Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.1-Quadratic-trinomial/1.2.1.2/89-1.2.1.2-a

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**344**]. This is test number [89].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (344)	0.00 (0)
Rubi	99.71 (343)	0.29 (1)
Maple	95.64 (329)	4.36 (15)
Fricas	95.64 (329)	4.36 (15)
Reduce	86.34 (297)	13.66 (47)
Giac	85.76 (295)	14.24 (49)
Mupad	75.00 (258)	25.00 (86)
Maxima	73.26 (252)	26.74 (92)
Sympy	61.34 (211)	38.66 (133)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

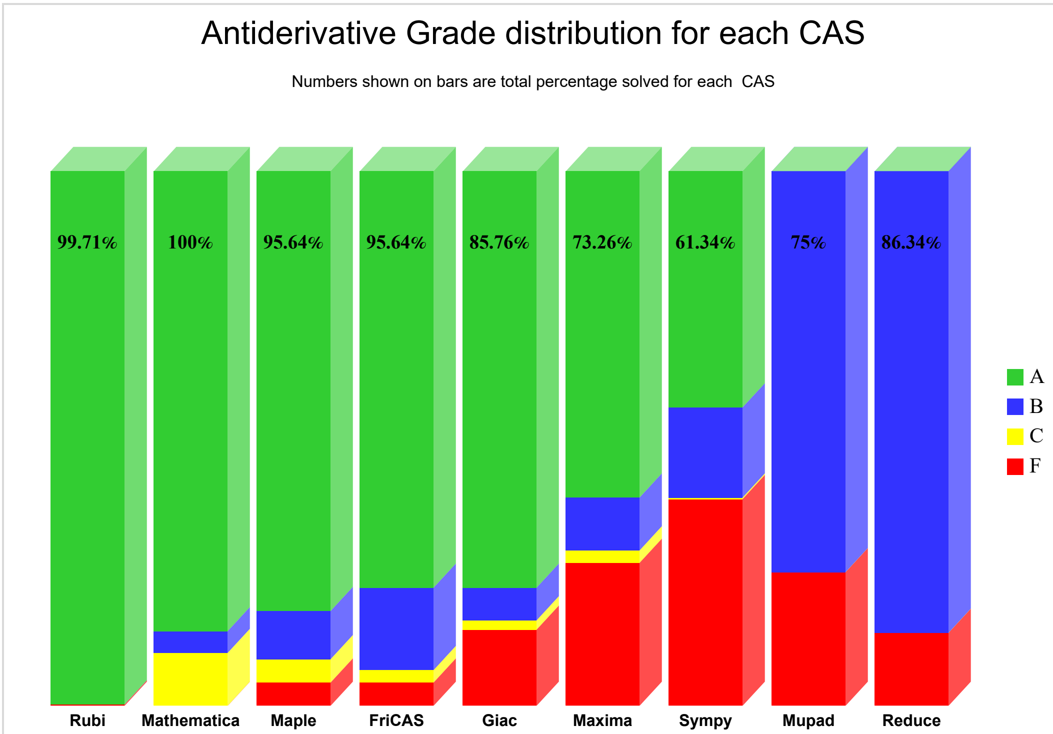
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

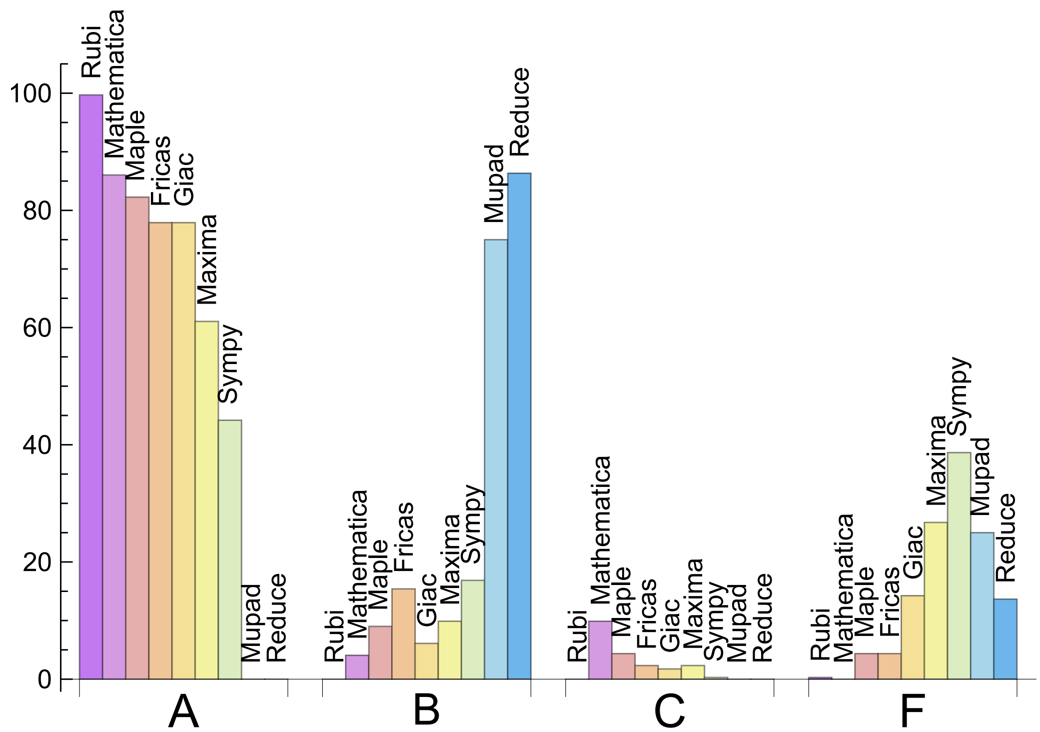
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.709	0.000	0.000	0.291
Mathematica	86.047	4.070	9.884	0.000
Maple	82.267	9.012	4.360	4.360
Fricas	77.907	15.407	2.326	4.360
Giac	77.907	6.105	1.744	14.244
Maxima	61.047	9.884	2.326	26.744
Sympy	44.186	16.860	0.291	38.663
Mupad	0.000	75.000	0.000	25.000
Reduce	0.000	86.337	0.000	13.663

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Rubi	1	100.00	0.00	0.00
Fricas	15	100.00	0.00	0.00
Maple	15	100.00	0.00	0.00
Reduce	47	100.00	0.00	0.00
Giac	49	100.00	0.00	0.00
Mupad	86	0.00	100.00	0.00
Maxima	92	55.43	0.00	44.57
Sympy	133	87.97	12.03	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.05
Fricas	0.11
Giac	0.22
Reduce	0.31
Rubi	0.43
Maple	0.83
Sympy	1.08
Mathematica	2.00
Mupad	4.78

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	65.73	1.06	42.00	0.87
Rubi	100.02	0.95	56.00	1.00
Giac	107.95	1.05	45.00	0.89
Mathematica	122.38	1.10	56.00	1.00
Maple	144.24	0.98	44.00	0.88
Sympy	177.57	1.94	46.00	1.00
Mupad	246.52	1.65	42.50	0.92
Fricas	248.05	1.71	50.00	0.88
Reduce	252.54	1.74	44.00	0.89

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

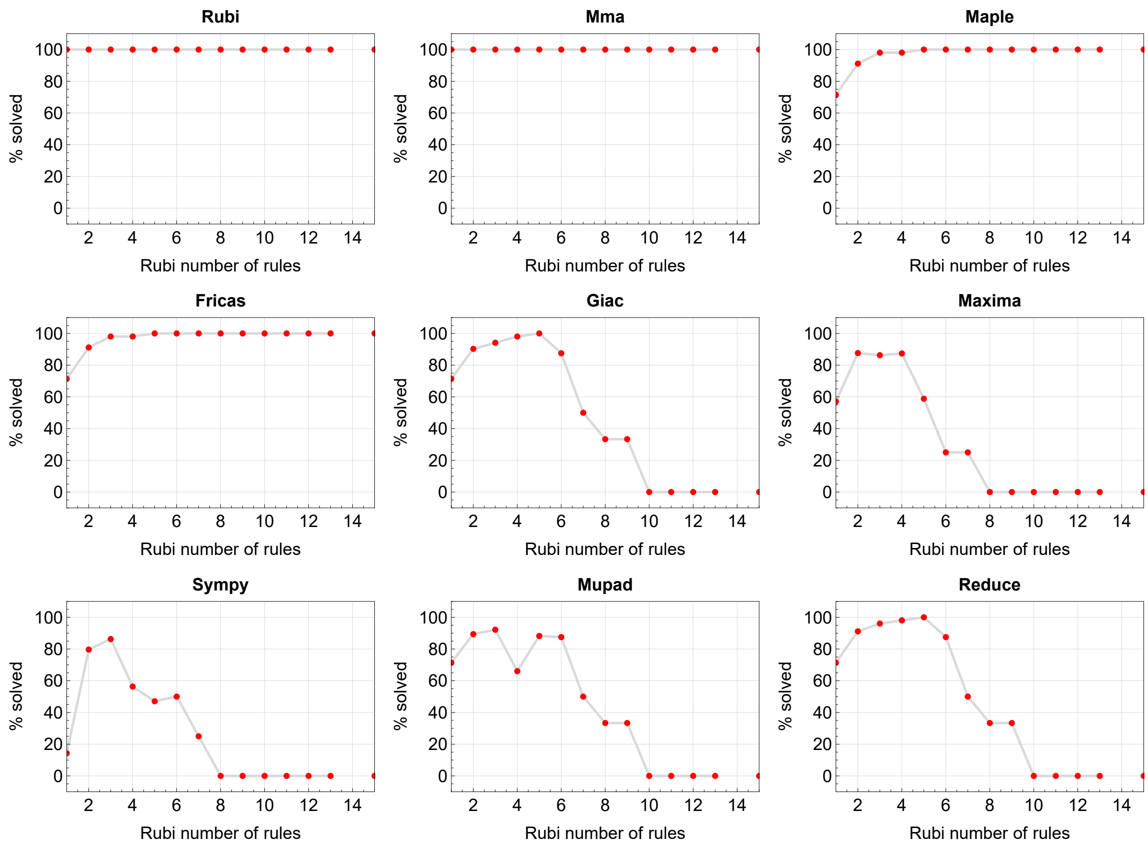


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

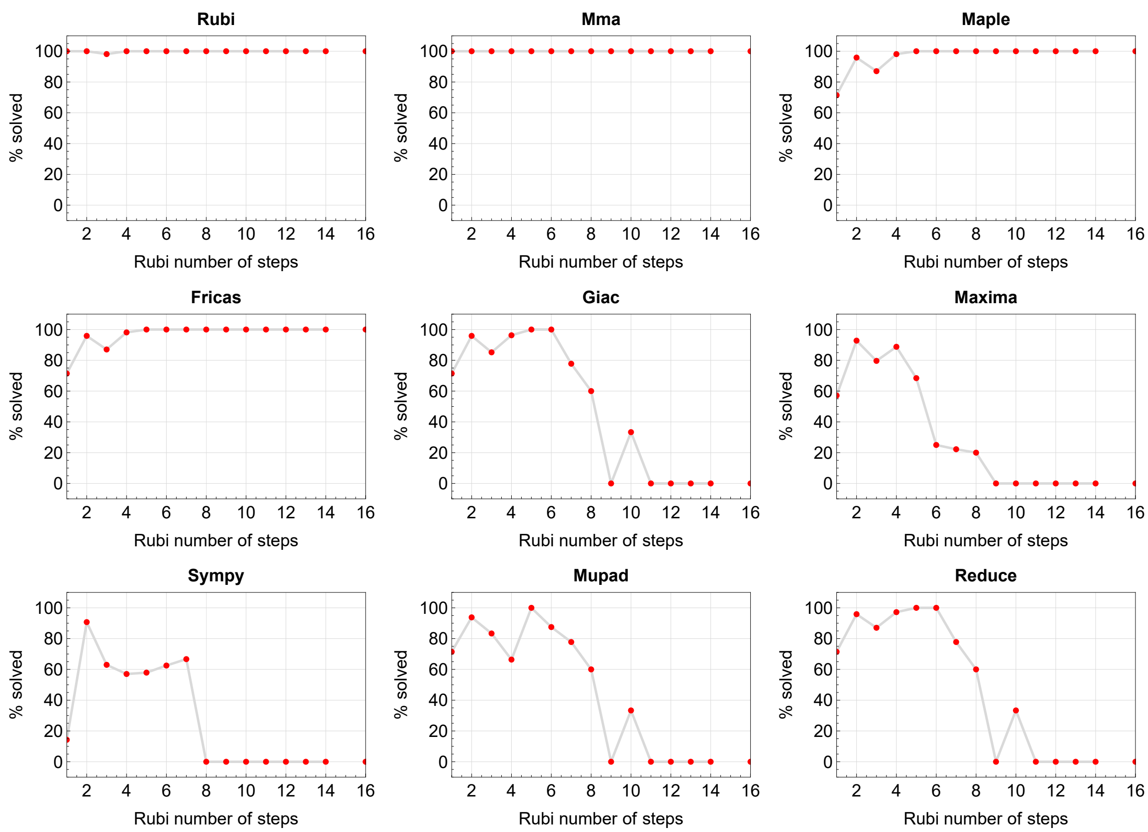


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

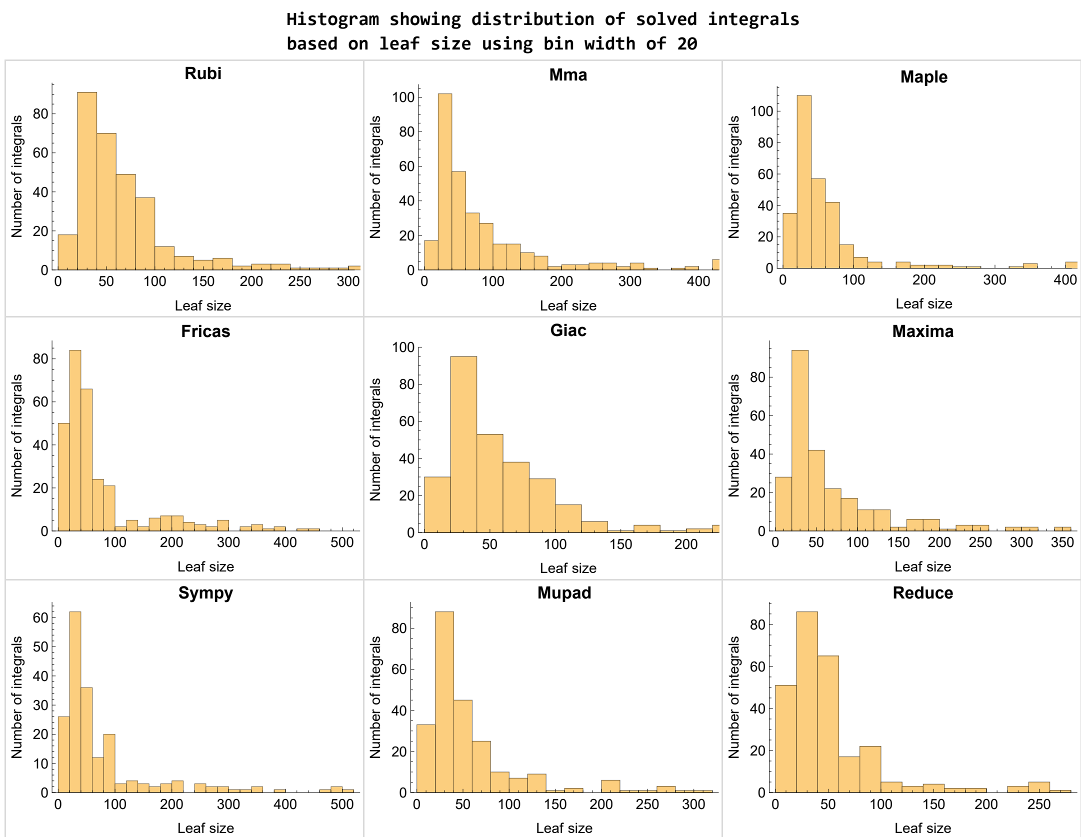


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

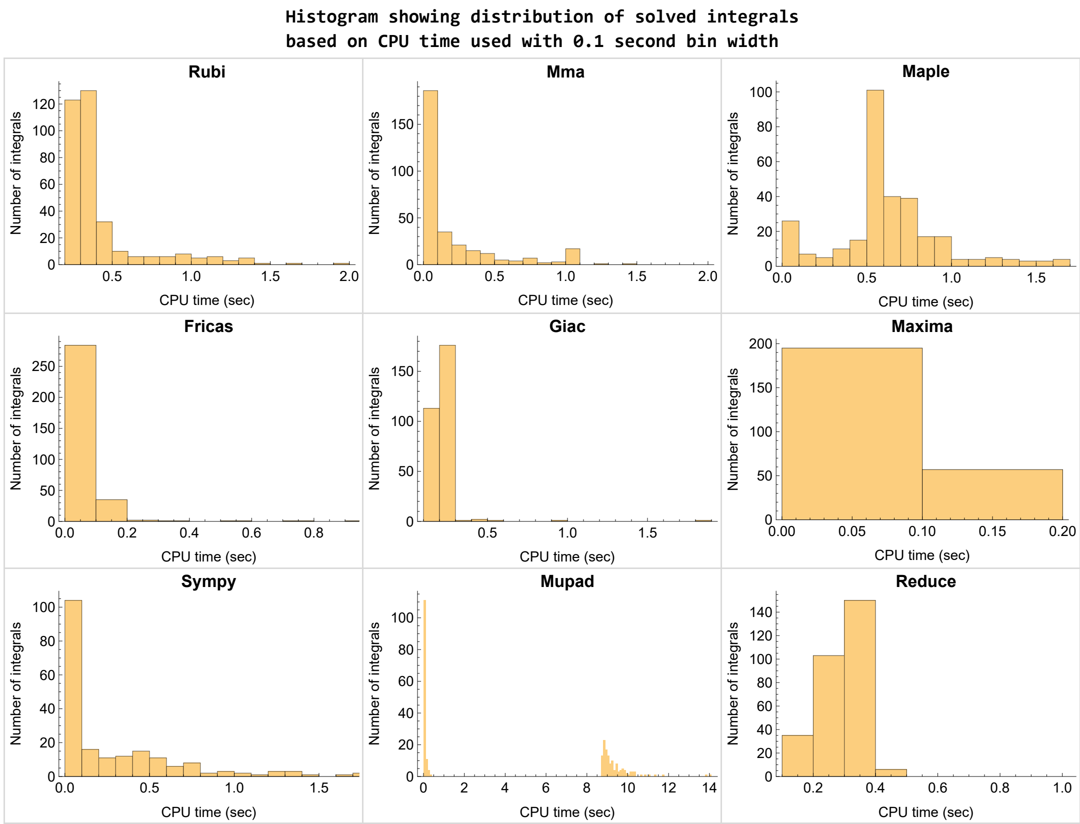


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

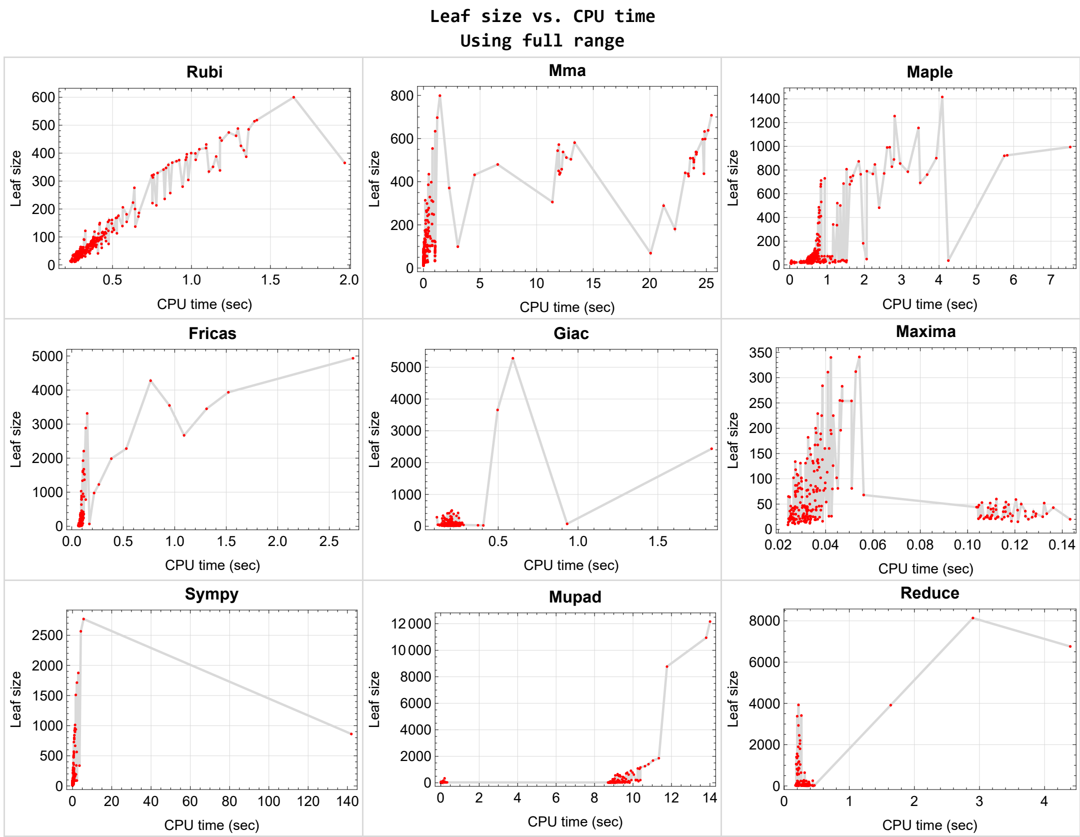


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {340}

Mathematica {172}

Maple {4, 5, 6, 7, 8, 9, 13, 14, 15, 19, 20, 21, 49, 50, 51, 52, 53, 54, 55, 56, 119, 183, 184}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

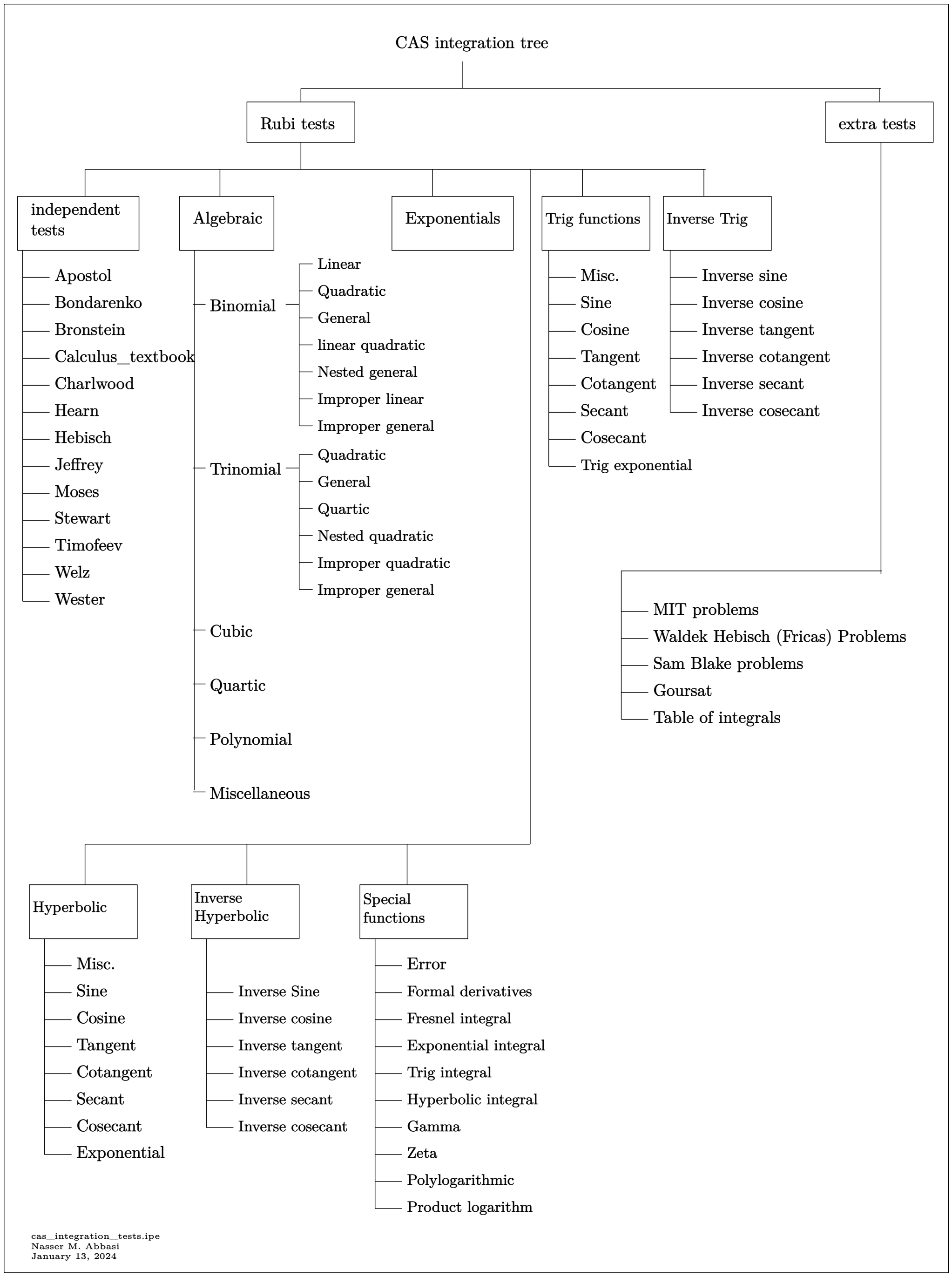
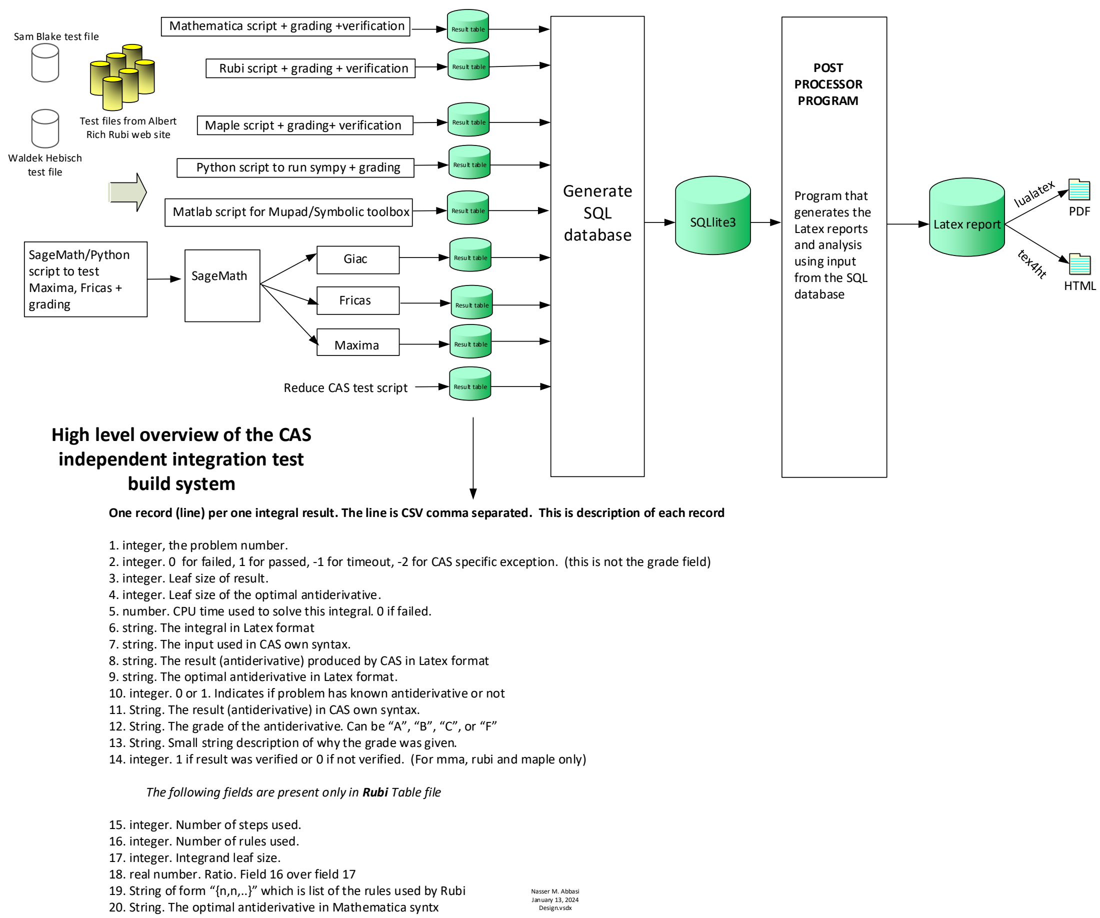


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344 }

B grade { }

C grade { }

F normal fail { 283 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 101, 102, 103, 106, 107, 108, 109, 111, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 339, 342, 343, 344 }

B grade { 17, 51, 52, 65, 82, 95, 104, 105, 110, 112, 132, 133, 171, 257 }

C grade { 172, 295, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 340, 341 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 128, 129, 132, 133, 134, 135, 137, 138, 139, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 249, 250, 251, 253, 254, 255, 262, 263, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 311, 312, 314, 315, 319, 321, 325, 326, 329, 331, 332, 333, 334, 336, 338 }

B grade { 12, 17, 18, 31, 84, 247, 248, 252, 256, 257, 258, 259, 260, 261, 264, 265, 309, 310, 313, 316, 317, 318, 320, 322, 323, 324, 327, 328, 330, 335, 337 }

C grade { 49, 50, 51, 52, 53, 54, 55, 56, 119, 126, 127, 130, 131, 171, 172 }

F normal fail { 136, 140, 141, 173, 174, 175, 176, 177, 178, 339, 340, 341, 342, 343, 344 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 37, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 128, 129, 132, 133, 137, 138, 139, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 294, 295, 296, 298, 299, 300, 301, 302, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338 }

B grade { 12, 17, 18, 31, 33, 34, 35, 36, 38, 39, 40, 41, 77, 110, 113, 163, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 279, 280, 281, 293, 297, 303 }

C grade { 126, 127, 130, 131, 134, 135, 171, 172 }

F normal fail { 136, 140, 141, 173, 174, 175, 176, 177, 178, 339, 340, 341, 342, 343, 344 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 35, 37, 40, 41, 42, 43, 44, 45, 49, 57, 58, 59, 60, 61, 63, 64, 69, 72, 73, 74, 75, 76, 78, 79, 80, 81, 87, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 128, 129, 132, 133, 137, 138, 139, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 304, 305, 306, 308 }

B grade { 12, 17, 18, 31, 33, 36, 38, 39, 46, 47, 48, 50, 51, 52, 53, 54, 55, 56, 62, 65, 66, 67, 68, 70, 71, 77, 82, 83, 84, 85, 86, 88, 89, 110 }

C grade { 126, 127, 130, 131, 134, 135, 295, 303 }

F normal fail { 136, 140, 141, 171, 172, 173, 174, 175, 176, 177, 178, 279, 280, 282, 283, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344 }

F(-1) timedout fail { }

F(-2) exception fail { 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 281, 307 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 121, 122, 123, 124, 125, 128, 129, 132, 133, 137, 138, 139, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 282, 283, 284, 285, 286, 287, 289, 290, 291, 292, 293, 294, 296, 297, 302, 304, 305, 306, 308 }

B grade { 12, 17, 18, 62, 67, 76, 77, 84, 85, 117, 257, 258, 279, 280, 281, 288, 298, 299, 300, 301, 307 }

C grade { 126, 127, 130, 131, 134, 135 }

F normal fail { 136, 140, 141, 171, 172, 173, 174, 175, 176, 177, 178, 295, 303, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 61, 62, 67, 68, 69, 70, 71, 77, 84, 85, 86, 87, 88, 89, 93, 94, 95, 96, 102, 103, 110, 111, 112, 113, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 307, 308 }

C grade { }

F normal fail { }

F(-1) timedout fail { 57, 58, 59, 60, 63, 64, 65, 66, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 90, 91, 92, 97, 98, 99, 100, 101, 104, 105, 106, 107, 108, 109, 114, 115, 116, 117, 136, 140, 141, 171, 172, 173, 174, 175, 176, 177, 178, 263, 306, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 4, 5, 6, 7, 8, 9, 10, 13, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 35, 37, 40, 41, 42, 43, 44, 45, 59, 60, 72, 73, 90, 91, 92, 94, 117, 118, 119, 120, 124, 125, 126, 127, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 284, 285, 286, 287, 290, 291, 292, 293, 294, 296, 297 }

B grade { 3, 11, 12, 17, 18, 31, 33, 36, 38, 39, 46, 47, 48, 49, 50, 57, 58, 61, 62, 74, 75, 76, 77, 93, 138, 139, 227, 228, 229, 230, 231, 232, 233, 235, 236, 237, 238, 239, 240, 244, 245, 246, 247, 248, 249, 250, 251, 255, 256, 257, 258, 259, 260, 261, 262, 263, 282, 283 }

C grade { 295 }

F normal fail { 51, 52, 53, 54, 55, 56, 63, 64, 65, 66, 67, 68, 69, 70, 71, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 121, 122, 123, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 140, 141, 171, 172, 175, 176, 177, 288, 289, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 340, 341, 342, 343, 344 }

F(-1) timedout fail { 173, 174, 178, 234, 241, 242, 243, 252, 253, 254, 264, 265, 279, 280, 281, 339 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308 }

C grade { }

F normal fail { 136, 140, 141, 171, 172, 173, 174, 175, 176, 177, 178, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.279	0.002	0.120	0.031	0.067	0.017	0.201	0.338	0.043

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80	0.80
time (sec)	N/A	0.286	0.002	0.119	0.026	0.073	0.015	0.202	0.325	0.033

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	22	13	20	20	19	20	21	20
N.S.	1	1.00	1.57	0.93	1.43	1.43	1.36	1.43	1.50	1.43
time (sec)	N/A	0.236	0.000	0.027	0.026	0.067	0.016	0.178	0.291	0.002

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	20	20	21	20	20
N.S.	1	1.00	1.00	0.95	0.91	0.91	0.91	0.95	0.91	0.91
time (sec)	N/A	0.277	0.001	0.045	0.033	0.068	0.034	0.181	0.325	0.030

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	24	17	21	24	20
N.S.	1	1.00	1.00	1.05	1.00	1.20	0.85	1.05	1.20	1.00
time (sec)	N/A	0.281	0.001	0.044	0.037	0.067	0.046	0.183	0.333	9.019

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	21	26	22	22	26	23
N.S.	1	1.00	1.00	0.96	0.88	1.08	0.92	0.92	1.08	0.96
time (sec)	N/A	0.284	0.004	0.037	0.029	0.068	0.066	0.190	0.298	0.046

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	26	17	26	23	22	22	24	22	24	22
N.S.	1	0.65	1.00	0.88	0.85	0.85	0.92	0.85	0.92	0.85
time (sec)	N/A	0.241	0.006	0.036	0.030	0.065	0.078	0.203	0.336	0.036

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	24	24	24	26	24	24	24
N.S.	1	1.00	1.00	0.80	0.80	0.80	0.87	0.80	0.80	0.80
time (sec)	N/A	0.280	0.003	0.036	0.026	0.069	0.074	0.206	0.326	0.036

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	24	24	24	26	24	24	24
N.S.	1	1.00	1.00	0.80	0.80	0.80	0.87	0.80	0.80	0.80
time (sec)	N/A	0.280	0.006	0.039	0.037	0.067	0.076	0.195	0.368	0.037

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	53	46	45	45	49	45	46	45
N.S.	1	1.00	1.13	0.98	0.96	0.96	1.04	0.96	0.98	0.96
time (sec)	N/A	0.327	0.002	0.542	0.032	0.067	0.021	0.201	0.396	0.026

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	56	47	46	46	53	46	46	46
N.S.	1	1.00	1.87	1.57	1.53	1.53	1.77	1.53	1.53	1.53
time (sec)	N/A	0.289	0.002	0.543	0.025	0.085	0.022	0.166	0.383	0.025

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	43	53	42	42	42	43	42
N.S.	1	1.00	1.00	3.07	3.79	3.00	3.00	3.00	3.07	3.00
time (sec)	N/A	0.233	0.002	0.498	0.029	0.075	0.022	0.179	0.302	0.002

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	43	42	42	46	43	42	42
N.S.	1	1.00	1.00	0.93	0.91	0.91	1.00	0.93	0.91	0.91
time (sec)	N/A	0.316	0.005	0.546	0.026	0.082	0.042	0.194	0.296	0.028

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	44	43	47	42	44	47	43
N.S.	1	1.00	1.00	0.98	0.96	1.04	0.93	0.98	1.04	0.96
time (sec)	N/A	0.318	0.007	0.514	0.031	0.071	0.050	0.244	0.280	0.030

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	44	42	47	46	43	47	44
N.S.	1	1.00	1.00	0.94	0.89	1.00	0.98	0.91	1.00	0.94
time (sec)	N/A	0.317	0.004	0.511	0.027	0.071	0.073	0.259	0.313	0.042

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	80	69	68	68	78	68	68	68
N.S.	1	1.00	1.70	1.47	1.45	1.45	1.66	1.45	1.45	1.45
time (sec)	N/A	0.332	0.003	0.549	0.056	0.071	0.029	0.234	0.263	0.031

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	78	69	68	68	76	68	68	68
N.S.	1	1.00	2.60	2.30	2.27	2.27	2.53	2.27	2.27	2.27
time (sec)	N/A	0.288	0.003	0.548	0.027	0.071	0.025	0.239	0.296	0.030

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	65	97	64	66	64	65	64
N.S.	1	1.00	1.00	4.64	6.93	4.57	4.71	4.57	4.64	4.57
time (sec)	N/A	0.242	0.001	0.520	0.032	0.081	0.027	0.227	0.299	0.028

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	65	64	64	75	65	64	64
N.S.	1	1.00	1.00	0.88	0.86	0.86	1.01	0.88	0.86	0.86
time (sec)	N/A	0.355	0.004	0.571	0.030	0.072	0.049	0.255	0.303	0.035

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	66	65	70	68	66	70	65
N.S.	1	1.00	1.00	0.96	0.94	1.01	0.99	0.96	1.01	0.94
time (sec)	N/A	0.384	0.008	0.525	0.024	0.087	0.068	0.215	0.304	0.036

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	66	64	69	71	65	69	66
N.S.	1	1.00	1.00	0.93	0.90	0.97	1.00	0.92	0.97	0.93
time (sec)	N/A	0.368	0.009	0.517	0.025	0.079	0.080	0.223	0.349	0.035

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	14	8	12	14	11
N.S.	1	1.00	1.00	1.09	1.00	1.27	0.73	1.09	1.27	1.00
time (sec)	N/A	0.260	0.001	0.042	0.027	0.079	0.026	0.201	0.306	0.030

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	15	15	15	15	15	15	11
N.S.	1	1.00	1.50	1.25	1.25	1.25	1.25	1.25	1.25	0.92
time (sec)	N/A	0.235	0.001	0.032	0.029	0.065	0.030	0.207	0.280	0.027

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	45	47	62	44	48	60	50
N.S.	1	1.00	0.93	0.98	1.02	1.35	0.96	1.04	1.30	1.09
time (sec)	N/A	0.347	0.014	0.602	0.030	0.071	0.084	0.225	0.326	0.050

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	34	36	47	31	34	46	36
N.S.	1	1.00	0.88	1.03	1.09	1.42	0.94	1.03	1.39	1.09
time (sec)	N/A	0.316	0.011	0.600	0.034	0.072	0.073	0.238	0.303	9.207

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	24	26	28	20	24	33	23
N.S.	1	1.00	0.87	1.04	1.13	1.22	0.87	1.04	1.43	1.00
time (sec)	N/A	0.289	0.006	0.625	0.029	0.082	0.056	0.236	0.302	0.040

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	13	13	10	12	12	12
N.S.	1	1.00	1.00	1.08	1.08	1.08	0.83	1.00	1.00	1.00
time (sec)	N/A	0.240	0.002	0.587	0.026	0.067	0.052	0.235	0.323	0.002

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	30	28	39	22	31	44	26
N.S.	1	1.00	0.83	1.03	0.97	1.34	0.76	1.07	1.52	0.90
time (sec)	N/A	0.307	0.008	0.597	0.030	0.075	0.090	0.192	0.292	0.058

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	35	43	45	63	37	45	70	41
N.S.	1	1.00	0.83	1.02	1.07	1.50	0.88	1.07	1.67	0.98
time (sec)	N/A	0.332	0.030	0.614	0.032	0.082	0.119	0.213	0.323	0.066

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	44	47	70	94	70	46	108	68
N.S.	1	1.00	0.76	0.81	1.21	1.62	1.21	0.79	1.86	1.17
time (sec)	N/A	0.372	0.014	0.596	0.033	0.082	0.132	0.204	0.301	9.169

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	31	33	54	54	56	29	37	56
N.S.	1	1.00	1.82	1.94	3.18	3.18	3.29	1.71	2.18	3.29
time (sec)	N/A	0.252	0.011	0.596	0.040	0.078	0.114	0.374	0.309	9.128

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	20	22	43	43	44	18	42	44
N.S.	1	1.00	0.67	0.73	1.43	1.43	1.47	0.60	1.40	1.47
time (sec)	N/A	0.298	0.005	0.593	0.030	0.071	0.115	0.217	0.283	9.702

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	35	35	37	12	34	37
N.S.	1	1.00	1.00	0.93	2.50	2.50	2.64	0.86	2.43	2.64
time (sec)	N/A	0.241	0.002	0.590	0.032	0.069	0.102	0.225	0.257	0.002

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	48	54	73	124	70	54	145	69
N.S.	1	1.00	0.84	0.95	1.28	2.18	1.23	0.95	2.54	1.21
time (sec)	N/A	0.361	0.025	0.605	0.027	0.090	0.168	0.269	0.369	9.446

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	64	69	91	153	90	71	171	85
N.S.	1	1.00	0.91	0.99	1.30	2.19	1.29	1.01	2.44	1.21
time (sec)	N/A	0.391	0.050	0.606	0.029	0.079	0.214	0.225	0.294	0.089

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	42	44	87	87	92	40	86	89
N.S.	1	1.00	1.20	1.26	2.49	2.49	2.63	1.14	2.46	2.54
time (sec)	N/A	0.270	0.014	0.595	0.034	0.074	0.170	0.256	0.308	0.065

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	31	33	76	76	80	29	75	77
N.S.	1	1.00	0.66	0.70	1.62	1.62	1.70	0.62	1.60	1.64
time (sec)	N/A	0.326	0.009	0.594	0.028	0.072	0.159	0.235	0.276	9.444

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	20	22	65	65	68	18	64	66
N.S.	1	1.00	0.67	0.73	2.17	2.17	2.27	0.60	2.13	2.20
time (sec)	N/A	0.296	0.006	0.599	0.032	0.087	0.183	0.241	0.287	9.751

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	57	57	61	12	56	59
N.S.	1	1.00	1.00	0.93	4.07	4.07	4.36	0.86	4.00	4.21
time (sec)	N/A	0.237	0.002	0.595	0.036	0.079	0.172	0.260	0.316	0.048

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	70	78	117	212	117	76	249	113
N.S.	1	1.00	0.82	0.92	1.38	2.49	1.38	0.89	2.93	1.33
time (sec)	N/A	0.409	0.041	0.602	0.037	0.091	0.245	0.251	0.310	0.095

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	86	94	135	241	138	93	275	129
N.S.	1	1.00	0.88	0.96	1.38	2.46	1.41	0.95	2.81	1.32
time (sec)	N/A	0.453	0.059	0.608	0.036	0.115	0.304	0.238	0.301	9.766

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	16	8	13	22	12
N.S.	1	1.00	1.00	1.08	1.00	1.33	0.67	1.08	1.83	1.00
time (sec)	N/A	0.266	0.004	0.473	0.033	0.076	0.028	0.252	0.313	9.820

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	26	21	20	29	19	21	33	20
N.S.	1	1.00	1.18	0.95	0.91	1.32	0.86	0.95	1.50	0.91
time (sec)	N/A	0.282	0.010	0.483	0.027	0.077	0.027	0.229	0.304	0.037

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	23	22	29	19	23	33	22
N.S.	1	1.00	0.96	0.88	0.85	1.12	0.73	0.88	1.27	0.85
time (sec)	N/A	0.289	0.009	0.517	0.025	0.069	0.027	0.239	0.256	0.040

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	30	28	27	34	24	28	38	27
N.S.	1	1.00	1.03	0.97	0.93	1.17	0.83	0.97	1.31	0.93
time (sec)	N/A	0.298	0.010	0.483	0.025	0.074	0.028	0.252	0.323	0.025

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	45	81	30	160	13	192	39	13	117
N.S.	1	0.63	1.14	0.42	2.25	0.18	2.70	0.55	0.18	1.65
time (sec)	N/A	0.326	0.438	0.365	0.041	0.074	1.791	0.238	0.311	10.285

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	45	81	30	131	13	160	39	13	92
N.S.	1	0.63	1.14	0.42	1.85	0.18	2.25	0.55	0.18	1.30
time (sec)	N/A	0.306	0.424	0.312	0.032	0.075	1.305	0.244	0.437	9.612

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	45	81	30	102	13	128	39	13	63
N.S.	1	0.63	1.14	0.42	1.44	0.18	1.80	0.55	0.18	0.89
time (sec)	N/A	0.314	0.281	0.294	0.045	0.080	0.925	0.260	0.448	9.726

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	79	25	75	13	95	39	13	55
N.S.	1	1.00	1.30	0.41	1.23	0.21	1.56	0.64	0.21	0.90
time (sec)	N/A	0.305	0.287	0.276	0.027	0.085	0.763	0.240	0.369	9.817

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	30	19	46	10	56	33	10	19
N.S.	1	1.00	0.94	0.59	1.44	0.31	1.75	1.03	0.31	0.59
time (sec)	N/A	0.250	0.008	0.282	0.038	0.077	0.503	0.218	0.317	9.576

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	62	36	398	20	82	8	0	21	8	98
N.S.	1	0.58	6.42	0.32	1.32	0.13	0.00	0.34	0.13	1.58
time (sec)	N/A	0.298	0.689	0.283	0.029	0.075	0.000	0.239	0.313	9.807

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	65	39	154	23	87	13	0	24	13	103
N.S.	1	0.60	2.37	0.35	1.34	0.20	0.00	0.37	0.20	1.58
time (sec)	N/A	0.304	0.192	0.274	0.027	0.079	0.000	0.407	0.312	9.104

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	31	18	80	11	0	39	13	27
N.S.	1	1.00	0.89	0.51	2.29	0.31	0.00	1.11	0.37	0.77
time (sec)	N/A	0.265	0.170	0.310	0.030	0.079	0.000	0.228	0.321	8.859

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	71	45	33	20	109	13	0	40	13	29
N.S.	1	0.63	0.46	0.28	1.54	0.18	0.00	0.56	0.18	0.41
time (sec)	N/A	0.307	0.131	0.338	0.028	0.074	0.000	0.250	0.319	8.958

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	71	45	33	20	138	13	0	40	13	29
N.S.	1	0.63	0.46	0.28	1.94	0.18	0.00	0.56	0.18	0.41
time (sec)	N/A	0.311	0.140	0.368	0.037	0.088	0.000	0.283	0.298	8.700

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	71	45	33	20	167	13	0	40	13	29
N.S.	1	0.63	0.46	0.28	2.35	0.18	0.00	0.56	0.18	0.41
time (sec)	N/A	0.309	0.155	0.399	0.037	0.084	0.000	0.257	0.334	8.800

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	71	103	52	189	35	250	73	35	0
N.S.	1	0.47	0.68	0.34	1.25	0.23	1.66	0.48	0.23	0.00
time (sec)	N/A	0.359	0.784	0.660	0.042	0.082	0.579	0.219	0.277	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	71	103	52	160	35	218	73	35	0
N.S.	1	0.47	0.68	0.34	1.06	0.23	1.44	0.48	0.23	0.00
time (sec)	N/A	0.352	0.657	0.599	0.040	0.090	0.547	0.256	0.341	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	71	103	52	131	35	184	73	35	0
N.S.	1	0.47	0.68	0.34	0.87	0.23	1.22	0.48	0.23	0.00
time (sec)	N/A	0.353	0.569	0.591	0.037	0.082	0.508	0.245	0.316	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	69	103	52	102	35	153	73	35	0
N.S.	1	0.46	0.68	0.34	0.68	0.23	1.01	0.48	0.23	0.00
time (sec)	N/A	0.360	0.616	0.566	0.030	0.089	0.478	0.233	0.305	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	103	52	75	34	121	72	35	42
N.S.	1	1.00	1.63	0.83	1.19	0.54	1.92	1.14	0.56	0.67
time (sec)	N/A	0.305	0.474	0.559	0.027	0.076	0.431	0.233	0.295	9.147

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	23	20	46	31	294	57	32	32
N.S.	1	1.00	0.68	0.59	1.35	0.91	8.65	1.68	0.94	0.94
time (sec)	N/A	0.255	0.008	0.547	0.037	0.082	0.777	0.214	0.330	0.044

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	63	204	51	134	31	0	56	31	0
N.S.	1	0.44	1.43	0.36	0.94	0.22	0.00	0.39	0.22	0.00
time (sec)	N/A	0.334	0.757	0.559	0.027	0.088	0.000	0.232	0.364	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	62	56	53	140	36	0	57	36	0
N.S.	1	0.44	0.39	0.37	0.99	0.25	0.00	0.40	0.25	0.00
time (sec)	N/A	0.339	1.024	0.574	0.032	0.090	0.000	0.253	0.352	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	61	554	54	200	37	0	56	37	0
N.S.	1	0.43	3.93	0.38	1.42	0.26	0.00	0.40	0.26	0.00
time (sec)	N/A	0.347	0.797	0.602	0.036	0.079	0.000	0.240	0.293	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	65	242	54	229	37	0	59	37	0
N.S.	1	0.45	1.67	0.37	1.58	0.26	0.00	0.41	0.26	0.00
time (sec)	N/A	0.344	0.358	0.586	0.037	0.083	0.000	0.219	0.337	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	37	28	50	138	33	0	73	35	135
N.S.	1	1.06	0.80	1.43	3.94	0.94	0.00	2.09	1.00	3.86
time (sec)	N/A	0.274	0.221	0.633	0.039	0.082	0.000	0.259	0.300	9.526

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	64	55	51	167	35	0	74	35	135
N.S.	1	0.84	0.72	0.67	2.20	0.46	0.00	0.97	0.46	1.78
time (sec)	N/A	0.300	0.250	0.672	0.036	0.085	0.000	0.228	0.343	9.485

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	71	55	51	196	35	0	74	35	135
N.S.	1	0.47	0.36	0.34	1.30	0.23	0.00	0.49	0.23	0.89
time (sec)	N/A	0.344	0.268	0.707	0.042	0.082	0.000	0.250	0.304	9.288

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	71	55	51	225	35	0	74	35	135
N.S.	1	0.47	0.36	0.34	1.49	0.23	0.00	0.49	0.23	0.89
time (sec)	N/A	0.352	0.290	0.753	0.038	0.086	0.000	0.218	0.286	9.273

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	71	55	51	254	35	0	74	35	135
N.S.	1	0.47	0.36	0.34	1.68	0.23	0.00	0.49	0.23	0.89
time (sec)	N/A	0.348	0.298	0.800	0.047	0.078	0.000	0.932	0.205	9.294

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	97	77	74	189	57	275	107	57	0
N.S.	1	0.42	0.33	0.32	0.82	0.25	1.19	0.46	0.25	0.00
time (sec)	N/A	0.420	1.024	0.638	0.038	0.081	0.781	0.239	0.195	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	97	125	74	160	57	245	107	57	0
N.S.	1	0.42	0.54	0.32	0.69	0.25	1.06	0.46	0.25	0.00
time (sec)	N/A	0.406	0.924	0.599	0.033	0.091	0.718	0.274	0.212	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	92	125	74	131	56	211	106	57	0
N.S.	1	0.64	0.87	0.51	0.91	0.39	1.47	0.74	0.40	0.00
time (sec)	N/A	0.401	0.982	0.588	0.035	0.078	0.657	0.274	0.211	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	75	125	74	102	57	180	107	57	0
N.S.	1	0.70	1.17	0.69	0.95	0.53	1.68	1.00	0.53	0.00
time (sec)	N/A	0.372	0.800	0.572	0.038	0.081	0.592	0.259	0.203	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	125	74	75	57	148	107	57	0
N.S.	1	1.00	1.98	1.17	1.19	0.90	2.35	1.70	0.90	0.00
time (sec)	N/A	0.316	0.788	0.560	0.028	0.088	0.552	0.231	0.200	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	23	20	46	53	666	83	54	32
N.S.	1	1.00	0.68	0.59	1.35	1.56	19.59	2.44	1.59	0.94
time (sec)	N/A	0.249	0.008	0.547	0.031	0.076	1.356	0.222	0.221	9.346

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	87	74	73	182	53	0	90	53	0
N.S.	1	0.39	0.33	0.33	0.82	0.24	0.00	0.41	0.24	0.00
time (sec)	N/A	0.375	1.025	0.564	0.033	0.080	0.000	0.245	0.221	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	86	79	76	191	59	0	91	59	0
N.S.	1	0.39	0.36	0.35	0.87	0.27	0.00	0.41	0.27	0.00
time (sec)	N/A	0.370	1.030	0.576	0.036	0.079	0.000	0.266	0.225	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	88	248	76	255	59	0	91	59	0
N.S.	1	0.40	1.12	0.34	1.15	0.27	0.00	0.41	0.27	0.00
time (sec)	N/A	0.383	0.907	0.595	0.046	0.086	0.000	0.224	0.225	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	88	251	76	284	59	0	92	59	0
N.S.	1	0.40	1.13	0.34	1.28	0.27	0.00	0.41	0.27	0.00
time (sec)	N/A	0.378	0.764	0.625	0.039	0.080	0.000	0.240	0.244	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	85	634	76	311	59	0	91	59	0
N.S.	1	0.39	2.89	0.35	1.42	0.27	0.00	0.42	0.27	0.00
time (sec)	N/A	0.374	1.032	0.648	0.041	0.082	0.000	0.227	0.216	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	89	248	76	340	59	0	93	59	0
N.S.	1	0.40	1.11	0.34	1.52	0.26	0.00	0.42	0.26	0.00
time (sec)	N/A	0.383	0.475	0.628	0.042	0.090	0.000	0.224	0.226	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	37	28	72	196	55	0	107	57	207
N.S.	1	1.06	0.80	2.06	5.60	1.57	0.00	3.06	1.63	5.91
time (sec)	N/A	0.274	0.387	0.713	0.046	0.091	0.000	0.211	0.229	9.916

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	64	36	73	225	57	0	108	57	207
N.S.	1	0.84	0.47	0.96	2.96	0.75	0.00	1.42	0.75	2.72
time (sec)	N/A	0.297	0.350	0.778	0.043	0.082	0.000	0.212	0.227	10.383

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	90	77	73	254	57	0	108	57	207
N.S.	1	0.78	0.66	0.63	2.19	0.49	0.00	0.93	0.49	1.78
time (sec)	N/A	0.321	0.485	0.856	0.051	0.081	0.000	0.209	0.244	10.150

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	95	77	73	283	57	0	108	57	207
N.S.	1	0.41	0.34	0.32	1.24	0.25	0.00	0.47	0.25	0.90
time (sec)	N/A	0.378	0.515	0.928	0.047	0.088	0.000	0.220	0.250	10.226

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	97	77	73	312	57	0	108	57	207
N.S.	1	0.42	0.33	0.32	1.35	0.25	0.00	0.47	0.25	0.90
time (sec)	N/A	0.388	0.558	1.012	0.053	0.087	0.000	0.243	0.254	10.143

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	97	77	73	341	57	0	108	57	207
N.S.	1	0.42	0.33	0.32	1.48	0.25	0.00	0.47	0.25	0.90
time (sec)	N/A	0.395	0.453	1.135	0.054	0.080	0.000	0.219	0.314	10.145

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	83	137	67	115	52	209	83	52	0
N.S.	1	0.46	0.75	0.37	0.63	0.29	1.15	0.46	0.29	0.00
time (sec)	N/A	0.384	0.631	0.528	0.035	0.079	1.057	0.214	0.222	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	70	57	56	87	41	172	67	41	0
N.S.	1	0.49	0.40	0.39	0.60	0.28	1.19	0.47	0.28	0.00
time (sec)	N/A	0.361	1.018	0.481	0.028	0.078	0.868	0.201	0.217	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	57	144	44	31	29	141	48	29	0
N.S.	1	0.54	1.36	0.42	0.29	0.27	1.33	0.45	0.27	0.00
time (sec)	N/A	0.337	0.384	0.463	0.026	0.077	0.722	0.205	0.221	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	97	33	37	17	105	31	17	57
N.S.	1	1.00	1.56	0.53	0.60	0.27	1.69	0.50	0.27	0.92
time (sec)	N/A	0.309	0.308	0.448	0.034	0.075	0.600	0.209	0.225	9.426

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	26	25	12	10	24	17	10	19
N.S.	1	1.00	0.74	0.71	0.34	0.29	0.69	0.49	0.29	0.54
time (sec)	N/A	0.259	0.006	0.425	0.029	0.076	0.467	0.203	0.218	9.433

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	44	120	30	38	16	0	28	15	46
N.S.	1	1.07	2.93	0.73	0.93	0.39	0.00	0.68	0.37	1.12
time (sec)	N/A	0.272	0.172	0.444	0.033	0.078	0.000	0.210	0.220	9.475

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	54	151	40	65	26	0	37	26	68
N.S.	1	0.52	1.47	0.39	0.63	0.25	0.00	0.36	0.25	0.66
time (sec)	N/A	0.331	0.211	0.467	0.027	0.075	0.000	0.219	0.206	9.290

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	68	146	58	95	41	0	54	43	0
N.S.	1	0.47	1.01	0.40	0.66	0.28	0.00	0.37	0.30	0.00
time (sec)	N/A	0.362	0.271	0.481	0.030	0.076	0.000	0.227	0.226	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	82	176	69	123	54	0	65	54	0
N.S.	1	0.45	0.96	0.38	0.67	0.29	0.00	0.35	0.29	0.00
time (sec)	N/A	0.373	0.394	0.500	0.037	0.081	0.000	0.217	0.200	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	90	83	98	131	95	0	89	97	0
N.S.	1	0.52	0.48	0.57	0.76	0.55	0.00	0.52	0.56	0.00
time (sec)	N/A	0.403	1.026	0.695	0.029	0.081	0.000	0.205	0.224	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	76	71	86	101	83	0	68	85	0
N.S.	1	0.57	0.53	0.65	0.76	0.62	0.00	0.51	0.64	0.00
time (sec)	N/A	0.380	1.022	0.678	0.030	0.076	0.000	0.207	0.233	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	67	171	61	46	61	0	53	71	0
N.S.	1	0.68	1.73	0.62	0.46	0.62	0.00	0.54	0.72	0.00
time (sec)	N/A	0.362	0.466	0.644	0.031	0.081	0.000	0.234	0.218	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	107	26	39	32	0	26	26	36
N.S.	1	1.00	1.75	0.43	0.64	0.52	0.00	0.43	0.43	0.59
time (sec)	N/A	0.312	0.322	0.743	0.032	0.079	0.000	0.216	0.216	9.227

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	23	20	14	24	0	20	23	30
N.S.	1	1.00	0.68	0.59	0.41	0.71	0.00	0.59	0.68	0.88
time (sec)	N/A	0.254	0.006	0.630	0.032	0.071	0.000	0.249	0.223	9.082

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	69	697	82	78	80	0	67	97	0
N.S.	1	0.55	5.53	0.65	0.62	0.63	0.00	0.53	0.77	0.00
time (sec)	N/A	0.373	1.215	0.680	0.033	0.082	0.000	0.238	0.209	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	83	799	101	107	109	0	84	119	0
N.S.	1	0.50	4.84	0.61	0.65	0.66	0.00	0.51	0.72	0.00
time (sec)	N/A	0.396	1.451	0.667	0.028	0.090	0.000	0.250	0.222	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	102	99	115	135	130	0	97	138	0
N.S.	1	0.49	0.47	0.55	0.65	0.62	0.00	0.46	0.66	0.00
time (sec)	N/A	0.435	1.046	0.685	0.035	0.086	0.000	0.255	0.200	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	122	308	118	126	162	0	111	172	0
N.S.	1	0.50	1.26	0.48	0.52	0.66	0.00	0.45	0.70	0.00
time (sec)	N/A	0.474	0.839	0.753	0.043	0.077	0.000	0.240	0.216	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	108	93	106	113	149	0	90	159	0
N.S.	1	0.53	0.45	0.52	0.55	0.73	0.00	0.44	0.78	0.00
time (sec)	N/A	0.447	1.030	0.729	0.041	0.091	0.000	0.247	0.229	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	99	313	83	92	127	0	73	147	0
N.S.	1	0.58	1.83	0.49	0.54	0.74	0.00	0.43	0.86	0.00
time (sec)	N/A	0.425	0.721	0.665	0.040	0.085	0.000	0.219	0.210	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	37	147	48	102	76	0	48	48	128
N.S.	1	1.06	4.20	1.37	2.91	2.17	0.00	1.37	1.37	3.66
time (sec)	N/A	0.287	0.568	0.670	0.026	0.079	0.000	0.236	0.208	8.968

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	73	187	37	47	65	0	37	64	47
N.S.	1	0.68	1.75	0.35	0.44	0.61	0.00	0.35	0.60	0.44
time (sec)	N/A	0.376	0.443	0.658	0.031	0.079	0.000	0.239	0.372	9.088

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	212	26	39	54	0	26	53	36
N.S.	1	1.00	3.37	0.41	0.62	0.86	0.00	0.41	0.84	0.57
time (sec)	N/A	0.311	0.450	0.644	0.026	0.082	0.000	0.233	0.379	9.075

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	23	20	14	46	0	20	45	30
N.S.	1	1.00	0.68	0.59	0.41	1.35	0.00	0.59	1.32	0.88
time (sec)	N/A	0.253	0.011	0.628	0.033	0.078	0.000	0.230	0.362	9.081

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	97	84	104	118	168	0	89	197	0
N.S.	1	0.50	0.43	0.54	0.61	0.87	0.00	0.46	1.02	0.00
time (sec)	N/A	0.421	1.033	0.645	0.027	0.084	0.000	0.216	0.377	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	113	103	123	148	197	0	106	223	0
N.S.	1	0.48	0.44	0.52	0.63	0.84	0.00	0.45	0.95	0.00
time (sec)	N/A	0.459	1.041	0.661	0.034	0.080	0.000	0.238	0.306	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	131	121	137	178	218	0	121	242	0
N.S.	1	0.47	0.44	0.49	0.64	0.78	0.00	0.44	0.87	0.00
time (sec)	N/A	0.505	1.038	0.674	0.038	0.080	0.000	0.234	0.339	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	27	47	44	31	54	75	30	0
N.S.	1	1.00	0.61	1.07	1.00	0.70	1.23	1.70	0.68	0.00
time (sec)	N/A	0.304	0.187	0.515	0.116	0.069	0.407	0.246	0.300	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	37	37	44	21	41	53	20	24
N.S.	1	1.00	0.84	0.84	1.00	0.48	0.93	1.20	0.45	0.55
time (sec)	N/A	0.295	0.110	0.486	0.104	0.071	0.334	0.227	0.378	0.053

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	30	21	44	11	24	31	10	23
N.S.	1	1.00	0.71	0.50	1.05	0.26	0.57	0.74	0.24	0.55
time (sec)	N/A	0.281	0.071	0.174	0.104	0.071	0.495	0.233	0.354	8.780

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	33	29	21	12	39	25	12	32
N.S.	1	1.00	0.69	0.60	0.44	0.25	0.81	0.52	0.25	0.67
time (sec)	N/A	0.284	0.080	0.375	0.109	0.070	0.458	0.245	0.336	8.849

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	25	13	24	19	0	22	16	26
N.S.	1	1.00	0.57	0.30	0.55	0.43	0.00	0.50	0.36	0.59
time (sec)	N/A	0.280	0.116	0.506	0.105	0.069	0.000	0.233	0.337	8.815

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	27	22	24	29	0	22	28	26
N.S.	1	1.00	0.61	0.50	0.55	0.66	0.00	0.50	0.64	0.59
time (sec)	N/A	0.274	0.163	0.501	0.115	0.067	0.000	0.201	0.384	8.834

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	27	22	24	39	0	22	38	26
N.S.	1	1.00	0.61	0.50	0.55	0.89	0.00	0.50	0.86	0.59
time (sec)	N/A	0.270	0.270	0.506	0.110	0.068	0.000	0.220	0.332	0.072

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	33	29	21	12	39	25	12	32
N.S.	1	1.00	0.69	0.60	0.44	0.25	0.81	0.52	0.25	0.67
time (sec)	N/A	0.273	0.091	0.376	0.105	0.069	0.467	0.202	0.356	8.768

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	33	29	21	12	39	25	12	32
N.S.	1	1.00	0.69	0.60	0.44	0.25	0.81	0.52	0.25	0.67
time (sec)	N/A	0.287	0.083	0.379	0.107	0.076	0.452	0.206	0.265	8.796

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	35	16	21	10	41	29	12	36
N.S.	1	1.00	0.73	0.33	0.44	0.21	0.85	0.60	0.25	0.75
time (sec)	N/A	0.281	1.011	0.516	0.110	0.072	0.462	0.224	0.305	8.879

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	35	16	21	10	44	29	12	36
N.S.	1	1.00	0.73	0.33	0.44	0.21	0.92	0.60	0.25	0.75
time (sec)	N/A	0.281	1.008	0.510	0.112	0.070	0.449	0.225	0.327	8.852

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	37	31	28	24	13	0	21	13	23
N.S.	1	1.37	1.15	1.04	0.89	0.48	0.00	0.78	0.48	0.85
time (sec)	N/A	0.263	0.080	0.398	0.108	0.073	0.000	0.223	0.352	8.996

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	37	31	28	24	13	0	21	13	23
N.S.	1	1.37	1.15	1.04	0.89	0.48	0.00	0.78	0.48	0.85
time (sec)	N/A	0.256	0.109	0.414	0.127	0.076	0.000	0.237	0.349	9.038

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	37	33	25	24	11	0	31	13	27
N.S.	1	1.37	1.22	0.93	0.89	0.41	0.00	1.15	0.48	1.00
time (sec)	N/A	0.261	1.008	0.524	0.113	0.079	0.000	0.225	0.340	9.137

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	37	33	21	24	11	0	31	13	27
N.S.	1	1.37	1.22	0.78	0.89	0.41	0.00	1.15	0.48	1.00
time (sec)	N/A	0.263	1.007	0.516	0.108	0.076	0.000	0.220	0.363	9.083

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	44	120	30	38	16	0	28	15	46
N.S.	1	1.07	2.93	0.73	0.93	0.39	0.00	0.68	0.37	1.12
time (sec)	N/A	0.280	0.038	0.444	0.029	0.087	0.000	0.220	0.316	0.002

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	46	103	34	38	17	0	31	16	47
N.S.	1	1.05	2.34	0.77	0.86	0.39	0.00	0.70	0.36	1.07
time (sec)	N/A	0.293	0.258	0.494	0.032	0.078	0.000	0.248	0.369	8.916

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	C	F	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	49	49	36	38	70	0	41	16	54
N.S.	1	1.04	1.04	0.77	0.81	1.49	0.00	0.87	0.34	1.15
time (sec)	N/A	0.293	0.089	0.761	0.038	0.077	0.000	0.211	0.328	8.812

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	C	F	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	47	49	33	38	66	0	43	15	55
N.S.	1	1.07	1.11	0.75	0.86	1.50	0.00	0.98	0.34	1.25
time (sec)	N/A	0.288	0.089	0.648	0.036	0.171	0.000	0.204	0.303	8.806

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	63	50	0	0	0	0	0	615	0
N.S.	1	1.05	0.83	0.00	0.00	0.00	0.00	0.00	10.25	0.00
time (sec)	N/A	0.314	0.031	0.000	0.000	0.000	0.000	0.000	0.383	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	114	68	87	74	108	0	218	99	133
N.S.	1	1.04	0.62	0.79	0.67	0.98	0.00	1.98	0.90	1.21
time (sec)	N/A	0.441	0.061	0.736	0.035	0.088	0.000	0.199	0.383	8.990

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	73	45	52	50	66	354	122	63	78
N.S.	1	1.06	0.65	0.75	0.72	0.96	5.13	1.77	0.91	1.13
time (sec)	N/A	0.341	0.035	0.737	0.033	0.083	0.610	0.199	0.389	8.890

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	38	25	26	25	32	100	51	34	41
N.S.	1	1.12	0.74	0.76	0.74	0.94	2.94	1.50	1.00	1.21
time (sec)	N/A	0.265	0.006	0.717	0.035	0.091	0.469	0.233	0.313	8.911

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	62	46	0	0	0	0	0	60	0
N.S.	1	1.13	0.84	0.00	0.00	0.00	0.00	0.00	1.09	0.00
time (sec)	N/A	0.309	0.026	0.000	0.000	0.000	0.000	0.000	0.294	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	58	46	0	0	0	0	0	62	0
N.S.	1	1.05	0.84	0.00	0.00	0.00	0.00	0.00	1.13	0.00
time (sec)	N/A	0.298	0.029	0.000	0.000	0.000	0.000	0.000	0.336	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	42	40	27	30	30	34	32	30	26
N.S.	1	1.05	1.00	0.68	0.75	0.75	0.85	0.80	0.75	0.65
time (sec)	N/A	0.305	0.005	0.569	0.032	0.075	0.049	0.181	0.336	8.766

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	35	33	22	25	25	27	27	25	21
N.S.	1	1.06	1.00	0.67	0.76	0.76	0.82	0.82	0.76	0.64
time (sec)	N/A	0.297	0.006	0.537	0.031	0.075	0.049	0.196	0.333	0.035

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	28	26	17	20	20	20	22	20	16
N.S.	1	1.08	1.00	0.65	0.77	0.77	0.77	0.85	0.77	0.62
time (sec)	N/A	0.274	0.004	0.548	0.028	0.082	0.046	0.212	0.360	0.071

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	23	21	14	17	17	17	19	17	13
N.S.	1	1.10	1.00	0.67	0.81	0.81	0.81	0.90	0.81	0.62
time (sec)	N/A	0.277	0.003	0.550	0.032	0.074	0.041	0.187	0.351	0.070

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	23	21	14	17	17	15	19	17	8
N.S.	1	1.10	1.00	0.67	0.81	0.81	0.71	0.90	0.81	0.38
time (sec)	N/A	0.275	0.004	0.544	0.025	0.069	0.046	0.203	0.355	0.088

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	29	27	18	21	21	24	24	21	17
N.S.	1	1.07	1.00	0.67	0.78	0.78	0.89	0.89	0.78	0.63
time (sec)	N/A	0.291	0.006	0.550	0.027	0.076	0.061	0.191	0.339	8.842

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	36	34	27	26	30	31	29	30	22
N.S.	1	1.06	1.00	0.79	0.76	0.88	0.91	0.85	0.88	0.65
time (sec)	N/A	0.303	0.006	0.556	0.025	0.076	0.085	0.200	0.321	0.035

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	43	41	31	31	39	36	34	39	26
N.S.	1	1.05	1.00	0.76	0.76	0.95	0.88	0.83	0.95	0.63
time (sec)	N/A	0.314	0.005	0.553	0.031	0.071	0.073	0.185	0.351	0.037

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	50	48	36	36	44	41	39	44	32
N.S.	1	1.04	1.00	0.75	0.75	0.92	0.85	0.81	0.92	0.67
time (sec)	N/A	0.321	0.005	0.559	0.026	0.073	0.077	0.220	0.341	0.042

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	42	40	27	30	30	34	32	30	26
N.S.	1	1.05	1.00	0.68	0.75	0.75	0.85	0.80	0.75	0.65
time (sec)	N/A	0.309	0.004	0.049	0.026	0.071	0.053	0.199	0.372	0.028

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	35	33	22	25	25	27	27	25	21
N.S.	1	1.06	1.00	0.67	0.76	0.76	0.82	0.82	0.76	0.64
time (sec)	N/A	0.299	0.004	0.049	0.030	0.072	0.057	0.202	0.363	0.029

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	28	26	17	20	20	20	22	20	16
N.S.	1	1.08	1.00	0.65	0.77	0.77	0.77	0.85	0.77	0.62
time (sec)	N/A	0.284	0.003	0.047	0.026	0.071	0.045	0.215	0.342	0.027

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	23	21	14	17	17	17	19	17	13
N.S.	1	1.10	1.00	0.67	0.81	0.81	0.81	0.90	0.81	0.62
time (sec)	N/A	0.276	0.002	0.037	0.031	0.069	0.042	0.213	0.335	0.025

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	23	21	14	17	17	15	19	17	8
N.S.	1	1.10	1.00	0.67	0.81	0.81	0.71	0.90	0.81	0.38
time (sec)	N/A	0.271	0.002	0.033	0.027	0.069	0.042	0.223	0.268	0.017

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	29	27	18	21	21	24	24	21	17
N.S.	1	1.07	1.00	0.67	0.78	0.78	0.89	0.89	0.78	0.63
time (sec)	N/A	0.300	0.004	0.039	0.034	0.076	0.061	0.227	0.339	0.027

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	36	34	27	26	30	31	29	30	22
N.S.	1	1.06	1.00	0.79	0.76	0.88	0.91	0.85	0.88	0.65
time (sec)	N/A	0.291	0.003	0.043	0.032	0.076	0.079	0.210	0.347	0.039

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	43	41	31	31	39	36	34	39	26
N.S.	1	1.05	1.00	0.76	0.76	0.95	0.88	0.83	0.95	0.63
time (sec)	N/A	0.305	0.003	0.044	0.025	0.080	0.077	0.227	0.318	0.038

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	50	48	36	36	44	41	39	44	32
N.S.	1	1.04	1.00	0.75	0.75	0.92	0.85	0.81	0.92	0.67
time (sec)	N/A	0.307	0.003	0.045	0.025	0.087	0.080	0.254	0.264	0.038

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	51	37	47	56	54	50	57	44
N.S.	1	1.00	0.84	0.61	0.77	0.92	0.89	0.82	0.93	0.72
time (sec)	N/A	0.432	0.117	0.568	0.112	0.078	0.047	0.189	0.342	0.097

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	50	30	40	50	53	43	50	37
N.S.	1	1.00	0.96	0.58	0.77	0.96	1.02	0.83	0.96	0.71
time (sec)	N/A	0.360	0.020	0.568	0.110	0.083	0.059	0.196	0.370	0.123

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	49	29	39	48	49	42	49	36
N.S.	1	1.00	0.96	0.57	0.76	0.94	0.96	0.82	0.96	0.71
time (sec)	N/A	0.356	0.015	0.559	0.112	0.087	0.048	0.180	0.362	8.817

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	38	31	17	27	37	39	29	25	16
N.S.	1	2.00	1.63	0.89	1.42	1.95	2.05	1.53	1.32	0.84
time (sec)	N/A	0.290	0.005	0.553	0.108	0.085	0.043	0.174	0.349	8.938

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	64	51	31	41	50	90	45	51	30
N.S.	1	1.21	0.96	0.58	0.77	0.94	1.70	0.85	0.96	0.57
time (sec)	N/A	0.396	0.020	0.569	0.111	0.082	0.254	0.195	0.338	0.087

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	75	58	38	48	62	95	52	64	39
N.S.	1	1.25	0.97	0.63	0.80	1.03	1.58	0.87	1.07	0.65
time (sec)	N/A	0.475	0.018	0.562	0.105	0.092	0.304	0.221	0.326	8.793

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	64	43	53	70	99	57	77	49
N.S.	1	1.00	0.85	0.57	0.71	0.93	1.32	0.76	1.03	0.65
time (sec)	N/A	0.430	0.044	0.565	0.106	0.087	0.306	0.214	0.354	8.853

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	15	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.88	0.76	0.76
time (sec)	N/A	0.281	0.006	0.565	0.027	0.076	0.040	0.205	0.320	8.710

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	12	16	14	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.67	0.89	0.78	0.78
time (sec)	N/A	0.275	0.004	0.578	0.028	0.080	0.044	0.198	0.323	0.045

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	14	14	17	16	14	14
N.S.	1	1.00	1.00	0.75	0.70	0.70	0.85	0.80	0.70	0.70
time (sec)	N/A	0.263	0.003	0.559	0.024	0.082	0.043	0.209	0.347	8.763

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	21	21	19	23	21	21
N.S.	1	1.00	1.00	0.81	0.78	0.78	0.70	0.85	0.78	0.78
time (sec)	N/A	0.306	0.005	0.579	0.024	0.078	0.046	0.217	0.351	0.031

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	41	69	58	0	20	0	0	28	0
N.S.	1	1.86	3.14	2.64	0.00	0.91	0.00	0.00	1.27	0.00
time (sec)	N/A	0.356	20.071	0.612	0.000	0.095	0.000	0.000	0.437	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	41	99	56	0	20	0	0	28	0
N.S.	1	1.86	4.50	2.55	0.00	0.91	0.00	0.00	1.27	0.00
time (sec)	N/A	0.408	3.034	0.632	0.000	0.084	0.000	0.000	0.490	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	78	0	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.384	0.223	0.000	0.000	0.000	0.000	0.000	0.453	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	193	223	153	0	0	0	0	0	0	0
N.S.	1	1.16	0.79	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.630	0.238	0.000	0.000	0.000	0.000	0.000	0.393	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	117	0	0	0	0	0	857	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	5.91	0.00
time (sec)	N/A	0.417	0.122	0.000	0.000	0.000	0.000	0.000	0.374	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	75	0	0	0	0	0	1347	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	14.80	0.00
time (sec)	N/A	0.319	0.013	0.000	0.000	0.000	0.000	0.000	0.358	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	81	0	0	0	0	0	126	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	1.35	0.00
time (sec)	N/A	0.379	0.230	0.000	0.000	0.000	0.000	0.000	0.414	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	100	87	0	0	0	0	0	0	0
N.S.	1	0.98	0.85	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.379	0.364	0.000	0.000	0.000	0.000	0.000	0.326	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.270	0.004	0.134	0.031	0.077	0.018	0.194	0.342	0.030

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.268	0.002	0.126	0.036	0.080	0.017	0.164	0.327	0.029

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.268	0.002	0.130	0.027	0.070	0.016	0.190	0.323	0.029

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	17	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.85	0.80
time (sec)	N/A	0.257	0.000	0.029	0.031	0.075	0.016	0.176	0.333	0.025

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	14	15	14	14
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.88	0.94	0.88	0.88
time (sec)	N/A	0.257	0.002	0.043	0.031	0.077	0.030	0.195	0.303	0.025

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	18	10	15	18	14
N.S.	1	1.00	1.00	1.07	1.00	1.29	0.71	1.07	1.29	1.00
time (sec)	N/A	0.258	0.005	0.046	0.028	0.080	0.038	0.166	0.355	0.027

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	16	21	17	17	21	17
N.S.	1	1.00	1.00	0.95	0.84	1.11	0.89	0.89	1.11	0.89
time (sec)	N/A	0.274	0.003	0.036	0.030	0.076	0.073	0.147	0.330	0.040

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	19	19	20	19	19	18
N.S.	1	1.00	1.00	0.83	0.83	0.83	0.87	0.83	0.83	0.78
time (sec)	N/A	0.268	0.003	0.033	0.035	0.070	0.091	0.190	0.320	0.032

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	19	19	19	20	19	19	19
N.S.	1	1.00	1.00	0.76	0.76	0.76	0.80	0.76	0.76	0.76
time (sec)	N/A	0.269	0.003	0.035	0.034	0.068	0.120	0.186	0.300	0.032

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	19	19	19	20	19	19	19
N.S.	1	1.00	1.00	0.76	0.76	0.76	0.80	0.76	0.76	0.76
time (sec)	N/A	0.272	0.003	0.036	0.027	0.087	0.124	0.159	0.327	0.032

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	48	46	46	45
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.89	0.85	0.85	0.83
time (sec)	N/A	0.340	0.007	0.655	0.031	0.077	0.017	0.188	0.340	0.031

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	46	45	44	44	49	46	46	45
N.S.	1	1.00	0.85	0.83	0.81	0.81	0.91	0.85	0.85	0.83
time (sec)	N/A	0.317	0.013	0.625	0.036	0.072	0.024	0.130	0.331	0.022

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	41	45	40	42	42	44	41
N.S.	1	1.00	1.00	0.89	0.98	0.87	0.91	0.91	0.96	0.89
time (sec)	N/A	0.307	0.008	0.571	0.027	0.078	0.019	0.176	0.342	0.022

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	41	40	40	42	42	41	40
N.S.	1	1.00	0.93	0.89	0.87	0.87	0.91	0.91	0.89	0.87
time (sec)	N/A	0.302	0.023	0.632	0.027	0.073	0.056	0.138	0.317	0.025

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	40	39	45	37	40	47	39
N.S.	1	1.00	1.00	0.98	0.95	1.10	0.90	0.98	1.15	0.95
time (sec)	N/A	0.304	0.033	0.573	0.035	0.072	0.056	0.131	0.343	0.028

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	39	41	39	45	42	40	49	41
N.S.	1	1.00	0.89	0.93	0.89	1.02	0.95	0.91	1.11	0.93
time (sec)	N/A	0.297	0.018	0.572	0.032	0.079	0.116	0.170	0.325	0.038

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	43	41	39	46	42	40	48	39
N.S.	1	1.00	1.02	0.98	0.93	1.10	1.00	0.95	1.14	0.93
time (sec)	N/A	0.303	0.024	0.569	0.036	0.077	0.178	0.139	0.370	9.002

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	47	43	43	46	46	44	48	43
N.S.	1	1.00	0.98	0.90	0.90	0.96	0.96	0.92	1.00	0.90
time (sec)	N/A	0.309	0.017	0.571	0.028	0.081	0.345	0.156	0.369	0.052

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	47	45	44	44	48	46	46	43
N.S.	1	1.00	0.94	0.90	0.88	0.88	0.96	0.92	0.92	0.86
time (sec)	N/A	0.313	0.015	0.559	0.032	0.079	0.464	0.167	0.321	8.843

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	49	45	44	44	48	46	46	45
N.S.	1	1.00	0.91	0.83	0.81	0.81	0.89	0.85	0.85	0.83
time (sec)	N/A	0.316	0.018	0.572	0.032	0.070	0.505	0.122	0.356	8.787

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	49	45	44	44	48	46	46	45
N.S.	1	1.00	0.91	0.83	0.81	0.81	0.89	0.85	0.85	0.83
time (sec)	N/A	0.315	0.014	0.568	0.027	0.071	0.607	0.194	0.319	0.034

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	85	81	81	92	87	87	76
N.S.	1	1.00	1.00	0.96	0.91	0.91	1.03	0.98	0.98	0.85
time (sec)	N/A	0.398	0.010	0.608	0.051	0.070	0.024	0.148	0.339	8.851

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	84	81	81	94	86	87	75
N.S.	1	1.00	1.00	0.94	0.91	0.91	1.06	0.97	0.98	0.84
time (sec)	N/A	0.373	0.010	0.599	0.028	0.067	0.025	0.178	0.312	0.032

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	84	80	80	90	86	87	75
N.S.	1	1.00	1.00	0.98	0.93	0.93	1.05	1.00	1.01	0.87
time (sec)	N/A	0.378	0.012	0.595	0.030	0.070	0.027	0.209	0.354	0.031

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	80	85	77	85	82	85	72
N.S.	1	1.00	1.00	0.99	1.05	0.95	1.05	1.01	1.05	0.89
time (sec)	N/A	0.384	0.008	0.560	0.036	0.087	0.022	0.217	0.344	0.031

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	81	77	77	90	84	83	72
N.S.	1	1.00	1.00	1.00	0.95	0.95	1.11	1.04	1.02	0.89
time (sec)	N/A	0.363	0.015	0.585	0.038	0.073	0.070	0.147	0.348	0.036

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	81	77	83	78	81	89	72
N.S.	1	1.00	1.00	1.05	1.00	1.08	1.01	1.05	1.16	0.94
time (sec)	N/A	0.367	0.027	0.573	0.032	0.077	0.082	0.203	0.329	0.038

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	76	75	82	80	78	90	75
N.S.	1	1.00	1.00	1.00	0.99	1.08	1.05	1.03	1.18	0.99
time (sec)	N/A	0.372	0.024	0.575	0.036	0.070	0.145	0.204	0.387	0.038

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	68	75	78	83	85	79	91	76
N.S.	1	1.00	0.86	0.95	0.99	1.05	1.08	1.00	1.15	0.96
time (sec)	N/A	0.384	0.025	0.583	0.035	0.077	0.274	0.190	0.332	8.919

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	73	77	83	82	78	91	79
N.S.	1	1.00	1.00	0.94	0.99	1.06	1.05	1.00	1.17	1.01
time (sec)	N/A	0.386	0.022	0.577	0.039	0.108	0.546	0.193	0.325	8.863

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	86	72	78	83	87	79	89	79
N.S.	1	1.00	1.12	0.94	1.01	1.08	1.13	1.03	1.16	1.03
time (sec)	N/A	0.376	0.025	0.574	0.036	0.079	0.737	0.208	0.357	8.983

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	87	74	80	83	88	81	89	83
N.S.	1	1.00	1.05	0.89	0.96	1.00	1.06	0.98	1.07	1.00
time (sec)	N/A	0.371	0.023	0.561	0.043	0.074	1.286	0.184	0.309	0.067

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	76	81	81	90	87	87	81
N.S.	1	1.00	1.00	0.89	0.95	0.95	1.06	1.02	1.02	0.95
time (sec)	N/A	0.369	0.020	0.595	0.045	0.071	1.442	0.171	0.292	0.053

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	76	81	81	90	87	87	83
N.S.	1	1.00	1.00	0.87	0.93	0.93	1.03	1.00	1.00	0.95
time (sec)	N/A	0.376	0.035	0.588	0.037	0.071	1.879	0.189	0.337	0.053

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	87	76	81	81	90	87	87	83
N.S.	1	1.00	0.98	0.85	0.91	0.91	1.01	0.98	0.98	0.93
time (sec)	N/A	0.379	0.017	0.587	0.040	0.070	2.252	0.175	0.305	8.859

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	26	26	26	26	26	25	26
N.S.	1	1.00	1.00	0.81	0.81	0.81	0.81	0.81	0.78	0.81
time (sec)	N/A	0.291	0.002	0.576	0.033	0.063	0.016	0.176	0.350	0.030

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	26	26	26	31	26	25	26
N.S.	1	1.00	1.00	0.72	0.72	0.72	0.86	0.72	0.69	0.72
time (sec)	N/A	0.292	0.001	0.575	0.041	0.075	0.021	0.178	0.336	0.020

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	26	26	26	27	26	25	26
N.S.	1	1.00	1.00	0.81	0.81	0.81	0.84	0.81	0.78	0.81
time (sec)	N/A	0.287	0.001	0.569	0.042	0.066	0.020	0.189	0.336	0.019

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	26	26	26	29	26	25	26
N.S.	1	1.00	1.00	0.76	0.76	0.76	0.85	0.76	0.74	0.76
time (sec)	N/A	0.283	0.002	0.578	0.034	0.072	0.020	0.178	0.359	0.019

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	30	28	24	24	24	24	24	23	24
N.S.	1	1.07	1.00	0.86	0.86	0.86	0.86	0.86	0.82	0.86
time (sec)	N/A	0.288	0.002	0.579	0.036	0.071	0.019	0.172	0.373	0.018

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	23	24	24	23	23
N.S.	1	1.00	1.00	0.89	0.85	0.85	0.89	0.89	0.85	0.85
time (sec)	N/A	0.291	0.001	0.589	0.032	0.066	0.028	0.170	0.433	0.022

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	23	25	20	24	25	23
N.S.	1	1.00	1.00	0.96	0.92	1.00	0.80	0.96	1.00	0.92
time (sec)	N/A	0.280	0.001	0.577	0.025	0.070	0.029	0.167	0.464	0.022

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	23	25	22	24	25	22
N.S.	1	1.00	1.00	0.85	0.85	0.93	0.81	0.89	0.93	0.81
time (sec)	N/A	0.277	0.001	0.586	0.025	0.067	0.034	0.193	0.391	0.028

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	21	21	24	19	22	24	21
N.S.	1	1.00	1.00	1.00	1.00	1.14	0.90	1.05	1.14	1.00
time (sec)	N/A	0.279	0.001	0.577	0.031	0.066	0.036	0.170	0.329	8.782

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	23	23	27	22	24	27	22
N.S.	1	1.00	1.00	0.92	0.92	1.08	0.88	0.96	1.08	0.88
time (sec)	N/A	0.281	0.001	0.569	0.026	0.071	0.039	0.219	0.428	8.772

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	25	25	24	25	25	23
N.S.	1	1.00	1.00	0.83	0.83	0.83	0.80	0.83	0.83	0.77
time (sec)	N/A	0.292	0.001	0.570	0.034	0.069	0.041	0.178	0.347	0.023

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	25	25	25	24	25	25	25
N.S.	1	1.00	1.00	0.69	0.69	0.69	0.67	0.69	0.69	0.69
time (sec)	N/A	0.282	0.001	0.562	0.033	0.071	0.046	0.154	0.392	0.022

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	10	9	9
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.91	0.82	0.82
time (sec)	N/A	0.243	0.001	0.039	0.024	0.064	0.024	0.164	0.340	0.022

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	112	128	0	383	498	113	255	151
N.S.	1	1.00	0.95	1.08	0.00	3.25	4.22	0.96	2.16	1.28
time (sec)	N/A	0.497	0.064	0.796	0.000	0.094	0.547	0.165	0.438	8.993

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	84	98	0	297	381	86	180	112
N.S.	1	1.00	0.94	1.10	0.00	3.34	4.28	0.97	2.02	1.26
time (sec)	N/A	0.417	0.070	0.757	0.000	0.083	0.445	0.177	0.380	0.136

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	73	75	0	235	306	67	136	172
N.S.	1	1.00	1.04	1.07	0.00	3.36	4.37	0.96	1.94	2.46
time (sec)	N/A	0.380	0.071	0.758	0.000	0.091	0.318	0.171	0.373	0.139

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	56	0	185	216	55	84	112
N.S.	1	1.00	1.02	1.00	0.00	3.30	3.86	0.98	1.50	2.00
time (sec)	N/A	0.340	0.038	0.709	0.000	0.082	0.151	0.173	0.212	8.825

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	38	35	0	120	124	34	46	46
N.S.	1	1.00	1.12	1.03	0.00	3.53	3.65	1.00	1.35	1.35
time (sec)	N/A	0.259	0.005	0.664	0.000	0.078	0.099	0.142	0.226	8.817

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	63	61	61	0	211	564	62	96	166
N.S.	1	1.02	0.98	0.98	0.00	3.40	9.10	1.00	1.55	2.68
time (sec)	N/A	0.376	0.063	0.736	0.000	0.093	2.141	0.194	0.204	9.192

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	85	77	81	0	269	862	79	158	222
N.S.	1	1.05	0.95	1.00	0.00	3.32	10.64	0.98	1.95	2.74
time (sec)	N/A	0.459	0.067	0.739	0.000	0.095	142.004	0.164	0.206	9.215

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	113	102	128	0	358	0	105	225	447
N.S.	1	1.09	0.98	1.23	0.00	3.44	0.00	1.01	2.16	4.30
time (sec)	N/A	0.528	0.104	0.764	0.000	0.117	0.000	0.193	0.211	9.474

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	186	163	238	0	1029	1012	188	846	382
N.S.	1	1.07	0.94	1.37	0.00	5.91	5.82	1.08	4.86	2.20
time (sec)	N/A	0.667	0.220	0.805	0.000	0.091	1.304	0.190	0.229	9.539

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	154	132	198	0	837	842	161	752	261
N.S.	1	1.11	0.95	1.42	0.00	6.02	6.06	1.16	5.41	1.88
time (sec)	N/A	0.590	0.134	0.794	0.000	0.095	0.951	0.185	0.211	9.432

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	125	109	169	0	635	729	125	547	279
N.S.	1	1.10	0.96	1.48	0.00	5.57	6.39	1.10	4.80	2.45
time (sec)	N/A	0.509	0.097	0.750	0.000	0.093	0.706	0.216	0.201	9.634

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	67	81	97	0	387	280	88	243	135
N.S.	1	0.87	1.05	1.26	0.00	5.03	3.64	1.14	3.16	1.75
time (sec)	N/A	0.349	0.062	0.700	0.000	0.085	0.306	0.208	0.201	8.928

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	69	70	0	338	253	76	223	110
N.S.	1	1.00	1.05	1.06	0.00	5.12	3.83	1.15	3.38	1.67
time (sec)	N/A	0.343	0.045	0.689	0.000	0.083	0.287	0.175	0.280	9.265

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	70	68	0	341	265	76	241	119
N.S.	1	1.00	1.06	1.03	0.00	5.17	4.02	1.15	3.65	1.80
time (sec)	N/A	0.336	0.050	0.701	0.000	0.082	0.293	0.151	0.255	0.140

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	139	107	177	0	781	0	126	644	620
N.S.	1	1.29	0.99	1.64	0.00	7.23	0.00	1.17	5.96	5.74
time (sec)	N/A	0.563	0.173	0.756	0.000	0.136	0.000	0.211	0.314	10.045

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	173	131	205	0	975	0	171	877	775
N.S.	1	1.25	0.95	1.49	0.00	7.07	0.00	1.24	6.36	5.62
time (sec)	N/A	0.664	0.177	0.707	0.000	0.217	0.000	0.191	0.223	9.975

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	213	175	255	0	1226	0	229	1039	914
N.S.	1	1.19	0.98	1.42	0.00	6.85	0.00	1.28	5.80	5.11
time (sec)	N/A	0.778	0.319	0.762	0.000	0.263	0.000	0.166	0.246	9.891

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	304	299	459	0	2207	1875	332	2048	762
N.S.	1	0.97	0.95	1.46	0.00	7.01	5.95	1.05	6.50	2.42
time (sec)	N/A	0.980	0.311	0.801	0.000	0.116	2.966	0.155	0.241	9.698

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	257	260	401	0	1926	1714	282	1896	705
N.S.	1	0.97	0.98	1.51	0.00	7.24	6.44	1.06	7.13	2.65
time (sec)	N/A	0.866	0.272	0.796	0.000	0.109	2.251	0.116	0.215	9.644

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	221	221	357	0	1603	1510	245	1555	620
N.S.	1	1.09	1.09	1.76	0.00	7.90	7.44	1.21	7.66	3.05
time (sec)	N/A	0.753	0.245	0.792	0.000	0.108	1.674	0.149	0.198	9.772

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	117	174	260	0	953	547	202	644	343
N.S.	1	1.01	1.50	2.24	0.00	8.22	4.72	1.74	5.55	2.96
time (sec)	N/A	0.453	0.128	0.734	0.000	0.109	0.739	0.175	0.259	0.202

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	118	126	223	0	872	513	163	604	271
N.S.	1	1.02	1.09	1.92	0.00	7.52	4.42	1.41	5.21	2.34
time (sec)	N/A	0.437	0.143	0.743	0.000	0.118	0.617	0.154	0.199	8.943

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	131	210	0	887	570	154	820	313
N.S.	1	1.00	1.02	1.63	0.00	6.88	4.42	1.19	6.36	2.43
time (sec)	N/A	0.461	0.152	0.772	0.000	0.095	0.669	0.161	0.195	9.048

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	115	102	118	0	788	481	135	575	253
N.S.	1	1.12	0.99	1.15	0.00	7.65	4.67	1.31	5.58	2.46
time (sec)	N/A	0.434	0.074	0.723	0.000	0.094	0.561	0.201	0.224	8.983

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	112	97	116	0	785	474	136	598	285
N.S.	1	1.11	0.96	1.15	0.00	7.77	4.69	1.35	5.92	2.82
time (sec)	N/A	0.411	0.067	0.747	0.000	0.095	0.565	0.233	0.210	9.034

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	236	178	352	0	1985	0	239	1801	1089
N.S.	1	1.28	0.96	1.90	0.00	10.73	0.00	1.29	9.74	5.89
time (sec)	N/A	0.830	0.243	0.759	0.000	0.384	0.000	0.209	0.243	10.248

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	280	221	404	0	2280	0	309	2201	1255
N.S.	1	1.21	0.95	1.74	0.00	9.83	0.00	1.33	9.49	5.41
time (sec)	N/A	0.944	0.355	0.775	0.000	0.530	0.000	0.202	0.243	10.621

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	334	269	462	0	2669	0	410	2450	1404
N.S.	1	1.19	0.96	1.65	0.00	9.53	0.00	1.46	8.75	5.01
time (sec)	N/A	1.111	0.386	0.795	0.000	1.091	0.000	0.193	0.230	10.805

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	441	387	435	730	0	3314	2769	467	3413	1159
N.S.	1	0.88	0.99	1.66	0.00	7.51	6.28	1.06	7.74	2.63
time (sec)	N/A	1.347	0.474	0.938	0.000	0.149	5.662	0.206	0.264	10.385

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	338	386	681	0	2884	2565	418	2933	1055
N.S.	1	0.99	1.13	1.99	0.00	8.43	7.50	1.22	8.58	3.08
time (sec)	N/A	1.181	0.425	0.833	0.000	0.135	4.240	0.249	0.218	10.336

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	169	314	531	0	1675	938	386	1181	656
N.S.	1	1.09	2.03	3.43	0.00	10.81	6.05	2.49	7.62	4.23
time (sec)	N/A	0.528	0.157	0.845	0.000	0.117	1.723	0.221	0.227	9.201

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	168	266	486	0	1594	898	326	1136	563
N.S.	1	1.09	1.73	3.16	0.00	10.35	5.83	2.12	7.38	3.66
time (sec)	N/A	0.522	0.252	0.780	0.000	0.106	1.297	0.239	0.217	9.076

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	181	221	438	0	1625	957	293	1443	588
N.S.	1	0.99	1.21	2.41	0.00	8.93	5.26	1.61	7.93	3.23
time (sec)	N/A	0.589	0.183	0.787	0.000	0.108	1.242	0.208	0.210	9.191

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	200	206	411	0	1522	925	258	1352	512
N.S.	1	1.14	1.18	2.35	0.00	8.70	5.29	1.47	7.73	2.93
time (sec)	N/A	0.642	0.194	0.835	0.000	0.124	1.111	0.254	0.196	9.271

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	176	166	409	0	1533	920	268	1404	574
N.S.	1	1.07	1.01	2.48	0.00	9.29	5.58	1.62	8.51	3.48
time (sec)	N/A	0.537	0.176	0.806	0.000	0.119	1.064	0.164	0.188	9.312

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	161	136	166	0	1364	799	227	1042	482
N.S.	1	1.18	0.99	1.21	0.00	9.96	5.83	1.66	7.61	3.52
time (sec)	N/A	0.497	0.098	0.800	0.000	0.123	0.878	0.211	0.193	9.353

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	160	134	164	0	1337	777	220	1056	0
N.S.	1	1.18	0.99	1.21	0.00	9.83	5.71	1.62	7.76	0.00
time (sec)	N/A	0.479	0.111	0.759	0.000	0.104	0.904	0.178	0.183	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	351	278	661	0	3548	0	405	3378	1680
N.S.	1	1.24	0.99	2.34	0.00	12.58	0.00	1.44	11.98	5.96
time (sec)	N/A	1.136	0.464	0.828	0.000	0.948	0.000	0.182	0.199	11.038

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	410	329	712	0	3934	0	496	3927	1856
N.S.	1	1.23	0.99	2.13	0.00	11.78	0.00	1.49	11.76	5.56
time (sec)	N/A	1.332	0.560	0.839	0.000	1.521	0.000	0.210	0.219	11.343

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	37	36	36	44	36	35	38
N.S.	1	1.00	0.93	0.82	0.80	0.80	0.98	0.80	0.78	0.84
time (sec)	N/A	0.313	0.014	1.524	0.124	0.081	0.047	0.161	0.179	0.047

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	26	25	25	32	25	24	27
N.S.	1	1.00	1.00	0.87	0.83	0.83	1.07	0.83	0.80	0.90
time (sec)	N/A	0.292	0.009	0.941	0.132	0.100	0.047	0.184	0.173	9.318

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	28	27	27	36	27	26	29
N.S.	1	1.00	1.00	0.88	0.84	0.84	1.12	0.84	0.81	0.91
time (sec)	N/A	0.296	0.007	0.826	0.119	0.086	0.059	0.133	0.197	0.032

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	26	26	34	26	25	28
N.S.	1	1.00	1.00	0.87	0.84	0.84	1.10	0.84	0.81	0.90
time (sec)	N/A	0.291	0.005	0.901	0.110	0.078	0.052	0.220	0.195	0.030

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	16	16	26	16	15	16
N.S.	1	1.00	1.00	0.89	0.84	0.84	1.37	0.84	0.79	0.84
time (sec)	N/A	0.242	0.003	0.819	0.119	0.096	0.044	0.173	0.192	0.026

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	29	28	28	37	29	27	28
N.S.	1	1.00	1.00	0.88	0.85	0.85	1.12	0.88	0.82	0.85
time (sec)	N/A	0.314	0.006	0.958	0.110	0.093	0.064	0.191	0.197	0.067

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	36	35	39	41	36	38	35
N.S.	1	1.00	1.00	0.90	0.88	0.98	1.02	0.90	0.95	0.88
time (sec)	N/A	0.328	0.014	4.250	0.115	0.101	0.065	0.206	0.175	8.803

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	27	29	36	27	28	26
N.S.	1	1.00	1.00	0.83	0.90	0.97	1.20	0.90	0.93	0.87
time (sec)	N/A	0.327	0.008	1.151	0.106	0.085	0.053	0.178	0.185	0.034

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	39	38	48	48	39	47	51
N.S.	1	1.00	1.00	0.83	0.81	1.02	1.02	0.83	1.00	1.09
time (sec)	N/A	0.348	0.015	1.215	0.121	0.090	0.075	0.202	0.183	8.782

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	23	15	16	15	26	14	15	34	15
N.S.	1	1.53	1.00	1.07	1.00	1.73	0.93	1.00	2.27	1.00
time (sec)	N/A	0.272	0.010	1.070	0.121	0.094	0.040	0.184	0.178	0.036

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	22	28	20	22	40	24
N.S.	1	1.00	1.08	0.92	0.85	1.08	0.77	0.85	1.54	0.92
time (sec)	N/A	0.268	0.015	0.792	0.110	0.079	0.045	0.154	0.189	8.841

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	28	26	21	20	20	20	20	20	20
N.S.	1	1.08	1.00	0.81	0.77	0.77	0.77	0.77	0.77	0.77
time (sec)	N/A	0.292	0.005	1.219	0.126	0.087	0.039	0.165	0.185	8.815

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	59	49	42	54	71	63	42	118	56
N.S.	1	1.09	0.91	0.78	1.00	1.31	1.17	0.78	2.19	1.04
time (sec)	N/A	0.302	0.034	0.918	0.116	0.086	0.057	0.211	0.186	0.047

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	365	371	335	0	3449	0	3656	3914	8768
N.S.	1	1.01	1.03	0.93	0.00	9.58	0.00	10.16	10.87	24.36
time (sec)	N/A	1.971	2.275	1.264	0.000	1.309	0.000	0.496	1.636	11.766

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	388	432	501	0	4275	0	2436	6759	10944
N.S.	1	0.99	1.10	1.28	0.00	10.91	0.00	6.21	17.24	27.92
time (sec)	N/A	1.156	4.513	1.355	0.000	0.766	0.000	1.837	4.395	13.804

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	426	480	521	0	4933	0	5281	8137	12164
N.S.	1	0.97	1.10	1.19	0.00	11.29	0.00	12.08	18.62	27.84
time (sec)	N/A	1.313	6.561	1.274	0.000	2.731	0.000	0.592	2.899	14.005

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	85	72	49	0	89	340	73	56	86
N.S.	1	1.29	1.09	0.74	0.00	1.35	5.15	1.11	0.85	1.30
time (sec)	N/A	0.365	0.143	2.060	0.000	0.083	1.947	0.211	0.185	0.110

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	A	B	A	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	0	72	49	0	89	337	73	56	86
N.S.	1	0.00	1.09	0.74	0.00	1.35	5.11	1.11	0.85	1.30
time (sec)	N/A	0.000	0.001	1.105	0.000	0.086	3.558	0.238	0.209	0.042

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	19	15	16	15	24	16	15	15
N.S.	1	1.00	0.68	0.54	0.57	0.54	0.86	0.57	0.54	0.54
time (sec)	N/A	0.261	0.015	0.124	0.039	0.074	0.386	0.209	0.209	0.031

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	50	40	52	52	32	32	53	47
N.S.	1	1.00	0.96	0.77	1.00	1.00	0.62	0.62	1.02	0.90
time (sec)	N/A	0.312	0.082	0.895	0.132	0.091	0.415	0.156	0.202	8.955

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	52	42	52	54	34	32	53	47
N.S.	1	1.00	0.93	0.75	0.93	0.96	0.61	0.57	0.95	0.84
time (sec)	N/A	0.315	0.082	0.892	0.110	0.095	0.373	0.206	0.196	0.076

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	55	47	33	49	39	37	40	58	39
N.S.	1	1.10	0.94	0.66	0.98	0.78	0.74	0.80	1.16	0.78
time (sec)	N/A	0.315	0.080	1.466	0.117	0.077	0.309	0.164	0.197	0.081

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	72	76	60	59	92	0	168	69	73
N.S.	1	1.16	1.23	0.97	0.95	1.48	0.00	2.71	1.11	1.18
time (sec)	N/A	0.399	0.144	1.084	0.120	0.084	0.000	0.209	0.215	0.092

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	92	83	71	60	78	0	77	65	77
N.S.	1	1.14	1.02	0.88	0.74	0.96	0.00	0.95	0.80	0.95
time (sec)	N/A	0.431	0.151	1.073	0.112	0.095	0.000	0.238	0.209	0.204

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	43	48	30	31	52	37	48	43	44
N.S.	1	1.08	1.20	0.75	0.78	1.30	0.92	1.20	1.08	1.10
time (sec)	N/A	0.301	0.104	1.352	0.129	0.081	0.288	0.181	0.199	8.947

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	43	60	30	31	55	37	31	29	46
N.S.	1	1.08	1.50	0.75	0.78	1.38	0.92	0.78	0.72	1.15
time (sec)	N/A	0.299	0.126	0.967	0.118	0.091	0.289	0.165	0.191	8.934

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	57	51	45	43	53	49	49	39	44
N.S.	1	1.10	0.98	0.87	0.83	1.02	0.94	0.94	0.75	0.85
time (sec)	N/A	0.299	0.123	0.918	0.109	0.084	0.280	0.204	0.206	0.115

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	51	27	26	55	32	26	24	46
N.S.	1	1.00	1.34	0.71	0.68	1.45	0.84	0.68	0.63	1.21
time (sec)	N/A	0.282	0.093	0.937	0.117	0.084	0.288	0.235	0.218	0.147

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	45	43	53	49	49	43	44
N.S.	1	1.00	0.89	0.83	0.80	0.98	0.91	0.91	0.80	0.81
time (sec)	N/A	0.299	0.080	0.949	0.136	0.093	0.285	0.178	0.207	8.847

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	106	41	31	81	54	0	30	46
N.S.	1	1.00	1.96	0.76	0.57	1.50	1.00	0.00	0.56	0.85
time (sec)	N/A	0.302	0.771	1.426	0.133	0.092	0.358	0.000	0.193	0.108

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	57	51	45	43	53	49	49	39	44
N.S.	1	1.10	0.98	0.87	0.83	1.02	0.94	0.94	0.75	0.85
time (sec)	N/A	0.294	0.109	0.941	0.115	0.084	0.420	0.180	0.199	8.787

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	51	27	26	55	29	26	24	46
N.S.	1	1.00	1.50	0.79	0.76	1.62	0.85	0.76	0.71	1.35
time (sec)	N/A	0.281	0.086	0.951	0.124	0.079	0.372	0.227	0.208	0.146

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	38	29	24	40	0	60	116	27
N.S.	1	1.00	1.23	0.94	0.77	1.29	0.00	1.94	3.74	0.87
time (sec)	N/A	0.270	0.084	1.203	0.113	0.090	0.000	0.199	0.206	0.191

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	29	35	39	0	98	80	27
N.S.	1	1.00	1.00	0.94	1.13	1.26	0.00	3.16	2.58	0.87
time (sec)	N/A	0.260	0.072	0.914	0.128	0.084	0.000	0.220	0.204	8.948

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	36	33	29	35	43	0	61	55	29
N.S.	1	1.16	1.06	0.94	1.13	1.39	0.00	1.97	1.77	0.94
time (sec)	N/A	0.265	0.075	0.858	0.111	0.088	0.000	0.245	0.197	9.133

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	36	27	29	35	43	0	84	43	29
N.S.	1	1.09	0.82	0.88	1.06	1.30	0.00	2.55	1.30	0.88
time (sec)	N/A	0.266	0.079	0.901	0.116	0.084	0.000	0.180	0.202	0.284

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	39	29	26	25	0	30	27	34
N.S.	1	1.00	1.18	0.88	0.79	0.76	0.00	0.91	0.82	1.03
time (sec)	N/A	0.266	0.062	0.940	0.129	0.079	0.000	0.181	0.198	9.051

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	27	29	25	64	0	0	117	34
N.S.	1	1.00	0.82	0.88	0.76	1.94	0.00	0.00	3.55	1.03
time (sec)	N/A	0.265	0.081	1.303	0.113	0.089	0.000	0.000	0.193	0.261

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	36	35	29	20	28	0	30	27	34
N.S.	1	1.16	1.13	0.94	0.65	0.90	0.00	0.97	0.87	1.10
time (sec)	N/A	0.264	0.067	0.888	0.143	0.082	0.000	0.188	0.193	0.332

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	36	30	29	20	40	0	44	23	34
N.S.	1	1.09	0.91	0.88	0.61	1.21	0.00	1.33	0.70	1.03
time (sec)	N/A	0.269	0.097	0.905	0.115	0.081	0.000	0.194	0.203	9.065

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	60	42	40	50	63	0	84	51	0
N.S.	1	1.05	0.74	0.70	0.88	1.11	0.00	1.47	0.89	0.00
time (sec)	N/A	0.340	0.108	1.398	0.123	0.092	0.000	0.195	0.197	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	27	30	24	0	30	0	50	125	27
N.S.	1	1.17	1.30	1.04	0.00	1.30	0.00	2.17	5.43	1.17
time (sec)	N/A	0.278	0.108	1.515	0.000	0.083	0.000	0.189	0.214	0.107

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	33	20	30	29	0	29	20	19
N.S.	1	1.00	1.43	0.87	1.30	1.26	0.00	1.26	0.87	0.83
time (sec)	N/A	0.244	0.087	0.906	0.027	0.087	0.000	0.190	0.210	0.054

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	575	485	597	990	0	267	0	0	319	0
N.S.	1	0.84	1.04	1.72	0.00	0.46	0.00	0.00	0.55	0.00
time (sec)	N/A	1.362	24.674	2.610	0.000	0.093	0.000	0.000	1.633	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	512	414	538	873	0	223	0	0	165	0
N.S.	1	0.81	1.05	1.71	0.00	0.44	0.00	0.00	0.32	0.00
time (sec)	N/A	1.050	24.079	1.848	0.000	0.096	0.000	0.000	1.028	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	478	376	491	764	0	185	0	0	144	0
N.S.	1	0.79	1.03	1.60	0.00	0.39	0.00	0.00	0.30	0.00
time (sec)	N/A	1.027	23.895	1.901	0.000	0.093	0.000	0.000	0.837	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	434	323	437	679	0	166	0	0	89	0
N.S.	1	0.74	1.01	1.56	0.00	0.38	0.00	0.00	0.21	0.00
time (sec)	N/A	0.761	23.388	1.609	0.000	0.103	0.000	0.000	0.707	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	320	426	740	0	161	0	0	93	0
N.S.	1	0.77	1.02	1.78	0.00	0.39	0.00	0.00	0.22	0.00
time (sec)	N/A	0.750	23.431	1.626	0.000	0.099	0.000	0.000	1.369	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	366	441	707	0	183	0	0	105	0
N.S.	1	0.82	0.98	1.58	0.00	0.41	0.00	0.00	0.23	0.00
time (sec)	N/A	0.878	12.083	1.656	0.000	0.080	0.000	0.000	1.330	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	504	431	509	790	0	217	0	0	178	0
N.S.	1	0.86	1.01	1.57	0.00	0.43	0.00	0.00	0.35	0.00
time (sec)	N/A	1.094	23.815	2.066	0.000	0.091	0.000	0.000	0.592	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	619	518	633	1255	0	297	0	0	357	0
N.S.	1	0.84	1.02	2.03	0.00	0.48	0.00	0.00	0.58	0.00
time (sec)	N/A	1.414	24.837	2.810	0.000	0.107	0.000	0.000	1.748	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	569	474	598	992	0	246	0	0	316	0
N.S.	1	0.83	1.05	1.74	0.00	0.43	0.00	0.00	0.56	0.00
time (sec)	N/A	1.236	24.890	2.679	0.000	0.102	0.000	0.000	1.628	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	505	399	527	847	0	216	0	0	164	0
N.S.	1	0.79	1.04	1.68	0.00	0.43	0.00	0.00	0.32	0.00
time (sec)	N/A	1.025	24.116	2.281	0.000	0.100	0.000	0.000	1.333	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	483	376	509	767	0	200	0	0	164	0
N.S.	1	0.78	1.05	1.59	0.00	0.41	0.00	0.00	0.34	0.00
time (sec)	N/A	0.963	23.579	2.226	0.000	0.101	0.000	0.000	2.576	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	471	375	463	828	0	190	0	0	143	0
N.S.	1	0.80	0.98	1.76	0.00	0.40	0.00	0.00	0.30	0.00
time (sec)	N/A	0.922	23.886	2.737	0.000	0.097	0.000	0.000	1.649	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	503	396	512	786	0	214	0	0	259	0
N.S.	1	0.79	1.02	1.56	0.00	0.43	0.00	0.00	0.51	0.00
time (sec)	N/A	0.971	12.627	3.164	0.000	0.104	0.000	0.000	0.913	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	455	538	900	0	251	0	0	227	0
N.S.	1	0.85	1.01	1.69	0.00	0.47	0.00	0.00	0.43	0.00
time (sec)	N/A	1.181	12.331	3.927	0.000	0.103	0.000	0.000	3.655	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	694	600	708	1416	0	331	0	0	544	0
N.S.	1	0.86	1.02	2.04	0.00	0.48	0.00	0.00	0.78	0.00
time (sec)	N/A	1.647	25.450	4.087	0.000	0.108	0.000	0.000	2.176	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	612	514	638	1155	0	290	0	0	359	0
N.S.	1	0.84	1.04	1.89	0.00	0.47	0.00	0.00	0.59	0.00
time (sec)	N/A	1.399	25.153	3.450	0.000	0.107	0.000	0.000	18.790	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	542	445	536	889	0	232	0	0	167	0
N.S.	1	0.82	0.99	1.64	0.00	0.43	0.00	0.00	0.31	0.00
time (sec)	N/A	1.191	24.138	2.792	0.000	0.105	0.000	0.000	1.226	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	488	385	501	771	0	199	0	0	151	0
N.S.	1	0.79	1.03	1.58	0.00	0.41	0.00	0.00	0.31	0.00
time (sec)	N/A	0.968	23.892	2.529	0.000	0.108	0.000	0.000	1.088	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	435	329	437	752	0	168	0	0	33	0
N.S.	1	0.76	1.00	1.73	0.00	0.39	0.00	0.00	0.08	0.00
time (sec)	N/A	0.785	24.784	1.694	0.000	0.104	0.000	0.000	0.713	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	276	289	341	0	133	0	0	31	0
N.S.	1	1.77	1.85	2.19	0.00	0.85	0.00	0.00	0.20	0.00
time (sec)	N/A	0.638	21.222	1.163	0.000	0.086	0.000	0.000	0.474	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	119	181	182	0	54	0	0	38	0
N.S.	1	0.76	1.15	1.16	0.00	0.34	0.00	0.00	0.24	0.00
time (sec)	N/A	0.386	22.218	1.967	0.000	0.087	0.000	0.000	0.438	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	313	306	482	0	164	0	0	59	0
N.S.	1	1.66	1.62	2.55	0.00	0.87	0.00	0.00	0.31	0.00
time (sec)	N/A	0.755	11.390	2.396	0.000	0.080	0.000	0.000	0.872	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	475	371	441	761	0	187	0	0	40	0
N.S.	1	0.78	0.93	1.60	0.00	0.39	0.00	0.00	0.08	0.00
time (sec)	N/A	0.905	23.130	3.682	0.000	0.095	0.000	0.000	0.539	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	591	462	572	920	0	420	0	0	64	0
N.S.	1	0.78	0.97	1.56	0.00	0.71	0.00	0.00	0.11	0.00
time (sec)	N/A	1.282	11.962	5.750	0.000	0.112	0.000	0.000	1.938	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	517	400	504	855	0	347	0	0	475	0
N.S.	1	0.77	0.97	1.65	0.00	0.67	0.00	0.00	0.92	0.00
time (sec)	N/A	0.998	13.025	2.958	0.000	0.115	0.000	0.000	1.943	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	462	347	458	686	0	282	0	0	652	0
N.S.	1	0.75	0.99	1.48	0.00	0.61	0.00	0.00	1.41	0.00
time (sec)	N/A	0.839	12.215	1.441	0.000	0.092	0.000	0.000	0.925	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	341	434	807	0	237	0	0	58	0
N.S.	1	0.78	0.99	1.84	0.00	0.54	0.00	0.00	0.13	0.00
time (sec)	N/A	0.823	12.019	1.522	0.000	0.090	0.000	0.000	0.475	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	478	358	450	692	0	286	0	0	72	0
N.S.	1	0.75	0.94	1.45	0.00	0.60	0.00	0.00	0.15	0.00
time (sec)	N/A	0.854	11.938	3.496	0.000	0.088	0.000	0.000	0.665	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	535	418	544	924	0	378	0	0	443	0
N.S.	1	0.78	1.02	1.73	0.00	0.71	0.00	0.00	0.83	0.00
time (sec)	N/A	1.092	11.854	5.828	0.000	0.102	0.000	0.000	1.204	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	604	488	581	994	0	456	0	0	968	0
N.S.	1	0.81	0.96	1.65	0.00	0.75	0.00	0.00	1.60	0.00
time (sec)	N/A	1.294	13.356	7.517	0.000	0.096	0.000	0.000	1.860	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	160	0	0	0	0	0	0	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.644	0.377	0.000	0.000	0.000	0.000	0.000	0.324	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	206	151	0	0	0	0	0	1399	0
N.S.	1	1.35	0.99	0.00	0.00	0.00	0.00	0.00	9.14	0.00
time (sec)	N/A	0.565	0.362	0.000	0.000	0.000	0.000	0.000	0.271	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	150	151	0	0	0	0	0	390	0
N.S.	1	1.32	1.32	0.00	0.00	0.00	0.00	0.00	3.42	0.00
time (sec)	N/A	0.419	0.243	0.000	0.000	0.000	0.000	0.000	0.292	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	122	126	0	0	0	0	0	243	0
N.S.	1	1.44	1.48	0.00	0.00	0.00	0.00	0.00	2.86	0.00
time (sec)	N/A	0.328	0.113	0.000	0.000	0.000	0.000	0.000	0.272	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	147	0	0	0	0	0	83	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.495	0.251	0.000	0.000	0.000	0.000	0.000	0.290	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	156	154	152	0	0	0	0	0	491	0
N.S.	1	0.99	0.97	0.00	0.00	0.00	0.00	0.00	3.15	0.00
time (sec)	N/A	0.471	0.301	0.000	0.000	0.000	0.000	0.000	0.252	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [323] had the largest ratio of [.6818180000000000035]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	4	1.00	20	0.200
2	A	4	4	1.00	18	0.222
3	A	2	2	1.00	16	0.125
4	A	4	4	1.00	20	0.200
5	A	4	4	1.00	20	0.200
6	A	4	4	1.00	20	0.200
7	A	3	3	0.65	20	0.150
8	A	4	4	1.00	20	0.200
9	A	4	4	1.00	20	0.200
10	A	4	4	1.00	22	0.182
11	A	4	4	1.00	20	0.200
12	A	2	2	1.00	18	0.111
13	A	4	4	1.00	22	0.182
14	A	4	4	1.00	22	0.182
15	A	4	4	1.00	22	0.182
16	A	4	4	1.00	22	0.182
17	A	4	4	1.00	20	0.200
18	A	2	2	1.00	18	0.111
19	A	4	4	1.00	22	0.182
20	A	4	4	1.00	22	0.182
21	A	4	4	1.00	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	3	3	1.00	12	0.250
23	A	2	2	1.00	12	0.167
24	A	4	4	1.00	22	0.182
25	A	4	4	1.00	22	0.182
26	A	4	4	1.00	20	0.200
27	A	2	2	1.00	18	0.111
28	A	4	4	1.00	22	0.182
29	A	4	4	1.00	22	0.182
30	A	4	4	1.00	22	0.182
31	A	3	3	1.00	22	0.136
32	A	4	4	1.00	20	0.200
33	A	2	2	1.00	18	0.111
34	A	4	4	1.00	22	0.182
35	A	4	4	1.00	22	0.182
36	A	4	4	1.00	22	0.182
37	A	4	4	1.00	22	0.182
38	A	4	4	1.00	20	0.200
39	A	2	2	1.00	18	0.111
40	A	4	4	1.00	22	0.182
41	A	4	4	1.00	22	0.182
42	A	3	3	1.00	12	0.250
43	A	3	3	1.00	14	0.214
44	A	3	3	1.00	14	0.214
45	A	3	3	1.00	14	0.214
46	A	4	4	0.63	24	0.167
47	A	4	4	0.63	24	0.167
48	A	4	4	0.63	24	0.167
49	A	3	3	1.00	22	0.136
50	A	2	2	1.00	20	0.100
51	A	4	4	0.58	24	0.167
52	A	4	4	0.60	24	0.167
53	A	3	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	4	4	0.63	24	0.167
55	A	4	4	0.63	24	0.167
56	A	4	4	0.63	24	0.167
57	A	4	4	0.47	24	0.167
58	A	4	4	0.47	24	0.167
59	A	4	4	0.47	24	0.167
60	A	2	2	0.46	24	0.083
61	A	3	3	1.00	22	0.136
62	A	2	2	1.00	20	0.100
63	A	4	4	0.44	24	0.167
64	A	4	4	0.44	24	0.167
65	A	4	4	0.43	24	0.167
66	A	4	4	0.45	24	0.167
67	A	3	3	1.06	24	0.125
68	A	4	4	0.84	24	0.167
69	A	4	4	0.47	24	0.167
70	A	4	4	0.47	24	0.167
71	A	4	4	0.47	24	0.167
72	A	4	4	0.42	24	0.167
73	A	2	2	0.42	24	0.083
74	A	4	4	0.64	24	0.167
75	A	4	4	0.70	24	0.167
76	A	3	3	1.00	22	0.136
77	A	2	2	1.00	20	0.100
78	A	4	4	0.39	24	0.167
79	A	4	4	0.39	24	0.167
80	A	4	4	0.40	24	0.167
81	A	4	4	0.40	24	0.167
82	A	4	4	0.39	24	0.167
83	A	4	4	0.40	24	0.167
84	A	3	3	1.06	24	0.125
85	A	4	4	0.84	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	5	5	0.78	24	0.208
87	A	4	4	0.41	24	0.167
88	A	4	4	0.42	24	0.167
89	A	4	4	0.42	24	0.167
90	A	4	4	0.46	24	0.167
91	A	4	4	0.49	24	0.167
92	A	4	4	0.54	24	0.167
93	A	3	3	1.00	22	0.136
94	A	2	2	1.00	20	0.100
95	A	5	5	1.07	24	0.208
96	A	4	4	0.52	24	0.167
97	A	4	4	0.47	24	0.167
98	A	4	4	0.45	24	0.167
99	A	4	4	0.52	24	0.167
100	A	4	4	0.57	24	0.167
101	A	4	4	0.68	24	0.167
102	A	2	2	1.00	22	0.091
103	A	1	1	1.00	20	0.050
104	A	4	4	0.55	24	0.167
105	A	4	4	0.50	24	0.167
106	A	4	4	0.49	24	0.167
107	A	4	4	0.50	24	0.167
108	A	4	4	0.53	24	0.167
109	A	4	4	0.58	24	0.167
110	A	3	3	1.06	24	0.125
111	A	4	4	0.68	24	0.167
112	A	2	2	1.00	22	0.091
113	A	1	1	1.00	20	0.050
114	A	4	4	0.50	24	0.167
115	A	4	4	0.48	24	0.167
116	A	4	4	0.47	24	0.167
117	A	3	3	1.00	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	3	3	1.00	16	0.188
119	A	3	3	1.00	16	0.188
120	A	3	3	1.00	16	0.188
121	A	2	2	1.00	16	0.125
122	A	2	2	1.00	16	0.125
123	A	2	2	1.00	16	0.125
124	A	3	3	1.00	16	0.188
125	A	3	3	1.00	16	0.188
126	A	3	3	1.00	16	0.188
127	A	3	3	1.00	16	0.188
128	A	5	5	1.37	18	0.278
129	A	5	5	1.37	18	0.278
130	A	5	5	1.37	18	0.278
131	A	5	5	1.37	18	0.278
132	A	5	5	1.07	24	0.208
133	A	6	6	1.05	24	0.250
134	A	5	5	1.04	27	0.185
135	A	6	6	1.07	27	0.222
136	A	3	3	1.05	24	0.125
137	A	3	3	1.04	22	0.136
138	A	3	3	1.06	20	0.150
139	A	2	2	1.12	18	0.111
140	A	2	2	1.13	22	0.091
141	A	2	2	1.05	22	0.091
142	A	2	2	1.05	16	0.125
143	A	2	2	1.06	16	0.125
144	A	2	2	1.08	16	0.125
145	A	2	2	1.10	14	0.143
146	A	2	2	1.10	12	0.167
147	A	2	2	1.07	16	0.125
148	A	2	2	1.06	16	0.125
149	A	2	2	1.05	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	2	2	1.04	16	0.125
151	A	3	3	1.05	20	0.150
152	A	3	3	1.06	20	0.150
153	A	3	3	1.08	20	0.150
154	A	3	3	1.10	20	0.150
155	A	3	3	1.10	18	0.167
156	A	3	3	1.07	16	0.188
157	A	3	3	1.06	20	0.150
158	A	3	3	1.05	20	0.150
159	A	3	3	1.04	20	0.150
160	A	2	2	1.00	14	0.143
161	A	2	2	1.00	14	0.143
162	A	2	2	1.00	12	0.167
163	A	2	2	2.00	10	0.200
164	A	2	2	1.21	14	0.143
165	A	2	2	1.25	14	0.143
166	A	2	2	1.00	14	0.143
167	A	2	2	1.00	12	0.167
168	A	2	2	1.00	14	0.143
169	A	2	2	1.00	12	0.167
170	A	2	2	1.00	14	0.143
171	A	7	6	1.86	20	0.300
172	A	8	7	1.86	19	0.368
173	A	3	2	1.00	27	0.074
174	A	4	4	1.16	25	0.160
175	A	2	2	1.00	23	0.087
176	A	1	1	1.00	21	0.048
177	A	3	2	1.00	25	0.080
178	A	3	2	0.98	25	0.080
179	A	2	2	1.00	14	0.143
180	A	2	2	1.00	14	0.143
181	A	2	2	1.00	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	1	1	1.00	10	0.100
183	A	2	2	1.00	14	0.143
184	A	2	2	1.00	14	0.143
185	A	2	2	1.00	14	0.143
186	A	2	2	1.00	14	0.143
187	A	2	2	1.00	14	0.143
188	A	2	2	1.00	14	0.143
189	A	2	2	1.00	16	0.125
190	A	2	2	1.00	14	0.143
191	A	2	2	1.00	12	0.167
192	A	2	2	1.00	16	0.125
193	A	2	2	1.00	16	0.125
194	A	2	2	1.00	16	0.125
195	A	2	2	1.00	16	0.125
196	A	2	2	1.00	16	0.125
197	A	2	2	1.00	16	0.125
198	A	2	2	1.00	16	0.125
199	A	2	2	1.00	16	0.125
200	A	2	2	1.00	16	0.125
201	A	2	2	1.00	16	0.125
202	A	2	2	1.00	14	0.143
203	A	2	2	1.00	12	0.167
204	A	2	2	1.00	16	0.125
205	A	2	2	1.00	16	0.125
206	A	2	2	1.00	16	0.125
207	A	2	2	1.00	16	0.125
208	A	2	2	1.00	16	0.125
209	A	2	2	1.00	16	0.125
210	A	2	2	1.00	16	0.125
211	A	2	2	1.00	16	0.125
212	A	2	2	1.00	16	0.125
213	A	2	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	2	2	1.00	14	0.143
215	A	2	2	1.00	14	0.143
216	A	2	2	1.00	14	0.143
217	A	2	2	1.00	12	0.167
218	A	2	2	1.07	10	0.200
219	A	2	2	1.00	14	0.143
220	A	2	2	1.00	14	0.143
221	A	2	2	1.00	14	0.143
222	A	2	2	1.00	14	0.143
223	A	2	2	1.00	14	0.143
224	A	2	2	1.00	14	0.143
225	A	2	2	1.00	14	0.143
226	A	2	2	1.00	10	0.200
227	A	2	2	1.00	16	0.125
228	A	2	2	1.00	16	0.125
229	A	2	2	1.00	16	0.125
230	A	5	4	1.00	14	0.286
231	A	3	2	1.00	12	0.167
232	A	7	6	1.02	16	0.375
233	A	4	4	1.05	16	0.250
234	A	4	4	1.09	16	0.250
235	A	3	3	1.07	16	0.188
236	A	4	4	1.11	16	0.250
237	A	3	3	1.10	16	0.188
238	A	4	3	0.87	16	0.188
239	A	4	3	1.00	14	0.214
240	A	4	3	1.00	12	0.250
241	A	4	4	1.29	16	0.250
242	A	4	4	1.25	16	0.250
243	A	4	4	1.19	16	0.250
244	A	6	6	0.97	16	0.375
245	A	6	6	0.97	16	0.375

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	5	5	1.09	16	0.312
247	A	5	4	1.01	16	0.250
248	A	5	4	1.02	16	0.250
249	A	6	5	1.00	16	0.312
250	A	5	4	1.12	14	0.286
251	A	5	4	1.11	12	0.333
252	A	6	6	1.28	16	0.375
253	A	6	6	1.21	16	0.375
254	A	6	6	1.19	16	0.375
255	A	7	7	0.88	16	0.438
256	A	6	6	0.99	16	0.375
257	A	6	5	1.09	16	0.312
258	A	6	5	1.09	16	0.312
259	A	7	6	0.99	16	0.375
260	A	7	6	1.14	16	0.375
261	A	7	6	1.07	16	0.375
262	A	6	5	1.18	14	0.357
263	A	6	5	1.18	12	0.417
264	A	8	8	1.24	16	0.500
265	A	8	8	1.23	16	0.500
266	A	2	2	1.00	12	0.167
267	A	2	2	1.00	12	0.167
268	A	2	2	1.00	12	0.167
269	A	5	4	1.00	10	0.400
270	A	3	2	1.00	8	0.250
271	A	7	6	1.00	12	0.500
272	A	4	4	1.00	12	0.333
273	A	4	4	1.00	12	0.333
274	A	4	4	1.00	12	0.333
275	A	4	3	1.53	14	0.214
276	A	4	3	1.00	12	0.250
277	A	6	5	1.08	12	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	5	4	1.09	10	0.400
279	A	10	9	1.01	18	0.500
280	A	10	9	0.99	18	0.500
281	A	10	9	0.97	18	0.500
282	A	6	5	1.29	19	0.263
283	F	0	0	N/A	0.000	N/A
284	A	2	2	1.00	14	0.143
285	A	5	4	1.00	16	0.250
286	A	5	4	1.00	16	0.250
287	A	5	4	1.10	14	0.286
288	A	8	7	1.16	18	0.389
289	A	7	6	1.14	18	0.333
290	A	4	3	1.08	16	0.188
291	A	4	3	1.08	16	0.188
292	A	4	3	1.10	16	0.188
293	A	4	3	1.00	16	0.188
294	A	4	3	1.00	16	0.188
295	A	4	3	1.00	16	0.188
296	A	4	3	1.10	16	0.188
297	A	4	3	1.00	16	0.188
298	A	3	2	1.00	18	0.111
299	A	3	2	1.00	18	0.111
300	A	3	2	1.16	18	0.111
301	A	3	2	1.09	18	0.111
302	A	3	2	1.00	18	0.111
303	A	3	2	1.00	18	0.111
304	A	3	2	1.16	18	0.111
305	A	3	2	1.09	18	0.111
306	A	6	5	1.05	14	0.357
307	A	1	1	1.17	23	0.043
308	A	1	1	1.00	16	0.062
309	A	13	12	0.84	22	0.545

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	11	10	0.81	22	0.455
311	A	12	11	0.79	22	0.500
312	A	10	9	0.74	22	0.409
313	A	8	7	0.77	22	0.318
314	A	10	9	0.82	22	0.409
315	A	12	11	0.86	22	0.500
316	A	13	12	0.84	22	0.545
317	A	14	13	0.83	22	0.591
318	A	12	11	0.79	22	0.500
319	A	10	9	0.78	22	0.409
320	A	10	9	0.80	22	0.409
321	A	10	9	0.79	22	0.409
322	A	12	11	0.85	22	0.500
323	A	16	15	0.86	22	0.682
324	A	14	13	0.84	22	0.591
325	A	13	12	0.82	22	0.545
326	A	11	10	0.79	22	0.455
327	A	9	8	0.76	22	0.364
328	A	7	6	1.77	22	0.273
329	A	4	3	0.76	22	0.136
330	A	10	9	1.66	22	0.409
331	A	11	10	0.78	22	0.455
332	A	13	12	0.78	22	0.545
333	A	11	10	0.77	22	0.455
334	A	9	8	0.75	22	0.364
335	A	9	8	0.78	22	0.364
336	A	9	8	0.75	22	0.364
337	A	11	10	0.78	22	0.455
338	A	13	12	0.81	22	0.545
339	A	3	2	1.00	18	0.111
340	A	4	4	1.35	16	0.250
341	A	2	2	1.32	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	1	1	1.44	12	0.083
343	A	3	2	1.00	16	0.125
344	A	3	2	0.99	16	0.125

CHAPTER 3

Listing of integrals

3.1 $\int x^2(a^2+2abx+b^2x^2)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 30

$$\int x^2\left(a^2+2abx+b^2x^2\right)\,dx=\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

output
```
1/3*a^2*x^3+1/2*a*b*x^4+1/5*b^2*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x^2\left(a^2+2abx+b^2x^2\right)\,dx=\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

input
```
Integrate[x^2*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a^2+2abx+b^2x^2\right) \, dx$$

$$\Big\downarrow \ 1098$$

$$\frac{\int b^2x^2(a+bx)^2dx}{b^2}$$

$$\Big\downarrow \ 27$$

$$\int x^2(a+bx)^2dx$$

$$\Big\downarrow \ 49$$

$$\int \left(a^2x^2+2abx^3+b^2x^4\right) \, dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

input
```
Int[x^2*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^{3}\left(6 b^{2} x^{2}+15 a b x+10 a^{2}\right)}{30}$	25
default	$\frac{1}{3} a^{2} x^{3}+\frac{1}{2} a b\, x^{4}+\frac{1}{5} b^{2} x^{5}$	25
norman	$\frac{1}{3} a^{2} x^{3}+\frac{1}{2} a b\, x^{4}+\frac{1}{5} b^{2} x^{5}$	25
risch	$\frac{1}{3} a^{2} x^{3}+\frac{1}{2} a b\, x^{4}+\frac{1}{5} b^{2} x^{5}$	25
parallelrisch	$\frac{1}{3} a^{2} x^{3}+\frac{1}{2} a b\, x^{4}+\frac{1}{5} b^{2} x^{5}$	25
orering	$\frac{x^{3}\left(6 b^{2} x^{2}+15 a b x+10 a^{2}\right)\left(b^{2} x^{2}+2 a b x+a^{2}\right)}{30\left(b x+a\right)^{2}}$	48

input
```
int(x^2*(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
1/30*x^3*(6*b^2*x^2+15*a*b*x+10*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2\left(a^2+2abx+b^2x^2\right)\,dx=\frac{1}{5}\,b^2x^5+\frac{1}{2}\,abx^4+\frac{1}{3}\,a^2x^3$$

input
```
integrate(x^2*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2\left(a^2+2abx+b^2x^2\right)\,dx=\frac{a^2x^3}{3}+\frac{abx^4}{2}+\frac{b^2x^5}{5}$$

input
```
integrate(x**2*(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
a**2*x**3/3 + a*b*x**4/2 + b**2*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2\left(a^2+2abx+b^2x^2\right)\,dx=\frac{1}{5}\,b^2x^5+\frac{1}{2}\,abx^4+\frac{1}{3}\,a^2x^3$$

input
```
integrate(x^2*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2\left(a^2+2abx+b^2x^2\right)\,dx=\frac{1}{5}\,b^2x^5+\frac{1}{2}\,abx^4+\frac{1}{3}\,a^2x^3$$

input
```
integrate(x^2*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2\left(a^2+2abx+b^2x^2\right)\,dx=\frac{a^2\,x^3}{3}+\frac{a\,b\,x^4}{2}+\frac{b^2\,x^5}{5}$$

input
```
int(x^2*(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
(a^2*x^3)/3 + (b^2*x^5)/5 + (a*b*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2\left(a^2+2abx+b^2x^2\right)\,dx=\frac{x^3(6b^2x^2+15abx+10a^2)}{30}$$

input
```
int(x^2*(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(x**3*(10*a**2 + 15*a*b*x + 6*b**2*x**2))/30
```

3.2 $\int x(a^2+2abx+b^2x^2)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 30

$$\int x\left(a^2+2abx+b^2x^2\right)\,dx=\frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}$$

output
```
1/2*a^2*x^2+2/3*a*b*x^3+1/4*b^2*x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x\left(a^2+2abx+b^2x^2\right)\,dx=\frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}$$

input
```
Integrate[x*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(a^2*x^2)/2 + (2*a*b*x^3)/3 + (b^2*x^4)/4
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a^2+2abx+b^2x^2\right)\,dx$$

$$\Big\downarrow \text{1098}$$

$$\frac{\int b^2x(a+bx)^2dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int x(a+bx)^2dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^2x+2abx^2+b^2x^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}$$

input
```
Int[x*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(a^2*x^2)/2 + (2*a*b*x^3)/3 + (b^2*x^4)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^{2}\left(3 b^{2} x^{2}+8 a b x+6 a^{2}\right)}{12}$	25
default	$\frac{1}{2} a^{2} x^{2}+\frac{2}{3} a b\, x^{3}+\frac{1}{4} b^{2} x^{4}$	25
norman	$\frac{1}{2} a^{2} x^{2}+\frac{2}{3} a b\, x^{3}+\frac{1}{4} b^{2} x^{4}$	25
risch	$\frac{1}{2} a^{2} x^{2}+\frac{2}{3} a b\, x^{3}+\frac{1}{4} b^{2} x^{4}$	25
parallelrisch	$\frac{1}{2} a^{2} x^{2}+\frac{2}{3} a b\, x^{3}+\frac{1}{4} b^{2} x^{4}$	25
orering	$\frac{x^{2}\left(3 b^{2} x^{2}+8 a b x+6 a^{2}\right)\left(b^{2} x^{2}+2 a b x+a^{2}\right)}{12\left(b x+a\right)^{2}}$	48

input
```
int(x*(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
1/12*x^2*(3*b^2*x^2+8*a*b*x+6*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(a^2 + 2abx + b^2x^2) \, dx = \frac{1}{4}b^2x^4 + \frac{2}{3}abx^3 + \frac{1}{2}a^2x^2$$

input
```
integrate(x*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
1/4*b^2*x^4 + 2/3*a*b*x^3 + 1/2*a^2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x(a^2 + 2abx + b^2x^2) \, dx = \frac{a^2x^2}{2} + \frac{2abx^3}{3} + \frac{b^2x^4}{4}$$

input
```
integrate(x*(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
a**2*x**2/2 + 2*a*b*x**3/3 + b**2*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(a^2 + 2abx + b^2x^2) \, dx = \frac{1}{4}b^2x^4 + \frac{2}{3}abx^3 + \frac{1}{2}a^2x^2$$

input
```
integrate(x*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
1/4*b^2*x^4 + 2/3*a*b*x^3 + 1/2*a^2*x^2
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(a^2 + 2abx + b^2x^2)\, dx = \frac{1}{4}b^2x^4 + \frac{2}{3}abx^3 + \frac{1}{2}a^2x^2$$

input
```
integrate(x*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
1/4*b^2*x^4 + 2/3*a*b*x^3 + 1/2*a^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(a^2 + 2abx + b^2x^2)\, dx = \frac{a^2\,x^2}{2} + \frac{2\,a\,b\,x^3}{3} + \frac{b^2\,x^4}{4}$$

input
```
int(x*(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
(a^2*x^2)/2 + (b^2*x^4)/4 + (2*a*b*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(a^2 + 2abx + b^2x^2)\, dx = \frac{x^2(3b^2x^2 + 8abx + 6a^2)}{12}$$

input
```
int(x*(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(x**2*(6*a**2 + 8*a*b*x + 3*b**2*x**2))/12
```

3.3 $\int \left(a^2+2abx+b^2x^2\right) dx$

Optimal result

Integrand size = 16, antiderivative size = 14

$$\int \left(a^2+2abx+b^2x^2\right) dx=\frac{(a+bx)^3}{3b}$$

output
```
1/3*(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \left(a^2+2abx+b^2x^2\right) dx=a^2x+abx^2+\frac{b^2x^3}{3}$$

input
```
Integrate[a^2 + 2*a*b*x + b^2*x^2,x]
```

output
```
a^2*x + a*b*x^2 + (b^2*x^3)/3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1077, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx+b^2x^2\right) \, dx$$

$$\Big\downarrow \text{1077}$$

$$\frac{\int \left(xb^2+ab\right)^2dx}{b^2}$$

$$\Big\downarrow \text{17}$$

$$\frac{(a+bx)^3}{3b}$$

input
```
Int[a^2 + 2*a*b*x + b^2*x^2,x]
```

output
```
(a + b*x)^3/(3*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1077
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int
[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] &&
 IntegerQ[p]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{(bx+a)^3}{3b}$	13
norman	$\frac{1}{3}b^2x^3+ab\,x^2+a^2x$	21
risch	$\frac{1}{3}b^2x^3+ab\,x^2+a^2x$	21
parallelrisch	$\frac{1}{3}b^2x^3+ab\,x^2+a^2x$	21
parts	$\frac{1}{3}b^2x^3+ab\,x^2+a^2x$	21
gosper	$\frac{x(b^2x^2+3abx+3a^2)}{3}$	22
orering	$\frac{x(b^2x^2+3abx+3a^2)(b^2x^2+2abx+a^2)}{3(bx+a)^2}$	45

input
```
int(b^2*x^2+2*a*b*x+a^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x+a)^3/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \left(a^2+2abx+b^2x^2\right)\,dx=\frac{1}{3}\,b^2x^3+abx^2+a^2x$$

input
```
integrate(b^2*x^2+2*a*b*x+a^2,x, algorithm="fricas")
```

output
```
1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \left(a^2 + 2abx + b^2x^2\right)\, dx = a^2x + abx^2 + \frac{b^2x^3}{3}$$

input
```
integrate(b**2*x**2+2*a*b*x+a**2,x)
```

output
```
a**2*x + a*b*x**2 + b**2*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \left(a^2 + 2abx + b^2x^2\right)\, dx = \frac{1}{3}\, b^2x^3 + abx^2 + a^2x$$

input
```
integrate(b^2*x^2+2*a*b*x+a^2,x, algorithm="maxima")
```

output
```
1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \left(a^2 + 2abx + b^2x^2\right)\, dx = \frac{1}{3}\, b^2x^3 + abx^2 + a^2x$$

input
```
integrate(b^2*x^2+2*a*b*x+a^2,x, algorithm="giac")
```

output
```
1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \left(a^2+2abx+b^2x^2\right)\,dx = a^2\,x+a\,b\,x^2+\frac{b^2\,x^3}{3}$$

input
```
int(a^2 + b^2*x^2 + 2*a*b*x,x)
```

output
```
a^2*x + (b^2*x^3)/3 + a*b*x^2
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \left(a^2+2abx+b^2x^2\right)\,dx = \frac{x(b^2x^2+3abx+3a^2)}{3}$$

input
```
int(b^2*x^2+2*a*b*x+a^2,x)
```

output
```
(x*(3*a**2 + 3*a*b*x + b**2*x**2))/3
```

3.4 $\int \frac{a^2+2abx+b^2x^2}{x} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 22

$$\int \frac{a^2+2abx+b^2x^2}{x} \, dx = 2abx + \frac{b^2x^2}{2} + a^2\log(x)$$

output
```
2*a*b*x+1/2*b^2*x^2+a^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{a^2+2abx+b^2x^2}{x} \, dx = 2abx + \frac{b^2x^2}{2} + a^2\log(x)$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)/x,x]
```

output
```
2*a*b*x + (b^2*x^2)/2 + a^2*Log[x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2+2abx+b^2x^2}{x} \, dx$$

$$\Big\downarrow \text{1098}$$

$$\frac{\int \frac{b^2(a+bx)^2}{x}dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^2}{x}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^2}{x}+2ab+b^2x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2\log(x)+2abx+\frac{b^2x^2}{2}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)/x,x]
```

output
```
2*a*b*x + (b^2*x^2)/2 + a^2*Log[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
default	$2abx+\frac{b^{2}x^{2}}{2}+a^{2}\ln\left(x\right)$	21
norman	$2abx+\frac{b^{2}x^{2}}{2}+a^{2}\ln\left(x\right)$	21
risch	$2abx+\frac{b^{2}x^{2}}{2}+a^{2}\ln\left(x\right)$	21
parallelrisch	$2abx+\frac{b^{2}x^{2}}{2}+a^{2}\ln\left(x\right)$	21

input
```
int((b^2*x^2+2*a*b*x+a^2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*a*b*x+1/2*b^2*x^2+a^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{a^2 + 2abx + b^2x^2}{x}\, dx = \frac{1}{2}\, b^2x^2 + 2\, abx + a^2 \log\left(x\right)$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)/x,x, algorithm="fricas")
```

output
```
1/2*b^2*x^2 + 2*a*b*x + a^2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{a^2 + 2abx + b^2x^2}{x}\, dx = a^2 \log\left(x\right) + 2abx + \frac{b^2x^2}{2}$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)/x,x)
```

output
```
a**2*log(x) + 2*a*b*x + b**2*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{a^2 + 2abx + b^2x^2}{x}\, dx = \frac{1}{2}\, b^2x^2 + 2\, abx + a^2 \log\left(x\right)$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)/x,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2 + 2*a*b*x + a^2*log(x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{a^2 + 2abx + b^2x^2}{x}\,dx = \frac{1}{2}\,b^2x^2 + 2\,abx + a^2\log\left(|x|\right)$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)/x,x, algorithm="giac")
```

output
```
1/2*b^2*x^2 + 2*a*b*x + a^2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{a^2 + 2abx + b^2x^2}{x}\,dx = a^2\,\ln\left(x\right) + \frac{b^2\,x^2}{2} + 2\,a\,b\,x$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)/x,x)
```

output
```
a^2*log(x) + (b^2*x^2)/2 + 2*a*b*x
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{a^2 + 2abx + b^2x^2}{x}\,dx = \log(x)\,a^2 + 2abx + \frac{b^2x^2}{2}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)/x,x)
```

output
```
(2*log(x)*a**2 + 4*a*b*x + b**2*x**2)/2
```

3.5 $\int \frac{a^2+2abx+b^2x^2}{x^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a^2+2abx+b^2x^2}{x^2} dx = -\frac{a^2}{x}+b^2x+2ab\log(x)$$

output
```
-a^2/x+b^2*x+2*a*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a^2+2abx+b^2x^2}{x^2} dx = -\frac{a^2}{x}+b^2x+2ab\log(x)$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)/x^2,x]
```

output
```
-(a^2/x) + b^2*x + 2*a*b*Log[x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2+2abx+b^2x^2}{x^2}\,dx$$

$$\Big\downarrow \text{1098}$$

$$\frac{\int \frac{b^2(a+bx)^2}{x^2}dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^2}{x^2}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^2}{x^2}+\frac{2ab}{x}+b^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{x}+2ab\log(x)+b^2x$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)/x^2,x]
```

output
```
-(a^2/x) + b^2*x + 2*a*b*Log[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
default	$-\frac{a^2}{x} + b^2 x + 2ab \ln(x)$	21
risch	$-\frac{a^2}{x} + b^2 x + 2ab \ln(x)$	21
norman	$\frac{b^2 x^2 - a^2}{x} + 2ab \ln(x)$	25
parallelrisch	$\frac{2ab \ln(x) x + b^2 x^2 - a^2}{x}$	25

input
```
int((b^2*x^2+2*a*b*x+a^2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/x+b^2*x+2*a*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{a^2 + 2abx + b^2x^2}{x^2} \, dx = \frac{b^2x^2 + 2\,abx\log(x) - a^2}{x}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)/x^2,x, algorithm="fricas")
```

output
```
(b^2*x^2 + 2*a*b*x*log(x) - a^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{a^2 + 2abx + b^2x^2}{x^2} \, dx = -\frac{a^2}{x} + 2ab\log(x) + b^2x$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)/x**2,x)
```

output
```
-a**2/x + 2*a*b*log(x) + b**2*x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a^2 + 2abx + b^2x^2}{x^2} \, dx = b^2x + 2\,ab\log(x) - \frac{a^2}{x}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)/x^2,x, algorithm="maxima")
```

output
```
b^2*x + 2*a*b*log(x) - a^2/x
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{a^2+2abx+b^2x^2}{x^2}\,dx = b^2x + 2\,ab\log\left(|x|\right) - \frac{a^2}{x}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)/x^2,x, algorithm="giac")
```

output
```
b^2*x + 2*a*b*log(abs(x)) - a^2/x
```

Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a^2+2abx+b^2x^2}{x^2}\,dx = b^2\,x - \frac{a^2}{x} + 2\,a\,b\,\ln\left(x\right)$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)/x^2,x)
```

output
```
b^2*x - a^2/x + 2*a*b*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{a^2+2abx+b^2x^2}{x^2}\,dx = \frac{2\log(x)\,abx - a^2 + b^2x^2}{x}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)/x^2,x)
```

output
```
(2*log(x)*a*b*x - a**2 + b**2*x**2)/x
```

3.6 $\int \frac{a^2+2abx+b^2x^2}{x^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \frac{a^2+2abx+b^2x^2}{x^3} dx = -\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2 \log(x)$$

output
```
-1/2*a^2/x^2-2*a*b/x+b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{a^2+2abx+b^2x^2}{x^3} dx = -\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2 \log(x)$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)/x^3,x]
```

output
```
-1/2*a^2/x^2 - (2*a*b)/x + b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2+2abx+b^2x^2}{x^3}\,dx$$

$$\Big\downarrow \text{1098}$$

$$\frac{\int \frac{b^2(a+bx)^2}{x^3}dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^2}{x^3}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{2x^2}-\frac{2ab}{x}+b^2\log(x)$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)/x^3,x]
```

output
```
-1/2*a^2/x^2 - (2*a*b)/x + b^2*Log[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{a^2}{2x^2}-\frac{2ab}{x}+b^2\ln\left(x\right)$	23
norman	$\frac{-\frac{1}{2}a^2-2abx}{x^2}+b^2\ln\left(x\right)$	23
risch	$\frac{-\frac{1}{2}a^2-2abx}{x^2}+b^2\ln\left(x\right)$	23
parallelrisch	$\frac{2b^2\ln\left(x\right)x^2-4abx-a^2}{2x^2}$	27

input
```
int((b^2*x^2+2*a*b*x+a^2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/x^2-2*a*b/x+b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{a^2+2abx+b^2x^2}{x^3}\,dx=\frac{2\,b^2x^2\log\left(x\right)-4\,abx-a^2}{2\,x^2}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)/x^3,x, algorithm="fricas")
```

output
```
1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{a^2+2abx+b^2x^2}{x^3}\,dx=b^2\log\left(x\right)+\frac{-a^2-4abx}{2x^2}$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)/x**3,x)
```

output
```
b**2*log(x) + (-a**2 - 4*a*b*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \frac{a^2+2abx+b^2x^2}{x^3}\,dx=b^2\log\left(x\right)-\frac{4\,abx+a^2}{2\,x^2}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)/x^3,x, algorithm="maxima")
```

output
```
b^2*log(x) - 1/2*(4*a*b*x + a^2)/x^2
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{a^2+2abx+b^2x^2}{x^3}\,dx = b^2\log(|x|) - \frac{4\,abx+a^2}{2\,x^2}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)/x^3,x, algorithm="giac")
```

output
```
b^2*log(abs(x)) - 1/2*(4*a*b*x + a^2)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{a^2+2abx+b^2x^2}{x^3}\,dx = b^2\,\ln(x) - \frac{\frac{a^2}{2}+2\,b\,x\,a}{x^2}$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)/x^3,x)
```

output
```
b^2*log(x) - (a^2/2 + 2*a*b*x)/x^2
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{a^2+2abx+b^2x^2}{x^3}\,dx = \frac{2\log(x)\,b^2x^2-a^2-4abx}{2x^2}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)/x^3,x)
```

output
```
(2*log(x)*b**2*x**2 - a**2 - 4*a*b*x)/(2*x**2)
```

3.7 $\int \frac{a^2+2abx+b^2x^2}{x^4} dx$

Optimal result

Integrand size = 20, antiderivative size = 26

$$\int \frac{a^2+2abx+b^2x^2}{x^4} dx = -\frac{a^2}{3x^3} - \frac{ab}{x^2} - \frac{b^2}{x}$$

output
```
-1/3*a^2/x^3-a*b/x^2-b^2/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{a^2+2abx+b^2x^2}{x^4} dx = -\frac{a^2}{3x^3} - \frac{ab}{x^2} - \frac{b^2}{x}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)/x^4,x]
```

output
```
-1/3*a^2/x^3 - (a*b)/x^2 - b^2/x
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1098, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2+2abx+b^2x^2}{x^4} dx$$

$$\Big\downarrow \text{1098}$$

$$\frac{\int \frac{b^2(a+bx)^2}{x^4}dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^2}{x^4}dx$$

$$\Big\downarrow \text{48}$$

$$-\frac{(a+bx)^3}{3ax^3}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)/x^4,x]
```

output
```
-1/3*(a + b*x)^3/(a*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
gosper	$-\frac{3b^2x^2+3abx+a^2}{3x^3}$	23
norman	$\frac{-b^2x^2-abx-\frac{1}{3}a^2}{x^3}$	24
risch	$\frac{-b^2x^2-abx-\frac{1}{3}a^2}{x^3}$	24
default	$-\frac{a^2}{3x^3}-\frac{ab}{x^2}-\frac{b^2}{x}$	25
parallelrisch	$\frac{-3b^2x^2-3abx-a^2}{3x^3}$	25
orering	$-\frac{(3b^2x^2+3abx+a^2)(b^2x^2+2abx+a^2)}{3x^3(bx+a)^2}$	46

input
```
int((b^2*x^2+2*a*b*x+a^2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3*b^2*x^2+3*a*b*x+a^2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{a^2+2abx+b^2x^2}{x^4}\,dx=-\frac{3\,b^2x^2+3\,abx+a^2}{3\,x^3}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)/x^4,x, algorithm="fricas")
```

output
```
-1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{a^2 + 2abx + b^2x^2}{x^4}\,dx = \frac{-a^2 - 3abx - 3b^2x^2}{3x^3}$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)/x**4,x)
```

output
```
(-a**2 - 3*a*b*x - 3*b**2*x**2)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{a^2 + 2abx + b^2x^2}{x^4}\,dx = -\frac{3\,b^2x^2 + 3\,abx + a^2}{3\,x^3}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)/x^4,x, algorithm="maxima")
```

output
```
-1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{a^2 + 2abx + b^2x^2}{x^4}\,dx = -\frac{3\,b^2x^2 + 3\,abx + a^2}{3\,x^3}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)/x^4,x, algorithm="giac")
```

output
```
-1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{a^2 + 2abx + b^2x^2}{x^4}\,dx = -\frac{\frac{a^2}{3} + a\,b\,x + b^2\,x^2}{x^3}$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)/x^4,x)
```

output
```
-(a^2/3 + b^2*x^2 + a*b*x)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{a^2 + 2abx + b^2x^2}{x^4}\,dx = \frac{-3b^2x^2 - 3abx - a^2}{3x^3}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)/x^4,x)
```

output
```
( - a**2 - 3*a*b*x - 3*b**2*x**2)/(3*x**3)
```

3.8 $\int \frac{a^2+2abx+b^2x^2}{x^5} dx$

Optimal result

Integrand size = 20, antiderivative size = 30

$$\int \frac{a^2+2abx+b^2x^2}{x^5} dx = -\frac{a^2}{4x^4} - \frac{2ab}{3x^3} - \frac{b^2}{2x^2}$$

output
```
-1/4*a^2/x^4-2/3*a*b/x^3-1/2*b^2/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{a^2+2abx+b^2x^2}{x^5} dx = -\frac{a^2}{4x^4} - \frac{2ab}{3x^3} - \frac{b^2}{2x^2}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)/x^5,x]
```

output
```
-1/4*a^2/x^4 - (2*a*b)/(3*x^3) - b^2/(2*x^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2+2abx+b^2x^2}{x^5}\,dx$$

$$\Big\downarrow \text{1098}$$

$$\frac{\int \frac{b^2(a+bx)^2}{x^5}dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^2}{x^5}dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{4x^4}-\frac{2ab}{3x^3}-\frac{b^2}{2x^2}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)/x^5,x]
```

output
```
-1/4*a^2/x^4 - (2*a*b)/(3*x^3) - b^2/(2*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result	size
norman	$\frac{-\frac{1}{2} b^{2} x^{2}-\frac{2}{3} a b x -\frac{1}{4} a^{2}}{x^{4}}$	24
risch	$\frac{-\frac{1}{2} b^{2} x^{2}-\frac{2}{3} a b x -\frac{1}{4} a^{2}}{x^{4}}$	24
gosper	$-\frac{6 b^{2} x^{2}+8 a b x +3 a^{2}}{12 x^{4}}$	25
default	$-\frac{a^{2}}{4 x^{4}}-\frac{2 a b}{3 x^{3}}-\frac{b^{2}}{2 x^{2}}$	25
parallelrisch	$\frac{-6 b^{2} x^{2}-8 a b x -3 a^{2}}{12 x^{4}}$	25
orering	$-\frac{\left(6 b^{2} x^{2}+8 a b x +3 a^{2}\right) \left(b^{2} x^{2}+2 a b x +a^{2}\right)}{12 x^{4} \left(b x +a \right)^{2}}$	48

input
```
int((b^2*x^2+2*a*b*x+a^2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*b^2*x^2-2/3*a*b*x-1/4*a^2)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{a^2+2abx+b^2x^2}{x^5}\,dx = -\frac{6\,b^2x^2+8\,abx+3\,a^2}{12\,x^4}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)/x^5,x, algorithm="fricas")
```

output
```
-1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{a^2+2abx+b^2x^2}{x^5}\,dx = \frac{-3a^2-8abx-6b^2x^2}{12x^4}$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)/x**5,x)
```

output
```
(-3*a**2 - 8*a*b*x - 6*b**2*x**2)/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{a^2+2abx+b^2x^2}{x^5}\,dx = -\frac{6\,b^2x^2+8\,abx+3\,a^2}{12\,x^4}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)/x^5,x, algorithm="maxima")
```

output
```
-1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{a^2 + 2abx + b^2x^2}{x^5}\,dx = -\frac{6\,b^2x^2 + 8\,abx + 3\,a^2}{12\,x^4}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)/x^5,x, algorithm="giac")
```

output
```
-1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{a^2 + 2abx + b^2x^2}{x^5}\,dx = -\frac{\frac{a^2}{4} + \frac{2\,a\,b\,x}{3} + \frac{b^2\,x^2}{2}}{x^4}$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)/x^5,x)
```

output
```
-(a^2/4 + (b^2*x^2)/2 + (2*a*b*x)/3)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{a^2 + 2abx + b^2x^2}{x^5}\,dx = \frac{-6b^2x^2 - 8abx - 3a^2}{12x^4}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)/x^5,x)
```

output
```
( - 3*a**2 - 8*a*b*x - 6*b**2*x**2)/(12*x**4)
```

3.9 $\int \frac{a^2+2abx+b^2x^2}{x^6} dx$

Optimal result

Integrand size = 20, antiderivative size = 30

$$\int \frac{a^2+2abx+b^2x^2}{x^6} dx = -\frac{a^2}{5x^5} - \frac{ab}{2x^4} - \frac{b^2}{3x^3}$$

output
```
-1/5*a^2/x^5-1/2*a*b/x^4-1/3*b^2/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{a^2+2abx+b^2x^2}{x^6} dx = -\frac{a^2}{5x^5} - \frac{ab}{2x^4} - \frac{b^2}{3x^3}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)/x^6,x]
```

output
```
-1/5*a^2/x^5 - (a*b)/(2*x^4) - b^2/(3*x^3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2+2abx+b^2x^2}{x^6}\,dx$$

$$\Big\downarrow \text{1098}$$

$$\frac{\int \frac{b^2(a+bx)^2}{x^6}dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^2}{x^6}dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^2}{x^6}+\frac{2ab}{x^5}+\frac{b^2}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{5x^5}-\frac{ab}{2x^4}-\frac{b^2}{3x^3}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)/x^6,x]
```

output
```
-1/5*a^2/x^5 - (a*b)/(2*x^4) - b^2/(3*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result	size
norman	$\frac{-\frac{1}{3}b^2x^2-\frac{1}{2}abx-\frac{1}{5}a^2}{x^5}$	24
risch	$\frac{-\frac{1}{3}b^2x^2-\frac{1}{2}abx-\frac{1}{5}a^2}{x^5}$	24
gosper	$-\frac{10b^2x^2+15abx+6a^2}{30x^5}$	25
default	$-\frac{a^2}{5x^5}-\frac{ab}{2x^4}-\frac{b^2}{3x^3}$	25
parallelrisch	$\frac{-10b^2x^2-15abx-6a^2}{30x^5}$	25
orering	$-\frac{(10b^2x^2+15abx+6a^2)(b^2x^2+2abx+a^2)}{30x^5(bx+a)^2}$	48

input
```
int((b^2*x^2+2*a*b*x+a^2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*b^2*x^2-1/2*a*b*x-1/5*a^2)/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{a^2+2abx+b^2x^2}{x^6}\,dx = -\frac{10\,b^2x^2+15\,abx+6\,a^2}{30\,x^5}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)/x^6,x, algorithm="fricas")
```

output
```
-1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{a^2+2abx+b^2x^2}{x^6}\,dx = \frac{-6a^2-15abx-10b^2x^2}{30x^5}$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)/x**6,x)
```

output
```
(-6*a**2 - 15*a*b*x - 10*b**2*x**2)/(30*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{a^2+2abx+b^2x^2}{x^6}\,dx = -\frac{10\,b^2x^2+15\,abx+6\,a^2}{30\,x^5}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)/x^6,x, algorithm="maxima")
```

output
```
-1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)/x^5
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{a^2 + 2abx + b^2x^2}{x^6}\,dx = -\frac{10\,b^2x^2 + 15\,abx + 6\,a^2}{30\,x^5}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)/x^6,x, algorithm="giac")
```

output
```
-1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{a^2 + 2abx + b^2x^2}{x^6}\,dx = -\frac{\frac{a^2}{5} + \frac{a\,b\,x}{2} + \frac{b^2\,x^2}{3}}{x^5}$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)/x^6,x)
```

output
```
-(a^2/5 + (b^2*x^2)/3 + (a*b*x)/2)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{a^2 + 2abx + b^2x^2}{x^6}\,dx = \frac{-10b^2x^2 - 15abx - 6a^2}{30x^5}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)/x^6,x)
```

output
```
( - 6*a**2 - 15*a*b*x - 10*b**2*x**2)/(30*x**5)
```

3.10 $\int x^2(a^2+2abx+b^2x^2)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 47

$$\int x^2\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{a^2(a+bx)^5}{5b^3}-\frac{a(a+bx)^6}{3b^3}+\frac{(a+bx)^7}{7b^3}$$

output
```
1/5*a^2*(b*x+a)^5/b^3-1/3*a*(b*x+a)^6/b^3+1/7*(b*x+a)^7/b^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int x^2\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{a^4x^3}{3}+a^3bx^4+\frac{6}{5}a^2b^2x^5+\frac{2}{3}ab^3x^6+\frac{b^4x^7}{7}$$

input
```
Integrate[x^2*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(a^4*x^3)/3 + a^3*b*x^4 + (6*a^2*b^2*x^5)/5 + (2*a*b^3*x^6)/3 + (b^4*x^7)/
7
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a^2+2abx+b^2x^2\right)^2 \, dx$$

$$\Big\downarrow \text{ 1098}$$

$$\frac{\int b^4x^2(a+bx)^4dx}{b^4}$$

$$\Big\downarrow \text{ 27}$$

$$\int x^2(a+bx)^4dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{a^2(a+bx)^4}{b^2}+\frac{(a+bx)^6}{b^2}-\frac{2a(a+bx)^5}{b^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2(a+bx)^5}{5b^3}+\frac{(a+bx)^7}{7b^3}-\frac{a(a+bx)^6}{3b^3}$$

input
```
Int[x^2*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(a^2*(a + b*x)^5)/(5*b^3) - (a*(a + b*x)^6)/(3*b^3) + (a + b*x)^7/(7*b^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{1}{7}b^4x^7+\frac{2}{3}a\,b^3x^6+\frac{6}{5}a^2b^2x^5+a^3b\,x^4+\frac{1}{3}a^4x^3$	46
norman	$\frac{1}{7}b^4x^7+\frac{2}{3}a\,b^3x^6+\frac{6}{5}a^2b^2x^5+a^3b\,x^4+\frac{1}{3}a^4x^3$	46
risch	$\frac{1}{7}b^4x^7+\frac{2}{3}a\,b^3x^6+\frac{6}{5}a^2b^2x^5+a^3b\,x^4+\frac{1}{3}a^4x^3$	46
parallelrisch	$\frac{1}{7}b^4x^7+\frac{2}{3}a\,b^3x^6+\frac{6}{5}a^2b^2x^5+a^3b\,x^4+\frac{1}{3}a^4x^3$	46
gosper	$\frac{x^3\left(15b^4x^4+70a\,b^3x^3+126a^2b^2x^2+105a^3bx+35a^4\right)}{105}$	47
orering	$\frac{x^3\left(15b^4x^4+70a\,b^3x^3+126a^2b^2x^2+105a^3bx+35a^4\right)\left(b^2x^2+2abx+a^2\right)^2}{105(bx+a)^4}$	72

input
```
int(x^2*(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*b^4*x^7+2/3*a*b^3*x^6+6/5*a^2*b^2*x^5+a^3*b*x^4+1/3*a^4*x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int x^2 (a^2 + 2abx + b^2x^2)^2 \, dx = \frac{1}{7} b^4 x^7 + \frac{2}{3} a b^3 x^6 + \frac{6}{5} a^2 b^2 x^5 + a^3 b x^4 + \frac{1}{3} a^4 x^3$$

input
```
integrate(x^2*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
1/7*b^4*x^7 + 2/3*a*b^3*x^6 + 6/5*a^2*b^2*x^5 + a^3*b*x^4 + 1/3*a^4*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int x^2 (a^2 + 2abx + b^2x^2)^2 \, dx = \frac{a^4 x^3}{3} + a^3 b x^4 + \frac{6a^2 b^2 x^5}{5} + \frac{2ab^3 x^6}{3} + \frac{b^4 x^7}{7}$$

input
```
integrate(x**2*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
a**4*x**3/3 + a**3*b*x**4 + 6*a**2*b**2*x**5/5 + 2*a*b**3*x**6/3 + b**4*x*
*7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int x^2 (a^2 + 2abx + b^2x^2)^2 \, dx = \frac{1}{7} b^4 x^7 + \frac{2}{3} a b^3 x^6 + \frac{6}{5} a^2 b^2 x^5 + a^3 b x^4 + \frac{1}{3} a^4 x^3$$

input
```
integrate(x^2*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
1/7*b^4*x^7 + 2/3*a*b^3*x^6 + 6/5*a^2*b^2*x^5 + a^3*b*x^4 + 1/3*a^4*x^3
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int x^2 \left(a^2+2abx+b^2x^2\right)^2 \, dx = \frac{1}{7}\,b^4x^7+\frac{2}{3}\,ab^3x^6+\frac{6}{5}\,a^2b^2x^5+a^3bx^4+\frac{1}{3}\,a^4x^3$$

input
```
integrate(x^2*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
1/7*b^4*x^7 + 2/3*a*b^3*x^6 + 6/5*a^2*b^2*x^5 + a^3*b*x^4 + 1/3*a^4*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int x^2 \left(a^2+2abx+b^2x^2\right)^2 \, dx = \frac{a^4\,x^3}{3}+a^3\,b\,x^4+\frac{6\,a^2\,b^2\,x^5}{5}+\frac{2\,a\,b^3\,x^6}{3}+\frac{b^4\,x^7}{7}$$

input
```
int(x^2*(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
(a^4*x^3)/3 + (b^4*x^7)/7 + a^3*b*x^4 + (2*a*b^3*x^6)/3 + (6*a^2*b^2*x^5)/
5
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int x^2 \left(a^2+2abx+b^2x^2\right)^2 \, dx = \frac{x^3(15b^4x^4+70a\,b^3x^3+126a^2b^2x^2+105a^3bx+35a^4)}{105}$$

input
```
int(x^2*(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
(x**3*(35*a**4 + 105*a**3*b*x + 126*a**2*b**2*x**2 + 70*a*b**3*x**3 + 15*b
**4*x**4))/105
```

3.11 $\int x(a^2+2abx+b^2x^2)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 30

$$\int x\left(a^2+2abx+b^2x^2\right)^2\,dx=-\frac{a(a+bx)^5}{5b^2}+\frac{(a+bx)^6}{6b^2}$$

output
```
-1/5*a*(b*x+a)^5/b^2+1/6*(b*x+a)^6/b^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.87

$$\int x\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{a^4x^2}{2}+\frac{4}{3}a^3bx^3+\frac{3}{2}a^2b^2x^4+\frac{4}{5}ab^3x^5+\frac{b^4x^6}{6}$$

input
```
Integrate[x*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(a^4*x^2)/2 + (4*a^3*b*x^3)/3 + (3*a^2*b^2*x^4)/2 + (4*a*b^3*x^5)/5 + (b^4
*x^6)/6
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \left(a^2+2 a b x+b^2 x^2\right)^2 \, dx$$

$$\Big\downarrow \text{1098}$$

$$\frac{\int b^4 x (a+b x)^4 dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int x (a+b x)^4 dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{(a+b x)^5}{b}-\frac{a (a+b x)^4}{b}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+b x)^6}{6 b^2}-\frac{a (a+b x)^5}{5 b^2}$$

input
```
Int[x*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
-1/5*(a*(a + b*x)^5)/b^2 + (a + b*x)^6/(6*b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.57

method	result	size
gosper	$\frac{x^{2}\left(5 b^{4} x^{4}+24 a\, b^{3} x^{3}+45 a^{2} b^{2} x^{2}+40 a^{3} b x +15 a^{4}\right)}{30}$	47
default	$\frac{1}{6} b^{4} x^{6}+\frac{4}{5} a\, b^{3} x^{5}+\frac{3}{2} a^{2} b^{2} x^{4}+\frac{4}{3} a^{3} b\, x^{3}+\frac{1}{2} a^{4} x^{2}$	47
norman	$\frac{1}{6} b^{4} x^{6}+\frac{4}{5} a\, b^{3} x^{5}+\frac{3}{2} a^{2} b^{2} x^{4}+\frac{4}{3} a^{3} b\, x^{3}+\frac{1}{2} a^{4} x^{2}$	47
risch	$\frac{1}{6} b^{4} x^{6}+\frac{4}{5} a\, b^{3} x^{5}+\frac{3}{2} a^{2} b^{2} x^{4}+\frac{4}{3} a^{3} b\, x^{3}+\frac{1}{2} a^{4} x^{2}$	47
parallelrisch	$\frac{1}{6} b^{4} x^{6}+\frac{4}{5} a\, b^{3} x^{5}+\frac{3}{2} a^{2} b^{2} x^{4}+\frac{4}{3} a^{3} b\, x^{3}+\frac{1}{2} a^{4} x^{2}$	47
orering	$\frac{x^{2}\left(5 b^{4} x^{4}+24 a\, b^{3} x^{3}+45 a^{2} b^{2} x^{2}+40 a^{3} b x +15 a^{4}\right)\left(b^{2} x^{2}+2 a b x +a^{2}\right)^{2}}{30\left(b x +a \right)^{4}}$	72

input
```
int(x*(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/30*x^2*(5*b^4*x^4+24*a*b^3*x^3+45*a^2*b^2*x^2+40*a^3*b*x+15*a^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int x\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{1}{6}\,b^4x^6+\frac{4}{5}\,ab^3x^5+\frac{3}{2}\,a^2b^2x^4+\frac{4}{3}\,a^3bx^3+\frac{1}{2}\,a^4x^2$$

input
```
integrate(x*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
1/6*b^4*x^6 + 4/5*a*b^3*x^5 + 3/2*a^2*b^2*x^4 + 4/3*a^3*b*x^3 + 1/2*a^4*x^
2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(24) = 48.

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.77

$$\int x\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{a^4x^2}{2}+\frac{4a^3bx^3}{3}+\frac{3a^2b^2x^4}{2}+\frac{4ab^3x^5}{5}+\frac{b^4x^6}{6}$$

input
```
integrate(x*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
a**4*x**2/2 + 4*a**3*b*x**3/3 + 3*a**2*b**2*x**4/2 + 4*a*b**3*x**5/5 + b**
4*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int x\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{1}{6}\,b^4x^6+\frac{4}{5}\,ab^3x^5+\frac{3}{2}\,a^2b^2x^4+\frac{4}{3}\,a^3bx^3+\frac{1}{2}\,a^4x^2$$

input
```
integrate(x*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
1/6*b^4*x^6 + 4/5*a*b^3*x^5 + 3/2*a^2*b^2*x^4 + 4/3*a^3*b*x^3 + 1/2*a^4*x^
2
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int x\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{1}{6}\,b^4x^6+\frac{4}{5}\,ab^3x^5+\frac{3}{2}\,a^2b^2x^4+\frac{4}{3}\,a^3bx^3+\frac{1}{2}\,a^4x^2$$

input
```
integrate(x*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
1/6*b^4*x^6 + 4/5*a*b^3*x^5 + 3/2*a^2*b^2*x^4 + 4/3*a^3*b*x^3 + 1/2*a^4*x^
2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int x\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{a^4\,x^2}{2}+\frac{4\,a^3\,b\,x^3}{3}+\frac{3\,a^2\,b^2\,x^4}{2}+\frac{4\,a\,b^3\,x^5}{5}+\frac{b^4\,x^6}{6}$$

input
```
int(x*(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
(a^4*x^2)/2 + (b^4*x^6)/6 + (4*a^3*b*x^3)/3 + (4*a*b^3*x^5)/5 + (3*a^2*b^2
*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int x(a^2 + 2abx + b^2x^2)^2 \, dx = \frac{x^2(5b^4x^4 + 24a\,b^3x^3 + 45a^2b^2x^2 + 40a^3bx + 15a^4)}{30}$$

input
```
int(x*(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
(x**2*(15*a**4 + 40*a**3*b*x + 45*a**2*b**2*x**2 + 24*a*b**3*x**3 + 5*b**4
*x**4))/30
```

3.12 $\int \left(a^2+2abx+b^2x^2\right)^2 \, dx$

Optimal result

Integrand size = 18, antiderivative size = 14

$$\int \left(a^2+2abx+b^2x^2\right)^2 \, dx = \frac{(a+bx)^5}{5b}$$

output
```
1/5*(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(a^2+2abx+b^2x^2\right)^2 \, dx = \frac{(a+bx)^5}{5b}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(a + b*x)^5/(5*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{1077, 17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx+b^2x^2\right)^2 \, dx$$

$$\Big\downarrow \text{1077}$$

$$\frac{\int \left(xb^2+ab\right)^4dx}{b^4}$$

$$\Big\downarrow \text{17}$$

$$\frac{(a+bx)^5}{5b}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(a + b*x)^5/(5*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1077
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int
[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] &&
 IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(12) = 24.

Time = 0.50 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.07

method	result	size
default	$\frac{1}{5}b^4x^5 + a\,b^3x^4 + 2a^2b^2x^3 + 2a^3b\,x^2 + a^4x$	43
norman	$\frac{1}{5}b^4x^5 + a\,b^3x^4 + 2a^2b^2x^3 + 2a^3b\,x^2 + a^4x$	43
risch	$\frac{1}{5}b^4x^5 + a\,b^3x^4 + 2a^2b^2x^3 + 2a^3b\,x^2 + a^4x$	43
parallelrisch	$\frac{1}{5}b^4x^5 + a\,b^3x^4 + 2a^2b^2x^3 + 2a^3b\,x^2 + a^4x$	43
gosper	$\frac{x\left(b^4x^4+5a\,b^3x^3+10a^2b^2x^2+10a^3bx+5a^4\right)}{5}$	44
orering	$\frac{x\left(b^4x^4+5a\,b^3x^3+10a^2b^2x^2+10a^3bx+5a^4\right)\left(b^2x^2+2abx+a^2\right)^2}{5(bx+a)^4}$	69

input
```
int((b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*b^4*x^5+a*b^3*x^4+2*a^2*b^2*x^3+2*a^3*b*x^2+a^4*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(12) = 24.

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.00

$$\int \left(a^2+2abx+b^2x^2\right)^2\,dx = \frac{1}{5}\,b^4x^5 + ab^3x^4 + 2\,a^2b^2x^3 + 2\,a^3bx^2 + a^4x$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
1/5*b^4*x^5 + a*b^3*x^4 + 2*a^2*b^2*x^3 + 2*a^3*b*x^2 + a^4*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(8) = 16$.

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.00

$$\int \left(a^2+2abx+b^2x^2\right)^2\,dx = a^4x+2a^3bx^2+2a^2b^2x^3+ab^3x^4+\frac{b^4x^5}{5}$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
a**4*x + 2*a**3*b*x**2 + 2*a**2*b**2*x**3 + a*b**3*x**4 + b**4*x**5/5
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(12) = 24$.

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.79

$$\int \left(a^2+2abx+b^2x^2\right)^2\,dx = \frac{1}{5}\,b^4x^5+ab^3x^4+\frac{4}{3}\,a^2b^2x^3+a^4x+\frac{2}{3}\,\left(b^2x^3+3\,abx^2\right)a^2$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
1/5*b^4*x^5 + a*b^3*x^4 + 4/3*a^2*b^2*x^3 + a^4*x + 2/3*(b^2*x^3 + 3*a*b*x
^2)*a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(12) = 24$.

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.00

$$\int \left(a^2+2abx+b^2x^2\right)^2\,dx = \frac{1}{5}\,b^4x^5+ab^3x^4+2\,a^2b^2x^3+2\,a^3bx^2+a^4x$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
1/5*b^4*x^5 + a*b^3*x^4 + 2*a^2*b^2*x^3 + 2*a^3*b*x^2 + a^4*x
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.00

$$\int \left(a^2+2abx+b^2x^2\right)^2\,dx=a^4\,x+2\,a^3\,b\,x^2+2\,a^2\,b^2\,x^3+a\,b^3\,x^4+\frac{b^4\,x^5}{5}$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
a^4*x + (b^4*x^5)/5 + 2*a^3*b*x^2 + a*b^3*x^4 + 2*a^2*b^2*x^3
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.07

$$\int \left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{x(b^4x^4+5a\,b^3x^3+10a^2b^2x^2+10a^3bx+5a^4)}{5}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
(x*(5*a**4 + 10*a**3*b*x + 10*a**2*b**2*x**2 + 5*a*b**3*x**3 + b**4*x**4))
/5
```

3.13 $\int \frac{\left(a^2+2abx+b^2x^2\right)^2}{x} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{(a^2+2abx+b^2x^2)^2}{x} \, dx = 4a^3bx+3a^2b^2x^2+\frac{4}{3}ab^3x^3+\frac{b^4x^4}{4}+a^4\log(x)$$

output
```
4*a^3*b*x+3*a^2*b^2*x^2+4/3*a*b^3*x^3+1/4*b^4*x^4+a^4*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx+b^2x^2)^2}{x} \, dx = 4a^3bx+3a^2b^2x^2+\frac{4}{3}ab^3x^3+\frac{b^4x^4}{4}+a^4\log(x)$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/x,x]
```

output
```
4*a^3*b*x + 3*a^2*b^2*x^2 + (4*a*b^3*x^3)/3 + (b^4*x^4)/4 + a^4*Log[x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^2}{x}\,dx$$

$$\Big\downarrow \text{1098}$$

$$\frac{\int \frac{b^4(a+bx)^4}{x}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^4}{x}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^4}{x}+4a^3b+6a^2b^2x+4ab^3x^2+b^4x^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^4\log(x)+4a^3bx+3a^2b^2x^2+\frac{4}{3}ab^3x^3+\frac{b^4x^4}{4}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^2/x,x]
```

output
```
4*a^3*b*x + 3*a^2*b^2*x^2 + (4*a*b^3*x^3)/3 + (b^4*x^4)/4 + a^4*Log[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

method	result	size
default	$4a^3bx+3a^2b^2x^2+\frac{4a\,b^3x^3}{3}+\frac{b^4x^4}{4}+a^4\ln\left(x\right)$	43
norman	$4a^3bx+3a^2b^2x^2+\frac{4a\,b^3x^3}{3}+\frac{b^4x^4}{4}+a^4\ln\left(x\right)$	43
risch	$4a^3bx+3a^2b^2x^2+\frac{4a\,b^3x^3}{3}+\frac{b^4x^4}{4}+a^4\ln\left(x\right)$	43
parallelrisch	$4a^3bx+3a^2b^2x^2+\frac{4a\,b^3x^3}{3}+\frac{b^4x^4}{4}+a^4\ln\left(x\right)$	43

input
```
int((b^2*x^2+2*a*b*x+a^2)^2/x,x,method=_RETURNVERBOSE)
```

output
```
4*a^3*b*x+3*a^2*b^2*x^2+4/3*a*b^3*x^3+1/4*b^4*x^4+a^4*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{x}\,dx = \frac{1}{4}\,b^4x^4 + \frac{4}{3}\,ab^3x^3 + 3\,a^2b^2x^2 + 4\,a^3bx + a^4\log\left(x\right)$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^2/x,x, algorithm="fricas")
```

output
```
1/4*b^4*x^4 + 4/3*a*b^3*x^3 + 3*a^2*b^2*x^2 + 4*a^3*b*x + a^4*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{x}\,dx = a^4\log\left(x\right) + 4a^3bx + 3a^2b^2x^2 + \frac{4ab^3x^3}{3} + \frac{b^4x^4}{4}$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**2/x,x)
```

output
```
a**4*log(x) + 4*a**3*b*x + 3*a**2*b**2*x**2 + 4*a*b**3*x**3/3 + b**4*x**4/
4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{x}\,dx = \frac{1}{4}\,b^4x^4 + \frac{4}{3}\,ab^3x^3 + 3\,a^2b^2x^2 + 4\,a^3bx + a^4\log\left(x\right)$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^2/x,x, algorithm="maxima")
```

output
```
1/4*b^4*x^4 + 4/3*a*b^3*x^3 + 3*a^2*b^2*x^2 + 4*a^3*b*x + a^4*log(x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{(a^2+2abx+b^2x^2)^2}{x}\,dx = \frac{1}{4}\,b^4x^4+\frac{4}{3}\,ab^3x^3+3\,a^2b^2x^2+4\,a^3bx+a^4\log\left(|x|\right)$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^2/x,x, algorithm="giac")
```

output
```
1/4*b^4*x^4 + 4/3*a*b^3*x^3 + 3*a^2*b^2*x^2 + 4*a^3*b*x + a^4*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{(a^2+2abx+b^2x^2)^2}{x}\,dx = a^4\,\ln\left(x\right)+\frac{b^4\,x^4}{4}+\frac{4\,a\,b^3\,x^3}{3}+3\,a^2\,b^2\,x^2+4\,a^3\,b\,x$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^2/x,x)
```

output
```
a^4*log(x) + (b^4*x^4)/4 + (4*a*b^3*x^3)/3 + 3*a^2*b^2*x^2 + 4*a^3*b*x
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{(a^2+2abx+b^2x^2)^2}{x}\,dx = \log(x)\,a^4+4a^3bx+3a^2b^2x^2+\frac{4a\,b^3x^3}{3}+\frac{b^4x^4}{4}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^2/x,x)
```

output
```
(12*log(x)*a**4 + 48*a**3*b*x + 36*a**2*b**2*x**2 + 16*a*b**3*x**3 + 3*b**
4*x**4)/12
```

3.14 $\int \frac{(a^2+2abx+b^2x^2)^2}{x^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 45

$$\int \frac{(a^2+2abx+b^2x^2)^2}{x^2} dx = -\frac{a^4}{x} + 6a^2b^2x + 2ab^3x^2 + \frac{b^4x^3}{3} + 4a^3b\log(x)$$

output
```
-a^4/x+6*a^2*b^2*x+2*a*b^3*x^2+1/3*b^4*x^3+4*a^3*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx+b^2x^2)^2}{x^2} dx = -\frac{a^4}{x} + 6a^2b^2x + 2ab^3x^2 + \frac{b^4x^3}{3} + 4a^3b\log(x)$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/x^2,x]
```

output
```
-(a^4/x) + 6*a^2*b^2*x + 2*a*b^3*x^2 + (b^4*x^3)/3 + 4*a^3*b*Log[x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^2}{x^2}\,dx$$

$$\Big\downarrow \text{1098}$$

$$\frac{\int \frac{b^4(a+bx)^4}{x^2}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^4}{x^2}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^4}{x^2}+\frac{4a^3b}{x}+6a^2b^2+4ab^3x+b^4x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4}{x}+4a^3b\log(x)+6a^2b^2x+2ab^3x^2+\frac{b^4x^3}{3}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^2/x^2,x]
```

output
```
-(a^4/x) + 6*a^2*b^2*x + 2*a*b^3*x^2 + (b^4*x^3)/3 + 4*a^3*b*Log[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{a^4}{x}+6x\,a^2b^2+2a\,b^3x^2+\frac{b^4x^3}{3}+4a^3b\ln\left(x\right)$	44
risch	$-\frac{a^4}{x}+6x\,a^2b^2+2a\,b^3x^2+\frac{b^4x^3}{3}+4a^3b\ln\left(x\right)$	44
norman	$\frac{-a^4+\frac{1}{3}b^4x^4+2a\,b^3x^3+6a^2b^2x^2}{x}+4a^3b\ln\left(x\right)$	48
parallelrisch	$\frac{b^4x^4+6a\,b^3x^3+12a^3b\ln\left(x\right)x+18a^2b^2x^2-3a^4}{3x}$	48

input
```
int((b^2*x^2+2*a*b*x+a^2)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^4/x+6*x*a^2*b^2+2*a*b^3*x^2+1/3*b^4*x^3+4*a^3*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^2}{x^2}\,dx=\frac{b^4x^4+6\,ab^3x^3+18\,a^2b^2x^2+12\,a^3bx\log\left(x\right)-3\,a^4}{3\,x}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^2/x^2,x, algorithm="fricas")
```

output
```
1/3*(b^4*x^4 + 6*a*b^3*x^3 + 18*a^2*b^2*x^2 + 12*a^3*b*x*log(x) - 3*a^4)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^2}{x^2}\,dx=-\frac{a^4}{x}+4a^3b\log\left(x\right)+6a^2b^2x+2ab^3x^2+\frac{b^4x^3}{3}$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**2/x**2,x)
```

output
```
-a**4/x + 4*a**3*b*log(x) + 6*a**2*b**2*x + 2*a*b**3*x**2 + b**4*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^2}{x^2}\,dx=\frac{1}{3}\,b^4x^3+2\,ab^3x^2+6\,a^2b^2x+4\,a^3b\log\left(x\right)-\frac{a^4}{x}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^2/x^2,x, algorithm="maxima")
```

output
```
1/3*b^4*x^3 + 2*a*b^3*x^2 + 6*a^2*b^2*x + 4*a^3*b*log(x) - a^4/x
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{(a^2+2abx+b^2x^2)^2}{x^2}\,dx = \frac{1}{3}\,b^4x^3+2\,ab^3x^2+6\,a^2b^2x+4\,a^3b\log\left(|x|\right)-\frac{a^4}{x}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^2/x^2,x, algorithm="giac")
```

output
```
1/3*b^4*x^3 + 2*a*b^3*x^2 + 6*a^2*b^2*x + 4*a^3*b*log(abs(x)) - a^4/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{(a^2+2abx+b^2x^2)^2}{x^2}\,dx = \frac{b^4\,x^3}{3}-\frac{a^4}{x}+6\,a^2\,b^2\,x+2\,a\,b^3\,x^2+4\,a^3\,b\,\ln\left(x\right)$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^2/x^2,x)
```

output
```
(b^4*x^3)/3 - a^4/x + 6*a^2*b^2*x + 2*a*b^3*x^2 + 4*a^3*b*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{(a^2+2abx+b^2x^2)^2}{x^2}\,dx = \frac{12\log(x)\,a^3bx-3a^4+18a^2b^2x^2+6a\,b^3x^3+b^4x^4}{3x}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^2/x^2,x)
```

output
```
(12*log(x)*a**3*b*x - 3*a**4 + 18*a**2*b**2*x**2 + 6*a*b**3*x**3 + b**4*x**
*4)/(3*x)
```

3.15 $\int \frac{(a^2+2abx+b^2x^2)^2}{x^3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 47

$$\int \frac{(a^2+2abx+b^2x^2)^2}{x^3} \, dx = -\frac{a^4}{2x^2} - \frac{4a^3b}{x} + 4ab^3x + \frac{b^4x^2}{2} + 6a^2b^2\log(x)$$

output
```
-1/2*a^4/x^2-4*a^3*b/x+4*a*b^3*x+1/2*b^4*x^2+6*a^2*b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx+b^2x^2)^2}{x^3} \, dx = -\frac{a^4}{2x^2} - \frac{4a^3b}{x} + 4ab^3x + \frac{b^4x^2}{2} + 6a^2b^2\log(x)$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/x^3,x]
```

output
```
-1/2*a^4/x^2 - (4*a^3*b)/x + 4*a*b^3*x + (b^4*x^2)/2 + 6*a^2*b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^2}{x^3}dx$$
$$\Big\downarrow \text{1098}$$
$$\frac{\int \frac{b^4(a+bx)^4}{x^3}dx}{b^4}$$
$$\Big\downarrow \text{27}$$
$$\int \frac{(a+bx)^4}{x^3}dx$$
$$\Big\downarrow \text{49}$$
$$\int \left(\frac{a^4}{x^3}+\frac{4a^3b}{x^2}+\frac{6a^2b^2}{x}+4ab^3+b^4x\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a^4}{2x^2}-\frac{4a^3b}{x}+6a^2b^2\log(x)+4ab^3x+\frac{b^4x^2}{2}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^2/x^3,x]
```

output
```
-1/2*a^4/x^2 - (4*a^3*b)/x + 4*a*b^3*x + (b^4*x^2)/2 + 6*a^2*b^2*Log[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{a^{4}}{2 x^{2}}-\frac{4 a^{3} b}{x}+4 a \,b^{3} x +\frac{b^{4} x^{2}}{2}+6 a^{2} b^{2} \ln \left(x \right)$	44
risch	$\frac{b^{4} x^{2}}{2}+4 a \,b^{3} x +\frac{-4 a^{3} b x -\frac{1}{2} a^{4}}{x^{2}}+6 a^{2} b^{2} \ln \left(x \right)$	44
norman	$\frac{-\frac{1}{2} a^{4}+\frac{1}{2} b^{4} x^{4}+4 a \,b^{3} x^{3}-4 a^{3} b x}{x^{2}}+6 a^{2} b^{2} \ln \left(x \right)$	46
parallelrisch	$\frac{b^{4} x^{4}+12 a^{2} b^{2} \ln \left(x \right) x^{2}+8 a \,b^{3} x^{3}-8 a^{3} b x -a^{4}}{2 x^{2}}$	48

input
```
int((b^2*x^2+2*a*b*x+a^2)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^4/x^2-4*a^3*b/x+4*a*b^3*x+1/2*b^4*x^2+6*a^2*b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx+b^2x^2)^2}{x^3}\,dx = \frac{b^4x^4+8\,ab^3x^3+12\,a^2b^2x^2\log(x)-8\,a^3bx-a^4}{2\,x^2}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^2/x^3,x, algorithm="fricas")
```

output
```
1/2*(b^4*x^4 + 8*a*b^3*x^3 + 12*a^2*b^2*x^2*log(x) - 8*a^3*b*x - a^4)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{(a^2+2abx+b^2x^2)^2}{x^3}\,dx = 6a^2b^2\log(x)+4ab^3x+\frac{b^4x^2}{2}+\frac{-a^4-8a^3bx}{2x^2}$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**2/x**3,x)
```

output
```
6*a**2*b**2*log(x) + 4*a*b**3*x + b**4*x**2/2 + (-a**4 - 8*a**3*b*x)/(2*x*
*2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{(a^2+2abx+b^2x^2)^2}{x^3}\,dx = \frac{1}{2}\,b^4x^2+4\,ab^3x+6\,a^2b^2\log(x)-\frac{8\,a^3bx+a^4}{2\,x^2}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^2/x^3,x, algorithm="maxima")
```

output
```
1/2*b^4*x^2 + 4*a*b^3*x + 6*a^2*b^2*log(x) - 1/2*(8*a^3*b*x + a^4)/x^2
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{(a^2+2abx+b^2x^2)^2}{x^3}\,dx = \frac{1}{2}\,b^4x^2 + 4\,ab^3x + 6\,a^2b^2\log\left(|x|\right) - \frac{8\,a^3bx + a^4}{2\,x^2}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^2/x^3,x, algorithm="giac")
```

output
```
1/2*b^4*x^2 + 4*a*b^3*x + 6*a^2*b^2*log(abs(x)) - 1/2*(8*a^3*b*x + a^4)/x^
2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{(a^2+2abx+b^2x^2)^2}{x^3}\,dx = \frac{b^4\,x^2}{2} - \frac{\frac{a^4}{2} + 4\,b\,x\,a^3}{x^2} + 6\,a^2\,b^2\,\ln\left(x\right) + 4\,a\,b^3\,x$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^2/x^3,x)
```

output
```
(b^4*x^2)/2 - (a^4/2 + 4*a^3*b*x)/x^2 + 6*a^2*b^2*log(x) + 4*a*b^3*x
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx+b^2x^2)^2}{x^3}\,dx = \frac{12\log(x)\,a^2b^2x^2 - a^4 - 8a^3bx + 8a\,b^3x^3 + b^4x^4}{2x^2}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^2/x^3,x)
```

output
```
(12*log(x)*a**2*b**2*x**2 - a**4 - 8*a**3*b*x + 8*a*b**3*x**3 + b**4*x**4)
/(2*x**2)
```

3.16 $\int x^2(a^2+2abx+b^2x^2)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 47

$$\int x^2\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{a^2(a+bx)^7}{7b^3}-\frac{a(a+bx)^8}{4b^3}+\frac{(a+bx)^9}{9b^3}$$

output
```
1/7*a^2*(b*x+a)^7/b^3-1/4*a*(b*x+a)^8/b^3+1/9*(b*x+a)^9/b^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.70

$$\int x^2\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{a^6x^3}{3}+\frac{3}{2}a^5bx^4+3a^4b^2x^5+\frac{10}{3}a^3b^3x^6+\frac{15}{7}a^2b^4x^7+\frac{3}{4}ab^5x^8+\frac{b^6x^9}{9}$$

input
```
Integrate[x^2*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(a^6*x^3)/3 + (3*a^5*b*x^4)/2 + 3*a^4*b^2*x^5 + (10*a^3*b^3*x^6)/3 + (15*a
^2*b^4*x^7)/7 + (3*a*b^5*x^8)/4 + (b^6*x^9)/9
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a^2+2abx+b^2x^2\right)^3 \, dx$$

$$\Big\downarrow \text{ 1098}$$

$$\frac{\int b^6x^2(a+bx)^6dx}{b^6}$$

$$\Big\downarrow \text{ 27}$$

$$\int x^2(a+bx)^6dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{a^2(a+bx)^6}{b^2}+\frac{(a+bx)^8}{b^2}-\frac{2a(a+bx)^7}{b^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2(a+bx)^7}{7b^3}+\frac{(a+bx)^9}{9b^3}-\frac{a(a+bx)^8}{4b^3}$$

input
```
Int[x^2*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(a^2*(a + b*x)^7)/(7*b^3) - (a*(a + b*x)^8)/(4*b^3) + (a + b*x)^9/(9*b^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.47

method	result	size
gosper	$\frac{x^{3}\left(28 b^{6} x^{6}+189 a\,b^{5} x^{5}+540 a^{2} b^{4} x^{4}+840 a^{3} b^{3} x^{3}+756 a^{4} b^{2} x^{2}+378 a^{5} b x+84 a^{6}\right)}{252}$	69
default	$\frac{1}{9} b^{6} x^{9}+\frac{3}{4} a\,b^{5} x^{8}+\frac{15}{7} a^{2} b^{4} x^{7}+\frac{10}{3} b^{3} x^{6} a^{3}+3 a^{4} b^{2} x^{5}+\frac{3}{2} a^{5} b\,x^{4}+\frac{1}{3} x^{3} a^{6}$	69
norman	$\frac{1}{9} b^{6} x^{9}+\frac{3}{4} a\,b^{5} x^{8}+\frac{15}{7} a^{2} b^{4} x^{7}+\frac{10}{3} b^{3} x^{6} a^{3}+3 a^{4} b^{2} x^{5}+\frac{3}{2} a^{5} b\,x^{4}+\frac{1}{3} x^{3} a^{6}$	69
risch	$\frac{1}{9} b^{6} x^{9}+\frac{3}{4} a\,b^{5} x^{8}+\frac{15}{7} a^{2} b^{4} x^{7}+\frac{10}{3} b^{3} x^{6} a^{3}+3 a^{4} b^{2} x^{5}+\frac{3}{2} a^{5} b\,x^{4}+\frac{1}{3} x^{3} a^{6}$	69
parallelrisch	$\frac{1}{9} b^{6} x^{9}+\frac{3}{4} a\,b^{5} x^{8}+\frac{15}{7} a^{2} b^{4} x^{7}+\frac{10}{3} b^{3} x^{6} a^{3}+3 a^{4} b^{2} x^{5}+\frac{3}{2} a^{5} b\,x^{4}+\frac{1}{3} x^{3} a^{6}$	69
orering	$\frac{x^{3}\left(28 b^{6} x^{6}+189 a\,b^{5} x^{5}+540 a^{2} b^{4} x^{4}+840 a^{3} b^{3} x^{3}+756 a^{4} b^{2} x^{2}+378 a^{5} b x+84 a^{6}\right)\left(b^{2} x^{2}+2 a b x+a^{2}\right)^{3}}{252\left(b x+a\right)^{6}}$	94

input
```
int(x^2*(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/252*x^3*(28*b^6*x^6+189*a*b^5*x^5+540*a^2*b^4*x^4+840*a^3*b^3*x^3+756*a^
4*b^2*x^2+378*a^5*b*x+84*a^6)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.45

$$\int x^2 \left(a^2+2abx+b^2x^2\right)^3 \, dx = \frac{1}{9} \, b^6 x^9 + \frac{3}{4} \, a b^5 x^8 + \frac{15}{7} \, a^2 b^4 x^7 + \frac{10}{3} \, a^3 b^3 x^6 + 3 \, a^4 b^2 x^5 + \frac{3}{2} \, a^5 b x^4 + \frac{1}{3} \, a^6 x^3$$

input
```
integrate(x^2*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
1/9*b^6*x^9 + 3/4*a*b^5*x^8 + 15/7*a^2*b^4*x^7 + 10/3*a^3*b^3*x^6 + 3*a^4*
b^2*x^5 + 3/2*a^5*b*x^4 + 1/3*a^6*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.66

$$\int x^2 \left(a^2+2abx+b^2x^2\right)^3 \, dx = \frac{a^6 x^3}{3} + \frac{3a^5 b x^4}{2} + 3a^4 b^2 x^5 + \frac{10a^3 b^3 x^6}{3} + \frac{15a^2 b^4 x^7}{7} + \frac{3ab^5 x^8}{4} + \frac{b^6 x^9}{9}$$

input
```
integrate(x**2*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
a**6*x**3/3 + 3*a**5*b*x**4/2 + 3*a**4*b**2*x**5 + 10*a**3*b**3*x**6/3 + 1
5*a**2*b**4*x**7/7 + 3*a*b**5*x**8/4 + b**6*x**9/9
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.45

$$\int x^2 \left(a^2+2abx+b^2x^2\right)^3 \, dx = \frac{1}{9}\,b^6x^9+\frac{3}{4}\,ab^5x^8+\frac{15}{7}\,a^2b^4x^7+\frac{10}{3}\,a^3b^3x^6 + 3\,a^4b^2x^5+\frac{3}{2}\,a^5bx^4+\frac{1}{3}\,a^6x^3$$

input
```
integrate(x^2*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
1/9*b^6*x^9 + 3/4*a*b^5*x^8 + 15/7*a^2*b^4*x^7 + 10/3*a^3*b^3*x^6 + 3*a^4*
b^2*x^5 + 3/2*a^5*b*x^4 + 1/3*a^6*x^3
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.45

$$\int x^2 \left(a^2+2abx+b^2x^2\right)^3 \, dx = \frac{1}{9}\,b^6x^9+\frac{3}{4}\,ab^5x^8+\frac{15}{7}\,a^2b^4x^7+\frac{10}{3}\,a^3b^3x^6 + 3\,a^4b^2x^5+\frac{3}{2}\,a^5bx^4+\frac{1}{3}\,a^6x^3$$

input
```
integrate(x^2*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
1/9*b^6*x^9 + 3/4*a*b^5*x^8 + 15/7*a^2*b^4*x^7 + 10/3*a^3*b^3*x^6 + 3*a^4*
b^2*x^5 + 3/2*a^5*b*x^4 + 1/3*a^6*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.45

$$\int x^2 \left(a^2+2abx+b^2x^2\right)^3 \, dx = \frac{a^6\,x^3}{3}+\frac{3\,a^5\,b\,x^4}{2}+3\,a^4\,b^2\,x^5+\frac{10\,a^3\,b^3\,x^6}{3} + \frac{15\,a^2\,b^4\,x^7}{7}+\frac{3\,a\,b^5\,x^8}{4}+\frac{b^6\,x^9}{9}$$

input
```
int(x^2*(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
(a^6*x^3)/3 + (b^6*x^9)/9 + (3*a^5*b*x^4)/2 + (3*a*b^5*x^8)/4 + 3*a^4*b^2*
x^5 + (10*a^3*b^3*x^6)/3 + (15*a^2*b^4*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.45

$$\int x^2\left(a^2+2abx+b^2x^2\right)^3\,dx = \frac{x^3(28b^6x^6+189a\,b^5x^5+540a^2b^4x^4+840a^3b^3x^3+756a^4b^2x^2+378a^5bx+84a^6)}{252}$$

input
```
int(x^2*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
(x**3*(84*a**6 + 378*a**5*b*x + 756*a**4*b**2*x**2 + 840*a**3*b**3*x**3 +
540*a**2*b**4*x**4 + 189*a*b**5*x**5 + 28*b**6*x**6))/252
```

3.17 $\int x(a^2+2abx+b^2x^2)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 30

$$\int x\left(a^2+2abx+b^2x^2\right)^3\,dx=-\frac{a(a+bx)^7}{7b^2}+\frac{(a+bx)^8}{8b^2}$$

output
```
-1/7*a*(b*x+a)^7/b^2+1/8*(b*x+a)^8/b^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 78 vs. 2(30) = 60.

Time = 0.00 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.60

$$\int x\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{a^6x^2}{2}+2a^5bx^3+\frac{15}{4}a^4b^2x^4+4a^3b^3x^5+\frac{5}{2}a^2b^4x^6+\frac{6}{7}ab^5x^7+\frac{b^6x^8}{8}$$

input
```
Integrate[x*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(a^6*x^2)/2 + 2*a^5*b*x^3 + (15*a^4*b^2*x^4)/4 + 4*a^3*b^3*x^5 + (5*a^2*b^
4*x^6)/2 + (6*a*b^5*x^7)/7 + (b^6*x^8)/8
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a^2+2abx+b^2x^2\right)^3\,dx$$

$$\Big\downarrow \text{1098}$$

$$\frac{\int b^6x(a+bx)^6dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int x(a+bx)^6dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{(a+bx)^7}{b}-\frac{a(a+bx)^6}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^8}{8b^2}-\frac{a(a+bx)^7}{7b^2}$$

input
```
Int[x*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
-1/7*(a*(a + b*x)^7)/b^2 + (a + b*x)^8/(8*b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(26) = 52$.

Time = 0.55 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.30

method	result	size
gosper	$\frac{x^{2} \left(7 b^{6} x^{6}+48 a \,b^{5} x^{5}+140 a^{2} b^{4} x^{4}+224 a^{3} b^{3} x^{3}+210 a^{4} b^{2} x^{2}+112 a^{5} b x +28 a^{6}\right)}{56}$	69
default	$\frac{1}{8} b^{6} x^{8}+\frac{6}{7} a \,b^{5} x^{7}+\frac{5}{2} a^{2} b^{4} x^{6}+4 a^{3} b^{3} x^{5}+\frac{15}{4} b^{2} x^{4} a^{4}+2 a^{5} b \,x^{3}+\frac{1}{2} a^{6} x^{2}$	69
norman	$\frac{1}{8} b^{6} x^{8}+\frac{6}{7} a \,b^{5} x^{7}+\frac{5}{2} a^{2} b^{4} x^{6}+4 a^{3} b^{3} x^{5}+\frac{15}{4} b^{2} x^{4} a^{4}+2 a^{5} b \,x^{3}+\frac{1}{2} a^{6} x^{2}$	69
risch	$\frac{1}{8} b^{6} x^{8}+\frac{6}{7} a \,b^{5} x^{7}+\frac{5}{2} a^{2} b^{4} x^{6}+4 a^{3} b^{3} x^{5}+\frac{15}{4} b^{2} x^{4} a^{4}+2 a^{5} b \,x^{3}+\frac{1}{2} a^{6} x^{2}$	69
parallelrisch	$\frac{1}{8} b^{6} x^{8}+\frac{6}{7} a \,b^{5} x^{7}+\frac{5}{2} a^{2} b^{4} x^{6}+4 a^{3} b^{3} x^{5}+\frac{15}{4} b^{2} x^{4} a^{4}+2 a^{5} b \,x^{3}+\frac{1}{2} a^{6} x^{2}$	69
orering	$\frac{x^{2} \left(7 b^{6} x^{6}+48 a \,b^{5} x^{5}+140 a^{2} b^{4} x^{4}+224 a^{3} b^{3} x^{3}+210 a^{4} b^{2} x^{2}+112 a^{5} b x +28 a^{6}\right) \left(b^{2} x^{2}+2 a b x +a^{2}\right)^{3}}{56 \left(b x +a \right)^{6}}$	94

input
```
int(x*(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/56*x^2*(7*b^6*x^6+48*a*b^5*x^5+140*a^2*b^4*x^4+224*a^3*b^3*x^3+210*a^4*b
^2*x^2+112*a^5*b*x+28*a^6)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(26) = 52$.

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.27

$$\int x(a^2 + 2abx + b^2x^2)^3 \, dx = \frac{1}{8}\,b^6x^8 + \frac{6}{7}\,ab^5x^7 + \frac{5}{2}\,a^2b^4x^6 + 4\,a^3b^3x^5 + \frac{15}{4}\,a^4b^2x^4 + 2\,a^5bx^3 + \frac{1}{2}\,a^6x^2$$

input
```
integrate(x*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
1/8*b^6*x^8 + 6/7*a*b^5*x^7 + 5/2*a^2*b^4*x^6 + 4*a^3*b^3*x^5 + 15/4*a^4*b
^2*x^4 + 2*a^5*b*x^3 + 1/2*a^6*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(24) = 48$.

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.53

$$\int x(a^2 + 2abx + b^2x^2)^3 \, dx = \frac{a^6x^2}{2} + 2a^5bx^3 + \frac{15a^4b^2x^4}{4} + 4a^3b^3x^5 + \frac{5a^2b^4x^6}{2} + \frac{6ab^5x^7}{7} + \frac{b^6x^8}{8}$$

input
```
integrate(x*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
a**6*x**2/2 + 2*a**5*b*x**3 + 15*a**4*b**2*x**4/4 + 4*a**3*b**3*x**5 + 5*a
**2*b**4*x**6/2 + 6*a*b**5*x**7/7 + b**6*x**8/8
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(26) = 52$.

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.27

$$\int x\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{1}{8}\,b^6x^8+\frac{6}{7}\,ab^5x^7+\frac{5}{2}\,a^2b^4x^6+4\,a^3b^3x^5+\frac{15}{4}\,a^4b^2x^4+2\,a^5bx^3+\frac{1}{2}\,a^6x^2$$

input
```
integrate(x*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
1/8*b^6*x^8 + 6/7*a*b^5*x^7 + 5/2*a^2*b^4*x^6 + 4*a^3*b^3*x^5 + 15/4*a^4*b
^2*x^4 + 2*a^5*b*x^3 + 1/2*a^6*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(26) = 52$.

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.27

$$\int x\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{1}{8}\,b^6x^8+\frac{6}{7}\,ab^5x^7+\frac{5}{2}\,a^2b^4x^6+4\,a^3b^3x^5+\frac{15}{4}\,a^4b^2x^4+2\,a^5bx^3+\frac{1}{2}\,a^6x^2$$

input
```
integrate(x*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
1/8*b^6*x^8 + 6/7*a*b^5*x^7 + 5/2*a^2*b^4*x^6 + 4*a^3*b^3*x^5 + 15/4*a^4*b
^2*x^4 + 2*a^5*b*x^3 + 1/2*a^6*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.27

$$\int x\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{a^6\,x^2}{2}+2\,a^5\,b\,x^3+\frac{15\,a^4\,b^2\,x^4}{4}+4\,a^3\,b^3\,x^5+\frac{5\,a^2\,b^4\,x^6}{2}+\frac{6\,a\,b^5\,x^7}{7}+\frac{b^6\,x^8}{8}$$

input
```
int(x*(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
(a^6*x^2)/2 + (b^6*x^8)/8 + 2*a^5*b*x^3 + (6*a*b^5*x^7)/7 + (15*a^4*b^2*x^
4)/4 + 4*a^3*b^3*x^5 + (5*a^2*b^4*x^6)/2
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.27

$$\int x\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{x^2(7b^6x^6+48a\,b^5x^5+140a^2b^4x^4+224a^3b^3x^3+210a^4b^2x^2+112a^5bx+28a^6)}{56}$$

input
```
int(x*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
(x**2*(28*a**6 + 112*a**5*b*x + 210*a**4*b**2*x**2 + 224*a**3*b**3*x**3 +
140*a**2*b**4*x**4 + 48*a*b**5*x**5 + 7*b**6*x**6))/56
```

3.18 $\int (a^2+2abx+b^2x^2)^3 \, dx$

Optimal result

Integrand size = 18, antiderivative size = 14

$$\int (a^2+2abx+b^2x^2)^3 \, dx = \frac{(a+bx)^7}{7b}$$

output
```
1/7*(b*x+a)^7/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a^2+2abx+b^2x^2)^3 \, dx = \frac{(a+bx)^7}{7b}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(a + b*x)^7/(7*b)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1077, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx+b^2x^2\right)^3 \, dx$$

$$\Big\downarrow \text{1077}$$

$$\frac{\int \left(xb^2+ab\right)^6dx}{b^6}$$

$$\Big\downarrow \text{17}$$

$$\frac{(a+bx)^7}{7b}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(a + b*x)^7/(7*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1077
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int
[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] &&
 IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(12) = 24.

Time = 0.52 (sec) , antiderivative size = 65, normalized size of antiderivative = 4.64

method	result	size
default	$\frac{1}{7}b^6x^7 + a\,b^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3b\,x^2a^5 + a^6x$	65
norman	$\frac{1}{7}b^6x^7 + a\,b^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3b\,x^2a^5 + a^6x$	65
risch	$\frac{1}{7}b^6x^7 + a\,b^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3b\,x^2a^5 + a^6x$	65
parallelrisch	$\frac{1}{7}b^6x^7 + a\,b^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3b\,x^2a^5 + a^6x$	65
gosper	$\frac{x\left(b^6x^6+7a\,b^5x^5+21a^2b^4x^4+35a^3b^3x^3+35a^4b^2x^2+21a^5bx+7a^6\right)}{7}$	66
orering	$\frac{x\left(b^6x^6+7a\,b^5x^5+21a^2b^4x^4+35a^3b^3x^3+35a^4b^2x^2+21a^5bx+7a^6\right)\left(b^2x^2+2abx+a^2\right)^3}{7(bx+a)^6}$	91

input
```
int((b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/7*b^6*x^7+a*b^5*x^6+3*a^2*b^4*x^5+5*a^3*b^3*x^4+5*a^4*b^2*x^3+3*b*x^2*a^
5+a^6*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(12) = 24.

