frac{2}{3}}a^2b^2+35x^{\frac{4}{3}}a^4-4x^{\frac{1}{3}}a\,b^3+50a^3bx+8b^4\right)}{105a^3}$$

input
```
int((b*x^(2/3)+a*x)^(3/2)/x,x)
```

output
```
(2*sqrt(x**(1/3)*a + b)*(3*x**(2/3)*a**2*b**2 + 35*x**(1/3)*a**4*x - 4*x**
(1/3)*a*b**3 + 50*a**3*b*x + 8*b**4))/(105*a**3)
```

3.159 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx$

Optimal result 1507
Mathematica [A] (verified) 1507
Rubi [A] (verified) 1508
Maple [A] (verified) 1509
Fricas [F(-1)] 1510
Sympy [F] 1510
Maxima [F] 1510
Giac [A] (verification not implemented) 1511
Mupad [F(-1)] 1511
Reduce [B] (verification not implemented) 1512

Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx=\frac{6b\sqrt{bx^{2/3}+ax}}{\sqrt[3]{x}}+\frac{2\left(bx^{2/3}+ax\right)^{3/2}}{x}-6b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)$$

output
```
6*b*(b*x^(2/3)+a*x)^(1/2)/x^(1/3)+2*(b*x^(2/3)+a*x)^(3/2)/x-6*b^(3/2)*arct
anh(b^(1/2)*x^(1/3)/(b*x^(2/3)+a*x)^(1/2))
```

Mathematica [A] (verified)

Time = 10.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx=\frac{2\sqrt{bx^{2/3}+ax}\left(\sqrt{b+a\sqrt[3]{x}}\left(4b+a\sqrt[3]{x}\right)-3b^{3/2}\text{arctanh}\left(\frac{\sqrt{b+a\sqrt[3]{x}}}{\sqrt{b}}\right)\right)}{\sqrt{b+a\sqrt[3]{x}}\sqrt[3]{x}}$$

input
```
Integrate[(b*x^(2/3) + a*x)^(3/2)/x^2,x]
```

output
```
(2*Sqrt[b*x^(2/3) + a*x]*(Sqrt[b + a*x^(1/3)]*(4*b + a*x^(1/3)) - 3*b^(3/2
)*ArcTanh[Sqrt[b + a*x^(1/3)]/Sqrt[b]]))/(Sqrt[b + a*x^(1/3)]*x^(1/3))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1927, 1927, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{1927}$$

$$b\int \frac{\sqrt{x^{2/3}b+ax}}{x^{4/3}}dx+\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{x}$$

$$\Big\downarrow \text{1927}$$

$$b\left(b\int \frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}dx+\frac{6\sqrt{ax+bx^{2/3}}}{\sqrt[3]{x}}\right)+\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{x}$$

$$\Big\downarrow \text{1935}$$

$$b\left(\frac{6\sqrt{ax+bx^{2/3}}}{\sqrt[3]{x}}-6b\int \frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}\right)+\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{x}$$

$$\Big\downarrow \text{219}$$

$$b\left(\frac{6\sqrt{ax+bx^{2/3}}}{\sqrt[3]{x}}-6\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)\right)+\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{x}$$

input
```
Int[(b*x^(2/3) + a*x)^(3/2)/x^2,x]
```

output
```
(2*(b*x^(2/3) + a*x)^(3/2))/x + b*((6*Sqrt[b*x^(2/3) + a*x])/x^(1/3) - 6*S
qrt[b]*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{2\left(b\,x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}} \left(-3 b^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{x^{\frac{1}{3}} a+b}}{\sqrt{b}}\right)+\left(x^{\frac{1}{3}} a+b\right)^{\frac{3}{2}}+3 b \sqrt{x^{\frac{1}{3}} a+b}\right)}{x \left(x^{\frac{1}{3}} a+b\right)^{\frac{3}{2}}}$	67
default	$\frac{2\left(b\,x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}} \left(-3 b^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{x^{\frac{1}{3}} a+b}}{\sqrt{b}}\right)+\left(x^{\frac{1}{3}} a+b\right)^{\frac{3}{2}}+3 b \sqrt{x^{\frac{1}{3}} a+b}\right)}{x \left(x^{\frac{1}{3}} a+b\right)^{\frac{3}{2}}}$	67

input
```
int((b*x^(2/3)+a*x)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2*(b*x^(2/3)+a*x)^(3/2)*(-3*b^(3/2)*arctanh((x^(1/3)*a+b)^(1/2)/b^(1/2))+(
x^(1/3)*a+b)^(3/2)+3*b*(x^(1/3)*a+b)^(1/2))/x/(x^(1/3)*a+b)^(3/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx = \int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x**(2/3)+a*x)**(3/2)/x**2,x)
```

output
```
Integral((a*x + b*x**(2/3))**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx = \int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(2/3))^(3/2)/x^2, x)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx=\frac{6\,b^2\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+2\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}+6\sqrt{ax^{\frac{1}{3}}+b}b-\frac{2\left(3\,b^2\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+4\sqrt{-b}b^{\frac{3}{2}}\right)}{\sqrt{-b}}$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^2,x, algorithm="giac")
```

output
```
6*b^2*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/sqrt(-b) + 2*(a*x^(1/3) + b)^(3
/2) + 6*sqrt(a*x^(1/3) + b)*b - 2*(3*b^2*arctan(sqrt(b)/sqrt(-b)) + 4*sqrt
(-b)*b^(3/2))/sqrt(-b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx=\int \frac{\left(a\,x+b\,x^{2/3}\right)^{3/2}}{x^2}\,dx$$

input
```
int((a*x + b*x^(2/3))^(3/2)/x^2,x)
```

output
```
int((a*x + b*x^(2/3))^(3/2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.79

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx = 2x^{\frac{1}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,a+8\sqrt{x^{\frac{1}{3}}a+b}\,b+3\sqrt{b}\log\left(\sqrt{x^{\frac{1}{3}}a+b}-\sqrt{b}\right)b-3\sqrt{b}\log\left(\sqrt{x^{\frac{1}{3}}a+b}+\sqrt{b}\right)b$$

input
```
int((b*x^(2/3)+a*x)^(3/2)/x^2,x)
```

output
```
2*x**(1/3)*sqrt(x**(1/3)*a + b)*a + 8*sqrt(x**(1/3)*a + b)*b + 3*sqrt(b)*l
og(sqrt(x**(1/3)*a + b) - sqrt(b))*b - 3*sqrt(b)*log(sqrt(x**(1/3)*a + b)
+ sqrt(b))*b
```

3.160 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx$

Optimal result 1513
Mathematica [C] (verified) 1513
Rubi [A] (verified) 1514
Maple [A] (verified) 1516
Fricas [F(-1)] 1516
Sympy [F] 1517
Maxima [F] 1517
Giac [A] (verification not implemented) 1517
Mupad [F(-1)] 1518
Reduce [B] (verification not implemented) 1518

Optimal result

Integrand size = 19, antiderivative size = 113

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx=-\frac{3a\sqrt{bx^{2/3}+ax}}{4x}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{8bx^{2/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}+\frac{3a^3\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{8b^{3/2}}$$

output
```
-3/4*a*(b*x^(2/3)+a*x)^(1/2)/x-3/8*a^2*(b*x^(2/3)+a*x)^(1/2)/b/x^(2/3)-(b*
x^(2/3)+a*x)^(3/2)/x^2+3/8*a^3*arctanh(b^(1/2)*x^(1/3)/(b*x^(2/3)+a*x)^(1/
2))/b^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx=\frac{6a^3\left(b+a\sqrt[3]{x}\right)^2\sqrt{bx^{2/3}+ax}\,\text{Hypergeometric2F1}\left(\frac{5}{2},4,\frac{7}{2},1+\frac{a\sqrt[3]{x}}{b}\right)}{5b^4\sqrt[3]{x}}$$

input
```
Integrate[(b*x^(2/3) + a*x)^(3/2)/x^3,x]
```

output
```
(6*a^3*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 4, 7
/2, 1 + (a*x^(1/3))/b])/(5*b^4*x^(1/3))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1926, 1926, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{ 1926}$$

$$\frac{1}{2}a\int \frac{\sqrt{x^{2/3}b+ax}}{x^2}dx-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^2}$$

$$\Big\downarrow \text{ 1926}$$

$$\frac{1}{2}a\left(\frac{1}{4}a\int \frac{1}{x\sqrt{x^{2/3}b+ax}}dx-\frac{3\sqrt{ax+bx^{2/3}}}{2x}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^2}$$

$$\Big\downarrow \text{ 1931}$$

$$\frac{1}{2}a\left(\frac{1}{4}a\left(-\frac{a\int \frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{2x}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^2}$$

$$\Big\downarrow \text{ 1935}$$

$$\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{3a\int \frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{2x}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{3a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{2x}\right)-\frac{(ax+bx^{2/3})^{3/2}}{x^2}$$

input
```
Int[(b*x^(2/3) + a*x)^(3/2)/x^3,x]
```

output
```
-((b*x^(2/3) + a*x)^(3/2)/x^2) + (a*((-3*Sqrt[b*x^(2/3) + a*x])/(2*x) + (a
*((-3*Sqrt[b*x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/
Sqrt[b*x^(2/3) + a*x]])/b^(3/2)))/4))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\left(b\,x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}} \left(3 b^{\frac{7}{2}} \sqrt{x^{\frac{1}{3}} a+b}-8 b^{\frac{5}{2}} \left(x^{\frac{1}{3}} a+b\right)^{\frac{3}{2}}-3 b^{\frac{3}{2}} \left(x^{\frac{1}{3}} a+b\right)^{\frac{5}{2}}+3 \,\operatorname{arctanh}\left(\frac{\sqrt{x^{\frac{1}{3}} a+b}}{\sqrt{b}}\right) b\,a^{3} x\right)}{8 x^{2} \left(x^{\frac{1}{3}} a+b\right)^{\frac{3}{2}} b^{\frac{5}{2}}}$	93
default	$\frac{\left(b\,x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}} \left(3 b^{\frac{7}{2}} \sqrt{x^{\frac{1}{3}} a+b}-8 b^{\frac{5}{2}} \left(x^{\frac{1}{3}} a+b\right)^{\frac{3}{2}}-3 b^{\frac{3}{2}} \left(x^{\frac{1}{3}} a+b\right)^{\frac{5}{2}}+3 \,\operatorname{arctanh}\left(\frac{\sqrt{x^{\frac{1}{3}} a+b}}{\sqrt{b}}\right) b\,a^{3} x\right)}{8 x^{2} \left(x^{\frac{1}{3}} a+b\right)^{\frac{3}{2}} b^{\frac{5}{2}}}$	93

input
```
int((b*x^(2/3)+a*x)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*(b*x^(2/3)+a*x)^(3/2)*(3*b^(7/2)*(x^(1/3)*a+b)^(1/2)-8*b^(5/2)*(x^(1/3
)*a+b)^(3/2)-3*b^(3/2)*(x^(1/3)*a+b)^(5/2)+3*arctanh((x^(1/3)*a+b)^(1/2)/b
^(1/2))*b*a^3*x)/x^2/(x^(1/3)*a+b)^(3/2)/b^(5/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx=\text{Timed out}$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(bx^{2/3}+ax)^{3/2}}{x^3}\,dx = \int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b*x**(2/3)+a*x)**(3/2)/x**3,x)
```

output
```
Integral((a*x + b*x**(2/3))**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{(bx^{2/3}+ax)^{3/2}}{x^3}\,dx = \int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(2/3))^(3/2)/x^3, x)
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.71

$$\int \frac{(bx^{2/3}+ax)^{3/2}}{x^3}\,dx = -\frac{1}{8}a^3\left(\frac{3\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b}+\frac{3\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}+8\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b-3\sqrt{ax^{\frac{1}{3}}+b}b^2}{a^3bx}\right)$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^3,x, algorithm="giac")
```

output
```
-1/8*a^3*(3*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) + (3*(a*x^(1
/3) + b)^(5/2) + 8*(a*x^(1/3) + b)^(3/2)*b - 3*sqrt(a*x^(1/3) + b)*b^2)/(a
^3*b*x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx=\int \frac{{\left(a\,x+b\,x^{2/3}\right)}^{3/2}}{x^3}\,dx$$

input
```
int((a*x + b*x^(2/3))^(3/2)/x^3,x)
```

output
```
int((a*x + b*x^(2/3))^(3/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.91

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx=\frac{-16x^{\frac{2}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,b^3-6x^{\frac{4}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,a^2b-28\sqrt{x^{\frac{1}{3}}a+b}\,a\,b^2x-3x^{\frac{5}{3}}\sqrt{b}\log\left(\sqrt{x^{\frac{1}{3}}a\right.}{16x^{\frac{5}{3}}b^2}$$

input
```
int((b*x^(2/3)+a*x)^(3/2)/x^3,x)
```

output
```
( - 16*x**(2/3)*sqrt(x**(1/3)*a + b)*b**3 - 6*x**(1/3)*sqrt(x**(1/3)*a + b
)*a**2*b*x - 28*sqrt(x**(1/3)*a + b)*a*b**2*x - 3*x**(2/3)*sqrt(b)*log(sqr
t(x**(1/3)*a + b) - sqrt(b))*a**3*x + 3*x**(2/3)*sqrt(b)*log(sqrt(x**(1/3)
*a + b) + sqrt(b))*a**3*x)/(16*x**(2/3)*b**2*x)
```

3.161 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4} \, dx$

Optimal result 1519
Mathematica [C] (verified) 1520
Rubi [A] (verified) 1520
Maple [A] (verified) 1525
Fricas [F(-1)] 1526
Sympy [F] 1526
Maxima [F] 1527
Giac [A] (verification not implemented) 1527
Mupad [F(-1)] 1528
Reduce [B] (verification not implemented) 1528

Optimal result

Integrand size = 19, antiderivative size = 203

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4} \, dx = -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}} + \frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}} - \frac{7a^4\sqrt{bx^{2/3}+ax}}{256b^3x} + \frac{21a^5\sqrt{bx^{2/3}+ax}}{512b^4x^{2/3}} - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3} - \frac{21a^6\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{512b^{9/2}}$$

output
```
-3/20*a*(b*x^(2/3)+a*x)^(1/2)/x^2-3/160*a^2*(b*x^(2/3)+a*x)^(1/2)/b/x^(5/3
)+7/320*a^3*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(4/3)-7/256*a^4*(b*x^(2/3)+a*x)^(1
/2)/b^3/x+21/512*a^5*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(2/3)-1/2*(b*x^(2/3)+a*x)
^(3/2)/x^3-21/512*a^6*arctanh(b^(1/2)*x^(1/3)/(b*x^(2/3)+a*x)^(1/2))/b^(9/
2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.30

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx = -\frac{6a^6\left(b+a\sqrt[3]{x}\right)^2\sqrt{bx^{2/3}+ax}\,\text{Hypergeometric2F1}\left(\frac{5}{2},7,\frac{7}{2},1+\frac{a\sqrt[3]{x}}{b}\right)}{5b^7\sqrt[3]{x}}$$

input
```
Integrate[(b*x^(2/3) + a*x)^(3/2)/x^4,x]
```

output
```
(-6*a^6*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 7,
7/2, 1 + (a*x^(1/3))/b])/(5*b^7*x^(1/3))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1926, 1926, 1931, 1931, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^4}\,dx$$

$$\Big\downarrow \text{ 1926}$$

$$\frac{1}{4}a\int \frac{\sqrt{x^{2/3}b+ax}}{x^3}dx-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{2x^3}$$

$$\Big\downarrow \text{ 1926}$$

$$\frac{1}{4}a\left(\frac{1}{10}a\int \frac{1}{x^2\sqrt{x^{2/3}b+ax}}dx-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{2x^3}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{4}a\left(\frac{1}{10}a\left(-\frac{7a\int \frac{1}{x^{5/3}\sqrt{x^{2/3}b+ax}}dx}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{2x^3}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{4}a\left(\frac{1}{10}a\left(-\frac{7a\left(-\frac{5a\int \frac{1}{x^{4/3}\sqrt{x^{2/3}b+ax}}dx}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{2x^3}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{4}a\left(\frac{1}{10}a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\int \frac{1}{x\sqrt{x^{2/3}b+ax}}dx}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{2x^3}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{4}a\left(\frac{1}{10}a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(-\frac{a\int\frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)-3\sqrt{\cdots}\right.$$

$$\frac{\left(ax+bx^{2/3}\right)^{3/2}}{2x^3}$$

$\big\downarrow$ 1935

$$\frac{1}{4}a\left(\frac{1}{10}a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\int\frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)\right.$$

$$\frac{\left(ax+bx^{2/3}\right)^{3/2}}{2x^3}$$

$\downarrow$ 219

$$\frac{1}{4}a\left(\frac{1}{10}a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)-\right.$$

$$\frac{\left(ax+bx^{2/3}\right)^{3/2}}{2x^3}$$

input
```
Int[(b*x^(2/3) + a*x)^(3/2)/x^4,x]
```

output
```
-1/2*(b*x^(2/3) + a*x)^(3/2)/x^3 + (a*((-3*Sqrt[b*x^(2/3) + a*x])/(5*x^2)
+ (a*((-3*Sqrt[b*x^(2/3) + a*x])/(4*b*x^(5/3)) - (7*a*(-(Sqrt[b*x^(2/3) +
a*x]/(b*x^(4/3))) - (5*a*((-3*Sqrt[b*x^(2/3) + a*x])/(2*b*x) - (3*a*((-3*S
qrt[b*x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*
x^(2/3) + a*x]])/b^(3/2)))/(4*b)))/(6*b)))/(8*b)))/10))/4
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.68

method	result
derivativedivides	$\frac{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(105b^{\frac{9}{2}}\left(x^{\frac{1}{3}}a+b\right)^{\frac{11}{2}}-595b^{\frac{11}{2}}\left(x^{\frac{1}{3}}a+b\right)^{\frac{9}{2}}+1386b^{\frac{13}{2}}\left(x^{\frac{1}{3}}a+b\right)^{\frac{7}{2}}-1686b^{\frac{15}{2}}\left(x^{\frac{1}{3}}a+b\right)^{\frac{5}{2}}-105\,\operatorname{arctanh}\left(\right.\right.}{2560x^{3}\left(x^{\frac{1}{3}}a+b\right)^{\frac{3}{2}}b^{\frac{17}{2}}}$
default	$-\frac{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(-105b^{\frac{9}{2}}\left(x^{\frac{1}{3}}a+b\right)^{\frac{11}{2}}+595b^{\frac{11}{2}}\left(x^{\frac{1}{3}}a+b\right)^{\frac{9}{2}}-1386b^{\frac{13}{2}}\left(x^{\frac{1}{3}}a+b\right)^{\frac{7}{2}}+1686b^{\frac{15}{2}}\left(x^{\frac{1}{3}}a+b\right)^{\frac{5}{2}}+595b^{\frac{17}{2}}\left(x\right.\right.}{2560x^{3}\left(x^{\frac{1}{3}}a+b\right)^{\frac{3}{2}}b^{\frac{17}{2}}}$

input
```
int((b*x^(2/3)+a*x)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/2560*(b*x^(2/3)+a*x)^(3/2)*(105*b^(9/2)*(x^(1/3)*a+b)^(11/2)-595*b^(11/2
)*(x^(1/3)*a+b)^(9/2)+1386*b^(13/2)*(x^(1/3)*a+b)^(7/2)-1686*b^(15/2)*(x^(
1/3)*a+b)^(5/2)-105*arctanh((x^(1/3)*a+b)^(1/2)/b^(1/2))*b^4*a^6*x^2-595*b
^(17/2)*(x^(1/3)*a+b)^(3/2)+105*b^(19/2)*(x^(1/3)*a+b)^(1/2))/x^3/(x^(1/3)
*a+b)^(3/2)/b^(17/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx = \int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x**(2/3)+a*x)**(3/2)/x**4,x)
```

output
```
Integral((a*x + b*x**(2/3))**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx = \int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(2/3))^(3/2)/x^4, x)
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.70

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx = \frac{\frac{105\,a^7\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^4} + \frac{105\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^7-595\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^7b+1386\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^7b^2-1686\left(ax^{\frac{1}{3}}+b\right)\cdots}{a^6b^4x^2}}{2560\,a}$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^4,x, algorithm="giac")
```

output
```
1/2560*(105*a^7*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) + (105
*(a*x^(1/3) + b)^(11/2)*a^7 - 595*(a*x^(1/3) + b)^(9/2)*a^7*b + 1386*(a*x^
(1/3) + b)^(7/2)*a^7*b^2 - 1686*(a*x^(1/3) + b)^(5/2)*a^7*b^3 - 595*(a*x^(
1/3) + b)^(3/2)*a^7*b^4 + 105*sqrt(a*x^(1/3) + b)*a^7*b^5)/(a^6*b^4*x^2))/
a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^{2/3}+ax)^{3/2}}{x^4}\,dx=\int \frac{(a\,x+b\,x^{2/3})^{3/2}}{x^4}\,dx$$

input
```
int((a*x + b*x^(2/3))^(3/2)/x^4,x)
```

output
```
int((a*x + b*x^(2/3))^(3/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.79

$$\int \frac{(bx^{2/3}+ax)^{3/2}}{x^4}\,dx=\frac{112x^{\frac{5}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,a^3b^3-2560x^{\frac{2}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,b^6+210x^{\frac{7}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,a^5b-96x^{\frac{4}{3}}\sqrt{x^{\frac{1}{3}}a}$$

input
```
int((b*x^(2/3)+a*x)^(3/2)/x^4,x)
```

output
```
(112*x**(2/3)*sqrt(x**(1/3)*a + b)*a**3*b**3*x - 2560*x**(2/3)*sqrt(x**(1/
3)*a + b)*b**6 + 210*x**(1/3)*sqrt(x**(1/3)*a + b)*a**5*b*x**2 - 96*x**(1/
3)*sqrt(x**(1/3)*a + b)*a**2*b**4*x - 140*sqrt(x**(1/3)*a + b)*a**4*b**2*x
**2 - 3328*sqrt(x**(1/3)*a + b)*a*b**5*x + 105*x**(2/3)*sqrt(b)*log(sqrt(x
**(1/3)*a + b) - sqrt(b))*a**6*x**2 - 105*x**(2/3)*sqrt(b)*log(sqrt(x**(1/
3)*a + b) + sqrt(b))*a**6*x**2)/(5120*x**(2/3)*b**5*x**2)
```

3.162 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx$

Optimal result . . . 1529
Mathematica [C] (verified) . . . 1530
Rubi [A] (verified) . . . 1530
Maple [A] (verified) . . . 1542
Fricas [F(-1)] . . . 1542
Sympy [F] . . . 1543
Maxima [F] . . . 1543
Giac [A] (verification not implemented) . . . 1543
Mupad [F(-1)] . . . 1544
Reduce [B] (verification not implemented) . . . 1544

Optimal result

Integrand size = 19, antiderivative size = 291

$$\begin{aligned}\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx = & -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}} \\ & +\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}+\frac{429a^5\sqrt{bx^{2/3}+ax}}{71680b^4x^{5/3}} \\ & -\frac{143a^6\sqrt{bx^{2/3}+ax}}{20480b^5x^{4/3}}+\frac{143a^7\sqrt{bx^{2/3}+ax}}{16384b^6x}-\frac{429a^8\sqrt{bx^{2/3}+ax}}{32768b^7x^{2/3}} \\ & -\frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4}+\frac{429a^9\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{32768b^{15/2}}\end{aligned}$$

output
```
-1/16*a*(b*x^(2/3)+a*x)^(1/2)/x^3-1/224*a^2*(b*x^(2/3)+a*x)^(1/2)/b/x^(8/3
)+13/2688*a^3*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(7/3)-143/26880*a^4*(b*x^(2/3)+a
*x)^(1/2)/b^3/x^2+429/71680*a^5*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(5/3)-143/2048
0*a^6*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(4/3)+143/16384*a^7*(b*x^(2/3)+a*x)^(1/2
)/b^6/x-429/32768*a^8*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(2/3)-1/3*(b*x^(2/3)+a*x
)^(3/2)/x^4+429/32768*a^9*arctanh(b^(1/2)*x^(1/3)/(b*x^(2/3)+a*x)^(1/2))/b
^(15/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.21

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx=\frac{6a^9\left(b+a\sqrt[3]{x}\right)^2\sqrt{bx^{2/3}+ax}\,\text{Hypergeometric2F1}\left(\frac{5}{2},10,\frac{7}{2},1+\frac{a\sqrt[3]{x}}{b}\right)}{5b^{10}\sqrt[3]{x}}$$

input
```
Integrate[(b*x^(2/3) + a*x)^(3/2)/x^5,x]
```

output
```
(6*a^9*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 10,
7/2, 1 + (a*x^(1/3))/b])/(5*b^10*x^(1/3))
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {1926, 1926, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^5}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{6}a\int \frac{\sqrt{x^{2/3}b+ax}}{x^4}dx-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{3x^4}$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{6}a\left(\frac{1}{16}a\int \frac{1}{x^3\sqrt{x^{2/3}b+ax}}dx-\frac{3\sqrt{ax+bx^{2/3}}}{8x^3}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{3x^4}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{6}a\left(\frac{1}{16}a\left(-\frac{13a\int \frac{1}{x^{8/3}\sqrt{x^{2/3}b+ax}}dx}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{8x^3}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{3x^4}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{6}a\left(\frac{1}{16}a\left(-\frac{13a\left(-\frac{11a\int \frac{1}{x^{7/3}\sqrt{x^{2/3}b+ax}}dx}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{8x^3}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{3x^4}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{6}a\left(\frac{1}{16}a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\int \frac{1}{x^{2}\sqrt{x^{2/3}b+ax}}dx}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{8x^3}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{3x^4}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{6}a\left(\frac{1}{16}a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\int\frac{1}{x^{5/3}\sqrt{x^{2/3}b+ax}}dx}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)-\right.$$

$$\frac{\left(ax+bx^{2/3}\right)^{3/2}}{3x^4}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{6}a\left(\frac{1}{16}a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\int\frac{1}{x^{4/3}\sqrt{x^{2/3}b+ax}}dx}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{}}{}\right.\right.$$

$$\frac{\left(ax+bx^{2/3}\right)^{3/2}}{3x^4}$$

$\downarrow$ 1931

$$\frac{1}{6}a\left(\frac{1}{16}a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\int\frac{1}{x\sqrt{x^{2/3}b+ax}}dx}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\frac{\sqrt{ax \cdots}}{2 \cdots}\right)}{14b} \cdots$$

↓ 1931

$$\left(\left(\left(13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(-\frac{a\int\frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\cdots}{12b}\cdots\right.\right.\right.\right.$$

↓ 1935

$$\left(\left(\left(\left(11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\int \frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx}}{4bx^{5/}}\cdots}{10b}\cdots\right.\right.\right.\right.\right.$$

$\downarrow$ 219

$$11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/\ldots}}}{4bx^{5/3}}\ldots}{10b}\ldots$$

input
```
Int[(b*x^(2/3) + a*x)^(3/2)/x^5,x]
```

output
```
-1/3*(b*x^(2/3) + a*x)^(3/2)/x^4 + (a*((-3*Sqrt[b*x^(2/3) + a*x])/(8*x^3)
+ (a*((-3*Sqrt[b*x^(2/3) + a*x])/(7*b*x^(8/3)) - (13*a*(-1/2*Sqrt[b*x^(2/3
) + a*x]/(b*x^(7/3)) - (11*a*((-3*Sqrt[b*x^(2/3) + a*x])/(5*b*x^2) - (9*a*
((-3*Sqrt[b*x^(2/3) + a*x])/(4*b*x^(5/3)) - (7*a*(-(Sqrt[b*x^(2/3) + a*x]/
(b*x^(4/3))) - (5*a*((-3*Sqrt[b*x^(2/3) + a*x])/(2*b*x) - (3*a*((-3*Sqrt[b
*x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/
3) + a*x]])/b^(3/2)))/(4*b)))/(6*b)))/(8*b)))/(10*b)))/(12*b)))/(14*b)))/1
6))/6
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.62

method	result
derivativedivides	$-\frac{\left(b\,x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}}\left(45045\left(x^{\frac{1}{3}}a+b\right)^{\frac{17}{2}}b^{\frac{15}{2}}-390390\left(x^{\frac{1}{3}}a+b\right)^{\frac{15}{2}}b^{\frac{17}{2}}+1495494\left(x^{\frac{1}{3}}a+b\right)^{\frac{13}{2}}b^{\frac{19}{2}}-3317886\left(x^{\frac{1}{3}}a+b\right)^{\frac{11}{2}}b\right.}{}$
default	$-\frac{\left(b\,x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}}\left(45045\left(x^{\frac{1}{3}}a+b\right)^{\frac{17}{2}}b^{\frac{15}{2}}-390390\left(x^{\frac{1}{3}}a+b\right)^{\frac{15}{2}}b^{\frac{17}{2}}+1495494\left(x^{\frac{1}{3}}a+b\right)^{\frac{13}{2}}b^{\frac{19}{2}}-3317886\left(x^{\frac{1}{3}}a+b\right)^{\frac{11}{2}}b\right.}{}$

input
```
int((b*x^(2/3)+a*x)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/3440640*(b*x^(2/3)+a*x)^(3/2)*(45045*(x^(1/3)*a+b)^(17/2)*b^(15/2)-3903
90*(x^(1/3)*a+b)^(15/2)*b^(17/2)+1495494*(x^(1/3)*a+b)^(13/2)*b^(19/2)-331
7886*(x^(1/3)*a+b)^(11/2)*b^(21/2)+4685824*(x^(1/3)*a+b)^(9/2)*b^(23/2)-43
49826*(x^(1/3)*a+b)^(7/2)*b^(25/2)+2633274*(x^(1/3)*a+b)^(5/2)*b^(27/2)-45
045*arctanh((x^(1/3)*a+b)^(1/2)/b^(1/2))*b^7*a^9*x^3+390390*(x^(1/3)*a+b)^
(3/2)*b^(29/2)-45045*(x^(1/3)*a+b)^(1/2)*b^(31/2))/x^4/(x^(1/3)*a+b)^(3/2)
/b^(29/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx=\text{Timed out}$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx = \int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x**(2/3)+a*x)**(3/2)/x**5,x)
```

output
```
Integral((a*x + b*x**(2/3))**(3/2)/x**5, x)
```

Maxima [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx = \int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(2/3))^(3/2)/x^5, x)
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.56

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx =$$

$$-\frac{1}{3440640}a^9\left(\frac{45045\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^7}+\frac{45045\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}-390390\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b+1495494\left(ax^{\frac{1}{3}}+\right.}{}\right.$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^5,x, algorithm="giac")
```

output
```
-1/3440640*a^9*(45045*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7)
+ (45045*(a*x^(1/3) + b)^(17/2) - 390390*(a*x^(1/3) + b)^(15/2)*b + 149549
4*(a*x^(1/3) + b)^(13/2)*b^2 - 3317886*(a*x^(1/3) + b)^(11/2)*b^3 + 468582
4*(a*x^(1/3) + b)^(9/2)*b^4 - 4349826*(a*x^(1/3) + b)^(7/2)*b^5 + 2633274*
(a*x^(1/3) + b)^(5/2)*b^6 + 390390*(a*x^(1/3) + b)^(3/2)*b^7 - 45045*sqrt(
a*x^(1/3) + b)*b^8)/(a^9*b^7*x^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx=\int \frac{\left(a\,x+b\,x^{2/3}\right)^{3/2}}{x^5}\,dx$$

input
```
int((a*x + b*x^(2/3))^(3/2)/x^5,x)
```

output
```
int((a*x + b*x^(2/3))^(3/2)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.75

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx=\frac{-48048x^{\frac{8}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,a^6b^3+33280x^{\frac{5}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,a^3b^6-2293760x^{\frac{2}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,b^9-9}{}$$

input
```
int((b*x^(2/3)+a*x)^(3/2)/x^5,x)
```

output
```
( - 48048*x**(2/3)*sqrt(x**(1/3)*a + b)*a**6*b**3*x**2 + 33280*x**(2/3)*sq
rt(x**(1/3)*a + b)*a**3*b**6*x - 2293760*x**(2/3)*sqrt(x**(1/3)*a + b)*b**
9 - 90090*x**(1/3)*sqrt(x**(1/3)*a + b)*a**8*b*x**3 + 41184*x**(1/3)*sqrt(
x**(1/3)*a + b)*a**5*b**4*x**2 - 30720*x**(1/3)*sqrt(x**(1/3)*a + b)*a**2*
b**7*x + 60060*sqrt(x**(1/3)*a + b)*a**7*b**2*x**3 - 36608*sqrt(x**(1/3)*a
 + b)*a**4*b**5*x**2 - 2723840*sqrt(x**(1/3)*a + b)*a*b**8*x - 45045*x**(2
/3)*sqrt(b)*log(sqrt(x**(1/3)*a + b) - sqrt(b))*a**9*x**3 + 45045*x**(2/3)
*sqrt(b)*log(sqrt(x**(1/3)*a + b) + sqrt(b))*a**9*x**3)/(6881280*x**(2/3)*
b**8*x**3)
```

3.163 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx$

Optimal result 1545
Mathematica [C] (verified) 1546
Rubi [A] (verified) 1546
Maple [A] (verified) 1564
Fricas [F(-1)] 1565
Sympy [F(-1)] 1565
Maxima [F] 1565
Giac [A] (verification not implemented) 1566
Mupad [F(-1)] 1566
Reduce [B] (verification not implemented) 1567

Optimal result

Integrand size = 19, antiderivative size = 379

$$\begin{aligned}\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx =& -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}\\&+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\sqrt{bx^{2/3}+ax}}{157696b^4x^{8/3}}\\&-\frac{4199a^6\sqrt{bx^{2/3}+ax}}{1892352b^5x^{7/3}}+\frac{4199a^7\sqrt{bx^{2/3}+ax}}{1720320b^6x^2}-\frac{12597a^8\sqrt{bx^{2/3}+ax}}{4587520b^7x^{5/3}}\\&+\frac{4199a^9\sqrt{bx^{2/3}+ax}}{1310720b^8x^{4/3}}-\frac{4199a^{10}\sqrt{bx^{2/3}+ax}}{1048576b^9x}+\frac{12597a^{11}\sqrt{bx^{2/3}+ax}}{2097152b^{10}x^{2/3}}\\&-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5}-\frac{12597a^{12}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{2097152b^{21/2}}\end{aligned}$$

output
```
-3/88*a*(b*x^(2/3)+a*x)^(1/2)/x^4-3/1760*a^2*(b*x^(2/3)+a*x)^(1/2)/b/x^(11
/3)+19/10560*a^3*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(10/3)-323/168960*a^4*(b*x^(2
/3)+a*x)^(1/2)/b^3/x^3+323/157696*a^5*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(8/3)-41
99/1892352*a^6*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(7/3)+4199/1720320*a^7*(b*x^(2/
3)+a*x)^(1/2)/b^6/x^2-12597/4587520*a^8*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(5/3)+
4199/1310720*a^9*(b*x^(2/3)+a*x)^(1/2)/b^8/x^(4/3)-4199/1048576*a^10*(b*x^
(2/3)+a*x)^(1/2)/b^9/x+12597/2097152*a^11*(b*x^(2/3)+a*x)^(1/2)/b^10/x^(2/
3)-1/4*(b*x^(2/3)+a*x)^(3/2)/x^5-12597/2097152*a^12*arctanh(b^(1/2)*x^(1/3
)/(b*x^(2/3)+a*x)^(1/2))/b^(21/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.16

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx = -\frac{6a^{12}\left(b+a\sqrt[3]{x}\right)^2\sqrt{bx^{2/3}+ax}\,\text{Hypergeometric2F1}\left(\frac{5}{2},13,\frac{7}{2},1+\frac{a\sqrt[3]{x}}{b}\right)}{5b^{13}\sqrt[3]{x}}$$

input
```
Integrate[(b*x^(2/3) + a*x)^(3/2)/x^6,x]
```

output
```
(-6*a^12*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 13
, 7/2, 1 + (a*x^(1/3))/b])/(5*b^13*x^(1/3))
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.14, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {1926, 1926, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^6}\,dx$$

$$\Big\downarrow \ 1926$$

$$\frac{1}{8}a\int \frac{\sqrt{x^{2/3}b+ax}}{x^5}dx-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{4x^5}$$

$$\Big\downarrow \ 1926$$

$$\frac{1}{8}a\left(\frac{1}{22}a\int \frac{1}{x^4\sqrt{x^{2/3}b+ax}}dx-\frac{3\sqrt{ax+bx^{2/3}}}{11x^4}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{4x^5}$$

$$\Big\downarrow \ 1931$$

$$\frac{1}{8}a\left(\frac{1}{22}a\left(-\frac{19a\int \frac{1}{x^{11/3}\sqrt{x^{2/3}b+ax}}dx}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{11x^4}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{4x^5}$$

$$\Big\downarrow \ 1931$$

$$\frac{1}{8}a\left(\frac{1}{22}a\left(-\frac{19a\left(-\frac{17a\int \frac{1}{x^{10/3}\sqrt{x^{2/3}b+ax}}dx}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{11x^4}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{4x^5}$$

$$\Big\downarrow \ 1931$$

$$\frac{1}{8}a\left(\frac{1}{22}a\left(-\frac{19a\left(-\frac{17a\left(-\frac{15a\int \frac{1}{x^3\sqrt{x^{2/3}b+ax}}dx}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{11x^4}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{4x^5}$$

$$\Big\downarrow \ 1931$$

$$\frac{1}{8}a\left(\frac{1}{22}a\left(-\frac{19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\int\frac{1}{x^{8/3}\sqrt{x^{2/3}b+ax}}dx}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^{3}}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}\right)\right.$$

$$\frac{\left(ax+bx^{2/3}\right)^{3/2}}{4x^{5}}$$

$$\Big\downarrow \ 1931$$

$$\frac{1}{8}a\left(\frac{1}{22}a\left(-\frac{19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\int\frac{1}{x^{7/3}\sqrt{x^{2/3}b+ax}}dx}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^{3}}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\right.\right.$$

$$\frac{\left(ax+bx^{2/3}\right)^{3/2}}{4x^5}$$

$\downarrow$ 1931

$$\frac{1}{8}a\left(\frac{1}{22}a\left(-\frac{19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\int\frac{1}{x^2\sqrt{x^{2/3}b+ax}}\,dx}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-}{20b}$$

↓ 1931

$$\left(\left(\left(19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\int\frac{1}{x^{5/3}\sqrt{x^{2/3}b+ax}}dx}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}}{16b}}{18b}$$

↓ 1931

$$17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\int\frac{1}{x^{4/3}\sqrt{x^{2/3}b+ax}}dx}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+}}{2bx}}{14b}}{16b}\right.$$

↓ 1931

$$\left(\left(\left(\left(\left(15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\int\frac{1}{x\sqrt{x^{2/3}b+ax}}dx}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{}}{}\right.}{12b}}{14b}\right.$$

↓ 1931

$$
\left(\left(\left(\left(\left(\left(13a \left(-\frac{11a \left(-\frac{9a \left(-\frac{7a \left(-\frac{5a \left(-\frac{3a \left(-\frac{a \int \frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}\,dx}{2b} - \frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}} \right)}{4b} - \frac{3\sqrt{ax+bx^{2/3}}}{2bx} \right)}{6b} - \frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}} \cdots}{8b} \cdots}{10b} \cdots}{12b} \cdots \right.\right.\right.\right.\right.\right.\right.
$$

↓ 1935

$$11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\int \frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}\,d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax}}{b}}{8b}}{10b}$$

↓ 219

$$5a\left(-\frac{3a\left(\frac{3a\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)$$

$$7a\left(-\frac{\cdots}{6b}-\frac{\sqrt{ax+\cdots}}{bx^{4\cdots}}\right.$$

$$9a\left(-\frac{\cdots}{8b}\right.$$

$$11a\left(-\frac{\cdots}{10b}\right.$$

input
```
Int[(b*x^(2/3) + a*x)^(3/2)/x^6,x]
```

output
```
-1/4*(b*x^(2/3) + a*x)^(3/2)/x^5 + (a*((-3*Sqrt[b*x^(2/3) + a*x])/(11*x^4)
 + (a*((-3*Sqrt[b*x^(2/3) + a*x])/(10*b*x^(11/3)) - (19*a*(-1/3*Sqrt[b*x^(
2/3) + a*x]/(b*x^(10/3)) - (17*a*((-3*Sqrt[b*x^(2/3) + a*x])/(8*b*x^3) - (
15*a*((-3*Sqrt[b*x^(2/3) + a*x])/(7*b*x^(8/3)) - (13*a*(-1/2*Sqrt[b*x^(2/3
) + a*x]/(b*x^(7/3)) - (11*a*((-3*Sqrt[b*x^(2/3) + a*x])/(5*b*x^2) - (9*a*
((-3*Sqrt[b*x^(2/3) + a*x])/(4*b*x^(5/3)) - (7*a*(-(Sqrt[b*x^(2/3) + a*x]/
(b*x^(4/3))) - (5*a*((-3*Sqrt[b*x^(2/3) + a*x])/(2*b*x) - (3*a*((-3*Sqrt[b
*x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/
3) + a*x]])/b^(3/2)))/(4*b)))/(6*b)))/(8*b)))/(10*b)))/(12*b)))/(14*b)))/(
16*b)))/(18*b)))/(20*b)))/22))/8
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(14549535b^{\frac{21}{2}}\left(x^{\frac{1}{3}}a+b\right)^{\frac{23}{2}}-169744575b^{\frac{23}{2}}\left(x^{\frac{1}{3}}a+b\right)^{\frac{21}{2}}+904981077b^{\frac{25}{2}}\left(x^{\frac{1}{3}}a+b\right)^{\frac{19}{2}}-29136483309b^{\frac{27}{2}}\right.}{}$
default	$\frac{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(14549535b^{\frac{21}{2}}\left(x^{\frac{1}{3}}a+b\right)^{\frac{23}{2}}-169744575b^{\frac{23}{2}}\left(x^{\frac{1}{3}}a+b\right)^{\frac{21}{2}}+904981077b^{\frac{25}{2}}\left(x^{\frac{1}{3}}a+b\right)^{\frac{19}{2}}-29136483309b^{\frac{27}{2}}\right.}{}$

input
```
int((b*x^(2/3)+a*x)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/2422210560*(b*x^(2/3)+a*x)^(3/2)*(14549535*b^(21/2)*(x^(1/3)*a+b)^(23/2)
-169744575*b^(23/2)*(x^(1/3)*a+b)^(21/2)+904981077*b^(25/2)*(x^(1/3)*a+b)^
(19/2)-2913648309*b^(27/2)*(x^(1/3)*a+b)^(17/2)+6303782342*b^(29/2)*(x^(1/
3)*a+b)^(15/2)-9643633350*b^(31/2)*(x^(1/3)*a+b)^(13/2)+10677769530*b^(33/
2)*(x^(1/3)*a+b)^(11/2)-8598579770*b^(35/2)*(x^(1/3)*a+b)^(9/2)+4975837515
*b^(37/2)*(x^(1/3)*a+b)^(7/2)-2001671595*b^(39/2)*(x^(1/3)*a+b)^(5/2)-1454
9535*arctanh((x^(1/3)*a+b)^(1/2)/b^(1/2))*b^10*a^12*x^4-169744575*b^(41/2)
*(x^(1/3)*a+b)^(3/2)+14549535*b^(43/2)*(x^(1/3)*a+b)^(1/2))/x^5/(x^(1/3)*a
+b)^(3/2)/b^(41/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((b*x**(2/3)+a*x)**(3/2)/x**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx = \int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(2/3))^(3/2)/x^6, x)
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.65

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx = \frac{\frac{14549535\,a^{13}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^{10}} + \frac{14549535\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}}a^{13} - 169744575\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}a^{13}b + 904981077\left(ax^{\frac{1}{3}}\right.}{}}{}$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^6,x, algorithm="giac")
```

output
```
1/2422210560*(14549535*a^13*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)
*b^10) + (14549535*(a*x^(1/3) + b)^(23/2)*a^13 - 169744575*(a*x^(1/3) + b)
^(21/2)*a^13*b + 904981077*(a*x^(1/3) + b)^(19/2)*a^13*b^2 - 2913648309*(a
*x^(1/3) + b)^(17/2)*a^13*b^3 + 6303782342*(a*x^(1/3) + b)^(15/2)*a^13*b^4
 - 9643633350*(a*x^(1/3) + b)^(13/2)*a^13*b^5 + 10677769530*(a*x^(1/3) + b
)^(11/2)*a^13*b^6 - 8598579770*(a*x^(1/3) + b)^(9/2)*a^13*b^7 + 4975837515
*(a*x^(1/3) + b)^(7/2)*a^13*b^8 - 2001671595*(a*x^(1/3) + b)^(5/2)*a^13*b^
9 - 169744575*(a*x^(1/3) + b)^(3/2)*a^13*b^10 + 14549535*sqrt(a*x^(1/3) +
b)*a^13*b^11)/(a^12*b^10*x^4))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx = \int \frac{\left(a\,x+b\,x^{2/3}\right)^{3/2}}{x^6}\,dx$$

input
```
int((a*x + b*x^(2/3))^(3/2)/x^6,x)
```

output
```
int((a*x + b*x^(2/3))^(3/2)/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.72

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx = \frac{15519504x^{\frac{11}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,a^9b^3-10749440x^{\frac{8}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,a^6b^6+8716288x^{\frac{5}{3}}\sqrt{x^{\frac{1}{3}}a+b}$$

input
```
int((b*x^(2/3)+a*x)^(3/2)/x^6,x)
```

output
```
(15519504*x**(2/3)*sqrt(x**(1/3)*a + b)*a**9*b**3*x**3 - 10749440*x**(2/3)
*sqrt(x**(1/3)*a + b)*a**6*b**6*x**2 + 8716288*x**(2/3)*sqrt(x**(1/3)*a +
b)*a**3*b**9*x - 1211105280*x**(2/3)*sqrt(x**(1/3)*a + b)*b**12 + 29099070
*x**(1/3)*sqrt(x**(1/3)*a + b)*a**11*b*x**4 - 13302432*x**(1/3)*sqrt(x**(1
/3)*a + b)*a**8*b**4*x**3 + 9922560*x**(1/3)*sqrt(x**(1/3)*a + b)*a**5*b**
7*x**2 - 8257536*x**(1/3)*sqrt(x**(1/3)*a + b)*a**2*b**10*x - 19399380*sqr
t(x**(1/3)*a + b)*a**10*b**2*x**4 + 11824384*sqrt(x**(1/3)*a + b)*a**7*b**
5*x**3 - 9261056*sqrt(x**(1/3)*a + b)*a**4*b**8*x**2 - 1376256000*sqrt(x**
(1/3)*a + b)*a*b**11*x + 14549535*x**(2/3)*sqrt(b)*log(sqrt(x**(1/3)*a + b
) - sqrt(b))*a**12*x**4 - 14549535*x**(2/3)*sqrt(b)*log(sqrt(x**(1/3)*a +
b) + sqrt(b))*a**12*x**4)/(4844421120*x**(2/3)*b**11*x**4)
```

3.164 $\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx$

Optimal result 1568
Mathematica [A] (verified) 1569
Rubi [A] (verified) 1569
Maple [A] (verified) 1590
Fricas [B] (verification not implemented) 1590
Sympy [F] 1591
Maxima [F] 1592
Giac [A] (verification not implemented) 1592
Mupad [F(-1)] 1593
Reduce [B] (verification not implemented) 1593

Optimal result

Integrand size = 19, antiderivative size = 401

$$\begin{aligned}\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx =& \frac{8388608b^{12}\sqrt{bx^{2/3}+ax}}{11700675a^{13}} - \frac{16777216b^{13}\sqrt{bx^{2/3}+ax}}{11700675a^{14}\sqrt[3]{x}} \\ &- \frac{2097152b^{11}\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{3900225a^{12}} + \frac{1048576b^{10}x^{2/3}\sqrt{bx^{2/3}+ax}}{2340135a^{11}} \\ &- \frac{131072b^9x\sqrt{bx^{2/3}+ax}}{334305a^{10}} + \frac{65536b^8x^{4/3}\sqrt{bx^{2/3}+ax}}{185725a^9} \\ &- \frac{180224b^7x^{5/3}\sqrt{bx^{2/3}+ax}}{557175a^8} + \frac{1171456b^6x^2\sqrt{bx^{2/3}+ax}}{3900225a^7} \\ &- \frac{73216b^5x^{7/3}\sqrt{bx^{2/3}+ax}}{260015a^6} + \frac{36608b^4x^{8/3}\sqrt{bx^{2/3}+ax}}{137655a^5} - \frac{9152b^3x^3\sqrt{bx^{2/3}+ax}}{36225a^4} \\ &+ \frac{416b^2x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^3} - \frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^2} + \frac{2x^4\sqrt{bx^{2/3}+ax}}{9a}\end{aligned}$$

output
```
8388608/11700675*b^12*(b*x^(2/3)+a*x)^(1/2)/a^13-16777216/11700675*b^13*(b
*x^(2/3)+a*x)^(1/2)/a^14/x^(1/3)-2097152/3900225*b^11*x^(1/3)*(b*x^(2/3)+a
*x)^(1/2)/a^12+1048576/2340135*b^10*x^(2/3)*(b*x^(2/3)+a*x)^(1/2)/a^11-131
072/334305*b^9*x*(b*x^(2/3)+a*x)^(1/2)/a^10+65536/185725*b^8*x^(4/3)*(b*x^
(2/3)+a*x)^(1/2)/a^9-180224/557175*b^7*x^(5/3)*(b*x^(2/3)+a*x)^(1/2)/a^8+1
171456/3900225*b^6*x^2*(b*x^(2/3)+a*x)^(1/2)/a^7-73216/260015*b^5*x^(7/3)*
(b*x^(2/3)+a*x)^(1/2)/a^6+36608/137655*b^4*x^(8/3)*(b*x^(2/3)+a*x)^(1/2)/a
^5-9152/36225*b^3*x^3*(b*x^(2/3)+a*x)^(1/2)/a^4+416/1725*b^2*x^(10/3)*(b*x
^(2/3)+a*x)^(1/2)/a^3-52/225*b*x^(11/3)*(b*x^(2/3)+a*x)^(1/2)/a^2+2/9*x^4*
(b*x^(2/3)+a*x)^(1/2)/a
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.46

$$\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2\sqrt{bx^{2/3}+ax}\left(-8388608b^{13}+4194304ab^{12}\sqrt[3]{x}-3145728a^2b^{11}x^{2/3}+2621440a^3b^{10}x\right.}{}$$

input
```
Integrate[x^4/Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(2/3) + a*x]*(-8388608*b^13 + 4194304*a*b^12*x^(1/3) - 3145728
*a^2*b^11*x^(2/3) + 2621440*a^3*b^10*x - 2293760*a^4*b^9*x^(4/3) + 2064384
*a^5*b^8*x^(5/3) - 1892352*a^6*b^7*x^2 + 1757184*a^7*b^6*x^(7/3) - 1647360
*a^8*b^5*x^(8/3) + 1555840*a^9*b^4*x^3 - 1478048*a^10*b^3*x^(10/3) + 14108
64*a^11*b^2*x^(11/3) - 1352078*a^12*b*x^4 + 1300075*a^13*x^(13/3)))/(11700
675*a^14*x^(1/3))
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.17, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {1922, 1922, 1922, 1922, 1922, 1922, 1922, 1922, 1922, 1922, 1922, 1922, 1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{ax+bx^{2/3}}}\,dx$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a}-\frac{26b\int \frac{x^{11/3}}{\sqrt{x^{2/3}b+ax}}dx}{27a}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a}-\frac{26b\left(\frac{6x^{11/3}\sqrt{ax+bx^{2/3}}}{25a}-\frac{24b\int \frac{x^{10/3}}{\sqrt{x^{2/3}b+ax}}dx}{25a}\right)}{27a}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a}-\frac{26b\left(\frac{6x^{11/3}\sqrt{ax+bx^{2/3}}}{25a}-\frac{24b\left(\frac{6x^{10/3}\sqrt{ax+bx^{2/3}}}{23a}-\frac{22b\int \frac{x^3}{\sqrt{x^{2/3}b+ax}}dx}{23a}\right)}{25a}\right)}{27a}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a}-\frac{26b\left(\frac{6x^{11/3}\sqrt{ax+bx^{2/3}}}{25a}-\frac{24b\left(\frac{6x^{10/3}\sqrt{ax+bx^{2/3}}}{23a}-\frac{22b\left(\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\int \frac{x^{8/3}}{\sqrt{x^{2/3}b+ax}}dx}{21a}\right)}{23a}\right)}{25a}\right)}{27a}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a}-\frac{26b\left(\frac{6x^{11/3}\sqrt{ax+bx^{2/3}}}{25a}-\frac{24b\left(\frac{6x^{10/3}\sqrt{ax+bx^{2/3}}}{23a}-\frac{22b\left(\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a}-\frac{18b\int\frac{x^{7/3}}{\sqrt{x^{2/3}b+ax}}dx}{19a}\right)}{21a}\right)}{23a}\right)}{25a}\right)}{27a}$$

$\downarrow$ 1922

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a} - \frac{26b\left(\frac{6x^{11/3}\sqrt{ax+bx^{2/3}}}{25a} - \frac{24b\left(\frac{6x^{10/3}\sqrt{ax+bx^{2/3}}}{23a} - \frac{22b\left(\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a} - \frac{20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a} - \frac{18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a} - \frac{16b\int\frac{x^2}{\sqrt{x^{2/3}b+ax}}dx}{17a}\right.}{19a}\right.}{21a}\right.}{23a}\right.}{25a}\right.}{27a}$$

$\downarrow$ 1922

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a} -$$

$$16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}\right.$$

$$18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a} - \frac{}{}\right.$$

$$20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a} - \frac{}{19a}\right.$$

$$22b\left(\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a} - \frac{}{21a}\right.$$

$$24b\left(\frac{6x^{10/3}\sqrt{ax+bx^{2/3}}}{23a} - \frac{}{23a}\right.$$

$$26b\left(\frac{6x^{11/3}\sqrt{ax+bx^{2/3}}}{25a} - \frac{}{25a}\right.$$

↓ 1922

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a} -$$

$$16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}\right.$$

$$18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a} - \right.$$

$$20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a} - \right.$$

$$22b\left(\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a} - \frac{\quad}{21a}\right.$$

$$24b\left(\frac{6x^{10/3}\sqrt{ax+bx^{2/3}}}{23a} - \frac{\quad}{23a}\right.$$

↓ 1922

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a} -$$

$$16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}\right.$$

$$18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a} - \right.$$

$$20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a} - \right.$$

$$22b\left(\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a} - \right.$$

↓ 1922

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a} -$$

$$16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}\right.$$

$$18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a} - \right.$$

$$20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a} - \right.$$

↓ 1922

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a}-$$

$$16b\ \frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}$$

↓ 1922

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a} -$$

$$16b \quad \frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}$$

↓ 1908

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a} -$$

↓ 1920

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a} -$$

input
```
Int[x^4/Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*x^4*Sqrt[b*x^(2/3) + a*x])/(9*a) - (26*b*((6*x^(11/3)*Sqrt[b*x^(2/3) +
a*x])/(25*a) - (24*b*((6*x^(10/3)*Sqrt[b*x^(2/3) + a*x])/(23*a) - (22*b*((
2*x^3*Sqrt[b*x^(2/3) + a*x])/(7*a) - (20*b*((6*x^(8/3)*Sqrt[b*x^(2/3) + a*
x])/(19*a) - (18*b*((6*x^(7/3)*Sqrt[b*x^(2/3) + a*x])/(17*a) - (16*b*((2*x
^2*Sqrt[b*x^(2/3) + a*x])/(5*a) - (14*b*((6*x^(5/3)*Sqrt[b*x^(2/3) + a*x])
/(13*a) - (12*b*((6*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(11*a) - (10*b*((2*x*Sq
rt[b*x^(2/3) + a*x])/(3*a) - (8*b*((6*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(7*a)
 - (6*b*((6*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(5*a) - (4*b*((2*Sqrt[b*x^(2/3)
 + a*x])/a - (4*b*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))))/(5*a)))/(7*a)))/(
9*a)))/(11*a)))/(13*a)))/(15*a)))/(17*a)))/(19*a)))/(21*a)))/(23*a)))/(25*
a)))/(27*a)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.42

method	result
derivativedivides	$\frac{2x^{\frac{1}{3}}\left(x^{\frac{1}{3}}a+b\right)\left(1300075a^{13}x^{\frac{13}{3}}-1352078a^{12}b\,x^{4}+1410864a^{11}b^{2}x^{\frac{11}{3}}-1478048a^{10}b^{3}x^{\frac{10}{3}}+1555840a^{9}b^{4}x^{3}-1647360a\right.}{}$
default	$\frac{2x^{\frac{1}{3}}\left(x^{\frac{1}{3}}a+b\right)\left(1300075a^{13}x^{\frac{13}{3}}-1352078a^{12}b\,x^{4}+1410864a^{11}b^{2}x^{\frac{11}{3}}-1478048a^{10}b^{3}x^{\frac{10}{3}}+1555840a^{9}b^{4}x^{3}-1647360a\right.}{}$

input
```
int(x^4/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/11700675*x^(1/3)*(x^(1/3)*a+b)*(1300075*a^13*x^(13/3)-1352078*a^12*b*x^4
+1410864*a^11*b^2*x^(11/3)-1478048*a^10*b^3*x^(10/3)+1555840*a^9*b^4*x^3-1
647360*a^8*b^5*x^(8/3)+1757184*a^7*b^6*x^(7/3)-1892352*a^6*b^7*x^2+2064384
*a^5*b^8*x^(5/3)-2293760*a^4*b^9*x^(4/3)+2621440*a^3*b^10*x-3145728*a^2*b^
11*x^(2/3)+4194304*a*b^12*x^(1/3)-8388608*b^13)/(b*x^(2/3)+a*x)^(1/2)/a^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1294 vs. 2(299) = 598.

Time = 133.15 (sec) , antiderivative size = 1294, normalized size of antiderivative = 3.23

$$\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
1/11700675*((211106232532992*b^19 + 43980465111040*b^18 + 206158430208*(64
*a^3 - 3)*b^16 - 4123168604160*b^17 - 1073741824*(11264*a^3 - 53)*b^15 + 1
5143273600*a^15 - 402653184*(5504*a^3 + 1)*b^14 + 12582912*(3194880*a^6 -
114688*a^3 - 3)*b^13 + 469762048*(18816*a^6 + 103*a^3)*b^12 - 50331648*(48
816*a^6 + 23*a^3)*b^11 - 786432*(45731840*a^9 - 495872*a^6 - 15*a^3)*b^10
- 7340032*(1349120*a^9 + 3439*a^6)*b^9 + 250478592*(5600*a^9 + 3*a^6)*b^8
+ 12288*(2616979456*a^12 - 21542400*a^9 - 693*a^6)*b^7 + 212992*(43743616*
a^12 + 89111*a^9)*b^6 - 638976*(1652476*a^12 + 935*a^9)*b^5 + 3264*(360854
3232*a^15 + 64599808*a^12 + 2145*a^9)*b^4 + 578816*(13049856*a^15 - 27313*
a^12)*b^3 + 217056*(6211584*a^15 + 2353*a^12)*b^2 - 156009*(2547712*a^15 +
 39*a^12)*b)*x + 2*(1300075*(16777216*a^13*b^6 + 6291456*a^13*b^5 + 196608
*a^13*b^4 - 262144*a^16 - 114688*a^13*b^3 - 2304*a^13*b^2 + 864*a^13*b - 2
7*a^13)*x^5 - 1478048*(16777216*a^10*b^9 + 6291456*a^10*b^8 + 196608*a^10*
b^7 - 114688*a^10*b^6 - 2304*a^10*b^5 + 864*a^10*b^4 - (262144*a^13 + 27*a
^10)*b^3)*x^4 + 1757184*(16777216*a^7*b^12 + 6291456*a^7*b^11 + 196608*a^7
*b^10 - 114688*a^7*b^9 - 2304*a^7*b^8 + 864*a^7*b^7 - (262144*a^10 + 27*a^
7)*b^6)*x^3 - 2293760*(16777216*a^4*b^15 + 6291456*a^4*b^14 + 196608*a^4*b
^13 - 114688*a^4*b^12 - 2304*a^4*b^11 + 864*a^4*b^10 - (262144*a^7 + 27*a^
4)*b^9)*x^2 + 4194304*(16777216*a*b^18 + 6291456*a*b^17 + 196608*a*b^16 -
114688*a*b^15 - 2304*a*b^14 + 864*a*b^13 - (262144*a^4 + 27*a)*b^12)*x ...
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx=\int \frac{x^4}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(x**4/(b*x**(2/3)+a*x)**(1/2),x)
```

output
```
Integral(x**4/sqrt(a*x + b*x**(2/3)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{x^4}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(a*x + b*x^(2/3)), x)
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.51

$$\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{16777216\,b^{\frac{27}{2}}}{11700675\,a^{14}} + \frac{2\left(1300075\left(ax^{\frac{1}{3}}+b\right)^{\frac{27}{2}} - 18253053\left(ax^{\frac{1}{3}}+b\right)^{\frac{25}{2}}b + 119041650\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}}b^2 - 478056150\left(ax^{\frac{1}{3}}+b\right)\right.}{}$$

input
```
integrate(x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
16777216/11700675*b^(27/2)/a^14 + 2/11700675*(1300075*(a*x^(1/3) + b)^(27/
2) - 18253053*(a*x^(1/3) + b)^(25/2)*b + 119041650*(a*x^(1/3) + b)^(23/2)*
b^2 - 478056150*(a*x^(1/3) + b)^(21/2)*b^3 + 1320944625*(a*x^(1/3) + b)^(1
9/2)*b^4 - 2657429775*(a*x^(1/3) + b)^(17/2)*b^5 + 4015671660*(a*x^(1/3) +
 b)^(15/2)*b^6 - 4633467300*(a*x^(1/3) + b)^(13/2)*b^7 + 4106936925*(a*x^(
1/3) + b)^(11/2)*b^8 - 2788660875*(a*x^(1/3) + b)^(9/2)*b^9 + 1434168450*(
a*x^(1/3) + b)^(7/2)*b^10 - 547591590*(a*x^(1/3) + b)^(5/2)*b^11 + 1521087
75*(a*x^(1/3) + b)^(3/2)*b^12 - 35102025*sqrt(a*x^(1/3) + b)*b^13)/a^14
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx=\int \frac{x^4}{\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

input
```
int(x^4/(a*x + b*x^(2/3))^(1/2),x)
```

output
```
int(x^4/(a*x + b*x^(2/3))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.38

$$\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx=\frac{2\sqrt{x^{\frac{1}{3}}a+b}\,\left(1410864x^{\frac{11}{3}}a^{11}b^2-1647360x^{\frac{8}{3}}a^8b^5+2064384x^{\frac{5}{3}}a^5b^8-3145728x^{\frac{2}{3}}a^2b^{11}\right.}{}$$

input
```
int(x^4/(b*x^(2/3)+a*x)^(1/2),x)
```

output
```
(2*sqrt(x**(1/3)*a + b)*(1410864*x**(2/3)*a**11*b**2*x**3 - 1647360*x**(2/
3)*a**8*b**5*x**2 + 2064384*x**(2/3)*a**5*b**8*x - 3145728*x**(2/3)*a**2*b
**11 + 1300075*x**(1/3)*a**13*x**4 - 1478048*x**(1/3)*a**10*b**3*x**3 + 17
57184*x**(1/3)*a**7*b**6*x**2 - 2293760*x**(1/3)*a**4*b**9*x + 4194304*x**
(1/3)*a*b**12 - 1352078*a**12*b*x**4 + 1555840*a**9*b**4*x**3 - 1892352*a*
*6*b**7*x**2 + 2621440*a**3*b**10*x - 8388608*b**13))/(11700675*a**14)
```

3.165 $\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx$

Optimal result 1594
Mathematica [A] (verified) 1595
Rubi [A] (verified) 1595
Maple [A] (verified) 1609
Fricas [B] (verification not implemented) 1609
Sympy [F] 1610
Maxima [F] 1611
Giac [A] (verification not implemented) 1611
Mupad [F(-1)] 1612
Reduce [B] (verification not implemented) 1612

Optimal result

Integrand size = 19, antiderivative size = 313

$$\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx = -\frac{262144b^9\sqrt{bx^{2/3}+ax}}{323323a^{10}} + \frac{524288b^{10}\sqrt{bx^{2/3}+ax}}{323323a^{11}\sqrt[3]{x}} + \frac{196608b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{323323a^9} - \frac{163840b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{323323a^8} + \frac{20480b^6x\sqrt{bx^{2/3}+ax}}{46189a^7} - \frac{18432b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{46189a^6} + \frac{1536b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^5} - \frac{768b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^4} + \frac{720b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^3} - \frac{40bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^2} + \frac{2x^3\sqrt{bx^{2/3}+ax}}{7a}$$

output
```
-262144/323323*b^9*(b*x^(2/3)+a*x)^(1/2)/a^10+524288/323323*b^10*(b*x^(2/3
)+a*x)^(1/2)/a^11/x^(1/3)+196608/323323*b^8*x^(1/3)*(b*x^(2/3)+a*x)^(1/2)/
a^9-163840/323323*b^7*x^(2/3)*(b*x^(2/3)+a*x)^(1/2)/a^8+20480/46189*b^6*x*
(b*x^(2/3)+a*x)^(1/2)/a^7-18432/46189*b^5*x^(4/3)*(b*x^(2/3)+a*x)^(1/2)/a^
6+1536/4199*b^4*x^(5/3)*(b*x^(2/3)+a*x)^(1/2)/a^5-768/2261*b^3*x^2*(b*x^(2
/3)+a*x)^(1/2)/a^4+720/2261*b^2*x^(7/3)*(b*x^(2/3)+a*x)^(1/2)/a^3-40/133*b
*x^(8/3)*(b*x^(2/3)+a*x)^(1/2)/a^2+2/7*x^3*(b*x^(2/3)+a*x)^(1/2)/a
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.47

$$\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2\sqrt{bx^{2/3}+ax}\left(262144b^{10}-131072ab^9\sqrt[3]{x}+98304a^2b^8x^{2/3}-81920a^3b^7x+71680a^4b\right.}{}$$

input
```
Integrate[x^3/Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(2/3) + a*x]*(262144*b^10 - 131072*a*b^9*x^(1/3) + 98304*a^2*b
^8*x^(2/3) - 81920*a^3*b^7*x + 71680*a^4*b^6*x^(4/3) - 64512*a^5*b^5*x^(5/
3) + 59136*a^6*b^4*x^2 - 54912*a^7*b^3*x^(7/3) + 51480*a^8*b^2*x^(8/3) - 4
8620*a^9*b*x^3 + 46189*a^10*x^(10/3)))/(323323*a^11*x^(1/3))
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {1922, 1922, 1922, 1922, 1922, 1922, 1922, 1922, 1922, 1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{ax+bx^{2/3}}}dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\int \frac{x^{8/3}}{\sqrt{x^{2/3}b+ax}}dx}{21a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a}-\frac{18b\int \frac{x^{7/3}}{\sqrt{x^{2/3}b+ax}}dx}{19a}\right)}{21a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a}-\frac{18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a}-\frac{16b\int\frac{x^2}{\sqrt{x^{2/3}b+ax}}dx}{17a}\right)}{19a}\right)}{21a}$$

$$\Big\downarrow \ 1922$$

$$\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a}-\frac{18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a}-\frac{16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\int\frac{x^{5/3}}{\sqrt{x^{2/3}b+ax}}dx}{15a}\right)}{17a}\right)}{19a}\right)}{21a}$$

$$\Big\downarrow \ 1922$$

$$\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a}-\frac{18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a}-\frac{16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\int\frac{x^{4/3}}{\sqrt{x^{2/3}b+ax}}dx}{13a}\right)}{15a}\right)}{17a}\right)}{19a}\right)}{21a}$$

$$\Big\downarrow \ 1922$$

$$\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a}-\frac{18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a}-\frac{16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\frac{10b\int\frac{x}{\sqrt{x^{2/3}b+ax}}dx}{11a}\right)}{13a}\right)}{15a}\right)}{17a}\right)}{19a}\right)}{21a}$$

$\downarrow$ 1922

$$\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a} -$$

$$10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a} - \cdots 8\right.$$

$$12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a} - \frac{\cdots}{11a}\right.$$

$$14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a} - \frac{\cdots}{13a}\right.$$

$$16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a} - \frac{\cdots}{15a}\right.$$

$$18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a} - \frac{\cdots}{17a}\right.$$

$$20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a} - \frac{\cdots}{19a}\right.$$

↓ 1922

$$\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a} -$$

$$10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a} - \cdots\right.$$

$$12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a} - \cdots\right.$$

$$14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a} - \cdots\right.$$

$$16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a} - \frac{\cdots}{15a}\right.$$

↓ 1922

$$\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-$$

$$16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\right.$$

$$14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\right.$$

$$12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\right.$$

$$10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\right.$$

8

↓ 1908

$$\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a} -$$

$$10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a} - \right.$$

$$12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a} - \right.$$

$$14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a} - \right.$$

↓ 1920

$$\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a} -$$

8

$$10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a} - \cdots\right.$$

$$12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a} - \right.$$

$$14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a} - \right.$$

input
```
Int[x^3/Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*x^3*Sqrt[b*x^(2/3) + a*x])/(7*a) - (20*b*((6*x^(8/3)*Sqrt[b*x^(2/3) + a
*x])/(19*a) - (18*b*((6*x^(7/3)*Sqrt[b*x^(2/3) + a*x])/(17*a) - (16*b*((2*
x^2*Sqrt[b*x^(2/3) + a*x])/(5*a) - (14*b*((6*x^(5/3)*Sqrt[b*x^(2/3) + a*x]
)/(13*a) - (12*b*((6*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(11*a) - (10*b*((2*x*S
qrt[b*x^(2/3) + a*x])/(3*a) - (8*b*((6*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(7*a
) - (6*b*((6*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(5*a) - (4*b*((2*Sqrt[b*x^(2/3
) + a*x])/a - (4*b*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))))/(5*a)))/(7*a)))/
(9*a)))/(11*a)))/(13*a)))/(15*a)))/(17*a)))/(19*a)))/(21*a)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.43

method	result
derivativedivides	$\frac{2x^{\frac{1}{3}}\left(x^{\frac{1}{3}}a+b\right)\left(46189a^{10}x^{\frac{10}{3}}-48620a^{9}b\,x^{3}+51480a^{8}b^{2}x^{\frac{8}{3}}-54912a^{7}b^{3}x^{\frac{7}{3}}+59136a^{6}b^{4}x^{2}-64512a^{5}b^{5}x^{\frac{5}{3}}+71680a^{4}b^{6}\right.}{323323\sqrt{b\,x^{\frac{2}{3}}+ax}\,a^{11}}$
default	$\frac{2x^{\frac{1}{3}}\left(x^{\frac{1}{3}}a+b\right)\left(46189a^{10}x^{\frac{10}{3}}-48620a^{9}b\,x^{3}+51480a^{8}b^{2}x^{\frac{8}{3}}-54912a^{7}b^{3}x^{\frac{7}{3}}+59136a^{6}b^{4}x^{2}-64512a^{5}b^{5}x^{\frac{5}{3}}+71680a^{4}b^{6}\right.}{323323\sqrt{b\,x^{\frac{2}{3}}+ax}\,a^{11}}$

input
```
int(x^3/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/323323*x^(1/3)*(x^(1/3)*a+b)*(46189*a^10*x^(10/3)-48620*a^9*b*x^3+51480*
a^8*b^2*x^(8/3)-54912*a^7*b^3*x^(7/3)+59136*a^6*b^4*x^2-64512*a^5*b^5*x^(5
/3)+71680*a^4*b^6*x^(4/3)-81920*a^3*b^7*x+98304*a^2*b^8*x^(2/3)-131072*a*b
^9*x^(1/3)+262144*b^10)/(b*x^(2/3)+a*x)^(1/2)/a^11
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. 2(233) = 466.

Time = 125.56 (sec) , antiderivative size = 1031, normalized size of antiderivative = 3.29

$$\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/323323*((3298534883328*b^16 + 687194767360*b^15 + 3221225472*(64*a^3 -
3)*b^13 - 64424509440*b^14 - 16777216*(11264*a^3 - 53)*b^12 - 269004736*a^
12 - 6291456*(5504*a^3 + 1)*b^11 + 196608*(3194880*a^6 - 114688*a^3 - 3)*b
^10 + 7340032*(18816*a^6 + 103*a^3)*b^9 - 786432*(48816*a^6 + 23*a^3)*b^8
- 12288*(45731840*a^9 - 495872*a^6 - 15*a^3)*b^7 - 114688*(1349120*a^9 + 3
439*a^6)*b^6 + 3913728*(5600*a^9 + 3*a^6)*b^5 - 2112*(101384192*a^12 + 195
8400*a^9 + 63*a^6)*b^4 - 36608*(3784704*a^12 - 8101*a^9)*b^3 - 109824*(226
688*a^12 + 85*a^9)*b^2 + 7293*(974848*a^12 + 15*a^9)*b)*x - (46189*(167772
16*a^10*b^6 + 6291456*a^10*b^5 + 196608*a^10*b^4 - 262144*a^13 - 114688*a^
10*b^3 - 2304*a^10*b^2 + 864*a^10*b - 27*a^10)*x^4 - 54912*(16777216*a^7*b
^9 + 6291456*a^7*b^8 + 196608*a^7*b^7 - 114688*a^7*b^6 - 2304*a^7*b^5 + 86
4*a^7*b^4 - (262144*a^10 + 27*a^7)*b^3)*x^3 + 71680*(16777216*a^4*b^12 + 6
291456*a^4*b^11 + 196608*a^4*b^10 - 114688*a^4*b^9 - 2304*a^4*b^8 + 864*a^
4*b^7 - (262144*a^7 + 27*a^4)*b^6)*x^2 - 131072*(16777216*a*b^15 + 6291456
*a*b^14 + 196608*a*b^13 - 114688*a*b^12 - 2304*a*b^11 + 864*a*b^10 - (2621
44*a^4 + 27*a)*b^9)*x + 4*(1099511627776*b^16 + 412316860416*b^15 + 128849
01888*b^14 - 7516192768*b^13 - 150994944*b^12 - 65536*(262144*a^3 + 27)*b^
10 + 56623104*b^11 - 12155*(16777216*a^9*b^7 + 6291456*a^9*b^6 + 196608*a^
9*b^5 - 114688*a^9*b^4 - 2304*a^9*b^3 + 864*a^9*b^2 - (262144*a^12 + 27*a^
9)*b)*x^3 + 14784*(16777216*a^6*b^10 + 6291456*a^6*b^9 + 196608*a^6*b^8...
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{bx^{2/3} + ax}}\,dx = \int \frac{x^3}{\sqrt{ax + bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(x**3/(b*x**(2/3)+a*x)**(1/2),x)
```

output
```
Integral(x**3/sqrt(a*x + b*x**(2/3)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{x^3}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(a*x + b*x^(2/3)), x)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.52

$$\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx = -\frac{524288\,b^{\frac{21}{2}}}{323323\,a^{11}} + \frac{2\left(46189\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}} - 510510\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}b + 2567565\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b^2 - 7759752\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^3 + 15\right.}{}$$

input
```
integrate(x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
-524288/323323*b^(21/2)/a^11 + 2/323323*(46189*(a*x^(1/3) + b)^(21/2) - 51
0510*(a*x^(1/3) + b)^(19/2)*b + 2567565*(a*x^(1/3) + b)^(17/2)*b^2 - 77597
52*(a*x^(1/3) + b)^(15/2)*b^3 + 15668730*(a*x^(1/3) + b)^(13/2)*b^4 - 2222
1108*(a*x^(1/3) + b)^(11/2)*b^5 + 22632610*(a*x^(1/3) + b)^(9/2)*b^6 - 166
28040*(a*x^(1/3) + b)^(7/2)*b^7 + 8729721*(a*x^(1/3) + b)^(5/2)*b^8 - 3233
230*(a*x^(1/3) + b)^(3/2)*b^9 + 969969*sqrt(a*x^(1/3) + b)*b^10)/a^11
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{x^3}{\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

input
```
int(x^3/(a*x + b*x^(2/3))^(1/2),x)
```

output
```
int(x^3/(a*x + b*x^(2/3))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.38

$$\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2\sqrt{x^{\frac{1}{3}}a+b}\,\left(51480x^{\frac{8}{3}}a^8b^2-64512x^{\frac{5}{3}}a^5b^5+98304x^{\frac{2}{3}}a^2b^8+46189x^{\frac{10}{3}}a^{10}-54912x^{\frac{7}{3}}\right.}{}$$

input
```
int(x^3/(b*x^(2/3)+a*x)^(1/2),x)
```

output
```
(2*sqrt(x**(1/3)*a + b)*(51480*x**(2/3)*a**8*b**2*x**2 - 64512*x**(2/3)*a*
*5*b**5*x + 98304*x**(2/3)*a**2*b**8 + 46189*x**(1/3)*a**10*x**3 - 54912*x
**(1/3)*a**7*b**3*x**2 + 71680*x**(1/3)*a**4*b**6*x - 131072*x**(1/3)*a*b*
*9 - 48620*a**9*b*x**3 + 59136*a**6*b**4*x**2 - 81920*a**3*b**7*x + 262144
*b**10))/(323323*a**11)
```

3.166 $\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx$

Optimal result 1613
Mathematica [A] (verified) 1614
Rubi [A] (verified) 1614
Maple [A] (verified) 1622
Fricas [B] (verification not implemented) 1622
Sympy [F] 1623
Maxima [F] 1624
Giac [A] (verification not implemented) 1624
Mupad [F(-1)] 1625
Reduce [B] (verification not implemented) 1625

Optimal result

Integrand size = 19, antiderivative size = 225

$$\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2048b^6\sqrt{bx^{2/3}+ax}}{2145a^7} - \frac{4096b^7\sqrt{bx^{2/3}+ax}}{2145a^8\sqrt[3]{x}} - \frac{512b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^6} + \frac{256b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^5} - \frac{224b^3x\sqrt{bx^{2/3}+ax}}{429a^4} + \frac{336b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^3} - \frac{28bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^2} + \frac{2x^2\sqrt{bx^{2/3}+ax}}{5a}$$

output
```
2048/2145*b^6*(b*x^(2/3)+a*x)^(1/2)/a^7-4096/2145*b^7*(b*x^(2/3)+a*x)^(1/2
)/a^8/x^(1/3)-512/715*b^5*x^(1/3)*(b*x^(2/3)+a*x)^(1/2)/a^6+256/429*b^4*x^
(2/3)*(b*x^(2/3)+a*x)^(1/2)/a^5-224/429*b^3*x*(b*x^(2/3)+a*x)^(1/2)/a^4+33
6/715*b^2*x^(4/3)*(b*x^(2/3)+a*x)^(1/2)/a^3-28/65*b*x^(5/3)*(b*x^(2/3)+a*x
)^(1/2)/a^2+2/5*x^2*(b*x^(2/3)+a*x)^(1/2)/a
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.49

$$\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2\sqrt{bx^{2/3}+ax}\left(-2048b^7+1024ab^6\sqrt[3]{x}-768a^2b^5x^{2/3}+640a^3b^4x-560a^4b^3x^{4/3}+50\right.}{2145a^8\sqrt[3]{x}}$$

input
```
Integrate[x^2/Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(2/3) + a*x]*(-2048*b^7 + 1024*a*b^6*x^(1/3) - 768*a^2*b^5*x^(
2/3) + 640*a^3*b^4*x - 560*a^4*b^3*x^(4/3) + 504*a^5*b^2*x^(5/3) - 462*a^6
*b*x^2 + 429*a^7*x^(7/3)))/(2145*a^8*x^(1/3))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1922, 1922, 1922, 1922, 1922, 1922, 1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{ax+bx^{2/3}}}dx$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\int \frac{x^{5/3}}{\sqrt{x^{2/3}b+ax}}dx}{15a}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\int \frac{x^{4/3}}{\sqrt{x^{2/3}b+ax}}dx}{13a}\right)}{15a}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\frac{10b\int\frac{x}{\sqrt{x^{2/3}b+ax}}dx}{11a}\right)}{13a}\right)}{15a}$$

$$\Big\downarrow \ \color{blue}{1922}$$

$$\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\frac{10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\int\frac{x^{2/3}}{\sqrt{x^{2/3}b+ax}}dx}{9a}\right)}{11a}\right)}{13a}\right)}{15a}$$

$$\Big\downarrow \ \color{blue}{1922}$$

$$\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\frac{10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\int\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}dx}{7a}\right)}{9a}\right)}{11a}\right)}{13a}\right)}{15a}$$

$\downarrow$ 1922

$$\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\frac{10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\left(\frac{6\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{5a}-\frac{4b\int\frac{1}{\sqrt{x^{2/3}b+ax}}dx}{5a}\right)}{7a}\right)}{9a}\right)}{11a}\right)}{13a}\right)}{15a}$$

↓ 1908

$$\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\frac{10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\left(\frac{6\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{5a}-\frac{4b\left(\frac{2\sqrt{ax+bx^{2/3}}}{a}-\frac{2b\int \cdots}{\cdots}\right)}{5a}\right)}{7a}\right)}{9a}\right)}{11a}\right)}{13a}\cdots\right)}{\cdots}$$

↓ 1920

$$\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a} -$$

$$\frac{6b\left(\frac{6\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{5a} - \frac{4b\left(\frac{2\sqrt{ax+bx^{2/3}}}{a} - \frac{4b\sqrt{ax}}{a^2}\right.}{5a}\right.}{}$$

$$8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a} - \frac{}{7a}\right.$$

$$10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a} - \frac{}{9a}\right.$$

$$12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a} - \frac{}{11a}\right.$$

$$14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a} - \frac{}{13a}\right.$$

input
```
Int[x^2/Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*x^2*Sqrt[b*x^(2/3) + a*x])/(5*a) - (14*b*((6*x^(5/3)*Sqrt[b*x^(2/3) + a
*x])/(13*a) - (12*b*((6*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(11*a) - (10*b*((2*
x*Sqrt[b*x^(2/3) + a*x])/(3*a) - (8*b*((6*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(
7*a) - (6*b*((6*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(5*a) - (4*b*((2*Sqrt[b*x^(
2/3) + a*x])/a - (4*b*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))))/(5*a)))/(7*a)
))/(9*a)))/(11*a)))/(13*a)))/(15*a)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.45

method	result	si
derivativedivides	$\frac{2x^{\frac{1}{3}}\left(x^{\frac{1}{3}}a+b\right)\left(429a^{7}x^{\frac{7}{3}}-462a^{6}b\,x^{2}+504a^{5}b^{2}x^{\frac{5}{3}}-560a^{4}b^{3}x^{\frac{4}{3}}+640a^{3}b^{4}x-768a^{2}b^{5}x^{\frac{2}{3}}+1024a\,b^{6}x^{\frac{1}{3}}-2048b^{7}\right)}{2145\sqrt{b\,x^{\frac{2}{3}}+ax}\,a^{8}}$	10
default	$\frac{2x^{\frac{1}{3}}\left(x^{\frac{1}{3}}a+b\right)\left(429a^{7}x^{\frac{7}{3}}-462a^{6}b\,x^{2}+504a^{5}b^{2}x^{\frac{5}{3}}-560a^{4}b^{3}x^{\frac{4}{3}}+640a^{3}b^{4}x-768a^{2}b^{5}x^{\frac{2}{3}}+1024a\,b^{6}x^{\frac{1}{3}}-2048b^{7}\right)}{2145\sqrt{b\,x^{\frac{2}{3}}+ax}\,a^{8}}$	10

input
```
int(x^2/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/2145*x^(1/3)*(x^(1/3)*a+b)*(429*a^7*x^(7/3)-462*a^6*b*x^2+504*a^5*b^2*x^
(5/3)-560*a^4*b^3*x^(4/3)+640*a^3*b^4*x-768*a^2*b^5*x^(2/3)+1024*a*b^6*x^(
1/3)-2048*b^7)/(b*x^(2/3)+a*x)^(1/2)/a^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 768 vs. 2(167) = 334.

Time = 136.13 (sec) , antiderivative size = 768, normalized size of antiderivative = 3.41

$$\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

output

```
1/2145*((51539607552*b^13 + 10737418240*b^12 + 50331648*(64*a^3 - 3)*b^10
- 1006632960*b^11 - 262144*(11264*a^3 - 53)*b^9 + 4996992*a^9 - 98304*(550
4*a^3 + 1)*b^8 + 3072*(3194880*a^6 - 114688*a^3 - 3)*b^7 + 114688*(18816*a
^6 + 103*a^3)*b^6 - 12288*(48816*a^6 + 23*a^3)*b^5 + 192*(21626880*a^9 + 4
95872*a^6 + 15*a^3)*b^4 + 256*(10690560*a^9 - 24073*a^6)*b^3 + 3744*(13312
0*a^9 + 49*a^6)*b^2 - 297*(450560*a^9 + 7*a^6)*b)*x + 2*(429*(16777216*a^7
*b^6 + 6291456*a^7*b^5 + 196608*a^7*b^4 - 262144*a^10 - 114688*a^7*b^3 - 2
304*a^7*b^2 + 864*a^7*b - 27*a^7)*x^3 - 560*(16777216*a^4*b^9 + 6291456*a^
4*b^8 + 196608*a^4*b^7 - 114688*a^4*b^6 - 2304*a^4*b^5 + 864*a^4*b^4 - (26
2144*a^7 + 27*a^4)*b^3)*x^2 + 1024*(16777216*a*b^12 + 6291456*a*b^11 + 196
608*a*b^10 - 114688*a*b^9 - 2304*a*b^8 + 864*a*b^7 - (262144*a^4 + 27*a)*b
^6)*x - 2*(17179869184*b^13 + 6442450944*b^12 + 201326592*b^11 - 117440512
*b^10 - 2359296*b^9 - 1024*(262144*a^3 + 27)*b^7 + 884736*b^8 + 231*(16777
216*a^6*b^7 + 6291456*a^6*b^6 + 196608*a^6*b^5 - 114688*a^6*b^4 - 2304*a^6
*b^3 + 864*a^6*b^2 - (262144*a^9 + 27*a^6)*b)*x^2 - 320*(16777216*a^3*b^10
 + 6291456*a^3*b^9 + 196608*a^3*b^8 - 114688*a^3*b^7 - 2304*a^3*b^6 + 864*
a^3*b^5 - (262144*a^6 + 27*a^3)*b^4)*x)*x^(2/3) + 24*(21*(16777216*a^5*b^8
 + 6291456*a^5*b^7 + 196608*a^5*b^6 - 114688*a^5*b^5 - 2304*a^5*b^4 + 864*
a^5*b^3 - (262144*a^8 + 27*a^5)*b^2)*x^2 - 32*(16777216*a^2*b^11 + 6291456
*a^2*b^10 + 196608*a^2*b^9 - 114688*a^2*b^8 - 2304*a^2*b^7 + 864*a^2*b^...
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{bx^{2/3} + ax}}\,dx = \int \frac{x^2}{\sqrt{ax + bx^{\frac{2}{3}}}}\,dx$$

input

```
integrate(x**2/(b*x**(2/3)+a*x)**(1/2),x)
```

output

```
Integral(x**2/sqrt(a*x + b*x**(2/3)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{x^2}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(a*x + b*x^(2/3)), x)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.54

$$\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{4096\,b^{\frac{15}{2}}}{2145\,a^8} + \frac{2\left(429\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}} - 3465\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b + 12285\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^2 - 25025\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^3 + 32175\left(ax^{\frac{1}{3}}\right.\right.}{2145\,a^8}$$

input
```
integrate(x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
4096/2145*b^(15/2)/a^8 + 2/2145*(429*(a*x^(1/3) + b)^(15/2) - 3465*(a*x^(1
/3) + b)^(13/2)*b + 12285*(a*x^(1/3) + b)^(11/2)*b^2 - 25025*(a*x^(1/3) +
b)^(9/2)*b^3 + 32175*(a*x^(1/3) + b)^(7/2)*b^4 - 27027*(a*x^(1/3) + b)^(5/
2)*b^5 + 15015*(a*x^(1/3) + b)^(3/2)*b^6 - 6435*sqrt(a*x^(1/3) + b)*b^7)/a
^8
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx=\int \frac{x^2}{\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

input
```
int(x^2/(a*x + b*x^(2/3))^(1/2),x)
```

output
```
int(x^2/(a*x + b*x^(2/3))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.39

$$\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx=\frac{2\sqrt{x^{\frac{1}{3}}a+b}\,\left(504x^{\frac{5}{3}}a^5b^2-768x^{\frac{2}{3}}a^2b^5+429x^{\frac{7}{3}}a^7-560x^{\frac{4}{3}}a^4b^3+1024x^{\frac{1}{3}}a\,b^6-462a^6\right.}{2145a^8}$$

input
```
int(x^2/(b*x^(2/3)+a*x)^(1/2),x)
```

output
```
(2*sqrt(x**(1/3)*a + b)*(504*x**(2/3)*a**5*b**2*x - 768*x**(2/3)*a**2*b**5
 + 429*x**(1/3)*a**7*x**2 - 560*x**(1/3)*a**4*b**3*x + 1024*x**(1/3)*a*b**
6 - 462*a**6*b*x**2 + 640*a**3*b**4*x - 2048*b**7))/(2145*a**8)
```

3.167 $\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx$

Optimal result 1626
Mathematica [A] (verified) 1626
Rubi [A] (verified) 1627
Maple [A] (verified) 1629
Fricas [B] (verification not implemented) 1630
Sympy [F] 1630
Maxima [F] 1631
Giac [A] (verification not implemented) 1631
Mupad [F(-1)] 1632
Reduce [B] (verification not implemented) 1632

Optimal result

Integrand size = 17, antiderivative size = 137

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx = -\frac{128b^3\sqrt{bx^{2/3}+ax}}{105a^4}+\frac{256b^4\sqrt{bx^{2/3}+ax}}{105a^5\sqrt[3]{x}}+\frac{32b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{35a^3}-\frac{16bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^2}+\frac{2x\sqrt{bx^{2/3}+ax}}{3a}$$

output
```
-128/105*b^3*(b*x^(2/3)+a*x)^(1/2)/a^4+256/105*b^4*(b*x^(2/3)+a*x)^(1/2)/a
^5/x^(1/3)+32/35*b^2*x^(1/3)*(b*x^(2/3)+a*x)^(1/2)/a^3-16/21*b*x^(2/3)*(b*
x^(2/3)+a*x)^(1/2)/a^2+2/3*x*(b*x^(2/3)+a*x)^(1/2)/a
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.54

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2\sqrt{bx^{2/3}+ax}\left(128b^4-64ab^3\sqrt[3]{x}+48a^2b^2x^{2/3}-40a^3bx+35a^4x^{4/3}\right)}{105a^5\sqrt[3]{x}}$$

input
```
Integrate[x/Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(2/3) + a*x]*(128*b^4 - 64*a*b^3*x^(1/3) + 48*a^2*b^2*x^(2/3)
- 40*a^3*b*x + 35*a^4*x^(4/3)))/(105*a^5*x^(1/3))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1922, 1922, 1922, 1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{ax+bx^{2/3}}}dx$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\int \frac{x^{2/3}}{\sqrt{x^{2/3}b+ax}}dx}{9a}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\int \frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}dx}{7a}\right)}{9a}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\left(\frac{6\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{5a}-\frac{4b\int \frac{1}{\sqrt{x^{2/3}b+ax}}dx}{5a}\right)}{7a}\right)}{9a}$$

$$\Big\downarrow \text{ 1908}$$

$$\frac{\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\left(\frac{6\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{5a}-\frac{4b\left(\frac{2\sqrt{ax+bx^{2/3}}}{a}-\frac{2b\int\frac{1}{\sqrt[3]{x}\sqrt{x^{2/3}b+ax}}dx}{3a}\right)}{5a}\right)}{7a}\right)}{9a}}{}$$

$$\Big\downarrow \; 1920$$

$$\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\left(\frac{6\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{5a}-\frac{4b\left(\frac{2\sqrt{ax+bx^{2/3}}}{a}-\frac{4b\sqrt{ax+bx^{2/3}}}{a^2\sqrt[3]{x}}\right)}{5a}\right)}{7a}\right)}{9a}$$

input
```
Int[x/Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*x*Sqrt[b*x^(2/3) + a*x])/(3*a) - (8*b*((6*x^(2/3)*Sqrt[b*x^(2/3) + a*x]
)/(7*a) - (6*b*((6*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(5*a) - (4*b*((2*Sqrt[b*
x^(2/3) + a*x])/a - (4*b*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))))/(5*a)))/(7
*a)))/(9*a)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.50

method	result	size
derivativedivides	$\frac{2x^{\frac{1}{3}}\left(x^{\frac{1}{3}}a+b\right)\left(35a^{4}x^{\frac{4}{3}}-40a^{3}bx+48a^{2}b^{2}x^{\frac{2}{3}}-64a\,b^{3}x^{\frac{1}{3}}+128b^{4}\right)}{105\sqrt{b\,x^{\frac{2}{3}}+ax}\,a^{5}}$	68
default	$\frac{2x^{\frac{1}{3}}\left(x^{\frac{1}{3}}a+b\right)\left(35a^{4}x^{\frac{4}{3}}-40a^{3}bx+48a^{2}b^{2}x^{\frac{2}{3}}-64a\,b^{3}x^{\frac{1}{3}}+128b^{4}\right)}{105\sqrt{b\,x^{\frac{2}{3}}+ax}\,a^{5}}$	68

input
```
int(x/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/105*x^(1/3)*(x^(1/3)*a+b)*(35*a^4*x^(4/3)-40*a^3*b*x+48*a^2*b^2*x^(2/3)-
64*a*b^3*x^(1/3)+128*b^4)/(b*x^(2/3)+a*x)^(1/2)/a^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. 2(101) = 202.

Time = 125.32 (sec) , antiderivative size = 502, normalized size of antiderivative = 3.66

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx = -\frac{2\left(2\left(805306368\,b^{10}+167772160\,b^9+786432\,(64\,a^3-3)b^7-15728640\,b^8-4096\,(11264\,a^3-53)b^6-10\right.\right.}{}$$

input
```
integrate(x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/105*(2*(805306368*b^10 + 167772160*b^9 + 786432*(64*a^3 - 3)*b^7 - 1572
8640*b^8 - 4096*(11264*a^3 - 53)*b^6 - 101920*a^6 - 1536*(5504*a^3 + 1)*b^
5 - 48*(1966080*a^6 + 114688*a^3 + 3)*b^4 - 1792*(36864*a^6 - 103*a^3)*b^3
 - 192*(65280*a^6 + 23*a^3)*b^2 + 15*(188416*a^6 + 3*a^3)*b)*x - (35*(1677
7216*a^4*b^6 + 6291456*a^4*b^5 + 196608*a^4*b^4 - 262144*a^7 - 114688*a^4*
b^3 - 2304*a^4*b^2 + 864*a^4*b - 27*a^4)*x^2 + 48*(16777216*a^2*b^8 + 6291
456*a^2*b^7 + 196608*a^2*b^6 - 114688*a^2*b^5 - 2304*a^2*b^4 + 864*a^2*b^3
 - (262144*a^5 + 27*a^2)*b^2)*x^(4/3) - 64*(16777216*a*b^9 + 6291456*a*b^8
 + 196608*a*b^7 - 114688*a*b^6 - 2304*a*b^5 + 864*a*b^4 - (262144*a^4 + 27
*a)*b^3)*x + 8*(268435456*b^10 + 100663296*b^9 + 3145728*b^8 - 1835008*b^7
 - 36864*b^6 - 16*(262144*a^3 + 27)*b^4 + 13824*b^5 - 5*(16777216*a^3*b^7
+ 6291456*a^3*b^6 + 196608*a^3*b^5 - 114688*a^3*b^4 - 2304*a^3*b^3 + 864*a
^3*b^2 - (262144*a^6 + 27*a^3)*b)*x)*x^(2/3))*sqrt(a*x + b*x^(2/3)))/((167
77216*a^5*b^6 + 6291456*a^5*b^5 + 196608*a^5*b^4 - 262144*a^8 - 114688*a^5
*b^3 - 2304*a^5*b^2 + 864*a^5*b - 27*a^5)*x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{x}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(x/(b*x**(2/3)+a*x)**(1/2),x)
```

output
```
Integral(x/sqrt(a*x + b*x**(2/3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{x}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(a*x + b*x^(2/3)), x)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.58

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx = -\frac{256\,b^{\frac{9}{2}}}{105\,a^5} + \frac{2\left(35\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}} - 180\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b + 378\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^2 - 420\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^3 + 315\sqrt{ax^{\frac{1}{3}}+b}\,b^4\right)}{105\,a^5}$$

input
```
integrate(x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
-256/105*b^(9/2)/a^5 + 2/105*(35*(a*x^(1/3) + b)^(9/2) - 180*(a*x^(1/3) +
b)^(7/2)*b + 378*(a*x^(1/3) + b)^(5/2)*b^2 - 420*(a*x^(1/3) + b)^(3/2)*b^3
 + 315*sqrt(a*x^(1/3) + b)*b^4)/a^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{x}{\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

input
```
int(x/(a*x + b*x^(2/3))^(1/2),x)
```

output
```
int(x/(a*x + b*x^(2/3))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.39

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2\sqrt{x^{\frac{1}{3}}a+b}\,\left(48x^{\frac{2}{3}}a^2b^2+35x^{\frac{4}{3}}a^4-64x^{\frac{1}{3}}a\,b^3-40a^3bx+128b^4\right)}{105a^5}$$

input
```
int(x/(b*x^(2/3)+a*x)^(1/2),x)
```

output
```
(2*sqrt(x**(1/3)*a + b)*(48*x**(2/3)*a**2*b**2 + 35*x**(1/3)*a**4*x - 64*x
**(1/3)*a*b**3 - 40*a**3*b*x + 128*b**4))/(105*a**5)
```

3.168 $\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx$

Optimal result 1633
Mathematica [A] (verified) 1633
Rubi [A] (verified) 1634
Maple [A] (verified) 1635
Fricas [B] (verification not implemented) 1635
Sympy [F] 1636
Maxima [F] 1636
Giac [A] (verification not implemented) 1636
Mupad [B] (verification not implemented) 1637
Reduce [B] (verification not implemented) 1637

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2\sqrt{bx^{2/3}+ax}}{a} - \frac{4b\sqrt{bx^{2/3}+ax}}{a^2\sqrt[3]{x}}$$

output
```
2*(b*x^(2/3)+a*x)^(1/2)/a-4*b*(b*x^(2/3)+a*x)^(1/2)/a^2/x^(1/3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2\left(-2b+a\sqrt[3]{x}\right)\sqrt{bx^{2/3}+ax}}{a^2\sqrt[3]{x}}$$

input
```
Integrate[1/Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*(-2*b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax+bx^{2/3}}}\,dx$$
$$\Big\downarrow \; 1908$$
$$\frac{2\sqrt{ax+bx^{2/3}}}{a}-\frac{2b\int \frac{1}{\sqrt[3]{x}\sqrt{x^{2/3}b+ax}}dx}{3a}$$
$$\Big\downarrow \; 1920$$
$$\frac{2\sqrt{ax+bx^{2/3}}}{a}-\frac{4b\sqrt{ax+bx^{2/3}}}{a^2\sqrt[3]{x}}$$

input
```
Int[1/Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(2/3) + a*x])/a - (4*b*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{2x^{\frac{1}{3}}\left(x^{\frac{1}{3}}a+b\right)\left(x^{\frac{1}{3}}a-2b\right)}{\sqrt{b\,x^{\frac{2}{3}}+ax}\,a^{2}}$	36
default	$\frac{2x^{\frac{1}{3}}\left(x^{\frac{1}{3}}a+b\right)\left(x^{\frac{1}{3}}a-2b\right)}{\sqrt{b\,x^{\frac{2}{3}}+ax}\,a^{2}}$	36

input
```
int(1/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/3)*(x^(1/3)*a+b)*(x^(1/3)*a-2*b)/(b*x^(2/3)+a*x)^(1/2)/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(37) = 74$.

Time = 136.85 (sec) , antiderivative size = 238, normalized size of antiderivative = 5.06

$$\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{(50331648\,b^7 + 10485760\,b^6 + 49152\,(512\,a^3-3)b^4 - 983040\,b^5 + 256\,(24576\,a^3+53$$

input
```
integrate(1/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
((50331648*b^7 + 10485760*b^6 + 49152*(512*a^3 - 3)*b^4 - 983040*b^5 + 256
*(24576*a^3 + 53)*b^3 + 11648*a^3 - 96*(2048*a^3 + 1)*b^2 - 3*(155648*a^3
+ 3)*b)*x + 2*((16777216*a*b^6 + 6291456*a*b^5 + 196608*a*b^4 - 262144*a^4
 - 114688*a*b^3 - 2304*a*b^2 + 864*a*b - 27*a)*x - 2*(16777216*b^7 + 62914
56*b^6 + 196608*b^5 - 114688*b^4 - 2304*b^3 - (262144*a^3 + 27)*b + 864*b^
2)*x^(2/3))*sqrt(a*x + b*x^(2/3)))/((16777216*a^2*b^6 + 6291456*a^2*b^5 +
196608*a^2*b^4 - 262144*a^5 - 114688*a^2*b^3 - 2304*a^2*b^2 + 864*a^2*b -
27*a^2)*x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(1/(b*x**(2/3)+a*x)**(1/2),x)
```

output
```
Integral(1/sqrt(a*x + b*x**(2/3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(1/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a*x + b*x^(2/3)), x)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{4\,b^{\frac{3}{2}}}{a^2} + \frac{2\left(\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}} - 3\sqrt{ax^{\frac{1}{3}}+b}\,b\right)}{a^2}$$

input
```
integrate(1/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
4*b^(3/2)/a^2 + 2*((a*x^(1/3) + b)^(3/2) - 3*sqrt(a*x^(1/3) + b)*b)/a^2
```

Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx=\frac{3\,x\,\sqrt{\frac{a\,x^{1/3}}{b}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{2},2;\ 3;\ -\frac{a\,x^{1/3}}{b}\right)}{2\,\sqrt{a\,x+b\,x^{2/3}}}$$

input
```
int(1/(a*x + b*x^(2/3))^(1/2),x)
```

output
```
(3*x*((a*x^(1/3))/b + 1)^(1/2)*hypergeom([1/2, 2], 3, -(a*x^(1/3))/b))/(2*
(a*x + b*x^(2/3))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.47

$$\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx=\frac{2\sqrt{x^{\frac{1}{3}}a+b}\,\left(x^{\frac{1}{3}}a-2b\right)}{a^2}$$

input
```
int(1/(b*x^(2/3)+a*x)^(1/2),x)
```

output
```
(2*sqrt(x**(1/3)*a + b)*(x**(1/3)*a - 2*b))/a**2
```

3.169 $\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx$

Optimal result 1638
Mathematica [A] (verified) 1638
Rubi [A] (verified) 1639
Maple [A] (verified) 1640
Fricas [F(-1)] 1641
Sympy [F] 1641
Maxima [F] 1641
Giac [A] (verification not implemented) 1642
Mupad [F(-1)] 1642
Reduce [B] (verification not implemented) 1642

Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{bx^{2/3}} + \frac{3a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b^{3/2}}$$

output
```
-3*(b*x^(2/3)+a*x)^(1/2)/b/x^(2/3)+3*a*arctanh(b^(1/2)*x^(1/3)/(b*x^(2/3)+
a*x)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{bx^{2/3}} + \frac{3a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b^{3/2}}$$

input
```
Integrate[1/(x*Sqrt[b*x^(2/3) + a*x]),x]
```

output
```
(-3*Sqrt[b*x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sq
rt[b*x^(2/3) + a*x]])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{ax+bx^{2/3}}}dx$$

$$\Big\downarrow \text{1931}$$

$$-\frac{a\int \frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{3a\int \frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{3a\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}$$

input
```
Int[1/(x*Sqrt[b*x^(2/3) + a*x]),x]
```

output
```
(-3*Sqrt[b*x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sq
rt[b*x^(2/3) + a*x]])/b^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\frac{3\sqrt{x^{\frac{1}{3}}a+b}\,\left(\sqrt{x^{\frac{1}{3}}a+b}\,b^{\frac{3}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{x^{\frac{1}{3}}a+b}}{\sqrt{b}}\right)b a\,x^{\frac{1}{3}}\right)}{\sqrt{b\,x^{\frac{2}{3}}+a x}\,b^{\frac{5}{2}}}$	61
default	$\frac{3\sqrt{x^{\frac{1}{3}}a+b}\,\left(\operatorname{arctanh}\left(\frac{\sqrt{x^{\frac{1}{3}}a+b}}{\sqrt{b}}\right)b a\,x^{\frac{1}{3}}-\sqrt{x^{\frac{1}{3}}a+b}\,b^{\frac{3}{2}}\right)}{\sqrt{b\,x^{\frac{2}{3}}+a x}\,b^{\frac{5}{2}}}$	61

input
```
int(1/x/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-3*(x^(1/3)*a+b)^(1/2)*((x^(1/3)*a+b)^(1/2)*b^(3/2)-arctanh((x^(1/3)*a+b)^
(1/2)/b^(1/2))*b*a*x^(1/3))/(b*x^(2/3)+a*x)^(1/2)/b^(5/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{x\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(1/x/(b*x**(2/3)+a*x)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a*x + b*x**(2/3))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}\,x}\,dx$$

input
```
integrate(1/x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*x^(2/3))*x), x)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.79

$$\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx = -3\,a\left(\frac{\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b} + \frac{\sqrt{ax^{\frac{1}{3}}+b}}{abx^{\frac{1}{3}}}\right)$$

input
```
integrate(1/x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
-3*a*(arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) + sqrt(a*x^(1/3) +
 b)/(a*b*x^(1/3)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{x\,\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

input
```
int(1/(x*(a*x + b*x^(2/3))^(1/2)),x)
```

output
```
int(1/(x*(a*x + b*x^(2/3))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx = \frac{-3\sqrt{x^{\frac{1}{3}}a+b}\,b - \frac{3x^{\frac{1}{3}}\sqrt{b}\log\left(\sqrt{x^{\frac{1}{3}}a+b}-\sqrt{b}\right)a}{2} + \frac{3x^{\frac{1}{3}}\sqrt{b}\log\left(\sqrt{x^{\frac{1}{3}}a+b}+\sqrt{b}\right)a}{2}}{x^{\frac{1}{3}}b^2}$$

input
```
int(1/x/(b*x^(2/3)+a*x)^(1/2),x)
```

output
```
(3*( - 2*sqrt(x**(1/3)*a + b)*b - x**(1/3)*sqrt(b)*log(sqrt(x**(1/3)*a + b
) - sqrt(b))*a + x**(1/3)*sqrt(b)*log(sqrt(x**(1/3)*a + b) + sqrt(b))*a))/
(2*x**(1/3)*b**2)
```

3.170 $\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx$

Optimal result 1644
Mathematica [A] (verified) 1644
Rubi [A] (verified) 1645
Maple [A] (verified) 1648
Fricas [F(-1)] 1648
Sympy [F] 1649
Maxima [F] 1649
Giac [A] (verification not implemented) 1649
Mupad [F(-1)] 1650
Reduce [B] (verification not implemented) 1650

Optimal result

Integrand size = 19, antiderivative size = 153

$$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}}+\frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} -\frac{35a^2\sqrt{bx^{2/3}+ax}}{32b^3x}+\frac{105a^3\sqrt{bx^{2/3}+ax}}{64b^4x^{2/3}}-\frac{105a^4\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{64b^{9/2}}$$

output

```
-3/4*(b*x^(2/3)+a*x)^(1/2)/b/x^(5/3)+7/8*a*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(4/
3)-35/32*a^2*(b*x^(2/3)+a*x)^(1/2)/b^3/x+105/64*a^3*(b*x^(2/3)+a*x)^(1/2)/
b^4/x^(2/3)-105/64*a^4*arctanh(b^(1/2)*x^(1/3)/(b*x^(2/3)+a*x)^(1/2))/b^(9
/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx = \frac{\sqrt{bx^{2/3}+ax}\left(-48b^3+56ab^2\sqrt[3]{x}-70a^2bx^{2/3}+105a^3x\right)}{64b^4x^{5/3}} -\frac{105a^4\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{64b^{9/2}}$$

input
```
Integrate[1/(x^2*Sqrt[b*x^(2/3) + a*x]),x]
```

output
```
(Sqrt[b*x^(2/3) + a*x]*(-48*b^3 + 56*a*b^2*x^(1/3) - 70*a^2*b*x^(2/3) + 10
5*a^3*x))/(64*b^4*x^(5/3)) - (105*a^4*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(
2/3) + a*x]])/(64*b^(9/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1931, 1931, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{ax+bx^{2/3}}}dx$$

$$\Big\downarrow \text{1931}$$

$$-\frac{7a\int \frac{1}{x^{5/3}\sqrt{x^{2/3}b+ax}}dx}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{7a\left(-\frac{5a\int \frac{1}{x^{4/3}\sqrt{x^{2/3}b+ax}}dx}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{7a\left(-\frac{5a\left(-\frac{3a\int \frac{1}{x\sqrt{x^{2/3}b+ax}}dx}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(-\frac{a\int \frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}$$

$$\Big\downarrow \text{ 1935}$$

$$-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\int \frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}$$

input
```
Int[1/(x^2*Sqrt[b*x^(2/3) + a*x]),x]
```

output
```
(-3*Sqrt[b*x^(2/3) + a*x])/(4*b*x^(5/3)) - (7*a*(-(Sqrt[b*x^(2/3) + a*x]/(
b*x^(4/3))) - (5*a*((-3*Sqrt[b*x^(2/3) + a*x])/(2*b*x) - (3*a*((-3*Sqrt[b*
x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3
) + a*x]])/b^(3/2)))/(4*b)))/(6*b)))/(8*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{\sqrt{x^{\frac{1}{3}}a+b}\left(48\sqrt{x^{\frac{1}{3}}a+b}\,b^{\frac{9}{2}}-56b^{\frac{7}{2}}\sqrt{x^{\frac{1}{3}}a+b}\,a\,x^{\frac{1}{3}}+70b^{\frac{5}{2}}\sqrt{x^{\frac{1}{3}}a+b}\,a^{2}x^{\frac{2}{3}}-105b^{\frac{3}{2}}\sqrt{x^{\frac{1}{3}}a+b}\,a^{3}x+105\,\text{arctanh}\left(\frac{\sqrt{x^{\frac{1}{3}}}}{\sqrt{}}\right)\right)}{64x\sqrt{b\,x^{\frac{2}{3}}+ax}\,b^{\frac{11}{2}}}$
default	$-\frac{\sqrt{x^{\frac{1}{3}}a+b}\left(-56b^{\frac{7}{2}}x^{\frac{4}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,a+70b^{\frac{5}{2}}x^{\frac{5}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,a^{2}+105x^{\frac{7}{3}}\,\text{arctanh}\left(\frac{\sqrt{x^{\frac{1}{3}}a+b}}{\sqrt{b}}\right)a^{4}b+48\sqrt{x^{\frac{1}{3}}a+b}\,b^{\frac{9}{2}}x-105b\right)}{64x^{2}\sqrt{b\,x^{\frac{2}{3}}+ax}\,b^{\frac{11}{2}}}$

input
```
int(1/x^2/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/64*(x^(1/3)*a+b)^(1/2)*(48*(x^(1/3)*a+b)^(1/2)*b^(9/2)-56*b^(7/2)*(x^(1
/3)*a+b)^(1/2)*a*x^(1/3)+70*b^(5/2)*(x^(1/3)*a+b)^(1/2)*a^2*x^(2/3)-105*b^
(3/2)*(x^(1/3)*a+b)^(1/2)*a^3*x+105*arctanh((x^(1/3)*a+b)^(1/2)/b^(1/2))*a
^4*b*x^(4/3))/x/(b*x^(2/3)+a*x)^(1/2)/b^(11/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{x^2\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(1/x**2/(b*x**(2/3)+a*x)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a*x + b*x**(2/3))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}\,x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*x^(2/3))*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx = \frac{\frac{105\,a^5\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^4} + \frac{105\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^5-385\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^5b+511\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^5b^2-279\sqrt{ax^{\frac{1}{3}}+b}a^5b^3}{a^4b^4x^{\frac{4}{3}}}}{64\,a}$$

input
```
integrate(1/x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/64*(105*a^5*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) + (105*(
a*x^(1/3) + b)^(7/2)*a^5 - 385*(a*x^(1/3) + b)^(5/2)*a^5*b + 511*(a*x^(1/3
) + b)^(3/2)*a^5*b^2 - 279*sqrt(a*x^(1/3) + b)*a^5*b^3)/(a^4*b^4*x^(4/3)))
/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{x^2\,\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

input
```
int(1/(x^2*(a*x + b*x^(2/3))^(1/2)),x)
```

output
```
int(1/(x^2*(a*x + b*x^(2/3))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx = \frac{-140x^{\frac{2}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,a^2b^2+112x^{\frac{1}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,a\,b^3+210\sqrt{x^{\frac{1}{3}}a+b}\,a^3bx-96\sqrt{x^{\frac{1}{3}}a+}{128x^{\frac{4}{3}}}$$

input
```
int(1/x^2/(b*x^(2/3)+a*x)^(1/2),x)
```

output
```
( - 140*x**(2/3)*sqrt(x**(1/3)*a + b)*a**2*b**2 + 112*x**(1/3)*sqrt(x**(1/
3)*a + b)*a*b**3 + 210*sqrt(x**(1/3)*a + b)*a**3*b*x - 96*sqrt(x**(1/3)*a
+ b)*b**4 + 105*x**(1/3)*sqrt(b)*log(sqrt(x**(1/3)*a + b) - sqrt(b))*a**4*
x - 105*x**(1/3)*sqrt(b)*log(sqrt(x**(1/3)*a + b) + sqrt(b))*a**4*x)/(128*
x**(1/3)*b**5*x)
```

3.171 $\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx$

Optimal result 1651
Mathematica [A] (verified) 1652
Rubi [A] (verified) 1652
Maple [A] (verified) 1662
Fricas [F(-1)] 1662
Sympy [F] 1663
Maxima [F] 1663
Giac [A] (verification not implemented) 1663
Mupad [F(-1)] 1664
Reduce [B] (verification not implemented) 1664

Optimal result

Integrand size = 19, antiderivative size = 241

$$\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx=-\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}}+\frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}}-\frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2}+\frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}}-\frac{429a^4\sqrt{bx^{2/3}+ax}}{640b^5x^{4/3}}+\frac{429a^5\sqrt{bx^{2/3}+ax}}{512b^6x}-\frac{1287a^6\sqrt{bx^{2/3}+ax}}{1024b^7x^{2/3}}+\frac{1287a^7\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{1024b^{15/2}}$$

output
```
-3/7*(b*x^(2/3)+a*x)^(1/2)/b/x^(8/3)+13/28*a*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(
7/3)-143/280*a^2*(b*x^(2/3)+a*x)^(1/2)/b^3/x^2+1287/2240*a^3*(b*x^(2/3)+a*
x)^(1/2)/b^4/x^(5/3)-429/640*a^4*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(4/3)+429/512
*a^5*(b*x^(2/3)+a*x)^(1/2)/b^6/x-1287/1024*a^6*(b*x^(2/3)+a*x)^(1/2)/b^7/x
^(2/3)+1287/1024*a^7*arctanh(b^(1/2)*x^(1/3)/(b*x^(2/3)+a*x)^(1/2))/b^(15/
2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx = \frac{\sqrt{bx^{2/3}+ax}\left(-15360b^6+16640ab^5\sqrt[3]{x}-18304a^2b^4x^{2/3}+20592a^3b^3x-24024a^4b\right.}{35840b^7x^{8/3}} + \frac{1287a^7\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{1024b^{15/2}}$$

input
```
Integrate[1/(x^3*Sqrt[b*x^(2/3) + a*x]),x]
```

output
```
(Sqrt[b*x^(2/3) + a*x]*(-15360*b^6 + 16640*a*b^5*x^(1/3) - 18304*a^2*b^4*x
^(2/3) + 20592*a^3*b^3*x - 24024*a^4*b^2*x^(4/3) + 30030*a^5*b*x^(5/3) - 4
5045*a^6*x^2))/(35840*b^7*x^(8/3)) + (1287*a^7*ArcTanh[(Sqrt[b]*x^(1/3))/S
qrt[b*x^(2/3) + a*x]])/(1024*b^(15/2))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1931, 1931, 1931, 1931, 1931, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{ax+bx^{2/3}}}dx$$

$$\Big\downarrow \ 1931$$

$$-\frac{13a\int \frac{1}{x^{8/3}\sqrt{x^{2/3}b+ax}}dx}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}$$

$$\Big\downarrow \ 1931$$

$$-\frac{13a\left(-\frac{11a\int \frac{1}{x^{7/3}\sqrt{x^{2/3}b+ax}}dx}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{13a\left(-\frac{11a\left(-\frac{9a\int \frac{1}{x^2\sqrt{x^{2/3}b+ax}}dx}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\int \frac{1}{x^{5/3}\sqrt{x^{2/3}b+ax}}dx}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\int \frac{1}{x^{4/3}\sqrt{x^{2/3}b+ax}}dx}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}$$

$$\Big\downarrow \; 1931$$

$$-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\int\frac{1}{x\sqrt{x^{2/3}b+ax}}dx}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}$$

$$\downarrow \text{1931}$$

$$\left(13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(-\frac{a\int \frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}\right.\right.$$

↓ 1935

$$\left(11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\int\frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+}}{5b}\right.\right.$$

↓ 219

$$\left(11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx}}{5bx^2}\right.\right.$$

input
```
Int[1/(x^3*Sqrt[b*x^(2/3) + a*x]),x]
```

output
```
(-3*Sqrt[b*x^(2/3) + a*x])/(7*b*x^(8/3)) - (13*a*(-1/2*Sqrt[b*x^(2/3) + a*
x]/(b*x^(7/3)) - (11*a*((-3*Sqrt[b*x^(2/3) + a*x])/(5*b*x^2) - (9*a*((-3*S
qrt[b*x^(2/3) + a*x])/(4*b*x^(5/3)) - (7*a*(-(Sqrt[b*x^(2/3) + a*x]/(b*x^(
4/3))) - (5*a*((-3*Sqrt[b*x^(2/3) + a*x])/(2*b*x) - (3*a*((-3*Sqrt[b*x^(2/
3) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a
*x]])/b^(3/2)))/(4*b)))/(6*b)))/(8*b)))/(10*b)))/(12*b)))/(14*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.76

method	result
derivativedivides	$-\frac{\sqrt{x^{\frac{1}{3}} a+b}\,\left(45045 \sqrt{x^{\frac{1}{3}} a+b}\, b^{\frac{3}{2}} a^{6} x^{2}-45045 \,\mathrm{arctanh}\left(\frac{\sqrt{x^{\frac{1}{3}} a+b}}{\sqrt{b}}\right) a^{7} b \,x^{\frac{7}{3}}-30030 \sqrt{x^{\frac{1}{3}} a+b}\, b^{\frac{5}{2}} a^{5} x^{\frac{5}{3}}+24024 \sqrt{x^{\frac{1}{3}} a+}}{35840 x^{2} \sqrt{b\,x}}$
default	$\frac{\sqrt{x^{\frac{1}{3}} a+b}\,\left(45045 \,\mathrm{arctanh}\left(\frac{\sqrt{x^{\frac{1}{3}} a+b}}{\sqrt{b}}\right) x^{\frac{13}{3}} a^{7} b-24024 x^{\frac{10}{3}} \sqrt{x^{\frac{1}{3}} a+b}\, b^{\frac{7}{2}} a^{4}-45045 x^{4} \sqrt{x^{\frac{1}{3}} a+b}\, b^{\frac{3}{2}} a^{6}+16640 x^{\frac{7}{3}} \sqrt{x^{\frac{1}{3}} a}}{35840 x^{4} \sqrt{b\,x}}$

input
```
int(1/x^3/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35840/x^2*(x^(1/3)*a+b)^(1/2)*(45045*(x^(1/3)*a+b)^(1/2)*b^(3/2)*a^6*x^
2-45045*arctanh((x^(1/3)*a+b)^(1/2)/b^(1/2))*a^7*b*x^(7/3)-30030*(x^(1/3)*
a+b)^(1/2)*b^(5/2)*a^5*x^(5/3)+24024*(x^(1/3)*a+b)^(1/2)*b^(7/2)*a^4*x^(4/
3)-20592*(x^(1/3)*a+b)^(1/2)*b^(9/2)*a^3*x+18304*(x^(1/3)*a+b)^(1/2)*b^(11
/2)*a^2*x^(2/3)-16640*(x^(1/3)*a+b)^(1/2)*b^(13/2)*a*x^(1/3)+15360*(x^(1/3
)*a+b)^(1/2)*b^(15/2))/(b*x^(2/3)+a*x)^(1/2)/b^(17/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{x^3\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(1/x**3/(b*x**(2/3)+a*x)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(a*x + b*x**(2/3))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*x^(2/3))*x^3), x)
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx = -\frac{1}{35840}a^7\left(\frac{45045\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^7}+\frac{45045\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}-300300\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b+849849\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}\right.$$

input
```
integrate(1/x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
-1/35840*a^7*(45045*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7) +
(45045*(a*x^(1/3) + b)^(13/2) - 300300*(a*x^(1/3) + b)^(11/2)*b + 849849*(
a*x^(1/3) + b)^(9/2)*b^2 - 1317888*(a*x^(1/3) + b)^(7/2)*b^3 + 1200199*(a*
x^(1/3) + b)^(5/2)*b^4 - 631540*(a*x^(1/3) + b)^(3/2)*b^5 + 169995*sqrt(a*
x^(1/3) + b)*b^6)/(a^7*b^7*x^(7/3)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{x^3\,\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

input
```
int(1/(x^3*(a*x + b*x^(2/3))^(1/2)),x)
```

output
```
int(1/(x^3*(a*x + b*x^(2/3))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx = \frac{60060x^{\frac{5}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,a^5b^2 - 36608x^{\frac{2}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,a^2b^5 - 48048x^{\frac{4}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,a^4b^3 + 332}{}$$

input
```
int(1/x^3/(b*x^(2/3)+a*x)^(1/2),x)
```

output
```
(60060*x**(2/3)*sqrt(x**(1/3)*a + b)*a**5*b**2*x - 36608*x**(2/3)*sqrt(x**
(1/3)*a + b)*a**2*b**5 - 48048*x**(1/3)*sqrt(x**(1/3)*a + b)*a**4*b**3*x +
 33280*x**(1/3)*sqrt(x**(1/3)*a + b)*a*b**6 - 90090*sqrt(x**(1/3)*a + b)*a
**6*b*x**2 + 41184*sqrt(x**(1/3)*a + b)*a**3*b**4*x - 30720*sqrt(x**(1/3)*
a + b)*b**7 - 45045*x**(1/3)*sqrt(b)*log(sqrt(x**(1/3)*a + b) - sqrt(b))*a
**7*x**2 + 45045*x**(1/3)*sqrt(b)*log(sqrt(x**(1/3)*a + b) + sqrt(b))*a**7
*x**2)/(71680*x**(1/3)*b**8*x**2)
```

3.172 $\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx$

Optimal result 1665
Mathematica [A] (verified) 1666
Rubi [A] (verified) 1666
Maple [A] (verified) 1682
Fricas [F(-1)] 1682
Sympy [F] 1683
Maxima [F] 1683
Giac [A] (verification not implemented) 1683
Mupad [F(-1)] 1684
Reduce [B] (verification not implemented) 1684

Optimal result

Integrand size = 19, antiderivative size = 329

$$\begin{aligned}\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx = & -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}}+\frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}}-\frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3}\\ & +\frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}-\frac{4199a^4\sqrt{bx^{2/3}+ax}}{10752b^5x^{7/3}}+\frac{46189a^5\sqrt{bx^{2/3}+ax}}{107520b^6x^2}\\ & -\frac{138567a^6\sqrt{bx^{2/3}+ax}}{286720b^7x^{5/3}}+\frac{46189a^7\sqrt{bx^{2/3}+ax}}{81920b^8x^{4/3}}-\frac{46189a^8\sqrt{bx^{2/3}+ax}}{65536b^9x}\\ & +\frac{138567a^9\sqrt{bx^{2/3}+ax}}{131072b^{10}x^{2/3}}-\frac{138567a^{10}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{131072b^{21/2}}\end{aligned}$$

output
```
-3/10*(b*x^(2/3)+a*x)^(1/2)/b/x^(11/3)+19/60*a*(b*x^(2/3)+a*x)^(1/2)/b^2/x
^(10/3)-323/960*a^2*(b*x^(2/3)+a*x)^(1/2)/b^3/x^3+323/896*a^3*(b*x^(2/3)+a
*x)^(1/2)/b^4/x^(8/3)-4199/10752*a^4*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(7/3)+461
89/107520*a^5*(b*x^(2/3)+a*x)^(1/2)/b^6/x^2-138567/286720*a^6*(b*x^(2/3)+a
*x)^(1/2)/b^7/x^(5/3)+46189/81920*a^7*(b*x^(2/3)+a*x)^(1/2)/b^8/x^(4/3)-46
189/65536*a^8*(b*x^(2/3)+a*x)^(1/2)/b^9/x+138567/131072*a^9*(b*x^(2/3)+a*x
)^(1/2)/b^10/x^(2/3)-138567/131072*a^10*arctanh(b^(1/2)*x^(1/3)/(b*x^(2/3)
+a*x)^(1/2))/b^(21/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx = \frac{\sqrt{bx^{2/3}+ax}\left(-4128768b^9+4358144ab^8\sqrt[3]{x}-4630528a^2b^7x^{2/3}+4961280a^3b^6x-\right.}{}$$

$$-\frac{138567a^{10}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{131072b^{21/2}}$$

input
```
Integrate[1/(x^4*Sqrt[b*x^(2/3) + a*x]),x]
```

output
```
(Sqrt[b*x^(2/3) + a*x]*(-4128768*b^9 + 4358144*a*b^8*x^(1/3) - 4630528*a^2
*b^7*x^(2/3) + 4961280*a^3*b^6*x - 5374720*a^4*b^5*x^(4/3) + 5912192*a^5*b
^4*x^(5/3) - 6651216*a^6*b^3*x^2 + 7759752*a^7*b^2*x^(7/3) - 9699690*a^8*b
*x^(8/3) + 14549535*a^9*x^3))/(13762560*b^10*x^(11/3)) - (138567*a^10*ArcT
anh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(131072*b^(21/2))
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.15, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{ax+bx^{2/3}}}dx$$

$$\Big\downarrow \text{ 1931}$$

$$-\frac{19a\int \frac{1}{x^{11/3}\sqrt{x^{2/3}b+ax}}dx}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}$$

$$\Big\downarrow \text{ 1931}$$

$$-\frac{19a\left(-\frac{17a\int \frac{1}{x^{10/3}\sqrt{x^{2/3}b+ax}}dx}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}$$

$\Big\downarrow$ 1931

$$-\frac{19a\left(-\frac{17a\left(-\frac{15a\int \frac{1}{x^{3}\sqrt{x^{2/3}b+ax}}dx}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^{3}}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}$$

$\Big\downarrow$ 1931

$$-\frac{19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\int \frac{1}{x^{8/3}\sqrt{x^{2/3}b+ax}}dx}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^{3}}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}$$

$\Big\downarrow$ 1931

$$-\frac{19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\int\frac{1}{x^{7/3}\sqrt{x^{2/3}b+ax}}dx}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}$$

$$\Big\downarrow \; 1931$$

$$-\frac{19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\int \frac{1}{x^2\sqrt{x^{2/3}b+ax}}dx}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}$$

$$\Big\downarrow \text{ 1931}$$

$$19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\int\frac{1}{x^{5/3}\sqrt{x^{2/3}b+ax}}dx}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx}}{8bx^{3}}}{18b}$$

↓ 1931

$$\Bigg(17a \Bigg(-\frac{15a \left(-\frac{13a \left(-\frac{11a \left(-\frac{9a \left(-\frac{7a \left(-\frac{5a \int \frac{1}{x^{4/3}\sqrt{x^{2/3}b+ax}}\,dx}{6b} - \frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}} \right)}{8b} - \frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}} \right)}{10b} - \frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}} \right)}{12b} - \frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}} \right)}{14b} - \frac{3\sqrt{ax}}{7b}}{16b}$$

↓ 1931

$$15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\int\frac{1}{x\sqrt{x^{2/3}b+ax}}dx}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\sqrt{\cdots}}{14b}\right.$$

↓ 1931

$$\Bigg(\Bigg(\Bigg(\Bigg(13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(-\frac{a\int\frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx}}{4bx^{5/3}}}{10b}}{12b}$$

↓ 1935

$$11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\int\frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{}}{}\right.}{10b}\right.$$

↓ 219

$$\left(\left(\left(\left(\left(11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax\cdots}}{4b\cdots}\right)}{10b}\cdots\right.\right.\right.\right.\right.\right.$$

input
```
Int[1/(x^4*Sqrt[b*x^(2/3) + a*x]),x]
```

output
```
(-3*Sqrt[b*x^(2/3) + a*x])/(10*b*x^(11/3)) - (19*a*(-1/3*Sqrt[b*x^(2/3) +
a*x]/(b*x^(10/3)) - (17*a*((-3*Sqrt[b*x^(2/3) + a*x])/(8*b*x^3) - (15*a*((
-3*Sqrt[b*x^(2/3) + a*x])/(7*b*x^(8/3)) - (13*a*(-1/2*Sqrt[b*x^(2/3) + a*x
]/(b*x^(7/3)) - (11*a*((-3*Sqrt[b*x^(2/3) + a*x])/(5*b*x^2) - (9*a*((-3*Sq
rt[b*x^(2/3) + a*x])/(4*b*x^(5/3)) - (7*a*(-(Sqrt[b*x^(2/3) + a*x]/(b*x^(4
/3))) - (5*a*((-3*Sqrt[b*x^(2/3) + a*x])/(2*b*x) - (3*a*((-3*Sqrt[b*x^(2/3
) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*
x]])/b^(3/2)))/(4*b)))/(6*b)))/(8*b)))/(10*b)))/(12*b)))/(14*b)))/(16*b)))
/(18*b)))/(20*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.74

method	result
derivativedivides	$-\frac{\sqrt{x^{\frac{1}{3}}a+b}\,\left(4128768\sqrt{x^{\frac{1}{3}}a+b}\,b\,b^{\frac{21}{2}}-4358144b^{\frac{19}{2}}\sqrt{x^{\frac{1}{3}}a+b}\,a\,x^{\frac{1}{3}}+4630528b^{\frac{17}{2}}\sqrt{x^{\frac{1}{3}}a+b}\,a^{2}x^{\frac{2}{3}}-4961280b^{\frac{15}{2}}\sqrt{x^{\frac{1}{3}}a+}\right.}{}$
default	$-\frac{\sqrt{x^{\frac{1}{3}}a+b}\,\left(9699690\sqrt{x^{\frac{1}{3}}a+b}\,x^{\frac{17}{3}}b^{\frac{5}{2}}a^{8}-7759752\sqrt{x^{\frac{1}{3}}a+b}\,x^{\frac{16}{3}}b^{\frac{7}{2}}a^{7}-5912192\sqrt{x^{\frac{1}{3}}a+b}\,x^{\frac{14}{3}}b^{\frac{11}{2}}a^{5}+5374720\sqrt{x^{\frac{1}{3}}}\right.}{}$

input
```
int(1/x^4/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/13762560*(x^(1/3)*a+b)^(1/2)*(4128768*(x^(1/3)*a+b)^(1/2)*b^(21/2)-4358
144*b^(19/2)*(x^(1/3)*a+b)^(1/2)*a*x^(1/3)+4630528*b^(17/2)*(x^(1/3)*a+b)^
(1/2)*a^2*x^(2/3)-4961280*b^(15/2)*(x^(1/3)*a+b)^(1/2)*a^3*x+5374720*b^(13
/2)*(x^(1/3)*a+b)^(1/2)*a^4*x^(4/3)-5912192*b^(11/2)*(x^(1/3)*a+b)^(1/2)*a
^5*x^(5/3)+6651216*b^(9/2)*(x^(1/3)*a+b)^(1/2)*a^6*x^2-7759752*b^(7/2)*(x^
(1/3)*a+b)^(1/2)*a^7*x^(7/3)+9699690*b^(5/2)*(x^(1/3)*a+b)^(1/2)*a^8*x^(8/
3)-14549535*b^(3/2)*(x^(1/3)*a+b)^(1/2)*a^9*x^3+14549535*arctanh((x^(1/3)*
a+b)^(1/2)/b^(1/2))*a^10*b*x^(10/3))/x^3/(b*x^(2/3)+a*x)^(1/2)/b^(23/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{x^4\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(1/x**4/(b*x**(2/3)+a*x)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt(a*x + b*x**(2/3))), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}\,x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*x^(2/3))*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx = \frac{\frac{14549535\,a^{11}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^{10}} + \frac{14549535\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}a^{11}-140645505\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{11}b+609140532\left(ax^{\frac{1}{3}}\right.}{}}{}$$

input
```
integrate(1/x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/13762560*(14549535*a^11*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b
^10) + (14549535*(a*x^(1/3) + b)^(19/2)*a^11 - 140645505*(a*x^(1/3) + b)^(
17/2)*a^11*b + 609140532*(a*x^(1/3) + b)^(15/2)*a^11*b^2 - 1554721740*(a*x
^(1/3) + b)^(13/2)*a^11*b^3 + 2585198330*(a*x^(1/3) + b)^(11/2)*a^11*b^4 -
 2918514950*(a*x^(1/3) + b)^(9/2)*a^11*b^5 + 2255541300*(a*x^(1/3) + b)^(7
/2)*a^11*b^6 - 1168982220*(a*x^(1/3) + b)^(5/2)*a^11*b^7 + 382331775*(a*x^
(1/3) + b)^(3/2)*a^11*b^8 - 68025825*sqrt(a*x^(1/3) + b)*a^11*b^9)/(a^10*b
^10*x^(10/3)))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{x^4\,\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

input
```
int(1/(x^4*(a*x + b*x^(2/3))^(1/2)),x)
```

output
```
int(1/(x^4*(a*x + b*x^(2/3))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx = \frac{-19399380x^{\frac{8}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,a^8b^2+11824384x^{\frac{5}{3}}\sqrt{x^{\frac{1}{3}}a+b}\,a^5b^5-9261056x^{\frac{2}{3}}\sqrt{x^{\frac{1}{3}}a+b}$$

input
```
int(1/x^4/(b*x^(2/3)+a*x)^(1/2),x)
```

output
```
( - 19399380*x**(2/3)*sqrt(x**(1/3)*a + b)*a**8*b**2*x**2 + 11824384*x**(2
/3)*sqrt(x**(1/3)*a + b)*a**5*b**5*x - 9261056*x**(2/3)*sqrt(x**(1/3)*a +
b)*a**2*b**8 + 15519504*x**(1/3)*sqrt(x**(1/3)*a + b)*a**7*b**3*x**2 - 107
49440*x**(1/3)*sqrt(x**(1/3)*a + b)*a**4*b**6*x + 8716288*x**(1/3)*sqrt(x*
*(1/3)*a + b)*a*b**9 + 29099070*sqrt(x**(1/3)*a + b)*a**9*b*x**3 - 1330243
2*sqrt(x**(1/3)*a + b)*a**6*b**4*x**2 + 9922560*sqrt(x**(1/3)*a + b)*a**3*
b**7*x - 8257536*sqrt(x**(1/3)*a + b)*b**10 + 14549535*x**(1/3)*sqrt(b)*lo
g(sqrt(x**(1/3)*a + b) - sqrt(b))*a**10*x**3 - 14549535*x**(1/3)*sqrt(b)*l
og(sqrt(x**(1/3)*a + b) + sqrt(b))*a**10*x**3)/(27525120*x**(1/3)*b**11*x*
*3)
```

3.173 $\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal result 1686
Mathematica [A] (verified) 1687
Rubi [A] (verified) 1687
Maple [A] (verified) 1704
Fricas [B] (verification not implemented) 1704
Sympy [F] 1705
Maxima [F] 1706
Giac [A] (verification not implemented) 1706
Mupad [F(-1)] 1707
Reduce [B] (verification not implemented) 1707

Optimal result

Integrand size = 19, antiderivative size = 336

$$\int \frac{x^4}{(bx^{2/3}+ax)^{3/2}}\,dx = -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} - \frac{524288b^9\sqrt{bx^{2/3}+ax}}{29393a^{11}} + \frac{1048576b^{10}\sqrt{bx^{2/3}+ax}}{29393a^{12}\sqrt[3]{x}} + \frac{393216b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{29393a^{10}} - \frac{327680b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{29393a^9} + \frac{40960b^6x\sqrt{bx^{2/3}+ax}}{4199a^8} - \frac{36864b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{4199a^7} + \frac{33792b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^6} - \frac{16896b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^5} + \frac{15840b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^4} - \frac{880bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^3} + \frac{44x^3\sqrt{bx^{2/3}+ax}}{7a^2}$$

output
```
-6*x^4/a/(b*x^(2/3)+a*x)^(1/2)-524288/29393*b^9*(b*x^(2/3)+a*x)^(1/2)/a^11
+1048576/29393*b^10*(b*x^(2/3)+a*x)^(1/2)/a^12/x^(1/3)+393216/29393*b^8*x^
(1/3)*(b*x^(2/3)+a*x)^(1/2)/a^10-327680/29393*b^7*x^(2/3)*(b*x^(2/3)+a*x)^
(1/2)/a^9+40960/4199*b^6*x*(b*x^(2/3)+a*x)^(1/2)/a^8-36864/4199*b^5*x^(4/3
)*(b*x^(2/3)+a*x)^(1/2)/a^7+33792/4199*b^4*x^(5/3)*(b*x^(2/3)+a*x)^(1/2)/a
^6-16896/2261*b^3*x^2*(b*x^(2/3)+a*x)^(1/2)/a^5+15840/2261*b^2*x^(7/3)*(b*
x^(2/3)+a*x)^(1/2)/a^4-880/133*b*x^(8/3)*(b*x^(2/3)+a*x)^(1/2)/a^3+44/7*x^
3*(b*x^(2/3)+a*x)^(1/2)/a^2
```

Mathematica [A] (verified)

Time = 4.51 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.48

$$\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{2\sqrt[3]{x}\left(524288b^{11}+262144ab^{10}\sqrt[3]{x}-65536a^2b^9x^{2/3}+32768a^3b^8x-20480a^4b^7x^{4/3}\right.}{}$$

input
```
Integrate[x^4/(b*x^(2/3) + a*x)^(3/2),x]
```

output
```
(2*x^(1/3)*(524288*b^11 + 262144*a*b^10*x^(1/3) - 65536*a^2*b^9*x^(2/3) +
32768*a^3*b^8*x - 20480*a^4*b^7*x^(4/3) + 14336*a^5*b^6*x^(5/3) - 10752*a^
6*b^5*x^2 + 8448*a^7*b^4*x^(7/3) - 6864*a^8*b^3*x^(8/3) + 5720*a^9*b^2*x^3
 - 4862*a^10*b*x^(10/3) + 4199*a^11*x^(11/3)))/(29393*a^12*Sqrt[b*x^(2/3)
+ a*x])
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.17, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {1921, 1922, 1922, 1922, 1922, 1922, 1922, 1922, 1922, 1922, 1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(ax+bx^{2/3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{22\int \frac{x^3}{\sqrt{x^{2/3}b+ax}}dx}{a}-\frac{6x^4}{a\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{22\left(\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\int \frac{x^{8/3}}{\sqrt{x^{2/3}b+ax}}dx}{21a}\right)}{a}-\frac{6x^4}{a\sqrt{ax+bx^{2/3}}}$$

$\big\downarrow$ 1922

$$\frac{22\left(\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a}-\frac{18b\int\frac{x^{7/3}}{\sqrt{x^{2/3}b+ax}}dx}{19a}\right)}{21a}\right)}{a}-\frac{6x^4}{a\sqrt{ax+bx^{2/3}}}$$

$\big\downarrow$ 1922

$$\frac{22\left(\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a}-\frac{18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a}-\frac{16b\int\frac{x^2}{\sqrt{x^{2/3}b+ax}}dx}{17a}\right)}{19a}\right)}{21a}\right)}{a}-\frac{6x^4}{a\sqrt{ax+bx^{2/3}}}$$

$\big\downarrow$ 1922

$$\frac{22\left(\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a}-\frac{18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a}-\frac{16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\int\frac{x^{5/3}}{\sqrt{x^{2/3}b+ax}}dx}{15a}\right)}{17a}\right)}{19a}\right)}{21a}\right)}{a}-\frac{6x^4}{a\sqrt{ax+bx^{2/3}}}$$

$\downarrow$ 1922

$$\frac{22\left(\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a}-\frac{18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a}-\frac{16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\int\frac{x^{4/3}}{\sqrt{x^{2/3}b+ax}}dx}{13a}\right)}{15a}\right)}{17a}\right)}{19a}\right)}{21a}\right)}{a}$$

$$\frac{6x^4}{a\sqrt{ax+bx^{2/3}}}$$

↓ 1922

$$22\left(\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a}-\frac{18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a}-\frac{16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\cdots\right.}{13a}\right.}{15a}\right.}{17a}\right.}{19a}\right.}{21a}\right.$$

↓ 1922

$$12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\right.$$

$$14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\right.$$

$$16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{\quad}{15a}\right.$$

$$18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a}-\frac{\quad}{17a}\right.$$

$$20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a}-\frac{\quad}{19a}\right.$$

↓ 1922

1

$12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\right.$

$14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\right.$

$16b\left(\frac{2x^{2}\sqrt{ax+bx^{2/3}}}{5a}-\right.$

↓ 1922

$$12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\cdots\right.$$

$$14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{\cdots}{\cdots}\right.$$

$$16b\left(\frac{2x^{2}\sqrt{ax+bx^{2/3}}}{5a}-\frac{\cdots}{\cdots}\right.$$

↓ 1908

1

$$12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\right.$$

$$14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\right.$$

↓ 1920

1

$$12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\right.$$

$$14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\right.$$

input
```
Int[x^4/(b*x^(2/3) + a*x)^(3/2),x]
```

output
```
(-6*x^4)/(a*Sqrt[b*x^(2/3) + a*x]) + (22*((2*x^3*Sqrt[b*x^(2/3) + a*x])/(7
*a) - (20*b*((6*x^(8/3)*Sqrt[b*x^(2/3) + a*x])/(19*a) - (18*b*((6*x^(7/3)*
Sqrt[b*x^(2/3) + a*x])/(17*a) - (16*b*((2*x^2*Sqrt[b*x^(2/3) + a*x])/(5*a)
 - (14*b*((6*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/(13*a) - (12*b*((6*x^(4/3)*Sqr
t[b*x^(2/3) + a*x])/(11*a) - (10*b*((2*x*Sqrt[b*x^(2/3) + a*x])/(3*a) - (8
*b*((6*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(7*a) - (6*b*((6*x^(1/3)*Sqrt[b*x^(2
/3) + a*x])/(5*a) - (4*b*((2*Sqrt[b*x^(2/3) + a*x])/a - (4*b*Sqrt[b*x^(2/3
) + a*x])/(a^2*x^(1/3))))/(5*a)))/(7*a)))/(9*a)))/(11*a)))/(13*a)))/(15*a)
))/(17*a)))/(19*a)))/(21*a)))/a
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.43

method	result
derivativedivides	$\frac{2x\left(x^{\frac{1}{3}}a+b\right)\left(4199a^{11}x^{\frac{11}{3}}-4862a^{10}b\,x^{\frac{10}{3}}+5720a^{9}b^{2}x^{3}-6864a^{8}b^{3}x^{\frac{8}{3}}+8448a^{7}b^{4}x^{\frac{7}{3}}-10752a^{6}b^{5}x^{2}+14336a^{5}b^{6}x^{\frac{5}{3}}-2}{29393\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^{12}}$
default	$\frac{2x\left(x^{\frac{1}{3}}a+b\right)\left(4199a^{11}x^{\frac{11}{3}}-4862a^{10}b\,x^{\frac{10}{3}}+5720a^{9}b^{2}x^{3}-6864a^{8}b^{3}x^{\frac{8}{3}}+8448a^{7}b^{4}x^{\frac{7}{3}}-10752a^{6}b^{5}x^{2}+14336a^{5}b^{6}x^{\frac{5}{3}}-2}{29393\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^{12}}$

input
```
int(x^4/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/29393*x*(x^(1/3)*a+b)*(4199*a^11*x^(11/3)-4862*a^10*b*x^(10/3)+5720*a^9*
b^2*x^3-6864*a^8*b^3*x^(8/3)+8448*a^7*b^4*x^(7/3)-10752*a^6*b^5*x^2+14336*
a^5*b^6*x^(5/3)-20480*a^4*b^7*x^(4/3)+32768*a^3*b^8*x-65536*a^2*b^9*x^(2/3
)+262144*a*b^10*x^(1/3)+524288*b^11)/(b*x^(2/3)+a*x)^(3/2)/a^12
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2566 vs. 2(252) = 504.

Time = 117.47 (sec) , antiderivative size = 2566, normalized size of antiderivative = 7.64

$$\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-1/29393*((6442450944*a^3*b^19 + 5368709120*a^3*b^18 - 2013265920*a^3*b^17
 - 6113744*a^18 + 402653184*(17*a^6 - 3*a^3)*b^16 + 8388608*(464*a^6 + 53*
a^3)*b^15 - 12582912*(246*a^6 + a^3)*b^14 + 1572864*(1036*a^9 - 2560*a^6 -
 3*a^3)*b^13 - 524288*(758*a^9 - 1569*a^6)*b^12 - 393216*(5803*a^9 + 124*a
^6)*b^11 + 98304*(1315*a^12 - 20924*a^9 - 33*a^6)*b^10 - 57344*(2264*a^12
- 3153*a^9)*b^9 - 6144*(83789*a^12 + 2066*a^9)*b^8 - 1536*(46256*a^15 - 15
9272*a^12 - 267*a^9)*b^7 - 128*(264488*a^15 + 382229*a^12)*b^6 + 9984*(155
47*a^15 + 482*a^12)*b^5 - 24*(2376192*a^18 + 4735792*a^15 + 7887*a^12)*b^4
 - 1664*(107856*a^18 - 16759*a^15)*b^3 - 156*(935424*a^18 + 17935*a^15)*b^
2 + 663*(97664*a^18 + 123*a^15)*b)*x^2 + (6442450944*b^22 + 5368709120*b^2
1 + 402653184*(17*a^3 - 3)*b^19 - 2013265920*b^20 + 8388608*(464*a^3 + 53)
*b^18 - 6113744*a^15*b^3 - 12582912*(246*a^3 + 1)*b^17 + 1572864*(1036*a^6
 - 2560*a^3 - 3)*b^16 - 524288*(758*a^6 - 1569*a^3)*b^15 - 393216*(5803*a^
6 + 124*a^3)*b^14 + 98304*(1315*a^9 - 20924*a^6 - 33*a^3)*b^13 - 57344*(22
64*a^9 - 3153*a^6)*b^12 - 6144*(83789*a^9 + 2066*a^6)*b^11 - 1536*(46256*a
^12 - 159272*a^9 - 267*a^6)*b^10 - 128*(264488*a^12 + 382229*a^9)*b^9 + 99
84*(15547*a^12 + 482*a^9)*b^8 - 24*(2376192*a^15 + 4735792*a^12 + 7887*a^9
)*b^7 - 1664*(107856*a^15 - 16759*a^12)*b^6 - 156*(935424*a^15 + 17935*a^1
2)*b^5 + 663*(97664*a^15 + 123*a^12)*b^4)*x - 2*(4199*(4096*a^13*b^9 + 614
4*a^13*b^8 + 768*a^13*b^7 - 4096*a^19 - 144*a^16*b^2 + 216*a^16*b - 27*...
```

Sympy [F]

$$\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{x^4}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4/(b*x**(2/3)+a*x)**(3/2),x)
```

output
```
Integral(x**4/(a*x + b*x**(2/3))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{x^4}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(a*x + b*x^(2/3))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.64

$$\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = -\frac{1048576\,b^{\frac{21}{2}}}{29393\,a^{12}} + \frac{6\,b^{11}}{\sqrt{ax^{\frac{1}{3}}+b}a^{12}} + \frac{2\left(4199\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}a^{240} - 51051\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}a^{240}b + 285285\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{240}b^2 - 969969\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}} \cdots\right.}{}$$

input
```
integrate(x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
-1048576/29393*b^(21/2)/a^12 + 6*b^11/(sqrt(a*x^(1/3) + b)*a^12) + 2/29393
*(4199*(a*x^(1/3) + b)^(21/2)*a^240 - 51051*(a*x^(1/3) + b)^(19/2)*a^240*b
 + 285285*(a*x^(1/3) + b)^(17/2)*a^240*b^2 - 969969*(a*x^(1/3) + b)^(15/2)
*a^240*b^3 + 2238390*(a*x^(1/3) + b)^(13/2)*a^240*b^4 - 3703518*(a*x^(1/3)
 + b)^(11/2)*a^240*b^5 + 4526522*(a*x^(1/3) + b)^(9/2)*a^240*b^6 - 4157010
*(a*x^(1/3) + b)^(7/2)*a^240*b^7 + 2909907*(a*x^(1/3) + b)^(5/2)*a^240*b^8
 - 1616615*(a*x^(1/3) + b)^(3/2)*a^240*b^9 + 969969*sqrt(a*x^(1/3) + b)*a^
240*b^10)/a^252
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{x^4}{{\left(a\,x+b\,x^{2/3}\right)}^{3/2}} \,d x$$

input
```
int(x^4/(a*x + b*x^(2/3))^(3/2),x)
```

output
```
int(x^4/(a*x + b*x^(2/3))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.40

$$\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{\frac{2x^{\frac{11}{3}}a^{11}}{7}-\frac{1056x^{\frac{8}{3}}a^{8}b^{3}}{2261}+\frac{4096x^{\frac{5}{3}}a^{5}b^{6}}{4199}-\frac{131072x^{\frac{2}{3}}a^{2}b^{9}}{29393}-\frac{44x^{\frac{10}{3}}a^{10}b}{133}+\frac{16896x^{\frac{7}{3}}a^{7}b^{4}}{29393}-\frac{40960x^{\frac{4}{3}}a^{4}}{29393}}{\sqrt{x^{\frac{1}{3}}a+b}\,a^{12}}$$

input
```
int(x^4/(b*x^(2/3)+a*x)^(3/2),x)
```

output
```
(2*(4199*x**(2/3)*a**11*x**3 - 6864*x**(2/3)*a**8*b**3*x**2 + 14336*x**(2/
3)*a**5*b**6*x - 65536*x**(2/3)*a**2*b**9 - 4862*x**(1/3)*a**10*b*x**3 + 8
448*x**(1/3)*a**7*b**4*x**2 - 20480*x**(1/3)*a**4*b**7*x + 262144*x**(1/3)
*a*b**10 + 5720*a**9*b**2*x**3 - 10752*a**6*b**5*x**2 + 32768*a**3*b**8*x
+ 524288*b**11))/(29393*sqrt(x**(1/3)*a + b)*a**12)
```

3.174 $\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal result . . . 1708
Mathematica [A] (verified) . . . 1709
Rubi [A] (verified) . . . 1709
Maple [A] (verified) . . . 1720
Fricas [B] (verification not implemented) . . . 1720
Sympy [F] . . . 1721
Maxima [F] . . . 1722
Giac [A] (verification not implemented) . . . 1722
Mupad [F(-1)] . . . 1723
Reduce [B] (verification not implemented) . . . 1723

Optimal result

Integrand size = 19, antiderivative size = 248

$$\begin{aligned}\int \frac{x^3}{(bx^{2/3}+ax)^{3/2}}\,dx = & -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}}+\frac{32768b^6\sqrt{bx^{2/3}+ax}}{2145a^8} \\ & -\frac{65536b^7\sqrt{bx^{2/3}+ax}}{2145a^9\sqrt[3]{x}}-\frac{8192b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^7} \\ & +\frac{4096b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^6}-\frac{3584b^3x\sqrt{bx^{2/3}+ax}}{429a^5} \\ & +\frac{5376b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^4}-\frac{448bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^3}+\frac{32x^2\sqrt{bx^{2/3}+ax}}{5a^2}\end{aligned}$$

output
```
-6*x^3/a/(b*x^(2/3)+a*x)^(1/2)+32768/2145*b^6*(b*x^(2/3)+a*x)^(1/2)/a^8-65
536/2145*b^7*(b*x^(2/3)+a*x)^(1/2)/a^9/x^(1/3)-8192/715*b^5*x^(1/3)*(b*x^(
2/3)+a*x)^(1/2)/a^7+4096/429*b^4*x^(2/3)*(b*x^(2/3)+a*x)^(1/2)/a^6-3584/42
9*b^3*x*(b*x^(2/3)+a*x)^(1/2)/a^5+5376/715*b^2*x^(4/3)*(b*x^(2/3)+a*x)^(1/
2)/a^4-448/65*b*x^(5/3)*(b*x^(2/3)+a*x)^(1/2)/a^3+32/5*x^2*(b*x^(2/3)+a*x)
^(1/2)/a^2
```

Mathematica [A] (verified)

Time = 4.50 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.49

$$\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{2\left(-32768b^8\sqrt[3]{x}-16384ab^7x^{2/3}+4096a^2b^6x-2048a^3b^5x^{4/3}+1280a^4b^4x^{5/3}-89\right.}{2145a^9\sqrt{bx^{2/3}+ax}}$$

input
```
Integrate[x^3/(b*x^(2/3) + a*x)^(3/2),x]
```

output
```
(2*(-32768*b^8*x^(1/3) - 16384*a*b^7*x^(2/3) + 4096*a^2*b^6*x - 2048*a^3*b
^5*x^(4/3) + 1280*a^4*b^4*x^(5/3) - 896*a^5*b^3*x^2 + 672*a^6*b^2*x^(7/3)
- 528*a^7*b*x^(8/3) + 429*a^8*x^3))/(2145*a^9*Sqrt[b*x^(2/3) + a*x])
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1921, 1922, 1922, 1922, 1922, 1922, 1922, 1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(ax+bx^{2/3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{16\int \frac{x^2}{\sqrt{x^{2/3}b+ax}}dx}{a}-\frac{6x^3}{a\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{16\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\int \frac{x^{5/3}}{\sqrt{x^{2/3}b+ax}}dx}{15a}\right)}{a}-\frac{6x^3}{a\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{16\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\int\frac{x^{4/3}}{\sqrt{x^{2/3}b+ax}}dx}{13a}\right)}{15a}\right)}{a}-\frac{6x^3}{a\sqrt{ax+bx^{2/3}}}$$

$\downarrow$ 1922

$$\frac{16\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\frac{10b\int\frac{x}{\sqrt{x^{2/3}b+ax}}dx}{11a}\right)}{13a}\right)}{15a}\right)}{a}-\frac{6x^3}{a\sqrt{ax+bx^{2/3}}}$$

$\downarrow$ 1922

$$\frac{16\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\frac{10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\int\frac{x^{2/3}}{\sqrt{x^{2/3}b+ax}}dx}{9a}\right)}{11a}\right)}{13a}\right)}{15a}\right)}{a}-$$

$$\frac{6x^3}{a\sqrt{ax+bx^{2/3}}}$$

$\downarrow$ 1922

$$16\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\frac{10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\int\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}\,dx}{7a}\right)}{9a}\right)}{11a}\right)}{13a}\right)}{15a}\right.$$

↓ 1922

$$16\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\frac{10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\left(\frac{6\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{5a}-\frac{4b\int \cdots}{\cdots}\right.}{7a}\right.}{9a}\right.}{11a}\right.}{13a}\right.}{15a}\right.$$

↓ 1908

$$4b$$

$$6b\left(\frac{6\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{5a}-\right.$$

$$8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\right.$$

$$10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{\qquad}{9a}\right.$$

$$12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\frac{\qquad}{11a}\right.$$

$$14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{\qquad}{13a}\right.$$

↓ 1920

$$14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\frac{10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\left(\frac{6\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{5a}-\frac{4b\left(\cdots\right.}{\cdots}\right.}{9a}\right.}{11a}\right.}{13a}\right.$$

input
```
Int[x^3/(b*x^(2/3) + a*x)^(3/2),x]
```

output
```
(-6*x^3)/(a*Sqrt[b*x^(2/3) + a*x]) + (16*((2*x^2*Sqrt[b*x^(2/3) + a*x])/(5
*a) - (14*b*((6*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/(13*a) - (12*b*((6*x^(4/3)*
Sqrt[b*x^(2/3) + a*x])/(11*a) - (10*b*((2*x*Sqrt[b*x^(2/3) + a*x])/(3*a) -
 (8*b*((6*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(7*a) - (6*b*((6*x^(1/3)*Sqrt[b*x
^(2/3) + a*x])/(5*a) - (4*b*((2*Sqrt[b*x^(2/3) + a*x])/a - (4*b*Sqrt[b*x^(
2/3) + a*x])/(a^2*x^(1/3))))/(5*a)))/(7*a)))/(9*a)))/(11*a)))/(13*a)))/(15
*a)))/a
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.44

method	result
derivativedivides	$\frac{2x\left(x^{\frac{1}{3}}a+b\right)\left(429a^{8}x^{\frac{8}{3}}-528a^{7}bx^{\frac{7}{3}}+672a^{6}b^{2}x^{2}-896a^{5}b^{3}x^{\frac{5}{3}}+1280x^{\frac{4}{3}}a^{4}b^{4}-2048a^{3}b^{5}x+4096a^{2}b^{6}x^{\frac{2}{3}}-16384x^{\frac{1}{3}}ab^{7}\right.}{2145\left(bx^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^{9}}$
default	$\frac{2x\left(x^{\frac{1}{3}}a+b\right)\left(429a^{8}x^{\frac{8}{3}}-528a^{7}bx^{\frac{7}{3}}+672a^{6}b^{2}x^{2}-896a^{5}b^{3}x^{\frac{5}{3}}+1280x^{\frac{4}{3}}a^{4}b^{4}-2048a^{3}b^{5}x+4096a^{2}b^{6}x^{\frac{2}{3}}-16384x^{\frac{1}{3}}ab^{7}\right.}{2145\left(bx^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^{9}}$

input
```
int(x^3/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/2145*x*(x^(1/3)*a+b)*(429*a^8*x^(8/3)-528*a^7*b*x^(7/3)+672*a^6*b^2*x^2-
896*a^5*b^3*x^(5/3)+1280*x^(4/3)*a^4*b^4-2048*a^3*b^5*x+4096*a^2*b^6*x^(2/
3)-16384*x^(1/3)*a*b^7-32768*b^8)/(b*x^(2/3)+a*x)^(3/2)/a^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2083 vs. 2(186) = 372.

Time = 124.49 (sec) , antiderivative size = 2083, normalized size of antiderivative = 8.40

$$\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
1/2145*((402653184*a^3*b^16 + 335544320*a^3*b^15 - 125829120*a^3*b^14 + 62
4624*a^15 + 25165824*(17*a^6 - 3*a^3)*b^13 + 524288*(464*a^6 + 53*a^3)*b^1
2 - 786432*(246*a^6 + a^3)*b^11 + 98304*(1036*a^9 - 2560*a^6 - 3*a^3)*b^10
 - 32768*(758*a^9 - 1569*a^6)*b^9 - 24576*(5803*a^9 + 124*a^6)*b^8 + 6144*
(600*a^12 - 20924*a^9 - 33*a^6)*b^7 - 1536*(7666*a^12 - 7357*a^9)*b^6 - 76
8*(40107*a^12 + 1033*a^9)*b^5 + 96*(63360*a^15 + 167852*a^12 + 267*a^9)*b^
4 + 32*(613440*a^15 - 105031*a^12)*b^3 + 468*(34560*a^15 + 661*a^12)*b^2 -
 99*(68480*a^15 + 87*a^12)*b)*x^2 + (402653184*b^19 + 335544320*b^18 + 251
65824*(17*a^3 - 3)*b^16 - 125829120*b^17 + 524288*(464*a^3 + 53)*b^15 + 62
4624*a^12*b^3 - 786432*(246*a^3 + 1)*b^14 + 98304*(1036*a^6 - 2560*a^3 - 3
)*b^13 - 32768*(758*a^6 - 1569*a^3)*b^12 - 24576*(5803*a^6 + 124*a^3)*b^11
 + 6144*(600*a^9 - 20924*a^6 - 33*a^3)*b^10 - 1536*(7666*a^9 - 7357*a^6)*b
^9 - 768*(40107*a^9 + 1033*a^6)*b^8 + 96*(63360*a^12 + 167852*a^9 + 267*a^
6)*b^7 + 32*(613440*a^12 - 105031*a^9)*b^6 + 468*(34560*a^12 + 661*a^9)*b^
5 - 99*(68480*a^12 + 87*a^9)*b^4)*x + 2*(429*(4096*a^10*b^9 + 6144*a^10*b^
8 + 768*a^10*b^7 - 4096*a^16 - 144*a^13*b^2 + 216*a^13*b - 27*a^13 + 256*(
16*a^13 - 7*a^10)*b^6 + 48*(128*a^13 - 3*a^10)*b^5 + 24*(32*a^13 + 9*a^10)
*b^4 - (5888*a^13 + 27*a^10)*b^3)*x^4 - 2096*(4096*a^7*b^12 + 6144*a^7*b^1
1 + 768*a^7*b^10 - 144*a^10*b^5 + 216*a^10*b^4 + 256*(16*a^10 - 7*a^7)*b^9
 + 48*(128*a^10 - 3*a^7)*b^8 + 24*(32*a^10 + 9*a^7)*b^7 - (5888*a^10 + ...
```

Sympy [F]

$$\int \frac{x^3}{(bx^{2/3}+ax)^{3/2}}\,dx=\int \frac{x^3}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(b*x**(2/3)+a*x)**(3/2),x)
```

output
```
Integral(x**3/(a*x + b*x**(2/3))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{x^3}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(a*x + b*x^(2/3))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.68

$$\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{65536\,b^{\frac{15}{2}}}{2145\,a^9} - \frac{2\left(\frac{6435\,b^8}{\sqrt{ax^{\frac{1}{3}}+b}a}-\frac{429\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{14}-3960\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^{14}b+16380\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^{14}b^2-40040\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^{14}b^3+64350\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^{14}b^4}{a^{15}}\right.}{2145\,a^8}$$

input
```
integrate(x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
65536/2145*b^(15/2)/a^9 - 2/2145*(6435*b^8/(sqrt(a*x^(1/3) + b)*a) - (429*
(a*x^(1/3) + b)^(15/2)*a^14 - 3960*(a*x^(1/3) + b)^(13/2)*a^14*b + 16380*(
a*x^(1/3) + b)^(11/2)*a^14*b^2 - 40040*(a*x^(1/3) + b)^(9/2)*a^14*b^3 + 64
350*(a*x^(1/3) + b)^(7/2)*a^14*b^4 - 72072*(a*x^(1/3) + b)^(5/2)*a^14*b^5
+ 60060*(a*x^(1/3) + b)^(3/2)*a^14*b^6 - 51480*sqrt(a*x^(1/3) + b)*a^14*b^
7)/a^15)/a^8
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{x^3}{\left(a\,x+b\,x^{2/3}\right)^{3/2}}\,dx$$

input
```
int(x^3/(a*x + b*x^(2/3))^(3/2),x)
```

output
```
int(x^3/(a*x + b*x^(2/3))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.40

$$\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{\frac{2x^{\frac{8}{3}}a^8}{5}-\frac{1792x^{\frac{5}{3}}a^5b^3}{2145}+\frac{8192x^{\frac{2}{3}}a^2b^6}{2145}-\frac{32x^{\frac{7}{3}}a^7b}{65}+\frac{512x^{\frac{4}{3}}a^4b^4}{429}-\frac{32768x^{\frac{1}{3}}a\,b^7}{2145}+\frac{448a^6b^2x^2}{715}-\frac{4096}{2}}{\sqrt{x^{\frac{1}{3}}a+b}\,a^9}$$

input
```
int(x^3/(b*x^(2/3)+a*x)^(3/2),x)
```

output
```
(2*(429*x**(2/3)*a**8*x**2 - 896*x**(2/3)*a**5*b**3*x + 4096*x**(2/3)*a**2
*b**6 - 528*x**(1/3)*a**7*b*x**2 + 1280*x**(1/3)*a**4*b**4*x - 16384*x**(1
/3)*a*b**7 + 672*a**6*b**2*x**2 - 2048*a**3*b**5*x - 32768*b**8))/(2145*sq
rt(x**(1/3)*a + b)*a**9)
```

3.175 $\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal result 1724
Mathematica [A] (verified) 1724
Rubi [A] (verified) 1725
Maple [A] (verified) 1728
Fricas [B] (verification not implemented) 1729
Sympy [F] 1730
Maxima [F] 1730
Giac [A] (verification not implemented) 1730
Mupad [F(-1)] 1731
Reduce [B] (verification not implemented) 1731

Optimal result

Integrand size = 19, antiderivative size = 160

$$\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=-\frac{6x^2}{a\sqrt{bx^{2/3}+ax}}-\frac{256b^3\sqrt{bx^{2/3}+ax}}{21a^5}+\frac{512b^4\sqrt{bx^{2/3}+ax}}{21a^6\sqrt[3]{x}}+\frac{64b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{7a^4}-\frac{160bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^3}+\frac{20x\sqrt{bx^{2/3}+ax}}{3a^2}$$

output
```
-6*x^2/a/(b*x^(2/3)+a*x)^(1/2)-256/21*b^3*(b*x^(2/3)+a*x)^(1/2)/a^5+512/21
*b^4*(b*x^(2/3)+a*x)^(1/2)/a^6/x^(1/3)+64/7*b^2*x^(1/3)*(b*x^(2/3)+a*x)^(1
/2)/a^4-160/21*b*x^(2/3)*(b*x^(2/3)+a*x)^(1/2)/a^3+20/3*x*(b*x^(2/3)+a*x)^
(1/2)/a^2
```

Mathematica [A] (verified)

Time = 4.87 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.53

$$\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{512b^5\sqrt[3]{x}+256ab^4x^{2/3}-64a^2b^3x+32a^3b^2x^{4/3}-20a^4bx^{5/3}+14a^5x^2}{21a^6\sqrt{bx^{2/3}+ax}}$$

input
```
Integrate[x^2/(b*x^(2/3) + a*x)^(3/2),x]
```

output
```
(512*b^5*x^(1/3) + 256*a*b^4*x^(2/3) - 64*a^2*b^3*x + 32*a^3*b^2*x^(4/3) -
 20*a^4*b*x^(5/3) + 14*a^5*x^2)/(21*a^6*Sqrt[b*x^(2/3) + a*x])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1921, 1922, 1922, 1922, 1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(ax+bx^{2/3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{10\int \frac{x}{\sqrt{x^{2/3}b+ax}}dx}{a}-\frac{6x^2}{a\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{10\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\int \frac{x^{2/3}}{\sqrt{x^{2/3}b+ax}}dx}{9a}\right)}{a}-\frac{6x^2}{a\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{10\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\int \frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}dx}{7a}\right)}{9a}\right)}{a}-\frac{6x^2}{a\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{10\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\left(\frac{6\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{5a}-\frac{4b\int\frac{1}{\sqrt{x^{2/3}b+ax}}dx}{5a}\right)}{7a}\right)}{9a}\right)}{a}-\frac{6x^2}{a\sqrt{ax+bx^{2/3}}}$$

$\downarrow$ 1908

$$\frac{10\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\left(\frac{6\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{5a}-\frac{4b\left(\frac{2\sqrt{ax+bx^{2/3}}}{a}-\frac{2b\int\frac{1}{\sqrt[3]{x}\sqrt{x^{2/3}b+ax}}dx}{3a}\right)}{5a}\right)}{7a}\right)}{9a}\right)}{a}-\frac{6x^2}{a\sqrt{ax+bx^{2/3}}}$$

$\downarrow$ 1920

$$\frac{10\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\left(\frac{6\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{5a}-\frac{4b\left(\frac{2\sqrt{ax+bx^{2/3}}}{a}-\frac{4b\sqrt{ax+bx^{2/3}}}{a^2\sqrt[3]{x}}\right)}{5a}\right)}{7a}\right)}{9a}\right)}{a}-\frac{6x^2}{a\sqrt{ax+bx^{2/3}}}$$

input
```
Int[x^2/(b*x^(2/3) + a*x)^(3/2),x]
```

output
```
(-6*x^2)/(a*Sqrt[b*x^(2/3) + a*x]) + (10*((2*x*Sqrt[b*x^(2/3) + a*x])/(3*a
) - (8*b*((6*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(7*a) - (6*b*((6*x^(1/3)*Sqrt[
b*x^(2/3) + a*x])/(5*a) - (4*b*((2*Sqrt[b*x^(2/3) + a*x])/a - (4*b*Sqrt[b*
x^(2/3) + a*x])/(a^2*x^(1/3))))/(5*a)))/(7*a)))/(9*a)))/a
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.48

method	result	size
derivativedivides	$\frac{2x\left(x^{\frac{1}{3}}a+b\right)\left(7a^{5}x^{\frac{5}{3}}-10x^{\frac{4}{3}}a^{4}b+16a^{3}b^{2}x-32a^{2}b^{3}x^{\frac{2}{3}}+128x^{\frac{1}{3}}a\,b^{4}+256b^{5}\right)}{21\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^{6}}$	77
default	$\frac{2x\left(x^{\frac{1}{3}}a+b\right)\left(7a^{5}x^{\frac{5}{3}}-10x^{\frac{4}{3}}a^{4}b+16a^{3}b^{2}x-32a^{2}b^{3}x^{\frac{2}{3}}+128x^{\frac{1}{3}}a\,b^{4}+256b^{5}\right)}{21\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^{6}}$	77

input
```
int(x^2/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/21*x*(x^(1/3)*a+b)*(7*a^5*x^(5/3)-10*x^(4/3)*a^4*b+16*a^3*b^2*x-32*a^2*b
^3*x^(2/3)+128*x^(1/3)*a*b^4+256*b^5)/(b*x^(2/3)+a*x)^(3/2)/a^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1598 vs. $2(120) = 240$.

Time = 114.81 (sec) , antiderivative size = 1598, normalized size of antiderivative = 9.99

$$\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-1/21*((3145728*a^3*b^13 + 2621440*a^3*b^12 - 983040*a^3*b^11 - 10192*a^12
 + 196608*(17*a^6 - 3*a^3)*b^10 + 4096*(464*a^6 + 53*a^3)*b^9 - 6144*(246*
a^6 + a^3)*b^8 + 768*(1120*a^9 - 2560*a^6 - 3*a^3)*b^7 - 256*(548*a^9 - 15
69*a^6)*b^6 - 768*(1477*a^9 + 31*a^6)*b^5 - 48*(2304*a^12 + 21176*a^9 + 33
*a^6)*b^4 - 4032*(96*a^12 - 23*a^9)*b^3 - 12*(27648*a^12 + 527*a^9)*b^2 +
3*(39296*a^12 + 51*a^9)*b)*x^2 + (3145728*b^16 + 2621440*b^15 + 196608*(17
*a^3 - 3)*b^13 - 983040*b^14 + 4096*(464*a^3 + 53)*b^12 - 10192*a^9*b^3 -
6144*(246*a^3 + 1)*b^11 + 768*(1120*a^6 - 2560*a^3 - 3)*b^10 - 256*(548*a^
6 - 1569*a^3)*b^9 - 768*(1477*a^6 + 31*a^3)*b^8 - 48*(2304*a^9 + 21176*a^6
 + 33*a^3)*b^7 - 4032*(96*a^9 - 23*a^6)*b^6 - 12*(27648*a^9 + 527*a^6)*b^5
 + 3*(39296*a^9 + 51*a^6)*b^4)*x - 2*(7*(4096*a^7*b^9 + 6144*a^7*b^8 + 768
*a^7*b^7 - 4096*a^13 - 144*a^10*b^2 + 216*a^10*b - 27*a^10 + 256*(16*a^10
- 7*a^7)*b^6 + 48*(128*a^10 - 3*a^7)*b^5 + 24*(32*a^10 + 9*a^7)*b^4 - (588
8*a^10 + 27*a^7)*b^3)*x^3 - 58*(4096*a^4*b^12 + 6144*a^4*b^11 + 768*a^4*b^
10 - 144*a^7*b^5 + 216*a^7*b^4 + 256*(16*a^7 - 7*a^4)*b^9 + 48*(128*a^7 -
3*a^4)*b^8 + 24*(32*a^7 + 9*a^4)*b^7 - (5888*a^7 + 27*a^4)*b^6 - (4096*a^1
0 + 27*a^7)*b^3)*x^2 - 128*(4096*a*b^15 + 6144*a*b^14 + 768*a*b^13 + 256*(
16*a^4 - 7*a)*b^12 - 144*a^4*b^8 + 48*(128*a^4 - 3*a)*b^11 + 216*a^4*b^7 +
 24*(32*a^4 + 9*a)*b^10 - (5888*a^4 + 27*a)*b^9 - (4096*a^7 + 27*a^4)*b^6)
*x + (1048576*b^16 + 1572864*b^15 + 65536*(16*a^3 - 7)*b^13 + 196608*b^...
```

Sympy [F]

$$\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{x^2}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(b*x**(2/3)+a*x)**(3/2),x)
```

output
```
Integral(x**2/(a*x + b*x**(2/3))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{x^2}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(a*x + b*x^(2/3))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = -\frac{512\,b^{\frac{9}{2}}}{21\,a^6} + \frac{6\,b^5}{\sqrt{ax^{\frac{1}{3}}+b}\,a^6} + \frac{2\left(7\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^{48} - 45\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^{48}b + 126\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^{48}b^2 - 210\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^{48}b^3 + 315\sqrt{ax^{\frac{1}{3}}+\cdots}\right.}{21\,a^{54}}$$

input
```
integrate(x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
-512/21*b^(9/2)/a^6 + 6*b^5/(sqrt(a*x^(1/3) + b)*a^6) + 2/21*(7*(a*x^(1/3)
 + b)^(9/2)*a^48 - 45*(a*x^(1/3) + b)^(7/2)*a^48*b + 126*(a*x^(1/3) + b)^(
5/2)*a^48*b^2 - 210*(a*x^(1/3) + b)^(3/2)*a^48*b^3 + 315*sqrt(a*x^(1/3) +
b)*a^48*b^4)/a^54
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{x^2}{\left(a\,x+b\,x^{2/3}\right)^{3/2}}\,dx$$

input
```
int(x^2/(a*x + b*x^(2/3))^(3/2),x)
```

output
```
int(x^2/(a*x + b*x^(2/3))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.42

$$\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{\frac{2x^{\frac{5}{3}}a^5}{3}-\frac{64x^{\frac{2}{3}}a^2b^3}{21}-\frac{20x^{\frac{4}{3}}a^4b}{21}+\frac{256x^{\frac{1}{3}}a\,b^4}{21}+\frac{32a^3b^2x}{21}+\frac{512b^5}{21}}{\sqrt{x^{\frac{1}{3}}a+b}\,a^6}$$

input
```
int(x^2/(b*x^(2/3)+a*x)^(3/2),x)
```

output
```
(2*(7*x**(2/3)*a**5*x - 32*x**(2/3)*a**2*b**3 - 10*x**(1/3)*a**4*b*x + 128
*x**(1/3)*a*b**4 + 16*a**3*b**2*x + 256*b**5))/(21*sqrt(x**(1/3)*a + b)*a*
*6)
```

3.176 $\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal result . . . 1732
Mathematica [A] (verified) . . . 1732
Rubi [A] (verified) . . . 1733
Maple [A] (verified) . . . 1734
Fricas [B] (verification not implemented) . . . 1735
Sympy [F] . . . 1736
Maxima [F] . . . 1736
Giac [A] (verification not implemented) . . . 1736
Mupad [F(-1)] . . . 1737
Reduce [B] (verification not implemented) . . . 1737

Optimal result

Integrand size = 17, antiderivative size = 68

$$\int \frac{x}{(bx^{2/3}+ax)^{3/2}}\,dx = -\frac{6x}{a\sqrt{bx^{2/3}+ax}}+\frac{8\sqrt{bx^{2/3}+ax}}{a^2}-\frac{16b\sqrt{bx^{2/3}+ax}}{a^3\sqrt[3]{x}}$$

output
```
-6*x/a/(b*x^(2/3)+a*x)^(1/2)+8*(b*x^(2/3)+a*x)^(1/2)/a^2-16*b*(b*x^(2/3)+a
*x)^(1/2)/a^3/x^(1/3)
```

Mathematica [A] (verified)

Time = 4.53 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.66

$$\int \frac{x}{(bx^{2/3}+ax)^{3/2}}\,dx = \frac{2\left(-8b^2\sqrt[3]{x}-4abx^{2/3}+a^2x\right)}{a^3\sqrt{bx^{2/3}+ax}}$$

input
```
Integrate[x/(b*x^(2/3) + a*x)^(3/2),x]
```

output
```
(2*(-8*b^2*x^(1/3) - 4*a*b*x^(2/3) + a^2*x))/(a^3*Sqrt[b*x^(2/3) + a*x])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1921, 1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(ax+bx^{2/3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{4\int \frac{1}{\sqrt{x^{2/3}b+ax}}dx}{a}-\frac{6x}{a\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1908}$$

$$\frac{4\left(\frac{2\sqrt{ax+bx^{2/3}}}{a}-\frac{2b\int \frac{1}{\sqrt[3]{x}\sqrt{x^{2/3}b+ax}}dx}{3a}\right)}{a}-\frac{6x}{a\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{4\left(\frac{2\sqrt{ax+bx^{2/3}}}{a}-\frac{4b\sqrt{ax+bx^{2/3}}}{a^2\sqrt[3]{x}}\right)}{a}-\frac{6x}{a\sqrt{ax+bx^{2/3}}}$$

input
```
Int[x/(b*x^(2/3) + a*x)^(3/2),x]
```

output
```
(-6*x)/(a*Sqrt[b*x^(2/3) + a*x]) + (4*((2*Sqrt[b*x^(2/3) + a*x])/a - (4*b*
Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))))/a
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.66

method	result	size
derivativedivides	$\frac{2x\left(x^{\frac{1}{3}}a+b\right)\left(x^{\frac{2}{3}}a^{2}-4x^{\frac{1}{3}}ab-8b^{2}\right)}{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^{3}}$	45
default	$\frac{2x\left(x^{\frac{1}{3}}a+b\right)\left(x^{\frac{2}{3}}a^{2}-4x^{\frac{1}{3}}ab-8b^{2}\right)}{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^{3}}$	45

input
```
int(x/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*x*(x^(1/3)*a+b)*(x^(2/3)*a^2-4*x^(1/3)*a*b-8*b^2)/(b*x^(2/3)+a*x)^(3/2)/
a^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1107 vs. $2(54) = 108$.