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 4.57

$$\int \left(a^2+2abx+b^2x^2\right)^3\,dx = \frac{1}{7}\,b^6x^7+ab^5x^6+3\,a^2b^4x^5+5\,a^3b^3x^4+5\,a^4b^2x^3+3\,a^5bx^2+a^6x$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
1/7*b^6*x^7 + a*b^5*x^6 + 3*a^2*b^4*x^5 + 5*a^3*b^3*x^4 + 5*a^4*b^2*x^3 +
3*a^5*b*x^2 + a^6*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(8) = 16$.

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 4.71

$$\int \left(a^2+2abx+b^2x^2\right)^3 dx = a^6x+3a^5bx^2+5a^4b^2x^3+5a^3b^3x^4+3a^2b^4x^5+ab^5x^6+\frac{b^6x^7}{7}$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
a**6*x + 3*a**5*b*x**2 + 5*a**4*b**2*x**3 + 5*a**3*b**3*x**4 + 3*a**2*b**4
*x**5 + a*b**5*x**6 + b**6*x**7/7
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(12) = 24$.

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 6.93

$$\int \left(a^2+2abx+b^2x^2\right)^3 dx = \frac{1}{7}b^6x^7+ab^5x^6+\frac{12}{5}a^2b^4x^5+2a^3b^3x^4+a^6x + \left(b^2x^3+3abx^2\right)a^4+\frac{1}{5}\left(3b^4x^5+15ab^3x^4+20a^2b^2x^3\right)a^2$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
1/7*b^6*x^7 + a*b^5*x^6 + 12/5*a^2*b^4*x^5 + 2*a^3*b^3*x^4 + a^6*x + (b^2*
x^3 + 3*a*b*x^2)*a^4 + 1/5*(3*b^4*x^5 + 15*a*b^3*x^4 + 20*a^2*b^2*x^3)*a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(12) = 24$.

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 4.57

$$\int \left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{1}{7}\,b^6x^7+ab^5x^6+3\,a^2b^4x^5+5\,a^3b^3x^4+5\,a^4b^2x^3+3\,a^5bx^2+a^6x$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
1/7*b^6*x^7 + a*b^5*x^6 + 3*a^2*b^4*x^5 + 5*a^3*b^3*x^4 + 5*a^4*b^2*x^3 +
3*a^5*b*x^2 + a^6*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 4.57

$$\int \left(a^2+2abx+b^2x^2\right)^3\,dx=a^6\,x+3\,a^5\,b\,x^2+5\,a^4\,b^2\,x^3+5\,a^3\,b^3\,x^4+3\,a^2\,b^4\,x^5+a\,b^5\,x^6+\frac{b^6\,x^7}{7}$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
a^6*x + (b^6*x^7)/7 + 3*a^5*b*x^2 + a*b^5*x^6 + 5*a^4*b^2*x^3 + 5*a^3*b^3*
x^4 + 3*a^2*b^4*x^5
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 4.64

$$\int \left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{x(b^6x^6+7a\,b^5x^5+21a^2b^4x^4+35a^3b^3x^3+35a^4b^2x^2+21a^5bx+7a^6)}{7}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
(x*(7*a**6 + 21*a**5*b*x + 35*a**4*b**2*x**2 + 35*a**3*b**3*x**3 + 21*a**2
*b**4*x**4 + 7*a*b**5*x**5 + b**6*x**6))/7
```

3.19 $\int \frac{(a^2+2abx+b^2x^2)^3}{x} dx$

Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{(a^2+2abx+b^2x^2)^3}{x} dx = 6a^5bx + \frac{15}{2}a^4b^2x^2 + \frac{20}{3}a^3b^3x^3 + \frac{15}{4}a^2b^4x^4 + \frac{6}{5}ab^5x^5 + \frac{b^6x^6}{6} + a^6\log(x)$$

output
```
6*a^5*b*x+15/2*a^4*b^2*x^2+20/3*a^3*b^3*x^3+15/4*a^2*b^4*x^4+6/5*a*b^5*x^5
+1/6*b^6*x^6+a^6*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx+b^2x^2)^3}{x} dx = 6a^5bx + \frac{15}{2}a^4b^2x^2 + \frac{20}{3}a^3b^3x^3 + \frac{15}{4}a^2b^4x^4 + \frac{6}{5}ab^5x^5 + \frac{b^6x^6}{6} + a^6\log(x)$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/x,x]
```

output
```
6*a^5*b*x + (15*a^4*b^2*x^2)/2 + (20*a^3*b^3*x^3)/3 + (15*a^2*b^4*x^4)/4 +
 (6*a*b^5*x^5)/5 + (b^6*x^6)/6 + a^6*Log[x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^3}{x} \, dx$$

$$\Big\downarrow \text{1098}$$

$$\frac{\int \frac{b^6(a+bx)^6}{x}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^6}{x}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^6}{x}+6a^5b+15a^4b^2x+20a^3b^3x^2+15a^2b^4x^3+6ab^5x^4+b^6x^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^6\log(x)+6a^5bx+\frac{15}{2}a^4b^2x^2+\frac{20}{3}a^3b^3x^3+\frac{15}{4}a^2b^4x^4+\frac{6}{5}ab^5x^5+\frac{b^6x^6}{6}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^3/x,x]
```

output

```
6*a^5*b*x + (15*a^4*b^2*x^2)/2 + (20*a^3*b^3*x^3)/3 + (15*a^2*b^4*x^4)/4 +
 (6*a*b^5*x^5)/5 + (b^6*x^6)/6 + a^6*Log[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

method	result	size
default	$6a^5bx + \frac{15a^4b^2x^2}{2} + \frac{20a^3b^3x^3}{3} + \frac{15a^2b^4x^4}{4} + \frac{6a\,b^5x^5}{5} + \frac{b^6x^6}{6} + a^6\ln\left(x\right)$	65
norman	$6a^5bx + \frac{15a^4b^2x^2}{2} + \frac{20a^3b^3x^3}{3} + \frac{15a^2b^4x^4}{4} + \frac{6a\,b^5x^5}{5} + \frac{b^6x^6}{6} + a^6\ln\left(x\right)$	65
risch	$6a^5bx + \frac{15a^4b^2x^2}{2} + \frac{20a^3b^3x^3}{3} + \frac{15a^2b^4x^4}{4} + \frac{6a\,b^5x^5}{5} + \frac{b^6x^6}{6} + a^6\ln\left(x\right)$	65
parallelrisch	$6a^5bx + \frac{15a^4b^2x^2}{2} + \frac{20a^3b^3x^3}{3} + \frac{15a^2b^4x^4}{4} + \frac{6a\,b^5x^5}{5} + \frac{b^6x^6}{6} + a^6\ln\left(x\right)$	65

input
```
int((b^2*x^2+2*a*b*x+a^2)^3/x,x,method=_RETURNVERBOSE)
```

output
```
6*a^5*b*x+15/2*a^4*b^2*x^2+20/3*a^3*b^3*x^3+15/4*a^2*b^4*x^4+6/5*a*b^5*x^5
+1/6*b^6*x^6+a^6*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx+b^2x^2)^3}{x}\,dx = \frac{1}{6}\,b^6x^6+\frac{6}{5}\,ab^5x^5+\frac{15}{4}\,a^2b^4x^4+\frac{20}{3}\,a^3b^3x^3+\frac{15}{2}\,a^4b^2x^2+6\,a^5bx+a^6\log\left(x\right)$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^3/x,x, algorithm="fricas")
```

output

```
1/6*b^6*x^6 + 6/5*a*b^5*x^5 + 15/4*a^2*b^4*x^4 + 20/3*a^3*b^3*x^3 + 15/2*a
^4*b^2*x^2 + 6*a^5*b*x + a^6*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \frac{(a^2+2abx+b^2x^2)^3}{x}\,dx = a^6\log\left(x\right)+6a^5bx+\frac{15a^4b^2x^2}{2}+\frac{20a^3b^3x^3}{3}+\frac{15a^2b^4x^4}{4}+\frac{6ab^5x^5}{5}+\frac{b^6x^6}{6}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**3/x,x)
```

output

```
a**6*log(x) + 6*a**5*b*x + 15*a**4*b**2*x**2/2 + 20*a**3*b**3*x**3/3 + 15*
a**2*b**4*x**4/4 + 6*a*b**5*x**5/5 + b**6*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{x} \, dx = \frac{1}{6} \, b^6x^6 + \frac{6}{5} \, ab^5x^5 + \frac{15}{4} \, a^2b^4x^4 + \frac{20}{3} \, a^3b^3x^3 + \frac{15}{2} \, a^4b^2x^2 + 6 \, a^5bx + a^6 \log{(x)}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^3/x,x, algorithm="maxima")
```

output
```
1/6*b^6*x^6 + 6/5*a*b^5*x^5 + 15/4*a^2*b^4*x^4 + 20/3*a^3*b^3*x^3 + 15/2*a
^4*b^2*x^2 + 6*a^5*b*x + a^6*log(x)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{x} \, dx = \frac{1}{6} \, b^6x^6 + \frac{6}{5} \, ab^5x^5 + \frac{15}{4} \, a^2b^4x^4 + \frac{20}{3} \, a^3b^3x^3 + \frac{15}{2} \, a^4b^2x^2 + 6 \, a^5bx + a^6 \log{(|x|)}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^3/x,x, algorithm="giac")
```

output
```
1/6*b^6*x^6 + 6/5*a*b^5*x^5 + 15/4*a^2*b^4*x^4 + 20/3*a^3*b^3*x^3 + 15/2*a
^4*b^2*x^2 + 6*a^5*b*x + a^6*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^3}{x}\,dx = a^6\,\ln\left(x\right)+\frac{b^6\,x^6}{6}+\frac{6\,a\,b^5\,x^5}{5}+\frac{15\,a^4\,b^2\,x^2}{2}+\frac{20\,a^3\,b^3\,x^3}{3}+\frac{15\,a^2\,b^4\,x^4}{4}+6\,a^5\,b\,x$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^3/x,x)
```

output
```
a^6*log(x) + (b^6*x^6)/6 + (6*a*b^5*x^5)/5 + (15*a^4*b^2*x^2)/2 + (20*a^3*
b^3*x^3)/3 + (15*a^2*b^4*x^4)/4 + 6*a^5*b*x
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^3}{x}\,dx = \log(x)\,a^6+6a^5bx+\frac{15a^4b^2x^2}{2}+\frac{20a^3b^3x^3}{3}+\frac{15a^2b^4x^4}{4}+\frac{6a\,b^5x^5}{5}+\frac{b^6x^6}{6}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^3/x,x)
```

output
```
(60*log(x)*a**6 + 360*a**5*b*x + 450*a**4*b**2*x**2 + 400*a**3*b**3*x**3 +
 225*a**2*b**4*x**4 + 72*a*b**5*x**5 + 10*b**6*x**6)/60
```

3.20 $\int \frac{(a^2+2abx+b^2x^2)^3}{x^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 69

$$\int \frac{(a^2+2abx+b^2x^2)^3}{x^2} dx = -\frac{a^6}{x}+15a^4b^2x+10a^3b^3x^2+5a^2b^4x^3+\frac{3}{2}ab^5x^4+\frac{b^6x^5}{5}+6a^5b\log(x)$$

output
```
-a^6/x+15*a^4*b^2*x+10*a^3*b^3*x^2+5*a^2*b^4*x^3+3/2*a*b^5*x^4+1/5*b^6*x^5
+6*a^5*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx+b^2x^2)^3}{x^2} dx = -\frac{a^6}{x}+15a^4b^2x+10a^3b^3x^2+5a^2b^4x^3+\frac{3}{2}ab^5x^4+\frac{b^6x^5}{5}+6a^5b\log(x)$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/x^2,x]
```

output
```
-(a^6/x) + 15*a^4*b^2*x + 10*a^3*b^3*x^2 + 5*a^2*b^4*x^3 + (3*a*b^5*x^4)/2
 + (b^6*x^5)/5 + 6*a^5*b*Log[x]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^3}{x^2}dx$$

$$\Big\downarrow \text{1098}$$

$$\frac{\int \frac{b^6(a+bx)^6}{x^2}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^6}{x^2}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^6}{x^2}+\frac{6a^5b}{x}+15a^4b^2+20a^3b^3x+15a^2b^4x^2+6ab^5x^3+b^6x^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^6}{x}+6a^5b\log(x)+15a^4b^2x+10a^3b^3x^2+5a^2b^4x^3+\frac{3}{2}ab^5x^4+\frac{b^6x^5}{5}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^3/x^2,x]
```

output
```
-(a^6/x) + 15*a^4*b^2*x + 10*a^3*b^3*x^2 + 5*a^2*b^4*x^3 + (3*a*b^5*x^4)/2
 + (b^6*x^5)/5 + 6*a^5*b*Log[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{a^{6}}{x}+15x\,a^{4}b^{2}+10a^{3}b^{3}x^{2}+5a^{2}b^{4}x^{3}+\frac{3a\,b^{5}x^{4}}{2}+\frac{b^{6}x^{5}}{5}+6a^{5}b\ln\left(x\right)$	66
risch	$-\frac{a^{6}}{x}+15x\,a^{4}b^{2}+10a^{3}b^{3}x^{2}+5a^{2}b^{4}x^{3}+\frac{3a\,b^{5}x^{4}}{2}+\frac{b^{6}x^{5}}{5}+6a^{5}b\ln\left(x\right)$	66
norman	$\frac{-a^{6}+\frac{1}{5}b^{6}x^{6}+\frac{3}{2}a\,b^{5}x^{5}+5a^{2}b^{4}x^{4}+10a^{3}b^{3}x^{3}+15a^{4}b^{2}x^{2}}{x}+6a^{5}b\ln\left(x\right)$	70
parallelrisch	$\frac{2b^{6}x^{6}+15a\,b^{5}x^{5}+50a^{2}b^{4}x^{4}+100a^{3}b^{3}x^{3}+60a^{5}b\ln\left(x\right)x+150a^{4}b^{2}x^{2}-10a^{6}}{10x}$	71

input
```
int((b^2*x^2+2*a*b*x+a^2)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^6/x+15*x*a^4*b^2+10*a^3*b^3*x^2+5*a^2*b^4*x^3+3/2*a*b^5*x^4+1/5*b^6*x^5
+6*a^5*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{(a^2+2abx+b^2x^2)^3}{x^2}\,dx = \frac{2\,b^6x^6+15\,ab^5x^5+50\,a^2b^4x^4+100\,a^3b^3x^3+150\,a^4b^2x^2+60\,a^5bx\log\left(x\right)-10\,a^6}{10\,x}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^3/x^2,x, algorithm="fricas")
```

output
```
1/10*(2*b^6*x^6 + 15*a*b^5*x^5 + 50*a^2*b^4*x^4 + 100*a^3*b^3*x^3 + 150*a^
4*b^2*x^2 + 60*a^5*b*x*log(x) - 10*a^6)/x
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{(a^2+2abx+b^2x^2)^3}{x^2}\,dx = -\frac{a^6}{x}+6a^5b\log\left(x\right)+15a^4b^2x+10a^3b^3x^2+5a^2b^4x^3+\frac{3ab^5x^4}{2}+\frac{b^6x^5}{5}$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**3/x**2,x)
```

output
```
-a**6/x + 6*a**5*b*log(x) + 15*a**4*b**2*x + 10*a**3*b**3*x**2 + 5*a**2*b*
*4*x**3 + 3*a*b**5*x**4/2 + b**6*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \frac{(a^2+2abx+b^2x^2)^3}{x^2}\,dx = \frac{1}{5}\,b^6x^5+\frac{3}{2}\,ab^5x^4+5\,a^2b^4x^3+10\,a^3b^3x^2 + 15\,a^4b^2x+6\,a^5b\log\left(x\right)-\frac{a^6}{x}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^3/x^2,x, algorithm="maxima")
```

output
```
1/5*b^6*x^5 + 3/2*a*b^5*x^4 + 5*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 15*a^4*b^2*
x + 6*a^5*b*log(x) - a^6/x
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{(a^2+2abx+b^2x^2)^3}{x^2}\,dx = \frac{1}{5}\,b^6x^5+\frac{3}{2}\,ab^5x^4+5\,a^2b^4x^3+10\,a^3b^3x^2 + 15\,a^4b^2x+6\,a^5b\log\left(|x|\right)-\frac{a^6}{x}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^3/x^2,x, algorithm="giac")
```

output
```
1/5*b^6*x^5 + 3/2*a*b^5*x^4 + 5*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 15*a^4*b^2*
x + 6*a^5*b*log(abs(x)) - a^6/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \frac{(a^2+2abx+b^2x^2)^3}{x^2}\,dx = \frac{b^6\,x^5}{5}-\frac{a^6}{x}+15\,a^4\,b^2\,x+\frac{3\,a\,b^5\,x^4}{2}+6\,a^5\,b\,\ln\left(x\right)+10\,a^3\,b^3\,x^2+5\,a^2\,b^4\,x^3$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^3/x^2,x)
```

output
```
(b^6*x^5)/5 - a^6/x + 15*a^4*b^2*x + (3*a*b^5*x^4)/2 + 6*a^5*b*log(x) + 10
*a^3*b^3*x^2 + 5*a^2*b^4*x^3
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{(a^2+2abx+b^2x^2)^3}{x^2}\,dx = \frac{60\log(x)\,a^5bx-10a^6+150a^4b^2x^2+100a^3b^3x^3+50a^2b^4x^4+15a\,b^5x^5+2b^6x^6}{10x}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^3/x^2,x)
```

output
```
(60*log(x)*a**5*b*x - 10*a**6 + 150*a**4*b**2*x**2 + 100*a**3*b**3*x**3 +
50*a**2*b**4*x**4 + 15*a*b**5*x**5 + 2*b**6*x**6)/(10*x)
```

3.21 $\int \frac{(a^2+2abx+b^2x^2)^3}{x^3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{(a^2+2abx+b^2x^2)^3}{x^3} \, dx = -\frac{a^6}{2x^2} - \frac{6a^5b}{x} + 20a^3b^3x + \frac{15}{2}a^2b^4x^2 + 2ab^5x^3 + \frac{b^6x^4}{4} + 15a^4b^2\log(x)$$

output
```
-1/2*a^6/x^2-6*a^5*b/x+20*a^3*b^3*x+15/2*a^2*b^4*x^2+2*a*b^5*x^3+1/4*b^6*x
^4+15*a^4*b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx+b^2x^2)^3}{x^3} \, dx = -\frac{a^6}{2x^2} - \frac{6a^5b}{x} + 20a^3b^3x + \frac{15}{2}a^2b^4x^2 + 2ab^5x^3 + \frac{b^6x^4}{4} + 15a^4b^2\log(x)$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/x^3,x]
```

output
```
-1/2*a^6/x^2 - (6*a^5*b)/x + 20*a^3*b^3*x + (15*a^2*b^4*x^2)/2 + 2*a*b^5*x
^3 + (b^6*x^4)/4 + 15*a^4*b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^3}{x^3}dx$$

$$\Big\downarrow \text{1098}$$

$$\frac{\int \frac{b^6(a+bx)^6}{x^3}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^6}{x^3}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^6}{x^3}+\frac{6a^5b}{x^2}+\frac{15a^4b^2}{x}+20a^3b^3+15a^2b^4x+6ab^5x^2+b^6x^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^6}{2x^2}-\frac{6a^5b}{x}+15a^4b^2\log(x)+20a^3b^3x+\frac{15}{2}a^2b^4x^2+2ab^5x^3+\frac{b^6x^4}{4}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^3/x^3,x]
```

output
```
-1/2*a^6/x^2 - (6*a^5*b)/x + 20*a^3*b^3*x + (15*a^2*b^4*x^2)/2 + 2*a*b^5*x
^3 + (b^6*x^4)/4 + 15*a^4*b^2*Log[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{a^{6}}{2x^{2}}-\frac{6a^{5}b}{x}+20b^{3}x\,a^{3}+\frac{15a^{2}b^{4}x^{2}}{2}+2a\,b^{5}x^{3}+\frac{b^{6}x^{4}}{4}+15a^{4}b^{2}\ln\left(x\right)$	66
risch	$\frac{b^{6}x^{4}}{4}+2a\,b^{5}x^{3}+\frac{15a^{2}b^{4}x^{2}}{2}+20b^{3}x\,a^{3}+\frac{-6a^{5}bx-\frac{1}{2}a^{6}}{x^{2}}+15a^{4}b^{2}\ln\left(x\right)$	66
norman	$\frac{-\frac{1}{2}a^{6}+\frac{1}{4}b^{6}x^{6}+2a\,b^{5}x^{5}+\frac{15}{2}a^{2}b^{4}x^{4}+20a^{3}b^{3}x^{3}-6a^{5}bx}{x^{2}}+15a^{4}b^{2}\ln\left(x\right)$	68
parallelrisch	$\frac{b^{6}x^{6}+8a\,b^{5}x^{5}+30a^{2}b^{4}x^{4}+60a^{4}b^{2}\ln\left(x\right)x^{2}+80a^{3}b^{3}x^{3}-24a^{5}bx-2a^{6}}{4x^{2}}$	70

input
```
int((b^2*x^2+2*a*b*x+a^2)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^6/x^2-6*a^5*b/x+20*b^3*x*a^3+15/2*a^2*b^4*x^2+2*a*b^5*x^3+1/4*b^6*x
^4+15*a^4*b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int \frac{(a^2+2abx+b^2x^2)^3}{x^3}\,dx = \frac{b^6x^6+8\,ab^5x^5+30\,a^2b^4x^4+80\,a^3b^3x^3+60\,a^4b^2x^2\log(x)-24\,a^5bx-2\,a^6}{4\,x^2}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^3/x^3,x, algorithm="fricas")
```

output
```
1/4*(b^6*x^6 + 8*a*b^5*x^5 + 30*a^2*b^4*x^4 + 80*a^3*b^3*x^3 + 60*a^4*b^2*
x^2*log(x) - 24*a^5*b*x - 2*a^6)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx+b^2x^2)^3}{x^3}\,dx = 15a^4b^2\log(x)+20a^3b^3x+\frac{15a^2b^4x^2}{2}+2ab^5x^3+\frac{b^6x^4}{4}+\frac{-a^6-12a^5bx}{2x^2}$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**3/x**3,x)
```

output
```
15*a**4*b**2*log(x) + 20*a**3*b**3*x + 15*a**2*b**4*x**2/2 + 2*a*b**5*x**3
 + b**6*x**4/4 + (-a**6 - 12*a**5*b*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

$$\int \frac{(a^2+2abx+b^2x^2)^3}{x^3}\,dx = \frac{1}{4}\,b^6x^4+2\,ab^5x^3+\frac{15}{2}\,a^2b^4x^2+20\,a^3b^3x+15\,a^4b^2\log\left(x\right)-\frac{12\,a^5bx+a^6}{2\,x^2}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^3/x^3,x, algorithm="maxima")
```

output
```
1/4*b^6*x^4 + 2*a*b^5*x^3 + 15/2*a^2*b^4*x^2 + 20*a^3*b^3*x + 15*a^4*b^2*l
og(x) - 1/2*(12*a^5*b*x + a^6)/x^2
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{(a^2+2abx+b^2x^2)^3}{x^3}\,dx = \frac{1}{4}\,b^6x^4+2\,ab^5x^3+\frac{15}{2}\,a^2b^4x^2+20\,a^3b^3x+15\,a^4b^2\log\left(|x|\right)-\frac{12\,a^5bx+a^6}{2\,x^2}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^3/x^3,x, algorithm="giac")
```

output
```
1/4*b^6*x^4 + 2*a*b^5*x^3 + 15/2*a^2*b^4*x^2 + 20*a^3*b^3*x + 15*a^4*b^2*l
og(abs(x)) - 1/2*(12*a^5*b*x + a^6)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \frac{(a^2+2abx+b^2x^2)^3}{x^3}\,dx = \frac{b^6\,x^4}{4}-\frac{\frac{a^6}{2}+6\,b\,x\,a^5}{x^2}+20\,a^3\,b^3\,x +2\,a\,b^5\,x^3+\frac{15\,a^2\,b^4\,x^2}{2}+15\,a^4\,b^2\,\ln\left(x\right)$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^3/x^3,x)
```

output
```
(b^6*x^4)/4 - (a^6/2 + 6*a^5*b*x)/x^2 + 20*a^3*b^3*x + 2*a*b^5*x^3 + (15*a
^2*b^4*x^2)/2 + 15*a^4*b^2*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int \frac{(a^2+2abx+b^2x^2)^3}{x^3}\,dx = \frac{60\log(x)\,a^4b^2x^2-2a^6-24a^5bx+80a^3b^3x^3+30a^2b^4x^4+8a\,b^5x^5+b^6x^6}{4x^2}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^3/x^3,x)
```

output
```
(60*log(x)*a**4*b**2*x**2 - 2*a**6 - 24*a**5*b*x + 80*a**3*b**3*x**3 + 30*
a**2*b**4*x**4 + 8*a*b**5*x**5 + b**6*x**6)/(4*x**2)
```

3.22 $\int \frac{9+6x+x^2}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 11

$$\int \frac{9+6x+x^2}{x^2}\,dx=-\frac{9}{x}+x+6\log(x)$$

output
```
-9/x+x+6*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{9+6x+x^2}{x^2}\,dx=-\frac{9}{x}+x+6\log(x)$$

input
```
Integrate[(9 + 6*x + x^2)/x^2,x]
```

output
```
-9/x + x + 6*Log[x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1098, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+6x+9}{x^2}\,dx$$

$$\Big\downarrow \text{ 1098}$$

$$\int \frac{(x+3)^2}{x^2}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{9}{x^2}+\frac{6}{x}+1\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$x-\frac{9}{x}+6\log(x)$$

input
```
Int[(9 + 6*x + x^2)/x^2,x]
```

output
```
-9/x + x + 6*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
default	$-\frac{9}{x}+x+6\ln\left(x\right)$	12
risch	$-\frac{9}{x}+x+6\ln\left(x\right)$	12
norman	$\frac{x^{2}-9}{x}+6\ln\left(x\right)$	15
parallelrisch	$\frac{6\ln\left(x\right)x+x^{2}-9}{x}$	15

input
```
int((x^2+6*x+9)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-9/x+x+6*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{9+6x+x^2}{x^2}\,dx=\frac{x^{2}+6\,x\log\left(x\right)-9}{x}$$

input
```
integrate((x^2+6*x+9)/x^2,x, algorithm="fricas")
```

output
```
(x^2 + 6*x*log(x) - 9)/x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{9+6x+x^2}{x^2}\,dx = x + 6\log{(x)} - \frac{9}{x}$$

input
```
integrate((x**2+6*x+9)/x**2,x)
```

output
```
x + 6*log(x) - 9/x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{9+6x+x^2}{x^2}\,dx = x - \frac{9}{x} + 6\,\log{(x)}$$

input
```
integrate((x^2+6*x+9)/x^2,x, algorithm="maxima")
```

output
```
x - 9/x + 6*log(x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{9+6x+x^2}{x^2}\,dx = x - \frac{9}{x} + 6\,\log{(|x|)}$$

input
```
integrate((x^2+6*x+9)/x^2,x, algorithm="giac")
```

output
```
x - 9/x + 6*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{9+6x+x^2}{x^2}\,dx = x + 6\ \ln(x) - \frac{9}{x}$$

input
```
int((6*x + x^2 + 9)/x^2,x)
```

output
```
x + 6*log(x) - 9/x
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{9+6x+x^2}{x^2}\,dx = \frac{6\log(x)\,x + x^2 - 9}{x}$$

input
```
int((x^2+6*x+9)/x^2,x)
```

output
```
(6*log(x)*x + x**2 - 9)/x
```

3.23 $\int \frac{1+2x+x^2}{x^4} dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1+2x+x^2}{x^4} \, dx = -\frac{(1+x)^3}{3x^3}$$

output
```
-1/3*(1+x)^3/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{1+2x+x^2}{x^4} \, dx = -\frac{1}{3x^3} - \frac{1}{x^2} - \frac{1}{x}$$

input
```
Integrate[(1 + 2*x + x^2)/x^4,x]
```

output
```
-1/3*1/x^3 - x^(-2) - x^(-1)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1098, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+2x+1}{x^4} \, dx$$

$$\Big\downarrow \ 1098$$

$$\int \frac{(x+1)^2}{x^4}dx$$

$$\Big\downarrow \ 48$$

$$-\frac{(x+1)^3}{3x^3}$$

input
```
Int[(1 + 2*x + x^2)/x^4,x]
```

output
```
-1/3*(1 + x)^3/x^3
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

method	result	size
norman	$\frac{-x^2-x-\frac{1}{3}}{x^3}$	15
risch	$\frac{-x^2-x-\frac{1}{3}}{x^3}$	15
gosper	$-\frac{3x^2+3x+1}{3x^3}$	16
parallelrisch	$\frac{-3x^2-3x-1}{3x^3}$	16
default	$-\frac{1}{3x^3}-\frac{1}{x^2}-\frac{1}{x}$	17
orering	$-\frac{(3x^2+3x+1)(x^2+2x+1)}{3x^3(x+1)^2}$	29

input
```
int((x^2+2*x+1)/x^4,x,method=_RETURNVERBOSE)
```

output
```
(-x^2-x-1/3)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{1+2x+x^2}{x^4}\,dx=-\frac{3\,x^2+3\,x+1}{3\,x^3}$$

input
```
integrate((x^2+2*x+1)/x^4,x, algorithm="fricas")
```

output
```
-1/3*(3*x^2 + 3*x + 1)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{1+2x+x^2}{x^4}\,dx = \frac{-3x^2-3x-1}{3x^3}$$

input
```
integrate((x**2+2*x+1)/x**4,x)
```

output
```
(-3*x**2 - 3*x - 1)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{1+2x+x^2}{x^4}\,dx = -\frac{3\,x^2+3\,x+1}{3\,x^3}$$

input
```
integrate((x^2+2*x+1)/x^4,x, algorithm="maxima")
```

output
```
-1/3*(3*x^2 + 3*x + 1)/x^3
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{1+2x+x^2}{x^4}\,dx = -\frac{3\,x^2+3\,x+1}{3\,x^3}$$

input
```
integrate((x^2+2*x+1)/x^4,x, algorithm="giac")
```

output
```
-1/3*(3*x^2 + 3*x + 1)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1+2x+x^2}{x^4}\,dx = -\frac{x^2+x+\frac{1}{3}}{x^3}$$

input
```
int((2*x + x^2 + 1)/x^4,x)
```

output
```
-(x + x^2 + 1/3)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{1+2x+x^2}{x^4}\,dx = \frac{-3x^2-3x-1}{3x^3}$$

input
```
int((x^2+2*x+1)/x^4,x)
```

output
```
( - 3*x**2 - 3*x - 1)/(3*x**3)
```

3.24 $\int \frac{x^3}{a^2+2abx+b^2x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{x^3}{a^2+2abx+b^2x^2}\,dx=-\frac{2ax}{b^3}+\frac{x^2}{2b^2}+\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}$$

output
```
-2*a*x/b^3+1/2*x^2/b^2+a^3/b^4/(b*x+a)+3*a^2*ln(b*x+a)/b^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{a^2+2abx+b^2x^2}\,dx=\frac{-4abx+b^2x^2+\frac{2a^3}{a+bx}+6a^2\log(a+bx)}{2b^4}$$

input
```
Integrate[x^3/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(-4*a*b*x + b^2*x^2 + (2*a^3)/(a + b*x) + 6*a^2*Log[a + b*x])/(2*b^4)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a^2+2abx+b^2x^2} \, dx$$

$$\Big\downarrow \text{1098}$$

$$b^2 \int \frac{x^3}{b^2(a+bx)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^3}{(a+bx)^2}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}-\frac{2a}{b^3}+\frac{x}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}$$

input
```
Int[x^3/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(-2*a*x)/b^3 + x^2/(2*b^2) + a^3/(b^4*(a + b*x)) + (3*a^2*Log[a + b*x])/b^
4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

method	result	size
risch	$-\frac{2ax}{b^3}+\frac{x^2}{2b^2}+\frac{a^3}{b^4(bx+a)}+\frac{3a^2\ln(bx+a)}{b^4}$	45
default	$-\frac{-\frac{1}{2}b\,x^2+2ax}{b^3}+\frac{a^3}{b^4(bx+a)}+\frac{3a^2\ln(bx+a)}{b^4}$	46
norman	$\frac{\frac{3a^3}{b^4}+\frac{x^3}{2b}-\frac{3a\,x^2}{2b^2}}{bx+a}+\frac{3a^2\ln(bx+a)}{b^4}$	50
parallelrisch	$\frac{b^3x^3+6\ln(bx+a)x\,a^2b-3a\,b^2x^2+6\ln(bx+a)a^3+6a^3}{2b^4(bx+a)}$	59

input
```
int(x^3/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
-2*a*x/b^3+1/2/b^2*x^2+a^3/b^4/(b*x+a)+3*a^2*ln(b*x+a)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.35

$$\int \frac{x^3}{a^2+2abx+b^2x^2}\,dx = \frac{b^3x^3-3\,ab^2x^2-4\,a^2bx+2\,a^3+6\,(a^2bx+a^3)\log{(bx+a)}}{2\,(b^5x+ab^4)}$$

input
```
integrate(x^3/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
 + a))/(b^5*x + a*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{x^3}{a^2+2abx+b^2x^2}\,dx = \frac{a^3}{ab^4+b^5x}+\frac{3a^2\log{(a+bx)}}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}$$

input
```
integrate(x**3/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
a**3/(a*b**4 + b**5*x) + 3*a**2*log(a + b*x)/b**4 - 2*a*x/b**3 + x**2/(2*b
**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{a^2+2abx+b^2x^2}\,dx = \frac{a^3}{b^5x+ab^4}+\frac{3\,a^2\log{(bx+a)}}{b^4}+\frac{bx^2-4\,ax}{2\,b^3}$$

input
```
integrate(x^3/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
a^3/(b^5*x + a*b^4) + 3*a^2*log(b*x + a)/b^4 + 1/2*(b*x^2 - 4*a*x)/b^3
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \frac{x^3}{a^2+2abx+b^2x^2}\,dx=\frac{3\,a^2\log\left(|bx+a|\right)}{b^4}+\frac{a^3}{(bx+a)b^4}+\frac{b^2x^2-4\,abx}{2\,b^4}$$

input
```
integrate(x^3/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
3*a^2*log(abs(b*x + a))/b^4 + a^3/((b*x + a)*b^4) + 1/2*(b^2*x^2 - 4*a*b*x
)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{a^2+2abx+b^2x^2}\,dx=\frac{x^2}{2\,b^2}+\frac{3\,a^2\,\ln\left(a+b\,x\right)}{b^4}+\frac{a^3}{b\,\left(x\,b^4+a\,b^3\right)}-\frac{2\,a\,x}{b^3}$$

input
```
int(x^3/(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
x^2/(2*b^2) + (3*a^2*log(a + b*x))/b^4 + a^3/(b*(a*b^3 + b^4*x)) - (2*a*x)
/b^3
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \frac{x^3}{a^2+2abx+b^2x^2}\,dx=\frac{6\log(bx+a)\,a^3+6\log(bx+a)\,a^2bx-6a^2bx-3a\,b^2x^2+b^3x^3}{2b^4\,(bx+a)}$$

input
```
int(x^3/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(6*log(a + b*x)*a**3 + 6*log(a + b*x)*a**2*b*x - 6*a**2*b*x - 3*a*b**2*x**
2 + b**3*x**3)/(2*b**4*(a + b*x))
```

3.25 $\int \frac{x^2}{a^2+2abx+b^2x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 33

$$\int \frac{x^2}{a^2+2abx+b^2x^2}\,dx=\frac{x}{b^2}-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}$$

output
```
x/b^2-a^2/b^3/(b*x+a)-2*a*ln(b*x+a)/b^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{a^2+2abx+b^2x^2}\,dx=\frac{bx-\frac{a^2}{a+bx}-2a\log(a+bx)}{b^3}$$

input
```
Integrate[x^2/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(b*x - a^2/(a + b*x) - 2*a*Log[a + b*x])/b^3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a^2+2abx+b^2x^2} \, dx$$

$$\Big\downarrow \text{1098}$$

$$b^2 \int \frac{x^2}{b^2(a+bx)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^2}{(a+bx)^2}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}+\frac{1}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}+\frac{x}{b^2}$$

input
```
Int[x^2/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*Log[a + b*x])/b^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{x}{b^2}-\frac{a^2}{b^3(bx+a)}-\frac{2a\ln(bx+a)}{b^3}$	34
risch	$\frac{x}{b^2}-\frac{a^2}{b^3(bx+a)}-\frac{2a\ln(bx+a)}{b^3}$	34
norman	$\frac{\frac{x^2}{b}-\frac{2a^2}{b^3}}{bx+a}-\frac{2a\ln(bx+a)}{b^3}$	38
parallelrisch	$-\frac{2\ln(bx+a)xab-b^2x^2+2\ln(bx+a)a^2+2a^2}{b^3(bx+a)}$	49

input
```
int(x^2/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
x/b^2-a^2/b^3/(b*x+a)-2*a*ln(b*x+a)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.42

$$\int \frac{x^2}{a^2+2abx+b^2x^2}\,dx=\frac{b^2x^2+abx-a^2-2\left(abx+a^2\right)\log\left(bx+a\right)}{b^4x+ab^3}$$

input
```
integrate(x^2/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
(b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))/(b^4*x + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{a^2+2abx+b^2x^2}\,dx=-\frac{a^2}{ab^3+b^4x}-\frac{2a\log\left(a+bx\right)}{b^3}+\frac{x}{b^2}$$

input
```
integrate(x**2/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
-a**2/(a*b**3 + b**4*x) - 2*a*log(a + b*x)/b**3 + x/b**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{a^2+2abx+b^2x^2}\,dx=-\frac{a^2}{b^4x+ab^3}+\frac{x}{b^2}-\frac{2\,a\log\left(bx+a\right)}{b^3}$$

input
```
integrate(x^2/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
-a^2/(b^4*x + a*b^3) + x/b^2 - 2*a*log(b*x + a)/b^3
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{x^2}{a^2+2abx+b^2x^2}\,dx = \frac{x}{b^2} - \frac{2\,a\log\left(|bx+a|\right)}{b^3} - \frac{a^2}{(bx+a)b^3}$$

input
```
integrate(x^2/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
x/b^2 - 2*a*log(abs(b*x + a))/b^3 - a^2/((b*x + a)*b^3)
```

Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{a^2+2abx+b^2x^2}\,dx = \frac{x}{b^2} - \frac{a^2}{x\,b^4+a\,b^3} - \frac{2\,a\,\ln\left(a+b\,x\right)}{b^3}$$

input
```
int(x^2/(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
x/b^2 - a^2/(a*b^3 + b^4*x) - (2*a*log(a + b*x))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \frac{x^2}{a^2+2abx+b^2x^2}\,dx = \frac{-2\log(bx+a)\,a^2-2\log(bx+a)\,abx+2abx+b^2x^2}{b^3\,(bx+a)}$$

input
```
int(x^2/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
( - 2*log(a + b*x)*a**2 - 2*log(a + b*x)*a*b*x + 2*a*b*x + b**2*x**2)/(b**
3*(a + b*x))
```

3.26 $\int \frac{x}{a^2+2abx+b^2x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 23

$$\int \frac{x}{a^2+2abx+b^2x^2}\,dx=\frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}$$

output
```
a/b^2/(b*x+a)+ln(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x}{a^2+2abx+b^2x^2}\,dx=\frac{\frac{a}{a+bx}+\log(a+bx)}{b^2}$$

input
```
Integrate[x/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(a/(a + b*x) + Log[a + b*x])/b^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a^2+2abx+b^2x^2}\,dx$$
$$\Big\downarrow \text{1098}$$
$$b^2\int \frac{x}{b^2(a+bx)^2}dx$$
$$\Big\downarrow \text{27}$$
$$\int \frac{x}{(a+bx)^2}dx$$
$$\Big\downarrow \text{49}$$
$$\int \left(\frac{1}{b(a+bx)}-\frac{a}{b(a+bx)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}$$

input
```
Int[x/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
a/(b^2*(a + b*x)) + Log[a + b*x]/b^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{a}{b^{2}(bx+a)}+\frac{\ln(bx+a)}{b^{2}}$	24
norman	$\frac{a}{b^{2}(bx+a)}+\frac{\ln(bx+a)}{b^{2}}$	24
risch	$\frac{a}{b^{2}(bx+a)}+\frac{\ln(bx+a)}{b^{2}}$	24
parallelrisch	$\frac{\ln(bx+a)xb+\ln(bx+a)a+a}{b^{2}(bx+a)}$	31

input
```
int(x/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
a/b^2/(b*x+a)+ln(b*x+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \frac{x}{a^2 + 2abx + b^2x^2}\, dx = \frac{(bx + a)\log(bx + a) + a}{b^3x + ab^2}$$

input
```
integrate(x/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
((b*x + a)*log(b*x + a) + a)/(b^3*x + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x}{a^2 + 2abx + b^2x^2}\, dx = \frac{a}{ab^2 + b^3x} + \frac{\log(a + bx)}{b^2}$$

input
```
integrate(x/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
a/(a*b**2 + b**3*x) + log(a + b*x)/b**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{x}{a^2 + 2abx + b^2x^2}\, dx = \frac{a}{b^3x + ab^2} + \frac{\log(bx + a)}{b^2}$$

input
```
integrate(x/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
a/(b^3*x + a*b^2) + log(b*x + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{x}{a^2+2abx+b^2x^2}\,dx=\frac{\log\left(|bx+a|\right)}{b^2}+\frac{a}{(bx+a)b^2}$$

input
```
integrate(x/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
log(abs(b*x + a))/b^2 + a/((b*x + a)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x}{a^2+2abx+b^2x^2}\,dx=\frac{\ln\left(a+b\,x\right)}{b^2}+\frac{a}{b^2\,\left(a+b\,x\right)}$$

input
```
int(x/(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
log(a + b*x)/b^2 + a/(b^2*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{x}{a^2+2abx+b^2x^2}\,dx=\frac{\log(bx+a)\,a+\log(bx+a)\,bx-bx}{b^2\,(bx+a)}$$

input
```
int(x/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(log(a + b*x)*a + log(a + b*x)*b*x - b*x)/(b**2*(a + b*x))
```

3.27 $\int \frac{1}{a^2+2abx+b^2x^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 12

$$\int \frac{1}{a^2+2abx+b^2x^2} dx = -\frac{1}{b(a+bx)}$$

output
```
-1/b/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+2abx+b^2x^2} dx = -\frac{1}{b(a+bx)}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(-1),x]
```

output
```
-(1/(b*(a + b*x)))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1077, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^2+2abx+b^2x^2}dx$$

$$\Big\downarrow \ 1077$$

$$b^2\int \frac{1}{(xb^2+ab)^2}dx$$

$$\Big\downarrow \ 17$$

$$-\frac{1}{b(a+bx)}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(-1),x]
```

output
```
-(1/(b*(a + b*x)))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1077
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int
[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] &&
 IntegerQ[p]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
gosper	$-\frac{1}{b(bx+a)}$	13
default	$-\frac{1}{b(bx+a)}$	13
norman	$\frac{x}{a(bx+a)}$	13
risch	$-\frac{1}{b(bx+a)}$	13
parallelrisch	$-\frac{1}{b(bx+a)}$	13
orering	$-\frac{bx+a}{b(b^2x^2+2abx+a^2)}$	29

input
```
int(1/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
-1/b/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{a^2+2abx+b^2x^2}\,dx=-\frac{1}{b^2x+ab}$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
-1/(b^2*x + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{a^2 + 2abx + b^2x^2} \, dx = -\frac{1}{ab + b^2x}$$

input
```
integrate(1/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
-1/(a*b + b**2*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{a^2 + 2abx + b^2x^2} \, dx = -\frac{1}{b^2x + ab}$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
-1/(b^2*x + a*b)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + 2abx + b^2x^2} \, dx = -\frac{1}{(bx + a)b}$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
-1/((b*x + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+2abx+b^2x^2}\,dx=-\frac{1}{b\,(a+b\,x)}$$

input
```
int(1/(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
-1/(b*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+2abx+b^2x^2}\,dx=\frac{x}{a\,(bx+a)}$$

input
```
int(1/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
x/(a*(a + b*x))
```

3.28 $\int \frac{1}{x(a^2+2abx+b^2x^2)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 29

$$\int \frac{1}{x(a^2+2abx+b^2x^2)} \, dx = \frac{1}{a(a+bx)} + \frac{\log(x)}{a^2} - \frac{\log(a+bx)}{a^2}$$

output
```
1/a/(b*x+a)+ln(x)/a^2-ln(b*x+a)/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{1}{x(a^2+2abx+b^2x^2)} \, dx = \frac{\frac{a}{a+bx} + \log(x) - \log(a+bx)}{a^2}$$

input
```
Integrate[1/(x*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output
```
(a/(a + b*x) + Log[x] - Log[a + b*x])/a^2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1098, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)}dx$$

$$\Big\downarrow \text{ 1098}$$

$$b^2\int \frac{1}{b^2x(a+bx)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{x(a+bx)^2}dx$$

$$\Big\downarrow \text{ 54}$$

$$\int \left(-\frac{b}{a^2(a+bx)}+\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\log(a+bx)}{a^2}+\frac{\log(x)}{a^2}+\frac{1}{a(a+bx)}$$

input
```
Int[1/(x*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output
```
1/(a*(a + b*x)) + Log[x]/a^2 - Log[a + b*x]/a^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{1}{a(bx+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(bx+a)}{a^2}$	30
risch	$\frac{1}{a(bx+a)}-\frac{\ln(bx+a)}{a^2}+\frac{\ln(-x)}{a^2}$	32
norman	$-\frac{bx}{a^2(bx+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(bx+a)}{a^2}$	33
parallelrisch	$\frac{\ln(x)xb-\ln(bx+a)xb+a\ln(x)-\ln(bx+a)a-bx}{a^2(bx+a)}$	45

input
```
int(1/x/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
1/a/(b*x+a)+ln(x)/a^2-ln(b*x+a)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)}\,dx = -\frac{(bx+a)\log(bx+a)-(bx+a)\log(x)-a}{a^2bx+a^3}$$

input
```
integrate(1/x/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
-((b*x + a)*log(b*x + a) - (b*x + a)*log(x) - a)/(a^2*b*x + a^3)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)}\,dx = \frac{1}{a^2+abx}+\frac{\log(x)-\log\left(\frac{a}{b}+x\right)}{a^2}$$

input
```
integrate(1/x/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
1/(a**2 + a*b*x) + (log(x) - log(a/b + x))/a**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)}\,dx = \frac{1}{abx+a^2}-\frac{\log(bx+a)}{a^2}+\frac{\log(x)}{a^2}$$

input
```
integrate(1/x/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
1/(a*b*x + a^2) - log(b*x + a)/a^2 + log(x)/a^2
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)}\,dx=-\frac{\log\left(|bx+a|\right)}{a^2}+\frac{\log\left(|x|\right)}{a^2}+\frac{1}{(bx+a)a}$$

input
```
integrate(1/x/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
-log(abs(b*x + a))/a^2 + log(abs(x))/a^2 + 1/((b*x + a)*a)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)}\,dx=\frac{1}{a^2+b\,x\,a}-\frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2}$$

input
```
int(1/(x*(a^2 + b^2*x^2 + 2*a*b*x)),x)
```

output
```
1/(a^2 + a*b*x) - (2*atanh((2*b*x)/a + 1))/a^2
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.52

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)}\,dx=\frac{-\log(bx+a)\,a-\log(bx+a)\,bx+\log(x)\,a+\log(x)\,bx-bx}{a^2\,(bx+a)}$$

input
```
int(1/x/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
( - log(a + b*x)*a - log(a + b*x)*b*x + log(x)*a + log(x)*b*x - b*x)/(a**2
*(a + b*x))
```

3.29 $\int \frac{1}{x^2(a^2+2abx+b^2x^2)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 42

$$\int \frac{1}{x^2\left(a^2+2abx+b^2x^2\right)}\,dx=-\frac{1}{a^2x}-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}$$

output
```
-1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2\left(a^2+2abx+b^2x^2\right)}\,dx=-\frac{a\left(\frac{1}{x}+\frac{b}{a+bx}\right)+2b\log(x)-2b\log(a+bx)}{a^3}$$

input
```
Integrate[1/(x^2*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output
```
-((a*(x^(-1) + b/(a + b*x)) + 2*b*Log[x] - 2*b*Log[a + b*x])/a^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1098, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)} dx$$

$$\Big\downarrow \text{1098}$$

$$b^2 \int \frac{1}{b^2x^2(a+bx)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{x^2(a+bx)^2}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{2b^2}{a^3(a+bx)} - \frac{2b}{a^3x} + \frac{b^2}{a^2(a+bx)^2} + \frac{1}{a^2x^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2b\log(x)}{a^3} + \frac{2b\log(a+bx)}{a^3} - \frac{b}{a^2(a+bx)} - \frac{1}{a^2x}$$

input
```
Int[1/(x^2*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output
```
-(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x])/a
^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{1}{a^{2} x}-\frac{b}{a^{2} \left(b x +a \right)}-\frac{2 b \ln \left(x \right)}{a^{3}}+\frac{2 b \ln \left(b x +a \right)}{a^{3}}$	43
norman	$\frac{-\frac{2 b x}{a^{2}}-\frac{1}{a}}{x \left(b x +a \right)}-\frac{2 b \ln \left(x \right)}{a^{3}}+\frac{2 b \ln \left(b x +a \right)}{a^{3}}$	46
risch	$\frac{-\frac{2 b x}{a^{2}}-\frac{1}{a}}{x \left(b x +a \right)}-\frac{2 b \ln \left(x \right)}{a^{3}}+\frac{2 b \ln \left(-b x -a \right)}{a^{3}}$	49
parallelrisch	$-\frac{2 b^{2} \ln \left(x \right) x^{2}-2 \ln \left(b x +a \right) x^{2} b^{2}+2 a b \ln \left(x \right) x -2 \ln \left(b x +a \right) x a b -2 b^{2} x^{2}+a^{2}}{a^{3} x \left(b x +a \right)}$	70

input
```
int(1/x^2/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.50

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)} \, dx = -\frac{2\,abx + a^2 - 2\,(b^2x^2 + abx)\log(bx + a) + 2\,(b^2x^2 + abx)\log(x)}{a^3bx^2 + a^4x}$$

input
```
integrate(1/x^2/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
-(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log(b*x + a) + 2*(b^2*x^2 + a*b*x)*l
og(x))/(a^3*b*x^2 + a^4*x)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)} \, dx = \frac{-a - 2bx}{a^3x + a^2bx^2} + \frac{2b\left(-\log(x) + \log\left(\frac{a}{b} + x\right)\right)}{a^3}$$

input
```
integrate(1/x**2/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
(-a - 2*b*x)/(a**3*x + a**2*b*x**2) + 2*b*(-log(x) + log(a/b + x))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)} \, dx = -\frac{2\,bx + a}{a^2bx^2 + a^3x} + \frac{2\,b\log(bx + a)}{a^3} - \frac{2\,b\log(x)}{a^3}$$

input
```
integrate(1/x^2/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
-(2*b*x + a)/(a^2*b*x^2 + a^3*x) + 2*b*log(b*x + a)/a^3 - 2*b*log(x)/a^3
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)} \, dx = \frac{2\, b \log (|bx + a|)}{a^3} - \frac{2\, b \log (|x|)}{a^3} - \frac{2\, bx + a}{(bx^2 + ax)a^2}$$

input
```
integrate(1/x^2/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
2*b*log(abs(b*x + a))/a^3 - 2*b*log(abs(x))/a^3 - (2*b*x + a)/((b*x^2 + a*
x)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)} \, dx = \frac{4\, b \operatorname{atanh}\left(\frac{2\, b\, x}{a} + 1\right)}{a^3} - \frac{\frac{1}{a} + \frac{2\, b\, x}{a^2}}{b\, x^2 + a\, x}$$

input
```
int(1/(x^2*(a^2 + b^2*x^2 + 2*a*b*x)),x)
```

output
```
(4*b*atanh((2*b*x)/a + 1))/a^3 - (1/a + (2*b*x)/a^2)/(a*x + b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.67

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)} \, dx$$
$$= \frac{2 \log(bx + a)\, abx + 2 \log(bx + a)\, b^2x^2 - 2 \log(x)\, abx - 2 \log(x)\, b^2x^2 - a^2 + 2b^2x^2}{a^3x\, (bx + a)}$$

input
```
int(1/x^2/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(2*log(a + b*x)*a*b*x + 2*log(a + b*x)*b**2*x**2 - 2*log(x)*a*b*x - 2*log(
x)*b**2*x**2 - a**2 + 2*b**2*x**2)/(a**3*x*(a + b*x))
```

3.30 $\int \frac{x^3}{(a^2+2abx+b^2x^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 58

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{a^3}{3b^4(a+bx)^3} - \frac{3a^2}{2b^4(a+bx)^2} + \frac{3a}{b^4(a+bx)} + \frac{\log(a+bx)}{b^4}$$

output
```
1/3*a^3/b^4/(b*x+a)^3-3/2*a^2/b^4/(b*x+a)^2+3*a/b^4/(b*x+a)+ln(b*x+a)/b^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{\frac{a(11a^2+27abx+18b^2x^2)}{(a+bx)^3} + 6\log(a+bx)}{6b^4}$$

input
```
Integrate[x^3/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
((a*(11*a^2 + 27*a*b*x + 18*b^2*x^2))/(a + b*x)^3 + 6*Log[a + b*x])/(6*b^4
)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^2}dx$$

$$\Big\downarrow \text{ 1098}$$

$$b^4\int \frac{x^3}{b^4(a+bx)^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x^3}{(a+bx)^4}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(-\frac{a^3}{b^3(a+bx)^4}+\frac{3a^2}{b^3(a+bx)^3}-\frac{3a}{b^3(a+bx)^2}+\frac{1}{b^3(a+bx)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^3}{3b^4(a+bx)^3}-\frac{3a^2}{2b^4(a+bx)^2}+\frac{3a}{b^4(a+bx)}+\frac{\log(a+bx)}{b^4}$$

input
```
Int[x^3/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
a^3/(3*b^4*(a + b*x)^3) - (3*a^2)/(2*b^4*(a + b*x)^2) + (3*a)/(b^4*(a + b*
x)) + Log[a + b*x]/b^4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

method	result	size
norman	$\frac{\frac{11a^3}{6b^4}+\frac{3a\,x^2}{b^2}+\frac{9a^2x}{2b^3}}{(bx+a)^3}+\frac{\ln(bx+a)}{b^4}$	47
default	$\frac{a^3}{3b^4(bx+a)^3}-\frac{3a^2}{2b^4(bx+a)^2}+\frac{3a}{b^4(bx+a)}+\frac{\ln(bx+a)}{b^4}$	55
risch	$\frac{\frac{11a^3}{6b^4}+\frac{3a\,x^2}{b^2}+\frac{9a^2x}{2b^3}}{(bx+a)(b^2x^2+2abx+a^2)}+\frac{\ln(bx+a)}{b^4}$	65
parallelrisch	$\frac{6\ln(bx+a)x^3b^3+18\ln(bx+a)x^2a\,b^2+18\ln(bx+a)x\,a^2b+18a\,b^2x^2+6\ln(bx+a)a^3+27a^2bx+11a^3}{6b^4(b^2x^2+2abx+a^2)(bx+a)}$	106

input
```
int(x^3/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(11/6*a^3/b^4+3*a*x^2/b^2+9/2*a^2*x/b^3)/(b*x+a)^3+ln(b*x+a)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.62

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{18\,ab^2x^2+27\,a^2bx+11\,a^3+6\,(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)\log{(bx+a)}}{6\,(b^7x^3+3\,ab^6x^2+3\,a^2b^5x+a^3b^4)}$$

input
```
integrate(x^3/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
1/6*(18*a*b^2*x^2 + 27*a^2*b*x + 11*a^3 + 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2
*b*x + a^3)*log(b*x + a))/(b^7*x^3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.21

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{11a^3+27a^2bx+18ab^2x^2}{6a^3b^4+18a^2b^5x+18ab^6x^2+6b^7x^3}+\frac{\log{(a+bx)}}{b^4}$$

input
```
integrate(x**3/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
(11*a**3 + 27*a**2*b*x + 18*a*b**2*x**2)/(6*a**3*b**4 + 18*a**2*b**5*x + 1
8*a*b**6*x**2 + 6*b**7*x**3) + log(a + b*x)/b**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.21

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{18\,ab^2x^2+27\,a^2bx+11\,a^3}{6\,(b^7x^3+3\,ab^6x^2+3\,a^2b^5x+a^3b^4)}+\frac{\log{(bx+a)}}{b^4}$$

input
```
integrate(x^3/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
1/6*(18*a*b^2*x^2 + 27*a^2*b*x + 11*a^3)/(b^7*x^3 + 3*a*b^6*x^2 + 3*a^2*b^
5*x + a^3*b^4) + log(b*x + a)/b^4
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{\log(|bx+a|)}{b^4} + \frac{18\,abx^2+27\,a^2x+\frac{11\,a^3}{b}}{6\,(bx+a)^3b^3}$$

input
```
integrate(x^3/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
log(abs(b*x + a))/b^4 + 1/6*(18*a*b*x^2 + 27*a^2*x + 11*a^3/b)/((b*x + a)^
3*b^3)
```

Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{\frac{11\,a^3}{6\,b^4}+\frac{3\,a\,x^2}{b^2}+\frac{9\,a^2\,x}{2\,b^3}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3} + \frac{\ln(a+b\,x)}{b^4}$$

input
```
int(x^3/(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
((11*a^3)/(6*b^4) + (3*a*x^2)/b^2 + (9*a^2*x)/(2*b^3))/(a^3 + b^3*x^3 + 3*
a*b^2*x^2 + 3*a^2*b*x) + log(a + b*x)/b^4
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.86

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^2}\,dx$$
$$= \frac{6\log(bx+a)\,a^3+18\log(bx+a)\,a^2bx+18\log(bx+a)\,a\,b^2x^2+6\log(bx+a)\,b^3x^3+5a^3+9a^2bx-6b^3x}{6b^4\left(b^3x^3+3a\,b^2x^2+3a^2bx+a^3\right)}$$

input
```
int(x^3/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
(6*log(a + b*x)*a**3 + 18*log(a + b*x)*a**2*b*x + 18*log(a + b*x)*a*b**2*x
**2 + 6*log(a + b*x)*b**3*x**3 + 5*a**3 + 9*a**2*b*x - 6*b**3*x**3)/(6*b**
4*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.31 $\int \frac{x^2}{(a^2+2abx+b^2x^2)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 17

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^2} dx = \frac{x^3}{3a(a+bx)^3}$$

output
```
1/3*x^3/a/(b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^2} dx = -\frac{a^2+3abx+3b^2x^2}{3b^3(a+bx)^3}$$

input
```
Integrate[x^2/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
-1/3*(a^2 + 3*a*b*x + 3*b^2*x^2)/(b^3*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1098, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^2}dx$$

$$\Big\downarrow \text{1098}$$

$$b^4\int \frac{x^2}{b^4(a+bx)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^2}{(a+bx)^4}dx$$

$$\Big\downarrow \text{48}$$

$$\frac{x^3}{3a(a+bx)^3}$$

input
```
Int[x^2/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
x^3/(3*a*(a + b*x)^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(15) = 30.

Time = 0.60 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

method	result	size
norman	$\frac{-\frac{x^2}{b}-\frac{ax}{b^2}-\frac{a^2}{3b^3}}{(bx+a)^3}$	33
default	$-\frac{a^2}{3b^3(bx+a)^3}-\frac{1}{(bx+a)b^3}+\frac{a}{b^3(bx+a)^2}$	41
orering	$-\frac{(3b^2x^2+3abx+a^2)(bx+a)}{3b^3(b^2x^2+2abx+a^2)^2}$	46
gosper	$-\frac{3b^2x^2+3abx+a^2}{3(bx+a)(b^2x^2+2abx+a^2)b^3}$	48
parallelrisch	$\frac{-3b^2x^2-3abx-a^2}{3b^3(b^2x^2+2abx+a^2)(bx+a)}$	50
risch	$\frac{-\frac{x^2}{b}-\frac{ax}{b^2}-\frac{a^2}{3b^3}}{(bx+a)(b^2x^2+2abx+a^2)}$	51

input
```
int(x^2/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(-x^2/b-a*x/b^2-1/3/b^3*a^2)/(b*x+a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(15) = 30.

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.18

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^2}\,dx = -\frac{3\,b^2x^2+3\,abx+a^2}{3\,(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3)}$$

input
```
integrate(x^2/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
-1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^
3*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(12) = 24.

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 3.29

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{-a^2-3abx-3b^2x^2}{3a^3b^3+9a^2b^4x+9ab^5x^2+3b^6x^3}$$

input
```
integrate(x**2/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
(-a**2 - 3*a*b*x - 3*b**2*x**2)/(3*a**3*b**3 + 9*a**2*b**4*x + 9*a*b**5*x*
*2 + 3*b**6*x**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(15) = 30.

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.18

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^2}\,dx = -\frac{3\,b^2x^2+3\,abx+a^2}{3\,(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3)}$$

input
```
integrate(x^2/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
-1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^
3*b^3)
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \frac{x^2}{(a^2 + 2abx + b^2x^2)^2}\, dx = -\frac{3\, b^2x^2 + 3\, abx + a^2}{3\, (bx + a)^3 b^3}$$

input
```
integrate(x^2/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
-1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/((b*x + a)^3*b^3)
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 3.29

$$\int \frac{x^2}{(a^2 + 2abx + b^2x^2)^2}\, dx = -\frac{a^2 + 3\, a\, b\, x + 3\, b^2\, x^2}{3\, a^3\, b^3 + 9\, a^2\, b^4\, x + 9\, a\, b^5\, x^2 + 3\, b^6\, x^3}$$

input
```
int(x^2/(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
-(a^2 + 3*b^2*x^2 + 3*a*b*x)/(3*a^3*b^3 + 3*b^6*x^3 + 9*a^2*b^4*x + 9*a*b^
5*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{x^2}{(a^2 + 2abx + b^2x^2)^2}\, dx = \frac{x^3}{3a\, (b^3x^3 + 3a\, b^2x^2 + 3a^2bx + a^3)}$$

input
```
int(x^2/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
x**3/(3*a*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.32 $\int \frac{x}{(a^2+2abx+b^2x^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 30

$$\int \frac{x}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{a}{3b^2(a+bx)^3} - \frac{1}{2b^2(a+bx)^2}$$

output
```
1/3*a/b^2/(b*x+a)^3-1/2/b^2/(b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{x}{(a^2+2abx+b^2x^2)^2}\,dx = -\frac{a+3bx}{6b^2(a+bx)^3}$$

input
```
Integrate[x/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
-1/6*(a + 3*b*x)/(b^2*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a^2+2abx+b^2x^2)^2}\,dx$$

$$\Big\downarrow \text{ 1098}$$

$$b^4\int \frac{x}{b^4(a+bx)^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x}{(a+bx)^4}dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{1}{b(a+bx)^3}-\frac{a}{b(a+bx)^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a}{3b^2(a+bx)^3}-\frac{1}{2b^2(a+bx)^2}$$

input
```
Int[x/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
a/(3*b^2*(a + b*x)^3) - 1/(2*b^2*(a + b*x)^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

method	result	size
norman	$\frac{-\frac{x}{2b}-\frac{a}{6b^2}}{(bx+a)^3}$	22
default	$\frac{a}{3b^2(bx+a)^3}-\frac{1}{2b^2(bx+a)^2}$	27
orering	$-\frac{(3bx+a)(bx+a)}{6b^2(b^2x^2+2abx+a^2)^2}$	35
gosper	$-\frac{3bx+a}{6(bx+a)(b^2x^2+2abx+a^2)b^2}$	37
risch	$\frac{-\frac{x}{2b}-\frac{a}{6b^2}}{(bx+a)(b^2x^2+2abx+a^2)}$	40
parallelrisch	$\frac{-3b^2x-ab}{6b^3(b^2x^2+2abx+a^2)(bx+a)}$	42

input
```
int(x/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*x/b-1/6*a/b^2)/(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int \frac{x}{(a^2 + 2abx + b^2x^2)^2}\,dx = -\frac{3\,bx + a}{6\,(b^5x^3 + 3\,ab^4x^2 + 3\,a^2b^3x + a^3b^2)}$$

input
```
integrate(x/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
-1/6*(3*b*x + a)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.47

$$\int \frac{x}{(a^2 + 2abx + b^2x^2)^2}\,dx = \frac{-a - 3bx}{6a^3b^2 + 18a^2b^3x + 18ab^4x^2 + 6b^5x^3}$$

input
```
integrate(x/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
(-a - 3*b*x)/(6*a**3*b**2 + 18*a**2*b**3*x + 18*a*b**4*x**2 + 6*b**5*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int \frac{x}{(a^2 + 2abx + b^2x^2)^2}\,dx = -\frac{3\,bx + a}{6\,(b^5x^3 + 3\,ab^4x^2 + 3\,a^2b^3x + a^3b^2)}$$

input
```
integrate(x/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(3*b*x + a)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int \frac{x}{(a^2+2abx+b^2x^2)^2}\,dx = -\frac{3\,bx+a}{6\,(bx+a)^3b^2}$$

input
```
integrate(x/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
-1/6*(3*b*x + a)/((b*x + a)^3*b^2)
```

Mupad [B] (verification not implemented)

Time = 9.70 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.47

$$\int \frac{x}{(a^2+2abx+b^2x^2)^2}\,dx = -\frac{\frac{a}{6\,b^2}+\frac{x}{2\,b}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}$$

input
```
int(x/(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
-(a/(6*b^2) + x/(2*b))/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \frac{x}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{-3bx-a}{6b^2\,(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)}$$

input
```
int(x/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
( - a - 3*b*x)/(6*b**2*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.33 $\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 14

$$\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^2} \, dx=-\frac{1}{3b(a+bx)^3}$$

output
```
-1/3/b/(b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^2} \, dx=-\frac{1}{3b(a+bx)^3}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(-2),x]
```

output
```
-1/3*1/(b*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1077, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2abx+b^2x^2)^2}\,dx$$

$$\Big\downarrow \ 1077$$

$$b^4\int \frac{1}{(xb^2+ab)^4}dx$$

$$\Big\downarrow \ 17$$

$$-\frac{1}{3b(a+bx)^3}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(-2),x]
```

output
```
-1/3*1/(b*(a + b*x)^3)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1077
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int
[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] &&
 IntegerQ[p]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{1}{3b(bx+a)^{3}}$	13
norman	$-\frac{1}{3b(bx+a)^{3}}$	13
risch	$-\frac{1}{3b(bx+a)^{3}}$	13
orering	$-\frac{bx+a}{3b(b^{2}x^{2}+2abx+a^{2})^{2}}$	29
gosper	$-\frac{1}{3(bx+a)(b^{2}x^{2}+2abx+a^{2})b}$	31
parallelrisch	$-\frac{1}{3(bx+a)(b^{2}x^{2}+2abx+a^{2})b}$	31

input
```
int(1/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/b/(b*x+a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(12) = 24$.

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.50

$$\int \frac{1}{(a^2+2abx+b^2x^2)^2}\,dx = -\frac{1}{3\,(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b)}$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
-1/3/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(12) = 24$.

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.64

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^2} \, dx = -\frac{1}{3a^3b + 9a^2b^2x + 9ab^3x^2 + 3b^4x^3}$$

input
```
integrate(1/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
-1/(3*a**3*b + 9*a**2*b**2*x + 9*a*b**3*x**2 + 3*b**4*x**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(12) = 24$.

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.50

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^2} \, dx = -\frac{1}{3\,(b^4x^3 + 3\,ab^3x^2 + 3\,a^2b^2x + a^3b)}$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
-1/3/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^2} \, dx = -\frac{1}{3\,(bx + a)^3b}$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
-1/3/((b*x + a)^3*b)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.64

$$\int \frac{1}{(a^2+2abx+b^2x^2)^2}\,dx=-\frac{1}{3\,a^3\,b+9\,a^2\,b^2\,x+9\,a\,b^3\,x^2+3\,b^4\,x^3}$$

input
```
int(1/(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
-1/(3*a^3*b + 3*b^4*x^3 + 9*a^2*b^2*x + 9*a*b^3*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.43

$$\int \frac{1}{(a^2+2abx+b^2x^2)^2}\,dx=-\frac{1}{3b\,(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)}$$

input
```
int(1/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
( - 1)/(3*b*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.34 $\int \frac{1}{x(a^2+2abx+b^2x^2)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 57

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^2} dx = \frac{1}{3a(a+bx)^3}+\frac{1}{2a^2(a+bx)^2} + \frac{1}{a^3(a+bx)}+\frac{\log(x)}{a^4}-\frac{\log(a+bx)}{a^4}$$

output
```
1/3/a/(b*x+a)^3+1/2/a^2/(b*x+a)^2+1/a^3/(b*x+a)+ln(x)/a^4-ln(b*x+a)/a^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^2} dx = \frac{\frac{a(11a^2+15abx+6b^2x^2)}{(a+bx)^3}+6\log(x)-6\log(a+bx)}{6a^4}$$

input
```
Integrate[1/(x*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

output
```
((a*(11*a^2 + 15*a*b*x + 6*b^2*x^2))/(a + b*x)^3 + 6*Log[x] - 6*Log[a + b*
x])/(6*a^4)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1098, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^2}dx$$

$$\Big\downarrow \text{ 1098}$$

$$b^4\int \frac{1}{b^4x(a+bx)^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{x(a+bx)^4}dx$$

$$\Big\downarrow \text{ 54}$$

$$\int \left(-\frac{b}{a^4(a+bx)}+\frac{1}{a^4x}-\frac{b}{a^3(a+bx)^2}-\frac{b}{a^2(a+bx)^3}-\frac{b}{a(a+bx)^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\log(a+bx)}{a^4}+\frac{\log(x)}{a^4}+\frac{1}{a^3(a+bx)}+\frac{1}{2a^2(a+bx)^2}+\frac{1}{3a(a+bx)^3}$$

input
```
Int[1/(x*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

output
```
1/(3*a*(a + b*x)^3) + 1/(2*a^2*(a + b*x)^2) + 1/(a^3*(a + b*x)) + Log[x]/a
^4 - Log[a + b*x]/a^4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

method	result
default	$\frac{1}{3a(bx+a)^{3}}+\frac{1}{2a^{2}(bx+a)^{2}}+\frac{1}{a^{3}(bx+a)}+\frac{\ln(x)}{a^{4}}-\frac{\ln(bx+a)}{a^{4}}$
norman	$\frac{-\frac{3bx}{a^{2}}-\frac{9b^{2}x^{2}}{2a^{3}}-\frac{11b^{3}x^{3}}{6a^{4}}}{(bx+a)^{3}}+\frac{\ln(x)}{a^{4}}-\frac{\ln(bx+a)}{a^{4}}$
risch	$\frac{\frac{b^{2}x^{2}}{a^{3}}+\frac{5bx}{2a^{2}}+\frac{11}{6a}}{(bx+a)(b^{2}x^{2}+2abx+a^{2})}+\frac{\ln(-x)}{a^{4}}-\frac{\ln(bx+a)}{a^{4}}$
parallelrisch	$\frac{6\ln(x)x^{3}b^{3}-6\ln(bx+a)x^{3}b^{3}+18\ln(x)x^{2}a\,b^{2}-18\ln(bx+a)x^{2}a\,b^{2}-11b^{3}x^{3}+18\ln(x)x\,a^{2}b-18\ln(bx+a)x\,a^{2}b-27a\,b^{2}x^{2}+6a^{3}}{6a^{4}(b^{2}x^{2}+2abx+a^{2})(bx+a)}$

input
```
int(1/x/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/a/(b*x+a)^3+1/2/a^2/(b*x+a)^2+1/a^3/(b*x+a)+ln(x)/a^4-ln(b*x+a)/a^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(53) = 106$.

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.18

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{6\,ab^2x^2+15\,a^2bx+11\,a^3-6\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)\log\left(bx+a\right)+6\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+\right.}{6\left(a^4b^3x^3+3\,a^5b^2x^2+3\,a^6bx+a^7\right)}$$

input
```
integrate(1/x/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
1/6*(6*a*b^2*x^2 + 15*a^2*b*x + 11*a^3 - 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*
b*x + a^3)*log(b*x + a) + 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*log(
x))/(a^4*b^3*x^3 + 3*a^5*b^2*x^2 + 3*a^6*b*x + a^7)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{11a^2+15abx+6b^2x^2}{6a^6+18a^5bx+18a^4b^2x^2+6a^3b^3x^3}+\frac{\log\left(x\right)-\log\left(\frac{a}{b}+x\right)}{a^4}$$

input
```
integrate(1/x/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
(11*a**2 + 15*a*b*x + 6*b**2*x**2)/(6*a**6 + 18*a**5*b*x + 18*a**4*b**2*x*
*2 + 6*a**3*b**3*x**3) + (log(x) - log(a/b + x))/a**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.28

$$\int \frac{1}{x(a^2+2abx+b^2x^2)^2}\,dx = \frac{6\,b^2x^2+15\,abx+11\,a^2}{6\,(a^3b^3x^3+3\,a^4b^2x^2+3\,a^5bx+a^6)} - \frac{\log(bx+a)}{a^4} + \frac{\log(x)}{a^4}$$

input
```
integrate(1/x/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
1/6*(6*b^2*x^2 + 15*a*b*x + 11*a^2)/(a^3*b^3*x^3 + 3*a^4*b^2*x^2 + 3*a^5*b
*x + a^6) - log(b*x + a)/a^4 + log(x)/a^4
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{1}{x(a^2+2abx+b^2x^2)^2}\,dx = -\frac{\log(|bx+a|)}{a^4} + \frac{\log(|x|)}{a^4} + \frac{6\,ab^2x^2+15\,a^2bx+11\,a^3}{6\,(bx+a)^3a^4}$$

input
```
integrate(1/x/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
-log(abs(b*x + a))/a^4 + log(abs(x))/a^4 + 1/6*(6*a*b^2*x^2 + 15*a^2*b*x +
 11*a^3)/((b*x + a)^3*a^4)
```

Mupad [B] (verification not implemented)

Time = 9.45 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.21

$$\int \frac{1}{x(a^2+2abx+b^2x^2)^2}\,dx = \frac{\frac{11}{6\,a}+\frac{b^2\,x^2}{a^3}+\frac{5\,b\,x}{2\,a^2}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3} - \frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^4}$$

input
```
int(1/(x*(a^2 + b^2*x^2 + 2*a*b*x)^2),x)
```

output
```
(11/(6*a) + (b^2*x^2)/a^3 + (5*b*x)/(2*a^2))/(a^3 + b^3*x^3 + 3*a*b^2*x^2
+ 3*a^2*b*x) - (2*atanh((2*b*x)/a + 1))/a^4
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.54

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^2}\,dx$$
$$=\frac{-6\log(bx+a)\,a^3-18\log(bx+a)\,a^2bx-18\log(bx+a)\,a\,b^2x^2-6\log(bx+a)\,b^3x^3+6\log(x)\,a^3+18\,\mathrm{lo}}{6a^4\left(b^3x^3+3a\,b^2x^2+3a^2bx+a^3\right)}$$

input
```
int(1/x/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
( - 6*log(a + b*x)*a**3 - 18*log(a + b*x)*a**2*b*x - 18*log(a + b*x)*a*b**
2*x**2 - 6*log(a + b*x)*b**3*x**3 + 6*log(x)*a**3 + 18*log(x)*a**2*b*x + 1
8*log(x)*a*b**2*x**2 + 6*log(x)*b**3*x**3 + 9*a**3 + 9*a**2*b*x - 2*b**3*x
**3)/(6*a**4*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.35 $\int \frac{1}{x^2\left(a^2+2abx+b^2x^2\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{1}{x^2\left(a^2+2abx+b^2x^2\right)^2}\,dx=-\frac{1}{a^4x}-\frac{b}{3a^2(a+bx)^3}-\frac{b}{a^3(a+bx)^2}-\frac{3b}{a^4(a+bx)}-\frac{4b\log(x)}{a^5}+\frac{4b\log(a+bx)}{a^5}$$

output
```
-1/a^4/x-1/3*b/a^2/(b*x+a)^3-b/a^3/(b*x+a)^2-3*b/a^4/(b*x+a)-4*b*ln(x)/a^5
+4*b*ln(b*x+a)/a^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2\left(a^2+2abx+b^2x^2\right)^2}\,dx=-\frac{\frac{a\left(3a^3+22a^2bx+30ab^2x^2+12b^3x^3\right)}{x(a+bx)^3}+12b\log(x)-12b\log(a+bx)}{3a^5}$$

input
```
Integrate[1/(x^2*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

output
```
-1/3*((a*(3*a^3 + 22*a^2*b*x + 30*a*b^2*x^2 + 12*b^3*x^3))/(x*(a + b*x)^3)
 + 12*b*Log[x] - 12*b*Log[a + b*x])/a^5
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1098, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a^2+2abx+b^2x^2)^2} \, dx$$

$$\Big\downarrow \text{1098}$$

$$b^4 \int \frac{1}{b^4x^2(a+bx)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{x^2(a+bx)^4}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{4b^2}{a^5(a+bx)}-\frac{4b}{a^5x}+\frac{3b^2}{a^4(a+bx)^2}+\frac{1}{a^4x^2}+\frac{2b^2}{a^3(a+bx)^3}+\frac{b^2}{a^2(a+bx)^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4b\log(x)}{a^5}+\frac{4b\log(a+bx)}{a^5}-\frac{3b}{a^4(a+bx)}-\frac{1}{a^4x}-\frac{b}{a^3(a+bx)^2}-\frac{b}{3a^2(a+bx)^3}$$

input
```
Int[1/(x^2*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

output
```
-(1/(a^4*x)) - b/(3*a^2*(a + b*x)^3) - b/(a^3*(a + b*x)^2) - (3*b)/(a^4*(a
 + b*x)) - (4*b*Log[x])/a^5 + (4*b*Log[a + b*x])/a^5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.99

method	result
default	$-\frac{1}{a^{4} x}-\frac{b}{3 a^{2} \left(b x +a \right)^{3}}-\frac{b}{a^{3} \left(b x +a \right)^{2}}-\frac{3 b}{a^{4} \left(b x +a \right)}-\frac{4 b \ln \left(x \right)}{a^{5}}+\frac{4 b \ln \left(b x +a \right)}{a^{5}}$
norman	$\frac{-\frac{1}{a}+\frac{12 b^{2} x^{2}}{a^{3}}+\frac{18 b^{3} x^{3}}{a^{4}}+\frac{22 b^{4} x^{4}}{3 a^{5}}}{x \left(b x +a \right)^{3}}-\frac{4 b \ln \left(x \right)}{a^{5}}+\frac{4 b \ln \left(b x +a \right)}{a^{5}}$
risch	$\frac{-\frac{4 b^{3} x^{3}}{a^{4}}-\frac{10 b^{2} x^{2}}{a^{3}}-\frac{22 b x}{3 a^{2}}-\frac{1}{a}}{x \left(b^{2} x^{2}+2 a b x +a^{2}\right) \left(b x +a \right)}-\frac{4 b \ln \left(x \right)}{a^{5}}+\frac{4 b \ln \left(-b x -a \right)}{a^{5}}$
parallelrisch	$-\frac{12 \ln \left(x \right) x^{4} b^{4}-12 \ln \left(b x +a \right) x^{4} b^{4}+36 \ln \left(x \right) x^{3} a \,b^{3}-36 \ln \left(b x +a \right) x^{3} a \,b^{3}-22 b^{4} x^{4}+36 a^{2} b^{2} \ln \left(x \right) x^{2}-36 \ln \left(b x +a \right) x^{2} a^{2} b^{2}-54 a \,b^{3}}{3 a^{5} x \left(b^{2} x^{2}+2 a b x +a^{2}\right) \left(b x +a \right)}$

input
```
int(1/x^2/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^4/x-1/3*b/a^2/(b*x+a)^3-b/a^3/(b*x+a)^2-3*b/a^4/(b*x+a)-4*b*ln(x)/a^5
+4*b*ln(b*x+a)/a^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(68) = 136$.

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.19

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)^2} \, dx = -\frac{12\,ab^3x^3 + 30\,a^2b^2x^2 + 22\,a^3bx + 3\,a^4 - 12\,(b^4x^4 + 3\,ab^3x^3 + 3\,a^2b^2x^2 + a^3bx)\log(bx + a) + 12\,(b^4x^4}{3\,(a^5b^3x^4 + 3\,a^6b^2x^3 + 3\,a^7bx^2 + a^8x)}$$

input
```
integrate(1/x^2/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
-1/3*(12*a*b^3*x^3 + 30*a^2*b^2*x^2 + 22*a^3*b*x + 3*a^4 - 12*(b^4*x^4 + 3
*a*b^3*x^3 + 3*a^2*b^2*x^2 + a^3*b*x)*log(b*x + a) + 12*(b^4*x^4 + 3*a*b^3
*x^3 + 3*a^2*b^2*x^2 + a^3*b*x)*log(x))/(a^5*b^3*x^4 + 3*a^6*b^2*x^3 + 3*a
^7*b*x^2 + a^8*x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)^2} \, dx = \frac{-3a^3 - 22a^2bx - 30ab^2x^2 - 12b^3x^3}{3a^7x + 9a^6bx^2 + 9a^5b^2x^3 + 3a^4b^3x^4} + \frac{4b\left(-\log(x) + \log\left(\frac{a}{b} + x\right)\right)}{a^5}$$

input
```
integrate(1/x**2/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
(-3*a**3 - 22*a**2*b*x - 30*a*b**2*x**2 - 12*b**3*x**3)/(3*a**7*x + 9*a**6
*b*x**2 + 9*a**5*b**2*x**3 + 3*a**4*b**3*x**4) + 4*b*(-log(x) + log(a/b +
x))/a**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.30

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)^2} dx = -\frac{12 b^3x^3 + 30 ab^2x^2 + 22 a^2bx + 3 a^3}{3 (a^4b^3x^4 + 3 a^5b^2x^3 + 3 a^6bx^2 + a^7x)} + \frac{4 b \log (bx + a)}{a^5} - \frac{4 b \log (x)}{a^5}$$

input
```
integrate(1/x^2/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
-1/3*(12*b^3*x^3 + 30*a*b^2*x^2 + 22*a^2*b*x + 3*a^3)/(a^4*b^3*x^4 + 3*a^5
*b^2*x^3 + 3*a^6*b*x^2 + a^7*x) + 4*b*log(b*x + a)/a^5 - 4*b*log(x)/a^5
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)^2} dx = \frac{4 b \log (|bx + a|)}{a^5} - \frac{4 b \log (|x|)}{a^5} - \frac{12 ab^3x^3 + 30 a^2b^2x^2 + 22 a^3bx + 3 a^4}{3 (bx + a)^3 a^5 x}$$

input
```
integrate(1/x^2/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
4*b*log(abs(b*x + a))/a^5 - 4*b*log(abs(x))/a^5 - 1/3*(12*a*b^3*x^3 + 30*a
^2*b^2*x^2 + 22*a^3*b*x + 3*a^4)/((b*x + a)^3*a^5*x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^2 \left(a^2+2abx+b^2x^2\right)^2} \, dx = \frac{8\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5} - \frac{\frac{1}{a}+\frac{10\,b^2\,x^2}{a^3}+\frac{4\,b^3\,x^3}{a^4}+\frac{22\,b\,x}{3\,a^2}}{a^3\,x+3\,a^2\,b\,x^2+3\,a\,b^2\,x^3+b^3\,x^4}$$

input
```
int(1/(x^2*(a^2 + b^2*x^2 + 2*a*b*x)^2),x)
```

output
```
(8*b*atanh((2*b*x)/a + 1))/a^5 - (1/a + (10*b^2*x^2)/a^3 + (4*b^3*x^3)/a^4
 + (22*b*x)/(3*a^2))/(a^3*x + b^3*x^4 + 3*a^2*b*x^2 + 3*a*b^2*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.44

$$\int \frac{1}{x^2 \left(a^2+2abx+b^2x^2\right)^2} \, dx$$
$$= \frac{12\log(bx+a)\,a^3bx+36\log(bx+a)\,a^2b^2x^2+36\log(bx+a)\,a\,b^3x^3+12\log(bx+a)\,b^4x^4-12\log(x)\,a^3b}{3a^5x\left(b^3x^3+3a\,b^2x^2+3\right.}$$

input
```
int(1/x^2/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
(12*log(a + b*x)*a**3*b*x + 36*log(a + b*x)*a**2*b**2*x**2 + 36*log(a + b*
x)*a*b**3*x**3 + 12*log(a + b*x)*b**4*x**4 - 12*log(x)*a**3*b*x - 36*log(x
)*a**2*b**2*x**2 - 36*log(x)*a*b**3*x**3 - 12*log(x)*b**4*x**4 - 3*a**4 -
18*a**3*b*x - 18*a**2*b**2*x**2 + 4*b**4*x**4)/(3*a**5*x*(a**3 + 3*a**2*b*
x + 3*a*b**2*x**2 + b**3*x**3))
```

3.36 $\int \frac{x^3}{(a^2+2abx+b^2x^2)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 35

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^3} dx = \frac{x^4}{5a(a+bx)^5} + \frac{x^4}{20a^2(a+bx)^4}$$

output
```
1/5*x^4/a/(b*x+a)^5+1/20*x^4/a^2/(b*x+a)^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^3} dx = -\frac{a^3+5a^2bx+10ab^2x^2+10b^3x^3}{20b^4(a+bx)^5}$$

input
```
Integrate[x^3/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
-1/20*(a^3 + 5*a^2*b*x + 10*a*b^2*x^2 + 10*b^3*x^3)/(b^4*(a + b*x)^5)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1098, 27, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^3}dx$$

$$\Big\downarrow \text{ 1098}$$

$$b^6\int \frac{x^3}{b^6(a+bx)^6}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x^3}{(a+bx)^6}dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{\int \frac{x^3}{(a+bx)^5}dx}{5a}+\frac{x^4}{5a(a+bx)^5}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{x^4}{20a^2(a+bx)^4}+\frac{x^4}{5a(a+bx)^5}$$

input
```
Int[x^3/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
x^4/(5*a*(a + b*x)^5) + x^4/(20*a^2*(a + b*x)^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

method	result	size
norman	$\frac{-\frac{x^{3}}{2 b}-\frac{a \,x^{2}}{2 b^{2}}-\frac{a^{2} x}{4 b^{3}}-\frac{a^{3}}{20 b^{4}}}{\left(b x +a \right)^{5}}$	44
default	$\frac{a^{3}}{5 b^{4} \left(b x +a \right)^{5}}+\frac{a}{b^{4} \left(b x +a \right)^{3}}-\frac{1}{2 b^{4} \left(b x +a \right)^{2}}-\frac{3 a^{2}}{4 b^{4} \left(b x +a \right)^{4}}$	56
orering	$-\frac{\left(10 b^{3} x^{3}+10 a \,b^{2} x^{2}+5 a^{2} b x +a^{3}\right) \left(b x +a \right)}{20 b^{4} \left(b^{2} x^{2}+2 a b x +a^{2}\right)^{3}}$	57
gosper	$-\frac{10 b^{3} x^{3}+10 a \,b^{2} x^{2}+5 a^{2} b x +a^{3}}{20 \left(b x +a \right) \left(b^{2} x^{2}+2 a b x +a^{2}\right)^{2} b^{4}}$	59
risch	$\frac{-\frac{x^{3}}{2 b}-\frac{a \,x^{2}}{2 b^{2}}-\frac{a^{2} x}{4 b^{3}}-\frac{a^{3}}{20 b^{4}}}{\left(b x +a \right) \left(b^{2} x^{2}+2 a b x +a^{2}\right)^{2}}$	62
parallelrisch	$\frac{-10 b^{4} x^{3}-10 a \,b^{3} x^{2}-5 x \,a^{2} b^{2}-a^{3} b}{20 b^{5} \left(b^{2} x^{2}+2 a b x +a^{2}\right)^{2} \left(b x +a \right)}$	64

input
```
int(x^3/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*x^3/b-1/2*a*x^2/b^2-1/4*a^2*x/b^3-1/20*a^3/b^4)/(b*x+a)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(31) = 62$.

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.49

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{10\,b^3x^3+10\,ab^2x^2+5\,a^2bx+a^3}{20\,(b^9x^5+5\,ab^8x^4+10\,a^2b^7x^3+10\,a^3b^6x^2+5\,a^4b^5x+a^5b^4)}$$

input
```
integrate(x^3/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
-1/20*(10*b^3*x^3 + 10*a*b^2*x^2 + 5*a^2*b*x + a^3)/(b^9*x^5 + 5*a*b^8*x^4
 + 10*a^2*b^7*x^3 + 10*a^3*b^6*x^2 + 5*a^4*b^5*x + a^5*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(27) = 54$.

Time = 0.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.63

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{-a^3-5a^2bx-10ab^2x^2-10b^3x^3}{20a^5b^4+100a^4b^5x+200a^3b^6x^2+200a^2b^7x^3+100ab^8x^4+20b^9x^5}$$

input
```
integrate(x**3/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
(-a**3 - 5*a**2*b*x - 10*a*b**2*x**2 - 10*b**3*x**3)/(20*a**5*b**4 + 100*a
**4*b**5*x + 200*a**3*b**6*x**2 + 200*a**2*b**7*x**3 + 100*a*b**8*x**4 + 2
0*b**9*x**5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(31) = 62$.

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.49

$$\int \frac{x^3}{(a^2 + 2abx + b^2x^2)^3}\,dx = -\frac{10\,b^3x^3 + 10\,ab^2x^2 + 5\,a^2bx + a^3}{20\,(b^9x^5 + 5\,ab^8x^4 + 10\,a^2b^7x^3 + 10\,a^3b^6x^2 + 5\,a^4b^5x + a^5b^4)}$$

input
```
integrate(x^3/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
-1/20*(10*b^3*x^3 + 10*a*b^2*x^2 + 5*a^2*b*x + a^3)/(b^9*x^5 + 5*a*b^8*x^4
 + 10*a^2*b^7*x^3 + 10*a^3*b^6*x^2 + 5*a^4*b^5*x + a^5*b^4)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \frac{x^3}{(a^2 + 2abx + b^2x^2)^3}\,dx = -\frac{10\,b^3x^3 + 10\,ab^2x^2 + 5\,a^2bx + a^3}{20\,(bx + a)^5b^4}$$

input
```
integrate(x^3/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
-1/20*(10*b^3*x^3 + 10*a*b^2*x^2 + 5*a^2*b*x + a^3)/((b*x + a)^5*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.54

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^3}\,dx$$
$$= -\frac{a^3+5\,a^2\,b\,x+10\,a\,b^2\,x^2+10\,b^3\,x^3}{20\,a^5\,b^4+100\,a^4\,b^5\,x+200\,a^3\,b^6\,x^2+200\,a^2\,b^7\,x^3+100\,a\,b^8\,x^4+20\,b^9\,x^5}$$

input
```
int(x^3/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
-(a^3 + 10*b^3*x^3 + 10*a*b^2*x^2 + 5*a^2*b*x)/(20*a^5*b^4 + 20*b^9*x^5 +
100*a^4*b^5*x + 100*a*b^8*x^4 + 200*a^3*b^6*x^2 + 200*a^2*b^7*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.46

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{-10b^3x^3-10a\,b^2x^2-5a^2bx-a^3}{20b^4\,(b^5x^5+5a\,b^4x^4+10a^2b^3x^3+10a^3b^2x^2+5a^4bx+a^5)}$$

input
```
int(x^3/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
( - a**3 - 5*a**2*b*x - 10*a*b**2*x**2 - 10*b**3*x**3)/(20*b**4*(a**5 + 5*
a**4*b*x + 10*a**3*b**2*x**2 + 10*a**2*b**3*x**3 + 5*a*b**4*x**4 + b**5*x*
*5))
```

3.37 $\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 47

$$\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^3}\,dx = -\frac{a^2}{5b^3(a+bx)^5}+\frac{a}{2b^3(a+bx)^4}-\frac{1}{3b^3(a+bx)^3}$$

output
```
-1/5*a^2/b^3/(b*x+a)^5+1/2*a/b^3/(b*x+a)^4-1/3/b^3/(b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^3}\,dx = -\frac{a^2+5abx+10b^2x^2}{30b^3(a+bx)^5}$$

input
```
Integrate[x^2/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
-1/30*(a^2 + 5*a*b*x + 10*b^2*x^2)/(b^3*(a + b*x)^5)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^3}dx$$

$$\Big\downarrow \text{ 1098}$$

$$b^6 \int \frac{x^2}{b^6(a+bx)^6}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x^2}{(a+bx)^6}dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{a^2}{b^2(a+bx)^6}-\frac{2a}{b^2(a+bx)^5}+\frac{1}{b^2(a+bx)^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2}{5b^3(a+bx)^5}+\frac{a}{2b^3(a+bx)^4}-\frac{1}{3b^3(a+bx)^3}$$

input
```
Int[x^2/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
-1/5*a^2/(b^3*(a + b*x)^5) + a/(2*b^3*(a + b*x)^4) - 1/(3*b^3*(a + b*x)^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

method	result	size
norman	$\frac{-\frac{x^{2}}{3 b}-\frac{a x}{6 b^{2}}-\frac{a^{2}}{30 b^{3}}}{\left(b x +a \right)^{5}}$	33
default	$-\frac{a^{2}}{5 b^{3} \left(b x +a \right)^{5}}+\frac{a}{2 b^{3} \left(b x +a \right)^{4}}-\frac{1}{3 b^{3} \left(b x +a \right)^{3}}$	42
orering	$-\frac{\left(10 b^{2} x^{2}+5 a b x +a^{2}\right) \left(b x +a \right)}{30 b^{3} \left(b^{2} x^{2}+2 a b x +a^{2}\right)^{3}}$	46
gosper	$-\frac{10 b^{2} x^{2}+5 a b x +a^{2}}{30 \left(b x +a \right) \left(b^{2} x^{2}+2 a b x +a^{2}\right)^{2} b^{3}}$	48
risch	$\frac{-\frac{x^{2}}{3 b}-\frac{a x}{6 b^{2}}-\frac{a^{2}}{30 b^{3}}}{\left(b x +a \right) \left(b^{2} x^{2}+2 a b x +a^{2}\right)^{2}}$	51
parallelrisch	$\frac{-10 b^{4} x^{2}-5 a \,b^{3} x -a^{2} b^{2}}{30 b^{5} \left(b^{2} x^{2}+2 a b x +a^{2}\right)^{2} \left(b x +a \right)}$	55

input
```
int(x^2/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*x^2/b-1/6*a*x/b^2-1/30/b^3*a^2)/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{10\,b^2x^2+5\,abx+a^2}{30\,(b^8x^5+5\,ab^7x^4+10\,a^2b^6x^3+10\,a^3b^5x^2+5\,a^4b^4x+a^5b^3)}$$

input
```
integrate(x^2/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
-1/30*(10*b^2*x^2 + 5*a*b*x + a^2)/(b^8*x^5 + 5*a*b^7*x^4 + 10*a^2*b^6*x^3
 + 10*a^3*b^5*x^2 + 5*a^4*b^4*x + a^5*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.70

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{-a^2-5abx-10b^2x^2}{30a^5b^3+150a^4b^4x+300a^3b^5x^2+300a^2b^6x^3+150ab^7x^4+30b^8x^5}$$

input
```
integrate(x**2/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
(-a**2 - 5*a*b*x - 10*b**2*x**2)/(30*a**5*b**3 + 150*a**4*b**4*x + 300*a**
3*b**5*x**2 + 300*a**2*b**6*x**3 + 150*a*b**7*x**4 + 30*b**8*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{10\,b^2x^2+5\,abx+a^2}{30\,(b^8x^5+5\,ab^7x^4+10\,a^2b^6x^3+10\,a^3b^5x^2+5\,a^4b^4x+a^5b^3)}$$

input
```
integrate(x^2/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
-1/30*(10*b^2*x^2 + 5*a*b*x + a^2)/(b^8*x^5 + 5*a*b^7*x^4 + 10*a^2*b^6*x^3
 + 10*a^3*b^5*x^2 + 5*a^4*b^4*x + a^5*b^3)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{10\,b^2x^2+5\,abx+a^2}{30\,(bx+a)^5b^3}$$

input
```
integrate(x^2/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
-1/30*(10*b^2*x^2 + 5*a*b*x + a^2)/((b*x + a)^5*b^3)
```

Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.64

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{\frac{a^2}{30\,b^3}+\frac{x^2}{3\,b}+\frac{a\,x}{6\,b^2}}{a^5+5\,a^4\,b\,x+10\,a^3\,b^2\,x^2+10\,a^2\,b^3\,x^3+5\,a\,b^4\,x^4+b^5\,x^5}$$

input
```
int(x^2/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
-(a^2/(30*b^3) + x^2/(3*b) + (a*x)/(6*b^2))/(a^5 + b^5*x^5 + 5*a*b^4*x^4 +
 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{-10b^2x^2-5abx-a^2}{30b^3\,(b^5x^5+5a\,b^4x^4+10a^2b^3x^3+10a^3b^2x^2+5a^4bx+a^5)}$$

input
```
int(x^2/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
( - a**2 - 5*a*b*x - 10*b**2*x**2)/(30*b**3*(a**5 + 5*a**4*b*x + 10*a**3*b
**2*x**2 + 10*a**2*b**3*x**3 + 5*a*b**4*x**4 + b**5*x**5))
```

3.38 $\int \frac{x}{\left(a^2+2abx+b^2x^2\right)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 30

$$\int \frac{x}{\left(a^2+2abx+b^2x^2\right)^3}\,dx=\frac{a}{5b^2(a+bx)^5}-\frac{1}{4b^2(a+bx)^4}$$

output
```
1/5*a/b^2/(b*x+a)^5-1/4/b^2/(b*x+a)^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{x}{\left(a^2+2abx+b^2x^2\right)^3}\,dx=-\frac{a+5bx}{20b^2(a+bx)^5}$$

input
```
Integrate[x/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
-1/20*(a + 5*b*x)/(b^2*(a + b*x)^5)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a^2+2abx+b^2x^2)^3}\,dx$$

$$\Big\downarrow \text{1098}$$

$$b^6\int \frac{x}{b^6(a+bx)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x}{(a+bx)^6}dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{1}{b(a+bx)^5}-\frac{a}{b(a+bx)^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a}{5b^2(a+bx)^5}-\frac{1}{4b^2(a+bx)^4}$$

input
```
Int[x/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
a/(5*b^2*(a + b*x)^5) - 1/(4*b^2*(a + b*x)^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

method	result	size
norman	$\frac{-\frac{x}{4b}-\frac{a}{20b^2}}{(bx+a)^5}$	22
default	$\frac{a}{5b^2(bx+a)^5}-\frac{1}{4b^2(bx+a)^4}$	27
orering	$-\frac{(5bx+a)(bx+a)}{20b^2(b^2x^2+2abx+a^2)^3}$	35
gosper	$-\frac{5bx+a}{20(bx+a)(b^2x^2+2abx+a^2)^2 b^2}$	37
risch	$\frac{-\frac{x}{4b}-\frac{a}{20b^2}}{(bx+a)(b^2x^2+2abx+a^2)^2}$	40
parallelrisch	$\frac{-5b^4x-a\,b^3}{20b^5(b^2x^2+2abx+a^2)^2(bx+a)}$	44

input
```
int(x/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/4*x/b-1/20*a/b^2)/(b*x+a)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(26) = 52$.

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.17

$$\int \frac{x}{(a^2 + 2abx + b^2x^2)^3} dx = -\frac{5 bx + a}{20 (b^7x^5 + 5 ab^6x^4 + 10 a^2b^5x^3 + 10 a^3b^4x^2 + 5 a^4b^3x + a^5b^2)}$$

input
```
integrate(x/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
-1/20*(5*b*x + a)/(b^7*x^5 + 5*a*b^6*x^4 + 10*a^2*b^5*x^3 + 10*a^3*b^4*x^2
 + 5*a^4*b^3*x + a^5*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(26) = 52$.

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.27

$$\int \frac{x}{(a^2 + 2abx + b^2x^2)^3} dx = \frac{-a - 5bx}{20a^5b^2 + 100a^4b^3x + 200a^3b^4x^2 + 200a^2b^5x^3 + 100ab^6x^4 + 20b^7x^5}$$

input
```
integrate(x/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
(-a - 5*b*x)/(20*a**5*b**2 + 100*a**4*b**3*x + 200*a**3*b**4*x**2 + 200*a*
*2*b**5*x**3 + 100*a*b**6*x**4 + 20*b**7*x**5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(26) = 52$.

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.17

$$\int \frac{x}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{5\,bx+a}{20\,(b^7x^5+5\,ab^6x^4+10\,a^2b^5x^3+10\,a^3b^4x^2+5\,a^4b^3x+a^5b^2)}$$

input
```
integrate(x/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
-1/20*(5*b*x + a)/(b^7*x^5 + 5*a*b^6*x^4 + 10*a^2*b^5*x^3 + 10*a^3*b^4*x^2
 + 5*a^4*b^3*x + a^5*b^2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int \frac{x}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{5\,bx+a}{20\,(bx+a)^5b^2}$$

input
```
integrate(x/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
-1/20*(5*b*x + a)/((b*x + a)^5*b^2)
```

Mupad [B] (verification not implemented)

Time = 9.75 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.20

$$\int \frac{x}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{\frac{a}{20\,b^2}+\frac{x}{4\,b}}{a^5+5\,a^4\,b\,x+10\,a^3\,b^2\,x^2+10\,a^2\,b^3\,x^3+5\,a\,b^4\,x^4+b^5\,x^5}$$

input
```
int(x/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
-(a/(20*b^2) + x/(4*b))/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10
*a^2*b^3*x^3 + 5*a^4*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.13

$$\int \frac{x}{(a^2 + 2abx + b^2x^2)^3} \, dx = \frac{-5bx - a}{20b^2 (b^5x^5 + 5a\, b^4x^4 + 10a^2b^3x^3 + 10a^3b^2x^2 + 5a^4bx + a^5)}$$

input
```
int(x/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
( - a - 5*b*x)/(20*b**2*(a**5 + 5*a**4*b*x + 10*a**3*b**2*x**2 + 10*a**2*b
**3*x**3 + 5*a*b**4*x**4 + b**5*x**5))
```

3.39 $\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 14

$$\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^3}\,dx=-\frac{1}{5b(a+bx)^5}$$

output
```
-1/5/b/(b*x+a)^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^3}\,dx=-\frac{1}{5b(a+bx)^5}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(-3),x]
```

output
```
-1/5*1/(b*(a + b*x)^5)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1077, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2abx+b^2x^2)^3}dx$$

$$\Big\downarrow \text{ 1077}$$

$$b^6 \int \frac{1}{(xb^2+ab)^6}dx$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{1}{5b(a+bx)^5}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(-3),x]
```

output
```
-1/5*1/(b*(a + b*x)^5)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1077
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int
[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] &&
 IntegerQ[p]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{1}{5b(bx+a)^5}$	13
norman	$-\frac{1}{5b(bx+a)^5}$	13
risch	$-\frac{1}{5b(bx+a)^5}$	13
orering	$-\frac{bx+a}{5b(b^2x^2+2abx+a^2)^3}$	29
gosper	$-\frac{1}{5(bx+a)(b^2x^2+2abx+a^2)^2b}$	31
parallelrisch	$-\frac{1}{5(bx+a)(b^2x^2+2abx+a^2)^2b}$	31

input
```
int(1/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/5/b/(b*x+a)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(12) = 24$.

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.07

$$\int \frac{1}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{1}{5\,(b^6x^5+5\,ab^5x^4+10\,a^2b^4x^3+10\,a^3b^3x^2+5\,a^4b^2x+a^5b)}$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
-1/5/(b^6*x^5 + 5*a*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5*a^4*b^2*
x + a^5*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(12) = 24$.

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 4.36

$$\int \frac{1}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{1}{5a^5b+25a^4b^2x+50a^3b^3x^2+50a^2b^4x^3+25ab^5x^4+5b^6x^5}$$

input
```
integrate(1/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
-1/(5*a**5*b + 25*a**4*b**2*x + 50*a**3*b**3*x**2 + 50*a**2*b**4*x**3 + 25
*a*b**5*x**4 + 5*b**6*x**5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(12) = 24$.

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.07

$$\int \frac{1}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{1}{5\,(b^6x^5+5\,ab^5x^4+10\,a^2b^4x^3+10\,a^3b^3x^2+5\,a^4b^2x+a^5b)}$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
-1/5/(b^6*x^5 + 5*a*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5*a^4*b^2*
x + a^5*b)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{1}{5\,(bx+a)^5b}$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
-1/5/((b*x + a)^5*b)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 4.21

$$\int \frac{1}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{1}{5\,a^5\,b+25\,a^4\,b^2\,x+50\,a^3\,b^3\,x^2+50\,a^2\,b^4\,x^3+25\,a\,b^5\,x^4+5\,b^6\,x^5}$$

input
```
int(1/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
-1/(5*a^5*b + 5*b^6*x^5 + 25*a^4*b^2*x + 25*a*b^5*x^4 + 50*a^3*b^3*x^2 + 5
0*a^2*b^4*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 4.00

$$\int \frac{1}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{1}{5b\,(b^5x^5+5a\,b^4x^4+10a^2b^3x^3+10a^3b^2x^2+5a^4bx+a^5)}$$

input
```
int(1/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
( - 1)/(5*b*(a**5 + 5*a**4*b*x + 10*a**3*b**2*x**2 + 10*a**2*b**3*x**3 + 5
*a*b**4*x**4 + b**5*x**5))
```

3.40 $\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{1}{5a(a+bx)^5}+\frac{1}{4a^2(a+bx)^4}+\frac{1}{3a^3(a+bx)^3}+\frac{1}{2a^4(a+bx)^2}+\frac{1}{a^5(a+bx)}+\frac{\log(x)}{a^6}-\frac{\log(a+bx)}{a^6}$$

output
```
1/5/a/(b*x+a)^5+1/4/a^2/(b*x+a)^4+1/3/a^3/(b*x+a)^3+1/2/a^4/(b*x+a)^2+1/a^
5/(b*x+a)+ln(x)/a^6-ln(b*x+a)/a^6
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{\frac{a\left(137a^4+385a^3bx+470a^2b^2x^2+270ab^3x^3+60b^4x^4\right)}{(a+bx)^5}+60\log(x)-60\log(a+bx)}{60a^6}$$

input
```
Integrate[1/(x*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

output
```
((a*(137*a^4 + 385*a^3*b*x + 470*a^2*b^2*x^2 + 270*a*b^3*x^3 + 60*b^4*x^4)
)/(a + b*x)^5 + 60*Log[x] - 60*Log[a + b*x])/(60*a^6)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1098, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^3}dx$$

$$\Big\downarrow \text{ 1098}$$

$$b^6\int \frac{1}{b^6x(a+bx)^6}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{x(a+bx)^6}dx$$

$$\Big\downarrow \text{ 54}$$

$$\int \left(-\frac{b}{a^6(a+bx)}+\frac{1}{a^6x}-\frac{b}{a^5(a+bx)^2}-\frac{b}{a^4(a+bx)^3}-\frac{b}{a^3(a+bx)^4}-\frac{b}{a^2(a+bx)^5}-\frac{b}{a(a+bx)^6}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\log(a+bx)}{a^6}+\frac{\log(x)}{a^6}+\frac{1}{a^5(a+bx)}+\frac{1}{2a^4(a+bx)^2}+\frac{1}{3a^3(a+bx)^3}+\frac{1}{4a^2(a+bx)^4}+\frac{1}{5a(a+bx)^5}$$

input
```
Int[1/(x*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

output
```
1/(5*a*(a + b*x)^5) + 1/(4*a^2*(a + b*x)^4) + 1/(3*a^3*(a + b*x)^3) + 1/(2
*a^4*(a + b*x)^2) + 1/(a^5*(a + b*x)) + Log[x]/a^6 - Log[a + b*x]/a^6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

method	result
default	$\frac{1}{5a(bx+a)^5}+\frac{1}{4a^2(bx+a)^4}+\frac{1}{3a^3(bx+a)^3}+\frac{1}{2a^4(bx+a)^2}+\frac{1}{a^5(bx+a)}+\frac{\ln(x)}{a^6}-\frac{\ln(bx+a)}{a^6}$
norman	$\frac{-\frac{5bx}{a^2}-\frac{15b^2x^2}{a^3}-\frac{55b^3x^3}{3a^4}-\frac{125b^4x^4}{12a^5}-\frac{137b^5x^5}{60a^6}}{(bx+a)^5}+\frac{\ln(x)}{a^6}-\frac{\ln(bx+a)}{a^6}$
risch	$\frac{\frac{b^4x^4}{a^5}+\frac{9b^3x^3}{2a^4}+\frac{47b^2x^2}{6a^3}+\frac{77bx}{12a^2}+\frac{137}{60a}}{(bx+a)(b^2x^2+2abx+a^2)^2}+\frac{\ln(-x)}{a^6}-\frac{\ln(bx+a)}{a^6}$
parallelrisch	$\frac{60\ln(x)x^5b^5-60\ln(bx+a)x^5b^5+300\ln(x)x^4ab^4-300\ln(bx+a)x^4ab^4-137b^5x^5+600\ln(x)x^3a^2b^3-600\ln(bx+a)x^3a^2b^3-62}{60a}$

input
```
int(1/x/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/5/a/(b*x+a)^5+1/4/a^2/(b*x+a)^4+1/3/a^3/(b*x+a)^3+1/2/a^4/(b*x+a)^2+1/a^
5/(b*x+a)+ln(x)/a^6-ln(b*x+a)/a^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(77) = 154$.

Time = 0.09 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.49

$$\int \frac{1}{x(a^2+2abx+b^2x^2)^3}\,dx = \frac{60\,ab^4x^4+270\,a^2b^3x^3+470\,a^3b^2x^2+385\,a^4bx+137\,a^5-60\,(b^5x^5+5\,ab^4x^4+10\,a^2b^3x^3+10\,a^3b^2x^2+5}{60\,(a^6b^5x^5+5\,a^7b^4x^4+10\,a^8b^3x^3+1}$$

input
```
integrate(1/x/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
1/60*(60*a*b^4*x^4 + 270*a^2*b^3*x^3 + 470*a^3*b^2*x^2 + 385*a^4*b*x + 137
*a^5 - 60*(b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4
*b*x + a^5)*log(b*x + a) + 60*(b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10
*a^3*b^2*x^2 + 5*a^4*b*x + a^5)*log(x))/(a^6*b^5*x^5 + 5*a^7*b^4*x^4 + 10*
a^8*b^3*x^3 + 10*a^9*b^2*x^2 + 5*a^10*b*x + a^11)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.38

$$\int \frac{1}{x(a^2+2abx+b^2x^2)^3}\,dx = \frac{137a^4+385a^3bx+470a^2b^2x^2+270ab^3x^3+60b^4x^4}{60a^{10}+300a^9bx+600a^8b^2x^2+600a^7b^3x^3+300a^6b^4x^4+60a^5b^5x^5} + \frac{\log(x)-\log\left(\frac{a}{b}+x\right)}{a^6}$$

input
```
integrate(1/x/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
(137*a**4 + 385*a**3*b*x + 470*a**2*b**2*x**2 + 270*a*b**3*x**3 + 60*b**4*
x**4)/(60*a**10 + 300*a**9*b*x + 600*a**8*b**2*x**2 + 600*a**7*b**3*x**3 +
 300*a**6*b**4*x**4 + 60*a**5*b**5*x**5) + (log(x) - log(a/b + x))/a**6
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.38

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{60\,b^4x^4+270\,ab^3x^3+470\,a^2b^2x^2+385\,a^3bx+137\,a^4}{60\left(a^5b^5x^5+5\,a^6b^4x^4+10\,a^7b^3x^3+10\,a^8b^2x^2+5\,a^9bx+a^{10}\right)} - \frac{\log\left(bx+a\right)}{a^6}+\frac{\log\left(x\right)}{a^6}$$

input
```
integrate(1/x/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
1/60*(60*b^4*x^4 + 270*a*b^3*x^3 + 470*a^2*b^2*x^2 + 385*a^3*b*x + 137*a^4
)/(a^5*b^5*x^5 + 5*a^6*b^4*x^4 + 10*a^7*b^3*x^3 + 10*a^8*b^2*x^2 + 5*a^9*b
*x + a^10) - log(b*x + a)/a^6 + log(x)/a^6
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^3}\,dx = -\frac{\log\left(|bx+a|\right)}{a^6}+\frac{\log\left(|x|\right)}{a^6} + \frac{60\,ab^4x^4+270\,a^2b^3x^3+470\,a^3b^2x^2+385\,a^4bx+137\,a^5}{60\left(bx+a\right)^5a^6}$$

input
```
integrate(1/x/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
-log(abs(b*x + a))/a^6 + log(abs(x))/a^6 + 1/60*(60*a*b^4*x^4 + 270*a^2*b^
3*x^3 + 470*a^3*b^2*x^2 + 385*a^4*b*x + 137*a^5)/((b*x + a)^5*a^6)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.33

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^3}\,dx=\frac{\frac{137}{60\,a}+\frac{47\,b^2\,x^2}{6\,a^3}+\frac{9\,b^3\,x^3}{2\,a^4}+\frac{b^4\,x^4}{a^5}+\frac{77\,b\,x}{12\,a^2}}{a^5+5\,a^4\,b\,x+10\,a^3\,b^2\,x^2+10\,a^2\,b^3\,x^3+5\,a\,b^4\,x^4+b^5\,x^5}-\frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^6}$$

input
```
int(1/(x*(a^2 + b^2*x^2 + 2*a*b*x)^3),x)
```

output
```
(137/(60*a) + (47*b^2*x^2)/(6*a^3) + (9*b^3*x^3)/(2*a^4) + (b^4*x^4)/a^5 +
 (77*b*x)/(12*a^2))/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2
*b^3*x^3 + 5*a^4*b*x) - (2*atanh((2*b*x)/a + 1))/a^6
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.93

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^3}\,dx$$
$$=\frac{-60\log(bx+a)\,a^5-300\log(bx+a)\,a^4bx-600\log(bx+a)\,a^3b^2x^2-600\log(bx+a)\,a^2b^3x^3-300\log(b}{}$$

input
```
int(1/x/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
( - 60*log(a + b*x)*a**5 - 300*log(a + b*x)*a**4*b*x - 600*log(a + b*x)*a*
*3*b**2*x**2 - 600*log(a + b*x)*a**2*b**3*x**3 - 300*log(a + b*x)*a*b**4*x
**4 - 60*log(a + b*x)*b**5*x**5 + 60*log(x)*a**5 + 300*log(x)*a**4*b*x + 6
00*log(x)*a**3*b**2*x**2 + 600*log(x)*a**2*b**3*x**3 + 300*log(x)*a*b**4*x
**4 + 60*log(x)*b**5*x**5 + 125*a**5 + 325*a**4*b*x + 350*a**3*b**2*x**2 +
 150*a**2*b**3*x**3 - 12*b**5*x**5)/(60*a**6*(a**5 + 5*a**4*b*x + 10*a**3*
b**2*x**2 + 10*a**2*b**3*x**3 + 5*a*b**4*x**4 + b**5*x**5))
```

3.41 $\int \frac{1}{x^2\left(a^2+2abx+b^2x^2\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{1}{x^2\left(a^2+2abx+b^2x^2\right)^3}\,dx=-\frac{1}{a^6x}-\frac{b}{5a^2(a+bx)^5}-\frac{b}{2a^3(a+bx)^4}-\frac{b}{a^4(a+bx)^3}-\frac{2b}{a^5(a+bx)^2}-\frac{5b}{a^6(a+bx)}-\frac{6b\log(x)}{a^7}+\frac{6b\log(a+bx)}{a^7}$$

output
```
-1/a^6/x-1/5*b/a^2/(b*x+a)^5-1/2*b/a^3/(b*x+a)^4-b/a^4/(b*x+a)^3-2*b/a^5/(
b*x+a)^2-5*b/a^6/(b*x+a)-6*b*ln(x)/a^7+6*b*ln(b*x+a)/a^7
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\left(a^2+2abx+b^2x^2\right)^3}\,dx=-\frac{\frac{a\left(10a^5+137a^4bx+385a^3b^2x^2+470a^2b^3x^3+270ab^4x^4+60b^5x^5\right)}{x(a+bx)^5}+60b\log(x)-60b\log(a+bx)}{10a^7}$$

input
```
Integrate[1/(x^2*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

output
```
-1/10*((a*(10*a^5 + 137*a^4*b*x + 385*a^3*b^2*x^2 + 470*a^2*b^3*x^3 + 270*
a*b^4*x^4 + 60*b^5*x^5))/(x*(a + b*x)^5) + 60*b*Log[x] - 60*b*Log[a + b*x]
)/a^7
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1098, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a^2+2abx+b^2x^2)^3} \, dx$$

$$\Big\downarrow \text{1098}$$

$$b^6 \int \frac{1}{b^6x^2(a+bx)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{x^2(a+bx)^6}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{6b^2}{a^7(a+bx)} - \frac{6b}{a^7x} + \frac{5b^2}{a^6(a+bx)^2} + \frac{1}{a^6x^2} + \frac{4b^2}{a^5(a+bx)^3} + \frac{3b^2}{a^4(a+bx)^4} + \frac{2b^2}{a^3(a+bx)^5} + \frac{b^2}{a^2(a+bx)^6}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6b\log(x)}{a^7} + \frac{6b\log(a+bx)}{a^7} - \frac{5b}{a^6(a+bx)} - \frac{1}{a^6x} - \frac{2b}{a^5(a+bx)^2} - \frac{b}{a^4(a+bx)^3} - \frac{b}{2a^3(a+bx)^4} - \frac{b}{5a^2(a+bx)^5}$$

input
```
Int[1/(x^2*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

output
```
-(1/(a^6*x)) - b/(5*a^2*(a + b*x)^5) - b/(2*a^3*(a + b*x)^4) - b/(a^4*(a +
 b*x)^3) - (2*b)/(a^5*(a + b*x)^2) - (5*b)/(a^6*(a + b*x)) - (6*b*Log[x])/
a^7 + (6*b*Log[a + b*x])/a^7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.96

method	result
norman	$\frac{-\frac{1}{a}+\frac{30b^2x^2}{a^3}+\frac{90b^3x^3}{a^4}+\frac{110b^4x^4}{a^5}+\frac{125b^5x^5}{2a^6}+\frac{137b^6x^6}{10a^7}}{x(bx+a)^5}-\frac{6b\ln(x)}{a^7}+\frac{6b\ln(bx+a)}{a^7}$
default	$-\frac{1}{a^6x}-\frac{b}{5a^2(bx+a)^5}-\frac{b}{2a^3(bx+a)^4}-\frac{b}{a^4(bx+a)^3}-\frac{2b}{a^5(bx+a)^2}-\frac{5b}{a^6(bx+a)}-\frac{6b\ln(x)}{a^7}+\frac{6b\ln(bx+a)}{a^7}$
risch	$\frac{-\frac{6b^5x^5}{a^6}-\frac{27b^4x^4}{a^5}-\frac{47b^3x^3}{a^4}-\frac{77b^2x^2}{2a^3}-\frac{137bx}{10a^2}-\frac{1}{a}}{x(b^2x^2+2abx+a^2)^2(bx+a)}-\frac{6b\ln(x)}{a^7}+\frac{6b\ln(-bx-a)}{a^7}$
parallelrisch	$-\frac{60\ln(x)x^6b^6-60\ln(bx+a)x^6b^6+300\ln(x)x^5a\,b^5-300\ln(bx+a)x^5a\,b^5-137b^6x^6+600\ln(x)x^4a^2b^4-600\ln(bx+a)x^4a^2b^4-}{}$

input
```
int(1/x^2/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/a+30*b^2/a^3*x^2+90*b^3/a^4*x^3+110*b^4/a^5*x^4+125/2*b^5/a^6*x^5+137/
10*b^6/a^7*x^6)/x/(b*x+a)^5-6*b*ln(x)/a^7+6*b*ln(b*x+a)/a^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(94) = 188$.

Time = 0.11 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.46

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)^3} \, dx = -\frac{60\, ab^5x^5 + 270\, a^2b^4x^4 + 470\, a^3b^3x^3 + 385\, a^4b^2x^2 + 137\, a^5bx + 10\, a^6 - 60\, (b^6x^6 + 5\, ab^5x^5 + 10\, a^2b^4x^4}{10\, (a^7b^5x^6 + 5\, a^8b^4x^5 + 10}$$

input
```
integrate(1/x^2/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
-1/10*(60*a*b^5*x^5 + 270*a^2*b^4*x^4 + 470*a^3*b^3*x^3 + 385*a^4*b^2*x^2
+ 137*a^5*b*x + 10*a^6 - 60*(b^6*x^6 + 5*a*b^5*x^5 + 10*a^2*b^4*x^4 + 10*a
^3*b^3*x^3 + 5*a^4*b^2*x^2 + a^5*b*x)*log(b*x + a) + 60*(b^6*x^6 + 5*a*b^5
*x^5 + 10*a^2*b^4*x^4 + 10*a^3*b^3*x^3 + 5*a^4*b^2*x^2 + a^5*b*x)*log(x))/
(a^7*b^5*x^6 + 5*a^8*b^4*x^5 + 10*a^9*b^3*x^4 + 10*a^10*b^2*x^3 + 5*a^11*b
*x^2 + a^12*x)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.41

$$\begin{aligned} \int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)^3} \, dx &= \frac{-10a^5 - 137a^4bx - 385a^3b^2x^2 - 470a^2b^3x^3 - 270ab^4x^4 - 60b^5x^5}{10a^{11}x + 50a^{10}bx^2 + 100a^9b^2x^3 + 100a^8b^3x^4 + 50a^7b^4x^5 + 10a^6b^5x^6} \\ &\quad + \frac{6b\left(-\log(x) + \log\left(\frac{a}{b} + x\right)\right)}{a^7} \end{aligned}$$

input
```
integrate(1/x**2/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
(-10*a**5 - 137*a**4*b*x - 385*a**3*b**2*x**2 - 470*a**2*b**3*x**3 - 270*a
*b**4*x**4 - 60*b**5*x**5)/(10*a**11*x + 50*a**10*b*x**2 + 100*a**9*b**2*x
**3 + 100*a**8*b**3*x**4 + 50*a**7*b**4*x**5 + 10*a**6*b**5*x**6) + 6*b*(-
log(x) + log(a/b + x))/a**7
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.38

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)^3} \, dx = -\frac{60 \, b^5x^5 + 270 \, ab^4x^4 + 470 \, a^2b^3x^3 + 385 \, a^3b^2x^2 + 137 \, a^4bx + 10 \, a^5}{10 \, (a^6b^5x^6 + 5 \, a^7b^4x^5 + 10 \, a^8b^3x^4 + 10 \, a^9b^2x^3 + 5 \, a^{10}bx^2 + a^{11}x)} + \frac{6 \, b \log(bx + a)}{a^7} - \frac{6 \, b \log(x)}{a^7}$$

input
```
integrate(1/x^2/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
-1/10*(60*b^5*x^5 + 270*a*b^4*x^4 + 470*a^2*b^3*x^3 + 385*a^3*b^2*x^2 + 13
7*a^4*b*x + 10*a^5)/(a^6*b^5*x^6 + 5*a^7*b^4*x^5 + 10*a^8*b^3*x^4 + 10*a^9
*b^2*x^3 + 5*a^10*b*x^2 + a^11*x) + 6*b*log(b*x + a)/a^7 - 6*b*log(x)/a^7
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)^3} \, dx = \frac{6 \, b \log(|bx + a|)}{a^7} - \frac{6 \, b \log(|x|)}{a^7} - \frac{60 \, ab^5x^5 + 270 \, a^2b^4x^4 + 470 \, a^3b^3x^3 + 385 \, a^4b^2x^2 + 137 \, a^5bx + 10 \, a^6}{10 \, (bx + a)^5a^7x}$$

input
```
integrate(1/x^2/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
6*b*log(abs(b*x + a))/a^7 - 6*b*log(abs(x))/a^7 - 1/10*(60*a*b^5*x^5 + 270
*a^2*b^4*x^4 + 470*a^3*b^3*x^3 + 385*a^4*b^2*x^2 + 137*a^5*b*x + 10*a^6)/(
(b*x + a)^5*a^7*x)
```

Mupad [B] (verification not implemented)

Time = 9.77 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)^3}\,dx$$
$$= \frac{12\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^7} - \frac{\frac{1}{a}+\frac{77\,b^2\,x^2}{2\,a^3}+\frac{47\,b^3\,x^3}{a^4}+\frac{27\,b^4\,x^4}{a^5}+\frac{6\,b^5\,x^5}{a^6}+\frac{137\,b\,x}{10\,a^2}}{a^5\,x+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^3+10\,a^2\,b^3\,x^4+5\,a\,b^4\,x^5+b^5\,x^6}$$

input
```
int(1/(x^2*(a^2 + b^2*x^2 + 2*a*b*x)^3),x)
```

output
```
(12*b*atanh((2*b*x)/a + 1))/a^7 - (1/a + (77*b^2*x^2)/(2*a^3) + (47*b^3*x^
3)/a^4 + (27*b^4*x^4)/a^5 + (6*b^5*x^5)/a^6 + (137*b*x)/(10*a^2))/(a^5*x +
 b^5*x^6 + 5*a^4*b*x^2 + 5*a*b^4*x^5 + 10*a^3*b^2*x^3 + 10*a^2*b^3*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.81

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)^3}\,dx$$
$$= \frac{60\log(bx+a)\,a^5bx + 300\log(bx+a)\,a^4b^2x^2 + 600\log(bx+a)\,a^3b^3x^3 + 600\log(bx+a)\,a^2b^4x^4 + 300\log}{}$$

input
```
int(1/x^2/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
(60*log(a + b*x)*a**5*b*x + 300*log(a + b*x)*a**4*b**2*x**2 + 600*log(a +
b*x)*a**3*b**3*x**3 + 600*log(a + b*x)*a**2*b**4*x**4 + 300*log(a + b*x)*a
*b**5*x**5 + 60*log(a + b*x)*b**6*x**6 - 60*log(x)*a**5*b*x - 300*log(x)*a
**4*b**2*x**2 - 600*log(x)*a**3*b**3*x**3 - 600*log(x)*a**2*b**4*x**4 - 30
0*log(x)*a*b**5*x**5 - 60*log(x)*b**6*x**6 - 10*a**6 - 125*a**5*b*x - 325*
a**4*b**2*x**2 - 350*a**3*b**3*x**3 - 150*a**2*b**4*x**4 + 12*b**6*x**6)/(
10*a**7*x*(a**5 + 5*a**4*b*x + 10*a**3*b**2*x**2 + 10*a**2*b**3*x**3 + 5*a
*b**4*x**4 + b**5*x**5))
```

3.42 $\int \frac{x}{4+4x+x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x}{4+4x+x^2}\,dx = \frac{2}{2+x} + \log(2+x)$$

output
```
2/(2+x)+ln(2+x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{4+4x+x^2}\,dx = \frac{2}{2+x} + \log(2+x)$$

input
```
Integrate[x/(4 + 4*x + x^2),x]
```

output
```
2/(2 + x) + Log[2 + x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1098, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^2+4x+4}dx$$
$$\Big\downarrow \ 1098$$
$$\int \frac{x}{(x+2)^2}dx$$
$$\Big\downarrow \ 49$$
$$\int \left(\frac{1}{x+2}-\frac{2}{(x+2)^2}\right)dx$$
$$\Big\downarrow \ 2009$$
$$\frac{2}{x+2}+\log(x+2)$$

input
```
Int[x/(4 + 4*x + x^2),x]
```

output
```
2/(2 + x) + Log[2 + x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{2}{2+x}+\ln\left(2+x\right)$	13
norman	$\frac{2}{2+x}+\ln\left(2+x\right)$	13
risch	$\frac{2}{2+x}+\ln\left(2+x\right)$	13
meijerg	$-\frac{x}{2\left(1+\frac{x}{2}\right)}+\ln\left(1+\frac{x}{2}\right)$	18
parallelrisch	$\frac{\ln(2+x)x+2+2\ln(2+x)}{2+x}$	21

input
```
int(x/(x^2+4*x+4),x,method=_RETURNVERBOSE)
```

output
```
2/(2+x)+ln(2+x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{x}{4+4x+x^2}\,dx=\frac{(x+2)\log\left(x+2\right)+2}{x+2}$$

input
```
integrate(x/(x^2+4*x+4),x, algorithm="fricas")
```

output
```
((x + 2)*log(x + 2) + 2)/(x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{4+4x+x^2}\,dx = \log(x+2) + \frac{2}{x+2}$$

input
```
integrate(x/(x**2+4*x+4),x)
```

output
```
log(x + 2) + 2/(x + 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{4+4x+x^2}\,dx = \frac{2}{x+2} + \log(x+2)$$

input
```
integrate(x/(x^2+4*x+4),x, algorithm="maxima")
```

output
```
2/(x + 2) + log(x + 2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{x}{4+4x+x^2}\,dx = \frac{2}{x+2} + \log(|x+2|)$$

input
```
integrate(x/(x^2+4*x+4),x, algorithm="giac")
```

output
```
2/(x + 2) + log(abs(x + 2))
```

Mupad [B] (verification not implemented)

Time = 9.82 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{4+4x+x^2}\,dx = \ln(x+2) + \frac{2}{x+2}$$

input
```
int(x/(4*x + x^2 + 4),x)
```

output
```
log(x + 2) + 2/(x + 2)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{x}{4+4x+x^2}\,dx = \frac{\log(x+2)\,x + 2\log(x+2) - x}{x+2}$$

input
```
int(x/(x^2+4*x+4),x)
```

output
```
(log(x + 2)*x + 2*log(x + 2) - x)/(x + 2)
```

3.43 $\int \frac{x^3}{1+2x+x^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 22

$$\int \frac{x^3}{1+2x+x^2} \, dx = -2x+\frac{x^2}{2}+\frac{1}{1+x}+3\log(1+x)$$

output
```
-2*x+1/2*x^2+1/(1+x)+3*ln(1+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{x^3}{1+2x+x^2} \, dx = \frac{1}{1+x}-3(1+x)+\frac{1}{2}(1+x)^2+3\log(1+x)$$

input
```
Integrate[x^3/(1 + 2*x + x^2),x]
```

output
```
(1 + x)^(-1) - 3*(1 + x) + (1 + x)^2/2 + 3*Log[1 + x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1098, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^2+2x+1} \, dx$$

$$\Big\downarrow \text{ 1098}$$

$$\int \frac{x^3}{(x+1)^2}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(x+\frac{3}{x+1}-\frac{1}{(x+1)^2}-2\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^2}{2}-2x+\frac{1}{x+1}+3\log(x+1)$$

input
```
Int[x^3/(1 + 2*x + x^2),x]
```

output
```
-2*x + x^2/2 + (1 + x)^(-1) + 3*Log[1 + x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
default	$-2x+\frac{x^{2}}{2}+\frac{1}{x+1}+3\ln\left(x+1\right)$	21
risch	$-2x+\frac{x^{2}}{2}+\frac{1}{x+1}+3\ln\left(x+1\right)$	21
norman	$\frac{-\frac{3}{2}x^{2}+\frac{1}{2}x^{3}+3}{x+1}+3\ln\left(x+1\right)$	26
meijerg	$-\frac{x\left(-2x^{2}+6x+12\right)}{4\left(x+1\right)}+3\ln\left(x+1\right)$	26
parallelrisch	$\frac{x^{3}+6\ln\left(x+1\right)x-3x^{2}+6+6\ln\left(x+1\right)}{2+2x}$	31

input
```
int(x^3/(x^2+2*x+1),x,method=_RETURNVERBOSE)
```

output
```
-2*x+1/2*x^2+1/(x+1)+3*ln(x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{x^3}{1+2x+x^2}\,dx=\frac{x^{3}-3\,x^{2}+6\,(x+1)\log\left(x+1\right)-4\,x+2}{2\,(x+1)}$$

input
```
integrate(x^3/(x^2+2*x+1),x, algorithm="fricas")
```

output
```
1/2*(x^3 - 3*x^2 + 6*(x + 1)*log(x + 1) - 4*x + 2)/(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{x^3}{1+2x+x^2}\,dx = \frac{x^2}{2} - 2x + 3\log{(x+1)} + \frac{1}{x+1}$$

input
```
integrate(x**3/(x**2+2*x+1),x)
```

output
```
x**2/2 - 2*x + 3*log(x + 1) + 1/(x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{1+2x+x^2}\,dx = \frac{1}{2}\,x^2 - 2\,x + \frac{1}{x+1} + 3\,\log\left(x+1\right)$$

input
```
integrate(x^3/(x^2+2*x+1),x, algorithm="maxima")
```

output
```
1/2*x^2 - 2*x + 1/(x + 1) + 3*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{1+2x+x^2}\,dx = \frac{1}{2}\,x^2 - 2\,x + \frac{1}{x+1} + 3\,\log\left(|x+1|\right)$$

input
```
integrate(x^3/(x^2+2*x+1),x, algorithm="giac")
```

output
```
1/2*x^2 - 2*x + 1/(x + 1) + 3*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{1+2x+x^2}\,dx = 3\,\ln\left(x+1\right) - 2\,x + \frac{1}{x+1} + \frac{x^2}{2}$$

input
```
int(x^3/(2*x + x^2 + 1),x)
```

output
```
3*log(x + 1) - 2*x + 1/(x + 1) + x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{x^3}{1+2x+x^2}\,dx = \frac{6\log(x+1)\,x + 6\log(x+1) + x^3 - 3x^2 - 6x}{2x+2}$$

input
```
int(x^3/(x^2+2*x+1),x)
```

output
```
(6*log(x + 1)*x + 6*log(x + 1) + x**3 - 3*x**2 - 6*x)/(2*(x + 1))
```

3.44 $\int \frac{x^3}{1-2x+x^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 26

$$\int \frac{x^3}{1-2x+x^2} \, dx = \frac{1}{1-x} + 2x + \frac{x^2}{2} + 3\log(1-x)$$

output
```
1/(1-x)+2*x+1/2*x^2+3*ln(1-x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{x^3}{1-2x+x^2} \, dx = \frac{1}{2}\left(-5 - \frac{2}{-1+x} + 4x + x^2 + 6\log(-1+x)\right)$$

input
```
Integrate[x^3/(1 - 2*x + x^2),x]
```

output
```
(-5 - 2/(-1 + x) + 4*x + x^2 + 6*Log[-1 + x])/2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1098, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^2-2x+1}dx$$

$$\Big\downarrow \text{ 1098}$$

$$\int \frac{x^3}{(1-x)^2}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(x+\frac{3}{x-1}+\frac{1}{(x-1)^2}+2\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^2}{2}+2x+\frac{1}{1-x}+3\log(1-x)$$

input
```
Int[x^3/(1 - 2*x + x^2),x]
```

output
```
(1 - x)^(-1) + 2*x + x^2/2 + 3*Log[1 - x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$2x+\frac{x^2}{2}-\frac{1}{x-1}+3\ln\left(x-1\right)$	23
risch	$2x+\frac{x^2}{2}-\frac{1}{x-1}+3\ln\left(x-1\right)$	23
norman	$\frac{\frac{3}{2}x^2+\frac{1}{2}x^3-3}{x-1}+3\ln\left(x-1\right)$	26
meijerg	$\frac{x\left(-2x^2-6x+12\right)}{4-4x}+3\ln\left(1-x\right)$	30
parallelrisch	$\frac{x^3+6\ln(x-1)x+3x^2-6-6\ln(x-1)}{2x-2}$	31

input
```
int(x^3/(x^2-2*x+1),x,method=_RETURNVERBOSE)
```

output
```
2*x+1/2*x^2-1/(x-1)+3*ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \frac{x^3}{1-2x+x^2}\,dx=\frac{x^3+3\,x^2+6\,(x-1)\log\left(x-1\right)-4\,x-2}{2\,(x-1)}$$

input
```
integrate(x^3/(x^2-2*x+1),x, algorithm="fricas")
```

output
```
1/2*(x^3 + 3*x^2 + 6*(x - 1)*log(x - 1) - 4*x - 2)/(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{1-2x+x^2}\,dx = \frac{x^2}{2} + 2x + 3\log{(x-1)} - \frac{1}{x-1}$$

input
```
integrate(x**3/(x**2-2*x+1),x)
```

output
```
x**2/2 + 2*x + 3*log(x - 1) - 1/(x - 1)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{1-2x+x^2}\,dx = \frac{1}{2}\,x^2 + 2\,x - \frac{1}{x-1} + 3\,\log{(x-1)}$$

input
```
integrate(x^3/(x^2-2*x+1),x, algorithm="maxima")
```

output
```
1/2*x^2 + 2*x - 1/(x - 1) + 3*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{1-2x+x^2}\,dx = \frac{1}{2}\,x^2 + 2\,x - \frac{1}{x-1} + 3\,\log{(|x-1|)}$$

input
```
integrate(x^3/(x^2-2*x+1),x, algorithm="giac")
```

output
```
1/2*x^2 + 2*x - 1/(x - 1) + 3*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{1-2x+x^2}\,dx = 2\,x + 3\,\ln\left(x-1\right) - \frac{1}{x-1} + \frac{x^2}{2}$$

input
```
int(x^3/(x^2 - 2*x + 1),x)
```

output
```
2*x + 3*log(x - 1) - 1/(x - 1) + x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \frac{x^3}{1-2x+x^2}\,dx = \frac{6\log(x-1)\,x - 6\log(x-1) + x^3 + 3x^2 - 6x}{2x-2}$$

input
```
int(x^3/(x^2-2*x+1),x)
```

output
```
(6*log(x - 1)*x - 6*log(x - 1) + x**3 + 3*x**2 - 6*x)/(2*(x - 1))
```

3.45 $\int \frac{x^4}{4+4x+x^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 29

$$\int \frac{x^4}{4+4x+x^2} dx = 12x - 2x^2 + \frac{x^3}{3} - \frac{16}{2+x} - 32\log(2+x)$$

output
```
12*x-2*x^2+1/3*x^3-16/(2+x)-32*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{x^4}{4+4x+x^2} dx = \frac{1}{3}\left(104 + 36x - 6x^2 + x^3 - \frac{48}{2+x} - 96\log(2+x)\right)$$

input
```
Integrate[x^4/(4 + 4*x + x^2),x]
```

output
```
(104 + 36*x - 6*x^2 + x^3 - 48/(2 + x) - 96*Log[2 + x])/3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1098, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^2+4x+4} \, dx$$
$$\Big\downarrow \text{ 1098}$$
$$\int \frac{x^4}{(x+2)^2}dx$$
$$\Big\downarrow \text{ 49}$$
$$\int \left(x^2-4x-\frac{32}{x+2}+\frac{16}{(x+2)^2}+12\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{x^3}{3}-2x^2+12x-\frac{16}{x+2}-32\log(x+2)$$

input
```
Int[x^4/(4 + 4*x + x^2),x]
```

output
```
12*x - 2*x^2 + x^3/3 - 16/(2 + x) - 32*Log[2 + x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

method	result	size
default	$12x-2x^{2}+\frac{x^{3}}{3}-\frac{16}{2+x}-32\ln\left(2+x\right)$	28
risch	$12x-2x^{2}+\frac{x^{3}}{3}-\frac{16}{2+x}-32\ln\left(2+x\right)$	28
norman	$\frac{8x^{2}-\frac{4}{3}x^{3}+\frac{1}{3}x^{4}-64}{2+x}-32\ln\left(2+x\right)$	31
meijerg	$\frac{4x\left(\frac{5}{8}x^{3}-\frac{5}{2}x^{2}+15x+60\right)}{15\left(1+\frac{x}{2}\right)}-32\ln\left(1+\frac{x}{2}\right)$	35
parallelrisch	$-\frac{-x^{4}+4x^{3}+96\ln(2+x)x-24x^{2}+192+192\ln(2+x)}{3(2+x)}$	38

input
```
int(x^4/(x^2+4*x+4),x,method=_RETURNVERBOSE)
```

output
```
12*x-2*x^2+1/3*x^3-16/(2+x)-32*ln(2+x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{x^4}{4+4x+x^2}\,dx=\frac{x^{4}-4\,x^{3}+24\,x^{2}-96\,(x+2)\log\left(x+2\right)+72\,x-48}{3\,(x+2)}$$

input
```
integrate(x^4/(x^2+4*x+4),x, algorithm="fricas")
```

output
```
1/3*(x^4 - 4*x^3 + 24*x^2 - 96*(x + 2)*log(x + 2) + 72*x - 48)/(x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{4+4x+x^2}\,dx = \frac{x^3}{3} - 2x^2 + 12x - 32\log{(x+2)} - \frac{16}{x+2}$$

input
```
integrate(x**4/(x**2+4*x+4),x)
```

output
```
x**3/3 - 2*x**2 + 12*x - 32*log(x + 2) - 16/(x + 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{x^4}{4+4x+x^2}\,dx = \frac{1}{3}\,x^3 - 2\,x^2 + 12\,x - \frac{16}{x+2} - 32\,\log\left(x+2\right)$$

input
```
integrate(x^4/(x^2+4*x+4),x, algorithm="maxima")
```

output
```
1/3*x^3 - 2*x^2 + 12*x - 16/(x + 2) - 32*log(x + 2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{x^4}{4+4x+x^2}\,dx = \frac{1}{3}\,x^3 - 2\,x^2 + 12\,x - \frac{16}{x+2} - 32\,\log\left(|x+2|\right)$$

input
```
integrate(x^4/(x^2+4*x+4),x, algorithm="giac")
```

output
```
1/3*x^3 - 2*x^2 + 12*x - 16/(x + 2) - 32*log(abs(x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{x^4}{4+4x+x^2}\,dx = 12\,x - 32\,\ln\left(x+2\right) - \frac{16}{x+2} - 2\,x^2 + \frac{x^3}{3}$$

input
```
int(x^4/(4*x + x^2 + 4),x)
```

output
```
12*x - 32*log(x + 2) - 16/(x + 2) - 2*x^2 + x^3/3
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.31

$$\int \frac{x^4}{4+4x+x^2}\,dx = \frac{-96\log(x+2)\,x - 192\log(x+2) + x^4 - 4x^3 + 24x^2 + 96x}{3x+6}$$

input
```
int(x^4/(x^2+4*x+4),x)
```

output
```
( - 96*log(x + 2)*x - 192*log(x + 2) + x**4 - 4*x**3 + 24*x**2 + 96*x)/(3*
(x + 2))
```

3.46 $\int x^4\sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 71

$$\int x^4\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{ax^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}+\frac{bx^6\sqrt{a^2+2abx+b^2x^2}}{6(a+bx)}$$

output
```
a*x^5*((b*x+a)^2)^(1/2)/(5*b*x+5*a)+b*x^6*((b*x+a)^2)^(1/2)/(6*b*x+6*a)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.14

$$\int x^4\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{x^5(6a+5bx)\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}-\sqrt{(a+bx)^2}\right)\right)}{30\left(-a^2-abx+\sqrt{a^2}\sqrt{(a+bx)^2}\right)}$$

input
```
Integrate[x^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(x^5*(6*a + 5*b*x)*(Sqrt[a^2]*b*x + a*(Sqrt[a^2] - Sqrt[(a + b*x)^2])))/(3
0*(-a^2 - a*b*x + Sqrt[a^2]*Sqrt[(a + b*x)^2]))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.63, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \sqrt{a^2+2abx+b^2x^2} \, dx$$

$$\Big\downarrow \text{ 1102}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2} \int bx^4(a+bx)dx}{b(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2} \int x^4(a+bx)dx}{a+bx}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2} \int \left(bx^5+ax^4\right) dx}{a+bx}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2} \left(\frac{ax^5}{5}+\frac{bx^6}{6}\right)}{a+bx}$$

input
```
Int[x^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((a*x^5)/5 + (b*x^6)/6))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.42

method	result	size
gosper	$\frac{x^{5}(5bx+6a)\sqrt{(bx+a)^{2}}}{30bx+30a}$	30
orering	$\frac{x^{5}(5bx+6a)\sqrt{(bx+a)^{2}}}{30bx+30a}$	30
risch	$\frac{a\,x^{5}\sqrt{(bx+a)^{2}}}{5bx+5a}+\frac{b\,x^{6}\sqrt{(bx+a)^{2}}}{6bx+6a}$	46
default	$\frac{\operatorname{csgn}(bx+a)(bx+a)^{2}\left(5b^{4}x^{4}-4a\,b^{3}x^{3}+3a^{2}b^{2}x^{2}-2a^{3}bx+a^{4}\right)}{30b^{5}}$	58

input
```
int(x^4*((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/30*x^5*(5*b*x+6*a)*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.18

$$\int x^4\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{1}{6}\,bx^6+\frac{1}{5}\,ax^5$$

input
```
integrate(x^4*((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*b*x^6 + 1/5*a*x^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(46) = 92$.

Time = 1.79 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.70

$$\int x^4\sqrt{a^2+2abx+b^2x^2}\,dx$$
$$=\begin{cases}\sqrt{a^2+2abx+b^2x^2}\left(\frac{a^5}{30b^5}-\frac{a^4x}{30b^4}+\frac{a^3x^2}{30b^3}-\frac{a^2x^3}{30b^2}+\frac{ax^4}{30b}+\frac{x^5}{6}\right) & \text{for } b^2\neq 0\\ \frac{\frac{a^8(a^2+2abx)^{\frac{3}{2}}}{3}-\frac{4a^6(a^2+2abx)^{\frac{5}{2}}}{5}+\frac{6a^4(a^2+2abx)^{\frac{7}{2}}}{7}-\frac{4a^2(a^2+2abx)^{\frac{9}{2}}}{9}+\frac{(a^2+2abx)^{\frac{11}{2}}}{11}}{16a^5b^5} & \text{for } ab\neq 0\\ \frac{x^5\sqrt{a^2}}{5} & \text{otherwise}\end{cases}$$

input
```
integrate(x**4*((b*x+a)**2)**(1/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(a**5/(30*b**5) - a**4*x/(30*b
**4) + a**3*x**2/(30*b**3) - a**2*x**3/(30*b**2) + a*x**4/(30*b) + x**5/6)
, Ne(b**2, 0)), ((a**8*(a**2 + 2*a*b*x)**(3/2)/3 - 4*a**6*(a**2 + 2*a*b*x)
**(5/2)/5 + 6*a**4*(a**2 + 2*a*b*x)**(7/2)/7 - 4*a**2*(a**2 + 2*a*b*x)**(9
/2)/9 + (a**2 + 2*a*b*x)**(11/2)/11)/(16*a**5*b**5), Ne(a*b, 0)), (x**5*sq
rt(a**2)/5, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(45) = 90$.

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.25

$$\int x^4\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}x^3}{6\,b^2} + \frac{\sqrt{b^2x^2+2\,abx+a^2}a^4x}{2\,b^4} - \frac{3\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}ax^2}{10\,b^3} + \frac{\sqrt{b^2x^2+2\,abx+a^2}a^5}{2\,b^5} + \frac{2\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}a^2x}{5\,b^4} - \frac{7\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}a^3}{15\,b^5}$$

input
```
integrate(x^4*((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*x^3/b^2 + 1/2*sqrt(b^2*x^2 + 2*a*b*x +
 a^2)*a^4*x/b^4 - 3/10*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a*x^2/b^3 + 1/2*sqr
t(b^2*x^2 + 2*a*b*x + a^2)*a^5/b^5 + 2/5*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a
^2*x/b^4 - 7/15*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a^3/b^5
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.55

$$\int x^4\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{1}{6}\,bx^6\mathrm{sgn}(bx+a) + \frac{1}{5}\,ax^5\mathrm{sgn}(bx+a) + \frac{a^6\mathrm{sgn}(bx+a)}{30\,b^5}$$

input
```
integrate(x^4*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/6*b*x^6*sgn(b*x + a) + 1/5*a*x^5*sgn(b*x + a) + 1/30*a^6*sgn(b*x + a)/b^
5
```

Mupad [B] (verification not implemented)

Time = 10.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.65

$$\int x^4\sqrt{a^2+2abx+b^2x^2}\,dx$$
$$=\frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}\,\left(a^5+5\,b^3\,x^3\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)-14\,a^3\,b^2\,x^2-13\,a^4\,b\,x-9\,a\,b^2\,x^2\,\left(a^2+2\,a\,b\,x+\right.\right.}{30\,b^5}$$

input
```
int(x^4*((a + b*x)^2)^(1/2),x)
```

output
```
((a^2 + b^2*x^2 + 2*a*b*x)^(1/2)*(a^5 + 5*b^3*x^3*(a^2 + b^2*x^2 + 2*a*b*x
) - 14*a^3*b^2*x^2 - 13*a^4*b*x - 9*a*b^2*x^2*(a^2 + b^2*x^2 + 2*a*b*x) +
12*a^2*b*x*(a^2 + b^2*x^2 + 2*a*b*x)))/(30*b^5)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.18

$$\int x^4\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{x^5(5bx+6a)}{30}$$

input
```
int(x^4*((b*x+a)^2)^(1/2),x)
```

output
```
(x**5*(6*a + 5*b*x))/30
```

3.47 $\int x^3\sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 71

$$\int x^3\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{ax^4\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)}+\frac{bx^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}$$

output
```
a*x^4*((b*x+a)^2)^(1/2)/(4*b*x+4*a)+b*x^5*((b*x+a)^2)^(1/2)/(5*b*x+5*a)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.14

$$\int x^3\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{x^4(5a+4bx)\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}-\sqrt{(a+bx)^2}\right)\right)}{20\left(-a^2-abx+\sqrt{a^2}\sqrt{(a+bx)^2}\right)}$$

input
```
Integrate[x^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(x^4*(5*a + 4*b*x)*(Sqrt[a^2]*b*x + a*(Sqrt[a^2] - Sqrt[(a + b*x)^2])))/(2
0*(-a^2 - a*b*x + Sqrt[a^2]*Sqrt[(a + b*x)^2]))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.63, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a^2+2abx+b^2x^2}\,dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int bx^3(a+bx)dx}{b(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^3(a+bx)dx}{a+bx}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(bx^4+ax^3\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{ax^4}{4}+\frac{bx^5}{5}\right)}{a+bx}$$

input
```
Int[x^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((a*x^4)/4 + (b*x^5)/5))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.42

method	result	size
gosper	$\frac{x^{4}(4bx+5a)\sqrt{(bx+a)^{2}}}{20bx+20a}$	30
orering	$\frac{x^{4}(4bx+5a)\sqrt{(bx+a)^{2}}}{20bx+20a}$	30
risch	$\frac{a\,x^{4}\sqrt{(bx+a)^{2}}}{4bx+4a}+\frac{b\,x^{5}\sqrt{(bx+a)^{2}}}{5bx+5a}$	46
default	$-\frac{\operatorname{csgn}(bx+a)(bx+a)^{2}\left(-4b^{3}x^{3}+3a\,b^{2}x^{2}-2a^{2}bx+a^{3}\right)}{20b^{4}}$	47

input
```
int(x^3*((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/20*x^4*(4*b*x+5*a)*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.18

$$\int x^3\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

input
```
integrate(x^3*((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/5*b*x^5 + 1/4*a*x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(46) = 92.

Time = 1.31 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.25

$$\int x^3\sqrt{a^2+2abx+b^2x^2}\,dx$$
$$=\begin{cases}\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^4}{20b^4}+\frac{a^3x}{20b^3}-\frac{a^2x^2}{20b^2}+\frac{ax^3}{20b}+\frac{x^4}{5}\right) & \text{for } b^2\neq 0\\ \frac{-\frac{a^6\left(a^2+2abx\right)^{\frac{3}{2}}}{3}+\frac{3a^4\left(a^2+2abx\right)^{\frac{5}{2}}}{5}-\frac{3a^2\left(a^2+2abx\right)^{\frac{7}{2}}}{7}+\frac{\left(a^2+2abx\right)^{\frac{9}{2}}}{9}}{8a^4b^4} & \text{for } ab\neq 0\\ \frac{x^4\sqrt{a^2}}{4} & \text{otherwise}\end{cases}$$

input
```
integrate(x**3*((b*x+a)**2)**(1/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(-a**4/(20*b**4) + a**3*x/(20*
b**3) - a**2*x**2/(20*b**2) + a*x**3/(20*b) + x**4/5), Ne(b**2, 0)), ((-a*
*6*(a**2 + 2*a*b*x)**(3/2)/3 + 3*a**4*(a**2 + 2*a*b*x)**(5/2)/5 - 3*a**2*(
a**2 + 2*a*b*x)**(7/2)/7 + (a**2 + 2*a*b*x)**(9/2)/9)/(8*a**4*b**4), Ne(a*
b, 0)), (x**4*sqrt(a**2)/4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(45) = 90.

Time = 0.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.85

$$\int x^3\sqrt{a^2+2abx+b^2x^2}\,dx = -\frac{\sqrt{b^2x^2+2\,abx+a^2}a^3x}{2\,b^3}+\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}x^2}{5\,b^2} - \frac{\sqrt{b^2x^2+2\,abx+a^2}a^4}{2\,b^4} - \frac{7\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}ax}{20\,b^3} + \frac{9\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}a^2}{20\,b^4}$$

input
```
integrate(x^3*((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^3*x/b^3 + 1/5*(b^2*x^2 + 2*a*b*x + a^
2)^(3/2)*x^2/b^2 - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^4/b^4 - 7/20*(b^2*x
^2 + 2*a*b*x + a^2)^(3/2)*a*x/b^3 + 9/20*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a
^2/b^4
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.55

$$\int x^3\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{1}{5}\,bx^5\mathrm{sgn}(bx+a)+\frac{1}{4}\,ax^4\mathrm{sgn}(bx+a)-\frac{a^5\mathrm{sgn}(bx+a)}{20\,b^4}$$

input
```
integrate(x^3*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/5*b*x^5*sgn(b*x + a) + 1/4*a*x^4*sgn(b*x + a) - 1/20*a^5*sgn(b*x + a)/b^
4
```

Mupad [B] (verification not implemented)

Time = 9.61 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.30

$$\int x^3\sqrt{a^2+2abx+b^2x^2}\,dx$$
$$=\frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}\,\left(4\,b^2\,x^2\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)-a^4+9\,a^2\,b^2\,x^2+8\,a^3\,b\,x-7\,a\,b\,x\,\left(a^2+2\,a\,b\,x+b^2\,x\right.}{20\,b^4}$$

input
```
int(x^3*((a + b*x)^2)^(1/2),x)
```

output
```
((a^2 + b^2*x^2 + 2*a*b*x)^(1/2)*(4*b^2*x^2*(a^2 + b^2*x^2 + 2*a*b*x) - a^
4 + 9*a^2*b^2*x^2 + 8*a^3*b*x - 7*a*b*x*(a^2 + b^2*x^2 + 2*a*b*x)))/(20*b^
4)
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.18

$$\int x^3\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{x^4(4bx+5a)}{20}$$

input
```
int(x^3*((b*x+a)^2)^(1/2),x)
```

output
```
(x**4*(5*a + 4*b*x))/20
```

3.48 $\int x^2\sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 71

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{ax^3\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)}+\frac{bx^4\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)}$$

output
```
a*x^3*((b*x+a)^2)^(1/2)/(3*b*x+3*a)+b*x^4*((b*x+a)^2)^(1/2)/(4*b*x+4*a)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.14

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{x^3(4a+3bx)\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}-\sqrt{(a+bx)^2}\right)\right)}{12\left(-a^2-abx+\sqrt{a^2}\sqrt{(a+bx)^2}\right)}$$

input
```
Integrate[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(x^3*(4*a + 3*b*x)*(Sqrt[a^2]*b*x + a*(Sqrt[a^2] - Sqrt[(a + b*x)^2])))/(1
2*(-a^2 - a*b*x + Sqrt[a^2]*Sqrt[(a + b*x)^2]))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.63, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\,dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int bx^2(a+bx)dx}{b(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^2(a+bx)dx}{a+bx}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(bx^3+ax^2\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{ax^3}{3}+\frac{bx^4}{4}\right)}{a+bx}$$

input
```
Int[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((a*x^3)/3 + (b*x^4)/4))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.42

method	result	size
gosper	$\frac{x^{3}(3bx+4a)\sqrt{(bx+a)^{2}}}{12bx+12a}$	30
orering	$\frac{x^{3}(3bx+4a)\sqrt{(bx+a)^{2}}}{12bx+12a}$	30
default	$\frac{\mathrm{csgn}(bx+a)(bx+a)^{2}(3b^{2}x^{2}-2abx+a^{2})}{12b^{3}}$	36
risch	$\frac{a\,x^{3}\sqrt{(bx+a)^{2}}}{3bx+3a}+\frac{b\,x^{4}\sqrt{(bx+a)^{2}}}{4bx+4a}$	46

input
```
int(x^2*((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*x^3*(3*b*x+4*a)*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.18

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

input
```
integrate(x^2*((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*b*x^4 + 1/3*a*x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(46) = 92.

Time = 0.92 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.80

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\,dx=\begin{cases}\sqrt{a^2+2abx+b^2x^2}\left(\frac{a^3}{12b^3}-\frac{a^2x}{12b^2}+\frac{ax^2}{12b}+\frac{x^3}{4}\right) & \text{for } b^2\neq 0\\ \frac{\frac{a^4(a^2+2abx)^{\frac{3}{2}}}{3}-\frac{2a^2(a^2+2abx)^{\frac{5}{2}}}{5}+\frac{(a^2+2abx)^{\frac{7}{2}}}{7}}{4a^3b^3} & \text{for } ab\neq 0\\ \frac{x^3\sqrt{a^2}}{3} & \text{otherwise}\end{cases}$$

input
```
integrate(x**2*((b*x+a)**2)**(1/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(a**3/(12*b**3) - a**2*x/(12*b
**2) + a*x**2/(12*b) + x**3/4), Ne(b**2, 0)), ((a**4*(a**2 + 2*a*b*x)**(3/
2)/3 - 2*a**2*(a**2 + 2*a*b*x)**(5/2)/5 + (a**2 + 2*a*b*x)**(7/2)/7)/(4*a*
*3*b**3), Ne(a*b, 0)), (x**3*sqrt(a**2)/3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(45) = 90$.

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.44

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{\sqrt{b^2x^2+2\,abx+a^2}a^2x}{2\,b^2} + \frac{\sqrt{b^2x^2+2\,abx+a^2}a^3}{2\,b^3} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}x}{4\,b^2} - \frac{5\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}a}{12\,b^3}$$

input
```
integrate(x^2*((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^2*x/b^2 + 1/2*sqrt(b^2*x^2 + 2*a*b*x +
 a^2)*a^3/b^3 + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*x/b^2 - 5/12*(b^2*x^2
+ 2*a*b*x + a^2)^(3/2)*a/b^3
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.55

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{1}{4}\,bx^4\mathrm{sgn}(bx+a) + \frac{1}{3}\,ax^3\mathrm{sgn}(bx+a) + \frac{a^4\mathrm{sgn}(bx+a)}{12\,b^3}$$

input
```
integrate(x^2*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*b*x^4*sgn(b*x + a) + 1/3*a*x^3*sgn(b*x + a) + 1/12*a^4*sgn(b*x + a)/b^
3
```

Mupad [B] (verification not implemented)

Time = 9.73 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\,dx$$
$$=\frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}\,\left(a^3-5\,a\,b^2\,x^2+3\,b\,x\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)-4\,a^2\,b\,x\right)}{12\,b^3}$$

input
```
int(x^2*((a + b*x)^2)^(1/2),x)
```

output
```
((a^2 + b^2*x^2 + 2*a*b*x)^(1/2)*(a^3 - 5*a*b^2*x^2 + 3*b*x*(a^2 + b^2*x^2
 + 2*a*b*x) - 4*a^2*b*x))/(12*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.18

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{x^3(3bx+4a)}{12}$$

input
```
int(x^2*((b*x+a)^2)^(1/2),x)
```

output
```
(x**3*(4*a + 3*b*x))/12
```

3.49 $\int x\sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 61

$$\int x\sqrt{a^2+2abx+b^2x^2}\,dx=-\frac{a(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2b^2}+\frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{3b^2}$$

output
```
-1/2*a*(b*x+a)*((b*x+a)^2)^(1/2)/b^2+1/3*(b^2*x^2+2*a*b*x+a^2)^(3/2)/b^2
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.30

$$\int x\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{x^2(3a+2bx)\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}-\sqrt{(a+bx)^2}\right)\right)}{-6a^2-6abx+6\sqrt{a^2}\sqrt{(a+bx)^2}}$$

input
```
Integrate[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(x^2*(3*a + 2*b*x)*(Sqrt[a^2]*b*x + a*(Sqrt[a^2] - Sqrt[(a + b*x)^2])))/(-
6*a^2 - 6*a*b*x + 6*Sqrt[a^2]*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a^2+2abx+b^2x^2}\,dx$$

$$\Big\downarrow \ 1100$$

$$\frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{3b^2}-\frac{a\int \sqrt{a^2+2bxa+b^2x^2}dx}{b}$$

$$\Big\downarrow \ 1079$$

$$\frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{3b^2}-\frac{a\sqrt{a^2+2abx+b^2x^2}\int (xb^2+ab)\,dx}{b^2(a+bx)}$$

$$\Big\downarrow \ 17$$

$$\frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{3b^2}-\frac{a(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2b^2}$$

input
```
Int[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
-1/2*(a*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/b^2 + (a^2 + 2*a*b*x + b^
2*x^2)^(3/2)/(3*b^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.41

method	result	size
default	$-\frac{\operatorname{csgn}\left(b x +a \right) \left(b x +a \right)^{2} \left(-2 b x +a \right)}{6 b^{2}}$	25
gosper	$\frac{x^{2} \left(2 b x +3 a \right) \sqrt{\left(b x +a \right)^{2}}}{6 b x +6 a}$	30
orering	$\frac{x^{2} \left(2 b x +3 a \right) \sqrt{\left(b x +a \right)^{2}}}{6 b x +6 a}$	30
risch	$\frac{\sqrt{\left(b x +a \right)^{2}}\, a \,x^{2}}{2 b x +2 a}+\frac{x^{3} b \sqrt{\left(b x +a \right)^{2}}}{3 b x +3 a}$	46

input
```
int(x*((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*csgn(b*x+a)*(b*x+a)^2*(-2*b*x+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.21

$$\int x\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

input
```
integrate(x*((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*b*x^3 + 1/2*a*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(46) = 92.

Time = 0.76 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.56

$$\int x\sqrt{a^2+2abx+b^2x^2}\,dx=\begin{cases}\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^2}{6b^2}+\frac{ax}{6b}+\frac{x^2}{3}\right) & \text{for } b^2\neq 0\\ \frac{-\frac{a^2(a^2+2abx)^{\frac{3}{2}}}{3}+\frac{(a^2+2abx)^{\frac{5}{2}}}{5}}{2a^2b^2} & \text{for } ab\neq 0\\ \frac{x^2\sqrt{a^2}}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*((b*x+a)**2)**(1/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(-a**2/(6*b**2) + a*x/(6*b) +
x**2/3), Ne(b**2, 0)), ((-a**2*(a**2 + 2*a*b*x)**(3/2)/3 + (a**2 + 2*a*b*x
)**(5/2)/5)/(2*a**2*b**2), Ne(a*b, 0)), (x**2*sqrt(a**2)/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\int x\sqrt{a^2+2abx+b^2x^2}\,dx = -\frac{\sqrt{b^2x^2+2\,abx+a^2}ax}{2\,b} - \frac{\sqrt{b^2x^2+2\,abx+a^2}a^2}{2\,b^2} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}}{3\,b^2}$$

input
```
integrate(x*((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a*x/b - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^
2)*a^2/b^2 + 1/3*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)/b^2
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64

$$\int x\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{1}{3}\,bx^3\mathrm{sgn}(bx+a) + \frac{1}{2}\,ax^2\mathrm{sgn}(bx+a) - \frac{a^3\mathrm{sgn}(bx+a)}{6\,b^2}$$

input
```
integrate(x*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*b*x^3*sgn(b*x + a) + 1/2*a*x^2*sgn(b*x + a) - 1/6*a^3*sgn(b*x + a)/b^2
```

Mupad [B] (verification not implemented)

Time = 9.82 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int x\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{\left(8\,b^2\,\left(a^2+b^2\,x^2\right)-12\,a^2\,b^2+4\,a\,b^3\,x\right)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{24\,b^4}$$

input
```
int(x*((a + b*x)^2)^(1/2),x)
```

output
```
((8*b^2*(a^2 + b^2*x^2) - 12*a^2*b^2 + 4*a*b^3*x)*(a^2 + b^2*x^2 + 2*a*b*x
)^(1/2))/(24*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.21

$$\int x\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{x^2(2bx+3a)}{6}$$

input
```
int(x*((b*x+a)^2)^(1/2),x)
```

output
```
(x**2*(3*a + 2*b*x))/6
```

3.50 $\int \sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 32

$$\int \sqrt{a^2+2abx+b^2x^2}\,dx=\frac{(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2b}$$

output
```
1/2*(b*x+a)*((b*x+a)^2)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \sqrt{a^2+2abx+b^2x^2}\,dx=\frac{x\sqrt{(a+bx)^2}(2a+bx)}{2(a+bx)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(x*Sqrt[(a + b*x)^2]*(2*a + b*x))/(2*(a + b*x))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+2abx+b^2x^2}\,dx$$

$$\Big\downarrow \text{ 1079}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(xb^2+ab\right)dx}{b(a+bx)}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2b}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{\operatorname{csgn}(bx+a)(bx+a)^2}{2b}$	19
gosper	$\frac{x(bx+2a)\sqrt{(bx+a)^2}}{2bx+2a}$	27
orering	$\frac{x(bx+2a)\sqrt{(bx+a)^2}}{2bx+2a}$	27
risch	$\frac{\sqrt{(bx+a)^2}\, b\, x^2}{2bx+2a} + \frac{\sqrt{(bx+a)^2}\, ax}{bx+a}$	43

input
```
int(((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*csgn(b*x+a)*(b*x+a)^2/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.31

$$\int \sqrt{a^2 + 2abx + b^2x^2}\, dx = \frac{1}{2}\, bx^2 + ax$$

input
```
integrate(((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*b*x^2 + a*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(19) = 38$.

Time = 0.50 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.75

$$\int \sqrt{a^2 + 2abx + b^2x^2}\,dx = \begin{cases} \left(\frac{a}{2b} + \frac{x}{2}\right)\sqrt{a^2 + 2abx + b^2x^2} & \text{for } b^2 \neq 0 \\ \frac{(a^2+2abx)^{\frac{3}{2}}}{3ab} & \text{for } ab \neq 0 \\ x\sqrt{a^2} & \text{otherwise} \end{cases}$$

input
```
integrate(((b*x+a)**2)**(1/2),x)
```

output
```
Piecewise(((a/(2*b) + x/2)*sqrt(a**2 + 2*a*b*x + b**2*x**2), Ne(b**2, 0)),
 ((a**2 + 2*a*b*x)**(3/2)/(3*a*b), Ne(a*b, 0)), (x*sqrt(a**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.44

$$\int \sqrt{a^2 + 2abx + b^2x^2}\,dx = \frac{1}{2}\sqrt{b^2x^2 + 2\,abx + a^2}x + \frac{\sqrt{b^2x^2 + 2\,abx + a^2}a}{2\,b}$$

input
```
integrate(((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*x + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a/
b
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \sqrt{a^2+2abx+b^2x^2}\,dx=\frac{1}{2}\left(bx^2+2\,ax\right)\mathrm{sgn}(bx+a)+\frac{a^2\mathrm{sgn}(bx+a)}{2\,b}$$

input
```
integrate(((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*(b*x^2 + 2*a*x)*sgn(b*x + a) + 1/2*a^2*sgn(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 9.58 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

$$\int \sqrt{a^2+2abx+b^2x^2}\,dx=\frac{\sqrt{(a+b\,x)^2}\,(a+b\,x)}{2\,b}$$

input
```
int(((a + b*x)^2)^(1/2),x)
```

output
```
(((a + b*x)^2)^(1/2)*(a + b*x))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.31

$$\int \sqrt{a^2+2abx+b^2x^2}\,dx=\frac{x(bx+2a)}{2}$$

input
```
int(((b*x+a)^2)^(1/2),x)
```

output
```
(x*(2*a + b*x))/2
```

3.51 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x}\,dx = \frac{bx\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{a\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}$$

output
```
b*x*((b*x+a)^2)^(1/2)/(b*x+a)+a*((b*x+a)^2)^(1/2)*ln(x)/(b*x+a)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 398 vs. 2(62) = 124.

Time = 0.69 (sec) , antiderivative size = 398, normalized size of antiderivative = 6.42

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x}\,dx = \frac{-2a\sqrt{a^2}bx-2\sqrt{a^2}b^2x^2+2abx\sqrt{(a+bx)^2}-2a\left(a^2+abx-\sqrt{a^2}\sqrt{(a+bx)^2}\right)\operatorname{arctanh}\left(\frac{bx}{\sqrt{a^2}-\sqrt{(a+bx)^2}}\right)}{}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/x,x]
```

output
```
(-2*a*Sqrt[a^2]*b*x - 2*Sqrt[a^2]*b^2*x^2 + 2*a*b*x*Sqrt[(a + b*x)^2] - 2*
a*(a^2 + a*b*x - Sqrt[a^2]*Sqrt[(a + b*x)^2])*ArcTanh[(b*x)/(Sqrt[a^2] - S
qrt[(a + b*x)^2])] - 2*((a^2)^(3/2) + a*Sqrt[a^2]*b*x - a^2*Sqrt[(a + b*x)
^2])*Log[x] + (a^2)^(3/2)*Log[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2]] + a*Sqr
t[a^2]*b*x*Log[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2]] - a^2*Sqrt[(a + b*x)^2
]*Log[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2]] + (a^2)^(3/2)*Log[Sqrt[a^2] + b
*x - Sqrt[(a + b*x)^2]] + a*Sqrt[a^2]*b*x*Log[Sqrt[a^2] + b*x - Sqrt[(a +
b*x)^2]] - a^2*Sqrt[(a + b*x)^2]*Log[Sqrt[a^2] + b*x - Sqrt[(a + b*x)^2]])
/(2*(a^2 + a*b*x - Sqrt[a^2]*Sqrt[(a + b*x)^2]))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.58, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x}dx$$

$$\Big\downarrow \text{ 1102}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b(a+bx)}{x}dx}{b(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{a+bx}{x}dx}{a+bx}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a}{x}+b\right)dx}{a+bx}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}(a\log(x)+bx)}{a+bx}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/x,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(b*x + a*Log[x]))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.32

method	result	size
default	$\operatorname{csgn}(bx+a)(bx+a+a\ln(-bx))$	20
risch	$\frac{bx\sqrt{(bx+a)^2}}{bx+a}+\frac{a\sqrt{(bx+a)^2}\,\ln(x)}{bx+a}$	41

input
```
int(((b*x+a)^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
csgn(b*x+a)*(b*x+a+a*ln(-b*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.13

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x}\,dx = bx + a\log\left(x\right)$$

input
```
integrate(((b*x+a)^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
b*x + a*log(x)
```

Sympy [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x}\,dx = \int \frac{\sqrt{(a+bx)^2}}{x}\,dx$$

input
```
integrate(((b*x+a)**2)**(1/2)/x,x)
```

output
```
Integral(sqrt((a + b*x)**2)/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(40) = 80.

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x}\,dx = (-1)^{2\,b^2x+2\,ab}\,a\log\left(2\,b^2x+2\,ab\right) - (-1)^{2\,abx+2\,a^2}\,a\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right) + \sqrt{b^2x^2+2\,abx+a^2}$$

input
```
integrate(((b*x+a)^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
(-1)^(2*b^2*x + 2*a*b)*a*log(2*b^2*x + 2*a*b) - (-1)^(2*a*b*x + 2*a^2)*a*l
og(2*a*b*x/abs(x) + 2*a^2/abs(x)) + sqrt(b^2*x^2 + 2*a*b*x + a^2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x}\,dx = bx\text{sgn}(bx+a)+a\log\left(|x|\right)\text{sgn}(bx+a)$$

input
```
integrate(((b*x+a)^2)^(1/2)/x,x, algorithm="giac")
```

output
```
b*x*sgn(b*x + a) + a*log(abs(x))*sgn(b*x + a)
```

Mupad [B] (verification not implemented)

Time = 9.81 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x}\,dx = \sqrt{a^2+2\,a\,b\,x+b^2\,x^2} - \ln\left(a\,b+\frac{a^2}{x}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x}\right)\sqrt{a^2} + \frac{a\,b\,\ln\left(a\,b+\sqrt{{(a+b\,x)}^2}\,\sqrt{b^2}+b^2\,x\right)}{\sqrt{b^2}}$$

input
```
int(((a + b*x)^2)^(1/2)/x,x)
```

output
```
(a^2 + b^2*x^2 + 2*a*b*x)^(1/2) - log(a*b + a^2/x + ((a^2)^(1/2)*(a^2 + b^
2*x^2 + 2*a*b*x)^(1/2))/x)*(a^2)^(1/2) + (a*b*log(a*b + ((a + b*x)^2)^(1/2
)*(b^2)^(1/2) + b^2*x))/(b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.13

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{x} \, dx = \log(x)\, a + bx$$

input
```
int(((b*x+a)^2)^(1/2)/x,x)
```

output
```
log(x)*a + b*x
```

3.52 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^2} \, dx = -\frac{a\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)} + \frac{b\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}$$

output
```
-a*((b*x+a)^2)^(1/2)/x/(b*x+a)+b*((b*x+a)^2)^(1/2)*ln(x)/(b*x+a)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 154 vs. 2(65) = 130.

Time = 0.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.37

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^2} \, dx = \frac{a\sqrt{a^2} - a\sqrt{(a+bx)^2} - 2abx\text{arctanh}\left(\frac{bx}{\sqrt{a^2}-\sqrt{(a+bx)^2}}\right) - 2\sqrt{a^2}bx\log(x) + \sqrt{a^2}bx\log\left(a\left(\sqrt{a^2} - bx - \sqrt{(\right.\right.}{2ax}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/x^2,x]
```

output
```
(a*Sqrt[a^2] - a*Sqrt[(a + b*x)^2] - 2*a*b*x*ArcTanh[(b*x)/(Sqrt[a^2] - Sq
rt[(a + b*x)^2])] - 2*Sqrt[a^2]*b*x*Log[x] + Sqrt[a^2]*b*x*Log[a*(Sqrt[a^2
] - b*x - Sqrt[(a + b*x)^2])] + Sqrt[a^2]*b*x*Log[a*(Sqrt[a^2] + b*x - Sqr
t[(a + b*x)^2])])/(2*a*x)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^2}dx$$

$$\Big\downarrow \text{ 1102}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b(a+bx)}{x^2}dx}{b(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{a+bx}{x^2}dx}{a+bx}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a}{x^2}+\frac{b}{x}\right)dx}{a+bx}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(b\log(x)-\frac{a}{x}\right)}{a+bx}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/x^2,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-(a/x) + b*Log[x]))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.35

method	result	size
default	$\frac{\operatorname{csgn}(bx+a)(\ln(-bx)bx-a)}{x}$	23
risch	$-\frac{a\sqrt{(bx+a)^2}}{x(bx+a)}+\frac{b\sqrt{(bx+a)^2}\,\ln(x)}{bx+a}$	44

input
```
int(((b*x+a)^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
csgn(b*x+a)*(ln(-b*x)*b*x-a)/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^2}\,dx = \frac{bx\log(x)-a}{x}$$

input
```
integrate(((b*x+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
(b*x*log(x) - a)/x
```

Sympy [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^2}\,dx = \int \frac{\sqrt{(a+bx)^2}}{x^2}\,dx$$

input
```
integrate(((b*x+a)**2)**(1/2)/x**2,x)
```

output
```
Integral(sqrt((a + b*x)**2)/x**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(43) = 86.

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^2}\,dx = (-1)^{2\,b^2x+2\,ab}\,b\log\left(2\,b^2x+2\,ab\right) - (-1)^{2\,abx+2\,a^2}\,b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right) - \frac{\sqrt{b^2x^2+2\,abx+a^2}}{x}$$

input
```
integrate(((b*x+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
(-1)^(2*b^2*x + 2*a*b)*b*log(2*b^2*x + 2*a*b) - (-1)^(2*a*b*x + 2*a^2)*b*l
og(2*a*b*x/abs(x) + 2*a^2/abs(x)) - sqrt(b^2*x^2 + 2*a*b*x + a^2)/x
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^2}\,dx = b\log\left(|x|\right)\operatorname{sgn}(bx+a) - \frac{a\operatorname{sgn}(bx+a)}{x}$$

input
```
integrate(((b*x+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
b*log(abs(x))*sgn(b*x + a) - a*sgn(b*x + a)/x
```

Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^2}\,dx = \ln\left(a\,b+\sqrt{(a+b\,x)^2}\,\sqrt{b^2}+b^2\,x\right)\sqrt{b^2} - \frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x} - \frac{a\,b\,\ln\left(a\,b+\frac{a^2}{x}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x}\right)}{\sqrt{a^2}}$$

input
```
int(((a + b*x)^2)^(1/2)/x^2,x)
```

output
```
log(a*b + ((a + b*x)^2)^(1/2)*(b^2)^(1/2) + b^2*x)*(b^2)^(1/2) - (a^2 + b^
2*x^2 + 2*a*b*x)^(1/2)/x - (a*b*log(a*b + a^2/x + ((a^2)^(1/2)*(a^2 + b^2*
x^2 + 2*a*b*x)^(1/2))/x))/(a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{x^2}\, dx = \frac{\log(x)\, bx - a}{x}$$

input
```
int(((b*x+a)^2)^(1/2)/x^2,x)
```

output
```
(log(x)*b*x - a)/x
```

3.53 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 35

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^3}\,dx=-\frac{(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2ax^2}$$

output
```
-1/2*(b*x+a)*((b*x+a)^2)^(1/2)/a/x^2
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^3}\,dx=-\frac{\sqrt{(a+bx)^2}(a+2bx)}{2x^2(a+bx)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/x^3,x]
```

output
```
-1/2*(Sqrt[(a + b*x)^2]*(a + 2*b*x))/(x^2*(a + b*x))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1102, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^3}\,dx$$

$$\Big\downarrow \ 1102$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b(a+bx)}{x^3}dx}{b(a+bx)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{a+bx}{x^3}dx}{a+bx}$$

$$\Big\downarrow \ 48$$

$$-\frac{(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2ax^2}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/x^3,x]
```

output
```
-1/2*((a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(a*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.51

method	result	size
default	$-\frac{\operatorname{csgn}(bx+a)(2bx+a)}{2x^{2}}$	18
gosper	$-\frac{(2bx+a)\sqrt{(bx+a)^{2}}}{2x^{2}(bx+a)}$	28
orering	$-\frac{(2bx+a)\sqrt{(bx+a)^{2}}}{2x^{2}(bx+a)}$	28
risch	$\frac{\left(-bx-\frac{a}{2}\right)\sqrt{(bx+a)^{2}}}{x^{2}(bx+a)}$	29

input
```
int(((b*x+a)^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*csgn(b*x+a)*(2*b*x+a)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.31

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^3}\,dx=-\frac{2\,bx+a}{2\,x^2}$$

input
```
integrate(((b*x+a)^2)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(2*b*x + a)/x^2
```

Sympy [F]

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{x^3}\,dx = \int \frac{\sqrt{(a+bx)^2}}{x^3}\,dx$$

input
```
integrate(((b*x+a)**2)**(1/2)/x**3,x)
```

output
```
Integral(sqrt((a + b*x)**2)/x**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(22) = 44.

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.29

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{x^3}\,dx = \frac{\sqrt{b^2x^2 + 2\,abx + a^2}b^2}{2\,a^2} + \frac{\sqrt{b^2x^2 + 2\,abx + a^2}b}{2\,ax} - \frac{(b^2x^2 + 2\,abx + a^2)^{\frac{3}{2}}}{2\,a^2x^2}$$

input
```
integrate(((b*x+a)^2)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^2/a^2 + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a
^2)*b/(a*x) - 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)/(a^2*x^2)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{x^3}\,dx = -\frac{b^2\mathrm{sgn}(bx+a)}{2\,a} - \frac{2\,bx\mathrm{sgn}(bx+a) + a\mathrm{sgn}(bx+a)}{2\,x^2}$$

input
```
integrate(((b*x+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

output
```
-1/2*b^2*sgn(b*x + a)/a - 1/2*(2*b*x*sgn(b*x + a) + a*sgn(b*x + a))/x^2
```

Mupad [B] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{x^3} \, dx = -\frac{\sqrt{(a + b\,x)^2}\,(a + 2\,b\,x)}{2\,x^2\,(a + b\,x)}$$

input
```
int(((a + b*x)^2)^(1/2)/x^3,x)
```

output
```
-(((a + b*x)^2)^(1/2)*(a + 2*b*x))/(2*x^2*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{x^3} \, dx = \frac{-2bx - a}{2x^2}$$

input
```
int(((b*x+a)^2)^(1/2)/x^3,x)
```

output
```
( - a - 2*b*x)/(2*x**2)
```

3.54 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 71

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^4}\,dx=-\frac{a\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)}-\frac{b\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)}$$

output
```
-1/3*a*((b*x+a)^2)^(1/2)/x^3/(b*x+a)-1/2*b*((b*x+a)^2)^(1/2)/x^2/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^4}\,dx=-\frac{\sqrt{(a+bx)^2}(2a+3bx)}{6x^3(a+bx)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/x^4,x]
```

output
```
-1/6*(Sqrt[(a + b*x)^2]*(2*a + 3*b*x))/(x^3*(a + b*x))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.63, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^4} \, dx$$

$$\Big\downarrow \text{ 1102}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2} \int \frac{b(a+bx)}{x^4}dx}{b(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2} \int \frac{a+bx}{x^4}dx}{a+bx}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2} \int \left(\frac{a}{x^4}+\frac{b}{x^3}\right)dx}{a+bx}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(-\frac{a}{3x^3}-\frac{b}{2x^2}\right)\sqrt{a^2+2abx+b^2x^2}}{a+bx}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/x^4,x]
```

output
```
((-1/3*a/x^3 - b/(2*x^2))*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.28

method	result	size
default	$-\frac{\operatorname{csgn}(bx+a)(3bx+2a)}{6x^3}$	20
risch	$\frac{\left(-\frac{bx}{2}-\frac{a}{3}\right)\sqrt{(bx+a)^2}}{x^3(bx+a)}$	29
gosper	$-\frac{(3bx+2a)\sqrt{(bx+a)^2}}{6x^3(bx+a)}$	30
orering	$-\frac{(3bx+2a)\sqrt{(bx+a)^2}}{6x^3(bx+a)}$	30

input
```
int(((b*x+a)^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*csgn(b*x+a)*(3*b*x+2*a)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^4}\,dx=-\frac{3\,bx+2\,a}{6\,x^3}$$

input
```
integrate(((b*x+a)^2)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-1/6*(3*b*x + 2*a)/x^3
```

Sympy [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^4}\,dx=\int \frac{\sqrt{(a+bx)^2}}{x^4}\,dx$$

input
```
integrate(((b*x+a)**2)**(1/2)/x**4,x)
```

output
```
Integral(sqrt((a + b*x)**2)/x**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(45) = 90.

Time = 0.03 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^4}\,dx=-\frac{\sqrt{b^2x^2+2\,abx+a^2}b^3}{2\,a^3}-\frac{\sqrt{b^2x^2+2\,abx+a^2}b^2}{2\,a^2x}+\frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b}{2\,a^3x^2}-\frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}}{3\,a^2x^3}$$

input
```
integrate(((b*x+a)^2)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
-1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^3/a^3 - 1/2*sqrt(b^2*x^2 + 2*a*b*x +
a^2)*b^2/(a^2*x) + 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b/(a^3*x^2) - 1/3*(
b^2*x^2 + 2*a*b*x + a^2)^(3/2)/(a^2*x^3)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^4}\,dx = \frac{b^3\mathrm{sgn}(bx+a)}{6\,a^2} - \frac{3\,bx\mathrm{sgn}(bx+a)+2\,a\mathrm{sgn}(bx+a)}{6\,x^3}$$

input
```
integrate(((b*x+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

output
```
1/6*b^3*sgn(b*x + a)/a^2 - 1/6*(3*b*x*sgn(b*x + a) + 2*a*sgn(b*x + a))/x^3
```

Mupad [B] (verification not implemented)

Time = 8.96 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^4}\,dx = -\frac{(2\,a+3\,b\,x)\,\sqrt{(a+b\,x)^2}}{6\,x^3\,(a+b\,x)}$$

input
```
int(((a + b*x)^2)^(1/2)/x^4,x)
```

output
```
-((2*a + 3*b*x)*((a + b*x)^2)^(1/2))/(6*x^3*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{x^4}\, dx = \frac{-3bx - 2a}{6x^3}$$

input
```
int(((b*x+a)^2)^(1/2)/x^4,x)
```

output
```
( - 2*a - 3*b*x)/(6*x**3)
```

3.55 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 71

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^5}\,dx = -\frac{a\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)} - \frac{b\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)}$$

output
```
-1/4*a*((b*x+a)^2)^(1/2)/x^4/(b*x+a)-1/3*b*((b*x+a)^2)^(1/2)/x^3/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^5}\,dx = -\frac{\sqrt{(a+bx)^2}(3a+4bx)}{12x^4(a+bx)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/x^5,x]
```

output
```
-1/12*(Sqrt[(a + b*x)^2]*(3*a + 4*b*x))/(x^4*(a + b*x))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.63, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^5}dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b(a+bx)}{x^5}dx}{b(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{a+bx}{x^5}dx}{a+bx}$$

$$\Big\downarrow \text{53}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a}{x^5}+\frac{b}{x^4}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a}{4x^4}-\frac{b}{3x^3}\right)\sqrt{a^2+2abx+b^2x^2}}{a+bx}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/x^5,x]
```

output
```
((-1/4*a/x^4 - b/(3*x^3))*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.28

method	result	size
default	$-\frac{\operatorname{csgn}(bx+a)(4bx+3a)}{12x^4}$	20
risch	$\frac{\left(-\frac{bx}{3}-\frac{a}{4}\right)\sqrt{(bx+a)^2}}{x^4(bx+a)}$	29
gosper	$-\frac{(4bx+3a)\sqrt{(bx+a)^2}}{12x^4(bx+a)}$	30
orering	$-\frac{(4bx+3a)\sqrt{(bx+a)^2}}{12x^4(bx+a)}$	30

input
```
int(((b*x+a)^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/12*csgn(b*x+a)*(4*b*x+3*a)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^5}\,dx=-\frac{4\,bx+3\,a}{12\,x^4}$$

input
```
integrate(((b*x+a)^2)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
-1/12*(4*b*x + 3*a)/x^4
```

Sympy [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^5}\,dx=\int \frac{\sqrt{(a+bx)^2}}{x^5}\,dx$$

input
```
integrate(((b*x+a)**2)**(1/2)/x**5,x)
```

output
```
Integral(sqrt((a + b*x)**2)/x**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(45) = 90.

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.94

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^5}\,dx=&\frac{\sqrt{b^2x^2+2\,abx+a^2}b^4}{2\,a^4}+\frac{\sqrt{b^2x^2+2\,abx+a^2}b^3}{2\,a^3x}\\&-\frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^2}{2\,a^4x^2}\\&+\frac{5\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b}{12\,a^3x^3}-\frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}}{4\,a^2x^4}\end{aligned}$$

input
```
integrate(((b*x+a)^2)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^4/a^4 + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a
^2)*b^3/(a^3*x) - 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^2/(a^4*x^2) + 5/12
*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b/(a^3*x^3) - 1/4*(b^2*x^2 + 2*a*b*x + a^
2)^(3/2)/(a^2*x^4)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^5}\,dx = -\frac{b^4\mathrm{sgn}(bx+a)}{12\,a^3} - \frac{4\,bx\mathrm{sgn}(bx+a)+3\,a\mathrm{sgn}(bx+a)}{12\,x^4}$$

input
```
integrate(((b*x+a)^2)^(1/2)/x^5,x, algorithm="giac")
```

output
```
-1/12*b^4*sgn(b*x + a)/a^3 - 1/12*(4*b*x*sgn(b*x + a) + 3*a*sgn(b*x + a))/
x^4
```

Mupad [B] (verification not implemented)

Time = 8.70 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^5}\,dx = -\frac{(3\,a+4\,b\,x)\,\sqrt{(a+b\,x)^2}}{12\,x^4\,(a+b\,x)}$$

input
```
int(((a + b*x)^2)^(1/2)/x^5,x)
```

output
```
-((3*a + 4*b*x)*((a + b*x)^2)^(1/2))/(12*x^4*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{x^5} \, dx = \frac{-4bx - 3a}{12x^4}$$

input
```
int(((b*x+a)^2)^(1/2)/x^5,x)
```

output
```
( - 3*a - 4*b*x)/(12*x**4)
```

3.56 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^6}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 71

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^6}\,dx=-\frac{a\sqrt{a^2+2abx+b^2x^2}}{5x^5(a+bx)}-\frac{b\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)}$$

output
```
-1/5*a*((b*x+a)^2)^(1/2)/x^5/(b*x+a)-1/4*b*((b*x+a)^2)^(1/2)/x^4/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^6}\,dx=-\frac{\sqrt{(a+bx)^2}(4a+5bx)}{20x^5(a+bx)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/x^6,x]
```

output
```
-1/20*(Sqrt[(a + b*x)^2]*(4*a + 5*b*x))/(x^5*(a + b*x))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.63, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^6}dx$$

$$\Big\downarrow \text{ 1102}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b(a+bx)}{x^6}dx}{b(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{a+bx}{x^6}dx}{a+bx}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a}{x^6}+\frac{b}{x^5}\right)dx}{a+bx}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(-\frac{a}{5x^5}-\frac{b}{4x^4}\right)\sqrt{a^2+2abx+b^2x^2}}{a+bx}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/x^6,x]
```

output
```
((-1/5*a/x^5 - b/(4*x^4))*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.28

method	result	size
default	$-\frac{\operatorname{csgn}(bx+a)(5bx+4a)}{20x^5}$	20
risch	$\frac{\left(-\frac{bx}{4}-\frac{a}{5}\right)\sqrt{(bx+a)^2}}{x^5(bx+a)}$	29
gosper	$-\frac{(5bx+4a)\sqrt{(bx+a)^2}}{20(bx+a)x^5}$	30
orering	$-\frac{(5bx+4a)\sqrt{(bx+a)^2}}{20(bx+a)x^5}$	30

input
```
int(((b*x+a)^2)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/20*csgn(b*x+a)*(5*b*x+4*a)/x^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^6}\,dx=-\frac{5\,bx+4\,a}{20\,x^5}$$

input
```
integrate(((b*x+a)^2)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
-1/20*(5*b*x + 4*a)/x^5
```

Sympy [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^6}\,dx=\int \frac{\sqrt{(a+bx)^2}}{x^6}\,dx$$

input
```
integrate(((b*x+a)**2)**(1/2)/x**6,x)
```

output
```
Integral(sqrt((a + b*x)**2)/x**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(45) = 90.

Time = 0.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.35

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^6}\,dx=&-\frac{\sqrt{b^2x^2+2\,abx+a^2}b^5}{2\,a^5}-\frac{\sqrt{b^2x^2+2\,abx+a^2}b^4}{2\,a^4x}\\&+\frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^3}{2\,a^5x^2}-\frac{9\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^2}{20\,a^4x^3}\\&+\frac{7\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b}{20\,a^3x^4}-\frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}}{5\,a^2x^5}\end{aligned}$$

input
```
integrate(((b*x+a)^2)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
-1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^5/a^5 - 1/2*sqrt(b^2*x^2 + 2*a*b*x +
a^2)*b^4/(a^4*x) + 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^3/(a^5*x^2) - 9/2
0*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^2/(a^4*x^3) + 7/20*(b^2*x^2 + 2*a*b*x
+ a^2)^(3/2)*b/(a^3*x^4) - 1/5*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)/(a^2*x^5)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^6}\,dx = \frac{b^5\mathrm{sgn}(bx+a)}{20\,a^4} - \frac{5\,bx\mathrm{sgn}(bx+a)+4\,a\mathrm{sgn}(bx+a)}{20\,x^5}$$

input
```
integrate(((b*x+a)^2)^(1/2)/x^6,x, algorithm="giac")
```

output
```
1/20*b^5*sgn(b*x + a)/a^4 - 1/20*(5*b*x*sgn(b*x + a) + 4*a*sgn(b*x + a))/x
^5
```

Mupad [B] (verification not implemented)

Time = 8.80 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^6}\,dx = -\frac{(4\,a+5\,b\,x)\,\sqrt{(a+b\,x)^2}}{20\,x^5\,(a+b\,x)}$$

input
```
int(((a + b*x)^2)^(1/2)/x^6,x)
```

output
```
-((4*a + 5*b*x)*((a + b*x)^2)^(1/2))/(20*x^5*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^6}\,dx=\frac{-5bx-4a}{20x^5}$$

input
```
int(((b*x+a)^2)^(1/2)/x^6,x)
```

output
```
( - 4*a - 5*b*x)/(20*x**5)
```

3.57 $\int x^5(a^2+2abx+b^2x^2)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 151

$$\int x^5\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{a^3x^6\sqrt{a^2+2abx+b^2x^2}}{6(a+bx)}+\frac{3a^2bx^7\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{3ab^2x^8\sqrt{a^2+2abx+b^2x^2}}{8(a+bx)}+\frac{b^3x^9\sqrt{a^2+2abx+b^2x^2}}{9(a+bx)}$$

output
```
a^3*x^6*((b*x+a)^2)^(1/2)/(6*b*x+6*a)+3*a^2*b*x^7*((b*x+a)^2)^(1/2)/(7*b*x
+7*a)+3*a*b^2*x^8*((b*x+a)^2)^(1/2)/(8*b*x+8*a)+b^3*x^9*((b*x+a)^2)^(1/2)/
(9*b*x+9*a)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.68

$$\int x^5\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{x^6(84a^3+216a^2bx+189ab^2x^2+56b^3x^3)\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}-\sqrt{(a+bx)^2}\right)\right)}{504\left(-a^2-abx+\sqrt{a^2}\sqrt{(a+bx)^2}\right)}$$

input
```
Integrate[x^5*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(x^6*(84*a^3 + 216*a^2*b*x + 189*a*b^2*x^2 + 56*b^3*x^3)*(Sqrt[a^2]*b*x +
a*(Sqrt[a^2] - Sqrt[(a + b*x)^2])))/(504*(-a^2 - a*b*x + Sqrt[a^2]*Sqrt[(a
 + b*x)^2]))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a^2+2abx+b^2x^2\right)^{3/2} \, dx$$

$$\Big\downarrow \text{ 1102}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2} \int b^3x^5(a+bx)^3dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2} \int x^5(a+bx)^3dx}{a+bx}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2} \int \left(b^3x^8+3ab^2x^7+3a^2bx^6+a^3x^5\right) dx}{a+bx}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2} \left(\frac{a^3x^6}{6}+\frac{3}{7}a^2bx^7+\frac{3}{8}ab^2x^8+\frac{b^3x^9}{9}\right)}{a+bx}$$

input
```
Int[x^5*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((a^3*x^6)/6 + (3*a^2*b*x^7)/7 + (3*a*b^2*x
^8)/8 + (b^3*x^9)/9))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.34

method	result	size
gosper	$\frac{x^{6}\left(56 b^{3} x^{3}+189 a \,b^{2} x^{2}+216 a^{2} b x +84 a^{3}\right) \left(\left(b x +a \right)^{2}\right)^{\frac{3}{2}}}{504 \left(b x +a \right)^{3}}$	52
default	$\frac{x^{6}\left(56 b^{3} x^{3}+189 a \,b^{2} x^{2}+216 a^{2} b x +84 a^{3}\right) \left(\left(b x +a \right)^{2}\right)^{\frac{3}{2}}}{504 \left(b x +a \right)^{3}}$	52
orering	$\frac{x^{6}\left(56 b^{3} x^{3}+189 a \,b^{2} x^{2}+216 a^{2} b x +84 a^{3}\right) \left(b^{2} x^{2}+2 a b x +a^{2}\right)^{\frac{3}{2}}}{504 \left(b x +a \right)^{3}}$	61
risch	$\frac{b^{3} x^{9} \sqrt{\left(b x +a \right)^{2}}}{9 b x +9 a}+\frac{3 \sqrt{\left(b x +a \right)^{2}}\, a \,b^{2} x^{8}}{8 \left(b x +a \right)}+\frac{3 \sqrt{\left(b x +a \right)^{2}}\, a^{2} b \,x^{7}}{7 \left(b x +a \right)}+\frac{a^{3} x^{6} \sqrt{\left(b x +a \right)^{2}}}{6 b x +6 a}$	100

input
```
int(x^5*(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/504*x^6*(56*b^3*x^3+189*a*b^2*x^2+216*a^2*b*x+84*a^3)*((b*x+a)^2)^(3/2)/
(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.23

$$\int x^5 \left(a^2+2abx+b^2x^2\right)^{3/2}\,dx = \frac{1}{9}\,b^3x^9+\frac{3}{8}\,ab^2x^8+\frac{3}{7}\,a^2bx^7+\frac{1}{6}\,a^3x^6$$

input
```
integrate(x^5*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/9*b^3*x^9 + 3/8*a*b^2*x^8 + 3/7*a^2*b*x^7 + 1/6*a^3*x^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(107) = 214.

Time = 0.58 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.66

$$\int x^5 \left(a^2+2abx+b^2x^2\right)^{3/2}\,dx = \begin{cases} \sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^8}{504b^6}+\frac{a^7x}{504b^5}-\frac{a^6x^2}{504b^4}+\frac{a^5x^3}{504b^3}-\frac{a^4x^4}{504b^2}+\frac{a^3x^5}{504b}+\frac{83a^2x^6}{504}+\frac{19abx^7}{72}+\frac{b^2x^8}{9}\right) \\ \frac{-\frac{a^{10}\left(a^2+2abx\right)^{\frac{5}{2}}}{5}+\frac{5a^8\left(a^2+2abx\right)^{\frac{7}{2}}}{7}-\frac{10a^6\left(a^2+2abx\right)^{\frac{9}{2}}}{9}+\frac{10a^4\left(a^2+2abx\right)^{\frac{11}{2}}}{11}-\frac{5a^2\left(a^2+2abx\right)^{\frac{13}{2}}}{13}+\frac{\left(a^2+2abx\right)^{\frac{15}{2}}}{15}}{32a^6b^6} \\ \frac{x^6\left(a^2\right)^{\frac{3}{2}}}{6} \end{cases}$$

input
```
integrate(x**5*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(-a**8/(504*b**6) + a**7*x/(50
4*b**5) - a**6*x**2/(504*b**4) + a**5*x**3/(504*b**3) - a**4*x**4/(504*b**
2) + a**3*x**5/(504*b) + 83*a**2*x**6/504 + 19*a*b*x**7/72 + b**2*x**8/9),
 Ne(b**2, 0)), ((-a**10*(a**2 + 2*a*b*x)**(5/2)/5 + 5*a**8*(a**2 + 2*a*b*x
)**(7/2)/7 - 10*a**6*(a**2 + 2*a*b*x)**(9/2)/9 + 10*a**4*(a**2 + 2*a*b*x)*
*(11/2)/11 - 5*a**2*(a**2 + 2*a*b*x)**(13/2)/13 + (a**2 + 2*a*b*x)**(15/2)
/15)/(32*a**6*b**6), Ne(a*b, 0)), (x**6*(a**2)**(3/2)/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.25

$$\int x^5\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}x^4}{9\,b^2}-\frac{13\,\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}ax^3}{72\,b^3}-\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}a^5x}{4\,b^5}+\frac{37\,\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}a^2x^2}{168\,b^4}-\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}a^6}{4\,b^6}-\frac{121\,\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}a^3x}{504\,b^5}+\frac{125\,\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}a^4}{504\,b^6}$$

input
```
integrate(x^5*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/9*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*x^4/b^2 - 13/72*(b^2*x^2 + 2*a*b*x + a
^2)^(5/2)*a*x^3/b^3 - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a^5*x/b^5 + 37/1
68*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^2*x^2/b^4 - 1/4*(b^2*x^2 + 2*a*b*x +
a^2)^(3/2)*a^6/b^6 - 121/504*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^3*x/b^5 + 1
25/504*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^4/b^6
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.48

$$\int x^5\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{1}{9}\,b^3x^9\mathrm{sgn}(bx+a)+\frac{3}{8}\,ab^2x^8\mathrm{sgn}(bx+a)+\frac{3}{7}\,a^2bx^7\mathrm{sgn}(bx+a)+\frac{1}{6}\,a^3x^6\mathrm{sgn}(bx+a)-\frac{a^9\mathrm{sgn}(bx+a)}{504\,b^6}$$

input
```
integrate(x^5*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/9*b^3*x^9*sgn(b*x + a) + 3/8*a*b^2*x^8*sgn(b*x + a) + 3/7*a^2*b*x^7*sgn(
b*x + a) + 1/6*a^3*x^6*sgn(b*x + a) - 1/504*a^9*sgn(b*x + a)/b^6
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(a^2+2abx+b^2x^2\right)^{3/2} \, dx = \int x^5 \, \left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2} \, dx$$

input
```
int(x^5*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
int(x^5*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.23

$$\int x^5 \left(a^2+2abx+b^2x^2\right)^{3/2} \, dx = \frac{x^6(56b^3x^3+189a\,b^2x^2+216a^2bx+84a^3)}{504}$$

input
```
int(x^5*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
(x**6*(84*a**3 + 216*a**2*b*x + 189*a*b**2*x**2 + 56*b**3*x**3))/504
```

3.58 $\int x^4(a^2+2abx+b^2x^2)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 151

$$\int x^4\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{a^3x^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}+\frac{a^2bx^6\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)}+\frac{3ab^2x^7\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{b^3x^8\sqrt{a^2+2abx+b^2x^2}}{8(a+bx)}$$

output
```
a^3*x^5*((b*x+a)^2)^(1/2)/(5*b*x+5*a)+a^2*b*x^6*((b*x+a)^2)^(1/2)/(2*b*x+2
*a)+3*a*b^2*x^7*((b*x+a)^2)^(1/2)/(7*b*x+7*a)+b^3*x^8*((b*x+a)^2)^(1/2)/(8
*b*x+8*a)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.68

$$\int x^4\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{x^5(56a^3+140a^2bx+120ab^2x^2+35b^3x^3)\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}-\sqrt{(a+bx)^2}\right)\right)}{280\left(-a^2-abx+\sqrt{a^2}\sqrt{(a+bx)^2}\right)}$$

input
```
Integrate[x^4*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(x^5*(56*a^3 + 140*a^2*b*x + 120*a*b^2*x^2 + 35*b^3*x^3)*(Sqrt[a^2]*b*x +
a*(Sqrt[a^2] - Sqrt[(a + b*x)^2])))/(280*(-a^2 - a*b*x + Sqrt[a^2]*Sqrt[(a
 + b*x)^2]))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a^2+2abx+b^2x^2\right)^{3/2} \, dx$$

$$\Big\downarrow \text{ 1102}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int b^3x^4(a+bx)^3dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^4(a+bx)^3dx}{a+bx}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(b^3x^7+3ab^2x^6+3a^2bx^5+a^3x^4\right)dx}{a+bx}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{a^3x^5}{5}+\frac{1}{2}a^2bx^6+\frac{3}{7}ab^2x^7+\frac{b^3x^8}{8}\right)}{a+bx}$$

input
```
Int[x^4*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((a^3*x^5)/5 + (a^2*b*x^6)/2 + (3*a*b^2*x^7
)/7 + (b^3*x^8)/8))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.34

method	result	size
gosper	$\frac{x^{5} \left(35 b^{3} x^{3}+120 a \,b^{2} x^{2}+140 a^{2} b x +56 a^{3}\right) \left(\left(b x +a \right)^{2}\right)^{\frac{3}{2}}}{280 \left(b x +a \right)^{3}}$	52
default	$\frac{x^{5} \left(35 b^{3} x^{3}+120 a \,b^{2} x^{2}+140 a^{2} b x +56 a^{3}\right) \left(\left(b x +a \right)^{2}\right)^{\frac{3}{2}}}{280 \left(b x +a \right)^{3}}$	52
orering	$\frac{x^{5} \left(35 b^{3} x^{3}+120 a \,b^{2} x^{2}+140 a^{2} b x +56 a^{3}\right) \left(b^{2} x^{2}+2 a b x +a^{2}\right)^{\frac{3}{2}}}{280 \left(b x +a \right)^{3}}$	61
risch	$\frac{b^{3} x^{8} \sqrt{\left(b x +a \right)^{2}}}{8 b x +8 a}+\frac{3 \sqrt{\left(b x +a \right)^{2}}\, a \,b^{2} x^{7}}{7 \left(b x +a \right)}+\frac{a^{2} b \,x^{6} \sqrt{\left(b x +a \right)^{2}}}{2 b x +2 a}+\frac{a^{3} x^{5} \sqrt{\left(b x +a \right)^{2}}}{5 b x +5 a}$	100

input
```
int(x^4*(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/280*x^5*(35*b^3*x^3+120*a*b^2*x^2+140*a^2*b*x+56*a^3)*((b*x+a)^2)^(3/2)/
(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.23

$$\int x^4 (a^2 + 2abx + b^2x^2)^{3/2} \, dx = \frac{1}{8} b^3x^8 + \frac{3}{7} ab^2x^7 + \frac{1}{2} a^2bx^6 + \frac{1}{5} a^3x^5$$

input
```
integrate(x^4*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/8*b^3*x^8 + 3/7*a*b^2*x^7 + 1/2*a^2*b*x^6 + 1/5*a^3*x^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(105) = 210.

Time = 0.55 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.44

$$\int x^4 (a^2 + 2abx + b^2x^2)^{3/2} \, dx = \begin{cases} \sqrt{a^2 + 2abx + b^2x^2} \left(\frac{a^7}{280b^5} - \frac{a^6x}{280b^4} + \frac{a^5x^2}{280b^3} - \frac{a^4x^3}{280b^2} + \frac{a^3x^4}{280b} + \frac{11a^2x^5}{56} + \frac{17abx^6}{56} + \frac{b^2x^7}{8}\right) & \text{for } b^2 \\ \frac{\frac{a^8(a^2+2abx)^{\frac{5}{2}}}{5} - \frac{4a^6(a^2+2abx)^{\frac{7}{2}}}{7} + \frac{2a^4(a^2+2abx)^{\frac{9}{2}}}{3} - \frac{4a^2(a^2+2abx)^{\frac{11}{2}}}{11} + \frac{(a^2+2abx)^{\frac{13}{2}}}{13}}{16a^5b^5} & \text{for } ab \\ \frac{x^5(a^2)^{\frac{3}{2}}}{5} & \text{otherv} \end{cases}$$

input
```
integrate(x**4*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(a**7/(280*b**5) - a**6*x/(280
*b**4) + a**5*x**2/(280*b**3) - a**4*x**3/(280*b**2) + a**3*x**4/(280*b) +
 11*a**2*x**5/56 + 17*a*b*x**6/56 + b**2*x**7/8), Ne(b**2, 0)), ((a**8*(a*
*2 + 2*a*b*x)**(5/2)/5 - 4*a**6*(a**2 + 2*a*b*x)**(7/2)/7 + 2*a**4*(a**2 +
 2*a*b*x)**(9/2)/3 - 4*a**2*(a**2 + 2*a*b*x)**(11/2)/11 + (a**2 + 2*a*b*x)
**(13/2)/13)/(16*a**5*b**5), Ne(a*b, 0)), (x**5*(a**2)**(3/2)/5, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.06

$$\int x^4 \left(a^2+2abx+b^2x^2\right)^{3/2} dx = \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}x^3}{8\,b^2} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}a^4x}{4\,b^4} - \frac{11\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}ax^2}{56\,b^3} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}a^5}{4\,b^5} + \frac{13\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}a^2x}{56\,b^4} - \frac{69\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}a^3}{280\,b^5}$$

input
```
integrate(x^4*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/8*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*x^3/b^2 + 1/4*(b^2*x^2 + 2*a*b*x + a^2
)^(3/2)*a^4*x/b^4 - 11/56*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a*x^2/b^3 + 1/4*
(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a^5/b^5 + 13/56*(b^2*x^2 + 2*a*b*x + a^2)^
(5/2)*a^2*x/b^4 - 69/280*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^3/b^5
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.48

$$\int x^4 \left(a^2+2abx+b^2x^2\right)^{3/2} dx = \frac{1}{8}\,b^3x^8\mathrm{sgn}(bx+a) + \frac{3}{7}\,ab^2x^7\mathrm{sgn}(bx+a) + \frac{1}{2}\,a^2bx^6\mathrm{sgn}(bx+a) + \frac{1}{5}\,a^3x^5\mathrm{sgn}(bx+a) + \frac{a^8\mathrm{sgn}(bx+a)}{280\,b^5}$$

input
```
integrate(x^4*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*b^3*x^8*sgn(b*x + a) + 3/7*a*b^2*x^7*sgn(b*x + a) + 1/2*a^2*b*x^6*sgn(
b*x + a) + 1/5*a^3*x^5*sgn(b*x + a) + 1/280*a^8*sgn(b*x + a)/b^5
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \left(a^2 + 2abx + b^2x^2\right)^{3/2} \, dx = \int x^4 \, \left(a^2 + 2\, a\, b\, x + b^2\, x^2\right)^{3/2} \, dx$$

input
```
int(x^4*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
int(x^4*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.23

$$\int x^4 \left(a^2 + 2abx + b^2x^2\right)^{3/2} \, dx = \frac{x^5(35b^3x^3 + 120a\, b^2x^2 + 140a^2bx + 56a^3)}{280}$$

input
```
int(x^4*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
(x**5*(56*a**3 + 140*a**2*b*x + 120*a*b**2*x**2 + 35*b**3*x**3))/280
```

3.59 $\int x^3(a^2+2abx+b^2x^2)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 151

$$\int x^3(a^2+2abx+b^2x^2)^{3/2}\,dx=\frac{a^3x^4\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)}+\frac{3a^2bx^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}+\frac{ab^2x^6\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)}+\frac{b^3x^7\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}$$

output
```
a^3*x^4*((b*x+a)^2)^(1/2)/(4*b*x+4*a)+3*a^2*b*x^5*((b*x+a)^2)^(1/2)/(5*b*x
+5*a)+a*b^2*x^6*((b*x+a)^2)^(1/2)/(2*b*x+2*a)+b^3*x^7*((b*x+a)^2)^(1/2)/(7
*b*x+7*a)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.68

$$\int x^3(a^2+2abx+b^2x^2)^{3/2}\,dx=\frac{x^4(35a^3+84a^2bx+70ab^2x^2+20b^3x^3)\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}-\sqrt{(a+bx)^2}\right)\right)}{140\left(-a^2-abx+\sqrt{a^2}\sqrt{(a+bx)^2}\right)}$$

input
```
Integrate[x^3*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(x^4*(35*a^3 + 84*a^2*b*x + 70*a*b^2*x^2 + 20*b^3*x^3)*(Sqrt[a^2]*b*x + a*
(Sqrt[a^2] - Sqrt[(a + b*x)^2])))/(140*(-a^2 - a*b*x + Sqrt[a^2]*Sqrt[(a +
 b*x)^2]))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a^2+2abx+b^2x^2\right)^{3/2} \, dx$$

$$\Big\downarrow \text{ 1102}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int b^3x^3(a+bx)^3dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^3(a+bx)^3dx}{a+bx}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(b^3x^6+3ab^2x^5+3a^2bx^4+a^3x^3\right)dx}{a+bx}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{a^3x^4}{4}+\frac{3}{5}a^2bx^5+\frac{1}{2}ab^2x^6+\frac{b^3x^7}{7}\right)}{a+bx}$$

input
```
Int[x^3*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((a^3*x^4)/4 + (3*a^2*b*x^5)/5 + (a*b^2*x^6
)/2 + (b^3*x^7)/7))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.34

method	result	size
gosper	$\frac{x^{4} \left(20 b^{3} x^{3}+70 a \,b^{2} x^{2}+84 a^{2} b x +35 a^{3}\right) \left(\left(b x +a \right)^{2}\right)^{\frac{3}{2}}}{140 \left(b x +a \right)^{3}}$	52
default	$\frac{x^{4} \left(20 b^{3} x^{3}+70 a \,b^{2} x^{2}+84 a^{2} b x +35 a^{3}\right) \left(\left(b x +a \right)^{2}\right)^{\frac{3}{2}}}{140 \left(b x +a \right)^{3}}$	52
orering	$\frac{x^{4} \left(20 b^{3} x^{3}+70 a \,b^{2} x^{2}+84 a^{2} b x +35 a^{3}\right) \left(b^{2} x^{2}+2 a b x +a^{2}\right)^{\frac{3}{2}}}{140 \left(b x +a \right)^{3}}$	61
risch	$\frac{b^{3} x^{7} \sqrt{\left(b x +a \right)^{2}}}{7 b x +7 a}+\frac{a \,b^{2} x^{6} \sqrt{\left(b x +a \right)^{2}}}{2 b x +2 a}+\frac{3 \sqrt{\left(b x +a \right)^{2}}\, a^{2} b \,x^{5}}{5 \left(b x +a \right)}+\frac{a^{3} x^{4} \sqrt{\left(b x +a \right)^{2}}}{4 b x +4 a}$	100

input
```
int(x^3*(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/140*x^4*(20*b^3*x^3+70*a*b^2*x^2+84*a^2*b*x+35*a^3)*((b*x+a)^2)^(3/2)/(b
*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.23

$$\int x^3\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{1}{7}\,b^3x^7+\frac{1}{2}\,ab^2x^6+\frac{3}{5}\,a^2bx^5+\frac{1}{4}\,a^3x^4$$

input
```
integrate(x^3*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 + 1/4*a^3*x^4
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.22

$$\int x^3\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\begin{cases}\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^6}{140b^4}+\frac{a^5x}{140b^3}-\frac{a^4x^2}{140b^2}+\frac{a^3x^3}{140b}+\frac{17a^2x^4}{70}+\frac{5abx^5}{14}+\frac{b^2x^6}{7}\right) & \text{for } b^2\neq 0\\ \frac{-\frac{a^6\left(a^2+2abx\right)^{\frac{5}{2}}}{5}+\frac{3a^4\left(a^2+2abx\right)^{\frac{7}{2}}}{7}-\frac{a^2\left(a^2+2abx\right)^{\frac{9}{2}}}{3}+\frac{\left(a^2+2abx\right)^{\frac{11}{2}}}{11}}{8a^4b^4} & \text{for } ab\neq 0\\ \frac{x^4\left(a^2\right)^{\frac{3}{2}}}{4} & \text{otherwise}\end{cases}$$

input
```
integrate(x**3*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(-a**6/(140*b**4) + a**5*x/(14
0*b**3) - a**4*x**2/(140*b**2) + a**3*x**3/(140*b) + 17*a**2*x**4/70 + 5*a
*b*x**5/14 + b**2*x**6/7), Ne(b**2, 0)), ((-a**6*(a**2 + 2*a*b*x)**(5/2)/5
 + 3*a**4*(a**2 + 2*a*b*x)**(7/2)/7 - a**2*(a**2 + 2*a*b*x)**(9/2)/3 + (a*
*2 + 2*a*b*x)**(11/2)/11)/(8*a**4*b**4), Ne(a*b, 0)), (x**4*(a**2)**(3/2)/
4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int x^3(a^2+2abx+b^2x^2)^{3/2}\,dx = -\frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}a^3x}{4\,b^3}+\frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}x^2}{7\,b^2} -\frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}a^4}{4\,b^4}-\frac{3\,(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}ax}{14\,b^3}+\frac{17\,(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}a^2}{70\,b^4}$$

input
```
integrate(x^3*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a^3*x/b^3 + 1/7*(b^2*x^2 + 2*a*b*x +
a^2)^(5/2)*x^2/b^2 - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a^4/b^4 - 3/14*(b
^2*x^2 + 2*a*b*x + a^2)^(5/2)*a*x/b^3 + 17/70*(b^2*x^2 + 2*a*b*x + a^2)^(5
/2)*a^2/b^4
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.48

$$\int x^3(a^2+2abx+b^2x^2)^{3/2}\,dx = \frac{1}{7}\,b^3x^7\mathrm{sgn}(bx+a)+\frac{1}{2}\,ab^2x^6\mathrm{sgn}(bx+a) +\frac{3}{5}\,a^2bx^5\mathrm{sgn}(bx+a)+\frac{1}{4}\,a^3x^4\mathrm{sgn}(bx+a)-\frac{a^7\mathrm{sgn}(bx+a)}{140\,b^4}$$

input
```
integrate(x^3*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/7*b^3*x^7*sgn(b*x + a) + 1/2*a*b^2*x^6*sgn(b*x + a) + 3/5*a^2*b*x^5*sgn(
b*x + a) + 1/4*a^3*x^4*sgn(b*x + a) - 1/140*a^7*sgn(b*x + a)/b^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\int x^3\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}\,dx$$

input
```
int(x^3*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
int(x^3*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.23

$$\int x^3 \left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{x^4(20b^3x^3+70a\,b^2x^2+84a^2bx+35a^3)}{140}$$

input
```
int(x^3*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
(x**4*(35*a**3 + 84*a**2*b*x + 70*a*b**2*x**2 + 20*b**3*x**3))/140
```

3.60 $\int x^2(a^2+2abx+b^2x^2)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 151

$$\int x^2\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{a^3x^3\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)}+\frac{3a^2bx^4\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)}+\frac{3ab^2x^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}+\frac{b^3x^6\sqrt{a^2+2abx+b^2x^2}}{6(a+bx)}$$

output
```
a^3*x^3*((b*x+a)^2)^(1/2)/(3*b*x+3*a)+3*a^2*b*x^4*((b*x+a)^2)^(1/2)/(4*b*x
+4*a)+3*a*b^2*x^5*((b*x+a)^2)^(1/2)/(5*b*x+5*a)+b^3*x^6*((b*x+a)^2)^(1/2)/
(6*b*x+6*a)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.68

$$\int x^2\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{x^3\left(20a^3+45a^2bx+36ab^2x^2+10b^3x^3\right)\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}-\sqrt{(a+bx)^2}\right)\right)}{60\left(-a^2-abx+\sqrt{a^2}\sqrt{(a+bx)^2}\right)}$$

input
```
Integrate[x^2*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(x^3*(20*a^3 + 45*a^2*b*x + 36*a*b^2*x^2 + 10*b^3*x^3)*(Sqrt[a^2]*b*x + a*
(Sqrt[a^2] - Sqrt[(a + b*x)^2])))/(60*(-a^2 - a*b*x + Sqrt[a^2]*Sqrt[(a +
b*x)^2]))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.46, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1101, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1101}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int\left(b(a+bx)^5-2ab(a+bx)^4+a^2b(a+bx)^3\right)dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{4}a^2(a+bx)^4+\frac{1}{6}(a+bx)^6-\frac{2}{5}a(a+bx)^5\right)}{b^3(a+bx)}$$

input
```
Int[x^2*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((a^2*(a + b*x)^4)/4 - (2*a*(a + b*x)^5)/5
+ (a + b*x)^6/6))/(b^3*(a + b*x))
```

Defintions of rubi rules used

rule 1101
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a
 + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   In
t[ExpandLinearProduct[(b/2 + c*x)^(2*p), x^m, b/2, c, x], x], x] /; FreeQ[{
a, b, c, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && IGtQ[m, 0] && EqQ[m - 2*p + 1,
 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.34

method	result	size
gosper	$\frac{x^{3}\left(10 b^{3} x^{3}+36 a\,b^{2} x^{2}+45 a^{2} b x +20 a^{3}\right) \left(\left(b x +a \right)^{2}\right)^{\frac{3}{2}}}{60 \left(b x +a \right)^{3}}$	52
default	$\frac{x^{3}\left(10 b^{3} x^{3}+36 a\,b^{2} x^{2}+45 a^{2} b x +20 a^{3}\right) \left(\left(b x +a \right)^{2}\right)^{\frac{3}{2}}}{60 \left(b x +a \right)^{3}}$	52
orering	$\frac{x^{3}\left(10 b^{3} x^{3}+36 a\,b^{2} x^{2}+45 a^{2} b x +20 a^{3}\right) \left(b^{2} x^{2}+2 a b x +a^{2}\right)^{\frac{3}{2}}}{60 \left(b x +a \right)^{3}}$	61
risch	$\frac{b^{3} x^{6} \sqrt{\left(b x +a \right)^{2}}}{6 b x +6 a}+\frac{3 \sqrt{\left(b x +a \right)^{2}}\, a\,b^{2} x^{5}}{5 \left(b x +a \right)}+\frac{3 \sqrt{\left(b x +a \right)^{2}}\, a^{2} b\,x^{4}}{4 \left(b x +a \right)}+\frac{a^{3} x^{3} \sqrt{\left(b x +a \right)^{2}}}{3 b x +3 a}$	100

input
```
int(x^2*(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/60*x^3*(10*b^3*x^3+36*a*b^2*x^2+45*a^2*b*x+20*a^3)*((b*x+a)^2)^(3/2)/(b*
x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.23

$$\int x^2 \left(a^2+2abx+b^2x^2\right)^{3/2} dx = \frac{1}{6}b^3x^6+\frac{3}{5}ab^2x^5+\frac{3}{4}a^2bx^4+\frac{1}{3}a^3x^3$$

input
```
integrate(x^2*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 + 1/3*a^3*x^3
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01

$$\int x^2 \left(a^2+2abx+b^2x^2\right)^{3/2} dx = \begin{cases} \sqrt{a^2+2abx+b^2x^2}\left(\frac{a^5}{60b^3}-\frac{a^4x}{60b^2}+\frac{a^3x^2}{60b}+\frac{19a^2x^3}{60}+\frac{13abx^4}{30}+\frac{b^2x^5}{6}\right) & \text{for } b^2 \neq 0 \\ \frac{\frac{a^4\left(a^2+2abx\right)^{\frac{5}{2}}}{5}-\frac{2a^2\left(a^2+2abx\right)^{\frac{7}{2}}}{7}+\frac{\left(a^2+2abx\right)^{\frac{9}{2}}}{9}}{4a^3b^3} & \text{for } ab \neq 0 \\ \frac{x^3\left(a^2\right)^{\frac{3}{2}}}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(a**5/(60*b**3) - a**4*x/(60*b
**2) + a**3*x**2/(60*b) + 19*a**2*x**3/60 + 13*a*b*x**4/30 + b**2*x**5/6),
 Ne(b**2, 0)), ((a**4*(a**2 + 2*a*b*x)**(5/2)/5 - 2*a**2*(a**2 + 2*a*b*x)*
*(7/2)/7 + (a**2 + 2*a*b*x)**(9/2)/9)/(4*a**3*b**3), Ne(a*b, 0)), (x**3*(a
**2)**(3/2)/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.68

$$\int x^2 \left(a^2+2abx+b^2x^2\right)^{3/2} \, dx = \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}a^2x}{4\,b^2} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}a^3}{4\,b^3} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}x}{6\,b^2} - \frac{7\,(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}a}{30\,b^3}$$

input
```
integrate(x^2*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a^2*x/b^2 + 1/4*(b^2*x^2 + 2*a*b*x + a
^2)^(3/2)*a^3/b^3 + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*x/b^2 - 7/30*(b^2*
x^2 + 2*a*b*x + a^2)^(5/2)*a/b^3
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.48

$$\int x^2 \left(a^2+2abx+b^2x^2\right)^{3/2} \, dx = \frac{1}{6}\,b^3x^6\mathrm{sgn}(bx+a) + \frac{3}{5}\,ab^2x^5\mathrm{sgn}(bx+a) + \frac{3}{4}\,a^2bx^4\mathrm{sgn}(bx+a) + \frac{1}{3}\,a^3x^3\mathrm{sgn}(bx+a) + \frac{a^6\mathrm{sgn}(bx+a)}{60\,b^3}$$

input
```
integrate(x^2*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/6*b^3*x^6*sgn(b*x + a) + 3/5*a*b^2*x^5*sgn(b*x + a) + 3/4*a^2*b*x^4*sgn(
b*x + a) + 1/3*a^3*x^3*sgn(b*x + a) + 1/60*a^6*sgn(b*x + a)/b^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (a^2 + 2abx + b^2x^2)^{3/2} \, dx = \int x^2 \left(a^2 + 2\,a\,b\,x + b^2\,x^2\right)^{3/2} \, dx$$

input
```
int(x^2*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
int(x^2*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.23

$$\int x^2 (a^2 + 2abx + b^2x^2)^{3/2} \, dx = \frac{x^3(10b^3x^3 + 36a\,b^2x^2 + 45a^2bx + 20a^3)}{60}$$

input
```
int(x^2*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
(x**3*(20*a**3 + 45*a**2*b*x + 36*a*b**2*x**2 + 10*b**3*x**3))/60
```

3.61 $\int x(a^2+2abx+b^2x^2)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int x\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=-\frac{a(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{4b^2}+\frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{5b^2}$$

output
```
-1/4*a*(b*x+a)^3*((b*x+a)^2)^(1/2)/b^2+1/5*(b^2*x^2+2*a*b*x+a^2)^(5/2)/b^2
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.63

$$\int x\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{x^2(10a^3+20a^2bx+15ab^2x^2+4b^3x^3)\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}-\sqrt{(a+bx)^2}\right)\right)}{20\left(-a^2-abx+\sqrt{a^2}\sqrt{(a+bx)^2}\right)}$$

input
```
Integrate[x*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(x^2*(10*a^3 + 20*a^2*b*x + 15*a*b^2*x^2 + 4*b^3*x^3)*(Sqrt[a^2]*b*x + a*(
Sqrt[a^2] - Sqrt[(a + b*x)^2])))/(20*(-a^2 - a*b*x + Sqrt[a^2]*Sqrt[(a + b
*x)^2]))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a^2+2abx+b^2x^2\right)^{3/2}dx$$

$$\Big\downarrow \text{1100}$$

$$\frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{5b^2}-\frac{a\int \left(a^2+2bxa+b^2x^2\right)^{3/2}dx}{b}$$

$$\Big\downarrow \text{1079}$$

$$\frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{5b^2}-\frac{a\sqrt{a^2+2abx+b^2x^2}\int \left(xb^2+ab\right)^3dx}{b^4(a+bx)}$$

$$\Big\downarrow \text{17}$$

$$\frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{5b^2}-\frac{a(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{4b^2}$$

input
```
Int[x*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
-1/4*(a*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/b^2 + (a^2 + 2*a*b*x +
b^2*x^2)^(5/2)/(5*b^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^{2}\left(4 b^{3} x^{3}+15 a\,b^{2} x^{2}+20 a^{2} b x +10 a^{3}\right)\left(\left(b x +a \right)^{2}\right)^{\frac{3}{2}}}{20 \left(b x +a \right)^{3}}$	52
default	$\frac{x^{2}\left(4 b^{3} x^{3}+15 a\,b^{2} x^{2}+20 a^{2} b x +10 a^{3}\right)\left(\left(b x +a \right)^{2}\right)^{\frac{3}{2}}}{20 \left(b x +a \right)^{3}}$	52
orering	$\frac{x^{2}\left(4 b^{3} x^{3}+15 a\,b^{2} x^{2}+20 a^{2} b x +10 a^{3}\right)\left(b^{2} x^{2}+2 a b x +a^{2}\right)^{\frac{3}{2}}}{20 \left(b x +a \right)^{3}}$	61
risch	$\frac{\sqrt{\left(b x +a \right)^{2}}\, b^{3} x^{5}}{5 b x +5 a}+\frac{3 \sqrt{\left(b x +a \right)^{2}}\, a\,b^{2} x^{4}}{4 \left(b x +a \right)}+\frac{\sqrt{\left(b x +a \right)^{2}}\, a^{2} b\,x^{3}}{b x +a}+\frac{\sqrt{\left(b x +a \right)^{2}}\, a^{3} x^{2}}{2 b x +2 a}$	99

input
```
int(x*(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/20*x^2*(4*b^3*x^3+15*a*b^2*x^2+20*a^2*b*x+10*a^3)*((b*x+a)^2)^(3/2)/(b*x
+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.54

$$\int x\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{1}{5}\,b^3x^5+\frac{3}{4}\,ab^2x^4+a^2bx^3+\frac{1}{2}\,a^3x^2$$

input
```
integrate(x*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 + 1/2*a^3*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(48) = 96.

Time = 0.43 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.92

$$\int x\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\begin{cases}\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^4}{20b^2}+\frac{a^3x}{20b}+\frac{9a^2x^2}{20}+\frac{11abx^3}{20}+\frac{b^2x^4}{5}\right) & \text{for } b^2\neq 0\\ \frac{-\frac{a^2\left(a^2+2abx\right)^{\frac{5}{2}}}{5}+\frac{\left(a^2+2abx\right)^{\frac{7}{2}}}{7}}{2a^2b^2} & \text{for } ab\neq 0\\ \frac{x^2\left(a^2\right)^{\frac{3}{2}}}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(-a**4/(20*b**2) + a**3*x/(20*
b) + 9*a**2*x**2/20 + 11*a*b*x**3/20 + b**2*x**4/5), Ne(b**2, 0)), ((-a**2
*(a**2 + 2*a*b*x)**(5/2)/5 + (a**2 + 2*a*b*x)**(7/2)/7)/(2*a**2*b**2), Ne(
a*b, 0)), (x**2*(a**2)**(3/2)/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19

$$\int x(a^2 + 2abx + b^2x^2)^{3/2} \, dx = -\frac{(b^2x^2 + 2\,abx + a^2)^{\frac{3}{2}}ax}{4\,b} - \frac{(b^2x^2 + 2\,abx + a^2)^{\frac{3}{2}}a^2}{4\,b^2} + \frac{(b^2x^2 + 2\,abx + a^2)^{\frac{5}{2}}}{5\,b^2}$$

input
```
integrate(x*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a*x/b - 1/4*(b^2*x^2 + 2*a*b*x + a^2)
^(3/2)*a^2/b^2 + 1/5*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)/b^2
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14

$$\int x(a^2 + 2abx + b^2x^2)^{3/2} \, dx = \frac{1}{5}\,b^3x^5\mathrm{sgn}(bx + a) + \frac{3}{4}\,ab^2x^4\mathrm{sgn}(bx + a) + a^2bx^3\mathrm{sgn}(bx + a) + \frac{1}{2}\,a^3x^2\mathrm{sgn}(bx + a) - \frac{a^5\mathrm{sgn}(bx + a)}{20\,b^2}$$

input
```
integrate(x*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/5*b^3*x^5*sgn(b*x + a) + 3/4*a*b^2*x^4*sgn(b*x + a) + a^2*b*x^3*sgn(b*x
+ a) + 1/2*a^3*x^2*sgn(b*x + a) - 1/20*a^5*sgn(b*x + a)/b^2
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67

$$\int x\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{\left(-a^2+3\,a\,b\,x+4\,b^2\,x^2\right)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}}{20\,b^2}$$

input
```
int(x*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
((4*b^2*x^2 - a^2 + 3*a*b*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/(20*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.56

$$\int x\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{x^2(4b^3x^3+15a\,b^2x^2+20a^2bx+10a^3)}{20}$$

input
```
int(x*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
(x**2*(10*a**3 + 20*a**2*b*x + 15*a*b**2*x**2 + 4*b**3*x**3))/20
```

3.62 $\int (a^2+2abx+b^2x^2)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 34

$$\int \left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{4b}$$

output
```
1/4*(b*x+a)^3*((b*x+a)^2)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{(a+bx)\left((a+bx)^2\right)^{3/2}}{4b}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
((a + b*x)*((a + b*x)^2)^(3/2))/(4*b)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx+b^2x^2\right)^{3/2}dx$$

$$\Big\downarrow \text{1079}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(xb^2+ab\right)^3dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{17}$$

$$\frac{(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{4b}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
((a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{\left((bx+a)^{2}\right)^{\frac{3}{2}}(bx+a)}{4b}$	20
risch	$\frac{(bx+a)^{3}\sqrt{(bx+a)^{2}}}{4b}$	22
gosper	$\frac{x\left(b^{3}x^{3}+4a\,b^{2}x^{2}+6a^{2}bx+4a^{3}\right)\left((bx+a)^{2}\right)^{\frac{3}{2}}}{4(bx+a)^{3}}$	49
orering	$\frac{x\left(b^{3}x^{3}+4a\,b^{2}x^{2}+6a^{2}bx+4a^{3}\right)\left(b^{2}x^{2}+2abx+a^{2}\right)^{\frac{3}{2}}}{4(bx+a)^{3}}$	58

input
```
int((b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*((b*x+a)^2)^(3/2)*(b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{1}{4}\,b^3x^4+ab^2x^3+\frac{3}{2}\,a^2bx^2+a^3x$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(19) = 38$.

Time = 0.78 (sec) , antiderivative size = 294, normalized size of antiderivative = 8.65

$$\int \left(a^2+2abx+b^2x^2\right)^{3/2}\,dx = a^2\left(\begin{cases}\left(\frac{a}{2b}+\frac{x}{2}\right)\sqrt{a^2+2abx+b^2x^2} & \text{for } b^2\neq 0\\ \frac{(a^2+2abx)^{\frac{3}{2}}}{3ab} & \text{for } ab\neq 0\\ x\sqrt{a^2} & \text{otherwise}\end{cases}\right) + 2ab\left(\begin{cases}\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^2}{6b^2}+\frac{ax}{6b}+\frac{x^2}{3}\right) & \text{for } b^2\neq 0\\ \frac{-\frac{a^2(a^2+2abx)^{\frac{3}{2}}}{3}+\frac{(a^2+2abx)^{\frac{5}{2}}}{5}}{2a^2b^2} & \text{for } ab\neq 0\\ \frac{x^2\sqrt{a^2}}{2} & \text{otherwise}\end{cases}\right) + b^2\left(\begin{cases}\sqrt{a^2+2abx+b^2x^2}\left(\frac{a^3}{12b^3}-\frac{a^2x}{12b^2}+\frac{ax^2}{12b}+\frac{x^3}{4}\right) & \text{for } b^2\neq 0\\ \frac{\frac{a^4(a^2+2abx)^{\frac{3}{2}}}{3}-\frac{2a^2(a^2+2abx)^{\frac{5}{2}}}{5}+\frac{(a^2+2abx)^{\frac{7}{2}}}{7}}{4a^3b^3} & \text{for } ab\neq 0\\ \frac{x^3\sqrt{a^2}}{3} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
a**2*Piecewise(((a/(2*b) + x/2)*sqrt(a**2 + 2*a*b*x + b**2*x**2), Ne(b**2,
 0)), ((a**2 + 2*a*b*x)**(3/2)/(3*a*b), Ne(a*b, 0)), (x*sqrt(a**2), True))
 + 2*a*b*Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(-a**2/(6*b**2) + a*x
/(6*b) + x**2/3), Ne(b**2, 0)), ((-a**2*(a**2 + 2*a*b*x)**(3/2)/3 + (a**2
+ 2*a*b*x)**(5/2)/5)/(2*a**2*b**2), Ne(a*b, 0)), (x**2*sqrt(a**2)/2, True)
) + b**2*Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(a**3/(12*b**3) - a**
2*x/(12*b**2) + a*x**2/(12*b) + x**3/4), Ne(b**2, 0)), ((a**4*(a**2 + 2*a*
b*x)**(3/2)/3 - 2*a**2*(a**2 + 2*a*b*x)**(5/2)/5 + (a**2 + 2*a*b*x)**(7/2)
/7)/(4*a**3*b**3), Ne(a*b, 0)), (x**3*sqrt(a**2)/3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(21) = 42$.

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{1}{4}\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}x+\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}a}{4\,b}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*x + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2
)*a/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(21) = 42$.

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68

$$\int \left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{1}{2}\left(bx^2+2\,ax\right)a^2\mathrm{sgn}(bx+a)+\frac{a^4\mathrm{sgn}(bx+a)}{4\,b}+\frac{1}{4}\left(bx^2+2\,ax\right)^2b\mathrm{sgn}(bx+a)$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*(b*x^2 + 2*a*x)*a^2*sgn(b*x + a) + 1/4*a^4*sgn(b*x + a)/b + 1/4*(b*x^2
 + 2*a*x)^2*b*sgn(b*x + a)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{\left(x\,b^2+a\,b\right)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}}{4\,b^2}$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
((a*b + b^2*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/(4*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{x(b^3x^3+4a\,b^2x^2+6a^2bx+4a^3)}{4}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
(x*(4*a**3 + 6*a**2*b*x + 4*a*b**2*x**2 + b**3*x**3))/4
```

3.63 $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 143

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x}\,dx = \frac{3a^2bx\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{3ab^2x^2\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)} + \frac{b^3x^3\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)} + \frac{a^3\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}$$

output
```
3*a^2*b*x*((b*x+a)^2)^(1/2)/(b*x+a)+3*a*b^2*x^2*((b*x+a)^2)^(1/2)/(2*b*x+2
*a)+b^3*x^3*((b*x+a)^2)^(1/2)/(3*b*x+3*a)+a^3*((b*x+a)^2)^(1/2)*ln(x)/(b*x
+a)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.43

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x}\,dx = \frac{1}{2}\left(\frac{bx(18a^2+9abx+2b^2x^2)\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}-\sqrt{(a+bx)^2}\right)\right)}{-3a^2-3abx+3\sqrt{a^2}\sqrt{(a+bx)^2}} - 2a^3\operatorname{arctanh}\left(\frac{bx}{\sqrt{a^2}-\sqrt{(a+bx)^2}}\right) - 2\left(a^2\right)^{3/2}\log(x)+\left(a^2\right)^{3/2}\log\left(\sqrt{a^2}-bx-\sqrt{(a+bx)^2}\right)+\left(a^2\right)^{3/2}\log\left(\sqrt{a^2}+bx-\sqrt{(a+bx)^2}\right)\right)$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x,x]
```

output
```
((b*x*(18*a^2 + 9*a*b*x + 2*b^2*x^2)*(Sqrt[a^2]*b*x + a*(Sqrt[a^2] - Sqrt[
(a + b*x)^2])))/(-3*a^2 - 3*a*b*x + 3*Sqrt[a^2]*Sqrt[(a + b*x)^2]) - 2*a^3
*ArcTanh[(b*x)/(Sqrt[a^2] - Sqrt[(a + b*x)^2])] - 2*(a^2)^(3/2)*Log[x] + (
a^2)^(3/2)*Log[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2]] + (a^2)^(3/2)*Log[Sqrt
[a^2] + b*x - Sqrt[(a + b*x)^2]])/2
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x}\,dx$$

$$\Big\downarrow \ 1102$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^3(a+bx)^3}{x}dx}{b^3(a+bx)}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^3}{x}dx}{a+bx}$$

$$\Big\downarrow 49$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a^3}{x}+3ba^2+3b^2xa+b^3x^2\right)dx}{a+bx}$$

$$\Big\downarrow 2009$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(a^3\log(x)+3a^2bx+\frac{3}{2}ab^2x^2+\frac{b^3x^3}{3}\right)}{a+bx}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(3*a^2*b*x + (3*a*b^2*x^2)/2 + (b^3*x^3)/3
+ a^3*Log[x]))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.36

method	result	size
default	$\frac{\left((bx+a)^2\right)^{\frac{3}{2}} \left(2b^3x^3+9a\,b^2x^2+6a^3\ln\left(x\right)+18a^2bx\right)}{6\left(bx+a\right)^3}$	51
risch	$\frac{\sqrt{(bx+a)^2}\,b\left(\frac{1}{3}b^2x^3+\frac{3}{2}ab\,x^2+3a^2x\right)}{bx+a}+\frac{a^3\sqrt{(bx+a)^2}\,\ln\left(x\right)}{bx+a}$	64

input
```
int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/6*((b*x+a)^2)^(3/2)*(2*b^3*x^3+9*a*b^2*x^2+6*a^3*ln(x)+18*a^2*b*x)/(b*x+
a)^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x}\,dx = \frac{1}{3}\,b^3x^3+\frac{3}{2}\,ab^2x^2+3\,a^2bx+a^3\log\left(x\right)$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x,x, algorithm="fricas")
```

output
```
1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(x)
```

Sympy [F]

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x}\,dx = \int \frac{\left((a+bx)^2\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/x,x)
```

output
```
Integral(((a + b*x)**2)**(3/2)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x}\,dx = (-1)^{2\,b^2x+2\,ab}\,a^3\log\left(2\,b^2x+2\,ab\right) - (-1)^{2\,abx+2\,a^2}\,a^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right) + \frac{1}{2}\,\sqrt{b^2x^2+2\,abx+a^2}abx + \frac{3}{2}\,\sqrt{b^2x^2+2\,abx+a^2}a^2 + \frac{1}{3}\,\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x,x, algorithm="maxima")
```

output
```
(-1)^(2*b^2*x + 2*a*b)*a^3*log(2*b^2*x + 2*a*b) - (-1)^(2*a*b*x + 2*a^2)*a
^3*log(2*a*b*x/abs(x) + 2*a^2/abs(x)) + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*
a*b*x + 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^2 + 1/3*(b^2*x^2 + 2*a*b*x + a
^2)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.39

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x}\,dx = \frac{1}{3}\,b^3x^3\mathrm{sgn}(bx+a) + \frac{3}{2}\,ab^2x^2\mathrm{sgn}(bx+a) + 3\,a^2bx\mathrm{sgn}(bx+a) + a^3\log\left(|x|\right)\mathrm{sgn}(bx+a)$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x,x, algorithm="giac")
```

output
```
1/3*b^3*x^3*sgn(b*x + a) + 3/2*a*b^2*x^2*sgn(b*x + a) + 3*a^2*b*x*sgn(b*x
+ a) + a^3*log(abs(x))*sgn(b*x + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x}\,dx = \int \frac{(a^2+2\,a\,b\,x+b^2\,x^2)^{3/2}}{x}\,dx$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/x,x)
```

output
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x}\,dx = \log(x)\,a^3+3a^2bx+\frac{3a\,b^2x^2}{2}+\frac{b^3x^3}{3}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x,x)
```

output
```
(6*log(x)*a**3 + 18*a**2*b*x + 9*a*b**2*x**2 + 2*b**3*x**3)/6
```

3.64 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 142

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^2}\,dx = -\frac{a^3\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}+\frac{3ab^2x\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{b^3x^2\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)}+\frac{3a^2b\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}$$

output
```
-a^3*((b*x+a)^2)^(1/2)/x/(b*x+a)+3*a*b^2*x*((b*x+a)^2)^(1/2)/(b*x+a)+b^3*x
^2*((b*x+a)^2)^(1/2)/(2*b*x+2*a)+3*a^2*b*((b*x+a)^2)^(1/2)*ln(x)/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.39

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^2}\,dx = \frac{\sqrt{(a+bx)^2}\left(-2a^3+6ab^2x^2+b^3x^3+6a^2bx\log(x)\right)}{2x(a+bx)}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^2,x]
```

output
```
(Sqrt[(a + b*x)^2]*(-2*a^3 + 6*a*b^2*x^2 + b^3*x^3 + 6*a^2*b*x*Log[x]))/(2
*x*(a + b*x))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^3(a+bx)^3}{x^2}dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^3}{x^2}dx}{a+bx}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a^3}{x^2}+\frac{3ba^2}{x}+3b^2a+b^3x\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^3}{x}+3a^2b\log(x)+3ab^2x+\frac{b^3x^2}{2}\right)}{a+bx}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^2,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-(a^3/x) + 3*a*b^2*x + (b^3*x^2)/2 + 3*a^2
*b*Log[x]))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.37

method	result	size
default	$\frac{\left((bx+a)^2\right)^{\frac{3}{2}} \left(b^3x^3+6\ln(x)x\,a^2b+6a\,b^2x^2-2a^3\right)}{2x(bx+a)^3}$	53
risch	$\frac{\sqrt{(bx+a)^2}\,b^2\left(\frac{1}{2}b\,x^2+3ax\right)}{bx+a}-\frac{a^3\sqrt{(bx+a)^2}}{x(bx+a)}+\frac{3a^2b\sqrt{(bx+a)^2}\,\ln(x)}{bx+a}$	81

input
```
int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*((b*x+a)^2)^(3/2)*(b^3*x^3+6*ln(x)*x*a^2*b+6*a*b^2*x^2-2*a^3)/x/(b*x+a
)^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.25

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^2}\,dx=\frac{b^3x^3+6\,ab^2x^2+6\,a^2bx\log(x)-2\,a^3}{2\,x}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
1/2*(b^3*x^3 + 6*a*b^2*x^2 + 6*a^2*b*x*log(x) - 2*a^3)/x
```

Sympy [F]

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^2}\,dx=\int \frac{\left((a+bx)^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/x**2,x)
```

output
```
Integral(((a + b*x)**2)**(3/2)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^2}\,dx &= 3\,(-1)^{2\,b^2x+2\,ab}\,a^2b\log\left(2\,b^2x+2\,ab\right)\\ &\quad -3\,(-1)^{2\,abx+2\,a^2}\,a^2b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)+\frac{3}{2}\,\sqrt{b^2x^2+2\,abx+a^2}b^2x\\ &\quad +\frac{9}{2}\,\sqrt{b^2x^2+2\,abx+a^2}ab-\frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}}{x}\end{aligned}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
3*(-1)^(2*b^2*x + 2*a*b)*a^2*b*log(2*b^2*x + 2*a*b) - 3*(-1)^(2*a*b*x + 2*
a^2)*a^2*b*log(2*a*b*x/abs(x) + 2*a^2/abs(x)) + 3/2*sqrt(b^2*x^2 + 2*a*b*x
 + a^2)*b^2*x + 9/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a*b - (b^2*x^2 + 2*a*b*x
 + a^2)^(3/2)/x
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.40

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{x^2} \, dx = \frac{1}{2} b^3 x^2 \text{sgn}(bx + a) + 3 ab^2 x \text{sgn}(bx + a) + 3 a^2 b \log(|x|) \text{sgn}(bx + a) - \frac{a^3 \text{sgn}(bx + a)}{x}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^2,x, algorithm="giac")
```

output
```
1/2*b^3*x^2*sgn(b*x + a) + 3*a*b^2*x*sgn(b*x + a) + 3*a^2*b*log(abs(x))*sg
n(b*x + a) - a^3*sgn(b*x + a)/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{x^2} \, dx = \int \frac{(a^2 + 2\,a\,b\,x + b^2\,x^2)^{3/2}}{x^2} \, dx$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/x^2,x)
```

output
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.25

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{x^2}\,dx = \frac{6\log(x)\,a^2bx - 2a^3 + 6a\,b^2x^2 + b^3x^3}{2x}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^2,x)
```

output
```
(6*log(x)*a**2*b*x - 2*a**3 + 6*a*b**2*x**2 + b**3*x**3)/(2*x)
```

3.65 $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 141

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^3}\,dx = -\frac{a^3\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)} - \frac{3a^2b\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)} + \frac{b^3x\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{3ab^2\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}$$

output
```
-1/2*a^3*((b*x+a)^2)^(1/2)/x^2/(b*x+a)-3*a^2*b*((b*x+a)^2)^(1/2)/x/(b*x+a)
+b^3*x*((b*x+a)^2)^(1/2)/(b*x+a)+3*a*b^2*((b*x+a)^2)^(1/2)*ln(x)/(b*x+a)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 554 vs. 2(141) = 282.

Time = 0.80 (sec) , antiderivative size = 554, normalized size of antiderivative = 3.93

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^3}\,dx = \frac{4a^4\sqrt{a^2}+28a^3\sqrt{a^2}bx+35(a^2)^{3/2}b^2x^2+3a\sqrt{a^2}b^3x^3-8\sqrt{a^2}b^4x^4-4a^4\sqrt{(a}{}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^3,x]
```

output
```
(4*a^4*Sqrt[a^2] + 28*a^3*Sqrt[a^2]*b*x + 35*(a^2)^(3/2)*b^2*x^2 + 3*a*Sqr
t[a^2]*b^3*x^3 - 8*Sqrt[a^2]*b^4*x^4 - 4*a^4*Sqrt[(a + b*x)^2] - 24*a^3*b*
x*Sqrt[(a + b*x)^2] - 11*a^2*b^2*x^2*Sqrt[(a + b*x)^2] + 8*a*b^3*x^3*Sqrt[
(a + b*x)^2] - 24*a*b^2*x^2*(a^2 + a*b*x - Sqrt[a^2]*Sqrt[(a + b*x)^2])*Ar
cTanh[(b*x)/(Sqrt[a^2] - Sqrt[(a + b*x)^2])] - 24*b^2*x^2*((a^2)^(3/2) + a
*Sqrt[a^2]*b*x - a^2*Sqrt[(a + b*x)^2])*Log[x] + 12*(a^2)^(3/2)*b^2*x^2*Lo
g[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2]] + 12*a*Sqrt[a^2]*b^3*x^3*Log[Sqrt[a
^2] - b*x - Sqrt[(a + b*x)^2]] - 12*a^2*b^2*x^2*Sqrt[(a + b*x)^2]*Log[Sqrt
[a^2] - b*x - Sqrt[(a + b*x)^2]] + 12*(a^2)^(3/2)*b^2*x^2*Log[Sqrt[a^2] +
b*x - Sqrt[(a + b*x)^2]] + 12*a*Sqrt[a^2]*b^3*x^3*Log[Sqrt[a^2] + b*x - Sq
rt[(a + b*x)^2]] - 12*a^2*b^2*x^2*Sqrt[(a + b*x)^2]*Log[Sqrt[a^2] + b*x -
Sqrt[(a + b*x)^2]])/(8*x^2*(a^2 + a*b*x - Sqrt[a^2]*Sqrt[(a + b*x)^2]))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.43, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^3(a+bx)^3}{x^3}dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^3}{x^3}dx}{a+bx}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a^3}{x^3}+\frac{3ba^2}{x^2}+\frac{3b^2a}{x}+b^3\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^3}{2x^2}-\frac{3a^2b}{x}+3ab^2\log(x)+b^3x\right)}{a+bx}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^3,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/2*a^3/x^2 - (3*a^2*b)/x + b^3*x + 3*a*b
^2*Log[x]))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.38

method	result	size
default	$\frac{\left((bx+a)^2\right)^{\frac{3}{2}}\left(6\ln(x)x^2a\,b^2+2b^3x^3-6a^2bx-a^3\right)}{2(bx+a)^3x^2}$	54
risch	$\frac{b^3x\sqrt{(bx+a)^2}}{bx+a}+\frac{\sqrt{(bx+a)^2}\,\left(-3a^2bx-\frac{1}{2}a^3\right)}{(bx+a)x^2}+\frac{3a\,b^2\sqrt{(bx+a)^2}\,\ln(x)}{bx+a}$	80

input
```
int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*((b*x+a)^2)^(3/2)*(6*ln(x)*x^2*a*b^2+2*b^3*x^3-6*a^2*b*x-a^3)/(b*x+a)^
3/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.26

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^3}\,dx=\frac{2\,b^3x^3+6\,ab^2x^2\log{(x)}-6\,a^2bx-a^3}{2\,x^2}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
1/2*(2*b^3*x^3 + 6*a*b^2*x^2*log(x) - 6*a^2*b*x - a^3)/x^2
```

Sympy [F]

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^3}\,dx=\int \frac{\left((a+bx)^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/x**3,x)
```

output
```
Integral(((a + b*x)**2)**(3/2)/x**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(95) = 190$.

Time = 0.04 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.42

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^3}\,dx = 3\,(-1)^{2\,b^2x+2\,ab}\,ab^2\log\left(2\,b^2x+2\,ab\right) - 3\,(-1)^{2\,abx+2\,a^2}\,ab^2\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right) + \frac{3\,\sqrt{b^2x^2+2\,abx+a^2}b^3x}{2\,a} + \frac{9}{2}\,\sqrt{b^2x^2+2\,abx+a^2}b^2 + \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^2}{2\,a^2} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b}{2\,ax} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}}{2\,a^2x^2}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
3*(-1)^(2*b^2*x + 2*a*b)*a*b^2*log(2*b^2*x + 2*a*b) - 3*(-1)^(2*a*b*x + 2*
a^2)*a*b^2*log(2*a*b*x/abs(x) + 2*a^2/abs(x)) + 3/2*sqrt(b^2*x^2 + 2*a*b*x
 + a^2)*b^3*x/a + 9/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^2 + 1/2*(b^2*x^2 + 2
*a*b*x + a^2)^(3/2)*b^2/a^2 - 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b/(a*x)
- 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)/(a^2*x^2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.40

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^3}\,dx = b^3x\text{sgn}(bx+a) + 3\,ab^2\log\left(|x|\right)\text{sgn}(bx+a) - \frac{6\,a^2bx\text{sgn}(bx+a)+a^3\text{sgn}(bx+a)}{2\,x^2}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^3,x, algorithm="giac")
```

output
```
b^3*x*sgn(b*x + a) + 3*a*b^2*log(abs(x))*sgn(b*x + a) - 1/2*(6*a^2*b*x*sgn
(b*x + a) + a^3*sgn(b*x + a))/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^3}\,dx = \int \frac{(a^2+2\,a\,b\,x+b^2\,x^2)^{3/2}}{x^3}\,dx$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/x^3,x)
```

output
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.26

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^3}\,dx = \frac{6\log(x)\,a\,b^2x^2-a^3-6a^2bx+2b^3x^3}{2x^2}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^3,x)
```

output
```
(6*log(x)*a*b**2*x**2 - a**3 - 6*a**2*b*x + 2*b**3*x**3)/(2*x**2)
```

3.66 $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^4} dx$

Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^4} dx = -\frac{a^3\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)} - \frac{3a^2b\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)} - \frac{3ab^2\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)} + \frac{b^3\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}$$

output
```
-1/3*a^3*((b*x+a)^2)^(1/2)/x^3/(b*x+a)-3/2*a^2*b*((b*x+a)^2)^(1/2)/x^2/(b*
x+a)-3*a*b^2*((b*x+a)^2)^(1/2)/x/(b*x+a)+b^3*((b*x+a)^2)^(1/2)*ln(x)/(b*x+
a)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.67

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^4} dx = \frac{2a^3\sqrt{a^2}+9(a^2)^{3/2}bx+18a\sqrt{a^2}b^2x^2-2a^3\sqrt{(a+bx)^2}-7a^2bx\sqrt{(a+bx)^2}\cdots}{\cdots}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^4,x]
```

output
```
(2*a^3*Sqrt[a^2] + 9*(a^2)^(3/2)*b*x + 18*a*Sqrt[a^2]*b^2*x^2 - 2*a^3*Sqrt
[(a + b*x)^2] - 7*a^2*b*x*Sqrt[(a + b*x)^2] - 11*a*b^2*x^2*Sqrt[(a + b*x)^
2] - 12*a*b^3*x^3*ArcTanh[(b*x)/(Sqrt[a^2] - Sqrt[(a + b*x)^2])] - 12*Sqrt
[a^2]*b^3*x^3*Log[x] + 6*Sqrt[a^2]*b^3*x^3*Log[a*(Sqrt[a^2] - b*x - Sqrt[(
a + b*x)^2])] + 6*Sqrt[a^2]*b^3*x^3*Log[a*(Sqrt[a^2] + b*x - Sqrt[(a + b*x
)^2])])/(12*a*x^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^3(a+bx)^3}{x^4}dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^3}{x^4}dx}{a+bx}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a^3}{x^4}+\frac{3ba^2}{x^3}+\frac{3b^2a}{x^2}+\frac{b^3}{x}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3ab^2}{x}+b^3\log(x)\right)}{a+bx}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^4,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/3*a^3/x^3 - (3*a^2*b)/(2*x^2) - (3*a*b^
2)/x + b^3*Log[x]))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.37

method	result	size
default	$\frac{\left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}}\left(6\ln\left(x\right)x^{3}b^{3}-18a\,b^{2}x^{2}-9a^{2}bx-2a^{3}\right)}{6x^{3}\left(bx+a\right)^{3}}$	54
risch	$\frac{\sqrt{\left(bx+a\right)^{2}}\,\left(-3a\,b^{2}x^{2}-\frac{3}{2}a^{2}bx-\frac{1}{3}a^{3}\right)}{\left(bx+a\right)x^{3}}+\frac{b^{3}\sqrt{\left(bx+a\right)^{2}}\,\ln\left(x\right)}{bx+a}$	66

input
```
int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*((b*x+a)^2)^(3/2)*(6*ln(x)*x^3*b^3-18*a*b^2*x^2-9*a^2*b*x-2*a^3)/x^3/(
b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.26

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{x^4} \, dx = \frac{6 \, b^3 x^3 \log{(x)} - 18 \, ab^2 x^2 - 9 \, a^2 bx - 2 \, a^3}{6 \, x^3}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
1/6*(6*b^3*x^3*log(x) - 18*a*b^2*x^2 - 9*a^2*b*x - 2*a^3)/x^3
```

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{x^4} \, dx = \int \frac{\left((a + bx)^2\right)^{\frac{3}{2}}}{x^4} \, dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/x**4,x)
```

output
```
Integral(((a + b*x)**2)**(3/2)/x**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(97) = 194$.

Time = 0.04 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.58

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^4}\,dx = (-1)^{2\,b^2x+2\,ab}\,b^3\log\left(2\,b^2x+2\,ab\right) - (-1)^{2\,abx+2\,a^2}\,b^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right) + \frac{\sqrt{b^2x^2+2\,abx+a^2}b^4x}{2\,a^2} + \frac{3\,\sqrt{b^2x^2+2\,abx+a^2}b^3}{2\,a} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^3}{6\,a^3} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^2}{2\,a^2x} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b}{6\,a^3x^2} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}}{3\,a^2x^3}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
(-1)^(2*b^2*x + 2*a*b)*b^3*log(2*b^2*x + 2*a*b) - (-1)^(2*a*b*x + 2*a^2)*b
^3*log(2*a*b*x/abs(x) + 2*a^2/abs(x)) + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*
b^4*x/a^2 + 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^3/a - 1/6*(b^2*x^2 + 2*a*b
*x + a^2)^(3/2)*b^3/a^3 - 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^2/(a^2*x)
+ 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*b/(a^3*x^2) - 1/3*(b^2*x^2 + 2*a*b*x
 + a^2)^(5/2)/(a^2*x^3)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.41

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^4}\,dx = b^3\log\left(|x|\right)\mathrm{sgn}(bx+a) - \frac{18\,ab^2x^2\mathrm{sgn}(bx+a)+9\,a^2bx\mathrm{sgn}(bx+a)+2\,a^3\mathrm{sgn}(bx+a)}{6\,x^3}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^4,x, algorithm="giac")
```

output
```
b^3*log(abs(x))*sgn(b*x + a) - 1/6*(18*a*b^2*x^2*sgn(b*x + a) + 9*a^2*b*x*
sgn(b*x + a) + 2*a^3*sgn(b*x + a))/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{x^4} \, dx = \int \frac{(a^2 + 2\,a\,b\,x + b^2\,x^2)^{3/2}}{x^4} \, dx$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/x^4,x)
```

output
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.26

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{x^4} \, dx = \frac{6 \log(x)\, b^3x^3 - 2a^3 - 9a^2bx - 18a\, b^2x^2}{6x^3}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^4,x)
```

output
```
(6*log(x)*b**3*x**3 - 2*a**3 - 9*a**2*b*x - 18*a*b**2*x**2)/(6*x**3)
```

3.67 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 35

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^5}\,dx=-\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{4ax^4}$$

output
```
-1/4*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/a/x^4
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^5}\,dx=-\frac{(a+bx)^3\sqrt{(a+bx)^2}}{4ax^4}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^5,x]
```

output
```
-1/4*((a + b*x)^3*Sqrt[(a + b*x)^2])/(a*x^4)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1102, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^5}dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^3(a+bx)^3}{x^5}dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^3}{x^5}dx}{a+bx}$$

$$\Big\downarrow \text{48}$$

$$-\frac{(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{4ax^4}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^5,x]
```

output
```
-1/4*((a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(a*x^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.43

method	result	size
gosper	$-\frac{\left(4b^{3}x^{3}+6a\,b^{2}x^{2}+4a^{2}bx+a^{3}\right)\left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}}}{4x^{4}\left(bx+a\right)^{3}}$	50
default	$-\frac{\left(4b^{3}x^{3}+6a\,b^{2}x^{2}+4a^{2}bx+a^{3}\right)\left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}}}{4x^{4}\left(bx+a\right)^{3}}$	50
risch	$\frac{\sqrt{\left(bx+a\right)^{2}}\,\left(-b^{3}x^{3}-\frac{3}{2}a\,b^{2}x^{2}-a^{2}bx-\frac{1}{4}a^{3}\right)}{\left(bx+a\right)x^{4}}$	51
orering	$-\frac{\left(4b^{3}x^{3}+6a\,b^{2}x^{2}+4a^{2}bx+a^{3}\right)\left(b^{2}x^{2}+2abx+a^{2}\right)^{\frac{3}{2}}}{4x^{4}\left(bx+a\right)^{3}}$	59

input
```
int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(4*b^3*x^3+6*a*b^2*x^2+4*a^2*b*x+a^3)*((b*x+a)^2)^(3/2)/x^4/(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^5}\,dx=-\frac{4\,b^3x^3+6\,ab^2x^2+4\,a^2bx+a^3}{4\,x^4}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
-1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

Sympy [F]

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^5}\,dx=\int \frac{\left((a+bx)^2\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/x**5,x)
```

output
```
Integral(((a + b*x)**2)**(3/2)/x**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(31) = 62$.

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.94

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^5}\,dx=\frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^4}{4\,a^4}+\frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^3}{4\,a^3x}-\frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^2}{4\,a^4x^2}+\frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b}{4\,a^3x^3}-\frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}}{4\,a^2x^4}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^4/a^4 + 1/4*(b^2*x^2 + 2*a*b*x + a^2
)^(3/2)*b^3/(a^3*x) - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*b^2/(a^4*x^2) +
1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*b/(a^3*x^3) - 1/4*(b^2*x^2 + 2*a*b*x +
 a^2)^(5/2)/(a^2*x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(31) = 62$.

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.09

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{x^5}\, dx = -\frac{b^4 \operatorname{sgn}(bx + a)}{4\,a} - \frac{4\,b^3x^3\operatorname{sgn}(bx + a) + 6\,ab^2x^2\operatorname{sgn}(bx + a) + 4\,a^2bx\operatorname{sgn}(bx + a) + a^3\operatorname{sgn}(bx + a)}{4\,x^4}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^5,x, algorithm="giac")
```

output
```
-1/4*b^4*sgn(b*x + a)/a - 1/4*(4*b^3*x^3*sgn(b*x + a) + 6*a*b^2*x^2*sgn(b*
x + a) + 4*a^2*b*x*sgn(b*x + a) + a^3*sgn(b*x + a))/x^4
```

Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.86

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{x^5}\, dx = -\frac{a^3\,\sqrt{a^2 + 2\,a\,b\,x + b^2\,x^2}}{4\,x^4\,(a + b\,x)} - \frac{b^3\,\sqrt{a^2 + 2\,a\,b\,x + b^2\,x^2}}{x\,(a + b\,x)} - \frac{3\,a\,b^2\,\sqrt{a^2 + 2\,a\,b\,x + b^2\,x^2}}{2\,x^2\,(a + b\,x)} - \frac{a^2\,b\,\sqrt{a^2 + 2\,a\,b\,x + b^2\,x^2}}{x^3\,(a + b\,x)}$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/x^5,x)
```

output
```
- (a^3*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(4*x^4*(a + b*x)) - (b^3*(a^2 + b^
2*x^2 + 2*a*b*x)^(1/2))/(x*(a + b*x)) - (3*a*b^2*(a^2 + b^2*x^2 + 2*a*b*x)
^(1/2))/(2*x^2*(a + b*x)) - (a^2*b*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^3*(
a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^5}\,dx=\frac{-4b^3x^3-6a\,b^2x^2-4a^2bx-a^3}{4x^4}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^5,x)
```

output
```
( - a**3 - 4*a**2*b*x - 6*a*b**2*x**2 - 4*b**3*x**3)/(4*x**4)
```

3.68 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^6} dx$

Optimal result

Integrand size = 24, antiderivative size = 76

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^6} dx = -\frac{(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{5ax^5} + \frac{b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{20a^2x^4}$$

output
```
-1/5*(b*x+a)^3*((b*x+a)^2)^(1/2)/a/x^5+1/20*b*(b*x+a)^3*((b*x+a)^2)^(1/2)/
a^2/x^4
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.72

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^6} dx = -\frac{\sqrt{(a+bx)^2}(4a^3+15a^2bx+20ab^2x^2+10b^3x^3)}{20x^5(a+bx)}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^6,x]
```

output
```
-1/20*(Sqrt[(a + b*x)^2]*(4*a^3 + 15*a^2*b*x + 20*a*b^2*x^2 + 10*b^3*x^3))
/(x^5*(a + b*x))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^6}dx$$

$$\Big\downarrow \text{ 1102}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^3(a+bx)^3}{x^6}dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^3}{x^6}dx}{a+bx}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{b\int \frac{(a+bx)^3}{x^5}dx}{5a}-\frac{(a+bx)^4}{5ax^5}\right)}{a+bx}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{b(a+bx)^4}{20a^2x^4}-\frac{(a+bx)^4}{5ax^5}\right)}{a+bx}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^6,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/5*(a + b*x)^4/(a*x^5) + (b*(a + b*x)^4)
/(20*a^2*x^4)))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{\sqrt{(bx+a)^{2}}\,\left(-\frac{1}{2}b^{3}x^{3}-a\,b^{2}x^{2}-\frac{3}{4}a^{2}bx-\frac{1}{5}a^{3}\right)}{(bx+a)x^{5}}$	51
gosper	$-\frac{\left(10b^{3}x^{3}+20a\,b^{2}x^{2}+15a^{2}bx+4a^{3}\right)\left((bx+a)^{2}\right)^{\frac{3}{2}}}{20x^{5}(bx+a)^{3}}$	52
default	$-\frac{\left(10b^{3}x^{3}+20a\,b^{2}x^{2}+15a^{2}bx+4a^{3}\right)\left((bx+a)^{2}\right)^{\frac{3}{2}}}{20x^{5}(bx+a)^{3}}$	52
orering	$-\frac{\left(10b^{3}x^{3}+20a\,b^{2}x^{2}+15a^{2}bx+4a^{3}\right)\left(b^{2}x^{2}+2abx+a^{2}\right)^{\frac{3}{2}}}{20x^{5}(bx+a)^{3}}$	61

input
```
int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*(-1/2*b^3*x^3-a*b^2*x^2-3/4*a^2*b*x-1/5*a^3)/x^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.46

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^6}\,dx=-\frac{10\,b^3x^3+20\,ab^2x^2+15\,a^2bx+4\,a^3}{20\,x^5}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
-1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

Sympy [F]

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^6}\,dx=\int \frac{\left(\left(a+bx\right)^2\right)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/x**6,x)
```

output
```
Integral(((a + b*x)**2)**(3/2)/x**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(50) = 100$.

Time = 0.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.20

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^6}\,dx = -\frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^5}{4\,a^5} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^4}{4\,a^4x} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^3}{4\,a^5x^2} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^2}{4\,a^4x^3} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b}{4\,a^3x^4} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}}{5\,a^2x^5}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
-1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^5/a^5 - 1/4*(b^2*x^2 + 2*a*b*x + a^
2)^(3/2)*b^4/(a^4*x) + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*b^3/(a^5*x^2) -
 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*b^2/(a^4*x^3) + 1/4*(b^2*x^2 + 2*a*b*
x + a^2)^(5/2)*b/(a^3*x^4) - 1/5*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)/(a^2*x^5)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.97

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^6}\,dx = \frac{b^5\mathrm{sgn}(bx+a)}{20\,a^2} - \frac{10\,b^3x^3\mathrm{sgn}(bx+a)+20\,ab^2x^2\mathrm{sgn}(bx+a)+15\,a^2bx\mathrm{sgn}(bx+a)+4\,a^3\mathrm{sgn}(bx+a)}{20\,x^5}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^6,x, algorithm="giac")
```

output
```
1/20*b^5*sgn(b*x + a)/a^2 - 1/20*(10*b^3*x^3*sgn(b*x + a) + 20*a*b^2*x^2*s
gn(b*x + a) + 15*a^2*b*x*sgn(b*x + a) + 4*a^3*sgn(b*x + a))/x^5
```

Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.78

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^6}\,dx = -\frac{a^3\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{5\,x^5\,\left(a+b\,x\right)}-\frac{b^3\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{2\,x^2\,\left(a+b\,x\right)} -\frac{a\,b^2\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^3\,\left(a+b\,x\right)}-\frac{3\,a^2\,b\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{4\,x^4\,\left(a+b\,x\right)}$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/x^6,x)
```

output
```
- (a^3*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(5*x^5*(a + b*x)) - (b^3*(a^2 + b^
2*x^2 + 2*a*b*x)^(1/2))/(2*x^2*(a + b*x)) - (a*b^2*(a^2 + b^2*x^2 + 2*a*b*
x)^(1/2))/(x^3*(a + b*x)) - (3*a^2*b*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(4*x
^4*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.46

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^6}\,dx = \frac{-10b^3x^3-20a\,b^2x^2-15a^2bx-4a^3}{20x^5}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^6,x)
```

output
```
( - 4*a**3 - 15*a**2*b*x - 20*a*b**2*x**2 - 10*b**3*x**3)/(20*x**5)
```

3.69 $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^7}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 151

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^7}\,dx = -\frac{a^3\sqrt{a^2+2abx+b^2x^2}}{6x^6(a+bx)} - \frac{3a^2b\sqrt{a^2+2abx+b^2x^2}}{5x^5(a+bx)} - \frac{3ab^2\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)} - \frac{b^3\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)}$$

output
```
-1/6*a^3*((b*x+a)^2)^(1/2)/x^6/(b*x+a)-3/5*a^2*b*((b*x+a)^2)^(1/2)/x^5/(b*
x+a)-3/4*a*b^2*((b*x+a)^2)^(1/2)/x^4/(b*x+a)-1/3*b^3*((b*x+a)^2)^(1/2)/x^3
/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.36

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^7}\,dx = -\frac{\sqrt{(a+bx)^2}(10a^3+36a^2bx+45ab^2x^2+20b^3x^3)}{60x^6(a+bx)}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^7,x]
```

output
```
-1/60*(Sqrt[(a + b*x)^2]*(10*a^3 + 36*a^2*b*x + 45*a*b^2*x^2 + 20*b^3*x^3)
)/(x^6*(a + b*x))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^7}dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^3(a+bx)^3}{x^7}dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^3}{x^7}dx}{a+bx}$$

$$\Big\downarrow \text{53}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a^3}{x^7}+\frac{3ba^2}{x^6}+\frac{3b^2a}{x^5}+\frac{b^3}{x^4}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^3}{6x^6}-\frac{3a^2b}{5x^5}-\frac{3ab^2}{4x^4}-\frac{b^3}{3x^3}\right)\sqrt{a^2+2abx+b^2x^2}}{a+bx}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^7,x]
```

output
```
((-1/6*a^3/x^6 - (3*a^2*b)/(5*x^5) - (3*a*b^2)/(4*x^4) - b^3/(3*x^3))*Sqrt
[a^2 + 2*a*b*x + b^2*x^2])/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{\sqrt{(bx+a)^2}\,\left(-\frac{1}{3}b^3x^3-\frac{3}{4}a\,b^2x^2-\frac{3}{5}a^2bx-\frac{1}{6}a^3\right)}{(bx+a)x^6}$	51
gosper	$-\frac{(20b^3x^3+45a\,b^2x^2+36a^2bx+10a^3)\left((bx+a)^2\right)^{\frac{3}{2}}}{60x^6(bx+a)^3}$	52
default	$-\frac{(20b^3x^3+45a\,b^2x^2+36a^2bx+10a^3)\left((bx+a)^2\right)^{\frac{3}{2}}}{60x^6(bx+a)^3}$	52
orering	$-\frac{(20b^3x^3+45a\,b^2x^2+36a^2bx+10a^3)\,(b^2x^2+2abx+a^2)^{\frac{3}{2}}}{60x^6(bx+a)^3}$	61

input
```
int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*(-1/3*b^3*x^3-3/4*a*b^2*x^2-3/5*a^2*b*x-1/6*a^3)
/x^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^7}\,dx = -\frac{20\,b^3x^3+45\,ab^2x^2+36\,a^2bx+10\,a^3}{60\,x^6}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
-1/60*(20*b^3*x^3 + 45*a*b^2*x^2 + 36*a^2*b*x + 10*a^3)/x^6
```

Sympy [F]

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^7}\,dx = \int \frac{\left((a+bx)^2\right)^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/x**7,x)
```

output
```
Integral(((a + b*x)**2)**(3/2)/x**7, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.30

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^7}\,dx = \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^6}{4\,a^6} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^5}{4\,a^5x} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^4}{4\,a^6x^2} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^3}{4\,a^5x^3} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^2}{4\,a^4x^4} + \frac{7\,(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b}{30\,a^3x^5} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}}{6\,a^2x^6}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^6/a^6 + 1/4*(b^2*x^2 + 2*a*b*x + a^2
)^(3/2)*b^5/(a^5*x) - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*b^4/(a^6*x^2) +
1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*b^3/(a^5*x^3) - 1/4*(b^2*x^2 + 2*a*b*x
 + a^2)^(5/2)*b^2/(a^4*x^4) + 7/30*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*b/(a^3*
x^5) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)/(a^2*x^6)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.49

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^7}\,dx = -\frac{b^6\text{sgn}(bx+a)}{60\,a^3} - \frac{20\,b^3x^3\text{sgn}(bx+a)+45\,ab^2x^2\text{sgn}(bx+a)+36\,a^2bx\text{sgn}(bx+a)+10\,a^3\text{sgn}(bx+a)}{60\,x^6}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^7,x, algorithm="giac")
```

output
```
-1/60*b^6*sgn(b*x + a)/a^3 - 1/60*(20*b^3*x^3*sgn(b*x + a) + 45*a*b^2*x^2*
sgn(b*x + a) + 36*a^2*b*x*sgn(b*x + a) + 10*a^3*sgn(b*x + a))/x^6
```

Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.89

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^7}\,dx = -\frac{a^3\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{6\,x^6\,(a+b\,x)} - \frac{b^3\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{3\,x^3\,(a+b\,x)} - \frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{4\,x^4\,(a+b\,x)} - \frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{5\,x^5\,(a+b\,x)}$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/x^7,x)
```

output
```
- (a^3*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(6*x^6*(a + b*x)) - (b^3*(a^2 + b^
2*x^2 + 2*a*b*x)^(1/2))/(3*x^3*(a + b*x)) - (3*a*b^2*(a^2 + b^2*x^2 + 2*a*
b*x)^(1/2))/(4*x^4*(a + b*x)) - (3*a^2*b*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/
(5*x^5*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.23

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{x^7}\,dx = \frac{-20b^3x^3 - 45a\,b^2x^2 - 36a^2bx - 10a^3}{60x^6}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^7,x)
```

output
```
( - 10*a**3 - 36*a**2*b*x - 45*a*b**2*x**2 - 20*b**3*x**3)/(60*x**6)
```

3.70 $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^8} dx$

Optimal result

Integrand size = 24, antiderivative size = 151

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^8} dx = -\frac{a^3\sqrt{a^2+2abx+b^2x^2}}{7x^7(a+bx)} - \frac{a^2b\sqrt{a^2+2abx+b^2x^2}}{2x^6(a+bx)} - \frac{3ab^2\sqrt{a^2+2abx+b^2x^2}}{5x^5(a+bx)} - \frac{b^3\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)}$$

output
```
-1/7*a^3*((b*x+a)^2)^(1/2)/x^7/(b*x+a)-1/2*a^2*b*((b*x+a)^2)^(1/2)/x^6/(b*
x+a)-3/5*a*b^2*((b*x+a)^2)^(1/2)/x^5/(b*x+a)-1/4*b^3*((b*x+a)^2)^(1/2)/x^4
/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.36

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^8} dx = -\frac{\sqrt{(a+bx)^2}(20a^3+70a^2bx+84ab^2x^2+35b^3x^3)}{140x^7(a+bx)}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^8,x]
```

output
```
-1/140*(Sqrt[(a + b*x)^2]*(20*a^3 + 70*a^2*b*x + 84*a*b^2*x^2 + 35*b^3*x^3
))/(x^7*(a + b*x))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^8}dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^3(a+bx)^3}{x^8}dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^3}{x^8}dx}{a+bx}$$

$$\Big\downarrow \text{53}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a^3}{x^8}+\frac{3ba^2}{x^7}+\frac{3b^2a}{x^6}+\frac{b^3}{x^5}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^3}{7x^7}-\frac{a^2b}{2x^6}-\frac{3ab^2}{5x^5}-\frac{b^3}{4x^4}\right)\sqrt{a^2+2abx+b^2x^2}}{a+bx}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^8,x]
```

output
```
((-1/7*a^3/x^7 - (a^2*b)/(2*x^6) - (3*a*b^2)/(5*x^5) - b^3/(4*x^4))*Sqrt[a
^2 + 2*a*b*x + b^2*x^2])/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{\sqrt{\left(bx+a\right)^{2}}\, \left(-\frac{1}{4} b^{3} x^{3}-\frac{3}{5} a\, b^{2} x^{2}-\frac{1}{2} a^{2} b x -\frac{1}{7} a^{3}\right)}{\left(bx+a\right) x^{7}}$	51
gosper	$-\frac{\left(35 b^{3} x^{3}+84 a\, b^{2} x^{2}+70 a^{2} b x +20 a^{3}\right) \left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}}}{140 x^{7} \left(bx+a\right)^{3}}$	52
default	$-\frac{\left(35 b^{3} x^{3}+84 a\, b^{2} x^{2}+70 a^{2} b x +20 a^{3}\right) \left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}}}{140 x^{7} \left(bx+a\right)^{3}}$	52
orering	$-\frac{\left(35 b^{3} x^{3}+84 a\, b^{2} x^{2}+70 a^{2} b x +20 a^{3}\right) \left(b^{2} x^{2}+2 a b x +a^{2}\right)^{\frac{3}{2}}}{140 x^{7} \left(bx+a\right)^{3}}$	61

input
```
int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*(-1/4*b^3*x^3-3/5*a*b^2*x^2-1/2*a^2*b*x-1/7*a^3)
/x^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^8}\,dx = -\frac{35\,b^3x^3+84\,ab^2x^2+70\,a^2bx+20\,a^3}{140\,x^7}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
-1/140*(35*b^3*x^3 + 84*a*b^2*x^2 + 70*a^2*b*x + 20*a^3)/x^7
```

Sympy [F]

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^8}\,dx = \int \frac{\left((a+bx)^2\right)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/x**8,x)
```

output
```
Integral(((a + b*x)**2)**(3/2)/x**8, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(99) = 198.

Time = 0.04 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.49

$$\begin{aligned}\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^8}\,dx = &-\frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^7}{4\,a^7}-\frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^6}{4\,a^6x}\\&+\frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^5}{4\,a^7x^2}-\frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^4}{4\,a^6x^3}+\frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^3}{4\,a^5x^4}\\&-\frac{17\,(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^2}{70\,a^4x^5}+\frac{3\,(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b}{14\,a^3x^6}-\frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}}{7\,a^2x^7}\end{aligned}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
-1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^7/a^7 - 1/4*(b^2*x^2 + 2*a*b*x + a^
2)^(3/2)*b^6/(a^6*x) + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*b^5/(a^7*x^2) -
 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*b^4/(a^6*x^3) + 1/4*(b^2*x^2 + 2*a*b*
x + a^2)^(5/2)*b^3/(a^5*x^4) - 17/70*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*b^2/(
a^4*x^5) + 3/14*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*b/(a^3*x^6) - 1/7*(b^2*x^2
 + 2*a*b*x + a^2)^(5/2)/(a^2*x^7)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.49

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{x^8}\,dx = \frac{b^7\mathrm{sgn}(bx+a)}{140\,a^4} - \frac{35\,b^3x^3\mathrm{sgn}(bx+a) + 84\,ab^2x^2\mathrm{sgn}(bx+a) + 70\,a^2bx\mathrm{sgn}(bx+a) + 20\,a^3\mathrm{sgn}(bx+a)}{140\,x^7}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^8,x, algorithm="giac")
```

output
```
1/140*b^7*sgn(b*x + a)/a^4 - 1/140*(35*b^3*x^3*sgn(b*x + a) + 84*a*b^2*x^2
*sgn(b*x + a) + 70*a^2*b*x*sgn(b*x + a) + 20*a^3*sgn(b*x + a))/x^7
```

Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.89

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{x^8}\,dx = -\frac{a^3\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{7\,x^7\,(a+b\,x)} - \frac{b^3\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{4\,x^4\,(a+b\,x)} - \frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{5\,x^5\,(a+b\,x)} - \frac{a^2\,b\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{2\,x^6\,(a+b\,x)}$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/x^8,x)
```

output
```
- (a^3*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(7*x^7*(a + b*x)) - (b^3*(a^2 + b^
2*x^2 + 2*a*b*x)^(1/2))/(4*x^4*(a + b*x)) - (3*a*b^2*(a^2 + b^2*x^2 + 2*a*
b*x)^(1/2))/(5*x^5*(a + b*x)) - (a^2*b*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(2
*x^6*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.23

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{x^8}\,dx = \frac{-35b^3x^3 - 84a\,b^2x^2 - 70a^2bx - 20a^3}{140x^7}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^8,x)
```

output
```
( - 20*a**3 - 70*a**2*b*x - 84*a*b**2*x**2 - 35*b**3*x**3)/(140*x**7)
```

3.71 $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^9} dx$

Optimal result

Integrand size = 24, antiderivative size = 151

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^9} dx = -\frac{a^3\sqrt{a^2+2abx+b^2x^2}}{8x^8(a+bx)} - \frac{3a^2b\sqrt{a^2+2abx+b^2x^2}}{7x^7(a+bx)} - \frac{ab^2\sqrt{a^2+2abx+b^2x^2}}{2x^6(a+bx)} - \frac{b^3\sqrt{a^2+2abx+b^2x^2}}{5x^5(a+bx)}$$

output
```
-1/8*a^3*((b*x+a)^2)^(1/2)/x^8/(b*x+a)-3/7*a^2*b*((b*x+a)^2)^(1/2)/x^7/(b*
x+a)-1/2*a*b^2*((b*x+a)^2)^(1/2)/x^6/(b*x+a)-1/5*b^3*((b*x+a)^2)^(1/2)/x^5
/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.36

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^9} dx = -\frac{\sqrt{(a+bx)^2}(35a^3+120a^2bx+140ab^2x^2+56b^3x^3)}{280x^8(a+bx)}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^9,x]
```

output
```
-1/280*(Sqrt[(a + b*x)^2]*(35*a^3 + 120*a^2*b*x + 140*a*b^2*x^2 + 56*b^3*x
^3))/(x^8*(a + b*x))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^9}dx$$

$$\Big\downarrow \ 1102$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^3(a+bx)^3}{x^9}dx}{b^3(a+bx)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^3}{x^9}dx}{a+bx}$$

$$\Big\downarrow \ 53$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a^3}{x^9}+\frac{3ba^2}{x^8}+\frac{3b^2a}{x^7}+\frac{b^3}{x^6}\right)dx}{a+bx}$$

$$\Big\downarrow \ 2009$$

$$\frac{\left(-\frac{a^3}{8x^8}-\frac{3a^2b}{7x^7}-\frac{ab^2}{2x^6}-\frac{b^3}{5x^5}\right)\sqrt{a^2+2abx+b^2x^2}}{a+bx}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^9,x]
```

output
```
((-1/8*a^3/x^8 - (3*a^2*b)/(7*x^7) - (a*b^2)/(2*x^6) - b^3/(5*x^5))*Sqrt[a
^2 + 2*a*b*x + b^2*x^2])/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{\sqrt{(bx+a)^2}\,\left(-\frac{1}{5}b^3x^3-\frac{1}{2}a\,b^2x^2-\frac{3}{7}a^2bx-\frac{1}{8}a^3\right)}{(bx+a)x^8}$	51
gosper	$-\frac{\left(56b^3x^3+140a\,b^2x^2+120a^2bx+35a^3\right)\left((bx+a)^2\right)^{\frac{3}{2}}}{280x^8(bx+a)^3}$	52
default	$-\frac{\left(56b^3x^3+140a\,b^2x^2+120a^2bx+35a^3\right)\left((bx+a)^2\right)^{\frac{3}{2}}}{280x^8(bx+a)^3}$	52
orering	$-\frac{\left(56b^3x^3+140a\,b^2x^2+120a^2bx+35a^3\right)\left(b^2x^2+2abx+a^2\right)^{\frac{3}{2}}}{280x^8(bx+a)^3}$	61

input
```
int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*(-1/5*b^3*x^3-1/2*a*b^2*x^2-3/7*a^2*b*x-1/8*a^3)
/x^8
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.23

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{x^9}\,dx = -\frac{56\,b^3x^3 + 140\,ab^2x^2 + 120\,a^2bx + 35\,a^3}{280\,x^8}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
-1/280*(56*b^3*x^3 + 140*a*b^2*x^2 + 120*a^2*b*x + 35*a^3)/x^8
```

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{x^9}\,dx = \int \frac{\left((a + bx)^2\right)^{\frac{3}{2}}}{x^9}\,dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/x**9,x)
```

output
```
Integral(((a + b*x)**2)**(3/2)/x**9, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(99) = 198$.

Time = 0.05 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.68

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^9}\,dx = \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^8}{4\,a^8} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^7}{4\,a^7x} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^6}{4\,a^8x^2} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^5}{4\,a^7x^3} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^4}{4\,a^6x^4} + \frac{69\,(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^3}{280\,a^5x^5} - \frac{13\,(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^2}{56\,a^4x^6} + \frac{11\,(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b}{56\,a^3x^7} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}}{8\,a^2x^8}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^8/a^8 + 1/4*(b^2*x^2 + 2*a*b*x + a^2
)^(3/2)*b^7/(a^7*x) - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*b^6/(a^8*x^2) +
1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*b^5/(a^7*x^3) - 1/4*(b^2*x^2 + 2*a*b*x
 + a^2)^(5/2)*b^4/(a^6*x^4) + 69/280*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*b^3/(
a^5*x^5) - 13/56*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*b^2/(a^4*x^6) + 11/56*(b^
2*x^2 + 2*a*b*x + a^2)^(5/2)*b/(a^3*x^7) - 1/8*(b^2*x^2 + 2*a*b*x + a^2)^(
5/2)/(a^2*x^8)
```

Giac [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.49

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^9}\,dx = -\frac{b^8\mathrm{sgn}(bx+a)}{280\,a^5} - \frac{56\,b^3x^3\mathrm{sgn}(bx+a)+140\,ab^2x^2\mathrm{sgn}(bx+a)+120\,a^2bx\mathrm{sgn}(bx+a)+35\,a^3\mathrm{sgn}(bx+a)}{280\,x^8}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^9,x, algorithm="giac")
```

output
```
-1/280*b^8*sgn(b*x + a)/a^5 - 1/280*(56*b^3*x^3*sgn(b*x + a) + 140*a*b^2*x
^2*sgn(b*x + a) + 120*a^2*b*x*sgn(b*x + a) + 35*a^3*sgn(b*x + a))/x^8
```

Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.89

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^9}\,dx = -\frac{a^3\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{8\,x^8\,(a+b\,x)} - \frac{b^3\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{5\,x^5\,(a+b\,x)} - \frac{a\,b^2\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{2\,x^6\,(a+b\,x)} - \frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{7\,x^7\,(a+b\,x)}$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/x^9,x)
```

output
```
- (a^3*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(8*x^8*(a + b*x)) - (b^3*(a^2 + b^
2*x^2 + 2*a*b*x)^(1/2))/(5*x^5*(a + b*x)) - (a*b^2*(a^2 + b^2*x^2 + 2*a*b*
x)^(1/2))/(2*x^6*(a + b*x)) - (3*a^2*b*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(7
*x^7*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^9}\,dx = \frac{-56b^3x^3-140a\,b^2x^2-120a^2bx-35a^3}{280x^8}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^9,x)
```

output
```
( - 35*a**3 - 120*a**2*b*x - 140*a*b**2*x**2 - 56*b**3*x**3)/(280*x**8)
```

3.72 $\int x^5(a^2+2abx+b^2x^2)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 231

$$\int x^5\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{a^5x^6\sqrt{a^2+2abx+b^2x^2}}{6(a+bx)}+\frac{5a^4bx^7\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{5a^3b^2x^8\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)}+\frac{10a^2b^3x^9\sqrt{a^2+2abx+b^2x^2}}{9(a+bx)}+\frac{ab^4x^{10}\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)}+\frac{b^5x^{11}\sqrt{a^2+2abx+b^2x^2}}{11(a+bx)}$$

output
```
a^5*x^6*((b*x+a)^2)^(1/2)/(6*b*x+6*a)+5*a^4*b*x^7*((b*x+a)^2)^(1/2)/(7*b*x
+7*a)+5*a^3*b^2*x^8*((b*x+a)^2)^(1/2)/(4*b*x+4*a)+10*a^2*b^3*x^9*((b*x+a)^
2)^(1/2)/(9*b*x+9*a)+a*b^4*x^10*((b*x+a)^2)^(1/2)/(2*b*x+2*a)+b^5*x^11*((b
*x+a)^2)^(1/2)/(11*b*x+11*a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.33

$$\int x^5 \left(a^2+2abx+b^2x^2\right)^{5/2} dx = \frac{x^6\sqrt{(a+bx)^2}(462a^5+1980a^4bx+3465a^3b^2x^2+3080a^2b^3x^3+1386ab^4x^4+252b^5x^5)}{2772(a+bx)}$$

input
```
Integrate[x^5*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(x^6*Sqrt[(a + b*x)^2]*(462*a^5 + 1980*a^4*b*x + 3465*a^3*b^2*x^2 + 3080*a
^2*b^3*x^3 + 1386*a*b^4*x^4 + 252*b^5*x^5))/(2772*(a + b*x))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.42, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a^2+2abx+b^2x^2\right)^{5/2} dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int b^5x^5(a+bx)^5dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^5(a+bx)^5dx}{a+bx}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(b^5x^{10}+5ab^4x^9+10a^2b^3x^8+10a^3b^2x^7+5a^4bx^6+a^5x^5\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{a^5x^6}{6}+\frac{5}{7}a^4bx^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{11}}{11}\right)}{a+bx}$$

input
```
Int[x^5*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((a^5*x^6)/6 + (5*a^4*b*x^7)/7 + (5*a^3*b^2
*x^8)/4 + (10*a^2*b^3*x^9)/9 + (a*b^4*x^10)/2 + (b^5*x^11)/11))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.32

method	result
gosper	$\frac{x^{6}\left(252b^{5}x^{5}+1386a\,b^{4}x^{4}+3080a^{2}b^{3}x^{3}+3465a^{3}b^{2}x^{2}+1980a^{4}bx+462a^{5}\right)\left(\left(bx+a\right)^{2}\right)^{\frac{5}{2}}}{2772\left(bx+a\right)^{5}}$
default	$\frac{x^{6}\left(252b^{5}x^{5}+1386a\,b^{4}x^{4}+3080a^{2}b^{3}x^{3}+3465a^{3}b^{2}x^{2}+1980a^{4}bx+462a^{5}\right)\left(\left(bx+a\right)^{2}\right)^{\frac{5}{2}}}{2772\left(bx+a\right)^{5}}$
orering	$\frac{x^{6}\left(252b^{5}x^{5}+1386a\,b^{4}x^{4}+3080a^{2}b^{3}x^{3}+3465a^{3}b^{2}x^{2}+1980a^{4}bx+462a^{5}\right)\left(b^{2}x^{2}+2abx+a^{2}\right)^{\frac{5}{2}}}{2772\left(bx+a\right)^{5}}$
risch	$\frac{b^{5}x^{11}\sqrt{\left(bx+a\right)^{2}}}{11bx+11a}+\frac{a\,b^{4}x^{10}\sqrt{\left(bx+a\right)^{2}}}{2bx+2a}+\frac{10\sqrt{\left(bx+a\right)^{2}}\,a^{2}b^{3}x^{9}}{9\left(bx+a\right)}+\frac{5\sqrt{\left(bx+a\right)^{2}}\,a^{3}b^{2}x^{8}}{4\left(bx+a\right)}+\frac{5\sqrt{\left(bx+a\right)^{2}}\,a^{4}b\,x^{7}}{7\left(bx+a\right)}+\frac{a^{5}x^{6}\sqrt{(bx+a}}{6bx+6a}$

input
```
int(x^5*(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2772*x^6*(252*b^5*x^5+1386*a*b^4*x^4+3080*a^2*b^3*x^3+3465*a^3*b^2*x^2+1
980*a^4*b*x+462*a^5)*((b*x+a)^2)^(5/2)/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.25

$$\int x^{5}\left(a^{2}+2abx+b^{2}x^{2}\right)^{5/2}\,dx=\frac{1}{11}\,b^{5}x^{11}+\frac{1}{2}\,ab^{4}x^{10}+\frac{10}{9}\,a^{2}b^{3}x^{9}+\frac{5}{4}\,a^{3}b^{2}x^{8}+\frac{5}{7}\,a^{4}bx^{7}+\frac{1}{6}\,a^{5}x^{6}$$

input
```
integrate(x^5*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/11*b^5*x^11 + 1/2*a*b^4*x^10 + 10/9*a^2*b^3*x^9 + 5/4*a^3*b^2*x^8 + 5/7*
a^4*b*x^7 + 1/6*a^5*x^6
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.19

$$\int x^5\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\begin{cases}\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^{10}}{2772b^6}+\frac{a^9x}{2772b^5}-\frac{a^8x^2}{2772b^4}+\frac{a^7x^3}{2772b^3}-\frac{a^6x^4}{2772b^2}+\frac{a^5x^5}{2772b}+\frac{461a^4x^6}{2772}+\frac{217a^3bx^7}{396}\right. \\ \frac{-\frac{a^{10}\left(a^2+2abx\right)^{\frac{7}{2}}}{7}+\frac{5a^8\left(a^2+2abx\right)^{\frac{9}{2}}}{9}-\frac{10a^6\left(a^2+2abx\right)^{\frac{11}{2}}}{11}+\frac{10a^4\left(a^2+2abx\right)^{\frac{13}{2}}}{13}-\frac{a^2\left(a^2+2abx\right)^{\frac{15}{2}}}{3}+\frac{\left(a^2+2abx\right)^{\frac{17}{2}}}{17}}{32a^6b^6} \\ \frac{x^6\left(a^2\right)^{\frac{5}{2}}}{6}\end{cases}$$

input
```
integrate(x**5*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(-a**10/(2772*b**6) + a**9*x/(
2772*b**5) - a**8*x**2/(2772*b**4) + a**7*x**3/(2772*b**3) - a**6*x**4/(27
72*b**2) + a**5*x**5/(2772*b) + 461*a**4*x**6/2772 + 217*a**3*b*x**7/396 +
 139*a**2*b**2*x**8/198 + 9*a*b**3*x**9/22 + b**4*x**10/11), Ne(b**2, 0)),
 ((-a**10*(a**2 + 2*a*b*x)**(7/2)/7 + 5*a**8*(a**2 + 2*a*b*x)**(9/2)/9 - 1
0*a**6*(a**2 + 2*a*b*x)**(11/2)/11 + 10*a**4*(a**2 + 2*a*b*x)**(13/2)/13 -
 a**2*(a**2 + 2*a*b*x)**(15/2)/3 + (a**2 + 2*a*b*x)**(17/2)/17)/(32*a**6*b
**6), Ne(a*b, 0)), (x**6*(a**2)**(5/2)/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.82

$$\int x^5\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}x^4}{11\,b^2}-\frac{3\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}ax^3}{22\,b^3}-\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}a^5x}{6\,b^5}+\frac{31\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}a^2x^2}{198\,b^4}-\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}a^6}{6\,b^6}-\frac{65\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}a^3x}{396\,b^5}+\frac{461\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}a^4}{2772\,b^6}$$

input
```
integrate(x^5*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/11*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*x^4/b^2 - 3/22*(b^2*x^2 + 2*a*b*x + a
^2)^(7/2)*a*x^3/b^3 - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^5*x/b^5 + 31/1
98*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*a^2*x^2/b^4 - 1/6*(b^2*x^2 + 2*a*b*x +
a^2)^(5/2)*a^6/b^6 - 65/396*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*a^3*x/b^5 + 46
1/2772*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*a^4/b^6
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.46

$$\int x^5(a^2+2abx+b^2x^2)^{5/2}\,dx=\frac{1}{11}b^5x^{11}\text{sgn}(bx+a)+\frac{1}{2}ab^4x^{10}\text{sgn}(bx+a)+\frac{10}{9}a^2b^3x^9\text{sgn}(bx+a)+\frac{5}{4}a^3b^2x^8\text{sgn}(bx+a)+\frac{5}{7}a^4bx^7\text{sgn}(bx+a)+\frac{1}{6}a^5x^6\text{sgn}(bx+a)-\frac{a^{11}\text{sgn}(bx+a)}{2772\,b^6}$$

input
```
integrate(x^5*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/11*b^5*x^11*sgn(b*x + a) + 1/2*a*b^4*x^10*sgn(b*x + a) + 10/9*a^2*b^3*x^
9*sgn(b*x + a) + 5/4*a^3*b^2*x^8*sgn(b*x + a) + 5/7*a^4*b*x^7*sgn(b*x + a)
 + 1/6*a^5*x^6*sgn(b*x + a) - 1/2772*a^11*sgn(b*x + a)/b^6
```

Mupad [F(-1)]

Timed out.

$$\int x^5(a^2+2abx+b^2x^2)^{5/2}\,dx=\int x^5\,{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}^{5/2}\,dx$$

input
```
int(x^5*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int(x^5*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.25

$$\int x^5 \left(a^2+2abx+b^2x^2\right)^{5/2}\,dx = \frac{x^6 (252b^5x^5+1386a\,b^4x^4+3080a^2b^3x^3+3465a^3b^2x^2+1980a^4bx+462a^5)}{2772}$$

input
```
int(x^5*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(x**6*(462*a**5 + 1980*a**4*b*x + 3465*a**3*b**2*x**2 + 3080*a**2*b**3*x**
3 + 1386*a*b**4*x**4 + 252*b**5*x**5))/2772
```

3.73 $\int x^4(a^2+2abx+b^2x^2)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 231

$$\int x^4\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{a^5x^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}+\frac{5a^4bx^6\sqrt{a^2+2abx+b^2x^2}}{6(a+bx)}+\frac{10a^3b^2x^7\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{5a^2b^3x^8\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)}+\frac{5ab^4x^9\sqrt{a^2+2abx+b^2x^2}}{9(a+bx)}+\frac{b^5x^{10}\sqrt{a^2+2abx+b^2x^2}}{10(a+bx)}$$

output
```
a^5*x^5*((b*x+a)^2)^(1/2)/(5*b*x+5*a)+5*a^4*b*x^6*((b*x+a)^2)^(1/2)/(6*b*x
+6*a)+10*a^3*b^2*x^7*((b*x+a)^2)^(1/2)/(7*b*x+7*a)+5*a^2*b^3*x^8*((b*x+a)^
2)^(1/2)/(4*b*x+4*a)+5*a*b^4*x^9*((b*x+a)^2)^(1/2)/(9*b*x+9*a)+b^5*x^10*((
b*x+a)^2)^(1/2)/(10*b*x+10*a)
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.54

$$\int x^4\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{x^5(252a^5+1050a^4bx+1800a^3b^2x^2+1575a^2b^3x^3+700ab^4x^4+126b^5x^5)\left(\sqrt{a^2}bx+a\left(\sqrt{a^{\cdots}}\right.\right.}{1260\left(-a^2-abx+\sqrt{a^2}\sqrt{(a+bx)^2}\right)}$$

input
```
Integrate[x^4*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(x^5*(252*a^5 + 1050*a^4*b*x + 1800*a^3*b^2*x^2 + 1575*a^2*b^3*x^3 + 700*a
*b^4*x^4 + 126*b^5*x^5)*(Sqrt[a^2]*b*x + a*(Sqrt[a^2] - Sqrt[(a + b*x)^2])
))/(1260*(-a^2 - a*b*x + Sqrt[a^2]*Sqrt[(a + b*x)^2]))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.42, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1101, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx$$

$$\Big\downarrow \text{1101}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(b(a+bx)^9-4ab(a+bx)^8+6a^2b(a+bx)^7-4a^3b(a+bx)^6+a^4b(a+bx)^5\right)dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{6}a^4(a+bx)^6-\frac{4}{7}a^3(a+bx)^7+\frac{3}{4}a^2(a+bx)^8+\frac{1}{10}(a+bx)^{10}-\frac{4}{9}a(a+bx)^9\right)}{b^5(a+bx)}$$

input
```
Int[x^4*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((a^4*(a + b*x)^6)/6 - (4*a^3*(a + b*x)^7)/
7 + (3*a^2*(a + b*x)^8)/4 - (4*a*(a + b*x)^9)/9 + (a + b*x)^10/10))/(b^5*(
a + b*x))
```

Defintions of rubi rules used

rule 1101
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a
 + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   In
t[ExpandLinearProduct[(b/2 + c*x)^(2*p), x^m, b/2, c, x], x], x] /; FreeQ[{
a, b, c, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && IGtQ[m, 0] && EqQ[m - 2*p + 1,
 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.32

method	result
gosper	$\frac{x^{5} \left(126 b^{5} x^{5}+700 a \,b^{4} x^{4}+1575 a^{2} b^{3} x^{3}+1800 a^{3} b^{2} x^{2}+1050 a^{4} b x +252 a^{5}\right) \left(\left(b x +a \right)^{2}\right)^{\frac{5}{2}}}{1260 \left(b x +a \right)^{5}}$
default	$\frac{x^{5} \left(126 b^{5} x^{5}+700 a \,b^{4} x^{4}+1575 a^{2} b^{3} x^{3}+1800 a^{3} b^{2} x^{2}+1050 a^{4} b x +252 a^{5}\right) \left(\left(b x +a \right)^{2}\right)^{\frac{5}{2}}}{1260 \left(b x +a \right)^{5}}$
orering	$\frac{x^{5} \left(126 b^{5} x^{5}+700 a \,b^{4} x^{4}+1575 a^{2} b^{3} x^{3}+1800 a^{3} b^{2} x^{2}+1050 a^{4} b x +252 a^{5}\right) \left(b^{2} x^{2}+2 a b x +a^{2}\right)^{\frac{5}{2}}}{1260 \left(b x +a \right)^{5}}$
risch	$\frac{b^{5} x^{10} \sqrt{\left(b x +a \right)^{2}}}{10 b x +10 a}+\frac{5 \sqrt{\left(b x +a \right)^{2}}\, a \,b^{4} x^{9}}{9 \left(b x +a \right)}+\frac{5 \sqrt{\left(b x +a \right)^{2}}\, a^{2} b^{3} x^{8}}{4 \left(b x +a \right)}+\frac{10 \sqrt{\left(b x +a \right)^{2}}\, a^{3} b^{2} x^{7}}{7 \left(b x +a \right)}+\frac{5 \sqrt{\left(b x +a \right)^{2}}\, a^{4} b \,x^{6}}{6 \left(b x +a \right)}+\frac{a^{5} x^{5} \sqrt{\left(b x +a \right)^{2}}}{5 b x +5 a}$

input
```
int(x^4*(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/1260*x^5*(126*b^5*x^5+700*a*b^4*x^4+1575*a^2*b^3*x^3+1800*a^3*b^2*x^2+10
50*a^4*b*x+252*a^5)*((b*x+a)^2)^(5/2)/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.25

$$\int x^4 \left(a^2+2abx+b^2x^2\right)^{5/2}\,dx = \frac{1}{10}\,b^5x^{10}+\frac{5}{9}\,ab^4x^9 + \frac{5}{4}\,a^2b^3x^8+\frac{10}{7}\,a^3b^2x^7+\frac{5}{6}\,a^4bx^6+\frac{1}{5}\,a^5x^5$$

input
```
integrate(x^4*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/10*b^5*x^10 + 5/9*a*b^4*x^9 + 5/4*a^2*b^3*x^8 + 10/7*a^3*b^2*x^7 + 5/6*a
^4*b*x^6 + 1/5*a^5*x^5
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.06

$$\int x^4 \left(a^2+2abx +b^2x^2\right)^{5/2}\,dx = \begin{cases} \sqrt{a^2+2abx+b^2x^2}\left(\frac{a^9}{1260b^5}-\frac{a^8x}{1260b^4}+\frac{a^7x^2}{1260b^3}-\frac{a^6x^3}{1260b^2}+\frac{a^5x^4}{1260b}+\frac{251a^4x^5}{1260}+\frac{799a^3bx^6}{1260}+\frac{143a^2b^2x}{180}\right. \\ \frac{\frac{a^8\left(a^2+2abx\right)^{\frac{7}{2}}}{7}-\frac{4a^6\left(a^2+2abx\right)^{\frac{9}{2}}}{9}+\frac{6a^4\left(a^2+2abx\right)^{\frac{11}{2}}}{11}-\frac{4a^2\left(a^2+2abx\right)^{\frac{13}{2}}}{13}+\frac{\left(a^2+2abx\right)^{\frac{15}{2}}}{15}}{16a^5b^5} \\ \frac{x^5\left(a^2\right)^{\frac{5}{2}}}{5} \end{cases}$$

input
```
integrate(x**4*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(a**9/(1260*b**5) - a**8*x/(12
60*b**4) + a**7*x**2/(1260*b**3) - a**6*x**3/(1260*b**2) + a**5*x**4/(1260
*b) + 251*a**4*x**5/1260 + 799*a**3*b*x**6/1260 + 143*a**2*b**2*x**7/180 +
 41*a*b**3*x**8/90 + b**4*x**9/10), Ne(b**2, 0)), ((a**8*(a**2 + 2*a*b*x)*
*(7/2)/7 - 4*a**6*(a**2 + 2*a*b*x)**(9/2)/9 + 6*a**4*(a**2 + 2*a*b*x)**(11
/2)/11 - 4*a**2*(a**2 + 2*a*b*x)**(13/2)/13 + (a**2 + 2*a*b*x)**(15/2)/15)
/(16*a**5*b**5), Ne(a*b, 0)), (x**5*(a**2)**(5/2)/5, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.69

$$\int x^4\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}x^3}{10\,b^2}+\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}a^4x}{6\,b^4}-\frac{13\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}ax^2}{90\,b^3}+\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}a^5}{6\,b^5}+\frac{29\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}a^2x}{180\,b^4}-\frac{209\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}a^3}{1260\,b^5}$$

input
```
integrate(x^4*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/10*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*x^3/b^2 + 1/6*(b^2*x^2 + 2*a*b*x + a^
2)^(5/2)*a^4*x/b^4 - 13/90*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*a*x^2/b^3 + 1/6
*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^5/b^5 + 29/180*(b^2*x^2 + 2*a*b*x + a^2
)^(7/2)*a^2*x/b^4 - 209/1260*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*a^3/b^5
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.46

$$\int x^4\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{1}{10}\,b^5x^{10}\mathrm{sgn}(bx+a)+\frac{5}{9}\,ab^4x^9\mathrm{sgn}(bx+a)+\frac{5}{4}\,a^2b^3x^8\mathrm{sgn}(bx+a)+\frac{10}{7}\,a^3b^2x^7\mathrm{sgn}(bx+a)+\frac{5}{6}\,a^4bx^6\mathrm{sgn}(bx+a)+\frac{1}{5}\,a^5x^5\mathrm{sgn}(bx+a)+\frac{a^{10}\mathrm{sgn}(bx+a)}{1260\,b^5}$$

input
```
integrate(x^4*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/10*b^5*x^10*sgn(b*x + a) + 5/9*a*b^4*x^9*sgn(b*x + a) + 5/4*a^2*b^3*x^8*
sgn(b*x + a) + 10/7*a^3*b^2*x^7*sgn(b*x + a) + 5/6*a^4*b*x^6*sgn(b*x + a)
+ 1/5*a^5*x^5*sgn(b*x + a) + 1/1260*a^10*sgn(b*x + a)/b^5
```

Mupad [F(-1)]

Timed out.

$$\int x^4(a^2+2abx+b^2x^2)^{5/2}\,dx=\int x^4\,(a^2+2\,a\,b\,x+b^2\,x^2)^{5/2}\,dx$$

input
```
int(x^4*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int(x^4*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.25

$$\int x^4(a^2+2abx+b^2x^2)^{5/2}\,dx=\frac{x^5(126b^5x^5+700a\,b^4x^4+1575a^2b^3x^3+1800a^3b^2x^2+1050a^4bx+252a^5)}{1260}$$

input
```
int(x^4*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(x**5*(252*a**5 + 1050*a**4*b*x + 1800*a**3*b**2*x**2 + 1575*a**2*b**3*x**
3 + 700*a*b**4*x**4 + 126*b**5*x**5))/1260
```

3.74 $\int x^3(a^2+2abx+b^2x^2)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 144

$$\int x^3\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=-\frac{a^3(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6b^4}+\frac{3a^2(a+bx)^6\sqrt{a^2+2abx+b^2x^2}}{7b^4}-\frac{3a(a+bx)^7\sqrt{a^2+2abx+b^2x^2}}{8b^4}+\frac{(a+bx)^8\sqrt{a^2+2abx+b^2x^2}}{9b^4}$$

output
```
-1/6*a^3*(b*x+a)^5*((b*x+a)^2)^(1/2)/b^4+3/7*a^2*(b*x+a)^6*((b*x+a)^2)^(1/
2)/b^4-3/8*a*(b*x+a)^7*((b*x+a)^2)^(1/2)/b^4+1/9*(b*x+a)^8*((b*x+a)^2)^(1/
2)/b^4
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.87

$$\int x^3\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{x^4(126a^5+504a^4bx+840a^3b^2x^2+720a^2b^3x^3+315ab^4x^4+56b^5x^5)\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}-\sqrt{}\right.\right.}{504\left(-a^2-abx+\sqrt{a^2}\sqrt{(a+bx)^2}\right)}$$

input
```
Integrate[x^3*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(x^4*(126*a^5 + 504*a^4*b*x + 840*a^3*b^2*x^2 + 720*a^2*b^3*x^3 + 315*a*b^
4*x^4 + 56*b^5*x^5)*(Sqrt[a^2]*b*x + a*(Sqrt[a^2] - Sqrt[(a + b*x)^2])))/(
504*(-a^2 - a*b*x + Sqrt[a^2]*Sqrt[(a + b*x)^2]))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.64, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a^2+2abx+b^2x^2\right)^{5/2} dx$$

$$\Big\downarrow \text{ 1102}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int b^5x^3(a+bx)^5dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^3(a+bx)^5dx}{a+bx}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{(a+bx)^8}{b^3}-\frac{3a(a+bx)^7}{b^3}+\frac{3a^2(a+bx)^6}{b^3}-\frac{a^3(a+bx)^5}{b^3}\right)dx}{a+bx}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^3(a+bx)^6}{6b^4}+\frac{3a^2(a+bx)^7}{7b^4}+\frac{(a+bx)^9}{9b^4}-\frac{3a(a+bx)^8}{8b^4}\right)}{a+bx}$$

input
```
Int[x^3*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/6*(a^3*(a + b*x)^6)/b^4 + (3*a^2*(a + b
*x)^7)/(7*b^4) - (3*a*(a + b*x)^8)/(8*b^4) + (a + b*x)^9/(9*b^4)))/(a + b*
x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.51

method	result
gosper	$\frac{x^{4} \left(56 b^{5} x^{5}+315 a \,b^{4} x^{4}+720 a^{2} b^{3} x^{3}+840 a^{3} b^{2} x^{2}+504 a^{4} b x +126 a^{5}\right) \left(\left(b x +a \right)^{2}\right)^{\frac{5}{2}}}{504 \left(b x +a \right)^{5}}$
default	$\frac{x^{4} \left(56 b^{5} x^{5}+315 a \,b^{4} x^{4}+720 a^{2} b^{3} x^{3}+840 a^{3} b^{2} x^{2}+504 a^{4} b x +126 a^{5}\right) \left(\left(b x +a \right)^{2}\right)^{\frac{5}{2}}}{504 \left(b x +a \right)^{5}}$
orering	$\frac{x^{4} \left(56 b^{5} x^{5}+315 a \,b^{4} x^{4}+720 a^{2} b^{3} x^{3}+840 a^{3} b^{2} x^{2}+504 a^{4} b x +126 a^{5}\right) \left(b^{2} x^{2}+2 a b x +a^{2}\right)^{\frac{5}{2}}}{504 \left(b x +a \right)^{5}}$
risch	$\frac{\sqrt{\left(b x +a \right)^{2}}\, b^{5} x^{9}}{9 b x +9 a}+\frac{5 \sqrt{\left(b x +a \right)^{2}}\, a \,b^{4} x^{8}}{8 \left(b x +a \right)}+\frac{10 \sqrt{\left(b x +a \right)^{2}}\, a^{2} b^{3} x^{7}}{7 \left(b x +a \right)}+\frac{5 \sqrt{\left(b x +a \right)^{2}}\, a^{3} b^{2} x^{6}}{3 \left(b x +a \right)}+\frac{\sqrt{\left(b x +a \right)^{2}}\, a^{4} b \,x^{5}}{b x +a}+\frac{\sqrt{\left(b x +a \right)^{2}}\, a^{5} x}{4 b x +4 a}$

input
```
int(x^3*(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/504*x^4*(56*b^5*x^5+315*a*b^4*x^4+720*a^2*b^3*x^3+840*a^3*b^2*x^2+504*a^
4*b*x+126*a^5)*((b*x+a)^2)^(5/2)/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.39

$$\int x^3 \left(a^2+2abx+b^2x^2\right)^{5/2}\,dx = \frac{1}{9}\,b^5x^9+\frac{5}{8}\,ab^4x^8+\frac{10}{7}\,a^2b^3x^7+\frac{5}{3}\,a^3b^2x^6+a^4bx^5+\frac{1}{4}\,a^5x^4$$

input
```
integrate(x^3*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/9*b^5*x^9 + 5/8*a*b^4*x^8 + 10/7*a^2*b^3*x^7 + 5/3*a^3*b^2*x^6 + a^4*b*x
^5 + 1/4*a^5*x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(99) = 198.

Time = 0.66 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.47

$$\int x^3 \left(a^2+2abx+b^2x^2\right)^{5/2}\,dx = \begin{cases} \sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^8}{504b^4}+\frac{a^7x}{504b^3}-\frac{a^6x^2}{504b^2}+\frac{a^5x^3}{504b}+\frac{125a^4x^4}{504}+\frac{379a^3bx^5}{504}+\frac{461a^2b^2x^6}{504}+\frac{37ab^3x^7}{72}\right. \\ \frac{-\frac{a^6\left(a^2+2abx\right)^{\frac{7}{2}}}{7}+\frac{a^4\left(a^2+2abx\right)^{\frac{9}{2}}}{3}-\frac{3a^2\left(a^2+2abx\right)^{\frac{11}{2}}}{11}+\frac{\left(a^2+2abx\right)^{\frac{13}{2}}}{13}}{8a^4b^4} \\ \frac{x^4\left(a^2\right)^{\frac{5}{2}}}{4} \end{cases}$$

input
```
integrate(x**3*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(-a**8/(504*b**4) + a**7*x/(50
4*b**3) - a**6*x**2/(504*b**2) + a**5*x**3/(504*b) + 125*a**4*x**4/504 + 3
79*a**3*b*x**5/504 + 461*a**2*b**2*x**6/504 + 37*a*b**3*x**7/72 + b**4*x**
8/9), Ne(b**2, 0)), ((-a**6*(a**2 + 2*a*b*x)**(7/2)/7 + a**4*(a**2 + 2*a*b
*x)**(9/2)/3 - 3*a**2*(a**2 + 2*a*b*x)**(11/2)/11 + (a**2 + 2*a*b*x)**(13/
2)/13)/(8*a**4*b**4), Ne(a*b, 0)), (x**4*(a**2)**(5/2)/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.91

$$\int x^3 \left(a^2+2abx+b^2x^2\right)^{5/2} \, dx = -\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}a^3x}{6\,b^3} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}x^2}{9\,b^2} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}a^4}{6\,b^4} - \frac{11\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}ax}{72\,b^3} + \frac{83\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}a^2}{504\,b^4}$$

input
```
integrate(x^3*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^3*x/b^3 + 1/9*(b^2*x^2 + 2*a*b*x +
a^2)^(7/2)*x^2/b^2 - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^4/b^4 - 11/72*(
b^2*x^2 + 2*a*b*x + a^2)^(7/2)*a*x/b^3 + 83/504*(b^2*x^2 + 2*a*b*x + a^2)^
(7/2)*a^2/b^4
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.74

$$\int x^3 \left(a^2+2abx+b^2x^2\right)^{5/2} \, dx = \frac{1}{9}\,b^5x^9\mathrm{sgn}(bx+a) + \frac{5}{8}\,ab^4x^8\mathrm{sgn}(bx+a) + \frac{10}{7}\,a^2b^3x^7\mathrm{sgn}(bx+a) + \frac{5}{3}\,a^3b^2x^6\mathrm{sgn}(bx+a) + a^4bx^5\mathrm{sgn}(bx+a) + \frac{1}{4}\,a^5x^4\mathrm{sgn}(bx+a) - \frac{a^9\mathrm{sgn}(bx+a)}{504\,b^4}$$

input
```
integrate(x^3*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/9*b^5*x^9*sgn(b*x + a) + 5/8*a*b^4*x^8*sgn(b*x + a) + 10/7*a^2*b^3*x^7*s
gn(b*x + a) + 5/3*a^3*b^2*x^6*sgn(b*x + a) + a^4*b*x^5*sgn(b*x + a) + 1/4*
a^5*x^4*sgn(b*x + a) - 1/504*a^9*sgn(b*x + a)/b^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (a^2 + 2abx + b^2x^2)^{5/2} \, dx = \int x^3 \left(a^2 + 2\,a\,b\,x + b^2\,x^2\right)^{5/2} dx$$

input
```
int(x^3*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int(x^3*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.40

$$\int x^3 (a^2 + 2abx + b^2x^2)^{5/2} \, dx = \frac{x^4(56b^5x^5 + 315a\,b^4x^4 + 720a^2b^3x^3 + 840a^3b^2x^2 + 504a^4bx + 126a^5)}{504}$$

input
```
int(x^3*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(x**4*(126*a**5 + 504*a**4*b*x + 840*a**3*b**2*x**2 + 720*a**2*b**3*x**3 +
 315*a*b**4*x**4 + 56*b**5*x**5))/504
```

3.75 $\int x^2(a^2 + 2abx + b^2x^2)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 107

$$\int x^2\left(a^2+2abx+b^2x^2\right)^{5/2}dx=\frac{a^2(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6b^3}-\frac{2a(a+bx)^6\sqrt{a^2+2abx+b^2x^2}}{7b^3}+\frac{(a+bx)^7\sqrt{a^2+2abx+b^2x^2}}{8b^3}$$

output
```
1/6*a^2*(b*x+a)^5*((b*x+a)^2)^(1/2)/b^3-2/7*a*(b*x+a)^6*((b*x+a)^2)^(1/2)/
b^3+1/8*(b*x+a)^7*((b*x+a)^2)^(1/2)/b^3
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.17

$$\int x^2\left(a^2+2abx+b^2x^2\right)^{5/2}dx=\frac{x^3(56a^5+210a^4bx+336a^3b^2x^2+280a^2b^3x^3+120ab^4x^4+21b^5x^5)\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}-\sqrt{\right.\right.}}{168\left(-a^2-abx+\sqrt{a^2}\sqrt{(a+bx)^2}\right)}$$

input
```
Integrate[x^2*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(x^3*(56*a^5 + 210*a^4*b*x + 336*a^3*b^2*x^2 + 280*a^2*b^3*x^3 + 120*a*b^4
*x^4 + 21*b^5*x^5)*(Sqrt[a^2]*b*x + a*(Sqrt[a^2] - Sqrt[(a + b*x)^2])))/(1
68*(-a^2 - a*b*x + Sqrt[a^2]*Sqrt[(a + b*x)^2]))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a^2+2abx+b^2x^2\right)^{5/2} dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int b^5x^2(a+bx)^5dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^2(a+bx)^5dx}{a+bx}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{(a+bx)^7}{b^2}-\frac{2a(a+bx)^6}{b^2}+\frac{a^2(a+bx)^5}{b^2}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{a^2(a+bx)^6}{6b^3}+\frac{(a+bx)^8}{8b^3}-\frac{2a(a+bx)^7}{7b^3}\right)}{a+bx}$$

input
```
Int[x^2*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((a^2*(a + b*x)^6)/(6*b^3) - (2*a*(a + b*x)
^7)/(7*b^3) + (a + b*x)^8/(8*b^3)))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.69

method	result
gosper	$\frac{x^{3}\left(21 b^{5} x^{5}+120 a\, b^{4} x^{4}+280 a^{2} b^{3} x^{3}+336 a^{3} b^{2} x^{2}+210 a^{4} b x+56 a^{5}\right)\left(\left(b x+a\right)^{2}\right)^{\frac{5}{2}}}{168 \left(b x+a\right)^{5}}$
default	$\frac{x^{3}\left(21 b^{5} x^{5}+120 a\, b^{4} x^{4}+280 a^{2} b^{3} x^{3}+336 a^{3} b^{2} x^{2}+210 a^{4} b x+56 a^{5}\right)\left(\left(b x+a\right)^{2}\right)^{\frac{5}{2}}}{168 \left(b x+a\right)^{5}}$
orering	$\frac{x^{3}\left(21 b^{5} x^{5}+120 a\, b^{4} x^{4}+280 a^{2} b^{3} x^{3}+336 a^{3} b^{2} x^{2}+210 a^{4} b x+56 a^{5}\right)\left(b^{2} x^{2}+2 a b x+a^{2}\right)^{\frac{5}{2}}}{168 \left(b x+a\right)^{5}}$
risch	$\frac{\sqrt{\left(b x+a\right)^{2}}\, b^{5} x^{8}}{8 b x+8 a}+\frac{5 \sqrt{\left(b x+a\right)^{2}}\, a\, b^{4} x^{7}}{7 \left(b x+a\right)}+\frac{5 \sqrt{\left(b x+a\right)^{2}}\, a^{2} x^{6} b^{3}}{3 \left(b x+a\right)}+\frac{2 \sqrt{\left(b x+a\right)^{2}}\, a^{3} b^{2} x^{5}}{b x+a}+\frac{5 \sqrt{\left(b x+a\right)^{2}}\, a^{4} b\, x^{4}}{4 \left(b x+a\right)}+\frac{\sqrt{\left(b x+a\right)^{2}}\, a^{5} x}{3 b x+3 a}$

input
```
int(x^2*(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/168*x^3*(21*b^5*x^5+120*a*b^4*x^4+280*a^2*b^3*x^3+336*a^3*b^2*x^2+210*a^
4*b*x+56*a^5)*((b*x+a)^2)^(5/2)/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.53

$$\int x^2 (a^2 + 2abx + b^2x^2)^{5/2}\, dx = \frac{1}{8}\, b^5x^8 + \frac{5}{7}\, ab^4x^7 + \frac{5}{3}\, a^2b^3x^6 + 2\, a^3b^2x^5 + \frac{5}{4}\, a^4bx^4 + \frac{1}{3}\, a^5x^3$$

input
```
integrate(x^2*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/8*b^5*x^8 + 5/7*a*b^4*x^7 + 5/3*a^2*b^3*x^6 + 2*a^3*b^2*x^5 + 5/4*a^4*b*
x^4 + 1/3*a^5*x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(71) = 142$.

Time = 0.59 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.68

$$\int x^2 (a^2 + 2abx + b^2x^2)^{5/2}\, dx = \begin{cases} \sqrt{a^2 + 2abx + b^2x^2} \left(\frac{a^7}{168b^3} - \frac{a^6x}{168b^2} + \frac{a^5x^2}{168b} + \frac{55a^4x^3}{168} + \frac{155a^3bx^4}{168} + \frac{181a^2b^2x^5}{168} + \frac{33ab^3x^6}{56} + \frac{b^4x^7}{8}\right) \\ \frac{\frac{a^4(a^2+2abx)^{\frac{7}{2}}}{7} - \frac{2a^2(a^2+2abx)^{\frac{9}{2}}}{9} + \frac{(a^2+2abx)^{\frac{11}{2}}}{11}}{4a^3b^3} \\ \frac{x^3(a^2)^{\frac{5}{2}}}{3} \end{cases}$$

input
```
integrate(x**2*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(a**7/(168*b**3) - a**6*x/(168
*b**2) + a**5*x**2/(168*b) + 55*a**4*x**3/168 + 155*a**3*b*x**4/168 + 181*
a**2*b**2*x**5/168 + 33*a*b**3*x**6/56 + b**4*x**7/8), Ne(b**2, 0)), ((a**
4*(a**2 + 2*a*b*x)**(7/2)/7 - 2*a**2*(a**2 + 2*a*b*x)**(9/2)/9 + (a**2 + 2
*a*b*x)**(11/2)/11)/(4*a**3*b**3), Ne(a*b, 0)), (x**3*(a**2)**(5/2)/3, Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.95

$$\int x^2\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}a^2x}{6\,b^2}+\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}a^3}{6\,b^3}+\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}x}{8\,b^2}-\frac{9\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}a}{56\,b^3}$$

input
```
integrate(x^2*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^2*x/b^2 + 1/6*(b^2*x^2 + 2*a*b*x + a
^2)^(5/2)*a^3/b^3 + 1/8*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*x/b^2 - 9/56*(b^2*
x^2 + 2*a*b*x + a^2)^(7/2)*a/b^3
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

$$\int x^2\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{1}{8}\,b^5x^8\mathrm{sgn}(bx+a)+\frac{5}{7}\,ab^4x^7\mathrm{sgn}(bx+a)+\frac{5}{3}\,a^2b^3x^6\mathrm{sgn}(bx+a)+2\,a^3b^2x^5\mathrm{sgn}(bx+a)+\frac{5}{4}\,a^4bx^4\mathrm{sgn}(bx+a)+\frac{1}{3}\,a^5x^3\mathrm{sgn}(bx+a)+\frac{a^8\mathrm{sgn}(bx+a)}{168\,b^3}$$

input
```
integrate(x^2*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/8*b^5*x^8*sgn(b*x + a) + 5/7*a*b^4*x^7*sgn(b*x + a) + 5/3*a^2*b^3*x^6*sg
n(b*x + a) + 2*a^3*b^2*x^5*sgn(b*x + a) + 5/4*a^4*b*x^4*sgn(b*x + a) + 1/3
*a^5*x^3*sgn(b*x + a) + 1/168*a^8*sgn(b*x + a)/b^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (a^2 + 2abx + b^2x^2)^{5/2} \, dx = \int x^2 \left(a^2 + 2\,a\,b\,x + b^2\,x^2\right)^{5/2} dx$$

input
```
int(x^2*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int(x^2*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.53

$$\int x^2 (a^2 + 2abx + b^2x^2)^{5/2} \, dx = \frac{x^3(21b^5x^5 + 120a\,b^4x^4 + 280a^2b^3x^3 + 336a^3b^2x^2 + 210a^4bx + 56a^5)}{168}$$

input
```
int(x^2*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(x**3*(56*a**5 + 210*a**4*b*x + 336*a**3*b**2*x**2 + 280*a**2*b**3*x**3 +
120*a*b**4*x**4 + 21*b**5*x**5))/168
```

3.76 $\int x(a^2+2abx+b^2x^2)^{5/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int x\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=-\frac{a(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6b^2}+\frac{\left(a^2+2abx+b^2x^2\right)^{7/2}}{7b^2}$$

output
```
-1/6*a*(b*x+a)^5*((b*x+a)^2)^(1/2)/b^2+1/7*(b^2*x^2+2*a*b*x+a^2)^(7/2)/b^2
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.98

$$\int x\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{x^2(21a^5+70a^4bx+105a^3b^2x^2+84a^2b^3x^3+35ab^4x^4+6b^5x^5)\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}-\sqrt{(a+\right.\right.}{42\left(-a^2-abx+\sqrt{a^2}\sqrt{(a+bx)^2}\right)}$$

input
```
Integrate[x*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(x^2*(21*a^5 + 70*a^4*b*x + 105*a^3*b^2*x^2 + 84*a^2*b^3*x^3 + 35*a*b^4*x^
4 + 6*b^5*x^5)*(Sqrt[a^2]*b*x + a*(Sqrt[a^2] - Sqrt[(a + b*x)^2])))/(42*(-
a^2 - a*b*x + Sqrt[a^2]*Sqrt[(a + b*x)^2]))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a^2+2abx+b^2x^2\right)^{5/2}dx$$

$$\Big\downarrow \text{1100}$$

$$\frac{\left(a^2+2abx+b^2x^2\right)^{7/2}}{7b^2}-\frac{a\int \left(a^2+2bxa+b^2x^2\right)^{5/2}dx}{b}$$

$$\Big\downarrow \text{1079}$$

$$\frac{\left(a^2+2abx+b^2x^2\right)^{7/2}}{7b^2}-\frac{a\sqrt{a^2+2abx+b^2x^2}\int \left(xb^2+ab\right)^5dx}{b^6(a+bx)}$$

$$\Big\downarrow \text{17}$$

$$\frac{\left(a^2+2abx+b^2x^2\right)^{7/2}}{7b^2}-\frac{a(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6b^2}$$

input
```
Int[x*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
-1/6*(a*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/b^2 + (a^2 + 2*a*b*x +
b^2*x^2)^(7/2)/(7*b^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.17

method	result
gosper	$\frac{x^2\left(6b^5x^5+35a\,b^4x^4+84a^2b^3x^3+105a^3b^2x^2+70a^4bx+21a^5\right)\left((bx+a)^2\right)^{\frac{5}{2}}}{42(bx+a)^5}$
default	$\frac{x^2\left(6b^5x^5+35a\,b^4x^4+84a^2b^3x^3+105a^3b^2x^2+70a^4bx+21a^5\right)\left((bx+a)^2\right)^{\frac{5}{2}}}{42(bx+a)^5}$
orering	$\frac{x^2\left(6b^5x^5+35a\,b^4x^4+84a^2b^3x^3+105a^3b^2x^2+70a^4bx+21a^5\right)\left(b^2x^2+2abx+a^2\right)^{\frac{5}{2}}}{42(bx+a)^5}$
risch	$\frac{\sqrt{(bx+a)^2}\,b^5x^7}{7bx+7a}+\frac{5\sqrt{(bx+a)^2}\,a\,b^4x^6}{6(bx+a)}+\frac{2\sqrt{(bx+a)^2}\,a^2b^3x^5}{bx+a}+\frac{5\sqrt{(bx+a)^2}\,a^3x^4b^2}{2(bx+a)}+\frac{5\sqrt{(bx+a)^2}\,a^4b\,x^3}{3(bx+a)}+\frac{\sqrt{(bx+a)^2}\,a^5x}{2bx+2a}$

input
```
int(x*(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/42*x^2*(6*b^5*x^5+35*a*b^4*x^4+84*a^2*b^3*x^3+105*a^3*b^2*x^2+70*a^4*b*x
+21*a^5)*((b*x+a)^2)^(5/2)/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int x\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{1}{7}\,b^5x^7+\frac{5}{6}\,ab^4x^6+2\,a^2b^3x^5+\frac{5}{2}\,a^3b^2x^4+\frac{5}{3}\,a^4bx^3+\frac{1}{2}\,a^5x^2$$

input
```
integrate(x*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/7*b^5*x^7 + 5/6*a*b^4*x^6 + 2*a^2*b^3*x^5 + 5/2*a^3*b^2*x^4 + 5/3*a^4*b*
x^3 + 1/2*a^5*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(48) = 96.

Time = 0.55 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.35

$$\int x\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\begin{cases}\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^6}{42b^2}+\frac{a^5x}{42b}+\frac{10a^4x^2}{21}+\frac{25a^3bx^3}{21}+\frac{55a^2b^2x^4}{42}+\frac{29ab^3x^5}{42}+\frac{b^4x^6}{7}\right) & \text{for } b^2\neq \\ \frac{-\frac{a^2\left(a^2+2abx\right)^{\frac{7}{2}}}{7}+\frac{\left(a^2+2abx\right)^{\frac{9}{2}}}{9}}{2a^2b^2} & \text{for } ab\neq \\ \frac{x^2\left(a^2\right)^{\frac{5}{2}}}{2} & \text{otherwis}\end{cases}$$

input
```
integrate(x*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(-a**6/(42*b**2) + a**5*x/(42*
b) + 10*a**4*x**2/21 + 25*a**3*b*x**3/21 + 55*a**2*b**2*x**4/42 + 29*a*b**
3*x**5/42 + b**4*x**6/7), Ne(b**2, 0)), ((-a**2*(a**2 + 2*a*b*x)**(7/2)/7
+ (a**2 + 2*a*b*x)**(9/2)/9)/(2*a**2*b**2), Ne(a*b, 0)), (x**2*(a**2)**(5/
2)/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19

$$\int x(a^2+2abx+b^2x^2)^{5/2}\,dx = -\frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}ax}{6\,b} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}a^2}{6\,b^2} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}}{7\,b^2}$$

input
```
integrate(x*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a*x/b - 1/6*(b^2*x^2 + 2*a*b*x + a^2)
^(5/2)*a^2/b^2 + 1/7*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(46) = 92.

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.70

$$\int x(a^2+2abx+b^2x^2)^{5/2}\,dx = \frac{1}{7}\,b^5x^7\mathrm{sgn}(bx+a) + \frac{5}{6}\,ab^4x^6\mathrm{sgn}(bx+a) + 2\,a^2b^3x^5\mathrm{sgn}(bx+a) + \frac{5}{2}\,a^3b^2x^4\mathrm{sgn}(bx+a) + \frac{5}{3}\,a^4bx^3\mathrm{sgn}(bx+a) + \frac{1}{2}\,a^5x^2\mathrm{sgn}(bx+a) - \frac{a^7\mathrm{sgn}(bx+a)}{42\,b^2}$$

input
```
integrate(x*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/7*b^5*x^7*sgn(b*x + a) + 5/6*a*b^4*x^6*sgn(b*x + a) + 2*a^2*b^3*x^5*sgn(
b*x + a) + 5/2*a^3*b^2*x^4*sgn(b*x + a) + 5/3*a^4*b*x^3*sgn(b*x + a) + 1/2
*a^5*x^2*sgn(b*x + a) - 1/42*a^7*sgn(b*x + a)/b^2
```

Mupad [F(-1)]

Timed out.

$$\int x(a^2 + 2abx + b^2x^2)^{5/2}\,dx = \int x\,(a^2 + 2\,a\,b\,x + b^2\,x^2)^{5/2}\,dx$$

input
```
int(x*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int(x*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int x(a^2 + 2abx + b^2x^2)^{5/2}\,dx = \frac{x^2(6b^5x^5 + 35a\,b^4x^4 + 84a^2b^3x^3 + 105a^3b^2x^2 + 70a^4bx + 21a^5)}{42}$$

input
```
int(x*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(x**2*(21*a**5 + 70*a**4*b*x + 105*a**3*b**2*x**2 + 84*a**2*b**3*x**3 + 35
*a*b**4*x**4 + 6*b**5*x**5))/42
```

3.77 $\int (a^2+2abx+b^2x^2)^{5/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 34

$$\int (a^2+2abx+b^2x^2)^{5/2}\,dx=\frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6b}$$

output
```
1/6*(b*x+a)^5*((b*x+a)^2)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int (a^2+2abx+b^2x^2)^{5/2}\,dx=\frac{(a+bx)\left((a+bx)^2\right)^{5/2}}{6b}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
((a + b*x)*((a + b*x)^2)^(5/2))/(6*b)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx+b^2x^2\right)^{5/2} dx$$

$$\Big\downarrow \text{ 1079}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(xb^2+ab\right)^5 dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6b}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
((a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{\left((bx+a)^2\right)^{\frac{5}{2}}(bx+a)}{6b}$	20
risch	$\frac{(bx+a)^5\sqrt{(bx+a)^2}}{6b}$	22
gosper	$\frac{x(b^5x^5+6a\,b^4x^4+15a^2b^3x^3+20a^3b^2x^2+15a^4bx+6a^5)\left((bx+a)^2\right)^{\frac{5}{2}}}{6(bx+a)^5}$	71
orering	$\frac{x(b^5x^5+6a\,b^4x^4+15a^2b^3x^3+20a^3b^2x^2+15a^4bx+6a^5)(b^2x^2+2abx+a^2)^{\frac{5}{2}}}{6(bx+a)^5}$	80

input
```
int((b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*((b*x+a)^2)^(5/2)*(b*x+a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(21) = 42.

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int (a^2+2abx+b^2x^2)^{5/2}\,dx = \frac{1}{6}\,b^5x^6+ab^4x^5+\frac{5}{2}\,a^2b^3x^4+\frac{10}{3}\,a^3b^2x^3+\frac{5}{2}\,a^4bx^2+a^5x$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x
^2 + a^5*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(19) = 38$.

Time = 1.36 (sec) , antiderivative size = 666, normalized size of antiderivative = 19.59

$$\int \left(a^2+2abx+b^2x^2\right)^{5/2} dx = \text{Too large to display}$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
a**4*Piecewise(((a/(2*b) + x/2)*sqrt(a**2 + 2*a*b*x + b**2*x**2), Ne(b**2,
 0)), ((a**2 + 2*a*b*x)**(3/2)/(3*a*b), Ne(a*b, 0)), (x*sqrt(a**2), True))
 + 4*a**3*b*Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(-a**2/(6*b**2) +
a*x/(6*b) + x**2/3), Ne(b**2, 0)), ((-a**2*(a**2 + 2*a*b*x)**(3/2)/3 + (a*
*2 + 2*a*b*x)**(5/2)/5)/(2*a**2*b**2), Ne(a*b, 0)), (x**2*sqrt(a**2)/2, Tr
ue)) + 6*a**2*b**2*Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(a**3/(12*b
**3) - a**2*x/(12*b**2) + a*x**2/(12*b) + x**3/4), Ne(b**2, 0)), ((a**4*(a
**2 + 2*a*b*x)**(3/2)/3 - 2*a**2*(a**2 + 2*a*b*x)**(5/2)/5 + (a**2 + 2*a*b
*x)**(7/2)/7)/(4*a**3*b**3), Ne(a*b, 0)), (x**3*sqrt(a**2)/3, True)) + 4*a
*b**3*Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(-a**4/(20*b**4) + a**3*
x/(20*b**3) - a**2*x**2/(20*b**2) + a*x**3/(20*b) + x**4/5), Ne(b**2, 0)),
 ((-a**6*(a**2 + 2*a*b*x)**(3/2)/3 + 3*a**4*(a**2 + 2*a*b*x)**(5/2)/5 - 3*
a**2*(a**2 + 2*a*b*x)**(7/2)/7 + (a**2 + 2*a*b*x)**(9/2)/9)/(8*a**4*b**4),
 Ne(a*b, 0)), (x**4*sqrt(a**2)/4, True)) + b**4*Piecewise((sqrt(a**2 + 2*a
*b*x + b**2*x**2)*(a**5/(30*b**5) - a**4*x/(30*b**4) + a**3*x**2/(30*b**3)
 - a**2*x**3/(30*b**2) + a*x**4/(30*b) + x**5/6), Ne(b**2, 0)), ((a**8*(a*
*2 + 2*a*b*x)**(3/2)/3 - 4*a**6*(a**2 + 2*a*b*x)**(5/2)/5 + 6*a**4*(a**2 +
 2*a*b*x)**(7/2)/7 - 4*a**2*(a**2 + 2*a*b*x)**(9/2)/9 + (a**2 + 2*a*b*x)**
(11/2)/11)/(16*a**5*b**5), Ne(a*b, 0)), (x**5*sqrt(a**2)/5, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(21) = 42$.

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \left(a^2+2abx+b^2x^2\right)^{5/2} dx = \frac{1}{6}\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}x+\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}a}{6\,b}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*x + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2
)*a/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(21) = 42$.

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.44

$$\int \left(a^2+2abx+b^2x^2\right)^{5/2} dx = \frac{1}{2}\left(bx^2+2\,ax\right)a^4\mathrm{sgn}(bx+a)+\frac{a^6\mathrm{sgn}(bx+a)}{6\,b} + \frac{1}{2}\left(bx^2+2\,ax\right)^2a^2b\mathrm{sgn}(bx+a)+\frac{1}{6}\left(bx^2+2\,ax\right)^3b^2\mathrm{sgn}(bx+a)$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/2*(b*x^2 + 2*a*x)*a^4*sgn(b*x + a) + 1/6*a^6*sgn(b*x + a)/b + 1/2*(b*x^2
 + 2*a*x)^2*a^2*b*sgn(b*x + a) + 1/6*(b*x^2 + 2*a*x)^3*b^2*sgn(b*x + a)
```

Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \left(a^2+2abx+b^2x^2\right)^{5/2} dx = \frac{\left(x\,b^2+a\,b\right)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{5/2}}{6\,b^2}$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
((a*b + b^2*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/(6*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

$$\int \left(a^2+2abx+b^2x^2\right)^{5/2} dx = \frac{x(b^5x^5+6a\,b^4x^4+15a^2b^3x^3+20a^3b^2x^2+15a^4bx+6a^5)}{6}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(x*(6*a**5 + 15*a**4*b*x + 20*a**3*b**2*x**2 + 15*a**2*b**3*x**3 + 6*a*b**
4*x**4 + b**5*x**5))/6
```

3.78 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 221

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x}\,dx = \frac{5a^4bx\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{5a^3b^2x^2\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{10a^2b^3x^3\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)} + \frac{5ab^4x^4\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)} + \frac{b^5x^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)} + \frac{a^5\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}$$

output
```
5*a^4*b*x*((b*x+a)^2)^(1/2)/(b*x+a)+5*a^3*b^2*x^2*((b*x+a)^2)^(1/2)/(b*x+a
)+10*a^2*b^3*x^3*((b*x+a)^2)^(1/2)/(3*b*x+3*a)+5*a*b^4*x^4*((b*x+a)^2)^(1/
2)/(4*b*x+4*a)+b^5*x^5*((b*x+a)^2)^(1/2)/(5*b*x+5*a)+a^5*((b*x+a)^2)^(1/2)
*ln(x)/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x}\,dx = \frac{\sqrt{(a+bx)^2}(bx(300a^4+300a^3bx+200a^2b^2x^2+75ab^3x^3+12b^4x^4)+60a^5\log(x))}{60(a+bx)}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x,x]
```

output
```
(Sqrt[(a + b*x)^2]*(b*x*(300*a^4 + 300*a^3*b*x + 200*a^2*b^2*x^2 + 75*a*b^
3*x^3 + 12*b^4*x^4) + 60*a^5*Log[x]))/(60*(a + b*x))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x}\,dx$$

$$\Big\downarrow \; 1102$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5}{x}dx}{b^5(a+bx)}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5}{x}dx}{a+bx}$$

$$\Big\downarrow \; 49$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a^5}{x}+5ba^4+10b^2xa^3+10b^3x^2a^2+5b^4x^3a+b^5x^4\right)dx}{a+bx}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(a^5\log(x)+5a^4bx+5a^3b^2x^2+\frac{10}{3}a^2b^3x^3+\frac{5}{4}ab^4x^4+\frac{b^5x^5}{5}\right)}{a+bx}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(5*a^4*b*x + 5*a^3*b^2*x^2 + (10*a^2*b^3*x^
3)/3 + (5*a*b^4*x^4)/4 + (b^5*x^5)/5 + a^5*Log[x]))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.33

method	result	size
default	$\frac{\left((bx+a)^2\right)^{\frac{5}{2}}\left(12b^5x^5+75a\,b^4x^4+200a^2b^3x^3+300a^3b^2x^2+60\ln(x)a^5+300a^4bx\right)}{60(bx+a)^5}$	73
risch	$\frac{\sqrt{(bx+a)^2}\,b\left(\frac{1}{5}b^4x^5+\frac{5}{4}a\,b^3x^4+\frac{10}{3}a^2b^2x^3+5a^3b\,x^2+5a^4x\right)}{bx+a}+\frac{a^5\sqrt{(bx+a)^2}\,\ln(x)}{bx+a}$	86

input
```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/60*((b*x+a)^2)^(5/2)*(12*b^5*x^5+75*a*b^4*x^4+200*a^2*b^3*x^3+300*a^3*b^
2*x^2+60*ln(x)*a^5+300*a^4*b*x)/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{x} \, dx = \frac{1}{5} \, b^5x^5 + \frac{5}{4} \, ab^4x^4 + \frac{10}{3} \, a^2b^3x^3 + 5 \, a^3b^2x^2 + 5 \, a^4bx + a^5 \log (x)$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x,x, algorithm="fricas")
```

output
```
1/5*b^5*x^5 + 5/4*a*b^4*x^4 + 10/3*a^2*b^3*x^3 + 5*a^3*b^2*x^2 + 5*a^4*b*x
 + a^5*log(x)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{x} \, dx = \int \frac{\left((a + bx)^2\right)^{\frac{5}{2}}}{x} \, dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/x,x)
```

output
```
Integral(((a + b*x)**2)**(5/2)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.82

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x}\,dx = (-1)^{2\,b^2x+2\,ab}\,a^5\log\left(2\,b^2x+2\,ab\right) - (-1)^{2\,abx+2\,a^2}\,a^5\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right) + \frac{1}{2}\,\sqrt{b^2x^2+2\,abx+a^2}a^3bx + \frac{3}{2}\,\sqrt{b^2x^2+2\,abx+a^2}a^4 + \frac{1}{4}\,\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}abx + \frac{7}{12}\,\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}a^2 + \frac{1}{5}\,\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x,x, algorithm="maxima")
```

output
```
(-1)^(2*b^2*x + 2*a*b)*a^5*log(2*b^2*x + 2*a*b) - (-1)^(2*a*b*x + 2*a^2)*a
^5*log(2*a*b*x/abs(x) + 2*a^2/abs(x)) + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*
a^3*b*x + 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^4 + 1/4*(b^2*x^2 + 2*a*b*x +
 a^2)^(3/2)*a*b*x + 7/12*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a^2 + 1/5*(b^2*x^
2 + 2*a*b*x + a^2)^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.41

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x}\,dx = \frac{1}{5}\,b^5x^5\mathrm{sgn}(bx+a) + \frac{5}{4}\,ab^4x^4\mathrm{sgn}(bx+a) + \frac{10}{3}\,a^2b^3x^3\mathrm{sgn}(bx+a) + 5\,a^3b^2x^2\mathrm{sgn}(bx+a) + 5\,a^4bx\mathrm{sgn}(bx+a) + a^5\log\left(|x|\right)\mathrm{sgn}(bx+a)$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x,x, algorithm="giac")
```

output
```
1/5*b^5*x^5*sgn(b*x + a) + 5/4*a*b^4*x^4*sgn(b*x + a) + 10/3*a^2*b^3*x^3*s
gn(b*x + a) + 5*a^3*b^2*x^2*sgn(b*x + a) + 5*a^4*b*x*sgn(b*x + a) + a^5*lo
g(abs(x))*sgn(b*x + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{x} \, dx = \int \frac{(a^2 + 2\,a\,b\,x + b^2\,x^2)^{5/2}}{x} \, dx$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/x,x)
```

output
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{x} \, dx = \log(x)\, a^5 + 5a^4bx + 5a^3b^2x^2 + \frac{10a^2b^3x^3}{3} + \frac{5a\,b^4x^4}{4} + \frac{b^5x^5}{5}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x,x)
```

output
```
(60*log(x)*a**5 + 300*a**4*b*x + 300*a**3*b**2*x**2 + 200*a**2*b**3*x**3 +
 75*a*b**4*x**4 + 12*b**5*x**5)/60
```

3.79 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 220

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^2} dx = -\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)} + \frac{10a^3b^2x\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{5a^2b^3x^2\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{5ab^4x^3\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)} + \frac{b^5x^4\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)} + \frac{5a^4b\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}$$

output
```
-a^5*((b*x+a)^2)^(1/2)/x/(b*x+a)+10*a^3*b^2*x*((b*x+a)^2)^(1/2)/(b*x+a)+5*
a^2*b^3*x^2*((b*x+a)^2)^(1/2)/(b*x+a)+5*a*b^4*x^3*((b*x+a)^2)^(1/2)/(3*b*x
+3*a)+b^5*x^4*((b*x+a)^2)^(1/2)/(4*b*x+4*a)+5*a^4*b*((b*x+a)^2)^(1/2)*ln(x
)/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.36

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^2}\,dx = \frac{\sqrt{(a+bx)^2}(-12a^5+120a^3b^2x^2+60a^2b^3x^3+20ab^4x^4+3b^5x^5+60a^4bx\log}{12x(a+bx)}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^2,x]
```

output
```
(Sqrt[(a + b*x)^2]*(-12*a^5 + 120*a^3*b^2*x^2 + 60*a^2*b^3*x^3 + 20*a*b^4*
x^4 + 3*b^5*x^5 + 60*a^4*b*x*Log[x]))/(12*x*(a + b*x))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^2}dx$$

$$\Big\downarrow \text{ 1102}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5}{x^2}dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5}{x^2}dx}{a+bx}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a^5}{x^2}+\frac{5ba^4}{x}+10b^2a^3+10b^3xa^2+5b^4x^2a+b^5x^3\right)dx}{a+bx}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^5}{x}+5a^4b\log(x)+10a^3b^2x+5a^2b^3x^2+\frac{5}{3}ab^4x^3+\frac{b^5x^4}{4}\right)}{a+bx}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^2,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-(a^5/x) + 10*a^3*b^2*x + 5*a^2*b^3*x^2 +
(5*a*b^4*x^3)/3 + (b^5*x^4)/4 + 5*a^4*b*Log[x]))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.35

method	result	size
default	$\frac{\left((bx+a)^{2}\right)^{\frac{5}{2}}\left(3b^{5}x^{5}+20a\,b^{4}x^{4}+60a^{2}b^{3}x^{3}+60\ln(x)x\,a^{4}b+120a^{3}b^{2}x^{2}-12a^{5}\right)}{12x\left(bx+a\right)^{5}}$	76
risch	$\frac{\sqrt{(bx+a)^{2}}\,b^{2}\left(\frac{1}{4}b^{3}x^{4}+\frac{5}{3}a\,b^{2}x^{3}+5a^{2}b\,x^{2}+10a^{3}x\right)}{bx+a}-\frac{a^{5}\sqrt{(bx+a)^{2}}}{x\left(bx+a\right)}+\frac{5a^{4}b\sqrt{(bx+a)^{2}}\,\ln(x)}{bx+a}$	103

input
```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*((b*x+a)^2)^(5/2)*(3*b^5*x^5+20*a*b^4*x^4+60*a^2*b^3*x^3+60*ln(x)*x*a
^4*b+120*a^3*b^2*x^2-12*a^5)/x/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.27

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{x^2} dx = \frac{3 b^5x^5 + 20 ab^4x^4 + 60 a^2b^3x^3 + 120 a^3b^2x^2 + 60 a^4bx \log(x) - 12 a^5}{12 x}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
1/12*(3*b^5*x^5 + 20*a*b^4*x^4 + 60*a^2*b^3*x^3 + 120*a^3*b^2*x^2 + 60*a^4
*b*x*log(x) - 12*a^5)/x
```

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{x^2} dx = \int \frac{\left((a + bx)^2\right)^{\frac{5}{2}}}{x^2} dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/x**2,x)
```

output
```
Integral(((a + b*x)**2)**(5/2)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.87

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^2}\,dx = 5\,(-1)^{2\,b^2x+2\,ab}\,a^4b\log\left(2\,b^2x+2\,ab\right) - 5\,(-1)^{2\,abx+2\,a^2}\,a^4b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right) + \frac{5}{2}\,\sqrt{b^2x^2+2\,abx+a^2}a^2b^2x + \frac{15}{2}\,\sqrt{b^2x^2+2\,abx+a^2}a^3b + \frac{5}{4}\,\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}b^2x + \frac{35}{12}\,\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}ab - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}}{x}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
5*(-1)^(2*b^2*x + 2*a*b)*a^4*b*log(2*b^2*x + 2*a*b) - 5*(-1)^(2*a*b*x + 2*
a^2)*a^4*b*log(2*a*b*x/abs(x) + 2*a^2/abs(x)) + 5/2*sqrt(b^2*x^2 + 2*a*b*x
 + a^2)*a^2*b^2*x + 15/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^3*b + 5/4*(b^2*x^
2 + 2*a*b*x + a^2)^(3/2)*b^2*x + 35/12*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a*b
 - (b^2*x^2 + 2*a*b*x + a^2)^(5/2)/x
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.41

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^2}\,dx = \frac{1}{4}\,b^5x^4\mathrm{sgn}(bx+a) + \frac{5}{3}\,ab^4x^3\mathrm{sgn}(bx+a) + 5\,a^2b^3x^2\mathrm{sgn}(bx+a) + 10\,a^3b^2x\mathrm{sgn}(bx+a) + 5\,a^4b\log\left(|x|\right)\mathrm{sgn}(bx+a) - \frac{a^5\mathrm{sgn}(bx+a)}{x}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^2,x, algorithm="giac")
```

output
```
1/4*b^5*x^4*sgn(b*x + a) + 5/3*a*b^4*x^3*sgn(b*x + a) + 5*a^2*b^3*x^2*sgn(
b*x + a) + 10*a^3*b^2*x*sgn(b*x + a) + 5*a^4*b*log(abs(x))*sgn(b*x + a) -
a^5*sgn(b*x + a)/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^2}\,dx = \int \frac{(a^2+2\,a\,b\,x+b^2\,x^2)^{5/2}}{x^2}\,dx$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/x^2,x)
```

output
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.27

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^2}\,dx = \frac{60\log(x)\,a^4bx-12a^5+120a^3b^2x^2+60a^2b^3x^3+20a\,b^4x^4+3b^5x^5}{12x}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^2,x)
```

output
```
(60*log(x)*a**4*b*x - 12*a**5 + 120*a**3*b**2*x**2 + 60*a**2*b**3*x**3 + 2
0*a*b**4*x**4 + 3*b**5*x**5)/(12*x)
```

3.80 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 222

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^3}\,dx = -\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)} - \frac{5a^4b\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)} + \frac{10a^2b^3x\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{5ab^4x^2\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)} + \frac{b^5x^3\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)} + \frac{10a^3b^2\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}$$

output
```
-1/2*a^5*((b*x+a)^2)^(1/2)/x^2/(b*x+a)-5*a^4*b*((b*x+a)^2)^(1/2)/x/(b*x+a)
+10*a^2*b^3*x*((b*x+a)^2)^(1/2)/(b*x+a)+5*a*b^4*x^2*((b*x+a)^2)^(1/2)/(2*b
*x+2*a)+b^5*x^3*((b*x+a)^2)^(1/2)/(3*b*x+3*a)+10*a^3*b^2*((b*x+a)^2)^(1/2)
*ln(x)/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.12

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^3}\,dx = \frac{(12a^5+120a^4bx+57a^3b^2x^2-240a^2b^3x^3-60ab^4x^4-8b^5x^5)\left(\sqrt{a^2}bx+a\left(\right.\right.}{24x^2\left(a^2+abx-\sqrt{a^2}\sqrt{(a+bx)^2}\right)}$$
$$-10a^3b^2\text{arctanh}\left(\frac{bx}{\sqrt{a^2}-\sqrt{(a+bx)^2}}\right)-10\left(a^2\right)^{3/2}b^2\log(x)$$
$$+5\left(a^2\right)^{3/2}b^2\log\left(\sqrt{a^2}-bx-\sqrt{(a+bx)^2}\right)+5\left(a^2\right)^{3/2}b^2\log\left(\sqrt{a^2}+bx-\sqrt{(a+bx)^2}\right)$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^3,x]
```

output
```
((12*a^5 + 120*a^4*b*x + 57*a^3*b^2*x^2 - 240*a^2*b^3*x^3 - 60*a*b^4*x^4 -
 8*b^5*x^5)*(Sqrt[a^2]*b*x + a*(Sqrt[a^2] - Sqrt[(a + b*x)^2])))/(24*x^2*(
a^2 + a*b*x - Sqrt[a^2]*Sqrt[(a + b*x)^2])) - 10*a^3*b^2*ArcTanh[(b*x)/(Sq
rt[a^2] - Sqrt[(a + b*x)^2])] - 10*(a^2)^(3/2)*b^2*Log[x] + 5*(a^2)^(3/2)*
b^2*Log[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2]] + 5*(a^2)^(3/2)*b^2*Log[Sqrt[
a^2] + b*x - Sqrt[(a + b*x)^2]]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^3}dx$$
$$\Big\downarrow \text{1102}$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5}{x^3}dx}{b^5(a+bx)}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5}{x^3}dx}{a+bx}$$

$$\Big\downarrow \; 49$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a^5}{x^3}+\frac{5ba^4}{x^2}+\frac{10b^2a^3}{x}+10b^3a^2+5b^4xa+b^5x^2\right)dx}{a+bx}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^5}{2x^2}-\frac{5a^4b}{x}+10a^3b^2\log(x)+10a^2b^3x+\frac{5}{2}ab^4x^2+\frac{b^5x^3}{3}\right)}{a+bx}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^3,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/2*a^5/x^2 - (5*a^4*b)/x + 10*a^2*b^3*x
+ (5*a*b^4*x^2)/2 + (b^5*x^3)/3 + 10*a^3*b^2*Log[x]))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.34

method	result	size
default	$\frac{\left((bx+a)^2\right)^{\frac{5}{2}}\left(2b^5x^5+15a\,b^4x^4+60\ln\left(x\right)x^2a^3b^2+60a^2b^3x^3-30a^4bx-3a^5\right)}{6(bx+a)^5x^2}$	76
risch	$\frac{\sqrt{(bx+a)^2}\,b^3\left(\frac{1}{3}b^2x^3+\frac{5}{2}ab\,x^2+10a^2x\right)}{bx+a}+\frac{\sqrt{(bx+a)^2}\left(-5a^4bx-\frac{1}{2}a^5\right)}{(bx+a)x^2}+\frac{10a^3b^2\sqrt{(bx+a)^2}\,\ln\left(x\right)}{bx+a}$	103

input
```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*((b*x+a)^2)^(5/2)*(2*b^5*x^5+15*a*b^4*x^4+60*ln(x)*x^2*a^3*b^2+60*a^2*
b^3*x^3-30*a^4*b*x-3*a^5)/(b*x+a)^5/x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.27

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^3}\,dx=\frac{2\,b^5x^5+15\,ab^4x^4+60\,a^2b^3x^3+60\,a^3b^2x^2\log\left(x\right)-30\,a^4bx-3\,a^5}{6\,x^2}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^3,x, algorithm="fricas")
```

output
```
1/6*(2*b^5*x^5 + 15*a*b^4*x^4 + 60*a^2*b^3*x^3 + 60*a^3*b^2*x^2*log(x) - 3
0*a^4*b*x - 3*a^5)/x^2
```

Sympy [F]

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^3}\,dx=\int \frac{\left((a+bx)^2\right)^{\frac{5}{2}}}{x^3}\,dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/x**3,x)
```

output
```
Integral(((a + b*x)**2)**(5/2)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.15

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^3}\,dx = 10\,(-1)^{2\,b^2x+2\,ab}\,a^3b^2\log\left(2\,b^2x+2\,ab\right) - 10\,(-1)^{2\,abx+2\,a^2}\,a^3b^2\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right) + 5\,\sqrt{b^2x^2+2\,abx+a^2}ab^3x + 15\,\sqrt{b^2x^2+2\,abx+a^2}a^2b^2 + \frac{5\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^3x}{2\,a} + \frac{35}{6}\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^2 + \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^2}{2\,a^2} - \frac{3\,(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b}{2\,ax} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}}{2\,a^2x^2}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^3,x, algorithm="maxima")
```

output
```
10*(-1)^(2*b^2*x + 2*a*b)*a^3*b^2*log(2*b^2*x + 2*a*b) - 10*(-1)^(2*a*b*x
+ 2*a^2)*a^3*b^2*log(2*a*b*x/abs(x) + 2*a^2/abs(x)) + 5*sqrt(b^2*x^2 + 2*a
*b*x + a^2)*a*b^3*x + 15*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^2*b^2 + 5/2*(b^2*
x^2 + 2*a*b*x + a^2)^(3/2)*b^3*x/a + 35/6*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*
b^2 + 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*b^2/a^2 - 3/2*(b^2*x^2 + 2*a*b*x
 + a^2)^(5/2)*b/(a*x) - 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)/(a^2*x^2)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.41

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^3}\,dx = \frac{1}{3}\,b^5x^3\mathrm{sgn}(bx+a) + \frac{5}{2}\,ab^4x^2\mathrm{sgn}(bx+a) + 10\,a^2b^3x\mathrm{sgn}(bx+a) + 10\,a^3b^2\log\left(|x|\right)\mathrm{sgn}(bx+a) - \frac{10\,a^4bx\mathrm{sgn}(bx+a)+a^5\mathrm{sgn}(bx+a)}{2\,x^2}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^3,x, algorithm="giac")
```

output
```
1/3*b^5*x^3*sgn(b*x + a) + 5/2*a*b^4*x^2*sgn(b*x + a) + 10*a^2*b^3*x*sgn(b
*x + a) + 10*a^3*b^2*log(abs(x))*sgn(b*x + a) - 1/2*(10*a^4*b*x*sgn(b*x +
a) + a^5*sgn(b*x + a))/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{x^3} \, dx = \int \frac{(a^2 + 2\,a\,b\,x + b^2\,x^2)^{5/2}}{x^3} \, dx$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/x^3,x)
```

output
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.27

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{x^3} \, dx = \frac{60 \log(x)\, a^3b^2x^2 - 3a^5 - 30a^4bx + 60a^2b^3x^3 + 15a\, b^4x^4 + 2b^5x^5}{6x^2}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^3,x)
```

output
```
(60*log(x)*a**3*b**2*x**2 - 3*a**5 - 30*a**4*b*x + 60*a**2*b**3*x**3 + 15*
a*b**4*x**4 + 2*b**5*x**5)/(6*x**2)
```

3.81 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 222

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^4}\,dx = -\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)} - \frac{5a^4b\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)} - \frac{10a^3b^2\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)} + \frac{5ab^4x\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{b^5x^2\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)} + \frac{10a^2b^3\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}$$

output
```
-1/3*a^5*((b*x+a)^2)^(1/2)/x^3/(b*x+a)-5/2*a^4*b*((b*x+a)^2)^(1/2)/x^2/(b*
x+a)-10*a^3*b^2*((b*x+a)^2)^(1/2)/x/(b*x+a)+5*a*b^4*x*((b*x+a)^2)^(1/2)/(b
*x+a)+b^5*x^2*((b*x+a)^2)^(1/2)/(2*b*x+2*a)+10*a^2*b^3*((b*x+a)^2)^(1/2)*l
n(x)/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.13

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^4}\,dx = \frac{(4a^5+30a^4bx+120a^3b^2x^2+53a^2b^3x^3-60ab^4x^4-6b^5x^5)\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}-\sqrt{(a+bx)^2}\right)\right)}{12x^3\left(a^2+abx-\sqrt{a^2}\sqrt{(a+bx)^2}\right)}$$
$$-10a^2b^3\text{arctanh}\left(\frac{bx}{\sqrt{a^2}-\sqrt{(a+bx)^2}}\right)-10a\sqrt{a^2}b^3\log(x)$$
$$+5a\sqrt{a^2}b^3\log\left(\sqrt{a^2}-bx-\sqrt{(a+bx)^2}\right)+5a\sqrt{a^2}b^3\log\left(\sqrt{a^2}+bx-\sqrt{(a+bx)^2}\right)$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^4,x]
```

output
```
((4*a^5 + 30*a^4*b*x + 120*a^3*b^2*x^2 + 53*a^2*b^3*x^3 - 60*a*b^4*x^4 - 6
*b^5*x^5)*(Sqrt[a^2]*b*x + a*(Sqrt[a^2] - Sqrt[(a + b*x)^2])))/(12*x^3*(a^
2 + a*b*x - Sqrt[a^2]*Sqrt[(a + b*x)^2])) - 10*a^2*b^3*ArcTanh[(b*x)/(Sqrt
[a^2] - Sqrt[(a + b*x)^2])] - 10*a*Sqrt[a^2]*b^3*Log[x] + 5*a*Sqrt[a^2]*b^
3*Log[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2]] + 5*a*Sqrt[a^2]*b^3*Log[Sqrt[a^
2] + b*x - Sqrt[(a + b*x)^2]]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^4}\,dx$$
$$\Big\downarrow \; 1102$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5}{x^4}dx}{b^5(a+bx)}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5}{x^4}dx}{a+bx}$$

$$\Big\downarrow \; 49$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a^5}{x^4}+\frac{5ba^4}{x^3}+\frac{10b^2a^3}{x^2}+\frac{10b^3a^2}{x}+5b^4a+b^5x\right)dx}{a+bx}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^5}{3x^3}-\frac{5a^4b}{2x^2}-\frac{10a^3b^2}{x}+10a^2b^3\log(x)+5ab^4x+\frac{b^5x^2}{2}\right)}{a+bx}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^4,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/3*a^5/x^3 - (5*a^4*b)/(2*x^2) - (10*a^3
*b^2)/x + 5*a*b^4*x + (b^5*x^2)/2 + 10*a^2*b^3*Log[x]))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.34

method	result	size
default	$\frac{\left((bx+a)^2\right)^{\frac{5}{2}}\left(3b^5x^5+60\ln(x)x^3a^2b^3+30a\,b^4x^4-60a^3b^2x^2-15a^4bx-2a^5\right)}{6(bx+a)^5x^3}$	76
risch	$\frac{\sqrt{(bx+a)^2}\,b^4\left(\frac{1}{2}b\,x^2+5ax\right)}{bx+a}+\frac{\sqrt{(bx+a)^2}\,\left(-10a^3b^2x^2-\frac{5}{2}a^4bx-\frac{1}{3}a^5\right)}{(bx+a)x^3}+\frac{10a^2b^3\sqrt{(bx+a)^2}\,\ln(x)}{bx+a}$	103

input
```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*((b*x+a)^2)^(5/2)*(3*b^5*x^5+60*ln(x)*x^3*a^2*b^3+30*a*b^4*x^4-60*a^3*
b^2*x^2-15*a^4*b*x-2*a^5)/(b*x+a)^5/x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.27

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^4}\,dx=\frac{3\,b^5x^5+30\,ab^4x^4+60\,a^2b^3x^3\log{(x)}-60\,a^3b^2x^2-15\,a^4bx-2\,a^5}{6\,x^3}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^4,x, algorithm="fricas")
```

output
```
1/6*(3*b^5*x^5 + 30*a*b^4*x^4 + 60*a^2*b^3*x^3*log(x) - 60*a^3*b^2*x^2 - 1
5*a^4*b*x - 2*a^5)/x^3
```

Sympy [F]

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^4}\,dx=\int \frac{\left((a+bx)^2\right)^{\frac{5}{2}}}{x^4}\,dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/x**4,x)
```

output
```
Integral(((a + b*x)**2)**(5/2)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.28

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^4}\,dx = 10\,(-1)^{2\,b^2x+2\,ab}\,a^2b^3\log\left(2\,b^2x+2\,ab\right) - 10\,(-1)^{2\,abx+2\,a^2}\,a^2b^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right) + 5\,\sqrt{b^2x^2+2\,abx+a^2}b^4x + 15\,\sqrt{b^2x^2+2\,abx+a^2}ab^3 + \frac{5\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^4x}{2\,a^2} + \frac{35\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^3}{6\,a} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^3}{6\,a^3} - \frac{11\,(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^2}{6\,a^2x} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}b}{6\,a^3x^2} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}}{3\,a^2x^3}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^4,x, algorithm="maxima")
```

output
```
10*(-1)^(2*b^2*x + 2*a*b)*a^2*b^3*log(2*b^2*x + 2*a*b) - 10*(-1)^(2*a*b*x
+ 2*a^2)*a^2*b^3*log(2*a*b*x/abs(x) + 2*a^2/abs(x)) + 5*sqrt(b^2*x^2 + 2*a
*b*x + a^2)*b^4*x + 15*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a*b^3 + 5/2*(b^2*x^2
+ 2*a*b*x + a^2)^(3/2)*b^4*x/a^2 + 35/6*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^
3/a + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*b^3/a^3 - 11/6*(b^2*x^2 + 2*a*b*
x + a^2)^(5/2)*b^2/(a^2*x) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b/(a^3*x^
2) - 1/3*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)/(a^2*x^3)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.41

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^4}\,dx = \frac{1}{2}\,b^5x^2\mathrm{sgn}(bx+a) + 5\,ab^4x\mathrm{sgn}(bx+a) + 10\,a^2b^3\log\left(|x|\right)\mathrm{sgn}(bx+a) - \frac{60\,a^3b^2x^2\mathrm{sgn}(bx+a)+15\,a^4bx\mathrm{sgn}(bx+a)+2\,a^5\mathrm{sgn}(bx+a)}{6\,x^3}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^4,x, algorithm="giac")
```

output
```
1/2*b^5*x^2*sgn(b*x + a) + 5*a*b^4*x*sgn(b*x + a) + 10*a^2*b^3*log(abs(x))
*sgn(b*x + a) - 1/6*(60*a^3*b^2*x^2*sgn(b*x + a) + 15*a^4*b*x*sgn(b*x + a)
 + 2*a^5*sgn(b*x + a))/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^4}\,dx=\int \frac{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{5/2}}{x^4}\,dx$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/x^4,x)
```

output
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.27

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^4}\,dx=\frac{60\log(x)\,a^2b^3x^3-2a^5-15a^4bx-60a^3b^2x^2+30a\,b^4x^4+3b^5x^5}{6x^3}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^4,x)
```

output
```
(60*log(x)*a**2*b**3*x**3 - 2*a**5 - 15*a**4*b*x - 60*a**3*b**2*x**2 + 30*
a*b**4*x**4 + 3*b**5*x**5)/(6*x**3)
```

3.82 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 219

$$\begin{aligned}\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^5}\,dx &= -\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)}-\frac{5a^4b\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)}\\&\quad-\frac{5a^3b^2\sqrt{a^2+2abx+b^2x^2}}{x^2(a+bx)}-\frac{10a^2b^3\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}\\&\quad+\frac{b^5x\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{5ab^4\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}\end{aligned}$$

output

```
-1/4*a^5*((b*x+a)^2)^(1/2)/x^4/(b*x+a)-5/3*a^4*b*((b*x+a)^2)^(1/2)/x^3/(b*
x+a)-5*a^3*b^2*((b*x+a)^2)^(1/2)/x^2/(b*x+a)-10*a^2*b^3*((b*x+a)^2)^(1/2)/
x/(b*x+a)+b^5*x*((b*x+a)^2)^(1/2)/(b*x+a)+5*a*b^4*((b*x+a)^2)^(1/2)*ln(x)/
(b*x+a)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 634 vs. 2(219) = 438.

Time = 1.03 (sec) , antiderivative size = 634, normalized size of antiderivative = 2.89

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^5}\,dx = \frac{48a^6\sqrt{a^2}+368a^5\sqrt{a^2}bx+1280a^4\sqrt{a^2}b^2x^2+2880a^3\sqrt{a^2}b^3x^3+2677(a^2)^{3/2}}{}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^5,x]
```

output
```
(48*a^6*Sqrt[a^2] + 368*a^5*Sqrt[a^2]*b*x + 1280*a^4*Sqrt[a^2]*b^2*x^2 + 2
880*a^3*Sqrt[a^2]*b^3*x^3 + 2677*(a^2)^(3/2)*b^4*x^4 + 565*a*Sqrt[a^2]*b^5
*x^5 - 192*Sqrt[a^2]*b^6*x^6 - 48*a^6*Sqrt[(a + b*x)^2] - 320*a^5*b*x*Sqrt
[(a + b*x)^2] - 960*a^4*b^2*x^2*Sqrt[(a + b*x)^2] - 1920*a^3*b^3*x^3*Sqrt[
(a + b*x)^2] - 757*a^2*b^4*x^4*Sqrt[(a + b*x)^2] + 192*a*b^5*x^5*Sqrt[(a +
 b*x)^2] - 960*a*b^4*x^4*(a^2 + a*b*x - Sqrt[a^2]*Sqrt[(a + b*x)^2])*ArcTa
nh[(b*x)/(Sqrt[a^2] - Sqrt[(a + b*x)^2])] - 960*b^4*x^4*((a^2)^(3/2) + a*S
qrt[a^2]*b*x - a^2*Sqrt[(a + b*x)^2])*Log[x] + 480*(a^2)^(3/2)*b^4*x^4*Log
[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2]] + 480*a*Sqrt[a^2]*b^5*x^5*Log[Sqrt[a
^2] - b*x - Sqrt[(a + b*x)^2]] - 480*a^2*b^4*x^4*Sqrt[(a + b*x)^2]*Log[Sqr
t[a^2] - b*x - Sqrt[(a + b*x)^2]] + 480*(a^2)^(3/2)*b^4*x^4*Log[Sqrt[a^2]
+ b*x - Sqrt[(a + b*x)^2]] + 480*a*Sqrt[a^2]*b^5*x^5*Log[Sqrt[a^2] + b*x -
 Sqrt[(a + b*x)^2]] - 480*a^2*b^4*x^4*Sqrt[(a + b*x)^2]*Log[Sqrt[a^2] + b*
x - Sqrt[(a + b*x)^2]])/(192*x^4*(a^2 + a*b*x - Sqrt[a^2]*Sqrt[(a + b*x)^2
]))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^5}\,dx$$
$$\Big\downarrow \; 1102$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5}{x^5}dx}{b^5(a+bx)}$$
$$\Big\downarrow \; 27$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5}{x^5}dx}{a+bx}$$
$$\Big\downarrow \; 49$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a^5}{x^5}+\frac{5ba^4}{x^4}+\frac{10b^2a^3}{x^3}+\frac{10b^3a^2}{x^2}+\frac{5b^4a}{x}+b^5\right)dx}{a+bx}$$
$$\Big\downarrow \; 2009$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^5}{4x^4}-\frac{5a^4b}{3x^3}-\frac{5a^3b^2}{x^2}-\frac{10a^2b^3}{x}+5ab^4\log(x)+b^5x\right)}{a+bx}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^5,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/4*a^5/x^4 - (5*a^4*b)/(3*x^3) - (5*a^3*
b^2)/x^2 - (10*a^2*b^3)/x + b^5*x + 5*a*b^4*Log[x]))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.35

method	result	size
default	$\frac{\left(\left(bx+a\right)^{2}\right)^{\frac{5}{2}}\left(60\ln\left(x\right)x^{4}a\,b^{4}+12b^{5}x^{5}-120a^{2}b^{3}x^{3}-60a^{3}b^{2}x^{2}-20a^{4}bx-3a^{5}\right)}{12\left(bx+a\right)^{5}x^{4}}$	76
risch	$\frac{b^{5}x\sqrt{\left(bx+a\right)^{2}}}{bx+a}+\frac{\sqrt{\left(bx+a\right)^{2}}\,\left(-10a^{2}b^{3}x^{3}-5a^{3}b^{2}x^{2}-\frac{5}{3}a^{4}bx-\frac{1}{4}a^{5}\right)}{\left(bx+a\right)x^{4}}+\frac{5a\,b^{4}\sqrt{\left(bx+a\right)^{2}}\,\ln\left(x\right)}{bx+a}$	102

input
```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/12*((b*x+a)^2)^(5/2)*(60*ln(x)*x^4*a*b^4+12*b^5*x^5-120*a^2*b^3*x^3-60*a
^3*b^2*x^2-20*a^4*b*x-3*a^5)/(b*x+a)^5/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.27

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^5}\,dx=\frac{12\,b^5x^5+60\,ab^4x^4\log\left(x\right)-120\,a^2b^3x^3-60\,a^3b^2x^2-20\,a^4bx-3\,a^5}{12\,x^4}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^5,x, algorithm="fricas")
```

output
```
1/12*(12*b^5*x^5 + 60*a*b^4*x^4*log(x) - 120*a^2*b^3*x^3 - 60*a^3*b^2*x^2
- 20*a^4*b*x - 3*a^5)/x^4
```

Sympy [F]

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^5}\,dx = \int \frac{\left((a+bx)^2\right)^{\frac{5}{2}}}{x^5}\,dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/x**5,x)
```

output
```
Integral(((a + b*x)**2)**(5/2)/x**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(149) = 298$.

Time = 0.04 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.42

$$\begin{aligned}\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^5}\,dx &= 5\,(-1)^{2\,b^2x+2\,ab}ab^4\log\left(2\,b^2x+2\,ab\right)\\
&- 5\,(-1)^{2\,abx+2\,a^2}ab^4\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)+\frac{5\,\sqrt{b^2x^2+2\,abx+a^2}b^5x}{2\,a}\\
&+ \frac{15}{2}\,\sqrt{b^2x^2+2\,abx+a^2}b^4+\frac{5\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^5x}{4\,a^3}\\
&+ \frac{35\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^4}{12\,a^2}+\frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^4}{3\,a^4}\\
&- \frac{2\,(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^3}{3\,a^3x}-\frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}b^2}{3\,a^4x^2}\\
&+ \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}b}{12\,a^3x^3}-\frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}}{4\,a^2x^4}\end{aligned}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^5,x, algorithm="maxima")
```

output
```
5*(-1)^(2*b^2*x + 2*a*b)*a*b^4*log(2*b^2*x + 2*a*b) - 5*(-1)^(2*a*b*x + 2*
a^2)*a*b^4*log(2*a*b*x/abs(x) + 2*a^2/abs(x)) + 5/2*sqrt(b^2*x^2 + 2*a*b*x
 + a^2)*b^5*x/a + 15/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^4 + 5/4*(b^2*x^2 +
2*a*b*x + a^2)^(3/2)*b^5*x/a^3 + 35/12*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^4
/a^2 + 1/3*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*b^4/a^4 - 2/3*(b^2*x^2 + 2*a*b*
x + a^2)^(5/2)*b^3/(a^3*x) - 1/3*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b^2/(a^4*
x^2) + 1/12*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b/(a^3*x^3) - 1/4*(b^2*x^2 + 2
*a*b*x + a^2)^(7/2)/(a^2*x^4)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.42

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^5}\,dx = b^5x\mathrm{sgn}(bx+a)+5\,ab^4\log(|x|)\,\mathrm{sgn}(bx+a) - \frac{120\,a^2b^3x^3\mathrm{sgn}(bx+a)+60\,a^3b^2x^2\mathrm{sgn}(bx+a)+20\,a^4bx\mathrm{sgn}(bx+a)+3\,a^5\mathrm{sgn}(bx+a)}{12\,x^4}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^5,x, algorithm="giac")
```

output
```
b^5*x*sgn(b*x + a) + 5*a*b^4*log(abs(x))*sgn(b*x + a) - 1/12*(120*a^2*b^3*
x^3*sgn(b*x + a) + 60*a^3*b^2*x^2*sgn(b*x + a) + 20*a^4*b*x*sgn(b*x + a) +
 3*a^5*sgn(b*x + a))/x^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^5}\,dx = \int \frac{(a^2+2\,a\,b\,x+b^2\,x^2)^{5/2}}{x^5}\,dx$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/x^5,x)
```

output
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.27

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^5}\,dx = \frac{60\log(x)\,a\,b^4x^4-3a^5-20a^4bx-60a^3b^2x^2-120a^2b^3x^3+12b^5x^5}{12x^4}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^5,x)
```

output
```
(60*log(x)*a*b**4*x**4 - 3*a**5 - 20*a**4*b*x - 60*a**3*b**2*x**2 - 120*a*
*2*b**3*x**3 + 12*b**5*x**5)/(12*x**4)
```

3.83 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^6}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 223

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^6}\,dx = -\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{5x^5(a+bx)} - \frac{5a^4b\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)} - \frac{10a^3b^2\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)} - \frac{5a^2b^3\sqrt{a^2+2abx+b^2x^2}}{x^2(a+bx)} - \frac{5ab^4\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)} + \frac{b^5\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}$$

output

```
-1/5*a^5*((b*x+a)^2)^(1/2)/x^5/(b*x+a)-5/4*a^4*b*((b*x+a)^2)^(1/2)/x^4/(b*
x+a)-10/3*a^3*b^2*((b*x+a)^2)^(1/2)/x^3/(b*x+a)-5*a^2*b^3*((b*x+a)^2)^(1/2
)/x^2/(b*x+a)-5*a*b^4*((b*x+a)^2)^(1/2)/x/(b*x+a)+b^5*((b*x+a)^2)^(1/2)*ln
(x)/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.11

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^6}\,dx = \frac{1}{120}\left(-\frac{\sqrt{(a+bx)^2}(12a^4+63a^3bx+137a^2b^2x^2+163ab^3x^3+137b^4x^4)}{x^5}\right.$$
$$+\frac{\sqrt{a^2}(12a^4+75a^3bx+200a^2b^2x^2+300ab^3x^3+300b^4x^4)}{x^5}$$
$$-120b^5\operatorname{arctanh}\left(\frac{bx}{\sqrt{a^2}-\sqrt{(a+bx)^2}}\right)-\frac{120\sqrt{a^2}b^5\log(x)}{a}$$
$$+\frac{60\sqrt{a^2}b^5\log\left(a\left(\sqrt{a^2}-bx-\sqrt{(a+bx)^2}\right)\right)}{a}$$
$$\left.+\frac{60\sqrt{a^2}b^5\log\left(a\left(\sqrt{a^2}+bx-\sqrt{(a+bx)^2}\right)\right)}{a}\right)$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^6,x]
```

output
```
(-((Sqrt[(a + b*x)^2]*(12*a^4 + 63*a^3*b*x + 137*a^2*b^2*x^2 + 163*a*b^3*x
^3 + 137*b^4*x^4))/x^5) + (Sqrt[a^2]*(12*a^4 + 75*a^3*b*x + 200*a^2*b^2*x^
2 + 300*a*b^3*x^3 + 300*b^4*x^4))/x^5 - 120*b^5*ArcTanh[(b*x)/(Sqrt[a^2] -
 Sqrt[(a + b*x)^2])] - (120*Sqrt[a^2]*b^5*Log[x])/a + (60*Sqrt[a^2]*b^5*Lo
g[a*(Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2])])/a + (60*Sqrt[a^2]*b^5*Log[a*(S
qrt[a^2] + b*x - Sqrt[(a + b*x)^2])])/a)/120
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^6} \, dx$$

$\Big\downarrow$ 1102

$$\frac{\sqrt{a^2+2abx+b^2x^2} \int \frac{b^5(a+bx)^5}{x^6}dx}{b^5(a+bx)}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{a^2+2abx+b^2x^2} \int \frac{(a+bx)^5}{x^6}dx}{a+bx}$$

$\Big\downarrow$ 49

$$\frac{\sqrt{a^2+2abx+b^2x^2} \int \left(\frac{a^5}{x^6}+\frac{5ba^4}{x^5}+\frac{10b^2a^3}{x^4}+\frac{10b^3a^2}{x^3}+\frac{5b^4a}{x^2}+\frac{b^5}{x}\right)dx}{a+bx}$$

$\Big\downarrow$ 2009

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^5}{5x^5}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{3x^3}-\frac{5a^2b^3}{x^2}-\frac{5ab^4}{x}+b^5\log(x)\right)}{a+bx}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^6,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/5*a^5/x^5 - (5*a^4*b)/(4*x^4) - (10*a^3
*b^2)/(3*x^3) - (5*a^2*b^3)/x^2 - (5*a*b^4)/x + b^5*Log[x]))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.34

method	result	size
default	$\frac{\left((bx+a)^{2}\right)^{\frac{5}{2}} \left(60 \ln \left(x \right) x^{5} b^{5}-300 a \,b^{4} x^{4}-300 a^{2} b^{3} x^{3}-200 a^{3} b^{2} x^{2}-75 a^{4} b x -12 a^{5}\right)}{60 \left(bx+a \right)^{5} x^{5}}$	76
risch	$\frac{\sqrt{(bx+a)^{2}}\, \left(-5 a \,b^{4} x^{4}-5 a^{2} b^{3} x^{3}-\frac{10}{3} a^{3} b^{2} x^{2}-\frac{5}{4} a^{4} b x -\frac{1}{5} a^{5}\right)}{\left(bx+a \right) x^{5}}+\frac{b^{5} \sqrt{(bx+a)^{2}}\, \ln \left(x \right)}{bx+a}$	88

input
```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/60*((b*x+a)^2)^(5/2)*(60*ln(x)*x^5*b^5-300*a*b^4*x^4-300*a^2*b^3*x^3-200
*a^3*b^2*x^2-75*a^4*b*x-12*a^5)/(b*x+a)^5/x^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.26

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^6}\,dx=\frac{60\,b^5x^5\log\left(x\right)-300\,ab^4x^4-300\,a^2b^3x^3-200\,a^3b^2x^2-75\,a^4bx-12\,a^5}{60\,x^5}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^6,x, algorithm="fricas")
```

output
```
1/60*(60*b^5*x^5*log(x) - 300*a*b^4*x^4 - 300*a^2*b^3*x^3 - 200*a^3*b^2*x^
2 - 75*a^4*b*x - 12*a^5)/x^5
```

Sympy [F]

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^6}\,dx=\int \frac{\left((a+bx)^2\right)^{\frac{5}{2}}}{x^6}\,dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/x**6,x)
```

output
```
Integral(((a + b*x)**2)**(5/2)/x**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(151) = 302$.

Time = 0.04 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.52

$$\begin{aligned}\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^6}\,dx &= (-1)^{2\,b^2x+2\,ab}b^5\log\left(2\,b^2x+2\,ab\right)\\ &\quad-(-1)^{2\,abx+2\,a^2}b^5\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)+\frac{\sqrt{b^2x^2+2\,abx+a^2}b^6x}{2\,a^2}\\ &\quad+\frac{3\sqrt{b^2x^2+2\,abx+a^2}b^5}{2\,a}+\frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^6x}{4\,a^4}+\frac{7\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^5}{12\,a^3}\\ &\quad-\frac{2\,(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^5}{15\,a^5}-\frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^4}{3\,a^4x}+\frac{2\,(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}b^3}{15\,a^5x^2}\\ &\quad-\frac{11\,(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}b^2}{60\,a^4x^3}+\frac{3\,(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}b}{20\,a^3x^4}-\frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}}{5\,a^2x^5}\end{aligned}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^6,x, algorithm="maxima")
```

output
```
(-1)^(2*b^2*x + 2*a*b)*b^5*log(2*b^2*x + 2*a*b) - (-1)^(2*a*b*x + 2*a^2)*b
^5*log(2*a*b*x/abs(x) + 2*a^2/abs(x)) + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*
b^6*x/a^2 + 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^5/a + 1/4*(b^2*x^2 + 2*a*b
*x + a^2)^(3/2)*b^6*x/a^4 + 7/12*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^5/a^3 -
 2/15*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*b^5/a^5 - 1/3*(b^2*x^2 + 2*a*b*x + a
^2)^(5/2)*b^4/(a^4*x) + 2/15*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b^3/(a^5*x^2)
 - 11/60*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b^2/(a^4*x^3) + 3/20*(b^2*x^2 + 2
*a*b*x + a^2)^(7/2)*b/(a^3*x^4) - 1/5*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)/(a^2
*x^5)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.42

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^6}\,dx = b^5\log(|x|)\,\mathrm{sgn}(bx+a) - \frac{300\,ab^4x^4\mathrm{sgn}(bx+a)+300\,a^2b^3x^3\mathrm{sgn}(bx+a)+200\,a^3b^2x^2\mathrm{sgn}(bx+a)+75\,a^4bx\mathrm{sgn}(bx+a)+12\,a^5\mathrm{sgn}}{60\,x^5}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^6,x, algorithm="giac")
```

output
```
b^5*log(abs(x))*sgn(b*x + a) - 1/60*(300*a*b^4*x^4*sgn(b*x + a) + 300*a^2*
b^3*x^3*sgn(b*x + a) + 200*a^3*b^2*x^2*sgn(b*x + a) + 75*a^4*b*x*sgn(b*x +
 a) + 12*a^5*sgn(b*x + a))/x^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^6}\,dx = \int \frac{(a^2+2\,a\,b\,x+b^2\,x^2)^{5/2}}{x^6}\,dx$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/x^6,x)
```

output
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.26

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^6}\,dx = \frac{60\log(x)\,b^5x^5-12a^5-75a^4bx-200a^3b^2x^2-300a^2b^3x^3-300a\,b^4x^4}{60x^5}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^6,x)
```

output
```
(60*log(x)*b**5*x**5 - 12*a**5 - 75*a**4*b*x - 200*a**3*b**2*x**2 - 300*a*
*2*b**3*x**3 - 300*a*b**4*x**4)/(60*x**5)
```

3.84 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^7} dx$

Optimal result

Integrand size = 24, antiderivative size = 35

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{x^7} dx = -\frac{(a + bx)(a^2 + 2abx + b^2x^2)^{5/2}}{6ax^6}$$

output
```
-1/6*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/a/x^6
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{x^7} dx = -\frac{(a + bx)^5\sqrt{(a + bx)^2}}{6ax^6}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^7,x]
```

output
```
-1/6*((a + b*x)^5*Sqrt[(a + b*x)^2])/(a*x^6)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1102, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^7}dx$$
$$\Big\downarrow \text{1102}$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5}{x^7}dx}{b^5(a+bx)}$$
$$\Big\downarrow \text{27}$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5}{x^7}dx}{a+bx}$$
$$\Big\downarrow \text{48}$$
$$-\frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6ax^6}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^7,x]
```

output
```
-1/6*((a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(a*x^6)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(31) = 62$.

Time = 0.71 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.06

method	result	size
gosper	$-\frac{(6b^5x^5+15a\,b^4x^4+20a^2b^3x^3+15a^3b^2x^2+6a^4bx+a^5)\left((bx+a)^2\right)^{\frac{5}{2}}}{6x^6(bx+a)^5}$	72
default	$-\frac{(6b^5x^5+15a\,b^4x^4+20a^2b^3x^3+15a^3b^2x^2+6a^4bx+a^5)\left((bx+a)^2\right)^{\frac{5}{2}}}{6x^6(bx+a)^5}$	72
risch	$\frac{\sqrt{(bx+a)^2}\,\left(-b^5x^5-\frac{5}{2}a\,b^4x^4-\frac{10}{3}a^2b^3x^3-\frac{5}{2}a^3b^2x^2-a^4bx-\frac{1}{6}a^5\right)}{(bx+a)x^6}$	73
orering	$-\frac{(6b^5x^5+15a\,b^4x^4+20a^2b^3x^3+15a^3b^2x^2+6a^4bx+a^5)\,(b^2x^2+2abx+a^2)^{\frac{5}{2}}}{6x^6(bx+a)^5}$	81

input
```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(6*b^5*x^5+15*a*b^4*x^4+20*a^2*b^3*x^3+15*a^3*b^2*x^2+6*a^4*b*x+a^5)*
((b*x+a)^2)^(5/2)/x^6/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.57

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^7}\,dx = -\frac{6\,b^5x^5+15\,ab^4x^4+20\,a^2b^3x^3+15\,a^3b^2x^2+6\,a^4bx+a^5}{6\,x^6}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^7,x, algorithm="fricas")
```

output
```
-1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b
*x + a^5)/x^6
```

Sympy [F]

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^7}\,dx=\int \frac{\left((a+bx)^2\right)^{\frac{5}{2}}}{x^7}\,dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/x**7,x)
```

output
```
Integral(((a + b*x)**2)**(5/2)/x**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(31) = 62$.

Time = 0.05 (sec) , antiderivative size = 196, normalized size of antiderivative = 5.60

$$\begin{aligned}\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^7}\,dx &= \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^6}{6\,a^6} \\ &+ \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^5}{6\,a^5x} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}b^4}{6\,a^6x^2} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}b^3}{6\,a^5x^3} \\ &- \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}b^2}{6\,a^4x^4} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}b}{6\,a^3x^5} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}}{6\,a^2x^6}\end{aligned}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^7,x, algorithm="maxima")
```

output
```
1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*b^6/a^6 + 1/6*(b^2*x^2 + 2*a*b*x + a^2
)^(5/2)*b^5/(a^5*x) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b^4/(a^6*x^2) +
1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b^3/(a^5*x^3) - 1/6*(b^2*x^2 + 2*a*b*x
 + a^2)^(7/2)*b^2/(a^4*x^4) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b/(a^3*x
^5) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)/(a^2*x^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(31) = 62$.

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.06

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{x^7} \, dx = -\frac{b^6 \operatorname{sgn}(bx + a)}{6a} - \frac{6b^5x^5 \operatorname{sgn}(bx + a) + 15ab^4x^4 \operatorname{sgn}(bx + a) + 20a^2b^3x^3 \operatorname{sgn}(bx + a) + 15a^3b^2x^2 \operatorname{sgn}(bx + a) + 6a^4bx \operatorname{sgn}(bx}{6x^6}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^7,x, algorithm="giac")
```

output
```
-1/6*b^6*sgn(b*x + a)/a - 1/6*(6*b^5*x^5*sgn(b*x + a) + 15*a*b^4*x^4*sgn(b
*x + a) + 20*a^2*b^3*x^3*sgn(b*x + a) + 15*a^3*b^2*x^2*sgn(b*x + a) + 6*a^
4*b*x*sgn(b*x + a) + a^5*sgn(b*x + a))/x^6
```

Mupad [B] (verification not implemented)

Time = 9.92 (sec) , antiderivative size = 207, normalized size of antiderivative = 5.91

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{x^7} \, dx = -\frac{a^5 \sqrt{a^2 + 2abx + b^2x^2}}{6x^6 (a + bx)} - \frac{b^5 \sqrt{a^2 + 2abx + b^2x^2}}{x (a + bx)} - \frac{10a^2b^3 \sqrt{a^2 + 2abx + b^2x^2}}{3x^3 (a + bx)} - \frac{5a^3b^2 \sqrt{a^2 + 2abx + b^2x^2}}{2x^4 (a + bx)} - \frac{5ab^4 \sqrt{a^2 + 2abx + b^2x^2}}{2x^2 (a + bx)} - \frac{a^4b \sqrt{a^2 + 2abx + b^2x^2}}{x^5 (a + bx)}$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/x^7,x)
```

output
```
- (a^5*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(6*x^6*(a + b*x)) - (b^5*(a^2 + b^
2*x^2 + 2*a*b*x)^(1/2))/(x*(a + b*x)) - (10*a^2*b^3*(a^2 + b^2*x^2 + 2*a*b
*x)^(1/2))/(3*x^3*(a + b*x)) - (5*a^3*b^2*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))
/(2*x^4*(a + b*x)) - (5*a*b^4*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(2*x^2*(a +
 b*x)) - (a^4*b*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^5*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.63

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{x^7} dx = \frac{-6b^5x^5 - 15a\,b^4x^4 - 20a^2b^3x^3 - 15a^3b^2x^2 - 6a^4bx - a^5}{6x^6}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^7,x)
```

output
```
( - a**5 - 6*a**4*b*x - 15*a**3*b**2*x**2 - 20*a**2*b**3*x**3 - 15*a*b**4*
x**4 - 6*b**5*x**5)/(6*x**6)
```

3.85 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^8} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 76

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^8} \, dx = -\frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{7ax^7} + \frac{b(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{42a^2x^6}$$

output
```
-1/7*(b*x+a)^5*((b*x+a)^2)^(1/2)/a/x^7+1/42*b*(b*x+a)^5*((b*x+a)^2)^(1/2)/
a^2/x^6
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.47

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^8} \, dx = -\frac{(6a-bx)(a+bx)^5\sqrt{(a+bx)^2}}{42a^2x^7}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^8,x]
```

output
```
-1/42*((6*a - b*x)*(a + b*x)^5*Sqrt[(a + b*x)^2])/(a^2*x^7)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^8}dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5}{x^8}dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5}{x^8}dx}{a+bx}$$

$$\Big\downarrow \text{55}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{b\int \frac{(a+bx)^5}{x^7}dx}{7a}-\frac{(a+bx)^6}{7ax^7}\right)}{a+bx}$$

$$\Big\downarrow \text{48}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{b(a+bx)^6}{42a^2x^6}-\frac{(a+bx)^6}{7ax^7}\right)}{a+bx}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^8,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/7*(a + b*x)^6/(a*x^7) + (b*(a + b*x)^6)
/(42*a^2*x^6)))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

method	result	size
risch	$\frac{\sqrt{(bx+a)^{2}}\,\left(-\frac{1}{2}b^{5}x^{5}-\frac{5}{3}a\,b^{4}x^{4}-\frac{5}{2}a^{2}b^{3}x^{3}-2a^{3}b^{2}x^{2}-\frac{5}{6}a^{4}bx-\frac{1}{7}a^{5}\right)}{(bx+a)x^{7}}$	73
gosper	$-\frac{\left(21b^{5}x^{5}+70a\,b^{4}x^{4}+105a^{2}b^{3}x^{3}+84a^{3}b^{2}x^{2}+35a^{4}bx+6a^{5}\right)\left((bx+a)^{2}\right)^{\frac{5}{2}}}{42x^{7}(bx+a)^{5}}$	74
default	$-\frac{\left(21b^{5}x^{5}+70a\,b^{4}x^{4}+105a^{2}b^{3}x^{3}+84a^{3}b^{2}x^{2}+35a^{4}bx+6a^{5}\right)\left((bx+a)^{2}\right)^{\frac{5}{2}}}{42x^{7}(bx+a)^{5}}$	74
orering	$-\frac{\left(21b^{5}x^{5}+70a\,b^{4}x^{4}+105a^{2}b^{3}x^{3}+84a^{3}b^{2}x^{2}+35a^{4}bx+6a^{5}\right)\left(b^{2}x^{2}+2abx+a^{2}\right)^{\frac{5}{2}}}{42x^{7}(bx+a)^{5}}$	83

input
```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*(-1/2*b^5*x^5-5/3*a*b^4*x^4-5/2*a^2*b^3*x^3-2*a^
3*b^2*x^2-5/6*a^4*b*x-1/7*a^5)/x^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{x^8} dx = -\frac{21 b^5x^5 + 70 ab^4x^4 + 105 a^2b^3x^3 + 84 a^3b^2x^2 + 35 a^4bx + 6 a^5}{42 x^7}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^8,x, algorithm="fricas")
```

output
```
-1/42*(21*b^5*x^5 + 70*a*b^4*x^4 + 105*a^2*b^3*x^3 + 84*a^3*b^2*x^2 + 35*a
^4*b*x + 6*a^5)/x^7
```

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{x^8} dx = \int \frac{\left((a + bx)^2\right)^{\frac{5}{2}}}{x^8} dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/x**8,x)
```

output
```
Integral(((a + b*x)**2)**(5/2)/x**8, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(50) = 100$.

Time = 0.04 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.96

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{x^8}\,dx = -\frac{(b^2x^2 + 2\,abx + a^2)^{\frac{5}{2}}b^7}{6\,a^7} - \frac{(b^2x^2 + 2\,abx + a^2)^{\frac{5}{2}}b^6}{6\,a^6x} + \frac{(b^2x^2 + 2\,abx + a^2)^{\frac{7}{2}}b^5}{6\,a^7x^2} - \frac{(b^2x^2 + 2\,abx + a^2)^{\frac{7}{2}}b^4}{6\,a^6x^3} + \frac{(b^2x^2 + 2\,abx + a^2)^{\frac{7}{2}}b^3}{6\,a^5x^4} - \frac{(b^2x^2 + 2\,abx + a^2)^{\frac{7}{2}}b^2}{6\,a^4x^5} + \frac{(b^2x^2 + 2\,abx + a^2)^{\frac{7}{2}}b}{6\,a^3x^6} - \frac{(b^2x^2 + 2\,abx + a^2)^{\frac{7}{2}}}{7\,a^2x^7}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^8,x, algorithm="maxima")
```

output
```
-1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*b^7/a^7 - 1/6*(b^2*x^2 + 2*a*b*x + a^
2)^(5/2)*b^6/(a^6*x) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b^5/(a^7*x^2) -
 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b^4/(a^6*x^3) + 1/6*(b^2*x^2 + 2*a*b*
x + a^2)^(7/2)*b^3/(a^5*x^4) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b^2/(a^
4*x^5) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b/(a^3*x^6) - 1/7*(b^2*x^2 +
2*a*b*x + a^2)^(7/2)/(a^2*x^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(50) = 100$.

Time = 0.21 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.42

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{x^8}\,dx = \frac{b^7\mathrm{sgn}(bx + a)}{42\,a^2} - \frac{21\,b^5x^5\mathrm{sgn}(bx + a) + 70\,ab^4x^4\mathrm{sgn}(bx + a) + 105\,a^2b^3x^3\mathrm{sgn}(bx + a) + 84\,a^3b^2x^2\mathrm{sgn}(bx + a) + 35\,a^4bx\mathrm{sgn}}{42\,x^7}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^8,x, algorithm="giac")
```

output
```
1/42*b^7*sgn(b*x + a)/a^2 - 1/42*(21*b^5*x^5*sgn(b*x + a) + 70*a*b^4*x^4*s
gn(b*x + a) + 105*a^2*b^3*x^3*sgn(b*x + a) + 84*a^3*b^2*x^2*sgn(b*x + a) +
 35*a^4*b*x*sgn(b*x + a) + 6*a^5*sgn(b*x + a))/x^7
```

Mupad [B] (verification not implemented)

Time = 10.38 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.72

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^8}\,dx = -\frac{a^5\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{7\,x^7\,(a+b\,x)} - \frac{b^5\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{2\,x^2\,(a+b\,x)} - \frac{5\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{2\,x^4\,(a+b\,x)} - \frac{2\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^5\,(a+b\,x)} - \frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{3\,x^3\,(a+b\,x)} - \frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{6\,x^6\,(a+b\,x)}$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/x^8,x)
```

output
```
- (a^5*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(7*x^7*(a + b*x)) - (b^5*(a^2 + b^
2*x^2 + 2*a*b*x)^(1/2))/(2*x^2*(a + b*x)) - (5*a^2*b^3*(a^2 + b^2*x^2 + 2*
a*b*x)^(1/2))/(2*x^4*(a + b*x)) - (2*a^3*b^2*(a^2 + b^2*x^2 + 2*a*b*x)^(1/
2))/(x^5*(a + b*x)) - (5*a*b^4*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(3*x^3*(a
+ b*x)) - (5*a^4*b*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(6*x^6*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^8}\,dx = \frac{-21b^5x^5-70a\,b^4x^4-105a^2b^3x^3-84a^3b^2x^2-35a^4bx-6a^5}{42x^7}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^8,x)
```

output
```
( - 6*a**5 - 35*a**4*b*x - 84*a**3*b**2*x**2 - 105*a**2*b**3*x**3 - 70*a*b
**4*x**4 - 21*b**5*x**5)/(42*x**7)
```

3.86 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^9}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^9}\,dx = -\frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{8ax^8} + \frac{b(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{28a^2x^7} - \frac{b^2(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{168a^3x^6}$$

output
```
-1/8*(b*x+a)^5*((b*x+a)^2)^(1/2)/a/x^8+1/28*b*(b*x+a)^5*((b*x+a)^2)^(1/2)/
a^2/x^7-1/168*b^2*(b*x+a)^5*((b*x+a)^2)^(1/2)/a^3/x^6
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.66

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^9}\,dx = -\frac{\sqrt{(a+bx)^2}\left(21a^5+120a^4bx+280a^3b^2x^2+336a^2b^3x^3+210ab^4x^4+56b^5x^5\right)}{168x^8(a+bx)}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^9,x]
```

output
```
-1/168*(Sqrt[(a + b*x)^2]*(21*a^5 + 120*a^4*b*x + 280*a^3*b^2*x^2 + 336*a^
2*b^3*x^3 + 210*a*b^4*x^4 + 56*b^5*x^5))/(x^8*(a + b*x))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.78, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1102, 27, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^9}\,dx$$

$$\Big\downarrow \ 1102$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5}{x^9}dx}{b^5(a+bx)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5}{x^9}dx}{a+bx}$$

$$\Big\downarrow \ 55$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{b\int \frac{(a+bx)^5}{x^8}dx}{4a}-\frac{(a+bx)^6}{8ax^8}\right)}{a+bx}$$

$$\Big\downarrow \ 55$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{b\left(-\frac{b\int \frac{(a+bx)^5}{x^7}dx}{7a}-\frac{(a+bx)^6}{7ax^7}\right)}{4a}-\frac{(a+bx)^6}{8ax^8}\right)}{a+bx}$$

$$\Big\downarrow \ 48$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{b\left(\frac{b(a+bx)^6}{42a^2x^6}-\frac{(a+bx)^6}{7ax^7}\right)}{4a}-\frac{(a+bx)^6}{8ax^8}\right)}{a+bx}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^9,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/8*(a + b*x)^6/(a*x^8) - (b*(-1/7*(a + b
*x)^6/(a*x^7) + (b*(a + b*x)^6)/(42*a^2*x^6)))/(4*a)))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.63

method	result	size
risch	$\frac{\sqrt{(bx+a)^2}\,\left(-\frac{1}{3}b^5x^5-\frac{5}{4}a\,b^4x^4-2a^2b^3x^3-\frac{5}{3}a^3b^2x^2-\frac{5}{7}a^4bx-\frac{1}{8}a^5\right)}{(bx+a)x^8}$	73
gosper	$-\frac{(56b^5x^5+210a\,b^4x^4+336a^2b^3x^3+280a^3b^2x^2+120a^4bx+21a^5)\left((bx+a)^2\right)^{\frac{5}{2}}}{168x^8(bx+a)^5}$	74
default	$-\frac{(56b^5x^5+210a\,b^4x^4+336a^2b^3x^3+280a^3b^2x^2+120a^4bx+21a^5)\left((bx+a)^2\right)^{\frac{5}{2}}}{168x^8(bx+a)^5}$	74
orering	$-\frac{(56b^5x^5+210a\,b^4x^4+336a^2b^3x^3+280a^3b^2x^2+120a^4bx+21a^5)\,(b^2x^2+2abx+a^2)^{\frac{5}{2}}}{168x^8(bx+a)^5}$	83

input
```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*(-1/3*b^5*x^5-5/4*a*b^4*x^4-2*a^2*b^3*x^3-5/3*a^
3*b^2*x^2-5/7*a^4*b*x-1/8*a^5)/x^8
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.49

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^9}\,dx = -\frac{56\,b^5x^5+210\,ab^4x^4+336\,a^2b^3x^3+280\,a^3b^2x^2+120\,a^4bx+21\,a^5}{168\,x^8}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^9,x, algorithm="fricas")
```

output
```
-1/168*(56*b^5*x^5 + 210*a*b^4*x^4 + 336*a^2*b^3*x^3 + 280*a^3*b^2*x^2 + 1
20*a^4*b*x + 21*a^5)/x^8
```

Sympy [F]

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^9}\,dx = \int \frac{\left((a+bx)^2\right)^{\frac{5}{2}}}{x^9}\,dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/x**9,x)
```

output
```
Integral(((a + b*x)**2)**(5/2)/x**9, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(77) = 154$.

Time = 0.05 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.19

$$\begin{aligned}\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^9}\,dx &= \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^8}{6\,a^8} \\ &+ \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^7}{6\,a^7x} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}b^6}{6\,a^8x^2} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}b^5}{6\,a^7x^3} \\ &- \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}b^4}{6\,a^6x^4} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}b^3}{6\,a^5x^5} \\ &- \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}b^2}{6\,a^4x^6} + \frac{9\,(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}b}{56\,a^3x^7} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}}{8\,a^2x^8}\end{aligned}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^9,x, algorithm="maxima")
```

output
```
1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*b^8/a^8 + 1/6*(b^2*x^2 + 2*a*b*x + a^2
)^(5/2)*b^7/(a^7*x) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b^6/(a^8*x^2) +
1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b^5/(a^7*x^3) - 1/6*(b^2*x^2 + 2*a*b*x
 + a^2)^(7/2)*b^4/(a^6*x^4) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b^3/(a^5
*x^5) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b^2/(a^4*x^6) + 9/56*(b^2*x^2
+ 2*a*b*x + a^2)^(7/2)*b/(a^3*x^7) - 1/8*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)/(
a^2*x^8)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^9}\,dx = -\frac{b^8\mathrm{sgn}(bx+a)}{168\,a^3} - \frac{56\,b^5x^5\mathrm{sgn}(bx+a)+210\,ab^4x^4\mathrm{sgn}(bx+a)+336\,a^2b^3x^3\mathrm{sgn}(bx+a)+280\,a^3b^2x^2\mathrm{sgn}(bx+a)+120\,a^4bx}{168\,x^8}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^9,x, algorithm="giac")
```

output
```
-1/168*b^8*sgn(b*x + a)/a^3 - 1/168*(56*b^5*x^5*sgn(b*x + a) + 210*a*b^4*x
^4*sgn(b*x + a) + 336*a^2*b^3*x^3*sgn(b*x + a) + 280*a^3*b^2*x^2*sgn(b*x +
 a) + 120*a^4*b*x*sgn(b*x + a) + 21*a^5*sgn(b*x + a))/x^8
```

Mupad [B] (verification not implemented)

Time = 10.15 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.78

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^9}\,dx = -\frac{a^5\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{8\,x^8\,(a+b\,x)} - \frac{b^5\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{3\,x^3\,(a+b\,x)} - \frac{2\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^5\,(a+b\,x)} - \frac{5\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{3\,x^6\,(a+b\,x)} - \frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{4\,x^4\,(a+b\,x)} - \frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{7\,x^7\,(a+b\,x)}$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/x^9,x)
```

output
```
- (a^5*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(8*x^8*(a + b*x)) - (b^5*(a^2 + b^
2*x^2 + 2*a*b*x)^(1/2))/(3*x^3*(a + b*x)) - (2*a^2*b^3*(a^2 + b^2*x^2 + 2*
a*b*x)^(1/2))/(x^5*(a + b*x)) - (5*a^3*b^2*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2)
)/(3*x^6*(a + b*x)) - (5*a*b^4*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(4*x^4*(a
+ b*x)) - (5*a^4*b*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(7*x^7*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.49

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^9}\,dx = \frac{-56b^5x^5-210a\,b^4x^4-336a^2b^3x^3-280a^3b^2x^2-120a^4bx-21a^5}{168x^8}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^9,x)
```

output
```
( - 21*a**5 - 120*a**4*b*x - 280*a**3*b**2*x**2 - 336*a**2*b**3*x**3 - 210
*a*b**4*x**4 - 56*b**5*x**5)/(168*x**8)
```

3.87 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{10}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 229

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{10}}\,dx = -\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{9x^9(a+bx)} - \frac{5a^4b\sqrt{a^2+2abx+b^2x^2}}{8x^8(a+bx)} - \frac{10a^3b^2\sqrt{a^2+2abx+b^2x^2}}{7x^7(a+bx)} - \frac{5a^2b^3\sqrt{a^2+2abx+b^2x^2}}{3x^6(a+bx)} - \frac{ab^4\sqrt{a^2+2abx+b^2x^2}}{x^5(a+bx)} - \frac{b^5\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)}$$

output
```
-1/9*a^5*((b*x+a)^2)^(1/2)/x^9/(b*x+a)-5/8*a^4*b*((b*x+a)^2)^(1/2)/x^8/(b*
x+a)-10/7*a^3*b^2*((b*x+a)^2)^(1/2)/x^7/(b*x+a)-5/3*a^2*b^3*((b*x+a)^2)^(1
/2)/x^6/(b*x+a)-a*b^4*((b*x+a)^2)^(1/2)/x^5/(b*x+a)-1/4*b^5*((b*x+a)^2)^(1
/2)/x^4/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.34

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{10}}\,dx = -\frac{\sqrt{(a+bx)^2}(56a^5+315a^4bx+720a^3b^2x^2+840a^2b^3x^3+504ab^4x^4+126b^5x^5)}{504x^9(a+bx)}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^10,x]
```

output
```
-1/504*(Sqrt[(a + b*x)^2]*(56*a^5 + 315*a^4*b*x + 720*a^3*b^2*x^2 + 840*a^
2*b^3*x^3 + 504*a*b^4*x^4 + 126*b^5*x^5))/(x^9*(a + b*x))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.41, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{10}}dx$$

$$\Big\downarrow \text{ 1102}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5}{x^{10}}dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5}{x^{10}}dx}{a+bx}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a^5}{x^{10}}+\frac{5ba^4}{x^9}+\frac{10b^2a^3}{x^8}+\frac{10b^3a^2}{x^7}+\frac{5b^4a}{x^6}+\frac{b^5}{x^5}\right)dx}{a+bx}$$

$$\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}\right)\sqrt{a^2+2abx+b^2x^2}}{a+bx}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^10,x]
```

output
```
((-1/9*a^5/x^9 - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3
*x^6) - (a*b^4)/x^5 - b^5/(4*x^4))*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(a + b*x
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.32

method	result	size
risch	$\frac{\sqrt{(bx+a)^2}\left(-\frac{1}{4}b^5x^5-a\,b^4x^4-\frac{5}{3}a^2b^3x^3-\frac{10}{7}a^3b^2x^2-\frac{5}{8}a^4bx-\frac{1}{9}a^5\right)}{(bx+a)x^9}$	73
gosper	$-\frac{\left(126b^5x^5+504a\,b^4x^4+840a^2b^3x^3+720a^3b^2x^2+315a^4bx+56a^5\right)\left((bx+a)^2\right)^{\frac{5}{2}}}{504x^9(bx+a)^5}$	74
default	$-\frac{\left(126b^5x^5+504a\,b^4x^4+840a^2b^3x^3+720a^3b^2x^2+315a^4bx+56a^5\right)\left((bx+a)^2\right)^{\frac{5}{2}}}{504x^9(bx+a)^5}$	74
orering	$-\frac{\left(126b^5x^5+504a\,b^4x^4+840a^2b^3x^3+720a^3b^2x^2+315a^4bx+56a^5\right)\left(b^2x^2+2abx+a^2\right)^{\frac{5}{2}}}{504x^9(bx+a)^5}$	83

input
```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*(-1/4*b^5*x^5-a*b^4*x^4-5/3*a^2*b^3*x^3-10/7*a^3
*b^2*x^2-5/8*a^4*b*x-1/9*a^5)/x^9
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.25

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{10}}\,dx = -\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^10,x, algorithm="fricas")
```

output
```
-1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 +
315*a^4*b*x + 56*a^5)/x^9
```

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{x^{10}}\,dx = \int \frac{\left((a + bx)^2\right)^{\frac{5}{2}}}{x^{10}}\,dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/x**10,x)
```

output
```
Integral(((a + b*x)**2)**(5/2)/x**10, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.24

$$\begin{aligned}\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{x^{10}}\,dx = &-\frac{(b^2x^2 + 2\,abx + a^2)^{\frac{5}{2}}b^9}{6\,a^9} \\ &- \frac{(b^2x^2 + 2\,abx + a^2)^{\frac{5}{2}}b^8}{6\,a^8x} + \frac{(b^2x^2 + 2\,abx + a^2)^{\frac{7}{2}}b^7}{6\,a^9x^2} - \frac{(b^2x^2 + 2\,abx + a^2)^{\frac{7}{2}}b^6}{6\,a^8x^3} \\ &+ \frac{(b^2x^2 + 2\,abx + a^2)^{\frac{7}{2}}b^5}{6\,a^7x^4} - \frac{(b^2x^2 + 2\,abx + a^2)^{\frac{7}{2}}b^4}{6\,a^6x^5} + \frac{(b^2x^2 + 2\,abx + a^2)^{\frac{7}{2}}b^3}{6\,a^5x^6} \\ &- \frac{83\,(b^2x^2 + 2\,abx + a^2)^{\frac{7}{2}}b^2}{504\,a^4x^7} + \frac{11\,(b^2x^2 + 2\,abx + a^2)^{\frac{7}{2}}b}{72\,a^3x^8} - \frac{(b^2x^2 + 2\,abx + a^2)^{\frac{7}{2}}}{9\,a^2x^9}\end{aligned}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^10,x, algorithm="maxima")
```

output
```
-1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*b^9/a^9 - 1/6*(b^2*x^2 + 2*a*b*x + a^
2)^(5/2)*b^8/(a^8*x) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b^7/(a^9*x^2) -
 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b^6/(a^8*x^3) + 1/6*(b^2*x^2 + 2*a*b*
x + a^2)^(7/2)*b^5/(a^7*x^4) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b^4/(a^
6*x^5) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b^3/(a^5*x^6) - 83/504*(b^2*x
^2 + 2*a*b*x + a^2)^(7/2)*b^2/(a^4*x^7) + 11/72*(b^2*x^2 + 2*a*b*x + a^2)^
(7/2)*b/(a^3*x^8) - 1/9*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)/(a^2*x^9)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.47

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{x^{10}}\,dx = \frac{b^9 \operatorname{sgn}(bx + a)}{504\,a^4} - \frac{126\,b^5x^5\operatorname{sgn}(bx + a) + 504\,ab^4x^4\operatorname{sgn}(bx + a) + 840\,a^2b^3x^3\operatorname{sgn}(bx + a) + 720\,a^3b^2x^2\operatorname{sgn}(bx + a) + 315\,a^4b \cdots}{504\,x^9}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^10,x, algorithm="giac")
```

output
```
1/504*b^9*sgn(b*x + a)/a^4 - 1/504*(126*b^5*x^5*sgn(b*x + a) + 504*a*b^4*x
^4*sgn(b*x + a) + 840*a^2*b^3*x^3*sgn(b*x + a) + 720*a^3*b^2*x^2*sgn(b*x +
 a) + 315*a^4*b*x*sgn(b*x + a) + 56*a^5*sgn(b*x + a))/x^9
```

Mupad [B] (verification not implemented)

Time = 10.23 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.90

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{x^{10}}\,dx = -\frac{a^5\,\sqrt{a^2 + 2\,a\,b\,x + b^2\,x^2}}{9\,x^9\,(a + b\,x)} - \frac{b^5\,\sqrt{a^2 + 2\,a\,b\,x + b^2\,x^2}}{4\,x^4\,(a + b\,x)} - \frac{5\,a^2\,b^3\,\sqrt{a^2 + 2\,a\,b\,x + b^2\,x^2}}{3\,x^6\,(a + b\,x)} - \frac{10\,a^3\,b^2\,\sqrt{a^2 + 2\,a\,b\,x + b^2\,x^2}}{7\,x^7\,(a + b\,x)} - \frac{a\,b^4\,\sqrt{a^2 + 2\,a\,b\,x + b^2\,x^2}}{x^5\,(a + b\,x)} - \frac{5\,a^4\,b\,\sqrt{a^2 + 2\,a\,b\,x + b^2\,x^2}}{8\,x^8\,(a + b\,x)}$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/x^10,x)
```

output
```
- (a^5*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(9*x^9*(a + b*x)) - (b^5*(a^2 + b^
2*x^2 + 2*a*b*x)^(1/2))/(4*x^4*(a + b*x)) - (5*a^2*b^3*(a^2 + b^2*x^2 + 2*
a*b*x)^(1/2))/(3*x^6*(a + b*x)) - (10*a^3*b^2*(a^2 + b^2*x^2 + 2*a*b*x)^(1
/2))/(7*x^7*(a + b*x)) - (a*b^4*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^5*(a +
 b*x)) - (5*a^4*b*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(8*x^8*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.25

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{x^{10}} dx = \frac{-126b^5x^5 - 504a\,b^4x^4 - 840a^2b^3x^3 - 720a^3b^2x^2 - 315a^4bx - 56a^5}{504x^9}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^10,x)
```

output
```
( - 56*a**5 - 315*a**4*b*x - 720*a**3*b**2*x**2 - 840*a**2*b**3*x**3 - 504
*a*b**4*x**4 - 126*b**5*x**5)/(504*x**9)
```

3.88 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{11}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 231

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{11}}\,dx = -\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{10x^{10}(a+bx)} - \frac{5a^4b\sqrt{a^2+2abx+b^2x^2}}{9x^9(a+bx)} - \frac{5a^3b^2\sqrt{a^2+2abx+b^2x^2}}{4x^8(a+bx)} - \frac{10a^2b^3\sqrt{a^2+2abx+b^2x^2}}{7x^7(a+bx)} - \frac{5ab^4\sqrt{a^2+2abx+b^2x^2}}{6x^6(a+bx)} - \frac{b^5\sqrt{a^2+2abx+b^2x^2}}{5x^5(a+bx)}$$

output
```
-1/10*a^5*((b*x+a)^2)^(1/2)/x^10/(b*x+a)-5/9*a^4*b*((b*x+a)^2)^(1/2)/x^9/(
b*x+a)-5/4*a^3*b^2*((b*x+a)^2)^(1/2)/x^8/(b*x+a)-10/7*a^2*b^3*((b*x+a)^2)^
(1/2)/x^7/(b*x+a)-5/6*a*b^4*((b*x+a)^2)^(1/2)/x^6/(b*x+a)-1/5*b^5*((b*x+a)
^2)^(1/2)/x^5/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{11}}\,dx = -\frac{\sqrt{(a+bx)^2}(126a^5+700a^4bx+1575a^3b^2x^2+1800a^2b^3x^3+1050ab^4x^4+252b^5x^5)}{1260x^{10}(a+bx)}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^11,x]
```

output
```
-1/1260*(Sqrt[(a + b*x)^2]*(126*a^5 + 700*a^4*b*x + 1575*a^3*b^2*x^2 + 180
0*a^2*b^3*x^3 + 1050*a*b^4*x^4 + 252*b^5*x^5))/(x^10*(a + b*x))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.42, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{11}}\,dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5}{x^{11}}dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5}{x^{11}}dx}{a+bx}$$

$$\Big\downarrow \text{53}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a^5}{x^{11}}+\frac{5ba^4}{x^{10}}+\frac{10b^2a^3}{x^9}+\frac{10b^3a^2}{x^8}+\frac{5b^4a}{x^7}+\frac{b^5}{x^6}\right)dx}{a+bx}$$

$$\Big\downarrow 2009$$

$$\frac{\left(-\frac{a^5}{10x^{10}}-\frac{5a^4b}{9x^9}-\frac{5a^3b^2}{4x^8}-\frac{10a^2b^3}{7x^7}-\frac{5ab^4}{6x^6}-\frac{b^5}{5x^5}\right)\sqrt{a^2+2abx+b^2x^2}}{a+bx}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^11,x]
```

output
```
((-1/10*a^5/x^10 - (5*a^4*b)/(9*x^9) - (5*a^3*b^2)/(4*x^8) - (10*a^2*b^3)/
(7*x^7) - (5*a*b^4)/(6*x^6) - b^5/(5*x^5))*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/
(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.32

method	result	size
risch	$\frac{\sqrt{(bx+a)^2}\,\left(-\frac{1}{5}b^5x^5-\frac{5}{6}a\,b^4x^4-\frac{10}{7}a^2b^3x^3-\frac{5}{4}a^3b^2x^2-\frac{5}{9}a^4bx-\frac{1}{10}a^5\right)}{(bx+a)x^{10}}$	73
gosper	$-\frac{\left(252b^5x^5+1050a\,b^4x^4+1800a^2b^3x^3+1575a^3b^2x^2+700a^4bx+126a^5\right)\left((bx+a)^2\right)^{\frac{5}{2}}}{1260x^{10}(bx+a)^5}$	74
default	$-\frac{\left(252b^5x^5+1050a\,b^4x^4+1800a^2b^3x^3+1575a^3b^2x^2+700a^4bx+126a^5\right)\left((bx+a)^2\right)^{\frac{5}{2}}}{1260x^{10}(bx+a)^5}$	74
orering	$-\frac{\left(252b^5x^5+1050a\,b^4x^4+1800a^2b^3x^3+1575a^3b^2x^2+700a^4bx+126a^5\right)\left(b^2x^2+2abx+a^2\right)^{\frac{5}{2}}}{1260x^{10}(bx+a)^5}$	83

input
```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*(-1/5*b^5*x^5-5/6*a*b^4*x^4-10/7*a^2*b^3*x^3-5/4
*a^3*b^2*x^2-5/9*a^4*b*x-1/10*a^5)/x^10
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.25

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{11}}\,dx = -\frac{252\,b^5x^5+1050\,ab^4x^4+1800\,a^2b^3x^3+1575\,a^3b^2x^2+700\,a^4bx+126\,a^5}{1260\,x^{10}}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^11,x, algorithm="fricas")
```

output
```
-1/1260*(252*b^5*x^5 + 1050*a*b^4*x^4 + 1800*a^2*b^3*x^3 + 1575*a^3*b^2*x^
2 + 700*a^4*b*x + 126*a^5)/x^10
```

Sympy [F]

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{11}}\,dx = \int \frac{\left((a+bx)^2\right)^{\frac{5}{2}}}{x^{11}}\,dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/x**11,x)
```

output
```
Integral(((a + b*x)**2)**(5/2)/x**11, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(153) = 306$.