Time = 132.05 (sec) , antiderivative size = 1107, normalized size of antiderivative = 16.28

$$\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
((98304*a^3*b^10 + 81920*a^3*b^9 - 30720*a^3*b^8 + 1456*a^9 + 6144*(16*a^6
 - 3*a^3)*b^7 + 6784*(8*a^6 + a^3)*b^6 - 192*(236*a^6 + a^3)*b^5 + 24*(153
6*a^9 - 2512*a^6 - 3*a^3)*b^4 + 32*(576*a^9 + 379*a^6)*b^3 - 12*(2304*a^9
+ 61*a^6)*b^2 - 3*(10112*a^9 + 15*a^6)*b)*x^2 + (98304*b^13 + 81920*b^12 +
 6144*(16*a^3 - 3)*b^10 - 30720*b^11 + 6784*(8*a^3 + 1)*b^9 + 1456*a^6*b^3
 - 192*(236*a^3 + 1)*b^8 + 24*(1536*a^6 - 2512*a^3 - 3)*b^7 + 32*(576*a^6
+ 379*a^3)*b^6 - 12*(2304*a^6 + 61*a^3)*b^5 - 3*(10112*a^6 + 15*a^3)*b^4)*
x + 2*((4096*a^4*b^9 + 6144*a^4*b^8 + 768*a^4*b^7 - 4096*a^10 - 144*a^7*b^
2 + 216*a^7*b - 27*a^7 + 256*(16*a^7 - 7*a^4)*b^6 + 48*(128*a^7 - 3*a^4)*b
^5 + 24*(32*a^7 + 9*a^4)*b^4 - (5888*a^7 + 27*a^4)*b^3)*x^2 - 3*(4096*a^2*
b^11 + 6144*a^2*b^10 + 768*a^2*b^9 - 144*a^5*b^4 + 256*(16*a^5 - 7*a^2)*b^
8 + 216*a^5*b^3 + 48*(128*a^5 - 3*a^2)*b^7 + 24*(32*a^5 + 9*a^2)*b^6 - (58
88*a^5 + 27*a^2)*b^5 - (4096*a^8 + 27*a^5)*b^2)*x^(4/3) + 4*(4096*a*b^12 +
 6144*a*b^11 + 768*a*b^10 + 256*(16*a^4 - 7*a)*b^9 - 144*a^4*b^5 + 48*(128
*a^4 - 3*a)*b^8 + 216*a^4*b^4 + 24*(32*a^4 + 9*a)*b^7 - (5888*a^4 + 27*a)*
b^6 - (4096*a^7 + 27*a^4)*b^3)*x - (32768*b^13 + 49152*b^12 + 2048*(16*a^3
 - 7)*b^10 + 6144*b^11 + 384*(128*a^3 - 3)*b^9 - 1152*a^3*b^6 + 192*(32*a^
3 + 9)*b^8 + 1728*a^3*b^5 - 8*(5888*a^3 + 27)*b^7 - 8*(4096*a^6 + 27*a^3)*
b^4 + 5*(4096*a^3*b^10 + 6144*a^3*b^9 + 768*a^3*b^8 - 144*a^6*b^3 + 216*a^
6*b^2 + 256*(16*a^6 - 7*a^3)*b^7 + 48*(128*a^6 - 3*a^3)*b^6 + 24*(32*a^...
```

Sympy [F]

$$\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{x}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x**(2/3)+a*x)**(3/2),x)
```

output
```
Integral(x/(a*x + b*x**(2/3))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{x}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(a*x + b*x^(2/3))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = -\frac{2\left(\frac{3\,b^2}{\sqrt{ax^{\frac{1}{3}}+b}\,a}-\frac{\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^2-6\,\sqrt{ax^{\frac{1}{3}}+b}\,a^2b}{a^3}\right)}{a^2}+\frac{16\,b^{\frac{3}{2}}}{a^3}$$

input
```
integrate(x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
-2*(3*b^2/(sqrt(a*x^(1/3) + b)*a) - ((a*x^(1/3) + b)^(3/2)*a^2 - 6*sqrt(a*
x^(1/3) + b)*a^2*b)/a^3)/a^2 + 16*b^(3/2)/a^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{x}{\left(a\,x+b\,x^{2/3}\right)^{3/2}}\,dx$$

input
```
int(x/(a*x + b*x^(2/3))^(3/2),x)
```

output
```
int(x/(a*x + b*x^(2/3))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.51

$$\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{2x^{\frac{2}{3}}a^2-8x^{\frac{1}{3}}ab-16b^2}{\sqrt{x^{\frac{1}{3}}a+b}\,a^3}$$

input
```
int(x/(b*x^(2/3)+a*x)^(3/2),x)
```

output
```
(2*(x**(2/3)*a**2 - 4*x**(1/3)*a*b - 8*b**2))/(sqrt(x**(1/3)*a + b)*a**3)
```

3.177 $\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal result 1738
Mathematica [A] (verified) 1738
Rubi [A] (verified) 1739
Maple [A] (verified) 1740
Fricas [F(-1)] 1741
Sympy [F] 1741
Maxima [F] 1741
Giac [A] (verification not implemented) 1742
Mupad [B] (verification not implemented) 1742
Reduce [B] (verification not implemented) 1742

Optimal result

Integrand size = 15, antiderivative size = 60

$$\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{6\sqrt[3]{x}}{b\sqrt{bx^{2/3}+ax}}-\frac{6\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b^{3/2}}$$

output
```
6*x^(1/3)/b/(b*x^(2/3)+a*x)^(1/2)-6*arctanh(b^(1/2)*x^(1/3)/(b*x^(2/3)+a*x
)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{6\sqrt{bx^{2/3}+ax}}{b\left(b+a\sqrt[3]{x}\right)\sqrt[3]{x}}-\frac{6\,\text{arctanh}\left(\frac{\sqrt{bx^{2/3}+ax}}{\sqrt{b}\sqrt[3]{x}}\right)}{b^{3/2}}$$

input
```
Integrate[(b*x^(2/3) + a*x)^(-3/2),x]
```

output
```
(6*Sqrt[b*x^(2/3) + a*x])/(b*(b + a*x^(1/3))*x^(1/3)) - (6*ArcTanh[Sqrt[b*
x^(2/3) + a*x]/(Sqrt[b]*x^(1/3))])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1912, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(ax+bx^{2/3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1912}$$

$$\frac{\int \frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}dx}{b}+\frac{6\sqrt[3]{x}}{b\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{ 1935}$$

$$\frac{6\sqrt[3]{x}}{b\sqrt{ax+bx^{2/3}}}-\frac{6\int \frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{6\sqrt[3]{x}}{b\sqrt{ax+bx^{2/3}}}-\frac{6\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}$$

input
```
Int[(b*x^(2/3) + a*x)^(-3/2),x]
```

output
```
(6*x^(1/3))/(b*Sqrt[b*x^(2/3) + a*x]) - (6*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[
b*x^(2/3) + a*x]])/b^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1912
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{6x\left(x^{\frac{1}{3}}a+b\right)\left(\operatorname{arctanh}\left(\frac{\sqrt{x^{\frac{1}{3}}a+b}}{\sqrt{b}}\right)b\sqrt{x^{\frac{1}{3}}a+b}-b^{\frac{3}{2}}\right)}{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}b^{\frac{5}{2}}}$	56
default	$-\frac{6x\left(x^{\frac{1}{3}}a+b\right)\left(\operatorname{arctanh}\left(\frac{\sqrt{x^{\frac{1}{3}}a+b}}{\sqrt{b}}\right)b\sqrt{x^{\frac{1}{3}}a+b}-b^{\frac{3}{2}}\right)}{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}b^{\frac{5}{2}}}$	56

input
```
int(1/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-6*x*(x^(1/3)*a+b)*(arctanh((x^(1/3)*a+b)^(1/2)/b^(1/2))*b*(x^(1/3)*a+b)^(
1/2)-b^(3/2))/(b*x^(2/3)+a*x)^(3/2)/b^(5/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**(2/3)+a*x)**(3/2),x)
```

output
```
Integral((a*x + b*x**(2/3))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(2/3))^(-3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{6\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b} - \frac{6\left(\sqrt{b}\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+\sqrt{-b}\right)}{\sqrt{-b}b^{\frac{3}{2}}} + \frac{6}{\sqrt{ax^{\frac{1}{3}}+b}b}$$

input
```
integrate(1/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
6*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) - 6*(sqrt(b)*arctan(sq
rt(b)/sqrt(-b)) + sqrt(-b))/(sqrt(-b)*b^(3/2)) + 6/(sqrt(a*x^(1/3) + b)*b)
```

Mupad [B] (verification not implemented)

Time = 8.83 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = -\frac{2\,x\,\left(\frac{b}{a\,x^{1/3}}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{3}{2};\ \frac{5}{2};\ -\frac{b}{a\,x^{1/3}}\right)}{\left(a\,x+b\,x^{2/3}\right)^{3/2}}$$

input
```
int(1/(a*x + b*x^(2/3))^(3/2),x)
```

output
```
-(2*x*(b/(a*x^(1/3)) + 1)^(3/2)*hypergeom([3/2, 3/2], 5/2, -b/(a*x^(1/3)))
)/(a*x + b*x^(2/3))^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13

$$\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{3\sqrt{b}\,\sqrt{x^{\frac{1}{3}}a+b}\log\left(\sqrt{x^{\frac{1}{3}}a+b}-\sqrt{b}\right)-3\sqrt{b}\,\sqrt{x^{\frac{1}{3}}a+b}\log\left(\sqrt{x^{\frac{1}{3}}a+b}+\sqrt{b}\right)+6b}{\sqrt{x^{\frac{1}{3}}a+b}\,b^{2}}$$

input
```
int(1/(b*x^(2/3)+a*x)^(3/2),x)
```

output
```
(3*(sqrt(b)*sqrt(x**(1/3)*a + b)*log(sqrt(x**(1/3)*a + b) - sqrt(b)) - sqr
t(b)*sqrt(x**(1/3)*a + b)*log(sqrt(x**(1/3)*a + b) + sqrt(b)) + 2*b))/(sqr
t(x**(1/3)*a + b)*b**2)
```

3.178 $\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal result . . . 1744
Mathematica [A] (verified) . . . 1744
Rubi [A] (verified) . . . 1745
Maple [A] (verified) . . . 1748
Fricas [F(-1)] . . . 1749
Sympy [F] . . . 1749
Maxima [F] . . . 1749
Giac [A] (verification not implemented) . . . 1750
Mupad [F(-1)] . . . 1750
Reduce [B] (verification not implemented) . . . 1751

Optimal result

Integrand size = 19, antiderivative size = 146

$$\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}}-\frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}}+\frac{35a\sqrt{bx^{2/3}+ax}}{4b^3x}-\frac{105a^2\sqrt{bx^{2/3}+ax}}{8b^4x^{2/3}}+\frac{105a^3\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{8b^{9/2}}$$

output
```
6/b/x^(2/3)/(b*x^(2/3)+a*x)^(1/2)-7*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(4/3)+35/4
*a*(b*x^(2/3)+a*x)^(1/2)/b^3/x-105/8*a^2*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(2/3)
+105/8*a^3*arctanh(b^(1/2)*x^(1/3)/(b*x^(2/3)+a*x)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 4.73 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{-\sqrt{b}\left(8b^3-14ab^2\sqrt[3]{x}+35a^2bx^{2/3}+105a^3x\right)+105a^3\sqrt{b+a\sqrt[3]{x}}\,x\,\text{arctanh}\left(\frac{\sqrt{b+a\sqrt[3]{x}}}{\sqrt{b}}\right)}{8b^{9/2}x^{2/3}\sqrt{bx^{2/3}+ax}}$$

input
```
Integrate[1/(x*(b*x^(2/3) + a*x)^(3/2)),x]
```

output
```
(-(Sqrt[b]*(8*b^3 - 14*a*b^2*x^(1/3) + 35*a^2*b*x^(2/3) + 105*a^3*x)) + 10
5*a^3*Sqrt[b + a*x^(1/3)]*x*ArcTanh[Sqrt[b + a*x^(1/3)]/Sqrt[b]])/(8*b^(9/
2)*x^(2/3)*Sqrt[b*x^(2/3) + a*x])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1929, 1931, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax+bx^{2/3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{7\int \frac{1}{x^{5/3}\sqrt{x^{2/3}b+ax}}dx}{b}+\frac{6}{bx^{2/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{7\left(-\frac{5a\int \frac{1}{x^{4/3}\sqrt{x^{2/3}b+ax}}dx}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{b}+\frac{6}{bx^{2/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{7\left(-\frac{5a\left(-\frac{3a\int \frac{1}{x\sqrt{x^{2/3}b+ax}}dx}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{b}+\frac{6}{bx^{2/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{7\left(-\frac{5a\left(-\frac{3a\left(-\frac{a\int \frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{b}+\frac{6}{bx^{2/3}\sqrt{ax+bx^{2/3}}}$$

$\downarrow$ 1935

$$\frac{7\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\int \frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{b}+\frac{6}{bx^{2/3}\sqrt{ax+bx^{2/3}}}$$

$\downarrow$ 219

$$\frac{7\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{b}+\frac{6}{bx^{2/3}\sqrt{ax+bx^{2/3}}}$$

input
```
Int[1/(x*(b*x^(2/3) + a*x)^(3/2)),x]
```

output
```
6/(b*x^(2/3)*Sqrt[b*x^(2/3) + a*x]) + (7*(-(Sqrt[b*x^(2/3) + a*x]/(b*x^(4/
3))) - (5*a*((-3*Sqrt[b*x^(2/3) + a*x])/(2*b*x) - (3*a*((-3*Sqrt[b*x^(2/3)
 + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x
]])/b^(3/2)))/(4*b)))/(6*b)))/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.60

method	result	size
derivativedivides	$\frac{\left(x^{\frac{1}{3}} a+b\right)\left(105 \operatorname{arctanh}\left(\frac{\sqrt{x^{\frac{1}{3}} a+b}}{\sqrt{b}}\right) \sqrt{x^{\frac{1}{3}} a+b}\, a^{3} x+14 b^{\frac{5}{2}} a\, x^{\frac{1}{3}}-35 b^{\frac{3}{2}} a^{2} x^{\frac{2}{3}}-105 x\, a^{3} \sqrt{b}-8 b^{\frac{7}{2}}\right)}{8\left(b\, x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}} b^{\frac{9}{2}}}$	88
default	$\frac{\left(x^{\frac{1}{3}} a+b\right)\left(105 \operatorname{arctanh}\left(\frac{\sqrt{x^{\frac{1}{3}} a+b}}{\sqrt{b}}\right) \sqrt{x^{\frac{1}{3}} a+b}\, a^{3} x+14 b^{\frac{5}{2}} a\, x^{\frac{1}{3}}-35 b^{\frac{3}{2}} a^{2} x^{\frac{2}{3}}-105 x\, a^{3} \sqrt{b}-8 b^{\frac{7}{2}}\right)}{8\left(b\, x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}} b^{\frac{9}{2}}}$	88

input
```
int(1/x/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(x^(1/3)*a+b)*(105*arctanh((x^(1/3)*a+b)^(1/2)/b^(1/2))*(x^(1/3)*a+b)^
(1/2)*a^3*x+14*b^(5/2)*a*x^(1/3)-35*b^(3/2)*a^2*x^(2/3)-105*x*a^3*b^(1/2)-
8*b^(7/2))/(b*x^(2/3)+a*x)^(3/2)/b^(9/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{1}{x\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(b*x**(2/3)+a*x)**(3/2),x)
```

output
```
Integral(1/(x*(a*x + b*x**(2/3))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*x^(2/3))^(3/2)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.72

$$\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx = -\frac{105\,a^3\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{8\,\sqrt{-b}b^4} - \frac{6\,a^3}{\sqrt{ax^{\frac{1}{3}}+b}b^4} - \frac{57\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^3 - 136\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^3b + 87\sqrt{ax^{\frac{1}{3}}+b}a^3b^2}{8\,a^3b^4x}$$

input
```
integrate(1/x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
-105/8*a^3*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) - 6*a^3/(sq
rt(a*x^(1/3) + b)*b^4) - 1/8*(57*(a*x^(1/3) + b)^(5/2)*a^3 - 136*(a*x^(1/3
) + b)^(3/2)*a^3*b + 87*sqrt(a*x^(1/3) + b)*a^3*b^2)/(a^3*b^4*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{1}{x\left(a\,x+b\,x^{2/3}\right)^{3/2}}\,dx$$

input
```
int(1/(x*(a*x + b*x^(2/3))^(3/2)),x)
```

output
```
int(1/(x*(a*x + b*x^(2/3))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{-105\sqrt{b}\,\sqrt{x^{\frac{1}{3}}a+b}\,\log\left(\sqrt{x^{\frac{1}{3}}a+b}-\sqrt{b}\right)a^{3}x+105\sqrt{b}\,\sqrt{x^{\frac{1}{3}}a+b}\,\log\left(\sqrt{x^{\frac{1}{3}}a+b}\right.}{16\sqrt{x^{\frac{1}{3}}a+b}\,b^{5}x}$$

input
```
int(1/x/(b*x^(2/3)+a*x)^(3/2),x)
```

output
```
( - 105*sqrt(b)*sqrt(x**(1/3)*a + b)*log(sqrt(x**(1/3)*a + b) - sqrt(b))*a
**3*x + 105*sqrt(b)*sqrt(x**(1/3)*a + b)*log(sqrt(x**(1/3)*a + b) + sqrt(b
))*a**3*x - 70*x**(2/3)*a**2*b**2 + 28*x**(1/3)*a*b**3 - 210*a**3*b*x - 16
*b**4)/(16*sqrt(x**(1/3)*a + b)*b**5*x)
```

3.179 $\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal result . . . 1752
Mathematica [C] (verified) . . . 1753
Rubi [A] (verified) . . . 1753
Maple [A] (verified) . . . 1763
Fricas [F(-1)] . . . 1763
Sympy [F] . . . 1764
Maxima [F] . . . 1764
Giac [A] (verification not implemented) . . . 1764
Mupad [F(-1)] . . . 1765
Reduce [B] (verification not implemented) . . . 1765

Optimal result

Integrand size = 19, antiderivative size = 236

$$\begin{aligned}\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx &= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}\\ &+\frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2}-\frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}}+\frac{3003a^3\sqrt{bx^{2/3}+ax}}{320b^5x^{4/3}}\\ &-\frac{3003a^4\sqrt{bx^{2/3}+ax}}{256b^6x}+\frac{9009a^5\sqrt{bx^{2/3}+ax}}{512b^7x^{2/3}}-\frac{9009a^6\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{512b^{15/2}}\end{aligned}$$

output
```
6/b/x^(5/3)/(b*x^(2/3)+a*x)^(1/2)-13/2*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(7/3)+1
43/20*a*(b*x^(2/3)+a*x)^(1/2)/b^3/x^2-1287/160*a^2*(b*x^(2/3)+a*x)^(1/2)/b
^4/x^(5/3)+3003/320*a^3*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(4/3)-3003/256*a^4*(b*
x^(2/3)+a*x)^(1/2)/b^6/x+9009/512*a^5*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(2/3)-90
09/512*a^6*arctanh(b^(1/2)*x^(1/3)/(b*x^(2/3)+a*x)^(1/2))/b^(15/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.20

$$\int \frac{1}{x^2 \left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{6a^6\sqrt[3]{x}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},7,\frac{1}{2},1+\frac{a\sqrt[3]{x}}{b}\right)}{b^7\sqrt{bx^{2/3}+ax}}$$

input
```
Integrate[1/(x^2*(b*x^(2/3) + a*x)^(3/2)),x]
```

output
```
(6*a^6*x^(1/3)*Hypergeometric2F1[-1/2, 7, 1/2, 1 + (a*x^(1/3))/b])/(b^7*Sq
rt[b*x^(2/3) + a*x])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1929, 1931, 1931, 1931, 1931, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(ax+bx^{2/3}\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{13\int \frac{1}{x^{8/3}\sqrt{x^{2/3}b+ax}}dx}{b}+\frac{6}{bx^{5/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{13\left(-\frac{11a\int \frac{1}{x^{7/3}\sqrt{x^{2/3}b+ax}}dx}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{b}+\frac{6}{bx^{5/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{13\left(-\frac{11a\left(-\frac{9a\int \frac{1}{x^2\sqrt{x^{2/3}b+ax}}dx}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{b}+\frac{6}{bx^{5/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \ 1931$$

$$\frac{13\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\int \frac{1}{x^{5/3}\sqrt{x^{2/3}b+ax}}dx}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{b}+\frac{6}{bx^{5/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \ 1931$$

$$\frac{13\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\int \frac{1}{x^{4/3}\sqrt{x^{2/3}b+ax}}dx}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{b}+\frac{6}{bx^{5/3}\sqrt{ax+bx^{2/3}}}$$

$\downarrow$ 1931

$$\frac{13\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\int\frac{1}{x\sqrt{x^{2/3}b+ax}}dx}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{b}+\frac{6}{bx^{5/3}\sqrt{ax+bx^{2/3}}}$$

↓ 1931

$$13\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(-\frac{a\int\frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-$$

↓ 1935

$$\Bigg(11a\Bigg(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\int \frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}$$

↓ 219

$$\left(11a \left(-\frac{9a \left(-\frac{7a \left(-\frac{5a \left(-\frac{3a \left(\frac{3a \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}} \right)}{b^{3/2}} - \frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}} \right)}{4b} - \frac{3\sqrt{ax+bx^{2/3}}}{2bx} \right)}{6b} - \frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}} \right)}{8b} - \frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}} \right)}{10b} - \frac{3\sqrt{ax+bx^{2/3}}}{5bx^2} \right. \right.$$

input
```
Int[1/(x^2*(b*x^(2/3) + a*x)^(3/2)),x]
```

output
```
6/(b*x^(5/3)*Sqrt[b*x^(2/3) + a*x]) + (13*(-1/2*Sqrt[b*x^(2/3) + a*x]/(b*x
^(7/3)) - (11*a*((-3*Sqrt[b*x^(2/3) + a*x])/(5*b*x^2) - (9*a*((-3*Sqrt[b*x
^(2/3) + a*x])/(4*b*x^(5/3)) - (7*a*(-(Sqrt[b*x^(2/3) + a*x]/(b*x^(4/3)))
- (5*a*((-3*Sqrt[b*x^(2/3) + a*x])/(2*b*x) - (3*a*((-3*Sqrt[b*x^(2/3) + a*
x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/b
^(3/2)))/(4*b)))/(6*b)))/(8*b)))/(10*b)))/(12*b)))/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.53

method	result
derivativedivides	$-\frac{(x^{\frac{1}{3}}a+b)(45045\,\text{arctanh}(\frac{\sqrt{x^{\frac{1}{3}}a+b}}{\sqrt{b}})\sqrt{x^{\frac{1}{3}}a+b}\,a^6x^2+1280b^{\frac{13}{2}}-1664b^{\frac{11}{2}}ax^{\frac{1}{3}}+2288b^{\frac{9}{2}}a^2x^{\frac{2}{3}}-3432b^{\frac{7}{2}}a^3x+6006}{2560x(bx^{\frac{2}{3}}+ax)^{\frac{3}{2}}b^{\frac{15}{2}}}$
default	$-\frac{(x^{\frac{1}{3}}a+b)(45045\,\text{arctanh}(\frac{\sqrt{x^{\frac{1}{3}}a+b}}{\sqrt{b}})\sqrt{x^{\frac{1}{3}}a+b}\,a^6x^2+1280b^{\frac{13}{2}}-1664b^{\frac{11}{2}}ax^{\frac{1}{3}}+2288b^{\frac{9}{2}}a^2x^{\frac{2}{3}}-3432b^{\frac{7}{2}}a^3x+6006}{2560x(bx^{\frac{2}{3}}+ax)^{\frac{3}{2}}b^{\frac{15}{2}}}$

input
```
int(1/x^2/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2560*(x^(1/3)*a+b)*(45045*arctanh((x^(1/3)*a+b)^(1/2)/b^(1/2))*(x^(1/3)
*a+b)^(1/2)*a^6*x^2+1280*b^(13/2)-1664*b^(11/2)*a*x^(1/3)+2288*b^(9/2)*a^2
*x^(2/3)-3432*b^(7/2)*a^3*x+6006*b^(5/2)*a^4*x^(4/3)-15015*b^(3/2)*a^5*x^(
5/3)-45045*a^6*x^2*b^(1/2))/x/(b*x^(2/3)+a*x)^(3/2)/b^(15/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^2\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(b*x**(2/3)+a*x)**(3/2),x)
```

output
```
Integral(1/(x**2*(a*x + b*x**(2/3))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*x^(2/3))^(3/2)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{9009\,a^6\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{512\sqrt{-b}b^7}+\frac{6\,a^6}{\sqrt{ax^{\frac{1}{3}}+b}b^7}+\frac{29685\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^6-163095\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^6b+364194\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^6b^2-416094\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^6b^3+2\cdots}{2560\,a^6b^7x^2}$$

input
```
integrate(1/x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
9009/512*a^6*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7) + 6*a^6/(
sqrt(a*x^(1/3) + b)*b^7) + 1/2560*(29685*(a*x^(1/3) + b)^(11/2)*a^6 - 1630
95*(a*x^(1/3) + b)^(9/2)*a^6*b + 364194*(a*x^(1/3) + b)^(7/2)*a^6*b^2 - 41
6094*(a*x^(1/3) + b)^(5/2)*a^6*b^3 + 246505*(a*x^(1/3) + b)^(3/2)*a^6*b^4
- 62475*sqrt(a*x^(1/3) + b)*a^6*b^5)/(a^6*b^7*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^2\left(a\,x+b\,x^{2/3}\right)^{3/2}}\,dx$$

input
```
int(1/(x^2*(a*x + b*x^(2/3))^(3/2)),x)
```

output
```
int(1/(x^2*(a*x + b*x^(2/3))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{45045\sqrt{b}\,\sqrt{x^{\frac{1}{3}}a+b}\log\left(\sqrt{x^{\frac{1}{3}}a+b}-\sqrt{b}\right)a^6x^2-45045\sqrt{b}\,\sqrt{x^{\frac{1}{3}}a+b}\log\left(\sqrt{x^{\frac{1}{3}}\right.}{}$$

input
```
int(1/x^2/(b*x^(2/3)+a*x)^(3/2),x)
```

output
```
(45045*sqrt(b)*sqrt(x**(1/3)*a + b)*log(sqrt(x**(1/3)*a + b) - sqrt(b))*a*
*6*x**2 - 45045*sqrt(b)*sqrt(x**(1/3)*a + b)*log(sqrt(x**(1/3)*a + b) + sq
rt(b))*a**6*x**2 + 30030*x**(2/3)*a**5*b**2*x - 4576*x**(2/3)*a**2*b**5 -
12012*x**(1/3)*a**4*b**3*x + 3328*x**(1/3)*a*b**6 + 90090*a**6*b*x**2 + 68
64*a**3*b**4*x - 2560*b**7)/(5120*sqrt(x**(1/3)*a + b)*b**8*x**2)
```

3.180 $\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal result . . . 1766
Mathematica [C] (verified) . . . 1767
Rubi [A] (verified) . . . 1767
Maple [A] (verified) . . . 1783
Fricas [F(-1)] . . . 1783
Sympy [F] . . . 1784
Maxima [F] . . . 1784
Giac [A] (verification not implemented) . . . 1784
Mupad [F(-1)] . . . 1785
Reduce [B] (verification not implemented) . . . 1785

Optimal result

Integrand size = 19, antiderivative size = 324

$$\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}-\frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}}+\frac{20995a^3\sqrt{bx^{2/3}+ax}}{2688b^5x^{7/3}}-\frac{46189a^4\sqrt{bx^{2/3}+ax}}{5376b^6x^2}+\frac{138567a^5\sqrt{bx^{2/3}+ax}}{14336b^7x^{5/3}}-\frac{46189a^6\sqrt{bx^{2/3}+ax}}{4096b^8x^{4/3}}+\frac{230945a^7\sqrt{bx^{2/3}+ax}}{16384b^9x}-\frac{692835a^8\sqrt{bx^{2/3}+ax}}{32768b^{10}x^{2/3}}+\frac{692835a^9\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{32768b^{21/2}}$$

output
```
6/b/x^(8/3)/(b*x^(2/3)+a*x)^(1/2)-19/3*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(10/3)+
323/48*a*(b*x^(2/3)+a*x)^(1/2)/b^3/x^3-1615/224*a^2*(b*x^(2/3)+a*x)^(1/2)/
b^4/x^(8/3)+20995/2688*a^3*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(7/3)-46189/5376*a^
4*(b*x^(2/3)+a*x)^(1/2)/b^6/x^2+138567/14336*a^5*(b*x^(2/3)+a*x)^(1/2)/b^7
/x^(5/3)-46189/4096*a^6*(b*x^(2/3)+a*x)^(1/2)/b^8/x^(4/3)+230945/16384*a^7
*(b*x^(2/3)+a*x)^(1/2)/b^9/x-692835/32768*a^8*(b*x^(2/3)+a*x)^(1/2)/b^10/x
^(2/3)+692835/32768*a^9*arctanh(b^(1/2)*x^(1/3)/(b*x^(2/3)+a*x)^(1/2))/b^(
21/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.15

$$\int \frac{1}{x^3 \left(bx^{2/3}+ax\right)^{3/2}}\,dx = -\frac{6a^9\sqrt[3]{x}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},10,\frac{1}{2},1+\frac{a\sqrt[3]{x}}{b}\right)}{b^{10}\sqrt{bx^{2/3}+ax}}$$

input
```
Integrate[1/(x^3*(b*x^(2/3) + a*x)^(3/2)),x]
```

output
```
(-6*a^9*x^(1/3)*Hypergeometric2F1[-1/2, 10, 1/2, 1 + (a*x^(1/3))/b])/(b^10
*Sqrt[b*x^(2/3) + a*x])
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.15, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {1929, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(ax+bx^{2/3}\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{19\int \frac{1}{x^{11/3}\sqrt{x^{2/3}b+ax}}dx}{b}+\frac{6}{bx^{8/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{19\left(-\frac{17a\int \frac{1}{x^{10/3}\sqrt{x^{2/3}b+ax}}dx}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{b}+\frac{6}{bx^{8/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{19\left(-\frac{17a\left(-\frac{15a\int\frac{1}{x^3\sqrt{x^{2/3}b+ax}}dx}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{b}+\frac{6}{bx^{8/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \ 1931$$

$$\frac{19\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\int\frac{1}{x^{8/3}\sqrt{x^{2/3}b+ax}}dx}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{b}+\frac{6}{bx^{8/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \ 1931$$

$$\frac{19\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\int\frac{1}{x^{7/3}\sqrt{x^{2/3}b+ax}}dx}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^{3}}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{b}+\frac{6}{bx^{8/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \; 1931$$

$$\frac{19\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\int \frac{1}{x^2\sqrt{x^{2/3}b+ax}}dx}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{b}+\frac{6}{bx^{8/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \; 1931$$

$$19\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\int\frac{1}{x^{5/3}\sqrt{x^{2/3}b+ax}}dx}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^{3}}\right)}{18b}\right.$$

↓ 1931

$$\left(17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\int\frac{1}{x^{4/3}\sqrt{x^{2/3}b+ax}}dx}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx}}{7bx^{8/3}}}{16b}\right.\right.$$

↓ 1931

$$\Bigg(\ \Bigg(\ 15a\Bigg(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\int\frac{1}{x\sqrt{x^{2/3}b+ax}}dx}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\frac{\sqrt{ax+\cdots}}{2ba\cdots}\right)}{14b}\cdots$$

↓ 1931

$$\left(\left(\left(13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(-\frac{a\int\frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}\,dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right.}{10b}\right.}{12b}\right.\right.\right.\right.$$

↓ 1935

$$\Bigg(\Bigg(\Bigg(\Bigg(\Bigg(11a-\frac{\left(9a-\frac{\left(7a-\frac{\left(5a-\frac{3a\left(\frac{3a\int\frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}\,d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+}}{4bx^{5}}\cdots\right)}{10b}\cdots$$

↓ 219

$$\Bigg(\Bigg(\Bigg(\Bigg(\Bigg(11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{\dots}}}{4bx^{5/3}}\dots}{10b}\dots$$

input
```
Int[1/(x^3*(b*x^(2/3) + a*x)^(3/2)),x]
```

output
```
6/(b*x^(8/3)*Sqrt[b*x^(2/3) + a*x]) + (19*(-1/3*Sqrt[b*x^(2/3) + a*x]/(b*x
^(10/3)) - (17*a*((-3*Sqrt[b*x^(2/3) + a*x])/(8*b*x^3) - (15*a*((-3*Sqrt[b
*x^(2/3) + a*x])/(7*b*x^(8/3)) - (13*a*(-1/2*Sqrt[b*x^(2/3) + a*x]/(b*x^(7
/3)) - (11*a*((-3*Sqrt[b*x^(2/3) + a*x])/(5*b*x^2) - (9*a*((-3*Sqrt[b*x^(2
/3) + a*x])/(4*b*x^(5/3)) - (7*a*(-(Sqrt[b*x^(2/3) + a*x]/(b*x^(4/3))) - (
5*a*((-3*Sqrt[b*x^(2/3) + a*x])/(2*b*x) - (3*a*((-3*Sqrt[b*x^(2/3) + a*x])
/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/b^(3
/2)))/(4*b)))/(6*b)))/(8*b)))/(10*b)))/(12*b)))/(14*b)))/(16*b)))/(18*b)))
/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.49

method	result
derivativedivides	$\frac{\left(x^{\frac{1}{3}} a+b\right)\left(-229376 b^{\frac{19}{2}}+14549535 \operatorname{arctanh}\left(\frac{\sqrt{x^{\frac{1}{3}} a+b}}{\sqrt{b}}\right) \sqrt{x^{\frac{1}{3}} a+b}\, a^{9} x^{3}-14549535 a^{9} x^{3} \sqrt{b}-330752 b^{\frac{15}{2}} a^{2} x^{\frac{2}{3}}+4134}{688128 x^{2}\left(b\right.}$
default	$-\frac{\left(x^{\frac{1}{3}} a+b\right)\left(229376 b^{\frac{19}{2}}-14549535 \operatorname{arctanh}\left(\frac{\sqrt{x^{\frac{1}{3}} a+b}}{\sqrt{b}}\right) \sqrt{x^{\frac{1}{3}} a+b}\, a^{9} x^{3}+14549535 a^{9} x^{3} \sqrt{b}-272384 b^{\frac{17}{2}} a\, x^{\frac{1}{3}}+3307}{688128 x^{2}\left(\right.}$

input
```
int(1/x^3/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/688128*(x^(1/3)*a+b)*(-229376*b^(19/2)+14549535*arctanh((x^(1/3)*a+b)^(1
/2)/b^(1/2))*(x^(1/3)*a+b)^(1/2)*a^9*x^3-14549535*a^9*x^3*b^(1/2)-330752*b
^(15/2)*a^2*x^(2/3)+413440*b^(13/2)*a^3*x-537472*b^(11/2)*a^4*x^(4/3)+7390
24*b^(9/2)*a^5*x^(5/3)-1108536*b^(7/2)*a^6*x^2+1939938*b^(5/2)*a^7*x^(7/3)
-4849845*b^(3/2)*a^8*x^(8/3)+272384*b^(17/2)*a*x^(1/3))/x^2/(b*x^(2/3)+a*x
)^(3/2)/b^(21/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(b x^{2/3}+a x\right)^{3/2}}\, dx = \text{Timed out}$$

input
```
integrate(1/x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^3 \left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^3 \left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**3/(b*x**(2/3)+a*x)**(3/2),x)
```

output
```
Integral(1/(x**3*(a*x + b*x**(2/3))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 \left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}} x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*x^(2/3))^(3/2)*x^3), x)
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^3 \left(bx^{2/3}+ax\right)^{3/2}}\,dx=-\frac{692835\,a^9 \arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{32768\,\sqrt{-b}b^{10}}-\frac{6\,a^9}{\sqrt{ax^{\frac{1}{3}}+b}b^{10}}$$
$$-\frac{10420767\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^9-88937058\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^9b+334408914\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^9b^2-724860666\left(ax^{\frac{1}{3}}\right.}{}$$

input
```
integrate(1/x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
-692835/32768*a^9*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^10) - 6
*a^9/(sqrt(a*x^(1/3) + b)*b^10) - 1/688128*(10420767*(a*x^(1/3) + b)^(17/2
)*a^9 - 88937058*(a*x^(1/3) + b)^(15/2)*a^9*b + 334408914*(a*x^(1/3) + b)^
(13/2)*a^9*b^2 - 724860666*(a*x^(1/3) + b)^(11/2)*a^9*b^3 + 993296384*(a*x
^(1/3) + b)^(9/2)*a^9*b^4 - 884769030*(a*x^(1/3) + b)^(7/2)*a^9*b^5 + 5037
30990*(a*x^(1/3) + b)^(5/2)*a^9*b^6 - 169799070*(a*x^(1/3) + b)^(3/2)*a^9*
b^7 + 26738145*sqrt(a*x^(1/3) + b)*a^9*b^8)/(a^9*b^10*x^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^3\,{\left(a\,x+b\,x^{2/3}\right)}^{3/2}}\,dx$$

input
```
int(1/(x^3*(a*x + b*x^(2/3))^(3/2)),x)
```

output
```
int(1/(x^3*(a*x + b*x^(2/3))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^3 \left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{-14549535\sqrt{b}\,\sqrt{x^{\frac{1}{3}}a+b}\log\left(\sqrt{x^{\frac{1}{3}}a+b}-\sqrt{b}\right)a^9x^3+14549535\sqrt{b}\,\sqrt{x^{\frac{1}{3}}a+b}\,\mathrm{l}}{}$$

input
```
int(1/x^3/(b*x^(2/3)+a*x)^(3/2),x)
```

output

```
( - 14549535*sqrt(b)*sqrt(x**(1/3)*a + b)*log(sqrt(x**(1/3)*a + b) - sqrt(
b))*a**9*x**3 + 14549535*sqrt(b)*sqrt(x**(1/3)*a + b)*log(sqrt(x**(1/3)*a
+ b) + sqrt(b))*a**9*x**3 - 9699690*x**(2/3)*a**8*b**2*x**2 + 1478048*x**(
2/3)*a**5*b**5*x - 661504*x**(2/3)*a**2*b**8 + 3879876*x**(1/3)*a**7*b**3*
x**2 - 1074944*x**(1/3)*a**4*b**6*x + 544768*x**(1/3)*a*b**9 - 29099070*a*
*9*b*x**3 - 2217072*a**6*b**4*x**2 + 826880*a**3*b**7*x - 458752*b**10)/(1
376256*sqrt(x**(1/3)*a + b)*b**11*x**3)
```

3.181 $\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal result 1787
Mathematica [C] (verified) 1788
Rubi [A] (verified) 1788
Maple [A] (verified) 1810
Fricas [F(-1)] 1810
Sympy [F] 1811
Maxima [F] 1811
Giac [A] (verification not implemented) 1811
Mupad [F(-1)] 1812
Reduce [B] (verification not implemented) 1812

Optimal result

Integrand size = 19, antiderivative size = 412

$$\begin{aligned}\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx &= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}\\
&+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\frac{15295a^3\sqrt{bx^{2/3}+ax}}{2112b^5x^{10/3}}\\
&-\frac{260015a^4\sqrt{bx^{2/3}+ax}}{33792b^6x^3}+\frac{185725a^5\sqrt{bx^{2/3}+ax}}{22528b^7x^{8/3}}-\frac{2414425a^6\sqrt{bx^{2/3}+ax}}{270336b^8x^{7/3}}\\
&+\frac{482885a^7\sqrt{bx^{2/3}+ax}}{49152b^9x^2}-\frac{1448655a^8\sqrt{bx^{2/3}+ax}}{131072b^{10}x^{5/3}}\\
&+\frac{3380195a^9\sqrt{bx^{2/3}+ax}}{262144b^{11}x^{4/3}}-\frac{16900975a^{10}\sqrt{bx^{2/3}+ax}}{1048576b^{12}x}\\
&+\frac{50702925a^{11}\sqrt{bx^{2/3}+ax}}{2097152b^{13}x^{2/3}}-\frac{50702925a^{12}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{2097152b^{27/2}}\end{aligned}$$

output
```
6/b/x^(11/3)/(b*x^(2/3)+a*x)^(1/2)-25/4*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(13/3)
+575/88*a*(b*x^(2/3)+a*x)^(1/2)/b^3/x^4-2415/352*a^2*(b*x^(2/3)+a*x)^(1/2)
/b^4/x^(11/3)+15295/2112*a^3*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(10/3)-260015/337
92*a^4*(b*x^(2/3)+a*x)^(1/2)/b^6/x^3+185725/22528*a^5*(b*x^(2/3)+a*x)^(1/2
)/b^7/x^(8/3)-2414425/270336*a^6*(b*x^(2/3)+a*x)^(1/2)/b^8/x^(7/3)+482885/
49152*a^7*(b*x^(2/3)+a*x)^(1/2)/b^9/x^2-1448655/131072*a^8*(b*x^(2/3)+a*x)
^(1/2)/b^10/x^(5/3)+3380195/262144*a^9*(b*x^(2/3)+a*x)^(1/2)/b^11/x^(4/3)-
16900975/1048576*a^10*(b*x^(2/3)+a*x)^(1/2)/b^12/x+50702925/2097152*a^11*(
b*x^(2/3)+a*x)^(1/2)/b^13/x^(2/3)-50702925/2097152*a^12*arctanh(b^(1/2)*x^
(1/3)/(b*x^(2/3)+a*x)^(1/2))/b^(27/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^4 \left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{6a^{12}\sqrt[3]{x}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},13,\frac{1}{2},1+\frac{a\sqrt[3]{x}}{b}\right)}{b^{13}\sqrt{bx^{2/3}+ax}}$$

input
```
Integrate[1/(x^4*(b*x^(2/3) + a*x)^(3/2)),x]
```

output
```
(6*a^12*x^(1/3)*Hypergeometric2F1[-1/2, 13, 1/2, 1 + (a*x^(1/3))/b])/(b^13
*Sqrt[b*x^(2/3) + a*x])
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.16, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.789, Rules used = {1929, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(ax+bx^{2/3}\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 1929$$

$$\frac{25\int \frac{1}{x^{14/3}\sqrt{x^{2/3}b+ax}}dx}{b}+\frac{6}{bx^{11/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \; 1931$$

$$\frac{25\left(-\frac{23a\int \frac{1}{x^{13/3}\sqrt{x^{2/3}b+ax}}dx}{24b}-\frac{\sqrt{ax+bx^{2/3}}}{4bx^{13/3}}\right)}{b}+\frac{6}{bx^{11/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \; 1931$$

$$\frac{25\left(-\frac{23a\left(-\frac{21a\int \frac{1}{x^4\sqrt{x^{2/3}b+ax}}dx}{22b}-\frac{3\sqrt{ax+bx^{2/3}}}{11bx^4}\right)}{24b}-\frac{\sqrt{ax+bx^{2/3}}}{4bx^{13/3}}\right)}{b}+\frac{6}{bx^{11/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \; 1931$$

$$\frac{25\left(-\frac{23a\left(-\frac{21a\left(-\frac{19a\int \frac{1}{x^{11/3}\sqrt{x^{2/3}b+ax}}dx}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}\right)}{22b}-\frac{3\sqrt{ax+bx^{2/3}}}{11bx^4}\right)}{24b}-\frac{\sqrt{ax+bx^{2/3}}}{4bx^{13/3}}\right)}{b}+\frac{6}{bx^{11/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \; 1931$$

$$\frac{25\left(-\frac{23a\left(-\frac{21a\left(-\frac{19a\left(-\frac{17a\int\frac{1}{x^{10/3}\sqrt{x^{2/3}b+ax}}dx}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}\right)}{22b}-\frac{3\sqrt{ax+bx^{2/3}}}{11bx^4}\right)}{24b}-\frac{\sqrt{ax+bx^{2/3}}}{4bx^{13/3}}\right)}{b}+$$

$$\frac{6}{bx^{11/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \; 1931$$

$$\frac{25\left(-\frac{23a\left(-\frac{21a\left(-\frac{19a\left(-\frac{17a\left(-\frac{15a\int \frac{1}{x^3\sqrt{x^{2/3}b+ax}}dx}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}\right)}{22b}-\frac{3\sqrt{ax+bx^{2/3}}}{11bx^4}\right)}{24b}-\frac{\sqrt{ax+bx^{2/3}}}{4bx^{13/3}}\right)}{b}+$$

$$\frac{6}{bx^{11/3}\sqrt{ax+bx^{2/3}}}$$

$\downarrow$ 1931

$$25\left(-\frac{23a\left(-\frac{21a\left(-\frac{19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\int\frac{1}{x^{8/3}\sqrt{x^{2/3}b+ax}}dx}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}\right)}{22b}-\frac{3\sqrt{ax+bx^{2/}}}{11bx^4}}{24b}\right.$$

↓ 1931

$$23a\left(-\frac{21a\left(-\frac{19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\int\frac{1}{x^{7/3}\sqrt{x^{2/3}b+ax}}\,dx}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^{3}}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax\cdots}}{10bx\cdots}}{22b}\cdots$$

↓ 1931

$$\left(\left(\left(21a\left(-\frac{19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\int\frac{1}{x^2\sqrt{x^{2/3}b+ax}}dx}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}\right)}{20b}\right.\right.\right.$$

↓ 1931

$$\left(\left(\left(\left(\left(19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\int\frac{1}{x^{5/3}\sqrt{x^{2/3}b+ax}}dx}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2}}}{7bx^{8/3}}\right)}{16b}}{18b}\right.\right.\right.\right.\right.$$

↓ 1931

$$17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\int\frac{1}{x^{4/3}\sqrt{x^{2/3}b+ax}}dx}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\frac{\sqrt{ax}}{2}}{14b}}{16b}\right.$$

↓ 1931

$$\Bigg(\Bigg(\Bigg(\Bigg(\Bigg(15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\int\frac{1}{x\sqrt{x^{2/3}b+ax}}dx}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-3\cdots}{12b}\cdots}{14b}\cdots$$

↓ 1931

$$\left(\left(\left(\left(\left(\left(13a\left(11a\left(9a\left(7a\left(5a\left(-\frac{3a\left(-\frac{a\int\frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/}}}{bx^{4/3}}\right.}{8b}\right.}{10b}\right.}{12b}\right.$$

↓ 1935

$$\left(\left(\left(\left(\left(\left(\frac{11a\left(\frac{9a\left(\frac{7a\left(\frac{5a\left(\frac{3a\left(\frac{3a\int \frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax\cdots}}{b\cdots}}{8b}}{10b}\right.\right.\right.\right.\right.\right.$$

↓ 219

$$\left(\left(\left(\left(\left(\left(11a\left(9a\left(7a\left(5a\left(-\frac{3a\left(\frac{3a\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+b}}{bx^{4/}}\right.\right.}{8b}\right.}{10b}\right.$$

input

```
Int[1/(x^4*(b*x^(2/3) + a*x)^(3/2)),x]
```

output

```
6/(b*x^(11/3)*Sqrt[b*x^(2/3) + a*x]) + (25*(-1/4*Sqrt[b*x^(2/3) + a*x]/(b*
x^(13/3)) - (23*a*((-3*Sqrt[b*x^(2/3) + a*x])/(11*b*x^4) - (21*a*((-3*Sqrt
[b*x^(2/3) + a*x])/(10*b*x^(11/3)) - (19*a*(-1/3*Sqrt[b*x^(2/3) + a*x]/(b*
x^(10/3)) - (17*a*((-3*Sqrt[b*x^(2/3) + a*x])/(8*b*x^3) - (15*a*((-3*Sqrt[
b*x^(2/3) + a*x])/(7*b*x^(8/3)) - (13*a*(-1/2*Sqrt[b*x^(2/3) + a*x]/(b*x^(
7/3)) - (11*a*((-3*Sqrt[b*x^(2/3) + a*x])/(5*b*x^2) - (9*a*((-3*Sqrt[b*x^(
2/3) + a*x])/(4*b*x^(5/3)) - (7*a*(-(Sqrt[b*x^(2/3) + a*x]/(b*x^(4/3))) -
(5*a*((-3*Sqrt[b*x^(2/3) + a*x])/(2*b*x) - (3*a*((-3*Sqrt[b*x^(2/3) + a*x]
)/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/b^(
3/2)))/(4*b)))/(6*b)))/(8*b)))/(10*b)))/(12*b)))/(14*b)))/(16*b)))/(18*b))
)/(20*b)))/(22*b)))/(24*b)))/b
```

Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1929

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.47

method	result
derivativedivides	$-\frac{\left(x^{\frac{1}{3}}a+b\right)\left(1673196525\sqrt{x^{\frac{1}{3}}a+b}\,\operatorname{arctanh}\left(\frac{\sqrt{x^{\frac{1}{3}}a+b}}{\sqrt{b}}\right)a^{12}x^4+17301504b^{\frac{25}{2}}+31324160b^{\frac{17}{2}}a^4x^{\frac{4}{3}}-38036480b^{\frac{15}{2}}a^5\right.}{}$
default	$-\frac{\left(x^{\frac{1}{3}}a+b\right)\left(1673196525\sqrt{x^{\frac{1}{3}}a+b}\,\operatorname{arctanh}\left(\frac{\sqrt{x^{\frac{1}{3}}a+b}}{\sqrt{b}}\right)a^{12}x^4+17301504b^{\frac{25}{2}}+31324160b^{\frac{17}{2}}a^4x^{\frac{4}{3}}-38036480b^{\frac{15}{2}}a^5\right.}{}$

input
```
int(1/x^4/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/69206016*(x^(1/3)*a+b)*(1673196525*(x^(1/3)*a+b)^(1/2)*arctanh((x^(1/3)
*a+b)^(1/2)/b^(1/2))*a^12*x^4+17301504*b^(25/2)+31324160*b^(17/2)*a^4*x^(4
/3)-38036480*b^(15/2)*a^5*x^(5/3)+47545600*b^(13/2)*a^6*x^2-61809280*b^(11
/2)*a^7*x^(7/3)+84987760*b^(9/2)*a^8*x^(8/3)-127481640*b^(7/2)*a^9*x^3+223
092870*b^(5/2)*a^10*x^(10/3)-557732175*b^(3/2)*a^11*x^(11/3)-19660800*b^(2
3/2)*a*x^(1/3)+22609920*b^(21/2)*a^2*x^(2/3)-26378240*b^(19/2)*a^3*x-16731
96525*a^12*x^4*b^(1/2))/x^3/(b*x^(2/3)+a*x)^(3/2)/b^(27/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^4 \left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^4 \left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**4/(b*x**(2/3)+a*x)**(3/2),x)
```

output
```
Integral(1/(x**4*(a*x + b*x**(2/3))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 \left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}} x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*x^(2/3))^(3/2)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^4 \left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{50702925\,a^{12}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{2097152\,\sqrt{-b}b^{13}} + \frac{6\,a^{12}}{\sqrt{ax^{\frac{1}{3}}+b}\,b^{13}}$$
$$+ \frac{1257960429\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}}a^{12} - 14537792973\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}a^{12}b + 76667241519\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}a^{12}b^2 - 24371}{}$$

input
```
integrate(1/x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
50702925/2097152*a^12*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^13)
 + 6*a^12/(sqrt(a*x^(1/3) + b)*b^13) + 1/69206016*(1257960429*(a*x^(1/3) +
 b)^(23/2)*a^12 - 14537792973*(a*x^(1/3) + b)^(21/2)*a^12*b + 76667241519*
(a*x^(1/3) + b)^(19/2)*a^12*b^2 - 243717614415*(a*x^(1/3) + b)^(17/2)*a^12
*b^3 + 519393101810*(a*x^(1/3) + b)^(15/2)*a^12*b^4 - 780150847218*(a*x^(1
/3) + b)^(13/2)*a^12*b^5 + 844265343246*(a*x^(1/3) + b)^(11/2)*a^12*b^6 -
659969685518*(a*x^(1/3) + b)^(9/2)*a^12*b^7 + 366679446705*(a*x^(1/3) + b)
^(7/2)*a^12*b^8 - 138840292305*(a*x^(1/3) + b)^(5/2)*a^12*b^9 + 3266070993
9*(a*x^(1/3) + b)^(3/2)*a^12*b^10 - 3724872723*sqrt(a*x^(1/3) + b)*a^12*b^
11)/(a^12*b^13*x^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(bx^{2/3}+ax\right)^{3/2}}\, dx = \int \frac{1}{x^4\, \left(a\, x+b\, x^{2/3}\right)^{3/2}} \,d x$$

input
```
int(1/(x^4*(a*x + b*x^(2/3))^(3/2)),x)
```

output
```
int(1/(x^4*(a*x + b*x^(2/3))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^4 \left(bx^{2/3}+ax\right)^{3/2}}\, dx = \frac{1673196525\sqrt{b}\,\sqrt{x^{\frac{1}{3}}a+b}\log\left(\sqrt{x^{\frac{1}{3}}a+b}-\sqrt{b}\right)a^{12}x^4-1673196525\sqrt{b}\,\sqrt{x^{\frac{1}{3}}a}$$

input
```
int(1/x^4/(b*x^(2/3)+a*x)^(3/2),x)
```

output

```
(1673196525*sqrt(b)*sqrt(x**(1/3)*a + b)*log(sqrt(x**(1/3)*a + b) - sqrt(b
))*a**12*x**4 - 1673196525*sqrt(b)*sqrt(x**(1/3)*a + b)*log(sqrt(x**(1/3)*
a + b) + sqrt(b))*a**12*x**4 + 1115464350*x**(2/3)*a**11*b**2*x**3 - 16997
5520*x**(2/3)*a**8*b**5*x**2 + 76072960*x**(2/3)*a**5*b**8*x - 45219840*x*
*(2/3)*a**2*b**11 - 446185740*x**(1/3)*a**10*b**3*x**3 + 123618560*x**(1/3
)*a**7*b**6*x**2 - 62648320*x**(1/3)*a**4*b**9*x + 39321600*x**(1/3)*a*b**
12 + 3346393050*a**12*b*x**4 + 254963280*a**9*b**4*x**3 - 95091200*a**6*b*
*7*x**2 + 52756480*a**3*b**10*x - 34603008*b**13)/(138412032*sqrt(x**(1/3)
*a + b)*b**14*x**4)
```

3.182 $\int x^2(ax^2+bx^3)\,dx$

Optimal result . . . 1814
Mathematica [A] (verified) . . . 1814
Rubi [A] (verified) . . . 1815
Maple [A] (verified) . . . 1816
Fricas [A] (verification not implemented) . . . 1816
Sympy [A] (verification not implemented) . . . 1817
Maxima [A] (verification not implemented) . . . 1817
Giac [A] (verification not implemented) . . . 1817
Mupad [B] (verification not implemented) . . . 1818
Reduce [B] (verification not implemented) . . . 1818

Optimal result

Integrand size = 15, antiderivative size = 17

$$\int x^2\left(ax^2+bx^3\right)\,dx=\frac{ax^5}{5}+\frac{bx^6}{6}$$

output
```
1/5*a*x^5+1/6*b*x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^2\left(ax^2+bx^3\right)\,dx=\frac{ax^5}{5}+\frac{bx^6}{6}$$

input
```
Integrate[x^2*(a*x^2 + b*x^3),x]
```

output
```
(a*x^5)/5 + (b*x^6)/6
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(ax^2+bx^3\right) \, dx$$

$$\Big\downarrow \; 9$$

$$\int x^4(a+bx)dx$$

$$\Big\downarrow \; 49$$

$$\int \left(ax^4+bx^5\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{ax^5}{5}+\frac{bx^6}{6}$$

input
```
Int[x^2*(a*x^2 + b*x^3),x]
```

output
```
(a*x^5)/5 + (b*x^6)/6
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{x^{5}(5bx+6a)}{30}$	14
default	$\frac{1}{5}a\,x^{5}+\frac{1}{6}b\,x^{6}$	14
norman	$\frac{1}{5}a\,x^{5}+\frac{1}{6}b\,x^{6}$	14
risch	$\frac{1}{5}a\,x^{5}+\frac{1}{6}b\,x^{6}$	14
parallelrisch	$\frac{1}{5}a\,x^{5}+\frac{1}{6}b\,x^{6}$	14
orering	$\frac{x^{3}(5bx+6a)\left(b\,x^{3}+a\,x^{2}\right)}{30bx+30a}$	32

input
```
int(x^2*(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/30*x^5*(5*b*x+6*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2\left(ax^2+bx^3\right)\,dx=\frac{1}{6}\,bx^6+\frac{1}{5}\,ax^5$$

input
```
integrate(x^2*(b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
1/6*b*x^6 + 1/5*a*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x^2(ax^2+bx^3)\,dx=\frac{ax^5}{5}+\frac{bx^6}{6}$$

input
```
integrate(x**2*(b*x**3+a*x**2),x)
```

output
```
a*x**5/5 + b*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2(ax^2+bx^3)\,dx=\frac{1}{6}\,bx^6+\frac{1}{5}\,ax^5$$

input
```
integrate(x^2*(b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
1/6*b*x^6 + 1/5*a*x^5
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2(ax^2+bx^3)\,dx=\frac{1}{6}\,bx^6+\frac{1}{5}\,ax^5$$

input
```
integrate(x^2*(b*x^3+a*x^2),x, algorithm="giac")
```

output
```
1/6*b*x^6 + 1/5*a*x^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2 (ax^2 + bx^3) \, dx = \frac{x^5 \, (6 \, a + 5 \, b \, x)}{30}$$

input
```
int(x^2*(a*x^2 + b*x^3),x)
```

output
```
(x^5*(6*a + 5*b*x))/30
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2 (ax^2 + bx^3) \, dx = \frac{x^5 (5bx + 6a)}{30}$$

input
```
int(x^2*(b*x^3+a*x^2),x)
```

output
```
(x**5*(6*a + 5*b*x))/30
```

3.183 $\int x(ax^2 + bx^3)\, dx$

Optimal result . . . 1819
Mathematica [A] (verified) . . . 1819
Rubi [A] (verified) . . . 1820
Maple [A] (verified) . . . 1821
Fricas [A] (verification not implemented) . . . 1821
Sympy [A] (verification not implemented) . . . 1822
Maxima [A] (verification not implemented) . . . 1822
Giac [A] (verification not implemented) . . . 1822
Mupad [B] (verification not implemented) . . . 1823
Reduce [B] (verification not implemented) . . . 1823

Optimal result

Integrand size = 13, antiderivative size = 17

$$\int x\left(ax^2 + bx^3\right)\, dx = \frac{ax^4}{4} + \frac{bx^5}{5}$$

output
```
1/4*a*x^4+1/5*b*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x\left(ax^2 + bx^3\right)\, dx = \frac{ax^4}{4} + \frac{bx^5}{5}$$

input
```
Integrate[x*(a*x^2 + b*x^3),x]
```

output
```
(a*x^4)/4 + (b*x^5)/5
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(ax^2+bx^3)\,dx$$
$$\Big\downarrow \text{9}$$
$$\int x^3(a+bx)dx$$
$$\Big\downarrow \text{49}$$
$$\int (ax^3+bx^4)\,dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

input
```
Int[x*(a*x^2 + b*x^3),x]
```

output
```
(a*x^4)/4 + (b*x^5)/5
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{x^4(4bx+5a)}{20}$	14
default	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5$	14
norman	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5$	14
risch	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5$	14
parallelrisch	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5$	14
orering	$\frac{x^2(4bx+5a)(b\,x^3+a\,x^2)}{20bx+20a}$	32

input
```
int(x*(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/20*x^4*(4*b*x+5*a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\left(ax^2+bx^3\right)\,dx=\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

input
```
integrate(x*(b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
1/5*b*x^5 + 1/4*a*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x(ax^2+bx^3)\,dx=\frac{ax^4}{4}+\frac{bx^5}{5}$$

input
```
integrate(x*(b*x**3+a*x**2),x)
```

output
```
a*x**4/4 + b*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(ax^2+bx^3)\,dx=\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

input
```
integrate(x*(b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
1/5*b*x^5 + 1/4*a*x^4
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(ax^2+bx^3)\,dx=\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

input
```
integrate(x*(b*x^3+a*x^2),x, algorithm="giac")
```

output
```
1/5*b*x^5 + 1/4*a*x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(ax^2 + bx^3)\ dx = \frac{x^4\,(5\,a + 4\,b\,x)}{20}$$

input
```
int(x*(a*x^2 + b*x^3),x)
```

output
```
(x^4*(5*a + 4*b*x))/20
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(ax^2 + bx^3)\ dx = \frac{x^4(4bx + 5a)}{20}$$

input
```
int(x*(b*x^3+a*x^2),x)
```

output
```
(x**4*(5*a + 4*b*x))/20
```

3.184 $\int (ax^2 + bx^3) \, dx$

Optimal result 1824
Mathematica [A] (verified) 1824
Rubi [A] (verified) 1825
Maple [A] (verified) 1826
Fricas [A] (verification not implemented) 1826
Sympy [A] (verification not implemented) 1827
Maxima [A] (verification not implemented) 1827
Giac [A] (verification not implemented) 1827
Mupad [B] (verification not implemented) 1828
Reduce [B] (verification not implemented) 1828

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int (ax^2 + bx^3) \, dx = \frac{ax^3}{3} + \frac{bx^4}{4}$$

output
```
1/3*a*x^3+1/4*b*x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int (ax^2 + bx^3) \, dx = \frac{ax^3}{3} + \frac{bx^4}{4}$$

input
```
Integrate[a*x^2 + b*x^3,x]
```

output
```
(a*x^3)/3 + (b*x^4)/4
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax^2 + bx^3) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^3}{3} + \frac{bx^4}{4}$$

input
```
Int[a*x^2 + b*x^3,x]
```

output
```
(a*x^3)/3 + (b*x^4)/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{x^3(3bx+4a)}{12}$	14
default	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14
norman	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14
risch	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14
parallelrisch	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14
parts	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14
orering	$\frac{x(3bx+4a)(b\,x^3+a\,x^2)}{12bx+12a}$	30

input
```
int(b*x^3+a*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*x^3*(3*b*x+4*a)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (ax^2+bx^3)\ dx = \frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

input
```
integrate(b*x^3+a*x^2,x, algorithm="fricas")
```

output
```
1/4*b*x^4 + 1/3*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \left(ax^2+bx^3\right) \, dx = \frac{ax^3}{3}+\frac{bx^4}{4}$$

input
```
integrate(b*x**3+a*x**2,x)
```

output
```
a*x**3/3 + b*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(ax^2+bx^3\right) \, dx = \frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

input
```
integrate(b*x^3+a*x^2,x, algorithm="maxima")
```

output
```
1/4*b*x^4 + 1/3*a*x^3
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(ax^2+bx^3\right) \, dx = \frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

input
```
integrate(b*x^3+a*x^2,x, algorithm="giac")
```

output
```
1/4*b*x^4 + 1/3*a*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (ax^2 + bx^3) \ dx = \frac{x^3 (4 a + 3 b x)}{12}$$

input
```
int(a*x^2 + b*x^3,x)
```

output
```
(x^3*(4*a + 3*b*x))/12
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (ax^2 + bx^3) \ dx = \frac{x^3(3bx + 4a)}{12}$$

input
```
int(b*x^3+a*x^2,x)
```

output
```
(x**3*(4*a + 3*b*x))/12
```

3.185 $\int \frac{ax^2+bx^3}{x} dx$

Optimal result 1829
Mathematica [A] (verified) 1829
Rubi [A] (verified) 1830
Maple [A] (warning: unable to verify) 1831
Fricas [A] (verification not implemented) 1831
Sympy [A] (verification not implemented) 1832
Maxima [A] (verification not implemented) 1832
Giac [A] (verification not implemented) 1832
Mupad [B] (verification not implemented) 1833
Reduce [B] (verification not implemented) 1833

Optimal result

Integrand size = 15, antiderivative size = 17

$$\int \frac{ax^2+bx^3}{x} dx = \frac{ax^2}{2} + \frac{bx^3}{3}$$

output
```
1/2*a*x^2+1/3*b*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{ax^2+bx^3}{x} dx = \frac{ax^2}{2} + \frac{bx^3}{3}$$

input
```
Integrate[(a*x^2 + b*x^3)/x,x]
```

output
```
(a*x^2)/2 + (b*x^3)/3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^2+bx^3}{x}\,dx$$

$$\Big\downarrow \; 9$$

$$\int x(a+bx)dx$$

$$\Big\downarrow \; 49$$

$$\int \left(ax+bx^2\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

input
```
Int[(a*x^2 + b*x^3)/x,x]
```

output
```
(a*x^2)/2 + (b*x^3)/3
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{x^2(2bx+3a)}{6}$	14
default	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3$	14
norman	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3$	14
risch	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3$	14
parallelrisch	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3$	14
orering	$\frac{(2bx+3a)(b\,x^3+a\,x^2)}{6bx+6a}$	29

input
```
int((b*x^3+a*x^2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/6*x^2*(2*b*x+3*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{ax^2+bx^3}{x}\,dx=\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

input
```
integrate((b*x^3+a*x^2)/x,x, algorithm="fricas")
```

output
```
1/3*b*x^3 + 1/2*a*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{ax^2 + bx^3}{x} dx = \frac{ax^2}{2} + \frac{bx^3}{3}$$

input
```
integrate((b*x**3+a*x**2)/x,x)
```

output
```
a*x**2/2 + b*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{ax^2 + bx^3}{x} dx = \frac{1}{3} bx^3 + \frac{1}{2} ax^2$$

input
```
integrate((b*x^3+a*x^2)/x,x, algorithm="maxima")
```

output
```
1/3*b*x^3 + 1/2*a*x^2
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{ax^2 + bx^3}{x} dx = \frac{1}{3} bx^3 + \frac{1}{2} ax^2$$

input
```
integrate((b*x^3+a*x^2)/x,x, algorithm="giac")
```

output
```
1/3*b*x^3 + 1/2*a*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{ax^2 + bx^3}{x} \, dx = \frac{x^2 \, (3 \, a + 2 \, b \, x)}{6}$$

input
```
int((a*x^2 + b*x^3)/x,x)
```

output
```
(x^2*(3*a + 2*b*x))/6
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{ax^2 + bx^3}{x} \, dx = \frac{x^2(2bx + 3a)}{6}$$

input
```
int((b*x^3+a*x^2)/x,x)
```

output
```
(x**2*(3*a + 2*b*x))/6
```

3.186 $\int \frac{ax^2+bx^3}{x^2} dx$

Optimal result . . . 1834
Mathematica [A] (verified) . . . 1834
Rubi [A] (verified) . . . 1835
Maple [A] (warning: unable to verify) . . . 1836
Fricas [A] (verification not implemented) . . . 1836
Sympy [A] (verification not implemented) . . . 1837
Maxima [A] (verification not implemented) . . . 1837
Giac [A] (verification not implemented) . . . 1837
Mupad [B] (verification not implemented) . . . 1838
Reduce [B] (verification not implemented) . . . 1838

Optimal result

Integrand size = 15, antiderivative size = 14

$$\int \frac{ax^2 + bx^3}{x^2} dx = \frac{(a + bx)^2}{2b}$$

output
```
1/2*(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{ax^2 + bx^3}{x^2} dx = ax + \frac{bx^2}{2}$$

input
```
Integrate[(a*x^2 + b*x^3)/x^2,x]
```

output
```
a*x + (b*x^2)/2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {9, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^2+bx^3}{x^2}\,dx$$

$$\Big\downarrow \; 9$$

$$\int (a+bx)dx$$

$$\Big\downarrow \; 17$$

$$\frac{(a+bx)^2}{2b}$$

input
```
Int[(a*x^2 + b*x^3)/x^2,x]
```

output
```
(a + b*x)^2/(2*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{x(bx+2a)}{2}$	11
default	$\frac{1}{2}b\,x^2+ax$	11
risch	$\frac{1}{2}b\,x^2+ax$	11
parallelrisch	$\frac{1}{2}b\,x^2+ax$	11
parts	$\frac{1}{2}b\,x^2+ax$	11
norman	$\frac{a\,x^2+\frac{1}{2}b\,x^3}{x}$	17
orering	$\frac{(bx+2a)\left(b\,x^3+a\,x^2\right)}{2x(bx+a)}$	31

input
```
int((b*x^3+a*x^2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(b*x+2*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{ax^2+bx^3}{x^2}\,dx=\frac{1}{2}\,bx^2+ax$$

input
```
integrate((b*x^3+a*x^2)/x^2,x, algorithm="fricas")
```

output
```
1/2*b*x^2 + a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{ax^2 + bx^3}{x^2} \, dx = ax + \frac{bx^2}{2}$$

input
```
integrate((b*x**3+a*x**2)/x**2,x)
```

output
```
a*x + b*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{ax^2 + bx^3}{x^2} \, dx = \frac{1}{2} bx^2 + ax$$

input
```
integrate((b*x^3+a*x^2)/x^2,x, algorithm="maxima")
```

output
```
1/2*b*x^2 + a*x
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{ax^2 + bx^3}{x^2} \, dx = \frac{1}{2} bx^2 + ax$$

input
```
integrate((b*x^3+a*x^2)/x^2,x, algorithm="giac")
```

output
```
1/2*b*x^2 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{ax^2 + bx^3}{x^2}\,dx = \frac{b\,x^2}{2} + a\,x$$

input
```
int((a*x^2 + b*x^3)/x^2,x)
```

output
```
a*x + (b*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{ax^2 + bx^3}{x^2}\,dx = \frac{x(bx + 2a)}{2}$$

input
```
int((b*x^3+a*x^2)/x^2,x)
```

output
```
(x*(2*a + b*x))/2
```

3.187 $\int x^2(ax^2+bx^3)^2\,dx$

Optimal result . . . 1839
Mathematica [A] (verified) . . . 1839
Rubi [A] (verified) . . . 1840
Maple [A] (verified) . . . 1841
Fricas [A] (verification not implemented) . . . 1841
Sympy [A] (verification not implemented) . . . 1842
Maxima [A] (verification not implemented) . . . 1842
Giac [A] (verification not implemented) . . . 1842
Mupad [B] (verification not implemented) . . . 1843
Reduce [B] (verification not implemented) . . . 1843

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int x^2\left(ax^2+bx^3\right)^2\,dx=\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{b^2x^9}{9}$$

output
```
1/7*a^2*x^7+1/4*a*b*x^8+1/9*b^2*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x^2\left(ax^2+bx^3\right)^2\,dx=\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{b^2x^9}{9}$$

input
```
Integrate[x^2*(a*x^2 + b*x^3)^2,x]
```

output
```
(a^2*x^7)/7 + (a*b*x^8)/4 + (b^2*x^9)/9
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(ax^2+bx^3\right)^2 \, dx$$

$$\Big\downarrow \; 9$$

$$\int x^6 (a+bx)^2 dx$$

$$\Big\downarrow \; 49$$

$$\int \left(a^2x^6+2abx^7+b^2x^8\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{b^2x^9}{9}$$

input
```
Int[x^2*(a*x^2 + b*x^3)^2,x]
```

output
```
(a^2*x^7)/7 + (a*b*x^8)/4 + (b^2*x^9)/9
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^{7}\left(28 b^{2} x^{2}+63 a b x+36 a^{2}\right)}{252}$	25
default	$\frac{1}{7} a^{2} x^{7}+\frac{1}{4} a b\, x^{8}+\frac{1}{9} b^{2} x^{9}$	25
norman	$\frac{1}{7} a^{2} x^{7}+\frac{1}{4} a b\, x^{8}+\frac{1}{9} b^{2} x^{9}$	25
risch	$\frac{1}{7} a^{2} x^{7}+\frac{1}{4} a b\, x^{8}+\frac{1}{9} b^{2} x^{9}$	25
parallelrisch	$\frac{1}{7} a^{2} x^{7}+\frac{1}{4} a b\, x^{8}+\frac{1}{9} b^{2} x^{9}$	25
orering	$\frac{x^{3}\left(28 b^{2} x^{2}+63 a b x+36 a^{2}\right)\left(b\, x^{3}+a\, x^{2}\right)^{2}}{252 \left(b x+a\right)^{2}}$	45

input
```
int(x^2*(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/252*x^7*(28*b^2*x^2+63*a*b*x+36*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2\left(ax^2+bx^3\right)^2\,dx=\frac{1}{9}\,b^2x^9+\frac{1}{4}\,abx^8+\frac{1}{7}\,a^2x^7$$

input
```
integrate(x^2*(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
1/9*b^2*x^9 + 1/4*a*b*x^8 + 1/7*a^2*x^7
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2 (ax^2 + bx^3)^2 \, dx = \frac{a^2x^7}{7} + \frac{abx^8}{4} + \frac{b^2x^9}{9}$$

input
```
integrate(x**2*(b*x**3+a*x**2)**2,x)
```

output
```
a**2*x**7/7 + a*b*x**8/4 + b**2*x**9/9
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2 (ax^2 + bx^3)^2 \, dx = \frac{1}{9}\,b^2x^9 + \frac{1}{4}\,abx^8 + \frac{1}{7}\,a^2x^7$$

input
```
integrate(x^2*(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
1/9*b^2*x^9 + 1/4*a*b*x^8 + 1/7*a^2*x^7
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2 (ax^2 + bx^3)^2 \, dx = \frac{1}{9}\,b^2x^9 + \frac{1}{4}\,abx^8 + \frac{1}{7}\,a^2x^7$$

input
```
integrate(x^2*(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
1/9*b^2*x^9 + 1/4*a*b*x^8 + 1/7*a^2*x^7
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2 (ax^2 + bx^3)^2 \, dx = \frac{a^2 x^7}{7} + \frac{a b x^8}{4} + \frac{b^2 x^9}{9}$$

input
```
int(x^2*(a*x^2 + b*x^3)^2,x)
```

output
```
(a^2*x^7)/7 + (b^2*x^9)/9 + (a*b*x^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2 (ax^2 + bx^3)^2 \, dx = \frac{x^7 (28b^2x^2 + 63abx + 36a^2)}{252}$$

input
```
int(x^2*(b*x^3+a*x^2)^2,x)
```

output
```
(x**7*(36*a**2 + 63*a*b*x + 28*b**2*x**2))/252
```

3.188 $\int x(ax^2+bx^3)^2\,dx$

Optimal result . . . 1844
Mathematica [A] (verified) . . . 1844
Rubi [A] (verified) . . . 1845
Maple [A] (verified) . . . 1846
Fricas [A] (verification not implemented) . . . 1846
Sympy [A] (verification not implemented) . . . 1847
Maxima [A] (verification not implemented) . . . 1847
Giac [A] (verification not implemented) . . . 1847
Mupad [B] (verification not implemented) . . . 1848
Reduce [B] (verification not implemented) . . . 1848

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int x(ax^2+bx^3)^2\,dx=\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}$$

output
```
1/6*a^2*x^6+2/7*a*b*x^7+1/8*b^2*x^8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x(ax^2+bx^3)^2\,dx=\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}$$

input
```
Integrate[x*(a*x^2 + b*x^3)^2,x]
```

output
```
(a^2*x^6)/6 + (2*a*b*x^7)/7 + (b^2*x^8)/8
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax^2+bx^3\right)^2 dx$$

$$\Big\downarrow \; 9$$

$$\int x^5(a+bx)^2dx$$

$$\Big\downarrow \; 49$$

$$\int \left(a^2x^5+2abx^6+b^2x^7\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}$$

input
```
Int[x*(a*x^2 + b*x^3)^2,x]
```

output
```
(a^2*x^6)/6 + (2*a*b*x^7)/7 + (b^2*x^8)/8
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^6(21b^2x^2+48abx+28a^2)}{168}$	25
default	$\frac{1}{6}a^2x^6+\frac{2}{7}ab\,x^7+\frac{1}{8}b^2x^8$	25
norman	$\frac{1}{6}a^2x^6+\frac{2}{7}ab\,x^7+\frac{1}{8}b^2x^8$	25
risch	$\frac{1}{6}a^2x^6+\frac{2}{7}ab\,x^7+\frac{1}{8}b^2x^8$	25
parallelrisch	$\frac{1}{6}a^2x^6+\frac{2}{7}ab\,x^7+\frac{1}{8}b^2x^8$	25
orering	$\frac{x^2(21b^2x^2+48abx+28a^2)(b\,x^3+a\,x^2)^2}{168(bx+a)^2}$	45

input
```
int(x*(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/168*x^6*(21*b^2*x^2+48*a*b*x+28*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(ax^2+bx^3)^2\,dx=\frac{1}{8}\,b^2x^8+\frac{2}{7}\,abx^7+\frac{1}{6}\,a^2x^6$$

input
```
integrate(x*(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
1/8*b^2*x^8 + 2/7*a*b*x^7 + 1/6*a^2*x^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x(ax^2+bx^3)^2\,dx = \frac{a^2x^6}{6}+\frac{2abx^7}{7}+\frac{b^2x^8}{8}$$

input
```
integrate(x*(b*x**3+a*x**2)**2,x)
```

output
```
a**2*x**6/6 + 2*a*b*x**7/7 + b**2*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(ax^2+bx^3)^2\,dx = \frac{1}{8}\,b^2x^8+\frac{2}{7}\,abx^7+\frac{1}{6}\,a^2x^6$$

input
```
integrate(x*(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
1/8*b^2*x^8 + 2/7*a*b*x^7 + 1/6*a^2*x^6
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(ax^2+bx^3)^2\,dx = \frac{1}{8}\,b^2x^8+\frac{2}{7}\,abx^7+\frac{1}{6}\,a^2x^6$$

input
```
integrate(x*(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
1/8*b^2*x^8 + 2/7*a*b*x^7 + 1/6*a^2*x^6
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(ax^2 + bx^3)^2 \, dx = \frac{a^2 x^6}{6} + \frac{2 a b x^7}{7} + \frac{b^2 x^8}{8}$$

input
```
int(x*(a*x^2 + b*x^3)^2,x)
```

output
```
(a^2*x^6)/6 + (b^2*x^8)/8 + (2*a*b*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(ax^2 + bx^3)^2 \, dx = \frac{x^6(21b^2x^2 + 48abx + 28a^2)}{168}$$

input
```
int(x*(b*x^3+a*x^2)^2,x)
```

output
```
(x**6*(28*a**2 + 48*a*b*x + 21*b**2*x**2))/168
```

3.189 $\int (ax^2+bx^3)^2 \, dx$

Optimal result . . . 1849
Mathematica [A] (verified) . . . 1849
Rubi [A] (verified) . . . 1850
Maple [A] (verified) . . . 1851
Fricas [A] (verification not implemented) . . . 1851
Sympy [A] (verification not implemented) . . . 1852
Maxima [A] (verification not implemented) . . . 1852
Giac [A] (verification not implemented) . . . 1852
Mupad [B] (verification not implemented) . . . 1853
Reduce [B] (verification not implemented) . . . 1853

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int (ax^2+bx^3)^2 \, dx = \frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}$$

output
```
1/5*a^2*x^5+1/3*a*b*x^6+1/7*b^2*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (ax^2+bx^3)^2 \, dx = \frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}$$

input
```
Integrate[(a*x^2 + b*x^3)^2,x]
```

output
```
(a^2*x^5)/5 + (a*b*x^6)/3 + (b^2*x^7)/7
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2027, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax^2+bx^3)^2\,dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int x^4(a+bx)^2dx$$

$$\Big\downarrow \text{ 49}$$

$$\int (a^2x^4+2abx^5+b^2x^6)\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}$$

input
```
Int[(a*x^2 + b*x^3)^2,x]
```

output
```
(a^2*x^5)/5 + (a*b*x^6)/3 + (b^2*x^7)/7
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^{5}\left(15 b^{2} x^{2}+35 a b x+21 a^{2}\right)}{105}$	25
default	$\frac{1}{5} a^{2} x^{5}+\frac{1}{3} a b\, x^{6}+\frac{1}{7} b^{2} x^{7}$	25
norman	$\frac{1}{5} a^{2} x^{5}+\frac{1}{3} a b\, x^{6}+\frac{1}{7} b^{2} x^{7}$	25
risch	$\frac{1}{5} a^{2} x^{5}+\frac{1}{3} a b\, x^{6}+\frac{1}{7} b^{2} x^{7}$	25
parallelrisch	$\frac{1}{5} a^{2} x^{5}+\frac{1}{3} a b\, x^{6}+\frac{1}{7} b^{2} x^{7}$	25
orering	$\frac{x\left(15 b^{2} x^{2}+35 a b x+21 a^{2}\right)\left(b\, x^{3}+a\, x^{2}\right)^{2}}{105\left(b x+a\right)^{2}}$	43

input
```
int((b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/105*x^5*(15*b^2*x^2+35*a*b*x+21*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(ax^2+bx^3\right)^2\,dx=\frac{1}{7}\,b^2x^7+\frac{1}{3}\,abx^6+\frac{1}{5}\,a^2x^5$$

input
```
integrate((b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
1/7*b^2*x^7 + 1/3*a*b*x^6 + 1/5*a^2*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (ax^2 + bx^3)^2 \, dx = \frac{a^2x^5}{5} + \frac{abx^6}{3} + \frac{b^2x^7}{7}$$

input
```
integrate((b*x**3+a*x**2)**2,x)
```

output
```
a**2*x**5/5 + a*b*x**6/3 + b**2*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (ax^2 + bx^3)^2 \, dx = \frac{1}{7}\,b^2x^7 + \frac{1}{3}\,abx^6 + \frac{1}{5}\,a^2x^5$$

input
```
integrate((b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
1/7*b^2*x^7 + 1/3*a*b*x^6 + 1/5*a^2*x^5
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (ax^2 + bx^3)^2 \, dx = \frac{1}{7}\,b^2x^7 + \frac{1}{3}\,abx^6 + \frac{1}{5}\,a^2x^5$$

input
```
integrate((b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
1/7*b^2*x^7 + 1/3*a*b*x^6 + 1/5*a^2*x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(ax^2 + bx^3\right)^2 \, dx = \frac{a^2 \, x^5}{5} + \frac{a \, b \, x^6}{3} + \frac{b^2 \, x^7}{7}$$

input
```
int((a*x^2 + b*x^3)^2,x)
```

output
```
(a^2*x^5)/5 + (b^2*x^7)/7 + (a*b*x^6)/3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(ax^2 + bx^3\right)^2 \, dx = \frac{x^5(15b^2x^2 + 35abx + 21a^2)}{105}$$

input
```
int((b*x^3+a*x^2)^2,x)
```

output
```
(x**5*(21*a**2 + 35*a*b*x + 15*b**2*x**2))/105
```

3.190 $\int \frac{(ax^2+bx^3)^2}{x} dx$

Optimal result . . . 1854
Mathematica [A] (verified) . . . 1854
Rubi [A] (verified) . . . 1855
Maple [A] (warning: unable to verify) . . . 1856
Fricas [A] (verification not implemented) . . . 1856
Sympy [A] (verification not implemented) . . . 1857
Maxima [A] (verification not implemented) . . . 1857
Giac [A] (verification not implemented) . . . 1857
Mupad [B] (verification not implemented) . . . 1858
Reduce [B] (verification not implemented) . . . 1858

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{(ax^2+bx^3)^2}{x} dx = \frac{a^2x^4}{4} + \frac{2}{5}abx^5 + \frac{b^2x^6}{6}$$

output
```
1/4*a^2*x^4+2/5*a*b*x^5+1/6*b^2*x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(ax^2+bx^3)^2}{x} dx = \frac{a^2x^4}{4} + \frac{2}{5}abx^5 + \frac{b^2x^6}{6}$$

input
```
Integrate[(a*x^2 + b*x^3)^2/x,x]
```

output
```
(a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^2}{x} \, dx$$
$$\Big\downarrow \text{ 9}$$
$$\int x^3(a+bx)^2dx$$
$$\Big\downarrow \text{ 49}$$
$$\int \left(a^2x^3+2abx^4+b^2x^5\right) dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

input
```
Int[(a*x^2 + b*x^3)^2/x,x]
```

output
```
(a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^{4}\left(10 b^{2} x^{2}+24 a b x+15 a^{2}\right)}{60}$	25
default	$\frac{1}{4} x^{4} a^{2}+\frac{2}{5} a b \,x^{5}+\frac{1}{6} b^{2} x^{6}$	25
norman	$\frac{1}{4} x^{4} a^{2}+\frac{2}{5} a b \,x^{5}+\frac{1}{6} b^{2} x^{6}$	25
risch	$\frac{1}{4} x^{4} a^{2}+\frac{2}{5} a b \,x^{5}+\frac{1}{6} b^{2} x^{6}$	25
parallelrisch	$\frac{1}{4} x^{4} a^{2}+\frac{2}{5} a b \,x^{5}+\frac{1}{6} b^{2} x^{6}$	25
orering	$\frac{\left(10 b^{2} x^{2}+24 a b x+15 a^{2}\right) \left(b \,x^{3}+a \,x^{2}\right)^{2}}{60 \left(b x+a\right)^{2}}$	42

input
```
int((b*x^3+a*x^2)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/60*x^4*(10*b^2*x^2+24*a*b*x+15*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^2}{x}\,dx = \frac{1}{6}\,b^2x^6 + \frac{2}{5}\,abx^5 + \frac{1}{4}\,a^2x^4$$

input
```
integrate((b*x^3+a*x^2)^2/x,x, algorithm="fricas")
```

output
```
1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(ax^2+bx^3)^2}{x}\,dx = \frac{a^2x^4}{4}+\frac{2abx^5}{5}+\frac{b^2x^6}{6}$$

input
```
integrate((b*x**3+a*x**2)**2/x,x)
```

output
```
a**2*x**4/4 + 2*a*b*x**5/5 + b**2*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^2}{x}\,dx = \frac{1}{6}\,b^2x^6+\frac{2}{5}\,abx^5+\frac{1}{4}\,a^2x^4$$

input
```
integrate((b*x^3+a*x^2)^2/x,x, algorithm="maxima")
```

output
```
1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^2}{x}\,dx = \frac{1}{6}\,b^2x^6+\frac{2}{5}\,abx^5+\frac{1}{4}\,a^2x^4$$

input
```
integrate((b*x^3+a*x^2)^2/x,x, algorithm="giac")
```

output
```
1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^2}{x}\,dx=\frac{a^2\,x^4}{4}+\frac{2\,a\,b\,x^5}{5}+\frac{b^2\,x^6}{6}$$

input
```
int((a*x^2 + b*x^3)^2/x,x)
```

output
```
(a^2*x^4)/4 + (b^2*x^6)/6 + (2*a*b*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^2}{x}\,dx=\frac{x^4(10b^2x^2+24abx+15a^2)}{60}$$

input
```
int((b*x^3+a*x^2)^2/x,x)
```

output
```
(x**4*(15*a**2 + 24*a*b*x + 10*b**2*x**2))/60
```

3.191 $\int \frac{(ax^2+bx^3)^2}{x^2} dx$

Optimal result 1859
Mathematica [A] (verified) 1859
Rubi [A] (verified) 1860
Maple [A] (warning: unable to verify) 1861
Fricas [A] (verification not implemented) 1861
Sympy [A] (verification not implemented) 1862
Maxima [A] (verification not implemented) 1862
Giac [A] (verification not implemented) 1862
Mupad [B] (verification not implemented) 1863
Reduce [B] (verification not implemented) 1863

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{(ax^2+bx^3)^2}{x^2} dx = \frac{a^2x^3}{3} + \frac{1}{2}abx^4 + \frac{b^2x^5}{5}$$

output
```
1/3*a^2*x^3+1/2*a*b*x^4+1/5*b^2*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(ax^2+bx^3)^2}{x^2} dx = \frac{a^2x^3}{3} + \frac{1}{2}abx^4 + \frac{b^2x^5}{5}$$

input
```
Integrate[(a*x^2 + b*x^3)^2/x^2,x]
```

output
```
(a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^2}{x^2} dx$$
$$\Big\downarrow \; 9$$
$$\int x^2(a+bx)^2 dx$$
$$\Big\downarrow \; 49$$
$$\int (a^2x^2+2abx^3+b^2x^4) \, dx$$
$$\Big\downarrow \; 2009$$
$$\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

input
```
Int[(a*x^2 + b*x^3)^2/x^2,x]
```

output
```
(a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^3(6b^2x^2+15abx+10a^2)}{30}$	25
default	$\frac{1}{3}a^2x^3+\frac{1}{2}ab\,x^4+\frac{1}{5}b^2x^5$	25
risch	$\frac{1}{3}a^2x^3+\frac{1}{2}ab\,x^4+\frac{1}{5}b^2x^5$	25
parallelrisch	$\frac{1}{3}a^2x^3+\frac{1}{2}ab\,x^4+\frac{1}{5}b^2x^5$	25
norman	$\frac{\frac{1}{5}b^2x^6+\frac{1}{3}x^4a^2+\frac{1}{2}ab\,x^5}{x}$	29
orering	$\frac{(6b^2x^2+15abx+10a^2)(b\,x^3+a\,x^2)^2}{30x(bx+a)^2}$	45

input
```
int((b*x^3+a*x^2)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/30*x^3*(6*b^2*x^2+15*a*b*x+10*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^2}{x^2}\,dx=\frac{1}{5}\,b^2x^5+\frac{1}{2}\,abx^4+\frac{1}{3}\,a^2x^3$$

input
```
integrate((b*x^3+a*x^2)^2/x^2,x, algorithm="fricas")
```

output
```
1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2 + bx^3)^2}{x^2} dx = \frac{a^2x^3}{3} + \frac{abx^4}{2} + \frac{b^2x^5}{5}$$

input
```
integrate((b*x**3+a*x**2)**2/x**2,x)
```

output
```
a**2*x**3/3 + a*b*x**4/2 + b**2*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2 + bx^3)^2}{x^2} dx = \frac{1}{5} b^2x^5 + \frac{1}{2} abx^4 + \frac{1}{3} a^2x^3$$

input
```
integrate((b*x^3+a*x^2)^2/x^2,x, algorithm="maxima")
```

output
```
1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2 + bx^3)^2}{x^2} dx = \frac{1}{5} b^2x^5 + \frac{1}{2} abx^4 + \frac{1}{3} a^2x^3$$

input
```
integrate((b*x^3+a*x^2)^2/x^2,x, algorithm="giac")
```

output
```
1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^2}{x^2}\,dx=\frac{a^2\,x^3}{3}+\frac{a\,b\,x^4}{2}+\frac{b^2\,x^5}{5}$$

input
```
int((a*x^2 + b*x^3)^2/x^2,x)
```

output
```
(a^2*x^3)/3 + (b^2*x^5)/5 + (a*b*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^2}{x^2}\,dx=\frac{x^3(6b^2x^2+15abx+10a^2)}{30}$$

input
```
int((b*x^3+a*x^2)^2/x^2,x)
```

output
```
(x**3*(10*a**2 + 15*a*b*x + 6*b**2*x**2))/30
```

3.192 $\int \frac{x^6}{ax^2+bx^3} dx$

Optimal result 1864
Mathematica [A] (verified) 1864
Rubi [A] (verified) 1865
Maple [A] (verified) 1866
Fricas [A] (verification not implemented) 1866
Sympy [A] (verification not implemented) 1867
Maxima [A] (verification not implemented) 1867
Giac [A] (verification not implemented) 1867
Mupad [B] (verification not implemented) 1868
Reduce [B] (verification not implemented) 1868

Optimal result

Integrand size = 17, antiderivative size = 57

$$\int \frac{x^6}{ax^2+bx^3} dx = -\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\log(a+bx)}{b^5}$$

output
```
-a^3*x/b^4+1/2*a^2*x^2/b^3-1/3*a*x^3/b^2+1/4*x^4/b+a^4*ln(b*x+a)/b^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{ax^2+bx^3} dx = -\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\log(a+bx)}{b^5}$$

input
```
Integrate[x^6/(a*x^2 + b*x^3),x]
```

output
```
-((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Lo
g[a + b*x])/b^5
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{ax^2+bx^3}\,dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{x^4}{a+bx}dx$$
$$\Big\downarrow \; 49$$
$$\int \left(\frac{a^4}{b^4(a+bx)}-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{a^4\log(a+bx)}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}$$

input
```
Int[x^6/(a*x^2 + b*x^3),x]
```

output
```
-((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Lo
g[a + b*x])/b^5
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{-\frac{1}{4}b^3x^4+\frac{1}{3}a\,b^2x^3-\frac{1}{2}a^2b\,x^2+a^3x}{b^4}+\frac{a^4\ln(bx+a)}{b^5}$	52
risch	$-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{a\,x^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\ln(bx+a)}{b^5}$	52
parallelrisch	$\frac{3b^4x^4-4a\,b^3x^3+6a^2b^2x^2+12a^4\ln(bx+a)-12a^3bx}{12b^5}$	53
norman	$\frac{\frac{x^5}{4b}-\frac{a\,x^4}{3b^2}-\frac{a^3x^2}{b^4}+\frac{a^2x^3}{2b^3}}{x}+\frac{a^4\ln(bx+a)}{b^5}$	59

input
```
int(x^6/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/b^4*(-1/4*b^3*x^4+1/3*a*b^2*x^3-1/2*a^2*b*x^2+a^3*x)+a^4*ln(b*x+a)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{x^6}{ax^2+bx^3}\,dx=\frac{3\,b^4x^4-4\,ab^3x^3+6\,a^2b^2x^2-12\,a^3bx+12\,a^4\log\left(bx+a\right)}{12\,b^5}$$

input
```
integrate(x^6/(b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
1/12*(3*b^4*x^4 - 4*a*b^3*x^3 + 6*a^2*b^2*x^2 - 12*a^3*b*x + 12*a^4*log(b*
x + a))/b^5
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{x^6}{ax^2 + bx^3} \, dx = \frac{a^4 \log(a + bx)}{b^5} - \frac{a^3x}{b^4} + \frac{a^2x^2}{2b^3} - \frac{ax^3}{3b^2} + \frac{x^4}{4b}$$

input
```
integrate(x**6/(b*x**3+a*x**2),x)
```

output
```
a**4*log(a + b*x)/b**5 - a**3*x/b**4 + a**2*x**2/(2*b**3) - a*x**3/(3*b**2
) + x**4/(4*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{x^6}{ax^2 + bx^3} \, dx = \frac{a^4 \log(bx + a)}{b^5} + \frac{3 \, b^3x^4 - 4 \, ab^2x^3 + 6 \, a^2bx^2 - 12 \, a^3x}{12 \, b^4}$$

input
```
integrate(x^6/(b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
a^4*log(b*x + a)/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 12*a^
3*x)/b^4
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{x^6}{ax^2 + bx^3} \, dx = \frac{a^4 \log(|bx + a|)}{b^5} + \frac{3 \, b^3x^4 - 4 \, ab^2x^3 + 6 \, a^2bx^2 - 12 \, a^3x}{12 \, b^4}$$

input
```
integrate(x^6/(b*x^3+a*x^2),x, algorithm="giac")
```

output
```
a^4*log(abs(b*x + a))/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 -
12*a^3*x)/b^4
```

Mupad [B] (verification not implemented)

Time = 8.69 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{x^6}{ax^2+bx^3}\,dx = \frac{x^4}{4\,b}+\frac{a^4\,\ln\left(a+b\,x\right)}{b^5}-\frac{a\,x^3}{3\,b^2}-\frac{a^3\,x}{b^4}+\frac{a^2\,x^2}{2\,b^3}$$

input
```
int(x^6/(a*x^2 + b*x^3),x)
```

output
```
x^4/(4*b) + (a^4*log(a + b*x))/b^5 - (a*x^3)/(3*b^2) - (a^3*x)/b^4 + (a^2*
x^2)/(2*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{x^6}{ax^2+bx^3}\,dx = \frac{12\log(bx+a)\,a^4-12a^3bx+6a^2b^2x^2-4a\,b^3x^3+3b^4x^4}{12b^5}$$

input
```
int(x^6/(b*x^3+a*x^2),x)
```

output
```
(12*log(a + b*x)*a**4 - 12*a**3*b*x + 6*a**2*b**2*x**2 - 4*a*b**3*x**3 + 3
*b**4*x**4)/(12*b**5)
```

3.193 $\int \frac{x^5}{ax^2+bx^3}\,dx$

Optimal result 1869
Mathematica [A] (verified) 1869
Rubi [A] (verified) 1870
Maple [A] (verified) 1871
Fricas [A] (verification not implemented) 1871
Sympy [A] (verification not implemented) 1872
Maxima [A] (verification not implemented) 1872
Giac [A] (verification not implemented) 1872
Mupad [B] (verification not implemented) 1873
Reduce [B] (verification not implemented) 1873

Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \frac{x^5}{ax^2+bx^3}\,dx = \frac{a^2x}{b^3} - \frac{ax^2}{2b^2} + \frac{x^3}{3b} - \frac{a^3\log(a+bx)}{b^4}$$

output
```
a^2*x/b^3-1/2*a*x^2/b^2+1/3*x^3/b-a^3*ln(b*x+a)/b^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{ax^2+bx^3}\,dx = \frac{a^2x}{b^3} - \frac{ax^2}{2b^2} + \frac{x^3}{3b} - \frac{a^3\log(a+bx)}{b^4}$$

input
```
Integrate[x^5/(a*x^2 + b*x^3),x]
```

output
```
(a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{ax^2+bx^3}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x^3}{a+bx}dx$$

$$\Big\downarrow \ 49$$

$$\int \left(-\frac{a^3}{b^3(a+bx)}+\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^3\log(a+bx)}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}$$

input
```
Int[x^5/(a*x^2 + b*x^3),x]
```

output
```
(a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\frac{1}{3}b^2x^3-\frac{1}{2}ab\,x^2+a^2x}{b^3}-\frac{a^3\ln(bx+a)}{b^4}$	41
risch	$\frac{a^2x}{b^3}-\frac{a\,x^2}{2b^2}+\frac{x^3}{3b}-\frac{a^3\ln(bx+a)}{b^4}$	41
parallelrisch	$-\frac{-2b^3x^3+3a\,b^2x^2+6a^3\ln(bx+a)-6a^2bx}{6b^4}$	42
norman	$\frac{\frac{a^2x^2}{b^3}+\frac{x^4}{3b}-\frac{a\,x^3}{2b^2}}{x}-\frac{a^3\ln(bx+a)}{b^4}$	48

input
```
int(x^5/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/3*b^2*x^3-1/2*a*b*x^2+a^2*x)-a^3*ln(b*x+a)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{ax^2+bx^3}\,dx=\frac{2\,b^3x^3-3\,ab^2x^2+6\,a^2bx-6\,a^3\log\left(bx+a\right)}{6\,b^4}$$

input
```
integrate(x^5/(b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))/b^4
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{x^5}{ax^2+bx^3}\,dx=-\frac{a^3\log(a+bx)}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}$$

input
```
integrate(x**5/(b*x**3+a*x**2),x)
```

output
```
-a**3*log(a + b*x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{x^5}{ax^2+bx^3}\,dx=-\frac{a^3\log(bx+a)}{b^4}+\frac{2\,b^2x^3-3\,abx^2+6\,a^2x}{6\,b^3}$$

input
```
integrate(x^5/(b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
-a^3*log(b*x + a)/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{x^5}{ax^2+bx^3}\,dx=-\frac{a^3\log(|bx+a|)}{b^4}+\frac{2\,b^2x^3-3\,abx^2+6\,a^2x}{6\,b^3}$$

input
```
integrate(x^5/(b*x^3+a*x^2),x, algorithm="giac")
```

output
```
-a^3*log(abs(b*x + a))/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int \frac{x^5}{ax^2+bx^3}\,dx=\frac{x^3}{3\,b}-\frac{a^3\,\ln\left(a+b\,x\right)}{b^4}-\frac{a\,x^2}{2\,b^2}+\frac{a^2\,x}{b^3}$$

input
```
int(x^5/(a*x^2 + b*x^3),x)
```

output
```
x^3/(3*b) - (a^3*log(a + b*x))/b^4 - (a*x^2)/(2*b^2) + (a^2*x)/b^3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{ax^2+bx^3}\,dx=\frac{-6\log(bx+a)\,a^3+6a^2bx-3a\,b^2x^2+2b^3x^3}{6b^4}$$

input
```
int(x^5/(b*x^3+a*x^2),x)
```

output
```
( - 6*log(a + b*x)*a**3 + 6*a**2*b*x - 3*a*b**2*x**2 + 2*b**3*x**3)/(6*b**
4)
```

3.194 $\int \frac{x^4}{ax^2+bx^3}\,dx$

Optimal result 1874
Mathematica [A] (verified) 1874
Rubi [A] (verified) 1875
Maple [A] (verified) 1876
Fricas [A] (verification not implemented) 1876
Sympy [A] (verification not implemented) 1877
Maxima [A] (verification not implemented) 1877
Giac [A] (verification not implemented) 1877
Mupad [B] (verification not implemented) 1878
Reduce [B] (verification not implemented) 1878

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \frac{x^4}{ax^2+bx^3}\,dx = -\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\log(a+bx)}{b^3}$$

output
```
-a*x/b^2+1/2*x^2/b+a^2*ln(b*x+a)/b^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{ax^2+bx^3}\,dx = -\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\log(a+bx)}{b^3}$$

input
```
Integrate[x^4/(a*x^2 + b*x^3),x]
```

output
```
-((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{ax^2+bx^3}\,dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{x^2}{a+bx}dx$$
$$\Big\downarrow \text{ 49}$$
$$\int \left(\frac{a^2}{b^2(a+bx)}-\frac{a}{b^2}+\frac{x}{b}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

input
```
Int[x^4/(a*x^2 + b*x^3),x]
```

output
```
-((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{-\frac{1}{2}b\,x^2+ax}{b^2}+\frac{a^2\ln(bx+a)}{b^3}$	30
risch	$-\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\ln(bx+a)}{b^3}$	30
parallelrisch	$\frac{b^2x^2+2a^2\ln(bx+a)-2abx}{2b^3}$	30
norman	$\frac{\frac{x^3}{2b}-\frac{a\,x^2}{b^2}}{x}+\frac{a^2\ln(bx+a)}{b^3}$	37

input
```
int(x^4/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/b^2*(-1/2*b*x^2+a*x)+a^2*ln(b*x+a)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{ax^2+bx^3}\,dx=\frac{b^2x^2-2\,abx+2\,a^2\log\left(bx+a\right)}{2\,b^3}$$

input
```
integrate(x^4/(b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^4}{ax^2+bx^3}\,dx = \frac{a^2\log{(a+bx)}}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

input
```
integrate(x**4/(b*x**3+a*x**2),x)
```

output
```
a**2*log(a + b*x)/b**3 - a*x/b**2 + x**2/(2*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{ax^2+bx^3}\,dx = \frac{a^2\log{(bx+a)}}{b^3} + \frac{bx^2-2\,ax}{2\,b^2}$$

input
```
integrate(x^4/(b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
a^2*log(b*x + a)/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{x^4}{ax^2+bx^3}\,dx = \frac{a^2\log{(|bx+a|)}}{b^3} + \frac{bx^2-2\,ax}{2\,b^2}$$

input
```
integrate(x^4/(b*x^3+a*x^2),x, algorithm="giac")
```

output
```
a^2*log(abs(b*x + a))/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{ax^2+bx^3}\,dx=\frac{2\,a^2\,\ln\left(a+b\,x\right)+b^2\,x^2-2\,a\,b\,x}{2\,b^3}$$

input
```
int(x^4/(a*x^2 + b*x^3),x)
```

output
```
(2*a^2*log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{ax^2+bx^3}\,dx=\frac{2\log(bx+a)\,a^2-2abx+b^2x^2}{2b^3}$$

input
```
int(x^4/(b*x^3+a*x^2),x)
```

output
```
(2*log(a + b*x)*a**2 - 2*a*b*x + b**2*x**2)/(2*b**3)
```

3.195 $\int \frac{x^3}{ax^2+bx^3} \, dx$

Optimal result 1879
Mathematica [A] (verified) 1879
Rubi [A] (verified) 1880
Maple [A] (verified) 1881
Fricas [A] (verification not implemented) 1881
Sympy [A] (verification not implemented) 1882
Maxima [A] (verification not implemented) 1882
Giac [A] (verification not implemented) 1882
Mupad [B] (verification not implemented) 1883
Reduce [B] (verification not implemented) 1883

Optimal result

Integrand size = 17, antiderivative size = 18

$$\int \frac{x^3}{ax^2+bx^3} \, dx = \frac{x}{b} - \frac{a\log(a+bx)}{b^2}$$

output
```
x/b-a*ln(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{ax^2+bx^3} \, dx = \frac{x}{b} - \frac{a\log(a+bx)}{b^2}$$

input
```
Integrate[x^3/(a*x^2 + b*x^3),x]
```

output
```
x/b - (a*Log[a + b*x])/b^2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{ax^2+bx^3}\,dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{x}{a+bx}dx$$
$$\Big\downarrow \text{ 49}$$
$$\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{x}{b}-\frac{a\log(a+bx)}{b^2}$$

input
```
Int[x^3/(a*x^2 + b*x^3),x]
```

output
```
x/b - (a*Log[a + b*x])/b^2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{x}{b}-\frac{a\ln\left(bx+a\right)}{b^{2}}$	19
norman	$\frac{x}{b}-\frac{a\ln\left(bx+a\right)}{b^{2}}$	19
risch	$\frac{x}{b}-\frac{a\ln\left(bx+a\right)}{b^{2}}$	19
parallelrisch	$-\frac{a\ln\left(bx+a\right)-bx}{b^{2}}$	19

input
```
int(x^3/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
x/b-a*ln(b*x+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{ax^2+bx^3}\,dx=\frac{bx-a\log\left(bx+a\right)}{b^2}$$

input
```
integrate(x^3/(b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
(b*x - a*log(b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{ax^2+bx^3}\,dx = -\frac{a\log{(a+bx)}}{b^2} + \frac{x}{b}$$

input
```
integrate(x**3/(b*x**3+a*x**2),x)
```

output
```
-a*log(a + b*x)/b**2 + x/b
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{ax^2+bx^3}\,dx = \frac{x}{b} - \frac{a\log{(bx+a)}}{b^2}$$

input
```
integrate(x^3/(b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
x/b - a*log(b*x + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{x^3}{ax^2+bx^3}\,dx = \frac{x}{b} - \frac{a\log{(|bx+a|)}}{b^2}$$

input
```
integrate(x^3/(b*x^3+a*x^2),x, algorithm="giac")
```

output
```
x/b - a*log(abs(b*x + a))/b^2
```

Mupad [B] (verification not implemented)

Time = 8.69 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{ax^2+bx^3}\,dx=-\frac{a\,\ln\left(a+b\,x\right)-b\,x}{b^2}$$

input
```
int(x^3/(a*x^2 + b*x^3),x)
```

output
```
-(a*log(a + b*x) - b*x)/b^2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{ax^2+bx^3}\,dx=\frac{-\log(bx+a)\,a+bx}{b^2}$$

input
```
int(x^3/(b*x^3+a*x^2),x)
```

output
```
( - log(a + b*x)*a + b*x)/b**2
```

3.196 $\int \frac{x^2}{ax^2+bx^3} \, dx$

Optimal result 1884
Mathematica [A] (verified) 1884
Rubi [A] (verified) 1885
Maple [A] (verified) 1886
Fricas [A] (verification not implemented) 1886
Sympy [A] (verification not implemented) 1886
Maxima [A] (verification not implemented) 1887
Giac [A] (verification not implemented) 1887
Mupad [B] (verification not implemented) 1887
Reduce [B] (verification not implemented) 1888

Optimal result

Integrand size = 17, antiderivative size = 10

$$\int \frac{x^2}{ax^2+bx^3} \, dx = \frac{\log(a+bx)}{b}$$

output
```
ln(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{ax^2+bx^3} \, dx = \frac{\log(a+bx)}{b}$$

input
```
Integrate[x^2/(a*x^2 + b*x^3),x]
```

output
```
Log[a + b*x]/b
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {9, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{ax^2+bx^3}\,dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{1}{a+bx}dx$$
$$\Big\downarrow \; 16$$
$$\frac{\log(a+bx)}{b}$$

input
```
Int[x^2/(a*x^2 + b*x^3),x]
```

output
```
Log[a + b*x]/b
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\ln(bx+a)}{b}$	11
norman	$\frac{\ln(bx+a)}{b}$	11
risch	$\frac{\ln(bx+a)}{b}$	11
parallelrisch	$\frac{\ln(bx+a)}{b}$	11

input
```
int(x^2/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
ln(b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{ax^2+bx^3}\,dx = \frac{\log(bx+a)}{b}$$

input
```
integrate(x^2/(b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
log(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{ax^2+bx^3}\,dx = \frac{\log(a+bx)}{b}$$

input
```
integrate(x**2/(b*x**3+a*x**2),x)
```

output
```
log(a + b*x)/b
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{ax^2+bx^3}\,dx=\frac{\log{(bx+a)}}{b}$$

input
```
integrate(x^2/(b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
log(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{ax^2+bx^3}\,dx=\frac{\log{(|bx+a|)}}{b}$$

input
```
integrate(x^2/(b*x^3+a*x^2),x, algorithm="giac")
```

output
```
log(abs(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{ax^2+bx^3}\,dx=\frac{\ln{(a+b\,x)}}{b}$$

input
```
int(x^2/(a*x^2 + b*x^3),x)
```

output
```
log(a + b*x)/b
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{ax^2 + bx^3} \, dx = \frac{\log(bx + a)}{b}$$

input
```
int(x^2/(b*x^3+a*x^2),x)
```

output
```
log(a + b*x)/b
```

3.197 $\int \frac{x}{ax^2+bx^3} dx$

Optimal result 1889
Mathematica [A] (verified) 1889
Rubi [A] (verified) 1890
Maple [A] (verified) 1891
Fricas [A] (verification not implemented) 1892
Sympy [A] (verification not implemented) 1892
Maxima [A] (verification not implemented) 1892
Giac [A] (verification not implemented) 1893
Mupad [B] (verification not implemented) 1893
Reduce [B] (verification not implemented) 1893

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{x}{ax^2+bx^3} dx = \frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

output
```
ln(x)/a-ln(b*x+a)/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{ax^2+bx^3} dx = \frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

input
```
Integrate[x/(a*x^2 + b*x^3),x]
```

output
```
Log[x]/a - Log[a + b*x]/a
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {9, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{ax^2+bx^3}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{1}{x(a+bx)}dx$$

$$\Big\downarrow \; 47$$

$$\frac{\int \frac{1}{x}dx}{a}-\frac{b\int \frac{1}{a+bx}dx}{a}$$

$$\Big\downarrow \; 14$$

$$\frac{\log(x)}{a}-\frac{b\int \frac{1}{a+bx}dx}{a}$$

$$\Big\downarrow \; 16$$

$$\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

input
```
Int[x/(a*x^2 + b*x^3),x]
```

output
```
Log[x]/a - Log[a + b*x]/a
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result	size
parallelrisch	$\frac{\ln(x)-\ln(bx+a)}{a}$	16
default	$\frac{\ln(x)}{a}-\frac{\ln(bx+a)}{a}$	19
norman	$\frac{\ln(x)}{a}-\frac{\ln(bx+a)}{a}$	19
risch	$-\frac{\ln(bx+a)}{a}+\frac{\ln(-x)}{a}$	21

input
```
int(x/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
(ln(x)-ln(b*x+a))/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{x}{ax^2 + bx^3} \, dx = -\frac{\log(bx + a) - \log(x)}{a}$$

input
```
integrate(x/(b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
-(log(b*x + a) - log(x))/a
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.56

$$\int \frac{x}{ax^2 + bx^3} \, dx = \frac{\log(x) - \log\left(\frac{a}{b} + x\right)}{a}$$

input
```
integrate(x/(b*x**3+a*x**2),x)
```

output
```
(log(x) - log(a/b + x))/a
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{ax^2 + bx^3} \, dx = -\frac{\log(bx + a)}{a} + \frac{\log(x)}{a}$$

input
```
integrate(x/(b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
-log(b*x + a)/a + log(x)/a
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x}{ax^2+bx^3}\,dx = -\frac{\log(|bx+a|)}{a}+\frac{\log(|x|)}{a}$$

input
```
integrate(x/(b*x^3+a*x^2),x, algorithm="giac")
```

output
```
-log(abs(b*x + a))/a + log(abs(x))/a
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{x}{ax^2+bx^3}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a}$$

input
```
int(x/(a*x^2 + b*x^3),x)
```

output
```
-(2*atanh((2*b*x)/a + 1))/a
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{x}{ax^2+bx^3}\,dx = \frac{-\log(bx+a)+\log(x)}{a}$$

input
```
int(x/(b*x^3+a*x^2),x)
```

output
```
( - log(a + b*x) + log(x))/a
```

3.198 $\int \frac{1}{ax^2+bx^3}\,dx$

Optimal result . . . 1894
Mathematica [A] (verified) . . . 1894
Rubi [A] (verified) . . . 1895
Maple [A] (verified) . . . 1896
Fricas [A] (verification not implemented) . . . 1896
Sympy [A] (verification not implemented) . . . 1897
Maxima [A] (verification not implemented) . . . 1897
Giac [A] (verification not implemented) . . . 1897
Mupad [B] (verification not implemented) . . . 1898
Reduce [B] (verification not implemented) . . . 1898

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{1}{ax^2+bx^3}\,dx=-\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}$$

output
```
-1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^2+bx^3}\,dx=-\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}$$

input
```
Integrate[(a*x^2 + b*x^3)^(-1),x]
```

output
```
-(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2026, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax^2+bx^3}dx$$

$$\Big\downarrow \text{ 2026}$$

$$\int \frac{1}{x^2(a+bx)}dx$$

$$\Big\downarrow \text{ 54}$$

$$\int \left(\frac{b^2}{a^2(a+bx)}-\frac{b}{a^2x}+\frac{1}{ax^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

input
```
Int[(a*x^2 + b*x^3)^(-1),x]
```

output
```
-(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result	size
parallelrisch	$-\frac{b\ln(x)x-b\ln(bx+a)x+a}{a^2x}$	26
default	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
norman	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
risch	$-\frac{1}{ax}+\frac{b\ln(-bx-a)}{a^2}-\frac{b\ln(x)}{a^2}$	32

input
```
int(1/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-(b*ln(x)*x-b*ln(b*x+a)*x+a)/a^2/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{ax^2+bx^3}\,dx=\frac{bx\log(bx+a)-bx\log(x)-a}{a^2x}$$

input
```
integrate(1/(b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
(b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{1}{ax^2 + bx^3} dx = -\frac{1}{ax} + \frac{b\left(-\log{(x)} + \log{\left(\frac{a}{b} + x\right)}\right)}{a^2}$$

input
```
integrate(1/(b*x**3+a*x**2),x)
```

output
```
-1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^2 + bx^3} dx = \frac{b\log{(bx + a)}}{a^2} - \frac{b\log{(x)}}{a^2} - \frac{1}{ax}$$

input
```
integrate(1/(b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{ax^2 + bx^3} dx = \frac{b\log{(|bx + a|)}}{a^2} - \frac{b\log{(|x|)}}{a^2} - \frac{1}{ax}$$

input
```
integrate(1/(b*x^3+a*x^2),x, algorithm="giac")
```

output
```
b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

Mupad [B] (verification not implemented)

Time = 8.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{ax^2+bx^3}\,dx = \frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2}-\frac{1}{a\,x}$$

input
```
int(1/(a*x^2 + b*x^3),x)
```

output
```
(2*b*atanh((2*b*x)/a + 1))/a^2 - 1/(a*x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{ax^2+bx^3}\,dx = \frac{\log(bx+a)\,bx-\log(x)\,bx-a}{a^2x}$$

input
```
int(1/(b*x^3+a*x^2),x)
```

output
```
(log(a + b*x)*b*x - log(x)*b*x - a)/(a**2*x)
```

3.199 $\int \frac{1}{x(ax^2+bx^3)}\,dx$

Optimal result 1899
Mathematica [A] (verified) 1899
Rubi [A] (verified) 1900
Maple [A] (verified) 1901
Fricas [A] (verification not implemented) 1901
Sympy [A] (verification not implemented) 1902
Maxima [A] (verification not implemented) 1902
Giac [A] (verification not implemented) 1902
Mupad [B] (verification not implemented) 1903
Reduce [B] (verification not implemented) 1903

Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{1}{x(ax^2+bx^3)}\,dx = -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}$$

output
```
-1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(ax^2+bx^3)}\,dx = -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}$$

input
```
Integrate[1/(x*(a*x^2 + b*x^3)),x]
```

output
```
-1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax^2+bx^3\right)}dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{1}{x^3(a+bx)}dx$$
$$\Big\downarrow \; 54$$
$$\int \left(-\frac{b^3}{a^3(a+bx)}+\frac{b^2}{a^3x}-\frac{b}{a^2x^2}+\frac{1}{ax^3}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

input
```
Int[1/(x*(a*x^2 + b*x^3)),x]
```

output
```
-1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{1}{2a\,x^{2}}+\frac{b}{a^{2}x}+\frac{b^{2}\ln\left(x\right)}{a^{3}}-\frac{b^{2}\ln\left(bx+a\right)}{a^{3}}$	41
norman	$\frac{\frac{bx}{a^{2}}-\frac{1}{2a}}{x^{2}}+\frac{b^{2}\ln\left(x\right)}{a^{3}}-\frac{b^{2}\ln\left(bx+a\right)}{a^{3}}$	41
risch	$\frac{\frac{bx}{a^{2}}-\frac{1}{2a}}{x^{2}}-\frac{b^{2}\ln\left(bx+a\right)}{a^{3}}+\frac{b^{2}\ln\left(-x\right)}{a^{3}}$	43
parallelrisch	$\frac{2b^{2}\ln\left(x\right)x^{2}-2b^{2}\ln\left(bx+a\right)x^{2}+2abx-a^{2}}{2a^{3}x^{2}}$	44

input
```
int(1/x/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{1}{x\left(ax^{2}+bx^{3}\right)}\,dx=-\frac{2\,b^{2}x^{2}\log\left(bx+a\right)-2\,b^{2}x^{2}\log\left(x\right)-2\,abx+a^{2}}{2\,a^{3}x^{2}}$$

input
```
integrate(1/x/(b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
-1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{1}{x\left(ax^2+bx^3\right)}\,dx = \frac{-a+2bx}{2a^2x^2}+\frac{b^2\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^3}$$

input
```
integrate(1/x/(b*x**3+a*x**2),x)
```

output
```
(-a + 2*b*x)/(2*a**2*x**2) + b**2*(log(x) - log(a/b + x))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{1}{x\left(ax^2+bx^3\right)}\,dx = -\frac{b^2\log\left(bx+a\right)}{a^3}+\frac{b^2\log\left(x\right)}{a^3}+\frac{2\,bx-a}{2\,a^2x^2}$$

input
```
integrate(1/x/(b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
-b^2*log(b*x + a)/a^3 + b^2*log(x)/a^3 + 1/2*(2*b*x - a)/(a^2*x^2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(ax^2+bx^3\right)}\,dx = -\frac{b^2\log\left(|bx+a|\right)}{a^3}+\frac{b^2\log\left(|x|\right)}{a^3}+\frac{2\,abx-a^2}{2\,a^3x^2}$$

input
```
integrate(1/x/(b*x^3+a*x^2),x, algorithm="giac")
```

output
```
-b^2*log(abs(b*x + a))/a^3 + b^2*log(abs(x))/a^3 + 1/2*(2*a*b*x - a^2)/(a^
3*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\left(ax^2+bx^3\right)}\,dx=-\frac{\frac{a^2}{2}-a\,b\,x}{a^3\,x^2}-\frac{2\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^3}$$

input
```
int(1/(x*(a*x^2 + b*x^3)),x)
```

output
```
- (a^2/2 - a*b*x)/(a^3*x^2) - (2*b^2*atanh((2*b*x)/a + 1))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\left(ax^2+bx^3\right)}\,dx=\frac{-2\log(bx+a)\,b^2x^2+2\log(x)\,b^2x^2-a^2+2abx}{2a^3x^2}$$

input
```
int(1/x/(b*x^3+a*x^2),x)
```

output
```
( - 2*log(a + b*x)*b**2*x**2 + 2*log(x)*b**2*x**2 - a**2 + 2*a*b*x)/(2*a**
3*x**2)
```

3.200 $\int \frac{1}{x^2(ax^2+bx^3)}\,dx$

Optimal result 1904
Mathematica [A] (verified) 1904
Rubi [A] (verified) 1905
Maple [A] (verified) 1906
Fricas [A] (verification not implemented) 1906
Sympy [A] (verification not implemented) 1907
Maxima [A] (verification not implemented) 1907
Giac [A] (verification not implemented) 1907
Mupad [B] (verification not implemented) 1908
Reduce [B] (verification not implemented) 1908

Optimal result

Integrand size = 17, antiderivative size = 56

$$\int \frac{1}{x^2(ax^2+bx^3)}\,dx = -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}$$

output
```
-1/3/a/x^3+1/2*b/a^2/x^2-b^2/a^3/x-b^3*ln(x)/a^4+b^3*ln(b*x+a)/a^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(ax^2+bx^3)}\,dx = -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}$$

input
```
Integrate[1/(x^2*(a*x^2 + b*x^3)),x]
```

output
```
-1/3*1/(a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log
[a + b*x])/a^4
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (ax^2+bx^3)} \, dx$$
$$\Big\downarrow \text{9}$$
$$\int \frac{1}{x^4(a+bx)}dx$$
$$\Big\downarrow \text{54}$$
$$\int \left(\frac{b^4}{a^4(a+bx)}-\frac{b^3}{a^4x}+\frac{b^2}{a^3x^2}-\frac{b}{a^2x^3}+\frac{1}{ax^4}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}-\frac{b^2}{a^3x}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

input
```
Int[1/(x^2*(a*x^2 + b*x^3)),x]
```

output
```
-1/3*1/(a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log
[a + b*x])/a^4
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{1}{3a\,x^{3}}+\frac{b}{2a^{2}x^{2}}-\frac{b^{2}}{a^{3}x}-\frac{b^{3}\ln\left(x\right)}{a^{4}}+\frac{b^{3}\ln\left(bx+a\right)}{a^{4}}$	53
norman	$\frac{-\frac{1}{3a}+\frac{bx}{2a^{2}}-\frac{b^{2}x^{2}}{a^{3}}}{x^{3}}+\frac{b^{3}\ln\left(bx+a\right)}{a^{4}}-\frac{b^{3}\ln\left(x\right)}{a^{4}}$	53
parallelrisch	$-\frac{6b^{3}\ln\left(x\right)x^{3}-6b^{3}\ln\left(bx+a\right)x^{3}+6a\,b^{2}x^{2}-3a^{2}bx+2a^{3}}{6a^{4}x^{3}}$	55
risch	$\frac{-\frac{1}{3a}+\frac{bx}{2a^{2}}-\frac{b^{2}x^{2}}{a^{3}}}{x^{3}}-\frac{b^{3}\ln\left(x\right)}{a^{4}}+\frac{b^{3}\ln\left(-bx-a\right)}{a^{4}}$	56

input
```
int(1/x^2/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a/x^3+1/2*b/a^2/x^2-b^2/a^3/x-b^3*ln(x)/a^4+b^3*ln(b*x+a)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)}\,dx=\frac{6\,b^3x^3\log\left(bx+a\right)-6\,b^3x^3\log\left(x\right)-6\,ab^2x^2+3\,a^2bx-2\,a^3}{6\,a^4x^3}$$

input
```
integrate(1/x^2/(b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
1/6*(6*b^3*x^3*log(b*x + a) - 6*b^3*x^3*log(x) - 6*a*b^2*x^2 + 3*a^2*b*x -
 2*a^3)/(a^4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^2 (ax^2 + bx^3)} dx = \frac{-2a^2 + 3abx - 6b^2x^2}{6a^3x^3} + \frac{b^3 \left(-\log(x) + \log\left(\frac{a}{b} + x\right)\right)}{a^4}$$

input
```
integrate(1/x**2/(b*x**3+a*x**2),x)
```

output
```
(-2*a**2 + 3*a*b*x - 6*b**2*x**2)/(6*a**3*x**3) + b**3*(-log(x) + log(a/b
+ x))/a**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2 (ax^2 + bx^3)} dx = \frac{b^3 \log(bx + a)}{a^4} - \frac{b^3 \log(x)}{a^4} - \frac{6 b^2 x^2 - 3 abx + 2 a^2}{6 a^3 x^3}$$

input
```
integrate(1/x^2/(b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
b^3*log(b*x + a)/a^4 - b^3*log(x)/a^4 - 1/6*(6*b^2*x^2 - 3*a*b*x + 2*a^2)/
(a^3*x^3)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (ax^2 + bx^3)} dx = \frac{b^3 \log(|bx + a|)}{a^4} - \frac{b^3 \log(|x|)}{a^4} - \frac{6 ab^2 x^2 - 3 a^2 bx + 2 a^3}{6 a^4 x^3}$$

input
```
integrate(1/x^2/(b*x^3+a*x^2),x, algorithm="giac")
```

output
```
b^3*log(abs(b*x + a))/a^4 - b^3*log(abs(x))/a^4 - 1/6*(6*a*b^2*x^2 - 3*a^2
*b*x + 2*a^3)/(a^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^2(ax^2+bx^3)}\,dx = \frac{2\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^4} - \frac{\frac{a^3}{3}-\frac{a^2\,b\,x}{2}+a\,b^2\,x^2}{a^4\,x^3}$$

input
```
int(1/(x^2*(a*x^2 + b*x^3)),x)
```

output
```
(2*b^3*atanh((2*b*x)/a + 1))/a^4 - (a^3/3 + a*b^2*x^2 - (a^2*b*x)/2)/(a^4*
x^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^2(ax^2+bx^3)}\,dx = \frac{6\log(bx+a)\,b^3x^3-6\log(x)\,b^3x^3-2a^3+3a^2bx-6a\,b^2x^2}{6a^4x^3}$$

input
```
int(1/x^2/(b*x^3+a*x^2),x)
```

output
```
(6*log(a + b*x)*b**3*x**3 - 6*log(x)*b**3*x**3 - 2*a**3 + 3*a**2*b*x - 6*a
*b**2*x**2)/(6*a**4*x**3)
```

3.201 $\int \frac{x^8}{(ax^2+bx^3)^2}\,dx$

Optimal result 1909
Mathematica [A] (verified) 1909
Rubi [A] (verified) 1910
Maple [A] (verified) 1911
Fricas [A] (verification not implemented) 1911
Sympy [A] (verification not implemented) 1912
Maxima [A] (verification not implemented) 1912
Giac [A] (verification not implemented) 1913
Mupad [B] (verification not implemented) 1913
Reduce [B] (verification not implemented) 1913

Optimal result

Integrand size = 17, antiderivative size = 58

$$\int \frac{x^8}{(ax^2+bx^3)^2}\,dx = \frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}$$

output
```
3*a^2*x/b^4-a*x^2/b^3+1/3*x^3/b^2-a^4/b^5/(b*x+a)-4*a^3*ln(b*x+a)/b^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{x^8}{(ax^2+bx^3)^2}\,dx = \frac{9a^2bx-3ab^2x^2+b^3x^3-\frac{3a^4}{a+bx}-12a^3\log(a+bx)}{3b^5}$$

input
```
Integrate[x^8/(a*x^2 + b*x^3)^2,x]
```

output
```
(9*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 - (3*a^4)/(a + b*x) - 12*a^3*Log[a + b*
x])/(3*b^5)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(ax^2+bx^3)^2} \, dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{x^4}{(a+bx)^2}dx$$
$$\Big\downarrow \text{ 49}$$
$$\int \left(\frac{a^4}{b^4(a+bx)^2}-\frac{4a^3}{b^4(a+bx)}+\frac{3a^2}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{b^2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}+\frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}$$

input
```
Int[x^8/(a*x^2 + b*x^3)^2,x]
```

output
```
(3*a^2*x)/b^4 - (a*x^2)/b^3 + x^3/(3*b^2) - a^4/(b^5*(a + b*x)) - (4*a^3*L
og[a + b*x])/b^5
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{\frac{1}{3}b^2x^3-ab\,x^2+3a^2x}{b^4}-\frac{a^4}{b^5(bx+a)}-\frac{4a^3\ln(bx+a)}{b^5}$	57
risch	$\frac{3a^2x}{b^4}-\frac{a\,x^2}{b^3}+\frac{x^3}{3b^2}-\frac{a^4}{b^5(bx+a)}-\frac{4a^3\ln(bx+a)}{b^5}$	57
norman	$\frac{\frac{x^7}{3b}-\frac{2a\,x^6}{3b^2}+\frac{2a^2x^5}{b^3}-\frac{4a^4x^3}{b^5}}{x^3(bx+a)}-\frac{4a^3\ln(bx+a)}{b^5}$	67
parallelrisch	$-\frac{-b^4x^4+2a\,b^3x^3+12\ln(bx+a)x\,a^3b-6a^2b^2x^2+12a^4\ln(bx+a)+12a^4}{3b^5(bx+a)}$	71

input
```
int(x^8/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(1/3*b^2*x^3-a*b*x^2+3*a^2*x)-a^4/b^5/(b*x+a)-4*a^3*ln(b*x+a)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.26

$$\int \frac{x^8}{(ax^2+bx^3)^2}\,dx = \frac{b^4x^4-2\,ab^3x^3+6\,a^2b^2x^2+9\,a^3bx-3\,a^4-12\,(a^3bx+a^4)\log(bx+a)}{3\,(b^6x+ab^5)}$$

input
```
integrate(x^8/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b
*x + a^4)*log(b*x + a))/(b^6*x + a*b^5)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{x^8}{(ax^2+bx^3)^2}\,dx = -\frac{a^4}{ab^5+b^6x} - \frac{4a^3\log(a+bx)}{b^5} + \frac{3a^2x}{b^4} - \frac{ax^2}{b^3} + \frac{x^3}{3b^2}$$

input
```
integrate(x**8/(b*x**3+a*x**2)**2,x)
```

output
```
-a**4/(a*b**5 + b**6*x) - 4*a**3*log(a + b*x)/b**5 + 3*a**2*x/b**4 - a*x**
2/b**3 + x**3/(3*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{x^8}{(ax^2+bx^3)^2}\,dx = -\frac{a^4}{b^6x+ab^5} - \frac{4\,a^3\log(bx+a)}{b^5} + \frac{b^2x^3-3\,abx^2+9\,a^2x}{3\,b^4}$$

input
```
integrate(x^8/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
-a^4/(b^6*x + a*b^5) - 4*a^3*log(b*x + a)/b^5 + 1/3*(b^2*x^3 - 3*a*b*x^2 +
 9*a^2*x)/b^4
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int \frac{x^8}{(ax^2+bx^3)^2}\,dx = -\frac{4\,a^3\log\left(|bx+a|\right)}{b^5} - \frac{a^4}{(bx+a)b^5} + \frac{b^4x^3-3\,ab^3x^2+9\,a^2b^2x}{3\,b^6}$$

input
```
integrate(x^8/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
-4*a^3*log(abs(b*x + a))/b^5 - a^4/((b*x + a)*b^5) + 1/3*(b^4*x^3 - 3*a*b^
3*x^2 + 9*a^2*b^2*x)/b^6
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int \frac{x^8}{(ax^2+bx^3)^2}\,dx = \frac{x^3}{3\,b^2} - \frac{4\,a^3\,\ln\left(a+b\,x\right)}{b^5} - \frac{a\,x^2}{b^3} + \frac{3\,a^2\,x}{b^4} - \frac{a^4}{b\,\left(x\,b^5+a\,b^4\right)}$$

input
```
int(x^8/(a*x^2 + b*x^3)^2,x)
```

output
```
x^3/(3*b^2) - (4*a^3*log(a + b*x))/b^5 - (a*x^2)/b^3 + (3*a^2*x)/b^4 - a^4
/(b*(a*b^4 + b^5*x))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22

$$\int \frac{x^8}{(ax^2+bx^3)^2}\,dx = \frac{-12\log(bx+a)\,a^4-12\log(bx+a)\,a^3bx+12a^3bx+6a^2b^2x^2-2a\,b^3x^3+b^4x^4}{3b^5\,(bx+a)}$$

input
```
int(x^8/(b*x^3+a*x^2)^2,x)
```

output
```
( - 12*log(a + b*x)*a**4 - 12*log(a + b*x)*a**3*b*x + 12*a**3*b*x + 6*a**2
*b**2*x**2 - 2*a*b**3*x**3 + b**4*x**4)/(3*b**5*(a + b*x))
```

3.202 $\int \frac{x^7}{(ax^2+bx^3)^2} dx$

Optimal result 1915
Mathematica [A] (verified) 1915
Rubi [A] (verified) 1916
Maple [A] (verified) 1917
Fricas [A] (verification not implemented) 1917
Sympy [A] (verification not implemented) 1918
Maxima [A] (verification not implemented) 1918
Giac [A] (verification not implemented) 1918
Mupad [B] (verification not implemented) 1919
Reduce [B] (verification not implemented) 1919

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \frac{x^7}{(ax^2+bx^3)^2} dx = -\frac{2ax}{b^3} + \frac{x^2}{2b^2} + \frac{a^3}{b^4(a+bx)} + \frac{3a^2\log(a+bx)}{b^4}$$

output
```
-2*a*x/b^3+1/2*x^2/b^2+a^3/b^4/(b*x+a)+3*a^2*ln(b*x+a)/b^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{x^7}{(ax^2+bx^3)^2} dx = \frac{-4abx + b^2x^2 + \frac{2a^3}{a+bx} + 6a^2\log(a+bx)}{2b^4}$$

input
```
Integrate[x^7/(a*x^2 + b*x^3)^2,x]
```

output
```
(-4*a*b*x + b^2*x^2 + (2*a^3)/(a + b*x) + 6*a^2*Log[a + b*x])/(2*b^4)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(ax^2+bx^3)^2}dx$$
$$\Big\downarrow 9$$
$$\int \frac{x^3}{(a+bx)^2}dx$$
$$\Big\downarrow 49$$
$$\int \left(-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}-\frac{2a}{b^3}+\frac{x}{b^2}\right)dx$$
$$\Big\downarrow 2009$$
$$\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}$$

input
```
Int[x^7/(a*x^2 + b*x^3)^2,x]
```

output
```
(-2*a*x)/b^3 + x^2/(2*b^2) + a^3/(b^4*(a + b*x)) + (3*a^2*Log[a + b*x])/b^
4
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

method	result	size
risch	$-\frac{2ax}{b^3}+\frac{x^2}{2b^2}+\frac{a^3}{b^4(bx+a)}+\frac{3a^2\ln(bx+a)}{b^4}$	45
default	$-\frac{-\frac{1}{2}b\,x^2+2ax}{b^3}+\frac{a^3}{b^4(bx+a)}+\frac{3a^2\ln(bx+a)}{b^4}$	46
norman	$\frac{\frac{3a^3x^3}{b^4}+\frac{x^6}{2b}-\frac{3a\,x^5}{2b^2}}{x^3(bx+a)}+\frac{3a^2\ln(bx+a)}{b^4}$	56
parallelrisch	$\frac{b^3x^3+6\ln(bx+a)x\,a^2b-3a\,b^2x^2+6a^3\ln(bx+a)+6a^3}{2b^4(bx+a)}$	59

input
```
int(x^7/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*a*x/b^3+1/2/b^2*x^2+a^3/b^4/(b*x+a)+3*a^2*ln(b*x+a)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.35

$$\int \frac{x^7}{(ax^2+bx^3)^2}\,dx=\frac{b^3x^3-3\,ab^2x^2-4\,a^2bx+2\,a^3+6\,(a^2bx+a^3)\log{(bx+a)}}{2\,(b^5x+ab^4)}$$

input
```
integrate(x^7/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
 + a))/(b^5*x + a*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{(ax^2+bx^3)^2}\,dx = \frac{a^3}{ab^4+b^5x} + \frac{3a^2\log(a+bx)}{b^4} - \frac{2ax}{b^3} + \frac{x^2}{2b^2}$$

input
```
integrate(x**7/(b*x**3+a*x**2)**2,x)
```

output
```
a**3/(a*b**4 + b**5*x) + 3*a**2*log(a + b*x)/b**4 - 2*a*x/b**3 + x**2/(2*b
**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \frac{x^7}{(ax^2+bx^3)^2}\,dx = \frac{a^3}{b^5x+ab^4} + \frac{3\,a^2\log(bx+a)}{b^4} + \frac{bx^2-4\,ax}{2\,b^3}$$

input
```
integrate(x^7/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
a^3/(b^5*x + a*b^4) + 3*a^2*log(b*x + a)/b^4 + 1/2*(b*x^2 - 4*a*x)/b^3
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \frac{x^7}{(ax^2+bx^3)^2}\,dx = \frac{3\,a^2\log(|bx+a|)}{b^4} + \frac{a^3}{(bx+a)b^4} + \frac{b^2x^2-4\,abx}{2\,b^4}$$

input
```
integrate(x^7/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
3*a^2*log(abs(b*x + a))/b^4 + a^3/((b*x + a)*b^4) + 1/2*(b^2*x^2 - 4*a*b*x
)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \frac{x^7}{(ax^2+bx^3)^2}\,dx = \frac{x^2}{2\,b^2}+\frac{3\,a^2\,\ln\left(a+b\,x\right)}{b^4}+\frac{a^3}{b\,\left(x\,b^4+a\,b^3\right)}-\frac{2\,a\,x}{b^3}$$

input
```
int(x^7/(a*x^2 + b*x^3)^2,x)
```

output
```
x^2/(2*b^2) + (3*a^2*log(a + b*x))/b^4 + a^3/(b*(a*b^3 + b^4*x)) - (2*a*x)
/b^3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \frac{x^7}{(ax^2+bx^3)^2}\,dx = \frac{6\log(bx+a)\,a^3+6\log(bx+a)\,a^2bx-6a^2bx-3a\,b^2x^2+b^3x^3}{2b^4\,(bx+a)}$$

input
```
int(x^7/(b*x^3+a*x^2)^2,x)
```

output
```
(6*log(a + b*x)*a**3 + 6*log(a + b*x)*a**2*b*x - 6*a**2*b*x - 3*a*b**2*x**
2 + b**3*x**3)/(2*b**4*(a + b*x))
```

3.203 $\int \frac{x^6}{(ax^2+bx^3)^2}\,dx$

Optimal result 1920
Mathematica [A] (verified) 1920
Rubi [A] (verified) 1921
Maple [A] (verified) 1922
Fricas [A] (verification not implemented) 1922
Sympy [A] (verification not implemented) 1923
Maxima [A] (verification not implemented) 1923
Giac [A] (verification not implemented) 1923
Mupad [B] (verification not implemented) 1924
Reduce [B] (verification not implemented) 1924

Optimal result

Integrand size = 17, antiderivative size = 33

$$\int \frac{x^6}{(ax^2+bx^3)^2}\,dx = \frac{x}{b^2} - \frac{a^2}{b^3(a+bx)} - \frac{2a\log(a+bx)}{b^3}$$

output
```
x/b^2-a^2/b^3/(b*x+a)-2*a*ln(b*x+a)/b^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{x^6}{(ax^2+bx^3)^2}\,dx = \frac{bx - \frac{a^2}{a+bx} - 2a\log(a+bx)}{b^3}$$

input
```
Integrate[x^6/(a*x^2 + b*x^3)^2,x]
```

output
```
(b*x - a^2/(a + b*x) - 2*a*Log[a + b*x])/b^3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(ax^2+bx^3)^2} \, dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{x^2}{(a+bx)^2}dx$$
$$\Big\downarrow \; 49$$
$$\int \left(\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}+\frac{1}{b^2}\right)dx$$
$$\Big\downarrow \; 2009$$
$$-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}+\frac{x}{b^2}$$

input
```
Int[x^6/(a*x^2 + b*x^3)^2,x]
```

output
```
x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*Log[a + b*x])/b^3
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{x}{b^{2}}-\frac{a^{2}}{b^{3}\left(b x +a \right)}-\frac{2 a \ln \left(b x +a \right)}{b^{3}}$	34
risch	$\frac{x}{b^{2}}-\frac{a^{2}}{b^{3}\left(b x +a \right)}-\frac{2 a \ln \left(b x +a \right)}{b^{3}}$	34
norman	$\frac{\frac{x^{5}}{b}-\frac{2 a^{2} x^{3}}{b^{3}}}{x^{3}\left(b x +a \right)}-\frac{2 a \ln \left(b x +a \right)}{b^{3}}$	44
parallelrisch	$-\frac{2 \ln \left(b x +a \right) x a b -b^{2} x^{2}+2 a^{2} \ln \left(b x +a \right)+2 a^{2}}{b^{3}\left(b x +a \right)}$	49

input
```
int(x^6/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
x/b^2-a^2/b^3/(b*x+a)-2*a*ln(b*x+a)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.42

$$\int \frac{x^6}{\left(ax^2+bx^3\right)^2}\,dx=\frac{b^2x^2+abx-a^2-2\left(abx+a^2\right)\log\left(bx+a\right)}{b^4x+ab^3}$$

input
```
integrate(x^6/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
(b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))/(b^4*x + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{x^6}{(ax^2+bx^3)^2}\,dx = -\frac{a^2}{ab^3+b^4x} - \frac{2a\log(a+bx)}{b^3} + \frac{x}{b^2}$$

input
```
integrate(x**6/(b*x**3+a*x**2)**2,x)
```

output
```
-a**2/(a*b**3 + b**4*x) - 2*a*log(a + b*x)/b**3 + x/b**2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{x^6}{(ax^2+bx^3)^2}\,dx = -\frac{a^2}{b^4x+ab^3} + \frac{x}{b^2} - \frac{2\,a\log(bx+a)}{b^3}$$

input
```
integrate(x^6/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
-a^2/(b^4*x + a*b^3) + x/b^2 - 2*a*log(b*x + a)/b^3
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{x^6}{(ax^2+bx^3)^2}\,dx = \frac{x}{b^2} - \frac{2\,a\log(|bx+a|)}{b^3} - \frac{a^2}{(bx+a)b^3}$$

input
```
integrate(x^6/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
x/b^2 - 2*a*log(abs(b*x + a))/b^3 - a^2/((b*x + a)*b^3)
```

Mupad [B] (verification not implemented)

Time = 8.66 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{x^6}{(ax^2+bx^3)^2}\,dx = \frac{x}{b^2} - \frac{a^2}{x\,b^4+a\,b^3} - \frac{2\,a\,\ln(a+b\,x)}{b^3}$$

input
```
int(x^6/(a*x^2 + b*x^3)^2,x)
```

output
```
x/b^2 - a^2/(a*b^3 + b^4*x) - (2*a*log(a + b*x))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \frac{x^6}{(ax^2+bx^3)^2}\,dx = \frac{-2\log(bx+a)\,a^2-2\log(bx+a)\,abx+2abx+b^2x^2}{b^3\,(bx+a)}$$

input
```
int(x^6/(b*x^3+a*x^2)^2,x)
```

output
```
( - 2*log(a + b*x)*a**2 - 2*log(a + b*x)*a*b*x + 2*a*b*x + b**2*x**2)/(b**
3*(a + b*x))
```

3.204 $\int \frac{x^5}{(ax^2+bx^3)^2}\,dx$

Optimal result 1925
Mathematica [A] (verified) 1925
Rubi [A] (verified) 1926
Maple [A] (verified) 1927
Fricas [A] (verification not implemented) 1927
Sympy [A] (verification not implemented) 1928
Maxima [A] (verification not implemented) 1928
Giac [A] (verification not implemented) 1928
Mupad [B] (verification not implemented) 1929
Reduce [B] (verification not implemented) 1929

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{x^5}{(ax^2+bx^3)^2}\,dx = \frac{a}{b^2(a+bx)} + \frac{\log(a+bx)}{b^2}$$

output
```
a/b^2/(b*x+a)+ln(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{(ax^2+bx^3)^2}\,dx = \frac{\frac{a}{a+bx} + \log(a+bx)}{b^2}$$

input
```
Integrate[x^5/(a*x^2 + b*x^3)^2,x]
```

output
```
(a/(a + b*x) + Log[a + b*x])/b^2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(ax^2+bx^3)^2}dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{x}{(a+bx)^2}dx$$
$$\Big\downarrow \text{ 49}$$
$$\int \left(\frac{1}{b(a+bx)}-\frac{a}{b(a+bx)^2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}$$

input
```
Int[x^5/(a*x^2 + b*x^3)^2,x]
```

output
```
a/(b^2*(a + b*x)) + Log[a + b*x]/b^2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{a}{b^2(bx+a)}+\frac{\ln(bx+a)}{b^2}$	24
norman	$\frac{a}{b^2(bx+a)}+\frac{\ln(bx+a)}{b^2}$	24
risch	$\frac{a}{b^2(bx+a)}+\frac{\ln(bx+a)}{b^2}$	24
parallelrisch	$\frac{b\ln(bx+a)x+a\ln(bx+a)+a}{b^2(bx+a)}$	31

input
```
int(x^5/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
a/b^2/(b*x+a)+ln(b*x+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \frac{x^5}{(ax^2+bx^3)^2}\,dx = \frac{(bx+a)\log(bx+a)+a}{b^3x+ab^2}$$

input
```
integrate(x^5/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
((b*x + a)*log(b*x + a) + a)/(b^3*x + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{(ax^2+bx^3)^2}\,dx = \frac{a}{ab^2+b^3x} + \frac{\log(a+bx)}{b^2}$$

input
```
integrate(x**5/(b*x**3+a*x**2)**2,x)
```

output
```
a/(a*b**2 + b**3*x) + log(a + b*x)/b**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{x^5}{(ax^2+bx^3)^2}\,dx = \frac{a}{b^3x+ab^2} + \frac{\log(bx+a)}{b^2}$$

input
```
integrate(x^5/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
a/(b^3*x + a*b^2) + log(b*x + a)/b^2
```

Giac [A] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{x^5}{(ax^2+bx^3)^2}\,dx = \frac{\log(|bx+a|)}{b^2} + \frac{a}{(bx+a)b^2}$$

input
```
integrate(x^5/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
log(abs(b*x + a))/b^2 + a/((b*x + a)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(ax^2+bx^3)^2}\,dx = \frac{\ln(a+b\,x)}{b^2} + \frac{a}{b^2\,(a+b\,x)}$$

input
```
int(x^5/(a*x^2 + b*x^3)^2,x)
```

output
```
log(a + b*x)/b^2 + a/(b^2*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{x^5}{(ax^2+bx^3)^2}\,dx = \frac{\log(bx+a)\,a+\log(bx+a)\,bx-bx}{b^2\,(bx+a)}$$

input
```
int(x^5/(b*x^3+a*x^2)^2,x)
```

output
```
(log(a + b*x)*a + log(a + b*x)*b*x - b*x)/(b**2*(a + b*x))
```

3.205 $\int \frac{x^4}{(ax^2+bx^3)^2} \, dx$

Optimal result 1930
Mathematica [A] (verified) 1930
Rubi [A] (verified) 1931
Maple [A] (verified) 1932
Fricas [A] (verification not implemented) 1932
Sympy [A] (verification not implemented) 1933
Maxima [A] (verification not implemented) 1933
Giac [A] (verification not implemented) 1933
Mupad [B] (verification not implemented) 1934
Reduce [B] (verification not implemented) 1934

Optimal result

Integrand size = 17, antiderivative size = 12

$$\int \frac{x^4}{(ax^2+bx^3)^2} \, dx = -\frac{1}{b(a+bx)}$$

output
```
-1/b/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(ax^2+bx^3)^2} \, dx = -\frac{1}{b(a+bx)}$$

input
```
Integrate[x^4/(a*x^2 + b*x^3)^2,x]
```

output
```
-(1/(b*(a + b*x)))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {9, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(ax^2+bx^3)^2} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{1}{(a+bx)^2}dx$$

$$\Big\downarrow \; 17$$

$$-\frac{1}{b(a+bx)}$$

input
```
Int[x^4/(a*x^2 + b*x^3)^2,x]
```

output
```
-(1/(b*(a + b*x)))
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
gosper	$-\frac{1}{b(bx+a)}$	13
default	$-\frac{1}{b(bx+a)}$	13
norman	$\frac{x}{a(bx+a)}$	13
risch	$-\frac{1}{b(bx+a)}$	13
parallelrisch	$-\frac{1}{b(bx+a)}$	13
orering	$-\frac{(bx+a)x^4}{b(b\,x^3+a\,x^2)^2}$	27

input
```
int(x^4/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{x^4}{(ax^2+bx^3)^2}\,dx = -\frac{1}{b^2x+ab}$$

input
```
integrate(x^4/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
-1/(b^2*x + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{(ax^2+bx^3)^2}\,dx=-\frac{1}{ab+b^2x}$$

input
```
integrate(x**4/(b*x**3+a*x**2)**2,x)
```

output
```
-1/(a*b + b**2*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{x^4}{(ax^2+bx^3)^2}\,dx=-\frac{1}{b^2x+ab}$$

input
```
integrate(x^4/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
-1/(b^2*x + a*b)
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(ax^2+bx^3)^2}\,dx=-\frac{1}{(bx+a)b}$$

input
```
integrate(x^4/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
-1/((b*x + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(ax^2+bx^3)^2}\,dx = -\frac{1}{b\,(a+b\,x)}$$

input
```
int(x^4/(a*x^2 + b*x^3)^2,x)
```

output
```
-1/(b*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(ax^2+bx^3)^2}\,dx = \frac{x}{a\,(bx+a)}$$

input
```
int(x^4/(b*x^3+a*x^2)^2,x)
```

output
```
x/(a*(a + b*x))
```

3.206 $\int \frac{x^3}{(ax^2+bx^3)^2}\,dx$

Optimal result . . . 1935
Mathematica [A] (verified) . . . 1935
Rubi [A] (verified) . . . 1936
Maple [A] (verified) . . . 1937
Fricas [A] (verification not implemented) . . . 1937
Sympy [A] (verification not implemented) . . . 1938
Maxima [A] (verification not implemented) . . . 1938
Giac [A] (verification not implemented) . . . 1938
Mupad [B] (verification not implemented) . . . 1939
Reduce [B] (verification not implemented) . . . 1939

Optimal result

Integrand size = 17, antiderivative size = 29

$$\int \frac{x^3}{(ax^2+bx^3)^2}\,dx = \frac{1}{a(a+bx)}+\frac{\log(x)}{a^2}-\frac{\log(a+bx)}{a^2}$$

output
```
1/a/(b*x+a)+ln(x)/a^2-ln(b*x+a)/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{(ax^2+bx^3)^2}\,dx = \frac{\frac{a}{a+bx}+\log(x)-\log(a+bx)}{a^2}$$

input
```
Integrate[x^3/(a*x^2 + b*x^3)^2,x]
```

output
```
(a/(a + b*x) + Log[x] - Log[a + b*x])/a^2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(ax^2+bx^3)^2}\,dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{1}{x(a+bx)^2}dx$$

$$\Big\downarrow \ 54$$

$$\int \left(-\frac{b}{a^2(a+bx)}+\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\log(a+bx)}{a^2}+\frac{\log(x)}{a^2}+\frac{1}{a(a+bx)}$$

input
```
Int[x^3/(a*x^2 + b*x^3)^2,x]
```

output
```
1/(a*(a + b*x)) + Log[x]/a^2 - Log[a + b*x]/a^2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{1}{a(bx+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(bx+a)}{a^2}$	30
risch	$\frac{1}{a(bx+a)}-\frac{\ln(bx+a)}{a^2}+\frac{\ln(-x)}{a^2}$	32
norman	$-\frac{bx}{a^2(bx+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(bx+a)}{a^2}$	33
parallelrisch	$\frac{b\ln(x)x-b\ln(bx+a)x+a\ln(x)-a\ln(bx+a)-bx}{a^2(bx+a)}$	45

input
```
int(x^3/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a/(b*x+a)+ln(x)/a^2-ln(b*x+a)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{x^3}{(ax^2+bx^3)^2}\,dx=-\frac{(bx+a)\log(bx+a)-(bx+a)\log(x)-a}{a^2bx+a^3}$$

input
```
integrate(x^3/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
-((b*x + a)*log(b*x + a) - (b*x + a)*log(x) - a)/(a^2*b*x + a^3)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{(ax^2+bx^3)^2}\,dx = \frac{1}{a^2+abx} + \frac{\log(x) - \log\left(\frac{a}{b}+x\right)}{a^2}$$

input
```
integrate(x**3/(b*x**3+a*x**2)**2,x)
```

output
```
1/(a**2 + a*b*x) + (log(x) - log(a/b + x))/a**2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{x^3}{(ax^2+bx^3)^2}\,dx = \frac{1}{abx+a^2} - \frac{\log(bx+a)}{a^2} + \frac{\log(x)}{a^2}$$

input
```
integrate(x^3/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
1/(a*b*x + a^2) - log(b*x + a)/a^2 + log(x)/a^2
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{(ax^2+bx^3)^2}\,dx = -\frac{\log(|bx+a|)}{a^2} + \frac{\log(|x|)}{a^2} + \frac{1}{(bx+a)a}$$

input
```
integrate(x^3/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
-log(abs(b*x + a))/a^2 + log(abs(x))/a^2 + 1/((b*x + a)*a)
```

Mupad [B] (verification not implemented)

Time = 8.73 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{(ax^2+bx^3)^2}\,dx = \frac{1}{a^2+b\,x\,a} - \frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2}$$

input
```
int(x^3/(a*x^2 + b*x^3)^2,x)
```

output
```
1/(a^2 + a*b*x) - (2*atanh((2*b*x)/a + 1))/a^2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.52

$$\int \frac{x^3}{(ax^2+bx^3)^2}\,dx = \frac{-\log(bx+a)\,a-\log(bx+a)\,bx+\log(x)\,a+\log(x)\,bx-bx}{a^2\,(bx+a)}$$

input
```
int(x^3/(b*x^3+a*x^2)^2,x)
```

output
```
( - log(a + b*x)*a - log(a + b*x)*b*x + log(x)*a + log(x)*b*x - b*x)/(a**2
*(a + b*x))
```

3.207 $\int \frac{x^2}{\left(ax^2+bx^3\right)^2}\,dx$

Optimal result 1940
Mathematica [A] (verified) 1940
Rubi [A] (verified) 1941
Maple [A] (verified) 1942
Fricas [A] (verification not implemented) 1942
Sympy [A] (verification not implemented) 1943
Maxima [A] (verification not implemented) 1943
Giac [A] (verification not implemented) 1943
Mupad [B] (verification not implemented) 1944
Reduce [B] (verification not implemented) 1944

Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{x^2}{\left(ax^2+bx^3\right)^2}\,dx=-\frac{1}{a^2x}-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}$$

output
```
-1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{\left(ax^2+bx^3\right)^2}\,dx=-\frac{a\left(\frac{1}{x}+\frac{b}{a+bx}\right)+2b\log(x)-2b\log(a+bx)}{a^3}$$

input
```
Integrate[x^2/(a*x^2 + b*x^3)^2,x]
```

output
```
-((a*(x^(-1) + b/(a + b*x)) + 2*b*Log[x] - 2*b*Log[a + b*x])/a^3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax^2+bx^3)^2}\,dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{1}{x^2(a+bx)^2}dx$$
$$\Big\downarrow \text{ 54}$$
$$\int \left(\frac{2b^2}{a^3(a+bx)}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{1}{a^2x^2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{b}{a^2(a+bx)}-\frac{1}{a^2x}$$

input
```
Int[x^2/(a*x^2 + b*x^3)^2,x]
```

output
```
-(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x])/a
^3
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{1}{a^{2} x}-\frac{b}{a^{2} \left(b x +a \right)}-\frac{2 b \ln \left(x \right)}{a^{3}}+\frac{2 b \ln \left(b x +a \right)}{a^{3}}$	43
risch	$\frac{-\frac{2 b x}{a^{2}}-\frac{1}{a}}{x \left(b x +a \right)}-\frac{2 b \ln \left(x \right)}{a^{3}}+\frac{2 b \ln \left(-b x -a \right)}{a^{3}}$	49
norman	$\frac{\frac{2 b^{2} x^{4}}{a^{3}}-\frac{x^{2}}{a}}{x^{3} \left(b x +a \right)}-\frac{2 b \ln \left(x \right)}{a^{3}}+\frac{2 b \ln \left(b x +a \right)}{a^{3}}$	53
parallelrisch	$-\frac{2 b^{2} \ln \left(x \right) x^{2}-2 b^{2} \ln \left(b x +a \right) x^{2}+2 \ln \left(x \right) x a b -2 \ln \left(b x +a \right) x a b -2 b^{2} x^{2}+a^{2}}{a^{3} x \left(b x +a \right)}$	70

input
```
int(x^2/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.50

$$\int \frac{x^2}{\left(ax^2+bx^3\right)^2}\,dx=-\frac{2\,abx+a^{2}-2\,\left(b^{2}x^{2}+abx\right)\log\left(bx+a\right)+2\,\left(b^{2}x^{2}+abx\right)\log\left(x\right)}{a^{3}bx^{2}+a^{4}x}$$

input
```
integrate(x^2/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
-(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log(b*x + a) + 2*(b^2*x^2 + a*b*x)*l
og(x))/(a^3*b*x^2 + a^4*x)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(ax^2+bx^3)^2}\,dx = \frac{-a-2bx}{a^3x+a^2bx^2} + \frac{2b\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^3}$$

input
```
integrate(x**2/(b*x**3+a*x**2)**2,x)
```

output
```
(-a - 2*b*x)/(a**3*x + a**2*b*x**2) + 2*b*(-log(x) + log(a/b + x))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(ax^2+bx^3)^2}\,dx = -\frac{2\,bx+a}{a^2bx^2+a^3x} + \frac{2\,b\log(bx+a)}{a^3} - \frac{2\,b\log(x)}{a^3}$$

input
```
integrate(x^2/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
-(2*b*x + a)/(a^2*b*x^2 + a^3*x) + 2*b*log(b*x + a)/a^3 - 2*b*log(x)/a^3
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(ax^2+bx^3)^2}\,dx = \frac{2\,b\log(|bx+a|)}{a^3} - \frac{2\,b\log(|x|)}{a^3} - \frac{2\,bx+a}{(bx^2+ax)a^2}$$

input
```
integrate(x^2/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
2*b*log(abs(b*x + a))/a^3 - 2*b*log(abs(x))/a^3 - (2*b*x + a)/((b*x^2 + a*
x)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{x^2}{(ax^2+bx^3)^2}\,dx = \frac{4\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^3} - \frac{\frac{1}{a}+\frac{2\,b\,x}{a^2}}{b\,x^2+a\,x}$$

input
```
int(x^2/(a*x^2 + b*x^3)^2,x)
```

output
```
(4*b*atanh((2*b*x)/a + 1))/a^3 - (1/a + (2*b*x)/a^2)/(a*x + b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.67

$$\int \frac{x^2}{(ax^2+bx^3)^2}\,dx = \frac{2\log(bx+a)\,abx + 2\log(bx+a)\,b^2x^2 - 2\log(x)\,abx - 2\log(x)\,b^2x^2 - a^2 + 2b^2x^2}{a^3x\,(bx+a)}$$

input
```
int(x^2/(b*x^3+a*x^2)^2,x)
```

output
```
(2*log(a + b*x)*a*b*x + 2*log(a + b*x)*b**2*x**2 - 2*log(x)*a*b*x - 2*log(
x)*b**2*x**2 - a**2 + 2*b**2*x**2)/(a**3*x*(a + b*x))
```

3.208 $\int \frac{x}{\left(ax^2+bx^3\right)^2} dx$

Optimal result 1945
Mathematica [A] (verified) 1945
Rubi [A] (verified) 1946
Maple [A] (verified) 1947
Fricas [A] (verification not implemented) 1947
Sympy [A] (verification not implemented) 1948
Maxima [A] (verification not implemented) 1948
Giac [A] (verification not implemented) 1949
Mupad [B] (verification not implemented) 1949
Reduce [B] (verification not implemented) 1949

Optimal result

Integrand size = 15, antiderivative size = 58

$$\int \frac{x}{(ax^2+bx^3)^2} dx = -\frac{1}{2a^2x^2} + \frac{2b}{a^3x} + \frac{b^2}{a^3(a+bx)} + \frac{3b^2\log(x)}{a^4} - \frac{3b^2\log(a+bx)}{a^4}$$

output
```
-1/2/a^2/x^2+2*b/a^3/x+b^2/a^3/(b*x+a)+3*b^2*ln(x)/a^4-3*b^2*ln(b*x+a)/a^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{x}{(ax^2+bx^3)^2} dx = \frac{a\left(-\frac{a}{x^2}+\frac{4b}{x}+\frac{2b^2}{a+bx}\right)+6b^2\log(x)-6b^2\log(a+bx)}{2a^4}$$

input
```
Integrate[x/(a*x^2 + b*x^3)^2,x]
```

output
```
(a*(-(a/x^2) + (4*b)/x + (2*b^2)/(a + b*x)) + 6*b^2*Log[x] - 6*b^2*Log[a +
 b*x])/(2*a^4)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {9, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(ax^2+bx^3)^2}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{1}{x^3(a+bx)^2}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(-\frac{3b^3}{a^4(a+bx)}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{2b}{a^3x^2}+\frac{1}{a^2x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}+\frac{b^2}{a^3(a+bx)}+\frac{2b}{a^3x}-\frac{1}{2a^2x^2}$$

input
```
Int[x/(a*x^2 + b*x^3)^2,x]
```

output
```
-1/2*1/(a^2*x^2) + (2*b)/(a^3*x) + b^2/(a^3*(a + b*x)) + (3*b^2*Log[x])/a^
4 - (3*b^2*Log[a + b*x])/a^4
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{1}{2a^{2}x^{2}}+\frac{2b}{a^{3}x}+\frac{b^{2}}{a^{3}\left(bx+a\right)}+\frac{3b^{2}\ln\left(x\right)}{a^{4}}-\frac{3b^{2}\ln\left(bx+a\right)}{a^{4}}$	57
risch	$\frac{\frac{3b^{2}x^{2}}{a^{3}}+\frac{3bx}{2a^{2}}-\frac{1}{2a}}{x^{2}\left(bx+a\right)}-\frac{3b^{2}\ln\left(bx+a\right)}{a^{4}}+\frac{3b^{2}\ln\left(-x\right)}{a^{4}}$	63
norman	$\frac{-\frac{3b^{3}x^{4}}{a^{4}}-\frac{x}{2a}+\frac{3b\,x^{2}}{2a^{2}}}{x^{3}\left(bx+a\right)}+\frac{3b^{2}\ln\left(x\right)}{a^{4}}-\frac{3b^{2}\ln\left(bx+a\right)}{a^{4}}$	64
parallelrisch	$\frac{6b^{3}\ln\left(x\right)x^{3}-6b^{3}\ln\left(bx+a\right)x^{3}+6\ln\left(x\right)x^{2}a\,b^{2}-6\ln\left(bx+a\right)x^{2}a\,b^{2}-6b^{3}x^{3}+3a^{2}bx-a^{3}}{2a^{4}x^{2}\left(bx+a\right)}$	87

input
```
int(x/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2/x^2+2*b/a^3/x+b^2/a^3/(b*x+a)+3*b^2*ln(x)/a^4-3*b^2*ln(b*x+a)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.48

$$\int \frac{x}{\left(ax^2+bx^3\right)^2}\,dx = \frac{6\,ab^2x^2+3\,a^2bx-a^3-6\,\left(b^3x^3+ab^2x^2\right)\log\left(bx+a\right)+6\,\left(b^3x^3+ab^2x^2\right)\log\left(x\right)}{2\,\left(a^4bx^3+a^5x^2\right)}$$

input
```
integrate(x/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 - 6*(b^3*x^3 + a*b^2*x^2)*log(b*x + a)
+ 6*(b^3*x^3 + a*b^2*x^2)*log(x))/(a^4*b*x^3 + a^5*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{x}{(ax^2 + bx^3)^2} \, dx = \frac{-a^2 + 3abx + 6b^2x^2}{2a^4x^2 + 2a^3bx^3} + \frac{3b^2\left(\log(x) - \log\left(\frac{a}{b} + x\right)\right)}{a^4}$$

input
```
integrate(x/(b*x**3+a*x**2)**2,x)
```

output
```
(-a**2 + 3*a*b*x + 6*b**2*x**2)/(2*a**4*x**2 + 2*a**3*b*x**3) + 3*b**2*(lo
g(x) - log(a/b + x))/a**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

$$\int \frac{x}{(ax^2 + bx^3)^2} \, dx = \frac{6\,b^2x^2 + 3\,abx - a^2}{2\,(a^3bx^3 + a^4x^2)} - \frac{3\,b^2\log(bx + a)}{a^4} + \frac{3\,b^2\log(x)}{a^4}$$

input
```
integrate(x/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
1/2*(6*b^2*x^2 + 3*a*b*x - a^2)/(a^3*b*x^3 + a^4*x^2) - 3*b^2*log(b*x + a)
/a^4 + 3*b^2*log(x)/a^4
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

$$\int \frac{x}{(ax^2+bx^3)^2}\,dx = -\frac{3\,b^2\log(|bx+a|)}{a^4} + \frac{3\,b^2\log(|x|)}{a^4} + \frac{6\,ab^2x^2+3\,a^2bx-a^3}{2\,(bx+a)a^4x^2}$$

input
```
integrate(x/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
-3*b^2*log(abs(b*x + a))/a^4 + 3*b^2*log(abs(x))/a^4 + 1/2*(6*a*b^2*x^2 +
3*a^2*b*x - a^3)/((b*x + a)*a^4*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \frac{x}{(ax^2+bx^3)^2}\,dx = \frac{\frac{3\,b^2\,x^2}{a^3}-\frac{1}{2\,a}+\frac{3\,b\,x}{2\,a^2}}{b\,x^3+a\,x^2} - \frac{6\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^4}$$

input
```
int(x/(a*x^2 + b*x^3)^2,x)
```

output
```
((3*b^2*x^2)/a^3 - 1/(2*a) + (3*b*x)/(2*a^2))/(a*x^2 + b*x^3) - (6*b^2*ata
nh((2*b*x)/a + 1))/a^4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.48

$$\int \frac{x}{(ax^2+bx^3)^2}\,dx = \frac{-6\log(bx+a)\,a\,b^2x^2-6\log(bx+a)\,b^3x^3+6\log(x)\,a\,b^2x^2+6\log(x)\,b^3x^3-a^3+3a^2bx-6b^3x^3}{2a^4x^2\,(bx+a)}$$

input
```
int(x/(b*x^3+a*x^2)^2,x)
```

output
```
( - 6*log(a + b*x)*a*b**2*x**2 - 6*log(a + b*x)*b**3*x**3 + 6*log(x)*a*b**
2*x**2 + 6*log(x)*b**3*x**3 - a**3 + 3*a**2*b*x - 6*b**3*x**3)/(2*a**4*x**
2*(a + b*x))
```

3.209 $\int \frac{1}{\left(ax^2+bx^3\right)^2} \, dx$

Optimal result 1951
Mathematica [A] (verified) 1951
Rubi [A] (verified) 1952
Maple [A] (verified) 1953
Fricas [A] (verification not implemented) 1953
Sympy [A] (verification not implemented) 1954
Maxima [A] (verification not implemented) 1954
Giac [A] (verification not implemented) 1955
Mupad [B] (verification not implemented) 1955
Reduce [B] (verification not implemented) 1955

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{1}{\left(ax^2+bx^3\right)^2} \, dx = -\frac{1}{3a^2x^3}+\frac{b}{a^3x^2}-\frac{3b^2}{a^4x}-\frac{b^3}{a^4(a+bx)}-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}$$

output
```
-1/3/a^2/x^3+b/a^3/x^2-3*b^2/a^4/x-b^3/a^4/(b*x+a)-4*b^3*ln(x)/a^5+4*b^3*l
n(b*x+a)/a^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(ax^2+bx^3\right)^2} \, dx = -\frac{\frac{a(a^3-2a^2bx+6ab^2x^2+12b^3x^3)}{x^3(a+bx)}+12b^3\log(x)-12b^3\log(a+bx)}{3a^5}$$

input
```
Integrate[(a*x^2 + b*x^3)^(-2),x]
```

output
```
-1/3*((a*(a^3 - 2*a^2*b*x + 6*a*b^2*x^2 + 12*b^3*x^3))/(x^3*(a + b*x)) + 1
2*b^3*Log[x] - 12*b^3*Log[a + b*x])/a^5
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2026, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^2+bx^3)^2} \, dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{1}{x^4(a+bx)^2}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{4b^4}{a^5(a+bx)}-\frac{4b^3}{a^5x}+\frac{b^4}{a^4(a+bx)^2}+\frac{3b^2}{a^4x^2}-\frac{2b}{a^3x^3}+\frac{1}{a^2x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}-\frac{b^3}{a^4(a+bx)}-\frac{3b^2}{a^4x}+\frac{b}{a^3x^2}-\frac{1}{3a^2x^3}$$

input
```
Int[(a*x^2 + b*x^3)^(-2),x]
```

output
```
-1/3*1/(a^2*x^3) + b/(a^3*x^2) - (3*b^2)/(a^4*x) - b^3/(a^4*(a + b*x)) - (
4*b^3*Log[x])/a^5 + (4*b^3*Log[a + b*x])/a^5
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

method	result	size
default	$-\frac{1}{3a^2x^3}+\frac{b}{a^3x^2}-\frac{3b^2}{a^4x}-\frac{b^3}{a^4(bx+a)}-\frac{4b^3\ln(x)}{a^5}+\frac{4b^3\ln(bx+a)}{a^5}$	68
norman	$\frac{\frac{4b^4x^4}{a^5}-\frac{1}{3a}+\frac{2bx}{3a^2}-\frac{2b^2x^2}{a^3}}{x^3(bx+a)}-\frac{4b^3\ln(x)}{a^5}+\frac{4b^3\ln(bx+a)}{a^5}$	72
risch	$\frac{-\frac{4b^3x^3}{a^4}-\frac{2b^2x^2}{a^3}+\frac{2bx}{3a^2}-\frac{1}{3a}}{x^3(bx+a)}+\frac{4b^3\ln(-bx-a)}{a^5}-\frac{4b^3\ln(x)}{a^5}$	75
parallelrisch	$-\frac{12\ln(x)x^4b^4-12\ln(bx+a)x^4b^4+12\ln(x)x^3a\,b^3-12\ln(bx+a)x^3a\,b^3-12b^4x^4+6a^2b^2x^2-2a^3bx+a^4}{3a^5(bx+a)x^3}$	96

input
```
int(1/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^2/x^3+b/a^3/x^2-3*b^2/a^4/x-b^3/a^4/(b*x+a)-4*b^3*ln(x)/a^5+4*b^3*l
n(b*x+a)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.38

$$\int\frac{1}{(ax^2+bx^3)^2}\,dx=-\frac{12\,ab^3x^3+6\,a^2b^2x^2-2\,a^3bx+a^4-12\,(b^4x^4+ab^3x^3)\log(bx+a)+12\,(b^4x^4+ab^3x^3)\log(x)}{3\,(a^5bx^4+a^6x^3)}$$

input
```
integrate(1/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
-1/3*(12*a*b^3*x^3 + 6*a^2*b^2*x^2 - 2*a^3*b*x + a^4 - 12*(b^4*x^4 + a*b^3
*x^3)*log(b*x + a) + 12*(b^4*x^4 + a*b^3*x^3)*log(x))/(a^5*b*x^4 + a^6*x^3
)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{1}{(ax^2+bx^3)^2}\,dx = \frac{-a^3+2a^2bx-6ab^2x^2-12b^3x^3}{3a^5x^3+3a^4bx^4} + \frac{4b^3\left(-\log{(x)}+\log{\left(\frac{a}{b}+x\right)}\right)}{a^5}$$

input
```
integrate(1/(b*x**3+a*x**2)**2,x)
```

output
```
(-a**3 + 2*a**2*b*x - 6*a*b**2*x**2 - 12*b**3*x**3)/(3*a**5*x**3 + 3*a**4*
b*x**4) + 4*b**3*(-log(x) + log(a/b + x))/a**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ax^2+bx^3)^2}\,dx = -\frac{12\,b^3x^3+6\,ab^2x^2-2\,a^2bx+a^3}{3\,(a^4bx^4+a^5x^3)} + \frac{4\,b^3\log{(bx+a)}}{a^5} - \frac{4\,b^3\log{(x)}}{a^5}$$

input
```
integrate(1/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
-1/3*(12*b^3*x^3 + 6*a*b^2*x^2 - 2*a^2*b*x + a^3)/(a^4*b*x^4 + a^5*x^3) +
4*b^3*log(b*x + a)/a^5 - 4*b^3*log(x)/a^5
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ax^2+bx^3)^2}\,dx = \frac{4\,b^3\log\left(|bx+a|\right)}{a^5} - \frac{4\,b^3\log\left(|x|\right)}{a^5} - \frac{12\,ab^3x^3+6\,a^2b^2x^2-2\,a^3bx+a^4}{3\,(bx+a)a^5x^3}$$

input
```
integrate(1/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
4*b^3*log(abs(b*x + a))/a^5 - 4*b^3*log(abs(x))/a^5 - 1/3*(12*a*b^3*x^3 +
6*a^2*b^2*x^2 - 2*a^3*b*x + a^4)/((b*x + a)*a^5*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ax^2+bx^3)^2}\,dx = \frac{8\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5} - \frac{\frac{1}{3\,a}+\frac{2\,b^2\,x^2}{a^3}+\frac{4\,b^3\,x^3}{a^4}-\frac{2\,b\,x}{3\,a^2}}{b\,x^4+a\,x^3}$$

input
```
int(1/(a*x^2 + b*x^3)^2,x)
```

output
```
(8*b^3*atanh((2*b*x)/a + 1))/a^5 - (1/(3*a) + (2*b^2*x^2)/a^3 + (4*b^3*x^3
)/a^4 - (2*b*x)/(3*a^2))/(a*x^3 + b*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.41

$$\int \frac{1}{(ax^2+bx^3)^2}\,dx = \frac{12\log(bx+a)\,a\,b^3x^3+12\log(bx+a)\,b^4x^4-12\log(x)\,a\,b^3x^3-12\log(x)\,b^4x^4-a^4+2a^3bx-6a^2b^2x^2+}{3a^5x^3\,(bx+a)}$$

input
```
int(1/(b*x^3+a*x^2)^2,x)
```

output
```
(12*log(a + b*x)*a*b**3*x**3 + 12*log(a + b*x)*b**4*x**4 - 12*log(x)*a*b**
3*x**3 - 12*log(x)*b**4*x**4 - a**4 + 2*a**3*b*x - 6*a**2*b**2*x**2 + 12*b
**4*x**4)/(3*a**5*x**3*(a + b*x))
```

3.210 $\int \frac{1}{x(ax^2+bx^3)^2} dx$

Optimal result 1957
Mathematica [A] (verified) 1957
Rubi [A] (verified) 1958
Maple [A] (verified) 1959
Fricas [A] (verification not implemented) 1960
Sympy [A] (verification not implemented) 1960
Maxima [A] (verification not implemented) 1961
Giac [A] (verification not implemented) 1961
Mupad [B] (verification not implemented) 1962
Reduce [B] (verification not implemented) 1962

Optimal result

Integrand size = 17, antiderivative size = 84

$$\int \frac{1}{x\left(ax^2+bx^3\right)^2} dx = -\frac{1}{4a^2x^4}+\frac{2b}{3a^3x^3}-\frac{3b^2}{2a^4x^2}+\frac{4b^3}{a^5x} + \frac{b^4}{a^5(a+bx)}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}$$

output
```
-1/4/a^2/x^4+2/3*b/a^3/x^3-3/2*b^2/a^4/x^2+4*b^3/a^5/x+b^4/a^5/(b*x+a)+5*b
^4*ln(x)/a^6-5*b^4*ln(b*x+a)/a^6
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(ax^2+bx^3\right)^2} dx = \frac{\frac{a\left(-3a^4+5a^3bx-10a^2b^2x^2+30ab^3x^3+60b^4x^4\right)}{x^4(a+bx)}+60b^4\log(x)-60b^4\log(a+bx)}{12a^6}$$

input
```
Integrate[1/(x*(a*x^2 + b*x^3)^2),x]
```

output
```
((a*(-3*a^4 + 5*a^3*b*x - 10*a^2*b^2*x^2 + 30*a*b^3*x^3 + 60*b^4*x^4))/(x^
4*(a + b*x)) + 60*b^4*Log[x] - 60*b^4*Log[a + b*x])/(12*a^6)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(ax^2+bx^3)^2}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x^5(a+bx)^2}dx$$

$$\Big\downarrow \text{ 54}$$

$$\int \left(-\frac{5b^5}{a^6(a+bx)}+\frac{5b^4}{a^6x}-\frac{b^5}{a^5(a+bx)^2}-\frac{4b^3}{a^5x^2}+\frac{3b^2}{a^4x^3}-\frac{2b}{a^3x^4}+\frac{1}{a^2x^5}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}+\frac{b^4}{a^5(a+bx)}+\frac{4b^3}{a^5x}-\frac{3b^2}{2a^4x^2}+\frac{2b}{3a^3x^3}-\frac{1}{4a^2x^4}$$

input
```
Int[1/(x*(a*x^2 + b*x^3)^2),x]
```

output
```
-1/4*1/(a^2*x^4) + (2*b)/(3*a^3*x^3) - (3*b^2)/(2*a^4*x^2) + (4*b^3)/(a^5*
x) + b^4/(a^5*(a + b*x)) + (5*b^4*Log[x])/a^6 - (5*b^4*Log[a + b*x])/a^6
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{1}{4a^2x^4}+\frac{2b}{3a^3x^3}-\frac{3b^2}{2a^4x^2}+\frac{4b^3}{a^5x}+\frac{b^4}{a^5(bx+a)}+\frac{5b^4\ln(x)}{a^6}-\frac{5b^4\ln(bx+a)}{a^6}$	79
norman	$\frac{-\frac{5b^5x^5}{a^6}-\frac{1}{4a}+\frac{5bx}{12a^2}-\frac{5b^2x^2}{6a^3}+\frac{5b^3x^3}{2a^4}}{x^4(bx+a)}+\frac{5b^4\ln(x)}{a^6}-\frac{5b^4\ln(bx+a)}{a^6}$	83
risch	$\frac{\frac{5b^4x^4}{a^5}+\frac{5b^3x^3}{2a^4}-\frac{5b^2x^2}{6a^3}+\frac{5bx}{12a^2}-\frac{1}{4a}}{x^4(bx+a)}+\frac{5b^4\ln(-x)}{a^6}-\frac{5b^4\ln(bx+a)}{a^6}$	85
parallelrisch	$\frac{60\ln(x)x^5b^5-60\ln(bx+a)x^5b^5+60\ln(x)x^4ab^4-60\ln(bx+a)x^4ab^4-60b^5x^5+30a^2b^3x^3-10a^3b^2x^2+5a^4bx-3a^5}{12a^6(bx+a)x^4}$	109

input
```
int(1/x/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^2/x^4+2/3*b/a^3/x^3-3/2*b^2/a^4/x^2+4*b^3/a^5/x+b^4/a^5/(b*x+a)+5*b
^4*ln(x)/a^6-5*b^4*ln(b*x+a)/a^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.29

$$\int \frac{1}{x\left(ax^{2}+bx^{3}\right)^{2}}\,dx = \frac{60\,ab^{4}x^{4}+30\,a^{2}b^{3}x^{3}-10\,a^{3}b^{2}x^{2}+5\,a^{4}bx-3\,a^{5}-60\,(b^{5}x^{5}+ab^{4}x^{4})\log\left(bx+a\right)+60\,(b^{5}x^{5}+ab^{4}x^{4})\log}{12\,(a^{6}bx^{5}+a^{7}x^{4})}$$

input
```
integrate(1/x/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
1/12*(60*a*b^4*x^4 + 30*a^2*b^3*x^3 - 10*a^3*b^2*x^2 + 5*a^4*b*x - 3*a^5 -
 60*(b^5*x^5 + a*b^4*x^4)*log(b*x + a) + 60*(b^5*x^5 + a*b^4*x^4)*log(x))/
(a^6*b*x^5 + a^7*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

$$\int \frac{1}{x\left(ax^{2}+bx^{3}\right)^{2}}\,dx = \frac{-3a^{4}+5a^{3}bx-10a^{2}b^{2}x^{2}+30ab^{3}x^{3}+60b^{4}x^{4}}{12a^{6}x^{4}+12a^{5}bx^{5}} + \frac{5b^{4}\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^{6}}$$

input
```
integrate(1/x/(b*x**3+a*x**2)**2,x)
```

output
```
(-3*a**4 + 5*a**3*b*x - 10*a**2*b**2*x**2 + 30*a*b**3*x**3 + 60*b**4*x**4)
/(12*a**6*x**4 + 12*a**5*b*x**5) + 5*b**4*(log(x) - log(a/b + x))/a**6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\left(ax^2+bx^3\right)^2}\,dx = \frac{60\,b^4x^4+30\,ab^3x^3-10\,a^2b^2x^2+5\,a^3bx-3\,a^4}{12\,\left(a^5bx^5+a^6x^4\right)} - \frac{5\,b^4\log\left(bx+a\right)}{a^6} + \frac{5\,b^4\log\left(x\right)}{a^6}$$

input
```
integrate(1/x/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
1/12*(60*b^4*x^4 + 30*a*b^3*x^3 - 10*a^2*b^2*x^2 + 5*a^3*b*x - 3*a^4)/(a^5
*b*x^5 + a^6*x^4) - 5*b^4*log(b*x + a)/a^6 + 5*b^4*log(x)/a^6
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\left(ax^2+bx^3\right)^2}\,dx = -\frac{5\,b^4\log\left(|bx+a|\right)}{a^6} + \frac{5\,b^4\log\left(|x|\right)}{a^6} + \frac{60\,ab^4x^4+30\,a^2b^3x^3-10\,a^3b^2x^2+5\,a^4bx-3\,a^5}{12\,(bx+a)a^6x^4}$$

input
```
integrate(1/x/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
-5*b^4*log(abs(b*x + a))/a^6 + 5*b^4*log(abs(x))/a^6 + 1/12*(60*a*b^4*x^4
+ 30*a^2*b^3*x^3 - 10*a^3*b^2*x^2 + 5*a^4*b*x - 3*a^5)/((b*x + a)*a^6*x^4)
```

Mupad [B] (verification not implemented)

Time = 8.65 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(ax^2+bx^3\right)^2}\,dx=\frac{\frac{5\,b^3\,x^3}{2\,a^4}-\frac{5\,b^2\,x^2}{6\,a^3}-\frac{1}{4\,a}+\frac{5\,b^4\,x^4}{a^5}+\frac{5\,b\,x}{12\,a^2}}{b\,x^5+a\,x^4}-\frac{10\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^6}$$

input
```
int(1/(x*(a*x^2 + b*x^3)^2),x)
```

output
```
((5*b^3*x^3)/(2*a^4) - (5*b^2*x^2)/(6*a^3) - 1/(4*a) + (5*b^4*x^4)/a^5 + (
5*b*x)/(12*a^2))/(a*x^4 + b*x^5) - (10*b^4*atanh((2*b*x)/a + 1))/a^6
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.29

$$\int \frac{1}{x\left(ax^2+bx^3\right)^2}\,dx =\frac{-60\log(bx+a)\,a\,b^4x^4-60\log(bx+a)\,b^5x^5+60\log(x)\,a\,b^4x^4+60\log(x)\,b^5x^5-3a^5+5a^4bx-10a^3b^2x^2+30a^2b^3x^3-60b^5x^5}{12a^6x^4\,(bx+a)}$$

input
```
int(1/x/(b*x^3+a*x^2)^2,x)
```

output
```
( - 60*log(a + b*x)*a*b**4*x**4 - 60*log(a + b*x)*b**5*x**5 + 60*log(x)*a*
b**4*x**4 + 60*log(x)*b**5*x**5 - 3*a**5 + 5*a**4*b*x - 10*a**3*b**2*x**2
+ 30*a**2*b**3*x**3 - 60*b**5*x**5)/(12*a**6*x**4*(a + b*x))
```

3.211 $\int x^2\sqrt{ax^2+bx^3}\,dx$

Optimal result . . . 1963
Mathematica [A] (verified) . . . 1963
Rubi [A] (verified) . . . 1964
Maple [A] (verified) . . . 1965
Fricas [A] (verification not implemented) . . . 1966
Sympy [F] . . . 1967
Maxima [A] (verification not implemented) . . . 1967
Giac [A] (verification not implemented) . . . 1967
Mupad [B] (verification not implemented) . . . 1968
Reduce [B] (verification not implemented) . . . 1968

Optimal result

Integrand size = 19, antiderivative size = 105

$$\int x^2\sqrt{ax^2+bx^3}\,dx=\frac{2(ax^2+bx^3)^{3/2}}{9b}-\frac{32a^3(ax^2+bx^3)^{3/2}}{315b^4x^3}+\frac{16a^2(ax^2+bx^3)^{3/2}}{105b^3x^2}-\frac{4a(ax^2+bx^3)^{3/2}}{21b^2x}$$

output
```
2/9*(b*x^3+a*x^2)^(3/2)/b-32/315*a^3*(b*x^3+a*x^2)^(3/2)/b^4/x^3+16/105*a^
2*(b*x^3+a*x^2)^(3/2)/b^3/x^2-4/21*a*(b*x^3+a*x^2)^(3/2)/b^2/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.50

$$\int x^2\sqrt{ax^2+bx^3}\,dx=\frac{2(x^2(a+bx))^{3/2}\,(-16a^3+24a^2bx-30ab^2x^2+35b^3x^3)}{315b^4x^3}$$

input
```
Integrate[x^2*Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*(x^2*(a + b*x))^(3/2)*(-16*a^3 + 24*a^2*b*x - 30*a*b^2*x^2 + 35*b^3*x^3
))/(315*b^4*x^3)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1922, 1922, 1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{ax^2+bx^3}\,dx$$

$$\Big\downarrow \ 1922$$

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}-\frac{2a\int x\sqrt{bx^3+ax^2}dx}{3b}$$

$$\Big\downarrow \ 1922$$

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}-\frac{2a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}-\frac{4a\int\sqrt{bx^3+ax^2}dx}{7b}\right)}{3b}$$

$$\Big\downarrow \ 1908$$

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}-\frac{2a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{2a\int\frac{\sqrt{bx^3+ax^2}}{x}dx}{5b}\right)}{7b}\right)}{3b}$$

$$\Big\downarrow \ 1920$$

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}-\frac{2a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{4a\left(ax^2+bx^3\right)^{3/2}}{15b^2x^3}\right)}{7b}\right)}{3b}$$

input
```
Int[x^2*Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*(a*x^2 + b*x^3)^(3/2))/(9*b) - (2*a*((2*(a*x^2 + b*x^3)^(3/2))/(7*b*x)
- (4*a*((-4*a*(a*x^2 + b*x^3)^(3/2))/(15*b^2*x^3) + (2*(a*x^2 + b*x^3)^(3/
2))/(5*b*x^2)))/(7*b)))/(3*b)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.30

method	result	size
pseudoelliptic	$\frac{2(bx+a)^{\frac{3}{2}}(15b^2x^2-12abx+8a^2)}{105b^3}$	32
gosper	$-\frac{2(bx+a)(-35b^3x^3+30a\,b^2x^2-24a^2bx+16a^3)\sqrt{b\,x^3+a\,x^2}}{315b^4x}$	57
default	$-\frac{2(bx+a)(-35b^3x^3+30a\,b^2x^2-24a^2bx+16a^3)\sqrt{b\,x^3+a\,x^2}}{315b^4x}$	57
orering	$-\frac{2(bx+a)(-35b^3x^3+30a\,b^2x^2-24a^2bx+16a^3)\sqrt{b\,x^3+a\,x^2}}{315b^4x}$	57
risch	$-\frac{2\sqrt{x^2(bx+a)}\,(-35b^4x^4-5a\,b^3x^3+6a^2b^2x^2-8a^3bx+16a^4)}{315x\,b^4}$	61
trager	$-\frac{2(-35b^4x^4-5a\,b^3x^3+6a^2b^2x^2-8a^3bx+16a^4)\sqrt{b\,x^3+a\,x^2}}{315b^4x}$	63

input
```
int(x^2*(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/105*(b*x+a)^(3/2)*(15*b^2*x^2-12*a*b*x+8*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.59

$$\int x^2\sqrt{ax^2+bx^3}\,dx=\frac{2\,(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4)\sqrt{bx^3+ax^2}}{315\,b^4x}$$

input
```
integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt
(b*x^3 + a*x^2)/(b^4*x)
```

Sympy [F]

$$\int x^2\sqrt{ax^2+bx^3}\,dx=\int x^2\sqrt{x^2\left(a+bx\right)}\,dx$$

input
```
integrate(x**2*(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**2*sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.50

$$\int x^2\sqrt{ax^2+bx^3}\,dx=\frac{2\,(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4)\sqrt{bx+a}}{315\,b^4}$$

input
```
integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt
(b*x + a)/b^4
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.25

$$\int x^2\sqrt{ax^2+bx^3}\,dx=\frac{32\,a^{\frac{9}{2}}\mathrm{sgn}(x)}{315\,b^4}+\frac{2\left(\frac{9\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)a\mathrm{sgn}(x)}{b^3}+\frac{\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a\right.}{b^3}\right.}{315\,b}$$

input
```
integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
32/315*a^(9/2)*sgn(x)/b^4 + 2/315*(9*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/
2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a*sgn(x)/b^3 + (35*(
b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*
x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*sgn(x)/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 8.70 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.59

$$\int x^2\sqrt{ax^2+bx^3}\,dx = \frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(-16\,a^4+8\,a^3\,b\,x-6\,a^2\,b^2\,x^2+5\,a\,b^3\,x^3+35\,b^4\,x^4\right)}{315\,b^4\,x}$$

input
```
int(x^2*(a*x^2 + b*x^3)^(1/2),x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(35*b^4*x^4 - 16*a^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^
2 + 8*a^3*b*x))/(315*b^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.50

$$\int x^2\sqrt{ax^2+bx^3}\,dx = \frac{2\sqrt{bx+a}\,(35b^4x^4+5a\,b^3x^3-6a^2b^2x^2+8a^3bx-16a^4)}{315b^4}$$

input
```
int(x^2*(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 16*a**4 + 8*a**3*b*x - 6*a**2*b**2*x**2 + 5*a*b**3*x*
*3 + 35*b**4*x**4))/(315*b**4)
```

3.212 $\int x\sqrt{ax^2+bx^3}\,dx$

Optimal result 1969
Mathematica [A] (verified) 1969
Rubi [A] (verified) 1970
Maple [A] (verified) 1971
Fricas [A] (verification not implemented) 1972
Sympy [F] 1972
Maxima [A] (verification not implemented) 1972
Giac [A] (verification not implemented) 1973
Mupad [B] (verification not implemented) 1973
Reduce [B] (verification not implemented) 1974

Optimal result

Integrand size = 17, antiderivative size = 80

$$\int x\sqrt{ax^2+bx^3}\,dx=\frac{16a^2(ax^2+bx^3)^{3/2}}{105b^3x^3}-\frac{8a(ax^2+bx^3)^{3/2}}{35b^2x^2}+\frac{2(ax^2+bx^3)^{3/2}}{7bx}$$

output
```
16/105*a^2*(b*x^3+a*x^2)^(3/2)/b^3/x^3-8/35*a*(b*x^3+a*x^2)^(3/2)/b^2/x^2+
2/7*(b*x^3+a*x^2)^(3/2)/b/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int x\sqrt{ax^2+bx^3}\,dx=\frac{2(x^2(a+bx))^{3/2}\,(8a^2-12abx+15b^2x^2)}{105b^3x^3}$$

input
```
Integrate[x*Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*(x^2*(a + b*x))^(3/2)*(8*a^2 - 12*a*b*x + 15*b^2*x^2))/(105*b^3*x^3)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1922, 1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{ax^2+bx^3}\,dx$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2(ax^2+bx^3)^{3/2}}{7bx}-\frac{4a\int \sqrt{bx^3+ax^2}dx}{7b}$$

$$\Big\downarrow \text{ 1908}$$

$$\frac{2(ax^2+bx^3)^{3/2}}{7bx}-\frac{4a\left(\frac{2(ax^2+bx^3)^{3/2}}{5bx^2}-\frac{2a\int \frac{\sqrt{bx^3+ax^2}}{x}dx}{5b}\right)}{7b}$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{2(ax^2+bx^3)^{3/2}}{7bx}-\frac{4a\left(\frac{2(ax^2+bx^3)^{3/2}}{5bx^2}-\frac{4a(ax^2+bx^3)^{3/2}}{15b^2x^3}\right)}{7b}$$

input
```
Int[x*Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*(a*x^2 + b*x^3)^(3/2))/(7*b*x) - (4*a*((-4*a*(a*x^2 + b*x^3)^(3/2))/(15
*b^2*x^3) + (2*(a*x^2 + b*x^3)^(3/2))/(5*b*x^2)))/(7*b)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.26

method	result	size
pseudoelliptic	$-\frac{2(bx+a)^{\frac{3}{2}}(-3bx+2a)}{15b^2}$	21
gosper	$\frac{2(bx+a)(15b^2x^2-12abx+8a^2)\sqrt{b\,x^3+a\,x^2}}{105b^3x}$	46
default	$\frac{2(bx+a)(15b^2x^2-12abx+8a^2)\sqrt{b\,x^3+a\,x^2}}{105b^3x}$	46
orering	$\frac{2(bx+a)(15b^2x^2-12abx+8a^2)\sqrt{b\,x^3+a\,x^2}}{105b^3x}$	46
risch	$\frac{2\sqrt{x^2(bx+a)}\,(15b^3x^3+3a\,b^2x^2-4a^2bx+8a^3)}{105x\,b^3}$	50
trager	$\frac{2(15b^3x^3+3a\,b^2x^2-4a^2bx+8a^3)\sqrt{b\,x^3+a\,x^2}}{105b^3x}$	52

input
```
int(x*(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(b*x+a)^(3/2)*(-3*b*x+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.64

$$\int x\sqrt{ax^2+bx^3}\,dx = \frac{2\,(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3)\sqrt{bx^3+ax^2}}{105\,b^3x}$$

input
```
integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2)/(
b^3*x)
```

Sympy [F]

$$\int x\sqrt{ax^2+bx^3}\,dx = \int x\sqrt{x^2\,(a+bx)}\,dx$$

input
```
integrate(x*(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x*sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int x\sqrt{ax^2+bx^3}\,dx = \frac{2\,(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3)\sqrt{bx+a}}{105\,b^3}$$

input
```
integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.35

$$\int x\sqrt{ax^2+bx^3}\,dx = -\frac{16\,a^{\frac{7}{2}}\mathrm{sgn}(x)}{105\,b^3} + \frac{2\left(\frac{7\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)a\mathrm{sgn}(x)}{b^2}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)\mathrm{sgn}(x)}{b^2}\right)}{105\,b}$$

input
```
integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-16/105*a^(7/2)*sgn(x)/b^3 + 2/105*(7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3
/2)*a + 15*sqrt(b*x + a)*a^2)*a*sgn(x)/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*
x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*sgn(x)/b^2
)/b
```

Mupad [B] (verification not implemented)

Time = 8.65 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.64

$$\int x\sqrt{ax^2+bx^3}\,dx = \frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(8\,a^3-4\,a^2\,b\,x+3\,a\,b^2\,x^2+15\,b^3\,x^3\right)}{105\,b^3\,x}$$

input
```
int(x*(a*x^2 + b*x^3)^(1/2),x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(8*a^3 + 15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x))/(
105*b^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.51

$$\int x\sqrt{ax^2+bx^3}\,dx=\frac{2\sqrt{bx+a}\,(15b^3x^3+3a\,b^2x^2-4a^2bx+8a^3)}{105b^3}$$

input

```
int(x*(b*x^3+a*x^2)^(1/2),x)
```

output

```
(2*sqrt(a + b*x)*(8*a**3 - 4*a**2*b*x + 3*a*b**2*x**2 + 15*b**3*x**3))/(10
5*b**3)
```

3.213 $\int \sqrt{ax^2+bx^3}\,dx$

Optimal result . . . 1975
Mathematica [A] (verified) . . . 1975
Rubi [A] (verified) . . . 1976
Maple [A] (verified) . . . 1977
Fricas [A] (verification not implemented) . . . 1977
Sympy [F] . . . 1978
Maxima [A] (verification not implemented) . . . 1978
Giac [A] (verification not implemented) . . . 1978
Mupad [B] (verification not implemented) . . . 1979
Reduce [B] (verification not implemented) . . . 1979

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \sqrt{ax^2+bx^3}\,dx=-\frac{4a(ax^2+bx^3)^{3/2}}{15b^2x^3}+\frac{2(ax^2+bx^3)^{3/2}}{5bx^2}$$

output
```
-4/15*a*(b*x^3+a*x^2)^(3/2)/b^2/x^3+2/5*(b*x^3+a*x^2)^(3/2)/b/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int \sqrt{ax^2+bx^3}\,dx=\frac{2\sqrt{x^2(a+bx)}(-2a^2+abx+3b^2x^2)}{15b^2x}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*Sqrt[x^2*(a + b*x)]*(-2*a^2 + a*b*x + 3*b^2*x^2))/(15*b^2*x)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ax^2+bx^3}\,dx$$
$$\Big\downarrow \text{1908}$$
$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{2a\int \frac{\sqrt{bx^3+ax^2}}{x}dx}{5b}$$
$$\Big\downarrow \text{1920}$$
$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{4a\left(ax^2+bx^3\right)^{3/2}}{15b^2x^3}$$

input
```
Int[Sqrt[a*x^2 + b*x^3],x]
```

output
```
(-4*a*(a*x^2 + b*x^3)^(3/2))/(15*b^2*x^3) + (2*(a*x^2 + b*x^3)^(3/2))/(5*b
*x^2)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.25

method	result	size
pseudoelliptic	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
gosper	$-\frac{2(bx+a)(-3bx+2a)\sqrt{b\,x^3+a\,x^2}}{15b^2x}$	35
default	$-\frac{2(bx+a)(-3bx+2a)\sqrt{b\,x^3+a\,x^2}}{15b^2x}$	35
orering	$-\frac{2(bx+a)(-3bx+2a)\sqrt{b\,x^3+a\,x^2}}{15b^2x}$	35
risch	$-\frac{2\sqrt{x^2(bx+a)}\,(-3b^2x^2-abx+2a^2)}{15x\,b^2}$	39
trager	$-\frac{2(-3b^2x^2-abx+2a^2)\sqrt{b\,x^3+a\,x^2}}{15b^2x}$	41

input
```
int((b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(b*x+a)^(3/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

$$\int \sqrt{ax^2+bx^3}\,dx = \frac{2\,(3\,b^2x^2+abx-2\,a^2)\sqrt{bx^3+ax^2}}{15\,b^2x}$$

input
```
integrate((b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x^3 + a*x^2)/(b^2*x)
```

Sympy [F]

$$\int \sqrt{ax^2 + bx^3}\,dx = \int \sqrt{ax^2 + bx^3}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(sqrt(a*x**2 + b*x**3), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.58

$$\int \sqrt{ax^2 + bx^3}\,dx = \frac{2\,(3\,b^2x^2 + abx - 2\,a^2)\sqrt{bx + a}}{15\,b^2}$$

input
```
integrate((b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.56

$$\int \sqrt{ax^2 + bx^3}\,dx$$
$$= \frac{4\,a^{\frac{5}{2}}\mathrm{sgn}(x)}{15\,b^2} + \frac{2\left(\frac{5\left((bx+a)^{\frac{3}{2}} - 3\,\sqrt{bx+a}a\right)a\mathrm{sgn}(x)}{b} + \frac{\left(3\,(bx+a)^{\frac{5}{2}} - 10\,(bx+a)^{\frac{3}{2}}a + 15\,\sqrt{bx+a}a^2\right)\mathrm{sgn}(x)}{b}\right)}{15\,b}$$

input
```
integrate((b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
4/15*a^(5/2)*sgn(x)/b^2 + 2/15*(5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a*
sgn(x)/b + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^
2)*sgn(x)/b)/b
```

Mupad [B] (verification not implemented)

Time = 8.79 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

$$\int \sqrt{ax^2+bx^3}\,dx=\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(-2\,a^2+a\,b\,x+3\,b^2\,x^2\right)}{15\,b^2\,x}$$

input
```
int((a*x^2 + b*x^3)^(1/2),x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(3*b^2*x^2 - 2*a^2 + a*b*x))/(15*b^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.56

$$\int \sqrt{ax^2+bx^3}\,dx=\frac{2\sqrt{bx+a}\,(3b^2x^2+abx-2a^2)}{15b^2}$$

input
```
int((b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 2*a**2 + a*b*x + 3*b**2*x**2))/(15*b**2)
```

3.214 $\int \frac{\sqrt{ax^2+bx^3}}{x} \, dx$

Optimal result 1980
Mathematica [A] (verified) 1980
Rubi [A] (verified) 1981
Maple [A] (verified) 1981
Fricas [A] (verification not implemented) 1982
Sympy [F] 1982
Maxima [A] (verification not implemented) 1983
Giac [B] (verification not implemented) 1983
Mupad [F(-1)] 1984
Reduce [B] (verification not implemented) 1984

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{\sqrt{ax^2+bx^3}}{x} \, dx = \frac{2(ax^2+bx^3)^{3/2}}{3bx^3}$$

output
```
2/3*(b*x^3+a*x^2)^(3/2)/b/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{ax^2+bx^3}}{x} \, dx = \frac{2(x^2(a+bx))^{3/2}}{3bx^3}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3]/x,x]
```

output
```
(2*(x^2*(a + b*x))^(3/2))/(3*b*x^3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}}{x} \, dx$$

$$\Big\downarrow \; 1920$$

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{3bx^3}$$

input
```
Int[Sqrt[a*x^2 + b*x^3]/x,x]
```

output
```
(2*(a*x^2 + b*x^3)^(3/2))/(3*b*x^3)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{2\sqrt{x^2(bx+a)}\,(bx+a)}{3xb}$	25
gosper	$\frac{2(bx+a)\sqrt{b\,x^3+a\,x^2}}{3bx}$	27
default	$\frac{2(bx+a)\sqrt{b\,x^3+a\,x^2}}{3bx}$	27
trager	$\frac{2(bx+a)\sqrt{b\,x^3+a\,x^2}}{3bx}$	27
orering	$\frac{2(bx+a)\sqrt{b\,x^3+a\,x^2}}{3bx}$	27
pseudoelliptic	$2\sqrt{bx+a}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)$	28

input
```
int((b*x^3+a*x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/3*(x^2*(b*x+a))^(1/2)/x*(b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx=\frac{2\,\sqrt{bx^3+ax^2}(bx+a)}{3\,bx}$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
2/3*sqrt(b*x^3 + a*x^2)*(b*x + a)/(b*x)
```

Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx=\int \frac{\sqrt{x^2\,(a+bx)}}{x}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(1/2)/x,x)
```

output
```
Integral(sqrt(x**2*(a + b*x))/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{ax^2 + bx^3}}{x}\,dx = \frac{2\,(bx + a)^{\frac{3}{2}}}{3\,b}$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
2/3*(b*x + a)^(3/2)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(21) = 42.

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{ax^2 + bx^3}}{x}\,dx = -\frac{2\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,b} + \frac{2\left(3\,\sqrt{bx + a}a\mathrm{sgn}(x) + \left((bx + a)^{\frac{3}{2}} - 3\,\sqrt{bx + a}a\right)\mathrm{sgn}(x)\right)}{3\,b}$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="giac")
```

output
```
-2/3*a^(3/2)*sgn(x)/b + 2/3*(3*sqrt(b*x + a)*a*sgn(x) + ((b*x + a)^(3/2) -
 3*sqrt(b*x + a)*a)*sgn(x))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx=\int \frac{\sqrt{b\,x^3+a\,x^2}}{x}\,dx$$

input
```
int((a*x^2 + b*x^3)^(1/2)/x,x)
```

output
```
int((a*x^2 + b*x^3)^(1/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx=\frac{2\sqrt{bx+a}\,(bx+a)}{3b}$$

input
```
int((b*x^3+a*x^2)^(1/2)/x,x)
```

output
```
(2*sqrt(a + b*x)*(a + b*x))/(3*b)
```

3.215 $\int \frac{\sqrt{ax^2+bx^3}}{x^2} dx$

Optimal result . . . 1985
Mathematica [A] (verified) . . . 1985
Rubi [A] (verified) . . . 1986
Maple [A] (verified) . . . 1987
Fricas [A] (verification not implemented) . . . 1987
Sympy [F] . . . 1988
Maxima [F] . . . 1988
Giac [A] (verification not implemented) . . . 1988
Mupad [B] (verification not implemented) . . . 1989
Reduce [B] (verification not implemented) . . . 1989

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2} dx = \frac{2\sqrt{ax^2+bx^3}}{x} - 2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)$$

output
```
2*(b*x^3+a*x^2)^(1/2)/x-2*a^(1/2)*arctanh(a^(1/2)*x/(b*x^3+a*x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2} dx = \frac{2x\left(a+bx-\sqrt{a}\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{x^2(a+bx)}}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3]/x^2,x]
```

output
```
(2*x*(a + b*x - Sqrt[a]*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/Sqr
t[x^2*(a + b*x)]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1927, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}dx$$

$$\Big\downarrow \text{1927}$$

$$a\int \frac{1}{\sqrt{bx^3+ax^2}}dx+\frac{2\sqrt{ax^2+bx^3}}{x}$$

$$\Big\downarrow \text{1914}$$

$$\frac{2\sqrt{ax^2+bx^3}}{x}-2a\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)$$

input
```
Int[Sqrt[a*x^2 + b*x^3]/x^2,x]
```

output
```
(2*Sqrt[a*x^2 + b*x^3])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x
^3]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
pseudoelliptic	$b\left(-\frac{\sqrt{bx+a}}{bx}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)$	36
default	$-\frac{2\sqrt{b\,x^{3}+a\,x^{2}}\,\left(\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-\sqrt{bx+a}\right)}{x\sqrt{bx+a}}$	52

input
```
int((b*x^3+a*x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
b*(-(b*x+a)^(1/2)/b/x-1/a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.27

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx = \left[\frac{\sqrt{a}x\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}}{x}, \frac{2\left(\sqrt{-a}x\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+\sqrt{bx^3+ax^2}\right)}{x}\right]$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[(sqrt(a)*x*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*s
qrt(b*x^3 + a*x^2))/x, 2*(sqrt(-a)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(
b*x^2 + a*x)) + sqrt(b*x^3 + a*x^2))/x]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx=\int \frac{\sqrt{x^2\,(a+bx)}}{x^2}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(x**2*(a + b*x))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx=\int \frac{\sqrt{bx^3+ax^2}}{x^2}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x^2)/x^2, x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx=\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}}+2\,\sqrt{bx+a}\operatorname{sgn}(x)-\frac{2\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}\right)\operatorname{sgn}(x)}{\sqrt{-a}}$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
2*a*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*sqrt(b*x + a)*sgn(x
) - 2*(a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)
```

Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx=\frac{2\,\sqrt{b\,x^3+a\,x^2}}{x}+\frac{\sqrt{a}\,\mathrm{asin}\left(\frac{\sqrt{a}\,\sqrt{\frac{1}{x}}\,1\mathrm{i}}{\sqrt{b}}\right)\,\sqrt{b\,x^3+a\,x^2}\,{\left(\frac{1}{x}\right)}^{3/2}\,2\mathrm{i}}{\sqrt{b}\,\sqrt{\frac{a}{b\,x}+1}}$$

input
```
int((a*x^2 + b*x^3)^(1/2)/x^2,x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2))/x + (a^(1/2)*asin((a^(1/2)*(1/x)^(1/2)*1i)/b^(1/
2))*(a*x^2 + b*x^3)^(1/2)*(1/x)^(3/2)*2i)/(b^(1/2)*(a/(b*x) + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx=2\sqrt{bx+a}+\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)-\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)$$

input
```
int((b*x^3+a*x^2)^(1/2)/x^2,x)
```

output
```
2*sqrt(a + b*x) + sqrt(a)*log(sqrt(a + b*x) - sqrt(a)) - sqrt(a)*log(sqrt(
a + b*x) + sqrt(a))
```

3.216 $\int \frac{\sqrt{ax^2+bx^3}}{x^3} dx$

Optimal result . . . 1990
Mathematica [A] (verified) . . . 1990
Rubi [A] (verified) . . . 1991
Maple [A] (verified) . . . 1992
Fricas [A] (verification not implemented) . . . 1992
Sympy [F] . . . 1993
Maxima [F] . . . 1993
Giac [A] (verification not implemented) . . . 1994
Mupad [F(-1)] . . . 1994
Reduce [B] (verification not implemented) . . . 1994

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{\sqrt{ax^2+bx^3}}{x^3} dx = -\frac{\sqrt{ax^2+bx^3}}{x^2} - \frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

output
```
-(b*x^3+a*x^2)^(1/2)/x^2-b*arctanh(a^(1/2)*x/(b*x^3+a*x^2)^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{ax^2+bx^3}}{x^3} dx = -\frac{\sqrt{a+bx}\left(\sqrt{a}\sqrt{a+bx}+bx\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{a}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3]/x^3,x]
```

output
```
-((Sqrt[a + b*x]*(Sqrt[a]*Sqrt[a + b*x] + b*x*ArcTanh[Sqrt[a + b*x]/Sqrt[a
]]))/(Sqrt[a]*Sqrt[x^2*(a + b*x)]))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1926, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx$$

$$\Big\downarrow \; 1926$$

$$\frac{1}{2}b\int \frac{1}{\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{x^2}$$

$$\Big\downarrow \; 1914$$

$$-b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}-\frac{\sqrt{ax^2+bx^3}}{x^2}$$

$$\Big\downarrow \; 219$$

$$-\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}-\frac{\sqrt{ax^2+bx^3}}{x^2}$$

input
```
Int[Sqrt[a*x^2 + b*x^3]/x^3,x]
```

output
```
-(Sqrt[a*x^2 + b*x^3]/x^2) - (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/
Sqrt[a]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{2}x^{2}-\left(2a^{\frac{3}{2}}+bx\sqrt{a}\right)\sqrt{bx+a}}{4a^{\frac{3}{2}}x^{2}}$	50
default	$-\frac{\sqrt{b\,x^{3}+a\,x^{2}}\,\left(\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)bx+\sqrt{bx+a}\,\sqrt{a}\right)}{x^{2}\sqrt{bx+a}\,\sqrt{a}}$	56
risch	$-\frac{\sqrt{x^{2}(bx+a)}}{x^{2}}-\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{x^{2}(bx+a)}}{\sqrt{a}\,x\sqrt{bx+a}}$	57

input
```
int((b*x^3+a*x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/a^(3/2)*(arctanh((b*x+a)^(1/2)/a^(1/2))*b^2*x^2-(2*a^(3/2)+b*x*a^(1/2)
)*(b*x+a)^(1/2))/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.54

$$\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx$$
$$=\left[\frac{\sqrt{a}bx^{2}\log\left(\frac{bx^{2}+2\,ax-2\,\sqrt{bx^{3}+ax^{2}}\sqrt{a}}{x^{2}}\right)-2\,\sqrt{bx^{3}+ax^{2}}a}{2\,ax^{2}},\frac{\sqrt{-a}bx^{2}\arctan\left(\frac{\sqrt{bx^{3}+ax^{2}}\sqrt{-a}}{bx^{2}+ax}\right)-\sqrt{bx^{3}+ax^{2}}a}{ax^{2}}\right]$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*b*x^2*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^
2) - 2*sqrt(b*x^3 + a*x^2)*a)/(a*x^2), (sqrt(-a)*b*x^2*arctan(sqrt(b*x^3 +
 a*x^2)*sqrt(-a)/(b*x^2 + a*x)) - sqrt(b*x^3 + a*x^2)*a)/(a*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2 + bx^3}}{x^3}\,dx = \int \frac{\sqrt{x^2\,(a + bx)}}{x^3}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(x**2*(a + b*x))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2 + bx^3}}{x^3}\,dx = \int \frac{\sqrt{bx^3 + ax^2}}{x^3}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x^2)/x^3, x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{ax^2 + bx^3}}{x^3}\, dx = \left(\frac{\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}} - \frac{\sqrt{bx + a}\mathrm{sgn}(x)}{bx}\right)b$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="giac")
```

output
```
(arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) - sqrt(b*x + a)*sgn(x)/(b*
x))*b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2 + bx^3}}{x^3}\, dx = \int \frac{\sqrt{b\,x^3 + a\,x^2}}{x^3}\, dx$$

input
```
int((a*x^2 + b*x^3)^(1/2)/x^3,x)
```

output
```
int((a*x^2 + b*x^3)^(1/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{ax^2 + bx^3}}{x^3}\, dx = \frac{-2\sqrt{bx + a}\,a + \sqrt{a}\log\left(\sqrt{bx + a} - \sqrt{a}\right)bx - \sqrt{a}\log\left(\sqrt{bx + a} + \sqrt{a}\right)bx}{2ax}$$

input
```
int((b*x^3+a*x^2)^(1/2)/x^3,x)
```

output
```
( - 2*sqrt(a + b*x)*a + sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b*x - sqrt(a)
*log(sqrt(a + b*x) + sqrt(a))*b*x)/(2*a*x)
```

3.217 $\int \frac{\sqrt{ax^2+bx^3}}{x^4} dx$

Optimal result 1996
Mathematica [A] (verified) 1996
Rubi [A] (verified) 1997
Maple [A] (verified) 1998
Fricas [A] (verification not implemented) 1999
Sympy [F] 1999
Maxima [F] 2000
Giac [A] (verification not implemented) 2000
Mupad [F(-1)] 2000
Reduce [B] (verification not implemented) 2001

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4} dx = -\frac{\sqrt{ax^2+bx^3}}{2x^3} - \frac{b\sqrt{ax^2+bx^3}}{4ax^2} + \frac{b^2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4a^{3/2}}$$

output
```
-1/2*(b*x^3+a*x^2)^(1/2)/x^3-1/4*b*(b*x^3+a*x^2)^(1/2)/a/x^2+1/4*b^2*arcta
nh(a^(1/2)*x/(b*x^3+a*x^2)^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4} dx = \frac{\sqrt{x^2(a+bx)}\left(-\sqrt{a}\sqrt{a+bx}(2a+bx)+b^2x^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{4a^{3/2}x^3\sqrt{a+bx}}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3]/x^4,x]
```

output
```
(Sqrt[x^2*(a + b*x)]*(-(Sqrt[a]*Sqrt[a + b*x]*(2*a + b*x)) + b^2*x^2*ArcTa
nh[Sqrt[a + b*x]/Sqrt[a]]))/(4*a^(3/2)*x^3*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1926, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{4}b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{2x^3}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{4}b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}$$

$$\Big\downarrow \text{1914}$$

$$\frac{1}{4}b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}$$

input
```
Int[Sqrt[a*x^2 + b*x^3]/x^4,x]
```

output
```
-1/2*Sqrt[a*x^2 + b*x^3]/x^3 + (b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*Arc
Tanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/4
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

method	result	size
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) b^{3} x^{3}-\left(\sqrt{a}\, b^{2} x^{2}-\frac{2 a^{\frac{3}{2}} b x}{3}-\frac{8 a^{\frac{5}{2}}}{3}\right) \sqrt{bx+a}}{8 a^{\frac{5}{2}} x^{3}}$	61
risch	$-\frac{(bx+2a) \sqrt{x^{2} (bx+a)}}{4 x^{3} a}+\frac{b^{2} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) \sqrt{x^{2} (bx+a)}}{4 a^{\frac{3}{2}} x \sqrt{bx+a}}$	69
default	$-\frac{\sqrt{b\,x^{3}+a\,x^{2}}\, \left(a^{\frac{3}{2}} (bx+a)^{\frac{3}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) a\, b^{2} x^{2}+\sqrt{bx+a}\, a^{\frac{5}{2}}\right)}{4 x^{3} \sqrt{bx+a}\, a^{\frac{5}{2}}}$	73

input
```
int((b*x^3+a*x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/8/a^(5/2)*(arctanh((b*x+a)^(1/2)/a^(1/2))*b^3*x^3-(a^(1/2)*b^2*x^2-2/3*
a^(3/2)*b*x-8/3*a^(5/2))*(b*x+a)^(1/2))/x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx = \left[\frac{\sqrt{a}b^2x^3\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}(abx+2\,a^2)}{8\,a^2x^3},\right.$$
$$\left.-\frac{\sqrt{-a}b^2x^3\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+\sqrt{bx^3+ax^2}(abx+2\,a^2)}{4\,a^2x^3}\right]$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a)*b^2*x^3*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/
x^2) - 2*sqrt(b*x^3 + a*x^2)*(a*b*x + 2*a^2))/(a^2*x^3), -1/4*(sqrt(-a)*b^
2*x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + sqrt(b*x^3 + a*
x^2)*(a*b*x + 2*a^2))/(a^2*x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx = \int \frac{\sqrt{x^2\,(a+bx)}}{x^4}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(x**2*(a + b*x))/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx = \int \frac{\sqrt{bx^3+ax^2}}{x^4}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x^2)/x^4, x)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx = -\frac{\frac{b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}a} + \frac{(bx+a)^{\frac{3}{2}}b^3\mathrm{sgn}(x)+\sqrt{bx+a}ab^3\mathrm{sgn}(x)}{ab^2x^2}}{4\,b}$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="giac")
```

output
```
-1/4*(b^3*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a) + ((b*x + a)^
(3/2)*b^3*sgn(x) + sqrt(b*x + a)*a*b^3*sgn(x))/(a*b^2*x^2))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx = \int \frac{\sqrt{b\,x^3+a\,x^2}}{x^4}\,dx$$

input
```
int((a*x^2 + b*x^3)^(1/2)/x^4,x)
```

output
```
int((a*x^2 + b*x^3)^(1/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx = \frac{-4\sqrt{bx+a}\,a^2-2\sqrt{bx+a}\,abx-\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2x^2+\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)b^2x^2}{8a^2x^2}$$

input
```
int((b*x^3+a*x^2)^(1/2)/x^4,x)
```

output
```
( - 4*sqrt(a + b*x)*a**2 - 2*sqrt(a + b*x)*a*b*x - sqrt(a)*log(sqrt(a + b*
x) - sqrt(a))*b**2*x**2 + sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**2*x**2)/
(8*a**2*x**2)
```

3.218 $\int \frac{\sqrt{ax^2+bx^3}}{x^5} dx$

Optimal result . . . 2002
Mathematica [A] (verified) . . . 2002
Rubi [A] (verified) . . . 2003
Maple [A] (verified) . . . 2005
Fricas [A] (verification not implemented) . . . 2005
Sympy [F] . . . 2006
Maxima [F] . . . 2006
Giac [A] (verification not implemented) . . . 2006
Mupad [F(-1)] . . . 2007
Reduce [B] (verification not implemented) . . . 2007

Optimal result

Integrand size = 19, antiderivative size = 112

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5} dx = -\frac{\sqrt{ax^2+bx^3}}{3x^4} - \frac{b\sqrt{ax^2+bx^3}}{12ax^3} + \frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2} - \frac{b^3\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{5/2}}$$

output
```
-1/3*(b*x^3+a*x^2)^(1/2)/x^4-1/12*b*(b*x^3+a*x^2)^(1/2)/a/x^3+1/8*b^2*(b*x
^3+a*x^2)^(1/2)/a^2/x^2-1/8*b^3*arctanh(a^(1/2)*x/(b*x^3+a*x^2)^(1/2))/a^(
5/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5} dx = -\frac{\sqrt{x^2(a+bx)}\left(\sqrt{a}\sqrt{a+bx}(8a^2+2abx-3b^2x^2)+3b^3x^3\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{24a^{5/2}x^4\sqrt{a+bx}}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3]/x^5,x]
```

output
```
-1/24*(Sqrt[x^2*(a + b*x)]*(Sqrt[a]*Sqrt[a + b*x]*(8*a^2 + 2*a*b*x - 3*b^2
*x^2) + 3*b^3*x^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(a^(5/2)*x^4*Sqrt[a + b
*x])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1926, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{6}b\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{3x^4}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{6}b\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}$$

$$\Big\downarrow \text{1914}$$

$$\frac{1}{6}b\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{6}b\left(-\frac{3b\left(\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}$$

input
```
Int[Sqrt[a*x^2 + b*x^3]/x^5,x]
```

output
```
-1/3*Sqrt[a*x^2 + b*x^3]/x^4 + (b*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (3*b
*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x
^3]])/a^(3/2)))/(4*a)))/6
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$-\frac{5\left(-\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{4}x^{4}+\sqrt{bx+a}\left(\sqrt{a}\,b^{3}x^{3}-\frac{2a^{\frac{3}{2}}b^{2}x^{2}}{3}+\frac{8a^{\frac{5}{2}}bx}{15}+\frac{16a^{\frac{7}{2}}}{5}\right)\right)}{64a^{\frac{7}{2}}x^{4}}$	72
risch	$-\frac{\left(-3b^{2}x^{2}+2abx+8a^{2}\right)\sqrt{x^{2}\left(bx+a\right)}}{24x^{4}a^{2}}-\frac{b^{3}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{x^{2}\left(bx+a\right)}}{8a^{\frac{5}{2}}x\sqrt{bx+a}}$	81
default	$-\frac{\sqrt{b\,x^{3}+a\,x^{2}}\left(3a^{\frac{9}{2}}\sqrt{bx+a}+8a^{\frac{7}{2}}\left(bx+a\right)^{\frac{3}{2}}-3a^{\frac{5}{2}}\left(bx+a\right)^{\frac{5}{2}}+3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^{2}b^{3}x^{3}\right)}{24x^{4}\sqrt{bx+a}\,a^{\frac{9}{2}}}$	89

input
```
int((b*x^3+a*x^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-5/64*(-arctanh((b*x+a)^(1/2)/a^(1/2))*b^4*x^4+(b*x+a)^(1/2)*(a^(1/2)*b^3*
x^3-2/3*a^(3/2)*b^2*x^2+8/15*a^(5/2)*b*x+16/5*a^(7/2)))/a^(7/2)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx$$
$$=\left[\frac{3\sqrt{a}b^3x^4\log\left(\frac{bx^2+2\,ax-2\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\left(3\,ab^2x^2-2\,a^2bx-8\,a^3\right)\sqrt{bx^3+ax^2}}{48\,a^3x^4},\frac{3\sqrt{-a}b^3x^4\arctan\left(\frac{\sqrt{bx^3}}{b}\right.}{}\right.$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
[1/48*(3*sqrt(a)*b^3*x^4*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a
))/x^2) + 2*(3*a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^3*x^
4), 1/24*(3*sqrt(-a)*b^3*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 +
a*x)) + (3*a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^3*x^4)]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx=\int \frac{\sqrt{x^2\,(a+bx)}}{x^5}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(1/2)/x**5,x)
```

output
```
Integral(sqrt(x**2*(a + b*x))/x**5, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx=\int \frac{\sqrt{bx^3+ax^2}}{x^5}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x^2)/x^5, x)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx = \frac{1}{24}\,b^3\left(\frac{3\,\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}a^2}+\frac{3\,(bx+a)^{\frac{5}{2}}\operatorname{sgn}(x)-8\,(bx+a)^{\frac{3}{2}}a\operatorname{sgn}(x)-3\,\sqrt{bx+a}a^2\operatorname{sgn}(x)}{a^2b^3x^3}\right)$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="giac")
```

output
```
1/24*b^3*(3*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^2) + (3*(b*x
 + a)^(5/2)*sgn(x) - 8*(b*x + a)^(3/2)*a*sgn(x) - 3*sqrt(b*x + a)*a^2*sgn(
x))/(a^2*b^3*x^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx=\int \frac{\sqrt{b\,x^3+a\,x^2}}{x^5}\,dx$$

input
```
int((a*x^2 + b*x^3)^(1/2)/x^5,x)
```

output
```
int((a*x^2 + b*x^3)^(1/2)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx = \frac{-16\sqrt{bx+a}\,a^3-4\sqrt{bx+a}\,a^2bx+6\sqrt{bx+a}\,a\,b^2x^2+3\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^3x^3-3\sqrt{a}\log\left(\sqrt{bx+\right.}{48a^3x^3}$$

input
```
int((b*x^3+a*x^2)^(1/2)/x^5,x)
```

output
```
( - 16*sqrt(a + b*x)*a**3 - 4*sqrt(a + b*x)*a**2*b*x + 6*sqrt(a + b*x)*a*b
**2*x**2 + 3*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**3*x**3 - 3*sqrt(a)*lo
g(sqrt(a + b*x) + sqrt(a))*b**3*x**3)/(48*a**3*x**3)
```

3.219 $\int x^2(ax^2+bx^3)^{3/2}\,dx$

Optimal result 2008
Mathematica [A] (verified) 2008
Rubi [A] (verified) 2009
Maple [A] (verified) 2012
Fricas [A] (verification not implemented) 2012
Sympy [F] 2013
Maxima [A] (verification not implemented) 2013
Giac [B] (verification not implemented) 2014
Mupad [B] (verification not implemented) 2014
Reduce [B] (verification not implemented) 2015

Optimal result

Integrand size = 19, antiderivative size = 161

$$\int x^2(ax^2+bx^3)^{3/2}\,dx=\frac{2(ax^2+bx^3)^{5/2}}{15b}-\frac{512a^5(ax^2+bx^3)^{5/2}}{45045b^6x^5}+\frac{256a^4(ax^2+bx^3)^{5/2}}{9009b^5x^4}-\frac{64a^3(ax^2+bx^3)^{5/2}}{1287b^4x^3}+\frac{32a^2(ax^2+bx^3)^{5/2}}{429b^3x^2}-\frac{4a(ax^2+bx^3)^{5/2}}{39b^2x}$$

output
```
2/15*(b*x^3+a*x^2)^(5/2)/b-512/45045*a^5*(b*x^3+a*x^2)^(5/2)/b^6/x^5+256/9
009*a^4*(b*x^3+a*x^2)^(5/2)/b^5/x^4-64/1287*a^3*(b*x^3+a*x^2)^(5/2)/b^4/x^
3+32/429*a^2*(b*x^3+a*x^2)^(5/2)/b^3/x^2-4/39*a*(b*x^3+a*x^2)^(5/2)/b^2/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.50

$$\int x^2(ax^2+bx^3)^{3/2}\,dx=\frac{2x(a+bx)^3(-256a^5+640a^4bx-1120a^3b^2x^2+1680a^2b^3x^3-2310ab^4x^4+3003b^5x^5)}{45045b^6\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x^2*(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*x*(a + b*x)^3*(-256*a^5 + 640*a^4*b*x - 1120*a^3*b^2*x^2 + 1680*a^2*b^3
*x^3 - 2310*a*b^4*x^4 + 3003*b^5*x^5))/(45045*b^6*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1922, 1922, 1908, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(ax^2+bx^3\right)^{3/2} dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}-\frac{2a\int x\left(bx^3+ax^2\right)^{3/2}dx}{3b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}-\frac{2a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\int \left(bx^3+ax^2\right)^{3/2}dx}{13b}\right)}{3b}$$

$$\Big\downarrow \text{1908}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}-\frac{2a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x}dx}{11b}\right)}{13b}\right)}{3b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}-\frac{2a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\int\frac{\left(bx^3+ax^2\right)^{3/2}}{x^2}dx}{9b}\right)}{11b}\right)}{13b}\right)}{3b}$$

$\Big\downarrow$ 1922

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}-\frac{2a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{2a\int\frac{\left(bx^3+ax^2\right)^{3/2}}{x^3}dx}{7b}\right)}{9b}\right)}{11b}\right)}{13b}\right)}{3b}$$

$\Big\downarrow$ 1920

$$\frac{2(ax^2+bx^3)^{5/2}}{15b} - \frac{2a\left(\frac{2(ax^2+bx^3)^{5/2}}{13bx} - \frac{8a\left(\frac{2(ax^2+bx^3)^{5/2}}{11bx^2} - \frac{6a\left(\frac{2(ax^2+bx^3)^{5/2}}{9bx^3} - \frac{4a\left(\frac{2(ax^2+bx^3)^{5/2}}{7bx^4} - \frac{4a(ax^2+bx^3)^{5/2}}{35b^2x^5}\right)}{9b}\right)}{11b}\right)}{13b}\right)}{3b}$$

input
```
Int[x^2*(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*(a*x^2 + b*x^3)^(5/2))/(15*b) - (2*a*((2*(a*x^2 + b*x^3)^(5/2))/(13*b*x
) - (8*a*((2*(a*x^2 + b*x^3)^(5/2))/(11*b*x^2) - (6*a*((2*(a*x^2 + b*x^3)^
(5/2))/(9*b*x^3) - (4*a*((-4*a*(a*x^2 + b*x^3)^(5/2))/(35*b^2*x^5) + (2*(a
*x^2 + b*x^3)^(5/2))/(7*b*x^4)))/(9*b)))/(11*b)))/(13*b)))/(3*b)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.20

method	result	size
pseudoelliptic	$\frac{2(bx+a)^{\frac{5}{2}}(35b^2x^2-20abx+8a^2)}{315b^3}$	32
gosper	$-\frac{2(bx+a)(-3003b^5x^5+2310a\,b^4x^4-1680a^2b^3x^3+1120a^3b^2x^2-640a^4bx+256a^5)(b\,x^3+a\,x^2)^{\frac{3}{2}}}{45045b^6x^3}$	79
default	$-\frac{2(bx+a)(-3003b^5x^5+2310a\,b^4x^4-1680a^2b^3x^3+1120a^3b^2x^2-640a^4bx+256a^5)(b\,x^3+a\,x^2)^{\frac{3}{2}}}{45045b^6x^3}$	79
orering	$-\frac{2(bx+a)(-3003b^5x^5+2310a\,b^4x^4-1680a^2b^3x^3+1120a^3b^2x^2-640a^4bx+256a^5)(b\,x^3+a\,x^2)^{\frac{3}{2}}}{45045b^6x^3}$	79
risch	$-\frac{2\sqrt{x^2(bx+a)}\,(-3003x^7b^7-3696b^6a\,x^6-63a^2x^5b^5+70b^4x^4a^3-80b^3x^3a^4+96x^2b^2a^5-128xb\,a^6+256a^7)}{45045x\,b^6}$	94
trager	$-\frac{2(-3003x^7b^7-3696b^6a\,x^6-63a^2x^5b^5+70b^4x^4a^3-80b^3x^3a^4+96x^2b^2a^5-128xb\,a^6+256a^7)\sqrt{b\,x^3+a\,x^2}}{45045b^6x}$	96

input
```
int(x^2*(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*(b*x+a)^(5/2)*(35*b^2*x^2-20*a*b*x+8*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.59

$$\int x^2\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2\left(3003\,b^7x^7+3696\,ab^6x^6+63\,a^2b^5x^5-70\,a^3b^4x^4+80\,a^4b^3x^3-96\,a^5b^2x^2+128\,a^6bx-256\right.}{45045\,b^6x}$$

input
```
integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2/45045*(3003*b^7*x^7 + 3696*a*b^6*x^6 + 63*a^2*b^5*x^5 - 70*a^3*b^4*x^4 +
 80*a^4*b^3*x^3 - 96*a^5*b^2*x^2 + 128*a^6*b*x - 256*a^7)*sqrt(b*x^3 + a*x
^2)/(b^6*x)
```

Sympy [F]

$$\int x^2 (ax^2 + bx^3)^{3/2} \, dx = \int x^2 (x^2(a + bx))^{\frac{3}{2}} \, dx$$

input
```
integrate(x**2*(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**2*(x**2*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.53

$$\int x^2 (ax^2 + bx^3)^{3/2} \, dx = \frac{2 \, (3003 \, b^7 x^7 + 3696 \, a b^6 x^6 + 63 \, a^2 b^5 x^5 - 70 \, a^3 b^4 x^4 + 80 \, a^4 b^3 x^3 - 96 \, a^5 b^2 x^2 + 128 \, a^6 b x - 256}{45045 \, b^6}$$

input
```
integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
2/45045*(3003*b^7*x^7 + 3696*a*b^6*x^6 + 63*a^2*b^5*x^5 - 70*a^3*b^4*x^4 +
 80*a^4*b^3*x^3 - 96*a^5*b^2*x^2 + 128*a^6*b*x - 256*a^7)*sqrt(b*x + a)/b^
6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(137) = 274$.