Time = 0.05 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.35

$$\begin{aligned}\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{11}}\,dx &= \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^{10}}{6\,a^{10}} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}b^9}{6\,a^9x} \\ &- \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}b^8}{6\,a^{10}x^2} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}b^7}{6\,a^9x^3} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}b^6}{6\,a^8x^4} \\ &+ \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}b^5}{6\,a^7x^5} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}b^4}{6\,a^6x^6} + \frac{209\,(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}b^3}{1260\,a^5x^7} \\ &- \frac{29\,(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}b^2}{180\,a^4x^8} + \frac{13\,(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}b}{90\,a^3x^9} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}}{10\,a^2x^{10}}\end{aligned}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^11,x, algorithm="maxima")
```

output
```
1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*b^10/a^10 + 1/6*(b^2*x^2 + 2*a*b*x + a
^2)^(5/2)*b^9/(a^9*x) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b^8/(a^10*x^2)
 + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b^7/(a^9*x^3) - 1/6*(b^2*x^2 + 2*a*
b*x + a^2)^(7/2)*b^6/(a^8*x^4) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b^5/(
a^7*x^5) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b^4/(a^6*x^6) + 209/1260*(b
^2*x^2 + 2*a*b*x + a^2)^(7/2)*b^3/(a^5*x^7) - 29/180*(b^2*x^2 + 2*a*b*x +
a^2)^(7/2)*b^2/(a^4*x^8) + 13/90*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b/(a^3*x^
9) - 1/10*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)/(a^2*x^10)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.47

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{11}}\,dx = -\frac{b^{10}\mathrm{sgn}(bx+a)}{1260\,a^5} - \frac{252\,b^5x^5\mathrm{sgn}(bx+a)+1050\,ab^4x^4\mathrm{sgn}(bx+a)+1800\,a^2b^3x^3\mathrm{sgn}(bx+a)+1575\,a^3b^2x^2\mathrm{sgn}(bx+a)+700}{1260\,x^{10}}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^11,x, algorithm="giac")
```

output
```
-1/1260*b^10*sgn(b*x + a)/a^5 - 1/1260*(252*b^5*x^5*sgn(b*x + a) + 1050*a*
b^4*x^4*sgn(b*x + a) + 1800*a^2*b^3*x^3*sgn(b*x + a) + 1575*a^3*b^2*x^2*sg
n(b*x + a) + 700*a^4*b*x*sgn(b*x + a) + 126*a^5*sgn(b*x + a))/x^10
```

Mupad [B] (verification not implemented)

Time = 10.14 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.90

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{11}}\,dx = -\frac{a^5\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{10\,x^{10}\,(a+b\,x)} - \frac{b^5\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{5\,x^5\,(a+b\,x)} - \frac{10\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{7\,x^7\,(a+b\,x)} - \frac{5\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{4\,x^8\,(a+b\,x)} - \frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{6\,x^6\,(a+b\,x)} - \frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{9\,x^9\,(a+b\,x)}$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/x^11,x)
```

output
```
- (a^5*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(10*x^10*(a + b*x)) - (b^5*(a^2 +
b^2*x^2 + 2*a*b*x)^(1/2))/(5*x^5*(a + b*x)) - (10*a^2*b^3*(a^2 + b^2*x^2 +
 2*a*b*x)^(1/2))/(7*x^7*(a + b*x)) - (5*a^3*b^2*(a^2 + b^2*x^2 + 2*a*b*x)^
(1/2))/(4*x^8*(a + b*x)) - (5*a*b^4*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(6*x^
6*(a + b*x)) - (5*a^4*b*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(9*x^9*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.25

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{11}}\,dx = \frac{-252b^5x^5-1050a\,b^4x^4-1800a^2b^3x^3-1575a^3b^2x^2-700a^4bx-126a^5}{1260x^{10}}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^11,x)
```

output
```
( - 126*a**5 - 700*a**4*b*x - 1575*a**3*b**2*x**2 - 1800*a**2*b**3*x**3 -
1050*a*b**4*x**4 - 252*b**5*x**5)/(1260*x**10)
```

3.89 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{12}} dx$

Optimal result

Integrand size = 24, antiderivative size = 231

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{12}} dx = -\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{11x^{11}(a+bx)} - \frac{a^4b\sqrt{a^2+2abx+b^2x^2}}{2x^{10}(a+bx)} - \frac{10a^3b^2\sqrt{a^2+2abx+b^2x^2}}{9x^9(a+bx)} - \frac{5a^2b^3\sqrt{a^2+2abx+b^2x^2}}{4x^8(a+bx)} - \frac{5ab^4\sqrt{a^2+2abx+b^2x^2}}{7x^7(a+bx)} - \frac{b^5\sqrt{a^2+2abx+b^2x^2}}{6x^6(a+bx)}$$

output
```
-1/11*a^5*((b*x+a)^2)^(1/2)/x^11/(b*x+a)-1/2*a^4*b*((b*x+a)^2)^(1/2)/x^10/
(b*x+a)-10/9*a^3*b^2*((b*x+a)^2)^(1/2)/x^9/(b*x+a)-5/4*a^2*b^3*((b*x+a)^2)
^(1/2)/x^8/(b*x+a)-5/7*a*b^4*((b*x+a)^2)^(1/2)/x^7/(b*x+a)-1/6*b^5*((b*x+a
)^2)^(1/2)/x^6/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{12}}\,dx = -\frac{\sqrt{(a+bx)^2}(252a^5+1386a^4bx+3080a^3b^2x^2+3465a^2b^3x^3+1980ab^4x^4+462b^5x^5)}{2772x^{11}(a+bx)}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^12,x]
```

output
```
-1/2772*(Sqrt[(a + b*x)^2]*(252*a^5 + 1386*a^4*b*x + 3080*a^3*b^2*x^2 + 34
65*a^2*b^3*x^3 + 1980*a*b^4*x^4 + 462*b^5*x^5))/(x^11*(a + b*x))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.42, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{12}}\,dx$$

$$\Big\downarrow \ 1102$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5}{x^{12}}dx}{b^5(a+bx)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5}{x^{12}}dx}{a+bx}$$

$$\Big\downarrow \ 53$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a^5}{x^{12}}+\frac{5ba^4}{x^{11}}+\frac{10b^2a^3}{x^{10}}+\frac{10b^3a^2}{x^9}+\frac{5b^4a}{x^8}+\frac{b^5}{x^7}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^5}{11x^{11}}-\frac{a^4b}{2x^{10}}-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{5ab^4}{7x^7}-\frac{b^5}{6x^6}\right)\sqrt{a^2+2abx+b^2x^2}}{a+bx}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^12,x]
```

output
```
((-1/11*a^5/x^11 - (a^4*b)/(2*x^10) - (10*a^3*b^2)/(9*x^9) - (5*a^2*b^3)/(
4*x^8) - (5*a*b^4)/(7*x^7) - b^5/(6*x^6))*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(
a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.32

method	result	size
risch	$\frac{\sqrt{(bx+a)^2}\left(-\frac{1}{6}b^5x^5-\frac{5}{7}a\,b^4x^4-\frac{5}{4}a^2b^3x^3-\frac{10}{9}a^3b^2x^2-\frac{1}{2}a^4bx-\frac{1}{11}a^5\right)}{(bx+a)x^{11}}$	73
gosper	$-\frac{(462b^5x^5+1980a\,b^4x^4+3465a^2b^3x^3+3080a^3b^2x^2+1386a^4bx+252a^5)\left((bx+a)^2\right)^{\frac{5}{2}}}{2772x^{11}(bx+a)^5}$	74
default	$-\frac{(462b^5x^5+1980a\,b^4x^4+3465a^2b^3x^3+3080a^3b^2x^2+1386a^4bx+252a^5)\left((bx+a)^2\right)^{\frac{5}{2}}}{2772x^{11}(bx+a)^5}$	74
orering	$-\frac{(462b^5x^5+1980a\,b^4x^4+3465a^2b^3x^3+3080a^3b^2x^2+1386a^4bx+252a^5)(b^2x^2+2abx+a^2)^{\frac{5}{2}}}{2772x^{11}(bx+a)^5}$	83

input
```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^12,x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*(-1/6*b^5*x^5-5/7*a*b^4*x^4-5/4*a^2*b^3*x^3-10/9
*a^3*b^2*x^2-1/2*a^4*b*x-1/11*a^5)/x^11
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.25

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{12}}\,dx = -\frac{462\,b^5x^5+1980\,ab^4x^4+3465\,a^2b^3x^3+3080\,a^3b^2x^2+1386\,a^4bx+252\,a^5}{2772\,x^{11}}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^12,x, algorithm="fricas")
```

output
```
-1/2772*(462*b^5*x^5 + 1980*a*b^4*x^4 + 3465*a^2*b^3*x^3 + 3080*a^3*b^2*x^
2 + 1386*a^4*b*x + 252*a^5)/x^11
```

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{x^{12}}\,dx = \int \frac{\left((a + bx)^2\right)^{\frac{5}{2}}}{x^{12}}\,dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/x**12,x)
```

output
```
Integral(((a + b*x)**2)**(5/2)/x**12, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(153) = 306$.

Time = 0.05 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.48

$$\begin{aligned}\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{x^{12}}\,dx &= -\frac{(b^2x^2 + 2\,abx + a^2)^{\frac{5}{2}}b^{11}}{6\,a^{11}} \\ &\quad - \frac{(b^2x^2 + 2\,abx + a^2)^{\frac{5}{2}}b^{10}}{6\,a^{10}x} + \frac{(b^2x^2 + 2\,abx + a^2)^{\frac{7}{2}}b^{9}}{6\,a^{11}x^2} - \frac{(b^2x^2 + 2\,abx + a^2)^{\frac{7}{2}}b^{8}}{6\,a^{10}x^3} \\ &\quad + \frac{(b^2x^2 + 2\,abx + a^2)^{\frac{7}{2}}b^{7}}{6\,a^{9}x^4} - \frac{(b^2x^2 + 2\,abx + a^2)^{\frac{7}{2}}b^{6}}{6\,a^{8}x^5} + \frac{(b^2x^2 + 2\,abx + a^2)^{\frac{7}{2}}b^{5}}{6\,a^{7}x^6} \\ &\quad - \frac{461\,(b^2x^2 + 2\,abx + a^2)^{\frac{7}{2}}b^{4}}{2772\,a^{6}x^7} + \frac{65\,(b^2x^2 + 2\,abx + a^2)^{\frac{7}{2}}b^{3}}{396\,a^{5}x^8} \\ &\quad - \frac{31\,(b^2x^2 + 2\,abx + a^2)^{\frac{7}{2}}b^{2}}{198\,a^{4}x^9} + \frac{3\,(b^2x^2 + 2\,abx + a^2)^{\frac{7}{2}}b}{22\,a^{3}x^{10}} - \frac{(b^2x^2 + 2\,abx + a^2)^{\frac{7}{2}}}{11\,a^{2}x^{11}}\end{aligned}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^12,x, algorithm="maxima")
```

output
```
-1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*b^11/a^11 - 1/6*(b^2*x^2 + 2*a*b*x +
a^2)^(5/2)*b^10/(a^10*x) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b^9/(a^11*x
^2) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b^8/(a^10*x^3) + 1/6*(b^2*x^2 +
2*a*b*x + a^2)^(7/2)*b^7/(a^9*x^4) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b
^6/(a^8*x^5) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b^5/(a^7*x^6) - 461/277
2*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b^4/(a^6*x^7) + 65/396*(b^2*x^2 + 2*a*b*
x + a^2)^(7/2)*b^3/(a^5*x^8) - 31/198*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b^2/
(a^4*x^9) + 3/22*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*b/(a^3*x^10) - 1/11*(b^2*
x^2 + 2*a*b*x + a^2)^(7/2)/(a^2*x^11)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.47

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{12}}\,dx = \frac{b^{11}\mathrm{sgn}(bx+a)}{2772\,a^6} - \frac{462\,b^5x^5\mathrm{sgn}(bx+a)+1980\,ab^4x^4\mathrm{sgn}(bx+a)+3465\,a^2b^3x^3\mathrm{sgn}(bx+a)+3080\,a^3b^2x^2\mathrm{sgn}(bx+a)+138}{2772\,x^{11}}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^12,x, algorithm="giac")
```

output
```
1/2772*b^11*sgn(b*x + a)/a^6 - 1/2772*(462*b^5*x^5*sgn(b*x + a) + 1980*a*b
^4*x^4*sgn(b*x + a) + 3465*a^2*b^3*x^3*sgn(b*x + a) + 3080*a^3*b^2*x^2*sgn
(b*x + a) + 1386*a^4*b*x*sgn(b*x + a) + 252*a^5*sgn(b*x + a))/x^11
```

Mupad [B] (verification not implemented)

Time = 10.15 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.90

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{12}}\,dx = -\frac{a^5\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{11\,x^{11}\,(a+b\,x)} - \frac{b^5\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{6\,x^6\,(a+b\,x)} - \frac{5\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{4\,x^8\,(a+b\,x)} - \frac{10\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{9\,x^9\,(a+b\,x)} - \frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{7\,x^7\,(a+b\,x)} - \frac{a^4\,b\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{2\,x^{10}\,(a+b\,x)}$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/x^12,x)
```

output
```
- (a^5*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(11*x^11*(a + b*x)) - (b^5*(a^2 +
b^2*x^2 + 2*a*b*x)^(1/2))/(6*x^6*(a + b*x)) - (5*a^2*b^3*(a^2 + b^2*x^2 +
2*a*b*x)^(1/2))/(4*x^8*(a + b*x)) - (10*a^3*b^2*(a^2 + b^2*x^2 + 2*a*b*x)^
(1/2))/(9*x^9*(a + b*x)) - (5*a*b^4*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(7*x^
7*(a + b*x)) - (a^4*b*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(2*x^10*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.25

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{x^{12}}\,dx = \frac{-462b^5x^5 - 1980a\,b^4x^4 - 3465a^2b^3x^3 - 3080a^3b^2x^2 - 1386a^4bx - 252a^5}{2772x^{11}}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^12,x)
```

output
```
( - 252*a**5 - 1386*a**4*b*x - 3080*a**3*b**2*x**2 - 3465*a**2*b**3*x**3 -
 1980*a*b**4*x**4 - 462*b**5*x**5)/(2772*x**11)
```

3.90 $\int \frac{x^4}{\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 182

$$\int \frac{x^4}{\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{a^3x(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}}+\frac{a^2x^2(a+bx)}{2b^3\sqrt{a^2+2abx+b^2x^2}} -\frac{ax^3(a+bx)}{3b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{x^4(a+bx)}{4b\sqrt{a^2+2abx+b^2x^2}} +\frac{a^4(a+bx)\log(a+bx)}{b^5\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-a^3*x*(b*x+a)/b^4/((b*x+a)^2)^(1/2)+1/2*a^2*x^2*(b*x+a)/b^3/((b*x+a)^2)^(
1/2)-1/3*a*x^3*(b*x+a)/b^2/((b*x+a)^2)^(1/2)+1/4*x^4*(b*x+a)/b/((b*x+a)^2)
^(1/2)+a^4*(b*x+a)*ln(b*x+a)/b^5/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.75

$$\int \frac{x^4}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{\frac{bx(12a^3-6a^2bx+4ab^2x^2-3b^3x^3)\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}-\sqrt{(a+bx)^2}\right)\right)}{a^2+abx-\sqrt{a^2}\sqrt{(a+bx)^2}}-24a^4\text{arctanh}\left(\frac{bx}{\sqrt{a^2}-\sqrt{(a+bx)^2}}\right)}{12b^5}$$

input
```
Integrate[x^4/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((b*x*(12*a^3 - 6*a^2*b*x + 4*a*b^2*x^2 - 3*b^3*x^3)*(Sqrt[a^2]*b*x + a*(S
qrt[a^2] - Sqrt[(a + b*x)^2])))/(a^2 + a*b*x - Sqrt[a^2]*Sqrt[(a + b*x)^2]
) - 24*a^4*ArcTanh[(b*x)/(Sqrt[a^2] - Sqrt[(a + b*x)^2])])/(12*b^5)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a^2+2abx+b^2x^2}}dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{b(a+bx)\int \frac{x^4}{b(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{x^4}{a+bx}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{(a+bx)\int \left(\frac{a^4}{b^4(a+bx)}-\frac{a^3}{b^4}+\frac{xa^2}{b^3}-\frac{x^2a}{b^2}+\frac{x^3}{b}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(\frac{a^4\log(a+bx)}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[x^4/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((a + b*x)*(-((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*
b) + (a^4*Log[a + b*x])/b^5))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.37

method	result	size
default	$\frac{(bx+a)\left(3b^{4}x^{4}-4a\,b^{3}x^{3}+6a^{2}b^{2}x^{2}+12a^{4}\ln\left(bx+a\right)-12a^{3}bx\right)}{12\sqrt{\left(bx+a\right)^{2}}\,b^{5}}$	67
risch	$\frac{\sqrt{\left(bx+a\right)^{2}}\,\left(\frac{1}{4}b^{3}x^{4}-\frac{1}{3}a\,b^{2}x^{3}+\frac{1}{2}a^{2}b\,x^{2}-a^{3}x\right)}{(bx+a)b^{4}}+\frac{\sqrt{\left(bx+a\right)^{2}}\,a^{4}\ln\left(bx+a\right)}{(bx+a)b^{5}}$	84

input
```
int(x^4/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(b*x+a)*(3*b^4*x^4-4*a*b^3*x^3+6*a^2*b^2*x^2+12*a^4*ln(b*x+a)-12*a^3*
b*x)/((b*x+a)^2)^(1/2)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.29

$$\int \frac{x^4}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\frac{3\,b^4x^4-4\,ab^3x^3+6\,a^2b^2x^2-12\,a^3bx+12\,a^4\log\left(bx+a\right)}{12\,b^5}$$

input
```
integrate(x^4/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(3*b^4*x^4 - 4*a*b^3*x^3 + 6*a^2*b^2*x^2 - 12*a^3*b*x + 12*a^4*log(b*
x + a))/b^5
```

Sympy [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.15

$$\int \frac{x^4}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \begin{cases} \frac{a^4\left(\frac{a}{b}+x\right)\log\left(\frac{a}{b}+x\right)}{b^4\sqrt{b^2\left(\frac{a}{b}+x\right)^2}}+\sqrt{a^2+2abx+b^2x^2}\left(-\frac{25a^3}{12b^5}+\frac{13a^2x}{12b^4}-\frac{7ax^2}{12b^3}+\frac{x^3}{4b^2}\right) & \text{for } b^2\neq 0 \\ \frac{a^8\sqrt{a^2+2abx}-\frac{4a^6\left(a^2+2abx\right)^{\frac{3}{2}}}{3}+\frac{6a^4\left(a^2+2abx\right)^{\frac{5}{2}}}{5}-\frac{4a^2\left(a^2+2abx\right)^{\frac{7}{2}}}{7}+\frac{\left(a^2+2abx\right)^{\frac{9}{2}}}{9}}{16a^5b^5} & \text{for } ab\neq 0 \\ \frac{x^5}{5\sqrt{a^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**4/((b*x+a)**2)**(1/2),x)
```

output
```
Piecewise((a**4*(a/b + x)*log(a/b + x)/(b**4*sqrt(b**2*(a/b + x)**2)) + sq
rt(a**2 + 2*a*b*x + b**2*x**2)*(-25*a**3/(12*b**5) + 13*a**2*x/(12*b**4) -
 7*a*x**2/(12*b**3) + x**3/(4*b**2)), Ne(b**2, 0)), ((a**8*sqrt(a**2 + 2*a
*b*x) - 4*a**6*(a**2 + 2*a*b*x)**(3/2)/3 + 6*a**4*(a**2 + 2*a*b*x)**(5/2)/
5 - 4*a**2*(a**2 + 2*a*b*x)**(7/2)/7 + (a**2 + 2*a*b*x)**(9/2)/9)/(16*a**5
*b**5), Ne(a*b, 0)), (x**5/(5*sqrt(a**2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.63

$$\int \frac{x^4}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{\sqrt{b^2x^2+2\,abx+a^2}x^3}{4\,b^2}+\frac{13\,a^2x^2}{12\,b^3}-\frac{7\,\sqrt{b^2x^2+2\,abx+a^2}ax^2}{12\,b^3} - \frac{13\,a^3x}{6\,b^4}+\frac{a^4\log\left(x+\frac{a}{b}\right)}{b^5}+\frac{7\,\sqrt{b^2x^2+2\,abx+a^2}a^3}{6\,b^5}$$

input
```
integrate(x^4/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(b^2*x^2 + 2*a*b*x + a^2)*x^3/b^2 + 13/12*a^2*x^2/b^3 - 7/12*sqrt(
b^2*x^2 + 2*a*b*x + a^2)*a*x^2/b^3 - 13/6*a^3*x/b^4 + a^4*log(x + a/b)/b^5
 + 7/6*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^3/b^5
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.46

$$\int \frac{x^4}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{a^4\log\left(|bx+a|\right)\operatorname{sgn}(bx+a)}{b^5} + \frac{3\,b^3x^4\operatorname{sgn}(bx+a)-4\,ab^2x^3\operatorname{sgn}(bx+a)+6\,a^2bx^2\operatorname{sgn}(bx+a)-12\,a^3x\operatorname{sgn}(bx+a)}{12\,b^4}$$

input
```
integrate(x^4/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
a^4*log(abs(b*x + a))*sgn(b*x + a)/b^5 + 1/12*(3*b^3*x^4*sgn(b*x + a) - 4*
a*b^2*x^3*sgn(b*x + a) + 6*a^2*b*x^2*sgn(b*x + a) - 12*a^3*x*sgn(b*x + a))
/b^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\int \frac{x^4}{\sqrt{(a+b\,x)^2}}\,dx$$

input
```
int(x^4/((a + b*x)^2)^(1/2),x)
```

output
```
int(x^4/((a + b*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.29

$$\int \frac{x^4}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\frac{12\log(bx+a)\,a^4-12a^3bx+6a^2b^2x^2-4a\,b^3x^3+3b^4x^4}{12b^5}$$

input
```
int(x^4/((b*x+a)^2)^(1/2),x)
```

output
```
(12*log(a + b*x)*a**4 - 12*a**3*b*x + 6*a**2*b**2*x**2 - 4*a*b**3*x**3 + 3
*b**4*x**4)/(12*b**5)
```

3.91 $\int \frac{x^3}{\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 144

$$\int \frac{x^3}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{a^2x(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}} - \frac{ax^2(a+bx)}{2b^2\sqrt{a^2+2abx+b^2x^2}} + \frac{x^3(a+bx)}{3b\sqrt{a^2+2abx+b^2x^2}} - \frac{a^3(a+bx)\log(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}}$$

output
```
a^2*x*(b*x+a)/b^3/((b*x+a)^2)^(1/2)-1/2*a*x^2*(b*x+a)/b^2/((b*x+a)^2)^(1/2
)+1/3*x^3*(b*x+a)/b/((b*x+a)^2)^(1/2)-a^3*(b*x+a)*ln(b*x+a)/b^4/((b*x+a)^2
)^(1/2)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.40

$$\int \frac{x^3}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{(a+bx)\left(bx(6a^2-3abx+2b^2x^2)-6a^3\log(a+bx)\right)}{6b^4\sqrt{(a+bx)^2}}$$

input
```
Integrate[x^3/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((a + b*x)*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/(6*b^
4*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a^2+2abx+b^2x^2}}dx$$

$$\Big\downarrow \text{ 1102}$$

$$\frac{b(a+bx)\int \frac{x^3}{b(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx)\int \frac{x^3}{a+bx}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{(a+bx)\int \left(-\frac{a^3}{b^3(a+bx)}+\frac{a^2}{b^3}-\frac{xa}{b^2}+\frac{x^2}{b}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(a+bx)\left(-\frac{a^3\log(a+bx)}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[x^3/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((a + b*x)*((a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])
/b^4))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.39

method	result	size
default	$-\frac{(bx+a)\left(-2b^{3}x^{3}+3a\,b^{2}x^{2}+6\ln(bx+a)a^{3}-6a^{2}bx\right)}{6\sqrt{(bx+a)^{2}}\,b^{4}}$	56
risch	$\frac{\sqrt{(bx+a)^{2}}\,\left(\frac{1}{3}b^{2}x^{3}-\frac{1}{2}ab\,x^{2}+a^{2}x\right)}{(bx+a)b^{3}}-\frac{\sqrt{(bx+a)^{2}}\,a^{3}\ln(bx+a)}{(bx+a)b^{4}}$	73

input
```
int(x^3/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*x+a)*(-2*b^3*x^3+3*a*b^2*x^2+6*ln(b*x+a)*a^3-6*a^2*b*x)/((b*x+a)^2
)^(1/2)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.28

$$\int \frac{x^3}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{2\,b^3x^3-3\,ab^2x^2+6\,a^2bx-6\,a^3\log\left(bx+a\right)}{6\,b^4}$$

input
```
integrate(x^3/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))/b^4
```

Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.19

$$\int \frac{x^3}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \begin{cases} -\frac{a^3\left(\frac{a}{b}+x\right)\log\left(\frac{a}{b}+x\right)}{b^3\sqrt{b^2\left(\frac{a}{b}+x\right)^2}} + \sqrt{a^2+2abx+b^2x^2}\cdot\left(\frac{11a^2}{6b^4}-\frac{5ax}{6b^3}+\frac{x^2}{3b^2}\right) & \text{for } b^2\neq 0 \\ \frac{-a^6\sqrt{a^2+2abx}+a^4\left(a^2+2abx\right)^{\frac{3}{2}}-\frac{3a^2\left(a^2+2abx\right)^{\frac{5}{2}}}{5}+\frac{\left(a^2+2abx\right)^{\frac{7}{2}}}{7}}{8a^4b^4} & \text{for } ab\neq 0 \\ \frac{x^4}{4\sqrt{a^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3/((b*x+a)**2)**(1/2),x)
```

output
```
Piecewise((-a**3*(a/b + x)*log(a/b + x)/(b**3*sqrt(b**2*(a/b + x)**2)) + s
qrt(a**2 + 2*a*b*x + b**2*x**2)*(11*a**2/(6*b**4) - 5*a*x/(6*b**3) + x**2/
(3*b**2)), Ne(b**2, 0)), ((-a**6*sqrt(a**2 + 2*a*b*x) + a**4*(a**2 + 2*a*b
*x)**(3/2) - 3*a**2*(a**2 + 2*a*b*x)**(5/2)/5 + (a**2 + 2*a*b*x)**(7/2)/7)
/(8*a**4*b**4), Ne(a*b, 0)), (x**4/(4*sqrt(a**2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.60

$$\int \frac{x^3}{\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{5\,ax^2}{6\,b^2}+\frac{\sqrt{b^2x^2+2\,abx+a^2}x^2}{3\,b^2}+\frac{5\,a^2x}{3\,b^3} - \frac{a^3\log\left(x+\frac{a}{b}\right)}{b^4}-\frac{2\,\sqrt{b^2x^2+2\,abx+a^2}a^2}{3\,b^4}$$

input
```
integrate(x^3/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-5/6*a*x^2/b^2 + 1/3*sqrt(b^2*x^2 + 2*a*b*x + a^2)*x^2/b^2 + 5/3*a^2*x/b^3
 - a^3*log(x + a/b)/b^4 - 2/3*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^2/b^4
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.47

$$\int \frac{x^3}{\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{a^3\log\left(|bx+a|\right)\operatorname{sgn}(bx+a)}{b^4} + \frac{2\,b^2x^3\operatorname{sgn}(bx+a)-3\,abx^2\operatorname{sgn}(bx+a)+6\,a^2x\operatorname{sgn}(bx+a)}{6\,b^3}$$

input
```
integrate(x^3/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-a^3*log(abs(b*x + a))*sgn(b*x + a)/b^4 + 1/6*(2*b^2*x^3*sgn(b*x + a) - 3*
a*b*x^2*sgn(b*x + a) + 6*a^2*x*sgn(b*x + a))/b^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\int \frac{x^3}{\sqrt{(a+b\,x)^2}}\,dx$$

input
```
int(x^3/((a + b*x)^2)^(1/2),x)
```

output
```
int(x^3/((a + b*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.28

$$\int \frac{x^3}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\frac{-6\log(bx+a)\,a^3+6a^2bx-3a\,b^2x^2+2b^3x^3}{6b^4}$$

input
```
int(x^3/((b*x+a)^2)^(1/2),x)
```

output
```
( - 6*log(a + b*x)*a**3 + 6*a**2*b*x - 3*a*b**2*x**2 + 2*b**3*x**3)/(6*b**
4)
```

3.92 $\int \frac{x^2}{\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 106

$$\int \frac{x^2}{\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{ax(a+bx)}{b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{x^2(a+bx)}{2b\sqrt{a^2+2abx+b^2x^2}} + \frac{a^2(a+bx)\log(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-a*x*(b*x+a)/b^2/((b*x+a)^2)^(1/2)+1/2*x^2*(b*x+a)/b/((b*x+a)^2)^(1/2)+a^2
*(b*x+a)*ln(b*x+a)/b^3/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.36

$$\int \frac{x^2}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{bx(2a-bx)\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}-\sqrt{(a+bx)^2}\right)\right)+4\left(-a^4-a^3bx+\left(a^2\right)^{3/2}\sqrt{(a+bx)^2}\right)\operatorname{arctanh}\left(\frac{b}{\sqrt{a^2}-\sqrt{}}\right.}{2b^3\left(a^2+abx-\sqrt{a^2}\sqrt{(a+bx)^2}\right)}$$

input
```
Integrate[x^2/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(b*x*(2*a - b*x)*(Sqrt[a^2]*b*x + a*(Sqrt[a^2] - Sqrt[(a + b*x)^2])) + 4*(
-a^4 - a^3*b*x + (a^2)^(3/2)*Sqrt[(a + b*x)^2])*ArcTanh[(b*x)/(Sqrt[a^2] -
 Sqrt[(a + b*x)^2])])/(2*b^3*(a^2 + a*b*x - Sqrt[a^2]*Sqrt[(a + b*x)^2]))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a^2+2abx+b^2x^2}}dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{b(a+bx)\int \frac{x^2}{b(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{x^2}{a+bx}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{(a+bx)\int \left(\frac{a^2}{b^2(a+bx)}-\frac{a}{b^2}+\frac{x}{b}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[x^2/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((a + b*x)*(-((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3))/Sqrt[a^2 +
 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.42

method	result	size
default	$\frac{(bx+a)\left(b^{2}x^{2}+2\ln\left(bx+a\right)a^{2}-2abx\right)}{2\sqrt{\left(bx+a\right)^{2}}\, b^{3}}$	44
risch	$\frac{\sqrt{\left(bx+a\right)^{2}}\, \left(\frac{1}{2} b\, x^{2}-a x\right)}{\left(bx+a\right) b^{2}}+\frac{\sqrt{\left(bx+a\right)^{2}}\, a^{2} \ln\left(bx+a\right)}{\left(bx+a\right) b^{3}}$	62

input
```
int(x^2/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*x+a)*(b^2*x^2+2*ln(b*x+a)*a^2-2*a*b*x)/((b*x+a)^2)^(1/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.27

$$\int \frac{x^2}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{b^2x^2-2\,abx+2\,a^2\log{(bx+a)}}{2\,b^3}$$

input
```
integrate(x^2/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.33

$$\int \frac{x^2}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \begin{cases} \frac{a^2\left(\frac{a}{b}+x\right)\log\left(\frac{a}{b}+x\right)}{b^2\sqrt{b^2\left(\frac{a}{b}+x\right)^2}} + \left(-\frac{3a}{2b^3}+\frac{x}{2b^2}\right)\sqrt{a^2+2abx+b^2x^2} & \text{for } b^2\neq 0 \\ \frac{a^4\sqrt{a^2+2abx}-\frac{2a^2\left(a^2+2abx\right)^{\frac{3}{2}}}{3}+\frac{\left(a^2+2abx\right)^{\frac{5}{2}}}{5}}{4a^3b^3} & \text{for } ab\neq 0 \\ \frac{x^3}{3\sqrt{a^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/((b*x+a)**2)**(1/2),x)
```

output
```
Piecewise((a**2*(a/b + x)*log(a/b + x)/(b**2*sqrt(b**2*(a/b + x)**2)) + (-
3*a/(2*b**3) + x/(2*b**2))*sqrt(a**2 + 2*a*b*x + b**2*x**2), Ne(b**2, 0)),
 ((a**4*sqrt(a**2 + 2*a*b*x) - 2*a**2*(a**2 + 2*a*b*x)**(3/2)/3 + (a**2 +
2*a*b*x)**(5/2)/5)/(4*a**3*b**3), Ne(a*b, 0)), (x**3/(3*sqrt(a**2)), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.29

$$\int \frac{x^2}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{x^2}{2\,b} - \frac{ax}{b^2} + \frac{a^2\log\left(x+\frac{a}{b}\right)}{b^3}$$

input
```
integrate(x^2/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*x^2/b - a*x/b^2 + a^2*log(x + a/b)/b^3
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.45

$$\int \frac{x^2}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{a^2\log\left(|bx+a|\right)\operatorname{sgn}(bx+a)}{b^3} + \frac{bx^2\operatorname{sgn}(bx+a) - 2\,ax\operatorname{sgn}(bx+a)}{2\,b^2}$$

input
```
integrate(x^2/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
a^2*log(abs(b*x + a))*sgn(b*x + a)/b^3 + 1/2*(b*x^2*sgn(b*x + a) - 2*a*x*s
gn(b*x + a))/b^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \int \frac{x^2}{\sqrt{(a+b\,x)^2}}\,dx$$

input
```
int(x^2/((a + b*x)^2)^(1/2),x)
```

output
```
int(x^2/((a + b*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.27

$$\int \frac{x^2}{\sqrt{a^2 + 2abx + b^2x^2}}\, dx = \frac{2\log(bx + a)\, a^2 - 2abx + b^2x^2}{2b^3}$$

input
```
int(x^2/((b*x+a)^2)^(1/2),x)
```

output
```
(2*log(a + b*x)*a**2 - 2*a*b*x + b**2*x**2)/(2*b**3)
```

3.93 $\int \frac{x}{\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 62

$$\int \frac{x}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}}{b^2} - \frac{a(a+bx)\log(a+bx)}{b^2\sqrt{a^2+2abx+b^2x^2}}$$

output
```
((b*x+a)^2)^(1/2)/b^2-a*(b*x+a)*ln(b*x+a)/b^2/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.56

$$\int \frac{x}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{\frac{b\left(-\sqrt{a^2}x(a+bx)+ax\sqrt{(a+bx)^2}\right)}{a^2+abx-\sqrt{a^2}\sqrt{(a+bx)^2}} + 2a\text{arctanh}\left(\frac{bx}{\sqrt{a^2}-\sqrt{(a+bx)^2}}\right)}{b^2}$$

input
```
Integrate[x/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((b*(-(Sqrt[a^2]*x*(a + b*x)) + a*x*Sqrt[(a + b*x)^2]))/(a^2 + a*b*x - Sqr
t[a^2]*Sqrt[(a + b*x)^2]) + 2*a*ArcTanh[(b*x)/(Sqrt[a^2] - Sqrt[(a + b*x)^
2])])/b^2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1100, 1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a^2+2abx+b^2x^2}}dx$$

$$\Big\downarrow \text{ 1100}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}}{b^2}-\frac{a\int \frac{1}{\sqrt{a^2+2bxa+b^2x^2}}dx}{b}$$

$$\Big\downarrow \text{ 1079}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}}{b^2}-\frac{a(a+bx)\int \frac{1}{xb^2+ab}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}}{b^2}-\frac{a(a+bx)\log(a+bx)}{b^2\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[x/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
Sqrt[a^2 + 2*a*b*x + b^2*x^2]/b^2 - (a*(a + b*x)*Log[a + b*x])/(b^2*Sqrt[a
^2 + 2*a*b*x + b^2*x^2])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.53

method	result	size
default	$-\frac{(bx+a)(\ln(bx+a)a-bx)}{\sqrt{(bx+a)^2}\, b^2}$	33
risch	$\frac{\sqrt{(bx+a)^2}\, x}{(bx+a)b}-\frac{\sqrt{(bx+a)^2}\, a\ln(bx+a)}{(bx+a)b^2}$	51

input
```
int(x/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*x+a)*(ln(b*x+a)*a-b*x)/((b*x+a)^2)^(1/2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.27

$$\int \frac{x}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\frac{bx-a\log{(bx+a)}}{b^2}$$

input
```
integrate(x/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
(b*x - a*log(b*x + a))/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(39) = 78.

Time = 0.60 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.69

$$\int \frac{x}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \begin{cases} -\frac{a\left(\frac{a}{b}+x\right)\log\left(\frac{a}{b}+x\right)}{b\sqrt{b^2\left(\frac{a}{b}+x\right)^2}} + \frac{\sqrt{a^2+2abx+b^2x^2}}{b^2} & \text{for } b^2 \neq 0 \\ \frac{-a^2\sqrt{a^2+2abx}+\frac{\left(a^2+2abx\right)^{\frac{3}{2}}}{3}}{2a^2b^2} & \text{for } ab \neq 0 \\ \frac{x^2}{2\sqrt{a^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(x/((b*x+a)**2)**(1/2),x)
```

output
```
Piecewise((-a*(a/b + x)*log(a/b + x)/(b*sqrt(b**2*(a/b + x)**2)) + sqrt(a*
*2 + 2*a*b*x + b**2*x**2)/b**2, Ne(b**2, 0)), ((-a**2*sqrt(a**2 + 2*a*b*x)
 + (a**2 + 2*a*b*x)**(3/2)/3)/(2*a**2*b**2), Ne(a*b, 0)), (x**2/(2*sqrt(a*
*2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.60

$$\int \frac{x}{\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{a\log\left(x+\frac{a}{b}\right)}{b^2} + \frac{\sqrt{b^2x^2+2\,abx+a^2}}{b^2}$$

input
```
integrate(x/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-a*log(x + a/b)/b^2 + sqrt(b^2*x^2 + 2*a*b*x + a^2)/b^2
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.50

$$\int \frac{x}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\frac{x\mathrm{sgn}(bx+a)}{b}-\frac{a\log(|bx+a|)\,\mathrm{sgn}(bx+a)}{b^2}$$

input
```
integrate(x/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
x*sgn(b*x + a)/b - a*log(abs(b*x + a))*sgn(b*x + a)/b^2
```

Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \frac{x}{\sqrt{a^2+2abx+b^2x^2}}\,dx$$
$$=\frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{b^2}-\frac{a\,b\,\ln\left(a\,b+\sqrt{(a+b\,x)^2}\,\sqrt{b^2}+b^2\,x\right)}{(b^2)^{3/2}}$$

input
```
int(x/((a + b*x)^2)^(1/2),x)
```

output
```
(a^2 + b^2*x^2 + 2*a*b*x)^(1/2)/b^2 - (a*b*log(a*b + ((a + b*x)^2)^(1/2)*(
b^2)^(1/2) + b^2*x))/(b^2)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.27

$$\int \frac{x}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\frac{-\log(bx+a)\,a+bx}{b^2}$$

input
```
int(x/((b*x+a)^2)^(1/2),x)
```

output
```
( - log(a + b*x)*a + b*x)/b**2
```

3.94 $\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 35

$$\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\frac{(a+bx)\log(a+bx)}{b\sqrt{a^2+2abx+b^2x^2}}$$

output
```
(b*x+a)*ln(b*x+a)/b/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\frac{(a+bx)\log(a+bx)}{b\sqrt{(a+bx)^2}}$$

input
```
Integrate[1/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((a + b*x)*Log[a + b*x])/(b*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}dx$$

$$\Big\downarrow \text{1079}$$

$$\frac{b(a+bx)\int \frac{1}{xb^2+ab}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{16}$$

$$\frac{(a+bx)\log(a+bx)}{b\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[1/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((a + b*x)*Log[a + b*x])/(b*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{(bx+a)\ln(bx+a)}{b\sqrt{(bx+a)^2}}$	25
risch	$\frac{\sqrt{(bx+a)^2}\ \ln(bx+a)}{(bx+a)b}$	27

input
```
int(1/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b*x+a)*ln(b*x+a)/b/((b*x+a)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.29

$$\int \frac{1}{\sqrt{a^2 + 2abx + b^2x^2}}\,dx = \frac{\log(bx + a)}{b}$$

input
```
integrate(1/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
log(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{a^2 + 2abx + b^2x^2}}\,dx = \frac{\left(\frac{a}{b} + x\right)\log\left(\frac{a}{b} + x\right)}{\sqrt{b^2\left(\frac{a}{b} + x\right)^2}}$$

input
```
integrate(1/((b*x+a)**2)**(1/2),x)
```

output
```
(a/b + x)*log(a/b + x)/sqrt(b**2*(a/b + x)**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.34

$$\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\frac{\log\left(x+\frac{a}{b}\right)}{b}$$

input
```
integrate(1/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
log(x + a/b)/b
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.49

$$\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\frac{\log\left(|bx+a|\right)\operatorname{sgn}(bx+a)}{b}$$

input
```
integrate(1/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
log(abs(b*x + a))*sgn(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\frac{\ln\left(a+b\,x+\sqrt{{\left(a+b\,x\right)}^2}\right)}{b}$$

input
```
int(1/((a + b*x)^2)^(1/2),x)
```

output
```
log(a + b*x + ((a + b*x)^2)^(1/2))/b
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.29

$$\int \frac{1}{\sqrt{a^2 + 2abx + b^2x^2}}\,dx = \frac{\log(bx + a)}{b}$$

input
```
int(1/((b*x+a)^2)^(1/2),x)
```

output
```
log(a + b*x)/b
```

3.95 $\int \frac{1}{x\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 41

$$\int \frac{1}{x\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{(a+bx)\log\left(\frac{b(a+bx)}{x}\right)}{a\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-(b*x+a)*ln(b*(b*x+a)/x)/a/((b*x+a)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 120 vs. 2(41) = 82.

Time = 0.17 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.93

$$\int \frac{1}{x\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{-2a\log(x)+\left(a-\sqrt{a^2}\right)\log\left(\sqrt{a^2}-bx-\sqrt{(a+bx)^2}\right)+a\log\left(\sqrt{a^2}+bx-\sqrt{(a+bx)^2}\right)+\sqrt{a^2}\log\left(\cdots\right.}{2a\sqrt{a^2}}$$

input
```
Integrate[1/(x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
(-2*a*Log[x] + (a - Sqrt[a^2])*Log[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2]] +
a*Log[Sqrt[a^2] + b*x - Sqrt[(a + b*x)^2]] + Sqrt[a^2]*Log[a*(Sqrt[a^2] +
b*x - Sqrt[(a + b*x)^2])])/(2*a*Sqrt[a^2])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1102, 27, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a^2+2abx+b^2x^2}}dx$$

$$\Big\downarrow \text{ 1102}$$

$$\frac{b(a+bx)\int \frac{1}{bx(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx)\int \frac{1}{x(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{(a+bx)\left(\frac{\int \frac{1}{x}dx}{a}-\frac{b\int \frac{1}{a+bx}dx}{a}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{(a+bx)\left(\frac{\log(x)}{a}-\frac{b\int \frac{1}{a+bx}dx}{a}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{(a+bx)\left(\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[1/(x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
((a + b*x)*(Log[x]/a - Log[a + b*x]/a))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{(bx+a)(\ln(x)-\ln(bx+a))}{\sqrt{(bx+a)^2}\,a}$	30
risch	$-\frac{\sqrt{(bx+a)^2}\,\ln(bx+a)}{(bx+a)a}+\frac{\sqrt{(bx+a)^2}\,\ln(-x)}{(bx+a)a}$	53

input
```
int(1/x/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b*x+a)*(ln(x)-ln(b*x+a))/((b*x+a)^2)^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.39

$$\int \frac{1}{x\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{\log\left(bx+a\right)-\log\left(x\right)}{a}$$

input
```
integrate(1/x/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
-(log(b*x + a) - log(x))/a
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a^2+2abx+b^2x^2}}\,dx = \int \frac{1}{x\sqrt{(a+bx)^2}}\,dx$$

input
```
integrate(1/x/((b*x+a)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt((a + b*x)**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{(-1)^{2\,abx+2\,a^2}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)}{a}$$

input
```
integrate(1/x/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-(-1)^(2*a*b*x + 2*a^2)*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int \frac{1}{x\sqrt{a^2+2abx+b^2x^2}}\,dx = -\left(\frac{\log\left(|bx+a|\right)}{a}-\frac{\log\left(|x|\right)}{a}\right)\mathrm{sgn}(bx+a)$$

input
```
integrate(1/x/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-(log(abs(b*x + a))/a - log(abs(x))/a)*sgn(b*x + a)
```

Mupad [B] (verification not implemented)

Time = 9.47 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{1}{x\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{\ln\left(a\,b+\frac{a^2}{x}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x}\right)}{\sqrt{a^2}}$$

input
```
int(1/(x*((a + b*x)^2)^(1/2)),x)
```

output
```
-log(a*b + a^2/x + ((a^2)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/x)/(a^2)^
(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.37

$$\int \frac{1}{x\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{-\log(bx+a)+\log(x)}{a}$$

input
```
int(1/x/((b*x+a)^2)^(1/2),x)
```

output
```
( - log(a + b*x) + log(x))/a
```

3.96 $\int \frac{1}{x^2\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 103

$$\int \frac{1}{x^2\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{a+bx}{ax\sqrt{a^2+2abx+b^2x^2}} - \frac{b(a+bx)\log(x)}{a^2\sqrt{a^2+2abx+b^2x^2}} + \frac{b(a+bx)\log(a+bx)}{a^2\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-(b*x+a)/a/x/((b*x+a)^2)^(1/2)-b*(b*x+a)*ln(x)/a^2/((b*x+a)^2)^(1/2)+b*(b*
x+a)*ln(b*x+a)/a^2/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.47

$$\int \frac{1}{x^2\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{a^2-\sqrt{a^2}\sqrt{(a+bx)^2}+2abx\log(x)+\left(-a+\sqrt{a^2}\right)bx\log\left(\sqrt{a^2}-bx-\sqrt{(a+bx)^2}\right)-abx\log\left(\sqrt{a^2}+\right.}{2\left(a^2\right)^{3/2}x}$$

input
```
Integrate[1/(x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
(a^2 - Sqrt[a^2]*Sqrt[(a + b*x)^2] + 2*a*b*x*Log[x] + (-a + Sqrt[a^2])*b*x
*Log[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2]] - a*b*x*Log[Sqrt[a^2] + b*x - Sq
rt[(a + b*x)^2]] - Sqrt[a^2]*b*x*Log[Sqrt[a^2] + b*x - Sqrt[(a + b*x)^2]])
/(2*(a^2)^(3/2)*x)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.52, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1102, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a^2+2abx+b^2x^2}}dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{b(a+bx)\int \frac{1}{bx^2(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{1}{x^2(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{54}$$

$$\frac{(a+bx)\int \left(\frac{b^2}{a^2(a+bx)}-\frac{b}{a^2x}+\frac{1}{ax^2}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[1/(x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
((a + b*x)*(-(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2))/Sqrt[a^2
+ 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.39

method	result	size
default	$-\frac{(bx+a)(\ln(x)xb-\ln(bx+a)xb+a)}{\sqrt{(bx+a)^2}\, a^2 x}$	40
risch	$-\frac{\sqrt{(bx+a)^2}}{(bx+a)ax}+\frac{\sqrt{(bx+a)^2}\, b\ln(-bx-a)}{(bx+a)a^2}-\frac{\sqrt{(bx+a)^2}\, b\ln(x)}{(bx+a)a^2}$	80

input
```
int(1/x^2/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*x+a)*(ln(x)*x*b-ln(b*x+a)*x*b+a)/((b*x+a)^2)^(1/2)/a^2/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.25

$$\int \frac{1}{x^2\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{bx\log(bx+a)-bx\log(x)-a}{a^2x}$$

input
```
integrate(1/x^2/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
(b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a^2+2abx+b^2x^2}}\,dx = \int \frac{1}{x^2\sqrt{(a+bx)^2}}\,dx$$

input
```
integrate(1/x**2/((b*x+a)**2)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt((a + b*x)**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^2\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{(-1)^{2\,abx+2\,a^2}\,b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)}{a^2}-\frac{\sqrt{b^2x^2+2\,abx+a^2}}{a^2x}$$

input
```
integrate(1/x^2/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
(-1)^(2*a*b*x + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a^2 - sqrt(b^2
*x^2 + 2*a*b*x + a^2)/(a^2*x)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.36

$$\int \frac{1}{x^2\sqrt{a^2+2abx+b^2x^2}}\,dx = \left(\frac{b\log\left(|bx+a|\right)}{a^2} - \frac{b\log\left(|x|\right)}{a^2} - \frac{1}{ax}\right)\operatorname{sgn}(bx+a)$$

input
```
integrate(1/x^2/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
(b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x))*sgn(b*x + a)
```

Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^2\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{a\,b\,\mathrm{atanh}\left(\frac{a^2+b\,x\,a}{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}\right)}{\left(a^2\right)^{3/2}} - \frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{a^2\,x}$$

input
```
int(1/(x^2*((a + b*x)^2)^(1/2)),x)
```

output
```
(a*b*atanh((a^2 + a*b*x)/((a^2)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))))/(
a^2)^(3/2) - (a^2 + b^2*x^2 + 2*a*b*x)^(1/2)/(a^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.25

$$\int \frac{1}{x^2\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{\log(bx+a)\,bx - \log(x)\,bx - a}{a^2x}$$

input
```
int(1/x^2/((b*x+a)^2)^(1/2),x)
```

output
```
(log(a + b*x)*b*x - log(x)*b*x - a)/(a**2*x)
```

3.97 $\int \frac{1}{x^3\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{1}{x^3\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{-a-bx}{2ax^2\sqrt{a^2+2abx+b^2x^2}}+\frac{b(a+bx)}{a^2x\sqrt{a^2+2abx+b^2x^2}} + \frac{b^2(a+bx)\log(x)}{a^3\sqrt{a^2+2abx+b^2x^2}}-\frac{b^2(a+bx)\log(a+bx)}{a^3\sqrt{a^2+2abx+b^2x^2}}$$

output
```
1/2*(-b*x-a)/a/x^2/((b*x+a)^2)^(1/2)+b*(b*x+a)/a^2/x/((b*x+a)^2)^(1/2)+b^2
*(b*x+a)*ln(x)/a^3/((b*x+a)^2)^(1/2)-b^2*(b*x+a)*ln(b*x+a)/a^3/((b*x+a)^2)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^3\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{\frac{a^3(a-2bx)}{\sqrt{a^2}x^2}-\frac{a(a-3bx)\sqrt{(a+bx)^2}}{x^2}-4\sqrt{a^2}b^2\log(x)+2\left(-a+\sqrt{a^2}\right)b^2\log\left(\sqrt{a^2}-bx-\sqrt{(a+bx)^2}\right)+2\left(a+\right.}{4a^4}$$

input
```
Integrate[1/(x^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
((a^3*(a - 2*b*x))/(Sqrt[a^2]*x^2) - (a*(a - 3*b*x)*Sqrt[(a + b*x)^2])/x^2
 - 4*Sqrt[a^2]*b^2*Log[x] + 2*(-a + Sqrt[a^2])*b^2*Log[Sqrt[a^2] - b*x - S
qrt[(a + b*x)^2]] + 2*(a + Sqrt[a^2])*b^2*Log[Sqrt[a^2] + b*x - Sqrt[(a +
b*x)^2]])/(4*a^4)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a^2+2abx+b^2x^2}}dx$$

$$\Big\downarrow \text{ 1102}$$

$$\frac{b(a+bx)\int \frac{1}{bx^3(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx)\int \frac{1}{x^3(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{(a+bx)\int \left(-\frac{b^3}{a^3(a+bx)}+\frac{b^2}{a^3x}-\frac{b}{a^2x^2}+\frac{1}{ax^3}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(a+bx)\left(\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[1/(x^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
((a + b*x)*(-1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b
*x])/a^3))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.40

method	result	size
default	$\frac{(bx+a)\left(2b^2\ln(x)x^2-2b^2\ln(bx+a)x^2+2abx-a^2\right)}{2\sqrt{(bx+a)^2}\,a^3x^2}$	58
risch	$\frac{\sqrt{(bx+a)^2}\left(\frac{bx}{a^2}-\frac{1}{2a}\right)}{(bx+a)x^2}-\frac{\sqrt{(bx+a)^2}\,b^2\ln(bx+a)}{(bx+a)a^3}+\frac{\sqrt{(bx+a)^2}\,b^2\ln(-x)}{(bx+a)a^3}$	91

input
```
int(1/x^3/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*x+a)*(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+2*a*b*x-a^2)/((b*x+a)^2)^
(1/2)/a^3/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.28

$$\int \frac{1}{x^3\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{2\,b^2x^2\log\left(bx+a\right)-2\,b^2x^2\log\left(x\right)-2\,abx+a^2}{2\,a^3x^2}$$

input
```
integrate(1/x^3/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{a^2+2abx+b^2x^2}}\,dx = \int \frac{1}{x^3\sqrt{(a+bx)^2}}\,dx$$

input
```
integrate(1/x**3/((b*x+a)**2)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt((a + b*x)**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^3\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{(-1)^{2\,abx+2\,a^2}\,b^2\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)}{a^3} + \frac{3\,\sqrt{b^2x^2+2\,abx+a^2}b}{2\,a^3x} - \frac{\sqrt{b^2x^2+2\,abx+a^2}}{2\,a^2x^2}$$

input
```
integrate(1/x^3/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-(-1)^(2*a*b*x + 2*a^2)*b^2*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a^3 + 3/2*s
qrt(b^2*x^2 + 2*a*b*x + a^2)*b/(a^3*x) - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)
/(a^2*x^2)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.37

$$\int \frac{1}{x^3\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{1}{2}\left(\frac{2\,b^2\log\left(|bx+a|\right)}{a^3}-\frac{2\,b^2\log\left(|x|\right)}{a^3}-\frac{2\,abx-a^2}{a^3x^2}\right)\mathrm{sgn}(bx+a)$$

input
```
integrate(1/x^3/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(2*b^2*log(abs(b*x + a))/a^3 - 2*b^2*log(abs(x))/a^3 - (2*a*b*x - a^2
)/(a^3*x^2))*sgn(b*x + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a^2+2abx+b^2x^2}}\,dx = \int \frac{1}{x^3\,\sqrt{{\left(a+b\,x\right)}^2}}\,dx$$

input
```
int(1/(x^3*((a + b*x)^2)^(1/2)),x)
```

output
```
int(1/(x^3*((a + b*x)^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.30

$$\int \frac{1}{x^3\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{-2\log(bx+a)\,b^2x^2+2\log(x)\,b^2x^2-a^2+2abx}{2a^3x^2}$$

input
```
int(1/x^3/((b*x+a)^2)^(1/2),x)
```

output
```
( - 2*log(a + b*x)*b**2*x**2 + 2*log(x)*b**2*x**2 - a**2 + 2*a*b*x)/(2*a**
3*x**2)
```

3.98 $\int \frac{1}{x^4\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 184

$$\int \frac{1}{x^4\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{-a-bx}{3ax^3\sqrt{a^2+2abx+b^2x^2}}+\frac{b(a+bx)}{2a^2x^2\sqrt{a^2+2abx+b^2x^2}} - \frac{b^2(a+bx)}{a^3x\sqrt{a^2+2abx+b^2x^2}} - \frac{b^3(a+bx)\log(x)}{a^4\sqrt{a^2+2abx+b^2x^2}}+\frac{b^3(a+bx)\log(a+bx)}{a^4\sqrt{a^2+2abx+b^2x^2}}$$

output
```
1/3*(-b*x-a)/a/x^3/((b*x+a)^2)^(1/2)+1/2*b*(b*x+a)/a^2/x^2/((b*x+a)^2)^(1/
2)-b^2*(b*x+a)/a^3/x/((b*x+a)^2)^(1/2)-b^3*(b*x+a)*ln(x)/a^4/((b*x+a)^2)^(
1/2)+b^3*(b*x+a)*ln(b*x+a)/a^4/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^4\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{1}{12}\left(\frac{\sqrt{(a+bx)^2}(-2a^2+5abx-11b^2x^2)}{a^4x^3} + \frac{2a^2-3abx+6b^2x^2}{(a^2)^{3/2}x^3} + \frac{12\sqrt{a^2}b^3\log(x)}{a^5} - \frac{6\left(-a+\sqrt{a^2}\right)b^3\log\left(\sqrt{a^2}-bx-\sqrt{(a+bx)^2}\right)}{a^5} - \frac{6\left(a+\sqrt{a^2}\right)b^3\log\left(\sqrt{a^2}+bx-\sqrt{(a+bx)^2}\right)}{a^5}\right)$$

input

```
Integrate[1/(x^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output

```
((Sqrt[(a + b*x)^2]*(-2*a^2 + 5*a*b*x - 11*b^2*x^2))/(a^4*x^3) + (2*a^2 -
3*a*b*x + 6*b^2*x^2)/((a^2)^(3/2)*x^3) + (12*Sqrt[a^2]*b^3*Log[x])/a^5 - (
6*(-a + Sqrt[a^2])*b^3*Log[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2]])/a^5 - (6*
(a + Sqrt[a^2])*b^3*Log[Sqrt[a^2] + b*x - Sqrt[(a + b*x)^2]])/a^5)/12
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a^2+2abx+b^2x^2}}dx$$

$$\Big\downarrow \; 1102$$

$$\frac{b(a+bx)\int \frac{1}{bx^4(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{1}{x^4(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{54}$$

$$\frac{(a+bx)\int \left(\frac{b^4}{a^4(a+bx)}-\frac{b^3}{a^4x}+\frac{b^2}{a^3x^2}-\frac{b}{a^2x^3}+\frac{1}{ax^4}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}-\frac{b^2}{a^3x}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[1/(x^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
((a + b*x)*(-1/3*1/(a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^
4 + (b^3*Log[a + b*x])/a^4))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.38

method	result	size
default	$-\frac{(bx+a)\left(6\ln(x)x^{3}b^{3}-6\ln(bx+a)x^{3}b^{3}+6a\,b^{2}x^{2}-3a^{2}bx+2a^{3}\right)}{6\sqrt{(bx+a)^{2}}\,a^{4}x^{3}}$	69
risch	$\frac{\sqrt{(bx+a)^{2}}\left(-\frac{1}{3a}+\frac{bx}{2a^{2}}-\frac{b^{2}x^{2}}{a^{3}}\right)}{(bx+a)x^{3}}-\frac{\sqrt{(bx+a)^{2}}\,b^{3}\ln(x)}{(bx+a)a^{4}}+\frac{\sqrt{(bx+a)^{2}}\,b^{3}\ln(-bx-a)}{(bx+a)a^{4}}$	104

input
```
int(1/x^4/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*x+a)*(6*ln(x)*x^3*b^3-6*ln(b*x+a)*x^3*b^3+6*a*b^2*x^2-3*a^2*b*x+2*
a^3)/((b*x+a)^2)^(1/2)/a^4/x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.29

$$\int \frac{1}{x^4\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{6\,b^3x^3\log\left(bx+a\right)-6\,b^3x^3\log\left(x\right)-6\,ab^2x^2+3\,a^2bx-2\,a^3}{6\,a^4x^3}$$

input
```
integrate(1/x^4/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(6*b^3*x^3*log(b*x + a) - 6*b^3*x^3*log(x) - 6*a*b^2*x^2 + 3*a^2*b*x -
 2*a^3)/(a^4*x^3)
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{a^2+2abx+b^2x^2}}\,dx=\int \frac{1}{x^4\sqrt{(a+bx)^2}}\,dx$$

input
```
integrate(1/x**4/((b*x+a)**2)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt((a + b*x)**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^4\sqrt{a^2+2abx+b^2x^2}}\,dx=\frac{(-1)^{2\,abx+2\,a^2}b^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)}{a^4}-\frac{11\,\sqrt{b^2x^2+2\,abx+a^2}b^2}{6\,a^4x}+\frac{5\,\sqrt{b^2x^2+2\,abx+a^2}b}{6\,a^3x^2}-\frac{\sqrt{b^2x^2+2\,abx+a^2}}{3\,a^2x^3}$$

input
```
integrate(1/x^4/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
(-1)^(2*a*b*x + 2*a^2)*b^3*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a^4 - 11/6*s
qrt(b^2*x^2 + 2*a*b*x + a^2)*b^2/(a^4*x) + 5/6*sqrt(b^2*x^2 + 2*a*b*x + a^
2)*b/(a^3*x^2) - 1/3*sqrt(b^2*x^2 + 2*a*b*x + a^2)/(a^2*x^3)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.35

$$\int \frac{1}{x^4\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{1}{6}\left(\frac{6\,b^3\log\left(|bx+a|\right)}{a^4}-\frac{6\,b^3\log\left(|x|\right)}{a^4}-\frac{6\,ab^2x^2-3\,a^2bx+2\,a^3}{a^4x^3}\right)\mathrm{sgn}(bx+a)$$

input
```
integrate(1/x^4/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/6*(6*b^3*log(abs(b*x + a))/a^4 - 6*b^3*log(abs(x))/a^4 - (6*a*b^2*x^2 -
3*a^2*b*x + 2*a^3)/(a^4*x^3))*sgn(b*x + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a^2+2abx+b^2x^2}}\,dx = \int \frac{1}{x^4\,\sqrt{{\left(a+b\,x\right)}^2}}\,dx$$

input
```
int(1/(x^4*((a + b*x)^2)^(1/2)),x)
```

output
```
int(1/(x^4*((a + b*x)^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.29

$$\int \frac{1}{x^4\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{6\log(bx+a)\,b^3x^3-6\log(x)\,b^3x^3-2a^3+3a^2bx-6a\,b^2x^2}{6a^4x^3}$$

input
```
int(1/x^4/((b*x+a)^2)^(1/2),x)
```

output
```
(6*log(a + b*x)*b**3*x**3 - 6*log(x)*b**3*x**3 - 2*a**3 + 3*a**2*b*x - 6*a
*b**2*x**2)/(6*a**4*x**3)
```

3.99 $\int \frac{x^4}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 172

$$\begin{aligned}\int \frac{x^4}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx &= \frac{4a^3}{b^5\sqrt{a^2+2abx+b^2x^2}} \\ &\quad -\frac{a^4}{2b^5(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{3ax(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}} \\ &\quad +\frac{x^2(a+bx)}{2b^3\sqrt{a^2+2abx+b^2x^2}}+\frac{6a^2(a+bx)\log(a+bx)}{b^5\sqrt{a^2+2abx+b^2x^2}}\end{aligned}$$

output
```
4*a^3/b^5/((b*x+a)^2)^(1/2)-1/2*a^4/b^5/(b*x+a)/((b*x+a)^2)^(1/2)-3*a*x*(b
*x+a)/b^4/((b*x+a)^2)^(1/2)+1/2*x^2*(b*x+a)/b^3/((b*x+a)^2)^(1/2)+6*a^2*(b
*x+a)*ln(b*x+a)/b^5/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.48

$$\int \frac{x^4}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\frac{7a^4+2a^3bx-11a^2b^2x^2-4ab^3x^3+b^4x^4+12a^2(a+bx)^2\log(a+bx)}{2b^5(a+bx)\sqrt{(a+bx)^2}}$$

input
```
Integrate[x^4/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(7*a^4 + 2*a^3*b*x - 11*a^2*b^2*x^2 - 4*a*b^3*x^3 + b^4*x^4 + 12*a^2*(a +
b*x)^2*Log[a + b*x])/(2*b^5*(a + b*x)*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.52, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a^2+2abx+b^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{b^3(a+bx)\int \frac{x^4}{b^3(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{x^4}{(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{(a+bx)\int \left(\frac{a^4}{b^4(a+bx)^3}-\frac{4a^3}{b^4(a+bx)^2}+\frac{6a^2}{b^4(a+bx)}-\frac{3a}{b^4}+\frac{x}{b^3}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(-\frac{a^4}{2b^5(a+bx)^2}+\frac{4a^3}{b^5(a+bx)}+\frac{6a^2\log(a+bx)}{b^5}-\frac{3ax}{b^4}+\frac{x^2}{2b^3}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[x^4/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
((a + b*x)*((-3*a*x)/b^4 + x^2/(2*b^3) - a^4/(2*b^5*(a + b*x)^2) + (4*a^3)
/(b^5*(a + b*x)) + (6*a^2*Log[a + b*x])/b^5))/Sqrt[a^2 + 2*a*b*x + b^2*x^2
]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.57

method	result	size
risch	$\frac{\sqrt{(bx+a)^2}\,(\frac{1}{2}b\,x^2-3ax)}{(bx+a)b^4}+\frac{\sqrt{(bx+a)^2}\,\left(4a^3x+\frac{7a^4}{2b}\right)}{(bx+a)^3b^4}+\frac{6\sqrt{(bx+a)^2}\,a^2\ln(bx+a)}{(bx+a)b^5}$	98
default	$\frac{(b^4x^4+12\ln(bx+a)x^2a^2b^2-4a\,b^3x^3+24\ln(bx+a)x\,a^3b-11a^2b^2x^2+12a^4\ln(bx+a)+2a^3bx+7a^4)(bx+a)}{2b^5\left((bx+a)^2\right)^{\frac{3}{2}}}$	101

input
```
int(x^4/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*(1/2*b*x^2-3*a*x)/b^4+((b*x+a)^2)^(1/2)/(b*x+a)^
3*(4*a^3*x+7/2*a^4/b)/b^4+6*((b*x+a)^2)^(1/2)/(b*x+a)*a^2/b^5*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.55

$$\int \frac{x^4}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{b^4x^4-4\,ab^3x^3-11\,a^2b^2x^2+2\,a^3bx+7\,a^4+12\,\left(a^2b^2x^2+2\,a^3bx+a^4\right)\log\left(b\right.}{2\,\left(b^7x^2+2\,ab^6x+a^2b^5\right)}$$

input
```
integrate(x^4/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(b^4*x^4 - 4*a*b^3*x^3 - 11*a^2*b^2*x^2 + 2*a^3*b*x + 7*a^4 + 12*(a^2*
b^2*x^2 + 2*a^3*b*x + a^4)*log(b*x + a))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Sympy [F]

$$\int \frac{x^4}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \int \frac{x^4}{\left(\left(a+bx\right)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Integral(x**4/((a + b*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{x^3}{2\sqrt{b^2x^2+2\,abx+a^2}b^2} - \frac{5\,ax^2}{2\sqrt{b^2x^2+2\,abx+a^2}b^3} + \frac{6\,a^2\log\left(x+\frac{a}{b}\right)}{b^5} - \frac{5\,a^3}{\sqrt{b^2x^2+2\,abx+a^2}b^5} + \frac{12\,a^3x}{b^6\left(x+\frac{a}{b}\right)^2} + \frac{23\,a^4}{2\,b^7\left(x+\frac{a}{b}\right)^2}$$

input
```
integrate(x^4/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*x^3/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^2) - 5/2*a*x^2/(sqrt(b^2*x^2 + 2*
a*b*x + a^2)*b^3) + 6*a^2*log(x + a/b)/b^5 - 5*a^3/(sqrt(b^2*x^2 + 2*a*b*x
 + a^2)*b^5) + 12*a^3*x/(b^6*(x + a/b)^2) + 23/2*a^4/(b^7*(x + a/b)^2)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.52

$$\int \frac{x^4}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{6\,a^2\log\left(|bx+a|\right)}{b^5\mathrm{sgn}\left(bx+a\right)} + \frac{b^3x^2\mathrm{sgn}(bx+a) - 6\,ab^2x\mathrm{sgn}(bx+a)}{2\,b^6} + \frac{8\,a^3bx+7\,a^4}{2\,(bx+a)^2b^5\mathrm{sgn}\left(bx+a\right)}$$

input
```
integrate(x^4/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
6*a^2*log(abs(b*x + a))/(b^5*sgn(b*x + a)) + 1/2*(b^3*x^2*sgn(b*x + a) - 6
*a*b^2*x*sgn(b*x + a))/b^6 + 1/2*(8*a^3*b*x + 7*a^4)/((b*x + a)^2*b^5*sgn(
b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\int \frac{x^4}{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}}\,dx$$

input
```
int(x^4/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
int(x^4/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.56

$$\int \frac{x^4}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\frac{12\log(bx+a)\,a^4+24\log(bx+a)\,a^3bx+12\log(bx+a)\,a^2b^2x^2+6a^4-12a}{2b^5\left(b^2x^2+2abx+a^2\right)}$$

input
```
int(x^4/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
(12*log(a + b*x)*a**4 + 24*log(a + b*x)*a**3*b*x + 12*log(a + b*x)*a**2*b*
*2*x**2 + 6*a**4 - 12*a**2*b**2*x**2 - 4*a*b**3*x**3 + b**4*x**4)/(2*b**5*
(a**2 + 2*a*b*x + b**2*x**2))
```

3.100 $\int \frac{x^3}{(a^2+2abx+b^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 133

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = -\frac{3a^2}{b^4\sqrt{a^2+2abx+b^2x^2}} + \frac{a^3}{2b^4(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{x(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}} - \frac{3a(a+bx)\log(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-3*a^2/b^4/((b*x+a)^2)^(1/2)+1/2*a^3/b^4/(b*x+a)/((b*x+a)^2)^(1/2)+x*(b*x+
a)/b^3/((b*x+a)^2)^(1/2)-3*a*(b*x+a)*ln(b*x+a)/b^4/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.53

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{-5a^3-4a^2bx+4ab^2x^2+2b^3x^3-6a(a+bx)^2\log(a+bx)}{2b^4(a+bx)\sqrt{(a+bx)^2}}$$

input
```
Integrate[x^3/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(-5*a^3 - 4*a^2*b*x + 4*a*b^2*x^2 + 2*b^3*x^3 - 6*a*(a + b*x)^2*Log[a + b*
x])/(2*b^4*(a + b*x)*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.57, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{b^3(a+bx)\int \frac{x^3}{b^3(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{x^3}{(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{(a+bx)\int \left(-\frac{a^3}{b^3(a+bx)^3}+\frac{3a^2}{b^3(a+bx)^2}-\frac{3a}{b^3(a+bx)}+\frac{1}{b^3}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(\frac{a^3}{2b^4(a+bx)^2}-\frac{3a^2}{b^4(a+bx)}-\frac{3a\log(a+bx)}{b^4}+\frac{x}{b^3}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[x^3/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
((a + b*x)*(x/b^3 + a^3/(2*b^4*(a + b*x)^2) - (3*a^2)/(b^4*(a + b*x)) - (3
*a*Log[a + b*x])/b^4))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{\sqrt{(bx+a)^2}\,x}{(bx+a)b^3}+\frac{\sqrt{(bx+a)^2}\left(-3a^2x-\frac{5a^3}{2b}\right)}{(bx+a)^3b^3}-\frac{3\sqrt{(bx+a)^2}\,a\ln(bx+a)}{(bx+a)b^4}$	86
default	$-\frac{(6\ln(bx+a)x^2a\,b^2-2b^3x^3+12\ln(bx+a)x\,a^2b-4a\,b^2x^2+6\ln(bx+a)a^3+4a^2bx+5a^3)(bx+a)}{2b^4\left((bx+a)^2\right)^{\frac{3}{2}}}$	89

input
```
int(x^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)/b^3*x+((b*x+a)^2)^(1/2)/(b*x+a)^3*(-3*a^2*x-5/2*
a^3/b)/b^3-3*((b*x+a)^2)^(1/2)/(b*x+a)*a/b^4*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.62

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{2\,b^3x^3+4\,ab^2x^2-4\,a^2bx-5\,a^3-6\,(ab^2x^2+2\,a^2bx+a^3)\log{(bx+a)}}{2\,(b^6x^2+2\,ab^5x+a^2b^4)}$$

input
```
integrate(x^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(2*b^3*x^3 + 4*a*b^2*x^2 - 4*a^2*b*x - 5*a^3 - 6*(a*b^2*x^2 + 2*a^2*b*
x + a^3)*log(b*x + a))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [F]

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{x^3}{\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Integral(x**3/((a + b*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{x^2}{\sqrt{b^2x^2+2\,abx+a^2}b^2} - \frac{3\,a\log{\left(x+\frac{a}{b}\right)}}{b^4} + \frac{2\,a^2}{\sqrt{b^2x^2+2\,abx+a^2}b^4} - \frac{6\,a^2x}{b^5\left(x+\frac{a}{b}\right)^2} - \frac{11\,a^3}{2\,b^6\left(x+\frac{a}{b}\right)^2}$$

input
```
integrate(x^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^2) - 3*a*log(x + a/b)/b^4 + 2*a^2/(sq
rt(b^2*x^2 + 2*a*b*x + a^2)*b^4) - 6*a^2*x/(b^5*(x + a/b)^2) - 11/2*a^3/(b
^6*(x + a/b)^2)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.51

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{x}{b^3\operatorname{sgn}(bx+a)} - \frac{3\,a\log(|bx+a|)}{b^4\operatorname{sgn}(bx+a)} - \frac{6\,a^2bx+5\,a^3}{2\,(bx+a)^2b^4\operatorname{sgn}(bx+a)}$$

input
```
integrate(x^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
x/(b^3*sgn(b*x + a)) - 3*a*log(abs(b*x + a))/(b^4*sgn(b*x + a)) - 1/2*(6*a
^2*b*x + 5*a^3)/((b*x + a)^2*b^4*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{x^3}{(a^2+2\,a\,b\,x+b^2\,x^2)^{3/2}}\,dx$$

input
```
int(x^3/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
int(x^3/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.64

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{-6\log(bx+a)\,a^3-12\log(bx+a)\,a^2bx-6\log(bx+a)\,a\,b^2x^2-3a^3+6a\,b^2}{2b^4\,(b^2x^2+2abx+a^2)}$$

input
```
int(x^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
( - 6*log(a + b*x)*a**3 - 12*log(a + b*x)*a**2*b*x - 6*log(a + b*x)*a*b**2
*x**2 - 3*a**3 + 6*a*b**2*x**2 + 2*b**3*x**3)/(2*b**4*(a**2 + 2*a*b*x + b*
*2*x**2))
```

3.101 $\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 99

$$\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{2a}{b^3\sqrt{a^2+2abx+b^2x^2}} - \frac{a^2}{2b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{(a+bx)\log(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}$$

output
```
2*a/b^3/((b*x+a)^2)^(1/2)-1/2*a^2/b^3/(b*x+a)/((b*x+a)^2)^(1/2)+(b*x+a)*ln
(b*x+a)/b^3/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.73

$$\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{\frac{b\left(ax\sqrt{(a+bx)^2}\left(-2a^2-abx+b^2x^2\right)+\sqrt{a^2}x\left(2a^3+3a^2bx+b^3x^3\right)\right)}{a^2(a+bx)\left(a^2+abx-\sqrt{a^2}\sqrt{(a+bx)^2}\right)}+2\log\left(\sqrt{a^2}-bx-\sqrt{(a+\cdots}\right.}{2b^3}$$

input
```
Integrate[x^2/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
((b*(a*x*Sqrt[(a + b*x)^2]*(-2*a^2 - a*b*x + b^2*x^2) + Sqrt[a^2]*x*(2*a^3
 + 3*a^2*b*x + b^3*x^3)))/(a^2*(a + b*x)*(a^2 + a*b*x - Sqrt[a^2]*Sqrt[(a
+ b*x)^2])) + 2*Log[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2]] - 2*Log[b^3*(Sqrt
[a^2] + b*x - Sqrt[(a + b*x)^2])])/(2*b^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.68, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{b^3(a+bx)\int \frac{x^2}{b^3(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{x^2}{(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{(a+bx)\int \left(\frac{a^2}{b^2(a+bx)^3}-\frac{2a}{b^2(a+bx)^2}+\frac{1}{b^2(a+bx)}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(-\frac{a^2}{2b^3(a+bx)^2}+\frac{2a}{b^3(a+bx)}+\frac{\log(a+bx)}{b^3}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[x^2/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
((a + b*x)*(-1/2*a^2/(b^3*(a + b*x)^2) + (2*a)/(b^3*(a + b*x)) + Log[a + b
*x]/b^3))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.62

method	result	size
risch	$\frac{\sqrt{(bx+a)^2}\left(\frac{2ax}{b^2}+\frac{3a^2}{2b^3}\right)}{(bx+a)^3}+\frac{\sqrt{(bx+a)^2}\,\ln(bx+a)}{(bx+a)b^3}$	61
default	$\frac{(2b^2\ln(bx+a)x^2+4\ln(bx+a)abx+2\ln(bx+a)a^2+4abx+3a^2)(bx+a)}{2b^3\left((bx+a)^2\right)^{\frac{3}{2}}}$	67

input
```
int(x^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)^3*(2*a*x/b^2+3/2/b^3*a^2)+((b*x+a)^2)^(1/2)/(b*x
+a)/b^3*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.62

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{4\,abx+3\,a^2+2\,(b^2x^2+2\,abx+a^2)\log\left(bx+a\right)}{2\,(b^5x^2+2\,ab^4x+a^2b^3)}$$

input
```
integrate(x^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(4*a*b*x + 3*a^2 + 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a))/(b^5*x^2
+ 2*a*b^4*x + a^2*b^3)
```

Sympy [F]

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{x^2}{\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Integral(x**2/((a + b*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.46

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^{3/2}}\,dx=\frac{\log\left(x+\frac{a}{b}\right)}{b^3}+\frac{2\,ax}{b^4\left(x+\frac{a}{b}\right)^2}+\frac{3\,a^2}{2\,b^5\left(x+\frac{a}{b}\right)^2}$$

input
```
integrate(x^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
log(x + a/b)/b^3 + 2*a*x/(b^4*(x + a/b)^2) + 3/2*a^2/(b^5*(x + a/b)^2)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.54

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^{3/2}}\,dx=\frac{\log\left(|bx+a|\right)}{b^3\mathrm{sgn}\left(bx+a\right)}+\frac{4\,ax+\frac{3\,a^2}{b}}{2\,(bx+a)^2b^2\mathrm{sgn}\left(bx+a\right)}$$

input
```
integrate(x^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
log(abs(b*x + a))/(b^3*sgn(b*x + a)) + 1/2*(4*a*x + 3*a^2/b)/((b*x + a)^2*
b^2*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^{3/2}}\,dx=\int \frac{x^2}{{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}^{3/2}}\,dx$$

input
```
int(x^2/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
int(x^2/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{2\log(bx+a)\,a^2+4\log(bx+a)\,abx+2\log(bx+a)\,b^2x^2+a^2-2b^2x^2}{2b^3\,(b^2x^2+2abx+a^2)}$$

input
```
int(x^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
(2*log(a + b*x)*a**2 + 4*log(a + b*x)*a*b*x + 2*log(a + b*x)*b**2*x**2 + a
**2 - 2*b**2*x**2)/(2*b**3*(a**2 + 2*a*b*x + b**2*x**2))
```

3.102 $\int \frac{x}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{x}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=-\frac{1}{b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{a}{2b^2(a+bx)\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-1/b^2/((b*x+a)^2)^(1/2)+1/2*a/b^2/(b*x+a)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.75

$$\int \frac{x}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=-\frac{x^2\left(a^3+ab^2x^2-a\sqrt{a^2}\sqrt{(a+bx)^2}+\sqrt{a^2}bx\sqrt{(a+bx)^2}\right)}{2a^3(a+bx)\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}-\sqrt{(a+bx)^2}\right)\right)}$$

input
```
Integrate[x/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
-1/2*(x^2*(a^3 + a*b^2*x^2 - a*Sqrt[a^2]*Sqrt[(a + b*x)^2] + Sqrt[a^2]*b*x
*Sqrt[(a + b*x)^2]))/(a^3*(a + b*x)*(Sqrt[a^2]*b*x + a*(Sqrt[a^2] - Sqrt[(
a + b*x)^2])))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1100, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a^2+2abx+b^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{1100}$$

$$-\frac{a\int \frac{1}{(a^2+2bxa+b^2x^2)^{3/2}}dx}{b}-\frac{1}{b^2\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{1078}$$

$$\frac{a}{2b^2(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{1}{b^2\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[x/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
-(1/(b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])) + a/(2*b^2*(a + b*x)*Sqrt[a^2 + 2
*a*b*x + b^2*x^2])
```

Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.43

method	result	size
gosper	$-\frac{(bx+a)(2bx+a)}{2b^{2}\left((bx+a)^{2}\right)^{\frac{3}{2}}}$	26
default	$-\frac{(bx+a)(2bx+a)}{2b^{2}\left((bx+a)^{2}\right)^{\frac{3}{2}}}$	26
risch	$\frac{\sqrt{(bx+a)^{2}}\left(-\frac{x}{b}-\frac{a}{2b^{2}}\right)}{(bx+a)^{3}}$	31
orering	$-\frac{(2bx+a)(bx+a)}{2b^{2}(b^{2}x^{2}+2abx+a^{2})^{\frac{3}{2}}}$	35

input
```
int(x/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x+a)*(2*b*x+a)/b^2/((b*x+a)^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.52

$$\int \frac{x}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = -\frac{2\,bx+a}{2\,(b^4x^2+2\,ab^3x+a^2b^2)}$$

input
```
integrate(x/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Sympy [F]

$$\int \frac{x}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{x}{\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Integral(x/((a + b*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64

$$\int \frac{x}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = -\frac{1}{\sqrt{b^2x^2+2\,abx+a^2}b^2} + \frac{a}{2\,b^4\left(x+\frac{a}{b}\right)^2}$$

input
```
integrate(x/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^2) + 1/2*a/(b^4*(x + a/b)^2)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.43

$$\int \frac{x}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = -\frac{2\,bx+a}{2\,(bx+a)^2b^2\mathrm{sgn}\,(bx+a)}$$

input
```
integrate(x/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(2*b*x + a)/((b*x + a)^2*b^2*sgn(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.59

$$\int \frac{x}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = -\frac{(a+2\,b\,x)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{2\,b^2\,(a+b\,x)^3}$$

input
```
int(x/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
-((a + 2*b*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(2*b^2*(a + b*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.43

$$\int \frac{x}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{x^2}{2a\,(b^2x^2+2abx+a^2)}$$

input
```
int(x/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
x**2/(2*a*(a**2 + 2*a*b*x + b**2*x**2))
```

3.103 $\int \frac{1}{(a^2+2abx+b^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 34

$$\int \frac{1}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = -\frac{1}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-1/2/b/(b*x+a)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \frac{1}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = -\frac{a+bx}{2b\left((a+bx)^2\right)^{3/2}}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(-3/2),x]
```

output
```
-1/2*(a + b*x)/(b*((a + b*x)^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2abx+b^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 1078}$$

$$-\frac{1}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(-3/2),x]
```

output
```
-1/2*1/(b*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{bx+a}{2b\left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}}}$	20
default	$-\frac{bx+a}{2b\left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}}}$	20
risch	$-\frac{\sqrt{\left(bx+a\right)^{2}}}{2\left(bx+a\right)^{3}b}$	22
orering	$-\frac{bx+a}{2b\left(b^{2}x^{2}+2abx+a^{2}\right)^{\frac{3}{2}}}$	29

input
```
int(1/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x+a)/b/((b*x+a)^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=-\frac{1}{2\left(b^3x^2+2\,ab^2x+a^2b\right)}$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Sympy [F]

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{3/2}}\,dx = \int \frac{1}{(a^2 + 2abx + b^2x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Integral((a**2 + 2*a*b*x + b**2*x**2)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.41

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{3/2}}\,dx = -\frac{1}{2\,b^3\left(x + \frac{a}{b}\right)^2}$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2/(b^3*(x + a/b)^2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.59

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{3/2}}\,dx = -\frac{1}{2\,(bx + a)^2 b\mathrm{sgn}\,(bx + a)}$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
-1/2/((b*x + a)^2*b*sgn(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{3/2}}\,dx = -\frac{\sqrt{a^2 + 2\,a\,b\,x + b^2\,x^2}}{2\,b\,(a + b\,x)^3}$$

input
```
int(1/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
-(a^2 + b^2*x^2 + 2*a*b*x)^(1/2)/(2*b*(a + b*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{3/2}}\,dx = -\frac{1}{2b\,(b^2x^2 + 2abx + a^2)}$$

input
```
int(1/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
( - 1)/(2*b*(a**2 + 2*a*b*x + b**2*x**2))
```

3.104 $\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 126

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{1}{a^2\sqrt{a^2+2abx+b^2x^2}} + \frac{1}{2a(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{(a+bx)\log(x)}{a^3\sqrt{a^2+2abx+b^2x^2}} - \frac{(a+bx)\log(a+bx)}{a^3\sqrt{a^2+2abx+b^2x^2}}$$

output
```
1/a^2/((b*x+a)^2)^(1/2)+1/2/a/(b*x+a)/((b*x+a)^2)^(1/2)+(b*x+a)*ln(x)/a^3/
((b*x+a)^2)^(1/2)-(b*x+a)*ln(b*x+a)/a^3/((b*x+a)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 697 vs. 2(126) = 252.

Time = 1.22 (sec) , antiderivative size = 697, normalized size of antiderivative = 5.53

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{4a^4bx+3a^3b^2x^2-ab^4x^4-4(a^2)^{3/2}\,bx\sqrt{(a+bx)^2}+a\sqrt{a^2}b^2x^2\sqrt{(a+bx)^2}}{}$$

input
```
Integrate[1/(x*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

output
```
(4*a^4*b*x + 3*a^3*b^2*x^2 - a*b^4*x^4 - 4*(a^2)^(3/2)*b*x*Sqrt[(a + b*x)^
2] + a*Sqrt[a^2]*b^2*x^2*Sqrt[(a + b*x)^2] - Sqrt[a^2]*b^3*x^3*Sqrt[(a + b
*x)^2] + 2*((a^2)^(3/2)*b^2*x^2 + a^4*(Sqrt[a^2] - Sqrt[(a + b*x)^2]) + a^
3*b*x*(2*Sqrt[a^2] - Sqrt[(a + b*x)^2]))*ArcTanh[(b*x)/(Sqrt[a^2] - Sqrt[(
a + b*x)^2])] - 2*(a^5 + 2*a^4*b*x - (a^2)^(3/2)*b*x*Sqrt[(a + b*x)^2] + a
^3*(b^2*x^2 - Sqrt[a^2]*Sqrt[(a + b*x)^2]))*Log[x] + a^5*Log[Sqrt[a^2] - b
*x - Sqrt[(a + b*x)^2]] + 2*a^4*b*x*Log[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2
]] + a^3*b^2*x^2*Log[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2]] - a^3*Sqrt[a^2]*
Sqrt[(a + b*x)^2]*Log[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2]] - (a^2)^(3/2)*b
*x*Sqrt[(a + b*x)^2]*Log[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2]] + a^5*Log[Sq
rt[a^2] + b*x - Sqrt[(a + b*x)^2]] + 2*a^4*b*x*Log[Sqrt[a^2] + b*x - Sqrt[
(a + b*x)^2]] + a^3*b^2*x^2*Log[Sqrt[a^2] + b*x - Sqrt[(a + b*x)^2]] - a^3
*Sqrt[a^2]*Sqrt[(a + b*x)^2]*Log[Sqrt[a^2] + b*x - Sqrt[(a + b*x)^2]] - (a
^2)^(3/2)*b*x*Sqrt[(a + b*x)^2]*Log[Sqrt[a^2] + b*x - Sqrt[(a + b*x)^2]])/
(a^3*Sqrt[a^2]*(a^2 + a*b*x - Sqrt[a^2]*Sqrt[(a + b*x)^2])^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.55, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{b^3(a+bx)\int \frac{1}{b^3x(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{1}{x(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{54}$$

$$\frac{(a+bx)\int\left(-\frac{b}{a^3(a+bx)}-\frac{b}{a^2(a+bx)^2}-\frac{b}{a(a+bx)^3}+\frac{1}{a^3x}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(-\frac{\log(a+bx)}{a^3}+\frac{\log(x)}{a^3}+\frac{1}{a^2(a+bx)}+\frac{1}{2a(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[1/(x*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

output
```
((a + b*x)*(1/(2*a*(a + b*x)^2) + 1/(a^2*(a + b*x)) + Log[x]/a^3 - Log[a +
 b*x]/a^3))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{\sqrt{(bx+a)^2}\left(\frac{bx}{a^2}+\frac{3}{2a}\right)}{(bx+a)^3}+\frac{\sqrt{(bx+a)^2}\,\ln(-x)}{(bx+a)a^3}-\frac{\sqrt{(bx+a)^2}\,\ln(bx+a)}{(bx+a)a^3}$	82
default	$-\frac{(2b^2\ln(bx+a)x^2-2b^2\ln(x)x^2+4\ln(bx+a)abx-4\ln(x)abx+2\ln(bx+a)a^2-2a^2\ln(x)-2abx-3a^2)(bx+a)}{2a^3\left((bx+a)^2\right)^{\frac{3}{2}}}$	91

input
```
int(1/x/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)^3*(b/a^2*x+3/2/a)+((b*x+a)^2)^(1/2)/(b*x+a)/a^3*
ln(-x)-((b*x+a)^2)^(1/2)/(b*x+a)/a^3*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.63

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\frac{2\,abx+3\,a^2-2\,(b^2x^2+2\,abx+a^2)\log{(bx+a)}+2\,(b^2x^2+2\,abx+a^2)\,\mathrm{lo}}{2\,(a^3b^2x^2+2\,a^4bx+a^5)}$$

input
```
integrate(1/x/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(2*a*b*x + 3*a^2 - 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a) + 2*(b^2*x
^2 + 2*a*b*x + a^2)*log(x))/(a^3*b^2*x^2 + 2*a^4*b*x + a^5)
```

Sympy [F]

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \int \frac{1}{x\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Integral(1/(x*((a + b*x)**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.62

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = -\frac{(-1)^{2\,abx+2\,a^2}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)}{a^3} + \frac{1}{\sqrt{b^2x^2+2\,abx+a^2}a^2} + \frac{1}{2\,ab^2\left(x+\frac{a}{b}\right)^2}$$

input
```
integrate(1/x/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-(-1)^(2*a*b*x + 2*a^2)*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a^3 + 1/(sqrt(b
^2*x^2 + 2*a*b*x + a^2)*a^2) + 1/2/(a*b^2*(x + a/b)^2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.53

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = -\frac{\log\left(|bx+a|\right)}{a^3\mathrm{sgn}\left(bx+a\right)} + \frac{\log\left(|x|\right)}{a^3\mathrm{sgn}\left(bx+a\right)} + \frac{2\,abx+3\,a^2}{2\,(bx+a)^2a^3\mathrm{sgn}\left(bx+a\right)}$$

input
```
integrate(1/x/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
-log(abs(b*x + a))/(a^3*sgn(b*x + a)) + log(abs(x))/(a^3*sgn(b*x + a)) + 1
/2*(2*a*b*x + 3*a^2)/((b*x + a)^2*a^3*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\int \frac{1}{x\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}} \,dx$$

input
```
int(1/(x*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)),x)
```

output
```
int(1/(x*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.77

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\frac{-2\log(bx+a)\,a^2-4\log(bx+a)\,abx-2\log(bx+a)\,b^2x^2+2\log(x)\,a^2+}{2a^3\left(b^2x^2+2abx+a^2\right)}$$

input
```
int(1/x/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
( - 2*log(a + b*x)*a**2 - 4*log(a + b*x)*a*b*x - 2*log(a + b*x)*b**2*x**2
+ 2*log(x)*a**2 + 4*log(x)*a*b*x + 2*log(x)*b**2*x**2 + 2*a**2 - b**2*x**2
)/(2*a**3*(a**2 + 2*a*b*x + b**2*x**2))
```

3.105 $\int \frac{1}{x^2(a^2+2abx+b^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 165

$$\int \frac{1}{x^2\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = -\frac{2b}{a^3\sqrt{a^2+2abx+b^2x^2}} - \frac{b}{2a^2(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{a+bx}{a^3x\sqrt{a^2+2abx+b^2x^2}} - \frac{3b(a+bx)\log(x)}{a^4\sqrt{a^2+2abx+b^2x^2}} + \frac{3b(a+bx)\log(a+bx)}{a^4\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-2*b/a^3/((b*x+a)^2)^(1/2)-1/2*b/a^2/(b*x+a)/((b*x+a)^2)^(1/2)-(b*x+a)/a^3
/x/((b*x+a)^2)^(1/2)-3*b*(b*x+a)*ln(x)/a^4/((b*x+a)^2)^(1/2)+3*b*(b*x+a)*l
n(b*x+a)/a^4/((b*x+a)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 799 vs. 2(165) = 330.

Time = 1.45 (sec) , antiderivative size = 799, normalized size of antiderivative = 4.84

$$\int \frac{1}{x^2\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{2a^6+5a^5bx-2a^4b^2x^2-4a^3b^3x^3+ab^5x^5-2a^4\sqrt{a^2}\sqrt{(a+bx)^2}-3a^3\sqrt{a}}{}$$

input
```
Integrate[1/(x^2*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

output
```
(2*a^6 + 5*a^5*b*x - 2*a^4*b^2*x^2 - 4*a^3*b^3*x^3 + a*b^5*x^5 - 2*a^4*Sqr
t[a^2]*Sqrt[(a + b*x)^2] - 3*a^3*Sqrt[a^2]*b*x*Sqrt[(a + b*x)^2] + 5*(a^2)
^(3/2)*b^2*x^2*Sqrt[(a + b*x)^2] - a*Sqrt[a^2]*b^3*x^3*Sqrt[(a + b*x)^2] +
 Sqrt[a^2]*b^4*x^4*Sqrt[(a + b*x)^2] - 6*b*x*((a^2)^(3/2)*b^2*x^2 + a^4*(S
qrt[a^2] - Sqrt[(a + b*x)^2]) + a^3*b*x*(2*Sqrt[a^2] - Sqrt[(a + b*x)^2]))
*ArcTanh[(b*x)/(Sqrt[a^2] - Sqrt[(a + b*x)^2])] + 6*b*x*(a^5 + 2*a^4*b*x -
 (a^2)^(3/2)*b*x*Sqrt[(a + b*x)^2] + a^3*(b^2*x^2 - Sqrt[a^2]*Sqrt[(a + b*
x)^2]))*Log[x] - 3*a^5*b*x*Log[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2]] - 6*a^
4*b^2*x^2*Log[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2]] - 3*a^3*b^3*x^3*Log[Sqr
t[a^2] - b*x - Sqrt[(a + b*x)^2]] + 3*a^3*Sqrt[a^2]*b*x*Sqrt[(a + b*x)^2]*
Log[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2]] + 3*(a^2)^(3/2)*b^2*x^2*Sqrt[(a +
 b*x)^2]*Log[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2]] - 3*a^5*b*x*Log[Sqrt[a^2
] + b*x - Sqrt[(a + b*x)^2]] - 6*a^4*b^2*x^2*Log[Sqrt[a^2] + b*x - Sqrt[(a
 + b*x)^2]] - 3*a^3*b^3*x^3*Log[Sqrt[a^2] + b*x - Sqrt[(a + b*x)^2]] + 3*a
^3*Sqrt[a^2]*b*x*Sqrt[(a + b*x)^2]*Log[Sqrt[a^2] + b*x - Sqrt[(a + b*x)^2]
] + 3*(a^2)^(3/2)*b^2*x^2*Sqrt[(a + b*x)^2]*Log[Sqrt[a^2] + b*x - Sqrt[(a
+ b*x)^2]])/(a^4*Sqrt[a^2]*x*(a^2 + a*b*x - Sqrt[a^2]*Sqrt[(a + b*x)^2])^2
)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)^{3/2}} dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{b^3(a+bx) \int \frac{1}{b^3x^2(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{1}{x^2(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$\big\downarrow$ 54

$$\frac{(a+bx)\int \left(\frac{3b^2}{a^4(a+bx)}+\frac{2b^2}{a^3(a+bx)^2}+\frac{b^2}{a^2(a+bx)^3}-\frac{3b}{a^4x}+\frac{1}{a^3x^2}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$\big\downarrow$ 2009

$$\frac{(a+bx)\left(-\frac{3b\log(x)}{a^4}+\frac{3b\log(a+bx)}{a^4}-\frac{2b}{a^3(a+bx)}-\frac{1}{a^3x}-\frac{b}{2a^2(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[1/(x^2*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

output
```
((a + b*x)*(-(1/(a^3*x)) - b/(2*a^2*(a + b*x)^2) - (2*b)/(a^3*(a + b*x)) -
 (3*b*Log[x])/a^4 + (3*b*Log[a + b*x])/a^4))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.61

method	result
risch	$\frac{\sqrt{(bx+a)^2}\left(-\frac{3b^2x^2}{a^3}-\frac{9bx}{2a^2}-\frac{1}{a}\right)}{(bx+a)^3 x}+\frac{3\sqrt{(bx+a)^2}\, b\ln(-bx-a)}{(bx+a)a^4}-\frac{3\sqrt{(bx+a)^2}\, b\ln(x)}{(bx+a)a^4}$
default	$\frac{(6\ln(bx+a)x^3b^3-6\ln(x)x^3b^3+12\ln(bx+a)x^2a\,b^2-12\ln(x)x^2a\,b^2+6\ln(bx+a)x\,a^2b-6\ln(x)x\,a^2b-6a\,b^2x^2-9a^2bx-2a^3)(bx+a)}{2x\,a^4\left((bx+a)^2\right)^{\frac{3}{2}}}$

input
```
int(1/x^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)^3*(-3*b^2/a^3*x^2-9/2*b/a^2*x-1/a)/x+3*((b*x+a)^
2)^(1/2)/(b*x+a)*b/a^4*ln(-b*x-a)-3*((b*x+a)^2)^(1/2)/(b*x+a)*b*ln(x)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^2 (a^2+2abx+b^2x^2)^{3/2}}\,dx = -\frac{6\,ab^2x^2+9\,a^2bx+2\,a^3-6\,(b^3x^3+2\,ab^2x^2+a^2bx)\log(bx+a)+6\,(b^3x^3+2\,ab^2x^2+a^2bx)\log(x)}{2\,(a^4b^2x^3+2\,a^5bx^2+a^6x)}$$

input
```
integrate(1/x^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(6*a*b^2*x^2 + 9*a^2*b*x + 2*a^3 - 6*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x
)*log(b*x + a) + 6*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)*log(x))/(a^4*b^2*x^3
+ 2*a^5*b*x^2 + a^6*x)
```

Sympy [F]

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)^{3/2}} \, dx = \int \frac{1}{x^2 \left((a + bx)^2\right)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x**2/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Integral(1/(x**2*((a + b*x)**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)^{3/2}} \, dx = \frac{3 (-1)^{2\,abx+2\,a^2} b \log\left(\frac{2\,abx}{|x|} + \frac{2\,a^2}{|x|}\right)}{a^4} - \frac{3\,b}{\sqrt{b^2x^2 + 2\,abx + a^2}a^3} - \frac{1}{\sqrt{b^2x^2 + 2\,abx + a^2}a^2x} - \frac{1}{2\,a^2b\left(x + \frac{a}{b}\right)^2}$$

input
```
integrate(1/x^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
3*(-1)^(2*a*b*x + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a^4 - 3*b/(s
qrt(b^2*x^2 + 2*a*b*x + a^2)*a^3) - 1/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^2*x
) - 1/2/(a^2*b*(x + a/b)^2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)^{3/2}} \, dx = \frac{3\,b \log\left(|bx + a|\right)}{a^4 \mathrm{sgn}\left(bx + a\right)} - \frac{3\,b \log\left(|x|\right)}{a^4 \mathrm{sgn}\left(bx + a\right)} - \frac{6\,ab^2x^2 + 9\,a^2bx + 2\,a^3}{2\,(bx + a)^2 a^4 x \mathrm{sgn}\left(bx + a\right)}$$

input
```
integrate(1/x^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
3*b*log(abs(b*x + a))/(a^4*sgn(b*x + a)) - 3*b*log(abs(x))/(a^4*sgn(b*x +
a)) - 1/2*(6*a*b^2*x^2 + 9*a^2*b*x + 2*a^3)/((b*x + a)^2*a^4*x*sgn(b*x + a
))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)^{3/2}} \, dx = \int \frac{1}{x^2 \left(a^2 + 2\,a\,b\,x + b^2\,x^2\right)^{3/2}} \, dx$$

input
```
int(1/(x^2*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)),x)
```

output
```
int(1/(x^2*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)^{3/2}} \, dx = \frac{6\log(bx+a)\,a^2bx + 12\log(bx+a)\,a\,b^2x^2 + 6\log(bx+a)\,b^3x^3 - 6\log(x)}{2a^4x\,(b^2x^2 + 2aba}$$

input
```
int(1/x^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
(6*log(a + b*x)*a**2*b*x + 12*log(a + b*x)*a*b**2*x**2 + 6*log(a + b*x)*b*
*3*x**3 - 6*log(x)*a**2*b*x - 12*log(x)*a*b**2*x**2 - 6*log(x)*b**3*x**3 -
 2*a**3 - 6*a**2*b*x + 3*b**3*x**3)/(2*a**4*x*(a**2 + 2*a*b*x + b**2*x**2)
)
```

3.106 $\int \frac{1}{x^3(a^2+2abx+b^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 209

$$\int \frac{1}{x^3\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{3b^2}{a^4\sqrt{a^2+2abx+b^2x^2}} + \frac{b^2}{2a^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{a+bx}{2a^3x^2\sqrt{a^2+2abx+b^2x^2}} + \frac{3b(a+bx)}{a^4x\sqrt{a^2+2abx+b^2x^2}} + \frac{6b^2(a+bx)\log(x)}{a^5\sqrt{a^2+2abx+b^2x^2}} - \frac{6b^2(a+bx)\log(a+bx)}{a^5\sqrt{a^2+2abx+b^2x^2}}$$

output
```
3*b^2/a^4/((b*x+a)^2)^(1/2)+1/2*b^2/a^3/(b*x+a)/((b*x+a)^2)^(1/2)-1/2*(b*x
+a)/a^3/x^2/((b*x+a)^2)^(1/2)+3*b*(b*x+a)/a^4/x/((b*x+a)^2)^(1/2)+6*b^2*(b
*x+a)*ln(x)/a^5/((b*x+a)^2)^(1/2)-6*b^2*(b*x+a)*ln(b*x+a)/a^5/((b*x+a)^2)^
(1/2)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^3 (a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{a(-a^3+4a^2bx+18ab^2x^2+12b^3x^3)+12b^2x^2(a+bx)^2\log(x)-12b^2x^2(a}{2a^5x^2(a+bx)\sqrt{(a+bx)^2}}$$

input
```
Integrate[1/(x^3*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

output
```
(a*(-a^3 + 4*a^2*b*x + 18*a*b^2*x^2 + 12*b^3*x^3) + 12*b^2*x^2*(a + b*x)^2
*Log[x] - 12*b^2*x^2*(a + b*x)^2*Log[a + b*x])/(2*a^5*x^2*(a + b*x)*Sqrt[(
a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a^2+2abx+b^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 1102}$$

$$\frac{b^3(a+bx)\int \frac{1}{b^3x^3(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx)\int \frac{1}{x^3(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{(a+bx)\int \left(-\frac{6b^3}{a^5(a+bx)}-\frac{3b^3}{a^4(a+bx)^2}-\frac{b^3}{a^3(a+bx)^3}+\frac{6b^2}{a^5x}-\frac{3b}{a^4x^2}+\frac{1}{a^3x^3}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(\frac{6b^2\log(x)}{a^5}-\frac{6b^2\log(a+bx)}{a^5}+\frac{3b^2}{a^4(a+bx)}+\frac{3b}{a^4x}+\frac{b^2}{2a^3(a+bx)^2}-\frac{1}{2a^3x^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[1/(x^3*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

output
```
((a + b*x)*(-1/2*1/(a^3*x^2) + (3*b)/(a^4*x) + b^2/(2*a^3*(a + b*x)^2) + (
3*b^2)/(a^4*(a + b*x)) + (6*b^2*Log[x])/a^5 - (6*b^2*Log[a + b*x])/a^5))/S
qrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.55

method	result
risch	$\frac{\sqrt{(bx+a)^2}\left(\frac{6b^3x^3}{a^4}+\frac{9b^2x^2}{a^3}+\frac{2bx}{a^2}-\frac{1}{2a}\right)}{(bx+a)^3x^2}-\frac{6\sqrt{(bx+a)^2}\,b^2\ln(bx+a)}{(bx+a)a^5}+\frac{6\sqrt{(bx+a)^2}\,b^2\ln(-x)}{(bx+a)a^5}$
default	$-\frac{(12\ln(bx+a)x^4b^4-12\ln(x)x^4b^4+24\ln(bx+a)x^3a\,b^3-24\ln(x)x^3a\,b^3+12\ln(bx+a)x^2a^2b^2-12a^2b^2\ln(x)x^2-12a\,b^3x^3-18a^2b^2x^2}{2x^2a^5\left((bx+a)^2\right)^{\frac{3}{2}}}$

input
```
int(1/x^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)^3*(6*b^3/a^4*x^3+9*b^2/a^3*x^2+2*b/a^2*x-1/2/a)/
x^2-6*((b*x+a)^2)^(1/2)/(b*x+a)*b^2/a^5*ln(b*x+a)+6*((b*x+a)^2)^(1/2)/(b*x
+a)*b^2/a^5*ln(-x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^3\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\frac{12\,ab^3x^3+18\,a^2b^2x^2+4\,a^3bx-a^4-12\left(b^4x^4+2\,ab^3x^3+a^2b^2x^2\right)\log\left(bx\right.}{2\left(a^5b^2x^4+2\,a^6bx^3+a^7x^2\right)}$$

input
```
integrate(1/x^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(12*a*b^3*x^3 + 18*a^2*b^2*x^2 + 4*a^3*b*x - a^4 - 12*(b^4*x^4 + 2*a*b
^3*x^3 + a^2*b^2*x^2)*log(b*x + a) + 12*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x
^2)*log(x))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)
```

Sympy [F]

$$\int \frac{1}{x^3 (a^2 + 2abx + b^2x^2)^{3/2}} \, dx = \int \frac{1}{x^3 \left((a + bx)^2\right)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x**3/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Integral(1/(x**3*((a + b*x)**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.65

$$\begin{aligned}\int \frac{1}{x^3 (a^2 + 2abx + b^2x^2)^{3/2}} \, dx &= -\frac{6 \, (-1)^{2 \, abx + 2 \, a^2} b^2 \log\left(\frac{2 \, abx}{|x|} + \frac{2 \, a^2}{|x|}\right)}{a^5} \\ &+ \frac{6 \, b^2}{\sqrt{b^2x^2 + 2 \, abx + a^2} a^4} + \frac{5 \, b}{2 \, \sqrt{b^2x^2 + 2 \, abx + a^2} a^3 x} \\ &- \frac{1}{2 \, \sqrt{b^2x^2 + 2 \, abx + a^2} a^2 x^2} + \frac{1}{2 \, a^3 \left(x + \frac{a}{b}\right)^2}\end{aligned}$$

input
```
integrate(1/x^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-6*(-1)^(2*a*b*x + 2*a^2)*b^2*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a^5 + 6*b
^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^4) + 5/2*b/(sqrt(b^2*x^2 + 2*a*b*x + a
^2)*a^3*x) - 1/2/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^2*x^2) + 1/2/(a^3*(x + a
/b)^2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.46

$$\int \frac{1}{x^3 (a^2+2abx+b^2x^2)^{3/2}}\,dx = -\frac{6\,b^2\log\left(|bx+a|\right)}{a^5\mathrm{sgn}\left(bx+a\right)} + \frac{6\,b^2\log\left(|x|\right)}{a^5\mathrm{sgn}\left(bx+a\right)} + \frac{12\,b^3x^3+18\,ab^2x^2+4\,a^2bx-a^3}{2\,(bx^2+ax)^2a^4\mathrm{sgn}\left(bx+a\right)}$$

input
```
integrate(1/x^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
-6*b^2*log(abs(b*x + a))/(a^5*sgn(b*x + a)) + 6*b^2*log(abs(x))/(a^5*sgn(b
*x + a)) + 1/2*(12*b^3*x^3 + 18*a*b^2*x^2 + 4*a^2*b*x - a^3)/((b*x^2 + a*x
)^2*a^4*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{1}{x^3\,{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}^{3/2}}\,dx$$

input
```
int(1/(x^3*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)),x)
```

output
```
int(1/(x^3*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^3 (a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{-12\log(bx+a)\,a^2b^2x^2-24\log(bx+a)\,a\,b^3x^3-12\log(bx+a)\,b^4x^4+12}{2a^5x^2\,(}$$

input
```
int(1/x^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
( - 12*log(a + b*x)*a**2*b**2*x**2 - 24*log(a + b*x)*a*b**3*x**3 - 12*log(
a + b*x)*b**4*x**4 + 12*log(x)*a**2*b**2*x**2 + 24*log(x)*a*b**3*x**3 + 12
*log(x)*b**4*x**4 - a**4 + 4*a**3*b*x + 12*a**2*b**2*x**2 - 6*b**4*x**4)/(
2*a**5*x**2*(a**2 + 2*a*b*x + b**2*x**2))
```

3.107 $\int \frac{x^6}{(a^2+2abx+b^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 244

$$\int \frac{x^6}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{20a^3}{b^7\sqrt{a^2+2abx+b^2x^2}} - \frac{a^6}{4b^7(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{2a^5}{b^7(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{15a^4}{2b^7(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{5ax(a+bx)}{b^6\sqrt{a^2+2abx+b^2x^2}} + \frac{x^2(a+bx)}{2b^5\sqrt{a^2+2abx+b^2x^2}} + \frac{15a^2(a+bx)\log(a+bx)}{b^7\sqrt{a^2+2abx+b^2x^2}}$$

output
```
20*a^3/b^7/((b*x+a)^2)^(1/2)-1/4*a^6/b^7/(b*x+a)^3/((b*x+a)^2)^(1/2)+2*a^5
/b^7/(b*x+a)^2/((b*x+a)^2)^(1/2)-15/2*a^4/b^7/(b*x+a)/((b*x+a)^2)^(1/2)-5*
a*x*(b*x+a)/b^6/((b*x+a)^2)^(1/2)+1/2*x^2*(b*x+a)/b^5/((b*x+a)^2)^(1/2)+15
*a^2*(b*x+a)*ln(b*x+a)/b^7/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.26

$$\int \frac{x^6}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{\frac{bx\left(-a^2b^5x^5\sqrt{(a+bx)^2}+ab^6x^6\sqrt{(a+bx)^2}+\sqrt{a^2}b^5x^5(-2a^2+b^2x^2)+10a^5b^2x^2\left(26\sqrt{a^2}-11\sqrt{(a+bx)^2}\right)+\cdots}{(a+bx)^3}}{\cdots}$$

input
```
Integrate[x^6/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
((b*x*(-(a^2*b^5*x^5*Sqrt[(a + b*x)^2]) + a*b^6*x^6*Sqrt[(a + b*x)^2] + Sq
rt[a^2]*b^5*x^5*(-2*a^2 + b^2*x^2) + 10*a^5*b^2*x^2*(26*Sqrt[a^2] - 11*Sqr
t[(a + b*x)^2]) + 30*a^6*b*x*(7*Sqrt[a^2] - 5*Sqrt[(a + b*x)^2]) + 5*a^4*b
^3*x^3*(25*Sqrt[a^2] - 3*Sqrt[(a + b*x)^2]) + 60*a^7*(Sqrt[a^2] - Sqrt[(a
+ b*x)^2]) + 3*a^3*b^4*x^4*(4*Sqrt[a^2] + Sqrt[(a + b*x)^2])))/((a + b*x)^
3*(a^2 + a*b*x - Sqrt[a^2]*Sqrt[(a + b*x)^2])) - 120*a^4*ArcTanh[(b*x)/(Sq
rt[a^2] - Sqrt[(a + b*x)^2])])/(4*a^2*b^7)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a^2+2abx+b^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{b^5(a+bx)\int \frac{x^6}{b^5(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{x^6}{(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\downarrow \text{49}$$

$$\frac{(a+bx)\int\left(\frac{a^6}{b^6(a+bx)^5}-\frac{6a^5}{b^6(a+bx)^4}+\frac{15a^4}{b^6(a+bx)^3}-\frac{20a^3}{b^6(a+bx)^2}+\frac{15a^2}{b^6(a+bx)}-\frac{5a}{b^6}+\frac{x}{b^5}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\downarrow \text{2009}$$

$$\frac{(a+bx)\left(-\frac{a^6}{4b^7(a+bx)^4}+\frac{2a^5}{b^7(a+bx)^3}-\frac{15a^4}{2b^7(a+bx)^2}+\frac{20a^3}{b^7(a+bx)}+\frac{15a^2\log(a+bx)}{b^7}-\frac{5ax}{b^6}+\frac{x^2}{2b^5}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[x^6/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
((a + b*x)*((-5*a*x)/b^6 + x^2/(2*b^5) - a^6/(4*b^7*(a + b*x)^4) + (2*a^5)
/(b^7*(a + b*x)^3) - (15*a^4)/(2*b^7*(a + b*x)^2) + (20*a^3)/(b^7*(a + b*x
)) + (15*a^2*Log[a + b*x])/b^7))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.48

method	result
risch	$\frac{\sqrt{(bx+a)^2}\,\left(\frac{1}{2} b\,x^2-5ax\right)}{(bx+a)b^6}+\frac{\sqrt{(bx+a)^2}\,\left(20a^3b^2x^3+\frac{105a^4x^2b}{2}+47a^5x+\frac{57a^6}{4b}\right)}{(bx+a)^5 b^6}+\frac{15\sqrt{(bx+a)^2}\,a^2\ln(bx+a)}{(bx+a)b^7}$
default	$\frac{(2b^6x^6+60\ln(bx+a)a^2b^4x^4-12a\,b^5x^5+240\ln(bx+a)a^3b^3x^3-68a^2b^4x^4+360\ln(bx+a)x^2a^4b^2-32a^3b^3x^3+240\ln(bx+a)a^5bx+132}{4b^7\left((bx+a)^2\right)^{\frac{5}{2}}}$

input
```
int(x^6/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*(1/2*b*x^2-5*a*x)/b^6+((b*x+a)^2)^(1/2)/(b*x+a)^
5*(20*a^3*b^2*x^3+105/2*a^4*x^2*b+47*a^5*x+57/4*a^6/b)/b^6+15*((b*x+a)^2)^
(1/2)/(b*x+a)/b^7*a^2*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.66

$$\int \frac{x^6}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{2\,b^6x^6-12\,ab^5x^5-68\,a^2b^4x^4-32\,a^3b^3x^3+132\,a^4b^2x^2+168\,a^5bx+57\,a^6+}{4\,(b^{11}x^4+4\,ab^{10}x^3+6\,a^2b^9x^2+}$$

input
```
integrate(x^6/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/4*(2*b^6*x^6 - 12*a*b^5*x^5 - 68*a^2*b^4*x^4 - 32*a^3*b^3*x^3 + 132*a^4*
b^2*x^2 + 168*a^5*b*x + 57*a^6 + 60*(a^2*b^4*x^4 + 4*a^3*b^3*x^3 + 6*a^4*b
^2*x^2 + 4*a^5*b*x + a^6)*log(b*x + a))/(b^11*x^4 + 4*a*b^10*x^3 + 6*a^2*b
^9*x^2 + 4*a^3*b^8*x + a^4*b^7)
```

Sympy [F]

$$\int \frac{x^6}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \int \frac{x^6}{\left((a+bx)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**6/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Integral(x**6/((a + b*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.52

$$\int \frac{x^6}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{2\,b^6x^6 - 12\,ab^5x^5 - 68\,a^2b^4x^4 - 32\,a^3b^3x^3 + 132\,a^4b^2x^2 + 168\,a^5bx + 57\,a^6}{4\,(b^{11}x^4 + 4\,ab^{10}x^3 + 6\,a^2b^9x^2 + 4\,a^3b^8x + a^4b^7)} + \frac{15\,a^2\log(bx+a)}{b^7}$$

input
```
integrate(x^6/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/4*(2*b^6*x^6 - 12*a*b^5*x^5 - 68*a^2*b^4*x^4 - 32*a^3*b^3*x^3 + 132*a^4*
b^2*x^2 + 168*a^5*b*x + 57*a^6)/(b^11*x^4 + 4*a*b^10*x^3 + 6*a^2*b^9*x^2 +
 4*a^3*b^8*x + a^4*b^7) + 15*a^2*log(b*x + a)/b^7
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.45

$$\int \frac{x^6}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{15\,a^2\log(|bx+a|)}{b^7\mathrm{sgn}(bx+a)} + \frac{b^5x^2\mathrm{sgn}(bx+a) - 10\,ab^4x\mathrm{sgn}(bx+a)}{2\,b^{10}} + \frac{80\,a^3b^3x^3 + 210\,a^4b^2x^2 + 188\,a^5bx + 57\,a^6}{4\,(bx+a)^4b^7\mathrm{sgn}(bx+a)}$$

input
```
integrate(x^6/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
15*a^2*log(abs(b*x + a))/(b^7*sgn(b*x + a)) + 1/2*(b^5*x^2*sgn(b*x + a) -
10*a*b^4*x*sgn(b*x + a))/b^10 + 1/4*(80*a^3*b^3*x^3 + 210*a^4*b^2*x^2 + 18
8*a^5*b*x + 57*a^6)/((b*x + a)^4*b^7*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a^2+2abx+b^2x^2)^{5/2}}\,dx=\int \frac{x^6}{(a^2+2\,a\,b\,x+b^2\,x^2)^{5/2}}\,dx$$

input
```
int(x^6/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int(x^6/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.70

$$\int \frac{x^6}{(a^2+2abx+b^2x^2)^{5/2}}\,dx=\frac{60\log(bx+a)\,a^6+240\log(bx+a)\,a^5bx+360\log(bx+a)\,a^4b^2x^2+240\log(}{4b^7\,(b^4x^4}$$

input
```
int(x^6/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(60*log(a + b*x)*a**6 + 240*log(a + b*x)*a**5*b*x + 360*log(a + b*x)*a**4*
b**2*x**2 + 240*log(a + b*x)*a**3*b**3*x**3 + 60*log(a + b*x)*a**2*b**4*x*
*4 + 65*a**6 + 200*a**5*b*x + 180*a**4*b**2*x**2 - 60*a**2*b**4*x**4 - 12*
a*b**5*x**5 + 2*b**6*x**6)/(4*b**7*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 +
 4*a*b**3*x**3 + b**4*x**4))
```

3.108 $\int \frac{x^5}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 205

$$\int \frac{x^5}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{10a^2}{b^6\sqrt{a^2+2abx+b^2x^2}} + \frac{a^5}{4b^6(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{5a^4}{3b^6(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} + \frac{5a^3}{b^6(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{x(a+bx)}{b^5\sqrt{a^2+2abx+b^2x^2}} - \frac{5a(a+bx)\log(a+bx)}{b^6\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-10*a^2/b^6/((b*x+a)^2)^(1/2)+1/4*a^5/b^6/(b*x+a)^3/((b*x+a)^2)^(1/2)-5/3*
a^4/b^6/(b*x+a)^2/((b*x+a)^2)^(1/2)+5*a^3/b^6/(b*x+a)/((b*x+a)^2)^(1/2)+x*
(b*x+a)/b^5/((b*x+a)^2)^(1/2)-5*a*(b*x+a)*ln(b*x+a)/b^6/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.45

$$\int \frac{x^5}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{-77a^5-248a^4bx-252a^3b^2x^2-48a^2b^3x^3+48ab^4x^4+12b^5x^5-60a(a+b}{12b^6(a+bx)^3\sqrt{(a+bx)^2}}$$

input
```
Integrate[x^5/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(-77*a^5 - 248*a^4*b*x - 252*a^3*b^2*x^2 - 48*a^2*b^3*x^3 + 48*a*b^4*x^4 +
 12*b^5*x^5 - 60*a*(a + b*x)^4*Log[a + b*x])/(12*b^6*(a + b*x)^3*Sqrt[(a +
 b*x)^2])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.53, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a^2+2abx+b^2x^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{b^5(a+bx)\int \frac{x^5}{b^5(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{x^5}{(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{(a+bx)\int \left(-\frac{a^5}{b^5(a+bx)^5}+\frac{5a^4}{b^5(a+bx)^4}-\frac{10a^3}{b^5(a+bx)^3}+\frac{10a^2}{b^5(a+bx)^2}-\frac{5a}{b^5(a+bx)}+\frac{1}{b^5}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(\frac{a^5}{4b^6(a+bx)^4}-\frac{5a^4}{3b^6(a+bx)^3}+\frac{5a^3}{b^6(a+bx)^2}-\frac{10a^2}{b^6(a+bx)}-\frac{5a\log(a+bx)}{b^6}+\frac{x}{b^5}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[x^5/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
((a + b*x)*(x/b^5 + a^5/(4*b^6*(a + b*x)^4) - (5*a^4)/(3*b^6*(a + b*x)^3)
+ (5*a^3)/(b^6*(a + b*x)^2) - (10*a^2)/(b^6*(a + b*x)) - (5*a*Log[a + b*x]
)/b^6))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.52

method	result
risch	$\frac{\sqrt{(bx+a)^2}\,x}{(bx+a)b^5}+\frac{\sqrt{(bx+a)^2}\left(-10a^2b^2x^3-25a^3b\,x^2-\frac{65a^4x}{3}-\frac{77a^5}{12b}\right)}{(bx+a)^5b^5}-\frac{5\sqrt{(bx+a)^2}\,a\ln(bx+a)}{(bx+a)b^6}$
default	$-\frac{(60\ln(bx+a)x^4a\,b^4-12b^5x^5+240\ln(bx+a)x^3a^2b^3-48a\,b^4x^4+360\ln(bx+a)x^2a^3b^2+48a^2b^3x^3+240\ln(bx+a)x\,a^4b+252a^3b^2x^2+}{12b^6\left((bx+a)^2\right)^{\frac{5}{2}}}$

input
```
int(x^5/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)/b^5*x+((b*x+a)^2)^(1/2)/(b*x+a)^5*(-10*a^2*b^2*x
^3-25*a^3*b*x^2-65/3*a^4*x-77/12*a^5/b)/b^5-5*((b*x+a)^2)^(1/2)/(b*x+a)/b^
6*a*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.73

$$\int \frac{x^5}{(a^2+2abx+b^2x^2)^{5/2}}\,dx=\frac{12\,b^5x^5+48\,ab^4x^4-48\,a^2b^3x^3-252\,a^3b^2x^2-248\,a^4bx-77\,a^5-60\,(ab^4x^4}{12\,(b^{10}x^4+4\,ab^9x^3+6\,a^2b^8x^2+4\,a^3b}$$

input
```
integrate(x^5/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/12*(12*b^5*x^5 + 48*a*b^4*x^4 - 48*a^2*b^3*x^3 - 252*a^3*b^2*x^2 - 248*a
^4*b*x - 77*a^5 - 60*(a*b^4*x^4 + 4*a^2*b^3*x^3 + 6*a^3*b^2*x^2 + 4*a^4*b*
x + a^5)*log(b*x + a))/(b^10*x^4 + 4*a*b^9*x^3 + 6*a^2*b^8*x^2 + 4*a^3*b^7
*x + a^4*b^6)
```

Sympy [F]

$$\int \frac{x^5}{(a^2 + 2abx + b^2x^2)^{5/2}} \, dx = \int \frac{x^5}{\left((a + bx)^2\right)^{\frac{5}{2}}} \, dx$$

input
```
integrate(x**5/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Integral(x**5/((a + b*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.55

$$\int \frac{x^5}{(a^2 + 2abx + b^2x^2)^{5/2}} \, dx = \frac{12\, b^5x^5 + 48\, ab^4x^4 - 48\, a^2b^3x^3 - 252\, a^3b^2x^2 - 248\, a^4bx - 77\, a^5}{12\, (b^{10}x^4 + 4\, ab^9x^3 + 6\, a^2b^8x^2 + 4\, a^3b^7x + a^4b^6)} - \frac{5\, a \log (bx + a)}{b^6}$$

input
```
integrate(x^5/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/12*(12*b^5*x^5 + 48*a*b^4*x^4 - 48*a^2*b^3*x^3 - 252*a^3*b^2*x^2 - 248*a
^4*b*x - 77*a^5)/(b^10*x^4 + 4*a*b^9*x^3 + 6*a^2*b^8*x^2 + 4*a^3*b^7*x + a
^4*b^6) - 5*a*log(b*x + a)/b^6
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.44

$$\int \frac{x^5}{(a^2 + 2abx + b^2x^2)^{5/2}} \, dx = \frac{x}{b^5 \mathrm{sgn}\,(bx + a)} - \frac{5\, a \log (|bx + a|)}{b^6 \mathrm{sgn}\,(bx + a)} - \frac{120\, a^2b^3x^3 + 300\, a^3b^2x^2 + 260\, a^4bx + 77\, a^5}{12\, (bx + a)^4 b^6 \mathrm{sgn}\,(bx + a)}$$

input
```
integrate(x^5/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
x/(b^5*sgn(b*x + a)) - 5*a*log(abs(b*x + a))/(b^6*sgn(b*x + a)) - 1/12*(12
0*a^2*b^3*x^3 + 300*a^3*b^2*x^2 + 260*a^4*b*x + 77*a^5)/((b*x + a)^4*b^6*s
gn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \int \frac{x^5}{(a^2+2\,a\,b\,x+b^2\,x^2)^{5/2}}\,dx$$

input
```
int(x^5/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int(x^5/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.78

$$\int \frac{x^5}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{-60\log(bx+a)\,a^5-240\log(bx+a)\,a^4bx-360\log(bx+a)\,a^3b^2x^2-240\,\mathrm{lo}}{12b^6\,(b^4x^4+4}$$

input
```
int(x^5/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
( - 60*log(a + b*x)*a**5 - 240*log(a + b*x)*a**4*b*x - 360*log(a + b*x)*a*
*3*b**2*x**2 - 240*log(a + b*x)*a**2*b**3*x**3 - 60*log(a + b*x)*a*b**4*x*
*4 - 65*a**5 - 200*a**4*b*x - 180*a**3*b**2*x**2 + 60*a*b**4*x**4 + 12*b**
5*x**5)/(12*b**6*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b
**4*x**4))
```

3.109 $\int \frac{x^4}{(a^2+2abx+b^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 171

$$\int \frac{x^4}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{4a}{b^5\sqrt{a^2+2abx+b^2x^2}} - \frac{a^4}{4b^5(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{4a^3}{3b^5(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{3a^2}{b^5(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{(a+bx)\log(a+bx)}{b^5\sqrt{a^2+2abx+b^2x^2}}$$

output
```
4*a/b^5/((b*x+a)^2)^(1/2)-1/4*a^4/b^5/(b*x+a)^3/((b*x+a)^2)^(1/2)+4/3*a^3/
b^5/(b*x+a)^2/((b*x+a)^2)^(1/2)-3*a^2/b^5/(b*x+a)/((b*x+a)^2)^(1/2)+(b*x+a
)*ln(b*x+a)/b^5/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.83

$$\int \frac{x^4}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{bx\left(3\sqrt{a^2}b^7x^7+3a^3b^4x^4\sqrt{(a+bx)^2}-3a^2b^5x^5\sqrt{(a+bx)^2}+3ab^6x^6\sqrt{(a+bx)^2}+2a^5b^2x^2\left(26\sqrt{a^2}-11\sqrt{(a+\right.\right.}{a^4(a+bx)^3\left(a^2+abx-\sqrt{\right.}}$$

input
```
Integrate[x^4/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
((b*x*(3*Sqrt[a^2]*b^7*x^7 + 3*a^3*b^4*x^4*Sqrt[(a + b*x)^2] - 3*a^2*b^5*x
^5*Sqrt[(a + b*x)^2] + 3*a*b^6*x^6*Sqrt[(a + b*x)^2] + 2*a^5*b^2*x^2*(26*S
qrt[a^2] - 11*Sqrt[(a + b*x)^2]) + 6*a^6*b*x*(7*Sqrt[a^2] - 5*Sqrt[(a + b*
x)^2]) + a^4*b^3*x^3*(25*Sqrt[a^2] - 3*Sqrt[(a + b*x)^2]) + 12*a^7*(Sqrt[a
^2] - Sqrt[(a + b*x)^2])))/(a^4*(a + b*x)^3*(a^2 + a*b*x - Sqrt[a^2]*Sqrt[
(a + b*x)^2])) + 12*Log[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2]] - 12*Log[b^5*
(Sqrt[a^2] + b*x - Sqrt[(a + b*x)^2])])/(12*b^5)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.58, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a^2+2abx+b^2x^2)^{5/2}}dx$$

$$\Big\downarrow \; 1102$$

$$\frac{b^5(a+bx)\int \frac{x^4}{b^5(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx)\int \frac{x^4}{(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 49$$

$$\frac{(a+bx)\int \left(\frac{a^4}{b^4(a+bx)^5}-\frac{4a^3}{b^4(a+bx)^4}+\frac{6a^2}{b^4(a+bx)^3}-\frac{4a}{b^4(a+bx)^2}+\frac{1}{b^4(a+bx)}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{(a+bx)\left(-\frac{a^4}{4b^5(a+bx)^4}+\frac{4a^3}{3b^5(a+bx)^3}-\frac{3a^2}{b^5(a+bx)^2}+\frac{4a}{b^5(a+bx)}+\frac{\log(a+bx)}{b^5}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[x^4/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
((a + b*x)*(-1/4*a^4/(b^5*(a + b*x)^4) + (4*a^3)/(3*b^5*(a + b*x)^3) - (3*
a^2)/(b^5*(a + b*x)^2) + (4*a)/(b^5*(a + b*x)) + Log[a + b*x]/b^5))/Sqrt[a
^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.49

method	result
risch	$\frac{\sqrt{(bx+a)^2}\left(\frac{4a\,x^3}{b^2}+\frac{9a^2x^2}{b^3}+\frac{22a^3x}{3b^4}+\frac{25a^4}{12b^5}\right)}{(bx+a)^5}+\frac{\sqrt{(bx+a)^2}\,\ln(bx+a)}{(bx+a)b^5}$
default	$\frac{(12\ln(bx+a)x^4b^4+48\ln(bx+a)x^3a\,b^3+72\ln(bx+a)x^2a^2b^2+48a\,b^3x^3+48\ln(bx+a)x\,a^3b+108a^2b^2x^2+12a^4\ln(bx+a)+88a^3bx+25}{12b^5\left((bx+a)^2\right)^{\frac{5}{2}}}$

input
```
int(x^4/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)^5*(4*a*x^3/b^2+9*a^2*x^2/b^3+22/3*a^3*x/b^4+25/1
2*a^4/b^5)+((b*x+a)^2)^(1/2)/(b*x+a)/b^5*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.74

$$\int \frac{x^4}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{48\,ab^3x^3+108\,a^2b^2x^2+88\,a^3bx+25\,a^4+12\,(b^4x^4+4\,ab^3x^3+6\,a^2b^2x^2+4}{12\,(b^9x^4+4\,ab^8x^3+6\,a^2b^7x^2+4\,a^3b^6x+a^4b^5)}$$

input
```
integrate(x^4/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/12*(48*a*b^3*x^3 + 108*a^2*b^2*x^2 + 88*a^3*b*x + 25*a^4 + 12*(b^4*x^4 +
 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*log(b*x + a))/(b^9*x^4 + 4
*a*b^8*x^3 + 6*a^2*b^7*x^2 + 4*a^3*b^6*x + a^4*b^5)
```

Sympy [F]

$$\int \frac{x^4}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \int \frac{x^4}{\left((a+bx)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Integral(x**4/((a + b*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.54

$$\int \frac{x^4}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{48\,ab^3x^3+108\,a^2b^2x^2+88\,a^3bx+25\,a^4}{12\,(b^9x^4+4\,ab^8x^3+6\,a^2b^7x^2+4\,a^3b^6x+a^4b^5)} + \frac{\log(bx+a)}{b^5}$$

input
```
integrate(x^4/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/12*(48*a*b^3*x^3 + 108*a^2*b^2*x^2 + 88*a^3*b*x + 25*a^4)/(b^9*x^4 + 4*a
*b^8*x^3 + 6*a^2*b^7*x^2 + 4*a^3*b^6*x + a^4*b^5) + log(b*x + a)/b^5
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.43

$$\int \frac{x^4}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{\log(|bx+a|)}{b^5\mathrm{sgn}(bx+a)} + \frac{48\,ab^2x^3+108\,a^2bx^2+88\,a^3x+\frac{25\,a^4}{b}}{12\,(bx+a)^4b^4\mathrm{sgn}(bx+a)}$$

input
```
integrate(x^4/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
log(abs(b*x + a))/(b^5*sgn(b*x + a)) + 1/12*(48*a*b^2*x^3 + 108*a^2*b*x^2
+ 88*a^3*x + 25*a^4/b)/((b*x + a)^4*b^4*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a^2+2abx+b^2x^2)^{5/2}}\,dx=\int \frac{x^4}{{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}^{5/2}}\,dx$$

input
```
int(x^4/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int(x^4/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.86

$$\int \frac{x^4}{(a^2+2abx+b^2x^2)^{5/2}}\,dx=\frac{12\log(bx+a)\,a^4+48\log(bx+a)\,a^3bx+72\log(bx+a)\,a^2b^2x^2+48\log(bx}{12b^5\,(b^4x^4+4a\,b^3x^3+6a}$$

input
```
int(x^4/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(12*log(a + b*x)*a**4 + 48*log(a + b*x)*a**3*b*x + 72*log(a + b*x)*a**2*b*
*2*x**2 + 48*log(a + b*x)*a*b**3*x**3 + 12*log(a + b*x)*b**4*x**4 + 13*a**
4 + 40*a**3*b*x + 36*a**2*b**2*x**2 - 12*b**4*x**4)/(12*b**5*(a**4 + 4*a**
3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4))
```

3.110 $\int \frac{x^3}{(a^2+2abx+b^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 35

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^{5/2}}\,dx=\frac{x^4(a+bx)}{4a\,(a^2+2abx+b^2x^2)^{5/2}}$$

output
```
1/4*x^4*(b*x+a)/a/(b^2*x^2+2*a*b*x+a^2)^(5/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 147 vs. 2(35) = 70.

Time = 0.57 (sec) , antiderivative size = 147, normalized size of antiderivative = 4.20

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{x^4\left(a^5+ab^4x^4-a^3\sqrt{a^2}\sqrt{(a+bx)^2}-a\sqrt{a^2}b^2x^2\sqrt{(a+bx)^2}+\sqrt{a^2}bx\sqrt{(a+bx)^2}(a^2+b^2x^2)\right)}{4a^5(a+bx)^3\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}-\sqrt{(a+bx)^2}\right)\right)}$$

input
```
Integrate[x^3/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
-1/4*(x^4*(a^5 + a*b^4*x^4 - a^3*Sqrt[a^2]*Sqrt[(a + b*x)^2] - a*Sqrt[a^2]
*b^2*x^2*Sqrt[(a + b*x)^2] + Sqrt[a^2]*b*x*Sqrt[(a + b*x)^2]*(a^2 + b^2*x^
2)))/(a^5*(a + b*x)^3*(Sqrt[a^2]*b*x + a*(Sqrt[a^2] - Sqrt[(a + b*x)^2]))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1102, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1102}$$

$$\frac{b^5(a+bx)\int \frac{x^3}{b^5(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx)\int \frac{x^3}{(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{x^4}{4a(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[x^3/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
x^4/(4*a*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

method	result	size
gosper	$-\frac{(bx+a)(4b^3x^3+6a\,b^2x^2+4a^2bx+a^3)}{4b^4\left((bx+a)^2\right)^{\frac{5}{2}}}$	48
default	$-\frac{(bx+a)(4b^3x^3+6a\,b^2x^2+4a^2bx+a^3)}{4b^4\left((bx+a)^2\right)^{\frac{5}{2}}}$	48
risch	$\frac{\sqrt{(bx+a)^2}\left(-\frac{x^3}{b}-\frac{3a\,x^2}{2b^2}-\frac{a^2x}{b^3}-\frac{a^3}{4b^4}\right)}{(bx+a)^5}$	53
orering	$-\frac{(4b^3x^3+6a\,b^2x^2+4a^2bx+a^3)(bx+a)}{4b^4(b^2x^2+2abx+a^2)^{\frac{5}{2}}}$	57

input
```
int(x^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b*x+a)*(4*b^3*x^3+6*a*b^2*x^2+4*a^2*b*x+a^3)/b^4/((b*x+a)^2)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(31) = 62$.

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.17

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{4\,b^3x^3+6\,ab^2x^2+4\,a^2bx+a^3}{4\,(b^8x^4+4\,ab^7x^3+6\,a^2b^6x^2+4\,a^3b^5x+a^4b^4)}$$

input
```
integrate(x^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/(b^8*x^4 + 4*a*b^7*x^3 +
6*a^2*b^6*x^2 + 4*a^3*b^5*x + a^4*b^4)
```

Sympy [F]

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \int \frac{x^3}{\left((a+bx)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Integral(x**3/((a + b*x)**2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(31) = 62$.

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.91

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{x^2}{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^2} - \frac{2\,a^2}{3\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^4} - \frac{a}{2\,b^6\left(x+\frac{a}{b}\right)^2} + \frac{2\,a^2}{3\,b^7\left(x+\frac{a}{b}\right)^3} + \frac{a^3}{4\,b^8\left(x+\frac{a}{b}\right)^4}$$

input
```
integrate(x^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-x^2/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^2) - 2/3*a^2/((b^2*x^2 + 2*a*b*x +
 a^2)^(3/2)*b^4) - 1/2*a/(b^6*(x + a/b)^2) + 2/3*a^2/(b^7*(x + a/b)^3) + 1
/4*a^3/(b^8*(x + a/b)^4)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{4\,b^3x^3+6\,ab^2x^2+4\,a^2bx+a^3}{4\,(bx+a)^4b^4\mathrm{sgn}\,(bx+a)}$$

input
```
integrate(x^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
-1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/((b*x + a)^4*b^4*sgn(b*x
+ a))
```

Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 128, normalized size of antiderivative = 3.66

$$\int \frac{x^3}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{a^3\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{4\,b^4\,(a+b\,x)^5} - \frac{a^2\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{b^4\,(a+b\,x)^4} - \frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{b^4\,(a+b\,x)^2} + \frac{3\,a\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{2\,b^4\,(a+b\,x)^3}$$

input
```
int(x^3/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
(a^3*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(4*b^4*(a + b*x)^5) - (a^2*(a^2 + b^
2*x^2 + 2*a*b*x)^(1/2))/(b^4*(a + b*x)^4) - (a^2 + b^2*x^2 + 2*a*b*x)^(1/2
)/(b^4*(a + b*x)^2) + (3*a*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(2*b^4*(a + b*
x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int \frac{x^3}{(a^2 + 2abx + b^2x^2)^{5/2}}\, dx = \frac{x^4}{4a\,(b^4x^4 + 4a\,b^3x^3 + 6a^2b^2x^2 + 4a^3bx + a^4)}$$

input
```
int(x^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
x**4/(4*a*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**
4))
```

3.111 $\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 107

$$\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx=-\frac{a^2}{4b^3(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}+\frac{2a}{3b^3(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}-\frac{1}{2b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-1/4*a^2/b^3/(b*x+a)^3/((b*x+a)^2)^(1/2)+2/3*a/b^3/(b*x+a)^2/((b*x+a)^2)^(
1/2)-1/2/b^3/(b*x+a)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.75

$$\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx=\frac{x^3\left(-4a^6-a^5bx+3ab^5x^5+4a^4\sqrt{a^2}\sqrt{(a+bx)^2}-3a^3\sqrt{a^2}bx\sqrt{(a+bx)^2}-\right.}{12a^6(a+bx)^3\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}\right.\right.}$$

input
```
Integrate[x^2/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(x^3*(-4*a^6 - a^5*b*x + 3*a*b^5*x^5 + 4*a^4*Sqrt[a^2]*Sqrt[(a + b*x)^2] -
 3*a^3*Sqrt[a^2]*b*x*Sqrt[(a + b*x)^2] - 3*a*Sqrt[a^2]*b^3*x^3*Sqrt[(a + b
*x)^2] + 3*Sqrt[a^2]*b^2*x^2*Sqrt[(a + b*x)^2]*(a^2 + b^2*x^2)))/(12*a^6*(
a + b*x)^3*(Sqrt[a^2]*b*x + a*(Sqrt[a^2] - Sqrt[(a + b*x)^2])))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.68, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{b^5(a+bx)\int \frac{x^2}{b^5(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{x^2}{(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{53}$$

$$\frac{(a+bx)\int \left(\frac{a^2}{b^2(a+bx)^5}-\frac{2a}{b^2(a+bx)^4}+\frac{1}{b^2(a+bx)^3}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(-\frac{a^2}{4b^3(a+bx)^4}+\frac{2a}{3b^3(a+bx)^3}-\frac{1}{2b^3(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[x^2/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
((a + b*x)*(-1/4*a^2/(b^3*(a + b*x)^4) + (2*a)/(3*b^3*(a + b*x)^3) - 1/(2*
b^3*(a + b*x)^2)))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.35

method	result	size
gosper	$-\frac{(bx+a)(6b^2x^2+4abx+a^2)}{12b^3\left((bx+a)^2\right)^{\frac{5}{2}}}$	37
default	$-\frac{(bx+a)(6b^2x^2+4abx+a^2)}{12b^3\left((bx+a)^2\right)^{\frac{5}{2}}}$	37
risch	$\frac{\sqrt{(bx+a)^2}\left(-\frac{x^2}{2b}-\frac{ax}{3b^2}-\frac{a^2}{12b^3}\right)}{(bx+a)^5}$	42
orering	$-\frac{(6b^2x^2+4abx+a^2)(bx+a)}{12b^3(b^2x^2+2abx+a^2)^{\frac{5}{2}}}$	46

input
```
int(x^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(b*x+a)*(6*b^2*x^2+4*a*b*x+a^2)/b^3/((b*x+a)^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{6\,b^2x^2+4\,abx+a^2}{12\,(b^7x^4+4\,ab^6x^3+6\,a^2b^5x^2+4\,a^3b^4x+a^4b^3)}$$

input
```
integrate(x^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(6*b^2*x^2 + 4*a*b*x + a^2)/(b^7*x^4 + 4*a*b^6*x^3 + 6*a^2*b^5*x^2 +
 4*a^3*b^4*x + a^4*b^3)
```

Sympy [F]

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \int \frac{x^2}{\left((a+bx)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Integral(x**2/((a + b*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{1}{2\,b^5\left(x+\frac{a}{b}\right)^2}+\frac{2\,a}{3\,b^6\left(x+\frac{a}{b}\right)^3}-\frac{a^2}{4\,b^7\left(x+\frac{a}{b}\right)^4}$$

input
```
integrate(x^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/2/(b^5*(x + a/b)^2) + 2/3*a/(b^6*(x + a/b)^3) - 1/4*a^2/(b^7*(x + a/b)^
4)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.35

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{6\,b^2x^2+4\,abx+a^2}{12\,(bx+a)^4b^3\mathrm{sgn}\,(bx+a)}$$

input
```
integrate(x^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
-1/12*(6*b^2*x^2 + 4*a*b*x + a^2)/((b*x + a)^4*b^3*sgn(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}\,\left(a^2+4\,a\,b\,x+6\,b^2\,x^2\right)}{12\,b^3\,\left(a+b\,x\right)^5}$$

input
```
int(x^2/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
-((a^2 + b^2*x^2 + 2*a*b*x)^(1/2)*(a^2 + 6*b^2*x^2 + 4*a*b*x))/(12*b^3*(a
+ b*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.60

$$\int \frac{x^2}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{-6b^2x^2-4abx-a^2}{12b^3\,(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)}$$

input
```
int(x^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
( - a**2 - 4*a*b*x - 6*b**2*x**2)/(12*b**3*(a**4 + 4*a**3*b*x + 6*a**2*b**
2*x**2 + 4*a*b**3*x**3 + b**4*x**4))
```

3.112 $\int \frac{x}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \frac{x}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = -\frac{1}{3b^2\left(a^2+2abx+b^2x^2\right)^{3/2}}+\frac{a}{4b^2(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-1/3/b^2/(b^2*x^2+2*a*b*x+a^2)^(3/2)+1/4*a/b^2/(b*x+a)^3/((b*x+a)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 212 vs. 2(63) = 126.

Time = 0.45 (sec) , antiderivative size = 212, normalized size of antiderivative = 3.37

$$\int \frac{x}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx =$$
$$-\frac{x^2\left(3\sqrt{a^2}b^6x^6+3a^3b^3x^3\sqrt{(a+bx)^2}-3a^2b^4x^4\sqrt{(a+bx)^2}+3ab^5x^5\sqrt{(a+bx)^2}+a^4b^2x^2\left(\sqrt{a^2}-3\sqrt{(a\cdots\right.\right.}{12a^7(a+bx)^3\left(a^2+abx-\sqrt{a^2}\sqrt{(a+b\cdots}\right.}$$

input
```
Integrate[x/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
-1/12*(x^2*(3*Sqrt[a^2]*b^6*x^6 + 3*a^3*b^3*x^3*Sqrt[(a + b*x)^2] - 3*a^2*
b^4*x^4*Sqrt[(a + b*x)^2] + 3*a*b^5*x^5*Sqrt[(a + b*x)^2] + a^4*b^2*x^2*(S
qrt[a^2] - 3*Sqrt[(a + b*x)^2]) + 6*a^6*(Sqrt[a^2] - Sqrt[(a + b*x)^2]) +
2*a^5*b*x*(2*Sqrt[a^2] + Sqrt[(a + b*x)^2])))/(a^7*(a + b*x)^3*(a^2 + a*b*
x - Sqrt[a^2]*Sqrt[(a + b*x)^2]))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1100, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a^2+2abx+b^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{1100}$$

$$-\frac{a\int \frac{1}{(a^2+2bxa+b^2x^2)^{5/2}}dx}{b}-\frac{1}{3b^2(a^2+2abx+b^2x^2)^{3/2}}$$

$$\Big\downarrow \text{1078}$$

$$\frac{a}{4b^2(a+bx)(a^2+2abx+b^2x^2)^{3/2}}-\frac{1}{3b^2(a^2+2abx+b^2x^2)^{3/2}}$$

input
```
Int[x/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
-1/3*1/(b^2*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)) + a/(4*b^2*(a + b*x)*(a^2 + 2
*a*b*x + b^2*x^2)^(3/2))
```

Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.41

method	result	size
gosper	$-\frac{(bx+a)(4bx+a)}{12b^{2}\left((bx+a)^{2}\right)^{\frac{5}{2}}}$	26
default	$-\frac{(bx+a)(4bx+a)}{12b^{2}\left((bx+a)^{2}\right)^{\frac{5}{2}}}$	26
risch	$\frac{\sqrt{(bx+a)^{2}}\left(-\frac{x}{3b}-\frac{a}{12b^{2}}\right)}{(bx+a)^{5}}$	31
orering	$-\frac{(4bx+a)(bx+a)}{12b^{2}(b^{2}x^{2}+2abx+a^{2})^{\frac{5}{2}}}$	35

input
```
int(x/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(b*x+a)*(4*b*x+a)/b^2/((b*x+a)^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int \frac{x}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{4\,bx+a}{12\,(b^6x^4+4\,ab^5x^3+6\,a^2b^4x^2+4\,a^3b^3x+a^4b^2)}$$

input
```
integrate(x/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(4*b*x + a)/(b^6*x^4 + 4*a*b^5*x^3 + 6*a^2*b^4*x^2 + 4*a^3*b^3*x + a
^4*b^2)
```

Sympy [F]

$$\int \frac{x}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \int \frac{x}{\left((a+bx)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Integral(x/((a + b*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.62

$$\int \frac{x}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{1}{3\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^2} + \frac{a}{4\,b^6\left(x+\frac{a}{b}\right)^4}$$

input
```
integrate(x/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^2) + 1/4*a/(b^6*(x + a/b)^4)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.41

$$\int \frac{x}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{4\,bx+a}{12\,(bx+a)^4b^2\mathrm{sgn}\,(bx+a)}$$

input
```
integrate(x/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
-1/12*(4*b*x + a)/((b*x + a)^4*b^2*sgn(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.57

$$\int \frac{x}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{(a+4\,b\,x)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{12\,b^2\,(a+b\,x)^5}$$

input
```
int(x/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
-((a + 4*b*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(12*b^2*(a + b*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{x}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{-4bx-a}{12b^2\,(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)}$$

input
```
int(x/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
( - a - 4*b*x)/(12*b**2*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x
**3 + b**4*x**4))
```

3.113 $\int \frac{1}{(a^2+2abx+b^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 34

$$\int \frac{1}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{1}{4b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-1/4/b/(b*x+a)^3/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \frac{1}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{a+bx}{4b\left((a+bx)^2\right)^{5/2}}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(-5/2),x]
```

output
```
-1/4*(a + b*x)/(b*((a + b*x)^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = $\{1078\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2abx+b^2x^2)^{5/2}}\,dx$$

$$\Big\downarrow \ 1078$$

$$-\frac{1}{4b(a+bx)\,(a^2+2abx+b^2x^2)^{3/2}}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(-5/2),x]
```

output
```
-1/4*1/(b*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))
```

Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{bx+a}{4b\left(\left(bx+a\right)^{2}\right)^{\frac{5}{2}}}$	20
default	$-\frac{bx+a}{4b\left(\left(bx+a\right)^{2}\right)^{\frac{5}{2}}}$	20
risch	$-\frac{\sqrt{\left(bx+a\right)^{2}}}{4\left(bx+a\right)^{5} b}$	22
orering	$-\frac{bx+a}{4b\left(b^{2}x^{2}+2abx+a^{2}\right)^{\frac{5}{2}}}$	29

input
```
int(1/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b*x+a)/b/((b*x+a)^2)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(21) = 42$.

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = -\frac{1}{4\left(b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b\right)}$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/4/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
```

Sympy [F]

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{5/2}}\,dx = \int \frac{1}{(a^2 + 2abx + b^2x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Integral((a**2 + 2*a*b*x + b**2*x**2)**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.41

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{5/2}}\,dx = -\frac{1}{4\,b^5\left(x + \frac{a}{b}\right)^4}$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/4/(b^5*(x + a/b)^4)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.59

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{5/2}}\,dx = -\frac{1}{4\,(bx + a)^4 b \operatorname{sgn}(bx + a)}$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
-1/4/((b*x + a)^4*b*sgn(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{4\,b\,{\left(a+b\,x\right)}^5}$$

input
```
int(1/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
-(a^2 + b^2*x^2 + 2*a*b*x)^(1/2)/(4*b*(a + b*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.32

$$\int \frac{1}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{1}{4b\left(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4\right)}$$

input
```
int(1/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
( - 1)/(4*b*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x
**4))
```

3.114 $\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 194

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{1}{a^4\sqrt{a^2+2abx+b^2x^2}} + \frac{1}{4a(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{1}{3a^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} + \frac{1}{2a^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{(a+bx)\log(x)}{a^5\sqrt{a^2+2abx+b^2x^2}} - \frac{(a+bx)\log(a+bx)}{a^5\sqrt{a^2+2abx+b^2x^2}}$$

output
```
1/a^4/((b*x+a)^2)^(1/2)+1/4/a/(b*x+a)^3/((b*x+a)^2)^(1/2)+1/3/a^2/(b*x+a)^
2/((b*x+a)^2)^(1/2)+1/2/a^3/(b*x+a)/((b*x+a)^2)^(1/2)+(b*x+a)*ln(x)/a^5/((
b*x+a)^2)^(1/2)-(b*x+a)*ln(b*x+a)/a^5/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.43

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{a(25a^3+52a^2bx+42ab^2x^2+12b^3x^3)+12(a+bx)^4\log(x)-12(a+bx)^4}{12a^5(a+bx)^3\sqrt{(a+bx)^2}}$$

input
```
Integrate[1/(x*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

output
```
(a*(25*a^3 + 52*a^2*b*x + 42*a*b^2*x^2 + 12*b^3*x^3) + 12*(a + b*x)^4*Log[
x] - 12*(a + b*x)^4*Log[a + b*x])/(12*a^5*(a + b*x)^3*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{b^5(a+bx)\int \frac{1}{b^5x(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{1}{x(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{54}$$

$$\frac{(a+bx)\int \left(-\frac{b}{a^5(a+bx)}-\frac{b}{a^4(a+bx)^2}-\frac{b}{a^3(a+bx)^3}-\frac{b}{a^2(a+bx)^4}-\frac{b}{a(a+bx)^5}+\frac{1}{a^5x}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(-\frac{\log(a+bx)}{a^5}+\frac{\log(x)}{a^5}+\frac{1}{a^4(a+bx)}+\frac{1}{2a^3(a+bx)^2}+\frac{1}{3a^2(a+bx)^3}+\frac{1}{4a(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[1/(x*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

output
```
((a + b*x)*(1/(4*a*(a + b*x)^4) + 1/(3*a^2*(a + b*x)^3) + 1/(2*a^3*(a + b*
x)^2) + 1/(a^4*(a + b*x)) + Log[x]/a^5 - Log[a + b*x]/a^5))/Sqrt[a^2 + 2*a
*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.54

method	result
risch	$\frac{\sqrt{(bx+a)^2}\left(\frac{b^3x^3}{a^4}+\frac{7b^2x^2}{2a^3}+\frac{13bx}{3a^2}+\frac{25}{12a}\right)}{(bx+a)^5}-\frac{\sqrt{(bx+a)^2}\,\ln(bx+a)}{(bx+a)a^5}+\frac{\sqrt{(bx+a)^2}\,\ln(-x)}{(bx+a)a^5}$
default	$-\frac{(12\ln(bx+a)x^4b^4-12\ln(x)x^4b^4+48\ln(bx+a)x^3a\,b^3-48\ln(x)x^3a\,b^3+72\ln(bx+a)x^2a^2b^2-72a^2b^2\ln(x)x^2-12a\,b^3x^3+48\ln(bx+}{12a^5\left((bx+a)^2\right)^{\frac{5}{2}}}$

input
```
int(1/x/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)^5*(b^3/a^4*x^3+7/2*b^2/a^3*x^2+13/3*b/a^2*x+25/1
2/a)-((b*x+a)^2)^(1/2)/(b*x+a)/a^5*ln(b*x+a)+((b*x+a)^2)^(1/2)/(b*x+a)/a^5
*ln(-x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{12\,ab^3x^3+42\,a^2b^2x^2+52\,a^3bx+25\,a^4-12\left(b^4x^4+4\,ab^3x^3+6\,a^2b^2x^2+4 \cdots\right.}{12\left(a^5b^4x^4+4\,a^6b^3x^3 \cdots\right.}$$

input
```
integrate(1/x/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/12*(12*a*b^3*x^3 + 42*a^2*b^2*x^2 + 52*a^3*b*x + 25*a^4 - 12*(b^4*x^4 +
4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*log(b*x + a) + 12*(b^4*x^4
+ 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*log(x))/(a^5*b^4*x^4 + 4*
a^6*b^3*x^3 + 6*a^7*b^2*x^2 + 4*a^8*b*x + a^9)
```

Sympy [F]

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \int \frac{1}{x\left(\left(a+bx\right)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Integral(1/(x*((a + b*x)**2)**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.61

$$\begin{aligned}\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx &= -\frac{(-1)^{2\,abx+2\,a^2}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)}{a^5} \\ &\quad+\frac{1}{3\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}a^2}+\frac{1}{\sqrt{b^2x^2+2\,abx+a^2}\,a^4} \\ &\quad+\frac{1}{2\,a^3b^2\left(x+\frac{a}{b}\right)^2}+\frac{1}{4\,ab^4\left(x+\frac{a}{b}\right)^4}\end{aligned}$$

input
```
integrate(1/x/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-(-1)^(2*a*b*x + 2*a^2)*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a^5 + 1/3/((b^2
*x^2 + 2*a*b*x + a^2)^(3/2)*a^2) + 1/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^4) +
 1/2/(a^3*b^2*(x + a/b)^2) + 1/4/(a*b^4*(x + a/b)^4)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.46

$$\int \frac{1}{x(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{\log(|bx+a|)}{a^5\mathrm{sgn}(bx+a)} + \frac{\log(|x|)}{a^5\mathrm{sgn}(bx+a)} + \frac{12\,ab^3x^3+42\,a^2b^2x^2+52\,a^3bx+25\,a^4}{12\,(bx+a)^4a^5\mathrm{sgn}(bx+a)}$$

input
```
integrate(1/x/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
-log(abs(b*x + a))/(a^5*sgn(b*x + a)) + log(abs(x))/(a^5*sgn(b*x + a)) + 1
/12*(12*a*b^3*x^3 + 42*a^2*b^2*x^2 + 52*a^3*b*x + 25*a^4)/((b*x + a)^4*a^5
*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(a^2+2abx+b^2x^2)^{5/2}}\,dx = \int \frac{1}{x\,(a^2+2\,a\,b\,x+b^2\,x^2)^{5/2}}\,dx$$

input
```
int(1/(x*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)),x)
```

output
```
int(1/(x*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{-12\log(bx+a)\,a^4-48\log(bx+a)\,a^3bx-72\log(bx+a)\,a^2b^2x^2-48\log(}{}$$

input
```
int(1/x/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
( - 12*log(a + b*x)*a**4 - 48*log(a + b*x)*a**3*b*x - 72*log(a + b*x)*a**2
*b**2*x**2 - 48*log(a + b*x)*a*b**3*x**3 - 12*log(a + b*x)*b**4*x**4 + 12*
log(x)*a**4 + 48*log(x)*a**3*b*x + 72*log(x)*a**2*b**2*x**2 + 48*log(x)*a*
b**3*x**3 + 12*log(x)*b**4*x**4 + 22*a**4 + 40*a**3*b*x + 24*a**2*b**2*x**
2 - 3*b**4*x**4)/(12*a**5*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3
*x**3 + b**4*x**4))
```

3.115 $\int \frac{1}{x^2(a^2+2abx+b^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 235

$$\int \frac{1}{x^2(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{4b}{a^5\sqrt{a^2+2abx+b^2x^2}} - \frac{b}{4a^2(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{2b}{3a^3(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{3b}{2a^4(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{a+bx}{a^5x\sqrt{a^2+2abx+b^2x^2}} - \frac{5b(a+bx)\log(x)}{a^6\sqrt{a^2+2abx+b^2x^2}} + \frac{5b(a+bx)\log(a+bx)}{a^6\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-4*b/a^5/((b*x+a)^2)^(1/2)-1/4*b/a^2/(b*x+a)^3/((b*x+a)^2)^(1/2)-2/3*b/a^3
/(b*x+a)^2/((b*x+a)^2)^(1/2)-3/2*b/a^4/(b*x+a)/((b*x+a)^2)^(1/2)-(b*x+a)/a
^5/x/((b*x+a)^2)^(1/2)-5*b*(b*x+a)*ln(x)/a^6/((b*x+a)^2)^(1/2)+5*b*(b*x+a)
*ln(b*x+a)/a^6/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^2 \left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{-a(12a^4+125a^3bx+260a^2b^2x^2+210ab^3x^3+60b^4x^4)-60bx(a+bx)^4}{12a^6x(a+bx)^3\sqrt{(a+bx)^2}}$$

input
```
Integrate[1/(x^2*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

output
```
(-(a*(12*a^4 + 125*a^3*b*x + 260*a^2*b^2*x^2 + 210*a*b^3*x^3 + 60*b^4*x^4)
) - 60*b*x*(a + b*x)^4*Log[x] + 60*b*x*(a + b*x)^4*Log[a + b*x])/(12*a^6*x
*(a + b*x)^3*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.48, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \ 1102$$

$$\frac{b^5(a+bx)\int \frac{1}{b^5x^2(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{(a+bx)\int \frac{1}{x^2(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \ 54$$

$$\frac{(a+bx)\int \left(\frac{5b^2}{a^6(a+bx)}+\frac{4b^2}{a^5(a+bx)^2}+\frac{3b^2}{a^4(a+bx)^3}+\frac{2b^2}{a^3(a+bx)^4}+\frac{b^2}{a^2(a+bx)^5}-\frac{5b}{a^6x}+\frac{1}{a^5x^2}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(-\frac{5b\log(x)}{a^6}+\frac{5b\log(a+bx)}{a^6}-\frac{4b}{a^5(a+bx)}-\frac{1}{a^5x}-\frac{3b}{2a^4(a+bx)^2}-\frac{2b}{3a^3(a+bx)^3}-\frac{b}{4a^2(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[1/(x^2*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

output
```
((a + b*x)*(-(1/(a^5*x)) - b/(4*a^2*(a + b*x)^4) - (2*b)/(3*a^3*(a + b*x)^
3) - (3*b)/(2*a^4*(a + b*x)^2) - (4*b)/(a^5*(a + b*x)) - (5*b*Log[x])/a^6
+ (5*b*Log[a + b*x])/a^6))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.52

method	result
risch	$\frac{\sqrt{(bx+a)^2}\left(-\frac{5b^4x^4}{a^5}-\frac{35b^3x^3}{2a^4}-\frac{65b^2x^2}{3a^3}-\frac{125bx}{12a^2}-\frac{1}{a}\right)}{(bx+a)^5 x}-\frac{5\sqrt{(bx+a)^2}\, b\ln(x)}{(bx+a)a^6}+\frac{5\sqrt{(bx+a)^2}\, b\ln(-bx-a)}{(bx+a)a^6}$
default	$\frac{(60\ln(bx+a)x^5b^5-60\ln(x)x^5b^5+240\ln(bx+a)x^4ab^4-240\ln(x)x^4ab^4+360\ln(bx+a)x^3a^2b^3-360\ln(x)x^3a^2b^3-60ab^4x^4+240\ln(}{12x\,a^6\left((bx+a)^2\right)}$

input
```
int(1/x^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)^5*(-5*b^4/a^5*x^4-35/2*b^3/a^4*x^3-65/3*b^2/a^3*
x^2-125/12*b/a^2*x-1/a)/x-5*((b*x+a)^2)^(1/2)/(b*x+a)*b/a^6*ln(x)+5*((b*x+
a)^2)^(1/2)/(b*x+a)*b/a^6*ln(-b*x-a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^2(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{60\,ab^4x^4+210\,a^2b^3x^3+260\,a^3b^2x^2+125\,a^4bx+12\,a^5-60\,(b^5x^5+4\,ab^4x^4+6\,a^2b^3x^3+4\,a^3b^2x^2+a^4bx}{12\,(a^6b^4x^5+4\,a^7b^3x^4+6\,a^8b^2x^3+4\,a^9bx}$$

input
```
integrate(1/x^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(60*a*b^4*x^4 + 210*a^2*b^3*x^3 + 260*a^3*b^2*x^2 + 125*a^4*b*x + 12
*a^5 - 60*(b^5*x^5 + 4*a*b^4*x^4 + 6*a^2*b^3*x^3 + 4*a^3*b^2*x^2 + a^4*b*x
)*log(b*x + a) + 60*(b^5*x^5 + 4*a*b^4*x^4 + 6*a^2*b^3*x^3 + 4*a^3*b^2*x^2
 + a^4*b*x)*log(x))/(a^6*b^4*x^5 + 4*a^7*b^3*x^4 + 6*a^8*b^2*x^3 + 4*a^9*b
*x^2 + a^10*x)
```

Sympy [F]

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)^{5/2}} \, dx = \int \frac{1}{x^2 \left((a + bx)^2\right)^{\frac{5}{2}}} \, dx$$

input
```
integrate(1/x**2/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Integral(1/(x**2*((a + b*x)**2)**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)^{5/2}} \, dx = \frac{5 \, (-1)^{2 \, abx + 2 \, a^2} b \log\left(\frac{2 \, abx}{|x|} + \frac{2 \, a^2}{|x|}\right)}{a^6} - \frac{5 \, b}{3 \, (b^2x^2 + 2 \, abx + a^2)^{\frac{3}{2}} a^3} - \frac{5 \, b}{\sqrt{b^2x^2 + 2 \, abx + a^2} a^5} - \frac{1}{(b^2x^2 + 2 \, abx + a^2)^{\frac{3}{2}} a^2 x} - \frac{5}{2 \, a^4 b \left(x + \frac{a}{b}\right)^2} - \frac{1}{4 \, a^2 b^3 \left(x + \frac{a}{b}\right)^4}$$

input
```
integrate(1/x^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
5*(-1)^(2*a*b*x + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a^6 - 5/3*b/
((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a^3) - 5*b/(sqrt(b^2*x^2 + 2*a*b*x + a^2)
*a^5) - 1/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a^2*x) - 5/2/(a^4*b*(x + a/b)^2
) - 1/4/(a^2*b^3*(x + a/b)^4)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)^{5/2}} dx = \frac{5\, b \log (|bx + a|)}{a^6 \mathrm{sgn}\,(bx + a)} - \frac{5\, b \log (|x|)}{a^6 \mathrm{sgn}\,(bx + a)} - \frac{60\, ab^4x^4 + 210\, a^2b^3x^3 + 260\, a^3b^2x^2 + 125\, a^4bx + 12\, a^5}{12\, (bx + a)^4 a^6 x \mathrm{sgn}\,(bx + a)}$$

input
```
integrate(1/x^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
5*b*log(abs(b*x + a))/(a^6*sgn(b*x + a)) - 5*b*log(abs(x))/(a^6*sgn(b*x +
a)) - 1/12*(60*a*b^4*x^4 + 210*a^2*b^3*x^3 + 260*a^3*b^2*x^2 + 125*a^4*b*x
 + 12*a^5)/((b*x + a)^4*a^6*x*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{1}{x^2 (a^2 + 2\, a\, b\, x + b^2\, x^2)^{5/2}} dx$$

input
```
int(1/(x^2*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)),x)
```

output
```
int(1/(x^2*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)^{5/2}} dx = \frac{60 \log(bx + a)\, a^4bx + 240 \log(bx + a)\, a^3b^2x^2 + 360 \log(bx + a)\, a^2b^3x^3 + 2}{}$$

input
```
int(1/x^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(60*log(a + b*x)*a**4*b*x + 240*log(a + b*x)*a**3*b**2*x**2 + 360*log(a +
b*x)*a**2*b**3*x**3 + 240*log(a + b*x)*a*b**4*x**4 + 60*log(a + b*x)*b**5*
x**5 - 60*log(x)*a**4*b*x - 240*log(x)*a**3*b**2*x**2 - 360*log(x)*a**2*b*
*3*x**3 - 240*log(x)*a*b**4*x**4 - 60*log(x)*b**5*x**5 - 12*a**5 - 110*a**
4*b*x - 200*a**3*b**2*x**2 - 120*a**2*b**3*x**3 + 15*b**5*x**5)/(12*a**6*x
*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4))
```

3.116 $\int \frac{1}{x^3(a^2+2abx+b^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 278

$$\int \frac{1}{x^3 (a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{10b^2}{a^6\sqrt{a^2+2abx+b^2x^2}} + \frac{b^2}{4a^3(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{b^2}{a^4(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} + \frac{3b^2}{a^5(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{a+bx}{2a^5x^2\sqrt{a^2+2abx+b^2x^2}} + \frac{5b(a+bx)}{a^6x\sqrt{a^2+2abx+b^2x^2}} + \frac{15b^2(a+bx)\log(x)}{a^7\sqrt{a^2+2abx+b^2x^2}} - \frac{15b^2(a+bx)\log(a+bx)}{a^7\sqrt{a^2+2abx+b^2x^2}}$$

output
```
10*b^2/a^6/((b*x+a)^2)^(1/2)+1/4*b^2/a^3/(b*x+a)^3/((b*x+a)^2)^(1/2)+b^2/a
^4/(b*x+a)^2/((b*x+a)^2)^(1/2)+3*b^2/a^5/(b*x+a)/((b*x+a)^2)^(1/2)-1/2*(b*
x+a)/a^5/x^2/((b*x+a)^2)^(1/2)+5*b*(b*x+a)/a^6/x/((b*x+a)^2)^(1/2)+15*b^2*
(b*x+a)*ln(x)/a^7/((b*x+a)^2)^(1/2)-15*b^2*(b*x+a)*ln(b*x+a)/a^7/((b*x+a)^
2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^3 (a^2 + 2abx + b^2x^2)^{5/2}} \, dx = \frac{a(-2a^5 + 12a^4bx + 125a^3b^2x^2 + 260a^2b^3x^3 + 210ab^4x^4 + 60b^5x^5) + 60b^2}{4a^7x^2(a + bx)^3 \sqrt{(a + bx)}}$$

input
```
Integrate[1/(x^3*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

output
```
(a*(-2*a^5 + 12*a^4*b*x + 125*a^3*b^2*x^2 + 260*a^2*b^3*x^3 + 210*a*b^4*x^
4 + 60*b^5*x^5) + 60*b^2*x^2*(a + b*x)^4*Log[x] - 60*b^2*x^2*(a + b*x)^4*L
og[a + b*x])/(4*a^7*x^2*(a + b*x)^3*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a^2 + 2abx + b^2x^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 1102}$$

$$\frac{b^5(a + bx) \int \frac{1}{b^5x^3(a+bx)^5}dx}{\sqrt{a^2 + 2abx + b^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a + bx) \int \frac{1}{x^3(a+bx)^5}dx}{\sqrt{a^2 + 2abx + b^2x^2}}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{(a + bx) \int \left(-\frac{15b^3}{a^7(a+bx)} - \frac{10b^3}{a^6(a+bx)^2} - \frac{6b^3}{a^5(a+bx)^3} - \frac{3b^3}{a^4(a+bx)^4} - \frac{b^3}{a^3(a+bx)^5} + \frac{15b^2}{a^7x} - \frac{5b}{a^6x^2} + \frac{1}{a^5x^3}\right)dx}{\sqrt{a^2 + 2abx + b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(\frac{15b^2\log(x)}{a^7}-\frac{15b^2\log(a+bx)}{a^7}+\frac{10b^2}{a^6(a+bx)}+\frac{5b}{a^6x}+\frac{3b^2}{a^5(a+bx)^2}-\frac{1}{2a^5x^2}+\frac{b^2}{a^4(a+bx)^3}+\frac{b^2}{4a^3(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[1/(x^3*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

output
```
((a + b*x)*(-1/2*1/(a^5*x^2) + (5*b)/(a^6*x) + b^2/(4*a^3*(a + b*x)^4) + b
^2/(a^4*(a + b*x)^3) + (3*b^2)/(a^5*(a + b*x)^2) + (10*b^2)/(a^6*(a + b*x)
) + (15*b^2*Log[x])/a^7 - (15*b^2*Log[a + b*x])/a^7))/Sqrt[a^2 + 2*a*b*x +
 b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.49

method	result
risch	$\frac{\sqrt{(bx+a)^2}\left(\frac{15b^5x^5}{a^6}+\frac{105b^4x^4}{2a^5}+\frac{65b^3x^3}{a^4}+\frac{125b^2x^2}{4a^3}+\frac{3bx}{a^2}-\frac{1}{2a}\right)}{(bx+a)^5x^2}+\frac{15\sqrt{(bx+a)^2}\,b^2\ln(-x)}{(bx+a)a^7}-\frac{15\sqrt{(bx+a)^2}\,b^2\ln(bx+a)}{(bx+a)a^7}$
default	$-\frac{(60\ln(bx+a)b^6x^6-60\ln(x)b^6x^6+240\ln(bx+a)a\,b^5x^5-240\ln(x)x^5a\,b^5+360\ln(bx+a)a^2b^4x^4-360\ln(x)a^2b^4x^4-60a\,b^5x^5+240\ln}{4x^2a^7(}$

input
```
int(1/x^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)^5*(15*b^5/a^6*x^5+105/2*b^4/a^5*x^4+65*b^3/a^4*x
^3+125/4*b^2/a^3*x^2+3*b/a^2*x-1/2/a)/x^2+15*((b*x+a)^2)^(1/2)/(b*x+a)/a^7
*b^2*ln(-x)-15*((b*x+a)^2)^(1/2)/(b*x+a)/a^7*b^2*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^3\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx=\frac{60\,ab^5x^5+210\,a^2b^4x^4+260\,a^3b^3x^3+125\,a^4b^2x^2+12\,a^5bx-2\,a^6-60\,(b}{4\,(a}$$

input
```
integrate(1/x^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/4*(60*a*b^5*x^5 + 210*a^2*b^4*x^4 + 260*a^3*b^3*x^3 + 125*a^4*b^2*x^2 +
12*a^5*b*x - 2*a^6 - 60*(b^6*x^6 + 4*a*b^5*x^5 + 6*a^2*b^4*x^4 + 4*a^3*b^3
*x^3 + a^4*b^2*x^2)*log(b*x + a) + 60*(b^6*x^6 + 4*a*b^5*x^5 + 6*a^2*b^4*x
^4 + 4*a^3*b^3*x^3 + a^4*b^2*x^2)*log(x))/(a^7*b^4*x^6 + 4*a^8*b^3*x^5 + 6
*a^9*b^2*x^4 + 4*a^10*b*x^3 + a^11*x^2)
```

Sympy [F]

$$\int \frac{1}{x^3 (a^2 + 2abx + b^2x^2)^{5/2}}\,dx = \int \frac{1}{x^3 \left((a+bx)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x**3/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Integral(1/(x**3*((a + b*x)**2)**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.64

$$\begin{aligned}\int \frac{1}{x^3 (a^2 + 2abx + b^2x^2)^{5/2}}\,dx = &-\frac{15\,(-1)^{2\,abx+2\,a^2}\,b^2 \log\left(\frac{2\,abx}{|x|} + \frac{2\,a^2}{|x|}\right)}{a^7} \\ &+ \frac{5\,b^2}{(b^2x^2 + 2\,abx + a^2)^{\frac{3}{2}}a^4} + \frac{15\,b^2}{\sqrt{b^2x^2 + 2\,abx + a^2}\,a^6} + \frac{7\,b}{2\,(b^2x^2 + 2\,abx + a^2)^{\frac{3}{2}}a^3x} \\ &- \frac{1}{2\,(b^2x^2 + 2\,abx + a^2)^{\frac{3}{2}}a^2x^2} + \frac{15}{2\,a^5\left(x + \frac{a}{b}\right)^2} + \frac{1}{4\,a^3b^2\left(x + \frac{a}{b}\right)^4}\end{aligned}$$

input
```
integrate(1/x^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-15*(-1)^(2*a*b*x + 2*a^2)*b^2*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a^7 + 5*
b^2/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a^4) + 15*b^2/(sqrt(b^2*x^2 + 2*a*b*x
 + a^2)*a^6) + 7/2*b/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a^3*x) - 1/2/((b^2*x
^2 + 2*a*b*x + a^2)^(3/2)*a^2*x^2) + 15/2/(a^5*(x + a/b)^2) + 1/4/(a^3*b^2
*(x + a/b)^4)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^3 (a^2 + 2abx + b^2x^2)^{5/2}} \, dx = -\frac{15 \, b^2 \log (|bx + a|)}{a^7 \text{sgn} (bx + a)} + \frac{15 \, b^2 \log (|x|)}{a^7 \text{sgn} (bx + a)} + \frac{60 \, ab^5x^5 + 210 \, a^2b^4x^4 + 260 \, a^3b^3x^3 + 125 \, a^4b^2x^2 + 12 \, a^5bx - 2 \, a^6}{4 \, (bx + a)^4 a^7 x^2 \text{sgn} (bx + a)}$$

input
```
integrate(1/x^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
-15*b^2*log(abs(b*x + a))/(a^7*sgn(b*x + a)) + 15*b^2*log(abs(x))/(a^7*sgn
(b*x + a)) + 1/4*(60*a*b^5*x^5 + 210*a^2*b^4*x^4 + 260*a^3*b^3*x^3 + 125*a
^4*b^2*x^2 + 12*a^5*b*x - 2*a^6)/((b*x + a)^4*a^7*x^2*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a^2 + 2abx + b^2x^2)^{5/2}} \, dx = \int \frac{1}{x^3 \, (a^2 + 2 \, a \, b \, x + b^2 \, x^2)^{5/2}} \, dx$$

input
```
int(1/(x^3*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)),x)
```

output
```
int(1/(x^3*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3 (a^2 + 2abx + b^2x^2)^{5/2}} \, dx = \frac{-60 \log(bx + a) \, a^4b^2x^2 - 240 \log(bx + a) \, a^3b^3x^3 - 360 \log(bx + a) \, a^2b^4x^4}{}$$

input
```
int(1/x^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
( - 60*log(a + b*x)*a**4*b**2*x**2 - 240*log(a + b*x)*a**3*b**3*x**3 - 360
*log(a + b*x)*a**2*b**4*x**4 - 240*log(a + b*x)*a*b**5*x**5 - 60*log(a + b
*x)*b**6*x**6 + 60*log(x)*a**4*b**2*x**2 + 240*log(x)*a**3*b**3*x**3 + 360
*log(x)*a**2*b**4*x**4 + 240*log(x)*a*b**5*x**5 + 60*log(x)*b**6*x**6 - 2*
a**6 + 12*a**5*b*x + 110*a**4*b**2*x**2 + 200*a**3*b**3*x**3 + 120*a**2*b*
*4*x**4 - 15*b**6*x**6)/(4*a**7*x**2*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2
 + 4*a*b**3*x**3 + b**4*x**4))
```

3.117 $\int x(9+12x+4x^2)^{5/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int x\left(9+12x+4x^2\right)^{5/2}\,dx=-\frac{1}{8}(3+2x)^5\sqrt{9+12x+4x^2}+\frac{1}{28}\left(9+12x+4x^2\right)^{7/2}$$

output
```
-1/8*(3+2*x)^5*((3+2*x)^2)^(1/2)+1/28*(4*x^2+12*x+9)^(7/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int x\left(9+12x+4x^2\right)^{5/2}\,dx=\frac{1}{56}(3+2x)^5\sqrt{(3+2x)^2}(-1+4x)$$

input
```
Integrate[x*(9 + 12*x + 4*x^2)^(5/2),x]
```

output
```
((3 + 2*x)^5*Sqrt[(3 + 2*x)^2]*(-1 + 4*x))/56
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(4x^2+12x+9)^{5/2}\,dx$$

$$\Big\downarrow \text{1100}$$

$$\frac{1}{28}(4x^2+12x+9)^{7/2}-\frac{3}{2}\int (4x^2+12x+9)^{5/2}\,dx$$

$$\Big\downarrow \text{1079}$$

$$\frac{1}{28}(4x^2+12x+9)^{7/2}-\frac{3\sqrt{4x^2+12x+9}\int (4x+6)^5dx}{64(2x+3)}$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{28}(4x^2+12x+9)^{7/2}-\frac{1}{8}(2x+3)^5\sqrt{4x^2+12x+9}$$

input
```
Int[x*(9 + 12*x + 4*x^2)^(5/2),x]
```

output
```
-1/8*((3 + 2*x)^5*Sqrt[9 + 12*x + 4*x^2]) + (9 + 12*x + 4*x^2)^(7/2)/28
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

method	result	si
gosper	$\frac{x^{2} \left(64 x^{5}+560 x^{4}+2016 x^{3}+3780 x^{2}+3780 x +1701\right) \left(\left(2 x +3\right)^{2}\right)^{\frac{5}{2}}}{14 \left(2 x +3\right)^{5}}$	4
default	$\frac{x^{2} \left(64 x^{5}+560 x^{4}+2016 x^{3}+3780 x^{2}+3780 x +1701\right) \left(\left(2 x +3\right)^{2}\right)^{\frac{5}{2}}}{14 \left(2 x +3\right)^{5}}$	4
orering	$\frac{x^{2} \left(64 x^{5}+560 x^{4}+2016 x^{3}+3780 x^{2}+3780 x +1701\right) \left(4 x^{2}+12 x +9\right)^{\frac{5}{2}}}{14 \left(2 x +3\right)^{5}}$	5
risch	$\frac{32 \sqrt{\left(2 x +3\right)^{2}}\, x^{7}}{7 \left(2 x +3\right)}+\frac{40 \sqrt{\left(2 x +3\right)^{2}}\, x^{6}}{2 x +3}+\frac{144 \sqrt{\left(2 x +3\right)^{2}}\, x^{5}}{2 x +3}+\frac{270 \sqrt{\left(2 x +3\right)^{2}}\, x^{4}}{2 x +3}+\frac{270 \sqrt{\left(2 x +3\right)^{2}}\, x^{3}}{2 x +3}+\frac{243 \sqrt{\left(2 x +3\right)^{2}}\, x^{2}}{2 \left(2 x +3\right)}$	1

input
```
int(x*(4*x^2+12*x+9)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/14*x^2*(64*x^5+560*x^4+2016*x^3+3780*x^2+3780*x+1701)*((2*x+3)^2)^(5/2)/
(2*x+3)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int x\left(9+12x+4x^2\right)^{5/2}\,dx=\frac{32}{7}\,x^7+40\,x^6+144\,x^5+270\,x^4+270\,x^3+\frac{243}{2}\,x^2$$

input
```
integrate(x*(4*x^2+12*x+9)^(5/2),x, algorithm="fricas")
```

output
```
32/7*x^7 + 40*x^6 + 144*x^5 + 270*x^4 + 270*x^3 + 243/2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.23

$$\int x(9+12x+4x^2)^{5/2}\,dx = \sqrt{4x^2+12x+9}\cdot\left(\frac{16x^6}{7}+\frac{116x^5}{7}+\frac{330x^4}{7}+\frac{450x^3}{7}+\frac{270x^2}{7}+\frac{81x}{28}-\frac{243}{56}\right)$$

input
```
integrate(x*(4*x**2+12*x+9)**(5/2),x)
```

output
```
sqrt(4*x**2 + 12*x + 9)*(16*x**6/7 + 116*x**5/7 + 330*x**4/7 + 450*x**3/7
+ 270*x**2/7 + 81*x/28 - 243/56)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int x(9+12x+4x^2)^{5/2}\,dx = \frac{1}{28}\left(4x^2+12x+9\right)^{\frac{7}{2}} - \frac{1}{4}\left(4x^2+12x+9\right)^{\frac{5}{2}}x - \frac{3}{8}\left(4x^2+12x+9\right)^{\frac{5}{2}}$$

input
```
integrate(x*(4*x^2+12*x+9)^(5/2),x, algorithm="maxima")
```

output
```
1/28*(4*x^2 + 12*x + 9)^(7/2) - 1/4*(4*x^2 + 12*x + 9)^(5/2)*x - 3/8*(4*x^
2 + 12*x + 9)^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(33) = 66$.