Time = 0.25 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.75

$$\int x^2 (ax^2 + bx^3)^{3/2} \, dx = \frac{512 \, a^{\frac{15}{2}} \operatorname{sgn}(x)}{45045 \, b^6} + \frac{2 \left(\frac{65 \left(63 (bx+a)^{\frac{11}{2}} - 385 (bx+a)^{\frac{9}{2}} a + 990 (bx+a)^{\frac{7}{2}} a^2 - 1386 (bx+a)^{\frac{5}{2}} a^3 + 1155 (bx+a)^{\frac{3}{2}} a^4 - 693 \sqrt{bx+a} a^5\right) a^2 \operatorname{sgn}(x)}{b^5} + \frac{30 \left(231 (bx+a)^{\frac{13}{2}} - 1\right.}{}\right.}{}$$

input
```
integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
512/45045*a^(15/2)*sgn(x)/b^6 + 2/45045*(65*(63*(b*x + a)^(11/2) - 385*(b*
x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155
*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a^2*sgn(x)/b^5 + 30*(231*(b*
x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*
(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5
+ 3003*sqrt(b*x + a)*a^6)*a*sgn(x)/b^5 + 7*(429*(b*x + a)^(15/2) - 3465*(b
*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3
+ 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^
(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*sgn(x)/b^5)/b
```

Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.50

$$\int x^2 (ax^2 + bx^3)^{3/2} \, dx = -\frac{2 \sqrt{b \, x^3 + a \, x^2} \, (a + b \, x)^2 \, (256 \, a^5 - 640 \, a^4 \, b \, x + 1120 \, a^3 \, b^2 \, x^2 - 1680 \, a^2 \, b^3 \, x^3 + 2310 \, a \, b^4 \, x^4 - 3003 \, b^5 \, x^5)}{45045 \, b^6 \, x}$$

input
```
int(x^2*(a*x^2 + b*x^3)^(3/2),x)
```

output
```
-(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(256*a^5 - 3003*b^5*x^5 + 2310*a*b^4
*x^4 + 1120*a^3*b^2*x^2 - 1680*a^2*b^3*x^3 - 640*a^4*b*x))/(45045*b^6*x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.53

$$\int x^2 \left(ax^2 + bx^3\right)^{3/2} dx = \frac{2\sqrt{bx+a}\,(3003b^7x^7 + 3696a\,b^6x^6 + 63a^2b^5x^5 - 70a^3b^4x^4 + 80a^4b^3x^3 - 96a^5b^2x^2 + 128a^6bx}{45045b^6}$$

input
```
int(x^2*(b*x^3+a*x^2)^(3/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 256*a**7 + 128*a**6*b*x - 96*a**5*b**2*x**2 + 80*a**4
*b**3*x**3 - 70*a**3*b**4*x**4 + 63*a**2*b**5*x**5 + 3696*a*b**6*x**6 + 30
03*b**7*x**7))/(45045*b**6)
```

3.220 $\int x(ax^2+bx^3)^{3/2}\,dx$

Optimal result . . . 2016
Mathematica [A] (verified) . . . 2016
Rubi [A] (verified) . . . 2017
Maple [A] (verified) . . . 2019
Fricas [A] (verification not implemented) . . . 2020
Sympy [F] . . . 2020
Maxima [A] (verification not implemented) . . . 2020
Giac [B] (verification not implemented) . . . 2021
Mupad [B] (verification not implemented) . . . 2021
Reduce [B] (verification not implemented) . . . 2022

Optimal result

Integrand size = 17, antiderivative size = 136

$$\int x\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{256a^4(ax^2+bx^3)^{5/2}}{15015b^5x^5}-\frac{128a^3(ax^2+bx^3)^{5/2}}{3003b^4x^4}+\frac{32a^2(ax^2+bx^3)^{5/2}}{429b^3x^3}-\frac{16a(ax^2+bx^3)^{5/2}}{143b^2x^2}+\frac{2(ax^2+bx^3)^{5/2}}{13bx}$$

output
```
256/15015*a^4*(b*x^3+a*x^2)^(5/2)/b^5/x^5-128/3003*a^3*(b*x^3+a*x^2)^(5/2)
/b^4/x^4+32/429*a^2*(b*x^3+a*x^2)^(5/2)/b^3/x^3-16/143*a*(b*x^3+a*x^2)^(5/
2)/b^2/x^2+2/13*(b*x^3+a*x^2)^(5/2)/b/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.51

$$\int x\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2x(a+bx)^3\left(128a^4-320a^3bx+560a^2b^2x^2-840ab^3x^3+1155b^4x^4\right)}{15015b^5\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x*(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*x*(a + b*x)^3*(128*a^4 - 320*a^3*b*x + 560*a^2*b^2*x^2 - 840*a*b^3*x^3
+ 1155*b^4*x^4))/(15015*b^5*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1922, 1908, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax^2+bx^3\right)^{3/2}dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\int \left(bx^3+ax^2\right)^{3/2}dx}{13b}$$

$$\Big\downarrow \text{1908}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x}dx}{11b}\right)}{13b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^2}dx}{9b}\right)}{11b}\right)}{13b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{2a\int\frac{\left(bx^3+ax^2\right)^{3/2}}{x^3}dx}{7b}\right)}{9b}\right)}{11b}\right)}{13b}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{35b^2x^5}\right)}{9b}\right)}{11b}\right)}{13b}$$

input
```
Int[x*(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*(a*x^2 + b*x^3)^(5/2))/(13*b*x) - (8*a*((2*(a*x^2 + b*x^3)^(5/2))/(11*b
*x^2) - (6*a*((2*(a*x^2 + b*x^3)^(5/2))/(9*b*x^3) - (4*a*((-4*a*(a*x^2 + b
*x^3)^(5/2))/(35*b^2*x^5) + (2*(a*x^2 + b*x^3)^(5/2))/(7*b*x^4)))/(9*b)))/
(11*b)))/(13*b)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.15

method	result	size
pseudoelliptic	$-\frac{2(bx+a)^{\frac{5}{2}}(-5bx+2a)}{35b^2}$	21
gosper	$\frac{2(bx+a)\left(1155b^4x^4-840a\,b^3x^3+560a^2b^2x^2-320a^3bx+128a^4\right)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{15015b^5x^3}$	68
default	$\frac{2(bx+a)\left(1155b^4x^4-840a\,b^3x^3+560a^2b^2x^2-320a^3bx+128a^4\right)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{15015b^5x^3}$	68
orering	$\frac{2(bx+a)\left(1155b^4x^4-840a\,b^3x^3+560a^2b^2x^2-320a^3bx+128a^4\right)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{15015b^5x^3}$	68
risch	$\frac{2\sqrt{x^2(bx+a)}\,\left(1155b^6x^6+1470a\,b^5x^5+35a^2b^4x^4-40a^3b^3x^3+48a^4b^2x^2-64a^5bx+128a^6\right)}{15015x\,b^5}$	83
trager	$\frac{2\left(1155b^6x^6+1470a\,b^5x^5+35a^2b^4x^4-40a^3b^3x^3+48a^4b^2x^2-64a^5bx+128a^6\right)\sqrt{b\,x^3+a\,x^2}}{15015b^5x}$	85

input
```
int(x*(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*(b*x+a)^(5/2)*(-5*b*x+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.62

$$\int x\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2\left(1155\,b^6x^6+1470\,ab^5x^5+35\,a^2b^4x^4-40\,a^3b^3x^3+48\,a^4b^2x^2-64\,a^5bx+128\,a^6\right)\sqrt{bx^3+a}}{15015\,b^5x}$$

input
```
integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2/15015*(1155*b^6*x^6 + 1470*a*b^5*x^5 + 35*a^2*b^4*x^4 - 40*a^3*b^3*x^3 +
 48*a^4*b^2*x^2 - 64*a^5*b*x + 128*a^6)*sqrt(b*x^3 + a*x^2)/(b^5*x)
```

Sympy [F]

$$\int x\left(ax^2+bx^3\right)^{3/2}\,dx=\int x\left(x^2(a+bx)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x*(x**2*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.55

$$\int x\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2\left(1155\,b^6x^6+1470\,ab^5x^5+35\,a^2b^4x^4-40\,a^3b^3x^3+48\,a^4b^2x^2-64\,a^5bx+128\,a^6\right)\sqrt{bx+a}}{15015\,b^5}$$

input
```
integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
2/15015*(1155*b^6*x^6 + 1470*a*b^5*x^5 + 35*a^2*b^4*x^4 - 40*a^3*b^3*x^3 +
 48*a^4*b^2*x^2 - 64*a^5*b*x + 128*a^6)*sqrt(b*x + a)/b^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(116) = 232$.

Time = 0.24 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.81

$$\int x\left(ax^2+bx^3\right)^{3/2}\,dx=-\frac{256\,a^{\frac{13}{2}}\operatorname{sgn}(x)}{15015\,b^5}$$
$$+\frac{2\left(\frac{143\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}a^4\right)a^2\operatorname{sgn}(x)}{b^4}+\frac{130\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(\right.}{}\right.}{}$$

input
```
integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
-256/15015*a^(13/2)*sgn(x)/b^5 + 2/45045*(143*(35*(b*x + a)^(9/2) - 180*(b
*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*
sqrt(b*x + a)*a^4)*a^2*sgn(x)/b^4 + 130*(63*(b*x + a)^(11/2) - 385*(b*x +
a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*
x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a*sgn(x)/b^4 + 15*(231*(b*x + a)
^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x +
 a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003
*sqrt(b*x + a)*a^6)*sgn(x)/b^4)/b
```

Mupad [B] (verification not implemented)

Time = 8.81 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.51

$$\int x\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2\,\sqrt{b\,x^3+a\,x^2}\,{\left(a+b\,x\right)}^2\,\left(128\,a^4-320\,a^3\,b\,x+560\,a^2\,b^2\,x^2-840\,a\,b^3\,x^3+1155\,b^4\,x^4\right)}{15015\,b^5\,x}$$

input
```
int(x*(a*x^2 + b*x^3)^(3/2),x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(128*a^4 + 1155*b^4*x^4 - 840*a*b^3*x
^3 + 560*a^2*b^2*x^2 - 320*a^3*b*x))/(15015*b^5*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.54

$$\int x(ax^2+bx^3)^{3/2}\,dx = \frac{2\sqrt{bx+a}\,(1155b^6x^6+1470a\,b^5x^5+35a^2b^4x^4-40a^3b^3x^3+48a^4b^2x^2-64a^5bx+128a^6)}{15015b^5}$$

input
```
int(x*(b*x^3+a*x^2)^(3/2),x)
```

output
```
(2*sqrt(a + b*x)*(128*a**6 - 64*a**5*b*x + 48*a**4*b**2*x**2 - 40*a**3*b**
3*x**3 + 35*a**2*b**4*x**4 + 1470*a*b**5*x**5 + 1155*b**6*x**6))/(15015*b*
*5)
```

3.221 $\int (ax^2+bx^3)^{3/2}\,dx$

Optimal result . . . 2023
Mathematica [A] (verified) . . . 2023
Rubi [A] (verified) . . . 2024
Maple [A] (verified) . . . 2025
Fricas [A] (verification not implemented) . . . 2026
Sympy [F] . . . 2027
Maxima [A] (verification not implemented) . . . 2027
Giac [B] (verification not implemented) . . . 2027
Mupad [B] (verification not implemented) . . . 2028
Reduce [B] (verification not implemented) . . . 2028

Optimal result

Integrand size = 15, antiderivative size = 108

$$\int (ax^2+bx^3)^{3/2}\,dx = -\frac{32a^3(ax^2+bx^3)^{5/2}}{1155b^4x^5} + \frac{16a^2(ax^2+bx^3)^{5/2}}{231b^3x^4} - \frac{4a(ax^2+bx^3)^{5/2}}{33b^2x^3} + \frac{2(ax^2+bx^3)^{5/2}}{11bx^2}$$

output
```
-32/1155*a^3*(b*x^3+a*x^2)^(5/2)/b^4/x^5+16/231*a^2*(b*x^3+a*x^2)^(5/2)/b^
3/x^4-4/33*a*(b*x^3+a*x^2)^(5/2)/b^2/x^3+2/11*(b*x^3+a*x^2)^(5/2)/b/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

$$\int (ax^2+bx^3)^{3/2}\,dx = \frac{2x(a+bx)^3(-16a^3+40a^2bx-70ab^2x^2+105b^3x^3)}{1155b^4\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*x*(a + b*x)^3*(-16*a^3 + 40*a^2*b*x - 70*a*b^2*x^2 + 105*b^3*x^3))/(115
5*b^4*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1908, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^2+bx^3\right)^{3/2} dx$$

$$\Big\downarrow \text{1908}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x}dx}{11b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^2}dx}{9b}\right)}{11b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{2a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^3}dx}{7b}\right)}{9b}\right)}{11b}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{35b^2x^5}\right)}{9b}\right)}{11b}$$

input
```
Int[(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*(a*x^2 + b*x^3)^(5/2))/(11*b*x^2) - (6*a*((2*(a*x^2 + b*x^3)^(5/2))/(9*
b*x^3) - (4*a*((-4*a*(a*x^2 + b*x^3)^(5/2))/(35*b^2*x^5) + (2*(a*x^2 + b*x
^3)^(5/2))/(7*b*x^4)))/(9*b)))/(11*b)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.12

method	result	size
pseudoelliptic	$\frac{2(bx+a)^{\frac{5}{2}}}{5b}$	13
gosper	$-\frac{2(bx+a)\left(-105b^{3}x^{3}+70a\,b^{2}x^{2}-40a^{2}bx+16a^{3}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{1155b^{4}x^{3}}$	57
default	$-\frac{2(bx+a)\left(-105b^{3}x^{3}+70a\,b^{2}x^{2}-40a^{2}bx+16a^{3}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{1155b^{4}x^{3}}$	57
orering	$-\frac{2(bx+a)\left(-105b^{3}x^{3}+70a\,b^{2}x^{2}-40a^{2}bx+16a^{3}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{1155b^{4}x^{3}}$	57
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(-105b^{5}x^{5}-140a\,b^{4}x^{4}-5a^{2}b^{3}x^{3}+6a^{3}b^{2}x^{2}-8a^{4}bx+16a^{5}\right)}{1155x\,b^{4}}$	72
trager	$-\frac{2\left(-105b^{5}x^{5}-140a\,b^{4}x^{4}-5a^{2}b^{3}x^{3}+6a^{3}b^{2}x^{2}-8a^{4}bx+16a^{5}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{1155b^{4}x}$	74

input
```
int((b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(b*x+a)^(5/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.68

$$\int \left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2\left(105\,b^5x^5+140\,ab^4x^4+5\,a^2b^3x^3-6\,a^3b^2x^2+8\,a^4bx-16\,a^5\right)\sqrt{bx^3+ax^2}}{1155\,b^4x}$$

input
```
integrate((b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^
4*b*x - 16*a^5)*sqrt(b*x^3 + a*x^2)/(b^4*x)
```

Sympy [F]

$$\int \left(ax^2+bx^3\right)^{3/2}\, dx = \int \left(ax^2+bx^3\right)^{\frac{3}{2}}\, dx$$

input
```
integrate((b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral((a*x**2 + b*x**3)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.59

$$\int \left(ax^2+bx^3\right)^{3/2}\, dx = \frac{2\,(105\,b^5x^5+140\,ab^4x^4+5\,a^2b^3x^3-6\,a^3b^2x^2+8\,a^4bx-16\,a^5)\sqrt{bx+a}}{1155\,b^4}$$

input
```
integrate((b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^
4*b*x - 16*a^5)*sqrt(b*x + a)/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(92) = 184$.

Time = 0.17 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.94

$$\int \left(ax^2+bx^3\right)^{3/2}\, dx = \frac{32\,a^{\frac{11}{2}}\,\mathrm{sgn}(x)}{1155\,b^4} + \frac{2\left(\frac{99\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)a^2\mathrm{sgn}(x)}{b^3} + \frac{22\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}\right.}{b^3}}{346}$$

input
```
integrate((b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
32/1155*a^(11/2)*sgn(x)/b^4 + 2/3465*(99*(5*(b*x + a)^(7/2) - 21*(b*x + a)
^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^2*sgn(x)/b^3 +
 22*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2
- 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a*sgn(x)/b^3 + 5*(63*(b
*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b
*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*sgn(
x)/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 8.78 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

$$\int \left(ax^2+bx^3\right)^{3/2}\,dx=-\frac{2\,\sqrt{b\,x^3+a\,x^2}\,{\left(a+b\,x\right)}^2\,\left(16\,a^3-40\,a^2\,b\,x+70\,a\,b^2\,x^2-105\,b^3\,x^3\right)}{1155\,b^4\,x}$$

input
```
int((a*x^2 + b*x^3)^(3/2),x)
```

output
```
-(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(16*a^3 - 105*b^3*x^3 + 70*a*b^2*x^2
 - 40*a^2*b*x))/(1155*b^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.58

$$\int \left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2\sqrt{bx+a}\,\left(105b^5x^5+140a\,b^4x^4+5a^2b^3x^3-6a^3b^2x^2+8a^4bx-16a^5\right)}{1155b^4}$$

input
```
int((b*x^3+a*x^2)^(3/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 16*a**5 + 8*a**4*b*x - 6*a**3*b**2*x**2 + 5*a**2*b**3
*x**3 + 140*a*b**4*x**4 + 105*b**5*x**5))/(1155*b**4)
```

3.222 $\int \frac{(ax^2+bx^3)^{3/2}}{x} dx$

Optimal result . . . 2029
Mathematica [A] (verified) . . . 2029
Rubi [A] (verified) . . . 2030
Maple [C] (verified) . . . 2031
Fricas [A] (verification not implemented) . . . 2032
Sympy [F] . . . 2032
Maxima [A] (verification not implemented) . . . 2032
Giac [B] (verification not implemented) . . . 2033
Mupad [B] (verification not implemented) . . . 2033
Reduce [B] (verification not implemented) . . . 2034

Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \frac{(ax^2+bx^3)^{3/2}}{x} dx = \frac{16a^2(ax^2+bx^3)^{5/2}}{315b^3x^5} - \frac{8a(ax^2+bx^3)^{5/2}}{63b^2x^4} + \frac{2(ax^2+bx^3)^{5/2}}{9bx^3}$$

output
```
16/315*a^2*(b*x^3+a*x^2)^(5/2)/b^3/x^5-8/63*a*(b*x^3+a*x^2)^(5/2)/b^2/x^4+
2/9*(b*x^3+a*x^2)^(5/2)/b/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

$$\int \frac{(ax^2+bx^3)^{3/2}}{x} dx = \frac{2x(a+bx)^3(8a^2-20abx+35b^2x^2)}{315b^3\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(3/2)/x,x]
```

output
```
(2*x*(a + b*x)^3*(8*a^2 - 20*a*b*x + 35*b^2*x^2))/(315*b^3*Sqrt[x^2*(a + b
*x)])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^2}dx}{9b}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{2a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^3}dx}{7b}\right)}{9b}$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{35b^2x^5}\right)}{9b}$$

input
```
Int[(a*x^2 + b*x^3)^(3/2)/x,x]
```

output
```
(2*(a*x^2 + b*x^3)^(5/2))/(9*b*x^3) - (4*a*((-4*a*(a*x^2 + b*x^3)^(5/2))/(
35*b^2*x^5) + (2*(a*x^2 + b*x^3)^(5/2))/(7*b*x^4)))/(9*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.44 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

method	result	size
pseudoelliptic	$-2a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\frac{2\sqrt{bx+a}\,(bx+4a)}{3}$	35
gosper	$\frac{2(bx+a)(35b^2x^2-20abx+8a^2)(b\,x^3+a\,x^2)^{\frac{3}{2}}}{315b^3x^3}$	46
default	$\frac{2(bx+a)(35b^2x^2-20abx+8a^2)(b\,x^3+a\,x^2)^{\frac{3}{2}}}{315b^3x^3}$	46
orering	$\frac{2(bx+a)(35b^2x^2-20abx+8a^2)(b\,x^3+a\,x^2)^{\frac{3}{2}}}{315b^3x^3}$	46
risch	$\frac{2\sqrt{x^2(bx+a)}\,(35b^4x^4+50a\,b^3x^3+3a^2b^2x^2-4a^3bx+8a^4)}{315x\,b^3}$	61
trager	$\frac{2(35b^4x^4+50a\,b^3x^3+3a^2b^2x^2-4a^3bx+8a^4)\sqrt{b\,x^3+a\,x^2}}{315b^3x}$	63

input
```
int((b*x^3+a*x^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2*a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2/3*(b*x+a)^(1/2)*(b*x+4*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int \frac{(ax^2+bx^3)^{3/2}}{x}\,dx = \frac{2\,(35\,b^4x^4+50\,ab^3x^3+3\,a^2b^2x^2-4\,a^3bx+8\,a^4)\sqrt{bx^3+ax^2}}{315\,b^3x}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="fricas")
```

output
```
2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt
(b*x^3 + a*x^2)/(b^3*x)
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x}\,dx = \int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(3/2)/x,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.66

$$\int \frac{(ax^2+bx^3)^{3/2}}{x}\,dx = \frac{2\,(35\,b^4x^4+50\,ab^3x^3+3\,a^2b^2x^2-4\,a^3bx+8\,a^4)\sqrt{bx+a}}{315\,b^3}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="maxima")
```

output
```
2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt
(b*x + a)/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(68) = 136$.

Time = 0.16 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.16

$$\int \frac{(ax^2+bx^3)^{3/2}}{x}\,dx = -\frac{16\,a^{\frac{9}{2}}\mathrm{sgn}(x)}{315\,b^3} + \frac{2\left(\frac{21\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)a^2\mathrm{sgn}(x)}{b^2}+\frac{18\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a a^3\right)a\mathrm{sgn}(x)}{b^2}+\frac{(35\,(b\cdots}{\cdots}\right.}{315\,b}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="giac")
```

output
```
-16/315*a^(9/2)*sgn(x)/b^3 + 2/315*(21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(
3/2)*a + 15*sqrt(b*x + a)*a^2)*a^2*sgn(x)/b^2 + 18*(5*(b*x + a)^(7/2) - 21
*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a*sgn(
x)/b^2 + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)
*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*sgn(x)/b^2)/b
```

Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

$$\int \frac{(ax^2+bx^3)^{3/2}}{x}\,dx = \frac{2\,\sqrt{b\,x^3+a\,x^2}\,(a+b\,x)^2\,(8\,a^2-20\,a\,b\,x+35\,b^2\,x^2)}{315\,b^3\,x}$$

input
```
int((a*x^2 + b*x^3)^(3/2)/x,x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(8*a^2 + 35*b^2*x^2 - 20*a*b*x))/(315
*b^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.65

$$\int \frac{(ax^2+bx^3)^{3/2}}{x}\,dx = \frac{2\sqrt{bx+a}\,(35b^4x^4+50a\,b^3x^3+3a^2b^2x^2-4a^3bx+8a^4)}{315b^3}$$

input
```
int((b*x^3+a*x^2)^(3/2)/x,x)
```

output
```
(2*sqrt(a + b*x)*(8*a**4 - 4*a**3*b*x + 3*a**2*b**2*x**2 + 50*a*b**3*x**3
+ 35*b**4*x**4))/(315*b**3)
```

3.223 $\int \frac{(ax^2+bx^3)^{3/2}}{x^2} dx$

Optimal result . . . 2035
Mathematica [A] (verified) . . . 2035
Rubi [A] (verified) . . . 2036
Maple [A] (verified) . . . 2037
Fricas [A] (verification not implemented) . . . 2037
Sympy [F] . . . 2038
Maxima [A] (verification not implemented) . . . 2038
Giac [B] (verification not implemented) . . . 2038
Mupad [B] (verification not implemented) . . . 2039
Reduce [B] (verification not implemented) . . . 2039

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2} dx = -\frac{4a(ax^2+bx^3)^{5/2}}{35b^2x^5} + \frac{2(ax^2+bx^3)^{5/2}}{7bx^4}$$

output
```
-4/35*a*(b*x^3+a*x^2)^(5/2)/b^2/x^5+2/7*(b*x^3+a*x^2)^(5/2)/b/x^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2} dx = \frac{2(x^2(a+bx))^{5/2}(-2a+5bx)}{35b^2x^5}$$

input
```
Integrate[(a*x^2 + b*x^3)^(3/2)/x^2,x]
```

output
```
(2*(x^2*(a + b*x))^(5/2)*(-2*a + 5*b*x))/(35*b^2*x^5)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx$$

$$\Big\downarrow \ 1922$$

$$\frac{2(ax^2+bx^3)^{5/2}}{7bx^4}-\frac{2a\int \frac{(bx^3+ax^2)^{3/2}}{x^3}dx}{7b}$$

$$\Big\downarrow \ 1920$$

$$\frac{2(ax^2+bx^3)^{5/2}}{7bx^4}-\frac{4a(ax^2+bx^3)^{5/2}}{35b^2x^5}$$

input
```
Int[(a*x^2 + b*x^3)^(3/2)/x^2,x]
```

output
```
(-4*a*(a*x^2 + b*x^3)^(5/2))/(35*b^2*x^5) + (2*(a*x^2 + b*x^3)^(5/2))/(7*b
*x^4)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

method	result	size
gosper	$-\frac{2(bx+a)(-5bx+2a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{35b^{2}x^{3}}$	35
default	$-\frac{2(bx+a)(-5bx+2a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{35b^{2}x^{3}}$	35
orering	$-\frac{2(bx+a)(-5bx+2a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{35b^{2}x^{3}}$	35
pseudoelliptic	$-\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)abx+\sqrt{bx+a}\,(-2bx+a)\sqrt{a}}{\sqrt{a}\,x}$	44
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(-5b^{3}x^{3}-8a\,b^{2}x^{2}-a^{2}bx+2a^{3}\right)}{35x\,b^{2}}$	50
trager	$-\frac{2\left(-5b^{3}x^{3}-8a\,b^{2}x^{2}-a^{2}bx+2a^{3}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{35b^{2}x}$	52

input
```
int((b*x^3+a*x^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2/35*(b*x+a)*(-5*b*x+2*a)*(b*x^3+a*x^2)^(3/2)/b^2/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx = \frac{2\,(5\,b^3x^3+8\,ab^2x^2+a^2bx-2\,a^3)\sqrt{bx^3+ax^2}}{35\,b^2x}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x^3 + a*x^2)/(b^2*
x)
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx = \int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(3/2)/x**2,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx = \frac{2\,(5\,b^3x^3+8\,ab^2x^2+a^2bx-2\,a^3)\sqrt{bx+a}}{35\,b^2}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x + a)/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(44) = 88.

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.62

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx = \frac{4\,a^{\frac{7}{2}}\mathrm{sgn}(x)}{35\,b^2} + \frac{2\left(\frac{35\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)a^2\mathrm{sgn}(x)}{b}+\frac{14\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)a\mathrm{sgn}(x)}{b}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)\right.}{b}\right.}{105\,b}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="giac")
```

output
```
4/35*a^(7/2)*sgn(x)/b^2 + 2/105*(35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*
a^2*sgn(x)/b + 14*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x
+ a)*a^2)*a*sgn(x)/b + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b
*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*sgn(x)/b)/b
```

Mupad [B] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx = -\frac{2\,(2\,a-5\,b\,x)\,\sqrt{b\,x^3+a\,x^2}\,(a+b\,x)^2}{35\,b^2\,x}$$

input
```
int((a*x^2 + b*x^3)^(3/2)/x^2,x)
```

output
```
-(2*(2*a - 5*b*x)*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2)/(35*b^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx = \frac{2\sqrt{bx+a}\,(5b^3x^3+8a\,b^2x^2+a^2bx-2a^3)}{35b^2}$$

input
```
int((b*x^3+a*x^2)^(3/2)/x^2,x)
```

output
```
(2*sqrt(a + b*x)*( - 2*a**3 + a**2*b*x + 8*a*b**2*x**2 + 5*b**3*x**3))/(35
*b**2)
```

3.224 $\int \frac{(ax^2+bx^3)^{3/2}}{x^3} dx$

Optimal result . . . 2040
Mathematica [A] (verified) . . . 2040
Rubi [A] (verified) . . . 2041
Maple [A] (verified) . . . 2041
Fricas [A] (verification not implemented) . . . 2042
Sympy [F] . . . 2043
Maxima [A] (verification not implemented) . . . 2043
Giac [B] (verification not implemented) . . . 2043
Mupad [B] (verification not implemented) . . . 2044
Reduce [B] (verification not implemented) . . . 2044

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3} dx = \frac{2(ax^2+bx^3)^{5/2}}{5bx^5}$$

output
```
2/5*(b*x^3+a*x^2)^(5/2)/b/x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3} dx = \frac{2(x^2(a+bx))^{5/2}}{5bx^5}$$

input
```
Integrate[(a*x^2 + b*x^3)^(3/2)/x^3,x]
```

output
```
(2*(x^2*(a + b*x))^(5/2))/(5*b*x^5)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3} \, dx$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{2(ax^2+bx^3)^{5/2}}{5bx^5}$$

input
```
Int[(a*x^2 + b*x^3)^(3/2)/x^3,x]
```

output
```
(2*(a*x^2 + b*x^3)^(5/2))/(5*b*x^5)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

method	result	size
gosper	$\frac{2(bx+a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{5b\,x^{3}}$	27
default	$\frac{2(bx+a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{5b\,x^{3}}$	27
orering	$\frac{2(bx+a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{5b\,x^{3}}$	27
risch	$\frac{2\sqrt{x^{2}(bx+a)}\,\left(b^{2}x^{2}+2abx+a^{2}\right)}{5xb}$	36
trager	$\frac{2\left(b^{2}x^{2}+2abx+a^{2}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{5bx}$	38
pseudoelliptic	$\frac{-3\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{2}x^{2}-5bx\sqrt{bx+a}\,\sqrt{a}-2a^{\frac{3}{2}}\sqrt{bx+a}}{4x^{2}\sqrt{a}}$	56

input
```
int((b*x^3+a*x^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
2/5*(b*x+a)/b*(b*x^3+a*x^2)^(3/2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx = \frac{2\,(b^2x^2+2\,abx+a^2)\sqrt{bx^3+ax^2}}{5\,bx}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x^3 + a*x^2)/(b*x)
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx=\int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(3/2)/x**3,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx=\frac{2\,(b^2x^2+2\,abx+a^2)\sqrt{bx+a}}{5\,b}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(21) = 42.

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.56

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx=-\frac{2\,a^{\frac{5}{2}}\mathrm{sgn}(x)}{5\,b}$$
$$+\frac{2\left(15\,\sqrt{bx+a}a^2\mathrm{sgn}(x)+10\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}a\right)a\mathrm{sgn}(x)+\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{b\right.}\right.}{15\,b}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="giac")
```

output
```
-2/5*a^(5/2)*sgn(x)/b + 2/15*(15*sqrt(b*x + a)*a^2*sgn(x) + 10*((b*x + a)^
(3/2) - 3*sqrt(b*x + a)*a)*a*sgn(x) + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3
/2)*a + 15*sqrt(b*x + a)*a^2)*sgn(x))/b
```

Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx = \frac{2\,\sqrt{b\,x^3+a\,x^2}\,(a+b\,x)^2}{5\,b\,x}$$

input
```
int((a*x^2 + b*x^3)^(3/2)/x^3,x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2)/(5*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx = \frac{2\sqrt{bx+a}\,(b^2x^2+2abx+a^2)}{5b}$$

input
```
int((b*x^3+a*x^2)^(3/2)/x^3,x)
```

output
```
(2*sqrt(a + b*x)*(a**2 + 2*a*b*x + b**2*x**2))/(5*b)
```

3.225 $\int \frac{(ax^2+bx^3)^{3/2}}{x^4} dx$

Optimal result 2045
Mathematica [A] (verified) 2045
Rubi [A] (verified) 2046
Maple [A] (verified) 2047
Fricas [A] (verification not implemented) 2048
Sympy [F] 2048
Maxima [F] 2048
Giac [A] (verification not implemented) 2049
Mupad [F(-1)] 2049
Reduce [B] (verification not implemented) 2049

Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4} dx = \frac{2a\sqrt{ax^2+bx^3}}{x} + \frac{2(ax^2+bx^3)^{3/2}}{3x^3} - 2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)$$

output
```
2*a*(b*x^3+a*x^2)^(1/2)/x+2/3*(b*x^3+a*x^2)^(3/2)/x^3-2*a^(3/2)*arctanh(a^
(1/2)*x/(b*x^3+a*x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4} dx = \frac{2x\sqrt{a+bx}\left(\sqrt{a+bx}(4a+bx) - 3a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{3\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(3/2)/x^4,x]
```

output
```
(2*x*Sqrt[a + b*x]*(Sqrt[a + b*x]*(4*a + b*x) - 3*a^(3/2)*ArcTanh[Sqrt[a +
 b*x]/Sqrt[a]]))/(3*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1927, 1927, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{1927}$$

$$a\int \frac{\sqrt{bx^3+ax^2}}{x^2}dx+\frac{2(ax^2+bx^3)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{1927}$$

$$a\left(a\int \frac{1}{\sqrt{bx^3+ax^2}}dx+\frac{2\sqrt{ax^2+bx^3}}{x}\right)+\frac{2(ax^2+bx^3)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{1914}$$

$$a\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2a\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}\right)+\frac{2(ax^2+bx^3)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{219}$$

$$a\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)\right)+\frac{2(ax^2+bx^3)^{3/2}}{3x^3}$$

input
```
Int[(a*x^2 + b*x^3)^(3/2)/x^4,x]
```

output
```
(2*(a*x^2 + b*x^3)^(3/2))/(3*x^3) + a*((2*Sqrt[a*x^2 + b*x^3])/x - 2*Sqrt[
a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{2\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}} \left(-3 a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right)+\left(b x +a \right)^{\frac{3}{2}}+3 a \sqrt{b x +a}\right)}{3 x^{3} \left(b x +a \right)^{\frac{3}{2}}}$	61
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) b^{3} x^{3}-\left(\sqrt{a}\, b^{2} x^{2}+\frac{14 a^{\frac{3}{2}} b x}{3}+\frac{8 a^{\frac{5}{2}}}{3}\right) \sqrt{b x +a}}{8 a^{\frac{3}{2}} x^{3}}$	61

input
```
int((b*x^3+a*x^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
2/3*(b*x^3+a*x^2)^(3/2)*(-3*a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+(b*x+a)
^(3/2)+3*a*(b*x+a)^(1/2))/x^3/(b*x+a)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.82

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx = \left[\frac{3\,a^{\frac{3}{2}}x\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}(bx+4\,a)}{3\,x}, \frac{2\left(3\,\sqrt{-a}ax\arctan\left(\right.\right.}{}\right.$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[1/3*(3*a^(3/2)*x*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2)
 + 2*sqrt(b*x^3 + a*x^2)*(b*x + 4*a))/x, 2/3*(3*sqrt(-a)*a*x*arctan(sqrt(b
*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + sqrt(b*x^3 + a*x^2)*(b*x + 4*a))/x
]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx = \int \frac{\left(x^2(a+bx)\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(3/2)/x**4,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)/x^4, x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx = \frac{2\,a^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}} + \frac{2}{3}\,(bx+a)^{\frac{3}{2}}\operatorname{sgn}(x) + 2\,\sqrt{bx+a}\,a\operatorname{sgn}(x) - \frac{2\left(3\,a^2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+4\,\sqrt{-a}a^{\frac{3}{2}}\right)\operatorname{sgn}(x)}{3\,\sqrt{-a}}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="giac")
```

output
```
2*a^2*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2/3*(b*x + a)^(3/2)
*sgn(x) + 2*sqrt(b*x + a)*a*sgn(x) - 2/3*(3*a^2*arctan(sqrt(a)/sqrt(-a)) +
 4*sqrt(-a)*a^(3/2))*sgn(x)/sqrt(-a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx = \int \frac{(b\,x^3+a\,x^2)^{3/2}}{x^4}\,dx$$

input
```
int((a*x^2 + b*x^3)^(3/2)/x^4,x)
```

output
```
int((a*x^2 + b*x^3)^(3/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.69

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx = \frac{8\sqrt{bx+a}\,a}{3} + \frac{2\sqrt{bx+a}\,bx}{3} + \sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)a - \sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)a$$

input
```
int((b*x^3+a*x^2)^(3/2)/x^4,x)
```

output
```
(8*sqrt(a + b*x)*a + 2*sqrt(a + b*x)*b*x + 3*sqrt(a)*log(sqrt(a + b*x) - s
qrt(a))*a - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*a)/3
```

3.226 $\int \frac{(ax^2+bx^3)^{3/2}}{x^5} dx$

Optimal result 2051
Mathematica [A] (verified) 2051
Rubi [A] (verified) 2052
Maple [A] (verified) 2053
Fricas [A] (verification not implemented) 2054
Sympy [F] 2054
Maxima [F] 2055
Giac [A] (verification not implemented) 2055
Mupad [F(-1)] 2055
Reduce [B] (verification not implemented) 2056

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5} dx = \frac{3b\sqrt{ax^2+bx^3}}{x} - \frac{(ax^2+bx^3)^{3/2}}{x^4} - 3\sqrt{a}b\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)$$

output
```
3*b*(b*x^3+a*x^2)^(1/2)/x-(b*x^3+a*x^2)^(3/2)/x^4-3*a^(1/2)*b*arctanh(a^(1
/2)*x/(b*x^3+a*x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5} dx = -\frac{\sqrt{a+bx}\left((a-2bx)\sqrt{a+bx}+3\sqrt{a}bx\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(3/2)/x^5,x]
```

output
```
-((Sqrt[a + b*x]*((a - 2*b*x)*Sqrt[a + b*x] + 3*Sqrt[a]*b*x*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]]))/Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1926, 1927, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}dx$$

$$\Big\downarrow \text{ 1926}$$

$$\frac{3}{2}b\int \frac{\sqrt{bx^3+ax^2}}{x^2}dx-\frac{(ax^2+bx^3)^{3/2}}{x^4}$$

$$\Big\downarrow \text{ 1927}$$

$$\frac{3}{2}b\left(a\int \frac{1}{\sqrt{bx^3+ax^2}}dx+\frac{2\sqrt{ax^2+bx^3}}{x}\right)-\frac{(ax^2+bx^3)^{3/2}}{x^4}$$

$$\Big\downarrow \text{ 1914}$$

$$\frac{3}{2}b\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2a\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}\right)-\frac{(ax^2+bx^3)^{3/2}}{x^4}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{3}{2}b\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)\right)-\frac{(ax^2+bx^3)^{3/2}}{x^4}$$

input
```
Int[(a*x^2 + b*x^3)^(3/2)/x^5,x]
```

output
```
-((a*x^2 + b*x^3)^(3/2)/x^4) + (3*b*((2*Sqrt[a*x^2 + b*x^3])/x - 2*Sqrt[a]
*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

method	result	size
risch	$-\frac{a\sqrt{x^{2}(bx+a)}}{x^{2}}+\frac{b\left(4\sqrt{bx+a}-6\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)\sqrt{x^{2}(bx+a)}}{2x\sqrt{bx+a}}$	70
default	$-\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(-2bx\sqrt{bx+a}\,\sqrt{a}+3\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)abx+a^{\frac{3}{2}}\sqrt{bx+a}\right)}{x^{4}(bx+a)^{\frac{3}{2}}\sqrt{a}}$	72
pseudoelliptic	$\frac{-\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{4}x^{4}}{64}+\frac{3\sqrt{bx+a}\,\left(\sqrt{a}\,b^{3}x^{3}-\frac{2a^{\frac{3}{2}}b^{2}x^{2}}{3}-8a^{\frac{5}{2}}bx-\frac{16a^{\frac{7}{2}}}{3}\right)}{64}}{a^{\frac{5}{2}}x^{4}}$	72

input
```
int((b*x^3+a*x^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-a/x^2*(x^2*(b*x+a))^(1/2)+1/2*b*(4*(b*x+a)^(1/2)-6*a^(1/2)*arctanh((b*x+a
)^(1/2)/a^(1/2)))*(x^2*(b*x+a))^(1/2)/x/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.93

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx = \left[\frac{3\sqrt{a}bx^2\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}(2\,bx-a)}{2\,x^2}, \frac{3\sqrt{-a}bx^2\arctan\left(\right.}{}\right.$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
[1/2*(3*sqrt(a)*b*x^2*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/
x^2) + 2*sqrt(b*x^3 + a*x^2)*(2*b*x - a))/x^2, (3*sqrt(-a)*b*x^2*arctan(sq
rt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + sqrt(b*x^3 + a*x^2)*(2*b*x - a
))/x^2]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx = \int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(3/2)/x**5,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)/x**5, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx=\int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)/x^5, x)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx=\left(\frac{3\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}}+2\sqrt{bx+a}\operatorname{sgn}(x)-\frac{\sqrt{bx+a}\,a\operatorname{sgn}(x)}{bx}\right)b$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="giac")
```

output
```
(3*a*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*sqrt(b*x + a)*sgn(
x) - sqrt(b*x + a)*a*sgn(x)/(b*x))*b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx=\int \frac{(b\,x^3+a\,x^2)^{3/2}}{x^5}\,dx$$

input
```
int((a*x^2 + b*x^3)^(3/2)/x^5,x)
```

output
```
int((a*x^2 + b*x^3)^(3/2)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \frac{(ax^2 + bx^3)^{3/2}}{x^5} \, dx = \frac{-2\sqrt{bx+a}\,a + 4\sqrt{bx+a}\,bx + 3\sqrt{a}\log\left(\sqrt{bx+a} - \sqrt{a}\right)bx - 3\sqrt{a}\log\left(\sqrt{bx+a} + \right.}{2x}$$

input

```
int((b*x^3+a*x^2)^(3/2)/x^5,x)
```

output

```
( - 2*sqrt(a + b*x)*a + 4*sqrt(a + b*x)*b*x + 3*sqrt(a)*log(sqrt(a + b*x)
- sqrt(a))*b*x - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b*x)/(2*x)
```

3.227 $\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx$

Optimal result 2057
Mathematica [A] (verified) 2057
Rubi [A] (verified) 2058
Maple [A] (verified) 2059
Fricas [A] (verification not implemented) 2060
Sympy [F] 2060
Maxima [F] 2060
Giac [A] (verification not implemented) 2061
Mupad [F(-1)] 2061
Reduce [B] (verification not implemented) 2061

Optimal result

Integrand size = 19, antiderivative size = 81

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx=-\frac{3b\sqrt{ax^2+bx^3}}{4x^2}-\frac{(ax^2+bx^3)^{3/2}}{2x^5}-\frac{3b^2\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4\sqrt{a}}$$

output
```
-3/4*b*(b*x^3+a*x^2)^(1/2)/x^2-1/2*(b*x^3+a*x^2)^(3/2)/x^5-3/4*b^2*arctanh
(a^(1/2)*x/(b*x^3+a*x^2)^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx=-\frac{\sqrt{x^2(a+bx)}\left(\sqrt{a}\sqrt{a+bx}(2a+5bx)+3b^2x^2\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{4\sqrt{a}x^3\sqrt{a+bx}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(3/2)/x^6,x]
```

output
```
-1/4*(Sqrt[x^2*(a + b*x)]*(Sqrt[a]*Sqrt[a + b*x]*(2*a + 5*b*x) + 3*b^2*x^2
*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(Sqrt[a]*x^3*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1926, 1926, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^6} \, dx$$

$$\Big\downarrow \text{ 1926}$$

$$\frac{3}{4} b \int \frac{\sqrt{bx^3+ax^2}}{x^3}dx-\frac{\left(ax^2+bx^3\right)^{3/2}}{2x^5}$$

$$\Big\downarrow \text{ 1926}$$

$$\frac{3}{4} b \left(\frac{1}{2} b \int \frac{1}{\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{x^2}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{2x^5}$$

$$\Big\downarrow \text{ 1914}$$

$$\frac{3}{4} b \left(-b \int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}-\frac{\sqrt{ax^2+bx^3}}{x^2}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{2x^5}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{3}{4} b \left(-\frac{b \text{arctanh}\left(\frac{\sqrt{a} x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}-\frac{\sqrt{ax^2+bx^3}}{x^2}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{2x^5}$$

input
```
Int[(a*x^2 + b*x^3)^(3/2)/x^6,x]
```

output
```
-1/2*(a*x^2 + b*x^3)^(3/2)/x^5 + (3*b*(-(Sqrt[a*x^2 + b*x^3]/x^2) - (b*Arc
Tanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/Sqrt[a]))/4
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{(5bx+2a)\sqrt{x^2(bx+a)}}{4x^3}-\frac{3b^2\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{x^2(bx+a)}}{4\sqrt{a}\,x\sqrt{bx+a}}$	67
default	$\frac{(b\,x^3+a\,x^2)^{\frac{3}{2}}\left(-3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^2x^2+3a^{\frac{3}{2}}\sqrt{bx+a}-5(bx+a)^{\frac{3}{2}}\sqrt{a}\right)}{4x^5(bx+a)^{\frac{3}{2}}\sqrt{a}}$	74
pseudoelliptic	$-\frac{-\frac{15\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^5x^5}{128}+\sqrt{bx+a}\left(\frac{15\sqrt{a}\,b^4x^4}{128}-\frac{5a^{\frac{3}{2}}b^3x^3}{64}+\frac{a^{\frac{5}{2}}b^2x^2}{16}+\frac{11a^{\frac{7}{2}}bx}{8}+a^{\frac{9}{2}}\right)}{5a^{\frac{7}{2}}x^5}$	82

input
```
int((b*x^3+a*x^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(5*b*x+2*a)/x^3*(x^2*(b*x+a))^(1/2)-3/4*b^2/a^(1/2)*arctanh((b*x+a)^(
1/2)/a^(1/2))*(x^2*(b*x+a))^(1/2)/x/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.96

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx = \left[\frac{3\sqrt{a}b^2x^3\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}(5\,abx+2\,a^2)}{8\,ax^3}, \frac{3\,\sqrt{-a}b^2x^3\,\mathrm{ar}}{}\right.$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
[1/8*(3*sqrt(a)*b^2*x^3*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a)
)/x^2) - 2*sqrt(b*x^3 + a*x^2)*(5*a*b*x + 2*a^2))/(a*x^3), 1/4*(3*sqrt(-a)
*b^2*x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) - sqrt(b*x^3 +
 a*x^2)*(5*a*b*x + 2*a^2))/(a*x^3)]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx = \int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(3/2)/x**6,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)/x**6, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)/x^6, x)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx = \frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}} - \frac{5\,(bx+a)^{\frac{3}{2}}b^3\mathrm{sgn}(x) - 3\,\sqrt{bx+a}ab^3\mathrm{sgn}(x)}{b^2x^2}}{4\,b}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="giac")
```

output
```
1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) - (5*(b*x + a)^(
3/2)*b^3*sgn(x) - 3*sqrt(b*x + a)*a*b^3*sgn(x))/(b^2*x^2))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx = \int \frac{(b\,x^3+a\,x^2)^{3/2}}{x^6}\,dx$$

input
```
int((a*x^2 + b*x^3)^(3/2)/x^6,x)
```

output
```
int((a*x^2 + b*x^3)^(3/2)/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx = \frac{-4\sqrt{bx+a}\,a^2 - 10\sqrt{bx+a}\,abx + 3\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2x^2 - 3\sqrt{a}\log\left(\sqrt{bx+\right.}{8a\,x^2}$$

input
```
int((b*x^3+a*x^2)^(3/2)/x^6,x)
```

output
```
( - 4*sqrt(a + b*x)*a**2 - 10*sqrt(a + b*x)*a*b*x + 3*sqrt(a)*log(sqrt(a +
 b*x) - sqrt(a))*b**2*x**2 - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**2*x
**2)/(8*a*x**2)
```

3.228 $\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx$

Optimal result 2062
Mathematica [A] (verified) 2062
Rubi [A] (verified) 2063
Maple [A] (verified) 2065
Fricas [A] (verification not implemented) 2065
Sympy [F] 2066
Maxima [F] 2066
Giac [A] (verification not implemented) 2066
Mupad [F(-1)] 2067
Reduce [B] (verification not implemented) 2067

Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx = -\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{b^2\sqrt{ax^2+bx^3}}{8ax^2} -\frac{(ax^2+bx^3)^{3/2}}{3x^6}+\frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{3/2}}$$

output
```
-1/4*b*(b*x^3+a*x^2)^(1/2)/x^3-1/8*b^2*(b*x^3+a*x^2)^(1/2)/a/x^2-1/3*(b*x^
3+a*x^2)^(3/2)/x^6+1/8*b^3*arctanh(a^(1/2)*x/(b*x^3+a*x^2)^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx = \frac{\sqrt{x^2(a+bx)}\left(-\sqrt{a}\sqrt{a+bx}(8a^2+14abx+3b^2x^2)+3b^3x^3\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{24a^{3/2}x^4\sqrt{a+bx}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(3/2)/x^7,x]
```

output
```
(Sqrt[x^2*(a + b*x)]*(-(Sqrt[a]*Sqrt[a + b*x]*(8*a^2 + 14*a*b*x + 3*b^2*x^
2)) + 3*b^3*x^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(24*a^(3/2)*x^4*Sqrt[a +
b*x])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1926, 1926, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{2}b\int \frac{\sqrt{bx^3+ax^2}}{x^4}dx-\frac{(ax^2+bx^3)^{3/2}}{3x^6}$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{2}b\left(\frac{1}{4}b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)-\frac{(ax^2+bx^3)^{3/2}}{3x^6}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)-\frac{(ax^2+bx^3)^{3/2}}{3x^6}$$

$$\Big\downarrow \text{1914}$$

$$\frac{1}{2}b\left(\frac{1}{4}b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)-\frac{(ax^2+bx^3)^{3/2}}{3x^6}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}b\left(\frac{1}{4}b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)-\frac{(ax^2+bx^3)^{3/2}}{3x^6}$$

input
```
Int[(a*x^2 + b*x^3)^(3/2)/x^7,x]
```

output
```
-1/3*(a*x^2 + b*x^3)^(3/2)/x^6 + (b*(-1/2*Sqrt[a*x^2 + b*x^3]/x^3 + (b*(-(
Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
)/a^(3/2)))/4))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{(3b^2x^2+14abx+8a^2)\sqrt{x^2(bx+a)}}{24x^4a}+\frac{b^3\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{x^2(bx+a)}}{8a^{\frac{3}{2}}x\sqrt{bx+a}}$	81
default	$-\frac{(b\,x^3+a\,x^2)^{\frac{3}{2}}\left(3(bx+a)^{\frac{5}{2}}a^{\frac{3}{2}}-3\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a\,b^3x^3+8(bx+a)^{\frac{3}{2}}a^{\frac{5}{2}}-3a^{\frac{7}{2}}\sqrt{bx+a}\right)}{24x^6(bx+a)^{\frac{3}{2}}a^{\frac{5}{2}}}$	87
pseudoelliptic	$-\frac{13\left(\frac{105\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^6x^6}{1664}+\sqrt{bx+a}\left(-\frac{105\sqrt{a}\,b^5x^5}{1664}+\frac{35a^{\frac{3}{2}}b^4x^4}{832}-\frac{7a^{\frac{5}{2}}b^3x^3}{208}+\frac{3a^{\frac{7}{2}}b^2x^2}{104}+a^{\frac{9}{2}}bx+\frac{10a^{\frac{11}{2}}}{13}\right)\right)}{60a^{\frac{9}{2}}x^6}$	94

input
```
int((b*x^3+a*x^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(3*b^2*x^2+14*a*b*x+8*a^2)/x^4/a*(x^2*(b*x+a))^(1/2)+1/8/a^(3/2)*b^3
*arctanh((b*x+a)^(1/2)/a^(1/2))*(x^2*(b*x+a))^(1/2)/x/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.65

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx = \left[\frac{3\sqrt{a}b^3x^4\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\left(3\,ab^2x^2+14\,a^2bx+8\,a^3\right)\sqrt{bx^3+ax^2}}{48\,a^2x^4},\right.$$
$$\left.-\frac{3\sqrt{-a}b^3x^4\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+\left(3\,ab^2x^2+14\,a^2bx+8\,a^3\right)\sqrt{bx^3+ax^2}}{24\,a^2x^4}\right]$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
[1/48*(3*sqrt(a)*b^3*x^4*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a
))/x^2) - 2*(3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^2*x
^4), -1/24*(3*sqrt(-a)*b^3*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2
+ a*x)) + (3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^2*x^4
)]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx = \int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(3/2)/x**7,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)/x**7, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)/x^7, x)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.73

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx = -\frac{1}{24}b^3\left(\frac{3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}a} + \frac{3(bx+a)^{\frac{5}{2}}\operatorname{sgn}(x)+8(bx+a)^{\frac{3}{2}}a\operatorname{sgn}(x)-3\sqrt{bx+a}a^2\operatorname{sgn}(x)}{ab^3x^3}\right)$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="giac")
```

output
```
-1/24*b^3*(3*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a) + (3*(b*x
+ a)^(5/2)*sgn(x) + 8*(b*x + a)^(3/2)*a*sgn(x) - 3*sqrt(b*x + a)*a^2*sgn(x
))/(a*b^3*x^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx = \int \frac{(b\,x^3+a\,x^2)^{3/2}}{x^7}\,dx$$

input
```
int((a*x^2 + b*x^3)^(3/2)/x^7,x)
```

output
```
int((a*x^2 + b*x^3)^(3/2)/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx = \frac{-16\sqrt{bx+a}\,a^3-28\sqrt{bx+a}\,a^2bx-6\sqrt{bx+a}\,a\,b^2x^2-3\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^3}{48a^2x^3}$$

input
```
int((b*x^3+a*x^2)^(3/2)/x^7,x)
```

output
```
( - 16*sqrt(a + b*x)*a**3 - 28*sqrt(a + b*x)*a**2*b*x - 6*sqrt(a + b*x)*a*
b**2*x**2 - 3*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**3*x**3 + 3*sqrt(a)*l
og(sqrt(a + b*x) + sqrt(a))*b**3*x**3)/(48*a**2*x**3)
```

3.229 $\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx$

Optimal result 2068
Mathematica [A] (verified) 2068
Rubi [A] (verified) 2069
Maple [A] (verified) 2071
Fricas [A] (verification not implemented) 2072
Sympy [F] 2072
Maxima [F] 2072
Giac [A] (verification not implemented) 2073
Mupad [F(-1)] 2073
Reduce [B] (verification not implemented) 2073

Optimal result

Integrand size = 19, antiderivative size = 137

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = -\frac{b\sqrt{ax^2+bx^3}}{8x^4} - \frac{b^2\sqrt{ax^2+bx^3}}{32ax^3} + \frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2} - \frac{(ax^2+bx^3)^{3/2}}{4x^7} - \frac{3b^4\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{64a^{5/2}}$$

output

```
-1/8*b*(b*x^3+a*x^2)^(1/2)/x^4-1/32*b^2*(b*x^3+a*x^2)^(1/2)/a/x^3+3/64*b^3
*(b*x^3+a*x^2)^(1/2)/a^2/x^2-1/4*(b*x^3+a*x^2)^(3/2)/x^7-3/64*b^4*arctanh(
a^(1/2)*x/(b*x^3+a*x^2)^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.76

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = -\frac{\sqrt{x^2(a+bx)}\left(\sqrt{a}\sqrt{a+bx}(16a^3+24a^2bx+2ab^2x^2-3b^3x^3)+3b^4x^4\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{64a^{5/2}x^5\sqrt{a+bx}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(3/2)/x^8,x]
```

output
```
-1/64*(Sqrt[x^2*(a + b*x)]*(Sqrt[a]*Sqrt[a + b*x]*(16*a^3 + 24*a^2*b*x + 2
*a*b^2*x^2 - 3*b^3*x^3) + 3*b^4*x^4*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(a^(5
/2)*x^5*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1926, 1926, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^8}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{3}{8}b\int \frac{\sqrt{bx^3+ax^2}}{x^5}dx-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}$$

$$\Big\downarrow \text{1926}$$

$$\frac{3}{8}b\left(\frac{1}{6}b\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}$$

$$\Big\downarrow \text{1931}$$

$$\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}$$

$$\Big\downarrow \text{1931}$$

$$\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}$$

$$\Big\downarrow \text{1914}$$

$$\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}$$

input
```
Int[(a*x^2 + b*x^3)^(3/2)/x^8,x]
```

output
```
-1/4*(a*x^2 + b*x^3)^(3/2)/x^7 + (3*b*(-1/3*Sqrt[a*x^2 + b*x^3]/x^4 + (b*(
-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) +
(b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/(4*a)))/6))/8
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.67

method	result
risch	$-\frac{\left(-3 b^{3} x^{3}+2 a \,b^{2} x^{2}+24 a^{2} b x +16 a^{3}\right) \sqrt{x^{2} \left(b x +a \right)}}{64 x^{5} a^{2}}-\frac{3 b^{4} \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) \sqrt{x^{2} \left(b x +a \right)}}{64 a^{\frac{5}{2}} x \sqrt{b x +a}}$
default	$\frac{\left(b \,x^{3}+a \,x^{2}\right)^{\frac{3}{2}} \left(3 \left(b x +a \right)^{\frac{7}{2}} a^{\frac{5}{2}}-11 \left(b x +a \right)^{\frac{5}{2}} a^{\frac{7}{2}}-3 \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) a^{2} b^{4} x^{4}-11 \left(b x +a \right)^{\frac{3}{2}} a^{\frac{9}{2}}+3 a^{\frac{11}{2}} \sqrt{b x +a}\right)}{64 x^{7} \left(b x +a \right)^{\frac{3}{2}} a^{\frac{9}{2}}}$
pseudoelliptic	$-\frac{5 \left(-\frac{63 \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) x^{7} b^{7}}{1280}+\sqrt{b x +a}\, \left(\frac{63 \sqrt{a}\, b^{6} x^{6}}{1280}-\frac{21 a^{\frac{3}{2}} b^{5} x^{5}}{640}+\frac{21 a^{\frac{5}{2}} b^{4} x^{4}}{800}-\frac{9 a^{\frac{7}{2}} b^{3} x^{3}}{400}+\frac{a^{\frac{9}{2}} b^{2} x^{2}}{50}+a^{\frac{11}{2}} b x +\frac{4 a^{\frac{13}{2}}}{5}\right)\right)}{28 a^{\frac{11}{2}} x^{7}}$

input
```
int((b*x^3+a*x^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/64*(-3*b^3*x^3+2*a*b^2*x^2+24*a^2*b*x+16*a^3)/x^5/a^2*(x^2*(b*x+a))^(1/
2)-3/64/a^(5/2)*b^4*arctanh((b*x+a)^(1/2)/a^(1/2))*(x^2*(b*x+a))^(1/2)/x/(
b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.47

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = \left[\frac{3\sqrt{a}b^4x^5\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(3\,ab^3x^3-2\,a^2b^2x^2-24\,a^3bx-16\,a^4)\sqrt{b}}{128\,a^3x^5}\right.$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
[1/128*(3*sqrt(a)*b^4*x^5*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(
a))/x^2) + 2*(3*a*b^3*x^3 - 2*a^2*b^2*x^2 - 24*a^3*b*x - 16*a^4)*sqrt(b*x^
3 + a*x^2))/(a^3*x^5), 1/64*(3*sqrt(-a)*b^4*x^5*arctan(sqrt(b*x^3 + a*x^2)
*sqrt(-a)/(b*x^2 + a*x)) + (3*a*b^3*x^3 - 2*a^2*b^2*x^2 - 24*a^3*b*x - 16*
a^4)*sqrt(b*x^3 + a*x^2))/(a^3*x^5)]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = \int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(3/2)/x**8,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)/x**8, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)/x^8, x)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = \frac{\frac{3\,b^5\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}a^2} + \frac{3\,(bx+a)^{\frac{7}{2}}b^5\mathrm{sgn}(x)-11\,(bx+a)^{\frac{5}{2}}ab^5\mathrm{sgn}(x)-11\,(bx+a)^{\frac{3}{2}}a^2b^5\mathrm{sgn}(x)+3\,\sqrt{bx+a}a^3b}{a^2b^4x^4}}{64\,b}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="giac")
```

output
```
1/64*(3*b^5*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^2) + (3*(b*x
 + a)^(7/2)*b^5*sgn(x) - 11*(b*x + a)^(5/2)*a*b^5*sgn(x) - 11*(b*x + a)^(3
/2)*a^2*b^5*sgn(x) + 3*sqrt(b*x + a)*a^3*b^5*sgn(x))/(a^2*b^4*x^4))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = \int \frac{{\left(b\,x^3+a\,x^2\right)}^{3/2}}{x^8}\,dx$$

input
```
int((a*x^2 + b*x^3)^(3/2)/x^8,x)
```

output
```
int((a*x^2 + b*x^3)^(3/2)/x^8, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.78

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = \frac{-32\sqrt{bx+a}\,a^4-48\sqrt{bx+a}\,a^3bx-4\sqrt{bx+a}\,a^2b^2x^2+6\sqrt{bx+a}\,a\,b^3x^3+3\sqrt{a}\log}{128a^3x^4}$$

input
```
int((b*x^3+a*x^2)^(3/2)/x^8,x)
```

output
```
( - 32*sqrt(a + b*x)*a**4 - 48*sqrt(a + b*x)*a**3*b*x - 4*sqrt(a + b*x)*a*
*2*b**2*x**2 + 6*sqrt(a + b*x)*a*b**3*x**3 + 3*sqrt(a)*log(sqrt(a + b*x) -
 sqrt(a))*b**4*x**4 - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**4*x**4)/(1
28*a**3*x**4)
```

3.230 $\int \frac{(ax^2+bx^3)^{3/2}}{x^9}\,dx$

Optimal result . . . 2075
Mathematica [A] (verified) . . . 2075
Rubi [A] (verified) . . . 2076
Maple [A] (verified) . . . 2079
Fricas [A] (verification not implemented) . . . 2079
Sympy [F] . . . 2080
Maxima [F] . . . 2080
Giac [A] (verification not implemented) . . . 2081
Mupad [F(-1)] . . . 2081
Reduce [B] (verification not implemented) . . . 2082

Optimal result

Integrand size = 19, antiderivative size = 165

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^9}\,dx = -\frac{3b\sqrt{ax^2+bx^3}}{40x^5} - \frac{b^2\sqrt{ax^2+bx^3}}{80ax^4} + \frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3} - \frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2} - \frac{(ax^2+bx^3)^{3/2}}{5x^8} + \frac{3b^5\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{128a^{7/2}}$$

output
```
-3/40*b*(b*x^3+a*x^2)^(1/2)/x^5-1/80*b^2*(b*x^3+a*x^2)^(1/2)/a/x^4+1/64*b^
3*(b*x^3+a*x^2)^(1/2)/a^2/x^3-3/128*b^4*(b*x^3+a*x^2)^(1/2)/a^3/x^2-1/5*(b
*x^3+a*x^2)^(3/2)/x^8+3/128*b^5*arctanh(a^(1/2)*x/(b*x^3+a*x^2)^(1/2))/a^(
7/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.70

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^9}\,dx = \frac{\sqrt{x^2(a+bx)}\left(-\sqrt{a}\sqrt{a+bx}(128a^4+176a^3bx+8a^2b^2x^2-10ab^3x^3+15b^4x^4)+15\right.}{640a^{7/2}x^6\sqrt{a+bx}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(3/2)/x^9,x]
```

output
```
(Sqrt[x^2*(a + b*x)]*(-(Sqrt[a]*Sqrt[a + b*x]*(128*a^4 + 176*a^3*b*x + 8*a
^2*b^2*x^2 - 10*a*b^3*x^3 + 15*b^4*x^4)) + 15*b^5*x^5*ArcTanh[Sqrt[a + b*x
]/Sqrt[a]]))/(640*a^(7/2)*x^6*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1926, 1926, 1931, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^9}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{3}{10}b\int \frac{\sqrt{bx^3+ax^2}}{x^6}dx-\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}$$

$$\Big\downarrow \text{1926}$$

$$\frac{3}{10}b\left(\frac{1}{8}b\int \frac{1}{x^3\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{4x^5}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}$$

$$\Big\downarrow \text{1931}$$

$$\frac{3}{10}b\left(\frac{1}{8}b\left(-\frac{5b\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3}}{4x^5}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}$$

$$\Big\downarrow \text{1931}$$

$$\frac{3}{10}b\left(\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3}}{4x^5}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}$$

$$\Big\downarrow \text{1931}$$

$$\frac{3}{10}b\left(\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3}}{4x^5}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}$$

$\big\downarrow$ 1914

$$\frac{3}{10}b\left(\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3}}{4x^5}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}$$

$\big\downarrow$ 219

$$\frac{3}{10}b\left(\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3}}{4x^5}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}$$

input
```
Int[(a*x^2 + b*x^3)^(3/2)/x^9,x]
```

output
```
-1/5*(a*x^2 + b*x^3)^(3/2)/x^8 + (3*b*(-1/4*Sqrt[a*x^2 + b*x^3]/x^5 + (b*(
-1/3*Sqrt[a*x^2 + b*x^3]/(a*x^4) - (5*b*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3)
- (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2
 + b*x^3]])/a^(3/2)))/(4*a)))/(6*a)))/8))/10
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.62

method	result
risch	$-\frac{\left(15 b^{4} x^{4}-10 a \,b^{3} x^{3}+8 a^{2} b^{2} x^{2}+176 a^{3} b x +128 a^{4}\right) \sqrt{x^{2} \left(b x +a \right)}}{640 x^{6} a^{3}}+\frac{3 b^{5} \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) \sqrt{x^{2} \left(b x +a \right)}}{128 a^{\frac{7}{2}} x \sqrt{b x +a}}$
default	$-\frac{\left(b \,x^{3}+a \,x^{2}\right)^{\frac{3}{2}} \left(15 \left(b x +a \right)^{\frac{9}{2}} a^{\frac{7}{2}}-70 \left(b x +a \right)^{\frac{7}{2}} a^{\frac{9}{2}}+128 \left(b x +a \right)^{\frac{5}{2}} a^{\frac{11}{2}}-15 \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) a^{3} b^{5} x^{5}+70 \left(b x +a \right)^{\frac{3}{2}} a^{\frac{13}{2}}-15 \sqrt{b}}{640 x^{8} \left(b x +a \right)^{\frac{3}{2}} a^{\frac{13}{2}}}$
pseudoelliptic	$-\frac{\frac{693 \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) b^{8} x^{8}}{256}+\sqrt{b x +a}\, \left(-\frac{693 \sqrt{a}\, b^{7} x^{7}}{256}+\frac{231 a^{\frac{3}{2}} b^{6} x^{6}}{128}-\frac{231 a^{\frac{5}{2}} b^{5} x^{5}}{160}+\frac{99 a^{\frac{7}{2}} b^{4} x^{4}}{80}-\frac{11 a^{\frac{9}{2}} b^{3} x^{3}}{10}+a^{\frac{11}{2}} b^{2} x^{2}+68 a^{\frac{13}{2}} b\right.}{448 a^{\frac{13}{2}} x^{8}}$

input
```
int((b*x^3+a*x^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/640*(15*b^4*x^4-10*a*b^3*x^3+8*a^2*b^2*x^2+176*a^3*b*x+128*a^4)/x^6/a^3
*(x^2*(b*x+a))^(1/2)+3/128*b^5/a^(7/2)*arctanh((b*x+a)^(1/2)/a^(1/2))*(x^2
*(b*x+a))^(1/2)/x/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.36

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^9}\,dx = \left[\frac{15\,\sqrt{a}b^5x^6\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,(15\,ab^4x^4-10\,a^2b^3x^3+8\,a^3b^2x^2+176\,}{1280\,a^4x^6}\right.$$
$$\left.-\frac{15\,\sqrt{-a}b^5x^6\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+(15\,ab^4x^4-10\,a^2b^3x^3+8\,a^3b^2x^2+176\,a^4bx+128\,a^5)\sqrt{bx^3+ax^2}}{640\,a^4x^6}\right]$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
[1/1280*(15*sqrt(a)*b^5*x^6*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqr
t(a))/x^2) - 2*(15*a*b^4*x^4 - 10*a^2*b^3*x^3 + 8*a^3*b^2*x^2 + 176*a^4*b*
x + 128*a^5)*sqrt(b*x^3 + a*x^2))/(a^4*x^6), -1/640*(15*sqrt(-a)*b^5*x^6*a
rctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + (15*a*b^4*x^4 - 10*a^2
*b^3*x^3 + 8*a^3*b^2*x^2 + 176*a^4*b*x + 128*a^5)*sqrt(b*x^3 + a*x^2))/(a^
4*x^6)]
```

Sympy [F]

$$\int \frac{(ax^2 + bx^3)^{3/2}}{x^9} \, dx = \int \frac{(x^2(a + bx))^{\frac{3}{2}}}{x^9} \, dx$$

input
```
integrate((b*x**3+a*x**2)**(3/2)/x**9,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)/x**9, x)
```

Maxima [F]

$$\int \frac{(ax^2 + bx^3)^{3/2}}{x^9} \, dx = \int \frac{(bx^3 + ax^2)^{\frac{3}{2}}}{x^9} \, dx$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)/x^9, x)
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.65

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^9}\,dx = -\frac{1}{640}b^5\left(\frac{15\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}a^3}+\frac{15\,(bx+a)^{\frac{9}{2}}\mathrm{sgn}(x)-70\,(bx+a)^{\frac{7}{2}}a\mathrm{sgn}(x)+128\,(bx+a)^{\frac{5}{2}}a^2\mathrm{sgn}(x$$

$$a^3b^5x^5$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="giac")
```

output
```
-1/640*b^5*(15*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^3) + (15*
(b*x + a)^(9/2)*sgn(x) - 70*(b*x + a)^(7/2)*a*sgn(x) + 128*(b*x + a)^(5/2)
*a^2*sgn(x) + 70*(b*x + a)^(3/2)*a^3*sgn(x) - 15*sqrt(b*x + a)*a^4*sgn(x))
/(a^3*b^5*x^5))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^9}\,dx = \int \frac{(b\,x^3+a\,x^2)^{3/2}}{x^9}\,dx$$

input
```
int((a*x^2 + b*x^3)^(3/2)/x^9,x)
```

output
```
int((a*x^2 + b*x^3)^(3/2)/x^9, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.75

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^9}\,dx = \frac{-256\sqrt{bx+a}\,a^5 - 352\sqrt{bx+a}\,a^4bx - 16\sqrt{bx+a}\,a^3b^2x^2 + 20\sqrt{bx+a}\,a^2b^3x^3 - 30}{1280}$$

input
```
int((b*x^3+a*x^2)^(3/2)/x^9,x)
```

output
```
( - 256*sqrt(a + b*x)*a**5 - 352*sqrt(a + b*x)*a**4*b*x - 16*sqrt(a + b*x)
*a**3*b**2*x**2 + 20*sqrt(a + b*x)*a**2*b**3*x**3 - 30*sqrt(a + b*x)*a*b**
4*x**4 - 15*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**5*x**5 + 15*sqrt(a)*lo
g(sqrt(a + b*x) + sqrt(a))*b**5*x**5)/(1280*a**4*x**5)
```

3.231 $\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result 2083
Mathematica [A] (verified) 2083
Rubi [A] (verified) 2084
Maple [A] (verified) 2085
Fricas [A] (verification not implemented) 2086
Sympy [F] 2086
Maxima [A] (verification not implemented) 2086
Giac [A] (verification not implemented) 2087
Mupad [B] (verification not implemented) 2087
Reduce [B] (verification not implemented) 2087

Optimal result

Integrand size = 19, antiderivative size = 103

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx = \frac{16a^2\sqrt{ax^2+bx^3}}{35b^3} - \frac{32a^3\sqrt{ax^2+bx^3}}{35b^4x} - \frac{12ax\sqrt{ax^2+bx^3}}{35b^2} + \frac{2x^2\sqrt{ax^2+bx^3}}{7b}$$

output
```
16/35*a^2*(b*x^3+a*x^2)^(1/2)/b^3-32/35*a^3*(b*x^3+a*x^2)^(1/2)/b^4/x-12/3
5*a*x*(b*x^3+a*x^2)^(1/2)/b^2+2/7*x^2*(b*x^3+a*x^2)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.51

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{x^2(a+bx)}(-16a^3+8a^2bx-6ab^2x^2+5b^3x^3)}{35b^4x}$$

input
```
Integrate[x^4/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*Sqrt[x^2*(a + b*x)]*(-16*a^3 + 8*a^2*b*x - 6*a*b^2*x^2 + 5*b^3*x^3))/(3
5*b^4*x)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^2\sqrt{ax^2+bx^3}}{7b}-\frac{6a\int \frac{x^3}{\sqrt{bx^3+ax^2}}dx}{7b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^2\sqrt{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\int \frac{x^2}{\sqrt{bx^3+ax^2}}dx}{5b}\right)}{7b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^2\sqrt{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{2a\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}\right)}{5b}\right)}{7b}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2x^2\sqrt{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\right)}{5b}\right)}{7b}$$

input
```
Int[x^4/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*x^2*Sqrt[a*x^2 + b*x^3])/(7*b) - (6*a*((2*x*Sqrt[a*x^2 + b*x^3])/(5*b)
- (4*a*((2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x
)))/(5*b)))/(7*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.50

method	result	size
trager	$-\frac{2\left(-5 b^{3} x^{3}+6 a \,b^{2} x^{2}-8 a^{2} b x +16 a^{3}\right) \sqrt{b \,x^{3}+a \,x^{2}}}{35 b^{4} x}$	52
risch	$-\frac{2 x \left(b x +a \right) \left(-5 b^{3} x^{3}+6 a \,b^{2} x^{2}-8 a^{2} b x +16 a^{3}\right)}{35 \sqrt{x^{2} \left(b x +a \right)}\, b^{4}}$	53
pseudoelliptic	$\frac{2 \sqrt{b x +a}\, \left(35 b^{4} x^{4}-40 a \,b^{3} x^{3}+48 a^{2} b^{2} x^{2}-64 a^{3} b x +128 a^{4}\right)}{315 b^{5}}$	54
gosper	$-\frac{2 \left(b x +a \right) \left(-5 b^{3} x^{3}+6 a \,b^{2} x^{2}-8 a^{2} b x +16 a^{3}\right) x}{35 b^{4} \sqrt{b \,x^{3}+a \,x^{2}}}$	55
default	$-\frac{2 \left(b x +a \right) \left(-5 b^{3} x^{3}+6 a \,b^{2} x^{2}-8 a^{2} b x +16 a^{3}\right) x}{35 b^{4} \sqrt{b \,x^{3}+a \,x^{2}}}$	55
orering	$-\frac{2 \left(b x +a \right) \left(-5 b^{3} x^{3}+6 a \,b^{2} x^{2}-8 a^{2} b x +16 a^{3}\right) x}{35 b^{4} \sqrt{b \,x^{3}+a \,x^{2}}}$	55

input
```
int(x^4/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*(-5*b^3*x^3+6*a*b^2*x^2-8*a^2*b*x+16*a^3)/b^4/x*(b*x^3+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.50

$$\int \frac{x^4}{\sqrt{ax^2 + bx^3}}\, dx = \frac{2\,(5\,b^3x^3 - 6\,ab^2x^2 + 8\,a^2bx - 16\,a^3)\sqrt{bx^3 + ax^2}}{35\,b^4x}$$

input
```
integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/35*(5*b^3*x^3 - 6*a*b^2*x^2 + 8*a^2*b*x - 16*a^3)*sqrt(b*x^3 + a*x^2)/(b
^4*x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{ax^2 + bx^3}}\, dx = \int \frac{x^4}{\sqrt{x^2\,(a + bx)}}\, dx$$

input
```
integrate(x**4/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**4/sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.51

$$\int \frac{x^4}{\sqrt{ax^2 + bx^3}}\, dx = \frac{2\,(5\,b^4x^4 - ab^3x^3 + 2\,a^2b^2x^2 - 8\,a^3bx - 16\,a^4)}{35\,\sqrt{bx + a}b^4}$$

input
```
integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/35*(5*b^4*x^4 - a*b^3*x^3 + 2*a^2*b^2*x^2 - 8*a^3*b*x - 16*a^4)/(sqrt(b*
x + a)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.62

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx = \frac{32\,a^{\frac{7}{2}}\mathrm{sgn}(x)}{35\,b^4} + \frac{2\left(5\,(bx+a)^{\frac{7}{2}} - 21\,(bx+a)^{\frac{5}{2}}a + 35\,(bx+a)^{\frac{3}{2}}a^2 - 35\,\sqrt{bx+a}a^3\right)}{35\,b^4\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
32/35*a^(7/2)*sgn(x)/b^4 + 2/35*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a
+ 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)/(b^4*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.50

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx = -\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(16\,a^3-8\,a^2\,b\,x+6\,a\,b^2\,x^2-5\,b^3\,x^3\right)}{35\,b^4\,x}$$

input
```
int(x^4/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
-(2*(a*x^2 + b*x^3)^(1/2)*(16*a^3 - 5*b^3*x^3 + 6*a*b^2*x^2 - 8*a^2*b*x))/
(35*b^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.40

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{bx+a}\,(5b^3x^3-6a\,b^2x^2+8a^2bx-16a^3)}{35b^4}$$

input
```
int(x^4/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 16*a**3 + 8*a**2*b*x - 6*a*b**2*x**2 + 5*b**3*x**3))/
(35*b**4)
```

3.232 $\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result . . . 2089
Mathematica [A] (verified) . . . 2089
Rubi [A] (verified) . . . 2090
Maple [A] (verified) . . . 2091
Fricas [A] (verification not implemented) . . . 2092
Sympy [F] . . . 2092
Maxima [A] (verification not implemented) . . . 2092
Giac [A] (verification not implemented) . . . 2093
Mupad [B] (verification not implemented) . . . 2093
Reduce [B] (verification not implemented) . . . 2093

Optimal result

Integrand size = 19, antiderivative size = 75

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx = -\frac{8a\sqrt{ax^2+bx^3}}{15b^2}+\frac{16a^2\sqrt{ax^2+bx^3}}{15b^3x}+\frac{2x\sqrt{ax^2+bx^3}}{5b}$$

output
```
-8/15*a*(b*x^3+a*x^2)^(1/2)/b^2+16/15*a^2*(b*x^3+a*x^2)^(1/2)/b^3/x+2/5*x*
(b*x^3+a*x^2)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.56

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{x^2(a+bx)}(8a^2-4abx+3b^2x^2)}{15b^3x}$$

input
```
Integrate[x^3/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*Sqrt[x^2*(a + b*x)]*(8*a^2 - 4*a*b*x + 3*b^2*x^2))/(15*b^3*x)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\int \frac{x^2}{\sqrt{bx^3+ax^2}}dx}{5b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{2a\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}\right)}{5b}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\right)}{5b}$$

input
```
Int[x^3/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*x*Sqrt[a*x^2 + b*x^3])/(5*b) - (4*a*((2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4
*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)))/(5*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

method	result	size
trager	$\frac{2\left(3b^{2}x^{2}-4abx+8a^{2}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{15b^{3}x}$	41
risch	$\frac{2x(bx+a)\left(3b^{2}x^{2}-4abx+8a^{2}\right)}{15\sqrt{x^{2}(bx+a)}\,b^{3}}$	42
pseudoelliptic	$-\frac{2\sqrt{bx+a}\,\left(-5b^{3}x^{3}+6a\,b^{2}x^{2}-8a^{2}bx+16a^{3}\right)}{35b^{4}}$	43
gosper	$\frac{2(bx+a)\left(3b^{2}x^{2}-4abx+8a^{2}\right)x}{15b^{3}\sqrt{b\,x^{3}+a\,x^{2}}}$	44
default	$\frac{2(bx+a)\left(3b^{2}x^{2}-4abx+8a^{2}\right)x}{15b^{3}\sqrt{b\,x^{3}+a\,x^{2}}}$	44
orering	$\frac{2(bx+a)\left(3b^{2}x^{2}-4abx+8a^{2}\right)x}{15b^{3}\sqrt{b\,x^{3}+a\,x^{2}}}$	44

input
```
int(x^3/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(3*b^2*x^2-4*a*b*x+8*a^2)/b^3/x*(b*x^3+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.53

$$\int \frac{x^3}{\sqrt{ax^2 + bx^3}}\,dx = \frac{2\,(3\,b^2x^2 - 4\,abx + 8\,a^2)\sqrt{bx^3 + ax^2}}{15\,b^3x}$$

input
```
integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*sqrt(b*x^3 + a*x^2)/(b^3*x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{ax^2 + bx^3}}\,dx = \int \frac{x^3}{\sqrt{x^2\,(a + bx)}}\,dx$$

input
```
integrate(x**3/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**3/sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.56

$$\int \frac{x^3}{\sqrt{ax^2 + bx^3}}\,dx = \frac{2\,(3\,b^3x^3 - ab^2x^2 + 4\,a^2bx + 8\,a^3)}{15\,\sqrt{bx + a}b^3}$$

input
```
integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(3*b^3*x^3 - a*b^2*x^2 + 4*a^2*b*x + 8*a^3)/(sqrt(b*x + a)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx = -\frac{16\,a^{\frac{5}{2}}\mathrm{sgn}(x)}{15\,b^3} + \frac{2\left(3\,(bx+a)^{\frac{5}{2}} - 10\,(bx+a)^{\frac{3}{2}}a + 15\,\sqrt{bx+a}a^2\right)}{15\,b^3\mathrm{sgn}\,(x)}$$

input
```
integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-16/15*a^(5/2)*sgn(x)/b^3 + 2/15*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a
 + 15*sqrt(b*x + a)*a^2)/(b^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 8.81 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.53

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{b\,x^3+a\,x^2}\,(8\,a^2-4\,a\,b\,x+3\,b^2\,x^2)}{15\,b^3\,x}$$

input
```
int(x^3/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(8*a^2 + 3*b^2*x^2 - 4*a*b*x))/(15*b^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.40

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{bx+a}\,(3b^2x^2-4abx+8a^2)}{15b^3}$$

input
```
int(x^3/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*(8*a**2 - 4*a*b*x + 3*b**2*x**2))/(15*b**3)
```

3.233 $\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result 2094
Mathematica [A] (verified) 2094
Rubi [A] (verified) 2095
Maple [A] (verified) 2096
Fricas [A] (verification not implemented) 2096
Sympy [F] 2097
Maxima [A] (verification not implemented) 2097
Giac [A] (verification not implemented) 2097
Mupad [B] (verification not implemented) 2098
Reduce [B] (verification not implemented) 2098

Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{ax^2+bx^3}}{3b} - \frac{4a\sqrt{ax^2+bx^3}}{3b^2x}$$

output
```
2/3*(b*x^3+a*x^2)^(1/2)/b-4/3*a*(b*x^3+a*x^2)^(1/2)/b^2/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{2(-2a+bx)\sqrt{x^2(a+bx)}}{3b^2x}$$

input
```
Integrate[x^2/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*(-2*a + b*x)*Sqrt[x^2*(a + b*x)])/(3*b^2*x)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx$$
$$\Big\downarrow \text{ 1922}$$
$$\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{2a\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}$$
$$\Big\downarrow \text{ 1920}$$
$$\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}$$

input
```
Int[x^2/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.61

method	result	size
trager	$-\frac{2(-bx+2a)\sqrt{b\,x^{3}+a\,x^{2}}}{3b^{2}x}$	30
risch	$-\frac{2x(bx+a)(-bx+2a)}{3\sqrt{x^{2}(bx+a)}\,b^{2}}$	31
pseudoelliptic	$\frac{2\sqrt{bx+a}\,(3b^{2}x^{2}-4abx+8a^{2})}{15b^{3}}$	32
gosper	$-\frac{2(bx+a)(-bx+2a)x}{3b^{2}\sqrt{b\,x^{3}+a\,x^{2}}}$	33
default	$-\frac{2(bx+a)(-bx+2a)x}{3b^{2}\sqrt{b\,x^{3}+a\,x^{2}}}$	33
orering	$-\frac{2(bx+a)(-bx+2a)x}{3b^{2}\sqrt{b\,x^{3}+a\,x^{2}}}$	33

input
```
int(x^2/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-b*x+2*a)/b^2/x*(b*x^3+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx=\frac{2\,\sqrt{bx^3+ax^2}(bx-2\,a)}{3\,b^2x}$$

input
```
integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(b*x^3 + a*x^2)*(b*x - 2*a)/(b^2*x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{ax^2 + bx^3}}\,dx = \int \frac{x^2}{\sqrt{x^2\,(a + bx)}}\,dx$$

input
```
integrate(x**2/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**2/sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{\sqrt{ax^2 + bx^3}}\,dx = \frac{2\,(b^2x^2 - abx - 2\,a^2)}{3\,\sqrt{bx + a}b^2}$$

input
```
integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(b^2*x^2 - a*b*x - 2*a^2)/(sqrt(b*x + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{\sqrt{ax^2 + bx^3}}\,dx = \frac{4\,a^{\frac{3}{2}}\text{sgn}(x)}{3\,b^2} + \frac{2\,\left((bx + a)^{\frac{3}{2}} - 3\,\sqrt{bx + a}a\right)}{3\,b^2\text{sgn}\,(x)}$$

input
```
integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
4/3*a^(3/2)*sgn(x)/b^2 + 2/3*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)/(b^2*sg
n(x))
```

Mupad [B] (verification not implemented)

Time = 8.74 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx = -\frac{\left(\frac{4\,a}{3\,b^2}-\frac{2\,x}{3\,b}\right)\,\sqrt{b\,x^3+a\,x^2}}{x}$$

input
```
int(x^2/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
-(((4*a)/(3*b^2) - (2*x)/(3*b))*(a*x^2 + b*x^3)^(1/2))/x
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.37

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{bx+a}\,(bx-2a)}{3b^2}$$

input
```
int(x^2/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 2*a + b*x))/(3*b**2)
```

3.234 $\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result . . . 2099
Mathematica [A] (verified) . . . 2099
Rubi [A] (verified) . . . 2100
Maple [A] (verified) . . . 2100
Fricas [A] (verification not implemented) . . . 2101
Sympy [F] . . . 2101
Maxima [A] (verification not implemented) . . . 2102
Giac [A] (verification not implemented) . . . 2102
Mupad [B] (verification not implemented) . . . 2102
Reduce [B] (verification not implemented) . . . 2103

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{ax^2+bx^3}}{bx}$$

output
```
2*(b*x^3+a*x^2)^(1/2)/b/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{x^2(a+bx)}}{bx}$$

input
```
Integrate[x/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*Sqrt[x^2*(a + b*x)])/(b*x)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{1920\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{2\sqrt{ax^2+bx^3}}{bx}$$

input
```
Int[x/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*Sqrt[a*x^2 + b*x^3])/(b*x)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

method	result	size
pseudoelliptic	$-\frac{2\sqrt{bx+a}\,(-bx+2a)}{3b^2}$	21
trager	$\frac{2\sqrt{b\,x^3+a\,x^2}}{bx}$	22
risch	$\frac{2x(bx+a)}{\sqrt{x^2(bx+a)}\,b}$	23
gosper	$\frac{2x(bx+a)}{b\sqrt{b\,x^3+a\,x^2}}$	25
default	$\frac{2x(bx+a)}{b\sqrt{b\,x^3+a\,x^2}}$	25
orering	$\frac{2x(bx+a)}{b\sqrt{b\,x^3+a\,x^2}}$	25

input
```
int(x/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x+a)^(1/2)*(-b*x+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{bx^3+ax^2}}{bx}$$

input
```
integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(b*x^3 + a*x^2)/(b*x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(x/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x/sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{bx+a}}{b}$$

input
```
integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx = -\frac{2\sqrt{a}\mathrm{sgn}(x)}{b} + \frac{2\sqrt{bx+a}}{b\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(a)*sgn(x)/b + 2*sqrt(b*x + a)/(b*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 8.80 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,|x|\,\sqrt{a+b\,x}}{b\,x}$$

input
```
int(x/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
(2*abs(x)*(a + b*x)^(1/2))/(b*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int \frac{x}{\sqrt{ax^2 + bx^3}}\,dx = \frac{2\sqrt{bx + a}}{b}$$

input
```
int(x/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x))/b
```

3.235 $\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result . . . 2104
Mathematica [A] (verified) . . . 2104
Rubi [A] (verified) . . . 2105
Maple [A] (verified) . . . 2106
Fricas [A] (verification not implemented) . . . 2106
Sympy [F] . . . 2106
Maxima [F] . . . 2107
Giac [A] (verification not implemented) . . . 2107
Mupad [F(-1)] . . . 2107
Reduce [B] (verification not implemented) . . . 2108

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx=-\frac{2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

output
```
-2*arctanh(a^(1/2)*x/(b*x^3+a*x^2)^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx=-\frac{2x\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[1/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(-2*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*Sqrt[x^2*(a +
 b*x)])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1914}$$

$$-2\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

input
```
Int[1/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(-2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/Sqrt[a]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.43

method	result	size
pseudoelliptic	$\frac{2\sqrt{bx+a}}{b}$	13
default	$-\frac{2x\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{b\,x^{3}+a\,x^{2}}\,\sqrt{a}}$	39

input
```
int(1/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(b*x+a)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.63

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx=\left[\frac{\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)}{\sqrt{a}},\frac{2\,\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)}{a}\right]$$

input
```
integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2)/sqrt(a), 2*sqrt(
-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x))/a]
```

Sympy [F]

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx=\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx$$

input
```
integrate(1/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a*x**2 + b*x**3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}} + \frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\operatorname{sgn}(x)}$$

input
```
integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-2*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*arctan(sqrt(b*x + a)/sqrt(
-a))/(sqrt(-a)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int(1/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
int(1/(a*x^2 + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{a}\,\left(\log\left(\sqrt{bx+a}-\sqrt{a}\right)-\log\left(\sqrt{bx+a}+\sqrt{a}\right)\right)}{a}$$

input
```
int(1/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(sqrt(a)*(log(sqrt(a + b*x) - sqrt(a)) - log(sqrt(a + b*x) + sqrt(a))))/a
```

3.236 $\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx$

Optimal result 2109
Mathematica [A] (verified) 2109
Rubi [A] (verified) 2110
Maple [A] (verified) 2111
Fricas [A] (verification not implemented) 2112
Sympy [F] 2112
Maxima [F] 2112
Giac [A] (verification not implemented) 2113
Mupad [F(-1)] 2113
Reduce [B] (verification not implemented) 2113

Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx=-\frac{\sqrt{ax^2+bx^3}}{ax^2}+\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}$$

output
```
-(b*x^3+a*x^2)^(1/2)/a/x^2+b*arctanh(a^(1/2)*x/(b*x^3+a*x^2)^(1/2))/a^(3/2
)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx=\frac{-\sqrt{a}(a+bx)+bx\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[1/(x*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-(Sqrt[a]*(a + b*x)) + b*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/
(a^(3/2)*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}dx$$
$$\Big\downarrow \text{1931}$$
$$-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}$$
$$\Big\downarrow \text{1914}$$
$$\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}$$
$$\Big\downarrow \text{219}$$
$$\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}$$

input
```
Int[1/(x*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3
]])/a^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.33

method	result	size
pseudoelliptic	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18
default	$-\frac{\sqrt{bx+a}\,\left(a^{\frac{3}{2}}\sqrt{bx+a}-\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)abx\right)}{\sqrt{b\,x^{3}+a\,x^{2}}\,a^{\frac{5}{2}}}$	55
risch	$-\frac{bx+a}{a\sqrt{x^{2}(bx+a)}}+\frac{b \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{bx+a}\,x}{a^{\frac{3}{2}}\sqrt{x^{2}(bx+a)}}$	59

input
```
int(1/x/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.44

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx = \left[\frac{\sqrt{a}bx^2\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}a}{2\,a^2x^2},\ -\frac{\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+\sqrt{bx^3+ax^2}a}{a^2x^2}\right]$$

input
```
integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*b*x^2*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^
2) - 2*sqrt(b*x^3 + a*x^2)*a)/(a^2*x^2), -(sqrt(-a)*b*x^2*arctan(sqrt(b*x^
3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + sqrt(b*x^3 + a*x^2)*a)/(a^2*x^2)]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(1/x/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}x}\,dx$$

input
```
integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x^2)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx=-\frac{b\left(\frac{\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{\sqrt{bx+a}}{abx}\right)}{\operatorname{sgn}(x)}$$

input
```
integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-b*(arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x + a)/(a*b*x))/s
gn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx=\int \frac{1}{x\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int(1/(x*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
int(1/(x*(a*x^2 + b*x^3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx=\frac{-2\sqrt{bx+a}\,a-\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)bx+\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)bx}{2a^2x}$$

input
```
int(1/x/(b*x^3+a*x^2)^(1/2),x)
```

output
```
( - 2*sqrt(a + b*x)*a - sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b*x + sqrt(a)
*log(sqrt(a + b*x) + sqrt(a))*b*x)/(2*a**2*x)
```

3.237 $\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx$

Optimal result 2115
Mathematica [A] (verified) 2115
Rubi [A] (verified) 2116
Maple [A] (verified) 2117
Fricas [A] (verification not implemented) 2118
Sympy [F] 2118
Maxima [F] 2118
Giac [A] (verification not implemented) 2119
Mupad [B] (verification not implemented) 2119
Reduce [B] (verification not implemented) 2120

Optimal result

Integrand size = 19, antiderivative size = 87

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx = -\frac{\sqrt{ax^2+bx^3}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2}-\frac{3b^2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4a^{5/2}}$$

output
```
-1/2*(b*x^3+a*x^2)^(1/2)/a/x^3+3/4*b*(b*x^3+a*x^2)^(1/2)/a^2/x^2-3/4*b^2*a
rctanh(a^(1/2)*x/(b*x^3+a*x^2)^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{a}(-2a^2+abx+3b^2x^2)-3b^2x^2\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}x\sqrt{x^2(a+bx)}}$$

input
```
Integrate[1/(x^2*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(Sqrt[a]*(-2*a^2 + a*b*x + 3*b^2*x^2) - 3*b^2*x^2*Sqrt[a + b*x]*ArcTanh[Sq
rt[a + b*x]/Sqrt[a]])/(4*a^(5/2)*x*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1931}$$

$$-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}$$

$$\Big\downarrow \text{1914}$$

$$-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}$$

$$\Big\downarrow \text{219}$$

$$-\frac{3b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}$$

input
```
Int[1/(x^2*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) +
(b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/(4*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.41

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) b x-\sqrt{bx+a}\,\sqrt{a}}{a^{\frac{3}{2}} x}$	36
risch	$-\frac{(bx+a)(-3bx+2a)}{4a^{2} x \sqrt{x^{2}(bx+a)}}-\frac{3b^{2} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) \sqrt{bx+a}\, x}{4a^{\frac{5}{2}} \sqrt{x^{2}(bx+a)}}$	73
default	$-\frac{\sqrt{bx+a}\left(-3a^{\frac{3}{2}} b x \sqrt{bx+a}+3 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) a\, b^{2} x^{2}+2\sqrt{bx+a}\, a^{\frac{5}{2}}\right)}{4x\sqrt{b\,x^{3}+a\,x^{2}}\, a^{\frac{7}{2}}}$	77

input
```
int(1/x^2/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(arctanh((b*x+a)^(1/2)/a^(1/2))*b*x-(b*x+a)^(1/2)*a^(1/2))/a^(3/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx = \left[\frac{3\,\sqrt{a}b^2x^3\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}(3\,abx-2\,a^2)}{8\,a^3x^3}, \frac{3\,\sqrt{-a}b^2x^3\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)}{4\,a^3x^3}\right.$$

input
```
integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(3*sqrt(a)*b^2*x^3*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a)
)/x^2) + 2*sqrt(b*x^3 + a*x^2)*(3*a*b*x - 2*a^2))/(a^3*x^3), 1/4*(3*sqrt(-
a)*b^2*x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + sqrt(b*x^3
 + a*x^2)*(3*a*b*x - 2*a^2))/(a^3*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^2\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(1/x**2/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x^2)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx = \frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{3\,(bx+a)^{\frac{3}{2}}b^3-5\,\sqrt{bx+a}ab^3}{a^2b^2x^2}}{4\,b\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x + a)^(3
/2)*b^3 - 5*sqrt(b*x + a)*a*b^3)/(a^2*b^2*x^2))/(b*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx = -\frac{2\,\sqrt{\frac{a}{b\,x}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{a}{b\,x}\right)}{5\,x\,\sqrt{b\,x^3+a\,x^2}}$$

input
```
int(1/(x^2*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
-(2*(a/(b*x) + 1)^(1/2)*hypergeom([1/2, 5/2], 7/2, -a/(b*x)))/(5*x*(a*x^2
+ b*x^3)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx = \frac{-4\sqrt{bx+a}\,a^2+6\sqrt{bx+a}\,abx+3\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2x^2-3\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)b^2x^2}{8a^3x^2}$$

input
```
int(1/x^2/(b*x^3+a*x^2)^(1/2),x)
```

output
```
( - 4*sqrt(a + b*x)*a**2 + 6*sqrt(a + b*x)*a*b*x + 3*sqrt(a)*log(sqrt(a +
b*x) - sqrt(a))*b**2*x**2 - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**2*x*
*2)/(8*a**3*x**2)
```

3.238 $\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx$

Optimal result 2121
Mathematica [A] (verified) 2121
Rubi [A] (verified) 2122
Maple [A] (verified) 2124
Fricas [A] (verification not implemented) 2124
Sympy [F] 2125
Maxima [F] 2125
Giac [A] (verification not implemented) 2126
Mupad [F(-1)] 2126
Reduce [B] (verification not implemented) 2126

Optimal result

Integrand size = 19, antiderivative size = 115

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx=-\frac{\sqrt{ax^2+bx^3}}{3ax^4}+\frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3}-\frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2}+\frac{5b^3\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{7/2}}$$

output
```
-1/3*(b*x^3+a*x^2)^(1/2)/a/x^4+5/12*b*(b*x^3+a*x^2)^(1/2)/a^2/x^3-5/8*b^2*
(b*x^3+a*x^2)^(1/2)/a^3/x^2+5/8*b^3*arctanh(a^(1/2)*x/(b*x^3+a*x^2)^(1/2))
/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx=\frac{-\sqrt{a}(8a^3-2a^2bx+5ab^2x^2+15b^3x^3)+15b^3x^3\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{24a^{7/2}x^2\sqrt{x^2(a+bx)}}$$

input
```
Integrate[1/(x^3*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-(Sqrt[a]*(8*a^3 - 2*a^2*b*x + 5*a*b^2*x^2 + 15*b^3*x^3)) + 15*b^3*x^3*Sq
rt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(24*a^(7/2)*x^2*Sqrt[x^2*(a +
b*x)])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1931, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1931}$$

$$-\frac{5b\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{5b\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{5b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}$$

$$\Big\downarrow \text{1914}$$

$$-\frac{5b\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}$$

$$\downarrow 219$$

$$-\frac{5b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}$$

input
```
Int[1/(x^3*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
-1/3*Sqrt[a*x^2 + b*x^3]/(a*x^4) - (5*b*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3)
- (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2
 + b*x^3]])/a^(3/2)))/(4*a)))/(6*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.49

method	result	size
pseudoelliptic	$\frac{-3\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{2}x^{2}+3bx\sqrt{bx+a}\,\sqrt{a}-2a^{\frac{3}{2}}\sqrt{bx+a}}{4a^{\frac{5}{2}}x^{2}}$	56
risch	$-\frac{(bx+a)\left(15b^{2}x^{2}-10abx+8a^{2}\right)}{24a^{3}x^{2}\sqrt{x^{2}(bx+a)}}+\frac{5b^{3}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{bx+a}\,x}{8a^{\frac{7}{2}}\sqrt{x^{2}(bx+a)}}$	84
default	$-\frac{\sqrt{bx+a}\,\left(15a^{\frac{3}{2}}b^{2}x^{2}\sqrt{bx+a}-15\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a\,b^{3}x^{3}-10a^{\frac{5}{2}}bx\sqrt{bx+a}+8a^{\frac{7}{2}}\sqrt{bx+a}\right)}{24x^{2}\sqrt{b\,x^{3}+a\,x^{2}}\,a^{\frac{9}{2}}}$	95

input
```
int(1/x^3/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-3*arctanh((b*x+a)^(1/2)/a^(1/2))*b^2*x^2+3*b*x*(b*x+a)^(1/2)*a^(1/2)
-2*a^(3/2)*(b*x+a)^(1/2))/a^(5/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.57

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx = \left[\frac{15\,\sqrt{a}b^3x^4\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,(15\,ab^2x^2-10\,a^2bx+8\,a^3)\sqrt{bx^3+ax^2}}{48\,a^4x^4},\right.$$
$$\left.-\frac{15\,\sqrt{-a}b^3x^4\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+(15\,ab^2x^2-10\,a^2bx+8\,a^3)\sqrt{bx^3+ax^2}}{24\,a^4x^4}\right]$$

input
```
integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(15*sqrt(a)*b^3*x^4*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(
a))/x^2) - 2*(15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^4
*x^4), -1/24*(15*sqrt(-a)*b^3*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x
^2 + a*x)) + (15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^4
*x^4)]
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx=\int \frac{1}{x^3\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(1/x**3/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx=\int \frac{1}{\sqrt{bx^3+ax^2}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x^2)*x^3), x)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx=-\frac{b^3\left(\frac{15\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3}+\frac{15\,(bx+a)^{\frac{5}{2}}-40\,(bx+a)^{\frac{3}{2}}a+33\,\sqrt{bx+a}a^2}{a^3b^3x^3}\right)}{24\,\mathrm{sgn}\,(x)}$$

input
```
integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/24*b^3*(15*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) + (15*(b*x + a
)^(5/2) - 40*(b*x + a)^(3/2)*a + 33*sqrt(b*x + a)*a^2)/(a^3*b^3*x^3))/sgn(
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx=\int \frac{1}{x^3\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int(1/(x^3*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
int(1/(x^3*(a*x^2 + b*x^3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx =\frac{-16\sqrt{bx+a}\,a^3+20\sqrt{bx+a}\,a^2bx-30\sqrt{bx+a}\,a\,b^2x^2-15\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^3x^3+15\sqrt{a}\log\left(\sqrt{b}\right.}{48a^4x^3}$$

input
```
int(1/x^3/(b*x^3+a*x^2)^(1/2),x)
```

output
```
( - 16*sqrt(a + b*x)*a**3 + 20*sqrt(a + b*x)*a**2*b*x - 30*sqrt(a + b*x)*a
*b**2*x**2 - 15*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**3*x**3 + 15*sqrt(a
)*log(sqrt(a + b*x) + sqrt(a))*b**3*x**3)/(48*a**4*x**3)
```

3.239 $\int \frac{x^6}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal result . . . 2128
Mathematica [A] (verified) . . . 2128
Rubi [A] (verified) . . . 2129
Maple [A] (verified) . . . 2131
Fricas [A] (verification not implemented) . . . 2131
Sympy [F] . . . 2132
Maxima [A] (verification not implemented) . . . 2132
Giac [A] (verification not implemented) . . . 2132
Mupad [B] (verification not implemented) . . . 2133
Reduce [B] (verification not implemented) . . . 2133

Optimal result

Integrand size = 19, antiderivative size = 98

$$\int \frac{x^6}{\left(ax^2+bx^3\right)^{3/2}}\,dx=-\frac{2x^4}{b\sqrt{ax^2+bx^3}}-\frac{16a\sqrt{ax^2+bx^3}}{5b^3}+\frac{32a^2\sqrt{ax^2+bx^3}}{5b^4x}+\frac{12x\sqrt{ax^2+bx^3}}{5b^2}$$

output
```
-2*x^4/b/(b*x^3+a*x^2)^(1/2)-16/5*a*(b*x^3+a*x^2)^(1/2)/b^3+32/5*a^2*(b*x^
3+a*x^2)^(1/2)/b^4/x+12/5*x*(b*x^3+a*x^2)^(1/2)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.51

$$\int \frac{x^6}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{2x(16a^3+8a^2bx-2ab^2x^2+b^3x^3)}{5b^4\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x^6/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*x*(16*a^3 + 8*a^2*b*x - 2*a*b^2*x^2 + b^3*x^3))/(5*b^4*Sqrt[x^2*(a + b*
x)])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1921, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{6\int \frac{x^3}{\sqrt{bx^3+ax^2}}dx}{b}-\frac{2x^4}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{6\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\int \frac{x^2}{\sqrt{bx^3+ax^2}}dx}{5b}\right)}{b}-\frac{2x^4}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{6\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{2a\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}\right)}{5b}\right)}{b}-\frac{2x^4}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{6\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\right)}{5b}\right)}{b}-\frac{2x^4}{b\sqrt{ax^2+bx^3}}$$

input
```
Int[x^6/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(-2*x^4)/(b*Sqrt[a*x^2 + b*x^3]) + (6*((2*x*Sqrt[a*x^2 + b*x^3])/(5*b) - (
4*a*((2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)))
/(5*b)))/b
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.57

method	result	size
gosper	$\frac{2(bx+a)(b^3x^3-2a\,b^2x^2+8a^2bx+16a^3)x^3}{5b^4(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	56
default	$\frac{2(bx+a)(b^3x^3-2a\,b^2x^2+8a^2bx+16a^3)x^3}{5b^4(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	56
orering	$\frac{2(bx+a)(b^3x^3-2a\,b^2x^2+8a^2bx+16a^3)x^3}{5b^4(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	56
trager	$\frac{2(b^3x^3-2a\,b^2x^2+8a^2bx+16a^3)\sqrt{b\,x^3+a\,x^2}}{5(bx+a)b^4x}$	58
risch	$\frac{2(b^2x^2-3abx+11a^2)(bx+a)x}{5b^4\sqrt{x^2(bx+a)}}+\frac{2a^3x}{b^4\sqrt{x^2(bx+a)}}$	62
pseudoelliptic	$\frac{\frac{2}{11}b^6x^6-\frac{8}{33}a\,b^5x^5+\frac{80}{231}a^2b^4x^4-\frac{128}{231}a^3b^3x^3+\frac{256}{231}a^4b^2x^2-\frac{1024}{231}a^5bx-\frac{2048}{231}a^6}{b^7\sqrt{bx+a}}$	76

input
```
int(x^6/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(b*x+a)*(b^3*x^3-2*a*b^2*x^2+8*a^2*b*x+16*a^3)*x^3/b^4/(b*x^3+a*x^2)^(
3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.61

$$\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,(b^3x^3-2\,ab^2x^2+8\,a^2bx+16\,a^3)\sqrt{bx^3+ax^2}}{5\,(b^5x^2+ab^4x)}$$

input
```
integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)*sqrt(b*x^3 + a*x^2)/(b^5*
x^2 + a*b^4*x)
```

Sympy [F]

$$\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^6}{\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**6/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**6/(x**2*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.42

$$\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\left(b^3x^3-2\,ab^2x^2+8\,a^2bx+16\,a^3\right)}{5\sqrt{bx+a}b^4}$$

input
```
integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)/(sqrt(b*x + a)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{32\,a^{\frac{5}{2}}\mathrm{sgn}(x)}{5\,b^4} + \frac{2\,a^3}{\sqrt{bx+a}b^4\mathrm{sgn}\left(x\right)} + \frac{2\left(\left(bx+a\right)^{\frac{5}{2}}b^{16}-5\left(bx+a\right)^{\frac{3}{2}}ab^{16}+15\sqrt{bx+a}a^2b^{16}\right)}{5\,b^{20}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
-32/5*a^(5/2)*sgn(x)/b^4 + 2*a^3/(sqrt(b*x + a)*b^4*sgn(x)) + 2/5*((b*x +
a)^(5/2)*b^16 - 5*(b*x + a)^(3/2)*a*b^16 + 15*sqrt(b*x + a)*a^2*b^16)/(b^2
0*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

$$\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\sqrt{b\,x^3+a\,x^2}\,(16\,a^3+8\,a^2\,b\,x-2\,a\,b^2\,x^2+b^3\,x^3)}{5\,b^4\,x\,(a+b\,x)}$$

input
```
int(x^6/(a*x^2 + b*x^3)^(3/2),x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(16*a^3 + b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x))/(5*
b^4*x*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.43

$$\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx = \frac{\frac{2}{5}b^3x^3-\frac{4}{5}a\,b^2x^2+\frac{16}{5}a^2bx+\frac{32}{5}a^3}{\sqrt{bx+a}\,b^4}$$

input
```
int(x^6/(b*x^3+a*x^2)^(3/2),x)
```

output
```
(2*(16*a**3 + 8*a**2*b*x - 2*a*b**2*x**2 + b**3*x**3))/(5*sqrt(a + b*x)*b*
*4)
```

3.240 $\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal result 2134
Mathematica [A] (verified) 2134
Rubi [A] (verified) 2135
Maple [A] (verified) 2136
Fricas [A] (verification not implemented) 2137
Sympy [F] 2137
Maxima [A] (verification not implemented) 2138
Giac [A] (verification not implemented) 2138
Mupad [B] (verification not implemented) 2138
Reduce [B] (verification not implemented) 2139

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2x^3}{b\sqrt{ax^2+bx^3}}+\frac{8\sqrt{ax^2+bx^3}}{3b^2}-\frac{16a\sqrt{ax^2+bx^3}}{3b^3x}$$

output
```
-2*x^3/b/(b*x^3+a*x^2)^(1/2)+8/3*(b*x^3+a*x^2)^(1/2)/b^2-16/3*a*(b*x^3+a*x
^2)^(1/2)/b^3/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.54

$$\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2x(-8a^2-4abx+b^2x^2)}{3b^3\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x^5/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*x*(-8*a^2 - 4*a*b*x + b^2*x^2))/(3*b^3*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1921, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{4\int \frac{x^2}{\sqrt{bx^3+ax^2}}dx}{b}-\frac{2x^3}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{4\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{2a\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}\right)}{b}-\frac{2x^3}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{4\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\right)}{b}-\frac{2x^3}{b\sqrt{ax^2+bx^3}}$$

input
```
Int[x^5/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(-2*x^3)/(b*Sqrt[a*x^2 + b*x^3]) + (4*((2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*
a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)))/b
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{2(bx+a)\left(-b^{2}x^{2}+4abx+8a^{2}\right)x^{3}}{3b^{3}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	46
default	$-\frac{2(bx+a)\left(-b^{2}x^{2}+4abx+8a^{2}\right)x^{3}}{3b^{3}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	46
orering	$-\frac{2(bx+a)\left(-b^{2}x^{2}+4abx+8a^{2}\right)x^{3}}{3b^{3}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	46
trager	$-\frac{2\left(-b^{2}x^{2}+4abx+8a^{2}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{3(bx+a)b^{3}x}$	48
risch	$-\frac{2(-bx+5a)(bx+a)x}{3b^{3}\sqrt{x^{2}(bx+a)}}-\frac{2a^{2}x}{b^{3}\sqrt{x^{2}(bx+a)}}$	52
pseudoelliptic	$\frac{\frac{2}{9}b^{5}x^{5}-\frac{20}{63}a\,b^{4}x^{4}+\frac{32}{63}a^{2}b^{3}x^{3}-\frac{64}{63}a^{3}b^{2}x^{2}+\frac{256}{63}a^{4}bx+\frac{512}{63}a^{5}}{b^{6}\sqrt{bx+a}}$	65

input
```
int(x^5/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x+a)*(-b^2*x^2+4*a*b*x+8*a^2)*x^3/b^3/(b*x^3+a*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.68

$$\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,(b^2x^2-4\,abx-8\,a^2)\sqrt{bx^3+ax^2}}{3\,(b^4x^2+ab^3x)}$$

input
```
integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)*sqrt(b*x^3 + a*x^2)/(b^4*x^2 + a*b^3*x)
```

Sympy [F]

$$\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^5}{(x^2\,(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**5/(x**2*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.42

$$\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,(b^2x^2-4\,abx-8\,a^2)}{3\,\sqrt{bx+a}b^3}$$

input
```
integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)/(sqrt(b*x + a)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx = \frac{16\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,b^3} - \frac{2\left(\frac{3\,a^2}{\sqrt{bx+a}b\mathrm{sgn}(x)} - \frac{(bx+a)^{\frac{3}{2}}b^2-6\,\sqrt{bx+a}ab^2}{b^3\mathrm{sgn}(x)}\right)}{3\,b^2}$$

input
```
integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
16/3*a^(3/2)*sgn(x)/b^3 - 2/3*(3*a^2/(sqrt(b*x + a)*b*sgn(x)) - ((b*x + a)
^(3/2)*b^2 - 6*sqrt(b*x + a)*a*b^2)/(b^3*sgn(x)))/b^2
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.65

$$\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2\,\sqrt{b\,x^3+a\,x^2}\,(8\,a^2+4\,a\,b\,x-b^2\,x^2)}{3\,b^3\,x\,(a+b\,x)}$$

input
```
int(x^5/(a*x^2 + b*x^3)^(3/2),x)
```

output
```
-(2*(a*x^2 + b*x^3)^(1/2)*(8*a^2 - b^2*x^2 + 4*a*b*x))/(3*b^3*x*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.43

$$\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx = \frac{\frac{2}{3}b^2x^2 - \frac{8}{3}abx - \frac{16}{3}a^2}{\sqrt{bx+a}\,b^3}$$

input
```
int(x^5/(b*x^3+a*x^2)^(3/2),x)
```

output
```
(2*( - 8*a**2 - 4*a*b*x + b**2*x**2))/(3*sqrt(a + b*x)*b**3)
```

3.241 $\int \frac{x^4}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal result 2140
Mathematica [A] (verified) 2140
Rubi [A] (verified) 2141
Maple [A] (verified) 2142
Fricas [A] (verification not implemented) 2143
Sympy [F] 2143
Maxima [A] (verification not implemented) 2143
Giac [A] (verification not implemented) 2144
Mupad [B] (verification not implemented) 2144
Reduce [B] (verification not implemented) 2144

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{x^4}{\left(ax^2+bx^3\right)^{3/2}}\,dx=-\frac{2x^2}{b\sqrt{ax^2+bx^3}}+\frac{4\sqrt{ax^2+bx^3}}{b^2x}$$

output
```
-2*x^2/b/(b*x^3+a*x^2)^(1/2)+4*(b*x^3+a*x^2)^(1/2)/b^2/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.55

$$\int \frac{x^4}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{2x(2a+bx)}{b^2\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x^4/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*x*(2*a + b*x))/(b^2*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(ax^2+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{2\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{b}-\frac{2x^2}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{4\sqrt{ax^2+bx^3}}{b^2x}-\frac{2x^2}{b\sqrt{ax^2+bx^3}}$$

input
```
Int[x^4/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(-2*x^2)/(b*Sqrt[a*x^2 + b*x^3]) + (4*Sqrt[a*x^2 + b*x^3])/(b^2*x)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{2(bx+a)(bx+2a)x^3}{b^2(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	34
default	$\frac{2(bx+a)(bx+2a)x^3}{b^2(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	34
orering	$\frac{2(bx+a)(bx+2a)x^3}{b^2(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	34
trager	$\frac{2(bx+2a)\sqrt{b\,x^3+a\,x^2}}{(bx+a)b^2x}$	36
risch	$\frac{2(bx+a)x}{b^2\sqrt{x^2(bx+a)}}+\frac{2ax}{b^2\sqrt{x^2(bx+a)}}$	42
pseudoelliptic	$\frac{\frac{2}{7}b^4x^4-\frac{16}{35}a\,b^3x^3+\frac{32}{35}a^2b^2x^2-\frac{128}{35}a^3bx-\frac{256}{35}a^4}{b^5\sqrt{bx+a}}$	54

input
```
int(x^4/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(b*x+a)*(b*x+2*a)*x^3/b^2/(b*x^3+a*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{x^4}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\sqrt{bx^3+ax^2}(bx+2a)}{b^3x^2+ab^2x}$$

input
```
integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2*sqrt(b*x^3 + a*x^2)*(b*x + 2*a)/(b^3*x^2 + a*b^2*x)
```

Sympy [F]

$$\int \frac{x^4}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^4}{(x^2(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**4/(x**2*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.40

$$\int \frac{x^4}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2(bx+2a)}{\sqrt{bx+a}b^2}$$

input
```
integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
2*(b*x + 2*a)/(sqrt(b*x + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{x^4}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\left(\frac{\sqrt{bx+a}}{b\mathrm{sgn}(x)}+\frac{a}{\sqrt{bx+a}b\mathrm{sgn}(x)}\right)}{b} - \frac{4\sqrt{a}\mathrm{sgn}(x)}{b^2}$$

input
```
integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
2*(sqrt(b*x + a)/(b*sgn(x)) + a/(sqrt(b*x + a)*b*sgn(x)))/b - 4*sqrt(a)*sg
n(x)/b^2
```

Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{x^4}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,(2\,a+b\,x)\,\sqrt{b\,x^3+a\,x^2}}{b^2\,x\,(a+b\,x)}$$

input
```
int(x^4/(a*x^2 + b*x^3)^(3/2),x)
```

output
```
(2*(2*a + b*x)*(a*x^2 + b*x^3)^(1/2))/(b^2*x*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.43

$$\int \frac{x^4}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2bx+4a}{\sqrt{bx+a}\,b^2}$$

input
```
int(x^4/(b*x^3+a*x^2)^(3/2),x)
```

output
```
(2*(2*a + b*x))/(sqrt(a + b*x)*b**2)
```

3.242 $\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal result 2145
Mathematica [A] (verified) 2145
Rubi [A] (verified) 2146
Maple [A] (verified) 2146
Fricas [A] (verification not implemented) 2147
Sympy [F] 2147
Maxima [A] (verification not implemented) 2148
Giac [A] (verification not implemented) 2148
Mupad [B] (verification not implemented) 2148
Reduce [B] (verification not implemented) 2149

Optimal result

Integrand size = 19, antiderivative size = 21

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2x}{b\sqrt{ax^2+bx^3}}$$

output
```
-2*x/b/(b*x^3+a*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2x}{b\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x^3/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(-2*x)/(b*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}} \, dx$$
$$\Big\downarrow \; 1920$$
$$-\frac{2x}{b\sqrt{ax^2+bx^3}}$$

input
```
Int[x^3/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(-2*x)/(b*Sqrt[a*x^2 + b*x^3])
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

method	result	size
gosper	$-\frac{2(bx+a)x^3}{b(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	27
default	$-\frac{2(bx+a)x^3}{b(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	27
orering	$-\frac{2(bx+a)x^3}{b(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	27
trager	$-\frac{2\sqrt{b\,x^3+a\,x^2}}{(bx+a)bx}$	29
pseudoelliptic	$\frac{\frac{2}{5}b^3x^3-\frac{4}{5}a\,b^2x^2+\frac{16}{5}a^2bx+\frac{32}{5}a^3}{b^4\sqrt{bx+a}}$	42

input
```
int(x^3/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(b*x+a)/b*x^3/(b*x^3+a*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2\sqrt{bx^3+ax^2}}{b^2x^2+abx}$$

input
```
integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(b*x^3 + a*x^2)/(b^2*x^2 + a*b*x)
```

Sympy [F]

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^3}{(x^2(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**3/(x**2*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.57

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2}{\sqrt{bx+a}b}$$

input
```
integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
-2/(sqrt(b*x + a)*b)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,\mathrm{sgn}(x)}{\sqrt{a}b} - \frac{2}{\sqrt{bx+a}b\mathrm{sgn}\,(x)}$$

input
```
integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
2*sgn(x)/(sqrt(a)*b) - 2/(sqrt(b*x + a)*b*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2\sqrt{b\,x^3+a\,x^2}}{b\,x\,(a+b\,x)}$$

input
```
int(x^3/(a*x^2 + b*x^3)^(3/2),x)
```

output
```
-(2*(a*x^2 + b*x^3)^(1/2))/(b*x*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2}{\sqrt{bx+a}\,b}$$

input
```
int(x^3/(b*x^3+a*x^2)^(3/2),x)
```

output
```
( - 2)/(sqrt(a + b*x)*b)
```

3.243 $\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal result 2150
Mathematica [A] (verified) 2150
Rubi [A] (verified) 2151
Maple [A] (verified) 2152
Fricas [A] (verification not implemented) 2153
Sympy [F] 2153
Maxima [F] 2153
Giac [A] (verification not implemented) 2154
Mupad [F(-1)] 2154
Reduce [B] (verification not implemented) 2154

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx=\frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}$$

output
```
2*x/a/(b*x^3+a*x^2)^(1/2)-2*arctanh(a^(1/2)*x/(b*x^3+a*x^2)^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx=\frac{2x\left(\sqrt{a}-\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{a^{3/2}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x^2/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*x*(Sqrt[a] - Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(a^(3/2)*Sq
rt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1929, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{a}+\frac{2x}{a\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1914}$$

$$\frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}$$

$$\Big\downarrow \text{219}$$

$$\frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}$$

input
```
Int[x^2/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*x)/(a*Sqrt[a*x^2 + b*x^3]) - (2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]
])/a^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

method	result	size
pseudoelliptic	$\frac{\frac{2}{3} b^{2} x^{2}-\frac{8}{3} a b x -\frac{16}{3} a^{2}}{b^{3} \sqrt{b x +a}}$	31
default	$-\frac{2 x^{3} \left(b x +a \right) \left(\operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) a \sqrt{b x +a}-a^{\frac{3}{2}}\right)}{\left(b \,x^{3}+a \,x^{2}\right)^{\frac{3}{2}} a^{\frac{5}{2}}}$	54

input
```
int(x^2/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(b^2*x^2-4*a*b*x-8*a^2)/b^3/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 3.10

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx = \left[\frac{(bx^2+ax)\sqrt{a}\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}a}{a^2bx^2+a^3x}, \frac{2\left((bx^2+ax)\sqrt{-a}\,\mathrm{ar}\right.}{}\right.$$

input
```
integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[((b*x^2 + a*x)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a)
)/x^2) + 2*sqrt(b*x^3 + a*x^2)*a)/(a^2*b*x^2 + a^3*x), 2*((b*x^2 + a*x)*sq
rt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + sqrt(b*x^3 + a
*x^2)*a)/(a^2*b*x^2 + a^3*x)]
```

Sympy [F]

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^2}{(x^2\,(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**2/(x**2*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^2}{(bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.48

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2\left(\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\right)\mathrm{sgn}(x)}{\sqrt{-a}a^{\frac{3}{2}}} + \frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a\mathrm{sgn}\left(x\right)} + \frac{2}{\sqrt{bx+a}a\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
-2*(sqrt(a)*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a))*sgn(x)/(sqrt(-a)*a^(3/2))
 + 2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a*sgn(x)) + 2/(sqrt(b*x + a)
*a*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^2}{\left(b\,x^3+a\,x^2\right)^{3/2}}\,dx$$

input
```
int(x^2/(a*x^2 + b*x^3)^(3/2),x)
```

output
```
int(x^2/(a*x^2 + b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx = \frac{\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}-\sqrt{a}\right)-\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}+\sqrt{a}\right)+2a}{\sqrt{bx+a}\,a^2}$$

input
```
int(x^2/(b*x^3+a*x^2)^(3/2),x)
```

output
```
(sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a)) - sqrt(a)*sqrt(a + b*x
)*log(sqrt(a + b*x) + sqrt(a)) + 2*a)/(sqrt(a + b*x)*a**2)
```

3.244 $\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal result . . . 2156
Mathematica [A] (verified) . . . 2156
Rubi [A] (verified) . . . 2157
Maple [A] (verified) . . . 2158
Fricas [A] (verification not implemented) . . . 2159
Sympy [F] . . . 2159
Maxima [F] . . . 2160
Giac [A] (verification not implemented) . . . 2160
Mupad [F(-1)] . . . 2160
Reduce [B] (verification not implemented) . . . 2161

Optimal result

Integrand size = 17, antiderivative size = 75

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2}{a\sqrt{ax^2+bx^3}} - \frac{3\sqrt{ax^2+bx^3}}{a^2x^2} + \frac{3b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{5/2}}$$

output
```
2/a/(b*x^3+a*x^2)^(1/2)-3*(b*x^3+a*x^2)^(1/2)/a^2/x^2+3*b*arctanh(a^(1/2)*
x/(b*x^3+a*x^2)^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx = \frac{-\sqrt{a}(a+3bx)+3bx\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(-(Sqrt[a]*(a + 3*b*x)) + 3*b*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a
]])/(a^(5/2)*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1929, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{3\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{a}+\frac{2}{a\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{3\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{a}+\frac{2}{a\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1914}$$

$$\frac{3\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{a}+\frac{2}{a\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{3\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{a}+\frac{2}{a\sqrt{ax^2+bx^3}}$$

input
```
Int[x/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
2/(a*Sqrt[a*x^2 + b*x^3]) + (3*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTan
h[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/a
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.27

method	result	size
pseudoelliptic	$\frac{2bx+4a}{b^{2}\sqrt{bx+a}}$	20
default	$-\frac{x^{2}(bx+a)\left(-3\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{bx+a}\,bx+3bx\sqrt{a}+a^{\frac{3}{2}}\right)}{(b\,x^{3}+a\,x^{2})^{\frac{3}{2}}\,a^{\frac{5}{2}}}$	61
risch	$-\frac{bx+a}{a^{2}\sqrt{x^{2}(bx+a)}}-\frac{b\left(-\frac{6\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{4}{\sqrt{bx+a}}\right)\sqrt{bx+a}\,x}{2a^{2}\sqrt{x^{2}(bx+a)}}$	75

input
```
int(x/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(2*b*x+4*a)/b^2/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.59

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx = \left[\frac{3\,(b^2x^3+abx^2)\sqrt{a}\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}(3\,abx+a^2)}{2\,(a^3bx^3+a^4x^2)},\right.$$
$$\left.-\frac{3\,(b^2x^3+abx^2)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+\sqrt{bx^3+ax^2}(3\,abx+a^2)}{a^3bx^3+a^4x^2}\right]$$

input
```
integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(3*(b^2*x^3 + a*b*x^2)*sqrt(a)*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*
x^2)*sqrt(a))/x^2) - 2*sqrt(b*x^3 + a*x^2)*(3*a*b*x + a^2))/(a^3*b*x^3 + a
^4*x^2), -(3*(b^2*x^3 + a*b*x^2)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(
-a)/(b*x^2 + a*x)) + sqrt(b*x^3 + a*x^2)*(3*a*b*x + a^2))/(a^3*b*x^3 + a^4
*x^2)]
```

Sympy [F]

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x}{(x^2\,(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x/(x**2*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x}{(bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{3\,b\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2\mathrm{sgn}\,(x)} - \frac{3\,(bx+a)b-2\,ab}{\left((bx+a)^{\frac{3}{2}}-\sqrt{bx+a}a\right)a^2\mathrm{sgn}\,(x)}$$

input
```
integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
-3*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2*sgn(x)) - (3*(b*x + a)*b
 - 2*a*b)/(((b*x + a)^(3/2) - sqrt(b*x + a)*a)*a^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x}{{\left(b\,x^3+a\,x^2\right)}^{3/2}}\,dx$$

input
```
int(x/(a*x^2 + b*x^3)^(3/2),x)
```

output
```
int(x/(a*x^2 + b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx = \frac{-3\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)bx+3\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)bx-2a}{2\sqrt{bx+a}\,a^3x}$$

input
```
int(x/(b*x^3+a*x^2)^(3/2),x)
```

output
```
( - 3*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b*x + 3*sqrt(a)*s
qrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b*x - 2*a**2 - 6*a*b*x)/(2*sqrt(
a + b*x)*a**3*x)
```

3.245 $\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal result . . . 2162
Mathematica [A] (verified) . . . 2162
Rubi [A] (verified) . . . 2163
Maple [A] (verified) . . . 2165
Fricas [A] (verification not implemented) . . . 2165
Sympy [F] . . . 2166
Maxima [F] . . . 2166
Giac [A] (verification not implemented) . . . 2167
Mupad [B] (verification not implemented) . . . 2167
Reduce [B] (verification not implemented) . . . 2168

Optimal result

Integrand size = 15, antiderivative size = 110

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2}{ax\sqrt{ax^2+bx^3}} - \frac{5\sqrt{ax^2+bx^3}}{2a^2x^3} + \frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2} - \frac{15b^2\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4a^{7/2}}$$

output
```
2/a/x/(b*x^3+a*x^2)^(1/2)-5/2*(b*x^3+a*x^2)^(1/2)/a^2/x^3+15/4*b*(b*x^3+a*
x^2)^(1/2)/a^3/x^2-15/4*b^2*arctanh(a^(1/2)*x/(b*x^3+a*x^2)^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx = \frac{\sqrt{a}(-2a^2+5abx+15b^2x^2) - 15b^2x^2\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}x\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(-3/2),x]
```

output
```
(Sqrt[a]*(-2*a^2 + 5*a*b*x + 15*b^2*x^2) - 15*b^2*x^2*Sqrt[a + b*x]*ArcTan
h[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(7/2)*x*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1912, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1912}$$

$$\frac{5\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{5\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{5\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1914}$$

$$\frac{5\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{5\left(-\frac{3b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

input
```
Int[(a*x^2 + b*x^3)^(-3/2),x]
```

output
```
2/(a*x*Sqrt[a*x^2 + b*x^3]) + (5*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (3*b*
(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^
3]])/a^(3/2)))/(4*a)))/a
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1912
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.12

method	result	size
pseudoelliptic	$-\frac{2}{\sqrt{bx+a}\,b}$	13
default	$-\frac{x(bx+a)\left(15\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^2x^2-5a^{\frac{3}{2}}bx-15\sqrt{a}\,b^2x^2+2a^{\frac{5}{2}}\right)}{4(b\,x^3+a\,x^2)^{\frac{3}{2}}a^{\frac{7}{2}}}$	76
risch	$-\frac{(bx+a)(-7bx+2a)}{4a^3x\sqrt{x^2(bx+a)}}+\frac{b^2\left(-\frac{30\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{16}{\sqrt{bx+a}}\right)\sqrt{bx+a}\,x}{8a^3\sqrt{x^2(bx+a)}}$	88

input
```
int(1/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(b*x+a)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.04

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx = \left[\frac{15\,(b^3x^4+ab^2x^3)\sqrt{a}\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(15\,ab^2x^2+5\,a^2bx-2\,a^3)\sqrt{b}}{8\,(a^4bx^4+a^5x^3)}\right.$$

input
```
integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(15*(b^3*x^4 + a*b^2*x^3)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 +
 a*x^2)*sqrt(a))/x^2) + 2*(15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x^3 +
a*x^2))/(a^4*b*x^4 + a^5*x^3), 1/4*(15*(b^3*x^4 + a*b^2*x^3)*sqrt(-a)*arct
an(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + (15*a*b^2*x^2 + 5*a^2*b*x
 - 2*a^3)*sqrt(b*x^3 + a*x^2))/(a^4*b*x^4 + a^5*x^3)]
```

Sympy [F]

$$\int \frac{1}{(ax^2 + bx^3)^{3/2}}\,dx = \int \frac{1}{(ax^2 + bx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral((a*x**2 + b*x**3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(ax^2 + bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3 + ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(-3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx = \frac{15\,b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\sqrt{-a}a^3\mathrm{sgn}\,(x)} + \frac{2\,b^2}{\sqrt{bx+a}a^3\mathrm{sgn}\,(x)} + \frac{7\,(bx+a)^{\frac{3}{2}}b^2 - 9\sqrt{bx+a}ab^2}{4\,a^3b^2x^2\mathrm{sgn}\,(x)}$$

input
```
integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
15/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3*sgn(x)) + 2*b^2/(sqr
t(b*x + a)*a^3*sgn(x)) + 1/4*(7*(b*x + a)^(3/2)*b^2 - 9*sqrt(b*x + a)*a*b^
2)/(a^3*b^2*x^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.38

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2\,x\,\left(\frac{a}{b\,x}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{a}{b\,x}\right)}{7\,(b\,x^3+a\,x^2)^{3/2}}$$

input
```
int(1/(a*x^2 + b*x^3)^(3/2),x)
```

output
```
-(2*x*(a/(b*x) + 1)^(3/2)*hypergeom([3/2, 7/2], 9/2, -a/(b*x)))/(7*(a*x^2
+ b*x^3)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{15\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2x^2-15\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}+\sqrt{a}\right)b^2x^2}{8\sqrt{bx+a}\,a^4x^2}$$

input
```
int(1/(b*x^3+a*x^2)^(3/2),x)
```

output
```
(15*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**2*x**2 - 15*sqrt
(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**2*x**2 - 4*a**3 + 10*a**
2*b*x + 30*a*b**2*x**2)/(8*sqrt(a + b*x)*a**4*x**2)
```

3.246 $\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal result 2169
Mathematica [A] (verified) 2169
Rubi [A] (verified) 2170
Maple [A] (verified) 2172
Fricas [A] (verification not implemented) 2173
Sympy [F] 2173
Maxima [F] 2174
Giac [A] (verification not implemented) 2174
Mupad [F(-1)] 2174
Reduce [B] (verification not implemented) 2175

Optimal result

Integrand size = 19, antiderivative size = 138

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{2}{ax^2\sqrt{ax^2+bx^3}}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}+\frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3}-\frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2}+\frac{35b^3\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{9/2}}$$

output
```
2/a/x^2/(b*x^3+a*x^2)^(1/2)-7/3*(b*x^3+a*x^2)^(1/2)/a^2/x^4+35/12*b*(b*x^3
+a*x^2)^(1/2)/a^3/x^3-35/8*b^2*(b*x^3+a*x^2)^(1/2)/a^4/x^2+35/8*b^3*arctan
h(a^(1/2)*x/(b*x^3+a*x^2)^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.70

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{-\sqrt{a}(8a^3-14a^2bx+35ab^2x^2+105b^3x^3)+105b^3x^3\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{24a^{9/2}x^2\sqrt{x^2(a+bx)}}$$

input
```
Integrate[1/(x*(a*x^2 + b*x^3)^(3/2)),x]
```

output
```
(-(Sqrt[a]*(8*a^3 - 14*a^2*b*x + 35*a*b^2*x^2 + 105*b^3*x^3)) + 105*b^3*x^
3*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(24*a^(9/2)*x^2*Sqrt[x^2*(
a + b*x)])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1929, 1931, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{7\int \frac{1}{x^3\sqrt{bx^3+ax^2}}dx}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{7\left(-\frac{5b\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{7\left(-\frac{5b\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{7\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int\frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

$\Big\downarrow$ 1914

$$\frac{7\left(-\frac{5b\left(-\frac{3b\left(\frac{b\int\frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

$\Big\downarrow$ 219

$$\frac{7\left(-\frac{5b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

input
```
Int[1/(x*(a*x^2 + b*x^3)^(3/2)),x]
```

output
```
2/(a*x^2*Sqrt[a*x^2 + b*x^3]) + (7*(-1/3*Sqrt[a*x^2 + b*x^3]/(a*x^4) - (5*
b*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2))
 + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/(4*a)))/(6*a)))/
a
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.22

method	result	size
pseudoelliptic	$-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{a\sqrt{bx+a}}$	31
default	$\frac{(bx+a)\left(105\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{3}x^{3}+14a^{\frac{5}{2}}bx-35a^{\frac{3}{2}}b^{2}x^{2}-105\sqrt{a}\,b^{3}x^{3}-8a^{\frac{7}{2}}\right)}{24\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}a^{\frac{9}{2}}}$	86
risch	$-\frac{(bx+a)\left(57b^{2}x^{2}-22abx+8a^{2}\right)}{24a^{4}x^{2}\sqrt{x^{2}(bx+a)}}-\frac{b^{3}\left(-\frac{70\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{32}{\sqrt{bx+a}}\right)\sqrt{bx+a}\,x}{16a^{4}\sqrt{x^{2}(bx+a)}}$	99

input
```
int(1/x/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2/a/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.78

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx = \left[\frac{105\left(b^4x^5+ab^3x^4\right)\sqrt{a}\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\left(105\,ab^3x^3+35\,a^2b^2x^2-1\right.}{48\left(a^5bx^5+a^6x^4\right)}\right.$$

$$\left.-\frac{105\left(b^4x^5+ab^3x^4\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+\left(105\,ab^3x^3+35\,a^2b^2x^2-14\,a^3bx+8\,a^4\right)\sqrt{bx^3+ax^2}}{24\left(a^5bx^5+a^6x^4\right)}\right]$$

input
```
integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/48*(105*(b^4*x^5 + a*b^3*x^4)*sqrt(a)*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3
 + a*x^2)*sqrt(a))/x^2) - 2*(105*a*b^3*x^3 + 35*a^2*b^2*x^2 - 14*a^3*b*x +
 8*a^4)*sqrt(b*x^3 + a*x^2))/(a^5*b*x^5 + a^6*x^4), -1/24*(105*(b^4*x^5 +
a*b^3*x^4)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + (
105*a*b^3*x^3 + 35*a^2*b^2*x^2 - 14*a^3*b*x + 8*a^4)*sqrt(b*x^3 + a*x^2))/
(a^5*b*x^5 + a^6*x^4)]
```

Sympy [F]

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx = \int \frac{1}{x\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(1/(x*(x**2*(a + b*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^3+ax^2\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a*x^2)^(3/2)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.78

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx=-\frac{35\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^4\mathrm{sgn}\left(x\right)}-\frac{2\,b^3}{\sqrt{bx+a}a^4\mathrm{sgn}\left(x\right)}-\frac{57\,(bx+a)^{\frac{5}{2}}b^3-136\,(bx+a)^{\frac{3}{2}}ab^3+87\,\sqrt{bx+a}a^2b^3}{24\,a^4b^3x^3\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
-35/8*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^4*sgn(x)) - 2*b^3/(sq
rt(b*x + a)*a^4*sgn(x)) - 1/24*(57*(b*x + a)^(5/2)*b^3 - 136*(b*x + a)^(3/
2)*a*b^3 + 87*sqrt(b*x + a)*a^2*b^3)/(a^4*b^3*x^3*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx=\int \frac{1}{x\,{\left(b\,x^3+a\,x^2\right)}^{3/2}}\,dx$$

input
```
int(1/(x*(a*x^2 + b*x^3)^(3/2)),x)
```

output
```
int(1/(x*(a*x^2 + b*x^3)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{-105\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^3x^3+105\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)}{48\sqrt{bx+a}\,a^5x^3}$$

input
```
int(1/x/(b*x^3+a*x^2)^(3/2),x)
```

output
```
( - 105*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**3*x**3 + 105
*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**3*x**3 - 16*a**4 +
28*a**3*b*x - 70*a**2*b**2*x**2 - 210*a*b**3*x**3)/(48*sqrt(a + b*x)*a**5*
x**3)
```

3.247 $\int \frac{1}{x^2(ax^2+bx^3)^{3/2}}\,dx$

Optimal result 2176
Mathematica [A] (verified) 2176
Rubi [A] (verified) 2177
Maple [A] (verified) 2181
Fricas [A] (verification not implemented) 2182
Sympy [F] 2182
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Giac [A] (verification not implemented) 2183
Mupad [B] (verification not implemented) 2183
Reduce [B] (verification not implemented) 2184

Optimal result

Integrand size = 19, antiderivative size = 166

$$\int \frac{1}{x^2(ax^2+bx^3)^{3/2}}\,dx = \frac{2}{ax^3\sqrt{ax^2+bx^3}} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4} - \frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3} + \frac{315b^3\sqrt{ax^2+bx^3}}{64a^5x^2} - \frac{315b^4\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{64a^{11/2}}$$

output
```
2/a/x^3/(b*x^3+a*x^2)^(1/2)-9/4*(b*x^3+a*x^2)^(1/2)/a^2/x^5+21/8*b*(b*x^3+
a*x^2)^(1/2)/a^3/x^4-105/32*b^2*(b*x^3+a*x^2)^(1/2)/a^4/x^3+315/64*b^3*(b*
x^3+a*x^2)^(1/2)/a^5/x^2-315/64*b^4*arctanh(a^(1/2)*x/(b*x^3+a*x^2)^(1/2))
/a^(11/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^2(ax^2+bx^3)^{3/2}}\,dx = \frac{\sqrt{a}(-16a^4+24a^3bx-42a^2b^2x^2+105ab^3x^3+315b^4x^4)-315b^4x^4\sqrt{a+bx}\text{arcta}}{64a^{11/2}x^3\sqrt{x^2(a+bx)}}$$

input
```
Integrate[1/(x^2*(a*x^2 + b*x^3)^(3/2)),x]
```

output
```
(Sqrt[a]*(-16*a^4 + 24*a^3*b*x - 42*a^2*b^2*x^2 + 105*a*b^3*x^3 + 315*b^4*
x^4) - 315*b^4*x^4*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(64*a^(11
/2)*x^3*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1929, 1931, 1931, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (ax^2+bx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{9 \int \frac{1}{x^4\sqrt{bx^3+ax^2}}dx}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{9\left(-\frac{7b\int \frac{1}{x^3\sqrt{bx^3+ax^2}}dx}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{9\left(-\frac{7b\left(-\frac{5b\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{9\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}$$

$\big\downarrow$ 1931

$$\frac{9\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}$$

$\big\downarrow$ 1914

$$\frac{9\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\int\frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+$$

$$\frac{2}{ax^3\sqrt{ax^2+bx^3}}$$

$\downarrow$ 219

$$\frac{9\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}$$

input
```
Int[1/(x^2*(a*x^2 + b*x^3)^(3/2)),x]
```

output
```
2/(a*x^3*Sqrt[a*x^2 + b*x^3]) + (9*(-1/4*Sqrt[a*x^2 + b*x^3]/(a*x^5) - (7*
b*(-1/3*Sqrt[a*x^2 + b*x^3]/(a*x^4) - (5*b*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^
3) - (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*
x^2 + b*x^3]])/a^(3/2)))/(4*a)))/(6*a)))/(8*a)))/a
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NegQ[n, 2]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.30

method	result	size
pseudoelliptic	$-\frac{-3 \,\operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) \sqrt{b x +a}\, b x +\left(3 b x +a \right) \sqrt{a}}{\sqrt{b x +a}\, x \,a^{\frac{5}{2}}}$	50
default	$-\frac{\left(b x +a \right) \left(315 \sqrt{b x +a}\, \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) b^{4} x^{4}-24 a^{\frac{7}{2}} b x +42 a^{\frac{5}{2}} b^{2} x^{2}-105 a^{\frac{3}{2}} b^{3} x^{3}-315 b^{4} x^{4} \sqrt{a}+16 a^{\frac{9}{2}}\right)}{64 x \left(b \,x^{3}+a \,x^{2}\right)^{\frac{3}{2}} a^{\frac{11}{2}}}$	100
risch	$-\frac{\left(b x +a \right) \left(-187 b^{3} x^{3}+82 a \,b^{2} x^{2}-40 a^{2} b x +16 a^{3}\right)}{64 a^{5} x^{3} \sqrt{x^{2} \left(b x +a \right)}}+\frac{b^{4} \left(-\frac{630 \,\operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{256}{\sqrt{b x +a}}\right) \sqrt{b x +a}\, x}{128 a^{5} \sqrt{x^{2} \left(b x +a \right)}}$	110

input
```
int(1/x^2/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(b*x+a)^(1/2)*(-3*arctanh((b*x+a)^(1/2)/a^(1/2))*(b*x+a)^(1/2)*b*x+(3*b
*x+a)*a^(1/2))/x/a^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.61

$$\int \frac{1}{x^2 (ax^2+bx^3)^{3/2}}\,dx = \left[\frac{315\,(b^5x^6+ab^4x^5)\sqrt{a}\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(315\,ab^4x^4+105\,a^2b^3x^3 -}{128\,(a^6bx^6+a^7x^5)}\right.$$

input
```
integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/128*(315*(b^5*x^6 + a*b^4*x^5)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^
3 + a*x^2)*sqrt(a))/x^2) + 2*(315*a*b^4*x^4 + 105*a^2*b^3*x^3 - 42*a^3*b^2
*x^2 + 24*a^4*b*x - 16*a^5)*sqrt(b*x^3 + a*x^2))/(a^6*b*x^6 + a^7*x^5), 1/
64*(315*(b^5*x^6 + a*b^4*x^5)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)
/(b*x^2 + a*x)) + (315*a*b^4*x^4 + 105*a^2*b^3*x^3 - 42*a^3*b^2*x^2 + 24*a
^4*b*x - 16*a^5)*sqrt(b*x^3 + a*x^2))/(a^6*b*x^6 + a^7*x^5)]
```

Sympy [F]

$$\int \frac{1}{x^2 (ax^2+bx^3)^{3/2}}\,dx = \int \frac{1}{x^2 (x^2 (a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(1/(x**2*(x**2*(a + b*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (ax^2+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a*x^2)^(3/2)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^2 (ax^2+bx^3)^{3/2}}\,dx = \frac{315\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{64\,\sqrt{-a}a^5\mathrm{sgn}\,(x)} + \frac{2\,b^4}{\sqrt{bx+a}a^5\mathrm{sgn}\,(x)} + \frac{187\,(bx+a)^{\frac{7}{2}}b^4 - 643\,(bx+a)^{\frac{5}{2}}ab^4 + 765\,(bx+a)^{\frac{3}{2}}a^2b^4 - 325\,\sqrt{bx+a}a^3b^4}{64\,a^5b^4x^4\mathrm{sgn}\,(x)}$$

input
```
integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
315/64*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^5*sgn(x)) + 2*b^4/(s
qrt(b*x + a)*a^5*sgn(x)) + 1/64*(187*(b*x + a)^(7/2)*b^4 - 643*(b*x + a)^(
5/2)*a*b^4 + 765*(b*x + a)^(3/2)*a^2*b^4 - 325*sqrt(b*x + a)*a^3*b^4)/(a^5
*b^4*x^4*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.72 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.27

$$\int \frac{1}{x^2 (ax^2+bx^3)^{3/2}}\,dx = -\frac{2\left(\frac{a}{b\,x}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{11}{2};\ \frac{13}{2};\ -\frac{a}{b\,x}\right)}{11\,x\,\left(b\,x^3+a\,x^2\right)^{3/2}}$$

input
```
int(1/(x^2*(a*x^2 + b*x^3)^(3/2)),x)
```

output
```
-(2*(a/(b*x) + 1)^(3/2)*hypergeom([3/2, 11/2], 13/2, -a/(b*x)))/(11*x*(a*x
^2 + b*x^3)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^2 (ax^2+bx^3)^{3/2}}\,dx = \frac{315\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^4x^4 - 315\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)}{128\sqrt{bx+a}\,a^6x^4}$$

input
```
int(1/x^2/(b*x^3+a*x^2)^(3/2),x)
```

output
```
(315*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**4*x**4 - 315*sq
rt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**4*x**4 - 32*a**5 + 48*
a**4*b*x - 84*a**3*b**2*x**2 + 210*a**2*b**3*x**3 + 630*a*b**4*x**4)/(128*
sqrt(a + b*x)*a**6*x**4)
```

3.248 $\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result 2185
Mathematica [A] (verified) 2185
Rubi [A] (verified) 2186
Maple [A] (verified) 2188
Fricas [A] (verification not implemented) 2188
Sympy [F] 2189
Maxima [F] 2189
Giac [A] (verification not implemented) 2190
Mupad [F(-1)] 2190
Reduce [B] (verification not implemented) 2190

Optimal result

Integrand size = 21, antiderivative size = 125

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx=\frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}}-\frac{5a\sqrt{x}\sqrt{ax^2+bx^3}}{12b^2}+\frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5a^3\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{8b^{7/2}}$$

output
```
5/8*a^2*(b*x^3+a*x^2)^(1/2)/b^3/x^(1/2)-5/12*a*x^(1/2)*(b*x^3+a*x^2)^(1/2)
/b^2+1/3*x^(3/2)*(b*x^3+a*x^2)^(1/2)/b-5/8*a^3*arctanh(b^(1/2)*x^(3/2)/(b*
x^3+a*x^2)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.88

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx=\frac{\sqrt{b}x^{3/2}(15a^3+5a^2bx-2ab^2x^2+8b^3x^3)+30a^3x\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{24b^{7/2}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x^(7/2)/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(Sqrt[b]*x^(3/2)*(15*a^3 + 5*a^2*b*x - 2*a*b^2*x^2 + 8*b^3*x^3) + 30*a^3*x
*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])])/(24*b
^(7/2)*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1930, 1930, 1930, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5a\int \frac{x^{5/2}}{\sqrt{bx^3+ax^2}}dx}{6b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5a\left(\frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}-\frac{3a\int \frac{x^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4b}\right)}{6b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5a\left(\frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}-\frac{3a\left(\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4b}\right)}{6b}$$

$$\Big\downarrow \text{1935}$$

$$\frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5a\left(\frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}-\frac{3a\left(\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b}\right)}{4b}\right)}{6b}$$

$\Big\downarrow$ 219

$$\frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5a\left(\frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}-\frac{3a\left(\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}\right)}{4b}\right)}{6b}$$

input
```
Int[x^(7/2)/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(x^(3/2)*Sqrt[a*x^2 + b*x^3])/(3*b) - (5*a*((Sqrt[x]*Sqrt[a*x^2 + b*x^3])/
(2*b) - (3*a*(Sqrt[a*x^2 + b*x^3]/(b*Sqrt[x]) - (a*ArcTanh[(Sqrt[b]*x^(3/2
))/Sqrt[a*x^2 + b*x^3]])/b^(3/2)))/(4*b)))/(6*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{\left(8 b^{2} x^{2}-10 a b x+15 a^{2}\right) x^{\frac{3}{2}} \left(b x+a\right)}{24 b^{3} \sqrt{x^{2} \left(b x+a\right)}}-\frac{5 a^{3} \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\,x^{2}+a x}\right) \sqrt{x}\, \sqrt{x \left(b x+a\right)}}{16 b^{\frac{7}{2}} \sqrt{x^{2} \left(b x+a\right)}}$	100
default	$\frac{\sqrt{x}\, \left(16 b^{\frac{9}{2}} x^{4}-4 b^{\frac{7}{2}} a\,x^{3}+10 b^{\frac{5}{2}} a^{2} x^{2}+30 b^{\frac{3}{2}} a^{3} x-15 \sqrt{x \left(b x+a\right)}\, \ln \left(\frac{2 \sqrt{b\,x^{2}+a x}\, \sqrt{b}+2 b x+a}{2 \sqrt{b}}\right) a^{3} b\right)}{48 \sqrt{b\,x^{3}+a\,x^{2}}\, b^{\frac{9}{2}}}$	103

input
```
int(x^(7/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*b^2*x^2-10*a*b*x+15*a^2)*x^(3/2)*(b*x+a)/b^3/(x^2*(b*x+a))^(1/2)-5
/16*a^3/b^(7/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/(x^2*(b*x+a))^(1
/2)*x^(1/2)*(x*(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.50

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx = \left[\frac{15\,a^3\sqrt{b}x\log\left(\frac{2\,bx^2+ax-2\,\sqrt{bx^3+ax^2}\sqrt{b}\sqrt{x}}{x}\right)+2\,(8\,b^3x^2-10\,ab^2x+15\,a^2b)\sqrt{bx^3+ax^2}\sqrt{}}{48\,b^4x}\right.$$

input
```
integrate(x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(15*a^3*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(b)
*sqrt(x))/x) + 2*(8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^3 + a*x^2)*s
qrt(x))/(b^4*x), 1/24*(15*a^3*sqrt(-b)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-
b)*sqrt(x)/(b*x^2 + a*x)) + (8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^3
 + a*x^2)*sqrt(x))/(b^4*x)]
```

Sympy [F]

$$\int \frac{x^{7/2}}{\sqrt{ax^2 + bx^3}}\, dx = \int \frac{x^{\frac{7}{2}}}{\sqrt{x^2\,(a + bx)}}\, dx$$

input
```
integrate(x**(7/2)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**(7/2)/sqrt(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x^{7/2}}{\sqrt{ax^2 + bx^3}}\, dx = \int \frac{x^{\frac{7}{2}}}{\sqrt{bx^3 + ax^2}}\, dx$$

input
```
integrate(x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(7/2)/sqrt(b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.66

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx = -\frac{5\,a^3\log\left(|a|\right)\operatorname{sgn}(x)}{16\,b^{\frac{7}{2}}} + \frac{\sqrt{bx+a}\left(2\,x\left(\frac{4\,x}{b}-\frac{5\,a}{b^2}\right)+\frac{15\,a^2}{b^3}\right)\sqrt{x}+\frac{15\,a^3\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{7}{2}}}}{24\operatorname{sgn}(x)}$$

input
```
integrate(x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-5/16*a^3*log(abs(a))*sgn(x)/b^(7/2) + 1/24*(sqrt(b*x + a)*(2*x*(4*x/b - 5
*a/b^2) + 15*a^2/b^3)*sqrt(x) + 15*a^3*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x
 + a)))/b^(7/2))/sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^{7/2}}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int(x^(7/2)/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
int(x^(7/2)/(a*x^2 + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.61

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{15\sqrt{x}\,\sqrt{bx+a}\,a^2b-10\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x+8\sqrt{x}\,\sqrt{bx+a}\,b^3x^2-15\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\cdots}{\sqrt{a}}\right.}{24b^4}$$

input
```
int(x^(7/2)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(15*sqrt(x)*sqrt(a + b*x)*a**2*b - 10*sqrt(x)*sqrt(a + b*x)*a*b**2*x + 8*s
qrt(x)*sqrt(a + b*x)*b**3*x**2 - 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*s
qrt(b))/sqrt(a))*a**3)/(24*b**4)
```

3.249 $\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}} dx$

Optimal result 2192
Mathematica [A] (verified) 2192
Rubi [A] (verified) 2193
Maple [A] (verified) 2194
Fricas [A] (verification not implemented) 2195
Sympy [F] 2195
Maxima [F] 2196
Giac [A] (verification not implemented) 2196
Mupad [F(-1)] 2196
Reduce [B] (verification not implemented) 2197

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}} dx = -\frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}} + \frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b} + \frac{3a^2 \text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{4b^{5/2}}$$

output

```
-3/4*a*(b*x^3+a*x^2)^(1/2)/b^2/x^(1/2)+1/2*x^(1/2)*(b*x^3+a*x^2)^(1/2)/b+3
/4*a^2*arctanh(b^(1/2)*x^(3/2)/(b*x^3+a*x^2)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}} dx = \frac{\sqrt{b}x^{3/2}(-3a^2-abx+2b^2x^2)+6a^2x\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{4b^{5/2}\sqrt{x^2(a+bx)}}$$

input

```
Integrate[x^(5/2)/Sqrt[a*x^2 + b*x^3],x]
```

output

```
(Sqrt[b]*x^(3/2)*(-3*a^2 - a*b*x + 2*b^2*x^2) + 6*a^2*x*Sqrt[a + b*x]*ArcT
anh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(4*b^(5/2)*Sqrt[x^2*(a
+ b*x)])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1930, 1930, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}-\frac{3a\int \frac{x^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4b}$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}-\frac{3a\left(\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4b}$$

$$\Big\downarrow \text{ 1935}$$

$$\frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}-\frac{3a\left(\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b}\right)}{4b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}-\frac{3a\left(\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}\right)}{4b}$$

input
```
Int[x^(5/2)/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(Sqrt[x]*Sqrt[a*x^2 + b*x^3])/(2*b) - (3*a*(Sqrt[a*x^2 + b*x^3]/(b*Sqrt[x]
) - (a*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/b^(3/2)))/(4*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

method	result	size
risch	$-\frac{(-2bx+3a)x^{\frac{3}{2}}(bx+a)}{4b^2\sqrt{x^2(bx+a)}}+\frac{3a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)\sqrt{x}\,\sqrt{x(bx+a)}}{8b^{\frac{5}{2}}\sqrt{x^2(bx+a)}}$	89
default	$\frac{\sqrt{x}\left(4b^{\frac{7}{2}}x^3-2a\,x^2b^{\frac{5}{2}}-6b^{\frac{3}{2}}a^2x+3\sqrt{x(bx+a)}\,\ln\left(\frac{2\sqrt{b\,x^2+ax}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a^2b\right)}{8\sqrt{b\,x^3+a\,x^2}\,b^{\frac{7}{2}}}$	92

input
```
int(x^(5/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*b*x+3*a)*x^(3/2)*(b*x+a)/b^2/(x^2*(b*x+a))^(1/2)+3/8*a^2/b^(5/2)*
ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/(x^2*(b*x+a))^(1/2)*x^(1/2)*(x*(
b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.76

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx = \left[\frac{3\,a^2\sqrt{b}x\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^3+ax^2}\sqrt{b}\sqrt{x}}{x}\right)+2\,\sqrt{bx^3+ax^2}(2\,b^2x-3\,ab)\sqrt{x}}{8\,b^3x},\right.$$
$$\left.-\frac{3\,a^2\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-b}\sqrt{x}}{bx^2+ax}\right)-\sqrt{bx^3+ax^2}(2\,b^2x-3\,ab)\sqrt{x}}{4\,b^3x}\right]$$

input
```
integrate(x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(3*a^2*sqrt(b)*x*log((2*b*x^2 + a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*s
qrt(x))/x) + 2*sqrt(b*x^3 + a*x^2)*(2*b^2*x - 3*a*b)*sqrt(x))/(b^3*x), -1/
4*(3*a^2*sqrt(-b)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b)*sqrt(x)/(b*x^2 + a
*x)) - sqrt(b*x^3 + a*x^2)*(2*b^2*x - 3*a*b)*sqrt(x))/(b^3*x)]
```

Sympy [F]

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^{\frac{5}{2}}}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(x**(5/2)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**(5/2)/sqrt(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^{\frac{5}{2}}}{\sqrt{bx^3+ax^2}}\,dx$$

input
```
integrate(x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(5/2)/sqrt(b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.75

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{3\,a^2\log\left(|a|\right)\operatorname{sgn}(x)}{8\,b^{\frac{5}{2}}} + \frac{\sqrt{bx+a}\sqrt{x}\left(\frac{2\,x}{b}-\frac{3\,a}{b^2}\right)-\frac{3\,a^2\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{5}{2}}}}{4\operatorname{sgn}\left(x\right)}$$

input
```
integrate(x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
3/8*a^2*log(abs(a))*sgn(x)/b^(5/2) + 1/4*(sqrt(b*x + a)*sqrt(x)*(2*x/b - 3
*a/b^2) - 3*a^2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(5/2))/sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^{5/2}}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int(x^(5/2)/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
int(x^(5/2)/(a*x^2 + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.60

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{-3\sqrt{x}\,\sqrt{bx+a}\,ab+2\sqrt{x}\,\sqrt{bx+a}\,b^2x+3\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2}{4b^3}$$

input
```
int(x^(5/2)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
( - 3*sqrt(x)*sqrt(a + b*x)*a*b + 2*sqrt(x)*sqrt(a + b*x)*b**2*x + 3*sqrt(
b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2)/(4*b**3)
```

3.250 $\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result 2198
Mathematica [A] (verified) 2198
Rubi [A] (verified) 2199
Maple [A] (verified) 2200
Fricas [A] (verification not implemented) 2201
Sympy [F] 2201
Maxima [F] 2201
Giac [A] (verification not implemented) 2202
Mupad [F(-1)] 2202
Reduce [B] (verification not implemented) 2202

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx=\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}$$

output
```
(b*x^3+a*x^2)^(1/2)/b/x^(1/2)-a*arctanh(b^(1/2)*x^(3/2)/(b*x^3+a*x^2)^(1/2
))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.33

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx=\frac{\sqrt{b}x^{3/2}(a+bx)+2ax\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{b^{3/2}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x^(3/2)/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(Sqrt[b]*x^(3/2)*(a + b*x) + 2*a*x*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])
/(Sqrt[a] - Sqrt[a + b*x])])/(b^(3/2)*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1930, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b}$$

$$\Big\downarrow \text{ 1935}$$

$$\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}$$

input
```
Int[x^(3/2)/Sqrt[a*x^2 + b*x^3],x]
```

output
```
Sqrt[a*x^2 + b*x^3]/(b*Sqrt[x]) - (a*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2
+ b*x^3]])/b^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.30

method	result	size
default	$-\frac{\sqrt{x}\left(-2b^{\frac{5}{2}}x^{2}-2b^{\frac{3}{2}}ax+a\sqrt{x(bx+a)}\,\ln\left(\frac{2\sqrt{b\,x^{2}+ax}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)b\right)}{2\sqrt{b\,x^{3}+a\,x^{2}}\,b^{\frac{5}{2}}}$	78
risch	$\frac{x^{\frac{3}{2}}(bx+a)}{b\sqrt{x^{2}(bx+a)}}-\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)\sqrt{x}\,\sqrt{x(bx+a)}}{2b^{\frac{3}{2}}\sqrt{x^{2}(bx+a)}}$	78

input
```
int(x^(3/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^(1/2)*(-2*b^(5/2)*x^2-2*b^(3/2)*a*x+a*(x*(b*x+a))^(1/2)*ln(1/2*(2*(
b*x^2+a*x)^(1/2)*b^(1/2)+2*b*x+a)/b^(1/2))*b)/(b*x^3+a*x^2)^(1/2)/b^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.32

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx = \left[\frac{a\sqrt{b}x\log\left(\frac{2\,bx^2+ax-2\,\sqrt{bx^3+ax^2}\sqrt{b}\sqrt{x}}{x}\right)+2\,\sqrt{bx^3+ax^2}b\sqrt{x}}{2\,b^2x},\ \frac{a\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-}}{bx^2+ax}\right.}{b^2x}\right.$$

input
```
integrate(x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(a*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*sqrt(
x))/x) + 2*sqrt(b*x^3 + a*x^2)*b*sqrt(x))/(b^2*x), (a*sqrt(-b)*x*arctan(sq
rt(b*x^3 + a*x^2)*sqrt(-b)*sqrt(x)/(b*x^2 + a*x)) + sqrt(b*x^3 + a*x^2)*b*
sqrt(x))/(b^2*x)]
```

Sympy [F]

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^{\frac{3}{2}}}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(x**(3/2)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**(3/2)/sqrt(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^{\frac{3}{2}}}{\sqrt{bx^3+ax^2}}\,dx$$

input
```
integrate(x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(3/2)/sqrt(b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx = -\frac{a\log(|a|)\,\mathrm{sgn}(x)}{2\,b^{\frac{3}{2}}} + \frac{\frac{a\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{3}{2}}} + \frac{\sqrt{bx+a}\sqrt{x}}{b}}{\mathrm{sgn}\,(x)}$$

input
```
integrate(x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*a*log(abs(a))*sgn(x)/b^(3/2) + (a*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x
 + a)))/b^(3/2) + sqrt(b*x + a)*sqrt(x)/b)/sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^{3/2}}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int(x^(3/2)/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
int(x^(3/2)/(a*x^2 + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.63

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{x}\,\sqrt{bx+a}\,b - \sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a}{b^2}$$

input
```
int(x^(3/2)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(sqrt(x)*sqrt(a + b*x)*b - sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/s
qrt(a))*a)/b**2
```

3.251 $\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result . . . 2203
Mathematica [A] (verified) . . . 2203
Rubi [A] (verified) . . . 2204
Maple [B] (verified) . . . 2205
Fricas [A] (verification not implemented) . . . 2205
Sympy [F] . . . 2206
Maxima [F] . . . 2206
Giac [A] (verification not implemented) . . . 2206
Mupad [F(-1)] . . . 2207
Reduce [B] (verification not implemented) . . . 2207

Optimal result

Integrand size = 21, antiderivative size = 34

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}}$$

output
```
2*arctanh(b^(1/2)*x^(3/2)/(b*x^3+a*x^2)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx=-\frac{2x\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[Sqrt[x]/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(-2*x*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]*Sqrt
[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \ 1935$$

$$2\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}$$

$$\Big\downarrow \ 219$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}}$$

input
```
Int[Sqrt[x]/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/Sqrt[b]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(26) = 52$.

Time = 0.40 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.71

method	result	size
default	$\frac{\sqrt{x}\,\sqrt{x(bx+a)}\,\ln\left(\frac{2\sqrt{b\,x^2+ax}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)}{\sqrt{b\,x^3+a\,x^2}\,\sqrt{b}}$	58

input
```
int(x^(1/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*x^3+a*x^2)^(1/2)*x^(1/2)*(x*(b*x+a))^(1/2)*ln(1/2*(2*(b*x^2+a*x)^(1/2
)*b^(1/2)+2*b*x+a)/b^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.50

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx = \left[\frac{\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^3+ax^2}\sqrt{b}\sqrt{x}}{x}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-b}\sqrt{x}}{bx^2+ax}\right)}{b}\right]$$

input
```
integrate(x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[log((2*b*x^2 + a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*sqrt(x))/x)/sqrt(b), -
2*sqrt(-b)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b)*sqrt(x)/(b*x^2 + a*x))/b]
```

Sympy [F]

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{\sqrt{x}}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(x**(1/2)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(sqrt(x)/sqrt(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}\,dx$$

input
```
integrate(x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/sqrt(b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx = \frac{\log\left(|a|\right)\mathrm{sgn}(x)}{\sqrt{b}} - \frac{2\,\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{\sqrt{b}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
log(abs(a))*sgn(x)/sqrt(b) - 2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/
(sqrt(b)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx=\int \frac{\sqrt{x}}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int(x^(1/2)/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
int(x^(1/2)/(a*x^2 + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx=\frac{2\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)}{b}$$

input
```
int(x^(1/2)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)))/b
```

3.252 $\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}} dx$

Optimal result 2208
Mathematica [A] (verified) 2208
Rubi [A] (verified) 2209
Maple [A] (verified) 2209
Fricas [A] (verification not implemented) 2210
Sympy [F] 2210
Maxima [F] 2211
Giac [A] (verification not implemented) 2211
Mupad [F(-1)] 2211
Reduce [B] (verification not implemented) 2212

Optimal result

Integrand size = 21, antiderivative size = 25

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}} dx = -\frac{2\sqrt{ax^2+bx^3}}{ax^{3/2}}$$

output
```
-2*(b*x^3+a*x^2)^(1/2)/a/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}} dx = -\frac{2\sqrt{x^2(a+bx)}}{ax^{3/2}}$$

input
```
Integrate[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-2*Sqrt[x^2*(a + b*x)])/(a*x^(3/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx$$
$$\Big\downarrow \text{1920}$$
$$-\frac{2\sqrt{ax^2+bx^3}}{ax^{3/2}}$$

input
```
Int[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-2*Sqrt[a*x^2 + b*x^3])/(a*x^(3/2))
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{2\sqrt{x}\,(bx+a)}{\sqrt{x^2(bx+a)}\,a}$	25
gosper	$-\frac{2\sqrt{x}\,(bx+a)}{a\sqrt{b\,x^3+a\,x^2}}$	27
default	$-\frac{2\sqrt{x}\,(bx+a)}{a\sqrt{b\,x^3+a\,x^2}}$	27
orering	$-\frac{2\sqrt{x}\,(bx+a)}{a\sqrt{b\,x^3+a\,x^2}}$	27

input
```
int(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(x^2*(b*x+a))^(1/2)*x^(1/2)/a*(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\sqrt{bx^3+ax^2}}{ax^{\frac{3}{2}}}$$

input
```
integrate(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(b*x^3 + a*x^2)/(a*x^(3/2))
```

Sympy [F]

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{x}\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(1/x**(1/2)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(x)*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x^2)*sqrt(x)), x)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx = \frac{4\sqrt{b}}{\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)\operatorname{sgn}(x)}$$

input
```
integrate(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
4*sqrt(b)/(((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{x}\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int(1/(x^(1/2)*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
int(1/(x^(1/2)*(a*x^2 + b*x^3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx = \frac{-2\sqrt{x}\,\sqrt{bx+a}-2\sqrt{b}\,x}{ax}$$

input
```
int(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
( - 2*(sqrt(x)*sqrt(a + b*x) + sqrt(b)*x))/(a*x)
```

3.253 $\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx$

Optimal result 2213
Mathematica [A] (verified) 2213
Rubi [A] (verified) 2214
Maple [A] (verified) 2215
Fricas [A] (verification not implemented) 2216
Sympy [F] 2216
Maxima [F] 2216
Giac [A] (verification not implemented) 2217
Mupad [F(-1)] 2217
Reduce [B] (verification not implemented) 2217

Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx=-\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}+\frac{4b\sqrt{ax^2+bx^3}}{3a^2x^{3/2}}$$

output
```
-2/3*(b*x^3+a*x^2)^(1/2)/a/x^(5/2)+4/3*b*(b*x^3+a*x^2)^(1/2)/a^2/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx=-\frac{2(a-2bx)\sqrt{x^2(a+bx)}}{3a^2x^{5/2}}$$

input
```
Integrate[1/(x^(3/2)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-2*(a - 2*b*x)*Sqrt[x^2*(a + b*x)])/(3*a^2*x^(5/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{2b\int \frac{1}{\sqrt{x}\sqrt{bx^3+ax^2}}dx}{3a}-\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{4b\sqrt{ax^2+bx^3}}{3a^2x^{3/2}}-\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}$$

input
```
Int[1/(x^(3/2)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-2*Sqrt[a*x^2 + b*x^3])/(3*a*x^(5/2)) + (4*b*Sqrt[a*x^2 + b*x^3])/(3*a^2*
x^(3/2))
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.55

method	result	size
risch	$-\frac{2(bx+a)(-2bx+a)}{3\sqrt{x^{2}(bx+a)}\,\sqrt{x}\,a^{2}}$	31
gosper	$-\frac{2(bx+a)(-2bx+a)}{3\sqrt{x}\,a^{2}\sqrt{b\,x^{3}+a\,x^{2}}}$	33
default	$-\frac{2(bx+a)(-2bx+a)}{3\sqrt{x}\,a^{2}\sqrt{b\,x^{3}+a\,x^{2}}}$	33
orering	$-\frac{2(bx+a)(-2bx+a)}{3\sqrt{x}\,a^{2}\sqrt{b\,x^{3}+a\,x^{2}}}$	33

input
```
int(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(x^2*(b*x+a))^(1/2)/x^(1/2)*(b*x+a)*(-2*b*x+a)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx = \frac{2\,\sqrt{bx^3+ax^2}(2\,bx-a)}{3\,a^2x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(b*x^3 + a*x^2)*(2*b*x - a)/(a^2*x^(5/2))
```

Sympy [F]

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^{\frac{3}{2}}\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(1/x**(3/2)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**(3/2)*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x^2)*x^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{bx+a}b^3\left(\frac{2(bx+a)b}{a^2}-\frac{3b}{a}\right)}{3\left((bx+a)b-ab\right)^{\frac{3}{2}}|b|\text{sgn}\left(x\right)}$$

input
```
integrate(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
2/3*sqrt(b*x + a)*b^3*(2*(b*x + a)*b/a^2 - 3*b/a)/(((b*x + a)*b - a*b)^(3/
2)*abs(b)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^{3/2}\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int(1/(x^(3/2)*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
int(1/(x^(3/2)*(a*x^2 + b*x^3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a}{3}+\frac{4\sqrt{x}\,\sqrt{bx+a}\,bx}{3}-\frac{4\sqrt{b}\,b\,x^2}{3}}{a^2x^2}$$

input
```
int(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*( - sqrt(x)*sqrt(a + b*x)*a + 2*sqrt(x)*sqrt(a + b*x)*b*x - 2*sqrt(b)*b
*x**2))/(3*a**2*x**2)
```

3.254 $\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx$

Optimal result 2218
Mathematica [A] (verified) 2218
Rubi [A] (verified) 2219
Maple [A] (verified) 2220
Fricas [A] (verification not implemented) 2221
Sympy [F] 2221
Maxima [F] 2221
Giac [A] (verification not implemented) 2222
Mupad [F(-1)] 2222
Reduce [B] (verification not implemented) 2222

Optimal result

Integrand size = 21, antiderivative size = 86

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}+\frac{8b\sqrt{ax^2+bx^3}}{15a^2x^{5/2}}-\frac{16b^2\sqrt{ax^2+bx^3}}{15a^3x^{3/2}}$$

output
```
-2/5*(b*x^3+a*x^2)^(1/2)/a/x^(7/2)+8/15*b*(b*x^3+a*x^2)^(1/2)/a^2/x^(5/2)-
16/15*b^2*(b*x^3+a*x^2)^(1/2)/a^3/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\sqrt{x^2(a+bx)}(3a^2-4abx+8b^2x^2)}{15a^3x^{7/2}}$$

input
```
Integrate[1/(x^(5/2)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-2*Sqrt[x^2*(a + b*x)]*(3*a^2 - 4*a*b*x + 8*b^2*x^2))/(15*a^3*x^(7/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{ 1922}$$

$$-\frac{4b\int \frac{1}{x^{3/2}\sqrt{bx^3+ax^2}}dx}{5a}-\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}$$

$$\Big\downarrow \text{ 1922}$$

$$-\frac{4b\left(-\frac{2b\int \frac{1}{\sqrt{x}\sqrt{bx^3+ax^2}}dx}{3a}-\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}\right)}{5a}-\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}$$

$$\Big\downarrow \text{ 1920}$$

$$-\frac{4b\left(\frac{4b\sqrt{ax^2+bx^3}}{3a^2x^{3/2}}-\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}\right)}{5a}-\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}$$

input
```
Int[1/(x^(5/2)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-2*Sqrt[a*x^2 + b*x^3])/(5*a*x^(7/2)) - (4*b*((-2*Sqrt[a*x^2 + b*x^3])/(3
*a*x^(5/2)) + (4*b*Sqrt[a*x^2 + b*x^3])/(3*a^2*x^(3/2))))/(5*a)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.51

method	result	size
risch	$-\frac{2(bx+a)\left(8b^{2}x^{2}-4abx+3a^{2}\right)}{15\sqrt{x^{2}(bx+a)}\, x^{\frac{3}{2}} a^{3}}$	44
gosper	$-\frac{2(bx+a)\left(8b^{2}x^{2}-4abx+3a^{2}\right)}{15x^{\frac{3}{2}} a^{3}\sqrt{b\,x^{3}+a\,x^{2}}}$	46
default	$-\frac{2(bx+a)\left(8b^{2}x^{2}-4abx+3a^{2}\right)}{15x^{\frac{3}{2}} a^{3}\sqrt{b\,x^{3}+a\,x^{2}}}$	46
orering	$-\frac{2(bx+a)\left(8b^{2}x^{2}-4abx+3a^{2}\right)}{15x^{\frac{3}{2}} a^{3}\sqrt{b\,x^{3}+a\,x^{2}}}$	46

input
```
int(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15/(x^2*(b*x+a))^(1/2)/x^(3/2)*(b*x+a)*(8*b^2*x^2-4*a*b*x+3*a^2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\,(8\,b^2x^2-4\,abx+3\,a^2)\sqrt{bx^3+ax^2}}{15\,a^3x^{\frac{7}{2}}}$$

input
```
integrate(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/15*(8*b^2*x^2 - 4*a*b*x + 3*a^2)*sqrt(b*x^3 + a*x^2)/(a^3*x^(7/2))
```

Sympy [F]

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^{\frac{5}{2}}\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(1/x**(5/2)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**(5/2)*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x^2)*x^(5/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx = \frac{32\left(10\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^4-5\,a\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2+a^2\right)b^{\frac{5}{2}}}{15\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^5\operatorname{sgn}(x)}$$

input
```
integrate(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
32/15*(10*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 - 5*a*(sqrt(b)*sqrt(x) - sqr
t(b*x + a))^2 + a^2)*b^(5/2)/(((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^5*
sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^{5/2}\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int(1/(x^(5/2)*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
int(1/(x^(5/2)*(a*x^2 + b*x^3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2}{5}+\frac{8\sqrt{x}\,\sqrt{bx+a}\,abx}{15}-\frac{16\sqrt{x}\,\sqrt{bx+a}\,b^2x^2}{15}+\frac{16\sqrt{b}\,b^2x^3}{15}}{a^3x^3}$$

input
```
int(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*( - 3*sqrt(x)*sqrt(a + b*x)*a**2 + 4*sqrt(x)*sqrt(a + b*x)*a*b*x - 8*sq
rt(x)*sqrt(a + b*x)*b**2*x**2 + 8*sqrt(b)*b**2*x**3))/(15*a**3*x**3)
```

3.255 $\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx$

Optimal result 2224
Mathematica [A] (verified) 2224
Rubi [A] (verified) 2225
Maple [A] (verified) 2226
Fricas [A] (verification not implemented) 2227
Sympy [F] 2227
Maxima [F] 2227
Giac [A] (verification not implemented) 2228
Mupad [F(-1)] 2228
Reduce [B] (verification not implemented) 2229

Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}} + \frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}} - \frac{16b^2\sqrt{ax^2+bx^3}}{35a^3x^{5/2}} + \frac{32b^3\sqrt{ax^2+bx^3}}{35a^4x^{3/2}}$$

output

```
-2/7*(b*x^3+a*x^2)^(1/2)/a/x^(9/2)+12/35*b*(b*x^3+a*x^2)^(1/2)/a^2/x^(7/2)
-16/35*b^2*(b*x^3+a*x^2)^(1/2)/a^3/x^(5/2)+32/35*b^3*(b*x^3+a*x^2)^(1/2)/a
^4/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{x^2(a+bx)}(-5a^3+6a^2bx-8ab^2x^2+16b^3x^3)}{35a^4x^{9/2}}$$

input

```
Integrate[1/(x^(7/2)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(2*Sqrt[x^2*(a + b*x)]*(-5*a^3 + 6*a^2*b*x - 8*a*b^2*x^2 + 16*b^3*x^3))/(3
5*a^4*x^(9/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{6b\int \frac{1}{x^{5/2}\sqrt{bx^3+ax^2}}dx}{7a}-\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{6b\left(-\frac{4b\int \frac{1}{x^{3/2}\sqrt{bx^3+ax^2}}dx}{5a}-\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}\right)}{7a}-\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{6b\left(-\frac{4b\left(-\frac{2b\int \frac{1}{\sqrt{x}\sqrt{bx^3+ax^2}}dx}{3a}-\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}\right)}{5a}-\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}\right)}{7a}-\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}$$

$$\Big\downarrow \text{1920}$$

$$-\frac{6b\left(-\frac{4b\left(\frac{4b\sqrt{ax^2+bx^3}}{3a^2x^{3/2}}-\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}\right)}{5a}-\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}\right)}{7a}-\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}$$

input
```
Int[1/(x^(7/2)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-2*Sqrt[a*x^2 + b*x^3])/(7*a*x^(9/2)) - (6*b*((-2*Sqrt[a*x^2 + b*x^3])/(5
*a*x^(7/2)) - (4*b*((-2*Sqrt[a*x^2 + b*x^3])/(3*a*x^(5/2)) + (4*b*Sqrt[a*x
^2 + b*x^3])/(3*a^2*x^(3/2))))/(5*a)))/(7*a)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.47

method	result	size
risch	$-\frac{2(bx+a)\left(-16b^{3}x^{3}+8a\,b^{2}x^{2}-6a^{2}bx+5a^{3}\right)}{35\sqrt{x^{2}(bx+a)}\,x^{\frac{5}{2}}\,a^{4}}$	55
gosper	$-\frac{2(bx+a)\left(-16b^{3}x^{3}+8a\,b^{2}x^{2}-6a^{2}bx+5a^{3}\right)}{35x^{\frac{5}{2}}\,a^{4}\sqrt{b\,x^{3}+a\,x^{2}}}$	57
default	$-\frac{2(bx+a)\left(-16b^{3}x^{3}+8a\,b^{2}x^{2}-6a^{2}bx+5a^{3}\right)}{35x^{\frac{5}{2}}\,a^{4}\sqrt{b\,x^{3}+a\,x^{2}}}$	57
orering	$-\frac{2(bx+a)\left(-16b^{3}x^{3}+8a\,b^{2}x^{2}-6a^{2}bx+5a^{3}\right)}{35x^{\frac{5}{2}}\,a^{4}\sqrt{b\,x^{3}+a\,x^{2}}}$	57

input
```
int(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35/(x^2*(b*x+a))^(1/2)/x^(5/2)*(b*x+a)*(-16*b^3*x^3+8*a*b^2*x^2-6*a^2*b
*x+5*a^3)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx = \frac{2\,(16\,b^3x^3-8\,ab^2x^2+6\,a^2bx-5\,a^3)\sqrt{bx^3+ax^2}}{35\,a^4x^{\frac{9}{2}}}$$

input
```
integrate(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x^3 + a*x^2)/(a
^4*x^(9/2))
```

Sympy [F]

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^{\frac{7}{2}}\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(1/x**(7/2)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**(7/2)*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}x^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x^2)*x^(7/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx = \frac{64\left(35\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^6-21\,a\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^4+7\,a^2\left(\sqrt{b}\sqrt{x}-\sqrt{bx+\cdots}\right.\right.}{35\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^7\operatorname{sgn}(x)}$$

input
```
integrate(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
64/35*(35*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^6 - 21*a*(sqrt(b)*sqrt(x) - sq
rt(b*x + a))^4 + 7*a^2*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a^3)*b^(7/2)/
(((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^7*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^{7/2}\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int(1/(x^(7/2)*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
int(1/(x^(7/2)*(a*x^2 + b*x^3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3}{7}+\frac{12\sqrt{x}\,\sqrt{bx+a}\,a^2bx}{35}-\frac{16\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x^2}{35}+\frac{32\sqrt{x}\,\sqrt{bx+a}\,b^3x^3}{35}-\frac{32\sqrt{b}\,b^3x^4}{35}}{a^4x^4}$$

input
```
int(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*( - 5*sqrt(x)*sqrt(a + b*x)*a**3 + 6*sqrt(x)*sqrt(a + b*x)*a**2*b*x - 8
*sqrt(x)*sqrt(a + b*x)*a*b**2*x**2 + 16*sqrt(x)*sqrt(a + b*x)*b**3*x**3 -
16*sqrt(b)*b**3*x**4))/(35*a**4*x**4)
```

3.256 $\int x^{1-3p}(ax^2+bx^3)^p\,dx$

Optimal result 2230
Mathematica [A] (verified) 2230
Rubi [A] (verified) 2231
Maple [F] 2232
Fricas [F] 2233
Sympy [F] 2233
Maxima [F] 2233
Giac [F] 2234
Mupad [F(-1)] 2234
Reduce [F] 2234

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int x^{1-3p}(ax^2+bx^3)^p\,dx = \frac{x^{2-3p}\left(1+\frac{bx}{a}\right)^{-p}(ax^2+bx^3)^p\,\text{Hypergeometric2F1}\left(2-p,-p,3-p,-\frac{bx}{a}\right)}{2-p}$$

output
```
x^(2-3*p)*(b*x^3+a*x^2)^p*hypergeom([-p, 2-p],[3-p],-b*x/a)/(2-p)/((1+b*x/
a)^p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int x^{1-3p}(ax^2+bx^3)^p\,dx = \frac{x^{2-3p}(x^2(a+bx))^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(2-p,-p,3-p,-\frac{bx}{a}\right)}{2-p}$$

input
```
Integrate[x^(1 - 3*p)*(a*x^2 + b*x^3)^p,x]
```

output
```
(x^(2 - 3*p)*(x^2*(a + b*x))^p*Hypergeometric2F1[2 - p, -p, 3 - p, -((b*x)
/a)])/((2 - p)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{1-3p}\left(ax^2+bx^3\right)^p\,dx$$

$$\Big\downarrow \; 1938$$

$$x^{-2p}(a+bx)^{-p}\left(ax^2+bx^3\right)^p\int x^{1-p}(a+bx)^pdx$$

$$\Big\downarrow \; 76$$

$$x^{-2p}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^2+bx^3\right)^p\int x^{1-p}\left(\frac{bx}{a}+1\right)^pdx$$

$$\Big\downarrow \; 74$$

$$\frac{x^{2-3p}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^2+bx^3\right)^p\text{Hypergeometric2F1}\left(2-p,-p,3-p,-\frac{bx}{a}\right)}{2-p}$$

input
```
Int[x^(1 - 3*p)*(a*x^2 + b*x^3)^p,x]
```

output
```
(x^(2 - 3*p)*(a*x^2 + b*x^3)^p*Hypergeometric2F1[2 - p, -p, 3 - p, -((b*x)
/a)])/((2 - p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !Integer
Q[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int x^{1-3p}\left(b\,x^{3}+a\,x^{2}\right)^{p}dx$$

input
```
int(x^(1-3*p)*(b*x^3+a*x^2)^p,x)
```

output
```
int(x^(1-3*p)*(b*x^3+a*x^2)^p,x)
```

Fricas [F]

$$\int x^{1-3p}\left(ax^2+bx^3\right)^p\,dx=\int \left(bx^3+ax^2\right)^p x^{-3\,p+1}\,dx$$

input
```
integrate(x^(1-3*p)*(b*x^3+a*x^2)^p,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a*x^2)^p*x^(-3*p + 1), x)
```

Sympy [F]

$$\int x^{1-3p}\left(ax^2+bx^3\right)^p\,dx=\int x^{1-3p}\left(x^2(a+bx)\right)^p\,dx$$

input
```
integrate(x**(1-3*p)*(b*x**3+a*x**2)**p,x)
```

output
```
Integral(x**(1 - 3*p)*(x**2*(a + b*x))**p, x)
```

Maxima [F]

$$\int x^{1-3p}\left(ax^2+bx^3\right)^p\,dx=\int \left(bx^3+ax^2\right)^p x^{-3\,p+1}\,dx$$

input
```
integrate(x^(1-3*p)*(b*x^3+a*x^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^p*x^(-3*p + 1), x)
```

Giac [F]

$$\int x^{1-3p}(ax^2+bx^3)^p\,dx = \int (bx^3+ax^2)^p x^{-3\,p+1}\,dx$$

input
```
integrate(x^(1-3*p)*(b*x^3+a*x^2)^p,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x^2)^p*x^(-3*p + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{1-3p}(ax^2+bx^3)^p\,dx = \int x^{1-3\,p}\,(b\,x^3+a\,x^2)^p\,dx$$

input
```
int(x^(1 - 3*p)*(a*x^2 + b*x^3)^p,x)
```

output
```
int(x^(1 - 3*p)*(a*x^2 + b*x^3)^p, x)
```

Reduce [F]

$$\int x^{1-3p}(ax^2+bx^3)^p\,dx = \frac{(b\,x^3+a\,x^2)^p\,apx+(b\,x^3+a\,x^2)^p\,b\,x^2+x^{3p}\left(\int \frac{(b\,x^3+a\,x^2)^p}{x^{3p}a+x^{3p}bx}dx\right)a^2p^2-x^{3p}\left(\int \frac{(b\,x^3+a\,x^2)^p}{x^{3p}a+x^{3p}bx}dx\right)a^2p}{2x^{3p}b}$$

input
```
int(x^(1-3*p)*(b*x^3+a*x^2)^p,x)
```

output
```
((a*x**2 + b*x**3)**p*a*p*x + (a*x**2 + b*x**3)**p*b*x**2 + x**(3*p)*int((
a*x**2 + b*x**3)**p/(x**(3*p)*a + x**(3*p)*b*x),x)*a**2*p**2 - x**(3*p)*in
t((a*x**2 + b*x**3)**p/(x**(3*p)*a + x**(3*p)*b*x),x)*a**2*p)/(2*x**(3*p)*
b)
```

3.257 $\int x^{-3p}(ax^2+bx^3)^p\,dx$

Optimal result . . . 2235
Mathematica [A] (verified) . . . 2235
Rubi [A] (verified) . . . 2236
Maple [F] . . . 2237
Fricas [F] . . . 2237
Sympy [F] . . . 2238
Maxima [F] . . . 2238
Giac [F] . . . 2238
Mupad [F(-1)] . . . 2239
Reduce [F] . . . 2239

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int x^{-3p}\left(ax^2+bx^3\right)^p\,dx=\frac{x^{-1-3p}\left(ax^2+bx^3\right)^{1+p}\text{Hypergeometric2F1}\left(1,2,2-p,-\frac{bx}{a}\right)}{a(1-p)}$$

output
```
x^(-1-3*p)*(b*x^3+a*x^2)^(p+1)*hypergeom([1, 2],[2-p],-b*x/a)/a/(1-p)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

$$\int x^{-3p}\left(ax^2+bx^3\right)^p\,dx=\frac{x^{1-3p}(x^2(a+bx))^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx}{a}\right)}{1-p}$$

input
```
Integrate[(a*x^2 + b*x^3)^p/x^(3*p),x]
```

output
```
(x^(1 - 3*p)*(x^2*(a + b*x))^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((b*x)
/a)])/((1 - p)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-3p}\left(ax^2+bx^3\right)^p\,dx$$

$$\Big\downarrow \text{1938}$$

$$x^{-2p}(a+bx)^{-p}\left(ax^2+bx^3\right)^p\int x^{-p}(a+bx)^p dx$$

$$\Big\downarrow \text{76}$$

$$x^{-2p}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^2+bx^3\right)^p\int x^{-p}\left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \text{74}$$

$$\frac{x^{1-3p}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^2+bx^3\right)^p \text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx}{a}\right)}{1-p}$$

input
```
Int[(a*x^2 + b*x^3)^p/x^(3*p),x]
```

output
```
(x^(1 - 3*p)*(a*x^2 + b*x^3)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((b*x)
/a)])/((1 - p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \left(b\,x^{3}+a\,x^{2}\right)^{p} x^{-3p}dx$$

input
```
int((b*x^3+a*x^2)^p/(x^(3*p)),x)
```

output
```
int((b*x^3+a*x^2)^p/(x^(3*p)),x)
```

Fricas [F]

$$\int x^{-3p}\left(ax^2+bx^3\right)^p\,dx=\int\frac{(bx^3+ax^2)^p}{x^{3\,p}}\,dx$$

input
```
integrate((b*x^3+a*x^2)^p/(x^(3*p)),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a*x^2)^p/x^(3*p), x)
```

Sympy [F]

$$\int x^{-3p}\left(ax^2+bx^3\right)^p\,dx=\int x^{-3p}\left(x^2(a+bx)\right)^p\,dx$$

input
```
integrate((b*x**3+a*x**2)**p/(x**(3*p)),x)
```

output
```
Integral((x**2*(a + b*x))**p/x**(3*p), x)
```

Maxima [F]

$$\int x^{-3p}\left(ax^2+bx^3\right)^p\,dx=\int \frac{(bx^3+ax^2)^p}{x^{3\,p}}\,dx$$

input
```
integrate((b*x^3+a*x^2)^p/(x^(3*p)),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^p/x^(3*p), x)
```

Giac [F]

$$\int x^{-3p}\left(ax^2+bx^3\right)^p\,dx=\int \frac{(bx^3+ax^2)^p}{x^{3\,p}}\,dx$$

input
```
integrate((b*x^3+a*x^2)^p/(x^(3*p)),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x^2)^p/x^(3*p), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-3p}\left(ax^2+bx^3\right)^p\,dx=\int \frac{(b\,x^3+a\,x^2)^p}{x^{3\,p}}\,dx$$

input
```
int((a*x^2 + b*x^3)^p/x^(3*p),x)
```

output
```
int((a*x^2 + b*x^3)^p/x^(3*p), x)
```

Reduce [F]

$$\int x^{-3p}\left(ax^2+bx^3\right)^p\,dx=\frac{(b\,x^3+a\,x^2)^p\,x+x^{3p}\left(\int \frac{(b\,x^3+a\,x^2)^p}{x^{3p}a+x^{3p}bx}dx\right)ap}{x^{3p}}$$

input
```
int((b*x^3+a*x^2)^p/(x^(3*p)),x)
```

output
```
((a*x**2 + b*x**3)**p*x + x**(3*p)*int((a*x**2 + b*x**3)**p/(x**(3*p)*a +
x**(3*p)*b*x),x)*a*p)/x**(3*p)
```

3.258 $\int x^{-1-3p}(ax^2+bx^3)^p\,dx$

Optimal result 2240
Mathematica [A] (verified) 2240
Rubi [A] (verified) 2241
Maple [F] 2242
Fricas [F] 2243
Sympy [F] 2243
Maxima [F] 2243
Giac [F] 2244
Mupad [F(-1)] 2244
Reduce [F] 2244

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int x^{-1-3p}(ax^2+bx^3)^p\,dx = -\frac{x^{-3p}\left(1+\frac{bx}{a}\right)^{-p}(ax^2+bx^3)^p\,\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx}{a}\right)}{p}$$

output
```
-(b*x^3+a*x^2)^p*hypergeom([-p, -p],[1-p],-b*x/a)/p/(x^(3*p))/((1+b*x/a)^p
)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int x^{-1-3p}(ax^2+bx^3)^p\,dx = -\frac{x^{-3p}(x^2(a+bx))^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx}{a}\right)}{p}$$

input
```
Integrate[x^(-1 - 3*p)*(a*x^2 + b*x^3)^p,x]
```

output
```
-(((x^2*(a + b*x))^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x)/a)])/(p*x^(3
*p)*(1 + (b*x)/a)^p))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-3p-1}\left(ax^2+bx^3\right)^p\,dx$$

$$\Big\downarrow \text{1938}$$

$$x^{-2p}(a+bx)^{-p}\left(ax^2+bx^3\right)^p\int x^{-p-1}(a+bx)^pdx$$

$$\Big\downarrow \text{76}$$

$$x^{-2p}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^2+bx^3\right)^p\int x^{-p-1}\left(\frac{bx}{a}+1\right)^p\,dx$$

$$\Big\downarrow \text{74}$$

$$-\frac{x^{-3p}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^2+bx^3\right)^p\,\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx}{a}\right)}{p}$$

input
```
Int[x^(-1 - 3*p)*(a*x^2 + b*x^3)^p,x]
```

output
```
-(((a*x^2 + b*x^3)^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x)/a)])/(p*x^(3
*p)*(1 + (b*x)/a)^p))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !Integer
Q[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int x^{-1-3p}\left(b\,x^{3}+a\,x^{2}\right)^{p}dx$$

input
```
int(x^(-1-3*p)*(b*x^3+a*x^2)^p,x)
```

output
```
int(x^(-1-3*p)*(b*x^3+a*x^2)^p,x)
```

Fricas [F]

$$\int x^{-1-3p}\left(ax^2+bx^3\right)^p\,dx=\int \left(bx^3+ax^2\right)^p x^{-3\,p-1}\,dx$$

input
```
integrate(x^(-1-3*p)*(b*x^3+a*x^2)^p,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a*x^2)^p*x^(-3*p - 1), x)
```

Sympy [F]

$$\int x^{-1-3p}\left(ax^2+bx^3\right)^p\,dx=\int x^{-3p-1}\left(x^2(a+bx)\right)^p\,dx$$

input
```
integrate(x**(-1-3*p)*(b*x**3+a*x**2)**p,x)
```

output
```
Integral(x**(-3*p - 1)*(x**2*(a + b*x))**p, x)
```

Maxima [F]

$$\int x^{-1-3p}\left(ax^2+bx^3\right)^p\,dx=\int \left(bx^3+ax^2\right)^p x^{-3\,p-1}\,dx$$

input
```
integrate(x^(-1-3*p)*(b*x^3+a*x^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^p*x^(-3*p - 1), x)
```

Giac [F]

$$\int x^{-1-3p}(ax^2+bx^3)^p\,dx=\int (bx^3+ax^2)^p x^{-3p-1}\,dx$$

input
```
integrate(x^(-1-3*p)*(b*x^3+a*x^2)^p,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x^2)^p*x^(-3*p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-3p}(ax^2+bx^3)^p\,dx=\int \frac{(b\,x^3+a\,x^2)^p}{x^{3\,p+1}}\,dx$$

input
```
int((a*x^2 + b*x^3)^p/x^(3*p + 1),x)
```

output
```
int((a*x^2 + b*x^3)^p/x^(3*p + 1), x)
```

Reduce [F]

$$\int x^{-1-3p}(ax^2+bx^3)^p\,dx=\int \frac{(b\,x^3+a\,x^2)^p}{x^{3p}x}dx$$

input
```
int(x^(-1-3*p)*(b*x^3+a*x^2)^p,x)
```

output
```
int((a*x**2 + b*x**3)**p/(x**(3*p)*x),x)
```

3.259 $\int x^{-2-3p}(ax^2+bx^3)^p\,dx$

Optimal result . . . 2245
Mathematica [A] (verified) . . . 2245
Rubi [A] (verified) . . . 2246
Maple [A] (verified) . . . 2246
Fricas [A] (verification not implemented) . . . 2247
Sympy [F] . . . 2247
Maxima [F] . . . 2248
Giac [F] . . . 2248
Mupad [B] (verification not implemented) . . . 2248
Reduce [B] (verification not implemented) . . . 2249

Optimal result

Integrand size = 21, antiderivative size = 32

$$\int x^{-2-3p}\left(ax^2+bx^3\right)^p\,dx=-\frac{x^{-3(1+p)}\left(ax^2+bx^3\right)^{1+p}}{a(1+p)}$$

output
```
-(b*x^3+a*x^2)^(p+1)/a/(p+1)/(x^(3*p+3))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int x^{-2-3p}\left(ax^2+bx^3\right)^p\,dx=-\frac{x^{-3(1+p)}\left(x^2(a+bx)\right)^{1+p}}{a(1+p)}$$

input
```
Integrate[x^(-2 - 3*p)*(a*x^2 + b*x^3)^p,x]
```

output
```
-((x^2*(a + b*x))^(1 + p)/(a*(1 + p)*x^(3*(1 + p))))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-3p-2}\left(ax^2+bx^3\right)^p\,dx$$

$$\Big\downarrow \text{ 1920}$$

$$-\frac{x^{-3(p+1)}\left(ax^2+bx^3\right)^{p+1}}{a(p+1)}$$

input
```
Int[x^(-2 - 3*p)*(a*x^2 + b*x^3)^p,x]
```

output
```
-((a*x^2 + b*x^3)^(1 + p)/(a*(1 + p)*x^(3*(1 + p))))
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

method	result	size
gosper	$-\frac{x^{-1-3p}(bx+a)\left(b\,x^{3}+a\,x^{2}\right)^{p}}{a(p+1)}$	36
orering	$-\frac{x(bx+a)x^{-3p-2}\left(b\,x^{3}+a\,x^{2}\right)^{p}}{a(p+1)}$	37
parallelrisch	$-\frac{x^{2}x^{-3p-2}\left(x^{2}(bx+a)\right)^{p}b+x\,x^{-3p-2}\left(x^{2}(bx+a)\right)^{p}a}{a(p+1)}$	56

input
```
int(x^(-3*p-2)*(b*x^3+a*x^2)^p,x,method=_RETURNVERBOSE)
```

output
```
-x^(-1-3*p)/a/(p+1)*(b*x+a)*(b*x^3+a*x^2)^p
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int x^{-2-3p}\left(ax^2+bx^3\right)^p\,dx=-\frac{(bx^2+ax)(bx^3+ax^2)^px^{-3\,p-2}}{ap+a}$$

input
```
integrate(x^(-2-3*p)*(b*x^3+a*x^2)^p,x, algorithm="fricas")
```

output
```
-(b*x^2 + a*x)*(b*x^3 + a*x^2)^p*x^(-3*p - 2)/(a*p + a)
```

Sympy [F]

$$\int x^{-2-3p}\left(ax^2+bx^3\right)^p\,dx=\int x^{-3p-2}\left(x^2(a+bx)\right)^p\,dx$$

input
```
integrate(x**(-2-3*p)*(b*x**3+a*x**2)**p,x)
```

output
```
Integral(x**(-3*p - 2)*(x**2*(a + b*x))**p, x)
```

Maxima [F]

$$\int x^{-2-3p}\left(ax^2+bx^3\right)^p\,dx=\int \left(bx^3+ax^2\right)^p x^{-3\,p-2}\,dx$$

input
```
integrate(x^(-2-3*p)*(b*x^3+a*x^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^p*x^(-3*p - 2), x)
```

Giac [F]

$$\int x^{-2-3p}\left(ax^2+bx^3\right)^p\,dx=\int \left(bx^3+ax^2\right)^p x^{-3\,p-2}\,dx$$

input
```
integrate(x^(-2-3*p)*(b*x^3+a*x^2)^p,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x^2)^p*x^(-3*p - 2), x)
```

Mupad [B] (verification not implemented)

Time = 9.39 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int x^{-2-3p}\left(ax^2+bx^3\right)^p\,dx=-\left(b\,x^3+a\,x^2\right)^p\left(\frac{x}{x^{3\,p+2}\,\left(p+1\right)}+\frac{b\,x^2}{a\,x^{3\,p+2}\,\left(p+1\right)}\right)$$

input
```
int((a*x^2 + b*x^3)^p/x^(3*p + 2),x)
```

output
```
-(a*x^2 + b*x^3)^p*(x/(x^(3*p + 2)*(p + 1)) + (b*x^2)/(a*x^(3*p + 2)*(p + 
1)))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int x^{-2-3p}(ax^2+bx^3)^p\,dx=-\frac{(b\,x^3+a\,x^2)^p\,(bx+a)}{x^{3p}ax\,(p+1)}$$

input
```
int(x^(-2-3*p)*(b*x^3+a*x^2)^p,x)
```

output
```
( - (a*x**2 + b*x**3)**p*(a + b*x))/(x**(3*p)*a*x*(p + 1))
```

3.260 $\int x^{-3-3p}(ax^2+bx^3)^p\,dx$

Optimal result . . . 2250
Mathematica [A] (verified) . . . 2250
Rubi [A] (verified) . . . 2251
Maple [A] (verified) . . . 2252
Fricas [A] (verification not implemented) . . . 2252
Sympy [F] . . . 2253
Maxima [F] . . . 2253
Giac [F] . . . 2253
Mupad [B] (verification not implemented) . . . 2254
Reduce [B] (verification not implemented) . . . 2254

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int x^{-3-3p}\left(ax^2+bx^3\right)^p\,dx=-\frac{x^{-4-3p}\left(ax^2+bx^3\right)^{1+p}}{a(2+p)}+\frac{bx^{-3(1+p)}\left(ax^2+bx^3\right)^{1+p}}{a^2(1+p)(2+p)}$$

output
```
-x^(-4-3*p)*(b*x^3+a*x^2)^(p+1)/a/(2+p)+b*(b*x^3+a*x^2)^(p+1)/a^2/(p+1)/(2
+p)/(x^(3*p+3))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.63

$$\int x^{-3-3p}\left(ax^2+bx^3\right)^p\,dx=-\frac{x^{-4-3p}(a+ap-bx)\left(x^2(a+bx)\right)^{1+p}}{a^2(1+p)(2+p)}$$

input
```
Integrate[x^(-3 - 3*p)*(a*x^2 + b*x^3)^p,x]
```

output
```
-((x^(-4 - 3*p)*(a + a*p - b*x)*(x^2*(a + b*x))^(1 + p))/(a^2*(1 + p)*(2 +
 p)))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-3p-3}\left(ax^2+bx^3\right)^p\,dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{b\int x^{-3p-2}\left(bx^3+ax^2\right)^p dx}{a(p+2)}-\frac{x^{-3p-4}\left(ax^2+bx^3\right)^{p+1}}{a(p+2)}$$

$$\Big\downarrow \text{1920}$$

$$\frac{bx^{-3(p+1)}\left(ax^2+bx^3\right)^{p+1}}{a^2(p+1)(p+2)}-\frac{x^{-3p-4}\left(ax^2+bx^3\right)^{p+1}}{a(p+2)}$$

input
```
Int[x^(-3 - 3*p)*(a*x^2 + b*x^3)^p,x]
```

output
```
-((x^(-4 - 3*p)*(a*x^2 + b*x^3)^(1 + p))/(a*(2 + p))) + (b*(a*x^2 + b*x^3)
^(1 + p))/(a^2*(1 + p)*(2 + p)*x^(3*(1 + p)))
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.71

method	result	size
gosper	$-\frac{x^{-3p-2}(bx+a)\left(b\,x^{3}+a\,x^{2}\right)^{p}(ap-bx+a)}{a^{2}(2+p)(p+1)}$	50
orering	$-\frac{\left(b\,x^{3}+a\,x^{2}\right)^{p}x^{-3-3p}(ap-bx+a)x(bx+a)}{(2+p)(p+1)a^{2}}$	51

input
```
int(x^(-3-3*p)*(b*x^3+a*x^2)^p,x,method=_RETURNVERBOSE)
```

output
```
-x^(-3*p-2)/a^2/(2+p)/(p+1)*(b*x+a)*(b*x^3+a*x^2)^p*(a*p-b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int x^{-3-3p}\left(ax^2+bx^3\right)^p\,dx=-\frac{(abpx^2-b^2x^3+(a^2p+a^2)x)(bx^3+ax^2)^p x^{-3\,p-3}}{a^2p^2+3\,a^2p+2\,a^2}$$

input
```
integrate(x^(-3-3*p)*(b*x^3+a*x^2)^p,x, algorithm="fricas")
```

output
```
-(a*b*p*x^2 - b^2*x^3 + (a^2*p + a^2)*x)*(b*x^3 + a*x^2)^p*x^(-3*p - 3)/(a
^2*p^2 + 3*a^2*p + 2*a^2)
```

Sympy [F]

$$\int x^{-3-3p}(ax^2+bx^3)^p\,dx=\int x^{-3p-3}(x^2(a+bx))^p\,dx$$

input
```
integrate(x**(-3-3*p)*(b*x**3+a*x**2)**p,x)
```

output
```
Integral(x**(-3*p - 3)*(x**2*(a + b*x))**p, x)
```

Maxima [F]

$$\int x^{-3-3p}(ax^2+bx^3)^p\,dx=\int (bx^3+ax^2)^p x^{-3p-3}\,dx$$

input
```
integrate(x^(-3-3*p)*(b*x^3+a*x^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^p*x^(-3*p - 3), x)
```

Giac [F]

$$\int x^{-3-3p}(ax^2+bx^3)^p\,dx=\int (bx^3+ax^2)^p x^{-3p-3}\,dx$$

input
```
integrate(x^(-3-3*p)*(b*x^3+a*x^2)^p,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x^2)^p*x^(-3*p - 3), x)
```

Mupad [B] (verification not implemented)

Time = 9.62 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.40

$$\int x^{-3-3p}(ax^2+bx^3)^p\,dx = -(b\,x^3+a\,x^2)^p\left(\frac{x\,(p+1)}{x^{3\,p+3}\,(p^2+3\,p+2)} - \frac{b^2\,x^3}{a^2\,x^{3\,p+3}\,(p^2+3\,p+2)} + \frac{b\,p\,x^2}{a\,x^{3\,p+3}\,(p^2+3\,p+2)}\right)$$

input
```
int((a*x^2 + b*x^3)^p/x^(3*p + 3),x)
```

output
```
-(a*x^2 + b*x^3)^p*((x*(p + 1))/(x^(3*p + 3)*(3*p + p^2 + 2)) - (b^2*x^3)/
(a^2*x^(3*p + 3)*(3*p + p^2 + 2)) + (b*p*x^2)/(a*x^(3*p + 3)*(3*p + p^2 +
2)))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int x^{-3-3p}(ax^2+bx^3)^p\,dx = \frac{(b\,x^3+a\,x^2)^p\,(-abpx+b^2x^2-a^2p-a^2)}{x^{3p}a^2x^2\,(p^2+3p+2)}$$

input
```
int(x^(-3-3*p)*(b*x^3+a*x^2)^p,x)
```

output
```
((a*x**2 + b*x**3)**p*( - a**2*p - a**2 - a*b*p*x + b**2*x**2))/(x**(3*p)*
a**2*x**2*(p**2 + 3*p + 2))
```

3.261 $\int x^{-4-3p}(ax^2+bx^3)^p\,dx$

Optimal result 2255
Mathematica [A] (verified) 2255
Rubi [A] (verified) 2256
Maple [A] (verified) 2257
Fricas [A] (verification not implemented) 2258
Sympy [F] 2258
Maxima [F] 2258
Giac [F] 2259
Mupad [B] (verification not implemented) 2259
Reduce [B] (verification not implemented) 2260

Optimal result

Integrand size = 21, antiderivative size = 116

$$\int x^{-4-3p}\left(ax^2+bx^3\right)^p\,dx=-\frac{x^{-5-3p}\left(ax^2+bx^3\right)^{1+p}}{a(3+p)}+\frac{2bx^{-4-3p}\left(ax^2+bx^3\right)^{1+p}}{a^2(2+p)(3+p)}-\frac{2b^2x^{-3(1+p)}\left(ax^2+bx^3\right)^{1+p}}{a^3(1+p)(2+p)(3+p)}$$

output
```
-x^(-5-3*p)*(b*x^3+a*x^2)^(p+1)/a/(3+p)+2*b*x^(-4-3*p)*(b*x^3+a*x^2)^(p+1)
/a^2/(2+p)/(3+p)-2*b^2*(b*x^3+a*x^2)^(p+1)/a^3/(p+1)/(2+p)/(3+p)/(x^(3*p+3
))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.62

$$\int x^{-4-3p}\left(ax^2+bx^3\right)^p\,dx=-\frac{x^{-3(1+p)}(a+bx)\left(x^2(a+bx)\right)^p\left(a^2(2+3p+p^2)-2ab(1+p)x+2b^2x^2\right)}{a^3(1+p)(2+p)(3+p)}$$

input
```
Integrate[x^(-4 - 3*p)*(a*x^2 + b*x^3)^p,x]
```

output
```
-(((a + b*x)*(x^2*(a + b*x))^p*(a^2*(2 + 3*p + p^2) - 2*a*b*(1 + p)*x + 2*
b^2*x^2))/(a^3*(1 + p)*(2 + p)*(3 + p)*x^(3*(1 + p))))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-3p-4} \left(ax^2+bx^3\right)^p \, dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{2b \int x^{-3(p+1)} \left(bx^3+ax^2\right)^p dx}{a(p+3)}-\frac{x^{-3p-5} \left(ax^2+bx^3\right)^{p+1}}{a(p+3)}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{2b \left(-\frac{b \int x^{-3p-2} \left(bx^3+ax^2\right)^p dx}{a(p+2)}-\frac{x^{-3p-4} \left(ax^2+bx^3\right)^{p+1}}{a(p+2)}\right)}{a(p+3)}-\frac{x^{-3p-5} \left(ax^2+bx^3\right)^{p+1}}{a(p+3)}$$

$$\Big\downarrow \text{1920}$$

$$-\frac{2b \left(\frac{b x^{-3(p+1)} \left(ax^2+bx^3\right)^{p+1}}{a^2(p+1)(p+2)}-\frac{x^{-3p-4} \left(ax^2+bx^3\right)^{p+1}}{a(p+2)}\right)}{a(p+3)}-\frac{x^{-3p-5} \left(ax^2+bx^3\right)^{p+1}}{a(p+3)}$$

input
```
Int[x^(-4 - 3*p)*(a*x^2 + b*x^3)^p,x]
```

output
```
-((x^(-5 - 3*p)*(a*x^2 + b*x^3)^(1 + p))/(a*(3 + p))) - (2*b*(-((x^(-4 - 3
*p)*(a*x^2 + b*x^3)^(1 + p))/(a*(2 + p))) + (b*(a*x^2 + b*x^3)^(1 + p))/(a
^2*(1 + p)*(2 + p)*x^(3*(1 + p)))))/(a*(3 + p))
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.72

method	result	size
gosper	$-\frac{x^{-3-3p} \left(b x+a \right) \left(b \,x^{3}+a \,x^{2}\right)^{p} \left(a^{2} p^{2}-2 a b p x+2 b^{2} x^{2}+3 a^{2} p-2 a b x+2 a^{2}\right)}{a^{3} \left(2+p \right) \left(3+p \right) \left(p+1\right)}$	84
orering	$-\frac{\left(b x+a \right) x \left(a^{2} p^{2}-2 a b p x+2 b^{2} x^{2}+3 a^{2} p-2 a b x+2 a^{2}\right) \left(b \,x^{3}+a \,x^{2}\right)^{p} x^{-4-3p}}{\left(3+p \right) \left(2+p \right) \left(p+1\right) a^{3}}$	85

input
```
int(x^(-4-3*p)*(b*x^3+a*x^2)^p,x,method=_RETURNVERBOSE)
```

output
```
-x^(-3-3*p)/a^3/(2+p)/(3+p)/(p+1)*(b*x+a)*(b*x^3+a*x^2)^p*(a^2*p^2-2*a*b*p
*x+2*b^2*x^2+3*a^2*p-2*a*b*x+2*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.96

$$\int x^{-4-3p}\left(ax^2+bx^3\right)^p\,dx$$
$$=\frac{\left(2\,ab^2px^3-2\,b^3x^4-(a^2bp^2+a^2bp)x^2-(a^3p^2+3\,a^3p+2\,a^3)x\right)(bx^3+ax^2)^px^{-3\,p-4}}{a^3p^3+6\,a^3p^2+11\,a^3p+6\,a^3}$$

input
```
integrate(x^(-4-3*p)*(b*x^3+a*x^2)^p,x, algorithm="fricas")
```

output
```
(2*a*b^2*p*x^3 - 2*b^3*x^4 - (a^2*b*p^2 + a^2*b*p)*x^2 - (a^3*p^2 + 3*a^3*
p + 2*a^3)*x)*(b*x^3 + a*x^2)^p*x^(-3*p - 4)/(a^3*p^3 + 6*a^3*p^2 + 11*a^3
*p + 6*a^3)
```

Sympy [F]

$$\int x^{-4-3p}\left(ax^2+bx^3\right)^p\,dx=\int x^{-3p-4}\left(x^2(a+bx)\right)^p\,dx$$

input
```
integrate(x**(-4-3*p)*(b*x**3+a*x**2)**p,x)
```

output
```
Integral(x**(-3*p - 4)*(x**2*(a + b*x))**p, x)
```

Maxima [F]

$$\int x^{-4-3p}\left(ax^2+bx^3\right)^p\,dx=\int \left(bx^3+ax^2\right)^px^{-3\,p-4}\,dx$$

input
```
integrate(x^(-4-3*p)*(b*x^3+a*x^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^p*x^(-3*p - 4), x)
```

Giac [F]

$$\int x^{-4-3p}\left(ax^2+bx^3\right)^p\,dx=\int \left(bx^3+ax^2\right)^p x^{-3p-4}\,dx$$

input
```
integrate(x^(-4-3*p)*(b*x^3+a*x^2)^p,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x^2)^p*x^(-3*p - 4), x)
```

Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.35

$$\int x^{-4-3p}\left(ax^2+bx^3\right)^p\,dx=-\left(b\,x^3+a\,x^2\right)^p\left(\frac{x\,\left(p^2+3\,p+2\right)}{x^{3\,p+4}\,\left(p^3+6\,p^2+11\,p+6\right)}+\frac{2\,b^3\,x^4}{a^3\,x^{3\,p+4}\,\left(p^3+6\,p^2+11\,p+6\right)}-\frac{2\,b^2\,p\,x^3}{a^2\,x^{3\,p+4}\,\left(p^3+6\,p^2+11\,p+6\right)}+\frac{b\,p\,x^2\,\left(p+1\right)}{a\,x^{3\,p+4}\,\left(p^3+6\,p^2+11\,p+6\right)}\right)$$

input
```
int((a*x^2 + b*x^3)^p/x^(3*p + 4),x)
```

output
```
-(a*x^2 + b*x^3)^p*((x*(3*p + p^2 + 2))/(x^(3*p + 4)*(11*p + 6*p^2 + p^3 +
 6)) + (2*b^3*x^4)/(a^3*x^(3*p + 4)*(11*p + 6*p^2 + p^3 + 6)) - (2*b^2*p*x
^3)/(a^2*x^(3*p + 4)*(11*p + 6*p^2 + p^3 + 6)) + (b*p*x^2*(p + 1))/(a*x^(3
*p + 4)*(11*p + 6*p^2 + p^3 + 6)))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

$$\int x^{-4-3p}(ax^2+bx^3)^p\,dx = \frac{(b\,x^3+a\,x^2)^p\,(-a^2b\,p^2x+2a\,b^2p\,x^2-2b^3x^3-a^3p^2-a^2bpx-3a^3p-2a^3)}{x^{3p}a^3x^3\,(p^3+6p^2+11p+6)}$$

input
```
int(x^(-4-3*p)*(b*x^3+a*x^2)^p,x)
```

output
```
((a*x**2 + b*x**3)**p*( - a**3*p**2 - 3*a**3*p - 2*a**3 - a**2*b*p**2*x -
a**2*b*p*x + 2*a*b**2*p*x**2 - 2*b**3*x**3))/(x**(3*p)*a**3*x**3*(p**3 + 6
*p**2 + 11*p + 6))
```

3.262 $\int \frac{x^{11}}{(ax^2+bx^5)^3} dx$

Optimal result . . . 2261
Mathematica [A] (verified) . . . 2261
Rubi [A] (verified) . . . 2262
Maple [A] (verified) . . . 2263
Fricas [B] (verification not implemented) . . . 2263
Sympy [B] (verification not implemented) . . . 2264
Maxima [B] (verification not implemented) . . . 2264
Giac [A] (verification not implemented) . . . 2264
Mupad [B] (verification not implemented) . . . 2265
Reduce [B] (verification not implemented) . . . 2265

Optimal result

Integrand size = 17, antiderivative size = 19

$$\int \frac{x^{11}}{(ax^2+bx^5)^3} dx = \frac{x^6}{6a(a+bx^3)^2}$$

output
```
1/6*x^6/a/(b*x^3+a)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{x^{11}}{(ax^2+bx^5)^3} dx = -\frac{a+2bx^3}{6b^2(a+bx^3)^2}$$

input
```
Integrate[x^11/(a*x^2 + b*x^5)^3,x]
```

output
```
-1/6*(a + 2*b*x^3)/(b^2*(a + b*x^3)^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {9, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(ax^2+bx^5)^3}\,dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x^5}{(a+bx^3)^3}dx$$

$$\Big\downarrow \ 796$$

$$\frac{x^6}{6a\,(a+bx^3)^2}$$

input
```
Int[x^11/(a*x^2 + b*x^5)^3,x]
```

output
```
x^6/(6*a*(a + b*x^3)^2)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

method	result	size
gosper	$-\frac{2b\,x^3+a}{6(b\,x^3+a)^2 b^2}$	23
parallelrisch	$\frac{-2b\,x^3-a}{6b^2(b\,x^3+a)^2}$	25
risch	$\frac{-\frac{x^3}{3b}-\frac{a}{6b^2}}{(b\,x^3+a)^2}$	26
default	$-\frac{1}{3b^2(b\,x^3+a)}+\frac{a}{6b^2(b\,x^3+a)^2}$	31
norman	$\frac{-\frac{x^8}{3b}-\frac{a\,x^5}{6b^2}}{x^5(b\,x^3+a)^2}$	32
orering	$-\frac{(2b\,x^3+a)(b\,x^3+a)x^6}{6b^2(b\,x^5+a\,x^2)^3}$	37

input
```
int(x^11/(b*x^5+a*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*b*x^3+a)/(b*x^3+a)^2/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(17) = 34.

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{x^{11}}{(ax^2+bx^5)^3}\,dx = -\frac{2\,bx^3+a}{6\,(b^4x^6+2\,ab^3x^3+a^2b^2)}$$

input
```
integrate(x^11/(b*x^5+a*x^2)^3,x, algorithm="fricas")
```

output
```
-1/6*(2*b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{x^{11}}{(ax^2+bx^5)^3}\,dx = \frac{-a-2bx^3}{6a^2b^2+12ab^3x^3+6b^4x^6}$$

input
```
integrate(x**11/(b*x**5+a*x**2)**3,x)
```

output
```
(-a - 2*b*x**3)/(6*a**2*b**2 + 12*a*b**3*x**3 + 6*b**4*x**6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{x^{11}}{(ax^2+bx^5)^3}\,dx = -\frac{2\,bx^3+a}{6\,(b^4x^6+2\,ab^3x^3+a^2b^2)}$$

input
```
integrate(x^11/(b*x^5+a*x^2)^3,x, algorithm="maxima")
```

output
```
-1/6*(2*b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{x^{11}}{(ax^2+bx^5)^3}\,dx = -\frac{2\,bx^3+a}{6\,(bx^3+a)^2b^2}$$

input
```
integrate(x^11/(b*x^5+a*x^2)^3,x, algorithm="giac")
```

output
```
-1/6*(2*b*x^3 + a)/((b*x^3 + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{x^{11}}{(ax^2+bx^5)^3}\,dx = -\frac{\frac{a}{6\,b^2}+\frac{x^3}{3\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$

input
```
int(x^11/(a*x^2 + b*x^5)^3,x)
```

output
```
-(a/(6*b^2) + x^3/(3*b))/(a^2 + b^2*x^6 + 2*a*b*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{x^{11}}{(ax^2+bx^5)^3}\,dx = \frac{x^6}{6a\,(b^2x^6+2ab\,x^3+a^2)}$$

input
```
int(x^11/(b*x^5+a*x^2)^3,x)
```

output
```
x**6/(6*a*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.263 $\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx$

Optimal result . . . 2266
Mathematica [A] (verified) . . . 2266
Rubi [A] (verified) . . . 2267
Maple [A] (verified) . . . 2268
Fricas [A] (verification not implemented) . . . 2269
Sympy [F] . . . 2269
Maxima [A] (verification not implemented) . . . 2269
Giac [A] (verification not implemented) . . . 2270
Mupad [B] (verification not implemented) . . . 2270
Reduce [B] (verification not implemented) . . . 2270

Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx=\frac{16a^2\sqrt{ax^2+bx^5}}{45b^3x}-\frac{8ax^2\sqrt{ax^2+bx^5}}{45b^2}+\frac{2x^5\sqrt{ax^2+bx^5}}{15b}$$

output
```
16/45*a^2*(b*x^5+a*x^2)^(1/2)/b^3/x-8/45*a*x^2*(b*x^5+a*x^2)^(1/2)/b^2+2/1
5*x^5*(b*x^5+a*x^2)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx=\frac{2\sqrt{x^2\left(a+bx^3\right)}(8a^2-4abx^3+3b^2x^6)}{45b^3x}$$

input
```
Integrate[x^9/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*Sqrt[x^2*(a + b*x^3)]*(8*a^2 - 4*a*b*x^3 + 3*b^2*x^6))/(45*b^3*x)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{\sqrt{ax^2+bx^5}}dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^5\sqrt{ax^2+bx^5}}{15b}-\frac{4a\int \frac{x^6}{\sqrt{bx^5+ax^2}}dx}{5b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^5\sqrt{ax^2+bx^5}}{15b}-\frac{4a\left(\frac{2x^2\sqrt{ax^2+bx^5}}{9b}-\frac{2a\int \frac{x^3}{\sqrt{bx^5+ax^2}}dx}{3b}\right)}{5b}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2x^5\sqrt{ax^2+bx^5}}{15b}-\frac{4a\left(\frac{2x^2\sqrt{ax^2+bx^5}}{9b}-\frac{4a\sqrt{ax^2+bx^5}}{9b^2x}\right)}{5b}$$

input
```
Int[x^9/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*x^5*Sqrt[a*x^2 + b*x^5])/(15*b) - (4*a*((-4*a*Sqrt[a*x^2 + b*x^5])/(9*b
^2*x) + (2*x^2*Sqrt[a*x^2 + b*x^5])/(9*b)))/(5*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.54

method	result	size
trager	$\frac{2\left(3b^{2}x^{6}-4ab\,x^{3}+8a^{2}\right)\sqrt{b\,x^{5}+a\,x^{2}}}{45b^{3}x}$	43
gosper	$\frac{2\left(b\,x^{3}+a\right)\left(3b^{2}x^{6}-4ab\,x^{3}+8a^{2}\right)x}{45b^{3}\sqrt{b\,x^{5}+a\,x^{2}}}$	48
default	$\frac{2\left(b\,x^{3}+a\right)\left(3b^{2}x^{6}-4ab\,x^{3}+8a^{2}\right)x}{45b^{3}\sqrt{b\,x^{5}+a\,x^{2}}}$	48
risch	$\frac{2x\left(b\,x^{3}+a\right)\left(3b^{2}x^{6}-4ab\,x^{3}+8a^{2}\right)}{45\sqrt{x^{2}\left(b\,x^{3}+a\right)}\,b^{3}}$	48
orering	$\frac{2\left(b\,x^{3}+a\right)\left(3b^{2}x^{6}-4ab\,x^{3}+8a^{2}\right)x}{45b^{3}\sqrt{b\,x^{5}+a\,x^{2}}}$	48

input
```
int(x^9/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/45*(3*b^2*x^6-4*a*b*x^3+8*a^2)/b^3/x*(b*x^5+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\,(3\,b^2x^6-4\,abx^3+8\,a^2)\sqrt{bx^5+ax^2}}{45\,b^3x}$$

input
```
integrate(x^9/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/45*(3*b^2*x^6 - 4*a*b*x^3 + 8*a^2)*sqrt(b*x^5 + a*x^2)/(b^3*x)
```

Sympy [F]

$$\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^9}{\sqrt{x^2\,(a+bx^3)}}\,dx$$

input
```
integrate(x**9/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x**9/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\,(3\,b^3x^9-ab^2x^6+4\,a^2bx^3+8\,a^3)}{45\,\sqrt{bx^3+a}b^3}$$

input
```
integrate(x^9/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/45*(3*b^3*x^9 - a*b^2*x^6 + 4*a^2*b*x^3 + 8*a^3)/(sqrt(b*x^3 + a)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.81

$$\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx = -\frac{16\,a^{\frac{5}{2}}\mathrm{sgn}(x)}{45\,b^3} + \frac{2\,\sqrt{bx^3+a}a^2}{3\,b^3\mathrm{sgn}\,(x)} + \frac{2\left(3\,(bx^3+a)^{\frac{5}{2}} - 10\,(bx^3+a)^{\frac{3}{2}}a\right)}{45\,b^3\mathrm{sgn}\,(x)}$$

input
```
integrate(x^9/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-16/45*a^(5/2)*sgn(x)/b^3 + 2/3*sqrt(b*x^3 + a)*a^2/(b^3*sgn(x)) + 2/45*(3
*(b*x^3 + a)^(5/2) - 10*(b*x^3 + a)^(3/2)*a)/(b^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.84 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\,\sqrt{b\,x^5+a\,x^2}\,(8\,a^2-4\,a\,b\,x^3+3\,b^2\,x^6)}{45\,b^3\,x}$$

input
```
int(x^9/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
(2*(a*x^2 + b*x^5)^(1/2)*(8*a^2 + 3*b^2*x^6 - 4*a*b*x^3))/(45*b^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.42

$$\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\sqrt{b\,x^3+a}\,(3b^2x^6-4ab\,x^3+8a^2)}{45b^3}$$

input
```
int(x^9/(b*x^5+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3)*(8*a**2 - 4*a*b*x**3 + 3*b**2*x**6))/(45*b**3)
```

3.264 $\int \frac{x^6}{\sqrt{ax^2+bx^5}} dx$

Optimal result 2271
Mathematica [A] (verified) 2271
Rubi [A] (verified) 2272
Maple [A] (verified) 2273
Fricas [A] (verification not implemented) 2274
Sympy [F] 2274
Maxima [A] (verification not implemented) 2274
Giac [A] (verification not implemented) 2275
Mupad [B] (verification not implemented) 2275
Reduce [B] (verification not implemented) 2275

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{x^6}{\sqrt{ax^2+bx^5}} dx = -\frac{4a\sqrt{ax^2+bx^5}}{9b^2x} + \frac{2x^2\sqrt{ax^2+bx^5}}{9b}$$

output
```
-4/9*a*(b*x^5+a*x^2)^(1/2)/b^2/x+2/9*x^2*(b*x^5+a*x^2)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.65

$$\int \frac{x^6}{\sqrt{ax^2+bx^5}} dx = \frac{2(-2a+bx^3)\sqrt{x^2(a+bx^3)}}{9b^2x}$$

input
```
Integrate[x^6/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*(-2*a + b*x^3)*Sqrt[x^2*(a + b*x^3)])/(9*b^2*x)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^2\sqrt{ax^2+bx^5}}{9b}-\frac{2a\int \frac{x^3}{\sqrt{bx^5+ax^2}}dx}{3b}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2x^2\sqrt{ax^2+bx^5}}{9b}-\frac{4a\sqrt{ax^2+bx^5}}{9b^2x}$$

input
```
Int[x^6/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(-4*a*Sqrt[a*x^2 + b*x^5])/(9*b^2*x) + (2*x^2*Sqrt[a*x^2 + b*x^5])/(9*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.62

method	result	size
trager	$-\frac{2\left(-b\,x^{3}+2a\right)\sqrt{b\,x^{5}+a\,x^{2}}}{9b^{2}x}$	32
gosper	$-\frac{2\left(b\,x^{3}+a\right)\left(-b\,x^{3}+2a\right)x}{9b^{2}\sqrt{b\,x^{5}+a\,x^{2}}}$	37
default	$-\frac{2\left(b\,x^{3}+a\right)\left(-b\,x^{3}+2a\right)x}{9b^{2}\sqrt{b\,x^{5}+a\,x^{2}}}$	37
risch	$-\frac{2x\left(b\,x^{3}+a\right)\left(-b\,x^{3}+2a\right)}{9\sqrt{x^{2}\left(b\,x^{3}+a\right)}\,b^{2}}$	37
orering	$-\frac{2\left(b\,x^{3}+a\right)\left(-b\,x^{3}+2a\right)x}{9b^{2}\sqrt{b\,x^{5}+a\,x^{2}}}$	37

input
```
int(x^6/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(-b*x^3+2*a)/b^2/x*(b*x^5+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.58

$$\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\sqrt{bx^5+ax^2}(bx^3-2a)}{9b^2x}$$

input
```
integrate(x^6/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/9*sqrt(b*x^5 + a*x^2)*(b*x^3 - 2*a)/(b^2*x)
```

Sympy [F]

$$\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^6}{\sqrt{x^2(a+bx^3)}}\,dx$$

input
```
integrate(x**6/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x**6/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.65

$$\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx = \frac{2(b^2x^6-abx^3-2a^2)}{9\sqrt{bx^3+a}b^2}$$

input
```
integrate(x^6/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/9*(b^2*x^6 - a*b*x^3 - 2*a^2)/(sqrt(b*x^3 + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx = \frac{4\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{9\,b^2} + \frac{2\,(bx^3+a)^{\frac{3}{2}}}{9\,b^2\mathrm{sgn}\,(x)} - \frac{2\,\sqrt{bx^3+a}a}{3\,b^2\mathrm{sgn}\,(x)}$$

input
```
integrate(x^6/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
4/9*a^(3/2)*sgn(x)/b^2 + 2/9*(b*x^3 + a)^(3/2)/(b^2*sgn(x)) - 2/3*sqrt(b*x
^3 + a)*a/(b^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 10.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.63

$$\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx = -\frac{\sqrt{b\,x^5+a\,x^2}\,\left(\frac{4\,a}{9\,b^2}-\frac{2\,x^3}{9\,b}\right)}{x}$$

input
```
int(x^6/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
-((a*x^2 + b*x^5)^(1/2)*((4*a)/(9*b^2) - (2*x^3)/(9*b)))/x
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.42

$$\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\sqrt{b\,x^3+a}\,(b\,x^3-2a)}{9b^2}$$

input
```
int(x^6/(b*x^5+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3)*( - 2*a + b*x**3))/(9*b**2)
```

3.265 $\int \frac{x^3}{\sqrt{ax^2+bx^5}} \, dx$

Optimal result . . . 2276
Mathematica [A] (verified) . . . 2276
Rubi [A] (verified) . . . 2277
Maple [A] (verified) . . . 2277
Fricas [A] (verification not implemented) . . . 2278
Sympy [F] . . . 2278
Maxima [A] (verification not implemented) . . . 2279
Giac [A] (verification not implemented) . . . 2279
Mupad [B] (verification not implemented) . . . 2279
Reduce [B] (verification not implemented) . . . 2280

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}} \, dx = \frac{2\sqrt{ax^2+bx^5}}{3bx}$$

output
```
2/3*(b*x^5+a*x^2)^(1/2)/b/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}} \, dx = \frac{2\sqrt{x^2(a+bx^3)}}{3bx}$$

input
```
Integrate[x^3/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*Sqrt[x^2*(a + b*x^3)])/(3*b*x)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx$$
$$\Big\downarrow \text{ 1920}$$
$$\frac{2\sqrt{ax^2+bx^5}}{3bx}$$

input
```
Int[x^3/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*Sqrt[a*x^2 + b*x^5])/(3*b*x)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
trager	$\frac{2\sqrt{b\,x^{5}+a\,x^{2}}}{3bx}$	22
gosper	$\frac{2x\left(b\,x^{3}+a\right)}{3b\sqrt{b\,x^{5}+a\,x^{2}}}$	27
default	$\frac{2x\left(b\,x^{3}+a\right)}{3b\sqrt{b\,x^{5}+a\,x^{2}}}$	27
risch	$\frac{2x\left(b\,x^{3}+a\right)}{3\sqrt{x^{2}\left(b\,x^{3}+a\right)}\,b}$	27
orering	$\frac{2x\left(b\,x^{3}+a\right)}{3b\sqrt{b\,x^{5}+a\,x^{2}}}$	27

input
```
int(x^3/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(b*x^5+a*x^2)^(1/2)/b/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\sqrt{bx^5+ax^2}}{3\,bx}$$

input
```
integrate(x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(b*x^5 + a*x^2)/(b*x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^3}{\sqrt{x^2\,(a+bx^3)}}\,dx$$

input
```
integrate(x**3/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x**3/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\sqrt{bx^3+a}}{3\,b}$$

input
```
integrate(x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/3*sqrt(b*x^3 + a)/b
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{a}\mathrm{sgn}(x)}{3\,b} + \frac{2\sqrt{bx^3+a}}{3\,b\mathrm{sgn}\,(x)}$$

input
```
integrate(x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(a)*sgn(x)/b + 2/3*sqrt(b*x^3 + a)/(b*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 10.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\sqrt{b\,x^5+a\,x^2}}{3\,b\,x}$$

input
```
int(x^3/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
(2*(a*x^2 + b*x^5)^(1/2))/(3*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.52

$$\int \frac{x^3}{\sqrt{ax^2 + bx^5}}\, dx = \frac{2\sqrt{b\, x^3 + a}}{3b}$$

input
```
int(x^3/(b*x^5+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3))/(3*b)
```

3.266 $\int \frac{1}{\sqrt{ax^2+bx^5}} \, dx$

Optimal result . . . 2281
Mathematica [A] (verified) . . . 2281
Rubi [A] (verified) . . . 2282
Maple [A] (verified) . . . 2283
Fricas [A] (verification not implemented) . . . 2283
Sympy [F] . . . 2283
Maxima [F] . . . 2284
Giac [A] (verification not implemented) . . . 2284
Mupad [F(-1)] . . . 2284
Reduce [B] (verification not implemented) . . . 2285

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{\sqrt{ax^2+bx^5}} \, dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{a}}$$

output
```
-2/3*arctanh(a^(1/2)*x/(b*x^5+a*x^2)^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int \frac{1}{\sqrt{ax^2+bx^5}} \, dx = -\frac{2x\sqrt{a+bx^3}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\sqrt{x^2(a+bx^3)}}$$

input
```
Integrate[1/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(-2*x*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]*Sqrt[x^
2*(a + b*x^3)])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax^2+bx^5}}dx$$

$$\Big\downarrow \text{1914}$$

$$-\frac{2}{3}\int \frac{1}{1-\frac{ax^2}{bx^5+ax^2}}d\frac{x}{\sqrt{bx^5+ax^2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{a}}$$

input
```
Int[1/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(-2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^5]])/(3*Sqrt[a])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.34

method	result	size
default	$-\frac{2x\sqrt{b\,x^{3}+a}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{b\,x^{5}+a\,x^{2}}\,\sqrt{a}}$	43

input
```
int(1/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(b*x^5+a*x^2)^(1/2)*x*(b*x^3+a)^(1/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)
)/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx = \left[\frac{\log\left(\frac{bx^4+2\,ax-2\,\sqrt{bx^5+ax^2}\sqrt{a}}{x^4}\right)}{3\,\sqrt{a}}, \frac{2\,\sqrt{-a}\arctan\left(\frac{\sqrt{bx^5+ax^2}\sqrt{-a}}{bx^4+ax}\right)}{3\,a}\right]$$

input
```
integrate(1/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/3*log((b*x^4 + 2*a*x - 2*sqrt(b*x^5 + a*x^2)*sqrt(a))/x^4)/sqrt(a), 2/3
*sqrt(-a)*arctan(sqrt(b*x^5 + a*x^2)*sqrt(-a)/(b*x^4 + a*x))/a]
```

Sympy [F]

$$\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{ax^2+bx^5}}\,dx$$

input
```
integrate(1/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a*x**2 + b*x**5), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{ax^2 + bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5 + ax^2}}\,dx$$

input
```
integrate(1/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*x^5 + a*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sqrt{ax^2 + bx^5}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{3\sqrt{-a}} + \frac{2\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}\operatorname{sgn}(x)}$$

input
```
integrate(1/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-2/3*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2/3*arctan(sqrt(b*x^3 + a)
/sqrt(-a))/(sqrt(-a)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ax^2 + bx^5}}\,dx = \int \frac{1}{\sqrt{b\,x^5 + a\,x^2}}\,dx$$

input
```
int(1/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
int(1/(a*x^2 + b*x^5)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{ax^2 + bx^5}}\,dx = \frac{\sqrt{a}\left(\log\left(\sqrt{b\,x^3 + a} - \sqrt{a}\right) - \log\left(\sqrt{b\,x^3 + a} + \sqrt{a}\right)\right)}{3a}$$

input
```
int(1/(b*x^5+a*x^2)^(1/2),x)
```

output
```
(sqrt(a)*(log(sqrt(a + b*x**3) - sqrt(a)) - log(sqrt(a + b*x**3) + sqrt(a)
)))/(3*a)
```

3.267 $\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx$

Optimal result . . . 2286
Mathematica [A] (verified) . . . 2286
Rubi [A] (verified) . . . 2287
Maple [A] (verified) . . . 2288
Fricas [A] (verification not implemented) . . . 2289
Sympy [F] . . . 2289
Maxima [F] . . . 2289
Giac [A] (verification not implemented) . . . 2290
Mupad [F(-1)] . . . 2290
Reduce [B] (verification not implemented) . . . 2291

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx = -\frac{\sqrt{ax^2+bx^5}}{3ax^4}+\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^5}}\right)}{3a^{3/2}}$$

output
```
-1/3*(b*x^5+a*x^2)^(1/2)/a/x^4+1/3*b*arctanh(a^(1/2)*x/(b*x^5+a*x^2)^(1/2)
)/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx = \frac{-\sqrt{a}(a+bx^3)+bx^3\sqrt{a+bx^3}\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}x^2\sqrt{x^2\,(a+bx^3)}}$$

input
```
Integrate[1/(x^3*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
(-(Sqrt[a]*(a + b*x^3)) + b*x^3*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sq
rt[a]])/(3*a^(3/2)*x^2*Sqrt[x^2*(a + b*x^3)])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{ax^2+bx^5}}dx$$

$$\Big\downarrow \text{1931}$$

$$-\frac{b\int \frac{1}{\sqrt{bx^5+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^5}}{3ax^4}$$

$$\Big\downarrow \text{1914}$$

$$\frac{b\int \frac{1}{1-\frac{ax^2}{bx^5+ax^2}}d\frac{x}{\sqrt{bx^5+ax^2}}}{3a}-\frac{\sqrt{ax^2+bx^5}}{3ax^4}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^5}}\right)}{3a^{3/2}}-\frac{\sqrt{ax^2+bx^5}}{3ax^4}$$

input
```
Int[1/(x^3*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
-1/3*Sqrt[a*x^2 + b*x^5]/(a*x^4) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x
^5]])/(3*a^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

method	result	size
default	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-b\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)a\,x^{3}+\sqrt{b\,x^{3}+a}\,a^{\frac{3}{2}}\right)}{3x^{2}\sqrt{b\,x^{5}+a\,x^{2}}\,a^{\frac{5}{2}}}$	66
risch	$-\frac{b\,x^{3}+a}{3a\,x^{2}\sqrt{x^{2}\left(b\,x^{3}+a\right)}}+\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)\sqrt{b\,x^{3}+a}\,x}{3a^{\frac{3}{2}}\sqrt{x^{2}\left(b\,x^{3}+a\right)}}$	73

input
```
int(1/x^3/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^2*(b*x^3+a)^(1/2)*(-b*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a*x^3+(b*x^3
+a)^(1/2)*a^(3/2))/(b*x^5+a*x^2)^(1/2)/a^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.24

$$\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx = \left[\frac{\sqrt{a}bx^4\log\left(\frac{bx^4+2\,ax+2\,\sqrt{bx^5+ax^2}\sqrt{a}}{x^4}\right)-2\,\sqrt{bx^5+ax^2}a}{6\,a^2x^4},\ -\frac{\sqrt{-a}bx^4\arctan\left(\frac{\sqrt{bx^5+ax^2}\sqrt{-a}}{bx^4+ax}\right)+\sqrt{bx^5+ax^2}a}{3\,a^2x^4}\right]$$

input
```
integrate(1/x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(sqrt(a)*b*x^4*log((b*x^4 + 2*a*x + 2*sqrt(b*x^5 + a*x^2)*sqrt(a))/x^
4) - 2*sqrt(b*x^5 + a*x^2)*a)/(a^2*x^4), -1/3*(sqrt(-a)*b*x^4*arctan(sqrt(
b*x^5 + a*x^2)*sqrt(-a)/(b*x^4 + a*x)) + sqrt(b*x^5 + a*x^2)*a)/(a^2*x^4)]
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^3\sqrt{x^2\,(a+bx^3)}}\,dx$$

input
```
integrate(1/x**3/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(x**2*(a + b*x**3))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^5 + a*x^2)*x^3), x)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx = -\frac{b\left(\frac{\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{\sqrt{bx^3+a}}{abx^3}\right)}{3\,\mathrm{sgn}\,(x)}$$

input
```
integrate(1/x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/3*b*(arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x^3 + a)/(a
*b*x^3))/sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^3\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(1/(x^3*(a*x^2 + b*x^5)^(1/2)),x)
```

output
```
int(1/(x^3*(a*x^2 + b*x^5)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx = \frac{-2\sqrt{b\,x^3+a}\,a-\sqrt{a}\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)b\,x^3+\sqrt{a}\log\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)b\,x^3}{6a^2x^3}$$

input

```
int(1/x^3/(b*x^5+a*x^2)^(1/2),x)
```

output

```
( - 2*sqrt(a + b*x**3)*a - sqrt(a)*log(sqrt(a + b*x**3) - sqrt(a))*b*x**3
+ sqrt(a)*log(sqrt(a + b*x**3) + sqrt(a))*b*x**3)/(6*a**2*x**3)
```

3.268 $\int \frac{x^4}{\sqrt{ax^2+bx^5}} \, dx$

Optimal result . . . 2292
Mathematica [C] (verified) . . . 2293
Rubi [A] (verified) . . . 2293
Maple [A] (verified) . . . 2295
Fricas [A] (verification not implemented) . . . 2296
Sympy [F] . . . 2296
Maxima [F] . . . 2296
Giac [F] . . . 2297
Mupad [F(-1)] . . . 2297
Reduce [F] . . . 2297

Optimal result

Integrand size = 19, antiderivative size = 238

$$\int \frac{x^4}{\sqrt{ax^2+bx^5}} \, dx = \frac{2\sqrt{ax^2+bx^5}}{5b} - \frac{4\sqrt{2+\sqrt{3}}ax\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output

```
2/5*(b*x^5+a*x^2)^(1/2)/b-4/15*(1/2*6^(1/2)+1/2*2^(1/2))*a*x*(a^(1/3)+b^(1
/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/
3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1
/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(4/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)
/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^5+a*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.29

$$\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx=\frac{2x^2\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)\right)}{5b\sqrt{x^2(a+bx^3)}}$$

input
```
Integrate[x^4/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*x^2*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3,
 -((b*x^3)/a)]))/(5*b*Sqrt[x^2*(a + b*x^3)])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1930, 1938, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx$$

$$\Big\downarrow \; 1930$$

$$\frac{2\sqrt{ax^2+bx^5}}{5b}-\frac{2a\int \frac{x}{\sqrt{bx^5+ax^2}}dx}{5b}$$

$$\Big\downarrow \; 1938$$

$$\frac{2\sqrt{ax^2+bx^5}}{5b}-\frac{2ax\sqrt{a+bx^3}\int \frac{1}{\sqrt{bx^3+a}}dx}{5b\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \; 759$$

$$\frac{2\sqrt{ax^2+bx^5}}{5b}-\frac{4\sqrt{2+\sqrt{3}}ax\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

input
```
Int[x^4/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*Sqrt[a*x^2 + b*x^5])/(5*b) - (4*Sqrt[2 + Sqrt[3]]*a*x*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a*x^2 + b*x^5])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.04

method	result
default	$\frac{2x\left(ia\sqrt{3}\left(-a b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{i\left(i\sqrt{3}\left(-a b^{2}\right)^{\frac{1}{3}}-2bx-\left(-a b^{2}\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a b^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{2\left(-bx+\left(-a b^{2}\right)^{\frac{1}{3}}\right)}{\left(-a b^{2}\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\sqrt{-\frac{i\left(i\sqrt{3}\left(-a b^{2}\right)^{\frac{1}{3}}+2bx+\left(-a b^{2}\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a b^{2}\right)^{\frac{1}{3}}}}\ \mathrm{Ellip}\cdots\right.}{15\sqrt{b x^{5}+a x^{2}}\, b^{2}}$
	$4ia\sqrt{3}\left(-a b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\, b}{\left(-a b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}+\cdots\right.}{\left(-a\cdots\right.}}$
risch	$\frac{2x^{2}\left(b x^{3}+a\right)}{5b\sqrt{x^{2}\left(b x^{3}+a\right)}}+\frac{\cdots}{15b^{2}\sqrt{x\cdots}}$

input
```
int(x^4/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*x*(I*a*3^(1/2)*(-a*b^2)^(1/3)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a
*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*(-2*(-b*x+(-a*b^2)^(1/3))/(-a*b
^2)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2
)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(-I*(
I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/
2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))+3*b^2*x^4+3*a*b*x)/(b*x^5+a*x^
2)^(1/2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.16

$$\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx=-\frac{2\left(2\,a\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-\sqrt{bx^5+ax^2}b\right)}{5\,b^2}$$

input
```
integrate(x^4/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/5*(2*a*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - sqrt(b*x^5 + a*x^2)*
b)/b^2
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^4}{\sqrt{x^2\,(a+bx^3)}}\,dx$$

input
```
integrate(x**4/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x**4/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^4}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^4/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(b*x^5 + a*x^2), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^4}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^4/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^4}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(x^4/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
int(x^4/(a*x^2 + b*x^5)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx = \frac{\frac{2\sqrt{b\,x^3+a}\,x}{5}-\frac{2\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)a}{5}}{b}$$

input
```
int(x^4/(b*x^5+a*x^2)^(1/2),x)
```

output
```
(2*(sqrt(a + b*x**3)*x - int(sqrt(a + b*x**3)/(a + b*x**3),x)*a))/(5*b)
```

3.269 $\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx$

Optimal result 2298
Mathematica [C] (verified) 2299
Rubi [A] (verified) 2299
Maple [A] (verified) 2300
Fricas [A] (verification not implemented) 2301
Sympy [F] 2301
Maxima [F] 2302
Giac [F] 2302
Mupad [F(-1)] 2302
Reduce [F] 2303

Optimal result

Integrand size = 17, antiderivative size = 212

$$\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\sqrt{2+\sqrt{3}}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output
```
2/3*(1/2*6^(1/2)+1/2*2^(1/2))*x*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-
3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)
*3^(3/4)/b^(1/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)
*x)^2)^(1/2)/(b*x^5+a*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.25

$$\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx=\frac{x^2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt{x^2\left(a+bx^3\right)}}$$

input
```
Integrate[x/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/S
qrt[x^2*(a + b*x^3)]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1938, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx$$
$$\Big\downarrow \text{1938}$$
$$\frac{x\sqrt{a+bx^3}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt{ax^2+bx^5}}$$
$$\Big\downarrow \text{759}$$
$$\frac{2\sqrt{2+\sqrt{3}}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

input
```
Int[x/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*Sqrt[2 + Sqrt[3]]*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.09

method	result
default	$-\frac{ix\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}} \sqrt{-\frac{i\left(i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}-2bx-\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{2\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\,\sqrt{-\frac{i\left(i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}+2bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\mathrm{Elliptic}}{3\sqrt{b\,x^{5}+a\,x^{2}}\,b}$

input
```
int(x/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I/(b*x^5+a*x^2)^(1/2)*x*3^(1/2)/b*(-a*b^2)^(1/3)*(-I*(I*3^(1/2)*(-a*b
^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*(-2*(-b*x+(-
a*b^2)^(1/3))/(-a*b^2)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^
(1/3)+2*b*x+(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*EllipticF(1/6*3^
(1/2)*2^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/
(-a*b^2)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.07

$$\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)}{\sqrt{b}}$$

input
```
integrate(x/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2*weierstrassPInverse(0, -4*a/b, x)/sqrt(b)
```

Sympy [F]

$$\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x}{\sqrt{x^2\,(a+bx^3)}}\,dx$$

input
```
integrate(x/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(b*x^5 + a*x^2), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(x/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
int(x/(a*x^2 + b*x^5)^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{ax^2 + bx^5}}\,dx = \int \frac{\sqrt{b\,x^3 + a}}{b\,x^3 + a}dx$$

input
```
int(x/(b*x^5+a*x^2)^(1/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a + b*x**3),x)
```

3.270 $\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx$

Optimal result 2304
Mathematica [C] (verified) 2305
Rubi [A] (verified) 2305
Maple [A] (verified) 2307
Fricas [A] (verification not implemented) 2308
Sympy [F] 2308
Maxima [F] 2308
Giac [F] 2309
Mupad [B] (verification not implemented) 2309
Reduce [F] 2309

Optimal result

Integrand size = 19, antiderivative size = 243

$$\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx = -\frac{\sqrt{ax^2+bx^5}}{2ax^3} - \frac{\sqrt{2+\sqrt{3}}\,b^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output
```
-1/2*(b*x^5+a*x^2)^(1/2)/a/x^3-1/6*(1/2*6^(1/2)+1/2*2^(1/2))*b^(2/3)*x*(a^
(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(
1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(
1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a/(a^(1/3)*(a^(1/3)+b^(1/3
)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^5+a*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.23

$$\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx = -\frac{\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a}\right)}{2x\sqrt{x^2\left(a+bx^3\right)}}$$

input
```
Integrate[1/(x^2*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
-1/2*(Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-2/3, 1/2, 1/3, -((b*x^3)/a)])
/(x*Sqrt[x^2*(a + b*x^3)])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1931, 1938, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx$$

$$\Big\downarrow \; 1931$$

$$-\frac{b\int \frac{x}{\sqrt{bx^5+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^5}}{2ax^3}$$

$$\Big\downarrow \; 1938$$

$$-\frac{bx\sqrt{a+bx^3}\int \frac{1}{\sqrt{bx^3+a}}dx}{4a\sqrt{ax^2+bx^5}}-\frac{\sqrt{ax^2+bx^5}}{2ax^3}$$

$$\Big\downarrow \; 759$$

$$-\frac{\sqrt{2+\sqrt{3}}b^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{\sqrt{ax^2+bx^5}}{2ax^3}$$

input
```
Int[1/(x^2*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
-1/2*Sqrt[a*x^2 + b*x^5]/(a*x^3) - (Sqrt[2 + Sqrt[3]]*b^(2/3)*x*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*S
qrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*Sqrt[a*x^2 + b*x^5])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.02

method	result
default	$\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{-\frac{i\left(i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}-2bx-(-a\,b^2)^{\frac{1}{3}}\right)\sqrt{3}}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{-\frac{2\left(-bx+(-a\,b^2)^{\frac{1}{3}}\right)}{(-a\,b^2)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\sqrt{-\frac{i\left(i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}+2bx+(-a\,b^2)^{\frac{1}{3}}\right)\sqrt{3}}{(-a\,b^2)^{\frac{1}{3}}}}\,\mathrm{EllipticF}\left(\ldots\right.}{12x\sqrt{b\,x^5+a\,x^2}\,a}$
risch	$-\frac{b\,x^3+a}{2ax\sqrt{x^2(b\,x^3+a)}}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\ldots\right.}{(-\ldots}}\ldots}{6a\sqrt{\ldots}}$

input
```
int(1/x^2/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/x*(I*3^(1/2)*(-a*b^2)^(1/3)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b
^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*(-2*(-b*x+(-a*b^2)^(1/3))/(-a*b^2
)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^
(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(-I*(I*
3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)
,2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*x^2-6*b*x^3-6*a)/(b*x^5+a*x^2)^(
1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.16

$$\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx = -\frac{\sqrt{b}x^3\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\sqrt{bx^5+ax^2}}{2\,ax^3}$$

input
```
integrate(1/x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(b)*x^3*weierstrassPInverse(0, -4*a/b, x) + sqrt(b*x^5 + a*x^2))
/(a*x^3)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^2\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

input
```
integrate(1/x**2/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(x**2*(a + b*x**3))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^5 + a*x^2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx=\int \frac{1}{\sqrt{bx^5+ax^2}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^5 + a*x^2)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 9.78 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx=-\frac{2\,\sqrt{\frac{a}{b\,x^3}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{6};\ \frac{13}{6};\ -\frac{a}{b\,x^3}\right)}{7\,x\,\sqrt{b\,x^5+a\,x^2}}$$

input
```
int(1/(x^2*(a*x^2 + b*x^5)^(1/2)),x)
```

output
```
-(2*(a/(b*x^3) + 1)^(1/2)*hypergeom([1/2, 7/6], 13/6, -a/(b*x^3)))/(7*x*(a
*x^2 + b*x^5)^(1/2))
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx=\int \frac{\sqrt{b\,x^3+a}}{b\,x^6+a\,x^3}dx$$

input
```
int(1/x^2/(b*x^5+a*x^2)^(1/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a*x**3 + b*x**6),x)
```

3.271 $\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx$

Optimal result 2310
Mathematica [C] (verified) 2311
Rubi [A] (warning: unable to verify) 2311
Maple [A] (verified) 2315
Fricas [A] (verification not implemented) 2316
Sympy [F] 2316
Maxima [F] 2316
Giac [F] 2317
Mupad [F(-1)] 2317
Reduce [F] 2317

Optimal result

Integrand size = 19, antiderivative size = 514

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx = -\frac{8ax(a+bx^3)}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}+\frac{2x\sqrt{ax^2+bx^5}}{7b}$$
$$+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$
$$-\frac{8\sqrt{2}a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output
```
-8/7*a*x*(b*x^3+a)/b^(5/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^5+a*x^2)^(
1/2)+2/7*x*(b*x^5+a*x^2)^(1/2)/b+4/7*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(
4/3)*x*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^
(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*
x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(5/3)/(a^(1/3)*(a^(1/3
)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^5+a*x^2)^(1/2)-
8/21*2^(1/2)*a^(4/3)*x*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*
a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/
b^(5/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1
/2)/(b*x^5+a*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.13

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx=\frac{2x^3\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{7b\sqrt{x^2\left(a+bx^3\right)}}$$

input
```
Integrate[x^5/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*x^3*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3,
 -((b*x^3)/a)]))/(7*b*Sqrt[x^2*(a + b*x^3)])
```

Rubi [A] (warning: unable to verify)

Time = 0.93 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1930, 1938, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx$$

$$\Big\downarrow \; 1930$$

$$\frac{2x\sqrt{ax^2+bx^5}}{7b}-\frac{4a\int \frac{x^2}{\sqrt{bx^5+ax^2}}dx}{7b}$$

$$\Big\downarrow \; 1938$$

$$\frac{2x\sqrt{ax^2+bx^5}}{7b}-\frac{4ax\sqrt{a+bx^3}\int \frac{x}{\sqrt{bx^3+a}}dx}{7b\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \; 832$$

$$\frac{2x\sqrt{ax^2+bx^5}}{7b}-\frac{4ax\sqrt{a+bx^3}\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{7b\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \; 759$$

$$\frac{2x\sqrt{ax^2+bx^5}}{7b}-$$

$$\frac{4ax\sqrt{a+bx^3}\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\cdots\right.}{\sqrt[3]{b}x+\left(1+\cdots\right.}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{7b\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \; 2416$$

$$\frac{2x\sqrt{ax^2+bx^5}}{7b}-\frac{4ax\sqrt{a+bx^3}\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.}{7b\sqrt{ax^2+bx}}$$

input
```
Int[x^5/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*x*Sqrt[a*x^2 + b*x^5])/(7*b) - (4*a*x*Sqrt[a + b*x^3]*(((2*Sqrt[a + b*x
^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqr
t[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*
Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 +
 Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(7*b*Sqrt[a*x^2 + b*x^
5])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.05

method	result
pseudoelliptic	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-b\,x^{3}+2a\right)}{9b^{2}}$
risch	$\frac{2x^{3}\left(b\,x^{3}+a\right)}{7b\sqrt{x^{2}\left(b\,x^{3}+a\right)}}+\frac{8ia\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-\cdots\right.}{\cdots}\right.}{\cdots}}\cdots}{\cdots}$
default	$-\frac{2x\left(3i\left(-a\,b^{2}\right)^{\frac{2}{3}}\sqrt{-\frac{i\left(i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}-2bx-\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{2\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\sqrt{-\frac{i\left(i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}+2bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\sqrt{\cdots}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\cdots\right.}{\cdots}$

input
```
int(x^5/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(b*x^3+a)^(1/2)*(-b*x^3+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.09

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\left(\sqrt{bx^5+ax^2}bx+4\,a\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)\right)}{7\,b^2}$$

input
```
integrate(x^5/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/7*(sqrt(b*x^5 + a*x^2)*b*x + 4*a*sqrt(b)*weierstrassZeta(0, -4*a/b, weie
rstrassPInverse(0, -4*a/b, x)))/b^2
```

Sympy [F]

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^5}{\sqrt{x^2\,(a+bx^3)}}\,dx$$

input
```
integrate(x**5/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x**5/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^5}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^5/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^5/sqrt(b*x^5 + a*x^2), x)
```

Giac [F]

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^5}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^5/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^5/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^5}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(x^5/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
int(x^5/(a*x^2 + b*x^5)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx = \frac{\frac{2\sqrt{b\,x^3+a}\,x^2}{7}-\frac{4\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)a}{7}}{b}$$

input
```
int(x^5/(b*x^5+a*x^2)^(1/2),x)
```

output
```
(2*(sqrt(a + b*x**3)*x**2 - 2*int((sqrt(a + b*x**3)*x)/(a + b*x**3),x)*a))
/(7*b)
```

3.272 $\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx$

Optimal result 2318
Mathematica [C] (verified) 2319
Rubi [A] (warning: unable to verify) 2319
Maple [A] (verified) 2322
Fricas [A] (verification not implemented) 2322
Sympy [F] 2323
Maxima [F] 2323
Giac [F] 2323
Mupad [F(-1)] 2324
Reduce [F] 2324

Optimal result

Integrand size = 19, antiderivative size = 484

$$\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx = \frac{2x(a+bx^3)}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$
$$+\frac{2\sqrt{2}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output
```
2*x*(b*x^3+a)/b^(2/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^5+a*x^2)^(1/2)-
3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(1/3)*x*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*El
lipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*
3^(1/2)+2*I)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(
1/3)*x)^2)^(1/2)/(b*x^5+a*x^2)^(1/2)+2/3*2^(1/2)*a^(1/3)*x*(a^(1/3)+b^(1/3
)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)
*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3
)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(
(1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^5+a*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.11

$$\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx = \frac{x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt{x^2\left(a+bx^3\right)}}$$

input
```
Integrate[x^2/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(
2*Sqrt[x^2*(a + b*x^3)])
```

Rubi [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1938, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx$$

$$\Big\downarrow \ 1938$$

$$\frac{x\sqrt{a+bx^3}\int \frac{x}{\sqrt{bx^3+a}}dx}{\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \ 832$$

$$\frac{x\sqrt{a+bx^3}\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \ 759$$

$$\frac{x\sqrt{a+bx^3}\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right.}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right.}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \ 2416$$

$$\frac{x\sqrt{a+bx^3}\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.}{\sqrt{ax^2+bx^5}}$$

input
```
Int[x^2/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(x*Sqrt[a + b*x^3]*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))
/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a
^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])))/Sqrt[a*x^2 + b*x^5]
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.03

method	result
pseudoelliptic	$\frac{2\sqrt{b\,x^{3}+a}}{3b}$
default	$-\frac{i x\sqrt{3}\left(-a\,b^{2}\right)^{\frac{2}{3}}\sqrt{-\frac{i\left(i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}-2bx-\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{2\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\sqrt{-\frac{i\left(i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}+2bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{}$

input
```
int(x^2/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/b*(b*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.05

$$\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx=-\frac{2\,\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)}{\sqrt{b}}$$

input
```
integrate(x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x))/sqrt(b)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^2}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

input
```
integrate(x**2/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x**2/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^2}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(b*x^5 + a*x^2), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^2}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^2}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(x^2/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
int(x^2/(a*x^2 + b*x^5)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx$$

input
```
int(x^2/(b*x^5+a*x^2)^(1/2),x)
```

output
```
int((sqrt(a + b*x**3)*x)/(a + b*x**3),x)
```

3.273 $\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx$

Optimal result . . . 2325
Mathematica [C] (verified) . . . 2326
Rubi [A] (warning: unable to verify) . . . 2326
Maple [A] (verified) . . . 2330
Fricas [A] (verification not implemented) . . . 2331
Sympy [F] . . . 2331
Maxima [F] . . . 2331
Giac [F] . . . 2332
Mupad [F(-1)] . . . 2332
Reduce [F] . . . 2332

Optimal result

Integrand size = 19, antiderivative size = 510

$$\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx = \frac{\sqrt[3]{b}x(a+bx^3)}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}-\frac{\sqrt{ax^2+bx^5}}{ax^2}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$
$$+\frac{\sqrt{2}\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output
```
b^(1/3)*x*(b*x^3+a)/a/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^5+a*x^2)^(1/2)-
(b*x^5+a*x^2)^(1/2)/a/x^2-1/2*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*b^(1/3)*x*
(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*
a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+
3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/a^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/
3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^5+a*x^2)^(1/2)+1/3*2^(
1/2)*b^(1/3)*x*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2
)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+
b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a^(2/3)/
(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x
^5+a*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.10

$$\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx = -\frac{\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a}\right)}{\sqrt{x^2\left(a+bx^3\right)}}$$

input
```
Integrate[1/(x*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
-((Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/3, 1/2, 2/3, -((b*x^3)/a)])/Sq
rt[x^2*(a + b*x^3)])
```

Rubi [A] (warning: unable to verify)

Time = 0.97 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1931, 1938, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx$$

$$\Big\downarrow \; 1931$$

$$\frac{b\int \frac{x^2}{\sqrt{bx^5+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^5}}{ax^2}$$

$$\Big\downarrow \; 1938$$

$$\frac{bx\sqrt{a+bx^3}\int \frac{x}{\sqrt{bx^3+a}}dx}{2a\sqrt{ax^2+bx^5}}-\frac{\sqrt{ax^2+bx^5}}{ax^2}$$

$$\Big\downarrow \; 832$$

$$\frac{bx\sqrt{a+bx^3}\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{2a\sqrt{ax^2+bx^5}}-\frac{\sqrt{ax^2+bx^5}}{ax^2}$$

$$\Big\downarrow \; 759$$

$$\frac{bx\sqrt{a+bx^3}\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{\ldots}\right.}{\sqrt[3]{b}x+\left(1+\sqrt{\ldots}\right.}\right)\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{2a\sqrt{ax^2+bx^5}}$$

$$\frac{\sqrt{ax^2+bx^5}}{ax^2}$$

$$\Big\downarrow \; 2416$$

$$\frac{bx\sqrt{a+bx^3}\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.}{2a\sqrt{ax^2+bx}}$$

$$\frac{\sqrt{ax^2+bx^5}}{ax^2}$$

input
```
Int[1/(x*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
-(Sqrt[a*x^2 + b*x^5]/(a*x^2)) + (b*x*Sqrt[a + b*x^3]*(((2*Sqrt[a + b*x^3]
)/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3
]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt
[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqr
t[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sq
rt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4
*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(2*a*Sqrt[a*x^2 + b*x^5])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.04

method	result
pseudoelliptic	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$
risch	$-\frac{b\,x^{3}+a}{a\sqrt{x^{2}(b\,x^{3}+a)}} - \frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a}{}\right.}{}}$
default	$\frac{3i\sqrt{-\frac{i\left(i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}-2bx-(-a\,b^{2})^{\frac{1}{3}}\right)\sqrt{3}}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{-\frac{2\left(-bx+(-a\,b^{2})^{\frac{1}{3}}\right)}{(-a\,b^{2})^{\frac{1}{3}}(i\sqrt{3}-3)}}\sqrt{-\frac{i\left(i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}+2bx+(-a\,b^{2})^{\frac{1}{3}}\right)\sqrt{3}}{(-a\,b^{2})^{\frac{1}{3}}}}\,\mathrm{EllipticE}\left(\frac{\sqrt{3}}{}\right.}{}$

input
```
int(1/x/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.09

$$\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx$$
$$= -\frac{\sqrt{b}x^2 \text{weierstrassZeta}\left(0, -\frac{4a}{b}, \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) + \sqrt{bx^5+ax^2}}{ax^2}$$

input
```
integrate(1/x/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(b)*x^2*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)
) + sqrt(b*x^5 + a*x^2))/(a*x^2)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

input
```
integrate(1/x/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(x**2*(a + b*x**3))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}x}\,dx$$

input
```
integrate(1/x/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^5 + a*x^2)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}x}\,dx$$

input
```
integrate(1/x/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^5 + a*x^2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(1/(x*(a*x^2 + b*x^5)^(1/2)),x)
```

output
```
int(1/(x*(a*x^2 + b*x^5)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx = \int \frac{\sqrt{b\,x^3+a}}{b\,x^5+a\,x^2}dx$$

input
```
int(1/x/(b*x^5+a*x^2)^(1/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a*x**2 + b*x**5),x)
```

3.274 $\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal result . . . 2333
Mathematica [C] (verified) . . . 2334
Rubi [A] (verified) . . . 2334
Maple [C] (verified) . . . 2336
Fricas [F] . . . 2337
Sympy [F] . . . 2338
Maxima [F] . . . 2338
Giac [F] . . . 2338
Mupad [F(-1)] . . . 2339
Reduce [F] . . . 2339

Optimal result

Integrand size = 21, antiderivative size = 265

$$\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx = -\frac{7a\sqrt{ax^2+bx^5}}{20b^2\sqrt{x}}+\frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b} + \frac{7a^{5/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40\sqrt[4]{3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output

```
-7/20*a*(b*x^5+a*x^2)^(1/2)/b^2/x^(1/2)+1/5*x^(5/2)*(b*x^5+a*x^2)^(1/2)/b+
7/120*a^(5/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((
a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2
)+1/4*2^(1/2))*3^(3/4)/b^2/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1
/2))*b^(1/3)*x)^2)^(1/2)/(b*x^5+a*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.32

$$\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{x^{3/2}\left(-7a^2-3abx^3+4b^2x^6+7a^2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)\right)}{20b^2\sqrt{x^2\left(a+bx^3\right)}}$$

input
```
Integrate[x^(13/2)/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(x^(3/2)*(-7*a^2 - 3*a*b*x^3 + 4*b^2*x^6 + 7*a^2*Sqrt[1 + (b*x^3)/a]*Hyper
geometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(20*b^2*Sqrt[x^2*(a + b*x^3)])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1930, 1930, 1938, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b}-\frac{7a\int \frac{x^{7/2}}{\sqrt{bx^5+ax^2}}dx}{10b}$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b}-\frac{7a\left(\frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}}-\frac{a\int \frac{\sqrt{x}}{\sqrt{bx^5+ax^2}}dx}{4b}\right)}{10b}$$

$$\Big\downarrow \text{ 1938}$$

$$\frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b}-\frac{7a\left(\frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}}-\frac{ax\sqrt{a+bx^3}\int\frac{1}{\sqrt{x}\sqrt{bx^3+a}}dx}{4b\sqrt{ax^2+bx^5}}\right)}{10b}$$

$$\Big\downarrow \text{851}$$

$$\frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b}-\frac{7a\left(\frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}}-\frac{ax\sqrt{a+bx^3}\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b\sqrt{ax^2+bx^5}}\right)}{10b}$$

$$\Big\downarrow \text{766}$$

$$\frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b}-\frac{7a\left(\frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}}-\frac{a^{2/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}\right)}{10b}$$

input
```
Int[x^(13/2)/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(x^(5/2)*Sqrt[a*x^2 + b*x^5])/(5*b) - (7*a*(Sqrt[a*x^2 + b*x^5]/(2*b*Sqrt[
x]) - (a^(2/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCo
s[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)]
, (2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a
^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])))/(10*b)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.57 (sec) , antiderivative size = 742, normalized size of antiderivative = 2.80

method	result
risch	$-\frac{(-4bx^3+7a)x^{\frac{3}{2}}(bx^3+a)}{20b^2\sqrt{x^2(bx^3+a)}}+\frac{7a^2\left(\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{(-ab^2)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{(-ab^2)^{\frac{1}{3}}}{b}\right.}{}$
default	Expression too large to display

input
```
int(x^(13/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20*(-4*b*x^3+7*a)/b^2*x^(3/2)*(b*x^3+a)/(x^2*(b*x^3+a))^(1/2)+7/20*a^2/
b*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^
2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)
^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x
-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))/(-a*b^2)^(1/3)/(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x
-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2))/(x^2*(b*x^3+a))^(1/2)*x^(1/2)*(x*(b*x^3+a))
^(1/2)
```

Fricas [F]

$$\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{13}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(13/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^5 + a*x^2)*x^(9/2)/(b*x^3 + a), x)
```

Sympy [F]

$$\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{13}{2}}}{\sqrt{x^2\,(a+bx^3)}}\,dx$$

input
```
integrate(x**(13/2)/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x**(13/2)/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{13}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(13/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(13/2)/sqrt(b*x^5 + a*x^2), x)
```

Giac [F]

$$\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{13}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(13/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(13/2)/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^{13/2}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(x^(13/2)/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
int(x^(13/2)/(a*x^2 + b*x^5)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx=\frac{-14\sqrt{x}\,\sqrt{b\,x^3+a}\,a+8\sqrt{x}\,\sqrt{b\,x^3+a}\,b\,x^3+7\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^4+ax}dx\right)a^2}{40b^2}$$

input
```
int(x^(13/2)/(b*x^5+a*x^2)^(1/2),x)
```

output
```
( - 14*sqrt(x)*sqrt(a + b*x**3)*a + 8*sqrt(x)*sqrt(a + b*x**3)*b*x**3 + 7*
int((sqrt(x)*sqrt(a + b*x**3))/(a*x + b*x**4),x)*a**2)/(40*b**2)
```

3.275 $\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal result 2340
Mathematica [C] (verified) 2341
Rubi [A] (verified) 2341
Maple [C] (verified) 2345
Fricas [F] 2346
Sympy [F] 2347
Maxima [F] 2347
Giac [F] 2347
Mupad [F(-1)] 2348
Reduce [F] 2348

Optimal result

Integrand size = 21, antiderivative size = 525

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx = -\frac{5\left(1+\sqrt{3}\right)ax^{3/2}\left(a+bx^3\right)}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}+\frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b}$$

$$+\frac{5\sqrt[4]{3}a^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{8b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

$$+\frac{5\left(1-\sqrt{3}\right)a^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right.\right.}{16\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output
```
-5/8*(1+3^(1/2))*a*x^(3/2)*(b*x^3+a)/b^(5/3)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*
x)/(b*x^5+a*x^2)^(1/2)+1/4*x^(3/2)*(b*x^5+a*x^2)^(1/2)/b+5/8*3^(1/4)*a^(4/
3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a
^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b
^(1/3)*x)^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/
2))/b^(5/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)
^2)^(1/2)/(b*x^5+a*x^2)^(1/2)+5/48*(1-3^(1/2))*a^(4/3)*x^(3/2)*(a^(1/3)+b^
(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(
1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a
^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/b^(5/3)/(b
^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x
^5+a*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.13

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx=\frac{x^{7/2}\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)\right)}{4b\sqrt{x^2\left(a+bx^3\right)}}$$

input
```
Integrate[x^(11/2)/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(x^(7/2)*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11
/6, -((b*x^3)/a)]))/(4*b*Sqrt[x^2*(a + b*x^3)])
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {1930, 1938, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx$$

$$\Big\downarrow \; 1930$$

$$\frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b}-\frac{5a\int \frac{x^{5/2}}{\sqrt{bx^5+ax^2}}dx}{8b}$$

$$\Big\downarrow \; 1938$$

$$\frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b}-\frac{5ax\sqrt{a+bx^3}\int \frac{x^{3/2}}{\sqrt{bx^3+a}}dx}{8b\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \; 851$$

$$\frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b}-\frac{5ax\sqrt{a+bx^3}\int \frac{x^2}{\sqrt{bx^3+a}}d\sqrt{x}}{4b\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \; 837$$

$$\frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b}-\frac{5ax\sqrt{a+bx^3}\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{4b\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \; 25$$

$$\frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b}-\frac{5ax\sqrt{a+bx^3}\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{4b\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \; 766$$

$$\frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b}-\frac{5ax\sqrt{a+bx^3}\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{4b\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \; 2420$$

$$\frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b} - \frac{5ax\sqrt{a+bx^3}\left(\frac{\frac{(1+\sqrt{3})\sqrt{x}\sqrt{a+bx^3}}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x} - \frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{(1-\sqrt{3})\sqrt[3]{b}x+\sqrt[3]{a}}{(1+\sqrt{3})\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{2b^{2/3}} - \cdots\right)}{4b\sqrt{ax^2+bx^5}}$$

input
```
Int[x^(11/2)/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(x^(3/2)*Sqrt[a*x^2 + b*x^5])/(4*b) - (5*a*x*Sqrt[a + b*x^3]*((((1 + Sqrt[
3])*Sqrt[x]*Sqrt[a + b*x^3])/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x) - (3^(1/4
)*a^(1/3)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^
(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2
+ Sqrt[3])/4])/(Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqr
t[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)
*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (
1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3]
)/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) +
 (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(4*b*Sqrt[a*x^2 + b*x^5])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.54 (sec) , antiderivative size = 1115, normalized size of antiderivative = 2.12

method	result	size
risch	Expression too large to display	1115
default	Expression too large to display	2586

input
```
int(x^(11/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*x^(7/2)*(b*x^3+a)/(x^2*(b*x^3+a))^(1/2)-5/8*a/b*(x*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1
/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*Ellipti
cF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a...
```

Fricas [F]

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^{\frac{11}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^5 + a*x^2)*x^(7/2)/(b*x^3 + a), x)
```

Sympy [F]

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{11}{2}}}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

input
```
integrate(x**(11/2)/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x**(11/2)/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{11}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(11/2)/sqrt(b*x^5 + a*x^2), x)
```

Giac [F]

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{11}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(11/2)/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^{11/2}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(x^(11/2)/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
int(x^(11/2)/(a*x^2 + b*x^5)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx=\frac{2\sqrt{x}\,\sqrt{b\,x^3+a}\,x^2-5\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)a}{8b}$$

input
```
int(x^(11/2)/(b*x^5+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(a + b*x**3)*x**2 - 5*int((sqrt(x)*sqrt(a + b*x**3)*x)/(a +
b*x**3),x)*a)/(8*b)
```

3.276 $\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}} \, dx$

Optimal result 2349
Mathematica [A] (verified) 2349
Rubi [A] (verified) 2350
Maple [A] (verified) 2351
Fricas [A] (verification not implemented) 2352
Sympy [F] 2352
Maxima [F] 2352
Giac [A] (verification not implemented) 2353
Mupad [F(-1)] 2353
Reduce [B] (verification not implemented) 2353

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}} \, dx = \frac{\sqrt{x}\sqrt{ax^2+bx^5}}{3b} - \frac{a \operatorname{arctanh}\left(\frac{\sqrt{b}x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3b^{3/2}}$$

output
```
1/3*x^(1/2)*(b*x^5+a*x^2)^(1/2)/b-1/3*a*arctanh(b^(1/2)*x^(5/2)/(b*x^5+a*x
^2)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26

$$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}} \, dx = \frac{\sqrt{b}x^{5/2}(a+bx^3) - ax\sqrt{a+bx^3} \log \left(\sqrt{b}x^{3/2} + \sqrt{a+bx^3}\right)}{3b^{3/2}\sqrt{x^2(a+bx^3)}}$$

input
```
Integrate[x^(9/2)/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(Sqrt[b]*x^(5/2)*(a + b*x^3) - a*x*Sqrt[a + b*x^3]*Log[Sqrt[b]*x^(3/2) + S
qrt[a + b*x^3]])/(3*b^(3/2)*Sqrt[x^2*(a + b*x^3)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1930, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{\sqrt{x}\sqrt{ax^2+bx^5}}{3b}-\frac{a\int \frac{x^{3/2}}{\sqrt{bx^5+ax^2}}dx}{2b}$$

$$\Big\downarrow \text{1935}$$

$$\frac{\sqrt{x}\sqrt{ax^2+bx^5}}{3b}-\frac{a\int \frac{1}{1-\frac{bx^5}{bx^5+ax^2}}d\frac{x^{5/2}}{\sqrt{bx^5+ax^2}}}{3b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{x}\sqrt{ax^2+bx^5}}{3b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3b^{3/2}}$$

input
```
Int[x^(9/2)/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(Sqrt[x]*Sqrt[a*x^2 + b*x^5])/(3*b) - (a*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a*
x^2 + b*x^5]])/(3*b^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{x^{\frac{3}{2}} \left(b \,x^{3}+a \right) \left(x \sqrt{x \left(b \,x^{3}+a \right)}\, \sqrt{b}-\operatorname{arctanh}\left(\frac{\sqrt{x \left(b \,x^{3}+a \right)}}{x^{2} \sqrt{b}}\right) a \right)}{3 \sqrt{b \,x^{5}+a \,x^{2}}\, \sqrt{x \left(b \,x^{3}+a \right)}\, b^{\frac{3}{2}}}$	79
risch	$\frac{x^{\frac{5}{2}} \left(b \,x^{3}+a \right)}{3 b \sqrt{x^{2} \left(b \,x^{3}+a \right)}}-\frac{a \,\operatorname{arctanh}\left(\frac{\sqrt{x \left(b \,x^{3}+a \right)}}{x^{2} \sqrt{b}}\right) \sqrt{x}\, \sqrt{x \left(b \,x^{3}+a \right)}}{3 b^{\frac{3}{2}} \sqrt{x^{2} \left(b \,x^{3}+a \right)}}$	82

input
```
int(x^(9/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^(3/2)*(b*x^3+a)*(x*(x*(b*x^3+a))^(1/2)*b^(1/2)-arctanh((x*(b*x^3+a))
^(1/2)/x^2/b^(1/2))*a)/(b*x^5+a*x^2)^(1/2)/(x*(b*x^3+a))^(1/2)/b^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.28

$$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx = \left[\frac{a\sqrt{b}\log\left(-8\,b^2x^6-8\,abx^3+4\,\sqrt{bx^5+ax^2}(2\,bx^3+a)\sqrt{b}\sqrt{x}-a^2\right)+4\,\sqrt{bx^5+ax^2}b}{12\,b^2}\right.$$

input
```
integrate(x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(a*sqrt(b)*log(-8*b^2*x^6 - 8*a*b*x^3 + 4*sqrt(b*x^5 + a*x^2)*(2*b*x
^3 + a)*sqrt(b)*sqrt(x) - a^2) + 4*sqrt(b*x^5 + a*x^2)*b*sqrt(x))/b^2, 1/6
*(a*sqrt(-b)*arctan(2*sqrt(b*x^5 + a*x^2)*sqrt(-b)*sqrt(x)/(2*b*x^3 + a))
+ 2*sqrt(b*x^5 + a*x^2)*b*sqrt(x))/b^2]
```

Sympy [F]

$$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{9}{2}}}{\sqrt{x^2\,(a+bx^3)}}\,dx$$

input
```
integrate(x**(9/2)/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x**(9/2)/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{9}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(9/2)/sqrt(b*x^5 + a*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

$$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{\sqrt{bx^3+a}x^{\frac{3}{2}}}{3\,b\text{sgn}\,(x)} + \frac{a\log\left(\left|-\sqrt{b}x^{\frac{3}{2}}+\sqrt{bx^3+a}\right|\right)}{3\,b^{\frac{3}{2}}\text{sgn}\,(x)}$$

input
```
integrate(x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(b*x^3 + a)*x^(3/2)/(b*sgn(x)) + 1/3*a*log(abs(-sqrt(b)*x^(3/2) +
sqrt(b*x^3 + a)))/(b^(3/2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{9/2}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(x^(9/2)/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
int(x^(9/2)/(a*x^2 + b*x^5)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\sqrt{x}\,\sqrt{b\,x^3+a}\,bx+\sqrt{b}\log\left(\sqrt{b\,x^3+a}-\sqrt{x}\,\sqrt{b}\,x\right)a-\sqrt{b}\log\left(\sqrt{b\,x^3+a}+\sqrt{x}\,\sqrt{b}\,x\right)}{6b^2}$$

input
```
int(x^(9/2)/(b*x^5+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(a + b*x**3)*b*x + sqrt(b)*log(sqrt(a + b*x**3) - sqrt(x)*s
qrt(b)*x)*a - sqrt(b)*log(sqrt(a + b*x**3) + sqrt(x)*sqrt(b)*x)*a)/(6*b**2
)
```

3.277 $\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal result 2354
Mathematica [C] (verified) 2355
Rubi [A] (verified) 2355
Maple [C] (verified) 2357
Fricas [F] 2358
Sympy [F] 2358
Maxima [F] 2359
Giac [F] 2359
Mupad [F(-1)] 2359
Reduce [F] 2360

Optimal result

Integrand size = 21, antiderivative size = 237

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}} - \frac{a^{2/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output

```
1/2*(b*x^5+a*x^2)^(1/2)/b/x^(1/2)-1/12*a^(2/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)
*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^
2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(
1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/b/(b^(1/3)*x*(a^(1
/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^5+a*x^2)^(1/2
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.30

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{x^{3/2}\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)\right)}{2b\sqrt{x^2\left(a+bx^3\right)}}$$

input
```
Integrate[x^(7/2)/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(x^(3/2)*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/
6, -((b*x^3)/a)]))/(2*b*Sqrt[x^2*(a + b*x^3)])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1930, 1938, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}}-\frac{a\int \frac{\sqrt{x}}{\sqrt{bx^5+ax^2}}dx}{4b}$$

$$\Big\downarrow \text{1938}$$

$$\frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}}-\frac{ax\sqrt{a+bx^3}\int \frac{1}{\sqrt{x}\sqrt{bx^3+a}}dx}{4b\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \text{851}$$

$$\frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}}-\frac{ax\sqrt{a+bx^3}\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \text{766}$$

$$\frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}}-\frac{a^{2/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

input
```
Int[x^(7/2)/Sqrt[a*x^2 + b*x^5],x]
```

output
```
Sqrt[a*x^2 + b*x^5]/(2*b*Sqrt[x]) - (a^(2/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])
*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/
3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/
3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*
x^2 + b*x^5])
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 727, normalized size of antiderivative = 3.07

method	result
risch	$\frac{x^{\frac{3}{2}}\left(b\,x^{3}+a\right)}{2b\sqrt{x^{2}\left(b\,x^{3}+a\right)}}-\frac{a\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2}\sqrt{\frac{\left(-a \cdots\right.}{b\left(-\frac{\left(-\cdots\right.}{}\right.}}}{2\left(-\frac{3\left(-a\,b \cdots\right.}{2b}\right.}$
default	Expression too large to display

input
```
int(x^(7/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*x^(3/2)*(b*x^3+a)/(x^2*(b*x^3+a))^(1/2)-1/2*a*(1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-
1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x
+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*
b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1
/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/
(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*E
llipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1
/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2))/(x^2*(b*x^3+a))^(1/2)*x^(1/2)*(x*(b*x^3+a))^(1/2)
```

Fricas [F]

$$\int \frac{x^{7/2}}{\sqrt{ax^2 + bx^5}}\,dx = \int \frac{x^{\frac{7}{2}}}{\sqrt{bx^5 + ax^2}}\,dx$$

input
```
integrate(x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^5 + a*x^2)*x^(3/2)/(b*x^3 + a), x)
```

Sympy [F]

$$\int \frac{x^{7/2}}{\sqrt{ax^2 + bx^5}}\,dx = \int \frac{x^{\frac{7}{2}}}{\sqrt{x^2\,(a + bx^3)}}\,dx$$

input
```
integrate(x**(7/2)/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x**(7/2)/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^{\frac{7}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(7/2)/sqrt(b*x^5 + a*x^2), x)
```

Giac [F]

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^{\frac{7}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(7/2)/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^{7/2}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(x^(7/2)/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
int(x^(7/2)/(a*x^2 + b*x^5)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\sqrt{x}\,\sqrt{b\,x^3+a}-\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^4+ax}dx\right)a}{4b}$$

input
```
int(x^(7/2)/(b*x^5+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(a + b*x**3) - int((sqrt(x)*sqrt(a + b*x**3))/(a*x + b*x**4
),x)*a)/(4*b)
```

3.278 $\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal result 2361
Mathematica [C] (verified) 2362
Rubi [A] (verified) 2362
Maple [C] (verified) 2366
Fricas [F] 2367
Sympy [F] 2367
Maxima [F] 2367
Giac [F] 2368
Mupad [F(-1)] 2368
Reduce [F] 2368

Optimal result

Integrand size = 21, antiderivative size = 492

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{\left(1+\sqrt{3}\right)x^{3/2}(a+bx^3)}{b^{2/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}$$
$$-\frac{\sqrt[4]{3}\sqrt[3]{a}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$
$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output
```
(1+3^(1/2))*x^(3/2)*(b*x^3+a)/b^(2/3)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)/(b*x
^5+a*x^2)^(1/2)-3^(1/4)*a^(1/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*Ellipt
icE((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)*x)^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2
)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/b^(2/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a
^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^5+a*x^2)^(1/2)-1/6*(1-3^(1/2))
*a^(1/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/
3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4
*2^(1/2))*3^(3/4)/b^(2/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/
2))*b^(1/3)*x)^2)^(1/2)/(b*x^5+a*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.12

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx=\frac{2x^{7/2}\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)}{5\sqrt{x^2\left(a+bx^3\right)}}$$

input
```
Integrate[x^(5/2)/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*x^(7/2)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/6, -((b*x^3)
/a)])/(5*Sqrt[x^2*(a + b*x^3)])
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1938, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx$$

$\downarrow$ 1938

$$\frac{x\sqrt{a+bx^3}\int \frac{x^{3/2}}{\sqrt{bx^3+a}}dx}{\sqrt{ax^2+bx^5}}$$

$\downarrow$ 851

$$\frac{2x\sqrt{a+bx^3}\int \frac{x^2}{\sqrt{bx^3+a}}d\sqrt{x}}{\sqrt{ax^2+bx^5}}$$

$\downarrow$ 837

$$\frac{2x\sqrt{a+bx^3}\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{\sqrt{ax^2+bx^5}}$$

$\downarrow$ 25

$$\frac{2x\sqrt{a+bx^3}\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{\sqrt{ax^2+bx^5}}$$

$\downarrow$ 766

$$\frac{2x\sqrt{a+bx^3}\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{\sqrt{ax^2+bx^5}}$$

$\downarrow$ 2420

$$\frac{2x\sqrt{a+bx^3}\left(\frac{\frac{(1+\sqrt{3})\sqrt{x}\sqrt{a+bx^3}}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}-\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{(1-\sqrt{3})\sqrt[3]{b}x+\sqrt[3]{a}}{(1+\sqrt{3})\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{2b^{2/3}}-\frac{(1\cdots}{\cdots}\right)}{\sqrt{ax^2+bx^5}}$$

input

```
Int[x^(5/2)/Sqrt[a*x^2 + b*x^5],x]
```

output

```
(2*x*Sqrt[a + b*x^3]*((((1 + Sqrt[3])*Sqrt[x]*Sqrt[a + b*x^3])/(a^(1/3) +
(1 + Sqrt[3])*b^(1/3)*x) - (3^(1/4)*a^(1/3)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*
b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3
) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*x*(a^(1/3)
 + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(2
*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 +
Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x
*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*
x^3])))/Sqrt[a*x^2 + b*x^5]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 2374, normalized size of antiderivative = 4.83

method	result	size
default	Expression too large to display	2374

input
```
int(x^(5/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/b^2*(-I*(-a*b^2)^(1/3)*3^(1/2)*(-
(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(
-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(
-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2
)-3))^(1/2))*b*x^2+2*I*(-a*b^2)^(2/3)*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/
2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^
2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^
(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ell
ipticE((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*
3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x-2*(-a*b^2)^
(1/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3
^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(
1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)
/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(
I*3^(1/2)-3))^(1/2))*b*x^2+3*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)
-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)
^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)...
```

Fricas [F]

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{5}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^5 + a*x^2)*sqrt(x)/(b*x^3 + a), x)
```

Sympy [F]

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{5}{2}}}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

input
```
integrate(x**(5/2)/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x**(5/2)/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{5}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(5/2)/sqrt(b*x^5 + a*x^2), x)
```

Giac [F]

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{5}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(5/2)/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{5/2}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(x^(5/2)/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
int(x^(5/2)/(a*x^2 + b*x^5)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}\,x}{b\,x^3+a}\,dx$$

input
```
int(x^(5/2)/(b*x^5+a*x^2)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(a + b*x**3)*x)/(a + b*x**3),x)
```

3.279 $\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal result 2369
Mathematica [A] (verified) 2369
Rubi [A] (verified) 2370
Maple [B] (verified) 2371
Fricas [A] (verification not implemented) 2371
Sympy [F] 2372
Maxima [F] 2372
Giac [A] (verification not implemented) 2372
Mupad [F(-1)] 2373
Reduce [B] (verification not implemented) 2373

Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{b}}$$

output
```
2/3*arctanh(b^(1/2)*x^(5/2)/(b*x^5+a*x^2)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.67

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{2x\sqrt{a+bx^3}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{3\sqrt{b}\sqrt{x^2(a+bx^3)}}$$

input
```
Integrate[x^(3/2)/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*x*Sqrt[a + b*x^3]*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]])/(3*Sqrt[b]*Sq
rt[x^2*(a + b*x^3)])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}dx$$

$$\Big\downarrow \text{ 1935}$$

$$\frac{2}{3}\int \frac{1}{1-\frac{bx^5}{bx^5+ax^2}}d\frac{x^{5/2}}{\sqrt{bx^5+ax^2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{b}}$$

input
```
Int[x^(3/2)/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a*x^2 + b*x^5]])/(3*Sqrt[b])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(26) = 52.

Time = 0.44 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.64

method	result	size
default	$\frac{2x^{\frac{3}{2}}\left(b\,x^{3}+a\right)\operatorname{arctanh}\left(\frac{\sqrt{x\left(b\,x^{3}+a\right)}}{x^{2}\sqrt{b}}\right)}{3\sqrt{b\,x^{5}+a\,x^{2}}\,\sqrt{x(b\,x^{3}+a)}\,\sqrt{b}}$	59

input
```
int(x^(3/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/(x*(b*x^3+a))^(1/2)/b^(1/2)*arct
anh((x*(b*x^3+a))^(1/2)/x^2/b^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.81

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx = \left[\frac{\log\left(-8\,b^2x^6-8\,abx^3-4\,\sqrt{bx^5+ax^2}(2\,bx^3+a)\sqrt{b}\sqrt{x}-a^2\right)}{6\,\sqrt{b}},\right.$$
$$\left.-\frac{\sqrt{-b}\arctan\left(\frac{2\,\sqrt{bx^5+ax^2}\sqrt{-b}\sqrt{x}}{2\,bx^3+a}\right)}{3\,b}\right]$$

input
```
integrate(x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*log(-8*b^2*x^6 - 8*a*b*x^3 - 4*sqrt(b*x^5 + a*x^2)*(2*b*x^3 + a)*sqrt
(b)*sqrt(x) - a^2)/sqrt(b), -1/3*sqrt(-b)*arctan(2*sqrt(b*x^5 + a*x^2)*sqr
t(-b)*sqrt(x)/(2*b*x^3 + a))/b]
```

Sympy [F]

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{3}{2}}}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

input
```
integrate(x**(3/2)/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x**(3/2)/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{3}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(3/2)/sqrt(b*x^5 + a*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{\log\left(|a|\right)\operatorname{sgn}(x)}{3\sqrt{b}} - \frac{2\log\left(\left|-\sqrt{b}x^{\frac{3}{2}}+\sqrt{bx^3+a}\right|\right)}{3\sqrt{b}\operatorname{sgn}\left(x\right)}$$

input
```
integrate(x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*log(abs(a))*sgn(x)/sqrt(b) - 2/3*log(abs(-sqrt(b)*x^(3/2) + sqrt(b*x^3
 + a)))/(sqrt(b)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^{3/2}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(x^(3/2)/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
int(x^(3/2)/(a*x^2 + b*x^5)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx=\frac{\sqrt{b}\left(-\log\left(\sqrt{b\,x^3+a}-\sqrt{x}\,\sqrt{b}\,x\right)+\log\left(\sqrt{b\,x^3+a}+\sqrt{x}\,\sqrt{b}\,x\right)\right)}{3b}$$

input
```
int(x^(3/2)/(b*x^5+a*x^2)^(1/2),x)
```

output
```
(sqrt(b)*( - log(sqrt(a + b*x**3) - sqrt(x)*sqrt(b)*x) + log(sqrt(a + b*x*
*3) + sqrt(x)*sqrt(b)*x)))/(3*b)
```

3.280 $\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal result 2374
Mathematica [C] (verified) 2375
Rubi [A] (verified) 2375
Maple [C] (verified) 2377
Fricas [A] (verification not implemented) 2377
Sympy [F] 2378
Maxima [F] 2378
Giac [F] 2378
Mupad [F(-1)] 2379
Reduce [F] 2379

Optimal result

Integrand size = 21, antiderivative size = 203

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx = \frac{x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output
```
1/3*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(
a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1
-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1
/2))*3^(3/4)/a^(1/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b
^(1/3)*x)^2)^(1/2)/(b*x^5+a*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.27

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx=\frac{2x^{3/2}\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)}{\sqrt{x^2\left(a+bx^3\right)}}$$

input
```
Integrate[Sqrt[x]/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*x^(3/2)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/
a)])/Sqrt[x^2*(a + b*x^3)]
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1938, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}dx$$

$$\Big\downarrow \text{ 1938}$$

$$\frac{x\sqrt{a+bx^3}\int \frac{1}{\sqrt{x}\sqrt{bx^3+a}}dx}{\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{2x\sqrt{a+bx^3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \text{ 766}$$

$$\frac{x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}(2+\sqrt{3})\right)}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

input
```
Int[Sqrt[x]/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (
1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3]
)/4])/(3^(1/4)*a^(1/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.15

method	result
default	$-\frac{4x^{\frac{3}{2}}(b\,x^3+a)\sqrt{-\frac{(i\sqrt{3}-3)xb}{(i\sqrt{3}-1)\left(-bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)}}\sqrt{\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}+2bx+\left(-a\,b^2\right)^{\frac{1}{3}}}{(1+i\sqrt{3})\left(-bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)}}\sqrt{\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}-2bx-\left(-a\,b^2\right)^{\frac{1}{3}}}{(i\sqrt{3}-1)\left(-bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)}}\operatorname{EllipticF}\left(\sqrt{-\ldots}\right.}{\sqrt{b\,x^5+a\,x^2}\,b(-a\,b^2)^{\frac{1}{3}}\sqrt{x(b\,x^3+a)}\,(i\sqrt{3}-3)\sqrt{\frac{x\left(-bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)\ldots}{\ldots}}}$

input
```
int(x^(1/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/b/(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*
x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+
2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))
)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(
I*3^(1/2)*b^2*x^2-2*I*(-a*b^2)^(1/3)*3^(1/2)*b*x+I*3^(1/2)*(-a*b^2)^(2/3)-
b^2*x^2+2*(-a*b^2)^(1/3)*b*x-(-a*b^2)^(2/3))/(x*(b*x^3+a))^(1/2)/(I*3^(1/2
)-3)/(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^
2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.08

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx = -\frac{2\,\text{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)}{\sqrt{a}}$$

input
```
integrate(x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2*weierstrassPInverse(0, -4*b/a, 1/x)/sqrt(a)
```

Sympy [F]

$$\int \frac{\sqrt{x}}{\sqrt{ax^2 + bx^5}}\,dx = \int \frac{\sqrt{x}}{\sqrt{x^2\,(a + bx^3)}}\,dx$$

input
```
integrate(x**(1/2)/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(sqrt(x)/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{ax^2 + bx^5}}\,dx = \int \frac{\sqrt{x}}{\sqrt{bx^5 + ax^2}}\,dx$$

input
```
integrate(x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/sqrt(b*x^5 + a*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{x}}{\sqrt{ax^2 + bx^5}}\,dx = \int \frac{\sqrt{x}}{\sqrt{bx^5 + ax^2}}\,dx$$

input
```
integrate(x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x)/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{\sqrt{x}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(x^(1/2)/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
int(x^(1/2)/(a*x^2 + b*x^5)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^4+ax}dx$$

input
```
int(x^(1/2)/(b*x^5+a*x^2)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(a + b*x**3))/(a*x + b*x**4),x)
```

3.281 $\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx$

Optimal result 2380
Mathematica [C] (verified) 2381
Rubi [A] (verified) 2381
Maple [C] (verified) 2385
Fricas [A] (verification not implemented) 2386
Sympy [F] 2387
Maxima [F] 2387
Giac [F] 2387
Mupad [F(-1)] 2388
Reduce [F] 2388

Optimal result

Integrand size = 21, antiderivative size = 519

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx = \frac{2\left(1+\sqrt{3}\right)\sqrt[3]{b}x^{3/2}(a+bx^3)}{a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}$$

$$-\frac{2\sqrt[4]{3}\sqrt[3]{b}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2+\sqrt{}\right.}{\sqrt[4]{3}a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output
```
2*(1+3^(1/2))*b^(1/3)*x^(3/2)*(b*x^3+a)/a/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)/
(b*x^5+a*x^2)^(1/2)-2*(b*x^5+a*x^2)^(1/2)/a/x^(3/2)-2*3^(1/4)*b^(1/3)*x^(3
/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+
(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)*
x)^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/a^(
2/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/
2)/(b*x^5+a*x^2)^(1/2)-1/3*(1-3^(1/2))*b^(1/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)
*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^
2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(
1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(2/3)/(b^(1/3)*x
*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^5+a*x^2
)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.11

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{x}\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},-\frac{bx^3}{a}\right)}{\sqrt{x^2\left(a+bx^3\right)}}$$

input
```
Integrate[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
(-2*Sqrt[x]*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/6, 1/2, 5/6, -((b*x^3
)/a)])/Sqrt[x^2*(a + b*x^3)]
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1931, 1938, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}dx$$

$$\Big\downarrow \text{ 1931}$$

$$\frac{2b\int \frac{x^{5/2}}{\sqrt{bx^5+ax^2}}dx}{a}-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}$$

$$\Big\downarrow \text{ 1938}$$

$$\frac{2bx\sqrt{a+bx^3}\int \frac{x^{3/2}}{\sqrt{bx^3+a}}dx}{a\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{4bx\sqrt{a+bx^3}\int \frac{x^2}{\sqrt{bx^3+a}}d\sqrt{x}}{a\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}$$

$$\Big\downarrow \text{ 837}$$

$$\frac{4bx\sqrt{a+bx^3}\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{a\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{4bx\sqrt{a+bx^3}\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{a\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}$$

$$\Big\downarrow \text{ 766}$$

$$\frac{4bx\sqrt{a+bx^3}\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{a\sqrt{ax^2+bx^5}}$$

$$\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}$$

$$\Big\downarrow \text{ 2420}$$

$$\frac{4bx\sqrt{a+bx^3}\left(\frac{\frac{(1+\sqrt{3})\sqrt{x}\sqrt{a+bx^3}}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}-\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt{x}(\sqrt[3]{a}+\sqrt[3]{b}x)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x)^2}}E\left(\arccos\left(\frac{(1-\sqrt{3})\sqrt[3]{b}x+\sqrt[3]{a}}{(1+\sqrt{3})\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{\sqrt{\frac{\sqrt[3]{b}x(\sqrt[3]{a}+\sqrt[3]{b}x)}{(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x)^2}}\sqrt{a+bx^3}}}{2b^{2/3}} - \cdots\right)}{a\sqrt{ax^2+bx^5}}$$

$$\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}$$

input
```
Int[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
(-2*Sqrt[a*x^2 + b*x^5])/(a*x^(3/2)) + (4*b*x*Sqrt[a + b*x^3]*((((1 + Sqrt
[3])*Sqrt[x]*Sqrt[a + b*x^3])/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x) - (3^(1/
4)*a^(1/3)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a
^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2
 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sq
rt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3
)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) +
(1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3
])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3)
+ (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]))))/(a*Sqrt[a*x^2 + b*x^5])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.47 (sec) , antiderivative size = 1115, normalized size of antiderivative = 2.15

method	result	size
risch	Expression too large to display	1115
default	Expression too large to display	2860

input
```
int(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(b*x^3+a)/a/(x^2*(b*x^3+a))^(1/2)*x^(1/2)+2*b/a*(x*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*
(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(
((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^...
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.05

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx = \frac{2\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,b}{a},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)\right)}{\sqrt{a}}$$

input
```
integrate(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2*weierstrassZeta(0, -4*b/a, weierstrassPInverse(0, -4*b/a, 1/x))/sqrt(a)
```

Sympy [F]

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{x}\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

input
```
integrate(1/x**(1/2)/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(x)*sqrt(x**2*(a + b*x**3))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^5 + a*x^2)*sqrt(x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^5 + a*x^2)*sqrt(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx=\int \frac{1}{\sqrt{x}\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(1/(x^(1/2)*(a*x^2 + b*x^5)^(1/2)),x)
```

output
```
int(1/(x^(1/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx=\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^5+a\,x^2}dx$$

input
```
int(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(a + b*x**3))/(a*x**2 + b*x**5),x)
```

3.282 $\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx$

Optimal result 2389
Mathematica [A] (verified) 2389
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Optimal result

Integrand size = 21, antiderivative size = 27

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{ax^2+bx^5}}{3ax^{5/2}}$$

output
```
-2/3*(b*x^5+a*x^2)^(1/2)/a/x^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{x^2(a+bx^3)}}{3ax^{5/2}}$$

input
```
Integrate[1/(x^(3/2)*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
(-2*Sqrt[x^2*(a + b*x^3)])/(3*a*x^(5/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx$$
$$\Big\downarrow \text{1920}$$
$$-\frac{2\sqrt{ax^2+bx^5}}{3ax^{5/2}}$$

input
```
Int[1/(x^(3/2)*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
(-2*Sqrt[a*x^2 + b*x^5])/(3*a*x^(5/2))
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

method	result	size
gosper	$-\frac{2(b\,x^{3}+a)}{3\sqrt{x}\,a\sqrt{b\,x^{5}+a\,x^{2}}}$	29
default	$-\frac{2(b\,x^{3}+a)}{3\sqrt{x}\,a\sqrt{b\,x^{5}+a\,x^{2}}}$	29
risch	$-\frac{2(b\,x^{3}+a)}{3a\sqrt{x^{2}(b\,x^{3}+a)}\,\sqrt{x}}$	29
orering	$-\frac{2(b\,x^{3}+a)}{3\sqrt{x}\,a\sqrt{b\,x^{5}+a\,x^{2}}}$	29

input
```
int(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/x^(1/2)*(b*x^3+a)/a/(b*x^5+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{bx^5+ax^2}}{3\,ax^{\frac{5}{2}}}$$

input
```
integrate(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(b*x^5 + a*x^2)/(a*x^(5/2))
```

Sympy [F]

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^{\frac{3}{2}}\sqrt{x^2\,(a+bx^3)}}\,dx$$

input
```
integrate(1/x**(3/2)/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**(3/2)*sqrt(x**2*(a + b*x**3))), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\,(bx^4+ax)}{3\,\sqrt{bx^3+a}ax^{\frac{5}{2}}}$$

input
```
integrate(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-2/3*(b*x^4 + a*x)/(sqrt(b*x^3 + a)*a*x^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\left(\frac{\sqrt{b+\frac{a}{x^3}}}{a}-\frac{\sqrt{b}}{a}\right)}{3\,\mathrm{sgn}\,(x)}$$

input
```
integrate(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-2/3*(sqrt(b + a/x^3)/a - sqrt(b)/a)/sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^{3/2}\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(1/(x^(3/2)*(a*x^2 + b*x^5)^(1/2)),x)
```

output
```
int(1/(x^(3/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{b\,x^3+a}}{3\sqrt{x}\,ax}$$

input
```
int(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x)
```

output
```
( - 2*sqrt(a + b*x**3))/(3*sqrt(x)*a*x)
```

3.283 $\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx$

Optimal result . . . 2394
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Sympy [F] . . . 2399
Maxima [F] . . . 2399
Giac [F] . . . 2399
Mupad [F(-1)] . . . 2400
Reduce [F] . . . 2400

Optimal result

Integrand size = 21, antiderivative size = 235

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}} - \frac{2bx^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output
```
-2/5*(b*x^5+a*x^2)^(1/2)/a/x^(7/2)-2/15*b*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^
(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1
/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(
1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(4/3)/(b^(1/3)*x*(a^(
1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^5+a*x^2)^(1/
2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.24

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},-\frac{bx^3}{a}\right)}{5x^{3/2}\sqrt{x^2\left(a+bx^3\right)}}$$

input
```
Integrate[1/(x^(5/2)*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
(-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-5/6, 1/2, 1/6, -((b*x^3)/a)])/(
5*x^(3/2)*Sqrt[x^2*(a + b*x^3)])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1931, 1938, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}dx$$

$$\Big\downarrow \text{ 1931}$$

$$-\frac{2b\int \frac{\sqrt{x}}{\sqrt{bx^5+ax^2}}dx}{5a}-\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}$$

$$\Big\downarrow \text{ 1938}$$

$$-\frac{2bx\sqrt{a+bx^3}\int \frac{1}{\sqrt{x}\sqrt{bx^3+a}}dx}{5a\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}$$

$$\Big\downarrow \text{ 851}$$

$$-\frac{4bx\sqrt{a+bx^3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{5a\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}$$

$$\Big\downarrow 766$$

$$-\frac{2bx^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}(2+\sqrt{3})\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}$$

input
```
Int[1/(x^(5/2)*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
(-2*Sqrt[a*x^2 + b*x^5])/(5*a*x^(7/2)) - (2*b*x^(3/2)*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1
/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(5*3^(1/4)*a^(4/3)*Sqrt
[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*
Sqrt[a*x^2 + b*x^5])
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.96 (sec) , antiderivative size = 732, normalized size of antiderivative = 3.11

method	result
risch	$-\frac{2(b\,x^3+a)}{5a\,x^{\frac{3}{2}}\sqrt{x^2(b\,x^3+a)}}-\frac{4b^2\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)^2 \sqrt{\cdots}}{5a\left(\cdots\right.}$
default	Expression too large to display

input
```
int(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/a*(b*x^3+a)/x^(3/2)/(x^2*(b*x^3+a))^(1/2)-4/5*b^2/a*(1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/
3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b
*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)
))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(
1/3)/(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)
))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2))/(x^2*(b*x^3+a))^(1/2)*x^(1/2)*(x*(b*x^3+a))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.20

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx = \frac{2\left(2\sqrt{a}bx^4\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)-\sqrt{bx^5+ax^2}a\sqrt{x}\right)}{5a^2x^4}$$

input
```
integrate(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(2*sqrt(a)*b*x^4*weierstrassPInverse(0, -4*b/a, 1/x) - sqrt(b*x^5 + a*
x^2)*a*sqrt(x))/(a^2*x^4)
```

Sympy [F]

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^{\frac{5}{2}}\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

input
```
integrate(1/x**(5/2)/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**(5/2)*sqrt(x**2*(a + b*x**3))), x)
```

Maxima [F]

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}\,x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^5 + a*x^2)*x^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}\,x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^5 + a*x^2)*x^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx=\int \frac{1}{x^{5/2}\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(1/(x^(5/2)*(a*x^2 + b*x^5)^(1/2)),x)
```

output
```
int(1/(x^(5/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx=\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^7+a\,x^4}dx$$

input
```
int(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(a + b*x**3))/(a*x**4 + b*x**7),x)
```

3.284 $\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx$

Optimal result . . . 2401
Mathematica [C] (verified) . . . 2402
Rubi [A] (verified) . . . 2402
Maple [C] (verified) . . . 2407
Fricas [A] (verification not implemented) . . . 2408
Sympy [F] . . . 2409
Maxima [F] . . . 2409
Giac [F] . . . 2409
Mupad [F(-1)] . . . 2410
Reduce [F] . . . 2410

Optimal result

Integrand size = 21, antiderivative size = 555

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{8(1+\sqrt{3})\,b^{4/3}x^{3/2}(a+bx^3)}{7a^2\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}} - \frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} + \frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}}$$
$$+ \frac{8\sqrt[4]{3}b^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}+(1-\sqrt{3})\sqrt[3]{b}x}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{7a^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$
$$+ \frac{4(1-\sqrt{3})\,b^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+(1-\sqrt{3})\sqrt[3]{b}x}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}\right),\frac{1}{4}(2+\sqrt{3}\right.}{7\sqrt[4]{3}a^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output
```
-8/7*(1+3^(1/2))*b^(4/3)*x^(3/2)*(b*x^3+a)/a^2/(a^(1/3)+(1+3^(1/2))*b^(1/3
)*x)/(b*x^5+a*x^2)^(1/2)-2/7*(b*x^5+a*x^2)^(1/2)/a/x^(9/2)+8/7*b*(b*x^5+a*
x^2)^(1/2)/a^2/x^(3/2)+8/7*3^(1/4)*b^(4/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(
1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)*x)^2/(a^(1/3)+(1+3^(1/2))*b
^(1/3)*x)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/a^(5/3)/(b^(1/3)*x*(a^(1/3)+b^
(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^5+a*x^2)^(1/2)+4/21
*(1-3^(1/2))*b^(4/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)
*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(a
rccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4
*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(5/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1
/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^5+a*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.10

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},-\frac{bx^3}{a}\right)}{7x^{5/2}\sqrt{x^2\left(a+bx^3\right)}}$$

input
```
Integrate[1/(x^(7/2)*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
(-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-7/6, 1/2, -1/6, -((b*x^3)/a)])/
(7*x^(5/2)*Sqrt[x^2*(a + b*x^3)])
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1931, 1931, 1938, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx$$

$\Big\downarrow$ 1931

$$-\frac{4b\int \frac{1}{\sqrt{x}\sqrt{bx^5+ax^2}}dx}{7a}-\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}}$$

$\Big\downarrow$ 1931

$$-\frac{4b\left(\frac{2b\int \frac{x^{5/2}}{\sqrt{bx^5+ax^2}}dx}{a}-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}\right)}{7a}-\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}}$$

$\Big\downarrow$ 1938

$$-\frac{4b\left(\frac{2bx\sqrt{a+bx^3}\int \frac{x^{3/2}}{\sqrt{bx^3+a}}dx}{a\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}\right)}{7a}-\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}}$$

$\Big\downarrow$ 851

$$-\frac{4b\left(\frac{4bx\sqrt{a+bx^3}\int \frac{x^2}{\sqrt{bx^3+a}}d\sqrt{x}}{a\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}\right)}{7a}-\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}}$$

$\Big\downarrow$ 837

$$-\frac{4b\left(\frac{4bx\sqrt{a+bx^3}\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{a\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}\right)}{7a}-\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}}$$

$\Big\downarrow$ 25

$$-\frac{4b\left(\frac{4bx\sqrt{a+bx^3}\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{a\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}\right)}{7a}-\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}}$$

$\Big\downarrow$ 766

$$-\frac{4b\Bigg(\frac{4bx\sqrt{a+bx^3}\Bigg(\frac{\int \frac{2b^{2/3}x^2+(1-\sqrt{3})a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{(1-\sqrt{3})\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\Bigg(\arccos\Bigg(\frac{(1-\sqrt{3})\sqrt[3]{b}x+\sqrt[3]{a}}{(1+\sqrt{3})\sqrt[3]{b}x+\sqrt[3]{a}}}{4\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{a\sqrt{ax^2+bx^5}}}{7a}$$

$$\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}}$$

$\downarrow$ 2420

$$-\frac{4b\left(\frac{4bx\sqrt{a+bx^3}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{x}\sqrt{a+bx^3}}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}-\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{2b^{2/3}}-}{a\sqrt{ax^2+bx^5}}}{7a}$$

$$\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}}$$

input
```
Int[1/(x^(7/2)*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
(-2*Sqrt[a*x^2 + b*x^5])/(7*a*x^(9/2)) - (4*b*((-2*Sqrt[a*x^2 + b*x^5])/(a
*x^(3/2)) + (4*b*x*Sqrt[a + b*x^3]*((((1 + Sqrt[3])*Sqrt[x]*Sqrt[a + b*x^3
])/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x) - (3^(1/4)*a^(1/3)*Sqrt[x]*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) +
(1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1
/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1
/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*Sqrt[x]*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt
[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a
^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*S
qrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^
2]*Sqrt[a + b*x^3])))/(a*Sqrt[a*x^2 + b*x^5])))/(7*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.35 (sec) , antiderivative size = 1125, normalized size of antiderivative = 2.03

method	result	size
risch	Expression too large to display	1125
default	Expression too large to display	3048

input
```
int(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-2/7*(b*x^3+a)*(-4*b*x^3+a)/a^2/x^(5/2)/(x^2*(b*x^3+a))^(1/2)-8/7*b^2/a^2*
(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2
)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(
-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-
a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b
^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.10

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\left(4\sqrt{a}bx^5\text{weierstrassZeta}\left(0,-\frac{4b}{a},\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)\right)+\sqrt{bx^5+ax^2}a\sqrt{x}\right)}{7a^2x^5}$$

input

```
integrate(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output

```
-2/7*(4*sqrt(a)*b*x^5*weierstrassZeta(0, -4*b/a, weierstrassPInverse(0, -4
*b/a, 1/x)) + sqrt(b*x^5 + a*x^2)*a*sqrt(x))/(a^2*x^5)
```

Sympy [F]

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^{\frac{7}{2}}\sqrt{x^2\,(a+bx^3)}}\,dx$$

input
```
integrate(1/x**(7/2)/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**(7/2)*sqrt(x**2*(a + b*x**3))), x)
```

Maxima [F]

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}x^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^5 + a*x^2)*x^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}x^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^5 + a*x^2)*x^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^{7/2}\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(1/(x^(7/2)*(a*x^2 + b*x^5)^(1/2)),x)
```

output
```
int(1/(x^(7/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^8+a\,x^5}dx$$

input
```
int(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(a + b*x**3))/(a*x**5 + b*x**8),x)
```

3.285 $\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx$

Optimal result . . . 2411
Mathematica [A] (verified) . . . 2411
Rubi [A] (verified) . . . 2412
Maple [A] (verified) . . . 2413
Fricas [A] (verification not implemented) . . . 2414
Sympy [F] . . . 2414
Maxima [A] (verification not implemented) . . . 2414
Giac [A] (verification not implemented) . . . 2415
Mupad [F(-1)] . . . 2415
Reduce [B] (verification not implemented) . . . 2415

Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx=-\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}}+\frac{4b\sqrt{ax^2+bx^5}}{9a^2x^{5/2}}$$

output
```
-2/9*(b*x^5+a*x^2)^(1/2)/a/x^(11/2)+4/9*b*(b*x^5+a*x^2)^(1/2)/a^2/x^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx=-\frac{2(a-2bx^3)\sqrt{x^2(a+bx^3)}}{9a^2x^{11/2}}$$

input
```
Integrate[1/(x^(9/2)*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
(-2*(a - 2*b*x^3)*Sqrt[x^2*(a + b*x^3)])/(9*a^2*x^(11/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}dx$$

$$\Big\downarrow \text{ 1922}$$

$$-\frac{2b\int \frac{1}{x^{3/2}\sqrt{bx^5+ax^2}}dx}{3a}-\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}}$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{4b\sqrt{ax^2+bx^5}}{9a^2x^{5/2}}-\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}}$$

input
```
Int[1/(x^(9/2)*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
(-2*Sqrt[a*x^2 + b*x^5])/(9*a*x^(11/2)) + (4*b*Sqrt[a*x^2 + b*x^5])/(9*a^2
*x^(5/2))
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{2\left(b\,x^{3}+a\right)\left(-2b\,x^{3}+a\right)}{9x^{\frac{7}{2}}\,a^{2}\sqrt{b\,x^{5}+a\,x^{2}}}$	37
default	$-\frac{2\left(b\,x^{3}+a\right)\left(-2b\,x^{3}+a\right)}{9x^{\frac{7}{2}}\,a^{2}\sqrt{b\,x^{5}+a\,x^{2}}}$	37
risch	$-\frac{2\left(b\,x^{3}+a\right)\left(-2b\,x^{3}+a\right)}{9\sqrt{x^{2}\left(b\,x^{3}+a\right)}\,x^{\frac{7}{2}}\,a^{2}}$	37
orering	$-\frac{2\left(b\,x^{3}+a\right)\left(-2b\,x^{3}+a\right)}{9x^{\frac{7}{2}}\,a^{2}\sqrt{b\,x^{5}+a\,x^{2}}}$	37

input
```
int(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(b*x^3+a)*(-2*b*x^3+a)/x^(7/2)/a^2/(b*x^5+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx = \frac{2\sqrt{bx^5+ax^2}(2\,bx^3-a)}{9\,a^2x^{\frac{11}{2}}}$$

input
```
integrate(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/9*sqrt(b*x^5 + a*x^2)*(2*b*x^3 - a)/(a^2*x^(11/2))
```

Sympy [F]

$$\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^{\frac{9}{2}}\sqrt{x^2\,(a+bx^3)}}\,dx$$

input
```
integrate(1/x**(9/2)/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**(9/2)*sqrt(x**2*(a + b*x**3))), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx = \frac{2\,(2\,b^2x^7+abx^4-a^2x)}{9\,\sqrt{bx^3+a}a^2x^{\frac{11}{2}}}$$

input
```
integrate(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/9*(2*b^2*x^7 + a*b*x^4 - a^2*x)/(sqrt(b*x^3 + a)*a^2*x^(11/2))
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx=-\frac{2\left(\frac{\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}}{a^2}-\frac{3\sqrt{b+\frac{a}{x^3}}\,b}{a^2}+\frac{2\,b^{\frac{3}{2}}}{a^2}\right)}{9\,\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-2/9*((b + a/x^3)^(3/2)/a^2 - 3*sqrt(b + a/x^3)*b/a^2 + 2*b^(3/2)/a^2)/sgn
(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx=\int \frac{1}{x^{9/2}\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(1/(x^(9/2)*(a*x^2 + b*x^5)^(1/2)),x)
```

output
```
int(1/(x^(9/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx=\frac{2\sqrt{b\,x^3+a}\,(2b\,x^3-a)}{9\sqrt{x}\,a^2x^4}$$

input
```
int(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3)*( - a + 2*b*x**3))/(9*sqrt(x)*a**2*x**4)
```

3.286 $\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx$

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Giac [F] . . . 2422
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Optimal result

Integrand size = 21, antiderivative size = 265

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx=-\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}+\frac{16b\sqrt{ax^2+bx^5}}{55a^2x^{7/2}}+\frac{16b^2x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{55\sqrt[4]{3}a^{7/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output

```
-2/11*(b*x^5+a*x^2)^(1/2)/a/x^(13/2)+16/55*b*(b*x^5+a*x^2)^(1/2)/a^2/x^(7/
2)+16/165*b^2*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((
a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2
)+1/4*2^(1/2))*3^(3/4)/a^(7/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+
3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^5+a*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx=-\frac{2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{11}{6},\frac{1}{2},-\frac{5}{6},-\frac{bx^3}{a}\right)}{11x^{9/2}\sqrt{x^2\left(a+bx^3\right)}}$$

input
```
Integrate[1/(x^(11/2)*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
(-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-11/6, 1/2, -5/6, -((b*x^3)/a)])
/(11*x^(9/2)*Sqrt[x^2*(a + b*x^3)])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1931, 1931, 1938, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}dx$$

$$\Big\downarrow \text{1931}$$

$$-\frac{8b\int \frac{1}{x^{5/2}\sqrt{bx^5+ax^2}}dx}{11a}-\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{8b\left(-\frac{2b\int \frac{\sqrt{x}}{\sqrt{bx^5+ax^2}}dx}{5a}-\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}\right)}{11a}-\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}$$

$$\Big\downarrow \text{1938}$$

$$-\frac{8b\left(-\frac{2bx\sqrt{a+bx^3}\int \frac{1}{\sqrt{x}\sqrt{bx^3+a}}dx}{5a\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}\right)}{11a}-\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}$$

$$\Big\downarrow \text{851}$$

$$-\frac{8b\left(-\frac{4bx\sqrt{a+bx^3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{5a\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}\right)}{11a}-\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}$$

$$\Big\downarrow \text{766}$$

$$-\frac{8b\left(-\frac{2bx^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}\right)}{11a}-\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}$$

input
```
Int[1/(x^(11/2)*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
(-2*Sqrt[a*x^2 + b*x^5])/(11*a*x^(13/2)) - (8*b*((-2*Sqrt[a*x^2 + b*x^5])/
(5*a*x^(7/2)) - (2*b*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF
[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/
3)*x)], (2 + Sqrt[3])/4])/(5*3^(1/4)*a^(4/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^
(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])))/(1
1*a)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.22 (sec) , antiderivative size = 742, normalized size of antiderivative = 2.80

method	result
risch	$-\frac{2(bx^3+a)(-8bx^3+5a)}{55a^2x^{\frac{9}{2}}\sqrt{x^2(bx^3+a)}}+\frac{32b^3\left(\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{(-ab^2)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{(-ab^2)^{\frac{1}{3}}}{b}\cdots\right.}{5\cdots}$
default	Expression too large to display

input
```
int(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/55*(b*x^3+a)*(-8*b*x^3+5*a)/a^2/x^(9/2)/(x^2*(b*x^3+a))^(1/2)+32/55*b^3
/a^2*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3
))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b
^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))/(-a*b^2)^(1/3)/(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))/(x^2*(b*x^3+a))^(1/2)*x^(1/2)*(x*(b*x^3+
a))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.23

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\left(16\sqrt{a}b^2x^7\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)-\sqrt{bx^5+ax^2}(8abx^3-5a^2)\sqrt{x}\right)}{55a^3x^7}$$

input
```
integrate(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/55*(16*sqrt(a)*b^2*x^7*weierstrassPInverse(0, -4*b/a, 1/x) - sqrt(b*x^5
 + a*x^2)*(8*a*b*x^3 - 5*a^2)*sqrt(x))/(a^3*x^7)
```

Sympy [F]

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^{\frac{11}{2}}\sqrt{x^2(a+bx^3)}}\,dx$$

input
```
integrate(1/x**(11/2)/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**(11/2)*sqrt(x**2*(a + b*x**3))), x)
```

Maxima [F]

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}x^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^5 + a*x^2)*x^(11/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}x^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^5 + a*x^2)*x^(11/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^{11/2}\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(1/(x^(11/2)*(a*x^2 + b*x^5)^(1/2)),x)
```

output
```
int(1/(x^(11/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^{10}+a\,x^7}dx$$

input
```
int(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(a + b*x**3))/(a*x**7 + b*x**10),x)
```

3.287 $\int \frac{x}{ax^3+bx^4} \, dx$

Optimal result 2423
Mathematica [A] (verified) 2423
Rubi [A] (verified) 2424
Maple [A] (verified) 2425
Fricas [A] (verification not implemented) 2425
Sympy [A] (verification not implemented) 2426
Maxima [A] (verification not implemented) 2426
Giac [A] (verification not implemented) 2426
Mupad [B] (verification not implemented) 2427
Reduce [B] (verification not implemented) 2427

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{x}{ax^3+bx^4} \, dx = -\frac{1}{ax} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx)}{a^2}$$

output
```
-1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x}{ax^3+bx^4} \, dx = -\frac{1}{ax} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx)}{a^2}$$

input
```
Integrate[x/(a*x^3 + b*x^4),x]
```

output
```
-(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {9, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{ax^3+bx^4}dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{1}{x^2(a+bx)}dx$$
$$\Big\downarrow \text{ 54}$$
$$\int \left(\frac{b^2}{a^2(a+bx)}-\frac{b}{a^2x}+\frac{1}{ax^2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

input
```
Int[x/(a*x^3 + b*x^4),x]
```

output
```
-(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result	size
parallelrisch	$-\frac{b\ln(x)x-b\ln(bx+a)x+a}{a^{2}x}$	26
default	$-\frac{1}{ax}-\frac{b\ln(x)}{a^{2}}+\frac{b\ln(bx+a)}{a^{2}}$	29
norman	$-\frac{1}{ax}-\frac{b\ln(x)}{a^{2}}+\frac{b\ln(bx+a)}{a^{2}}$	29
risch	$-\frac{1}{ax}+\frac{b\ln(-bx-a)}{a^{2}}-\frac{b\ln(x)}{a^{2}}$	32

input
```
int(x/(b*x^4+a*x^3),x,method=_RETURNVERBOSE)
```

output
```
-(b*ln(x)*x-b*ln(b*x+a)*x+a)/a^2/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x}{ax^3+bx^4}\,dx=\frac{bx\log(bx+a)-bx\log(x)-a}{a^2x}$$

input
```
integrate(x/(b*x^4+a*x^3),x, algorithm="fricas")
```

output
```
(b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{x}{ax^3 + bx^4} \, dx = -\frac{1}{ax} + \frac{b\left(-\log\left(x\right) + \log\left(\frac{a}{b} + x\right)\right)}{a^2}$$

input
```
integrate(x/(b*x**4+a*x**3),x)
```

output
```
-1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x}{ax^3 + bx^4} \, dx = \frac{b\log\left(bx + a\right)}{a^2} - \frac{b\log\left(x\right)}{a^2} - \frac{1}{ax}$$

input
```
integrate(x/(b*x^4+a*x^3),x, algorithm="maxima")
```

output
```
b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x}{ax^3 + bx^4} \, dx = \frac{b\log\left(|bx + a|\right)}{a^2} - \frac{b\log\left(|x|\right)}{a^2} - \frac{1}{ax}$$

input
```
integrate(x/(b*x^4+a*x^3),x, algorithm="giac")
```

output
```
b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{x}{ax^3+bx^4}\,dx=\frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2}-\frac{1}{a\,x}$$

input
```
int(x/(a*x^3 + b*x^4),x)
```

output
```
(2*b*atanh((2*b*x)/a + 1))/a^2 - 1/(a*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x}{ax^3+bx^4}\,dx=\frac{\log(bx+a)\,bx-\log(x)\,bx-a}{a^2x}$$

input
```
int(x/(b*x^4+a*x^3),x)
```

output
```
(log(a + b*x)*b*x - log(x)*b*x - a)/(a**2*x)
```

3.288 $\int \frac{1}{ax^3+bx^4} dx$

Optimal result . . . 2428
Mathematica [A] (verified) . . . 2428
Rubi [A] (verified) . . . 2429
Maple [A] (verified) . . . 2430
Fricas [A] (verification not implemented) . . . 2430
Sympy [A] (verification not implemented) . . . 2431
Maxima [A] (verification not implemented) . . . 2431
Giac [A] (verification not implemented) . . . 2431
Mupad [B] (verification not implemented) . . . 2432
Reduce [B] (verification not implemented) . . . 2432

Optimal result

Integrand size = 13, antiderivative size = 42

$$\int \frac{1}{ax^3+bx^4} dx = -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}$$

output
```
-1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^3+bx^4} dx = -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}$$

input
```
Integrate[(a*x^3 + b*x^4)^(-1),x]
```

output
```
-1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2026, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax^3+bx^4} \, dx$$
$$\Big\downarrow \text{ 2026}$$
$$\int \frac{1}{x^3(a+bx)}dx$$
$$\Big\downarrow \text{ 54}$$
$$\int \left(-\frac{b^3}{a^3(a+bx)}+\frac{b^2}{a^3x}-\frac{b}{a^2x^2}+\frac{1}{ax^3}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

input
```
Int[(a*x^3 + b*x^4)^(-1),x]
```

output
```
-1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{1}{2a\,x^2}+\frac{b}{a^2x}+\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln(bx+a)}{a^3}$	41
norman	$\frac{\frac{bx}{a^2}-\frac{1}{2a}}{x^2}+\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln(bx+a)}{a^3}$	41
risch	$\frac{\frac{bx}{a^2}-\frac{1}{2a}}{x^2}-\frac{b^2\ln(bx+a)}{a^3}+\frac{b^2\ln(-x)}{a^3}$	43
parallelrisch	$\frac{2b^2\ln(x)x^2-2b^2\ln(bx+a)x^2+2abx-a^2}{2a^3x^2}$	44

input
```
int(1/(b*x^4+a*x^3),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int\frac{1}{ax^3+bx^4}\,dx=-\frac{2\,b^2x^2\log\left(bx+a\right)-2\,b^2x^2\log\left(x\right)-2\,abx+a^2}{2\,a^3x^2}$$

input
```
integrate(1/(b*x^4+a*x^3),x, algorithm="fricas")
```

output
```
-1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{1}{ax^3 + bx^4}\,dx = \frac{-a + 2bx}{2a^2x^2} + \frac{b^2\left(\log\left(x\right) - \log\left(\frac{a}{b} + x\right)\right)}{a^3}$$

input
```
integrate(1/(b*x**4+a*x**3),x)
```

output
```
(-a + 2*b*x)/(2*a**2*x**2) + b**2*(log(x) - log(a/b + x))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{1}{ax^3 + bx^4}\,dx = -\frac{b^2\log\left(bx + a\right)}{a^3} + \frac{b^2\log\left(x\right)}{a^3} + \frac{2\,bx - a}{2\,a^2x^2}$$

input
```
integrate(1/(b*x^4+a*x^3),x, algorithm="maxima")
```

output
```
-b^2*log(b*x + a)/a^3 + b^2*log(x)/a^3 + 1/2*(2*b*x - a)/(a^2*x^2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{1}{ax^3 + bx^4}\,dx = -\frac{b^2\log\left(|bx + a|\right)}{a^3} + \frac{b^2\log\left(|x|\right)}{a^3} + \frac{2\,abx - a^2}{2\,a^3x^2}$$

input
```
integrate(1/(b*x^4+a*x^3),x, algorithm="giac")
```

output
```
-b^2*log(abs(b*x + a))/a^3 + b^2*log(abs(x))/a^3 + 1/2*(2*a*b*x - a^2)/(a^
3*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{1}{ax^3+bx^4}\,dx = -\frac{\frac{a^2}{2}-a\,b\,x}{a^3\,x^2}-\frac{2\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^3}$$

input
```
int(1/(a*x^3 + b*x^4),x)
```

output
```
- (a^2/2 - a*b*x)/(a^3*x^2) - (2*b^2*atanh((2*b*x)/a + 1))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{1}{ax^3+bx^4}\,dx = \frac{-2\log(bx+a)\,b^2x^2+2\log(x)\,b^2x^2-a^2+2abx}{2a^3x^2}$$

input
```
int(1/(b*x^4+a*x^3),x)
```

output
```
( - 2*log(a + b*x)*b**2*x**2 + 2*log(x)*b**2*x**2 - a**2 + 2*a*b*x)/(2*a**
3*x**2)
```

3.289 $\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx$

Optimal result . . . 2433
Mathematica [A] (verified) . . . 2433
Rubi [A] (verified) . . . 2434
Maple [A] (verified) . . . 2436
Fricas [A] (verification not implemented) . . . 2436
Sympy [F] . . . 2437
Maxima [F] . . . 2437
Giac [A] (verification not implemented) . . . 2437
Mupad [F(-1)] . . . 2438
Reduce [B] (verification not implemented) . . . 2438

Optimal result

Integrand size = 19, antiderivative size = 112

$$\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx = -\frac{5a\sqrt{ax^3+bx^4}}{12b^2}+\frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x}+\frac{x\sqrt{ax^3+bx^4}}{3b}-\frac{5a^3\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{8b^{7/2}}$$

output
```
-5/12*a*(b*x^4+a*x^3)^(1/2)/b^2+5/8*a^2*(b*x^4+a*x^3)^(1/2)/b^3/x+1/3*x*(b
*x^4+a*x^3)^(1/2)/b-5/8*a^3*arctanh(b^(1/2)*x^2/(b*x^4+a*x^3)^(1/2))/b^(7/
2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx = \frac{\sqrt{b}x^2(15a^3+5a^2bx-2ab^2x^2+8b^3x^3)+30a^3x^{3/2}\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{24b^{7/2}\sqrt{x^3(a+bx)}}$$

input
```
Integrate[x^4/Sqrt[a*x^3 + b*x^4],x]
```

output
```
(Sqrt[b]*x^2*(15*a^3 + 5*a^2*b*x - 2*a*b^2*x^2 + 8*b^3*x^3) + 30*a^3*x^(3/
2)*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])])/(24
*b^(7/2)*Sqrt[x^3*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1930, 1930, 1930, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{ax^3+bx^4}}dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{x\sqrt{ax^3+bx^4}}{3b}-\frac{5a\int \frac{x^3}{\sqrt{bx^4+ax^3}}dx}{6b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{x\sqrt{ax^3+bx^4}}{3b}-\frac{5a\left(\frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\int \frac{x^2}{\sqrt{bx^4+ax^3}}dx}{4b}\right)}{6b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{x\sqrt{ax^3+bx^4}}{3b}-\frac{5a\left(\frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\left(\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\int \frac{x}{\sqrt{bx^4+ax^3}}dx}{2b}\right)}{4b}\right)}{6b}$$

$$\Big\downarrow \text{1935}$$

$$\frac{x\sqrt{ax^3+bx^4}}{3b}-\frac{5a\left(\frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\left(\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\int \frac{1}{1-\frac{bx^4}{bx^4+ax^3}}d\frac{x^2}{\sqrt{bx^4+ax^3}}}{b}\right)}{4b}\right)}{6b}$$

$$\Big\downarrow \text{219}$$

$$\frac{x\sqrt{ax^3+bx^4}}{3b}-\frac{5a\left(\frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\left(\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{b^{3/2}}\right)}{4b}\right)}{6b}$$

input
```
Int[x^4/Sqrt[a*x^3 + b*x^4],x]
```

output
```
(x*Sqrt[a*x^3 + b*x^4])/(3*b) - (5*a*(Sqrt[a*x^3 + b*x^4]/(2*b) - (3*a*(Sq
rt[a*x^3 + b*x^4]/(b*x) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/b
^(3/2)))/(4*b)))/(6*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$\frac{8b^{\frac{5}{2}}x^{2}\sqrt{x^{3}(bx+a)}-10a\,b^{\frac{3}{2}}x\sqrt{x^{3}(bx+a)}-15\,\operatorname{arctanh}\left(\frac{\sqrt{x^{3}(bx+a)}}{x^{2}\sqrt{b}}\right)a^{3}x+15a^{2}\sqrt{b}\,\sqrt{x^{3}(bx+a)}}{24b^{\frac{7}{2}}x}$	91
risch	$\frac{(8b^{2}x^{2}-10abx+15a^{2})x^{2}(bx+a)}{24b^{3}\sqrt{x^{3}(bx+a)}}-\frac{5a^{3}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)x\sqrt{x(bx+a)}}{16b^{\frac{7}{2}}\sqrt{x^{3}(bx+a)}}$	98
default	$\frac{x\sqrt{x(bx+a)}\left(16x^{2}\sqrt{b\,x^{2}+ax}\,b^{\frac{7}{2}}-20\sqrt{b\,x^{2}+ax}\,b^{\frac{5}{2}}ax+30\sqrt{b\,x^{2}+ax}\,b^{\frac{3}{2}}a^{2}-15\ln\left(\frac{2\sqrt{b\,x^{2}+ax}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a^{3}b\right)}{48\sqrt{b\,x^{4}+a\,x^{3}}\,b^{\frac{9}{2}}}$	120

input
```
int(x^4/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*b^(5/2)*x^2*(x^3*(b*x+a))^(1/2)-10*a*b^(3/2)*x*(x^3*(b*x+a))^(1/2)
-15*arctanh((x^3*(b*x+a))^(1/2)/x^2/b^(1/2))*a^3*x+15*a^2*b^(1/2)*(x^3*(b*
x+a))^(1/2))/b^(7/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.57

$$\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx = \left[\frac{15\,a^3\sqrt{b}x\log\left(\frac{2\,bx^2+ax-2\,\sqrt{bx^4+ax^3}\sqrt{b}}{x}\right)+2\,(8\,b^3x^2-10\,ab^2x+15\,a^2b)\sqrt{bx^4+ax^3}}{48\,b^4x}, \frac{15\,a^3\sqrt{-b}x\arctan\left(\frac{\sqrt{}\right.}{}\right.}{}\right.$$

input
```
integrate(x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(15*a^3*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^4 + a*x^3)*sqrt(b)
)/x) + 2*(8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^4 + a*x^3))/(b^4*x),
 1/24*(15*a^3*sqrt(-b)*x*arctan(sqrt(b*x^4 + a*x^3)*sqrt(-b)/(b*x^2 + a*x)
) + (8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^4 + a*x^3))/(b^4*x)]
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x^4}{\sqrt{x^3(a+bx)}}\,dx$$

input
```
integrate(x**4/(b*x**4+a*x**3)**(1/2),x)
```

output
```
Integral(x**4/sqrt(x**3*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x^4}{\sqrt{bx^4+ax^3}}\,dx$$

input
```
integrate(x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(b*x^4 + a*x^3), x)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.93

$$\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx = \frac{1}{24}\sqrt{bx^2+ax}\left(2\,x\left(\frac{4\,x}{b\mathrm{sgn}\,(x)}-\frac{5\,a}{b^2\mathrm{sgn}\,(x)}\right)+\frac{15\,a^2}{b^3\mathrm{sgn}\,(x)}\right) - \frac{5\,a^3\log\left(|a|\right)\mathrm{sgn}(x)}{16\,b^{\frac{7}{2}}} + \frac{5\,a^3\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{16\,b^{\frac{7}{2}}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(b*x^2 + a*x)*(2*x*(4*x/(b*sgn(x)) - 5*a/(b^2*sgn(x))) + 15*a^2/(
b^3*sgn(x))) - 5/16*a^3*log(abs(a))*sgn(x)/b^(7/2) + 5/16*a^3*log(abs(2*(s
qrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/(b^(7/2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x^4}{\sqrt{b\,x^4+a\,x^3}}\,dx$$

input
```
int(x^4/(a*x^3 + b*x^4)^(1/2),x)
```

output
```
int(x^4/(a*x^3 + b*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.68

$$\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx = \frac{15\sqrt{x}\,\sqrt{bx+a}\,a^2b-10\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x+8\sqrt{x}\,\sqrt{bx+a}\,b^3x^2-15\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^3}{24b^4}$$

input
```
int(x^4/(b*x^4+a*x^3)^(1/2),x)
```

output
```
(15*sqrt(x)*sqrt(a + b*x)*a**2*b - 10*sqrt(x)*sqrt(a + b*x)*a*b**2*x + 8*s
qrt(x)*sqrt(a + b*x)*b**3*x**2 - 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*s
qrt(b))/sqrt(a))*a**3)/(24*b**4)
```

3.290 $\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx$

Optimal result . . . 2439
Mathematica [A] (verified) . . . 2439
Rubi [A] (verified) . . . 2440
Maple [A] (verified) . . . 2441
Fricas [A] (verification not implemented) . . . 2442
Sympy [F] . . . 2442
Maxima [F] . . . 2443
Giac [A] (verification not implemented) . . . 2443
Mupad [F(-1)] . . . 2443
Reduce [B] (verification not implemented) . . . 2444

Optimal result

Integrand size = 19, antiderivative size = 86

$$\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx=\frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\sqrt{ax^3+bx^4}}{4b^2x}+\frac{3a^2\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{4b^{5/2}}$$

output
```
1/2*(b*x^4+a*x^3)^(1/2)/b-3/4*a*(b*x^4+a*x^3)^(1/2)/b^2/x+3/4*a^2*arctanh(
b^(1/2)*x^2/(b*x^4+a*x^3)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.17

$$\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx=\frac{\sqrt{b}x^2(-3a^2-abx+2b^2x^2)+6a^2x^{3/2}\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{4b^{5/2}\sqrt{x^3(a+bx)}}$$

input
```
Integrate[x^3/Sqrt[a*x^3 + b*x^4],x]
```

output
```
(Sqrt[b]*x^2*(-3*a^2 - a*b*x + 2*b^2*x^2) + 6*a^2*x^(3/2)*Sqrt[a + b*x]*Ar
cTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(4*b^(5/2)*Sqrt[x^3*(
a + b*x)])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1930, 1930, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{ax^3+bx^4}}dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\int \frac{x^2}{\sqrt{bx^4+ax^3}}dx}{4b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\left(\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\int \frac{x}{\sqrt{bx^4+ax^3}}dx}{2b}\right)}{4b}$$

$$\Big\downarrow \text{1935}$$

$$\frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\left(\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\int \frac{1}{1-\frac{bx^4}{bx^4+ax^3}}d\frac{x^2}{\sqrt{bx^4+ax^3}}}{b}\right)}{4b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\left(\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{b^{3/2}}\right)}{4b}$$

input
```
Int[x^3/Sqrt[a*x^3 + b*x^4],x]
```

output
```
Sqrt[a*x^3 + b*x^4]/(2*b) - (3*a*(Sqrt[a*x^3 + b*x^4]/(b*x) - (a*ArcTanh[(
Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/b^(3/2)))/(4*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

method	result	size
pseudoelliptic	$\frac{3 \operatorname{arctanh}\left(\frac{\sqrt{x^{3}(bx+a)}}{x^{2}\sqrt{b}}\right) a^{2} x+2 b^{\frac{3}{2}} x \sqrt{x^{3}(bx+a)}-3 a \sqrt{b}\, \sqrt{x^{3}(bx+a)}}{4 b^{\frac{5}{2}} x}$	69
risch	$-\frac{(-2bx+3a)x^{2}(bx+a)}{4b^{2}\sqrt{x^{3}(bx+a)}}+\frac{3a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)x\sqrt{x(bx+a)}}{8b^{\frac{5}{2}}\sqrt{x^{3}(bx+a)}}$	87
default	$\frac{x\sqrt{x(bx+a)}\left(4x\sqrt{b\,x^{2}+ax}\,b^{\frac{5}{2}}-6\sqrt{b\,x^{2}+ax}\,b^{\frac{3}{2}}a+3\ln\left(\frac{2\sqrt{b\,x^{2}+ax}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a^{2}b\right)}{8\sqrt{b\,x^{4}+a\,x^{3}}\,b^{\frac{7}{2}}}$	98

input
```
int(x^3/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/b^(5/2)*(3*arctanh((x^3*(b*x+a))^(1/2)/x^2/b^(1/2))*a^2*x+2*b^(3/2)*x*
(x^3*(b*x+a))^(1/2)-3*a*b^(1/2)*(x^3*(b*x+a))^(1/2))/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.80

$$\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx = \left[\frac{3\,a^2\sqrt{b}x\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^4+ax^3}\sqrt{b}}{x}\right)+2\,\sqrt{bx^4+ax^3}(2\,b^2x-3\,ab)}{8\,b^3x},\right.$$
$$\left.-\frac{3\,a^2\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^4+ax^3}\sqrt{-b}}{bx^2+ax}\right)-\sqrt{bx^4+ax^3}(2\,b^2x-3\,ab)}{4\,b^3x}\right]$$

input
```
integrate(x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(3*a^2*sqrt(b)*x*log((2*b*x^2 + a*x + 2*sqrt(b*x^4 + a*x^3)*sqrt(b))/
x) + 2*sqrt(b*x^4 + a*x^3)*(2*b^2*x - 3*a*b))/(b^3*x), -1/4*(3*a^2*sqrt(-b
)*x*arctan(sqrt(b*x^4 + a*x^3)*sqrt(-b)/(b*x^2 + a*x)) - sqrt(b*x^4 + a*x^
3)*(2*b^2*x - 3*a*b))/(b^3*x)]
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x^3}{\sqrt{x^3\,(a+bx)}}\,dx$$

input
```
integrate(x**3/(b*x**4+a*x**3)**(1/2),x)
```

output
```
Integral(x**3/sqrt(x**3*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x^3}{\sqrt{bx^4+ax^3}}\,dx$$

input
```
integrate(x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(b*x^4 + a*x^3), x)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx = \frac{1}{4}\sqrt{bx^2+ax}\left(\frac{2\,x}{b\mathrm{sgn}\left(x\right)} - \frac{3\,a}{b^2\mathrm{sgn}\left(x\right)}\right) + \frac{3\,a^2\log\left(|a|\right)\mathrm{sgn}(x)}{8\,b^{\frac{5}{2}}} - \frac{3\,a^2\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,b^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(b*x^2 + a*x)*(2*x/(b*sgn(x)) - 3*a/(b^2*sgn(x))) + 3/8*a^2*log(ab
s(a))*sgn(x)/b^(5/2) - 3/8*a^2*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*s
qrt(b) + a))/(b^(5/2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x^3}{\sqrt{b\,x^4+a\,x^3}}\,dx$$

input
```
int(x^3/(a*x^3 + b*x^4)^(1/2),x)
```

output
```
int(x^3/(a*x^3 + b*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.66

$$\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx = \frac{-3\sqrt{x}\,\sqrt{bx+a}\,ab+2\sqrt{x}\,\sqrt{bx+a}\,b^2x+3\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2}{4b^3}$$

input
```
int(x^3/(b*x^4+a*x^3)^(1/2),x)
```

output
```
( - 3*sqrt(x)*sqrt(a + b*x)*a*b + 2*sqrt(x)*sqrt(a + b*x)*b**2*x + 3*sqrt(
b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2)/(4*b**3)
```

3.291 $\int \frac{x^2}{\sqrt{ax^3+bx^4}} dx$

Optimal result 2445
Mathematica [A] (verified) 2445
Rubi [A] (verified) 2446
Maple [A] (verified) 2447
Fricas [A] (verification not implemented) 2448
Sympy [F] 2448
Maxima [F] 2448
Giac [A] (verification not implemented) 2449
Mupad [F(-1)] 2449
Reduce [B] (verification not implemented) 2449

Optimal result

Integrand size = 19, antiderivative size = 56

$$\int \frac{x^2}{\sqrt{ax^3+bx^4}} dx = \frac{\sqrt{ax^3+bx^4}}{bx} - \frac{a\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{b^{3/2}}$$

output
```
(b*x^4+a*x^3)^(1/2)/b/x-a*arctanh(b^(1/2)*x^2/(b*x^4+a*x^3)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.46

$$\int \frac{x^2}{\sqrt{ax^3+bx^4}} dx = \frac{\sqrt{b}x^2(a+bx) + 2ax^{3/2}\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{b^{3/2}\sqrt{x^3(a+bx)}}$$

input
```
Integrate[x^2/Sqrt[a*x^3 + b*x^4],x]
```

output
```
(Sqrt[b]*x^2*(a + b*x) + 2*a*x^(3/2)*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x
])/(Sqrt[a] - Sqrt[a + b*x])])/(b^(3/2)*Sqrt[x^3*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1930, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{ax^3+bx^4}}dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\int \frac{x}{\sqrt{bx^4+ax^3}}dx}{2b}$$

$$\Big\downarrow \text{1935}$$

$$\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\int \frac{1}{1-\frac{bx^4}{bx^4+ax^3}}d\frac{x^2}{\sqrt{bx^4+ax^3}}}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{b^{3/2}}$$

input
```
Int[x^2/Sqrt[a*x^3 + b*x^4],x]
```

output
```
Sqrt[a*x^3 + b*x^4]/(b*x) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])
/b^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{-\operatorname{arctanh}\left(\frac{\sqrt{x^{3}(bx+a)}}{x^{2}\sqrt{b}}\right)ax+\sqrt{x^{3}(bx+a)}\,\sqrt{b}}{b^{\frac{3}{2}}x}$	47
risch	$\frac{x^{2}(bx+a)}{b\sqrt{x^{3}(bx+a)}}-\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)x\sqrt{x(bx+a)}}{2b^{\frac{3}{2}}\sqrt{x^{3}(bx+a)}}$	76
default	$\frac{x\sqrt{x(bx+a)}\left(2\sqrt{b\,x^{2}+ax}\,b^{\frac{3}{2}}-a\ln\left(\frac{2\sqrt{b\,x^{2}+ax}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)b\right)}{2\sqrt{b\,x^{4}+a\,x^{3}}\,b^{\frac{5}{2}}}$	78

input
```
int(x^2/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-arctanh((x^3*(b*x+a))^(1/2)/x^2/b^(1/2))*a*x+(x^3*(b*x+a))^(1/2)*b^(1/2)
)/b^(3/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.27

$$\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx = \left[\frac{a\sqrt{b}x\log\left(\frac{2\,bx^2+ax-2\,\sqrt{bx^4+ax^3}\sqrt{b}}{x}\right)+2\,\sqrt{bx^4+ax^3}b}{2\,b^2x}, \frac{a\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^4+ax^3}\sqrt{-b}}{bx^2+ax}\right)+\sqrt{bx^4+ax^3}b}{b^2x}\right]$$

input
```
integrate(x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(a*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^4 + a*x^3)*sqrt(b))/x) +
 2*sqrt(b*x^4 + a*x^3)*b)/(b^2*x), (a*sqrt(-b)*x*arctan(sqrt(b*x^4 + a*x^3
)*sqrt(-b)/(b*x^2 + a*x)) + sqrt(b*x^4 + a*x^3)*b)/(b^2*x)]
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x^2}{\sqrt{x^3\,(a+bx)}}\,dx$$

input
```
integrate(x**2/(b*x**4+a*x**3)**(1/2),x)
```

output
```
Integral(x**2/sqrt(x**3*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x^2}{\sqrt{bx^4+ax^3}}\,dx$$

input
```
integrate(x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(b*x^4 + a*x^3), x)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.23

$$\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx = -\frac{a\log(|a|)\,\mathrm{sgn}(x)}{2\,b^{\frac{3}{2}}} + \frac{a\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{2\,b^{\frac{3}{2}}\mathrm{sgn}\,(x)} + \frac{\sqrt{bx^2+ax}}{b\mathrm{sgn}\,(x)}$$

input
```
integrate(x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

output
```
-1/2*a*log(abs(a))*sgn(x)/b^(3/2) + 1/2*a*log(abs(2*(sqrt(b)*x - sqrt(b*x^
2 + a*x))*sqrt(b) + a))/(b^(3/2)*sgn(x)) + sqrt(b*x^2 + a*x)/(b*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x^2}{\sqrt{b\,x^4+a\,x^3}}\,dx$$

input
```
int(x^2/(a*x^3 + b*x^4)^(1/2),x)
```

output
```
int(x^2/(a*x^3 + b*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx = \frac{\sqrt{x}\,\sqrt{bx+a}\,b-\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a}{b^2}$$

input
```
int(x^2/(b*x^4+a*x^3)^(1/2),x)
```

output
```
(sqrt(x)*sqrt(a + b*x)*b - sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/s
qrt(a))*a)/b**2
```

3.292 $\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx$

Optimal result . . . 2451
Mathematica [A] (verified) . . . 2451
Rubi [A] (verified) . . . 2452
Maple [A] (verified) . . . 2453
Fricas [A] (verification not implemented) . . . 2453
Sympy [F] . . . 2454
Maxima [F] . . . 2454
Giac [A] (verification not implemented) . . . 2454
Mupad [F(-1)] . . . 2455
Reduce [B] (verification not implemented) . . . 2455

Optimal result

Integrand size = 17, antiderivative size = 32

$$\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{\sqrt{b}}$$

output
```
2*arctanh(b^(1/2)*x^2/(b*x^4+a*x^3)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.78

$$\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx=-\frac{2x^{3/2}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{x^3(a+bx)}}$$

input
```
Integrate[x/Sqrt[a*x^3 + b*x^4],x]
```

output
```
(-2*x^(3/2)*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b
]*Sqrt[x^3*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{ax^3+bx^4}}dx$$

$$\Big\downarrow \text{1935}$$

$$2\int \frac{1}{1-\frac{bx^4}{bx^4+ax^3}}d\frac{x^2}{\sqrt{bx^4+ax^3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{\sqrt{b}}$$

input
```
Int[x/Sqrt[a*x^3 + b*x^4],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/Sqrt[b]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
pseudoelliptic	$\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{x^{3}(b x+a)}}{x^{2} \sqrt{b}}\right)}{\sqrt{b}}$	25
default	$\frac{x \sqrt{x(b x+a)}\, \ln \left(\frac{2 \sqrt{b\, x^{2}+a x}\, \sqrt{b}+2 b x+a}{2 \sqrt{b}}\right)}{\sqrt{b\, x^{4}+a\, x^{3}}\, \sqrt{b}}$	56

input
```
int(x/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^(1/2)*arctanh((x^3*(b*x+a))^(1/2)/x^2/b^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.47

$$\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx = \left[\frac{\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^4+ax^3}\sqrt{b}}{x}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^4+ax^3}\sqrt{-b}}{bx^2+ax}\right)}{b}\right]$$

input
```
integrate(x/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

output
```
[log((2*b*x^2 + a*x + 2*sqrt(b*x^4 + a*x^3)*sqrt(b))/x)/sqrt(b), -2*sqrt(-
b)*arctan(sqrt(b*x^4 + a*x^3)*sqrt(-b)/(b*x^2 + a*x))/b]
```

Sympy [F]

$$\int \frac{x}{\sqrt{ax^3 + bx^4}} \, dx = \int \frac{x}{\sqrt{x^3 (a + bx)}} \, dx$$

input
```
integrate(x/(b*x**4+a*x**3)**(1/2),x)
```

output
```
Integral(x/sqrt(x**3*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{ax^3 + bx^4}} \, dx = \int \frac{x}{\sqrt{bx^4 + ax^3}} \, dx$$

input
```
integrate(x/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(b*x^4 + a*x^3), x)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \frac{x}{\sqrt{ax^3 + bx^4}} \, dx = \frac{\log (|a|) \operatorname{sgn}(x)}{\sqrt{b}} - \frac{\log \left(\left| 2 \left(\sqrt{b}x - \sqrt{bx^2 + ax} \right) \sqrt{b} + a \right| \right)}{\sqrt{b}\operatorname{sgn}(x)}$$

input
```
integrate(x/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

output
```
log(abs(a))*sgn(x)/sqrt(b) - log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqr
t(b) + a))/(sqrt(b)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x}{\sqrt{b\,x^4+a\,x^3}}\,dx$$

input
```
int(x/(a*x^3 + b*x^4)^(1/2),x)
```

output
```
int(x/(a*x^3 + b*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx = \frac{2\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)}{b}$$

input
```
int(x/(b*x^4+a*x^3)^(1/2),x)
```

output
```
(2*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)))/b
```

3.293 $\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx$

Optimal result 2456
Mathematica [A] (verified) 2456
Rubi [A] (verified) 2457
Maple [A] (verified) 2458
Fricas [A] (verification not implemented) 2458
Sympy [F] 2459
Maxima [F] 2459
Giac [A] (verification not implemented) 2459
Mupad [B] (verification not implemented) 2460
Reduce [B] (verification not implemented) 2460

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx = -\frac{2\sqrt{ax^3+bx^4}}{ax^2}$$

output
```
-2*(b*x^4+a*x^3)^(1/2)/a/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx = -\frac{2\sqrt{x^3(a+bx)}}{ax^2}$$

input
```
Integrate[1/Sqrt[a*x^3 + b*x^4],x]
```

output
```
(-2*Sqrt[x^3*(a + b*x)])/(a*x^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{1906\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax^3+bx^4}}dx$$
$$\Big\downarrow \text{ 1906}$$
$$-\frac{2\sqrt{ax^3+bx^4}}{ax^2}$$

input
```
Int[1/Sqrt[a*x^3 + b*x^4],x]
```

output
```
(-2*Sqrt[a*x^3 + b*x^4])/(a*x^2)
```

Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
pseudoelliptic	$-\frac{2\sqrt{x^3(bx+a)}}{a\,x^2}$	20
trager	$-\frac{2\sqrt{b\,x^4+a\,x^3}}{a\,x^2}$	22
risch	$-\frac{2x(bx+a)}{\sqrt{x^3(bx+a)}\,a}$	23
gosper	$-\frac{2x(bx+a)}{a\sqrt{b\,x^4+a\,x^3}}$	25
orering	$-\frac{2x(bx+a)}{a\sqrt{b\,x^4+a\,x^3}}$	25
default	$-\frac{2\sqrt{x(bx+a)}\,\sqrt{b\,x^2+ax}}{\sqrt{b\,x^4+a\,x^3}\,a}$	39

input
```
int(1/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(x^3*(b*x+a))^(1/2)/a/x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx = -\frac{2\sqrt{bx^4+ax^3}}{ax^2}$$

input
```
integrate(1/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(b*x^4 + a*x^3)/(a*x^2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx=\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx$$

input
```
integrate(1/(b*x**4+a*x**3)**(1/2),x)
```

output
```
Integral(1/sqrt(a*x**3 + b*x**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx=\int \frac{1}{\sqrt{bx^4+ax^3}}\,dx$$

input
```
integrate(1/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*x^4 + a*x^3), x)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx=\frac{2}{\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\operatorname{sgn}(x)}$$

input
```
integrate(1/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

output
```
2/((sqrt(b)*x - sqrt(b*x^2 + a*x))*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx = -\frac{2\sqrt{b\,x^4+a\,x^3}}{a\,x^2}$$

input
```
int(1/(a*x^3 + b*x^4)^(1/2),x)
```

output
```
-(2*(a*x^3 + b*x^4)^(1/2))/(a*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx = \frac{-2\sqrt{x}\,\sqrt{bx+a}-2\sqrt{b}\,x}{ax}$$

input
```
int(1/(b*x^4+a*x^3)^(1/2),x)
```

output
```
( - 2*(sqrt(x)*sqrt(a + b*x) + sqrt(b)*x))/(a*x)
```

3.294 $\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx$

Optimal result 2461
Mathematica [A] (verified) 2461
Rubi [A] (verified) 2462
Maple [A] (verified) 2463
Fricas [A] (verification not implemented) 2463
Sympy [F] 2464
Maxima [F] 2464
Giac [A] (verification not implemented) 2464
Mupad [B] (verification not implemented) 2465
Reduce [B] (verification not implemented) 2465

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx = -\frac{2\sqrt{ax^3+bx^4}}{3ax^3}+\frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2}$$

output
```
-2/3*(b*x^4+a*x^3)^(1/2)/a/x^3+4/3*b*(b*x^4+a*x^3)^(1/2)/a^2/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx = -\frac{2(a-2bx)(a+bx)}{3a^2\sqrt{x^3(a+bx)}}$$

input
```
Integrate[1/(x*Sqrt[a*x^3 + b*x^4]),x]
```

output
```
(-2*(a - 2*b*x)*(a + b*x))/(3*a^2*Sqrt[x^3*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1922, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{ax^3+bx^4}}dx$$
$$\Big\downarrow \ 1922$$
$$-\frac{2b\int \frac{1}{\sqrt{bx^4+ax^3}}dx}{3a}-\frac{2\sqrt{ax^3+bx^4}}{3ax^3}$$
$$\Big\downarrow \ 1906$$
$$\frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2}-\frac{2\sqrt{ax^3+bx^4}}{3ax^3}$$

input
```
Int[1/(x*Sqrt[a*x^3 + b*x^4]),x]
```

output
```
(-2*Sqrt[a*x^3 + b*x^4])/(3*a*x^3) + (4*b*Sqrt[a*x^3 + b*x^4])/(3*a^2*x^2)
```

Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.50

method	result	size
pseudoelliptic	$-\frac{2(-2bx+a)\sqrt{x^3(bx+a)}}{3a^2x^3}$	26
trager	$-\frac{2(-2bx+a)\sqrt{b\,x^4+a\,x^3}}{3a^2x^3}$	28
risch	$-\frac{2(bx+a)(-2bx+a)}{3\sqrt{x^3(bx+a)}\,a^2}$	28
gosper	$-\frac{2(bx+a)(-2bx+a)}{3a^2\sqrt{b\,x^4+a\,x^3}}$	30
orering	$-\frac{2(bx+a)(-2bx+a)}{3a^2\sqrt{b\,x^4+a\,x^3}}$	30
default	$-\frac{2\sqrt{x(bx+a)}\,\sqrt{b\,x^2+ax}\,(-2bx+a)}{3x\sqrt{b\,x^4+a\,x^3}\,a^2}$	48

input
```
int(1/x/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-2*b*x+a)/a^2/x^3*(x^3*(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.56

$$\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx = \frac{2\sqrt{bx^4+ax^3}(2\,bx-a)}{3\,a^2x^3}$$

input
```
integrate(1/x/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(b*x^4 + a*x^3)*(2*b*x - a)/(a^2*x^3)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx = \int \frac{1}{x\sqrt{x^3\,(a+bx)}}\,dx$$

input
```
integrate(1/x/(b*x**4+a*x**3)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(x**3*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+ax^3}x}\,dx$$

input
```
integrate(1/x/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^4 + a*x^3)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx = \frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right)}{3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3\operatorname{sgn}(x)}$$

input
```
integrate(1/x/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

output
```
2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)/((sqrt(b)*x - sqrt(b*x
^2 + a*x))^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx = -\frac{2\,a\,\sqrt{b\,x^4+a\,x^3}-4\,b\,x\,\sqrt{b\,x^4+a\,x^3}}{3\,a^2\,x^3}$$

input
```
int(1/(x*(a*x^3 + b*x^4)^(1/2)),x)
```

output
```
-(2*a*(a*x^3 + b*x^4)^(1/2) - 4*b*x*(a*x^3 + b*x^4)^(1/2))/(3*a^2*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a}{3}+\frac{4\sqrt{x}\,\sqrt{bx+a}\,bx}{3}-\frac{4\sqrt{b}\,b\,x^2}{3}}{a^2x^2}$$

input
```
int(1/x/(b*x^4+a*x^3)^(1/2),x)
```

output
```
(2*( - sqrt(x)*sqrt(a + b*x)*a + 2*sqrt(x)*sqrt(a + b*x)*b*x - 2*sqrt(b)*b
*x**2))/(3*a**2*x**2)
```

3.295 $\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx$

Optimal result 2466
Mathematica [A] (verified) 2466
Rubi [A] (verified) 2467
Maple [A] (verified) 2468
Fricas [A] (verification not implemented) 2469
Sympy [F] 2469
Maxima [F] 2469
Giac [A] (verification not implemented) 2470
Mupad [B] (verification not implemented) 2470
Reduce [B] (verification not implemented) 2470

Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx=-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}+\frac{8b\sqrt{ax^3+bx^4}}{15a^2x^3}-\frac{16b^2\sqrt{ax^3+bx^4}}{15a^3x^2}$$

output
```
-2/5*(b*x^4+a*x^3)^(1/2)/a/x^4+8/15*b*(b*x^4+a*x^3)^(1/2)/a^2/x^3-16/15*b^
2*(b*x^4+a*x^3)^(1/2)/a^3/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx=-\frac{2\sqrt{x^3(a+bx)}(3a^2-4abx+8b^2x^2)}{15a^3x^4}$$

input
```
Integrate[1/(x^2*Sqrt[a*x^3 + b*x^4]),x]
```

output
```
(-2*Sqrt[x^3*(a + b*x)]*(3*a^2 - 4*a*b*x + 8*b^2*x^2))/(15*a^3*x^4)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1922, 1922, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{ax^3+bx^4}}dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{4b\int \frac{1}{x\sqrt{bx^4+ax^3}}dx}{5a}-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{4b\left(-\frac{2b\int \frac{1}{\sqrt{bx^4+ax^3}}dx}{3a}-\frac{2\sqrt{ax^3+bx^4}}{3ax^3}\right)}{5a}-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}$$

$$\Big\downarrow \text{1906}$$

$$-\frac{4b\left(\frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2}-\frac{2\sqrt{ax^3+bx^4}}{3ax^3}\right)}{5a}-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}$$

input
```
Int[1/(x^2*Sqrt[a*x^3 + b*x^4]),x]
```

output
```
(-2*Sqrt[a*x^3 + b*x^4])/(5*a*x^4) - (4*b*((-2*Sqrt[a*x^3 + b*x^4])/(3*a*x
^3) + (4*b*Sqrt[a*x^3 + b*x^4])/(3*a^2*x^2)))/(5*a)
```

Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

method	result	size
pseudoelliptic	$-\frac{2\left(8b^{2}x^{2}-4abx+3a^{2}\right)\sqrt{x^{3}\left(bx+a\right)}}{15a^{3}x^{4}}$	39
trager	$-\frac{2\left(8b^{2}x^{2}-4abx+3a^{2}\right)\sqrt{b\,x^{4}+a\,x^{3}}}{15a^{3}x^{4}}$	41
risch	$-\frac{2\left(bx+a\right)\left(8b^{2}x^{2}-4abx+3a^{2}\right)}{15x\sqrt{x^{3}\left(bx+a\right)}\,a^{3}}$	44
gosper	$-\frac{2\left(bx+a\right)\left(8b^{2}x^{2}-4abx+3a^{2}\right)}{15x\,a^{3}\sqrt{b\,x^{4}+a\,x^{3}}}$	46
orering	$-\frac{2\left(bx+a\right)\left(8b^{2}x^{2}-4abx+3a^{2}\right)}{15x\,a^{3}\sqrt{b\,x^{4}+a\,x^{3}}}$	46
default	$-\frac{2\sqrt{x\left(bx+a\right)}\,\sqrt{b\,x^{2}+ax}\,\left(8b^{2}x^{2}-4abx+3a^{2}\right)}{15x^{2}\sqrt{b\,x^{4}+a\,x^{3}}\,a^{3}}$	61

input
```
int(1/x^2/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(8*b^2*x^2-4*a*b*x+3*a^2)/a^3/x^4*(x^3*(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx = -\frac{2\,\sqrt{bx^4+ax^3}(8\,b^2x^2-4\,abx+3\,a^2)}{15\,a^3x^4}$$

input
```
integrate(1/x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

output
```
-2/15*sqrt(b*x^4 + a*x^3)*(8*b^2*x^2 - 4*a*b*x + 3*a^2)/(a^3*x^4)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx = \int \frac{1}{x^2\,\sqrt{x^3\,(a+bx)}}\,dx$$

input
```
integrate(1/x**2/(b*x**4+a*x**3)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(x**3*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+ax^3}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^4 + a*x^3)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx = \frac{2\left(20\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2b+15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)a\sqrt{b}+3\,a^2\right)}{15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5\operatorname{sgn}(x)}$$

input
```
integrate(1/x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

output
```
2/15*(20*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*b + 15*(sqrt(b)*x - sqrt(b*x^2
+ a*x))*a*sqrt(b) + 3*a^2)/((sqrt(b)*x - sqrt(b*x^2 + a*x))^5*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx = -\frac{2\sqrt{b\,x^4+a\,x^3}\,\left(3\,a^2-4\,a\,b\,x+8\,b^2\,x^2\right)}{15\,a^3\,x^4}$$

input
```
int(1/(x^2*(a*x^3 + b*x^4)^(1/2)),x)
```

output
```
-(2*(a*x^3 + b*x^4)^(1/2)*(3*a^2 + 8*b^2*x^2 - 4*a*b*x))/(15*a^3*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2}{5}+\frac{8\sqrt{x}\,\sqrt{bx+a}\,abx}{15}-\frac{16\sqrt{x}\,\sqrt{bx+a}\,b^2x^2}{15}+\frac{16\sqrt{b}\,b^2x^3}{15}}{a^3x^3}$$

input
```
int(1/x^2/(b*x^4+a*x^3)^(1/2),x)
```

output
```
(2*( - 3*sqrt(x)*sqrt(a + b*x)*a**2 + 4*sqrt(x)*sqrt(a + b*x)*a*b*x - 8*sq
rt(x)*sqrt(a + b*x)*b**2*x**2 + 8*sqrt(b)*b**2*x**3))/(15*a**3*x**3)
```

3.296 $\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx$

Optimal result . . . 2472
Mathematica [A] (verified) . . . 2472
Rubi [A] (verified) . . . 2473
Maple [A] (verified) . . . 2474
Fricas [A] (verification not implemented) . . . 2475
Sympy [F] . . . 2475
Maxima [F] . . . 2475
Giac [A] (verification not implemented) . . . 2476
Mupad [B] (verification not implemented) . . . 2476
Reduce [B] (verification not implemented) . . . 2477

Optimal result

Integrand size = 19, antiderivative size = 108

$$\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx = -\frac{2\sqrt{ax^3+bx^4}}{7ax^5}+\frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4}-\frac{16b^2\sqrt{ax^3+bx^4}}{35a^3x^3}+\frac{32b^3\sqrt{ax^3+bx^4}}{35a^4x^2}$$

output
```
-2/7*(b*x^4+a*x^3)^(1/2)/a/x^5+12/35*b*(b*x^4+a*x^3)^(1/2)/a^2/x^4-16/35*b
^2*(b*x^4+a*x^3)^(1/2)/a^3/x^3+32/35*b^3*(b*x^4+a*x^3)^(1/2)/a^4/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx = \frac{2\sqrt{x^3(a+bx)}(-5a^3+6a^2bx-8ab^2x^2+16b^3x^3)}{35a^4x^5}$$

input
```
Integrate[1/(x^3*Sqrt[a*x^3 + b*x^4]),x]
```

output
```
(2*Sqrt[x^3*(a + b*x)]*(-5*a^3 + 6*a^2*b*x - 8*a*b^2*x^2 + 16*b^3*x^3))/(3
5*a^4*x^5)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1922, 1922, 1922, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{ax^3+bx^4}}dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{6b\int \frac{1}{x^2\sqrt{bx^4+ax^3}}dx}{7a}-\frac{2\sqrt{ax^3+bx^4}}{7ax^5}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{6b\left(-\frac{4b\int \frac{1}{x\sqrt{bx^4+ax^3}}dx}{5a}-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}\right)}{7a}-\frac{2\sqrt{ax^3+bx^4}}{7ax^5}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{6b\left(-\frac{4b\left(-\frac{2b\int \frac{1}{\sqrt{bx^4+ax^3}}dx}{3a}-\frac{2\sqrt{ax^3+bx^4}}{3ax^3}\right)}{5a}-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}\right)}{7a}-\frac{2\sqrt{ax^3+bx^4}}{7ax^5}$$

$$\Big\downarrow \text{1906}$$

$$-\frac{6b\left(-\frac{4b\left(\frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2}-\frac{2\sqrt{ax^3+bx^4}}{3ax^3}\right)}{5a}-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}\right)}{7a}-\frac{2\sqrt{ax^3+bx^4}}{7ax^5}$$

input
```
Int[1/(x^3*Sqrt[a*x^3 + b*x^4]),x]
```

output
```
(-2*Sqrt[a*x^3 + b*x^4])/(7*a*x^5) - (6*b*((-2*Sqrt[a*x^3 + b*x^4])/(5*a*x
^4) - (4*b*((-2*Sqrt[a*x^3 + b*x^4])/(3*a*x^3) + (4*b*Sqrt[a*x^3 + b*x^4])
/(3*a^2*x^2)))/(5*a)))/(7*a)
```

Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.46

method	result	size
pseudoelliptic	$-\frac{2\left(-16 b^{3} x^{3}+8 a\, b^{2} x^{2}-6 a^{2} b x +5 a^{3}\right) \sqrt{x^{3} \left(b x +a \right)}}{35 a^{4} x^{5}}$	50
trager	$-\frac{2\left(-16 b^{3} x^{3}+8 a\, b^{2} x^{2}-6 a^{2} b x +5 a^{3}\right) \sqrt{b\,x^{4}+a\,x^{3}}}{35 a^{4} x^{5}}$	52
risch	$-\frac{2\left(b x +a \right) \left(-16 b^{3} x^{3}+8 a\, b^{2} x^{2}-6 a^{2} b x +5 a^{3}\right)}{35 x^{2} \sqrt{x^{3} \left(b x +a \right)}\, a^{4}}$	55
gosper	$-\frac{2\left(b x +a \right) \left(-16 b^{3} x^{3}+8 a\, b^{2} x^{2}-6 a^{2} b x +5 a^{3}\right)}{35 x^{2} a^{4} \sqrt{b\,x^{4}+a\,x^{3}}}$	57
orering	$-\frac{2\left(b x +a \right) \left(-16 b^{3} x^{3}+8 a\, b^{2} x^{2}-6 a^{2} b x +5 a^{3}\right)}{35 x^{2} a^{4} \sqrt{b\,x^{4}+a\,x^{3}}}$	57
default	$-\frac{2 \sqrt{x \left(b x +a \right)}\, \sqrt{b\,x^{2}+a x}\, \left(-16 b^{3} x^{3}+8 a\, b^{2} x^{2}-6 a^{2} b x +5 a^{3}\right)}{35 x^{3} \sqrt{b\,x^{4}+a\,x^{3}}\, a^{4}}$	72

input
```
int(1/x^3/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*(-16*b^3*x^3+8*a*b^2*x^2-6*a^2*b*x+5*a^3)/a^4/x^5*(x^3*(b*x+a))^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx = \frac{2\,(16\,b^3x^3-8\,ab^2x^2+6\,a^2bx-5\,a^3)\sqrt{bx^4+ax^3}}{35\,a^4x^5}$$

input
```
integrate(1/x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

output
```
2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x^4 + a*x^3)/(a
^4*x^5)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx = \int \frac{1}{x^3\sqrt{x^3\,(a+bx)}}\,dx$$

input
```
integrate(1/x**3/(b*x**4+a*x**3)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(x**3*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+ax^3}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^4 + a*x^3)*x^3), x)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx = \frac{2\left(70\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3 b^{\frac{3}{2}}+84\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2 ab+35\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)a^2\sqrt{b}+5\,a^3\right)}{35\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^7 \operatorname{sgn}(x)}$$

input
```
integrate(1/x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

output
```
2/35*(70*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*b^(3/2) + 84*(sqrt(b)*x - sqrt(
b*x^2 + a*x))^2*a*b + 35*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^2*sqrt(b) + 5*a
^3)/((sqrt(b)*x - sqrt(b*x^2 + a*x))^7*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx = \frac{12\,b\,\sqrt{b\,x^4+a\,x^3}}{35\,a^2\,x^4}-\frac{2\,\sqrt{b\,x^4+a\,x^3}}{7\,a\,x^5} - \frac{16\,b^2\,\sqrt{b\,x^4+a\,x^3}}{35\,a^3\,x^3}+\frac{32\,b^3\,\sqrt{b\,x^4+a\,x^3}}{35\,a^4\,x^2}$$

input
```
int(1/(x^3*(a*x^3 + b*x^4)^(1/2)),x)
```

output
```
(12*b*(a*x^3 + b*x^4)^(1/2))/(35*a^2*x^4) - (2*(a*x^3 + b*x^4)^(1/2))/(7*a
*x^5) - (16*b^2*(a*x^3 + b*x^4)^(1/2))/(35*a^3*x^3) + (32*b^3*(a*x^3 + b*x
^4)^(1/2))/(35*a^4*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3}{7}+\frac{12\sqrt{x}\,\sqrt{bx+a}\,a^2bx}{35}-\frac{16\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x^2}{35}+\frac{32\sqrt{x}\,\sqrt{bx+a}\,b^3x^3}{35}-\frac{32\sqrt{b}\,b^3x^4}{35}}{a^4x^4}$$

input
```
int(1/x^3/(b*x^4+a*x^3)^(1/2),x)
```

output
```
(2*( - 5*sqrt(x)*sqrt(a + b*x)*a**3 + 6*sqrt(x)*sqrt(a + b*x)*a**2*b*x - 8
*sqrt(x)*sqrt(a + b*x)*a*b**2*x**2 + 16*sqrt(x)*sqrt(a + b*x)*b**3*x**3 -
16*sqrt(b)*b**3*x**4))/(35*a**4*x**4)
```

3.297 $\int \frac{1}{x^3+bx^5} dx$

Optimal result 2478
Mathematica [A] (verified) 2478
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Sympy [A] (verification not implemented) 2481
Maxima [A] (verification not implemented) 2481
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Mupad [B] (verification not implemented) 2482
Reduce [B] (verification not implemented) 2482

Optimal result

Integrand size = 11, antiderivative size = 26

$$\int \frac{1}{x^3+bx^5} dx = -\frac{1}{2x^2} - b\log(x) + \frac{1}{2}b\log\left(1+bx^2\right)$$

output
```
-1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3+bx^5} dx = -\frac{1}{2x^2} - b\log(x) + \frac{1}{2}b\log\left(1+bx^2\right)$$

input
```
Integrate[(x^3 + b*x^5)^(-1),x]
```

output
```
-1/2*1/x^2 - b*Log[x] + (b*Log[1 + b*x^2])/2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2026, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{bx^5+x^3}\,dx$$
$$\Big\downarrow \text{ 2026}$$
$$\int \frac{1}{x^3\left(bx^2+1\right)}dx$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2}\int \frac{1}{x^4\left(bx^2+1\right)}dx^2$$
$$\Big\downarrow \text{ 54}$$
$$\frac{1}{2}\int \left(\frac{b^2}{bx^2+1}-\frac{b}{x^2}+\frac{1}{x^4}\right)dx^2$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2}\left(-b\log\left(x^2\right)+b\log\left(bx^2+1\right)-\frac{1}{x^2}\right)$$

input
```
Int[(x^3 + b*x^5)^(-1),x]
```

output
```
(-x^(-2) - b*Log[x^2] + b*Log[1 + b*x^2])/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{1}{2x^{2}}-b\ln\left(x\right)+\frac{b\ln\left(b\,x^{2}+1\right)}{2}$	23
norman	$-\frac{1}{2x^{2}}-b\ln\left(x\right)+\frac{b\ln\left(b\,x^{2}+1\right)}{2}$	23
risch	$-\frac{1}{2x^{2}}-b\ln\left(x\right)+\frac{b\ln\left(-b\,x^{2}-1\right)}{2}$	24
meijerg	$\frac{b\left(\ln\left(b\,x^{2}+1\right)-2\ln\left(x\right)-\ln\left(b\right)-\frac{1}{b\,x^{2}}\right)}{2}$	29
parallelrisch	$-\frac{2b\ln\left(x\right)x^{2}-b\ln\left(b\,x^{2}+1\right)x^{2}+1}{2x^{2}}$	30

input
```
int(1/(b*x^5+x^3),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3 + bx^5} \, dx = \frac{bx^2 \log(bx^2 + 1) - 2\, bx^2 \log(x) - 1}{2\, x^2}$$

input
```
integrate(1/(b*x^5+x^3),x, algorithm="fricas")
```

output
```
1/2*(b*x^2*log(b*x^2 + 1) - 2*b*x^2*log(x) - 1)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3 + bx^5} \, dx = -b \log(x) + \frac{b \log\left(x^2 + \frac{1}{b}\right)}{2} - \frac{1}{2x^2}$$

input
```
integrate(1/(b*x**5+x**3),x)
```

output
```
-b*log(x) + b*log(x**2 + 1/b)/2 - 1/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3 + bx^5} \, dx = \frac{1}{2}\, b \log\left(bx^2 + 1\right) - b \log(x) - \frac{1}{2\, x^2}$$

input
```
integrate(1/(b*x^5+x^3),x, algorithm="maxima")
```

output
```
1/2*b*log(b*x^2 + 1) - b*log(x) - 1/2/x^2
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^3+bx^5}\,dx = -\frac{1}{2}\,b\log\left(x^2\right)+\frac{1}{2}\,b\log\left(\left|bx^2+1\right|\right)+\frac{bx^2-1}{2\,x^2}$$

input
```
integrate(1/(b*x^5+x^3),x, algorithm="giac")
```

output
```
-1/2*b*log(x^2) + 1/2*b*log(abs(b*x^2 + 1)) + 1/2*(b*x^2 - 1)/x^2
```

Mupad [B] (verification not implemented)

Time = 8.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3+bx^5}\,dx = \frac{b\,\ln\left(b\,x^2+1\right)}{2}-b\,\ln\left(x\right)-\frac{1}{2\,x^2}$$

input
```
int(1/(b*x^5 + x^3),x)
```

output
```
(b*log(b*x^2 + 1))/2 - b*log(x) - 1/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3+bx^5}\,dx = \frac{\log(b\,x^2+1)\,b\,x^2-2\log(x)\,b\,x^2-1}{2x^2}$$

input
```
int(1/(b*x^5+x^3),x)
```

output
```
(log(b*x**2 + 1)*b*x**2 - 2*log(x)*b*x**2 - 1)/(2*x**2)
```

3.298 $\int \frac{1}{-x^3+bx^5} dx$

Optimal result 2483
Mathematica [A] (verified) 2483
Rubi [A] (verified) 2484
Maple [A] (verified) 2485
Fricas [A] (verification not implemented) 2486
Sympy [A] (verification not implemented) 2486
Maxima [A] (verification not implemented) 2486
Giac [A] (verification not implemented) 2487
Mupad [B] (verification not implemented) 2487
Reduce [B] (verification not implemented) 2487

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{1}{-x^3+bx^5} dx = \frac{1}{2x^2} - b\log(x) + \frac{1}{2}b\log\left(1-bx^2\right)$$

output
```
1/2/x^2-b*ln(x)+1/2*b*ln(-b*x^2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{-x^3+bx^5} dx = \frac{1}{2x^2} - b\log(x) + \frac{1}{2}b\log\left(1-bx^2\right)$$

input
```
Integrate[(-x^3 + b*x^5)^(-1),x]
```

output
```
1/(2*x^2) - b*Log[x] + (b*Log[1 - b*x^2])/2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2026, 243, 25, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{bx^5 - x^3} \, dx$$
$$\Big\downarrow \text{2026}$$
$$\int \frac{1}{x^3 (bx^2 - 1)} dx$$
$$\Big\downarrow \text{243}$$
$$\frac{1}{2} \int -\frac{1}{x^4 (1 - bx^2)} dx^2$$
$$\Big\downarrow \text{25}$$
$$-\frac{1}{2} \int \frac{1}{x^4 (1 - bx^2)} dx^2$$
$$\Big\downarrow \text{54}$$
$$-\frac{1}{2} \int \left(-\frac{b^2}{bx^2 - 1} + \frac{b}{x^2} + \frac{1}{x^4} \right) dx^2$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2} \left(-b \log (x^2) + b \log (1 - bx^2) + \frac{1}{x^2} \right)$$

input
```
Int[(-x^3 + b*x^5)^(-1),x]
```

output
```
(x^(-2) - b*Log[x^2] + b*Log[1 - b*x^2])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{b\ln\left(b\,x^{2}-1\right)}{2}+\frac{1}{2x^{2}}-b\ln\left(x\right)$	23
norman	$\frac{b\ln\left(b\,x^{2}-1\right)}{2}+\frac{1}{2x^{2}}-b\ln\left(x\right)$	23
risch	$\frac{1}{2x^{2}}-b\ln\left(x\right)+\frac{b\ln\left(-b\,x^{2}+1\right)}{2}$	24
parallelrisch	$-\frac{2b\ln\left(x\right)x^{2}-b\ln\left(b\,x^{2}-1\right)x^{2}-1}{2x^{2}}$	30
meijerg	$\frac{b\left(\ln\left(-b\,x^{2}+1\right)-2\ln\left(x\right)-\ln\left(-b\right)+\frac{1}{b\,x^{2}}\right)}{2}$	31

input
```
int(1/(b*x^5-x^3),x,method=_RETURNVERBOSE)
```

output
```
1/2*b*ln(b*x^2-1)+1/2/x^2-b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{1}{-x^3+bx^5}\,dx=\frac{bx^2\log\left(bx^2-1\right)-2\,bx^2\log\left(x\right)+1}{2\,x^2}$$

input
```
integrate(1/(b*x^5-x^3),x, algorithm="fricas")
```

output
```
1/2*(b*x^2*log(b*x^2 - 1) - 2*b*x^2*log(x) + 1)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{-x^3+bx^5}\,dx=-b\log\left(x\right)+\frac{b\log\left(x^2-\frac{1}{b}\right)}{2}+\frac{1}{2x^2}$$

input
```
integrate(1/(b*x**5-x**3),x)
```

output
```
-b*log(x) + b*log(x**2 - 1/b)/2 + 1/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{-x^3+bx^5}\,dx=\frac{1}{2}\,b\log\left(bx^2-1\right)-b\log\left(x\right)+\frac{1}{2\,x^2}$$

input
```
integrate(1/(b*x^5-x^3),x, algorithm="maxima")
```

output
```
1/2*b*log(b*x^2 - 1) - b*log(x) + 1/2/x^2
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{1}{-x^3+bx^5}\,dx = -\frac{1}{2}\,b\log\left(x^2\right)+\frac{1}{2}\,b\log\left(\left|bx^2-1\right|\right)+\frac{bx^2+1}{2\,x^2}$$

input
```
integrate(1/(b*x^5-x^3),x, algorithm="giac")
```

output
```
-1/2*b*log(x^2) + 1/2*b*log(abs(b*x^2 - 1)) + 1/2*(b*x^2 + 1)/x^2
```

Mupad [B] (verification not implemented)

Time = 8.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{-x^3+bx^5}\,dx = \frac{b\,\ln\left(b\,x^2-1\right)}{2}-b\,\ln\left(x\right)+\frac{1}{2\,x^2}$$

input
```
int(1/(b*x^5 - x^3),x)
```

output
```
(b*log(b*x^2 - 1))/2 - b*log(x) + 1/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{1}{-x^3+bx^5}\,dx = \frac{\log\left(-\sqrt{b}+bx\right)b\,x^2+\log\left(\sqrt{b}+bx\right)b\,x^2-2\log(x)\,b\,x^2+1}{2x^2}$$

input
```
int(1/(b*x^5-x^3),x)
```

output
```
(log( - sqrt(b) + b*x)*b*x**2 + log(sqrt(b) + b*x)*b*x**2 - 2*log(x)*b*x**
2 + 1)/(2*x**2)
```

3.299 $\int \frac{1}{ax+bx} \, dx$

Optimal result . . . 2488
Mathematica [A] (verified) . . . 2488
Rubi [A] (verified) . . . 2489
Maple [A] (verified) . . . 2490
Fricas [A] (verification not implemented) . . . 2490
Sympy [A] (verification not implemented) . . . 2490
Maxima [A] (verification not implemented) . . . 2491
Giac [A] (verification not implemented) . . . 2491
Mupad [B] (verification not implemented) . . . 2491
Reduce [B] (verification not implemented) . . . 2492

Optimal result

Integrand size = 9, antiderivative size = 8

$$\int \frac{1}{ax+bx} \, dx = \frac{\log(x)}{a+b}$$

output
```
ln(x)/(a+b)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1}{ax+bx} \, dx = \frac{\log(ax+bx)}{a+b}$$

input
```
Integrate[(a*x + b*x)^(-1),x]
```

output
```
Log[a*x + b*x]/(a + b)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax+bx}\,dx$$
$$\Big\downarrow \text{ 6}$$
$$\int \frac{1}{x(a+b)}dx$$
$$\Big\downarrow \text{ 14}$$
$$\frac{\log(x)}{a+b}$$

input
```
Int[(a*x + b*x)^(-1),x]
```

output
```
Log[x]/(a + b)
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{\ln(x)}{b+a}$	9
norman	$\frac{\ln(x)}{b+a}$	9
risch	$\frac{\ln(x)}{b+a}$	9
parallelrisch	$\frac{\ln(x)}{b+a}$	9

input
```
int(1/(a*x+b*x),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/(b+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax+bx}\,dx = \frac{\log{(x)}}{a+b}$$

input
```
integrate(1/(a*x+b*x),x, algorithm="fricas")
```

output
```
log(x)/(a + b)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1}{ax+bx}\,dx = \frac{\log{(x)}}{a+b}$$

input
```
integrate(1/(a*x+b*x),x)
```

output
```
log(x)/(a + b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1}{ax+bx}\,dx = \frac{\log(ax+bx)}{a+b}$$

input
```
integrate(1/(a*x+b*x),x, algorithm="maxima")
```

output
```
log(a*x + b*x)/(a + b)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{1}{ax+bx}\,dx = \frac{\log(|ax+bx|)}{a+b}$$

input
```
integrate(1/(a*x+b*x),x, algorithm="giac")
```

output
```
log(abs(a*x + b*x))/(a + b)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax+bx}\,dx = \frac{\ln(x)}{a+b}$$

input
```
int(1/(a*x + b*x),x)
```

output
```
log(x)/(a + b)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax+bx}\,dx = \frac{\log(x)}{a+b}$$

input
```
int(1/(a*x+b*x),x)
```

output
```
log(x)/(a + b)
```

3.300 $\int \frac{1}{(ax+bx)^2}\,dx$

Optimal result . . . 2493
Mathematica [A] (verified) . . . 2493
Rubi [A] (verified) . . . 2494
Maple [A] (verified) . . . 2495
Fricas [A] (verification not implemented) . . . 2495
Sympy [A] (verification not implemented) . . . 2496
Maxima [A] (verification not implemented) . . . 2496
Giac [A] (verification not implemented) . . . 2496
Mupad [B] (verification not implemented) . . . 2497
Reduce [B] (verification not implemented) . . . 2497

Optimal result

Integrand size = 9, antiderivative size = 10

$$\int \frac{1}{(ax+bx)^2}\,dx = -\frac{1}{(a+b)^2x}$$

output
```
-1/(a+b)^2/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ax+bx)^2}\,dx = -\frac{1}{(a+b)^2x}$$

input
```
Integrate[(a*x + b*x)^(-2),x]
```

output
```
-(1/((a + b)^2*x))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax+bx)^2} dx$$

$$\Big\downarrow \; 6$$

$$\int \frac{1}{x^2(a+b)^2} dx$$

$$\Big\downarrow \; 15$$

$$-\frac{1}{x(a+b)^2}$$

input
```
Int[(a*x + b*x)^(-2),x]
```

output
```
-(1/((a + b)^2*x))
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
 + (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
gosper	$-\frac{1}{(b+a)^2 x}$	11
default	$-\frac{1}{(b+a)^2 x}$	11
norman	$-\frac{1}{(b+a)^2 x}$	11
risch	$-\frac{1}{(b+a)^2 x}$	11
parallelrisch	$-\frac{1}{(b+a)^2 x}$	11
orering	$-\frac{x}{(ax+bx)^2}$	13

input
```
int(1/(a*x+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(b+a)^2/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{1}{(ax+bx)^2}\,dx = -\frac{1}{(a^2+2\,ab+b^2)x}$$

input
```
integrate(1/(a*x+b*x)^2,x, algorithm="fricas")
```

output
```
-1/((a^2 + 2*a*b + b^2)*x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{1}{(ax+bx)^2}\,dx = -\frac{1}{x\,(a^2+2ab+b^2)}$$

input
```
integrate(1/(a*x+b*x)**2,x)
```

output
```
-1/(x*(a**2 + 2*a*b + b**2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{(ax+bx)^2}\,dx = -\frac{1}{(ax+bx)(a+b)}$$

input
```
integrate(1/(a*x+b*x)^2,x, algorithm="maxima")
```

output
```
-1/((a*x + b*x)*(a + b))
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{(ax+bx)^2}\,dx = -\frac{1}{(ax+bx)(a+b)}$$

input
```
integrate(1/(a*x+b*x)^2,x, algorithm="giac")
```

output
```
-1/((a*x + b*x)*(a + b))
```

Mupad [B] (verification not implemented)

Time = 8.77 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ax+bx)^2}\,dx = -\frac{1}{x\,(a+b)^2}$$

input
```
int(1/(a*x + b*x)^2,x)
```

output
```
-1/(x*(a + b)^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{1}{(ax+bx)^2}\,dx = -\frac{1}{x\,(a^2+2ab+b^2)}$$

input
```
int(1/(a*x+b*x)^2,x)
```

output
```
( - 1)/(x*(a**2 + 2*a*b + b**2))
```

3.301 $\int \frac{1}{(ax+bx)^3} dx$

Optimal result 2498
Mathematica [A] (verified) 2498
Rubi [A] (verified) 2499
Maple [A] (verified) 2500
Fricas [B] (verification not implemented) 2500
Sympy [B] (verification not implemented) 2501
Maxima [A] (verification not implemented) 2501
Giac [A] (verification not implemented) 2501
Mupad [B] (verification not implemented) 2502
Reduce [B] (verification not implemented) 2502

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{1}{(ax+bx)^3} dx = -\frac{1}{2(a+b)^3x^2}$$

output
```
-1/2/(a+b)^3/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ax+bx)^3} dx = -\frac{1}{2(a+b)^3x^2}$$

input
```
Integrate[(a*x + b*x)^(-3),x]
```

output
```
-1/2*1/((a + b)^3*x^2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax+bx)^3}\,dx$$

$$\Big\downarrow \; 6$$

$$\int \frac{1}{x^3(a+b)^3}dx$$

$$\Big\downarrow \; 15$$

$$-\frac{1}{2x^2(a+b)^3}$$

input
```
Int[(a*x + b*x)^(-3),x]
```

output
```
-1/2*1/((a + b)^3*x^2)
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[Fx, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$-\frac{1}{2(b+a)^{3} x^{2}}$	11
default	$-\frac{1}{2(b+a)^{3} x^{2}}$	11
norman	$-\frac{1}{2(b+a)^{3} x^{2}}$	11
risch	$-\frac{1}{2(b+a)^{3} x^{2}}$	11
parallelrisch	$-\frac{1}{2(b+a)^{3} x^{2}}$	11
orering	$-\frac{x}{2(ax+bx)^{3}}$	13

input
```
int(1/(a*x+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(b+a)^3/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(10) = 20.

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{1}{(ax+bx)^3}\,dx = -\frac{1}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)x^2}$$

input
```
integrate(1/(a*x+b*x)^3,x, algorithm="fricas")
```

output
```
-1/2/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(12) = 24$.