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.70

$$\int x(9+12x+4x^2)^{5/2}\,dx = \frac{32}{7}x^7\text{sgn}(2x+3)+40x^6\text{sgn}(2x+3)+144x^5\text{sgn}(2x+3)+270x^4\text{sgn}(2x+3)+270x^3\text{sgn}(2x+3)+\frac{243}{2}x^2\text{sgn}(2x+3)-\frac{729}{56}\text{sgn}(2x+3)$$

input
```
integrate(x*(4*x^2+12*x+9)^(5/2),x, algorithm="giac")
```

output
```
32/7*x^7*sgn(2*x + 3) + 40*x^6*sgn(2*x + 3) + 144*x^5*sgn(2*x + 3) + 270*x
^4*sgn(2*x + 3) + 270*x^3*sgn(2*x + 3) + 243/2*x^2*sgn(2*x + 3) - 729/56*s
gn(2*x + 3)
```

Mupad [F(-1)]

Timed out.

$$\int x(9+12x+4x^2)^{5/2}\,dx = \int x\left(4x^2+12x+9\right)^{5/2}\,dx$$

input
```
int(x*(12*x + 4*x^2 + 9)^(5/2),x)
```

output
```
int(x*(12*x + 4*x^2 + 9)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

$$\int x(9+12x+4x^2)^{5/2}\,dx = \frac{x^2(64x^5+560x^4+2016x^3+3780x^2+3780x+1701)}{14}$$

input
```
int(x*(4*x^2+12*x+9)^(5/2),x)
```

output
```
(x**2*(64*x**5 + 560*x**4 + 2016*x**3 + 3780*x**2 + 3780*x + 1701))/14
```

3.118 $\int x(9+12x+4x^2)^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int x\left(9+12x+4x^2\right)^{3/2}\,dx=-\frac{3}{16}(3+2x)^3\sqrt{9+12x+4x^2}+\frac{1}{20}\left(9+12x+4x^2\right)^{5/2}$$

output
```
-3/16*(3+2*x)^3*((3+2*x)^2)^(1/2)+1/20*(4*x^2+12*x+9)^(5/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int x\left(9+12x+4x^2\right)^{3/2}\,dx=\frac{x^2\sqrt{(3+2x)^2}(135+180x+90x^2+16x^3)}{30+20x}$$

input
```
Integrate[x*(9 + 12*x + 4*x^2)^(3/2),x]
```

output
```
(x^2*Sqrt[(3 + 2*x)^2]*(135 + 180*x + 90*x^2 + 16*x^3))/(30 + 20*x)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(4x^2+12x+9)^{3/2}\,dx$$

$$\Big\downarrow \text{ 1100}$$

$$\frac{1}{20}(4x^2+12x+9)^{5/2}-\frac{3}{2}\int (4x^2+12x+9)^{3/2}dx$$

$$\Big\downarrow \text{ 1079}$$

$$\frac{1}{20}(4x^2+12x+9)^{5/2}-\frac{3\sqrt{4x^2+12x+9}\int (4x+6)^3dx}{16(2x+3)}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{1}{20}(4x^2+12x+9)^{5/2}-\frac{3}{16}(2x+3)^3\sqrt{4x^2+12x+9}$$

input
```
Int[x*(9 + 12*x + 4*x^2)^(3/2),x]
```

output
```
(-3*(3 + 2*x)^3*Sqrt[9 + 12*x + 4*x^2])/16 + (9 + 12*x + 4*x^2)^(5/2)/20
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{x^{2}\left(16 x^{3}+90 x^{2}+180 x+135\right)\left(\left(2 x+3\right)^{2}\right)^{\frac{3}{2}}}{10\left(2 x+3\right)^{3}}$	37
default	$\frac{x^{2}\left(16 x^{3}+90 x^{2}+180 x+135\right)\left(\left(2 x+3\right)^{2}\right)^{\frac{3}{2}}}{10\left(2 x+3\right)^{3}}$	37
orering	$\frac{x^{2}\left(16 x^{3}+90 x^{2}+180 x+135\right)\left(4 x^{2}+12 x+9\right)^{\frac{3}{2}}}{10\left(2 x+3\right)^{3}}$	40
risch	$\frac{8 \sqrt{\left(2 x+3\right)^{2}}\, x^{5}}{5 \left(2 x+3\right)}+\frac{9 \sqrt{\left(2 x+3\right)^{2}}\, x^{4}}{2 x+3}+\frac{18 \sqrt{\left(2 x+3\right)^{2}}\, x^{3}}{2 x+3}+\frac{27 \sqrt{\left(2 x+3\right)^{2}}\, x^{2}}{2 \left(2 x+3\right)}$	86

input
```
int(x*(4*x^2+12*x+9)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/10*x^2*(16*x^3+90*x^2+180*x+135)*((2*x+3)^2)^(3/2)/(2*x+3)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

$$\int x\left(9+12x+4x^2\right)^{3/2}\,dx=\frac{8}{5}\,x^{5}+9\,x^{4}+18\,x^{3}+\frac{27}{2}\,x^{2}$$

input
```
integrate(x*(4*x^2+12*x+9)^(3/2),x, algorithm="fricas")
```

output
```
8/5*x^5 + 9*x^4 + 18*x^3 + 27/2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int x\left(9+12x+4x^2\right)^{3/2}\,dx=\sqrt{4x^2+12x+9}\cdot\left(\frac{4x^4}{5}+\frac{33x^3}{10}+\frac{81x^2}{20}+\frac{27x}{40}-\frac{81}{80}\right)$$

input
```
integrate(x*(4*x**2+12*x+9)**(3/2),x)
```

output
```
sqrt(4*x**2 + 12*x + 9)*(4*x**4/5 + 33*x**3/10 + 81*x**2/20 + 27*x/40 - 81
/80)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int x\left(9+12x+4x^2\right)^{3/2}\,dx=\frac{1}{20}\left(4x^2+12x+9\right)^{\frac{5}{2}}-\frac{3}{8}\left(4x^2+12x+9\right)^{\frac{3}{2}}x-\frac{9}{16}\left(4x^2+12x+9\right)^{\frac{3}{2}}$$

input
```
integrate(x*(4*x^2+12*x+9)^(3/2),x, algorithm="maxima")
```

output
```
1/20*(4*x^2 + 12*x + 9)^(5/2) - 3/8*(4*x^2 + 12*x + 9)^(3/2)*x - 9/16*(4*x
^2 + 12*x + 9)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int x\left(9+12x+4x^2\right)^{3/2}\,dx=\frac{8}{5}x^5\operatorname{sgn}(2x+3)+9x^4\operatorname{sgn}(2x+3)+18x^3\operatorname{sgn}(2x+3)+\frac{27}{2}x^2\operatorname{sgn}(2x+3)-\frac{243}{80}\operatorname{sgn}(2x+3)$$

input
```
integrate(x*(4*x^2+12*x+9)^(3/2),x, algorithm="giac")
```

output
```
8/5*x^5*sgn(2*x + 3) + 9*x^4*sgn(2*x + 3) + 18*x^3*sgn(2*x + 3) + 27/2*x^2
*sgn(2*x + 3) - 243/80*sgn(2*x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.55

$$\int x(9+12x+4x^2)^{3/2}\,dx = \frac{(4\,x^2+12\,x+9)^{3/2}\,(16\,x^2+18\,x-9)}{80}$$

input
```
int(x*(12*x + 4*x^2 + 9)^(3/2),x)
```

output
```
((12*x + 4*x^2 + 9)^(3/2)*(18*x + 16*x^2 - 9))/80
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.45

$$\int x(9+12x+4x^2)^{3/2}\,dx = \frac{x^2(16x^3+90x^2+180x+135)}{10}$$

input
```
int(x*(4*x^2+12*x+9)^(3/2),x)
```

output
```
(x**2*(16*x**3 + 90*x**2 + 180*x + 135))/10
```

3.119 $\int x\sqrt{9+12x+4x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 42

$$\int x\sqrt{9+12x+4x^2}\,dx=-\frac{3}{8}(3+2x)\sqrt{9+12x+4x^2}+\frac{1}{12}\left(9+12x+4x^2\right)^{3/2}$$

output
```
-3/8*(3+2*x)*((3+2*x)^2)^(1/2)+1/12*(4*x^2+12*x+9)^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int x\sqrt{9+12x+4x^2}\,dx=\frac{x^2\sqrt{(3+2x)^2}(9+4x)}{6(3+2x)}$$

input
```
Integrate[x*Sqrt[9 + 12*x + 4*x^2],x]
```

output
```
(x^2*Sqrt[(3 + 2*x)^2]*(9 + 4*x))/(6*(3 + 2*x))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{4x^2+12x+9}\,dx$$

$$\Big\downarrow \text{ 1100}$$

$$\frac{1}{12}\left(4x^2+12x+9\right)^{3/2}-\frac{3}{2}\int \sqrt{4x^2+12x+9}dx$$

$$\Big\downarrow \text{ 1079}$$

$$\frac{1}{12}\left(4x^2+12x+9\right)^{3/2}-\frac{3\sqrt{4x^2+12x+9}\int (4x+6)dx}{4(2x+3)}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{1}{12}\left(4x^2+12x+9\right)^{3/2}-\frac{3}{8}(2x+3)\sqrt{4x^2+12x+9}$$

input
```
Int[x*Sqrt[9 + 12*x + 4*x^2],x]
```

output
```
(-3*(3 + 2*x)*Sqrt[9 + 12*x + 4*x^2])/8 + (9 + 12*x + 4*x^2)^(3/2)/12
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.50

method	result	size
default	$\frac{\operatorname{csgn}(2x+3)(2x+3)^2(4x-3)}{24}$	21
gosper	$\frac{x^2(4x+9)\sqrt{(2x+3)^2}}{12x+18}$	27
orering	$\frac{x^2(4x+9)\sqrt{(2x+3)^2}}{12x+18}$	27
risch	$\frac{2\sqrt{(2x+3)^2}\,x^3}{3(2x+3)}+\frac{3\sqrt{(2x+3)^2}\,x^2}{2(2x+3)}$	44

input
```
int(x*((2*x+3)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*csgn(2*x+3)*(2*x+3)^2*(4*x-3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.26

$$\int x\sqrt{9+12x+4x^2}\,dx=\frac{2}{3}\,x^3+\frac{3}{2}\,x^2$$

input
```
integrate(x*((3+2*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
2/3*x^3 + 3/2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

$$\int x\sqrt{9+12x+4x^2}\,dx = \left(\frac{x^2}{3}+\frac{x}{4}-\frac{3}{8}\right)\sqrt{4x^2+12x+9}$$

input
```
integrate(x*((3+2*x)**2)**(1/2),x)
```

output
```
(x**2/3 + x/4 - 3/8)*sqrt(4*x**2 + 12*x + 9)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int x\sqrt{9+12x+4x^2}\,dx = \frac{1}{12}\left(4\,x^2+12\,x+9\right)^{\frac{3}{2}} - \frac{3}{4}\sqrt{4\,x^2+12\,x+9}x - \frac{9}{8}\sqrt{4\,x^2+12\,x+9}$$

input
```
integrate(x*((3+2*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/12*(4*x^2 + 12*x + 9)^(3/2) - 3/4*sqrt(4*x^2 + 12*x + 9)*x - 9/8*sqrt(4*
x^2 + 12*x + 9)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int x\sqrt{9+12x+4x^2}\,dx = \frac{2}{3}\,x^3\mathrm{sgn}(2\,x+3) + \frac{3}{2}\,x^2\mathrm{sgn}(2\,x+3) - \frac{9}{8}\,\mathrm{sgn}(2\,x+3)$$

input
```
integrate(x*((3+2*x)^2)^(1/2),x, algorithm="giac")
```

output
```
2/3*x^3*sgn(2*x + 3) + 3/2*x^2*sgn(2*x + 3) - 9/8*sgn(2*x + 3)
```

Mupad [B] (verification not implemented)

Time = 8.78 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.55

$$\int x\sqrt{9+12x+4x^2}\,dx = \left(\frac{x^2}{3}+\frac{x}{4}-\frac{3}{8}\right)\,\sqrt{4\,x^2+12\,x+9}$$

input
```
int(x*((2*x + 3)^2)^(1/2),x)
```

output
```
(x/4 + x^2/3 - 3/8)*(12*x + 4*x^2 + 9)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.24

$$\int x\sqrt{9+12x+4x^2}\,dx = \frac{x^2(4x+9)}{6}$$

input
```
int(x*((3+2*x)^2)^(1/2),x)
```

output
```
(x**2*(4*x + 9))/6
```

3.120 $\int \frac{x}{\sqrt{9+12x+4x^2}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 48

$$\int \frac{x}{\sqrt{9+12x+4x^2}} \, dx = \frac{1}{4}\sqrt{9+12x+4x^2} - \frac{3(3+2x)\log(3+2x)}{4\sqrt{9+12x+4x^2}}$$

output
```
1/4*((3+2*x)^2)^(1/2)-3/4*(3+2*x)*ln(3+2*x)/((3+2*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.69

$$\int \frac{x}{\sqrt{9+12x+4x^2}} \, dx = \frac{(3+2x)\left(\frac{x}{2} - \frac{3}{4}\log(3+2x)\right)}{\sqrt{(3+2x)^2}}$$

input
```
Integrate[x/Sqrt[9 + 12*x + 4*x^2],x]
```

output
```
((3 + 2*x)*(x/2 - (3*Log[3 + 2*x])/4))/Sqrt[(3 + 2*x)^2]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1100, 1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{4x^2+12x+9}}dx$$

$$\Big\downarrow \text{1100}$$

$$\frac{1}{4}\sqrt{4x^2+12x+9}-\frac{3}{2}\int \frac{1}{\sqrt{4x^2+12x+9}}dx$$

$$\Big\downarrow \text{1079}$$

$$\frac{1}{4}\sqrt{4x^2+12x+9}-\frac{3(2x+3)\int \frac{1}{4x+6}dx}{\sqrt{4x^2+12x+9}}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{4}\sqrt{4x^2+12x+9}-\frac{3(2x+3)\log(2x+3)}{4\sqrt{4x^2+12x+9}}$$

input
```
Int[x/Sqrt[9 + 12*x + 4*x^2],x]
```

output
```
Sqrt[9 + 12*x + 4*x^2]/4 - (3*(3 + 2*x)*Log[3 + 2*x])/(4*Sqrt[9 + 12*x + 4
*x^2])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.60

method	result	size
default	$-\frac{(2x+3)(-2x+3\ln(2x+3))}{4\sqrt{(2x+3)^{2}}}$	29
risch	$\frac{x\sqrt{(2x+3)^{2}}}{4x+6}-\frac{3\sqrt{(2x+3)^{2}}\,\ln(2x+3)}{4(2x+3)}$	45
meijerg	$\frac{3x\left(\frac{2x}{3}-\ln\left(1+\frac{2x}{3}\right)\right)}{2\sqrt{(2x+3)^{2}}}+\frac{\frac{3x}{2}-\frac{9\ln\left(1+\frac{2x}{3}\right)}{4}}{\sqrt{(2x+3)^{2}}}$	49

input
```
int(x/((2*x+3)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*x+3)*(-2*x+3*ln(2*x+3))/((2*x+3)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.25

$$\int \frac{x}{\sqrt{9+12x+4x^2}}\,dx=\frac{1}{2}\,x-\frac{3}{4}\,\log\left(2\,x+3\right)$$

input
```
integrate(x/((3+2*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*x - 3/4*log(2*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{x}{\sqrt{9+12x+4x^2}}\,dx = -\frac{3\left(x+\frac{3}{2}\right)\log\left(x+\frac{3}{2}\right)}{4\sqrt{\left(x+\frac{3}{2}\right)^2}} + \frac{\sqrt{4x^2+12x+9}}{4}$$

input
```
integrate(x/((3+2*x)**2)**(1/2),x)
```

output
```
-3*(x + 3/2)*log(x + 3/2)/(4*sqrt((x + 3/2)**2)) + sqrt(4*x**2 + 12*x + 9)
/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.44

$$\int \frac{x}{\sqrt{9+12x+4x^2}}\,dx = \frac{1}{4}\sqrt{4x^2+12x+9} - \frac{3}{4}\log\left(x+\frac{3}{2}\right)$$

input
```
integrate(x/((3+2*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(4*x^2 + 12*x + 9) - 3/4*log(x + 3/2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.52

$$\int \frac{x}{\sqrt{9+12x+4x^2}}\,dx = \frac{1}{2}x\mathrm{sgn}(2x+3) - \frac{3}{4}\log\left(|2x+3|\right)\mathrm{sgn}(2x+3)$$

input
```
integrate(x/((3+2*x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*x*sgn(2*x + 3) - 3/4*log(abs(2*x + 3))*sgn(2*x + 3)
```

Mupad [B] (verification not implemented)

Time = 8.85 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.67

$$\int \frac{x}{\sqrt{9+12x+4x^2}}\,dx=\frac{\sqrt{4\,x^2+12\,x+9}}{4}-\frac{3\,\ln\left(x+\frac{\sqrt{{\left(2\,x+3\right)}^2}}{2}+\frac{3}{2}\right)}{4}$$

input
```
int(x/((2*x + 3)^2)^(1/2),x)
```

output
```
(12*x + 4*x^2 + 9)^(1/2)/4 - (3*log(x + ((2*x + 3)^2)^(1/2)/2 + 3/2))/4
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.25

$$\int \frac{x}{\sqrt{9+12x+4x^2}}\,dx=-\frac{3\log(2x+3)}{4}+\frac{x}{2}$$

input
```
int(x/((3+2*x)^2)^(1/2),x)
```

output
```
( - 3*log(2*x + 3) + 2*x)/4
```

3.121 $\int \frac{x}{(9+12x+4x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{x}{(9+12x+4x^2)^{3/2}}\,dx = -\frac{1}{4\sqrt{9+12x+4x^2}} + \frac{3}{8(3+2x)\sqrt{9+12x+4x^2}}$$

output
```
-1/4/((3+2*x)^2)^(1/2)+3/8/(3+2*x)/((3+2*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.57

$$\int \frac{x}{(9+12x+4x^2)^{3/2}}\,dx = \frac{x^2\sqrt{(3+2x)^2}}{6(3+2x)^3}$$

input
```
Integrate[x/(9 + 12*x + 4*x^2)^(3/2),x]
```

output
```
(x^2*Sqrt[(3 + 2*x)^2])/(6*(3 + 2*x)^3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1100, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(4x^2+12x+9)^{3/2}}dx$$

$$\Big\downarrow \text{1100}$$

$$-\frac{3}{2}\int \frac{1}{(4x^2+12x+9)^{3/2}}dx-\frac{1}{4\sqrt{4x^2+12x+9}}$$

$$\Big\downarrow \text{1078}$$

$$\frac{3}{8(2x+3)\sqrt{4x^2+12x+9}}-\frac{1}{4\sqrt{4x^2+12x+9}}$$

input
```
Int[x/(9 + 12*x + 4*x^2)^(3/2),x]
```

output
```
-1/4*1/Sqrt[9 + 12*x + 4*x^2] + 3/(8*(3 + 2*x)*Sqrt[9 + 12*x + 4*x^2])
```

Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.30

method	result	size
meijerg	$\frac{x^{2}}{54\left(1+\frac{2x}{3}\right)^{2}}$	13
gosper	$-\frac{(2x+3)(3+4x)}{8\left((2x+3)^{2}\right)^{\frac{3}{2}}}$	22
default	$-\frac{(2x+3)(3+4x)}{8\left((2x+3)^{2}\right)^{\frac{3}{2}}}$	22
risch	$\frac{4\sqrt{(2x+3)^{2}}\,\left(-\frac{x}{8}-\frac{3}{32}\right)}{(2x+3)^{3}}$	24
orering	$-\frac{(2x+3)(3+4x)}{8(4x^{2}+12x+9)^{\frac{3}{2}}}$	25

input
```
int(x/(4*x^2+12*x+9)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/54*x^2/(1+2/3*x)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.43

$$\int \frac{x}{(9+12x+4x^2)^{3/2}}\,dx = -\frac{4\,x+3}{8\,(4\,x^2+12\,x+9)}$$

input
```
integrate(x/(4*x^2+12*x+9)^(3/2),x, algorithm="fricas")
```

output
```
-1/8*(4*x + 3)/(4*x^2 + 12*x + 9)
```

Sympy [F]

$$\int \frac{x}{(9+12x+4x^2)^{3/2}}\,dx = \int \frac{x}{\left((2x+3)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(4*x**2+12*x+9)**(3/2),x)
```

output
```
Integral(x/((2*x + 3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.55

$$\int \frac{x}{(9+12x+4x^2)^{3/2}}\,dx = -\frac{1}{4\sqrt{4x^2+12x+9}} + \frac{3}{8(2x+3)^2}$$

input
```
integrate(x/(4*x^2+12*x+9)^(3/2),x, algorithm="maxima")
```

output
```
-1/4/sqrt(4*x^2 + 12*x + 9) + 3/8/(2*x + 3)^2
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

$$\int \frac{x}{(9+12x+4x^2)^{3/2}}\,dx = -\frac{4x+3}{8(2x+3)^2\operatorname{sgn}(2x+3)}$$

input
```
integrate(x/(4*x^2+12*x+9)^(3/2),x, algorithm="giac")
```

output
```
-1/8*(4*x + 3)/((2*x + 3)^2*sgn(2*x + 3))
```

Mupad [B] (verification not implemented)

Time = 8.82 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

$$\int \frac{x}{(9+12x+4x^2)^{3/2}}\,dx = -\frac{(4\,x+3)\,\sqrt{4\,x^2+12\,x+9}}{8\,(2\,x+3)^3}$$

input
```
int(x/(12*x + 4*x^2 + 9)^(3/2),x)
```

output
```
-((4*x + 3)*(12*x + 4*x^2 + 9)^(1/2))/(8*(2*x + 3)^3)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.36

$$\int \frac{x}{(9+12x+4x^2)^{3/2}}\,dx = \frac{x^2}{24x^2+72x+54}$$

input
```
int(x/(4*x^2+12*x+9)^(3/2),x)
```

output
```
x**2/(6*(4*x**2 + 12*x + 9))
```

3.122 $\int \frac{x}{(9+12x+4x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{x}{(9+12x+4x^2)^{5/2}}\,dx = -\frac{1}{12\,(9+12x+4x^2)^{3/2}} + \frac{3}{16(3+2x)^3\sqrt{9+12x+4x^2}}$$

output
```
-1/12/(4*x^2+12*x+9)^(3/2)+3/16/(3+2*x)^3/((3+2*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int \frac{x}{(9+12x+4x^2)^{5/2}}\,dx = \frac{-3-8x}{48(3+2x)^3\sqrt{(3+2x)^2}}$$

input
```
Integrate[x/(9 + 12*x + 4*x^2)^(5/2),x]
```

output
```
(-3 - 8*x)/(48*(3 + 2*x)^3*Sqrt[(3 + 2*x)^2])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1100, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(4x^2+12x+9)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 1100}$$

$$-\frac{3}{2}\int \frac{1}{(4x^2+12x+9)^{5/2}}dx - \frac{1}{12\,(4x^2+12x+9)^{3/2}}$$

$$\Big\downarrow \text{ 1078}$$

$$\frac{3}{16(2x+3)\,(4x^2+12x+9)^{3/2}} - \frac{1}{12\,(4x^2+12x+9)^{3/2}}$$

input
```
Int[x/(9 + 12*x + 4*x^2)^(5/2),x]
```

output
```
-1/12*1/(9 + 12*x + 4*x^2)^(3/2) + 3/(16*(3 + 2*x)*(9 + 12*x + 4*x^2)^(3/2
))
```

Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

method	result	size
gosper	$-\frac{(2x+3)(8x+3)}{48\left((2x+3)^2\right)^{\frac{5}{2}}}$	22
default	$-\frac{(2x+3)(8x+3)}{48\left((2x+3)^2\right)^{\frac{5}{2}}}$	22
meijerg	$\frac{x^2\left(\frac{4}{9}x^2+\frac{8}{3}x+6\right)}{2916\left(1+\frac{2x}{3}\right)^4}$	23
risch	$\frac{16\sqrt{(2x+3)^2}\,\left(-\frac{x}{96}-\frac{1}{256}\right)}{(2x+3)^5}$	24
orering	$-\frac{(2x+3)(8x+3)}{48(4x^2+12x+9)^{\frac{5}{2}}}$	25

input
```
int(x/(4*x^2+12*x+9)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*(2*x+3)*(8*x+3)/((2*x+3)^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int \frac{x}{(9+12x+4x^2)^{5/2}}\,dx = -\frac{8\,x+3}{48\,(16\,x^4+96\,x^3+216\,x^2+216\,x+81)}$$

input
```
integrate(x/(4*x^2+12*x+9)^(5/2),x, algorithm="fricas")
```

output
```
-1/48*(8*x + 3)/(16*x^4 + 96*x^3 + 216*x^2 + 216*x + 81)
```

Sympy [F]

$$\int \frac{x}{(9+12x+4x^2)^{5/2}}\,dx = \int \frac{x}{\left((2x+3)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(4*x**2+12*x+9)**(5/2),x)
```

output
```
Integral(x/((2*x + 3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.55

$$\int \frac{x}{(9+12x+4x^2)^{5/2}}\,dx = -\frac{1}{12\,(4\,x^2+12\,x+9)^{\frac{3}{2}}} + \frac{3}{16\,(2\,x+3)^4}$$

input
```
integrate(x/(4*x^2+12*x+9)^(5/2),x, algorithm="maxima")
```

output
```
-1/12/(4*x^2 + 12*x + 9)^(3/2) + 3/16/(2*x + 3)^4
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

$$\int \frac{x}{(9+12x+4x^2)^{5/2}}\,dx = -\frac{8\,x+3}{48\,(2\,x+3)^4\mathrm{sgn}\,(2\,x+3)}$$

input
```
integrate(x/(4*x^2+12*x+9)^(5/2),x, algorithm="giac")
```

output
```
-1/48*(8*x + 3)/((2*x + 3)^4*sgn(2*x + 3))
```

Mupad [B] (verification not implemented)

Time = 8.83 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

$$\int \frac{x}{(9+12x+4x^2)^{5/2}}\,dx = -\frac{(8\,x+3)\,\sqrt{4\,x^2+12\,x+9}}{48\,(2\,x+3)^5}$$

input
```
int(x/(12*x + 4*x^2 + 9)^(5/2),x)
```

output
```
-((8*x + 3)*(12*x + 4*x^2 + 9)^(1/2))/(48*(2*x + 3)^5)
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

$$\int \frac{x}{(9+12x+4x^2)^{5/2}}\,dx = \frac{-8x-3}{768x^4+4608x^3+10368x^2+10368x+3888}$$

input
```
int(x/(4*x^2+12*x+9)^(5/2),x)
```

output
```
( - 8*x - 3)/(48*(16*x**4 + 96*x**3 + 216*x**2 + 216*x + 81))
```

3.123 $\int \frac{x}{(9+12x+4x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{x}{(9+12x+4x^2)^{7/2}}\,dx = -\frac{1}{20\,(9+12x+4x^2)^{5/2}} + \frac{1}{8(3+2x)^5\sqrt{9+12x+4x^2}}$$

output
```
-1/20/(4*x^2+12*x+9)^(5/2)+1/8/(3+2*x)^5/((3+2*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int \frac{x}{(9+12x+4x^2)^{7/2}}\,dx = \frac{-1-4x}{40(3+2x)^5\sqrt{(3+2x)^2}}$$

input
```
Integrate[x/(9 + 12*x + 4*x^2)^(7/2),x]
```

output
```
(-1 - 4*x)/(40*(3 + 2*x)^5*Sqrt[(3 + 2*x)^2])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1100, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(4x^2+12x+9)^{7/2}} \, dx$$

$$\Big\downarrow \text{1100}$$

$$-\frac{3}{2}\int \frac{1}{(4x^2+12x+9)^{7/2}}dx-\frac{1}{20\,(4x^2+12x+9)^{5/2}}$$

$$\Big\downarrow \text{1078}$$

$$\frac{1}{8(2x+3)\,(4x^2+12x+9)^{5/2}}-\frac{1}{20\,(4x^2+12x+9)^{5/2}}$$

input
```
Int[x/(9 + 12*x + 4*x^2)^(7/2),x]
```

output
```
-1/20*1/(9 + 12*x + 4*x^2)^(5/2) + 1/(8*(3 + 2*x)*(9 + 12*x + 4*x^2)^(5/2)
)
```

Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

method	result	size
gosper	$-\frac{(2x+3)(1+4x)}{40\left((2x+3)^2\right)^{\frac{7}{2}}}$	22
default	$-\frac{(2x+3)(1+4x)}{40\left((2x+3)^2\right)^{\frac{7}{2}}}$	22
risch	$\frac{64\sqrt{(2x+3)^2}\,\left(-\frac{x}{640}-\frac{1}{2560}\right)}{(2x+3)^7}$	24
orering	$-\frac{(2x+3)(1+4x)}{40(4x^2+12x+9)^{\frac{7}{2}}}$	25
meijerg	$\frac{x^2\left(\frac{16}{81}x^4+\frac{16}{9}x^3+\frac{20}{3}x^2+\frac{40}{3}x+15\right)}{65610\left(1+\frac{2x}{3}\right)^6}$	33

input
```
int(x/(4*x^2+12*x+9)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/40*(2*x+3)*(1+4*x)/((2*x+3)^2)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{x}{(9+12x+4x^2)^{7/2}}\,dx = -\frac{4\,x+1}{40\,(64\,x^6+576\,x^5+2160\,x^4+4320\,x^3+4860\,x^2+2916\,x+729)}$$

input
```
integrate(x/(4*x^2+12*x+9)^(7/2),x, algorithm="fricas")
```

output
```
-1/40*(4*x + 1)/(64*x^6 + 576*x^5 + 2160*x^4 + 4320*x^3 + 4860*x^2 + 2916*
x + 729)
```

Sympy [F]

$$\int \frac{x}{(9+12x+4x^2)^{7/2}}\,dx = \int \frac{x}{\left((2x+3)^2\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/(4*x**2+12*x+9)**(7/2),x)
```

output
```
Integral(x/((2*x + 3)**2)**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.55

$$\int \frac{x}{(9+12x+4x^2)^{7/2}}\,dx = -\frac{1}{20\,(4\,x^2+12\,x+9)^{\frac{5}{2}}} + \frac{1}{8\,(2\,x+3)^6}$$

input
```
integrate(x/(4*x^2+12*x+9)^(7/2),x, algorithm="maxima")
```

output
```
-1/20/(4*x^2 + 12*x + 9)^(5/2) + 1/8/(2*x + 3)^6
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

$$\int \frac{x}{(9+12x+4x^2)^{7/2}}\,dx = -\frac{4\,x+1}{40\,(2\,x+3)^6\mathrm{sgn}\,(2\,x+3)}$$

input
```
integrate(x/(4*x^2+12*x+9)^(7/2),x, algorithm="giac")
```

output
```
-1/40*(4*x + 1)/((2*x + 3)^6*sgn(2*x + 3))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

$$\int \frac{x}{(9+12x+4x^2)^{7/2}}\,dx = -\frac{(4\,x+1)\,\sqrt{4\,x^2+12\,x+9}}{40\,(2\,x+3)^7}$$

input
```
int(x/(12*x + 4*x^2 + 9)^(7/2),x)
```

output
```
-((4*x + 1)*(12*x + 4*x^2 + 9)^(1/2))/(40*(2*x + 3)^7)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{x}{(9+12x+4x^2)^{7/2}}\,dx = \frac{-4x-1}{2560x^6+23040x^5+86400x^4+172800x^3+194400x^2+116640x+29160}$$

input
```
int(x/(4*x^2+12*x+9)^(7/2),x)
```

output
```
( - 4*x - 1)/(40*(64*x**6 + 576*x**5 + 2160*x**4 + 4320*x**3 + 4860*x**2 +
 2916*x + 729))
```

3.124 $\int \frac{x}{\sqrt{4+12x+9x^2}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 48

$$\int \frac{x}{\sqrt{4+12x+9x^2}} \, dx = \frac{1}{9}\sqrt{4+12x+9x^2} - \frac{2(2+3x)\log(2+3x)}{9\sqrt{4+12x+9x^2}}$$

output
```
1/9*((2+3*x)^2)^(1/2)-2/9*(2+3*x)*ln(2+3*x)/((2+3*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.69

$$\int \frac{x}{\sqrt{4+12x+9x^2}} \, dx = \frac{(2+3x)\left(\frac{x}{3} - \frac{2}{9}\log(2+3x)\right)}{\sqrt{(2+3x)^2}}$$

input
```
Integrate[x/Sqrt[4 + 12*x + 9*x^2],x]
```

output
```
((2 + 3*x)*(x/3 - (2*Log[2 + 3*x])/9))/Sqrt[(2 + 3*x)^2]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1100, 1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{9x^2+12x+4}}dx$$

$$\Big\downarrow \text{ 1100}$$

$$\frac{1}{9}\sqrt{9x^2+12x+4}-\frac{2}{3}\int \frac{1}{\sqrt{9x^2+12x+4}}dx$$

$$\Big\downarrow \text{ 1079}$$

$$\frac{1}{9}\sqrt{9x^2+12x+4}-\frac{2(3x+2)\int \frac{1}{9x+6}dx}{\sqrt{9x^2+12x+4}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{9}\sqrt{9x^2+12x+4}-\frac{2(3x+2)\log(3x+2)}{9\sqrt{9x^2+12x+4}}$$

input
```
Int[x/Sqrt[4 + 12*x + 9*x^2],x]
```

output
```
Sqrt[4 + 12*x + 9*x^2]/9 - (2*(2 + 3*x)*Log[2 + 3*x])/(9*Sqrt[4 + 12*x + 9
*x^2])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.60

method	result	size
default	$-\frac{(3x+2)(-3x+2\ln(3x+2))}{9\sqrt{(3x+2)^2}}$	29
risch	$\frac{x\sqrt{(3x+2)^2}}{9x+6}-\frac{2\sqrt{(3x+2)^2}\,\ln(3x+2)}{9(3x+2)}$	45
meijerg	$\frac{2x\left(\frac{3x}{2}-\ln\left(1+\frac{3x}{2}\right)\right)}{3\sqrt{(3x+2)^2}}+\frac{\frac{2x}{3}-\frac{4\ln\left(1+\frac{3x}{2}\right)}{9}}{\sqrt{(3x+2)^2}}$	49

input
```
int(x/((3*x+2)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*(3*x+2)*(-3*x+2*ln(3*x+2))/((3*x+2)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.25

$$\int \frac{x}{\sqrt{4+12x+9x^2}}\,dx=\frac{1}{3}\,x-\frac{2}{9}\,\log\left(3\,x+2\right)$$

input
```
integrate(x/((2+3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*x - 2/9*log(3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{x}{\sqrt{4+12x+9x^2}}\,dx = -\frac{2\left(x+\frac{2}{3}\right)\log\left(x+\frac{2}{3}\right)}{9\sqrt{\left(x+\frac{2}{3}\right)^2}} + \frac{\sqrt{9x^2+12x+4}}{9}$$

input
```
integrate(x/((2+3*x)**2)**(1/2),x)
```

output
```
-2*(x + 2/3)*log(x + 2/3)/(9*sqrt((x + 2/3)**2)) + sqrt(9*x**2 + 12*x + 4)
/9
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.44

$$\int \frac{x}{\sqrt{4+12x+9x^2}}\,dx = \frac{1}{9}\sqrt{9x^2+12x+4} - \frac{2}{9}\log\left(x+\frac{2}{3}\right)$$

input
```
integrate(x/((2+3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/9*sqrt(9*x^2 + 12*x + 4) - 2/9*log(x + 2/3)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.52

$$\int \frac{x}{\sqrt{4+12x+9x^2}}\,dx = \frac{1}{3}x\mathrm{sgn}(3x+2) - \frac{2}{9}\log\left(|3x+2|\right)\mathrm{sgn}(3x+2)$$

input
```
integrate(x/((2+3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*x*sgn(3*x + 2) - 2/9*log(abs(3*x + 2))*sgn(3*x + 2)
```

Mupad [B] (verification not implemented)

Time = 8.77 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.67

$$\int \frac{x}{\sqrt{4+12x+9x^2}}\,dx = \frac{\sqrt{9\,x^2+12\,x+4}}{9} - \frac{2\,\ln\left(x+\frac{\sqrt{{\left(3\,x+2\right)}^2}}{3}+\frac{2}{3}\right)}{9}$$

input
```
int(x/((3*x + 2)^2)^(1/2),x)
```

output
```
(12*x + 9*x^2 + 4)^(1/2)/9 - (2*log(x + ((3*x + 2)^2)^(1/2)/3 + 2/3))/9
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.25

$$\int \frac{x}{\sqrt{4+12x+9x^2}}\,dx = -\frac{2\log(3x+2)}{9}+\frac{x}{3}$$

input
```
int(x/((2+3*x)^2)^(1/2),x)
```

output
```
( - 2*log(3*x + 2) + 3*x)/9
```

3.125 $\int \frac{x}{\sqrt{4-12x+9x^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 48

$$\int \frac{x}{\sqrt{4-12x+9x^2}}\,dx=\frac{1}{9}\sqrt{4-12x+9x^2}-\frac{2(2-3x)\log(2-3x)}{9\sqrt{4-12x+9x^2}}$$

output
```
1/9*((-2+3*x)^2)^(1/2)-2/9*(2-3*x)*ln(2-3*x)/((-2+3*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.69

$$\int \frac{x}{\sqrt{4-12x+9x^2}}\,dx=\frac{(-2+3x)\left(\frac{x}{3}+\frac{2}{9}\log(2-3x)\right)}{\sqrt{(-2+3x)^2}}$$

input
```
Integrate[x/Sqrt[4 - 12*x + 9*x^2],x]
```

output
```
((-2 + 3*x)*(x/3 + (2*Log[2 - 3*x])/9))/Sqrt[(-2 + 3*x)^2]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1100, 1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{9x^2-12x+4}}dx$$

$$\Big\downarrow \; 1100$$

$$\frac{2}{3}\int \frac{1}{\sqrt{9x^2-12x+4}}dx+\frac{1}{9}\sqrt{9x^2-12x+4}$$

$$\Big\downarrow \; 1079$$

$$\frac{1}{9}\sqrt{9x^2-12x+4}-\frac{2(2-3x)\int \frac{1}{9x-6}dx}{\sqrt{9x^2-12x+4}}$$

$$\Big\downarrow \; 16$$

$$\frac{1}{9}\sqrt{9x^2-12x+4}-\frac{2(2-3x)\log(2-3x)}{9\sqrt{9x^2-12x+4}}$$

input
```
Int[x/Sqrt[4 - 12*x + 9*x^2],x]
```

output
```
Sqrt[4 - 12*x + 9*x^2]/9 - (2*(2 - 3*x)*Log[2 - 3*x])/(9*Sqrt[4 - 12*x + 9
*x^2])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{(-2+3x)(3x+2\ln(-2+3x))}{9\sqrt{(-2+3x)^2}}$	29
risch	$\frac{\sqrt{(-2+3x)^2}\,x}{-6+9x}+\frac{2\sqrt{(-2+3x)^2}\,\ln(-2+3x)}{9(-2+3x)}$	45
meijerg	$\frac{-\frac{2x}{3}-\frac{4\ln\left(1-\frac{3x}{2}\right)}{9}}{\sqrt{(-2+3x)^2}}-\frac{2x\left(-\frac{3x}{2}-\ln\left(1-\frac{3x}{2}\right)\right)}{3\sqrt{(-2+3x)^2}}$	49

input
```
int(x/((-2+3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*(-2+3*x)*(3*x+2*ln(-2+3*x))/((-2+3*x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.25

$$\int \frac{x}{\sqrt{4-12x+9x^2}}\,dx=\frac{1}{3}\,x+\frac{2}{9}\,\log\left(3\,x-2\right)$$

input
```
integrate(x/((-2+3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*x + 2/9*log(3*x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{x}{\sqrt{4-12x+9x^2}}\,dx = \frac{2\left(x-\frac{2}{3}\right)\log\left(x-\frac{2}{3}\right)}{9\sqrt{\left(x-\frac{2}{3}\right)^2}} + \frac{\sqrt{9x^2-12x+4}}{9}$$

input
```
integrate(x/((-2+3*x)**2)**(1/2),x)
```

output
```
2*(x - 2/3)*log(x - 2/3)/(9*sqrt((x - 2/3)**2)) + sqrt(9*x**2 - 12*x + 4)/
9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.44

$$\int \frac{x}{\sqrt{4-12x+9x^2}}\,dx = \frac{1}{9}\sqrt{9\,x^2-12\,x+4} + \frac{2}{9}\log\left(x-\frac{2}{3}\right)$$

input
```
integrate(x/((-2+3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/9*sqrt(9*x^2 - 12*x + 4) + 2/9*log(x - 2/3)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.52

$$\int \frac{x}{\sqrt{4-12x+9x^2}}\,dx = \frac{1}{3}\,x\mathrm{sgn}(3\,x-2) + \frac{2}{9}\log\left(|3\,x-2|\right)\mathrm{sgn}(3\,x-2)$$

input
```
integrate(x/((-2+3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*x*sgn(3*x - 2) + 2/9*log(abs(3*x - 2))*sgn(3*x - 2)
```

Mupad [B] (verification not implemented)

Time = 8.80 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.67

$$\int \frac{x}{\sqrt{4-12x+9x^2}}\,dx = \frac{2\,\ln\left(x+\frac{\sqrt{(3\,x-2)^2}}{3}-\frac{2}{3}\right)}{9}+\frac{\sqrt{9\,x^2-12\,x+4}}{9}$$

input
```
int(x/((3*x - 2)^2)^(1/2),x)
```

output
```
(2*log(x + ((3*x - 2)^2)^(1/2)/3 - 2/3))/9 + (9*x^2 - 12*x + 4)^(1/2)/9
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.25

$$\int \frac{x}{\sqrt{4-12x+9x^2}}\,dx = \frac{2\log(3x-2)}{9}+\frac{x}{3}$$

input
```
int(x/((-2+3*x)^2)^(1/2),x)
```

output
```
(2*log(3*x - 2) + 3*x)/9
```

3.126 $\int \frac{x}{\sqrt{-4+12x-9x^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 48

$$\int \frac{x}{\sqrt{-4+12x-9x^2}}\,dx=-\frac{1}{9}\sqrt{-4+12x-9x^2}-\frac{2(2-3x)\log(2-3x)}{9\sqrt{-4+12x-9x^2}}$$

output
```
-1/9*(-(-2+3*x)^2)^(1/2)-2/9*(2-3*x)*ln(2-3*x)/(-(-2+3*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.73

$$\int \frac{x}{\sqrt{-4+12x-9x^2}}\,dx=\frac{(-2+3x)(-2+3x+2\log(2-3x))}{9\sqrt{-(2-3x)^2}}$$

input
```
Integrate[x/Sqrt[-4 + 12*x - 9*x^2],x]
```

output
```
((-2 + 3*x)*(-2 + 3*x + 2*Log[2 - 3*x]))/(9*Sqrt[-(2 - 3*x)^2])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1100, 1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{-9x^2+12x-4}}dx$$

$$\Big\downarrow \ 1100$$

$$\frac{2}{3}\int \frac{1}{\sqrt{-9x^2+12x-4}}dx-\frac{1}{9}\sqrt{-9x^2+12x-4}$$

$$\Big\downarrow \ 1079$$

$$\frac{2(2-3x)\int \frac{1}{6-9x}dx}{\sqrt{-9x^2+12x-4}}-\frac{1}{9}\sqrt{-9x^2+12x-4}$$

$$\Big\downarrow \ 16$$

$$-\frac{1}{9}\sqrt{-9x^2+12x-4}-\frac{2(2-3x)\log(2-3x)}{9\sqrt{-9x^2+12x-4}}$$

input
```
Int[x/Sqrt[-4 + 12*x - 9*x^2],x]
```

output
```
-1/9*Sqrt[-4 + 12*x - 9*x^2] - (2*(2 - 3*x)*Log[2 - 3*x])/(9*Sqrt[-4 + 12*
x - 9*x^2])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.33

method	result	size
meijerg	$\frac{2i\left(-\frac{3x}{2}-\ln\left(1-\frac{3x}{2}\right)\right)}{9}$	16
default	$\frac{(-2+3x)(3x+2\ln(-2+3x))}{9\sqrt{-(-2+3x)^2}}$	31
risch	$\frac{(-2+3x)x}{3\sqrt{-(-2+3x)^2}}+\frac{2(-2+3x)\ln(-2+3x)}{9\sqrt{-(-2+3x)^2}}$	45

input
```
int(x/(-(-2+3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*I*(-3/2*x-ln(1-3/2*x))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.21

$$\int \frac{x}{\sqrt{-4+12x-9x^2}}\,dx = -\frac{1}{3}i\,x-\frac{2}{9}i\,\log\left(x-\frac{2}{3}\right)$$

input
```
integrate(x/(-(-2+3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*I*x - 2/9*I*log(x - 2/3)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{-4+12x-9x^2}}\,dx = -\frac{\sqrt{-9x^2+12x-4}}{9} + \frac{2\left(x-\frac{2}{3}\right)\log\left(x-\frac{2}{3}\right)}{9\sqrt{-\left(x-\frac{2}{3}\right)^2}}$$

input
```
integrate(x/(-(-2+3*x)**2)**(1/2),x)
```

output
```
-sqrt(-9*x**2 + 12*x - 4)/9 + 2*(x - 2/3)*log(x - 2/3)/(9*sqrt(-(x - 2/3)*
*2))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.44

$$\int \frac{x}{\sqrt{-4+12x-9x^2}}\,dx = -\frac{1}{9}\sqrt{-9x^2+12x-4} + \frac{2}{9}i\log\left(x-\frac{2}{3}\right)$$

input
```
integrate(x/(-(-2+3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/9*sqrt(-9*x^2 + 12*x - 4) + 2/9*I*log(x - 2/3)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.60

$$\int \frac{x}{\sqrt{-4+12x-9x^2}}\,dx = \frac{ix}{3\,\mathrm{sgn}(-3x+2)} + \frac{2i\log(|3x-2|)}{9\,\mathrm{sgn}(-3x+2)}$$

input
```
integrate(x/(-(-2+3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*I*x/sgn(-3*x + 2) + 2/9*I*log(abs(3*x - 2))/sgn(-3*x + 2)
```

Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{x}{\sqrt{-4+12x-9x^2}}\,dx = -\frac{\sqrt{-9\,x^2+12\,x-4}}{9}-\frac{\ln\left(x-\frac{2}{3}-\frac{\sqrt{-\left(3\,x-2\right)^2}\,1\mathrm{i}}{3}\right)\,2\mathrm{i}}{9}$$

input
```
int(x/(-(3*x - 2)^2)^(1/2),x)
```

output
```
- (log(x - ((-(3*x - 2)^2)^(1/2)*1i)/3 - 2/3)*2i)/9 - (12*x - 9*x^2 - 4)^(
1/2)/9
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.25

$$\int \frac{x}{\sqrt{-4+12x-9x^2}}\,dx = \frac{2\log(3x-2)}{9}+\frac{x}{3}$$

input
```
int(x/(-(-2+3*x)^2)^(1/2),x)
```

output
```
(2*log(3*x - 2) + 3*x)/9
```

3.127 $\int \frac{x}{\sqrt{-4-12x-9x^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 48

$$\int \frac{x}{\sqrt{-4-12x-9x^2}}\,dx = -\frac{1}{9}\sqrt{-4-12x-9x^2} - \frac{2(2+3x)\log(2+3x)}{9\sqrt{-4-12x-9x^2}}$$

output
```
-1/9*(-(2+3*x)^2)^(1/2)-2/9*(2+3*x)*ln(2+3*x)/(-(2+3*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.73

$$\int \frac{x}{\sqrt{-4-12x-9x^2}}\,dx = \frac{(2+3x)(2+3x-2\log(2+3x))}{9\sqrt{-(2+3x)^2}}$$

input
```
Integrate[x/Sqrt[-4 - 12*x - 9*x^2],x]
```

output
```
((2 + 3*x)*(2 + 3*x - 2*Log[2 + 3*x]))/(9*Sqrt[-(2 + 3*x)^2])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1100, 1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{-9x^2-12x-4}}dx$$

$$\Big\downarrow \text{1100}$$

$$-\frac{2}{3}\int \frac{1}{\sqrt{-9x^2-12x-4}}dx-\frac{1}{9}\sqrt{-9x^2-12x-4}$$

$$\Big\downarrow \text{1079}$$

$$\frac{2(3x+2)\int \frac{1}{-9x-6}dx}{\sqrt{-9x^2-12x-4}}-\frac{1}{9}\sqrt{-9x^2-12x-4}$$

$$\Big\downarrow \text{16}$$

$$-\frac{1}{9}\sqrt{-9x^2-12x-4}-\frac{2(3x+2)\log(3x+2)}{9\sqrt{-9x^2-12x-4}}$$

input
```
Int[x/Sqrt[-4 - 12*x - 9*x^2],x]
```

output
```
-1/9*Sqrt[-4 - 12*x - 9*x^2] - (2*(2 + 3*x)*Log[2 + 3*x])/(9*Sqrt[-4 - 12*
x - 9*x^2])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.33

method	result	size
meijerg	$-\frac{2i\left(\frac{3x}{2}-\ln\left(1+\frac{3x}{2}\right)\right)}{9}$	16
default	$-\frac{(3x+2)(-3x+2\ln(3x+2))}{9\sqrt{-(3x+2)^{2}}}$	31
risch	$\frac{(3x+2)x}{3\sqrt{-(3x+2)^{2}}}-\frac{2(3x+2)\ln(3x+2)}{9\sqrt{-(3x+2)^{2}}}$	45

input
```
int(x/(-(3*x+2)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*I*(3/2*x-ln(1+3/2*x))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.21

$$\int \frac{x}{\sqrt{-4-12x-9x^2}}\,dx = -\frac{1}{3}i\,x+\frac{2}{9}i\,\log\left(x+\frac{2}{3}\right)$$

input
```
integrate(x/(-(2+3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*I*x + 2/9*I*log(x + 2/3)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{x}{\sqrt{-4-12x-9x^2}}\,dx = -\frac{\sqrt{-9x^2-12x-4}}{9} - \frac{2\left(x+\frac{2}{3}\right)\log\left(x+\frac{2}{3}\right)}{9\sqrt{-\left(x+\frac{2}{3}\right)^2}}$$

input
```
integrate(x/(-(2+3*x)**2)**(1/2),x)
```

output
```
-sqrt(-9*x**2 - 12*x - 4)/9 - 2*(x + 2/3)*log(x + 2/3)/(9*sqrt(-(x + 2/3)*
*2))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.44

$$\int \frac{x}{\sqrt{-4-12x-9x^2}}\,dx = -\frac{1}{9}\sqrt{-9\,x^2-12\,x-4} - \frac{2}{9}i\,\log\left(x+\frac{2}{3}\right)$$

input
```
integrate(x/(-(2+3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/9*sqrt(-9*x^2 - 12*x - 4) - 2/9*I*log(x + 2/3)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.60

$$\int \frac{x}{\sqrt{-4-12x-9x^2}}\,dx = \frac{i\,x}{3\,\mathrm{sgn}\left(-3\,x-2\right)} - \frac{2i\,\log\left(|3\,x+2|\right)}{9\,\mathrm{sgn}\left(-3\,x-2\right)}$$

input
```
integrate(x/(-(2+3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*I*x/sgn(-3*x - 2) - 2/9*I*log(abs(3*x + 2))/sgn(-3*x - 2)
```

Mupad [B] (verification not implemented)

Time = 8.85 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{x}{\sqrt{-4-12x-9x^2}}\,dx = -\frac{\sqrt{-9\,x^2-12\,x-4}}{9}+\frac{\ln\left(x+\frac{2}{3}-\frac{\sqrt{-(3\,x+2)^2}\,1\mathrm{i}}{3}\right)\,2\mathrm{i}}{9}$$

input
```
int(x/(-(3*x + 2)^2)^(1/2),x)
```

output
```
(log(x - ((-(3*x + 2)^2)^(1/2)*1i)/3 + 2/3)*2i)/9 - (- 12*x - 9*x^2 - 4)^(
1/2)/9
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.25

$$\int \frac{x}{\sqrt{-4-12x-9x^2}}\,dx = -\frac{2\log(3x+2)}{9}+\frac{x}{3}$$

input
```
int(x/(-(2+3*x)^2)^(1/2),x)
```

output
```
( - 2*log(3*x + 2) + 3*x)/9
```

3.128 $\int \frac{1}{x\sqrt{4+12x+9x^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 27

$$\int \frac{1}{x\sqrt{4+12x+9x^2}}\,dx = -\frac{(2+3x)\text{arctanh}(1+3x)}{\sqrt{4+12x+9x^2}}$$

output
```
-(2+3*x)*arctanh(1+3*x)/((2+3*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{1}{x\sqrt{4+12x+9x^2}}\,dx = \frac{(2+3x)(\log(x)-\log(2+3x))}{2\sqrt{(2+3x)^2}}$$

input
```
Integrate[1/(x*Sqrt[4 + 12*x + 9*x^2]),x]
```

output
```
((2 + 3*x)*(Log[x] - Log[2 + 3*x]))/(2*Sqrt[(2 + 3*x)^2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1102, 27, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{9x^2+12x+4}}dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{3(3x+2)\int \frac{1}{3x(3x+2)}dx}{\sqrt{9x^2+12x+4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(3x+2)\int \frac{1}{x(3x+2)}dx}{\sqrt{9x^2+12x+4}}$$

$$\Big\downarrow \text{47}$$

$$\frac{(3x+2)\left(\frac{\int \frac{1}{x}dx}{2}-\frac{3}{2}\int \frac{1}{3x+2}dx\right)}{\sqrt{9x^2+12x+4}}$$

$$\Big\downarrow \text{14}$$

$$\frac{(3x+2)\left(\frac{\log(x)}{2}-\frac{3}{2}\int \frac{1}{3x+2}dx\right)}{\sqrt{9x^2+12x+4}}$$

$$\Big\downarrow \text{16}$$

$$\frac{(3x+2)\left(\frac{\log(x)}{2}-\frac{1}{2}\log(3x+2)\right)}{\sqrt{9x^2+12x+4}}$$

input
```
Int[1/(x*Sqrt[4 + 12*x + 9*x^2]),x]
```

output
```
((2 + 3*x)*(Log[x]/2 - Log[2 + 3*x]/2))/Sqrt[4 + 12*x + 9*x^2]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result	size
default	$-\frac{(3x+2)(\ln(3x+2)-\ln(x))}{2\sqrt{(3x+2)^{2}}}$	28
risch	$-\frac{\sqrt{(3x+2)^{2}}\,\ln(3x+2)}{2(3x+2)}+\frac{\sqrt{(3x+2)^{2}}\,\ln(x)}{6x+4}$	46
meijerg	$\frac{3x\left(-\ln\left(1+\frac{3x}{2}\right)+\ln(x)-\ln(2)+\ln(3)\right)}{2\sqrt{(3x+2)^{2}}}+\frac{-\ln\left(1+\frac{3x}{2}\right)+\ln(x)-\ln(2)+\ln(3)}{\sqrt{(3x+2)^{2}}}$	58

input
```
int(1/x/((3*x+2)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(3*x+2)*(ln(3*x+2)-ln(x))/((3*x+2)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.48

$$\int \frac{1}{x\sqrt{4+12x+9x^2}}\,dx = -\frac{1}{2}\log\left(3\,x+2\right)+\frac{1}{2}\log\left(x\right)$$

input
```
integrate(1/x/((2+3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log(3*x + 2) + 1/2*log(x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{4+12x+9x^2}}\,dx = \int \frac{1}{x\sqrt{(3x+2)^2}}\,dx$$

input
```
integrate(1/x/((2+3*x)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt((3*x + 2)**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\sqrt{4+12x+9x^2}}\,dx = -\frac{1}{2}\left(-1\right)^{12\,x+8}\log\left(\frac{12\,x}{|x|}+\frac{8}{|x|}\right)$$

input
```
integrate(1/x/((2+3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(-1)^(12*x + 8)*log(12*x/abs(x) + 8/abs(x))
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{x\sqrt{4+12x+9x^2}}\,dx = -\frac{1}{2}\left(\log\left(|3\,x+2|\right) - \log\left(|x|\right)\right)\mathrm{sgn}(3\,x+2)$$

input
```
integrate(1/x/((2+3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(log(abs(3*x + 2)) - log(abs(x)))*sgn(3*x + 2)
```

Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\sqrt{4+12x+9x^2}}\,dx = -\frac{\ln\left(\frac{6\,x+2\,\sqrt{(3\,x+2)^2}+4}{x}\right)}{2}$$

input
```
int(1/(x*((3*x + 2)^2)^(1/2)),x)
```

output
```
-log((6*x + 2*((3*x + 2)^2)^(1/2) + 4)/x)/2
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.48

$$\int \frac{1}{x\sqrt{4+12x+9x^2}}\,dx = -\frac{\log(3x+2)}{2} + \frac{\log(x)}{2}$$

input
```
int(1/x/((2+3*x)^2)^(1/2),x)
```

output
```
( - log(3*x + 2) + log(x))/2
```

3.129 $\int \frac{1}{x\sqrt{4-12x+9x^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 27

$$\int \frac{1}{x\sqrt{4-12x+9x^2}}\,dx = -\frac{(2-3x)\text{arctanh}(1-3x)}{\sqrt{4-12x+9x^2}}$$

output
```
(2-3*x)*arctanh(-1+3*x)/((-2+3*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{1}{x\sqrt{4-12x+9x^2}}\,dx = \frac{(-2+3x)(\log(2-3x)-\log(x))}{2\sqrt{(2-3x)^2}}$$

input
```
Integrate[1/(x*Sqrt[4 - 12*x + 9*x^2]),x]
```

output
```
((-2 + 3*x)*(Log[2 - 3*x] - Log[x]))/(2*Sqrt[(2 - 3*x)^2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1102, 27, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{9x^2-12x+4}}dx$$

$$\Big\downarrow \text{ 1102}$$

$$-\frac{3(2-3x)\int -\frac{1}{3(2-3x)x}dx}{\sqrt{9x^2-12x+4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(2-3x)\int \frac{1}{(2-3x)x}dx}{\sqrt{9x^2-12x+4}}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{(2-3x)\left(\frac{3}{2}\int \frac{1}{2-3x}dx+\frac{\int \frac{1}{x}dx}{2}\right)}{\sqrt{9x^2-12x+4}}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{(2-3x)\left(\frac{3}{2}\int \frac{1}{2-3x}dx+\frac{\log(x)}{2}\right)}{\sqrt{9x^2-12x+4}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{(2-3x)\left(\frac{\log(x)}{2}-\frac{1}{2}\log(2-3x)\right)}{\sqrt{9x^2-12x+4}}$$

input
```
Int[1/(x*Sqrt[4 - 12*x + 9*x^2]),x]
```

output
```
((2 - 3*x)*(-1/2*Log[2 - 3*x] + Log[x]/2))/Sqrt[4 - 12*x + 9*x^2]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{(-2+3x)(-\ln(x)+\ln(-2+3x))}{2\sqrt{(-2+3x)^2}}$	28
risch	$-\frac{\sqrt{(-2+3x)^2}\,\ln(x)}{2(-2+3x)}+\frac{\sqrt{(-2+3x)^2}\,\ln(-2+3x)}{6x-4}$	46
meijerg	$\frac{-\ln(1-\frac{3x}{2})+\ln(x)-\ln(2)+\ln(3)+i\pi}{\sqrt{(-2+3x)^2}}-\frac{3x(-\ln(1-\frac{3x}{2})+\ln(x)-\ln(2)+\ln(3)+i\pi)}{2\sqrt{(-2+3x)^2}}$	66

input
```
int(1/x/((-2+3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-2+3*x)*(-ln(x)+ln(-2+3*x))/((-2+3*x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.48

$$\int \frac{1}{x\sqrt{4-12x+9x^2}}\,dx = \frac{1}{2}\,\log\left(3\,x-2\right)-\frac{1}{2}\,\log\left(x\right)$$

input
```
integrate(1/x/((-2+3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*log(3*x - 2) - 1/2*log(x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{4-12x+9x^2}}\,dx = \int \frac{1}{x\sqrt{\left(3x-2\right)^2}}\,dx$$

input
```
integrate(1/x/((-2+3*x)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt((3*x - 2)**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\sqrt{4-12x+9x^2}}\,dx = -\frac{1}{2}\left(-1\right)^{-12\,x+8}\log\left(-\frac{12\,x}{|x|}+\frac{8}{|x|}\right)$$

input
```
integrate(1/x/((-2+3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(-1)^(-12*x + 8)*log(-12*x/abs(x) + 8/abs(x))
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{x\sqrt{4-12x+9x^2}}\,dx = \frac{1}{2}\left(\log\left(|3\,x-2|\right)-\log\left(|x|\right)\right)\mathrm{sgn}(3\,x-2)$$

input
```
integrate(1/x/((-2+3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*(log(abs(3*x - 2)) - log(abs(x)))*sgn(3*x - 2)
```

Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\sqrt{4-12x+9x^2}}\,dx = -\frac{\ln\left(\frac{2\,\sqrt{(3\,x-2)^2}-6\,x+4}{x}\right)}{2}$$

input
```
int(1/(x*((3*x - 2)^2)^(1/2)),x)
```

output
```
-log((2*((3*x - 2)^2)^(1/2) - 6*x + 4)/x)/2
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.48

$$\int \frac{1}{x\sqrt{4-12x+9x^2}}\,dx = \frac{\log(3x-2)}{2}-\frac{\log(x)}{2}$$

input
```
int(1/x/((-2+3*x)^2)^(1/2),x)
```

output
```
(log(3*x - 2) - log(x))/2
```

3.130 $\int \frac{1}{x\sqrt{-4+12x-9x^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 27

$$\int \frac{1}{x\sqrt{-4+12x-9x^2}}\,dx = -\frac{(2-3x)\operatorname{arctanh}(1-3x)}{\sqrt{-4+12x-9x^2}}$$

output
```
(2-3*x)*arctanh(-1+3*x)/(-(-2+3*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\sqrt{-4+12x-9x^2}}\,dx = \frac{(-2+3x)(\log(2-3x)-\log(x))}{2\sqrt{-(2-3x)^2}}$$

input
```
Integrate[1/(x*Sqrt[-4 + 12*x - 9*x^2]),x]
```

output
```
((-2 + 3*x)*(Log[2 - 3*x] - Log[x]))/(2*Sqrt[-(2 - 3*x)^2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1102, 27, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{-9x^2+12x-4}}dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{3(2-3x)\int \frac{1}{3(2-3x)x}dx}{\sqrt{-9x^2+12x-4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(2-3x)\int \frac{1}{(2-3x)x}dx}{\sqrt{-9x^2+12x-4}}$$

$$\Big\downarrow \text{47}$$

$$\frac{(2-3x)\left(\frac{3}{2}\int \frac{1}{2-3x}dx+\frac{\int \frac{1}{x}dx}{2}\right)}{\sqrt{-9x^2+12x-4}}$$

$$\Big\downarrow \text{14}$$

$$\frac{(2-3x)\left(\frac{3}{2}\int \frac{1}{2-3x}dx+\frac{\log(x)}{2}\right)}{\sqrt{-9x^2+12x-4}}$$

$$\Big\downarrow \text{16}$$

$$\frac{(2-3x)\left(\frac{\log(x)}{2}-\frac{1}{2}\log(2-3x)\right)}{\sqrt{-9x^2+12x-4}}$$

input
```
Int[1/(x*Sqrt[-4 + 12*x - 9*x^2]),x]
```

output
```
((2 - 3*x)*(-1/2*Log[2 - 3*x] + Log[x]/2))/Sqrt[-4 + 12*x - 9*x^2]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

method	result	size
meijerg	$\frac{i\left(-\ln\left(1-\frac{3x}{2}\right)+\ln(x)-\ln(2)+\ln(3)+i\pi\right)}{2}$	25
default	$-\frac{(-2+3x)(\ln(x)-\ln(-2+3x))}{2\sqrt{-(-2+3x)^2}}$	30
risch	$-\frac{(-2+3x)\ln(x)}{2\sqrt{-(-2+3x)^2}}+\frac{(-2+3x)\ln(-2+3x)}{2\sqrt{-(-2+3x)^2}}$	46

input
```
int(1/x/(-(-2+3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*(-ln(1-3/2*x)+ln(x)-ln(2)+ln(3)+I*Pi)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.41

$$\int \frac{1}{x\sqrt{-4+12x-9x^2}}\,dx = -\frac{1}{2}i\log\left(x-\frac{2}{3}\right)+\frac{1}{2}i\log(x)$$

input
```
integrate(1/x/(-(-2+3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*I*log(x - 2/3) + 1/2*I*log(x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{-4+12x-9x^2}}\,dx = \int \frac{1}{x\sqrt{-(3x-2)^2}}\,dx$$

input
```
integrate(1/x/(-(-2+3*x)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(-(3*x - 2)**2)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\sqrt{-4+12x-9x^2}}\,dx = -\frac{1}{2}i\,(-1)^{-12\,x+8}\log\left(-\frac{12\,x}{|x|}+\frac{8}{|x|}\right)$$

input
```
integrate(1/x/(-(-2+3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*I*(-1)^(-12*x + 8)*log(-12*x/abs(x) + 8/abs(x))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{1}{x\sqrt{-4+12x-9x^2}}\,dx = \frac{i\,\log\left(|3\,x-2|\right)}{2\,\mathrm{sgn}\left(-3\,x+2\right)} - \frac{i\,\log\left(|x|\right)}{2\,\mathrm{sgn}\left(-3\,x+2\right)}$$

input
```
integrate(1/x/(-(-2+3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*I*log(abs(3*x - 2))/sgn(-3*x + 2) - 1/2*I*log(abs(x))/sgn(-3*x + 2)
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{-4+12x-9x^2}}\,dx = \frac{\ln\left(\frac{6\,x-4+\sqrt{-{\left(3\,x-2\right)}^2}\,2{}\mathrm{i}}{x}\right)\,1{}\mathrm{i}}{2}$$

input
```
int(1/(x*(-(3*x - 2)^2)^(1/2)),x)
```

output
```
(log((6*x + (-(3*x - 2)^2)^(1/2)*2i - 4)/x)*1i)/2
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.48

$$\int \frac{1}{x\sqrt{-4+12x-9x^2}}\,dx=\frac{\log(3x-2)}{2}-\frac{\log(x)}{2}$$

input
```
int(1/x/(-(-2+3*x)^2)^(1/2),x)
```

output
```
(log(3*x - 2) - log(x))/2
```

3.131 $\int \frac{1}{x\sqrt{-4-12x-9x^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 27

$$\int \frac{1}{x\sqrt{-4-12x-9x^2}}\,dx = -\frac{(2+3x)\text{arctanh}(1+3x)}{\sqrt{-4-12x-9x^2}}$$

output
```
-(2+3*x)*arctanh(1+3*x)/(-(2+3*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\sqrt{-4-12x-9x^2}}\,dx = \frac{(2+3x)(\log(x)-\log(2+3x))}{2\sqrt{-(2+3x)^2}}$$

input
```
Integrate[1/(x*Sqrt[-4 - 12*x - 9*x^2]),x]
```

output
```
((2 + 3*x)*(Log[x] - Log[2 + 3*x]))/(2*Sqrt[-(2 + 3*x)^2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1102, 27, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{-9x^2-12x-4}}dx$$

$$\Big\downarrow \text{ 1102}$$

$$-\frac{3(3x+2)\int -\frac{1}{3x(3x+2)}dx}{\sqrt{-9x^2-12x-4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(3x+2)\int \frac{1}{x(3x+2)}dx}{\sqrt{-9x^2-12x-4}}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{(3x+2)\left(\frac{\int \frac{1}{x}dx}{2}-\frac{3}{2}\int \frac{1}{3x+2}dx\right)}{\sqrt{-9x^2-12x-4}}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{(3x+2)\left(\frac{\log(x)}{2}-\frac{3}{2}\int \frac{1}{3x+2}dx\right)}{\sqrt{-9x^2-12x-4}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{(3x+2)\left(\frac{\log(x)}{2}-\frac{1}{2}\log(3x+2)\right)}{\sqrt{-9x^2-12x-4}}$$

input
```
Int[1/(x*Sqrt[-4 - 12*x - 9*x^2]),x]
```

output
```
((2 + 3*x)*(Log[x]/2 - Log[2 + 3*x]/2))/Sqrt[-4 - 12*x - 9*x^2]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

method	result	size
meijerg	$-\frac{i\left(-\ln\left(1+\frac{3x}{2}\right)+\ln\left(x\right)-\ln\left(2\right)+\ln\left(3\right)\right)}{2}$	21
default	$-\frac{\left(3x+2\right)\left(\ln\left(3x+2\right)-\ln\left(x\right)\right)}{2\sqrt{-\left(3x+2\right)^{2}}}$	30
risch	$-\frac{\left(3x+2\right)\ln\left(3x+2\right)}{2\sqrt{-\left(3x+2\right)^{2}}}+\frac{\left(3x+2\right)\ln\left(x\right)}{2\sqrt{-\left(3x+2\right)^{2}}}$	46

input
```
int(1/x/(-(3*x+2)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*(-ln(1+3/2*x)+ln(x)-ln(2)+ln(3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.41

$$\int \frac{1}{x\sqrt{-4-12x-9x^2}}\,dx = \frac{1}{2}i\,\log\left(x+\frac{2}{3}\right) - \frac{1}{2}i\,\log\left(x\right)$$

input
```
integrate(1/x/(-(2+3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*I*log(x + 2/3) - 1/2*I*log(x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{-4-12x-9x^2}}\,dx = \int \frac{1}{x\sqrt{-\left(3x+2\right)^2}}\,dx$$

input
```
integrate(1/x/(-(2+3*x)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(-(3*x + 2)**2)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\sqrt{-4-12x-9x^2}}\,dx = -\frac{1}{2}i\left(-1\right)^{12\,x+8}\log\left(\frac{12\,x}{|x|}+\frac{8}{|x|}\right)$$

input
```
integrate(1/x/(-(2+3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*I*(-1)^(12*x + 8)*log(12*x/abs(x) + 8/abs(x))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{1}{x\sqrt{-4-12x-9x^2}}\,dx = -\frac{i\log(|3x+2|)}{2\,\mathrm{sgn}(-3x-2)} + \frac{i\log(|x|)}{2\,\mathrm{sgn}(-3x-2)}$$

input
```
integrate(1/x/(-(2+3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*I*log(abs(3*x + 2))/sgn(-3*x - 2) + 1/2*I*log(abs(x))/sgn(-3*x - 2)
```

Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{-4-12x-9x^2}}\,dx = \frac{\ln\left(\frac{6x+4-\sqrt{-(3x+2)^2}\,2\mathrm{i}}{x}\right)\,1\mathrm{i}}{2}$$

input
```
int(1/(x*(-(3*x + 2)^2)^(1/2)),x)
```

output
```
(log((6*x - (-(3*x + 2)^2)^(1/2)*2i + 4)/x)*1i)/2
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.48

$$\int \frac{1}{x\sqrt{-4-12x-9x^2}}\,dx = -\frac{\log(3x+2)}{2} + \frac{\log(x)}{2}$$

input
```
int(1/x/(-(2+3*x)^2)^(1/2),x)
```

output
```
( - log(3*x + 2) + log(x))/2
```

3.132 $\int \frac{1}{x\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 41

$$\int \frac{1}{x\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{(a+bx)\log\left(\frac{b(a+bx)}{x}\right)}{a\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-(b*x+a)*ln(b*(b*x+a)/x)/a/((b*x+a)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 120 vs. 2(41) = 82.

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.93

$$\int \frac{1}{x\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{-2a\log(x)+\left(a-\sqrt{a^2}\right)\log\left(\sqrt{a^2}-bx-\sqrt{(a+bx)^2}\right)+a\log\left(\sqrt{a^2}+bx-\sqrt{(a+bx)^2}\right)+\sqrt{a^2}\log\left(\cdots\right.}{2a\sqrt{a^2}}$$

input
```
Integrate[1/(x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
(-2*a*Log[x] + (a - Sqrt[a^2])*Log[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2]] +
a*Log[Sqrt[a^2] + b*x - Sqrt[(a + b*x)^2]] + Sqrt[a^2]*Log[a*(Sqrt[a^2] +
b*x - Sqrt[(a + b*x)^2])])/(2*a*Sqrt[a^2])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1102, 27, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a^2+2abx+b^2x^2}}dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{b(a+bx)\int \frac{1}{bx(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{1}{x(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{47}$$

$$\frac{(a+bx)\left(\frac{\int \frac{1}{x}dx}{a}-\frac{b\int \frac{1}{a+bx}dx}{a}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{14}$$

$$\frac{(a+bx)\left(\frac{\log(x)}{a}-\frac{b\int \frac{1}{a+bx}dx}{a}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{16}$$

$$\frac{(a+bx)\left(\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[1/(x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
((a + b*x)*(Log[x]/a - Log[a + b*x]/a))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{(bx+a)(\ln(x)-\ln(bx+a))}{\sqrt{(bx+a)^2}\,a}$	30
risch	$-\frac{\sqrt{(bx+a)^2}\,\ln(bx+a)}{(bx+a)a}+\frac{\sqrt{(bx+a)^2}\,\ln(-x)}{(bx+a)a}$	53

input
```
int(1/x/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b*x+a)*(ln(x)-ln(b*x+a))/((b*x+a)^2)^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.39

$$\int \frac{1}{x\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{\log{(bx+a)}-\log{(x)}}{a}$$

input
```
integrate(1/x/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
-(log(b*x + a) - log(x))/a
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a^2+2abx+b^2x^2}}\,dx = \int \frac{1}{x\sqrt{(a+bx)^2}}\,dx$$

input
```
integrate(1/x/((b*x+a)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt((a + b*x)**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{(-1)^{2\,abx+2\,a^2}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)}{a}$$

input
```
integrate(1/x/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-(-1)^(2*a*b*x + 2*a^2)*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int \frac{1}{x\sqrt{a^2+2abx+b^2x^2}}\,dx = -\left(\frac{\log(|bx+a|)}{a}-\frac{\log(|x|)}{a}\right)\operatorname{sgn}(bx+a)$$

input
```
integrate(1/x/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-(log(abs(b*x + a))/a - log(abs(x))/a)*sgn(b*x + a)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{1}{x\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{\ln\left(a\,b+\frac{a^2}{x}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x}\right)}{\sqrt{a^2}}$$

input
```
int(1/(x*((a + b*x)^2)^(1/2)),x)
```

output
```
-log(a*b + a^2/x + ((a^2)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/x)/(a^2)^
(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.37

$$\int \frac{1}{x\sqrt{a^2+2abx+b^2x^2}}\,dx=\frac{-\log(bx+a)+\log(x)}{a}$$

input
```
int(1/x/((b*x+a)^2)^(1/2),x)
```

output
```
( - log(a + b*x) + log(x))/a
```

3.133 $\int \frac{1}{x\sqrt{a^2-2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 44

$$\int \frac{1}{x\sqrt{a^2-2abx+b^2x^2}}\,dx=-\frac{(a-bx)\log\left(-\frac{b(a-bx)}{x}\right)}{a\sqrt{a^2-2abx+b^2x^2}}$$

output
```
-(-b*x+a)*ln(-b*(-b*x+a)/x)/a/((b*x-a)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 103 vs. 2(44) = 88.

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.34

$$\int \frac{1}{x\sqrt{a^2-2abx+b^2x^2}}\,dx$$
$$=-\frac{\operatorname{arctanh}\left(\frac{bx}{\sqrt{a^2}-\sqrt{(a-bx)^2}}\right)}{a}+\frac{-2\log(x)+\log\left(\sqrt{a^2}-bx-\sqrt{(a-bx)^2}\right)+\log\left(\sqrt{a^2}+bx-\sqrt{(a-bx)^2}\right)}{2\sqrt{a^2}}$$

input
```
Integrate[1/(x*Sqrt[a^2 - 2*a*b*x + b^2*x^2]),x]
```

output
```
-(ArcTanh[(b*x)/(Sqrt[a^2] - Sqrt[(a - b*x)^2])]/a) + (-2*Log[x] + Log[Sqr
t[a^2] - b*x - Sqrt[(a - b*x)^2]] + Log[Sqrt[a^2] + b*x - Sqrt[(a - b*x)^2
]])/(2*Sqrt[a^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1102, 25, 27, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a^2-2abx+b^2x^2}}dx$$

$$\Big\downarrow \text{1102}$$

$$-\frac{b(a-bx)\int -\frac{1}{bx(a-bx)}dx}{\sqrt{a^2-2abx+b^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{b(a-bx)\int \frac{1}{bx(a-bx)}dx}{\sqrt{a^2-2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a-bx)\int \frac{1}{x(a-bx)}dx}{\sqrt{a^2-2abx+b^2x^2}}$$

$$\Big\downarrow \text{47}$$

$$\frac{(a-bx)\left(\frac{b\int \frac{1}{a-bx}dx}{a}+\frac{\int \frac{1}{x}dx}{a}\right)}{\sqrt{a^2-2abx+b^2x^2}}$$

$$\Big\downarrow \text{14}$$

$$\frac{(a-bx)\left(\frac{b\int \frac{1}{a-bx}dx}{a}+\frac{\log(x)}{a}\right)}{\sqrt{a^2-2abx+b^2x^2}}$$

$$\Big\downarrow \text{16}$$

$$\frac{(a-bx)\left(\frac{\log(x)}{a}-\frac{\log(a-bx)}{a}\right)}{\sqrt{a^2-2abx+b^2x^2}}$$

input
```
Int[1/(x*Sqrt[a^2 - 2*a*b*x + b^2*x^2]),x]
```

output
```
((a - b*x)*(Log[x]/a - Log[a - b*x]/a))/Sqrt[a^2 - 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{(-bx+a)(\ln(-bx+a)-\ln(x))}{\sqrt{(-bx+a)^2}\, a}$	34
risch	$-\frac{\sqrt{(-bx+a)^2}\,\ln(-bx+a)}{(-bx+a)a}+\frac{\sqrt{(-bx+a)^2}\,\ln(x)}{(-bx+a)a}$	56

input
```
int(1/x/((b*x-a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-b*x+a)*(ln(-b*x+a)-ln(x))/((-b*x+a)^2)^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.39

$$\int \frac{1}{x\sqrt{a^2-2abx+b^2x^2}}\,dx=\frac{\log(bx-a)-\log(x)}{a}$$

input
```
integrate(1/x/((b*x-a)^2)^(1/2),x, algorithm="fricas")
```

output
```
(log(b*x - a) - log(x))/a
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a^2-2abx+b^2x^2}}\,dx=\int \frac{1}{x\sqrt{(-a+bx)^2}}\,dx$$

input
```
integrate(1/x/((b*x-a)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt((-a + b*x)**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\sqrt{a^2-2abx+b^2x^2}}\,dx = -\frac{(-1)^{-2\,abx+2\,a^2}\log\left(-\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)}{a}$$

input
```
integrate(1/x/((b*x-a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-(-1)^(-2*a*b*x + 2*a^2)*log(-2*a*b*x/abs(x) + 2*a^2/abs(x))/a
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{1}{x\sqrt{a^2-2abx+b^2x^2}}\,dx = \left(\frac{\log(|bx-a|)}{a}-\frac{\log(|x|)}{a}\right)\operatorname{sgn}(bx-a)$$

input
```
integrate(1/x/((b*x-a)^2)^(1/2),x, algorithm="giac")
```

output
```
(log(abs(b*x - a))/a - log(abs(x))/a)*sgn(b*x - a)
```

Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\sqrt{a^2-2abx+b^2x^2}}\,dx = -\frac{\ln\left(\frac{a^2}{x}-a\,b+\frac{\sqrt{a^2}\,\sqrt{a^2-2\,a\,b\,x+b^2\,x^2}}{x}\right)}{\sqrt{a^2}}$$

input
```
int(1/(x*((a - b*x)^2)^(1/2)),x)
```

output
```
-log(a^2/x - a*b + ((a^2)^(1/2)*(a^2 + b^2*x^2 - 2*a*b*x)^(1/2))/x)/(a^2)^
(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.36

$$\int \frac{1}{x\sqrt{a^2-2abx+b^2x^2}}\,dx = \frac{-\log(-bx+a)+\log(x)}{a}$$

input
```
int(1/x/((b*x-a)^2)^(1/2),x)
```

output
```
( - log(a - b*x) + log(x))/a
```

3.134 $\int \frac{1}{x\sqrt{-a^2+2abx-b^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 47

$$\int \frac{1}{x\sqrt{-a^2+2abx-b^2x^2}}\,dx = -\frac{(a-bx)\log\left(-\frac{b(a-bx)}{x}\right)}{a\sqrt{-a^2+2abx-b^2x^2}}$$

output
```
-(-b*x+a)*ln(-b*(-b*x+a)/x)/a/(-(b*x-a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\sqrt{-a^2+2abx-b^2x^2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{-b^2}x}{a}-\frac{\sqrt{-a^2+2abx-b^2x^2}}{a}\right)}{a}$$

input
```
Integrate[1/(x*Sqrt[-a^2 + 2*a*b*x - b^2*x^2]),x]
```

output
```
(-2*ArcTan[(Sqrt[-b^2]*x)/a - Sqrt[-a^2 + 2*a*b*x - b^2*x^2]/a])/a
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1102, 27, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{-a^2+2abx-b^2x^2}}dx$$

$$\Big\downarrow \text{1102}$$

$$\frac{b(a-bx)\int \frac{1}{bx(a-bx)}dx}{\sqrt{-a^2+2abx-b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a-bx)\int \frac{1}{x(a-bx)}dx}{\sqrt{-a^2+2abx-b^2x^2}}$$

$$\Big\downarrow \text{47}$$

$$\frac{(a-bx)\left(\frac{b\int \frac{1}{a-bx}dx}{a}+\frac{\int \frac{1}{x}dx}{a}\right)}{\sqrt{-a^2+2abx-b^2x^2}}$$

$$\Big\downarrow \text{14}$$

$$\frac{(a-bx)\left(\frac{b\int \frac{1}{a-bx}dx}{a}+\frac{\log(x)}{a}\right)}{\sqrt{-a^2+2abx-b^2x^2}}$$

$$\Big\downarrow \text{16}$$

$$\frac{(a-bx)\left(\frac{\log(x)}{a}-\frac{\log(a-bx)}{a}\right)}{\sqrt{-a^2+2abx-b^2x^2}}$$

input
```
Int[1/(x*Sqrt[-a^2 + 2*a*b*x - b^2*x^2]),x]
```

output
```
((a - b*x)*(Log[x]/a - Log[a - b*x]/a))/Sqrt[-a^2 + 2*a*b*x - b^2*x^2]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{(-bx+a)(\ln(-bx+a)-\ln(x))}{\sqrt{-(-bx+a)^{2}}\,a}$	36
risch	$\frac{(-bx+a)\ln(-x)}{\sqrt{-(-bx+a)^{2}}\,a}-\frac{(-bx+a)\ln(bx-a)}{\sqrt{-(-bx+a)^{2}}\,a}$	59

input
```
int(1/x/(-(b*x-a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-b*x+a)*(ln(-b*x+a)-ln(x))/(-(-b*x+a)^2)^(1/2)/a
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.49

$$\int \frac{1}{x\sqrt{-a^2+2abx-b^2x^2}}\,dx = -\sqrt{-\frac{1}{a^2}}\log\left(\frac{i\,a^2\sqrt{-\frac{1}{a^2}}+2\,bx-a}{2\,b}\right) + \sqrt{-\frac{1}{a^2}}\log\left(\frac{-i\,a^2\sqrt{-\frac{1}{a^2}}+2\,bx-a}{2\,b}\right)$$

input
```
integrate(1/x/(-(b*x-a)^2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-1/a^2)*log(1/2*(I*a^2*sqrt(-1/a^2) + 2*b*x - a)/b) + sqrt(-1/a^2)*l
og(1/2*(-I*a^2*sqrt(-1/a^2) + 2*b*x - a)/b)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{-a^2+2abx-b^2x^2}}\,dx = \int \frac{1}{x\sqrt{-\left(-a+bx\right)^2}}\,dx$$

input
```
integrate(1/x/(-(b*x-a)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(-(-a + b*x)**2)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{1}{x\sqrt{-a^2+2abx-b^2x^2}}\,dx = -\frac{i\,(-1)^{-2\,abx+2\,a^2}\log\left(-\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)}{a}$$

input
```
integrate(1/x/(-(b*x-a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-I*(-1)^(-2*a*b*x + 2*a^2)*log(-2*a*b*x/abs(x) + 2*a^2/abs(x))/a
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\sqrt{-a^2+2abx-b^2x^2}}\,dx = \frac{i\,\log\left(|bx-a|\right)}{a\mathrm{sgn}\left(-bx+a\right)} - \frac{i\,\log\left(|x|\right)}{a\mathrm{sgn}\left(-bx+a\right)}$$

input
```
integrate(1/x/(-(b*x-a)^2)^(1/2),x, algorithm="giac")
```

output
```
I*log(abs(b*x - a))/(a*sgn(-b*x + a)) - I*log(abs(x))/(a*sgn(-b*x + a))
```

Mupad [B] (verification not implemented)

Time = 8.81 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

$$\int \frac{1}{x\sqrt{-a^2+2abx-b^2x^2}}\,dx = -\frac{\ln\left(a\,b-\frac{a^2}{x}+\frac{\sqrt{-a^2}\,\sqrt{-a^2+2\,a\,b\,x-b^2\,x^2}}{x}\right)}{\sqrt{-a^2}}$$

input
```
int(1/(x*(-(a - b*x)^2)^(1/2)),x)
```

output
```
-log(a*b - a^2/x + ((-a^2)^(1/2)*(2*a*b*x - b^2*x^2 - a^2)^(1/2))/x)/(-a^2
)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.34

$$\int \frac{1}{x\sqrt{-a^2 + 2abx - b^2x^2}}\,dx = \frac{-\log(-bx + a) + \log(x)}{a}$$

input
```
int(1/x/(-(b*x-a)^2)^(1/2),x)
```

output
```
( - log(a - b*x) + log(x))/a
```

3.135 $\int \frac{1}{x\sqrt{-a^2-2abx-b^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 44

$$\int \frac{1}{x\sqrt{-a^2-2abx-b^2x^2}}\,dx=-\frac{(a+bx)\log\left(\frac{b(a+bx)}{x}\right)}{a\sqrt{-a^2-2abx-b^2x^2}}$$

output
```
-(b*x+a)*ln(b*(b*x+a)/x)/a/(-(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\sqrt{-a^2-2abx-b^2x^2}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{-b^2}x}{a}-\frac{\sqrt{-a^2-2abx-b^2x^2}}{a}\right)}{a}$$

input
```
Integrate[1/(x*Sqrt[-a^2 - 2*a*b*x - b^2*x^2]),x]
```

output
```
(-2*ArcTan[(Sqrt[-b^2]*x)/a - Sqrt[-a^2 - 2*a*b*x - b^2*x^2]/a])/a
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1102, 25, 27, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{-a^2-2abx-b^2x^2}}dx$$

$$\Big\downarrow \text{ 1102}$$

$$-\frac{b(a+bx)\int -\frac{1}{bx(a+bx)}dx}{\sqrt{-a^2-2abx-b^2x^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{b(a+bx)\int \frac{1}{bx(a+bx)}dx}{\sqrt{-a^2-2abx-b^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx)\int \frac{1}{x(a+bx)}dx}{\sqrt{-a^2-2abx-b^2x^2}}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{(a+bx)\left(\frac{\int \frac{1}{x}dx}{a}-\frac{b\int \frac{1}{a+bx}dx}{a}\right)}{\sqrt{-a^2-2abx-b^2x^2}}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{(a+bx)\left(\frac{\log(x)}{a}-\frac{b\int \frac{1}{a+bx}dx}{a}\right)}{\sqrt{-a^2-2abx-b^2x^2}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{(a+bx)\left(\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}\right)}{\sqrt{-a^2-2abx-b^2x^2}}$$

input
```
Int[1/(x*Sqrt[-a^2 - 2*a*b*x - b^2*x^2]),x]
```

output
```
((a + b*x)*(Log[x]/a - Log[a + b*x]/a))/Sqrt[-a^2 - 2*a*b*x - b^2*x^2]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{(bx+a)(-\ln(x)+\ln(bx+a))}{\sqrt{-(bx+a)^2 a}}$	33
risch	$-\frac{(bx+a)\ln(bx+a)}{\sqrt{-(bx+a)^2 a}}+\frac{(bx+a)\ln(-x)}{\sqrt{-(bx+a)^2 a}}$	53

input
```
int(1/x/(-(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*x+a)*(-ln(x)+ln(b*x+a))/(-(b*x+a)^2)^(1/2)/a
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.50

$$\int \frac{1}{x\sqrt{-a^2-2abx-b^2x^2}}\,dx = -\sqrt{-\frac{1}{a^2}}\log\left(\frac{i\,a^2\sqrt{-\frac{1}{a^2}}+2\,bx+a}{2\,b}\right) + \sqrt{-\frac{1}{a^2}}\log\left(\frac{-i\,a^2\sqrt{-\frac{1}{a^2}}+2\,bx+a}{2\,b}\right)$$

input
```
integrate(1/x/(-(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-1/a^2)*log(1/2*(I*a^2*sqrt(-1/a^2) + 2*b*x + a)/b) + sqrt(-1/a^2)*l
og(1/2*(-I*a^2*sqrt(-1/a^2) + 2*b*x + a)/b)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{-a^2-2abx-b^2x^2}}\,dx = \int \frac{1}{x\sqrt{-\left(a+bx\right)^2}}\,dx$$

input
```
integrate(1/x/(-(b*x+a)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(-(a + b*x)**2)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\sqrt{-a^2-2abx-b^2x^2}}\,dx = -\frac{i\,(-1)^{2\,abx+2\,a^2}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)}{a}$$

input
```
integrate(1/x/(-(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-I*(-1)^(2*a*b*x + 2*a^2)*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{1}{x\sqrt{-a^2-2abx-b^2x^2}}\,dx = -\frac{i\,\log\left(|bx+a|\right)}{a\mathrm{sgn}\left(-bx-a\right)}+\frac{i\,\log\left(|x|\right)}{a\mathrm{sgn}\left(-bx-a\right)}$$

input
```
integrate(1/x/(-(b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-I*log(abs(b*x + a))/(a*sgn(-b*x - a)) + I*log(abs(x))/(a*sgn(-b*x - a))
```

Mupad [B] (verification not implemented)

Time = 8.81 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

$$\int \frac{1}{x\sqrt{-a^2-2abx-b^2x^2}}\,dx = -\frac{\ln\left(\frac{\sqrt{-a^2}\,\sqrt{-a^2-2\,a\,b\,x-b^2\,x^2}}{x}-\frac{a^2}{x}-a\,b\right)}{\sqrt{-a^2}}$$

input
```
int(1/(x*(-(a + b*x)^2)^(1/2)),x)
```

output
```
-log(((-a^2)^(1/2)*(- a^2 - b^2*x^2 - 2*a*b*x)^(1/2))/x - a^2/x - a*b)/(-a
^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.34

$$\int \frac{1}{x\sqrt{-a^2-2abx-b^2x^2}}\,dx=\frac{-\log(bx+a)+\log(x)}{a}$$

input
```
int(1/x/(-(b*x+a)^2)^(1/2),x)
```

output
```
( - log(a + b*x) + log(x))/a
```

3.136 $\int (dx)^m \left(a^2+2abx+b^2x^2\right)^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int (dx)^m \left(a^2+2abx+b^2x^2\right)^p \, dx = \frac{(dx)^{1+m}(a+bx)\left(a^2+2abx+b^2x^2\right)^p \text{Hypergeometric2F1}\left(1,2+m+2p,2+m,-\frac{bx}{a}\right)}{ad(1+m)}$$

output
```
(d*x)^(1+m)*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^p*hypergeom([1, 2+m+2*p],[2+m],-
b*x/a)/a/d/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (dx)^m \left(a^2+2abx+b^2x^2\right)^p \, dx = \frac{x(dx)^m \left((a+bx)^2\right)^p \left(1+\frac{bx}{a}\right)^{-2p} \text{Hypergeometric2F1}\left(1+m,-2p,2+m,-\frac{bx}{a}\right)}{1+m}$$

input
```
Integrate[(d*x)^m*(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

output
```
(x*(d*x)^m*((a + b*x)^2)^p*Hypergeometric2F1[1 + m, -2*p, 2 + m, -((b*x)/a
)])/((1 + m)*(1 + (b*x)/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1102, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a^2+2abx+b^2x^2\right)^p \, dx$$

$$\Big\downarrow \text{1102}$$

$$\left(ab+b^2x\right)^{-2p} \left(a^2+2abx+b^2x^2\right)^p \int (dx)^m \left(xb^2+ab\right)^{2p} dx$$

$$\Big\downarrow \text{76}$$

$$\left(\frac{bx}{a}+1\right)^{-2p} \left(a^2+2abx+b^2x^2\right)^p \int (dx)^m \left(\frac{bx}{a}+1\right)^{2p} dx$$

$$\Big\downarrow \text{74}$$

$$\frac{(dx)^{m+1} \left(\frac{bx}{a}+1\right)^{-2p} \left(a^2+2abx+b^2x^2\right)^p \text{Hypergeometric2F1}\left(m+1,-2p,m+2,-\frac{bx}{a}\right)}{d(m+1)}$$

input
```
Int[(d*x)^m*(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

output
```
((d*x)^(1 + m)*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometric2F1[1 + m, -2*p,
2 + m, -((b*x)/a)])/(d*(1 + m)*(1 + (b*x)/a)^(2*p))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int (dx)^m \left(b^2x^2+2abx+a^2\right)^p dx$$

input
```
int((d*x)^m*(b^2*x^2+2*a*b*x+a^2)^p,x)
```

output
```
int((d*x)^m*(b^2*x^2+2*a*b*x+a^2)^p,x)
```

Fricas [F]

$$\int (dx)^m \left(a^2+2abx+b^2x^2\right)^p \, dx = \int \left(b^2x^2+2\,abx+a^2\right)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="fricas")
```

output
```
integral((b^2*x^2 + 2*a*b*x + a^2)^p*(d*x)^m, x)
```

Sympy [F]

$$\int (dx)^m \left(a^2+2abx+b^2x^2\right)^p\,dx=\int (dx)^m \left((a+bx)^2\right)^p\,dx$$

input
```
integrate((d*x)**m*(b**2*x**2+2*a*b*x+a**2)**p,x)
```

output
```
Integral((d*x)**m*((a + b*x)**2)**p, x)
```

Maxima [F]

$$\int (dx)^m \left(a^2+2abx+b^2x^2\right)^p\,dx=\int \left(b^2x^2+2\,abx+a^2\right)^p(dx)^m\,dx$$

input
```
integrate((d*x)^m*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="maxima")
```

output
```
integrate((b^2*x^2 + 2*a*b*x + a^2)^p*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m \left(a^2+2abx+b^2x^2\right)^p\,dx=\int \left(b^2x^2+2\,abx+a^2\right)^p(dx)^m\,dx$$

input
```
integrate((d*x)^m*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="giac")
```

output
```
integrate((b^2*x^2 + 2*a*b*x + a^2)^p*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a^2+2abx+b^2x^2\right)^p\,dx=\int (d\,x)^m\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^p\,dx$$

input
```
int((d*x)^m*(a^2 + b^2*x^2 + 2*a*b*x)^p,x)
```

output
```
int((d*x)^m*(a^2 + b^2*x^2 + 2*a*b*x)^p, x)
```

Reduce [F]

$$\int (dx)^m \left(a^2+2abx+b^2x^2\right)^p\,dx$$
$$=\frac{d^m\Big(2x^m(b^2x^2+2abx+a^2)^p\,ap+x^m(b^2x^2+2abx+a^2)^p\,bmx+2x^m(b^2x^2+2abx+a^2)^p\,bpx-2\Big(\int \frac{}{bm^2}$$

input
```
int((d*x)^m*(b^2*x^2+2*a*b*x+a^2)^p,x)
```

output
```
(d**m*(2*x**m*(a**2 + 2*a*b*x + b**2*x**2)**p*a*p + x**m*(a**2 + 2*a*b*x +
 b**2*x**2)**p*b*m*x + 2*x**m*(a**2 + 2*a*b*x + b**2*x**2)**p*b*p*x - 2*in
t((x**m*(a**2 + 2*a*b*x + b**2*x**2)**p)/(a*m**2*x + 4*a*m*p*x + a*m*x + 4
*a*p**2*x + 2*a*p*x + b*m**2*x**2 + 4*b*m*p*x**2 + b*m*x**2 + 4*b*p**2*x**
2 + 2*b*p*x**2),x)*a**2*m**3*p - 8*int((x**m*(a**2 + 2*a*b*x + b**2*x**2)*
*p)/(a*m**2*x + 4*a*m*p*x + a*m*x + 4*a*p**2*x + 2*a*p*x + b*m**2*x**2 + 4
*b*m*p*x**2 + b*m*x**2 + 4*b*p**2*x**2 + 2*b*p*x**2),x)*a**2*m**2*p**2 - 2
*int((x**m*(a**2 + 2*a*b*x + b**2*x**2)**p)/(a*m**2*x + 4*a*m*p*x + a*m*x
+ 4*a*p**2*x + 2*a*p*x + b*m**2*x**2 + 4*b*m*p*x**2 + b*m*x**2 + 4*b*p**2*
x**2 + 2*b*p*x**2),x)*a**2*m**2*p - 8*int((x**m*(a**2 + 2*a*b*x + b**2*x**
2)**p)/(a*m**2*x + 4*a*m*p*x + a*m*x + 4*a*p**2*x + 2*a*p*x + b*m**2*x**2
+ 4*b*m*p*x**2 + b*m*x**2 + 4*b*p**2*x**2 + 2*b*p*x**2),x)*a**2*m*p**3 - 4
*int((x**m*(a**2 + 2*a*b*x + b**2*x**2)**p)/(a*m**2*x + 4*a*m*p*x + a*m*x
+ 4*a*p**2*x + 2*a*p*x + b*m**2*x**2 + 4*b*m*p*x**2 + b*m*x**2 + 4*b*p**2*
x**2 + 2*b*p*x**2),x)*a**2*m*p**2))/(b*(m**2 + 4*m*p + m + 4*p**2 + 2*p))
```

3.137 $\int x^2(a^2+2abx+b^2x^2)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 110

$$\int x^2(a^2+2abx+b^2x^2)^p\,dx=\frac{a^2(a+bx)\left(a^2+2abx+b^2x^2\right)^p}{b^3(1+2p)}-\frac{a(a+bx)^2\left(a^2+2abx+b^2x^2\right)^p}{b^3(1+p)}+\frac{(a+bx)^3\left(a^2+2abx+b^2x^2\right)^p}{b^3(3+2p)}$$

output
```
a^2*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^p/b^3/(1+2*p)-a*(b*x+a)^2*(b^2*x^2+2*a*b
*x+a^2)^p/b^3/(p+1)+(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2)^p/b^3/(3+2*p)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.62

$$\int x^2(a^2+2abx+b^2x^2)^p\,dx=\frac{(a+bx)\left((a+bx)^2\right)^p\left(a^2-ab(1+2p)x+b^2(1+3p+2p^2)\,x^2\right)}{b^3(1+p)(1+2p)(3+2p)}$$

input
```
Integrate[x^2*(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

output
```
((a + b*x)*((a + b*x)^2)^p*(a^2 - a*b*(1 + 2*p)*x + b^2*(1 + 3*p + 2*p^2)*
x^2))/(b^3*(1 + p)*(1 + 2*p)*(3 + 2*p))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1102, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a^2+2abx+b^2x^2\right)^p dx$$

$$\Big\downarrow \text{1102}$$

$$\left(ab+b^2x\right)^{-2p}\left(a^2+2abx+b^2x^2\right)^p \int x^2 \left(xb^2+ab\right)^{2p}dx$$

$$\Big\downarrow \text{53}$$

$$\left(ab+b^2x\right)^{-2p}\left(a^2+2abx+b^2x^2\right)^p \int \left(\frac{a^2\left(xb^2+ab\right)^{2p}}{b^2}-\frac{2a\left(xb^2+ab\right)^{2p+1}}{b^3}+\frac{\left(xb^2+ab\right)^{2p+2}}{b^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\left(ab+b^2x\right)^{-2p}\left(a^2+2abx+b^2x^2\right)^p \left(\frac{a^2\left(ab+b^2x\right)^{2p+1}}{b^4(2p+1)}+\frac{\left(ab+b^2x\right)^{2p+3}}{b^6(2p+3)}-\frac{a\left(ab+b^2x\right)^{2(p+1)}}{b^5(p+1)}\right)$$

input
```
Int[x^2*(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

output
```
((a^2 + 2*a*b*x + b^2*x^2)^p*(-((a*(a*b + b^2*x)^(2*(1 + p)))/(b^5*(1 + p)
)) + (a^2*(a*b + b^2*x)^(1 + 2*p))/(b^4*(1 + 2*p)) + (a*b + b^2*x)^(3 + 2*
p)/(b^6*(3 + 2*p))))/(a*b + b^2*x)^(2*p)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.79

method	result
gosper	$\frac{(bx+a)\left(2b^{2}p^{2}x^{2}+3b^{2}p\,x^{2}-2abpx+b^{2}x^{2}-abx+a^{2}\right)\left(b^{2}x^{2}+2abx+a^{2}\right)^{p}}{b^{3}\left(4p^{3}+12p^{2}+11p+3\right)}$
orering	$\frac{(bx+a)\left(2b^{2}p^{2}x^{2}+3b^{2}p\,x^{2}-2abpx+b^{2}x^{2}-abx+a^{2}\right)\left(b^{2}x^{2}+2abx+a^{2}\right)^{p}}{b^{3}\left(4p^{3}+12p^{2}+11p+3\right)}$
risch	$\frac{\left(2b^{3}p^{2}x^{3}+2a\,b^{2}p^{2}x^{2}+3b^{3}p\,x^{3}+x^{2}ap\,b^{2}+b^{3}x^{3}-2a^{2}pxb+a^{3}\right)\left((bx+a)^{2}\right)^{p}}{(p+1)(3+2p)(2p+1)b^{3}}$
norman	$\frac{x^{3}\mathrm{e}^{p\ln\left(b^{2}x^{2}+2abx+a^{2}\right)}}{3+2p}+\frac{a^{3}\mathrm{e}^{p\ln\left(b^{2}x^{2}+2abx+a^{2}\right)}}{b^{3}\left(4p^{3}+12p^{2}+11p+3\right)}+\frac{ap\,x^{2}\mathrm{e}^{p\ln\left(b^{2}x^{2}+2abx+a^{2}\right)}}{b\left(2p^{2}+5p+3\right)}-\frac{2p\,a^{2}x\,\mathrm{e}^{p\ln\left(b^{2}x^{2}+2abx+a^{2}\right)}}{b^{2}\left(4p^{3}+12p^{2}+11p+3\right)}$
parallelrisch	$\frac{2x^{3}\left(b^{2}x^{2}+2abx+a^{2}\right)^{p}a\,b^{3}p^{2}+3x^{3}\left(b^{2}x^{2}+2abx+a^{2}\right)^{p}a\,b^{3}p+2x^{2}\left(b^{2}x^{2}+2abx+a^{2}\right)^{p}a^{2}b^{2}p^{2}+x^{3}\left(b^{2}x^{2}+2abx+a^{2}\right)^{p}a\,b^{3}+x^{2}(b^{2}}{(3+2p)(p+1)(2p+1)a\,b^{3}}$

input
```
int(x^2*(b^2*x^2+2*a*b*x+a^2)^p,x,method=_RETURNVERBOSE)
```

output
```
(b*x+a)*(2*b^2*p^2*x^2+3*b^2*p*x^2-2*a*b*p*x+b^2*x^2-a*b*x+a^2)*(b^2*x^2+2
*a*b*x+a^2)^p/b^3/(4*p^3+12*p^2+11*p+3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.98

$$\int x^2 \left(a^2+2abx+b^2x^2\right)^p\,dx = -\frac{(2\,a^2bpx-(2\,b^3p^2+3\,b^3p+b^3)x^3-a^3-(2\,ab^2p^2+ab^2p)x^2)(b^2x^2+2\,abx+a^2)^p}{4\,b^3p^3+12\,b^3p^2+11\,b^3p+3\,b^3}$$

input
```
integrate(x^2*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="fricas")
```

output
```
-(2*a^2*b*p*x - (2*b^3*p^2 + 3*b^3*p + b^3)*x^3 - a^3 - (2*a*b^2*p^2 + a*b
^2*p)*x^2)*(b^2*x^2 + 2*a*b*x + a^2)^p/(4*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p
+ 3*b^3)
```

Sympy [F]

$$\int x^2 \left(a^2+2abx+b^2x^2\right)^p\,dx = \begin{cases} \frac{x^3(a^2)^p}{3} \\ \int \frac{x^2}{\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx \\ -\frac{2a^2\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x} - \frac{2a^2}{ab^3+b^4x} - \frac{2abx\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x} + \frac{b^2x^2}{ab^3+b^4x} \\ \begin{cases} \frac{a^2\left(\frac{a}{b}+x\right)\log\left(\frac{a}{b}+x\right)}{b^2\sqrt{b^2\left(\frac{a}{b}+x\right)^2}} + \left(-\frac{3a}{2b^3}+\frac{x}{2b^2}\right)\sqrt{a^2+2abx+b^2x^2} & \text{for}\ b^2\neq 0 \\ \frac{a^4\sqrt{a^2+2abx}-\frac{2a^2\left(a^2+2abx\right)^{\frac{3}{2}}}{3}+\frac{\left(a^2+2abx\right)^{\frac{5}{2}}}{5}}{4a^3b^3} & \text{for}\ ab\neq 0 \\ \frac{x^3}{3\sqrt{a^2}} & \text{otherwise} \end{cases} \\ \frac{a^3\left(a^2+2abx+b^2x^2\right)^p}{4b^3p^3+12b^3p^2+11b^3p+3b^3} - \frac{2a^2bpx\left(a^2+2abx+b^2x^2\right)^p}{4b^3p^3+12b^3p^2+11b^3p+3b^3} + \frac{2ab^2p^2x^2\left(a^2+2abx+b^2x^2\right)^p}{4b^3p^3+12b^3p^2+11b^3p+3b^3} + \frac{ab^2px^2\left(a^2+2abx+b^2x^2\right)^p}{4b^3p^3+12b^3p^2+11b^3p+3b^3} + \frac{2b^3p^2x^3(a^2}{4b^3p^3+12b^3} \end{cases}$$

input
```
integrate(x**2*(b**2*x**2+2*a*b*x+a**2)**p,x)
```

output
```
Piecewise((x**3*(a**2)**p/3, Eq(b, 0)), (Integral(x**2/((a + b*x)**2)**(3/
2), x), Eq(p, -3/2)), (-2*a**2*log(a/b + x)/(a*b**3 + b**4*x) - 2*a**2/(a*
b**3 + b**4*x) - 2*a*b*x*log(a/b + x)/(a*b**3 + b**4*x) + b**2*x**2/(a*b**
3 + b**4*x), Eq(p, -1)), (Piecewise((a**2*(a/b + x)*log(a/b + x)/(b**2*sqr
t(b**2*(a/b + x)**2)) + (-3*a/(2*b**3) + x/(2*b**2))*sqrt(a**2 + 2*a*b*x +
 b**2*x**2), Ne(b**2, 0)), ((a**4*sqrt(a**2 + 2*a*b*x) - 2*a**2*(a**2 + 2*
a*b*x)**(3/2)/3 + (a**2 + 2*a*b*x)**(5/2)/5)/(4*a**3*b**3), Ne(a*b, 0)), (
x**3/(3*sqrt(a**2)), True)), Eq(p, -1/2)), (a**3*(a**2 + 2*a*b*x + b**2*x*
*2)**p/(4*b**3*p**3 + 12*b**3*p**2 + 11*b**3*p + 3*b**3) - 2*a**2*b*p*x*(a
**2 + 2*a*b*x + b**2*x**2)**p/(4*b**3*p**3 + 12*b**3*p**2 + 11*b**3*p + 3*
b**3) + 2*a*b**2*p**2*x**2*(a**2 + 2*a*b*x + b**2*x**2)**p/(4*b**3*p**3 +
12*b**3*p**2 + 11*b**3*p + 3*b**3) + a*b**2*p*x**2*(a**2 + 2*a*b*x + b**2*
x**2)**p/(4*b**3*p**3 + 12*b**3*p**2 + 11*b**3*p + 3*b**3) + 2*b**3*p**2*x
**3*(a**2 + 2*a*b*x + b**2*x**2)**p/(4*b**3*p**3 + 12*b**3*p**2 + 11*b**3*
p + 3*b**3) + 3*b**3*p*x**3*(a**2 + 2*a*b*x + b**2*x**2)**p/(4*b**3*p**3 +
 12*b**3*p**2 + 11*b**3*p + 3*b**3) + b**3*x**3*(a**2 + 2*a*b*x + b**2*x**
2)**p/(4*b**3*p**3 + 12*b**3*p**2 + 11*b**3*p + 3*b**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.67

$$\int x^2 \left(a^2+2abx+b^2x^2\right)^p \, dx = \frac{\left((2\,p^2+3\,p+1)b^3x^3+(2\,p^2+p)ab^2x^2-2\,a^2bpx+a^3\right)(bx+a)^{2\,p}}{(4\,p^3+12\,p^2+11\,p+3)b^3}$$

input
```
integrate(x^2*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="maxima")
```

output
```
((2*p^2 + 3*p + 1)*b^3*x^3 + (2*p^2 + p)*a*b^2*x^2 - 2*a^2*b*p*x + a^3)*(b
*x + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.98

$$\int x^2(a^2+2abx+b^2x^2)^p\,dx$$
$$=\frac{2\left(b^2x^2+2\,abx+a^2\right)^pb^3p^2x^3+2\left(b^2x^2+2\,abx+a^2\right)^pab^2p^2x^2+3\left(b^2x^2+2\,abx+a^2\right)^pb^3px^3+\left(b^2x^2+2\right.}{4\,b^3p^3+12\,b^3p^2+}$$

input
```
integrate(x^2*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="giac")
```

output
```
(2*(b^2*x^2 + 2*a*b*x + a^2)^p*b^3*p^2*x^3 + 2*(b^2*x^2 + 2*a*b*x + a^2)^p
*a*b^2*p^2*x^2 + 3*(b^2*x^2 + 2*a*b*x + a^2)^p*b^3*p*x^3 + (b^2*x^2 + 2*a*
b*x + a^2)^p*a*b^2*p*x^2 + (b^2*x^2 + 2*a*b*x + a^2)^p*b^3*x^3 - 2*(b^2*x^
2 + 2*a*b*x + a^2)^p*a^2*b*p*x + (b^2*x^2 + 2*a*b*x + a^2)^p*a^3)/(4*b^3*p
^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.21

$$\int x^2(a^2+2abx+b^2x^2)^p\,dx=\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^p\left(\frac{x^3\,\left(2\,p^2+3\,p+1\right)}{4\,p^3+12\,p^2+11\,p+3}+\frac{a^3}{b^3\,\left(4\,p^3+12\,p^2+11\,p+3\right)}-\frac{2\,a^2\,p\,x}{b^2\,\left(4\,p^3+12\,p^2+11\,p+3\right)}+\frac{a\,p\,x^2\,\left(2\,p+1\right)}{b\,\left(4\,p^3+12\,p^2+11\,p+3\right)}\right)$$

input
```
int(x^2*(a^2 + b^2*x^2 + 2*a*b*x)^p,x)
```

output
```
(a^2 + b^2*x^2 + 2*a*b*x)^p*((x^3*(3*p + 2*p^2 + 1))/(11*p + 12*p^2 + 4*p^
3 + 3) + a^3/(b^3*(11*p + 12*p^2 + 4*p^3 + 3)) - (2*a^2*p*x)/(b^2*(11*p +
12*p^2 + 4*p^3 + 3)) + (a*p*x^2*(2*p + 1))/(b*(11*p + 12*p^2 + 4*p^3 + 3))
)
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.90

$$\int x^2\left(a^2+2abx+b^2x^2\right)^p\,dx$$
$$=\frac{\left(b^2x^2+2abx+a^2\right)^p\left(2b^3p^2x^3+2a\,b^2p^2x^2+3b^3p\,x^3+a\,b^2p\,x^2+b^3x^3-2a^2bpx+a^3\right)}{b^3\left(4p^3+12p^2+11p+3\right)}$$

input
```
int(x^2*(b^2*x^2+2*a*b*x+a^2)^p,x)
```

output
```
((a**2 + 2*a*b*x + b**2*x**2)**p*(a**3 - 2*a**2*b*p*x + 2*a*b**2*p**2*x**2
 + a*b**2*p*x**2 + 2*b**3*p**2*x**3 + 3*b**3*p*x**3 + b**3*x**3))/(b**3*(4
*p**3 + 12*p**2 + 11*p + 3))
```

3.138 $\int x(a^2+2abx+b^2x^2)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 69

$$\int x\left(a^2+2abx+b^2x^2\right)^p\,dx=-\frac{a(a+bx)\left(a^2+2abx+b^2x^2\right)^p}{b^2(1+2p)}+\frac{\left(a^2+2abx+b^2x^2\right)^{1+p}}{2b^2(1+p)}$$

output
```
-a*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^p/b^2/(1+2*p)+1/2*(b^2*x^2+2*a*b*x+a^2)^(
p+1)/b^2/(p+1)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

$$\int x\left(a^2+2abx+b^2x^2\right)^p\,dx=\frac{(a+bx)\left((a+bx)^2\right)^p(-a+b(1+2p)x)}{2b^2(1+p)(1+2p)}$$

input
```
Integrate[x*(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

output
```
((a + b*x)*((a + b*x)^2)^p*(-a + b*(1 + 2*p)*x))/(2*b^2*(1 + p)*(1 + 2*p))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a^2+2abx+b^2x^2)^p\,dx$$

$$\Big\downarrow \text{ 1100}$$

$$\frac{(a^2+2abx+b^2x^2)^{p+1}}{2b^2(p+1)}-\frac{a\int (a^2+2bxa+b^2x^2)^p\,dx}{b}$$

$$\Big\downarrow \text{ 1079}$$

$$\frac{(a^2+2abx+b^2x^2)^{p+1}}{2b^2(p+1)}-\frac{a(ab+b^2x)^{-2p}(a^2+2abx+b^2x^2)^p\int (xb^2+ab)^{2p}\,dx}{b}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(a^2+2abx+b^2x^2)^{p+1}}{2b^2(p+1)}-\frac{a(ab+b^2x)(a^2+2abx+b^2x^2)^p}{b^3(2p+1)}$$

input
```
Int[x*(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

output
```
-((a*(a*b + b^2*x)*(a^2 + 2*a*b*x + b^2*x^2)^p)/(b^3*(1 + 2*p))) + (a^2 +
2*a*b*x + b^2*x^2)^(1 + p)/(2*b^2*(1 + p))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

method	result	size
gosper	$-\frac{\left(b^{2} x^{2}+2 a b x+a^{2}\right)^{p} \left(-2 p b x-b x+a\right) \left(b x+a\right)}{2 b^{2} \left(2 p^{2}+3 p+1\right)}$	52
orering	$-\frac{\left(b^{2} x^{2}+2 a b x+a^{2}\right)^{p} \left(-2 p b x-b x+a\right) \left(b x+a\right)}{2 b^{2} \left(2 p^{2}+3 p+1\right)}$	52
risch	$-\frac{\left(-2 b^{2} p \,x^{2}-2 a b p x-b^{2} x^{2}+a^{2}\right) \left(\left(b x+a\right)^{2}\right)^{p}}{2 b^{2} \left(p+1\right) \left(2 p+1\right)}$	54
norman	$\frac{p a x \,{\mathrm e}^{p \ln \left(b^{2} x^{2}+2 a b x+a^{2}\right)}}{b \left(2 p^{2}+3 p+1\right)}+\frac{x^{2} {\mathrm e}^{p \ln \left(b^{2} x^{2}+2 a b x+a^{2}\right)}}{2+2 p}-\frac{a^{2} {\mathrm e}^{p \ln \left(b^{2} x^{2}+2 a b x+a^{2}\right)}}{2 b^{2} \left(2 p^{2}+3 p+1\right)}$	111
parallelrisch	$\frac{2 x^{2} \left(b^{2} x^{2}+2 a b x+a^{2}\right)^{p} b^{2} p+x^{2} \left(b^{2} x^{2}+2 a b x+a^{2}\right)^{p} b^{2}+2 x \left(b^{2} x^{2}+2 a b x+a^{2}\right)^{p} a b p-\left(b^{2} x^{2}+2 a b x+a^{2}\right)^{p} a^{2}}{2 b^{2} \left(2 p^{2}+3 p+1\right)}$	118

input
```
int(x*(b^2*x^2+2*a*b*x+a^2)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b^2*x^2+2*a*b*x+a^2)^p*(-2*b*p*x-b*x+a)*(b*x+a)/b^2/(2*p^2+3*p+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int x(a^2+2abx+b^2x^2)^p\,dx=\frac{(2\,abpx+(2\,b^2p+b^2)x^2-a^2)(b^2x^2+2\,abx+a^2)^p}{2\,(2\,b^2p^2+3\,b^2p+b^2)}$$

input
```
integrate(x*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="fricas")
```

output
```
1/2*(2*a*b*p*x + (2*b^2*p + b^2)*x^2 - a^2)*(b^2*x^2 + 2*a*b*x + a^2)^p/(2
*b^2*p^2 + 3*b^2*p + b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(61) = 122.

Time = 0.61 (sec) , antiderivative size = 354, normalized size of antiderivative = 5.13

$$\int x(a^2+2abx+b^2x^2)^p\,dx$$

$$=\begin{cases}\frac{x^2(a^2)^p}{2} & \text{for } b=0\\ \frac{a\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x}+\frac{a}{ab^2+b^3x}+\frac{bx\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} & \text{for } p=-1\\ \begin{cases}-\frac{a\left(\frac{a}{b}+x\right)\log\left(\frac{a}{b}+x\right)}{b\sqrt{b^2\left(\frac{a}{b}+x\right)^2}}+\frac{\sqrt{a^2+2abx+b^2x^2}}{b^2} & \text{for } b^2\neq 0\\ \frac{-a^2\sqrt{a^2+2abx}+\frac{(a^2+2abx)^{\frac{3}{2}}}{3}}{2a^2b^2} & \text{for } ab\neq 0\\ \frac{x^2}{2\sqrt{a^2}} & \text{otherwise}\end{cases} & \text{for } p=-\frac{1}{2}\\ -\frac{a^2(a^2+2abx+b^2x^2)^p}{4b^2p^2+6b^2p+2b^2}+\frac{2abpx(a^2+2abx+b^2x^2)^p}{4b^2p^2+6b^2p+2b^2}+\frac{2b^2px^2(a^2+2abx+b^2x^2)^p}{4b^2p^2+6b^2p+2b^2}+\frac{b^2x^2(a^2+2abx+b^2x^2)^p}{4b^2p^2+6b^2p+2b^2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*(b**2*x**2+2*a*b*x+a**2)**p,x)
```

output

```
Piecewise((x**2*(a**2)**p/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**3*x)
+ a/(a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(p, -1)), (P
iecewise((-a*(a/b + x)*log(a/b + x)/(b*sqrt(b**2*(a/b + x)**2)) + sqrt(a**
2 + 2*a*b*x + b**2*x**2)/b**2, Ne(b**2, 0)), ((-a**2*sqrt(a**2 + 2*a*b*x)
+ (a**2 + 2*a*b*x)**(3/2)/3)/(2*a**2*b**2), Ne(a*b, 0)), (x**2/(2*sqrt(a**
2)), True)), Eq(p, -1/2)), (-a**2*(a**2 + 2*a*b*x + b**2*x**2)**p/(4*b**2*
p**2 + 6*b**2*p + 2*b**2) + 2*a*b*p*x*(a**2 + 2*a*b*x + b**2*x**2)**p/(4*b
**2*p**2 + 6*b**2*p + 2*b**2) + 2*b**2*p*x**2*(a**2 + 2*a*b*x + b**2*x**2)
**p/(4*b**2*p**2 + 6*b**2*p + 2*b**2) + b**2*x**2*(a**2 + 2*a*b*x + b**2*x
**2)**p/(4*b**2*p**2 + 6*b**2*p + 2*b**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.72

$$\int x\left(a^2+2abx+b^2x^2\right)^p\,dx=\frac{\left(b^2(2\,p+1)x^2+2\,abpx-a^2\right)(bx+a)^{2\,p}}{2\,(2\,p^2+3\,p+1)b^2}$$

input

```
integrate(x*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="maxima")
```

output

```
1/2*(b^2*(2*p + 1)*x^2 + 2*a*b*p*x - a^2)*(b*x + a)^(2*p)/((2*p^2 + 3*p +
1)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.77

$$\int x\left(a^2+2abx+b^2x^2\right)^p\,dx$$
$$=\frac{2\,(b^2x^2+2\,abx+a^2)^pb^2px^2+2\,(b^2x^2+2\,abx+a^2)^pabpx+(b^2x^2+2\,abx+a^2)^pb^2x^2-(b^2x^2+2\,abx+}{2\,(2\,b^2p^2+3\,b^2p+b^2)}$$

input

```
integrate(x*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="giac")
```

output
```
1/2*(2*(b^2*x^2 + 2*a*b*x + a^2)^p*b^2*p*x^2 + 2*(b^2*x^2 + 2*a*b*x + a^2)
^p*a*b*p*x + (b^2*x^2 + 2*a*b*x + a^2)^p*b^2*x^2 - (b^2*x^2 + 2*a*b*x + a^
2)^p*a^2)/(2*b^2*p^2 + 3*b^2*p + b^2)
```

Mupad [B] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int x(a^2 + 2abx + b^2x^2)^p \, dx = (a^2 + 2\,a\,b\,x + b^2\,x^2)^p \left(\frac{x^2\,(p + \frac{1}{2})}{2\,p^2 + 3\,p + 1} - \frac{a^2}{2\,b^2\,(2\,p^2 + 3\,p + 1)} + \frac{a\,p\,x}{b\,(2\,p^2 + 3\,p + 1)}\right)$$

input
```
int(x*(a^2 + b^2*x^2 + 2*a*b*x)^p,x)
```

output
```
(a^2 + b^2*x^2 + 2*a*b*x)^p*((x^2*(p + 1/2))/(3*p + 2*p^2 + 1) - a^2/(2*b^
2*(3*p + 2*p^2 + 1)) + (a*p*x)/(b*(3*p + 2*p^2 + 1)))
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int x(a^2 + 2abx + b^2x^2)^p \, dx = \frac{(b^2x^2 + 2abx + a^2)^p\,(2b^2p\,x^2 + 2abpx + b^2x^2 - a^2)}{2b^2\,(2p^2 + 3p + 1)}$$

input
```
int(x*(b^2*x^2+2*a*b*x+a^2)^p,x)
```

output
```
((a**2 + 2*a*b*x + b**2*x**2)**p*( - a**2 + 2*a*b*p*x + 2*b**2*p*x**2 + b*
*2*x**2))/(2*b**2*(2*p**2 + 3*p + 1))
```

3.139 $\int (a^2+2abx+b^2x^2)^p \, dx$

Optimal result

Integrand size = 18, antiderivative size = 34

$$\int \left(a^2+2abx+b^2x^2\right)^p \, dx=\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^p}{b(1+2p)}$$

output
```
(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^p/b/(1+2*p)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \left(a^2+2abx+b^2x^2\right)^p \, dx=\frac{(a+bx)\left((a+bx)^2\right)^p}{b(1+2p)}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

output
```
((a + b*x)*((a + b*x)^2)^p)/(b*(1 + 2*p))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx+b^2x^2\right)^p\,dx$$

$$\Big\downarrow \text{ 1079}$$

$$\left(ab+b^2x\right)^{-2p}\left(a^2+2abx+b^2x^2\right)^p\int \left(xb^2+ab\right)^{2p}dx$$

$$\Big\downarrow \text{ 17}$$

$$\frac{\left(ab+b^2x\right)\left(a^2+2abx+b^2x^2\right)^p}{b^2(2p+1)}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

output
```
((a*b + b^2*x)*(a^2 + 2*a*b*x + b^2*x^2)^p)/(b^2*(1 + 2*p))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{(bx+a)\left((bx+a)^2\right)^p}{b(2p+1)}$	26
gosper	$\frac{(bx+a)(b^2x^2+2abx+a^2)^p}{b(2p+1)}$	35
orering	$\frac{(bx+a)(b^2x^2+2abx+a^2)^p}{b(2p+1)}$	35
parallelrisch	$\frac{x(b^2x^2+2abx+a^2)^p ab+(b^2x^2+2abx+a^2)^p a^2}{(2p+1)ab}$	60
norman	$\frac{x\,e^{p\ln\left(b^2x^2+2abx+a^2\right)}}{2p+1}+\frac{a\,e^{p\ln\left(b^2x^2+2abx+a^2\right)}}{b(2p+1)}$	63

input
```
int((b^2*x^2+2*a*b*x+a^2)^p,x,method=_RETURNVERBOSE)
```

output
```
(b*x+a)/b/(2*p+1)*((b*x+a)^2)^p
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \left(a^2+2abx+b^2x^2\right)^p\,dx=\frac{(bx+a)(b^2x^2+2\,abx+a^2)^p}{2\,bp+b}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="fricas")
```

output
```
(b*x + a)*(b^2*x^2 + 2*a*b*x + a^2)^p/(2*b*p + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(29) = 58$.

Time = 0.47 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.94

$$\int \left(a^2+2abx+b^2x^2\right)^p\,dx = \begin{cases} \frac{x}{\sqrt{a^2}} & \text{for } b=0 \wedge p=-\frac{1}{2} \\ x\left(a^2\right)^p & \text{for } b=0 \\ \frac{\left(\frac{a}{b}+x\right)\log\left(\frac{a}{b}+x\right)}{\sqrt{b^2\left(\frac{a}{b}+x\right)^2}} & \text{for } p=-\frac{1}{2} \\ \frac{a\left(a^2+2abx+b^2x^2\right)^p}{2bp+b}+\frac{bx\left(a^2+2abx+b^2x^2\right)^p}{2bp+b} & \text{otherwise} \end{cases}$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**p,x)
```

output
```
Piecewise((x/sqrt(a**2), Eq(b, 0) & Eq(p, -1/2)), (x*(a**2)**p, Eq(b, 0)),
 ((a/b + x)*log(a/b + x)/sqrt(b**2*(a/b + x)**2), Eq(p, -1/2)), (a*(a**2 +
 2*a*b*x + b**2*x**2)**p/(2*b*p + b) + b*x*(a**2 + 2*a*b*x + b**2*x**2)**p
/(2*b*p + b), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \left(a^2+2abx+b^2x^2\right)^p\,dx = \frac{(bx+a)(bx+a)^{2p}}{b(2p+1)}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="maxima")
```

output
```
(b*x + a)*(b*x + a)^(2*p)/(b*(2*p + 1))
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.50

$$\int \left(a^2+2abx+b^2x^2\right)^p\,dx=\frac{\left(b^2x^2+2\,abx+a^2\right)^pbx+\left(b^2x^2+2\,abx+a^2\right)^pa}{2\,bp+b}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="giac")
```

output
```
((b^2*x^2 + 2*a*b*x + a^2)^p*b*x + (b^2*x^2 + 2*a*b*x + a^2)^p*a)/(2*b*p +
 b)
```

Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int \left(a^2+2abx+b^2x^2\right)^p\,dx=\left(\frac{x}{2\,p+1}+\frac{a}{b\,\left(2\,p+1\right)}\right)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^p$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^p,x)
```

output
```
(x/(2*p + 1) + a/(b*(2*p + 1)))*(a^2 + b^2*x^2 + 2*a*b*x)^p
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \left(a^2+2abx+b^2x^2\right)^p\,dx=\frac{\left(b^2x^2+2abx+a^2\right)^p\left(bx+a\right)}{b\left(2p+1\right)}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^p,x)
```

output
```
((a**2 + 2*a*b*x + b**2*x**2)**p*(a + b*x))/(b*(2*p + 1))
```

3.140 $\int \frac{(a^2+2abx+b^2x^2)^p}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{(a^2+2abx+b^2x^2)^p}{x}\,dx = -\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^p \text{Hypergeometric2F1}\left(1,1+2p,2(1+p),1+\frac{bx}{a}\right)}{a(1+2p)}$$

output
```
-(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^p*hypergeom([1, 1+2*p],[2*p+2],1+b*x/a)/a/(
1+2*p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{(a^2+2abx+b^2x^2)^p}{x}\,dx = -\frac{(a+bx)\left((a+bx)^2\right)^p \text{Hypergeometric2F1}\left(1,1+2p,2+2p,1+\frac{bx}{a}\right)}{a(1+2p)}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^p/x,x]
```

output
```
-(((a + b*x)*((a + b*x)^2)^p*Hypergeometric2F1[1, 1 + 2*p, 2 + 2*p, 1 + (b
*x)/a])/(a*(1 + 2*p)))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1102, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^p}{x}\,dx$$

$$\Big\downarrow \text{1102}$$

$$(ab+b^2x)^{-2p}\,(a^2+2abx+b^2x^2)^p\int \frac{(xb^2+ab)^{2p}}{x}dx$$

$$\Big\downarrow \text{75}$$

$$-\frac{(ab+b^2x)\,(a^2+2abx+b^2x^2)^p\,\text{Hypergeometric2F1}\left(1,2p+1,2(p+1),\frac{bx}{a}+1\right)}{ab(2p+1)}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^p/x,x]
```

output
```
-(((a*b + b^2*x)*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometric2F1[1, 1 + 2*p,
 2*(1 + p), 1 + (b*x)/a])/(a*b*(1 + 2*p)))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \frac{(b^2x^2 + 2abx + a^2)^p}{x} dx$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^p/x,x)
```

output
```
int((b^2*x^2+2*a*b*x+a^2)^p/x,x)
```

Fricas [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{x} dx = \int \frac{(b^2x^2 + 2\,abx + a^2)^p}{x} dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^p/x,x, algorithm="fricas")
```

output
```
integral((b^2*x^2 + 2*a*b*x + a^2)^p/x, x)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{x}\,dx = \int \frac{\left((a+bx)^2\right)^p}{x}\,dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**p/x,x)
```

output
```
Integral(((a + b*x)**2)**p/x, x)
```

Maxima [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{x}\,dx = \int \frac{(b^2x^2 + 2\,abx + a^2)^p}{x}\,dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^p/x,x, algorithm="maxima")
```

output
```
integrate((b^2*x^2 + 2*a*b*x + a^2)^p/x, x)
```

Giac [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{x}\,dx = \int \frac{(b^2x^2 + 2\,abx + a^2)^p}{x}\,dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^p/x,x, algorithm="giac")
```

output
```
integrate((b^2*x^2 + 2*a*b*x + a^2)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{x}\,dx = \int \frac{(a^2 + 2\,a\,b\,x + b^2\,x^2)^p}{x}\,dx$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^p/x,x)
```

output
```
int((a^2 + b^2*x^2 + 2*a*b*x)^p/x, x)
```

Reduce [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{x}\,dx = \frac{(b^2x^2 + 2abx + a^2)^p + 2\left(\int \frac{(b^2x^2+2abx+a^2)^p}{b\,x^2+ax}dx\right)ap}{2p}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^p/x,x)
```

output
```
((a**2 + 2*a*b*x + b**2*x**2)**p + 2*int((a**2 + 2*a*b*x + b**2*x**2)**p/(
a*x + b*x**2),x)*a*p)/(2*p)
```

3.141 $\int \frac{(a^2+2abx+b^2x^2)^p}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{(a^2+2abx+b^2x^2)^p}{x^2}\,dx = \frac{b(a+bx)\,(a^2+2abx+b^2x^2)^p \operatorname{Hypergeometric2F1}\left(2,1+2p,2(1+p),1+\frac{bx}{a}\right)}{a^2(1+2p)}$$

output
```
b*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^p*hypergeom([2, 1+2*p],[2*p+2],1+b*x/a)/a^
2/(1+2*p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{(a^2+2abx+b^2x^2)^p}{x^2}\,dx = \frac{b(a+bx)\,((a+bx)^2)^p \operatorname{Hypergeometric2F1}\left(2,1+2p,2+2p,1+\frac{bx}{a}\right)}{a^2(1+2p)}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^p/x^2,x]
```

output
```
(b*(a + b*x)*((a + b*x)^2)^p*Hypergeometric2F1[2, 1 + 2*p, 2 + 2*p, 1 + (b
*x)/a])/(a^2*(1 + 2*p))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1102, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{x^2} dx$$

$$\Big\downarrow \text{ 1102}$$

$$(ab + b^2x)^{-2p} (a^2 + 2abx + b^2x^2)^p \int \frac{(xb^2 + ab)^{2p}}{x^2} dx$$

$$\Big\downarrow \text{ 75}$$

$$\frac{(ab + b^2x) (a^2 + 2abx + b^2x^2)^p \text{Hypergeometric2F1}\left(2, 2p + 1, 2(p + 1), \frac{bx}{a} + 1\right)}{a^2(2p + 1)}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^p/x^2,x]
```

output
```
((a*b + b^2*x)*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometric2F1[2, 1 + 2*p, 2
*(1 + p), 1 + (b*x)/a])/(a^2*(1 + 2*p))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \frac{(b^2x^2 + 2abx + a^2)^p}{x^2} dx$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^p/x^2,x)
```

output
```
int((b^2*x^2+2*a*b*x+a^2)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{x^2} dx = \int \frac{(b^2x^2 + 2\,abx + a^2)^p}{x^2} dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^p/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^2 + 2*a*b*x + a^2)^p/x^2, x)
```

Sympy [F]

$$\int \frac{(a^2+2abx+b^2x^2)^p}{x^2}\,dx = \int \frac{\left((a+bx)^2\right)^p}{x^2}\,dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**p/x**2,x)
```

output
```
Integral(((a + b*x)**2)**p/x**2, x)
```

Maxima [F]

$$\int \frac{(a^2+2abx+b^2x^2)^p}{x^2}\,dx = \int \frac{(b^2x^2+2\,abx+a^2)^p}{x^2}\,dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b^2*x^2 + 2*a*b*x + a^2)^p/x^2, x)
```

Giac [F]

$$\int \frac{(a^2+2abx+b^2x^2)^p}{x^2}\,dx = \int \frac{(b^2x^2+2\,abx+a^2)^p}{x^2}\,dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^p/x^2,x, algorithm="giac")
```

output
```
integrate((b^2*x^2 + 2*a*b*x + a^2)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{x^2}\,dx = \int \frac{(a^2 + 2\,a\,b\,x + b^2\,x^2)^p}{x^2}\,dx$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^p/x^2,x)
```

output
```
int((a^2 + b^2*x^2 + 2*a*b*x)^p/x^2, x)
```

Reduce [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{x^2}\,dx = \frac{-(b^2x^2 + 2abx + a^2)^p + 2\left(\int \frac{(b^2x^2+2abx+a^2)^p}{b\,x^2+ax}dx\right) bpx}{x}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^p/x^2,x)
```

output
```
( - (a**2 + 2*a*b*x + b**2*x**2)**p + 2*int((a**2 + 2*a*b*x + b**2*x**2)**
p/(a*x + b*x**2),x)*b*p*x)/x
```

3.142 $\int \frac{x^4}{2+13x+15x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 40

$$\int \frac{x^4}{2+13x+15x^2}\,dx=\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16}{567}\log(2+3x)+\frac{\log(1+5x)}{4375}$$

output
```
139/3375*x-13/450*x^2+1/45*x^3-16/567*ln(2+3*x)+1/4375*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{2+13x+15x^2}\,dx=\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16}{567}\log(2+3x)+\frac{\log(1+5x)}{4375}$$

input
```
Integrate[x^4/(2 + 13*x + 15*x^2),x]
```

output
```
(139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x
]/4375
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{15x^2+13x+2}dx$$

$$\Big\downarrow \text{1141}$$

$$15\int \left(\frac{x^2}{225}-\frac{13x}{3375}-\frac{16}{2835(3x+2)}+\frac{1}{13125(5x+1)}+\frac{139}{50625}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$15\left(\frac{x^3}{675}-\frac{13x^2}{6750}+\frac{139x}{50625}-\frac{16\log(3x+2)}{8505}+\frac{\log(5x+1)}{65625}\right)$$

input
```
Int[x^4/(2 + 13*x + 15*x^2),x]
```

output
```
15*((139*x)/50625 - (13*x^2)/6750 + x^3/675 - (16*Log[2 + 3*x])/8505 + Log
[1 + 5*x]/65625)
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{x^{3}}{45}-\frac{13 x^{2}}{450}+\frac{139 x}{3375}+\frac{\ln \left(x+\frac{1}{5}\right)}{4375}-\frac{16 \ln \left(x+\frac{2}{3}\right)}{567}$	27
default	$\frac{139 x}{3375}-\frac{13 x^{2}}{450}+\frac{x^{3}}{45}-\frac{16 \ln \left(3 x+2\right)}{567}+\frac{\ln \left(1+5 x\right)}{4375}$	31
norman	$\frac{139 x}{3375}-\frac{13 x^{2}}{450}+\frac{x^{3}}{45}-\frac{16 \ln \left(3 x+2\right)}{567}+\frac{\ln \left(1+5 x\right)}{4375}$	31
risch	$\frac{139 x}{3375}-\frac{13 x^{2}}{450}+\frac{x^{3}}{45}-\frac{16 \ln \left(3 x+2\right)}{567}+\frac{\ln \left(1+5 x\right)}{4375}$	31

input
```
int(x^4/(15*x^2+13*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/45*x^3-13/450*x^2+139/3375*x+1/4375*ln(x+1/5)-16/567*ln(x+2/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^4}{2+13x+15x^2}\,dx=\frac{1}{45}\,x^{3}-\frac{13}{450}\,x^{2}+\frac{139}{3375}\,x+\frac{1}{4375}\,\log\left(5\,x+1\right)-\frac{16}{567}\,\log\left(3\,x+2\right)$$

input
```
integrate(x^4/(15*x^2+13*x+2),x, algorithm="fricas")
```

output
```
1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x
+ 2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{2+13x+15x^2}\,dx = \frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} + \frac{\log\left(x+\frac{1}{5}\right)}{4375} - \frac{16\log\left(x+\frac{2}{3}\right)}{567}$$

input
```
integrate(x**4/(15*x**2+13*x+2),x)
```

output
```
x**3/45 - 13*x**2/450 + 139*x/3375 + log(x + 1/5)/4375 - 16*log(x + 2/3)/5
67
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^4}{2+13x+15x^2}\,dx = \frac{1}{45}\,x^3 - \frac{13}{450}\,x^2 + \frac{139}{3375}\,x + \frac{1}{4375}\,\log(5\,x+1) - \frac{16}{567}\,\log(3\,x+2)$$

input
```
integrate(x^4/(15*x^2+13*x+2),x, algorithm="maxima")
```

output
```
1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x
+ 2)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{x^4}{2+13x+15x^2}\,dx = \frac{1}{45}\,x^3 - \frac{13}{450}\,x^2 + \frac{139}{3375}\,x + \frac{1}{4375}\,\log(|5\,x+1|) - \frac{16}{567}\,\log(|3\,x+2|)$$

input
```
integrate(x^4/(15*x^2+13*x+2),x, algorithm="giac")
```

output
```
1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(abs(5*x + 1)) - 16/567*log
(abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 8.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

$$\int \frac{x^4}{2+13x+15x^2}\,dx = \frac{139\,x}{3375} - \frac{16\,\ln\left(x+\frac{2}{3}\right)}{567} + \frac{\ln\left(x+\frac{1}{5}\right)}{4375} - \frac{13\,x^2}{450} + \frac{x^3}{45}$$

input
```
int(x^4/(13*x + 15*x^2 + 2),x)
```

output
```
(139*x)/3375 - (16*log(x + 2/3))/567 + log(x + 1/5)/4375 - (13*x^2)/450 +
x^3/45
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^4}{2+13x+15x^2}\,dx = \frac{\log(5x+1)}{4375} - \frac{16\log(3x+2)}{567} + \frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375}$$

input
```
int(x^4/(15*x^2+13*x+2),x)
```

output
```
(162*log(5*x + 1) - 20000*log(3*x + 2) + 15750*x**3 - 20475*x**2 + 29190*x
)/708750
```

3.143 $\int \frac{x^3}{2+13x+15x^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{x^3}{2+13x+15x^2} dx = -\frac{13x}{225} + \frac{x^2}{30} + \frac{8}{189}\log(2+3x) - \frac{1}{875}\log(1+5x)$$

output
```
-13/225*x+1/30*x^2+8/189*ln(2+3*x)-1/875*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{2+13x+15x^2} dx = -\frac{13x}{225} + \frac{x^2}{30} + \frac{8}{189}\log(2+3x) - \frac{1}{875}\log(1+5x)$$

input
```
Integrate[x^3/(2 + 13*x + 15*x^2),x]
```

output
```
(-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{15x^2+13x+2}\,dx$$
$$\Big\downarrow \text{1141}$$
$$15\int \left(\frac{x}{225}+\frac{8}{945(3x+2)}-\frac{1}{2625(5x+1)}-\frac{13}{3375}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$15\left(\frac{x^2}{450}-\frac{13x}{3375}+\frac{8\log(3x+2)}{2835}-\frac{\log(5x+1)}{13125}\right)$$

input
```
Int[x^3/(2 + 13*x + 15*x^2),x]
```

output
```
15*((-13*x)/3375 + x^2/450 + (8*Log[2 + 3*x])/2835 - Log[1 + 5*x]/13125)
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{x^2}{30}-\frac{13x}{225}-\frac{\ln\left(x+\frac{1}{5}\right)}{875}+\frac{8\ln\left(x+\frac{2}{3}\right)}{189}$	22
default	$-\frac{13x}{225}+\frac{x^2}{30}+\frac{8\ln(3x+2)}{189}-\frac{\ln(1+5x)}{875}$	26
norman	$-\frac{13x}{225}+\frac{x^2}{30}+\frac{8\ln(3x+2)}{189}-\frac{\ln(1+5x)}{875}$	26
risch	$-\frac{13x}{225}+\frac{x^2}{30}+\frac{8\ln(3x+2)}{189}-\frac{\ln(1+5x)}{875}$	26

input
```
int(x^3/(15*x^2+13*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/30*x^2-13/225*x-1/875*ln(x+1/5)+8/189*ln(x+2/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{2+13x+15x^2}\,dx=\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(5\,x+1\right)+\frac{8}{189}\,\log\left(3\,x+2\right)$$

input
```
integrate(x^3/(15*x^2+13*x+2),x, algorithm="fricas")
```

output
```
1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{2+13x+15x^2}\,dx=\frac{x^2}{30}-\frac{13x}{225}-\frac{\log\left(x+\frac{1}{5}\right)}{875}+\frac{8\log\left(x+\frac{2}{3}\right)}{189}$$

input
```
integrate(x**3/(15*x**2+13*x+2),x)
```

output
```
x**2/30 - 13*x/225 - log(x + 1/5)/875 + 8*log(x + 2/3)/189
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{2+13x+15x^2}\,dx = \frac{1}{30}\,x^2 - \frac{13}{225}\,x - \frac{1}{875}\,\log\left(5\,x+1\right) + \frac{8}{189}\,\log\left(3\,x+2\right)$$

input
```
integrate(x^3/(15*x^2+13*x+2),x, algorithm="maxima")
```

output
```
1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{2+13x+15x^2}\,dx = \frac{1}{30}\,x^2 - \frac{13}{225}\,x - \frac{1}{875}\,\log\left(|5\,x+1|\right) + \frac{8}{189}\,\log\left(|3\,x+2|\right)$$

input
```
integrate(x^3/(15*x^2+13*x+2),x, algorithm="giac")
```

output
```
1/30*x^2 - 13/225*x - 1/875*log(abs(5*x + 1)) + 8/189*log(abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.64

$$\int \frac{x^3}{2+13x+15x^2}\,dx = \frac{8\,\ln\left(x+\frac{2}{3}\right)}{189} - \frac{13\,x}{225} - \frac{\ln\left(x+\frac{1}{5}\right)}{875} + \frac{x^2}{30}$$

input
```
int(x^3/(13*x + 15*x^2 + 2),x)
```

output
```
(8*log(x + 2/3))/189 - (13*x)/225 - log(x + 1/5)/875 + x^2/30
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{2+13x+15x^2}\,dx = -\frac{\log(5x+1)}{875} + \frac{8\log(3x+2)}{189} + \frac{x^2}{30} - \frac{13x}{225}$$

input
```
int(x^3/(15*x^2+13*x+2),x)
```

output
```
( - 54*log(5*x + 1) + 2000*log(3*x + 2) + 1575*x**2 - 2730*x)/47250
```

3.144 $\int \frac{x^2}{2+13x+15x^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 26

$$\int \frac{x^2}{2+13x+15x^2} \, dx = \frac{x}{15} - \frac{4}{63}\log(2+3x) + \frac{1}{175}\log(1+5x)$$

output
```
1/15*x-4/63*ln(2+3*x)+1/175*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{2+13x+15x^2} \, dx = \frac{x}{15} - \frac{4}{63}\log(2+3x) + \frac{1}{175}\log(1+5x)$$

input
```
Integrate[x^2/(2 + 13*x + 15*x^2),x]
```

output
```
x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{15x^2+13x+2}\,dx$$

$$\Big\downarrow \text{1141}$$

$$15\int \left(\frac{1}{525(5x+1)}+\frac{1}{225}-\frac{4}{315(3x+2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$15\left(\frac{x}{225}-\frac{4}{945}\log(3x+2)+\frac{\log(5x+1)}{2625}\right)$$

input
```
Int[x^2/(2 + 13*x + 15*x^2),x]
```

output
```
15*(x/225 - (4*Log[2 + 3*x])/945 + Log[1 + 5*x]/2625)
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$\frac{x}{15}+\frac{\ln\left(x+\frac{1}{5}\right)}{175}-\frac{4\ln\left(x+\frac{2}{3}\right)}{63}$	17
default	$\frac{x}{15}-\frac{4\ln(3x+2)}{63}+\frac{\ln(1+5x)}{175}$	21
norman	$\frac{x}{15}-\frac{4\ln(3x+2)}{63}+\frac{\ln(1+5x)}{175}$	21
risch	$\frac{x}{15}-\frac{4\ln(3x+2)}{63}+\frac{\ln(1+5x)}{175}$	21

input
```
int(x^2/(15*x^2+13*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x+1/175*ln(x+1/5)-4/63*ln(x+2/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x^2}{2+13x+15x^2}\,dx=\frac{1}{15}\,x+\frac{1}{175}\,\log\left(5\,x+1\right)-\frac{4}{63}\,\log\left(3\,x+2\right)$$

input
```
integrate(x^2/(15*x^2+13*x+2),x, algorithm="fricas")
```

output
```
1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x^2}{2+13x+15x^2}\,dx=\frac{x}{15}+\frac{\log\left(x+\frac{1}{5}\right)}{175}-\frac{4\log\left(x+\frac{2}{3}\right)}{63}$$

input
```
integrate(x**2/(15*x**2+13*x+2),x)
```

output
```
x/15 + log(x + 1/5)/175 - 4*log(x + 2/3)/63
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x^2}{2+13x+15x^2}\,dx = \frac{1}{15}\,x + \frac{1}{175}\,\log\left(5\,x+1\right) - \frac{4}{63}\,\log\left(3\,x+2\right)$$

input
```
integrate(x^2/(15*x^2+13*x+2),x, algorithm="maxima")
```

output
```
1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{2+13x+15x^2}\,dx = \frac{1}{15}\,x + \frac{1}{175}\,\log\left(|5\,x+1|\right) - \frac{4}{63}\,\log\left(|3\,x+2|\right)$$

input
```
integrate(x^2/(15*x^2+13*x+2),x, algorithm="giac")
```

output
```
1/15*x + 1/175*log(abs(5*x + 1)) - 4/63*log(abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \frac{x^2}{2+13x+15x^2}\,dx = \frac{x}{15} - \frac{4\,\ln\left(x+\frac{2}{3}\right)}{63} + \frac{\ln\left(x+\frac{1}{5}\right)}{175}$$

input
```
int(x^2/(13*x + 15*x^2 + 2),x)
```

output
```
x/15 - (4*log(x + 2/3))/63 + log(x + 1/5)/175
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x^2}{2+13x+15x^2}\,dx = \frac{\log(5x+1)}{175} - \frac{4\log(3x+2)}{63} + \frac{x}{15}$$

input
```
int(x^2/(15*x^2+13*x+2),x)
```

output
```
(9*log(5*x + 1) - 100*log(3*x + 2) + 105*x)/1575
```

3.145 $\int \frac{x}{2+13x+15x^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 21

$$\int \frac{x}{2+13x+15x^2} \, dx = \frac{2}{21}\log(2+3x) - \frac{1}{35}\log(1+5x)$$

output
```
2/21*ln(2+3*x)-1/35*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x}{2+13x+15x^2} \, dx = \frac{2}{21}\log(2+3x) - \frac{1}{35}\log(1+5x)$$

input
```
Integrate[x/(2 + 13*x + 15*x^2),x]
```

output
```
(2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{15x^2+13x+2}\,dx$$

$$\Big\downarrow \text{1141}$$

$$15\int \left(\frac{2}{105(3x+2)}-\frac{1}{105(5x+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$15\left(\frac{2}{315}\log(3x+2)-\frac{1}{525}\log(5x+1)\right)$$

input
```
Int[x/(2 + 13*x + 15*x^2),x]
```

output
```
15*((2*Log[2 + 3*x])/315 - Log[1 + 5*x]/525)
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$-\frac{\ln\left(x+\frac{1}{5}\right)}{35}+\frac{2\ln\left(x+\frac{2}{3}\right)}{21}$	14
default	$\frac{2\ln(3x+2)}{21}-\frac{\ln(1+5x)}{35}$	18
norman	$\frac{2\ln(3x+2)}{21}-\frac{\ln(1+5x)}{35}$	18
risch	$\frac{2\ln(3x+2)}{21}-\frac{\ln(1+5x)}{35}$	18

input
```
int(x/(15*x^2+13*x+2),x,method=_RETURNVERBOSE)
```

output
```
-1/35*ln(x+1/5)+2/21*ln(x+2/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x}{2+13x+15x^2}\,dx=-\frac{1}{35}\,\log\left(5\,x+1\right)+\frac{2}{21}\,\log\left(3\,x+2\right)$$

input
```
integrate(x/(15*x^2+13*x+2),x, algorithm="fricas")
```

output
```
-1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x}{2+13x+15x^2}\,dx=-\frac{\log\left(x+\frac{1}{5}\right)}{35}+\frac{2\log\left(x+\frac{2}{3}\right)}{21}$$

input
```
integrate(x/(15*x**2+13*x+2),x)
```

output
```
-log(x + 1/5)/35 + 2*log(x + 2/3)/21
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x}{2+13x+15x^2} \, dx = -\frac{1}{35} \log(5x+1) + \frac{2}{21} \log(3x+2)$$

input
```
integrate(x/(15*x^2+13*x+2),x, algorithm="maxima")
```

output
```
-1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{x}{2+13x+15x^2} \, dx = -\frac{1}{35} \log(|5x+1|) + \frac{2}{21} \log(|3x+2|)$$

input
```
integrate(x/(15*x^2+13*x+2),x, algorithm="giac")
```

output
```
-1/35*log(abs(5*x + 1)) + 2/21*log(abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{x}{2+13x+15x^2} \, dx = \frac{2 \ln\left(x+\frac{2}{3}\right)}{21} - \frac{\ln\left(x+\frac{1}{5}\right)}{35}$$

input
```
int(x/(13*x + 15*x^2 + 2),x)
```

output
```
(2*log(x + 2/3))/21 - log(x + 1/5)/35
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x}{2+13x+15x^2} \, dx = -\frac{\log(5x+1)}{35} + \frac{2\log(3x+2)}{21}$$

input
```
int(x/(15*x^2+13*x+2),x)
```

output
```
( - 3*log(5*x + 1) + 10*log(3*x + 2))/105
```

3.146 $\int \frac{1}{2+13x+15x^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 21

$$\int \frac{1}{2+13x+15x^2} dx = -\frac{1}{7}\log(2+3x) + \frac{1}{7}\log(1+5x)$$

output
```
-1/7*ln(2+3*x)+1/7*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+13x+15x^2} dx = -\frac{1}{7}\log(2+3x) + \frac{1}{7}\log(1+5x)$$

input
```
Integrate[(2 + 13*x + 15*x^2)^(-1),x]
```

output
```
-1/7*Log[2 + 3*x] + Log[1 + 5*x]/7
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{15x^2+13x+2}dx$$

$$\Big\downarrow \text{1081}$$

$$15\int \left(\frac{1}{21(5x+1)}-\frac{1}{35(3x+2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$15\left(\frac{1}{105}\log(5x+1)-\frac{1}{105}\log(3x+2)\right)$$

input
```
Int[(2 + 13*x + 15*x^2)^(-1),x]
```

output
```
15*(-1/105*Log[2 + 3*x] + Log[1 + 5*x]/105)
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{\ln\left(x+\frac{1}{5}\right)}{7}-\frac{\ln\left(x+\frac{2}{3}\right)}{7}$	14
default	$-\frac{\ln(3x+2)}{7}+\frac{\ln(1+5x)}{7}$	18
norman	$-\frac{\ln(3x+2)}{7}+\frac{\ln(1+5x)}{7}$	18
risch	$-\frac{\ln(3x+2)}{7}+\frac{\ln(1+5x)}{7}$	18

input
```
int(1/(15*x^2+13*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/7*ln(x+1/5)-1/7*ln(x+2/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{2+13x+15x^2}\,dx=\frac{1}{7}\log\left(5\,x+1\right)-\frac{1}{7}\log\left(3\,x+2\right)$$

input
```
integrate(1/(15*x^2+13*x+2),x, algorithm="fricas")
```

output
```
1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{2+13x+15x^2}\,dx=\frac{\log\left(x+\frac{1}{5}\right)}{7}-\frac{\log\left(x+\frac{2}{3}\right)}{7}$$

input
```
integrate(1/(15*x**2+13*x+2),x)
```

output
```
log(x + 1/5)/7 - log(x + 2/3)/7
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{2+13x+15x^2}\,dx = \frac{1}{7}\log\left(5\,x+1\right) - \frac{1}{7}\log\left(3\,x+2\right)$$

input
```
integrate(1/(15*x^2+13*x+2),x, algorithm="maxima")
```

output
```
1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{2+13x+15x^2}\,dx = \frac{1}{7}\log\left(|5\,x+1|\right) - \frac{1}{7}\log\left(|3\,x+2|\right)$$

input
```
integrate(1/(15*x^2+13*x+2),x, algorithm="giac")
```

output
```
1/7*log(abs(5*x + 1)) - 1/7*log(abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.38

$$\int \frac{1}{2+13x+15x^2}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{30\,x}{7}+\frac{13}{7}\right)}{7}$$

input
```
int(1/(13*x + 15*x^2 + 2),x)
```

output
```
-(2*atanh((30*x)/7 + 13/7))/7
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{2+13x+15x^2}\,dx = \frac{\log(5x+1)}{7} - \frac{\log(3x+2)}{7}$$

input
```
int(1/(15*x^2+13*x+2),x)
```

output
```
(log(5*x + 1) - log(3*x + 2))/7
```

3.147 $\int \frac{1}{x(2+13x+15x^2)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 27

$$\int \frac{1}{x(2+13x+15x^2)}\,dx = \frac{\log(x)}{2} + \frac{3}{14}\log(2+3x) - \frac{5}{7}\log(1+5x)$$

output
```
1/2*ln(x)+3/14*ln(2+3*x)-5/7*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(2+13x+15x^2)}\,dx = \frac{\log(x)}{2} + \frac{3}{14}\log(2+3x) - \frac{5}{7}\log(1+5x)$$

input
```
Integrate[1/(x*(2 + 13*x + 15*x^2)),x]
```

output
```
Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(15x^2+13x+2)}dx$$
$$\Big\downarrow \text{1141}$$
$$15\int \left(\frac{3}{70(3x+2)}-\frac{5}{21(5x+1)}+\frac{1}{30x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$15\left(\frac{\log(x)}{30}+\frac{1}{70}\log(3x+2)-\frac{1}{21}\log(5x+1)\right)$$

input
```
Int[1/(x*(2 + 13*x + 15*x^2)),x]
```

output
```
15*(Log[x]/30 + Log[2 + 3*x]/70 - Log[1 + 5*x]/21)
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{\ln(x)}{2}-\frac{5\ln\left(x+\frac{1}{5}\right)}{7}+\frac{3\ln\left(x+\frac{2}{3}\right)}{14}$	18
default	$\frac{\ln(x)}{2}+\frac{3\ln(3x+2)}{14}-\frac{5\ln(1+5x)}{7}$	22
norman	$\frac{\ln(x)}{2}+\frac{3\ln(3x+2)}{14}-\frac{5\ln(1+5x)}{7}$	22
risch	$\frac{\ln(x)}{2}+\frac{3\ln(3x+2)}{14}-\frac{5\ln(1+5x)}{7}$	22

input
```
int(1/x/(15*x^2+13*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x)-5/7*ln(x+1/5)+3/14*ln(x+2/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{x\left(2+13x+15x^2\right)}\,dx=-\frac{5}{7}\log\left(5x+1\right)+\frac{3}{14}\log\left(3x+2\right)+\frac{1}{2}\log\left(x\right)$$

input
```
integrate(1/x/(15*x^2+13*x+2),x, algorithm="fricas")
```

output
```
-5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(2+13x+15x^2\right)}\,dx=\frac{\log\left(x\right)}{2}-\frac{5\log\left(x+\frac{1}{5}\right)}{7}+\frac{3\log\left(x+\frac{2}{3}\right)}{14}$$

input
```
integrate(1/x/(15*x**2+13*x+2),x)
```

output
```
log(x)/2 - 5*log(x + 1/5)/7 + 3*log(x + 2/3)/14
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{x (2+13x+15x^2)} \, dx = -\frac{5}{7} \log(5x+1) + \frac{3}{14} \log(3x+2) + \frac{1}{2} \log(x)$$

input
```
integrate(1/x/(15*x^2+13*x+2),x, algorithm="maxima")
```

output
```
-5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{x (2+13x+15x^2)} \, dx = -\frac{5}{7} \log(|5x+1|) + \frac{3}{14} \log(|3x+2|) + \frac{1}{2} \log(|x|)$$

input
```
integrate(1/x/(15*x^2+13*x+2),x, algorithm="giac")
```

output
```
-5/7*log(abs(5*x + 1)) + 3/14*log(abs(3*x + 2)) + 1/2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 8.84 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{x (2+13x+15x^2)} \, dx = \frac{3 \ln\left(x+\frac{2}{3}\right)}{14} - \frac{5 \ln\left(x+\frac{1}{5}\right)}{7} + \frac{\ln(x)}{2}$$

input
```
int(1/(x*(13*x + 15*x^2 + 2)),x)
```

output
```
(3*log(x + 2/3))/14 - (5*log(x + 1/5))/7 + log(x)/2
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{x\,(2+13x+15x^2)}\,dx = -\frac{5\log(5x+1)}{7}+\frac{3\log(3x+2)}{14}+\frac{\log(x)}{2}$$

input
```
int(1/x/(15*x^2+13*x+2),x)
```

output
```
( - 10*log(5*x + 1) + 3*log(3*x + 2) + 7*log(x))/14
```

3.148 $\int \frac{1}{x^2(2+13x+15x^2)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 34

$$\int \frac{1}{x^2\left(2+13x+15x^2\right)}\,dx=-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)$$

output
```
-1/2/x-13/4*ln(x)-9/28*ln(2+3*x)+25/7*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(2+13x+15x^2\right)}\,dx=-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)$$

input
```
Integrate[1/(x^2*(2 + 13*x + 15*x^2)),x]
```

output
```
-1/2*1/x - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (15x^2+13x+2)} dx$$

$$\Big\downarrow \text{1141}$$

$$15 \int \left(-\frac{9}{140(3x+2)}+\frac{25}{21(5x+1)}-\frac{13}{60x}+\frac{1}{30x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$15\left(-\frac{1}{30x}-\frac{13\log(x)}{60}-\frac{3}{140}\log(3x+2)+\frac{5}{21}\log(5x+1)\right)$$

input
```
Int[1/(x^2*(2 + 13*x + 15*x^2)),x]
```

output
```
15*(-1/30*1/x - (13*Log[x])/60 - (3*Log[2 + 3*x])/140 + (5*Log[1 + 5*x])/2
1)
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(3x+2)}{28}+\frac{25\ln(1+5x)}{7}$	27
norman	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(3x+2)}{28}+\frac{25\ln(1+5x)}{7}$	27
risch	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(3x+2)}{28}+\frac{25\ln(1+5x)}{7}$	27
parallelrisch	$-\frac{91\ln(x)x-100\ln\left(x+\frac{1}{5}\right)x+9\ln\left(x+\frac{2}{3}\right)x+14}{28x}$	27

input
```
int(1/x^2/(15*x^2+13*x+2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x-13/4*ln(x)-9/28*ln(3*x+2)+25/7*ln(1+5*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\left(2+13x+15x^2\right)}\,dx=\frac{100\,x\log\left(5\,x+1\right)-9\,x\log\left(3\,x+2\right)-91\,x\log\left(x\right)-14}{28\,x}$$

input
```
integrate(1/x^2/(15*x^2+13*x+2),x, algorithm="fricas")
```

output
```
1/28*(100*x*log(5*x + 1) - 9*x*log(3*x + 2) - 91*x*log(x) - 14)/x
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2\left(2+13x+15x^2\right)}\,dx=-\frac{13\log\left(x\right)}{4}+\frac{25\log\left(x+\frac{1}{5}\right)}{7}-\frac{9\log\left(x+\frac{2}{3}\right)}{28}-\frac{1}{2x}$$

input
```
integrate(1/x**2/(15*x**2+13*x+2),x)
```

output
```
-13*log(x)/4 + 25*log(x + 1/5)/7 - 9*log(x + 2/3)/28 - 1/(2*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^2\left(2+13x+15x^2\right)}\,dx=-\frac{1}{2\,x}+\frac{25}{7}\,\log\left(5\,x+1\right)-\frac{9}{28}\,\log\left(3\,x+2\right)-\frac{13}{4}\,\log\left(x\right)$$

input
```
integrate(1/x^2/(15*x^2+13*x+2),x, algorithm="maxima")
```

output
```
-1/2/x + 25/7*log(5*x + 1) - 9/28*log(3*x + 2) - 13/4*log(x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2\left(2+13x+15x^2\right)}\,dx=-\frac{1}{2\,x}+\frac{25}{7}\,\log\left(\left|5\,x+1\right|\right)-\frac{9}{28}\,\log\left(\left|3\,x+2\right|\right)-\frac{13}{4}\,\log\left(\left|x\right|\right)$$

input
```
integrate(1/x^2/(15*x^2+13*x+2),x, algorithm="giac")
```

output
```
-1/2/x + 25/7*log(abs(5*x + 1)) - 9/28*log(abs(3*x + 2)) - 13/4*log(abs(x)
)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^2\left(2+13x+15x^2\right)}\,dx=\frac{25\,\ln\left(x+\frac{1}{5}\right)}{7}-\frac{9\,\ln\left(x+\frac{2}{3}\right)}{28}-\frac{13\,\ln\left(x\right)}{4}-\frac{1}{2\,x}$$

input
```
int(1/(x^2*(13*x + 15*x^2 + 2)),x)
```

output
```
(25*log(x + 1/5))/7 - (9*log(x + 2/3))/28 - (13*log(x))/4 - 1/(2*x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2 (2+13x+15x^2)}\,dx = \frac{100\log(5x+1)\,x-9\log(3x+2)\,x-91\log(x)\,x-14}{28x}$$

input
```
int(1/x^2/(15*x^2+13*x+2),x)
```

output
```
(100*log(5*x + 1)*x - 9*log(3*x + 2)*x - 91*log(x)*x - 14)/(28*x)
```

3.149 $\int \frac{1}{x^3(2+13x+15x^2)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 41

$$\int \frac{1}{x^3\left(2+13x+15x^2\right)}\,dx=-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)$$

output
```
-1/4/x^2+13/4/x+139/8*ln(x)+27/56*ln(2+3*x)-125/7*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\left(2+13x+15x^2\right)}\,dx=-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)$$

input
```
Integrate[1/(x^3*(2 + 13*x + 15*x^2)),x]
```

output
```
-1/4*1/x^2 + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
 + 5*x])/7
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (15x^2 + 13x + 2)} \, dx$$

$$\Big\downarrow \text{1141}$$

$$15 \int \left(\frac{27}{280(3x+2)} - \frac{125}{21(5x+1)} + \frac{139}{120x} - \frac{13}{60x^2} + \frac{1}{30x^3} \right) dx$$

$$\Big\downarrow \text{2009}$$

$$15 \left(-\frac{1}{60x^2} + \frac{13}{60x} + \frac{139 \log(x)}{120} + \frac{9}{280} \log(3x+2) - \frac{25}{21} \log(5x+1) \right)$$

input
```
Int[1/(x^3*(2 + 13*x + 15*x^2)),x]
```

output
```
15*(-1/60*1/x^2 + 13/(60*x) + (139*Log[x])/120 + (9*Log[2 + 3*x])/280 - (2
5*Log[1 + 5*x])/21)
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

method	result	size
norman	$\frac{-\frac{1}{4}+\frac{13x}{4}}{x^2}+\frac{139\ln(x)}{8}+\frac{27\ln(3x+2)}{56}-\frac{125\ln(1+5x)}{7}$	31
risch	$\frac{-\frac{1}{4}+\frac{13x}{4}}{x^2}+\frac{139\ln(x)}{8}+\frac{27\ln(3x+2)}{56}-\frac{125\ln(1+5x)}{7}$	31
default	$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\ln(x)}{8}+\frac{27\ln(3x+2)}{56}-\frac{125\ln(1+5x)}{7}$	32
parallelrisch	$\frac{973\ln(x)x^2-1000\ln\left(x+\frac{1}{5}\right)x^2+27\ln\left(x+\frac{2}{3}\right)x^2-14+182x}{56x^2}$	36

input
```
int(1/x^3/(15*x^2+13*x+2),x,method=_RETURNVERBOSE)
```

output
```
(-1/4+13/4*x)/x^2+139/8*ln(x)+27/56*ln(3*x+2)-125/7*ln(1+5*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3(2+13x+15x^2)}\,dx = -\frac{1000\,x^2\log(5\,x+1)-27\,x^2\log(3\,x+2)-973\,x^2\log(x)-182\,x+14}{56\,x^2}$$

input
```
integrate(1/x^3/(15*x^2+13*x+2),x, algorithm="fricas")
```

output
```
-1/56*(1000*x^2*log(5*x + 1) - 27*x^2*log(3*x + 2) - 973*x^2*log(x) - 182*
x + 14)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3 (2+13x+15x^2)} \, dx = \frac{139 \log(x)}{8} - \frac{125 \log\left(x+\frac{1}{5}\right)}{7} + \frac{27 \log\left(x+\frac{2}{3}\right)}{56} + \frac{13x-1}{4x^2}$$

input
```
integrate(1/x**3/(15*x**2+13*x+2),x)
```

output
```
139*log(x)/8 - 125*log(x + 1/5)/7 + 27*log(x + 2/3)/56 + (13*x - 1)/(4*x**
2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^3 (2+13x+15x^2)} \, dx = \frac{13\,x-1}{4\,x^2} - \frac{125}{7} \log(5\,x+1) + \frac{27}{56} \log(3\,x+2) + \frac{139}{8} \log(x)$$

input
```
integrate(1/x^3/(15*x^2+13*x+2),x, algorithm="maxima")
```

output
```
1/4*(13*x - 1)/x^2 - 125/7*log(5*x + 1) + 27/56*log(3*x + 2) + 139/8*log(x
)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3 (2+13x+15x^2)} \, dx = \frac{13\,x-1}{4\,x^2} - \frac{125}{7} \log(|5\,x+1|) + \frac{27}{56} \log(|3\,x+2|) + \frac{139}{8} \log(|x|)$$

input
```
integrate(1/x^3/(15*x^2+13*x+2),x, algorithm="giac")
```

output
```
1/4*(13*x - 1)/x^2 - 125/7*log(abs(5*x + 1)) + 27/56*log(abs(3*x + 2)) + 1
39/8*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^3 (2 + 13x + 15x^2)} \, dx = \frac{27 \ln \left(x + \frac{2}{3}\right)}{56} - \frac{125 \ln \left(x + \frac{1}{5}\right)}{7} + \frac{139 \ln (x)}{8} + \frac{\frac{13\,x}{4} - \frac{1}{4}}{x^2}$$

input
```
int(1/(x^3*(13*x + 15*x^2 + 2)),x)
```

output
```
(27*log(x + 2/3))/56 - (125*log(x + 1/5))/7 + (139*log(x))/8 + ((13*x)/4 -
 1/4)/x^2
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3 (2 + 13x + 15x^2)} \, dx = \frac{-1000 \log(5x+1)\, x^2 + 27 \log(3x+2)\, x^2 + 973 \log(x)\, x^2 + 182x - 14}{56x^2}$$

input
```
int(1/x^3/(15*x^2+13*x+2),x)
```

output
```
( - 1000*log(5*x + 1)*x**2 + 27*log(3*x + 2)*x**2 + 973*log(x)*x**2 + 182*
x - 14)/(56*x**2)
```

3.150 $\int \frac{1}{x^4(2+13x+15x^2)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 48

$$\int \frac{1}{x^4(2+13x+15x^2)}\,dx = -\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16} -\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)$$

output
```
-1/6/x^3+13/8/x^2-139/8/x-1417/16*ln(x)-81/112*ln(2+3*x)+625/7*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4(2+13x+15x^2)}\,dx = -\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16} -\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)$$

input
```
Integrate[1/(x^4*(2 + 13*x + 15*x^2)),x]
```

output
```
-1/6*1/x^3 + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])
/112 + (625*Log[1 + 5*x])/7
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (15x^2 + 13x + 2)} \, dx$$

$$\Big\downarrow \text{1141}$$

$$15 \int \left(-\frac{81}{560(3x+2)} + \frac{625}{21(5x+1)} - \frac{1417}{240x} + \frac{139}{120x^2} - \frac{13}{60x^3} + \frac{1}{30x^4}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$15\left(-\frac{1}{90x^3} + \frac{13}{120x^2} - \frac{139}{120x} - \frac{1417\log(x)}{240} - \frac{27}{560}\log(3x+2) + \frac{125}{21}\log(5x+1)\right)$$

input
```
Int[1/(x^4*(2 + 13*x + 15*x^2)),x]
```

output
```
15*(-1/90*1/x^3 + 13/(120*x^2) - 139/(120*x) - (1417*Log[x])/240 - (27*Log
[2 + 3*x])/560 + (125*Log[1 + 5*x])/21)
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

method	result	size
norman	$\frac{-\frac{1}{6}+\frac{13}{8}x-\frac{139}{8}x^2}{x^3}-\frac{1417\ln(x)}{16}-\frac{81\ln(3x+2)}{112}+\frac{625\ln(1+5x)}{7}$	36
risch	$\frac{-\frac{1}{6}+\frac{13}{8}x-\frac{139}{8}x^2}{x^3}-\frac{1417\ln(x)}{16}-\frac{81\ln(3x+2)}{112}+\frac{625\ln(1+5x)}{7}$	36
default	$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\ln(x)}{16}-\frac{81\ln(3x+2)}{112}+\frac{625\ln(1+5x)}{7}$	37
parallelrisch	$-\frac{29757\ln(x)x^3-30000\ln\left(x+\frac{1}{5}\right)x^3+243\ln\left(x+\frac{2}{3}\right)x^3+56+5838x^2-546x}{336x^3}$	41

input
```
int(1/x^4/(15*x^2+13*x+2),x,method=_RETURNVERBOSE)
```

output
```
(-1/6+13/8*x-139/8*x^2)/x^3-1417/16*ln(x)-81/112*ln(3*x+2)+625/7*ln(1+5*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4(2+13x+15x^2)}\,dx = \frac{30000\,x^3\log(5\,x+1)-243\,x^3\log(3\,x+2)-29757\,x^3\log(x)-5838\,x^2+546\,x-56}{336\,x^3}$$

input
```
integrate(1/x^4/(15*x^2+13*x+2),x, algorithm="fricas")
```

output
```
1/336*(30000*x^3*log(5*x + 1) - 243*x^3*log(3*x + 2) - 29757*x^3*log(x) -
5838*x^2 + 546*x - 56)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4 (2+13x+15x^2)} \, dx = -\frac{1417 \log (x)}{16} + \frac{625 \log \left(x+\frac{1}{5}\right)}{7} - \frac{81 \log \left(x+\frac{2}{3}\right)}{112} + \frac{-417x^2+39x-4}{24x^3}$$

input
```
integrate(1/x**4/(15*x**2+13*x+2),x)
```

output
```
-1417*log(x)/16 + 625*log(x + 1/5)/7 - 81*log(x + 2/3)/112 + (-417*x**2 +
39*x - 4)/(24*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^4 (2+13x+15x^2)} \, dx = -\frac{417 \, x^2 - 39 \, x + 4}{24 \, x^3} + \frac{625}{7} \, \log (5 \, x + 1) - \frac{81}{112} \, \log (3 \, x + 2) - \frac{1417}{16} \, \log (x)$$

input
```
integrate(1/x^4/(15*x^2+13*x+2),x, algorithm="maxima")
```

output
```
-1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(5*x + 1) - 81/112*log(3*x + 2)
- 1417/16*log(x)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^4 (2+13x+15x^2)}\,dx = -\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\log\left(|5\,x+1|\right) - \frac{81}{112}\log\left(|3\,x+2|\right)-\frac{1417}{16}\log\left(|x|\right)$$

input
```
integrate(1/x^4/(15*x^2+13*x+2),x, algorithm="giac")
```

output
```
-1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(abs(5*x + 1)) - 81/112*log(abs(
3*x + 2)) - 1417/16*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^4 (2+13x+15x^2)}\,dx = \frac{625\,\ln\left(x+\frac{1}{5}\right)}{7}-\frac{81\,\ln\left(x+\frac{2}{3}\right)}{112} - \frac{1417\,\ln\left(x\right)}{16}-\frac{\frac{139\,x^2}{8}-\frac{13\,x}{8}+\frac{1}{6}}{x^3}$$

input
```
int(1/(x^4*(13*x + 15*x^2 + 2)),x)
```

output
```
(625*log(x + 1/5))/7 - (81*log(x + 2/3))/112 - (1417*log(x))/16 - ((139*x^
2)/8 - (13*x)/8 + 1/6)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4 (2 + 13x + 15x^2)} dx = \frac{30000 \log(5x + 1) x^3 - 243 \log(3x + 2) x^3 - 29757 \log(x) x^3 - 5838x^2 + 546x - 56}{336x^3}$$

input
```
int(1/x^4/(15*x^2+13*x+2),x)
```

output
```
(30000*log(5*x + 1)*x**3 - 243*log(3*x + 2)*x**3 - 29757*log(x)*x**3 - 583
8*x**2 + 546*x - 56)/(336*x**3)
```

3.151 $\int \frac{x^5}{2x+13x^2+15x^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 40

$$\int \frac{x^5}{2x+13x^2+15x^3} \, dx = \frac{139x}{3375} - \frac{13x^2}{450} + \frac{x^3}{45} - \frac{16}{567}\log(2+3x) + \frac{\log(1+5x)}{4375}$$

output
```
139/3375*x-13/450*x^2+1/45*x^3-16/567*ln(2+3*x)+1/4375*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{2x+13x^2+15x^3} \, dx = \frac{139x}{3375} - \frac{13x^2}{450} + \frac{x^3}{45} - \frac{16}{567}\log(2+3x) + \frac{\log(1+5x)}{4375}$$

input
```
Integrate[x^5/(2*x + 13*x^2 + 15*x^3),x]
```

output
```
(139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x
]/4375
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{15x^3+13x^2+2x}dx$$
$$\Big\downarrow 9$$
$$\int \frac{x^4}{15x^2+13x+2}dx$$
$$\Big\downarrow 1141$$
$$15\int \left(\frac{x^2}{225}-\frac{13x}{3375}-\frac{16}{2835(3x+2)}+\frac{1}{13125(5x+1)}+\frac{139}{50625}\right)dx$$
$$\Big\downarrow 2009$$
$$15\left(\frac{x^3}{675}-\frac{13x^2}{6750}+\frac{139x}{50625}-\frac{16\log(3x+2)}{8505}+\frac{\log(5x+1)}{65625}\right)$$

input
```
Int[x^5/(2*x + 13*x^2 + 15*x^3),x]
```

output
```
15*((139*x)/50625 - (13*x^2)/6750 + x^3/675 - (16*Log[2 + 3*x])/8505 + Log
[1 + 5*x]/65625)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}+\frac{\ln\left(x+\frac{1}{5}\right)}{4375}-\frac{16\ln\left(x+\frac{2}{3}\right)}{567}$	27
default	$\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16\ln(3x+2)}{567}+\frac{\ln(1+5x)}{4375}$	31
norman	$\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16\ln(3x+2)}{567}+\frac{\ln(1+5x)}{4375}$	31
risch	$\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16\ln(3x+2)}{567}+\frac{\ln(1+5x)}{4375}$	31

input
```
int(x^5/(15*x^3+13*x^2+2*x),x,method=_RETURNVERBOSE)
```

output
```
1/45*x^3-13/450*x^2+139/3375*x+1/4375*ln(x+1/5)-16/567*ln(x+2/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{2x+13x^2+15x^3}\,dx=\frac{1}{45}\,x^3-\frac{13}{450}\,x^2+\frac{139}{3375}\,x+\frac{1}{4375}\,\log\left(5\,x+1\right)-\frac{16}{567}\,\log\left(3\,x+2\right)$$

input
```
integrate(x^5/(15*x^3+13*x^2+2*x),x, algorithm="fricas")
```

output
```
1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x
+ 2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{2x+13x^2+15x^3}\,dx = \frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} + \frac{\log\left(x+\frac{1}{5}\right)}{4375} - \frac{16\log\left(x+\frac{2}{3}\right)}{567}$$

input
```
integrate(x**5/(15*x**3+13*x**2+2*x),x)
```

output
```
x**3/45 - 13*x**2/450 + 139*x/3375 + log(x + 1/5)/4375 - 16*log(x + 2/3)/5
67
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{2x+13x^2+15x^3}\,dx = \frac{1}{45}\,x^3 - \frac{13}{450}\,x^2 + \frac{139}{3375}\,x + \frac{1}{4375}\,\log\left(5\,x+1\right) - \frac{16}{567}\,\log\left(3\,x+2\right)$$

input
```
integrate(x^5/(15*x^3+13*x^2+2*x),x, algorithm="maxima")
```

output
```
1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x
+ 2)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{2x+13x^2+15x^3}\,dx = \frac{1}{45}\,x^3 - \frac{13}{450}\,x^2 + \frac{139}{3375}\,x + \frac{1}{4375}\,\log\left(|5\,x+1|\right) - \frac{16}{567}\,\log\left(|3\,x+2|\right)$$

input
```
integrate(x^5/(15*x^3+13*x^2+2*x),x, algorithm="giac")
```

output
```
1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(abs(5*x + 1)) - 16/567*log
(abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

$$\int \frac{x^5}{2x+13x^2+15x^3}\,dx = \frac{139\,x}{3375} - \frac{16\,\ln\left(x+\frac{2}{3}\right)}{567} + \frac{\ln\left(x+\frac{1}{5}\right)}{4375} - \frac{13\,x^2}{450} + \frac{x^3}{45}$$

input
```
int(x^5/(2*x + 13*x^2 + 15*x^3),x)
```

output
```
(139*x)/3375 - (16*log(x + 2/3))/567 + log(x + 1/5)/4375 - (13*x^2)/450 +
x^3/45
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{2x+13x^2+15x^3}\,dx = \frac{\log(5x+1)}{4375} - \frac{16\log(3x+2)}{567} + \frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375}$$

input
```
int(x^5/(15*x^3+13*x^2+2*x),x)
```

output

```
(162*log(5*x + 1) - 20000*log(3*x + 2) + 15750*x**3 - 20475*x**2 + 29190*x
)/708750
```

3.152 $\int \frac{x^4}{2x+13x^2+15x^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 33

$$\int \frac{x^4}{2x+13x^2+15x^3} dx = -\frac{13x}{225} + \frac{x^2}{30} + \frac{8}{189}\log(2+3x) - \frac{1}{875}\log(1+5x)$$

output
```
-13/225*x+1/30*x^2+8/189*ln(2+3*x)-1/875*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{2x+13x^2+15x^3} dx = -\frac{13x}{225} + \frac{x^2}{30} + \frac{8}{189}\log(2+3x) - \frac{1}{875}\log(1+5x)$$

input
```
Integrate[x^4/(2*x + 13*x^2 + 15*x^3),x]
```

output
```
(-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {9, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{15x^3+13x^2+2x}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^3}{15x^2+13x+2}dx$$

$$\Big\downarrow \; 1141$$

$$15\int \left(\frac{x}{225}+\frac{8}{945(3x+2)}-\frac{1}{2625(5x+1)}-\frac{13}{3375}\right)dx$$

$$\Big\downarrow \; 2009$$

$$15\left(\frac{x^2}{450}-\frac{13x}{3375}+\frac{8\log(3x+2)}{2835}-\frac{\log(5x+1)}{13125}\right)$$

input
```
Int[x^4/(2*x + 13*x^2 + 15*x^3),x]
```

output
```
15*((-13*x)/3375 + x^2/450 + (8*Log[2 + 3*x])/2835 - Log[1 + 5*x]/13125)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{x^{2}}{30}-\frac{13 x}{225}-\frac{\ln \left(x+\frac{1}{5}\right)}{875}+\frac{8 \ln \left(x+\frac{2}{3}\right)}{189}$	22
default	$-\frac{13 x}{225}+\frac{x^{2}}{30}+\frac{8 \ln \left(3 x+2\right)}{189}-\frac{\ln \left(1+5 x\right)}{875}$	26
norman	$-\frac{13 x}{225}+\frac{x^{2}}{30}+\frac{8 \ln \left(3 x+2\right)}{189}-\frac{\ln \left(1+5 x\right)}{875}$	26
risch	$-\frac{13 x}{225}+\frac{x^{2}}{30}+\frac{8 \ln \left(3 x+2\right)}{189}-\frac{\ln \left(1+5 x\right)}{875}$	26

input
```
int(x^4/(15*x^3+13*x^2+2*x),x,method=_RETURNVERBOSE)
```

output
```
1/30*x^2-13/225*x-1/875*ln(x+1/5)+8/189*ln(x+2/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{2x+13x^2+15x^3}\,dx=\frac{1}{30}\,x^{2}-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(5\,x+1\right)+\frac{8}{189}\,\log\left(3\,x+2\right)$$

input
```
integrate(x^4/(15*x^3+13*x^2+2*x),x, algorithm="fricas")
```

output
```
1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{x^4}{2x+13x^2+15x^3}\,dx = \frac{x^2}{30} - \frac{13x}{225} - \frac{\log\left(x+\frac{1}{5}\right)}{875} + \frac{8\log\left(x+\frac{2}{3}\right)}{189}$$

input
```
integrate(x**4/(15*x**3+13*x**2+2*x),x)
```

output
```
x**2/30 - 13*x/225 - log(x + 1/5)/875 + 8*log(x + 2/3)/189
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{2x+13x^2+15x^3}\,dx = \frac{1}{30}\,x^2 - \frac{13}{225}\,x - \frac{1}{875}\,\log\left(5\,x+1\right) + \frac{8}{189}\,\log\left(3\,x+2\right)$$

input
```
integrate(x^4/(15*x^3+13*x^2+2*x),x, algorithm="maxima")
```

output
```
1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{x^4}{2x+13x^2+15x^3}\,dx = \frac{1}{30}\,x^2 - \frac{13}{225}\,x - \frac{1}{875}\,\log\left(|5\,x+1|\right) + \frac{8}{189}\,\log\left(|3\,x+2|\right)$$

input
```
integrate(x^4/(15*x^3+13*x^2+2*x),x, algorithm="giac")
```

output
```
1/30*x^2 - 13/225*x - 1/875*log(abs(5*x + 1)) + 8/189*log(abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.64

$$\int \frac{x^4}{2x + 13x^2 + 15x^3} \, dx = \frac{8 \ln\left(x + \frac{2}{3}\right)}{189} - \frac{13\,x}{225} - \frac{\ln\left(x + \frac{1}{5}\right)}{875} + \frac{x^2}{30}$$

input
```
int(x^4/(2*x + 13*x^2 + 15*x^3),x)
```

output
```
(8*log(x + 2/3))/189 - (13*x)/225 - log(x + 1/5)/875 + x^2/30
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{2x + 13x^2 + 15x^3} \, dx = -\frac{\log(5x + 1)}{875} + \frac{8 \log(3x + 2)}{189} + \frac{x^2}{30} - \frac{13x}{225}$$

input
```
int(x^4/(15*x^3+13*x^2+2*x),x)
```

output
```
( - 54*log(5*x + 1) + 2000*log(3*x + 2) + 1575*x**2 - 2730*x)/47250
```

3.153 $\int \frac{x^3}{2x+13x^2+15x^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 26

$$\int \frac{x^3}{2x+13x^2+15x^3} dx = \frac{x}{15} - \frac{4}{63}\log(2+3x) + \frac{1}{175}\log(1+5x)$$

output
```
1/15*x-4/63*ln(2+3*x)+1/175*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{2x+13x^2+15x^3} dx = \frac{x}{15} - \frac{4}{63}\log(2+3x) + \frac{1}{175}\log(1+5x)$$

input
```
Integrate[x^3/(2*x + 13*x^2 + 15*x^3),x]
```

output
```
x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {9, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{15x^3+13x^2+2x} \, dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{x^2}{15x^2+13x+2}dx$$

$$\Big\downarrow \text{1141}$$

$$15 \int \left(\frac{1}{525(5x+1)}+\frac{1}{225}-\frac{4}{315(3x+2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$15\left(\frac{x}{225}-\frac{4}{945}\log(3x+2)+\frac{\log(5x+1)}{2625}\right)$$

input
```
Int[x^3/(2*x + 13*x^2 + 15*x^3),x]
```

output
```
15*(x/225 - (4*Log[2 + 3*x])/945 + Log[1 + 5*x]/2625)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px, 
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p, 
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$\frac{x}{15}+\frac{\ln\left(x+\frac{1}{5}\right)}{175}-\frac{4\ln\left(x+\frac{2}{3}\right)}{63}$	17
default	$\frac{x}{15}-\frac{4\ln(3x+2)}{63}+\frac{\ln(1+5x)}{175}$	21
norman	$\frac{x}{15}-\frac{4\ln(3x+2)}{63}+\frac{\ln(1+5x)}{175}$	21
risch	$\frac{x}{15}-\frac{4\ln(3x+2)}{63}+\frac{\ln(1+5x)}{175}$	21

input
```
int(x^3/(15*x^3+13*x^2+2*x),x,method=_RETURNVERBOSE)
```

output
```
1/15*x+1/175*ln(x+1/5)-4/63*ln(x+2/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{2x+13x^2+15x^3}\,dx=\frac{1}{15}\,x+\frac{1}{175}\,\log\left(5\,x+1\right)-\frac{4}{63}\,\log\left(3\,x+2\right)$$

input
```
integrate(x^3/(15*x^3+13*x^2+2*x),x, algorithm="fricas")
```

output
```
1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{2x+13x^2+15x^3}\,dx = \frac{x}{15}+\frac{\log\left(x+\frac{1}{5}\right)}{175}-\frac{4\log\left(x+\frac{2}{3}\right)}{63}$$

input
```
integrate(x**3/(15*x**3+13*x**2+2*x),x)
```

output
```
x/15 + log(x + 1/5)/175 - 4*log(x + 2/3)/63
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{2x+13x^2+15x^3}\,dx = \frac{1}{15}\,x+\frac{1}{175}\,\log\left(5\,x+1\right)-\frac{4}{63}\,\log\left(3\,x+2\right)$$

input
```
integrate(x^3/(15*x^3+13*x^2+2*x),x, algorithm="maxima")
```

output
```
1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{2x+13x^2+15x^3}\,dx = \frac{1}{15}\,x+\frac{1}{175}\,\log\left(|5\,x+1|\right)-\frac{4}{63}\,\log\left(|3\,x+2|\right)$$

input
```
integrate(x^3/(15*x^3+13*x^2+2*x),x, algorithm="giac")
```

output
```
1/15*x + 1/175*log(abs(5*x + 1)) - 4/63*log(abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \frac{x^3}{2x + 13x^2 + 15x^3} \, dx = \frac{x}{15} - \frac{4 \ln \left(x + \frac{2}{3}\right)}{63} + \frac{\ln \left(x + \frac{1}{5}\right)}{175}$$

input
```
int(x^3/(2*x + 13*x^2 + 15*x^3),x)
```

output
```
x/15 - (4*log(x + 2/3))/63 + log(x + 1/5)/175
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{2x + 13x^2 + 15x^3} \, dx = \frac{\log(5x + 1)}{175} - \frac{4 \log(3x + 2)}{63} + \frac{x}{15}$$

input
```
int(x^3/(15*x^3+13*x^2+2*x),x)
```

output
```
(9*log(5*x + 1) - 100*log(3*x + 2) + 105*x)/1575
```

3.154 $\int \frac{x^2}{2x+13x^2+15x^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 21

$$\int \frac{x^2}{2x+13x^2+15x^3} \, dx = \frac{2}{21}\log(2+3x) - \frac{1}{35}\log(1+5x)$$

output
```
2/21*ln(2+3*x)-1/35*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{2x+13x^2+15x^3} \, dx = \frac{2}{21}\log(2+3x) - \frac{1}{35}\log(1+5x)$$

input
```
Integrate[x^2/(2*x + 13*x^2 + 15*x^3),x]
```

output
```
(2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {9, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{15x^3+13x^2+2x}dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{x}{15x^2+13x+2}dx$$
$$\Big\downarrow \; 1141$$
$$15\int \left(\frac{2}{105(3x+2)}-\frac{1}{105(5x+1)}\right)dx$$
$$\Big\downarrow \; 2009$$
$$15\left(\frac{2}{315}\log(3x+2)-\frac{1}{525}\log(5x+1)\right)$$

input
```
Int[x^2/(2*x + 13*x^2 + 15*x^3),x]
```

output
```
15*((2*Log[2 + 3*x])/315 - Log[1 + 5*x]/525)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$-\frac{\ln\left(x+\frac{1}{5}\right)}{35}+\frac{2\ln\left(x+\frac{2}{3}\right)}{21}$	14
default	$\frac{2\ln(3x+2)}{21}-\frac{\ln(1+5x)}{35}$	18
norman	$\frac{2\ln(3x+2)}{21}-\frac{\ln(1+5x)}{35}$	18
risch	$\frac{2\ln(3x+2)}{21}-\frac{\ln(1+5x)}{35}$	18

input
```
int(x^2/(15*x^3+13*x^2+2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/35*ln(x+1/5)+2/21*ln(x+2/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{2x+13x^2+15x^3}\,dx=-\frac{1}{35}\,\log\left(5\,x+1\right)+\frac{2}{21}\,\log\left(3\,x+2\right)$$

input
```
integrate(x^2/(15*x^3+13*x^2+2*x),x, algorithm="fricas")
```

output
```
-1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{2x+13x^2+15x^3}\,dx = -\frac{\log\left(x+\frac{1}{5}\right)}{35}+\frac{2\log\left(x+\frac{2}{3}\right)}{21}$$

input
```
integrate(x**2/(15*x**3+13*x**2+2*x),x)
```

output
```
-log(x + 1/5)/35 + 2*log(x + 2/3)/21
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{2x+13x^2+15x^3}\,dx = -\frac{1}{35}\,\log\left(5\,x+1\right)+\frac{2}{21}\,\log\left(3\,x+2\right)$$

input
```
integrate(x^2/(15*x^3+13*x^2+2*x),x, algorithm="maxima")
```

output
```
-1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{2x+13x^2+15x^3}\,dx = -\frac{1}{35}\,\log\left(|5\,x+1|\right)+\frac{2}{21}\,\log\left(|3\,x+2|\right)$$

input
```
integrate(x^2/(15*x^3+13*x^2+2*x),x, algorithm="giac")
```

output
```
-1/35*log(abs(5*x + 1)) + 2/21*log(abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{x^2}{2x+13x^2+15x^3}\,dx = \frac{2\,\ln\left(x+\frac{2}{3}\right)}{21} - \frac{\ln\left(x+\frac{1}{5}\right)}{35}$$

input
```
int(x^2/(2*x + 13*x^2 + 15*x^3),x)
```

output
```
(2*log(x + 2/3))/21 - log(x + 1/5)/35
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{2x+13x^2+15x^3}\,dx = -\frac{\log(5x+1)}{35} + \frac{2\log(3x+2)}{21}$$

input
```
int(x^2/(15*x^3+13*x^2+2*x),x)
```

output
```
( - 3*log(5*x + 1) + 10*log(3*x + 2))/105
```

3.155 $\int \frac{x}{2x+13x^2+15x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 21

$$\int \frac{x}{2x+13x^2+15x^3}\,dx = -\frac{1}{7}\log(2+3x)+\frac{1}{7}\log(1+5x)$$

output
```
-1/7*ln(2+3*x)+1/7*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x}{2x+13x^2+15x^3}\,dx = -\frac{1}{7}\log(2+3x)+\frac{1}{7}\log(1+5x)$$

input
```
Integrate[x/(2*x + 13*x^2 + 15*x^3),x]
```

output
```
-1/7*Log[2 + 3*x] + Log[1 + 5*x]/7
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{15x^3+13x^2+2x}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{15x^2+13x+2}dx$$

$$\Big\downarrow \text{ 1081}$$

$$15\int \left(\frac{1}{21(5x+1)}-\frac{1}{35(3x+2)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$15\left(\frac{1}{105}\log(5x+1)-\frac{1}{105}\log(3x+2)\right)$$

input
```
Int[x/(2*x + 13*x^2 + 15*x^3),x]
```

output
```
15*(-1/105*Log[2 + 3*x] + Log[1 + 5*x]/105)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{\ln\left(x+\frac{1}{5}\right)}{7}-\frac{\ln\left(x+\frac{2}{3}\right)}{7}$	14
default	$-\frac{\ln(3x+2)}{7}+\frac{\ln(1+5x)}{7}$	18
norman	$-\frac{\ln(3x+2)}{7}+\frac{\ln(1+5x)}{7}$	18
risch	$-\frac{\ln(3x+2)}{7}+\frac{\ln(1+5x)}{7}$	18

input
```
int(x/(15*x^3+13*x^2+2*x),x,method=_RETURNVERBOSE)
```

output
```
1/7*ln(x+1/5)-1/7*ln(x+2/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x}{2x+13x^2+15x^3}\,dx=\frac{1}{7}\,\log\left(5\,x+1\right)-\frac{1}{7}\,\log\left(3\,x+2\right)$$

input
```
integrate(x/(15*x^3+13*x^2+2*x),x, algorithm="fricas")
```

output
```
1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{x}{2x+13x^2+15x^3}\,dx=\frac{\log\left(x+\frac{1}{5}\right)}{7}-\frac{\log\left(x+\frac{2}{3}\right)}{7}$$

input
```
integrate(x/(15*x**3+13*x**2+2*x),x)
```

output
```
log(x + 1/5)/7 - log(x + 2/3)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x}{2x+13x^2+15x^3}\,dx=\frac{1}{7}\log\left(5x+1\right)-\frac{1}{7}\log\left(3x+2\right)$$

input
```
integrate(x/(15*x^3+13*x^2+2*x),x, algorithm="maxima")
```

output
```
1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{x}{2x+13x^2+15x^3}\,dx=\frac{1}{7}\log\left(|5x+1|\right)-\frac{1}{7}\log\left(|3x+2|\right)$$

input
```
integrate(x/(15*x^3+13*x^2+2*x),x, algorithm="giac")
```

output
```
1/7*log(abs(5*x + 1)) - 1/7*log(abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.38

$$\int \frac{x}{2x+13x^2+15x^3}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{30\,x}{7}+\frac{13}{7}\right)}{7}$$

input
```
int(x/(2*x + 13*x^2 + 15*x^3),x)
```

output
```
-(2*atanh((30*x)/7 + 13/7))/7
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x}{2x+13x^2+15x^3}\,dx = \frac{\log(5x+1)}{7}-\frac{\log(3x+2)}{7}$$

input
```
int(x/(15*x^3+13*x^2+2*x),x)
```

output
```
(log(5*x + 1) - log(3*x + 2))/7
```

3.156 $\int \frac{1}{2x+13x^2+15x^3} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 27

$$\int \frac{1}{2x+13x^2+15x^3} \, dx = \frac{\log(x)}{2} + \frac{3}{14}\log(2+3x) - \frac{5}{7}\log(1+5x)$$

output
```
1/2*ln(x)+3/14*ln(2+3*x)-5/7*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{2x+13x^2+15x^3} \, dx = \frac{\log(x)}{2} + \frac{3}{14}\log(2+3x) - \frac{5}{7}\log(1+5x)$$

input
```
Integrate[(2*x + 13*x^2 + 15*x^3)^(-1),x]
```

output
```
Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1949, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{15x^3+13x^2+2x}dx$$

$$\Big\downarrow \text{1949}$$

$$\int \frac{1}{x(15x^2+13x+2)}dx$$

$$\Big\downarrow \text{1141}$$

$$15\int \left(\frac{3}{70(3x+2)}-\frac{5}{21(5x+1)}+\frac{1}{30x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$15\left(\frac{\log(x)}{30}+\frac{1}{70}\log(3x+2)-\frac{1}{21}\log(5x+1)\right)$$

input
```
Int[(2*x + 13*x^2 + 15*x^3)^(-1),x]
```

output
```
15*(Log[x]/30 + Log[2 + 3*x]/70 - Log[1 + 5*x]/21)
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1949
```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Int[x^(p*q)*(a + b*x^(n - q) + c*x^(2*(n - q)))^p, x] /; FreeQ[{a, b,
c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{\ln(x)}{2}-\frac{5\ln\left(x+\frac{1}{5}\right)}{7}+\frac{3\ln\left(x+\frac{2}{3}\right)}{14}$	18
default	$\frac{\ln(x)}{2}+\frac{3\ln(3x+2)}{14}-\frac{5\ln(1+5x)}{7}$	22
norman	$\frac{\ln(x)}{2}+\frac{3\ln(3x+2)}{14}-\frac{5\ln(1+5x)}{7}$	22
risch	$\frac{\ln(x)}{2}+\frac{3\ln(3x+2)}{14}-\frac{5\ln(1+5x)}{7}$	22

input
```
int(1/(15*x^3+13*x^2+2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x)-5/7*ln(x+1/5)+3/14*ln(x+2/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{2x+13x^2+15x^3}\,dx=-\frac{5}{7}\log\left(5\,x+1\right)+\frac{3}{14}\log\left(3\,x+2\right)+\frac{1}{2}\log\left(x\right)$$

input
```
integrate(1/(15*x^3+13*x^2+2*x),x, algorithm="fricas")
```

output
```
-5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{2x+13x^2+15x^3}\,dx = \frac{\log(x)}{2} - \frac{5\log\left(x+\frac{1}{5}\right)}{7} + \frac{3\log\left(x+\frac{2}{3}\right)}{14}$$

input
```
integrate(1/(15*x**3+13*x**2+2*x),x)
```

output
```
log(x)/2 - 5*log(x + 1/5)/7 + 3*log(x + 2/3)/14
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{2x+13x^2+15x^3}\,dx = -\frac{5}{7}\log(5x+1) + \frac{3}{14}\log(3x+2) + \frac{1}{2}\log(x)$$

input
```
integrate(1/(15*x^3+13*x^2+2*x),x, algorithm="maxima")
```

output
```
-5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{2x+13x^2+15x^3}\,dx = -\frac{5}{7}\log(|5x+1|) + \frac{3}{14}\log(|3x+2|) + \frac{1}{2}\log(|x|)$$

input
```
integrate(1/(15*x^3+13*x^2+2*x),x, algorithm="giac")
```

output
```
-5/7*log(abs(5*x + 1)) + 3/14*log(abs(3*x + 2)) + 1/2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{2x+13x^2+15x^3}\,dx = \frac{3\,\ln\left(x+\frac{2}{3}\right)}{14} - \frac{5\,\ln\left(x+\frac{1}{5}\right)}{7} + \frac{\ln\left(x\right)}{2}$$

input
```
int(1/(2*x + 13*x^2 + 15*x^3),x)
```

output
```
(3*log(x + 2/3))/14 - (5*log(x + 1/5))/7 + log(x)/2
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{2x+13x^2+15x^3}\,dx = -\frac{5\log(5x+1)}{7} + \frac{3\log(3x+2)}{14} + \frac{\log(x)}{2}$$

input
```
int(1/(15*x^3+13*x^2+2*x),x)
```

output
```
( - 10*log(5*x + 1) + 3*log(3*x + 2) + 7*log(x))/14
```

3.157 $\int \frac{1}{x(2x+13x^2+15x^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 34

$$\int \frac{1}{x(2x+13x^2+15x^3)}\,dx=-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)$$

output
```
-1/2/x-13/4*ln(x)-9/28*ln(2+3*x)+25/7*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(2x+13x^2+15x^3)}\,dx=-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)$$

input
```
Integrate[1/(x*(2*x + 13*x^2 + 15*x^3)),x]
```

output
```
-1/2*1/x - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (15x^3+13x^2+2x)} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{1}{x^2 (15x^2+13x+2)}dx$$

$$\Big\downarrow \; 1141$$

$$15 \int \left(-\frac{9}{140(3x+2)}+\frac{25}{21(5x+1)}-\frac{13}{60x}+\frac{1}{30x^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$15 \left(-\frac{1}{30x}-\frac{13\log(x)}{60}-\frac{3}{140}\log(3x+2)+\frac{5}{21}\log(5x+1)\right)$$

input
```
Int[1/(x*(2*x + 13*x^2 + 15*x^3)),x]
```

output
```
15*(-1/30*1/x - (13*Log[x])/60 - (3*Log[2 + 3*x])/140 + (5*Log[1 + 5*x])/2
1)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(3x+2)}{28}+\frac{25\ln(1+5x)}{7}$	27
norman	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(3x+2)}{28}+\frac{25\ln(1+5x)}{7}$	27
risch	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(3x+2)}{28}+\frac{25\ln(1+5x)}{7}$	27
parallelrisch	$\frac{100\ln\left(x+\frac{1}{5}\right)x-9\ln\left(x+\frac{2}{3}\right)x-91\ln(x)x-14}{28x}$	27

input
```
int(1/x/(15*x^3+13*x^2+2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x-13/4*ln(x)-9/28*ln(3*x+2)+25/7*ln(1+5*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\left(2x+13x^2+15x^3\right)}\,dx=\frac{100\,x\log\left(5\,x+1\right)-9\,x\log\left(3\,x+2\right)-91\,x\log\left(x\right)-14}{28\,x}$$

input
```
integrate(1/x/(15*x^3+13*x^2+2*x),x, algorithm="fricas")
```

output
```
1/28*(100*x*log(5*x + 1) - 9*x*log(3*x + 2) - 91*x*log(x) - 14)/x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\left(2x+13x^2+15x^3\right)}\,dx=-\frac{13\log\left(x\right)}{4}+\frac{25\log\left(x+\frac{1}{5}\right)}{7}-\frac{9\log\left(x+\frac{2}{3}\right)}{28}-\frac{1}{2x}$$

input
```
integrate(1/x/(15*x**3+13*x**2+2*x),x)
```

output
```
-13*log(x)/4 + 25*log(x + 1/5)/7 - 9*log(x + 2/3)/28 - 1/(2*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\left(2x+13x^2+15x^3\right)}\,dx=-\frac{1}{2\,x}+\frac{25}{7}\,\log\left(5\,x+1\right)-\frac{9}{28}\,\log\left(3\,x+2\right)-\frac{13}{4}\,\log\left(x\right)$$

input
```
integrate(1/x/(15*x^3+13*x^2+2*x),x, algorithm="maxima")
```

output
```
-1/2/x + 25/7*log(5*x + 1) - 9/28*log(3*x + 2) - 13/4*log(x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\left(2x+13x^2+15x^3\right)}\,dx=-\frac{1}{2\,x}+\frac{25}{7}\,\log\left(|5\,x+1|\right)-\frac{9}{28}\,\log\left(|3\,x+2|\right)-\frac{13}{4}\,\log\left(|x|\right)$$

input
```
integrate(1/x/(15*x^3+13*x^2+2*x),x, algorithm="giac")
```

output
```
-1/2/x + 25/7*log(abs(5*x + 1)) - 9/28*log(abs(3*x + 2)) - 13/4*log(abs(x)
)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \frac{1}{x(2x+13x^2+15x^3)}\,dx = \frac{25\,\ln\left(x+\frac{1}{5}\right)}{7} - \frac{9\,\ln\left(x+\frac{2}{3}\right)}{28} - \frac{13\,\ln(x)}{4} - \frac{1}{2\,x}$$

input
```
int(1/(x*(2*x + 13*x^2 + 15*x^3)),x)
```

output
```
(25*log(x + 1/5))/7 - (9*log(x + 2/3))/28 - (13*log(x))/4 - 1/(2*x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{x(2x+13x^2+15x^3)}\,dx = \frac{100\log(5x+1)\,x - 9\log(3x+2)\,x - 91\log(x)\,x - 14}{28x}$$

input
```
int(1/x/(15*x^3+13*x^2+2*x),x)
```

output
```
(100*log(5*x + 1)*x - 9*log(3*x + 2)*x - 91*log(x)*x - 14)/(28*x)
```

3.158 $\int \frac{1}{x^2(2x+13x^2+15x^3)} dx$

Optimal result

Integrand size = 20, antiderivative size = 41

$$\int \frac{1}{x^2(2x+13x^2+15x^3)} dx = -\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)$$

output
```
-1/4/x^2+13/4/x+139/8*ln(x)+27/56*ln(2+3*x)-125/7*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(2x+13x^2+15x^3)} dx = -\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)$$

input
```
Integrate[1/(x^2*(2*x + 13*x^2 + 15*x^3)),x]
```

output
```
-1/4*1/x^2 + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
 + 5*x])/7
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {9, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(15x^3+13x^2+2x)}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{1}{x^3(15x^2+13x+2)}dx$$

$$\Big\downarrow \text{1141}$$

$$15\int \left(\frac{27}{280(3x+2)}-\frac{125}{21(5x+1)}+\frac{139}{120x}-\frac{13}{60x^2}+\frac{1}{30x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$15\left(-\frac{1}{60x^2}+\frac{13}{60x}+\frac{139\log(x)}{120}+\frac{9}{280}\log(3x+2)-\frac{25}{21}\log(5x+1)\right)$$

input
```
Int[1/(x^2*(2*x + 13*x^2 + 15*x^3)),x]
```

output
```
15*(-1/60*1/x^2 + 13/(60*x) + (139*Log[x])/120 + (9*Log[2 + 3*x])/280 - (2
5*Log[1 + 5*x])/21)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

method	result	size
norman	$\frac{-\frac{1}{4}+\frac{13x}{4}}{x^2}+\frac{139\ln(x)}{8}+\frac{27\ln(3x+2)}{56}-\frac{125\ln(1+5x)}{7}$	31
risch	$\frac{-\frac{1}{4}+\frac{13x}{4}}{x^2}+\frac{139\ln(x)}{8}+\frac{27\ln(3x+2)}{56}-\frac{125\ln(1+5x)}{7}$	31
default	$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\ln(x)}{8}+\frac{27\ln(3x+2)}{56}-\frac{125\ln(1+5x)}{7}$	32
parallelrisch	$\frac{973\ln(x)x^2-1000\ln\left(x+\frac{1}{5}\right)x^2+27\ln\left(x+\frac{2}{3}\right)x^2-14+182x}{56x^2}$	36

input
```
int(1/x^2/(15*x^3+13*x^2+2*x),x,method=_RETURNVERBOSE)
```

output
```
(-1/4+13/4*x)/x^2+139/8*ln(x)+27/56*ln(3*x+2)-125/7*ln(1+5*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2(2x+13x^2+15x^3)}\,dx = -\frac{1000\,x^2\log(5\,x+1)-27\,x^2\log(3\,x+2)-973\,x^2\log(x)-182\,x+14}{56\,x^2}$$

input
```
integrate(1/x^2/(15*x^3+13*x^2+2*x),x, algorithm="fricas")
```

output
```
-1/56*(1000*x^2*log(5*x + 1) - 27*x^2*log(3*x + 2) - 973*x^2*log(x) - 182*
x + 14)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\left(2x+13x^2+15x^3\right)}\,dx=\frac{139\log\left(x\right)}{8}-\frac{125\log\left(x+\frac{1}{5}\right)}{7}+\frac{27\log\left(x+\frac{2}{3}\right)}{56}+\frac{13x-1}{4x^2}$$

input
```
integrate(1/x**2/(15*x**3+13*x**2+2*x),x)
```

output
```
139*log(x)/8 - 125*log(x + 1/5)/7 + 27*log(x + 2/3)/56 + (13*x - 1)/(4*x**
2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^2\left(2x+13x^2+15x^3\right)}\,dx=\frac{13\,x-1}{4\,x^2}-\frac{125}{7}\,\log\left(5\,x+1\right)+\frac{27}{56}\,\log\left(3\,x+2\right)+\frac{139}{8}\,\log\left(x\right)$$

input
```
integrate(1/x^2/(15*x^3+13*x^2+2*x),x, algorithm="maxima")
```

output
```
1/4*(13*x - 1)/x^2 - 125/7*log(5*x + 1) + 27/56*log(3*x + 2) + 139/8*log(x
)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2\,(2x+13x^2+15x^3)}\,dx = \frac{13\,x-1}{4\,x^2} - \frac{125}{7}\,\log\left(|5\,x+1|\right) + \frac{27}{56}\,\log\left(|3\,x+2|\right) + \frac{139}{8}\,\log\left(|x|\right)$$

input
```
integrate(1/x^2/(15*x^3+13*x^2+2*x),x, algorithm="giac")
```

output
```
1/4*(13*x - 1)/x^2 - 125/7*log(abs(5*x + 1)) + 27/56*log(abs(3*x + 2)) + 1
39/8*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^2\,(2x+13x^2+15x^3)}\,dx = \frac{27\,\ln\left(x+\frac{2}{3}\right)}{56} - \frac{125\,\ln\left(x+\frac{1}{5}\right)}{7} + \frac{139\,\ln\left(x\right)}{8} + \frac{\frac{13\,x}{4}-\frac{1}{4}}{x^2}$$

input
```
int(1/(x^2*(2*x + 13*x^2 + 15*x^3)),x)
```

output
```
(27*log(x + 2/3))/56 - (125*log(x + 1/5))/7 + (139*log(x))/8 + ((13*x)/4 -
 1/4)/x^2
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2\,(2x+13x^2+15x^3)}\,dx = \frac{-1000\log(5x+1)\,x^2+27\log(3x+2)\,x^2+973\log(x)\,x^2+182x-14}{56x^2}$$

input
```
int(1/x^2/(15*x^3+13*x^2+2*x),x)
```

output
```
( - 1000*log(5*x + 1)*x**2 + 27*log(3*x + 2)*x**2 + 973*log(x)*x**2 + 182*
x - 14)/(56*x**2)
```

3.159 $\int \frac{1}{x^3(2x+13x^2+15x^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 48

$$\int \frac{1}{x^3(2x+13x^2+15x^3)}\,dx=-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)$$

output
```
-1/6/x^3+13/8/x^2-139/8/x-1417/16*ln(x)-81/112*ln(2+3*x)+625/7*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3(2x+13x^2+15x^3)}\,dx=-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)$$

input
```
Integrate[1/(x^3*(2*x + 13*x^2 + 15*x^3)),x]
```

output
```
-1/6*1/x^3 + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])
/112 + (625*Log[1 + 5*x])/7
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {9, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (15x^3+13x^2+2x)} \, dx$$
$$\Big\downarrow \ 9$$
$$\int \frac{1}{x^4 (15x^2+13x+2)}dx$$
$$\Big\downarrow \ 1141$$
$$15 \int \left(-\frac{81}{560(3x+2)}+\frac{625}{21(5x+1)}-\frac{1417}{240x}+\frac{139}{120x^2}-\frac{13}{60x^3}+\frac{1}{30x^4}\right)dx$$
$$\Big\downarrow \ 2009$$
$$15\left(-\frac{1}{90x^3}+\frac{13}{120x^2}-\frac{139}{120x}-\frac{1417\log(x)}{240}-\frac{27}{560}\log(3x+2)+\frac{125}{21}\log(5x+1)\right)$$

input
```
Int[1/(x^3*(2*x + 13*x^2 + 15*x^3)),x]
```

output
```
15*(-1/90*1/x^3 + 13/(120*x^2) - 139/(120*x) - (1417*Log[x])/240 - (27*Log
[2 + 3*x])/560 + (125*Log[1 + 5*x])/21)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

method	result	size
norman	$\frac{-\frac{1}{6}+\frac{13}{8}x-\frac{139}{8}x^2}{x^3}-\frac{1417\ln(x)}{16}-\frac{81\ln(3x+2)}{112}+\frac{625\ln(1+5x)}{7}$	36
risch	$\frac{-\frac{1}{6}+\frac{13}{8}x-\frac{139}{8}x^2}{x^3}-\frac{1417\ln(x)}{16}-\frac{81\ln(3x+2)}{112}+\frac{625\ln(1+5x)}{7}$	36
default	$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\ln(x)}{16}-\frac{81\ln(3x+2)}{112}+\frac{625\ln(1+5x)}{7}$	37
parallelrisch	$-\frac{29757\ln(x)x^3-30000\ln(x+\frac{1}{5})x^3+243\ln(x+\frac{2}{3})x^3+56+5838x^2-546x}{336x^3}$	41

input
```
int(1/x^3/(15*x^3+13*x^2+2*x),x,method=_RETURNVERBOSE)
```

output
```
(-1/6+13/8*x-139/8*x^2)/x^3-1417/16*ln(x)-81/112*ln(3*x+2)+625/7*ln(1+5*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3(2x+13x^2+15x^3)}\,dx = \frac{30000\,x^3\log(5\,x+1)-243\,x^3\log(3\,x+2)-29757\,x^3\log(x)-5838\,x^2+546\,x-56}{336\,x^3}$$

input
```
integrate(1/x^3/(15*x^3+13*x^2+2*x),x, algorithm="fricas")
```

output
```
1/336*(30000*x^3*log(5*x + 1) - 243*x^3*log(3*x + 2) - 29757*x^3*log(x) -
5838*x^2 + 546*x - 56)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3(2x+13x^2+15x^3)}\,dx = -\frac{1417\log(x)}{16} + \frac{625\log\left(x+\frac{1}{5}\right)}{7} - \frac{81\log\left(x+\frac{2}{3}\right)}{112} + \frac{-417x^2+39x-4}{24x^3}$$

input
```
integrate(1/x**3/(15*x**3+13*x**2+2*x),x)
```

output
```
-1417*log(x)/16 + 625*log(x + 1/5)/7 - 81*log(x + 2/3)/112 + (-417*x**2 +
39*x - 4)/(24*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^3(2x+13x^2+15x^3)}\,dx = -\frac{417\,x^2-39\,x+4}{24\,x^3} + \frac{625}{7}\log(5\,x+1) - \frac{81}{112}\log(3\,x+2) - \frac{1417}{16}\log(x)$$

input
```
integrate(1/x^3/(15*x^3+13*x^2+2*x),x, algorithm="maxima")
```

output
```
-1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(5*x + 1) - 81/112*log(3*x + 2)
- 1417/16*log(x)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3\left(2x+13x^2+15x^3\right)}\,dx = -\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\,\log\left(|5\,x+1|\right)-\frac{81}{112}\,\log\left(|3\,x+2|\right)-\frac{1417}{16}\,\log\left(|x|\right)$$

input
```
integrate(1/x^3/(15*x^3+13*x^2+2*x),x, algorithm="giac")
```

output
```
-1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(abs(5*x + 1)) - 81/112*log(abs(
3*x + 2)) - 1417/16*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^3\left(2x+13x^2+15x^3\right)}\,dx = \frac{625\,\ln\left(x+\frac{1}{5}\right)}{7}-\frac{81\,\ln\left(x+\frac{2}{3}\right)}{112}-\frac{1417\,\ln\left(x\right)}{16}-\frac{\frac{139\,x^2}{8}-\frac{13\,x}{8}+\frac{1}{6}}{x^3}$$

input
```
int(1/(x^3*(2*x + 13*x^2 + 15*x^3)),x)
```

output
```
(625*log(x + 1/5))/7 - (81*log(x + 2/3))/112 - (1417*log(x))/16 - ((139*x^
2)/8 - (13*x)/8 + 1/6)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3 (2x + 13x^2 + 15x^3)} dx$$
$$= \frac{30000 \log(5x + 1) x^3 - 243 \log(3x + 2) x^3 - 29757 \log(x) x^3 - 5838x^2 + 546x - 56}{336x^3}$$

input
```
int(1/x^3/(15*x^3+13*x^2+2*x),x)
```

output
```
(30000*log(5*x + 1)*x**3 - 243*log(3*x + 2)*x**3 - 29757*log(x)*x**3 - 583
8*x**2 + 546*x - 56)/(336*x**3)
```

3.160 $\int \frac{x^3}{1+3x+x^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 61

$$\int \frac{x^3}{1+3x+x^2} \, dx = -3x + \frac{x^2}{2} + \frac{1}{5}\left(20-9\sqrt{5}\right)\log\left(3-\sqrt{5}+2x\right) + \frac{1}{5}\left(20+9\sqrt{5}\right)\log\left(3+\sqrt{5}+2x\right)$$

output
```
-3*x+1/2*x^2+1/5*(20-9*5^(1/2))*ln(3-5^(1/2)+2*x)+1/5*(20+9*5^(1/2))*ln(3+
5^(1/2)+2*x)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{1+3x+x^2} \, dx = \frac{1}{2}(-6+x)x + \left(4-\frac{9}{\sqrt{5}}\right)\log\left(-3+\sqrt{5}-2x\right) + \left(4+\frac{9}{\sqrt{5}}\right)\log\left(3+\sqrt{5}+2x\right)$$

input
```
Integrate[x^3/(1 + 3*x + x^2),x]
```

output
```
((-6 + x)*x)/2 + (4 - 9/Sqrt[5])*Log[-3 + Sqrt[5] - 2*x] + (4 + 9/Sqrt[5])
*Log[3 + Sqrt[5] + 2*x]
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^2+3x+1}\,dx$$

$$\Big\downarrow \text{1141}$$

$$\int \left(x+\frac{2\left(20-9\sqrt{5}\right)}{5\left(2x-\sqrt{5}+3\right)}+\frac{2\left(20+9\sqrt{5}\right)}{5\left(2x+\sqrt{5}+3\right)}-3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2}{2}-3x+\frac{1}{5}\left(20-9\sqrt{5}\right)\log\left(2x-\sqrt{5}+3\right)+\frac{1}{5}\left(20+9\sqrt{5}\right)\log\left(2x+\sqrt{5}+3\right)$$

input
```
Int[x^3/(1 + 3*x + x^2),x]
```

output
```
-3*x + x^2/2 + ((20 - 9*Sqrt[5])*Log[3 - Sqrt[5] + 2*x])/5 + ((20 + 9*Sqrt
[5])*Log[3 + Sqrt[5] + 2*x])/5
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.61

method	result	size
default	$-3x+\frac{x^2}{2}+4\ln\left(x^2+3x+1\right)+\frac{18\,\mathrm{arctanh}\left(\frac{(2x+3)\sqrt{5}}{5}\right)\sqrt{5}}{5}$	37
risch	$-3x+\frac{x^2}{2}+4\ln\left(3+\sqrt{5}+2x\right)+\frac{9\ln\left(3+\sqrt{5}+2x\right)\sqrt{5}}{5}+4\ln\left(3-\sqrt{5}+2x\right)-\frac{9\ln\left(3-\sqrt{5}+2x\right)\sqrt{5}}{5}$	64

input
```
int(x^3/(x^2+3*x+1),x,method=_RETURNVERBOSE)
```

output
```
-3*x+1/2*x^2+4*ln(x^2+3*x+1)+18/5*arctanh(1/5*(2*x+3)*5^(1/2))*5^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{1+3x+x^2}\,dx=\frac{1}{2}\,x^2+\frac{9}{5}\,\sqrt{5}\log\left(\frac{2\,x^2+\sqrt{5}(2\,x+3)+6\,x+7}{x^2+3\,x+1}\right)-3\,x+4\,\log\left(x^2+3\,x+1\right)$$

input
```
integrate(x^3/(x^2+3*x+1),x, algorithm="fricas")
```

output
```
1/2*x^2 + 9/5*sqrt(5)*log((2*x^2 + sqrt(5)*(2*x + 3) + 6*x + 7)/(x^2 + 3*x
 + 1)) - 3*x + 4*log(x^2 + 3*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{1+3x+x^2}\,dx = \frac{x^2}{2} - 3x + \left(4 - \frac{9\sqrt{5}}{5}\right)\log\left(x - \frac{\sqrt{5}}{2} + \frac{3}{2}\right) + \left(4 + \frac{9\sqrt{5}}{5}\right)\log\left(x + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)$$

input
```
integrate(x**3/(x**2+3*x+1),x)
```

output
```
x**2/2 - 3*x + (4 - 9*sqrt(5)/5)*log(x - sqrt(5)/2 + 3/2) + (4 + 9*sqrt(5)
/5)*log(x + sqrt(5)/2 + 3/2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{1+3x+x^2}\,dx = \frac{1}{2}\,x^2 - \frac{9}{5}\,\sqrt{5}\log\left(\frac{2\,x-\sqrt{5}+3}{2\,x+\sqrt{5}+3}\right) - 3\,x + 4\,\log\left(x^2+3\,x+1\right)$$

input
```
integrate(x^3/(x^2+3*x+1),x, algorithm="maxima")
```

output
```
1/2*x^2 - 9/5*sqrt(5)*log((2*x - sqrt(5) + 3)/(2*x + sqrt(5) + 3)) - 3*x +
 4*log(x^2 + 3*x + 1)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{1+3x+x^2}\,dx = \frac{1}{2}\,x^2 - \frac{9}{5}\,\sqrt{5}\log\left(\frac{\left|2\,x-\sqrt{5}+3\right|}{\left|2\,x+\sqrt{5}+3\right|}\right) - 3\,x + 4\,\log\left(\left|x^2+3\,x+1\right|\right)$$

input
```
integrate(x^3/(x^2+3*x+1),x, algorithm="giac")
```

output
```
1/2*x^2 - 9/5*sqrt(5)*log(abs(2*x - sqrt(5) + 3)/abs(2*x + sqrt(5) + 3)) -
 3*x + 4*log(abs(x^2 + 3*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{x^3}{1+3x+x^2}\,dx = \ln\left(x+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\left(\frac{9\,\sqrt{5}}{5}+4\right) - \ln\left(x-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\left(\frac{9\,\sqrt{5}}{5}-4\right) - 3\,x + \frac{x^2}{2}$$

input
```
int(x^3/(3*x + x^2 + 1),x)
```

output
```
log(x + 5^(1/2)/2 + 3/2)*((9*5^(1/2))/5 + 4) - log(x - 5^(1/2)/2 + 3/2)*((
9*5^(1/2))/5 - 4) - 3*x + x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{1+3x+x^2}\,dx = -\frac{9\sqrt{5}\log\left(-\sqrt{5}+2x+3\right)}{5}+\frac{9\sqrt{5}\log\left(\sqrt{5}+2x+3\right)}{5} + 4\log\left(-\sqrt{5}+2x+3\right)+4\log\left(\sqrt{5}+2x+3\right)+\frac{x^2}{2}-3x$$

input
```
int(x^3/(x^2+3*x+1),x)
```

output
```
( - 18*sqrt(5)*log( - sqrt(5) + 2*x + 3) + 18*sqrt(5)*log(sqrt(5) + 2*x +
3) + 40*log( - sqrt(5) + 2*x + 3) + 40*log(sqrt(5) + 2*x + 3) + 5*x**2 - 3
0*x)/10
```

3.161 $\int \frac{x^2}{1+3x+x^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 52

$$\int \frac{x^2}{1+3x+x^2} \, dx = x-\frac{1}{10}\left(15-7\sqrt{5}\right)\log\left(3-\sqrt{5}+2x\right) -\frac{1}{10}\left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}+2x\right)$$

output
```
x-1/10*(15-7*5^(1/2))*ln(3-5^(1/2)+2*x)-1/10*(15+7*5^(1/2))*ln(3+5^(1/2)+2
*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{1+3x+x^2} \, dx = x+\frac{1}{10}\left(-15+7\sqrt{5}\right)\log\left(-3+\sqrt{5}-2x\right) +\frac{1}{10}\left(-15-7\sqrt{5}\right)\log\left(3+\sqrt{5}+2x\right)$$

input
```
Integrate[x^2/(1 + 3*x + x^2),x]
```

output
```
x + ((-15 + 7*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x])/10 + ((-15 - 7*Sqrt[5])*Lo
g[3 + Sqrt[5] + 2*x])/10
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^2+3x+1} \, dx$$
$$\Big\downarrow \text{1141}$$
$$\int \left(-\frac{15+7\sqrt{5}}{5\left(2x+\sqrt{5}+3\right)}-\frac{15-7\sqrt{5}}{5\left(2x-\sqrt{5}+3\right)}+1\right)dx$$
$$\Big\downarrow \text{2009}$$
$$x-\frac{1}{10}\left(15-7\sqrt{5}\right)\log\left(2x-\sqrt{5}+3\right)-\frac{1}{10}\left(15+7\sqrt{5}\right)\log\left(2x+\sqrt{5}+3\right)$$

input
```
Int[x^2/(1 + 3*x + x^2),x]
```

output
```
x - ((15 - 7*Sqrt[5])*Log[3 - Sqrt[5] + 2*x])/10 - ((15 + 7*Sqrt[5])*Log[3
 + Sqrt[5] + 2*x])/10
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.58

method	result	size
default	$x-\frac{3\ln\left(x^{2}+3x+1\right)}{2}-\frac{7\,\mathrm{arctanh}\left(\frac{(2x+3)\sqrt{5}}{5}\right)\sqrt{5}}{5}$	30
risch	$x-\frac{3\ln\left(3-\sqrt{5}+2x\right)}{2}+\frac{7\ln\left(3-\sqrt{5}+2x\right)\sqrt{5}}{10}-\frac{3\ln\left(3+\sqrt{5}+2x\right)}{2}-\frac{7\ln\left(3+\sqrt{5}+2x\right)\sqrt{5}}{10}$	57

input
```
int(x^2/(x^2+3*x+1),x,method=_RETURNVERBOSE)
```

output
```
x-3/2*ln(x^2+3*x+1)-7/5*arctanh(1/5*(2*x+3)*5^(1/2))*5^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{1+3x+x^2}\,dx=\frac{7}{10}\,\sqrt{5}\log\left(\frac{2\,x^{2}-\sqrt{5}(2\,x+3)+6\,x+7}{x^{2}+3\,x+1}\right)+x-\frac{3}{2}\,\log\left(x^{2}+3\,x+1\right)$$

input
```
integrate(x^2/(x^2+3*x+1),x, algorithm="fricas")
```

output
```
7/10*sqrt(5)*log((2*x^2 - sqrt(5)*(2*x + 3) + 6*x + 7)/(x^2 + 3*x + 1)) +
x - 3/2*log(x^2 + 3*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{x^2}{1+3x+x^2}\,dx = x + \left(-\frac{3}{2}+\frac{7\sqrt{5}}{10}\right)\log\left(x-\frac{\sqrt{5}}{2}+\frac{3}{2}\right) + \left(-\frac{7\sqrt{5}}{10}-\frac{3}{2}\right)\log\left(x+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)$$

input
```
integrate(x**2/(x**2+3*x+1),x)
```

output
```
x + (-3/2 + 7*sqrt(5)/10)*log(x - sqrt(5)/2 + 3/2) + (-7*sqrt(5)/10 - 3/2)
*log(x + sqrt(5)/2 + 3/2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int \frac{x^2}{1+3x+x^2}\,dx = \frac{7}{10}\sqrt{5}\log\left(\frac{2x-\sqrt{5}+3}{2x+\sqrt{5}+3}\right) + x - \frac{3}{2}\log\left(x^2+3x+1\right)$$

input
```
integrate(x^2/(x^2+3*x+1),x, algorithm="maxima")
```

output
```
7/10*sqrt(5)*log((2*x - sqrt(5) + 3)/(2*x + sqrt(5) + 3)) + x - 3/2*log(x^
2 + 3*x + 1)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{1+3x+x^2}\,dx = \frac{7}{10}\sqrt{5}\log\left(\frac{|2\,x-\sqrt{5}+3|}{|2\,x+\sqrt{5}+3|}\right)+x-\frac{3}{2}\,\log\left(|x^2+3\,x+1|\right)$$

input
```
integrate(x^2/(x^2+3*x+1),x, algorithm="giac")
```

output
```
7/10*sqrt(5)*log(abs(2*x - sqrt(5) + 3)/abs(2*x + sqrt(5) + 3)) + x - 3/2*
log(abs(x^2 + 3*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{1+3x+x^2}\,dx = x+\ln\left(x-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\left(\frac{7\sqrt{5}}{10}-\frac{3}{2}\right)-\ln\left(x+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\left(\frac{7\sqrt{5}}{10}+\frac{3}{2}\right)$$

input
```
int(x^2/(3*x + x^2 + 1),x)
```

output
```
x + log(x - 5^(1/2)/2 + 3/2)*((7*5^(1/2))/10 - 3/2) - log(x + 5^(1/2)/2 +
3/2)*((7*5^(1/2))/10 + 3/2)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{1+3x+x^2}\,dx = \frac{7\sqrt{5}\log\left(-\sqrt{5}+2x+3\right)}{10}-\frac{7\sqrt{5}\log\left(\sqrt{5}+2x+3\right)}{10}-\frac{3\log\left(-\sqrt{5}+2x+3\right)}{2}-\frac{3\log\left(\sqrt{5}+2x+3\right)}{2}+x$$

input
```
int(x^2/(x^2+3*x+1),x)
```

output
```
(7*sqrt(5)*log( - sqrt(5) + 2*x + 3) - 7*sqrt(5)*log(sqrt(5) + 2*x + 3) -
15*log( - sqrt(5) + 2*x + 3) - 15*log(sqrt(5) + 2*x + 3) + 10*x)/10
```

3.162 $\int \frac{x}{1+3x+x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 51

$$\int \frac{x}{1+3x+x^2}\,dx=\frac{1}{10}\left(5-3\sqrt{5}\right)\log\left(3-\sqrt{5}+2x\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x\right)$$

output
```
1/10*(5-3*5^(1/2))*ln(3-5^(1/2)+2*x)+1/10*(5+3*5^(1/2))*ln(3+5^(1/2)+2*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{x}{1+3x+x^2}\,dx=\frac{1}{10}\left(5-3\sqrt{5}\right)\log\left(-3+\sqrt{5}-2x\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x\right)$$

input
```
Integrate[x/(1 + 3*x + x^2),x]
```

output
```
((5 - 3*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x])/10 + ((5 + 3*Sqrt[5])*Log[3 + Sq
rt[5] + 2*x])/10
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^2+3x+1}\,dx$$

$$\Big\downarrow \ 1141$$

$$\int \left(\frac{5+3\sqrt{5}}{5\left(2x+\sqrt{5}+3\right)}+\frac{5-3\sqrt{5}}{5\left(2x-\sqrt{5}+3\right)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\log\left(2x-\sqrt{5}+3\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\log\left(2x+\sqrt{5}+3\right)$$

input
```
Int[x/(1 + 3*x + x^2),x]
```

output
```
((5 - 3*Sqrt[5])*Log[3 - Sqrt[5] + 2*x])/10 + ((5 + 3*Sqrt[5])*Log[3 + Sqr
t[5] + 2*x])/10
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{\ln(x^2+3x+1)}{2}+\frac{3\,\operatorname{arctanh}\left(\frac{(2x+3)\sqrt{5}}{5}\right)\sqrt{5}}{5}$	29
risch	$\frac{\ln\left(3+\sqrt{5}+2x\right)}{2}+\frac{3\ln\left(3+\sqrt{5}+2x\right)\sqrt{5}}{10}+\frac{\ln\left(3-\sqrt{5}+2x\right)}{2}-\frac{3\ln\left(3-\sqrt{5}+2x\right)\sqrt{5}}{10}$	56

input
```
int(x/(x^2+3*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^2+3*x+1)+3/5*arctanh(1/5*(2*x+3)*5^(1/2))*5^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{x}{1+3x+x^2}\,dx=\frac{3}{10}\sqrt{5}\log\left(\frac{2\,x^2+\sqrt{5}(2\,x+3)+6\,x+7}{x^2+3\,x+1}\right)+\frac{1}{2}\log\left(x^2+3\,x+1\right)$$

input
```
integrate(x/(x^2+3*x+1),x, algorithm="fricas")
```

output
```
3/10*sqrt(5)*log((2*x^2 + sqrt(5)*(2*x + 3) + 6*x + 7)/(x^2 + 3*x + 1)) +
1/2*log(x^2 + 3*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{x}{1+3x+x^2}\,dx=\left(\frac{1}{2}-\frac{3\sqrt{5}}{10}\right)\log\left(x-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)+\left(\frac{1}{2}+\frac{3\sqrt{5}}{10}\right)\log\left(x+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)$$

input
```
integrate(x/(x**2+3*x+1),x)
```

output
```
(1/2 - 3*sqrt(5)/10)*log(x - sqrt(5)/2 + 3/2) + (1/2 + 3*sqrt(5)/10)*log(x
 + sqrt(5)/2 + 3/2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{x}{1+3x+x^2}\,dx = -\frac{3}{10}\,\sqrt{5}\log\left(\frac{2\,x-\sqrt{5}+3}{2\,x+\sqrt{5}+3}\right)+\frac{1}{2}\,\log\left(x^2+3\,x+1\right)$$

input
```
integrate(x/(x^2+3*x+1),x, algorithm="maxima")
```

output
```
-3/10*sqrt(5)*log((2*x - sqrt(5) + 3)/(2*x + sqrt(5) + 3)) + 1/2*log(x^2 +
 3*x + 1)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{x}{1+3x+x^2}\,dx = -\frac{3}{10}\,\sqrt{5}\log\left(\frac{\left|2\,x-\sqrt{5}+3\right|}{\left|2\,x+\sqrt{5}+3\right|}\right)+\frac{1}{2}\,\log\left(\left|x^2+3\,x+1\right|\right)$$

input
```
integrate(x/(x^2+3*x+1),x, algorithm="giac")
```

output
```
-3/10*sqrt(5)*log(abs(2*x - sqrt(5) + 3)/abs(2*x + sqrt(5) + 3)) + 1/2*log
(abs(x^2 + 3*x + 1))
```

Mupad [B] (verification not implemented)

Time = 8.82 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

$$\int \frac{x}{1+3x+x^2}\,dx = \ln\left(x+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\,\left(\frac{3\,\sqrt{5}}{10}+\frac{1}{2}\right)-\ln\left(x-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\,\left(\frac{3\,\sqrt{5}}{10}-\frac{1}{2}\right)$$

input
```
int(x/(3*x + x^2 + 1),x)
```

output
```
log(x + 5^(1/2)/2 + 3/2)*((3*5^(1/2))/10 + 1/2) - log(x - 5^(1/2)/2 + 3/2)
*((3*5^(1/2))/10 - 1/2)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{x}{1+3x+x^2}\,dx = -\frac{3\sqrt{5}\log\left(-\sqrt{5}+2x+3\right)}{10}+\frac{3\sqrt{5}\log\left(\sqrt{5}+2x+3\right)}{10} + \frac{\log\left(-\sqrt{5}+2x+3\right)}{2}+\frac{\log\left(\sqrt{5}+2x+3\right)}{2}$$

input
```
int(x/(x^2+3*x+1),x)
```

output
```
( - 3*sqrt(5)*log( - sqrt(5) + 2*x + 3) + 3*sqrt(5)*log(sqrt(5) + 2*x + 3)
 + 5*log( - sqrt(5) + 2*x + 3) + 5*log(sqrt(5) + 2*x + 3))/10
```

3.163 $\int \frac{1}{1+3x+x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 19

$$\int \frac{1}{1+3x+x^2} dx = -\frac{2\text{arctanh}\left(\frac{3+2x}{\sqrt{5}}\right)}{\sqrt{5}}$$

output
```
-2/5*arctanh(1/5*(3+2*x)*5^(1/2))*5^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int \frac{1}{1+3x+x^2} dx = \frac{\log\left(-3+\sqrt{5}-2x\right)-\log\left(3+\sqrt{5}+2x\right)}{\sqrt{5}}$$

input
```
Integrate[(1 + 3*x + x^2)^(-1),x]
```

output
```
(Log[-3 + Sqrt[5] - 2*x] - Log[3 + Sqrt[5] + 2*x])/Sqrt[5]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2+3x+1}dx$$

$$\Big\downarrow \text{1081}$$

$$\int \left(\frac{2}{\sqrt{5}\left(2x-\sqrt{5}+3\right)}-\frac{2}{\sqrt{5}\left(2x+\sqrt{5}+3\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log\left(2x-\sqrt{5}+3\right)}{\sqrt{5}}-\frac{\log\left(2x+\sqrt{5}+3\right)}{\sqrt{5}}$$

input
```
Int[(1 + 3*x + x^2)^(-1),x]
```

output
```
Log[3 - Sqrt[5] + 2*x]/Sqrt[5] - Log[3 + Sqrt[5] + 2*x]/Sqrt[5]
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{2 \operatorname{arctanh}\left(\frac{(2x+3)\sqrt{5}}{5}\right)\sqrt{5}}{5}$	17
risch	$\frac{\ln\left(3-\sqrt{5}+2x\right)\sqrt{5}}{5}-\frac{\ln\left(3+\sqrt{5}+2x\right)\sqrt{5}}{5}$	32

input
```
int(1/(x^2+3*x+1),x,method=_RETURNVERBOSE)
```

output
```
-2/5*arctanh(1/5*(2*x+3)*5^(1/2))*5^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(16) = 32.