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.25

$$\int \frac{1}{(ax+bx)^3}\,dx = -\frac{1}{2x^2\,(a^3+3a^2b+3ab^2+b^3)}$$

input
```
integrate(1/(a*x+b*x)**3,x)
```

output
```
-1/(2*x**2*(a**3 + 3*a**2*b + 3*a*b**2 + b**3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{1}{(ax+bx)^3}\,dx = -\frac{1}{2\,(ax+bx)^2(a+b)}$$

input
```
integrate(1/(a*x+b*x)^3,x, algorithm="maxima")
```

output
```
-1/2/((a*x + b*x)^2*(a + b))
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{1}{(ax+bx)^3}\,dx = -\frac{1}{2\,(ax+bx)^2(a+b)}$$

input
```
integrate(1/(a*x+b*x)^3,x, algorithm="giac")
```

output
```
-1/2/((a*x + b*x)^2*(a + b))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{1}{(ax+bx)^3}\,dx = -\frac{1}{2\,x^2\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}$$

input
```
int(1/(a*x + b*x)^3,x)
```

output
```
-1/(2*x^2*(3*a*b^2 + 3*a^2*b + a^3 + b^3))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{1}{(ax+bx)^3}\,dx = -\frac{1}{2x^2\,(a^3+3a^2b+3a\,b^2+b^3)}$$

input
```
int(1/(a*x+b*x)^3,x)
```

output
```
( - 1)/(2*x**2*(a**3 + 3*a**2*b + 3*a*b**2 + b**3))
```

3.302 $\int \frac{1}{ax^2+bx^2}\,dx$

Optimal result . . . 2503
Mathematica [A] (verified) . . . 2503
Rubi [A] (verified) . . . 2504
Maple [A] (verified) . . . 2505
Fricas [A] (verification not implemented) . . . 2505
Sympy [A] (verification not implemented) . . . 2506
Maxima [A] (verification not implemented) . . . 2506
Giac [A] (verification not implemented) . . . 2506
Mupad [B] (verification not implemented) . . . 2507
Reduce [B] (verification not implemented) . . . 2507

Optimal result

Integrand size = 13, antiderivative size = 10

$$\int \frac{1}{ax^2+bx^2}\,dx=-\frac{1}{(a+b)x}$$

output
```
-1/(a+b)/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^2+bx^2}\,dx=-\frac{1}{(a+b)x}$$

input
```
Integrate[(a*x^2 + b*x^2)^(-1),x]
```

output
```
-(1/((a + b)*x))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax^2+bx^2} \, dx$$
$$\Big\downarrow \; 6$$
$$\int \frac{1}{x^2(a+b)}dx$$
$$\Big\downarrow \; 15$$
$$-\frac{1}{x(a+b)}$$

input
```
Int[(a*x^2 + b*x^2)^(-1),x]
```

output
```
-(1/((a + b)*x))
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[Fx, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
gosper	$-\frac{1}{(b+a)x}$	11
default	$-\frac{1}{(b+a)x}$	11
norman	$-\frac{1}{(b+a)x}$	11
risch	$-\frac{1}{(b+a)x}$	11
parallelrisch	$-\frac{1}{(b+a)x}$	11
orering	$-\frac{x}{a\,x^{2}+b\,x^{2}}$	17

input
```
int(1/(a*x^2+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/(b+a)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^2+bx^2}\,dx=-\frac{1}{(a+b)x}$$

input
```
integrate(1/(a*x^2+b*x^2),x, algorithm="fricas")
```

output
```
-1/((a + b)*x)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{ax^2+bx^2}\,dx=-\frac{1}{x\,(a+b)}$$

input
```
integrate(1/(a*x**2+b*x**2),x)
```

output
```
-1/(x*(a + b))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^2+bx^2}\,dx=-\frac{1}{(a+b)x}$$

input
```
integrate(1/(a*x^2+b*x^2),x, algorithm="maxima")
```

output
```
-1/((a + b)*x)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^2+bx^2}\,dx=-\frac{1}{(a+b)x}$$

input
```
integrate(1/(a*x^2+b*x^2),x, algorithm="giac")
```

output
```
-1/((a + b)*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^2 + bx^2} \, dx = -\frac{1}{x\,(a+b)}$$

input
```
int(1/(a*x^2 + b*x^2),x)
```

output
```
-1/(x*(a + b))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^2 + bx^2} \, dx = -\frac{1}{x\,(a+b)}$$

input
```
int(1/(a*x^2+b*x^2),x)
```

output
```
( - 1)/(x*(a + b))
```

3.303 $\int \frac{1}{ax^n+bx^n} dx$

Optimal result . . . 2508
Mathematica [A] (verified) . . . 2508
Rubi [A] (verified) . . . 2509
Maple [A] (verified) . . . 2510
Fricas [A] (verification not implemented) . . . 2510
Sympy [B] (verification not implemented) . . . 2511
Maxima [A] (verification not implemented) . . . 2511
Giac [F] . . . 2511
Mupad [B] (verification not implemented) . . . 2512
Reduce [B] (verification not implemented) . . . 2512

Optimal result

Integrand size = 13, antiderivative size = 20

$$\int \frac{1}{ax^n+bx^n} dx = \frac{x^{1-n}}{(a+b)(1-n)}$$

output
```
x^(1-n)/(a+b)/(1-n)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^n+bx^n} dx = \frac{x^{1-n}}{(a+b)(1-n)}$$

input
```
Integrate[(a*x^n + b*x^n)^(-1),x]
```

output
```
x^(1 - n)/((a + b)*(1 - n))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax^n + bx^n} \, dx$$

$$\Big\downarrow \; 6$$

$$\int \frac{x^{-n}}{a+b} dx$$

$$\Big\downarrow \; 15$$

$$\frac{x^{1-n}}{(1-n)(a+b)}$$

input
```
Int[(a*x^n + b*x^n)^(-1),x]
```

output
```
x^(1 - n)/((a + b)*(1 - n))
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
gosper	$-\frac{x\,x^{-n}}{(-1+n)(b+a)}$	19
risch	$-\frac{x\,x^{-n}}{(-1+n)(b+a)}$	19
parallelrisch	$-\frac{x\,x^{-n}}{(-1+n)(b+a)}$	19
orering	$-\frac{x}{(-1+n)(a\,x^{n}+b\,x^{n})}$	22
norman	$-\frac{x\,\mathrm{e}^{-n\ln(x)}}{an+bn-a-b}$	26

input
```
int(1/(a*x^n+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
-x/(-1+n)/(x^n)/(b+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{ax^n+bx^n}\,dx=-\frac{x}{((a+b)n-a-b)x^n}$$

input
```
integrate(1/(a*x^n+b*x^n),x, algorithm="fricas")
```

output
```
-x/(((a + b)*n - a - b)*x^n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(10) = 20$.

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{1}{ax^n + bx^n} \, dx = \begin{cases} -\frac{x}{anx^n - ax^n + bnx^n - bx^n} & \text{for } n \neq 1 \\ \frac{\log(x)}{a+b} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x**n+b*x**n),x)
```

output
```
Piecewise((-x/(a*n*x**n - a*x**n + b*n*x**n - b*x**n), Ne(n, 1)), (log(x)/
(a + b), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{1}{ax^n + bx^n} \, dx = -\frac{x}{(a(n-1) + b(n-1))x^n}$$

input
```
integrate(1/(a*x^n+b*x^n),x, algorithm="maxima")
```

output
```
-x/((a*(n - 1) + b*(n - 1))*x^n)
```

Giac [F]

$$\int \frac{1}{ax^n + bx^n} \, dx = \int \frac{1}{ax^n + bx^n} \, dx$$

input
```
integrate(1/(a*x^n+b*x^n),x, algorithm="giac")
```

output
```
integrate(1/(a*x^n + b*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 8.79 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{ax^n + bx^n} \, dx = -\frac{x^{1-n}}{(a+b)\ (n-1)}$$

input
```
int(1/(a*x^n + b*x^n),x)
```

output
```
-x^(1 - n)/((a + b)*(n - 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{1}{ax^n + bx^n} \, dx = -\frac{x}{x^n\,(an + bn - a - b)}$$

input
```
int(1/(a*x^n+b*x^n),x)
```

output
```
( - x)/(x**n*(a*n - a + b*n - b))
```

3.304 $\int \frac{1}{(ax^n+bx^n)^2}\,dx$

Optimal result 2513
Mathematica [A] (verified) 2513
Rubi [A] (verified) 2514
Maple [A] (verified) 2515
Fricas [A] (verification not implemented) 2515
Sympy [B] (verification not implemented) 2516
Maxima [A] (verification not implemented) 2516
Giac [F] 2516
Mupad [B] (verification not implemented) 2517
Reduce [B] (verification not implemented) 2517

Optimal result

Integrand size = 13, antiderivative size = 20

$$\int \frac{1}{(ax^n+bx^n)^2}\,dx = \frac{x^{1-2n}}{(a+b)^2(1-2n)}$$

output
```
x^(1-2*n)/(a+b)^2/(1-2*n)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ax^n+bx^n)^2}\,dx = \frac{x^{1-2n}}{(a+b)^2(1-2n)}$$

input
```
Integrate[(a*x^n + b*x^n)^(-2),x]
```

output
```
x^(1 - 2*n)/((a + b)^2*(1 - 2*n))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^n+bx^n)^2}\,dx$$
$$\Big\downarrow \ 6$$
$$\int \frac{x^{-2n}}{(a+b)^2}dx$$
$$\Big\downarrow \ 15$$
$$\frac{x^{1-2n}}{(1-2n)(a+b)^2}$$

input
```
Int[(a*x^n + b*x^n)^(-2),x]
```

output
```
x^(1 - 2*n)/((a + b)^2*(1 - 2*n))
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
gosper	$-\frac{x\,x^{-2n}}{(-1+2n)(b+a)^2}$	21
parallelrisch	$-\frac{x\,x^{-2n}}{(-1+2n)(b+a)^2}$	21
orering	$-\frac{x}{(-1+2n)(a\,x^n+b\,x^n)^2}$	24
risch	$-\frac{x\,x^{-2n}}{(a^2+2ab+b^2)(-1+2n)}$	29
norman	$-\frac{x\,\mathrm{e}^{-2n\ln(x)}}{(2an+2bn-a-b)(b+a)}$	33

input
```
int(1/(a*x^n+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
-x/(-1+2*n)/(x^n)^2/(b+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{1}{(ax^n+bx^n)^2}\,dx = \frac{x}{(a^2+2\,ab+b^2-2\,(a^2+2\,ab+b^2)n)x^{2\,n}}$$

input
```
integrate(1/(a*x^n+b*x^n)^2,x, algorithm="fricas")
```

output
```
x/((a^2 + 2*a*b + b^2 - 2*(a^2 + 2*a*b + b^2)*n)*x^(2*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(15) = 30$.

Time = 0.36 (sec) , antiderivative size = 82, normalized size of antiderivative = 4.10

$$\int \frac{1}{(ax^n + bx^n)^2}\,dx = \begin{cases} -\frac{x}{2a^2nx^{2n} - a^2x^{2n} + 4abnx^{2n} - 2abx^{2n} + 2b^2nx^{2n} - b^2x^{2n}} & \text{for } n \neq \frac{1}{2} \\ \frac{\log(x)}{a^2 + 2ab + b^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x**n+b*x**n)**2,x)
```

output
```
Piecewise((-x/(2*a**2*n*x**(2*n) - a**2*x**(2*n) + 4*a*b*n*x**(2*n) - 2*a*
b*x**(2*n) + 2*b**2*n*x**(2*n) - b**2*x**(2*n)), Ne(n, 1/2)), (log(x)/(a**
2 + 2*a*b + b**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int \frac{1}{(ax^n + bx^n)^2}\,dx = -\frac{x}{(a^2(2n-1) + 2ab(2n-1) + b^2(2n-1))x^{2n}}$$

input
```
integrate(1/(a*x^n+b*x^n)^2,x, algorithm="maxima")
```

output
```
-x/((a^2*(2*n - 1) + 2*a*b*(2*n - 1) + b^2*(2*n - 1))*x^(2*n))
```

Giac [F]

$$\int \frac{1}{(ax^n + bx^n)^2}\,dx = \int \frac{1}{(ax^n + bx^n)^2}\,dx$$

input
```
integrate(1/(a*x^n+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((a*x^n + b*x^n)^(-2), x)
```

Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{1}{(ax^n+bx^n)^2}\,dx=-\frac{x^{1-2\,n}}{(a+b)^2\,(2\,n-1)}$$

input
```
int(1/(a*x^n + b*x^n)^2,x)
```

output
```
-x^(1 - 2*n)/((a + b)^2*(2*n - 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{1}{(ax^n+bx^n)^2}\,dx=-\frac{x}{x^{2n}\,(2a^2n+4abn+2b^2n-a^2-2ab-b^2)}$$

input
```
int(1/(a*x^n+b*x^n)^2,x)
```

output
```
( - x)/(x**(2*n)*(2*a**2*n - a**2 + 4*a*b*n - 2*a*b + 2*b**2*n - b**2))
```

3.305 $\int \frac{1}{(ax^n+bx^n)^3} dx$

Optimal result 2518
Mathematica [A] (verified) 2518
Rubi [A] (verified) 2519
Maple [A] (verified) 2520
Fricas [B] (verification not implemented) 2520
Sympy [B] (verification not implemented) 2521
Maxima [B] (verification not implemented) 2521
Giac [F] 2522
Mupad [B] (verification not implemented) 2522
Reduce [B] (verification not implemented) 2522

Optimal result

Integrand size = 13, antiderivative size = 20

$$\int \frac{1}{(ax^n+bx^n)^3} dx = \frac{x^{1-3n}}{(a+b)^3(1-3n)}$$

output
```
x^(1-3*n)/(a+b)^3/(1-3*n)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ax^n+bx^n)^3} dx = \frac{x^{1-3n}}{(a+b)^3(1-3n)}$$

input
```
Integrate[(a*x^n + b*x^n)^(-3),x]
```

output
```
x^(1 - 3*n)/((a + b)^3*(1 - 3*n))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^n+bx^n)^3}dx$$
$$\Big\downarrow 6$$
$$\int \frac{x^{-3n}}{(a+b)^3}dx$$
$$\Big\downarrow 15$$
$$\frac{x^{1-3n}}{(1-3n)(a+b)^3}$$

input
```
Int[(a*x^n + b*x^n)^(-3),x]
```

output
```
x^(1 - 3*n)/((a + b)^3*(1 - 3*n))
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
gosper	$-\frac{x\,x^{-3n}}{(-1+3n)(b+a)^{3}}$	21
parallelrisch	$-\frac{x\,x^{-3n}}{(-1+3n)(b+a)^{3}}$	21
orering	$-\frac{x}{(-1+3n)(a\,x^{n}+b\,x^{n})^{3}}$	24
norman	$-\frac{x\,\mathrm{e}^{-3n\ln(x)}}{(3an+3bn-a-b)(b+a)^{2}}$	33
risch	$-\frac{x\,x^{-3n}}{(a^{3}+3a^{2}b+3a\,b^{2}+b^{3})(-1+3n)}$	37

input
```
int(1/(a*x^n+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
-x/(-1+3*n)/(x^n)^3/(b+a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(21) = 42.

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int \frac{1}{(ax^n+bx^n)^3}\,dx = \frac{x}{(a^3+3\,a^2b+3\,ab^2+b^3-3\,(a^3+3\,a^2b+3\,ab^2+b^3)n)x^{3\,n}}$$

input
```
integrate(1/(a*x^n+b*x^n)^3,x, algorithm="fricas")
```

output
```
x/((a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*n)*x
^(3*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(15) = 30$.

Time = 0.42 (sec) , antiderivative size = 119, normalized size of antiderivative = 5.95

$$\int \frac{1}{(ax^n + bx^n)^3} \, dx = \begin{cases} -\frac{x}{3a^3nx^{3n} - a^3x^{3n} + 9a^2bnx^{3n} - 3a^2bx^{3n} + 9ab^2nx^{3n} - 3ab^2x^{3n} + 3b^3nx^{3n} - b^3x^{3n}} & \text{for } n \neq \frac{1}{3} \\ \frac{\log(x)}{a^3 + 3a^2b + 3ab^2 + b^3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x**n+b*x**n)**3,x)
```

output
```
Piecewise((-x/(3*a**3*n*x**(3*n) - a**3*x**(3*n) + 9*a**2*b*n*x**(3*n) - 3
*a**2*b*x**(3*n) + 9*a*b**2*n*x**(3*n) - 3*a*b**2*x**(3*n) + 3*b**3*n*x**(
3*n) - b**3*x**(3*n)), Ne(n, 1/3)), (log(x)/(a**3 + 3*a**2*b + 3*a*b**2 +
b**3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(21) = 42$.

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.65

$$\int \frac{1}{(ax^n + bx^n)^3} \, dx = -\frac{x}{(a^3(3n-1) + 3a^2b(3n-1) + 3ab^2(3n-1) + b^3(3n-1))x^{3n}}$$

input
```
integrate(1/(a*x^n+b*x^n)^3,x, algorithm="maxima")
```

output
```
-x/((a^3*(3*n - 1) + 3*a^2*b*(3*n - 1) + 3*a*b^2*(3*n - 1) + b^3*(3*n - 1)
)*x^(3*n))
```

Giac [F]

$$\int \frac{1}{(ax^n+bx^n)^3}\,dx=\int \frac{1}{(ax^n+bx^n)^3}\,dx$$

input
```
integrate(1/(a*x^n+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((a*x^n + b*x^n)^(-3), x)
```

Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{1}{(ax^n+bx^n)^3}\,dx=-\frac{x^{1-3n}}{(a+b)^3\,(3n-1)}$$

input
```
int(1/(a*x^n + b*x^n)^3,x)
```

output
```
-x^(1 - 3*n)/((a + b)^3*(3*n - 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.05

$$\int \frac{1}{(ax^n+bx^n)^3}\,dx=-\frac{x}{x^{3n}\,(3a^3n+9a^2bn+9a\,b^2n+3b^3n-a^3-3a^2b-3a\,b^2-b^3)}$$

input
```
int(1/(a*x^n+b*x^n)^3,x)
```

output
```
( - x)/(x**(3*n)*(3*a**3*n - a**3 + 9*a**2*b*n - 3*a**2*b + 9*a*b**2*n - 3
*a*b**2 + 3*b**3*n - b**3))
```

3.306 $\int (ax+bx^{14})^{12}\,dx$

Optimal result . . . 2523
Mathematica [B] (verified) . . . 2523
Rubi [A] (verified) . . . 2524
Maple [B] (verified) . . . 2525
Fricas [B] (verification not implemented) . . . 2526
Sympy [B] (verification not implemented) . . . 2526
Maxima [B] (verification not implemented) . . . 2527
Giac [B] (verification not implemented) . . . 2527
Mupad [B] (verification not implemented) . . . 2528
Reduce [B] (verification not implemented) . . . 2528

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int (ax+bx^{14})^{12}\,dx=\frac{(a+bx^{13})^{13}}{169b}$$

output
```
1/169*(b*x^13+a)^13/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int (ax+bx^{14})^{12}\,dx=\frac{a^{12}x^{13}}{13}+\frac{6}{13}a^{11}bx^{26}+\frac{22}{13}a^{10}b^2x^{39}+\frac{55}{13}a^9b^3x^{52}+\frac{99}{13}a^8b^4x^{65}+\frac{132}{13}a^7b^5x^{78}+\frac{132}{13}a^6b^6x^{91}+\frac{99}{13}a^5b^7x^{104}+\frac{55}{13}a^4b^8x^{117}+\frac{22}{13}a^3b^9x^{130}+\frac{6}{13}a^2b^{10}x^{143}+\frac{1}{13}ab^{11}x^{156}+\frac{b^{12}x^{169}}{169}$$

input
```
Integrate[(a*x + b*x^14)^12,x]
```

output
```
(a^12*x^13)/13 + (6*a^11*b*x^26)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*
x^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^9
1)/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)
/13 + (6*a^2*b^10*x^143)/13 + (a*b^11*x^156)/13 + (b^12*x^169)/169
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2027, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^{14}\right)^{12} dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int x^{12}\left(a+bx^{13}\right)^{12} dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

input
```
Int[(a*x + b*x^14)^12,x]
```

output
```
(a + b*x^13)^13/(169*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.50 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{132}{13}a^7b^5x^{78}+\frac{55}{13}a^4b^8x^{117}+\frac{132}{13}a^6b^6x^{91}+\frac{1}{13}a\,b^{11}x^{156}+\frac{6}{13}a^{11}b\,x^{26}+\frac{22}{13}a^{10}b^2x^{39}+\frac{6}{13}a^2b^{10}x^{143}$
parallelrisch	$\frac{132}{13}a^7b^5x^{78}+\frac{55}{13}a^4b^8x^{117}+\frac{132}{13}a^6b^6x^{91}+\frac{1}{13}a\,b^{11}x^{156}+\frac{6}{13}a^{11}b\,x^{26}+\frac{22}{13}a^{10}b^2x^{39}+\frac{6}{13}a^2b^{10}x^{143}$
gosper	$\frac{x^{13}(b^{12}x^{156}+13a\,b^{11}x^{143}+78a^2b^{10}x^{130}+286a^3b^9x^{117}+715a^4b^8x^{104}+1287a^5b^7x^{91}+1716a^6b^6x^{78}+1716a^7b^5x^{65}+1287a^8b^4}{169}$
risch	$\frac{b^{12}x^{169}}{169}+\frac{a\,b^{11}x^{156}}{13}+\frac{6a^2b^{10}x^{143}}{13}+\frac{22a^3b^9x^{130}}{13}+\frac{55a^4b^8x^{117}}{13}+\frac{99a^5b^7x^{104}}{13}+\frac{132a^6b^6x^{91}}{13}+\frac{132a^7b^5x^{78}}{13}+$
orering	$\frac{x(b^{12}x^{156}+13a\,b^{11}x^{143}+78a^2b^{10}x^{130}+286a^3b^9x^{117}+715a^4b^8x^{104}+1287a^5b^7x^{91}+1716a^6b^6x^{78}+1716a^7b^5x^{65}+1287a^8b^4x}{169(b\,x^{13}+a)^{12}}$

input
```
int((b*x^14+a*x)^12,x,method=_RETURNVERBOSE)
```

output
```
132/13*a^7*b^5*x^78+55/13*a^4*b^8*x^117+132/13*a^6*b^6*x^91+1/13*a*b^11*x^
156+6/13*a^11*b*x^26+22/13*a^10*b^2*x^39+6/13*a^2*b^10*x^143+22/13*a^3*b^9
*x^130+1/169*b^12*x^169+99/13*a^5*b^7*x^104+99/13*a^8*b^4*x^65+55/13*a^9*b
^3*x^52+1/13*a^12*x^13
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int \left(ax+bx^{14}\right)^{12} dx = \frac{1}{169}b^{12}x^{169} + \frac{1}{13}ab^{11}x^{156} + \frac{6}{13}a^2b^{10}x^{143} + \frac{22}{13}a^3b^9x^{130} + \frac{55}{13}a^4b^8x^{117} + \frac{99}{13}a^5b^7x^{104} + \frac{132}{13}a^6b^6x^{91} + \frac{132}{13}a^7b^5x^{78} + \frac{99}{13}a^8b^4x^{65} + \frac{55}{13}a^9b^3x^{52} + \frac{22}{13}a^{10}b^2x^{39} + \frac{6}{13}a^{11}bx^{26} + \frac{1}{13}a^{12}x^{13}$$

input
```
integrate((b*x^14+a*x)^12,x, algorithm="fricas")
```

output
```
1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9
*x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 +
 132/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^1
0*b^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int \left(ax+bx^{14}\right)^{12} dx = \frac{a^{12}x^{13}}{13} + \frac{6a^{11}bx^{26}}{13} + \frac{22a^{10}b^2x^{39}}{13} + \frac{55a^9b^3x^{52}}{13} + \frac{99a^8b^4x^{65}}{13} + \frac{132a^7b^5x^{78}}{13} + \frac{132a^6b^6x^{91}}{13} + \frac{99a^5b^7x^{104}}{13} + \frac{55a^4b^8x^{117}}{13} + \frac{22a^3b^9x^{130}}{13} + \frac{6a^2b^{10}x^{143}}{13} + \frac{ab^{11}x^{156}}{13} + \frac{b^{12}x^{169}}{169}$$

input
```
integrate((b*x**14+a*x)**12,x)
```

output
```
a**12*x**13/13 + 6*a**11*b*x**26/13 + 22*a**10*b**2*x**39/13 + 55*a**9*b**
3*x**52/13 + 99*a**8*b**4*x**65/13 + 132*a**7*b**5*x**78/13 + 132*a**6*b**
6*x**91/13 + 99*a**5*b**7*x**104/13 + 55*a**4*b**8*x**117/13 + 22*a**3*b**
9*x**130/13 + 6*a**2*b**10*x**143/13 + a*b**11*x**156/13 + b**12*x**169/16
9
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(14) = 28.

Time = 0.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int (ax+bx^{14})^{12}\,dx = \frac{1}{169}\,b^{12}x^{169} + \frac{1}{13}\,ab^{11}x^{156} + \frac{6}{13}\,a^2b^{10}x^{143} + \frac{22}{13}\,a^3b^9x^{130} + \frac{55}{13}\,a^4b^8x^{117} + \frac{99}{13}\,a^5b^7x^{104} + \frac{132}{13}\,a^6b^6x^{91} + \frac{132}{13}\,a^7b^5x^{78} + \frac{99}{13}\,a^8b^4x^{65} + \frac{55}{13}\,a^9b^3x^{52} + \frac{22}{13}\,a^{10}b^2x^{39} + \frac{6}{13}\,a^{11}bx^{26} + \frac{1}{13}\,a^{12}x^{13}$$

input
```
integrate((b*x^14+a*x)^12,x, algorithm="maxima")
```

output
```
1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9
*x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 +
 132/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^1
0*b^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(14) = 28.

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int (ax+bx^{14})^{12}\,dx = \frac{1}{169}\,b^{12}x^{169} + \frac{1}{13}\,ab^{11}x^{156} + \frac{6}{13}\,a^2b^{10}x^{143} + \frac{22}{13}\,a^3b^9x^{130} + \frac{55}{13}\,a^4b^8x^{117} + \frac{99}{13}\,a^5b^7x^{104} + \frac{132}{13}\,a^6b^6x^{91} + \frac{132}{13}\,a^7b^5x^{78} + \frac{99}{13}\,a^8b^4x^{65} + \frac{55}{13}\,a^9b^3x^{52} + \frac{22}{13}\,a^{10}b^2x^{39} + \frac{6}{13}\,a^{11}bx^{26} + \frac{1}{13}\,a^{12}x^{13}$$

input
```
integrate((b*x^14+a*x)^12,x, algorithm="giac")
```

output
```
1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9
*x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 +
 132/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^1
0*b^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int \left(ax+bx^{14}\right)^{12}\,dx = \frac{a^{12}\,x^{13}}{13}+\frac{6\,a^{11}\,b\,x^{26}}{13}+\frac{22\,a^{10}\,b^2\,x^{39}}{13}+\frac{55\,a^9\,b^3\,x^{52}}{13}+\frac{99\,a^8\,b^4\,x^{65}}{13}+\frac{132\,a^7\,b^5\,x^{78}}{13}+\frac{132\,a^6\,b^6\,x^{91}}{13}+\frac{99\,a^5\,b^7\,x^{104}}{13}+\frac{55\,a^4\,b^8\,x^{117}}{13}+\frac{22\,a^3\,b^9\,x^{130}}{13}+\frac{6\,a^2\,b^{10}\,x^{143}}{13}+\frac{a\,b^{11}\,x^{156}}{13}+\frac{b^{12}\,x^{169}}{169}$$

input
```
int((a*x + b*x^14)^12,x)
```

output
```
(a^12*x^13)/13 + (b^12*x^169)/169 + (6*a^11*b*x^26)/13 + (a*b^11*x^156)/13
 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x^52)/13 + (99*a^8*b^4*x^65)/13 + (
132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)/13 + (99*a^5*b^7*x^104)/13 + (55
*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13 + (6*a^2*b^10*x^143)/13
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

$$\int \left(ax+bx^{14}\right)^{12}\,dx = \frac{x^{13}(b^{12}x^{156}+13a\,b^{11}x^{143}+78a^2b^{10}x^{130}+286a^3b^9x^{117}+715a^4b^8x^{104}+1287a^5b^7x^{91}+1716a^6b^6x^{78}+17}{169}$$

input
```
int((b*x^14+a*x)^12,x)
```

output

```
(x**13*(13*a**12 + 78*a**11*b*x**13 + 286*a**10*b**2*x**26 + 715*a**9*b**3
*x**39 + 1287*a**8*b**4*x**52 + 1716*a**7*b**5*x**65 + 1716*a**6*b**6*x**7
8 + 1287*a**5*b**7*x**91 + 715*a**4*b**8*x**104 + 286*a**3*b**9*x**117 + 7
8*a**2*b**10*x**130 + 13*a*b**11*x**143 + b**12*x**156))/169
```

3.307 $\int x^{12}(ax+bx^{26})^{12}\,dx$

Optimal result 2530
Mathematica [B] (verified) 2530
Rubi [A] (verified) 2531
Maple [B] (verified) 2532
Fricas [B] (verification not implemented) 2533
Sympy [B] (verification not implemented) 2533
Maxima [B] (verification not implemented) 2534
Giac [B] (verification not implemented) 2534
Mupad [B] (verification not implemented) 2535
Reduce [B] (verification not implemented) 2536

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

output
```
1/325*(b*x^25+a)^13/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{a^{12}x^{25}}{25}+\frac{6}{25}a^{11}bx^{50}+\frac{22}{25}a^{10}b^2x^{75}+\frac{11}{5}a^9b^3x^{100}+\frac{99}{25}a^8b^4x^{125}+\frac{132}{25}a^7b^5x^{150}+\frac{132}{25}a^6b^6x^{175}+\frac{99}{25}a^5b^7x^{200}+\frac{11}{5}a^4b^8x^{225}+\frac{22}{25}a^3b^9x^{250}+\frac{6}{25}a^2b^{10}x^{275}+\frac{1}{25}ab^{11}x^{300}+\frac{b^{12}x^{325}}{325}$$

input
```
Integrate[x^12*(a*x + b*x^26)^12,x]
```

output
```
(a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*
x^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x
^175)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^25
0)/25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {9, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{12}\left(ax+bx^{26}\right)^{12}dx$$
$$\Big\downarrow \text{ 9}$$
$$\int x^{24}\left(a+bx^{25}\right)^{12}dx$$
$$\Big\downarrow \text{ 793}$$
$$\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

input
```
Int[x^12*(a*x + b*x^26)^12,x]
```

output
```
(a + b*x^25)^13/(325*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.93 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{11}{5}a^9b^3x^{100}+\frac{132}{25}a^6b^6x^{175}+\frac{1}{25}a^{12}x^{25}+\frac{1}{325}b^{12}x^{325}+\frac{99}{25}a^5b^7x^{200}+\frac{6}{25}a^{11}b\,x^{50}+\frac{132}{25}a^7b^5x^{150}$
parallelrisch	$\frac{11}{5}a^9b^3x^{100}+\frac{132}{25}a^6b^6x^{175}+\frac{1}{25}a^{12}x^{25}+\frac{1}{325}b^{12}x^{325}+\frac{99}{25}a^5b^7x^{200}+\frac{6}{25}a^{11}b\,x^{50}+\frac{132}{25}a^7b^5x^{150}$
gosper	$\frac{x^{25}(b^{12}x^{300}+13a\,b^{11}x^{275}+78a^2b^{10}x^{250}+286a^3b^9x^{225}+715a^4b^8x^{200}+1287a^5b^7x^{175}+1716a^6b^6x^{150}+1716a^7b^5x^{125}+1287a}{325}$
risch	$\frac{b^{12}x^{325}}{325}+\frac{a\,b^{11}x^{300}}{25}+\frac{6a^2b^{10}x^{275}}{25}+\frac{22a^3b^9x^{250}}{25}+\frac{11a^4b^8x^{225}}{5}+\frac{99a^5b^7x^{200}}{25}+\frac{132a^6b^6x^{175}}{25}+\frac{132a^7b^5x^{150}}{25}+$
orering	$\frac{x^{13}(b^{12}x^{300}+13a\,b^{11}x^{275}+78a^2b^{10}x^{250}+286a^3b^9x^{225}+715a^4b^8x^{200}+1287a^5b^7x^{175}+1716a^6b^6x^{150}+1716a^7b^5x^{125}+1287a}{325(b\,x^{25}+a)^{12}}$

input
```
int(x^12*(b*x^26+a*x)^12,x,method=_RETURNVERBOSE)
```

output
```
11/5*a^9*b^3*x^100+132/25*a^6*b^6*x^175+1/25*a^12*x^25+1/325*b^12*x^325+99
/25*a^5*b^7*x^200+6/25*a^11*b*x^50+132/25*a^7*b^5*x^150+1/25*a*b^11*x^300+
99/25*a^8*b^4*x^125+11/5*a^4*b^8*x^225+22/25*a^10*b^2*x^75+22/25*a^3*b^9*x
^250+6/25*a^2*b^10*x^275
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^{2}b^{10}x^{275}+\frac{22}{25}\,a^{3}b^{9}x^{250}+\frac{11}{5}\,a^{4}b^{8}x^{225}+\frac{99}{25}\,a^{5}b^{7}x^{200}+\frac{132}{25}\,a^{6}b^{6}x^{175}+\frac{132}{25}\,a^{7}b^{5}x^{150}+\frac{99}{25}\,a^{8}b^{4}x^{125}+\frac{11}{5}\,a^{9}b^{3}x^{100}+\frac{22}{25}\,a^{10}b^{2}x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}$$

input
```
integrate(x^12*(b*x^26+a*x)^12,x, algorithm="fricas")
```

output
```
1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9
*x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 +
 132/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a
^10*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{a^{12}x^{25}}{25}+\frac{6a^{11}bx^{50}}{25}+\frac{22a^{10}b^{2}x^{75}}{25}+\frac{11a^{9}b^{3}x^{100}}{5}+\frac{99a^{8}b^{4}x^{125}}{25}+\frac{132a^{7}b^{5}x^{150}}{25}+\frac{132a^{6}b^{6}x^{175}}{25}+\frac{99a^{5}b^{7}x^{200}}{25}+\frac{11a^{4}b^{8}x^{225}}{5}+\frac{22a^{3}b^{9}x^{250}}{25}+\frac{6a^{2}b^{10}x^{275}}{25}+\frac{ab^{11}x^{300}}{25}+\frac{b^{12}x^{325}}{325}$$

input
```
integrate(x**12*(b*x**26+a*x)**12,x)
```

output
```
a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**
3*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b
**6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b
**9*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/
325
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx &= \frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^2b^{10}x^{275}\\&+\frac{22}{25}\,a^3b^9x^{250}+\frac{11}{5}\,a^4b^8x^{225}+\frac{99}{25}\,a^5b^7x^{200}\\&+\frac{132}{25}\,a^6b^6x^{175}+\frac{132}{25}\,a^7b^5x^{150}+\frac{99}{25}\,a^8b^4x^{125}\\&+\frac{11}{5}\,a^9b^3x^{100}+\frac{22}{25}\,a^{10}b^2x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}\end{aligned}$$

input
```
integrate(x^12*(b*x^26+a*x)^12,x, algorithm="maxima")
```

output
```
1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9
*x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 +
 132/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a
^10*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^2b^{10}x^{275}+\frac{22}{25}\,a^3b^9x^{250}+\frac{11}{5}\,a^4b^8x^{225}+\frac{99}{25}\,a^5b^7x^{200}+\frac{132}{25}\,a^6b^6x^{175}+\frac{132}{25}\,a^7b^5x^{150}+\frac{99}{25}\,a^8b^4x^{125}+\frac{11}{5}\,a^9b^3x^{100}+\frac{22}{25}\,a^{10}b^2x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}$$

input

```
integrate(x^12*(b*x^26+a*x)^12,x, algorithm="giac")
```

output

```
1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9
*x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 +
 132/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a
^10*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{a^{12}\,x^{25}}{25}+\frac{6\,a^{11}\,b\,x^{50}}{25}+\frac{22\,a^{10}\,b^2\,x^{75}}{25}+\frac{11\,a^9\,b^3\,x^{100}}{5}+\frac{99\,a^8\,b^4\,x^{125}}{25}+\frac{132\,a^7\,b^5\,x^{150}}{25}+\frac{132\,a^6\,b^6\,x^{175}}{25}+\frac{99\,a^5\,b^7\,x^{200}}{25}+\frac{11\,a^4\,b^8\,x^{225}}{5}+\frac{22\,a^3\,b^9\,x^{250}}{25}+\frac{6\,a^2\,b^{10}\,x^{275}}{25}+\frac{a\,b^{11}\,x^{300}}{25}+\frac{b^{12}\,x^{325}}{325}$$

input

```
int(x^12*(a*x + b*x^26)^12,x)
```

output

```
(a^12*x^25)/25 + (b^12*x^325)/325 + (6*a^11*b*x^50)/25 + (a*b^11*x^300)/25
 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x^100)/5 + (99*a^8*b^4*x^125)/25 +
(132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^175)/25 + (99*a^5*b^7*x^200)/25 +
(11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/25 + (6*a^2*b^10*x^275)/25
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx$$
$$= \frac{x^{25}(b^{12}x^{300}+13a\,b^{11}x^{275}+78a^2b^{10}x^{250}+286a^3b^9x^{225}+715a^4b^8x^{200}+1287a^5b^7x^{175}+1716a^6b^6x^{150}+1}{325}$$

input
```
int(x^12*(b*x^26+a*x)^12,x)
```

output
```
(x**25*(13*a**12 + 78*a**11*b*x**25 + 286*a**10*b**2*x**50 + 715*a**9*b**3
*x**75 + 1287*a**8*b**4*x**100 + 1716*a**7*b**5*x**125 + 1716*a**6*b**6*x*
*150 + 1287*a**5*b**7*x**175 + 715*a**4*b**8*x**200 + 286*a**3*b**9*x**225
 + 78*a**2*b**10*x**250 + 13*a*b**11*x**275 + b**12*x**300))/325
```

3.308 $\int x^{24}(ax+bx^{38})^{12}\,dx$

Optimal result . . . 2537
Mathematica [B] (verified) . . . 2537
Rubi [A] (verified) . . . 2538
Maple [B] (verified) . . . 2539
Fricas [B] (verification not implemented) . . . 2540
Sympy [B] (verification not implemented) . . . 2540
Maxima [B] (verification not implemented) . . . 2541
Giac [B] (verification not implemented) . . . 2541
Mupad [B] (verification not implemented) . . . 2542
Reduce [B] (verification not implemented) . . . 2543

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx=\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

output
```
1/481*(b*x^37+a)^13/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx=\frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8x^{333}+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^2b^{10}x^{407}+\frac{1}{37}ab^{11}x^{444}+\frac{b^{12}x^{481}}{481}$$

input
```
Integrate[x^24*(a*x + b*x^38)^12,x]
```

output
```
(a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3
*x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6
*x^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x
^370)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = $\{9, 793\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{24}\left(ax+bx^{38}\right)^{12}dx$$

$$\Big\downarrow \text{9}$$

$$\int x^{36}\left(a+bx^{37}\right)^{12}dx$$

$$\Big\downarrow \text{793}$$

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

input
```
Int[x^24*(a*x + b*x^38)^12,x]
```

output
```
(a + b*x^37)^13/(481*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 1.76 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{99}{37}a^8b^4x^{185}+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^{11}b\,x^{74}+\frac{1}{37}a\,b^{11}x^{444}+\frac{132}{37}a^6b^6x^{259}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^4b^8x^{33}$
parallelrisch	$\frac{99}{37}a^8b^4x^{185}+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^{11}b\,x^{74}+\frac{1}{37}a\,b^{11}x^{444}+\frac{132}{37}a^6b^6x^{259}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^4b^8x^{33}$
gosper	$\frac{x^{37}(b^{12}x^{444}+13a\,b^{11}x^{407}+78a^2b^{10}x^{370}+286a^3b^9x^{333}+715a^4b^8x^{296}+1287a^5b^7x^{259}+1716a^6b^6x^{222}+1716a^7b^5x^{185}+1287a}{481}$
risch	$\frac{b^{12}x^{481}}{481}+\frac{a\,b^{11}x^{444}}{37}+\frac{6a^2b^{10}x^{407}}{37}+\frac{22a^3b^9x^{370}}{37}+\frac{55a^4b^8x^{333}}{37}+\frac{99a^5b^7x^{296}}{37}+\frac{132a^6b^6x^{259}}{37}+\frac{132a^7b^5x^{222}}{37}+$
orering	$\frac{x^{25}(b^{12}x^{444}+13a\,b^{11}x^{407}+78a^2b^{10}x^{370}+286a^3b^9x^{333}+715a^4b^8x^{296}+1287a^5b^7x^{259}+1716a^6b^6x^{222}+1716a^7b^5x^{185}+1287a}{481(b\,x^{37}+a)^{12}}$

input
```
int(x^24*(b*x^38+a*x)^12,x,method=_RETURNVERBOSE)
```

output
```
99/37*a^8*b^4*x^185+22/37*a^3*b^9*x^370+6/37*a^11*b*x^74+1/37*a*b^11*x^444
+132/37*a^6*b^6*x^259+22/37*a^10*b^2*x^111+55/37*a^4*b^8*x^333+1/481*b^12*
x^481+132/37*a^7*b^5*x^222+55/37*a^9*b^3*x^148+99/37*a^5*b^7*x^296+1/37*a^
12*x^37+6/37*a^2*b^10*x^407
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.06 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx = \frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}+\frac{99}{37}\,a^{8}b^{4}x^{185}+\frac{55}{37}\,a^{9}b^{3}x^{148}+\frac{22}{37}\,a^{10}b^{2}x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}$$

input
```
integrate(x^24*(b*x^38+a*x)^12,x, algorithm="fricas")
```

output
```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx = \frac{a^{12}x^{37}}{37}+\frac{6a^{11}bx^{74}}{37}+\frac{22a^{10}b^{2}x^{111}}{37}+\frac{55a^{9}b^{3}x^{148}}{37}+\frac{99a^{8}b^{4}x^{185}}{37}+\frac{132a^{7}b^{5}x^{222}}{37}+\frac{132a^{6}b^{6}x^{259}}{37}+\frac{99a^{5}b^{7}x^{296}}{37}+\frac{55a^{4}b^{8}x^{333}}{37}+\frac{22a^{3}b^{9}x^{370}}{37}+\frac{6a^{2}b^{10}x^{407}}{37}+\frac{ab^{11}x^{444}}{37}+\frac{b^{12}x^{481}}{481}$$

input
```
integrate(x**24*(b*x**38+a*x)**12,x)
```

output
```
a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b*
*3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6
*b**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**
3*b**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**4
81/481
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx = {} & \frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}\\ & +\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}\\ & +\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}+\frac{99}{37}\,a^{8}b^{4}x^{185}\\ & +\frac{55}{37}\,a^{9}b^{3}x^{148}+\frac{22}{37}\,a^{10}b^{2}x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}\end{aligned}$$

input
```
integrate(x^24*(b*x^38+a*x)^12,x, algorithm="maxima")
```

output
```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{24}(ax+bx^{38})^{12}\,dx = \frac{1}{481}b^{12}x^{481}+\frac{1}{37}ab^{11}x^{444}+\frac{6}{37}a^2b^{10}x^{407}+\frac{22}{37}a^3b^9x^{370}+\frac{55}{37}a^4b^8x^{333}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^6b^6x^{259}+\frac{132}{37}a^7b^5x^{222}+\frac{99}{37}a^8b^4x^{185}+\frac{55}{37}a^9b^3x^{148}+\frac{22}{37}a^{10}b^2x^{111}+\frac{6}{37}a^{11}bx^{74}+\frac{1}{37}a^{12}x^{37}$$

input
```
integrate(x^24*(b*x^38+a*x)^12,x, algorithm="giac")
```

output
```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{24}(ax+bx^{38})^{12}\,dx = \frac{a^{12}x^{37}}{37}+\frac{6a^{11}bx^{74}}{37}+\frac{22a^{10}b^2x^{111}}{37}+\frac{55a^9b^3x^{148}}{37}+\frac{99a^8b^4x^{185}}{37}+\frac{132a^7b^5x^{222}}{37}+\frac{132a^6b^6x^{259}}{37}+\frac{99a^5b^7x^{296}}{37}+\frac{55a^4b^8x^{333}}{37}+\frac{22a^3b^9x^{370}}{37}+\frac{6a^2b^{10}x^{407}}{37}+\frac{ab^{11}x^{444}}{37}+\frac{b^{12}x^{481}}{481}$$

input
```
int(x^24*(a*x + b*x^38)^12,x)
```

output
```
(a^12*x^37)/37 + (b^12*x^481)/481 + (6*a^11*b*x^74)/37 + (a*b^11*x^444)/37
 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37
+ (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37
+ (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx = \frac{x^{37}(b^{12}x^{444}+13a\,b^{11}x^{407}+78a^2b^{10}x^{370}+286a^3b^9x^{333}+715a^4b^8x^{296}+1287a^5b^7x^{259}+1716a^6b^6x^{222}+1}{481}$$

input
```
int(x^24*(b*x^38+a*x)^12,x)
```

output
```
(x**37*(13*a**12 + 78*a**11*b*x**37 + 286*a**10*b**2*x**74 + 715*a**9*b**3
*x**111 + 1287*a**8*b**4*x**148 + 1716*a**7*b**5*x**185 + 1716*a**6*b**6*x
**222 + 1287*a**5*b**7*x**259 + 715*a**4*b**8*x**296 + 286*a**3*b**9*x**33
3 + 78*a**2*b**10*x**370 + 13*a*b**11*x**407 + b**12*x**444))/481
```

3.309 $\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx$

Optimal result 2544
Mathematica [A] (verified) 2544
Rubi [A] (verified) 2545
Maple [B] (verified) 2546
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Giac [B] (verification not implemented) 2548
Mupad [B] (verification not implemented) 2549
Reduce [B] (verification not implemented) 2550

Optimal result

Integrand size = 23, antiderivative size = 27

$$\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx=\frac{\left(a+bx^{1+12m}\right)^{13}}{13b(1+12m)}$$

output
```
1/13*(a+b*x^(1+12*m))^13/b/(1+12*m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx=\frac{\left(a+bx^{1+12m}\right)^{13}}{13b+156bm}$$

input
```
Integrate[x^(12*(-1 + m))*(a*x + b*x^(2 + 12*m))^12,x]
```

output
```
(a + b*x^(1 + 12*m))^13/(13*b + 156*b*m)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {10, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{12(m-1)}\left(ax+bx^{12m+2}\right)^{12}dx$$

$$\Big\downarrow \text{ 10}$$

$$\int x^{12m}\left(a+bx^{12m+1}\right)^{12}dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{\left(a+bx^{12m+1}\right)^{13}}{13b(12m+1)}$$

input
```
Int[x^(12*(-1 + m))*(a*x + b*x^(2 + 12*m))^12,x]
```

output
```
(a + b*x^(1 + 12*m))^13/(13*b*(1 + 12*m))
```

Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(25) = 50$.

Time = 69.34 (sec) , antiderivative size = 338, normalized size of antiderivative = 12.52

method	result
parallelrisch	$\frac{13a^{12}x^{-12+12m}x^{13}+78a^{11}b\,x^{-12+12m}x^{2+12m}x^{12}+286a^{10}b^2x^{-12+12m}x^{4+24m}x^{11}+715a^9b^3x^{-12+12m}x^{6+36m}x^{10}+1287a^{8}}{}$
risch	$\frac{b^{12}x^{26+156m}}{13(1+12m)x^{13}}+\frac{a\,b^{11}x^{24+144m}}{(1+12m)x^{12}}+\frac{6a^2b^{10}x^{22+132m}}{(1+12m)x^{11}}+\frac{22a^3b^9x^{20+120m}}{(1+12m)x^{10}}+\frac{55a^4b^8x^{18+108m}}{(1+12m)x^9}+\frac{99a^5b^7x^{16+96m}}{(1+12m)x^8}+$
orering	Expression too large to display

input
```
int(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x,method=_RETURNVERBOSE)
```

output
```
1/13*(13*a^12*x^(-12+12*m)*x^13+78*a^11*b*x^(-12+12*m)*x^(2+12*m)*x^12+286
*a^10*b^2*x^(-12+12*m)*(x^(2+12*m))^2*x^11+715*a^9*b^3*x^(-12+12*m)*(x^(2+
12*m))^3*x^10+1287*a^8*b^4*x^(-12+12*m)*(x^(2+12*m))^4*x^9+1716*a^7*b^5*x^
(-12+12*m)*(x^(2+12*m))^5*x^8+1716*a^6*b^6*x^(-12+12*m)*(x^(2+12*m))^6*x^7
+1287*a^5*b^7*x^(-12+12*m)*(x^(2+12*m))^7*x^6+715*a^4*b^8*x^(-12+12*m)*(x^
(2+12*m))^8*x^5+286*a^3*b^9*x^(-12+12*m)*(x^(2+12*m))^9*x^4+78*a^2*b^10*x^
(-12+12*m)*(x^(2+12*m))^10*x^3+13*a*b^11*x^(-12+12*m)*(x^(2+12*m))^11*x^2+
b^12*x^(-12+12*m)*(x^(2+12*m))^12*x)/(1+12*m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(25) = 50$.

Time = 0.11 (sec) , antiderivative size = 231, normalized size of antiderivative = 8.56

$$\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx = \frac{13\,a^{12}x^{12}x^{12\,m+2}+78\,a^{11}bx^{11}x^{24\,m+4}+286\,a^{10}b^2x^{10}x^{36\,m+6}+715\,a^9b^3x^9x^{48\,m+8}+1287\,a^8b^4x^8x^{60\,m+10}+}{}$$

input
```
integrate(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x, algorithm="fricas")
```

output
```
1/13*(13*a^12*x^12*x^(12*m + 2) + 78*a^11*b*x^11*x^(24*m + 4) + 286*a^10*b
^2*x^10*x^(36*m + 6) + 715*a^9*b^3*x^9*x^(48*m + 8) + 1287*a^8*b^4*x^8*x^(
60*m + 10) + 1716*a^7*b^5*x^7*x^(72*m + 12) + 1716*a^6*b^6*x^6*x^(84*m + 1
4) + 1287*a^5*b^7*x^5*x^(96*m + 16) + 715*a^4*b^8*x^4*x^(108*m + 18) + 286
*a^3*b^9*x^3*x^(120*m + 20) + 78*a^2*b^10*x^2*x^(132*m + 22) + 13*a*b^11*x
*x^(144*m + 24) + b^12*x^(156*m + 26))/((12*m + 1)*x^13)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(19) = 38$.

Time = 6.29 (sec) , antiderivative size = 520, normalized size of antiderivative = 19.26

$$\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx$$
$$= \begin{cases} \frac{13a^{12}x^{13}x^{12m-12}}{156m+13} + \frac{78a^{11}bx^{12}x^{12m-12}x^{12m+2}}{156m+13} + \frac{286a^{10}b^2x^{11}x^{12m-12}x^{24m+4}}{156m+13} + \frac{715a^9b^3x^{10}x^{12m-12}x^{36m+6}}{156m+13} + \frac{1287a^8b^4x^9x^{12m-}}{156m+1} \\ a^{12}\log(x) + 12a^{11}b\log(x) + 66a^{10}b^2\log(x) + 220a^9b^3\log(x) + 495a^8b^4\log(x) + 792a^7b^5\log(x) + 92 \end{cases}$$

input
```
integrate(x**(-12+12*m)*(a*x+b*x**(2+12*m))**12,x)
```

output
```
Piecewise((13*a**12*x**13*x**(12*m - 12)/(156*m + 13) + 78*a**11*b*x**12*x
**(12*m - 12)*x**(12*m + 2)/(156*m + 13) + 286*a**10*b**2*x**11*x**(12*m -
 12)*x**(24*m + 4)/(156*m + 13) + 715*a**9*b**3*x**10*x**(12*m - 12)*x**(3
6*m + 6)/(156*m + 13) + 1287*a**8*b**4*x**9*x**(12*m - 12)*x**(48*m + 8)/(
156*m + 13) + 1716*a**7*b**5*x**8*x**(12*m - 12)*x**(60*m + 10)/(156*m + 1
3) + 1716*a**6*b**6*x**7*x**(12*m - 12)*x**(72*m + 12)/(156*m + 13) + 1287
*a**5*b**7*x**6*x**(12*m - 12)*x**(84*m + 14)/(156*m + 13) + 715*a**4*b**8
*x**5*x**(12*m - 12)*x**(96*m + 16)/(156*m + 13) + 286*a**3*b**9*x**4*x**(
12*m - 12)*x**(108*m + 18)/(156*m + 13) + 78*a**2*b**10*x**3*x**(12*m - 12
)*x**(120*m + 20)/(156*m + 13) + 13*a*b**11*x**2*x**(12*m - 12)*x**(132*m
+ 22)/(156*m + 13) + b**12*x*x**(12*m - 12)*x**(144*m + 24)/(156*m + 13),
Ne(m, -1/12)), (a**12*log(x) + 12*a**11*b*log(x) + 66*a**10*b**2*log(x) +
220*a**9*b**3*log(x) + 495*a**8*b**4*log(x) + 792*a**7*b**5*log(x) + 924*a
**6*b**6*log(x) + 792*a**5*b**7*log(x) + 495*a**4*b**8*log(x) + 220*a**3*b
**9*log(x) + 66*a**2*b**10*log(x) + 12*a*b**11*log(x) + b**12*log(x), True
))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(25) = 50$.

Time = 0.04 (sec) , antiderivative size = 275, normalized size of antiderivative = 10.19

$$\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx = \frac{b^{12}x^{156\,m+13}}{13\,(12\,m+1)} + \frac{ab^{11}x^{144\,m+12}}{12\,m+1} + \frac{6\,a^{2}b^{10}x^{132\,m+11}}{12\,m+1} + \frac{22\,a^{3}b^{9}x^{120\,m+10}}{12\,m+1} + \frac{55\,a^{4}b^{8}x^{108\,m+9}}{12\,m+1} + \frac{99\,a^{5}b^{7}x^{96\,m+8}}{12\,m+1} + \frac{132\,a^{6}b^{6}x^{84\,m+7}}{12\,m+1} + \frac{132\,a^{7}b^{5}x^{72\,m+6}}{12\,m+1} + \frac{99\,a^{8}b^{4}x^{60\,m+5}}{12\,m+1} + \frac{55\,a^{9}b^{3}x^{48\,m+4}}{12\,m+1} + \frac{22\,a^{10}b^{2}x^{36\,m+3}}{12\,m+1} + \frac{6\,a^{11}bx^{24\,m+2}}{12\,m+1} + \frac{a^{12}x^{12\,m+1}}{12\,m+1}$$

input
```
integrate(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x, algorithm="maxima")
```

output
```
1/13*b^12*x^(156*m + 13)/(12*m + 1) + a*b^11*x^(144*m + 12)/(12*m + 1) + 6
*a^2*b^10*x^(132*m + 11)/(12*m + 1) + 22*a^3*b^9*x^(120*m + 10)/(12*m + 1)
 + 55*a^4*b^8*x^(108*m + 9)/(12*m + 1) + 99*a^5*b^7*x^(96*m + 8)/(12*m + 1
) + 132*a^6*b^6*x^(84*m + 7)/(12*m + 1) + 132*a^7*b^5*x^(72*m + 6)/(12*m +
 1) + 99*a^8*b^4*x^(60*m + 5)/(12*m + 1) + 55*a^9*b^3*x^(48*m + 4)/(12*m +
 1) + 22*a^10*b^2*x^(36*m + 3)/(12*m + 1) + 6*a^11*b*x^(24*m + 2)/(12*m +
1) + a^12*x^(12*m + 1)/(12*m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(25) = 50$.

Time = 0.33 (sec) , antiderivative size = 285, normalized size of antiderivative = 10.56

$$\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx$$

$$= \frac{13\,a^{12}x^{12}e^{(12\,m\log(x)+2\,\log(x))} + 78\,a^{11}bx^{11}e^{(24\,m\log(x)+4\,\log(x))} + 286\,a^{10}b^{2}x^{10}e^{(36\,m\log(x)+6\,\log(x))} + 715\,a^{9}b^{3}x^{9}\cdots}{}$$

input
```
integrate(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x, algorithm="giac")
```

output
```
1/13*(13*a^12*x^12*e^(12*m*log(x) + 2*log(x)) + 78*a^11*b*x^11*e^(24*m*log
(x) + 4*log(x)) + 286*a^10*b^2*x^10*e^(36*m*log(x) + 6*log(x)) + 715*a^9*b
^3*x^9*e^(48*m*log(x) + 8*log(x)) + 1287*a^8*b^4*x^8*e^(60*m*log(x) + 10*l
og(x)) + 1716*a^7*b^5*x^7*e^(72*m*log(x) + 12*log(x)) + 1716*a^6*b^6*x^6*e
^(84*m*log(x) + 14*log(x)) + 1287*a^5*b^7*x^5*e^(96*m*log(x) + 16*log(x))
+ 715*a^4*b^8*x^4*e^(108*m*log(x) + 18*log(x)) + 286*a^3*b^9*x^3*e^(120*m*
log(x) + 20*log(x)) + 78*a^2*b^10*x^2*e^(132*m*log(x) + 22*log(x)) + 13*a*
b^11*x*e^(144*m*log(x) + 24*log(x)) + b^12*e^(156*m*log(x) + 26*log(x)))/(
12*m*x^13 + x^13)
```

Mupad [B] (verification not implemented)

Time = 9.72 (sec) , antiderivative size = 287, normalized size of antiderivative = 10.63

$$\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx=\frac{b^{12}\,x^{156\,m}\,x^{13}}{156\,m+13}+\frac{13\,a^{12}\,x\,x^{12\,m}}{156\,m+13}+\frac{78\,a^{11}\,b\,x^{24\,m}\,x^2}{156\,m+13}+\frac{13\,a\,b^{11}\,x^{144\,m}\,x^{12}}{156\,m+13}+\frac{286\,a^{10}\,b^2\,x^{36\,m}\,x^3}{156\,m+13}+\frac{715\,a^9\,b^3\,x^{48\,m}\,x^4}{156\,m+13}+\frac{1287\,a^8\,b^4\,x^{60\,m}\,x^5}{156\,m+13}+\frac{1716\,a^7\,b^5\,x^{72\,m}\,x^6}{156\,m+13}+\frac{1716\,a^6\,b^6\,x^{84\,m}\,x^7}{156\,m+13}+\frac{1287\,a^5\,b^7\,x^{96\,m}\,x^8}{156\,m+13}+\frac{715\,a^4\,b^8\,x^{108\,m}\,x^9}{156\,m+13}+\frac{286\,a^3\,b^9\,x^{120\,m}\,x^{10}}{156\,m+13}+\frac{78\,a^2\,b^{10}\,x^{132\,m}\,x^{11}}{156\,m+13}$$

input
```
int(x^(12*m - 12)*(a*x + b*x^(12*m + 2))^12,x)
```

output
```
(b^12*x^(156*m)*x^13)/(156*m + 13) + (13*a^12*x*x^(12*m))/(156*m + 13) + (
78*a^11*b*x^(24*m)*x^2)/(156*m + 13) + (13*a*b^11*x^(144*m)*x^12)/(156*m +
 13) + (286*a^10*b^2*x^(36*m)*x^3)/(156*m + 13) + (715*a^9*b^3*x^(48*m)*x^
4)/(156*m + 13) + (1287*a^8*b^4*x^(60*m)*x^5)/(156*m + 13) + (1716*a^7*b^5
*x^(72*m)*x^6)/(156*m + 13) + (1716*a^6*b^6*x^(84*m)*x^7)/(156*m + 13) + (
1287*a^5*b^7*x^(96*m)*x^8)/(156*m + 13) + (715*a^4*b^8*x^(108*m)*x^9)/(156
*m + 13) + (286*a^3*b^9*x^(120*m)*x^10)/(156*m + 13) + (78*a^2*b^10*x^(132
*m)*x^11)/(156*m + 13)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 202, normalized size of antiderivative = 7.48

$$\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx = \frac{x^{12m}x(x^{144m}b^{12}x^{12}+13x^{132m}a\,b^{11}x^{11}+78x^{120m}a^2b^{10}x^{10}+286x^{108m}a^3b^9x^9+715x^{96m}a^4b^8x^8+1287x^{84m}}{}$$

input
```
int(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x)
```

output
```
(x**(12*m)*x*(x**(144*m)*b**12*x**12 + 13*x**(132*m)*a*b**11*x**11 + 78*x*
*(120*m)*a**2*b**10*x**10 + 286*x**(108*m)*a**3*b**9*x**9 + 715*x**(96*m)*
a**4*b**8*x**8 + 1287*x**(84*m)*a**5*b**7*x**7 + 1716*x**(72*m)*a**6*b**6*
x**6 + 1716*x**(60*m)*a**7*b**5*x**5 + 1287*x**(48*m)*a**8*b**4*x**4 + 715
*x**(36*m)*a**9*b**3*x**3 + 286*x**(24*m)*a**10*b**2*x**2 + 78*x**(12*m)*a
**11*b*x + 13*a**12))/(13*(12*m + 1))
```

3.310 $\int (ax + bx^{14})^{12} \, dx$

Optimal result . . . 2551
Mathematica [B] (verified) . . . 2551
Rubi [A] (verified) . . . 2552
Maple [B] (verified) . . . 2553
Fricas [B] (verification not implemented) . . . 2554
Sympy [B] (verification not implemented) . . . 2554
Maxima [B] (verification not implemented) . . . 2555
Giac [B] (verification not implemented) . . . 2555
Mupad [B] (verification not implemented) . . . 2556
Reduce [B] (verification not implemented) . . . 2556

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int (ax + bx^{14})^{12} \, dx = \frac{(a + bx^{13})^{13}}{169b}$$

output
```
1/169*(b*x^13+a)^13/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int (ax + bx^{14})^{12} \, dx = \frac{a^{12}x^{13}}{13} + \frac{6}{13}a^{11}bx^{26} + \frac{22}{13}a^{10}b^2x^{39} + \frac{55}{13}a^9b^3x^{52} + \frac{99}{13}a^8b^4x^{65} + \frac{132}{13}a^7b^5x^{78} + \frac{132}{13}a^6b^6x^{91} + \frac{99}{13}a^5b^7x^{104} + \frac{55}{13}a^4b^8x^{117} + \frac{22}{13}a^3b^9x^{130} + \frac{6}{13}a^2b^{10}x^{143} + \frac{1}{13}ab^{11}x^{156} + \frac{b^{12}x^{169}}{169}$$

input
```
Integrate[(a*x + b*x^14)^12,x]
```

output
```
(a^12*x^13)/13 + (6*a^11*b*x^26)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*
x^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^9
1)/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)
/13 + (6*a^2*b^10*x^143)/13 + (a*b^11*x^156)/13 + (b^12*x^169)/169
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2027, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^{14}\right)^{12} dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int x^{12}\left(a+bx^{13}\right)^{12} dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

input
```
Int[(a*x + b*x^14)^12,x]
```

output
```
(a + b*x^13)^13/(169*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(14) = 28.

Time = 0.47 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{132}{13}a^7b^5x^{78}+\frac{55}{13}a^4b^8x^{117}+\frac{132}{13}a^6b^6x^{91}+\frac{1}{13}a\,b^{11}x^{156}+\frac{6}{13}a^{11}b\,x^{26}+\frac{22}{13}a^{10}b^2x^{39}+\frac{6}{13}a^2b^{10}x^{143}$
parallelrisch	$\frac{132}{13}a^7b^5x^{78}+\frac{55}{13}a^4b^8x^{117}+\frac{132}{13}a^6b^6x^{91}+\frac{1}{13}a\,b^{11}x^{156}+\frac{6}{13}a^{11}b\,x^{26}+\frac{22}{13}a^{10}b^2x^{39}+\frac{6}{13}a^2b^{10}x^{143}$
gosper	$\frac{x^{13}(b^{12}x^{156}+13a\,b^{11}x^{143}+78a^2b^{10}x^{130}+286a^3b^9x^{117}+715a^4b^8x^{104}+1287a^5b^7x^{91}+1716a^6b^6x^{78}+1716a^7b^5x^{65}+1287a^8b^4}{169}$
risch	$\frac{b^{12}x^{169}}{169}+\frac{a\,b^{11}x^{156}}{13}+\frac{6a^2b^{10}x^{143}}{13}+\frac{22a^3b^9x^{130}}{13}+\frac{55a^4b^8x^{117}}{13}+\frac{99a^5b^7x^{104}}{13}+\frac{132a^6b^6x^{91}}{13}+\frac{132a^7b^5x^{78}}{13}+$
orering	$\frac{x(b^{12}x^{156}+13a\,b^{11}x^{143}+78a^2b^{10}x^{130}+286a^3b^9x^{117}+715a^4b^8x^{104}+1287a^5b^7x^{91}+1716a^6b^6x^{78}+1716a^7b^5x^{65}+1287a^8b^4x^5}{169(b\,x^{13}+a)^{12}}$

input
```
int((b*x^14+a*x)^12,x,method=_RETURNVERBOSE)
```

output
```
132/13*a^7*b^5*x^78+55/13*a^4*b^8*x^117+132/13*a^6*b^6*x^91+1/13*a*b^11*x^
156+6/13*a^11*b*x^26+22/13*a^10*b^2*x^39+6/13*a^2*b^10*x^143+22/13*a^3*b^9
*x^130+1/169*b^12*x^169+99/13*a^5*b^7*x^104+99/13*a^8*b^4*x^65+55/13*a^9*b
^3*x^52+1/13*a^12*x^13
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int \left(ax+bx^{14}\right)^{12} dx = \frac{1}{169}b^{12}x^{169} + \frac{1}{13}ab^{11}x^{156} + \frac{6}{13}a^2b^{10}x^{143} + \frac{22}{13}a^3b^9x^{130} + \frac{55}{13}a^4b^8x^{117} + \frac{99}{13}a^5b^7x^{104} + \frac{132}{13}a^6b^6x^{91} + \frac{132}{13}a^7b^5x^{78} + \frac{99}{13}a^8b^4x^{65} + \frac{55}{13}a^9b^3x^{52} + \frac{22}{13}a^{10}b^2x^{39} + \frac{6}{13}a^{11}bx^{26} + \frac{1}{13}a^{12}x^{13}$$

input
```
integrate((b*x^14+a*x)^12,x, algorithm="fricas")
```

output
```
1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9
*x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 +
 132/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^1
0*b^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int \left(ax+bx^{14}\right)^{12} dx = \frac{a^{12}x^{13}}{13} + \frac{6a^{11}bx^{26}}{13} + \frac{22a^{10}b^2x^{39}}{13} + \frac{55a^9b^3x^{52}}{13} + \frac{99a^8b^4x^{65}}{13} + \frac{132a^7b^5x^{78}}{13} + \frac{132a^6b^6x^{91}}{13} + \frac{99a^5b^7x^{104}}{13} + \frac{55a^4b^8x^{117}}{13} + \frac{22a^3b^9x^{130}}{13} + \frac{6a^2b^{10}x^{143}}{13} + \frac{ab^{11}x^{156}}{13} + \frac{b^{12}x^{169}}{169}$$

input
```
integrate((b*x**14+a*x)**12,x)
```

output
```
a**12*x**13/13 + 6*a**11*b*x**26/13 + 22*a**10*b**2*x**39/13 + 55*a**9*b**
3*x**52/13 + 99*a**8*b**4*x**65/13 + 132*a**7*b**5*x**78/13 + 132*a**6*b**
6*x**91/13 + 99*a**5*b**7*x**104/13 + 55*a**4*b**8*x**117/13 + 22*a**3*b**
9*x**130/13 + 6*a**2*b**10*x**143/13 + a*b**11*x**156/13 + b**12*x**169/16
9
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(14) = 28.

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int (ax+bx^{14})^{12}\,dx = \frac{1}{169}\,b^{12}x^{169} + \frac{1}{13}\,ab^{11}x^{156} + \frac{6}{13}\,a^{2}b^{10}x^{143} + \frac{22}{13}\,a^{3}b^{9}x^{130} + \frac{55}{13}\,a^{4}b^{8}x^{117} + \frac{99}{13}\,a^{5}b^{7}x^{104} + \frac{132}{13}\,a^{6}b^{6}x^{91} + \frac{132}{13}\,a^{7}b^{5}x^{78} + \frac{99}{13}\,a^{8}b^{4}x^{65} + \frac{55}{13}\,a^{9}b^{3}x^{52} + \frac{22}{13}\,a^{10}b^{2}x^{39} + \frac{6}{13}\,a^{11}bx^{26} + \frac{1}{13}\,a^{12}x^{13}$$

input
```
integrate((b*x^14+a*x)^12,x, algorithm="maxima")
```

output
```
1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9
*x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 +
 132/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^1
0*b^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(14) = 28.

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int (ax+bx^{14})^{12}\,dx = \frac{1}{169}\,b^{12}x^{169} + \frac{1}{13}\,ab^{11}x^{156} + \frac{6}{13}\,a^{2}b^{10}x^{143} + \frac{22}{13}\,a^{3}b^{9}x^{130} + \frac{55}{13}\,a^{4}b^{8}x^{117} + \frac{99}{13}\,a^{5}b^{7}x^{104} + \frac{132}{13}\,a^{6}b^{6}x^{91} + \frac{132}{13}\,a^{7}b^{5}x^{78} + \frac{99}{13}\,a^{8}b^{4}x^{65} + \frac{55}{13}\,a^{9}b^{3}x^{52} + \frac{22}{13}\,a^{10}b^{2}x^{39} + \frac{6}{13}\,a^{11}bx^{26} + \frac{1}{13}\,a^{12}x^{13}$$

input
```
integrate((b*x^14+a*x)^12,x, algorithm="giac")
```

output
```
1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9
*x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 +
 132/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^1
0*b^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int \left(ax+bx^{14}\right)^{12}\,dx=\frac{a^{12}\,x^{13}}{13}+\frac{6\,a^{11}\,b\,x^{26}}{13}+\frac{22\,a^{10}\,b^2\,x^{39}}{13}+\frac{55\,a^9\,b^3\,x^{52}}{13}+\frac{99\,a^8\,b^4\,x^{65}}{13}+\frac{132\,a^7\,b^5\,x^{78}}{13}+\frac{132\,a^6\,b^6\,x^{91}}{13}+\frac{99\,a^5\,b^7\,x^{104}}{13}+\frac{55\,a^4\,b^8\,x^{117}}{13}+\frac{22\,a^3\,b^9\,x^{130}}{13}+\frac{6\,a^2\,b^{10}\,x^{143}}{13}+\frac{a\,b^{11}\,x^{156}}{13}+\frac{b^{12}\,x^{169}}{169}$$

input
```
int((a*x + b*x^14)^12,x)
```

output
```
(a^12*x^13)/13 + (b^12*x^169)/169 + (6*a^11*b*x^26)/13 + (a*b^11*x^156)/13
 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x^52)/13 + (99*a^8*b^4*x^65)/13 + (
132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)/13 + (99*a^5*b^7*x^104)/13 + (55
*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13 + (6*a^2*b^10*x^143)/13
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

$$\int \left(ax+bx^{14}\right)^{12}\,dx=\frac{x^{13}(b^{12}x^{156}+13a\,b^{11}x^{143}+78a^2b^{10}x^{130}+286a^3b^9x^{117}+715a^4b^8x^{104}+1287a^5b^7x^{91}+1716a^6b^6x^{78}+17}{169}$$

input
```
int((b*x^14+a*x)^12,x)
```

output
```
(x**13*(13*a**12 + 78*a**11*b*x**13 + 286*a**10*b**2*x**26 + 715*a**9*b**3
*x**39 + 1287*a**8*b**4*x**52 + 1716*a**7*b**5*x**65 + 1716*a**6*b**6*x**7
8 + 1287*a**5*b**7*x**91 + 715*a**4*b**8*x**104 + 286*a**3*b**9*x**117 + 7
8*a**2*b**10*x**130 + 13*a*b**11*x**143 + b**12*x**156))/169
```

3.311 $\int (ax^2+bx^{27})^{12}\,dx$

Optimal result . . . 2558
Mathematica [B] (verified) . . . 2558
Rubi [A] (verified) . . . 2559
Maple [B] (verified) . . . 2560
Fricas [B] (verification not implemented) . . . 2561
Sympy [B] (verification not implemented) . . . 2561
Maxima [B] (verification not implemented) . . . 2562
Giac [B] (verification not implemented) . . . 2562
Mupad [B] (verification not implemented) . . . 2563
Reduce [B] (verification not implemented) . . . 2563

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int (ax^2+bx^{27})^{12}\,dx=\frac{(a+bx^{25})^{13}}{325b}$$

output
```
1/325*(b*x^25+a)^13/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.01 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\begin{aligned}\int (ax^2+bx^{27})^{12}\,dx=&\frac{a^{12}x^{25}}{25}+\frac{6}{25}a^{11}bx^{50}+\frac{22}{25}a^{10}b^2x^{75}+\frac{11}{5}a^9b^3x^{100}+\frac{99}{25}a^8b^4x^{125}\\&+\frac{132}{25}a^7b^5x^{150}+\frac{132}{25}a^6b^6x^{175}+\frac{99}{25}a^5b^7x^{200}+\frac{11}{5}a^4b^8x^{225}\\&+\frac{22}{25}a^3b^9x^{250}+\frac{6}{25}a^2b^{10}x^{275}+\frac{1}{25}ab^{11}x^{300}+\frac{b^{12}x^{325}}{325}\end{aligned}$$

input
```
Integrate[(a*x^2 + b*x^27)^12,x]
```

output
```
(a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*
x^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x
^175)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^25
0)/25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2027, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^2+bx^{27}\right)^{12} dx$$

$$\Big\downarrow \text{2027}$$

$$\int x^{24}\left(a+bx^{25}\right)^{12} dx$$

$$\Big\downarrow \text{793}$$

$$\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

input
```
Int[(a*x^2 + b*x^27)^12,x]
```

output
```
(a + b*x^25)^13/(325*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.86 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{11}{5}a^9b^3x^{100}+\frac{132}{25}a^6b^6x^{175}+\frac{1}{25}a^{12}x^{25}+\frac{1}{325}b^{12}x^{325}+\frac{99}{25}a^5b^7x^{200}+\frac{6}{25}a^{11}b\,x^{50}+\frac{132}{25}a^7b^5x^{150}$
parallelrisch	$\frac{11}{5}a^9b^3x^{100}+\frac{132}{25}a^6b^6x^{175}+\frac{1}{25}a^{12}x^{25}+\frac{1}{325}b^{12}x^{325}+\frac{99}{25}a^5b^7x^{200}+\frac{6}{25}a^{11}b\,x^{50}+\frac{132}{25}a^7b^5x^{150}$
gosper	$\frac{x^{25}(b^{12}x^{300}+13a\,b^{11}x^{275}+78a^2b^{10}x^{250}+286a^3b^9x^{225}+715a^4b^8x^{200}+1287a^5b^7x^{175}+1716a^6b^6x^{150}+1716a^7b^5x^{125}+1287a}{325}$
risch	$\frac{b^{12}x^{325}}{325}+\frac{a\,b^{11}x^{300}}{25}+\frac{6a^2b^{10}x^{275}}{25}+\frac{22a^3b^9x^{250}}{25}+\frac{11a^4b^8x^{225}}{5}+\frac{99a^5b^7x^{200}}{25}+\frac{132a^6b^6x^{175}}{25}+\frac{132a^7b^5x^{150}}{25}+$
orering	$\frac{x(b^{12}x^{300}+13a\,b^{11}x^{275}+78a^2b^{10}x^{250}+286a^3b^9x^{225}+715a^4b^8x^{200}+1287a^5b^7x^{175}+1716a^6b^6x^{150}+1716a^7b^5x^{125}+1287a^8b}{325(b\,x^{25}+a)^{12}}$

input
```
int((b*x^27+a*x^2)^12,x,method=_RETURNVERBOSE)
```

output
```
11/5*a^9*b^3*x^100+132/25*a^6*b^6*x^175+1/25*a^12*x^25+1/325*b^12*x^325+99
/25*a^5*b^7*x^200+6/25*a^11*b*x^50+132/25*a^7*b^5*x^150+1/25*a*b^11*x^300+
99/25*a^8*b^4*x^125+11/5*a^4*b^8*x^225+22/25*a^10*b^2*x^75+22/25*a^3*b^9*x
^250+6/25*a^2*b^10*x^275
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int \left(ax^2+bx^{27}\right)^{12} dx = \frac{1}{325}b^{12}x^{325} + \frac{1}{25}ab^{11}x^{300} + \frac{6}{25}a^2b^{10}x^{275} + \frac{22}{25}a^3b^9x^{250} + \frac{11}{5}a^4b^8x^{225} + \frac{99}{25}a^5b^7x^{200} + \frac{132}{25}a^6b^6x^{175} + \frac{132}{25}a^7b^5x^{150} + \frac{99}{25}a^8b^4x^{125} + \frac{11}{5}a^9b^3x^{100} + \frac{22}{25}a^{10}b^2x^{75} + \frac{6}{25}a^{11}bx^{50} + \frac{1}{25}a^{12}x^{25}$$

input
```
integrate((b*x^27+a*x^2)^12,x, algorithm="fricas")
```

output
```
1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9
*x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 +
 132/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a
^10*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int \left(ax^2+bx^{27}\right)^{12} dx = \frac{a^{12}x^{25}}{25} + \frac{6a^{11}bx^{50}}{25} + \frac{22a^{10}b^2x^{75}}{25} + \frac{11a^9b^3x^{100}}{5} + \frac{99a^8b^4x^{125}}{25} + \frac{132a^7b^5x^{150}}{25} + \frac{132a^6b^6x^{175}}{25} + \frac{99a^5b^7x^{200}}{25} + \frac{11a^4b^8x^{225}}{5} + \frac{22a^3b^9x^{250}}{25} + \frac{6a^2b^{10}x^{275}}{25} + \frac{ab^{11}x^{300}}{25} + \frac{b^{12}x^{325}}{325}$$

input
```
integrate((b*x**27+a*x**2)**12,x)
```

output

```
a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**
3*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b
**6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b
**9*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/
325
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int (ax^2 + bx^{27})^{12} \, dx = \frac{1}{325} b^{12}x^{325} + \frac{1}{25} ab^{11}x^{300} + \frac{6}{25} a^2b^{10}x^{275} + \frac{22}{25} a^3b^9x^{250} + \frac{11}{5} a^4b^8x^{225} + \frac{99}{25} a^5b^7x^{200} + \frac{132}{25} a^6b^6x^{175} + \frac{132}{25} a^7b^5x^{150} + \frac{99}{25} a^8b^4x^{125} + \frac{11}{5} a^9b^3x^{100} + \frac{22}{25} a^{10}b^2x^{75} + \frac{6}{25} a^{11}bx^{50} + \frac{1}{25} a^{12}x^{25}$$

input

```
integrate((b*x^27+a*x^2)^12,x, algorithm="maxima")
```

output

```
1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9
*x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 +
 132/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a
^10*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int (ax^2 + bx^{27})^{12} \, dx = \frac{1}{325} b^{12}x^{325} + \frac{1}{25} ab^{11}x^{300} + \frac{6}{25} a^2b^{10}x^{275} + \frac{22}{25} a^3b^9x^{250} + \frac{11}{5} a^4b^8x^{225} + \frac{99}{25} a^5b^7x^{200} + \frac{132}{25} a^6b^6x^{175} + \frac{132}{25} a^7b^5x^{150} + \frac{99}{25} a^8b^4x^{125} + \frac{11}{5} a^9b^3x^{100} + \frac{22}{25} a^{10}b^2x^{75} + \frac{6}{25} a^{11}bx^{50} + \frac{1}{25} a^{12}x^{25}$$

input
```
integrate((b*x^27+a*x^2)^12,x, algorithm="giac")
```

output
```
1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9
*x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 +
 132/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a
^10*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int \left(ax^2+bx^{27}\right)^{12}\,dx=\frac{a^{12}\,x^{25}}{25}+\frac{6\,a^{11}\,b\,x^{50}}{25}+\frac{22\,a^{10}\,b^2\,x^{75}}{25}+\frac{11\,a^9\,b^3\,x^{100}}{5}+\frac{99\,a^8\,b^4\,x^{125}}{25}+\frac{132\,a^7\,b^5\,x^{150}}{25}+\frac{132\,a^6\,b^6\,x^{175}}{25}+\frac{99\,a^5\,b^7\,x^{200}}{25}+\frac{11\,a^4\,b^8\,x^{225}}{5}+\frac{22\,a^3\,b^9\,x^{250}}{25}+\frac{6\,a^2\,b^{10}\,x^{275}}{25}+\frac{a\,b^{11}\,x^{300}}{25}+\frac{b^{12}\,x^{325}}{325}$$

input
```
int((a*x^2 + b*x^27)^12,x)
```

output
```
(a^12*x^25)/25 + (b^12*x^325)/325 + (6*a^11*b*x^50)/25 + (a*b^11*x^300)/25
 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x^100)/5 + (99*a^8*b^4*x^125)/25 +
(132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^175)/25 + (99*a^5*b^7*x^200)/25 +
(11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/25 + (6*a^2*b^10*x^275)/25
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

$$\int \left(ax^2+bx^{27}\right)^{12}\,dx$$
$$=\frac{x^{25}(b^{12}x^{300}+13a\,b^{11}x^{275}+78a^2b^{10}x^{250}+286a^3b^9x^{225}+715a^4b^8x^{200}+1287a^5b^7x^{175}+1716a^6b^6x^{150}+1}{325}$$

input
```
int((b*x^27+a*x^2)^12,x)
```

output
```
(x**25*(13*a**12 + 78*a**11*b*x**25 + 286*a**10*b**2*x**50 + 715*a**9*b**3
*x**75 + 1287*a**8*b**4*x**100 + 1716*a**7*b**5*x**125 + 1716*a**6*b**6*x*
*150 + 1287*a**5*b**7*x**175 + 715*a**4*b**8*x**200 + 286*a**3*b**9*x**225
 + 78*a**2*b**10*x**250 + 13*a*b**11*x**275 + b**12*x**300))/325
```

3.312 $\int (ax^3+bx^{40})^{12}\, dx$

Optimal result 2565
Mathematica [B] (verified) 2565
Rubi [A] (verified) 2566
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Reduce [B] (verification not implemented) 2571

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int (ax^3+bx^{40})^{12}\, dx = \frac{(a+bx^{37})^{13}}{481b}$$

output
```
1/481*(b*x^37+a)^13/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.01 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int (ax^3+bx^{40})^{12}\, dx = \frac{a^{12}x^{37}}{37} + \frac{6}{37}a^{11}bx^{74} + \frac{22}{37}a^{10}b^2x^{111} + \frac{55}{37}a^9b^3x^{148} + \frac{99}{37}a^8b^4x^{185} + \frac{132}{37}a^7b^5x^{222} + \frac{132}{37}a^6b^6x^{259} + \frac{99}{37}a^5b^7x^{296} + \frac{55}{37}a^4b^8x^{333} + \frac{22}{37}a^3b^9x^{370} + \frac{6}{37}a^2b^{10}x^{407} + \frac{1}{37}ab^{11}x^{444} + \frac{b^{12}x^{481}}{481}$$

input
```
Integrate[(a*x^3 + b*x^40)^12,x]
```

output
```
(a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3
*x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6
*x^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x
^370)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2027, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^3+bx^{40}\right)^{12} dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int x^{36}\left(a+bx^{37}\right)^{12} dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

input
```
Int[(a*x^3 + b*x^40)^12,x]
```

output
```
(a + b*x^37)^13/(481*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 1.69 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{99}{37}a^8b^4x^{185}+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^{11}b\,x^{74}+\frac{1}{37}a\,b^{11}x^{444}+\frac{132}{37}a^6b^6x^{259}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^4b^8x^{33}$
parallelrisch	$\frac{99}{37}a^8b^4x^{185}+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^{11}b\,x^{74}+\frac{1}{37}a\,b^{11}x^{444}+\frac{132}{37}a^6b^6x^{259}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^4b^8x^{33}$
gosper	$\frac{x^{37}(b^{12}x^{444}+13a\,b^{11}x^{407}+78a^2b^{10}x^{370}+286a^3b^9x^{333}+715a^4b^8x^{296}+1287a^5b^7x^{259}+1716a^6b^6x^{222}+1716a^7b^5x^{185}+1287a}{481}$
risch	$\frac{b^{12}x^{481}}{481}+\frac{a\,b^{11}x^{444}}{37}+\frac{6a^2b^{10}x^{407}}{37}+\frac{22a^3b^9x^{370}}{37}+\frac{55a^4b^8x^{333}}{37}+\frac{99a^5b^7x^{296}}{37}+\frac{132a^6b^6x^{259}}{37}+\frac{132a^7b^5x^{222}}{37}+$
orering	$\frac{x(b^{12}x^{444}+13a\,b^{11}x^{407}+78a^2b^{10}x^{370}+286a^3b^9x^{333}+715a^4b^8x^{296}+1287a^5b^7x^{259}+1716a^6b^6x^{222}+1716a^7b^5x^{185}+1287a^8b}{481(b\,x^{37}+a)^{12}}$

input
```
int((b*x^40+a*x^3)^12,x,method=_RETURNVERBOSE)
```

output
```
99/37*a^8*b^4*x^185+22/37*a^3*b^9*x^370+6/37*a^11*b*x^74+1/37*a*b^11*x^444
+132/37*a^6*b^6*x^259+22/37*a^10*b^2*x^111+55/37*a^4*b^8*x^333+1/481*b^12*
x^481+132/37*a^7*b^5*x^222+55/37*a^9*b^3*x^148+99/37*a^5*b^7*x^296+1/37*a^
12*x^37+6/37*a^2*b^10*x^407
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int \left(ax^3+bx^{40}\right)^{12}\,dx = \frac{1}{481}\,b^{12}x^{481} + \frac{1}{37}\,ab^{11}x^{444} + \frac{6}{37}\,a^2b^{10}x^{407} + \frac{22}{37}\,a^3b^9x^{370} + \frac{55}{37}\,a^4b^8x^{333} + \frac{99}{37}\,a^5b^7x^{296} + \frac{132}{37}\,a^6b^6x^{259} + \frac{132}{37}\,a^7b^5x^{222} + \frac{99}{37}\,a^8b^4x^{185} + \frac{55}{37}\,a^9b^3x^{148} + \frac{22}{37}\,a^{10}b^2x^{111} + \frac{6}{37}\,a^{11}bx^{74} + \frac{1}{37}\,a^{12}x^{37}$$

input

```
integrate((b*x^40+a*x^3)^12,x, algorithm="fricas")
```

output

```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int \left(ax^3+bx^{40}\right)^{12}\,dx = \frac{a^{12}x^{37}}{37} + \frac{6a^{11}bx^{74}}{37} + \frac{22a^{10}b^2x^{111}}{37} + \frac{55a^9b^3x^{148}}{37} + \frac{99a^8b^4x^{185}}{37} + \frac{132a^7b^5x^{222}}{37} + \frac{132a^6b^6x^{259}}{37} + \frac{99a^5b^7x^{296}}{37} + \frac{55a^4b^8x^{333}}{37} + \frac{22a^3b^9x^{370}}{37} + \frac{6a^2b^{10}x^{407}}{37} + \frac{ab^{11}x^{444}}{37} + \frac{b^{12}x^{481}}{481}$$

input

```
integrate((b*x**40+a*x**3)**12,x)
```

output
```
a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b*
*3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6
*b**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**
3*b**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**4
81/481
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int (ax^3+bx^{40})^{12}\,dx =& \frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^2b^{10}x^{407}+\frac{22}{37}\,a^3b^9x^{370}\\&+\frac{55}{37}\,a^4b^8x^{333}+\frac{99}{37}\,a^5b^7x^{296}+\frac{132}{37}\,a^6b^6x^{259}\\&+\frac{132}{37}\,a^7b^5x^{222}+\frac{99}{37}\,a^8b^4x^{185}+\frac{55}{37}\,a^9b^3x^{148}\\&+\frac{22}{37}\,a^{10}b^2x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}\end{aligned}$$

input
```
integrate((b*x^40+a*x^3)^12,x, algorithm="maxima")
```

output
```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int \left(ax^3+bx^{40}\right)^{12}\,dx=\frac{1}{481}b^{12}x^{481}+\frac{1}{37}ab^{11}x^{444}+\frac{6}{37}a^2b^{10}x^{407}+\frac{22}{37}a^3b^9x^{370}+\frac{55}{37}a^4b^8x^{333}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^6b^6x^{259}+\frac{132}{37}a^7b^5x^{222}+\frac{99}{37}a^8b^4x^{185}+\frac{55}{37}a^9b^3x^{148}+\frac{22}{37}a^{10}b^2x^{111}+\frac{6}{37}a^{11}bx^{74}+\frac{1}{37}a^{12}x^{37}$$

input
```
integrate((b*x^40+a*x^3)^12,x, algorithm="giac")
```

output
```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int \left(ax^3+bx^{40}\right)^{12}\,dx=\frac{a^{12}\,x^{37}}{37}+\frac{6\,a^{11}\,b\,x^{74}}{37}+\frac{22\,a^{10}\,b^2\,x^{111}}{37}+\frac{55\,a^9\,b^3\,x^{148}}{37}+\frac{99\,a^8\,b^4\,x^{185}}{37}+\frac{132\,a^7\,b^5\,x^{222}}{37}+\frac{132\,a^6\,b^6\,x^{259}}{37}+\frac{99\,a^5\,b^7\,x^{296}}{37}+\frac{55\,a^4\,b^8\,x^{333}}{37}+\frac{22\,a^3\,b^9\,x^{370}}{37}+\frac{6\,a^2\,b^{10}\,x^{407}}{37}+\frac{a\,b^{11}\,x^{444}}{37}+\frac{b^{12}\,x^{481}}{481}$$

input
```
int((a*x^3 + b*x^40)^12,x)
```

output
```
(a^12*x^37)/37 + (b^12*x^481)/481 + (6*a^11*b*x^74)/37 + (a*b^11*x^444)/37
 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37
+ (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37
+ (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

$$\int \left(ax^3 + bx^{40}\right)^{12} dx$$
$$= \frac{x^{37}(b^{12}x^{444} + 13a\,b^{11}x^{407} + 78a^2b^{10}x^{370} + 286a^3b^9x^{333} + 715a^4b^8x^{296} + 1287a^5b^7x^{259} + 1716a^6b^6x^{222} + 1}{481}$$

input
```
int((b*x^40+a*x^3)^12,x)
```

output
```
(x**37*(13*a**12 + 78*a**11*b*x**37 + 286*a**10*b**2*x**74 + 715*a**9*b**3
*x**111 + 1287*a**8*b**4*x**148 + 1716*a**7*b**5*x**185 + 1716*a**6*b**6*x
**222 + 1287*a**5*b**7*x**259 + 715*a**4*b**8*x**296 + 286*a**3*b**9*x**33
3 + 78*a**2*b**10*x**370 + 13*a*b**11*x**407 + b**12*x**444))/481
```

3.313 $\int (ax^m + bx^{1+13m})^{12}\, dx$

Optimal result . . . 2572
Mathematica [B] (verified) . . . 2572
Rubi [A] (verified) . . . 2573
Maple [B] (verified) . . . 2574
Fricas [B] (verification not implemented) . . . 2575
Sympy [B] (verification not implemented) . . . 2575
Maxima [B] (verification not implemented) . . . 2576
Giac [B] (verification not implemented) . . . 2577
Mupad [B] (verification not implemented) . . . 2578
Reduce [B] (verification not implemented) . . . 2579

Optimal result

Integrand size = 17, antiderivative size = 27

$$\int (ax^m + bx^{1+13m})^{12}\, dx = \frac{(a + bx^{1+12m})^{13}}{13b(1+12m)}$$

output
```
1/13*(a+b*x^(1+12*m))^13/b/(1+12*m)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 193 vs. 2(27) = 54.

Time = 0.08 (sec) , antiderivative size = 193, normalized size of antiderivative = 7.15

$$\int (ax^m + bx^{1+13m})^{12}\, dx$$
$$= \frac{x^{1+12m}(13a^{12} + 78a^{11}bx^{1+12m} + 286a^{10}b^2x^{2+24m} + 715a^9b^3x^{3+36m} + 1287a^8b^4x^{4+48m} + 1716a^7b^5x^{5+60m} -}{}$$

input
```
Integrate[(a*x^m + b*x^(1 + 13*m))^12,x]
```

output
```
(x^(1 + 12*m)*(13*a^12 + 78*a^11*b*x^(1 + 12*m) + 286*a^10*b^2*x^(2 + 24*m
) + 715*a^9*b^3*x^(3 + 36*m) + 1287*a^8*b^4*x^(4 + 48*m) + 1716*a^7*b^5*x^
(5 + 60*m) + 1716*a^6*b^6*x^(6 + 72*m) + 1287*a^5*b^7*x^(7 + 84*m) + 715*a
^4*b^8*x^(8 + 96*m) + 286*a^3*b^9*x^(9 + 108*m) + 78*a^2*b^10*x^(10 + 120*
m) + 13*a*b^11*x^(11 + 132*m) + b^12*x^(12 + 144*m)))/(13 + 156*m)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = $\{2027, 793\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^m+bx^{13m+1}\right)^{12}dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int x^{12m}\left(a+bx^{12m+1}\right)^{12}dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{\left(a+bx^{12m+1}\right)^{13}}{13b(12m+1)}$$

input
```
Int[(a*x^m + b*x^(1 + 13*m))^12,x]
```

output
```
(a + b*x^(1 + 12*m))^13/(13*b*(1 + 12*m))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(25) = 50$.

Time = 54.63 (sec) , antiderivative size = 281, normalized size of antiderivative = 10.41

method	result
parallelrisch	$13a^{12}x^{12m}x+78a^{11}b\,x^{1+13m}x^{11m}x+286a^{10}b^{2}x^{2+26m}x^{10m}x+715a^{9}b^{3}x^{3+39m}x^{9m}x+1287a^{8}b^{4}x^{4+52m}x^{8m}x+1716a^{7}b^{5}x^{5}$
risch	$\frac{b^{12}x^{13}x^{156m}}{13+156m}+\frac{a\,b^{11}x^{12}x^{144m}}{1+12m}+\frac{6a^{2}b^{10}x^{11}x^{132m}}{1+12m}+\frac{22a^{3}b^{9}x^{10}x^{120m}}{1+12m}+\frac{55a^{4}b^{8}x^{9}x^{108m}}{1+12m}+\frac{99a^{5}b^{7}x^{8}x^{96m}}{1+12m}+132$
orering	Expression too large to display

input
```
int((a*x^m+b*x^(1+13*m))^12,x,method=_RETURNVERBOSE)
```

output
```
1/13*(13*a^12*(x^m)^12*x+78*a^11*b*x^(1+13*m)*(x^m)^11*x+286*a^10*b^2*(x^(
1+13*m))^2*(x^m)^10*x+715*a^9*b^3*(x^(1+13*m))^3*(x^m)^9*x+1287*a^8*b^4*(x
^(1+13*m))^4*(x^m)^8*x+1716*a^7*b^5*(x^(1+13*m))^5*(x^m)^7*x+1716*a^6*b^6*
(x^(1+13*m))^6*(x^m)^6*x+1287*a^5*b^7*(x^(1+13*m))^7*(x^m)^5*x+715*a^4*b^8
*(x^(1+13*m))^8*(x^m)^4*x+286*a^3*b^9*(x^(1+13*m))^9*(x^m)^3*x+78*a^2*b^10
*(x^(1+13*m))^10*(x^m)^2*x+13*a*b^11*(x^(1+13*m))^11*x^m*x+b^12*(x^(1+13*m
))^12*x)/(1+12*m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(25) = 50$.

Time = 0.09 (sec) , antiderivative size = 205, normalized size of antiderivative = 7.59

$$\int \left(ax^m + bx^{1+13m}\right)^{12} dx = \frac{b^{12}x^{13}x^{156\,m} + 13\,ab^{11}x^{12}x^{144\,m} + 78\,a^2b^{10}x^{11}x^{132\,m} + 286\,a^3b^9x^{10}x^{120\,m} + 715\,a^4b^8x^9x^{108\,m} + 1287\,a^5b^7x^8}{}$$

input
```
integrate((a*x^m+b*x^(1+13*m))^12,x, algorithm="fricas")
```

output
```
1/13*(b^12*x^13*x^(156*m) + 13*a*b^11*x^12*x^(144*m) + 78*a^2*b^10*x^11*x^
(132*m) + 286*a^3*b^9*x^10*x^(120*m) + 715*a^4*b^8*x^9*x^(108*m) + 1287*a^
5*b^7*x^8*x^(96*m) + 1716*a^6*b^6*x^7*x^(84*m) + 1716*a^7*b^5*x^6*x^(72*m)
 + 1287*a^8*b^4*x^5*x^(60*m) + 715*a^9*b^3*x^4*x^(48*m) + 286*a^10*b^2*x^3
*x^(36*m) + 78*a^11*b*x^2*x^(24*m) + 13*a^12*x*x^(12*m))/(12*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(19) = 38$.

Time = 5.09 (sec) , antiderivative size = 471, normalized size of antiderivative = 17.44

$$\int \left(ax^m + bx^{1+13m}\right)^{12} dx = \begin{cases} \frac{13a^{12}xx^{12m}}{156m+13} + \frac{78a^{11}bxx^{11m}x^{13m+1}}{156m+13} + \frac{286a^{10}b^2xx^{10m}x^{26m+2}}{156m+13} + \frac{715a^9b^3xx^{9m}x^{39m+3}}{156m+13} + \frac{1287a^8b^4xx^{8m}x^{52m+4}}{156m+13} + \frac{1716a^7b^5xx^{7m}}{156m+1} \\ a^{12}\log(x) + 12a^{11}b\log(x) + 66a^{10}b^2\log(x) + 220a^9b^3\log(x) + 495a^8b^4\log(x) + 792a^7b^5\log(x) + 92 \end{cases}$$

input
```
integrate((a*x**m+b*x**(1+13*m))**12,x)
```

output
```
Piecewise((13*a**12*x*x**(12*m)/(156*m + 13) + 78*a**11*b*x*x**(11*m)*x**(
13*m + 1)/(156*m + 13) + 286*a**10*b**2*x*x**(10*m)*x**(26*m + 2)/(156*m +
 13) + 715*a**9*b**3*x*x**(9*m)*x**(39*m + 3)/(156*m + 13) + 1287*a**8*b**
4*x*x**(8*m)*x**(52*m + 4)/(156*m + 13) + 1716*a**7*b**5*x*x**(7*m)*x**(65
*m + 5)/(156*m + 13) + 1716*a**6*b**6*x*x**(6*m)*x**(78*m + 6)/(156*m + 13
) + 1287*a**5*b**7*x*x**(5*m)*x**(91*m + 7)/(156*m + 13) + 715*a**4*b**8*x
*x**(4*m)*x**(104*m + 8)/(156*m + 13) + 286*a**3*b**9*x*x**(3*m)*x**(117*m
 + 9)/(156*m + 13) + 78*a**2*b**10*x*x**(2*m)*x**(130*m + 10)/(156*m + 13)
 + 13*a*b**11*x*x**m*x**(143*m + 11)/(156*m + 13) + b**12*x*x**(156*m + 12
)/(156*m + 13), Ne(m, -1/12)), (a**12*log(x) + 12*a**11*b*log(x) + 66*a**1
0*b**2*log(x) + 220*a**9*b**3*log(x) + 495*a**8*b**4*log(x) + 792*a**7*b**
5*log(x) + 924*a**6*b**6*log(x) + 792*a**5*b**7*log(x) + 495*a**4*b**8*log
(x) + 220*a**3*b**9*log(x) + 66*a**2*b**10*log(x) + 12*a*b**11*log(x) + b*
*12*log(x), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(25) = 50$.

Time = 0.04 (sec) , antiderivative size = 275, normalized size of antiderivative = 10.19

$$\int \left(ax^m + bx^{1+13m}\right)^{12} dx = \frac{b^{12}x^{156\,m+13}}{13\,(12\,m+1)} + \frac{ab^{11}x^{144\,m+12}}{12\,m+1} + \frac{6\,a^2b^{10}x^{132\,m+11}}{12\,m+1} + \frac{22\,a^3b^9x^{120\,m+10}}{12\,m+1} + \frac{55\,a^4b^8x^{108\,m+9}}{12\,m+1} + \frac{99\,a^5b^7x^{96\,m+8}}{12\,m+1} + \frac{132\,a^6b^6x^{84\,m+7}}{12\,m+1} + \frac{132\,a^7b^5x^{72\,m+6}}{12\,m+1} + \frac{99\,a^8b^4x^{60\,m+5}}{12\,m+1} + \frac{55\,a^9b^3x^{48\,m+4}}{12\,m+1} + \frac{22\,a^{10}b^2x^{36\,m+3}}{12\,m+1} + \frac{6\,a^{11}bx^{24\,m+2}}{12\,m+1} + \frac{a^{12}x^{12\,m+1}}{12\,m+1}$$

input
```
integrate((a*x^m+b*x^(1+13*m))^12,x, algorithm="maxima")
```

output
```
1/13*b^12*x^(156*m + 13)/(12*m + 1) + a*b^11*x^(144*m + 12)/(12*m + 1) + 6
*a^2*b^10*x^(132*m + 11)/(12*m + 1) + 22*a^3*b^9*x^(120*m + 10)/(12*m + 1)
 + 55*a^4*b^8*x^(108*m + 9)/(12*m + 1) + 99*a^5*b^7*x^(96*m + 8)/(12*m + 1
) + 132*a^6*b^6*x^(84*m + 7)/(12*m + 1) + 132*a^7*b^5*x^(72*m + 6)/(12*m +
 1) + 99*a^8*b^4*x^(60*m + 5)/(12*m + 1) + 55*a^9*b^3*x^(48*m + 4)/(12*m +
 1) + 22*a^10*b^2*x^(36*m + 3)/(12*m + 1) + 6*a^11*b*x^(24*m + 2)/(12*m +
1) + a^12*x^(12*m + 1)/(12*m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62225 vs. $2(25) = 50$.

Time = 6.77 (sec) , antiderivative size = 62225, normalized size of antiderivative = 2304.63

$$\int \left(ax^m + bx^{1+13m}\right)^{12} dx = \text{Too large to display}$$

input
```
integrate((a*x^m+b*x^(1+13*m))^12,x, algorithm="giac")
```

output

```
1/516792671179609850285798050743837318394721352172304883731466447462921824
61678212195512812969327305874098880017201709248885135017652458118963024850
50960406291251721993420556288886359619022280199069073831268029705887083006
43428435998184455748480991205357250120080001470299251101685759440052974111
55728275215854822884322447802943519587193876170616396367658960957920584551
87085060407497887581187669244111709118190190960860489863026473129163200921
60000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000*(95407877756235664668147332445016120319025480401040901
61196303645469325992925208405325442394337348776756716310868007861332640310
95122303734701997240177305776846471752631487314871327929665651729058905938
08025163800992301657248326645818668753565721453296723099091692579132169434
15737097354824137444134450809080890378644144209774188231481946369959950098
64473125376995407111461857305999610014988492783528315529512035254312705820
86642580846089863168000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000*b^12*m^2*x^13*x^(144*m)*log(x)^144
- 105682572283830274709332429785248625584151301367306844862482132688275610
99855615464360490036804447875792054990499947169476155421361354749061314430
96811784860506861018299493641088240168245029607572941962181201814410991649
12644300284599140773180491455959447125147721010731018450143549386084205830
15041237819289601650190436663134485425641540594417175493883394619560564...
```

Mupad [B] (verification not implemented)

Time = 9.72 (sec) , antiderivative size = 285, normalized size of antiderivative = 10.56

$$\int (ax^m + bx^{1+13m})^{12} \, dx = \frac{b^{12}\,x^{156\,m}\,x^{13}}{156\,m+13} + \frac{a^{12}\,x\,x^{12\,m}}{12\,m+1} + \frac{6\,a^{11}\,b\,x^{24\,m}\,x^2}{12\,m+1} + \frac{a\,b^{11}\,x^{144\,m}\,x^{12}}{12\,m+1} + \frac{22\,a^{10}\,b^2\,x^{36\,m}\,x^3}{12\,m+1} + \frac{55\,a^9\,b^3\,x^{48\,m}\,x^4}{12\,m+1} + \frac{99\,a^8\,b^4\,x^{60\,m}\,x^5}{12\,m+1} + \frac{132\,a^7\,b^5\,x^{72\,m}\,x^6}{12\,m+1} + \frac{132\,a^6\,b^6\,x^{84\,m}\,x^7}{12\,m+1} + \frac{99\,a^5\,b^7\,x^{96\,m}\,x^8}{12\,m+1} + \frac{55\,a^4\,b^8\,x^{108\,m}\,x^9}{12\,m+1} + \frac{22\,a^3\,b^9\,x^{120\,m}\,x^{10}}{12\,m+1} + \frac{6\,a^2\,b^{10}\,x^{132\,m}\,x^{11}}{12\,m+1}$$

input

```
int((a*x^m + b*x^(13*m + 1))^12,x)
```

output
```
(b^12*x^(156*m)*x^13)/(156*m + 13) + (a^12*x*x^(12*m))/(12*m + 1) + (6*a^1
1*b*x^(24*m)*x^2)/(12*m + 1) + (a*b^11*x^(144*m)*x^12)/(12*m + 1) + (22*a^
10*b^2*x^(36*m)*x^3)/(12*m + 1) + (55*a^9*b^3*x^(48*m)*x^4)/(12*m + 1) + (
99*a^8*b^4*x^(60*m)*x^5)/(12*m + 1) + (132*a^7*b^5*x^(72*m)*x^6)/(12*m + 1
) + (132*a^6*b^6*x^(84*m)*x^7)/(12*m + 1) + (99*a^5*b^7*x^(96*m)*x^8)/(12*
m + 1) + (55*a^4*b^8*x^(108*m)*x^9)/(12*m + 1) + (22*a^3*b^9*x^(120*m)*x^1
0)/(12*m + 1) + (6*a^2*b^10*x^(132*m)*x^11)/(12*m + 1)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 202, normalized size of antiderivative = 7.48

$$\int \left(ax^m + bx^{1+13m}\right)^{12} dx$$
$$= \frac{x^{12m}x(x^{144m}b^{12}x^{12} + 13x^{132m}a\,b^{11}x^{11} + 78x^{120m}a^2b^{10}x^{10} + 286x^{108m}a^3b^9x^9 + 715x^{96m}a^4b^8x^8 + 1287x^{84m}}{}$$

input
```
int((a*x^m+b*x^(1+13*m))^12,x)
```

output
```
(x**(12*m)*x*(x**(144*m)*b**12*x**12 + 13*x**(132*m)*a*b**11*x**11 + 78*x*
*(120*m)*a**2*b**10*x**10 + 286*x**(108*m)*a**3*b**9*x**9 + 715*x**(96*m)*
a**4*b**8*x**8 + 1287*x**(84*m)*a**5*b**7*x**7 + 1716*x**(72*m)*a**6*b**6*
x**6 + 1716*x**(60*m)*a**7*b**5*x**5 + 1287*x**(48*m)*a**8*b**4*x**4 + 715
*x**(36*m)*a**9*b**3*x**3 + 286*x**(24*m)*a**10*b**2*x**2 + 78*x**(12*m)*a
**11*b*x + 13*a**12))/(13*(12*m + 1))
```

3.314 $\int (ax^m + bx^{1+6m})^5 \, dx$

Optimal result 2580
Mathematica [B] (verified) 2580
Rubi [A] (verified) 2581
Maple [B] (verified) 2582
Fricas [B] (verification not implemented) 2582
Sympy [B] (verification not implemented) 2583
Maxima [B] (verification not implemented) 2583
Giac [B] (verification not implemented) 2584
Mupad [B] (verification not implemented) 2584
Reduce [B] (verification not implemented) 2585

Optimal result

Integrand size = 17, antiderivative size = 27

$$\int (ax^m + bx^{1+6m})^5 \, dx = \frac{(a + bx^{1+5m})^6}{6b(1 + 5m)}$$

output
```
1/6*(a+b*x^(1+5*m))^6/b/(1+5*m)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 88 vs. 2(27) = 54.

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.26

$$\int (ax^m + bx^{1+6m})^5 \, dx = \frac{x^{1+5m}(6a^5 + 15a^4bx^{1+5m} + 20a^3b^2x^{2+10m} + 15a^2b^3x^{3+15m} + 6ab^4x^{4+20m} + b^5x^{5+25m})}{6 + 30m}$$

input
```
Integrate[(a*x^m + b*x^(1 + 6*m))^5,x]
```

output
```
(x^(1 + 5*m)*(6*a^5 + 15*a^4*b*x^(1 + 5*m) + 20*a^3*b^2*x^(2 + 10*m) + 15*
a^2*b^3*x^(3 + 15*m) + 6*a*b^4*x^(4 + 20*m) + b^5*x^(5 + 25*m)))/(6 + 30*m
)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2027, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^m + bx^{6m+1}\right)^5 dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int x^{5m} \left(a + bx^{5m+1}\right)^5 dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{\left(a + bx^{5m+1}\right)^6}{6b(5m+1)}$$

input
```
Int[(a*x^m + b*x^(1 + 6*m))^5,x]
```

output
```
(a + b*x^(1 + 5*m))^6/(6*b*(1 + 5*m))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(25) = 50.

Time = 1.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 4.44

method	result	size
parallelrisch	$\frac{6x\,x^{5m}a^5+15x\,x^{4m}x^{1+6m}a^4b+20x\,x^{3m}x^{2+12m}a^3b^2+15x\,x^{2m}x^{3+18m}a^2b^3+6x\,x^m x^{4+24m}a\,b^4+x\,x^{5+30m}b^5}{6+30m}$	120
risch	$\frac{b^5x^6x^{30m}}{6+30m}+\frac{a\,b^4x^5x^{25m}}{1+5m}+\frac{5a^2b^3x^4x^{20m}}{2(1+5m)}+\frac{10a^3b^2x^3x^{15m}}{3(1+5m)}+\frac{5a^4x^2b\,x^{10m}}{2(1+5m)}+\frac{a^5x\,x^{5m}}{1+5m}$	126
orering	Expression too large to display	2157

input
```
int((a*x^m+b*x^(1+6*m))^5,x,method=_RETURNVERBOSE)
```

output
```
1/6*(6*x*(x^m)^5*a^5+15*x*(x^m)^4*x^(1+6*m)*a^4*b+20*x*(x^m)^3*(x^(1+6*m))
^2*a^3*b^2+15*x*(x^m)^2*(x^(1+6*m))^3*a^2*b^3+6*x*x^m*(x^(1+6*m))^4*a*b^4+
x*(x^(1+6*m))^5*b^5)/(1+5*m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(25) = 50.

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.44

$$\int \left(ax^m+bx^{1+6m}\right)^5\,dx$$
$$=\frac{b^5x^6x^{30\,m}+6\,ab^4x^5x^{25\,m}+15\,a^2b^3x^4x^{20\,m}+20\,a^3b^2x^3x^{15\,m}+15\,a^4bx^2x^{10\,m}+6\,a^5xx^{5\,m}}{6\,(5\,m+1)}$$

input
```
integrate((a*x^m+b*x^(1+6*m))^5,x, algorithm="fricas")
```

output
```
1/6*(b^5*x^6*x^(30*m) + 6*a*b^4*x^5*x^(25*m) + 15*a^2*b^3*x^4*x^(20*m) + 2
0*a^3*b^2*x^3*x^(15*m) + 15*a^4*b*x^2*x^(10*m) + 6*a^5*x*x^(5*m))/(5*m + 1
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(19) = 38$.

Time = 0.60 (sec) , antiderivative size = 197, normalized size of antiderivative = 7.30

$$\int \left(ax^m + bx^{1+6m}\right)^5 dx$$
$$= \begin{cases} \frac{6a^5xx^{5m}}{30m+6} + \frac{15a^4bxx^{4m}x^{6m+1}}{30m+6} + \frac{20a^3b^2xx^{3m}x^{12m+2}}{30m+6} + \frac{15a^2b^3xx^{2m}x^{18m+3}}{30m+6} + \frac{6ab^4xx^mx^{24m+4}}{30m+6} + \frac{b^5xx^{30m+5}}{30m+6} & \text{for } m \neq -\frac{1}{5} \\ a^5 \log(x) + 5a^4b\log(x) + 10a^3b^2\log(x) + 10a^2b^3\log(x) + 5ab^4\log(x) + b^5\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((a*x**m+b*x**(1+6*m))**5,x)
```

output
```
Piecewise((6*a**5*x*x**(5*m)/(30*m + 6) + 15*a**4*b*x*x**(4*m)*x**(6*m + 1
)/(30*m + 6) + 20*a**3*b**2*x*x**(3*m)*x**(12*m + 2)/(30*m + 6) + 15*a**2*
b**3*x*x**(2*m)*x**(18*m + 3)/(30*m + 6) + 6*a*b**4*x*x**m*x**(24*m + 4)/(
30*m + 6) + b**5*x*x**(30*m + 5)/(30*m + 6), Ne(m, -1/5)), (a**5*log(x) +
5*a**4*b*log(x) + 10*a**3*b**2*log(x) + 10*a**2*b**3*log(x) + 5*a*b**4*log
(x) + b**5*log(x), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(25) = 50$.

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 4.48

$$\int \left(ax^m + bx^{1+6m}\right)^5 dx = \frac{b^5x^{30m+6}}{6(5m+1)} + \frac{ab^4x^{25m+5}}{5m+1} + \frac{5a^2b^3x^{20m+4}}{2(5m+1)} + \frac{10a^3b^2x^{15m+3}}{3(5m+1)} + \frac{5a^4bx^{10m+2}}{2(5m+1)} + \frac{a^5x^{5m+1}}{5m+1}$$

input
```
integrate((a*x^m+b*x^(1+6*m))^5,x, algorithm="maxima")
```

output
```
1/6*b^5*x^(30*m + 6)/(5*m + 1) + a*b^4*x^(25*m + 5)/(5*m + 1) + 5/2*a^2*b^
3*x^(20*m + 4)/(5*m + 1) + 10/3*a^3*b^2*x^(15*m + 3)/(5*m + 1) + 5/2*a^4*b
*x^(10*m + 2)/(5*m + 1) + a^5*x^(5*m + 1)/(5*m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(25) = 50$.

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.44

$$\int \left(ax^m + bx^{1+6m}\right)^5 \, dx = \frac{b^5x^6x^{30\,m} + 6\,ab^4x^5x^{25\,m} + 15\,a^2b^3x^4x^{20\,m} + 20\,a^3b^2x^3x^{15\,m} + 15\,a^4bx^2x^{10\,m} + 6\,a^5xx^{5\,m}}{6\,(5\,m + 1)}$$

input
```
integrate((a*x^m+b*x^(1+6*m))^5,x, algorithm="giac")
```

output
```
1/6*(b^5*x^6*x^(30*m) + 6*a*b^4*x^5*x^(25*m) + 15*a^2*b^3*x^4*x^(20*m) + 2
0*a^3*b^2*x^3*x^(15*m) + 15*a^4*b*x^2*x^(10*m) + 6*a^5*x*x^(5*m))/(5*m + 1
)
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 4.59

$$\int \left(ax^m + bx^{1+6m}\right)^5 \, dx = \frac{b^5\,x^{30\,m}\,x^6}{30\,m + 6} + \frac{a^5\,x\,x^{5\,m}}{5\,m + 1} + \frac{5\,a^4\,b\,x^{10\,m}\,x^2}{10\,m + 2} + \frac{a\,b^4\,x^{25\,m}\,x^5}{5\,m + 1} + \frac{5\,a^2\,b^3\,x^{20\,m}\,x^4}{10\,m + 2} + \frac{10\,a^3\,b^2\,x^{15\,m}\,x^3}{15\,m + 3}$$

input
```
int((a*x^m + b*x^(6*m + 1))^5,x)
```

output
```
(b^5*x^(30*m)*x^6)/(30*m + 6) + (a^5*x*x^(5*m))/(5*m + 1) + (5*a^4*b*x^(10
*m)*x^2)/(10*m + 2) + (a*b^4*x^(25*m)*x^5)/(5*m + 1) + (5*a^2*b^3*x^(20*m)
*x^4)/(10*m + 2) + (10*a^3*b^2*x^(15*m)*x^3)/(15*m + 3)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.33

$$\int (ax^m + bx^{1+6m})^5 \, dx = \frac{x^{5m}x(x^{25m}b^5x^5 + 6x^{20m}a\,b^4x^4 + 15x^{15m}a^2b^3x^3 + 20x^{10m}a^3b^2x^2 + 15x^{5m}a^4bx + 6a^5)}{30m + 6}$$

input
```
int((a*x^m+b*x^(1+6*m))^5,x)
```

output
```
(x**(5*m)*x*(x**(25*m)*b**5*x**5 + 6*x**(20*m)*a*b**4*x**4 + 15*x**(15*m)*
a**2*b**3*x**3 + 20*x**(10*m)*a**3*b**2*x**2 + 15*x**(5*m)*a**4*b*x + 6*a*
*5))/(6*(5*m + 1))
```

3.315 $\int \frac{1}{(bx^{1-2m}+ax^m)^3} dx$

Optimal result . . . 2586
Mathematica [A] (verified) . . . 2586
Rubi [A] (verified) . . . 2587
Maple [A] (verified) . . . 2588
Fricas [B] (verification not implemented) . . . 2588
Sympy [F(-1)] . . . 2589
Maxima [B] (verification not implemented) . . . 2589
Giac [F] . . . 2589
Mupad [B] (verification not implemented) . . . 2590
Reduce [B] (verification not implemented) . . . 2590

Optimal result

Integrand size = 17, antiderivative size = 27

$$\int \frac{1}{(bx^{1-2m}+ax^m)^3} dx = -\frac{1}{2b(1-3m)(a+bx^{1-3m})^2}$$

output
```
-1/2/b/(1-3*m)/(a+b*x^(1-3*m))^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bx^{1-2m}+ax^m)^3} dx = -\frac{1}{2b(1-3m)(a+bx^{1-3m})^2}$$

input
```
Integrate[(b*x^(1 - 2*m) + a*x^m)^(-3),x]
```

output
```
-1/2*1/(b*(1 - 3*m)*(a + b*x^(1 - 3*m))^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = $\{2027, 793\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^m+bx^{1-2m})^3}dx$$
$$\Big\downarrow \ 2027$$
$$\int \frac{x^{-3m}}{(a+bx^{1-3m})^3}dx$$
$$\Big\downarrow \ 793$$
$$-\frac{1}{2b(1-3m)\left(a+bx^{1-3m}\right)^2}$$

input
```
Int[(b*x^(1 - 2*m) + a*x^m)^(-3),x]
```

output
```
-1/2*1/(b*(1 - 3*m)*(a + b*x^(1 - 3*m))^2)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

method	result	size
risch	$-\frac{x(2a\,x^{3m}+bx)}{2(3m-1)a^2(a\,x^{3m}+bx)^2}$	39

input
```
int(1/(b*x^(1-2*m)+a*x^m)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*(2*a*(x^m)^3+b*x)/(3*m-1)/a^2/(a*(x^m)^3+b*x)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(25) = 50$.

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.04

$$\int \frac{1}{(bx^{1-2m}+ax^m)^3}\,dx = -\frac{2\,axx^{3\,m}+bx^2}{2\,(2\,(3\,a^3bm-a^3b)xx^{3\,m}+(3\,a^2b^2m-a^2b^2)x^2+(3\,a^4m-a^4)x^{6\,m})}$$

input
```
integrate(1/(b*x^(1-2*m)+a*x^m)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*a*x*x^(3*m) + b*x^2)/(2*(3*a^3*b*m - a^3*b)*x*x^(3*m) + (3*a^2*b^2
*m - a^2*b^2)*x^2 + (3*a^4*m - a^4)*x^(6*m))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(bx^{1-2m}+ax^m)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**(1-2*m)+a*x**m)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(25) = 50$.

Time $= 0.04$ (sec) , antiderivative size $= 66$, normalized size of antiderivative $= 2.44$

$$\int \frac{1}{(bx^{1-2m}+ax^m)^3}\,dx = -\frac{2\,axx^{3\,m}+bx^2}{2\,(2\,a^3b(3\,m-1)xx^{3\,m}+a^2b^2(3\,m-1)x^2+a^4(3\,m-1)x^{6\,m})}$$

input
```
integrate(1/(b*x^(1-2*m)+a*x^m)^3,x, algorithm="maxima")
```

output
```
-1/2*(2*a*x*x^(3*m) + b*x^2)/(2*a^3*b*(3*m - 1)*x*x^(3*m) + a^2*b^2*(3*m -
 1)*x^2 + a^4*(3*m - 1)*x^(6*m))
```

Giac [F]

$$\int \frac{1}{(bx^{1-2m}+ax^m)^3}\,dx = \int \frac{1}{(ax^m+bx^{-2\,m+1})^3}\,dx$$

input
```
integrate(1/(b*x^(1-2*m)+a*x^m)^3,x, algorithm="giac")
```

output
```
integrate((a*x^m + b*x^(-2*m + 1))^(-3), x)
```

Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{1}{(bx^{1-2m}+ax^m)^3}\,dx = -\frac{x\,(b\,x+2\,a\,x^{3\,m})}{2\,a^2\,(3\,m-1)\,(b\,x+a\,x^{3\,m})^2}$$

input
```
int(1/(a*x^m + b*x^(1 - 2*m))^3,x)
```

output
```
-(x*(b*x + 2*a*x^(3*m)))/(2*a^2*(3*m - 1)*(b*x + a*x^(3*m))^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.67

$$\int \frac{1}{(bx^{1-2m}+ax^m)^3}\,dx = \frac{x^{6m}}{2b\,(3x^{6m}a^2m-x^{6m}a^2+6x^{3m}abmx-2x^{3m}abx+3b^2m\,x^2-b^2x^2)}$$

input
```
int(1/(b*x^(1-2*m)+a*x^m)^3,x)
```

output
```
x**(6*m)/(2*b*(3*x**(6*m)*a**2*m - x**(6*m)*a**2 + 6*x**(3*m)*a*b*m*x - 2*
x**(3*m)*a*b*x + 3*b**2*m*x**2 - b**2*x**2))
```

3.316 $\int \frac{1}{\frac{b}{x}+ax}\,dx$

Optimal result 2591
Mathematica [A] (verified) 2591
Rubi [A] (verified) 2592
Maple [A] (verified) 2593
Fricas [A] (verification not implemented) 2593
Sympy [A] (verification not implemented) 2593
Maxima [A] (verification not implemented) 2594
Giac [A] (verification not implemented) 2594
Mupad [B] (verification not implemented) 2594
Reduce [B] (verification not implemented) 2595

Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{1}{\frac{b}{x}+ax}\,dx=\frac{\log(b+ax^2)}{2a}$$

output
```
1/2*ln(a*x^2+b)/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\frac{b}{x}+ax}\,dx=\frac{\log(b+ax^2)}{2a}$$

input
```
Integrate[(b/x + a*x)^(-1),x]
```

output
```
Log[b + a*x^2]/(2*a)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2027, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax+\frac{b}{x}}dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{x}{ax^2+b}dx$$

$$\Big\downarrow \text{ 240}$$

$$\frac{\log \left(ax^2+b\right)}{2a}$$

input
```
Int[(b/x + a*x)^(-1),x]
```

output
```
Log[b + a*x^2]/(2*a)
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln(a\,x^2+b)}{2a}$	14
norman	$\frac{\ln(a\,x^2+b)}{2a}$	14
risch	$\frac{\ln(a\,x^2+b)}{2a}$	14
parallelrisch	$\frac{\ln(a\,x^2+b)}{2a}$	14

input
```
int(1/(b/x+a*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(a*x^2+b)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x}+ax}\,dx = \frac{\log\left(ax^2+b\right)}{2\,a}$$

input
```
integrate(1/(b/x+a*x),x, algorithm="fricas")
```

output
```
1/2*log(a*x^2 + b)/a
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1}{\frac{b}{x}+ax}\,dx = \frac{\log\left(ax^2+b\right)}{2a}$$

input
```
integrate(1/(b/x+a*x),x)
```

output
```
log(a*x**2 + b)/(2*a)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x}+ax}\,dx=\frac{\log\left(ax^2+b\right)}{2\,a}$$

input
```
integrate(1/(b/x+a*x),x, algorithm="maxima")
```

output
```
1/2*log(a*x^2 + b)/a
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{1}{\frac{b}{x}+ax}\,dx=\frac{\log\left(|ax^2+b|\right)}{2\,a}$$

input
```
integrate(1/(b/x+a*x),x, algorithm="giac")
```

output
```
1/2*log(abs(a*x^2 + b))/a
```

Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x}+ax}\,dx=\frac{\ln\left(a\,x^2+b\right)}{2\,a}$$

input
```
int(1/(a*x + b/x),x)
```

output
```
log(b + a*x^2)/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x} + ax} \, dx = \frac{\log(a\,x^2 + b)}{2a}$$

input
```
int(1/(b/x+a*x),x)
```

output
```
log(a*x**2 + b)/(2*a)
```

3.317 $\int \frac{1}{\frac{b}{x^2}+ax}\,dx$

Optimal result . . . 2596
Mathematica [A] (verified) . . . 2596
Rubi [A] (verified) . . . 2597
Maple [A] (verified) . . . 2598
Fricas [A] (verification not implemented) . . . 2598
Sympy [A] (verification not implemented) . . . 2598
Maxima [A] (verification not implemented) . . . 2599
Giac [A] (verification not implemented) . . . 2599
Mupad [B] (verification not implemented) . . . 2599
Reduce [B] (verification not implemented) . . . 2600

Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{1}{\frac{b}{x^2}+ax}\,dx = \frac{\log(b+ax^3)}{3a}$$

output
```
1/3*ln(a*x^3+b)/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\frac{b}{x^2}+ax}\,dx = \frac{\log(b+ax^3)}{3a}$$

input
```
Integrate[(b/x^2 + a*x)^(-1),x]
```

output
```
Log[b + a*x^3]/(3*a)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2027, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax+\frac{b}{x^2}}dx$$
$$\Big\downarrow \text{ 2027}$$
$$\int \frac{x^2}{ax^3+b}dx$$
$$\Big\downarrow \text{ 792}$$
$$\frac{\log(ax^3+b)}{3a}$$

input
```
Int[(b/x^2 + a*x)^(-1),x]
```

output
```
Log[b + a*x^3]/(3*a)
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln(a\,x^3+b)}{3a}$	14
norman	$\frac{\ln(a\,x^3+b)}{3a}$	14
risch	$\frac{\ln(a\,x^3+b)}{3a}$	14
parallelrisch	$\frac{\ln(a\,x^3+b)}{3a}$	14

input
```
int(1/(b/x^2+a*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(a*x^3+b)/a
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x^2}+ax}\,dx=\frac{\log(ax^3+b)}{3\,a}$$

input
```
integrate(1/(b/x^2+a*x),x, algorithm="fricas")
```

output
```
1/3*log(a*x^3 + b)/a
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1}{\frac{b}{x^2}+ax}\,dx=\frac{\log(ax^3+b)}{3a}$$

input
```
integrate(1/(b/x**2+a*x),x)
```

output
```
log(a*x**3 + b)/(3*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x^2}+ax}\,dx = \frac{\log(ax^3+b)}{3\,a}$$

input
```
integrate(1/(b/x^2+a*x),x, algorithm="maxima")
```

output
```
1/3*log(a*x^3 + b)/a
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{1}{\frac{b}{x^2}+ax}\,dx = \frac{\log(|ax^3+b|)}{3\,a}$$

input
```
integrate(1/(b/x^2+a*x),x, algorithm="giac")
```

output
```
1/3*log(abs(a*x^3 + b))/a
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x^2}+ax}\,dx = \frac{\ln(a\,x^3+b)}{3\,a}$$

input
```
int(1/(a*x + b/x^2),x)
```

output
```
log(b + a*x^3)/(3*a)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.47

$$\int \frac{1}{\frac{b}{x^2}+ax}\,dx=\frac{\log\left(a^{\frac{2}{3}}x^2-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)+\log\left(a^{\frac{1}{3}}x+b^{\frac{1}{3}}\right)}{3a}$$

input
```
int(1/(b/x^2+a*x),x)
```

output
```
(log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3)) + log(a**(1/3)*x + b*
*(1/3)))/(3*a)
```

3.318 $\int \frac{1}{\frac{b}{x^3}+ax}\,dx$

Optimal result . . . 2601
Mathematica [A] (verified) . . . 2601
Rubi [A] (verified) . . . 2602
Maple [A] (verified) . . . 2603
Fricas [A] (verification not implemented) . . . 2603
Sympy [A] (verification not implemented) . . . 2603
Maxima [A] (verification not implemented) . . . 2604
Giac [A] (verification not implemented) . . . 2604
Mupad [B] (verification not implemented) . . . 2604
Reduce [B] (verification not implemented) . . . 2605

Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{1}{\frac{b}{x^3}+ax}\,dx=\frac{\log\left(b+ax^4\right)}{4a}$$

output
```
1/4*ln(a*x^4+b)/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\frac{b}{x^3}+ax}\,dx=\frac{\log\left(b+ax^4\right)}{4a}$$

input
```
Integrate[(b/x^3 + a*x)^(-1),x]
```

output
```
Log[b + a*x^4]/(4*a)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2027, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax+\frac{b}{x^3}}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x^3}{ax^4+b}dx$$

$$\Big\downarrow \text{792}$$

$$\frac{\log(ax^4+b)}{4a}$$

input
```
Int[(b/x^3 + a*x)^(-1),x]
```

output
```
Log[b + a*x^4]/(4*a)
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln(a\,x^4+b)}{4a}$	14
norman	$\frac{\ln(a\,x^4+b)}{4a}$	14
risch	$\frac{\ln(a\,x^4+b)}{4a}$	14
parallelrisch	$\frac{\ln(a\,x^4+b)}{4a}$	14

input
```
int(1/(b/x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(a*x^4+b)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x^3}+ax}\,dx=\frac{\log(ax^4+b)}{4\,a}$$

input
```
integrate(1/(b/x^3+a*x),x, algorithm="fricas")
```

output
```
1/4*log(a*x^4 + b)/a
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1}{\frac{b}{x^3}+ax}\,dx=\frac{\log(ax^4+b)}{4a}$$

input
```
integrate(1/(b/x**3+a*x),x)
```

output
```
log(a*x**4 + b)/(4*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x^3}+ax}\,dx=\frac{\log\left(ax^4+b\right)}{4\,a}$$

input
```
integrate(1/(b/x^3+a*x),x, algorithm="maxima")
```

output
```
1/4*log(a*x^4 + b)/a
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{1}{\frac{b}{x^3}+ax}\,dx=\frac{\log\left(|ax^4+b|\right)}{4\,a}$$

input
```
integrate(1/(b/x^3+a*x),x, algorithm="giac")
```

output
```
1/4*log(abs(a*x^4 + b))/a
```

Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x^3}+ax}\,dx=\frac{\ln\left(a\,x^4+b\right)}{4\,a}$$

input
```
int(1/(a*x + b/x^3),x)
```

output
```
log(b + a*x^4)/(4*a)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.13

$$\int \frac{1}{\frac{b}{x^3}+ax}\,dx=\frac{\log\left(-b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}\,x^2+\sqrt{b}\right)+\log\left(b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}\,x^2+\sqrt{b}\right)}{4a}$$

input
```
int(1/(b/x^3+a*x),x)
```

output
```
(log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a)*x**2 + sqrt(b)) + log(b**(1/
4)*a**(1/4)*sqrt(2)*x + sqrt(a)*x**2 + sqrt(b)))/(4*a)
```

3.319 $\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx$

Optimal result 2606
Mathematica [A] (verified) 2606
Rubi [A] (verified) 2607
Maple [A] (verified) 2608
Fricas [B] (verification not implemented) 2608
Sympy [B] (verification not implemented) 2609
Maxima [B] (verification not implemented) 2609
Giac [A] (verification not implemented) 2609
Mupad [B] (verification not implemented) 2610
Reduce [B] (verification not implemented) 2610

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx=\frac{x^4}{4b\left(b+ax^2\right)^2}$$

output
```
1/4*x^4/b/(a*x^2+b)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx=-\frac{b+2ax^2}{4a^2\left(b+ax^2\right)^2}$$

input
```
Integrate[(b/x + a*x)^(-3),x]
```

output
```
-1/4*(b + 2*a*x^2)/(a^2*(b + a*x^2)^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2027, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(ax+\frac{b}{x}\right)^3}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x^3}{(ax^2+b)^3}dx$$

$$\Big\downarrow \text{242}$$

$$\frac{x^4}{4b\left(ax^2+b\right)^2}$$

input
```
Int[(b/x + a*x)^(-3),x]
```

output
```
x^4/(4*b*(b + a*x^2)^2)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

method	result	size
gosper	$-\frac{2a\,x^{2}+b}{4\left(a\,x^{2}+b\right)^{2}a^{2}}$	23
parallelrisch	$\frac{-2a\,x^{2}-b}{4a^{2}\left(a\,x^{2}+b\right)^{2}}$	25
norman	$\frac{-\frac{x^{2}}{2a}-\frac{b}{4a^{2}}}{\left(a\,x^{2}+b\right)^{2}}$	26
risch	$\frac{-\frac{x^{2}}{2a}-\frac{b}{4a^{2}}}{\left(a\,x^{2}+b\right)^{2}}$	26
default	$-\frac{1}{2a^{2}\left(a\,x^{2}+b\right)}+\frac{b}{4a^{2}\left(a\,x^{2}+b\right)^{2}}$	31
orering	$-\frac{\left(2a\,x^{2}+b\right)\left(a\,x^{2}+b\right)}{4a^{2}x^{3}\left(\frac{b}{x}+ax\right)^{3}}$	35

input
```
int(1/(b/x+a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*a*x^2+b)/(a*x^2+b)^2/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x}+ax\right)^{3}}\,dx=-\frac{2\,ax^{2}+b}{4\,\left(a^{4}x^{4}+2\,a^{3}bx^{2}+a^{2}b^{2}\right)}$$

input
```
integrate(1/(b/x+a*x)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*a*x^2 + b)/(a^4*x^4 + 2*a^3*b*x^2 + a^2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx = \frac{-2ax^2-b}{4a^4x^4+8a^3bx^2+4a^2b^2}$$

input
```
integrate(1/(b/x+a*x)**3,x)
```

output
```
(-2*a*x**2 - b)/(4*a**4*x**4 + 8*a**3*b*x**2 + 4*a**2*b**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx = -\frac{2\,ax^2+b}{4\,(a^4x^4+2\,a^3bx^2+a^2b^2)}$$

input
```
integrate(1/(b/x+a*x)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*a*x^2 + b)/(a^4*x^4 + 2*a^3*b*x^2 + a^2*b^2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx = -\frac{2\,ax^2+b}{4\,(ax^2+b)^2a^2}$$

input
```
integrate(1/(b/x+a*x)^3,x, algorithm="giac")
```

output
```
-1/4*(2*a*x^2 + b)/((a*x^2 + b)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx = -\frac{\frac{b}{4\,a^2}+\frac{x^2}{2\,a}}{a^2\,x^4+2\,a\,b\,x^2+b^2}$$

input
```
int(1/(a*x + b/x)^3,x)
```

output
```
-(b/(4*a^2) + x^2/(2*a))/(b^2 + a^2*x^4 + 2*a*b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx = \frac{x^4}{4b\left(a^2x^4+2ab\,x^2+b^2\right)}$$

input
```
int(1/(b/x+a*x)^3,x)
```

output
```
x**4/(4*b*(a**2*x**4 + 2*a*b*x**2 + b**2))
```

3.320 $\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx$

Optimal result 2611
Mathematica [A] (verified) 2611
Rubi [A] (verified) 2612
Maple [A] (verified) 2613
Fricas [B] (verification not implemented) 2613
Sympy [B] (verification not implemented) 2614
Maxima [B] (verification not implemented) 2614
Giac [A] (verification not implemented) 2614
Mupad [B] (verification not implemented) 2615
Reduce [B] (verification not implemented) 2615

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx=\frac{x^{10}}{10b\left(b+ax^5\right)^2}$$

output
```
1/10*x^10/b/(a*x^5+b)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx=-\frac{b+2ax^5}{10a^2\left(b+ax^5\right)^2}$$

input
```
Integrate[(b/x^3 + a*x^2)^(-3),x]
```

output
```
-1/10*(b + 2*a*x^5)/(a^2*(b + a*x^5)^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2027, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(ax^2+\frac{b}{x^3}\right)^3}dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{x^9}{(ax^5+b)^3}dx$$

$$\Big\downarrow \text{ 796}$$

$$\frac{x^{10}}{10b\,(ax^5+b)^2}$$

input
```
Int[(b/x^3 + a*x^2)^(-3),x]
```

output
```
x^10/(10*b*(b + a*x^5)^2)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

method	result	size
gosper	$-\frac{2a\,x^{5}+b}{10\left(a\,x^{5}+b\right)^{2} a^{2}}$	23
parallelrisch	$\frac{-2a\,x^{5}-b}{10a^{2} \left(a\,x^{5}+b\right)^{2}}$	25
norman	$\frac{-\frac{x^{5}}{5a}-\frac{b}{10a^{2}}}{\left(a\,x^{5}+b\right)^{2}}$	26
risch	$\frac{-\frac{x^{5}}{5a}-\frac{b}{10a^{2}}}{\left(a\,x^{5}+b\right)^{2}}$	26
default	$-\frac{1}{5a^{2} \left(a\,x^{5}+b\right)}+\frac{b}{10a^{2} \left(a\,x^{5}+b\right)^{2}}$	31
orering	$-\frac{\left(2a\,x^{5}+b\right) \left(a\,x^{5}+b\right)}{10a^{2} x^{9} \left(\frac{b}{x^{3}}+a\,x^{2}\right)^{3}}$	37

input
```
int(1/(b/x^3+a*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/10*(2*a*x^5+b)/(a*x^5+b)^2/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(17) = 34.

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx = -\frac{2\,ax^5+b}{10\,\left(a^4x^{10}+2\,a^3bx^5+a^2b^2\right)}$$

input
```
integrate(1/(b/x^3+a*x^2)^3,x, algorithm="fricas")
```

output
```
-1/10*(2*a*x^5 + b)/(a^4*x^10 + 2*a^3*b*x^5 + a^2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx = \frac{-2ax^5-b}{10a^4x^{10}+20a^3bx^5+10a^2b^2}$$

input
```
integrate(1/(b/x**3+a*x**2)**3,x)
```

output
```
(-2*a*x**5 - b)/(10*a**4*x**10 + 20*a**3*b*x**5 + 10*a**2*b**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx = -\frac{2\,ax^5+b}{10\,(a^4x^{10}+2\,a^3bx^5+a^2b^2)}$$

input
```
integrate(1/(b/x^3+a*x^2)^3,x, algorithm="maxima")
```

output
```
-1/10*(2*a*x^5 + b)/(a^4*x^10 + 2*a^3*b*x^5 + a^2*b^2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx = -\frac{2\,ax^5+b}{10\,(ax^5+b)^2a^2}$$

input
```
integrate(1/(b/x^3+a*x^2)^3,x, algorithm="giac")
```

output
```
-1/10*(2*a*x^5 + b)/((a*x^5 + b)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx = -\frac{\frac{b}{10\,a^2}+\frac{x^5}{5\,a}}{a^2\,x^{10}+2\,a\,b\,x^5+b^2}$$

input
```
int(1/(a*x^2 + b/x^3)^3,x)
```

output
```
-(b/(10*a^2) + x^5/(5*a))/(b^2 + a^2*x^10 + 2*a*b*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx = \frac{x^{10}}{10b\,(a^2x^{10}+2ab\,x^5+b^2)}$$

input
```
int(1/(b/x^3+a*x^2)^3,x)
```

output
```
x**10/(10*b*(a**2*x**10 + 2*a*b*x**5 + b**2))
```

3.321 $\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx$

Optimal result 2616
Mathematica [A] (verified) 2616
Rubi [A] (verified) 2617
Maple [A] (verified) 2618
Fricas [B] (verification not implemented) 2618
Sympy [B] (verification not implemented) 2619
Maxima [B] (verification not implemented) 2619
Giac [A] (verification not implemented) 2619
Mupad [B] (verification not implemented) 2620
Reduce [B] (verification not implemented) 2620

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx=\frac{x^{16}}{16b\left(b+ax^8\right)^2}$$

output
```
1/16*x^16/b/(a*x^8+b)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx=-\frac{b+2ax^8}{16a^2\left(b+ax^8\right)^2}$$

input
```
Integrate[(b/x^5 + a*x^3)^(-3),x]
```

output
```
-1/16*(b + 2*a*x^8)/(a^2*(b + a*x^8)^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2027, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(ax^3+\frac{b}{x^5}\right)^3}dx$$
$$\Big\downarrow \text{ 2027}$$
$$\int \frac{x^{15}}{(ax^8+b)^3}dx$$
$$\Big\downarrow \text{ 796}$$
$$\frac{x^{16}}{16b\,(ax^8+b)^2}$$

input
```
Int[(b/x^5 + a*x^3)^(-3),x]
```

output
```
x^16/(16*b*(b + a*x^8)^2)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

method	result	size
gosper	$-\frac{2a\,x^{8}+b}{16\left(a\,x^{8}+b\right)^{2} a^{2}}$	23
parallelrisch	$\frac{-2a\,x^{8}-b}{16a^{2} \left(a\,x^{8}+b\right)^{2}}$	25
norman	$\frac{-\frac{b}{16a^{2}}-\frac{x^{8}}{8a}}{\left(a\,x^{8}+b\right)^{2}}$	26
risch	$\frac{-\frac{b}{16a^{2}}-\frac{x^{8}}{8a}}{\left(a\,x^{8}+b\right)^{2}}$	26
default	$-\frac{1}{8a^{2} \left(a\,x^{8}+b\right)}+\frac{b}{16a^{2} \left(a\,x^{8}+b\right)^{2}}$	31
orering	$-\frac{\left(2a\,x^{8}+b\right) \left(a\,x^{8}+b\right)}{16a^{2} x^{15} \left(\frac{b}{x^{5}}+a\,x^{3}\right)^{3}}$	37

input
```
int(1/(b/x^5+a*x^3)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*(2*a*x^8+b)/(a*x^8+b)^2/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx=-\frac{2\,ax^8+b}{16\,\left(a^4x^{16}+2\,a^3bx^8+a^2b^2\right)}$$

input
```
integrate(1/(b/x^5+a*x^3)^3,x, algorithm="fricas")
```

output
```
-1/16*(2*a*x^8 + b)/(a^4*x^16 + 2*a^3*b*x^8 + a^2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx = \frac{-2ax^8-b}{16a^4x^{16}+32a^3bx^8+16a^2b^2}$$

input
```
integrate(1/(b/x**5+a*x**3)**3,x)
```

output
```
(-2*a*x**8 - b)/(16*a**4*x**16 + 32*a**3*b*x**8 + 16*a**2*b**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx = -\frac{2\,ax^8+b}{16\,(a^4x^{16}+2\,a^3bx^8+a^2b^2)}$$

input
```
integrate(1/(b/x^5+a*x^3)^3,x, algorithm="maxima")
```

output
```
-1/16*(2*a*x^8 + b)/(a^4*x^16 + 2*a^3*b*x^8 + a^2*b^2)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx = -\frac{2\,ax^8+b}{16\,(ax^8+b)^2a^2}$$

input
```
integrate(1/(b/x^5+a*x^3)^3,x, algorithm="giac")
```

output
```
-1/16*(2*a*x^8 + b)/((a*x^8 + b)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx = -\frac{\frac{b}{16\,a^2}+\frac{x^8}{8\,a}}{a^2\,x^{16}+2\,a\,b\,x^8+b^2}$$

input
```
int(1/(a*x^3 + b/x^5)^3,x)
```

output
```
-(b/(16*a^2) + x^8/(8*a))/(b^2 + a^2*x^16 + 2*a*b*x^8)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx = \frac{x^{16}}{16b\,(a^2x^{16}+2ab\,x^8+b^2)}$$

input
```
int(1/(b/x^5+a*x^3)^3,x)
```

output
```
x**16/(16*b*(a**2*x**16 + 2*a*b*x**8 + b**2))
```

3.322 $\int \left(\frac{a}{x}+bx\right)^2 dx$

Optimal result 2621
Mathematica [A] (verified) 2621
Rubi [A] (verified) 2622
Maple [A] (warning: unable to verify) 2623
Fricas [A] (verification not implemented) 2623
Sympy [A] (verification not implemented) 2624
Maxima [A] (verification not implemented) 2624
Giac [A] (verification not implemented) 2624
Mupad [B] (verification not implemented) 2625
Reduce [B] (verification not implemented) 2625

Optimal result

Integrand size = 11, antiderivative size = 24

$$\int \left(\frac{a}{x}+bx\right)^2 dx = -\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

output
```
-a^2/x+2*a*b*x+1/3*b^2*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \left(\frac{a}{x}+bx\right)^2 dx = -\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

input
```
Integrate[(a/x + b*x)^2,x]
```

output
```
-(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2027, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{a}{x}+bx\right)^2 dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{(a+bx^2)^2}{x^2}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^2}{x^2}+2ab+b^2x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

input
```
Int[(a/x + b*x)^2,x]
```

output
```
-(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$	23
risch	$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$	23
norman	$\frac{\frac{1}{3}b^2x^4+2ab\,x^2-a^2}{x}$	26
parallelrisch	$\frac{b^2x^4+6ab\,x^2-3a^2}{3x}$	26
gosper	$-\frac{-b^2x^4-6ab\,x^2+3a^2}{3x}$	27
orering	$-\frac{\left(-b^2x^4-6ab\,x^2+3a^2\right)x\left(\frac{a}{x}+bx\right)^2}{3\left(b\,x^2+a\right)^2}$	45

input
```
int((a/x+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/x+2*a*b*x+1/3*b^2*x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \left(\frac{a}{x}+bx\right)^2\,dx=\frac{b^2x^4+6\,abx^2-3\,a^2}{3\,x}$$

input
```
integrate((a/x+b*x)^2,x, algorithm="fricas")
```

output
```
1/3*(b^2*x^4 + 6*a*b*x^2 - 3*a^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \left(\frac{a}{x}+bx\right)^2 dx = -\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

input
```
integrate((a/x+b*x)**2,x)
```

output
```
-a**2/x + 2*a*b*x + b**2*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \left(\frac{a}{x}+bx\right)^2 dx = \frac{1}{3}\,b^2x^3 + 2\,abx - \frac{a^2}{x}$$

input
```
integrate((a/x+b*x)^2,x, algorithm="maxima")
```

output
```
1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \left(\frac{a}{x}+bx\right)^2 dx = \frac{1}{3}\,b^2x^3 + 2\,abx - \frac{a^2}{x}$$

input
```
integrate((a/x+b*x)^2,x, algorithm="giac")
```

output
```
1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \left(\frac{a}{x}+bx\right)^2 dx = \frac{b^2 x^3}{3} - \frac{a^2}{x} + 2 a b x$$

input
```
int((b*x + a/x)^2,x)
```

output
```
(b^2*x^3)/3 - a^2/x + 2*a*b*x
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \left(\frac{a}{x}+bx\right)^2 dx = \frac{b^2x^4+6ab\,x^2-3a^2}{3x}$$

input
```
int((a/x+b*x)^2,x)
```

output
```
( - 3*a**2 + 6*a*b*x**2 + b**2*x**4)/(3*x)
```

3.323 $\int \left(\frac{a}{x}+bx\right)^3 dx$

Optimal result 2626
Mathematica [A] (verified) 2626
Rubi [A] (verified) 2627
Maple [A] (warning: unable to verify) 2628
Fricas [A] (verification not implemented) 2629
Sympy [A] (verification not implemented) 2629
Maxima [A] (verification not implemented) 2629
Giac [A] (verification not implemented) 2630
Mupad [B] (verification not implemented) 2630
Reduce [B] (verification not implemented) 2630

Optimal result

Integrand size = 11, antiderivative size = 40

$$\int \left(\frac{a}{x}+bx\right)^3 dx = -\frac{a^3}{2x^2}+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}+3a^2b\log(x)$$

output
```
-1/2*a^3/x^2+3/2*a*b^2*x^2+1/4*b^3*x^4+3*a^2*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \left(\frac{a}{x}+bx\right)^3 dx = -\frac{a^3}{2x^2}+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}+3a^2b\log(x)$$

input
```
Integrate[(a/x + b*x)^3,x]
```

output
```
-1/2*a^3/x^2 + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*Log[x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2027, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{a}{x}+bx\right)^3 dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{(a+bx^2)^3}{x^3}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^3}{x^4}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^3}{x^4}+\frac{3ba^2}{x^2}+3b^2a+b^3x^2\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^3}{x^2}+3a^2b\log\left(x^2\right)+3ab^2x^2+\frac{b^3x^4}{2}\right)$$

input
```
Int[(a/x + b*x)^3,x]
```

output
```
(-(a^3/x^2) + 3*a*b^2*x^2 + (b^3*x^4)/2 + 3*a^2*b*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{a^{3}}{2x^{2}}+\frac{3a\,b^{2}x^{2}}{2}+\frac{b^{3}x^{4}}{4}+3a^{2}b\ln\left(x\right)$	35
norman	$\frac{-\frac{1}{2}a^{3}+\frac{1}{4}b^{3}x^{6}+\frac{3}{2}a\,b^{2}x^{4}}{x^{2}}+3a^{2}b\ln\left(x\right)$	37
parallelrisch	$\frac{b^{3}x^{6}+6a\,b^{2}x^{4}+12a^{2}b\ln\left(x\right)x^{2}-2a^{3}}{4x^{2}}$	39
risch	$\frac{b^{3}x^{4}}{4}+\frac{3a\,b^{2}x^{2}}{2}+\frac{9a^{2}b}{4}-\frac{a^{3}}{2x^{2}}+3a^{2}b\ln\left(x\right)$	41

input
```
int((a/x+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^3/x^2+3/2*a*b^2*x^2+1/4*b^3*x^4+3*a^2*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \left(\frac{a}{x}+bx\right)^3 dx = \frac{b^3x^6 + 6\,ab^2x^4 + 12\,a^2bx^2\log(x) - 2\,a^3}{4\,x^2}$$

input
```
integrate((a/x+b*x)^3,x, algorithm="fricas")
```

output
```
1/4*(b^3*x^6 + 6*a*b^2*x^4 + 12*a^2*b*x^2*log(x) - 2*a^3)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \left(\frac{a}{x}+bx\right)^3 dx = -\frac{a^3}{2x^2} + 3a^2b\log(x) + \frac{3ab^2x^2}{2} + \frac{b^3x^4}{4}$$

input
```
integrate((a/x+b*x)**3,x)
```

output
```
-a**3/(2*x**2) + 3*a**2*b*log(x) + 3*a*b**2*x**2/2 + b**3*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \left(\frac{a}{x}+bx\right)^3 dx = \frac{1}{4}\,b^3x^4 + \frac{3}{2}\,ab^2x^2 + 3\,a^2b\log(x) - \frac{a^3}{2\,x^2}$$

input
```
integrate((a/x+b*x)^3,x, algorithm="maxima")
```

output
```
1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3*a^2*b*log(x) - 1/2*a^3/x^2
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \left(\frac{a}{x}+bx\right)^3 dx = \frac{1}{4}b^3x^4 + \frac{3}{2}ab^2x^2 + \frac{3}{2}a^2b\log\left(x^2\right) - \frac{3a^2bx^2+a^3}{2x^2}$$

input
```
integrate((a/x+b*x)^3,x, algorithm="giac")
```

output
```
1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3/2*a^2*b*log(x^2) - 1/2*(3*a^2*b*x^2 + a^3)
/x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \left(\frac{a}{x}+bx\right)^3 dx = \frac{b^3x^4}{4} - \frac{a^3}{2x^2} + \frac{3ab^2x^2}{2} + 3a^2b\ln(x)$$

input
```
int((b*x + a/x)^3,x)
```

output
```
(b^3*x^4)/4 - a^3/(2*x^2) + (3*a*b^2*x^2)/2 + 3*a^2*b*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \left(\frac{a}{x}+bx\right)^3 dx = \frac{12\log(x)a^2bx^2 - 2a^3 + 6ab^2x^4 + b^3x^6}{4x^2}$$

input
```
int((a/x+b*x)^3,x)
```

output
```
(12*log(x)*a**2*b*x**2 - 2*a**3 + 6*a*b**2*x**4 + b**3*x**6)/(4*x**2)
```

3.324 $\int \left(\frac{a}{x}+bx\right)^4 \, dx$

Optimal result . . . 2631
Mathematica [A] (verified) . . . 2631
Rubi [A] (verified) . . . 2632
Maple [A] (warning: unable to verify) . . . 2633
Fricas [A] (verification not implemented) . . . 2633
Sympy [A] (verification not implemented) . . . 2634
Maxima [A] (verification not implemented) . . . 2634
Giac [A] (verification not implemented) . . . 2634
Mupad [B] (verification not implemented) . . . 2635
Reduce [B] (verification not implemented) . . . 2635

Optimal result

Integrand size = 11, antiderivative size = 50

$$\int \left(\frac{a}{x}+bx\right)^4 \, dx=-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

output
```
-1/3*a^4/x^3-4*a^3*b/x+6*a^2*b^2*x+4/3*a*b^3*x^3+1/5*b^4*x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \left(\frac{a}{x}+bx\right)^4 \, dx=-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

input
```
Integrate[(a/x + b*x)^4,x]
```

output
```
-1/3*a^4/x^3 - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2027, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{a}{x}+bx\right)^4 dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{(a+bx^2)^4}{x^4}dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(\frac{a^4}{x^4}+\frac{4a^3b}{x^2}+6a^2b^2+4ab^3x^2+b^4x^4\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

input
```
Int[(a/x + b*x)^4,x]
```

output
```
-1/3*a^4/x^3 - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6x\,a^2b^2+\frac{4a\,b^3x^3}{3}+\frac{b^4x^5}{5}$	45
risch	$\frac{b^4x^5}{5}+\frac{4a\,b^3x^3}{3}+6x\,a^2b^2+\frac{-4a^3b\,x^2-\frac{1}{3}a^4}{x^3}$	47
norman	$\frac{\frac{1}{5}b^4x^8+\frac{4}{3}a\,b^3x^6+6a^2b^2x^4-4a^3b\,x^2-\frac{1}{3}a^4}{x^3}$	48
gosper	$-\frac{-3b^4x^8-20a\,b^3x^6-90a^2b^2x^4+60a^3b\,x^2+5a^4}{15x^3}$	49
parallelrisch	$\frac{3b^4x^8+20a\,b^3x^6+90a^2b^2x^4-60a^3b\,x^2-5a^4}{15x^3}$	49
orering	$-\frac{\left(-3b^4x^8-20a\,b^3x^6-90a^2b^2x^4+60a^3b\,x^2+5a^4\right)x\left(\frac{a}{x}+bx\right)^4}{15\left(b\,x^2+a\right)^4}$	67

input
```
int((a/x+b*x)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^4/x^3-4*a^3*b/x+6*x*a^2*b^2+4/3*a*b^3*x^3+1/5*b^4*x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(\frac{a}{x}+bx\right)^4\,dx=\frac{3\,b^4x^8+20\,ab^3x^6+90\,a^2b^2x^4-60\,a^3bx^2-5\,a^4}{15\,x^3}$$

input
```
integrate((a/x+b*x)^4,x, algorithm="fricas")
```

output
```
1/15*(3*b^4*x^8 + 20*a*b^3*x^6 + 90*a^2*b^2*x^4 - 60*a^3*b*x^2 - 5*a^4)/x^
3
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \left(\frac{a}{x}+bx\right)^4 dx = 6a^2b^2x + \frac{4ab^3x^3}{3} + \frac{b^4x^5}{5} + \frac{-a^4-12a^3bx^2}{3x^3}$$

input
```
integrate((a/x+b*x)**4,x)
```

output
```
6*a**2*b**2*x + 4*a*b**3*x**3/3 + b**4*x**5/5 + (-a**4 - 12*a**3*b*x**2)/(
3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \left(\frac{a}{x}+bx\right)^4 dx = \frac{1}{5}b^4x^5 + \frac{4}{3}ab^3x^3 + 6a^2b^2x - \frac{4a^3b}{x} - \frac{a^4}{3x^3}$$

input
```
integrate((a/x+b*x)^4,x, algorithm="maxima")
```

output
```
1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 4*a^3*b/x - 1/3*a^4/x^3
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \left(\frac{a}{x}+bx\right)^4 dx = \frac{1}{5}b^4x^5 + \frac{4}{3}ab^3x^3 + 6a^2b^2x - \frac{12a^3bx^2+a^4}{3x^3}$$

input
```
integrate((a/x+b*x)^4,x, algorithm="giac")
```

output
```
1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 1/3*(12*a^3*b*x^2 + a^4)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \left(\frac{a}{x}+bx\right)^4 dx = \frac{b^4 x^5}{5} - \frac{\frac{a^4}{3} + 4 b a^3 x^2}{x^3} + 6 a^2 b^2 x + \frac{4 a b^3 x^3}{3}$$

input
```
int((b*x + a/x)^4,x)
```

output
```
(b^4*x^5)/5 - (a^4/3 + 4*a^3*b*x^2)/x^3 + 6*a^2*b^2*x + (4*a*b^3*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(\frac{a}{x}+bx\right)^4 dx = \frac{3b^4x^8 + 20a b^3x^6 + 90a^2b^2x^4 - 60a^3b x^2 - 5a^4}{15x^3}$$

input
```
int((a/x+b*x)^4,x)
```

output
```
( - 5*a**4 - 60*a**3*b*x**2 + 90*a**2*b**2*x**4 + 20*a*b**3*x**6 + 3*b**4*
x**8)/(15*x**3)
```

3.325 $\int \frac{x}{\frac{1}{x}+x}\,dx$

Optimal result 2636
Mathematica [A] (verified) 2636
Rubi [A] (verified) 2637
Maple [A] (verified) 2638
Fricas [A] (verification not implemented) 2638
Sympy [A] (verification not implemented) 2639
Maxima [A] (verification not implemented) 2639
Giac [A] (verification not implemented) 2639
Mupad [B] (verification not implemented) 2640
Reduce [B] (verification not implemented) 2640

Optimal result

Integrand size = 9, antiderivative size = 6

$$\int \frac{x}{\frac{1}{x}+x}\,dx = x-\arctan(x)$$

output
```
x-arctan(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{x}{\frac{1}{x}+x}\,dx = x-\arctan(x)$$

input
```
Integrate[x/(x^(-1) + x),x]
```

output
```
x - ArcTan[x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {10, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x+\frac{1}{x}}dx$$

$$\Big\downarrow \text{ 10}$$

$$\int \frac{x^2}{x^2+1}dx$$

$$\Big\downarrow \text{ 262}$$

$$x-\int \frac{1}{x^2+1}dx$$

$$\Big\downarrow \text{ 216}$$

$$x-\arctan (x)$$

input
```
Int[x/(x^(-1) + x),x]
```

output
```
x - ArcTan[x]
```

Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
default	$x-\arctan\left(x\right)$	7
risch	$x-\arctan\left(x\right)$	7
parallelrisch	$x+\frac{i\ln\left(x-i\right)}{2}-\frac{i\ln\left(x+i\right)}{2}$	19

input
```
int(x/(1/x+x),x,method=_RETURNVERBOSE)
```

output
```
x-arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{x}{\frac{1}{x}+x}\,dx=x-\arctan\left(x\right)$$

input
```
integrate(x/(1/x+x),x, algorithm="fricas")
```

output
```
x - arctan(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50

$$\int \frac{x}{\frac{1}{x}+x}\,dx = x - \operatorname{atan}(x)$$

input
```
integrate(x/(1/x+x),x)
```

output
```
x - atan(x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{x}{\frac{1}{x}+x}\,dx = x - \arctan(x)$$

input
```
integrate(x/(1/x+x),x, algorithm="maxima")
```

output
```
x - arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{x}{\frac{1}{x}+x}\,dx = x - \arctan(x)$$

input
```
integrate(x/(1/x+x),x, algorithm="giac")
```

output
```
x - arctan(x)
```

Mupad [B] (verification not implemented)

Time = 8.82 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{x}{\frac{1}{x}+x}\,dx = x - \operatorname{atan}(x)$$

input
```
int(x/(x + 1/x),x)
```

output
```
x - atan(x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{x}{\frac{1}{x}+x}\,dx = -atan(x) + x$$

input
```
int(x/(1/x+x),x)
```

output
```
- atan(x) + x
```

3.326 $\int \frac{1}{\frac{1}{x^2}+x^3}\,dx$

Optimal result 2641
Mathematica [A] (verified) 2642
Rubi [A] (verified) 2642
Maple [C] (verified) 2646
Fricas [A] (verification not implemented) 2646
Sympy [A] (verification not implemented) 2647
Maxima [A] (verification not implemented) 2647
Giac [A] (verification not implemented) 2648
Mupad [B] (verification not implemented) 2649
Reduce [F] 2650

Optimal result

Integrand size = 9, antiderivative size = 168

$$
\begin{aligned}
\int \frac{1}{\frac{1}{x^2}+x^3}\,dx = {} & \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{1-\sqrt{5}-4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) \\
& -\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}-4x\right)\right) \\
& +\frac{1}{5}\log(1+x)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(1-\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2\right) \\
& -\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(1-\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2\right)
\end{aligned}
$$

output
```
1/10*(10-2*5^(1/2))^(1/2)*arctan((1-5^(1/2)-4*x)/(10+2*5^(1/2))^(1/2))-1/1
0*(10+2*5^(1/2))^(1/2)*arctan(1/20*(50+10*5^(1/2))^(1/2)*(1+5^(1/2)-4*x))+
1/5*ln(1+x)-1/20*(5^(1/2)+1)*ln(1-1/2*x*(-5^(1/2)+1)+x^2)-1/20*(-5^(1/2)+1
)*ln(1-1/2*(5^(1/2)+1)*x+x^2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.86

$$\int \frac{1}{\frac{1}{x^2}+x^3}\,dx = \frac{1}{20}\left(-2\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{1+\sqrt{5}-4x}{\sqrt{10-2\sqrt{5}}}\right) - 2\sqrt{10-2\sqrt{5}}\arctan\left(\frac{-1+\sqrt{5}+4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) + 4\log(1+x) - \left(1+\sqrt{5}\right)\log\left(1+\frac{1}{2}\left(-1+\sqrt{5}\right)x+x^2\right) + \left(-1+\sqrt{5}\right)\log\left(1-\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2\right)\right)$$

input
```
Integrate[(x^(-2) + x^3)^(-1),x]
```

output
```
(-2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 + Sqrt[5] - 4*x)/Sqrt[10 - 2*Sqrt[5]]]
 - 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-1 + Sqrt[5] + 4*x)/Sqrt[2*(5 + Sqrt[5])
]] + 4*Log[1 + x] - (1 + Sqrt[5])*Log[1 + ((-1 + Sqrt[5])*x)/2 + x^2] + (-
1 + Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])/20
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2027, 822, 16, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3+\frac{1}{x^2}}dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{x^2}{x^5+1}dx$$

$$\Big\downarrow \text{822}$$

$$\frac{2}{5}\int -\frac{\left(1+\sqrt{5}\right)x+\sqrt{5}+1}{2\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)}dx+\frac{2}{5}\int -\frac{\left(1-\sqrt{5}\right)x-\sqrt{5}+1}{2\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)}dx+\frac{1}{5}\int \frac{1}{x+1}dx$$

$$\Big\downarrow \text{16}$$

$$\frac{2}{5}\int -\frac{\left(1+\sqrt{5}\right)x+\sqrt{5}+1}{2\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)}dx+\frac{2}{5}\int -\frac{\left(1-\sqrt{5}\right)x-\sqrt{5}+1}{2\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)}dx+\frac{1}{5}\log(x+1)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{5}\int \frac{\left(1+\sqrt{5}\right)x+\sqrt{5}+1}{2x^2-\left(1-\sqrt{5}\right)x+2}dx-\frac{1}{5}\int \frac{\left(1-\sqrt{5}\right)x-\sqrt{5}+1}{2x^2-\left(1+\sqrt{5}\right)x+2}dx+\frac{1}{5}\log(x+1)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{5}\left(-\sqrt{5}\int \frac{1}{2x^2-\left(1-\sqrt{5}\right)x+2}dx-\frac{1}{4}\left(1+\sqrt{5}\right)\int -\frac{-4x-\sqrt{5}+1}{2x^2-\left(1-\sqrt{5}\right)x+2}dx\right)+\frac{1}{5}\left(\sqrt{5}\int \frac{1}{2x^2-\left(1+\sqrt{5}\right)x+2}dx-\frac{1}{4}\left(1-\sqrt{5}\right)\int -\frac{-4x+\sqrt{5}+1}{2x^2-\left(1+\sqrt{5}\right)x+2}dx\right)+\frac{1}{5}\log(x+1)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{-4x-\sqrt{5}+1}{2x^2-\left(1-\sqrt{5}\right)x+2}dx-\sqrt{5}\int \frac{1}{2x^2-\left(1-\sqrt{5}\right)x+2}dx\right)+\frac{1}{5}\left(\sqrt{5}\int \frac{1}{2x^2-\left(1+\sqrt{5}\right)x+2}dx+\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{-4x+\sqrt{5}+1}{2x^2-\left(1+\sqrt{5}\right)x+2}dx\right)+\frac{1}{5}\log(x+1)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{-4x+\sqrt{5}+1}{2x^2-\left(1+\sqrt{5}\right)x+2}dx-2\sqrt{5}\int \frac{1}{-\left(4x-\sqrt{5}-1\right)^2-2\left(5-\sqrt{5}\right)}d\left(4x-\sqrt{5}-1\right)\right)+\frac{1}{5}\left(\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{-4x-\sqrt{5}+1}{2x^2-\left(1-\sqrt{5}\right)x+2}dx+2\sqrt{5}\int \frac{1}{-\left(4x+\sqrt{5}-1\right)^2-2\left(5+\sqrt{5}\right)}d\left(4x+\sqrt{5}-1\right)\right)+\frac{1}{5}\log(x+1)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{-4x-\sqrt{5}+1}{2x^2-(1-\sqrt{5})\,x+2}dx-\sqrt{\frac{10}{5+\sqrt{5}}}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\,(5+\sqrt{5})}}\right)\right)+$$
$$\frac{1}{5}\left(\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{-4x+\sqrt{5}+1}{2x^2-(1+\sqrt{5})\,x+2}dx+\sqrt{\frac{10}{5-\sqrt{5}}}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{2\,(5-\sqrt{5})}}\right)\right)+$$
$$\frac{1}{5}\log(x+1)$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{5}\left(-\sqrt{\frac{10}{5+\sqrt{5}}}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\,(5+\sqrt{5})}}\right)-\frac{1}{4}\left(1+\sqrt{5}\right)\log\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)\right)+$$
$$\frac{1}{5}\left(\sqrt{\frac{10}{5-\sqrt{5}}}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{2\,(5-\sqrt{5})}}\right)-\frac{1}{4}\left(1-\sqrt{5}\right)\log\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)\right)+$$
$$\frac{1}{5}\log(x+1)$$

input
```
Int[(x^(-2) + x^3)^(-1),x]
```

output
```
Log[1 + x]/5 + (-(Sqrt[10/(5 + Sqrt[5])]*ArcTan[(-1 + Sqrt[5] + 4*x)/Sqrt[
2*(5 + Sqrt[5])]]) - ((1 + Sqrt[5])*Log[2 - (1 - Sqrt[5])*x + 2*x^2])/4)/5
 + (Sqrt[10/(5 - Sqrt[5])]*ArcTan[(-1 - Sqrt[5] + 4*x)/Sqrt[2*(5 - Sqrt[5]
)]] - ((1 - Sqrt[5])*Log[2 - (1 + Sqrt[5])*x + 2*x^2])/4)/5
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 822
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x]; -(-r)^(m + 1)/(a*n*s^m)   Int[1/(r + s*x), x]
 + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 1)/2}], x]] /; FreeQ[{a, b}
, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.20

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}+\_Z^{3}+\_Z^{2}+\_Z+1\right)}\_R\ln\left(\_R^{2}+x\right)\right)}{5}+\frac{\ln\left(x+1\right)}{5}$
default	$\frac{\ln\left(x+1\right)}{5}-\frac{\left(-\sqrt{5}+1\right)\ln\left(2-x-\sqrt{5}\,x+2x^{2}\right)}{20}-\frac{2\left(-\sqrt{5}+1-\frac{\left(-\sqrt{5}+1\right)\left(-\sqrt{5}-1\right)}{4}\right)\arctan\left(\frac{-1-\sqrt{5}+4x}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}+\frac{\left(-\sqrt{5}-1\right)\ln\left(2-x-\right.}{20}$

input
```
int(1/(1/x^2+x^3),x,method=_RETURNVERBOSE)
```

output
```
1/5*sum(_R*ln(_R^2+x),_R=RootOf(_Z^4+_Z^3+_Z^2+_Z+1))+1/5*ln(x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

$$
\begin{aligned}
\int \frac{1}{\frac{1}{x^2}+x^3}\,dx = &-\frac{1}{20}\left(\sqrt{5}+1\right)\log\left(2\,x^{2}+\sqrt{5}x-x+2\right)\\
&+\frac{1}{20}\left(\sqrt{5}-1\right)\log\left(2\,x^{2}-\sqrt{5}x-x+2\right)\\
&+\frac{1}{5}\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}\arctan\left(\frac{1}{10}\left(\sqrt{5}(4\,x-1)-5\right)\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}\right)\\
&-\frac{1}{5}\sqrt{-\frac{1}{2}\sqrt{5}+\frac{5}{2}}\arctan\left(\frac{1}{10}\left(\sqrt{5}(4\,x-1)+5\right)\sqrt{-\frac{1}{2}\sqrt{5}+\frac{5}{2}}\right)\\
&+\frac{1}{5}\log\left(x+1\right)
\end{aligned}
$$

input
```
integrate(1/(1/x^2+x^3),x, algorithm="fricas")
```

output
```
-1/20*(sqrt(5) + 1)*log(2*x^2 + sqrt(5)*x - x + 2) + 1/20*(sqrt(5) - 1)*lo
g(2*x^2 - sqrt(5)*x - x + 2) + 1/5*sqrt(1/2*sqrt(5) + 5/2)*arctan(1/10*(sq
rt(5)*(4*x - 1) - 5)*sqrt(1/2*sqrt(5) + 5/2)) - 1/5*sqrt(-1/2*sqrt(5) + 5/
2)*arctan(1/10*(sqrt(5)*(4*x - 1) + 5)*sqrt(-1/2*sqrt(5) + 5/2)) + 1/5*log
(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.21

$$\int \frac{1}{\frac{1}{x^2}+x^3}\,dx = \frac{\log(x+1)}{5} + \operatorname{RootSum}\left(625t^4+125t^3+25t^2+5t+1,\left(t\mapsto t\log\left(25t^2+x\right)\right)\right)$$

input
```
integrate(1/(1/x**2+x**3),x)
```

output
```
log(x + 1)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t + 1, Lambda
(_t, _t*log(25*_t**2 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.74

$$\int \frac{1}{\frac{1}{x^2}+x^3}\,dx = -\frac{2\sqrt{5}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right)}{5\sqrt{2\sqrt{5}+10}} + \frac{2\sqrt{5}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)}{5\sqrt{-2\sqrt{5}+10}} + \frac{\log\left(2x^2-x(\sqrt{5}+1)+2\right)}{5(\sqrt{5}+1)} - \frac{\log\left(2x^2+x(\sqrt{5}-1)+2\right)}{5(\sqrt{5}-1)} + \frac{1}{5}\log(x+1)$$

input
```
integrate(1/(1/x^2+x^3),x, algorithm="maxima")
```

output
```
-2/5*sqrt(5)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10))/sqrt(2*sqrt(
5) + 10) + 2/5*sqrt(5)*arctan((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10))/s
qrt(-2*sqrt(5) + 10) + 1/5*log(2*x^2 - x*(sqrt(5) + 1) + 2)/(sqrt(5) + 1)
- 1/5*log(2*x^2 + x*(sqrt(5) - 1) + 2)/(sqrt(5) - 1) + 1/5*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.67

$$\int \frac{1}{\frac{1}{x^2}+x^3}\,dx = \frac{1}{20}\left(\sqrt{5}-1\right)\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{5}+1\right)+1\right) - \frac{1}{20}\left(\sqrt{5}+1\right)\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{5}-1\right)+1\right) - \frac{1}{10}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{4\,x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right) + \frac{1}{10}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{4\,x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right) + \frac{1}{5}\log\left(|x+1|\right)$$

input
```
integrate(1/(1/x^2+x^3),x, algorithm="giac")
```

output
```
1/20*(sqrt(5) - 1)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) - 1/20*(sqrt(5) + 1)
*log(x^2 + 1/2*x*(sqrt(5) - 1) + 1) - 1/10*sqrt(-2*sqrt(5) + 10)*arctan((4
*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10)) + 1/10*sqrt(2*sqrt(5) + 10)*arctan
((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10)) + 1/5*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 9.65 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.17

$$\int \frac{1}{\frac{1}{x^2}+x^3}\,dx = \frac{\ln(x+1)}{5} - \ln\left(1-\frac{x\left(\sqrt{2}\sqrt{-\sqrt{5}-5}-\sqrt{5}+1\right)^3}{64}\right)\left(\frac{\sqrt{2}\sqrt{-\sqrt{5}-5}}{20}-\frac{\sqrt{5}}{20}+\frac{1}{20}\right) + \ln\left(\frac{x\left(\sqrt{2}\sqrt{-\sqrt{5}-5}+\sqrt{5}-1\right)^3}{64}+1\right)\left(\frac{\sqrt{2}\sqrt{-\sqrt{5}-5}}{20}+\frac{\sqrt{5}}{20}-\frac{1}{20}\right) - \ln\left(1-\frac{x\left(\sqrt{5}+\sqrt{2}\sqrt{\sqrt{5}-5}+1\right)^3}{64}\right)\left(\frac{\sqrt{5}}{20}+\frac{\sqrt{2}\sqrt{\sqrt{5}-5}}{20}+\frac{1}{20}\right) - \ln\left(1-\frac{x\left(\sqrt{5}-\sqrt{2}\sqrt{\sqrt{5}-5}+1\right)^3}{64}\right)\left(\frac{\sqrt{5}}{20}-\frac{\sqrt{2}\sqrt{\sqrt{5}-5}}{20}+\frac{1}{20}\right)$$

input
```
int(1/(1/x^2 + x^3),x)
```

output
```
log(x + 1)/5 - log(1 - (x*(2^(1/2)*(- 5^(1/2) - 5)^(1/2) - 5^(1/2) + 1)^3)
/64)*((2^(1/2)*(- 5^(1/2) - 5)^(1/2))/20 - 5^(1/2)/20 + 1/20) + log((x*(2^
(1/2)*(- 5^(1/2) - 5)^(1/2) + 5^(1/2) - 1)^3)/64 + 1)*((2^(1/2)*(- 5^(1/2)
 - 5)^(1/2))/20 + 5^(1/2)/20 - 1/20) - log(1 - (x*(5^(1/2) + 2^(1/2)*(5^(1
/2) - 5)^(1/2) + 1)^3)/64)*(5^(1/2)/20 + (2^(1/2)*(5^(1/2) - 5)^(1/2))/20
+ 1/20) - log(1 - (x*(5^(1/2) - 2^(1/2)*(5^(1/2) - 5)^(1/2) + 1)^3)/64)*(5
^(1/2)/20 - (2^(1/2)*(5^(1/2) - 5)^(1/2))/20 + 1/20)
```

Reduce [F]

$$\int \frac{1}{\frac{1}{x^2} + x^3} \, dx = \int \frac{x^2}{x^5 + 1} dx$$

input
```
int(1/(1/x^2+x^3),x)
```

output
```
int(x**2/(x**5 + 1),x)
```

3.327 $\int x^p(ax^n + bx^{1+13n+p})^{12} dx$

Optimal result . . . 2651
Mathematica [B] (verified) . . . 2651
Rubi [A] (verified) . . . 2652
Maple [B] (verified) . . . 2653
Fricas [B] (verification not implemented) . . . 2654
Sympy [B] (verification not implemented) . . . 2654
Maxima [B] (verification not implemented) . . . 2655
Giac [B] (verification not implemented) . . . 2656
Mupad [B] (verification not implemented) . . . 2657
Reduce [B] (verification not implemented) . . . 2658

Optimal result

Integrand size = 22, antiderivative size = 29

$$\int x^p \left(ax^n + bx^{1+13n+p}\right)^{12} dx = \frac{\left(a + bx^{1+12n+p}\right)^{13}}{13b(1 + 12n + p)}$$

output
```
1/13*(a+b*x^(1+12*n+p))^13/b/(1+12*n+p)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 232 vs. 2(29) = 58.

Time = 0.13 (sec) , antiderivative size = 232, normalized size of antiderivative = 8.00

$$\int x^p \left(ax^n + bx^{1+13n+p}\right)^{12} dx = \frac{x^{1+12n+p}(13a^{12} + 78a^{11}bx^{1+12n+p} + 286a^{10}b^2x^{2+24n+2p} + 715a^9b^3x^{3+36n+3p} + 1287a^8b^4x^{4+48n+4p} + 1716a}{}$$

input
```
Integrate[x^p*(a*x^n + b*x^(1 + 13*n + p))^12,x]
```

output
```
(x^(1 + 12*n + p)*(13*a^12 + 78*a^11*b*x^(1 + 12*n + p) + 286*a^10*b^2*x^(
2 + 24*n + 2*p) + 715*a^9*b^3*x^(3 + 36*n + 3*p) + 1287*a^8*b^4*x^(4 + 48*
n + 4*p) + 1716*a^7*b^5*x^(5 + 60*n + 5*p) + 1716*a^6*b^6*x^(6 + 72*n + 6*
p) + 1287*a^5*b^7*x^(7 + 84*n + 7*p) + 715*a^4*b^8*x^(8 + 96*n + 8*p) + 28
6*a^3*b^9*x^(9 + 108*n + 9*p) + 78*a^2*b^10*x^(10 + 120*n + 10*p) + 13*a*b
^11*x^(11 + 132*n + 11*p) + b^12*x^(12 + 144*n + 12*p)))/(13*(1 + 12*n + p
))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {10, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^p \left(ax^n+bx^{13n+p+1}\right)^{12} dx$$

$$\Big\downarrow \ 10$$

$$\int x^{12n+p} \left(a+bx^{12n+p+1}\right)^{12} dx$$

$$\Big\downarrow \ 793$$

$$\frac{\left(a+bx^{12n+p+1}\right)^{13}}{13b(12n+p+1)}$$

input
```
Int[x^p*(a*x^n + b*x^(1 + 13*n + p))^12,x]
```

output
```
(a + b*x^(1 + 12*n + p))^13/(13*b*(1 + 12*n + p))
```

Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(27) = 54$.

Time = 0.04 (sec) , antiderivative size = 363, normalized size of antiderivative = 12.52

$$\frac{b^{12}x^{13}x^{156n}x^{13p}}{13+156n+13p}+\frac{a\,b^{11}x^{12}x^{144n}x^{12p}}{1+12n+p}+\frac{6a^2b^{10}x^{11}x^{132n}x^{11p}}{1+12n+p}+\frac{22a^3b^9x^{10}x^{120n}x^{10p}}{1+12n+p}+\frac{55a^4b^8x^9x^{108n}x^{9p}}{1+12n+p}+\frac{99a^5b^7}{1+1}$$

input
```
int(x^p*(a*x^n+b*x^(1+13*n+p))^12,x)
```

output
```
1/13*b^12*x^13*(x^n)^156/(1+12*n+p)*(x^p)^13+a*b^11*x^12*(x^n)^144/(1+12*n
+p)*(x^p)^12+6*a^2*b^10*x^11*(x^n)^132/(1+12*n+p)*(x^p)^11+22*a^3*b^9*x^10
*(x^n)^120/(1+12*n+p)*(x^p)^10+55*a^4*b^8*x^9*(x^n)^108/(1+12*n+p)*(x^p)^9
+99*a^5*b^7*x^8*(x^n)^96/(1+12*n+p)*(x^p)^8+132*a^6*b^6*x^7*(x^n)^84/(1+12
*n+p)*(x^p)^7+132*a^7*b^5*x^6*(x^n)^72/(1+12*n+p)*(x^p)^6+99*a^8*b^4*x^5*(
x^n)^60/(1+12*n+p)*(x^p)^5+55*a^9*b^3*x^4*(x^n)^48/(1+12*n+p)*(x^p)^4+22*a
^10*b^2*x^3*(x^n)^36/(1+12*n+p)*(x^p)^3+6*a^11*b*x^2*(x^n)^24/(1+12*n+p)*(
x^p)^2+a^12/(1+12*n+p)*x*(x^n)^12*x^p
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(27) = 54$.

Time = 0.10 (sec) , antiderivative size = 297, normalized size of antiderivative = 10.24

$$\int x^p \left(ax^n + bx^{1+13n+p}\right)^{12} dx$$
$$= \frac{78\, a^2 b^{10} x^{2\,n} x^{143\,n+11\,p+11} + 286\, a^3 b^9 x^{3\,n} x^{130\,n+10\,p+10} + 715\, a^4 b^8 x^{4\,n} x^{117\,n+9\,p+9} + 1287\, a^5 b^7 x^{5\,n} x^{104\,n+8\,p+8}}{}$$

input
```
integrate(x^p*(a*x^n+b*x^(1+13*n+p))^12,x, algorithm="fricas")
```

output
```
1/13*(78*a^2*b^10*x^(2*n)*x^(143*n + 11*p + 11) + 286*a^3*b^9*x^(3*n)*x^(1
30*n + 10*p + 10) + 715*a^4*b^8*x^(4*n)*x^(117*n + 9*p + 9) + 1287*a^5*b^7
*x^(5*n)*x^(104*n + 8*p + 8) + 1716*a^6*b^6*x^(6*n)*x^(91*n + 7*p + 7) + 1
716*a^7*b^5*x^(7*n)*x^(78*n + 6*p + 6) + 1287*a^8*b^4*x^(8*n)*x^(65*n + 5*
p + 5) + 715*a^9*b^3*x^(9*n)*x^(52*n + 4*p + 4) + 286*a^10*b^2*x^(10*n)*x^
(39*n + 3*p + 3) + 78*a^11*b*x^(11*n)*x^(26*n + 2*p + 2) + 13*a^12*x^(12*n
)*x^(13*n + p + 1) + 13*a*b^11*x^(156*n + 12*p + 12)*x^n + b^12*x^(169*n +
 13*p + 13))/((12*n + p + 1)*x^(13*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 690 vs. $2(22) = 44$.

Time = 147.77 (sec) , antiderivative size = 690, normalized size of antiderivative = 23.79

$$\int x^p \left(ax^n + bx^{1+13n+p}\right)^{12} dx = \text{Too large to display}$$

input
```
integrate(x**p*(a*x**n+b*x**(1+13*n+p))**12,x)
```

output

```
Piecewise((13*a**12*x*x**(12*n)*x**p/(156*n + 13*p + 13) + 78*a**11*b*x*x*
*(11*n)*x**p*x**(13*n + p + 1)/(156*n + 13*p + 13) + 286*a**10*b**2*x*x**(
10*n)*x**p*x**(26*n + 2*p + 2)/(156*n + 13*p + 13) + 715*a**9*b**3*x*x**(9
*n)*x**p*x**(39*n + 3*p + 3)/(156*n + 13*p + 13) + 1287*a**8*b**4*x*x**(8*
n)*x**p*x**(52*n + 4*p + 4)/(156*n + 13*p + 13) + 1716*a**7*b**5*x*x**(7*n
)*x**p*x**(65*n + 5*p + 5)/(156*n + 13*p + 13) + 1716*a**6*b**6*x*x**(6*n)
*x**p*x**(78*n + 6*p + 6)/(156*n + 13*p + 13) + 1287*a**5*b**7*x*x**(5*n)*
x**p*x**(91*n + 7*p + 7)/(156*n + 13*p + 13) + 715*a**4*b**8*x*x**(4*n)*x*
*p*x**(104*n + 8*p + 8)/(156*n + 13*p + 13) + 286*a**3*b**9*x*x**(3*n)*x**
p*x**(117*n + 9*p + 9)/(156*n + 13*p + 13) + 78*a**2*b**10*x*x**(2*n)*x**p
*x**(130*n + 10*p + 10)/(156*n + 13*p + 13) + 13*a*b**11*x*x**n*x**p*x**(1
43*n + 11*p + 11)/(156*n + 13*p + 13) + b**12*x*x**p*x**(156*n + 12*p + 12
)/(156*n + 13*p + 13), Ne(n, -p/12 - 1/12)), (a**12*Piecewise((log(x), Eq(
p, 0)), (log(x**p)/p, True)) + 12*a**11*b*Piecewise((log(x), Eq(p, 0)), (l
og(x**p)/p, True)) + 66*a**10*b**2*Piecewise((log(x), Eq(p, 0)), (log(x**p
)/p, True)) + 220*a**9*b**3*Piecewise((log(x), Eq(p, 0)), (log(x**p)/p, Tr
ue)) + 495*a**8*b**4*Piecewise((log(x), Eq(p, 0)), (log(x**p)/p, True)) +
792*a**7*b**5*Piecewise((log(x), Eq(p, 0)), (log(x**p)/p, True)) + 924*a**
6*b**6*Piecewise((log(x), Eq(p, 0)), (log(x**p)/p, True)) + 792*a**5*b**7*
Piecewise((log(x), Eq(p, 0)), (log(x**p)/p, True)) + 495*a**4*b**8*Piec...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(27) = 54$.

Time = 0.04 (sec) , antiderivative size = 325, normalized size of antiderivative = 11.21

$$\int x^p\left(ax^n+bx^{1+13n+p}\right)^{12}\,dx=\frac{b^{12}x^{156\,n+13\,p+13}}{13\,(12\,n+p+1)}+\frac{ab^{11}x^{144\,n+12\,p+12}}{12\,n+p+1}+\frac{6\,a^2b^{10}x^{132\,n+11\,p+11}}{12\,n+p+1}+\frac{22\,a^3b^9x^{120\,n+10\,p+10}}{12\,n+p+1}+\frac{55\,a^4b^8x^{108\,n+9\,p+9}}{12\,n+p+1}+\frac{99\,a^5b^7x^{96\,n+8\,p+8}}{12\,n+p+1}+\frac{132\,a^6b^6x^{84\,n+7\,p+7}}{12\,n+p+1}+\frac{132\,a^7b^5x^{72\,n+6\,p+6}}{12\,n+p+1}+\frac{99\,a^8b^4x^{60\,n+5\,p+5}}{12\,n+p+1}+\frac{55\,a^9b^3x^{48\,n+4\,p+4}}{12\,n+p+1}+\frac{22\,a^{10}b^2x^{36\,n+3\,p+3}}{12\,n+p+1}+\frac{6\,a^{11}bx^{24\,n+2\,p+2}}{12\,n+p+1}+\frac{a^{12}x^{12\,n+p+1}}{12\,n+p+1}$$

input
```
integrate(x^p*(a*x^n+b*x^(1+13*n+p))^12,x, algorithm="maxima")
```

output
```
1/13*b^12*x^(156*n + 13*p + 13)/(12*n + p + 1) + a*b^11*x^(144*n + 12*p +
12)/(12*n + p + 1) + 6*a^2*b^10*x^(132*n + 11*p + 11)/(12*n + p + 1) + 22*
a^3*b^9*x^(120*n + 10*p + 10)/(12*n + p + 1) + 55*a^4*b^8*x^(108*n + 9*p +
 9)/(12*n + p + 1) + 99*a^5*b^7*x^(96*n + 8*p + 8)/(12*n + p + 1) + 132*a^
6*b^6*x^(84*n + 7*p + 7)/(12*n + p + 1) + 132*a^7*b^5*x^(72*n + 6*p + 6)/(
12*n + p + 1) + 99*a^8*b^4*x^(60*n + 5*p + 5)/(12*n + p + 1) + 55*a^9*b^3*
x^(48*n + 4*p + 4)/(12*n + p + 1) + 22*a^10*b^2*x^(36*n + 3*p + 3)/(12*n +
 p + 1) + 6*a^11*b*x^(24*n + 2*p + 2)/(12*n + p + 1) + a^12*x^(12*n + p +
1)/(12*n + p + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50971 vs. $2(27) = 54$.

Time = 1.45 (sec) , antiderivative size = 50971, normalized size of antiderivative = 1757.62

$$\int x^p \left(ax^n + bx^{1+13n+p}\right)^{12} dx = \text{Too large to display}$$

input
```
integrate(x^p*(a*x^n+b*x^(1+13*n+p))^12,x, algorithm="giac")
```

output
```
(31408819200*a^2*b^10*n^10*p*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) +
 35*log(x)) + 331405966080*a^2*b^10*n^9*p^2*x*x^(2*n)*x^p*e^(455*n*log(x)
+ 35*p*log(x) + 35*log(x)) + 1230778965888*a^2*b^10*n^8*p^3*x*x^(2*n)*x^p*
e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 2139674600448*a^2*b^10*n^7*p^
4*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 1890383812992
*a^2*b^10*n^6*p^5*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x))
 + 874552702464*a^2*b^10*n^5*p^6*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(
x) + 35*log(x)) + 222844093056*a^2*b^10*n^4*p^7*x*x^(2*n)*x^p*e^(455*n*log
(x) + 35*p*log(x) + 35*log(x)) + 32330382336*a^2*b^10*n^3*p^8*x*x^(2*n)*x^
p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 2661766272*a^2*b^10*n^2*p^9
*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 115879680*a^2*
b^10*n*p^10*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 207
3600*a^2*b^10*p^11*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)
) + 3156586329600*a^5*b^7*n^10*p*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(
x) + 34*log(x)) + 33302016570240*a^5*b^7*n^9*p^2*x*x^(5*n)*x^p*e^(442*n*lo
g(x) + 34*p*log(x) + 34*log(x)) + 123648483714624*a^5*b^7*n^8*p^3*x*x^(5*n
)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 214873536791232*a^5*b^7
*n^7*p^4*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 189706
686719616*a^5*b^7*n^6*p^5*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34
*log(x)) + 87659938485504*a^5*b^7*n^5*p^6*x*x^(5*n)*x^p*e^(442*n*log(x)...
```

Mupad [B] (verification not implemented)

Time = 10.86 (sec) , antiderivative size = 363, normalized size of antiderivative = 12.52

$$\int x^p\left(ax^n+bx^{1+13n+p}\right)^{12}\,dx=\frac{a^{12}\,x\,x^p\,x^{12\,n}}{12\,n+p+1}+\frac{b^{12}\,x^{156\,n}\,x^{13\,p}\,x^{13}}{156\,n+13\,p+13}+\frac{22\,a^{10}\,b^2\,x^{36\,n}\,x^{3\,p}\,x^3}{12\,n+p+1}+\frac{55\,a^9\,b^3\,x^{48\,n}\,x^{4\,p}\,x^4}{12\,n+p+1}+\frac{99\,a^8\,b^4\,x^{60\,n}\,x^{5\,p}\,x^5}{12\,n+p+1}+\frac{132\,a^7\,b^5\,x^{72\,n}\,x^{6\,p}\,x^6}{12\,n+p+1}+\frac{132\,a^6\,b^6\,x^{84\,n}\,x^{7\,p}\,x^7}{12\,n+p+1}+\frac{99\,a^5\,b^7\,x^{96\,n}\,x^{8\,p}\,x^8}{12\,n+p+1}+\frac{55\,a^4\,b^8\,x^{108\,n}\,x^{9\,p}\,x^9}{12\,n+p+1}+\frac{22\,a^3\,b^9\,x^{120\,n}\,x^{10\,p}\,x^{10}}{12\,n+p+1}+\frac{6\,a^2\,b^{10}\,x^{132\,n}\,x^{11\,p}\,x^{11}}{12\,n+p+1}+\frac{6\,a^{11}\,b\,x^{24\,n}\,x^{2\,p}\,x^2}{12\,n+p+1}+\frac{a\,b^{11}\,x^{144\,n}\,x^{12\,p}\,x^{12}}{12\,n+p+1}$$

input
```
int(x^p*(a*x^n + b*x^(13*n + p + 1))^12,x)
```

output
```
(a^12*x*x^p*x^(12*n))/(12*n + p + 1) + (b^12*x^(156*n)*x^(13*p)*x^13)/(156
*n + 13*p + 13) + (22*a^10*b^2*x^(36*n)*x^(3*p)*x^3)/(12*n + p + 1) + (55*
a^9*b^3*x^(48*n)*x^(4*p)*x^4)/(12*n + p + 1) + (99*a^8*b^4*x^(60*n)*x^(5*p
)*x^5)/(12*n + p + 1) + (132*a^7*b^5*x^(72*n)*x^(6*p)*x^6)/(12*n + p + 1)
+ (132*a^6*b^6*x^(84*n)*x^(7*p)*x^7)/(12*n + p + 1) + (99*a^5*b^7*x^(96*n)
*x^(8*p)*x^8)/(12*n + p + 1) + (55*a^4*b^8*x^(108*n)*x^(9*p)*x^9)/(12*n +
p + 1) + (22*a^3*b^9*x^(120*n)*x^(10*p)*x^10)/(12*n + p + 1) + (6*a^2*b^10
*x^(132*n)*x^(11*p)*x^11)/(12*n + p + 1) + (6*a^11*b*x^(24*n)*x^(2*p)*x^2)
/(12*n + p + 1) + (a*b^11*x^(144*n)*x^(12*p)*x^12)/(12*n + p + 1)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 253, normalized size of antiderivative = 8.72

$$\int x^p(ax^n+bx^{1+13n+p})^{12}\,dx$$
$$= \frac{x^{12n+p}x(x^{144n+12p}b^{12}x^{12}+13x^{132n+11p}a\,b^{11}x^{11}+78x^{120n+10p}a^2b^{10}x^{10}+286x^{108n+9p}a^3b^9x^9+715x^{96n+8p}a}{}$$

input
```
int(x^p*(a*x^n+b*x^(1+13*n+p))^12,x)
```

output
```
(x**(12*n + p)*x*(x**(144*n + 12*p)*b**12*x**12 + 13*x**(132*n + 11*p)*a*b
**11*x**11 + 78*x**(120*n + 10*p)*a**2*b**10*x**10 + 286*x**(108*n + 9*p)*
a**3*b**9*x**9 + 715*x**(96*n + 8*p)*a**4*b**8*x**8 + 1287*x**(84*n + 7*p)
*a**5*b**7*x**7 + 1716*x**(72*n + 6*p)*a**6*b**6*x**6 + 1716*x**(60*n + 5*
p)*a**7*b**5*x**5 + 1287*x**(48*n + 4*p)*a**8*b**4*x**4 + 715*x**(36*n + 3
*p)*a**9*b**3*x**3 + 286*x**(24*n + 2*p)*a**10*b**2*x**2 + 78*x**(12*n + p
)*a**11*b*x + 13*a**12))/(13*(12*n + p + 1))
```

3.328 $\int x^{12}(a+bx^{13})^{12}\,dx$

Optimal result . . . 2659
Mathematica [B] (verified) . . . 2659
Rubi [A] (verified) . . . 2660
Maple [A] (verified) . . . 2661
Fricas [B] (verification not implemented) . . . 2661
Sympy [B] (verification not implemented) . . . 2662
Maxima [A] (verification not implemented) . . . 2662
Giac [A] (verification not implemented) . . . 2663
Mupad [B] (verification not implemented) . . . 2663
Reduce [B] (verification not implemented) . . . 2663

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int x^{12}\left(a+bx^{13}\right)^{12}\,dx=\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

output
```
1/169*(b*x^13+a)^13/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\begin{aligned}\int x^{12}\left(a+bx^{13}\right)^{12}\,dx=&\frac{a^{12}x^{13}}{13}+\frac{6}{13}a^{11}bx^{26}+\frac{22}{13}a^{10}b^2x^{39}+\frac{55}{13}a^9b^3x^{52}+\frac{99}{13}a^8b^4x^{65}\\&+\frac{132}{13}a^7b^5x^{78}+\frac{132}{13}a^6b^6x^{91}+\frac{99}{13}a^5b^7x^{104}+\frac{55}{13}a^4b^8x^{117}\\&+\frac{22}{13}a^3b^9x^{130}+\frac{6}{13}a^2b^{10}x^{143}+\frac{1}{13}ab^{11}x^{156}+\frac{b^{12}x^{169}}{169}\end{aligned}$$

input
```
Integrate[x^12*(a + b*x^13)^12,x]
```

output
```
(a^12*x^13)/13 + (6*a^11*b*x^26)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*
x^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^9
1)/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)
/13 + (6*a^2*b^10*x^143)/13 + (a*b^11*x^156)/13 + (b^12*x^169)/169
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{793\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{12}\left(a+bx^{13}\right)^{12} dx$$

$$\Big\downarrow \text{793}$$

$$\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

input
```
Int[x^12*(a + b*x^13)^12,x]
```

output
```
(a + b*x^13)^13/(169*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
default	$\frac{(b\,x^{13}+a)^{13}}{169b}$
gosper	$\frac{132}{13}a^7b^5x^{78}+\frac{55}{13}a^4b^8x^{117}+\frac{132}{13}a^6b^6x^{91}+\frac{1}{13}a\,b^{11}x^{156}+\frac{6}{13}a^{11}b\,x^{26}+\frac{22}{13}a^{10}b^2x^{39}+\frac{6}{13}a^2b^{10}x^{143}$
parallelrisch	$\frac{132}{13}a^7b^5x^{78}+\frac{55}{13}a^4b^8x^{117}+\frac{132}{13}a^6b^6x^{91}+\frac{1}{13}a\,b^{11}x^{156}+\frac{6}{13}a^{11}b\,x^{26}+\frac{22}{13}a^{10}b^2x^{39}+\frac{6}{13}a^2b^{10}x^{143}$
orering	$\frac{x^{13}(b^{12}x^{156}+13a\,b^{11}x^{143}+78a^2b^{10}x^{130}+286a^3b^9x^{117}+715a^4b^8x^{104}+1287a^5b^7x^{91}+1716a^6b^6x^{78}+1716a^7b^5x^{65}+1287a^8b^4}{169}$
risch	$\frac{b^{12}x^{169}}{169}+\frac{a\,b^{11}x^{156}}{13}+\frac{6a^2b^{10}x^{143}}{13}+\frac{22a^3b^9x^{130}}{13}+\frac{55a^4b^8x^{117}}{13}+\frac{99a^5b^7x^{104}}{13}+\frac{132a^6b^6x^{91}}{13}+\frac{132a^7b^5x^{78}}{13}+$

input
```
int(x^12*(b*x^13+a)^12,x,method=_RETURNVERBOSE)
```

output
```
1/169*(b*x^13+a)^13/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(14) = 28.