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{1}{1+3x+x^2}\,dx = \frac{1}{5}\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}(2\,x+3)+6\,x+7}{x^2+3\,x+1}\right)$$

input
```
integrate(1/(x^2+3*x+1),x, algorithm="fricas")
```

output
```
1/5*sqrt(5)*log((2*x^2 - sqrt(5)*(2*x + 3) + 6*x + 7)/(x^2 + 3*x + 1))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{1}{1+3x+x^2}\,dx = \frac{\sqrt{5}\log\left(x-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{5}-\frac{\sqrt{5}\log\left(x+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{5}$$

input
```
integrate(1/(x**2+3*x+1),x)
```

output
```
sqrt(5)*log(x - sqrt(5)/2 + 3/2)/5 - sqrt(5)*log(x + sqrt(5)/2 + 3/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{1}{1+3x+x^2}\,dx = \frac{1}{5}\,\sqrt{5}\log\left(\frac{2\,x-\sqrt{5}+3}{2\,x+\sqrt{5}+3}\right)$$

input
```
integrate(1/(x^2+3*x+1),x, algorithm="maxima")
```

output
```
1/5*sqrt(5)*log((2*x - sqrt(5) + 3)/(2*x + sqrt(5) + 3))
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \frac{1}{1+3x+x^2}\,dx = \frac{1}{5}\,\sqrt{5}\log\left(\frac{\left|2\,x-\sqrt{5}+3\right|}{\left|2\,x+\sqrt{5}+3\right|}\right)$$

input
```
integrate(1/(x^2+3*x+1),x, algorithm="giac")
```

output
```
1/5*sqrt(5)*log(abs(2*x - sqrt(5) + 3)/abs(2*x + sqrt(5) + 3))
```

Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{1+3x+x^2}\,dx = -\frac{2\,\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,(2\,x+3)}{5}\right)}{5}$$

input
```
int(1/(3*x + x^2 + 1),x)
```

output
```
-(2*5^(1/2)*atanh((5^(1/2)*(2*x + 3))/5))/5
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{1}{1+3x+x^2}\,dx = \frac{\sqrt{5}\left(\log\left(-\sqrt{5}+2x+3\right)-\log\left(\sqrt{5}+2x+3\right)\right)}{5}$$

input
```
int(1/(x^2+3*x+1),x)
```

output
```
(sqrt(5)*(log( - sqrt(5) + 2*x + 3) - log(sqrt(5) + 2*x + 3)))/5
```

3.164 $\int \frac{1}{x(1+3x+x^2)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 53

$$\int \frac{1}{x(1+3x+x^2)}\,dx = \log(x) - \frac{1}{10}\left(5+3\sqrt{5}\right)\log\left(3-\sqrt{5}+2x\right) - \frac{1}{10}\left(5-3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x\right)$$

output
```
ln(x)-1/10*(5+3*5^(1/2))*ln(3-5^(1/2)+2*x)-1/10*(5-3*5^(1/2))*ln(3+5^(1/2)
+2*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(1+3x+x^2)}\,dx = \frac{1}{10}\left(-5-3\sqrt{5}\right)\log\left(-3+\sqrt{5}-2x\right) + \log(x) + \frac{1}{10}\left(-5+3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x\right)$$

input
```
Integrate[1/(x*(1 + 3*x + x^2)),x]
```

output
```
((-5 - 3*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x])/10 + Log[x] + ((-5 + 3*Sqrt[5])
*Log[3 + Sqrt[5] + 2*x])/10
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(x^2+3x+1)}dx$$

$$\Big\downarrow \text{1141}$$

$$\int \left(\frac{4}{\sqrt{5}(3+\sqrt{5})(2x+\sqrt{5}+3)}+\frac{2}{(5-3\sqrt{5})x-7\sqrt{5}+15}+\frac{1}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\log(x)+\frac{2\log(2x+\sqrt{5}+3)}{\sqrt{5}(3+\sqrt{5})}+\frac{2\log((5-3\sqrt{5})x-7\sqrt{5}+15)}{5-3\sqrt{5}}$$

input
```
Int[1/(x*(1 + 3*x + x^2)),x]
```

output
```
Log[x] + (2*Log[3 + Sqrt[5] + 2*x])/(Sqrt[5]*(3 + Sqrt[5])) + (2*Log[15 -
7*Sqrt[5] + (5 - 3*Sqrt[5])*x])/(5 - 3*Sqrt[5])
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

method	result	size
default	$\ln\left(x\right)-\frac{\ln\left(x^{2}+3x+1\right)}{2}+\frac{3\,\operatorname{arctanh}\left(\frac{\left(2x+3\right)\sqrt{5}}{5}\right)\sqrt{5}}{5}$	31
risch	$\ln\left(x\right)+\frac{3\ln\left(3x+\frac{3\sqrt{5}}{2}+\frac{9}{2}\right)\sqrt{5}}{10}-\frac{\ln\left(3x+\frac{3\sqrt{5}}{2}+\frac{9}{2}\right)}{2}-\frac{\ln\left(3x+\frac{9}{2}-\frac{3\sqrt{5}}{2}\right)}{2}-\frac{3\ln\left(3x+\frac{9}{2}-\frac{3\sqrt{5}}{2}\right)\sqrt{5}}{10}$	62

input
```
int(1/x/(x^2+3*x+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/2*ln(x^2+3*x+1)+3/5*arctanh(1/5*(2*x+3)*5^(1/2))*5^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(1+3x+x^{2}\right)}\,dx=\frac{3}{10}\,\sqrt{5}\log\left(\frac{2\,x^{2}+\sqrt{5}(2\,x+3)+6\,x+7}{x^{2}+3\,x+1}\right)-\frac{1}{2}\,\log\left(x^{2}+3\,x+1\right)+\log\left(x\right)$$

input
```
integrate(1/x/(x^2+3*x+1),x, algorithm="fricas")
```

output
```
3/10*sqrt(5)*log((2*x^2 + sqrt(5)*(2*x + 3) + 6*x + 7)/(x^2 + 3*x + 1)) -
1/2*log(x^2 + 3*x + 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.70

$$\int \frac{1}{x\left(1+3x+x^{2}\right)}\,dx = \log{\left(x \right)} + \left(-\frac{1}{2} + \frac{3\sqrt{5}}{10}\right)\log{\left(x - \frac{\sqrt{5}}{6} - \frac{20\left(-\frac{1}{2} + \frac{3\sqrt{5}}{10}\right)^{2}}{9} + \frac{55}{18} \right)} + \left(-\frac{3\sqrt{5}}{10} - \frac{1}{2}\right)\log{\left(x - \frac{20\left(-\frac{3\sqrt{5}}{10} - \frac{1}{2}\right)^{2}}{9} + \frac{\sqrt{5}}{6} + \frac{55}{18} \right)}$$

input
```
integrate(1/x/(x**2+3*x+1),x)
```

output
```
log(x) + (-1/2 + 3*sqrt(5)/10)*log(x - sqrt(5)/6 - 20*(-1/2 + 3*sqrt(5)/10
)**2/9 + 55/18) + (-3*sqrt(5)/10 - 1/2)*log(x - 20*(-3*sqrt(5)/10 - 1/2)**
2/9 + sqrt(5)/6 + 55/18)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \frac{1}{x\left(1+3x+x^{2}\right)}\,dx = -\frac{3}{10}\sqrt{5}\log\left(\frac{2\,x-\sqrt{5}+3}{2\,x+\sqrt{5}+3}\right) - \frac{1}{2}\log\left(x^{2}+3\,x+1\right) + \log\left(x\right)$$

input
```
integrate(1/x/(x^2+3*x+1),x, algorithm="maxima")
```

output
```
-3/10*sqrt(5)*log((2*x - sqrt(5) + 3)/(2*x + sqrt(5) + 3)) - 1/2*log(x^2 +
 3*x + 1) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(1+3x+x^2)}\,dx$$
$$= -\frac{3}{10}\sqrt{5}\log\left(\frac{|2x-\sqrt{5}+3|}{|2x+\sqrt{5}+3|}\right) - \frac{1}{2}\log\left(|x^2+3x+1|\right) + \log\left(|x|\right)$$

input
```
integrate(1/x/(x^2+3*x+1),x, algorithm="giac")
```

output
```
-3/10*sqrt(5)*log(abs(2*x - sqrt(5) + 3)/abs(2*x + sqrt(5) + 3)) - 1/2*log
(abs(x^2 + 3*x + 1)) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.57

$$\int \frac{1}{x(1+3x+x^2)}\,dx = \ln(x) - \frac{\ln(x^2+3x+1)}{2} + \frac{3\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}(2x+3)}{5}\right)}{5}$$

input
```
int(1/(x*(3*x + x^2 + 1)),x)
```

output
```
log(x) - log(3*x + x^2 + 1)/2 + (3*5^(1/2)*atanh((5^(1/2)*(2*x + 3))/5))/5
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(1+3x+x^2)}\,dx = -\frac{3\sqrt{5}\log(-\sqrt{5}+2x+3)}{10} + \frac{3\sqrt{5}\log(\sqrt{5}+2x+3)}{10} - \frac{\log(-\sqrt{5}+2x+3)}{2} - \frac{\log(\sqrt{5}+2x+3)}{2} + \log(x)$$

input
```
int(1/x/(x^2+3*x+1),x)
```

output
```
( - 3*sqrt(5)*log( - sqrt(5) + 2*x + 3) + 3*sqrt(5)*log(sqrt(5) + 2*x + 3)
 - 5*log( - sqrt(5) + 2*x + 3) - 5*log(sqrt(5) + 2*x + 3) + 10*log(x))/10
```

3.165 $\int \frac{1}{x^2(1+3x+x^2)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 60

$$\int \frac{1}{x^2(1+3x+x^2)}\,dx = -\frac{1}{x} - 3\log(x) + \frac{1}{10}\left(15+7\sqrt{5}\right)\log\left(3-\sqrt{5}+2x\right) + \frac{1}{10}\left(15-7\sqrt{5}\right)\log\left(3+\sqrt{5}+2x\right)$$

output
```
-1/x-3*ln(x)+1/10*(15+7*5^(1/2))*ln(3-5^(1/2)+2*x)+1/10*(15-7*5^(1/2))*ln(
3+5^(1/2)+2*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^2(1+3x+x^2)}\,dx = -\frac{1}{x} + \frac{1}{10}\left(15+7\sqrt{5}\right)\log\left(-3+\sqrt{5}-2x\right) - 3\log(x) + \frac{1}{10}\left(15-7\sqrt{5}\right)\log\left(3+\sqrt{5}+2x\right)$$

input
```
Integrate[1/(x^2*(1 + 3*x + x^2)),x]
```

output
```
-x^(-1) + ((15 + 7*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x])/10 - 3*Log[x] + ((15
- 7*Sqrt[5])*Log[3 + Sqrt[5] + 2*x])/10
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (x^2+3x+1)} \, dx$$

$$\Big\downarrow \ 1141$$

$$\int \left(\frac{1}{x^2} - \frac{8}{\sqrt{5}\left(3+\sqrt{5}\right)^2 \left(2x+\sqrt{5}+3\right)} - \frac{2}{\left(15-7\sqrt{5}\right)x+2\left(20-9\sqrt{5}\right)} - \frac{3}{x}\right) dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{1}{x} - 3\log(x) - \frac{4\log\left(2x+\sqrt{5}+3\right)}{\sqrt{5}\left(3+\sqrt{5}\right)^2} - \frac{2\log\left(-\left(\left(15-7\sqrt{5}\right)x\right)-2\left(20-9\sqrt{5}\right)\right)}{15-7\sqrt{5}}$$

input
```
Int[1/(x^2*(1 + 3*x + x^2)),x]
```

output
```
-x^(-1) - 3*Log[x] - (4*Log[3 + Sqrt[5] + 2*x])/(Sqrt[5]*(3 + Sqrt[5])^2)
- (2*Log[-2*(20 - 9*Sqrt[5]) - (15 - 7*Sqrt[5])*x])/(15 - 7*Sqrt[5])
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.63

method	result	size
default	$-\frac{1}{x}-3\ln\left(x\right)+\frac{3\ln\left(x^{2}+3x+1\right)}{2}-\frac{7\,\operatorname{arctanh}\left(\frac{(2x+3)\sqrt{5}}{5}\right)\sqrt{5}}{5}$	38
risch	$-\frac{1}{x}+\frac{3\ln\left(3-\sqrt{5}+2x\right)}{2}+\frac{7\ln\left(3-\sqrt{5}+2x\right)\sqrt{5}}{10}+\frac{3\ln\left(3+\sqrt{5}+2x\right)}{2}-\frac{7\ln\left(3+\sqrt{5}+2x\right)\sqrt{5}}{10}-3\ln\left(x\right)$	65

input
```
int(1/x^2/(x^2+3*x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/x-3*ln(x)+3/2*ln(x^2+3*x+1)-7/5*arctanh(1/5*(2*x+3)*5^(1/2))*5^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^2\left(1+3x+x^2\right)}\,dx = \frac{7\sqrt{5}x\log\left(\frac{2\,x^2-\sqrt{5}(2\,x+3)+6\,x+7}{x^2+3\,x+1}\right)+15\,x\log\left(x^2+3\,x+1\right)-30\,x\log\left(x\right)-10}{10\,x}$$

input
```
integrate(1/x^2/(x^2+3*x+1),x, algorithm="fricas")
```

output
```
1/10*(7*sqrt(5)*x*log((2*x^2 - sqrt(5)*(2*x + 3) + 6*x + 7)/(x^2 + 3*x + 1
)) + 15*x*log(x^2 + 3*x + 1) - 30*x*log(x) - 10)/x
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^2(1+3x+x^2)}\,dx = -3\log(x) + \left(\frac{3}{2}-\frac{7\sqrt{5}}{10}\right)\log\left(x-\frac{5\sqrt{5}}{89}-\frac{165\left(\frac{3}{2}-\frac{7\sqrt{5}}{10}\right)^2}{623}+\frac{1710}{623}\right) + \left(\frac{3}{2}+\frac{7\sqrt{5}}{10}\right)\log\left(x-\frac{165\left(\frac{3}{2}+\frac{7\sqrt{5}}{10}\right)^2}{623}+\frac{5\sqrt{5}}{89}+\frac{1710}{623}\right)-\frac{1}{x}$$

input
```
integrate(1/x**2/(x**2+3*x+1),x)
```

output
```
-3*log(x) + (3/2 - 7*sqrt(5)/10)*log(x - 5*sqrt(5)/89 - 165*(3/2 - 7*sqrt(
5)/10)**2/623 + 1710/623) + (3/2 + 7*sqrt(5)/10)*log(x - 165*(3/2 + 7*sqrt
(5)/10)**2/623 + 5*sqrt(5)/89 + 1710/623) - 1/x
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^2(1+3x+x^2)}\,dx = \frac{7}{10}\sqrt{5}\log\left(\frac{2x-\sqrt{5}+3}{2x+\sqrt{5}+3}\right)-\frac{1}{x}+\frac{3}{2}\log\left(x^2+3x+1\right)-3\log(x)$$

input
```
integrate(1/x^2/(x^2+3*x+1),x, algorithm="maxima")
```

output
```
7/10*sqrt(5)*log((2*x - sqrt(5) + 3)/(2*x + sqrt(5) + 3)) - 1/x + 3/2*log(
x^2 + 3*x + 1) - 3*log(x)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^2(1+3x+x^2)}\,dx = \frac{7}{10}\sqrt{5}\log\left(\frac{|2\,x-\sqrt{5}+3|}{|2\,x+\sqrt{5}+3|}\right)-\frac{1}{x}+\frac{3}{2}\,\log\left(|x^2+3\,x+1|\right)-3\,\log\left(|x|\right)$$

input
```
integrate(1/x^2/(x^2+3*x+1),x, algorithm="giac")
```

output
```
7/10*sqrt(5)*log(abs(2*x - sqrt(5) + 3)/abs(2*x + sqrt(5) + 3)) - 1/x + 3/
2*log(abs(x^2 + 3*x + 1)) - 3*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 8.79 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^2(1+3x+x^2)}\,dx = \frac{3\,\ln\left(x^2+3\,x+1\right)}{2}-3\,\ln\left(x\right)-\frac{1}{x}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\,(2\,x+3)\,1\mathrm{i}}{5}\right)\,7\mathrm{i}}{5}$$

input
```
int(1/(x^2*(3*x + x^2 + 1)),x)
```

output
```
(3*log(3*x + x^2 + 1))/2 - 3*log(x) + (5^(1/2)*atan((5^(1/2)*(2*x + 3)*1i)
/5)*7i)/5 - 1/x
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2(1+3x+x^2)}\,dx = \frac{7\sqrt{5}\log\left(-\sqrt{5}+2x+3\right)x-7\sqrt{5}\log\left(\sqrt{5}+2x+3\right)x+15\log\left(-\sqrt{5}+2x+3\right)x+15\log\left(\sqrt{5}+2x+3\right.}{10x}$$

input
```
int(1/x^2/(x^2+3*x+1),x)
```

output
```
(7*sqrt(5)*log( - sqrt(5) + 2*x + 3)*x - 7*sqrt(5)*log(sqrt(5) + 2*x + 3)*
x + 15*log( - sqrt(5) + 2*x + 3)*x + 15*log(sqrt(5) + 2*x + 3)*x - 30*log(
x)*x - 10)/(10*x)
```

3.166 $\int \frac{1}{x^3(1+3x+x^2)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 75

$$\int \frac{1}{x^3(1+3x+x^2)}\,dx = -\frac{1}{2x^2}+\frac{3}{x}+8\log(x)-\frac{8\log\left(3-\sqrt{5}+2x\right)}{\sqrt{5}\left(3-\sqrt{5}\right)^3}+\frac{8\log\left(3+\sqrt{5}+2x\right)}{\sqrt{5}\left(3+\sqrt{5}\right)^3}$$

output
```
-1/2/x^2+3/x+8*ln(x)-8/5*ln(3-5^(1/2)+2*x)*5^(1/2)/(3-5^(1/2))^3+8/5*ln(3+
5^(1/2)+2*x)*5^(1/2)/(3+5^(1/2))^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3(1+3x+x^2)}\,dx = -\frac{1}{2x^2}+\frac{3}{x}+\frac{1}{5}\left(-20-9\sqrt{5}\right)\log\left(-3+\sqrt{5}-2x\right)+8\log(x)+\frac{\log\left(3+\sqrt{5}+2x\right)}{20+9\sqrt{5}}$$

input
```
Integrate[1/(x^3*(1 + 3*x + x^2)),x]
```

output
```
-1/2*1/x^2 + 3/x + ((-20 - 9*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x])/5 + 8*Log[x
] + Log[3 + Sqrt[5] + 2*x]/(20 + 9*Sqrt[5])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (x^2+3x+1)} \, dx$$

$$\Big\downarrow \text{1141}$$

$$\int \left(\frac{1}{x^3}-\frac{3}{x^2}-\frac{16}{\sqrt{5}\left(3-\sqrt{5}\right)^3\left(2x-\sqrt{5}+3\right)}+\frac{16}{\sqrt{5}\left(3+\sqrt{5}\right)^3\left(2x+\sqrt{5}+3\right)}+\frac{8}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2x^2}+\frac{3}{x}+8\log(x)-\frac{8\log\left(2x-\sqrt{5}+3\right)}{\sqrt{5}\left(3-\sqrt{5}\right)^3}+\frac{8\log\left(2x+\sqrt{5}+3\right)}{\sqrt{5}\left(3+\sqrt{5}\right)^3}$$

input
```
Int[1/(x^3*(1 + 3*x + x^2)),x]
```

output
```
-1/2*1/x^2 + 3/x + 8*Log[x] - (8*Log[3 - Sqrt[5] + 2*x])/(Sqrt[5]*(3 - Sqr
t[5])^3) + (8*Log[3 + Sqrt[5] + 2*x])/(Sqrt[5]*(3 + Sqrt[5])^3)
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.57

method	result
default	$-\frac{1}{2x^2}+\frac{3}{x}+8\ln\left(x\right)-4\ln\left(x^2+3x+1\right)+\frac{18\,\operatorname{arctanh}\left(\frac{(2x+3)\sqrt{5}}{5}\right)\sqrt{5}}{5}$
risch	$\frac{3x-\frac{1}{2}}{x^2}-4\ln\left(3+\sqrt{5}+2x\right)+\frac{9\ln\left(3+\sqrt{5}+2x\right)\sqrt{5}}{5}-4\ln\left(3-\sqrt{5}+2x\right)-\frac{9\ln\left(3-\sqrt{5}+2x\right)\sqrt{5}}{5}+8\ln\left(x\right)$

input
```
int(1/x^3/(x^2+3*x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2+3/x+8*ln(x)-4*ln(x^2+3*x+1)+18/5*arctanh(1/5*(2*x+3)*5^(1/2))*5^(
1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

$$\int\frac{1}{x^3\left(1+3x+x^2\right)}\,dx = \frac{18\,\sqrt{5}x^2\log\left(\frac{2\,x^2+\sqrt{5}(2\,x+3)+6\,x+7}{x^2+3\,x+1}\right)-40\,x^2\log\left(x^2+3\,x+1\right)+80\,x^2\log\left(x\right)+30\,x-5}{10\,x^2}$$

input
```
integrate(1/x^3/(x^2+3*x+1),x, algorithm="fricas")
```

output
```
1/10*(18*sqrt(5)*x^2*log((2*x^2 + sqrt(5)*(2*x + 3) + 6*x + 7)/(x^2 + 3*x
+ 1)) - 40*x^2*log(x^2 + 3*x + 1) + 80*x^2*log(x) + 30*x - 5)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^3(1+3x+x^2)}\,dx = 8\log(x) + \left(-4+\frac{9\sqrt{5}}{5}\right)\log\left(x-\frac{19\sqrt{5}}{426}-\frac{145\left(-4+\frac{9\sqrt{5}}{5}\right)^2}{3834}+\frac{5210}{1917}\right) + \left(-\frac{9\sqrt{5}}{5}-4\right)\log\left(x-\frac{145\left(-\frac{9\sqrt{5}}{5}-4\right)^2}{3834}+\frac{19\sqrt{5}}{426}+\frac{5210}{1917}\right) + \frac{6x-1}{2x^2}$$

input
```
integrate(1/x**3/(x**2+3*x+1),x)
```

output
```
8*log(x) + (-4 + 9*sqrt(5)/5)*log(x - 19*sqrt(5)/426 - 145*(-4 + 9*sqrt(5)
/5)**2/3834 + 5210/1917) + (-9*sqrt(5)/5 - 4)*log(x - 145*(-9*sqrt(5)/5 -
4)**2/3834 + 19*sqrt(5)/426 + 5210/1917) + (6*x - 1)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^3(1+3x+x^2)}\,dx = -\frac{9}{5}\sqrt{5}\log\left(\frac{2x-\sqrt{5}+3}{2x+\sqrt{5}+3}\right)+\frac{6x-1}{2x^2} - 4\log\left(x^2+3x+1\right)+8\log(x)$$

input
```
integrate(1/x^3/(x^2+3*x+1),x, algorithm="maxima")
```

output
```
-9/5*sqrt(5)*log((2*x - sqrt(5) + 3)/(2*x + sqrt(5) + 3)) + 1/2*(6*x - 1)/
x^2 - 4*log(x^2 + 3*x + 1) + 8*log(x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^3(1+3x+x^2)}\,dx = -\frac{9}{5}\sqrt{5}\log\left(\frac{|2x-\sqrt{5}+3|}{|2x+\sqrt{5}+3|}\right)+\frac{6x-1}{2x^2} - 4\log\left(|x^2+3x+1|\right)+8\log\left(|x|\right)$$

input
```
integrate(1/x^3/(x^2+3*x+1),x, algorithm="giac")
```

output
```
-9/5*sqrt(5)*log(abs(2*x - sqrt(5) + 3)/abs(2*x + sqrt(5) + 3)) + 1/2*(6*x
 - 1)/x^2 - 4*log(abs(x^2 + 3*x + 1)) + 8*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 8.85 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^3(1+3x+x^2)}\,dx = 8\ln(x)+\frac{3x-\frac{1}{2}}{x^2}-\ln\left(x-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\left(\frac{9\sqrt{5}}{5}+4\right) + \ln\left(x+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\left(\frac{9\sqrt{5}}{5}-4\right)$$

input
```
int(1/(x^3*(3*x + x^2 + 1)),x)
```

output
```
8*log(x) + (3*x - 1/2)/x^2 - log(x - 5^(1/2)/2 + 3/2)*((9*5^(1/2))/5 + 4)
+ log(x + 5^(1/2)/2 + 3/2)*((9*5^(1/2))/5 - 4)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3 (1+3x+x^2)} \, dx$$
$$= \frac{-18\sqrt{5}\log\left(-\sqrt{5}+2x+3\right)x^2+18\sqrt{5}\log\left(\sqrt{5}+2x+3\right)x^2-40\log\left(-\sqrt{5}+2x+3\right)x^2-40\log\left(\sqrt{5}+\right.}{10x^2}$$

input
```
int(1/x^3/(x^2+3*x+1),x)
```

output
```
( - 18*sqrt(5)*log( - sqrt(5) + 2*x + 3)*x**2 + 18*sqrt(5)*log(sqrt(5) + 2
*x + 3)*x**2 - 40*log( - sqrt(5) + 2*x + 3)*x**2 - 40*log(sqrt(5) + 2*x +
3)*x**2 + 80*log(x)*x**2 + 30*x - 5)/(10*x**2)
```

3.167 $\int \frac{x}{6-5x+x^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 17

$$\int \frac{x}{6-5x+x^2} \, dx = -2\log(2-x) + 3\log(3-x)$$

output
```
-2*ln(2-x)+3*ln(3-x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x}{6-5x+x^2} \, dx = -2\log(2-x) + 3\log(3-x)$$

input
```
Integrate[x/(6 - 5*x + x^2),x]
```

output
```
-2*Log[2 - x] + 3*Log[3 - x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^2-5x+6} \, dx$$

$$\Big\downarrow \text{ 1141}$$

$$\int \left(\frac{2}{2-x}-\frac{3}{3-x}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$3\log(3-x)-2\log(2-x)$$

input
```
Int[x/(6 - 5*x + x^2),x]
```

output
```
-2*Log[2 - x] + 3*Log[3 - x]
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$3\ln(-3+x)-2\ln(x-2)$	14
norman	$3\ln(-3+x)-2\ln(x-2)$	14
risch	$3\ln(-3+x)-2\ln(x-2)$	14
parallelrisch	$3\ln(-3+x)-2\ln(x-2)$	14

input
```
int(x/(x^2-5*x+6),x,method=_RETURNVERBOSE)
```

output
```
3*ln(-3+x)-2*ln(x-2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{x}{6-5x+x^2}\,dx = -2\,\log(x-2)+3\,\log(x-3)$$

input
```
integrate(x/(x^2-5*x+6),x, algorithm="fricas")
```

output
```
-2*log(x - 2) + 3*log(x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{x}{6-5x+x^2}\,dx = 3\log(x-3)-2\log(x-2)$$

input
```
integrate(x/(x**2-5*x+6),x)
```

output
```
3*log(x - 3) - 2*log(x - 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{x}{6-5x+x^2}\,dx = -2\,\log\left(x-2\right)+3\,\log\left(x-3\right)$$

input
```
integrate(x/(x^2-5*x+6),x, algorithm="maxima")
```

output
```
-2*log(x - 2) + 3*log(x - 3)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x}{6-5x+x^2}\,dx = -2\,\log\left(|x-2|\right)+3\,\log\left(|x-3|\right)$$

input
```
integrate(x/(x^2-5*x+6),x, algorithm="giac")
```

output
```
-2*log(abs(x - 2)) + 3*log(abs(x - 3))
```

Mupad [B] (verification not implemented)

Time = 8.71 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{x}{6-5x+x^2}\,dx = 3\,\ln\left(x-3\right)-2\,\ln\left(x-2\right)$$

input
```
int(x/(x^2 - 5*x + 6),x)
```

output
```
3*log(x - 3) - 2*log(x - 2)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{x}{6-5x+x^2}\,dx = 3\log(x-3)-2\log(x-2)$$

input
```
int(x/(x^2-5*x+6),x)
```

output
```
3*log(x - 3) - 2*log(x - 2)
```

3.168 $\int \frac{x^2}{2-3x+x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 18

$$\int \frac{x^2}{2-3x+x^2}\,dx = x-\log(1-x)+4\log(2-x)$$

output
```
x-ln(1-x)+4*ln(2-x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{2-3x+x^2}\,dx = x-\log(1-x)+4\log(2-x)$$

input
```
Integrate[x^2/(2 - 3*x + x^2),x]
```

output
```
x - Log[1 - x] + 4*Log[2 - x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^2-3x+2}\,dx$$

$$\Big\downarrow \text{ 1141}$$

$$\int \left(-\frac{4}{2-x}+\frac{1}{1-x}+1\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$x-\log(1-x)+4\log(2-x)$$

input
```
Int[x^2/(2 - 3*x + x^2),x]
```

output
```
x - Log[1 - x] + 4*Log[2 - x]
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
default	$x + 4\ln(x-2) - \ln(x-1)$	15
norman	$x + 4\ln(x-2) - \ln(x-1)$	15
risch	$x + 4\ln(x-2) - \ln(x-1)$	15
parallelrisch	$x + 4\ln(x-2) - \ln(x-1)$	15

input
```
int(x^2/(x^2-3*x+2),x,method=_RETURNVERBOSE)
```

output
```
x+4*ln(x-2)-ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{2-3x+x^2}\,dx = x - \log(x-1) + 4\,\log(x-2)$$

input
```
integrate(x^2/(x^2-3*x+2),x, algorithm="fricas")
```

output
```
x - log(x - 1) + 4*log(x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{2-3x+x^2}\,dx = x + 4\log(x-2) - \log(x-1)$$

input
```
integrate(x**2/(x**2-3*x+2),x)
```

output
```
x + 4*log(x - 2) - log(x - 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{2-3x+x^2}\,dx = x-\log(x-1)+4\log(x-2)$$

input
```
integrate(x^2/(x^2-3*x+2),x, algorithm="maxima")
```

output
```
x - log(x - 1) + 4*log(x - 2)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{2-3x+x^2}\,dx = x-\log(|x-1|)+4\log(|x-2|)$$

input
```
integrate(x^2/(x^2-3*x+2),x, algorithm="giac")
```

output
```
x - log(abs(x - 1)) + 4*log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{2-3x+x^2}\,dx = x-\ln(x-1)+4\ln(x-2)$$

input
```
int(x^2/(x^2 - 3*x + 2),x)
```

output
```
x - log(x - 1) + 4*log(x - 2)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{2-3x+x^2}\,dx = 4\log(x-2)-\log(x-1)+x$$

input
```
int(x^2/(x^2-3*x+2),x)
```

output
```
4*log(x - 2) - log(x - 1) + x
```

3.169 $\int \frac{x^2}{-6+x+x^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 20

$$\int \frac{x^2}{-6+x+x^2} \, dx = x + \frac{4}{5} \log(2-x) - \frac{9}{5} \log(3+x)$$

output
```
x+4/5*ln(2-x)-9/5*ln(3+x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{-6+x+x^2} \, dx = x + \frac{4}{5} \log(2-x) - \frac{9}{5} \log(3+x)$$

input
```
Integrate[x^2/(-6 + x + x^2),x]
```

output
```
x + (4*Log[2 - x])/5 - (9*Log[3 + x])/5
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^2+x-6} \, dx$$

$$\Big\downarrow \text{1141}$$

$$\int \left(-\frac{9}{5(x+3)}-\frac{4}{5(2-x)}+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x+\frac{4}{5}\log(2-x)-\frac{9}{5}\log(x+3)$$

input
```
Int[x^2/(-6 + x + x^2),x]
```

output
```
x + (4*Log[2 - x])/5 - (9*Log[3 + x])/5
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
default	$x+\frac{4\ln(x-2)}{5}-\frac{9\ln(3+x)}{5}$	15
norman	$x+\frac{4\ln(x-2)}{5}-\frac{9\ln(3+x)}{5}$	15
risch	$x+\frac{4\ln(x-2)}{5}-\frac{9\ln(3+x)}{5}$	15
parallelrisch	$x+\frac{4\ln(x-2)}{5}-\frac{9\ln(3+x)}{5}$	15

input
```
int(x^2/(x^2+x-6),x,method=_RETURNVERBOSE)
```

output
```
x+4/5*ln(x-2)-9/5*ln(3+x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{-6+x+x^2}\,dx = x-\frac{9}{5}\,\log\left(x+3\right)+\frac{4}{5}\,\log\left(x-2\right)$$

input
```
integrate(x^2/(x^2+x-6),x, algorithm="fricas")
```

output
```
x - 9/5*log(x + 3) + 4/5*log(x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{-6+x+x^2}\,dx = x+\frac{4\log\left(x-2\right)}{5}-\frac{9\log\left(x+3\right)}{5}$$

input
```
integrate(x**2/(x**2+x-6),x)
```

output
```
x + 4*log(x - 2)/5 - 9*log(x + 3)/5
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{-6+x+x^2}\,dx = x - \frac{9}{5}\log\left(x+3\right) + \frac{4}{5}\log\left(x-2\right)$$

input
```
integrate(x^2/(x^2+x-6),x, algorithm="maxima")
```

output
```
x - 9/5*log(x + 3) + 4/5*log(x - 2)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{-6+x+x^2}\,dx = x - \frac{9}{5}\log\left(|x+3|\right) + \frac{4}{5}\log\left(|x-2|\right)$$

input
```
integrate(x^2/(x^2+x-6),x, algorithm="giac")
```

output
```
x - 9/5*log(abs(x + 3)) + 4/5*log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 8.76 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{-6+x+x^2}\,dx = x + \frac{4\ln\left(x-2\right)}{5} - \frac{9\ln\left(x+3\right)}{5}$$

input
```
int(x^2/(x + x^2 - 6),x)
```

output
```
x + (4*log(x - 2))/5 - (9*log(x + 3))/5
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{-6+x+x^2}\,dx = \frac{4\log(x-2)}{5} - \frac{9\log(x+3)}{5} + x$$

input
```
int(x^2/(x^2+x-6),x)
```

output
```
(4*log(x - 2) - 9*log(x + 3) + 5*x)/5
```

3.170 $\int \frac{x^3}{2-3x+x^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \frac{x^3}{2-3x+x^2} dx = 3x + \frac{x^2}{2} - \log(1-x) + 8\log(2-x)$$

output
```
3*x+1/2*x^2-ln(1-x)+8*ln(2-x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{2-3x+x^2} dx = 3x + \frac{x^2}{2} - \log(1-x) + 8\log(2-x)$$

input
```
Integrate[x^3/(2 - 3*x + x^2),x]
```

output
```
3*x + x^2/2 - Log[1 - x] + 8*Log[2 - x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^2-3x+2}\,dx$$

$$\Big\downarrow \text{1141}$$

$$\int \left(x+\frac{1}{1-x}-\frac{8}{2-x}+3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2}{2}+3x-\log(1-x)+8\log(2-x)$$

input
```
Int[x^3/(2 - 3*x + x^2),x]
```

output
```
3*x + x^2/2 - Log[1 - x] + 8*Log[2 - x]
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
default	$3x+\frac{x^2}{2}+8\ln\left(x-2\right)-\ln\left(x-1\right)$	22
norman	$3x+\frac{x^2}{2}+8\ln\left(x-2\right)-\ln\left(x-1\right)$	22
risch	$3x+\frac{x^2}{2}+8\ln\left(x-2\right)-\ln\left(x-1\right)$	22
parallelrisch	$3x+\frac{x^2}{2}+8\ln\left(x-2\right)-\ln\left(x-1\right)$	22

input
```
int(x^3/(x^2-3*x+2),x,method=_RETURNVERBOSE)
```

output
```
3*x+1/2*x^2+8*ln(x-2)-ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{2-3x+x^2}\,dx=\frac{1}{2}\,x^2+3\,x-\log\left(x-1\right)+8\,\log\left(x-2\right)$$

input
```
integrate(x^3/(x^2-3*x+2),x, algorithm="fricas")
```

output
```
1/2*x^2 + 3*x - log(x - 1) + 8*log(x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x^3}{2-3x+x^2}\,dx=\frac{x^2}{2}+3x+8\log\left(x-2\right)-\log\left(x-1\right)$$

input
```
integrate(x**3/(x**2-3*x+2),x)
```

output
```
x**2/2 + 3*x + 8*log(x - 2) - log(x - 1)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{2-3x+x^2}\,dx = \frac{1}{2}\,x^2 + 3\,x - \log\left(x-1\right) + 8\,\log\left(x-2\right)$$

input
```
integrate(x^3/(x^2-3*x+2),x, algorithm="maxima")
```

output
```
1/2*x^2 + 3*x - log(x - 1) + 8*log(x - 2)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{2-3x+x^2}\,dx = \frac{1}{2}\,x^2 + 3\,x - \log\left(|x-1|\right) + 8\,\log\left(|x-2|\right)$$

input
```
integrate(x^3/(x^2-3*x+2),x, algorithm="giac")
```

output
```
1/2*x^2 + 3*x - log(abs(x - 1)) + 8*log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{2-3x+x^2}\,dx = 3\,x - \ln\left(x-1\right) + 8\,\ln\left(x-2\right) + \frac{x^2}{2}$$

input
```
int(x^3/(x^2 - 3*x + 2),x)
```

output
```
3*x - log(x - 1) + 8*log(x - 2) + x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{2-3x+x^2}\,dx = 8\log(x-2) - \log(x-1) + \frac{x^2}{2} + 3x$$

input
```
int(x^3/(x^2-3*x+2),x)
```

output
```
(16*log(x - 2) - 2*log(x - 1) + x**2 + 6*x)/2
```

3.171 $\int \frac{\sqrt{x}}{\sqrt{-8+6x-x^2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 22

$$\int \frac{\sqrt{x}}{\sqrt{-8+6x-x^2}} \, dx = -4E\left(\arcsin\left(\frac{\sqrt{4-x}}{\sqrt{2}}\right)|\frac{1}{2}\right)$$

output
```
-4*EllipticE(1/2*(4-x)^(1/2)*2^(1/2),1/2*2^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 69 vs. 2(22) = 44.

Time = 20.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.14

$$\int \frac{\sqrt{x}}{\sqrt{-8+6x-x^2}} \, dx = -\frac{4\sqrt{2-x}\sqrt{4-x}\left(E\left(\arcsin\left(\frac{\sqrt{x}}{\sqrt{2}}\right)|\frac{1}{2}\right)-\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{x}}{\sqrt{2}}\right),\frac{1}{2}\right)\right)}{\sqrt{-8+6x-x^2}}$$

input
```
Integrate[Sqrt[x]/Sqrt[-8 + 6*x - x^2],x]
```

output
```
(-4*Sqrt[2 - x]*Sqrt[4 - x]*(EllipticE[ArcSin[Sqrt[x]/Sqrt[2]], 1/2] - Ell
ipticF[ArcSin[Sqrt[x]/Sqrt[2]], 1/2]))/Sqrt[-8 + 6*x - x^2]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1170, 1452, 27, 389, 322, 328}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{-x^2+6x-8}}dx$$
$$\Big\downarrow \ 1170$$
$$2\int \frac{x}{\sqrt{-x^2+6x-8}}d\sqrt{x}$$
$$\Big\downarrow \ 1452$$
$$4\int \frac{x}{2\sqrt{4-x}\sqrt{x-2}}d\sqrt{x}$$
$$\Big\downarrow \ 27$$
$$2\int \frac{x}{\sqrt{4-x}\sqrt{x-2}}d\sqrt{x}$$
$$\Big\downarrow \ 389$$
$$2\left(2\int \frac{1}{\sqrt{4-x}\sqrt{x-2}}d\sqrt{x}+\int \frac{\sqrt{x-2}}{\sqrt{4-x}}d\sqrt{x}\right)$$
$$\Big\downarrow \ 322$$
$$2\left(\int \frac{\sqrt{x-2}}{\sqrt{4-x}}d\sqrt{x}-\sqrt{2}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt{x}}{2}\right),2\right)\right)$$
$$\Big\downarrow \ 328$$
$$2\left(-\sqrt{2}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt{x}}{2}\right),2\right)-\sqrt{2}E\left(\arccos\left(\frac{\sqrt{x}}{2}\right)\middle|2\right)\right)$$

input
```
Int[Sqrt[x]/Sqrt[-8 + 6*x - x^2],x]
```

output
```
2*(-(Sqrt[2]*EllipticE[ArcCos[Sqrt[x]/2], 2]) - Sqrt[2]*EllipticF[ArcCos[S
qrt[x]/2], 2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 322
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

rule 328
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(-Sqrt[a - b*(c/d)]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcCos[Rt[-d/c, 2]*x],
 b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0]
 && GtQ[a - b*(c/d), 0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

rule 1170
```
Int[(x_)^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2
 Subst[Int[x^(2*m + 1)/Sqrt[a + b*x^2 + c*x^4], x], x, Sqrt[x]], x] /; Free
Q[{a, b, c}, x] && EqQ[m^2, 1/4]
```

rule 1452
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[x^2/(Sqrt[b + q + 2*c*x^2]*Sqrt
[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
&& LtQ[c, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.64

method	result	size
default	$\frac{\left(\operatorname{EllipticE}\left(\frac{\sqrt{-2x+4}}{2}, i\right)-2\operatorname{EllipticF}\left(\frac{\sqrt{-2x+4}}{2}, i\right)\right)\sqrt{-2x+4}\,\sqrt{-2x+8}\,\sqrt{2}}{\sqrt{-x^2+6x-8}}$	58
elliptic	$-\frac{\sqrt{-(x^2-6x+8)x}\,\sqrt{-2x+4}\,\sqrt{-2x+8}\,\sqrt{2}\left(-2\operatorname{EllipticE}\left(\frac{\sqrt{-2x+4}}{2}, i\right)+4\operatorname{EllipticF}\left(\frac{\sqrt{-2x+4}}{2}, i\right)\right)}{2\sqrt{-x^2+6x-8}\,\sqrt{-x^3+6x^2-8x}}$	90

input
```
int(x^(1/2)/(-x^2+6*x-8)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(EllipticE(1/2*(-2*x+4)^(1/2),I)-2*EllipticF(1/2*(-2*x+4)^(1/2),I))*(-2*x+
4)^(1/2)*(-2*x+8)^(1/2)*2^(1/2)/(-x^2+6*x-8)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{x}}{\sqrt{-8+6x-x^2}}\,dx = -4i\,\text{weierstrassPInverse}(16,0,x-2) + 2i\,\text{weierstrassZeta}(16,0,\text{weierstrassPInverse}(16,0,x-2))$$

input
```
integrate(x^(1/2)/(-x^2+6*x-8)^(1/2),x, algorithm="fricas")
```

output
```
-4*I*weierstrassPInverse(16, 0, x - 2) + 2*I*weierstrassZeta(16, 0, weiers
trassPInverse(16, 0, x - 2))
```

Sympy [F]

$$\int \frac{\sqrt{x}}{\sqrt{-8+6x-x^2}}\,dx = \int \frac{\sqrt{x}}{\sqrt{-(x-4)(x-2)}}\,dx$$

input
```
integrate(x**(1/2)/(-x**2+6*x-8)**(1/2),x)
```

output
```
Integral(sqrt(x)/sqrt(-(x - 4)*(x - 2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{-8+6x-x^2}}\,dx = \int \frac{\sqrt{x}}{\sqrt{-x^2+6\,x-8}}\,dx$$

input
```
integrate(x^(1/2)/(-x^2+6*x-8)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/sqrt(-x^2 + 6*x - 8), x)
```

Giac [F]

$$\int \frac{\sqrt{x}}{\sqrt{-8+6x-x^2}}\,dx = \int \frac{\sqrt{x}}{\sqrt{-x^2+6\,x-8}}\,dx$$

input
```
integrate(x^(1/2)/(-x^2+6*x-8)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x)/sqrt(-x^2 + 6*x - 8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{-8+6x-x^2}}\,dx = \int \frac{\sqrt{x}}{\sqrt{-x^2+6\,x-8}}\,dx$$

input
```
int(x^(1/2)/(6*x - x^2 - 8)^(1/2),x)
```

output
```
int(x^(1/2)/(6*x - x^2 - 8)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{x}}{\sqrt{-8+6x-x^2}}\,dx = -\left(\int \frac{\sqrt{x}\,\sqrt{-x^2+6x-8}}{x^2-6x+8}dx\right)$$

input
```
int(x^(1/2)/(-x^2+6*x-8)^(1/2),x)
```

output
```
- int((sqrt(x)*sqrt( - x**2 + 6*x - 8))/(x**2 - 6*x + 8),x)
```

3.172 $\int \frac{\sqrt{x}}{\sqrt{(4-x)(-2+x)}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \frac{\sqrt{x}}{\sqrt{(4-x)(-2+x)}} \, dx = -4E\left(\arcsin\left(\frac{\sqrt{4-x}}{\sqrt{2}}\right)|\frac{1}{2}\right)$$

output
```
-4*EllipticE(1/2*(4-x)^(1/2)*2^(1/2),1/2*2^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 4.50

$$\int \frac{\sqrt{x}}{\sqrt{(4-x)(-2+x)}} \, dx = \frac{2\sqrt{\frac{-4+x}{x}}\sqrt{\frac{x}{-4+x}}\left(\sqrt{-4+x}(-2+x)\sqrt{\frac{x}{-4+x}}+2i(-4+x)\sqrt{\frac{-2+x}{-4+x}}E\left(i\text{arcsinh}\left(\frac{2}{\sqrt{-4+x}}\right)|\frac{1}{2}\right)\right)}{\sqrt{-8+6x-x^2}}$$

input
```
Integrate[Sqrt[x]/Sqrt[(4 - x)*(-2 + x)],x]
```

output
```
(2*Sqrt[(-4 + x)/x]*Sqrt[x/(-4 + x)]*(Sqrt[-4 + x]*(-2 + x)*Sqrt[x/(-4 + x
)] + (2*I)*(-4 + x)*Sqrt[(-2 + x)/(-4 + x)]*EllipticE[I*ArcSinh[2/Sqrt[-4
+ x]], 1/2]))/Sqrt[-8 + 6*x - x^2]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2035, 2048, 1452, 27, 389, 322, 328}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{(4-x)(x-2)}}dx$$

$$\Big\downarrow \text{2035}$$

$$2\int \frac{x}{\sqrt{-((2-x)(4-x))}}d\sqrt{x}$$

$$\Big\downarrow \text{2048}$$

$$2\int \frac{x}{\sqrt{-x^2+6x-8}}d\sqrt{x}$$

$$\Big\downarrow \text{1452}$$

$$4\int \frac{x}{2\sqrt{4-x}\sqrt{x-2}}d\sqrt{x}$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{x}{\sqrt{4-x}\sqrt{x-2}}d\sqrt{x}$$

$$\Big\downarrow \text{389}$$

$$2\left(2\int \frac{1}{\sqrt{4-x}\sqrt{x-2}}d\sqrt{x}+\int \frac{\sqrt{x-2}}{\sqrt{4-x}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{322}$$

$$2\left(\int \frac{\sqrt{x-2}}{\sqrt{4-x}}d\sqrt{x}-\sqrt{2}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt{x}}{2}\right),2\right)\right)$$

$$\Big\downarrow \text{328}$$

$$2\left(-\sqrt{2}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt{x}}{2}\right),2\right)-\sqrt{2}E\left(\arccos\left(\frac{\sqrt{x}}{2}\right)\Big|2\right)\right)$$

input
```
Int[Sqrt[x]/Sqrt[(4 - x)*(-2 + x)],x]
```

output
```
2*(-(Sqrt[2]*EllipticE[ArcCos[Sqrt[x]/2], 2]) - Sqrt[2]*EllipticF[ArcCos[S
qrt[x]/2], 2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 322
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

rule 328
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(-Sqrt[a - b*(c/d)]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcCos[Rt[-d/c, 2]*x],
 b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0]
 && GtQ[a - b*(c/d), 0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

rule 1452
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[x^2/(Sqrt[b + q + 2*c*x^2]*Sqrt
[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
&& LtQ[c, 0]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.55

method	result	size
default	$\frac{\left(\operatorname{EllipticE}\left(\frac{\sqrt{-2x+4}}{2},i\right)-2\operatorname{EllipticF}\left(\frac{\sqrt{-2x+4}}{2},i\right)\right)\sqrt{2}\,\sqrt{-2x+8}\,\sqrt{-2x+4}}{\sqrt{-(x-2)(x-4)}}$	56
elliptic	$-\frac{\sqrt{-(x-2)(x-4)x}\,\sqrt{-2x+4}\,\sqrt{-2x+8}\,\sqrt{2}\left(-2\operatorname{EllipticE}\left(\frac{\sqrt{-2x+4}}{2},i\right)+4\operatorname{EllipticF}\left(\frac{\sqrt{-2x+4}}{2},i\right)\right)}{2\sqrt{-(x-2)(x-4)}\,\sqrt{-x^3+6x^2-8x}}$	86

input
```
int(x^(1/2)/((4-x)*(x-2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(EllipticE(1/2*(-2*x+4)^(1/2),I)-2*EllipticF(1/2*(-2*x+4)^(1/2),I))*2^(1/2
)*(-2*x+8)^(1/2)*(-2*x+4)^(1/2)/(-(x-2)*(x-4))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{x}}{\sqrt{(4-x)(-2+x)}}\,dx = -4i\,\text{weierstrassPInverse}(16,0,x-2) + 2i\,\text{weierstrassZeta}(16,0,\text{weierstrassPInverse}(16,0,x-2))$$

input
```
integrate(x^(1/2)/((4-x)*(-2+x))^(1/2),x, algorithm="fricas")
```

output
```
-4*I*weierstrassPInverse(16, 0, x - 2) + 2*I*weierstrassZeta(16, 0, weiers
trassPInverse(16, 0, x - 2))
```

Sympy [F]

$$\int \frac{\sqrt{x}}{\sqrt{(4-x)(-2+x)}}\,dx = \int \frac{\sqrt{x}}{\sqrt{-(x-4)(x-2)}}\,dx$$

input
```
integrate(x**(1/2)/((4-x)*(-2+x))**(1/2),x)
```

output
```
Integral(sqrt(x)/sqrt(-(x - 4)*(x - 2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{(4-x)(-2+x)}}\,dx = \int \frac{\sqrt{x}}{\sqrt{-(x-2)(x-4)}}\,dx$$

input
```
integrate(x^(1/2)/((4-x)*(-2+x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/sqrt(-(x - 2)*(x - 4)), x)
```

Giac [F]

$$\int \frac{\sqrt{x}}{\sqrt{(4-x)(-2+x)}}\,dx = \int \frac{\sqrt{x}}{\sqrt{-(x-2)(x-4)}}\,dx$$

input
```
integrate(x^(1/2)/((4-x)*(-2+x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x)/sqrt(-(x - 2)*(x - 4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{(4-x)(-2+x)}}\,dx = \int \frac{\sqrt{x}}{\sqrt{-(x-2)\,(x-4)}}\,dx$$

input
```
int(x^(1/2)/(-(x - 2)*(x - 4))^(1/2),x)
```

output
```
int(x^(1/2)/(-(x - 2)*(x - 4))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{x}}{\sqrt{(4-x)(-2+x)}}\,dx = -\left(\int \frac{\sqrt{x}\,\sqrt{-x^2+6x-8}}{x^2-6x+8}dx\right)$$

input
```
int(x^(1/2)/((4-x)*(-2+x))^(1/2),x)
```

output
```
- int((sqrt(x)*sqrt( - x**2 + 6*x - 8))/(x**2 - 6*x + 8),x)
```

3.173 $\int (dx)^m (ac + (bc + ad)x + bdx^2)^p \, dx$

Optimal result

Integrand size = 27, antiderivative size = 88

$$\int (dx)^m (ac + (bc + ad)x + bdx^2)^p \, dx = \frac{(dx)^{1+m} \left(1 + \frac{bx}{a}\right)^{-p} \left(1 + \frac{dx}{c}\right)^{-p} (ac + (bc + ad)x + bdx^2)^p \operatorname{AppellF1}\left(1 + m, -p, -p, 2 + m, -\frac{dx}{c}, -\frac{bx}{a}\right)}{d(1 + m)}$$

output
```
(d*x)^(1+m)*(a*c+(a*d+b*c)*x+b*d*x^2)^p*AppellF1(1+m,-p,-p,2+m,-b*x/a,-d*x
/c)/d/(1+m)/((1+b*x/a)^p)/((1+d*x/c)^p)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

$$\int (dx)^m (ac + (bc + ad)x + bdx^2)^p \, dx = \frac{x(dx)^m \left(\frac{a+bx}{a}\right)^{-p} \left(\frac{c+dx}{c}\right)^{-p} ((a + bx)(c + dx))^p \operatorname{AppellF1}\left(1 + m, -p, -p, 2 + m, -\frac{bx}{a}, -\frac{dx}{c}\right)}{1 + m}$$

input
```
Integrate[(d*x)^m*(a*c + (b*c + a*d)*x + b*d*x^2)^p,x]
```

output
```
(x*(d*x)^m*((a + b*x)*(c + d*x))^p*AppellF1[1 + m, -p, -p, 2 + m, -((b*x)/
a), -((d*x)/c)])/((1 + m)*((a + b*x)/a)^p*((c + d*x)/c)^p)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(x(ad+bc)+ac+bdx^2\right)^p \, dx$$

$$\Big\downarrow \text{1179}$$

$$\frac{\left(\frac{bx}{a}+1\right)^{-p}\left(\frac{dx}{c}+1\right)^{-p}\left(x(ad+bc)+ac+bdx^2\right)^p \int (dx)^m \left(\frac{bx}{a}+1\right)^p \left(\frac{dx}{c}+1\right)^p d(dx)}{d}$$

$$\Big\downarrow \text{150}$$

$$\frac{(dx)^{m+1}\left(\frac{bx}{a}+1\right)^{-p}\left(\frac{dx}{c}+1\right)^{-p}\left(x(ad+bc)+ac+bdx^2\right)^p \text{AppellF1}\left(m+1,-p,-p,m+2,-\frac{dx}{c},-\frac{bx}{a}\right)}{d(m+1)}$$

input
```
Int[(d*x)^m*(a*c + (b*c + a*d)*x + b*d*x^2)^p,x]
```

output
```
((d*x)^(1 + m)*(a*c + (b*c + a*d)*x + b*d*x^2)^p*AppellF1[1 + m, -p, -p, 2
 + m, -((d*x)/c), -((b*x)/a)])/(d*(1 + m)*(1 + (b*x)/a)^p*(1 + (d*x)/c)^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

Maple [F]

$$\int (dx)^m \left(ac+(ad+bc)\,x+bd\,x^2\right)^p dx$$

input
```
int((d*x)^m*(a*c+(a*d+b*c)*x+b*d*x^2)^p,x)
```

output
```
int((d*x)^m*(a*c+(a*d+b*c)*x+b*d*x^2)^p,x)
```

Fricas [F]

$$\int (dx)^m \left(ac+(bc+ad)x+bdx^2\right)^p \, dx = \int \left(bdx^2+ac+(bc+ad)x\right)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a*c+(a*d+b*c)*x+b*d*x^2)^p,x, algorithm="fricas")
```

output
```
integral((b*d*x^2 + a*c + (b*c + a*d)*x)^p*(d*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(ac+(bc+ad)x+bdx^2\right)^p\,dx=\text{Timed out}$$

input
```
integrate((d*x)**m*(a*c+(a*d+b*c)*x+b*d*x**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (dx)^m \left(ac+(bc+ad)x+bdx^2\right)^p\,dx=\int \left(bdx^2+ac+(bc+ad)x\right)^p(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a*c+(a*d+b*c)*x+b*d*x^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*d*x^2 + a*c + (b*c + a*d)*x)^p*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m \left(ac+(bc+ad)x+bdx^2\right)^p\,dx=\int \left(bdx^2+ac+(bc+ad)x\right)^p(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a*c+(a*d+b*c)*x+b*d*x^2)^p,x, algorithm="giac")
```

output
```
integrate((b*d*x^2 + a*c + (b*c + a*d)*x)^p*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(ac+(bc+ad)x+bdx^2\right)^p \, dx = \int (d\,x)^m \left(b\,d\,x^2+(a\,d+b\,c)\,x+a\,c\right)^p dx$$

input
```
int((d*x)^m*(a*c + x*(a*d + b*c) + b*d*x^2)^p,x)
```

output
```
int((d*x)^m*(a*c + x*(a*d + b*c) + b*d*x^2)^p, x)
```

Reduce [F]

$$\int (dx)^m \left(ac+(bc+ad)x+bdx^2\right)^p \, dx = \text{too large to display}$$

input
```
int((d*x)^m*(a*c+(a*d+b*c)*x+b*d*x^2)^p,x)
```

output

```
(d**m*(2*x**m*(a*c + a*d*x + b*c*x + b*d*x**2)**p*a*c*p + x**m*(a*c + a*d*
x + b*c*x + b*d*x**2)**p*a*d*m*x + x**m*(a*c + a*d*x + b*c*x + b*d*x**2)**
p*a*d*p*x + x**m*(a*c + a*d*x + b*c*x + b*d*x**2)**p*b*c*m*x + x**m*(a*c +
 a*d*x + b*c*x + b*d*x**2)**p*b*c*p*x + int((x**m*(a*c + a*d*x + b*c*x + b
*d*x**2)**p*x)/(a**2*c*d*m**2 + 3*a**2*c*d*m*p + a**2*c*d*m + 2*a**2*c*d*p
**2 + a**2*c*d*p + a**2*d**2*m**2*x + 3*a**2*d**2*m*p*x + a**2*d**2*m*x +
2*a**2*d**2*p**2*x + a**2*d**2*p*x + a*b*c**2*m**2 + 3*a*b*c**2*m*p + a*b*
c**2*m + 2*a*b*c**2*p**2 + a*b*c**2*p + 2*a*b*c*d*m**2*x + 6*a*b*c*d*m*p*x
 + 2*a*b*c*d*m*x + 4*a*b*c*d*p**2*x + 2*a*b*c*d*p*x + a*b*d**2*m**2*x**2 +
 3*a*b*d**2*m*p*x**2 + a*b*d**2*m*x**2 + 2*a*b*d**2*p**2*x**2 + a*b*d**2*p
*x**2 + b**2*c**2*m**2*x + 3*b**2*c**2*m*p*x + b**2*c**2*m*x + 2*b**2*c**2
*p**2*x + b**2*c**2*p*x + b**2*c*d*m**2*x**2 + 3*b**2*c*d*m*p*x**2 + b**2*
c*d*m*x**2 + 2*b**2*c*d*p**2*x**2 + b**2*c*d*p*x**2),x)*a**3*d**3*m**3*p +
 4*int((x**m*(a*c + a*d*x + b*c*x + b*d*x**2)**p*x)/(a**2*c*d*m**2 + 3*a**
2*c*d*m*p + a**2*c*d*m + 2*a**2*c*d*p**2 + a**2*c*d*p + a**2*d**2*m**2*x +
 3*a**2*d**2*m*p*x + a**2*d**2*m*x + 2*a**2*d**2*p**2*x + a**2*d**2*p*x +
a*b*c**2*m**2 + 3*a*b*c**2*m*p + a*b*c**2*m + 2*a*b*c**2*p**2 + a*b*c**2*p
 + 2*a*b*c*d*m**2*x + 6*a*b*c*d*m*p*x + 2*a*b*c*d*m*x + 4*a*b*c*d*p**2*x +
 2*a*b*c*d*p*x + a*b*d**2*m**2*x**2 + 3*a*b*d**2*m*p*x**2 + a*b*d**2*m*x**
2 + 2*a*b*d**2*p**2*x**2 + a*b*d**2*p*x**2 + b**2*c**2*m**2*x + 3*b**2*...
```

3.174 $\int x^2(ac+(bc+ad)x+bdx^2)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 193

$$\int x^2\left(ac+(bc+ad)x+bdx^2\right)^p\,dx$$
$$=-\frac{((bc+ad)(2+p)-2bd(1+p)x)\left(ac+(bc+ad)x+bdx^2\right)^{1+p}}{2b^2d^2(1+p)(3+2p)}$$
$$+\frac{(2abcd-(bc+ad)^2(2+p))\left(-\frac{d(a+bx)}{bc-ad}\right)^{-1-p}\left(ac+(bc+ad)x+bdx^2\right)^{1+p}\text{Hypergeometric2F1}\left(-p,1\right.}{2b^2d^2(bc-ad)(1+p)(3+2p)}$$

output
```
-1/2*((a*d+b*c)*(2+p)-2*b*d*(p+1)*x)*(a*c+(a*d+b*c)*x+b*d*x^2)^(p+1)/b^2/d
^2/(p+1)/(3+2*p)+1/2*(2*a*b*c*d-(a*d+b*c)^2*(2+p))*(-d*(b*x+a)/(-a*d+b*c))
^(-1-p)*(a*c+(a*d+b*c)*x+b*d*x^2)^(p+1)*hypergeom([-p, p+1],[2+p],b*(d*x+c
)/(-a*d+b*c))/b^2/d^2/(-a*d+b*c)/(p+1)/(3+2*p)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.79

$$\int x^2 (ac + (bc + ad)x + bdx^2)^p \, dx$$
$$= \frac{(a + bx)((a + bx)(c + dx))^p \left(-b(bc + ad)(2 + p)(c + dx) + 2b^2d(1 + p)x(c + dx) + (2abcd(1 + p) + b^2c\right.}{2b^3d^2(1 + p)(3 + 2p}$$

input
```
Integrate[x^2*(a*c + (b*c + a*d)*x + b*d*x^2)^p,x]
```

output
```
((a + b*x)*((a + b*x)*(c + d*x))^p*(-(b*(b*c + a*d)*(2 + p)*(c + d*x)) + 2
*b^2*d*(1 + p)*x*(c + d*x) + ((2*a*b*c*d*(1 + p) + b^2*c^2*(2 + p) + a^2*d
^2*(2 + p))*Hypergeometric2F1[-p, 1 + p, 2 + p, (d*(a + b*x))/(-(b*c) + a*
d)])/((b*(c + d*x))/(b*c - a*d))^p))/(2*b^3*d^2*(1 + p)*(3 + 2*p))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1166, 25, 1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (x(ad + bc) + ac + bdx^2)^p \, dx$$

$$\Big\downarrow \text{ 1166}$$

$$\frac{\int -((ac + (bc + ad)(p + 2)x) (bdx^2 + (bc + ad)x + ac)^p) \, dx}{bd(2p + 3)} + \frac{x (x(ad + bc) + ac + bdx^2)^{p+1}}{bd(2p + 3)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{x (x(ad + bc) + ac + bdx^2)^{p+1}}{bd(2p + 3)} - \frac{\int (ac + (bc + ad)(p + 2)x) (bdx^2 + (bc + ad)x + ac)^p \, dx}{bd(2p + 3)}$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{x\left(x(ad+bc)+ac+bdx^2\right)^{p+1}}{bd(2p+3)}-\frac{\frac{\left(2abcd-(p+2)(ad+bc)^2\right)\int\left(bdx^2+(bc+ad)x+ac\right)^p dx}{2bd}+\frac{(p+2)(ad+bc)\left(x(ad+bc)+ac+bdx^2\right)^{p+1}}{2bd(p+1)}}{bd(2p+3)}$$

$$\Big\downarrow \text{ 1096}$$

$$\frac{x\left(x(ad+bc)+ac+bdx^2\right)^{p+1}}{bd(2p+3)}-\frac{\frac{(p+2)(ad+bc)\left(x(ad+bc)+ac+bdx^2\right)^{p+1}}{2bd(p+1)}-\frac{\left(2abcd-(p+2)(ad+bc)^2\right)\left(-\frac{d(a+bx)}{bc-ad}\right)^{-p-1}\left(x(ad+bc)+ac+bdx^2\right)^{p+1}\text{Hypergeometric2F1}\left(-p,p+1\right.}{2bd(p+1)(bc-ad)}}{bd(2p+3)}$$

input
```
Int[x^2*(a*c + (b*c + a*d)*x + b*d*x^2)^p,x]
```

output
```
(x*(a*c + (b*c + a*d)*x + b*d*x^2)^(1 + p))/(b*d*(3 + 2*p)) - (((b*c + a*d
)*(2 + p)*(a*c + (b*c + a*d)*x + b*d*x^2)^(1 + p))/(2*b*d*(1 + p)) - ((2*a
*b*c*d - (b*c + a*d)^2*(2 + p))*(-((d*(a + b*x))/(b*c - a*d)))^(-1 - p)*(a
*c + (b*c + a*d)*x + b*d*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p,
(b*(c + d*x))/(b*c - a*d)])/(2*b*d*(b*c - a*d)*(1 + p)))/(b*d*(3 + 2*p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

Maple [F]

$$\int x^{2} \left(a c +\left(a d +b c \right) x +b d \,x^{2}\right)^{p}d x$$

input
```
int(x^2*(a*c+(a*d+b*c)*x+b*d*x^2)^p,x)
```

output
```
int(x^2*(a*c+(a*d+b*c)*x+b*d*x^2)^p,x)
```

Fricas [F]

$$\int x^2\left(ac+(bc+ad)x+bdx^2\right)^p\,dx=\int\left(bdx^2+ac+(bc+ad)x\right)^px^2\,dx$$

input
```
integrate(x^2*(a*c+(a*d+b*c)*x+b*d*x^2)^p,x, algorithm="fricas")
```

output
```
integral((b*d*x^2 + a*c + (b*c + a*d)*x)^p*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(ac + (bc + ad)x + bdx^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate(x**2*(a*c+(a*d+b*c)*x+b*d*x**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2 \left(ac + (bc + ad)x + bdx^2\right)^p \, dx = \int \left(bdx^2 + ac + (bc + ad)x\right)^p x^2 \, dx$$

input
```
integrate(x^2*(a*c+(a*d+b*c)*x+b*d*x^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*d*x^2 + a*c + (b*c + a*d)*x)^p*x^2, x)
```

Giac [F]

$$\int x^2 \left(ac + (bc + ad)x + bdx^2\right)^p \, dx = \int \left(bdx^2 + ac + (bc + ad)x\right)^p x^2 \, dx$$

input
```
integrate(x^2*(a*c+(a*d+b*c)*x+b*d*x^2)^p,x, algorithm="giac")
```

output
```
integrate((b*d*x^2 + a*c + (b*c + a*d)*x)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(ac+(bc+ad)x+bdx^2)^p\,dx=\int x^2\,\left(b\,d\,x^2+(a\,d+b\,c)\,x+a\,c\right)^p\,dx$$

input
```
int(x^2*(a*c + x*(a*d + b*c) + b*d*x^2)^p,x)
```

output
```
int(x^2*(a*c + x*(a*d + b*c) + b*d*x^2)^p, x)
```

Reduce [F]

$$\int x^2(ac+(bc+ad)x+bdx^2)^p\,dx=\text{too large to display}$$

input
```
int(x^2*(a*c+(a*d+b*c)*x+b*d*x^2)^p,x)
```

output
```
((a*c + a*d*x + b*c*x + b*d*x**2)**p*a**3*c*d**2*p + 2*(a*c + a*d*x + b*c*
x + b*d*x**2)**p*a**3*c*d**2 - (a*c + a*d*x + b*c*x + b*d*x**2)**p*a**3*d*
*3*p**2*x - 2*(a*c + a*d*x + b*c*x + b*d*x**2)**p*a**3*d**3*p*x - 2*(a*c +
 a*d*x + b*c*x + b*d*x**2)**p*a**2*b*c**2*d*p + (a*c + a*d*x + b*c*x + b*d
*x**2)**p*a**2*b*c*d**2*p**2*x - 2*(a*c + a*d*x + b*c*x + b*d*x**2)**p*a**
2*b*c*d**2*p*x + 2*(a*c + a*d*x + b*c*x + b*d*x**2)**p*a**2*b*d**3*p**2*x*
*2 + (a*c + a*d*x + b*c*x + b*d*x**2)**p*a**2*b*d**3*p*x**2 + (a*c + a*d*x
 + b*c*x + b*d*x**2)**p*a*b**2*c**3*p + 2*(a*c + a*d*x + b*c*x + b*d*x**2)
**p*a*b**2*c**3 + (a*c + a*d*x + b*c*x + b*d*x**2)**p*a*b**2*c**2*d*p**2*x
 - 2*(a*c + a*d*x + b*c*x + b*d*x**2)**p*a*b**2*c**2*d*p*x + 4*(a*c + a*d*
x + b*c*x + b*d*x**2)**p*a*b**2*c*d**2*p**2*x**2 + 2*(a*c + a*d*x + b*c*x
+ b*d*x**2)**p*a*b**2*c*d**2*p*x**2 + 4*(a*c + a*d*x + b*c*x + b*d*x**2)**
p*a*b**2*d**3*p**2*x**3 + 6*(a*c + a*d*x + b*c*x + b*d*x**2)**p*a*b**2*d**
3*p*x**3 + 2*(a*c + a*d*x + b*c*x + b*d*x**2)**p*a*b**2*d**3*x**3 - (a*c +
 a*d*x + b*c*x + b*d*x**2)**p*b**3*c**3*p**2*x - 2*(a*c + a*d*x + b*c*x +
b*d*x**2)**p*b**3*c**3*p*x + 2*(a*c + a*d*x + b*c*x + b*d*x**2)**p*b**3*c*
*2*d*p**2*x**2 + (a*c + a*d*x + b*c*x + b*d*x**2)**p*b**3*c**2*d*p*x**2 +
4*(a*c + a*d*x + b*c*x + b*d*x**2)**p*b**3*c*d**2*p**2*x**3 + 6*(a*c + a*d
*x + b*c*x + b*d*x**2)**p*b**3*c*d**2*p*x**3 + 2*(a*c + a*d*x + b*c*x + b*
d*x**2)**p*b**3*c*d**2*x**3 + 4*int(((a*c + a*d*x + b*c*x + b*d*x**2)**...
```

3.175 $\int x(ac+(bc+ad)x+bdx^2)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 145

$$\int x\left(ac+(bc+ad)x+bdx^2\right)^p\,dx=\frac{\left(ac+(bc+ad)x+bdx^2\right)^{1+p}}{2bd(1+p)}+\frac{(bc+ad)\left(-\frac{d(a+bx)}{bc-ad}\right)^{-1-p}\left(ac+(bc+ad)x+bdx^2\right)^{1+p}\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{b(c+dx)}{bc-ad}\right)}{2bd(bc-ad)(1+p)}$$

output
```
1/2*(a*c+(a*d+b*c)*x+b*d*x^2)^(p+1)/b/d/(p+1)+1/2*(a*d+b*c)*(-d*(b*x+a)/(-
a*d+b*c))^(-1-p)*(a*c+(a*d+b*c)*x+b*d*x^2)^(p+1)*hypergeom([-p, p+1],[2+p]
,b*(d*x+c)/(-a*d+b*c))/b/d/(-a*d+b*c)/(p+1)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.81

$$\int x\left(ac+(bc+ad)x+bdx^2\right)^p\,dx$$

$$=\frac{(a+bx)\left(\frac{b(c+dx)}{bc-ad}\right)^{-p}((a+bx)(c+dx))^p\left(b(c+dx)\left(\frac{b(c+dx)}{bc-ad}\right)^p-(bc+ad)\,\text{Hypergeometric2F1}\left(-p,1+\right.\right.}{2b^2d(1+p)}$$

input
```
Integrate[x*(a*c + (b*c + a*d)*x + b*d*x^2)^p,x]
```

output
```
((a + b*x)*((a + b*x)*(c + d*x))^p*(b*(c + d*x)*((b*(c + d*x))/(b*c - a*d)
)^p - (b*c + a*d)*Hypergeometric2F1[-p, 1 + p, 2 + p, (d*(a + b*x))/(-(b*c
) + a*d)]))/(2*b^2*d*(1 + p)*((b*(c + d*x))/(b*c - a*d))^p)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(x(ad+bc)+ac+bdx^2\right)^p\,dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{\left(x(ad+bc)+ac+bdx^2\right)^{p+1}}{2bd(p+1)}-\frac{(ad+bc)\int \left(bdx^2+(bc+ad)x+ac\right)^p dx}{2bd}$$

$$\Big\downarrow \text{1096}$$

$$\frac{(ad+bc)\left(x(ad+bc)+ac+bdx^2\right)^{p+1}\left(-\frac{d(a+bx)}{bc-ad}\right)^{-p-1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{b(c+dx)}{bc-ad}\right)}{2bd(p+1)(bc-ad)}+\frac{\left(x(ad+bc)+ac+bdx^2\right)^{p+1}}{2bd(p+1)}$$

input
```
Int[x*(a*c + (b*c + a*d)*x + b*d*x^2)^p,x]
```

output
```
(a*c + (b*c + a*d)*x + b*d*x^2)^(1 + p)/(2*b*d*(1 + p)) + ((b*c + a*d)*(-(
(d*(a + b*x))/(b*c - a*d)))^(-1 - p)*(a*c + (b*c + a*d)*x + b*d*x^2)^(1 +
p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b*(c + d*x))/(b*c - a*d)])/(2*b*d*
(b*c - a*d)*(1 + p))
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [F]

$$\int x\left(ac+\left(ad+bc\right)x+bd\,x^{2}\right)^{p}dx$$

input
```
int(x*(a*c+(a*d+b*c)*x+b*d*x^2)^p,x)
```

output
```
int(x*(a*c+(a*d+b*c)*x+b*d*x^2)^p,x)
```

Fricas [F]

$$\int x\left(ac+(bc+ad)x+bdx^2\right)^p\,dx=\int\left(bdx^2+ac+(bc+ad)x\right)^px\,dx$$

input
```
integrate(x*(a*c+(a*d+b*c)*x+b*d*x^2)^p,x, algorithm="fricas")
```

output
```
integral((b*d*x^2 + a*c + (b*c + a*d)*x)^p*x, x)
```

Sympy [F]

$$\int x\left(ac+(bc+ad)x+bdx^2\right)^p\,dx=\int x((a+bx)\,(c+dx))^p\,dx$$

input
```
integrate(x*(a*c+(a*d+b*c)*x+b*d*x**2)**p,x)
```

output
```
Integral(x*((a + b*x)*(c + d*x))**p, x)
```

Maxima [F]

$$\int x\left(ac+(bc+ad)x+bdx^2\right)^p\,dx=\int\left(bdx^2+ac+(bc+ad)x\right)^p x\,dx$$

input
```
integrate(x*(a*c+(a*d+b*c)*x+b*d*x^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*d*x^2 + a*c + (b*c + a*d)*x)^p*x, x)
```

Giac [F]

$$\int x\left(ac+(bc+ad)x+bdx^2\right)^p\,dx=\int\left(bdx^2+ac+(bc+ad)x\right)^p x\,dx$$

input
```
integrate(x*(a*c+(a*d+b*c)*x+b*d*x^2)^p,x, algorithm="giac")
```

output
```
integrate((b*d*x^2 + a*c + (b*c + a*d)*x)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(ac+(bc+ad)x+bdx^2\right)^p\,dx=\int x\,\left(b\,d\,x^2+\left(a\,d+b\,c\right)\,x+a\,c\right)^p\,dx$$

input
```
int(x*(a*c + x*(a*d + b*c) + b*d*x^2)^p,x)
```

output
```
int(x*(a*c + x*(a*d + b*c) + b*d*x^2)^p, x)
```

Reduce [F]

$$\int x\left(ac+(bc+ad)x+bdx^2\right)^p\,dx$$
$$=\frac{-(bd\,x^2+adx+bcx+ac)^p\,ac+(bd\,x^2+adx+bcx+ac)^p\,adpx+(bd\,x^2+adx+bcx+ac)^p\,bcpx+2(}{}$$

input
```
int(x*(a*c+(a*d+b*c)*x+b*d*x^2)^p,x)
```

output
```
( - (a*c + a*d*x + b*c*x + b*d*x**2)**p*a*c + (a*c + a*d*x + b*c*x + b*d*x
**2)**p*a*d*p*x + (a*c + a*d*x + b*c*x + b*d*x**2)**p*b*c*p*x + 2*(a*c + a
*d*x + b*c*x + b*d*x**2)**p*b*d*p*x**2 + (a*c + a*d*x + b*c*x + b*d*x**2)*
*p*b*d*x**2 - 2*int(((a*c + a*d*x + b*c*x + b*d*x**2)**p*x)/(2*a*c*p + a*c
 + 2*a*d*p*x + a*d*x + 2*b*c*p*x + b*c*x + 2*b*d*p*x**2 + b*d*x**2),x)*a**
2*d**2*p**3 - 3*int(((a*c + a*d*x + b*c*x + b*d*x**2)**p*x)/(2*a*c*p + a*c
 + 2*a*d*p*x + a*d*x + 2*b*c*p*x + b*c*x + 2*b*d*p*x**2 + b*d*x**2),x)*a**
2*d**2*p**2 - int(((a*c + a*d*x + b*c*x + b*d*x**2)**p*x)/(2*a*c*p + a*c +
 2*a*d*p*x + a*d*x + 2*b*c*p*x + b*c*x + 2*b*d*p*x**2 + b*d*x**2),x)*a**2*
d**2*p + 4*int(((a*c + a*d*x + b*c*x + b*d*x**2)**p*x)/(2*a*c*p + a*c + 2*
a*d*p*x + a*d*x + 2*b*c*p*x + b*c*x + 2*b*d*p*x**2 + b*d*x**2),x)*a*b*c*d*
p**3 + 6*int(((a*c + a*d*x + b*c*x + b*d*x**2)**p*x)/(2*a*c*p + a*c + 2*a*
d*p*x + a*d*x + 2*b*c*p*x + b*c*x + 2*b*d*p*x**2 + b*d*x**2),x)*a*b*c*d*p*
*2 + 2*int(((a*c + a*d*x + b*c*x + b*d*x**2)**p*x)/(2*a*c*p + a*c + 2*a*d*
p*x + a*d*x + 2*b*c*p*x + b*c*x + 2*b*d*p*x**2 + b*d*x**2),x)*a*b*c*d*p -
2*int(((a*c + a*d*x + b*c*x + b*d*x**2)**p*x)/(2*a*c*p + a*c + 2*a*d*p*x +
 a*d*x + 2*b*c*p*x + b*c*x + 2*b*d*p*x**2 + b*d*x**2),x)*b**2*c**2*p**3 -
3*int(((a*c + a*d*x + b*c*x + b*d*x**2)**p*x)/(2*a*c*p + a*c + 2*a*d*p*x +
 a*d*x + 2*b*c*p*x + b*c*x + 2*b*d*p*x**2 + b*d*x**2),x)*b**2*c**2*p**2 -
int(((a*c + a*d*x + b*c*x + b*d*x**2)**p*x)/(2*a*c*p + a*c + 2*a*d*p*x ...
```

3.176 $\int (ac+(bc+ad)x+bdx^2)^p\, dx$

Optimal result

Integrand size = 21, antiderivative size = 91

$$\int (ac+(bc+ad)x+bdx^2)^p\, dx = -\frac{\left(-\frac{d(a+bx)}{bc-ad}\right)^{-1-p}(ac+(bc+ad)x+bdx^2)^{1+p}\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{b(c+dx)}{bc-ad}\right)}{(bc-ad)(1+p)}$$

output
```
-(-d*(b*x+a)/(-a*d+b*c))^(-1-p)*(a*c+(a*d+b*c)*x+b*d*x^2)^(p+1)*hypergeom(
[-p, p+1],[2+p],b*(d*x+c)/(-a*d+b*c))/(-a*d+b*c)/(p+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int (ac+(bc+ad)x+bdx^2)^p\, dx = \frac{(a+bx)\left(\frac{b(c+dx)}{bc-ad}\right)^{-p}((a+bx)(c+dx))^p\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{d(a+bx)}{-bc+ad}\right)}{b(1+p)}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^p,x]
```

output
```
((a + b*x)*((a + b*x)*(c + d*x))^p*Hypergeometric2F1[-p, 1 + p, 2 + p, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*(1 + p)*((b*(c + d*x))/(b*c - a*d))^p)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = $\{1096\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x(ad+bc)+ac+bdx^2\right)^p\,dx$$

$$\Big\downarrow \text{1096}$$

$$-\frac{\left(-\frac{d(a+bx)}{bc-ad}\right)^{-p-1}\left(x(ad+bc)+ac+bdx^2\right)^{p+1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{b(c+dx)}{bc-ad}\right)}{(p+1)(bc-ad)}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^p,x]
```

output
```
-(((-((d*(a + b*x))/(b*c - a*d)))^(-1 - p)*(a*c + (b*c + a*d)*x + b*d*x^2)
^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b*(c + d*x))/(b*c - a*d)])/(
(b*c - a*d)*(1 + p)))
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && !IntegerQ[4*p] && !IntegerQ[3*p]
```

Maple [F]

$$\int \left(ac+(ad+bc)\,x+bd\,x^{2}\right)^{p}dx$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^p,x)
```

output
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^p,x)
```

Fricas [F]

$$\int \left(ac+(bc+ad)x+bdx^2\right)^p\,dx=\int \left(bdx^2+ac+(bc+ad)x\right)^p\,dx$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^p,x, algorithm="fricas")
```

output
```
integral((b*d*x^2 + a*c + (b*c + a*d)*x)^p, x)
```

Sympy [F]

$$\int \left(ac+(bc+ad)x+bdx^2\right)^p\,dx=\int \left(ac+bdx^2+x(ad+bc)\right)^p\,dx$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**p,x)
```

output
```
Integral((a*c + b*d*x**2 + x*(a*d + b*c))**p, x)
```

Maxima [F]

$$\int \left(ac+(bc+ad)x+bdx^2\right)^p \, dx = \int \left(bdx^2+ac+(bc+ad)x\right)^p \, dx$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*d*x^2 + a*c + (b*c + a*d)*x)^p, x)
```

Giac [F]

$$\int \left(ac+(bc+ad)x+bdx^2\right)^p \, dx = \int \left(bdx^2+ac+(bc+ad)x\right)^p \, dx$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^p,x, algorithm="giac")
```

output
```
integrate((b*d*x^2 + a*c + (b*c + a*d)*x)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(ac+(bc+ad)x+bdx^2\right)^p \, dx = \int \left(b\,d\,x^2+(a\,d+b\,c)\,x+a\,c\right)^p \, dx$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^p,x)
```

output
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^p, x)
```

Reduce [F]

$$\int \left(ac+(bc+ad)x+bdx^2\right)^p\,dx=\text{Too large to display}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^p,x)
```

output
```
(2*(a*c + a*d*x + b*c*x + b*d*x**2)**p*a*c + (a*c + a*d*x + b*c*x + b*d*x*
*2)**p*a*d*x + (a*c + a*d*x + b*c*x + b*d*x**2)**p*b*c*x + 2*int(((a*c + a
*d*x + b*c*x + b*d*x**2)**p*x)/(2*a**2*c*d*p + a**2*c*d + 2*a**2*d**2*p*x
+ a**2*d**2*x + 2*a*b*c**2*p + a*b*c**2 + 4*a*b*c*d*p*x + 2*a*b*c*d*x + 2*
a*b*d**2*p*x**2 + a*b*d**2*x**2 + 2*b**2*c**2*p*x + b**2*c**2*x + 2*b**2*c
*d*p*x**2 + b**2*c*d*x**2),x)*a**3*d**3*p**2 + int(((a*c + a*d*x + b*c*x +
 b*d*x**2)**p*x)/(2*a**2*c*d*p + a**2*c*d + 2*a**2*d**2*p*x + a**2*d**2*x
+ 2*a*b*c**2*p + a*b*c**2 + 4*a*b*c*d*p*x + 2*a*b*c*d*x + 2*a*b*d**2*p*x**
2 + a*b*d**2*x**2 + 2*b**2*c**2*p*x + b**2*c**2*x + 2*b**2*c*d*p*x**2 + b*
*2*c*d*x**2),x)*a**3*d**3*p - 2*int(((a*c + a*d*x + b*c*x + b*d*x**2)**p*x
)/(2*a**2*c*d*p + a**2*c*d + 2*a**2*d**2*p*x + a**2*d**2*x + 2*a*b*c**2*p
+ a*b*c**2 + 4*a*b*c*d*p*x + 2*a*b*c*d*x + 2*a*b*d**2*p*x**2 + a*b*d**2*x*
*2 + 2*b**2*c**2*p*x + b**2*c**2*x + 2*b**2*c*d*p*x**2 + b**2*c*d*x**2),x)
*a**2*b*c*d**2*p**2 - int(((a*c + a*d*x + b*c*x + b*d*x**2)**p*x)/(2*a**2*
c*d*p + a**2*c*d + 2*a**2*d**2*p*x + a**2*d**2*x + 2*a*b*c**2*p + a*b*c**2
 + 4*a*b*c*d*p*x + 2*a*b*c*d*x + 2*a*b*d**2*p*x**2 + a*b*d**2*x**2 + 2*b**
2*c**2*p*x + b**2*c**2*x + 2*b**2*c*d*p*x**2 + b**2*c*d*x**2),x)*a**2*b*c*
d**2*p - 2*int(((a*c + a*d*x + b*c*x + b*d*x**2)**p*x)/(2*a**2*c*d*p + a**
2*c*d + 2*a**2*d**2*p*x + a**2*d**2*x + 2*a*b*c**2*p + a*b*c**2 + 4*a*b*c*
d*p*x + 2*a*b*c*d*x + 2*a*b*d**2*p*x**2 + a*b*d**2*x**2 + 2*b**2*c**2*p...
```

3.177 $\int \frac{(ac+(bc+ad)x+bdx^2)^p}{x} dx$

Optimal result

Integrand size = 25, antiderivative size = 93

$$\int \frac{(ac+(bc+ad)x+bdx^2)^p}{x} dx = \frac{\left(\frac{a+bx}{bx}\right)^{-p}\left(\frac{c+dx}{dx}\right)^{-p}(ac+(bc+ad)x+bdx^2)^p \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{a}{bx},-\frac{c}{dx}\right)}{2p}$$

output
```
1/2*(a*c+(a*d+b*c)*x+b*d*x^2)^p*AppellF1(-2*p,-p,-p,1-2*p,-a/b/x,-c/d/x)/p
/(((b*x+a)/b/x)^p)/(((d*x+c)/d/x)^p)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.87

$$\int \frac{(ac+(bc+ad)x+bdx^2)^p}{x} dx = \frac{\left(1+\frac{a}{bx}\right)^{-p}\left(1+\frac{c}{dx}\right)^{-p}((a+bx)(c+dx))^p \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{a}{bx},-\frac{c}{dx}\right)}{2p}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^p/x,x]
```

output
```
(((a + b*x)*(c + d*x))^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -(a/(b*x)), -(c/(
d*x))])/(2*p*(1 + a/(b*x))^p*(1 + c/(d*x))^p)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ad+bc)+ac+bdx^2)^p}{x}dx$$

$$\Big\downarrow \text{1178}$$

$$-\left(\frac{1}{x}\right)^{2p}\left(\frac{a+bx}{bx}\right)^{-p}\left(\frac{c+dx}{dx}\right)^{-p}(x(ad+bc)+ac+bdx^2)^p\int \left(\frac{a}{bx}+1\right)^p\left(\frac{c}{dx}+1\right)^p\left(\frac{1}{x}\right)^{-2p-1}d\frac{1}{x}$$

$$\Big\downarrow \text{150}$$

$$\frac{\left(\frac{a+bx}{bx}\right)^{-p}\left(\frac{c+dx}{dx}\right)^{-p}(x(ad+bc)+ac+bdx^2)^p\operatorname{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{a}{bx},-\frac{c}{dx}\right)}{2p}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^p/x,x]
```

output
```
((a*c + (b*c + a*d)*x + b*d*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -(a/(b*
x)), -(c/(d*x))])/(2*p*((a + b*x)/(b*x))^p*((c + d*x)/(d*x))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))    Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

Maple [F]

$$\int \frac{(ac + (ad + bc)\,x + bd\,x^2)^p}{x}dx$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^p/x,x)
```

output
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^p/x,x)
```

Fricas [F]

$$\int \frac{(ac + (bc + ad)x + bdx^2)^p}{x}\,dx = \int \frac{(bdx^2 + ac + (bc + ad)x)^p}{x}\,dx$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^p/x,x, algorithm="fricas")
```

output
```
integral((b*d*x^2 + a*c + (b*c + a*d)*x)^p/x, x)
```

Sympy [F]

$$\int \frac{(ac + (bc + ad)x + bdx^2)^p}{x} \, dx = \int \frac{((a + bx)(c + dx))^p}{x} \, dx$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**p/x,x)
```

output
```
Integral(((a + b*x)*(c + d*x))**p/x, x)
```

Maxima [F]

$$\int \frac{(ac + (bc + ad)x + bdx^2)^p}{x} \, dx = \int \frac{(bdx^2 + ac + (bc + ad)x)^p}{x} \, dx$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^p/x,x, algorithm="maxima")
```

output
```
integrate((b*d*x^2 + a*c + (b*c + a*d)*x)^p/x, x)
```

Giac [F]

$$\int \frac{(ac + (bc + ad)x + bdx^2)^p}{x} \, dx = \int \frac{(bdx^2 + ac + (bc + ad)x)^p}{x} \, dx$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^p/x,x, algorithm="giac")
```

output
```
integrate((b*d*x^2 + a*c + (b*c + a*d)*x)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ac+(bc+ad)x+bdx^2)^p}{x}\,dx=\int \frac{\left(b\,d\,x^2+\left(a\,d+b\,c\right)\,x+a\,c\right)^p}{x}\,dx$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^p/x,x)
```

output
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^p/x, x)
```

Reduce [F]

$$\int \frac{(ac+(bc+ad)x+bdx^2)^p}{x}\,dx$$
$$=\frac{(bd\,x^2+adx+bcx+ac)^p+\left(\int \frac{(bd\,x^2+adx+bcx+ac)^p}{bd\,x^3+ad\,x^2+bc\,x^2+acx}dx\right)acp-\left(\int \frac{(bd\,x^2+adx+bcx+ac)^p x}{bd\,x^2+adx+bcx+ac}dx\right)bdp}{p}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^p/x,x)
```

output
```
((a*c + a*d*x + b*c*x + b*d*x**2)**p + int((a*c + a*d*x + b*c*x + b*d*x**2
)**p/(a*c*x + a*d*x**2 + b*c*x**2 + b*d*x**3),x)*a*c*p - int(((a*c + a*d*x
 + b*c*x + b*d*x**2)**p*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b*d*p)/p
```

3.178 $\int \frac{(ac+(bc+ad)x+bdx^2)^p}{x^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 102

$$\int \frac{(ac+(bc+ad)x+bdx^2)^p}{x^2} dx = -\frac{\left(\frac{a+bx}{bx}\right)^{-p}\left(\frac{c+dx}{dx}\right)^{-p}(ac+(bc+ad)x+bdx^2)^p \operatorname{AppellF1}\left(1-2p,-p,-p,2(1-p),-\frac{a}{bx},-\frac{c}{dx}\right)}{(1-2p)x}$$

output
```
-(a*c+(a*d+b*c)*x+b*d*x^2)^p*AppellF1(1-2*p,-p,-p,2-2*p,-a/b/x,-c/d/x)/(1-
2*p)/x/(((b*x+a)/b/x)^p)/(((d*x+c)/d/x)^p)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.85

$$\int \frac{(ac+(bc+ad)x+bdx^2)^p}{x^2} dx = \frac{\left(1+\frac{a}{bx}\right)^{-p}\left(1+\frac{c}{dx}\right)^{-p}((a+bx)(c+dx))^p \operatorname{AppellF1}\left(1-2p,-p,-p,2-2p,-\frac{a}{bx},-\frac{c}{dx}\right)}{(-1+2p)x}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^p/x^2,x]
```

output
```
(((a + b*x)*(c + d*x))^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, -(a/(b*x)), -(
c/(d*x))])/((-1 + 2*p)*(1 + a/(b*x))^p*(1 + c/(d*x))^p*x)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ad+bc)+ac+bdx^2)^p}{x^2}\,dx$$

$$\Big\downarrow \text{1178}$$

$$-\left(\frac{1}{x}\right)^{2p}\left(\frac{a+bx}{bx}\right)^{-p}\left(\frac{c+dx}{dx}\right)^{-p}(x(ad+bc)+ac+bdx^2)^p\int\left(\frac{a}{bx}+1\right)^p\left(\frac{c}{dx}+1\right)^p\left(\frac{1}{x}\right)^{-2p}d\frac{1}{x}$$

$$\Big\downarrow \text{150}$$

$$-\frac{\left(\frac{a+bx}{bx}\right)^{-p}\left(\frac{c+dx}{dx}\right)^{-p}(x(ad+bc)+ac+bdx^2)^p\,\text{AppellF1}\left(1-2p,-p,-p,2-2p,-\frac{a}{bx},-\frac{c}{dx}\right)}{(1-2p)x}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^p/x^2,x]
```

output
```
-(((a*c + (b*c + a*d)*x + b*d*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, -(
a/(b*x)), -(c/(d*x))])/((1 - 2*p)*x*((a + b*x)/(b*x))^p*((c + d*x)/(d*x))^
p))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

Maple [F]

$$\int \frac{(ac + (ad + bc)\,x + bd\,x^2)^p}{x^2}dx$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^p/x^2,x)
```

output
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(ac + (bc + ad)x + bdx^2)^p}{x^2}\,dx = \int \frac{(bdx^2 + ac + (bc + ad)x)^p}{x^2}\,dx$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^p/x^2,x, algorithm="fricas")
```

output
```
integral((b*d*x^2 + a*c + (b*c + a*d)*x)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ac + (bc + ad)x + bdx^2)^p}{x^2} \, dx = \text{Timed out}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**p/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ac + (bc + ad)x + bdx^2)^p}{x^2} \, dx = \int \frac{(bdx^2 + ac + (bc + ad)x)^p}{x^2} \, dx$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b*d*x^2 + a*c + (b*c + a*d)*x)^p/x^2, x)
```

Giac [F]

$$\int \frac{(ac + (bc + ad)x + bdx^2)^p}{x^2} \, dx = \int \frac{(bdx^2 + ac + (bc + ad)x)^p}{x^2} \, dx$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^p/x^2,x, algorithm="giac")
```

output
```
integrate((b*d*x^2 + a*c + (b*c + a*d)*x)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ac+(bc+ad)x+bdx^2)^p}{x^2}\,dx = \int \frac{\left(b\,d\,x^2+\left(a\,d+b\,c\right)\,x+a\,c\right)^p}{x^2}\,dx$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^p/x^2,x)
```

output
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^p/x^2, x)
```

Reduce [F]

$$\int \frac{(ac+(bc+ad)x+bdx^2)^p}{x^2}\,dx = \text{too large to display}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^p/x^2,x)
```

output

```
( - (a*c + a*d*x + b*c*x + b*d*x**2)**p*a*d*p + (a*c + a*d*x + b*c*x + b*d
*x**2)**p*a*d - (a*c + a*d*x + b*c*x + b*d*x**2)**p*b*c*p + (a*c + a*d*x +
 b*c*x + b*d*x**2)**p*b*c - (a*c + a*d*x + b*c*x + b*d*x**2)**p*b*d*x + in
t((a*c + a*d*x + b*c*x + b*d*x**2)**p/(a**2*c*d*p*x - a**2*c*d*x + a**2*d*
*2*p*x**2 - a**2*d**2*x**2 + a*b*c**2*p*x - a*b*c**2*x + 2*a*b*c*d*p*x**2
- 2*a*b*c*d*x**2 + a*b*d**2*p*x**3 - a*b*d**2*x**3 + b**2*c**2*p*x**2 - b*
*2*c**2*x**2 + b**2*c*d*p*x**3 - b**2*c*d*x**3),x)*a**3*d**3*p**3*x - 2*in
t((a*c + a*d*x + b*c*x + b*d*x**2)**p/(a**2*c*d*p*x - a**2*c*d*x + a**2*d*
*2*p*x**2 - a**2*d**2*x**2 + a*b*c**2*p*x - a*b*c**2*x + 2*a*b*c*d*p*x**2
- 2*a*b*c*d*x**2 + a*b*d**2*p*x**3 - a*b*d**2*x**3 + b**2*c**2*p*x**2 - b*
*2*c**2*x**2 + b**2*c*d*p*x**3 - b**2*c*d*x**3),x)*a**3*d**3*p**2*x + int(
(a*c + a*d*x + b*c*x + b*d*x**2)**p/(a**2*c*d*p*x - a**2*c*d*x + a**2*d**2
*p*x**2 - a**2*d**2*x**2 + a*b*c**2*p*x - a*b*c**2*x + 2*a*b*c*d*p*x**2 -
2*a*b*c*d*x**2 + a*b*d**2*p*x**3 - a*b*d**2*x**3 + b**2*c**2*p*x**2 - b**2
*c**2*x**2 + b**2*c*d*p*x**3 - b**2*c*d*x**3),x)*a**3*d**3*p*x + 3*int((a*
c + a*d*x + b*c*x + b*d*x**2)**p/(a**2*c*d*p*x - a**2*c*d*x + a**2*d**2*p*
x**2 - a**2*d**2*x**2 + a*b*c**2*p*x - a*b*c**2*x + 2*a*b*c*d*p*x**2 - 2*a
*b*c*d*x**2 + a*b*d**2*p*x**3 - a*b*d**2*x**3 + b**2*c**2*p*x**2 - b**2*c*
*2*x**2 + b**2*c*d*p*x**3 - b**2*c*d*x**3),x)*a**2*b*c*d**2*p**3*x - 6*int
((a*c + a*d*x + b*c*x + b*d*x**2)**p/(a**2*c*d*p*x - a**2*c*d*x + a**2*...
```

3.179 $\int x^3(a+bx+cx^2)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int x^3\left(a+bx+cx^2\right)\,dx=\frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}$$

output
```
1/4*a*x^4+1/5*b*x^5+1/6*c*x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^3\left(a+bx+cx^2\right)\,dx=\frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}$$

input
```
Integrate[x^3*(a + b*x + c*x^2),x]
```

output
```
(a*x^4)/4 + (b*x^5)/5 + (c*x^6)/6
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a + bx + cx^2) \, dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int (ax^3 + bx^4 + cx^5) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ax^4}{4} + \frac{bx^5}{5} + \frac{cx^6}{6}$$

input
```
Int[x^3*(a + b*x + c*x^2),x]
```

output
```
(a*x^4)/4 + (b*x^5)/5 + (c*x^6)/6
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{1}{4}a\,x^{4}+\frac{1}{5}b\,x^{5}+\frac{1}{6}c\,x^{6}$	20
default	$\frac{1}{4}a\,x^{4}+\frac{1}{5}b\,x^{5}+\frac{1}{6}c\,x^{6}$	20
norman	$\frac{1}{4}a\,x^{4}+\frac{1}{5}b\,x^{5}+\frac{1}{6}c\,x^{6}$	20
risch	$\frac{1}{4}a\,x^{4}+\frac{1}{5}b\,x^{5}+\frac{1}{6}c\,x^{6}$	20
parallelrisch	$\frac{1}{4}a\,x^{4}+\frac{1}{5}b\,x^{5}+\frac{1}{6}c\,x^{6}$	20
orering	$\frac{x^{4}\left(10c\,x^{2}+12bx+15a\right)}{60}$	20

input
```
int(x^3*(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^4+1/5*b*x^5+1/6*c*x^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^3(a+bx+cx^2)\ dx=\frac{1}{6}\,cx^6+\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

input
```
integrate(x^3*(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
1/6*c*x^6 + 1/5*b*x^5 + 1/4*a*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^3 \left(a + bx + cx^2\right) \, dx = \frac{ax^4}{4} + \frac{bx^5}{5} + \frac{cx^6}{6}$$

input
```
integrate(x**3*(c*x**2+b*x+a),x)
```

output
```
a*x**4/4 + b*x**5/5 + c*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^3 \left(a + bx + cx^2\right) \, dx = \frac{1}{6} cx^6 + \frac{1}{5} bx^5 + \frac{1}{4} ax^4$$

input
```
integrate(x^3*(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
1/6*c*x^6 + 1/5*b*x^5 + 1/4*a*x^4
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^3 \left(a + bx + cx^2\right) \, dx = \frac{1}{6} cx^6 + \frac{1}{5} bx^5 + \frac{1}{4} ax^4$$

input
```
integrate(x^3*(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/6*c*x^6 + 1/5*b*x^5 + 1/4*a*x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^3 (a + bx + cx^2) \, dx = \frac{x^4 (10\,c\,x^2 + 12\,b\,x + 15\,a)}{60}$$

input
```
int(x^3*(a + b*x + c*x^2),x)
```

output
```
(x^4*(15*a + 12*b*x + 10*c*x^2))/60
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^3 (a + bx + cx^2) \, dx = \frac{x^4(10c\,x^2 + 12bx + 15a)}{60}$$

input
```
int(x^3*(c*x^2+b*x+a),x)
```

output
```
(x**4*(15*a + 12*b*x + 10*c*x**2))/60
```

3.180 $\int x^2(a + bx + cx^2)\, dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int x^2\left(a + bx + cx^2\right)\, dx = \frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

output
```
1/3*a*x^3+1/4*b*x^4+1/5*c*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^2\left(a + bx + cx^2\right)\, dx = \frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

input
```
Integrate[x^2*(a + b*x + c*x^2),x]
```

output
```
(a*x^3)/3 + (b*x^4)/4 + (c*x^5)/5
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx+cx^2\right) \, dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int \left(ax^2+bx^3+cx^4\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ax^3}{3}+\frac{bx^4}{4}+\frac{cx^5}{5}$$

input
```
Int[x^2*(a + b*x + c*x^2),x]
```

output
```
(a*x^3)/3 + (b*x^4)/4 + (c*x^5)/5
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{1}{3}a\,x^{3}+\frac{1}{4}b\,x^{4}+\frac{1}{5}c\,x^{5}$	20
default	$\frac{1}{3}a\,x^{3}+\frac{1}{4}b\,x^{4}+\frac{1}{5}c\,x^{5}$	20
norman	$\frac{1}{3}a\,x^{3}+\frac{1}{4}b\,x^{4}+\frac{1}{5}c\,x^{5}$	20
risch	$\frac{1}{3}a\,x^{3}+\frac{1}{4}b\,x^{4}+\frac{1}{5}c\,x^{5}$	20
parallelrisch	$\frac{1}{3}a\,x^{3}+\frac{1}{4}b\,x^{4}+\frac{1}{5}c\,x^{5}$	20
orering	$\frac{x^{3}\left(12c\,x^{2}+15bx+20a\right)}{60}$	20

input
```
int(x^2*(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3+1/4*b*x^4+1/5*c*x^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2\left(a+bx+cx^2\right)\,dx=\frac{1}{5}\,cx^{5}+\frac{1}{4}\,bx^{4}+\frac{1}{3}\,ax^{3}$$

input
```
integrate(x^2*(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
1/5*c*x^5 + 1/4*b*x^4 + 1/3*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2(a+bx+cx^2)\ dx=\frac{ax^3}{3}+\frac{bx^4}{4}+\frac{cx^5}{5}$$

input
```
integrate(x**2*(c*x**2+b*x+a),x)
```

output
```
a*x**3/3 + b*x**4/4 + c*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2(a+bx+cx^2)\ dx=\frac{1}{5}\,cx^5+\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

input
```
integrate(x^2*(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
1/5*c*x^5 + 1/4*b*x^4 + 1/3*a*x^3
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2(a+bx+cx^2)\ dx=\frac{1}{5}\,cx^5+\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

input
```
integrate(x^2*(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/5*c*x^5 + 1/4*b*x^4 + 1/3*a*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2(a+bx+cx^2)\ dx=\frac{x^3\,(12\,c\,x^2+15\,b\,x+20\,a)}{60}$$

input
```
int(x^2*(a + b*x + c*x^2),x)
```

output
```
(x^3*(20*a + 15*b*x + 12*c*x^2))/60
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2(a+bx+cx^2)\ dx=\frac{x^3(12c\,x^2+15bx+20a)}{60}$$

input
```
int(x^2*(c*x^2+b*x+a),x)
```

output
```
(x**3*(20*a + 15*b*x + 12*c*x**2))/60
```

3.181 $\int x(a+bx+cx^2)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 25

$$\int x(a+bx+cx^2)\,dx=\frac{ax^2}{2}+\frac{bx^3}{3}+\frac{cx^4}{4}$$

output
```
1/2*a*x^2+1/3*b*x^3+1/4*c*x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x(a+bx+cx^2)\,dx=\frac{ax^2}{2}+\frac{bx^3}{3}+\frac{cx^4}{4}$$

input
```
Integrate[x*(a + b*x + c*x^2),x]
```

output
```
(a*x^2)/2 + (b*x^3)/3 + (c*x^4)/4
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx+cx^2)\,dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int (ax+bx^2+cx^3)\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ax^2}{2}+\frac{bx^3}{3}+\frac{cx^4}{4}$$

input
```
Int[x*(a + b*x + c*x^2),x]
```

output
```
(a*x^2)/2 + (b*x^3)/3 + (c*x^4)/4
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{1}{2}a\,x^{2}+\frac{1}{3}b\,x^{3}+\frac{1}{4}c\,x^{4}$	20
default	$\frac{1}{2}a\,x^{2}+\frac{1}{3}b\,x^{3}+\frac{1}{4}c\,x^{4}$	20
norman	$\frac{1}{2}a\,x^{2}+\frac{1}{3}b\,x^{3}+\frac{1}{4}c\,x^{4}$	20
risch	$\frac{1}{2}a\,x^{2}+\frac{1}{3}b\,x^{3}+\frac{1}{4}c\,x^{4}$	20
parallelrisch	$\frac{1}{2}a\,x^{2}+\frac{1}{3}b\,x^{3}+\frac{1}{4}c\,x^{4}$	20
orering	$\frac{x^{2}\left(3c\,x^{2}+4bx+6a\right)}{12}$	20

input
```
int(x*(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+1/3*b*x^3+1/4*c*x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x\left(a+bx+cx^2\right)\,dx=\frac{1}{4}\,cx^4+\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

input
```
integrate(x*(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(a+bx+cx^2)\,dx=\frac{ax^2}{2}+\frac{bx^3}{3}+\frac{cx^4}{4}$$

input
```
integrate(x*(c*x**2+b*x+a),x)
```

output
```
a*x**2/2 + b*x**3/3 + c*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(a+bx+cx^2)\,dx=\frac{1}{4}\,cx^4+\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

input
```
integrate(x*(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(a+bx+cx^2)\,dx=\frac{1}{4}\,cx^4+\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

input
```
integrate(x*(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(a+bx+cx^2)\,dx=\frac{x^2\,(3\,c\,x^2+4\,b\,x+6\,a)}{12}$$

input
```
int(x*(a + b*x + c*x^2),x)
```

output
```
(x^2*(6*a + 4*b*x + 3*c*x^2))/12
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(a+bx+cx^2)\,dx=\frac{x^2(3c\,x^2+4bx+6a)}{12}$$

input
```
int(x*(c*x^2+b*x+a),x)
```

output
```
(x**2*(6*a + 4*b*x + 3*c*x**2))/12
```

3.182 $\int (a+bx+cx^2)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 20

$$\int (a+bx+cx^2)\,dx = ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

output
```
a*x+1/2*b*x^2+1/3*c*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (a+bx+cx^2)\,dx = ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

input
```
Integrate[a + b*x + c*x^2,x]
```

output
```
a*x + (b*x^2)/2 + (c*x^3)/3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = $\{2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

input
```
Int[a + b*x + c*x^2,x]
```

output
```
a*x + (b*x^2)/2 + (c*x^3)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
gosper	$ax+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}$	17
default	$ax+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}$	17
norman	$ax+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}$	17
risch	$ax+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}$	17
parallelrisch	$ax+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}$	17
parts	$ax+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}$	17
orering	$\frac{x\left(2c\,x^{2}+3bx+6a\right)}{6}$	18

input
```
int(c*x^2+b*x+a,x,method=_RETURNVERBOSE)
```

output
```
a*x+1/2*b*x^2+1/3*c*x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \left(a+bx+cx^2\right)\,dx=\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax$$

input
```
integrate(c*x^2+b*x+a,x, algorithm="fricas")
```

output
```
1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \left(a+bx+cx^2\right)\,dx=ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

input
```
integrate(c*x**2+b*x+a,x)
```

output
```
a*x + b*x**2/2 + c*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \left(a+bx+cx^2\right)\,dx=\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax$$

input
```
integrate(c*x^2+b*x+a,x, algorithm="maxima")
```

output
```
1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \left(a+bx+cx^2\right)\,dx=\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax$$

input
```
integrate(c*x^2+b*x+a,x, algorithm="giac")
```

output
```
1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int (a+bx+cx^2)\ dx=\frac{c\,x^3}{3}+\frac{b\,x^2}{2}+a\,x$$

input
```
int(a + b*x + c*x^2,x)
```

output
```
a*x + (b*x^2)/2 + (c*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int (a+bx+cx^2)\ dx=\frac{x(2c\,x^2+3bx+6a)}{6}$$

input
```
int(c*x^2+b*x+a,x)
```

output
```
(x*(6*a + 3*b*x + 2*c*x**2))/6
```

3.183 $\int \frac{a+bx+cx^2}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 16

$$\int \frac{a+bx+cx^2}{x}\,dx = bx+\frac{cx^2}{2}+a\log(x)$$

output
```
b*x+1/2*c*x^2+a*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{a+bx+cx^2}{x}\,dx = bx+\frac{cx^2}{2}+a\log(x)$$

input
```
Integrate[(a + b*x + c*x^2)/x,x]
```

output
```
b*x + (c*x^2)/2 + a*Log[x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x}\,dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int \left(\frac{a}{x}+b+cx\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$a\log(x)+bx+\frac{cx^2}{2}$$

input
```
Int[(a + b*x + c*x^2)/x,x]
```

output
```
b*x + (c*x^2)/2 + a*Log[x]
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$bx + \frac{c x^2}{2} + a \ln (x)$	15
norman	$bx + \frac{c x^2}{2} + a \ln (x)$	15
risch	$bx + \frac{c x^2}{2} + a \ln (x)$	15
parallelrisch	$bx + \frac{c x^2}{2} + a \ln (x)$	15

input
```
int((c*x^2+b*x+a)/x,x,method=_RETURNVERBOSE)
```

output
```
b*x+1/2*c*x^2+a*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{a+bx+cx^2}{x}\,dx = \frac{1}{2}\,cx^2 + bx + a\log\left(x\right)$$

input
```
integrate((c*x^2+b*x+a)/x,x, algorithm="fricas")
```

output
```
1/2*c*x^2 + b*x + a*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{a+bx+cx^2}{x}\,dx = a\log\left(x\right) + bx + \frac{cx^2}{2}$$

input
```
integrate((c*x**2+b*x+a)/x,x)
```

output
```
a*log(x) + b*x + c*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{a+bx+cx^2}{x}\,dx = \frac{1}{2}\,cx^2 + bx + a\log{(x)}$$

input
```
integrate((c*x^2+b*x+a)/x,x, algorithm="maxima")
```

output
```
1/2*c*x^2 + b*x + a*log(x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{a+bx+cx^2}{x}\,dx = \frac{1}{2}\,cx^2 + bx + a\log{(|x|)}$$

input
```
integrate((c*x^2+b*x+a)/x,x, algorithm="giac")
```

output
```
1/2*c*x^2 + b*x + a*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{a+bx+cx^2}{x}\,dx = b\,x + \frac{c\,x^2}{2} + a\,\ln{(x)}$$

input
```
int((a + b*x + c*x^2)/x,x)
```

output
```
b*x + (c*x^2)/2 + a*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{a + bx + cx^2}{x} \, dx = \log(x) \, a + bx + \frac{c \, x^2}{2}$$

input
```
int((c*x^2+b*x+a)/x,x)
```

output
```
(2*log(x)*a + 2*b*x + c*x**2)/2
```

3.184 $\int \frac{a+bx+cx^2}{x^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{a+bx+cx^2}{x^2} dx = -\frac{a}{x} + cx + b\log(x)$$

output
```
-a/x+c*x+b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{a+bx+cx^2}{x^2} dx = -\frac{a}{x} + cx + b\log(x)$$

input
```
Integrate[(a + b*x + c*x^2)/x^2,x]
```

output
```
-(a/x) + c*x + b*Log[x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x^2} \, dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int \left(\frac{a}{x^2}+\frac{b}{x}+c\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a}{x}+b\log(x)+cx$$

input
```
Int[(a + b*x + c*x^2)/x^2,x]
```

output
```
-(a/x) + c*x + b*Log[x]
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$-\frac{a}{x}+cx+b\ln\left(x\right)$	15
risch	$-\frac{a}{x}+cx+b\ln\left(x\right)$	15
norman	$\frac{c\,x^{2}-a}{x}+b\ln\left(x\right)$	19
parallelrisch	$\frac{\ln\left(x\right)xb+c\,x^{2}-a}{x}$	19

input
```
int((c*x^2+b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x+c*x+b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{a+bx+cx^2}{x^2}\,dx=\frac{cx^2+bx\log\left(x\right)-a}{x}$$

input
```
integrate((c*x^2+b*x+a)/x^2,x, algorithm="fricas")
```

output
```
(c*x^2 + b*x*log(x) - a)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{a+bx+cx^2}{x^2}\,dx=-\frac{a}{x}+b\log\left(x\right)+cx$$

input
```
integrate((c*x**2+b*x+a)/x**2,x)
```

output
```
-a/x + b*log(x) + c*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{a+bx+cx^2}{x^2}\,dx = cx + b\log(x) - \frac{a}{x}$$

input
```
integrate((c*x^2+b*x+a)/x^2,x, algorithm="maxima")
```

output
```
c*x + b*log(x) - a/x
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{a+bx+cx^2}{x^2}\,dx = cx + b\log(|x|) - \frac{a}{x}$$

input
```
integrate((c*x^2+b*x+a)/x^2,x, algorithm="giac")
```

output
```
c*x + b*log(abs(x)) - a/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{a+bx+cx^2}{x^2}\,dx = c\,x - \frac{a}{x} + b\,\ln(x)$$

input
```
int((a + b*x + c*x^2)/x^2,x)
```

output
```
c*x - a/x + b*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{a+bx+cx^2}{x^2}\,dx = \frac{\log(x)\,bx-a+c\,x^2}{x}$$

input
```
int((c*x^2+b*x+a)/x^2,x)
```

output
```
(log(x)*b*x - a + c*x**2)/x
```

3.185 $\int \frac{a+bx+cx^2}{x^3} dx$

Optimal result

Integrand size = 14, antiderivative size = 19

$$\int \frac{a+bx+cx^2}{x^3} dx = -\frac{a}{2x^2} - \frac{b}{x} + c\log(x)$$

output
```
-1/2*a/x^2-b/x+c*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{a+bx+cx^2}{x^3} dx = -\frac{a}{2x^2} - \frac{b}{x} + c\log(x)$$

input
```
Integrate[(a + b*x + c*x^2)/x^3,x]
```

output
```
-1/2*a/x^2 - b/x + c*Log[x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x^3} \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{a}{x^3}+\frac{b}{x^2}+\frac{c}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{2x^2}-\frac{b}{x}+c\log(x)$$

input
```
Int[(a + b*x + c*x^2)/x^3,x]
```

output
```
-1/2*a/x^2 - b/x + c*Log[x]
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{a}{2x^2}-\frac{b}{x}+c\ln\left(x\right)$	18
norman	$\frac{-bx-\frac{a}{2}}{x^2}+c\ln\left(x\right)$	18
risch	$\frac{-bx-\frac{a}{2}}{x^2}+c\ln\left(x\right)$	18
parallelrisch	$\frac{2c\ln(x)x^2-2bx-a}{2x^2}$	22

input
```
int((c*x^2+b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/x^2-b/x+c*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{a+bx+cx^2}{x^3}\,dx=\frac{2\,cx^2\log\left(x\right)-2\,bx-a}{2\,x^2}$$

input
```
integrate((c*x^2+b*x+a)/x^3,x, algorithm="fricas")
```

output
```
1/2*(2*c*x^2*log(x) - 2*b*x - a)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a+bx+cx^2}{x^3}\,dx=c\log\left(x\right)+\frac{-a-2bx}{2x^2}$$

input
```
integrate((c*x**2+b*x+a)/x**3,x)
```

output
```
c*log(x) + (-a - 2*b*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{a+bx+cx^2}{x^3}\,dx = c\log(x) - \frac{2\,bx+a}{2\,x^2}$$

input
```
integrate((c*x^2+b*x+a)/x^3,x, algorithm="maxima")
```

output
```
c*log(x) - 1/2*(2*b*x + a)/x^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a+bx+cx^2}{x^3}\,dx = c\log(|x|) - \frac{2\,bx+a}{2\,x^2}$$

input
```
integrate((c*x^2+b*x+a)/x^3,x, algorithm="giac")
```

output
```
c*log(abs(x)) - 1/2*(2*b*x + a)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a+bx+cx^2}{x^3}\,dx = c\,\ln(x) - \frac{\frac{a}{2}+b\,x}{x^2}$$

input
```
int((a + b*x + c*x^2)/x^3,x)
```

output
```
c*log(x) - (a/2 + b*x)/x^2
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{a+bx+cx^2}{x^3}\,dx = \frac{2\log(x)\,c\,x^2-a-2bx}{2x^2}$$

input
```
int((c*x^2+b*x+a)/x^3,x)
```

output
```
(2*log(x)*c*x**2 - a - 2*b*x)/(2*x**2)
```

3.186 $\int \frac{a+bx+cx^2}{x^4} dx$

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{a+bx+cx^2}{x^4} dx = -\frac{a}{3x^3} - \frac{b}{2x^2} - \frac{c}{x}$$

output
```
-1/3*a/x^3-1/2*b/x^2-c/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+bx+cx^2}{x^4} dx = -\frac{a}{3x^3} - \frac{b}{2x^2} - \frac{c}{x}$$

input
```
Integrate[(a + b*x + c*x^2)/x^4,x]
```

output
```
-1/3*a/x^3 - b/(2*x^2) - c/x
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x^4} \, dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int \left(\frac{a}{x^4}+\frac{b}{x^3}+\frac{c}{x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a}{3x^3}-\frac{b}{2x^2}-\frac{c}{x}$$

input
```
Int[(a + b*x + c*x^2)/x^4,x]
```

output
```
-1/3*a/x^3 - b/(2*x^2) - c/x
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
norman	$\frac{-c\,x^{2}-\frac{1}{2}bx-\frac{1}{3}a}{x^{3}}$	19
risch	$\frac{-c\,x^{2}-\frac{1}{2}bx-\frac{1}{3}a}{x^{3}}$	19
gosper	$-\frac{6c\,x^{2}+3bx+2a}{6x^{3}}$	20
default	$-\frac{a}{3x^{3}}-\frac{b}{2x^{2}}-\frac{c}{x}$	20
parallelrisch	$\frac{-6c\,x^{2}-3bx-2a}{6x^{3}}$	20
orering	$-\frac{6c\,x^{2}+3bx+2a}{6x^{3}}$	20

input
```
int((c*x^2+b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/x^3*(-c*x^2-1/2*b*x-1/3*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{a+bx+cx^2}{x^4}\,dx=-\frac{6\,cx^2+3\,bx+2\,a}{6\,x^3}$$

input
```
integrate((c*x^2+b*x+a)/x^4,x, algorithm="fricas")
```

output
```
-1/6*(6*c*x^2 + 3*b*x + 2*a)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a+bx+cx^2}{x^4}\,dx = \frac{-2a-3bx-6cx^2}{6x^3}$$

input
```
integrate((c*x**2+b*x+a)/x**4,x)
```

output
```
(-2*a - 3*b*x - 6*c*x**2)/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{a+bx+cx^2}{x^4}\,dx = -\frac{6\,cx^2+3\,bx+2\,a}{6\,x^3}$$

input
```
integrate((c*x^2+b*x+a)/x^4,x, algorithm="maxima")
```

output
```
-1/6*(6*c*x^2 + 3*b*x + 2*a)/x^3
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{a+bx+cx^2}{x^4}\,dx = -\frac{6\,cx^2+3\,bx+2\,a}{6\,x^3}$$

input
```
integrate((c*x^2+b*x+a)/x^4,x, algorithm="giac")
```

output
```
-1/6*(6*c*x^2 + 3*b*x + 2*a)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{a+bx+cx^2}{x^4}\,dx = -\frac{c\,x^2+\frac{b\,x}{2}+\frac{a}{3}}{x^3}$$

input
```
int((a + b*x + c*x^2)/x^4,x)
```

output
```
-(a/3 + (b*x)/2 + c*x^2)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{a+bx+cx^2}{x^4}\,dx = \frac{-6c\,x^2-3bx-2a}{6x^3}$$

input
```
int((c*x^2+b*x+a)/x^4,x)
```

output
```
( - 2*a - 3*b*x - 6*c*x**2)/(6*x**3)
```

3.187 $\int \frac{a+bx+cx^2}{x^5} dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \frac{a+bx+cx^2}{x^5} dx = -\frac{a}{4x^4} - \frac{b}{3x^3} - \frac{c}{2x^2}$$

output
```
-1/4*a/x^4-1/3*b/x^3-1/2*c/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{a+bx+cx^2}{x^5} dx = -\frac{a}{4x^4} - \frac{b}{3x^3} - \frac{c}{2x^2}$$

input
```
Integrate[(a + b*x + c*x^2)/x^5,x]
```

output
```
-1/4*a/x^4 - b/(3*x^3) - c/(2*x^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x^5} \, dx$$

$$\Big\downarrow \; 1140$$

$$\int \left(\frac{a}{x^5}+\frac{b}{x^4}+\frac{c}{x^3}\right) dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{c}{2x^2}$$

input
```
Int[(a + b*x + c*x^2)/x^5,x]
```

output
```
-1/4*a/x^4 - b/(3*x^3) - c/(2*x^2)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

method	result	size
norman	$\frac{-\frac{1}{2}c\,x^2-\frac{1}{3}bx-\frac{1}{4}a}{x^4}$	19
risch	$\frac{-\frac{1}{2}c\,x^2-\frac{1}{3}bx-\frac{1}{4}a}{x^4}$	19
gosper	$-\frac{6c\,x^2+4bx+3a}{12x^4}$	20
default	$-\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{c}{2x^2}$	20
parallelrisch	$\frac{-6c\,x^2-4bx-3a}{12x^4}$	20
orering	$-\frac{6c\,x^2+4bx+3a}{12x^4}$	20

input
```
int((c*x^2+b*x+a)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/x^4*(-1/2*c*x^2-1/3*b*x-1/4*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{a+bx+cx^2}{x^5}\,dx=-\frac{6\,cx^2+4\,bx+3\,a}{12\,x^4}$$

input
```
integrate((c*x^2+b*x+a)/x^5,x, algorithm="fricas")
```

output
```
-1/12*(6*c*x^2 + 4*b*x + 3*a)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{a+bx+cx^2}{x^5}\,dx = \frac{-3a-4bx-6cx^2}{12x^4}$$

input
```
integrate((c*x**2+b*x+a)/x**5,x)
```

output
```
(-3*a - 4*b*x - 6*c*x**2)/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{a+bx+cx^2}{x^5}\,dx = -\frac{6\,cx^2+4\,bx+3\,a}{12\,x^4}$$

input
```
integrate((c*x^2+b*x+a)/x^5,x, algorithm="maxima")
```

output
```
-1/12*(6*c*x^2 + 4*b*x + 3*a)/x^4
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{a+bx+cx^2}{x^5}\,dx = -\frac{6\,cx^2+4\,bx+3\,a}{12\,x^4}$$

input
```
integrate((c*x^2+b*x+a)/x^5,x, algorithm="giac")
```

output
```
-1/12*(6*c*x^2 + 4*b*x + 3*a)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{a + bx + cx^2}{x^5} \, dx = -\frac{\frac{c x^2}{2} + \frac{b x}{3} + \frac{a}{4}}{x^4}$$

input
```
int((a + b*x + c*x^2)/x^5,x)
```

output
```
-(a/4 + (b*x)/3 + (c*x^2)/2)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{a + bx + cx^2}{x^5} \, dx = \frac{-6c x^2 - 4bx - 3a}{12x^4}$$

input
```
int((c*x^2+b*x+a)/x^5,x)
```

output
```
( - 3*a - 4*b*x - 6*c*x**2)/(12*x**4)
```

3.188 $\int \frac{a+bx+cx^2}{x^6} dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \frac{a+bx+cx^2}{x^6} dx = -\frac{a}{5x^5} - \frac{b}{4x^4} - \frac{c}{3x^3}$$

output
```
-1/5*a/x^5-1/4*b/x^4-1/3*c/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{a+bx+cx^2}{x^6} dx = -\frac{a}{5x^5} - \frac{b}{4x^4} - \frac{c}{3x^3}$$

input
```
Integrate[(a + b*x + c*x^2)/x^6,x]
```

output
```
-1/5*a/x^5 - b/(4*x^4) - c/(3*x^3)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x^6} \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{a}{x^6}+\frac{b}{x^5}+\frac{c}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{5x^5}-\frac{b}{4x^4}-\frac{c}{3x^3}$$

input
```
Int[(a + b*x + c*x^2)/x^6,x]
```

output
```
-1/5*a/x^5 - b/(4*x^4) - c/(3*x^3)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

method	result	size
norman	$\frac{-\frac{1}{3}c\,x^2-\frac{1}{4}bx-\frac{1}{5}a}{x^5}$	19
risch	$\frac{-\frac{1}{3}c\,x^2-\frac{1}{4}bx-\frac{1}{5}a}{x^5}$	19
gosper	$-\frac{20c\,x^2+15bx+12a}{60x^5}$	20
default	$-\frac{a}{5x^5}-\frac{b}{4x^4}-\frac{c}{3x^3}$	20
parallelrisch	$\frac{-20c\,x^2-15bx-12a}{60x^5}$	20
orering	$-\frac{20c\,x^2+15bx+12a}{60x^5}$	20

input
```
int((c*x^2+b*x+a)/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/x^5*(-1/3*c*x^2-1/4*b*x-1/5*a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{a+bx+cx^2}{x^6}\,dx=-\frac{20\,cx^2+15\,bx+12\,a}{60\,x^5}$$

input
```
integrate((c*x^2+b*x+a)/x^6,x, algorithm="fricas")
```

output
```
-1/60*(20*c*x^2 + 15*b*x + 12*a)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{a+bx+cx^2}{x^6}\,dx = \frac{-12a-15bx-20cx^2}{60x^5}$$

input
```
integrate((c*x**2+b*x+a)/x**6,x)
```

output
```
(-12*a - 15*b*x - 20*c*x**2)/(60*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{a+bx+cx^2}{x^6}\,dx = -\frac{20\,cx^2+15\,bx+12\,a}{60\,x^5}$$

input
```
integrate((c*x^2+b*x+a)/x^6,x, algorithm="maxima")
```

output
```
-1/60*(20*c*x^2 + 15*b*x + 12*a)/x^5
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{a+bx+cx^2}{x^6}\,dx = -\frac{20\,cx^2+15\,bx+12\,a}{60\,x^5}$$

input
```
integrate((c*x^2+b*x+a)/x^6,x, algorithm="giac")
```

output
```
-1/60*(20*c*x^2 + 15*b*x + 12*a)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{a+bx+cx^2}{x^6}\,dx = -\frac{\frac{c\,x^2}{3}+\frac{b\,x}{4}+\frac{a}{5}}{x^5}$$

input
```
int((a + b*x + c*x^2)/x^6,x)
```

output
```
-(a/5 + (b*x)/4 + (c*x^2)/3)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{a+bx+cx^2}{x^6}\,dx = \frac{-20c\,x^2-15bx-12a}{60x^5}$$

input
```
int((c*x^2+b*x+a)/x^6,x)
```

output
```
( - 12*a - 15*b*x - 20*c*x**2)/(60*x**5)
```

3.189 $\int x^2(a+bx+cx^2)^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 54

$$\int x^2\left(a+bx+cx^2\right)^2\,dx=\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{1}{3}bcx^6+\frac{c^2x^7}{7}$$

output
```
1/3*a^2*x^3+1/2*a*b*x^4+1/5*(2*a*c+b^2)*x^5+1/3*b*c*x^6+1/7*c^2*x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int x^2\left(a+bx+cx^2\right)^2\,dx=\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{1}{3}bcx^6+\frac{c^2x^7}{7}$$

input
```
Integrate[x^2*(a + b*x + c*x^2)^2,x]
```

output
```
(a^2*x^3)/3 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^5)/5 + (b*c*x^6)/3 + (c^2*x^7
)/7
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a + bx + cx^2)^2 \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int (a^2x^2 + x^4(2ac + b^2) + 2abx^3 + 2bcx^5 + c^2x^6) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2x^3}{3} + \frac{1}{5}x^5(2ac + b^2) + \frac{1}{2}abx^4 + \frac{1}{3}bcx^6 + \frac{c^2x^7}{7}$$

input
```
Int[x^2*(a + b*x + c*x^2)^2,x]
```

output
```
(a^2*x^3)/3 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^5)/5 + (b*c*x^6)/3 + (c^2*x^7
)/7
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^{2} x^{3}}{3}+\frac{a b \,x^{4}}{2}+\frac{\left(2 a c +b^{2}\right) x^{5}}{5}+\frac{b c \,x^{6}}{3}+\frac{c^{2} x^{7}}{7}$	45
norman	$\frac{c^{2} x^{7}}{7}+\frac{b c \,x^{6}}{3}+\left(\frac{2 a c}{5}+\frac{b^{2}}{5}\right) x^{5}+\frac{a b \,x^{4}}{2}+\frac{a^{2} x^{3}}{3}$	46
gosper	$\frac{1}{7} c^{2} x^{7}+\frac{1}{3} b c \,x^{6}+\frac{2}{5} a c \,x^{5}+\frac{1}{5} b^{2} x^{5}+\frac{1}{2} a b \,x^{4}+\frac{1}{3} a^{2} x^{3}$	47
risch	$\frac{1}{7} c^{2} x^{7}+\frac{1}{3} b c \,x^{6}+\frac{2}{5} a c \,x^{5}+\frac{1}{5} b^{2} x^{5}+\frac{1}{2} a b \,x^{4}+\frac{1}{3} a^{2} x^{3}$	47
parallelrisch	$\frac{1}{7} c^{2} x^{7}+\frac{1}{3} b c \,x^{6}+\frac{2}{5} a c \,x^{5}+\frac{1}{5} b^{2} x^{5}+\frac{1}{2} a b \,x^{4}+\frac{1}{3} a^{2} x^{3}$	47
orering	$\frac{x^{3} \left(30 c^{2} x^{4}+70 b c \,x^{3}+84 a c \,x^{2}+42 b^{2} x^{2}+105 a b x +70 a^{2}\right)}{210}$	47

input

```
int(x^2*(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output

```
1/3*a^2*x^3+1/2*a*b*x^4+1/5*(2*a*c+b^2)*x^5+1/3*b*c*x^6+1/7*c^2*x^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x^2 \left(a + bx + cx^2\right)^2 \, dx = \frac{1}{7}\, c^2 x^7 + \frac{1}{3}\, bcx^6 + \frac{1}{2}\, abx^4 + \frac{1}{5}\, \left(b^2 + 2\, ac\right)x^5 + \frac{1}{3}\, a^2 x^3$$

input

```
integrate(x^2*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output

```
1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/2*a*b*x^4 + 1/5*(b^2 + 2*a*c)*x^5 + 1/3*a^2*
x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int x^2 \left(a + bx + cx^2\right)^2 \, dx = \frac{a^2x^3}{3} + \frac{abx^4}{2} + \frac{bcx^6}{3} + \frac{c^2x^7}{7} + x^5 \cdot \left(\frac{2ac}{5} + \frac{b^2}{5}\right)$$

input
```
integrate(x**2*(c*x**2+b*x+a)**2,x)
```

output
```
a**2*x**3/3 + a*b*x**4/2 + b*c*x**6/3 + c**2*x**7/7 + x**5*(2*a*c/5 + b**2
/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x^2 \left(a + bx + cx^2\right)^2 \, dx = \frac{1}{7} c^2x^7 + \frac{1}{3} bcx^6 + \frac{1}{2} abx^4 + \frac{1}{5} \left(b^2 + 2\,ac\right)x^5 + \frac{1}{3} a^2x^3$$

input
```
integrate(x^2*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/2*a*b*x^4 + 1/5*(b^2 + 2*a*c)*x^5 + 1/3*a^2*
x^3
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int x^2 \left(a + bx + cx^2\right)^2 \, dx = \frac{1}{7} c^2x^7 + \frac{1}{3} bcx^6 + \frac{1}{5} b^2x^5 + \frac{2}{5} acx^5 + \frac{1}{2} abx^4 + \frac{1}{3} a^2x^3$$

input
```
integrate(x^2*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 1/2*a*b*x^4 + 1/3*
a^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int x^2 (a + bx + cx^2)^2 \, dx = x^5 \left(\frac{b^2}{5} + \frac{2\,a\,c}{5}\right) + \frac{a^2\,x^3}{3} + \frac{c^2\,x^7}{7} + \frac{a\,b\,x^4}{2} + \frac{b\,c\,x^6}{3}$$

input
```
int(x^2*(a + b*x + c*x^2)^2,x)
```

output
```
x^5*((2*a*c)/5 + b^2/5) + (a^2*x^3)/3 + (c^2*x^7)/7 + (a*b*x^4)/2 + (b*c*x
^6)/3
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int x^2 (a + bx + cx^2)^2 \, dx = \frac{x^3(30c^2x^4 + 70bc\,x^3 + 84ac\,x^2 + 42b^2x^2 + 105abx + 70a^2)}{210}$$

input
```
int(x^2*(c*x^2+b*x+a)^2,x)
```

output
```
(x**3*(70*a**2 + 105*a*b*x + 84*a*c*x**2 + 42*b**2*x**2 + 70*b*c*x**3 + 30
*c**2*x**4))/210
```

3.190 $\int x(a+bx+cx^2)^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int x\left(a+bx+cx^2\right)^2\,dx=\frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{1}{4}\left(b^2+2ac\right)x^4+\frac{2}{5}bcx^5+\frac{c^2x^6}{6}$$

output
```
1/2*a^2*x^2+2/3*a*b*x^3+1/4*(2*a*c+b^2)*x^4+2/5*b*c*x^5+1/6*c^2*x^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int x\left(a+bx+cx^2\right)^2\,dx=\frac{1}{60}x^2\left(30a^2+40abx+15\left(b^2+2ac\right)x^2+24bcx^3+10c^2x^4\right)$$

input
```
Integrate[x*(a + b*x + c*x^2)^2,x]
```

output
```
(x^2*(30*a^2 + 40*a*b*x + 15*(b^2 + 2*a*c)*x^2 + 24*b*c*x^3 + 10*c^2*x^4))
/60
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx+cx^2\right)^2 \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(a^2x+x^3\left(2ac+b^2\right)+2abx^2+2bcx^4+c^2x^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2x^2}{2}+\frac{1}{4}x^4\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{5}bcx^5+\frac{c^2x^6}{6}$$

input
```
Int[x*(a + b*x + c*x^2)^2,x]
```

output
```
(a^2*x^2)/2 + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^4)/4 + (2*b*c*x^5)/5 + (c^2
*x^6)/6
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^{2} x^{2}}{2}+\frac{2 a b \,x^{3}}{3}+\frac{\left(2 a c +b^{2}\right) x^{4}}{4}+\frac{2 x^{5} b c}{5}+\frac{c^{2} x^{6}}{6}$	45
norman	$\frac{c^{2} x^{6}}{6}+\frac{2 x^{5} b c}{5}+\left(\frac{a c}{2}+\frac{b^{2}}{4}\right) x^{4}+\frac{2 a b \,x^{3}}{3}+\frac{a^{2} x^{2}}{2}$	46
gosper	$\frac{1}{6} c^{2} x^{6}+\frac{2}{5} x^{5} b c +\frac{1}{2} a c \,x^{4}+\frac{1}{4} b^{2} x^{4}+\frac{2}{3} a b \,x^{3}+\frac{1}{2} a^{2} x^{2}$	47
risch	$\frac{1}{6} c^{2} x^{6}+\frac{2}{5} x^{5} b c +\frac{1}{2} a c \,x^{4}+\frac{1}{4} b^{2} x^{4}+\frac{2}{3} a b \,x^{3}+\frac{1}{2} a^{2} x^{2}$	47
parallelrisch	$\frac{1}{6} c^{2} x^{6}+\frac{2}{5} x^{5} b c +\frac{1}{2} a c \,x^{4}+\frac{1}{4} b^{2} x^{4}+\frac{2}{3} a b \,x^{3}+\frac{1}{2} a^{2} x^{2}$	47
orering	$\frac{x^{2} \left(10 c^{2} x^{4}+24 b c \,x^{3}+30 a c \,x^{2}+15 b^{2} x^{2}+40 a b x +30 a^{2}\right)}{60}$	47

input
```
int(x*(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x^2+2/3*a*b*x^3+1/4*(2*a*c+b^2)*x^4+2/5*x^5*b*c+1/6*c^2*x^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x\left(a+bx+cx^2\right)^2\,dx=\frac{1}{6}\,c^{2}x^{6}+\frac{2}{5}\,bcx^{5}+\frac{2}{3}\,abx^{3}+\frac{1}{4}\,\left(b^{2}+2\,ac\right)x^{4}+\frac{1}{2}\,a^{2}x^{2}$$

input
```
integrate(x*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
1/6*c^2*x^6 + 2/5*b*c*x^5 + 2/3*a*b*x^3 + 1/4*(b^2 + 2*a*c)*x^4 + 1/2*a^2*
x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int x(a+bx+cx^2)^2\,dx=\frac{a^2x^2}{2}+\frac{2abx^3}{3}+\frac{2bcx^5}{5}+\frac{c^2x^6}{6}+x^4\left(\frac{ac}{2}+\frac{b^2}{4}\right)$$

input
```
integrate(x*(c*x**2+b*x+a)**2,x)
```

output
```
a**2*x**2/2 + 2*a*b*x**3/3 + 2*b*c*x**5/5 + c**2*x**6/6 + x**4*(a*c/2 + b*
*2/4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x(a+bx+cx^2)^2\,dx=\frac{1}{6}\,c^2x^6+\frac{2}{5}\,bcx^5+\frac{2}{3}\,abx^3+\frac{1}{4}\,(b^2+2\,ac)x^4+\frac{1}{2}\,a^2x^2$$

input
```
integrate(x*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
1/6*c^2*x^6 + 2/5*b*c*x^5 + 2/3*a*b*x^3 + 1/4*(b^2 + 2*a*c)*x^4 + 1/2*a^2*
x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int x(a+bx+cx^2)^2\,dx=\frac{1}{6}\,c^2x^6+\frac{2}{5}\,bcx^5+\frac{1}{4}\,b^2x^4+\frac{1}{2}\,acx^4+\frac{2}{3}\,abx^3+\frac{1}{2}\,a^2x^2$$

input
```
integrate(x*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
1/6*c^2*x^6 + 2/5*b*c*x^5 + 1/4*b^2*x^4 + 1/2*a*c*x^4 + 2/3*a*b*x^3 + 1/2*
a^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int x(a+bx+cx^2)^2\,dx=x^4\left(\frac{b^2}{4}+\frac{a\,c}{2}\right)+\frac{a^2\,x^2}{2}+\frac{c^2\,x^6}{6}+\frac{2\,a\,b\,x^3}{3}+\frac{2\,b\,c\,x^5}{5}$$

input
```
int(x*(a + b*x + c*x^2)^2,x)
```

output
```
x^4*((a*c)/2 + b^2/4) + (a^2*x^2)/2 + (c^2*x^6)/6 + (2*a*b*x^3)/3 + (2*b*c
*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int x(a+bx+cx^2)^2\,dx=\frac{x^2(10c^2x^4+24bc\,x^3+30ac\,x^2+15b^2x^2+40abx+30a^2)}{60}$$

input
```
int(x*(c*x^2+b*x+a)^2,x)
```

output
```
(x**2*(30*a**2 + 40*a*b*x + 30*a*c*x**2 + 15*b**2*x**2 + 24*b*c*x**3 + 10*
c**2*x**4))/60
```

3.191 $\int (a+bx+cx^2)^2 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 46

$$\int (a+bx+cx^2)^2 \, dx = a^2x+abx^2+\frac{1}{3}(b^2+2ac)\,x^3+\frac{1}{2}bcx^4+\frac{c^2x^5}{5}$$

output
```
a^2*x+a*b*x^2+1/3*(2*a*c+b^2)*x^3+1/2*b*c*x^4+1/5*c^2*x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (a+bx+cx^2)^2 \, dx = a^2x+abx^2+\frac{1}{3}(b^2+2ac)\,x^3+\frac{1}{2}bcx^4+\frac{c^2x^5}{5}$$

input
```
Integrate[(a + b*x + c*x^2)^2,x]
```

output
```
a^2*x + a*b*x^2 + ((b^2 + 2*a*c)*x^3)/3 + (b*c*x^4)/2 + (c^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right)^2 \, dx$$

$$\Big\downarrow \text{1085}$$

$$\int \left(a^2+b^2x^2\left(\frac{2ac}{b^2}+1\right)+2abx+2bcx^3+c^2x^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2x+\frac{1}{3}x^3\left(2ac+b^2\right)+abx^2+\frac{1}{2}bcx^4+\frac{c^2x^5}{5}$$

input
```
Int[(a + b*x + c*x^2)^2,x]
```

output
```
a^2*x + a*b*x^2 + ((b^2 + 2*a*c)*x^3)/3 + (b*c*x^4)/2 + (c^2*x^5)/5
```

Defintions of rubi rules used

rule 1085
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && IntegerQ[p] && (G
tQ[p, 0] || EqQ[a, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

method	result	size
default	$a^{2} x +a b \,x^{2}+\frac{\left(2 a c +b^{2}\right) x^{3}}{3}+\frac{b c \,x^{4}}{2}+\frac{c^{2} x^{5}}{5}$	41
norman	$\frac{c^{2} x^{5}}{5}+\frac{b c \,x^{4}}{2}+\left(\frac{2 a c}{3}+\frac{b^{2}}{3}\right) x^{3}+a b \,x^{2}+a^{2} x$	42
gosper	$\frac{1}{5} c^{2} x^{5}+\frac{1}{2} b c \,x^{4}+\frac{2}{3} a c \,x^{3}+\frac{1}{3} b^{2} x^{3}+a b \,x^{2}+a^{2} x$	43
risch	$\frac{1}{5} c^{2} x^{5}+\frac{1}{2} b c \,x^{4}+\frac{2}{3} a c \,x^{3}+\frac{1}{3} b^{2} x^{3}+a b \,x^{2}+a^{2} x$	43
parallelrisch	$\frac{1}{5} c^{2} x^{5}+\frac{1}{2} b c \,x^{4}+\frac{2}{3} a c \,x^{3}+\frac{1}{3} b^{2} x^{3}+a b \,x^{2}+a^{2} x$	43
orering	$\frac{x \left(6 c^{2} x^{4}+15 b c \,x^{3}+20 a c \,x^{2}+10 b^{2} x^{2}+30 a b x +30 a^{2}\right)}{30}$	45

input
```
int((c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+a*b*x^2+1/3*(2*a*c+b^2)*x^3+1/2*b*c*x^4+1/5*c^2*x^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \left(a+bx+cx^2\right)^2 \, dx=\frac{1}{5} \, c^{2} x^{5} + \frac{1}{2} \, b c x^{4} + a b x^{2} + \frac{1}{3} \, {\left(b^{2} + 2 \, a c\right)} x^{3} + a^{2} x$$

input
```
integrate((c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
1/5*c^2*x^5 + 1/2*b*c*x^4 + a*b*x^2 + 1/3*(b^2 + 2*a*c)*x^3 + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \left(a+bx+cx^2\right)^2\,dx=a^2x+abx^2+\frac{bcx^4}{2}+\frac{c^2x^5}{5}+x^3\cdot\left(\frac{2ac}{3}+\frac{b^2}{3}\right)$$

input
```
integrate((c*x**2+b*x+a)**2,x)
```

output
```
a**2*x + a*b*x**2 + b*c*x**4/2 + c**2*x**5/5 + x**3*(2*a*c/3 + b**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \left(a+bx+cx^2\right)^2\,dx=\frac{1}{5}\,c^2x^5+\frac{1}{2}\,bcx^4+\frac{1}{3}\,b^2x^3+a^2x+\frac{1}{3}\,\left(2\,cx^3+3\,bx^2\right)a$$

input
```
integrate((c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
1/5*c^2*x^5 + 1/2*b*c*x^4 + 1/3*b^2*x^3 + a^2*x + 1/3*(2*c*x^3 + 3*b*x^2)*
a
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \left(a+bx+cx^2\right)^2\,dx=\frac{1}{5}\,c^2x^5+\frac{1}{2}\,bcx^4+\frac{1}{3}\,b^2x^3+\frac{2}{3}\,acx^3+abx^2+a^2x$$

input
```
integrate((c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
1/5*c^2*x^5 + 1/2*b*c*x^4 + 1/3*b^2*x^3 + 2/3*a*c*x^3 + a*b*x^2 + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \left(a+bx+cx^2\right)^2\,dx=a^2\,x+x^3\,\left(\frac{b^2}{3}+\frac{2\,a\,c}{3}\right)+\frac{c^2\,x^5}{5}+a\,b\,x^2+\frac{b\,c\,x^4}{2}$$

input
```
int((a + b*x + c*x^2)^2,x)
```

output
```
a^2*x + x^3*((2*a*c)/3 + b^2/3) + (c^2*x^5)/5 + a*b*x^2 + (b*c*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \left(a+bx+cx^2\right)^2\,dx=\frac{x(6c^2x^4+15bc\,x^3+20ac\,x^2+10b^2x^2+30abx+30a^2)}{30}$$

input
```
int((c*x^2+b*x+a)^2,x)
```

output
```
(x*(30*a**2 + 30*a*b*x + 20*a*c*x**2 + 10*b**2*x**2 + 15*b*c*x**3 + 6*c**2
*x**4))/30
```

3.192 $\int \frac{(a+bx+cx^2)^2}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int \frac{(a+bx+cx^2)^2}{x}\,dx = 2abx + \frac{1}{2}(b^2+2ac)\,x^2 + \frac{2}{3}bcx^3 + \frac{c^2x^4}{4} + a^2\log(x)$$

output
```
2*a*b*x+1/2*(2*a*c+b^2)*x^2+2/3*b*c*x^3+1/4*c^2*x^4+a^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx+cx^2)^2}{x}\,dx = ax(2b+cx) + \frac{1}{12}x^2(6b^2+8bcx+3c^2x^2) + a^2\log(x)$$

input
```
Integrate[(a + b*x + c*x^2)^2/x,x]
```

output
```
a*x*(2*b + c*x) + (x^2*(6*b^2 + 8*b*c*x + 3*c^2*x^2))/12 + a^2*Log[x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^2}{x}\,dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int \left(\frac{a^2}{x}+x(2ac+b^2)+2ab+2bcx^2+c^2x^3\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^2\log(x)+\frac{1}{2}x^2(2ac+b^2)+2abx+\frac{2}{3}bcx^3+\frac{c^2x^4}{4}$$

input
```
Int[(a + b*x + c*x^2)^2/x,x]
```

output
```
2*a*b*x + ((b^2 + 2*a*c)*x^2)/2 + (2*b*c*x^3)/3 + (c^2*x^4)/4 + a^2*Log[x]
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

method	result	size
norman	$\left(ac+\frac{b^2}{2}\right)x^2+\frac{c^2x^4}{4}+2abx+\frac{2bc\,x^3}{3}+a^2\ln\left(x\right)$	41
default	$\frac{c^2x^4}{4}+\frac{2bc\,x^3}{3}+ac\,x^2+\frac{b^2x^2}{2}+2abx+a^2\ln\left(x\right)$	42
risch	$\frac{c^2x^4}{4}+\frac{2bc\,x^3}{3}+ac\,x^2+\frac{b^2x^2}{2}+2abx+a^2\ln\left(x\right)$	42
parallelrisch	$\frac{c^2x^4}{4}+\frac{2bc\,x^3}{3}+ac\,x^2+\frac{b^2x^2}{2}+2abx+a^2\ln\left(x\right)$	42

input
```
int((c*x^2+b*x+a)^2/x,x,method=_RETURNVERBOSE)
```

output
```
(a*c+1/2*b^2)*x^2+1/4*c^2*x^4+2*a*b*x+2/3*b*c*x^3+a^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx+cx^2)^2}{x}\,dx=\frac{1}{4}\,c^2x^4+\frac{2}{3}\,bcx^3+2\,abx+\frac{1}{2}\,(b^2+2\,ac)x^2+a^2\log\left(x\right)$$

input
```
integrate((c*x^2+b*x+a)^2/x,x, algorithm="fricas")
```

output
```
1/4*c^2*x^4 + 2/3*b*c*x^3 + 2*a*b*x + 1/2*(b^2 + 2*a*c)*x^2 + a^2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx+cx^2)^2}{x}\,dx=a^2\log\left(x\right)+2abx+\frac{2bcx^3}{3}+\frac{c^2x^4}{4}+x^2\left(ac+\frac{b^2}{2}\right)$$

input
```
integrate((c*x**2+b*x+a)**2/x,x)
```

output
```
a**2*log(x) + 2*a*b*x + 2*b*c*x**3/3 + c**2*x**4/4 + x**2*(a*c + b**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx+cx^2)^2}{x}\,dx = \frac{1}{4}\,c^2x^4 + \frac{2}{3}\,bcx^3 + 2\,abx + \frac{1}{2}\,(b^2+2\,ac)x^2 + a^2\log{(x)}$$

input
```
integrate((c*x^2+b*x+a)^2/x,x, algorithm="maxima")
```

output
```
1/4*c^2*x^4 + 2/3*b*c*x^3 + 2*a*b*x + 1/2*(b^2 + 2*a*c)*x^2 + a^2*log(x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx+cx^2)^2}{x}\,dx = \frac{1}{4}\,c^2x^4 + \frac{2}{3}\,bcx^3 + \frac{1}{2}\,b^2x^2 + acx^2 + 2\,abx + a^2\log{(|x|)}$$

input
```
integrate((c*x^2+b*x+a)^2/x,x, algorithm="giac")
```

output
```
1/4*c^2*x^4 + 2/3*b*c*x^3 + 1/2*b^2*x^2 + a*c*x^2 + 2*a*b*x + a^2*log(abs(
x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx+cx^2)^2}{x}\,dx = a^2\,\ln{(x)} + x^2\left(\frac{b^2}{2} + a\,c\right) + \frac{c^2\,x^4}{4} + 2\,a\,b\,x + \frac{2\,b\,c\,x^3}{3}$$

input
```
int((a + b*x + c*x^2)^2/x,x)
```

output
```
a^2*log(x) + x^2*(a*c + b^2/2) + (c^2*x^4)/4 + 2*a*b*x + (2*b*c*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx+cx^2)^2}{x}\,dx = \log(x)\,a^2 + 2abx + ac\,x^2 + \frac{b^2x^2}{2} + \frac{2bc\,x^3}{3} + \frac{c^2x^4}{4}$$

input
```
int((c*x^2+b*x+a)^2/x,x)
```

output
```
(12*log(x)*a**2 + 24*a*b*x + 12*a*c*x**2 + 6*b**2*x**2 + 8*b*c*x**3 + 3*c*
*2*x**4)/12
```

3.193 $\int \frac{(a+bx+cx^2)^2}{x^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 41

$$\int \frac{(a+bx+cx^2)^2}{x^2} dx = -\frac{a^2}{x} + (b^2+2ac)\, x + bcx^2 + \frac{c^2x^3}{3} + 2ab\log(x)$$

output
```
-a^2/x+(2*a*c+b^2)*x+b*c*x^2+1/3*c^2*x^3+2*a*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2)^2}{x^2} dx = -\frac{a^2}{x} + (b^2+2ac)\, x + bcx^2 + \frac{c^2x^3}{3} + 2ab\log(x)$$

input
```
Integrate[(a + b*x + c*x^2)^2/x^2,x]
```

output
```
-(a^2/x) + (b^2 + 2*a*c)*x + b*c*x^2 + (c^2*x^3)/3 + 2*a*b*Log[x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^2}{x^2}\,dx$$

$$\Big\downarrow \ 1140$$

$$\int \left(\frac{a^2}{x^2}+b^2\left(\frac{2ac}{b^2}+1\right)+\frac{2ab}{x}+2bcx+c^2x^2\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^2}{x}+x(2ac+b^2)+2ab\log(x)+bcx^2+\frac{c^2x^3}{3}$$

input
```
Int[(a + b*x + c*x^2)^2/x^2,x]
```

output
```
-(a^2/x) + (b^2 + 2*a*c)*x + b*c*x^2 + (c^2*x^3)/3 + 2*a*b*Log[x]
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{x^{3} c^{2}}{3}+b c \,x^{2}+2 a c x +b^{2} x +2 a b \ln \left(x \right)-\frac{a^{2}}{x}$	40
risch	$\frac{x^{3} c^{2}}{3}+b c \,x^{2}+2 a c x +b^{2} x +2 a b \ln \left(x \right)-\frac{a^{2}}{x}$	40
norman	$\frac{\left(2 a c +b^{2}\right) x^{2}+b c \,x^{3}-a^{2}+\frac{c^{2} x^{4}}{3}}{x}+2 a b \ln \left(x \right)$	44
parallelrisch	$\frac{c^{2} x^{4}+3 b c \,x^{3}+6 \ln \left(x \right) a b x +6 a c \,x^{2}+3 b^{2} x^{2}-3 a^{2}}{3 x}$	48

input
```
int((c*x^2+b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*c^2+b*c*x^2+2*a*c*x+b^2*x+2*a*b*ln(x)-a^2/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+bx+cx^2\right)^2}{x^2}\,dx=\frac{c^{2} x^{4}+3 \, b c x^{3}+6 \, a b x \log \left(x\right)+3 \,{\left(b^{2}+2 \, a c\right)} x^{2}-3 \, a^{2}}{3 \, x}$$

input
```
integrate((c*x^2+b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
1/3*(c^2*x^4 + 3*b*c*x^3 + 6*a*b*x*log(x) + 3*(b^2 + 2*a*c)*x^2 - 3*a^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+bx+cx^2\right)^2}{x^2}\,dx=-\frac{a^{2}}{x}+2ab\log{\left(x \right)}+bcx^{2}+\frac{c^{2}x^{3}}{3}+x\left(2ac+b^{2}\right)$$

input
```
integrate((c*x**2+b*x+a)**2/x**2,x)
```

output
```
-a**2/x + 2*a*b*log(x) + b*c*x**2 + c**2*x**3/3 + x*(2*a*c + b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx+cx^2)^2}{x^2}\,dx = \frac{1}{3}\,c^2x^3 + bcx^2 + 2\,ab\log{(x)} + \left(b^2+2\,ac\right)x - \frac{a^2}{x}$$

input
```
integrate((c*x^2+b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
1/3*c^2*x^3 + b*c*x^2 + 2*a*b*log(x) + (b^2 + 2*a*c)*x - a^2/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx+cx^2)^2}{x^2}\,dx = \frac{1}{3}\,c^2x^3 + bcx^2 + b^2x + 2\,acx + 2\,ab\log{(|x|)} - \frac{a^2}{x}$$

input
```
integrate((c*x^2+b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
1/3*c^2*x^3 + b*c*x^2 + b^2*x + 2*a*c*x + 2*a*b*log(abs(x)) - a^2/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx+cx^2)^2}{x^2}\,dx = x\,\left(b^2+2\,a\,c\right) - \frac{a^2}{x} + \frac{c^2\,x^3}{3} + 2\,a\,b\,\ln{(x)} + b\,c\,x^2$$

input
```
int((a + b*x + c*x^2)^2/x^2,x)
```

output
```
x*(2*a*c + b^2) - a^2/x + (c^2*x^3)/3 + 2*a*b*log(x) + b*c*x^2
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx+cx^2)^2}{x^2}\,dx = \frac{6\log(x)\,abx - 3a^2 + 6ac\,x^2 + 3b^2x^2 + 3bc\,x^3 + c^2x^4}{3x}$$

input
```
int((c*x^2+b*x+a)^2/x^2,x)
```

output
```
(6*log(x)*a*b*x - 3*a**2 + 6*a*c*x**2 + 3*b**2*x**2 + 3*b*c*x**3 + c**2*x**
*4)/(3*x)
```

3.194 $\int \frac{(a+bx+cx^2)^2}{x^3} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{(a+bx+cx^2)^2}{x^3} \, dx = -\frac{a^2}{2x^2} - \frac{2ab}{x} + 2bcx + \frac{c^2x^2}{2} + (b^2+2ac)\log(x)$$

output
```
-1/2*a^2/x^2-2*a*b/x+2*b*c*x+1/2*c^2*x^2+(2*a*c+b^2)*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx+cx^2)^2}{x^3} \, dx = \frac{(-a+cx^2)(a+x(4b+cx))}{2x^2} + (b^2+2ac)\log(x)$$

input
```
Integrate[(a + b*x + c*x^2)^2/x^3,x]
```

output
```
((-a + c*x^2)*(a + x*(4*b + c*x)))/(2*x^2) + (b^2 + 2*a*c)*Log[x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^2}{x^3}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{a^2}{x^3}+\frac{2ac+b^2}{x}+\frac{2ab}{x^2}+2bc+c^2x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{2x^2}+\log(x)\left(2ac+b^2\right)-\frac{2ab}{x}+2bcx+\frac{c^2x^2}{2}$$

input
```
Int[(a + b*x + c*x^2)^2/x^3,x]
```

output
```
-1/2*a^2/x^2 - (2*a*b)/x + 2*b*c*x + (c^2*x^2)/2 + (b^2 + 2*a*c)*Log[x]
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{a^2}{2x^2}-\frac{2ab}{x}+2cbx+\frac{c^2x^2}{2}+(2ac+b^2)\ln(x)$	41
risch	$\frac{c^2x^2}{2}+2cbx+\frac{-\frac{1}{2}a^2-2abx}{x^2}+2ac\ln(x)+b^2\ln(x)$	42
norman	$\frac{-\frac{1}{2}a^2+\frac{1}{2}c^2x^4-2abx+2bc\,x^3}{x^2}+(2ac+b^2)\ln(x)$	43
parallelrisch	$\frac{c^2x^4+4\ln(x)x^2ac+2b^2\ln(x)x^2+4bc\,x^3-4abx-a^2}{2x^2}$	50

input
```
int((c*x^2+b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/x^2-2*a*b/x+2*c*b*x+1/2*c^2*x^2+(2*a*c+b^2)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int\frac{(a+bx+cx^2)^2}{x^3}\,dx=\frac{c^2x^4+4\,bcx^3+2\,(b^2+2\,ac)x^2\log(x)-4\,abx-a^2}{2\,x^2}$$

input
```
integrate((c*x^2+b*x+a)^2/x^3,x, algorithm="fricas")
```

output
```
1/2*(c^2*x^4 + 4*b*c*x^3 + 2*(b^2 + 2*a*c)*x^2*log(x) - 4*a*b*x - a^2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int\frac{(a+bx+cx^2)^2}{x^3}\,dx=2bcx+\frac{c^2x^2}{2}+(2ac+b^2)\log(x)+\frac{-a^2-4abx}{2x^2}$$

input
```
integrate((c*x**2+b*x+a)**2/x**3,x)
```

output
```
2*b*c*x + c**2*x**2/2 + (2*a*c + b**2)*log(x) + (-a**2 - 4*a*b*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx+cx^2)^2}{x^3}\,dx = \frac{1}{2}\,c^2x^2 + 2\,bcx + \left(b^2+2\,ac\right)\log\left(x\right) - \frac{4\,abx+a^2}{2\,x^2}$$

input
```
integrate((c*x^2+b*x+a)^2/x^3,x, algorithm="maxima")
```

output
```
1/2*c^2*x^2 + 2*b*c*x + (b^2 + 2*a*c)*log(x) - 1/2*(4*a*b*x + a^2)/x^2
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx+cx^2)^2}{x^3}\,dx = \frac{1}{2}\,c^2x^2 + 2\,bcx + \left(b^2+2\,ac\right)\log\left(|x|\right) - \frac{4\,abx+a^2}{2\,x^2}$$

input
```
integrate((c*x^2+b*x+a)^2/x^3,x, algorithm="giac")
```

output
```
1/2*c^2*x^2 + 2*b*c*x + (b^2 + 2*a*c)*log(abs(x)) - 1/2*(4*a*b*x + a^2)/x^
2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx+cx^2)^2}{x^3}\,dx = \ln\left(x\right)\,\left(b^2+2\,a\,c\right) - \frac{\frac{a^2}{2}+2\,b\,x\,a}{x^2} + \frac{c^2\,x^2}{2} + 2\,b\,c\,x$$

input
```
int((a + b*x + c*x^2)^2/x^3,x)
```

output
```
log(x)*(2*a*c + b^2) - (a^2/2 + 2*a*b*x)/x^2 + (c^2*x^2)/2 + 2*b*c*x
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx+cx^2)^2}{x^3}\,dx = \frac{4\log(x)\,ac\,x^2+2\log(x)\,b^2x^2-a^2-4abx+4bc\,x^3+c^2x^4}{2x^2}$$

input
```
int((c*x^2+b*x+a)^2/x^3,x)
```

output
```
(4*log(x)*a*c*x**2 + 2*log(x)*b**2*x**2 - a**2 - 4*a*b*x + 4*b*c*x**3 + c*
*2*x**4)/(2*x**2)
```

3.195 $\int \frac{(a+bx+cx^2)^2}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 42

$$\int \frac{(a+bx+cx^2)^2}{x^4}\,dx = -\frac{a^2}{3x^3}-\frac{ab}{x^2}-\frac{b^2+2ac}{x}+c^2x+2bc\log(x)$$

output
```
-1/3*a^2/x^3-a*b/x^2-(2*a*c+b^2)/x+c^2*x+2*b*c*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx+cx^2)^2}{x^4}\,dx = -\frac{a^2}{3x^3}-\frac{ab}{x^2}+\frac{-b^2-2ac}{x}+c^2x+2bc\log(x)$$

input
```
Integrate[(a + b*x + c*x^2)^2/x^4,x]
```

output
```
-1/3*a^2/x^3 - (a*b)/x^2 + (-b^2 - 2*a*c)/x + c^2*x + 2*b*c*Log[x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^2}{x^4}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{a^2}{x^4}+\frac{2ac+b^2}{x^2}+\frac{2ab}{x^3}+\frac{2bc}{x}+c^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{3x^3}-\frac{2ac+b^2}{x}-\frac{ab}{x^2}+2bc\log(x)+c^2x$$

input
```
Int[(a + b*x + c*x^2)^2/x^4,x]
```

output
```
-1/3*a^2/x^3 - (a*b)/x^2 - (b^2 + 2*a*c)/x + c^2*x + 2*b*c*Log[x]
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{a^{2}}{3x^{3}}-\frac{ab}{x^{2}}-\frac{2ac+b^{2}}{x}+c^{2}x+2bc\ln\left(x\right)$	41
risch	$c^{2}x+\frac{\left(-2ac-b^{2}\right)x^{2}-abx-\frac{a^{2}}{3}}{x^{3}}+2bc\ln\left(x\right)$	42
norman	$\frac{c^{2}x^{4}+\left(-2ac-b^{2}\right)x^{2}-\frac{a^{2}}{3}-abx}{x^{3}}+2bc\ln\left(x\right)$	44
parallelrisch	$\frac{6bc\ln\left(x\right)x^{3}+3c^{2}x^{4}-6ac\,x^{2}-3b^{2}x^{2}-3abx-a^{2}}{3x^{3}}$	49

input
```
int((c*x^2+b*x+a)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2/x^3-a*b/x^2-(2*a*c+b^2)/x+c^2*x+2*b*c*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int\frac{\left(a+bx+cx^{2}\right)^{2}}{x^{4}}\,dx=\frac{3\,c^{2}x^{4}+6\,bcx^{3}\log\left(x\right)-3\,abx-3\,(b^{2}+2\,ac)x^{2}-a^{2}}{3\,x^{3}}$$

input
```
integrate((c*x^2+b*x+a)^2/x^4,x, algorithm="fricas")
```

output
```
1/3*(3*c^2*x^4 + 6*b*c*x^3*log(x) - 3*a*b*x - 3*(b^2 + 2*a*c)*x^2 - a^2)/x
^3
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2)^2}{x^4}\,dx = 2bc\log(x) + c^2x + \frac{-a^2-3abx+x^2(-6ac-3b^2)}{3x^3}$$

input
```
integrate((c*x**2+b*x+a)**2/x**4,x)
```

output
```
2*b*c*log(x) + c**2*x + (-a**2 - 3*a*b*x + x**2*(-6*a*c - 3*b**2))/(3*x**3
)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx+cx^2)^2}{x^4}\,dx = c^2x + 2\,bc\log(x) - \frac{3\,abx+3\,(b^2+2\,ac)x^2+a^2}{3\,x^3}$$

input
```
integrate((c*x^2+b*x+a)^2/x^4,x, algorithm="maxima")
```

output
```
c^2*x + 2*b*c*log(x) - 1/3*(3*a*b*x + 3*(b^2 + 2*a*c)*x^2 + a^2)/x^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx+cx^2)^2}{x^4}\,dx = c^2x + 2\,bc\log(|x|) - \frac{3\,abx+3\,(b^2+2\,ac)x^2+a^2}{3\,x^3}$$

input
```
integrate((c*x^2+b*x+a)^2/x^4,x, algorithm="giac")
```

output
```
c^2*x + 2*b*c*log(abs(x)) - 1/3*(3*a*b*x + 3*(b^2 + 2*a*c)*x^2 + a^2)/x^3
```

Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx+cx^2)^2}{x^4}\,dx = c^2\,x - \frac{x^2\,(b^2+2\,a\,c) + \frac{a^2}{3} + a\,b\,x}{x^3} + 2\,b\,c\,\ln(x)$$

input
```
int((a + b*x + c*x^2)^2/x^4,x)
```

output
```
c^2*x - (x^2*(2*a*c + b^2) + a^2/3 + a*b*x)/x^3 + 2*b*c*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx+cx^2)^2}{x^4}\,dx = \frac{6\log(x)\,bc\,x^3 - a^2 - 3abx - 6ac\,x^2 - 3b^2x^2 + 3c^2x^4}{3x^3}$$

input
```
int((c*x^2+b*x+a)^2/x^4,x)
```

output
```
(6*log(x)*b*c*x**3 - a**2 - 3*a*b*x - 6*a*c*x**2 - 3*b**2*x**2 + 3*c**2*x*
*4)/(3*x**3)
```

3.196 $\int \frac{(a+bx+cx^2)^2}{x^5} dx$

Optimal result

Integrand size = 16, antiderivative size = 48

$$\int \frac{(a+bx+cx^2)^2}{x^5}\,dx = -\frac{a^2}{4x^4}-\frac{2ab}{3x^3}-\frac{b^2+2ac}{2x^2}-\frac{2bc}{x}+c^2\log(x)$$

output
```
-1/4*a^2/x^4-2/3*a*b/x^3-1/2*(2*a*c+b^2)/x^2-2*b*c/x+c^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx+cx^2)^2}{x^5}\,dx = -\frac{3a^2+4ax(2b+3cx)+6bx^2(b+4cx)-12c^2x^4\log(x)}{12x^4}$$

input
```
Integrate[(a + b*x + c*x^2)^2/x^5,x]
```

output
```
-1/12*(3*a^2 + 4*a*x*(2*b + 3*c*x) + 6*b*x^2*(b + 4*c*x) - 12*c^2*x^4*Log[
x])/x^4
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^2}{x^5}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{a^2}{x^5}+\frac{2ac+b^2}{x^3}+\frac{2ab}{x^4}+\frac{2bc}{x^2}+\frac{c^2}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{4x^4}-\frac{2ac+b^2}{2x^2}-\frac{2ab}{3x^3}-\frac{2bc}{x}+c^2\log(x)$$

input
```
Int[(a + b*x + c*x^2)^2/x^5,x]
```

output
```
-1/4*a^2/x^4 - (2*a*b)/(3*x^3) - (b^2 + 2*a*c)/(2*x^2) - (2*b*c)/x + c^2*L
og[x]
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{a^2}{4x^4}-\frac{2ab}{3x^3}-\frac{2ac+b^2}{2x^2}-\frac{2cb}{x}+c^2\ln(x)$	43
norman	$\frac{\left(-ac-\frac{b^2}{2}\right)x^2-\frac{a^2}{4}-\frac{2abx}{3}-2bc\,x^3}{x^4}+c^2\ln(x)$	44
risch	$\frac{\left(-ac-\frac{b^2}{2}\right)x^2-\frac{a^2}{4}-\frac{2abx}{3}-2bc\,x^3}{x^4}+c^2\ln(x)$	44
parallelrisch	$\frac{12c^2\ln(x)x^4-24bc\,x^3-12ac\,x^2-6b^2x^2-8abx-3a^2}{12x^4}$	49

input
```
int((c*x^2+b*x+a)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^4*a^2-2/3*a*b/x^3-1/2*(2*a*c+b^2)/x^2-2*c*b/x+c^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx+cx^2)^2}{x^5}\,dx=\frac{12\,c^2x^4\log(x)-24\,bcx^3-8\,abx-6\,(b^2+2\,ac)x^2-3\,a^2}{12\,x^4}$$

input
```
integrate((c*x^2+b*x+a)^2/x^5,x, algorithm="fricas")
```

output
```
1/12*(12*c^2*x^4*log(x) - 24*b*c*x^3 - 8*a*b*x - 6*(b^2 + 2*a*c)*x^2 - 3*a
^2)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx+cx^2)^2}{x^5}\,dx = c^2\log(x) + \frac{-3a^2-8abx-24bcx^3+x^2(-12ac-6b^2)}{12x^4}$$

input
```
integrate((c*x**2+b*x+a)**2/x**5,x)
```

output
```
c**2*log(x) + (-3*a**2 - 8*a*b*x - 24*b*c*x**3 + x**2*(-12*a*c - 6*b**2))/
(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx+cx^2)^2}{x^5}\,dx = c^2\log(x) - \frac{24\,bcx^3+8\,abx+6\,(b^2+2\,ac)x^2+3\,a^2}{12\,x^4}$$

input
```
integrate((c*x^2+b*x+a)^2/x^5,x, algorithm="maxima")
```

output
```
c^2*log(x) - 1/12*(24*b*c*x^3 + 8*a*b*x + 6*(b^2 + 2*a*c)*x^2 + 3*a^2)/x^4
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx+cx^2)^2}{x^5}\,dx = c^2\log(|x|) - \frac{24\,bcx^3+8\,abx+6\,(b^2+2\,ac)x^2+3\,a^2}{12\,x^4}$$

input
```
integrate((c*x^2+b*x+a)^2/x^5,x, algorithm="giac")
```

output
```
c^2*log(abs(x)) - 1/12*(24*b*c*x^3 + 8*a*b*x + 6*(b^2 + 2*a*c)*x^2 + 3*a^2
)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx+cx^2)^2}{x^5}\,dx = c^2\,\ln(x) - \frac{\frac{a^2}{4} + x^2\left(\frac{b^2}{2} + a\,c\right) + \frac{2\,a\,b\,x}{3} + 2\,b\,c\,x^3}{x^4}$$

input
```
int((a + b*x + c*x^2)^2/x^5,x)
```

output
```
c^2*log(x) - (a^2/4 + x^2*(a*c + b^2/2) + (2*a*b*x)/3 + 2*b*c*x^3)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2)^2}{x^5}\,dx = \frac{12\log(x)\,c^2x^4 - 3a^2 - 8abx - 12ac\,x^2 - 6b^2x^2 - 24bc\,x^3}{12x^4}$$

input
```
int((c*x^2+b*x+a)^2/x^5,x)
```

output
```
(12*log(x)*c**2*x**4 - 3*a**2 - 8*a*b*x - 12*a*c*x**2 - 6*b**2*x**2 - 24*b
*c*x**3)/(12*x**4)
```

3.197 $\int \frac{(a+bx+cx^2)^2}{x^6} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 50

$$\int \frac{(a+bx+cx^2)^2}{x^6} \, dx = -\frac{a^2}{5x^5} - \frac{ab}{2x^4} - \frac{b^2+2ac}{3x^3} - \frac{bc}{x^2} - \frac{c^2}{x}$$

output
```
-1/5*a^2/x^5-1/2*a*b/x^4-1/3*(2*a*c+b^2)/x^3-b*c/x^2-c^2/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx+cx^2)^2}{x^6} \, dx = -\frac{6a^2+5ax(3b+4cx)+10x^2(b^2+3bcx+3c^2x^2)}{30x^5}$$

input
```
Integrate[(a + b*x + c*x^2)^2/x^6,x]
```

output
```
-1/30*(6*a^2 + 5*a*x*(3*b + 4*c*x) + 10*x^2*(b^2 + 3*b*c*x + 3*c^2*x^2))/x
^5
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^2}{x^6} \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{a^2}{x^6}+\frac{2ac+b^2}{x^4}+\frac{2ab}{x^5}+\frac{2bc}{x^3}+\frac{c^2}{x^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{5x^5}-\frac{2ac+b^2}{3x^3}-\frac{ab}{2x^4}-\frac{bc}{x^2}-\frac{c^2}{x}$$

input
```
Int[(a + b*x + c*x^2)^2/x^6,x]
```

output
```
-1/5*a^2/x^5 - (a*b)/(2*x^4) - (b^2 + 2*a*c)/(3*x^3) - (b*c)/x^2 - c^2/x
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{a^2}{5x^5}-\frac{ab}{2x^4}-\frac{2ac+b^2}{3x^3}-\frac{bc}{x^2}-\frac{c^2}{x}$	45
norman	$\frac{-c^2x^4-bc\,x^3+\left(-\frac{2ac}{3}-\frac{b^2}{3}\right)x^2-\frac{abx}{2}-\frac{a^2}{5}}{x^5}$	45
risch	$\frac{-c^2x^4-bc\,x^3+\left(-\frac{2ac}{3}-\frac{b^2}{3}\right)x^2-\frac{abx}{2}-\frac{a^2}{5}}{x^5}$	45
gosper	$-\frac{30c^2x^4+30bc\,x^3+20ac\,x^2+10b^2x^2+15abx+6a^2}{30x^5}$	47
parallelrisch	$\frac{-30c^2x^4-30bc\,x^3-20ac\,x^2-10b^2x^2-15abx-6a^2}{30x^5}$	47
orering	$-\frac{30c^2x^4+30bc\,x^3+20ac\,x^2+10b^2x^2+15abx+6a^2}{30x^5}$	47

input
```
int((c*x^2+b*x+a)^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^2/x^5-1/2*a*b/x^4-1/3*(2*a*c+b^2)/x^3-b*c/x^2-c^2/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx+cx^2)^2}{x^6}\,dx=-\frac{30\,c^2x^4+30\,bcx^3+15\,abx+10\,(b^2+2\,ac)x^2+6\,a^2}{30\,x^5}$$

input
```
integrate((c*x^2+b*x+a)^2/x^6,x, algorithm="fricas")
```

output
```
-1/30*(30*c^2*x^4 + 30*b*c*x^3 + 15*a*b*x + 10*(b^2 + 2*a*c)*x^2 + 6*a^2)/
x^5
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx+cx^2)^2}{x^6}\,dx = \frac{-6a^2-15abx-30bcx^3-30c^2x^4+x^2(-20ac-10b^2)}{30x^5}$$

input
```
integrate((c*x**2+b*x+a)**2/x**6,x)
```

output
```
(-6*a**2 - 15*a*b*x - 30*b*c*x**3 - 30*c**2*x**4 + x**2*(-20*a*c - 10*b**2
))/(30*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx+cx^2)^2}{x^6}\,dx = -\frac{30\,c^2x^4+30\,bcx^3+15\,abx+10\,(b^2+2\,ac)x^2+6\,a^2}{30\,x^5}$$

input
```
integrate((c*x^2+b*x+a)^2/x^6,x, algorithm="maxima")
```

output
```
-1/30*(30*c^2*x^4 + 30*b*c*x^3 + 15*a*b*x + 10*(b^2 + 2*a*c)*x^2 + 6*a^2)/
x^5
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx+cx^2)^2}{x^6}\,dx = -\frac{30\,c^2x^4+30\,bcx^3+10\,b^2x^2+20\,acx^2+15\,abx+6\,a^2}{30\,x^5}$$

input
```
integrate((c*x^2+b*x+a)^2/x^6,x, algorithm="giac")
```

output
```
-1/30*(30*c^2*x^4 + 30*b*c*x^3 + 10*b^2*x^2 + 20*a*c*x^2 + 15*a*b*x + 6*a^
2)/x^5
```

Mupad [B] (verification not implemented)

Time = 8.84 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx+cx^2)^2}{x^6}\,dx = -\frac{\frac{a^2}{5}+x^2\left(\frac{b^2}{3}+\frac{2\,a\,c}{3}\right)+c^2\,x^4+\frac{a\,b\,x}{2}+b\,c\,x^3}{x^5}$$

input
```
int((a + b*x + c*x^2)^2/x^6,x)
```

output
```
-(a^2/5 + x^2*((2*a*c)/3 + b^2/3) + c^2*x^4 + (a*b*x)/2 + b*c*x^3)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx+cx^2)^2}{x^6}\,dx = \frac{-30c^2x^4-30bc\,x^3-20ac\,x^2-10b^2x^2-15abx-6a^2}{30x^5}$$

input
```
int((c*x^2+b*x+a)^2/x^6,x)
```

output
```
( - 6*a**2 - 15*a*b*x - 20*a*c*x**2 - 10*b**2*x**2 - 30*b*c*x**3 - 30*c**2
*x**4)/(30*x**5)
```

3.198 $\int \frac{(a+bx+cx^2)^2}{x^7} dx$

Optimal result

Integrand size = 16, antiderivative size = 54

$$\int \frac{(a+bx+cx^2)^2}{x^7} dx = -\frac{a^2}{6x^6} - \frac{2ab}{5x^5} - \frac{b^2+2ac}{4x^4} - \frac{2bc}{3x^3} - \frac{c^2}{2x^2}$$

output
```
-1/6*a^2/x^6-2/5*a*b/x^5-1/4*(2*a*c+b^2)/x^4-2/3*b*c/x^3-1/2*c^2/x^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx+cx^2)^2}{x^7} dx = -\frac{10a^2+6ax(4b+5cx)+5x^2(3b^2+8bcx+6c^2x^2)}{60x^6}$$

input
```
Integrate[(a + b*x + c*x^2)^2/x^7,x]
```

output
```
-1/60*(10*a^2 + 6*a*x*(4*b + 5*c*x) + 5*x^2*(3*b^2 + 8*b*c*x + 6*c^2*x^2))
/x^6
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^2}{x^7} \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{a^2}{x^7}+\frac{2ac+b^2}{x^5}+\frac{2ab}{x^6}+\frac{2bc}{x^4}+\frac{c^2}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{6x^6}-\frac{2ac+b^2}{4x^4}-\frac{2ab}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{2x^2}$$

input
```
Int[(a + b*x + c*x^2)^2/x^7,x]
```

output
```
-1/6*a^2/x^6 - (2*a*b)/(5*x^5) - (b^2 + 2*a*c)/(4*x^4) - (2*b*c)/(3*x^3) -
 c^2/(2*x^2)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{a^{2}}{6x^{6}}-\frac{2ab}{5x^{5}}-\frac{2ac+b^{2}}{4x^{4}}-\frac{2bc}{3x^{3}}-\frac{c^{2}}{2x^{2}}$	45
norman	$\frac{-\frac{c^{2}x^{4}}{2}-\frac{2bc\,x^{3}}{3}+\left(-\frac{ac}{2}-\frac{b^{2}}{4}\right)x^{2}-\frac{2abx}{5}-\frac{a^{2}}{6}}{x^{6}}$	45
risch	$\frac{-\frac{c^{2}x^{4}}{2}-\frac{2bc\,x^{3}}{3}+\left(-\frac{ac}{2}-\frac{b^{2}}{4}\right)x^{2}-\frac{2abx}{5}-\frac{a^{2}}{6}}{x^{6}}$	45
gosper	$-\frac{30c^{2}x^{4}+40bc\,x^{3}+30ac\,x^{2}+15b^{2}x^{2}+24abx+10a^{2}}{60x^{6}}$	47
parallelrisch	$\frac{-30c^{2}x^{4}-40bc\,x^{3}-30ac\,x^{2}-15b^{2}x^{2}-24abx-10a^{2}}{60x^{6}}$	47
orering	$-\frac{30c^{2}x^{4}+40bc\,x^{3}+30ac\,x^{2}+15b^{2}x^{2}+24abx+10a^{2}}{60x^{6}}$	47

input
```
int((c*x^2+b*x+a)^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^2/x^6-2/5*a*b/x^5-1/4*(2*a*c+b^2)/x^4-2/3*b*c/x^3-1/2*c^2/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx+cx^2)^2}{x^7}\,dx=-\frac{30\,c^2x^4+40\,bcx^3+24\,abx+15\,(b^2+2\,ac)x^2+10\,a^2}{60\,x^6}$$

input
```
integrate((c*x^2+b*x+a)^2/x^7,x, algorithm="fricas")
```

output
```
-1/60*(30*c^2*x^4 + 40*b*c*x^3 + 24*a*b*x + 15*(b^2 + 2*a*c)*x^2 + 10*a^2)
/x^6
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx+cx^2)^2}{x^7}\,dx = \frac{-10a^2-24abx-40bcx^3-30c^2x^4+x^2(-30ac-15b^2)}{60x^6}$$

input
```
integrate((c*x**2+b*x+a)**2/x**7,x)
```

output
```
(-10*a**2 - 24*a*b*x - 40*b*c*x**3 - 30*c**2*x**4 + x**2*(-30*a*c - 15*b**
2))/(60*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx+cx^2)^2}{x^7}\,dx = -\frac{30\,c^2x^4+40\,bcx^3+24\,abx+15\,(b^2+2\,ac)x^2+10\,a^2}{60\,x^6}$$

input
```
integrate((c*x^2+b*x+a)^2/x^7,x, algorithm="maxima")
```

output
```
-1/60*(30*c^2*x^4 + 40*b*c*x^3 + 24*a*b*x + 15*(b^2 + 2*a*c)*x^2 + 10*a^2)
/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx+cx^2)^2}{x^7}\,dx = -\frac{30\,c^2x^4+40\,bcx^3+15\,b^2x^2+30\,acx^2+24\,abx+10\,a^2}{60\,x^6}$$

input
```
integrate((c*x^2+b*x+a)^2/x^7,x, algorithm="giac")
```

output
```
-1/60*(30*c^2*x^4 + 40*b*c*x^3 + 15*b^2*x^2 + 30*a*c*x^2 + 24*a*b*x + 10*a
^2)/x^6
```

Mupad [B] (verification not implemented)

Time = 8.79 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx+cx^2)^2}{x^7}\,dx = -\frac{\frac{a^2}{6}+x^2\left(\frac{b^2}{4}+\frac{a\,c}{2}\right)+\frac{c^2\,x^4}{2}+\frac{2\,a\,b\,x}{5}+\frac{2\,b\,c\,x^3}{3}}{x^6}$$

input
```
int((a + b*x + c*x^2)^2/x^7,x)
```

output
```
-(a^2/6 + x^2*((a*c)/2 + b^2/4) + (c^2*x^4)/2 + (2*a*b*x)/5 + (2*b*c*x^3)/
3)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx+cx^2)^2}{x^7}\,dx = \frac{-30c^2x^4-40bc\,x^3-30ac\,x^2-15b^2x^2-24abx-10a^2}{60x^6}$$

input
```
int((c*x^2+b*x+a)^2/x^7,x)
```

output
```
( - 10*a**2 - 24*a*b*x - 30*a*c*x**2 - 15*b**2*x**2 - 40*b*c*x**3 - 30*c**
2*x**4)/(60*x**6)
```

3.199 $\int \frac{(a+bx+cx^2)^2}{x^8} dx$

Optimal result

Integrand size = 16, antiderivative size = 54

$$\int \frac{(a+bx+cx^2)^2}{x^8} dx = -\frac{a^2}{7x^7} - \frac{ab}{3x^6} - \frac{b^2+2ac}{5x^5} - \frac{bc}{2x^4} - \frac{c^2}{3x^3}$$

output
```
-1/7*a^2/x^7-1/3*a*b/x^6-1/5*(2*a*c+b^2)/x^5-1/2*b*c/x^4-1/3*c^2/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx+cx^2)^2}{x^8} dx = -\frac{30a^2+14ax(5b+6cx)+7x^2(6b^2+15bcx+10c^2x^2)}{210x^7}$$

input
```
Integrate[(a + b*x + c*x^2)^2/x^8,x]
```

output
```
-1/210*(30*a^2 + 14*a*x*(5*b + 6*c*x) + 7*x^2*(6*b^2 + 15*b*c*x + 10*c^2*x
^2))/x^7
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^2}{x^8}\,dx$$

$$\Big\downarrow \; 1140$$

$$\int \left(\frac{a^2}{x^8}+\frac{2ac+b^2}{x^6}+\frac{2ab}{x^7}+\frac{2bc}{x^5}+\frac{c^2}{x^4}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^2}{7x^7}-\frac{2ac+b^2}{5x^5}-\frac{ab}{3x^6}-\frac{bc}{2x^4}-\frac{c^2}{3x^3}$$

input
```
Int[(a + b*x + c*x^2)^2/x^8,x]
```

output
```
-1/7*a^2/x^7 - (a*b)/(3*x^6) - (b^2 + 2*a*c)/(5*x^5) - (b*c)/(2*x^4) - c^2
/(3*x^3)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{a^{2}}{7x^{7}}-\frac{ab}{3x^{6}}-\frac{2ac+b^{2}}{5x^{5}}-\frac{bc}{2x^{4}}-\frac{c^{2}}{3x^{3}}$	45
norman	$\frac{-\frac{c^{2}x^{4}}{3}-\frac{bc\,x^{3}}{2}+\left(-\frac{2ac}{5}-\frac{b^{2}}{5}\right)x^{2}-\frac{abx}{3}-\frac{a^{2}}{7}}{x^{7}}$	45
risch	$\frac{-\frac{c^{2}x^{4}}{3}-\frac{bc\,x^{3}}{2}+\left(-\frac{2ac}{5}-\frac{b^{2}}{5}\right)x^{2}-\frac{abx}{3}-\frac{a^{2}}{7}}{x^{7}}$	45
gosper	$-\frac{70c^{2}x^{4}+105bc\,x^{3}+84ac\,x^{2}+42b^{2}x^{2}+70abx+30a^{2}}{210x^{7}}$	47
parallelrisch	$\frac{-70c^{2}x^{4}-105bc\,x^{3}-84ac\,x^{2}-42b^{2}x^{2}-70abx-30a^{2}}{210x^{7}}$	47
orering	$-\frac{70c^{2}x^{4}+105bc\,x^{3}+84ac\,x^{2}+42b^{2}x^{2}+70abx+30a^{2}}{210x^{7}}$	47

input
```
int((c*x^2+b*x+a)^2/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^2/x^7-1/3*a*b/x^6-1/5*(2*a*c+b^2)/x^5-1/2*b*c/x^4-1/3*c^2/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx+cx^2)^2}{x^8}\,dx = -\frac{70\,c^2x^4+105\,bcx^3+70\,abx+42\,(b^2+2\,ac)x^2+30\,a^2}{210\,x^7}$$

input
```
integrate((c*x^2+b*x+a)^2/x^8,x, algorithm="fricas")
```

output
```
-1/210*(70*c^2*x^4 + 105*b*c*x^3 + 70*a*b*x + 42*(b^2 + 2*a*c)*x^2 + 30*a^
2)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx+cx^2)^2}{x^8}\,dx = \frac{-30a^2-70abx-105bcx^3-70c^2x^4+x^2(-84ac-42b^2)}{210x^7}$$

input
```
integrate((c*x**2+b*x+a)**2/x**8,x)
```

output
```
(-30*a**2 - 70*a*b*x - 105*b*c*x**3 - 70*c**2*x**4 + x**2*(-84*a*c - 42*b*
*2))/(210*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx+cx^2)^2}{x^8}\,dx = -\frac{70\,c^2x^4+105\,bcx^3+70\,abx+42\,(b^2+2\,ac)x^2+30\,a^2}{210\,x^7}$$

input
```
integrate((c*x^2+b*x+a)^2/x^8,x, algorithm="maxima")
```

output
```
-1/210*(70*c^2*x^4 + 105*b*c*x^3 + 70*a*b*x + 42*(b^2 + 2*a*c)*x^2 + 30*a^
2)/x^7
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx+cx^2)^2}{x^8}\,dx = -\frac{70\,c^2x^4+105\,bcx^3+42\,b^2x^2+84\,acx^2+70\,abx+30\,a^2}{210\,x^7}$$

input
```
integrate((c*x^2+b*x+a)^2/x^8,x, algorithm="giac")
```

output
```
-1/210*(70*c^2*x^4 + 105*b*c*x^3 + 42*b^2*x^2 + 84*a*c*x^2 + 70*a*b*x + 30
*a^2)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx+cx^2)^2}{x^8} \, dx = -\frac{\frac{a^2}{7} + x^2 \left(\frac{b^2}{5} + \frac{2\,a\,c}{5}\right) + \frac{c^2\,x^4}{3} + \frac{a\,b\,x}{3} + \frac{b\,c\,x^3}{2}}{x^7}$$

input
```
int((a + b*x + c*x^2)^2/x^8,x)
```

output
```
-(a^2/7 + x^2*((2*a*c)/5 + b^2/5) + (c^2*x^4)/3 + (a*b*x)/3 + (b*c*x^3)/2)
/x^7
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx+cx^2)^2}{x^8} \, dx = \frac{-70c^2x^4 - 105bc\,x^3 - 84ac\,x^2 - 42b^2x^2 - 70abx - 30a^2}{210x^7}$$

input
```
int((c*x^2+b*x+a)^2/x^8,x)
```

output
```
( - 30*a**2 - 70*a*b*x - 84*a*c*x**2 - 42*b**2*x**2 - 105*b*c*x**3 - 70*c*
*2*x**4)/(210*x**7)
```

3.200 $\int x^3(a+bx+cx^2)^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 89

$$\int x^3\left(a+bx+cx^2\right)^3\,dx=\frac{a^3x^4}{4}+\frac{3}{5}a^2bx^5+\frac{1}{2}a\left(b^2+ac\right)x^6+\frac{1}{7}b\left(b^2+6ac\right)x^7+\frac{3}{8}c\left(b^2+ac\right)x^8+\frac{1}{3}bc^2x^9+\frac{c^3x^{10}}{10}$$

output
```
1/4*a^3*x^4+3/5*a^2*b*x^5+1/2*a*(a*c+b^2)*x^6+1/7*b*(6*a*c+b^2)*x^7+3/8*c*
(a*c+b^2)*x^8+1/3*b*c^2*x^9+1/10*c^3*x^10
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int x^3\left(a+bx+cx^2\right)^3\,dx=\frac{a^3x^4}{4}+\frac{3}{5}a^2bx^5+\frac{1}{2}a\left(b^2+ac\right)x^6+\frac{1}{7}b\left(b^2+6ac\right)x^7+\frac{3}{8}c\left(b^2+ac\right)x^8+\frac{1}{3}bc^2x^9+\frac{c^3x^{10}}{10}$$

input
```
Integrate[x^3*(a + b*x + c*x^2)^3,x]
```

output
```
(a^3*x^4)/4 + (3*a^2*b*x^5)/5 + (a*(b^2 + a*c)*x^6)/2 + (b*(b^2 + 6*a*c)*x
^7)/7 + (3*c*(b^2 + a*c)*x^8)/8 + (b*c^2*x^9)/3 + (c^3*x^10)/10
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a + bx + cx^2)^3 \, dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int \left(a^3x^3 + 3a^2bx^4 + 3cx^7(ac + b^2) + bx^6(6ac + b^2) + 3ax^5(ac + b^2) + 3bc^2x^8 + c^3x^9\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^3x^4}{4} + \frac{3}{5}a^2bx^5 + \frac{3}{8}cx^8(ac + b^2) + \frac{1}{7}bx^7(6ac + b^2) + \frac{1}{2}ax^6(ac + b^2) + \frac{1}{3}bc^2x^9 + \frac{c^3x^{10}}{10}$$

input
```
Int[x^3*(a + b*x + c*x^2)^3,x]
```

output
```
(a^3*x^4)/4 + (3*a^2*b*x^5)/5 + (a*(b^2 + a*c)*x^6)/2 + (b*(b^2 + 6*a*c)*x
^7)/7 + (3*c*(b^2 + a*c)*x^8)/8 + (b*c^2*x^9)/3 + (c^3*x^10)/10
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

method	result
norman	$\frac{c^3x^{10}}{10}+\frac{b\,c^2x^9}{3}+\left(\frac{3}{8}a\,c^2+\frac{3}{8}b^2c\right)x^8+\left(\frac{6}{7}abc+\frac{1}{7}b^3\right)x^7+\left(\frac{1}{2}a^2c+\frac{1}{2}a\,b^2\right)x^6+\frac{3a^2b\,x^5}{5}+\frac{a^3x^4}{4}$
gosper	$\frac{1}{10}c^3x^{10}+\frac{1}{3}b\,c^2x^9+\frac{3}{8}x^8a\,c^2+\frac{3}{8}b^2c\,x^8+\frac{6}{7}x^7abc+\frac{1}{7}b^3x^7+\frac{1}{2}x^6a^2c+\frac{1}{2}a\,b^2x^6+\frac{3}{5}a^2b\,x^5+\frac{1}{4}$
risch	$\frac{1}{10}c^3x^{10}+\frac{1}{3}b\,c^2x^9+\frac{3}{8}x^8a\,c^2+\frac{3}{8}b^2c\,x^8+\frac{6}{7}x^7abc+\frac{1}{7}b^3x^7+\frac{1}{2}x^6a^2c+\frac{1}{2}a\,b^2x^6+\frac{3}{5}a^2b\,x^5+\frac{1}{4}$
parallelrisch	$\frac{1}{10}c^3x^{10}+\frac{1}{3}b\,c^2x^9+\frac{3}{8}x^8a\,c^2+\frac{3}{8}b^2c\,x^8+\frac{6}{7}x^7abc+\frac{1}{7}b^3x^7+\frac{1}{2}x^6a^2c+\frac{1}{2}a\,b^2x^6+\frac{3}{5}a^2b\,x^5+\frac{1}{4}$
orering	$\frac{x^4\left(84c^3x^6+280b\,c^2x^5+315a\,c^2x^4+315b^2c\,x^4+720abc\,x^3+120b^3x^3+420a^2c\,x^2+420a\,b^2x^2+504a^2bx+210a^3\right)}{840}$
default	$\frac{c^3x^{10}}{10}+\frac{b\,c^2x^9}{3}+\frac{\left(a\,c^2+2b^2c+c\left(2ac+b^2\right)\right)x^8}{8}+\frac{\left(4abc+b\left(2ac+b^2\right)\right)x^7}{7}+\frac{\left(a\left(2ac+b^2\right)+2a\,b^2+a^2c\right)x^6}{6}+\frac{3a^2b\,x^5}{5}+$

input
```
int(x^3*(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10*c^3*x^10+1/3*b*c^2*x^9+(3/8*a*c^2+3/8*b^2*c)*x^8+(6/7*a*b*c+1/7*b^3)*
x^7+(1/2*a^2*c+1/2*a*b^2)*x^6+3/5*a^2*b*x^5+1/4*a^3*x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int x^3\left(a+bx+cx^2\right)^3\,dx=\frac{1}{10}\,c^3x^{10}+\frac{1}{3}\,bc^2x^9+\frac{3}{8}\,\left(b^2c+ac^2\right)x^8+\frac{3}{5}\,a^2bx^5+\frac{1}{7}\,\left(b^3+6\,abc\right)x^7+\frac{1}{4}\,a^3x^4+\frac{1}{2}\,\left(ab^2+a^2c\right)x^6$$

input
```
integrate(x^3*(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
1/10*c^3*x^10 + 1/3*b*c^2*x^9 + 3/8*(b^2*c + a*c^2)*x^8 + 3/5*a^2*b*x^5 +
1/7*(b^3 + 6*a*b*c)*x^7 + 1/4*a^3*x^4 + 1/2*(a*b^2 + a^2*c)*x^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int x^3 \left(a+bx+cx^2\right)^3 \, dx = \frac{a^3x^4}{4} + \frac{3a^2bx^5}{5} + \frac{bc^2x^9}{3} + \frac{c^3x^{10}}{10} + x^8 \cdot \left(\frac{3ac^2}{8} + \frac{3b^2c}{8}\right) + x^7 \cdot \left(\frac{6abc}{7} + \frac{b^3}{7}\right) + x^6\left(\frac{a^2c}{2} + \frac{ab^2}{2}\right)$$

input
```
integrate(x**3*(c*x**2+b*x+a)**3,x)
```

output
```
a**3*x**4/4 + 3*a**2*b*x**5/5 + b*c**2*x**9/3 + c**3*x**10/10 + x**8*(3*a*
c**2/8 + 3*b**2*c/8) + x**7*(6*a*b*c/7 + b**3/7) + x**6*(a**2*c/2 + a*b**2
/2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int x^3 \left(a+bx+cx^2\right)^3 \, dx = \frac{1}{10}\,c^3x^{10} + \frac{1}{3}\,bc^2x^9 + \frac{3}{8}\,\left(b^2c+ac^2\right)x^8 + \frac{3}{5}\,a^2bx^5 + \frac{1}{7}\,\left(b^3+6\,abc\right)x^7 + \frac{1}{4}\,a^3x^4 + \frac{1}{2}\,\left(ab^2+a^2c\right)x^6$$

input
```
integrate(x^3*(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
1/10*c^3*x^10 + 1/3*b*c^2*x^9 + 3/8*(b^2*c + a*c^2)*x^8 + 3/5*a^2*b*x^5 +
1/7*(b^3 + 6*a*b*c)*x^7 + 1/4*a^3*x^4 + 1/2*(a*b^2 + a^2*c)*x^6
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98

$$\int x^3(a+bx+cx^2)^3\,dx = \frac{1}{10}c^3x^{10} + \frac{1}{3}bc^2x^9 + \frac{3}{8}b^2cx^8 + \frac{3}{8}ac^2x^8 + \frac{1}{7}b^3x^7 + \frac{6}{7}abcx^7 + \frac{1}{2}ab^2x^6 + \frac{1}{2}a^2cx^6 + \frac{3}{5}a^2bx^5 + \frac{1}{4}a^3x^4$$

input
```
integrate(x^3*(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
1/10*c^3*x^10 + 1/3*b*c^2*x^9 + 3/8*b^2*c*x^8 + 3/8*a*c^2*x^8 + 1/7*b^3*x^
7 + 6/7*a*b*c*x^7 + 1/2*a*b^2*x^6 + 1/2*a^2*c*x^6 + 3/5*a^2*b*x^5 + 1/4*a^
3*x^4
```

Mupad [B] (verification not implemented)

Time = 8.85 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

$$\int x^3(a+bx+cx^2)^3\,dx = x^7\left(\frac{b^3}{7}+\frac{6\,a\,c\,b}{7}\right) + \frac{a^3\,x^4}{4} + \frac{c^3\,x^{10}}{10} + \frac{3\,a^2\,b\,x^5}{5} + \frac{b\,c^2\,x^9}{3} + \frac{a\,x^6\,(b^2+a\,c)}{2} + \frac{3\,c\,x^8\,(b^2+a\,c)}{8}$$

input
```
int(x^3*(a + b*x + c*x^2)^3,x)
```

output
```
x^7*(b^3/7 + (6*a*b*c)/7) + (a^3*x^4)/4 + (c^3*x^10)/10 + (3*a^2*b*x^5)/5
+ (b*c^2*x^9)/3 + (a*x^6*(a*c + b^2))/2 + (3*c*x^8*(a*c + b^2))/8
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98

$$\int x^3(a + bx + cx^2)^3 \, dx$$
$$= \frac{x^4(84c^3x^6 + 280b\,c^2x^5 + 315a\,c^2x^4 + 315b^2c\,x^4 + 720abc\,x^3 + 120b^3x^3 + 420a^2c\,x^2 + 420a\,b^2x^2 + 504a^2b}{840}$$

input
```
int(x^3*(c*x^2+b*x+a)^3,x)
```

output
```
(x**4*(210*a**3 + 504*a**2*b*x + 420*a**2*c*x**2 + 420*a*b**2*x**2 + 720*a
*b*c*x**3 + 315*a*c**2*x**4 + 120*b**3*x**3 + 315*b**2*c*x**4 + 280*b*c**2
*x**5 + 84*c**3*x**6))/840
```

3.201 $\int x^2(a+bx+cx^2)^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 89

$$\int x^2(a+bx+cx^2)^3\,dx=\frac{a^3x^3}{3}+\frac{3}{4}a^2bx^4+\frac{3}{5}a(b^2+ac)\,x^5+\frac{1}{6}b(b^2+6ac)\,x^6+\frac{3}{7}c(b^2+ac)\,x^7+\frac{3}{8}bc^2x^8+\frac{c^3x^9}{9}$$

output
```
1/3*a^3*x^3+3/4*a^2*b*x^4+3/5*a*(a*c+b^2)*x^5+1/6*b*(6*a*c+b^2)*x^6+3/7*c*
(a*c+b^2)*x^7+3/8*b*c^2*x^8+1/9*c^3*x^9
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int x^2(a+bx+cx^2)^3\,dx=\frac{a^3x^3}{3}+\frac{3}{4}a^2bx^4+\frac{3}{5}a(b^2+ac)\,x^5+\frac{1}{6}b(b^2+6ac)\,x^6+\frac{3}{7}c(b^2+ac)\,x^7+\frac{3}{8}bc^2x^8+\frac{c^3x^9}{9}$$

input
```
Integrate[x^2*(a + b*x + c*x^2)^3,x]
```

output
```
(a^3*x^3)/3 + (3*a^2*b*x^4)/4 + (3*a*(b^2 + a*c)*x^5)/5 + (b*(b^2 + 6*a*c)
*x^6)/6 + (3*c*(b^2 + a*c)*x^7)/7 + (3*b*c^2*x^8)/8 + (c^3*x^9)/9
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+bx+cx^2\right)^3dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(a^3x^2+3a^2bx^3+3cx^6\left(ac+b^2\right)+bx^5\left(6ac+b^2\right)+3ax^4\left(ac+b^2\right)+3bc^2x^7+c^3x^8\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^3}{3}+\frac{3}{4}a^2bx^4+\frac{3}{7}cx^7\left(ac+b^2\right)+\frac{1}{6}bx^6\left(6ac+b^2\right)+\frac{3}{5}ax^5\left(ac+b^2\right)+\frac{3}{8}bc^2x^8+\frac{c^3x^9}{9}$$

input
```
Int[x^2*(a + b*x + c*x^2)^3,x]
```

output
```
(a^3*x^3)/3 + (3*a^2*b*x^4)/4 + (3*a*(b^2 + a*c)*x^5)/5 + (b*(b^2 + 6*a*c)
*x^6)/6 + (3*c*(b^2 + a*c)*x^7)/7 + (3*b*c^2*x^8)/8 + (c^3*x^9)/9
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

method	result
norman	$\frac{c^3x^9}{9}+\frac{3b\,c^2x^8}{8}+\left(\frac{3}{7}a\,c^2+\frac{3}{7}b^2c\right)x^7+\left(abc+\frac{1}{6}b^3\right)x^6+\left(\frac{3}{5}a^2c+\frac{3}{5}a\,b^2\right)x^5+\frac{3a^2b\,x^4}{4}+\frac{a^3x^3}{3}$
gosper	$\frac{1}{9}c^3x^9+\frac{3}{8}b\,c^2x^8+\frac{3}{7}x^7a\,c^2+\frac{3}{7}b^2c\,x^7+abc\,x^6+\frac{1}{6}b^3x^6+\frac{3}{5}a^2c\,x^5+\frac{3}{5}a\,b^2x^5+\frac{3}{4}a^2b\,x^4+\frac{1}{3}a^3$
risch	$\frac{1}{9}c^3x^9+\frac{3}{8}b\,c^2x^8+\frac{3}{7}x^7a\,c^2+\frac{3}{7}b^2c\,x^7+abc\,x^6+\frac{1}{6}b^3x^6+\frac{3}{5}a^2c\,x^5+\frac{3}{5}a\,b^2x^5+\frac{3}{4}a^2b\,x^4+\frac{1}{3}a^3$
parallelrisch	$\frac{1}{9}c^3x^9+\frac{3}{8}b\,c^2x^8+\frac{3}{7}x^7a\,c^2+\frac{3}{7}b^2c\,x^7+abc\,x^6+\frac{1}{6}b^3x^6+\frac{3}{5}a^2c\,x^5+\frac{3}{5}a\,b^2x^5+\frac{3}{4}a^2b\,x^4+\frac{1}{3}a^3$
orering	$\frac{x^3\left(280c^3x^6+945b\,c^2x^5+1080a\,c^2x^4+1080b^2c\,x^4+2520abc\,x^3+420b^3x^3+1512a^2c\,x^2+1512a\,b^2x^2+1890a^2bx+840a^3\right)}{2520}$
default	$\frac{c^3x^9}{9}+\frac{3b\,c^2x^8}{8}+\frac{\left(a\,c^2+2b^2c+c\left(2ac+b^2\right)\right)x^7}{7}+\frac{\left(4abc+b\left(2ac+b^2\right)\right)x^6}{6}+\frac{\left(a\left(2ac+b^2\right)+2a\,b^2+a^2c\right)x^5}{5}+\frac{3a^2b\,x^4}{4}+$

input
```
int(x^2*(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/9*c^3*x^9+3/8*b*c^2*x^8+(3/7*a*c^2+3/7*b^2*c)*x^7+(a*b*c+1/6*b^3)*x^6+(3
/5*a^2*c+3/5*a*b^2)*x^5+3/4*a^2*b*x^4+1/3*a^3*x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int x^2\left(a+bx+cx^2\right)^3\,dx=\frac{1}{9}\,c^3x^9+\frac{3}{8}\,bc^2x^8+\frac{3}{7}\left(b^2c+ac^2\right)x^7+\frac{3}{4}\,a^2bx^4+\frac{1}{6}\left(b^3+6\,abc\right)x^6+\frac{1}{3}\,a^3x^3+\frac{3}{5}\left(ab^2+a^2c\right)x^5$$

input
```
integrate(x^2*(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
1/9*c^3*x^9 + 3/8*b*c^2*x^8 + 3/7*(b^2*c + a*c^2)*x^7 + 3/4*a^2*b*x^4 + 1/
6*(b^3 + 6*a*b*c)*x^6 + 1/3*a^3*x^3 + 3/5*(a*b^2 + a^2*c)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int x^2(a+bx+cx^2)^3\,dx=\frac{a^3x^3}{3}+\frac{3a^2bx^4}{4}+\frac{3bc^2x^8}{8}+\frac{c^3x^9}{9}+x^7\cdot\left(\frac{3ac^2}{7}+\frac{3b^2c}{7}\right)+x^6\left(abc+\frac{b^3}{6}\right)+x^5\cdot\left(\frac{3a^2c}{5}+\frac{3ab^2}{5}\right)$$

input
```
integrate(x**2*(c*x**2+b*x+a)**3,x)
```

output
```
a**3*x**3/3 + 3*a**2*b*x**4/4 + 3*b*c**2*x**8/8 + c**3*x**9/9 + x**7*(3*a*
c**2/7 + 3*b**2*c/7) + x**6*(a*b*c + b**3/6) + x**5*(3*a**2*c/5 + 3*a*b**2
/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int x^2(a+bx+cx^2)^3\,dx=\frac{1}{9}\,c^3x^9+\frac{3}{8}\,bc^2x^8+\frac{3}{7}\,(b^2c+ac^2)x^7+\frac{3}{4}\,a^2bx^4+\frac{1}{6}\,(b^3+6\,abc)x^6+\frac{1}{3}\,a^3x^3+\frac{3}{5}\,(ab^2+a^2c)x^5$$

input
```
integrate(x^2*(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
1/9*c^3*x^9 + 3/8*b*c^2*x^8 + 3/7*(b^2*c + a*c^2)*x^7 + 3/4*a^2*b*x^4 + 1/
6*(b^3 + 6*a*b*c)*x^6 + 1/3*a^3*x^3 + 3/5*(a*b^2 + a^2*c)*x^5
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97

$$\int x^2(a+bx+cx^2)^3\,dx = \frac{1}{9}c^3x^9+\frac{3}{8}bc^2x^8+\frac{3}{7}b^2cx^7+\frac{3}{7}ac^2x^7+\frac{1}{6}b^3x^6+abcx^6+\frac{3}{5}ab^2x^5+\frac{3}{5}a^2cx^5+\frac{3}{4}a^2bx^4+\frac{1}{3}a^3x^3$$

input
```
integrate(x^2*(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
1/9*c^3*x^9 + 3/8*b*c^2*x^8 + 3/7*b^2*c*x^7 + 3/7*a*c^2*x^7 + 1/6*b^3*x^6
+ a*b*c*x^6 + 3/5*a*b^2*x^5 + 3/5*a^2*c*x^5 + 3/4*a^2*b*x^4 + 1/3*a^3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.84

$$\int x^2(a+bx+cx^2)^3\,dx = x^6\left(\frac{b^3}{6}+a\,c\,b\right)+\frac{a^3\,x^3}{3}+\frac{c^3\,x^9}{9}+\frac{3\,a^2\,b\,x^4}{4}+\frac{3\,b\,c^2\,x^8}{8}+\frac{3\,a\,x^5\,(b^2+a\,c)}{5}+\frac{3\,c\,x^7\,(b^2+a\,c)}{7}$$

input
```
int(x^2*(a + b*x + c*x^2)^3,x)
```

output
```
x^6*(b^3/6 + a*b*c) + (a^3*x^3)/3 + (c^3*x^9)/9 + (3*a^2*b*x^4)/4 + (3*b*c
^2*x^8)/8 + (3*a*x^5*(a*c + b^2))/5 + (3*c*x^7*(a*c + b^2))/7
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98

$$\int x^2(a+bx+cx^2)^3\,dx$$
$$=\frac{x^3(280c^3x^6+945b\,c^2x^5+1080a\,c^2x^4+1080b^2c\,x^4+2520abc\,x^3+420b^3x^3+1512a^2c\,x^2+1512a\,b^2x^2+}{2520}$$

input
```
int(x^2*(c*x^2+b*x+a)^3,x)
```

output
```
(x**3*(840*a**3 + 1890*a**2*b*x + 1512*a**2*c*x**2 + 1512*a*b**2*x**2 + 25
20*a*b*c*x**3 + 1080*a*c**2*x**4 + 420*b**3*x**3 + 1080*b**2*c*x**4 + 945*
b*c**2*x**5 + 280*c**3*x**6))/2520
```

3.202 $\int x(a+bx+cx^2)^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 86

$$\int x(a+bx+cx^2)^3\,dx=\frac{a^3x^2}{2}+a^2bx^3+\frac{3}{4}a(b^2+ac)\,x^4+\frac{1}{5}b(b^2+6ac)\,x^5+\frac{1}{2}c(b^2+ac)\,x^6+\frac{3}{7}bc^2x^7+\frac{c^3x^8}{8}$$

output
```
1/2*a^3*x^2+a^2*b*x^3+3/4*a*(a*c+b^2)*x^4+1/5*b*(6*a*c+b^2)*x^5+1/2*c*(a*c
+b^2)*x^6+3/7*b*c^2*x^7+1/8*c^3*x^8
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int x(a+bx+cx^2)^3\,dx=\frac{a^3x^2}{2}+a^2bx^3+\frac{3}{4}a(b^2+ac)\,x^4+\frac{1}{5}b(b^2+6ac)\,x^5+\frac{1}{2}c(b^2+ac)\,x^6+\frac{3}{7}bc^2x^7+\frac{c^3x^8}{8}$$

input
```
Integrate[x*(a + b*x + c*x^2)^3,x]
```

output
```
(a^3*x^2)/2 + a^2*b*x^3 + (3*a*(b^2 + a*c)*x^4)/4 + (b*(b^2 + 6*a*c)*x^5)/
5 + (c*(b^2 + a*c)*x^6)/2 + (3*b*c^2*x^7)/7 + (c^3*x^8)/8
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx+cx^2\right)^3\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(a^3x+3a^2bx^2+3cx^5\left(ac+b^2\right)+bx^4\left(6ac+b^2\right)+3ax^3\left(ac+b^2\right)+3bc^2x^6+c^3x^7\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^2}{2}+a^2bx^3+\frac{1}{2}cx^6\left(ac+b^2\right)+\frac{1}{5}bx^5\left(6ac+b^2\right)+\frac{3}{4}ax^4\left(ac+b^2\right)+\frac{3}{7}bc^2x^7+\frac{c^3x^8}{8}$$

input
```
Int[x*(a + b*x + c*x^2)^3,x]
```

output
```
(a^3*x^2)/2 + a^2*b*x^3 + (3*a*(b^2 + a*c)*x^4)/4 + (b*(b^2 + 6*a*c)*x^5)/
5 + (c*(b^2 + a*c)*x^6)/2 + (3*b*c^2*x^7)/7 + (c^3*x^8)/8
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.98

method	result
norman	$\frac{c^3x^8}{8}+\frac{3b\,c^2x^7}{7}+\left(\frac{1}{2}a\,c^2+\frac{1}{2}b^2c\right)x^6+\left(\frac{6}{5}abc+\frac{1}{5}b^3\right)x^5+\left(\frac{3}{4}a^2c+\frac{3}{4}a\,b^2\right)x^4+a^2b\,x^3+\frac{a^3x^2}{2}$
gosper	$\frac{1}{8}c^3x^8+\frac{3}{7}b\,c^2x^7+\frac{1}{2}x^6a\,c^2+\frac{1}{2}b^2c\,x^6+\frac{6}{5}abc\,x^5+\frac{1}{5}b^3x^5+\frac{3}{4}a^2c\,x^4+\frac{3}{4}a\,b^2x^4+a^2b\,x^3+\frac{1}{2}a^3$
risch	$\frac{1}{8}c^3x^8+\frac{3}{7}b\,c^2x^7+\frac{1}{2}x^6a\,c^2+\frac{1}{2}b^2c\,x^6+\frac{6}{5}abc\,x^5+\frac{1}{5}b^3x^5+\frac{3}{4}a^2c\,x^4+\frac{3}{4}a\,b^2x^4+a^2b\,x^3+\frac{1}{2}a^3$
parallelrisch	$\frac{1}{8}c^3x^8+\frac{3}{7}b\,c^2x^7+\frac{1}{2}x^6a\,c^2+\frac{1}{2}b^2c\,x^6+\frac{6}{5}abc\,x^5+\frac{1}{5}b^3x^5+\frac{3}{4}a^2c\,x^4+\frac{3}{4}a\,b^2x^4+a^2b\,x^3+\frac{1}{2}a^3$
orering	$\frac{x^2\left(35c^3x^6+120b\,c^2x^5+140a\,c^2x^4+140b^2c\,x^4+336abc\,x^3+56b^3x^3+210a^2c\,x^2+210a\,b^2x^2+280a^2bx+140a^3\right)}{280}$
default	$\frac{c^3x^8}{8}+\frac{3b\,c^2x^7}{7}+\frac{\left(a\,c^2+2b^2c+c\left(2ac+b^2\right)\right)x^6}{6}+\frac{\left(4abc+b\left(2ac+b^2\right)\right)x^5}{5}+\frac{\left(a\left(2ac+b^2\right)+2a\,b^2+a^2c\right)x^4}{4}+a^2b\,x^3+$

input
```
int(x*(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*c^3*x^8+3/7*b*c^2*x^7+(1/2*a*c^2+1/2*b^2*c)*x^6+(6/5*a*b*c+1/5*b^3)*x^
5+(3/4*a^2*c+3/4*a*b^2)*x^4+a^2*b*x^3+1/2*a^3*x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int x\left(a+bx+cx^2\right)^3\,dx=\frac{1}{8}\,c^3x^8+\frac{3}{7}\,bc^2x^7+\frac{1}{2}\,\left(b^2c+ac^2\right)x^6+a^2bx^3+\frac{1}{5}\,\left(b^3+6\,abc\right)x^5+\frac{1}{2}\,a^3x^2+\frac{3}{4}\,\left(ab^2+a^2c\right)x^4$$

input
```
integrate(x*(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
1/8*c^3*x^8 + 3/7*b*c^2*x^7 + 1/2*(b^2*c + a*c^2)*x^6 + a^2*b*x^3 + 1/5*(b
^3 + 6*a*b*c)*x^5 + 1/2*a^3*x^2 + 3/4*(a*b^2 + a^2*c)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05

$$\int x(a+bx+cx^2)^3\,dx = \frac{a^3x^2}{2} + a^2bx^3 + \frac{3bc^2x^7}{7} + \frac{c^3x^8}{8} + x^6\left(\frac{ac^2}{2} + \frac{b^2c}{2}\right) + x^5\cdot\left(\frac{6abc}{5} + \frac{b^3}{5}\right) + x^4\cdot\left(\frac{3a^2c}{4} + \frac{3ab^2}{4}\right)$$

input
```
integrate(x*(c*x**2+b*x+a)**3,x)
```

output
```
a**3*x**2/2 + a**2*b*x**3 + 3*b*c**2*x**7/7 + c**3*x**8/8 + x**6*(a*c**2/2
 + b**2*c/2) + x**5*(6*a*b*c/5 + b**3/5) + x**4*(3*a**2*c/4 + 3*a*b**2/4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int x(a+bx+cx^2)^3\,dx = \frac{1}{8}c^3x^8 + \frac{3}{7}bc^2x^7 + \frac{1}{2}(b^2c+ac^2)x^6 + a^2bx^3 + \frac{1}{5}(b^3+6\,abc)x^5 + \frac{1}{2}a^3x^2 + \frac{3}{4}(ab^2+a^2c)x^4$$

input
```
integrate(x*(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
1/8*c^3*x^8 + 3/7*b*c^2*x^7 + 1/2*(b^2*c + a*c^2)*x^6 + a^2*b*x^3 + 1/5*(b
^3 + 6*a*b*c)*x^5 + 1/2*a^3*x^2 + 3/4*(a*b^2 + a^2*c)*x^4
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int x(a+bx+cx^2)^3\,dx = \frac{1}{8}c^3x^8 + \frac{3}{7}bc^2x^7 + \frac{1}{2}b^2cx^6 + \frac{1}{2}ac^2x^6 + \frac{1}{5}b^3x^5 + \frac{6}{5}abcx^5 + \frac{3}{4}ab^2x^4 + \frac{3}{4}a^2cx^4 + a^2bx^3 + \frac{1}{2}a^3x^2$$

input
```
integrate(x*(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
1/8*c^3*x^8 + 3/7*b*c^2*x^7 + 1/2*b^2*c*x^6 + 1/2*a*c^2*x^6 + 1/5*b^3*x^5
+ 6/5*a*b*c*x^5 + 3/4*a*b^2*x^4 + 3/4*a^2*c*x^4 + a^2*b*x^3 + 1/2*a^3*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87

$$\int x(a+bx+cx^2)^3\,dx = x^5\left(\frac{b^3}{5} + \frac{6\,a\,c\,b}{5}\right) + \frac{a^3\,x^2}{2} + \frac{c^3\,x^8}{8} + a^2\,b\,x^3 + \frac{3\,b\,c^2\,x^7}{7} + \frac{3\,a\,x^4\,(b^2+a\,c)}{4} + \frac{c\,x^6\,(b^2+a\,c)}{2}$$

input
```
int(x*(a + b*x + c*x^2)^3,x)
```

output
```
x^5*(b^3/5 + (6*a*b*c)/5) + (a^3*x^2)/2 + (c^3*x^8)/8 + a^2*b*x^3 + (3*b*c
^2*x^7)/7 + (3*a*x^4*(a*c + b^2))/4 + (c*x^6*(a*c + b^2))/2
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.01

$$\int x(a+bx+cx^2)^3\,dx$$
$$= \frac{x^2(35c^3x^6+120b\,c^2x^5+140a\,c^2x^4+140b^2c\,x^4+336abc\,x^3+56b^3x^3+210a^2c\,x^2+210a\,b^2x^2+280a^2bx}{280}$$

input
```
int(x*(c*x^2+b*x+a)^3,x)
```

output
```
(x**2*(140*a**3 + 280*a**2*b*x + 210*a**2*c*x**2 + 210*a*b**2*x**2 + 336*a
*b*c*x**3 + 140*a*c**2*x**4 + 56*b**3*x**3 + 140*b**2*c*x**4 + 120*b*c**2*
x**5 + 35*c**3*x**6))/280
```

3.203 $\int (a+bx+cx^2)^3 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 81

$$\int (a+bx+cx^2)^3 \, dx = a^3x + \frac{3}{2}a^2bx^2 + a(b^2+ac)\,x^3 + \frac{1}{4}b(b^2+6ac)\,x^4 + \frac{3}{5}c(b^2+ac)\,x^5 + \frac{1}{2}bc^2x^6 + \frac{c^3x^7}{7}$$

output
```
a^3*x+3/2*a^2*b*x^2+a*(a*c+b^2)*x^3+1/4*b*(6*a*c+b^2)*x^4+3/5*c*(a*c+b^2)*
x^5+1/2*b*c^2*x^6+1/7*c^3*x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int (a+bx+cx^2)^3 \, dx = a^3x + \frac{3}{2}a^2bx^2 + a(b^2+ac)\,x^3 + \frac{1}{4}b(b^2+6ac)\,x^4 + \frac{3}{5}c(b^2+ac)\,x^5 + \frac{1}{2}bc^2x^6 + \frac{c^3x^7}{7}$$

input
```
Integrate[(a + b*x + c*x^2)^3,x]
```

output
```
a^3*x + (3*a^2*b*x^2)/2 + a*(b^2 + a*c)*x^3 + (b*(b^2 + 6*a*c)*x^4)/4 + (3
*c*(b^2 + a*c)*x^5)/5 + (b*c^2*x^6)/2 + (c^3*x^7)/7
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right)^3 \, dx$$

$$\Big\downarrow \text{1085}$$

$$\int \left(a^3+3a^2bx+3b^2cx^4\left(\frac{ac}{b^2}+1\right)+3ab^2x^2\left(\frac{ac}{b^2}+1\right)+b^3x^3\left(\frac{6ac}{b^2}+1\right)+3bc^2x^5+c^3x^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^3x+\frac{3}{2}a^2bx^2+\frac{3}{5}cx^5\left(ac+b^2\right)+\frac{1}{4}bx^4\left(6ac+b^2\right)+ax^3\left(ac+b^2\right)+\frac{1}{2}bc^2x^6+\frac{c^3x^7}{7}$$

input
```
Int[(a + b*x + c*x^2)^3,x]
```

output
```
a^3*x + (3*a^2*b*x^2)/2 + a*(b^2 + a*c)*x^3 + (b*(b^2 + 6*a*c)*x^4)/4 + (3
*c*(b^2 + a*c)*x^5)/5 + (b*c^2*x^6)/2 + (c^3*x^7)/7
```

Defintions of rubi rules used

rule 1085
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && IntegerQ[p] && (G
tQ[p, 0] || EqQ[a, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

method	result
norman	$\frac{c^3x^7}{7}+\frac{b\,c^2x^6}{2}+\left(\frac{3}{5}a\,c^2+\frac{3}{5}b^2c\right)x^5+\left(\frac{3}{2}abc+\frac{1}{4}b^3\right)x^4+\left(a^2c+a\,b^2\right)x^3+\frac{3a^2b\,x^2}{2}+a^3x$
gosper	$\frac{1}{7}c^3x^7+\frac{1}{2}b\,c^2x^6+\frac{3}{5}x^5a\,c^2+\frac{3}{5}x^5b^2c+\frac{3}{2}abc\,x^4+\frac{1}{4}b^3x^4+a^2c\,x^3+a\,b^2x^3+\frac{3}{2}a^2b\,x^2+a^3x$
risch	$\frac{1}{7}c^3x^7+\frac{1}{2}b\,c^2x^6+\frac{3}{5}x^5a\,c^2+\frac{3}{5}x^5b^2c+\frac{3}{2}abc\,x^4+\frac{1}{4}b^3x^4+a^2c\,x^3+a\,b^2x^3+\frac{3}{2}a^2b\,x^2+a^3x$
parallelrisch	$\frac{1}{7}c^3x^7+\frac{1}{2}b\,c^2x^6+\frac{3}{5}x^5a\,c^2+\frac{3}{5}x^5b^2c+\frac{3}{2}abc\,x^4+\frac{1}{4}b^3x^4+a^2c\,x^3+a\,b^2x^3+\frac{3}{2}a^2b\,x^2+a^3x$
orering	$\frac{x\left(20c^3x^6+70b\,c^2x^5+84a\,c^2x^4+84b^2c\,x^4+210abc\,x^3+35b^3x^3+140a^2c\,x^2+140a\,b^2x^2+210a^2bx+140a^3\right)}{140}$
default	$\frac{c^3x^7}{7}+\frac{b\,c^2x^6}{2}+\frac{\left(a\,c^2+2b^2c+c\left(2ac+b^2\right)\right)x^5}{5}+\frac{\left(4abc+b\left(2ac+b^2\right)\right)x^4}{4}+\frac{\left(a\left(2ac+b^2\right)+2a\,b^2+a^2c\right)x^3}{3}+\frac{3a^2b\,x^2}{2}+a$

input
```
int((c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/7*c^3*x^7+1/2*b*c^2*x^6+(3/5*a*c^2+3/5*b^2*c)*x^5+(3/2*a*b*c+1/4*b^3)*x^
4+(a^2*c+a*b^2)*x^3+3/2*a^2*b*x^2+a^3*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \left(a+bx+cx^2\right)^3\,dx=\frac{1}{7}\,c^3x^7+\frac{1}{2}\,bc^2x^6+\frac{3}{5}\,\left(b^2c+ac^2\right)x^5+\frac{3}{2}\,a^2bx^2+\frac{1}{4}\,\left(b^3+6\,abc\right)x^4+a^3x+\left(ab^2+a^2c\right)x^3$$

input
```
integrate((c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
1/7*c^3*x^7 + 1/2*b*c^2*x^6 + 3/5*(b^2*c + a*c^2)*x^5 + 3/2*a^2*b*x^2 + 1/
4*(b^3 + 6*a*b*c)*x^4 + a^3*x + (a*b^2 + a^2*c)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \left(a+bx+cx^2\right)^3\,dx=a^3x+\frac{3a^2bx^2}{2}+\frac{bc^2x^6}{2}+\frac{c^3x^7}{7}+x^5\cdot\left(\frac{3ac^2}{5}+\frac{3b^2c}{5}\right)+x^4\cdot\left(\frac{3abc}{2}+\frac{b^3}{4}\right)+x^3\left(a^2c+ab^2\right)$$

input
```
integrate((c*x**2+b*x+a)**3,x)
```

output
```
a**3*x + 3*a**2*b*x**2/2 + b*c**2*x**6/2 + c**3*x**7/7 + x**5*(3*a*c**2/5
+ 3*b**2*c/5) + x**4*(3*a*b*c/2 + b**3/4) + x**3*(a**2*c + a*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \left(a+bx+cx^2\right)^3\,dx=\frac{1}{7}\,c^3x^7+\frac{1}{2}\,bc^2x^6+\frac{3}{5}\,b^2cx^5+\frac{1}{4}\,b^3x^4+a^3x+\frac{1}{2}\,\left(2\,cx^3+3\,bx^2\right)a^2+\frac{1}{10}\,\left(6\,c^2x^5+15\,bcx^4+10\,b^2x^3\right)a$$

input
```
integrate((c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
1/7*c^3*x^7 + 1/2*b*c^2*x^6 + 3/5*b^2*c*x^5 + 1/4*b^3*x^4 + a^3*x + 1/2*(2
*c*x^3 + 3*b*x^2)*a^2 + 1/10*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*a
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

$$\int \left(a+bx+cx^2\right)^3 \, dx = \frac{1}{7}c^3x^7 + \frac{1}{2}bc^2x^6 + \frac{3}{5}b^2cx^5 + \frac{3}{5}ac^2x^5 + \frac{1}{4}b^3x^4 + \frac{3}{2}abcx^4 + ab^2x^3 + a^2cx^3 + \frac{3}{2}a^2bx^2 + a^3x$$

input
```
integrate((c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
1/7*c^3*x^7 + 1/2*b*c^2*x^6 + 3/5*b^2*c*x^5 + 3/5*a*c^2*x^5 + 1/4*b^3*x^4
+ 3/2*a*b*c*x^4 + a*b^2*x^3 + a^2*c*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

$$\int \left(a+bx+cx^2\right)^3 \, dx = x^4\left(\frac{b^3}{4}+\frac{3\,a\,c\,b}{2}\right) + a^3\,x + \frac{c^3\,x^7}{7} + \frac{3\,a^2\,b\,x^2}{2} + \frac{b\,c^2\,x^6}{2} + a\,x^3\left(b^2+a\,c\right) + \frac{3\,c\,x^5\left(b^2+a\,c\right)}{5}$$

input
```
int((a + b*x + c*x^2)^3,x)
```

output
```
x^4*(b^3/4 + (3*a*b*c)/2) + a^3*x + (c^3*x^7)/7 + (3*a^2*b*x^2)/2 + (b*c^2
*x^6)/2 + a*x^3*(a*c + b^2) + (3*c*x^5*(a*c + b^2))/5
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int (a + bx + cx^2)^3 \, dx$$
$$= \frac{x(20c^3x^6 + 70b\,c^2x^5 + 84a\,c^2x^4 + 84b^2c\,x^4 + 210abc\,x^3 + 35b^3x^3 + 140a^2c\,x^2 + 140a\,b^2x^2 + 210a^2bx + 1}{140}$$

input
```
int((c*x^2+b*x+a)^3,x)
```

output
```
(x*(140*a**3 + 210*a**2*b*x + 140*a**2*c*x**2 + 140*a*b**2*x**2 + 210*a*b*
c*x**3 + 84*a*c**2*x**4 + 35*b**3*x**3 + 84*b**2*c*x**4 + 70*b*c**2*x**5 +
 20*c**3*x**6))/140
```

3.204 $\int \frac{(a+bx+cx^2)^3}{x} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 81

$$\int \frac{(a+bx+cx^2)^3}{x} \, dx = 3a^2bx + \frac{3}{2}a(b^2+ac)\,x^2 + \frac{1}{3}b(b^2+6ac)\,x^3 + \frac{3}{4}c(b^2+ac)\,x^4 + \frac{3}{5}bc^2x^5 + \frac{c^3x^6}{6} + a^3\log(x)$$

output
```
3*a^2*b*x+3/2*a*(a*c+b^2)*x^2+1/3*b*(6*a*c+b^2)*x^3+3/4*c*(a*c+b^2)*x^4+3/
5*b*c^2*x^5+1/6*c^3*x^6+a^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2)^3}{x} \, dx = 3a^2bx + \frac{3}{2}a(b^2+ac)\,x^2 + \frac{1}{3}b(b^2+6ac)\,x^3 + \frac{3}{4}c(b^2+ac)\,x^4 + \frac{3}{5}bc^2x^5 + \frac{c^3x^6}{6} + a^3\log(x)$$

input
```
Integrate[(a + b*x + c*x^2)^3/x,x]
```

output
```
3*a^2*b*x + (3*a*(b^2 + a*c)*x^2)/2 + (b*(b^2 + 6*a*c)*x^3)/3 + (3*c*(b^2
+ a*c)*x^4)/4 + (3*b*c^2*x^5)/5 + (c^3*x^6)/6 + a^3*Log[x]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^3}{x}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{a^3}{x}+3a^2b+3cx^3\left(ac+b^2\right)+bx^2\left(6ac+b^2\right)+3ax\left(ac+b^2\right)+3bc^2x^4+c^3x^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^3\log(x)+3a^2bx+\frac{3}{4}cx^4\left(ac+b^2\right)+\frac{1}{3}bx^3\left(6ac+b^2\right)+\frac{3}{2}ax^2\left(ac+b^2\right)+\frac{3}{5}bc^2x^5+\frac{c^3x^6}{6}$$

input
```
Int[(a + b*x + c*x^2)^3/x,x]
```

output

```
3*a^2*b*x + (3*a*(b^2 + a*c)*x^2)/2 + (b*(b^2 + 6*a*c)*x^3)/3 + (3*c*(b^2
+ a*c)*x^4)/4 + (3*b*c^2*x^5)/5 + (c^3*x^6)/6 + a^3*Log[x]
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

method	result
norman	$\left(\frac{3}{4}a\,c^{2}+\frac{3}{4}b^{2}c\right)x^{4}+\left(\frac{3}{2}a^{2}c+\frac{3}{2}a\,b^{2}\right)x^{2}+\left(2abc+\frac{1}{3}b^{3}\right)x^{3}+\frac{c^{3}x^{6}}{6}+3a^{2}bx+\frac{3b\,c^{2}x^{5}}{5}+a^{3}\ln\left(x\right.$
default	$\frac{c^{3}x^{6}}{6}+\frac{3b\,c^{2}x^{5}}{5}+\frac{3a\,c^{2}x^{4}}{4}+\frac{3b^{2}c\,x^{4}}{4}+2abc\,x^{3}+\frac{b^{3}x^{3}}{3}+\frac{3a^{2}c\,x^{2}}{2}+\frac{3a\,b^{2}x^{2}}{2}+3a^{2}bx+a^{3}\ln\left(x\right)$
risch	$\frac{c^{3}x^{6}}{6}+\frac{3b\,c^{2}x^{5}}{5}+\frac{3a\,c^{2}x^{4}}{4}+\frac{3b^{2}c\,x^{4}}{4}+2abc\,x^{3}+\frac{b^{3}x^{3}}{3}+\frac{3a^{2}c\,x^{2}}{2}+\frac{3a\,b^{2}x^{2}}{2}+3a^{2}bx+a^{3}\ln\left(x\right)$
parallelrisch	$\frac{c^{3}x^{6}}{6}+\frac{3b\,c^{2}x^{5}}{5}+\frac{3a\,c^{2}x^{4}}{4}+\frac{3b^{2}c\,x^{4}}{4}+2abc\,x^{3}+\frac{b^{3}x^{3}}{3}+\frac{3a^{2}c\,x^{2}}{2}+\frac{3a\,b^{2}x^{2}}{2}+3a^{2}bx+a^{3}\ln\left(x\right)$

input
```
int((c*x^2+b*x+a)^3/x,x,method=_RETURNVERBOSE)
```

output
```
(3/4*a*c^2+3/4*b^2*c)*x^4+(3/2*a^2*c+3/2*a*b^2)*x^2+(2*a*b*c+1/3*b^3)*x^3+
1/6*c^3*x^6+3*a^2*b*x+3/5*b*c^2*x^5+a^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx+cx^2)^3}{x}\,dx = \frac{1}{6}\,c^3x^6+\frac{3}{5}\,bc^2x^5+\frac{3}{4}\,\left(b^2c+ac^2\right)x^4+3\,a^2bx+\frac{1}{3}\,\left(b^3+6\,abc\right)x^3+a^3\log\left(x\right)+\frac{3}{2}\,\left(ab^2+a^2c\right)x^2$$

input
```
integrate((c*x^2+b*x+a)^3/x,x, algorithm="fricas")
```

output
```
1/6*c^3*x^6 + 3/5*b*c^2*x^5 + 3/4*(b^2*c + a*c^2)*x^4 + 3*a^2*b*x + 1/3*(b
^3 + 6*a*b*c)*x^3 + a^3*log(x) + 3/2*(a*b^2 + a^2*c)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx+cx^2)^3}{x}\,dx = a^3\log(x) + 3a^2bx + \frac{3bc^2x^5}{5} + \frac{c^3x^6}{6} + x^4\cdot\left(\frac{3ac^2}{4}+\frac{3b^2c}{4}\right) + x^3\cdot\left(2abc+\frac{b^3}{3}\right) + x^2\cdot\left(\frac{3a^2c}{2}+\frac{3ab^2}{2}\right)$$

input
```
integrate((c*x**2+b*x+a)**3/x,x)
```

output
```
a**3*log(x) + 3*a**2*b*x + 3*b*c**2*x**5/5 + c**3*x**6/6 + x**4*(3*a*c**2/
4 + 3*b**2*c/4) + x**3*(2*a*b*c + b**3/3) + x**2*(3*a**2*c/2 + 3*a*b**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx+cx^2)^3}{x}\,dx = \frac{1}{6}\,c^3x^6 + \frac{3}{5}\,bc^2x^5 + \frac{3}{4}\,(b^2c+ac^2)x^4 + 3\,a^2bx + \frac{1}{3}\,(b^3+6\,abc)x^3 + a^3\log(x) + \frac{3}{2}\,(ab^2+a^2c)x^2$$

input
```
integrate((c*x^2+b*x+a)^3/x,x, algorithm="maxima")
```

output
```
1/6*c^3*x^6 + 3/5*b*c^2*x^5 + 3/4*(b^2*c + a*c^2)*x^4 + 3*a^2*b*x + 1/3*(b
^3 + 6*a*b*c)*x^3 + a^3*log(x) + 3/2*(a*b^2 + a^2*c)*x^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx+cx^2)^3}{x}\,dx = \frac{1}{6}\,c^3x^6+\frac{3}{5}\,bc^2x^5+\frac{3}{4}\,b^2cx^4+\frac{3}{4}\,ac^2x^4+\frac{1}{3}\,b^3x^3 +2\,abcx^3+\frac{3}{2}\,ab^2x^2+\frac{3}{2}\,a^2cx^2+3\,a^2bx+a^3\log\left(|x|\right)$$

input
```
integrate((c*x^2+b*x+a)^3/x,x, algorithm="giac")
```

output
```
1/6*c^3*x^6 + 3/5*b*c^2*x^5 + 3/4*b^2*c*x^4 + 3/4*a*c^2*x^4 + 1/3*b^3*x^3
+ 2*a*b*c*x^3 + 3/2*a*b^2*x^2 + 3/2*a^2*c*x^2 + 3*a^2*b*x + a^3*log(abs(x)
)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx+cx^2)^3}{x}\,dx = a^3\,\ln\left(x\right)+x^3\left(\frac{b^3}{3}+2\,a\,c\,b\right)+\frac{c^3\,x^6}{6}+\frac{3\,b\,c^2\,x^5}{5} +\frac{3\,a\,x^2\,\left(b^2+a\,c\right)}{2}+\frac{3\,c\,x^4\,\left(b^2+a\,c\right)}{4}+3\,a^2\,b\,x$$

input
```
int((a + b*x + c*x^2)^3/x,x)
```

output
```
a^3*log(x) + x^3*(b^3/3 + 2*a*b*c) + (c^3*x^6)/6 + (3*b*c^2*x^5)/5 + (3*a*
x^2*(a*c + b^2))/2 + (3*c*x^4*(a*c + b^2))/4 + 3*a^2*b*x
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx+cx^2)^3}{x}\,dx = \log(x)\,a^3 + 3a^2bx + \frac{3a^2c\,x^2}{2} + \frac{3a\,b^2x^2}{2} + 2abc\,x^3 + \frac{3a\,c^2x^4}{4} + \frac{b^3x^3}{3} + \frac{3b^2c\,x^4}{4} + \frac{3b\,c^2x^5}{5} + \frac{c^3x^6}{6}$$

input
```
int((c*x^2+b*x+a)^3/x,x)
```

output
```
(60*log(x)*a**3 + 180*a**2*b*x + 90*a**2*c*x**2 + 90*a*b**2*x**2 + 120*a*b
*c*x**3 + 45*a*c**2*x**4 + 20*b**3*x**3 + 45*b**2*c*x**4 + 36*b*c**2*x**5
+ 10*c**3*x**6)/60
```

3.205 $\int \frac{(a+bx+cx^2)^3}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int \frac{(a+bx+cx^2)^3}{x^2}\,dx = -\frac{a^3}{x}+3a(b^2+ac)\,x+\frac{1}{2}b(b^2+6ac)\,x^2 + c(b^2+ac)\,x^3+\frac{3}{4}bc^2x^4+\frac{c^3x^5}{5}+3a^2b\log(x)$$

output
```
-a^3/x+3*a*(a*c+b^2)*x+1/2*b*(6*a*c+b^2)*x^2+c*(a*c+b^2)*x^3+3/4*b*c^2*x^4
+1/5*c^3*x^5+3*a^2*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2)^3}{x^2}\,dx = -\frac{a^3}{x}+3a(b^2+ac)\,x+\frac{1}{2}b(b^2+6ac)\,x^2 + c(b^2+ac)\,x^3+\frac{3}{4}bc^2x^4+\frac{c^3x^5}{5}+3a^2b\log(x)$$

input
```
Integrate[(a + b*x + c*x^2)^3/x^2,x]
```

output
```
-(a^3/x) + 3*a*(b^2 + a*c)*x + (b*(b^2 + 6*a*c)*x^2)/2 + c*(b^2 + a*c)*x^3
 + (3*b*c^2*x^4)/4 + (c^3*x^5)/5 + 3*a^2*b*Log[x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^3}{x^2}dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{a^3}{x^2}+\frac{3a^2b}{x}+3cx^2(ac+b^2)+bx(6ac+b^2)+3a(ac+b^2)+3bc^2x^3+c^3x^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3}{x}+3a^2b\log(x)+cx^3(ac+b^2)+\frac{1}{2}bx^2(6ac+b^2)+3ax(ac+b^2)+\frac{3}{4}bc^2x^4+\frac{c^3x^5}{5}$$

input
```
Int[(a + b*x + c*x^2)^3/x^2,x]
```

output

```
-(a^3/x) + 3*a*(b^2 + a*c)*x + (b*(b^2 + 6*a*c)*x^2)/2 + c*(b^2 + a*c)*x^3
 + (3*b*c^2*x^4)/4 + (c^3*x^5)/5 + 3*a^2*b*Log[x]
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

method	result	size
default	$\frac{c^3x^5}{5}+\frac{3b\,c^2x^4}{4}+a\,c^2x^3+b^2c\,x^3+3abc\,x^2+\frac{b^3x^2}{2}+3a^2cx+3xa\,b^2+3a^2b\ln\left(x\right)-\frac{a^3}{x}$	81
risch	$\frac{c^3x^5}{5}+\frac{3b\,c^2x^4}{4}+a\,c^2x^3+b^2c\,x^3+3abc\,x^2+\frac{b^3x^2}{2}+3a^2cx+3xa\,b^2+3a^2b\ln\left(x\right)-\frac{a^3}{x}$	81
norman	$\frac{\left(3abc+\frac{1}{2}b^3\right)x^3+\left(a\,c^2+b^2c\right)x^4+\left(3a^2c+3a\,b^2\right)x^2-a^3+\frac{c^3x^6}{5}+\frac{3b\,c^2x^5}{4}}{x}+3a^2b\ln\left(x\right)$	84
parallelrisch	$\frac{4c^3x^6+15b\,c^2x^5+20a\,c^2x^4+20b^2c\,x^4+60abc\,x^3+10b^3x^3+60\ln\left(x\right)x\,a^2b+60a^2c\,x^2+60a\,b^2x^2-20a^3}{20x}$	90

input
```
int((c*x^2+b*x+a)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*c^3*x^5+3/4*b*c^2*x^4+a*c^2*x^3+b^2*c*x^3+3*a*b*c*x^2+1/2*b^3*x^2+3*a^
2*c*x+3*x*a*b^2+3*a^2*b*ln(x)-a^3/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08

$$\int\frac{\left(a+bx+cx^2\right)^3}{x^2}\,dx=\frac{4\,c^3x^6+15\,bc^2x^5+20\,(b^2c+ac^2)x^4+60\,a^2bx\log\left(x\right)+10\,(b^3+6\,abc)x^3-20\,a^3+60\,(ab^2+a^2c)x^2}{20\,x}$$

input
```
integrate((c*x^2+b*x+a)^3/x^2,x, algorithm="fricas")
```

output
```
1/20*(4*c^3*x^6 + 15*b*c^2*x^5 + 20*(b^2*c + a*c^2)*x^4 + 60*a^2*b*x*log(x
) + 10*(b^3 + 6*a*b*c)*x^3 - 20*a^3 + 60*(a*b^2 + a^2*c)*x^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx+cx^2)^3}{x^2}\,dx = -\frac{a^3}{x} + 3a^2b\log{(x)} + \frac{3bc^2x^4}{4} + \frac{c^3x^5}{5} + x^3(ac^2+b^2c) + x^2\cdot\left(3abc+\frac{b^3}{2}\right) + x(3a^2c+3ab^2)$$

input
```
integrate((c*x**2+b*x+a)**3/x**2,x)
```

output
```
-a**3/x + 3*a**2*b*log(x) + 3*b*c**2*x**4/4 + c**3*x**5/5 + x**3*(a*c**2 +
 b**2*c) + x**2*(3*a*b*c + b**3/2) + x*(3*a**2*c + 3*a*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2)^3}{x^2}\,dx = \frac{1}{5}c^3x^5 + \frac{3}{4}bc^2x^4 + (b^2c+ac^2)x^3 + 3a^2b\log{(x)} + \frac{1}{2}(b^3+6abc)x^2 - \frac{a^3}{x} + 3(ab^2+a^2c)x$$

input
```
integrate((c*x^2+b*x+a)^3/x^2,x, algorithm="maxima")
```

output
```
1/5*c^3*x^5 + 3/4*b*c^2*x^4 + (b^2*c + a*c^2)*x^3 + 3*a^2*b*log(x) + 1/2*(
b^3 + 6*a*b*c)*x^2 - a^3/x + 3*(a*b^2 + a^2*c)*x
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx+cx^2)^3}{x^2}\,dx = \frac{1}{5}c^3x^5 + \frac{3}{4}bc^2x^4 + b^2cx^3 + ac^2x^3 + \frac{1}{2}b^3x^2 + 3\,abcx^2 + 3\,ab^2x + 3\,a^2cx + 3\,a^2b\log\left(|x|\right) - \frac{a^3}{x}$$

input
```
integrate((c*x^2+b*x+a)^3/x^2,x, algorithm="giac")
```

output
```
1/5*c^3*x^5 + 3/4*b*c^2*x^4 + b^2*c*x^3 + a*c^2*x^3 + 1/2*b^3*x^2 + 3*a*b*
c*x^2 + 3*a*b^2*x + 3*a^2*c*x + 3*a^2*b*log(abs(x)) - a^3/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx+cx^2)^3}{x^2}\,dx = x^2\left(\frac{b^3}{2} + 3\,a\,c\,b\right) - \frac{a^3}{x} + \frac{c^3\,x^5}{5} + \frac{3\,b\,c^2\,x^4}{4} + 3\,a^2\,b\,\ln\left(x\right) + 3\,a\,x\,\left(b^2 + a\,c\right) + c\,x^3\,\left(b^2 + a\,c\right)$$

input
```
int((a + b*x + c*x^2)^3/x^2,x)
```

output
```
x^2*(b^3/2 + 3*a*b*c) - a^3/x + (c^3*x^5)/5 + (3*b*c^2*x^4)/4 + 3*a^2*b*lo
g(x) + 3*a*x*(a*c + b^2) + c*x^3*(a*c + b^2)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx+cx^2)^3}{x^2}\,dx = \frac{60\log(x)\,a^2bx - 20a^3 + 60a^2c\,x^2 + 60a\,b^2x^2 + 60abc\,x^3 + 20a\,c^2x^4 + 10b^3x^3 + 20b^2c\,x^4 + 15b\,c^2x^5 + 4c^3}{20x}$$

input
```
int((c*x^2+b*x+a)^3/x^2,x)
```

output
```
(60*log(x)*a**2*b*x - 20*a**3 + 60*a**2*c*x**2 + 60*a*b**2*x**2 + 60*a*b*c
*x**3 + 20*a*c**2*x**4 + 10*b**3*x**3 + 20*b**2*c*x**4 + 15*b*c**2*x**5 +
4*c**3*x**6)/(20*x)
```

3.206 $\int \frac{(a+bx+cx^2)^3}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 76

$$\int \frac{(a+bx+cx^2)^3}{x^3}\,dx = -\frac{a^3}{2x^2}-\frac{3a^2b}{x}+b(b^2+6ac)\,x+\frac{3}{2}c(b^2+ac)\,x^2+bc^2x^3+\frac{c^3x^4}{4}+3a(b^2+ac)\log(x)$$

output
```
-1/2*a^3/x^2-3*a^2*b/x+b*(6*a*c+b^2)*x+3/2*c*(a*c+b^2)*x^2+b*c^2*x^3+1/4*c
^3*x^4+3*a*(a*c+b^2)*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2)^3}{x^3}\,dx = -\frac{a^3}{2x^2}-\frac{3a^2b}{x}+b(b^2+6ac)\,x+\frac{3}{2}c(b^2+ac)\,x^2+bc^2x^3+\frac{c^3x^4}{4}+3a(b^2+ac)\log(x)$$

input
```
Integrate[(a + b*x + c*x^2)^3/x^3,x]
```

output
```
-1/2*a^3/x^2 - (3*a^2*b)/x + b*(b^2 + 6*a*c)*x + (3*c*(b^2 + a*c)*x^2)/2 +
 b*c^2*x^3 + (c^3*x^4)/4 + 3*a*(b^2 + a*c)*Log[x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^3}{x^3}dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{a^3}{x^3}+\frac{3a^2b}{x^2}+\frac{3a(ac+b^2)}{x}+3cx(ac+b^2)+b^3\left(\frac{6ac}{b^2}+1\right)+3bc^2x^2+c^3x^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3}{2x^2}-\frac{3a^2b}{x}+\frac{3}{2}cx^2(ac+b^2)+bx(6ac+b^2)+3a\log(x)\left(ac+b^2\right)+bc^2x^3+\frac{c^3x^4}{4}$$

input
```
Int[(a + b*x + c*x^2)^3/x^3,x]
```

output

```
-1/2*a^3/x^2 - (3*a^2*b)/x + b*(b^2 + 6*a*c)*x + (3*c*(b^2 + a*c)*x^2)/2 +
 b*c^2*x^3 + (c^3*x^4)/4 + 3*a*(b^2 + a*c)*Log[x]
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

method	result	siz
default	$\frac{c^3x^4}{4}+b\,c^2x^3+\frac{3a\,c^2x^2}{2}+\frac{3b^2c\,x^2}{2}+6abcx+b^3x-\frac{a^3}{2x^2}+3a(ac+b^2)\ln(x)-\frac{3a^2b}{x}$	76
risch	$\frac{c^3x^4}{4}+b\,c^2x^3+\frac{3a\,c^2x^2}{2}+\frac{3b^2c\,x^2}{2}+6abcx+b^3x+\frac{-3a^2bx-\frac{1}{2}a^3}{x^2}+3a^2c\ln(x)+3\ln(x)\,a\,b^2$	80
norman	$\frac{(\frac{3}{2}a\,c^2+\frac{3}{2}b^2c)x^4+(6abc+b^3)x^3+b\,c^2x^5-\frac{a^3}{2}+\frac{c^3x^6}{4}-3a^2bx}{x^2}+(3a^2c+3a\,b^2)\ln(x)$	81
parallelrisch	$\frac{c^3x^6+4b\,c^2x^5+6a\,c^2x^4+6b^2c\,x^4+12\ln(x)x^2a^2c+12\ln(x)x^2a\,b^2+24abc\,x^3+4b^3x^3-12a^2bx-2a^3}{4x^2}$	91

input
```
int((c*x^2+b*x+a)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*c^3*x^4+b*c^2*x^3+3/2*a*c^2*x^2+3/2*b^2*c*x^2+6*a*b*c*x+b^3*x-1/2*a^3/
x^2+3*a*(a*c+b^2)*ln(x)-3*a^2*b/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int\frac{(a+bx+cx^2)^3}{x^3}\,dx$$
$$=\frac{c^3x^6+4\,bc^2x^5+6\,(b^2c+ac^2)x^4-12\,a^2bx+4\,(b^3+6\,abc)x^3+12\,(ab^2+a^2c)x^2\log(x)-2\,a^3}{4\,x^2}$$

input
```
integrate((c*x^2+b*x+a)^3/x^3,x, algorithm="fricas")
```

output
```
1/4*(c^3*x^6 + 4*b*c^2*x^5 + 6*(b^2*c + a*c^2)*x^4 - 12*a^2*b*x + 4*(b^3 +
 6*a*b*c)*x^3 + 12*(a*b^2 + a^2*c)*x^2*log(x) - 2*a^3)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx+cx^2)^3}{x^3}\,dx = 3a(ac+b^2)\log(x)+bc^2x^3+\frac{c^3x^4}{4}+x^2\cdot\left(\frac{3ac^2}{2}+\frac{3b^2c}{2}\right)+x(6abc+b^3)+\frac{-a^3-6a^2bx}{2x^2}$$

input
```
integrate((c*x**2+b*x+a)**3/x**3,x)
```

output
```
3*a*(a*c + b**2)*log(x) + b*c**2*x**3 + c**3*x**4/4 + x**2*(3*a*c**2/2 + 3
*b**2*c/2) + x*(6*a*b*c + b**3) + (-a**3 - 6*a**2*b*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx+cx^2)^3}{x^3}\,dx = \frac{1}{4}c^3x^4+bc^2x^3+\frac{3}{2}(b^2c+ac^2)x^2+(b^3+6abc)x+3(ab^2+a^2c)\log(x)-\frac{6a^2bx+a^3}{2x^2}$$

input
```
integrate((c*x^2+b*x+a)^3/x^3,x, algorithm="maxima")
```

output
```
1/4*c^3*x^4 + b*c^2*x^3 + 3/2*(b^2*c + a*c^2)*x^2 + (b^3 + 6*a*b*c)*x + 3*
(a*b^2 + a^2*c)*log(x) - 1/2*(6*a^2*b*x + a^3)/x^2
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx+cx^2)^3}{x^3}\,dx = \frac{1}{4}c^3x^4 + bc^2x^3 + \frac{3}{2}b^2cx^2 + \frac{3}{2}ac^2x^2 + b^3x + 6\,abcx + 3\left(ab^2+a^2c\right)\log\left(|x|\right) - \frac{6\,a^2bx+a^3}{2\,x^2}$$

input
```
integrate((c*x^2+b*x+a)^3/x^3,x, algorithm="giac")
```

output
```
1/4*c^3*x^4 + b*c^2*x^3 + 3/2*b^2*c*x^2 + 3/2*a*c^2*x^2 + b^3*x + 6*a*b*c*
x + 3*(a*b^2 + a^2*c)*log(abs(x)) - 1/2*(6*a^2*b*x + a^3)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx+cx^2)^3}{x^3}\,dx = x\left(b^3+6\,a\,c\,b\right) + \ln\left(x\right)\,\left(3\,c\,a^2+3\,a\,b^2\right) - \frac{\frac{a^3}{2}+3\,b\,x\,a^2}{x^2} + \frac{c^3\,x^4}{4} + b\,c^2\,x^3 + \frac{3\,c\,x^2\,\left(b^2+a\,c\right)}{2}$$

input
```
int((a + b*x + c*x^2)^3/x^3,x)
```

output
```
x*(b^3 + 6*a*b*c) + log(x)*(3*a*b^2 + 3*a^2*c) - (a^3/2 + 3*a^2*b*x)/x^2 +
 (c^3*x^4)/4 + b*c^2*x^3 + (3*c*x^2*(a*c + b^2))/2
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx+cx^2)^3}{x^3}\,dx$$
$$= \frac{12\log(x)\,a^2c\,x^2+12\log(x)\,a\,b^2x^2-2a^3-12a^2bx+24abc\,x^3+6a\,c^2x^4+4b^3x^3+6b^2c\,x^4+4b\,c^2x^5+c^3}{4x^2}$$

input
```
int((c*x^2+b*x+a)^3/x^3,x)
```

output
```
(12*log(x)*a**2*c*x**2 + 12*log(x)*a*b**2*x**2 - 2*a**3 - 12*a**2*b*x + 24
*a*b*c*x**3 + 6*a*c**2*x**4 + 4*b**3*x**3 + 6*b**2*c*x**4 + 4*b*c**2*x**5
+ c**3*x**6)/(4*x**2)
```

3.207 $\int \frac{(a+bx+cx^2)^3}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 79

$$\int \frac{(a+bx+cx^2)^3}{x^4}\,dx = -\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3a(b^2+ac)}{x}+3c(b^2+ac)\,x+\frac{3}{2}bc^2x^2+\frac{c^3x^3}{3}+b(b^2+6ac)\log(x)$$

output
```
-1/3*a^3/x^3-3/2*a^2*b/x^2-3*a*(a*c+b^2)/x+3*c*(a*c+b^2)*x+3/2*b*c^2*x^2+1
/3*c^3*x^3+b*(6*a*c+b^2)*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx+cx^2)^3}{x^4}\,dx = -\frac{(a-cx^2)\,(2a^2+ax(9b+20cx)+x^2(18b^2+9bcx+2c^2x^2))}{6x^3}+(b^3+6abc)\log(x)$$

input
```
Integrate[(a + b*x + c*x^2)^3/x^4,x]
```

output
```
-1/6*((a - c*x^2)*(2*a^2 + a*x*(9*b + 20*c*x) + x^2*(18*b^2 + 9*b*c*x + 2*
c^2*x^2)))/x^3 + (b^3 + 6*a*b*c)*Log[x]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^3}{x^4}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{a^3}{x^4}+\frac{3a^2b}{x^3}+\frac{6abc+b^3}{x}+\frac{3a(ac+b^2)}{x^2}+3c(ac+b^2)+3bc^2x+c^3x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3a(ac+b^2)}{x}+3cx(ac+b^2)+b\log(x)\left(6ac+b^2\right)+\frac{3}{2}bc^2x^2+\frac{c^3x^3}{3}$$

input
```
Int[(a + b*x + c*x^2)^3/x^4,x]
```

output
```
-1/3*a^3/x^3 - (3*a^2*b)/(2*x^2) - (3*a*(b^2 + a*c))/x + 3*c*(b^2 + a*c)*x
 + (3*b*c^2*x^2)/2 + (c^3*x^3)/3 + b*(b^2 + 6*a*c)*Log[x]
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{c^3x^3}{3}+\frac{3b\,c^2x^2}{2}+3a\,c^2x+3b^2cx-\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}+b(6ac+b^2)\ln(x)-\frac{3a(ac+b^2)}{x}$	75
risch	$\frac{c^3x^3}{3}+\frac{3b\,c^2x^2}{2}+3a\,c^2x+3b^2cx+\frac{(-3a^2c-3a\,b^2)x^2-\frac{3a^2bx}{2}-\frac{a^3}{3}}{x^3}+6abc\ln(x)+\ln(x)\,b^3$	80
norman	$\frac{(3a\,c^2+3b^2c)x^4+(-3a^2c-3a\,b^2)x^2-\frac{a^3}{3}+\frac{c^3x^6}{3}-\frac{3a^2bx}{2}+\frac{3b\,c^2x^5}{2}}{x^3}+(6abc+b^3)\ln(x)$	82
parallelrisch	$\frac{2c^3x^6+9b\,c^2x^5+36\ln(x)x^3abc+6\ln(x)x^3b^3+18a\,c^2x^4+18b^2c\,x^4-18a^2c\,x^2-18a\,b^2x^2-9a^2bx-2a^3}{6x^3}$	92

input
```
int((c*x^2+b*x+a)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*c^3*x^3+3/2*b*c^2*x^2+3*a*c^2*x+3*b^2*c*x-1/3*a^3/x^3-3/2*a^2*b/x^2+b*
(6*a*c+b^2)*ln(x)-3*a*(a*c+b^2)/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.05

$$\int\frac{(a+bx+cx^2)^3}{x^4}\,dx = \frac{2\,c^3x^6+9\,bc^2x^5+18\,(b^2c+ac^2)x^4+6\,(b^3+6\,abc)x^3\log(x)-9\,a^2bx-2\,a^3-18\,(ab^2+a^2c)x^2}{6\,x^3}$$

input
```
integrate((c*x^2+b*x+a)^3/x^4,x, algorithm="fricas")
```

output
```
1/6*(2*c^3*x^6 + 9*b*c^2*x^5 + 18*(b^2*c + a*c^2)*x^4 + 6*(b^3 + 6*a*b*c)*
x^3*log(x) - 9*a^2*b*x - 2*a^3 - 18*(a*b^2 + a^2*c)*x^2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx+cx^2)^3}{x^4}\,dx = \frac{3bc^2x^2}{2} + b(6ac+b^2)\log(x) + \frac{c^3x^3}{3} + x(3ac^2+3b^2c) + \frac{-2a^3-9a^2bx+x^2(-18a^2c-18ab^2)}{6x^3}$$

input
```
integrate((c*x**2+b*x+a)**3/x**4,x)
```

output
```
3*b*c**2*x**2/2 + b*(6*a*c + b**2)*log(x) + c**3*x**3/3 + x*(3*a*c**2 + 3*
b**2*c) + (-2*a**3 - 9*a**2*b*x + x**2*(-18*a**2*c - 18*a*b**2))/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx+cx^2)^3}{x^4}\,dx = \frac{1}{3}c^3x^3 + \frac{3}{2}bc^2x^2 + 3(b^2c+ac^2)x + (b^3+6abc)\log(x) - \frac{9a^2bx+2a^3+18(ab^2+a^2c)x^2}{6x^3}$$

input
```
integrate((c*x^2+b*x+a)^3/x^4,x, algorithm="maxima")
```

output
```
1/3*c^3*x^3 + 3/2*b*c^2*x^2 + 3*(b^2*c + a*c^2)*x + (b^3 + 6*a*b*c)*log(x)
 - 1/6*(9*a^2*b*x + 2*a^3 + 18*(a*b^2 + a^2*c)*x^2)/x^3
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2)^3}{x^4}\,dx = \frac{1}{3}\,c^3x^3 + \frac{3}{2}\,bc^2x^2 + 3\,b^2cx + 3\,ac^2x + \left(b^3 + 6\,abc\right)\log\left(|x|\right) - \frac{9\,a^2bx + 2\,a^3 + 18\,(ab^2 + a^2c)x^2}{6\,x^3}$$

input
```
integrate((c*x^2+b*x+a)^3/x^4,x, algorithm="giac")
```

output
```
1/3*c^3*x^3 + 3/2*b*c^2*x^2 + 3*b^2*c*x + 3*a*c^2*x + (b^3 + 6*a*b*c)*log(
abs(x)) - 1/6*(9*a^2*b*x + 2*a^3 + 18*(a*b^2 + a^2*c)*x^2)/x^3
```

Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx+cx^2)^3}{x^4}\,dx = \ln\left(x\right)\,\left(b^3 + 6\,a\,c\,b\right) - \frac{x^2\,\left(3\,c\,a^2 + 3\,a\,b^2\right) + \frac{a^3}{3} + \frac{3\,a^2\,b\,x}{2}}{x^3} + \frac{c^3\,x^3}{3} + \frac{3\,b\,c^2\,x^2}{2} + 3\,c\,x\,\left(b^2 + a\,c\right)$$

input
```
int((a + b*x + c*x^2)^3/x^4,x)
```

output
```
log(x)*(b^3 + 6*a*b*c) - (x^2*(3*a*b^2 + 3*a^2*c) + a^3/3 + (3*a^2*b*x)/2)
/x^3 + (c^3*x^3)/3 + (3*b*c^2*x^2)/2 + 3*c*x*(a*c + b^2)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx+cx^2)^3}{x^4}\,dx$$
$$= \frac{36\log(x)\,abc\,x^3+6\log(x)\,b^3x^3-2a^3-9a^2bx-18a^2c\,x^2-18a\,b^2x^2+18a\,c^2x^4+18b^2c\,x^4+9b\,c^2x^5+2}{6x^3}$$

input
```
int((c*x^2+b*x+a)^3/x^4,x)
```

output
```
(36*log(x)*a*b*c*x**3 + 6*log(x)*b**3*x**3 - 2*a**3 - 9*a**2*b*x - 18*a**2
*c*x**2 - 18*a*b**2*x**2 + 18*a*c**2*x**4 + 18*b**2*c*x**4 + 9*b*c**2*x**5
 + 2*c**3*x**6)/(6*x**3)
```

3.208 $\int \frac{(a+bx+cx^2)^3}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 78

$$\int \frac{(a+bx+cx^2)^3}{x^5}\,dx = -\frac{a^3}{4x^4}-\frac{a^2b}{x^3}-\frac{3a(b^2+ac)}{2x^2}-\frac{b(b^2+6ac)}{x} + 3bc^2x+\frac{c^3x^2}{2}+3c(b^2+ac)\log(x)$$

output
```
-1/4*a^3/x^4-a^2*b/x^3-3/2*a*(a*c+b^2)/x^2-b*(6*a*c+b^2)/x+3*b*c^2*x+1/2*c
^3*x^2+3*c*(a*c+b^2)*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2)^3}{x^5}\,dx = -\frac{a^3+4b^3x^3-12bc^2x^5-2c^3x^6+2a^2x(2b+3cx)+6abx^2(b+4cx)-12c(b^2+ac)\,x^4\log(x)}{4x^4}$$

input
```
Integrate[(a + b*x + c*x^2)^3/x^5,x]
```

output
```
-1/4*(a^3 + 4*b^3*x^3 - 12*b*c^2*x^5 - 2*c^3*x^6 + 2*a^2*x*(2*b + 3*c*x) +
 6*a*b*x^2*(b + 4*c*x) - 12*c*(b^2 + a*c)*x^4*Log[x])/x^4
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^3}{x^5}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{a^3}{x^5}+\frac{3a^2b}{x^4}+\frac{6abc+b^3}{x^2}+\frac{3a(ac+b^2)}{x^3}+\frac{3c(ac+b^2)}{x}+3bc^2+c^3x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3}{4x^4}-\frac{a^2b}{x^3}-\frac{3a(ac+b^2)}{2x^2}-\frac{b(6ac+b^2)}{x}+3c\log(x)\left(ac+b^2\right)+3bc^2x+\frac{c^3x^2}{2}$$

input
```
Int[(a + b*x + c*x^2)^3/x^5,x]
```

output
```
-1/4*a^3/x^4 - (a^2*b)/x^3 - (3*a*(b^2 + a*c))/(2*x^2) - (b*(b^2 + 6*a*c))
/x + 3*b*c^2*x + (c^3*x^2)/2 + 3*c*(b^2 + a*c)*Log[x]
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{a^3}{4x^4}-\frac{a^2b}{x^3}-\frac{3a(ac+b^2)}{2x^2}-\frac{b(6ac+b^2)}{x}+3c^2bx+\frac{c^3x^2}{2}+3c(ac+b^2)\ln(x)$	73
risch	$\frac{c^3x^2}{2}+3c^2bx+\frac{(-6abc-b^3)x^3+(-\frac{3}{2}a^2c-\frac{3}{2}a\,b^2)x^2-a^2bx-\frac{a^3}{4}}{x^4}+3a\,c^2\ln(x)+3\ln(x)\,b^2c$	82
norman	$\frac{(-\frac{3}{2}a^2c-\frac{3}{2}a\,b^2)x^2+(-6abc-b^3)x^3-\frac{a^3}{4}+\frac{c^3x^6}{2}-a^2bx+3b\,c^2x^5}{x^4}+(3a\,c^2+3b^2c)\ln(x)$	84
parallelrisch	$\frac{2c^3x^6+12\ln(x)x^4a\,c^2+12\ln(x)x^4b^2c+12b\,c^2x^5-24abc\,x^3-4b^3x^3-6a^2c\,x^2-6a\,b^2x^2-4a^2bx-a^3}{4x^4}$	92

input
```
int((c*x^2+b*x+a)^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^3/x^4-a^2*b/x^3-3/2*a*(a*c+b^2)/x^2-b*(6*a*c+b^2)/x+3*c^2*b*x+1/2*c
^3*x^2+3*c*(a*c+b^2)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06

$$\int\frac{(a+bx+cx^2)^3}{x^5}\,dx$$
$$=\frac{2\,c^3x^6+12\,bc^2x^5+12\,(b^2c+ac^2)x^4\log(x)-4\,a^2bx-4\,(b^3+6\,abc)x^3-a^3-6\,(ab^2+a^2c)x^2}{4\,x^4}$$

input
```
integrate((c*x^2+b*x+a)^3/x^5,x, algorithm="fricas")
```

output
```
1/4*(2*c^3*x^6 + 12*b*c^2*x^5 + 12*(b^2*c + a*c^2)*x^4*log(x) - 4*a^2*b*x
- 4*(b^3 + 6*a*b*c)*x^3 - a^3 - 6*(a*b^2 + a^2*c)*x^2)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx+cx^2)^3}{x^5}\,dx = 3bc^2x + \frac{c^3x^2}{2} + 3c\left(ac+b^2\right)\log(x) + \frac{-a^3 - 4a^2bx + x^3(-24abc - 4b^3) + x^2(-6a^2c - 6ab^2)}{4x^4}$$

input
```
integrate((c*x**2+b*x+a)**3/x**5,x)
```

output
```
3*b*c**2*x + c**3*x**2/2 + 3*c*(a*c + b**2)*log(x) + (-a**3 - 4*a**2*b*x +
 x**3*(-24*a*b*c - 4*b**3) + x**2*(-6*a**2*c - 6*a*b**2))/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx+cx^2)^3}{x^5}\,dx = \frac{1}{2}\,c^3x^2 + 3\,bc^2x + 3\,\left(b^2c + ac^2\right)\log(x) - \frac{4\,a^2bx + 4\,(b^3 + 6\,abc)x^3 + a^3 + 6\,(ab^2 + a^2c)x^2}{4\,x^4}$$

input
```
integrate((c*x^2+b*x+a)^3/x^5,x, algorithm="maxima")
```

output
```
1/2*c^3*x^2 + 3*b*c^2*x + 3*(b^2*c + a*c^2)*log(x) - 1/4*(4*a^2*b*x + 4*(b
^3 + 6*a*b*c)*x^3 + a^3 + 6*(a*b^2 + a^2*c)*x^2)/x^4
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2)^3}{x^5}\,dx = \frac{1}{2}\,c^3x^2+3\,bc^2x+3\left(b^2c+ac^2\right)\log\left(|x|\right) - \frac{4\,a^2bx+4\,(b^3+6\,abc)x^3+a^3+6\,(ab^2+a^2c)x^2}{4\,x^4}$$

input
```
integrate((c*x^2+b*x+a)^3/x^5,x, algorithm="giac")
```

output
```
1/2*c^3*x^2 + 3*b*c^2*x + 3*(b^2*c + a*c^2)*log(abs(x)) - 1/4*(4*a^2*b*x +
 4*(b^3 + 6*a*b*c)*x^3 + a^3 + 6*(a*b^2 + a^2*c)*x^2)/x^4
```

Mupad [B] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx+cx^2)^3}{x^5}\,dx = \ln(x)\,\left(3\,b^2\,c+3\,a\,c^2\right) - \frac{x^2\left(\frac{3\,c\,a^2}{2}+\frac{3\,a\,b^2}{2}\right)+\frac{a^3}{4}+x^3\,\left(b^3+6\,a\,c\,b\right)+a^2\,b\,x}{x^4} + \frac{c^3\,x^2}{2}+3\,b\,c^2\,x$$

input
```
int((a + b*x + c*x^2)^3/x^5,x)
```

output
```
log(x)*(3*a*c^2 + 3*b^2*c) - (x^2*((3*a*b^2)/2 + (3*a^2*c)/2) + a^3/4 + x^
3*(b^3 + 6*a*b*c) + a^2*b*x)/x^4 + (c^3*x^2)/2 + 3*b*c^2*x
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx+cx^2)^3}{x^5}\,dx = \frac{12\log(x)\,a\,c^2x^4+12\log(x)\,b^2c\,x^4-a^3-4a^2bx-6a^2c\,x^2-6a\,b^2x^2-24abc\,x^3-4b^3x^3+12b\,c^2x^5+2c^3}{4x^4}$$

input
```
int((c*x^2+b*x+a)^3/x^5,x)
```

output
```
(12*log(x)*a*c**2*x**4 + 12*log(x)*b**2*c*x**4 - a**3 - 4*a**2*b*x - 6*a**
2*c*x**2 - 6*a*b**2*x**2 - 24*a*b*c*x**3 - 4*b**3*x**3 + 12*b*c**2*x**5 +
2*c**3*x**6)/(4*x**4)
```

3.209 $\int \frac{(a+bx+cx^2)^3}{x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int \frac{(a+bx+cx^2)^3}{x^6}\,dx = -\frac{a^3}{5x^5} - \frac{3a^2b}{4x^4} - \frac{a(b^2+ac)}{x^3} - \frac{b(b^2+6ac)}{2x^2} - \frac{3c(b^2+ac)}{x} + c^3x + 3bc^2\log(x)$$

output
```
-1/5*a^3/x^5-3/4*a^2*b/x^4-a*(a*c+b^2)/x^3-1/2*b*(6*a*c+b^2)/x^2-3*c*(a*c+
b^2)/x+c^3*x+3*b*c^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx+cx^2)^3}{x^6}\,dx = -\frac{a^3}{5x^5} - \frac{3a^2b}{4x^4} + \frac{-ab^2-a^2c}{x^3} + \frac{-b^3-6abc}{2x^2} - \frac{3(b^2c+ac^2)}{x} + c^3x + 3bc^2\log(x)$$

input
```
Integrate[(a + b*x + c*x^2)^3/x^6,x]
```

output
```
-1/5*a^3/x^5 - (3*a^2*b)/(4*x^4) + (-(a*b^2) - a^2*c)/x^3 + (-b^3 - 6*a*b*
c)/(2*x^2) - (3*(b^2*c + a*c^2))/x + c^3*x + 3*b*c^2*Log[x]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^3}{x^6}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{a^3}{x^6}+\frac{3a^2b}{x^5}+\frac{6abc+b^3}{x^3}+\frac{3a(ac+b^2)}{x^4}+\frac{3c(ac+b^2)}{x^2}+\frac{3bc^2}{x}+c^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3}{5x^5}-\frac{3a^2b}{4x^4}-\frac{a(ac+b^2)}{x^3}-\frac{b(6ac+b^2)}{2x^2}-\frac{3c(ac+b^2)}{x}+3bc^2\log(x)+c^3x$$

input
```
Int[(a + b*x + c*x^2)^3/x^6,x]
```

output
```
-1/5*a^3/x^5 - (3*a^2*b)/(4*x^4) - (a*(b^2 + a*c))/x^3 - (b*(b^2 + 6*a*c))
/(2*x^2) - (3*c*(b^2 + a*c))/x + c^3*x + 3*b*c^2*Log[x]
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{a^{3}}{5x^{5}}-\frac{3a^{2}b}{4x^{4}}-\frac{a(ac+b^{2})}{x^{3}}-\frac{b(6ac+b^{2})}{2x^{2}}-\frac{3c(ac+b^{2})}{x}+c^{3}x+3b\,c^{2}\ln\left(x\right)$	72
risch	$c^{3}x+\frac{(-3a\,c^{2}-3b^{2}c)x^{4}+(-3abc-\frac{1}{2}b^{3})x^{3}+(-a^{2}c-a\,b^{2})x^{2}-\frac{3a^{2}bx}{4}-\frac{a^{3}}{5}}{x^{5}}+3b\,c^{2}\ln\left(x\right)$	81
norman	$\frac{c^{3}x^{6}+(-3abc-\frac{1}{2}b^{3})x^{3}+(-3a\,c^{2}-3b^{2}c)x^{4}+(-a^{2}c-a\,b^{2})x^{2}-\frac{a^{3}}{5}-\frac{3a^{2}bx}{4}}{x^{5}}+3b\,c^{2}\ln\left(x\right)$	83
parallelrisch	$\frac{60b\,c^{2}\ln(x)x^{5}+20c^{3}x^{6}-60a\,c^{2}x^{4}-60b^{2}c\,x^{4}-60abc\,x^{3}-10b^{3}x^{3}-20a^{2}c\,x^{2}-20a\,b^{2}x^{2}-15a^{2}bx-4a^{3}}{20x^{5}}$	90

input
```
int((c*x^2+b*x+a)^3/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^3/x^5-3/4*a^2*b/x^4-a*(a*c+b^2)/x^3-1/2*b*(6*a*c+b^2)/x^2-3*c*(a*c+
b^2)/x+c^3*x+3*b*c^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx+cx^2)^3}{x^6}\,dx = \frac{20\,c^3x^6+60\,bc^2x^5\log\left(x\right)-60\,(b^2c+ac^2)x^4-15\,a^2bx-10\,(b^3+6\,abc)x^3-4\,a^3-20\,(ab^2+a^2c)x^2}{20\,x^5}$$

input
```
integrate((c*x^2+b*x+a)^3/x^6,x, algorithm="fricas")
```

output
```
1/20*(20*c^3*x^6 + 60*b*c^2*x^5*log(x) - 60*(b^2*c + a*c^2)*x^4 - 15*a^2*b
*x - 10*(b^3 + 6*a*b*c)*x^3 - 4*a^3 - 20*(a*b^2 + a^2*c)*x^2)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx+cx^2)^3}{x^6}\,dx = 3bc^2\log{(x)} + c^3x + \frac{-4a^3 - 15a^2bx + x^4(-60ac^2 - 60b^2c) + x^3(-60abc - 10b^3) + x^2(-20a^2c - 20ab^2)}{20x^5}$$

input
```
integrate((c*x**2+b*x+a)**3/x**6,x)
```

output
```
3*b*c**2*log(x) + c**3*x + (-4*a**3 - 15*a**2*b*x + x**4*(-60*a*c**2 - 60*
b**2*c) + x**3*(-60*a*b*c - 10*b**3) + x**2*(-20*a**2*c - 20*a*b**2))/(20*
x**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx+cx^2)^3}{x^6}\,dx = c^3x + 3\,bc^2\log{(x)} - \frac{60\,(b^2c + ac^2)x^4 + 15\,a^2bx + 10\,(b^3 + 6\,abc)x^3 + 4\,a^3 + 20\,(ab^2 + a^2c)x^2}{20\,x^5}$$

input
```
integrate((c*x^2+b*x+a)^3/x^6,x, algorithm="maxima")
```

output
```
c^3*x + 3*b*c^2*log(x) - 1/20*(60*(b^2*c + a*c^2)*x^4 + 15*a^2*b*x + 10*(b
^3 + 6*a*b*c)*x^3 + 4*a^3 + 20*(a*b^2 + a^2*c)*x^2)/x^5
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx+cx^2)^3}{x^6}\,dx$$
$$= c^3x + 3\,bc^2\log(|x|) - \frac{60\,(b^2c+ac^2)x^4 + 15\,a^2bx + 10\,(b^3+6\,abc)x^3 + 4\,a^3 + 20\,(ab^2+a^2c)x^2}{20\,x^5}$$

input
```
integrate((c*x^2+b*x+a)^3/x^6,x, algorithm="giac")
```

output
```
c^3*x + 3*b*c^2*log(abs(x)) - 1/20*(60*(b^2*c + a*c^2)*x^4 + 15*a^2*b*x +
10*(b^3 + 6*a*b*c)*x^3 + 4*a^3 + 20*(a*b^2 + a^2*c)*x^2)/x^5
```

Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx+cx^2)^3}{x^6}\,dx$$
$$= c^3\,x - \frac{x^3\,\left(\frac{b^3}{2}+3\,a\,c\,b\right) + x^2\,(c\,a^2+a\,b^2) + x^4\,(3\,b^2\,c+3\,a\,c^2) + \frac{a^3}{5} + \frac{3\,a^2\,b\,x}{4}}{x^5} + 3\,b\,c^2\,\ln(x)$$

input
```
int((a + b*x + c*x^2)^3/x^6,x)
```

output
```
c^3*x - (x^3*(b^3/2 + 3*a*b*c) + x^2*(a*b^2 + a^2*c) + x^4*(3*a*c^2 + 3*b^
2*c) + a^3/5 + (3*a^2*b*x)/4)/x^5 + 3*b*c^2*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.16

$$\int \frac{(a + bx + cx^2)^3}{x^6} \, dx$$
$$= \frac{60 \log(x) \, b \, c^2 x^5 - 4a^3 - 15a^2 bx - 20a^2 c \, x^2 - 20a \, b^2 x^2 - 60abc \, x^3 - 60a \, c^2 x^4 - 10b^3 x^3 - 60b^2 c \, x^4 + 20c^3}{20x^5}$$

input
```
int((c*x^2+b*x+a)^3/x^6,x)
```

output
```
(60*log(x)*b*c**2*x**5 - 4*a**3 - 15*a**2*b*x - 20*a**2*c*x**2 - 20*a*b**2
*x**2 - 60*a*b*c*x**3 - 60*a*c**2*x**4 - 10*b**3*x**3 - 60*b**2*c*x**4 + 2
0*c**3*x**6)/(20*x**5)
```

3.210 $\int \frac{(a+bx+cx^2)^3}{x^7} dx$

Optimal result

Integrand size = 16, antiderivative size = 83

$$\int \frac{(a+bx+cx^2)^3}{x^7} dx = -\frac{a^3}{6x^6} - \frac{3a^2b}{5x^5} - \frac{3a(b^2+ac)}{4x^4} - \frac{b(b^2+6ac)}{3x^3} - \frac{3c(b^2+ac)}{2x^2} - \frac{3bc^2}{x} + c^3\log(x)$$

output
```
-1/6*a^3/x^6-3/5*a^2*b/x^5-3/4*a*(a*c+b^2)/x^4-1/3*b*(6*a*c+b^2)/x^3-3/2*c
*(a*c+b^2)/x^2-3*b*c^2/x+c^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx+cx^2)^3}{x^7} dx = -\frac{10a^3 + 9a^2x(4b+5cx) + 15ax^2(3b^2+8bcx+6c^2x^2) + 10bx^3(2b^2+9bcx+18c^2x^2) - 60c^3x^6\log(x)}{60x^6}$$

input
```
Integrate[(a + b*x + c*x^2)^3/x^7,x]
```

output
```
-1/60*(10*a^3 + 9*a^2*x*(4*b + 5*c*x) + 15*a*x^2*(3*b^2 + 8*b*c*x + 6*c^2*
x^2) + 10*b*x^3*(2*b^2 + 9*b*c*x + 18*c^2*x^2) - 60*c^3*x^6*Log[x])/x^6
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^3}{x^7}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{a^3}{x^7}+\frac{3a^2b}{x^6}+\frac{6abc+b^3}{x^4}+\frac{3a(ac+b^2)}{x^5}+\frac{3c(ac+b^2)}{x^3}+\frac{3bc^2}{x^2}+\frac{c^3}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3}{6x^6}-\frac{3a^2b}{5x^5}-\frac{3a(ac+b^2)}{4x^4}-\frac{b(6ac+b^2)}{3x^3}-\frac{3c(ac+b^2)}{2x^2}-\frac{3bc^2}{x}+c^3\log(x)$$

input
```
Int[(a + b*x + c*x^2)^3/x^7,x]
```

output
```
-1/6*a^3/x^6 - (3*a^2*b)/(5*x^5) - (3*a*(b^2 + a*c))/(4*x^4) - (b*(b^2 + 6
*a*c))/(3*x^3) - (3*c*(b^2 + a*c))/(2*x^2) - (3*b*c^2)/x + c^3*Log[x]
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a^3}{6x^6}-\frac{3a^2b}{5x^5}-\frac{3a(ac+b^2)}{4x^4}-\frac{b(6ac+b^2)}{3x^3}-\frac{3c(ac+b^2)}{2x^2}-\frac{3b\,c^2}{x}+c^3\ln(x)$	74
norman	$\frac{\left(-\frac{3}{2}a\,c^2-\frac{3}{2}b^2c\right)x^4+\left(-\frac{3}{4}a^2c-\frac{3}{4}a\,b^2\right)x^2+\left(-2abc-\frac{1}{3}b^3\right)x^3-\frac{a^3}{6}-\frac{3a^2bx}{5}-3b\,c^2x^5}{x^6}+c^3\ln(x)$	83
risch	$\frac{\left(-\frac{3}{2}a\,c^2-\frac{3}{2}b^2c\right)x^4+\left(-\frac{3}{4}a^2c-\frac{3}{4}a\,b^2\right)x^2+\left(-2abc-\frac{1}{3}b^3\right)x^3-\frac{a^3}{6}-\frac{3a^2bx}{5}-3b\,c^2x^5}{x^6}+c^3\ln(x)$	83
parallelrisch	$\frac{60c^3\ln(x)x^6-180b\,c^2x^5-90a\,c^2x^4-90b^2c\,x^4-120abc\,x^3-20b^3x^3-45a^2c\,x^2-45a\,b^2x^2-36a^2bx-10a^3}{60x^6}$	90

input
```
int((c*x^2+b*x+a)^3/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^3/x^6-3/5*a^2*b/x^5-3/4*a*(a*c+b^2)/x^4-1/3*b*(6*a*c+b^2)/x^3-3/2*c
*(a*c+b^2)/x^2-3*b*c^2/x+c^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2)^3}{x^7}\,dx = \frac{60\,c^3x^6\log(x)-180\,bc^2x^5-90\,(b^2c+ac^2)x^4-36\,a^2bx-20\,(b^3+6\,abc)x^3-10\,a^3-45\,(ab^2+a^2c)x^2}{60\,x^6}$$

input
```
integrate((c*x^2+b*x+a)^3/x^7,x, algorithm="fricas")
```

output
```
1/60*(60*c^3*x^6*log(x) - 180*b*c^2*x^5 - 90*(b^2*c + a*c^2)*x^4 - 36*a^2*
b*x - 20*(b^3 + 6*a*b*c)*x^3 - 10*a^3 - 45*(a*b^2 + a^2*c)*x^2)/x^6
```

Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx+cx^2)^3}{x^7}\,dx = c^3\log(x) + \frac{-10a^3-36a^2bx-180bc^2x^5+x^4(-90ac^2-90b^2c)+x^3(-120abc-20b^3)+x^2(-45a^2c-45ab^2)}{60x^6}$$

input
```
integrate((c*x**2+b*x+a)**3/x**7,x)
```

output
```
c**3*log(x) + (-10*a**3 - 36*a**2*b*x - 180*b*c**2*x**5 + x**4*(-90*a*c**2
 - 90*b**2*c) + x**3*(-120*a*b*c - 20*b**3) + x**2*(-45*a**2*c - 45*a*b**2
))/(60*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx+cx^2)^3}{x^7}\,dx = c^3\log(x) - \frac{180\,bc^2x^5+90\,(b^2c+ac^2)x^4+36\,a^2bx+20\,(b^3+6\,abc)x^3+10\,a^3+45\,(ab^2+a^2c)x^2}{60\,x^6}$$

input
```
integrate((c*x^2+b*x+a)^3/x^7,x, algorithm="maxima")
```

output
```
c^3*log(x) - 1/60*(180*b*c^2*x^5 + 90*(b^2*c + a*c^2)*x^4 + 36*a^2*b*x + 2
0*(b^3 + 6*a*b*c)*x^3 + 10*a^3 + 45*(a*b^2 + a^2*c)*x^2)/x^6
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx+cx^2)^3}{x^7}\,dx = c^3\log(|x|) - \frac{180\,bc^2x^5 + 90\,(b^2c+ac^2)x^4 + 36\,a^2bx + 20\,(b^3+6\,abc)x^3 + 10\,a^3 + 45\,(ab^2+a^2c)x^2}{60\,x^6}$$

input
```
integrate((c*x^2+b*x+a)^3/x^7,x, algorithm="giac")
```

output
```
c^3*log(abs(x)) - 1/60*(180*b*c^2*x^5 + 90*(b^2*c + a*c^2)*x^4 + 36*a^2*b*
x + 20*(b^3 + 6*a*b*c)*x^3 + 10*a^3 + 45*(a*b^2 + a^2*c)*x^2)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2)^3}{x^7}\,dx = c^3\,\ln(x) - \frac{x^3\left(\frac{b^3}{3}+2\,a\,c\,b\right)+x^2\left(\frac{3\,c\,a^2}{4}+\frac{3\,a\,b^2}{4}\right)+x^4\left(\frac{3\,b^2\,c}{2}+\frac{3\,a\,c^2}{2}\right)+\frac{a^3}{6}+3\,b\,c^2\,x^5+\frac{3\,a^2\,b\,x}{5}}{x^6}$$

input
```
int((a + b*x + c*x^2)^3/x^7,x)
```

output
```
c^3*log(x) - (x^3*(b^3/3 + 2*a*b*c) + x^2*((3*a*b^2)/4 + (3*a^2*c)/4) + x^
4*((3*a*c^2)/2 + (3*b^2*c)/2) + a^3/6 + 3*b*c^2*x^5 + (3*a^2*b*x)/5)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx+cx^2)^3}{x^7}\,dx$$
$$= \frac{60\log(x)\,c^3x^6 - 10a^3 - 36a^2bx - 45a^2c\,x^2 - 45a\,b^2x^2 - 120abc\,x^3 - 90a\,c^2x^4 - 20b^3x^3 - 90b^2c\,x^4 - 180}{60x^6}$$

input
```
int((c*x^2+b*x+a)^3/x^7,x)
```

output
```
(60*log(x)*c**3*x**6 - 10*a**3 - 36*a**2*b*x - 45*a**2*c*x**2 - 45*a*b**2*
x**2 - 120*a*b*c*x**3 - 90*a*c**2*x**4 - 20*b**3*x**3 - 90*b**2*c*x**4 - 1
80*b*c**2*x**5)/(60*x**6)
```

3.211 $\int \frac{(a+bx+cx^2)^3}{x^8} dx$

Optimal result

Integrand size = 16, antiderivative size = 85

$$\int \frac{(a+bx+cx^2)^3}{x^8} dx = -\frac{a^3}{7x^7} - \frac{a^2b}{2x^6} - \frac{3a(b^2+ac)}{5x^5} - \frac{b(b^2+6ac)}{4x^4} - \frac{c(b^2+ac)}{x^3} - \frac{3bc^2}{2x^2} - \frac{c^3}{x}$$

output
```
-1/7*a^3/x^7-1/2*a^2*b/x^6-3/5*a*(a*c+b^2)/x^5-1/4*b*(6*a*c+b^2)/x^4-c*(a*
c+b^2)/x^3-3/2*b*c^2/x^2-c^3/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2)^3}{x^8} dx = -\frac{20a^3 + 14a^2x(5b+6cx) + 14ax^2(6b^2+15bcx+10c^2x^2) + 35x^3(b^3+4b^2cx+6bc^2x^2+4c^3x^3)}{140x^7}$$

input
```
Integrate[(a + b*x + c*x^2)^3/x^8,x]
```

output
```
-1/140*(20*a^3 + 14*a^2*x*(5*b + 6*c*x) + 14*a*x^2*(6*b^2 + 15*b*c*x + 10*
c^2*x^2) + 35*x^3*(b^3 + 4*b^2*c*x + 6*b*c^2*x^2 + 4*c^3*x^3))/x^7
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^3}{x^8}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{a^3}{x^8}+\frac{3a^2b}{x^7}+\frac{6abc+b^3}{x^5}+\frac{3a(ac+b^2)}{x^6}+\frac{3c(ac+b^2)}{x^4}+\frac{3bc^2}{x^3}+\frac{c^3}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3}{7x^7}-\frac{a^2b}{2x^6}-\frac{3a(ac+b^2)}{5x^5}-\frac{b(6ac+b^2)}{4x^4}-\frac{c(ac+b^2)}{x^3}-\frac{3bc^2}{2x^2}-\frac{c^3}{x}$$

input
```
Int[(a + b*x + c*x^2)^3/x^8,x]
```

output
```
-1/7*a^3/x^7 - (a^2*b)/(2*x^6) - (3*a*(b^2 + a*c))/(5*x^5) - (b*(b^2 + 6*a
*c))/(4*x^4) - (c*(b^2 + a*c))/x^3 - (3*b*c^2)/(2*x^2) - c^3/x
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a^{3}}{7x^{7}}-\frac{a^{2}b}{2x^{6}}-\frac{3a\left(ac+b^{2}\right)}{5x^{5}}-\frac{b\left(6ac+b^{2}\right)}{4x^{4}}-\frac{c\left(ac+b^{2}\right)}{x^{3}}-\frac{3b\,c^{2}}{2x^{2}}-\frac{c^{3}}{x}$	76
norman	$\frac{-c^{3}x^{6}-\frac{3b\,c^{2}x^{5}}{2}+\left(-a\,c^{2}-b^{2}c\right)x^{4}+\left(-\frac{3}{2}abc-\frac{1}{4}b^{3}\right)x^{3}+\left(-\frac{3}{5}a^{2}c-\frac{3}{5}a\,b^{2}\right)x^{2}-\frac{a^{2}bx}{2}-\frac{a^{3}}{7}}{x^{7}}$	84
risch	$\frac{-c^{3}x^{6}-\frac{3b\,c^{2}x^{5}}{2}+\left(-a\,c^{2}-b^{2}c\right)x^{4}+\left(-\frac{3}{2}abc-\frac{1}{4}b^{3}\right)x^{3}+\left(-\frac{3}{5}a^{2}c-\frac{3}{5}a\,b^{2}\right)x^{2}-\frac{a^{2}bx}{2}-\frac{a^{3}}{7}}{x^{7}}$	84
gosper	$-\frac{140c^{3}x^{6}+210b\,c^{2}x^{5}+140a\,c^{2}x^{4}+140b^{2}c\,x^{4}+210abc\,x^{3}+35b^{3}x^{3}+84a^{2}c\,x^{2}+84a\,b^{2}x^{2}+70a^{2}bx+20a^{3}}{140x^{7}}$	88
parallelrisch	$\frac{-140c^{3}x^{6}-210b\,c^{2}x^{5}-140a\,c^{2}x^{4}-140b^{2}c\,x^{4}-210abc\,x^{3}-35b^{3}x^{3}-84a^{2}c\,x^{2}-84a\,b^{2}x^{2}-70a^{2}bx-20a^{3}}{140x^{7}}$	88
orering	$-\frac{140c^{3}x^{6}+210b\,c^{2}x^{5}+140a\,c^{2}x^{4}+140b^{2}c\,x^{4}+210abc\,x^{3}+35b^{3}x^{3}+84a^{2}c\,x^{2}+84a\,b^{2}x^{2}+70a^{2}bx+20a^{3}}{140x^{7}}$	88

input
```
int((c*x^2+b*x+a)^3/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^3/x^7-1/2*a^2*b/x^6-3/5*a*(a*c+b^2)/x^5-1/4*b*(6*a*c+b^2)/x^4-c*(a*
c+b^2)/x^3-3/2*b*c^2/x^2-c^3/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx+cx^2)^3}{x^8}\,dx = -\frac{140\,c^3x^6+210\,bc^2x^5+140\,(b^2c+ac^2)x^4+70\,a^2bx+35\,(b^3+6\,abc)x^3+20\,a^3+84\,(ab^2+a^2c)x^2}{140\,x^7}$$

input
```
integrate((c*x^2+b*x+a)^3/x^8,x, algorithm="fricas")
```

output
```
-1/140*(140*c^3*x^6 + 210*b*c^2*x^5 + 140*(b^2*c + a*c^2)*x^4 + 70*a^2*b*x
 + 35*(b^3 + 6*a*b*c)*x^3 + 20*a^3 + 84*(a*b^2 + a^2*c)*x^2)/x^7
```

Sympy [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx+cx^2)^3}{x^8}\,dx$$
$$= \frac{-20a^3-70a^2bx-210bc^2x^5-140c^3x^6+x^4(-140ac^2-140b^2c)+x^3(-210abc-35b^3)+x^2(-84a^2c-}{140x^7}$$

input
```
integrate((c*x**2+b*x+a)**3/x**8,x)
```

output
```
(-20*a**3 - 70*a**2*b*x - 210*b*c**2*x**5 - 140*c**3*x**6 + x**4*(-140*a*c
**2 - 140*b**2*c) + x**3*(-210*a*b*c - 35*b**3) + x**2*(-84*a**2*c - 84*a*
b**2))/(140*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx+cx^2)^3}{x^8}\,dx =$$
$$-\frac{140\,c^3x^6+210\,bc^2x^5+140\,(b^2c+ac^2)x^4+70\,a^2bx+35\,(b^3+6\,abc)x^3+20\,a^3+84\,(ab^2+a^2c)x^2}{140\,x^7}$$

input
```
integrate((c*x^2+b*x+a)^3/x^8,x, algorithm="maxima")
```

output
```
-1/140*(140*c^3*x^6 + 210*b*c^2*x^5 + 140*(b^2*c + a*c^2)*x^4 + 70*a^2*b*x
 + 35*(b^3 + 6*a*b*c)*x^3 + 20*a^3 + 84*(a*b^2 + a^2*c)*x^2)/x^7
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx+cx^2)^3}{x^8}\,dx = -\frac{140\,c^3x^6+210\,bc^2x^5+140\,b^2cx^4+140\,ac^2x^4+35\,b^3x^3+210\,abcx^3+84\,ab^2x^2+84\,a^2cx^2+70\,a^2bx+}{140\,x^7}$$

input
```
integrate((c*x^2+b*x+a)^3/x^8,x, algorithm="giac")
```

output
```
-1/140*(140*c^3*x^6 + 210*b*c^2*x^5 + 140*b^2*c*x^4 + 140*a*c^2*x^4 + 35*b
^3*x^3 + 210*a*b*c*x^3 + 84*a*b^2*x^2 + 84*a^2*c*x^2 + 70*a^2*b*x + 20*a^3
)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx+cx^2)^3}{x^8}\,dx = -\frac{x^3\left(\frac{b^3}{4}+\frac{3\,a\,c\,b}{2}\right)+x^2\left(\frac{3\,c\,a^2}{5}+\frac{3\,a\,b^2}{5}\right)+x^4\left(b^2\,c+a\,c^2\right)+\frac{a^3}{7}+c^3\,x^6+\frac{3\,b\,c^2\,x^5}{2}+\frac{a^2\,b\,x}{2}}{x^7}$$

input
```
int((a + b*x + c*x^2)^3/x^8,x)
```

output
```
-(x^3*(b^3/4 + (3*a*b*c)/2) + x^2*((3*a*b^2)/5 + (3*a^2*c)/5) + x^4*(a*c^2
 + b^2*c) + a^3/7 + c^3*x^6 + (3*b*c^2*x^5)/2 + (a^2*b*x)/2)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx+cx^2)^3}{x^8}\,dx$$
$$= \frac{-140c^3x^6 - 210b\,c^2x^5 - 140a\,c^2x^4 - 140b^2c\,x^4 - 210abc\,x^3 - 35b^3x^3 - 84a^2c\,x^2 - 84a\,b^2x^2 - 70a^2bx -}{140x^7}$$

input
```
int((c*x^2+b*x+a)^3/x^8,x)
```

output
```
( - 20*a**3 - 70*a**2*b*x - 84*a**2*c*x**2 - 84*a*b**2*x**2 - 210*a*b*c*x*
*3 - 140*a*c**2*x**4 - 35*b**3*x**3 - 140*b**2*c*x**4 - 210*b*c**2*x**5 -
140*c**3*x**6)/(140*x**7)
```

3.212 $\int \frac{(a+bx+cx^2)^3}{x^9} dx$

Optimal result

Integrand size = 16, antiderivative size = 87

$$\int \frac{(a+bx+cx^2)^3}{x^9} dx = -\frac{a^3}{8x^8} - \frac{3a^2b}{7x^7} - \frac{a(b^2+ac)}{2x^6} - \frac{b(b^2+6ac)}{5x^5} - \frac{3c(b^2+ac)}{4x^4} - \frac{bc^2}{x^3} - \frac{c^3}{2x^2}$$

output
```
-1/8*a^3/x^8-3/7*a^2*b/x^7-1/2*a*(a*c+b^2)/x^6-1/5*b*(6*a*c+b^2)/x^5-3/4*c
*(a*c+b^2)/x^4-b*c^2/x^3-1/2*c^3/x^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2)^3}{x^9} dx = -\frac{35a^3 + 20a^2x(6b+7cx) + 14ax^2(10b^2+24bcx+15c^2x^2) + 14x^3(4b^3+15b^2cx+20bc^2x^2+10c^3x^3)}{280x^8}$$

input
```
Integrate[(a + b*x + c*x^2)^3/x^9,x]
```

output
```
-1/280*(35*a^3 + 20*a^2*x*(6*b + 7*c*x) + 14*a*x^2*(10*b^2 + 24*b*c*x + 15
*c^2*x^2) + 14*x^3*(4*b^3 + 15*b^2*c*x + 20*b*c^2*x^2 + 10*c^3*x^3))/x^8
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^3}{x^9}\,dx$$

$$\Big\downarrow \; 1140$$

$$\int \left(\frac{a^3}{x^9}+\frac{3a^2b}{x^8}+\frac{6abc+b^3}{x^6}+\frac{3a(ac+b^2)}{x^7}+\frac{3c(ac+b^2)}{x^5}+\frac{3bc^2}{x^4}+\frac{c^3}{x^3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^3}{8x^8}-\frac{3a^2b}{7x^7}-\frac{a(ac+b^2)}{2x^6}-\frac{b(6ac+b^2)}{5x^5}-\frac{3c(ac+b^2)}{4x^4}-\frac{bc^2}{x^3}-\frac{c^3}{2x^2}$$

input
```
Int[(a + b*x + c*x^2)^3/x^9,x]
```

output
```
-1/8*a^3/x^8 - (3*a^2*b)/(7*x^7) - (a*(b^2 + a*c))/(2*x^6) - (b*(b^2 + 6*a
*c))/(5*x^5) - (3*c*(b^2 + a*c))/(4*x^4) - (b*c^2)/x^3 - c^3/(2*x^2)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{a^{3}}{8x^{8}}-\frac{3a^{2}b}{7x^{7}}-\frac{a\left(ac+b^{2}\right)}{2x^{6}}-\frac{b\left(6ac+b^{2}\right)}{5x^{5}}-\frac{3c\left(ac+b^{2}\right)}{4x^{4}}-\frac{b\,c^{2}}{x^{3}}-\frac{c^{3}}{2x^{2}}$	76
norman	$\frac{-\frac{c^{3}x^{6}}{2}-b\,c^{2}x^{5}+\left(-\frac{3}{4}a\,c^{2}-\frac{3}{4}b^{2}c\right)x^{4}+\left(-\frac{6}{5}abc-\frac{1}{5}b^{3}\right)x^{3}+\left(-\frac{1}{2}a^{2}c-\frac{1}{2}a\,b^{2}\right)x^{2}-\frac{3a^{2}bx}{7}-\frac{a^{3}}{8}}{x^{8}}$	84
risch	$\frac{-\frac{c^{3}x^{6}}{2}-b\,c^{2}x^{5}+\left(-\frac{3}{4}a\,c^{2}-\frac{3}{4}b^{2}c\right)x^{4}+\left(-\frac{6}{5}abc-\frac{1}{5}b^{3}\right)x^{3}+\left(-\frac{1}{2}a^{2}c-\frac{1}{2}a\,b^{2}\right)x^{2}-\frac{3a^{2}bx}{7}-\frac{a^{3}}{8}}{x^{8}}$	84
gosper	$-\frac{140c^{3}x^{6}+280b\,c^{2}x^{5}+210a\,c^{2}x^{4}+210b^{2}c\,x^{4}+336abc\,x^{3}+56b^{3}x^{3}+140a^{2}c\,x^{2}+140a\,b^{2}x^{2}+120a^{2}bx+35a^{3}}{280x^{8}}$	88
parallelrisch	$\frac{-140c^{3}x^{6}-280b\,c^{2}x^{5}-210a\,c^{2}x^{4}-210b^{2}c\,x^{4}-336abc\,x^{3}-56b^{3}x^{3}-140a^{2}c\,x^{2}-140a\,b^{2}x^{2}-120a^{2}bx-35a^{3}}{280x^{8}}$	88
orering	$-\frac{140c^{3}x^{6}+280b\,c^{2}x^{5}+210a\,c^{2}x^{4}+210b^{2}c\,x^{4}+336abc\,x^{3}+56b^{3}x^{3}+140a^{2}c\,x^{2}+140a\,b^{2}x^{2}+120a^{2}bx+35a^{3}}{280x^{8}}$	88

input
```
int((c*x^2+b*x+a)^3/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*a^3/x^8-3/7*a^2*b/x^7-1/2*a*(a*c+b^2)/x^6-1/5*b*(6*a*c+b^2)/x^5-3/4*c
*(a*c+b^2)/x^4-b*c^2/x^3-1/2*c^3/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx+cx^2)^3}{x^9}\,dx = -\frac{140\,c^3x^6+280\,bc^2x^5+210\,(b^2c+ac^2)x^4+120\,a^2bx+56\,(b^3+6\,abc)x^3+35\,a^3+140\,(ab^2+a^2c)x^2}{280\,x^8}$$

input
```
integrate((c*x^2+b*x+a)^3/x^9,x, algorithm="fricas")
```

output
```
-1/280*(140*c^3*x^6 + 280*b*c^2*x^5 + 210*(b^2*c + a*c^2)*x^4 + 120*a^2*b*
x + 56*(b^3 + 6*a*b*c)*x^3 + 35*a^3 + 140*(a*b^2 + a^2*c)*x^2)/x^8
```

Sympy [A] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx+cx^2)^3}{x^9}\,dx = \frac{-35a^3-120a^2bx-280bc^2x^5-140c^3x^6+x^4(-210ac^2-210b^2c)+x^3(-336abc-56b^3)+x^2(-140a^2c}{280x^8}$$

input
```
integrate((c*x**2+b*x+a)**3/x**9,x)
```

output
```
(-35*a**3 - 120*a**2*b*x - 280*b*c**2*x**5 - 140*c**3*x**6 + x**4*(-210*a*
c**2 - 210*b**2*c) + x**3*(-336*a*b*c - 56*b**3) + x**2*(-140*a**2*c - 140
*a*b**2))/(280*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx+cx^2)^3}{x^9}\,dx = -\frac{140\,c^3x^6+280\,bc^2x^5+210\,(b^2c+ac^2)x^4+120\,a^2bx+56\,(b^3+6\,abc)x^3+35\,a^3+140\,(ab^2+a^2c)x^2}{280\,x^8}$$

input
```
integrate((c*x^2+b*x+a)^3/x^9,x, algorithm="maxima")
```

output
```
-1/280*(140*c^3*x^6 + 280*b*c^2*x^5 + 210*(b^2*c + a*c^2)*x^4 + 120*a^2*b*
x + 56*(b^3 + 6*a*b*c)*x^3 + 35*a^3 + 140*(a*b^2 + a^2*c)*x^2)/x^8
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2)^3}{x^9}\,dx = -\frac{140\,c^3x^6+280\,bc^2x^5+210\,b^2cx^4+210\,ac^2x^4+56\,b^3x^3+336\,abcx^3+140\,ab^2x^2+140\,a^2cx^2+120\,a^2}{280\,x^8}$$

input
```
integrate((c*x^2+b*x+a)^3/x^9,x, algorithm="giac")
```

output
```
-1/280*(140*c^3*x^6 + 280*b*c^2*x^5 + 210*b^2*c*x^4 + 210*a*c^2*x^4 + 56*b
^3*x^3 + 336*a*b*c*x^3 + 140*a*b^2*x^2 + 140*a^2*c*x^2 + 120*a^2*b*x + 35*
a^3)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx+cx^2)^3}{x^9}\,dx = -\frac{x^3\left(\frac{b^3}{5}+\frac{6\,a\,c\,b}{5}\right)+x^2\left(\frac{c\,a^2}{2}+\frac{a\,b^2}{2}\right)+x^4\left(\frac{3\,b^2\,c}{4}+\frac{3\,a\,c^2}{4}\right)+\frac{a^3}{8}+\frac{c^3\,x^6}{2}+b\,c^2\,x^5+\frac{3\,a^2\,b\,x}{7}}{x^8}$$

input
```
int((a + b*x + c*x^2)^3/x^9,x)
```

output
```
-(x^3*(b^3/5 + (6*a*b*c)/5) + x^2*((a*b^2)/2 + (a^2*c)/2) + x^4*((3*a*c^2)
/4 + (3*b^2*c)/4) + a^3/8 + (c^3*x^6)/2 + b*c^2*x^5 + (3*a^2*b*x)/7)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2)^3}{x^9}\,dx$$
$$= \frac{-140c^3x^6 - 280b\,c^2x^5 - 210a\,c^2x^4 - 210b^2c\,x^4 - 336abc\,x^3 - 56b^3x^3 - 140a^2c\,x^2 - 140a\,b^2x^2 - 120a^2bx}{280x^8}$$

input
```
int((c*x^2+b*x+a)^3/x^9,x)
```

output
```
( - 35*a**3 - 120*a**2*b*x - 140*a**2*c*x**2 - 140*a*b**2*x**2 - 336*a*b*c
*x**3 - 210*a*c**2*x**4 - 56*b**3*x**3 - 210*b**2*c*x**4 - 280*b*c**2*x**5
 - 140*c**3*x**6)/(280*x**8)
```

3.213 $\int \frac{(a+bx+cx^2)^3}{x^{10}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 89

$$\int \frac{(a+bx+cx^2)^3}{x^{10}}\,dx = -\frac{a^3}{9x^9}-\frac{3a^2b}{8x^8}-\frac{3a(b^2+ac)}{7x^7}-\frac{b(b^2+6ac)}{6x^6} -\frac{3c(b^2+ac)}{5x^5}-\frac{3bc^2}{4x^4}-\frac{c^3}{3x^3}$$

output
```
-1/9*a^3/x^9-3/8*a^2*b/x^8-3/7*a*(a*c+b^2)/x^7-1/6*b*(6*a*c+b^2)/x^6-3/5*c
*(a*c+b^2)/x^5-3/4*b*c^2/x^4-1/3*c^3/x^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx+cx^2)^3}{x^{10}}\,dx = -\frac{280a^3+135a^2x(7b+8cx)+72ax^2(15b^2+35bcx+21c^2x^2)+42x^3(10b^3+36b^2cx+45bc^2x^2+20c^3x^3}{2520x^9}$$

input
```
Integrate[(a + b*x + c*x^2)^3/x^10,x]
```

output
```
-1/2520*(280*a^3 + 135*a^2*x*(7*b + 8*c*x) + 72*a*x^2*(15*b^2 + 35*b*c*x +
 21*c^2*x^2) + 42*x^3*(10*b^3 + 36*b^2*c*x + 45*b*c^2*x^2 + 20*c^3*x^3))/x
^9
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^3}{x^{10}}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{a^3}{x^{10}}+\frac{3a^2b}{x^9}+\frac{6abc+b^3}{x^7}+\frac{3a(ac+b^2)}{x^8}+\frac{3c(ac+b^2)}{x^6}+\frac{3bc^2}{x^5}+\frac{c^3}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3}{9x^9}-\frac{3a^2b}{8x^8}-\frac{3a(ac+b^2)}{7x^7}-\frac{b(6ac+b^2)}{6x^6}-\frac{3c(ac+b^2)}{5x^5}-\frac{3bc^2}{4x^4}-\frac{c^3}{3x^3}$$

input
```
Int[(a + b*x + c*x^2)^3/x^10,x]
```

output
```
-1/9*a^3/x^9 - (3*a^2*b)/(8*x^8) - (3*a*(b^2 + a*c))/(7*x^7) - (b*(b^2 + 6
*a*c))/(6*x^6) - (3*c*(b^2 + a*c))/(5*x^5) - (3*b*c^2)/(4*x^4) - c^3/(3*x^
3)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{a^{3}}{9x^{9}}-\frac{3a^{2}b}{8x^{8}}-\frac{3a\left(ac+b^{2}\right)}{7x^{7}}-\frac{b\left(6ac+b^{2}\right)}{6x^{6}}-\frac{3c\left(ac+b^{2}\right)}{5x^{5}}-\frac{3b\,c^{2}}{4x^{4}}-\frac{c^{3}}{3x^{3}}$	76
norman	$\frac{-\frac{c^{3}x^{6}}{3}-\frac{3b\,c^{2}x^{5}}{4}+\left(-\frac{3}{5}a\,c^{2}-\frac{3}{5}b^{2}c\right)x^{4}+\left(-abc-\frac{1}{6}b^{3}\right)x^{3}+\left(-\frac{3}{7}a^{2}c-\frac{3}{7}a\,b^{2}\right)x^{2}-\frac{3a^{2}bx}{8}-\frac{a^{3}}{9}}{x^{9}}$	84
risch	$\frac{-\frac{c^{3}x^{6}}{3}-\frac{3b\,c^{2}x^{5}}{4}+\left(-\frac{3}{5}a\,c^{2}-\frac{3}{5}b^{2}c\right)x^{4}+\left(-abc-\frac{1}{6}b^{3}\right)x^{3}+\left(-\frac{3}{7}a^{2}c-\frac{3}{7}a\,b^{2}\right)x^{2}-\frac{3a^{2}bx}{8}-\frac{a^{3}}{9}}{x^{9}}$	84
gosper	$-\frac{840c^{3}x^{6}+1890b\,c^{2}x^{5}+1512a\,c^{2}x^{4}+1512b^{2}c\,x^{4}+2520abc\,x^{3}+420b^{3}x^{3}+1080a^{2}c\,x^{2}+1080a\,b^{2}x^{2}+945a^{2}bx+280a^{3}}{2520x^{9}}$	88
parallelrisch	$\frac{-840c^{3}x^{6}-1890b\,c^{2}x^{5}-1512a\,c^{2}x^{4}-1512b^{2}c\,x^{4}-2520abc\,x^{3}-420b^{3}x^{3}-1080a^{2}c\,x^{2}-1080a\,b^{2}x^{2}-945a^{2}bx-280a^{3}}{2520x^{9}}$	88
orering	$-\frac{840c^{3}x^{6}+1890b\,c^{2}x^{5}+1512a\,c^{2}x^{4}+1512b^{2}c\,x^{4}+2520abc\,x^{3}+420b^{3}x^{3}+1080a^{2}c\,x^{2}+1080a\,b^{2}x^{2}+945a^{2}bx+280a^{3}}{2520x^{9}}$	88

input
```
int((c*x^2+b*x+a)^3/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a^3/x^9-3/8*a^2*b/x^8-3/7*a*(a*c+b^2)/x^7-1/6*b*(6*a*c+b^2)/x^6-3/5*c
*(a*c+b^2)/x^5-3/4*b*c^2/x^4-1/3*c^3/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx+cx^2)^3}{x^{10}}\,dx = -\frac{840\,c^3x^6+1890\,bc^2x^5+1512\,(b^2c+ac^2)x^4+945\,a^2bx+420\,(b^3+6\,abc)x^3+280\,a^3+1080\,(ab^2+a^2}{2520\,x^9}$$

input
```
integrate((c*x^2+b*x+a)^3/x^10,x, algorithm="fricas")
```

output
```
-1/2520*(840*c^3*x^6 + 1890*b*c^2*x^5 + 1512*(b^2*c + a*c^2)*x^4 + 945*a^2
*b*x + 420*(b^3 + 6*a*b*c)*x^3 + 280*a^3 + 1080*(a*b^2 + a^2*c)*x^2)/x^9
```

Sympy [A] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx+cx^2)^3}{x^{10}}\,dx = \frac{-280a^3-945a^2bx-1890bc^2x^5-840c^3x^6+x^4(-1512ac^2-1512b^2c)+x^3(-2520abc-420b^3)+x^2(-}{2520x^9}$$

input
```
integrate((c*x**2+b*x+a)**3/x**10,x)
```

output
```
(-280*a**3 - 945*a**2*b*x - 1890*b*c**2*x**5 - 840*c**3*x**6 + x**4*(-1512
*a*c**2 - 1512*b**2*c) + x**3*(-2520*a*b*c - 420*b**3) + x**2*(-1080*a**2*
c - 1080*a*b**2))/(2520*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx+cx^2)^3}{x^{10}}\,dx = -\frac{840\,c^3x^6 + 1890\,bc^2x^5 + 1512\,(b^2c+ac^2)x^4 + 945\,a^2bx + 420\,(b^3+6\,abc)x^3 + 280\,a^3 + 1080\,(ab^2 + a^2}{2520\,x^9}$$

input
```
integrate((c*x^2+b*x+a)^3/x^10,x, algorithm="maxima")
```

output
```
-1/2520*(840*c^3*x^6 + 1890*b*c^2*x^5 + 1512*(b^2*c + a*c^2)*x^4 + 945*a^2
*b*x + 420*(b^3 + 6*a*b*c)*x^3 + 280*a^3 + 1080*(a*b^2 + a^2*c)*x^2)/x^9
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx+cx^2)^3}{x^{10}}\,dx = -\frac{840\,c^3x^6 + 1890\,bc^2x^5 + 1512\,b^2cx^4 + 1512\,ac^2x^4 + 420\,b^3x^3 + 2520\,abcx^3 + 1080\,ab^2x^2 + 1080\,a^2cx^2 +}{2520\,x^9}$$

input
```
integrate((c*x^2+b*x+a)^3/x^10,x, algorithm="giac")
```

output
```
-1/2520*(840*c^3*x^6 + 1890*b*c^2*x^5 + 1512*b^2*c*x^4 + 1512*a*c^2*x^4 +
420*b^3*x^3 + 2520*a*b*c*x^3 + 1080*a*b^2*x^2 + 1080*a^2*c*x^2 + 945*a^2*b
*x + 280*a^3)/x^9
```

Mupad [B] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx+cx^2)^3}{x^{10}}\,dx = -\frac{x^3\left(\frac{b^3}{6}+a\,c\,b\right)+x^2\left(\frac{3\,c\,a^2}{7}+\frac{3\,a\,b^2}{7}\right)+x^4\left(\frac{3\,b^2\,c}{5}+\frac{3\,a\,c^2}{5}\right)+\frac{a^3}{9}+\frac{c^3\,x^6}{3}+\frac{3\,b\,c^2\,x^5}{4}+\frac{3\,a^2\,b\,x}{8}}{x^9}$$

input
```
int((a + b*x + c*x^2)^3/x^10,x)
```

output
```
-(x^3*(b^3/6 + a*b*c) + x^2*((3*a*b^2)/7 + (3*a^2*c)/7) + x^4*((3*a*c^2)/5
 + (3*b^2*c)/5) + a^3/9 + (c^3*x^6)/3 + (3*b*c^2*x^5)/4 + (3*a^2*b*x)/8)/x
^9
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx+cx^2)^3}{x^{10}}\,dx = \frac{-840c^3x^6-1890b\,c^2x^5-1512a\,c^2x^4-1512b^2c\,x^4-2520abc\,x^3-420b^3x^3-1080a^2c\,x^2-1080a\,b^2x^2-}{2520x^9}$$

input
```
int((c*x^2+b*x+a)^3/x^10,x)
```

output
```
( - 280*a**3 - 945*a**2*b*x - 1080*a**2*c*x**2 - 1080*a*b**2*x**2 - 2520*a
*b*c*x**3 - 1512*a*c**2*x**4 - 420*b**3*x**3 - 1512*b**2*c*x**4 - 1890*b*c
**2*x**5 - 840*c**3*x**6)/(2520*x**9)
```

3.214 $\int x^4(3-4x+x^2)^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 32

$$\int x^4(3-4x+x^2)^2\,dx=\frac{9x^5}{5}-4x^6+\frac{22x^7}{7}-x^8+\frac{x^9}{9}$$

output
```
9/5*x^5-4*x^6+22/7*x^7-x^8+1/9*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int x^4(3-4x+x^2)^2\,dx=\frac{9x^5}{5}-4x^6+\frac{22x^7}{7}-x^8+\frac{x^9}{9}$$

input
```
Integrate[x^4*(3 - 4*x + x^2)^2,x]
```

output
```
(9*x^5)/5 - 4*x^6 + (22*x^7)/7 - x^8 + x^9/9
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(x^2-4x+3\right)^2 \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(x^8-8x^7+22x^6-24x^5+9x^4\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^9}{9}-x^8+\frac{22x^7}{7}-4x^6+\frac{9x^5}{5}$$

input
```
Int[x^4*(3 - 4*x + x^2)^2,x]
```

output
```
(9*x^5)/5 - 4*x^6 + (22*x^7)/7 - x^8 + x^9/9
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{x^5(35x^4-315x^3+990x^2-1260x+567)}{315}$	26
default	$\frac{9}{5}x^5-4x^6+\frac{22}{7}x^7-x^8+\frac{1}{9}x^9$	27
norman	$\frac{9}{5}x^5-4x^6+\frac{22}{7}x^7-x^8+\frac{1}{9}x^9$	27
risch	$\frac{9}{5}x^5-4x^6+\frac{22}{7}x^7-x^8+\frac{1}{9}x^9$	27
parallelrisch	$\frac{9}{5}x^5-4x^6+\frac{22}{7}x^7-x^8+\frac{1}{9}x^9$	27
orering	$\frac{x^5(35x^4-315x^3+990x^2-1260x+567)(x^2-4x+3)^2}{315(x-1)^2(-3+x)^2}$	46

input
```
int(x^4*(x^2-4*x+3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/315*x^5*(35*x^4-315*x^3+990*x^2-1260*x+567)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int x^4(3-4x+x^2)^2\,dx=\frac{1}{9}\,x^9-x^8+\frac{22}{7}\,x^7-4\,x^6+\frac{9}{5}\,x^5$$

input
```
integrate(x^4*(x^2-4*x+3)^2,x, algorithm="fricas")
```

output
```
1/9*x^9 - x^8 + 22/7*x^7 - 4*x^6 + 9/5*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int x^4 (3 - 4x + x^2)^2 \, dx = \frac{x^9}{9} - x^8 + \frac{22x^7}{7} - 4x^6 + \frac{9x^5}{5}$$

input
```
integrate(x**4*(x**2-4*x+3)**2,x)
```

output
```
x**9/9 - x**8 + 22*x**7/7 - 4*x**6 + 9*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int x^4 (3 - 4x + x^2)^2 \, dx = \frac{1}{9} x^9 - x^8 + \frac{22}{7} x^7 - 4 x^6 + \frac{9}{5} x^5$$

input
```
integrate(x^4*(x^2-4*x+3)^2,x, algorithm="maxima")
```

output
```
1/9*x^9 - x^8 + 22/7*x^7 - 4*x^6 + 9/5*x^5
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int x^4 (3 - 4x + x^2)^2 \, dx = \frac{1}{9} x^9 - x^8 + \frac{22}{7} x^7 - 4 x^6 + \frac{9}{5} x^5$$

input
```
integrate(x^4*(x^2-4*x+3)^2,x, algorithm="giac")
```

output
```
1/9*x^9 - x^8 + 22/7*x^7 - 4*x^6 + 9/5*x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int x^4 (3 - 4x + x^2)^2 \, dx = \frac{x^9}{9} - x^8 + \frac{22\,x^7}{7} - 4\,x^6 + \frac{9\,x^5}{5}$$

input
```
int(x^4*(x^2 - 4*x + 3)^2,x)
```

output
```
(9*x^5)/5 - 4*x^6 + (22*x^7)/7 - x^8 + x^9/9
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int x^4 (3 - 4x + x^2)^2 \, dx = \frac{x^5(35x^4 - 315x^3 + 990x^2 - 1260x + 567)}{315}$$

input
```
int(x^4*(x^2-4*x+3)^2,x)
```

output
```
(x**5*(35*x**4 - 315*x**3 + 990*x**2 - 1260*x + 567))/315
```

3.215 $\int x^3(3-4x+x^2)^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 36

$$\int x^3(3-4x+x^2)^2\,dx=\frac{9x^4}{4}-\frac{24x^5}{5}+\frac{11x^6}{3}-\frac{8x^7}{7}+\frac{x^8}{8}$$

output
```
9/4*x^4-24/5*x^5+11/3*x^6-8/7*x^7+1/8*x^8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int x^3(3-4x+x^2)^2\,dx=\frac{9x^4}{4}-\frac{24x^5}{5}+\frac{11x^6}{3}-\frac{8x^7}{7}+\frac{x^8}{8}$$

input
```
Integrate[x^3*(3 - 4*x + x^2)^2,x]
```

output
```
(9*x^4)/4 - (24*x^5)/5 + (11*x^6)/3 - (8*x^7)/7 + x^8/8
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(x^2-4 x+3\right)^2 \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(x^7-8 x^6+22 x^5-24 x^4+9 x^3\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^8}{8}-\frac{8 x^7}{7}+\frac{11 x^6}{3}-\frac{24 x^5}{5}+\frac{9 x^4}{4}$$

input
```
Int[x^3*(3 - 4*x + x^2)^2,x]
```

output
```
(9*x^4)/4 - (24*x^5)/5 + (11*x^6)/3 - (8*x^7)/7 + x^8/8
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{x^4(105x^4-960x^3+3080x^2-4032x+1890)}{840}$	26
default	$\frac{9}{4}x^4-\frac{24}{5}x^5+\frac{11}{3}x^6-\frac{8}{7}x^7+\frac{1}{8}x^8$	27
norman	$\frac{9}{4}x^4-\frac{24}{5}x^5+\frac{11}{3}x^6-\frac{8}{7}x^7+\frac{1}{8}x^8$	27
risch	$\frac{9}{4}x^4-\frac{24}{5}x^5+\frac{11}{3}x^6-\frac{8}{7}x^7+\frac{1}{8}x^8$	27
parallelrisch	$\frac{9}{4}x^4-\frac{24}{5}x^5+\frac{11}{3}x^6-\frac{8}{7}x^7+\frac{1}{8}x^8$	27
orering	$\frac{x^4(105x^4-960x^3+3080x^2-4032x+1890)(x^2-4x+3)^2}{840(x-1)^2(-3+x)^2}$	46

input
```
int(x^3*(x^2-4*x+3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/840*x^4*(105*x^4-960*x^3+3080*x^2-4032*x+1890)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int x^3(3-4x+x^2)^2\,dx=\frac{1}{8}x^8-\frac{8}{7}x^7+\frac{11}{3}x^6-\frac{24}{5}x^5+\frac{9}{4}x^4$$

input
```
integrate(x^3*(x^2-4*x+3)^2,x, algorithm="fricas")
```

output
```
1/8*x^8 - 8/7*x^7 + 11/3*x^6 - 24/5*x^5 + 9/4*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int x^3(3-4x+x^2)^2\,dx=\frac{x^8}{8}-\frac{8x^7}{7}+\frac{11x^6}{3}-\frac{24x^5}{5}+\frac{9x^4}{4}$$

input
```
integrate(x**3*(x**2-4*x+3)**2,x)
```

output
```
x**8/8 - 8*x**7/7 + 11*x**6/3 - 24*x**5/5 + 9*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int x^3(3-4x+x^2)^2\,dx=\frac{1}{8}\,x^8-\frac{8}{7}\,x^7+\frac{11}{3}\,x^6-\frac{24}{5}\,x^5+\frac{9}{4}\,x^4$$

input
```
integrate(x^3*(x^2-4*x+3)^2,x, algorithm="maxima")
```

output
```
1/8*x^8 - 8/7*x^7 + 11/3*x^6 - 24/5*x^5 + 9/4*x^4
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int x^3(3-4x+x^2)^2\,dx=\frac{1}{8}\,x^8-\frac{8}{7}\,x^7+\frac{11}{3}\,x^6-\frac{24}{5}\,x^5+\frac{9}{4}\,x^4$$

input
```
integrate(x^3*(x^2-4*x+3)^2,x, algorithm="giac")
```

output
```
1/8*x^8 - 8/7*x^7 + 11/3*x^6 - 24/5*x^5 + 9/4*x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int x^3(3-4x+x^2)^2\,dx=\frac{x^8}{8}-\frac{8\,x^7}{7}+\frac{11\,x^6}{3}-\frac{24\,x^5}{5}+\frac{9\,x^4}{4}$$

input
```
int(x^3*(x^2 - 4*x + 3)^2,x)
```

output
```
(9*x^4)/4 - (24*x^5)/5 + (11*x^6)/3 - (8*x^7)/7 + x^8/8
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int x^3(3-4x+x^2)^2\,dx=\frac{x^4(105x^4-960x^3+3080x^2-4032x+1890)}{840}$$

input
```
int(x^3*(x^2-4*x+3)^2,x)
```

output
```
(x**4*(105*x**4 - 960*x**3 + 3080*x**2 - 4032*x + 1890))/840
```

3.216 $\int x^2(3-4x+x^2)^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 32

$$\int x^2(3-4x+x^2)^2\,dx=3x^3-6x^4+\frac{22x^5}{5}-\frac{4x^6}{3}+\frac{x^7}{7}$$

output
```
3*x^3-6*x^4+22/5*x^5-4/3*x^6+1/7*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int x^2(3-4x+x^2)^2\,dx=3x^3-6x^4+\frac{22x^5}{5}-\frac{4x^6}{3}+\frac{x^7}{7}$$

input
```
Integrate[x^2*(3 - 4*x + x^2)^2,x]
```

output
```
3*x^3 - 6*x^4 + (22*x^5)/5 - (4*x^6)/3 + x^7/7
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(x^2-4x+3\right)^2\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(x^6-8x^5+22x^4-24x^3+9x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^7}{7}-\frac{4x^6}{3}+\frac{22x^5}{5}-6x^4+3x^3$$

input
```
Int[x^2*(3 - 4*x + x^2)^2,x]
```

output
```
3*x^3 - 6*x^4 + (22*x^5)/5 - (4*x^6)/3 + x^7/7
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{x^{3}\left(15x^{4}-140x^{3}+462x^{2}-630x+315\right)}{105}$	26
default	$3x^{3}-6x^{4}+\frac{22}{5}x^{5}-\frac{4}{3}x^{6}+\frac{1}{7}x^{7}$	27
norman	$3x^{3}-6x^{4}+\frac{22}{5}x^{5}-\frac{4}{3}x^{6}+\frac{1}{7}x^{7}$	27
risch	$3x^{3}-6x^{4}+\frac{22}{5}x^{5}-\frac{4}{3}x^{6}+\frac{1}{7}x^{7}$	27
parallelrisch	$3x^{3}-6x^{4}+\frac{22}{5}x^{5}-\frac{4}{3}x^{6}+\frac{1}{7}x^{7}$	27
orering	$\frac{x^{3}\left(15x^{4}-140x^{3}+462x^{2}-630x+315\right)\left(x^{2}-4x+3\right)^{2}}{105\left(x-1\right)^{2}\left(-3+x\right)^{2}}$	46

input
```
int(x^2*(x^2-4*x+3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/105*x^3*(15*x^4-140*x^3+462*x^2-630*x+315)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int x^{2}\left(3-4x+x^{2}\right)^{2}\,dx=\frac{1}{7}\,x^{7}-\frac{4}{3}\,x^{6}+\frac{22}{5}\,x^{5}-6\,x^{4}+3\,x^{3}$$

input
```
integrate(x^2*(x^2-4*x+3)^2,x, algorithm="fricas")
```

output
```
1/7*x^7 - 4/3*x^6 + 22/5*x^5 - 6*x^4 + 3*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int x^2(3-4x+x^2)^2\,dx = \frac{x^7}{7} - \frac{4x^6}{3} + \frac{22x^5}{5} - 6x^4 + 3x^3$$

input
```
integrate(x**2*(x**2-4*x+3)**2,x)
```

output
```
x**7/7 - 4*x**6/3 + 22*x**5/5 - 6*x**4 + 3*x**3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int x^2(3-4x+x^2)^2\,dx = \frac{1}{7}\,x^7 - \frac{4}{3}\,x^6 + \frac{22}{5}\,x^5 - 6\,x^4 + 3\,x^3$$

input
```
integrate(x^2*(x^2-4*x+3)^2,x, algorithm="maxima")
```

output
```
1/7*x^7 - 4/3*x^6 + 22/5*x^5 - 6*x^4 + 3*x^3
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int x^2(3-4x+x^2)^2\,dx = \frac{1}{7}\,x^7 - \frac{4}{3}\,x^6 + \frac{22}{5}\,x^5 - 6\,x^4 + 3\,x^3$$

input
```
integrate(x^2*(x^2-4*x+3)^2,x, algorithm="giac")
```

output
```
1/7*x^7 - 4/3*x^6 + 22/5*x^5 - 6*x^4 + 3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int x^2(3-4x+x^2)^2\,dx=\frac{x^7}{7}-\frac{4\,x^6}{3}+\frac{22\,x^5}{5}-6\,x^4+3\,x^3$$

input
```
int(x^2*(x^2 - 4*x + 3)^2,x)
```

output
```
3*x^3 - 6*x^4 + (22*x^5)/5 - (4*x^6)/3 + x^7/7
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int x^2(3-4x+x^2)^2\,dx=\frac{x^3(15x^4-140x^3+462x^2-630x+315)}{105}$$

input
```
int(x^2*(x^2-4*x+3)^2,x)
```

output
```
(x**3*(15*x**4 - 140*x**3 + 462*x**2 - 630*x + 315))/105
```

3.217 $\int x(3-4x+x^2)^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 34

$$\int x\left(3-4x+x^2\right)^2\,dx=\frac{9x^2}{2}-8x^3+\frac{11x^4}{2}-\frac{8x^5}{5}+\frac{x^6}{6}$$

output
```
9/2*x^2-8*x^3+11/2*x^4-8/5*x^5+1/6*x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int x\left(3-4x+x^2\right)^2\,dx=\frac{9x^2}{2}-8x^3+\frac{11x^4}{2}-\frac{8x^5}{5}+\frac{x^6}{6}$$

input
```
Integrate[x*(3 - 4*x + x^2)^2,x]
```

output
```
(9*x^2)/2 - 8*x^3 + (11*x^4)/2 - (8*x^5)/5 + x^6/6
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(x^2-4x+3)^2\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(x^5-8x^4+22x^3-24x^2+9x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^6}{6}-\frac{8x^5}{5}+\frac{11x^4}{2}-8x^3+\frac{9x^2}{2}$$

input
```
Int[x*(3 - 4*x + x^2)^2,x]
```

output
```
(9*x^2)/2 - 8*x^3 + (11*x^4)/2 - (8*x^5)/5 + x^6/6
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

method	result	size
gosper	$\frac{x^{2}\left(5x^{4}-48x^{3}+165x^{2}-240x+135\right)}{30}$	26
default	$\frac{9}{2}x^{2}-8x^{3}+\frac{11}{2}x^{4}-\frac{8}{5}x^{5}+\frac{1}{6}x^{6}$	27
norman	$\frac{9}{2}x^{2}-8x^{3}+\frac{11}{2}x^{4}-\frac{8}{5}x^{5}+\frac{1}{6}x^{6}$	27
risch	$\frac{9}{2}x^{2}-8x^{3}+\frac{11}{2}x^{4}-\frac{8}{5}x^{5}+\frac{1}{6}x^{6}$	27
parallelrisch	$\frac{9}{2}x^{2}-8x^{3}+\frac{11}{2}x^{4}-\frac{8}{5}x^{5}+\frac{1}{6}x^{6}$	27
orering	$\frac{x^{2}\left(5x^{4}-48x^{3}+165x^{2}-240x+135\right)\left(x^{2}-4x+3\right)^{2}}{30\left(x-1\right)^{2}\left(-3+x\right)^{2}}$	46

input
```
int(x*(x^2-4*x+3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/30*x^2*(5*x^4-48*x^3+165*x^2-240*x+135)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int x(3-4x+x^2)^2\,dx=\frac{1}{6}\,x^{6}-\frac{8}{5}\,x^{5}+\frac{11}{2}\,x^{4}-8\,x^{3}+\frac{9}{2}\,x^{2}$$

input
```
integrate(x*(x^2-4*x+3)^2,x, algorithm="fricas")
```

output
```
1/6*x^6 - 8/5*x^5 + 11/2*x^4 - 8*x^3 + 9/2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int x(3-4x+x^2)^2\,dx = \frac{x^6}{6} - \frac{8x^5}{5} + \frac{11x^4}{2} - 8x^3 + \frac{9x^2}{2}$$

input
```
integrate(x*(x**2-4*x+3)**2,x)
```

output
```
x**6/6 - 8*x**5/5 + 11*x**4/2 - 8*x**3 + 9*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int x(3-4x+x^2)^2\,dx = \frac{1}{6}\,x^6 - \frac{8}{5}\,x^5 + \frac{11}{2}\,x^4 - 8\,x^3 + \frac{9}{2}\,x^2$$

input
```
integrate(x*(x^2-4*x+3)^2,x, algorithm="maxima")
```

output
```
1/6*x^6 - 8/5*x^5 + 11/2*x^4 - 8*x^3 + 9/2*x^2
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int x(3-4x+x^2)^2\,dx = \frac{1}{6}\,x^6 - \frac{8}{5}\,x^5 + \frac{11}{2}\,x^4 - 8\,x^3 + \frac{9}{2}\,x^2$$

input
```
integrate(x*(x^2-4*x+3)^2,x, algorithm="giac")
```

output
```
1/6*x^6 - 8/5*x^5 + 11/2*x^4 - 8*x^3 + 9/2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int x(3-4x+x^2)^2\,dx = \frac{x^6}{6} - \frac{8\,x^5}{5} + \frac{11\,x^4}{2} - 8\,x^3 + \frac{9\,x^2}{2}$$

input
```
int(x*(x^2 - 4*x + 3)^2,x)
```

output
```
(9*x^2)/2 - 8*x^3 + (11*x^4)/2 - (8*x^5)/5 + x^6/6
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int x(3-4x+x^2)^2\,dx = \frac{x^2(5x^4-48x^3+165x^2-240x+135)}{30}$$

input
```
int(x*(x^2-4*x+3)^2,x)
```

output
```
(x**2*(5*x**4 - 48*x**3 + 165*x**2 - 240*x + 135))/30
```

3.218 $\int (3-4x+x^2)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 28

$$\int (3-4x+x^2)^2 \, dx = -\frac{4}{3}(3-x)^3 - \frac{1}{5}(3-x)^5 + (-3+x)^4$$

output
```
-4/3*(3-x)^3-1/5*(3-x)^5+(-3+x)^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (3-4x+x^2)^2 \, dx = 9x - 12x^2 + \frac{22x^3}{3} - 2x^4 + \frac{x^5}{5}$$

input
```
Integrate[(3 - 4*x + x^2)^2,x]
```

output
```
9*x - 12*x^2 + (22*x^3)/3 - 2*x^4 + x^5/5
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^2-4x+3\right)^2 dx$$

$$\Big\downarrow \text{1084}$$

$$\int \left((x-3)^4-4(3-x)^3+4(3-x)^2\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{5}(3-x)^5+(3-x)^4-\frac{4}{3}(3-x)^3$$

input
```
Int[(3 - 4*x + x^2)^2,x]
```

output
```
(-4*(3 - x)^3)/3 + (3 - x)^4 - (3 - x)^5/5
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

method	result	size
gosper	$\frac{x(3x^4-30x^3+110x^2-180x+135)}{15}$	24
default	$\frac{1}{5}x^5-2x^4+\frac{22}{3}x^3-12x^2+9x$	25
norman	$\frac{1}{5}x^5-2x^4+\frac{22}{3}x^3-12x^2+9x$	25
risch	$\frac{1}{5}x^5-2x^4+\frac{22}{3}x^3-12x^2+9x$	25
parallelrisch	$\frac{1}{5}x^5-2x^4+\frac{22}{3}x^3-12x^2+9x$	25
orering	$\frac{x(3x^4-30x^3+110x^2-180x+135)(x^2-4x+3)^2}{15(x-1)^2(-3+x)^2}$	44

input
```
int((x^2-4*x+3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(3*x^4-30*x^3+110*x^2-180*x+135)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(3-4x+x^2\right)^2\,dx=\frac{1}{5}\,x^5-2\,x^4+\frac{22}{3}\,x^3-12\,x^2+9\,x$$

input
```
integrate((x^2-4*x+3)^2,x, algorithm="fricas")
```

output
```
1/5*x^5 - 2*x^4 + 22/3*x^3 - 12*x^2 + 9*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(3-4x+x^2\right)^2\,dx=\frac{x^5}{5}-2x^4+\frac{22x^3}{3}-12x^2+9x$$

input
```
integrate((x**2-4*x+3)**2,x)
```

output
```
x**5/5 - 2*x**4 + 22*x**3/3 - 12*x**2 + 9*x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(3-4x+x^2\right)^2\,dx=\frac{1}{5}\,x^5-2\,x^4+\frac{22}{3}\,x^3-12\,x^2+9\,x$$

input
```
integrate((x^2-4*x+3)^2,x, algorithm="maxima")
```

output
```
1/5*x^5 - 2*x^4 + 22/3*x^3 - 12*x^2 + 9*x
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(3-4x+x^2\right)^2\,dx=\frac{1}{5}\,x^5-2\,x^4+\frac{22}{3}\,x^3-12\,x^2+9\,x$$

input
```
integrate((x^2-4*x+3)^2,x, algorithm="giac")
```

output
```
1/5*x^5 - 2*x^4 + 22/3*x^3 - 12*x^2 + 9*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(3-4x+x^2\right)^2 \, dx = \frac{x^5}{5} - 2\,x^4 + \frac{22\,x^3}{3} - 12\,x^2 + 9\,x$$

input
```
int((x^2 - 4*x + 3)^2,x)
```

output
```
9*x - 12*x^2 + (22*x^3)/3 - 2*x^4 + x^5/5
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \left(3-4x+x^2\right)^2 \, dx = \frac{x(3x^4 - 30x^3 + 110x^2 - 180x + 135)}{15}$$

input
```
int((x^2-4*x+3)^2,x)
```

output
```
(x*(3*x**4 - 30*x**3 + 110*x**2 - 180*x + 135))/15
```

3.219 $\int \frac{(3-4x+x^2)^2}{x} dx$

Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \frac{(3-4x+x^2)^2}{x} dx = -24x + 11x^2 - \frac{8x^3}{3} + \frac{x^4}{4} + 9\log(x)$$

output
```
-24*x+11*x^2-8/3*x^3+1/4*x^4+9*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(3-4x+x^2)^2}{x} dx = -24x + 11x^2 - \frac{8x^3}{3} + \frac{x^4}{4} + 9\log(x)$$

input
```
Integrate[(3 - 4*x + x^2)^2/x,x]
```

output
```
-24*x + 11*x^2 - (8*x^3)/3 + x^4/4 + 9*Log[x]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2-4x+3)^2}{x} \, dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int \left(x^3-8x^2+22x+\frac{9}{x}-24\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^4}{4}-\frac{8x^3}{3}+11x^2-24x+9\log(x)$$

input
```
Int[(3 - 4*x + x^2)^2/x,x]
```

output
```
-24*x + 11*x^2 - (8*x^3)/3 + x^4/4 + 9*Log[x]
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
default	$-24x+11x^2-\frac{8x^3}{3}+\frac{x^4}{4}+9\ln(x)$	24
norman	$-24x+11x^2-\frac{8x^3}{3}+\frac{x^4}{4}+9\ln(x)$	24
risch	$-24x+11x^2-\frac{8x^3}{3}+\frac{x^4}{4}+9\ln(x)$	24
parallelrisch	$-24x+11x^2-\frac{8x^3}{3}+\frac{x^4}{4}+9\ln(x)$	24

input
```
int((x^2-4*x+3)^2/x,x,method=_RETURNVERBOSE)
```

output
```
-24*x+11*x^2-8/3*x^3+1/4*x^4+9*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{(3-4x+x^2)^2}{x}\,dx = \frac{1}{4}\,x^4 - \frac{8}{3}\,x^3 + 11\,x^2 - 24\,x + 9\,\log(x)$$

input
```
integrate((x^2-4*x+3)^2/x,x, algorithm="fricas")
```

output
```
1/4*x^4 - 8/3*x^3 + 11*x^2 - 24*x + 9*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(3-4x+x^2)^2}{x}\,dx = \frac{x^4}{4} - \frac{8x^3}{3} + 11x^2 - 24x + 9\log(x)$$

input
```
integrate((x**2-4*x+3)**2/x,x)
```

output
```
x**4/4 - 8*x**3/3 + 11*x**2 - 24*x + 9*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{(3-4x+x^2)^2}{x} \, dx = \frac{1}{4}\,x^4 - \frac{8}{3}\,x^3 + 11\,x^2 - 24\,x + 9\,\log\left(x\right)$$

input
```
integrate((x^2-4*x+3)^2/x,x, algorithm="maxima")
```

output
```
1/4*x^4 - 8/3*x^3 + 11*x^2 - 24*x + 9*log(x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(3-4x+x^2)^2}{x} \, dx = \frac{1}{4}\,x^4 - \frac{8}{3}\,x^3 + 11\,x^2 - 24\,x + 9\,\log\left(|x|\right)$$

input
```
integrate((x^2-4*x+3)^2/x,x, algorithm="giac")
```

output
```
1/4*x^4 - 8/3*x^3 + 11*x^2 - 24*x + 9*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{(3-4x+x^2)^2}{x} \, dx = 9\,\ln\left(x\right) - 24\,x + 11\,x^2 - \frac{8\,x^3}{3} + \frac{x^4}{4}$$

input
```
int((x^2 - 4*x + 3)^2/x,x)
```

output
```
9*log(x) - 24*x + 11*x^2 - (8*x^3)/3 + x^4/4
```

Reduce [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{(3-4x+x^2)^2}{x}\,dx = 9\log(x)+\frac{x^4}{4}-\frac{8x^3}{3}+11x^2-24x$$

input
```
int((x^2-4*x+3)^2/x,x)
```

output
```
(108*log(x) + 3*x**4 - 32*x**3 + 132*x**2 - 288*x)/12
```

3.220 $\int \frac{(3-4x+x^2)^2}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \frac{(3-4x+x^2)^2}{x^2}\,dx = -\frac{9}{x}+22x-4x^2+\frac{x^3}{3}-24\log(x)$$

output
```
-9/x+22*x-4*x^2+1/3*x^3-24*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(3-4x+x^2)^2}{x^2}\,dx = -\frac{9}{x}+22x-4x^2+\frac{x^3}{3}-24\log(x)$$

input
```
Integrate[(3 - 4*x + x^2)^2/x^2,x]
```

output
```
-9/x + 22*x - 4*x^2 + x^3/3 - 24*Log[x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2-4x+3)^2}{x^2} \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(x^2+\frac{9}{x^2}-8x-\frac{24}{x}+22\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3}{3}-4x^2+22x-\frac{9}{x}-24\log(x)$$

input
```
Int[(3 - 4*x + x^2)^2/x^2,x]
```

output
```
-9/x + 22*x - 4*x^2 + x^3/3 - 24*Log[x]
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{9}{x}+22x-4x^{2}+\frac{x^{3}}{3}-24\ln\left(x\right)$	24
risch	$-\frac{9}{x}+22x-4x^{2}+\frac{x^{3}}{3}-24\ln\left(x\right)$	24
norman	$\frac{-9+22x^{2}-4x^{3}+\frac{1}{3}x^{4}}{x}-24\ln\left(x\right)$	27
parallelrisch	$-\frac{-x^{4}+12x^{3}+72\ln\left(x\right)x-66x^{2}+27}{3x}$	28

input
```
int((x^2-4*x+3)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-9/x+22*x-4*x^2+1/3*x^3-24*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\left(3-4x+x^{2}\right)^{2}}{x^{2}}\,dx=\frac{x^{4}-12\,x^{3}+66\,x^{2}-72\,x\log\left(x\right)-27}{3\,x}$$

input
```
integrate((x^2-4*x+3)^2/x^2,x, algorithm="fricas")
```

output
```
1/3*(x^4 - 12*x^3 + 66*x^2 - 72*x*log(x) - 27)/x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{\left(3-4x+x^{2}\right)^{2}}{x^{2}}\,dx=\frac{x^{3}}{3}-4x^{2}+22x-24\log\left(x\right)-\frac{9}{x}$$

input
```
integrate((x**2-4*x+3)**2/x**2,x)
```

output
```
x**3/3 - 4*x**2 + 22*x - 24*log(x) - 9/x
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(3-4x+x^2)^2}{x^2}\,dx = \frac{1}{3}\,x^3 - 4\,x^2 + 22\,x - \frac{9}{x} - 24\,\log\left(x\right)$$

input
```
integrate((x^2-4*x+3)^2/x^2,x, algorithm="maxima")
```

output
```
1/3*x^3 - 4*x^2 + 22*x - 9/x - 24*log(x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(3-4x+x^2)^2}{x^2}\,dx = \frac{1}{3}\,x^3 - 4\,x^2 + 22\,x - \frac{9}{x} - 24\,\log\left(|x|\right)$$

input
```
integrate((x^2-4*x+3)^2/x^2,x, algorithm="giac")
```

output
```
1/3*x^3 - 4*x^2 + 22*x - 9/x - 24*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(3-4x+x^2)^2}{x^2}\,dx = 22\,x - 24\,\ln\left(x\right) - \frac{9}{x} - 4\,x^2 + \frac{x^3}{3}$$

input
```
int((x^2 - 4*x + 3)^2/x^2,x)
```

output
```
22*x - 24*log(x) - 9/x - 4*x^2 + x^3/3
```

Reduce [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(3-4x+x^2)^2}{x^2}\,dx = \frac{-72\log(x)\,x + x^4 - 12x^3 + 66x^2 - 27}{3x}$$

input
```
int((x^2-4*x+3)^2/x^2,x)
```

output
```
( - 72*log(x)*x + x**4 - 12*x**3 + 66*x**2 - 27)/(3*x)
```

3.221 $\int \frac{(3-4x+x^2)^2}{x^3} dx$

Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \frac{(3-4x+x^2)^2}{x^3} dx = -\frac{9}{2x^2} + \frac{24}{x} - 8x + \frac{x^2}{2} + 22\log(x)$$

output
```
-9/2/x^2+24/x-8*x+1/2*x^2+22*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(3-4x+x^2)^2}{x^3} dx = -\frac{9}{2x^2} + \frac{24}{x} - 8x + \frac{x^2}{2} + 22\log(x)$$

input
```
Integrate[(3 - 4*x + x^2)^2/x^3,x]
```

output
```
-9/(2*x^2) + 24/x - 8*x + x^2/2 + 22*Log[x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2-4x+3)^2}{x^3}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{9}{x^3}-\frac{24}{x^2}+x+\frac{22}{x}-8\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2}{2}-\frac{9}{2x^2}-8x+\frac{24}{x}+22\log(x)$$

input
```
Int[(3 - 4*x + x^2)^2/x^3,x]
```

output
```
-9/(2*x^2) + 24/x - 8*x + x^2/2 + 22*Log[x]
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
risch	$-8x+\frac{x^2}{2}+\frac{24x-\frac{9}{2}}{x^2}+22\ln(x)$	23
default	$-\frac{9}{2x^2}+\frac{24}{x}-8x+\frac{x^2}{2}+22\ln(x)$	24
norman	$\frac{-\frac{9}{2}+24x-8x^3+\frac{1}{2}x^4}{x^2}+22\ln(x)$	25
parallelrisch	$\frac{x^4+44\ln(x)x^2-16x^3-9+48x}{2x^2}$	26

input
```
int((x^2-4*x+3)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-8*x+1/2*x^2+(24*x-9/2)/x^2+22*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(3-4x+x^2)^2}{x^3}\,dx=\frac{x^4-16\,x^3+44\,x^2\log(x)+48\,x-9}{2\,x^2}$$

input
```
integrate((x^2-4*x+3)^2/x^3,x, algorithm="fricas")
```

output
```
1/2*(x^4 - 16*x^3 + 44*x^2*log(x) + 48*x - 9)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{(3-4x+x^2)^2}{x^3}\,dx=\frac{x^2}{2}-8x+22\log(x)+\frac{48x-9}{2x^2}$$

input
```
integrate((x**2-4*x+3)**2/x**3,x)
```

output
```
x**2/2 - 8*x + 22*log(x) + (48*x - 9)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{(3-4x+x^2)^2}{x^3}\,dx = \frac{1}{2}\,x^2 - 8\,x + \frac{3\,(16\,x-3)}{2\,x^2} + 22\,\log\left(x\right)$$

input
```
integrate((x^2-4*x+3)^2/x^3,x, algorithm="maxima")
```

output
```
1/2*x^2 - 8*x + 3/2*(16*x - 3)/x^2 + 22*log(x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(3-4x+x^2)^2}{x^3}\,dx = \frac{1}{2}\,x^2 - 8\,x + \frac{3\,(16\,x-3)}{2\,x^2} + 22\,\log\left(|x|\right)$$

input
```
integrate((x^2-4*x+3)^2/x^3,x, algorithm="giac")
```

output
```
1/2*x^2 - 8*x + 3/2*(16*x - 3)/x^2 + 22*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{(3-4x+x^2)^2}{x^3}\,dx = 22\,\ln\left(x\right) - 8\,x + \frac{24\,x-\frac{9}{2}}{x^2} + \frac{x^2}{2}$$

input
```
int((x^2 - 4*x + 3)^2/x^3,x)
```

output
```
22*log(x) - 8*x + (24*x - 9/2)/x^2 + x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(3-4x+x^2)^2}{x^3}\,dx=\frac{44\log(x)\,x^2+x^4-16x^3+48x-9}{2x^2}$$

input
```
int((x^2-4*x+3)^2/x^3,x)
```

output
```
(44*log(x)*x**2 + x**4 - 16*x**3 + 48*x - 9)/(2*x**2)
```

3.222 $\int \frac{(3-4x+x^2)^2}{x^4} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 21

$$\int \frac{(3-4x+x^2)^2}{x^4} \, dx = -\frac{3}{x^3} + \frac{12}{x^2} - \frac{22}{x} + x - 8\log(x)$$

output
```
-3/x^3+12/x^2-22/x+x-8*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(3-4x+x^2)^2}{x^4} \, dx = -\frac{3}{x^3} + \frac{12}{x^2} - \frac{22}{x} + x - 8\log(x)$$

input
```
Integrate[(3 - 4*x + x^2)^2/x^4,x]
```

output
```
-3/x^3 + 12/x^2 - 22/x + x - 8*Log[x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2-4x+3)^2}{x^4} \, dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int \left(\frac{9}{x^4}-\frac{24}{x^3}+\frac{22}{x^2}-\frac{8}{x}+1\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{3}{x^3}+\frac{12}{x^2}+x-\frac{22}{x}-8\log(x)$$

input
```
Int[(3 - 4*x + x^2)^2/x^4,x]
```

output
```
-3/x^3 + 12/x^2 - 22/x + x - 8*Log[x]
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

method	result	size
risch	$x+\frac{-22x^2+12x-3}{x^3}-8\ln(x)$	21
default	$-\frac{3}{x^3}+\frac{12}{x^2}-\frac{22}{x}+x-8\ln(x)$	22
norman	$\frac{x^4-22x^2+12x-3}{x^3}-8\ln(x)$	23
parallelrisch	$-\frac{8\ln(x)x^3-x^4+3+22x^2-12x}{x^3}$	28

input
```
int((x^2-4*x+3)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
x+(-22*x^2+12*x-3)/x^3-8*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{(3-4x+x^2)^2}{x^4}\,dx = \frac{x^4-8\,x^3\log(x)-22\,x^2+12\,x-3}{x^3}$$

input
```
integrate((x^2-4*x+3)^2/x^4,x, algorithm="fricas")
```

output
```
(x^4 - 8*x^3*log(x) - 22*x^2 + 12*x - 3)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(3-4x+x^2)^2}{x^4}\,dx = x-8\log(x)+\frac{-22x^2+12x-3}{x^3}$$

input
```
integrate((x**2-4*x+3)**2/x**4,x)
```

output
```
x - 8*log(x) + (-22*x**2 + 12*x - 3)/x**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(3-4x+x^2)^2}{x^4}\,dx = x - \frac{22\,x^2 - 12\,x + 3}{x^3} - 8\,\log\left(x\right)$$

input
```
integrate((x^2-4*x+3)^2/x^4,x, algorithm="maxima")
```

output
```
x - (22*x^2 - 12*x + 3)/x^3 - 8*log(x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{(3-4x+x^2)^2}{x^4}\,dx = x - \frac{22\,x^2 - 12\,x + 3}{x^3} - 8\,\log\left(|x|\right)$$

input
```
integrate((x^2-4*x+3)^2/x^4,x, algorithm="giac")
```

output
```
x - (22*x^2 - 12*x + 3)/x^3 - 8*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 8.78 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(3-4x+x^2)^2}{x^4}\,dx = x - 8\,\ln\left(x\right) - \frac{22\,x^2 - 12\,x + 3}{x^3}$$

input
```
int((x^2 - 4*x + 3)^2/x^4,x)
```

output
```
x - 8*log(x) - (22*x^2 - 12*x + 3)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{(3-4x+x^2)^2}{x^4}\,dx = \frac{-8\log(x)\,x^3+x^4-22x^2+12x-3}{x^3}$$

input
```
int((x^2-4*x+3)^2/x^4,x)
```

output
```
( - 8*log(x)*x**3 + x**4 - 22*x**2 + 12*x - 3)/x**3
```

3.223 $\int \frac{(3-4x+x^2)^2}{x^5} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \frac{(3-4x+x^2)^2}{x^5} \, dx = -\frac{9}{4x^4} + \frac{8}{x^3} - \frac{11}{x^2} + \frac{8}{x} + \log(x)$$

output
```
-9/4/x^4+8/x^3-11/x^2+8/x+ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(3-4x+x^2)^2}{x^5} \, dx = -\frac{9}{4x^4} + \frac{8}{x^3} - \frac{11}{x^2} + \frac{8}{x} + \log(x)$$

input
```
Integrate[(3 - 4*x + x^2)^2/x^5,x]
```

output
```
-9/(4*x^4) + 8/x^3 - 11/x^2 + 8/x + Log[x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2-4x+3)^2}{x^5} \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{9}{x^5}-\frac{24}{x^4}+\frac{22}{x^3}-\frac{8}{x^2}+\frac{1}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{9}{4x^4}+\frac{8}{x^3}-\frac{11}{x^2}+\frac{8}{x}+\log(x)$$

input
```
Int[(3 - 4*x + x^2)^2/x^5,x]
```

output
```
-9/(4*x^4) + 8/x^3 - 11/x^2 + 8/x + Log[x]
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

method	result	size
norman	$\frac{-\frac{9}{4}+8x^3-11x^2+8x}{x^4}+\ln\left(x\right)$	23
risch	$\frac{-\frac{9}{4}+8x^3-11x^2+8x}{x^4}+\ln\left(x\right)$	23
default	$-\frac{9}{4x^4}+\frac{8}{x^3}-\frac{11}{x^2}+\frac{8}{x}+\ln\left(x\right)$	24
parallelrisch	$\frac{4\ln(x)x^4-9+32x^3-44x^2+32x}{4x^4}$	28

input
```
int((x^2-4*x+3)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
(-9/4+8*x^3-11*x^2+8*x)/x^4+ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\left(3-4x+x^2\right)^2}{x^5}\,dx=\frac{4\,x^4\log\left(x\right)+32\,x^3-44\,x^2+32\,x-9}{4\,x^4}$$

input
```
integrate((x^2-4*x+3)^2/x^5,x, algorithm="fricas")
```

output
```
1/4*(4*x^4*log(x) + 32*x^3 - 44*x^2 + 32*x - 9)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{\left(3-4x+x^2\right)^2}{x^5}\,dx=\log\left(x\right)+\frac{32x^3-44x^2+32x-9}{4x^4}$$

input
```
integrate((x**2-4*x+3)**2/x**5,x)
```

output
```
log(x) + (32*x**3 - 44*x**2 + 32*x - 9)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(3-4x+x^2)^2}{x^5}\,dx = \frac{32\,x^3-44\,x^2+32\,x-9}{4\,x^4}+\log{(x)}$$

input
```
integrate((x^2-4*x+3)^2/x^5,x, algorithm="maxima")
```

output
```
1/4*(32*x^3 - 44*x^2 + 32*x - 9)/x^4 + log(x)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(3-4x+x^2)^2}{x^5}\,dx = \frac{32\,x^3-44\,x^2+32\,x-9}{4\,x^4}+\log{(|x|)}$$

input
```
integrate((x^2-4*x+3)^2/x^5,x, algorithm="giac")
```

output
```
1/4*(32*x^3 - 44*x^2 + 32*x - 9)/x^4 + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 8.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(3-4x+x^2)^2}{x^5}\,dx = \ln{(x)}+\frac{8\,x^3-11\,x^2+8\,x-\frac{9}{4}}{x^4}$$

input
```
int((x^2 - 4*x + 3)^2/x^5,x)
```

output
```
log(x) + (8*x - 11*x^2 + 8*x^3 - 9/4)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(3-4x+x^2)^2}{x^5}\,dx = \frac{4\log(x)\,x^4+32x^3-44x^2+32x-9}{4x^4}$$

input
```
int((x^2-4*x+3)^2/x^5,x)
```

output
```
(4*log(x)*x**4 + 32*x**3 - 44*x**2 + 32*x - 9)/(4*x**4)
```

3.224 $\int \frac{(3-4x+x^2)^2}{x^6} dx$

Optimal result

Integrand size = 14, antiderivative size = 30

$$\int \frac{(3-4x+x^2)^2}{x^6} dx = -\frac{9}{5x^5} + \frac{6}{x^4} - \frac{22}{3x^3} + \frac{4}{x^2} - \frac{1}{x}$$

output
```
-9/5/x^5+6/x^4-22/3/x^3+4/x^2-1/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(3-4x+x^2)^2}{x^6} dx = -\frac{9}{5x^5} + \frac{6}{x^4} - \frac{22}{3x^3} + \frac{4}{x^2} - \frac{1}{x}$$

input
```
Integrate[(3 - 4*x + x^2)^2/x^6,x]
```

output
```
-9/(5*x^5) + 6/x^4 - 22/(3*x^3) + 4/x^2 - x^(-1)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2-4x+3)^2}{x^6}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{9}{x^6}-\frac{24}{x^5}+\frac{22}{x^4}-\frac{8}{x^3}+\frac{1}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{9}{5x^5}+\frac{6}{x^4}-\frac{22}{3x^3}+\frac{4}{x^2}-\frac{1}{x}$$

input
```
Int[(3 - 4*x + x^2)^2/x^6,x]
```

output
```
-9/(5*x^5) + 6/x^4 - 22/(3*x^3) + 4/x^2 - x^(-1)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
norman	$\frac{-x^4+4x^3-\frac{22}{3}x^2+6x-\frac{9}{5}}{x^5}$	25
risch	$\frac{-x^4+4x^3-\frac{22}{3}x^2+6x-\frac{9}{5}}{x^5}$	25
gosper	$-\frac{15x^4-60x^3+110x^2-90x+27}{15x^5}$	26
parallelrisch	$\frac{-15x^4+60x^3-110x^2+90x-27}{15x^5}$	26
default	$-\frac{9}{5x^5}+\frac{6}{x^4}-\frac{22}{3x^3}+\frac{4}{x^2}-\frac{1}{x}$	27
orering	$-\frac{(15x^4-60x^3+110x^2-90x+27)(x^2-4x+3)^2}{15x^5(x-1)^2(-3+x)^2}$	46

input
```
int((x^2-4*x+3)^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
(-x^4+4*x^3-22/3*x^2+6*x-9/5)/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{(3-4x+x^2)^2}{x^6}\,dx = -\frac{15\,x^4-60\,x^3+110\,x^2-90\,x+27}{15\,x^5}$$

input
```
integrate((x^2-4*x+3)^2/x^6,x, algorithm="fricas")
```

output
```
-1/15*(15*x^4 - 60*x^3 + 110*x^2 - 90*x + 27)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(3-4x+x^2)^2}{x^6}\,dx = \frac{-15x^4+60x^3-110x^2+90x-27}{15x^5}$$

input
```
integrate((x**2-4*x+3)**2/x**6,x)
```

output
```
(-15*x**4 + 60*x**3 - 110*x**2 + 90*x - 27)/(15*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{(3-4x+x^2)^2}{x^6}\,dx = -\frac{15\,x^4-60\,x^3+110\,x^2-90\,x+27}{15\,x^5}$$

input
```
integrate((x^2-4*x+3)^2/x^6,x, algorithm="maxima")
```

output
```
-1/15*(15*x^4 - 60*x^3 + 110*x^2 - 90*x + 27)/x^5
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{(3-4x+x^2)^2}{x^6}\,dx = -\frac{15\,x^4-60\,x^3+110\,x^2-90\,x+27}{15\,x^5}$$

input
```
integrate((x^2-4*x+3)^2/x^6,x, algorithm="giac")
```

output
```
-1/15*(15*x^4 - 60*x^3 + 110*x^2 - 90*x + 27)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{(3-4x+x^2)^2}{x^6}\,dx = -\frac{x^4-4\,x^3+\frac{22\,x^2}{3}-6\,x+\frac{9}{5}}{x^5}$$

input
```
int((x^2 - 4*x + 3)^2/x^6,x)
```

output
```
-((22*x^2)/3 - 6*x - 4*x^3 + x^4 + 9/5)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{(3-4x+x^2)^2}{x^6}\,dx = \frac{-15x^4+60x^3-110x^2+90x-27}{15x^5}$$

input
```
int((x^2-4*x+3)^2/x^6,x)
```

output
```
( - 15*x**4 + 60*x**3 - 110*x**2 + 90*x - 27)/(15*x**5)
```

3.225 $\int \frac{(3-4x+x^2)^2}{x^7} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 36

$$\int \frac{(3-4x+x^2)^2}{x^7} \, dx = -\frac{3}{2x^6} + \frac{24}{5x^5} - \frac{11}{2x^4} + \frac{8}{3x^3} - \frac{1}{2x^2}$$

output
```
-3/2/x^6+24/5/x^5-11/2/x^4+8/3/x^3-1/2/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{(3-4x+x^2)^2}{x^7} \, dx = -\frac{3}{2x^6} + \frac{24}{5x^5} - \frac{11}{2x^4} + \frac{8}{3x^3} - \frac{1}{2x^2}$$

input
```
Integrate[(3 - 4*x + x^2)^2/x^7,x]
```

output
```
-3/(2*x^6) + 24/(5*x^5) - 11/(2*x^4) + 8/(3*x^3) - 1/(2*x^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2-4x+3)^2}{x^7} \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{9}{x^7}-\frac{24}{x^6}+\frac{22}{x^5}-\frac{8}{x^4}+\frac{1}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{2x^6}+\frac{24}{5x^5}-\frac{11}{2x^4}+\frac{8}{3x^3}-\frac{1}{2x^2}$$

input
```
Int[(3 - 4*x + x^2)^2/x^7,x]
```

output
```
-3/(2*x^6) + 24/(5*x^5) - 11/(2*x^4) + 8/(3*x^3) - 1/(2*x^2)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
norman	$\frac{-\frac{1}{2}x^4+\frac{8}{3}x^3-\frac{11}{2}x^2+\frac{24}{5}x-\frac{3}{2}}{x^6}$	25
risch	$\frac{-\frac{1}{2}x^4+\frac{8}{3}x^3-\frac{11}{2}x^2+\frac{24}{5}x-\frac{3}{2}}{x^6}$	25
gosper	$-\frac{15x^4-80x^3+165x^2-144x+45}{30x^6}$	26
parallelrisch	$\frac{-15x^4+80x^3-165x^2+144x-45}{30x^6}$	26
default	$-\frac{3}{2x^6}+\frac{24}{5x^5}-\frac{11}{2x^4}+\frac{8}{3x^3}-\frac{1}{2x^2}$	27
orering	$-\frac{(15x^4-80x^3+165x^2-144x+45)(x^2-4x+3)^2}{30x^6(x-1)^2(-3+x)^2}$	46

input
```
int((x^2-4*x+3)^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*x^4+8/3*x^3-11/2*x^2+24/5*x-3/2)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \frac{(3-4x+x^2)^2}{x^7}\,dx = -\frac{15\,x^4-80\,x^3+165\,x^2-144\,x+45}{30\,x^6}$$

input
```
integrate((x^2-4*x+3)^2/x^7,x, algorithm="fricas")
```

output
```
-1/30*(15*x^4 - 80*x^3 + 165*x^2 - 144*x + 45)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{(3-4x+x^2)^2}{x^7}\,dx = \frac{-15x^4+80x^3-165x^2+144x-45}{30x^6}$$

input
```
integrate((x**2-4*x+3)**2/x**7,x)
```

output
```
(-15*x**4 + 80*x**3 - 165*x**2 + 144*x - 45)/(30*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \frac{(3-4x+x^2)^2}{x^7}\,dx = -\frac{15\,x^4-80\,x^3+165\,x^2-144\,x+45}{30\,x^6}$$

input
```
integrate((x^2-4*x+3)^2/x^7,x, algorithm="maxima")
```

output
```
-1/30*(15*x^4 - 80*x^3 + 165*x^2 - 144*x + 45)/x^6
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \frac{(3-4x+x^2)^2}{x^7}\,dx = -\frac{15\,x^4-80\,x^3+165\,x^2-144\,x+45}{30\,x^6}$$

input
```
integrate((x^2-4*x+3)^2/x^7,x, algorithm="giac")
```

output
```
-1/30*(15*x^4 - 80*x^3 + 165*x^2 - 144*x + 45)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \frac{(3-4x+x^2)^2}{x^7}\,dx = -\frac{15\,x^4-80\,x^3+165\,x^2-144\,x+45}{30\,x^6}$$

input
```
int((x^2 - 4*x + 3)^2/x^7,x)
```

output
```
-(165*x^2 - 144*x - 80*x^3 + 15*x^4 + 45)/(30*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \frac{(3-4x+x^2)^2}{x^7}\,dx = \frac{-15x^4+80x^3-165x^2+144x-45}{30x^6}$$

input
```
int((x^2-4*x+3)^2/x^7,x)
```

output
```
( - 15*x**4 + 80*x**3 - 165*x**2 + 144*x - 45)/(30*x**6)
```

3.226 $\int \frac{1+x+x^2}{x} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 11

$$\int \frac{1+x+x^2}{x} \, dx = x + \frac{x^2}{2} + \log(x)$$

output
```
x+1/2*x^2+ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{x} \, dx = x + \frac{x^2}{2} + \log(x)$$

input
```
Integrate[(1 + x + x^2)/x,x]
```

output
```
x + x^2/2 + Log[x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x+1}{x} \, dx$$
$$\Big\downarrow \text{1140}$$
$$\int \left(x+\frac{1}{x}+1\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{x^2}{2}+x+\log(x)$$

input
```
Int[(1 + x + x^2)/x,x]
```

output
```
x + x^2/2 + Log[x]
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
default	$x+\frac{x^2}{2}+\ln(x)$	10
norman	$x+\frac{x^2}{2}+\ln(x)$	10
risch	$x+\frac{x^2}{2}+\ln(x)$	10
parallelrisch	$x+\frac{x^2}{2}+\ln(x)$	10

input
```
int((x^2+x+1)/x,x,method=_RETURNVERBOSE)
```

output
```
x+1/2*x^2+ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1+x+x^2}{x}\,dx=\frac{1}{2}\,x^2+x+\log(x)$$

input
```
integrate((x^2+x+1)/x,x, algorithm="fricas")
```

output
```
1/2*x^2 + x + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1+x+x^2}{x}\,dx=\frac{x^2}{2}+x+\log(x)$$

input
```
integrate((x**2+x+1)/x,x)
```

output
```
x**2/2 + x + log(x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1+x+x^2}{x}\,dx = \frac{1}{2}\,x^2 + x + \log\left(x\right)$$

input
```
integrate((x^2+x+1)/x,x, algorithm="maxima")
```

output
```
1/2*x^2 + x + log(x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1+x+x^2}{x}\,dx = \frac{1}{2}\,x^2 + x + \log\left(|x|\right)$$

input
```
integrate((x^2+x+1)/x,x, algorithm="giac")
```

output
```
1/2*x^2 + x + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1+x+x^2}{x}\,dx = x + \ln\left(x\right) + \frac{x^2}{2}$$

input
```
int((x + x^2 + 1)/x,x)
```

output
```
x + log(x) + x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1+x+x^2}{x}\,dx = \log(x) + \frac{x^2}{2} + x$$

input
```
int((x^2+x+1)/x,x)
```

output
```
(2*log(x) + x**2 + 2*x)/2
```

3.227 $\int \frac{x^4}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 118

$$\int \frac{x^4}{a+bx+cx^2}\,dx=\frac{(b^2-ac)\,x}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}-\frac{(b^4-4ab^2c+2a^2c^2)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}}-\frac{b(b^2-2ac)\log\left(a+bx+cx^2\right)}{2c^4}$$

output
```
(-a*c+b^2)*x/c^3-1/2*b*x^2/c^2+1/3*x^3/c-(2*a^2*c^2-4*a*b^2*c+b^4)*arctanh
((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^4/(-4*a*c+b^2)^(1/2)-1/2*b*(-2*a*c+b^2)*l
n(c*x^2+b*x+a)/c^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.95

$$\int \frac{x^4}{a+bx+cx^2}\,dx=\frac{cx(6b^2-6ac-3bcx+2c^2x^2)+\frac{6(b^4-4ab^2c+2a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-3(b^3-2abc)\log(a+x(b+cx))}{6c^4}$$

input
```
Integrate[x^4/(a + b*x + c*x^2),x]
```

output
```
(c*x*(6*b^2 - 6*a*c - 3*b*c*x + 2*c^2*x^2) + (6*(b^4 - 4*a*b^2*c + 2*a^2*c
^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 3*(b^3 -
2*a*b*c)*Log[a + x*(b + c*x)])/(6*c^4)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{a+bx+cx^2} \, dx$$

$$\Big\downarrow \text{1143}$$

$$\int \left(-\frac{bx(b^2-2ac)+a(b^2-ac)}{c^3\,(a+bx+cx^2)}+\frac{b^2-ac}{c^3}-\frac{bx}{c^2}+\frac{x^2}{c}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(2a^2c^2-4ab^2c+b^4)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}}-\frac{b(b^2-2ac)\log\left(a+bx+cx^2\right)}{2c^4}+\frac{x(b^2-ac)}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}$$

input
```
Int[x^4/(a + b*x + c*x^2),x]
```

output
```
((b^2 - a*c)*x)/c^3 - (b*x^2)/(2*c^2) + x^3/(3*c) - ((b^4 - 4*a*b^2*c + 2*
a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*Sqrt[b^2 - 4*a*c]) -
 (b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*c^4)
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{-\frac{1}{3}x^3c^2+\frac{1}{2}bc\,x^2+acx-b^2x}{c^3}+\frac{\frac{\left(2abc-b^3\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(a^2c-a\,b^2-\frac{\left(2abc-b^3\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c^3}$	128
risch	Expression too large to display	1138

input
```
int(x^4/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/c^3*(-1/3*x^3*c^2+1/2*b*c*x^2+a*c*x-b^2*x)+1/c^3*(1/2*(2*a*b*c-b^3)/c*l
n(c*x^2+b*x+a)+2*(a^2*c-a*b^2-1/2*(2*a*b*c-b^3)*b/c)/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.25

$$\int \frac{x^4}{a+bx+cx^2}\,dx$$
$$= \left[\frac{2\,(b^2c^3-4\,ac^4)x^3-3\,(b^3c^2-4\,abc^3)x^2+3\,(b^4-4\,ab^2c+2\,a^2c^2)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-}}{cx^2+bx+a}\right.}{6\,(b^2c^4-4\,ac^5)}\right.$$

input
```
integrate(x^4/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[1/6*(2*(b^2*c^3 - 4*a*c^4)*x^3 - 3*(b^3*c^2 - 4*a*b*c^3)*x^2 + 3*(b^4 - 4
*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2
*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 6*(b^4*c - 5*a*
b^2*c^2 + 4*a^2*c^3)*x - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*log(c*x^2 + b*x
 + a))/(b^2*c^4 - 4*a*c^5), 1/6*(2*(b^2*c^3 - 4*a*c^4)*x^3 - 3*(b^3*c^2 -
4*a*b*c^3)*x^2 - 6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*arctan
(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 6*(b^4*c - 5*a*b^2*c^2 +
 4*a^2*c^3)*x - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*log(c*x^2 + b*x + a))/(b
^2*c^4 - 4*a*c^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(110) = 220$.

Time = 0.55 (sec) , antiderivative size = 498, normalized size of antiderivative = 4.22

$$\int \frac{x^4}{a+bx+cx^2}\,dx = -\frac{bx^2}{2c^2} + x\left(-\frac{a}{c^2}+\frac{b^2}{c^3}\right) + \left(\frac{b(2ac-b^2)}{2c^4} - \frac{\sqrt{-4ac+b^2}\cdot(2a^2c^2-4ab^2c+b^4)}{2c^4\cdot(4ac-b^2)}\right)\log\left(x + \frac{-3a^2bc+ab^3+4ac^4\left(\frac{b(2ac-b^2)}{2c^4} - \frac{\sqrt{-4ac+b^2}\cdot(2a^2c^2-4ab^2c+}{2c^4\cdot(4ac-b^2)}\right.}{2a^2c^2-4ab^2}\right. + \left(\frac{b(2ac-b^2)}{2c^4} + \frac{\sqrt{-4ac+b^2}\cdot(2a^2c^2-4ab^2c+b^4)}{2c^4\cdot(4ac-b^2)}\right)\log\left(x + \frac{-3a^2bc+ab^3+4ac^4\left(\frac{b(2ac-b^2)}{2c^4} + \frac{\sqrt{-4ac+b^2}\cdot(2a^2c^2-4ab^2c+}{2c^4\cdot(4ac-b^2)}\right.}{2a^2c^2-4ab^2}\right. + \frac{x^3}{3c}$$

input
```
integrate(x**4/(c*x**2+b*x+a),x)
```

output
```
-b*x**2/(2*c**2) + x*(-a/c**2 + b**2/c**3) + (b*(2*a*c - b**2)/(2*c**4) -
sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**
2)))*log(x + (-3*a**2*b*c + a*b**3 + 4*a*c**4*(b*(2*a*c - b**2)/(2*c**4) -
 sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b*
*2))) - b**2*c**3*(b*(2*a*c - b**2)/(2*c**4) - sqrt(-4*a*c + b**2)*(2*a**2
*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2))))/(2*a**2*c**2 - 4*a*b*
*2*c + b**4)) + (b*(2*a*c - b**2)/(2*c**4) + sqrt(-4*a*c + b**2)*(2*a**2*c
**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2)))*log(x + (-3*a**2*b*c + a
*b**3 + 4*a*c**4*(b*(2*a*c - b**2)/(2*c**4) + sqrt(-4*a*c + b**2)*(2*a**2*
c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2))) - b**2*c**3*(b*(2*a*c -
 b**2)/(2*c**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2
*c**4*(4*a*c - b**2))))/(2*a**2*c**2 - 4*a*b**2*c + b**4)) + x**3/(3*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96

$$\int \frac{x^4}{a+bx+cx^2}\,dx = \frac{2\,c^2x^3 - 3\,bcx^2 + 6\,b^2x - 6\,acx}{6\,c^3} - \frac{(b^3 - 2\,abc)\log\left(cx^2 + bx + a\right)}{2\,c^4} + \frac{(b^4 - 4\,ab^2c + 2\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^4}$$

input
```
integrate(x^4/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/6*(2*c^2*x^3 - 3*b*c*x^2 + 6*b^2*x - 6*a*c*x)/c^3 - 1/2*(b^3 - 2*a*b*c)*
log(c*x^2 + b*x + a)/c^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*arctan((2*c*x + b
)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.28

$$\int \frac{x^4}{a+bx+cx^2}\,dx = \frac{x^3}{3\,c}-x\left(\frac{a}{c^2}-\frac{b^2}{c^3}\right)-\frac{b\,x^2}{2\,c^2}+\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(8\,a^2\,b\,c^2-6\,a\,b^3\,c+b^5\right)}{2\,\left(4\,a\,c^5-b^2\,c^4\right)}+\frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(2\,a^2\,c^2-4\,a\,b^2\,c+b^4\right)}{c^4\,\sqrt{4\,a\,c-b^2}}$$

input
```
int(x^4/(a + b*x + c*x^2),x)
```

output
```
x^3/(3*c) - x*(a/c^2 - b^2/c^3) - (b*x^2)/(2*c^2) + (log(a + b*x + c*x^2)*
(b^5 + 8*a^2*b*c^2 - 6*a*b^3*c))/(2*(4*a*c^5 - b^2*c^4)) + (atan(b/(4*a*c
- b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2))*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))
/(c^4*(4*a*c - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.16

$$\int \frac{x^4}{a+bx+cx^2}\,dx = \frac{12\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2c^2-24\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^2c+6\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^4+\cdots}{}$$

input
```
int(x^4/(c*x^2+b*x+a),x)
```

output

```
(12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*c**2 - 24
*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c + 6*sqrt
(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**4 + 24*log(a + b*x
+ c*x**2)*a**2*b*c**2 - 18*log(a + b*x + c*x**2)*a*b**3*c + 3*log(a + b*x
+ c*x**2)*b**5 - 24*a**2*c**3*x + 30*a*b**2*c**2*x - 12*a*b*c**3*x**2 + 8*
a*c**4*x**3 - 6*b**4*c*x + 3*b**3*c**2*x**2 - 2*b**2*c**3*x**3)/(6*c**4*(4
*a*c - b**2))
```

3.228 $\int \frac{x^3}{a+bx+cx^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 89

$$\int \frac{x^3}{a+bx+cx^2} \, dx = -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{b(b^2-3ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}} + \frac{(b^2-ac)\log(a+bx+cx^2)}{2c^3}$$

output
```
-b*x/c^2+1/2*x^2/c+b*(-3*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^
3/(-4*a*c+b^2)^(1/2)+1/2*(-a*c+b^2)*ln(c*x^2+b*x+a)/c^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{a+bx+cx^2} \, dx = \frac{cx(-2b+cx)-\frac{2b(b^2-3ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(b^2-ac)\log(a+x(b+cx))}{2c^3}$$

input
```
Integrate[x^3/(a + b*x + c*x^2),x]
```

output
```
(c*x*(-2*b + c*x) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + x*(b + c*x)])/(2*c^3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+bx+cx^2} \, dx$$

$$\Big\downarrow \text{1143}$$

$$\int \left(\frac{x\left(b^2-ac\right)+ab}{c^2\left(a+bx+cx^2\right)}-\frac{b}{c^2}+\frac{x}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(b^2-3ac\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2c^3}-\frac{bx}{c^2}+\frac{x^2}{2c}$$

input
```
Int[x^3/(a + b*x + c*x^2),x]
```

output
```
-((b*x)/c^2) + x^2/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*
c^3)
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.10

method	result
default	$-\frac{-\frac{1}{2}c\,x^2+bx}{c^2}+\frac{\frac{\left(-ac+b^2\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(ab-\frac{\left(-ac+b^2\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c^2}$
risch	$\frac{x^2}{2c}-\frac{bx}{c^2}-\frac{2\ln\left(12a^2b\,c^2-7a\,b^3c+b^5-2\sqrt{-b^2(4ac-b^2)(3ac-b^2)^2}\,cx-\sqrt{-b^2(4ac-b^2)(3ac-b^2)^2}\,b\right)a^2}{c(4ac-b^2)}+\frac{5\ln\left(12a^2b\,c^2-7a\,b^3\right.}{}$

input
```
int(x^3/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/c^2*(-1/2*c*x^2+b*x)+1/c^2*(1/2*(-a*c+b^2)/c*ln(c*x^2+b*x+a)+2*(a*b-1/2
*(-a*c+b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.34

$$\int\frac{x^3}{a+bx+cx^2}\,dx$$
$$=\left[\frac{(b^2c^2-4\,ac^3)x^2-(b^3-3\,abc)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\,(b^3c-4\,abc^2)x+}{2\,(b^2c^3-4\,ac^4)}\right.$$

input
```
integrate(x^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[1/2*((b^2*c^2 - 4*a*c^3)*x^2 - (b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c
^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b
*x + a)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x
^2 + b*x + a))/(b^2*c^3 - 4*a*c^4), 1/2*((b^2*c^2 - 4*a*c^3)*x^2 + 2*(b^3
- 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
- 4*a*c)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*
x^2 + b*x + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(83) = 166$.

Time = 0.44 (sec) , antiderivative size = 381, normalized size of antiderivative = 4.28

$$\int \frac{x^3}{a+bx+cx^2}\,dx = -\frac{bx}{c^2} + \left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right)\log\left(x + \frac{2a^2c - ab^2 + 4ac^3\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right) - b^2c^2\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)} - \frac{ac-b}{2c^3}\right.}{3abc-b^3}\right) + \left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right)\log\left(x + \frac{2a^2c - ab^2 + 4ac^3\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right) - b^2c^2\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right)}{3abc-b^3}\right) + \frac{x^2}{2c}$$

input
```
integrate(x**3/(c*x**2+b*x+a),x)
```

output
```
-b*x/c**2 + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2))
 - (a*c - b**2)/(2*c**3))*log(x + (2*a**2*c - a*b**2 + 4*a*c**3*(-b*sqrt(-
4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**
3)) - b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b*
*2)) - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c + b**2)*
(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))*log(x + (2
*a**2*c - a*b**2 + 4*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*
(4*a*c - b**2)) - (a*c - b**2)/(2*c**3)) - b**2*c**2*(b*sqrt(-4*a*c + b**2
)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3)))/(3*a*b*
c - b**3)) + x**2/(2*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97

$$\int \frac{x^3}{a+bx+cx^2}\,dx = \frac{cx^2-2\,bx}{2\,c^2} + \frac{(b^2-ac)\log\left(cx^2+bx+a\right)}{2\,c^3} - \frac{(b^3-3\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^3}$$

input
```
integrate(x^3/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/2*(c*x^2 - 2*b*x)/c^2 + 1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/c^3 - (b^3
- 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26

$$\int \frac{x^3}{a+bx+cx^2}\,dx = \frac{x^2}{2\,c} - \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(4\,a^2\,c^2-5\,a\,b^2\,c+b^4\right)}{2\,\left(4\,a\,c^4-b^2\,c^3\right)} - \frac{b\,x}{c^2} + \frac{b\,\mathrm{atan}\left(\frac{b+2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(3\,a\,c-b^2\right)}{c^3\,\sqrt{4\,a\,c-b^2}}$$

input
```
int(x^3/(a + b*x + c*x^2),x)
```

output
```
x^2/(2*c) - (log(a + b*x + c*x^2)*(b^4 + 4*a^2*c^2 - 5*a*b^2*c))/(2*(4*a*c
^4 - b^2*c^3)) - (b*x)/c^2 + (b*atan((b + 2*c*x)/(4*a*c - b^2)^(1/2))*(3*a
*c - b^2))/(c^3*(4*a*c - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.02

$$\int \frac{x^3}{a+bx+cx^2}\,dx = \frac{6\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)abc-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^3-4\log(c\,x^2+bx+a)\,a^2c^2+5\log(c\,x^2+}{2c^3\left(4ac-b^2\right)}$$

input
```
int(x^3/(c*x^2+b*x+a),x)
```

output
```
(6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c - 2*sqrt(
4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3 - 4*log(a + b*x +
c*x**2)*a**2*c**2 + 5*log(a + b*x + c*x**2)*a*b**2*c - log(a + b*x + c*x**
2)*b**4 - 8*a*b*c**2*x + 4*a*c**3*x**2 + 2*b**3*c*x - b**2*c**2*x**2)/(2*c
**3*(4*a*c - b**2))
```

3.229 $\int \frac{x^2}{a+bx+cx^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 70

$$\int \frac{x^2}{a+bx+cx^2} dx = \frac{x}{c} - \frac{(b^2-2ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}} - \frac{b\log(a+bx+cx^2)}{2c^2}$$

output
```
x/c-(-2*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1
/2)-1/2*b*ln(c*x^2+b*x+a)/c^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{a+bx+cx^2} dx = \frac{x}{c} + \frac{(b^2-2ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{c^2\sqrt{-b^2+4ac}} - \frac{b\log(a+bx+cx^2)}{2c^2}$$

input
```
Integrate[x^2/(a + b*x + c*x^2),x]
```

output
```
x/c + ((b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^
2 + 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+bx+cx^2} \, dx$$

$$\Big\downarrow \text{1143}$$

$$\int \left(\frac{1}{c}-\frac{a+bx}{c\left(a+bx+cx^2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(b^2-2ac\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

input
```
Int[x^2/(a + b*x + c*x^2),x]
```

output
```
x/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2
 - 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07

method	result
default	$\frac{x}{c}+\frac{-\frac{b\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(-a+\frac{b^2}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c}$
risch	$\frac{x}{c}-\frac{2\ln\left(-8a^2c^2+6ca\,b^2-b^4-2\sqrt{-(4ac-b^2)(2ac-b^2)^2}\,cx-\sqrt{-(4ac-b^2)(2ac-b^2)^2}\,b\right)ab}{c(4ac-b^2)}+\frac{\ln\left(-8a^2c^2+6ca\,b^2-b^4-2\sqrt{-(4a\right.}{}$

input
```
int(x^2/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
x/c+1/c*(-1/2*b/c*ln(c*x^2+b*x+a)+2*(-a+1/2/c*b^2)/(4*a*c-b^2)^(1/2)*arcta
n((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.36

$$\int \frac{x^2}{a+bx+cx^2}\,dx = \left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\,(b^2c-4\,ac^2)x+(b^3-4\,abc)\log\left(cx^2\right.}{2\,(b^2c^2-4\,ac^3)}\right.$$
$$-\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)-2\,(b^2c-4\,ac^2)x+(b^3-4\,abc)\log\left(cx^2+bx+\right.}{2\,(b^2c^2-4\,ac^3)}$$

input
```
integrate(x^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*
a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^2*c - 4*a*c
^2)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3), -1/2*(2
*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(
b^2 - 4*a*c)) - 2*(b^2*c - 4*a*c^2)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x +
a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(65) = 130$.

Time = 0.32 (sec) , antiderivative size = 306, normalized size of antiderivative = 4.37

$$\int \frac{x^2}{a+bx+cx^2}\,dx = \left(-\frac{b}{2c^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)\log\left(x + \frac{-ab-4ac^2\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)+b^2c\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2a}{2c^2\cdot(4ac-b}\cdots}{2ac-b^2}\right.$$
$$+\left(-\frac{b}{2c^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)\log\left(x + \frac{-ab-4ac^2\left(-\frac{b}{2c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)+b^2c\left(-\frac{b}{2c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2a}{2c^2\cdot(4ac-b}\cdots}{2ac-b^2}\right.$$
$$+\frac{x}{c}$$

input
```
integrate(x**2/(c*x**2+b*x+a),x)
```

output
```
(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))
*log(x + (-a*b - 4*a*c**2*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2
)/(2*c**2*(4*a*c - b**2))) + b**2*c*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*
a*c - b**2)/(2*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(2*c**2) + sqr
t(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*log(x + (-a*b - 4
*a*c**2*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c -
 b**2))) + b**2*c*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**
2*(4*a*c - b**2))))/(2*a*c - b**2)) + x/c
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{a+bx+cx^2}\,dx = \frac{x}{c} - \frac{b\log\left(cx^2+bx+a\right)}{2\,c^2} + \frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^2}$$

input
```
integrate(x^2/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
x/c - 1/2*b*log(c*x^2 + b*x + a)/c^2 + (b^2 - 2*a*c)*arctan((2*c*x + b)/sq
rt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.46

$$\int \frac{x^2}{a+bx+cx^2}\,dx = \frac{x}{c} + \frac{b^3\,\ln\left(c\,x^2+b\,x+a\right)}{2\,\left(4\,a\,c^3-b^2\,c^2\right)} - \frac{2\,a\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}} + \frac{b^2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c^2\,\sqrt{4\,a\,c-b^2}} - \frac{2\,a\,b\,c\,\ln\left(c\,x^2+b\,x+a\right)}{4\,a\,c^3-b^2\,c^2}$$

input
```
int(x^2/(a + b*x + c*x^2),x)
```

output
```
x/c + (b^3*log(a + b*x + c*x^2))/(2*(4*a*c^3 - b^2*c^2)) - (2*a*atan(b/(4*
a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) +
 (b^2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c^2*(4*a
*c - b^2)^(1/2)) - (2*a*b*c*log(a + b*x + c*x^2))/(4*a*c^3 - b^2*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.94

$$\int \frac{x^2}{a+bx+cx^2}\,dx = \frac{-4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)ac+2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^2-4\log(c\,x^2+bx+a)\,abc+\log(c\,x^2+b}{2c^2\,(4ac-b^2)}$$

input
```
int(x^2/(c*x^2+b*x+a),x)
```

output
```
( - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c + 2*sqrt
(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2 - 4*log(a + b*x +
 c*x**2)*a*b*c + log(a + b*x + c*x**2)*b**3 + 8*a*c**2*x - 2*b**2*c*x)/(2*
c**2*(4*a*c - b**2))
```

3.230 $\int \frac{x}{a+bx+cx^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 56

$$\int \frac{x}{a+bx+cx^2} dx = \frac{b\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} + \frac{\log(a+bx+cx^2)}{2c}$$

output
```
b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2)^(1/2)+1/2*ln(c*x^2+
b*x+a)/c
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{x}{a+bx+cx^2} dx = \frac{-\frac{2b\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + \log(a+x(b+cx))}{2c}$$

input
```
Integrate[x/(a + b*x + c*x^2),x]
```

output
```
((-2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
+ x*(b + c*x)])/(2*c)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{\int \frac{b+2cx}{cx^2+bx+a}dx}{2c}-\frac{b\int \frac{1}{cx^2+bx+a}dx}{2c}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{b\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{c}+\frac{\int \frac{b+2cx}{cx^2+bx+a}dx}{2c}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\int \frac{b+2cx}{cx^2+bx+a}dx}{2c}+\frac{b\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{b\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

input
```
Int[x/(a + b*x + c*x^2),x]
```

output
```
(b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[a +
 b*x + c*x^2]/(2*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

method	result
default	$\frac{\ln\left(c\,x^{2}+b x+a\right)}{2 c}-\frac{b \arctan\left(\frac{2 c x+b}{\sqrt{4 a c-b^{2}}}\right)}{c \sqrt{4 a c-b^{2}}}$
risch	$\frac{2 \ln\left(-2 \sqrt{-b^{2}\left(4 a c-b^{2}\right)}\, c x-4 a b c+b^{3}-\sqrt{-b^{2}\left(4 a c-b^{2}\right)}\, b\right) a}{4 a c-b^{2}}-\frac{\ln\left(-2 \sqrt{-b^{2}\left(4 a c-b^{2}\right)}\, c x-4 a b c+b^{3}-\sqrt{-b^{2}\left(4 a c-b^{2}\right)}\, b\right) b^{2}}{2 c\left(4 a c-b^{2}\right)}+\ln\Big(-$

input
```
int(x/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(c*x^2+b*x+a)/c-b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.30

$$\int \frac{x}{a+bx+cx^2}\,dx$$
$$=\left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+(b^2-4\,ac)\log\left(cx^2+bx+a\right)}{2\,(b^2c-4\,ac^2)},\frac{2\,\sqrt{-b^2+4\,ac}b\,\mathrm{arc}\right.$$

input
```
integrate(x/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^
2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^2 - 4*a*c)*log(c*x^2 + b*x
 + a))/(b^2*c - 4*a*c^2), 1/2*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-sqrt(-b^2 +
4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^2 + b*x + a))/(b
^2*c - 4*a*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(49) = 98$.

Time = 0.15 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.86

$$\int \frac{x}{a+bx+cx^2}\,dx$$
$$=\left(-\frac{b\sqrt{-4ac+b^2}}{2c\,(4ac-b^2)}+\frac{1}{2c}\right)\log\left(x+\frac{-4ac\left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)+2a+b^2\left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)}{b}\right)$$
$$+\left(\frac{b\sqrt{-4ac+b^2}}{2c\,(4ac-b^2)}+\frac{1}{2c}\right)\log\left(x+\frac{-4ac\left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)+2a+b^2\left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)}{b}\right)$$

input
```
integrate(x/(c*x**2+b*x+a),x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(-
b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(-b*sqr
t(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)))/b) + (b*sqrt(-4*a*c + b*
*2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(b*sqrt(-4*a*c + b**2)
/(2*c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(b*sqrt(-4*a*c + b**2)/(2*c*
(4*a*c - b**2)) + 1/(2*c)))/b)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \frac{x}{a+bx+cx^2}\,dx = -\frac{b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c} + \frac{\log\left(cx^2+bx+a\right)}{2\,c}$$

input
```
integrate(x/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/2*log
(c*x^2 + b*x + a)/c
```

Mupad [B] (verification not implemented)

Time = 8.83 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.00

$$\int \frac{x}{a+bx+cx^2}\,dx = \frac{2\,a\,c\,\ln\left(c\,x^2+b\,x+a\right)}{4\,a\,c^2-b^2\,c} - \frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}} - \frac{b^2\,\ln\left(c\,x^2+b\,x+a\right)}{2\,\left(4\,a\,c^2-b^2\,c\right)}$$

input
```
int(x/(a + b*x + c*x^2),x)
```

output
```
(2*a*c*log(a + b*x + c*x^2))/(4*a*c^2 - b^2*c) - (b*atan(b/(4*a*c - b^2)^(
1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) - (b^2*log(a
+ b*x + c*x^2))/(2*(4*a*c^2 - b^2*c))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.50

$$\int \frac{x}{a+bx+cx^2}\,dx = \frac{-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b+4\log\left(c\,x^2+bx+a\right)ac-\log\left(c\,x^2+bx+a\right)b^2}{2c\left(4ac-b^2\right)}$$

input
```
int(x/(c*x^2+b*x+a),x)
```

output
```
( - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b + 4*log(a
+ b*x + c*x**2)*a*c - log(a + b*x + c*x**2)*b**2)/(2*c*(4*a*c - b**2))
```

3.231 $\int \frac{1}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 34

$$\int \frac{1}{a+bx+cx^2}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

output
```
-2*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{1}{a+bx+cx^2}\,dx = \frac{2\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

input
```
Integrate[(a + b*x + c*x^2)^(-1),x]
```

output
```
(2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{1083}$$

$$-2\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[(a + b*x + c*x^2)^(-1),x]
```

output
```
(-2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}$	35
risch	$-\frac{\ln\left(2cx+\sqrt{-4ac+b^{2}}+b\right)}{\sqrt{-4ac+b^{2}}}+\frac{\ln\left(-2cx+\sqrt{-4ac+b^{2}}-b\right)}{\sqrt{-4ac+b^{2}}}$	61

input
```
int(1/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.53

$$\int \frac{1}{a+bx+cx^2}\,dx = \left[\frac{\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)}{\sqrt{b^2-4\,ac}},\ -\frac{2\,\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

input
```
integrate(1/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(
c*x^2 + b*x + a))/sqrt(b^2 - 4*a*c), -2*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b
^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(34) = 68$.

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.65

$$\int \frac{1}{a+bx+cx^2}\,dx = -\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right) + \sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)$$

input
```
integrate(1/(c*x**2+b*x+a),x)
```

output
```
-sqrt(-1/(4*a*c - b**2))*log(x + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*sq
rt(-1/(4*a*c - b**2)) + b)/(2*c)) + sqrt(-1/(4*a*c - b**2))*log(x + (4*a*c
*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx+cx^2}\,dx = \frac{2\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

input
```
integrate(1/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

Mupad [B] (verification not implemented)

Time = 8.82 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{1}{a+bx+cx^2}\,dx = \frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}$$

input
```
int(1/(a + b*x + c*x^2),x)
```

output
```
(2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(4*a*c - b^2
)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{1}{a+bx+cx^2}\,dx = \frac{2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{4ac-b^2}$$

input
```
int(1/(c*x^2+b*x+a),x)
```

output
```
(2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2)))/(4*a*c - b**2)
```

3.232 $\int \frac{1}{x(a+bx+cx^2)} dx$

Optimal result

Integrand size = 16, antiderivative size = 62

$$\int \frac{1}{x(a+bx+cx^2)} dx = \frac{b\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}} + \frac{\log(x)}{a} - \frac{\log(a+bx+cx^2)}{2a}$$

output
```
b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)+ln(x)/a-1/2*l
n(c*x^2+b*x+a)/a
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{1}{x(a+bx+cx^2)} dx = -\frac{\frac{2b\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - 2\log(x) + \log(a+x(b+cx))}{2a}$$

input
```
Integrate[1/(x*(a + b*x + c*x^2)),x]
```

output
```
-1/2*((2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 2*
Log[x] + Log[a + x*(b + c*x)])/a
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1144, 25, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx+cx^2\right)}dx$$

$$\Big\downarrow \text{1144}$$

$$\frac{\int -\frac{b+cx}{cx^2+bx+a}dx}{a}+\frac{\log(x)}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\log(x)}{a}-\frac{\int \frac{b+cx}{cx^2+bx+a}dx}{a}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\log(x)}{a}-\frac{\frac{1}{2}b\int \frac{1}{cx^2+bx+a}dx+\frac{1}{2}\int \frac{b+2cx}{cx^2+bx+a}dx}{a}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\log(x)}{a}-\frac{\frac{1}{2}\int \frac{b+2cx}{cx^2+bx+a}dx-b\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{a}$$

$$\Big\downarrow \text{219}$$

$$\frac{\log(x)}{a}-\frac{\frac{1}{2}\int \frac{b+2cx}{cx^2+bx+a}dx-\frac{b\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\log(x)}{a}-\frac{\frac{1}{2}\log\left(a+bx+cx^2\right)-\frac{b\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}$$

input
```
Int[1/(x*(a + b*x + c*x^2)),x]
```

output
```
Log[x]/a - (-((b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
) + Log[a + b*x + c*x^2]/2)/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

method	result	s
default	$\frac{\ln(x)}{a}+\frac{-\frac{\ln\left(c\,x^{2}+bx+a\right)}{2}-\frac{b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{a}$	6
risch	$\frac{\ln(x)}{a}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{2}c-a\,b^{2}\right)\_Z^{2}+\left(4ac-b^{2}\right)\_Z+c\right)}\_R\ln\left(\left(\left(6ac-2b^{2}\right)\_R+3c\right)x-ab\_R+b\right)\right)$	7

input
```
int(1/x/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a+1/a*(-1/2*ln(c*x^2+b*x+a)-b/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*
a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.40

$$\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx = \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-(b^2-4\,ac)\log\left(cx^2+bx+a\right)+2\,(b^2-4\,ac)\log\left(\right.}{2\,(ab^2-4\,a^2c)}\right.$$

input
```
integrate(1/x/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^
2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^2 - 4*a*c)*log(c*x^2 + b*x
 + a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/2*(2*sqrt(-b^2 + 4*a*
c)*b*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)
*log(c*x^2 + b*x + a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(54) = 108$.

Time = 2.14 (sec) , antiderivative size = 564, normalized size of antiderivative = 9.10

$$\int \frac{1}{x(a+bx+cx^2)}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)} - \frac{1}{2a}\right)\log\left(x + \frac{24a^4c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2 - 14a^3b^2c\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2 - 12a^3c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)}{9abc^2 -}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)} - \frac{1}{2a}\right)\log\left(x + \frac{24a^4c^2\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2 - 14a^3b^2c\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2 - 12a^3c^2\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right) + 2a}{9abc^2 - 2b^3c}\right) + \frac{\log(x)}{a}$$

input
```
integrate(1/x/(c*x**2+b*x+a),x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c
**2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b
**2*c*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3
*c**2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**
4*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 + 3*a**2*b**2
*c*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) - 12*a**2*c**2
+ 11*a*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b**3*c)) + (b*sqrt(-4*a*c + b**2)/
(2*a*(4*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c**2*(b*sqrt(-4*a*c + b**
2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**2*c*(b*sqrt(-4*a*c + b*
*2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c**2*(b*sqrt(-4*a*c + b**
2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(b*sqrt(-4*a*c + b**2)/(2
*a*(4*a*c - b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(b*sqrt(-4*a*c + b**2)/(2
*a*(4*a*c - b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a*b**2*c - 2*b**4)/(9*a*
b*c**2 - 2*b**3*c)) + log(x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\,(a+bx+cx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,(a+bx+cx^2)}\,dx = -\frac{b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a} - \frac{\log\left(cx^2+bx+a\right)}{2\,a} + \frac{\log\left(|x|\right)}{a}$$

input
```
integrate(1/x/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/2*log
(c*x^2 + b*x + a)/a + log(abs(x))/a
```

Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.68

$$\int \frac{1}{x\,(a+bx+cx^2)}\,dx = \begin{cases} \frac{2\,c\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)}{b^2}-\frac{1}{b\,x} & \text{if } a=0 \\ -\frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a} & \text{if } b\neq 0\land c=0 \\ \frac{2}{2\,a+b\,x}-\frac{2\,\mathrm{atanh}\left(\frac{b\,x}{a}+1\right)}{a} & \text{if } b^2=4\,a\,c\land a\neq 0 \\ \frac{\ln(x)}{a}-\frac{\ln\left(c\,x^2+b\,x+a\right)}{2\,a}-\frac{b\,\mathrm{atan}\left(\frac{b+2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{a\,\sqrt{4\,a\,c-b^2}} & \text{if } b^2\neq 4\,a\,c\land a\neq 0\land c\neq 0 \end{cases}$$

input
```
int(1/(x*(a + b*x + c*x^2)),x)
```

output
```
piecewise(a == 0, - 1/(b*x) + (2*c*atanh((2*c*x)/b + 1))/b^2, b ~= 0 & c =
= 0, -(2*atanh((2*b*x)/a + 1))/a, b^2 == 4*a*c & a ~= 0, 2/(2*a + b*x) - (
2*atanh((b*x)/a + 1))/a, b^2 ~= 4*a*c & a ~= 0 & c ~= 0, log(x)/a - log(a
+ b*x + c*x^2)/(2*a) - (b*atan((b + 2*c*x)/(4*a*c - b^2)^(1/2)))/(a*(4*a*c
 - b^2)^(1/2)))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.55

$$\int \frac{1}{x\,(a+bx+cx^2)}\,dx = \frac{-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b-4\log(c\,x^2+bx+a)\,ac+\log(c\,x^2+bx+a)\,b^2+8\log(x)\,ac-2\log(x)\,b}{2a\,(4ac-b^2)}$$

input
```
int(1/x/(c*x^2+b*x+a),x)
```

output
```
( - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b - 4*log(a
+ b*x + c*x**2)*a*c + log(a + b*x + c*x**2)*b**2 + 8*log(x)*a*c - 2*log(x)
*b**2)/(2*a*(4*a*c - b**2))
```

3.233 $\int \frac{1}{x^2(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 81

$$\int \frac{1}{x^2(a+bx+cx^2)}\,dx = -\frac{1}{ax} - \frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx+cx^2)}{2a^2}$$

output
```
-1/a/x-(-2*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)
^(1/2)-b*ln(x)/a^2+1/2*b*ln(c*x^2+b*x+a)/a^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2(a+bx+cx^2)}\,dx = \frac{-\frac{2a}{x} + \frac{2(b^2-2ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - 2b\log(x) + b\log(a+x(b+cx))}{2a^2}$$

input
```
Integrate[1/(x^2*(a + b*x + c*x^2)),x]
```

output
```
((-2*a)/x + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[
-b^2 + 4*a*c] - 2*b*Log[x] + b*Log[a + x*(b + c*x)])/(2*a^2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+bx+cx^2\right)}dx$$

$$\Big\downarrow \text{ 1145}$$

$$\frac{\int -\frac{b+cx}{x(cx^2+bx+a)}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{b+cx}{x(cx^2+bx+a)}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{ 1200}$$

$$-\frac{\int \left(\frac{b}{ax}+\frac{-b^2-cxb+ac}{a(cx^2+bx+a)}\right)dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{(b^2-2ac)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{b\log(a+bx+cx^2)}{2a}+\frac{b\log(x)}{a}}{a}-\frac{1}{ax}$$

input
```
Int[1/(x^2*(a + b*x + c*x^2)),x]
```

output
```
-(1/(a*x)) - (((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sq
rt[b^2 - 4*a*c]) + (b*Log[x])/a - (b*Log[a + b*x + c*x^2])/(2*a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

method	result
default	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{\frac{b\ln\left(c\,x^2+bx+a\right)}{2}+\frac{2\left(-ac+\frac{b^2}{2}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a^2}$
risch	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4c\,a^3-a^2b^2\right)\_Z^2+\left(-4abc+b^3\right)\_Z+c^2\right)}\_R\ln\left(\left(\left(6c\,a^3-2a^2b^2\right)\_R^2-2\_Rab\right.\right.\right.$

input
```
int(1/x^2/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/a/x-b*ln(x)/a^2+1/a^2*(1/2*b*ln(c*x^2+b*x+a)+2*(-a*c+1/2*b^2)/(4*a*c-b^
2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.32

$$\int \frac{1}{x^2(a+bx+cx^2)}\,dx = \left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}x\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+2\,ab^2-8\,a^2c-(b^3-4\,abc)x\log\left(cx^2\right.}{2\,(a^2b^2-4\,a^3c)x}\right.$$

$$-\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}x\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+2\,ab^2-8\,a^2c-(b^3-4\,abc)x\log\left(cx^2+bx+\right.}{2\,(a^2b^2-4\,a^3c)x}$$

input
```
integrate(1/x^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x*log((2*c^2*x^2 + 2*b*c*x + b^2 -
2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*a*b^2 - 8*a^
2*c - (b^3 - 4*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))
/((a^2*b^2 - 4*a^3*c)*x), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x*arcta
n(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a*b^2 - 8*a^2*c - (b^
3 - 4*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((a^2*b^
2 - 4*a^3*c)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 862 vs. 2(75) = 150.

Time = 142.00 (sec) , antiderivative size = 862, normalized size of antiderivative = 10.64

$$\int \frac{1}{x^2(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**2/(c*x**2+b*x+a),x)
```

output
```
(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))*
log(x + (-28*a**6*b*c**2*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/
(2*a**2*(4*a*c - b**2)))**2 + 15*a**5*b**3*c*(b/(2*a**2) - sqrt(-4*a*c + b
**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))**2 - 4*a**5*c**3*(b/(2*a**2)
- sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))) - 2*a**4*b**
5*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))
)**2 - 3*a**4*b**2*c**2*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(
2*a**2*(4*a*c - b**2))) + a**3*b**4*c*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2
*a*c - b**2)/(2*a**2*(4*a*c - b**2))) - 4*a**3*b*c**3 + 25*a**2*b**3*c**2
- 14*a*b**5*c + 2*b**7)/(2*a**3*c**4 + 15*a**2*b**2*c**3 - 12*a*b**4*c**2
+ 2*b**6*c)) + (b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4
*a*c - b**2)))*log(x + (-28*a**6*b*c**2*(b/(2*a**2) + sqrt(-4*a*c + b**2)*
(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))**2 + 15*a**5*b**3*c*(b/(2*a**2) +
sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))**2 - 4*a**5*c*
*3*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)
)) - 2*a**4*b**5*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*
(4*a*c - b**2)))**2 - 3*a**4*b**2*c**2*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(
2*a*c - b**2)/(2*a**2*(4*a*c - b**2))) + a**3*b**4*c*(b/(2*a**2) + sqrt(-4
*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))) - 4*a**3*b*c**3 + 25*
a**2*b**3*c**2 - 14*a*b**5*c + 2*b**7)/(2*a**3*c**4 + 15*a**2*b**2*c**3...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (a + bx + cx^2)} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^2 (a + bx + cx^2)} \, dx = \frac{b \log (cx^2 + bx + a)}{2 a^2} - \frac{b \log (|x|)}{a^2} + \frac{(b^2 - 2 ac) \arctan \left(\frac{2 cx + b}{\sqrt{-b^2 + 4 ac}} \right)}{\sqrt{-b^2 + 4 ac} a^2} - \frac{1}{ax}$$

input
```
integrate(1/x^2/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/2*b*log(c*x^2 + b*x + a)/a^2 - b*log(abs(x))/a^2 + (b^2 - 2*a*c)*arctan(
(2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/(a*x)
```

Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.74

$$\int \frac{1}{x^2 (a + bx + cx^2)} \, dx = \begin{cases} -\frac{\frac{b^2}{2} - b\,c\,x}{b^3\,x^2} - \frac{2\,c^2\,\mathrm{atanh}\left(\frac{2\,c\,x}{b} + 1\right)}{b^3} & \text{if } a = 0 \\ \frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a} + 1\right)}{a^2} - \frac{1}{a\,x} & \text{if } c = 0 \\ -\frac{1}{a\,x} - \frac{b\,\ln(x)}{a^2} & \text{if } b^2 = 4\,a\,c \\ \frac{b\,\ln(c\,x^2 + b\,x + a)}{2\,a^2} - \frac{b\,\ln(x)}{a^2} - \frac{1}{a\,x} - \frac{2\,c\,\mathrm{atan}\left(\frac{b + 2\,c\,x}{\sqrt{4\,a\,c - b^2}}\right)}{a\,\sqrt{4\,a\,c - b^2}} + \frac{b^2\,\mathrm{atan}\left(\frac{b + 2\,c\,x}{\sqrt{4\,a\,c - b^2}}\right)}{a^2\,\sqrt{4\,a\,c - b^2}} & \text{if } b^2 \neq 4\,a\,c \wedge a \neq 0 \wedge c \neq 0 \end{cases}$$

input
```
int(1/(x^2*(a + b*x + c*x^2)),x)
```

output
```
piecewise(a == 0, - (b^2/2 - b*c*x)/(b^3*x^2) - (2*c^2*atanh((2*c*x)/b + 1
))/b^3, c == 0, - 1/(a*x) + (2*b*atanh((2*b*x)/a + 1))/a^2, b^2 == 4*a*c,
- 1/(a*x) - (b*log(x))/a^2, b^2 ~= 4*a*c & a ~= 0 & c ~= 0, - 1/(a*x) - (b
*log(x))/a^2 + (b*log(a + b*x + c*x^2))/(2*a^2) - (2*c*atan((b + 2*c*x)/(4
*a*c - b^2)^(1/2)))/(a*(4*a*c - b^2)^(1/2)) + (b^2*atan((b + 2*c*x)/(4*a*c
 - b^2)^(1/2)))/(a^2*(4*a*c - b^2)^(1/2)))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.95

$$\int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx$$
$$= \frac{-4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)acx+2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^2x+4\log(c\,x^2+bx+a)\,abcx-\log(c\,x^2}{2a^2x\left(4ac-b^2\right)}$$

input
```
int(1/x^2/(c*x^2+b*x+a),x)
```

output
```
( - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c*x + 2*sq
rt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*x + 4*log(a + b
*x + c*x**2)*a*b*c*x - log(a + b*x + c*x**2)*b**3*x - 8*log(x)*a*b*c*x + 2
*log(x)*b**3*x - 8*a**2*c + 2*a*b**2)/(2*a**2*x*(4*a*c - b**2))
```

3.234 $\int \frac{1}{x^3(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 104

$$\int \frac{1}{x^3(a+bx+cx^2)}\,dx = -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b(b^2-3ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log(x)}{a^3}-\frac{(b^2-ac)\log(a+bx+cx^2)}{2a^3}$$

output
```
-1/2/a/x^2+b/a^2/x+b*(-3*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^
3/(-4*a*c+b^2)^(1/2)+(-a*c+b^2)*ln(x)/a^3-1/2*(-a*c+b^2)*ln(c*x^2+b*x+a)/a
^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^3(a+bx+cx^2)}\,dx = \frac{-\frac{a^2}{x^2}+\frac{2ab}{x}-\frac{2b(b^2-3ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+2(b^2-ac)\log(x)+(-b^2+ac)\log(a+x(b+cx))}{2a^3}$$

input
```
Integrate[1/(x^3*(a + b*x + c*x^2)),x]
```

output
```
(-(a^2/x^2) + (2*a*b)/x - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2
+ 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(b^2 - a*c)*Log[x] + (-b^2 + a*c)*Log[a
+ x*(b + c*x)])/(2*a^3)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx+cx^2)} \, dx$$

$$\Big\downarrow \text{1145}$$

$$\frac{\int -\frac{b+cx}{x^2(cx^2+bx+a)}dx}{a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{b+cx}{x^2(cx^2+bx+a)}dx}{a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \text{1200}$$

$$-\frac{\int \left(\frac{b}{ax^2}+\frac{ac-b^2}{a^2x}+\frac{b(b^2-2ac)+c(b^2-ac)x}{a^2(cx^2+bx+a)}\right)dx}{a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{b(b^2-3ac)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log(a+bx+cx^2)}{2a^2}-\frac{\log(x)(b^2-ac)}{a^2}-\frac{b}{ax}}{a}-\frac{1}{2ax^2}$$

input
```
Int[1/(x^3*(a + b*x + c*x^2)),x]
```

output
```
-1/2*1/(a*x^2) - (-(b/(a*x)) - (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b
^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - ((b^2 - a*c)*Log[x])/a^2 + ((b^2 -
 a*c)*Log[a + b*x + c*x^2])/(2*a^2))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.23

method	result
default	$-\frac{1}{2a\,x^{2}}+\frac{\left(-ac+b^{2}\right)\ln\left(x\right)}{a^{3}}+\frac{b}{a^{2}x}+\frac{\frac{\left(a\,c^{2}-b^{2}c\right)\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(2abc-b^{3}-\frac{\left(a\,c^{2}-b^{2}c\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{a^{3}}$
risch	$\frac{\frac{bx}{a^{2}}-\frac{1}{2a}}{x^{2}}-\frac{c\ln\left(x\right)}{a^{2}}+\frac{b^{2}\ln\left(x\right)}{a^{3}}+\left(\sum_{\_R=\operatorname{RootOf}\left(\left(4c\,a^{4}-a^{3}b^{2}\right)\_Z^{2}+\left(-4a^{2}c^{2}+5ca\,b^{2}-b^{4}\right)\_Z+c^{3}\right)}\_R\ln\left(\left(\left(6c\,a^{5}-2a^{4}\right.\right.\right.\right.$

input
```
int(1/x^3/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/x^2+(-a*c+b^2)*ln(x)/a^3+b/a^2/x+1/a^3*(1/2*(a*c^2-b^2*c)/c*ln(c*x^
2+b*x+a)+2*(2*a*b*c-b^3-1/2*(a*c^2-b^2*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 358, normalized size of antiderivative = 3.44

$$\int \frac{1}{x^3(a+bx+cx^2)}\,dx$$
$$= \left[-\frac{(b^3-3abc)\sqrt{b^2-4ac}x^2\log\left(\frac{2c^2x^2+2bcx+b^2-2ac-\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right)+a^2b^2-4a^3c+(b^4-5ab^2c+4a^2c}{2(a^3b^2-4a^4c)x^2}\right.$$

input
```
integrate(1/x^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[-1/2*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^2 + 2*b*c*x + b^
2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + a^2*b^2 -
4*a^3*c + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) - 2*(b^4
- 5*a*b^2*c + 4*a^2*c^2)*x^2*log(x) - 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 -
 4*a^4*c)*x^2), 1/2*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^2*arctan(-sqrt
(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - a^2*b^2 + 4*a^3*c - (b^4 - 5*a
*b^2*c + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) + 2*(b^4 - 5*a*b^2*c + 4*a^2*
c^2)*x^2*log(x) + 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a^4*c)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3(a+bx+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**3/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (a+bx+cx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^3 (a+bx+cx^2)}\,dx = -\frac{(b^2-ac)\log(cx^2+bx+a)}{2\,a^3} + \frac{(b^2-ac)\log(|x|)}{a^3} - \frac{(b^3-3\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a^3} + \frac{2\,abx-a^2}{2\,a^3x^2}$$

input
```
integrate(1/x^3/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/a^3 + (b^2 - a*c)*log(abs(x))/a^3 -
(b^3 - 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)
*a^3) + 1/2*(2*a*b*x - a^2)/(a^3*x^2)
```

Mupad [B] (verification not implemented)

Time = 9.47 (sec) , antiderivative size = 447, normalized size of antiderivative = 4.30

$$\int \frac{1}{x^3 (a+bx+cx^2)}\,dx$$

$$= \frac{\ln\left(2\,a\,b^4+2\,b^5\,x+6\,a^3\,c^2+2\,a\,b^3\,\sqrt{b^2-4\,a\,c}+2\,b^4\,x\,\sqrt{b^2-4\,a\,c}-9\,a^2\,b^2\,c-10\,a\,b^3\,c\,x-3\,a^2\,b\,c\,\sqrt{b^2-\right.}{}$$

$$-\frac{\ln\left(2\,a\,b^4+2\,b^5\,x+6\,a^3\,c^2-2\,a\,b^3\,\sqrt{b^2-4\,a\,c}-2\,b^4\,x\,\sqrt{b^2-4\,a\,c}-9\,a^2\,b^2\,c-10\,a\,b^3\,c\,x+3\,a^2\,b\,c\,\sqrt{b\right.}{}$$

$$-\frac{\frac{1}{2\,a}-\frac{b\,x}{a^2}}{x^2}-\frac{\ln(x)\,\left(a\,c-b^2\right)}{a^3}$$

input
```
int(1/(x^3*(a + b*x + c*x^2)),x)
```

output
```
(log(2*a*b^4 + 2*b^5*x + 6*a^3*c^2 + 2*a*b^3*(b^2 - 4*a*c)^(1/2) + 2*b^4*x
*(b^2 - 4*a*c)^(1/2) - 9*a^2*b^2*c - 10*a*b^3*c*x - 3*a^2*b*c*(b^2 - 4*a*c
)^(1/2) + 9*a^2*b*c^2*x + 3*a^2*c^2*x*(b^2 - 4*a*c)^(1/2) - 6*a*b^2*c*x*(b
^2 - 4*a*c)^(1/2))*(b^4/2 - a*((5*b^2*c)/2 + (3*b*c*(b^2 - 4*a*c)^(1/2))/2
) + (b^3*(b^2 - 4*a*c)^(1/2))/2 + 2*a^2*c^2))/(4*a^4*c - a^3*b^2) - (log(2
*a*b^4 + 2*b^5*x + 6*a^3*c^2 - 2*a*b^3*(b^2 - 4*a*c)^(1/2) - 2*b^4*x*(b^2
- 4*a*c)^(1/2) - 9*a^2*b^2*c - 10*a*b^3*c*x + 3*a^2*b*c*(b^2 - 4*a*c)^(1/2
) + 9*a^2*b*c^2*x - 3*a^2*c^2*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c*x*(b^2 - 4
*a*c)^(1/2))*(a*((5*b^2*c)/2 - (3*b*c*(b^2 - 4*a*c)^(1/2))/2) - b^4/2 + (b
^3*(b^2 - 4*a*c)^(1/2))/2 - 2*a^2*c^2))/(4*a^4*c - a^3*b^2) - (1/(2*a) - (
b*x)/a^2)/x^2 - (log(x)*(a*c - b^2))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.16

$$\int \frac{1}{x^3 (a+bx+cx^2)}\,dx$$

$$= \frac{6\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)abc\,x^2-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^3x^2+4\log(c\,x^2+bx+a)\,a^2c^2x^2-5\log}{}$$

input
```
int(1/x^3/(c*x^2+b*x+a),x)
```

output

```
(6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c*x**2 - 2*
sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*x**2 + 4*log(
a + b*x + c*x**2)*a**2*c**2*x**2 - 5*log(a + b*x + c*x**2)*a*b**2*c*x**2 +
 log(a + b*x + c*x**2)*b**4*x**2 - 8*log(x)*a**2*c**2*x**2 + 10*log(x)*a*b
**2*c*x**2 - 2*log(x)*b**4*x**2 - 4*a**3*c + a**2*b**2 + 8*a**2*b*c*x - 2*
a*b**3*x)/(2*a**3*x**2*(4*a*c - b**2))
```

3.235 $\int \frac{x^5}{\left(a+bx+cx^2\right)^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 174

$$\int \frac{x^5}{\left(a+bx+cx^2\right)^2} dx = -\frac{2bx}{c^3}+\frac{x^2}{2c^2}+\frac{a(b^4-4ab^2c+2a^2c^2)+b(b^4-5ab^2c+5a^2c^2)\,x}{c^4\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{b(3b^4-20ab^2c+30a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\left(b^2-4ac\right)^{3/2}} + \frac{(3b^2-2ac)\log\left(a+bx+cx^2\right)}{2c^4}$$

output
```
-2*b*x/c^3+1/2*x^2/c^2+(a*(2*a^2*c^2-4*a*b^2*c+b^4)+b*(5*a^2*c^2-5*a*b^2*c
+b^4)*x)/c^4/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(30*a^2*c^2-20*a*b^2*c+3*b^4)*ar
ctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^4/(-4*a*c+b^2)^(3/2)+1/2*(-2*a*c+3*b
^2)*ln(c*x^2+b*x+a)/c^4
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{(a+bx+cx^2)^2}\,dx$$
$$= \frac{-4bcx+c^2x^2+\frac{2(2a^3c^2+b^5x+ab^3(b-5cx)+a^2bc(-4b+5cx))}{(b^2-4ac)(a+x(b+cx))}+\frac{2b(3b^4-20ab^2c+30a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+(3b^2-2ac)\log}{2c^4}$$

input
```
Integrate[x^5/(a + b*x + c*x^2)^2,x]
```

output
```
(-4*b*c*x + c^2*x^2 + (2*(2*a^3*c^2 + b^5*x + a*b^3*(b - 5*c*x) + a^2*b*c*
(-4*b + 5*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*b^4 - 20*a*b^
2*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(
3/2) + (3*b^2 - 2*a*c)*Log[a + x*(b + c*x)])/(2*c^4)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1164, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx+cx^2)^2}\,dx$$
$$\Big\downarrow \; 1164$$
$$\frac{x^4(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{\int \frac{x^3(8a+3bx)}{cx^2+bx+a}dx}{b^2-4ac}$$
$$\Big\downarrow \; 1200$$

$$\frac{x^4(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{\int \left(\frac{3bx^2}{c}-\frac{(3b^2-8ac)x}{c^2}+\frac{b(3b^2-11ac)}{c^3}-\frac{ab(3b^2-11ac)+(b^2-4ac)(3b^2-2ac)x}{c^3(cx^2+bx+a)}\right)dx}{b^2-4ac}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^4(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{-\frac{b(30a^2c^2-20ab^2c+3b^4)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}}-\frac{(b^2-4ac)(3b^2-2ac)\log(a+bx+cx^2)}{2c^4}+\frac{bx(3b^2-11ac)}{c^3}-\frac{x^2(3b^2-8ac)}{2c^2}+\frac{bx^3}{c}}{b^2-4ac}$$

input
```
Int[x^5/(a + b*x + c*x^2)^2,x]
```

output
```
(x^4*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - ((b*(3*b^2 - 11*a*c)
*x)/c^3 - ((3*b^2 - 8*a*c)*x^2)/(2*c^2) + (b*x^3)/c - (b*(3*b^4 - 20*a*b^2
*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*Sqrt[b^2 - 4
*a*c]) - ((b^2 - 4*a*c)*(3*b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*c^4))/(b^
2 - 4*a*c)
```

Defintions of rubi rules used

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
 + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
 + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.37

method	result
default	$-\frac{-\frac{1}{2}c\,x^{2}+2bx}{c^{3}}+\frac{\frac{-\frac{b\left(5a^{2}c^{2}-5ca\,b^{2}+b^{4}\right)x}{c\left(4ac-b^{2}\right)}-\frac{a\left(2a^{2}c^{2}-4ca\,b^{2}+b^{4}\right)}{c\left(4ac-b^{2}\right)}}{c\,x^{2}+bx+a}+\frac{\frac{\left(-8a^{2}c^{2}+14ca\,b^{2}-3b^{4}\right)\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(11c\,a^{2}b-3a\,b^{3}-\frac{\left(-8a^{2}c \cdots\right.}{\cdots}\right.}{\cdots}}{4ac-b^{2}}}{c^{3}}$
risch	Expression too large to display

input
```
int(x^5/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/c^3*(-1/2*c*x^2+2*b*x)+1/c^3*((-b*(5*a^2*c^2-5*a*b^2*c+b^4)/c/(4*a*c-b^
2)*x-a/c*(2*a^2*c^2-4*a*b^2*c+b^4)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4*a*c-b^2
)*(1/2*(-8*a^2*c^2+14*a*b^2*c-3*b^4)/c*ln(c*x^2+b*x+a)+2*(11*c*a^2*b-3*a*b
^3-1/2*(-8*a^2*c^2+14*a*b^2*c-3*b^4)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. 2(166) = 332.