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}(a+bx^{13})^{12}\,dx=\frac{1}{169}b^{12}x^{169}+\frac{1}{13}ab^{11}x^{156}+\frac{6}{13}a^2b^{10}x^{143}+\frac{22}{13}a^3b^9x^{130}+\frac{55}{13}a^4b^8x^{117}+\frac{99}{13}a^5b^7x^{104}+\frac{132}{13}a^6b^6x^{91}+\frac{132}{13}a^7b^5x^{78}+\frac{99}{13}a^8b^4x^{65}+\frac{55}{13}a^9b^3x^{52}+\frac{22}{13}a^{10}b^2x^{39}+\frac{6}{13}a^{11}bx^{26}+\frac{1}{13}a^{12}x^{13}$$

input
```
integrate(x^12*(b*x^13+a)^12,x, algorithm="fricas")
```

output
```
1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9
*x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 +
 132/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^1
0*b^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{12}\left(a+bx^{13}\right)^{12}\,dx = \frac{a^{12}x^{13}}{13} + \frac{6a^{11}bx^{26}}{13} + \frac{22a^{10}b^2x^{39}}{13} + \frac{55a^9b^3x^{52}}{13} + \frac{99a^8b^4x^{65}}{13} + \frac{132a^7b^5x^{78}}{13} + \frac{132a^6b^6x^{91}}{13} + \frac{99a^5b^7x^{104}}{13} + \frac{55a^4b^8x^{117}}{13} + \frac{22a^3b^9x^{130}}{13} + \frac{6a^2b^{10}x^{143}}{13} + \frac{ab^{11}x^{156}}{13} + \frac{b^{12}x^{169}}{169}$$

input
```
integrate(x**12*(b*x**13+a)**12,x)
```

output
```
a**12*x**13/13 + 6*a**11*b*x**26/13 + 22*a**10*b**2*x**39/13 + 55*a**9*b**
3*x**52/13 + 99*a**8*b**4*x**65/13 + 132*a**7*b**5*x**78/13 + 132*a**6*b**
6*x**91/13 + 99*a**5*b**7*x**104/13 + 55*a**4*b**8*x**117/13 + 22*a**3*b**
9*x**130/13 + 6*a**2*b**10*x**143/13 + a*b**11*x**156/13 + b**12*x**169/16
9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{12}\left(a+bx^{13}\right)^{12}\,dx = \frac{\left(bx^{13}+a\right)^{13}}{169\,b}$$

input
```
integrate(x^12*(b*x^13+a)^12,x, algorithm="maxima")
```

output
```
1/169*(b*x^13 + a)^13/b
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{12}\left(a+bx^{13}\right)^{12}\,dx=\frac{\left(bx^{13}+a\right)^{13}}{169\,b}$$

input
```
integrate(x^12*(b*x^13+a)^12,x, algorithm="giac")
```

output
```
1/169*(b*x^13 + a)^13/b
```

Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{12}\left(a+bx^{13}\right)^{12}\,dx=\frac{\left(b\,x^{13}+a\right)^{13}}{169\,b}$$

input
```
int(x^12*(a + b*x^13)^12,x)
```

output
```
(a + b*x^13)^13/(169*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

$$\int x^{12}\left(a+bx^{13}\right)^{12}\,dx$$
$$=\frac{x^{13}(b^{12}x^{156}+13a\,b^{11}x^{143}+78a^{2}b^{10}x^{130}+286a^{3}b^{9}x^{117}+715a^{4}b^{8}x^{104}+1287a^{5}b^{7}x^{91}+1716a^{6}b^{6}x^{78}+17}{169}$$

input
```
int(x^12*(b*x^13+a)^12,x)
```

output
```
(x**13*(13*a**12 + 78*a**11*b*x**13 + 286*a**10*b**2*x**26 + 715*a**9*b**3
*x**39 + 1287*a**8*b**4*x**52 + 1716*a**7*b**5*x**65 + 1716*a**6*b**6*x**7
8 + 1287*a**5*b**7*x**91 + 715*a**4*b**8*x**104 + 286*a**3*b**9*x**117 + 7
8*a**2*b**10*x**130 + 13*a*b**11*x**143 + b**12*x**156))/169
```

3.329 $\int x^{12}(ax+bx^{26})^{12}\,dx$

Optimal result 2665
Mathematica [B] (verified) 2665
Rubi [A] (verified) 2666
Maple [B] (verified) 2667
Fricas [B] (verification not implemented) 2668
Sympy [B] (verification not implemented) 2668
Maxima [B] (verification not implemented) 2669
Giac [B] (verification not implemented) 2669
Mupad [B] (verification not implemented) 2670
Reduce [B] (verification not implemented) 2671

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

output
```
1/325*(b*x^25+a)^13/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{a^{12}x^{25}}{25}+\frac{6}{25}a^{11}bx^{50}+\frac{22}{25}a^{10}b^2x^{75}+\frac{11}{5}a^9b^3x^{100}+\frac{99}{25}a^8b^4x^{125}+\frac{132}{25}a^7b^5x^{150}+\frac{132}{25}a^6b^6x^{175}+\frac{99}{25}a^5b^7x^{200}+\frac{11}{5}a^4b^8x^{225}+\frac{22}{25}a^3b^9x^{250}+\frac{6}{25}a^2b^{10}x^{275}+\frac{1}{25}ab^{11}x^{300}+\frac{b^{12}x^{325}}{325}$$

input
```
Integrate[x^12*(a*x + b*x^26)^12,x]
```

output
```
(a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*
x^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x
^175)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^25
0)/25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {9, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{12}\left(ax+bx^{26}\right)^{12}dx$$
$$\Big\downarrow \; 9$$
$$\int x^{24}\left(a+bx^{25}\right)^{12}dx$$
$$\Big\downarrow \; 793$$
$$\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

input
```
Int[x^12*(a*x + b*x^26)^12,x]
```

output
```
(a + b*x^25)^13/(325*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(14) = 28.

Time = 0.89 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{11}{5}a^9b^3x^{100}+\frac{132}{25}a^6b^6x^{175}+\frac{1}{25}a^{12}x^{25}+\frac{1}{325}b^{12}x^{325}+\frac{99}{25}a^5b^7x^{200}+\frac{6}{25}a^{11}b\,x^{50}+\frac{132}{25}a^7b^5x^{150}$
parallelrisch	$\frac{11}{5}a^9b^3x^{100}+\frac{132}{25}a^6b^6x^{175}+\frac{1}{25}a^{12}x^{25}+\frac{1}{325}b^{12}x^{325}+\frac{99}{25}a^5b^7x^{200}+\frac{6}{25}a^{11}b\,x^{50}+\frac{132}{25}a^7b^5x^{150}$
gosper	$\frac{x^{25}(b^{12}x^{300}+13a\,b^{11}x^{275}+78a^2b^{10}x^{250}+286a^3b^9x^{225}+715a^4b^8x^{200}+1287a^5b^7x^{175}+1716a^6b^6x^{150}+1716a^7b^5x^{125}+1287a}{325}$
risch	$\frac{b^{12}x^{325}}{325}+\frac{a\,b^{11}x^{300}}{25}+\frac{6a^2b^{10}x^{275}}{25}+\frac{22a^3b^9x^{250}}{25}+\frac{11a^4b^8x^{225}}{5}+\frac{99a^5b^7x^{200}}{25}+\frac{132a^6b^6x^{175}}{25}+\frac{132a^7b^5x^{150}}{25}+$
orering	$\frac{x^{13}(b^{12}x^{300}+13a\,b^{11}x^{275}+78a^2b^{10}x^{250}+286a^3b^9x^{225}+715a^4b^8x^{200}+1287a^5b^7x^{175}+1716a^6b^6x^{150}+1716a^7b^5x^{125}+1287a}{325(b\,x^{25}+a)^{12}}$

input
```
int(x^12*(b*x^26+a*x)^12,x,method=_RETURNVERBOSE)
```

output
```
11/5*a^9*b^3*x^100+132/25*a^6*b^6*x^175+1/25*a^12*x^25+1/325*b^12*x^325+99
/25*a^5*b^7*x^200+6/25*a^11*b*x^50+132/25*a^7*b^5*x^150+1/25*a*b^11*x^300+
99/25*a^8*b^4*x^125+11/5*a^4*b^8*x^225+22/25*a^10*b^2*x^75+22/25*a^3*b^9*x
^250+6/25*a^2*b^10*x^275
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^{2}b^{10}x^{275}+\frac{22}{25}\,a^{3}b^{9}x^{250}+\frac{11}{5}\,a^{4}b^{8}x^{225}+\frac{99}{25}\,a^{5}b^{7}x^{200}+\frac{132}{25}\,a^{6}b^{6}x^{175}+\frac{132}{25}\,a^{7}b^{5}x^{150}+\frac{99}{25}\,a^{8}b^{4}x^{125}+\frac{11}{5}\,a^{9}b^{3}x^{100}+\frac{22}{25}\,a^{10}b^{2}x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}$$

input
```
integrate(x^12*(b*x^26+a*x)^12,x, algorithm="fricas")
```

output
```
1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9
*x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 +
 132/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a
^10*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{a^{12}x^{25}}{25}+\frac{6a^{11}bx^{50}}{25}+\frac{22a^{10}b^{2}x^{75}}{25}+\frac{11a^{9}b^{3}x^{100}}{5}+\frac{99a^{8}b^{4}x^{125}}{25}+\frac{132a^{7}b^{5}x^{150}}{25}+\frac{132a^{6}b^{6}x^{175}}{25}+\frac{99a^{5}b^{7}x^{200}}{25}+\frac{11a^{4}b^{8}x^{225}}{5}+\frac{22a^{3}b^{9}x^{250}}{25}+\frac{6a^{2}b^{10}x^{275}}{25}+\frac{ab^{11}x^{300}}{25}+\frac{b^{12}x^{325}}{325}$$

input
```
integrate(x**12*(b*x**26+a*x)**12,x)
```

output
```
a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**
3*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b
**6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b
**9*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/
325
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx = \frac{1}{325}\,b^{12}x^{325} + \frac{1}{25}\,ab^{11}x^{300} + \frac{6}{25}\,a^2b^{10}x^{275} + \frac{22}{25}\,a^3b^9x^{250} + \frac{11}{5}\,a^4b^8x^{225} + \frac{99}{25}\,a^5b^7x^{200} + \frac{132}{25}\,a^6b^6x^{175} + \frac{132}{25}\,a^7b^5x^{150} + \frac{99}{25}\,a^8b^4x^{125} + \frac{11}{5}\,a^9b^3x^{100} + \frac{22}{25}\,a^{10}b^2x^{75} + \frac{6}{25}\,a^{11}bx^{50} + \frac{1}{25}\,a^{12}x^{25}$$

input
```
integrate(x^12*(b*x^26+a*x)^12,x, algorithm="maxima")
```

output
```
1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9
*x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 +
 132/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a
^10*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.23 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^2b^{10}x^{275}+\frac{22}{25}\,a^3b^9x^{250}+\frac{11}{5}\,a^4b^8x^{225}+\frac{99}{25}\,a^5b^7x^{200}+\frac{132}{25}\,a^6b^6x^{175}+\frac{132}{25}\,a^7b^5x^{150}+\frac{99}{25}\,a^8b^4x^{125}+\frac{11}{5}\,a^9b^3x^{100}+\frac{22}{25}\,a^{10}b^2x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}$$

input
```
integrate(x^12*(b*x^26+a*x)^12,x, algorithm="giac")
```

output
```
1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9
*x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 +
 132/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a
^10*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{a^{12}\,x^{25}}{25}+\frac{6\,a^{11}\,b\,x^{50}}{25}+\frac{22\,a^{10}\,b^2\,x^{75}}{25}+\frac{11\,a^9\,b^3\,x^{100}}{5}+\frac{99\,a^8\,b^4\,x^{125}}{25}+\frac{132\,a^7\,b^5\,x^{150}}{25}+\frac{132\,a^6\,b^6\,x^{175}}{25}+\frac{99\,a^5\,b^7\,x^{200}}{25}+\frac{11\,a^4\,b^8\,x^{225}}{5}+\frac{22\,a^3\,b^9\,x^{250}}{25}+\frac{6\,a^2\,b^{10}\,x^{275}}{25}+\frac{a\,b^{11}\,x^{300}}{25}+\frac{b^{12}\,x^{325}}{325}$$

input
```
int(x^12*(a*x + b*x^26)^12,x)
```

output
```
(a^12*x^25)/25 + (b^12*x^325)/325 + (6*a^11*b*x^50)/25 + (a*b^11*x^300)/25
 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x^100)/5 + (99*a^8*b^4*x^125)/25 +
(132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^175)/25 + (99*a^5*b^7*x^200)/25 +
(11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/25 + (6*a^2*b^10*x^275)/25
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx = \frac{x^{25}(b^{12}x^{300}+13a\,b^{11}x^{275}+78a^2b^{10}x^{250}+286a^3b^9x^{225}+715a^4b^8x^{200}+1287a^5b^7x^{175}+1716a^6b^6x^{150}+1}{325}$$

input
```
int(x^12*(b*x^26+a*x)^12,x)
```

output
```
(x**25*(13*a**12 + 78*a**11*b*x**25 + 286*a**10*b**2*x**50 + 715*a**9*b**3
*x**75 + 1287*a**8*b**4*x**100 + 1716*a**7*b**5*x**125 + 1716*a**6*b**6*x*
*150 + 1287*a**5*b**7*x**175 + 715*a**4*b**8*x**200 + 286*a**3*b**9*x**225
 + 78*a**2*b**10*x**250 + 13*a*b**11*x**275 + b**12*x**300))/325
```

3.330 $\int x^{12}(ax^2+bx^{39})^{12}\,dx$

Optimal result 2672
Mathematica [B] (verified) 2672
Rubi [A] (verified) 2673
Maple [B] (verified) 2674
Fricas [B] (verification not implemented) 2675
Sympy [B] (verification not implemented) 2675
Maxima [B] (verification not implemented) 2676
Giac [B] (verification not implemented) 2676
Mupad [B] (verification not implemented) 2677
Reduce [B] (verification not implemented) 2678

Optimal result

Integrand size = 17, antiderivative size = 16

$$\int x^{12}\left(ax^2+bx^{39}\right)^{12}\,dx=\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

output
```
1/481*(b*x^37+a)^13/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{12}\left(ax^2+bx^{39}\right)^{12}\,dx=\frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8x^{333}+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^2b^{10}x^{407}+\frac{1}{37}ab^{11}x^{444}+\frac{b^{12}x^{481}}{481}$$

input
```
Integrate[x^12*(a*x^2 + b*x^39)^12,x]
```

output
```
(a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3
*x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6
*x^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x
^370)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {9, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{12}\left(ax^2+bx^{39}\right)^{12}dx$$

$$\Big\downarrow \text{9}$$

$$\int x^{36}\left(a+bx^{37}\right)^{12}dx$$

$$\Big\downarrow \text{793}$$

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

input
```
Int[x^12*(a*x^2 + b*x^39)^12,x]
```

output
```
(a + b*x^37)^13/(481*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 1.75 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{99}{37}a^8b^4x^{185} + \frac{22}{37}a^3b^9x^{370} + \frac{6}{37}a^{11}b\,x^{74} + \frac{1}{37}a\,b^{11}x^{444} + \frac{132}{37}a^6b^6x^{259} + \frac{22}{37}a^{10}b^2x^{111} + \frac{55}{37}a^4b^8x^{33}$
parallelrisch	$\frac{99}{37}a^8b^4x^{185} + \frac{22}{37}a^3b^9x^{370} + \frac{6}{37}a^{11}b\,x^{74} + \frac{1}{37}a\,b^{11}x^{444} + \frac{132}{37}a^6b^6x^{259} + \frac{22}{37}a^{10}b^2x^{111} + \frac{55}{37}a^4b^8x^{33}$
gosper	$\frac{x^{37}(b^{12}x^{444}+13a\,b^{11}x^{407}+78a^2b^{10}x^{370}+286a^3b^9x^{333}+715a^4b^8x^{296}+1287a^5b^7x^{259}+1716a^6b^6x^{222}+1716a^7b^5x^{185}+1287a}{481}$
risch	$\frac{b^{12}x^{481}}{481} + \frac{a\,b^{11}x^{444}}{37} + \frac{6a^2b^{10}x^{407}}{37} + \frac{22a^3b^9x^{370}}{37} + \frac{55a^4b^8x^{333}}{37} + \frac{99a^5b^7x^{296}}{37} + \frac{132a^6b^6x^{259}}{37} + \frac{132a^7b^5x^{222}}{37} +$
orering	$\frac{x^{13}(b^{12}x^{444}+13a\,b^{11}x^{407}+78a^2b^{10}x^{370}+286a^3b^9x^{333}+715a^4b^8x^{296}+1287a^5b^7x^{259}+1716a^6b^6x^{222}+1716a^7b^5x^{185}+1287a}{481(b\,x^{37}+a)^{12}}$

input
```
int(x^12*(b*x^39+a*x^2)^12,x,method=_RETURNVERBOSE)
```

output
```
99/37*a^8*b^4*x^185+22/37*a^3*b^9*x^370+6/37*a^11*b*x^74+1/37*a*b^11*x^444
+132/37*a^6*b^6*x^259+22/37*a^10*b^2*x^111+55/37*a^4*b^8*x^333+1/481*b^12*
x^481+132/37*a^7*b^5*x^222+55/37*a^9*b^3*x^148+99/37*a^5*b^7*x^296+1/37*a^
12*x^37+6/37*a^2*b^10*x^407
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax^2+bx^{39}\right)^{12}\,dx = \frac{1}{481}\,b^{12}x^{481} + \frac{1}{37}\,ab^{11}x^{444} + \frac{6}{37}\,a^{2}b^{10}x^{407} + \frac{22}{37}\,a^{3}b^{9}x^{370} + \frac{55}{37}\,a^{4}b^{8}x^{333} + \frac{99}{37}\,a^{5}b^{7}x^{296} + \frac{132}{37}\,a^{6}b^{6}x^{259} + \frac{132}{37}\,a^{7}b^{5}x^{222} + \frac{99}{37}\,a^{8}b^{4}x^{185} + \frac{55}{37}\,a^{9}b^{3}x^{148} + \frac{22}{37}\,a^{10}b^{2}x^{111} + \frac{6}{37}\,a^{11}bx^{74} + \frac{1}{37}\,a^{12}x^{37}$$

input
```
integrate(x^12*(b*x^39+a*x^2)^12,x, algorithm="fricas")
```

output
```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{12}\left(ax^2+bx^{39}\right)^{12}\,dx = \frac{a^{12}x^{37}}{37} + \frac{6a^{11}bx^{74}}{37} + \frac{22a^{10}b^{2}x^{111}}{37} + \frac{55a^{9}b^{3}x^{148}}{37} + \frac{99a^{8}b^{4}x^{185}}{37} + \frac{132a^{7}b^{5}x^{222}}{37} + \frac{132a^{6}b^{6}x^{259}}{37} + \frac{99a^{5}b^{7}x^{296}}{37} + \frac{55a^{4}b^{8}x^{333}}{37} + \frac{22a^{3}b^{9}x^{370}}{37} + \frac{6a^{2}b^{10}x^{407}}{37} + \frac{ab^{11}x^{444}}{37} + \frac{b^{12}x^{481}}{481}$$

input
```
integrate(x**12*(b*x**39+a*x**2)**12,x)
```

output
```
a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b*
*3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6
*b**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**
3*b**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**4
81/481
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}(ax^2+bx^{39})^{12}\,dx = \frac{1}{481}\,b^{12}x^{481} + \frac{1}{37}\,ab^{11}x^{444} + \frac{6}{37}\,a^2b^{10}x^{407} + \frac{22}{37}\,a^3b^9x^{370} + \frac{55}{37}\,a^4b^8x^{333} + \frac{99}{37}\,a^5b^7x^{296} + \frac{132}{37}\,a^6b^6x^{259} + \frac{132}{37}\,a^7b^5x^{222} + \frac{99}{37}\,a^8b^4x^{185} + \frac{55}{37}\,a^9b^3x^{148} + \frac{22}{37}\,a^{10}b^2x^{111} + \frac{6}{37}\,a^{11}bx^{74} + \frac{1}{37}\,a^{12}x^{37}$$

input
```
integrate(x^12*(b*x^39+a*x^2)^12,x, algorithm="maxima")
```

output
```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax^2+bx^{39}\right)^{12}\,dx=\frac{1}{481}b^{12}x^{481}+\frac{1}{37}ab^{11}x^{444}+\frac{6}{37}a^2b^{10}x^{407}+\frac{22}{37}a^3b^9x^{370}+\frac{55}{37}a^4b^8x^{333}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^6b^6x^{259}+\frac{132}{37}a^7b^5x^{222}+\frac{99}{37}a^8b^4x^{185}+\frac{55}{37}a^9b^3x^{148}+\frac{22}{37}a^{10}b^2x^{111}+\frac{6}{37}a^{11}bx^{74}+\frac{1}{37}a^{12}x^{37}$$

input
```
integrate(x^12*(b*x^39+a*x^2)^12,x, algorithm="giac")
```

output
```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax^2+bx^{39}\right)^{12}\,dx=\frac{a^{12}\,x^{37}}{37}+\frac{6\,a^{11}\,b\,x^{74}}{37}+\frac{22\,a^{10}\,b^2\,x^{111}}{37}+\frac{55\,a^9\,b^3\,x^{148}}{37}+\frac{99\,a^8\,b^4\,x^{185}}{37}+\frac{132\,a^7\,b^5\,x^{222}}{37}+\frac{132\,a^6\,b^6\,x^{259}}{37}+\frac{99\,a^5\,b^7\,x^{296}}{37}+\frac{55\,a^4\,b^8\,x^{333}}{37}+\frac{22\,a^3\,b^9\,x^{370}}{37}+\frac{6\,a^2\,b^{10}\,x^{407}}{37}+\frac{a\,b^{11}\,x^{444}}{37}+\frac{b^{12}\,x^{481}}{481}$$

input
```
int(x^12*(a*x^2 + b*x^39)^12,x)
```

output
```
(a^12*x^37)/37 + (b^12*x^481)/481 + (6*a^11*b*x^74)/37 + (a*b^11*x^444)/37
 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37
+ (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37
+ (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

$$\int x^{12}\left(ax^2+bx^{39}\right)^{12}\,dx$$
$$=\frac{x^{37}(b^{12}x^{444}+13a\,b^{11}x^{407}+78a^2b^{10}x^{370}+286a^3b^9x^{333}+715a^4b^8x^{296}+1287a^5b^7x^{259}+1716a^6b^6x^{222}+1}{481}$$

input
```
int(x^12*(b*x^39+a*x^2)^12,x)
```

output
```
(x**37*(13*a**12 + 78*a**11*b*x**37 + 286*a**10*b**2*x**74 + 715*a**9*b**3
*x**111 + 1287*a**8*b**4*x**148 + 1716*a**7*b**5*x**185 + 1716*a**6*b**6*x
**222 + 1287*a**5*b**7*x**259 + 715*a**4*b**8*x**296 + 286*a**3*b**9*x**33
3 + 78*a**2*b**10*x**370 + 13*a*b**11*x**407 + b**12*x**444))/481
```

3.331 $\int x^{24}(a+bx^{25})^{12}\,dx$

Optimal result . . . 2679
Mathematica [B] (verified) . . . 2679
Rubi [A] (verified) . . . 2680
Maple [A] (verified) . . . 2681
Fricas [B] (verification not implemented) . . . 2681
Sympy [B] (verification not implemented) . . . 2682
Maxima [A] (verification not implemented) . . . 2682
Giac [A] (verification not implemented) . . . 2683
Mupad [B] (verification not implemented) . . . 2683
Reduce [B] (verification not implemented) . . . 2683

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int x^{24}\left(a+bx^{25}\right)^{12}\,dx=\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

output
```
1/325*(b*x^25+a)^13/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\begin{aligned}\int x^{24}\left(a+bx^{25}\right)^{12}\,dx=&\frac{a^{12}x^{25}}{25}+\frac{6}{25}a^{11}bx^{50}+\frac{22}{25}a^{10}b^2x^{75}+\frac{11}{5}a^9b^3x^{100}+\frac{99}{25}a^8b^4x^{125}\\&+\frac{132}{25}a^7b^5x^{150}+\frac{132}{25}a^6b^6x^{175}+\frac{99}{25}a^5b^7x^{200}+\frac{11}{5}a^4b^8x^{225}\\&+\frac{22}{25}a^3b^9x^{250}+\frac{6}{25}a^2b^{10}x^{275}+\frac{1}{25}ab^{11}x^{300}+\frac{b^{12}x^{325}}{325}\end{aligned}$$

input
```
Integrate[x^24*(a + b*x^25)^12,x]
```

output
```
(a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*
x^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x
^175)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^25
0)/25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{793\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{24}\left(a+bx^{25}\right)^{12}\,dx$$
$$\Big\downarrow \; 793$$
$$\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

input
```
Int[x^24*(a + b*x^25)^12,x]
```

output
```
(a + b*x^25)^13/(325*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
default	$\frac{\left(b\,x^{25}+a\right)^{13}}{325b}$
gosper	$\frac{11}{5}a^{9}b^{3}x^{100}+\frac{132}{25}a^{6}b^{6}x^{175}+\frac{1}{25}a^{12}x^{25}+\frac{1}{325}b^{12}x^{325}+\frac{99}{25}a^{5}b^{7}x^{200}+\frac{6}{25}a^{11}b\,x^{50}+\frac{132}{25}a^{7}b^{5}x^{150}$ -
parallelrisch	$\frac{11}{5}a^{9}b^{3}x^{100}+\frac{132}{25}a^{6}b^{6}x^{175}+\frac{1}{25}a^{12}x^{25}+\frac{1}{325}b^{12}x^{325}+\frac{99}{25}a^{5}b^{7}x^{200}+\frac{6}{25}a^{11}b\,x^{50}+\frac{132}{25}a^{7}b^{5}x^{150}$ -
orering	$\frac{x^{25}(b^{12}x^{300}+13a\,b^{11}x^{275}+78a^{2}b^{10}x^{250}+286a^{3}b^{9}x^{225}+715a^{4}b^{8}x^{200}+1287a^{5}b^{7}x^{175}+1716a^{6}b^{6}x^{150}+1716a^{7}b^{5}x^{125}+1287a}{325}$
risch	$\frac{b^{12}x^{325}}{325}+\frac{a\,b^{11}x^{300}}{25}+\frac{6a^{2}b^{10}x^{275}}{25}+\frac{22a^{3}b^{9}x^{250}}{25}+\frac{11a^{4}b^{8}x^{225}}{5}+\frac{99a^{5}b^{7}x^{200}}{25}+\frac{132a^{6}b^{6}x^{175}}{25}+\frac{132a^{7}b^{5}x^{150}}{25}$ +

input
```
int(x^24*(b*x^25+a)^12,x,method=_RETURNVERBOSE)
```

output
```
1/325*(b*x^25+a)^13/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(14) = 28.

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int x^{24}\left(a+bx^{25}\right)^{12}\,dx &= \frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^{2}b^{10}x^{275}\\&+\frac{22}{25}\,a^{3}b^{9}x^{250}+\frac{11}{5}\,a^{4}b^{8}x^{225}+\frac{99}{25}\,a^{5}b^{7}x^{200}\\&+\frac{132}{25}\,a^{6}b^{6}x^{175}+\frac{132}{25}\,a^{7}b^{5}x^{150}+\frac{99}{25}\,a^{8}b^{4}x^{125}\\&+\frac{11}{5}\,a^{9}b^{3}x^{100}+\frac{22}{25}\,a^{10}b^{2}x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}\end{aligned}$$

input
```
integrate(x^24*(b*x^25+a)^12,x, algorithm="fricas")
```

output
```
1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9
*x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 +
 132/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a
^10*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{24}(a+bx^{25})^{12}\,dx = \frac{a^{12}x^{25}}{25}+\frac{6a^{11}bx^{50}}{25}+\frac{22a^{10}b^2x^{75}}{25}+\frac{11a^9b^3x^{100}}{5}+\frac{99a^8b^4x^{125}}{25}+\frac{132a^7b^5x^{150}}{25}+\frac{132a^6b^6x^{175}}{25}+\frac{99a^5b^7x^{200}}{25}+\frac{11a^4b^8x^{225}}{5}+\frac{22a^3b^9x^{250}}{25}+\frac{6a^2b^{10}x^{275}}{25}+\frac{ab^{11}x^{300}}{25}+\frac{b^{12}x^{325}}{325}$$

input
```
integrate(x**24*(b*x**25+a)**12,x)
```

output
```
a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**
3*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b
**6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b
**9*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/
325
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{24}(a+bx^{25})^{12}\,dx = \frac{(bx^{25}+a)^{13}}{325\,b}$$

input
```
integrate(x^24*(b*x^25+a)^12,x, algorithm="maxima")
```

output
```
1/325*(b*x^25 + a)^13/b
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{24}(a+bx^{25})^{12}\,dx=\frac{(bx^{25}+a)^{13}}{325\,b}$$

input
```
integrate(x^24*(b*x^25+a)^12,x, algorithm="giac")
```

output
```
1/325*(b*x^25 + a)^13/b
```

Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{24}(a+bx^{25})^{12}\,dx=\frac{(b\,x^{25}+a)^{13}}{325\,b}$$

input
```
int(x^24*(a + b*x^25)^12,x)
```

output
```
(a + b*x^25)^13/(325*b)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

$$\int x^{24}(a+bx^{25})^{12}\,dx$$
$$=\frac{x^{25}(b^{12}x^{300}+13a\,b^{11}x^{275}+78a^2b^{10}x^{250}+286a^3b^9x^{225}+715a^4b^8x^{200}+1287a^5b^7x^{175}+1716a^6b^6x^{150}+1}{325}$$

input
```
int(x^24*(b*x^25+a)^12,x)
```

output
```
(x**25*(13*a**12 + 78*a**11*b*x**25 + 286*a**10*b**2*x**50 + 715*a**9*b**3
*x**75 + 1287*a**8*b**4*x**100 + 1716*a**7*b**5*x**125 + 1716*a**6*b**6*x*
*150 + 1287*a**5*b**7*x**175 + 715*a**4*b**8*x**200 + 286*a**3*b**9*x**225
 + 78*a**2*b**10*x**250 + 13*a*b**11*x**275 + b**12*x**300))/325
```

3.332 $\int x^{24}(ax+bx^{38})^{12}\,dx$

Optimal result . . . 2685
Mathematica [B] (verified) . . . 2685
Rubi [A] (verified) . . . 2686
Maple [B] (verified) . . . 2687
Fricas [B] (verification not implemented) . . . 2688
Sympy [B] (verification not implemented) . . . 2688
Maxima [B] (verification not implemented) . . . 2689
Giac [B] (verification not implemented) . . . 2689
Mupad [B] (verification not implemented) . . . 2690
Reduce [B] (verification not implemented) . . . 2691

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx=\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

output
```
1/481*(b*x^37+a)^13/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx=\frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8x^{333}+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^2b^{10}x^{407}+\frac{1}{37}ab^{11}x^{444}+\frac{b^{12}x^{481}}{481}$$

input
```
Integrate[x^24*(a*x + b*x^38)^12,x]
```

output
```
(a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3
*x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6
*x^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x
^370)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = $\{9, 793\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{24}\left(ax+bx^{38}\right)^{12}dx$$
$$\Big\downarrow \ 9$$
$$\int x^{36}\left(a+bx^{37}\right)^{12}dx$$
$$\Big\downarrow \ 793$$
$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

input
```
Int[x^24*(a*x + b*x^38)^12,x]
```

output
```
(a + b*x^37)^13/(481*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 1.72 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{99}{37}a^8b^4x^{185}+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^{11}b\,x^{74}+\frac{1}{37}a\,b^{11}x^{444}+\frac{132}{37}a^6b^6x^{259}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^4b^8x^{33}$
parallelrisch	$\frac{99}{37}a^8b^4x^{185}+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^{11}b\,x^{74}+\frac{1}{37}a\,b^{11}x^{444}+\frac{132}{37}a^6b^6x^{259}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^4b^8x^{33}$
gosper	$\frac{x^{37}(b^{12}x^{444}+13a\,b^{11}x^{407}+78a^2b^{10}x^{370}+286a^3b^9x^{333}+715a^4b^8x^{296}+1287a^5b^7x^{259}+1716a^6b^6x^{222}+1716a^7b^5x^{185}+1287a}{481}$
risch	$\frac{b^{12}x^{481}}{481}+\frac{a\,b^{11}x^{444}}{37}+\frac{6a^2b^{10}x^{407}}{37}+\frac{22a^3b^9x^{370}}{37}+\frac{55a^4b^8x^{333}}{37}+\frac{99a^5b^7x^{296}}{37}+\frac{132a^6b^6x^{259}}{37}+\frac{132a^7b^5x^{222}}{37}+$
orering	$\frac{x^{25}(b^{12}x^{444}+13a\,b^{11}x^{407}+78a^2b^{10}x^{370}+286a^3b^9x^{333}+715a^4b^8x^{296}+1287a^5b^7x^{259}+1716a^6b^6x^{222}+1716a^7b^5x^{185}+1287a}{481(b\,x^{37}+a)^{12}}$

input
```
int(x^24*(b*x^38+a*x)^12,x,method=_RETURNVERBOSE)
```

output
```
99/37*a^8*b^4*x^185+22/37*a^3*b^9*x^370+6/37*a^11*b*x^74+1/37*a*b^11*x^444
+132/37*a^6*b^6*x^259+22/37*a^10*b^2*x^111+55/37*a^4*b^8*x^333+1/481*b^12*
x^481+132/37*a^7*b^5*x^222+55/37*a^9*b^3*x^148+99/37*a^5*b^7*x^296+1/37*a^
12*x^37+6/37*a^2*b^10*x^407
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx = \frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}+\frac{99}{37}\,a^{8}b^{4}x^{185}+\frac{55}{37}\,a^{9}b^{3}x^{148}+\frac{22}{37}\,a^{10}b^{2}x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}$$

input
```
integrate(x^24*(b*x^38+a*x)^12,x, algorithm="fricas")
```

output
```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx = \frac{a^{12}x^{37}}{37}+\frac{6a^{11}bx^{74}}{37}+\frac{22a^{10}b^{2}x^{111}}{37}+\frac{55a^{9}b^{3}x^{148}}{37}+\frac{99a^{8}b^{4}x^{185}}{37}+\frac{132a^{7}b^{5}x^{222}}{37}+\frac{132a^{6}b^{6}x^{259}}{37}+\frac{99a^{5}b^{7}x^{296}}{37}+\frac{55a^{4}b^{8}x^{333}}{37}+\frac{22a^{3}b^{9}x^{370}}{37}+\frac{6a^{2}b^{10}x^{407}}{37}+\frac{ab^{11}x^{444}}{37}+\frac{b^{12}x^{481}}{481}$$

input
```
integrate(x**24*(b*x**38+a*x)**12,x)
```

output
```
a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b*
*3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6
*b**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**
3*b**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**4
81/481
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx &= \frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^2b^{10}x^{407}\\ &+\frac{22}{37}\,a^3b^9x^{370}+\frac{55}{37}\,a^4b^8x^{333}+\frac{99}{37}\,a^5b^7x^{296}\\ &+\frac{132}{37}\,a^6b^6x^{259}+\frac{132}{37}\,a^7b^5x^{222}+\frac{99}{37}\,a^8b^4x^{185}\\ &+\frac{55}{37}\,a^9b^3x^{148}+\frac{22}{37}\,a^{10}b^2x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}\end{aligned}$$

input
```
integrate(x^24*(b*x^38+a*x)^12,x, algorithm="maxima")
```

output
```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx=\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}+\frac{99}{37}\,a^{8}b^{4}x^{185}+\frac{55}{37}\,a^{9}b^{3}x^{148}+\frac{22}{37}\,a^{10}b^{2}x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}$$

input
```
integrate(x^24*(b*x^38+a*x)^12,x, algorithm="giac")
```

output
```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx=\frac{a^{12}\,x^{37}}{37}+\frac{6\,a^{11}\,b\,x^{74}}{37}+\frac{22\,a^{10}\,b^2\,x^{111}}{37}+\frac{55\,a^9\,b^3\,x^{148}}{37}+\frac{99\,a^8\,b^4\,x^{185}}{37}+\frac{132\,a^7\,b^5\,x^{222}}{37}+\frac{132\,a^6\,b^6\,x^{259}}{37}+\frac{99\,a^5\,b^7\,x^{296}}{37}+\frac{55\,a^4\,b^8\,x^{333}}{37}+\frac{22\,a^3\,b^9\,x^{370}}{37}+\frac{6\,a^2\,b^{10}\,x^{407}}{37}+\frac{a\,b^{11}\,x^{444}}{37}+\frac{b^{12}\,x^{481}}{481}$$

input
```
int(x^24*(a*x + b*x^38)^12,x)
```

output
```
(a^12*x^37)/37 + (b^12*x^481)/481 + (6*a^11*b*x^74)/37 + (a*b^11*x^444)/37
 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37
+ (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37
+ (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx$$
$$=\frac{x^{37}(b^{12}x^{444}+13a\,b^{11}x^{407}+78a^2b^{10}x^{370}+286a^3b^9x^{333}+715a^4b^8x^{296}+1287a^5b^7x^{259}+1716a^6b^6x^{222}+1}{481}$$

input
```
int(x^24*(b*x^38+a*x)^12,x)
```

output
```
(x**37*(13*a**12 + 78*a**11*b*x**37 + 286*a**10*b**2*x**74 + 715*a**9*b**3
*x**111 + 1287*a**8*b**4*x**148 + 1716*a**7*b**5*x**185 + 1716*a**6*b**6*x
**222 + 1287*a**5*b**7*x**259 + 715*a**4*b**8*x**296 + 286*a**3*b**9*x**33
3 + 78*a**2*b**10*x**370 + 13*a*b**11*x**407 + b**12*x**444))/481
```

3.333 $\int x^{36}(a+bx^{37})^{12}\,dx$

Optimal result . . . 2692
Mathematica [B] (verified) . . . 2692
Rubi [A] (verified) . . . 2693
Maple [A] (verified) . . . 2694
Fricas [B] (verification not implemented) . . . 2694
Sympy [B] (verification not implemented) . . . 2695
Maxima [A] (verification not implemented) . . . 2695
Giac [A] (verification not implemented) . . . 2696
Mupad [B] (verification not implemented) . . . 2696
Reduce [B] (verification not implemented) . . . 2696

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx=\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

output
```
1/481*(b*x^37+a)^13/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx=\frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8x^{333}+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^2b^{10}x^{407}+\frac{1}{37}ab^{11}x^{444}+\frac{b^{12}x^{481}}{481}$$

input
```
Integrate[x^36*(a + b*x^37)^12,x]
```

output
```
(a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3
*x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6
*x^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x
^370)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{793\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{36} \left(a + b x^{37}\right)^{12} dx$$

$$\Big\downarrow \; 793$$

$$\frac{\left(a + b x^{37}\right)^{13}}{481 b}$$

input
```
Int[x^36*(a + b*x^37)^12,x]
```

output
```
(a + b*x^37)^13/(481*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
default	$\frac{(b\,x^{37}+a)^{13}}{481b}$
gosper	$\frac{99}{37}a^8b^4x^{185}+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^{11}b\,x^{74}+\frac{1}{37}a\,b^{11}x^{444}+\frac{132}{37}a^6b^6x^{259}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^4b^8x^{3}$
parallelrisch	$\frac{99}{37}a^8b^4x^{185}+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^{11}b\,x^{74}+\frac{1}{37}a\,b^{11}x^{444}+\frac{132}{37}a^6b^6x^{259}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^4b^8x^{3}$
orering	$\frac{x^{37}(b^{12}x^{444}+13a\,b^{11}x^{407}+78a^2b^{10}x^{370}+286a^3b^9x^{333}+715a^4b^8x^{296}+1287a^5b^7x^{259}+1716a^6b^6x^{222}+1716a^7b^5x^{185}+1287a}{481}$
risch	$\frac{b^{12}x^{481}}{481}+\frac{a\,b^{11}x^{444}}{37}+\frac{6a^2b^{10}x^{407}}{37}+\frac{22a^3b^9x^{370}}{37}+\frac{55a^4b^8x^{333}}{37}+\frac{99a^5b^7x^{296}}{37}+\frac{132a^6b^6x^{259}}{37}+\frac{132a^7b^5x^{222}}{37}+$

input
```
int(x^36*(b*x^37+a)^12,x,method=_RETURNVERBOSE)
```

output
```
1/481*(b*x^37+a)^13/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{36}(a+bx^{37})^{12}\,dx=\frac{1}{481}b^{12}x^{481}+\frac{1}{37}ab^{11}x^{444}+\frac{6}{37}a^2b^{10}x^{407}+\frac{22}{37}a^3b^9x^{370}+\frac{55}{37}a^4b^8x^{333}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^6b^6x^{259}+\frac{132}{37}a^7b^5x^{222}+\frac{99}{37}a^8b^4x^{185}+\frac{55}{37}a^9b^3x^{148}+\frac{22}{37}a^{10}b^2x^{111}+\frac{6}{37}a^{11}bx^{74}+\frac{1}{37}a^{12}x^{37}$$

input
```
integrate(x^36*(b*x^37+a)^12,x, algorithm="fricas")
```

output
```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx=\frac{a^{12}x^{37}}{37}+\frac{6a^{11}bx^{74}}{37}+\frac{22a^{10}b^{2}x^{111}}{37}+\frac{55a^{9}b^{3}x^{148}}{37}+\frac{99a^{8}b^{4}x^{185}}{37}+\frac{132a^{7}b^{5}x^{222}}{37}+\frac{132a^{6}b^{6}x^{259}}{37}+\frac{99a^{5}b^{7}x^{296}}{37}+\frac{55a^{4}b^{8}x^{333}}{37}+\frac{22a^{3}b^{9}x^{370}}{37}+\frac{6a^{2}b^{10}x^{407}}{37}+\frac{ab^{11}x^{444}}{37}+\frac{b^{12}x^{481}}{481}$$

input
```
integrate(x**36*(b*x**37+a)**12,x)
```

output
```
a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b*
*3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6
*b**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**
3*b**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**4
81/481
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx=\frac{\left(bx^{37}+a\right)^{13}}{481\,b}$$

input
```
integrate(x^36*(b*x^37+a)^12,x, algorithm="maxima")
```

output
```
1/481*(b*x^37 + a)^13/b
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx=\frac{\left(bx^{37}+a\right)^{13}}{481\,b}$$

input
```
integrate(x^36*(b*x^37+a)^12,x, algorithm="giac")
```

output
```
1/481*(b*x^37 + a)^13/b
```

Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx=\frac{\left(b\,x^{37}+a\right)^{13}}{481\,b}$$

input
```
int(x^36*(a + b*x^37)^12,x)
```

output
```
(a + b*x^37)^13/(481*b)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx$$
$$=\frac{x^{37}(b^{12}x^{444}+13a\,b^{11}x^{407}+78a^2b^{10}x^{370}+286a^3b^9x^{333}+715a^4b^8x^{296}+1287a^5b^7x^{259}+1716a^6b^6x^{222}+1}{481}$$

input
```
int(x^36*(b*x^37+a)^12,x)
```

output
```
(x**37*(13*a**12 + 78*a**11*b*x**37 + 286*a**10*b**2*x**74 + 715*a**9*b**3
*x**111 + 1287*a**8*b**4*x**148 + 1716*a**7*b**5*x**185 + 1716*a**6*b**6*x
**222 + 1287*a**5*b**7*x**259 + 715*a**4*b**8*x**296 + 286*a**3*b**9*x**33
3 + 78*a**2*b**10*x**370 + 13*a*b**11*x**407 + b**12*x**444))/481
```

3.334 $\int \frac{1}{ax+bx^n} dx$

Optimal result 2698
Mathematica [A] (verified) 2698
Rubi [A] (verified) 2699
Maple [A] (verified) 2700
Fricas [A] (verification not implemented) 2700
Sympy [B] (verification not implemented) 2700
Maxima [A] (verification not implemented) 2701
Giac [F] 2702
Mupad [B] (verification not implemented) 2702
Reduce [B] (verification not implemented) 2702

Optimal result

Integrand size = 11, antiderivative size = 23

$$\int \frac{1}{ax+bx^n} dx = \frac{\log(b+ax^{1-n})}{a(1-n)}$$

output
```
ln(b+a*x^(1-n))/a/(1-n)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax+bx^n} dx = \frac{\log(b+ax^{1-n})}{a(1-n)}$$

input
```
Integrate[(a*x + b*x^n)^(-1),x]
```

output
```
Log[b + a*x^(1 - n)]/(a*(1 - n))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2027, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax+bx^n}dx$$
$$\Big\downarrow \text{ 2027}$$
$$\int \frac{x^{-n}}{ax^{1-n}+b}dx$$
$$\Big\downarrow \text{ 792}$$
$$\frac{\log\left(ax^{1-n}+b\right)}{a(1-n)}$$

input
```
Int[(a*x + b*x^n)^(-1),x]
```

output
```
Log[b + a*x^(1 - n)]/(a*(1 - n))
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

method	result	size
parallelrisch	$\frac{n\ln(x)-\ln(ax+b\,x^{n})}{a(-1+n)}$	27
risch	$\frac{n\ln(x)}{a(-1+n)}-\frac{\ln\left(x^{n}+\frac{ax}{b}\right)}{a(-1+n)}$	35
norman	$\frac{n\ln(x)}{a(-1+n)}-\frac{\ln\left(ax+b\,\mathrm{e}^{n\ln(x)}\right)}{a(-1+n)}$	36

input
```
int(1/(a*x+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
(n*ln(x)-ln(a*x+b*x^n))/a/(-1+n)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{1}{ax+bx^n}\,dx=\frac{n\log(x)-\log(ax+bx^n)}{an-a}$$

input
```
integrate(1/(a*x+b*x^n),x, algorithm="fricas")
```

output
```
(n*log(x) - log(a*x + b*x^n))/(a*n - a)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(14) = 28.

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \frac{1}{ax+bx^n}\,dx = \begin{cases} \tilde{\infty}\log(x) & \text{for } a=0 \wedge b=0 \wedge n=1 \\ \frac{\log(x)}{a} & \text{for } b=0 \\ -\frac{x}{b(nx^n-x^n)} & \text{for } a=0 \\ \frac{\log(x)}{a+b} & \text{for } n=1 \\ \frac{n\log(x)}{an-a} - \frac{\log\left(\frac{ax}{b}+x^n\right)}{an-a} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x+b*x**n),x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 1)), (log(x)/a, Eq(b, 0
)), (-x/(b*(n*x**n - x**n)), Eq(a, 0)), (log(x)/(a + b), Eq(n, 1)), (n*log
(x)/(a*n - a) - log(a*x/b + x**n)/(a*n - a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \frac{1}{ax+bx^n}\,dx = \frac{n\log(x)}{a(n-1)} - \frac{\log\left(\frac{ax+bx^n}{b}\right)}{a(n-1)}$$

input
```
integrate(1/(a*x+b*x^n),x, algorithm="maxima")
```

output
```
n*log(x)/(a*(n - 1)) - log((a*x + b*x^n)/b)/(a*(n - 1))
```

Giac [F]

$$\int \frac{1}{ax + bx^n} \, dx = \int \frac{1}{ax + bx^n} \, dx$$

input
```
integrate(1/(a*x+b*x^n),x, algorithm="giac")
```

output
```
integrate(1/(a*x + b*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 9.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{1}{ax + bx^n} \, dx = -\frac{\ln (b\,x^n + a\,x) - n\,\ln (x)}{a\,(n-1)}$$

input
```
int(1/(b*x^n + a*x),x)
```

output
```
-(log(b*x^n + a*x) - n*log(x))/(a*(n - 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{1}{ax + bx^n} \, dx = \frac{-\log(x^n b + ax) + \log(x)\,n}{a\,(n-1)}$$

input
```
int(1/(a*x+b*x^n),x)
```

output
```
( - log(x**n*b + a*x) + log(x)*n)/(a*(n - 1))
```

3.335 $\int \frac{1}{ax+bx^{1+n}}\,dx$

Optimal result . . . 2703
Mathematica [A] (verified) . . . 2703
Rubi [A] (verified) . . . 2704
Maple [A] (verified) . . . 2705
Fricas [A] (verification not implemented) . . . 2706
Sympy [B] (verification not implemented) . . . 2706
Maxima [A] (verification not implemented) . . . 2707
Giac [F] . . . 2707
Mupad [B] (verification not implemented) . . . 2707
Reduce [B] (verification not implemented) . . . 2708

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{1}{ax+bx^{1+n}}\,dx=\frac{\log(x)}{a}-\frac{\log\left(a+bx^n\right)}{an}$$

output
```
ln(x)/a-ln(a+b*x^n)/a/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{ax+bx^{1+n}}\,dx=\frac{\log\left(x^n\right)-\log\left(an(a+bx^n)\right)}{an}$$

input
```
Integrate[(a*x + b*x^(1 + n))^(-1),x]
```

output
```
(Log[x^n] - Log[a*n*(a + b*x^n)])/(a*n)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2027, 798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax+bx^{n+1}}dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{1}{x(a+bx^n)}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\int \frac{x^{-n}}{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{\frac{\int x^{-n}dx^n}{a}-\frac{b\int \frac{1}{bx^n+a}dx^n}{a}}{n}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{\frac{\log(x^n)}{a}-\frac{b\int \frac{1}{bx^n+a}dx^n}{a}}{n}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\frac{\log(x^n)}{a}-\frac{\log(a+bx^n)}{a}}{n}$$

input
```
Int[(a*x + b*x^(1 + n))^(-1),x]
```

output
```
(Log[x^n]/a - Log[a + b*x^n]/a)/n
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

method	result	size
parallelrisch	$\frac{n\ln(x)+\ln(x)-\ln\left(ax+b\,x^{1+n}\right)}{an}$	29
norman	$\frac{(1+n)\ln(x)}{an}-\frac{\ln\left(ax+b\,\mathrm{e}^{(1+n)\ln(x)}\right)}{an}$	36
risch	$\frac{\ln(x)}{an}+\frac{\ln(x)}{a}-\frac{\ln\left(x^{1+n}+\frac{ax}{b}\right)}{an}$	38

input
```
int(1/(a*x+b*x^(1+n)),x,method=_RETURNVERBOSE)
```

output
```
(n*ln(x)+ln(x)-ln(a*x+b*x^(1+n)))/a/n
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \frac{1}{ax+bx^{1+n}}\,dx = \frac{(n+1)\log(x) - \log(ax+bx^{n+1})}{an}$$

input
```
integrate(1/(a*x+b*x^(1+n)),x, algorithm="fricas")
```

output
```
((n + 1)*log(x) - log(a*x + b*x^(n + 1)))/(a*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(15) = 30.

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.13

$$\int \frac{1}{ax+bx^{1+n}}\,dx = \begin{cases} \frac{\log(x)}{b} & \text{for } a=0 \wedge n=0 \\ -\frac{xx^{-n-1}}{bn} & \text{for } a=0 \\ \frac{\log(x)}{a+b} & \text{for } n=0 \\ \frac{\log(x)}{a} + \frac{\log(x)}{an} - \frac{\log\left(x+\frac{bx^{n+1}}{a}\right)}{an} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x+b*x**(1+n)),x)
```

output
```
Piecewise((log(x)/b, Eq(a, 0) & Eq(n, 0)), (-x*x**(-n - 1)/(b*n), Eq(a, 0)
), (log(x)/(a + b), Eq(n, 0)), (log(x)/a + log(x)/(a*n) - log(x + b*x**(n
+ 1)/a)/(a*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{1}{ax+bx^{1+n}}\,dx=\frac{\log\left(x\right)}{a}-\frac{\log\left(\frac{bx^{n}+a}{b}\right)}{an}$$

input
```
integrate(1/(a*x+b*x^(1+n)),x, algorithm="maxima")
```

output
```
log(x)/a - log((b*x^n + a)/b)/(a*n)
```

Giac [F]

$$\int \frac{1}{ax+bx^{1+n}}\,dx=\int \frac{1}{ax+bx^{n+1}}\,dx$$

input
```
integrate(1/(a*x+b*x^(1+n)),x, algorithm="giac")
```

output
```
integrate(1/(a*x + b*x^(n + 1)), x)
```

Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{1}{ax+bx^{1+n}}\,dx=\frac{\ln\left(x\right)\,\left(n+1\right)}{a\,n}-\frac{\ln\left(x\,\left(a+b\,x^{n}\right)\right)}{a\,n}$$

input
```
int(1/(a*x + b*x^(n + 1)),x)
```

output
```
(log(x)*(n + 1))/(a*n) - log(x*(a + b*x^n))/(a*n)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{1}{ax + bx^{1+n}}\, dx = \frac{-\log(x^n b + a) + \log(x)\, n}{an}$$

input
```
int(1/(a*x+b*x^(1+n)),x)
```

output
```
( - log(x**n*b + a) + log(x)*n)/(a*n)
```

3.336 $\int \frac{1}{ax+bx^{1-n}}\,dx$

Optimal result . . . 2709
Mathematica [A] (verified) . . . 2709
Rubi [A] (verified) . . . 2710
Maple [A] (verified) . . . 2711
Fricas [A] (verification not implemented) . . . 2711
Sympy [B] (verification not implemented) . . . 2711
Maxima [A] (verification not implemented) . . . 2712
Giac [F] . . . 2713
Mupad [B] (verification not implemented) . . . 2713
Reduce [B] (verification not implemented) . . . 2713

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{1}{ax+bx^{1-n}}\,dx=\frac{\log(b+ax^n)}{an}$$

output
```
ln(b+a*x^n)/a/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax+bx^{1-n}}\,dx=\frac{\log(b+ax^n)}{an}$$

input
```
Integrate[(a*x + b*x^(1 - n))^(-1),x]
```

output
```
Log[b + a*x^n]/(a*n)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2027, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax+bx^{1-n}}dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{x^{n-1}}{ax^n+b}dx$$

$$\Big\downarrow \text{ 792}$$

$$\frac{\log(ax^n+b)}{an}$$

input
```
Int[(a*x + b*x^(1 - n))^(-1),x]
```

output
```
Log[b + a*x^n]/(a*n)
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

method	result	size
parallelrisch	$\frac{n\ln(x)-\ln(x)+\ln(ax+b\,x^{1-n})}{an}$	31
norman	$\frac{(-1+n)\ln(x)}{an}+\frac{\ln(ax+b\,\mathrm{e}^{(1-n)\ln(x)})}{an}$	37
risch	$-\frac{\ln(x)}{an}+\frac{\ln(x)}{a}+\frac{\ln(x^{1-n}+\frac{ax}{b})}{an}$	40

input
```
int(1/(a*x+b*x^(1-n)),x,method=_RETURNVERBOSE)
```

output
```
(n*ln(x)-ln(x)+ln(a*x+b*x^(1-n)))/a/n
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{1}{ax+bx^{1-n}}\,dx=\frac{(n-1)\log(x)+\log(ax+bx^{-n+1})}{an}$$

input
```
integrate(1/(a*x+b*x^(1-n)),x, algorithm="fricas")
```

output
```
((n - 1)*log(x) + log(a*x + b*x^(-n + 1)))/(a*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(10) = 20.

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.40

$$\int \frac{1}{ax + bx^{1-n}} \, dx = \begin{cases} \tilde{\infty} \log(x) & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ \frac{x x^{n-1}}{bn} & \text{for } a = 0 \\ \frac{\log(x)}{a} & \text{for } b = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 0 \\ \frac{\log(x)}{a} - \frac{\log(x)}{an} + \frac{\log\left(\frac{ax}{b} + x^{1-n}\right)}{an} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x+b*x**(1-n)),x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (x*x**(n - 1)/(b*n
), Eq(a, 0)), (log(x)/a, Eq(b, 0)), (log(x)/(a + b), Eq(n, 0)), (log(x)/a
- log(x)/(a*n) + log(a*x/b + x**(1 - n))/(a*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{1}{ax + bx^{1-n}} \, dx = \frac{\log\left(\frac{ax^n + b}{a}\right)}{an}$$

input
```
integrate(1/(a*x+b*x^(1-n)),x, algorithm="maxima")
```

output
```
log((a*x^n + b)/a)/(a*n)
```

Giac [F]

$$\int \frac{1}{ax+bx^{1-n}}\,dx=\int \frac{1}{ax+bx^{-n+1}}\,dx$$

input
```
integrate(1/(a*x+b*x^(1-n)),x, algorithm="giac")
```

output
```
integrate(1/(a*x + b*x^(-n + 1)), x)
```

Mupad [B] (verification not implemented)

Time = 9.51 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.27

$$\int \frac{1}{ax+bx^{1-n}}\,dx=\frac{\ln\left(a\,x+b\,x^{1-n}\right)}{a\,n}+\frac{\ln\left(x\right)\,\left(n-1\right)}{a\,n}$$

input
```
int(1/(a*x + b*x^(1 - n)),x)
```

output
```
log(a*x + b*x^(1 - n))/(a*n) + (log(x)*(n - 1))/(a*n)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax+bx^{1-n}}\,dx=\frac{\log(x^n a+b)}{an}$$

input
```
int(1/(a*x+b*x^(1-n)),x)
```

output
```
log(x**n*a + b)/(a*n)
```

3.337 $\int \frac{1}{2x+3x^{1+n}}\,dx$

Optimal result . . . 2714
Mathematica [A] (verified) . . . 2714
Rubi [A] (verified) . . . 2715
Maple [A] (verified) . . . 2716
Fricas [A] (verification not implemented) . . . 2717
Sympy [B] (verification not implemented) . . . 2717
Maxima [A] (verification not implemented) . . . 2718
Giac [F] . . . 2718
Mupad [B] (verification not implemented) . . . 2718
Reduce [B] (verification not implemented) . . . 2719

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1}{2x+3x^{1+n}}\,dx=\frac{\log(x)}{2}-\frac{\log\left(2+3x^n\right)}{2n}$$

output
```
1/2*ln(x)-1/2*ln(2+3*x^n)/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{2x+3x^{1+n}}\,dx=\frac{\log\left(x^n\right)-\log\left(n(2+3x^n)\right)}{2n}$$

input
```
Integrate[(2*x + 3*x^(1 + n))^(-1),x]
```

output
```
(Log[x^n] - Log[n*(2 + 3*x^n)])/(2*n)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2027, 798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3x^{n+1}+2x}dx$$

$$\Big\downarrow \ 2027$$

$$\int \frac{1}{x(3x^n+2)}dx$$

$$\Big\downarrow \ 798$$

$$\frac{\int \frac{x^{-n}}{3x^n+2}dx^n}{n}$$

$$\Big\downarrow \ 47$$

$$\frac{\frac{\int x^{-n}dx^n}{2}-\frac{3}{2}\int \frac{1}{3x^n+2}dx^n}{n}$$

$$\Big\downarrow \ 14$$

$$\frac{\frac{\log(x^n)}{2}-\frac{3}{2}\int \frac{1}{3x^n+2}dx^n}{n}$$

$$\Big\downarrow \ 16$$

$$\frac{\frac{\log(x^n)}{2}-\frac{1}{2}\log(3x^n+2)}{n}$$

input
```
Int[(2*x + 3*x^(1 + n))^(-1),x]
```

output
```
(Log[x^n]/2 - Log[2 + 3*x^n]/2)/n
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

method	result	size
parallelrisch	$\frac{n\ln(x)+\ln(x)-\ln\left(x+\frac{3x^{1+n}}{2}\right)}{2n}$	25
meijerg	$\frac{-\ln\left(1+\frac{3x^{n}}{2}\right)+n\ln(x)-\ln(2)+\ln(3)}{2n}$	27
risch	$\frac{\ln(x)}{2n}+\frac{\ln(x)}{2}-\frac{\ln\left(\frac{2x}{3}+x^{1+n}\right)}{2n}$	28
norman	$\frac{(1+n)\ln(x)}{2n}-\frac{\ln\left(2x+3\,\mathrm{e}^{(1+n)\ln(x)}\right)}{2n}$	31

input
```
int(1/(2*x+3*x^(1+n)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(n*ln(x)+ln(x)-ln(x+3/2*x^(1+n)))/n
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1}{2x+3x^{1+n}}\,dx = \frac{(n+1)\log(x) - \log\left(3\,x^{n+1}+2\,x\right)}{2\,n}$$

input
```
integrate(1/(2*x+3*x^(1+n)),x, algorithm="fricas")
```

output
```
1/2*((n + 1)*log(x) - log(3*x^(n + 1) + 2*x))/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$.

Time = 0.39 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \frac{1}{2x+3x^{1+n}}\,dx = \begin{cases} \frac{\log(x)}{2} + \frac{\log(x)}{2n} - \frac{\log(2x+3x^{n+1})}{2n} & \text{for } n \neq 0 \\ \frac{\log(x)}{5} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(2*x+3*x**(1+n)),x)
```

output
```
Piecewise((log(x)/2 + log(x)/(2*n) - log(2*x + 3*x**(n + 1))/(2*n), Ne(n,
0)), (log(x)/5, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1}{2x+3x^{1+n}}\,dx = -\frac{\log\left(x^n+\frac{2}{3}\right)}{2\,n}+\frac{1}{2}\log\left(x\right)$$

input
```
integrate(1/(2*x+3*x^(1+n)),x, algorithm="maxima")
```

output
```
-1/2*log(x^n + 2/3)/n + 1/2*log(x)
```

Giac [F]

$$\int \frac{1}{2x+3x^{1+n}}\,dx = \int \frac{1}{3\,x^{n+1}+2\,x}\,dx$$

input
```
integrate(1/(2*x+3*x^(1+n)),x, algorithm="giac")
```

output
```
integrate(1/(3*x^(n + 1) + 2*x), x)
```

Mupad [B] (verification not implemented)

Time = 9.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1}{2x+3x^{1+n}}\,dx = \frac{\ln\left(x\right)\,\left(n+1\right)}{2\,n}-\frac{\ln\left(\frac{2\,x}{3}+x^{n+1}\right)}{2\,n}$$

input
```
int(1/(2*x + 3*x^(n + 1)),x)
```

output
```
(log(x)*(n + 1))/(2*n) - log((2*x)/3 + x^(n + 1))/(2*n)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{2x+3x^{1+n}}\,dx = \frac{-\log(3x^n+2)+\log(x)\,n}{2n}$$

input
```
int(1/(2*x+3*x^(1+n)),x)
```

output
```
( - log(3*x**n + 2) + log(x)*n)/(2*n)
```

3.338 $\int \frac{1}{2x+3x^{1-n}} dx$

Optimal result . . . 2720
Mathematica [A] (verified) . . . 2720
Rubi [A] (verified) . . . 2721
Maple [A] (verified) . . . 2722
Fricas [A] (verification not implemented) . . . 2722
Sympy [B] (verification not implemented) . . . 2723
Maxima [A] (verification not implemented) . . . 2723
Giac [F] . . . 2723
Mupad [B] (verification not implemented) . . . 2724
Reduce [B] (verification not implemented) . . . 2724

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{1}{2x+3x^{1-n}} dx = \frac{\log(3+2x^n)}{2n}$$

output
```
1/2*ln(3+2*x^n)/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{2x+3x^{1-n}} dx = \frac{\log(3+2x^n)}{2n}$$

input
```
Integrate[(2*x + 3*x^(1 - n))^(-1),x]
```

output
```
Log[3 + 2*x^n]/(2*n)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2027, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3x^{1-n}+2x}dx$$
$$\Big\downarrow \text{ 2027}$$
$$\int \frac{x^{n-1}}{2x^n+3}dx$$
$$\Big\downarrow \text{ 792}$$
$$\frac{\log(2x^n+3)}{2n}$$

input
```
Int[(2*x + 3*x^(1 - n))^(-1),x]
```

output
```
Log[3 + 2*x^n]/(2*n)
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

method	result	size
parallelrisch	$\frac{n\ln(x)-\ln(x)+\ln\left(x+\frac{3x^{1-n}}{2}\right)}{2n}$	27
meijerg	$-\frac{-\ln\left(1+\frac{3x^{-n}}{2}\right)-n\ln(x)-\ln(2)+\ln(3)}{2n}$	30
risch	$-\frac{\ln(x)}{2n}+\frac{\ln(x)}{2}+\frac{\ln\left(\frac{2x}{3}+x^{1-n}\right)}{2n}$	30
norman	$\frac{(-1+n)\ln(x)}{2n}+\frac{\ln\left(2x+3\,\mathrm{e}^{(1-n)\ln(x)}\right)}{2n}$	33

input
```
int(1/(2*x+3*x^(1-n)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(n*ln(x)-ln(x)+ln(x+3/2*x^(1-n)))/n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \frac{1}{2x+3x^{1-n}}\,dx=\frac{(n-1)\log{(x)}+\log{(3\,x^{-n+1}+2\,x)}}{2\,n}$$

input
```
integrate(1/(2*x+3*x^(1-n)),x, algorithm="fricas")
```

output
```
1/2*((n - 1)*log(x) + log(3*x^(-n + 1) + 2*x))/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(10) = 20$.

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \frac{1}{2x + 3x^{1-n}}\,dx = \begin{cases} \frac{\log(x)}{2} - \frac{\log(x)}{2n} + \frac{\log(2x+3x^{1-n})}{2n} & \text{for } n \neq 0 \\ \frac{\log(x)}{5} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(2*x+3*x**(1-n)),x)
```

output
```
Piecewise((log(x)/2 - log(x)/(2*n) + log(2*x + 3*x**(1 - n))/(2*n), Ne(n,
0)), (log(x)/5, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1}{2x + 3x^{1-n}}\,dx = \frac{\log\left(x^n + \frac{3}{2}\right)}{2\,n}$$

input
```
integrate(1/(2*x+3*x^(1-n)),x, algorithm="maxima")
```

output
```
1/2*log(x^n + 3/2)/n
```

Giac [F]

$$\int \frac{1}{2x + 3x^{1-n}}\,dx = \int \frac{1}{3\,x^{-n+1} + 2\,x}\,dx$$

input
```
integrate(1/(2*x+3*x^(1-n)),x, algorithm="giac")
```

output
```
integrate(1/(3*x^(-n + 1) + 2*x), x)
```

Mupad [B] (verification not implemented)

Time = 9.58 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{1}{2x + 3x^{1-n}}\,dx = \frac{\ln\left(\frac{2\,x}{3} + x^{1-n}\right)}{2\,n} + \frac{\ln(x)\,\,(n-1)}{2\,n}$$

input
```
int(1/(2*x + 3*x^(1 - n)),x)
```

output
```
log((2*x)/3 + x^(1 - n))/(2*n) + (log(x)*(n - 1))/(2*n)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{2x + 3x^{1-n}}\,dx = \frac{\log(2x^n + 3)}{2n}$$

input
```
int(1/(2*x+3*x^(1-n)),x)
```

output
```
log(2*x**n + 3)/(2*n)
```

3.339 $\int \frac{1}{-\sqrt{x}+x} dx$

Optimal result 2725
Mathematica [A] (verified) 2725
Rubi [A] (verified) 2726
Maple [A] (verified) 2727
Fricas [A] (verification not implemented) 2727
Sympy [A] (verification not implemented) 2727
Maxima [A] (verification not implemented) 2728
Giac [A] (verification not implemented) 2728
Mupad [B] (verification not implemented) 2728
Reduce [B] (verification not implemented) 2729

Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{1}{-\sqrt{x}+x} dx = 2\log\left(1-\sqrt{x}\right)$$

output
```
2*ln(1-x^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{-\sqrt{x}+x} dx = 2\log\left(-1+\sqrt{x}\right)$$

input
```
Integrate[(-Sqrt[x] + x)^(-1),x]
```

output
```
2*Log[-1 + Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2027, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x-\sqrt{x}}dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{1}{\left(\sqrt{x}-1\right)\sqrt{x}}dx$$

$$\Big\downarrow \text{ 792}$$

$$2\log\left(1-\sqrt{x}\right)$$

input
```
Int[(-Sqrt[x] + x)^(-1),x]
```

output
```
2*Log[1 - Sqrt[x]]
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$2\ln\left(-1+\sqrt{x}\right)$	9
meijerg	$2\ln\left(1-\sqrt{x}\right)$	11
default	$\ln\left(x-1\right)-2\,\mathrm{arctanh}\left(\sqrt{x}\right)$	12
trager	$\ln\left(2\sqrt{x}-1-x\right)$	12

input
```
int(1/(-x^(1/2)+x),x,method=_RETURNVERBOSE)
```

output
```
2*ln(-1+x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{-\sqrt{x}+x}\,dx = 2\,\log\left(\sqrt{x}-1\right)$$

input
```
integrate(1/(-x^(1/2)+x),x, algorithm="fricas")
```

output
```
2*log(sqrt(x) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{-\sqrt{x}+x}\,dx = 2\log\left(\sqrt{x}-1\right)$$

input
```
integrate(1/(-x**(1/2)+x),x)
```

output
```
2*log(sqrt(x) - 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{-\sqrt{x}+x}\,dx = 2\,\log\left(\sqrt{x}-1\right)$$

input
```
integrate(1/(-x^(1/2)+x),x, algorithm="maxima")
```

output
```
2*log(sqrt(x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{-\sqrt{x}+x}\,dx = 2\,\log\left(\left|\sqrt{x}-1\right|\right)$$

input
```
integrate(1/(-x^(1/2)+x),x, algorithm="giac")
```

output
```
2*log(abs(sqrt(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{-\sqrt{x}+x}\,dx = 2\,\ln\left(\sqrt{x}-1\right)$$

input
```
int(1/(x - x^(1/2)),x)
```

output
```
2*log(x^(1/2) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{1}{-\sqrt{x}+x}\,dx = 2\log\left(\sqrt{x}-1\right)$$

input
```
int(1/(-x^(1/2)+x),x)
```

output
```
2*log(sqrt(x) - 1)
```

3.340 $\int \frac{1}{-x^{3/5}+x} \, dx$

Optimal result 2730
Mathematica [A] (verified) 2730
Rubi [A] (verified) 2731
Maple [A] (verified) 2732
Fricas [A] (verification not implemented) 2732
Sympy [B] (verification not implemented) 2733
Maxima [A] (verification not implemented) 2733
Giac [A] (verification not implemented) 2733
Mupad [B] (verification not implemented) 2734
Reduce [B] (verification not implemented) 2734

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{1}{-x^{3/5}+x} \, dx = \frac{5}{2} \log \left(1-x^{2/5}\right)$$

output
```
5/2*ln(1-x^(2/5))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{1}{-x^{3/5}+x} \, dx = \frac{5}{2} \log \left(-1+\sqrt[5]{x}\right)+\frac{5}{2} \log \left(1+\sqrt[5]{x}\right)$$

input
```
Integrate[(-x^(3/5) + x)^(-1),x]
```

output
```
(5*Log[-1 + x^(1/5)])/2 + (5*Log[1 + x^(1/5)])/2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2027, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x-x^{3/5}} \, dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{1}{\left(x^{2/5}-1\right) x^{3/5}}dx$$

$$\Big\downarrow \text{792}$$

$$\frac{5}{2} \log \left(1-x^{2/5}\right)$$

input
```
Int[(-x^(3/5) + x)^(-1),x]
```

output
```
(5*Log[1 - x^(2/5)])/2
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result
meijerg	$\frac{5\ln\left(1-x^{\frac{2}{5}}\right)}{2}$
derivativedivides	$\frac{5\ln\left(x^{\frac{1}{5}}-1\right)}{2}+\frac{5\ln\left(x^{\frac{1}{5}}+1\right)}{2}$
trager	$\frac{\ln\left(10x^{\frac{4}{5}}+5x^{\frac{8}{5}}-5x^{\frac{2}{5}}-10x^{\frac{6}{5}}-x^{2}+1\right)}{2}$
default	$-\frac{\ln\left(\sqrt{5}\,x^{\frac{1}{5}}+2x^{\frac{2}{5}}+x^{\frac{1}{5}}+2\right)}{4}+\frac{\ln(x-1)}{2}-\frac{\ln\left(-\sqrt{5}\,x^{\frac{1}{5}}+2x^{\frac{2}{5}}-x^{\frac{1}{5}}+2\right)}{4}-\frac{\ln\left(-\sqrt{5}\,x^{\frac{1}{5}}+2x^{\frac{2}{5}}+x^{\frac{1}{5}}+2\right)}{4}+2\ln\Big($

input
```
int(1/(-x^(3/5)+x),x,method=_RETURNVERBOSE)
```

output
```
5/2*ln(1-x^(2/5))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{1}{-x^{3/5}+x}\,dx=\frac{5}{2}\log\left(x^{\frac{2}{5}}-1\right)$$

input
```
integrate(1/(-x^(3/5)+x),x, algorithm="fricas")
```

output
```
5/2*log(x^(2/5) - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{1}{-x^{3/5}+x}\,dx = \frac{5\log\left(\sqrt[5]{x}-1\right)}{2} + \frac{5\log\left(\sqrt[5]{x}+1\right)}{2}$$

input
```
integrate(1/(-x**(3/5)+x),x)
```

output
```
5*log(x**(1/5) - 1)/2 + 5*log(x**(1/5) + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \frac{1}{-x^{3/5}+x}\,dx = \frac{5}{2}\log\left(x^{\frac{1}{5}}+1\right) + \frac{5}{2}\log\left(x^{\frac{1}{5}}-1\right)$$

input
```
integrate(1/(-x^(3/5)+x),x, algorithm="maxima")
```

output
```
5/2*log(x^(1/5) + 1) + 5/2*log(x^(1/5) - 1)
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{1}{-x^{3/5}+x}\,dx = \frac{5}{2}\log\left(x^{\frac{1}{5}}+1\right) + \frac{5}{2}\log\left(\left|x^{\frac{1}{5}}-1\right|\right)$$

input
```
integrate(1/(-x^(3/5)+x),x, algorithm="giac")
```

output
```
5/2*log(x^(1/5) + 1) + 5/2*log(abs(x^(1/5) - 1))
```

Mupad [B] (verification not implemented)

Time = 9.50 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{1}{-x^{3/5}+x}\,dx=\frac{5\ \ln\left(x^{2/5}-1\right)}{2}$$

input
```
int(1/(x - x^(3/5)),x)
```

output
```
(5*log(x^(2/5) - 1))/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \frac{1}{-x^{3/5}+x}\,dx=\frac{5\log\left(x^{\frac{1}{5}}-1\right)}{2}+\frac{5\log\left(x^{\frac{1}{5}}+1\right)}{2}$$

input
```
int(1/(-x^(3/5)+x),x)
```

output
```
(5*(log(x**(1/5) - 1) + log(x**(1/5) + 1)))/2
```

3.341 $\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x} \, dx$

Optimal result . . . 2735
Mathematica [A] (verified) . . . 2735
Rubi [A] (verified) . . . 2736
Maple [A] (verified) . . . 2737
Fricas [A] (verification not implemented) . . . 2737
Sympy [A] (verification not implemented) . . . 2738
Maxima [A] (verification not implemented) . . . 2738
Giac [B] (verification not implemented) . . . 2738
Mupad [B] (verification not implemented) . . . 2739
Reduce [B] (verification not implemented) . . . 2739

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x} \, dx=\frac{3}{4} \log \left(1+x^{4/3}\right)$$

output
```
3/4*ln(1+x^(4/3))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x} \, dx=\frac{3}{4} \log \left(1+x^{4/3}\right)$$

input
```
Integrate[(x^(-1/3) + x)^(-1),x]
```

output
```
(3*Log[1 + x^(4/3)])/4
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2027, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x+\frac{1}{\sqrt[3]{x}}}dx$$
$$\Big\downarrow \text{2027}$$
$$\int \frac{\sqrt[3]{x}}{x^{4/3}+1}dx$$
$$\Big\downarrow \text{792}$$
$$\frac{3}{4}\log\left(x^{4/3}+1\right)$$

input
```
Int[(x^(-1/3) + x)^(-1),x]
```

output
```
(3*Log[1 + x^(4/3)])/4
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{3\ln\left(1+x^{\frac{4}{3}}\right)}{4}$	9
default	$\frac{3\ln\left(1+x^{\frac{4}{3}}\right)}{4}$	9
meijerg	$\frac{3\ln\left(1+x^{\frac{4}{3}}\right)}{4}$	9
trager	$-\frac{\ln\left(\frac{3x^{\frac{20}{3}}-x^{8}-6x^{\frac{16}{3}}-6x^{\frac{8}{3}}+7x^{4}+3x^{\frac{4}{3}}-1}{\left(x^{4}+1\right)^{3}}\right)}{4}$	44

input
```
int(1/(1/x^(1/3)+x),x,method=_RETURNVERBOSE)
```

output
```
3/4*ln(1+x^(4/3))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx=\frac{3}{4}\,\log\left(x^{\frac{4}{3}}+1\right)$$

input
```
integrate(1/(1/x^(1/3)+x),x, algorithm="fricas")
```

output
```
3/4*log(x^(4/3) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx = \frac{3\log\left(x^{\frac{4}{3}}+1\right)}{4}$$

input
```
integrate(1/(1/x**(1/3)+x),x)
```

output
```
3*log(x**(4/3) + 1)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx = \frac{3}{4}\log\left(x^{\frac{4}{3}}+1\right)$$

input
```
integrate(1/(1/x^(1/3)+x),x, algorithm="maxima")
```

output
```
3/4*log(x^(4/3) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(8) = 16.

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.67

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx = \frac{3}{4}\log\left(\sqrt{2}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)+\frac{3}{4}\log\left(-\sqrt{2}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)$$

input
```
integrate(1/(1/x^(1/3)+x),x, algorithm="giac")
```

output
```
3/4*log(sqrt(2)*x^(1/3) + x^(2/3) + 1) + 3/4*log(-sqrt(2)*x^(1/3) + x^(2/3
) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx=\frac{3\ \ln\left(x^{4/3}+1\right)}{4}$$

input
```
int(1/(x + 1/x^(1/3)),x)
```

output
```
(3*log(x^(4/3) + 1))/4
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.50

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx=\frac{3\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\sqrt{2}+1\right)}{4}+\frac{3\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\sqrt{2}+1\right)}{4}$$

input
```
int(1/(1/x^(1/3)+x),x)
```

output
```
(3*(log(x**(2/3) - x**(1/3)*sqrt(2) + 1) + log(x**(2/3) + x**(1/3)*sqrt(2)
 + 1)))/4
```

3.342 $\int \frac{1}{x+x^{\sqrt{2}}} dx$

Optimal result 2740
Mathematica [A] (verified) 2740
Rubi [A] (verified) 2741
Maple [A] (verified) 2742
Fricas [A] (verification not implemented) 2743
Sympy [A] (verification not implemented) 2743
Maxima [A] (verification not implemented) 2743
Giac [F] 2744
Mupad [B] (verification not implemented) 2744
Reduce [B] (verification not implemented) 2744

Optimal result

Integrand size = 11, antiderivative size = 24

$$\int \frac{1}{x+x^{\sqrt{2}}} dx = \log(x) - \left(1+\sqrt{2}\right) \log \left(1+x^{-1+\sqrt{2}}\right)$$

output
```
ln(x)-(1+2^(1/2))*ln(1+x^(2^(1/2)-1))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x+x^{\sqrt{2}}} dx = \log(x) - \left(1+\sqrt{2}\right) \log \left(1+x^{-1+\sqrt{2}}\right)$$

input
```
Integrate[(x + x^Sqrt[2])^(-1),x]
```

output
```
Log[x] - (1 + Sqrt[2])*Log[1 + x^(-1 + Sqrt[2])]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.38, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {2027, 798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{\sqrt{2}}+x}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{1}{x\left(x^{\sqrt{2}-1}+1\right)}dx$$

$$\Big\downarrow \text{798}$$

$$\left(1+\sqrt{2}\right)\int \frac{x^{1-\sqrt{2}}}{x^{-1+\sqrt{2}}+1}dx^{-1+\sqrt{2}}$$

$$\Big\downarrow \text{47}$$

$$\left(1+\sqrt{2}\right)\left(\int x^{1-\sqrt{2}}dx^{-1+\sqrt{2}}-\int \frac{1}{x^{-1+\sqrt{2}}+1}dx^{-1+\sqrt{2}}\right)$$

$$\Big\downarrow \text{14}$$

$$\left(1+\sqrt{2}\right)\left(\log\left(x^{\sqrt{2}-1}\right)-\int \frac{1}{x^{-1+\sqrt{2}}+1}dx^{-1+\sqrt{2}}\right)$$

$$\Big\downarrow \text{16}$$

$$\left(1+\sqrt{2}\right)\left(\log\left(x^{\sqrt{2}-1}\right)-\log\left(x^{\sqrt{2}-1}+1\right)\right)$$

input
```
Int[(x + x^Sqrt[2])^(-1),x]
```

output
```
(1 + Sqrt[2])*(Log[x^(-1 + Sqrt[2])] - Log[1 + x^(-1 + Sqrt[2])])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

method	result	size
norman	$\left(2+\sqrt{2}\right)\ln\left(x\right)+\left(-1-\sqrt{2}\right)\ln\left(x+{\mathrm e}^{\sqrt{2}\,\ln\left(x\right)}\right)$	28
meijerg	$\frac{-\ln\left(1+x^{\sqrt{2}-1}\right)+\left(\sqrt{2}-1\right)\ln\left(x\right)}{\sqrt{2}-1}$	30
risch	$2\ln\left(x\right)+\sqrt{2}\,\ln\left(x\right)-\ln\left(x+x^{\sqrt{2}}\right)\sqrt{2}-\ln\left(x+x^{\sqrt{2}}\right)$	35
parallelrisch	$2\ln\left(x\right)+\sqrt{2}\,\ln\left(x\right)-\ln\left(x+x^{\sqrt{2}}\right)\sqrt{2}-\ln\left(x+x^{\sqrt{2}}\right)$	35

input
```
int(1/(x+x^(2^(1/2))),x,method=_RETURNVERBOSE)
```

output
```
(2+2^(1/2))*ln(x)+(-1-2^(1/2))*ln(x+exp(2^(1/2)*ln(x)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x + x^{\sqrt{2}}}\,dx = -\left(\sqrt{2} + 1\right)\log\left(x + x^{\left(\sqrt{2}\right)}\right) + \left(\sqrt{2} + 2\right)\log\left(x\right)$$

input
```
integrate(1/(x+x^(2^(1/2))),x, algorithm="fricas")
```

output
```
-(sqrt(2) + 1)*log(x + x^sqrt(2)) + (sqrt(2) + 2)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{1}{x + x^{\sqrt{2}}}\,dx = -\frac{2\log\left(x\right)}{-2 + \sqrt{2}} + \frac{\sqrt{2}\log\left(x + x^{\sqrt{2}}\right)}{-2 + \sqrt{2}}$$

input
```
integrate(1/(x+x**(2**(1/2))),x)
```

output
```
-2*log(x)/(-2 + sqrt(2)) + sqrt(2)*log(x + x**(sqrt(2)))/(-2 + sqrt(2))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{1}{x + x^{\sqrt{2}}}\,dx = \frac{\sqrt{2}\log\left(x\right)}{\sqrt{2} - 1} - \frac{\log\left(x + x^{\left(\sqrt{2}\right)}\right)}{\sqrt{2} - 1}$$

input
```
integrate(1/(x+x^(2^(1/2))),x, algorithm="maxima")
```

output
```
sqrt(2)*log(x)/(sqrt(2) - 1) - log(x + x^sqrt(2))/(sqrt(2) - 1)
```

Giac [F]

$$\int \frac{1}{x+x^{\sqrt{2}}}\,dx = \int \frac{1}{x+x^{\left(\sqrt{2}\right)}}\,dx$$

input
```
integrate(1/(x+x^(2^(1/2))),x, algorithm="giac")
```

output
```
integrate(1/(x + x^sqrt(2)), x)
```

Mupad [B] (verification not implemented)

Time = 9.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x+x^{\sqrt{2}}}\,dx = \ln(x)\,\left(\sqrt{2}+2\right) - \frac{\ln\left(x+x^{\sqrt{2}}\right)}{\sqrt{2}-1}$$

input
```
int(1/(x + x^(2^(1/2))),x)
```

output
```
log(x)*(2^(1/2) + 2) - log(x + x^(2^(1/2)))/(2^(1/2) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int \frac{1}{x+x^{\sqrt{2}}}\,dx = -\sqrt{2}\log\left(x^{\sqrt{2}}+x\right)+\sqrt{2}\log(x)-\log\left(x^{\sqrt{2}}+x\right)+2\log(x)$$

input
```
int(1/(x+x^(2^(1/2))),x)
```

output
```
- sqrt(2)*log(x**sqrt(2) + x) + sqrt(2)*log(x) - log(x**sqrt(2) + x) + 2*
log(x)
```

3.343 $\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx$

Optimal result 2746
Mathematica [A] (verified) 2746
Rubi [A] (verified) 2747
Maple [F] 2748
Fricas [F(-2)] 2748
Sympy [F] 2749
Maxima [F] 2749
Giac [F] 2749
Mupad [F(-1)] 2750
Reduce [F] 2750

Optimal result

Integrand size = 25, antiderivative size = 75

$$\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx=-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}+\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}$$

output
```
-2*(a*x^j+b*x^n)^(1/2)/(j-n)/(x^(1/2*j))+2*a^(1/2)*arctanh(a^(1/2)*x^(1/2*
j)/(a*x^j+b*x^n)^(1/2))/(j-n)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.39

$$\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx$$
$$=-\frac{2x^{-j/2}\left(ax^j+bx^n-\sqrt{a}\sqrt{b}x^{\frac{j+n}{2}}\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{arcsinh}\left(\frac{\sqrt{a}x^{\frac{j-n}{2}}}{\sqrt{b}}\right)\right)}{(j-n)\sqrt{ax^j+bx^n}}$$

input
```
Integrate[x^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

output
```
(-2*(a*x^j + b*x^n - Sqrt[a]*Sqrt[b]*x^((j + n)/2)*Sqrt[1 + (a*x^(j - n))/
b]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[b]]))/((j - n)*x^(j/2)*Sqrt[a*x^j
+ b*x^n])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1934, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-\frac{j}{2}-1}\sqrt{ax^j+bx^n}\,dx$$

$$\Big\downarrow \text{1934}$$

$$a\int \frac{x^{\frac{j-2}{2}}}{\sqrt{ax^j+bx^n}}dx-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}$$

$$\Big\downarrow \text{1935}$$

$$\frac{2a\int \frac{1}{1-\frac{ax^j}{ax^j+bx^n}}d\frac{x^{j/2}}{\sqrt{ax^j+bx^n}}}{j-n}-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}$$

input
```
Int[x^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

output
```
(-2*Sqrt[a*x^j + b*x^n])/((j - n)*x^(j/2)) + (2*Sqrt[a]*ArcTanh[(Sqrt[a]*x
^(j/2))/Sqrt[a*x^j + b*x^n]])/(j - n)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [F]

$$\int x^{-1-\frac{j}{2}}\sqrt{a\,x^{j}+b\,x^{n}}dx$$

input
```
int(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)
```

output
```
int(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx=\int x^{-\frac{j}{2}-1}\sqrt{ax^j+bx^n}\,dx$$

input
```
integrate(x**(-1-1/2*j)*(a*x**j+b*x**n)**(1/2),x)
```

output
```
Integral(x**(-j/2 - 1)*sqrt(a*x**j + b*x**n), x)
```

Maxima [F]

$$\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx=\int \sqrt{ax^j+bx^n}x^{-\frac{1}{2}\,j-1}\,dx$$

input
```
integrate(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^j + b*x^n)*x^(-1/2*j - 1), x)
```

Giac [F]

$$\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx=\int \sqrt{ax^j+bx^n}x^{-\frac{1}{2}\,j-1}\,dx$$

input
```
integrate(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^j + b*x^n)*x^(-1/2*j - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx=\int \frac{\sqrt{a\,x^j+b\,x^n}}{x^{\frac{j}{2}+1}}\,dx$$

input
```
int((a*x^j + b*x^n)^(1/2)/x^(j/2 + 1),x)
```

output
```
int((a*x^j + b*x^n)^(1/2)/x^(j/2 + 1), x)
```

Reduce [F]

$$\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx$$
$$=\frac{-2\sqrt{x^ja+x^nb}+x^{\frac{j}{2}}\left(\int\frac{x^j\sqrt{x^ja+x^nb}}{x^{\frac{3j}{2}}ax+x^{\frac{j}{2}+n}bx}dx\right)aj-x^{\frac{j}{2}}\left(\int\frac{x^j\sqrt{x^ja+x^nb}}{x^{\frac{3j}{2}}ax+x^{\frac{j}{2}+n}bx}dx\right)an}{x^{\frac{j}{2}}\,(j-n)}$$

input
```
int(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)
```

output
```
( - 2*sqrt(x**j*a + x**n*b) + x**(j/2)*int((x**j*sqrt(x**j*a + x**n*b))/(x
**((3*j)/2)*a*x + x**((j + 2*n)/2)*b*x),x)*a*j - x**(j/2)*int((x**j*sqrt(x
**j*a + x**n*b))/(x**((3*j)/2)*a*x + x**((j + 2*n)/2)*b*x),x)*a*n)/(x**(j/
2)*(j - n))
```

3.344 $\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx$

Optimal result . . . 2751
Mathematica [A] (verified) . . . 2751
Rubi [A] (verified) . . . 2752
Maple [F] . . . 2753
Fricas [F(-2)] . . . 2754
Sympy [F] . . . 2754
Maxima [F] . . . 2754
Giac [F] . . . 2755
Mupad [F(-1)] . . . 2755
Reduce [F] . . . 2755

Optimal result

Integrand size = 27, antiderivative size = 99

$$\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx = -\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)}+\frac{2\sqrt{a}x^{j/2}(cx)^{-j/2}\text{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}$$

output
```
-2*(a*x^j+b*x^n)^(1/2)/c/(j-n)/((c*x)^(1/2*j))+2*a^(1/2)*x^(1/2*j)*arctanh
(a^(1/2)*x^(1/2*j)/(a*x^j+b*x^n)^(1/2))/c/(j-n)/((c*x)^(1/2*j))
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.10

$$\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx = -\frac{2(cx)^{-j/2}\left(ax^j+bx^n-\sqrt{a}\sqrt{b}x^{\frac{j+n}{2}}\sqrt{1+\frac{ax^{j-n}}{b}}\text{arcsinh}\left(\frac{\sqrt{a}x^{\frac{j-n}{2}}}{\sqrt{b}}\right)\right)}{c(j-n)\sqrt{ax^j+bx^n}}$$

input
```
Integrate[(c*x)^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

output
```
(-2*(a*x^j + b*x^n - Sqrt[a]*Sqrt[b]*x^((j + n)/2)*Sqrt[1 + (a*x^(j - n))/
b]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[b]]))/(c*(j - n)*(c*x)^(j/2)*Sqrt[
a*x^j + b*x^n])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1937, 1934, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-\frac{j}{2}-1}\sqrt{ax^j+bx^n}\,dx$$

$$\Big\downarrow \text{1937}$$

$$\frac{x^{j/2}(cx)^{-j/2}\int x^{-\frac{j}{2}-1}\sqrt{ax^j+bx^n}dx}{c}$$

$$\Big\downarrow \text{1934}$$

$$\frac{x^{j/2}(cx)^{-j/2}\left(a\int \frac{x^{\frac{j-2}{2}}}{\sqrt{ax^j+bx^n}}dx-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}\right)}{c}$$

$$\Big\downarrow \text{1935}$$

$$\frac{x^{j/2}(cx)^{-j/2}\left(\frac{2a\int \frac{1}{1-\frac{ax^j}{ax^j+bx^n}}d\frac{x^{j/2}}{\sqrt{ax^j+bx^n}}}{j-n}-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}\right)}{c}$$

$$\Big\downarrow \text{219}$$

$$\frac{x^{j/2}(cx)^{-j/2}\left(\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}\right)}{c}$$

input
```
Int[(c*x)^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

output
```
(x^(j/2)*((-2*Sqrt[a*x^j + b*x^n])/((j - n)*x^(j/2)) + (2*Sqrt[a]*ArcTanh[
(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(j - n)))/(c*(c*x)^(j/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [F]

$$\int (cx)^{-1-\frac{j}{2}} \sqrt{a\,x^{j}+b\,x^{n}}dx$$

input
```
int((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)
```

output
```
int((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx = \int (cx)^{-\frac{j}{2}-1}\sqrt{ax^j+bx^n}\,dx$$

input
```
integrate((c*x)**(-1-1/2*j)*(a*x**j+b*x**n)**(1/2),x)
```

output
```
Integral((c*x)**(-j/2 - 1)*sqrt(a*x**j + b*x**n), x)
```

Maxima [F]

$$\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx = \int \sqrt{ax^j+bx^n}(cx)^{-\frac{1}{2}j-1}\,dx$$

input
```
integrate((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^j + b*x^n)*(c*x)^(-1/2*j - 1), x)
```

Giac [F]

$$\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx = \int \sqrt{ax^j+bx^n}(cx)^{-\frac{1}{2}j-1}\,dx$$

input
```
integrate((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^j + b*x^n)*(c*x)^(-1/2*j - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx = \int \frac{\sqrt{a\,x^j+b\,x^n}}{(c\,x)^{\frac{j}{2}+1}}\,dx$$

input
```
int((a*x^j + b*x^n)^(1/2)/(c*x)^(j/2 + 1),x)
```

output
```
int((a*x^j + b*x^n)^(1/2)/(c*x)^(j/2 + 1), x)
```

Reduce [F]

$$\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx = \frac{-2\sqrt{x^ja+x^nb}+x^{\frac{j}{2}}\left(\int \frac{x^j\sqrt{x^ja+x^nb}}{x^{\frac{3j}{2}}ax+x^{\frac{j}{2}+n}bx}dx\right)aj-x^{\frac{j}{2}}\left(\int \frac{x^j\sqrt{x^ja+x^nb}}{x^{\frac{3j}{2}}ax+x^{\frac{j}{2}+n}bx}dx\right)an}{x^{\frac{j}{2}}c^{\frac{j}{2}}c\,(j-n)}$$

input
```
int((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)
```

output
```
( - 2*sqrt(x**j*a + x**n*b) + x**(j/2)*int((x**j*sqrt(x**j*a + x**n*b))/(x
**((3*j)/2)*a*x + x**((j + 2*n)/2)*b*x),x)*a*j - x**(j/2)*int((x**j*sqrt(x
**j*a + x**n*b))/(x**((3*j)/2)*a*x + x**((j + 2*n)/2)*b*x),x)*a*n)/(x**(j/
2)*c**(j/2)*c*(j - n))
```

3.345 $\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx$

Optimal result 2757
Mathematica [A] (verified) 2757
Rubi [A] (verified) 2758
Maple [F] 2759
Fricas [F(-2)] 2760
Sympy [F] 2760
Maxima [F] 2760
Giac [F] 2761
Mupad [F(-1)] 2761
Reduce [F] 2761

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx=-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}+\frac{2\sqrt{a}\sqrt{cx}\,\text{arctanh}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}}$$

output
```
-2*(a*x^3+b*x^n)^(1/2)/c/(3-n)/(c*x)^(3/2)+2*a^(1/2)*(c*x)^(1/2)*arctanh(a
^(1/2)*x^(3/2)/(a*x^3+b*x^n)^(1/2))/c^3/(3-n)/x^(1/2)
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx=\frac{2x\left(ax^3+bx^n-\sqrt{a}\sqrt{b}x^{\frac{3+n}{2}}\sqrt{1+\frac{ax^{3-n}}{b}}\,\text{arcsinh}\left(\frac{\sqrt{a}x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{(-3+n)(cx)^{5/2}\sqrt{ax^3+bx^n}}$$

input
```
Integrate[Sqrt[a*x^3 + b*x^n]/(c*x)^(5/2),x]
```

output
```
(2*x*(a*x^3 + b*x^n - Sqrt[a]*Sqrt[b]*x^((3 + n)/2)*Sqrt[1 + (a*x^(3 - n))
/b]*ArcSinh[(Sqrt[a]*x^(3/2 - n/2))/Sqrt[b]]))/((-3 + n)*(c*x)^(5/2)*Sqrt[
a*x^3 + b*x^n])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1934, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}dx$$

$$\Big\downarrow \text{ 1934}$$

$$\frac{a\int \frac{\sqrt{cx}}{\sqrt{bx^n+ax^3}}dx}{c^3}-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}$$

$$\Big\downarrow \text{ 1937}$$

$$\frac{a\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{bx^n+ax^3}}dx}{c^3\sqrt{x}}-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}$$

$$\Big\downarrow \text{ 1935}$$

$$\frac{2a\sqrt{cx}\int \frac{1}{1-\frac{ax^3}{bx^n+ax^3}}d\frac{x^{3/2}}{\sqrt{bx^n+ax^3}}}{c^3(3-n)\sqrt{x}}-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\sqrt{a}\sqrt{cx}\,\text{arctanh}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}}-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}$$

input
```
Int[Sqrt[a*x^3 + b*x^n]/(c*x)^(5/2),x]
```

output
```
(-2*Sqrt[a*x^3 + b*x^n])/(c*(3 - n)*(c*x)^(3/2)) + (2*Sqrt[a]*Sqrt[c*x]*Ar
cTanh[(Sqrt[a]*x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(c^3*(3 - n)*Sqrt[x])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [F]

$$\int \frac{\sqrt{a\,x^{3}+b\,x^{n}}}{(cx)^{\frac{5}{2}}}dx$$

input
```
int((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x)
```

output
```
int((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{5/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{5/2}} \, dx = \int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{\frac{5}{2}}} \, dx$$

input
```
integrate((a*x**3+b*x**n)**(1/2)/(c*x)**(5/2),x)
```

output
```
Integral(sqrt(a*x**3 + b*x**n)/(c*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{5/2}} \, dx = \int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{\frac{5}{2}}} \, dx$$

input
```
integrate((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^3 + b*x^n)/(c*x)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{5/2}}\,dx = \int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^3 + b*x^n)/(c*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^n + a\,x^3}}{(c\,x)^{5/2}}\,dx$$

input
```
int((b*x^n + a*x^3)^(1/2)/(c*x)^(5/2),x)
```

output
```
int((b*x^n + a*x^3)^(1/2)/(c*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{5/2}}\,dx = \frac{\int \frac{\sqrt{x^n b + a\,x^3}}{\sqrt{x}\,x^2}dx}{\sqrt{c}\,c^2}$$

input
```
int((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x)
```

output
```
int(sqrt(x**n*b + a*x**3)/(sqrt(x)*x**2),x)/(sqrt(c)*c**2)
```

3.346 $\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx$

Optimal result 2762
Mathematica [A] (verified) 2762
Rubi [A] (verified) 2763
Maple [F] 2764
Fricas [F(-2)] 2765
Sympy [F] 2765
Maxima [F] 2765
Giac [F] 2766
Mupad [F(-1)] 2766
Reduce [F] 2766

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx = -\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x}+\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{c^2(2-n)}$$

output
```
-2*(a*x^2+b*x^n)^(1/2)/c^2/(2-n)/x+2*a^(1/2)*arctanh(a^(1/2)*x/(a*x^2+b*x^
n)^(1/2))/c^2/(2-n)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx = \frac{2\left(ax^2+bx^n-\sqrt{a}\sqrt{b}x^{1+\frac{n}{2}}\sqrt{1+\frac{ax^{2-n}}{b}}\,\text{arcsinh}\left(\frac{\sqrt{a}x^{1-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{c^2(-2+n)x\sqrt{ax^2+bx^n}}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^n]/(c^2*x^2),x]
```

output
```
(2*(a*x^2 + b*x^n - Sqrt[a]*Sqrt[b]*x^(1 + n/2)*Sqrt[1 + (a*x^(2 - n))/b]*
ArcSinh[(Sqrt[a]*x^(1 - n/2))/Sqrt[b]]))/(c^2*(-2 + n)*x*Sqrt[a*x^2 + b*x^
n])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {27, 1934, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{bx^n+ax^2}}{x^2}dx}{c^2}$$

$$\Big\downarrow \text{1934}$$

$$\frac{a\int \frac{1}{\sqrt{bx^n+ax^2}}dx-\frac{2\sqrt{ax^2+bx^n}}{(2-n)x}}{c^2}$$

$$\Big\downarrow \text{1914}$$

$$\frac{\frac{2a\int \frac{1}{1-\frac{ax^2}{bx^n+ax^2}}d\frac{x}{\sqrt{bx^n+ax^2}}}{2-n}-\frac{2\sqrt{ax^2+bx^n}}{(2-n)x}}{c^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{2-n}-\frac{2\sqrt{ax^2+bx^n}}{(2-n)x}}{c^2}$$

input
```
Int[Sqrt[a*x^2 + b*x^n]/(c^2*x^2),x]
```

output
```
((-2*Sqrt[a*x^2 + b*x^n])/((2 - n)*x) + (2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqr
t[a*x^2 + b*x^n]])/(2 - n))/c^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{\sqrt{a\,x^2+b\,x^n}}{c^2x^2}dx$$

input
```
int((a*x^2+b*x^n)^(1/2)/c^2/x^2,x)
```

output
```
int((a*x^2+b*x^n)^(1/2)/c^2/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax^2 + bx^n}}{c^2x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x^2+b*x^n)^(1/2)/c^2/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{ax^2 + bx^n}}{c^2x^2}\,dx = \frac{\int \frac{\sqrt{ax^2+bx^n}}{x^2}\,dx}{c^2}$$

input
```
integrate((a*x**2+b*x**n)**(1/2)/c**2/x**2,x)
```

output
```
Integral(sqrt(a*x**2 + b*x**n)/x**2, x)/c**2
```

Maxima [F]

$$\int \frac{\sqrt{ax^2 + bx^n}}{c^2x^2}\,dx = \int \frac{\sqrt{ax^2 + bx^n}}{c^2x^2}\,dx$$

input
```
integrate((a*x^2+b*x^n)^(1/2)/c^2/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^2 + b*x^n)/x^2, x)/c^2
```

Giac [F]

$$\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx=\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx$$

input
```
integrate((a*x^2+b*x^n)^(1/2)/c^2/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^2 + b*x^n)/(c^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx=\int \frac{\sqrt{b\,x^n+a\,x^2}}{c^2\,x^2}\,dx$$

input
```
int((b*x^n + a*x^2)^(1/2)/(c^2*x^2),x)
```

output
```
int((b*x^n + a*x^2)^(1/2)/(c^2*x^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx=\frac{\int \frac{\sqrt{x^n b+a\,x^2}}{x^2}dx}{c^2}$$

input
```
int((a*x^2+b*x^n)^(1/2)/c^2/x^2,x)
```

output
```
int(sqrt(x**n*b + a*x**2)/x**2,x)/c**2
```

3.347 $\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx$

Optimal result 2767
Mathematica [A] (verified) 2767
Rubi [A] (verified) 2768
Maple [F] 2769
Fricas [F(-2)] 2770
Sympy [F] 2770
Maxima [F] 2770
Giac [F] 2771
Mupad [F(-1)] 2771
Reduce [F] 2771

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx = -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}} + \frac{2\sqrt{a}\sqrt{x}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}}$$

output
```
-2*(a*x+b*x^n)^(1/2)/c/(1-n)/(c*x)^(1/2)+2*a^(1/2)*x^(1/2)*arctanh(a^(1/2)
*x^(1/2)/(a*x+b*x^n)^(1/2))/c/(1-n)/(c*x)^(1/2)
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx = \frac{x\left(2ax+2bx^n-2\sqrt{a}\sqrt{b}x^{\frac{1+n}{2}}\sqrt{1+\frac{ax^{1-n}}{b}}\,\text{arcsinh}\left(\frac{\sqrt{a}x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{(-1+n)(cx)^{3/2}\sqrt{ax+bx^n}}$$

input
```
Integrate[Sqrt[a*x + b*x^n]/(c*x)^(3/2),x]
```

output
```
(x*(2*a*x + 2*b*x^n - 2*Sqrt[a]*Sqrt[b]*x^((1 + n)/2)*Sqrt[1 + (a*x^(1 - n
))/b]*ArcSinh[(Sqrt[a]*x^(1/2 - n/2))/Sqrt[b]]))/((-1 + n)*(c*x)^(3/2)*Sqr
t[a*x + b*x^n])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1934, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}dx$$

$$\Big\downarrow \text{1934}$$

$$\frac{a\int \frac{1}{\sqrt{cx}\sqrt{bx^n+ax}}dx}{c}-\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{a\sqrt{x}\int \frac{1}{\sqrt{x}\sqrt{bx^n+ax}}dx}{c\sqrt{cx}}-\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{2a\sqrt{x}\int \frac{1}{1-\frac{ax}{bx^n+ax}}d\frac{\sqrt{x}}{\sqrt{bx^n+ax}}}{c(1-n)\sqrt{cx}}-\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{a}\sqrt{x}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}}-\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}$$

input
```
Int[Sqrt[a*x + b*x^n]/(c*x)^(3/2),x]
```

output
```
(-2*Sqrt[a*x + b*x^n])/(c*(1 - n)*Sqrt[c*x]) + (2*Sqrt[a]*Sqrt[x]*ArcTanh[
(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^n]])/(c*(1 - n)*Sqrt[c*x])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [F]

$$\int \frac{\sqrt{ax+b\,x^{n}}}{(cx)^{\frac{3}{2}}}dx$$

input
```
int((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x)
```

output
```
int((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx = \int \frac{\sqrt{ax+bx^n}}{(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*x+b*x**n)**(1/2)/(c*x)**(3/2),x)
```

output
```
Integral(sqrt(a*x + b*x**n)/(c*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx = \int \frac{\sqrt{ax+bx^n}}{(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^n)/(c*x)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx = \int \frac{\sqrt{ax+bx^n}}{(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x + b*x^n)/(c*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^n+a\,x}}{(c\,x)^{3/2}}\,dx$$

input
```
int((b*x^n + a*x)^(1/2)/(c*x)^(3/2),x)
```

output
```
int((b*x^n + a*x)^(1/2)/(c*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx = \frac{\sqrt{c}\left(2\sqrt{x}\,\sqrt{x^n b+ax}+\left(\int \frac{\sqrt{x}\,\sqrt{x^n b+ax}}{x^n bx+a\,x^2}dx\right)anx-\left(\int \frac{\sqrt{x}\,\sqrt{x^n b+ax}}{x^n bx+a\,x^2}dx\right)ax\right)}{c^2x\,(n-1)}$$

input
```
int((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x)
```

output
```
(sqrt(c)*(2*sqrt(x)*sqrt(x**n*b + a*x) + int((sqrt(x)*sqrt(x**n*b + a*x))/
(x**n*b*x + a*x**2),x)*a*n*x - int((sqrt(x)*sqrt(x**n*b + a*x))/(x**n*b*x
 + a*x**2),x)*a*x))/(c**2*x*(n - 1))
```

3.348 $\int \frac{\sqrt{a+bx^n}}{cx}\,dx$

Optimal result 2772
Mathematica [A] (verified) 2772
Rubi [A] (verified) 2773
Maple [A] (verified) 2775
Fricas [A] (verification not implemented) 2775
Sympy [A] (verification not implemented) 2776
Maxima [A] (verification not implemented) 2776
Giac [F] 2776
Mupad [F(-1)] 2777
Reduce [F] 2777

Optimal result

Integrand size = 18, antiderivative size = 51

$$\int \frac{\sqrt{a+bx^n}}{cx}\,dx=\frac{2\sqrt{a+bx^n}}{cn}-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

output
```
2*(a+b*x^n)^(1/2)/c/n-2*a^(1/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2))/c/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+bx^n}}{cx}\,dx=\frac{2\left(\sqrt{a+bx^n}-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)\right)}{cn}$$

input
```
Integrate[Sqrt[a + b*x^n]/(c*x),x]
```

output
```
(2*(Sqrt[a + b*x^n] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]]))/(c*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {27, 798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^n}}{cx} \, dx$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{bx^n+a}}{x}dx}{c}$$

$$\Big\downarrow \; 798$$

$$\frac{\int x^{-n}\sqrt{bx^n+a}dx^n}{cn}$$

$$\Big\downarrow \; 60$$

$$\frac{a\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n+2\sqrt{a+bx^n}}{cn}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{2a\int \frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{b}+2\sqrt{a+bx^n}}{cn}$$

$$\Big\downarrow \; 221$$

$$\frac{2\sqrt{a+bx^n}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

input
```
Int[Sqrt[a + b*x^n]/(c*x),x]
```

output
```
(2*Sqrt[a + b*x^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(c*n)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{2\sqrt{a+b\,x^{n}}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{cn}$	39
default	$\frac{2\sqrt{a+b\,x^{n}}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{cn}$	39
risch	$\frac{2\sqrt{a+b\,{\mathrm e}^{n\ln(x)}}}{nc}-\frac{2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,{\mathrm e}^{n\ln(x)}}}{\sqrt{a}}\right)}{nc}$	48

input
```
int((a+b*x^n)^(1/2)/c/x,x,method=_RETURNVERBOSE)
```

output
```
1/c/n*(2*(a+b*x^n)^(1/2)-2*a^(1/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt{a+bx^n}}{cx}\,dx = \left[\frac{\sqrt{a}\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)+2\sqrt{bx^n+a}}{cn}, \frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^n+a}}\right)+\sqrt{bx^n+a}\right)}{cn}\right]$$

input
```
integrate((a+b*x^n)^(1/2)/c/x,x, algorithm="fricas")
```

output
```
[(sqrt(a)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*sqrt(b*x^
n + a))/(c*n), 2*(sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^n + a)) + sqrt(b*x^n +
 a))/(c*n)]
```

Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{a+bx^n}}{cx}\,dx = \frac{-\frac{2\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{n} + \frac{2ax^{-\frac{n}{2}}}{\sqrt{b}n\sqrt{\frac{ax^{-n}}{b}+1}} + \frac{2\sqrt{b}x^{\frac{n}{2}}}{n\sqrt{\frac{ax^{-n}}{b}+1}}}{c}$$

input
```
integrate((a+b*x**n)**(1/2)/c/x,x)
```

output
```
(-2*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x**(n/2)))/n + 2*a/(sqrt(b)*n*x**(n/2)*
sqrt(a/(b*x**n) + 1)) + 2*sqrt(b)*x**(n/2)/(n*sqrt(a/(b*x**n) + 1)))/c
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a+bx^n}}{cx}\,dx = \frac{\frac{\sqrt{a}\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{n} + \frac{2\sqrt{bx^n+a}}{n}}{c}$$

input
```
integrate((a+b*x^n)^(1/2)/c/x,x, algorithm="maxima")
```

output
```
(sqrt(a)*log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/n +
2*sqrt(b*x^n + a)/n)/c
```

Giac [F]

$$\int \frac{\sqrt{a+bx^n}}{cx}\,dx = \int \frac{\sqrt{bx^n+a}}{cx}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)/c/x,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a)/(c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^n}}{cx}\,dx=\int \frac{\sqrt{a+b\,x^n}}{c\,x}\,dx$$

input
```
int((a + b*x^n)^(1/2)/(c*x),x)
```

output
```
int((a + b*x^n)^(1/2)/(c*x), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^n}}{cx}\,dx=\frac{2\sqrt{x^nb+a}+\left(\int \frac{\sqrt{x^nb+a}}{x^nbx+ax}dx\right)an}{cn}$$

input
```
int((a+b*x^n)^(1/2)/c/x,x)
```

output
```
(2*sqrt(x**n*b + a) + int(sqrt(x**n*b + a)/(x**n*b*x + a*x),x)*a*n)/(c*n)
```

3.349 $\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx$

Optimal result . . . 2778
Mathematica [A] (verified) . . . 2778
Rubi [A] (verified) . . . 2779
Maple [F] . . . 2780
Fricas [F(-2)] . . . 2781
Sympy [F] . . . 2781
Maxima [F] . . . 2781
Giac [F] . . . 2782
Mupad [F(-1)] . . . 2782
Reduce [F] . . . 2782

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx=\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(1+n)}-\frac{2\sqrt{a}\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(1+n)\sqrt{cx}}$$

output
```
2*(c*x)^(1/2)*(a/x+b*x^n)^(1/2)/c/(1+n)-2*a^(1/2)*x^(1/2)*arctanh(a^(1/2)/
x^(1/2)/(a/x+b*x^n)^(1/2))/(1+n)/(c*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx=\frac{2x\sqrt{\frac{a}{x}+bx^n}\left(\sqrt{a+bx^{1+n}}-\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^{1+n}}}{\sqrt{a}}\right)\right)}{(1+n)\sqrt{cx}\sqrt{a+bx^{1+n}}}$$

input
```
Integrate[Sqrt[a/x + b*x^n]/Sqrt[c*x],x]
```

output
```
(2*x*Sqrt[a/x + b*x^n]*(Sqrt[a + b*x^(1 + n)] - Sqrt[a]*ArcTanh[Sqrt[a + b
*x^(1 + n)]/Sqrt[a]]))/((1 + n)*Sqrt[c*x]*Sqrt[a + b*x^(1 + n)])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1934, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}dx$$

$$\Big\downarrow \text{1934}$$

$$ac\int \frac{1}{(cx)^{3/2}\sqrt{bx^n+\frac{a}{x}}}dx+\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(n+1)}$$

$$\Big\downarrow \text{1937}$$

$$\frac{a\sqrt{x}\int \frac{1}{x^{3/2}\sqrt{bx^n+\frac{a}{x}}}dx}{\sqrt{cx}}+\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(n+1)}$$

$$\Big\downarrow \text{1935}$$

$$\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(n+1)}-\frac{2a\sqrt{x}\int \frac{1}{1-\frac{a}{x\left(bx^n+\frac{a}{x}\right)}}d\frac{1}{\sqrt{x}\sqrt{bx^n+\frac{a}{x}}}}{(n+1)\sqrt{cx}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(n+1)}-\frac{2\sqrt{a}\sqrt{x}\text{arctanh}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(n+1)\sqrt{cx}}$$

input
```
Int[Sqrt[a/x + b*x^n]/Sqrt[c*x],x]
```

output
```
(2*Sqrt[c*x]*Sqrt[a/x + b*x^n])/(c*(1 + n)) - (2*Sqrt[a]*Sqrt[x]*ArcTanh[S
qrt[a]/(Sqrt[x]*Sqrt[a/x + b*x^n])])/((1 + n)*Sqrt[c*x])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [F]

$$\int \frac{\sqrt{\frac{a}{x} + b\,x^{n}}}{\sqrt{cx}}dx$$

input
```
int((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x)
```

output
```
int((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx = \int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx$$

input
```
integrate((a/x+b*x**n)**(1/2)/(c*x)**(1/2),x)
```

output
```
Integral(sqrt(a/x + b*x**n)/sqrt(c*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx = \int \frac{\sqrt{bx^n+\frac{a}{x}}}{\sqrt{cx}}\,dx$$

input
```
integrate((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^n + a/x)/sqrt(c*x), x)
```

Giac [F]

$$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx=\int \frac{\sqrt{bx^n+\frac{a}{x}}}{\sqrt{cx}}\,dx$$

input
```
integrate((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a/x)/sqrt(c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx=\int \frac{\sqrt{b\,x^n+\frac{a}{x}}}{\sqrt{c\,x}}\,dx$$

input
```
int((b*x^n + a/x)^(1/2)/(c*x)^(1/2),x)
```

output
```
int((b*x^n + a/x)^(1/2)/(c*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx=\frac{\sqrt{c}\left(2\sqrt{x^nbx+a}+\left(\int \frac{\sqrt{x^nbx+a}}{x^nb\,x^2+ax}dx\right)an+\left(\int \frac{\sqrt{x^nbx+a}}{x^nb\,x^2+ax}dx\right)a\right)}{c\,(n+1)}$$

input
```
int((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x)
```

output
```
(sqrt(c)*(2*sqrt(x**n*b*x + a) + int(sqrt(x**n*b*x + a)/(x**n*b*x**2 + a*x
),x)*a*n + int(sqrt(x**n*b*x + a)/(x**n*b*x**2 + a*x),x)*a))/(c*(n + 1))
```

3.350 $\int \sqrt{\frac{a}{x^2}+bx^n}\,dx$

Optimal result 2783
Mathematica [A] (verified) 2783
Rubi [A] (verified) 2784
Maple [F] 2785
Fricas [F(-2)] 2785
Sympy [F] 2786
Maxima [F] 2786
Giac [F] 2786
Mupad [B] (verification not implemented) 2787
Reduce [F] 2787

Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \sqrt{\frac{a}{x^2}+bx^n}\,dx=\frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}-\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}$$

output
```
2*x*(a/x^2+b*x^n)^(1/2)/(2+n)-2*a^(1/2)*arctanh(a^(1/2)/x/(a/x^2+b*x^n)^(1
/2))/(2+n)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26

$$\int \sqrt{\frac{a}{x^2}+bx^n}\,dx=\frac{2x\sqrt{\frac{a}{x^2}+bx^n}\left(\sqrt{a+bx^{2+n}}-\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+bx^{2+n}}}{\sqrt{a}}\right)\right)}{(2+n)\sqrt{a+bx^{2+n}}}$$

input
```
Integrate[Sqrt[a/x^2 + b*x^n],x]
```

output
```
(2*x*Sqrt[a/x^2 + b*x^n]*(Sqrt[a + b*x^(2 + n)] - Sqrt[a]*ArcTanh[Sqrt[a +
 b*x^(2 + n)]/Sqrt[a]]))/((2 + n)*Sqrt[a + b*x^(2 + n)])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1913, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{a}{x^2}+bx^n}\,dx$$

$$\Big\downarrow \ 1913$$

$$a\int \frac{1}{x^2\sqrt{bx^n+\frac{a}{x^2}}}dx+\frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}$$

$$\Big\downarrow \ 1935$$

$$\frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}-\frac{2a\int \frac{1}{1-\frac{a}{x^2\left(bx^n+\frac{a}{x^2}\right)}}d\frac{1}{x\sqrt{bx^n+\frac{a}{x^2}}}}{n+2}$$

$$\Big\downarrow \ 219$$

$$\frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{n+2}$$

input
```
Int[Sqrt[a/x^2 + b*x^n],x]
```

output
```
(2*x*Sqrt[a/x^2 + b*x^n])/(2 + n) - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x
^2 + b*x^n])])/(2 + n)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1913
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Simp[a   Int[x^j*(a*x^j + b*x^n)^(p - 1), x]
, x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simp
lify[j*p + 1], 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [F]

$$\int \sqrt{\frac{a}{x^2} + b\,x^n}dx$$

input
```
int((a/x^2+b*x^n)^(1/2),x)
```

output
```
int((a/x^2+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\frac{a}{x^2} + bx^n}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a/x^2+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sqrt{\frac{a}{x^2}+bx^n}\,dx=\int \sqrt{\frac{a}{x^2}+bx^n}\,dx$$

input
```
integrate((a/x**2+b*x**n)**(1/2),x)
```

output
```
Integral(sqrt(a/x**2 + b*x**n), x)
```

Maxima [F]

$$\int \sqrt{\frac{a}{x^2}+bx^n}\,dx=\int \sqrt{bx^n+\frac{a}{x^2}}\,dx$$

input
```
integrate((a/x^2+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^n + a/x^2), x)
```

Giac [F]

$$\int \sqrt{\frac{a}{x^2}+bx^n}\,dx=\int \sqrt{bx^n+\frac{a}{x^2}}\,dx$$

input
```
integrate((a/x^2+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a/x^2), x)
```

Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.59

$$\int \sqrt{\frac{a}{x^2}+bx^n}\,dx = \frac{x\sqrt{b\,x^n+\frac{a}{x^2}}}{\frac{n}{2}+1}+\frac{\sqrt{a}\,x\,\mathrm{asin}\left(\frac{\sqrt{a}\,1\mathrm{i}}{\sqrt{b}\,x^{\frac{n}{2}+1}}\right)\,\sqrt{b\,x^n+\frac{a}{x^2}}\,1\mathrm{i}}{\sqrt{b}\,x^{\frac{n}{2}+1}\,\left(\frac{n}{2}+1\right)\,\sqrt{\frac{a}{b\,x^{n+2}}+1}}$$

input
```
int((b*x^n + a/x^2)^(1/2),x)
```

output
```
(x*(b*x^n + a/x^2)^(1/2))/(n/2 + 1) + (a^(1/2)*x*asin((a^(1/2)*1i)/(b^(1/2
)*x^(n/2 + 1)))*(b*x^n + a/x^2)^(1/2)*1i)/(b^(1/2)*x^(n/2 + 1)*(n/2 + 1)*(
a/(b*x^(n + 2)) + 1)^(1/2))
```

Reduce [F]

$$\int \sqrt{\frac{a}{x^2}+bx^n}\,dx = \int \frac{\sqrt{x^n b\,x^2+a}}{x}dx$$

input
```
int((a/x^2+b*x^n)^(1/2),x)
```

output
```
int(sqrt(x**n*b*x**2 + a)/x,x)
```

3.351 $\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx$

Optimal result . . . 2788
Mathematica [A] (verified) . . . 2788
Rubi [A] (verified) . . . 2789
Maple [F] . . . 2790
Fricas [F(-2)] . . . 2791
Sympy [F] . . . 2791
Maxima [F] . . . 2791
Giac [F] . . . 2792
Mupad [F(-1)] . . . 2792
Reduce [F] . . . 2792

Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx=\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(3+n)}-\frac{2\sqrt{a}c\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(3+n)\sqrt{cx}}$$

output
```
2*(c*x)^(3/2)*(a/x^3+b*x^n)^(1/2)/c/(3+n)-2*a^(1/2)*c*x^(1/2)*arctanh(a^(1
/2)/x^(3/2)/(a/x^3+b*x^n)^(1/2))/(3+n)/(c*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx=\frac{2x\sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\left(\sqrt{a+bx^{3+n}}-\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^{3+n}}}{\sqrt{a}}\right)\right)}{(3+n)\sqrt{a+bx^{3+n}}}$$

input
```
Integrate[Sqrt[c*x]*Sqrt[a/x^3 + b*x^n],x]
```

output
```
(2*x*Sqrt[c*x]*Sqrt[a/x^3 + b*x^n]*(Sqrt[a + b*x^(3 + n)] - Sqrt[a]*ArcTan
h[Sqrt[a + b*x^(3 + n)]/Sqrt[a]]))/((3 + n)*Sqrt[a + b*x^(3 + n)])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1934, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx$$

$$\Big\downarrow \text{1934}$$

$$ac^3\int \frac{1}{(cx)^{5/2}\sqrt{bx^n+\frac{a}{x^3}}}dx+\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(n+3)}$$

$$\Big\downarrow \text{1937}$$

$$\frac{ac\sqrt{x}\int \frac{1}{x^{5/2}\sqrt{bx^n+\frac{a}{x^3}}}dx}{\sqrt{cx}}+\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(n+3)}$$

$$\Big\downarrow \text{1935}$$

$$\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(n+3)}-\frac{2ac\sqrt{x}\int \frac{1}{1-\frac{a}{x^3\left(bx^n+\frac{a}{x^3}\right)}}d\frac{1}{x^{3/2}\sqrt{bx^n+\frac{a}{x^3}}}}{(n+3)\sqrt{cx}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(n+3)}-\frac{2\sqrt{a}c\sqrt{x}\text{arctanh}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(n+3)\sqrt{cx}}$$

input
```
Int[Sqrt[c*x]*Sqrt[a/x^3 + b*x^n],x]
```

output
```
(2*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n])/(c*(3 + n)) - (2*Sqrt[a]*c*Sqrt[x]*Arc
Tanh[Sqrt[a]/(x^(3/2)*Sqrt[a/x^3 + b*x^n])])/((3 + n)*Sqrt[c*x])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [F]

$$\int \sqrt{cx}\,\sqrt{\frac{a}{x^3}+b\,x^n}dx$$

input
```
int((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x)
```

output
```
int((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx = \int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx$$

input
```
integrate((c*x)**(1/2)*(a/x**3+b*x**n)**(1/2),x)
```

output
```
Integral(sqrt(c*x)*sqrt(a/x**3 + b*x**n), x)
```

Maxima [F]

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx = \int \sqrt{bx^n+\frac{a}{x^3}}\sqrt{cx}\,dx$$

input
```
integrate((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^n + a/x^3)*sqrt(c*x), x)
```

Giac [F]

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx=\int \sqrt{bx^n+\frac{a}{x^3}}\sqrt{cx}\,dx$$

input
```
integrate((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a/x^3)*sqrt(c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx=\int \sqrt{c\,x}\,\sqrt{b\,x^n+\frac{a}{x^3}}\,dx$$

input
```
int((c*x)^(1/2)*(b*x^n + a/x^3)^(1/2),x)
```

output
```
int((c*x)^(1/2)*(b*x^n + a/x^3)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx=\sqrt{c}\left(\int \frac{\sqrt{x^n b\,x^3+a}}{x}dx\right)$$

input
```
int((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x)
```

output
```
sqrt(c)*int(sqrt(x**n*b*x**3 + a)/x,x)
```

3.352 $\int (cx)^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2} dx$

Optimal result . . . 2793
Mathematica [A] (verified) . . . 2793
Rubi [A] (verified) . . . 2794
Maple [F] . . . 2796
Fricas [F(-2)] . . . 2796
Sympy [F] . . . 2796
Maxima [F] . . . 2797
Giac [F] . . . 2797
Mupad [F(-1)] . . . 2797
Reduce [F] . . . 2798

Optimal result

Integrand size = 27, antiderivative size = 141

$$\int (cx)^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2} dx = -\frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)} - \frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)} + \frac{2a^{3/2}x^{3j/2}(cx)^{-3j/2}\text{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}$$

output
```
-2*a*x^j*(a*x^j+b*x^n)^(1/2)/c/(j-n)/((c*x)^(3/2*j))-2/3*(a*x^j+b*x^n)^(3/
2)/c/(j-n)/((c*x)^(3/2*j))+2*a^(3/2)*x^(3/2*j)*arctanh(a^(1/2)*x^(1/2*j)/(
a*x^j+b*x^n)^(1/2))/c/(j-n)/((c*x)^(3/2*j))
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.93

$$\int (cx)^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2} dx = -\frac{2(cx)^{-3j/2}\left(4a^2x^{2j}+b^2x^{2n}+5abx^{j+n}-3a^{3/2}\sqrt{b}x^{\frac{1}{2}(3j+n)}\sqrt{1+\frac{ax^{j-n}}{b}}\text{arcsinh}\left(\frac{\sqrt{a}x^{\frac{j-n}{2}}}{\sqrt{b}}\right)\right)}{3c(j-n)\sqrt{ax^j+bx^n}}$$

input
```
Integrate[(c*x)^(-1 - (3*j)/2)*(a*x^j + b*x^n)^(3/2),x]
```

output
```
(-2*(4*a^2*x^(2*j) + b^2*x^(2*n) + 5*a*b*x^(j + n) - 3*a^(3/2)*Sqrt[b]*x^(
(3*j + n)/2)*Sqrt[1 + (a*x^(j - n))/b]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqr
t[b]]))/(3*c*(j - n)*(c*x)^((3*j)/2)*Sqrt[a*x^j + b*x^n])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1937, 1934, 1934, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-\frac{3j}{2}-1} \left(ax^j+bx^n\right)^{3/2} dx$$

$$\Big\downarrow \text{1937}$$

$$\frac{x^{3j/2}(cx)^{-3j/2}\int x^{-\frac{3j}{2}-1}\left(ax^j+bx^n\right)^{3/2}dx}{c}$$

$$\Big\downarrow \text{1934}$$

$$\frac{x^{3j/2}(cx)^{-3j/2}\left(a\int x^{-\frac{j}{2}-1}\sqrt{ax^j+bx^n}dx-\frac{2x^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3(j-n)}\right)}{c}$$

$$\Big\downarrow \text{1934}$$

$$\frac{x^{3j/2}(cx)^{-3j/2}\left(a\left(a\int \frac{x^{\frac{j-2}{2}}}{\sqrt{ax^j+bx^n}}dx-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}\right)-\frac{2x^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3(j-n)}\right)}{c}$$

$$\Big\downarrow \text{1935}$$

$$\frac{x^{3j/2}(cx)^{-3j/2}\left(a\left(\frac{2a\int \frac{1}{1-\frac{ax^j}{ax^j+bx^n}}d\frac{x^{j/2}}{\sqrt{ax^j+bx^n}}}{j-n}-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}\right)-\frac{2x^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3(j-n)}\right)}{c}$$

$$\Big\downarrow \text{219}$$

$$\frac{x^{3j/2}(cx)^{-3j/2}\left(a\left(\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}\right)-\frac{2x^{-3j/2}(ax^j+bx^n)^{3/2}}{3(j-n)}\right)}{c}$$

input
```
Int[(c*x)^(-1 - (3*j)/2)*(a*x^j + b*x^n)^(3/2),x]
```

output
```
(x^((3*j)/2)*((-2*(a*x^j + b*x^n)^(3/2))/(3*(j - n)*x^((3*j)/2)) + a*((-2*
Sqrt[a*x^j + b*x^n])/((j - n)*x^(j/2)) + (2*Sqrt[a]*ArcTanh[(Sqrt[a]*x^(j/
2))/Sqrt[a*x^j + b*x^n]])/(j - n))))/(c*(c*x)^((3*j)/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [F]

$$\int (cx)^{-1-\frac{3j}{2}} \left(a\, x^{j}+b\, x^{n}\right)^{\frac{3}{2}} dx$$

input
```
int((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x)
```

output
```
int((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (cx)^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (cx)^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2} dx = \int (cx)^{-\frac{3j}{2}-1} \left(ax^j+bx^n\right)^{\frac{3}{2}} dx$$

input
```
integrate((c*x)**(-1-3/2*j)*(a*x**j+b*x**n)**(3/2),x)
```

output
```
Integral((c*x)**(-3*j/2 - 1)*(a*x**j + b*x**n)**(3/2), x)
```

Maxima [F]

$$\int (cx)^{-1-\frac{3j}{2}} \left(ax^j + bx^n\right)^{3/2} dx = \int \left(ax^j + bx^n\right)^{\frac{3}{2}} (cx)^{-\frac{3}{2}j-1} dx$$

input
```
integrate((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x^j + b*x^n)^(3/2)*(c*x)^(-3/2*j - 1), x)
```

Giac [F]

$$\int (cx)^{-1-\frac{3j}{2}} \left(ax^j + bx^n\right)^{3/2} dx = \int \left(ax^j + bx^n\right)^{\frac{3}{2}} (cx)^{-\frac{3}{2}j-1} dx$$

input
```
integrate((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*x^j + b*x^n)^(3/2)*(c*x)^(-3/2*j - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{-1-\frac{3j}{2}} \left(ax^j + bx^n\right)^{3/2} dx = \int \frac{\left(a\,x^j + b\,x^n\right)^{3/2}}{(c\,x)^{\frac{3\,j}{2}+1}} \,dx$$

input
```
int((a*x^j + b*x^n)^(3/2)/(c*x)^((3*j)/2 + 1),x)
```

output
```
int((a*x^j + b*x^n)^(3/2)/(c*x)^((3*j)/2 + 1), x)
```

Reduce [F]

$$\int (cx)^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2} dx = \frac{-8x^j\sqrt{x^ja+x^nb}\,a-2x^n\sqrt{x^ja+x^nb}\,b+3x^{\frac{3j}{2}}\left(\int \frac{x^{2j}\sqrt{x^ja+x^nb}}{x^{\frac{5j}{2}}ax+x^{\frac{3j}{2}+n}bx}dx\right)a^2j-3x^{\frac{3j}{2}}\left(\int \frac{x^{2j}\sqrt{x^ja-\cdots}}{x^{\frac{5j}{2}}ax+x^{\frac{3}{2}\cdots}}\right.\cdots}{3x^{\frac{3j}{2}}c^{\frac{3j}{2}}c\,(j-n)}$$

input
```
int((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x)
```

output
```
( - 8*x**j*sqrt(x**j*a + x**n*b)*a - 2*x**n*sqrt(x**j*a + x**n*b)*b + 3*x*
*((3*j)/2)*int((x**(2*j)*sqrt(x**j*a + x**n*b))/(x**((5*j)/2)*a*x + x**((3
*j + 2*n)/2)*b*x),x)*a**2*j - 3*x**((3*j)/2)*int((x**(2*j)*sqrt(x**j*a + x
**n*b))/(x**((5*j)/2)*a*x + x**((3*j + 2*n)/2)*b*x),x)*a**2*n)/(3*x**((3*j
)/2)*c**((3*j)/2)*c*(j - n))
```

3.353 $\int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}}\,dx$

Optimal result 2799
Mathematica [A] (verified) 2799
Rubi [A] (verified) 2800
Maple [F] 2802
Fricas [F(-2)] 2802
Sympy [F(-1)] 2802
Maxima [F] 2803
Giac [F] 2803
Mupad [F(-1)] 2803
Reduce [F] 2804

Optimal result

Integrand size = 23, antiderivative size = 128

$$\int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}}\,dx = -\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}} - \frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}} + \frac{2a^{3/2}\sqrt{cx}\,\text{arctanh}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^6(3-n)\sqrt{x}}$$

output
```
-2*a*(a*x^3+b*x^n)^(1/2)/c^4/(3-n)/(c*x)^(3/2)-2/3*(a*x^3+b*x^n)^(3/2)/c/(
3-n)/(c*x)^(9/2)+2*a^(3/2)*(c*x)^(1/2)*arctanh(a^(1/2)*x^(3/2)/(a*x^3+b*x^
n)^(1/2))/c^6/(3-n)/x^(1/2)
```

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98

$$\int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}}\,dx = \frac{2\sqrt{cx}\left(4a^2x^6+b^2x^{2n}+5abx^{3+n}-3a^{3/2}\sqrt{b}x^{\frac{9+n}{2}}\sqrt{1+\frac{ax^{3-n}}{b}}\,\text{arcsinh}\left(\frac{\sqrt{a}x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{3c^6(-3+n)x^5\sqrt{ax^3+bx^n}}$$

input
```
Integrate[(a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2),x]
```

output
```
(2*Sqrt[c*x]*(4*a^2*x^6 + b^2*x^(2*n) + 5*a*b*x^(3 + n) - 3*a^(3/2)*Sqrt[b
]*x^((9 + n)/2)*Sqrt[1 + (a*x^(3 - n))/b]*ArcSinh[(Sqrt[a]*x^(3/2 - n/2))/
Sqrt[b]]))/(3*c^6*(-3 + n)*x^5*Sqrt[a*x^3 + b*x^n])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1934, 1934, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}}\,dx$$

$$\Big\downarrow \text{1934}$$

$$\frac{a\int \frac{\sqrt{bx^n+ax^3}}{(cx)^{5/2}}dx}{c^3}-\frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}}$$

$$\Big\downarrow \text{1934}$$

$$\frac{a\left(\frac{a\int \frac{\sqrt{cx}}{\sqrt{bx^n+ax^3}}dx}{c^3}-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}\right)}{c^3}-\frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{a\left(\frac{a\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{bx^n+ax^3}}dx}{c^3\sqrt{x}}-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}\right)}{c^3}-\frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{a\left(\frac{2a\sqrt{cx}\int \frac{1}{1-\frac{ax^3}{bx^n+ax^3}}d\frac{x^{3/2}}{\sqrt{bx^n+ax^3}}}{c^3(3-n)\sqrt{x}}-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}\right)}{c^3}-\frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{a\left(\frac{2\sqrt{a}\sqrt{cx}\text{arctanh}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}}-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}\right)}{c^3}-\frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}}$$

input
```
Int[(a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2),x]
```

output
```
(-2*(a*x^3 + b*x^n)^(3/2))/(3*c*(3 - n)*(c*x)^(9/2)) + (a*((-2*Sqrt[a*x^3
+ b*x^n])/(c*(3 - n)*(c*x)^(3/2)) + (2*Sqrt[a]*Sqrt[c*x]*ArcTanh[(Sqrt[a]*
x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(c^3*(3 - n)*Sqrt[x])))/c^3
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [F]

$$\int \frac{(a\,x^{3}+b\,x^{n})^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}}dx$$

input
```
int((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x)
```

output
```
int((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*x**3+b*x**n)**(3/2)/(c*x)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ax^3 + bx^n)^{3/2}}{(cx)^{11/2}} \, dx = \int \frac{(ax^3 + bx^n)^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}} \, dx$$

input
```
integrate((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x, algorithm="maxima")
```

output
```
integrate((a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2), x)
```

Giac [F]

$$\int \frac{(ax^3 + bx^n)^{3/2}}{(cx)^{11/2}} \, dx = \int \frac{(ax^3 + bx^n)^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}} \, dx$$

input
```
integrate((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x, algorithm="giac")
```

output
```
integrate((a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^3 + bx^n)^{3/2}}{(cx)^{11/2}} \, dx = \int \frac{(b\,x^n + a\,x^3)^{3/2}}{(c\,x)^{11/2}} \, dx$$

input
```
int((b*x^n + a*x^3)^(3/2)/(c*x)^(11/2),x)
```

output
```
int((b*x^n + a*x^3)^(3/2)/(c*x)^(11/2), x)
```

Reduce [F]

$$\int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}}\,dx=\frac{\left(\int \frac{\sqrt{x^n b+a\,x^3}}{\sqrt{x}\,x^2}dx\right)a+\left(\int \frac{x^n\sqrt{x^n b+a\,x^3}}{\sqrt{x}\,x^5}dx\right)b}{\sqrt{c}\,c^5}$$

input
```
int((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x)
```

output
```
(int(sqrt(x**n*b + a*x**3)/(sqrt(x)*x**2),x)*a + int((x**n*sqrt(x**n*b + a
*x**3))/(sqrt(x)*x**5),x)*b)/(sqrt(c)*c**5)
```

3.354 $\int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4}\,dx$

Optimal result . . . 2805
Mathematica [A] (verified) . . . 2805
Rubi [A] (verified) . . . 2806
Maple [F] . . . 2807
Fricas [F(-2)] . . . 2808
Sympy [F] . . . 2808
Maxima [F] . . . 2808
Giac [F] . . . 2809
Mupad [F(-1)] . . . 2809
Reduce [F] . . . 2809

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4}\,dx = -\frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x} - \frac{2(ax^2+bx^n)^{3/2}}{3c^4(2-n)x^3} + \frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{c^4(2-n)}$$

output
```
-2*a*(a*x^2+b*x^n)^(1/2)/c^4/(2-n)/x-2/3*(a*x^2+b*x^n)^(3/2)/c^4/(2-n)/x^3
+2*a^(3/2)*arctanh(a^(1/2)*x/(a*x^2+b*x^n)^(1/2))/c^4/(2-n)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.12

$$\int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4}\,dx = \frac{2\left(4a^2x^4+b^2x^{2n}+5abx^{2+n}-3a^{3/2}\sqrt{b}x^{3+\frac{n}{2}}\sqrt{1+\frac{ax^{2-n}}{b}}\,\text{arcsinh}\left(\frac{\sqrt{a}x^{1-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{3c^4(-2+n)x^3\sqrt{ax^2+bx^n}}$$

input
```
Integrate[(a*x^2 + b*x^n)^(3/2)/(c^4*x^4),x]
```

output
```
(2*(4*a^2*x^4 + b^2*x^(2*n) + 5*a*b*x^(2 + n) - 3*a^(3/2)*Sqrt[b]*x^(3 + n
/2)*Sqrt[1 + (a*x^(2 - n))/b]*ArcSinh[(Sqrt[a]*x^(1 - n/2))/Sqrt[b]]))/(3*
c^4*(-2 + n)*x^3*Sqrt[a*x^2 + b*x^n])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {27, 1934, 1934, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4}dx$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{(bx^n+ax^2)^{3/2}}{x^4}dx}{c^4}$$
$$\Big\downarrow \text{1934}$$
$$\frac{a\int \frac{\sqrt{bx^n+ax^2}}{x^2}dx-\frac{2(ax^2+bx^n)^{3/2}}{3(2-n)x^3}}{c^4}$$
$$\Big\downarrow \text{1934}$$
$$\frac{a\left(a\int \frac{1}{\sqrt{bx^n+ax^2}}dx-\frac{2\sqrt{ax^2+bx^n}}{(2-n)x}\right)-\frac{2(ax^2+bx^n)^{3/2}}{3(2-n)x^3}}{c^4}$$
$$\Big\downarrow \text{1914}$$
$$\frac{a\left(\frac{2a\int \frac{1}{1-\frac{ax^2}{bx^n+ax^2}}d\frac{x}{\sqrt{bx^n+ax^2}}}{2-n}-\frac{2\sqrt{ax^2+bx^n}}{(2-n)x}\right)-\frac{2(ax^2+bx^n)^{3/2}}{3(2-n)x^3}}{c^4}$$
$$\Big\downarrow \text{219}$$
$$\frac{a\left(\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{2-n}-\frac{2\sqrt{ax^2+bx^n}}{(2-n)x}\right)-\frac{2(ax^2+bx^n)^{3/2}}{3(2-n)x^3}}{c^4}$$

input
```
Int[(a*x^2 + b*x^n)^(3/2)/(c^4*x^4),x]
```

output
```
((-2*(a*x^2 + b*x^n)^(3/2))/(3*(2 - n)*x^3) + a*((-2*Sqrt[a*x^2 + b*x^n])/
((2 - n)*x) + (2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^n]])/(2 - n)
))/c^4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{(a\,x^2+b\,x^n)^{\frac{3}{2}}}{c^4x^4}dx$$

input
```
int((a*x^2+b*x^n)^(3/2)/c^4/x^4,x)
```

output
```
int((a*x^2+b*x^n)^(3/2)/c^4/x^4,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\left(ax^2+bx^n\right)^{3/2}}{c^4x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x^2+b*x^n)^(3/2)/c^4/x^4,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\left(ax^2+bx^n\right)^{3/2}}{c^4x^4}\,dx = \frac{\int \frac{a\sqrt{ax^2+bx^n}}{x^2}\,dx + \int \frac{bx^n\sqrt{ax^2+bx^n}}{x^4}\,dx}{c^4}$$

input
```
integrate((a*x**2+b*x**n)**(3/2)/c**4/x**4,x)
```

output
```
(Integral(a*sqrt(a*x**2 + b*x**n)/x**2, x) + Integral(b*x**n*sqrt(a*x**2 +
 b*x**n)/x**4, x))/c**4
```

Maxima [F]

$$\int \frac{\left(ax^2+bx^n\right)^{3/2}}{c^4x^4}\,dx = \int \frac{\left(ax^2+bx^n\right)^{\frac{3}{2}}}{c^4x^4}\,dx$$

input
```
integrate((a*x^2+b*x^n)^(3/2)/c^4/x^4,x, algorithm="maxima")
```

output
```
integrate((a*x^2 + b*x^n)^(3/2)/x^4, x)/c^4
```

Giac [F]

$$\int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4}\,dx=\int \frac{(ax^2+bx^n)^{\frac{3}{2}}}{c^4x^4}\,dx$$

input
```
integrate((a*x^2+b*x^n)^(3/2)/c^4/x^4,x, algorithm="giac")
```

output
```
integrate((a*x^2 + b*x^n)^(3/2)/(c^4*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4}\,dx=\int \frac{(b\,x^n+a\,x^2)^{3/2}}{c^4\,x^4}\,dx$$

input
```
int((b*x^n + a*x^2)^(3/2)/(c^4*x^4),x)
```

output
```
int((b*x^n + a*x^2)^(3/2)/(c^4*x^4), x)
```

Reduce [F]

$$\int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4}\,dx=\frac{2x^n\sqrt{x^nb+a\,x^2}\,b+2\sqrt{x^nb+a\,x^2}\,a\,x^2+3\left(\int\frac{\sqrt{x^nb+a\,x^2}}{x^2}dx\right)an\,x^3-6\left(\int\frac{\sqrt{x^nb+a\,x^2}}{x^2}d\right.}{3c^4x^3\,(n-2)}$$

input
```
int((a*x^2+b*x^n)^(3/2)/c^4/x^4,x)
```

output
```
(2*x**n*sqrt(x**n*b + a*x**2)*b + 2*sqrt(x**n*b + a*x**2)*a*x**2 + 3*int(s
qrt(x**n*b + a*x**2)/x**2,x)*a*n*x**3 - 6*int(sqrt(x**n*b + a*x**2)/x**2,x
)*a*x**3)/(3*c**4*x**3*(n - 2))
```

3.355 $\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}\,dx$

Optimal result . . . 2811
Mathematica [A] (verified) . . . 2811
Rubi [A] (verified) . . . 2812
Maple [F] . . . 2814
Fricas [F(-2)] . . . 2814
Sympy [F] . . . 2814
Maxima [F] . . . 2815
Giac [F] . . . 2815
Mupad [F(-1)] . . . 2815
Reduce [F] . . . 2816

Optimal result

Integrand size = 21, antiderivative size = 122

$$\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}\,dx = -\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}} - \frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}} + \frac{2a^{3/2}\sqrt{x}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c^2(1-n)\sqrt{cx}}$$

output
```
-2*a*(a*x+b*x^n)^(1/2)/c^2/(1-n)/(c*x)^(1/2)-2/3*(a*x+b*x^n)^(3/2)/c/(1-n)
/(c*x)^(3/2)+2*a^(3/2)*x^(1/2)*arctanh(a^(1/2)*x^(1/2)/(a*x+b*x^n)^(1/2))/
c^2/(1-n)/(c*x)^(1/2)
```

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98

$$\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}\,dx = \frac{x\left(8a^2x^2+2b^2x^{2n}+10abx^{1+n}-6a^{3/2}\sqrt{b}x^{\frac{3+n}{2}}\sqrt{1+\frac{ax^{1-n}}{b}}\,\text{arcsinh}\left(\frac{\sqrt{a}x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{3(-1+n)(cx)^{5/2}\sqrt{ax+bx^n}}$$

input
```
Integrate[(a*x + b*x^n)^(3/2)/(c*x)^(5/2),x]
```

output
```
(x*(8*a^2*x^2 + 2*b^2*x^(2*n) + 10*a*b*x^(1 + n) - 6*a^(3/2)*Sqrt[b]*x^((3
 + n)/2)*Sqrt[1 + (a*x^(1 - n))/b]*ArcSinh[(Sqrt[a]*x^(1/2 - n/2))/Sqrt[b]
]))/(3*(-1 + n)*(c*x)^(5/2)*Sqrt[a*x + b*x^n])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1934, 1934, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}dx$$

$$\Big\downarrow \text{1934}$$

$$\frac{a\int \frac{\sqrt{bx^n+ax}}{(cx)^{3/2}}dx}{c}-\frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}}$$

$$\Big\downarrow \text{1934}$$

$$\frac{a\left(\frac{a\int \frac{1}{\sqrt{cx}\sqrt{bx^n+ax}}dx}{c}-\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}\right)}{c}-\frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{a\left(\frac{a\sqrt{x}\int \frac{1}{\sqrt{x}\sqrt{bx^n+ax}}dx}{c\sqrt{cx}}-\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}\right)}{c}-\frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{a\left(\frac{2a\sqrt{x}\int \frac{1}{1-\frac{ax}{bx^n+ax}}d\frac{\sqrt{x}}{\sqrt{bx^n+ax}}}{c(1-n)\sqrt{cx}}-\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}\right)}{c}-\frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{a\left(\frac{2\sqrt{a}\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}}-\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}\right)}{c}-\frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}}$$

input
```
Int[(a*x + b*x^n)^(3/2)/(c*x)^(5/2),x]
```

output
```
(-2*(a*x + b*x^n)^(3/2))/(3*c*(1 - n)*(c*x)^(3/2)) + (a*((-2*Sqrt[a*x + b*
x^n])/(c*(1 - n)*Sqrt[c*x]) + (2*Sqrt[a]*Sqrt[x]*ArcTanh[(Sqrt[a]*Sqrt[x])
/Sqrt[a*x + b*x^n]])/(c*(1 - n)*Sqrt[c*x])))/c
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [F]

$$\int \frac{(ax+b\,x^{n})^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}dx$$

input
```
int((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x)
```

output
```
int((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}\,dx = \int \frac{(ax+bx^n)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*x+b*x**n)**(3/2)/(c*x)**(5/2),x)
```

output
```
Integral((a*x + b*x**n)**(3/2)/(c*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{(ax + bx^n)^{3/2}}{(cx)^{5/2}}\,dx = \int \frac{(ax + bx^n)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^n)^(3/2)/(c*x)^(5/2), x)
```

Giac [F]

$$\int \frac{(ax + bx^n)^{3/2}}{(cx)^{5/2}}\,dx = \int \frac{(ax + bx^n)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*x + b*x^n)^(3/2)/(c*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax + bx^n)^{3/2}}{(cx)^{5/2}}\,dx = \int \frac{(b\,x^n + a\,x)^{3/2}}{(c\,x)^{5/2}}\,dx$$

input
```
int((b*x^n + a*x)^(3/2)/(c*x)^(5/2),x)
```

output
```
int((b*x^n + a*x)^(3/2)/(c*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}\,dx = \frac{\sqrt{c}\left(2x^{n+\frac{1}{2}}\sqrt{x^n b+ax}\,b+8\sqrt{x}\,\sqrt{x^n b+ax}\,ax+3\left(\int \frac{\sqrt{x}\,\sqrt{x^n b+ax}}{x^n bx+a\,x^2}dx\right)a^2 n\,x^2-3\left(\int \cdots\right.\right.}{3c^3x^2\,(n-1)}$$

input

```
int((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x)
```

output

```
(sqrt(c)*(2*x**((2*n + 1)/2)*sqrt(x**n*b + a*x)*b + 8*sqrt(x)*sqrt(x**n*b
+ a*x)*a*x + 3*int((sqrt(x)*sqrt(x**n*b + a*x))/(x**n*b*x + a*x**2),x)*a**
2*n*x**2 - 3*int((sqrt(x)*sqrt(x**n*b + a*x))/(x**n*b*x + a*x**2),x)*a**2*
x**2))/(3*c**3*x**2*(n - 1))
```

3.356 $\int \frac{(a+bx^n)^{3/2}}{cx} dx$

Optimal result 2817
Mathematica [A] (verified) 2817
Rubi [A] (verified) 2818
Maple [A] (verified) 2820
Fricas [A] (verification not implemented) 2820
Sympy [A] (verification not implemented) 2821
Maxima [A] (verification not implemented) 2821
Giac [F] 2821
Mupad [F(-1)] 2822
Reduce [F] 2822

Optimal result

Integrand size = 18, antiderivative size = 73

$$\int \frac{(a+bx^n)^{3/2}}{cx} dx = \frac{2a\sqrt{a+bx^n}}{cn} + \frac{2(a+bx^n)^{3/2}}{3cn} - \frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

output
```
2*a*(a+b*x^n)^(1/2)/c/n+2/3*(a+b*x^n)^(3/2)/c/n-2*a^(3/2)*arctanh((a+b*x^n
)^(1/2)/a^(1/2))/c/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^n)^{3/2}}{cx} dx = \frac{2\sqrt{a+bx^n}(4a+bx^n) - 6a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{3cn}$$

input
```
Integrate[(a + b*x^n)^(3/2)/(c*x),x]
```

output
```
(2*Sqrt[a + b*x^n]*(4*a + b*x^n) - 6*a^(3/2)*ArcTanh[Sqrt[a + b*x^n]/Sqrt[
a]])/(3*c*n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {27, 798, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^{3/2}}{cx}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(bx^n+a)^{3/2}}{x}dx}{c}$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-n}(bx^n+a)^{3/2}dx^n}{cn}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\int x^{-n}\sqrt{bx^n+a}dx^n+\frac{2}{3}(a+bx^n)^{3/2}}{cn}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\left(a\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n+2\sqrt{a+bx^n}\right)+\frac{2}{3}(a+bx^n)^{3/2}}{cn}$$

$$\Big\downarrow \text{73}$$

$$\frac{a\left(\frac{2a\int \frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{b}+2\sqrt{a+bx^n}\right)+\frac{2}{3}(a+bx^n)^{3/2}}{cn}$$

$$\Big\downarrow \text{221}$$

$$\frac{a\left(2\sqrt{a+bx^n}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+bx^n)^{3/2}}{cn}$$

input
```
Int[(a + b*x^n)^(3/2)/(c*x),x]
```

output
```
((2*(a + b*x^n)^(3/2))/3 + a*(2*Sqrt[a + b*x^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a
 + b*x^n]/Sqrt[a]]))/(c*n)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{\frac{2(a+b\,x^{n})^{\frac{3}{2}}}{3}+2a\sqrt{a+b\,x^{n}}-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{cn}$	51
default	$\frac{\frac{2(a+b\,x^{n})^{\frac{3}{2}}}{3}+2a\sqrt{a+b\,x^{n}}-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{cn}$	51
risch	$\frac{2\left(b\,\mathrm{e}^{n\ln(x)}+4a\right)\sqrt{a+b\,\mathrm{e}^{n\ln(x)}}}{3nc}-\frac{2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{e}^{n\ln(x)}}}{\sqrt{a}}\right)}{nc}$	59

input
```
int((a+b*x^n)^(3/2)/c/x,x,method=_RETURNVERBOSE)
```

output
```
1/c/n*(2/3*(a+b*x^n)^(3/2)+2*a*(a+b*x^n)^(1/2)-2*a^(3/2)*arctanh((a+b*x^n)
^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.60

$$\int \frac{(a+bx^n)^{3/2}}{cx}\,dx = \left[\frac{3\,a^{\frac{3}{2}}\log\left(\frac{bx^n-2\,\sqrt{bx^n+a}\sqrt{a}+2\,a}{x^n}\right)+2\,(bx^n+4\,a)\sqrt{bx^n+a}}{3\,cn}, \frac{2\left(3\,\sqrt{-a}a\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^n+a}}\right)\right.}{3\,cn}\right.$$

input
```
integrate((a+b*x^n)^(3/2)/c/x,x, algorithm="fricas")
```

output
```
[1/3*(3*a^(3/2)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*(b*
x^n + 4*a)*sqrt(b*x^n + a))/(c*n), 2/3*(3*sqrt(-a)*a*arctan(sqrt(-a)/sqrt(
b*x^n + a)) + (b*x^n + 4*a)*sqrt(b*x^n + a))/(c*n)]
```

Sympy [A] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^n)^{3/2}}{cx}\,dx = \frac{\frac{8a^{\frac{3}{2}}\sqrt{1+\frac{bx^n}{a}}}{3n} + \frac{a^{\frac{3}{2}}\log\left(\frac{bx^n}{a}\right)}{n} - \frac{2a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{bx^n}{a}}+1\right)}{n} + \frac{2\sqrt{a}bx^n\sqrt{1+\frac{bx^n}{a}}}{3n}}{c}$$

input
```
integrate((a+b*x**n)**(3/2)/c/x,x)
```

output
```
(8*a**(3/2)*sqrt(1 + b*x**n/a)/(3*n) + a**(3/2)*log(b*x**n/a)/n - 2*a**(3/
2)*log(sqrt(1 + b*x**n/a) + 1)/n + 2*sqrt(a)*b*x**n*sqrt(1 + b*x**n/a)/(3*
n))/c
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^n)^{3/2}}{cx}\,dx = \frac{\frac{3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{n} + \frac{2\left((bx^n+a)^{\frac{3}{2}}+3\sqrt{bx^n+a}a\right)}{n}}{3\,c}$$

input
```
integrate((a+b*x^n)^(3/2)/c/x,x, algorithm="maxima")
```

output
```
1/3*(3*a^(3/2)*log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a))
)/n + 2*((b*x^n + a)^(3/2) + 3*sqrt(b*x^n + a)*a)/n)/c
```

Giac [F]

$$\int \frac{(a+bx^n)^{3/2}}{cx}\,dx = \int \frac{(bx^n+a)^{\frac{3}{2}}}{cx}\,dx$$

input
```
integrate((a+b*x^n)^(3/2)/c/x,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(3/2)/(c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^{3/2}}{cx}\,dx=\int \frac{(a+b\,x^n)^{3/2}}{c\,x}\,dx$$

input
```
int((a + b*x^n)^(3/2)/(c*x),x)
```

output
```
int((a + b*x^n)^(3/2)/(c*x), x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^{3/2}}{cx}\,dx=\frac{2x^n\sqrt{x^nb+a}\,b+8\sqrt{x^nb+a}\,a+3\left(\int \frac{\sqrt{x^nb+a}}{x^nbx+ax}dx\right)a^2n}{3cn}$$

input
```
int((a+b*x^n)^(3/2)/c/x,x)
```

output
```
(2*x**n*sqrt(x**n*b + a)*b + 8*sqrt(x**n*b + a)*a + 3*int(sqrt(x**n*b + a)
/(x**n*b*x + a*x),x)*a**2*n)/(3*c*n)
```

3.357 $\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}\,dx$

Optimal result . . . 2823
Mathematica [A] (verified) . . . 2823
Rubi [A] (verified) . . . 2824
Maple [F] . . . 2826
Fricas [F(-2)] . . . 2826
Sympy [F] . . . 2826
Maxima [F] . . . 2827
Giac [F] . . . 2827
Mupad [F(-1)] . . . 2827
Reduce [F] . . . 2828

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}\,dx=\frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{1+n}+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)}-\frac{2a^{3/2}c\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(1+n)\sqrt{cx}}$$

output
```
2*a*(c*x)^(1/2)*(a/x+b*x^n)^(1/2)/(1+n)+2/3*(c*x)^(3/2)*(a/x+b*x^n)^(3/2)/
c/(1+n)-2*a^(3/2)*c*x^(1/2)*arctanh(a^(1/2)/x^(1/2)/(a/x+b*x^n)^(1/2))/(1+
n)/(c*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.83

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}\,dx=\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}\left(\sqrt{a+bx^{1+n}}(4a+bx^{1+n})-3a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^{1+n}}}{\sqrt{a}}\right)\right)}{3(1+n)\sqrt{a+bx^{1+n}}}$$

input
```
Integrate[Sqrt[c*x]*(a/x + b*x^n)^(3/2),x]
```

output
```
(2*Sqrt[c*x]*Sqrt[a/x + b*x^n]*(Sqrt[a + b*x^(1 + n)]*(4*a + b*x^(1 + n))
- 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(1 + n)]/Sqrt[a]]))/(3*(1 + n)*Sqrt[a + b
*x^(1 + n)])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1934, 1934, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}dx$$

$$\Big\downarrow \text{1934}$$

$$ac\int \frac{\sqrt{bx^n+\frac{a}{x}}}{\sqrt{cx}}dx+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(n+1)}$$

$$\Big\downarrow \text{1934}$$

$$ac\left(ac\int \frac{1}{(cx)^{3/2}\sqrt{bx^n+\frac{a}{x}}}dx+\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(n+1)}\right)+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(n+1)}$$

$$\Big\downarrow \text{1937}$$

$$ac\left(\frac{a\sqrt{x}\int \frac{1}{x^{3/2}\sqrt{bx^n+\frac{a}{x}}}dx}{\sqrt{cx}}+\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(n+1)}\right)+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(n+1)}$$

$$\Big\downarrow \text{1935}$$

$$ac\left(\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(n+1)}-\frac{2a\sqrt{x}\int \frac{1}{1-\frac{a}{x\left(bx^n+\frac{a}{x}\right)}}d\frac{1}{\sqrt{x}\sqrt{bx^n+\frac{a}{x}}}}{(n+1)\sqrt{cx}}\right)+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(n+1)}$$

$$\Big\downarrow \text{219}$$

$$ac\left(\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(n+1)}-\frac{2\sqrt{a}\sqrt{x}\text{arctanh}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(n+1)\sqrt{cx}}\right)+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(n+1)}$$

input
```
Int[Sqrt[c*x]*(a/x + b*x^n)^(3/2),x]
```

output
```
(2*(c*x)^(3/2)*(a/x + b*x^n)^(3/2))/(3*c*(1 + n)) + a*c*((2*Sqrt[c*x]*Sqrt
[a/x + b*x^n])/(c*(1 + n)) - (2*Sqrt[a]*Sqrt[x]*ArcTanh[Sqrt[a]/(Sqrt[x]*S
qrt[a/x + b*x^n])])/((1 + n)*Sqrt[c*x]))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [F]

$$\int \sqrt{cx}\left(\frac{a}{x}+b\,x^{n}\right)^{\frac{3}{2}}dx$$

input
```
int((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x)
```

output
```
int((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}dx = \int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{\frac{3}{2}}dx$$

input
```
integrate((c*x)**(1/2)*(a/x+b*x**n)**(3/2),x)
```

output
```
Integral(sqrt(c*x)*(a/x + b*x**n)**(3/2), x)
```

Maxima [F]

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}\,dx=\int \left(bx^n+\frac{a}{x}\right)^{\frac{3}{2}}\sqrt{cx}\,dx$$

input
```
integrate((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a/x)^(3/2)*sqrt(c*x), x)
```

Giac [F]

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}\,dx=\int \left(bx^n+\frac{a}{x}\right)^{\frac{3}{2}}\sqrt{cx}\,dx$$

input
```
integrate((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^n + a/x)^(3/2)*sqrt(c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}\,dx=\int \sqrt{c\,x}\,\left(b\,x^n+\frac{a}{x}\right)^{3/2}\,dx$$

input
```
int((c*x)^(1/2)*(b*x^n + a/x)^(3/2),x)
```

output
```
int((c*x)^(1/2)*(b*x^n + a/x)^(3/2), x)
```

Reduce [F]

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2} dx = \frac{\sqrt{c}\left(2x^n\sqrt{x^nbx+a}\,bx+8\sqrt{x^nbx+a}\,a+3\left(\int \frac{\sqrt{x^nbx+a}}{x^nb\,x^2+ax}dx\right)a^2n+3\left(\int \frac{\sqrt{x^nbx+a}}{x^nb\,x^2+ax}dx\right)a^2\right)}{3n+3}$$

input
```
int((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x)
```

output
```
(sqrt(c)*(2*x**n*sqrt(x**n*b*x + a)*b*x + 8*sqrt(x**n*b*x + a)*a + 3*int(s
qrt(x**n*b*x + a)/(x**n*b*x**2 + a*x),x)*a**2*n + 3*int(sqrt(x**n*b*x + a)
/(x**n*b*x**2 + a*x),x)*a**2))/(3*(n + 1))
```

3.358 $\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}\,dx$

Optimal result . . . 2829
Mathematica [A] (verified) . . . 2829
Rubi [A] (verified) . . . 2830
Maple [F] . . . 2832
Fricas [F(-2)] . . . 2832
Sympy [F] . . . 2832
Maxima [F] . . . 2833
Giac [F] . . . 2833
Mupad [F(-1)] . . . 2833
Reduce [F] . . . 2834

Optimal result

Integrand size = 22, antiderivative size = 98

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}\,dx=\frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}+\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(2+n)}-\frac{2a^{3/2}c^2\operatorname{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}$$

output
```
2*a*c^2*x*(a/x^2+b*x^n)^(1/2)/(2+n)+2*c^2*x^3*(a/x^2+b*x^n)^(3/2)/(6+3*n)-
2*a^(3/2)*c^2*arctanh(a^(1/2)/x/(a/x^2+b*x^n)^(1/2))/(2+n)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.96

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}\,dx=\frac{2c^2x\sqrt{\frac{a}{x^2}+bx^n}\left(\sqrt{a+bx^{2+n}}(4a+bx^{2+n})-3a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^{2+n}}}{\sqrt{a}}\right)\right)}{3(2+n)\sqrt{a+bx^{2+n}}}$$

input
```
Integrate[c^2*x^2*(a/x^2 + b*x^n)^(3/2),x]
```

output
```
(2*c^2*x*Sqrt[a/x^2 + b*x^n]*(Sqrt[a + b*x^(2 + n)]*(4*a + b*x^(2 + n)) -
3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(2 + n)]/Sqrt[a]]))/(3*(2 + n)*Sqrt[a + b*x
^(2 + n)])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {27, 1934, 1913, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int x^2\left(bx^n+\frac{a}{x^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{1934}$$

$$c^2\left(a\int \sqrt{bx^n+\frac{a}{x^2}}dx+\frac{2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(n+2)}\right)$$

$$\Big\downarrow \text{1913}$$

$$c^2\left(a\left(a\int \frac{1}{x^2\sqrt{bx^n+\frac{a}{x^2}}}dx+\frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}\right)+\frac{2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(n+2)}\right)$$

$$\Big\downarrow \text{1935}$$

$$c^2\left(a\left(\frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}-\frac{2a\int \frac{1}{1-\frac{a}{x^2\left(bx^n+\frac{a}{x^2}\right)}}d\frac{1}{x\sqrt{bx^n+\frac{a}{x^2}}}}{n+2}\right)+\frac{2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(n+2)}\right)$$

$$\Big\downarrow \text{219}$$

$$c^2\left(a\left(\frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{n+2}\right)+\frac{2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(n+2)}\right)$$

input
```
Int[c^2*x^2*(a/x^2 + b*x^n)^(3/2),x]
```

output
```
c^2*((2*x^3*(a/x^2 + b*x^n)^(3/2))/(3*(2 + n)) + a*((2*x*Sqrt[a/x^2 + b*x^
n])/(2 + n) - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x^n])])/(2 + n)
))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1913
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Simp[a   Int[x^j*(a*x^j + b*x^n)^(p - 1), x]
, x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simp
lify[j*p + 1], 0]
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [F]

$$\int c^2x^2\left(\frac{a}{x^2}+b\,x^n\right)^{\frac{3}{2}}dx$$

input
```
int(c^2*x^2*(a/x^2+b*x^n)^(3/2),x)
```

output
```
int(c^2*x^2*(a/x^2+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}dx=\text{Exception raised: TypeError}$$

input
```
integrate(c^2*x^2*(a/x^2+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}dx=c^2\left(\int a\sqrt{\frac{a}{x^2}+bx^n}\,dx+\int bx^2x^n\sqrt{\frac{a}{x^2}+bx^n}\,dx\right)$$

input
```
integrate(c**2*x**2*(a/x**2+b*x**n)**(3/2),x)
```

output
```
c**2*(Integral(a*sqrt(a/x**2 + b*x**n), x) + Integral(b*x**2*x**n*sqrt(a/x
**2 + b*x**n), x))
```

Maxima [F]

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}\,dx=\int\left(bx^n+\frac{a}{x^2}\right)^{\frac{3}{2}}c^2x^2\,dx$$

input
```
integrate(c^2*x^2*(a/x^2+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
c^2*integrate((b*x^n + a/x^2)^(3/2)*x^2, x)
```

Giac [F]

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}\,dx=\int\left(bx^n+\frac{a}{x^2}\right)^{\frac{3}{2}}c^2x^2\,dx$$

input
```
integrate(c^2*x^2*(a/x^2+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^n + a/x^2)^(3/2)*c^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}\,dx=\int c^2\,x^2\,\left(b\,x^n+\frac{a}{x^2}\right)^{3/2}\,dx$$

input
```
int(c^2*x^2*(b*x^n + a/x^2)^(3/2),x)
```

output
```
int(c^2*x^2*(b*x^n + a/x^2)^(3/2), x)
```

Reduce [F]

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}\,dx=c^2\left(\left(\int\frac{\sqrt{x^nb\,x^2+a}}{x}dx\right)a+\left(\int x^n\sqrt{x^nb\,x^2+a}\,xdx\right)b\right)$$

input
```
int(c^2*x^2*(a/x^2+b*x^n)^(3/2),x)
```

output
```
c**2*(int(sqrt(x**n*b*x**2 + a)/x,x)*a + int(x**n*sqrt(x**n*b*x**2 + a)*x,
x)*b)
```

3.359 $\int (cx)^{7/2} \left(\frac{a}{x^3}+bx^n\right)^{3/2} dx$

Optimal result 2835
Mathematica [A] (verified) 2835
Rubi [A] (verified) 2836
Maple [F] 2838
Fricas [F(-2)] 2838
Sympy [F(-1)] 2838
Maxima [F] 2839
Giac [F] 2839
Mupad [F(-1)] 2839
Reduce [F] 2840

Optimal result

Integrand size = 23, antiderivative size = 122

$$\int (cx)^{7/2} \left(\frac{a}{x^3}+bx^n\right)^{3/2} dx = \frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{3+n} + \frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)} - \frac{2a^{3/2}c^4\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(3+n)\sqrt{cx}}$$

output
```
2*a*c^2*(c*x)^(3/2)*(a/x^3+b*x^n)^(1/2)/(3+n)+2/3*(c*x)^(9/2)*(a/x^3+b*x^n
)^(3/2)/c/(3+n)-2*a^(3/2)*c^4*x^(1/2)*arctanh(a^(1/2)/x^(3/2)/(a/x^3+b*x^n
)^(1/2))/(3+n)/(c*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.82

$$\int (cx)^{7/2} \left(\frac{a}{x^3}+bx^n\right)^{3/2} dx = \frac{2c^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}\left(\sqrt{a+bx^{3+n}}(4a+bx^{3+n})-3a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^{3+n}}}{\sqrt{a}}\right)\right)}{3(3+n)\sqrt{a+bx^{3+n}}}$$

input
```
Integrate[(c*x)^(7/2)*(a/x^3 + b*x^n)^(3/2),x]
```

output
```
(2*c^2*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n]*(Sqrt[a + b*x^(3 + n)]*(4*a + b*x^(
3 + n)) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(3 + n)]/Sqrt[a]]))/(3*(3 + n)*Sq
rt[a + b*x^(3 + n)])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1934, 1934, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{7/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}dx$$

$$\Big\downarrow \text{1934}$$

$$ac^3\int \sqrt{cx}\sqrt{bx^n+\frac{a}{x^3}}dx+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(n+3)}$$

$$\Big\downarrow \text{1934}$$

$$ac^3\left(ac^3\int \frac{1}{(cx)^{5/2}\sqrt{bx^n+\frac{a}{x^3}}}dx+\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(n+3)}\right)+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(n+3)}$$

$$\Big\downarrow \text{1937}$$

$$ac^3\left(\frac{ac\sqrt{x}\int \frac{1}{x^{5/2}\sqrt{bx^n+\frac{a}{x^3}}}dx}{\sqrt{cx}}+\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(n+3)}\right)+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(n+3)}$$

$$\Big\downarrow \text{1935}$$

$$ac^3\left(\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(n+3)}-\frac{2ac\sqrt{x}\int \frac{1}{1-\frac{a}{x^3\left(bx^n+\frac{a}{x^3}\right)}}d\frac{1}{x^{3/2}\sqrt{bx^n+\frac{a}{x^3}}}}{(n+3)\sqrt{cx}}\right)+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(n+3)}$$

$$\Big\downarrow \text{219}$$

$$ac^3\left(\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(n+3)}-\frac{2\sqrt{a}c\sqrt{x}\,\text{arctanh}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(n+3)\sqrt{cx}}\right)+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(n+3)}$$

input
```
Int[(c*x)^(7/2)*(a/x^3 + b*x^n)^(3/2),x]
```

output
```
(2*(c*x)^(9/2)*(a/x^3 + b*x^n)^(3/2))/(3*c*(3 + n)) + a*c^3*((2*(c*x)^(3/2
)*Sqrt[a/x^3 + b*x^n])/(c*(3 + n)) - (2*Sqrt[a]*c*Sqrt[x]*ArcTanh[Sqrt[a]/
(x^(3/2)*Sqrt[a/x^3 + b*x^n])])/((3 + n)*Sqrt[c*x]))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [F]

$$\int (cx)^{\frac{7}{2}} \left(\frac{a}{x^3}+b\,x^n\right)^{\frac{3}{2}} dx$$

input
```
int((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x)
```

output
```
int((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (cx)^{7/2} \left(\frac{a}{x^3}+bx^n\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int (cx)^{7/2} \left(\frac{a}{x^3}+bx^n\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate((c*x)**(7/2)*(a/x**3+b*x**n)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (cx)^{7/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}\,dx=\int \left(bx^n+\frac{a}{x^3}\right)^{\frac{3}{2}}(cx)^{\frac{7}{2}}\,dx$$

input
```
integrate((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a/x^3)^(3/2)*(c*x)^(7/2), x)
```

Giac [F]

$$\int (cx)^{7/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}\,dx=\int \left(bx^n+\frac{a}{x^3}\right)^{\frac{3}{2}}(cx)^{\frac{7}{2}}\,dx$$

input
```
integrate((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^n + a/x^3)^(3/2)*(c*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{7/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}\,dx=\int (c\,x)^{7/2}\left(b\,x^n+\frac{a}{x^3}\right)^{3/2}\,dx$$

input
```
int((c*x)^(7/2)*(b*x^n + a/x^3)^(3/2),x)
```

output
```
int((c*x)^(7/2)*(b*x^n + a/x^3)^(3/2), x)
```

Reduce [F]

$$\int (cx)^{7/2} \left(\frac{a}{x^3} + bx^n\right)^{3/2} dx = \sqrt{c}\, c^3 \left(\left(\int \frac{\sqrt{x^n b\, x^3 + a}}{x} dx \right) a + \left(\int x^n \sqrt{x^n b\, x^3 + a}\, x^2 dx \right) b \right)$$

input
```
int((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x)
```

output
```
sqrt(c)*c**3*(int(sqrt(x**n*b*x**3 + a)/x,x)*a + int(x**n*sqrt(x**n*b*x**3
 + a)*x**2,x)*b)
```

3.360 $\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx$

Optimal result . . . 2841
Mathematica [A] (verified) . . . 2841
Rubi [A] (verified) . . . 2842
Maple [F] . . . 2844
Fricas [F(-2)] . . . 2844
Sympy [F] . . . 2844
Maxima [F] . . . 2845
Giac [F] . . . 2845
Mupad [F(-1)] . . . 2845
Reduce [F] . . . 2846

Optimal result

Integrand size = 22, antiderivative size = 100

$$\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx=\frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{4+n}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(4+n)}-\frac{2a^{3/2}c^5\text{arctanh}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{4+n}$$

output
```
2*a*c^5*x^2*(a/x^4+b*x^n)^(1/2)/(4+n)+2*c^5*x^6*(a/x^4+b*x^n)^(3/2)/(12+3*
n)-2*a^(3/2)*c^5*arctanh(a^(1/2)/x^2/(a/x^4+b*x^n)^(1/2))/(4+n)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96

$$\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx=\frac{2c^5x^2\sqrt{\frac{a}{x^4}+bx^n}\left(\sqrt{a+bx^{4+n}}(4a+bx^{4+n})-3a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^{4+n}}}{\sqrt{a}}\right)\right)}{3(4+n)\sqrt{a+bx^{4+n}}}$$

input
```
Integrate[c^5*x^5*(a/x^4 + b*x^n)^(3/2),x]
```

output
```
(2*c^5*x^2*Sqrt[a/x^4 + b*x^n]*(Sqrt[a + b*x^(4 + n)]*(4*a + b*x^(4 + n))
- 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(4 + n)]/Sqrt[a]]))/(3*(4 + n)*Sqrt[a + b
*x^(4 + n)])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {27, 1934, 1934, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int c^5 x^5 \left(\frac{a}{x^4}+bx^n\right)^{3/2} dx$$

$$\Big\downarrow \text{27}$$

$$c^5 \int x^5 \left(bx^n+\frac{a}{x^4}\right)^{3/2} dx$$

$$\Big\downarrow \text{1934}$$

$$c^5 \left(a \int x\sqrt{bx^n+\frac{a}{x^4}}dx+\frac{2x^6 \left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(n+4)}\right)$$

$$\Big\downarrow \text{1934}$$

$$c^5 \left(a \left(a \int \frac{1}{x^3\sqrt{bx^n+\frac{a}{x^4}}}dx+\frac{2x^2\sqrt{\frac{a}{x^4}+bx^n}}{n+4}\right)+\frac{2x^6 \left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(n+4)}\right)$$

$$\Big\downarrow \text{1935}$$

$$c^5 \left(a \left(\frac{2x^2\sqrt{\frac{a}{x^4}+bx^n}}{n+4}-\frac{2a\int \frac{1}{1-\frac{a}{x^4\left(bx^n+\frac{a}{x^4}\right)}}d\frac{1}{x^2\sqrt{bx^n+\frac{a}{x^4}}}}{n+4}\right)+\frac{2x^6 \left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(n+4)}\right)$$

$$\Big\downarrow \text{219}$$

$$c^5\left(a\left(\frac{2x^2\sqrt{\frac{a}{x^4}+bx^n}}{n+4}-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{n+4}\right)+\frac{2x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(n+4)}\right)$$

input
```
Int[c^5*x^5*(a/x^4 + b*x^n)^(3/2),x]
```

output
```
c^5*((2*x^6*(a/x^4 + b*x^n)^(3/2))/(3*(4 + n)) + a*((2*x^2*Sqrt[a/x^4 + b*
x^n])/(4 + n) - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x^2*Sqrt[a/x^4 + b*x^n])])/(4
+ n)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [F]

$$\int c^5x^5\left(\frac{a}{x^4}+b\,x^n\right)^{\frac{3}{2}}dx$$

input
```
int(c^5*x^5*(a/x^4+b*x^n)^(3/2),x)
```

output
```
int(c^5*x^5*(a/x^4+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(c^5*x^5*(a/x^4+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx=c^5\left(\int ax\sqrt{\frac{a}{x^4}+bx^n}\,dx+\int bx^5x^n\sqrt{\frac{a}{x^4}+bx^n}\,dx\right)$$

input
```
integrate(c**5*x**5*(a/x**4+b*x**n)**(3/2),x)
```

output
```
c**5*(Integral(a*x*sqrt(a/x**4 + b*x**n), x) + Integral(b*x**5*x**n*sqrt(a
/x**4 + b*x**n), x))
```

Maxima [F]

$$\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx=\int \left(bx^n+\frac{a}{x^4}\right)^{\frac{3}{2}}c^5x^5\,dx$$

input
```
integrate(c^5*x^5*(a/x^4+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
c^5*integrate((b*x^n + a/x^4)^(3/2)*x^5, x)
```

Giac [F]

$$\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx=\int \left(bx^n+\frac{a}{x^4}\right)^{\frac{3}{2}}c^5x^5\,dx$$

input
```
integrate(c^5*x^5*(a/x^4+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^n + a/x^4)^(3/2)*c^5*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx=\int c^5\,x^5\,\left(b\,x^n+\frac{a}{x^4}\right)^{3/2}\,dx$$

input
```
int(c^5*x^5*(b*x^n + a/x^4)^(3/2),x)
```

output
```
int(c^5*x^5*(b*x^n + a/x^4)^(3/2), x)
```

Reduce [F]

$$\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx = c^5\left(\left(\int\frac{\sqrt{x^nb\,x^4+a}}{x}dx\right)a+\left(\int x^n\sqrt{x^nb\,x^4+a}\,x^3dx\right)b\right)$$

input
```
int(c^5*x^5*(a/x^4+b*x^n)^(3/2),x)
```

output
```
c**5*(int(sqrt(x**n*b*x**4 + a)/x,x)*a + int(x**n*sqrt(x**n*b*x**4 + a)*x*
*3,x)*b)
```

3.361 $\int \sqrt{\frac{a+bx}{x^2}}\,dx$

Optimal result . . . 2847
Mathematica [A] (verified) . . . 2847
Rubi [A] (verified) . . . 2848
Maple [A] (verified) . . . 2849
Fricas [A] (verification not implemented) . . . 2850
Sympy [F] . . . 2850
Maxima [F] . . . 2851
Giac [A] (verification not implemented) . . . 2851
Mupad [B] (verification not implemented) . . . 2851
Reduce [B] (verification not implemented) . . . 2852

Optimal result

Integrand size = 13, antiderivative size = 45

$$\int \sqrt{\frac{a+bx}{x^2}}\,dx = 2x\sqrt{\frac{a+bx}{x^2}} - 2\sqrt{a}\operatorname{arctanh}\left(\frac{x\sqrt{\frac{a+bx}{x^2}}}{\sqrt{a}}\right)$$

output
```
2*x*((b*x+a)/x^2)^(1/2)-2*a^(1/2)*arctanh(x*((b*x+a)/x^2)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int \sqrt{\frac{a+bx}{x^2}}\,dx = \frac{2x\sqrt{\frac{a+bx}{x^2}}\left(\sqrt{a+bx}-\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{a+bx}}$$

input
```
Integrate[Sqrt[(a + b*x)/x^2],x]
```

output
```
(2*x*Sqrt[(a + b*x)/x^2]*(Sqrt[a + b*x] - Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sq
rt[a]]))/Sqrt[a + b*x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2078, 1913, 1919, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{a+bx}{x^2}}dx$$

$$\Big\downarrow \text{2078}$$

$$\int \sqrt{\frac{a}{x^2}+\frac{b}{x}}dx$$

$$\Big\downarrow \text{1913}$$

$$a\int \frac{1}{\sqrt{\frac{a}{x^2}+\frac{b}{x}}x^2}dx+2x\sqrt{\frac{a}{x^2}+\frac{b}{x}}$$

$$\Big\downarrow \text{1919}$$

$$2x\sqrt{\frac{a}{x^2}+\frac{b}{x}}-a\int \frac{1}{\sqrt{\frac{a}{x^2}+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{1091}$$

$$2x\sqrt{\frac{a}{x^2}+\frac{b}{x}}-2a\int \frac{1}{1-\frac{a}{x^2}}d\frac{1}{\sqrt{\frac{a}{x^2}+\frac{b}{x}}x}$$

$$\Big\downarrow \text{219}$$

$$2x\sqrt{\frac{a}{x^2}+\frac{b}{x}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+\frac{b}{x}}}\right)$$

input
```
Int[Sqrt[(a + b*x)/x^2],x]
```

output
```
2*Sqrt[a/x^2 + b/x]*x - 2*Sqrt[a]*ArcTanh[Sqrt[a]/(Sqrt[a/x^2 + b/x]*x)]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1913
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Simp[a   Int[x^j*(a*x^j + b*x^n)^(p - 1), x]
, x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simp
lify[j*p + 1], 0]
```

rule 1919
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
 && EqQ[Simplify[m - n + 1], 0]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07

method	result	size
default	$-\frac{2\sqrt{\frac{bx+a}{x^{2}}}\, x \left(\sqrt{a}\, \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-\sqrt{bx+a}\right)}{\sqrt{bx+a}}$	48

input
```
int(((b*x+a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((b*x+a)/x^2)^(1/2)*x*(a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))-(b*x+a)^(
1/2))/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.16

$$\int \sqrt{\frac{a+bx}{x^2}}\,dx = \left[2\,x\sqrt{\frac{bx+a}{x^2}}+\sqrt{a}\log\left(\frac{bx-2\,\sqrt{a}x\sqrt{\frac{bx+a}{x^2}}+2\,a}{x}\right),2\,x\sqrt{\frac{bx+a}{x^2}} + 2\,\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{bx+a}{x^2}}}{bx+a}\right)\right]$$

input
```
integrate(((b*x+a)/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[2*x*sqrt((b*x + a)/x^2) + sqrt(a)*log((b*x - 2*sqrt(a)*x*sqrt((b*x + a)/x
^2) + 2*a)/x), 2*x*sqrt((b*x + a)/x^2) + 2*sqrt(-a)*arctan(sqrt(-a)*x*sqrt
((b*x + a)/x^2)/(b*x + a))]
```

Sympy [F]

$$\int \sqrt{\frac{a+bx}{x^2}}\,dx = \int \sqrt{\frac{a+bx}{x^2}}\,dx$$

input
```
integrate(((b*x+a)/x**2)**(1/2),x)
```

output
```
Integral(sqrt((a + b*x)/x**2), x)
```

Maxima [F]

$$\int \sqrt{\frac{a+bx}{x^2}}\,dx = \int \sqrt{\frac{bx+a}{x^2}}\,dx$$

input
```
integrate(((b*x+a)/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x + a)/x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.49

$$\int \sqrt{\frac{a+bx}{x^2}}\,dx = \frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}} + 2\sqrt{bx+a}\mathrm{sgn}(x) - \frac{2\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}\right)\mathrm{sgn}(x)}{\sqrt{-a}}$$

input
```
integrate(((b*x+a)/x^2)^(1/2),x, algorithm="giac")
```

output
```
2*a*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*sqrt(b*x + a)*sgn(x
) - 2*(a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)
```

Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.49

$$\int \sqrt{\frac{a+bx}{x^2}}\,dx = 2\,x\sqrt{\frac{a}{x^2}+\frac{b}{x}} + \frac{\sqrt{a}\,\sqrt{x}\,\mathrm{asin}\left(\frac{\sqrt{a}\,1\mathrm{i}}{\sqrt{b}\,\sqrt{x}}\right)\,\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,2\mathrm{i}}{\sqrt{b}\,\sqrt{\frac{a}{b\,x}+1}}$$

input
```
int(((a + b*x)/x^2)^(1/2),x)
```

output
```
2*x*(a/x^2 + b/x)^(1/2) + (a^(1/2)*x^(1/2)*asin((a^(1/2)*1i)/(b^(1/2)*x^(1
/2)))*(a/x^2 + b/x)^(1/2)*2i)/(b^(1/2)*(a/(b*x) + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \sqrt{\frac{a+bx}{x^2}}\,dx = 2\sqrt{bx+a}+\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)-\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)$$

input
```
int(((b*x+a)/x^2)^(1/2),x)
```

output
```
2*sqrt(a + b*x) + sqrt(a)*log(sqrt(a + b*x) - sqrt(a)) - sqrt(a)*log(sqrt(
a + b*x) + sqrt(a))
```

3.362 $\int \sqrt{\frac{a+bx^2}{x^2}}\,dx$

Optimal result . . . 2853
Mathematica [A] (verified) . . . 2853
Rubi [A] (verified) . . . 2854
Maple [A] (verified) . . . 2855
Fricas [A] (verification not implemented) . . . 2856
Sympy [F] . . . 2856
Maxima [A] (verification not implemented) . . . 2857
Giac [B] (verification not implemented) . . . 2857
Mupad [B] (verification not implemented) . . . 2858
Reduce [B] (verification not implemented) . . . 2858

Optimal result

Integrand size = 15, antiderivative size = 42

$$\int \sqrt{\frac{a+bx^2}{x^2}}\,dx = \sqrt{b+\frac{a}{x^2}}x - \sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}}{\sqrt{b+\frac{a}{x^2}}x}\right)$$

output
```
(b+a/x^2)^(1/2)*x-a^(1/2)*arctanh(a^(1/2)/(b+a/x^2)^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.48

$$\int \sqrt{\frac{a+bx^2}{x^2}}\,dx = \sqrt{b+\frac{a}{x^2}}x - \frac{\sqrt{a}\sqrt{b+\frac{a}{x^2}}x\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a+bx^2}}$$

input
```
Integrate[Sqrt[(a + b*x^2)/x^2],x]
```

output
```
Sqrt[b + a/x^2]*x - (Sqrt[a]*Sqrt[b + a/x^2]*x*ArcTanh[Sqrt[a + b*x^2]/Sqr
t[a]])/Sqrt[a + b*x^2]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2072, 773, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{a+bx^2}{x^2}}dx$$
$$\Big\downarrow \text{ 2072}$$
$$\int \sqrt{\frac{a}{x^2}+b}dx$$
$$\Big\downarrow \text{ 773}$$
$$-\int \sqrt{\frac{a}{x^2}+bx^2}d\frac{1}{x}$$
$$\Big\downarrow \text{ 247}$$
$$x\sqrt{\frac{a}{x^2}+b}-a\int \frac{1}{\sqrt{\frac{a}{x^2}+b}}d\frac{1}{x}$$
$$\Big\downarrow \text{ 224}$$
$$x\sqrt{\frac{a}{x^2}+b}-a\int \frac{1}{1-\frac{a}{x^2}}d\frac{1}{\sqrt{\frac{a}{x^2}+b}x}$$
$$\Big\downarrow \text{ 219}$$
$$x\sqrt{\frac{a}{x^2}+b}-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+b}}\right)$$

input
```
Int[Sqrt[(a + b*x^2)/x^2],x]
```

output
```
Sqrt[b + a/x^2]*x - Sqrt[a]*ArcTanh[Sqrt[a]/(Sqrt[b + a/x^2]*x)]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 2072
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45

method	result	size
default	$\frac{\sqrt{\frac{b\,x^{2}+a}{x^{2}}}\, x \left(\sqrt{b\,x^{2}+a}-\sqrt{a}\, \ln\left(\frac{2 a+2 \sqrt{a}\, \sqrt{b\,x^{2}+a}}{x}\right)\right)}{\sqrt{b\,x^{2}+a}}$	61

input
```
int(((b*x^2+a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)/x^2)^(1/2)*x/(b*x^2+a)^(1/2)*((b*x^2+a)^(1/2)-a^(1/2)*ln(2*(a^(
1/2)*(b*x^2+a)^(1/2)+a)/x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.43

$$\int \sqrt{\frac{a+bx^2}{x^2}}\,dx = \left[x\sqrt{\frac{bx^2+a}{x^2}} + \frac{1}{2}\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{a}x\sqrt{\frac{bx^2+a}{x^2}}+2a}{x^2}\right), x\sqrt{\frac{bx^2+a}{x^2}} + \sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{bx^2+a}{x^2}}}{a}\right)\right]$$

input
```
integrate(((b*x^2+a)/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[x*sqrt((b*x^2 + a)/x^2) + 1/2*sqrt(a)*log(-(b*x^2 - 2*sqrt(a)*x*sqrt((b*x
^2 + a)/x^2) + 2*a)/x^2), x*sqrt((b*x^2 + a)/x^2) + sqrt(-a)*arctan(sqrt(-
a)*x*sqrt((b*x^2 + a)/x^2)/a)]
```

Sympy [F]

$$\int \sqrt{\frac{a+bx^2}{x^2}}\,dx = \int \sqrt{\frac{a+bx^2}{x^2}}\,dx$$

input
```
integrate(((b*x**2+a)/x**2)**(1/2),x)
```

output
```
Integral(sqrt((a + b*x**2)/x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int \sqrt{\frac{a+bx^2}{x^2}}\,dx = \sqrt{b+\frac{a}{x^2}}x + \frac{1}{2}\sqrt{a}\log\left(\frac{\sqrt{b+\frac{a}{x^2}}x-\sqrt{a}}{\sqrt{b+\frac{a}{x^2}}x+\sqrt{a}}\right)$$

input
```
integrate(((b*x^2+a)/x^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(b + a/x^2)*x + 1/2*sqrt(a)*log((sqrt(b + a/x^2)*x - sqrt(a))/(sqrt(b
+ a/x^2)*x + sqrt(a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(34) = 68.

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.64

$$\int \sqrt{\frac{a+bx^2}{x^2}}\,dx = \frac{a\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}} + \sqrt{bx^2+a}\operatorname{sgn}(x) - \frac{\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}\right)\operatorname{sgn}(x)}{\sqrt{-a}}$$

input
```
integrate(((b*x^2+a)/x^2)^(1/2),x, algorithm="giac")
```

output
```
a*arctan(sqrt(b*x^2 + a)/sqrt(-a))*sgn(x)/sqrt(-a) + sqrt(b*x^2 + a)*sgn(x
) - (a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)
```

Mupad [B] (verification not implemented)

Time = 9.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.31

$$\int \sqrt{\frac{a+bx^2}{x^2}}\,dx = x\,\sqrt{b+\frac{a}{x^2}}+\frac{\sqrt{a}\,\mathrm{asin}\left(\frac{\sqrt{a}\,1\mathrm{i}}{\sqrt{b}\,x}\right)\,\sqrt{b+\frac{a}{x^2}}\,1\mathrm{i}}{\sqrt{b}\,\sqrt{\frac{a}{b\,x^2}+1}}$$

input
```
int(((a + b*x^2)/x^2)^(1/2),x)
```

output
```
x*(b + a/x^2)^(1/2) + (a^(1/2)*asin((a^(1/2)*1i)/(b^(1/2)*x))*(b + a/x^2)^
(1/2)*1i)/(b^(1/2)*(a/(b*x^2) + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.43

$$\int \sqrt{\frac{a+bx^2}{x^2}}\,dx = \sqrt{b\,x^2+a}+\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right) - \sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)$$

input
```
int(((b*x^2+a)/x^2)^(1/2),x)
```

output
```
sqrt(a + b*x**2) + sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sq
rt(a)) - sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))
```

3.363 $\int \sqrt{\frac{a+bx^3}{x^2}}\,dx$

Optimal result 2859
Mathematica [A] (verified) 2859
Rubi [A] (verified) 2860
Maple [A] (verified) 2861
Fricas [A] (verification not implemented) 2862
Sympy [F(-1)] 2862
Maxima [F] 2863
Giac [A] (verification not implemented) 2863
Mupad [B] (verification not implemented) 2863
Reduce [B] (verification not implemented) 2864

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \sqrt{\frac{a+bx^3}{x^2}}\,dx=\frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}-\frac{2}{3}\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx}}\right)$$

output
```
2/3*x*(a/x^2+b*x)^(1/2)-2/3*a^(1/2)*arctanh(a^(1/2)/x/(a/x^2+b*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \sqrt{\frac{a+bx^3}{x^2}}\,dx=\frac{2x\sqrt{\frac{a}{x^2}+bx}\left(\sqrt{a+bx^3}-\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)}{3\sqrt{a+bx^3}}$$

input
```
Integrate[Sqrt[(a + b*x^3)/x^2],x]
```

output
```
(2*x*Sqrt[a/x^2 + b*x]*(Sqrt[a + b*x^3] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^3]/
Sqrt[a]]))/(3*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2078, 1913, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{a+bx^3}{x^2}}dx$$
$$\Big\downarrow \text{ 2078}$$
$$\int \sqrt{\frac{a}{x^2}+bx}dx$$
$$\Big\downarrow \text{ 1913}$$
$$a\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx}}dx+\frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}$$
$$\Big\downarrow \text{ 1935}$$
$$\frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}-\frac{2}{3}a\int \frac{1}{1-\frac{a}{x^2\left(\frac{a}{x^2}+bx\right)}}d\frac{1}{x\sqrt{\frac{a}{x^2}+bx}}$$
$$\Big\downarrow \text{ 219}$$
$$\frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}-\frac{2}{3}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx}}\right)$$

input
```
Int[Sqrt[(a + b*x^3)/x^2],x]
```

output
```
(2*x*Sqrt[a/x^2 + b*x])/3 - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x
])])/3
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1913
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Simp[a   Int[x^j*(a*x^j + b*x^n)^(p - 1), x]
, x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simp
lify[j*p + 1], 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

method	result	size
default	$-\frac{2\sqrt{\frac{b\,x^{3}+a}{x^{2}}}\,x\left(\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)-\sqrt{b\,x^{3}+a}\right)}{3\sqrt{b\,x^{3}+a}}$	56

input
```
int(((b*x^3+a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((b*x^3+a)/x^2)^(1/2)*x*(a^(1/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2))-(b*
x^3+a)^(1/2))/(b*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.16

$$\int \sqrt{\frac{a+bx^3}{x^2}}\,dx = \left[\frac{2}{3}\,x\sqrt{\frac{bx^3+a}{x^2}} + \frac{1}{3}\,\sqrt{a}\log\left(\frac{bx^3-2\,\sqrt{a}x\sqrt{\frac{bx^3+a}{x^2}}+2\,a}{x^3}\right), \frac{2}{3}\,x\sqrt{\frac{bx^3+a}{x^2}} + \frac{2}{3}\,\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{bx^3+a}{x^2}}}{bx^3+a}\right)\right]$$

input
```
integrate(((b*x^3+a)/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[2/3*x*sqrt((b*x^3 + a)/x^2) + 1/3*sqrt(a)*log((b*x^3 - 2*sqrt(a)*x*sqrt((
b*x^3 + a)/x^2) + 2*a)/x^3), 2/3*x*sqrt((b*x^3 + a)/x^2) + 2/3*sqrt(-a)*ar
ctan(sqrt(-a)*x*sqrt((b*x^3 + a)/x^2)/(b*x^3 + a))]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\frac{a+bx^3}{x^2}}\,dx = \text{Timed out}$$

input
```
integrate(((b*x**3+a)/x**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{\frac{a+bx^3}{x^2}}\,dx = \int \sqrt{\frac{bx^3+a}{x^2}}\,dx$$

input
```
integrate(((b*x^3+a)/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x^3 + a)/x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.39

$$\int \sqrt{\frac{a+bx^3}{x^2}}\,dx = \frac{2\,a\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{3\,\sqrt{-a}} + \frac{2}{3}\,\sqrt{bx^3+a}\operatorname{sgn}(x) - \frac{2\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}\right)\operatorname{sgn}(x)}{3\,\sqrt{-a}}$$

input
```
integrate(((b*x^3+a)/x^2)^(1/2),x, algorithm="giac")
```

output
```
2/3*a*arctan(sqrt(b*x^3 + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2/3*sqrt(b*x^3 +
a)*sgn(x) - 2/3*(a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqr
t(-a)
```

Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

$$\int \sqrt{\frac{a+bx^3}{x^2}}\,dx = \frac{2\,x\,\sqrt{b\,x+\frac{a}{x^2}}}{3} + \frac{\sqrt{a}\,\mathrm{asin}\left(\frac{\sqrt{a}\,1\mathrm{i}}{\sqrt{b}\,x^{3/2}}\right)\,\sqrt{b\,x+\frac{a}{x^2}}\,2\mathrm{i}}{3\,\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{a}{b\,x^3}+1}}$$

input
```
int(((a + b*x^3)/x^2)^(1/2),x)
```

output
```
(2*x*(b*x + a/x^2)^(1/2))/3 + (a^(1/2)*asin((a^(1/2)*1i)/(b^(1/2)*x^(3/2))
)*(b*x + a/x^2)^(1/2)*2i)/(3*b^(1/2)*x^(1/2)*(a/(b*x^3) + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \sqrt{\frac{a+bx^3}{x^2}}\,dx = \frac{2\sqrt{b\,x^3+a}}{3} + \frac{\sqrt{a}\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{3} - \frac{\sqrt{a}\log\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{3}$$

input
```
int(((b*x^3+a)/x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3) + sqrt(a)*log(sqrt(a + b*x**3) - sqrt(a)) - sqrt(a)*lo
g(sqrt(a + b*x**3) + sqrt(a)))/3
```

3.364 $\int \sqrt{\frac{a+bx^n}{x^2}}\,dx$

Optimal result 2865
Mathematica [A] (verified) 2865
Rubi [A] (verified) 2866
Maple [A] (verified) 2867
Fricas [A] (verification not implemented) 2868
Sympy [F] 2868
Maxima [F] 2869
Giac [F] 2869
Mupad [F(-1)] 2869
Reduce [F] 2870

Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \sqrt{\frac{a+bx^n}{x^2}}\,dx=\frac{2x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}{n}-\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}\right)}{n}$$

output
```
2*x*(a/x^2+b*x^(-2+n))^(1/2)/n-2*a^(1/2)*arctanh(a^(1/2)/x/(a/x^2+b*x^(-2+
n))^(1/2))/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \sqrt{\frac{a+bx^n}{x^2}}\,dx=\frac{2x\sqrt{\frac{a+bx^n}{x^2}}\left(\sqrt{a+bx^n}-\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)\right)}{n\sqrt{a+bx^n}}$$

input
```
Integrate[Sqrt[(a + b*x^n)/x^2],x]
```

output
```
(2*x*Sqrt[(a + b*x^n)/x^2]*(Sqrt[a + b*x^n] - Sqrt[a]*ArcTanh[Sqrt[a + b*x
^n]/Sqrt[a]]))/(n*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2078, 1913, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{a+bx^n}{x^2}}dx$$

$$\Big\downarrow \text{2078}$$

$$\int \sqrt{\frac{a}{x^2}+bx^{n-2}}dx$$

$$\Big\downarrow \text{1913}$$

$$a\int \frac{1}{x^2\sqrt{bx^{n-2}+\frac{a}{x^2}}}dx+\frac{2x\sqrt{\frac{a}{x^2}+bx^{n-2}}}{n}$$

$$\Big\downarrow \text{1935}$$

$$\frac{2x\sqrt{\frac{a}{x^2}+bx^{n-2}}}{n}-\frac{2a\int \frac{1}{1-\frac{a}{x^2\left(bx^{n-2}+\frac{a}{x^2}\right)}}d\frac{1}{x\sqrt{bx^{n-2}+\frac{a}{x^2}}}}{n}$$

$$\Big\downarrow \text{219}$$

$$\frac{2x\sqrt{\frac{a}{x^2}+bx^{n-2}}}{n}-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^{n-2}}}\right)}{n}$$

input
```
Int[Sqrt[(a + b*x^n)/x^2],x]
```

output
```
(2*x*Sqrt[a/x^2 + b*x^(-2 + n)])/n - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/
x^2 + b*x^(-2 + n)])])/n
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1913
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Simp[a   Int[x^j*(a*x^j + b*x^n)^(p - 1), x]
, x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simp
lify[j*p + 1], 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.21

method	result	size
risch	$\frac{2\sqrt{\frac{a+b\,\mathrm{e}^{n\ln(x)}}{x^2}}\,x}{n}-\frac{2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{e}^{n\ln(x)}}}{\sqrt{a}}\right)\sqrt{\frac{a+b\,\mathrm{e}^{n\ln(x)}}{x^2}}\,x}{n\sqrt{a+b\,\mathrm{e}^{n\ln(x)}}}$	74

input
```
int(((a+b*x^n)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/n*((a+b*exp(n*ln(x)))/x^2)^(1/2)*x-2*a^(1/2)/n*arctanh((a+b*exp(n*ln(x))
)^(1/2)/a^(1/2))*((a+b*exp(n*ln(x)))/x^2)^(1/2)/(a+b*exp(n*ln(x)))^(1/2)*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.93

$$\int \sqrt{\frac{a+bx^n}{x^2}}\,dx = \left[\frac{2\,x\sqrt{\frac{bx^n+a}{x^2}}+\sqrt{a}\log\left(\frac{bx^n-2\,\sqrt{a}x\sqrt{\frac{bx^n+a}{x^2}}+2\,a}{x^n}\right)}{n}, \frac{2\left(x\sqrt{\frac{bx^n+a}{x^2}}+\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{bx^n+a}{x^2}}}{bx^n+a}\right)\right)}{n}\right]$$

input
```
integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[(2*x*sqrt((b*x^n + a)/x^2) + sqrt(a)*log((b*x^n - 2*sqrt(a)*x*sqrt((b*x^n
 + a)/x^2) + 2*a)/x^n))/n, 2*(x*sqrt((b*x^n + a)/x^2) + sqrt(-a)*arctan(sq
rt(-a)*x*sqrt((b*x^n + a)/x^2)/(b*x^n + a)))/n]
```

Sympy [F]

$$\int \sqrt{\frac{a+bx^n}{x^2}}\,dx = \int \sqrt{\frac{a+bx^n}{x^2}}\,dx$$

input
```
integrate(((a+b*x**n)/x**2)**(1/2),x)
```

output
```
Integral(sqrt((a + b*x**n)/x**2), x)
```

Maxima [F]

$$\int \sqrt{\frac{a+bx^n}{x^2}}\,dx = \int \sqrt{\frac{bx^n+a}{x^2}}\,dx$$

input
```
integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x^n + a)/x^2), x)
```

Giac [F]

$$\int \sqrt{\frac{a+bx^n}{x^2}}\,dx = \int \sqrt{\frac{bx^n+a}{x^2}}\,dx$$

input
```
integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((b*x^n + a)/x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{a+bx^n}{x^2}}\,dx = \int \sqrt{\frac{a+b\,x^n}{x^2}}\,dx$$

input
```
int(((a + b*x^n)/x^2)^(1/2),x)
```

output
```
int(((a + b*x^n)/x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\frac{a+bx^n}{x^2}}\,dx = \frac{2\sqrt{x^n b+a}+\left(\int \frac{\sqrt{x^n b+a}}{x^n bx+ax}dx\right)an}{n}$$

input
```
int(((a+b*x^n)/x^2)^(1/2),x)
```

output
```
(2*sqrt(x**n*b + a) + int(sqrt(x**n*b + a)/(x**n*b*x + a*x),x)*a*n)/n
```

3.365 $\int \sqrt{\frac{-a+bx}{x^2}}\,dx$

Optimal result . . . 2871
Mathematica [A] (verified) . . . 2871
Rubi [A] (verified) . . . 2872
Maple [A] (verified) . . . 2873
Fricas [A] (verification not implemented) . . . 2874
Sympy [F] . . . 2874
Maxima [F] . . . 2875
Giac [A] (verification not implemented) . . . 2875
Mupad [B] (verification not implemented) . . . 2875
Reduce [B] (verification not implemented) . . . 2876

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \sqrt{\frac{-a+bx}{x^2}}\,dx = 2x\sqrt{\frac{-a+bx}{x^2}} - 2\sqrt{a}\arctan\left(\frac{x\sqrt{\frac{-a+bx}{x^2}}}{\sqrt{a}}\right)$$

output
```
2*x*((b*x-a)/x^2)^(1/2)-2*a^(1/2)*arctan(x*((b*x-a)/x^2)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.35

$$\int \sqrt{\frac{-a+bx}{x^2}}\,dx = \frac{2x\sqrt{\frac{-a+bx}{x^2}}\left(\sqrt{-a+bx} - \sqrt{a}\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{-a+bx}}$$

input
```
Integrate[Sqrt[(-a + b*x)/x^2],x]
```

output
```
(2*x*Sqrt[(-a + b*x)/x^2]*(Sqrt[-a + b*x] - Sqrt[a]*ArcTan[Sqrt[-a + b*x]/
Sqrt[a]]))/Sqrt[-a + b*x]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2078, 1913, 1919, 1091, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{bx-a}{x^2}}dx$$
$$\Big\downarrow \ 2078$$
$$\int \sqrt{\frac{b}{x}-\frac{a}{x^2}}dx$$
$$\Big\downarrow \ 1913$$
$$2x\sqrt{\frac{b}{x}-\frac{a}{x^2}}-a\int \frac{1}{\sqrt{\frac{b}{x}-\frac{a}{x^2}}x^2}dx$$
$$\Big\downarrow \ 1919$$
$$a\int \frac{1}{\sqrt{\frac{b}{x}-\frac{a}{x^2}}}d\frac{1}{x}+2x\sqrt{\frac{b}{x}-\frac{a}{x^2}}$$
$$\Big\downarrow \ 1091$$
$$2a\int \frac{1}{\frac{a}{x^2}+1}d\frac{1}{\sqrt{\frac{b}{x}-\frac{a}{x^2}}x}+2x\sqrt{\frac{b}{x}-\frac{a}{x^2}}$$
$$\Big\downarrow \ 216$$
$$2\sqrt{a}\arctan\left(\frac{\sqrt{a}}{x\sqrt{\frac{b}{x}-\frac{a}{x^2}}}\right)+2x\sqrt{\frac{b}{x}-\frac{a}{x^2}}$$

input
```
Int[Sqrt[(-a + b*x)/x^2],x]
```

output
```
2*Sqrt[-(a/x^2) + b/x]*x + 2*Sqrt[a]*ArcTan[Sqrt[a]/(Sqrt[-(a/x^2) + b/x]*
x)]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1913
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Simp[a   Int[x^j*(a*x^j + b*x^n)^(p - 1), x]
, x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simp
lify[j*p + 1], 0]
```

rule 1919
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
 && EqQ[Simplify[m - n + 1], 0]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

method	result	size
default	$-\frac{2\sqrt{-\frac{-bx+a}{x^{2}}}\, x\left(\sqrt{a}\, \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\sqrt{bx-a}\right)}{\sqrt{bx-a}}$	56

input
```
int(((b*x-a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-(-b*x+a)/x^2)^(1/2)*x*(a^(1/2)*arctan((b*x-a)^(1/2)/a^(1/2))-(b*x-a)^
(1/2))/(b*x-a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.18

$$\int \sqrt{\frac{-a+bx}{x^2}}\,dx = \left[2\,x\sqrt{\frac{bx-a}{x^2}} + \sqrt{-a}\log\left(\frac{bx-2\,\sqrt{-a}x\sqrt{\frac{bx-a}{x^2}}-2\,a}{x}\right), 2\,x\sqrt{\frac{bx-a}{x^2}} + 2\,\sqrt{a}\arctan\left(\frac{\sqrt{a}x\sqrt{\frac{bx-a}{x^2}}}{bx-a}\right)\right]$$

input
```
integrate(((b*x-a)/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[2*x*sqrt((b*x - a)/x^2) + sqrt(-a)*log((b*x - 2*sqrt(-a)*x*sqrt((b*x - a)
/x^2) - 2*a)/x), 2*x*sqrt((b*x - a)/x^2) + 2*sqrt(a)*arctan(sqrt(a)*x*sqrt
((b*x - a)/x^2)/(b*x - a))]
```

Sympy [F]

$$\int \sqrt{\frac{-a+bx}{x^2}}\,dx = \int \sqrt{\frac{-a+bx}{x^2}}\,dx$$

input
```
integrate(((b*x-a)/x**2)**(1/2),x)
```

output
```
Integral(sqrt((-a + b*x)/x**2), x)
```

Maxima [F]

$$\int \sqrt{\frac{-a+bx}{x^2}}\,dx = \int \sqrt{\frac{bx-a}{x^2}}\,dx$$

input
```
integrate(((b*x-a)/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x - a)/x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.24

$$\int \sqrt{\frac{-a+bx}{x^2}}\,dx = -2\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)\operatorname{sgn}(x) + 2\left(\sqrt{a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{a}}\right) - \sqrt{-a}\right)\operatorname{sgn}(x) + 2\sqrt{bx-a}\operatorname{sgn}(x)$$

input
```
integrate(((b*x-a)/x^2)^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a))*sgn(x) + 2*(sqrt(a)*arctan(sqrt(-
a)/sqrt(a)) - sqrt(-a))*sgn(x) + 2*sqrt(b*x - a)*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 10.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.37

$$\int \sqrt{\frac{-a+bx}{x^2}}\,dx = 2x\sqrt{\frac{b}{x}-\frac{a}{x^2}} + \frac{2\sqrt{a}\sqrt{x}\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)\sqrt{\frac{b}{x}-\frac{a}{x^2}}}{\sqrt{b}\sqrt{1-\frac{a}{bx}}}$$

input
```
int((-(a - b*x)/x^2)^(1/2),x)
```

output
```
2*x*(b/x - a/x^2)^(1/2) + (2*a^(1/2)*x^(1/2)*asin(a^(1/2)/(b^(1/2)*x^(1/2)
))*(b/x - a/x^2)^(1/2))/(b^(1/2)*(1 - a/(b*x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.59

$$\int \sqrt{\frac{-a+bx}{x^2}}\,dx = -2\sqrt{a}\,atan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\sqrt{bx-a}$$

input
```
int(((b*x-a)/x^2)^(1/2),x)
```

output
```
2*( - sqrt(a)*atan(sqrt( - a + b*x)/sqrt(a)) + sqrt( - a + b*x))
```

3.366 $\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx$

Optimal result 2877
Mathematica [A] (verified) 2877
Rubi [A] (verified) 2878
Maple [B] (verified) 2879
Fricas [A] (verification not implemented) 2880
Sympy [F] 2880
Maxima [A] (verification not implemented) 2881
Giac [A] (verification not implemented) 2881
Mupad [B] (verification not implemented) 2881
Reduce [B] (verification not implemented) 2882

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx = \sqrt{b-\frac{a}{x^2}}\,x + \sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{b-\frac{a}{x^2}}\,x}\right)$$

output
```
(b-a/x^2)^(1/2)*x+a^(1/2)*arctan(a^(1/2)/(b-a/x^2)^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx = \sqrt{b-\frac{a}{x^2}}\,x - \frac{\sqrt{a}\sqrt{b-\frac{a}{x^2}}\,x\arctan\left(\frac{\sqrt{-a+bx^2}}{\sqrt{a}}\right)}{\sqrt{-a+bx^2}}$$

input
```
Integrate[Sqrt[(-a + b*x^2)/x^2],x]
```

output
```
Sqrt[b - a/x^2]*x - (Sqrt[a]*Sqrt[b - a/x^2]*x*ArcTan[Sqrt[-a + b*x^2]/Sqr
t[a]])/Sqrt[-a + b*x^2]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2072, 773, 247, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{bx^2-a}{x^2}}dx$$
$$\Big\downarrow \text{2072}$$
$$\int \sqrt{b-\frac{a}{x^2}}dx$$
$$\Big\downarrow \text{773}$$
$$-\int \sqrt{b-\frac{a}{x^2}}x^2d\frac{1}{x}$$
$$\Big\downarrow \text{247}$$
$$a\int \frac{1}{\sqrt{b-\frac{a}{x^2}}}d\frac{1}{x}+x\sqrt{b-\frac{a}{x^2}}$$
$$\Big\downarrow \text{224}$$
$$a\int \frac{1}{\frac{a}{x^2}+1}d\frac{1}{\sqrt{b-\frac{a}{x^2}}x}+x\sqrt{b-\frac{a}{x^2}}$$
$$\Big\downarrow \text{216}$$
$$\sqrt{a}\arctan\left(\frac{\sqrt{a}}{x\sqrt{b-\frac{a}{x^2}}}\right)+x\sqrt{b-\frac{a}{x^2}}$$

input
```
Int[Sqrt[(-a + b*x^2)/x^2],x]
```

output
```
Sqrt[b - a/x^2]*x + Sqrt[a]*ArcTan[Sqrt[a]/(Sqrt[b - a/x^2]*x)]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 2072
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(35) = 70$.

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.88

method	result	size
default	$\frac{\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}\,x\left(a\ln\left(\frac{-2a+2\sqrt{-a}\,\sqrt{b\,x^{2}-a}}{x}\right)+\sqrt{-a}\,\sqrt{b\,x^{2}-a}\right)}{\sqrt{-a}\,\sqrt{b\,x^{2}-a}}$	81

input
```
int(((b*x^2-a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b*x^2+a)/x^2)^(1/2)*x*(a*ln(2*((-a)^(1/2)*(b*x^2-a)^(1/2)-a)/x)+(-a)^(
1/2)*(b*x^2-a)^(1/2))/(-a)^(1/2)/(b*x^2-a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.51

$$\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx = \left[x\sqrt{\frac{bx^2-a}{x^2}} + \frac{1}{2}\sqrt{-a}\log\left(-\frac{bx^2-2\sqrt{-a}x\sqrt{\frac{bx^2-a}{x^2}}-2a}{x^2}\right), x\sqrt{\frac{bx^2-a}{x^2}} - \sqrt{a}\arctan\left(\frac{x\sqrt{\frac{bx^2-a}{x^2}}}{\sqrt{a}}\right)\right]$$

input
```
integrate(((b*x^2-a)/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[x*sqrt((b*x^2 - a)/x^2) + 1/2*sqrt(-a)*log(-(b*x^2 - 2*sqrt(-a)*x*sqrt((b
*x^2 - a)/x^2) - 2*a)/x^2), x*sqrt((b*x^2 - a)/x^2) - sqrt(a)*arctan(x*sqr
t((b*x^2 - a)/x^2)/sqrt(a))]
```

Sympy [F]

$$\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx = \int \sqrt{\frac{-a+bx^2}{x^2}}\,dx$$

input
```
integrate(((b*x**2-a)/x**2)**(1/2),x)
```

output
```
Integral(sqrt((-a + b*x**2)/x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx=\sqrt{b-\frac{a}{x^2}}\,x-\sqrt{a}\arctan\left(\frac{\sqrt{b-\frac{a}{x^2}}\,x}{\sqrt{a}}\right)$$

input
```
integrate(((b*x^2-a)/x^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(b - a/x^2)*x - sqrt(a)*arctan(sqrt(b - a/x^2)*x/sqrt(a))
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.47

$$\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx=-\sqrt{a}\arctan\left(\frac{\sqrt{bx^2-a}}{\sqrt{a}}\right)\mathrm{sgn}(x)+\left(\sqrt{a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{a}}\right)-\sqrt{-a}\right)\mathrm{sgn}(x)+\sqrt{bx^2-a}\,\mathrm{sgn}(x)$$

input
```
integrate(((b*x^2-a)/x^2)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(a)*arctan(sqrt(b*x^2 - a)/sqrt(a))*sgn(x) + (sqrt(a)*arctan(sqrt(-a)
/sqrt(a)) - sqrt(-a))*sgn(x) + sqrt(b*x^2 - a)*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 9.38 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx=x\,\sqrt{b-\frac{a}{x^2}}+\frac{\sqrt{a}\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)\,\sqrt{b-\frac{a}{x^2}}}{\sqrt{b}\,\sqrt{1-\frac{a}{b\,x^2}}}$$

input
```
int((-(a - b*x^2)/x^2)^(1/2),x)
```

output
```
x*(b - a/x^2)^(1/2) + (a^(1/2)*asin(a^(1/2)/(b^(1/2)*x))*(b - a/x^2)^(1/2)
)/(b^(1/2)*(1 - a/(b*x^2))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx = -2\sqrt{a}\,atan\left(\frac{\sqrt{b\,x^2-a}+\sqrt{b}\,x}{\sqrt{a}}\right)+\sqrt{b\,x^2-a}$$

input
```
int(((b*x^2-a)/x^2)^(1/2),x)
```

output
```
- 2*sqrt(a)*atan((sqrt( - a + b*x**2) + sqrt(b)*x)/sqrt(a)) + sqrt( - a +
b*x**2)
```

3.367 $\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx$

Optimal result 2883
Mathematica [A] (verified) 2883
Rubi [A] (verified) 2884
Maple [A] (verified) 2885
Fricas [A] (verification not implemented) 2886
Sympy [F(-1)] 2886
Maxima [F] 2887
Giac [A] (verification not implemented) 2887
Mupad [B] (verification not implemented) 2887
Reduce [B] (verification not implemented) 2888

Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx = \frac{2}{3}x\sqrt{-\frac{a}{x^2}+bx}+\frac{2}{3}\sqrt{a}\arctan\left(\frac{\sqrt{a}}{x\sqrt{-\frac{a}{x^2}+bx}}\right)$$

output
```
2/3*x*(-a/x^2+b*x)^(1/2)+2/3*a^(1/2)*arctan(a^(1/2)/x/(-a/x^2+b*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx = \frac{2x\sqrt{-\frac{a}{x^2}+bx}\left(\sqrt{-a+bx^3}-\sqrt{a}\arctan\left(\frac{\sqrt{-a+bx^3}}{\sqrt{a}}\right)\right)}{3\sqrt{-a+bx^3}}$$

input
```
Integrate[Sqrt[(-a + b*x^3)/x^2],x]
```

output
```
(2*x*Sqrt[-(a/x^2) + b*x]*(Sqrt[-a + b*x^3] - Sqrt[a]*ArcTan[Sqrt[-a + b*x
^3]/Sqrt[a]]))/(3*Sqrt[-a + b*x^3])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2078, 1913, 1935, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{bx^3-a}{x^2}}dx$$

$$\Big\downarrow \text{2078}$$

$$\int \sqrt{bx-\frac{a}{x^2}}dx$$

$$\Big\downarrow \text{1913}$$

$$\frac{2}{3}x\sqrt{bx-\frac{a}{x^2}}-a\int \frac{1}{x^2\sqrt{bx-\frac{a}{x^2}}}dx$$

$$\Big\downarrow \text{1935}$$

$$\frac{2}{3}a\int \frac{1}{\frac{a}{x^2\left(bx-\frac{a}{x^2}\right)}+1}d\frac{1}{x\sqrt{bx-\frac{a}{x^2}}}+\frac{2}{3}x\sqrt{bx-\frac{a}{x^2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2}{3}\sqrt{a}\arctan\left(\frac{\sqrt{a}}{x\sqrt{bx-\frac{a}{x^2}}}\right)+\frac{2}{3}x\sqrt{bx-\frac{a}{x^2}}$$

input
```
Int[Sqrt[(-a + b*x^3)/x^2],x]
```

output
```
(2*x*Sqrt[-(a/x^2) + b*x])/3 + (2*Sqrt[a]*ArcTan[Sqrt[a]/(x*Sqrt[-(a/x^2)
+ b*x])])/3
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1913
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Simp[a   Int[x^j*(a*x^j + b*x^n)^(p - 1), x]
, x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simp
lify[j*p + 1], 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

method	result	size
default	$\frac{2\sqrt{-\frac{-b\,x^{3}+a}{x^{2}}}\,x\left(\sqrt{b\,x^{3}-a}\,\sqrt{-a}+a\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}-a}}{\sqrt{-a}}\right)\right)}{3\sqrt{b\,x^{3}-a}\,\sqrt{-a}}$	73

input
```
int(((b*x^3-a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(-(-b*x^3+a)/x^2)^(1/2)*x*((b*x^3-a)^(1/2)*(-a)^(1/2)+a*arctanh((b*x^3
-a)^(1/2)/(-a)^(1/2)))/(b*x^3-a)^(1/2)/(-a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.26

$$\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx = \left[\frac{2}{3}\,x\sqrt{\frac{bx^3-a}{x^2}} + \frac{1}{3}\,\sqrt{-a}\log\left(\frac{bx^3-2\,\sqrt{-a}x\sqrt{\frac{bx^3-a}{x^2}}-2\,a}{x^3}\right),\ \frac{2}{3}\,x\sqrt{\frac{bx^3-a}{x^2}} + \frac{2}{3}\,\sqrt{a}\arctan\left(\frac{\sqrt{a}x\sqrt{\frac{bx^3-a}{x^2}}}{bx^3-a}\right)\right]$$

input
```
integrate(((b*x^3-a)/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[2/3*x*sqrt((b*x^3 - a)/x^2) + 1/3*sqrt(-a)*log((b*x^3 - 2*sqrt(-a)*x*sqrt
((b*x^3 - a)/x^2) - 2*a)/x^3), 2/3*x*sqrt((b*x^3 - a)/x^2) + 2/3*sqrt(a)*a
rctan(sqrt(a)*x*sqrt((b*x^3 - a)/x^2)/(b*x^3 - a))]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx = \text{Timed out}$$

input
```
integrate(((b*x**3-a)/x**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx = \int \sqrt{\frac{bx^3-a}{x^2}}\,dx$$

input
```
integrate(((b*x^3-a)/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x^3 - a)/x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.23

$$\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx = -\frac{2}{3}\sqrt{a}\arctan\left(\frac{\sqrt{bx^3-a}}{\sqrt{a}}\right)\mathrm{sgn}(x) + \frac{2}{3}\left(\sqrt{a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{a}}\right)-\sqrt{-a}\right)\mathrm{sgn}(x) + \frac{2}{3}\sqrt{bx^3-a}\mathrm{sgn}(x)$$

input
```
integrate(((b*x^3-a)/x^2)^(1/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(a)*arctan(sqrt(b*x^3 - a)/sqrt(a))*sgn(x) + 2/3*(sqrt(a)*arctan(
sqrt(-a)/sqrt(a)) - sqrt(-a))*sgn(x) + 2/3*sqrt(b*x^3 - a)*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 9.63 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19

$$\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx = \frac{2\,x\,\sqrt{b\,x-\frac{a}{x^2}}}{3} + \frac{2\,\sqrt{a}\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{3/2}}\right)\,\sqrt{b\,x-\frac{a}{x^2}}}{3\,\sqrt{b}\,\sqrt{x}\,\sqrt{1-\frac{a}{b\,x^3}}}$$

input
```
int((-(a - b*x^3)/x^2)^(1/2),x)
```

output
```
(2*x*(b*x - a/x^2)^(1/2))/3 + (2*a^(1/2)*asin(a^(1/2)/(b^(1/2)*x^(3/2)))*(
b*x - a/x^2)^(1/2))/(3*b^(1/2)*x^(1/2)*(1 - a/(b*x^3))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.28

$$\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx = -\frac{\sqrt{a}\,atan\left(\frac{2\sqrt{a}\,\sqrt{b\,x^3-a}\,a-\sqrt{a}\,\sqrt{b\,x^3-a}\,b\,x^3}{-2ab\,x^3+2a^2}\right)}{3}+\frac{2\sqrt{b\,x^3-a}}{3}$$

input
```
int(((b*x^3-a)/x^2)^(1/2),x)
```

output
```
( - sqrt(a)*atan((2*sqrt(a)*sqrt( - a + b*x**3)*a - sqrt(a)*sqrt( - a + b*
x**3)*b*x**3)/(2*a**2 - 2*a*b*x**3)) + 2*sqrt( - a + b*x**3))/3
```

3.368 $\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx$

Optimal result 2889
Mathematica [A] (verified) 2889
Rubi [A] (verified) 2890
Maple [A] (verified) 2891
Fricas [A] (verification not implemented) 2892
Sympy [F] 2892
Maxima [F] 2893
Giac [F] 2893
Mupad [F(-1)] 2893
Reduce [F] 2894

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx=\frac{2x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}{n}+\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}}{x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}\right)}{n}$$

output
```
2*x*(-a/x^2+b*x^(-2+n))^(1/2)/n+2*a^(1/2)*arctan(a^(1/2)/x/(-a/x^2+b*x^(-2
+n))^(1/2))/n
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.22

$$\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx=\frac{2x\sqrt{\frac{-a+bx^n}{x^2}}\left(\sqrt{-a+bx^n}-\sqrt{a}\arctan\left(\frac{\sqrt{-a+bx^n}}{\sqrt{a}}\right)\right)}{n\sqrt{-a+bx^n}}$$

input
```
Integrate[Sqrt[(-a + b*x^n)/x^2],x]
```

output
```
(2*x*Sqrt[(-a + b*x^n)/x^2]*(Sqrt[-a + b*x^n] - Sqrt[a]*ArcTan[Sqrt[-a + b
*x^n]/Sqrt[a]]))/(n*Sqrt[-a + b*x^n])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2078, 1913, 1935, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{bx^n-a}{x^2}}dx$$

$$\Big\downarrow \text{2078}$$

$$\int \sqrt{bx^{n-2}-\frac{a}{x^2}}dx$$

$$\Big\downarrow \text{1913}$$

$$\frac{2x\sqrt{bx^{n-2}-\frac{a}{x^2}}}{n}-a\int \frac{1}{x^2\sqrt{bx^{n-2}-\frac{a}{x^2}}}dx$$

$$\Big\downarrow \text{1935}$$

$$\frac{2a\int \frac{1}{\frac{a}{x^2\left(bx^{n-2}-\frac{a}{x^2}\right)}+1}d\frac{1}{x\sqrt{bx^{n-2}-\frac{a}{x^2}}}}{n}+\frac{2x\sqrt{bx^{n-2}-\frac{a}{x^2}}}{n}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}}{x\sqrt{bx^{n-2}-\frac{a}{x^2}}}\right)}{n}+\frac{2x\sqrt{bx^{n-2}-\frac{a}{x^2}}}{n}$$

input
```
Int[Sqrt[(-a + b*x^n)/x^2],x]
```

output
```
(2*x*Sqrt[-(a/x^2) + b*x^(-2 + n)])/n + (2*Sqrt[a]*ArcTan[Sqrt[a]/(x*Sqrt[
-(a/x^2) + b*x^(-2 + n)])])/n
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1913
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Simp[a   Int[x^j*(a*x^j + b*x^n)^(p - 1), x]
, x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simp
lify[j*p + 1], 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.67

method	result	size
risch	$-\frac{2\left(a-b\,{\mathrm e}^{n\ln\left(x\right)}\right)\sqrt{\frac{-a+b\,{\mathrm e}^{n\ln\left(x\right)}}{x^{2}}}\,x}{n\left(-a+b\,{\mathrm e}^{n\ln\left(x\right)}\right)}-\frac{2\sqrt{a}\,\arctan\left(\frac{\sqrt{-a+b\,{\mathrm e}^{n\ln\left(x\right)}}}{\sqrt{a}}\right)\sqrt{\frac{-a+b\,{\mathrm e}^{n\ln\left(x\right)}}{x^{2}}}\,x}{n\sqrt{-a+b\,{\mathrm e}^{n\ln\left(x\right)}}}$	105

input
```
int(((-a+b*x^n)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(a-b*exp(n*ln(x)))/n/(-a+b*exp(n*ln(x)))*((-a+b*exp(n*ln(x)))/x^2)^(1/2
)*x-2*a^(1/2)/n*arctan((-a+b*exp(n*ln(x)))^(1/2)/a^(1/2))*((-a+b*exp(n*ln(
x)))/x^2)^(1/2)/(-a+b*exp(n*ln(x)))^(1/2)*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.03

$$\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx = \left[\frac{2\,x\sqrt{\frac{bx^n-a}{x^2}}+\sqrt{-a}\log\left(\frac{bx^n-2\,\sqrt{-a}x\sqrt{\frac{bx^n-a}{x^2}}-2\,a}{x^n}\right)}{n}, \frac{2\left(x\sqrt{\frac{bx^n-a}{x^2}}+\sqrt{a}\arctan\left(\frac{\sqrt{a}x\sqrt{\frac{bx^n-a}{x^2}}}{bx^n-a}\right)\right)}{n}\right]$$

input
```
integrate(((-a+b*x^n)/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[(2*x*sqrt((b*x^n - a)/x^2) + sqrt(-a)*log((b*x^n - 2*sqrt(-a)*x*sqrt((b*x
^n - a)/x^2) - 2*a)/x^n))/n, 2*(x*sqrt((b*x^n - a)/x^2) + sqrt(a)*arctan(s
qrt(a)*x*sqrt((b*x^n - a)/x^2)/(b*x^n - a)))/n]
```

Sympy [F]

$$\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx = \int \sqrt{\frac{-a+bx^n}{x^2}}\,dx$$

input
```
integrate(((-a+b*x**n)/x**2)**(1/2),x)
```

output
```
Integral(sqrt((-a + b*x**n)/x**2), x)
```

Maxima [F]

$$\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx = \int \sqrt{\frac{bx^n-a}{x^2}}\,dx$$

input
```
integrate(((-a+b*x^n)/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x^n - a)/x^2), x)
```

Giac [F]

$$\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx = \int \sqrt{\frac{bx^n-a}{x^2}}\,dx$$

input
```
integrate(((-a+b*x^n)/x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((b*x^n - a)/x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx = \int \sqrt{-\frac{a-b\,x^n}{x^2}}\,dx$$

input
```
int((-(a - b*x^n)/x^2)^(1/2),x)
```

output
```
int((-(a - b*x^n)/x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx = \frac{2\sqrt{x^n b-a}-\left(\int \frac{\sqrt{x^n b-a}}{x^n bx-ax}dx\right)an}{n}$$

input
```
int(((-a+b*x^n)/x^2)^(1/2),x)
```

output
```
(2*sqrt(x**n*b - a) - int(sqrt(x**n*b - a)/(x**n*b*x - a*x),x)*a*n)/n
```

3.369 $\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx$

Optimal result 2895
Mathematica [A] (verified) 2895
Rubi [A] (verified) 2896
Maple [F] 2897
Fricas [F(-2)] 2897
Sympy [F] 2898
Maxima [F] 2898
Giac [F] 2898
Mupad [F(-1)] 2899
Reduce [F] 2899

Optimal result

Integrand size = 27, antiderivative size = 62

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx=\frac{2x^{-j/2}(cx)^{j/2}\text{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{\sqrt{a}c(j-n)}$$

output
```
2*(c*x)^(1/2*j)*arctanh(a^(1/2)*x^(1/2*j)/(a*x^j+b*x^n)^(1/2))/a^(1/2)/c/(
j-n)/(x^(1/2*j))
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.58

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx=\frac{2\sqrt{b}x^{\frac{1}{2}(-j+n)}(cx)^{j/2}\sqrt{1+\frac{ax^{j-n}}{b}}\text{arcsinh}\left(\frac{\sqrt{a}x^{\frac{j-n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}c(j-n)\sqrt{ax^j+bx^n}}$$

input
```
Integrate[(c*x)^(-1 + j/2)/Sqrt[a*x^j + b*x^n],x]
```

output
```
(2*Sqrt[b]*x^((-j + n)/2)*(c*x)^(j/2)*Sqrt[1 + (a*x^(j - n))/b]*ArcSinh[(S
qrt[a]*x^((j - n)/2))/Sqrt[b]])/(Sqrt[a]*c*(j - n)*Sqrt[a*x^j + b*x^n])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{\frac{j}{2}-1}}{\sqrt{ax^j+bx^n}}dx$$

$$\Big\downarrow \text{1937}$$

$$\frac{x^{-j/2}(cx)^{j/2}\int \frac{x^{\frac{j-2}{2}}}{\sqrt{ax^j+bx^n}}dx}{c}$$

$$\Big\downarrow \text{1935}$$

$$\frac{2x^{-j/2}(cx)^{j/2}\int \frac{1}{1-\frac{ax^j}{ax^j+bx^n}}d\frac{x^{j/2}}{\sqrt{ax^j+bx^n}}}{c(j-n)}$$

$$\Big\downarrow \text{219}$$

$$\frac{2x^{-j/2}(cx)^{j/2}\text{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{\sqrt{a}c(j-n)}$$

input
```
Int[(c*x)^(-1 + j/2)/Sqrt[a*x^j + b*x^n],x]
```

output
```
(2*(c*x)^(j/2)*ArcTanh[(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(Sqrt[a]*c*
(j - n)*x^(j/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [F]

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{a\,x^{j}+b\,x^{n}}}dx$$

input
```
int((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x)
```

output
```
int((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx = \int \frac{(cx)^{\frac{j}{2}-1}}{\sqrt{ax^j+bx^n}}\,dx$$

input
```
integrate((c*x)**(-1+1/2*j)/(a*x**j+b*x**n)**(1/2),x)
```

output
```
Integral((c*x)**(j/2 - 1)/sqrt(a*x**j + b*x**n), x)
```

Maxima [F]

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx = \int \frac{(cx)^{\frac{1}{2}\,j-1}}{\sqrt{ax^j+bx^n}}\,dx$$

input
```
integrate((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(1/2*j - 1)/sqrt(a*x^j + b*x^n), x)
```

Giac [F]

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx = \int \frac{(cx)^{\frac{1}{2}\,j-1}}{\sqrt{ax^j+bx^n}}\,dx$$

input
```
integrate((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(1/2*j - 1)/sqrt(a*x^j + b*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx=\int \frac{(c\,x)^{\frac{j}{2}-1}}{\sqrt{a\,x^j+b\,x^n}}\,dx$$

input
```
int((c*x)^(j/2 - 1)/(a*x^j + b*x^n)^(1/2),x)
```

output
```
int((c*x)^(j/2 - 1)/(a*x^j + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx=\frac{c^{\frac{j}{2}}\left(\int \frac{x^{\frac{j}{2}}\sqrt{x^ja+x^nb}}{x^jax+x^nbx}dx\right)}{c}$$

input
```
int((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x)
```

output
```
(c**(j/2)*int((x**(j/2)*sqrt(x**j*a + x**n*b))/(x**j*a*x + x**n*b*x),x))/c
```

3.370 $\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx$

Optimal result 2900
Mathematica [A] (verified) 2900
Rubi [A] (verified) 2901
Maple [F] 2902
Fricas [F(-2)] 2902
Sympy [F] 2903
Maxima [F] 2903
Giac [F] 2903
Mupad [F(-1)] 2904
Reduce [F] 2904

Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx = \frac{2\sqrt{cx}\,\text{arctanh}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{\sqrt{a}(3-n)\sqrt{x}}$$

output
```
2*(c*x)^(1/2)*arctanh(a^(1/2)*x^(3/2)/(a*x^3+b*x^n)^(1/2))/a^(1/2)/(3-n)/x
^(1/2)
```

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx = -\frac{2\sqrt{b}x^{\frac{1}{2}(-1+n)}\sqrt{cx}\sqrt{1+\frac{ax^{3-n}}{b}}\,\text{arcsinh}\left(\frac{\sqrt{a}x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}(-3+n)\sqrt{ax^3+bx^n}}$$

input
```
Integrate[Sqrt[c*x]/Sqrt[a*x^3 + b*x^n],x]
```

output
```
(-2*Sqrt[b]*x^((-1 + n)/2)*Sqrt[c*x]*Sqrt[1 + (a*x^(3 - n))/b]*ArcSinh[(Sq
rt[a]*x^(3/2 - n/2))/Sqrt[b]])/(Sqrt[a]*(-3 + n)*Sqrt[a*x^3 + b*x^n])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}dx$$

$$\Big\downarrow \text{1937}$$

$$\frac{\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{bx^n+ax^3}}dx}{\sqrt{x}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{2\sqrt{cx}\int \frac{1}{1-\frac{ax^3}{bx^n+ax^3}}d\frac{x^{3/2}}{\sqrt{bx^n+ax^3}}}{(3-n)\sqrt{x}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{cx}\text{arctanh}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{\sqrt{a}(3-n)\sqrt{x}}$$

input
```
Int[Sqrt[c*x]/Sqrt[a*x^3 + b*x^n],x]
```

output
```
(2*Sqrt[c*x]*ArcTanh[(Sqrt[a]*x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(Sqrt[a]*(3 -
 n)*Sqrt[x])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [F]

$$\int \frac{\sqrt{cx}}{\sqrt{a\,x^{3}+b\,x^{n}}}dx$$

input
```
int((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x)
```

output
```
int((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3 + bx^n}}\,dx = \int \frac{\sqrt{cx}}{\sqrt{ax^3 + bx^n}}\,dx$$

input
```
integrate((c*x)**(1/2)/(a*x**3+b*x**n)**(1/2),x)
```

output
```
Integral(sqrt(c*x)/sqrt(a*x**3 + b*x**n), x)
```

Maxima [F]

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3 + bx^n}}\,dx = \int \frac{\sqrt{cx}}{\sqrt{ax^3 + bx^n}}\,dx$$

input
```
integrate((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x)/sqrt(a*x^3 + b*x^n), x)
```

Giac [F]

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3 + bx^n}}\,dx = \int \frac{\sqrt{cx}}{\sqrt{ax^3 + bx^n}}\,dx$$

input
```
integrate((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x)/sqrt(a*x^3 + b*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx=\int \frac{\sqrt{c\,x}}{\sqrt{b\,x^n+a\,x^3}}\,dx$$

input
```
int((c*x)^(1/2)/(b*x^n + a*x^3)^(1/2),x)
```

output
```
int((c*x)^(1/2)/(b*x^n + a*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx=\sqrt{c}\left(\int \frac{\sqrt{x}}{\sqrt{x^n b+a\,x^3}}dx\right)$$

input
```
int((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x)
```

output
```
sqrt(c)*int(sqrt(x)/sqrt(x**n*b + a*x**3),x)
```

3.371 $\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx$

Optimal result . . . 2905
Mathematica [B] (verified) . . . 2905
Rubi [A] (verified) . . . 2906
Maple [F] . . . 2907
Fricas [F(-2)] . . . 2907
Sympy [F] . . . 2907
Maxima [F] . . . 2908
Giac [F] . . . 2908
Mupad [B] (verification not implemented) . . . 2908
Reduce [F] . . . 2909

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{\sqrt{a}(2-n)}$$

output
```
2*arctanh(a^(1/2)*x/(a*x^2+b*x^n)^(1/2))/a^(1/2)/(2-n)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 78 vs. 2(37) = 74.

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx=-\frac{2\sqrt{b}x^{n/2}\sqrt{1+\frac{ax^{2-n}}{b}}\text{arcsinh}\left(\frac{\sqrt{a}x^{1-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}(-2+n)\sqrt{ax^2+bx^n}}$$

input
```
Integrate[1/Sqrt[a*x^2 + b*x^n],x]
```

output
```
(-2*Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(2 - n))/b]*ArcSinh[(Sqrt[a]*x^(1 - n/2)
)/Sqrt[b]])/(Sqrt[a]*(-2 + n)*Sqrt[a*x^2 + b*x^n])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax^2+bx^n}}dx$$

$$\Big\downarrow \text{1914}$$

$$\frac{2\int \frac{1}{1-\frac{ax^2}{bx^n+ax^2}}d\frac{x}{\sqrt{bx^n+ax^2}}}{2-n}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{\sqrt{a}(2-n)}$$

input
```
Int[1/Sqrt[a*x^2 + b*x^n],x]
```

output
```
(2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^n]])/(Sqrt[a]*(2 - n))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

Maple [F]

$$\int \frac{1}{\sqrt{a\,x^2+b\,x^n}}dx$$

input
```
int(1/(a*x^2+b*x^n)^(1/2),x)
```

output
```
int(1/(a*x^2+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x^2+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx = \int \frac{1}{\sqrt{ax^2+bx^n}}\,dx$$

input
```
integrate(1/(a*x**2+b*x**n)**(1/2),x)
```

output
```
Integral(1/sqrt(a*x**2 + b*x**n), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{ax^2 + bx^n}}\,dx = \int \frac{1}{\sqrt{ax^2 + bx^n}}\,dx$$

input
```
integrate(1/(a*x^2+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a*x^2 + b*x^n), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{ax^2 + bx^n}}\,dx = \int \frac{1}{\sqrt{ax^2 + bx^n}}\,dx$$

input
```
integrate(1/(a*x^2+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(a*x^2 + b*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.81

$$\int \frac{1}{\sqrt{ax^2 + bx^n}}\,dx = \frac{\sqrt{b}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,x^{1-\frac{n}{2}}\,1\mathrm{i}}{\sqrt{b}}\right)\,\sqrt{\frac{a\,x^{2-n}}{b}+1}\,1\mathrm{i}}{\sqrt{a}\,\left(\frac{n}{2}-1\right)\,\sqrt{b\,x^n+a\,x^2}}$$

input
```
int(1/(b*x^n + a*x^2)^(1/2),x)
```

output
```
(b^(1/2)*x^(n/2)*asin((a^(1/2)*x^(1 - n/2)*1i)/b^(1/2))*((a*x^(2 - n))/b +
 1)^(1/2)*1i)/(a^(1/2)*(n/2 - 1)*(b*x^n + a*x^2)^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{ax^2 + bx^n}}\,dx = \int \frac{1}{\sqrt{x^n b + a\,x^2}}dx$$

input
```
int(1/(a*x^2+b*x^n)^(1/2),x)
```

output
```
int(1/sqrt(x**n*b + a*x**2),x)
```

3.372 $\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx$

Optimal result . . . 2910
Mathematica [A] (verified) . . . 2910
Rubi [A] (verified) . . . 2911
Maple [F] . . . 2912
Fricas [F(-2)] . . . 2912
Sympy [F] . . . 2913
Maxima [F] . . . 2913
Giac [F] . . . 2913
Mupad [F(-1)] . . . 2914
Reduce [F] . . . 2914

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx=\frac{2\sqrt{x}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{\sqrt{a}(1-n)\sqrt{cx}}$$

output
```
2*x^(1/2)*arctanh(a^(1/2)*x^(1/2)/(a*x+b*x^n)^(1/2))/a^(1/2)/(1-n)/(c*x)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.71

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx=-\frac{2\sqrt{b}x^{\frac{1+n}{2}}\sqrt{1+\frac{ax^{1-n}}{b}}\text{arcsinh}\left(\frac{\sqrt{a}x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}(-1+n)\sqrt{cx}\sqrt{ax+bx^n}}$$

input
```
Integrate[1/(Sqrt[c*x]*Sqrt[a*x + b*x^n]),x]
```

output
```
(-2*Sqrt[b]*x^((1 + n)/2)*Sqrt[1 + (a*x^(1 - n))/b]*ArcSinh[(Sqrt[a]*x^(1/
2 - n/2))/Sqrt[b]])/(Sqrt[a]*(-1 + n)*Sqrt[c*x]*Sqrt[a*x + b*x^n])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}dx$$

$$\Big\downarrow \text{1937}$$

$$\frac{\sqrt{x}\int \frac{1}{\sqrt{x}\sqrt{bx^n+ax}}dx}{\sqrt{cx}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{2\sqrt{x}\int \frac{1}{1-\frac{ax}{bx^n+ax}}d\frac{\sqrt{x}}{\sqrt{bx^n+ax}}}{(1-n)\sqrt{cx}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{x}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{\sqrt{a}(1-n)\sqrt{cx}}$$

input
```
Int[1/(Sqrt[c*x]*Sqrt[a*x + b*x^n]),x]
```

output
```
(2*Sqrt[x]*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^n]])/(Sqrt[a]*(1 - n)*
Sqrt[c*x])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [F]

$$\int \frac{1}{\sqrt{cx}\,\sqrt{ax+b\,x^{n}}}dx$$

input
```
int(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x)
```

output
```
int(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx = \int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx$$

input
```
integrate(1/(c*x)**(1/2)/(a*x+b*x**n)**(1/2),x)
```

output
```
Integral(1/(sqrt(c*x)*sqrt(a*x + b*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx = \int \frac{1}{\sqrt{ax+bx^n}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*x^n)*sqrt(c*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx = \int \frac{1}{\sqrt{ax+bx^n}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*x + b*x^n)*sqrt(c*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx = \int \frac{1}{\sqrt{c\,x}\,\sqrt{b\,x^n+a\,x}}\,dx$$

input
```
int(1/((c*x)^(1/2)*(b*x^n + a*x)^(1/2)),x)
```

output
```
int(1/((c*x)^(1/2)*(b*x^n + a*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{x}\,\sqrt{x^n b+ax}}{x^n bx+a\,x^2}dx\right)}{c}$$

input
```
int(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(x)*sqrt(x**n*b + a*x))/(x**n*b*x + a*x**2),x))/c
```

3.373 $\int \frac{1}{cx\sqrt{a+bx^n}}\,dx$

Optimal result . . . 2915
Mathematica [A] (verified) . . . 2915
Rubi [A] (verified) . . . 2916
Maple [A] (verified) . . . 2917
Fricas [A] (verification not implemented) . . . 2918
Sympy [A] (verification not implemented) . . . 2918
Maxima [A] (verification not implemented) . . . 2918
Giac [F] . . . 2919
Mupad [F(-1)] . . . 2919
Reduce [F] . . . 2919

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \frac{1}{cx\sqrt{a+bx^n}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}cn}$$

output
```
-2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(1/2)/c/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{cx\sqrt{a+bx^n}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}cn}$$

input
```
Integrate[1/(c*x*Sqrt[a + b*x^n]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt[a]*c*n)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {27, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{cx\sqrt{a+bx^n}}dx$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{1}{x\sqrt{bx^n+a}}dx}{c}$$
$$\Big\downarrow \text{798}$$
$$\frac{\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n}{cn}$$
$$\Big\downarrow \text{73}$$
$$\frac{2\int \frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{bcn}$$
$$\Big\downarrow \text{221}$$
$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}cn}$$

input
```
Int[1/(c*x*Sqrt[a + b*x^n]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt[a]*c*n)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{\sqrt{a}\,c n}$	26
default	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{\sqrt{a}\,c n}$	26

input
```
int(1/c/x/(a+b*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(1/2)/c/n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.35

$$\int \frac{1}{cx\sqrt{a+bx^n}}\,dx = \left[\frac{\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)}{\sqrt{a}cn}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^n+a}}\right)}{acn}\right]$$

input
```
integrate(1/c/x/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
[log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n)/(sqrt(a)*c*n), 2*sqrt(
-a)*arctan(sqrt(-a)/sqrt(b*x^n + a))/(a*c*n)]
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{cx\sqrt{a+bx^n}}\,dx = -\frac{2\operatorname{asinh}\left(\frac{\sqrt{a}x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}cn}$$

input
```
integrate(1/c/x/(a+b*x**n)**(1/2),x)
```

output
```
-2*asinh(sqrt(a)/(sqrt(b)*x**(n/2)))/(sqrt(a)*c*n)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{1}{cx\sqrt{a+bx^n}}\,dx = \frac{\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{\sqrt{a}cn}$$

input
```
integrate(1/c/x/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/(sqrt(a)*c*n)
```

Giac [F]

$$\int \frac{1}{cx\sqrt{a+bx^n}}\,dx = \int \frac{1}{\sqrt{bx^n+acx}}\,dx$$

input
```
integrate(1/c/x/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^n + a)*c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{cx\sqrt{a+bx^n}}\,dx = \int \frac{1}{c\,x\,\sqrt{a+b\,x^n}}\,dx$$

input
```
int(1/(c*x*(a + b*x^n)^(1/2)),x)
```

output
```
int(1/(c*x*(a + b*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{cx\sqrt{a+bx^n}}\,dx = \frac{\int \frac{\sqrt{x^n b+a}}{x^n bx+ax}dx}{c}$$

input
```
int(1/c/x/(a+b*x^n)^(1/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**n*b*x + a*x),x)/c
```

3.374 $\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx$

Optimal result 2920
Mathematica [A] (verified) 2920
Rubi [A] (verified) 2921
Maple [F] 2922
Fricas [F(-2)] 2922
Sympy [F] 2923
Maxima [F] 2923
Giac [F] 2923
Mupad [F(-1)] 2924
Reduce [F] 2924

Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx=-\frac{2\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{\sqrt{a}c(1+n)\sqrt{cx}}$$

output
```
-2*x^(1/2)*arctanh(a^(1/2)/x^(1/2)/(a/x+b*x^n)^(1/2))/a^(1/2)/c/(1+n)/(c*x
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx=-\frac{2x\sqrt{a+bx^{1+n}}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^{1+n}}}{\sqrt{a}}\right)}{\sqrt{a}(1+n)(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}$$

input
```
Integrate[1/((c*x)^(3/2)*Sqrt[a/x + b*x^n]),x]
```

output
```
(-2*x*Sqrt[a + b*x^(1 + n)]*ArcTanh[Sqrt[a + b*x^(1 + n)]/Sqrt[a]])/(Sqrt[
a]*(1 + n)*(c*x)^(3/2)*Sqrt[a/x + b*x^n])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}dx$$

$$\Big\downarrow \text{1937}$$

$$\frac{\sqrt{x}\int \frac{1}{x^{3/2}\sqrt{bx^n+\frac{a}{x}}}dx}{c\sqrt{cx}}$$

$$\Big\downarrow \text{1935}$$

$$-\frac{2\sqrt{x}\int \frac{1}{1-\frac{a}{x\left(bx^n+\frac{a}{x}\right)}}d\frac{1}{\sqrt{x}\sqrt{bx^n+\frac{a}{x}}}}{c(n+1)\sqrt{cx}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\sqrt{x}\text{arctanh}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{\sqrt{a}c(n+1)\sqrt{cx}}$$

input
```
Int[1/((c*x)^(3/2)*Sqrt[a/x + b*x^n]),x]
```

output
```
(-2*Sqrt[x]*ArcTanh[Sqrt[a]/(Sqrt[x]*Sqrt[a/x + b*x^n])])/(Sqrt[a]*c*(1 +
n)*Sqrt[c*x])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{3}{2}} \sqrt{\frac{a}{x}+b\,x^{n}}}dx$$

input
```
int(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x)
```

output
```
int(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx=\int \frac{1}{(cx)^{\frac{3}{2}}\sqrt{\frac{a}{x}+bx^n}}\,dx$$

input
```
integrate(1/(c*x)**(3/2)/(a/x+b*x**n)**(1/2),x)
```

output
```
Integral(1/((c*x)**(3/2)*sqrt(a/x + b*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx=\int \frac{1}{\sqrt{bx^n+\frac{a}{x}}\,(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^n + a/x)*(c*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx=\int \frac{1}{\sqrt{bx^n+\frac{a}{x}}\,(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^n + a/x)*(c*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx=\int \frac{1}{(c\,x)^{3/2}\,\sqrt{b\,x^n+\frac{a}{x}}}\,dx$$

input
```
int(1/((c*x)^(3/2)*(b*x^n + a/x)^(1/2)),x)
```

output
```
int(1/((c*x)^(3/2)*(b*x^n + a/x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx=\frac{\sqrt{c}\left(\int \frac{\sqrt{x^n b x+a}}{x^n b\,x^2+a x}dx\right)}{c^2}$$

input
```
int(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x)
```

output
```
(sqrt(c)*int(sqrt(x**n*b*x + a)/(x**n*b*x**2 + a*x),x))/c**2
```

3.375 $\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx$

Optimal result . . . 2925
Mathematica [A] (verified) . . . 2925
Rubi [A] (verified) . . . 2926
Maple [F] . . . 2927
Fricas [F(-2)] . . . 2927
Sympy [F] . . . 2928
Maxima [F] . . . 2928
Giac [F] . . . 2928
Mupad [F(-1)] . . . 2929
Reduce [F] . . . 2929

Optimal result

Integrand size = 22, antiderivative size = 40

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{\sqrt{a}c^2(2+n)}$$

output
```
-2*arctanh(a^(1/2)/x/(a/x^2+b*x^n)^(1/2))/a^(1/2)/c^2/(2+n)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx=-\frac{2\sqrt{a+bx^{2+n}}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^{2+n}}}{\sqrt{a}}\right)}{\sqrt{a}c^2(2+n)x\sqrt{\frac{a}{x^2}+bx^n}}$$

input
```
Integrate[1/(c^2*x^2*Sqrt[a/x^2 + b*x^n]),x]
```

output
```
(-2*Sqrt[a + b*x^(2 + n)]*ArcTanh[Sqrt[a + b*x^(2 + n)]/Sqrt[a]])/(Sqrt[a]
*c^2*(2 + n)*x*Sqrt[a/x^2 + b*x^n])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {27, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}dx$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{1}{x^2\sqrt{bx^n+\frac{a}{x^2}}}dx}{c^2}$$

$$\Big\downarrow \ 1935$$

$$-\frac{2\int \frac{1}{1-\frac{a}{x^2\left(bx^n+\frac{a}{x^2}\right)}}d\frac{1}{x\sqrt{bx^n+\frac{a}{x^2}}}}{c^2(n+2)}$$

$$\Big\downarrow \ 219$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{\sqrt{a}c^2(n+2)}$$

input
```
Int[1/(c^2*x^2*Sqrt[a/x^2 + b*x^n]),x]
```

output
```
(-2*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x^n])])/(Sqrt[a]*c^2*(2 + n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Maple [F]

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+b\,x^n}}dx$$

input
```
int(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x)
```

output
```
int(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx = \frac{\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx}{c^2}$$

input
```
integrate(1/c**2/x**2/(a/x**2+b*x**n)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a/x**2 + b*x**n)), x)/c**2
```

Maxima [F]

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx = \int \frac{1}{\sqrt{bx^n+\frac{a}{x^2}}c^2x^2}\,dx$$

input
```
integrate(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^n + a/x^2)*x^2), x)/c^2
```

Giac [F]

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx = \int \frac{1}{\sqrt{bx^n+\frac{a}{x^2}}c^2x^2}\,dx$$

input
```
integrate(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^n + a/x^2)*c^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx=\int \frac{1}{c^2\,x^2\,\sqrt{b\,x^n+\frac{a}{x^2}}}\,dx$$

input
```
int(1/(c^2*x^2*(b*x^n + a/x^2)^(1/2)),x)
```

output
```
int(1/(c^2*x^2*(b*x^n + a/x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx=\frac{\int \frac{1}{\sqrt{x^n b\,x^2+a}\,x}dx}{c^2}$$

input
```
int(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x)
```

output
```
int(1/(sqrt(x**n*b*x**2 + a)*x),x)/c**2
```

3.376 $\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx$

Optimal result . . . 2930
Mathematica [A] (verified) . . . 2930
Rubi [A] (verified) . . . 2931
Maple [F] . . . 2932
Fricas [F(-2)] . . . 2932
Sympy [F] . . . 2933
Maxima [F] . . . 2933
Giac [F] . . . 2933
Mupad [F(-1)] . . . 2934
Reduce [F] . . . 2934

Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx=-\frac{2\sqrt{x}\,\text{arctanh}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{\sqrt{a}c^2(3+n)\sqrt{cx}}$$

output
```
-2*x^(1/2)*arctanh(a^(1/2)/x^(3/2)/(a/x^3+b*x^n)^(1/2))/a^(1/2)/c^2/(3+n)/
(c*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx=-\frac{2x\sqrt{a+bx^{3+n}}\,\text{arctanh}\left(\frac{\sqrt{a+bx^{3+n}}}{\sqrt{a}}\right)}{\sqrt{a}(3+n)(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}$$

input
```
Integrate[1/((c*x)^(5/2)*Sqrt[a/x^3 + b*x^n]),x]
```

output
```
(-2*x*Sqrt[a + b*x^(3 + n)]*ArcTanh[Sqrt[a + b*x^(3 + n)]/Sqrt[a]])/(Sqrt[
a]*(3 + n)*(c*x)^(5/2)*Sqrt[a/x^3 + b*x^n])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx$$

$$\Big\downarrow \; 1937$$

$$\frac{\sqrt{x}\int \frac{1}{x^{5/2}\sqrt{bx^n+\frac{a}{x^3}}}dx}{c^2\sqrt{cx}}$$

$$\Big\downarrow \; 1935$$

$$-\frac{2\sqrt{x}\int \frac{1}{1-\frac{a}{x^3\left(bx^n+\frac{a}{x^3}\right)}}d\frac{1}{x^{3/2}\sqrt{bx^n+\frac{a}{x^3}}}}{c^2(n+3)\sqrt{cx}}$$

$$\Big\downarrow \; 219$$

$$-\frac{2\sqrt{x}\text{arctanh}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{\sqrt{a}c^2(n+3)\sqrt{cx}}$$

input
```
Int[1/((c*x)^(5/2)*Sqrt[a/x^3 + b*x^n]),x]
```

output
```
(-2*Sqrt[x]*ArcTanh[Sqrt[a]/(x^(3/2)*Sqrt[a/x^3 + b*x^n])])/(Sqrt[a]*c^2*(
3 + n)*Sqrt[c*x])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{5}{2}} \sqrt{\frac{a}{x^3}+b\,x^n}}dx$$

input
```
int(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x)
```

output
```
int(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx = \int \frac{1}{(cx)^{\frac{5}{2}}\sqrt{\frac{a}{x^3}+bx^n}}\,dx$$

input
```
integrate(1/(c*x)**(5/2)/(a/x**3+b*x**n)**(1/2),x)
```

output
```
Integral(1/((c*x)**(5/2)*sqrt(a/x**3 + b*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx = \int \frac{1}{\sqrt{bx^n+\frac{a}{x^3}}\,(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^n + a/x^3)*(c*x)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx = \int \frac{1}{\sqrt{bx^n+\frac{a}{x^3}}\,(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^n + a/x^3)*(c*x)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx = \int \frac{1}{(c\,x)^{5/2}\,\sqrt{b\,x^n+\frac{a}{x^3}}}\,dx$$

input
```
int(1/((c*x)^(5/2)*(b*x^n + a/x^3)^(1/2)),x)
```

output
```
int(1/((c*x)^(5/2)*(b*x^n + a/x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx = \frac{\int \frac{1}{\sqrt{x^n b\,x^3+a}\,x}dx}{\sqrt{c}\,c^2}$$

input
```
int(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x)
```

output
```
int(1/(sqrt(x**n*b*x**3 + a)*x),x)/(sqrt(c)*c**2)
```

3.377 $\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx$

Optimal result 2935
Mathematica [A] (verified) 2935
Rubi [A] (verified) 2936
Maple [F] 2938
Fricas [F(-2)] 2938
Sympy [F] 2938
Maxima [F] 2939
Giac [F] 2939
Mupad [F(-1)] 2939
Reduce [F] 2940

Optimal result

Integrand size = 27, antiderivative size = 107

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx = -\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}}+\frac{2x^{-3j/2}(cx)^{3j/2}\operatorname{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{a^{3/2}c(j-n)}$$

output
```
-2*(c*x)^(3/2*j)/a/c/(j-n)/(x^j)/(a*x^j+b*x^n)^(1/2)+2*(c*x)^(3/2*j)*arcta
nh(a^(1/2)*x^(1/2*j)/(a*x^j+b*x^n)^(1/2))/a^(3/2)/c/(j-n)/(x^(3/2*j))
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx = -\frac{2x^{-3j/2}(cx)^{3j/2}\left(\sqrt{a}x^{j/2}-\sqrt{b}x^{n/2}\sqrt{1+\frac{ax^{j-n}}{b}}\operatorname{arcsinh}\left(\frac{\sqrt{a}x^{\frac{j-n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}c(j-n)\sqrt{ax^j+bx^n}}$$

input
```
Integrate[(c*x)^(-1 + (3*j)/2)/(a*x^j + b*x^n)^(3/2),x]
```

output
```
(-2*(c*x)^((3*j)/2)*(Sqrt[a]*x^(j/2) - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(j -
n))/b]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[b]]))/(a^(3/2)*c*(j - n)*x^((3
*j)/2)*Sqrt[a*x^j + b*x^n])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1937, 1936, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{\frac{3j}{2}-1}}{(ax^j+bx^n)^{3/2}}dx$$

$$\Big\downarrow \text{1937}$$

$$\frac{x^{-3j/2}(cx)^{3j/2}\int \frac{x^{\frac{3j}{2}-1}}{(ax^j+bx^n)^{3/2}}dx}{c}$$

$$\Big\downarrow \text{1936}$$

$$\frac{x^{-3j/2}(cx)^{3j/2}\left(\frac{\int \frac{x^{\frac{j-2}{2}}}{\sqrt{ax^j+bx^n}}dx}{a}-\frac{2x^{j/2}}{a(j-n)\sqrt{ax^j+bx^n}}\right)}{c}$$

$$\Big\downarrow \text{1935}$$

$$\frac{x^{-3j/2}(cx)^{3j/2}\left(\frac{2\int \frac{1}{1-\frac{ax^j}{ax^j+bx^n}}d\frac{x^{j/2}}{\sqrt{ax^j+bx^n}}}{a(j-n)}-\frac{2x^{j/2}}{a(j-n)\sqrt{ax^j+bx^n}}\right)}{c}$$

$$\Big\downarrow \text{219}$$

$$\frac{x^{-3j/2}(cx)^{3j/2}\left(\frac{2\text{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{a^{3/2}(j-n)}-\frac{2x^{j/2}}{a(j-n)\sqrt{ax^j+bx^n}}\right)}{c}$$

input
```
Int[(c*x)^(-1 + (3*j)/2)/(a*x^j + b*x^n)^(3/2),x]
```

output
```
((c*x)^((3*j)/2)*((-2*x^(j/2))/(a*(j - n)*Sqrt[a*x^j + b*x^n]) + (2*ArcTan
h[(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(a^(3/2)*(j - n))))/(c*x^((3*j)/
2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1936
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (
IntegerQ[j] || GtQ[c, 0])
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [F]

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(a\,x^j+b\,x^n)^{\frac{3}{2}}}dx$$

input
```
int((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x)
```

output
```
int((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx = \int \frac{(cx)^{\frac{3j}{2}-1}}{(ax^j+bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)**(-1+3/2*j)/(a*x**j+b*x**n)**(3/2),x)
```

output
```
Integral((c*x)**(3*j/2 - 1)/(a*x**j + b*x**n)**(3/2), x)
```

Maxima [F]

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx=\int \frac{(cx)^{\frac{3}{2}j-1}}{(ax^j+bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(3/2*j - 1)/(a*x^j + b*x^n)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx=\int \frac{(cx)^{\frac{3}{2}j-1}}{(ax^j+bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(3/2*j - 1)/(a*x^j + b*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx=\int \frac{(c\,x)^{\frac{3\,j}{2}-1}}{(a\,x^j+b\,x^n)^{3/2}}\,dx$$

input
```
int((c*x)^((3*j)/2 - 1)/(a*x^j + b*x^n)^(3/2),x)
```

output
```
int((c*x)^((3*j)/2 - 1)/(a*x^j + b*x^n)^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx = \frac{c^{\frac{3j}{2}}\left(\int \frac{x^{\frac{3j}{2}}\sqrt{x^j a+x^n b}}{x^{2j}a^2x+2x^{j+n}abx+x^{2n}b^2x}dx\right)}{c}$$

input
```
int((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x)
```

output
```
(c**((3*j)/2)*int((x**((3*j)/2)*sqrt(x**j*a + x**n*b))/(x**(2*j)*a**2*x +
2*x**(j + n)*a*b*x + x**(2*n)*b**2*x),x))/c
```

3.378 $\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}} dx$

Optimal result . . . 2941
Mathematica [A] (verified) . . . 2941
Rubi [A] (verified) . . . 2942
Maple [F] . . . 2943
Fricas [F(-2)] . . . 2944
Sympy [F(-1)] . . . 2944
Maxima [F] . . . 2944
Giac [F] . . . 2945
Mupad [F(-1)] . . . 2945
Reduce [F] . . . 2945

Optimal result

Integrand size = 23, antiderivative size = 94

$$\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}}\,dx = -\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}+\frac{2c^3\sqrt{cx}\,\text{arctanh}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{a^{3/2}(3-n)\sqrt{x}}$$

output
```
-2*c^2*(c*x)^(3/2)/a/(3-n)/(a*x^3+b*x^n)^(1/2)+2*c^3*(c*x)^(1/2)*arctanh(a
^(1/2)*x^(3/2)/(a*x^3+b*x^n)^(1/2))/a^(3/2)/(3-n)/x^(1/2)
```

Mathematica [A] (verified)

Time = 2.73 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.16

$$\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}}\,dx = \frac{2c^3\sqrt{cx}\left(\sqrt{a}x^{3/2}-\sqrt{b}x^{n/2}\sqrt{1+\frac{ax^{3-n}}{b}}\,\text{arcsinh}\left(\frac{\sqrt{a}x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}(-3+n)\sqrt{x}\sqrt{ax^3+bx^n}}$$

input
```
Integrate[(c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2),x]
```

output
```
(2*c^3*Sqrt[c*x]*(Sqrt[a]*x^(3/2) - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(3 - n))
/b]*ArcSinh[(Sqrt[a]*x^(3/2 - n/2))/Sqrt[b]]))/(a^(3/2)*(-3 + n)*Sqrt[x]*S
qrt[a*x^3 + b*x^n])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1936, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1936}$$

$$\frac{c^3\int \frac{\sqrt{cx}}{\sqrt{bx^n+ax^3}}dx}{a}-\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}$$

$$\Big\downarrow \text{ 1937}$$

$$\frac{c^3\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{bx^n+ax^3}}dx}{a\sqrt{x}}-\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}$$

$$\Big\downarrow \text{ 1935}$$

$$\frac{2c^3\sqrt{cx}\int \frac{1}{1-\frac{ax^3}{bx^n+ax^3}}d\frac{x^{3/2}}{\sqrt{bx^n+ax^3}}}{a(3-n)\sqrt{x}}-\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2c^3\sqrt{cx}\text{arctanh}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{a^{3/2}(3-n)\sqrt{x}}-\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}$$

input
```
Int[(c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2),x]
```

output
```
(-2*c^2*(c*x)^(3/2))/(a*(3 - n)*Sqrt[a*x^3 + b*x^n]) + (2*c^3*Sqrt[c*x]*Ar
cTanh[(Sqrt[a]*x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(a^(3/2)*(3 - n)*Sqrt[x])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1936
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (
IntegerQ[j] || GtQ[c, 0])
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [F]

$$\int \frac{(cx)^{\frac{7}{2}}}{(a\,x^3+b\,x^n)^{\frac{3}{2}}}dx$$

input
```
int((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x)
```

output
```
int((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^{7/2}}{(ax^3 + bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(cx)^{7/2}}{(ax^3 + bx^n)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*x)**(7/2)/(a*x**3+b*x**n)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(cx)^{7/2}}{(ax^3 + bx^n)^{3/2}}\,dx = \int \frac{(cx)^{\frac{7}{2}}}{(ax^3 + bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}}\,dx=\int \frac{(cx)^{\frac{7}{2}}}{(ax^3+bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}}\,dx=\int \frac{(c\,x)^{7/2}}{(b\,x^n+a\,x^3)^{3/2}}\,dx$$

input
```
int((c*x)^(7/2)/(b*x^n + a*x^3)^(3/2),x)
```

output
```
int((c*x)^(7/2)/(b*x^n + a*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}}\,dx=\sqrt{c}\left(\int \frac{\sqrt{x}\,x^3}{x^n\sqrt{x^nb+a\,x^3}\,b+\sqrt{x^nb+a\,x^3}\,a\,x^3}dx\right)c^3$$

input
```
int((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x)
```

output
```
sqrt(c)*int((sqrt(x)*x**3)/(x**n*sqrt(x**n*b + a*x**3)*b + sqrt(x**n*b + a
*x**3)*a*x**3),x)*c**3
```

3.379 $\int \frac{c^2x^2}{\left(ax^2+bx^n\right)^{3/2}}\,dx$

Optimal result . . . 2946
Mathematica [A] (verified) . . . 2946
Rubi [A] (verified) . . . 2947
Maple [F] . . . 2948
Fricas [F(-2)] . . . 2949
Sympy [F] . . . 2949
Maxima [F] . . . 2949
Giac [F] . . . 2950
Mupad [F(-1)] . . . 2950
Reduce [F] . . . 2950

Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{c^2x^2}{\left(ax^2+bx^n\right)^{3/2}}\,dx=-\frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}}+\frac{2c^2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{a^{3/2}(2-n)}$$

output
```
-2*c^2*x/a/(2-n)/(a*x^2+b*x^n)^(1/2)+2*c^2*arctanh(a^(1/2)*x/(a*x^2+b*x^n)
^(1/2))/a^(3/2)/(2-n)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

$$\int \frac{c^2x^2}{\left(ax^2+bx^n\right)^{3/2}}\,dx=\frac{2c^2\left(\sqrt{a}x-\sqrt{b}x^{n/2}\sqrt{1+\frac{ax^{2-n}}{b}}\text{arcsinh}\left(\frac{\sqrt{a}x^{1-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}(-2+n)\sqrt{ax^2+bx^n}}$$

input
```
Integrate[(c^2*x^2)/(a*x^2 + b*x^n)^(3/2),x]
```

output
```
(2*c^2*(Sqrt[a]*x - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(2 - n))/b]*ArcSinh[(Sqr
t[a]*x^(1 - n/2))/Sqrt[b]]))/(a^(3/2)*(-2 + n)*Sqrt[a*x^2 + b*x^n])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {27, 1936, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx$$

$$\Big\downarrow \ 27$$

$$c^2\int \frac{x^2}{(bx^n+ax^2)^{3/2}}dx$$

$$\Big\downarrow \ 1936$$

$$c^2\left(\frac{\int \frac{1}{\sqrt{bx^n+ax^2}}dx}{a}-\frac{2x}{a(2-n)\sqrt{ax^2+bx^n}}\right)$$

$$\Big\downarrow \ 1914$$

$$c^2\left(\frac{2\int \frac{1}{1-\frac{ax^2}{bx^n+ax^2}}d\frac{x}{\sqrt{bx^n+ax^2}}}{a(2-n)}-\frac{2x}{a(2-n)\sqrt{ax^2+bx^n}}\right)$$

$$\Big\downarrow \ 219$$

$$c^2\left(\frac{2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{a^{3/2}(2-n)}-\frac{2x}{a(2-n)\sqrt{ax^2+bx^n}}\right)$$

input
```
Int[(c^2*x^2)/(a*x^2 + b*x^n)^(3/2),x]
```

output
```
c^2*((-2*x)/(a*(2 - n)*Sqrt[a*x^2 + b*x^n]) + (2*ArcTanh[(Sqrt[a]*x)/Sqrt[
a*x^2 + b*x^n]])/(a^(3/2)*(2 - n)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1936
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (
IntegerQ[j] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{c^2x^2}{\left(a\,x^2+b\,x^n\right)^{\frac{3}{2}}}dx$$

input
```
int(c^2*x^2/(a*x^2+b*x^n)^(3/2),x)
```

output
```
int(c^2*x^2/(a*x^2+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(c^2*x^2/(a*x^2+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx = c^2\int \frac{x^2}{ax^2\sqrt{ax^2+bx^n}+bx^n\sqrt{ax^2+bx^n}}\,dx$$

input
```
integrate(c**2*x**2/(a*x**2+b*x**n)**(3/2),x)
```

output
```
c**2*Integral(x**2/(a*x**2*sqrt(a*x**2 + b*x**n) + b*x**n*sqrt(a*x**2 + b*
x**n)), x)
```

Maxima [F]

$$\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx = \int \frac{c^2x^2}{(ax^2+bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate(c^2*x^2/(a*x^2+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
c^2*integrate(x^2/(a*x^2 + b*x^n)^(3/2), x)
```

Giac [F]

$$\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx = \int \frac{c^2x^2}{(ax^2+bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate(c^2*x^2/(a*x^2+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(c^2*x^2/(a*x^2 + b*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx = \int \frac{c^2\,x^2}{(b\,x^n+a\,x^2)^{3/2}}\,dx$$

input
```
int((c^2*x^2)/(b*x^n + a*x^2)^(3/2),x)
```

output
```
int((c^2*x^2)/(b*x^n + a*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx = \left(\int \frac{x^2}{x^n\sqrt{x^nb+a\,x^2}\,b+\sqrt{x^nb+a\,x^2}\,a\,x^2}dx\right)c^2$$

input
```
int(c^2*x^2/(a*x^2+b*x^n)^(3/2),x)
```

output
```
int(x**2/(x**n*sqrt(x**n*b + a*x**2)*b + sqrt(x**n*b + a*x**2)*a*x**2),x)*
c**2
```

3.380 $\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx$

Optimal result 2951
Mathematica [A] (verified) 2951
Rubi [A] (verified) 2952
Maple [F] 2953
Fricas [F(-2)] 2954
Sympy [F] 2954
Maxima [F] 2954
Giac [F] 2955
Mupad [F(-1)] 2955
Reduce [F] 2955

Optimal result

Integrand size = 21, antiderivative size = 85

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx = -\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}} + \frac{2c\sqrt{x}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a^{3/2}(1-n)\sqrt{cx}}$$

output
```
-2*(c*x)^(1/2)/a/(1-n)/(a*x+b*x^n)^(1/2)+2*c*x^(1/2)*arctanh(a^(1/2)*x^(1/
2)/(a*x+b*x^n)^(1/2))/a^(3/2)/(1-n)/(c*x)^(1/2)
```

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx = \frac{2\sqrt{cx}\left(\sqrt{a}\sqrt{x} - \sqrt{b}x^{n/2}\sqrt{1+\frac{ax^{1-n}}{b}}\,\text{arcsinh}\left(\frac{\sqrt{a}x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}(-1+n)\sqrt{x}\sqrt{ax+bx^n}}$$

input
```
Integrate[Sqrt[c*x]/(a*x + b*x^n)^(3/2),x]
```

output
```
(2*Sqrt[c*x]*(Sqrt[a]*Sqrt[x] - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(1 - n))/b]*
ArcSinh[(Sqrt[a]*x^(1/2 - n/2))/Sqrt[b]]))/(a^(3/2)*(-1 + n)*Sqrt[x]*Sqrt[
a*x + b*x^n])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1936, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}dx$$

$$\Big\downarrow \text{1936}$$

$$\frac{c\int \frac{1}{\sqrt{cx}\sqrt{bx^n+ax}}dx}{a}-\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{c\sqrt{x}\int \frac{1}{\sqrt{x}\sqrt{bx^n+ax}}dx}{a\sqrt{cx}}-\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{2c\sqrt{x}\int \frac{1}{1-\frac{ax}{bx^n+ax}}d\frac{\sqrt{x}}{\sqrt{bx^n+ax}}}{a(1-n)\sqrt{cx}}-\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2c\sqrt{x}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a^{3/2}(1-n)\sqrt{cx}}-\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}$$

input
```
Int[Sqrt[c*x]/(a*x + b*x^n)^(3/2),x]
```

output
```
(-2*Sqrt[c*x])/(a*(1 - n)*Sqrt[a*x + b*x^n]) + (2*c*Sqrt[x]*ArcTanh[(Sqrt[
a]*Sqrt[x])/Sqrt[a*x + b*x^n]])/(a^(3/2)*(1 - n)*Sqrt[c*x])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1936
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (
IntegerQ[j] || GtQ[c, 0])
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [F]

$$\int \frac{\sqrt{cx}}{(ax+b\,x^n)^{\frac{3}{2}}}dx$$

input
```
int((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x)
```

output
```
int((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx = \int \frac{\sqrt{cx}}{(ax+bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)**(1/2)/(a*x+b*x**n)**(3/2),x)
```

output
```
Integral(sqrt(c*x)/(a*x + b*x**n)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx = \int \frac{\sqrt{cx}}{(ax+bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x)/(a*x + b*x^n)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx = \int \frac{\sqrt{cx}}{(ax+bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x)/(a*x + b*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx = \int \frac{\sqrt{c\,x}}{(b\,x^n+a\,x)^{3/2}}\,dx$$

input
```
int((c*x)^(1/2)/(b*x^n + a*x)^(3/2),x)
```

output
```
int((c*x)^(1/2)/(b*x^n + a*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{x}\,\sqrt{x^nb+ax}}{x^{2n}b^2+2x^nabx+a^2x^2}dx\right)$$

input
```
int((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x)
```

output
```
sqrt(c)*int((sqrt(x)*sqrt(x**n*b + a*x))/(x**(2*n)*b**2 + 2*x**n*a*b*x + a
**2*x**2),x)
```

3.381 $\int \frac{1}{cx(a+bx^n)^{3/2}}\,dx$

Optimal result . . . 2956
Mathematica [A] (verified) . . . 2956
Rubi [A] (verified) . . . 2957
Maple [A] (verified) . . . 2958
Fricas [A] (verification not implemented) . . . 2959
Sympy [B] (verification not implemented) . . . 2959
Maxima [A] (verification not implemented) . . . 2960
Giac [F] . . . 2960
Mupad [F(-1)] . . . 2961
Reduce [F] . . . 2961

Optimal result

Integrand size = 18, antiderivative size = 54

$$\int \frac{1}{cx\,(a+bx^n)^{3/2}}\,dx=\frac{2}{acn\sqrt{a+bx^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}cn}$$

output
```
2/a/c/n/(a+b*x^n)^(1/2)-2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(3/2)/c/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{1}{cx\,(a+bx^n)^{3/2}}\,dx=\frac{\frac{2}{an\sqrt{a+bx^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}n}}{c}$$

input
```
Integrate[1/(c*x*(a + b*x^n)^(3/2)),x]
```

output
```
(2/(a*n*Sqrt[a + b*x^n]) - (2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(a^(3/2)*n
))/c
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {27, 798, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{cx\left(a+bx^n\right)^{3/2}}dx$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{1}{x(bx^n+a)^{3/2}}dx}{c}$$

$$\Big\downarrow \ 798$$

$$\frac{\int \frac{x^{-n}}{(bx^n+a)^{3/2}}dx^n}{cn}$$

$$\Big\downarrow \ 61$$

$$\frac{\frac{\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n}{a}+\frac{2}{a\sqrt{a+bx^n}}}{cn}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{2\int \frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{ab}+\frac{2}{a\sqrt{a+bx^n}}}{cn}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{2}{a\sqrt{a+bx^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}}}{cn}$$

input
```
Int[1/(c*x*(a + b*x^n)^(3/2)),x]
```

output
```
(2/(a*Sqrt[a + b*x^n]) - (2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/a^(3/2))/(c*
n)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{2}{a\sqrt{a+b\,x^{n}}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{cn}$	42
default	$\frac{\frac{2}{a\sqrt{a+b\,x^{n}}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{cn}$	42

input
```
int(1/c/x/(a+b*x^n)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/c/n*(2/a/(a+b*x^n)^(1/2)-2/a^(3/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.69

$$\int \frac{1}{cx\left(a+bx^n\right)^{3/2}}\,dx = \left[\frac{\left(\sqrt{a}bx^n + a^{\frac{3}{2}}\right)\log\left(\frac{bx^n - 2\sqrt{bx^n+a}\sqrt{a} + 2a}{x^n}\right) + 2\sqrt{bx^n+a}a}{a^2bcnx^n + a^3cn}, \frac{2\left(\left(\sqrt{-a}bx^n + \sqrt{-a}a\right)\right.}{a^2bc}\right.$$

input
```
integrate(1/c/x/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
[((sqrt(a)*b*x^n + a^(3/2))*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/
x^n) + 2*sqrt(b*x^n + a)*a)/(a^2*b*c*n*x^n + a^3*c*n), 2*((sqrt(-a)*b*x^n
+ sqrt(-a)*a)*arctan(sqrt(-a)/sqrt(b*x^n + a)) + sqrt(b*x^n + a)*a)/(a^2*b
*c*n*x^n + a^3*c*n)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(42) = 84.

Time = 1.17 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.43

$$\int \frac{1}{cx\left(a+bx^n\right)^{3/2}}\,dx = \frac{\frac{2a^3\sqrt{1+\frac{bx^n}{a}}}{a^{\frac{9}{2}}n + a^{\frac{7}{2}}bnx^n} + \frac{a^3\log\left(\frac{bx^n}{a}\right)}{a^{\frac{9}{2}}n + a^{\frac{7}{2}}bnx^n} - \frac{2a^3\log\left(\sqrt{1+\frac{bx^n}{a}}+1\right)}{a^{\frac{9}{2}}n + a^{\frac{7}{2}}bnx^n} + \frac{a^2bx^n\log\left(\frac{bx^n}{a}\right)}{a^{\frac{9}{2}}n + a^{\frac{7}{2}}bnx^n} - \frac{2a^2bx^n\log\left(\sqrt{1+\frac{bx^n}{a}}+\right.}{a^{\frac{9}{2}}n + a^{\frac{7}{2}}bnx^n}}{c}$$

input
```
integrate(1/c/x/(a+b*x**n)**(3/2),x)
```

output
```
(2*a**3*sqrt(1 + b*x**n/a)/(a**(9/2)*n + a**(7/2)*b*n*x**n) + a**3*log(b*x
**n/a)/(a**(9/2)*n + a**(7/2)*b*n*x**n) - 2*a**3*log(sqrt(1 + b*x**n/a) +
1)/(a**(9/2)*n + a**(7/2)*b*n*x**n) + a**2*b*x**n*log(b*x**n/a)/(a**(9/2)*
n + a**(7/2)*b*n*x**n) - 2*a**2*b*x**n*log(sqrt(1 + b*x**n/a) + 1)/(a**(9/
2)*n + a**(7/2)*b*n*x**n))/c
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

$$\int \frac{1}{cx\left(a+bx^n\right)^{3/2}}\,dx = \frac{\frac{\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}n} + \frac{2}{\sqrt{bx^n+a}an}}{c}$$

input
```
integrate(1/c/x/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
(log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/(a^(3/2)*n)
+ 2/(sqrt(b*x^n + a)*a*n))/c
```

Giac [F]

$$\int \frac{1}{cx\left(a+bx^n\right)^{3/2}}\,dx = \int \frac{1}{\left(bx^n+a\right)^{\frac{3}{2}}cx}\,dx$$

input
```
integrate(1/c/x/(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^(3/2)*c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{cx\left(a+bx^n\right)^{3/2}}\,dx=\int \frac{1}{c\,x\,\left(a+b\,x^n\right)^{3/2}}\,dx$$

input
```
int(1/(c*x*(a + b*x^n)^(3/2)),x)
```

output
```
int(1/(c*x*(a + b*x^n)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{cx\left(a+bx^n\right)^{3/2}}\,dx=\frac{\int \frac{\sqrt{x^n b+a}}{x^{2n}b^2x+2x^n abx+a^2x}dx}{c}$$

input
```
int(1/c/x/(a+b*x^n)^(3/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**(2*n)*b**2*x + 2*x**n*a*b*x + a**2*x),x)/c
```

3.382 $\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx$

Optimal result . . . 2962
Mathematica [A] (verified) . . . 2962
Rubi [A] (verified) . . . 2963
Maple [F] . . . 2964
Fricas [F(-2)] . . . 2965
Sympy [F] . . . 2965
Maxima [F] . . . 2965
Giac [F] . . . 2966
Mupad [F(-1)] . . . 2966
Reduce [F] . . . 2966

Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx=\frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}-\frac{2\sqrt{x}\,\text{arctanh}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{a^{3/2}c^2(1+n)\sqrt{cx}}$$

output
```
2/a/c^2/(1+n)/(c*x)^(1/2)/(a/x+b*x^n)^(1/2)-2*x^(1/2)*arctanh(a^(1/2)/x^(1
/2)/(a/x+b*x^n)^(1/2))/a^(3/2)/c^2/(1+n)/(c*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx=\frac{2\left(\sqrt{a}-\sqrt{a+bx^{1+n}}\,\text{arctanh}\left(\frac{\sqrt{a+bx^{1+n}}}{\sqrt{a}}\right)\right)}{a^{3/2}c^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}$$

input
```
Integrate[1/((c*x)^(5/2)*(a/x + b*x^n)^(3/2)),x]
```

output
```
(2*(Sqrt[a] - Sqrt[a + b*x^(1 + n)]*ArcTanh[Sqrt[a + b*x^(1 + n)]/Sqrt[a]]
))/(a^(3/2)*c^2*(1 + n)*Sqrt[c*x]*Sqrt[a/x + b*x^n])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1936, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}dx$$

$$\Big\downarrow \text{1936}$$

$$\frac{\int \frac{1}{(cx)^{3/2}\sqrt{bx^n+\frac{a}{x}}}dx}{ac}+\frac{2}{ac^2(n+1)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{\sqrt{x}\int \frac{1}{x^{3/2}\sqrt{bx^n+\frac{a}{x}}}dx}{ac^2\sqrt{cx}}+\frac{2}{ac^2(n+1)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{2}{ac^2(n+1)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}-\frac{2\sqrt{x}\int \frac{1}{1-\frac{a}{x\left(bx^n+\frac{a}{x}\right)}}d\frac{1}{\sqrt{x}\sqrt{bx^n+\frac{a}{x}}}}{ac^2(n+1)\sqrt{cx}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2}{ac^2(n+1)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}-\frac{2\sqrt{x}\text{arctanh}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{a^{3/2}c^2(n+1)\sqrt{cx}}$$

input
```
Int[1/((c*x)^(5/2)*(a/x + b*x^n)^(3/2)),x]
```

output
```
2/(a*c^2*(1 + n)*Sqrt[c*x]*Sqrt[a/x + b*x^n]) - (2*Sqrt[x]*ArcTanh[Sqrt[a]
/(Sqrt[x]*Sqrt[a/x + b*x^n])])/(a^(3/2)*c^2*(1 + n)*Sqrt[c*x])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1936
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (
IntegerQ[j] || GtQ[c, 0])
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{5}{2}} \left(\frac{a}{x} + b\,x^{n}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x)
```

output
```
int(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx = \int \frac{1}{(cx)^{\frac{5}{2}}\left(\frac{a}{x}+bx^n\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)**(5/2)/(a/x+b*x**n)**(3/2),x)
```

output
```
Integral(1/((c*x)**(5/2)*(a/x + b*x**n)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx = \int \frac{1}{\left(bx^n+\frac{a}{x}\right)^{\frac{3}{2}}(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a/x)^(3/2)*(c*x)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^n+\frac{a}{x}\right)^{\frac{3}{2}}(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a/x)^(3/2)*(c*x)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{(c\,x)^{5/2}\,\left(b\,x^n+\frac{a}{x}\right)^{3/2}}\,dx$$

input
```
int(1/((c*x)^(5/2)*(b*x^n + a/x)^(3/2)),x)
```

output
```
int(1/((c*x)^(5/2)*(b*x^n + a/x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx=\frac{\sqrt{c}\left(\int \frac{\sqrt{x^n bx+a}}{x^{2n}b^2x^3+2x^n ab\,x^2+a^2x}dx\right)}{c^3}$$

input
```
int(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x)
```

output
```
(sqrt(c)*int(sqrt(x**n*b*x + a)/(x**(2*n)*b**2*x**3 + 2*x**n*a*b*x**2 + a*
*2*x),x))/c**3
```

3.383 $\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx$

Optimal result 2967
Mathematica [A] (verified) 2967
Rubi [A] (verified) 2968
Maple [F] 2969
Fricas [F(-2)] 2970
Sympy [F] 2970
Maxima [F] 2970
Giac [F] 2971
Mupad [F(-1)] 2971
Reduce [F] 2971

Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx=\frac{2}{ac^4(2+n)x\sqrt{\frac{a}{x^2}+bx^n}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{a^{3/2}c^4(2+n)}$$

output
```
2/a/c^4/(2+n)/x/(a/x^2+b*x^n)^(1/2)-2*arctanh(a^(1/2)/x/(a/x^2+b*x^n)^(1/2
))/a^(3/2)/c^4/(2+n)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx=\frac{2\left(\sqrt{a}-\sqrt{a+bx^{2+n}}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^{2+n}}}{\sqrt{a}}\right)\right)}{a^{3/2}c^4(2+n)x\sqrt{\frac{a}{x^2}+bx^n}}$$

input
```
Integrate[1/(c^4*x^4*(a/x^2 + b*x^n)^(3/2)),x]
```

output
```
(2*(Sqrt[a] - Sqrt[a + b*x^(2 + n)]*ArcTanh[Sqrt[a + b*x^(2 + n)]/Sqrt[a]]
))/(a^(3/2)*c^4*(2 + n)*x*Sqrt[a/x^2 + b*x^n])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {27, 1936, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{x^4\left(bx^n+\frac{a}{x^2}\right)^{3/2}}dx}{c^4}$$

$$\Big\downarrow \text{ 1936}$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{bx^n+\frac{a}{x^2}}}dx}{a}+\frac{2}{a(n+2)x\sqrt{\frac{a}{x^2}+bx^n}}}{c^4}$$

$$\Big\downarrow \text{ 1935}$$

$$\frac{\frac{2}{a(n+2)x\sqrt{\frac{a}{x^2}+bx^n}}-\frac{2\int \frac{1}{1-\frac{a}{x^2\left(bx^n+\frac{a}{x^2}\right)}}d\frac{1}{x\sqrt{bx^n+\frac{a}{x^2}}}}{a(n+2)}}{c^4}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{2}{a(n+2)x\sqrt{\frac{a}{x^2}+bx^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{a^{3/2}(n+2)}}{c^4}$$

input
```
Int[1/(c^4*x^4*(a/x^2 + b*x^n)^(3/2)),x]
```

output
```
(2/(a*(2 + n)*x*Sqrt[a/x^2 + b*x^n]) - (2*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 +
b*x^n])])/(a^(3/2)*(2 + n)))/c^4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1936
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (
IntegerQ[j] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{1}{c^4 x^4 \left(\frac{a}{x^2}+b\,x^n\right)^{\frac{3}{2}}}dx$$

input
```
int(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x)
```

output
```
int(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx = \frac{\int \frac{1}{ax^2\sqrt{\frac{a}{x^2}+bx^n}+bx^4x^n\sqrt{\frac{a}{x^2}+bx^n}}\,dx}{c^4}$$

input
```
integrate(1/c**4/x**4/(a/x**2+b*x**n)**(3/2),x)
```

output
```
Integral(1/(a*x**2*sqrt(a/x**2 + b*x**n) + b*x**4*x**n*sqrt(a/x**2 + b*x**
n)), x)/c**4
```

Maxima [F]

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx = \int \frac{1}{\left(bx^n+\frac{a}{x^2}\right)^{\frac{3}{2}}c^4x^4}\,dx$$

input
```
integrate(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a/x^2)^(3/2)*x^4), x)/c^4
```

Giac [F]

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^n+\frac{a}{x^2}\right)^{\frac{3}{2}}c^4x^4}\,dx$$

input
```
integrate(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a/x^2)^(3/2)*c^4*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{c^4\,x^4\left(b\,x^n+\frac{a}{x^2}\right)^{3/2}}\,dx$$

input
```
int(1/(c^4*x^4*(b*x^n + a/x^2)^(3/2)),x)
```

output
```
int(1/(c^4*x^4*(b*x^n + a/x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx=\frac{\int \frac{\sqrt{x^n b\,x^2+a}}{x^{2n}b^2x^5+2x^n ab\,x^3+a^2x}dx}{c^4}$$

input
```
int(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x)
```

output
```
int(sqrt(x**n*b*x**2 + a)/(x**(2*n)*b**2*x**5 + 2*x**n*a*b*x**3 + a**2*x),
x)/c**4
```

3.384 $\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx$

Optimal result . . . 2972
Mathematica [A] (verified) . . . 2972
Rubi [A] (verified) . . . 2973
Maple [F] . . . 2974
Fricas [F(-2)] . . . 2975
Sympy [F(-1)] . . . 2975
Maxima [F] . . . 2975
Giac [F] . . . 2976
Mupad [F(-1)] . . . 2976
Reduce [F] . . . 2976

Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx=\frac{2}{ac^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}-\frac{2\sqrt{x}\,\mathrm{arctanh}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{a^{3/2}c^5(3+n)\sqrt{cx}}$$

output
```
2/a/c^4/(3+n)/(c*x)^(3/2)/(a/x^3+b*x^n)^(1/2)-2*x^(1/2)*arctanh(a^(1/2)/x^
(3/2)/(a/x^3+b*x^n)^(1/2))/a^(3/2)/c^5/(3+n)/(c*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx=\frac{2\left(\sqrt{a}-\sqrt{a+bx^{3+n}}\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^{3+n}}}{\sqrt{a}}\right)\right)}{a^{3/2}c^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}$$

input
```
Integrate[1/((c*x)^(11/2)*(a/x^3 + b*x^n)^(3/2)),x]
```

output
```
(2*(Sqrt[a] - Sqrt[a + b*x^(3 + n)]*ArcTanh[Sqrt[a + b*x^(3 + n)]/Sqrt[a]]
))/(a^(3/2)*c^4*(3 + n)*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1936, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}dx$$

$$\Big\downarrow \text{1936}$$

$$\frac{\int \frac{1}{(cx)^{5/2}\sqrt{bx^n+\frac{a}{x^3}}}dx}{ac^3}+\frac{2}{ac^4(n+3)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{\sqrt{x}\int \frac{1}{x^{5/2}\sqrt{bx^n+\frac{a}{x^3}}}dx}{ac^5\sqrt{cx}}+\frac{2}{ac^4(n+3)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{2}{ac^4(n+3)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}-\frac{2\sqrt{x}\int \frac{1}{1-\frac{a}{x^3\left(bx^n+\frac{a}{x^3}\right)}}d\frac{1}{x^{3/2}\sqrt{bx^n+\frac{a}{x^3}}}}{ac^5(n+3)\sqrt{cx}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2}{ac^4(n+3)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}-\frac{2\sqrt{x}\text{arctanh}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{a^{3/2}c^5(n+3)\sqrt{cx}}$$

input
```
Int[1/((c*x)^(11/2)*(a/x^3 + b*x^n)^(3/2)),x]
```

output
```
2/(a*c^4*(3 + n)*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n]) - (2*Sqrt[x]*ArcTanh[Sqr
t[a]/(x^(3/2)*Sqrt[a/x^3 + b*x^n])])/(a^(3/2)*c^5*(3 + n)*Sqrt[c*x])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1936
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (
IntegerQ[j] || GtQ[c, 0])
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{11}{2}} \left(\frac{a}{x^3} + b\,x^n\right)^{\frac{3}{2}}} dx$$

input
```
int(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x)
```

output
```
int(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x)**(11/2)/(a/x**3+b*x**n)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx = \int \frac{1}{\left(bx^n+\frac{a}{x^3}\right)^{\frac{3}{2}}(cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a/x^3)^(3/2)*(c*x)^(11/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^n+\frac{a}{x^3}\right)^{\frac{3}{2}}(cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a/x^3)^(3/2)*(c*x)^(11/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{(c\,x)^{11/2}\,\left(b\,x^n+\frac{a}{x^3}\right)^{3/2}}\,dx$$

input
```
int(1/((c*x)^(11/2)*(b*x^n + a/x^3)^(3/2)),x)
```

output
```
int(1/((c*x)^(11/2)*(b*x^n + a/x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx=\frac{\int \frac{1}{x^n\sqrt{x^n b\,x^3+a}\,b\,x^4+\sqrt{x^n b\,x^3+a}\,a x}dx}{\sqrt{c}\,c^5}$$

input
```
int(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x)
```

output
```
int(1/(x**n*sqrt(x**n*b*x**3 + a)*b*x**4 + sqrt(x**n*b*x**3 + a)*a*x),x)/(
sqrt(c)*c**5)
```

3.385 $\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx$

Optimal result . . . 2977
Mathematica [A] (verified) . . . 2977
Rubi [A] (verified) . . . 2978
Maple [F] . . . 2979
Fricas [F(-2)] . . . 2980
Sympy [F] . . . 2980
Maxima [F] . . . 2980
Giac [F] . . . 2981
Mupad [F(-1)] . . . 2981
Reduce [F] . . . 2981

Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx=\frac{2}{ac^7(4+n)x^2\sqrt{\frac{a}{x^4}+bx^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{a^{3/2}c^7(4+n)}$$

output
```
2/a/c^7/(4+n)/x^2/(a/x^4+b*x^n)^(1/2)-2*arctanh(a^(1/2)/x^2/(a/x^4+b*x^n)^
(1/2))/a^(3/2)/c^7/(4+n)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx=\frac{2\left(\sqrt{a}-\sqrt{a+bx^{4+n}}\text{arctanh}\left(\frac{\sqrt{a+bx^{4+n}}}{\sqrt{a}}\right)\right)}{a^{3/2}c^7(4+n)x^2\sqrt{\frac{a}{x^4}+bx^n}}$$

input
```
Integrate[1/(c^7*x^7*(a/x^4 + b*x^n)^(3/2)),x]
```

output
```
(2*(Sqrt[a] - Sqrt[a + b*x^(4 + n)]*ArcTanh[Sqrt[a + b*x^(4 + n)]/Sqrt[a]]
))/(a^(3/2)*c^7*(4 + n)*x^2*Sqrt[a/x^4 + b*x^n])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = $\{27, 1936, 1935, 219\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{c^7 x^7 \left(\frac{a}{x^4}+b x^n\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{x^7 \left(b x^n+\frac{a}{x^4}\right)^{3/2}}dx}{c^7}$$

$$\Big\downarrow \text{1936}$$

$$\frac{\frac{\int \frac{1}{x^3 \sqrt{b x^n+\frac{a}{x^4}}}dx}{a}+\frac{2}{a (n+4) x^2 \sqrt{\frac{a}{x^4}+b x^n}}}{c^7}$$

$$\Big\downarrow \text{1935}$$

$$\frac{\frac{2}{a (n+4) x^2 \sqrt{\frac{a}{x^4}+b x^n}}-\frac{2 \int \frac{1}{1-\frac{a}{x^4 \left(b x^n+\frac{a}{x^4}\right)}}d\frac{1}{x^2 \sqrt{b x^n+\frac{a}{x^4}}}}{a (n+4)}}{c^7}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{2}{a (n+4) x^2 \sqrt{\frac{a}{x^4}+b x^n}}-\frac{2 \text{arctanh}\left(\frac{\sqrt{a}}{x^2 \sqrt{\frac{a}{x^4}+b x^n}}\right)}{a^{3/2} (n+4)}}{c^7}$$

input
```
Int[1/(c^7*x^7*(a/x^4 + b*x^n)^(3/2)),x]
```

output
```
(2/(a*(4 + n)*x^2*Sqrt[a/x^4 + b*x^n]) - (2*ArcTanh[Sqrt[a]/(x^2*Sqrt[a/x^
4 + b*x^n])])/(a^(3/2)*(4 + n)))/c^7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1936
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (
IntegerQ[j] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{1}{c^7 x^7 \left(\frac{a}{x^4}+b\,x^n\right)^{\frac{3}{2}}}dx$$

input
```
int(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x)
```

output
```
int(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx = \frac{\int \frac{1}{ax^3\sqrt{\frac{a}{x^4}+bx^n}+bx^7x^n\sqrt{\frac{a}{x^4}+bx^n}}\,dx}{c^7}$$

input
```
integrate(1/c**7/x**7/(a/x**4+b*x**n)**(3/2),x)
```

output
```
Integral(1/(a*x**3*sqrt(a/x**4 + b*x**n) + b*x**7*x**n*sqrt(a/x**4 + b*x**
n)), x)/c**7
```

Maxima [F]

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx = \int \frac{1}{\left(bx^n+\frac{a}{x^4}\right)^{\frac{3}{2}}c^7x^7}\,dx$$

input
```
integrate(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a/x^4)^(3/2)*x^7), x)/c^7
```

Giac [F]

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^n+\frac{a}{x^4}\right)^{\frac{3}{2}}c^7x^7}\,dx$$

input
```
integrate(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a/x^4)^(3/2)*c^7*x^7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{c^7\,x^7\,\left(b\,x^n+\frac{a}{x^4}\right)^{3/2}}\,dx$$

input
```
int(1/(c^7*x^7*(b*x^n + a/x^4)^(3/2)),x)
```

output
```
int(1/(c^7*x^7*(b*x^n + a/x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx=\frac{\int \frac{1}{x^n\sqrt{x^n b\,x^4+a}\,b\,x^5+\sqrt{x^n b\,x^4+a}\,a x}dx}{c^7}$$

input
```
int(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x)
```

output
```
int(1/(x**n*sqrt(x**n*b*x**4 + a)*b*x**5 + sqrt(x**n*b*x**4 + a)*a*x),x)/c
**7
```

3.386 $\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx$

Optimal result . . . 2982
Mathematica [A] (verified) . . . 2982
Rubi [A] (verified) . . . 2983
Maple [B] (verified) . . . 2984
Fricas [B] (verification not implemented) . . . 2985
Sympy [F] . . . 2985
Maxima [F] . . . 2986
Giac [F(-2)] . . . 2986
Mupad [F(-1)] . . . 2986
Reduce [B] (verification not implemented) . . . 2987

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x}+bx^2}}\right)}{3\sqrt{b}}$$

output
```
2/3*arctanh(b^(1/2)*x/(a/x+b*x^2)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx=\frac{2\sqrt{a+bx^3}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{3\sqrt{b}\sqrt{x}\sqrt{\frac{a+bx^3}{x}}}$$

input
```
Integrate[1/Sqrt[(a + b*x^3)/x],x]
```

output
```
(2*Sqrt[a + b*x^3]*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]])/(3*Sqrt[b]*Sqrt
[x]*Sqrt[(a + b*x^3)/x])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2078, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}dx$$

$$\Big\downarrow \text{ 2078}$$

$$\int \frac{1}{\sqrt{\frac{a}{x}+bx^2}}dx$$

$$\Big\downarrow \text{ 1914}$$

$$\frac{2}{3}\int \frac{1}{1-\frac{bx^2}{bx^2+\frac{a}{x}}}d\frac{x}{\sqrt{bx^2+\frac{a}{x}}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x}+bx^2}}\right)}{3\sqrt{b}}$$

input
```
Int[1/Sqrt[(a + b*x^3)/x],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x)/Sqrt[a/x + b*x^2]])/(3*Sqrt[b])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(24) = 48$.

Time = 0.35 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.75

method	result	size
default	$\frac{2(b\,x^3+a)\,\operatorname{arctanh}\left(\frac{\sqrt{x\left(b\,x^3+a\right)}}{x^2\sqrt{b}}\right)}{3\sqrt{\frac{b\,x^3+a}{x}}\,\sqrt{x(b\,x^3+a)}\,\sqrt{b}}$	56

input
```
int(1/((b*x^3+a)/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/((b*x^3+a)/x)^(1/2)*(b*x^3+a)/(x*(b*x^3+a))^(1/2)/b^(1/2)*arctanh((x*(
b*x^3+a))^(1/2)/x^2/b^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(24) = 48$.

Time = 0.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.47

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx = \left[\frac{\log\left(-8\,b^2x^6 - 8\,abx^3 - a^2 - 4\,(2\,bx^5 + ax^2)\sqrt{b}\sqrt{\frac{bx^3+a}{x}}\right)}{6\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(\frac{(2\,bx^3+a)\sqrt{-b}\sqrt{\frac{bx^3+a}{x}}}{2\,(b^2x^4+abx)}\right)}{3\,b}\right]$$

input
```
integrate(1/((b*x^3+a)/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*log(-8*b^2*x^6 - 8*a*b*x^3 - a^2 - 4*(2*b*x^5 + a*x^2)*sqrt(b)*sqrt((
b*x^3 + a)/x))/sqrt(b), -1/3*sqrt(-b)*arctan(1/2*(2*b*x^3 + a)*sqrt(-b)*sq
rt((b*x^3 + a)/x)/(b^2*x^4 + a*b*x))/b]
```

Sympy [F]

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx = \int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx$$

input
```
integrate(1/((b*x**3+a)/x)**(1/2),x)
```

output
```
Integral(1/sqrt((a + b*x**3)/x), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx = \int \frac{1}{\sqrt{\frac{bx^3+a}{x}}}\,dx$$

input
```
integrate(1/((b*x^3+a)/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((b*x^3 + a)/x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((b*x^3+a)/x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx = \int \frac{1}{\sqrt{\frac{b\,x^3+a}{x}}}\,dx$$

input
```
int(1/((a + b*x^3)/x)^(1/2),x)
```

output
```
int(1/((a + b*x^3)/x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.34

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx = \frac{\sqrt{b}\left(-\log\left(\sqrt{b\,x^3+a}-\sqrt{x}\,\sqrt{b}\,x\right)+\log\left(\sqrt{b\,x^3+a}+\sqrt{x}\,\sqrt{b}\,x\right)\right)}{3b}$$

input
```
int(1/((b*x^3+a)/x)^(1/2),x)
```

output
```
(sqrt(b)*( - log(sqrt(a + b*x**3) - sqrt(x)*sqrt(b)*x) + log(sqrt(a + b*x*
*3) + sqrt(x)*sqrt(b)*x)))/(3*b)
```

3.387 $\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx$

Optimal result 2988
Mathematica [A] (verified) 2988
Rubi [A] (verified) 2989
Maple [A] (verified) 2990
Fricas [A] (verification not implemented) 2990
Sympy [F] 2991
Maxima [F] 2991
Giac [A] (verification not implemented) 2992
Mupad [F(-1)] 2992
Reduce [B] (verification not implemented) 2992

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^2}+bx^2}}\right)}{2\sqrt{b}}$$

output
```
1/2*arctanh(b^(1/2)*x/(a/x^2+b*x^2)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.88

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx=\frac{\sqrt{a+bx^4}\log\left(\sqrt{b}x^2+\sqrt{a+bx^4}\right)}{2\sqrt{b}x\sqrt{\frac{a+bx^4}{x^2}}}$$

input
```
Integrate[1/Sqrt[(a + b*x^4)/x^2],x]
```

output
```
(Sqrt[a + b*x^4]*Log[Sqrt[b]*x^2 + Sqrt[a + b*x^4]])/(2*Sqrt[b]*x*Sqrt[(a
+ b*x^4)/x^2])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2078, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}dx$$

$$\Big\downarrow \text{2078}$$

$$\int \frac{1}{\sqrt{\frac{a}{x^2}+bx^2}}dx$$

$$\Big\downarrow \text{1914}$$

$$\frac{1}{2}\int \frac{1}{1-\frac{bx^2}{bx^2+\frac{a}{x^2}}}d\frac{x}{\sqrt{bx^2+\frac{a}{x^2}}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^2}+bx^2}}\right)}{2\sqrt{b}}$$

input
```
Int[1/Sqrt[(a + b*x^4)/x^2],x]
```

output
```
ArcTanh[(Sqrt[b]*x)/Sqrt[a/x^2 + b*x^2]]/(2*Sqrt[b])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.53

method	result	size
default	$\frac{\sqrt{b\,x^{4}+a}\,\ln\left(\sqrt{b}\,x^{2}+\sqrt{b\,x^{4}+a}\right)}{2\sqrt{\frac{b\,x^{4}+a}{x^{2}}}\,x\sqrt{b}}$	49

input
```
int(1/((b*x^4+a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((b*x^4+a)/x^2)^(1/2)/x*(b*x^4+a)^(1/2)*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2)
)/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.31

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx = \left[\frac{\log\left(-2\,bx^4 - 2\,\sqrt{b}x^3\sqrt{\frac{bx^4+a}{x^2}} - a\right)}{4\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{bx^4+a}{x^2}}}{bx}\right)}{2\,b}\right]$$

input
```
integrate(1/((b*x^4+a)/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log(-2*b*x^4 - 2*sqrt(b)*x^3*sqrt((b*x^4 + a)/x^2) - a)/sqrt(b), -1/2
*sqrt(-b)*arctan(sqrt(-b)*sqrt((b*x^4 + a)/x^2)/(b*x))/b]
```

Sympy [F]

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx = \int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx$$

input
```
integrate(1/((b*x**4+a)/x**2)**(1/2),x)
```

output
```
Integral(1/sqrt((a + b*x**4)/x**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx = \int \frac{1}{\sqrt{\frac{bx^4+a}{x^2}}}\,dx$$

input
```
integrate(1/((b*x^4+a)/x^2)^(1/2),x, algorithm="maxima")
```

output
```
b*integrate(x^5/(b*x^4 + a)^(3/2), x) + 1/2*x^2/sqrt(b*x^4 + a)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx = \frac{\log\left(|a|\right)\operatorname{sgn}(x)}{4\sqrt{b}} - \frac{\log\left(\left|-\sqrt{b}x^2+\sqrt{bx^4+a}\right|\right)}{2\sqrt{b}\operatorname{sgn}(x)}$$

input
```
integrate(1/((b*x^4+a)/x^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*log(abs(a))*sgn(x)/sqrt(b) - 1/2*log(abs(-sqrt(b)*x^2 + sqrt(b*x^4 + a
)))/(sqrt(b)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx = \int \frac{1}{\sqrt{\frac{b\,x^4+a}{x^2}}}\,dx$$

input
```
int(1/((a + b*x^4)/x^2)^(1/2),x)
```

output
```
int(1/((a + b*x^4)/x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx = \frac{\log\left(\frac{\sqrt{b\,x^4+a}+\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}}$$

input
```
int(1/((b*x^4+a)/x^2)^(1/2),x)
```

output
```
log((sqrt(a + b*x**4) + sqrt(b)*x**2)/sqrt(a))/(2*sqrt(b))
```

3.388 $\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}} \, dx$

Optimal result 2993
Mathematica [A] (verified) 2993
Rubi [A] (verified) 2994
Maple [F] 2995
Fricas [A] (verification not implemented) 2995
Sympy [F(-1)] 2996
Maxima [F] 2996
Giac [F(-2)] 2997
Mupad [F(-1)] 2997
Reduce [B] (verification not implemented) 2997

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}} \, dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^3}+bx^2}}\right)}{5\sqrt{b}}$$

output
```
2/5*arctanh(b^(1/2)*x/(a/x^3+b*x^2)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}} \, dx = \frac{2\sqrt{a+bx^5}\log\left(\sqrt{b}x^{5/2}+\sqrt{a+bx^5}\right)}{5\sqrt{b}x^{3/2}\sqrt{\frac{a+bx^5}{x^3}}}$$

input
```
Integrate[1/Sqrt[(a + b*x^5)/x^3],x]
```

output
```
(2*Sqrt[a + b*x^5]*Log[Sqrt[b]*x^(5/2) + Sqrt[a + b*x^5]])/(5*Sqrt[b]*x^(3
/2)*Sqrt[(a + b*x^5)/x^3])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2078, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}dx$$

$$\Big\downarrow \text{2078}$$

$$\int \frac{1}{\sqrt{\frac{a}{x^3}+bx^2}}dx$$

$$\Big\downarrow \text{1914}$$

$$\frac{2}{5}\int \frac{1}{1-\frac{bx^2}{bx^2+\frac{a}{x^3}}}d\frac{x}{\sqrt{bx^2+\frac{a}{x^3}}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^3}+bx^2}}\right)}{5\sqrt{b}}$$

input
```
Int[1/Sqrt[(a + b*x^5)/x^3],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x)/Sqrt[a/x^3 + b*x^2]])/(5*Sqrt[b])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{\frac{b\,x^{5}+a}{x^{3}}}}dx$$

input
```
int(1/((b*x^5+a)/x^3)^(1/2),x)
```

output
```
int(1/((b*x^5+a)/x^3)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.19

$$\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx = \left[\frac{\log\left(-8\,b^2x^{10} - 8\,abx^5 - a^2 - 4\,(2\,bx^9 + ax^4)\sqrt{b}\sqrt{\frac{bx^5+a}{x^3}}\right)}{10\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(\frac{2\,\sqrt{-b}x^4\sqrt{\frac{bx^5+a}{x^3}}}{2\,bx^5+a}\right)}{5\,b}\right]$$

input
```
integrate(1/((b*x^5+a)/x^3)^(1/2),x, algorithm="fricas")
```

output
```
[1/10*log(-8*b^2*x^10 - 8*a*b*x^5 - a^2 - 4*(2*b*x^9 + a*x^4)*sqrt(b)*sqrt
((b*x^5 + a)/x^3))/sqrt(b), -1/5*sqrt(-b)*arctan(2*sqrt(-b)*x^4*sqrt((b*x^
5 + a)/x^3)/(2*b*x^5 + a))/b]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx = \text{Timed out}$$

input
```
integrate(1/((b*x**5+a)/x**3)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx = \int \frac{1}{\sqrt{\frac{bx^5+a}{x^3}}}\,dx$$

input
```
integrate(1/((b*x^5+a)/x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((b*x^5 + a)/x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((b*x^5+a)/x^3)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx = \int \frac{1}{\sqrt{\frac{b\,x^5+a}{x^3}}}\,dx$$

input
```
int(1/((a + b*x^5)/x^3)^(1/2),x)
```

output
```
int(1/((a + b*x^5)/x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx = \frac{\sqrt{b}\left(-\log\left(\sqrt{b\,x^5+a}-\sqrt{x}\,\sqrt{b}\,x^2\right)+\log\left(\sqrt{b\,x^5+a}+\sqrt{x}\,\sqrt{b}\,x^2\right)\right)}{5b}$$

input
```
int(1/((b*x^5+a)/x^3)^(1/2),x)
```

output
```
(sqrt(b)*( - log(sqrt(a + b*x**5) - sqrt(x)*sqrt(b)*x**2) + log(sqrt(a + b
*x**5) + sqrt(x)*sqrt(b)*x**2)))/(5*b)
```

3.389 $\int \frac{1}{\sqrt{x^{2-n}(a+bx^n)}}\,dx$

Optimal result 2999
Mathematica [B] (verified) 2999
Rubi [A] (verified) 3000
Maple [F] 3001
Fricas [A] (verification not implemented) 3001
Sympy [F] 3002
Maxima [F] 3002
Giac [F] 3002
Mupad [F(-1)] 3003
Reduce [F] 3003

Optimal result

Integrand size = 19, antiderivative size = 37

$$\int \frac{1}{\sqrt{x^{2-n}\left(a+bx^n\right)}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+ax^{2-n}}}\right)}{\sqrt{b}n}$$

output
```
2*arctanh(b^(1/2)*x/(b*x^2+a*x^(2-n))^(1/2))/b^(1/2)/n
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 76 vs. 2(37) = 74.

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.05

$$\int \frac{1}{\sqrt{x^{2-n}\left(a+bx^n\right)}}\,dx=\frac{2\sqrt{a}x^{1-\frac{n}{2}}\sqrt{1+\frac{bx^n}{a}}\text{arcsinh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a}}\right)}{\sqrt{b}n\sqrt{x^{2-n}\left(a+bx^n\right)}}$$

input
```
Integrate[1/Sqrt[x^(2 - n)*(a + b*x^n)],x]
```

output
```
(2*Sqrt[a]*x^(1 - n/2)*Sqrt[1 + (b*x^n)/a]*ArcSinh[(Sqrt[b]*x^(n/2))/Sqrt[
a]])/(Sqrt[b]*n*Sqrt[x^(2 - n)*(a + b*x^n)])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2078, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^{2-n}(a+bx^n)}}dx$$
$$\Big\downarrow \text{2078}$$
$$\int \frac{1}{\sqrt{ax^{2-n}+bx^2}}dx$$
$$\Big\downarrow \text{1914}$$
$$\frac{2\int \frac{1}{1-\frac{bx^2}{ax^{2-n}+bx^2}}d\frac{x}{\sqrt{ax^{2-n}+bx^2}}}{n}$$
$$\Big\downarrow \text{219}$$
$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax^{2-n}+bx^2}}\right)}{\sqrt{b}n}$$

input
```
Int[1/Sqrt[x^(2 - n)*(a + b*x^n)],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^(2 - n)]])/(Sqrt[b]*n)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{x^{2-n}\,(a+b\,x^{n})}}dx$$

input
```
int(1/(x^(2-n)*(a+b*x^n))^(1/2),x)
```

output
```
int(1/(x^(2-n)*(a+b*x^n))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.00

$$\int \frac{1}{\sqrt{x^{2-n}\,(a+bx^{n})}}\,dx = \left[\frac{\log\left(\frac{2\,bxx^{n}+ax+2\,\sqrt{b}x^{n}\sqrt{\frac{bx^{2}x^{n}+ax^{2}}{x^{n}}}}{x}\right)}{\sqrt{b}n}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x^{n}\sqrt{\frac{bx^{2}x^{n}+ax^{2}}{x^{n}}}}{bxx^{n}+ax}\right)}{bn}\right]$$

input
```
integrate(1/(x^(2-n)*(a+b*x^n))^(1/2),x, algorithm="fricas")
```

output
```
[log((2*b*x*x^n + a*x + 2*sqrt(b)*x^n*sqrt((b*x^2*x^n + a*x^2)/x^n))/x)/(s
qrt(b)*n), -2*sqrt(-b)*arctan(sqrt(-b)*x^n*sqrt((b*x^2*x^n + a*x^2)/x^n)/(
b*x*x^n + a*x))/(b*n)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{x^{2-n}\left(a+bx^n\right)}}\,dx = \int \frac{1}{\sqrt{x^{2-n}\left(a+bx^n\right)}}\,dx$$

input
```
integrate(1/(x**(2-n)*(a+b*x**n))**(1/2),x)
```

output
```
Integral(1/sqrt(x**(2 - n)*(a + b*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x^{2-n}\left(a+bx^n\right)}}\,dx = \int \frac{1}{\sqrt{(bx^n+a)x^{-n+2}}}\,dx$$

input
```
integrate(1/(x^(2-n)*(a+b*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((b*x^n + a)*x^(-n + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x^{2-n}\left(a+bx^n\right)}}\,dx = \int \frac{1}{\sqrt{(bx^n+a)x^{-n+2}}}\,dx$$

input
```
integrate(1/(x^(2-n)*(a+b*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt((b*x^n + a)*x^(-n + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x^{2-n}\left(a+bx^n\right)}}\,dx=\int \frac{1}{\sqrt{x^{2-n}\,\left(a+b\,x^n\right)}}\,dx$$

input
```
int(1/(x^(2 - n)*(a + b*x^n))^(1/2),x)
```

output
```
int(1/(x^(2 - n)*(a + b*x^n))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{x^{2-n}\left(a+bx^n\right)}}\,dx=\int \frac{x^{\frac{n}{2}}\sqrt{x^nb+a}}{x^nbx+ax}dx$$

input
```
int(1/(x^(2-n)*(a+b*x^n))^(1/2),x)
```

output
```
int((x**(n/2)*sqrt(x**n*b + a))/(x**n*b*x + a*x),x)
```

3.390 $\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}} dx$

Optimal result . . . 3004
Mathematica [C] (verified) . . . 3004
Rubi [A] (verified) . . . 3005
Maple [B] (verified) . . . 3006
Fricas [B] (verification not implemented) . . . 3007
Sympy [F] . . . 3007
Maxima [F] . . . 3008
Giac [F(-2)] . . . 3008
Mupad [F(-1)] . . . 3008
Reduce [B] (verification not implemented) . . . 3009

Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}} dx = \frac{2 \arctan\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x}-bx^2}}\right)}{3\sqrt{b}}$$

output
```
2/3*arctan(b^(1/2)*x/(a/x-b*x^2)^(1/2))/b^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.18

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}} dx = -\frac{2i\sqrt{a-bx^3} \log\left(i\sqrt{b}x^{3/2}+\sqrt{a-bx^3}\right)}{3\sqrt{b}\sqrt{x}\sqrt{\frac{a-bx^3}{x}}}$$

input
```
Integrate[1/Sqrt[(a - b*x^3)/x],x]
```

output
```
(((-2*I)/3)*Sqrt[a - b*x^3]*Log[I*Sqrt[b]*x^(3/2) + Sqrt[a - b*x^3]])/(Sqr
t[b]*Sqrt[x]*Sqrt[(a - b*x^3)/x])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2078, 1914, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}dx$$

$$\Big\downarrow \text{ 2078}$$

$$\int \frac{1}{\sqrt{\frac{a}{x}-bx^2}}dx$$

$$\Big\downarrow \text{ 1914}$$

$$\frac{2}{3}\int \frac{1}{\frac{bx^2}{\frac{a}{x}-bx^2}+1}d\frac{x}{\sqrt{\frac{a}{x}-bx^2}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x}-bx^2}}\right)}{3\sqrt{b}}$$

input
```
Int[1/Sqrt[(a - b*x^3)/x],x]
```

output
```
(2*ArcTan[(Sqrt[b]*x)/Sqrt[a/x - b*x^2]])/(3*Sqrt[b])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(25) = 50$.

Time = 2.79 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.82

method	result	size
default	$-\frac{2(-b\,x^3+a)\arctan\left(\frac{\sqrt{x(-b\,x^3+a)}}{x^2\sqrt{b}}\right)}{3\sqrt{\frac{-b\,x^3+a}{x}}\,\sqrt{x(-b\,x^3+a)}\,\sqrt{b}}$	60

input
```
int(1/((-b*x^3+a)/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/((-b*x^3+a)/x)^(1/2)*(-b*x^3+a)/(x*(-b*x^3+a))^(1/2)/b^(1/2)*arctan((
x*(-b*x^3+a))^(1/2)/x^2/b^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(25) = 50$.

Time = 0.18 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.67

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx = \left[-\frac{\sqrt{-b}\log\left(-8\,b^2x^6 + 8\,abx^3 - a^2 + 4\,(2\,bx^5 - ax^2)\sqrt{-b}\sqrt{-\frac{bx^3-a}{x}}\right)}{6\,b}, -\frac{\arctan\left(\frac{(2\,bx^3-a)\sqrt{b}\sqrt{-\frac{bx^3-a}{x}}}{2\,(b^2x^4-abx)}\right)}{3\,\sqrt{b}}\right]$$

input
```
integrate(1/((-b*x^3+a)/x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/6*sqrt(-b)*log(-8*b^2*x^6 + 8*a*b*x^3 - a^2 + 4*(2*b*x^5 - a*x^2)*sqrt
(-b)*sqrt(-(b*x^3 - a)/x))/b, -1/3*arctan(1/2*(2*b*x^3 - a)*sqrt(b)*sqrt(-
(b*x^3 - a)/x)/(b^2*x^4 - a*b*x))/sqrt(b)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx = \int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx$$

input
```
integrate(1/((-b*x**3+a)/x)**(1/2),x)
```

output
```
Integral(1/sqrt((a - b*x**3)/x), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx = \int \frac{1}{\sqrt{-\frac{bx^3-a}{x}}}\,dx$$

input
```
integrate(1/((-b*x^3+a)/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-(b*x^3 - a)/x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((-b*x^3+a)/x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx = \int \frac{1}{\sqrt{\frac{a-b\,x^3}{x}}}\,dx$$

input
```
int(1/((a - b*x^3)/x)^(1/2),x)
```

output
```
int(1/((a - b*x^3)/x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.88

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx = -\frac{\sqrt{b}\,atan\left(\frac{\sqrt{x}\,\sqrt{b}\,\sqrt{-b\,x^3+a}\,a-2\sqrt{x}\,\sqrt{b}\,\sqrt{-b\,x^3+a}\,b\,x^3}{-2b^2x^5+2ab\,x^2}\right)}{3b}$$

input
```
int(1/((-b*x^3+a)/x)^(1/2),x)
```

output
```
( - sqrt(b)*atan((sqrt(x)*sqrt(b)*sqrt(a - b*x**3)*a - 2*sqrt(x)*sqrt(b)*s
qrt(a - b*x**3)*b*x**3)/(2*a*b*x**2 - 2*b**2*x**5)))/(3*b)
```

3.391 $\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx$

Optimal result . . . 3010
Mathematica [C] (verified) . . . 3010
Rubi [A] (verified) . . . 3011
Maple [A] (verified) . . . 3012
Fricas [A] (verification not implemented) . . . 3013
Sympy [F] . . . 3013
Maxima [F] . . . 3013
Giac [A] (verification not implemented) . . . 3014
Mupad [F(-1)] . . . 3014
Reduce [B] (verification not implemented) . . . 3014

Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^2}-bx^2}}\right)}{2\sqrt{b}}$$

output
```
1/2*arctan(b^(1/2)*x/(a/x^2-b*x^2)^(1/2))/b^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.06

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx=-\frac{i\sqrt{a-bx^4}\log\left(i\sqrt{b}x^2+\sqrt{a-bx^4}\right)}{2\sqrt{b}x\sqrt{\frac{a-bx^4}{x^2}}}$$

input
```
Integrate[1/Sqrt[(a - b*x^4)/x^2],x]
```

output
```
((-1/2*I)*Sqrt[a - b*x^4]*Log[I*Sqrt[b]*x^2 + Sqrt[a - b*x^4]])/(Sqrt[b]*x
*Sqrt[(a - b*x^4)/x^2])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2078, 1914, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}dx$$

$$\Big\downarrow \text{ 2078}$$

$$\int \frac{1}{\sqrt{\frac{a}{x^2}-bx^2}}dx$$

$$\Big\downarrow \text{ 1914}$$

$$\frac{1}{2}\int \frac{1}{\frac{bx^2}{\frac{a}{x^2}-bx^2}+1}d\frac{x}{\sqrt{\frac{a}{x^2}-bx^2}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^2}-bx^2}}\right)}{2\sqrt{b}}$$

input
```
Int[1/Sqrt[(a - b*x^4)/x^2],x]
```

output
```
ArcTan[(Sqrt[b]*x)/Sqrt[a/x^2 - b*x^2]]/(2*Sqrt[b])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

method	result	size
default	$\frac{\sqrt{-b\,x^{4}+a}\,\arctan\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{-b\,x^{4}+a}}\right)}{2\sqrt{\frac{-b\,x^{4}+a}{x^{2}}}\,x\sqrt{b}}$	51

input
```
int(1/((-b*x^4+a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((-b*x^4+a)/x^2)^(1/2)/x*(-b*x^4+a)^(1/2)/b^(1/2)*arctan(b^(1/2)*x^2/(
-b*x^4+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.33

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx = \left[-\frac{\sqrt{-b}\log\left(2\,bx^4-2\,\sqrt{-b}x^3\sqrt{-\frac{bx^4-a}{x^2}}-a\right)}{4\,b},\,-\frac{\arctan\left(\frac{\sqrt{-\frac{bx^4-a}{x^2}}}{\sqrt{b}x}\right)}{2\,\sqrt{b}}\right]$$

input
```
integrate(1/((-b*x^4+a)/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-b)*log(2*b*x^4 - 2*sqrt(-b)*x^3*sqrt(-(b*x^4 - a)/x^2) - a)/b,
 -1/2*arctan(sqrt(-(b*x^4 - a)/x^2)/(sqrt(b)*x))/sqrt(b)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx = \int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx$$

input
```
integrate(1/((-b*x**4+a)/x**2)**(1/2),x)
```

output
```
Integral(1/sqrt((a - b*x**4)/x**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx = \int \frac{1}{\sqrt{-\frac{bx^4-a}{x^2}}}\,dx$$

input
```
integrate(1/((-b*x^4+a)/x^2)^(1/2),x, algorithm="maxima")
```

output
```
b*integrate(x^5/((b*x^4 - a)*sqrt(-b*x^4 + a)), x) + 1/2*x^2/sqrt(-b*x^4 +
 a)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.42

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx = \frac{\log\left(|a|\right)\mathrm{sgn}(x)}{4\sqrt{-b}} - \frac{\log\left(\left|-\sqrt{-b}x^2+\sqrt{-bx^4+a}\right|\right)}{2\sqrt{-b}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/((-b*x^4+a)/x^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*log(abs(a))*sgn(x)/sqrt(-b) - 1/2*log(abs(-sqrt(-b)*x^2 + sqrt(-b*x^4
+ a)))/(sqrt(-b)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx = \int \frac{1}{\sqrt{\frac{a-b\,x^4}{x^2}}}\,dx$$

input
```
int(1/((a - b*x^4)/x^2)^(1/2),x)
```

output
```
int(1/((a - b*x^4)/x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx = \frac{asin\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}}$$

input
```
int(1/((-b*x^4+a)/x^2)^(1/2),x)
```

output
```
asin((sqrt(b)*x**2)/sqrt(a))/(2*sqrt(b))
```

3.392 $\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}} dx$

Optimal result . . . 3015
Mathematica [C] (verified) . . . 3015
Rubi [A] (verified) . . . 3016
Maple [F] . . . 3017
Fricas [A] (verification not implemented) . . . 3017
Sympy [F(-1)] . . . 3018
Maxima [F] . . . 3018
Giac [F(-2)] . . . 3019
Mupad [F(-1)] . . . 3019
Reduce [B] (verification not implemented) . . . 3019

Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}} dx = \frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^3}-bx^2}}\right)}{5\sqrt{b}}$$

output
```
2/5*arctan(b^(1/2)*x/(a/x^3-b*x^2)^(1/2))/b^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.18

$$\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}} dx = -\frac{2i\sqrt{a-bx^5}\log\left(i\sqrt{b}x^{5/2}+\sqrt{a-bx^5}\right)}{5\sqrt{b}x^{3/2}\sqrt{\frac{a-bx^5}{x^3}}}$$

input
```
Integrate[1/Sqrt[(a - b*x^5)/x^3],x]
```

output
```
(((-2*I)/5)*Sqrt[a - b*x^5]*Log[I*Sqrt[b]*x^(5/2) + Sqrt[a - b*x^5]])/(Sqr
t[b]*x^(3/2)*Sqrt[(a - b*x^5)/x^3])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2078, 1914, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}dx$$

$$\Big\downarrow \text{2078}$$

$$\int \frac{1}{\sqrt{\frac{a}{x^3}-bx^2}}dx$$

$$\Big\downarrow \text{1914}$$

$$\frac{2}{5}\int \frac{1}{\frac{bx^2}{\frac{a}{x^3}-bx^2}+1}d\frac{x}{\sqrt{\frac{a}{x^3}-bx^2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^3}-bx^2}}\right)}{5\sqrt{b}}$$

input
```
Int[1/Sqrt[(a - b*x^5)/x^3],x]
```

output
```
(2*ArcTan[(Sqrt[b]*x)/Sqrt[a/x^3 - b*x^2]])/(5*Sqrt[b])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{\frac{-b\,x^{5}+a}{x^{3}}}}dx$$

input
```
int(1/((-b*x^5+a)/x^3)^(1/2),x)
```

output
```
int(1/((-b*x^5+a)/x^3)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.36

$$\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx = \left[-\frac{\sqrt{-b}\log\left(-8\,b^2x^{10}+8\,abx^5-a^2+4\,(2\,bx^9-ax^4)\sqrt{-b}\sqrt{-\frac{bx^5-a}{x^3}}\right)}{10\,b}, -\frac{\arctan\left(\frac{2\,\sqrt{b}x^4\sqrt{-\frac{bx^5-a}{x^3}}}{2\,bx^5-a}\right)}{5\,\sqrt{b}}\right]$$

input
```
integrate(1/((-b*x^5+a)/x^3)^(1/2),x, algorithm="fricas")
```

output
```
[-1/10*sqrt(-b)*log(-8*b^2*x^10 + 8*a*b*x^5 - a^2 + 4*(2*b*x^9 - a*x^4)*sq
rt(-b)*sqrt(-(b*x^5 - a)/x^3))/b, -1/5*arctan(2*sqrt(b)*x^4*sqrt(-(b*x^5 -
 a)/x^3)/(2*b*x^5 - a))/sqrt(b)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx = \text{Timed out}$$

input
```
integrate(1/((-b*x**5+a)/x**3)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx = \int \frac{1}{\sqrt{-\frac{bx^5-a}{x^3}}}\,dx$$

input
```
integrate(1/((-b*x^5+a)/x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-(b*x^5 - a)/x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((-b*x^5+a)/x^3)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx = \int \frac{1}{\sqrt{\frac{a-b\,x^5}{x^3}}}\,dx$$

input
```
int(1/((a - b*x^5)/x^3)^(1/2),x)
```

output
```
int(1/((a - b*x^5)/x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.88

$$\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx = -\frac{\sqrt{b}\,atan\left(\frac{\sqrt{b}\,\sqrt{-b\,x^5+a}\,a-2\sqrt{b}\,\sqrt{-b\,x^5+a}\,b\,x^5}{2\sqrt{x}\,ab\,x^2-2\sqrt{x}\,b^2x^7}\right)}{5b}$$

input
```
int(1/((-b*x^5+a)/x^3)^(1/2),x)
```

output
```
( - sqrt(b)*atan((sqrt(b)*sqrt(a - b*x**5)*a - 2*sqrt(b)*sqrt(a - b*x**5)*
b*x**5)/(2*sqrt(x)*a*b*x**2 - 2*sqrt(x)*b**2*x**7)))/(5*b)
```

3.393 $\int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}}\,dx$

Optimal result . . . 3021
Mathematica [B] (verified) . . . 3021
Rubi [A] (verified) . . . 3022
Maple [F] . . . 3023
Fricas [A] (verification not implemented) . . . 3023
Sympy [F] . . . 3024
Maxima [F] . . . 3024
Giac [F] . . . 3024
Mupad [F(-1)] . . . 3025
Reduce [F] . . . 3025

Optimal result

Integrand size = 20, antiderivative size = 38

$$\int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}}\,dx = \frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{-bx^2+ax^{2-n}}}\right)}{\sqrt{b}n}$$

output
```
2*arctan(b^(1/2)*x/(-b*x^2+a*x^(2-n))^(1/2))/b^(1/2)/n
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 78 vs. 2(38) = 76.

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.05

$$\int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}}\,dx = \frac{2\sqrt{a}x^{1-\frac{n}{2}}\sqrt{1-\frac{bx^n}{a}}\arcsin\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a}}\right)}{\sqrt{b}n\sqrt{x^{2-n}(a-bx^n)}}$$

input
```
Integrate[1/Sqrt[x^(2 - n)*(a - b*x^n)],x]
```

output
```
(2*Sqrt[a]*x^(1 - n/2)*Sqrt[1 - (b*x^n)/a]*ArcSin[(Sqrt[b]*x^(n/2))/Sqrt[a
]])/(Sqrt[b]*n*Sqrt[x^(2 - n)*(a - b*x^n)])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2078, 1914, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}}dx$$

$$\Big\downarrow \text{2078}$$

$$\int \frac{1}{\sqrt{ax^{2-n}-bx^2}}dx$$

$$\Big\downarrow \text{1914}$$

$$\frac{2\int \frac{1}{\frac{bx^2}{ax^{2-n}-bx^2}+1}d\frac{x}{\sqrt{ax^{2-n}-bx^2}}}{n}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{ax^{2-n}-bx^2}}\right)}{\sqrt{b}n}$$

input
```
Int[1/Sqrt[x^(2 - n)*(a - b*x^n)],x]
```

output
```
(2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^(2 - n)]])/(Sqrt[b]*n)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{x^{2-n} \left(a-b\,x^{n}\right)}}dx$$

input
```
int(1/(x^(2-n)*(a-b*x^n))^(1/2),x)
```

output
```
int(1/(x^(2-n)*(a-b*x^n))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 3.13

$$\int \frac{1}{\sqrt{x^{2-n} (a-bx^n)}}\,dx = \left[-\frac{\sqrt{-b}\log\left(-\frac{2\,bxx^n-ax-2\,\sqrt{-b}x^n\sqrt{-\frac{bx^2x^n-ax^2}{x^n}}}{x}\right)}{bn}, -\frac{2\,\arctan\left(\frac{\sqrt{b}x^n\sqrt{-\frac{bx^2x^n-ax^2}{x^n}}}{bxx^n-ax}\right)}{\sqrt{b}n}\right]$$

input
```
integrate(1/(x^(2-n)*(a-b*x^n))^(1/2),x, algorithm="fricas")
```

output
```
[-sqrt(-b)*log(-(2*b*x*x^n - a*x - 2*sqrt(-b)*x^n*sqrt(-(b*x^2*x^n - a*x^2
)/x^n))/x)/(b*n), -2*arctan(sqrt(b)*x^n*sqrt(-(b*x^2*x^n - a*x^2)/x^n)/(b*
x*x^n - a*x))/(sqrt(b)*n)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}}\,dx = \int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}}\,dx$$

input
```
integrate(1/(x**(2-n)*(a-b*x**n))**(1/2),x)
```

output
```
Integral(1/sqrt(x**(2 - n)*(a - b*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}}\,dx = \int \frac{1}{\sqrt{-(bx^n-a)x^{-n+2}}}\,dx$$

input
```
integrate(1/(x^(2-n)*(a-b*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-(b*x^n - a)*x^(-n + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}}\,dx = \int \frac{1}{\sqrt{-(bx^n-a)x^{-n+2}}}\,dx$$

input
```
integrate(1/(x^(2-n)*(a-b*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-(b*x^n - a)*x^(-n + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x^{2-n}\,(a-bx^n)}}\,dx = \int \frac{1}{\sqrt{x^{2-n}\,(a-b\,x^n)}}\,dx$$

input
```
int(1/(x^(2 - n)*(a - b*x^n))^(1/2),x)
```

output
```
int(1/(x^(2 - n)*(a - b*x^n))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{x^{2-n}\,(a-bx^n)}}\,dx = -\left(\int \frac{x^{\frac{n}{2}}\sqrt{-x^n b+a}}{x^n bx-ax}dx\right)$$

input
```
int(1/(x^(2-n)*(a-b*x^n))^(1/2),x)
```

output
```
- int((x**(n/2)*sqrt( - x**n*b + a))/(x**n*b*x - a*x),x)
```

3.394 $\int \frac{1}{\sqrt{x^n(a+bx^{2-n})}}\,dx$

Optimal result 3026
Mathematica [B] (verified) 3026
Rubi [A] (verified) 3027
Maple [F] 3028
Fricas [F(-2)] 3028
Sympy [F] 3029
Maxima [F] 3029
Giac [F] 3029
Mupad [B] (verification not implemented) 3030
Reduce [F] 3030

Optimal result

Integrand size = 19, antiderivative size = 37

$$\int \frac{1}{\sqrt{x^n\,(a+bx^{2-n})}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}$$

output
```
2*arctanh(b^(1/2)*x/(b*x^2+a*x^n)^(1/2))/b^(1/2)/(2-n)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 78 vs. 2(37) = 74.

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{x^n\,(a+bx^{2-n})}}\,dx=-\frac{2\sqrt{a}x^{n/2}\sqrt{1+\frac{bx^{2-n}}{a}}\text{arcsinh}\left(\frac{\sqrt{b}x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(-2+n)\sqrt{bx^2+ax^n}}$$

input
```
Integrate[1/Sqrt[x^n*(a + b*x^(2 - n))],x]
```

output
```
(-2*Sqrt[a]*x^(n/2)*Sqrt[1 + (b*x^(2 - n))/a]*ArcSinh[(Sqrt[b]*x^(1 - n/2)
)/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[b*x^2 + a*x^n])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2078, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^n\left(a+bx^{2-n}\right)}}dx$$
$$\Big\downarrow \text{ 2078}$$
$$\int \frac{1}{\sqrt{ax^n+bx^2}}dx$$
$$\Big\downarrow \text{ 1914}$$
$$\frac{2\int \frac{1}{1-\frac{bx^2}{ax^n+bx^2}}d\frac{x}{\sqrt{ax^n+bx^2}}}{2-n}$$
$$\Big\downarrow \text{ 219}$$
$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}(2-n)}$$

input
```
Int[1/Sqrt[x^n*(a + b*x^(2 - n))],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{x^{n} \left(a +b \,x^{2-n}\right)}}d x$$

input
```
int(1/(x^n*(a+b*x^(2-n)))^(1/2),x)
```

output
```
int(1/(x^n*(a+b*x^(2-n)))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{x^n (a+bx^{2-n})}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(x^n*(a+b*x^(2-n)))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{x^n\left(a+bx^{2-n}\right)}}\,dx = \int \frac{1}{\sqrt{x^n\left(a+bx^{2-n}\right)}}\,dx$$

input
```
integrate(1/(x**n*(a+b*x**(2-n)))**(1/2),x)
```

output
```
Integral(1/sqrt(x**n*(a + b*x**(2 - n))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x^n\left(a+bx^{2-n}\right)}}\,dx = \int \frac{1}{\sqrt{(bx^{-n+2}+a)x^n}}\,dx$$

input
```
integrate(1/(x^n*(a+b*x^(2-n)))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((b*x^(-n + 2) + a)*x^n), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x^n\left(a+bx^{2-n}\right)}}\,dx = \int \frac{1}{\sqrt{(bx^{-n+2}+a)x^n}}\,dx$$

input
```
integrate(1/(x^n*(a+b*x^(2-n)))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt((b*x^(-n + 2) + a)*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 9.92 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.81

$$\int \frac{1}{\sqrt{x^n (a+bx^{2-n})}}\,dx = \frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{\frac{b\,x^{2-n}}{a}+1}\,1\mathrm{i}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\,\sqrt{a\,x^n+b\,x^2}}$$

input
```
int(1/(x^n*(a + b*x^(2 - n)))^(1/2),x)
```

output
```
(a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2)*1i)/a^(1/2))*((b*x^(2 - n))/a +
 1)^(1/2)*1i)/(b^(1/2)*(n/2 - 1)*(a*x^n + b*x^2)^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{x^n (a+bx^{2-n})}}\,dx = \int \frac{1}{\sqrt{x^n a+b\,x^2}}dx$$

input
```
int(1/(x^n*(a+b*x^(2-n)))^(1/2),x)
```

output
```
int(1/sqrt(x**n*a + b*x**2),x)
```

3.395 $\int \frac{1}{\sqrt{x^2(b+ax^{-2+n})}}\,dx$

Optimal result 3031
Mathematica [B] (verified) 3031
Rubi [A] (verified) 3032
Maple [F] 3033
Fricas [A] (verification not implemented) 3033
Sympy [F] 3034
Maxima [F] 3034
Giac [F] 3034
Mupad [B] (verification not implemented) 3035
Reduce [F] 3035

Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \frac{1}{\sqrt{x^2\left(b+ax^{-2+n}\right)}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}$$

output
```
2*arctanh(b^(1/2)*x/(b*x^2+a*x^n)^(1/2))/b^(1/2)/(2-n)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 78 vs. 2(37) = 74.

Time = 0.01 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{x^2\left(b+ax^{-2+n}\right)}}\,dx=-\frac{2\sqrt{a}x^{n/2}\sqrt{1+\frac{bx^{2-n}}{a}}\operatorname{arcsinh}\left(\frac{\sqrt{b}x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(-2+n)\sqrt{bx^2+ax^n}}$$

input
```
Integrate[1/Sqrt[x^2*(b + a*x^(-2 + n))],x]
```

output
```
(-2*Sqrt[a]*x^(n/2)*Sqrt[1 + (b*x^(2 - n))/a]*ArcSinh[(Sqrt[b]*x^(1 - n/2)
)/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[b*x^2 + a*x^n])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2078, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2 (ax^{n-2}+b)}} \, dx$$

$$\Big\downarrow \text{2078}$$

$$\int \frac{1}{\sqrt{ax^n+bx^2}}dx$$

$$\Big\downarrow \text{1914}$$

$$\frac{2\int \frac{1}{1-\frac{bx^2}{ax^n+bx^2}}d\frac{x}{\sqrt{ax^n+bx^2}}}{2-n}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}(2-n)}$$

input
```
Int[1/Sqrt[x^2*(b + a*x^(-2 + n))],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{x^2 \left(b+a\,x^{-2+n}\right)}}dx$$

input
```
int(1/(x^2*(b+a*x^(-2+n)))^(1/2),x)
```

output
```
int(1/(x^2*(b+a*x^(-2+n)))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.16

$$\int \frac{1}{\sqrt{x^2 \left(b+ax^{-2+n}\right)}}\,dx = \left[\frac{\sqrt{b}\log\left(\frac{axx^{n-2}+2\,bx-2\,\sqrt{ax^2x^{n-2}+bx^2}\sqrt{b}}{xx^{n-2}}\right)}{bn-2\,b}, \frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{ax^2x^{n-2}+bx^2}\sqrt{-b}}{axx^{n-2}+bx}\right)}{bn-2\,b}\right]$$

input
```
integrate(1/(x^2*(b+a*x^(-2+n)))^(1/2),x, algorithm="fricas")
```

output
```
[sqrt(b)*log((a*x*x^(n - 2) + 2*b*x - 2*sqrt(a*x^2*x^(n - 2) + b*x^2)*sqrt
(b))/(x*x^(n - 2)))/(b*n - 2*b), 2*sqrt(-b)*arctan(sqrt(a*x^2*x^(n - 2) +
b*x^2)*sqrt(-b)/(a*x*x^(n - 2) + b*x))/(b*n - 2*b)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{x^2 (b + a x^{-2+n})}}\,dx = \int \frac{1}{\sqrt{x^2 (a x^{n-2} + b)}}\,dx$$

input
```
integrate(1/(x**2*(b+a*x**(-2+n)))**(1/2),x)
```

output
```
Integral(1/sqrt(x**2*(a*x**(n - 2) + b)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x^2 (b + a x^{-2+n})}}\,dx = \int \frac{1}{\sqrt{(a x^{n-2} + b) x^2}}\,dx$$

input
```
integrate(1/(x^2*(b+a*x^(-2+n)))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((a*x^(n - 2) + b)*x^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x^2 (b + a x^{-2+n})}}\,dx = \int \frac{1}{\sqrt{(a x^{n-2} + b) x^2}}\,dx$$

input
```
integrate(1/(x^2*(b+a*x^(-2+n)))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt((a*x^(n - 2) + b)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.81

$$\int \frac{1}{\sqrt{x^2 (b + ax^{-2+n})}} \, dx = \frac{\sqrt{a}\, x^{n/2} \operatorname{asin}\left(\frac{\sqrt{b}\, x^{1-\frac{n}{2}}\, 1\mathrm{i}}{\sqrt{a}}\right) \sqrt{\frac{b\, x^{2-n}}{a} + 1}\, 1\mathrm{i}}{\sqrt{b}\, \left(\frac{n}{2} - 1\right) \sqrt{a\, x^n + b\, x^2}}$$

input
```
int(1/(x^2*(b + a*x^(n - 2)))^(1/2),x)
```

output
```
(a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2)*1i)/a^(1/2))*((b*x^(2 - n))/a +
 1)^(1/2)*1i)/(b^(1/2)*(n/2 - 1)*(a*x^n + b*x^2)^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{x^2 (b + ax^{-2+n})}} \, dx = \int \frac{1}{\sqrt{x^n a + b\, x^2}} dx$$

input
```
int(1/(x^2*(b+a*x^(-2+n)))^(1/2),x)
```

output
```
int(1/sqrt(x**n*a + b*x**2),x)
```

3.396 $\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx$

Optimal result . . . 3036
Mathematica [B] (verified) . . . 3036
Rubi [A] (verified) . . . 3037
Maple [F] . . . 3038
Fricas [F(-2)] . . . 3038
Sympy [F] . . . 3039
Maxima [F] . . . 3039
Giac [F] . . . 3039
Mupad [B] (verification not implemented) . . . 3040
Reduce [F] . . . 3040

Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}$$

output
```
2*arctanh(b^(1/2)*x/(b*x^2+a*x^n)^(1/2))/b^(1/2)/(2-n)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 78 vs. 2(37) = 74.

Time = 0.01 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx=-\frac{2\sqrt{a}x^{n/2}\sqrt{1+\frac{bx^{2-n}}{a}}\text{arcsinh}\left(\frac{\sqrt{b}x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(-2+n)\sqrt{bx^2+ax^n}}$$

input
```
Integrate[1/Sqrt[x*(b*x + a*x^(-1 + n))],x]
```

output
```
(-2*Sqrt[a]*x^(n/2)*Sqrt[1 + (b*x^(2 - n))/a]*ArcSinh[(Sqrt[b]*x^(1 - n/2)
)/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[b*x^2 + a*x^n])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2078, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x\left(ax^{n-1}+bx\right)}}dx$$
$$\Big\downarrow \text{2078}$$
$$\int \frac{1}{\sqrt{ax^n+bx^2}}dx$$
$$\Big\downarrow \text{1914}$$
$$\frac{2\int \frac{1}{1-\frac{bx^2}{ax^n+bx^2}}d\frac{x}{\sqrt{ax^n+bx^2}}}{2-n}$$
$$\Big\downarrow \text{219}$$
$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}(2-n)}$$

input
```
Int[1/Sqrt[x*(b*x + a*x^(-1 + n))],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{x\,(bx + a\,x^{-1+n})}}dx$$

input
```
int(1/(x*(b*x+a*x^(-1+n)))^(1/2),x)
```

output
```
int(1/(x*(b*x+a*x^(-1+n)))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{x\,(bx + ax^{-1+n})}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(x*(b*x+a*x^(-1+n)))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx=\int \frac{1}{\sqrt{x\left(ax^{n-1}+bx\right)}}\,dx$$

input
```
integrate(1/(x*(b*x+a*x**(-1+n)))**(1/2),x)
```

output
```
Integral(1/sqrt(x*(a*x**(n - 1) + b*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx=\int \frac{1}{\sqrt{(ax^{n-1}+bx)x}}\,dx$$

input
```
integrate(1/(x*(b*x+a*x^(-1+n)))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((a*x^(n - 1) + b*x)*x), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx=\int \frac{1}{\sqrt{(ax^{n-1}+bx)x}}\,dx$$

input
```
integrate(1/(x*(b*x+a*x^(-1+n)))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt((a*x^(n - 1) + b*x)*x), x)
```

Mupad [B] (verification not implemented)

Time = 9.41 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.81

$$\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx=\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{\frac{b\,x^{2-n}}{a}+1}\,1\mathrm{i}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\,\sqrt{a\,x^n+b\,x^2}}$$

input
```
int(1/(x*(b*x + a*x^(n - 1)))^(1/2),x)
```

output
```
(a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2)*1i)/a^(1/2))*((b*x^(2 - n))/a +
 1)^(1/2)*1i)/(b^(1/2)*(n/2 - 1)*(a*x^n + b*x^2)^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx=\int \frac{1}{\sqrt{x^n a+b\,x^2}}dx$$

input
```
int(1/(x*(b*x+a*x^(-1+n)))^(1/2),x)
```

output
```
int(1/sqrt(x**n*a + b*x**2),x)
```

3.397 $\int \frac{1}{\sqrt{x^n(a-bx^{2-n})}}\,dx$

Optimal result 3041
Mathematica [B] (verified) 3041
Rubi [A] (verified) 3042
Maple [F] 3043
Fricas [F(-2)] 3043
Sympy [F] 3044
Maxima [F] 3044
Giac [F] 3044
Mupad [B] (verification not implemented) 3045
Reduce [F] 3045

Optimal result

Integrand size = 20, antiderivative size = 38

$$\int \frac{1}{\sqrt{x^n(a-bx^{2-n})}}\,dx = \frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}$$

output
```
2*arctan(b^(1/2)*x/(-b*x^2+a*x^n)^(1/2))/b^(1/2)/(2-n)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 80 vs. 2(38) = 76.

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{x^n(a-bx^{2-n})}}\,dx = -\frac{2\sqrt{a}x^{n/2}\sqrt{1-\frac{bx^{2-n}}{a}}\arcsin\left(\frac{\sqrt{b}x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(-2+n)\sqrt{-bx^2+ax^n}}$$

input
```
Integrate[1/Sqrt[x^n*(a - b*x^(2 - n))],x]
```

output
```
(-2*Sqrt[a]*x^(n/2)*Sqrt[1 - (b*x^(2 - n))/a]*ArcSin[(Sqrt[b]*x^(1 - n/2))
/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[-(b*x^2) + a*x^n])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2078, 1914, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^n (a-bx^{2-n})}}dx$$

$$\Big\downarrow \text{2078}$$

$$\int \frac{1}{\sqrt{ax^n-bx^2}}dx$$

$$\Big\downarrow \text{1914}$$

$$\frac{2\int \frac{1}{\frac{bx^2}{ax^n-bx^2}+1}d\frac{x}{\sqrt{ax^n-bx^2}}}{2-n}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}(2-n)}$$

input
```
Int[1/Sqrt[x^n*(a - b*x^(2 - n))],x]
```

output
```
(2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{x^{n} \left(a-b\,x^{2-n}\right)}}dx$$

input
```
int(1/(x^n*(a-b*x^(2-n)))^(1/2),x)
```

output
```
int(1/(x^n*(a-b*x^(2-n)))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{x^{n}\left(a-bx^{2-n}\right)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(x^n*(a-b*x^(2-n)))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx=\int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx$$

input
```
integrate(1/(x**n*(a-b*x**(2-n)))**(1/2),x)
```

output
```
Integral(1/sqrt(x**n*(a - b*x**(2 - n))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx=\int \frac{1}{\sqrt{-(bx^{-n+2}-a)x^n}}\,dx$$

input
```
integrate(1/(x^n*(a-b*x^(2-n)))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-(b*x^(-n + 2) - a)*x^n), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx=\int \frac{1}{\sqrt{-(bx^{-n+2}-a)x^n}}\,dx$$

input
```
integrate(1/(x^n*(a-b*x^(2-n)))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-(b*x^(-n + 2) - a)*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx=-\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)\,\sqrt{1-\frac{b\,x^{2-n}}{a}}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\,\sqrt{a\,x^n-b\,x^2}}$$

input
```
int(1/(x^n*(a - b*x^(2 - n)))^(1/2),x)
```

output
```
-(a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2))/a^(1/2))*(1 - (b*x^(2 - n))/a
)^(1/2))/(b^(1/2)*(n/2 - 1)*(a*x^n - b*x^2)^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx=\int \frac{\sqrt{x^n a-b\,x^2}}{x^n a-b\,x^2}dx$$

input
```
int(1/(x^n*(a-b*x^(2-n)))^(1/2),x)
```

output
```
int(sqrt(x**n*a - b*x**2)/(x**n*a - b*x**2),x)
```

3.398 $\int \frac{1}{\sqrt{x^2(-b+ax^{-2+n})}}\,dx$

Optimal result . . . 3046
Mathematica [B] (verified) . . . 3046
Rubi [A] (verified) . . . 3047
Maple [F] . . . 3048
Fricas [A] (verification not implemented) . . . 3048
Sympy [F] . . . 3049
Maxima [F] . . . 3049
Giac [F] . . . 3049
Mupad [B] (verification not implemented) . . . 3050
Reduce [F] . . . 3050

Optimal result

Integrand size = 19, antiderivative size = 38

$$\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx=\frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}$$

output
```
2*arctan(b^(1/2)*x/(-b*x^2+a*x^n)^(1/2))/b^(1/2)/(2-n)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 80 vs. 2(38) = 76.

Time = 0.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx=-\frac{2\sqrt{a}x^{n/2}\sqrt{1-\frac{bx^{2-n}}{a}}\arcsin\left(\frac{\sqrt{b}x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(-2+n)\sqrt{-bx^2+ax^n}}$$

input
```
Integrate[1/Sqrt[x^2*(-b + a*x^(-2 + n))],x]
```

output
```
(-2*Sqrt[a]*x^(n/2)*Sqrt[1 - (b*x^(2 - n))/a]*ArcSin[(Sqrt[b]*x^(1 - n/2))
/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[-(b*x^2) + a*x^n])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2078, 1914, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2 (ax^{n-2} - b)}} dx$$

$$\Big\downarrow \text{ 2078}$$

$$\int \frac{1}{\sqrt{ax^n - bx^2}} dx$$

$$\Big\downarrow \text{ 1914}$$

$$\frac{2\int \frac{1}{\frac{bx^2}{ax^n-bx^2}+1}d\frac{x}{\sqrt{ax^n-bx^2}}}{2-n}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}(2-n)}$$

input
```
Int[1/Sqrt[x^2*(-b + a*x^(-2 + n))],x]
```

output
```
(2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{x^2\left(-b+a\,x^{-2+n}\right)}}dx$$

input
```
int(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x)
```

output
```
int(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.18

$$\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx = \left[-\frac{\sqrt{-b}\log\left(\frac{axx^{n-2}-2\,bx-2\,\sqrt{ax^2x^{n-2}-bx^2}\sqrt{-b}}{xx^{n-2}}\right)}{bn-2\,b}, -\frac{2\,\sqrt{b}\arctan\left(\frac{\sqrt{ax^2x^{n-2}-bx^2}\sqrt{b}}{axx^{n-2}-bx}\right)}{bn-2\,b}\right]$$

input
```
integrate(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x, algorithm="fricas")
```

output
```
[-sqrt(-b)*log((a*x*x^(n - 2) - 2*b*x - 2*sqrt(a*x^2*x^(n - 2) - b*x^2)*sq
rt(-b))/(x*x^(n - 2)))/(b*n - 2*b), -2*sqrt(b)*arctan(sqrt(a*x^2*x^(n - 2)
 - b*x^2)*sqrt(b)/(a*x*x^(n - 2) - b*x))/(b*n - 2*b)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx=\int \frac{1}{\sqrt{x^2\left(ax^{n-2}-b\right)}}\,dx$$

input
```
integrate(1/(x**2*(-b+a*x**(-2+n)))**(1/2),x)
```

output
```
Integral(1/sqrt(x**2*(a*x**(n - 2) - b)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx=\int \frac{1}{\sqrt{(ax^{n-2}-b)x^2}}\,dx$$

input
```
integrate(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((a*x^(n - 2) - b)*x^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx=\int \frac{1}{\sqrt{(ax^{n-2}-b)x^2}}\,dx$$

input
```
integrate(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt((a*x^(n - 2) - b)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 9.50 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{x^2(-b+ax^{-2+n})}}\,dx = -\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)\sqrt{1-\frac{b\,x^{2-n}}{a}}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\sqrt{a\,x^n-b\,x^2}}$$

input
```
int(1/(-x^2*(b - a*x^(n - 2)))^(1/2),x)
```

output
```
-(a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2))/a^(1/2))*(1 - (b*x^(2 - n))/a
)^(1/2))/(b^(1/2)*(n/2 - 1)*(a*x^n - b*x^2)^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{x^2(-b+ax^{-2+n})}}\,dx = \int \frac{\sqrt{x^n a-b\,x^2}}{x^n a-b\,x^2}dx$$

input
```
int(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x)
```

output
```
int(sqrt(x**n*a - b*x**2)/(x**n*a - b*x**2),x)
```

3.399 $\int \frac{1}{\sqrt{x(-bx+ax^{-1+n})}}\,dx$

Optimal result . . . 3051
Mathematica [B] (verified) . . . 3051
Rubi [A] (verified) . . . 3052
Maple [F] . . . 3053
Fricas [F(-2)] . . . 3053
Sympy [F] . . . 3054
Maxima [F] . . . 3054
Giac [F] . . . 3054
Mupad [B] (verification not implemented) . . . 3055
Reduce [F] . . . 3055

Optimal result

Integrand size = 18, antiderivative size = 38

$$\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx=\frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}$$

output
```
2*arctan(b^(1/2)*x/(-b*x^2+a*x^n)^(1/2))/b^(1/2)/(2-n)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 80 vs. 2(38) = 76.

Time = 0.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx=-\frac{2\sqrt{a}x^{n/2}\sqrt{1-\frac{bx^{2-n}}{a}}\arcsin\left(\frac{\sqrt{b}x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(-2+n)\sqrt{-bx^2+ax^n}}$$

input
```
Integrate[1/Sqrt[x*(-(b*x) + a*x^(-1 + n))],x]
```

output
```
(-2*Sqrt[a]*x^(n/2)*Sqrt[1 - (b*x^(2 - n))/a]*ArcSin[(Sqrt[b]*x^(1 - n/2))
/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[-(b*x^2) + a*x^n])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2078, 1914, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x(ax^{n-1}-bx)}}dx$$
$$\Big\downarrow \ 2078$$
$$\int \frac{1}{\sqrt{ax^n-bx^2}}dx$$
$$\Big\downarrow \ 1914$$
$$\frac{2\int \frac{1}{\frac{bx^2}{ax^n-bx^2}+1}d\frac{x}{\sqrt{ax^n-bx^2}}}{2-n}$$
$$\Big\downarrow \ 216$$
$$\frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}(2-n)}$$

input
```
Int[1/Sqrt[x*(-(b*x) + a*x^(-1 + n))],x]
```

output
```
(2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{x\left(-bx+a\,x^{-1+n}\right)}}dx$$

input
```
int(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x)
```

output
```
int(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{x\,(-bx + ax^{-1+n})}}\,dx = \int \frac{1}{\sqrt{x\,(ax^{n-1} - bx)}}\,dx$$

input
```
integrate(1/(x*(-b*x+a*x**(-1+n)))**(1/2),x)
```

output
```
Integral(1/sqrt(x*(a*x**(n - 1) - b*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x\,(-bx + ax^{-1+n})}}\,dx = \int \frac{1}{\sqrt{(ax^{n-1} - bx)x}}\,dx$$

input
```
integrate(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((a*x^(n - 1) - b*x)*x), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x\,(-bx + ax^{-1+n})}}\,dx = \int \frac{1}{\sqrt{(ax^{n-1} - bx)x}}\,dx$$

input
```
integrate(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt((a*x^(n - 1) - b*x)*x), x)
```

Mupad [B] (verification not implemented)

Time = 9.79 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx = -\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)\,\sqrt{1-\frac{b\,x^{2-n}}{a}}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\,\sqrt{a\,x^n-b\,x^2}}$$

input
```
int(1/(-x*(b*x - a*x^(n - 1)))^(1/2),x)
```

output
```
-(a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2))/a^(1/2))*(1 - (b*x^(2 - n))/a
)^(1/2))/(b^(1/2)*(n/2 - 1)*(a*x^n - b*x^2)^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx = \int \frac{\sqrt{x^n a-b\,x^2}}{x^n a-b\,x^2}dx$$

input
```
int(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x)
```

output
```
int(sqrt(x**n*a - b*x**2)/(x**n*a - b*x**2),x)
```

3.400 $\int (cx)^m \left(ax^j+bx^n\right)^{3/2} dx$

Optimal result . . . 3056
Mathematica [B] (verified) . . . 3056
Rubi [A] (verified) . . . 3057
Maple [F] . . . 3058
Fricas [F(-2)] . . . 3059
Sympy [F] . . . 3059
Maxima [F] . . . 3059
Giac [F] . . . 3060
Mupad [F(-1)] . . . 3060
Reduce [F] . . . 3060

Optimal result

Integrand size = 21, antiderivative size = 107

$$\int (cx)^m \left(ax^j+bx^n\right)^{3/2} dx=\frac{2bx^{1+n}(cx)^m\sqrt{ax^j+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m+\frac{3n}{2}}{j-n},1+\frac{1+m+\frac{3n}{2}}{j-n},-\frac{ax^{j-n}}{b}\right)}{(2+2m+3n)\sqrt{1+\frac{ax^{j-n}}{b}}}$$

output
```
2*b*x^(1+n)*(c*x)^m*(a*x^j+b*x^n)^(1/2)*hypergeom([-3/2, (1+m+3/2*n)/(j-n)
],[1+(1+m+3/2*n)/(j-n)],-a*x^(j-n)/b)/(2+2*m+3*n)/(1+a*x^(j-n)/b)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 218 vs. 2(107) = 214.

Time = 0.41 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.04

$$\int (cx)^m \left(ax^j+bx^n\right)^{3/2} dx=\frac{2(cx)^m\left((2+4j+2m-n)x^{-m}(ax^j+bx^n)\left(a(2-j+2m+4n)x^{1+j+m}+b(2+2j+2m-\right.\right.}{(2+4j+2m-n)(2+2j}$$

input
```
Integrate[(c*x)^m*(a*x^j + b*x^n)^(3/2),x]
```

output
```
(2*(c*x)^m*(((2 + 4*j + 2*m - n)*(a*x^j + b*x^n)*(a*(2 - j + 2*m + 4*n)*x^
(1 + j + m) + b*(2 + 2*j + 2*m + n)*x^(1 + m + n)))/x^m + 3*a^2*(j - n)^2*
x^(1 + 2*j)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 + 4*j + 2*
m - n)/(2*j - 2*n), (2 + 6*j + 2*m - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]
))/((2 + 4*j + 2*m - n)*(2 + 2*j + 2*m + n)*(2 + 2*m + 3*n)*Sqrt[a*x^j + b
*x^n])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1938, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \left(ax^j+bx^n\right)^{3/2} dx$$

$$\Big\downarrow \text{1938}$$

$$\frac{(cx)^m x^{-m-\frac{n}{2}} \sqrt{ax^j+bx^n} \int x^{m+\frac{3n}{2}} \left(ax^{j-n}+b\right)^{3/2} dx}{\sqrt{ax^{j-n}+b}}$$

$$\Big\downarrow \text{889}$$

$$\frac{b(cx)^m x^{-m-\frac{n}{2}} \sqrt{ax^j+bx^n} \int x^{m+\frac{3n}{2}} \left(\frac{ax^{j-n}}{b}+1\right)^{3/2} dx}{\sqrt{\frac{ax^{j-n}}{b}+1}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2bx^{n+1}(cx)^m \sqrt{ax^j+bx^n} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m+\frac{3n}{2}+1}{j-n},\frac{m+\frac{3n}{2}+1}{j-n}+1,-\frac{ax^{j-n}}{b}\right)}{(2m+3n+2)\sqrt{\frac{ax^{j-n}}{b}+1}}$$

input
```
Int[(c*x)^m*(a*x^j + b*x^n)^(3/2),x]
```

output
```
(2*b*x^(1 + n)*(c*x)^m*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-3/2, (1 + m
+ (3*n)/2)/(j - n), 1 + (1 + m + (3*n)/2)/(j - n), -((a*x^(j - n))/b)])/((
2 + 2*m + 3*n)*Sqrt[1 + (a*x^(j - n))/b])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int (cx)^m \left(a\,x^j+b\,x^n\right)^{\frac{3}{2}}dx$$

input
```
int((c*x)^m*(a*x^j+b*x^n)^(3/2),x)
```

output
```
int((c*x)^m*(a*x^j+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (cx)^m \left(ax^j + bx^n\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m*(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (cx)^m \left(ax^j + bx^n\right)^{3/2} dx = \int (cx)^m \left(ax^j + bx^n\right)^{\frac{3}{2}} dx$$

input
```
integrate((c*x)**m*(a*x**j+b*x**n)**(3/2),x)
```

output
```
Integral((c*x)**m*(a*x**j + b*x**n)**(3/2), x)
```

Maxima [F]

$$\int (cx)^m \left(ax^j + bx^n\right)^{3/2} dx = \int \left(ax^j + bx^n\right)^{\frac{3}{2}} (cx)^m dx$$

input
```
integrate((c*x)^m*(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x^j + b*x^n)^(3/2)*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m \left(ax^j+bx^n\right)^{3/2} dx = \int \left(ax^j+bx^n\right)^{\frac{3}{2}}(cx)^m\, dx$$

input
```
integrate((c*x)^m*(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*x^j + b*x^n)^(3/2)*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(ax^j+bx^n\right)^{3/2} dx = \int (c\,x)^m \left(a\,x^j+b\,x^n\right)^{3/2} dx$$

input
```
int((c*x)^m*(a*x^j + b*x^n)^(3/2),x)
```

output
```
int((c*x)^m*(a*x^j + b*x^n)^(3/2), x)
```

Reduce [F]

$$\int (cx)^m \left(ax^j+bx^n\right)^{3/2} dx = \text{too large to display}$$

input
```
int((c*x)^m*(a*x^j+b*x^n)^(3/2),x)
```

output
```
(c**m*(2*x**(j + m)*sqrt(x**j*a + x**n*b)*a*j*x + 4*x**(j + m)*sqrt(x**j*a
 + x**n*b)*a*m*x + 4*x**(j + m)*sqrt(x**j*a + x**n*b)*a*n*x + 4*x**(j + m)
*sqrt(x**j*a + x**n*b)*a*x + 8*x**(m + n)*sqrt(x**j*a + x**n*b)*b*j*x + 4*
x**(m + n)*sqrt(x**j*a + x**n*b)*b*m*x - 2*x**(m + n)*sqrt(x**j*a + x**n*b
)*b*n*x + 4*x**(m + n)*sqrt(x**j*a + x**n*b)*b*x + 9*int((x**(m + 2*n)*sqr
t(x**j*a + x**n*b))/(3*x**j*a*j**2 + 8*x**j*a*j*m + 6*x**j*a*j*n + 8*x**j*
a*j + 4*x**j*a*m**2 + 4*x**j*a*m*n + 8*x**j*a*m + 4*x**j*a*n + 4*x**j*a +
3*x**n*b*j**2 + 8*x**n*b*j*m + 6*x**n*b*j*n + 8*x**n*b*j + 4*x**n*b*m**2 +
 4*x**n*b*m*n + 8*x**n*b*m + 4*x**n*b*n + 4*x**n*b),x)*b**2*j**4 + 24*int(
(x**(m + 2*n)*sqrt(x**j*a + x**n*b))/(3*x**j*a*j**2 + 8*x**j*a*j*m + 6*x**
j*a*j*n + 8*x**j*a*j + 4*x**j*a*m**2 + 4*x**j*a*m*n + 8*x**j*a*m + 4*x**j*
a*n + 4*x**j*a + 3*x**n*b*j**2 + 8*x**n*b*j*m + 6*x**n*b*j*n + 8*x**n*b*j
+ 4*x**n*b*m**2 + 4*x**n*b*m*n + 8*x**n*b*m + 4*x**n*b*n + 4*x**n*b),x)*b*
*2*j**3*m + 24*int((x**(m + 2*n)*sqrt(x**j*a + x**n*b))/(3*x**j*a*j**2 + 8
*x**j*a*j*m + 6*x**j*a*j*n + 8*x**j*a*j + 4*x**j*a*m**2 + 4*x**j*a*m*n + 8
*x**j*a*m + 4*x**j*a*n + 4*x**j*a + 3*x**n*b*j**2 + 8*x**n*b*j*m + 6*x**n*
b*j*n + 8*x**n*b*j + 4*x**n*b*m**2 + 4*x**n*b*m*n + 8*x**n*b*m + 4*x**n*b*
n + 4*x**n*b),x)*b**2*j**3 + 12*int((x**(m + 2*n)*sqrt(x**j*a + x**n*b))/(
3*x**j*a*j**2 + 8*x**j*a*j*m + 6*x**j*a*j*n + 8*x**j*a*j + 4*x**j*a*m**2 +
 4*x**j*a*m*n + 8*x**j*a*m + 4*x**j*a*n + 4*x**j*a + 3*x**n*b*j**2 + 8*...
```

3.401 $\int (cx)^m \sqrt{ax^j+bx^n}\,dx$

Optimal result . . . 3062
Mathematica [A] (verified) . . . 3062
Rubi [A] (verified) . . . 3063
Maple [F] . . . 3064
Fricas [F(-2)] . . . 3064
Sympy [F] . . . 3065
Maxima [F] . . . 3065
Giac [F] . . . 3065
Mupad [F(-1)] . . . 3066
Reduce [F] . . . 3066

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int (cx)^m \sqrt{ax^j+bx^n}\,dx = \frac{2x(cx)^m\sqrt{ax^j+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m+\frac{n}{2}}{j-n},1+\frac{2+2m+n}{2j-2n},-\frac{ax^{j-n}}{b}\right)}{(2+2m+n)\sqrt{1+\frac{ax^{j-n}}{b}}}$$

output
```
2*x*(c*x)^m*(a*x^j+b*x^n)^(1/2)*hypergeom([-1/2, (1+m+1/2*n)/(j-n)],[1+(2+
2*m+n)/(2*j-2*n)],-a*x^(j-n)/b)/(2+2*m+n)/(1+a*x^(j-n)/b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.56

$$\int (cx)^m \sqrt{ax^j+bx^n}\,dx = \frac{2x(cx)^m\left(\left(2+2j+2m-n\right)\left(ax^j+bx^n\right)-a(j-n)x^j\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+2j+2m-n}{2j-2n},\right.\right.}{(2+2j+2m-n)(2+2m+n)\sqrt{ax^j+bx^n}}$$

input
```
Integrate[(c*x)^m*Sqrt[a*x^j + b*x^n],x]
```

output
```
(2*x*(c*x)^m*((2 + 2*j + 2*m - n)*(a*x^j + b*x^n) - a*(j - n)*x^j*Sqrt[1 +
 (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 + 2*j + 2*m - n)/(2*j - 2*n),
(2 + 4*j + 2*m - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]))/((2 + 2*j + 2*m -
 n)*(2 + 2*m + n)*Sqrt[a*x^j + b*x^n])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1938, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \sqrt{ax^j + bx^n}\, dx$$

$$\Big\downarrow \text{1938}$$

$$\frac{(cx)^m x^{-m-\frac{n}{2}} \sqrt{ax^j + bx^n} \int x^{m+\frac{n}{2}} \sqrt{ax^{j-n} + b} dx}{\sqrt{ax^{j-n} + b}}$$

$$\Big\downarrow \text{889}$$

$$\frac{(cx)^m x^{-m-\frac{n}{2}} \sqrt{ax^j + bx^n} \int x^{m+\frac{n}{2}} \sqrt{\frac{ax^{j-n}}{b} + 1} dx}{\sqrt{\frac{ax^{j-n}}{b} + 1}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x(cx)^m \sqrt{ax^j + bx^n} \, \text{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{m+\frac{n}{2}+1}{j-n}, \frac{2m+n+2}{2j-2n} + 1, -\frac{ax^{j-n}}{b}\right)}{(2m+n+2)\sqrt{\frac{ax^{j-n}}{b} + 1}}$$

input
```
Int[(c*x)^m*Sqrt[a*x^j + b*x^n],x]
```

output
```
(2*x*(c*x)^m*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-1/2, (1 + m + n/2)/(j
 - n), 1 + (2 + 2*m + n)/(2*j - 2*n), -((a*x^(j - n))/b)])/((2 + 2*m + n)*S
qrt[1 + (a*x^(j - n))/b])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int (cx)^m \sqrt{a\,x^j + b\,x^n}dx$$

input
```
int((c*x)^m*(a*x^j+b*x^n)^(1/2),x)
```

output
```
int((c*x)^m*(a*x^j+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (cx)^m \sqrt{ax^j + bx^n}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m*(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (cx)^m\sqrt{ax^j+bx^n}\,dx=\int (cx)^m\,\sqrt{ax^j+bx^n}\,dx$$

input
```
integrate((c*x)**m*(a*x**j+b*x**n)**(1/2),x)
```

output
```
Integral((c*x)**m*sqrt(a*x**j + b*x**n), x)
```

Maxima [F]

$$\int (cx)^m\sqrt{ax^j+bx^n}\,dx=\int \sqrt{ax^j+bx^n}(cx)^m\,dx$$

input
```
integrate((c*x)^m*(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^j + b*x^n)*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m\sqrt{ax^j+bx^n}\,dx=\int \sqrt{ax^j+bx^n}(cx)^m\,dx$$

input
```
integrate((c*x)^m*(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^j + b*x^n)*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m \sqrt{ax^j + bx^n}\, dx = \int (c\,x)^m\, \sqrt{a\,x^j + b\,x^n}\, dx$$

input
```
int((c*x)^m*(a*x^j + b*x^n)^(1/2),x)
```

output
```
int((c*x)^m*(a*x^j + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int (cx)^m \sqrt{ax^j + bx^n}\, dx$$
$$= \frac{c^m \left(2x^m \sqrt{x^j a + x^n b}\, x - 2\left(\int \frac{x^{j+m}\sqrt{x^j a+x^n b}}{2x^j am+x^j an+2x^j a+2x^n bm+x^n bn+2x^n b}dx\right) ajm - \left(\int \frac{x^{j+m}\sqrt{x^j a+x^n b}}{2x^j am+x^j an+2x^j a+2x^n bm+x^n bn-}\right.\right.}{}$$

input
```
int((c*x)^m*(a*x^j+b*x^n)^(1/2),x)
```

output
```
(c**m*(2*x**m*sqrt(x**j*a + x**n*b)*x - 2*int((x**(j + m)*sqrt(x**j*a + x*
*n*b))/(2*x**j*a*m + x**j*a*n + 2*x**j*a + 2*x**n*b*m + x**n*b*n + 2*x**n*
b),x)*a*j*m - int((x**(j + m)*sqrt(x**j*a + x**n*b))/(2*x**j*a*m + x**j*a*
n + 2*x**j*a + 2*x**n*b*m + x**n*b*n + 2*x**n*b),x)*a*j*n - 2*int((x**(j +
 m)*sqrt(x**j*a + x**n*b))/(2*x**j*a*m + x**j*a*n + 2*x**j*a + 2*x**n*b*m
+ x**n*b*n + 2*x**n*b),x)*a*j + 2*int((x**(j + m)*sqrt(x**j*a + x**n*b))/(
2*x**j*a*m + x**j*a*n + 2*x**j*a + 2*x**n*b*m + x**n*b*n + 2*x**n*b),x)*a*
m*n + int((x**(j + m)*sqrt(x**j*a + x**n*b))/(2*x**j*a*m + x**j*a*n + 2*x*
*j*a + 2*x**n*b*m + x**n*b*n + 2*x**n*b),x)*a*n**2 + 2*int((x**(j + m)*sqr
t(x**j*a + x**n*b))/(2*x**j*a*m + x**j*a*n + 2*x**j*a + 2*x**n*b*m + x**n*
b*n + 2*x**n*b),x)*a*n))/(2*m + n + 2)
```

3.402 $\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx$

Optimal result . . . 3067
Mathematica [A] (verified) . . . 3067
Rubi [A] (verified) . . . 3068
Maple [F] . . . 3069
Fricas [F(-2)] . . . 3069
Sympy [F] . . . 3070
Maxima [F] . . . 3070
Giac [F] . . . 3070
Mupad [F(-1)] . . . 3071
Reduce [F] . . . 3071

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx = \frac{2x(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m-\frac{n}{2}}{j-n},1+\frac{1+m-\frac{n}{2}}{j-n},-\frac{ax^{j-n}}{b}\right)}{(2+2m-n)\sqrt{ax^j+bx^n}}$$

output
```
2*x*(c*x)^m*(1+a*x^(j-n)/b)^(1/2)*hypergeom([1/2, (1+m-1/2*n)/(j-n)],[1+(1
+m-1/2*n)/(j-n)],-a*x^(j-n)/b)/(2+2*m-n)/(a*x^j+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.04

$$\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx = \frac{2x(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+2m-n}{2j-2n},1+\frac{2+2m-n}{2j-2n},-\frac{ax^{j-n}}{b}\right)}{(2+2m-n)\sqrt{ax^j+bx^n}}$$

input
```
Integrate[(c*x)^m/Sqrt[a*x^j + b*x^n],x]
```

output
```
(2*x*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 + 2*m - n
)/(2*j - 2*n), 1 + (2 + 2*m - n)/(2*j - 2*n), -((a*x^(j - n))/b)])/((2 + 2
*m - n)*Sqrt[a*x^j + b*x^n])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.18, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1938, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}dx$$

$$\Big\downarrow \text{ 1938}$$

$$\frac{(cx)^m x^{\frac{1}{2}(n-2m)}\sqrt{ax^{j-n}+b}\int \frac{x^{m-\frac{n}{2}}}{\sqrt{ax^{j-n}+b}}dx}{\sqrt{ax^j+bx^n}}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{(cx)^m x^{\frac{1}{2}(n-2m)}\sqrt{\frac{ax^{j-n}}{b}+1}\int \frac{x^{m-\frac{n}{2}}}{\sqrt{\frac{ax^{j-n}}{b}+1}}dx}{\sqrt{ax^j+bx^n}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{2(cx)^m x^{\frac{1}{2}(n-2m)+m-\frac{n}{2}+1}\sqrt{\frac{ax^{j-n}}{b}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m-\frac{n}{2}+1}{j-n},\frac{m-\frac{n}{2}+1}{j-n}+1,-\frac{ax^{j-n}}{b}\right)}{(2m-n+2)\sqrt{ax^j+bx^n}}$$

input
```
Int[(c*x)^m/Sqrt[a*x^j + b*x^n],x]
```

output
```
(2*x^(1 + m - n/2 + (-2*m + n)/2)*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hyperg
eometric2F1[1/2, (1 + m - n/2)/(j - n), 1 + (1 + m - n/2)/(j - n), -((a*x^
(j - n))/b)])/((2 + 2*m - n)*Sqrt[a*x^j + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{(cx)^m}{\sqrt{a\,x^j+b\,x^n}}dx$$

input
```
int((c*x)^m/(a*x^j+b*x^n)^(1/2),x)
```

output
```
int((c*x)^m/(a*x^j+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m/(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(cx)^m}{\sqrt{ax^j + bx^n}}\,dx = \int \frac{(cx)^m}{\sqrt{ax^j + bx^n}}\,dx$$

input
```
integrate((c*x)**m/(a*x**j+b*x**n)**(1/2),x)
```

output
```
Integral((c*x)**m/sqrt(a*x**j + b*x**n), x)
```

Maxima [F]

$$\int \frac{(cx)^m}{\sqrt{ax^j + bx^n}}\,dx = \int \frac{(cx)^m}{\sqrt{ax^j + bx^n}}\,dx$$

input
```
integrate((c*x)^m/(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^m/sqrt(a*x^j + b*x^n), x)
```

Giac [F]

$$\int \frac{(cx)^m}{\sqrt{ax^j + bx^n}}\,dx = \int \frac{(cx)^m}{\sqrt{ax^j + bx^n}}\,dx$$

input
```
integrate((c*x)^m/(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^m/sqrt(a*x^j + b*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx=\int \frac{(c\,x)^m}{\sqrt{a\,x^j+b\,x^n}}\,dx$$

input
```
int((c*x)^m/(a*x^j + b*x^n)^(1/2),x)
```

output
```
int((c*x)^m/(a*x^j + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx=c^m\left(\int \frac{x^m\sqrt{x^ja+x^nb}}{x^ja+x^nb}dx\right)$$

input
```
int((c*x)^m/(a*x^j+b*x^n)^(1/2),x)
```

output
```
c**m*int((x**m*sqrt(x**j*a + x**n*b))/(x**j*a + x**n*b),x)
```

3.403 $\int \frac{(cx)^m}{\left(ax^j+bx^n\right)^{3/2}} \, dx$

Optimal result 3072
Mathematica [A] (verified) 3072
Rubi [A] (verified) 3073
Maple [F] 3074
Fricas [F(-2)] 3075
Sympy [F] 3075
Maxima [F] 3075
Giac [F] 3076
Mupad [F(-1)] 3076
Reduce [F] 3076

Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \frac{(cx)^m}{\left(ax^j+bx^n\right)^{3/2}} \, dx = \frac{2x^{1-n}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m-\frac{3n}{2}}{j-n},1+\frac{1+m-\frac{3n}{2}}{j-n},-\frac{ax^{j-n}}{b}\right)}{b(2+2m-3n)\sqrt{ax^j+bx^n}}$$

output
```
2*x^(1-n)*(c*x)^m*(1+a*x^(j-n)/b)^(1/2)*hypergeom([3/2, (1+m-3/2*n)/(j-n)]
,[1+(1+m-3/2*n)/(j-n)],-a*x^(j-n)/b)/b/(2+2*m-3*n)/(a*x^j+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

$$\int \frac{(cx)^m}{\left(ax^j+bx^n\right)^{3/2}} \, dx = \frac{2x^{1-j}(cx)^m\left(-1+\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-2j+2m-n}{2j-2n},\frac{2+2m-3n}{2j-2n},-\frac{ax^{j-}}{b}\right.\right.}{a(j-n)\sqrt{ax^j+bx^n}}$$

input
```
Integrate[(c*x)^m/(a*x^j + b*x^n)^(3/2),x]
```

output
```
(2*x^(1 - j)*(c*x)^m*(-1 + Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2
, (2 - 2*j + 2*m - n)/(2*j - 2*n), (2 + 2*m - 3*n)/(2*j - 2*n), -((a*x^(j
- n))/b)]))/(a*(j - n)*Sqrt[a*x^j + b*x^n])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1938, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{\left(ax^j+bx^n\right)^{3/2}}dx$$

$$\Big\downarrow \text{1938}$$

$$\frac{(cx)^m x^{\frac{1}{2}(n-2m)}\sqrt{ax^{j-n}+b}\int \frac{x^{m-\frac{3n}{2}}}{\left(ax^{j-n}+b\right)^{3/2}}dx}{\sqrt{ax^j+bx^n}}$$

$$\Big\downarrow \text{889}$$

$$\frac{(cx)^m x^{\frac{1}{2}(n-2m)}\sqrt{\frac{ax^{j-n}}{b}+1}\int \frac{x^{m-\frac{3n}{2}}}{\left(\frac{ax^{j-n}}{b}+1\right)^{3/2}}dx}{b\sqrt{ax^j+bx^n}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2(cx)^m x^{\frac{1}{2}(n-2m)+m-\frac{3n}{2}+1}\sqrt{\frac{ax^{j-n}}{b}+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m-\frac{3n}{2}+1}{j-n},\frac{m-\frac{3n}{2}+1}{j-n}+1,-\frac{ax^{j-n}}{b}\right)}{b(2m-3n+2)\sqrt{ax^j+bx^n}}$$

input
```
Int[(c*x)^m/(a*x^j + b*x^n)^(3/2),x]
```

output
```
(2*x^(1 + m - (3*n)/2 + (-2*m + n)/2)*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hy
pergeometric2F1[3/2, (1 + m - (3*n)/2)/(j - n), 1 + (1 + m - (3*n)/2)/(j -
 n), -((a*x^(j - n))/b)])/(b*(2 + 2*m - 3*n)*Sqrt[a*x^j + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{(cx)^m}{(a\,x^j+b\,x^n)^{\frac{3}{2}}}dx$$

input
```
int((c*x)^m/(a*x^j+b*x^n)^(3/2),x)
```

output
```
int((c*x)^m/(a*x^j+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^m}{(ax^j + bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m/(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(cx)^m}{(ax^j + bx^n)^{3/2}}\,dx = \int \frac{(cx)^m}{(ax^j + bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)**m/(a*x**j+b*x**n)**(3/2),x)
```

output
```
Integral((c*x)**m/(a*x**j + b*x**n)**(3/2), x)
```

Maxima [F]

$$\int \frac{(cx)^m}{(ax^j + bx^n)^{3/2}}\,dx = \int \frac{(cx)^m}{(ax^j + bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^m/(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^m/(a*x^j + b*x^n)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{3/2}}\,dx=\int \frac{(cx)^m}{(ax^j+bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^m/(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^m/(a*x^j + b*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{3/2}}\,dx=\int \frac{(c\,x)^m}{(a\,x^j+b\,x^n)^{3/2}}\,dx$$

input
```
int((c*x)^m/(a*x^j + b*x^n)^(3/2),x)
```

output
```
int((c*x)^m/(a*x^j + b*x^n)^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{3/2}}\,dx=c^m\left(\int \frac{x^m\sqrt{x^ja+x^nb}}{x^{2j}a^2+2x^{j+n}ab+x^{2n}b^2}dx\right)$$

input
```
int((c*x)^m/(a*x^j+b*x^n)^(3/2),x)
```

output
```
c**m*int((x**m*sqrt(x**j*a + x**n*b))/(x**(2*j)*a**2 + 2*x**(j + n)*a*b +
x**(2*n)*b**2),x)
```

3.404 $\int \frac{(cx)^m}{\left(ax^j+bx^n\right)^{5/2}}\,dx$

Optimal result 3077
Mathematica [A] (verified) 3077
Rubi [A] (verified) 3078
Maple [F] 3079
Fricas [F(-2)] 3080
Sympy [F] 3080
Maxima [F] 3080
Giac [F] 3081
Mupad [F(-1)] 3081
Reduce [F] 3081

Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \frac{(cx)^m}{\left(ax^j+bx^n\right)^{5/2}}\,dx = \frac{2x^{1-2n}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+m-\frac{5n}{2}}{j-n},1+\frac{1+m-\frac{5n}{2}}{j-n},-\frac{ax^{j-n}}{b}\right)}{b^2(2+2m-5n)\sqrt{ax^j+bx^n}}$$

output
```
2*x^(1-2*n)*(c*x)^m*(1+a*x^(j-n)/b)^(1/2)*hypergeom([5/2, (1+m-5/2*n)/(j-n
)],[1+(1+m-5/2*n)/(j-n)],-a*x^(j-n)/b)/b^2/(2+2*m-5*n)/(a*x^j+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.50

$$\int \frac{(cx)^m}{\left(ax^j+bx^n\right)^{5/2}}\,dx = \frac{2x^{1-2j}(cx)^m\left(-2+2j-2m+3n-\frac{a(j-n)x^j}{ax^j+bx^n}-(-2+2j-2m+3n)\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{H}\right.}{3a^2(j-n)^2\sqrt{ax^j+bx^n}}$$

input
```
Integrate[(c*x)^m/(a*x^j + b*x^n)^(5/2),x]
```

output
```
(2*x^(1 - 2*j)*(c*x)^m*(-2 + 2*j - 2*m + 3*n - (a*(j - n)*x^j)/(a*x^j + b*
x^n) - (-2 + 2*j - 2*m + 3*n)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[
1/2, (2 - 4*j + 2*m - n)/(2*j - 2*n), (2 - 2*j + 2*m - 3*n)/(2*j - 2*n), -
((a*x^(j - n))/b)]))/(3*a^2*(j - n)^2*Sqrt[a*x^j + b*x^n])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1938, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{5/2}}\,dx$$

$$\Big\downarrow \text{1938}$$

$$\frac{(cx)^m x^{\frac{1}{2}(n-2m)}\sqrt{ax^{j-n}+b}\int \frac{x^{m-\frac{5n}{2}}}{(ax^{j-n}+b)^{5/2}}dx}{\sqrt{ax^j+bx^n}}$$

$$\Big\downarrow \text{889}$$

$$\frac{(cx)^m x^{\frac{1}{2}(n-2m)}\sqrt{\frac{ax^{j-n}}{b}+1}\int \frac{x^{m-\frac{5n}{2}}}{\left(\frac{ax^{j-n}}{b}+1\right)^{5/2}}dx}{b^2\sqrt{ax^j+bx^n}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2(cx)^m x^{\frac{1}{2}(n-2m)+m-\frac{5n}{2}+1}\sqrt{\frac{ax^{j-n}}{b}+1}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m-\frac{5n}{2}+1}{j-n},\frac{m-\frac{5n}{2}+1}{j-n}+1,-\frac{ax^{j-n}}{b}\right)}{b^2(2m-5n+2)\sqrt{ax^j+bx^n}}$$

input
```
Int[(c*x)^m/(a*x^j + b*x^n)^(5/2),x]
```

output
```
(2*x^(1 + m - (5*n)/2 + (-2*m + n)/2)*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hy
pergeometric2F1[5/2, (1 + m - (5*n)/2)/(j - n), 1 + (1 + m - (5*n)/2)/(j -
 n), -((a*x^(j - n))/b)])/(b^2*(2 + 2*m - 5*n)*Sqrt[a*x^j + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{(cx)^m}{(a\,x^j+b\,x^n)^{\frac{5}{2}}}dx$$

input
```
int((c*x)^m/(a*x^j+b*x^n)^(5/2),x)
```

output
```
int((c*x)^m/(a*x^j+b*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m/(a*x^j+b*x^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{5/2}}\,dx = \int \frac{(cx)^m}{(ax^j+bx^n)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)**m/(a*x**j+b*x**n)**(5/2),x)
```

output
```
Integral((c*x)**m/(a*x**j + b*x**n)**(5/2), x)
```

Maxima [F]

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{5/2}}\,dx = \int \frac{(cx)^m}{(ax^j+bx^n)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^m/(a*x^j+b*x^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^m/(a*x^j + b*x^n)^(5/2), x)
```

Giac [F]

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{5/2}}\,dx = \int \frac{(cx)^m}{(ax^j+bx^n)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^m/(a*x^j+b*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*x)^m/(a*x^j + b*x^n)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{5/2}}\,dx = \int \frac{(c\,x)^m}{(a\,x^j+b\,x^n)^{5/2}}\,dx$$

input
```
int((c*x)^m/(a*x^j + b*x^n)^(5/2),x)
```

output
```
int((c*x)^m/(a*x^j + b*x^n)^(5/2), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{5/2}}\,dx = c^m\left(\int \frac{x^m\sqrt{x^ja+x^nb}}{x^{3j}a^3+3x^{2j+n}a^2b+3x^{j+2n}a\,b^2+x^{3n}b^3}dx\right)$$

input
```
int((c*x)^m/(a*x^j+b*x^n)^(5/2),x)
```

output
```
c**m*int((x**m*sqrt(x**j*a + x**n*b))/(x**(3*j)*a**3 + 3*x**(2*j + n)*a**2
*b + 3*x**(j + 2*n)*a*b**2 + x**(3*n)*b**3),x)
```

3.405 $\int \left(ax^j+bx^n\right)^{3/2} dx$

Optimal result . . . 3082
Mathematica [A] (verified) . . . 3082
Rubi [A] (verified) . . . 3083
Maple [F] . . . 3084
Fricas [F(-2)] . . . 3084
Sympy [F] . . . 3085
Maxima [F] . . . 3085
Giac [F] . . . 3085
Mupad [B] (verification not implemented) . . . 3086
Reduce [F] . . . 3086

Optimal result

Integrand size = 15, antiderivative size = 97

$$\int \left(ax^j+bx^n\right)^{3/2} dx=\frac{2bx^{1+n}\sqrt{ax^j+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+\frac{3n}{2}}{j-n},\frac{2+2j+n}{2(j-n)},-\frac{ax^{j-n}}{b}\right)}{(2+3n)\sqrt{1+\frac{ax^{j-n}}{b}}}$$

output
```
2*b*x^(1+n)*(a*x^j+b*x^n)^(1/2)*hypergeom([-3/2, (1+3/2*n)/(j-n)],[(2+2*j+
n)/(2*j-2*n)],-a*x^(j-n)/b)/(2+3*n)/(1+a*x^(j-n)/b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.82

$$\int \left(ax^j+bx^n\right)^{3/2} dx=\frac{2x\left(\left(2+4j-n\right)\left(ax^j+bx^n\right)\left(a(2-j+4n)x^j+b(2+2j+n)x^n\right)+3a^2(j-n)^2x^{2j}\sqrt{1+\cdots}\right.}{(2+4j-n)(2+2j+n)(2+3n)\sqrt{ax^j+\cdots}}$$

input
```
Integrate[(a*x^j + b*x^n)^(3/2),x]
```

output
```
(2*x*((2 + 4*j - n)*(a*x^j + b*x^n)*(a*(2 - j + 4*n)*x^j + b*(2 + 2*j + n)
*x^n) + 3*a^2*(j - n)^2*x^(2*j)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F
1[1/2, (2 + 4*j - n)/(2*j - 2*n), (2 + 6*j - 3*n)/(2*j - 2*n), -((a*x^(j -
 n))/b)]))/((2 + 4*j - n)*(2 + 2*j + n)*(2 + 3*n)*Sqrt[a*x^j + b*x^n])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1917, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax^j + bx^n)^{3/2} \, dx$$

$$\Big\downarrow \text{1917}$$

$$\frac{x^{-n/2}\sqrt{ax^j+bx^n}\int x^{3n/2}\left(ax^{j-n}+b\right)^{3/2}dx}{\sqrt{ax^{j-n}+b}}$$

$$\Big\downarrow \text{889}$$

$$\frac{bx^{-n/2}\sqrt{ax^j+bx^n}\int x^{3n/2}\left(\frac{ax^{j-n}}{b}+1\right)^{3/2}dx}{\sqrt{\frac{ax^{j-n}}{b}+1}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2bx^{n+1}\sqrt{ax^j+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{\frac{3n}{2}+1}{j-n},\frac{2j+n+2}{2(j-n)},-\frac{ax^{j-n}}{b}\right)}{(3n+2)\sqrt{\frac{ax^{j-n}}{b}+1}}$$

input
```
Int[(a*x^j + b*x^n)^(3/2),x]
```

output
```
(2*b*x^(1 + n)*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-3/2, (1 + (3*n)/2)/(
j - n), (2 + 2*j + n)/(2*(j - n)), -((a*x^(j - n))/b)])/((2 + 3*n)*Sqrt[1
+ (a*x^(j - n))/b])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \left(a\,x^{j}+b\,x^{n}\right)^{\frac{3}{2}}dx$$

input
```
int((a*x^j+b*x^n)^(3/2),x)
```

output
```
int((a*x^j+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(ax^j+bx^n\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(ax^j + bx^n\right)^{3/2} dx = \int \left(ax^j + bx^n\right)^{\frac{3}{2}} dx$$

input
```
integrate((a*x**j+b*x**n)**(3/2),x)
```

output
```
Integral((a*x**j + b*x**n)**(3/2), x)
```

Maxima [F]

$$\int \left(ax^j + bx^n\right)^{3/2} dx = \int \left(ax^j + bx^n\right)^{\frac{3}{2}} dx$$

input
```
integrate((a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x^j + b*x^n)^(3/2), x)
```

Giac [F]

$$\int \left(ax^j + bx^n\right)^{3/2} dx = \int \left(ax^j + bx^n\right)^{\frac{3}{2}} dx$$

input
```
integrate((a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*x^j + b*x^n)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 9.84 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \left(ax^j+bx^n\right)^{3/2}\,dx=\frac{x\,\left(a\,x^j+b\,x^n\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{\frac{3\,n}{2}+1}{j-n};\ \frac{\frac{3\,n}{2}+1}{j-n}+1;\ -\frac{a\,x^{j-n}}{b}\right)}{\left(\frac{3\,n}{2}+1\right)\,\left(\frac{a\,x^{j-n}}{b}+1\right)^{3/2}}$$

input
```
int((a*x^j + b*x^n)^(3/2),x)
```

output
```
(x*(a*x^j + b*x^n)^(3/2)*hypergeom([-3/2, ((3*n)/2 + 1)/(j - n)], ((3*n)/2
 + 1)/(j - n) + 1, -(a*x^(j - n))/b))/(((3*n)/2 + 1)*((a*x^(j - n))/b + 1)
^(3/2))
```

Reduce [F]

$$\int \left(ax^j+bx^n\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
int((a*x^j+b*x^n)^(3/2),x)
```

output

```
(2*x**j*sqrt(x**j*a + x**n*b)*a*j*x + 4*x**j*sqrt(x**j*a + x**n*b)*a*n*x +
 4*x**j*sqrt(x**j*a + x**n*b)*a*x + 8*x**n*sqrt(x**j*a + x**n*b)*b*j*x - 2
*x**n*sqrt(x**j*a + x**n*b)*b*n*x + 4*x**n*sqrt(x**j*a + x**n*b)*b*x + 9*i
nt((x**(2*n)*sqrt(x**j*a + x**n*b))/(3*x**j*a*j**2 + 6*x**j*a*j*n + 8*x**j
*a*j + 4*x**j*a*n + 4*x**j*a + 3*x**n*b*j**2 + 6*x**n*b*j*n + 8*x**n*b*j +
 4*x**n*b*n + 4*x**n*b),x)*b**2*j**4 + 24*int((x**(2*n)*sqrt(x**j*a + x**n
*b))/(3*x**j*a*j**2 + 6*x**j*a*j*n + 8*x**j*a*j + 4*x**j*a*n + 4*x**j*a +
3*x**n*b*j**2 + 6*x**n*b*j*n + 8*x**n*b*j + 4*x**n*b*n + 4*x**n*b),x)*b**2
*j**3 - 27*int((x**(2*n)*sqrt(x**j*a + x**n*b))/(3*x**j*a*j**2 + 6*x**j*a*
j*n + 8*x**j*a*j + 4*x**j*a*n + 4*x**j*a + 3*x**n*b*j**2 + 6*x**n*b*j*n +
8*x**n*b*j + 4*x**n*b*n + 4*x**n*b),x)*b**2*j**2*n**2 - 36*int((x**(2*n)*s
qrt(x**j*a + x**n*b))/(3*x**j*a*j**2 + 6*x**j*a*j*n + 8*x**j*a*j + 4*x**j*
a*n + 4*x**j*a + 3*x**n*b*j**2 + 6*x**n*b*j*n + 8*x**n*b*j + 4*x**n*b*n +
4*x**n*b),x)*b**2*j**2*n + 12*int((x**(2*n)*sqrt(x**j*a + x**n*b))/(3*x**j
*a*j**2 + 6*x**j*a*j*n + 8*x**j*a*j + 4*x**j*a*n + 4*x**j*a + 3*x**n*b*j**
2 + 6*x**n*b*j*n + 8*x**n*b*j + 4*x**n*b*n + 4*x**n*b),x)*b**2*j**2 + 18*i
nt((x**(2*n)*sqrt(x**j*a + x**n*b))/(3*x**j*a*j**2 + 6*x**j*a*j*n + 8*x**j
*a*j + 4*x**j*a*n + 4*x**j*a + 3*x**n*b*j**2 + 6*x**n*b*j*n + 8*x**n*b*j +
 4*x**n*b*n + 4*x**n*b),x)*b**2*j*n**3 - 24*int((x**(2*n)*sqrt(x**j*a + x*
*n*b))/(3*x**j*a*j**2 + 6*x**j*a*j*n + 8*x**j*a*j + 4*x**j*a*n + 4*x**j...
```

3.406 $\int \sqrt{ax^j+bx^n}\,dx$

Optimal result . . . 3088
Mathematica [A] (verified) . . . 3088
Rubi [A] (verified) . . . 3089
Maple [F] . . . 3090
Fricas [F(-2)] . . . 3090
Sympy [F] . . . 3091
Maxima [F] . . . 3091
Giac [F] . . . 3091
Mupad [B] (verification not implemented) . . . 3092
Reduce [F] . . . 3092

Optimal result

Integrand size = 15, antiderivative size = 87

$$\int \sqrt{ax^j+bx^n}\,dx=\frac{2x\sqrt{ax^j+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2+n}{2(j-n)},1+\frac{2+n}{2j-2n},-\frac{ax^{j-n}}{b}\right)}{(2+n)\sqrt{1+\frac{ax^{j-n}}{b}}}$$

output
```
2*x*(a*x^j+b*x^n)^(1/2)*hypergeom([-1/2, (2+n)/(2*j-2*n)],[1+(2+n)/(2*j-2*
n)],-a*x^(j-n)/b)/(2+n)/(1+a*x^(j-n)/b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.54

$$\int \sqrt{ax^j+bx^n}\,dx$$
$$=\frac{2x\left(-((2+2j-n)\left(ax^j+bx^n\right))+a(j-n)x^j\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+2j-n}{2j-2n},\frac{2+4j-3n}{2j-2n},-\ldots\right.\right.}{(2+n)(-2-2j+n)\sqrt{ax^j+bx^n}}$$

input
```
Integrate[Sqrt[a*x^j + b*x^n],x]
```

output
```
(2*x*(-((2 + 2*j - n)*(a*x^j + b*x^n)) + a*(j - n)*x^j*Sqrt[1 + (a*x^(j -
n))/b]*Hypergeometric2F1[1/2, (2 + 2*j - n)/(2*j - 2*n), (2 + 4*j - 3*n)/(
2*j - 2*n), -((a*x^(j - n))/b)]))/((2 + n)*(-2 - 2*j + n)*Sqrt[a*x^j + b*x
^n])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1917, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ax^j + bx^n}\,dx$$

$$\Big\downarrow \text{1917}$$

$$\frac{x^{-n/2}\sqrt{ax^j + bx^n}\int x^{n/2}\sqrt{ax^{j-n} + b}dx}{\sqrt{ax^{j-n} + b}}$$

$$\Big\downarrow \text{889}$$

$$\frac{x^{-n/2}\sqrt{ax^j + bx^n}\int x^{n/2}\sqrt{\frac{ax^{j-n}}{b} + 1}dx}{\sqrt{\frac{ax^{j-n}}{b} + 1}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x^{\frac{n+2}{2}-\frac{n}{2}}\sqrt{ax^j + bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{n+2}{2(j-n)}, \frac{n+2}{2j-2n} + 1, -\frac{ax^{j-n}}{b}\right)}{(n+2)\sqrt{\frac{ax^{j-n}}{b} + 1}}$$

input
```
Int[Sqrt[a*x^j + b*x^n],x]
```

output
```
(2*x^(-1/2*n + (2 + n)/2)*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-1/2, (2 +
 n)/(2*(j - n)), 1 + (2 + n)/(2*j - 2*n), -((a*x^(j - n))/b)])/((2 + n)*Sq
rt[1 + (a*x^(j - n))/b])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \sqrt{a\,x^{j}+b\,x^{n}}dx$$

input
```
int((a*x^j+b*x^n)^(1/2),x)
```

output
```
int((a*x^j+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{ax^j+bx^n}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sqrt{ax^j + bx^n}\, dx = \int \sqrt{ax^j + bx^n}\, dx$$

input
```
integrate((a*x**j+b*x**n)**(1/2),x)
```

output
```
Integral(sqrt(a*x**j + b*x**n), x)
```

Maxima [F]

$$\int \sqrt{ax^j + bx^n}\, dx = \int \sqrt{ax^j + bx^n}\, dx$$

input
```
integrate((a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^j + b*x^n), x)
```

Giac [F]

$$\int \sqrt{ax^j + bx^n}\, dx = \int \sqrt{ax^j + bx^n}\, dx$$

input
```
integrate((a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^j + b*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 9.85 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int \sqrt{ax^j + bx^n}\, dx = \frac{x\sqrt{a\,x^j + b\,x^n}\;{}_2\mathrm{F}_1\left(-\frac{1}{2}, \frac{\frac{n}{2}+1}{j-n};\ \frac{\frac{n}{2}+1}{j-n} + 1;\ -\frac{a\,x^{j-n}}{b}\right)}{\left(\frac{n}{2}+1\right)\sqrt{\frac{a\,x^{j-n}}{b} + 1}}$$

input
```
int((a*x^j + b*x^n)^(1/2),x)
```

output
```
(x*(a*x^j + b*x^n)^(1/2)*hypergeom([-1/2, (n/2 + 1)/(j - n)], (n/2 + 1)/(j
 - n) + 1, -(a*x^(j - n))/b))/((n/2 + 1)*((a*x^(j - n))/b + 1)^(1/2))
```

Reduce [F]

$$\int \sqrt{ax^j + bx^n}\, dx$$
$$= \frac{2\sqrt{x^j a + x^n b}\,x - \left(\int \frac{x^j\sqrt{x^j a+x^n b}}{x^j an+2x^j a+x^n bn+2x^n b}dx\right) ajn - 2\left(\int \frac{x^j\sqrt{x^j a+x^n b}}{x^j an+2x^j a+x^n bn+2x^n b}dx\right) aj + \left(\int \frac{x^j\sqrt{x^j a+x^n b}}{x^j an+2x^j a+x^n bn+2}\right.}{n+2}$$

input
```
int((a*x^j+b*x^n)^(1/2),x)
```

output
```
(2*sqrt(x**j*a + x**n*b)*x - int((x**j*sqrt(x**j*a + x**n*b))/(x**j*a*n +
2*x**j*a + x**n*b*n + 2*x**n*b),x)*a*j*n - 2*int((x**j*sqrt(x**j*a + x**n*
b))/(x**j*a*n + 2*x**j*a + x**n*b*n + 2*x**n*b),x)*a*j + int((x**j*sqrt(x*
*j*a + x**n*b))/(x**j*a*n + 2*x**j*a + x**n*b*n + 2*x**n*b),x)*a*n**2 + 2*
int((x**j*sqrt(x**j*a + x**n*b))/(x**j*a*n + 2*x**j*a + x**n*b*n + 2*x**n*
b),x)*a*n)/(n + 2)
```

3.407 $\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx$

Optimal result . . . 3093
Mathematica [A] (verified) . . . 3093
Rubi [A] (verified) . . . 3094
Maple [F] . . . 3095
Fricas [F(-2)] . . . 3095
Sympy [F] . . . 3096
Maxima [F] . . . 3096
Giac [F] . . . 3096
Mupad [B] (verification not implemented) . . . 3097
Reduce [F] . . . 3097

Optimal result

Integrand size = 15, antiderivative size = 93

$$\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx=\frac{2x\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-n}{2(j-n)},1+\frac{1-\frac{n}{2}}{j-n},-\frac{ax^{j-n}}{b}\right)}{(2-n)\sqrt{ax^j+bx^n}}$$

output
```
2*x*(1+a*x^(j-n)/b)^(1/2)*hypergeom([1/2, (2-n)/(2*j-2*n)],[1+(1-1/2*n)/(j
-n)],-a*x^(j-n)/b)/(2-n)/(a*x^j+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx=-\frac{2x\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{-2+n}{2(-j+n)},1+\frac{-2+n}{2(-j+n)},-\frac{ax^{j-n}}{b}\right)}{(-2+n)\sqrt{ax^j+bx^n}}$$

input
```
Integrate[1/Sqrt[a*x^j + b*x^n],x]
```

output
```
(-2*x*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (-2 + n)/(2*(-j + n
)), 1 + (-2 + n)/(2*(-j + n)), -((a*x^(j - n))/b)])/((-2 + n)*Sqrt[a*x^j +
 b*x^n])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1917, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx$$

$$\Big\downarrow \text{1917}$$

$$\frac{x^{n/2}\sqrt{ax^{j-n}+b}\int \frac{x^{-n/2}}{\sqrt{ax^{j-n}+b}}dx}{\sqrt{ax^j+bx^n}}$$

$$\Big\downarrow \text{889}$$

$$\frac{x^{n/2}\sqrt{\frac{ax^{j-n}}{b}+1}\int \frac{x^{-n/2}}{\sqrt{\frac{ax^{j-n}}{b}+1}}dx}{\sqrt{ax^j+bx^n}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x\sqrt{\frac{ax^{j-n}}{b}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-n}{2(j-n)},\frac{1-\frac{n}{2}}{j-n}+1,-\frac{ax^{j-n}}{b}\right)}{(2-n)\sqrt{ax^j+bx^n}}$$

input
```
Int[1/Sqrt[a*x^j + b*x^n],x]
```

output
```
(2*x*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 - n)/(2*(j - n)),
 1 + (1 - n/2)/(j - n), -((a*x^(j - n))/b)])/((2 - n)*Sqrt[a*x^j + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{1}{\sqrt{a\,x^{j}+b\,x^{n}}}dx$$

input
```
int(1/(a*x^j+b*x^n)^(1/2),x)
```

output
```
int(1/(a*x^j+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{ax^j + bx^n}} \, dx = \int \frac{1}{\sqrt{ax^j + bx^n}} \, dx$$

input
```
integrate(1/(a*x**j+b*x**n)**(1/2),x)
```

output
```
Integral(1/sqrt(a*x**j + b*x**n), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{ax^j + bx^n}} \, dx = \int \frac{1}{\sqrt{ax^j + bx^n}} \, dx$$

input
```
integrate(1/(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a*x^j + b*x^n), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{ax^j + bx^n}} \, dx = \int \frac{1}{\sqrt{ax^j + bx^n}} \, dx$$

input
```
integrate(1/(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(a*x^j + b*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 9.47 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{ax^j + bx^n}}\,dx = -\frac{x\sqrt{\frac{b\,x^{n-j}}{a}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{\frac{j}{2}-1}{j-n};\ \frac{\frac{j}{2}-1}{j-n}+1;\ -\frac{b\,x^{n-j}}{a}\right)}{\left(\frac{j}{2}-1\right)\sqrt{a\,x^j+b\,x^n}}$$

input
```
int(1/(a*x^j + b*x^n)^(1/2),x)
```

output
```
-(x*((b*x^(n - j))/a + 1)^(1/2)*hypergeom([1/2, (j/2 - 1)/(j - n)], (j/2 -
 1)/(j - n) + 1, -(b*x^(n - j))/a))/((j/2 - 1)*(a*x^j + b*x^n)^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{ax^j + bx^n}}\,dx = \int \frac{\sqrt{x^j a + x^n b}}{x^j a + x^n b}dx$$

input
```
int(1/(a*x^j+b*x^n)^(1/2),x)
```

output
```
int(sqrt(x**j*a + x**n*b)/(x**j*a + x**n*b),x)
```

3.408 $\int \frac{1}{(ax^j+bx^n)^{3/2}}\,dx$

Optimal result . . . 3098
Mathematica [A] (verified) . . . 3098
Rubi [A] (verified) . . . 3099
Maple [F] . . . 3100
Fricas [F(-2)] . . . 3100
Sympy [F] . . . 3101
Maxima [F] . . . 3101
Giac [F] . . . 3101
Mupad [B] (verification not implemented) . . . 3102
Reduce [F] . . . 3102

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{1}{(ax^j+bx^n)^{3/2}}\,dx=\frac{2x^{1-n}\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1-\frac{3n}{2}}{j-n},1+\frac{1-\frac{3n}{2}}{j-n},-\frac{ax^{j-n}}{b}\right)}{b(2-3n)\sqrt{ax^j+bx^n}}$$

output
```
2*x^(1-n)*(1+a*x^(j-n)/b)^(1/2)*hypergeom([3/2, (1-3/2*n)/(j-n)],[1+(1-3/2
*n)/(j-n)],-a*x^(j-n)/b)/b/(2-3*n)/(a*x^j+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03

$$\int \frac{1}{(ax^j+bx^n)^{3/2}}\,dx=\frac{2x^{1-j}\left(-1+\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{-2+2j+n}{2(j-n)},\frac{2-3n}{2j-2n},-\frac{ax^{j-n}}{b}\right)\right)}{a(j-n)\sqrt{ax^j+bx^n}}$$

input
```
Integrate[(a*x^j + b*x^n)^(-3/2),x]
```

output
```
(2*x^(1 - j)*(-1 + Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, -1/2*(
-2 + 2*j + n)/(j - n), (2 - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]))/(a*(j
- n)*Sqrt[a*x^j + b*x^n])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1917, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^j+bx^n)^{3/2}}\,dx$$

$$\Big\downarrow \; 1917$$

$$\frac{x^{n/2}\sqrt{ax^{j-n}+b}\int \frac{x^{-3n/2}}{(ax^{j-n}+b)^{3/2}}dx}{\sqrt{ax^j+bx^n}}$$

$$\Big\downarrow \; 889$$

$$\frac{x^{n/2}\sqrt{\frac{ax^{j-n}}{b}+1}\int \frac{x^{-3n/2}}{\left(\frac{ax^{j-n}}{b}+1\right)^{3/2}}dx}{b\sqrt{ax^j+bx^n}}$$

$$\Big\downarrow \; 888$$

$$\frac{2x^{1-n}\sqrt{\frac{ax^{j-n}}{b}+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1-\frac{3n}{2}}{j-n},\frac{1-\frac{3n}{2}}{j-n}+1,-\frac{ax^{j-n}}{b}\right)}{b(2-3n)\sqrt{ax^j+bx^n}}$$

input
```
Int[(a*x^j + b*x^n)^(-3/2),x]
```

output
```
(2*x^(1 - n)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[3/2, (1 - (3*n)/2
)/(j - n), 1 + (1 - (3*n)/2)/(j - n), -((a*x^(j - n))/b)])/(b*(2 - 3*n)*Sq
rt[a*x^j + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{1}{\left(a\,x^{j}+b\,x^{n}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(a*x^j+b*x^n)^(3/2),x)
```

output
```
int(1/(a*x^j+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(ax^j+bx^n)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(ax^j+bx^n)^{3/2}}\,dx=\int \frac{1}{(ax^j+bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*x**j+b*x**n)**(3/2),x)
```

output
```
Integral((a*x**j + b*x**n)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(ax^j+bx^n)^{3/2}}\,dx=\int \frac{1}{(ax^j+bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x^j + b*x^n)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(ax^j+bx^n)^{3/2}}\,dx=\int \frac{1}{(ax^j+bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*x^j + b*x^n)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 9.56 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

$$\int \frac{1}{(ax^j+bx^n)^{3/2}}\,dx=-\frac{x\,{\left(\frac{b\,x^{n-j}}{a}+1\right)}^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{\frac{3\,j}{2}-1}{j-n};\ \frac{\frac{3\,j}{2}-1}{j-n}+1;\ -\frac{b\,x^{n-j}}{a}\right)}{\left(\frac{3\,j}{2}-1\right)\,{\left(a\,x^j+b\,x^n\right)}^{3/2}}$$

input
```
int(1/(a*x^j + b*x^n)^(3/2),x)
```

output
```
-(x*((b*x^(n - j))/a + 1)^(3/2)*hypergeom([3/2, ((3*j)/2 - 1)/(j - n)], ((
3*j)/2 - 1)/(j - n) + 1, -(b*x^(n - j))/a))/(((3*j)/2 - 1)*(a*x^j + b*x^n)
^(3/2))
```

Reduce [F]

$$\int \frac{1}{(ax^j+bx^n)^{3/2}}\,dx=\int \frac{\sqrt{x^j a+x^n b}}{x^{2j}a^2+2x^{j+n}ab+x^{2n}b^2}dx$$

input
```
int(1/(a*x^j+b*x^n)^(3/2),x)
```

output
```
int(sqrt(x**j*a + x**n*b)/(x**(2*j)*a**2 + 2*x**(j + n)*a*b + x**(2*n)*b**
2),x)
```

3.409 $\int \frac{1}{(ax^j+bx^n)^{5/2}}\,dx$

Optimal result 3103
Mathematica [A] (verified) 3103
Rubi [A] (verified) 3104
Maple [F] 3105
Fricas [F(-2)] 3105
Sympy [F] 3106
Maxima [F] 3106
Giac [F] 3106
Mupad [B] (verification not implemented) 3107
Reduce [F] 3107

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{1}{(ax^j+bx^n)^{5/2}}\,dx=\frac{2x^{1-2n}\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1-\frac{5n}{2}}{j-n},1+\frac{1-\frac{5n}{2}}{j-n},-\frac{ax^{j-n}}{b}\right)}{b^2(2-5n)\sqrt{ax^j+bx^n}}$$

output
```
2*x^(1-2*n)*(1+a*x^(j-n)/b)^(1/2)*hypergeom([5/2, (1-5/2*n)/(j-n)],[1+(1-5
/2*n)/(j-n)],-a*x^(j-n)/b)/b^2/(2-5*n)/(a*x^j+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.83

$$\int \frac{1}{(ax^j+bx^n)^{5/2}}\,dx=\frac{2x^{1-2j}\left(-((-2+4j+n)\,(a(-2+j+4n)x^j+b(-2+2j+3n)x^n))+(4+8j^2-\right.}{3a^2(2-}$$

input
```
Integrate[(a*x^j + b*x^n)^(-5/2),x]
```

output
```
(2*x^(1 - 2*j)*(-((-2 + 4*j + n)*(a*(-2 + j + 4*n)*x^j + b*(-2 + 2*j + 3*n
)*x^n)) + (4 + 8*j^2 - 8*n + 3*n^2 + 2*j*(-6 + 7*n))*Sqrt[1 + (a*x^(j - n)
)/b]*(a*x^j + b*x^n)*Hypergeometric2F1[1/2, -1/2*(-2 + 4*j + n)/(j - n), (
2 - 2*j - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]))/(3*a^2*(2 - 4*j - n)*(j
- n)^2*(a*x^j + b*x^n)^(3/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1917, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^j+bx^n)^{5/2}}\,dx$$

$$\Big\downarrow \text{1917}$$

$$\frac{x^{n/2}\sqrt{ax^{j-n}+b}\int \frac{x^{-5n/2}}{(ax^{j-n}+b)^{5/2}}dx}{\sqrt{ax^j+bx^n}}$$

$$\Big\downarrow \text{889}$$

$$\frac{x^{n/2}\sqrt{\frac{ax^{j-n}}{b}+1}\int \frac{x^{-5n/2}}{\left(\frac{ax^{j-n}}{b}+1\right)^{5/2}}dx}{b^2\sqrt{ax^j+bx^n}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x^{1-2n}\sqrt{\frac{ax^{j-n}}{b}+1}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1-\frac{5n}{2}}{j-n},\frac{1-\frac{5n}{2}}{j-n}+1,-\frac{ax^{j-n}}{b}\right)}{b^2(2-5n)\sqrt{ax^j+bx^n}}$$

input
```
Int[(a*x^j + b*x^n)^(-5/2),x]
```

output
```
(2*x^(1 - 2*n)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[5/2, (1 - (5*n)
/2)/(j - n), 1 + (1 - (5*n)/2)/(j - n), -((a*x^(j - n))/b)])/(b^2*(2 - 5*n
)*Sqrt[a*x^j + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{1}{(a\,x^{j}+b\,x^{n})^{\frac{5}{2}}}dx$$

input
```
int(1/(a*x^j+b*x^n)^(5/2),x)
```

output
```
int(1/(a*x^j+b*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(ax^j+bx^n)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x^j+b*x^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(ax^j + bx^n)^{5/2}}\,dx = \int \frac{1}{(ax^j + bx^n)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*x**j+b*x**n)**(5/2),x)
```

output
```
Integral((a*x**j + b*x**n)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(ax^j + bx^n)^{5/2}}\,dx = \int \frac{1}{(ax^j + bx^n)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*x^j+b*x^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*x^j + b*x^n)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(ax^j + bx^n)^{5/2}}\,dx = \int \frac{1}{(ax^j + bx^n)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*x^j+b*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*x^j + b*x^n)^(-5/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

$$\int \frac{1}{(ax^j + bx^n)^{5/2}} \, dx = -\frac{x \left(\frac{b\,x^{n-j}}{a} + 1\right)^{5/2} {}_2F_1\left(\frac{5}{2}, \frac{\frac{5\,j}{2}-1}{j-n};\ \frac{\frac{5\,j}{2}-1}{j-n} + 1;\ -\frac{b\,x^{n-j}}{a}\right)}{\left(\frac{5\,j}{2} - 1\right)\,\left(a\,x^j + b\,x^n\right)^{5/2}}$$

input
```
int(1/(a*x^j + b*x^n)^(5/2),x)
```

output
```
-(x*((b*x^(n - j))/a + 1)^(5/2)*hypergeom([5/2, ((5*j)/2 - 1)/(j - n)], ((
5*j)/2 - 1)/(j - n) + 1, -(b*x^(n - j))/a))/(((5*j)/2 - 1)*(a*x^j + b*x^n)
^(5/2))
```

Reduce [F]

$$\int \frac{1}{(ax^j + bx^n)^{5/2}} \, dx = \int \frac{\sqrt{x^j a + x^n b}}{x^{3j} a^3 + 3x^{2j+n} a^2 b + 3x^{j+2n} a\, b^2 + x^{3n} b^3} dx$$

input
```
int(1/(a*x^j+b*x^n)^(5/2),x)
```

output
```
int(sqrt(x**j*a + x**n*b)/(x**(3*j)*a**3 + 3*x**(2*j + n)*a**2*b + 3*x**(j
 + 2*n)*a*b**2 + x**(3*n)*b**3),x)
```

3.410 $\int \sqrt{\frac{1+x}{x^5}}\,dx$

Optimal result . . . 3108
Mathematica [A] (verified) . . . 3108
Rubi [A] (verified) . . . 3109
Maple [A] (verified) . . . 3110
Fricas [A] (verification not implemented) . . . 3110
Sympy [F] . . . 3111
Maxima [F] . . . 3111
Giac [B] (verification not implemented) . . . 3111
Mupad [B] (verification not implemented) . . . 3112
Reduce [B] (verification not implemented) . . . 3112

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \sqrt{\frac{1+x}{x^5}}\,dx = -\frac{2}{3}x^6\left(\frac{1+x}{x^5}\right)^{3/2}$$

output
```
-2/3*x^6*((1+x)/x^5)^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \sqrt{\frac{1+x}{x^5}}\,dx = -\frac{2}{3}x(1+x)\sqrt{\frac{1+x}{x^5}}$$

input
```
Integrate[Sqrt[(1 + x)/x^5],x]
```

output
```
(-2*x*(1 + x)*Sqrt[(1 + x)/x^5])/3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2078, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{x+1}{x^5}}dx$$

$$\Big\downarrow \text{ 2078}$$

$$\int \sqrt{\frac{1}{x^5}+\frac{1}{x^4}}dx$$

$$\Big\downarrow \text{ 1906}$$

$$-\frac{2}{3}\left(\frac{1}{x^5}+\frac{1}{x^4}\right)^{3/2}x^6$$

input
```
Int[Sqrt[(1 + x)/x^5],x]
```

output
```
(-2*(x^(-5) + x^(-4))^(3/2)*x^6)/3
```

Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result	size
gosper	$-\frac{2(x+1)x\sqrt{\frac{x+1}{x^5}}}{3}$	16
orering	$-\frac{2(x+1)x\sqrt{\frac{x+1}{x^5}}}{3}$	16
trager	$-\frac{2(x+1)x\sqrt{-\frac{-x-1}{x^5}}}{3}$	19
default	$-\frac{2\sqrt{\frac{x+1}{x^5}}\,(x^2+x)^{\frac{3}{2}}}{3\sqrt{(x+1)x}}$	26
risch	$-\frac{2\sqrt{\frac{x+1}{x^5}}\,x(x^2+2x+1)}{3(x+1)}$	26

input
```
int(((x+1)/x^5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(x+1)*x*((x+1)/x^5)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \sqrt{\frac{1+x}{x^5}}\,dx = -\frac{2}{3}\left(x^2+x\right)\sqrt{\frac{x+1}{x^5}}$$

input
```
integrate(((1+x)/x^5)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(x^2 + x)*sqrt((x + 1)/x^5)
```

Sympy [F]

$$\int \sqrt{\frac{1+x}{x^5}}\,dx = \int \sqrt{\frac{x+1}{x^5}}\,dx$$

input
```
integrate(((1+x)/x**5)**(1/2),x)
```

output
```
Integral(sqrt((x + 1)/x**5), x)
```

Maxima [F]

$$\int \sqrt{\frac{1+x}{x^5}}\,dx = \int \sqrt{\frac{x+1}{x^5}}\,dx$$

input
```
integrate(((1+x)/x^5)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((x + 1)/x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(14) = 28$.

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int \sqrt{\frac{1+x}{x^5}}\,dx = \frac{2\left(3\left(x-\sqrt{x^2+x}\right)^2\operatorname{sgn}(x)+3\left(x-\sqrt{x^2+x}\right)\operatorname{sgn}(x)+\operatorname{sgn}(x)\right)}{3\left(x-\sqrt{x^2+x}\right)^3}$$

input
```
integrate(((1+x)/x^5)^(1/2),x, algorithm="giac")
```

output
```
2/3*(3*(x - sqrt(x^2 + x))^2*sgn(x) + 3*(x - sqrt(x^2 + x))*sgn(x) + sgn(x
))/(x - sqrt(x^2 + x))^3
```

Mupad [B] (verification not implemented)

Time = 9.57 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \sqrt{\frac{1+x}{x^5}}\,dx = -\frac{2\,x\,\sqrt{\frac{x+1}{x^5}}\,(x+1)}{3}$$

input
```
int(((x + 1)/x^5)^(1/2),x)
```

output
```
-(2*x*((x + 1)/x^5)^(1/2)*(x + 1))/3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

$$\int \sqrt{\frac{1+x}{x^5}}\,dx = \frac{-2\sqrt{x}\,\sqrt{x+1}\,x-2\sqrt{x}\,\sqrt{x+1}-2x^2}{3x^2}$$

input
```
int(((1+x)/x^5)^(1/2),x)
```

output
```
( - 2*(sqrt(x)*sqrt(x + 1)*x + sqrt(x)*sqrt(x + 1) + x**2))/(3*x**2)
```

3.411 $\int \sqrt{x+x^{5/2}}\,dx$

Optimal result . . . 3113
Mathematica [A] (verified) . . . 3113
Rubi [A] (verified) . . . 3114
Maple [A] (verified) . . . 3115
Fricas [A] (verification not implemented) . . . 3115
Sympy [F] . . . 3116
Maxima [F] . . . 3116
Giac [A] (verification not implemented) . . . 3116
Mupad [B] (verification not implemented) . . . 3117
Reduce [B] (verification not implemented) . . . 3117

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \sqrt{x+x^{5/2}}\,dx=\frac{4\left(x+x^{5/2}\right)^{3/2}}{9x^{3/2}}$$

output

```
4/9*(x+x^(5/2))^(3/2)/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{x+x^{5/2}}\,dx=\frac{4\left(x+x^{5/2}\right)^{3/2}}{9x^{3/2}}$$

input

```
Integrate[Sqrt[x + x^(5/2)],x]
```

output

```
(4*(x + x^(5/2))^(3/2))/(9*x^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{1906\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^{5/2}+x}\,dx$$

$$\Big\downarrow \text{ 1906}$$

$$\frac{4\left(x^{5/2}+x\right)^{3/2}}{9x^{3/2}}$$

input
```
Int[Sqrt[x + x^(5/2)],x]
```

output
```
(4*(x + x^(5/2))^(3/2))/(9*x^(3/2))
```

Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j + 
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p}, 
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{4\sqrt{x+x^{\frac{5}{2}}}\,\left(x^{\frac{3}{2}}+1\right)}{9\sqrt{x}}$	18
default	$\frac{4\sqrt{x+x^{\frac{5}{2}}}\,\left(x^{\frac{3}{2}}+1\right)}{9\sqrt{x}}$	18
meijerg	$-\frac{\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\left(2+2x^{\frac{3}{2}}\right)\sqrt{x^{\frac{3}{2}}+1}}{3}}{3\sqrt{\pi}}$	31

input
```
int((x+x^(5/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/9*(x+x^(5/2))^(1/2)/x^(1/2)*(x^(3/2)+1)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \sqrt{x+x^{5/2}}\,dx = \frac{4\,\sqrt{x^{\frac{5}{2}}+x}\left(x^2+\sqrt{x}\right)}{9\,x}$$

input
```
integrate((x+x^(5/2))^(1/2),x, algorithm="fricas")
```

output
```
4/9*sqrt(x^(5/2) + x)*(x^2 + sqrt(x))/x
```

Sympy [F]

$$\int \sqrt{x+x^{5/2}}\,dx = \int \sqrt{x^{\frac{5}{2}}+x}\,dx$$

input
```
integrate((x+x**(5/2))**(1/2),x)
```

output
```
Integral(sqrt(x**(5/2) + x), x)
```

Maxima [F]

$$\int \sqrt{x+x^{5/2}}\,dx = \int \sqrt{x^{\frac{5}{2}}+x}\,dx$$

input
```
integrate((x+x^(5/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^(5/2) + x), x)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.55

$$\int \sqrt{x+x^{5/2}}\,dx = \frac{4}{9}\left(x^{\frac{3}{2}}+1\right)^{\frac{3}{2}} - \frac{4}{9}$$

input
```
integrate((x+x^(5/2))^(1/2),x, algorithm="giac")
```

output
```
4/9*(x^(3/2) + 1)^(3/2) - 4/9
```

Mupad [B] (verification not implemented)

Time = 9.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \sqrt{x+x^{5/2}}\,dx=\frac{2\,x\,\sqrt{x+x^{5/2}}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},1;\ 2;\ -x^{3/2}\right)}{3\,\sqrt{x^{3/2}+1}}$$

input
```
int((x + x^(5/2))^(1/2),x)
```

output
```
(2*x*(x + x^(5/2))^(1/2)*hypergeom([-1/2, 1], 2, -x^(3/2)))/(3*(x^(3/2) +
1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \sqrt{x+x^{5/2}}\,dx=\frac{4\sqrt{\sqrt{x}\,x+1}\,\left(\sqrt{x}\,x+1\right)}{9}$$

input
```
int((x+x^(5/2))^(1/2),x)
```

output
```
(4*sqrt(sqrt(x)*x + 1)*(sqrt(x)*x + 1))/9
```

3.412 $\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx$

Optimal result . . . 3118
Mathematica [A] (verified) . . . 3118
Rubi [A] (verified) . . . 3119
Maple [A] (verified) . . . 3120
Fricas [A] (verification not implemented) . . . 3120
Sympy [A] (verification not implemented) . . . 3121
Maxima [A] (verification not implemented) . . . 3121
Giac [A] (verification not implemented) . . . 3121
Mupad [B] (verification not implemented) . . . 3122
Reduce [B] (verification not implemented) . . . 3122

Optimal result

Integrand size = 13, antiderivative size = 8

$$\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx = 2\arctan\left(\sqrt{x}\right)$$

output
```
2*arctan(x^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx = 2\arctan\left(\sqrt{x}\right)$$

input
```
Integrate[(Sqrt[x] + x^(3/2))^(-1),x]
```

output
```
2*ArcTan[Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2027, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}+\sqrt{x}}dx$$
$$\Big\downarrow \text{ 2027}$$
$$\int \frac{1}{\sqrt{x}(x+1)}dx$$
$$\Big\downarrow \text{ 73}$$
$$2\int \frac{1}{x+1}d\sqrt{x}$$
$$\Big\downarrow \text{ 216}$$
$$2\arctan\left(\sqrt{x}\right)$$

input
```
Int[(Sqrt[x] + x^(3/2))^(-1),x]
```

output
```
2*ArcTan[Sqrt[x]]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$2\arctan\left(\sqrt{x}\right)$	7
default	$2\arctan\left(\sqrt{x}\right)$	7
meijerg	$2\arctan\left(\sqrt{x}\right)$	7
trager	$\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{2\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{x}+x-1}{x+1}\right)$	29

input
```
int(1/(x^(1/2)+x^(3/2)),x,method=_RETURNVERBOSE)
```

output
```
2*arctan(x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx=2\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(1/(x^(1/2)+x^(3/2)),x, algorithm="fricas")
```

output
```
2*arctan(sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx = 2\,\text{atan}\left(\sqrt{x}\right)$$

input
```
integrate(1/(x**(1/2)+x**(3/2)),x)
```

output
```
2*atan(sqrt(x))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx = 2\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(1/(x^(1/2)+x^(3/2)),x, algorithm="maxima")
```

output
```
2*arctan(sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx = 2\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(1/(x^(1/2)+x^(3/2)),x, algorithm="giac")
```

output
```
2*arctan(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx = 2\,\mathrm{atan}\left(\sqrt{x}\right)$$

input
```
int(1/(x^(1/2) + x^(3/2)),x)
```

output
```
2*atan(x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx = 2atan\left(\sqrt{x}\right)$$

input
```
int(1/(x^(1/2)+x^(3/2)),x)
```

output
```
2*atan(sqrt(x))
```

3.413 $\int x\sqrt{x^2\left(a+bx^3\right)}\,dx$

Optimal result . . . 3123
Mathematica [A] (verified) . . . 3123
Rubi [A] (verified) . . . 3124
Maple [A] (verified) . . . 3124
Fricas [A] (verification not implemented) . . . 3125
Sympy [F(-1)] . . . 3125
Maxima [A] (verification not implemented) . . . 3126
Giac [A] (verification not implemented) . . . 3126
Mupad [B] (verification not implemented) . . . 3126
Reduce [B] (verification not implemented) . . . 3127

Optimal result

Integrand size = 17, antiderivative size = 25

$$\int x\sqrt{x^2\left(a+bx^3\right)}\,dx=\frac{2\left(ax^2+bx^5\right)^{3/2}}{9bx^3}$$

output
```
2/9*(b*x^5+a*x^2)^(3/2)/b/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x\sqrt{x^2\left(a+bx^3\right)}\,dx=\frac{2\left(x^2\left(a+bx^3\right)\right)^{3/2}}{9bx^3}$$

input
```
Integrate[x*Sqrt[x^2*(a + b*x^3)],x]
```

output
```
(2*(x^2*(a + b*x^3))^(3/2))/(9*b*x^3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{x^2\left(a+bx^3\right)}\,dx$$

$$\Big\downarrow \text{ 2021}$$

$$\frac{2\left(x^2\left(a+bx^3\right)\right)^{3/2}}{9bx^3}$$

input
```
Int[x*Sqrt[x^2*(a + b*x^3)],x]
```

output
```
(2*(x^2*(a + b*x^3))^(3/2))/(9*b*x^3)
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

method	result	size
gosper	$\frac{2(b\,x^{3}+a)\,\sqrt{x^{2}(b\,x^{3}+a)}}{9bx}$	29
default	$\frac{2(b\,x^{3}+a)\,\sqrt{x^{2}(b\,x^{3}+a)}}{9bx}$	29
trager	$\frac{2(b\,x^{3}+a)\,\sqrt{b\,x^{5}+a\,x^{2}}}{9bx}$	29
risch	$\frac{2(b\,x^{3}+a)\,\sqrt{x^{2}(b\,x^{3}+a)}}{9bx}$	29
orering	$\frac{2(b\,x^{3}+a)\,\sqrt{x^{2}(b\,x^{3}+a)}}{9bx}$	29

input
```
int(x*(x^2*(b*x^3+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*(b*x^3+a)*(x^2*(b*x^3+a))^(1/2)/b/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int x\sqrt{x^2\,(a+bx^3)}\,dx=\frac{2\,\sqrt{bx^5+ax^2}(bx^3+a)}{9\,bx}$$

input
```
integrate(x*(x^2*(b*x^3+a))^(1/2),x, algorithm="fricas")
```

output
```
2/9*sqrt(b*x^5 + a*x^2)*(b*x^3 + a)/(b*x)
```

Sympy [F(-1)]

Timed out.

$$\int x\sqrt{x^2\,(a+bx^3)}\,dx=\text{Timed out}$$

input
```
integrate(x*(x**2*(b*x**3+a))**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int x\sqrt{x^2\left(a+bx^3\right)}\,dx=\frac{2\left(bx^3+a\right)^{\frac{3}{2}}}{9\,b}$$

input
```
integrate(x*(x^2*(b*x^3+a))^(1/2),x, algorithm="maxima")
```

output
```
2/9*(b*x^3 + a)^(3/2)/b
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x\sqrt{x^2\left(a+bx^3\right)}\,dx=\frac{2\left(bx^3+a\right)^{\frac{3}{2}}\mathrm{sgn}(x)}{9\,b}-\frac{2\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{9\,b}$$

input
```
integrate(x*(x^2*(b*x^3+a))^(1/2),x, algorithm="giac")
```

output
```
2/9*(b*x^3 + a)^(3/2)*sgn(x)/b - 2/9*a^(3/2)*sgn(x)/b
```

Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int x\sqrt{x^2\left(a+bx^3\right)}\,dx=\frac{2\left(b\,x^3+a\right)^{3/2}\sqrt{x^2}}{9\,b\,x}$$

input
```
int(x*(x^2*(a + b*x^3))^(1/2),x)
```

output
```
(2*(a + b*x^3)^(3/2)*(x^2)^(1/2))/(9*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int x\sqrt{x^2\left(a+bx^3\right)}\,dx=\frac{2\sqrt{b\,x^3+a}\,(b\,x^3+a)}{9b}$$

input
```
int(x*(x^2*(b*x^3+a))^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3)*(a + b*x**3))/(9*b)
```

3.414 $\int x\sqrt{ax^2+bx^5}\,dx$

Optimal result . . . 3128
Mathematica [A] (verified) . . . 3128
Rubi [A] (verified) . . . 3129
Maple [A] (verified) . . . 3129
Fricas [A] (verification not implemented) . . . 3130
Sympy [F] . . . 3130
Maxima [A] (verification not implemented) . . . 3131
Giac [A] (verification not implemented) . . . 3131
Mupad [B] (verification not implemented) . . . 3131
Reduce [B] (verification not implemented) . . . 3132

Optimal result

Integrand size = 17, antiderivative size = 25

$$\int x\sqrt{ax^2+bx^5}\,dx=\frac{2(ax^2+bx^5)^{3/2}}{9bx^3}$$

output
```
2/9*(b*x^5+a*x^2)^(3/2)/b/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x\sqrt{ax^2+bx^5}\,dx=\frac{2(x^2(a+bx^3))^{3/2}}{9bx^3}$$

input
```
Integrate[x*Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*(x^2*(a + b*x^3))^(3/2))/(9*b*x^3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{ax^2+bx^5}\,dx$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{2(ax^2+bx^5)^{3/2}}{9bx^3}$$

input
```
Int[x*Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*(a*x^2 + b*x^5)^(3/2))/(9*b*x^3)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

method	result	size
gosper	$\frac{2(b\,x^3+a)\sqrt{b\,x^5+a\,x^2}}{9bx}$	29
default	$\frac{2(b\,x^3+a)\sqrt{b\,x^5+a\,x^2}}{9bx}$	29
trager	$\frac{2(b\,x^3+a)\sqrt{b\,x^5+a\,x^2}}{9bx}$	29
risch	$\frac{2(b\,x^3+a)\sqrt{x^2(b\,x^3+a)}}{9bx}$	29
orering	$\frac{2(b\,x^3+a)\sqrt{b\,x^5+a\,x^2}}{9bx}$	29

input
```
int(x*(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*(b*x^3+a)/b/x*(b*x^5+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int x\sqrt{ax^2+bx^5}\,dx=\frac{2\sqrt{bx^5+ax^2}(bx^3+a)}{9\,bx}$$

input
```
integrate(x*(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/9*sqrt(b*x^5 + a*x^2)*(b*x^3 + a)/(b*x)
```

Sympy [F]

$$\int x\sqrt{ax^2+bx^5}\,dx=\int x\sqrt{x^2\left(a+bx^3\right)}\,dx$$

input
```
integrate(x*(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x*sqrt(x**2*(a + b*x**3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int x\sqrt{ax^2+bx^5}\,dx=\frac{2\,(bx^3+a)^{\frac{3}{2}}}{9\,b}$$

input
```
integrate(x*(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/9*(b*x^3 + a)^(3/2)/b
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x\sqrt{ax^2+bx^5}\,dx=\frac{2\,(bx^3+a)^{\frac{3}{2}}\mathrm{sgn}(x)}{9\,b}-\frac{2\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{9\,b}$$

input
```
integrate(x*(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
2/9*(b*x^3 + a)^(3/2)*sgn(x)/b - 2/9*a^(3/2)*sgn(x)/b
```

Mupad [B] (verification not implemented)

Time = 9.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int x\sqrt{ax^2+bx^5}\,dx=\frac{\left(\frac{2\,a}{9\,b}+\frac{2\,x^3}{9}\right)\,\sqrt{b\,x^5+a\,x^2}}{x}$$

input
```
int(x*(a*x^2 + b*x^5)^(1/2),x)
```

output
```
(((2*a)/(9*b) + (2*x^3)/9)*(a*x^2 + b*x^5)^(1/2))/x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int x\sqrt{ax^2+bx^5}\,dx=\frac{2\sqrt{b\,x^3+a}\,(b\,x^3+a)}{9b}$$

input
```
int(x*(b*x^5+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3)*(a + b*x**3))/(9*b)
```

3.415 $\int \sqrt{x^4 \left(a+b x^3\right)} \, dx$

Optimal result . . . 3133
Mathematica [A] (verified) . . . 3133
Rubi [A] (verified) . . . 3134
Maple [A] (verified) . . . 3135
Fricas [A] (verification not implemented) . . . 3135
Sympy [F] . . . 3136
Maxima [A] (verification not implemented) . . . 3136
Giac [A] (verification not implemented) . . . 3136
Mupad [B] (verification not implemented) . . . 3137
Reduce [B] (verification not implemented) . . . 3137

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \sqrt{x^4 \left(a+b x^3\right)} \, dx=\frac{2 \left(a x^4+b x^7\right)^{3/2}}{9 b x^6}$$

output
```
2/9*(b*x^7+a*x^4)^(3/2)/b/x^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{x^4 \left(a+b x^3\right)} \, dx=\frac{2 \left(x^4 \left(a+b x^3\right)\right)^{3/2}}{9 b x^6}$$

input
```
Integrate[Sqrt[x^4*(a + b*x^3)],x]
```

output
```
(2*(x^4*(a + b*x^3))^(3/2))/(9*b*x^6)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2078, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^4 \left(a+b x^3\right)} \, dx$$

$$\Big\downarrow \ 2078$$

$$\int \sqrt{a x^4+b x^7}dx$$

$$\Big\downarrow \ 1906$$

$$\frac{2 \left(a x^4+b x^7\right)^{3/2}}{9 b x^6}$$

input
```
Int[Sqrt[x^4*(a + b*x^3)],x]
```

output
```
(2*(a*x^4 + b*x^7)^(3/2))/(9*b*x^6)
```

Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

method	result	size
gosper	$\frac{2(b\,x^{3}+a)\,\sqrt{x^{4}(b\,x^{3}+a)}}{9b\,x^{2}}$	29
default	$\frac{2(b\,x^{3}+a)\,\sqrt{x^{4}(b\,x^{3}+a)}}{9b\,x^{2}}$	29
trager	$\frac{2(b\,x^{3}+a)\sqrt{b\,x^{7}+a\,x^{4}}}{9b\,x^{2}}$	29
risch	$\frac{2(b\,x^{3}+a)\,\sqrt{x^{4}(b\,x^{3}+a)}}{9b\,x^{2}}$	29
orering	$\frac{2(b\,x^{3}+a)\,\sqrt{x^{4}(b\,x^{3}+a)}}{9b\,x^{2}}$	29

input
```
int((x^4*(b*x^3+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*(b*x^3+a)*(x^4*(b*x^3+a))^(1/2)/b/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \sqrt{x^4\,(a+bx^3)}\,dx = \frac{2\,\sqrt{bx^7+ax^4}(bx^3+a)}{9\,bx^2}$$

input
```
integrate((x^4*(b*x^3+a))^(1/2),x, algorithm="fricas")
```

output
```
2/9*sqrt(b*x^7 + a*x^4)*(b*x^3 + a)/(b*x^2)
```

Sympy [F]

$$\int \sqrt{x^4 (a + bx^3)}\, dx = \int \sqrt{x^4 (a + bx^3)}\, dx$$

input
```
integrate((x**4*(b*x**3+a))**(1/2),x)
```

output
```
Integral(sqrt(x**4*(a + b*x**3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \sqrt{x^4 (a + bx^3)}\, dx = \frac{2 (bx^3 + a)^{\frac{3}{2}}}{9 b}$$

input
```
integrate((x^4*(b*x^3+a))^(1/2),x, algorithm="maxima")
```

output
```
2/9*(b*x^3 + a)^(3/2)/b
```

Giac [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \sqrt{x^4 (a + bx^3)}\, dx = \frac{2 (bx^3 + a)^{\frac{3}{2}}}{9 b}$$

input
```
integrate((x^4*(b*x^3+a))^(1/2),x, algorithm="giac")
```

output
```
2/9*(b*x^3 + a)^(3/2)/b
```

Mupad [B] (verification not implemented)

Time = 8.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \sqrt{x^4 (a+bx^3)}\,dx = \frac{2\,(b\,x^3+a)^{3/2}\,\sqrt{x^4}}{9\,b\,x^2}$$

input
```
int((x^4*(a + b*x^3))^(1/2),x)
```

output
```
(2*(a + b*x^3)^(3/2)*(x^4)^(1/2))/(9*b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \sqrt{x^4 (a+bx^3)}\,dx = \frac{2\sqrt{b\,x^3+a}\,(b\,x^3+a)}{9b}$$

input
```
int((x^4*(b*x^3+a))^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3)*(a + b*x**3))/(9*b)
```

3.416 $\int (cx)^m (ax^q + bx^r)^3 \, dx$

Optimal result . . . 3138
Mathematica [A] (verified) . . . 3138
Rubi [A] (verified) . . . 3139
Maple [C] (warning: unable to verify) . . . 3140
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Maxima [A] (verification not implemented) . . . 3143
Giac [B] (verification not implemented) . . . 3144
Mupad [B] (verification not implemented) . . . 3145
Reduce [B] (verification not implemented) . . . 3145

Optimal result

Integrand size = 19, antiderivative size = 105

$$\int (cx)^m (ax^q+bx^r)^3 \, dx = \frac{a^3x^{1+3q}(cx)^m}{1+m+3q} + \frac{3a^2bx^{1+2q+r}(cx)^m}{1+m+2q+r} + \frac{3ab^2x^{1+q+2r}(cx)^m}{1+m+q+2r} + \frac{b^3x^{1+3r}(cx)^m}{1+m+3r}$$

output
```
a^3*x^(1+3*q)*(c*x)^m/(1+m+3*q)+3*a^2*b*x^(1+2*q+r)*(c*x)^m/(1+m+2*q+r)+3*
a*b^2*x^(1+q+2*r)*(c*x)^m/(1+m+q+2*r)+b^3*x^(1+3*r)*(c*x)^m/(1+m+3*r)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.82

$$\int (cx)^m (ax^q+bx^r)^3 \, dx = x(cx)^m \left(\frac{a^3x^{3q}}{1+m+3q} + \frac{b^3x^{3r}}{1+m+3r} + \frac{3a^2bx^{2q+r}}{1+m+2q+r} + \frac{3ab^2x^{q+2r}}{1+m+q+2r}\right)$$

input
```
Integrate[(c*x)^m*(a*x^q + b*x^r)^3,x]
```

output
```
x*(c*x)^m*((a^3*x^(3*q))/(1 + m + 3*q) + (b^3*x^(3*r))/(1 + m + 3*r) + (3*
a^2*b*x^(2*q + r))/(1 + m + 2*q + r) + (3*a*b^2*x^(q + 2*r))/(1 + m + q +
2*r))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1939, 10, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m (ax^q + bx^r)^3 \, dx$$

$$\Big\downarrow \text{1939}$$

$$x^{-m}(cx)^m \int x^m (ax^q + bx^r)^3 dx$$

$$\Big\downarrow \text{10}$$

$$x^{-m}(cx)^m \int x^{m+3r} \left(ax^{q-r} + b\right)^3 dx$$

$$\Big\downarrow \text{802}$$

$$x^{-m}(cx)^m \int \left(a^3 x^{m+3q} + 3a^2 b x^{m+2q+r} + 3ab^2 x^{m+q+2r} + b^3 x^{m+3r}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$x^{-m}(cx)^m \left(\frac{a^3 x^{m+3q+1}}{m+3q+1} + \frac{3a^2 b x^{m+2q+r+1}}{m+2q+r+1} + \frac{3ab^2 x^{m+q+2r+1}}{m+q+2r+1} + \frac{b^3 x^{m+3r+1}}{m+3r+1}\right)$$

input
```
Int[(c*x)^m*(a*x^q + b*x^r)^3,x]
```

output
```
((c*x)^m*((a^3*x^(1 + m + 3*q))/(1 + m + 3*q) + (3*a^2*b*x^(1 + m + 2*q +
r))/(1 + m + 2*q + r) + (3*a*b^2*x^(1 + m + q + 2*r))/(1 + m + q + 2*r) +
(b^3*x^(1 + m + 3*r))/(1 + m + 3*r)))/x^m
```

Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1939
```
Int[(u_)^(m_.)*((a_.)*(v_)^(j_.) + (b_.)*(v_)^(n_.))^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a*x^j + b*x^n)^p, x], x,
 v], x] /; FreeQ[{a, b, j, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.96 (sec) , antiderivative size = 1162, normalized size of antiderivative = 11.07

method	result	size
risch	Expression too large to display	1162
parallelrisch	Expression too large to display	1559
orering	Expression too large to display	1618

input
```
int((c*x)^m*(a*x^q+b*x^r)^3,x,method=_RETURNVERBOSE)
```

output
```
x*(54*a^2*b*m*q*r*(x^q)^2*x^r+12*a*b^2*x^q*(x^r)^2*r+54*a*b^2*q*r*x^q*(x^r
)^2+27*a*b^2*q*r^2*x^q*(x^r)^2+9*a^2*b*m*q^2*(x^q)^2*x^r+18*a^2*b*m*r^2*(x
^q)^2*x^r+54*a^2*b*q*r*(x^q)^2*x^r+18*a*b^2*m*q^2*x^q*(x^r)^2+24*a^2*b*m*q
*(x^q)^2*x^r+12*a*b^2*m^2*r*x^q*(x^r)^2+54*a*b^2*m*q*r*x^q*(x^r)^2+9*a*b^2
*m*r^2*x^q*(x^r)^2+12*a^2*b*m^2*q*(x^q)^2*x^r+9*a*b^2*r^2*x^q*(x^r)^2+9*m*
a^2*b*(x^q)^2*x^r+12*a^2*b*(x^q)^2*x^r*q+15*a^2*b*(x^q)^2*r*x^r+9*m*a*b^2*
x^q*(x^r)^2+15*a*b^2*q*x^q*(x^r)^2+b^3*m^3*(x^r)^3+3*(x^r)^2*x^q*a*b^2+15*
a^3*q*r^2*(x^q)^3+6*b^3*m^2*q*(x^r)^3+3*b^3*m^2*r*(x^r)^3+11*b^3*m*q^2*(x^
r)^3+2*b^3*m*r^2*(x^r)^3+15*b^3*q^2*r*(x^r)^3+6*b^3*q*r^2*(x^r)^3+6*a^3*m*
q*(x^q)^3+12*a^3*m*r*(x^q)^3+14*a^3*q*r*(x^q)^3+12*b^3*m*q*(x^r)^3+6*b^3*m
*r*(x^r)^3+14*b^3*q*r*(x^r)^3+3*a^3*m^2*q*(x^q)^3+6*a^3*m^2*r*(x^q)^3+30*a
*b^2*m*q*x^q*(x^r)^2+54*a*b^2*q^2*r*x^q*(x^r)^2+2*b^3*r^2*(x^r)^3+15*a*b^2
*m^2*q*x^q*(x^r)^2+24*a*b^2*m*r*x^q*(x^r)^2+30*a^2*b*m*r*(x^q)^2*x^r+54*a^
2*b*q*r^2*(x^q)^2*x^r+27*a^2*b*q^2*r*(x^q)^2*x^r+15*a^2*b*m^2*r*(x^q)^2*x^
r+2*a^3*m*q^2*(x^q)^3+11*a^3*m*r^2*(x^q)^3+6*a^3*q^2*r*(x^q)^3+3*m*a^3*(x^
q)^3+3*a^3*(x^q)^3*q+3*a^2*b*m^3*(x^q)^2*x^r+3*a*b^2*m^3*x^q*(x^r)^2+14*b^
3*m*q*r*(x^r)^3+9*a^2*b*m^2*(x^q)^2*x^r+9*a^2*b*q^2*(x^q)^2*x^r+18*a^2*b*r
^2*(x^q)^2*x^r+9*a*b^2*m^2*x^q*(x^r)^2+18*a*b^2*q^2*x^q*(x^r)^2+14*a^3*m*q
*r*(x^q)^3+3*a^3*m^2*(x^q)^3+2*a^3*q^2*(x^q)^3+11*a^3*r^2*(x^q)^3+3*b^3*m^
2*(x^r)^3+(x^q)^3*a^3+3*x^r*(x^q)^2*a^2*b+(x^r)^3*b^3+6*a^3*r*(x^q)^3+1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 763 vs. $2(105) = 210$.

Time = 0.11 (sec) , antiderivative size = 763, normalized size of antiderivative = 7.27

$$\int (cx)^m (ax^q + bx^r)^3 \, dx = \text{Too large to display}$$

input
```
integrate((c*x)^m*(a*x^q+b*x^r)^3,x, algorithm="fricas")
```

output
```
(3*(a*b^2*m^3 + 3*a*b^2*m^2 + 3*a*b^2*m + a*b^2 + 6*(a*b^2*m + a*b^2)*q^2
+ 3*(a*b^2*m + 3*a*b^2*q + a*b^2)*r^2 + 5*(a*b^2*m^2 + 2*a*b^2*m + a*b^2)*
q + 2*(2*a*b^2*m^2 + 9*a*b^2*q^2 + 4*a*b^2*m + 2*a*b^2 + 9*(a*b^2*m + a*b^
2)*q)*r)*x*x^q*x^(2*r)*e^(m*log(c) + m*log(x)) + 3*(a^2*b*m^3 + 3*a^2*b*m^
2 + 3*a^2*b*m + a^2*b + 3*(a^2*b*m + a^2*b)*q^2 + 6*(a^2*b*m + 3*a^2*b*q +
 a^2*b)*r^2 + 4*(a^2*b*m^2 + 2*a^2*b*m + a^2*b)*q + (5*a^2*b*m^2 + 9*a^2*b
*q^2 + 10*a^2*b*m + 5*a^2*b + 18*(a^2*b*m + a^2*b)*q)*r)*x*x^(2*q)*x^r*e^(
m*log(c) + m*log(x)) + (a^3*m^3 + 6*a^3*r^3 + 3*a^3*m^2 + 3*a^3*m + a^3 +
2*(a^3*m + a^3)*q^2 + (11*a^3*m + 15*a^3*q + 11*a^3)*r^2 + 3*(a^3*m^2 + 2*
a^3*m + a^3)*q + 2*(3*a^3*m^2 + 3*a^3*q^2 + 6*a^3*m + 3*a^3 + 7*(a^3*m + a
^3)*q)*r)*x*x^(3*q)*e^(m*log(c) + m*log(x)) + (b^3*m^3 + 6*b^3*q^3 + 3*b^3
*m^2 + 3*b^3*m + b^3 + 11*(b^3*m + b^3)*q^2 + 2*(b^3*m + 3*b^3*q + b^3)*r^
2 + 6*(b^3*m^2 + 2*b^3*m + b^3)*q + (3*b^3*m^2 + 15*b^3*q^2 + 6*b^3*m + 3*
b^3 + 14*(b^3*m + b^3)*q)*r)*x*x^(3*r)*e^(m*log(c) + m*log(x)))/(m^4 + 6*(
m + 1)*q^3 + 6*(m + 3*q + 1)*r^3 + 4*m^3 + 11*(m^2 + 2*m + 1)*q^2 + (11*m^
2 + 48*(m + 1)*q + 45*q^2 + 22*m + 11)*r^2 + 6*m^2 + 6*(m^3 + 3*m^2 + 3*m
+ 1)*q + 2*(3*m^3 + 24*(m + 1)*q^2 + 9*q^3 + 9*m^2 + 16*(m^2 + 2*m + 1)*q
+ 9*m + 3)*r + 4*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17454 vs. $2(100) = 200$.

Time = 15.87 (sec) , antiderivative size = 17454, normalized size of antiderivative = 166.23

$$\int (cx)^m (ax^q + bx^r)^3 \, dx = \text{Too large to display}$$

input
```
integrate((c*x)**m*(a*x**q+b*x**r)**3,x)
```

output
```
Piecewise((a**3*x*x**(3*r)*(c*x)**(-3*r - 1)*log(x) + 3*a**2*b*x*x**(3*r)*
(c*x)**(-3*r - 1)*log(x) + 3*a*b**2*x*x**(3*r)*(c*x)**(-3*r - 1)*log(x) +
b**3*x*x**(3*r)*(c*x)**(-3*r - 1)*log(x), Eq(q, r) & Eq(m, -3*r - 1)), (a*
*3*x*x**(3*q)*(c*x)**(-3*q - 1)*log(x) + 3*a**2*b*Piecewise((x*x**(2*q)*x*
*r*(c*x)**(-3*q - 1)/(-q + r), Ne(q - r, 0)), (x*x**(2*q)*x**r*(c*x)**(-3*
q - 1)*log(x), True)) + 3*a*b**2*Piecewise((x*x**q*x**(2*r)*(c*x)**(-3*q -
 1)/(-2*q + 2*r), Ne(2*q - 2*r, 0)), (x*x**q*x**(2*r)*(c*x)**(-3*q - 1)*lo
g(x), True)) + b**3*Piecewise((x*x**(3*r)*(c*x)**(-3*q - 1)/(-3*q + 3*r),
Ne(3*q - 3*r, 0)), (x*x**(3*r)*(c*x)**(-3*q - 1)*log(x), True)), Eq(m, -3*
q - 1)), (a**3*Piecewise((x*x**(3*q)*(c*x)**(-3*r - 1)/(3*q - 3*r), Ne(3*q
 - 3*r, 0)), (x*x**(3*q)*(c*x)**(-3*r - 1)*log(x), True)) + 3*a**2*b*Piece
wise((x*x**(2*q)*x**r*(c*x)**(-3*r - 1)/(2*q - 2*r), Ne(2*q - 2*r, 0)), (x
*x**(2*q)*x**r*(c*x)**(-3*r - 1)*log(x), True)) + 3*a*b**2*Piecewise((x*x*
*q*x**(2*r)*(c*x)**(-3*r - 1)/(q - r), Ne(q - r, 0)), (x*x**q*x**(2*r)*(c*
x)**(-3*r - 1)*log(x), True)) + b**3*x*x**(3*r)*(c*x)**(-3*r - 1)*log(x),
Eq(m, -3*r - 1)), (a**3*Piecewise((x*x**(3*q)*(c*x)**(-2*q - r - 1)/(q - r
), Ne(q - r, 0)), (x*x**(3*q)*(c*x)**(-2*q - r - 1)*log(x), True)) + 3*a**
2*b*x*x**(2*q)*x**r*(c*x)**(-2*q - r - 1)*log(x) + 3*a*b**2*Piecewise((x*x
**q*x**(2*r)*(c*x)**(-2*q - r - 1)/(-q + r), Ne(q - r, 0)), (x*x**q*x**(2*
r)*(c*x)**(-2*q - r - 1)*log(x), True)) + b**3*Piecewise((x*x**(3*r)*(c...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.17

$$\int (cx)^m (ax^q + bx^r)^3 \, dx = \frac{a^3 c^m x e^{(m \log(x) + 3\, q \log(x))}}{m + 3\, q + 1} + \frac{3\, a^2 b c^m x e^{(m \log(x) + 2\, q \log(x) + r \log(x))}}{m + 2\, q + r + 1} + \frac{3\, a b^2 c^m x e^{(m \log(x) + q \log(x) + 2\, r \log(x))}}{m + q + 2\, r + 1} + \frac{b^3 c^m x e^{(m \log(x) + 3\, r \log(x))}}{m + 3\, r + 1}$$

input
```
integrate((c*x)^m*(a*x^q+b*x^r)^3,x, algorithm="maxima")
```

output
```
a^3*c^m*x*e^(m*log(x) + 3*q*log(x))/(m + 3*q + 1) + 3*a^2*b*c^m*x*e^(m*log
(x) + 2*q*log(x) + r*log(x))/(m + 2*q + r + 1) + 3*a*b^2*c^m*x*e^(m*log(x)
 + q*log(x) + 2*r*log(x))/(m + q + 2*r + 1) + b^3*c^m*x*e^(m*log(x) + 3*r*
log(x))/(m + 3*r + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4599 vs. $2(105) = 210$.

Time = 0.30 (sec) , antiderivative size = 4599, normalized size of antiderivative = 43.80

$$\int (cx)^m (ax^q + bx^r)^3 \, dx = \text{Too large to display}$$

input
```
integrate((c*x)^m*(a*x^q+b*x^r)^3,x, algorithm="giac")
```

output
```
(3*a*b^2*m^3*x*x^q*x^(2*r)*e^(m*log(c) + m*log(x)) + 15*a*b^2*m^2*q*x*x^q*
x^(2*r)*e^(m*log(c) + m*log(x)) + 18*a*b^2*m*q^2*x*x^q*x^(2*r)*e^(m*log(c)
 + m*log(x)) + 12*a*b^2*m^2*r*x*x^q*x^(2*r)*e^(m*log(c) + m*log(x)) + 54*a
*b^2*m*q*r*x*x^q*x^(2*r)*e^(m*log(c) + m*log(x)) + 54*a*b^2*q^2*r*x*x^q*x^
(2*r)*e^(m*log(c) + m*log(x)) + 9*a*b^2*m*r^2*x*x^q*x^(2*r)*e^(m*log(c) +
m*log(x)) + 27*a*b^2*q*r^2*x*x^q*x^(2*r)*e^(m*log(c) + m*log(x)) + 3*a^2*b
*m^3*x*x^(2*q)*x^r*e^(m*log(c) + m*log(x)) + 12*a^2*b*m^2*q*x*x^(2*q)*x^r*
e^(m*log(c) + m*log(x)) + 9*a^2*b*m*q^2*x*x^(2*q)*x^r*e^(m*log(c) + m*log(
x)) + 15*a^2*b*m^2*r*x*x^(2*q)*x^r*e^(m*log(c) + m*log(x)) + 54*a^2*b*m*q*
r*x*x^(2*q)*x^r*e^(m*log(c) + m*log(x)) + 27*a^2*b*q^2*r*x*x^(2*q)*x^r*e^(
m*log(c) + m*log(x)) + 18*a^2*b*m*r^2*x*x^(2*q)*x^r*e^(m*log(c) + m*log(x)
) + 54*a^2*b*q*r^2*x*x^(2*q)*x^r*e^(m*log(c) + m*log(x)) + 3*a*b^2*m^3*x*x
^q*x^r*e^(m*log(c) + m*log(x)) + 15*a*b^2*m^2*q*x*x^q*x^r*e^(m*log(c) + m*
log(x)) + 18*a*b^2*m*q^2*x*x^q*x^r*e^(m*log(c) + m*log(x)) + 12*a*b^2*m^2*
r*x*x^q*x^r*e^(m*log(c) + m*log(x)) + 54*a*b^2*m*q*r*x*x^q*x^r*e^(m*log(c)
 + m*log(x)) + 54*a*b^2*q^2*r*x*x^q*x^r*e^(m*log(c) + m*log(x)) + 9*a*b^2*
m*r^2*x*x^q*x^r*e^(m*log(c) + m*log(x)) + 27*a*b^2*q*r^2*x*x^q*x^r*e^(m*lo
g(c) + m*log(x)) + a^3*m^3*x*x^(3*q)*e^(m*log(c) + m*log(x)) + 3*a^3*m^2*q
*x*x^(3*q)*e^(m*log(c) + m*log(x)) + 2*a^3*m*q^2*x*x^(3*q)*e^(m*log(c) + m
*log(x)) + 6*a^3*m^2*r*x*x^(3*q)*e^(m*log(c) + m*log(x)) + 14*a^3*m*q*r...
```

Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.55

$$\int (cx)^m (ax^q + bx^r)^3 \, dx$$
$$= (c\,x)^m \left(\frac{a^3\,x^{3\,q+1}}{m+3\,q+1} + \frac{b^3\,x^{3\,r+1}}{m+3\,r+1} + \frac{3\,a\,b^2\,x^{q+2\,r+1}\,(m+2\,q+r+1)}{m^2+3\,m\,q+3\,m\,r+2\,m+2\,q^2+5\,q\,r+3\,q+2\,r^2+3\,r+1} + \frac{3\,a^2\,b\,x^{2\,q+r+1}\,(m+q+2\,r+1)}{m^2+3\,m\,q+3\,m\,r+2\,m+2\,q^2+5\,q\,r+3\,q+2\,r^2+3\,r+1}\right)$$

input
```
int((c*x)^m*(a*x^q + b*x^r)^3,x)
```

output
```
(c*x)^m*((a^3*x^(3*q + 1))/(m + 3*q + 1) + (b^3*x^(3*r + 1))/(m + 3*r + 1)
 + (3*a*b^2*x^(q + 2*r + 1)*(m + 2*q + r + 1))/(2*m + 3*q + 3*r + 3*m*q +
3*m*r + 5*q*r + m^2 + 2*q^2 + 2*r^2 + 1) + (3*a^2*b*x^(2*q + r + 1)*(m + q
 + 2*r + 1))/(2*m + 3*q + 3*r + 3*m*q + 3*m*r + 5*q*r + m^2 + 2*q^2 + 2*r^
2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1231, normalized size of antiderivative = 11.72

$$\int (cx)^m (ax^q + bx^r)^3 \, dx = \text{Too large to display}$$

input
```
int((c*x)^m*(a*x^q+b*x^r)^3,x)
```

output

```
(x**m*c**m*x*(x**(3*q)*a**3*m**3 + 3*x**(3*q)*a**3*m**2*q + 6*x**(3*q)*a**
3*m**2*r + 3*x**(3*q)*a**3*m**2 + 2*x**(3*q)*a**3*m*q**2 + 14*x**(3*q)*a**
3*m*q*r + 6*x**(3*q)*a**3*m*q + 11*x**(3*q)*a**3*m*r**2 + 12*x**(3*q)*a**3
*m*r + 3*x**(3*q)*a**3*m + 6*x**(3*q)*a**3*q**2*r + 2*x**(3*q)*a**3*q**2 +
 15*x**(3*q)*a**3*q*r**2 + 14*x**(3*q)*a**3*q*r + 3*x**(3*q)*a**3*q + 6*x*
*(3*q)*a**3*r**3 + 11*x**(3*q)*a**3*r**2 + 6*x**(3*q)*a**3*r + x**(3*q)*a*
*3 + 3*x**(2*q + r)*a**2*b*m**3 + 12*x**(2*q + r)*a**2*b*m**2*q + 15*x**(2
*q + r)*a**2*b*m**2*r + 9*x**(2*q + r)*a**2*b*m**2 + 9*x**(2*q + r)*a**2*b
*m*q**2 + 54*x**(2*q + r)*a**2*b*m*q*r + 24*x**(2*q + r)*a**2*b*m*q + 18*x
**(2*q + r)*a**2*b*m*r**2 + 30*x**(2*q + r)*a**2*b*m*r + 9*x**(2*q + r)*a*
*2*b*m + 27*x**(2*q + r)*a**2*b*q**2*r + 9*x**(2*q + r)*a**2*b*q**2 + 54*x
**(2*q + r)*a**2*b*q*r**2 + 54*x**(2*q + r)*a**2*b*q*r + 12*x**(2*q + r)*a
**2*b*q + 18*x**(2*q + r)*a**2*b*r**2 + 15*x**(2*q + r)*a**2*b*r + 3*x**(2
*q + r)*a**2*b + 3*x**(q + 2*r)*a*b**2*m**3 + 15*x**(q + 2*r)*a*b**2*m**2*
q + 12*x**(q + 2*r)*a*b**2*m**2*r + 9*x**(q + 2*r)*a*b**2*m**2 + 18*x**(q
+ 2*r)*a*b**2*m*q**2 + 54*x**(q + 2*r)*a*b**2*m*q*r + 30*x**(q + 2*r)*a*b*
*2*m*q + 9*x**(q + 2*r)*a*b**2*m*r**2 + 24*x**(q + 2*r)*a*b**2*m*r + 9*x**
(q + 2*r)*a*b**2*m + 54*x**(q + 2*r)*a*b**2*q**2*r + 18*x**(q + 2*r)*a*b**
2*q**2 + 27*x**(q + 2*r)*a*b**2*q*r**2 + 54*x**(q + 2*r)*a*b**2*q*r + 15*x
**(q + 2*r)*a*b**2*q + 9*x**(q + 2*r)*a*b**2*r**2 + 12*x**(q + 2*r)*a*b...
```

3.417 $\int (cx)^m (ax^q + bx^r)^2 \, dx$

Optimal result . . . 3147
Mathematica [A] (verified) . . . 3147
Rubi [A] (verified) . . . 3148
Maple [C] (warning: unable to verify) . . . 3149
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Optimal result

Integrand size = 19, antiderivative size = 71

$$\int (cx)^m (ax^q + bx^r)^2 \, dx = \frac{a^2 x^{1+2q} (cx)^m}{1+m+2q} + \frac{2abx^{1+q+r} (cx)^m}{1+m+q+r} + \frac{b^2 x^{1+2r} (cx)^m}{1+m+2r}$$

output
```
a^2*x^(1+2*q)*(c*x)^m/(1+m+2*q)+2*a*b*x^(1+q+r)*(c*x)^m/(1+m+q+r)+b^2*x^(1
+2*r)*(c*x)^m/(1+m+2*r)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

$$\int (cx)^m (ax^q + bx^r)^2 \, dx = x(cx)^m \left(\frac{a^2 x^{2q}}{1+m+2q} + \frac{b^2 x^{2r}}{1+m+2r} + \frac{2abx^{q+r}}{1+m+q+r} \right)$$

input
```
Integrate[(c*x)^m*(a*x^q + b*x^r)^2,x]
```

output
```
x*(c*x)^m*((a^2*x^(2*q))/(1 + m + 2*q) + (b^2*x^(2*r))/(1 + m + 2*r) + (2*
a*b*x^(q + r))/(1 + m + q + r))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1939, 10, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \left(ax^q+bx^r\right)^2 \, dx$$

$$\Big\downarrow \; 1939$$

$$x^{-m}(cx)^m \int x^m\left(ax^q+bx^r\right)^2dx$$

$$\Big\downarrow \; 10$$

$$x^{-m}(cx)^m \int x^{m+2r}\left(ax^{q-r}+b\right)^2dx$$

$$\Big\downarrow \; 802$$

$$x^{-m}(cx)^m \int \left(a^2x^{m+2q}+2abx^{m+q+r}+b^2x^{m+2r}\right)dx$$

$$\Big\downarrow \; 2009$$

$$x^{-m}(cx)^m\left(\frac{a^2x^{m+2q+1}}{m+2q+1}+\frac{2abx^{m+q+r+1}}{m+q+r+1}+\frac{b^2x^{m+2r+1}}{m+2r+1}\right)$$

input
```
Int[(c*x)^m*(a*x^q + b*x^r)^2,x]
```

output
```
((c*x)^m*((a^2*x^(1 + m + 2*q))/(1 + m + 2*q) + (2*a*b*x^(1 + m + q + r))/
(1 + m + q + r) + (b^2*x^(1 + m + 2*r))/(1 + m + 2*r)))/x^m
```

Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1939
```
Int[(u_)^(m_.)*((a_.)*(v_)^(j_.) + (b_.)*(v_)^(n_.))^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a*x^j + b*x^n)^p, x], x,
 v], x] /; FreeQ[{a, b, j, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.78 (sec) , antiderivative size = 367, normalized size of antiderivative = 5.17

method	result
risch	$\frac{x\left(8abqr\,x^{q}x^{r}+a^{2}mq\,x^{2q}+3a^{2}mr\,x^{2q}+2a^{2}qr\,x^{2q}+3b^{2}mq\,x^{2r}+b^{2}mr\,x^{2r}+2b^{2}qr\,x^{2r}+2ab\,m^{2}x^{q}x^{r}+4abmr\,x^{q}x^{r}+4abmq\,x^{q}x\right.}{}$
orering	$\frac{x\left(3m^{2}+6mq+6mr+2q^{2}+8qr+2r^{2}+3m+3q+3r+1\right)(cx)^{m}\left(a\,x^{q}+b\,x^{r}\right)^{2}}{\left(m^{2}+2mq+2mr+4qr+2m+2q+2r+1\right)\left(1+m+q+r\right)}-\frac{3x^{2}(m+q+r)\left(\frac{(cx)^{m}m\left(a\,x^{q}+b\,x^{r}\right)^{2}}{x}+2(cx)^{m}(a\right.}{\left(m^{2}+2mq+2mr+4qr+2m+2q+2r+\right.}$
parallelrisch	$\frac{4x\,x^{q}x^{r}(cx)^{m}abr+2x\,x^{q}x^{r}(cx)^{m}ab\,m^{2}+4x\,x^{q}x^{r}(cx)^{m}abm+4x\,x^{q}x^{r}(cx)^{m}abq+4x\,x^{q}x^{r}(cx)^{m}abmq+4x\,x^{q}x^{r}(cx)^{m}abmr+8}{}$

input
```
int((c*x)^m*(a*x^q+b*x^r)^2,x,method=_RETURNVERBOSE)
```

output
```
x*(8*a*b*q*r*x^q*x^r+b^2*(x^r)^2+2*a*b*m^2*x^q*x^r+4*a*b*m*r*x^q*x^r+4*a*b
*m*q*x^q*x^r+a^2*(x^q)^2+4*m*a*b*x^q*x^r+a^2*m^2*(x^q)^2+2*a^2*r^2*(x^q)^2
+b^2*m^2*(x^r)^2+2*b*a*x^r*x^q+a^2*m*q*(x^q)^2+3*a^2*m*r*(x^q)^2+2*a^2*q*r
*(x^q)^2+3*b^2*m*q*(x^r)^2+b^2*m*r*(x^r)^2+2*b^2*q*r*(x^r)^2+(x^q)^2*a^2*q
+3*(x^q)^2*a^2*r+3*(x^r)^2*b^2*q+(x^r)^2*b^2*r+4*x^q*x^r*a*b*q+4*x^q*x^r*a
*b*r+2*m*b^2*(x^r)^2+2*b^2*q^2*(x^r)^2+2*m*a^2*(x^q)^2)/(1+m+2*q)/(1+m+q+r
)/(1+m+2*r)*x^m*c^m*exp(1/2*I*Pi*csgn(I*c*x)*m*(csgn(I*c*x)-csgn(I*x))*(-c
sgn(I*c*x)+csgn(I*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(71) = 142$.

Time = 0.09 (sec) , antiderivative size = 275, normalized size of antiderivative = 3.87

$$\int (cx)^m (ax^q+bx^r)^2\, dx = \frac{2\,(abm^2+2\,abm+ab+2\,(abm+ab)q+2\,(abm+2\,abq+ab)r)xx^qx^re^{(m\log(c)+m\log(x))}+(a^2m^2+2\,a^2r^2 \cdots}{m^3+2\,(m+1)q^2+2\,(m \cdots}$$

input
```
integrate((c*x)^m*(a*x^q+b*x^r)^2,x, algorithm="fricas")
```

output
```
(2*(a*b*m^2 + 2*a*b*m + a*b + 2*(a*b*m + a*b)*q + 2*(a*b*m + 2*a*b*q + a*b
)*r)*x*x^q*x^r*e^(m*log(c) + m*log(x)) + (a^2*m^2 + 2*a^2*r^2 + 2*a^2*m +
a^2 + (a^2*m + a^2)*q + (3*a^2*m + 2*a^2*q + 3*a^2)*r)*x*x^(2*q)*e^(m*log(
c) + m*log(x)) + (b^2*m^2 + 2*b^2*q^2 + 2*b^2*m + b^2 + 3*(b^2*m + b^2)*q
+ (b^2*m + 2*b^2*q + b^2)*r)*x*x^(2*r)*e^(m*log(c) + m*log(x)))/(m^3 + 2*(
m + 1)*q^2 + 2*(m + 2*q + 1)*r^2 + 3*m^2 + 3*(m^2 + 2*m + 1)*q + (3*m^2 +
8*(m + 1)*q + 4*q^2 + 6*m + 3)*r + 3*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3398 vs. $2(66) = 132$.

Time = 7.17 (sec) , antiderivative size = 3398, normalized size of antiderivative = 47.86

$$\int (cx)^m \left(ax^q+bx^r\right)^2 \, dx = \text{Too large to display}$$

input

```
integrate((c*x)**m*(a*x**q+b*x**r)**2,x)
```

output

```
Piecewise((a**2*x*x**(2*r)*(c*x)**(-2*r - 1)*log(x) + 2*a*b*x*x**(2*r)*(c*
x)**(-2*r - 1)*log(x) + b**2*x*x**(2*r)*(c*x)**(-2*r - 1)*log(x), Eq(q, r)
 & Eq(m, -2*r - 1)), (a**2*x*x**(2*q)*(c*x)**(-2*q - 1)*log(x) + 2*a*b*Pie
cewise((x*x**q*x**r*(c*x)**(-2*q - 1)/(-q + r), Ne(q - r, 0)), (x*x**q*x**
r*(c*x)**(-2*q - 1)*log(x), True)) + b**2*Piecewise((x*x**(2*r)*(c*x)**(-2
*q - 1)/(-2*q + 2*r), Ne(2*q - 2*r, 0)), (x*x**(2*r)*(c*x)**(-2*q - 1)*log
(x), True)), Eq(m, -2*q - 1)), (a**2*Piecewise((x*x**(2*q)*(c*x)**(-2*r -
1)/(2*q - 2*r), Ne(2*q - 2*r, 0)), (x*x**(2*q)*(c*x)**(-2*r - 1)*log(x), T
rue)) + 2*a*b*Piecewise((x*x**q*x**r*(c*x)**(-2*r - 1)/(q - r), Ne(q - r,
0)), (x*x**q*x**r*(c*x)**(-2*r - 1)*log(x), True)) + b**2*x*x**(2*r)*(c*x)
**(-2*r - 1)*log(x), Eq(m, -2*r - 1)), (a**2*Piecewise((x*x**(2*q)*(c*x)**
(-q - r - 1)/(q - r), Ne(q - r, 0)), (x*x**(2*q)*(c*x)**(-q - r - 1)*log(x
), True)) + 2*a*b*x*x**q*x**r*(c*x)**(-q - r - 1)*log(x) + b**2*Piecewise(
(x*x**(2*r)*(c*x)**(-q - r - 1)/(-q + r), Ne(q - r, 0)), (x*x**(2*r)*(c*x)
**(-q - r - 1)*log(x), True)), Eq(m, -q - r - 1)), (a**2*m**2*x*x**(2*q)*(
c*x)**m/(m**3 + 3*m**2*q + 3*m**2*r + 3*m**2 + 2*m*q**2 + 8*m*q*r + 6*m*q
+ 2*m*r**2 + 6*m*r + 3*m + 4*q**2*r + 2*q**2 + 4*q*r**2 + 8*q*r + 3*q + 2*
r**2 + 3*r + 1) + a**2*m*q*x*x**(2*q)*(c*x)**m/(m**3 + 3*m**2*q + 3*m**2*r
 + 3*m**2 + 2*m*q**2 + 8*m*q*r + 6*m*q + 2*m*r**2 + 6*m*r + 3*m + 4*q**2*r
 + 2*q**2 + 4*q*r**2 + 8*q*r + 3*q + 2*r**2 + 3*r + 1) + 3*a**2*m*r*x*x...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.18

$$\int (cx)^m (ax^q + bx^r)^2 \, dx = \frac{a^2 c^m x e^{(m \log(x) + 2 q \log(x))}}{m + 2q + 1} + \frac{2 abc^m x e^{(m \log(x) + q \log(x) + r \log(x))}}{m + q + r + 1} + \frac{b^2 c^m x e^{(m \log(x) + 2 r \log(x))}}{m + 2r + 1}$$

input
```
integrate((c*x)^m*(a*x^q+b*x^r)^2,x, algorithm="maxima")
```

output
```
a^2*c^m*x*e^(m*log(x) + 2*q*log(x))/(m + 2*q + 1) + 2*a*b*c^m*x*e^(m*log(x
) + q*log(x) + r*log(x))/(m + q + r + 1) + b^2*c^m*x*e^(m*log(x) + 2*r*log
(x))/(m + 2*r + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1159 vs. 2(71) = 142.

Time = 0.23 (sec) , antiderivative size = 1159, normalized size of antiderivative = 16.32

$$\int (cx)^m (ax^q + bx^r)^2 \, dx = \text{Too large to display}$$

input
```
integrate((c*x)^m*(a*x^q+b*x^r)^2,x, algorithm="giac")
```

output
```
(2*a*b*m^2*x*x^q*x^r*e^(m*log(c) + m*log(x)) + 4*a*b*m*q*x*x^q*x^r*e^(m*lo
g(c) + m*log(x)) + 4*a*b*m*r*x*x^q*x^r*e^(m*log(c) + m*log(x)) + 8*a*b*q*r
*x*x^q*x^r*e^(m*log(c) + m*log(x)) + a^2*m^2*x*x^(2*q)*e^(m*log(c) + m*log
(x)) + a^2*m*q*x*x^(2*q)*e^(m*log(c) + m*log(x)) + 3*a^2*m*r*x*x^(2*q)*e^(
m*log(c) + m*log(x)) + 2*a^2*q*r*x*x^(2*q)*e^(m*log(c) + m*log(x)) + 2*a^2
*r^2*x*x^(2*q)*e^(m*log(c) + m*log(x)) + 2*a*b*m^2*x*x^q*e^(m*log(c) + m*l
og(x)) + 4*a*b*m*q*x*x^q*e^(m*log(c) + m*log(x)) + 4*a*b*m*r*x*x^q*e^(m*lo
g(c) + m*log(x)) + 8*a*b*q*r*x*x^q*e^(m*log(c) + m*log(x)) + b^2*m^2*x*x^(
2*r)*e^(m*log(c) + m*log(x)) + 3*b^2*m*q*x*x^(2*r)*e^(m*log(c) + m*log(x))
 + 2*b^2*q^2*x*x^(2*r)*e^(m*log(c) + m*log(x)) + b^2*m*r*x*x^(2*r)*e^(m*lo
g(c) + m*log(x)) + 2*b^2*q*r*x*x^(2*r)*e^(m*log(c) + m*log(x)) + b^2*m^2*x
*x^r*e^(m*log(c) + m*log(x)) + 3*b^2*m*q*x*x^r*e^(m*log(c) + m*log(x)) + 2
*b^2*q^2*x*x^r*e^(m*log(c) + m*log(x)) + b^2*m*r*x*x^r*e^(m*log(c) + m*log
(x)) + 2*b^2*q*r*x*x^r*e^(m*log(c) + m*log(x)) + 4*a*b*m*x*x^q*x^r*e^(m*lo
g(c) + m*log(x)) + 4*a*b*q*x*x^q*x^r*e^(m*log(c) + m*log(x)) + 4*a*b*r*x*x
^q*x^r*e^(m*log(c) + m*log(x)) + b^2*m^2*x*e^(m*log(c) + m*log(x)) + 3*b^2
*m*q*x*e^(m*log(c) + m*log(x)) + 2*b^2*q^2*x*e^(m*log(c) + m*log(x)) + b^2
*m*r*x*e^(m*log(c) + m*log(x)) + 2*b^2*q*r*x*e^(m*log(c) + m*log(x)) + 2*a
^2*m*x*x^(2*q)*e^(m*log(c) + m*log(x)) + a^2*q*x*x^(2*q)*e^(m*log(c) + m*l
og(x)) + 3*a^2*r*x*x^(2*q)*e^(m*log(c) + m*log(x)) + 4*a*b*m*x*x^q*e^(m...
```

Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

$$\int (cx)^m \left(ax^q+bx^r\right)^2\,dx=(c\,x)^m\,\left(\frac{a^2\,x^{2\,q+1}}{m+2\,q+1}+\frac{b^2\,x^{2\,r+1}}{m+2\,r+1}+\frac{2\,a\,b\,x^{q+r+1}}{m+q+r+1}\right)$$

input
```
int((c*x)^m*(a*x^q + b*x^r)^2,x)
```

output
```
(c*x)^m*((a^2*x^(2*q + 1))/(m + 2*q + 1) + (b^2*x^(2*r + 1))/(m + 2*r + 1)
 + (2*a*b*x^(q + r + 1))/(m + q + r + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 379, normalized size of antiderivative = 5.34

$$\int (cx)^m (ax^q + bx^r)^2 \, dx$$
$$= \frac{x^m c^m x(4x^{q+r}abmq + 4x^{q+r}abmr + 2x^{2q}a^2m + 2x^{2q}a^2r^2 + 3x^{2q}a^2r + 2x^{q+r}ab + 2x^{2r}b^2m + 2x^{2r}b^2q^2 + 3}{m^3 + 3m^2q +}$$

input
```
int((c*x)^m*(a*x^q+b*x^r)^2,x)
```

output
```
(x**m*c**m*x*(x**(2*q)*a**2*m**2 + x**(2*q)*a**2*m*q + 3*x**(2*q)*a**2*m*r
 + 2*x**(2*q)*a**2*m + 2*x**(2*q)*a**2*q*r + x**(2*q)*a**2*q + 2*x**(2*q)*
a**2*r**2 + 3*x**(2*q)*a**2*r + x**(2*q)*a**2 + 2*x**(q + r)*a*b*m**2 + 4*
x**(q + r)*a*b*m*q + 4*x**(q + r)*a*b*m*r + 4*x**(q + r)*a*b*m + 8*x**(q +
 r)*a*b*q*r + 4*x**(q + r)*a*b*q + 4*x**(q + r)*a*b*r + 2*x**(q + r)*a*b +
 x**(2*r)*b**2*m**2 + 3*x**(2*r)*b**2*m*q + x**(2*r)*b**2*m*r + 2*x**(2*r)
*b**2*m + 2*x**(2*r)*b**2*q**2 + 2*x**(2*r)*b**2*q*r + 3*x**(2*r)*b**2*q +
 x**(2*r)*b**2*r + x**(2*r)*b**2))/(m**3 + 3*m**2*q + 3*m**2*r + 3*m**2 +
2*m*q**2 + 8*m*q*r + 6*m*q + 2*m*r**2 + 6*m*r + 3*m + 4*q**2*r + 2*q**2 +
4*q*r**2 + 8*q*r + 3*q + 2*r**2 + 3*r + 1)
```

3.418 $\int (cx)^m (ax^q + bx^r) \, dx$

Optimal result 3155
Mathematica [A] (verified) 3155
Rubi [A] (verified) 3156
Maple [B] (verified) 3157
Fricas [A] (verification not implemented) 3158
Sympy [B] (verification not implemented) 3158
Maxima [A] (verification not implemented) 3159
Giac [B] (verification not implemented) 3159
Mupad [B] (verification not implemented) 3160
Reduce [B] (verification not implemented) 3160

Optimal result

Integrand size = 17, antiderivative size = 37

$$\int (cx)^m (ax^q + bx^r) \, dx = \frac{ax^{1+q}(cx)^m}{1+m+q} + \frac{bx^{1+r}(cx)^m}{1+m+r}$$

output
```
a*x^(1+q)*(c*x)^m/(1+m+q)+b*x^(1+r)*(c*x)^m/(1+m+r)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int (cx)^m (ax^q + bx^r) \, dx = x(cx)^m \left(\frac{ax^q}{1+m+q} + \frac{bx^r}{1+m+r}\right)$$

input
```
Integrate[(c*x)^m*(a*x^q + b*x^r),x]
```

output
```
x*(c*x)^m*((a*x^q)/(1 + m + q) + (b*x^r)/(1 + m + r))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1939, 10, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \left(ax^q+bx^r\right) \, dx$$

$$\Big\downarrow \text{ 1939}$$

$$x^{-m}(cx)^m \int x^m (ax^q+bx^r) \, dx$$

$$\Big\downarrow \text{ 10}$$

$$x^{-m}(cx)^m \int x^{m+r}\left(ax^{q-r}+b\right) dx$$

$$\Big\downarrow \text{ 802}$$

$$x^{-m}(cx)^m \int \left(ax^{m+q}+bx^{m+r}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$x^{-m}(cx)^m \left(\frac{ax^{m+q+1}}{m+q+1}+\frac{bx^{m+r+1}}{m+r+1}\right)$$

input
```
Int[(c*x)^m*(a*x^q + b*x^r),x]
```

output
```
((c*x)^m*((a*x^(1 + m + q))/(1 + m + q) + (b*x^(1 + m + r))/(1 + m + r)))/
x^m
```

Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1939
```
Int[(u_)^(m_.)*((a_.)*(v_)^(j_.) + (b_.)*(v_)^(n_.))^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a*x^j + b*x^n)^p, x], x,
 v], x] /; FreeQ[{a, b, j, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(37) = 74$.

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.30

method	result	size
parallelrisch	$\frac{x\,x^{q} \left(c x \right)^{m} a m +x\,x^{q} \left(c x \right)^{m} a r +x\,x^{r} \left(c x \right)^{m} b m +x\,x^{r} \left(c x \right)^{m} b q +x\,x^{q} \left(c x \right)^{m} a +x\,x^{r} \left(c x \right)^{m} b}{\left(1+m +q \right) \left(1+m +r \right)}$	85
risch	$\frac{x \left(x^{q} a m +x^{q} a r +x^{r} b m +x^{r} b q +a\,x^{q}+b\,x^{r}\right) x^{m} c^{m} {\mathrm e}^{\frac{i \pi \,\mathrm{csgn}\left(i c x \right) m \left(\mathrm{csgn}\left(i c x \right)-\mathrm{csgn}\left(i x \right)\right) \left(-\mathrm{csgn}\left(i c x \right)+\mathrm{csgn}\left(i c \right)\right)}{2}}}{\left(1+m +q \right) \left(1+m +r \right)}$	96
orering	$\frac{x \left(2 m +q +r +1\right) \left(c x \right)^{m} \left(a\,x^{q}+b\,x^{r}\right)}{m^{2}+m q +m r +q r +2 m +q +r +1}-\frac{x^{2} \left(\frac{\left(c x \right)^{m} m \left(a\,x^{q}+b\,x^{r}\right)}{x}+\left(c x \right)^{m} \left(\frac{a\,x^{q} q}{x}+\frac{b\,x^{r} r}{x}\right)\right)}{m^{2}+m q +m r +q r +2 m +q +r +1}$	121

input
```
int((c*x)^m*(a*x^q+b*x^r),x,method=_RETURNVERBOSE)
```

output
```
(x*x^q*(c*x)^m*a*m+x*x^q*(c*x)^m*a*r+x*x^r*(c*x)^m*b*m+x*x^r*(c*x)^m*b*q+x
*x^q*(c*x)^m*a+x*x^r*(c*x)^m*b)/(1+m+q)/(1+m+r)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.86

$$\int (cx)^m (ax^q + bx^r) \, dx = \frac{(am + ar + a)xx^q e^{(m\log(c)+m\log(x))} + (bm + bq + b)xx^r e^{(m\log(c)+m\log(x))}}{m^2 + (m+1)q + (m+q+1)r + 2m + 1}$$

input
```
integrate((c*x)^m*(a*x^q+b*x^r),x, algorithm="fricas")
```

output
```
((a*m + a*r + a)*x*x^q*e^(m*log(c) + m*log(x)) + (b*m + b*q + b)*x*x^r*e^(
m*log(c) + m*log(x)))/(m^2 + (m + 1)*q + (m + q + 1)*r + 2*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(32) = 64.

Time = 2.25 (sec) , antiderivative size = 367, normalized size of antiderivative = 9.92

$$\int (cx)^m (ax^q + bx^r) \, dx = \begin{cases} axx^r (cx)^{-r-1} \log(x) + bxx^r (cx)^{-r-1} \log(x) \\ axx^q (cx)^{-q-1} \log(x) + b \left(\begin{cases} \frac{xx^r (cx)^{-q-1}}{-q+r} & \text{for } q - r \neq 0 \\ xx^r (cx)^{-q-1} \log(x) & \text{otherwise} \end{cases} \right) \\ a \left(\begin{cases} \frac{xx^q (cx)^{-r-1}}{q-r} & \text{for } q - r \neq 0 \\ xx^q (cx)^{-r-1} \log(x) & \text{otherwise} \end{cases} \right) + bxx^r (cx)^{-r-1} \log(x) \\ \frac{amxx^q (cx)^m}{m^2+mq+mr+2m+qr+q+r+1} + \frac{arxx^q (cx)^m}{m^2+mq+mr+2m+qr+q+r+1} + \frac{axx^q (cx)^m}{m^2+mq+mr+2m+qr+q+r+1} + \frac{bmxx^r (cx)^m}{m^2+mq+mr+2m+qr+q+r+1} \end{cases}$$

input
```
integrate((c*x)**m*(a*x**q+b*x**r),x)
```

output
```
Piecewise((a*x*x**r*(c*x)**(-r - 1)*log(x) + b*x*x**r*(c*x)**(-r - 1)*log(
x), Eq(q, r) & Eq(m, -r - 1)), (a*x*x**q*(c*x)**(-q - 1)*log(x) + b*Piecew
ise((x*x**r*(c*x)**(-q - 1)/(-q + r), Ne(q - r, 0)), (x*x**r*(c*x)**(-q -
1)*log(x), True)), Eq(m, -q - 1)), (a*Piecewise((x*x**q*(c*x)**(-r - 1)/(q
 - r), Ne(q - r, 0)), (x*x**q*(c*x)**(-r - 1)*log(x), True)) + b*x*x**r*(c
*x)**(-r - 1)*log(x), Eq(m, -r - 1)), (a*m*x*x**q*(c*x)**m/(m**2 + m*q + m
*r + 2*m + q*r + q + r + 1) + a*r*x*x**q*(c*x)**m/(m**2 + m*q + m*r + 2*m
+ q*r + q + r + 1) + a*x*x**q*(c*x)**m/(m**2 + m*q + m*r + 2*m + q*r + q +
 r + 1) + b*m*x*x**r*(c*x)**m/(m**2 + m*q + m*r + 2*m + q*r + q + r + 1) +
 b*q*x*x**r*(c*x)**m/(m**2 + m*q + m*r + 2*m + q*r + q + r + 1) + b*x*x**r
*(c*x)**m/(m**2 + m*q + m*r + 2*m + q*r + q + r + 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int (cx)^m (ax^q + bx^r) \, dx = \frac{ac^m x e^{(m\log(x)+q\log(x))}}{m+q+1} + \frac{bc^m x e^{(m\log(x)+r\log(x))}}{m+r+1}$$

input
```
integrate((c*x)^m*(a*x^q+b*x^r),x, algorithm="maxima")
```

output
```
a*c^m*x*e^(m*log(x) + q*log(x))/(m + q + 1) + b*c^m*x*e^(m*log(x) + r*log(
x))/(m + r + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(37) = 74.

Time = 0.22 (sec) , antiderivative size = 164, normalized size of antiderivative = 4.43

$$\int (cx)^m (ax^q + bx^r) \, dx = \frac{amxx^q e^{(m\log(c)+m\log(x))} + arxx^q e^{(m\log(c)+m\log(x))} + bmxx^r e^{(m\log(c)+m\log(x))} + bqxx^r e^{(m\log(c)+m\log(x))} + br}{m^2 + mq + mr}$$

input
```
integrate((c*x)^m*(a*x^q+b*x^r),x, algorithm="giac")
```

output
```
(a*m*x*x^q*e^(m*log(c) + m*log(x)) + a*r*x*x^q*e^(m*log(c) + m*log(x)) + b
*m*x*x^r*e^(m*log(c) + m*log(x)) + b*q*x*x^r*e^(m*log(c) + m*log(x)) + b*m
*x*e^(m*log(c) + m*log(x)) + b*q*x*e^(m*log(c) + m*log(x)) + a*x*x^q*e^(m*
log(c) + m*log(x)) + b*x*x^r*e^(m*log(c) + m*log(x)) + b*x*e^(m*log(c) + m
*log(x)))/(m^2 + m*q + m*r + q*r + 2*m + q + r + 1)
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int (cx)^m (ax^q + bx^r) \, dx = (c\,x)^m \left(\frac{a\,x^{q+1}}{m+q+1} + \frac{b\,x^{r+1}}{m+r+1}\right)$$

input
```
int((c*x)^m*(a*x^q + b*x^r),x)
```

output
```
(c*x)^m*((a*x^(q + 1))/(m + q + 1) + (b*x^(r + 1))/(m + r + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.73

$$\int (cx)^m (ax^q + bx^r) \, dx = \frac{x^m c^m x (x^q am + x^q ar + x^q a + x^r bm + x^r bq + x^r b)}{m^2 + mq + mr + qr + 2m + q + r + 1}$$

input
```
int((c*x)^m*(a*x^q+b*x^r),x)
```

output
```
(x**m*c**m*x*(x**q*a*m + x**q*a*r + x**q*a + x**r*b*m + x**r*b*q + x**r*b)
)/(m**2 + m*q + m*r + 2*m + q*r + q + r + 1)
```

3.419 $\int \frac{(cx)^m}{ax^q+bx^r}\,dx$

Optimal result . . . 3161
Mathematica [A] (verified) . . . 3161
Rubi [A] (verified) . . . 3162
Maple [F] . . . 3163
Fricas [F] . . . 3163
Sympy [F] . . . 3164
Maxima [F] . . . 3164
Giac [F] . . . 3164
Mupad [F(-1)] . . . 3165
Reduce [F] . . . 3165

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \frac{(cx)^m}{ax^q+bx^r}\,dx=\frac{x^{1-r}(cx)^m \text{Hypergeometric2F1}\left(1,\frac{1+m-r}{q-r},\frac{1+m+q-2r}{q-r},-\frac{ax^{q-r}}{b}\right)}{b(1+m-r)}$$

output
```
x^(1-r)*(c*x)^m*hypergeom([1, (1+m-r)/(q-r)],[(1+m+q-2*r)/(q-r)],-a*x^(q-r
)/b)/b/(1+m-r)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \frac{(cx)^m}{ax^q+bx^r}\,dx=\frac{x^{1-r}(cx)^m \text{Hypergeometric2F1}\left(1,\frac{1+m-r}{q-r},1+\frac{1+m-r}{q-r},-\frac{ax^{q-r}}{b}\right)}{b(1+m-r)}$$

input
```
Integrate[(c*x)^m/(a*x^q + b*x^r),x]
```

output
```
(x^(1 - r)*(c*x)^m*Hypergeometric2F1[1, (1 + m - r)/(q - r), 1 + (1 + m -
r)/(q - r), -((a*x^(q - r))/b)])/(b*(1 + m - r))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1939, 10, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{ax^q + bx^r} \, dx$$
$$\Big\downarrow \text{1939}$$
$$x^{-m}(cx)^m \int \frac{x^m}{ax^q + bx^r} dx$$
$$\Big\downarrow \text{10}$$
$$x^{-m}(cx)^m \int \frac{x^{m-r}}{ax^{q-r} + b} dx$$
$$\Big\downarrow \text{888}$$
$$\frac{x^{1-r}(cx)^m \,\text{Hypergeometric2F1}\left(1, \frac{m-r+1}{q-r}, \frac{m+q-2r+1}{q-r}, -\frac{ax^{q-r}}{b}\right)}{b(m-r+1)}$$

input
```
Int[(c*x)^m/(a*x^q + b*x^r),x]
```

output
```
(x^(1 - r)*(c*x)^m*Hypergeometric2F1[1, (1 + m - r)/(q - r), (1 + m + q -
2*r)/(q - r), -((a*x^(q - r))/b)])/(b*(1 + m - r))
```

Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1939
```
Int[(u_)^(m_.)*((a_.)*(v_)^(j_.) + (b_.)*(v_)^(n_.))^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a*x^j + b*x^n)^p, x], x,
 v], x] /; FreeQ[{a, b, j, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [F]

$$\int \frac{(cx)^m}{a\,x^q + b\,x^r}dx$$

input
```
int((c*x)^m/(a*x^q+b*x^r),x)
```

output
```
int((c*x)^m/(a*x^q+b*x^r),x)
```

Fricas [F]

$$\int \frac{(cx)^m}{ax^q + bx^r}\,dx = \int \frac{(cx)^m}{ax^q + bx^r}\,dx$$

input
```
integrate((c*x)^m/(a*x^q+b*x^r),x, algorithm="fricas")
```

output
```
integral((c*x)^m/(a*x^q + b*x^r), x)
```

Sympy [F]

$$\int \frac{(cx)^m}{ax^q + bx^r}\,dx = \int \frac{(cx)^m}{ax^q + bx^r}\,dx$$

input
```
integrate((c*x)**m/(a*x**q+b*x**r),x)
```

output
```
Integral((c*x)**m/(a*x**q + b*x**r), x)
```

Maxima [F]

$$\int \frac{(cx)^m}{ax^q + bx^r}\,dx = \int \frac{(cx)^m}{ax^q + bx^r}\,dx$$

input
```
integrate((c*x)^m/(a*x^q+b*x^r),x, algorithm="maxima")
```

output
```
integrate((c*x)^m/(a*x^q + b*x^r), x)
```

Giac [F]

$$\int \frac{(cx)^m}{ax^q + bx^r}\,dx = \int \frac{(cx)^m}{ax^q + bx^r}\,dx$$

input
```
integrate((c*x)^m/(a*x^q+b*x^r),x, algorithm="giac")
```

output
```
integrate((c*x)^m/(a*x^q + b*x^r), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{ax^q + bx^r} \, dx = \int \frac{(c\,x)^m}{a\,x^q + b\,x^r} \, dx$$

input
```
int((c*x)^m/(a*x^q + b*x^r),x)
```

output
```
int((c*x)^m/(a*x^q + b*x^r), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{ax^q + bx^r} \, dx = c^m \left(\int \frac{x^m}{x^q a + x^r b} dx \right)$$

input
```
int((c*x)^m/(a*x^q+b*x^r),x)
```

output
```
c**m*int(x**m/(x**q*a + x**r*b),x)
```

3.420 $\int \frac{(cx)^m}{(ax^q+bx^r)^2}\,dx$

Optimal result . . . 3166
Mathematica [A] (verified) . . . 3166
Rubi [A] (verified) . . . 3167
Maple [F] . . . 3168
Fricas [F] . . . 3168
Sympy [F] . . . 3169
Maxima [F] . . . 3169
Giac [F] . . . 3169
Mupad [F(-1)] . . . 3170
Reduce [F] . . . 3170

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \frac{(cx)^m}{(ax^q+bx^r)^2}\,dx=\frac{x^{1-2r}(cx)^m \operatorname{Hypergeometric2F1}\left(2,\frac{1+m-2r}{q-r},\frac{1+m+q-3r}{q-r},-\frac{ax^{q-r}}{b}\right)}{b^2(1+m-2r)}$$

output
```
x^(1-2*r)*(c*x)^m*hypergeom([2, (1+m-2*r)/(q-r)],[(1+m+q-3*r)/(q-r)],-a*x^
(q-r)/b)/b^2/(1+m-2*r)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \frac{(cx)^m}{(ax^q+bx^r)^2}\,dx=\frac{x^{1-2r}(cx)^m \operatorname{Hypergeometric2F1}\left(2,\frac{1+m-2r}{q-r},1+\frac{1+m-2r}{q-r},-\frac{ax^{q-r}}{b}\right)}{b^2(1+m-2r)}$$

input
```
Integrate[(c*x)^m/(a*x^q + b*x^r)^2,x]
```

output
```
(x^(1 - 2*r)*(c*x)^m*Hypergeometric2F1[2, (1 + m - 2*r)/(q - r), 1 + (1 +
m - 2*r)/(q - r), -((a*x^(q - r))/b)])/(b^2*(1 + m - 2*r))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1939, 10, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(ax^q+bx^r)^2}dx$$
$$\Big\downarrow \ 1939$$
$$x^{-m}(cx)^m\int \frac{x^m}{(ax^q+bx^r)^2}dx$$
$$\Big\downarrow \ 10$$
$$x^{-m}(cx)^m\int \frac{x^{m-2r}}{(ax^{q-r}+b)^2}dx$$
$$\Big\downarrow \ 888$$
$$\frac{x^{1-2r}(cx)^m \text{Hypergeometric2F1}\left(2,\frac{m-2r+1}{q-r},\frac{m+q-3r+1}{q-r},-\frac{ax^{q-r}}{b}\right)}{b^2(m-2r+1)}$$

input
```
Int[(c*x)^m/(a*x^q + b*x^r)^2,x]
```

output
```
(x^(1 - 2*r)*(c*x)^m*Hypergeometric2F1[2, (1 + m - 2*r)/(q - r), (1 + m +
q - 3*r)/(q - r), -((a*x^(q - r))/b)])/(b^2*(1 + m - 2*r))
```

Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1939
```
Int[(u_)^(m_.)*((a_.)*(v_)^(j_.) + (b_.)*(v_)^(n_.))^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a*x^j + b*x^n)^p, x], x,
 v], x] /; FreeQ[{a, b, j, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [F]

$$\int \frac{(cx)^m}{(a\,x^q+b\,x^r)^2}dx$$

input
```
int((c*x)^m/(a*x^q+b*x^r)^2,x)
```

output
```
int((c*x)^m/(a*x^q+b*x^r)^2,x)
```

Fricas [F]

$$\int \frac{(cx)^m}{(ax^q+bx^r)^2}\,dx=\int \frac{(cx)^m}{(ax^q+bx^r)^2}\,dx$$

input
```
integrate((c*x)^m/(a*x^q+b*x^r)^2,x, algorithm="fricas")
```

output
```
integral((c*x)^m/(2*a*b*x^q*x^r + a^2*x^(2*q) + b^2*x^(2*r)), x)
```

Sympy [F]

$$\int \frac{(cx)^m}{(ax^q+bx^r)^2}\,dx = \int \frac{(cx)^m}{(ax^q+bx^r)^2}\,dx$$

input
```
integrate((c*x)**m/(a*x**q+b*x**r)**2,x)
```

output
```
Integral((c*x)**m/(a*x**q + b*x**r)**2, x)
```

Maxima [F]

$$\int \frac{(cx)^m}{(ax^q+bx^r)^2}\,dx = \int \frac{(cx)^m}{(ax^q+bx^r)^2}\,dx$$

input
```
integrate((c*x)^m/(a*x^q+b*x^r)^2,x, algorithm="maxima")
```

output
```
c^m*(m - q - r + 1)*integrate(x^m/(a^2*(q - r)*x^(2*q) + a*b*(q - r)*e^(q*
log(x) + r*log(x))), x) - c^m*x*x^m/(a^2*(q - r)*x^(2*q) + a*b*(q - r)*e^(
q*log(x) + r*log(x)))
```

Giac [F]

$$\int \frac{(cx)^m}{(ax^q+bx^r)^2}\,dx = \int \frac{(cx)^m}{(ax^q+bx^r)^2}\,dx$$

input
```
integrate((c*x)^m/(a*x^q+b*x^r)^2,x, algorithm="giac")
```

output
```
integrate((c*x)^m/(a*x^q + b*x^r)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{(ax^q+bx^r)^2}\,dx=\int \frac{(c\,x)^m}{(a\,x^q+b\,x^r)^2}\,dx$$

input
```
int((c*x)^m/(a*x^q + b*x^r)^2,x)
```

output
```
int((c*x)^m/(a*x^q + b*x^r)^2, x)
```

Reduce [F]

$$\int \frac{(cx)^m}{(ax^q+bx^r)^2}\,dx=c^m\left(\int \frac{x^m}{x^{2q}a^2+2x^{q+r}ab+x^{2r}b^2}dx\right)$$

input
```
int((c*x)^m/(a*x^q+b*x^r)^2,x)
```

output
```
c**m*int(x**m/(x**(2*q)*a**2 + 2*x**(q + r)*a*b + x**(2*r)*b**2),x)
```

3.421 $\int \frac{(cx)^m}{(ax^q+bx^r)^3} dx$

Optimal result 3171
Mathematica [A] (verified) 3171
Rubi [A] (verified) 3172
Maple [F] 3173
Fricas [F] 3173
Sympy [F] 3174
Maxima [F] 3174
Giac [F] 3174
Mupad [F(-1)] 3175
Reduce [F] 3175

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \frac{(cx)^m}{(ax^q+bx^r)^3}\,dx=\frac{x^{1-3r}(cx)^m \,\text{Hypergeometric2F1}\left(3,\frac{1+m-3r}{q-r},\frac{1+m+q-4r}{q-r},-\frac{ax^{q-r}}{b}\right)}{b^3(1+m-3r)}$$

output
```
x^(1-3*r)*(c*x)^m*hypergeom([3, (1+m-3*r)/(q-r)],[(1+m+q-4*r)/(q-r)],-a*x^
(q-r)/b)/b^3/(1+m-3*r)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \frac{(cx)^m}{(ax^q+bx^r)^3}\,dx=\frac{x^{1-3r}(cx)^m \,\text{Hypergeometric2F1}\left(3,\frac{1+m-3r}{q-r},1+\frac{1+m-3r}{q-r},-\frac{ax^{q-r}}{b}\right)}{b^3(1+m-3r)}$$

input
```
Integrate[(c*x)^m/(a*x^q + b*x^r)^3,x]
```

output
```
(x^(1 - 3*r)*(c*x)^m*Hypergeometric2F1[3, (1 + m - 3*r)/(q - r), 1 + (1 +
m - 3*r)/(q - r), -((a*x^(q - r))/b)])/(b^3*(1 + m - 3*r))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1939, 10, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(ax^q+bx^r)^3}\,dx$$

$$\Big\downarrow \text{1939}$$

$$x^{-m}(cx)^m\int \frac{x^m}{(ax^q+bx^r)^3}dx$$

$$\Big\downarrow \text{10}$$

$$x^{-m}(cx)^m\int \frac{x^{m-3r}}{(ax^{q-r}+b)^3}dx$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{1-3r}(cx)^m \text{Hypergeometric2F1}\left(3,\frac{m-3r+1}{q-r},\frac{m+q-4r+1}{q-r},-\frac{ax^{q-r}}{b}\right)}{b^3(m-3r+1)}$$

input
```
Int[(c*x)^m/(a*x^q + b*x^r)^3,x]
```

output
```
(x^(1 - 3*r)*(c*x)^m*Hypergeometric2F1[3, (1 + m - 3*r)/(q - r), (1 + m +
q - 4*r)/(q - r), -((a*x^(q - r))/b)])/(b^3*(1 + m - 3*r))
```

Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1939
```
Int[(u_)^(m_.)*((a_.)*(v_)^(j_.) + (b_.)*(v_)^(n_.))^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a*x^j + b*x^n)^p, x], x,
 v], x] /; FreeQ[{a, b, j, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [F]

$$\int \frac{(cx)^m}{(a\,x^q+b\,x^r)^3}dx$$

input
```
int((c*x)^m/(a*x^q+b*x^r)^3,x)
```

output
```
int((c*x)^m/(a*x^q+b*x^r)^3,x)
```

Fricas [F]

$$\int \frac{(cx)^m}{(ax^q+bx^r)^3}\,dx=\int \frac{(cx)^m}{(ax^q+bx^r)^3}\,dx$$

input
```
integrate((c*x)^m/(a*x^q+b*x^r)^3,x, algorithm="fricas")
```

output
```
integral((c*x)^m/(3*a*b^2*x^q*x^(2*r) + 3*a^2*b*x^(2*q)*x^r + a^3*x^(3*q)
+ b^3*x^(3*r)), x)
```

Sympy [F]

$$\int \frac{(cx)^m}{(ax^q+bx^r)^3}\,dx=\int \frac{(cx)^m}{(ax^q+bx^r)^3}\,dx$$

input
```
integrate((c*x)**m/(a*x**q+b*x**r)**3,x)
```

output
```
Integral((c*x)**m/(a*x**q + b*x**r)**3, x)
```

Maxima [F]

$$\int \frac{(cx)^m}{(ax^q+bx^r)^3}\,dx=\int \frac{(cx)^m}{(ax^q+bx^r)^3}\,dx$$

input
```
integrate((c*x)^m/(a*x^q+b*x^r)^3,x, algorithm="maxima")
```

output
```
(m^2 - m*(3*q + 3*r - 2) + 2*q^2 + q*(5*r - 3) + 2*r^2 - 3*r + 1)*c^m*inte
grate(1/2*x^m/((q^2 - 2*q*r + r^2)*a^3*x^(3*q) + (q^2 - 2*q*r + r^2)*a^2*b
*e^(2*q*log(x) + r*log(x))), x) - 1/2*(a*c^m*(m - 3*r + 1)*x*e^(m*log(x) +
 q*log(x)) + b*c^m*(m - q - 2*r + 1)*x*e^(m*log(x) + r*log(x)))/((q^2 - 2*
q*r + r^2)*a^4*x^(4*q) + 2*(q^2 - 2*q*r + r^2)*a^3*b*e^(3*q*log(x) + r*log
(x)) + (q^2 - 2*q*r + r^2)*a^2*b^2*e^(2*q*log(x) + 2*r*log(x)))
```

Giac [F]

$$\int \frac{(cx)^m}{(ax^q+bx^r)^3}\,dx=\int \frac{(cx)^m}{(ax^q+bx^r)^3}\,dx$$

input
```
integrate((c*x)^m/(a*x^q+b*x^r)^3,x, algorithm="giac")
```

output
```
integrate((c*x)^m/(a*x^q + b*x^r)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{(ax^q+bx^r)^3}\,dx = \int \frac{(c\,x)^m}{(a\,x^q+b\,x^r)^3}\,dx$$

input
```
int((c*x)^m/(a*x^q + b*x^r)^3,x)
```

output
```
int((c*x)^m/(a*x^q + b*x^r)^3, x)
```

Reduce [F]

$$\int \frac{(cx)^m}{(ax^q+bx^r)^3}\,dx = c^m\left(\int \frac{x^m}{x^{3q}a^3+3x^{2q+r}a^2b+3x^{q+2r}a\,b^2+x^{3r}b^3}dx\right)$$

input
```
int((c*x)^m/(a*x^q+b*x^r)^3,x)
```

output
```
c**m*int(x**m/(x**(3*q)*a**3 + 3*x**(2*q + r)*a**2*b + 3*x**(q + 2*r)*a*b*
*2 + x**(3*r)*b**3),x)
```

3.422 $\int x^m \left(ax^j + bx^n\right)^p \, dx$

Optimal result . . . 3176
Mathematica [A] (verified) . . . 3176
Rubi [A] (verified) . . . 3177
Maple [F] . . . 3178
Fricas [F] . . . 3178
Sympy [F] . . . 3179
Maxima [F] . . . 3179
Giac [F] . . . 3179
Mupad [F(-1)] . . . 3180
Reduce [F] . . . 3180

Optimal result

Integrand size = 17, antiderivative size = 92

$$\int x^m \left(ax^j + bx^n\right)^p \, dx = \frac{x^{1+m}\left(1+\frac{ax^{j-n}}{b}\right)^{-p}\left(ax^j+bx^n\right)^p \text{Hypergeometric2F1}\left(-p,\frac{1+m+np}{j-n},1+\frac{1+m+np}{j-n},-\frac{ax^{j-n}}{b}\right)}{1+m+np}$$

output
```
x^(1+m)*(a*x^j+b*x^n)^p*hypergeom([-p, (n*p+m+1)/(j-n)],[1+(n*p+m+1)/(j-n)
],-a*x^(j-n)/b)/(n*p+m+1)/((1+a*x^(j-n)/b)^p)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int x^m \left(ax^j + bx^n\right)^p \, dx = \frac{x^{1+m}\left(1+\frac{ax^{j-n}}{b}\right)^{-p}\left(ax^j+bx^n\right)^p \text{Hypergeometric2F1}\left(-p,\frac{1+m+np}{j-n},1+\frac{1+m+np}{j-n},-\frac{ax^{j-n}}{b}\right)}{1+m+np}$$

input
```
Integrate[x^m*(a*x^j + b*x^n)^p,x]
```

output
```
(x^(1 + m)*(a*x^j + b*x^n)^p*Hypergeometric2F1[-p, (1 + m + n*p)/(j - n),
1 + (1 + m + n*p)/(j - n), -((a*x^(j - n))/b)])/((1 + m + n*p)*(1 + (a*x^(
j - n))/b)^p)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1938, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(ax^j+bx^n\right)^p dx$$

$$\Big\downarrow \text{1938}$$

$$x^{-np}\left(ax^{j-n}+b\right)^{-p}\left(ax^j+bx^n\right)^p\int x^{m+np}\left(ax^{j-n}+b\right)^p dx$$

$$\Big\downarrow \text{889}$$

$$x^{-np}\left(\frac{ax^{j-n}}{b}+1\right)^{-p}\left(ax^j+bx^n\right)^p\int x^{m+np}\left(\frac{ax^{j-n}}{b}+1\right)^p dx$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{m+1}\left(\frac{ax^{j-n}}{b}+1\right)^{-p}\left(ax^j+bx^n\right)^p \text{Hypergeometric2F1}\left(-p,\frac{m+np+1}{j-n},\frac{m+np+1}{j-n}+1,-\frac{ax^{j-n}}{b}\right)}{m+np+1}$$

input
```
Int[x^m*(a*x^j + b*x^n)^p,x]
```

output
```
(x^(1 + m)*(a*x^j + b*x^n)^p*Hypergeometric2F1[-p, (1 + m + n*p)/(j - n),
1 + (1 + m + n*p)/(j - n), -((a*x^(j - n))/b)])/((1 + m + n*p)*(1 + (a*x^(
j - n))/b)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int x^m (a x^j + b x^n)^p \, dx$$

input
```
int(x^m*(a*x^j+b*x^n)^p,x)
```

output
```
int(x^m*(a*x^j+b*x^n)^p,x)
```

Fricas [F]

$$\int x^m (ax^j + bx^n)^p \, dx = \int (ax^j + bx^n)^p x^m \, dx$$

input
```
integrate(x^m*(a*x^j+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((a*x^j + b*x^n)^p*x^m, x)
```

Sympy [F]

$$\int x^m\left(ax^j+bx^n\right)^p\,dx=\int x^m\left(ax^j+bx^n\right)^p\,dx$$

input
```
integrate(x**m*(a*x**j+b*x**n)**p,x)
```

output
```
Integral(x**m*(a*x**j + b*x**n)**p, x)
```

Maxima [F]

$$\int x^m\left(ax^j+bx^n\right)^p\,dx=\int \left(ax^j+bx^n\right)^p x^m\,dx$$

input
```
integrate(x^m*(a*x^j+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((a*x^j + b*x^n)^p*x^m, x)
```

Giac [F]

$$\int x^m\left(ax^j+bx^n\right)^p\,dx=\int \left(ax^j+bx^n\right)^p x^m\,dx$$

input
```
integrate(x^m*(a*x^j+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((a*x^j + b*x^n)^p*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \left(ax^j+bx^n\right)^p \, dx = \int x^m \, \left(a\,x^j+b\,x^n\right)^p \,dx$$

input
```
int(x^m*(a*x^j + b*x^n)^p,x)
```

output
```
int(x^m*(a*x^j + b*x^n)^p, x)
```

Reduce [F]

$$\int x^m \left(ax^j+bx^n\right)^p \, dx = \frac{x^m (x^j a+x^n b)^p x - \left(\int \frac{x^{j+m} (x^j a+x^n b)^p}{x^j am+x^j anp+x^j a+x^n bm+x^n bnp+x^n b} dx\right) ajmp - \left(\int \frac{x^{j+m} (x^j a+x^n b)^p}{x^j am+x^j anp+x^j a+x^n bm+x^n bnp+x^n b} dx\right)}{}$$

input
```
int(x^m*(a*x^j+b*x^n)^p,x)
```

output
```
(x**m*(x**j*a + x**n*b)**p*x - int((x**(j + m)*(x**j*a + x**n*b)**p)/(x**j
*a*m + x**j*a*n*p + x**j*a + x**n*b*m + x**n*b*n*p + x**n*b),x)*a*j*m*p -
int((x**(j + m)*(x**j*a + x**n*b)**p)/(x**j*a*m + x**j*a*n*p + x**j*a + x*
*n*b*m + x**n*b*n*p + x**n*b),x)*a*j*n*p**2 - int((x**(j + m)*(x**j*a + x*
*n*b)**p)/(x**j*a*m + x**j*a*n*p + x**j*a + x**n*b*m + x**n*b*n*p + x**n*b
),x)*a*j*p + int((x**(j + m)*(x**j*a + x**n*b)**p)/(x**j*a*m + x**j*a*n*p
+ x**j*a + x**n*b*m + x**n*b*n*p + x**n*b),x)*a*m*n*p + int((x**(j + m)*(x
**j*a + x**n*b)**p)/(x**j*a*m + x**j*a*n*p + x**j*a + x**n*b*m + x**n*b*n*
p + x**n*b),x)*a*n**2*p**2 + int((x**(j + m)*(x**j*a + x**n*b)**p)/(x**j*a
*m + x**j*a*n*p + x**j*a + x**n*b*m + x**n*b*n*p + x**n*b),x)*a*n*p)/(m +
n*p + 1)
```

3.423 $\int x^{-1-pq}(bx^n+ax^q)^p\,dx$

Optimal result . . . 3181
Mathematica [A] (verified) . . . 3181
Rubi [A] (verified) . . . 3182
Maple [F] . . . 3183
Fricas [F] . . . 3183
Sympy [F] . . . 3184
Maxima [F] . . . 3184
Giac [F] . . . 3184
Mupad [F(-1)] . . . 3185
Reduce [F] . . . 3185

Optimal result

Integrand size = 22, antiderivative size = 69

$$\int x^{-1-pq}(bx^n+ax^q)^p\,dx = -\frac{x^{-pq}(a+bx^{n-q})\,(bx^n+ax^q)^p\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^{n-q}}{a}\right)}{a(1+p)(n-q)}$$

output
```
-(a+b*x^(n-q))*(b*x^n+a*x^q)^p*hypergeom([1, p+1],[2+p],1+b*x^(n-q)/a)/a/(
p+1)/(n-q)/(x^(p*q))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int x^{-1-pq}(bx^n+ax^q)^p\,dx = \frac{x^{-pq}(bx^n+ax^q)^p\left(1+\frac{ax^{-n+q}}{b}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{ax^{-n+q}}{b}\right)}{p(n-q)}$$

input
```
Integrate[x^(-1 - p*q)*(b*x^n + a*x^q)^p,x]
```

output
```
((b*x^n + a*x^q)^p*Hypergeometric2F1[-p, -p, 1 - p, -((a*x^(-n + q))/b)])/
(p*(n - q)*x^(p*q)*(1 + (a*x^(-n + q))/b)^p)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1938, 798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-pq-1}(ax^q+bx^n)^p\,dx$$

$$\Big\downarrow \text{ 1938}$$

$$x^{-pq}\left(a+bx^{n-q}\right)^{-p}(ax^q+bx^n)^p\int \frac{\left(bx^{n-q}+a\right)^p}{x}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{x^{-pq}(a+bx^{n-q})^{-p}(ax^q+bx^n)^p\int x^{q-n}\left(bx^{n-q}+a\right)^p dx^{n-q}}{n-q}$$

$$\Big\downarrow \text{ 75}$$

$$-\frac{x^{-pq}(a+bx^{n-q})(ax^q+bx^n)^p\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^{n-q}}{a}+1\right)}{a(p+1)(n-q)}$$

input
```
Int[x^(-1 - p*q)*(b*x^n + a*x^q)^p,x]
```

output
```
-(((a + b*x^(n - q))*(b*x^n + a*x^q)^p*Hypergeometric2F1[1, 1 + p, 2 + p,
1 + (b*x^(n - q))/a])/(a*(1 + p)*(n - q)*x^(p*q)))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int x^{-pq-1}(b\,x^{n}+a\,x^{q})^{p}\,dx$$

input
```
int(x^(-p*q-1)*(b*x^n+a*x^q)^p,x)
```

output
```
int(x^(-p*q-1)*(b*x^n+a*x^q)^p,x)
```

Fricas [F]

$$\int x^{-1-pq}(bx^n+ax^q)^p\,dx=\int (bx^n+ax^q)^p x^{-pq-1}\,dx$$

input
```
integrate(x^(-p*q-1)*(b*x^n+a*x^q)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a*x^q)^p*x^(-p*q - 1), x)
```

Sympy [F]

$$\int x^{-1-pq}(bx^n + ax^q)^p \, dx = \int x^{-pq-1}(ax^q + bx^n)^p \, dx$$

input
```
integrate(x**(-p*q-1)*(b*x**n+a*x**q)**p,x)
```

output
```
Integral(x**(-p*q - 1)*(a*x**q + b*x**n)**p, x)
```

Maxima [F]

$$\int x^{-1-pq}(bx^n + ax^q)^p \, dx = \int (bx^n + ax^q)^p x^{-pq-1} \, dx$$

input
```
integrate(x^(-p*q-1)*(b*x^n+a*x^q)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a*x^q)^p*x^(-p*q - 1), x)
```

Giac [F]

$$\int x^{-1-pq}(bx^n + ax^q)^p \, dx = \int (bx^n + ax^q)^p x^{-pq-1} \, dx$$

input
```
integrate(x^(-p*q-1)*(b*x^n+a*x^q)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a*x^q)^p*x^(-p*q - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-pq}(bx^n + ax^q)^p \, dx = \int \frac{(b\,x^n + a\,x^q)^p}{x^{p\,q+1}} \, dx$$

input
```
int((b*x^n + a*x^q)^p/x^(p*q + 1),x)
```

output
```
int((b*x^n + a*x^q)^p/x^(p*q + 1), x)
```

Reduce [F]

$$\int x^{-1-pq}(bx^n + ax^q)^p \, dx = \frac{(x^n b + x^q a)^p + x^{pq}\left(\int \frac{x^q (x^n b+x^q a)^p}{x^{pq+n}bx+x^{pq+q}ax}dx\right) anp - x^{pq}\left(\int \frac{x^q (x^n b+x^q a)^p}{x^{pq+n}bx+x^{pq+q}ax}dx\right) apq}{x^{pq}p\,(n-q)}$$

input
```
int(x^(-p*q-1)*(b*x^n+a*x^q)^p,x)
```

output
```
((x**n*b + x**q*a)**p + x**(p*q)*int((x**q*(x**n*b + x**q*a)**p)/(x**(n +
p*q)*b*x + x**(p*q + q)*a*x),x)*a*n*p - x**(p*q)*int((x**q*(x**n*b + x**q*
a)**p)/(x**(n + p*q)*b*x + x**(p*q + q)*a*x),x)*a*p*q)/(x**(p*q)*p*(n - q)
)
```

3.424 $\int x^{-1-np}(bx^n + ax^q)^p \, dx$

Optimal result . . . 3186
Mathematica [A] (verified) . . . 3186
Rubi [A] (verified) . . . 3187
Maple [F] . . . 3188
Fricas [F] . . . 3189
Sympy [F] . . . 3189
Maxima [F] . . . 3189
Giac [F] . . . 3190
Mupad [F(-1)] . . . 3190
Reduce [F] . . . 3190

Optimal result

Integrand size = 22, antiderivative size = 74

$$\int x^{-1-np}(bx^n + ax^q)^p \, dx = -\frac{x^{-np}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}(bx^n+ax^q)^p \text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^{n-q}}{a}\right)}{p(n-q)}$$

output
```
-(b*x^n+a*x^q)^p*hypergeom([-p, -p],[1-p],-b*x^(n-q)/a)/p/(n-q)/(x^(n*p))/
((1+b*x^(n-q)/a)^p)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int x^{-1-np}(bx^n + ax^q)^p \, dx = -\frac{x^{-np}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}(bx^n+ax^q)^p \text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^{n-q}}{a}\right)}{p(n-q)}$$

input
```
Integrate[x^(-1 - n*p)*(b*x^n + a*x^q)^p,x]
```

output
```
-(((b*x^n + a*x^q)^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^(n - q))/a)])
/(p*(n - q)*x^(n*p)*(1 + (b*x^(n - q))/a)^p))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1938, 882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-np-1}(ax^q+bx^n)^p\,dx$$

$$\Big\downarrow \text{1938}$$

$$x^{-pq}\left(a+bx^{n-q}\right)^{-p}(ax^q+bx^n)^p\int x^{-np+qp-1}\left(bx^{n-q}+a\right)^p dx$$

$$\Big\downarrow \text{882}$$

$$\frac{x^{-p(n-q)-pq}\left(\frac{x^{n-q}}{a+bx^{n-q}}\right)^p(ax^q+bx^n)^p\int\frac{\left(\frac{x^{n-q}}{bx^{n-q}+a}\right)^{-p-1}}{1-\frac{bx^{n-q}}{bx^{n-q}+a}}d\frac{x^{n-q}}{bx^{n-q}+a}}{n-q}$$

$$\Big\downarrow \text{74}$$

$$-\frac{x^{-p(n-q)-pq}(ax^q+bx^n)^p\,\text{Hypergeometric2F1}\left(1,-p,1-p,\frac{bx^{n-q}}{bx^{n-q}+a}\right)}{p(n-q)}$$

input
```
Int[x^(-1 - n*p)*(b*x^n + a*x^q)^p,x]
```

output
```
-((x^(-(p*(n - q)) - p*q)*(b*x^n + a*x^q)^p*Hypergeometric2F1[1, -p, 1 - p
, (b*x^(n - q))/(a + b*x^(n - q))])/(p*(n - q)))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int x^{-np-1}(b\,x^n + a\,x^q)^p\,dx$$

input
```
int(x^(-n*p-1)*(b*x^n+a*x^q)^p,x)
```

output
```
int(x^(-n*p-1)*(b*x^n+a*x^q)^p,x)
```

Fricas [F]

$$\int x^{-1-np}(bx^n + ax^q)^p \, dx = \int (bx^n + ax^q)^p x^{-np-1} \, dx$$

input
```
integrate(x^(-n*p-1)*(b*x^n+a*x^q)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a*x^q)^p*x^(-n*p - 1), x)
```

Sympy [F]

$$\int x^{-1-np}(bx^n + ax^q)^p \, dx = \int x^{-np-1}(ax^q + bx^n)^p \, dx$$

input
```
integrate(x**(-n*p-1)*(b*x**n+a*x**q)**p,x)
```

output
```
Integral(x**(-n*p - 1)*(a*x**q + b*x**n)**p, x)
```

Maxima [F]

$$\int x^{-1-np}(bx^n + ax^q)^p \, dx = \int (bx^n + ax^q)^p x^{-np-1} \, dx$$

input
```
integrate(x^(-n*p-1)*(b*x^n+a*x^q)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a*x^q)^p*x^(-n*p - 1), x)
```

Giac [F]

$$\int x^{-1-np}(bx^n+ax^q)^p\,dx=\int (bx^n+ax^q)^p x^{-np-1}\,dx$$

input
```
integrate(x^(-n*p-1)*(b*x^n+a*x^q)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a*x^q)^p*x^(-n*p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-np}(bx^n+ax^q)^p\,dx=\int \frac{(b\,x^n+a\,x^q)^p}{x^{n\,p+1}}\,dx$$

input
```
int((b*x^n + a*x^q)^p/x^(n*p + 1),x)
```

output
```
int((b*x^n + a*x^q)^p/x^(n*p + 1), x)
```

Reduce [F]

$$\int x^{-1-np}(bx^n+ax^q)^p\,dx$$
$$=\frac{-(x^nb+x^qa)^p+x^{np}\left(\int\frac{x^n(x^nb+x^qa)^p}{x^{np+n}bx+x^{np+q}ax}dx\right)bnp-x^{np}\left(\int\frac{x^n(x^nb+x^qa)^p}{x^{np+n}bx+x^{np+q}ax}dx\right)bpq}{x^{np}p\,(n-q)}$$

input
```
int(x^(-n*p-1)*(b*x^n+a*x^q)^p,x)
```

output
```
( - (x**n*b + x**q*a)**p + x**(n*p)*int((x**n*(x**n*b + x**q*a)**p)/(x**(n
*p + n)*b*x + x**(n*p + q)*a*x),x)*b*n*p - x**(n*p)*int((x**n*(x**n*b + x*
*q*a)**p)/(x**(n*p + n)*b*x + x**(n*p + q)*a*x),x)*b*p*q)/(x**(n*p)*p*(n -
 q))
```

3.425 $\int x^{-1-n-(-1+p)q}(bx^n+ax^q)^p\,dx$

Optimal result 3191
Mathematica [A] (verified) 3191
Rubi [A] (verified) 3192
Maple [F] 3193
Fricas [F] 3193
Sympy [F] 3194
Maxima [F] 3194
Giac [F] 3194
Mupad [F(-1)] 3195
Reduce [F] 3195

Optimal result

Integrand size = 27, antiderivative size = 69

$$\int x^{-1-n-(-1+p)q}(bx^n+ax^q)^p\,dx = \frac{bx^{-pq}(a+bx^{n-q})\,(bx^n+ax^q)^p\,\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{bx^{n-q}}{a}\right)}{a^2(1+p)(n-q)}$$

output
```
b*(a+b*x^(n-q))*(b*x^n+a*x^q)^p*hypergeom([2, p+1],[2+p],1+b*x^(n-q)/a)/a^
2/(p+1)/(n-q)/(x^(p*q))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int x^{-1-n-(-1+p)q}(bx^n+ax^q)^p\,dx = \frac{x^{-n+q-pq}(bx^n+ax^q)^p\left(1+\frac{ax^{-n+q}}{b}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{ax^{-n+q}}{b}\right)}{(-1+p)(n-q)}$$

input
```
Integrate[x^(-1 - n - (-1 + p)*q)*(b*x^n + a*x^q)^p,x]
```

output
```
(x^(-n + q - p*q)*(b*x^n + a*x^q)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -(
(a*x^(-n + q))/b)])/((-1 + p)*(n - q)*(1 + (a*x^(-n + q))/b)^p)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1938, 798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n-(p-1)q-1}(ax^q+bx^n)^p\,dx$$

$$\Big\downarrow \text{1938}$$

$$x^{-pq}\left(a+bx^{n-q}\right)^{-p}(ax^q+bx^n)^p\int x^{-n+q-1}\left(bx^{n-q}+a\right)^p dx$$

$$\Big\downarrow \text{798}$$

$$\frac{x^{-pq}\left(a+bx^{n-q}\right)^{-p}(ax^q+bx^n)^p\int x^{2q-2n}\left(bx^{n-q}+a\right)^p dx^{n-q}}{n-q}$$

$$\Big\downarrow \text{75}$$

$$\frac{bx^{-pq}\left(a+bx^{n-q}\right)(ax^q+bx^n)^p\,\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{bx^{n-q}}{a}+1\right)}{a^2(p+1)(n-q)}$$

input
```
Int[x^(-1 - n - (-1 + p)*q)*(b*x^n + a*x^q)^p,x]
```

output
```
(b*(a + b*x^(n - q))*(b*x^n + a*x^q)^p*Hypergeometric2F1[2, 1 + p, 2 + p,
1 + (b*x^(n - q))/a])/(a^2*(1 + p)*(n - q)*x^(p*q))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int x^{-1-n-(p-1)q}(b\,x^n + a\,x^q)^p\,dx$$

input
```
int(x^(-1-n-(p-1)*q)*(b*x^n+a*x^q)^p,x)
```

output
```
int(x^(-1-n-(p-1)*q)*(b*x^n+a*x^q)^p,x)
```

Fricas [F]

$$\int x^{-1-n-(-1+p)q}(bx^n + ax^q)^p\,dx = \int (bx^n + ax^q)^p x^{-(p-1)q-n-1}\,dx$$

input
```
integrate(x^(-1-n-(-1+p)*q)*(b*x^n+a*x^q)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a*x^q)^p*x^(-(p - 1)*q - n - 1), x)
```

Sympy [F]

$$\int x^{-1-n-(-1+p)q}(bx^n + ax^q)^p \, dx = \int x^{-n-q(p-1)-1}(ax^q + bx^n)^p \, dx$$

input
```
integrate(x**(-1-n-(-1+p)*q)*(b*x**n+a*x**q)**p,x)
```

output
```
Integral(x**(-n - q*(p - 1) - 1)*(a*x**q + b*x**n)**p, x)
```

Maxima [F]

$$\int x^{-1-n-(-1+p)q}(bx^n + ax^q)^p \, dx = \int (bx^n + ax^q)^p x^{-(p-1)q-n-1} \, dx$$

input
```
integrate(x^(-1-n-(-1+p)*q)*(b*x^n+a*x^q)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a*x^q)^p*x^(-(p - 1)*q - n - 1), x)
```

Giac [F]

$$\int x^{-1-n-(-1+p)q}(bx^n + ax^q)^p \, dx = \int (bx^n + ax^q)^p x^{-(p-1)q-n-1} \, dx$$

input
```
integrate(x^(-1-n-(-1+p)*q)*(b*x^n+a*x^q)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a*x^q)^p*x^(-(p - 1)*q - n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n-(-1+p)q}(bx^n+ax^q)^p\,dx=\int \frac{(b\,x^n+a\,x^q)^p}{x^{n+q\,(p-1)+1}}\,dx$$

input
```
int((b*x^n + a*x^q)^p/x^(n + q*(p - 1) + 1),x)
```

output
```
int((b*x^n + a*x^q)^p/x^(n + q*(p - 1) + 1), x)
```

Reduce [F]

$$\int x^{-1-n-(-1+p)q}(bx^n+ax^q)^p\,dx$$
$$=\frac{-x^q(x^nb+x^qa)^p+x^{pq+n}\left(\int\frac{x^q(x^nb+x^qa)^p}{x^{pq+n}bx+x^{pq+q}ax}dx\right)bnp-x^{pq+n}\left(\int\frac{x^q(x^nb+x^qa)^p}{x^{pq+n}bx+x^{pq+q}ax}dx\right)bpq}{x^{pq+n}\,(n-q)}$$

input
```
int(x^(-1-n-(-1+p)*q)*(b*x^n+a*x^q)^p,x)
```

output
```
( - x**q*(x**n*b + x**q*a)**p + x**(n + p*q)*int((x**q*(x**n*b + x**q*a)**
p)/(x**(n + p*q)*b*x + x**(p*q + q)*a*x),x)*b*n*p - x**(n + p*q)*int((x**q
*(x**n*b + x**q*a)**p)/(x**(n + p*q)*b*x + x**(p*q + q)*a*x),x)*b*p*q)/(x*
*(n + p*q)*(n - q))
```

3.426 $\int x^{-1-n(-1+p)-q}(bx^n+ax^q)^p\,dx$

Optimal result 3196
Mathematica [A] (verified) 3196
Rubi [A] (verified) 3197
Maple [F] 3198
Fricas [F] 3199
Sympy [F] 3199
Maxima [F] 3199
Giac [F] 3200
Mupad [F(-1)] 3200
Reduce [F] 3200

Optimal result

Integrand size = 27, antiderivative size = 84

$$\int x^{-1-n(-1+p)-q}(bx^n+ax^q)^p\,dx = \frac{x^{n-np-q}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}(bx^n+ax^q)^p\,\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^{n-q}}{a}\right)}{(1-p)(n-q)}$$

output
```
x^(-n*p+n-q)*(b*x^n+a*x^q)^p*hypergeom([-p, 1-p],[2-p],-b*x^(n-q)/a)/(1-p)
/(n-q)/((1+b*x^(n-q)/a)^p)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int x^{-1-n(-1+p)-q}(bx^n+ax^q)^p\,dx = -\frac{x^{n-np-q}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}(bx^n+ax^q)^p\,\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^{n-q}}{a}\right)}{(-1+p)(n-q)}$$

input
```
Integrate[x^(-1 - n*(-1 + p) - q)*(b*x^n + a*x^q)^p,x]
```

output
```
-((x^(n - n*p - q)*(b*x^n + a*x^q)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -
((b*x^(n - q))/a)])/((-1 + p)*(n - q)*(1 + (b*x^(n - q))/a)^p))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1938, 882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p-1)-q-1}(ax^q+bx^n)^p\,dx$$

$$\Big\downarrow \text{1938}$$

$$x^{-pq}\left(a+bx^{n-q}\right)^{-p}(ax^q+bx^n)^p\int x^{-pn+n-(1-p)q-1}\left(bx^{n-q}+a\right)^p dx$$

$$\Big\downarrow \text{882}$$

$$\frac{ax^{-p(n-q)-pq}\left(\frac{x^{n-q}}{a+bx^{n-q}}\right)^p(ax^q+bx^n)^p\int \frac{\left(\frac{x^{n-q}}{bx^{n-q}+a}\right)^{-p}}{\left(1-\frac{bx^{n-q}}{bx^{n-q}+a}\right)^2}d\frac{x^{n-q}}{bx^{n-q}+a}}{n-q}$$

$$\Big\downarrow \text{74}$$

$$\frac{ax^{-p(n-q)+n-pq-q}(ax^q+bx^n)^p\,\text{Hypergeometric2F1}\left(2,1-p,2-p,\frac{bx^{n-q}}{bx^{n-q}+a}\right)}{(1-p)(n-q)\left(a+bx^{n-q}\right)}$$

input
```
Int[x^(-1 - n*(-1 + p) - q)*(b*x^n + a*x^q)^p,x]
```

output
```
(a*x^(n - p*(n - q) - q - p*q)*(b*x^n + a*x^q)^p*Hypergeometric2F1[2, 1 -
p, 2 - p, (b*x^(n - q))/(a + b*x^(n - q))])/((1 - p)*(n - q)*(a + b*x^(n -
q)))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int x^{-1-n(p-1)-q}(b\,x^n + a\,x^q)^p\,dx$$

input
```
int(x^(-1-n*(p-1)-q)*(b*x^n+a*x^q)^p,x)
```

output
```
int(x^(-1-n*(p-1)-q)*(b*x^n+a*x^q)^p,x)
```

Fricas [F]

$$\int x^{-1-n(-1+p)-q}(bx^n+ax^q)^p\,dx=\int (bx^n+ax^q)^p x^{-n(p-1)-q-1}\,dx$$

input
```
integrate(x^(-1-n*(-1+p)-q)*(b*x^n+a*x^q)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a*x^q)^p*x^(-n*p + n - q - 1), x)
```

Sympy [F]

$$\int x^{-1-n(-1+p)-q}(bx^n+ax^q)^p\,dx=\int x^{-n(p-1)-q-1}(ax^q+bx^n)^p\,dx$$

input
```
integrate(x**(-1-n*(-1+p)-q)*(b*x**n+a*x**q)**p,x)
```

output
```
Integral(x**(-n*(p - 1) - q - 1)*(a*x**q + b*x**n)**p, x)
```

Maxima [F]

$$\int x^{-1-n(-1+p)-q}(bx^n+ax^q)^p\,dx=\int (bx^n+ax^q)^p x^{-n(p-1)-q-1}\,dx$$

input
```
integrate(x^(-1-n*(-1+p)-q)*(b*x^n+a*x^q)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a*x^q)^p*x^(-n*(p - 1) - q - 1), x)
```

Giac [F]

$$\int x^{-1-n(-1+p)-q}(bx^n+ax^q)^p\,dx=\int (bx^n+ax^q)^p x^{-n(p-1)-q-1}\,dx$$

input
```
integrate(x^(-1-n*(-1+p)-q)*(b*x^n+a*x^q)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a*x^q)^p*x^(-n*(p - 1) - q - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(-1+p)-q}(bx^n+ax^q)^p\,dx=\int \frac{(b\,x^n+a\,x^q)^p}{x^{q+n\,(p-1)+1}}\,dx$$

input
```
int((b*x^n + a*x^q)^p/x^(q + n*(p - 1) + 1),x)
```

output
```
int((b*x^n + a*x^q)^p/x^(q + n*(p - 1) + 1), x)
```

Reduce [F]

$$\int x^{-1-n(-1+p)-q}(bx^n+ax^q)^p\,dx$$
$$=\frac{x^n(x^nb+x^qa)^p+x^{np+q}\left(\int\frac{x^n(x^nb+x^qa)^p}{x^{np+n}bx+x^{np+q}ax}dx\right)anp-x^{np+q}\left(\int\frac{x^n(x^nb+x^qa)^p}{x^{np+n}bx+x^{np+q}ax}dx\right)apq}{x^{np+q}\,(n-q)}$$

input
```
int(x^(-1-n*(-1+p)-q)*(b*x^n+a*x^q)^p,x)
```

output
```
(x**n*(x**n*b + x**q*a)**p + x**(n*p + q)*int((x**n*(x**n*b + x**q*a)**p)/
(x**(n*p + n)*b*x + x**(n*p + q)*a*x),x)*a*n*p - x**(n*p + q)*int((x**n*(x
**n*b + x**q*a)**p)/(x**(n*p + n)*b*x + x**(n*p + q)*a*x),x)*a*p*q)/(x**(n
*p + q)*(n - q))
```

3.427 $\int \frac{\left(-\frac{1}{x}+x\right)^p}{x} \, dx$

Optimal result 3201
Mathematica [A] (verified) 3201
Rubi [A] (verified) 3202
Maple [F] 3203
Fricas [F] 3203
Sympy [F] 3204
Maxima [F] 3204
Giac [F] 3204
Mupad [F(-1)] 3205
Reduce [F] 3205

Optimal result

Integrand size = 13, antiderivative size = 44

$$\int \frac{\left(-\frac{1}{x}+x\right)^p}{x} \, dx = -\frac{\left(-\frac{1}{x}+x\right)^p \left(1-x^2\right)^{-p} \text{Hypergeometric2F1}\left(-p,-\frac{p}{2},1-\frac{p}{2},x^2\right)}{p}$$

output
```
-(-1/x+x)^p*hypergeom([-p, -1/2*p],[1-1/2*p],x^2)/p/((-x^2+1)^p)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{\left(-\frac{1}{x}+x\right)^p}{x} \, dx = -\frac{\left(-\frac{1}{x}+x\right)^p \left(1-x^2\right)^{-p} \text{Hypergeometric2F1}\left(-p,-\frac{p}{2},1-\frac{p}{2},x^2\right)}{p}$$

input
```
Integrate[(-x^(-1) + x)^p/x,x]
```

output
```
-(((-x^(-1) + x)^p*Hypergeometric2F1[-p, -1/2*p, 1 - p/2, x^2])/(p*(1 - x^
2)^p))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1938, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x-\frac{1}{x}\right)^p}{x} \, dx$$

$$\Big\downarrow \text{1938}$$

$$x^p \left(x-\frac{1}{x}\right)^p \left(x^2-1\right)^{-p} \int x^{-p-1} \left(x^2-1\right)^p dx$$

$$\Big\downarrow \text{279}$$

$$x^p \left(x-\frac{1}{x}\right)^p \left(1-x^2\right)^{-p} \int x^{-p-1} \left(1-x^2\right)^p dx$$

$$\Big\downarrow \text{278}$$

$$-\frac{\left(x-\frac{1}{x}\right)^p \left(1-x^2\right)^{-p} \text{Hypergeometric2F1}\left(-p,-\frac{p}{2},1-\frac{p}{2},x^2\right)}{p}$$

input
```
Int[(-x^(-1) + x)^p/x,x]
```

output
```
-(((-x^(-1) + x)^p*Hypergeometric2F1[-p, -1/2*p, 1 - p/2, x^2])/(p*(1 - x^
2)^p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{\left(-\frac{1}{x}+x\right)^{p}}{x}dx$$

input
```
int((-1/x+x)^p/x,x)
```

output
```
int((-1/x+x)^p/x,x)
```

Fricas [F]

$$\int \frac{\left(-\frac{1}{x}+x\right)^{p}}{x}\,dx=\int \frac{\left(x-\frac{1}{x}\right)^{p}}{x}\,dx$$

input
```
integrate((-1/x+x)^p/x,x, algorithm="fricas")
```

output
```
integral(((x^2 - 1)/x)^p/x, x)
```

Sympy [F]

$$\int \frac{\left(-\frac{1}{x}+x\right)^p}{x}\,dx=\int \frac{\left(x-\frac{1}{x}\right)^p}{x}\,dx$$

input
```
integrate((-1/x+x)**p/x,x)
```

output
```
Integral((x - 1/x)**p/x, x)
```

Maxima [F]

$$\int \frac{\left(-\frac{1}{x}+x\right)^p}{x}\,dx=\int \frac{\left(x-\frac{1}{x}\right)^p}{x}\,dx$$

input
```
integrate((-1/x+x)^p/x,x, algorithm="maxima")
```

output
```
integrate((x - 1/x)^p/x, x)
```

Giac [F]

$$\int \frac{\left(-\frac{1}{x}+x\right)^p}{x}\,dx=\int \frac{\left(x-\frac{1}{x}\right)^p}{x}\,dx$$

input
```
integrate((-1/x+x)^p/x,x, algorithm="giac")
```

output
```
integrate((x - 1/x)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(-\frac{1}{x}+x\right)^p}{x}\,dx=\int \frac{\left(x-\frac{1}{x}\right)^p}{x}\,dx$$

input
```
int((x - 1/x)^p/x,x)
```

output
```
int((x - 1/x)^p/x, x)
```

Reduce [F]

$$\int \frac{\left(-\frac{1}{x}+x\right)^p}{x}\,dx=\frac{(x^2-1)^p-2x^p\left(\int \frac{(x^2-1)^p}{x^p x^3-x^p x}dx\right)p}{x^p p}$$

input
```
int((-1/x+x)^p/x,x)
```

output
```
((x**2 - 1)**p - 2*x**p*int((x**2 - 1)**p/(x**p*x**3 - x**p*x),x)*p)/(x**p
*p)
```

3.428 $\int (ax^m + bx^{1+m+mp})^p \, dx$

Optimal result . . . 3206
Mathematica [A] (verified) . . . 3206
Rubi [A] (verified) . . . 3207
Maple [F] . . . 3207
Fricas [A] (verification not implemented) . . . 3208
Sympy [F] . . . 3208
Maxima [F] . . . 3208
Giac [F] . . . 3209
Mupad [B] (verification not implemented) . . . 3209
Reduce [F] . . . 3209

Optimal result

Integrand size = 18, antiderivative size = 44

$$\int (ax^m + bx^{1+m+mp})^p \, dx = \frac{x^{-m(1+p)}(ax^m + bx^{1+m+mp})^{1+p}}{b(1+p)(1+mp)}$$

output
```
(a*x^m+b*x^(m*p+m+1))^(p+1)/b/(p+1)/(m*p+1)/(x^(m*(p+1)))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int (ax^m + bx^{1+m+mp})^p \, dx = \frac{x^{-m(1+p)}(x^m(a + bx^{1+mp}))^{1+p}}{b(1+p)(1+mp)}$$

input
```
Integrate[(a*x^m + b*x^(1 + m + m*p))^p,x]
```

output
```
(x^m*(a + b*x^(1 + m*p)))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^m + bx^{mp+m+1}\right)^p dx$$
$$\Big\downarrow \text{ 1906}$$
$$\frac{x^{-m(p+1)}\left(ax^m + bx^{mp+m+1}\right)^{p+1}}{b(p+1)(mp+1)}$$

input
```
Int[(a*x^m + b*x^(1 + m + m*p))^p,x]
```

output
```
(a*x^m + b*x^(1 + m + m*p))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Maple [F]

$$\int \left(a\,x^m + b\,x^{mp+m+1}\right)^p dx$$

input
```
int((a*x^m+b*x^(m*p+m+1))^p,x)
```

output
```
int((a*x^m+b*x^(m*p+m+1))^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.45

$$\int \left(ax^m + bx^{1+m+mp}\right)^p \, dx = \frac{(bxx^{mp+m+1} + axx^m)(bx^{mp+m+1} + ax^m)^p}{(bmp^2 + (bm+b)p + b)x^{mp+m+1}}$$

input
```
integrate((a*x^m+b*x^(m*p+m+1))^p,x, algorithm="fricas")
```

output
```
(b*x*x^(m*p + m + 1) + a*x*x^m)*(b*x^(m*p + m + 1) + a*x^m)^p/((b*m*p^2 +
(b*m + b)*p + b)*x^(m*p + m + 1))
```

Sympy [F]

$$\int \left(ax^m + bx^{1+m+mp}\right)^p \, dx = \int \left(ax^m + bx^{mp+m+1}\right)^p \, dx$$

input
```
integrate((a*x**m+b*x**(m*p+m+1))**p,x)
```

output
```
Integral((a*x**m + b*x**(m*p + m + 1))**p, x)
```

Maxima [F]

$$\int \left(ax^m + bx^{1+m+mp}\right)^p \, dx = \int \left(bx^{mp+m+1} + ax^m\right)^p \, dx$$

input
```
integrate((a*x^m+b*x^(m*p+m+1))^p,x, algorithm="maxima")
```

output
```
integrate((b*x^(m*p + m + 1) + a*x^m)^p, x)
```

Giac [F]

$$\int \left(ax^m+bx^{1+m+mp}\right)^p\,dx=\int \left(bx^{mp+m+1}+ax^m\right)^p\,dx$$

input
```
integrate((a*x^m+b*x^(m*p+m+1))^p,x, algorithm="giac")
```

output
```
integrate((b*x^(m*p + m + 1) + a*x^m)^p, x)
```

Mupad [B] (verification not implemented)

Time = 9.91 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.73

$$\int \left(ax^m+bx^{1+m+mp}\right)^p\,dx=\frac{a\,\left(a\,x^m+b\,x^{m+m\,p+1}\right)^p\,\left(\frac{b\,x^{m\,p+1}}{a}-\frac{1}{{\left(\frac{b\,x^{m\,p+1}}{a}+1\right)}^p}+1\right)}{b\,x^{m\,p}\,\left(m\,p+1\right)\,\left(p+1\right)}$$

input
```
int((a*x^m + b*x^(m + m*p + 1))^p,x)
```

output
```
(a*(a*x^m + b*x^(m + m*p + 1))^p*((b*x^(m*p + 1))/a - 1/((b*x^(m*p + 1))/a
 + 1)^p + 1))/(b*x^(m*p)*(m*p + 1)*(p + 1))
```

Reduce [F]

$$\int \left(ax^m+bx^{1+m+mp}\right)^p\,dx=\int \left(x^{mp+m}bx+x^m a\right)^p\,dx$$

input
```
int((a*x^m+b*x^(m*p+m+1))^p,x)
```

output
```
int((x**(m*p + m)*b*x + x**m*a)**p,x)
```

3.429 $\int (x^m(a+bx^{1+mp}))^p \, dx$

Optimal result . . . 3210
Mathematica [A] (verified) . . . 3210
Rubi [A] (verified) . . . 3211
Maple [F] . . . 3212
Fricas [A] (verification not implemented) . . . 3212
Sympy [F] . . . 3212
Maxima [F] . . . 3213
Giac [F] . . . 3213
Mupad [F(-1)] . . . 3213
Reduce [B] (verification not implemented) . . . 3214

Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \left(x^m\left(a+bx^{1+mp}\right)\right)^p \, dx=\frac{x^{-m(1+p)}\left(ax^m+bx^{1+m+mp}\right)^{1+p}}{b(1+p)(1+mp)}$$

output
```
(a*x^m+b*x^(m*p+m+1))^(p+1)/b/(p+1)/(m*p+1)/(x^(m*(p+1)))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \left(x^m\left(a+bx^{1+mp}\right)\right)^p \, dx=\frac{x^{-m(1+p)}\left(x^m\left(a+bx^{1+mp}\right)\right)^{1+p}}{b(1+p)(1+mp)}$$

input
```
Integrate[(x^m*(a + b*x^(1 + m*p)))^p,x]
```

output
```
(x^m*(a + b*x^(1 + m*p)))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2078, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^m\left(a+bx^{mp+1}\right)\right)^p\,dx$$

$$\Big\downarrow \text{ 2078}$$

$$\int \left(ax^m+bx^{mp+m+1}\right)^p\,dx$$

$$\Big\downarrow \text{ 1906}$$

$$\frac{x^{-m(p+1)}\left(ax^m+bx^{mp+m+1}\right)^{p+1}}{b(p+1)(mp+1)}$$

input
```
Int[(x^m*(a + b*x^(1 + m*p)))^p,x]
```

output
```
(a*x^m + b*x^(1 + m + m*p))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Maple [F]

$$\int \left(x^m \left(a+b\,x^{mp+1}\right)\right)^p dx$$

input
```
int((x^m*(a+b*x^(m*p+1)))^p,x)
```

output
```
int((x^m*(a+b*x^(m*p+1)))^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.39

$$\int \left(x^m \left(a+bx^{1+mp}\right)\right)^p \, dx = \frac{(bxx^{mp+1}+ax)(bx^{mp+1}x^m+ax^m)^p}{(bmp^2+(bm+b)p+b)x^{mp+1}}$$

input
```
integrate((x^m*(a+b*x^(m*p+1)))^p,x, algorithm="fricas")
```

output
```
(b*x*x^(m*p + 1) + a*x)*(b*x^(m*p + 1)*x^m + a*x^m)^p/((b*m*p^2 + (b*m + b
)*p + b)*x^(m*p + 1))
```

Sympy [F]

$$\int \left(x^m \left(a+bx^{1+mp}\right)\right)^p \, dx = \int \left(x^m \left(a+bx^{mp+1}\right)\right)^p \, dx$$

input
```
integrate((x**m*(a+b*x**(m*p+1)))**p,x)
```

output
```
Integral((x**m*(a + b*x**(m*p + 1)))**p, x)
```

Maxima [F]

$$\int \left(x^m \left(a+bx^{1+mp}\right)\right)^p \, dx = \int \left(\left(bx^{mp+1}+a\right)x^m\right)^p \, dx$$

input
```
integrate((x^m*(a+b*x^(m*p+1)))^p,x, algorithm="maxima")
```

output
```
integrate(((b*x^(m*p + 1) + a)*x^m)^p, x)
```

Giac [F]

$$\int \left(x^m \left(a+bx^{1+mp}\right)\right)^p \, dx = \int \left(\left(bx^{mp+1}+a\right)x^m\right)^p \, dx$$

input
```
integrate((x^m*(a+b*x^(m*p+1)))^p,x, algorithm="giac")
```

output
```
integrate(((b*x^(m*p + 1) + a)*x^m)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(x^m \left(a+bx^{1+mp}\right)\right)^p \, dx = \int \left(x^m \left(a+b\,x^{m\,p+1}\right)\right)^p \, dx$$

input
```
int((x^m*(a + b*x^(m*p + 1)))^p,x)
```

output
```
int((x^m*(a + b*x^(m*p + 1)))^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \left(x^m \left(a + b x^{1+mp}\right)\right)^p \, dx = \frac{\left(x^{mp} b x + a\right)^p \left(x^{mp} b x + a\right)}{b \left(m\, p^2 + mp + p + 1\right)}$$

input
```
int((x^m*(a+b*x^(m*p+1)))^p,x)
```

output
```
((x**(m*p)*b*x + a)**p*(x**(m*p)*b*x + a))/(b*(m*p**2 + m*p + p + 1))
```

3.430 $\int x^n(x^m(a+bx^{1+n+mp}))^p\,dx$

Optimal result . . . 3215
Mathematica [A] (verified) . . . 3215
Rubi [A] (verified) . . . 3216
Maple [F] . . . 3217
Fricas [A] (verification not implemented) . . . 3217
Sympy [F(-1)] . . . 3217
Maxima [F] . . . 3218
Giac [F] . . . 3218
Mupad [F(-1)] . . . 3218
Reduce [B] (verification not implemented) . . . 3219

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int x^n\left(x^m\left(a+bx^{1+n+mp}\right)\right)^p\,dx=\frac{x^{-m(1+p)}\left(ax^m+bx^{1+m+n+mp}\right)^{1+p}}{b(1+p)(1+n+mp)}$$

output
```
(a*x^m+b*x^(m*p+m+n+1))^(p+1)/b/(p+1)/(m*p+n+1)/(x^(m*(p+1)))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int x^n\left(x^m\left(a+bx^{1+n+mp}\right)\right)^p\,dx=\frac{x^{-m(1+p)}\left(x^m\left(a+bx^{1+n+mp}\right)\right)^{1+p}}{b(1+p)(1+n+mp)}$$

input
```
Integrate[x^n*(x^m*(a + b*x^(1 + n + m*p)))^p,x]
```

output
```
(x^m*(a + b*x^(1 + n + m*p)))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 + p
)))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2079, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^n \left(x^m \left(a+b x^{m p+n+1}\right)\right)^p \, dx$$

$$\Big\downarrow \text{2079}$$

$$\int x^n \left(a x^m+b x^{m p+m+n+1}\right)^p \, dx$$

$$\Big\downarrow \text{1920}$$

$$\frac{x^{-m(p+1)} \left(a x^m+b x^{m p+m+n+1}\right)^{p+1}}{b(p+1)(m p+n+1)}$$

input
```
Int[x^n*(x^m*(a + b*x^(1 + n + m*p)))^p,x]
```

output
```
(a*x^m + b*x^(1 + m + n + m*p))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 +
 p)))
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 2079
```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] && !Generali
zedBinomialMatchQ[u, x]
```

Maple [F]

$$\int x^n \left(x^m \left(a+b\,x^{mp+n+1}\right)\right)^p dx$$

input
```
int(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x)
```

output
```
int(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.65

$$\int x^n \left(x^m \left(a+bx^{1+n+mp}\right)\right)^p \, dx = \frac{(bxx^{mp+n+1}x^n+axx^n)(bx^{mp+n+1}x^m+ax^m)^p}{(bmp^2+bn+(bm+bn+b)p+b)x^{mp+n+1}}$$

input
```
integrate(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x, algorithm="fricas")
```

output
```
(b*x*x^(m*p + n + 1)*x^n + a*x*x^n)*(b*x^(m*p + n + 1)*x^m + a*x^m)^p/((b*
m*p^2 + b*n + (b*m + b*n + b)*p + b)*x^(m*p + n + 1))
```

Sympy [F(-1)]

Timed out.

$$\int x^n \left(x^m \left(a+bx^{1+n+mp}\right)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(x**n*(x**m*(a+b*x**(m*p+n+1)))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^n (x^m (a + bx^{1+n+mp}))^p \, dx = \int ((bx^{mp+n+1} + a)x^m)^p \, x^n \, dx$$

input
```
integrate(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x, algorithm="maxima")
```

output
```
integrate(((b*x^(m*p + n + 1) + a)*x^m)^p*x^n, x)
```

Giac [F]

$$\int x^n (x^m (a + bx^{1+n+mp}))^p \, dx = \int ((bx^{mp+n+1} + a)x^m)^p \, x^n \, dx$$

input
```
integrate(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x, algorithm="giac")
```

output
```
integrate(((b*x^(m*p + n + 1) + a)*x^m)^p*x^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^n (x^m (a + bx^{1+n+mp}))^p \, dx = \int x^n \, (x^m \, (a + b \, x^{n+m\,p+1}))^p \, dx$$

input
```
int(x^n*(x^m*(a + b*x^(n + m*p + 1)))^p,x)
```

output
```
int(x^n*(x^m*(a + b*x^(n + m*p + 1)))^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int x^n \left(x^m \left(a+bx^{1+n+mp}\right)\right)^p \, dx = \frac{(x^{mp+n}bx+a)^p \, (x^{mp+n}bx+a)}{b\,(m\,p^2+mp+np+n+p+1)}$$

input
```
int(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x)
```

output
```
((x**(m*p + n)*b*x + a)**p*(x**(m*p + n)*b*x + a))/(b*(m*p**2 + m*p + n*p
+ n + p + 1))
```

3.431 $\int x^n(ax^m + bx^{1+m+n+mp})^p \, dx$

Optimal result . . . 3220
Mathematica [A] (verified) . . . 3220
Rubi [A] (verified) . . . 3221
Maple [F] . . . 3221
Fricas [A] (verification not implemented) . . . 3222
Sympy [F] . . . 3222
Maxima [F] . . . 3222
Giac [F] . . . 3223
Mupad [F(-1)] . . . 3223
Reduce [F] . . . 3223

Optimal result

Integrand size = 23, antiderivative size = 46

$$\int x^n \left(ax^m + bx^{1+m+n+mp}\right)^p \, dx = \frac{x^{-m(1+p)}\left(ax^m + bx^{1+m+n+mp}\right)^{1+p}}{b(1+p)(1+n+mp)}$$

output
```
(a*x^m+b*x^(m*p+m+n+1))^(p+1)/b/(p+1)/(m*p+n+1)/(x^(m*(p+1)))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int x^n \left(ax^m + bx^{1+m+n+mp}\right)^p \, dx = \frac{x^{-m(1+p)}\left(x^m\left(a + bx^{1+n+mp}\right)\right)^{1+p}}{b(1+p)(1+n+mp)}$$

input
```
Integrate[x^n*(a*x^m + b*x^(1 + m + n + m*p))^p,x]
```

output
```
(x^m*(a + b*x^(1 + n + m*p)))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 + p
)))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^n\left(ax^m+bx^{mp+m+n+1}\right)^p\,dx$$

$$\Big\downarrow \ 1920$$

$$\frac{x^{-m(p+1)}\left(ax^m+bx^{mp+m+n+1}\right)^{p+1}}{b(p+1)(mp+n+1)}$$

input
```
Int[x^n*(a*x^m + b*x^(1 + m + n + m*p))^p,x]
```

output
```
(a*x^m + b*x^(1 + m + n + m*p))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 +
 p)))
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [F]

$$\int x^n\left(a\,x^m+b\,x^{mp+m+n+1}\right)^p dx$$

input
```
int(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x)
```

output
```
int(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.72

$$\int x^n \left(ax^m + bx^{1+m+n+mp}\right)^p \, dx = \frac{(bxx^{mp+m+n+1}x^n + axx^m x^n)(bx^{mp+m+n+1} + ax^m)^p}{(bmp^2 + bn + (bm + bn + b)p + b)x^{mp+m+n+1}}$$

input
```
integrate(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x, algorithm="fricas")
```

output
```
(b*x*x^(m*p + m + n + 1)*x^n + a*x*x^m*x^n)*(b*x^(m*p + m + n + 1) + a*x^m
)^p/((b*m*p^2 + b*n + (b*m + b*n + b)*p + b)*x^(m*p + m + n + 1))
```

Sympy [F]

$$\int x^n \left(ax^m + bx^{1+m+n+mp}\right)^p \, dx = \int x^n \left(ax^m + bx^{mp+m+n+1}\right)^p \, dx$$

input
```
integrate(x**n*(a*x**m+b*x**(m*p+m+n+1))**p,x)
```

output
```
Integral(x**n*(a*x**m + b*x**(m*p + m + n + 1))**p, x)
```

Maxima [F]

$$\int x^n \left(ax^m + bx^{1+m+n+mp}\right)^p \, dx = \int \left(bx^{mp+m+n+1} + ax^m\right)^p x^n \, dx$$

input
```
integrate(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x, algorithm="maxima")
```

output
```
integrate((b*x^(m*p + m + n + 1) + a*x^m)^p*x^n, x)
```

Giac [F]

$$\int x^n \left(ax^m + bx^{1+m+n+mp}\right)^p \, dx = \int \left(bx^{mp+m+n+1} + ax^m\right)^p x^n \, dx$$

input
```
integrate(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x, algorithm="giac")
```

output
```
integrate((b*x^(m*p + m + n + 1) + a*x^m)^p*x^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^n \left(ax^m + bx^{1+m+n+mp}\right)^p \, dx = \int x^n \left(a\,x^m + b\,x^{m+n+m\,p+1}\right)^p \, dx$$

input
```
int(x^n*(a*x^m + b*x^(m + n + m*p + 1))^p,x)
```

output
```
int(x^n*(a*x^m + b*x^(m + n + m*p + 1))^p, x)
```

Reduce [F]

$$\int x^n \left(ax^m + bx^{1+m+n+mp}\right)^p \, dx = \int x^n \left(x^{mp+m+n}bx + x^m a\right)^p \, dx$$

input
```
int(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x)
```

output
```
int(x**n*(x**(m*p + m + n)*b*x + x**m*a)**p,x)
```

3.432 $\int \sqrt{x^{2(-1+n)}\left(a+bx^n\right)}\,dx$

Optimal result 3224
Mathematica [A] (verified) 3224
Rubi [A] (verified) 3225
Maple [A] (verified) 3226
Fricas [A] (verification not implemented) 3226
Sympy [F(-1)] 3226
Maxima [A] (verification not implemented) 3227
Giac [F] 3227
Mupad [F(-1)] 3227
Reduce [B] (verification not implemented) 3228

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \sqrt{x^{2(-1+n)}\left(a+bx^n\right)}\,dx=\frac{2x^{3(1-n)}\left(ax^{-2(1-n)}+bx^{-2+3n}\right)^{3/2}}{3bn}$$

output
```
2/3*x^(3-3*n)*(a/(x^(2-2*n))+b*x^(-2+3*n))^(3/2)/b/n
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \sqrt{x^{2(-1+n)}\left(a+bx^n\right)}\,dx=\frac{2x^{3-3n}\left(x^{-2+2n}(a+bx^n)\right)^{3/2}}{3bn}$$

input
```
Integrate[Sqrt[x^(2*(-1 + n))*(a + b*x^n)],x]
```

output
```
(2*x^(3 - 3*n)*(x^(-2 + 2*n)*(a + b*x^n))^(3/2))/(3*b*n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2078, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^{2(n-1)}\left(a+bx^n\right)}dx$$

$$\Big\downarrow \ 2078$$

$$\int \sqrt{ax^{2(n-1)}+bx^{2(n-1)+n}}dx$$

$$\Big\downarrow \ 1906$$

$$\frac{2x^{3(1-n)}\left(ax^{-2(1-n)}+bx^{3n-2}\right)^{3/2}}{3bn}$$

input
```
Int[Sqrt[x^(2*(-1 + n))*(a + b*x^n)],x]
```

output
```
(2*x^(3*(1 - n))*(a/x^(2*(1 - n)) + b*x^(-2 + 3*n))^(3/2))/(3*b*n)
```

Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{2\sqrt{\frac{x^{2n}(a+b\,x^{n})}{x^{2}}}\,(a+b\,x^{n})x^{-n}x}{3bn}$	40

input
```
int((x^(-2+2*n)*(a+b*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(1/x^2*(x^n)^2*(a+b*x^n))^(1/2)*(a+b*x^n)/(x^n)*x/b/n
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int\sqrt{x^{2(-1+n)}\left(a+bx^{n}\right)}\,dx=\frac{2\left(bxx^{n}+ax\right)\sqrt{\frac{bx^{3\,n}+ax^{2\,n}}{x^{2}}}}{3\,bnx^{n}}$$

input
```
integrate((x^(-2+2*n)*(a+b*x^n))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(b*x*x^n + a*x)*sqrt((b*x^(3*n) + a*x^(2*n))/x^2)/(b*n*x^n)
```

Sympy [F(-1)]

Timed out.

$$\int\sqrt{x^{2(-1+n)}\left(a+bx^{n}\right)}\,dx=\text{Timed out}$$

input
```
integrate((x**(-2+2*n)*(a+b*x**n))**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.39

$$\int \sqrt{x^{2(-1+n)}\left(a+bx^n\right)}\,dx = \frac{2\left(bx^n+a\right)^{\frac{3}{2}}}{3\,bn}$$

input
```
integrate((x^(-2+2*n)*(a+b*x^n))^(1/2),x, algorithm="maxima")
```

output
```
2/3*(b*x^n + a)^(3/2)/(b*n)
```

Giac [F]

$$\int \sqrt{x^{2(-1+n)}\left(a+bx^n\right)}\,dx = \int \sqrt{(bx^n+a)x^{2\,n-2}}\,dx$$

input
```
integrate((x^(-2+2*n)*(a+b*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((b*x^n + a)*x^(2*n - 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x^{2(-1+n)}\left(a+bx^n\right)}\,dx = \int \sqrt{x^{2\,n-2}\,\left(a+b\,x^n\right)}\,dx$$

input
```
int((x^(2*n - 2)*(a + b*x^n))^(1/2),x)
```

output
```
int((x^(2*n - 2)*(a + b*x^n))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.52

$$\int \sqrt{x^{2(-1+n)}\left(a+bx^{n}\right)}\,dx=\frac{2\sqrt{x^{n}b+a}\,\left(x^{n}b+a\right)}{3bn}$$

input
```
int((x^(-2+2*n)*(a+b*x^n))^(1/2),x)
```

output
```
(2*sqrt(x**n*b + a)*(x**n*b + a))/(3*b*n)
```

3.433 $\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^n\right)}\,dx$

Optimal result . . . 3229
Mathematica [A] (verified) . . . 3229
Rubi [A] (verified) . . . 3230
Maple [A] (verified) . . . 3231
Fricas [A] (verification not implemented) . . . 3231
Sympy [F(-1)] . . . 3231
Maxima [A] (verification not implemented) . . . 3232
Giac [F] . . . 3232
Mupad [F(-1)] . . . 3232
Reduce [B] (verification not implemented) . . . 3233

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^n\right)}\,dx=\frac{3x^{4(1-n)}\left(ax^{-3(1-n)}+bx^{-3+4n}\right)^{4/3}}{4bn}$$

output
```
3/4*x^(4-4*n)*(a/(x^(3-3*n))+b*x^(-3+4*n))^(4/3)/b/n
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^n\right)}\,dx=\frac{3x^{4-4n}\left(x^{-3+3n}(a+bx^n)\right)^{4/3}}{4bn}$$

input
```
Integrate[(x^(3*(-1 + n))*(a + b*x^n))^(1/3),x]
```

output
```
(3*x^(4 - 4*n)*(x^(-3 + 3*n)*(a + b*x^n))^(4/3))/(4*b*n)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2078, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{x^{3(n-1)}\left(a+bx^n\right)}dx$$

$$\Big\downarrow \ 2078$$

$$\int \sqrt[3]{ax^{3(n-1)}+bx^{3(n-1)+n}}dx$$

$$\Big\downarrow \ 1906$$

$$\frac{3x^{4(1-n)}\left(ax^{-3(1-n)}+bx^{4n-3}\right)^{4/3}}{4bn}$$

input
```
Int[(x^(3*(-1 + n))*(a + b*x^n))^(1/3),x]
```

output
```
(3*x^(4*(1 - n))*(a/x^(3*(1 - n)) + b*x^(-3 + 4*n))^(4/3))/(4*b*n)
```

Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{3\left(\frac{x^{3n}(a+b\,x^{n})}{x^{3}}\right)^{\frac{1}{3}} x\,x^{-n}(a+b\,x^{n})}{4bn}$	40

input
```
int((x^(-3+3*n)*(a+b*x^n))^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/4*(1/x^3*(x^n)^3*(a+b*x^n))^(1/3)*x/(x^n)*(a+b*x^n)/b/n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^{n}\right)}\,dx=\frac{3\left(bxx^{n}+ax\right)\left(\frac{bx^{4\,n}+ax^{3\,n}}{x^{3}}\right)^{\frac{1}{3}}}{4\,bnx^{n}}$$

input
```
integrate((x^(-3+3*n)*(a+b*x^n))^(1/3),x, algorithm="fricas")
```

output
```
3/4*(b*x*x^n + a*x)*((b*x^(4*n) + a*x^(3*n))/x^3)^(1/3)/(b*n*x^n)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^{n}\right)}\,dx=\text{Timed out}$$

input
```
integrate((x**(-3+3*n)*(a+b*x**n))**(1/3),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.39

$$\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^n\right)}\,dx = \frac{3\left(bx^n+a\right)^{\frac{4}{3}}}{4\,bn}$$

input
```
integrate((x^(-3+3*n)*(a+b*x^n))^(1/3),x, algorithm="maxima")
```

output
```
3/4*(b*x^n + a)^(4/3)/(b*n)
```

Giac [F]

$$\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^n\right)}\,dx = \int \left((bx^n+a)x^{3\,n-3}\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((x^(-3+3*n)*(a+b*x^n))^(1/3),x, algorithm="giac")
```

output
```
integrate(((b*x^n + a)*x^(3*n - 3))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^n\right)}\,dx = \int \left(x^{3\,n-3}\,\left(a+b\,x^n\right)\right)^{1/3}\,dx$$

input
```
int((x^(3*n - 3)*(a + b*x^n))^(1/3),x)
```

output
```
int((x^(3*n - 3)*(a + b*x^n))^(1/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.39

$$\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^n\right)}\,dx=\frac{3(x^nb+a)^{\frac{4}{3}}}{4bn}$$

input
```
int((x^(-3+3*n)*(a+b*x^n))^(1/3),x)
```

output
```
(3*(x**n*b + a)**(1/3)*(x**n*b + a))/(4*b*n)
```

3.434 $\int \sqrt[4]{x^{4(-1+n)}\left(a+bx^n\right)}\,dx$

Optimal result . . . 3234
Mathematica [A] (verified) . . . 3234
Rubi [A] (verified) . . . 3235
Maple [A] (verified) . . . 3236
Fricas [A] (verification not implemented) . . . 3236
Sympy [F(-1)] . . . 3236
Maxima [A] (verification not implemented) . . . 3237
Giac [F] . . . 3237
Mupad [F(-1)] . . . 3237
Reduce [B] (verification not implemented) . . . 3238

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \sqrt[4]{x^{4(-1+n)}\left(a+bx^n\right)}\,dx=\frac{4x^{5(1-n)}\left(ax^{-4(1-n)}+bx^{-4+5n}\right)^{5/4}}{5bn}$$

output
```
4/5*x^(5-5*n)*(a/(x^(4-4*n))+b*x^(-4+5*n))^(5/4)/b/n
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \sqrt[4]{x^{4(-1+n)}\left(a+bx^n\right)}\,dx=\frac{4x^{5-5n}\left(x^{-4+4n}\left(a+bx^n\right)\right)^{5/4}}{5bn}$$

input
```
Integrate[(x^(4*(-1 + n))*(a + b*x^n))^(1/4),x]
```

output
```
(4*x^(5 - 5*n)*(x^(-4 + 4*n)*(a + b*x^n))^(5/4))/(5*b*n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2078, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[4]{x^{4(n-1)}(a+bx^n)}dx$$

$$\Big\downarrow \text{ 2078}$$

$$\int \sqrt[4]{ax^{4(n-1)}+bx^{4(n-1)+n}}dx$$

$$\Big\downarrow \text{ 1906}$$

$$\frac{4x^{5(1-n)}\left(ax^{-4(1-n)}+bx^{5n-4}\right)^{5/4}}{5bn}$$

input
```
Int[(x^(4*(-1 + n))*(a + b*x^n))^(1/4),x]
```

output
```
(4*x^(5*(1 - n))*(a/x^(4*(1 - n)) + b*x^(-4 + 5*n))^(5/4))/(5*b*n)
```

Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{4\left(\frac{x^{4n}(a+b\,x^{n})}{x^{4}}\right)^{\frac{1}{4}} x\,x^{-n}(a+b\,x^{n})}{5bn}$	40

input
```
int((x^(-4+4*n)*(a+b*x^n))^(1/4),x,method=_RETURNVERBOSE)
```

output
```
4/5*(1/x^4*(x^n)^4*(a+b*x^n))^(1/4)*x/(x^n)*(a+b*x^n)/b/n
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \sqrt[4]{x^{4(-1+n)}\,(a+bx^n)}\,dx = \frac{4\,(bxx^n + ax)\left(\frac{bx^{5\,n}+ax^{4\,n}}{x^4}\right)^{\frac{1}{4}}}{5\,bnx^n}$$

input
```
integrate((x^(-4+4*n)*(a+b*x^n))^(1/4),x, algorithm="fricas")
```

output
```
4/5*(b*x*x^n + a*x)*((b*x^(5*n) + a*x^(4*n))/x^4)^(1/4)/(b*n*x^n)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt[4]{x^{4(-1+n)}\,(a+bx^n)}\,dx = \text{Timed out}$$

input
```
integrate((x**(-4+4*n)*(a+b*x**n))**(1/4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.39

$$\int \sqrt[4]{x^{4(-1+n)}\left(a+bx^n\right)}\,dx=\frac{4\left(bx^n+a\right)^{\frac{5}{4}}}{5\,bn}$$

input
```
integrate((x^(-4+4*n)*(a+b*x^n))^(1/4),x, algorithm="maxima")
```

output
```
4/5*(b*x^n + a)^(5/4)/(b*n)
```

Giac [F]

$$\int \sqrt[4]{x^{4(-1+n)}\left(a+bx^n\right)}\,dx=\int \left((bx^n+a)x^{4\,n-4}\right)^{\frac{1}{4}}\,dx$$

input
```
integrate((x^(-4+4*n)*(a+b*x^n))^(1/4),x, algorithm="giac")
```

output
```
integrate(((b*x^n + a)*x^(4*n - 4))^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[4]{x^{4(-1+n)}\left(a+bx^n\right)}\,dx=\int \left(x^{4\,n-4}\,\left(a+b\,x^n\right)\right)^{1/4}\,dx$$

input
```
int((x^(4*n - 4)*(a + b*x^n))^(1/4),x)
```

output
```
int((x^(4*n - 4)*(a + b*x^n))^(1/4), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.39

$$\int \sqrt[4]{x^{4(-1+n)}\left(a+bx^n\right)}\,dx=\frac{4(x^nb+a)^{\frac{5}{4}}}{5bn}$$

input
```
int((x^(-4+4*n)*(a+b*x^n))^(1/4),x)
```

output
```
(4*(x**n*b + a)**(1/4)*(x**n*b + a))/(5*b*n)
```

3.435 $\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}}\,dx$

Optimal result 3239
Mathematica [A] (verified) 3239
Rubi [A] (verified) 3240
Maple [F] 3241
Fricas [A] (verification not implemented) 3241
Sympy [F] 3241
Maxima [F] 3242
Giac [F] 3242
Mupad [F(-1)] 3242
Reduce [B] (verification not implemented) 3243

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}}\,dx=\frac{px^{(1-n)(1+p)}\left(ax^{-((1-n)p)}+bx^{n-(1-n)p}\right)^{1+\frac{1}{p}}}{bn(1+p)}$$

output
```
p*x^((1-n)*(p+1))*(a/(x^((1-n)*p))+b*x^(n-(1-n)*p))^(1+1/p)/b/n/(p+1)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}}\,dx=\frac{x^{1-n}(a+bx^n)\left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}}}{bn\left(1+\frac{1}{p}\right)}$$

input
```
Integrate[(x^((-1 + n)*p)*(a + b*x^n))^p^(-1),x]
```

output
```
(x^(1 - n)*(a + b*x^n)*(x^((-1 + n)*p)*(a + b*x^n))^p^(-1))/(b*n*(1 + p^(-
1)))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2078, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^{(n-1)p}(a+bx^n)\right)^{\frac{1}{p}}dx$$
$$\Big\downarrow \text{ 2078}$$
$$\int \left(ax^{(n-1)p}+bx^{(n-1)p+n}\right)^{\frac{1}{p}}dx$$
$$\Big\downarrow \text{ 1906}$$
$$\frac{px^{(1-n)(p+1)}\left(ax^{-((1-n)p)}+bx^{n-(1-n)p}\right)^{\frac{1}{p}+1}}{bn(p+1)}$$

input
```
Int[(x^((-1 + n)*p)*(a + b*x^n))^p^(-1),x]
```

output
```
(p*x^((1 - n)*(1 + p))*(a/x^((1 - n)*p) + b*x^(n - (1 - n)*p))^(1 + p^(-1)
))/(b*n*(1 + p))
```

Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Maple [F]

$$\int \left(x^{(-1+n)p}(a+b\,x^n)\right)^{\frac{1}{p}}dx$$

input
```
int((x^((-1+n)*p)*(a+b*x^n))^(1/p),x)
```

output
```
int((x^((-1+n)*p)*(a+b*x^n))^(1/p),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}}\,dx=\frac{(bpxx^n+apx)\left((bx^n+a)x^{(n-1)p}\right)^{\left(\frac{1}{p}\right)}}{(bnp+bn)x^n}$$

input
```
integrate((x^((-1+n)*p)*(a+b*x^n))^(1/p),x, algorithm="fricas")
```

output
```
(b*p*x*x^n + a*p*x)*((b*x^n + a)*x^((n - 1)*p))^(1/p)/((b*n*p + b*n)*x^n)
```

Sympy [F]

$$\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}}\,dx=\int \left(x^{p(n-1)}(a+bx^n)\right)^{\frac{1}{p}}\,dx$$

input
```
integrate((x**((-1+n)*p)*(a+b*x**n))**(1/p),x)
```

output
```
Integral((x**(p*(n - 1))*(a + b*x**n))**(1/p), x)
```

Maxima [F]

$$\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}}\,dx=\int \left((bx^n+a)x^{(n-1)p}\right)^{\left(\frac{1}{p}\right)}\,dx$$

input
```
integrate((x^((-1+n)*p)*(a+b*x^n))^(1/p),x, algorithm="maxima")
```

output
```
integrate(((b*x^n + a)*x^((n - 1)*p))^(1/p), x)
```

Giac [F]

$$\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}}\,dx=\int \left((bx^n+a)x^{(n-1)p}\right)^{\left(\frac{1}{p}\right)}\,dx$$

input
```
integrate((x^((-1+n)*p)*(a+b*x^n))^(1/p),x, algorithm="giac")
```

output
```
integrate(((b*x^n + a)*x^((n - 1)*p))^(1/p), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}}\,dx=\int \left(x^{p\,(n-1)}\,\left(a+b\,x^n\right)\right)^{1/p}\,dx$$

input
```
int((x^(p*(n - 1))*(a + b*x^n))^(1/p),x)
```

output
```
int((x^(p*(n - 1))*(a + b*x^n))^(1/p), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.54

$$\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}}\,dx=\frac{(x^nb+a)^{\frac{1}{p}}\,p(x^nb+a)}{bn\,(p+1)}$$

input
```
int((x^((-1+n)*p)*(a+b*x^n))^(1/p),x)
```

output
```
((x**n*b + a)**(1/p)*p*(x**n*b + a))/(b*n*(p + 1))
```

3.436 $\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p dx$

Optimal result . . . 3244
Mathematica [A] (verified) . . . 3244
Rubi [A] (verified) . . . 3245
Maple [F] . . . 3246
Fricas [A] (verification not implemented) . . . 3246
Sympy [F] . . . 3246
Maxima [F] . . . 3247
Giac [F] . . . 3247
Mupad [F(-1)] . . . 3247
Reduce [B] (verification not implemented) . . . 3248

Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p dx = \frac{x^{\frac{(1-n)(1+p)}{p}}\left(bx^{n-\frac{1-n}{p}}+ax^{-\frac{1-n}{p}}\right)^{1+p}}{bn(1+p)}$$

output
```
x^((1-n)*(p+1)/p)*(b*x^(n-(1-n)/p)+a/(x^((1-n)/p)))^(p+1)/b/n/(p+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p dx = \frac{x^{1-n}(a+bx^n)\left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p}{bn(1+p)}$$

input
```
Integrate[(x^((-1 + n)/p)*(a + b*x^n))^p,x]
```

output
```
(x^(1 - n)*(a + b*x^n)*(x^((-1 + n)/p)*(a + b*x^n))^p)/(b*n*(1 + p))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2078, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^{\frac{n-1}{p}}(a+bx^n)\right)^p dx$$

$$\Big\downarrow \text{2078}$$

$$\int \left(ax^{\frac{n-1}{p}}+bx^{\frac{n-1}{p}+n}\right)^p dx$$

$$\Big\downarrow \text{1906}$$

$$\frac{x^{\frac{(1-n)(p+1)}{p}}\left(ax^{-\frac{1-n}{p}}+bx^{n-\frac{1-n}{p}}\right)^{p+1}}{bn(p+1)}$$

input
```
Int[(x^((-1 + n)/p)*(a + b*x^n))^p,x]
```

output
```
(x^(((1 - n)*(1 + p))/p)*(b*x^(n - (1 - n)/p) + a/x^((1 - n)/p))^(1 + p))/
(b*n*(1 + p))
```

Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Maple [F]

$$\int \left(x^{\frac{-1+n}{p}}(a+b\,x^{n})\right)^{p} dx$$

input
```
int((x^((-1+n)/p)*(a+b*x^n))^p,x)
```

output
```
int((x^((-1+n)/p)*(a+b*x^n))^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p dx = \frac{(bxx^n+ax)\left(bx^nx^{\frac{n-1}{p}}+ax^{\frac{n-1}{p}}\right)^p}{(bnp+bn)x^n}$$

input
```
integrate((x^((-1+n)/p)*(a+b*x^n))^p,x, algorithm="fricas")
```

output
```
(b*x*x^n + a*x)*(b*x^n*x^((n - 1)/p) + a*x^((n - 1)/p))^p/((b*n*p + b*n)*x
^n)
```

Sympy [F]

$$\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p dx = \int \left(x^{\frac{n-1}{p}}(a+bx^n)\right)^p dx$$

input
```
integrate((x**((-1+n)/p)*(a+b*x**n))**p,x)
```

output
```
Integral((x**((n - 1)/p)*(a + b*x**n))**p, x)
```

Maxima [F]

$$\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p dx = \int \left((bx^n+a)x^{\frac{n-1}{p}}\right)^p dx$$

input
```
integrate((x^((-1+n)/p)*(a+b*x^n))^p,x, algorithm="maxima")
```

output
```
integrate(((b*x^n + a)*x^((n - 1)/p))^p, x)
```

Giac [F]

$$\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p dx = \int \left((bx^n+a)x^{\frac{n-1}{p}}\right)^p dx$$

input
```
integrate((x^((-1+n)/p)*(a+b*x^n))^p,x, algorithm="giac")
```

output
```
integrate(((b*x^n + a)*x^((n - 1)/p))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p dx = \int \left(x^{\frac{n-1}{p}}\,(a+b\,x^n)\right)^p dx$$

input
```
int((x^((n - 1)/p)*(a + b*x^n))^p,x)
```

output
```
int((x^((n - 1)/p)*(a + b*x^n))^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.46

$$\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p \, dx = \frac{(x^n b+a)^p (x^n b+a)}{bn (p+1)}$$

input
```
int((x^((-1+n)/p)*(a+b*x^n))^p,x)
```

output
```
((x**n*b + a)**p*(x**n*b + a))/(b*n*(p + 1))
```

3.437 $\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx$

Optimal result . . . 3249
Mathematica [A] (verified) . . . 3249
Rubi [A] (verified) . . . 3250
Maple [F] . . . 3250
Fricas [A] (verification not implemented) . . . 3251
Sympy [F] . . . 3251
Maxima [F] . . . 3251
Giac [F] . . . 3252
Mupad [F(-1)] . . . 3252
Reduce [B] (verification not implemented) . . . 3252

Optimal result

Integrand size = 25, antiderivative size = 39

$$\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx=\frac{x^{-p(1+q)}(ax^n+bx^p)^{1+q}}{a(n-p)(1+q)}$$

output
```
(a*x^n+b*x^p)^(1+q)/a/(n-p)/(1+q)/(x^(p*(1+q)))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx=-\frac{x^{-p(1+q)}(ax^n+bx^p)^{1+q}}{a(-n+p)(1+q)}$$

input
```
Integrate[x^(-1 + n - p*(1 + q))*(a*x^n + b*x^p)^q,x]
```

output
```
-((a*x^n + b*x^p)^(1 + q)/(a*(-n + p)*(1 + q)*x^(p*(1 + q))))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-p(q+1)-1}(ax^n+bx^p)^q\,dx$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{x^{-p(q+1)}(ax^n+bx^p)^{q+1}}{a(q+1)(n-p)}$$

input
```
Int[x^(-1 + n - p*(1 + q))*(a*x^n + b*x^p)^q,x]
```

output
```
(a*x^n + b*x^p)^(1 + q)/(a*(n - p)*(1 + q)*x^(p*(1 + q)))
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [F]

$$\int x^{-1+n-p(q+1)}(a\,x^n+b\,x^p)^q\,dx$$

input
```
int(x^(-1+n-p*(q+1))*(a*x^n+b*x^p)^q,x)
```

output
```
int(x^(-1+n-p*(q+1))*(a*x^n+b*x^p)^q,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.95

$$\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx=\frac{(axx^{-pq+n-p-1}x^n+bxx^{-pq+n-p-1}x^p)(ax^n+bx^p)^q}{(an-ap+(an-ap)q)x^n}$$

input
```
integrate(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x, algorithm="fricas")
```

output
```
(a*x*x^(-p*q + n - p - 1)*x^n + b*x*x^(-p*q + n - p - 1)*x^p)*(a*x^n + b*x
^p)^q/((a*n - a*p + (a*n - a*p)*q)*x^n)
```

Sympy [F]

$$\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx=\int x^{n-p(q+1)-1}(ax^n+bx^p)^q\,dx$$

input
```
integrate(x**(-1+n-p*(1+q))*(a*x**n+b*x**p)**q,x)
```

output
```
Integral(x**(n - p*(q + 1) - 1)*(a*x**n + b*x**p)**q, x)
```

Maxima [F]

$$\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx=\int (ax^n+bx^p)^q x^{-p(q+1)+n-1}\,dx$$

input
```
integrate(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x, algorithm="maxima")
```

output
```
integrate((a*x^n + b*x^p)^q*x^(-p*(q + 1) + n - 1), x)
```

Giac [F]

$$\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx=\int (ax^n+bx^p)^q x^{-p(q+1)+n-1}\,dx$$

input
```
integrate(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x, algorithm="giac")
```

output
```
integrate((a*x^n + b*x^p)^q*x^(-p*(q + 1) + n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx=\int x^{n-p\,(q+1)-1}\,(a\,x^n+b\,x^p)^q\,dx$$

input
```
int(x^(n - p*(q + 1) - 1)*(a*x^n + b*x^p)^q,x)
```

output
```
int(x^(n - p*(q + 1) - 1)*(a*x^n + b*x^p)^q, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.31

$$\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx=\frac{(x^na+x^pb)^q\,(x^na+x^pb)}{x^{pq+p}a\,(nq-pq+n-p)}$$

input
```
int(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x)
```

output
```
((x**n*a + x**p*b)**q*(x**n*a + x**p*b))/(x**(p*q + p)*a*(n*q + n - p*q -
p))
```

3.438 $\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx$

Optimal result . . . 3253
Mathematica [A] (verified) . . . 3253
Rubi [A] (verified) . . . 3254
Maple [B] (verified) . . . 3255
Fricas [A] (verification not implemented) . . . 3255
Sympy [F(-1)] . . . 3256
Maxima [F] . . . 3256
Giac [F] . . . 3256
Mupad [F(-1)] . . . 3257
Reduce [B] (verification not implemented) . . . 3257

Optimal result

Integrand size = 28, antiderivative size = 40

$$\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx=-\frac{x^{-((n+p)(1+q))}(ax^n+bx^{n+p})^{1+q}}{ap(1+q)}$$

output
```
-(a*x^n+b*x^(n+p))^(1+q)/a/p/(1+q)/(x^((n+p)*(1+q)))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx=-\frac{x^{-((n+p)(1+q))}(x^n(a+bx^p))^{1+q}}{ap(1+q)}$$

input
```
Integrate[x^(-1 - n*q - p*(1 + q))*(x^n*(a + b*x^p))^q,x]
```

output
```
-((x^n*(a + b*x^p))^(1 + q)/(a*p*(1 + q)*x^((n + p)*(1 + q))))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2079, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-nq-p(q+1)-1}(x^n(a+bx^p))^q\,dx$$
$$\Big\downarrow \text{2079}$$
$$\int x^{-nq-p(q+1)-1}\left(ax^n+bx^{n+p}\right)^q dx$$
$$\Big\downarrow \text{1920}$$
$$-\frac{x^{-((q+1)(n+p))}\left(ax^n+bx^{n+p}\right)^{q+1}}{ap(q+1)}$$

input
```
Int[x^(-1 - n*q - p*(1 + q))*(x^n*(a + b*x^p))^q,x]
```

output
```
-((a*x^n + b*x^(n + p))^(1 + q)/(a*p*(1 + q)*x^((n + p)*(1 + q))))
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 2079
```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] && !Generali
zedBinomialMatchQ[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(41) = 82$.

Time = 1.65 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.15

method	result	size
parallelrisch	$-\frac{x\,x^{p} x^{-q n -p q -p -1} \left(x^{n} \left(a +b\,x^{p}\right)\right)^{q} b^{2}+x\,x^{-q n -p q -p -1} \left(x^{n} \left(a +b\,x^{p}\right)\right)^{q} a b}{b p \left(q +1\right) a}$	86

input
```
int(x^(-1-q*n-p*(q+1))*(x^n*(a+b*x^p))^q,x,method=_RETURNVERBOSE)
```

output
```
-(x*x^p*x^(-n*q-p*q-p-1)*(x^n*(a+b*x^p))^q*b^2+x*x^(-n*q-p*q-p-1)*(x^n*(a+
b*x^p))^q*a*b)/b/p/(q+1)/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.60

$$\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx = -\frac{\left(bxx^{-(n+p)q-p-1}x^p+axx^{-(n+p)q-p-1}\right)\left(bx^nx^p+ax^n\right)^q}{apq+ap}$$

input
```
integrate(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x, algorithm="fricas")
```

output
```
-(b*x*x^(-(n + p)*q - p - 1)*x^p + a*x*x^(-(n + p)*q - p - 1))*(b*x^n*x^p
+ a*x^n)^q/(a*p*q + a*p)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q \, dx = \text{Timed out}$$

input
```
integrate(x**(-1-n*q-p*(1+q))*(x**n*(a+b*x**p))**q,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q \, dx = \int ((bx^p+a)x^n)^q \, x^{-p(q+1)-nq-1} \, dx$$

input
```
integrate(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x, algorithm="maxima")
```

output
```
integrate(((b*x^p + a)*x^n)^q*x^(-p*(q + 1) - n*q - 1), x)
```

Giac [F]

$$\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q \, dx = \int ((bx^p+a)x^n)^q \, x^{-p(q+1)-nq-1} \, dx$$

input
```
integrate(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x, algorithm="giac")
```

output
```
integrate(((b*x^p + a)*x^n)^q*x^(-p*(q + 1) - n*q - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx = \int \frac{(x^n\,(a+b\,x^p))^q}{x^{n\,q+p\,(q+1)+1}}\,dx$$

input
```
int((x^n*(a + b*x^p))^q/x^(n*q + p*(q + 1) + 1),x)
```

output
```
int((x^n*(a + b*x^p))^q/x^(n*q + p*(q + 1) + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx = -\frac{(x^pb+a)^q\,(x^pb+a)}{x^{pq+p}ap\,(q+1)}$$

input
```
int(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x)
```

output
```
( - (x**p*b + a)**q*(x**p*b + a))/(x**(p*q + p)*a*p*(q + 1))
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions 3258
4.2 Links to plain text integration problems used in this report for each CAS . 3276

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#    "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                    convert(leaf_count_result,string)," vs. $2 (",
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Exp
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
       return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
       if expn.parent() is SR:
          return expn.operator() is None
       if expn.parent() in (ZZ, QQ, AA, QQbar):
          return expn in expn.parent() # Should always return True
       if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
          return expn in expn.parent().base_ring() or expn in expn.parent().gens()

       return False

    except AttributeError as error:
       print("Exception,AttributeError in  is_atom")
       print ("cought exception" , type(error).__name__ )
       return False


def expnType(expn):

    if debug:
       print (">>>>>Enter expnType, expn=", expn)
       print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
       return 1
    elif type(expn)==list:   #isinstance(expn,list):
       return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
       if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
          return 1
       else:
          return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
       if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file