Time = 0.09 (sec) , antiderivative size = 1029, normalized size of antiderivative = 5.91

$$\int \frac{x^5}{\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
[1/2*(2*a*b^6 - 16*a^2*b^4*c + 36*a^3*b^2*c^2 - 16*a^4*c^3 + (b^4*c^3 - 8*
a*b^2*c^4 + 16*a^2*c^5)*x^4 - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^3
 - (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b^2*c^3 - 16*a^3*c^4)*x^2 - (3*a*b^5 -
 20*a^2*b^3*c + 30*a^3*b*c^2 + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^2
 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x
^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x +
 a)) + 2*(b^7 - 11*a*b^5*c + 41*a^2*b^3*c^2 - 52*a^3*b*c^3)*x + (3*a*b^6 -
 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3 + (3*b^6*c - 26*a*b^4*c^2 + 64
*a^2*b^2*c^3 - 32*a^3*c^4)*x^2 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32
*a^3*b*c^3)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c
^6 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^2 + (b^5*c^4 - 8*a*b^3*c^5 + 1
6*a^2*b*c^6)*x), 1/2*(2*a*b^6 - 16*a^2*b^4*c + 36*a^3*b^2*c^2 - 16*a^4*c^3
 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 - 3*(b^5*c^2 - 8*a*b^3*c^3 + 1
6*a^2*b*c^4)*x^3 - (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b^2*c^3 - 16*a^3*c^4)*
x^2 + 2*(3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2 + (3*b^5*c - 20*a*b^3*c^2 +
 30*a^2*b*c^3)*x^2 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x)*sqrt(-b^2 +
4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^7 - 11
*a*b^5*c + 41*a^2*b^3*c^2 - 52*a^3*b*c^3)*x + (3*a*b^6 - 26*a^2*b^4*c + 64
*a^3*b^2*c^2 - 32*a^4*c^3 + (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*
a^3*c^4)*x^2 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x)*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1012 vs. $2(168) = 336$.

Time = 1.30 (sec) , antiderivative size = 1012, normalized size of antiderivative = 5.82

$$\int \frac{x^5}{(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**5/(c*x**2+b*x+a)**2,x)
```

output
```
-2*b*x/c**3 + (-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3
*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) -
(2*a*c - 3*b**2)/(2*c**4))*log(x + (16*a**3*c**2 - 17*a**2*b**2*c + 16*a**
2*c**5*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/
(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c
- 3*b**2)/(2*c**4)) + 3*a*b**4 - 8*a*b**2*c**4*(-b*sqrt(-(4*a*c - b**2)**3
)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b*
*2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4)) + b**4*c**3*(-
b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(
64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)
/(2*c**4)))/(30*a**2*b*c**2 - 20*a*b**3*c + 3*b**5)) + (b*sqrt(-(4*a*c - b
**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*
a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4))*log(x +
 (16*a**3*c**2 - 17*a**2*b**2*c + 16*a**2*c**5*(b*sqrt(-(4*a*c - b**2)**3)
*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4)) + 3*a*b**4 - 8*
a*b**2*c**4*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b*
*4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*
a*c - 3*b**2)/(2*c**4)) + b**4*c**3*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c
**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 +...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx+cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(a+bx+cx^2)^2}\,dx = -\frac{(3\,b^5-20\,ab^3c+30\,a^2bc^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^4-4\,ac^5)\sqrt{-b^2+4\,ac}} + \frac{(3\,b^2-2\,ac)\log\left(cx^2+bx+a\right)}{2\,c^4} + \frac{c^2x^2-4\,bcx}{2\,c^4} + \frac{ab^4-4\,a^2b^2c+2\,a^3c^2+(b^5-5\,ab^3c+5\,a^2bc^2)x}{(cx^2+bx+a)(b^2-4\,ac)c^4}$$

input
```
integrate(x^5/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
-(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c)
)/((b^2*c^4 - 4*a*c^5)*sqrt(-b^2 + 4*a*c)) + 1/2*(3*b^2 - 2*a*c)*log(c*x^2
 + b*x + a)/c^4 + 1/2*(c^2*x^2 - 4*b*c*x)/c^4 + (a*b^4 - 4*a^2*b^2*c + 2*a
^3*c^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*
c)*c^4)
```

Mupad [B] (verification not implemented)

Time = 9.54 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.20

$$\int \frac{x^5}{(a+bx+cx^2)^2}\,dx = \frac{x^2}{2\,c^2} - \frac{\frac{a\,\left(2\,a^2\,c^2-4\,a\,b^2\,c+b^4\right)}{c\,\left(4\,a\,c-b^2\right)}+\frac{b\,x\,\left(5\,a^2\,c^2-5\,a\,b^2\,c+b^4\right)}{c\,\left(4\,a\,c-b^2\right)}}{c^4\,x^2+b\,c^3\,x+a\,c^3} - \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(128\,a^4\,c^4-288\,a^3\,b^2\,c^3+168\,a^2\,b^4\,c^2-38\,a\,b^6\,c+3\,b^8\right)}{2\,\left(64\,a^3\,c^7-48\,a^2\,b^2\,c^6+12\,a\,b^4\,c^5-b^6\,c^4\right)} - \frac{2\,b\,x}{c^3} + \frac{b\,\mathrm{atan}\left(\frac{c^4\,\left(\frac{2\,b\,x\,\left(30\,a^2\,c^2-20\,a\,b^2\,c+3\,b^4\right)}{c^3\,{\left(4\,a\,c-b^2\right)}^3}-\frac{b\,\left(b^3\,c^3-4\,a\,b\,c^4\right)\,\left(30\,a^2\,c^2-20\,a\,b^2\,c+3\,b^4\right)}{c^7\,{\left(4\,a\,c-b^2\right)}^4}\right)\,{\left(4\,a\,c-b^2\right)}^{5/2}}{30\,a^2\,b\,c^2-20\,a\,b^3\,c+3\,b^5}\right)\,\left(30\,a^2\,c^2-20\,a\,b^2\,c+3\,b\right.}{c^4\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

input
```
int(x^5/(a + b*x + c*x^2)^2,x)
```

output
```
x^2/(2*c^2) - ((a*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/(c*(4*a*c - b^2)) + (b*x*
(b^4 + 5*a^2*c^2 - 5*a*b^2*c))/(c*(4*a*c - b^2)))/(a*c^3 + c^4*x^2 + b*c^3
*x) - (log(a + b*x + c*x^2)*(3*b^8 + 128*a^4*c^4 + 168*a^2*b^4*c^2 - 288*a
^3*b^2*c^3 - 38*a*b^6*c))/(2*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2
*b^2*c^6)) - (2*b*x)/c^3 + (b*atan((c^4*((2*b*x*(3*b^4 + 30*a^2*c^2 - 20*a
*b^2*c))/(c^3*(4*a*c - b^2)^3) - (b*(b^3*c^3 - 4*a*b*c^4)*(3*b^4 + 30*a^2*
c^2 - 20*a*b^2*c))/(c^7*(4*a*c - b^2)^4))*(4*a*c - b^2)^(5/2))/(3*b^5 + 30
*a^2*b*c^2 - 20*a*b^3*c))*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(c^4*(4*a*c -
 b^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 846, normalized size of antiderivative = 4.86

$$\int \frac{x^5}{(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(x^5/(c*x^2+b*x+a)^2,x)
```

output
```
(60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**2 -
40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3*c + 6
0*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2*c**2*x
 + 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c**3*
x**2 + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**5 -
40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**4*c*x - 40
*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3*c**2*x**2
+ 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**6*x + 6*sqr
t(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**5*c*x**2 - 32*log(
a + b*x + c*x**2)*a**4*c**3 + 64*log(a + b*x + c*x**2)*a**3*b**2*c**2 - 32
*log(a + b*x + c*x**2)*a**3*b*c**3*x - 32*log(a + b*x + c*x**2)*a**3*c**4*
x**2 - 26*log(a + b*x + c*x**2)*a**2*b**4*c + 64*log(a + b*x + c*x**2)*a**
2*b**3*c**2*x + 64*log(a + b*x + c*x**2)*a**2*b**2*c**3*x**2 + 3*log(a + b
*x + c*x**2)*a*b**6 - 26*log(a + b*x + c*x**2)*a*b**5*c*x - 26*log(a + b*x
 + c*x**2)*a*b**4*c**2*x**2 + 3*log(a + b*x + c*x**2)*b**7*x + 3*log(a + b
*x + c*x**2)*b**6*c*x**2 + 88*a**4*c**3 - 46*a**3*b**2*c**2 + 120*a**3*c**
4*x**2 + 6*a**2*b**4*c - 154*a**2*b**2*c**3*x**2 - 48*a**2*b*c**4*x**3 + 1
6*a**2*c**5*x**4 + 55*a*b**4*c**2*x**2 + 24*a*b**3*c**3*x**3 - 8*a*b**2*c*
*4*x**4 - 6*b**6*c*x**2 - 3*b**5*c**2*x**3 + b**4*c**3*x**4)/(2*c**4*(16*a
**3*c**2 - 8*a**2*b**2*c + 16*a**2*b*c**2*x + 16*a**2*c**3*x**2 + a*b**...
```

3.236 $\int \frac{x^4}{(a+bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 139

$$\int \frac{x^4}{(a+bx+cx^2)^2}\,dx = \frac{x}{c^2} - \frac{ab(b^2-3ac)+(b^4-4ab^2c+2a^2c^2)\,x}{c^3\,(b^2-4ac)\,(a+bx+cx^2)} - \frac{2(b^4-6ab^2c+6a^2c^2)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\,(b^2-4ac)^{3/2}} - \frac{b\log\left(a+bx+cx^2\right)}{c^3}$$

output
```
x/c^2-(a*b*(-3*a*c+b^2)+(2*a^2*c^2-4*a*b^2*c+b^4)*x)/c^3/(-4*a*c+b^2)/(c*x
^2+b*x+a)-2*(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2)
)/c^3/(-4*a*c+b^2)^(3/2)-b*ln(c*x^2+b*x+a)/c^3
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.95

$$\int \frac{x^4}{\left(a+bx+cx^2\right)^2} dx = \frac{cx+\frac{-b^4x-ab^2(b-4cx)+a^2c(3b-2cx)}{(b^2-4ac)(a+x(b+cx))}-\frac{2(b^4-6ab^2c+6a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}-b\log(a+x(b+cx))}{c^3}$$

input
```
Integrate[x^4/(a + b*x + c*x^2)^2,x]
```

output
```
(c*x + (-(b^4*x) - a*b^2*(b - 4*c*x) + a^2*c*(3*b - 2*c*x))/((b^2 - 4*a*c)
*(a + x*(b + c*x))) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/
Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) - b*Log[a + x*(b + c*x)])/c^3
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1164, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(a+bx+cx^2\right)^2} dx$$

$$\Big\downarrow \text{ 1164}$$

$$\frac{x^3(2a+bx)}{(b^2-4ac)\left(a+bx+cx^2\right)}-\frac{\int \frac{2x^2(3a+bx)}{cx^2+bx+a}dx}{b^2-4ac}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^3(2a+bx)}{(b^2-4ac)\left(a+bx+cx^2\right)}-\frac{2\int \frac{x^2(3a+bx)}{cx^2+bx+a}dx}{b^2-4ac}$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{2\int\left(-\frac{b^2-3ac}{c^2}+\frac{bx}{c}+\frac{a(b^2-3ac)+b(b^2-4ac)x}{c^2(cx^2+bx+a)}\right)dx}{b^2-4ac}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{2\left(\frac{(6a^2c^2-6ab^2c+b^4)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{b(b^2-4ac)\log(a+bx+cx^2)}{2c^3}-\frac{x(b^2-3ac)}{c^2}+\frac{bx^2}{2c}\right)}{b^2-4ac}$$

input
```
Int[x^4/(a + b*x + c*x^2)^2,x]
```

output
```
(x^3*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*(-(((b^2 - 3*a*c)
*x)/c^2) + (b*x^2)/(2*c) + ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c
*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + (b*(b^2 - 4*a*c)*Log[a +
 b*x + c*x^2])/(2*c^3)))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.42

method	result	size
default	$\frac{x}{c^{2}}-\frac{\frac{-\frac{\left(2 a^{2} c^{2}-4 c a \,b^{2}+b^{4}\right) x}{c \left(4 a c -b^{2}\right)}+\frac{b a \left(3 a c -b^{2}\right)}{c \left(4 a c -b^{2}\right)}}{c \,x^{2}+b x +a}+\frac{\frac{\left(4 a b c -b^{3}\right) \ln \left(c \,x^{2}+b x +a \right)}{c}+\frac{4 \left(3 a^{2} c -a \,b^{2}-\frac{\left(4 a b c -b^{3}\right) b}{2 c}\right) \arctan \left(\frac{2 c x +b}{\sqrt{4 a c -b^{2}}}\right)}{\sqrt{4 a c -b^{2}}}}{4 a c -b^{2}}}{c^{2}}$	198
risch	Expression too large to display	1176

input
```
int(x^4/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*x-1/c^2*((-(2*a^2*c^2-4*a*b^2*c+b^4)/c/(4*a*c-b^2)*x+b*a/c*(3*a*c-b^
2)/(4*a*c-b^2))/(c*x^2+b*x+a)+2/(4*a*c-b^2)*(1/2*(4*a*b*c-b^3)/c*ln(c*x^2+
b*x+a)+2*(3*a^2*c-a*b^2-1/2*(4*a*b*c-b^3)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. 2(135) = 270.

Time = 0.10 (sec) , antiderivative size = 837, normalized size of antiderivative = 6.02

$$\int \frac{x^4}{(a+bx+cx^2)^2}\,dx=\text{Too large to display}$$

input
```
integrate(x^4/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
[-(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*x^3 - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (a*b^4 - 6*a^2*b^2*c +
 6*a^3*c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*
a^2*b*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + s
qrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^6 - 9*a*b^4*c + 26*a
^2*b^2*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c
- 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x)*
log(c*x^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 -
 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x)
, -(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*x^3 - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + 2*(a*b^4 - 6*a^2*b^2*
c + 6*a^3*c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c +
 6*a^2*b*c^2)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)
/(b^2 - 4*a*c)) + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*x + (a*b
^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2
 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^3
- 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 +
(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 842 vs. $2(134) = 268$.

Time = 0.95 (sec) , antiderivative size = 842, normalized size of antiderivative = 6.06

$$\int \frac{x^4}{(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**4/(c*x**2+b*x+a)**2,x)
```

output
```
(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**
3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a
**2*b*c - 16*a**2*c**4*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 -
6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
 b**6))) + 2*a*b**3 + 8*a*b**2*c**3*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6
*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6))) - b**4*c**2*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a
**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12
*a*b**4*c - b**6))))/(12*a**2*c**2 - 12*a*b**2*c + 2*b**4)) + (-b/c**3 + s
qrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c
**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**2*b*c - 16
*a**2*c**4*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c +
 b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 2
*a*b**3 + 8*a*b**2*c**3*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 -
 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6))) - b**4*c**2*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6
*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
b**6))))/(12*a**2*c**2 - 12*a*b**2*c + 2*b**4)) + (-3*a**2*b*c + a*b**3 +
x*(2*a**2*c**2 - 4*a*b**2*c + b**4))/(4*a**2*c**4 - a*b**2*c**3 + x**2*(4*
a*c**5 - b**2*c**4) + x*(4*a*b*c**4 - b**3*c**3)) + x/c**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(a + bx + cx^2)^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.16

$$\int \frac{x^4}{(a+bx+cx^2)^2}\,dx = \frac{2\,(b^4-6\,ab^2c+6\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^3-4\,ac^4)\sqrt{-b^2+4\,ac}}+\frac{x}{c^2} - \frac{b\log\left(cx^2+bx+a\right)}{c^3} - \frac{\frac{(b^4-4\,ab^2c+2\,a^2c^2)x}{c}+\frac{ab^3-3\,a^2bc}{c}}{(cx^2+bx+a)(b^2-4\,ac)c^2}$$

input
```
integrate(x^4/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b
^2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4*a*c)) + x/c^2 - b*log(c*x^2 + b*x + a)/c^3
 - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*x/c + (a*b^3 - 3*a^2*b*c)/c)/((c*x^2 + b
*x + a)*(b^2 - 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.88

$$\int \frac{x^4}{(a+bx+cx^2)^2}\,dx = \frac{x}{c^2}+\frac{\frac{a\,(b^3-3\,a\,b\,c)}{c\,(4\,a\,c-b^2)}+\frac{x\,(2\,a^2\,c^2-4\,a\,b^2\,c+b^4)}{c\,(4\,a\,c-b^2)}}{c^3\,x^2+b\,c^2\,x+a\,c^2} + \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-128\,a^3\,b\,c^3+96\,a^2\,b^3\,c^2-24\,a\,b^5\,c+2\,b^7\right)}{2\,\left(64\,a^3\,c^6-48\,a^2\,b^2\,c^5+12\,a\,b^4\,c^4-b^6\,c^3\right)} - \frac{2\,\mathrm{atan}\left(\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}-\frac{b^3\,c^2-4\,a\,b\,c^3}{c^2\,{(4\,a\,c-b^2)}^{3/2}}\right)\,\left(6\,a^2\,c^2-6\,a\,b^2\,c+b^4\right)}{c^3\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

input
```
int(x^4/(a + b*x + c*x^2)^2,x)
```

output
```
x/c^2 + ((a*(b^3 - 3*a*b*c))/(c*(4*a*c - b^2)) + (x*(b^4 + 2*a^2*c^2 - 4*a
*b^2*c))/(c*(4*a*c - b^2)))/(a*c^2 + c^3*x^2 + b*c^2*x) + (log(a + b*x + c
*x^2)*(2*b^7 - 128*a^3*b*c^3 + 96*a^2*b^3*c^2 - 24*a*b^5*c))/(2*(64*a^3*c^
6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (2*atan((2*c*x)/(4*a*c - b
^2)^(1/2) - (b^3*c^2 - 4*a*b*c^3)/(c^2*(4*a*c - b^2)^(3/2)))*(b^4 + 6*a^2*
c^2 - 6*a*b^2*c))/(c^3*(4*a*c - b^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 752, normalized size of antiderivative = 5.41

$$\int \frac{x^4}{(a+bx+cx^2)^2}\,dx$$

$$= \frac{-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^5-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^6x-16\log(c\,x^2+bx+a)\,a^3b^2c^2+8\log}{}$$

input
```
int(x^4/(c*x^2+b*x+a)^2,x)
```

output
```
( - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**2
 + 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3*c
- 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2*c**
2*x - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c*
*3*x**2 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**5
 + 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**4*c*x +
 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3*c**2*x*
*2 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**6*x - 2*
sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**5*c*x**2 - 16*l
og(a + b*x + c*x**2)*a**3*b**2*c**2 + 8*log(a + b*x + c*x**2)*a**2*b**4*c
- 16*log(a + b*x + c*x**2)*a**2*b**3*c**2*x - 16*log(a + b*x + c*x**2)*a**
2*b**2*c**3*x**2 - log(a + b*x + c*x**2)*a*b**6 + 8*log(a + b*x + c*x**2)*
a*b**5*c*x + 8*log(a + b*x + c*x**2)*a*b**4*c**2*x**2 - log(a + b*x + c*x*
*2)*b**7*x - log(a + b*x + c*x**2)*b**6*c*x**2 - 24*a**4*c**3 + 14*a**3*b*
*2*c**2 - 24*a**3*c**4*x**2 - 2*a**2*b**4*c + 42*a**2*b**2*c**3*x**2 + 16*
a**2*b*c**4*x**3 - 17*a*b**4*c**2*x**2 - 8*a*b**3*c**3*x**3 + 2*b**6*c*x**
2 + b**5*c**2*x**3)/(b*c**3*(16*a**3*c**2 - 8*a**2*b**2*c + 16*a**2*b*c**2
*x + 16*a**2*c**3*x**2 + a*b**4 - 8*a*b**3*c*x - 8*a*b**2*c**2*x**2 + b**5
*x + b**4*c*x**2))
```

3.237 $\int \frac{x^3}{(a+bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 114

$$\int \frac{x^3}{(a+bx+cx^2)^2}\,dx = -\frac{bx}{c(b^2-4ac)}+\frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{b(b^2-6ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2-4ac)^{3/2}}+\frac{\log(a+bx+cx^2)}{2c^2}$$

output
```
-b*x/c/(-4*a*c+b^2)+x^2*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(-6*a*c+b^2
)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/2*ln(c*x^
2+b*x+a)/c^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{x^3}{(a+bx+cx^2)^2}\,dx = \frac{\frac{2(-2a^2c+b^3x+ab(b-3cx))}{(b^2-4ac)(a+x(b+cx))}+\frac{2b(b^2-6ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+\log(a+x(b+cx))}{2c^2}$$

input
```
Integrate[x^3/(a + b*x + c*x^2)^2,x]
```

output
```
((2*(-2*a^2*c + b^3*x + a*b*(b - 3*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))
) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a
*c)^(3/2) + Log[a + x*(b + c*x)])/(2*c^2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1164, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx+cx^2)^2}\,dx$$

$$\Big\downarrow \text{1164}$$

$$\frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{\int \frac{x(4a+bx)}{cx^2+bx+a}dx}{b^2-4ac}$$

$$\Big\downarrow \text{1200}$$

$$\frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{\int \left(\frac{b}{c}-\frac{ab+(b^2-4ac)x}{c(cx^2+bx+a)}\right)dx}{b^2-4ac}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{-\frac{b(b^2-6ac)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{(b^2-4ac)\log(a+bx+cx^2)}{2c^2}+\frac{bx}{c}}{b^2-4ac}$$

input
```
Int[x^3/(a + b*x + c*x^2)^2,x]
```

output
```
(x^2*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - ((b*x)/c - (b*(b^2 -
 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) -
((b^2 - 4*a*c)*Log[a + b*x + c*x^2])/(2*c^2))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.48

method	result
default	$\frac{\frac{b\left(3ac-b^2\right)x}{c^2\left(4ac-b^2\right)}+\frac{a\left(2ac-b^2\right)}{\left(4ac-b^2\right)c^2}}{c\,x^2+bx+a}+\frac{\frac{\left(4ac-b^2\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(-ab-\frac{\left(4ac-b^2\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c(4ac-b^2)}$
risch	$\frac{\frac{b\left(3ac-b^2\right)x}{c^2\left(4ac-b^2\right)}+\frac{a\left(2ac-b^2\right)}{\left(4ac-b^2\right)c^2}}{c\,x^2+bx+a}+\frac{8\ln\left(-24a^2b\,c^2+10a\,b^3c-b^5-2\sqrt{-b^2(4ac-b^2)(6ac-b^2)^2}\,cx-\sqrt{-b^2(4ac-b^2)(6ac-b^2)^2}\,b\right)a^2}{(4ac-b^2)^2}-\frac{4}{}$

input
```
int(x^3/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(b*(3*a*c-b^2)/c^2/(4*a*c-b^2)*x+a*(2*a*c-b^2)/(4*a*c-b^2)/c^2)/(c*x^2+b*x
+a)+1/c/(4*a*c-b^2)*(1/2*(4*a*c-b^2)/c*ln(c*x^2+b*x+a)+2*(-a*b-1/2*(4*a*c-
b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(108) = 216$.

Time = 0.09 (sec) , antiderivative size = 635, normalized size of antiderivative = 5.57

$$\int \frac{x^3}{(a+bx+cx^2)^2}\,dx$$
$$= \left[\frac{2\,ab^4 - 12\,a^2b^2c + 16\,a^3c^2 + (ab^3 - 6\,a^2bc + (b^3c - 6\,abc^2)x^2 + (b^4 - 6\,ab^2c)x)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bc\cdots}{\cdots}\right.}{2\,(ab^4c^2 - 8\,a^2b^2c\cdots}\right.$$

input
```
integrate(x^3/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
[1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c -
6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2
*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) +
 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
+ (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2
)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*
c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^
4)*x), 1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (
b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(-b^2 + 4*a*c)*arctan(-s
qrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^5 - 7*a*b^3*c + 12*a^2
*b*c^2)*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*
a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(c*x^2 + b*x + a))/(
a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c
^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(104) = 208$.

Time = 0.71 (sec) , antiderivative size = 729, normalized size of antiderivative = 6.39

$$\int \frac{x^3}{(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**3/(c*x**2+b*x+a)**2,x)
```

output
```
(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**
2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(-
b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*
(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**
2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(-b*sqr
t(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*
c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (b*sqrt(-(4
*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(b*sqrt(-(4*a*
c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(b*sqrt(-(4*a
*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(b*sqrt(-(4*a*c - b**
2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**
4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c - a*b**2 + x*(3*
a*b*c - b**3))/(4*a**2*c**3 - a*b**2*c**2 + x**2*(4*a*c**4 - b**2*c**3) +
x*(4*a*b*c**3 - b**3*c**2))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx+cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.10

$$\int \frac{x^3}{(a+bx+cx^2)^2}\,dx = -\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^2-4\,ac^3)\sqrt{-b^2+4\,ac}} + \frac{\log\left(cx^2+bx+a\right)}{2\,c^2} + \frac{ab^2-2\,a^2c+(b^3-3\,abc)x}{(cx^2+bx+a)(b^2-4\,ac)c^2}$$

input
```
integrate(x^3/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
-(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^
3)*sqrt(-b^2 + 4*a*c)) + 1/2*log(c*x^2 + b*x + a)/c^2 + (a*b^2 - 2*a^2*c +
 (b^3 - 3*a*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 9.63 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.45

$$\int \frac{x^3}{(a+bx+cx^2)^2}\,dx = \frac{\frac{a\,(2\,a\,c-b^2)}{c^2\,(4\,a\,c-b^2)}+\frac{b\,x\,(3\,a\,c-b^2)}{c^2\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a} - \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6\right)}{2\,\left(64\,a^3\,c^5-48\,a^2\,b^2\,c^4+12\,a\,b^4\,c^3-b^6\,c^2\right)} + \frac{b\,\mathrm{atan}\left(\frac{c^2\,{\left(4\,a\,c-b^2\right)}^{5/2}\,\left(\frac{2\,b\,x\,\left(6\,a\,c-b^2\right)}{c\,{\left(4\,a\,c-b^2\right)}^3}+\frac{b^2\,\left(4\,a\,c^2-b^2\,c\right)\,\left(6\,a\,c-b^2\right)}{c^3\,{\left(4\,a\,c-b^2\right)}^4}\right)}{b^3-6\,a\,b\,c}\right)\,\left(6\,a\,c-b^2\right)}{c^2\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

input
```
int(x^3/(a + b*x + c*x^2)^2,x)
```

output
```
((a*(2*a*c - b^2))/(c^2*(4*a*c - b^2)) + (b*x*(3*a*c - b^2))/(c^2*(4*a*c -
 b^2)))/(a + b*x + c*x^2) - (log(a + b*x + c*x^2)*(b^6 - 64*a^3*c^3 + 48*a
^2*b^2*c^2 - 12*a*b^4*c))/(2*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2
*b^2*c^4)) + (b*atan((c^2*(4*a*c - b^2)^(5/2)*((2*b*x*(6*a*c - b^2))/(c*(4
*a*c - b^2)^3) + (b^2*(4*a*c^2 - b^2*c)*(6*a*c - b^2))/(c^3*(4*a*c - b^2)^
4)))/(b^3 - 6*a*b*c))*(6*a*c - b^2))/(c^2*(4*a*c - b^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 547, normalized size of antiderivative = 4.80

$$\int \frac{x^3}{(a+bx+cx^2)^2}\,dx = \frac{-12\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2bc+2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^3-12\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^2c}{}$$

input
```
int(x^3/(c*x^2+b*x+a)^2,x)
```

output
```
( - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c +
2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3 - 12*sqrt
(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c*x - 12*sqrt(4
*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c**2*x**2 + 2*sqrt(4
*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**4*x + 2*sqrt(4*a*c -
b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*c*x**2 + 16*log(a + b*x +
c*x**2)*a**3*c**2 - 8*log(a + b*x + c*x**2)*a**2*b**2*c + 16*log(a + b*x +
 c*x**2)*a**2*b*c**2*x + 16*log(a + b*x + c*x**2)*a**2*c**3*x**2 + log(a +
 b*x + c*x**2)*a*b**4 - 8*log(a + b*x + c*x**2)*a*b**3*c*x - 8*log(a + b*x
 + c*x**2)*a*b**2*c**2*x**2 + log(a + b*x + c*x**2)*b**5*x + log(a + b*x +
 c*x**2)*b**4*c*x**2 - 8*a**3*c**2 + 2*a**2*b**2*c - 24*a**2*c**3*x**2 + 1
4*a*b**2*c**2*x**2 - 2*b**4*c*x**2)/(2*c**2*(16*a**3*c**2 - 8*a**2*b**2*c
+ 16*a**2*b*c**2*x + 16*a**2*c**3*x**2 + a*b**4 - 8*a*b**3*c*x - 8*a*b**2*
c**2*x**2 + b**5*x + b**4*c*x**2))
```

3.238 $\int \frac{x^2}{(a+bx+cx^2)^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int \frac{x^2}{(a+bx+cx^2)^2} dx = -\frac{ab+(b^2-2ac)x}{c(b^2-4ac)(a+bx+cx^2)} + \frac{4a\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
-(a*b+(-2*a*c+b^2)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)+4*a*arctanh((2*c*x+b)/(
-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{(a+bx+cx^2)^2} dx = \frac{b^2x+a(b-2cx)}{c(-b^2+4ac)(a+x(b+cx))} + \frac{4a\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

input
```
Integrate[x^2/(a + b*x + c*x^2)^2,x]
```

output
```
(b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))) + (4*a*ArcTan
[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1153, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx+cx^2)^2}\,dx$$

$$\Big\downarrow \text{1153}$$

$$\frac{x(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)}-\frac{2a\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}$$

$$\Big\downarrow \text{1083}$$

$$\frac{4a\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}+\frac{x(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)}$$

$$\Big\downarrow \text{219}$$

$$\frac{4a\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{x(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)}$$

input
```
Int[x^2/(a + b*x + c*x^2)^2,x]
```

output
```
(x*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (4*a*ArcTanh[(b + 2*c*
x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1153
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m - 2)*(a + b*x +
c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
&& LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.26

method	result
default	$\frac{-\frac{\left(2ac-b^2\right)x}{c\left(4ac-b^2\right)}+\frac{ab}{c\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{4a\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}$
risch	$\frac{-\frac{\left(2ac-b^2\right)x}{c\left(4ac-b^2\right)}+\frac{ab}{c\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{2a\ln\left(\left(-8a\,c^2+2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{2a\ln\left(\left(8a\,c^2-2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b\right.}{\left(-4ac+b^2\right)^{\frac{3}{2}}}$

input
```
int(x^2/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-(2*a*c-b^2)/c/(4*a*c-b^2)*x+a*b/c/(4*a*c-b^2))/(c*x^2+b*x+a)+4*a/(4*a*c-
b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(73) = 146$.

Time = 0.09 (sec) , antiderivative size = 387, normalized size of antiderivative = 5.03

$$\int \frac{x^2}{(a+bx+cx^2)^2}\,dx$$
$$= \left[-\frac{ab^3 - 4\,a^2bc + 2\,(ac^2x^2 + abcx + a^2c)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right) + (b^4 - 6\,ab^2}{ab^4c - 8\,a^2b^2c^2 + 16\,a^3c^3 + (b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4)x^2 + (b^5c - 8\,ab^3c^2 + 16\,a^2bc^3)x}\right.$$
$$-\frac{ab^3 - 4\,a^2bc - 4\,(ac^2x^2 + abcx + a^2c)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right) + (b^4 - 6\,ab^2c + 8\,a^2c^2)}{ab^4c - 8\,a^2b^2c^2 + 16\,a^3c^3 + (b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4)x^2 + (b^5c - 8\,ab^3c^2 + 16\,a^2bc^3)x}$$

input
```
integrate(x^2/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
[-(a*b^3 - 4*a^2*b*c + 2*(a*c^2*x^2 + a*b*c*x + a^2*c)*sqrt(b^2 - 4*a*c)*l
og((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*
x^2 + b*x + a)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^
2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b
^3*c^2 + 16*a^2*b*c^3)*x), -(a*b^3 - 4*a^2*b*c - 4*(a*c^2*x^2 + a*b*c*x +
a^2*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*
a*c)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3
*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 1
6*a^2*b*c^3)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(70) = 140$.

Time = 0.31 (sec) , antiderivative size = 280, normalized size of antiderivative = 3.64

$$\int \frac{x^2}{(a+bx+cx^2)^2}\,dx = -2a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{-32a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+16a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}-2ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab}{4ac}\right) +2a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{32a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-16a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab}{4ac}\right) + \frac{ab+x(-2ac+b^2)}{4a^2c^2-ab^2c+x^2\cdot(4ac^3-b^2c^2)+x(4abc^2-b^3c)}$$

input
```
integrate(x**2/(c*x**2+b*x+a)**2,x)
```

output
```
-2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**3*c**2*sqrt(-1/(4*a*c - b*
*2)**3) + 16*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - 2*a*b**4*sqrt(-1/(4*
a*c - b**2)**3) + 2*a*b)/(4*a*c)) + 2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x +
 (32*a**3*c**2*sqrt(-1/(4*a*c - b**2)**3) - 16*a**2*b**2*c*sqrt(-1/(4*a*c
- b**2)**3) + 2*a*b**4*sqrt(-1/(4*a*c - b**2)**3) + 2*a*b)/(4*a*c)) + (a*b
 + x*(-2*a*c + b**2))/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2
) + x*(4*a*b*c**2 - b**3*c))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx+cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14

$$\int \frac{x^2}{(a+bx+cx^2)^2}\,dx = -\frac{4\,a\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}-\frac{b^2x-2\,acx+ab}{(b^2c-4\,ac^2)(cx^2+bx+a)}$$

input
```
integrate(x^2/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
-4*a*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a
*c)) - (b^2*x - 2*a*c*x + a*b)/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a))
```

Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.75

$$\int \frac{x^2}{(a+bx+cx^2)^2}\,dx = -\frac{\frac{x\,(2\,a\,c-b^2)}{c\,(4\,a\,c-b^2)}-\frac{a\,b}{c\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a} - \frac{4\,a\,\mathrm{atan}\left(\frac{\left(\frac{2\,a\,\left(b^3-4\,a\,b\,c\right)}{\left(4\,a\,c-b^2\right)^{5/2}}-\frac{4\,a\,c\,x}{\left(4\,a\,c-b^2\right)^{3/2}}\right)\,(4\,a\,c-b^2)}{2\,a}\right)}{\left(4\,a\,c-b^2\right)^{3/2}}$$

input
```
int(x^2/(a + b*x + c*x^2)^2,x)
```

output
```
- ((x*(2*a*c - b^2))/(c*(4*a*c - b^2)) - (a*b)/(c*(4*a*c - b^2)))/(a + b*x
 + c*x^2) - (4*a*atan((((2*a*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*a*c
*x)/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/(2*a)))/(4*a*c - b^2)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.16

$$\int \frac{x^2}{(a+bx+cx^2)^2}\,dx$$
$$= \frac{4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2b+4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^2x+4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)abc\,x^2+}{b\left(16a^2c^3x^2-8a\,b^2c^2x^2+b^4c\,x^2+16a^2b\,c^2x-8a\,b^3cx+b^5x+16a^3c^2-8\right.}$$

input
```
int(x^2/(c*x^2+b*x+a)^2,x)
```

output
```
(4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b + 4*sqrt
(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*x + 4*sqrt(4*a*
c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c*x**2 + 8*a**3*c - 2*a
**2*b**2 + 8*a**2*c**2*x**2 - 6*a*b**2*c*x**2 + b**4*x**2)/(b*(16*a**3*c**
2 - 8*a**2*b**2*c + 16*a**2*b*c**2*x + 16*a**2*c**3*x**2 + a*b**4 - 8*a*b*
*3*c*x - 8*a*b**2*c**2*x**2 + b**5*x + b**4*c*x**2))
```

3.239 $\int \frac{x}{(a+bx+cx^2)^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 66

$$\int \frac{x}{(a+bx+cx^2)^2} dx = \frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)} - \frac{2b\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)-2*b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

$$\int \frac{x}{(a+bx+cx^2)^2} dx = \frac{2a+bx}{(b^2-4ac)(a+x(b+cx))} - \frac{2b\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

input
```
Integrate[x/(a + b*x + c*x^2)^2,x]
```

output
```
(2*a + b*x)/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*b*ArcTan[(b + 2*c*x)/Sq
rt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1159, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx+cx^2)^2} \, dx$$

$$\Big\downarrow \text{1159}$$

$$\frac{b\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}+\frac{2a+bx}{(b^2-4ac)\,(a+bx+cx^2)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{2a+bx}{(b^2-4ac)\,(a+bx+cx^2)}-\frac{2b\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}$$

$$\Big\downarrow \text{219}$$

$$\frac{2a+bx}{(b^2-4ac)\,(a+bx+cx^2)}-\frac{2b\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

input
```
Int[x/(a + b*x + c*x^2)^2,x]
```

output
```
(2*a + b*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*b*ArcTanh[(b + 2*c*x)/S
qrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

method	result	siz
default	$\frac{-bx-2a}{(4ac-b^2)(c\,x^2+bx+a)}-\frac{2b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$	70
risch	$\frac{-\frac{bx}{4ac-b^2}-\frac{2a}{4ac-b^2}}{c\,x^2+bx+a}+\frac{b\ln\left(\left(-8a\,c^2+2b^2c\right)x-\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{b\ln\left(\left(8a\,c^2-2b^2c\right)x-\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}$	14

input
```
int(x/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)-2*b/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(62) = 124$.

Time = 0.08 (sec) , antiderivative size = 338, normalized size of antiderivative = 5.12

$$\int \frac{x}{(a+bx+cx^2)^2}\,dx = \left[\frac{2\,ab^2 - 8\,a^2c - (bcx^2 + b^2x + ab)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right) + (b^3-4\,abc)x}{ab^4 - 8\,a^2b^2c + 16\,a^3c^2 + (b^4c - 8\,ab^2c^2 + 16\,a^2c^3)x^2 + (b^5 - 8\,ab^3c + 16\,a^2bc^2)x}, 2\,a\right.$$

input
```
integrate(x/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
[(2*a*b^2 - 8*a^2*c - (b*c*x^2 + b^2*x + a*b)*sqrt(b^2 - 4*a*c)*log((2*c^2
*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
 + a)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), (2*a*b
^2 - 8*a^2*c - 2*(b*c*x^2 + b^2*x + a*b)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-
b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^
2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a
*b^3*c + 16*a^2*b*c^2)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(60) = 120$.

Time = 0.29 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.83

$$\int \frac{x}{(a+bx+cx^2)^2}\,dx = b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x + \frac{-16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}} + 8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}} - b^5\sqrt{-\frac{1}{(4ac-b^2)^3}} + b^2}{2bc}\right) - b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x + \frac{16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}} - 8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}} + b^5\sqrt{-\frac{1}{(4ac-b^2)^3}} + b^2}{2bc}\right) + \frac{-2a-bx}{4a^2c - ab^2 + x^2\cdot(4ac^2-b^2c) + x\,(4abc-b^3)}$$

input
```
integrate(x/(c*x**2+b*x+a)**2,x)
```

output
```
b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-16*a**2*b*c**2*sqrt(-1/(4*a*c - b**
2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c - b**
2)**3) + b**2)/(2*b*c)) - b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (16*a**2*b*
c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) +
b**5*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c)) + (-2*a - b*x)/(4*a**2*c
- a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx+cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \frac{x}{(a+bx+cx^2)^2}\,dx = \frac{2\,b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} + \frac{bx+2\,a}{(cx^2+bx+a)(b^2-4\,ac)}$$

input
```
integrate(x/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
2*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*
c)) + (b*x + 2*a)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.67

$$\int \frac{x}{(a+bx+cx^2)^2}\,dx = -\frac{\frac{2\,a}{4\,a\,c-b^2}+\frac{b\,x}{4\,a\,c-b^2}}{c\,x^2+b\,x+a} - \frac{2\,b\,\mathrm{atan}\left(\frac{\left(\frac{b^2}{\left(4\,a\,c-b^2\right)^{3/2}}+\frac{2\,b\,c\,x}{\left(4\,a\,c-b^2\right)^{3/2}}\right)\,\left(4\,a\,c-b^2\right)}{b}\right)}{\left(4\,a\,c-b^2\right)^{3/2}}$$

input
```
int(x/(a + b*x + c*x^2)^2,x)
```

output
```
- ((2*a)/(4*a*c - b^2) + (b*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (2*b*ata
n(((b^2/(4*a*c - b^2)^(3/2) + (2*b*c*x)/(4*a*c - b^2)^(3/2))*(4*a*c - b^2)
)/b))/(4*a*c - b^2)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.38

$$\int \frac{x}{(a+bx+cx^2)^2}\,dx$$
$$= \frac{-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)ab - 2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^2x - 2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)bc\,x^2 - 4}{16a^2c^3x^2 - 8a\,b^2c^2x^2 + b^4c\,x^2 + 16a^2b\,c^2x - 8a\,b^3cx + b^5x + 16a^3c^2 - 8a^2b^2c +}$$

input
```
int(x/(c*x^2+b*x+a)^2,x)
```

output
```
( - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b - 2*sqrt
(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*x - 2*sqrt(4*a*c
- b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c*x**2 - 4*a**2*c + a*b**2
+ 4*a*c**2*x**2 - b**2*c*x**2)/(16*a**3*c**2 - 8*a**2*b**2*c + 16*a**2*b*c
**2*x + 16*a**2*c**3*x**2 + a*b**4 - 8*a*b**3*c*x - 8*a*b**2*c**2*x**2 + b
**5*x + b**4*c*x**2)
```

3.240 $\int \frac{1}{(a+bx+cx^2)^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 66

$$\int \frac{1}{(a+bx+cx^2)^2} \, dx = -\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)} + \frac{4c\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
-(2*c*x+b)/(-4*a*c+b^2)/(c*x^2+b*x+a)+4*c*arctanh((2*c*x+b)/(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+bx+cx^2)^2} \, dx = -\frac{\frac{b+2cx}{a+x(b+cx)} + \frac{4c\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{b^2-4ac}$$

input
```
Integrate[(a + b*x + c*x^2)^(-2),x]
```

output
```
-(((b + 2*c*x)/(a + x*(b + c*x)) + (4*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a
*c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx+cx^2)^2} \, dx$$

$$\Big\downarrow \text{1086}$$

$$-\frac{2c\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{4c\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \text{219}$$

$$\frac{4c\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

input
```
Int[(a + b*x + c*x^2)^(-2),x]
```

output
```
-((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

method	result	si
default	$\frac{2cx+b}{(4ac-b^2)(c\,x^2+bx+a)}+\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$	68
risch	$\frac{\frac{2cx}{4ac-b^2}+\frac{b}{4ac-b^2}}{c\,x^2+bx+a}+\frac{2c\ln\left(\left(-8a\,c^2+2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{2c\ln\left(\left(8a\,c^2-2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}$	14

input
```
int(1/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(62) = 124$.

Time = 0.08 (sec) , antiderivative size = 341, normalized size of antiderivative = 5.17

$$\int \frac{1}{(a+bx+cx^2)^2}\,dx$$
$$= \left[-\frac{b^3-4\,abc+2\,(c^2x^2+bcx+ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+2\,(b^2c-4\,ac^2)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x},\right.$$
$$\left.-\frac{b^3-4\,abc-4\,(c^2x^2+bcx+ac)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+2\,(b^2c-4\,ac^2)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x}\right]$$

input
```
integrate(1/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
[-(b^3 - 4*a*b*c + 2*(c^2*x^2 + b*c*x + a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
+ a)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), -(b
^3 - 4*a*b*c - 4*(c^2*x^2 + b*c*x + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-
b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 -
8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 -
 8*a*b^3*c + 16*a^2*b*c^2)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(61) = 122$.

Time = 0.29 (sec) , antiderivative size = 265, normalized size of antiderivative = 4.02

$$\int \frac{1}{(a+bx+cx^2)^2}\,dx = -2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{-32a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+16ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-2b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2bc}{4c^2}\right) +2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{32a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}-16ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+2b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2bc}{4c^2}\right) + \frac{b+2cx}{4a^2c-ab^2+x^2\cdot(4ac^2-b^2c)+x\,(4abc-b^3)}$$

input
```
integrate(1/(c*x**2+b*x+a)**2,x)
```

output
```
-2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**2*c**3*sqrt(-1/(4*a*c - b*
*2)**3) + 16*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - 2*b**4*c*sqrt(-1/(4*
a*c - b**2)**3) + 2*b*c)/(4*c**2)) + 2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x
+ (32*a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 16*a*b**2*c**2*sqrt(-1/(4*a*c
 - b**2)**3) + 2*b**4*c*sqrt(-1/(4*a*c - b**2)**3) + 2*b*c)/(4*c**2)) + (b
 + 2*c*x)/(4*a**2*c - a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**
3))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx+cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \frac{1}{(a+bx+cx^2)^2}\,dx=-\frac{4\,c\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}-\frac{2\,cx+b}{(cx^2+bx+a)(b^2-4\,ac)}$$

input
```
integrate(1/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
-4*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a
*c)) - (2*c*x + b)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.80

$$\int \frac{1}{(a+bx+cx^2)^2}\,dx=\frac{\frac{b}{4\,a\,c-b^2}+\frac{2\,c\,x}{4\,a\,c-b^2}}{c\,x^2+b\,x+a}-\frac{4\,c\,\mathrm{atan}\left(\frac{\left(\frac{2\,c\,\left(b^3-4\,a\,b\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}-\frac{4\,c^2\,x}{{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(4\,a\,c-b^2\right)}{2\,c}\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

input
```
int(1/(a + b*x + c*x^2)^2,x)
```

output
```
(b/(4*a*c - b^2) + (2*c*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (4*c*atan(((
(2*c*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*c^2*x)/(4*a*c - b^2)^(3/2))
*(4*a*c - b^2))/(2*c)))/(4*a*c - b^2)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.65

$$\int \frac{1}{(a+bx+cx^2)^2}\,dx$$
$$= \frac{4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)abc + 4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^2cx + 4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b\,c^2x^2 -}{b\,(16a^2c^3x^2 - 8a\,b^2c^2x^2 + b^4c\,x^2 + 16a^2b\,c^2x - 8a\,b^3cx + b^5x + 16a^3c^2 - 8a^2}$$

input
```
int(1/(c*x^2+b*x+a)^2,x)
```

output
```
(4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c + 4*sqrt(
4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c*x + 4*sqrt(4*a*c
 - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c**2*x**2 - 8*a**2*c**2 +
6*a*b**2*c - 8*a*c**3*x**2 - b**4 + 2*b**2*c**2*x**2)/(b*(16*a**3*c**2 - 8
*a**2*b**2*c + 16*a**2*b*c**2*x + 16*a**2*c**3*x**2 + a*b**4 - 8*a*b**3*c*
x - 8*a*b**2*c**2*x**2 + b**5*x + b**4*c*x**2))
```

3.241 $\int \frac{1}{x(a+bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 108

$$\int \frac{1}{x(a+bx+cx^2)^2}\,dx = \frac{b^2-2ac+bcx}{a(b^2-4ac)(a+bx+cx^2)} + \frac{b(b^2-6ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2(b^2-4ac)^{3/2}} + \frac{\log(x)}{a^2} - \frac{\log(a+bx+cx^2)}{2a^2}$$

output
```
(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(-6*a*c+b^2)*arctanh((2*c
*x+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+ln(x)/a^2-1/2*ln(c*x^2+b*
x+a)/a^2
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

$$\int \frac{1}{x(a+bx+cx^2)^2}\,dx = \frac{\frac{2a(b^2-2ac+bcx)}{(b^2-4ac)(a+x(b+cx))} + \frac{2b(b^2-6ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + 2\log(x) - \log(a+x(b+cx))}{2a^2}$$

input
```
Integrate[1/(x*(a + b*x + c*x^2)^2),x]
```

output
```
((2*a*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(b^2
 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2
*Log[x] - Log[a + x*(b + c*x)])/(2*a^2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.29, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1165, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+bx+cx^2)^2} \, dx$$

$$\Big\downarrow \; 1165$$

$$\frac{-2ac+b^2+bcx}{a (b^2-4ac) (a+bx+cx^2)}-\frac{\int -\frac{b^2+cxb-4ac}{x(cx^2+bx+a)}dx}{a (b^2-4ac)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{b^2+cxb-4ac}{x(cx^2+bx+a)}dx}{a (b^2-4ac)}+\frac{-2ac+b^2+bcx}{a (b^2-4ac) (a+bx+cx^2)}$$

$$\Big\downarrow \; 1200$$

$$\frac{\int \left(\frac{b^2-4ac}{ax}+\frac{-b(b^2-5ac)-c(b^2-4ac)x}{a(cx^2+bx+a)}\right)dx}{a (b^2-4ac)}+\frac{-2ac+b^2+bcx}{a (b^2-4ac) (a+bx+cx^2)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{b(b^2-6ac)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{(b^2-4ac)\log(a+bx+cx^2)}{2a}+\frac{\log(x)(b^2-4ac)}{a}}{a (b^2-4ac)}+$$
$$\frac{-2ac+b^2+bcx}{a (b^2-4ac) (a+bx+cx^2)}$$

input
```
Int[1/(x*(a + b*x + c*x^2)^2),x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + ((b*(b^2 - 6*a
*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + ((b^2
- 4*a*c)*Log[x])/a - ((b^2 - 4*a*c)*Log[a + b*x + c*x^2])/(2*a))/(a*(b^2 -
 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.64

method	result	size
default	$\frac{\ln(x)}{a^2}-\frac{\frac{\frac{abcx}{4ac-b^2}-\frac{a\left(2ac-b^2\right)}{4ac-b^2}}{c\,x^2+bx+a}+\frac{\frac{\left(4a\,c^2-b^2c\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(5abc-b^3-\frac{\left(4a\,c^2-b^2c\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{a^2}$	177
risch	Expression too large to display	2292

input
```
int(1/x/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a^2-1/a^2*((a*b*c/(4*a*c-b^2)*x-a*(2*a*c-b^2)/(4*a*c-b^2))/(c*x^2+b*
x+a)+1/(4*a*c-b^2)*(1/2*(4*a*c^2-b^2*c)/c*ln(c*x^2+b*x+a)+2*(5*a*b*c-b^3-1
/2*(4*a*c^2-b^2*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/
2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(102) = 204.

Time = 0.14 (sec) , antiderivative size = 781, normalized size of antiderivative = 7.23

$$\int \frac{1}{x\left(a+bx+cx^2\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate(1/x/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
[1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c -
6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2
*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) +
 2*(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(
c*x^2 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*
c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(x))/(a^3*b
^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x
^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x), 1/2*(2*a*b^4 - 12*a^2*b^2*
c + 16*a^3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6
*a*b^2*c)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^
2 - 4*a*c)) + 2*(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*
c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b
*c^2)*x)*log(c*x^2 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4
*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*l
og(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 +
16*a^4*c^3)*x^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx+cx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(c*x**2+b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(a+bx+cx^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.17

$$\int \frac{1}{x(a+bx+cx^2)^2}\,dx = -\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^2b^2-4\,a^3c)\sqrt{-b^2+4\,ac}} - \frac{\log\left(cx^2+bx+a\right)}{2\,a^2} + \frac{\log\left(|x|\right)}{a^2} + \frac{abcx+ab^2-2\,a^2c}{(cx^2+bx+a)(b^2-4\,ac)a^2}$$

input
```
integrate(1/x/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
-(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2 - 4*a^3*
c)*sqrt(-b^2 + 4*a*c)) - 1/2*log(c*x^2 + b*x + a)/a^2 + log(abs(x))/a^2 +
(a*b*c*x + a*b^2 - 2*a^2*c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^2)
```

Mupad [B] (verification not implemented)

Time = 10.04 (sec) , antiderivative size = 620, normalized size of antiderivative = 5.74

$$\int \frac{1}{x(a+bx+cx^2)^2}\,dx = \frac{\ln(x)}{a^2} + \frac{\frac{2\,a\,c-b^2}{a\,(4\,a\,c-b^2)} - \frac{b\,c\,x}{a\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a} + \frac{\ln\left(2\,a\,b^6+2\,b^7\,x-96\,a^4\,c^3+2\,a\,b^3\,\sqrt{-(4\,a\,c-b^2)^3}-23\,a^2\,b^4\,c+2\,b^4\,x\,\sqrt{-(4\,a\,c-b^2)^3}+84\,a^3\,b^2\,c^2\right.}{} + \frac{\ln\left(96\,a^4\,c^3-2\,b^7\,x-2\,a\,b^6+2\,a\,b^3\,\sqrt{-(4\,a\,c-b^2)^3}+23\,a^2\,b^4\,c+2\,b^4\,x\,\sqrt{-(4\,a\,c-b^2)^3}-84\,a^3\,b^2\,c^2\right.}{}$$

input
```
int(1/(x*(a + b*x + c*x^2)^2),x)
```

output
```
log(x)/a^2 + ((2*a*c - b^2)/(a*(4*a*c - b^2)) - (b*c*x)/(a*(4*a*c - b^2)))
/(a + b*x + c*x^2) + (log(2*a*b^6 + 2*b^7*x - 96*a^4*c^3 + 2*a*b^3*(-(4*a*
c - b^2)^3)^(1/2) - 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^(1/2) + 84*a
^3*b^2*c^2 + 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2) - 24
*a*b^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2) - 120*a^3*b*c^3*x - 12*a*b
^2*c*x*(-(4*a*c - b^2)^3)^(1/2))*(b^6 - 64*a^3*c^3 + b^3*(-(4*a*c - b^2)^3
)^(1/2) + 48*a^2*b^2*c^2 - 12*a*b^4*c - 6*a*b*c*(-(4*a*c - b^2)^3)^(1/2)))
/(2*a^2*(4*a*c - b^2)^3) + (log(96*a^4*c^3 - 2*b^7*x - 2*a*b^6 + 2*a*b^3*(
-(4*a*c - b^2)^3)^(1/2) + 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^(1/2)
- 84*a^3*b^2*c^2 - 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2
) + 24*a*b^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2) + 120*a^3*b*c^3*x -
12*a*b^2*c*x*(-(4*a*c - b^2)^3)^(1/2))*(b^6 - 64*a^3*c^3 - b^3*(-(4*a*c -
b^2)^3)^(1/2) + 48*a^2*b^2*c^2 - 12*a*b^4*c + 6*a*b*c*(-(4*a*c - b^2)^3)^(
1/2)))/(2*a^2*(4*a*c - b^2)^3)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 644, normalized size of antiderivative = 5.96

$$\int \frac{1}{x\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{32\log(x)\,a^3c^2+2\log(x)\,a\,b^4+2\log(x)\,b^5x-12\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^2cx-12\sqrt{4ac-b^2}\,atan\left(\frac{}{\sqrt{}}\right.}{}$$

input
```
int(1/x/(c*x^2+b*x+a)^2,x)
```

output
```
( - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c +
2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3 - 12*sqrt
(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c*x - 12*sqrt(4
*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c**2*x**2 + 2*sqrt(4
*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**4*x + 2*sqrt(4*a*c -
b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*c*x**2 - 16*log(a + b*x +
c*x**2)*a**3*c**2 + 8*log(a + b*x + c*x**2)*a**2*b**2*c - 16*log(a + b*x +
 c*x**2)*a**2*b*c**2*x - 16*log(a + b*x + c*x**2)*a**2*c**3*x**2 - log(a +
 b*x + c*x**2)*a*b**4 + 8*log(a + b*x + c*x**2)*a*b**3*c*x + 8*log(a + b*x
 + c*x**2)*a*b**2*c**2*x**2 - log(a + b*x + c*x**2)*b**5*x - log(a + b*x +
 c*x**2)*b**4*c*x**2 + 32*log(x)*a**3*c**2 - 16*log(x)*a**2*b**2*c + 32*lo
g(x)*a**2*b*c**2*x + 32*log(x)*a**2*c**3*x**2 + 2*log(x)*a*b**4 - 16*log(x
)*a*b**3*c*x - 16*log(x)*a*b**2*c**2*x**2 + 2*log(x)*b**5*x + 2*log(x)*b**
4*c*x**2 + 24*a**3*c**2 - 14*a**2*b**2*c + 8*a**2*c**3*x**2 + 2*a*b**4 - 2
*a*b**2*c**2*x**2)/(2*a**2*(16*a**3*c**2 - 8*a**2*b**2*c + 16*a**2*b*c**2*
x + 16*a**2*c**3*x**2 + a*b**4 - 8*a*b**3*c*x - 8*a*b**2*c**2*x**2 + b**5*
x + b**4*c*x**2))
```

3.242 $\int \frac{1}{x^2(a+bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 138

$$\int \frac{1}{x^2(a+bx+cx^2)^2}\,dx = -\frac{1}{a^2x} - \frac{b(b^2-3ac)+c(b^2-2ac)x}{a^2(b^2-4ac)(a+bx+cx^2)} - \frac{2(b^4-6ab^2c+6a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}} - \frac{2b\log(x)}{a^3} + \frac{b\log(a+bx+cx^2)}{a^3}$$

output
```
-1/a^2/x-(b*(-3*a*c+b^2)+c*(-2*a*c+b^2)*x)/a^2/(-4*a*c+b^2)/(c*x^2+b*x+a)-
2*(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*
a*c+b^2)^(3/2)-2*b*ln(x)/a^3+b*ln(c*x^2+b*x+a)/a^3
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2 (a+bx+cx^2)^2} \, dx = -\frac{\frac{a}{x}+\frac{a(b^3-3abc+b^2cx-2ac^2x)}{(b^2-4ac)(a+x(b+cx))}+\frac{2(b^4-6ab^2c+6a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+2b\log(x)-b\log(a+x(b+cx))}{a^3}$$

input
```
Integrate[1/(x^2*(a + b*x + c*x^2)^2),x]
```

output
```
-((a/x + (a*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((b^2 - 4*a*c)*(a + x*(
b + c*x))) + (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2
 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*b*Log[x] - b*Log[a + x*(b + c*x)])/a^
3)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.25, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1165, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx+cx^2)^2} \, dx$$

$$\Big\downarrow \text{ 1165}$$

$$\frac{-2ac+b^2+bcx}{ax(b^2-4ac)(a+bx+cx^2)}-\frac{\int -\frac{2(b^2+cxb-3ac)}{x^2(cx^2+bx+a)}dx}{a(b^2-4ac)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\int \frac{b^2+cxb-3ac}{x^2(cx^2+bx+a)}dx}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{ax(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \text{1200}$$

$$\frac{2\int\left(\frac{b^2-3ac}{ax^2}+\frac{4abc-b^3}{a^2x}+\frac{b^4-5acb^2+c(b^2-4ac)xb+3a^2c^2}{a^2(cx^2+bx+a)}\right)dx}{a\,(b^2-4ac)}+\frac{-2ac+b^2+bcx}{ax\,(b^2-4ac)\,(a+bx+cx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(-\frac{(6a^2c^2-6ab^2c+b^4)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{b(b^2-4ac)\log(a+bx+cx^2)}{2a^2}-\frac{b\log(x)(b^2-4ac)}{a^2}-\frac{b^2-3ac}{ax}\right)}{a\,(b^2-4ac)}+\frac{-2ac+b^2+bcx}{ax\,(b^2-4ac)\,(a+bx+cx^2)}$$

input
```
Int[1/(x^2*(a + b*x + c*x^2)^2),x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)) + (2*(-((b^2 -
 3*a*c)/(a*x)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b
^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - (b*(b^2 - 4*a*c)*Log[x])/a^2 + (b*
(b^2 - 4*a*c)*Log[a + b*x + c*x^2])/(2*a^2)))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.49

method	result
default	$-\frac{1}{a^2 x}-\frac{2b\ln(x)}{a^3}-\frac{\frac{\frac{ac(2ac-b^2)x}{4ac-b^2}+\frac{ab(3ac-b^2)}{4ac-b^2}}{cx^2+bx+a}+\frac{\frac{(-4abc^2+b^3c)\ln(cx^2+bx+a)}{c}+\frac{4\left(3a^2c^2-5cab^2+b^4-\frac{(-4abc^2+b^3c)b}{2c}\right)\arctan\left(\right.}{\sqrt{4ac-b^2}}}{4ac-b^2}}{a^3}$
risch	Expression too large to display

input
```
int(1/x^2/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2/x-2*b*ln(x)/a^3-1/a^3*((a*c*(2*a*c-b^2)/(4*a*c-b^2)*x+a*b*(3*a*c-b^
2)/(4*a*c-b^2))/(c*x^2+b*x+a)+2/(4*a*c-b^2)*(1/2*(-4*a*b*c^2+b^3*c)/c*ln(c
*x^2+b*x+a)+2*(3*a^2*c^2-5*c*a*b^2+b^4-1/2*(-4*a*b*c^2+b^3*c)*b/c)/(4*a*c-
b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(134) = 268$.

Time = 0.22 (sec) , antiderivative size = 975, normalized size of antiderivative = 7.07

$$\int \frac{1}{x^2(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
[-(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^
3*c^3)*x^2 + ((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^3 + (b^5 - 6*a*b^3*c + 6
*a^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x)*sqrt(b^2 - 4*a*c)*l
og((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*
x^2 + b*x + a)) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*
a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (
a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 +
 (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c
^2 + 16*a^5*c^3)*x^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^4*b
^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x), -(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 +
2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6
*a^2*c^3)*x^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c
 + 6*a^3*c^2)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)
/(b^2 - 4*a*c)) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*
a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (
a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 +
 (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c
^2 + 16*a^5*c^3)*x^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx + cx^2)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/x**2/(c*x**2+b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (a + bx + cx^2)^2} dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^2/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^2 (a + bx + cx^2)^2} dx = \frac{2 (b^4 - 6 ab^2c + 6 a^2c^2) \arctan\left(\frac{2 cx+b}{\sqrt{-b^2+4 ac}}\right)}{(a^3b^2 - 4 a^4c)\sqrt{-b^2 + 4 ac}} - \frac{2 b^2cx^2 - 6 ac^2x^2 + 2 b^3x - 7 abcx + ab^2 - 4 a^2c}{(a^2b^2 - 4 a^3c)(cx^3 + bx^2 + ax)} + \frac{b \log (cx^2 + bx + a)}{a^3} - \frac{2 b \log (|x|)}{a^3}$$

input
```
integrate(1/x^2/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a
^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - (2*b^2*c*x^2 - 6*a*c^2*x^2 + 2*b^3
*x - 7*a*b*c*x + a*b^2 - 4*a^2*c)/((a^2*b^2 - 4*a^3*c)*(c*x^3 + b*x^2 + a*
x)) + b*log(c*x^2 + b*x + a)/a^3 - 2*b*log(abs(x))/a^3
```

Mupad [B] (verification not implemented)

Time = 9.98 (sec) , antiderivative size = 775, normalized size of antiderivative = 5.62

$$\int \frac{1}{x^2\left(a+bx+cx^2\right)^2}\,dx = \ln\Bigg(2\,a\,b^7+2\,b^8\,x+2\,a\,b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-23\,a^2\,b^5\,c-108\,a^4\,b\,c^3+24\,a^4\,c^4\,x+2\,b^5\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+87\,a^3\,b^3\,c^2+3\,a^3\,c^2\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-9\,a^2\,b^2\,c\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+97\,a^2\,b^4\,c^2\,x-138\,a^3\,b^2\,c^3\,x-24\,a\,b^6\,c\,x-12\,a\,b^3\,c\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+15\,a^2\,b\,c^2\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}\Bigg)\Bigg(\frac{b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+6\,a^2\,c^2\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-6\,a\,b^2\,c\,\sqrt{-(4\,a\,c-b^2)}{-64\,a^6\,c^3+48\,a^5\,b^2\,c^2-12\,a^4\,b^4\,c+a^3\,b^6}+\frac{b}{a^3}\Bigg)-\frac{\frac{1}{a}-\frac{x\,\left(2\,b^3-7\,a\,b\,c\right)}{a^2\,\left(4\,a\,c-b^2\right)}+\frac{2\,c\,x^2\,\left(3\,a\,c-b^2\right)}{a^2\,\left(4\,a\,c-b^2\right)}}{c\,x^3+b\,x^2+a\,x}-\ln\Bigg(2\,a\,b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-2\,b^8\,x-2\,a\,b^7+23\,a^2\,b^5\,c+108\,a^4\,b\,c^3-24\,a^4\,c^4\,x+2\,b^5\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-87\,a^3\,b^3\,c^2+3\,a^3\,c^2\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-9\,a^2\,b^2\,c\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-97\,a^2\,b^4\,c^2\,x+138\,a^3\,b^2\,c^3\,x+24\,a\,b^6\,c\,x-12\,a\,b^3\,c\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+15\,a^2\,b\,c^2\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}\Bigg)\Bigg(\frac{b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+6\,a^2\,c^2\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-6\,a\,b^2\,c\,\sqrt{-(4\,a\,c-b^2)}{-64\,a^6\,c^3+48\,a^5\,b^2\,c^2-12\,a^4\,b^4\,c+a^3\,b^6}-\frac{b}{a^3}\Bigg)-\frac{2\,b\,\ln\left(x\right)}{a^3}$$

input

```
int(1/(x^2*(a + b*x + c*x^2)^2),x)
```

output
```
log(2*a*b^7 + 2*b^8*x + 2*a*b^4*(-(4*a*c - b^2)^3)^(1/2) - 23*a^2*b^5*c -
108*a^4*b*c^3 + 24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^(1/2) + 87*a^3*b
^3*c^2 + 3*a^3*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*(-(4*a*c - b^2)^
3)^(1/2) + 97*a^2*b^4*c^2*x - 138*a^3*b^2*c^3*x - 24*a*b^6*c*x - 12*a*b^3*
c*x*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*((
b^4*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^
2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^
5*b^2*c^2) + b/a^3) - (1/a - (x*(2*b^3 - 7*a*b*c))/(a^2*(4*a*c - b^2)) + (
2*c*x^2*(3*a*c - b^2))/(a^2*(4*a*c - b^2)))/(a*x + b*x^2 + c*x^3) - log(2*
a*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*b^8*x - 2*a*b^7 + 23*a^2*b^5*c + 108*a^
4*b*c^3 - 24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^(1/2) - 87*a^3*b^3*c^2
 + 3*a^3*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*(-(4*a*c - b^2)^3)^(1/
2) - 97*a^2*b^4*c^2*x + 138*a^3*b^2*c^3*x + 24*a*b^6*c*x - 12*a*b^3*c*x*(-
(4*a*c - b^2)^3)^(1/2) + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*((b^4*(-
(4*a*c - b^2)^3)^(1/2) + 6*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*(-
(4*a*c - b^2)^3)^(1/2))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*
c^2) - b/a^3) - (2*b*log(x))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 877, normalized size of antiderivative = 6.36

$$\int \frac{1}{x^2 (a+bx+cx^2)^2} \, dx = \text{Too large to display}$$

input
```
int(1/x^2/(c*x^2+b*x+a)^2,x)
```

output

```
( - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**2
*x + 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3*
c*x - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2
*c**2*x**2 - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*
*2*b*c**3*x**3 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))
*a*b**5*x + 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b
**4*c*x**2 + 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*
b**3*c**2*x**3 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))
*b**6*x**2 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**
5*c*x**3 + 16*log(a + b*x + c*x**2)*a**3*b**2*c**2*x - 8*log(a + b*x + c*x
**2)*a**2*b**4*c*x + 16*log(a + b*x + c*x**2)*a**2*b**3*c**2*x**2 + 16*log
(a + b*x + c*x**2)*a**2*b**2*c**3*x**3 + log(a + b*x + c*x**2)*a*b**6*x -
8*log(a + b*x + c*x**2)*a*b**5*c*x**2 - 8*log(a + b*x + c*x**2)*a*b**4*c**
2*x**3 + log(a + b*x + c*x**2)*b**7*x**2 + log(a + b*x + c*x**2)*b**6*c*x*
*3 - 32*log(x)*a**3*b**2*c**2*x + 16*log(x)*a**2*b**4*c*x - 32*log(x)*a**2
*b**3*c**2*x**2 - 32*log(x)*a**2*b**2*c**3*x**3 - 2*log(x)*a*b**6*x + 16*l
og(x)*a*b**5*c*x**2 + 16*log(x)*a*b**4*c**2*x**3 - 2*log(x)*b**7*x**2 - 2*
log(x)*b**6*c*x**3 - 16*a**4*b*c**2 + 24*a**4*c**3*x + 8*a**3*b**3*c - 42*
a**3*b**2*c**2*x + 24*a**3*c**4*x**3 - a**2*b**5 + 17*a**2*b**4*c*x - 14*a
**2*b**2*c**3*x**3 - 2*a*b**6*x + 2*a*b**4*c**2*x**3)/(a**3*b*x*(16*a**...
```

3.243 $\int \frac{1}{x^3 (a+bx+cx^2)^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 179

$$\int \frac{1}{x^3 (a+bx+cx^2)^2} \, dx = -\frac{1}{2a^2x^2} + \frac{2b}{a^3x} + \frac{b^4-4ab^2c+2a^2c^2+bc(b^2-3ac)\,x}{a^3(b^2-4ac)\,(a+bx+cx^2)} + \frac{b(3b^4-20ab^2c+30a^2c^2)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4(b^2-4ac)^{3/2}} + \frac{(3b^2-2ac)\log(x)}{a^4} - \frac{(3b^2-2ac)\log(a+bx+cx^2)}{2a^4}$$

output
```
-1/2/a^2/x^2+2*b/a^3/x+(b^4-4*a*b^2*c+2*a^2*c^2+b*c*(-3*a*c+b^2)*x)/a^3/(-
4*a*c+b^2)/(c*x^2+b*x+a)+b*(30*a^2*c^2-20*a*b^2*c+3*b^4)*arctanh((2*c*x+b)
/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(3/2)+(-2*a*c+3*b^2)*ln(x)/a^4-1/2*(
-2*a*c+3*b^2)*ln(c*x^2+b*x+a)/a^4
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^3 (a+bx+cx^2)^2} \, dx$$
$$= \frac{-\frac{a^2}{x^2}+\frac{4ab}{x}+\frac{2a(b^4-4ab^2c+2a^2c^2+b^3cx-3abc^2x)}{(b^2-4ac)(a+x(b+cx))}+\frac{2b(3b^4-20ab^2c+30a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+2(3b^2-2ac)\log(x)+(\cdot}{2a^4}$$

input
```
Integrate[1/(x^3*(a + b*x + c*x^2)^2),x]
```

output
```
(-(a^2/x^2) + (4*a*b)/x + (2*a*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x - 3*
a*b*c^2*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*b^4 - 20*a*b^2*c +
 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
+ 2*(3*b^2 - 2*a*c)*Log[x] + (-3*b^2 + 2*a*c)*Log[a + x*(b + c*x)])/(2*a^4
)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1165, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx+cx^2)^2} \, dx$$
$$\Big\downarrow \text{1165}$$
$$\frac{-2ac+b^2+bcx}{ax^2(b^2-4ac)(a+bx+cx^2)}-\frac{\int -\frac{3b^2+3cxb-8ac}{x^3(cx^2+bx+a)}dx}{a(b^2-4ac)}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{3b^2+3cxb-8ac}{x^3(cx^2+bx+a)}dx}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{ax^2(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \text{1200}$$

$$\frac{\int \left(\frac{3b^2-8ac}{ax^3}+\frac{(b^2-4ac)(3b^2-2ac)}{a^3x}+\frac{-b(3b^4-17acb^2+19a^2c^2)-c(3b^4-14acb^2+8a^2c^2)x}{a^3(cx^2+bx+a)}+\frac{b(11ac-3b^2)}{a^2x^2}\right)dx}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{ax^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{(b^2-4ac)(3b^2-2ac)\log(a+bx+cx^2)}{2a^3}+\frac{\log(x)(b^2-4ac)(3b^2-2ac)}{a^3}+\frac{b(3b^2-11ac)}{a^2x}+\frac{b(30a^2c^2-20ab^2c+3b^4)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}-3}{a\left(b^2-4ac\right)}$$

$$\frac{-2ac+b^2+bcx}{ax^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

input
```
Int[1/(x^3*(a + b*x + c*x^2)^2),x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^2*(a + b*x + c*x^2)) + (-1/2*(3*b
^2 - 8*a*c)/(a*x^2) + (b*(3*b^2 - 11*a*c))/(a^2*x) + (b*(3*b^4 - 20*a*b^2*
c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*
a*c]) + ((b^2 - 4*a*c)*(3*b^2 - 2*a*c)*Log[x])/a^3 - ((b^2 - 4*a*c)*(3*b^2
 - 2*a*c)*Log[a + b*x + c*x^2])/(2*a^3))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.42

method	result
default	$-\frac{1}{2a^2x^2}+\frac{(-2ac+3b^2)\ln(x)}{a^4}+\frac{2b}{a^3x}+\frac{\frac{\frac{acb(3ac-b^2)x}{4ac-b^2}-\frac{a(2a^2c^2-4ca\,b^2+b^4)}{4ac-b^2}}{c\,x^2+bx+a}+\frac{\frac{(8a^2c^3-14a\,c^2b^2+3b^4c)\ln(c\,x^2+bx+a)}{2c}+\frac{2(19a^2b}{}}{}}{a^4}$
risch	Expression too large to display

input
```
int(1/x^3/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2/x^2+(-2*a*c+3*b^2)*ln(x)/a^4+2*b/a^3/x+1/a^4*((a*c*b*(3*a*c-b^2)/
(4*a*c-b^2)*x-a*(2*a^2*c^2-4*a*b^2*c+b^4)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4*
a*c-b^2)*(1/2*(8*a^2*c^3-14*a*b^2*c^2+3*b^4*c)/c*ln(c*x^2+b*x+a)+2*(19*a^2
*b*c^2-17*a*b^3*c+3*b^5-1/2*(8*a^2*c^3-14*a*b^2*c^2+3*b^4*c)*b/c)/(4*a*c-b
^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. 2(171) = 342.

Time = 0.26 (sec) , antiderivative size = 1226, normalized size of antiderivative = 6.85

$$\int \frac{1}{x^3(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
[-1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2
+ 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c
^3)*x^2 + ((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a*b^4
*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)*sq
rt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)
*(2*c*x + b))/(c*x^2 + b*x + a)) - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2
)*x + ((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7
 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4
*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(c*x^2 + b*x + a) - 2*((3*b^6*c
- 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c +
64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*
c^2 - 32*a^4*c^3)*x^2)*log(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x
^4 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c +
 16*a^7*c^2)*x^2), -1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c
 - 23*a^2*b^3*c^2 + 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*
b^2*c^2 - 32*a^4*c^3)*x^2 - 2*((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4
 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 3
0*a^3*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b
)/(b^2 - 4*a*c)) - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x + ((3*b^6*c
- 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx+cx^2)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/x**3/(c*x**2+b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (a + bx + cx^2)^2} dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^3/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^3 (a + bx + cx^2)^2} dx = -\frac{(3 b^5 - 20 ab^3c + 30 a^2bc^2) \arctan\left(\frac{2 cx+b}{\sqrt{-b^2+4 ac}}\right)}{(a^4b^2 - 4 a^5c)\sqrt{-b^2 + 4 ac}} - \frac{(3 b^2 - 2 ac) \log(cx^2 + bx + a)}{2 a^4} + \frac{(3 b^2 - 2 ac) \log(|x|)}{a^4} - \frac{a^3b^2 - 4 a^4c - 2 (3 ab^3c - 11 a^2bc^2)x^3 - (6 ab^4 - 25 a^2b^2c + 8 a^3c^2)x^2 - 3 (a^2b^3 - 4 a^3bc)x}{2 (cx^2 + bx + a)(b^2 - 4 ac)a^4x^2}$$

input
```
integrate(1/x^3/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
-(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c)
)/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(3*b^2 - 2*a*c)*log(c*x^2
 + b*x + a)/a^4 + (3*b^2 - 2*a*c)*log(abs(x))/a^4 - 1/2*(a^3*b^2 - 4*a^4*c
 - 2*(3*a*b^3*c - 11*a^2*b*c^2)*x^3 - (6*a*b^4 - 25*a^2*b^2*c + 8*a^3*c^2)
*x^2 - 3*(a^2*b^3 - 4*a^3*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^4*x^2
)
```

Mupad [B] (verification not implemented)

Time = 9.89 (sec) , antiderivative size = 914, normalized size of antiderivative = 5.11

$$\int \frac{1}{x^3 (a+bx+cx^2)^2}\,dx$$
$$= \frac{\ln\Big(6\,a\,b^8+6\,b^9\,x+192\,a^5\,c^4-6\,a\,b^5\,\sqrt{-(4\,a\,c-b^2)^3}-73\,a^2\,b^6\,c-6\,b^6\,x\,\sqrt{-(4\,a\,c-b^2)^3}+307\,a^3\,b^4\,c^2}{}$$
$$-\frac{\ln(x)\,(2\,a\,c-3\,b^2)}{a^4}-\frac{\frac{1}{2\,a}-\frac{3\,b\,x}{2\,a^2}+\frac{x^2\,(8\,a^2\,c^2-25\,a\,b^2\,c+6\,b^4)}{2\,a^3\,(4\,a\,c-b^2)}-\frac{b\,c\,x^3\,(11\,a\,c-3\,b^2)}{a^3\,(4\,a\,c-b^2)}}{c\,x^4+b\,x^3+a\,x^2}$$
$$+\frac{\ln\Big(6\,a\,b^8+6\,b^9\,x+192\,a^5\,c^4+6\,a\,b^5\,\sqrt{-(4\,a\,c-b^2)^3}-73\,a^2\,b^6\,c+6\,b^6\,x\,\sqrt{-(4\,a\,c-b^2)^3}+307\,a^3\,b^4}{}$$

input
```
int(1/(x^3*(a + b*x + c*x^2)^2),x)
```

output
```
(log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 - 6*a*b^5*(-(4*a*c - b^2)^3)^(1/2) -
73*a^2*b^6*c - 6*b^6*x*(-(4*a*c - b^2)^3)^(1/2) + 307*a^3*b^4*c^2 - 492*a^
4*b^2*c^3 + 31*a^2*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 27*a^3*b*c^2*(-(4*a*c
- b^2)^3)^(1/2) + 339*a^2*b^5*c^2*x - 602*a^3*b^3*c^3*x + 24*a^3*c^3*x*(-(
4*a*c - b^2)^3)^(1/2) - 76*a*b^7*c*x + 312*a^4*b*c^4*x + 40*a*b^4*c*x*(-(4
*a*c - b^2)^3)^(1/2) - 69*a^2*b^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*(3*b^8 +
 128*a^4*c^4 - 3*b^5*(-(4*a*c - b^2)^3)^(1/2) + 168*a^2*b^4*c^2 - 288*a^3*
b^2*c^3 - 38*a*b^6*c - 30*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 20*a*b^3*c*
(-(4*a*c - b^2)^3)^(1/2)))/(2*a^4*(4*a*c - b^2)^3) - (log(x)*(2*a*c - 3*b^
2))/a^4 - (1/(2*a) - (3*b*x)/(2*a^2) + (x^2*(6*b^4 + 8*a^2*c^2 - 25*a*b^2*
c))/(2*a^3*(4*a*c - b^2)) - (b*c*x^3*(11*a*c - 3*b^2))/(a^3*(4*a*c - b^2))
)/(a*x^2 + b*x^3 + c*x^4) + (log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 + 6*a*b^5
*(-(4*a*c - b^2)^3)^(1/2) - 73*a^2*b^6*c + 6*b^6*x*(-(4*a*c - b^2)^3)^(1/2
) + 307*a^3*b^4*c^2 - 492*a^4*b^2*c^3 - 31*a^2*b^3*c*(-(4*a*c - b^2)^3)^(1
/2) + 27*a^3*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 339*a^2*b^5*c^2*x - 602*a^3*
b^3*c^3*x - 24*a^3*c^3*x*(-(4*a*c - b^2)^3)^(1/2) - 76*a*b^7*c*x + 312*a^4
*b*c^4*x - 40*a*b^4*c*x*(-(4*a*c - b^2)^3)^(1/2) + 69*a^2*b^2*c^2*x*(-(4*a
*c - b^2)^3)^(1/2))*(3*b^8 + 128*a^4*c^4 + 3*b^5*(-(4*a*c - b^2)^3)^(1/2)
+ 168*a^2*b^4*c^2 - 288*a^3*b^2*c^3 - 38*a*b^6*c + 30*a^2*b*c^2*(-(4*a*c -
 b^2)^3)^(1/2) - 20*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*a^4*(4*a*c - ...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1039, normalized size of antiderivative = 5.80

$$\int \frac{1}{x^3 (a+bx+cx^2)^2} \, dx = \text{Too large to display}$$

input
```
int(1/x^3/(c*x^2+b*x+a)^2,x)
```

output
```
(60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**2*x*
*2 - 40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3*
c*x**2 + 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b
**2*c**2*x**3 + 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))
*a**2*b*c**3*x**4 + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**
2))*a*b**5*x**2 - 40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2
))*a*b**4*c*x**3 - 40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**
2))*a*b**3*c**2*x**4 + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c -
b**2))*b**6*x**3 + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2
))*b**5*c*x**4 + 32*log(a + b*x + c*x**2)*a**4*c**3*x**2 - 64*log(a + b*x
+ c*x**2)*a**3*b**2*c**2*x**2 + 32*log(a + b*x + c*x**2)*a**3*b*c**3*x**3
+ 32*log(a + b*x + c*x**2)*a**3*c**4*x**4 + 26*log(a + b*x + c*x**2)*a**2*
b**4*c*x**2 - 64*log(a + b*x + c*x**2)*a**2*b**3*c**2*x**3 - 64*log(a + b*
x + c*x**2)*a**2*b**2*c**3*x**4 - 3*log(a + b*x + c*x**2)*a*b**6*x**2 + 26
*log(a + b*x + c*x**2)*a*b**5*c*x**3 + 26*log(a + b*x + c*x**2)*a*b**4*c**
2*x**4 - 3*log(a + b*x + c*x**2)*b**7*x**3 - 3*log(a + b*x + c*x**2)*b**6*
c*x**4 - 64*log(x)*a**4*c**3*x**2 + 128*log(x)*a**3*b**2*c**2*x**2 - 64*lo
g(x)*a**3*b*c**3*x**3 - 64*log(x)*a**3*c**4*x**4 - 52*log(x)*a**2*b**4*c*x
**2 + 128*log(x)*a**2*b**3*c**2*x**3 + 128*log(x)*a**2*b**2*c**3*x**4 + 6*
log(x)*a*b**6*x**2 - 52*log(x)*a*b**5*c*x**3 - 52*log(x)*a*b**4*c**2*x*...
```

3.244 $\int \frac{x^7}{(a+bx+cx^2)^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 315

$$\begin{aligned}\int \frac{x^7}{(a+bx+cx^2)^3}\,dx &= -\frac{3bx}{c^4}+\frac{x^2}{2c^3}+\frac{a(b^2-2ac)\left(b^4-4ab^2c+a^2c^2\right)+b(b^6-7ab^4c+14a^2b^2c^2-7a^3c^3)\,x}{2c^6\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} \\ &\quad+\frac{17ab^6-\frac{b^8}{c}-88a^2b^4c+153a^3b^2c^2-48a^4c^3+2b(4b^6-35ab^4c+91a^2b^2c^2-63a^3c^3)\,x}{2c^5\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} \\ &\quad+\frac{3b(2b^6-21ab^4c+70a^2b^2c^2-70a^3c^3)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5\left(b^2-4ac\right)^{5/2}} \\ &\quad+\frac{3(2b^2-ac)\log\left(a+bx+cx^2\right)}{2c^5}\end{aligned}$$

output
```
-3*b*x/c^4+1/2*x^2/c^3+1/2*(a*(-2*a*c+b^2)*(a^2*c^2-4*a*b^2*c+b^4)+b*(-7*a
^3*c^3+14*a^2*b^2*c^2-7*a*b^4*c+b^6)*x)/c^6/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+1
/2*(17*a*b^6-b^8/c-88*a^2*b^4*c+153*a^3*b^2*c^2-48*a^4*c^3+2*b*(-63*a^3*c^
3+91*a^2*b^2*c^2-35*a*b^4*c+4*b^6)*x)/c^5/(-4*a*c+b^2)^2/(c*x^2+b*x+a)+3*b
*(-70*a^3*c^3+70*a^2*b^2*c^2-21*a*b^4*c+2*b^6)*arctanh((2*c*x+b)/(-4*a*c+b
^2)^(1/2))/c^5/(-4*a*c+b^2)^(5/2)+3/2*(-a*c+2*b^2)*ln(c*x^2+b*x+a)/c^5
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.95

$$\int \frac{x^7}{(a+bx+cx^2)^3}\,dx$$

$$= \frac{-6bc^2x+c^3x^2-\frac{b^8-17ab^6c+88a^2b^4c^2-153a^3b^2c^3+48a^4c^4-8b^7cx+70ab^5c^2x-182a^2b^3c^3x+126a^3bc^4x}{(b^2-4ac)^2(a+x(b+cx))}+\frac{-2a^4c^3+b^7x+ab^5(b-7cx)\cdots}{(b^2-4a\cdots}}{\cdots}$$

input
```
Integrate[x^7/(a + b*x + c*x^2)^3,x]
```

output
```
(-6*b*c^2*x + c^3*x^2 - (b^8 - 17*a*b^6*c + 88*a^2*b^4*c^2 - 153*a^3*b^2*c
^3 + 48*a^4*c^4 - 8*b^7*c*x + 70*a*b^5*c^2*x - 182*a^2*b^3*c^3*x + 126*a^3
*b*c^4*x)/((b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (-2*a^4*c^3 + b^7*x + a*b^
5*(b - 7*c*x) + a^3*b*c^2*(9*b - 7*c*x) + 2*a^2*b^3*c*(-3*b + 7*c*x))/((b^
2 - 4*a*c)*(a + x*(b + c*x))^2) + (6*b*c*(-2*b^6 + 21*a*b^4*c - 70*a^2*b^2
*c^2 + 70*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^
(5/2) - 3*c*(-2*b^2 + a*c)*Log[a + x*(b + c*x)])/(2*c^6)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1164, 27, 1233, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx+cx^2)^3}\,dx$$

$$\Big\downarrow \ 1164$$

$$\frac{x^6(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\int \frac{3x^5(4a+bx)}{(cx^2+bx+a)^2}dx}{2(b^2-4ac)}$$

$$\Big\downarrow \ 27$$

$$\frac{x^6(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{3\int \frac{x^5(4a+bx)}{(cx^2+bx+a)^2}dx}{2(b^2-4ac)}$$

$$\Big\downarrow \textbf{1233}$$

$$\frac{x^6(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{3\left(\frac{\int \frac{2x^3\left(2a\left(b^2-8ac\right)+b\left(2b^2-11ac\right)x\right)}{cx^2+bx+a}dx}{c(b^2-4ac)}-\frac{x^4(bx(b^2-6ac)+a(b^2-8ac))}{c(b^2-4ac)(a+bx+cx^2)}\right)}{2(b^2-4ac)}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{x^6(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{3\left(\frac{2\int \frac{x^3\left(2a\left(b^2-8ac\right)+b\left(2b^2-11ac\right)x\right)}{cx^2+bx+a}dx}{c(b^2-4ac)}-\frac{x^4(bx(b^2-6ac)+a(b^2-8ac))}{c(b^2-4ac)(a+bx+cx^2)}\right)}{2(b^2-4ac)}$$

$$\Big\downarrow \textbf{1200}$$

$$\frac{x^6(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-$$

$$\frac{3\left(\frac{2\int \left(\frac{b\left(2b^2-11ac\right)x^2}{c}-\frac{\left(2b^4-13acb^2+16a^2c^2\right)x}{c^2}+\frac{b\left(2b^2-9ac\right)\left(b^2-3ac\right)}{c^3}-\frac{\left(2b^2-ac\right)x\left(b^2-4ac\right)^2+ab\left(2b^2-9ac\right)\left(b^2-3ac\right)}{c^3\left(cx^2+bx+a\right)}\right)dx}{c(b^2-4ac)}-\frac{x^4(bx(b^2-6ac)+a\cdots}{c(b^2-4ac)(a+b\cdots}\right.}{2(b^2-4ac)}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{x^6(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-$$

$$\frac{3\left(\frac{2\left(-\frac{x^2\left(16a^2c^2-13ab^2c+2b^4\right)}{2c^2}-\frac{b\left(-70a^3c^3+70a^2b^2c^2-21ab^4c+2b^6\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}}-\frac{\left(b^2-4ac\right)^2\left(2b^2-ac\right)\log\left(a+bx+cx^2\right)}{2c^4}+\frac{bx\left(2b^2-9ac\right)\left(\cdots\right.}{c^3}\right.}{c(b^2-4ac)}\right.}{2(b^2-4ac)}$$

input
```
Int[x^7/(a + b*x + c*x^2)^3,x]
```

output
```
(x^6*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (3*(-((x^4*(a*(b
^2 - 8*a*c) + b*(b^2 - 6*a*c)*x))/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2))) + (
2*((b*(2*b^2 - 9*a*c)*(b^2 - 3*a*c)*x)/c^3 - ((2*b^4 - 13*a*b^2*c + 16*a^2
*c^2)*x^2)/(2*c^2) + (b*(2*b^2 - 11*a*c)*x^3)/(3*c) - (b*(2*b^6 - 21*a*b^4
*c + 70*a^2*b^2*c^2 - 70*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/
(c^4*Sqrt[b^2 - 4*a*c]) - ((b^2 - 4*a*c)^2*(2*b^2 - a*c)*Log[a + b*x + c*x
^2])/(2*c^4)))/(c*(b^2 - 4*a*c))))/(2*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.46

method	result
default	$-\frac{-\frac{1}{2}c\,x^2+3bx}{c^4}+\frac{\frac{-\frac{b\left(63a^3c^3-91a^2b^2c^2+35a\,b^4c-4b^6\right)x^3}{16a^2c^2-8ca\,b^2+b^4}-\frac{\left(48a^4c^4-27a^3b^2c^3-94a^2b^4c^2+53a\,b^6c-7b^8\right)x^2}{2c\left(16a^2c^2-8ca\,b^2+b^4\right)}-\frac{ab\left(73a^3c^3-136a^2b^2c^2+58\cdots\right.}{c\left(16a^2c^2-8ca\,b^2+\cdots\right.}}{\left(c\,x^2+bx+a\right)^2}$
risch	Expression too large to display

input
```
int(x^7/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/c^4*(-1/2*c*x^2+3*b*x)+1/c^4*((-b*(63*a^3*c^3-91*a^2*b^2*c^2+35*a*b^4*c
-4*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*(48*a^4*c^4-27*a^3*b^2*c^3-94*a
^2*b^4*c^2+53*a*b^6*c-7*b^8)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-a/c*b*(73*a^
3*c^3-136*a^2*b^2*c^2+58*a*b^4*c-7*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x-1/2*a
^2*(40*a^3*c^3-115*a^2*b^2*c^2+55*a*b^4*c-7*b^6)/c/(16*a^2*c^2-8*a*b^2*c+b
^4))/(c*x^2+b*x+a)^2+3/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(-16*a^3*c^3+40*a^2
*b^2*c^2-17*a*b^4*c+2*b^6)/c*ln(c*x^2+b*x+a)+2*(27*a^3*b*c^2-15*a^2*b^3*c+
2*a*b^5-1/2*(-16*a^3*c^3+40*a^2*b^2*c^2-17*a*b^4*c+2*b^6)*b/c)/(4*a*c-b^2)
^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1093 vs. $2(303) = 606$.

Time = 0.12 (sec) , antiderivative size = 2207, normalized size of antiderivative = 7.01

$$\int \frac{x^7}{(a + bx + cx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(x^7/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
[1/2*(7*a^2*b^8 - 83*a^3*b^6*c + 335*a^4*b^4*c^2 - 500*a^5*b^2*c^3 + 160*a
^6*c^4 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*x^6 - 4*(b
^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*x^5 - (11*b^8*c^2 -
 134*a*b^6*c^3 + 552*a^2*b^4*c^4 - 800*a^3*b^2*c^5 + 128*a^4*c^6)*x^4 + 2*
(b^9*c - 20*a*b^7*c^2 + 147*a^2*b^5*c^3 - 475*a^3*b^3*c^4 + 572*a^4*b*c^5)
*x^3 + (7*b^10 - 93*a*b^8*c + 451*a^2*b^6*c^2 - 937*a^3*b^4*c^3 + 660*a^4*
b^2*c^4 + 128*a^5*c^5)*x^2 - 3*(2*a^2*b^7 - 21*a^3*b^5*c + 70*a^4*b^3*c^2
- 70*a^5*b*c^3 + (2*b^7*c^2 - 21*a*b^5*c^3 + 70*a^2*b^3*c^4 - 70*a^3*b*c^5
)*x^4 + 2*(2*b^8*c - 21*a*b^6*c^2 + 70*a^2*b^4*c^3 - 70*a^3*b^2*c^4)*x^3 +
 (2*b^9 - 17*a*b^7*c + 28*a^2*b^5*c^2 + 70*a^3*b^3*c^3 - 140*a^4*b*c^4)*x^
2 + 2*(2*a*b^8 - 21*a^2*b^6*c + 70*a^3*b^4*c^2 - 70*a^4*b^2*c^3)*x)*sqrt(b
^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*
c*x + b))/(c*x^2 + b*x + a)) + 2*(7*a*b^9 - 89*a^2*b^7*c + 404*a^3*b^5*c^2
 - 761*a^4*b^3*c^3 + 484*a^5*b*c^4)*x + 3*(2*a^2*b^8 - 25*a^3*b^6*c + 108*
a^4*b^4*c^2 - 176*a^5*b^2*c^3 + 64*a^6*c^4 + (2*b^8*c^2 - 25*a*b^6*c^3 + 1
08*a^2*b^4*c^4 - 176*a^3*b^2*c^5 + 64*a^4*c^6)*x^4 + 2*(2*b^9*c - 25*a*b^7
*c^2 + 108*a^2*b^5*c^3 - 176*a^3*b^3*c^4 + 64*a^4*b*c^5)*x^3 + (2*b^10 - 2
1*a*b^8*c + 58*a^2*b^6*c^2 + 40*a^3*b^4*c^3 - 288*a^4*b^2*c^4 + 128*a^5*c^
5)*x^2 + 2*(2*a*b^9 - 25*a^2*b^7*c + 108*a^3*b^5*c^2 - 176*a^4*b^3*c^3 + 6
4*a^5*b*c^4)*x)*log(c*x^2 + b*x + a))/(a^2*b^6*c^5 - 12*a^3*b^4*c^6 + 4...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1875 vs. $2(316) = 632$.

Time = 2.97 (sec) , antiderivative size = 1875, normalized size of antiderivative = 5.95

$$\int \frac{x^7}{(a + bx + cx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(x**7/(c*x**2+b*x+a)**3,x)
```

output
```
-3*b*x/c**4 + (-3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*
c**2 + 21*a*b**4*c - 2*b**6)/(2*c**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4
 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 3*(a*
c - 2*b**2)/(2*c**5))*log(x + (96*a**4*c**3 - 159*a**3*b**2*c**2 + 64*a**3
*c**7*(-3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 2
1*a*b**4*c - 2*b**6)/(2*c**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a
**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 3*(a*c - 2*b*
*2)/(2*c**5)) + 57*a**2*b**4*c - 48*a**2*b**2*c**6*(-3*b*sqrt(-(4*a*c - b*
*2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*c**5*
(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6
*c**2 + 20*a*b**8*c - b**10)) - 3*(a*c - 2*b**2)/(2*c**5)) - 6*a*b**6 + 12
*a*b**4*c**5*(-3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c
**2 + 21*a*b**4*c - 2*b**6)/(2*c**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4
+ 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 3*(a*c
 - 2*b**2)/(2*c**5)) - b**6*c**4*(-3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c
**3 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*c**5*(1024*a**5*c**5 -
1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*
c - b**10)) - 3*(a*c - 2*b**2)/(2*c**5)))/(210*a**3*b*c**3 - 210*a**2*b**3
*c**2 + 63*a*b**5*c - 6*b**7)) + (3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c*
*3 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*c**5*(1024*a**5*c**5 ...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{(a+bx+cx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.05

$$\int \frac{x^7}{(a+bx+cx^2)^3}\,dx = -\frac{3\,(2\,b^7-21\,ab^5c+70\,a^2b^3c^2-70\,a^3bc^3)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4c^5-8\,ab^2c^6+16\,a^2c^7)\sqrt{-b^2+4\,ac}} + \frac{3\,(2\,b^2-ac)\log\left(cx^2+bx+a\right)}{2\,c^5} + \frac{c^3x^2-6\,bc^2x}{2\,c^6} + \frac{7\,a^2b^6-55\,a^3b^4c+115\,a^4b^2c^2-40\,a^5c^3+2\,(4\,b^7c-35\,ab^5c^2+91\,a^2b^3c^3-63\,a^3bc^4)x^3+(7\,b^8-53\,ab}{2\,(cx^2+bx+a)^2(b^2-}$$

input
```
integrate(x^7/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
-3*(2*b^7 - 21*a*b^5*c + 70*a^2*b^3*c^2 - 70*a^3*b*c^3)*arctan((2*c*x + b)
/sqrt(-b^2 + 4*a*c))/((b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt(-b^2 + 4*a
*c)) + 3/2*(2*b^2 - a*c)*log(c*x^2 + b*x + a)/c^5 + 1/2*(c^3*x^2 - 6*b*c^2
*x)/c^6 + 1/2*(7*a^2*b^6 - 55*a^3*b^4*c + 115*a^4*b^2*c^2 - 40*a^5*c^3 + 2
*(4*b^7*c - 35*a*b^5*c^2 + 91*a^2*b^3*c^3 - 63*a^3*b*c^4)*x^3 + (7*b^8 - 5
3*a*b^6*c + 94*a^2*b^4*c^2 + 27*a^3*b^2*c^3 - 48*a^4*c^4)*x^2 + 2*(7*a*b^7
 - 58*a^2*b^5*c + 136*a^3*b^3*c^2 - 73*a^4*b*c^3)*x)/((c*x^2 + b*x + a)^2*
(b^2 - 4*a*c)^2*c^5)
```

Mupad [B] (verification not implemented)

Time = 9.70 (sec) , antiderivative size = 762, normalized size of antiderivative = 2.42

$$\int \frac{x^7}{(a+bx+cx^2)^3}\,dx = \frac{\frac{a(-40a^4c^3+115a^3b^2c^2-55a^2b^4c+7ab^6)}{2c(16a^2c^2-8ab^2c+b^4)}+\frac{bx^3(-63a^3c^3+91a^2b^2c^2-35ab^4c+4b^6)}{16a^2c^2-8ab^2c+b^4}+\frac{x^2(-48a^4c^4+27a^3b^2c^3+94a^2b^4c^2-53ab}{2c(16a^2c^2-8ab^2c+b^4)}}{a^2c^4+c^6x^4+x^2(b^2c^4+2ac^5)+2bc^5x^3+2abc^4x}$$
$$+\frac{x^2}{2c^3}$$
$$-\frac{\ln(cx^2+bx+a)\,(3072a^6c^6-9984a^5b^2c^5+9600a^4b^4c^4-4320a^3b^6c^3+1020a^2b^8c^2-123ab^{10}c}{2\,(1024a^5c^{10}-1280a^4b^2c^9+640a^3b^4c^8-160a^2b^6c^7+20ab^8c^6-b^{10}c^5)}$$
$$-\frac{3bx}{c^4}$$
$$-\frac{3b\,\mathrm{atan}\left(\frac{\left(\frac{3bx(-70a^3c^3+70a^2b^2c^2-21ab^4c+2b^6)}{c^4(4ac-b^2)^5}+\frac{3b^2(16a^2c^6-8ab^2c^5+b^4c^4)(-70a^3c^3+70a^2b^2c^2-21ab^4c+2b^6)}{2c^9(4ac-b^2)^5(16a^2c^2-8ab^2c+b^4)}\right)(32a^2c^7(4a}{-210a^3bc^3+210a^2b^3c^2-63ab^5c+6b^7}\right.}{c^5(4ac-b^2)^{5/2}}$$

input
```
int(x^7/(a + b*x + c*x^2)^3,x)
```

output
```
((a*(7*a*b^6 - 40*a^4*c^3 - 55*a^2*b^4*c + 115*a^3*b^2*c^2))/(2*c*(b^4 + 1
6*a^2*c^2 - 8*a*b^2*c)) + (b*x^3*(4*b^6 - 63*a^3*c^3 + 91*a^2*b^2*c^2 - 35
*a*b^4*c))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (x^2*(7*b^8 - 48*a^4*c^4 + 94*
a^2*b^4*c^2 + 27*a^3*b^2*c^3 - 53*a*b^6*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b
^2*c)) + (b*x*(7*a*b^6 - 73*a^4*c^3 - 58*a^2*b^4*c + 136*a^3*b^2*c^2))/(c*
(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(a^2*c^4 + c^6*x^4 + x^2*(2*a*c^5 + b^2*c
^4) + 2*b*c^5*x^3 + 2*a*b*c^4*x) + x^2/(2*c^3) - (log(a + b*x + c*x^2)*(6*
b^12 + 3072*a^6*c^6 + 1020*a^2*b^8*c^2 - 4320*a^3*b^6*c^3 + 9600*a^4*b^4*c
^4 - 9984*a^5*b^2*c^5 - 123*a*b^10*c))/(2*(1024*a^5*c^10 - b^10*c^5 + 20*a
*b^8*c^6 - 160*a^2*b^6*c^7 + 640*a^3*b^4*c^8 - 1280*a^4*b^2*c^9)) - (3*b*x
)/c^4 - (3*b*atan((((3*b*x*(2*b^6 - 70*a^3*c^3 + 70*a^2*b^2*c^2 - 21*a*b^4
*c))/(c^4*(4*a*c - b^2)^5) + (3*b^2*(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5)*(
2*b^6 - 70*a^3*c^3 + 70*a^2*b^2*c^2 - 21*a*b^4*c))/(2*c^9*(4*a*c - b^2)^5*
(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(32*a^2*c^7*(4*a*c - b^2)^(5/2) + 2*b^4*c
^5*(4*a*c - b^2)^(5/2) - 16*a*b^2*c^6*(4*a*c - b^2)^(5/2)))/(6*b^7 - 210*a
^3*b*c^3 + 210*a^2*b^3*c^2 - 63*a*b^5*c))*(2*b^6 - 70*a^3*c^3 + 70*a^2*b^2
*c^2 - 21*a*b^4*c))/(c^5*(4*a*c - b^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 2048, normalized size of antiderivative = 6.50

$$\int \frac{x^7}{(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input

```
int(x^7/(c*x^2+b*x+a)^3,x)
```

output

```
(420*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**5*b*c**3 -
 420*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b**3*c**
2 + 840*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b**2*
c**3*x + 840*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*
b*c**4*x**2 + 126*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*
a**3*b**5*c - 840*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*
a**3*b**4*c**2*x - 420*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b*
*2))*a**3*b**3*c**3*x**2 + 840*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*
a*c - b**2))*a**3*b**2*c**4*x**3 + 420*sqrt(4*a*c - b**2)*atan((b + 2*c*x)
/sqrt(4*a*c - b**2))*a**3*b*c**5*x**4 - 12*sqrt(4*a*c - b**2)*atan((b + 2*
c*x)/sqrt(4*a*c - b**2))*a**2*b**7 + 252*sqrt(4*a*c - b**2)*atan((b + 2*c*
x)/sqrt(4*a*c - b**2))*a**2*b**6*c*x - 168*sqrt(4*a*c - b**2)*atan((b + 2*
c*x)/sqrt(4*a*c - b**2))*a**2*b**5*c**2*x**2 - 840*sqrt(4*a*c - b**2)*atan
((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**4*c**3*x**3 - 420*sqrt(4*a*c - b*
*2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3*c**4*x**4 - 24*sqrt(4*a
*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**8*x + 102*sqrt(4*a*c
- b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**7*c*x**2 + 252*sqrt(4*a*
c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**6*c**2*x**3 + 126*sqrt
(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**5*c**3*x**4 - 12*
sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**9*x**2 - 24*...
```

3.245 $\int \frac{x^6}{(a+bx+cx^2)^3} dx$

Optimal result

Integrand size = 16, antiderivative size = 266

$$\begin{aligned}\int \frac{x^6}{(a+bx+cx^2)^3}\,dx &= \frac{x}{c^3}-\frac{ab(b^4-5ab^2c+5a^2c^2)+(b^2-2ac)\left(b^4-4ab^2c+a^2c^2\right)x}{2c^5\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}\\ &\quad-\frac{b\left(14ab^4-\frac{b^6}{c}-61a^2b^2c+78a^3c^2\right)+6(b^2-3ac)\left(b^4-5ab^2c+2a^2c^2\right)x}{2c^4\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}\\ &\quad-\frac{3(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\left(b^2-4ac\right)^{5/2}}-\frac{3b\log\left(a+bx+cx^2\right)}{2c^4}\end{aligned}$$

output
```
x/c^3-1/2*(a*b*(5*a^2*c^2-5*a*b^2*c+b^4)+(-2*a*c+b^2)*(a^2*c^2-4*a*b^2*c+b
^4)*x)/c^5/(-4*a*c+b^2)/(c*x^2+b*x+a)^2-1/2*(b*(14*a*b^4-b^6/c-61*a^2*b^2*
c+78*a^3*c^2)+6*(-3*a*c+b^2)*(2*a^2*c^2-5*a*b^2*c+b^4)*x)/c^4/(-4*a*c+b^2)
^2/(c*x^2+b*x+a)-3*(-20*a^3*c^3+30*a^2*b^2*c^2-10*a*b^4*c+b^6)*arctanh((2*
c*x+b)/(-4*a*c+b^2)^(1/2))/c^4/(-4*a*c+b^2)^(5/2)-3/2*b*ln(c*x^2+b*x+a)/c^
4
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.98

$$\int \frac{x^6}{(a+bx+cx^2)^3}\,dx$$
$$= \frac{2c^2x+\frac{b^7-14ab^5c+61a^2b^3c^2-78a^3bc^3-6b^6cx+48ab^4c^2x-102a^2b^2c^3x+36a^3c^4x}{(b^2-4ac)^2(a+x(b+cx))}+\frac{-b^6x+a^2b^2c(5b-9cx)-ab^4(b-6cx)+a^3c^2(-5b+2cx)}{(b^2-4ac)(a+x(b+cx))^2}}{2c^5}$$

input
```
Integrate[x^6/(a + b*x + c*x^2)^3,x]
```

output
```
(2*c^2*x + (b^7 - 14*a*b^5*c + 61*a^2*b^3*c^2 - 78*a^3*b*c^3 - 6*b^6*c*x +
 48*a*b^4*c^2*x - 102*a^2*b^2*c^3*x + 36*a^3*c^4*x)/((b^2 - 4*a*c)^2*(a +
x*(b + c*x))) + (-(b^6*x) + a^2*b^2*c*(5*b - 9*c*x) - a*b^4*(b - 6*c*x) +
a^3*c^2*(-5*b + 2*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (6*c*(b^6 -
10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4
*a*c]])/(-b^2 + 4*a*c)^(5/2) - 3*b*c*Log[a + x*(b + c*x)])/(2*c^5)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1164, 27, 1233, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx+cx^2)^3}\,dx$$
$$\Big\downarrow \text{1164}$$
$$\frac{x^5(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\int \frac{2x^4(5a+bx)}{(cx^2+bx+a)^2}dx}{2(b^2-4ac)}$$
$$\Big\downarrow \text{27}$$

$$\frac{x^5(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\int \frac{x^4(5a+bx)}{(cx^2+bx+a)^2}dx}{b^2-4ac}$$

$$\Big\downarrow \; 1233$$

$$\frac{x^5(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{\int \frac{3x^2\left(a\left(b^2-10ac\right)+b\left(b^2-6ac\right)x\right)}{cx^2+bx+a}dx}{c(b^2-4ac)}-\frac{x^3(bx(b^2-7ac)+a(b^2-10ac))}{c(b^2-4ac)(a+bx+cx^2)}}{b^2-4ac}$$

$$\Big\downarrow \; 27$$

$$\frac{x^5(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{3\int \frac{x^2\left(a\left(b^2-10ac\right)+b\left(b^2-6ac\right)x\right)}{cx^2+bx+a}dx}{c(b^2-4ac)}-\frac{x^3(bx(b^2-7ac)+a(b^2-10ac))}{c(b^2-4ac)(a+bx+cx^2)}}{b^2-4ac}$$

$$\Big\downarrow \; 1200$$

$$\frac{x^5(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{3\int \left(-\frac{b^4-7acb^2+10a^2c^2}{c^2}+\frac{b\left(b^2-6ac\right)x}{c}+\frac{bx\left(b^2-4ac\right)^2+a\left(b^4-7acb^2+10a^2c^2\right)}{c^2\left(cx^2+bx+a\right)}\right)dx}{c(b^2-4ac)}-\frac{x^3(bx(b^2-7ac)+a(b^2-10ac))}{c(b^2-4ac)(a+bx+cx^2)}}{b^2-4ac}$$

$$\Big\downarrow \; 2009$$

$$\frac{x^5(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{3\left(-\frac{x\left(10a^2c^2-7ab^2c+b^4\right)}{c^2}+\frac{\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{b\left(b^2-4ac\right)^2\log\left(a+bx+cx^2\right)}{2c^3}+\frac{bx^2\left(b^2-6ac\right)}{2c}\right)}{c(b^2-4ac)}-\frac{x^3(bx(b^2-7}{c(b^2-4a}}{b^2-4ac}$$

input
```
Int[x^6/(a + b*x + c*x^2)^3,x]
```

output
```
(x^5*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (-((x^3*(a*(b^2
- 10*a*c) + b*(b^2 - 7*a*c)*x))/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2))) + (3*
(-(((b^4 - 7*a*b^2*c + 10*a^2*c^2)*x)/c^2) + (b*(b^2 - 6*a*c)*x^2)/(2*c) +
 ((b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqr
t[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + (b*(b^2 - 4*a*c)^2*Log[a + b*x
+ c*x^2])/(2*c^3)))/(c*(b^2 - 4*a*c)))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.51

method	result
default	$\frac{x}{c^3} - \frac{\frac{-\frac{3\left(6a^3c^3-17a^2b^2c^2+8a\,b^4c-b^6\right)x^3}{16a^2c^2-8ca\,b^2+b^4}+\frac{b\left(42a^3c^3+41a^2b^2c^2-34a\,b^4c+5b^6\right)x^2}{2\left(16a^2c^2-8ca\,b^2+b^4\right)c}-\frac{a\left(14a^3c^3-71a^2b^2c^2+38a\,b^4c-5b^6\right)x}{c\left(16a^2c^2-8ca\,b^2+b^4\right)}+\frac{b\,a^2\left(58a^2c^2-3\right.}{2c\left(16a^2c^2-8\right.}}{\left(c\,x^2+bx+a\right)^2}}{c^3}$
risch	Expression too large to display

input
```
int(x^6/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
x/c^3-1/c^3*((-3*(6*a^3*c^3-17*a^2*b^2*c^2+8*a*b^4*c-b^6)/(16*a^2*c^2-8*a*
b^2*c+b^4)*x^3+1/2*b*(42*a^3*c^3+41*a^2*b^2*c^2-34*a*b^4*c+5*b^6)/(16*a^2*
c^2-8*a*b^2*c+b^4)/c*x^2-a/c*(14*a^3*c^3-71*a^2*b^2*c^2+38*a*b^4*c-5*b^6)/
(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/2*b*a^2/c*(58*a^2*c^2-36*a*b^2*c+5*b^4)/(16
*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+3/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2
*(16*a^2*b*c^2-8*a*b^3*c+b^5)/c*ln(c*x^2+b*x+a)+2*(10*c^2*a^3-7*a^2*b^2*c+
a*b^4-1/2*(16*a^2*b*c^2-8*a*b^3*c+b^5)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*
x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 953 vs. $2(256) = 512$.

Time = 0.11 (sec) , antiderivative size = 1926, normalized size of antiderivative = 7.24

$$\int \frac{x^6}{\left(a+bx+cx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
[-1/2*(5*a^2*b^7 - 56*a^3*b^5*c + 202*a^4*b^3*c^2 - 232*a^5*b*c^3 - 2*(b^6
*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^5 - 4*(b^7*c^2 - 12*a
*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^4 + 2*(2*b^8*c - 26*a*b^6*c^2
+ 123*a^2*b^4*c^3 - 254*a^3*b^2*c^4 + 200*a^4*c^5)*x^3 + (5*b^9 - 58*a*b^7
*c + 225*a^2*b^5*c^2 - 314*a^3*b^3*c^3 + 88*a^4*b*c^4)*x^2 + 3*(a^2*b^6 -
10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 + (b^6*c^2 - 10*a*b^4*c^3 + 30*
a^2*b^2*c^4 - 20*a^3*c^5)*x^4 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 -
 20*a^3*b*c^4)*x^3 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 -
40*a^4*c^4)*x^2 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)
*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 -
4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(5*a*b^8 - 59*a^2*b^6*c + 235*a
^3*b^4*c^2 - 346*a^4*b^2*c^3 + 120*a^5*c^4)*x + 3*(a^2*b^7 - 12*a^3*b^5*c
+ 48*a^4*b^3*c^2 - 64*a^5*b*c^3 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4
 - 64*a^3*b*c^5)*x^4 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b
^2*c^4)*x^3 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^
4*b*c^4)*x^2 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*
x)*log(c*x^2 + b*x + a))/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 -
64*a^5*c^7 + (b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*x^4 +
2*(b^7*c^5 - 12*a*b^5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*x^3 + (b^8*c^4
- 10*a*b^6*c^5 + 24*a^2*b^4*c^6 + 32*a^3*b^2*c^7 - 128*a^4*c^8)*x^2 + 2...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1714 vs. $2(265) = 530$.

Time = 2.25 (sec) , antiderivative size = 1714, normalized size of antiderivative = 6.44

$$\int \frac{x^6}{(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**6/(c*x**2+b*x+a)**3,x)
```

output
```
(-3*b/(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c
**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 +
640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))*log(x + (
-66*a**3*b*c**2 - 64*a**3*c**6*(-3*b/(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)
*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**
5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 2
0*a*b**8*c - b**10))) + 27*a**2*b**3*c + 48*a**2*b**2*c**5*(-3*b/(2*c**4)
- 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4
*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c
**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) - 3*a*b**5 - 12*a*b**4*c
**4*(-3*b/(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b*
*2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**
4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + b**
6*c**3*(-3*b/(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2
*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*
c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))))/(
60*a**3*c**3 - 90*a**2*b**2*c**2 + 30*a*b**4*c - 3*b**6)) + (-3*b/(2*c**4)
 + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**
4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*
c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))*log(x + (-66*a**3*b*...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6}{(a+bx+cx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^6/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.06

$$\int \frac{x^6}{(a+bx+cx^2)^3}\,dx$$
$$= \frac{3\,(b^6-10\,ab^4c+30\,a^2b^2c^2-20\,a^3c^3)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4c^4-8\,ab^2c^5+16\,a^2c^6)\sqrt{-b^2+4\,ac}}+\frac{x}{c^3}-\frac{3\,b\log\left(cx^2+bx+a\right)}{2\,c^4}$$
$$-\frac{5\,a^2b^5-36\,a^3b^3c+58\,a^4bc^2+6\,(b^6c-8\,ab^4c^2+17\,a^2b^2c^3-6\,a^3c^4)x^3+(5\,b^7-34\,ab^5c+41\,a^2b^3c^2+\cdots}{2\,(cx^2+bx+a)^2(b^2-4\,ac)^2c^4}$$

input
```
integrate(x^6/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*x + b)/sqrt
(-b^2 + 4*a*c))/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*sqrt(-b^2 + 4*a*c))
+ x/c^3 - 3/2*b*log(c*x^2 + b*x + a)/c^4 - 1/2*(5*a^2*b^5 - 36*a^3*b^3*c +
 58*a^4*b*c^2 + 6*(b^6*c - 8*a*b^4*c^2 + 17*a^2*b^2*c^3 - 6*a^3*c^4)*x^3 +
 (5*b^7 - 34*a*b^5*c + 41*a^2*b^3*c^2 + 42*a^3*b*c^3)*x^2 + 2*(5*a*b^6 - 3
8*a^2*b^4*c + 71*a^3*b^2*c^2 - 14*a^4*c^3)*x)/((c*x^2 + b*x + a)^2*(b^2 -
4*a*c)^2*c^4)
```

Mupad [B] (verification not implemented)

Time = 9.64 (sec) , antiderivative size = 705, normalized size of antiderivative = 2.65

$$\int \frac{x^6}{(a+bx+cx^2)^3}\,dx = \frac{x}{c^3}$$
$$-\frac{\frac{3\,x^3\,\left(-6\,a^3\,c^3+17\,a^2\,b^2\,c^2-8\,a\,b^4\,c+b^6\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{x^2\,\left(42\,a^3\,b\,c^3+41\,a^2\,b^3\,c^2-34\,a\,b^5\,c+5\,b^7\right)}{2\,c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a^2\,\left(58\,a^2\,b\,c^2-36\,a\,b^3\,c+5\,b^5\right)}{2\,c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a\,x\,\left(-14\,a^3\,c^3\cdots\right)}{c\,(16\cdots}}{a^2\,c^3+c^5\,x^4+x^2\,\left(b^2\,c^3+2\,a\,c^4\right)+2\,b\,c^4\,x^3+2\,a\,b\,c^3\,x}$$
$$+\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-3072\,a^5\,b\,c^5+3840\,a^4\,b^3\,c^4-1920\,a^3\,b^5\,c^3+480\,a^2\,b^7\,c^2-60\,a\,b^9\,c+3\,b^{11}\right)}{2\,\left(1024\,a^5\,c^9-1280\,a^4\,b^2\,c^8+640\,a^3\,b^4\,c^7-160\,a^2\,b^6\,c^6+20\,a\,b^8\,c^5-b^{10}\,c^4\right)}$$
$$+\frac{3\,\mathrm{atan}\left(\frac{\left(\frac{3\,x\,\left(-20\,a^3\,c^3+30\,a^2\,b^2\,c^2-10\,a\,b^4\,c+b^6\right)}{c^3\,\left(4\,a\,c-b^2\right)^5}+\frac{3\,\left(16\,a^2\,b\,c^5-8\,a\,b^3\,c^4+b^5\,c^3\right)\,\left(-20\,a^3\,c^3+30\,a^2\,b^2\,c^2-10\,a\,b^4\,c+b^6\right)}{2\,c^7\,\left(4\,a\,c-b^2\right)^5\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)\,\left(32\,a^2\,c^6\,\left(4\,a\,c-b^2\right)^{5/\cdots}\cdots\right.}{-60\,a^3\,c^3+90\,a^2\,b^2\,c^2-30\,a\,b^4\,c+3\,b^6}\cdots\right)\cdots}{c^4\,\left(4\,a\,c-b^2\right)^{5/2}}$$

input
```
int(x^6/(a + b*x + c*x^2)^3,x)
```

output
```
x/c^3 - ((3*x^3*(b^6 - 6*a^3*c^3 + 17*a^2*b^2*c^2 - 8*a*b^4*c))/(b^4 + 16*
a^2*c^2 - 8*a*b^2*c) + (x^2*(5*b^7 + 42*a^3*b*c^3 + 41*a^2*b^3*c^2 - 34*a*
b^5*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2*(5*b^5 + 58*a^2*b*c^2
- 36*a*b^3*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*x*(5*b^6 - 14*a^3
*c^3 + 71*a^2*b^2*c^2 - 38*a*b^4*c))/(c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(
a^2*c^3 + c^5*x^4 + x^2*(2*a*c^4 + b^2*c^3) + 2*b*c^4*x^3 + 2*a*b*c^3*x) +
 (log(a + b*x + c*x^2)*(3*b^11 - 3072*a^5*b*c^5 + 480*a^2*b^7*c^2 - 1920*a
^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 60*a*b^9*c))/(2*(1024*a^5*c^9 - b^10*c^4 +
 20*a*b^8*c^5 - 160*a^2*b^6*c^6 + 640*a^3*b^4*c^7 - 1280*a^4*b^2*c^8)) + (
3*atan((((3*x*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(c^3*(4*a*
c - b^2)^5) + (3*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*(b^6 - 20*a^3*c^3
+ 30*a^2*b^2*c^2 - 10*a*b^4*c))/(2*c^7*(4*a*c - b^2)^5*(b^4 + 16*a^2*c^2 -
 8*a*b^2*c)))*(32*a^2*c^6*(4*a*c - b^2)^(5/2) + 2*b^4*c^4*(4*a*c - b^2)^(5
/2) - 16*a*b^2*c^5*(4*a*c - b^2)^(5/2)))/(3*b^6 - 60*a^3*c^3 + 90*a^2*b^2*
c^2 - 30*a*b^4*c))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(c^4*
(4*a*c - b^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1896, normalized size of antiderivative = 7.13

$$\int \frac{x^6}{(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(x^6/(c*x^2+b*x+a)^3,x)
```

output

```
( - 120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**5*b*c**
3 + 180*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b**3*
c**2 - 240*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b*
*2*c**3*x - 240*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*
*4*b*c**4*x**2 - 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2)
)*a**3*b**5*c + 360*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2)
)*a**3*b**4*c**2*x + 240*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c -
b**2))*a**3*b**3*c**3*x**2 - 240*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(
4*a*c - b**2))*a**3*b**2*c**4*x**3 - 120*sqrt(4*a*c - b**2)*atan((b + 2*c*
x)/sqrt(4*a*c - b**2))*a**3*b*c**5*x**4 + 6*sqrt(4*a*c - b**2)*atan((b + 2
*c*x)/sqrt(4*a*c - b**2))*a**2*b**7 - 120*sqrt(4*a*c - b**2)*atan((b + 2*c
*x)/sqrt(4*a*c - b**2))*a**2*b**6*c*x + 60*sqrt(4*a*c - b**2)*atan((b + 2*
c*x)/sqrt(4*a*c - b**2))*a**2*b**5*c**2*x**2 + 360*sqrt(4*a*c - b**2)*atan
((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**4*c**3*x**3 + 180*sqrt(4*a*c - b*
*2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3*c**4*x**4 + 12*sqrt(4*a
*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**8*x - 48*sqrt(4*a*c -
 b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**7*c*x**2 - 120*sqrt(4*a*c
 - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**6*c**2*x**3 - 60*sqrt(4
*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**5*c**3*x**4 + 6*sqr
t(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**9*x**2 + 12*sqr...
```

3.246 $\int \frac{x^5}{(a+bx+cx^2)^3} dx$

Optimal result

Integrand size = 16, antiderivative size = 203

$$\begin{aligned}\int \frac{x^5}{(a+bx+cx^2)^3}\,dx =& -\frac{bx}{2c^2(b^2-4ac)}+\frac{x^4(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}\\&+\frac{a(b^4-11ab^2c+16a^2c^2)+b(b^4-12ab^2c+26a^2c^2)x}{2c^3(b^2-4ac)^2(a+bx+cx^2)}\\&+\frac{b(b^4-10ab^2c+30a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3(b^2-4ac)^{5/2}}\\&+\frac{\log(a+bx+cx^2)}{2c^3}\end{aligned}$$

output
```
-1/2*b*x/c^2/(-4*a*c+b^2)+1/2*x^4*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+1
/2*(a*(16*a^2*c^2-11*a*b^2*c+b^4)+b*(26*a^2*c^2-12*a*b^2*c+b^4)*x)/c^3/(-4
*a*c+b^2)^2/(c*x^2+b*x+a)+b*(30*a^2*c^2-10*a*b^2*c+b^4)*arctanh((2*c*x+b)/
(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(5/2)+1/2*ln(c*x^2+b*x+a)/c^3
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.09

$$\int \frac{x^5}{(a+bx+cx^2)^3}\,dx$$

$$= \frac{\frac{-b^6+11ab^4c-39a^2b^2c^2+32a^3c^3+4b^5cx-30ab^3c^2x+50a^2bc^3x}{(b^2-4ac)^2(a+x(b+cx))} + \frac{2a^3c^2+b^5x+ab^3(b-5cx)+a^2bc(-4b+5cx)}{(b^2-4ac)(a+x(b+cx))^2} - \frac{2bc(b^4-10ab^2c+30a^2c^2)\,\mathrm{arct}}{(-b^2+4ac)^{5/}}}{2c^4}$$

input
```
Integrate[x^5/(a + b*x + c*x^2)^3,x]
```

output
```
((-b^6 + 11*a*b^4*c - 39*a^2*b^2*c^2 + 32*a^3*c^3 + 4*b^5*c*x - 30*a*b^3*c
^2*x + 50*a^2*b*c^3*x)/((b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (2*a^3*c^2 +
b^5*x + a*b^3*(b - 5*c*x) + a^2*b*c*(-4*b + 5*c*x))/((b^2 - 4*a*c)*(a + x*
(b + c*x))^2) - (2*b*c*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)/
Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*Log[a + x*(b + c*x)])/(2*c^4
)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1164, 1233, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx+cx^2)^3}\,dx$$

$$\Big\downarrow \; 1164$$

$$\frac{x^4(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2} - \frac{\int \frac{x^3(8a+bx)}{(cx^2+bx+a)^2}dx}{2(b^2-4ac)}$$

$$\Big\downarrow \; 1233$$

$$\frac{x^4(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{\int \frac{2x\left(a\left(b^2-16ac\right)+b\left(b^2-7ac\right)x\right)}{cx^2+bx+a}dx}{c(b^2-4ac)}-\frac{x^2(bx(b^2-10ac)+a(b^2-16ac))}{c(b^2-4ac)(a+bx+cx^2)}}{2(b^2-4ac)}$$

$$\Big\downarrow \; 27$$

$$\frac{x^4(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{2\int \frac{x\left(a\left(b^2-16ac\right)+b\left(b^2-7ac\right)x\right)}{cx^2+bx+a}dx}{c(b^2-4ac)}-\frac{x^2(bx(b^2-10ac)+a(b^2-16ac))}{c(b^2-4ac)(a+bx+cx^2)}}{2(b^2-4ac)}$$

$$\Big\downarrow \; 1200$$

$$\frac{x^4(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{2\int \left(-b\left(7a-\frac{b^2}{c}\right)-\frac{x\left(b^2-4ac\right)^2+ab\left(b^2-7ac\right)}{c\left(cx^2+bx+a\right)}\right)dx}{c(b^2-4ac)}-\frac{x^2(bx(b^2-10ac)+a(b^2-16ac))}{c(b^2-4ac)(a+bx+cx^2)}}{2(b^2-4ac)}$$

$$\Big\downarrow \; 2009$$

$$\frac{x^4(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{2\left(-\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{\left(b^2-4ac\right)^2\log\left(a+bx+cx^2\right)}{2c^2}-bx\left(7a-\frac{b^2}{c}\right)\right)}{c(b^2-4ac)}-\frac{x^2(bx(b^2-10ac)+a(b^2-16ac))}{c(b^2-4ac)(a+bx+cx^2)}}{2(b^2-4ac)}$$

input
```
Int[x^5/(a + b*x + c*x^2)^3,x]
```

output
```
(x^4*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (-((x^2*(a*(b^2
- 16*a*c) + b*(b^2 - 10*a*c)*x))/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2))) + (2
*(-(b*(7*a - b^2/c)*x) - (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2
*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) - ((b^2 - 4*a*c)^2*Log[a
 + b*x + c*x^2])/(2*c^2)))/(c*(b^2 - 4*a*c)))/(2*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.76

method	result
default	$\frac{\frac{b\left(25a^2c^2-15ca\,b^2+2b^4\right)x^3}{c^2\left(16a^2c^2-8ca\,b^2+b^4\right)}+\frac{\left(32a^3c^3+11a^2b^2c^2-19a\,b^4c+3b^6\right)x^2}{2c^3\left(16a^2c^2-8ca\,b^2+b^4\right)}+\frac{ab\left(31a^2c^2-22ca\,b^2+3b^4\right)x}{\left(16a^2c^2-8ca\,b^2+b^4\right)c^3}+\frac{3a^2\left(8a^2c^2-7ca\,b^2+b^4\right)}{2c^3\left(16a^2c^2-8ca\,b^2+b^4\right)}}{\left(c\,x^2+bx+a\right)^2}+ \cdots$
risch	Expression too large to display

input
```
int(x^5/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/c^2*b*(25*a^2*c^2-15*a*b^2*c+2*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*
(32*a^3*c^3+11*a^2*b^2*c^2-19*a*b^4*c+3*b^6)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4
)*x^2+a*b*(31*a^2*c^2-22*a*b^2*c+3*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x+3
/2*a^2*(8*a^2*c^2-7*a*b^2*c+b^4)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*
x+a)^2+1/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(16*a^2*c^2-8*a*b^2*c+b^4)/c*
ln(c*x^2+b*x+a)+2*(-7*c*a^2*b+a*b^3-1/2*(16*a^2*c^2-8*a*b^2*c+b^4)*b/c)/(4
*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 792 vs. 2(191) = 382.

Time = 0.11 (sec) , antiderivative size = 1603, normalized size of antiderivative = 7.90

$$\int \frac{x^5}{\left(a+bx+cx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
[1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(2*b^7*c
 - 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*x^3 + (3*b^8 - 31*a*b^6*
c + 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + (a^2*b^5 - 10*a^3
*b^3*c + 30*a^4*b*c^2 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b
^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*
c^2 + 60*a^3*b*c^3)*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*sqr
t(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*
(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*
c^2 - 124*a^4*b*c^3)*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5
*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*
b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)*log(c*x^2 + b*x + a))/(a^2*b^6*c
^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^
6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^4 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*
b^3*c^6 - 64*a^3*b*c^7)*x^3 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 3
2*a^3*b^2*c^6 - 128*a^4*c^7)*x^2 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*
b^3*c^5 - 64*a^4*b*c^6)*x), 1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^
2 - 96*a^5*c^3 + 2*(2*b^7*c - 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^
4)*x^3 + (3*b^8 - 31*a*b^6*c + 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1510 vs. $2(196) = 392$.

Time = 1.67 (sec) , antiderivative size = 1510, normalized size of antiderivative = 7.44

$$\int \frac{x^5}{(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**5/(c*x**2+b*x+a)**3,x)
```

output
```
(-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(
1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*
c**2 + 20*a*b**8*c - b**10)) + 1/(2*c**3))*log(x + (-64*a**3*c**5*(-b*sqrt
(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**
5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 2
0*a*b**8*c - b**10)) + 1/(2*c**3)) + 32*a**3*c**2 + 48*a**2*b**2*c**4*(-b*
sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024
*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2
 + 20*a*b**8*c - b**10)) + 1/(2*c**3)) - 9*a**2*b**2*c - 12*a*b**4*c**3*(-
b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(10
24*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c*
*2 + 20*a*b**8*c - b**10)) + 1/(2*c**3)) + a*b**4 + b**6*c**2*(-b*sqrt(-(4
*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c*
*5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*
b**8*c - b**10)) + 1/(2*c**3)))/(30*a**2*b*c**2 - 10*a*b**3*c + b**5)) + (
b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(10
24*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c*
*2 + 20*a*b**8*c - b**10)) + 1/(2*c**3))*log(x + (-64*a**3*c**5*(b*sqrt(-(
4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c
**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 2...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx+cx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.21

$$\int \frac{x^5}{(a+bx+cx^2)^3}\,dx$$
$$= -\frac{(b^5-10\,ab^3c+30\,a^2bc^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4c^3-8\,ab^2c^4+16\,a^2c^5)\sqrt{-b^2+4\,ac}}+\frac{\log\left(cx^2+bx+a\right)}{2\,c^3}$$
$$+\frac{3\,a^2b^4-21\,a^3b^2c+24\,a^4c^2+2\,(2\,b^5c-15\,ab^3c^2+25\,a^2bc^3)x^3+(3\,b^6-19\,ab^4c+11\,a^2b^2c^2+32\,a^3c^3)}{2\,(cx^2+bx+a)^2(b^2-4\,ac)^2c^3}$$

input
```
integrate(x^5/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
-(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/
((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(-b^2 + 4*a*c)) + 1/2*log(c*x^2
+ b*x + a)/c^3 + 1/2*(3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*c^2 + 2*(2*b^5*c -
 15*a*b^3*c^2 + 25*a^2*b*c^3)*x^3 + (3*b^6 - 19*a*b^4*c + 11*a^2*b^2*c^2 +
 32*a^3*c^3)*x^2 + 2*(3*a*b^5 - 22*a^2*b^3*c + 31*a^3*b*c^2)*x)/((c*x^2 +
b*x + a)^2*(b^2 - 4*a*c)^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 9.77 (sec) , antiderivative size = 620, normalized size of antiderivative = 3.05

$$\int \frac{x^5}{(a+bx+cx^2)^3}\,dx$$
$$= \frac{\frac{3\,a^2\,\left(8\,a^2\,c^2-7\,a\,b^2\,c+b^4\right)}{2\,c^3\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{x^2\,\left(32\,a^3\,c^3+11\,a^2\,b^2\,c^2-19\,a\,b^4\,c+3\,b^6\right)}{2\,c^3\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{b\,x^3\,\left(25\,a^2\,c^2-15\,a\,b^2\,c+2\,b^4\right)}{c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a\,b\,x\,\left(31\,a^2\,c^2-22\,a\,b^2\,c+3\,b^4\right)}{c^3\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3}$$
$$-\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-1024\,a^5\,c^5+1280\,a^4\,b^2\,c^4-640\,a^3\,b^4\,c^3+160\,a^2\,b^6\,c^2-20\,a\,b^8\,c+b^{10}\right)}{2\,\left(1024\,a^5\,c^8-1280\,a^4\,b^2\,c^7+640\,a^3\,b^4\,c^6-160\,a^2\,b^6\,c^5+20\,a\,b^8\,c^4-b^{10}\,c^3\right)}$$
$$-\frac{b\,\mathrm{atan}\left(\frac{\left(\frac{b\,x\,\left(30\,a^2\,c^2-10\,a\,b^2\,c+b^4\right)}{c^2\,{\left(4\,a\,c-b^2\right)}^5}+\frac{b^2\,\left(16\,a^2\,c^4-8\,a\,b^2\,c^3+b^4\,c^2\right)\,\left(30\,a^2\,c^2-10\,a\,b^2\,c+b^4\right)}{2\,c^5\,{\left(4\,a\,c-b^2\right)}^5\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)\,\left(32\,a^2\,c^5\,{\left(4\,a\,c-b^2\right)}^{5/2}+2\,b^4\,c^3\,{\left(4\,a\,c-b^2\right)}^{5/2}-\right.}{30\,a^2\,b\,c^2-10\,a\,b^3\,c+b^5}\right.}{c^3\,{\left(4\,a\,c-b^2\right)}^{5/2}}$$

input
```
int(x^5/(a + b*x + c*x^2)^3,x)
```

output
```
((3*a^2*(b^4 + 8*a^2*c^2 - 7*a*b^2*c))/(2*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*
c)) + (x^2*(3*b^6 + 32*a^3*c^3 + 11*a^2*b^2*c^2 - 19*a*b^4*c))/(2*c^3*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*x^3*(2*b^4 + 25*a^2*c^2 - 15*a*b^2*c))/(c
^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*b*x*(3*b^4 + 31*a^2*c^2 - 22*a*b^2
*c))/(c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*
x^4 + 2*a*b*x + 2*b*c*x^3) - (log(a + b*x + c*x^2)*(b^10 - 1024*a^5*c^5 +
160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 20*a*b^8*c))/(2*(10
24*a^5*c^8 - b^10*c^3 + 20*a*b^8*c^4 - 160*a^2*b^6*c^5 + 640*a^3*b^4*c^6 -
 1280*a^4*b^2*c^7)) - (b*atan((((b*x*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(c^2
*(4*a*c - b^2)^5) + (b^2*(16*a^2*c^4 + b^4*c^2 - 8*a*b^2*c^3)*(b^4 + 30*a^
2*c^2 - 10*a*b^2*c))/(2*c^5*(4*a*c - b^2)^5*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
))*(32*a^2*c^5*(4*a*c - b^2)^(5/2) + 2*b^4*c^3*(4*a*c - b^2)^(5/2) - 16*a*
b^2*c^4*(4*a*c - b^2)^(5/2)))/(b^5 + 30*a^2*b*c^2 - 10*a*b^3*c))*(b^4 + 30
*a^2*c^2 - 10*a*b^2*c))/(c^3*(4*a*c - b^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1555, normalized size of antiderivative = 7.66

$$\int \frac{x^5}{(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(x^5/(c*x^2+b*x+a)^3,x)
```

output
```
( - 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b*c**2
 + 20*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**3*c
- 120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**2*c*
*2*x - 120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*
c**3*x**2 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2
*b**5 + 40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*
*4*c*x - 20*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b
**3*c**2*x**2 - 120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2)
)*a**2*b**2*c**3*x**3 - 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
- b**2))*a**2*b*c**4*x**4 - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a
*c - b**2))*a*b**6*x + 16*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c -
 b**2))*a*b**5*c*x**2 + 40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
- b**2))*a*b**4*c**2*x**3 + 20*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*
a*c - b**2))*a*b**3*c**3*x**4 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt
(4*a*c - b**2))*b**7*x**2 - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a
*c - b**2))*b**6*c*x**3 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
 - b**2))*b**5*c**2*x**4 + 64*log(a + b*x + c*x**2)*a**5*c**3 - 48*log(a +
 b*x + c*x**2)*a**4*b**2*c**2 + 128*log(a + b*x + c*x**2)*a**4*b*c**3*x +
128*log(a + b*x + c*x**2)*a**4*c**4*x**2 + 12*log(a + b*x + c*x**2)*a**3*b
**4*c - 96*log(a + b*x + c*x**2)*a**3*b**3*c**2*x - 32*log(a + b*x + c*...
```

3.247 $\int \frac{x^4}{(a+bx+cx^2)^3} dx$

Optimal result

Integrand size = 16, antiderivative size = 116

$$\int \frac{x^4}{(a+bx+cx^2)^3} dx = \frac{x^3(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2} + \frac{3a(ab+(b^2-2ac)x)}{c(b^2-4ac)^2(a+bx+cx^2)} - \frac{12a^2\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

output
```
1/2*x^3*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+3*a*(a*b+(-2*a*c+b^2)*x)/c/
(-4*a*c+b^2)^2/(c*x^2+b*x+a)-12*a^2*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/
(-4*a*c+b^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.50

$$\int \frac{x^4}{(a+bx+cx^2)^3}\,dx = \frac{1}{2}\left(\frac{b^5-8ab^3c+22a^2bc^2-2b^4cx+16ab^2c^2x-20a^2c^3x}{c^3(b^2-4ac)^2(a+x(b+cx))} + \frac{b^4x+ab^2(b-4cx)+a^2c(-3b+2cx)}{c^3(-b^2+4ac)(a+x(b+cx))^2} + \frac{24a^2\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}}\right)$$

input
```
Integrate[x^4/(a + b*x + c*x^2)^3,x]
```

output
```
((b^5 - 8*a*b^3*c + 22*a^2*b*c^2 - 2*b^4*c*x + 16*a*b^2*c^2*x - 20*a^2*c^3
*x)/(c^3*(b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (b^4*x + a*b^2*(b - 4*c*x) +
 a^2*c*(-3*b + 2*c*x))/(c^3*(-b^2 + 4*a*c)*(a + x*(b + c*x))^2) + (24*a^2*
ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/2
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1153, 1153, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx+cx^2)^3}\,dx$$

$$\Big\downarrow \ 1153$$

$$\frac{x^3(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2} - \frac{3a\int \frac{x^2}{(cx^2+bx+a)^2}dx}{b^2-4ac}$$

$$\Big\downarrow \ 1153$$

$$\frac{x^3(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{3a\left(\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{2a\int\frac{1}{cx^2+bx+a}dx}{b^2-4ac}\right)}{b^2-4ac}$$

$$\Big\downarrow \text{1083}$$

$$\frac{x^3(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{3a\left(\frac{4a\int\frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}+\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}$$

$$\Big\downarrow \text{219}$$

$$\frac{x^3(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{3a\left(\frac{4a\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}$$

input
```
Int[x^4/(a + b*x + c*x^2)^3,x]
```

output
```
(x^3*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (3*a*((x*(2*a +
b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (4*a*ArcTanh[(b + 2*c*x)/Sqrt[b^
2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1153
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m - 2)*(a + b*x +
c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
&& LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(110) = 220$.

Time = 0.73 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.24

method	result
default	$\frac{-\frac{(10a^2c^2-8ca\,b^2+b^4)x^3}{c(16a^2c^2-8ca\,b^2+b^4)}+\frac{b(2a^2c^2+8ca\,b^2-b^4)x^2}{2c^2(16a^2c^2-8ca\,b^2+b^4)}-\frac{a(6a^2c^2-10ca\,b^2+b^4)x}{(16a^2c^2-8ca\,b^2+b^4)c^2}+\frac{a^2b(10ac-b^2)}{2c^2(16a^2c^2-8ca\,b^2+b^4)}}{(c\,x^2+bx+a)^2}+\frac{12a^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(16a^2c^2-8ca\,b^2+b^4)\sqrt{4ac-}}$
risch	$\frac{-\frac{(10a^2c^2-8ca\,b^2+b^4)x^3}{c(16a^2c^2-8ca\,b^2+b^4)}+\frac{b(2a^2c^2+8ca\,b^2-b^4)x^2}{2c^2(16a^2c^2-8ca\,b^2+b^4)}-\frac{a(6a^2c^2-10ca\,b^2+b^4)x}{(16a^2c^2-8ca\,b^2+b^4)c^2}+\frac{a^2b(10ac-b^2)}{2c^2(16a^2c^2-8ca\,b^2+b^4)}}{(c\,x^2+bx+a)^2}-\frac{6a^2\ln\left((32a^2c^3-16a\,c^2b^2+\right.}{}$

input
```
int(x^4/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/c*(10*a^2*c^2-8*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*b*(2*a
^2*c^2+8*a*b^2*c-b^4)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-a*(6*a^2*c^2-10*a
*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x+1/2*a^2*b*(10*a*c-b^2)/c^2/(1
6*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+12*a^2/(16*a^2*c^2-8*a*b^2*c+b^4
)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(110) = 220$.

Time = 0.11 (sec) , antiderivative size = 953, normalized size of antiderivative = 8.22

$$\int \frac{x^4}{(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
[-1/2*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*c - 12*a*b^4*c^2 + 4
2*a^2*b^2*c^3 - 40*a^3*c^4)*x^3 + (b^7 - 12*a*b^5*c + 30*a^2*b^3*c^2 + 8*a
^3*b*c^3)*x^2 - 12*(a^2*c^4*x^4 + 2*a^2*b*c^3*x^3 + 2*a^3*b*c^2*x + a^4*c^
2 + (a^2*b^2*c^2 + 2*a^3*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*
c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*
(a*b^6 - 14*a^2*b^4*c + 46*a^3*b^2*c^2 - 24*a^4*c^3)*x)/(a^2*b^6*c^2 - 12*
a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a
^2*b^2*c^6 - 64*a^3*c^7)*x^4 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5
- 64*a^3*b*c^6)*x^3 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^
2*c^5 - 128*a^4*c^6)*x^2 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4
- 64*a^4*b*c^5)*x), -1/2*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*c
 - 12*a*b^4*c^2 + 42*a^2*b^2*c^3 - 40*a^3*c^4)*x^3 + (b^7 - 12*a*b^5*c + 3
0*a^2*b^3*c^2 + 8*a^3*b*c^3)*x^2 + 24*(a^2*c^4*x^4 + 2*a^2*b*c^3*x^3 + 2*a
^3*b*c^2*x + a^4*c^2 + (a^2*b^2*c^2 + 2*a^3*c^3)*x^2)*sqrt(-b^2 + 4*a*c)*a
rctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(a*b^6 - 14*a^2*b
^4*c + 46*a^3*b^2*c^2 - 24*a^4*c^3)*x)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*
a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a
^3*c^7)*x^4 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*x
^3 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 128*a^4*c
^6)*x^2 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b*c^5...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(107) = 214$.

Time = 0.74 (sec) , antiderivative size = 547, normalized size of antiderivative = 4.72

$$\int \frac{x^4}{(a+bx+cx^2)^3}\,dx =$$

$$-6a^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{-384a^5c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+288a^4b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}-72a^3b^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}+}{12a^2c}\right.$$

$$+6a^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{384a^5c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-288a^4b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+72a^3b^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}-6}{12a^2c}\right.$$

$$+\frac{10a^3bc-a^2b^3+x^3(-20a^2c^3+16ab^2c^2-2b^4c)+x^2\cdot(2a^2bc^2+8ab^3c-}{32a^4c^4-16a^3b^2c^3+2a^2b^4c^2+x^4\cdot(32a^2c^6-16ab^2c^5+2b^4c^4)+x^3\cdot(64a^2bc^5-32ab^3c^4+4b^5c^3)+x^2}$$

input
```
integrate(x**4/(c*x**2+b*x+a)**3,x)
```

output
```
-6*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-384*a**5*c**3*sqrt(-1/(4*a*c
- b**2)**5) + 288*a**4*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5) - 72*a**3*b**4
*c*sqrt(-1/(4*a*c - b**2)**5) + 6*a**2*b**6*sqrt(-1/(4*a*c - b**2)**5) + 6
*a**2*b)/(12*a**2*c)) + 6*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (384*a**
5*c**3*sqrt(-1/(4*a*c - b**2)**5) - 288*a**4*b**2*c**2*sqrt(-1/(4*a*c - b*
*2)**5) + 72*a**3*b**4*c*sqrt(-1/(4*a*c - b**2)**5) - 6*a**2*b**6*sqrt(-1/
(4*a*c - b**2)**5) + 6*a**2*b)/(12*a**2*c)) + (10*a**3*b*c - a**2*b**3 + x
**3*(-20*a**2*c**3 + 16*a*b**2*c**2 - 2*b**4*c) + x**2*(2*a**2*b*c**2 + 8*
a*b**3*c - b**5) + x*(-12*a**3*c**2 + 20*a**2*b**2*c - 2*a*b**4))/(32*a**4
*c**4 - 16*a**3*b**2*c**3 + 2*a**2*b**4*c**2 + x**4*(32*a**2*c**6 - 16*a*b
**2*c**5 + 2*b**4*c**4) + x**3*(64*a**2*b*c**5 - 32*a*b**3*c**4 + 4*b**5*c
**3) + x**2*(64*a**3*c**5 - 12*a*b**4*c**3 + 2*b**6*c**2) + x*(64*a**3*b*c
**4 - 32*a**2*b**3*c**3 + 4*a*b**5*c**2))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(a + bx + cx^2)^3} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.74

$$\int \frac{x^4}{(a+bx+cx^2)^3}\,dx = \frac{12\,a^2\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} - \frac{2\,b^4cx^3-16\,ab^2c^2x^3+20\,a^2c^3x^3+b^5x^2-8\,ab^3cx^2-2\,a^2bc^2x^2+2\,ab^4x-20\,a^2b^2cx+12\,a^3c^2x+a^2b^3}{2\,(b^4c^2-8\,ab^2c^3+16\,a^2c^4)(cx^2+bx+a)^2}$$

input
```
integrate(x^4/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
12*a^2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c
^2)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*b^4*c*x^3 - 16*a*b^2*c^2*x^3 + 20*a^2*c^3
*x^3 + b^5*x^2 - 8*a*b^3*c*x^2 - 2*a^2*b*c^2*x^2 + 2*a*b^4*x - 20*a^2*b^2*
c*x + 12*a^3*c^2*x + a^2*b^3 - 10*a^3*b*c)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^
2*c^4)*(c*x^2 + b*x + a)^2)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.96

$$\int \frac{x^4}{(a+bx+cx^2)^3}\,dx = \frac{12\,a^2\,\mathrm{atan}\left(\frac{\left(\frac{6\,a^2\,\left(16\,a^2\,b\,c^2-8\,a\,b^3\,c+b^5\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{12\,a^2\,c\,x}{{\left(4\,a\,c-b^2\right)}^{5/2}}\right)\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{6\,a^2}\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}} - \frac{\frac{x^3\,\left(10\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a^2\,\left(b^3-10\,a\,b\,c\right)}{2\,c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}-\frac{x^2\,\left(2\,a^2\,b\,c^2+8\,a\,b^3\,c-b^5\right)}{2\,c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a\,x\,\left(6\,a^2\,c^2-10\,a\,b^2\,c+b^4\right)}{c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3}$$

input
```
int(x^4/(a + b*x + c*x^2)^3,x)
```

output
```
(12*a^2*atan((((6*a^2*(b^5 + 16*a^2*b*c^2 - 8*a*b^3*c))/((4*a*c - b^2)^(5/
2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (12*a^2*c*x)/(4*a*c - b^2)^(5/2))*(b^
4 + 16*a^2*c^2 - 8*a*b^2*c))/(6*a^2)))/(4*a*c - b^2)^(5/2) - ((x^3*(b^4 +
10*a^2*c^2 - 8*a*b^2*c))/(c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2*(b^3 -
10*a*b*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^2*(2*a^2*b*c^2 - b^
5 + 8*a*b^3*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*x*(b^4 + 6*a^2
*c^2 - 10*a*b^2*c))/(c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^
2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 644, normalized size of antiderivative = 5.55

$$\int \frac{x^4}{(a+bx+cx^2)^3}\,dx$$
$$= \frac{24\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^4bc+48\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^3b^2cx+48\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^3}{2bc\,(64a^3c^5x^4-48a^2b^2c^4x^4+12a\,b^4}$$

input
```
int(x^4/(c*x^2+b*x+a)^3,x)
```

output
```
(24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b*c + 48*
sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**2*c*x + 48
*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**2*x**2
+ 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3*c*x
**2 + 48*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2
*c**2*x**3 + 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*
*2*b*c**3*x**4 + 40*a**5*c**2 - 2*a**4*b**2*c + 32*a**4*b*c**2*x + 80*a**4
*c**3*x**2 - 2*a**3*b**4 + 8*a**3*b**3*c*x - 36*a**3*b**2*c**2*x**2 + 40*a
**3*c**4*x**4 - 4*a**2*b**5*x + 12*a**2*b**4*c*x**2 - 42*a**2*b**2*c**3*x*
*4 - 2*a*b**6*x**2 + 12*a*b**4*c**2*x**4 - b**6*c*x**4)/(2*b*c*(64*a**5*c*
*3 - 48*a**4*b**2*c**2 + 128*a**4*b*c**3*x + 128*a**4*c**4*x**2 + 12*a**3*
b**4*c - 96*a**3*b**3*c**2*x - 32*a**3*b**2*c**3*x**2 + 128*a**3*b*c**4*x*
*3 + 64*a**3*c**5*x**4 - a**2*b**6 + 24*a**2*b**5*c*x - 24*a**2*b**4*c**2*
x**2 - 96*a**2*b**3*c**3*x**3 - 48*a**2*b**2*c**4*x**4 - 2*a*b**7*x + 10*a
*b**6*c*x**2 + 24*a*b**5*c**2*x**3 + 12*a*b**4*c**3*x**4 - b**8*x**2 - 2*b
**7*c*x**3 - b**6*c**2*x**4))
```

3.248 $\int \frac{x^3}{(a+bx+cx^2)^3} dx$

Optimal result

Integrand size = 16, antiderivative size = 116

$$\int \frac{x^3}{(a+bx+cx^2)^3} dx = -\frac{x^3(b+2cx)}{2(b^2-4ac)(a+bx+cx^2)^2} - \frac{3b(ab+(b^2-2ac)x)}{2c(b^2-4ac)^2(a+bx+cx^2)} + \frac{6ab\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

output
```
-1/2*x^3*(2*c*x+b)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2-3/2*b*(a*b+(-2*a*c+b^2)*x)
/c/(-4*a*c+b^2)^2/(c*x^2+b*x+a)+6*a*b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2)
)/(-4*a*c+b^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(a+bx+cx^2)^3} dx = -\frac{8a^3c+b^4x^2+abx(2b^2+bcx+6c^2x^2)+a^2(b^2+10bcx+16c^2x^2)}{2c(b^2-4ac)^2(a+x(b+cx))^2} - \frac{6ab\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}}$$

input
```
Integrate[x^3/(a + b*x + c*x^2)^3,x]
```

output
```
-1/2*(8*a^3*c + b^4*x^2 + a*b*x*(2*b^2 + b*c*x + 6*c^2*x^2) + a^2*(b^2 + 1
0*b*c*x + 16*c^2*x^2))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^2) - (6*a*b*Ar
cTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1156, 1153, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx+cx^2)^3} \, dx$$

$$\Big\downarrow \text{1156}$$

$$\frac{3b \int \frac{x^2}{(cx^2+bx+a)^2}dx}{2(b^2-4ac)} - \frac{x^3(b+2cx)}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{1153}$$

$$\frac{3b\left(\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} - \frac{2a\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}\right)}{2(b^2-4ac)} - \frac{x^3(b+2cx)}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{3b\left(\frac{4a\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac} + \frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}\right)}{2(b^2-4ac)} - \frac{x^3(b+2cx)}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{3b\left(\frac{4a\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} + \frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}\right)}{2(b^2-4ac)} - \frac{x^3(b+2cx)}{2(b^2-4ac)(a+bx+cx^2)^2}$$

input
```
Int[x^3/(a + b*x + c*x^2)^3,x]
```

output
```
-1/2*(x^3*(b + 2*c*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*b*((x*(2*a
 + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (4*a*ArcTanh[(b + 2*c*x)/Sqrt
[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/(2*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1153
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m - 2)*(a + b*x +
c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
&& LtQ[p, -1]
```

rule 1156
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] + Simp[m*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*c)))   Int[
(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(108) = 216$.

Time = 0.74 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.92

method	result
default	$\frac{-\frac{3abc\,x^3}{16a^2c^2-8ca\,b^2+b^4}-\frac{\left(16a^2c^2+ca\,b^2+b^4\right)x^2}{2c\left(16a^2c^2-8ca\,b^2+b^4\right)}-\frac{\left(5ac+b^2\right)abx}{c\left(16a^2c^2-8ca\,b^2+b^4\right)}-\frac{a^2\left(8ac+b^2\right)}{2c\left(16a^2c^2-8ca\,b^2+b^4\right)}}{\left(c\,x^2+bx+a\right)^2}-\frac{6ab\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8ca\,b^2+b^4\right)\sqrt{4ac-b^2}}$
risch	$\frac{-\frac{3abc\,x^3}{16a^2c^2-8ca\,b^2+b^4}-\frac{\left(16a^2c^2+ca\,b^2+b^4\right)x^2}{2c\left(16a^2c^2-8ca\,b^2+b^4\right)}-\frac{\left(5ac+b^2\right)abx}{c\left(16a^2c^2-8ca\,b^2+b^4\right)}-\frac{a^2\left(8ac+b^2\right)}{2c\left(16a^2c^2-8ca\,b^2+b^4\right)}}{\left(c\,x^2+bx+a\right)^2}-\frac{3ba\ln\left(\left(32a^2c^3-16a\,c^2b^2+2b^4c\right)x-\right.}{\left(-4a\right.}$

input
```
int(x^3/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-3*a*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*(16*a^2*c^2+a*b^2*c+b^4)/c/(1
6*a^2*c^2-8*a*b^2*c+b^4)*x^2-(5*a*c+b^2)*a*b/c/(16*a^2*c^2-8*a*b^2*c+b^4)*
x-1/2*a^2*(8*a*c+b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2-6*a*b/
(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(108) = 216$.

Time = 0.12 (sec) , antiderivative size = 872, normalized size of antiderivative = 7.52

$$\int \frac{x^3}{(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
[-1/2*(a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^2 + 6*(a*b^3*c^2 - 4*a^2*b*c^3)*x^
3 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 - 64*a^3*c^3)*x^2 - 6*(a*b*c^3*x^4 +
 2*a*b^2*c^2*x^3 + 2*a^2*b^2*c*x + a^3*b*c + (a*b^3*c + 2*a^2*b*c^2)*x^2)*
sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*
c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(a*b^5 + a^2*b^3*c - 20*a^3*b*c^2)*
x)/(a^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 -
12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^4 + 2*(b^7*c^2 - 12*a*b^5*c^
3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^3 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^
4*c^3 + 32*a^3*b^2*c^4 - 128*a^4*c^5)*x^2 + 2*(a*b^7*c - 12*a^2*b^5*c^2 +
48*a^3*b^3*c^3 - 64*a^4*b*c^4)*x), -1/2*(a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^
2 + 6*(a*b^3*c^2 - 4*a^2*b*c^3)*x^3 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 -
64*a^3*c^3)*x^2 - 12*(a*b*c^3*x^4 + 2*a*b^2*c^2*x^3 + 2*a^2*b^2*c*x + a^3*
b*c + (a*b^3*c + 2*a^2*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(a*b^5 + a^2*b^3*c - 20*a^3*b*c^2)*x
)/(a^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 - 1
2*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^4 + 2*(b^7*c^2 - 12*a*b^5*c^3
 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^3 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^4
*c^3 + 32*a^3*b^2*c^4 - 128*a^4*c^5)*x^2 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 4
8*a^3*b^3*c^3 - 64*a^4*b*c^4)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(109) = 218$.

Time = 0.62 (sec) , antiderivative size = 513, normalized size of antiderivative = 4.42

$$\int \frac{x^3}{(a+bx+cx^2)^3}\,dx$$
$$= 3ab\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x + \frac{-192a^4bc^3\sqrt{-\frac{1}{(4ac-b^2)^5}} + 144a^3b^3c^2\sqrt{-\frac{1}{(4ac-b^2)^5}} - 36a^2b^5c\sqrt{-\frac{1}{(4ac-b^2)^5}} + 3}{6abc}\right.$$
$$-3ab\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x + \frac{192a^4bc^3\sqrt{-\frac{1}{(4ac-b^2)^5}} - 144a^3b^3c^2\sqrt{-\frac{1}{(4ac-b^2)^5}} + 36a^2b^5c\sqrt{-\frac{1}{(4ac-b^2)^5}} -}{6abc}\right.$$
$$+\frac{-8a^3c - a^2b^2 - 6abc^2x^3 + x^2(-16a^2c^2 - ab^2c - b^4) + x(}{32a^4c^3 - 16a^3b^2c^2 + 2a^2b^4c + x^4\cdot(32a^2c^5 - 16ab^2c^4 + 2b^4c^3) + x^3\cdot(64a^2bc^4 - 32ab^3c^3 + 4b^5c^2) + x^2}$$

input
```
integrate(x**3/(c*x**2+b*x+a)**3,x)
```

output
```
3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-192*a**4*b*c**3*sqrt(-1/(4*a*c
- b**2)**5) + 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**2)**5) - 36*a**2*b**5
*c*sqrt(-1/(4*a*c - b**2)**5) + 3*a*b**7*sqrt(-1/(4*a*c - b**2)**5) + 3*a*
b**2)/(6*a*b*c)) - 3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x + (192*a**4*b*c*
*3*sqrt(-1/(4*a*c - b**2)**5) - 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**2)*
*5) + 36*a**2*b**5*c*sqrt(-1/(4*a*c - b**2)**5) - 3*a*b**7*sqrt(-1/(4*a*c
- b**2)**5) + 3*a*b**2)/(6*a*b*c)) + (-8*a**3*c - a**2*b**2 - 6*a*b*c**2*x
**3 + x**2*(-16*a**2*c**2 - a*b**2*c - b**4) + x*(-10*a**2*b*c - 2*a*b**3)
)/(32*a**4*c**3 - 16*a**3*b**2*c**2 + 2*a**2*b**4*c + x**4*(32*a**2*c**5 -
 16*a*b**2*c**4 + 2*b**4*c**3) + x**3*(64*a**2*b*c**4 - 32*a*b**3*c**3 + 4
*b**5*c**2) + x**2*(64*a**3*c**4 - 12*a*b**4*c**2 + 2*b**6*c) + x*(64*a**3
*b*c**3 - 32*a**2*b**3*c**2 + 4*a*b**5*c))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx+cx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.41

$$\int \frac{x^3}{(a+bx+cx^2)^3}\,dx = -\frac{6\,ab\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} - \frac{6\,abc^2x^3+b^4x^2+ab^2cx^2+16\,a^2c^2x^2+2\,ab^3x+10\,a^2bcx+a^2b^2+8\,a^3c}{2\,(b^4c-8\,ab^2c^2+16\,a^2c^3)(cx^2+bx+a)^2}$$

input
```
integrate(x^3/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
-6*a*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c
^2)*sqrt(-b^2 + 4*a*c)) - 1/2*(6*a*b*c^2*x^3 + b^4*x^2 + a*b^2*c*x^2 + 16*
a^2*c^2*x^2 + 2*a*b^3*x + 10*a^2*b*c*x + a^2*b^2 + 8*a^3*c)/((b^4*c - 8*a*
b^2*c^2 + 16*a^2*c^3)*(c*x^2 + b*x + a)^2)
```

Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.34

$$\int \frac{x^3}{(a+bx+cx^2)^3}\,dx = -\frac{\frac{x^2\,(16\,a^2\,c^2+a\,b^2\,c+b^4)}{2\,c\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{a^2\,(b^2+8\,a\,c)}{2\,c\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{3\,a\,b\,c\,x^3}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{a\,b\,x\,(b^2+5\,a\,c)}{c\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}}{x^2\,(b^2+2\,a\,c)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3} - \frac{6\,a\,b\,\mathrm{atan}\left(\frac{\left(\frac{3\,a\,b^2}{{(4\,a\,c-b^2)}^{5/2}}+\frac{6\,a\,b\,c\,x}{{(4\,a\,c-b^2)}^{5/2}}\right)\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}{3\,a\,b}\right)}{{(4\,a\,c-b^2)}^{5/2}}$$

input
```
int(x^3/(a + b*x + c*x^2)^3,x)
```

output
```
- ((x^2*(b^4 + 16*a^2*c^2 + a*b^2*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
 + (a^2*(8*a*c + b^2))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a*b*c*x^3
)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (a*b*x*(5*a*c + b^2))/(c*(b^4 + 16*a^2*
c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^
3) - (6*a*b*atan((((3*a*b^2)/(4*a*c - b^2)^(5/2) + (6*a*b*c*x)/(4*a*c - b^
2)^(5/2))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(3*a*b)))/(4*a*c - b^2)^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 604, normalized size of antiderivative = 5.21

$$\int \frac{x^3}{(a+bx+cx^2)^3}\,dx$$
$$= \frac{-12\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^3bc-24\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2b^2cx-24\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{2c\,(64a^3c^5x^4-48a^2b^2c^4x^4+12a\,b^4c^3x^4-b^6c^2x^4+}$$

input
```
int(x^3/(c*x^2+b*x+a)^3,x)
```

output
```
( - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c -
24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2*c*x -
 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c**2*x*
*2 - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3*c*x
**2 - 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c*
*2*x**3 - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c
**3*x**4 - 20*a**4*c**2 + a**3*b**2*c - 16*a**3*b*c**2*x - 40*a**3*c**3*x*
*2 + a**2*b**4 - 4*a**2*b**3*c*x + 18*a**2*b**2*c**2*x**2 + 12*a**2*c**4*x
**4 + 2*a*b**5*x - 6*a*b**4*c*x**2 - 3*a*b**2*c**3*x**4 + b**6*x**2)/(2*c*
(64*a**5*c**3 - 48*a**4*b**2*c**2 + 128*a**4*b*c**3*x + 128*a**4*c**4*x**2
 + 12*a**3*b**4*c - 96*a**3*b**3*c**2*x - 32*a**3*b**2*c**3*x**2 + 128*a**
3*b*c**4*x**3 + 64*a**3*c**5*x**4 - a**2*b**6 + 24*a**2*b**5*c*x - 24*a**2
*b**4*c**2*x**2 - 96*a**2*b**3*c**3*x**3 - 48*a**2*b**2*c**4*x**4 - 2*a*b*
*7*x + 10*a*b**6*c*x**2 + 24*a*b**5*c**2*x**3 + 12*a*b**4*c**3*x**4 - b**8
*x**2 - 2*b**7*c*x**3 - b**6*c**2*x**4))
```

3.249 $\int \frac{x^2}{(a+bx+cx^2)^3} dx$

Optimal result

Integrand size = 16, antiderivative size = 129

$$\int \frac{x^2}{(a+bx+cx^2)^3} dx = -\frac{ab+(b^2-2ac)x}{2c(b^2-4ac)(a+bx+cx^2)^2} + \frac{(b^2+2ac)(b+2cx)}{2c(b^2-4ac)^2(a+bx+cx^2)} - \frac{2(b^2+2ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

output
```
-1/2*(a*b+(-2*a*c+b^2)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+1/2*(2*a*c+b^2)*(
2*c*x+b)/c/(-4*a*c+b^2)^2/(c*x^2+b*x+a)-2*(2*a*c+b^2)*arctanh((2*c*x+b)/(-
4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02

$$\int \frac{x^2}{(a+bx+cx^2)^3}\,dx = \frac{1}{2}\left(\frac{b^2x+a(b-2cx)}{c(-b^2+4ac)(a+x(b+cx))^2} + \frac{(b^2+2ac)(b+2cx)}{c(b^2-4ac)^2(a+x(b+cx))} + \frac{4(b^2+2ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}}\right)$$

input
```
Integrate[x^2/(a + b*x + c*x^2)^3,x]
```

output
```
((b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))^2) + ((b^2 +
2*a*c)*(b + 2*c*x))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (4*(b^2 + 2*a*
c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/2
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1164, 27, 1159, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx+cx^2)^3}\,dx$$

$$\Big\downarrow \text{1164}$$

$$\frac{x(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2} - \frac{\int \frac{2(a-bx)}{(cx^2+bx+a)^2}dx}{2(b^2-4ac)}$$

$$\Big\downarrow \text{27}$$

$$\frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{\int \frac{a-bx}{\left(cx^2+bx+a\right)^2}dx}{b^2-4ac}$$

$\Big\downarrow$ 1159

$$\frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{-\frac{\left(2ac+b^2\right)\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}-\frac{x\left(2ac+b^2\right)+3ab}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}}{b^2-4ac}$$

$\Big\downarrow$ 1083

$$\frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{\frac{2\left(2ac+b^2\right)\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}-\frac{x\left(2ac+b^2\right)+3ab}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}}{b^2-4ac}$$

$\Big\downarrow$ 219

$$\frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{\frac{2\left(2ac+b^2\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{x\left(2ac+b^2\right)+3ab}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}}{b^2-4ac}$$

input
```
Int[x^2/(a + b*x + c*x^2)^3,x]
```

output
```
(x*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (-((3*a*b + (b^2 +
 2*a*c)*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (2*(b^2 + 2*a*c)*ArcTanh[(
b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.63

method	result
default	$\frac{\frac{c\left(2ac+b^2\right)x^3}{16a^2c^2-8ca\,b^2+b^4}+\frac{3b\left(2ac+b^2\right)x^2}{2\left(16a^2c^2-8ca\,b^2+b^4\right)}-\frac{a\left(2ac-5b^2\right)x}{16a^2c^2-8ca\,b^2+b^4}+\frac{3a^2b}{16a^2c^2-8ca\,b^2+b^4}}{\left(c\,x^2+bx+a\right)^2}+\frac{2(2ac+b^2)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(16a^2c^2-8ca\,b^2+b^4)\sqrt{4ac-b^2}}$
risch	$\frac{\frac{c\left(2ac+b^2\right)x^3}{16a^2c^2-8ca\,b^2+b^4}+\frac{3b\left(2ac+b^2\right)x^2}{2\left(16a^2c^2-8ca\,b^2+b^4\right)}-\frac{a\left(2ac-5b^2\right)x}{16a^2c^2-8ca\,b^2+b^4}+\frac{3a^2b}{16a^2c^2-8ca\,b^2+b^4}}{\left(c\,x^2+bx+a\right)^2}-\frac{2\ln\left(\left(32a^2c^3-16a\,c^2b^2+2b^4c\right)x+\left(-4ac+b^2\right)\right.}{\left(-4ac+b^2\right)^{\frac{5}{2}}}$

input
```
int(x^2/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(c*(2*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+3/2*b*(2*a*c+b^2)/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^2-a*(2*a*c-5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+3*a^2*b/
(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+2*(2*a*c+b^2)/(16*a^2*c^2-8*a*
b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(121) = 242$.

Time = 0.09 (sec) , antiderivative size = 887, normalized size of antiderivative = 6.88

$$\int \frac{x^2}{(a+bx+cx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(x^2/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
[1/2*(6*a^2*b^3 - 24*a^3*b*c + 2*(b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*x^3 + 3
*(b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*x^2 + 2*((b^2*c^2 + 2*a*c^3)*x^4 + a^2*b^
2 + 2*a^3*c + 2*(b^3*c + 2*a*b*c^2)*x^3 + (b^4 + 4*a*b^2*c + 4*a^2*c^2)*x^
2 + 2*(a*b^3 + 2*a^2*b*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x +
b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(5*a*b
^4 - 22*a^2*b^2*c + 8*a^3*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4
 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 1
0*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7
- 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), 1/2*(6*a^2*b^3 - 24*a^
3*b*c + 2*(b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*x^3 + 3*(b^5 - 2*a*b^3*c - 8*a
^2*b*c^2)*x^2 - 4*((b^2*c^2 + 2*a*c^3)*x^4 + a^2*b^2 + 2*a^3*c + 2*(b^3*c
+ 2*a*b*c^2)*x^3 + (b^4 + 4*a*b^2*c + 4*a^2*c^2)*x^2 + 2*(a*b^3 + 2*a^2*b*
c)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a
*c)) + 2*(5*a*b^4 - 22*a^2*b^2*c + 8*a^3*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)
*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*
x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(119) = 238$.

Time = 0.67 (sec) , antiderivative size = 570, normalized size of antiderivative = 4.42

$$\int \frac{x^2}{(a+bx+cx^2)^3}\,dx = -\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)\log\left(x+\frac{-64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)+48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)-12ab^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}}{4ac^2+2b^2c}\right. + \sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)\log\left(x+\frac{64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)-48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)+12ab^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}}{4ac^2+2b^2c}\right. + \frac{6a^2b+x^3\cdot(4ac^2+2b^2c)+x^2\cdot(6abc+3b^3)+x(-4a^2c}{32a^4c^2-16a^3b^2c+2a^2b^4+x^4\cdot(32a^2c^4-16ab^2c^3+2b^4c^2)+x^3\cdot(64a^2bc^3-32ab^3c^2+4b^5c)+x^2\cdot(6}$$

input
```
integrate(x**2/(c*x**2+b*x+a)**3,x)
```

output
```
-sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x + (-64*a**3*c**3*sqrt(-1/
(4*a*c - b**2)**5)*(2*a*c + b**2) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**
2)**5)*(2*a*c + b**2) - 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b*
*2) + 2*a*b*c + b**6*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) + b**3)/(4*
a*c**2 + 2*b**2*c)) + sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x + (6
4*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) - 48*a**2*b**2*c**2*
sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) + 12*a*b**4*c*sqrt(-1/(4*a*c - b
**2)**5)*(2*a*c + b**2) + 2*a*b*c - b**6*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c
 + b**2) + b**3)/(4*a*c**2 + 2*b**2*c)) + (6*a**2*b + x**3*(4*a*c**2 + 2*b
**2*c) + x**2*(6*a*b*c + 3*b**3) + x*(-4*a**2*c + 10*a*b**2))/(32*a**4*c**
2 - 16*a**3*b**2*c + 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2
*b**4*c**2) + x**3*(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64
*a**3*c**3 - 12*a*b**4*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c +
4*a*b**5))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx+cx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.19

$$\int \frac{x^2}{(a+bx+cx^2)^3}\,dx = \frac{2\,(b^2+2\,ac)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} + \frac{2\,b^2cx^3+4\,ac^2x^3+3\,b^3x^2+6\,abcx^2+10\,ab^2x-4\,a^2cx+6\,a^2b}{2\,(b^4-8\,ab^2c+16\,a^2c^2)(cx^2+bx+a)^2}$$

input
```
integrate(x^2/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
2*(b^2 + 2*a*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c +
 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(2*b^2*c*x^3 + 4*a*c^2*x^3 + 3*b^3*
x^2 + 6*a*b*c*x^2 + 10*a*b^2*x - 4*a^2*c*x + 6*a^2*b)/((b^4 - 8*a*b^2*c +
16*a^2*c^2)*(c*x^2 + b*x + a)^2)
```

Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.43

$$\int \frac{x^2}{(a+bx+cx^2)^3}\,dx = \frac{\frac{3a^2b}{16a^2c^2-8ab^2c+b^4} - \frac{ax(2ac-5b^2)}{16a^2c^2-8ab^2c+b^4} + \frac{3bx^2(b^2+2ac)}{2(16a^2c^2-8ab^2c+b^4)} + \frac{cx^3(b^2+2ac)}{16a^2c^2-8ab^2c+b^4}}{x^2(b^2+2ac)+a^2+c^2x^4+2abx+2bcx^3} + \frac{2\,\mathrm{atan}\left(\frac{\left(\frac{(b^2+2ac)(16a^2bc^2-8ab^3c+b^5)}{(4ac-b^2)^{5/2}(16a^2c^2-8ab^2c+b^4)}+\frac{2cx(b^2+2ac)}{(4ac-b^2)^{5/2}}\right)(16a^2c^2-8ab^2c+b^4)}{b^2+2ac}\right)(b^2+2ac)}{(4ac-b^2)^{5/2}}$$

input
```
int(x^2/(a + b*x + c*x^2)^3,x)
```

output
```
((3*a^2*b)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) - (a*x*(2*a*c - 5*b^2))/(b^4 + 1
6*a^2*c^2 - 8*a*b^2*c) + (3*b*x^2*(2*a*c + b^2))/(2*(b^4 + 16*a^2*c^2 - 8*
a*b^2*c)) + (c*x^3*(2*a*c + b^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(2*
a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) + (2*atan(((((2*a*c + b^
2)*(b^5 + 16*a^2*b*c^2 - 8*a*b^3*c))/((4*a*c - b^2)^(5/2)*(b^4 + 16*a^2*c^
2 - 8*a*b^2*c)) + (2*c*x*(2*a*c + b^2))/(4*a*c - b^2)^(5/2))*(b^4 + 16*a^2
*c^2 - 8*a*b^2*c))/(2*a*c + b^2))*(2*a*c + b^2))/(4*a*c - b^2)^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 820, normalized size of antiderivative = 6.36

$$\int \frac{x^2}{(a+bx+cx^2)^3}\,dx = \frac{8\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^3bc+4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2b^3+16\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2b^2cx}{}$$

input
```
int(x^2/(c*x^2+b*x+a)^3,x)
```

output
```
(8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c + 4*sq
rt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3 + 16*sqrt(
4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2*c*x + 16*sqrt
(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c**2*x**2 + 8*s
qrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**4*x + 16*sqrt(
4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3*c*x**2 + 16*sqrt
(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c**2*x**3 + 8*s
qrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c**3*x**4 + 4*s
qrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**5*x**2 + 8*sqrt(
4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**4*c*x**3 + 4*sqrt(4*
a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*c**2*x**4 - 8*a**4*c
**2 + 22*a**3*b**2*c - 32*a**3*b*c**2*x - 16*a**3*c**3*x**2 - 5*a**2*b**4
+ 40*a**2*b**3*c*x + 12*a**2*b**2*c**2*x**2 - 8*a**2*c**4*x**4 - 8*a*b**5*
x + 6*a*b**4*c*x**2 - 2*a*b**2*c**3*x**4 - 2*b**6*x**2 + b**4*c**2*x**4)/(
2*b*(64*a**5*c**3 - 48*a**4*b**2*c**2 + 128*a**4*b*c**3*x + 128*a**4*c**4*
x**2 + 12*a**3*b**4*c - 96*a**3*b**3*c**2*x - 32*a**3*b**2*c**3*x**2 + 128
*a**3*b*c**4*x**3 + 64*a**3*c**5*x**4 - a**2*b**6 + 24*a**2*b**5*c*x - 24*
a**2*b**4*c**2*x**2 - 96*a**2*b**3*c**3*x**3 - 48*a**2*b**2*c**4*x**4 - 2*
a*b**7*x + 10*a*b**6*c*x**2 + 24*a*b**5*c**2*x**3 + 12*a*b**4*c**3*x**4 -
b**8*x**2 - 2*b**7*c*x**3 - b**6*c**2*x**4))
```

3.250 $\int \frac{x}{(a+bx+cx^2)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 103

$$\int \frac{x}{(a+bx+cx^2)^3}\,dx = \frac{2a+bx}{2(b^2-4ac)(a+bx+cx^2)^2} - \frac{3b(b+2cx)}{2(b^2-4ac)^2(a+bx+cx^2)} + \frac{6b\text{carctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

output
```
1/2*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2-3/2*b*(2*c*x+b)/(-4*a*c+b^2)^2/
(c*x^2+b*x+a)+6*b*c*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/
2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99

$$\int \frac{x}{(a+bx+cx^2)^3}\,dx = \frac{\frac{(b^2-4ac)(2a+bx)}{(a+x(b+cx))^2} - \frac{3b(b+2cx)}{a+x(b+cx)} - \frac{12bc\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{2(b^2-4ac)^2}$$

input
```
Integrate[x/(a + b*x + c*x^2)^3,x]
```

output
```
(((b^2 - 4*a*c)*(2*a + b*x))/(a + x*(b + c*x))^2 - (3*b*(b + 2*c*x))/(a +
x*(b + c*x)) - (12*b*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 +
 4*a*c])/(2*(b^2 - 4*a*c)^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1159, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx+cx^2)^3}\,dx$$

$$\Big\downarrow \text{1159}$$

$$\frac{3b\int \frac{1}{(cx^2+bx+a)^2}dx}{2(b^2-4ac)}+\frac{2a+bx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{1086}$$

$$\frac{3b\left(-\frac{2c\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{2(b^2-4ac)}+\frac{2a+bx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{3b\left(\frac{4c\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{2(b^2-4ac)}+\frac{2a+bx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{3b\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{2(b^2-4ac)}+\frac{2a+bx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

input
```
Int[x/(a + b*x + c*x^2)^3,x]
```

output
```
(2*a + b*x)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*b*(-((b + 2*c*x)/((
b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a
*c]])/(b^2 - 4*a*c)^(3/2)))/(2*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

method	result
default	$\frac{-bx-2a}{2(4ac-b^2)(cx^2+bx+a)^2}-\frac{3b\left(\frac{2cx+b}{(4ac-b^2)(cx^2+bx+a)}+\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}\right)}{2(4ac-b^2)}$
risch	$\frac{-\frac{3b\,c^2x^3}{16a^2c^2-8ca\,b^2+b^4}-\frac{9b^2c\,x^2}{2(16a^2c^2-8ca\,b^2+b^4)}-\frac{(5ac+b^2)bx}{16a^2c^2-8ca\,b^2+b^4}-\frac{a(8ac+b^2)}{2(16a^2c^2-8ca\,b^2+b^4)}}{(c\,x^2+bx+a)^2}-\frac{3cb\ln\left((32a^2c^3-16a\,c^2b^2+2b^4c)x-(-4\right.}{(-4ac+b^2}$

input
```
int(x/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)^2-3/2*b/(4*a*c-b^2)*((2*c*x+b)/(4
*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)
^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(95) = 190.

Time = 0.09 (sec) , antiderivative size = 788, normalized size of antiderivative = 7.65

$$\int \frac{x}{(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
[-1/2*(a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 6*(b^3*c^2 - 4*a*b*c^3)*x^3 + 9*
(b^4*c - 4*a*b^2*c^2)*x^2 - 6*(b*c^3*x^4 + 2*b^2*c^2*x^3 + 2*a*b^2*c*x + a
^2*b*c + (b^3*c + 2*a*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c
*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(
b^5 + a*b^3*c - 20*a^2*b*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
- 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4
+ 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10
*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 -
 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), -1/2*(a*b^4 + 4*a^2*b^2
*c - 32*a^3*c^2 + 6*(b^3*c^2 - 4*a*b*c^3)*x^3 + 9*(b^4*c - 4*a*b^2*c^2)*x^
2 - 12*(b*c^3*x^4 + 2*b^2*c^2*x^3 + 2*a*b^2*c*x + a^2*b*c + (b^3*c + 2*a*b
*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
- 4*a*c)) + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c +
48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 6
4*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*
x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x
^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(95) = 190$.

Time = 0.56 (sec) , antiderivative size = 481, normalized size of antiderivative = 4.67

$$\int \frac{x}{(a+bx+cx^2)^3}\,dx$$
$$= 3bc\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{-192a^3bc^4\sqrt{-\frac{1}{(4ac-b^2)^5}}+144a^2b^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-36ab^5c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+3}{6bc^2}\right.$$
$$-3bc\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{192a^3bc^4\sqrt{-\frac{1}{(4ac-b^2)^5}}-144a^2b^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+36ab^5c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}-3}{6bc^2}\right.$$
$$+\frac{-8a^2c-ab^2-9b^2cx^2-6bc^2x^3+x(-10abc-2b}{32a^4c^2-16a^3b^2c+2a^2b^4+x^4\cdot(32a^2c^4-16ab^2c^3+2b^4c^2)+x^3\cdot(64a^2bc^3-32ab^3c^2+4b^5c)+x^2\cdot(6}$$

input
```
integrate(x/(c*x**2+b*x+a)**3,x)
```

output
```
3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-192*a**3*b*c**4*sqrt(-1/(4*a*c
- b**2)**5) + 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) - 36*a*b**5*c*
*2*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*sqrt(-1/(4*a*c - b**2)**5) + 3*b*
*2*c)/(6*b*c**2)) - 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x + (192*a**3*b*c
**4*sqrt(-1/(4*a*c - b**2)**5) - 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)
**5) + 36*a*b**5*c**2*sqrt(-1/(4*a*c - b**2)**5) - 3*b**7*c*sqrt(-1/(4*a*c
 - b**2)**5) + 3*b**2*c)/(6*b*c**2)) + (-8*a**2*c - a*b**2 - 9*b**2*c*x**2
 - 6*b*c**2*x**3 + x*(-10*a*b*c - 2*b**3))/(32*a**4*c**2 - 16*a**3*b**2*c
+ 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2) + x**3*
(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 - 12*a*b
**4*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx+cx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.31

$$\int \frac{x}{(a+bx+cx^2)^3}\,dx = -\frac{6\,bc\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} - \frac{6\,bc^2x^3+9\,b^2cx^2+2\,b^3x+10\,abcx+ab^2+8\,a^2c}{2\,(b^4-8\,ab^2c+16\,a^2c^2)(cx^2+bx+a)^2}$$

input
```
integrate(x/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
-6*b*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c
^2)*sqrt(-b^2 + 4*a*c)) - 1/2*(6*b*c^2*x^3 + 9*b^2*c*x^2 + 2*b^3*x + 10*a*
b*c*x + a*b^2 + 8*a^2*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a)
^2)
```

Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.46

$$\int \frac{x}{(a+bx+cx^2)^3}\,dx$$
$$= -\frac{\frac{8\,c\,a^2+a\,b^2}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{9\,b^2\,c\,x^2}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{3\,b\,c^2\,x^3}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{b\,x\,(b^2+5\,a\,c)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^2\,(b^2+2\,a\,c)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3}$$
$$-\frac{6\,b\,c\,\mathrm{atan}\left(\frac{\left(\frac{3\,b^2\,c}{(4\,a\,c-b^2)^{5/2}}+\frac{6\,b\,c^2\,x}{(4\,a\,c-b^2)^{5/2}}\right)(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}{3\,b\,c}\right)}{(4\,a\,c-b^2)^{5/2}}$$

input
```
int(x/(a + b*x + c*x^2)^3,x)
```

output
```
- ((a*b^2 + 8*a^2*c)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^2*c*x^2)/(2
*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*c^2*x^3)/(b^4 + 16*a^2*c^2 - 8*a*b
^2*c) + (b*x*(5*a*c + b^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(2*a*c +
b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) - (6*b*c*atan((((3*b^2*c)/(4*a
*c - b^2)^(5/2) + (6*b*c^2*x)/(4*a*c - b^2)^(5/2))*(b^4 + 16*a^2*c^2 - 8*a
*b^2*c))/(3*b*c)))/(4*a*c - b^2)^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 575, normalized size of antiderivative = 5.58

$$\int \frac{x}{(a+bx+cx^2)^3}\,dx$$
$$= \frac{-12\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2bc-24\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^2cx-24\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a}{128a^3c^5x^4-96a^2b^2c^4x^4+24a\,b^4c^3x^4-2b^6c^2x^4+256a^3b\,c^4x^3-}$$

input
```
int(x/(c*x^2+b*x+a)^3,x)
```

output
```
( - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c -
24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c*x - 24
*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c**2*x**2 - 1
2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*c*x**2 - 24
*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c**2*x**3 -
12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c**3*x**4 - 2
0*a**3*c**2 + a**2*b**2*c - 16*a**2*b*c**2*x + 24*a**2*c**3*x**2 + a*b**4
- 4*a*b**3*c*x - 30*a*b**2*c**2*x**2 + 12*a*c**4*x**4 + 2*b**5*x + 6*b**4*
c*x**2 - 3*b**2*c**3*x**4)/(2*(64*a**5*c**3 - 48*a**4*b**2*c**2 + 128*a**4
*b*c**3*x + 128*a**4*c**4*x**2 + 12*a**3*b**4*c - 96*a**3*b**3*c**2*x - 32
*a**3*b**2*c**3*x**2 + 128*a**3*b*c**4*x**3 + 64*a**3*c**5*x**4 - a**2*b**
6 + 24*a**2*b**5*c*x - 24*a**2*b**4*c**2*x**2 - 96*a**2*b**3*c**3*x**3 - 4
8*a**2*b**2*c**4*x**4 - 2*a*b**7*x + 10*a*b**6*c*x**2 + 24*a*b**5*c**2*x**
3 + 12*a*b**4*c**3*x**4 - b**8*x**2 - 2*b**7*c*x**3 - b**6*c**2*x**4))
```

3.251 $\int \frac{1}{(a+bx+cx^2)^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 101

$$\int \frac{1}{(a+bx+cx^2)^3}\,dx = -\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2} + \frac{3c(b+2cx)}{(b^2-4ac)^2(a+bx+cx^2)} - \frac{12c^2\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

output
```
-1/2*(2*c*x+b)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+3*c*(2*c*x+b)/(-4*a*c+b^2)^2/(
c*x^2+b*x+a)-12*c^2*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/
2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int \frac{1}{(a+bx+cx^2)^3}\,dx = \frac{-\frac{(b+2cx)(b^2-6bcx-2c(5a+3cx^2))}{(a+x(b+cx))^2} + \frac{24c^2\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{2(b^2-4ac)^2}$$

input
```
Integrate[(a + b*x + c*x^2)^(-3),x]
```

output
```
(-(((b + 2*c*x)*(b^2 - 6*b*c*x - 2*c*(5*a + 3*c*x^2)))/(a + x*(b + c*x))^2
) + (24*c^2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2
*(b^2 - 4*a*c)^2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1086, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx+cx^2)^3}dx$$

$$\Big\downarrow \text{ 1086}$$

$$-\frac{3c\int \frac{1}{(cx^2+bx+a)^2}dx}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{ 1086}$$

$$-\frac{3c\left(-\frac{2c\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{ 1083}$$

$$-\frac{3c\left(\frac{4c\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{3c\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

input
```
Int[(a + b*x + c*x^2)^(-3),x]
```

output
```
-1/2*(b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (3*c*(-((b + 2*c*x)
/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/(b^2 - 4*a*c)^(3/2)))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.15

method	result
default	$\frac{2cx+b}{2(4ac-b^2)(c\,x^2+bx+a)^2}+\frac{3c\left(\frac{2cx+b}{\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)}+\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}\right)}{4ac-b^2}$
risch	$\frac{\frac{6c^3x^3}{16a^2c^2-8ca\,b^2+b^4}+\frac{9b\,c^2x^2}{16a^2c^2-8ca\,b^2+b^4}+\frac{2\left(5ac+b^2\right)cx}{16a^2c^2-8ca\,b^2+b^4}+\frac{b\left(10ac-b^2\right)}{32a^2c^2-16ca\,b^2+2b^4}}{(c\,x^2+bx+a)^2}-\frac{6c^2\ln\left(\left(32a^2c^3-16a\,c^2b^2+2b^4c\right)x+(-4ac+b^2\right.}{(-4ac+b^2)^{\frac{5}{2}}}$

input
```
int(1/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^2+3*c/(4*a*c-b^2)*((2*c*x+b)/(4*a*
c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(95) = 190$.

Time = 0.10 (sec) , antiderivative size = 785, normalized size of antiderivative = 7.77

$$\int \frac{1}{(a+bx+cx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
[-1/2*(b^5 - 14*a*b^3*c + 40*a^2*b*c^2 - 12*(b^2*c^3 - 4*a*c^4)*x^3 - 18*(
b^3*c^2 - 4*a*b*c^3)*x^2 - 12*(c^4*x^4 + 2*b*c^3*x^3 + 2*a*b*c^2*x + a^2*c
^2 + (b^2*c^2 + 2*a*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x +
 b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 4*(b^4*
c + a*b^2*c^2 - 20*a^2*c^3)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 +
2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a
*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 1
2*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), -1/2*(b^5 - 14*a*b^3*c +
40*a^2*b*c^2 - 12*(b^2*c^3 - 4*a*c^4)*x^3 - 18*(b^3*c^2 - 4*a*b*c^3)*x^2 +
 24*(c^4*x^4 + 2*b*c^3*x^3 + 2*a*b*c^2*x + a^2*c^2 + (b^2*c^2 + 2*a*c^3)*x
^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c
)) - 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^
4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3
*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 +
 (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 +
2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(95) = 190$.

Time = 0.57 (sec) , antiderivative size = 474, normalized size of antiderivative = 4.69

$$\int \frac{1}{(a+bx+cx^2)^3}\,dx =$$

$$-6c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{-384a^3c^5\sqrt{-\frac{1}{(4ac-b^2)^5}}+288a^2b^2c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}-72ab^4c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+}{12c^3}\right.$$

$$+6c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{384a^3c^5\sqrt{-\frac{1}{(4ac-b^2)^5}}-288a^2b^2c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}+72ab^4c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-6}{12c^3}\right.$$

$$+\frac{10abc-b^3+18bc^2x^2+12c^3x^3+x(20ac^2+4b^2c}{32a^4c^2-16a^3b^2c+2a^2b^4+x^4\cdot(32a^2c^4-16ab^2c^3+2b^4c^2)+x^3\cdot(64a^2bc^3-32ab^3c^2+4b^5c)+x^2\cdot(6}$$

input

```
integrate(1/(c*x**2+b*x+a)**3,x)
```

output

```
-6*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-384*a**3*c**5*sqrt(-1/(4*a*c
- b**2)**5) + 288*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) - 72*a*b**4*c*
*3*sqrt(-1/(4*a*c - b**2)**5) + 6*b**6*c**2*sqrt(-1/(4*a*c - b**2)**5) + 6
*b*c**2)/(12*c**3)) + 6*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (384*a**3*
c**5*sqrt(-1/(4*a*c - b**2)**5) - 288*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2
)**5) + 72*a*b**4*c**3*sqrt(-1/(4*a*c - b**2)**5) - 6*b**6*c**2*sqrt(-1/(4
*a*c - b**2)**5) + 6*b*c**2)/(12*c**3)) + (10*a*b*c - b**3 + 18*b*c**2*x**
2 + 12*c**3*x**3 + x*(20*a*c**2 + 4*b**2*c))/(32*a**4*c**2 - 16*a**3*b**2*
c + 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2) + x**
3*(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 - 12*a
*b**4*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx+cx^2)^3} dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.35

$$\int \frac{1}{(a+bx+cx^2)^3} dx = \frac{12\,c^2 \arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} + \frac{12\,c^3x^3+18\,bc^2x^2+4\,b^2cx+20\,ac^2x-b^3+10\,abc}{2\,(b^4-8\,ab^2c+16\,a^2c^2)(cx^2+bx+a)^2}$$

input
```
integrate(1/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
12*c^2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c
^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3*x^3 + 18*b*c^2*x^2 + 4*b^2*c*x + 20*
a*c^2*x - b^3 + 10*a*b*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a
)^2)
```

Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.82

$$\int \frac{1}{(a+bx+cx^2)^3}\,dx = \frac{\frac{6\,c^3\,x^3}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}-\frac{b^3-10\,a\,b\,c}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{9\,b\,c^2\,x^2}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{2\,c\,x\,(b^2+5\,a\,c)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^2\,(b^2+2\,a\,c)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3} + \frac{12\,c^2\,\mathrm{atan}\left(\frac{\left(\frac{12\,c^3\,x}{(4\,a\,c-b^2)^{5/2}}+\frac{6\,c^2\,(16\,a^2\,b\,c^2-8\,a\,b^3\,c+b^5)}{(4\,a\,c-b^2)^{5/2}\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}\right)(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}{6\,c^2}\right)}{(4\,a\,c-b^2)^{5/2}}$$

input
```
int(1/(a + b*x + c*x^2)^3,x)
```

output
```
((6*c^3*x^3)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) - (b^3 - 10*a*b*c)/(2*(b^4 + 1
6*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*x^2)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (
2*c*x*(5*a*c + b^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(2*a*c + b^2) +
a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) + (12*c^2*atan((((12*c^3*x)/(4*a*c -
b^2)^(5/2) + (6*c^2*(b^5 + 16*a^2*b*c^2 - 8*a*b^3*c))/((4*a*c - b^2)^(5/2)
*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(6*c^2))
)/(4*a*c - b^2)^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 598, normalized size of antiderivative = 5.92

$$\int \frac{1}{(a+bx+cx^2)^3}\,dx = \frac{24\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2b\,c^2+48\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^2c^2x+48\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)\cdots}{2b\,(64a^3c^5x^4-48a^2b^2c^4x^4+12a\,b^4c^3x^4-b^6c^2x^4+1\cdots}$$

input
```
int(1/(c*x^2+b*x+a)^3,x)
```

output
```
(24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c**2 +
48*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c**2*x +
 48*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c**3*x**2
+ 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*c**2*x**
2 + 48*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c**3*x
**3 + 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c**4*x*
*4 - 24*a**3*c**3 + 46*a**2*b**2*c**2 + 32*a**2*b*c**3*x - 48*a**2*c**4*x*
*2 - 14*a*b**4*c + 8*a*b**3*c**2*x + 60*a*b**2*c**3*x**2 - 24*a*c**5*x**4
+ b**6 - 4*b**5*c*x - 12*b**4*c**2*x**2 + 6*b**2*c**4*x**4)/(2*b*(64*a**5*
c**3 - 48*a**4*b**2*c**2 + 128*a**4*b*c**3*x + 128*a**4*c**4*x**2 + 12*a**
3*b**4*c - 96*a**3*b**3*c**2*x - 32*a**3*b**2*c**3*x**2 + 128*a**3*b*c**4*
x**3 + 64*a**3*c**5*x**4 - a**2*b**6 + 24*a**2*b**5*c*x - 24*a**2*b**4*c**
2*x**2 - 96*a**2*b**3*c**3*x**3 - 48*a**2*b**2*c**4*x**4 - 2*a*b**7*x + 10
*a*b**6*c*x**2 + 24*a*b**5*c**2*x**3 + 12*a*b**4*c**3*x**4 - b**8*x**2 - 2
*b**7*c*x**3 - b**6*c**2*x**4))
```

3.252 $\int \frac{1}{x(a+bx+cx^2)^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 185

$$\int \frac{1}{x(a+bx+cx^2)^3}\,dx = \frac{b^2-2ac+bcx}{2a(b^2-4ac)(a+bx+cx^2)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\,x}{2a^2(b^2-4ac)^2(a+bx+cx^2)} + \frac{b(b^4-10ab^2c+30a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{5/2}} + \frac{\log(x)}{a^3} - \frac{\log(a+bx+cx^2)}{2a^3}$$

output
```
1/2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+1/2*(2*b^4-15*a*b^2*c
+16*a^2*c^2+2*b*c*(-7*a*c+b^2)*x)/a^2/(-4*a*c+b^2)^2/(c*x^2+b*x+a)+b*(30*a
^2*c^2-10*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b
^2)^(5/2)+ln(x)/a^3-1/2*ln(c*x^2+b*x+a)/a^3
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\left(a+bx+cx^2\right)^3}\,dx$$
$$=\frac{\frac{a^2(b^2-2ac+bcx)}{(b^2-4ac)(a+x(b+cx))^2}+\frac{a(2b^4-15ab^2c+16a^2c^2+2b^3cx-14abc^2x)}{(b^2-4ac)^2(a+x(b+cx))}-\frac{2b(b^4-10ab^2c+30a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}}+2\log(x)-\log}{2a^3}$$

input
```
Integrate[1/(x*(a + b*x + c*x^2)^3),x]
```

output
```
((a^2*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (a*(2*b
^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*x - 14*a*b*c^2*x))/((b^2 - 4*a*c)^2
*(a + x*(b + c*x))) - (2*b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTan[(b + 2*c
*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + 2*Log[x] - Log[a + x*(b + 
c*x)])/(2*a^3)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1165, 25, 1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx+cx^2\right)^3}\,dx$$
$$\Big\downarrow \text{1165}$$
$$\frac{-2ac+b^2+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{\int -\frac{2(b^2-4ac)+3bcx}{x(cx^2+bx+a)^2}dx}{2a\left(b^2-4ac\right)}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{2(b^2-4ac)+3bcx}{x(cx^2+bx+a)^2}dx}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$$\Big\downarrow \text{1235}$$

$$\frac{\frac{16a^2c^2+2bcx(b^2-7ac)-15ab^2c+2b^4}{a(b^2-4ac)(a+bx+cx^2)}-\frac{\int -\frac{2\left(\left(b^2-4ac\right)^2+bc\left(b^2-7ac\right)x\right)}{x\left(cx^2+bx+a\right)}dx}{a(b^2-4ac)}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2\int \frac{\left(b^2-4ac\right)^2+bc\left(b^2-7ac\right)x}{x\left(cx^2+bx+a\right)}dx}{a(b^2-4ac)}+\frac{16a^2c^2+2bcx(b^2-7ac)-15ab^2c+2b^4}{a(b^2-4ac)(a+bx+cx^2)}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$$\Big\downarrow \text{1200}$$

$$\frac{\frac{2\int \left(\frac{\left(4ac-b^2\right)^2}{ax}+\frac{-cx\left(b^2-4ac\right)^2-b\left(b^4-9acb^2+23a^2c^2\right)}{a\left(cx^2+bx+a\right)}\right)dx}{a(b^2-4ac)}+\frac{16a^2c^2+2bcx(b^2-7ac)-15ab^2c+2b^4}{a(b^2-4ac)(a+bx+cx^2)}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2\left(\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{\left(b^2-4ac\right)^2\log\left(a+bx+cx^2\right)}{2a}+\frac{\log(x)\left(b^2-4ac\right)^2}{a}\right)}{a(b^2-4ac)}+\frac{16a^2c^2+2bcx(b^2-7ac)-15ab^2c+2b^4}{a(b^2-4ac)(a+bx+cx^2)}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

input

```
Int[1/(x*(a + b*x + c*x^2)^3),x]
```

output

```
(b^2 - 2*a*c + b*c*x)/(2*a*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + ((2*b^4 -
15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*x)/(a*(b^2 - 4*a*c)*(a + b*x
 + c*x^2)) + (2*((b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + ((b^2 - 4*a*c)^2*Log[x])/a - ((b
^2 - 4*a*c)^2*Log[a + b*x + c*x^2])/(2*a)))/(a*(b^2 - 4*a*c)))/(2*a*(b^2 -
 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(175) = 350.

Time = 0.76 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.90

method	result
default	$\frac{\ln(x)}{a^3}-\frac{\frac{\frac{ab\,c^2\left(7ac-b^2\right)x^3}{16a^2c^2-8ca\,b^2+b^4}-\frac{ac\left(16a^2c^2-29ca\,b^2+4b^4\right)x^2}{2\left(16a^2c^2-8ca\,b^2+b^4\right)}+\frac{ab\left(a^2c^2+6ca\,b^2-b^4\right)x}{16a^2c^2-8ca\,b^2+b^4}-\frac{3a^2\left(8a^2c^2-7ca\,b^2+b^4\right)}{2\left(16a^2c^2-8ca\,b^2+b^4\right)}}{\left(c\,x^2+bx+a\right)^2}+\frac{\frac{\left(16a^2c^3-8a\,c^2b^2+b^4c\right)\ln(}{2c}}{}}{a^3}$
risch	$\frac{-\frac{b\,c^2\left(7ac-b^2\right)x^3}{a^2\left(16a^2c^2-8ca\,b^2+b^4\right)}+\frac{c\left(16a^2c^2-29ca\,b^2+4b^4\right)x^2}{2\left(16a^2c^2-8ca\,b^2+b^4\right)a^2}-\frac{b\left(a^2c^2+6ca\,b^2-b^4\right)x}{a^2\left(16a^2c^2-8ca\,b^2+b^4\right)}+\frac{12a^2c^2-\frac{21}{2}ca\,b^2+\frac{3}{2}b^4}{a\left(16a^2c^2-8ca\,b^2+b^4\right)}}{\left(c\,x^2+bx+a\right)^2}+\frac{\ln(x)}{a^3}+\Bigg(\sum_{\_R=\mathrm{RootOf}(}$

input
```
int(1/x/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a^3-1/a^3*((a*b*c^2*(7*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*a
*c*(16*a^2*c^2-29*a*b^2*c+4*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+a*b*(a^2*c
^2+6*a*b^2*c-b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x-3/2*a^2*(8*a^2*c^2-7*a*b^2*
c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+1/(16*a^2*c^2-8*a*b^2*c
+b^4)*(1/2*(16*a^2*c^3-8*a*b^2*c^2+b^4*c)/c*ln(c*x^2+b*x+a)+2*(23*a^2*b*c^
2-9*a*b^3*c+b^5-1/2*(16*a^2*c^3-8*a*b^2*c^2+b^4*c)*b/c)/(4*a*c-b^2)^(1/2)*
arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. 2(175) = 350.

Time = 0.38 (sec) , antiderivative size = 1985, normalized size of antiderivative = 10.73

$$\int \frac{1}{x\left(a+bx+cx^2\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate(1/x/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
[1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*b^5*c
^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 13
2*a^3*b^2*c^3 - 64*a^4*c^4)*x^2 + (a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 +
 (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b^6*c - 10*a*b^4*c^2 + 3
0*a^2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*x^2
 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c
^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b
*x + a)) + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*x - (a^
2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c
^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b
^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b
^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*
a^4*b*c^3)*x)*log(c*x^2 + b*x + a) + 2*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5
)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^
8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a
*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)*log(x))/(a^5*b^6 -
 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^4*c^
3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*x^4 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*
a^5*b^3*c^3 - 64*a^6*b*c^4)*x^3 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx+cx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(c*x**2+b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(a+bx+cx^2\right)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.29

$$\int \frac{1}{x\left(a+bx+cx^2\right)^3}\,dx$$
$$= -\frac{\left(b^5-10\,ab^3c+30\,a^2bc^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^3b^4-8\,a^4b^2c+16\,a^5c^2\right)\sqrt{-b^2+4\,ac}} - \frac{\log\left(cx^2+bx+a\right)}{2\,a^3} + \frac{\log\left(|x|\right)}{a^3}$$
$$+ \frac{3\,a^2b^4-21\,a^3b^2c+24\,a^4c^2+2\left(ab^3c^2-7\,a^2bc^3\right)x^3+\left(4\,ab^4c-29\,a^2b^2c^2+16\,a^3c^3\right)x^2+2\left(ab^5-6\,a^2b^3\right.}{2\left(cx^2+bx+a\right)^2\left(b^2-4\,ac\right)^2a^3}$$

input
```
integrate(1/x/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
-(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/
((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt(-b^2 + 4*a*c)) - 1/2*log(c*x^2
+ b*x + a)/a^3 + log(abs(x))/a^3 + 1/2*(3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*
c^2 + 2*(a*b^3*c^2 - 7*a^2*b*c^3)*x^3 + (4*a*b^4*c - 29*a^2*b^2*c^2 + 16*a
^3*c^3)*x^2 + 2*(a*b^5 - 6*a^2*b^3*c - a^3*b*c^2)*x)/((c*x^2 + b*x + a)^2*
(b^2 - 4*a*c)^2*a^3)
```

Mupad [B] (verification not implemented)

Time = 10.25 (sec) , antiderivative size = 1089, normalized size of antiderivative = 5.89

$$\int \frac{1}{x\left(a+bx+cx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(a + b*x + c*x^2)^3),x)
```

output
```
log(x)/a^3 + ((3*(b^4 + 8*a^2*c^2 - 7*a*b^2*c))/(2*a*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)) + (x^2*(4*b^4*c + 16*a^2*c^3 - 29*a*b^2*c^2))/(2*a^2*(b^4 + 16*
a^2*c^2 - 8*a*b^2*c)) - (b*x*(a^2*c^2 - b^4 + 6*a*b^2*c))/(a^2*(b^4 + 16*a
^2*c^2 - 8*a*b^2*c)) - (b*c^2*x^3*(7*a*c - b^2))/(a^2*(b^4 + 16*a^2*c^2 -
8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) - (
log(1536*a^6*c^5 - 2*b^11*x - 2*a*b^10 + 2*a*b^5*(-(4*a*c - b^2)^5)^(1/2)
+ 39*a^2*b^8*c + 2*b^6*x*(-(4*a*c - b^2)^5)^(1/2) - 303*a^3*b^6*c^2 + 1160
*a^4*b^4*c^3 - 2160*a^5*b^2*c^4 - 17*a^2*b^3*c*(-(4*a*c - b^2)^5)^(1/2) +
39*a^3*b*c^2*(-(4*a*c - b^2)^5)^(1/2) - 321*a^2*b^7*c^2*x + 1286*a^3*b^5*c
^3*x - 2560*a^4*b^3*c^4*x - 48*a^3*c^3*x*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b
^9*c*x + 2016*a^5*b*c^5*x - 20*a*b^4*c*x*(-(4*a*c - b^2)^5)^(1/2) + 63*a^2
*b^2*c^2*x*(-(4*a*c - b^2)^5)^(1/2))*(1024*a^5*c^5 - b^10 + b^5*(-(4*a*c -
 b^2)^5)^(1/2) - 160*a^2*b^6*c^2 + 640*a^3*b^4*c^3 - 1280*a^4*b^2*c^4 + 20
*a*b^8*c + 30*a^2*b*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a*b^3*c*(-(4*a*c - b
^2)^5)^(1/2)))/(2*a^3*(4*a*c - b^2)^5) + (log(2*a*b^10 + 2*b^11*x - 1536*a
^6*c^5 + 2*a*b^5*(-(4*a*c - b^2)^5)^(1/2) - 39*a^2*b^8*c + 2*b^6*x*(-(4*a*
c - b^2)^5)^(1/2) + 303*a^3*b^6*c^2 - 1160*a^4*b^4*c^3 + 2160*a^5*b^2*c^4
- 17*a^2*b^3*c*(-(4*a*c - b^2)^5)^(1/2) + 39*a^3*b*c^2*(-(4*a*c - b^2)^5)^
(1/2) + 321*a^2*b^7*c^2*x - 1286*a^3*b^5*c^3*x + 2560*a^4*b^3*c^4*x - 48*a
^3*c^3*x*(-(4*a*c - b^2)^5)^(1/2) - 40*a*b^9*c*x - 2016*a^5*b*c^5*x - 2...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1801, normalized size of antiderivative = 9.74

$$\int \frac{1}{x\left(a+bx+cx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/x/(c*x^2+b*x+a)^3,x)
```

output
```
( - 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b*c**2
 + 20*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**3*c
- 120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**2*c*
*2*x - 120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*
c**3*x**2 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2
*b**5 + 40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*
*4*c*x - 20*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b
**3*c**2*x**2 - 120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2)
)*a**2*b**2*c**3*x**3 - 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
- b**2))*a**2*b*c**4*x**4 - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a
*c - b**2))*a*b**6*x + 16*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c -
 b**2))*a*b**5*c*x**2 + 40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
- b**2))*a*b**4*c**2*x**3 + 20*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*
a*c - b**2))*a*b**3*c**3*x**4 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt
(4*a*c - b**2))*b**7*x**2 - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a
*c - b**2))*b**6*c*x**3 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
 - b**2))*b**5*c**2*x**4 - 64*log(a + b*x + c*x**2)*a**5*c**3 + 48*log(a +
 b*x + c*x**2)*a**4*b**2*c**2 - 128*log(a + b*x + c*x**2)*a**4*b*c**3*x -
128*log(a + b*x + c*x**2)*a**4*c**4*x**2 - 12*log(a + b*x + c*x**2)*a**3*b
**4*c + 96*log(a + b*x + c*x**2)*a**3*b**3*c**2*x + 32*log(a + b*x + c*...
```

3.253 $\int \frac{1}{x^2(a+bx+cx^2)^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 232

$$\int \frac{1}{x^2(a+bx+cx^2)^3}\,dx = -\frac{1}{a^3x} - \frac{b(b^2-3ac)+c(b^2-2ac)\,x}{2a^2(b^2-4ac)\,(a+bx+cx^2)^2} - \frac{b(4b^4-29ab^2c+46a^2c^2)+2c(2b^4-13ab^2c+14a^2c^2)\,x}{2a^3(b^2-4ac)^2\,(a+bx+cx^2)} - \frac{3(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4(b^2-4ac)^{5/2}} - \frac{3b\log(x)}{a^4} + \frac{3b\log(a+bx+cx^2)}{2a^4}$$

output

```
-1/a^3/x-1/2*(b*(-3*a*c+b^2)+c*(-2*a*c+b^2)*x)/a^2/(-4*a*c+b^2)/(c*x^2+b*x
+a)^2-1/2*(b*(46*a^2*c^2-29*a*b^2*c+4*b^4)+2*c*(14*a^2*c^2-13*a*b^2*c+2*b^
4)*x)/a^3/(-4*a*c+b^2)^2/(c*x^2+b*x+a)-3*(-20*a^3*c^3+30*a^2*b^2*c^2-10*a*
b^4*c+b^6)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(5/2)-3*
b*ln(x)/a^4+3/2*b*ln(c*x^2+b*x+a)/a^4
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2 (a+bx+cx^2)^3} \, dx$$
$$= \frac{-\frac{2a}{x} + \frac{a^2(b^3-3abc+b^2cx-2ac^2x)}{(-b^2+4ac)(a+x(b+cx))^2} - \frac{a(4b^5-29ab^3c+46a^2bc^2+4b^4cx-26ab^2c^2x+28a^2c^3x)}{(b^2-4ac)^2(a+x(b+cx))} + \frac{6(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3)\arctan\Big(}{(-b^2+4ac)^{5/2}}}{2a^4}$$

input
```
Integrate[1/(x^2*(a + b*x + c*x^2)^3),x]
```

output
```
((-2*a)/x + (a^2*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((-b^2 + 4*a*c)*(a
 + x*(b + c*x))^2) - (a*(4*b^5 - 29*a*b^3*c + 46*a^2*b*c^2 + 4*b^4*c*x - 2
6*a*b^2*c^2*x + 28*a^2*c^3*x))/((b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (6*(b
^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^
2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) - 6*b*Log[x] + 3*b*Log[a + x*(b + c*x)])
/(2*a^4)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1165, 25, 1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx+cx^2)^3} \, dx$$
$$\Big\downarrow \; 1165$$
$$\frac{-2ac+b^2+bcx}{2ax(b^2-4ac)(a+bx+cx^2)^2} - \frac{\int -\frac{3b^2+4cxb-10ac}{x^2(cx^2+bx+a)^2}dx}{2a(b^2-4ac)}$$
$$\Big\downarrow \; 25$$

$$\frac{\int \frac{3b^2+4cxb-10ac}{x^2(cx^2+bx+a)^2}dx}{2a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{2ax(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \; 1235$$

$$\frac{\frac{20a^2c^2+3bcx(b^2-6ac)-20ab^2c+3b^4}{ax(b^2-4ac)(a+bx+cx^2)}-\frac{\int -\frac{6\left(\left(b^2-5ac\right)\left(b^2-2ac\right)+bc\left(b^2-6ac\right)x\right)}{x^2\left(cx^2+bx+a\right)}dx}{a(b^2-4ac)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{2ax(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{6\int \frac{\left(b^2-5ac\right)\left(b^2-2ac\right)+bc\left(b^2-6ac\right)x}{x^2\left(cx^2+bx+a\right)}dx}{a(b^2-4ac)}+\frac{20a^2c^2+3bcx(b^2-6ac)-20ab^2c+3b^4}{ax(b^2-4ac)(a+bx+cx^2)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{2ax(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \; 1200$$

$$\frac{\frac{6\int \left(-\frac{b\left(4ac-b^2\right)^2}{a^2x}+\frac{b^6-9acb^4+23a^2c^2b^2+c\left(b^2-4ac\right)^2xb-10a^3c^3}{a^2\left(cx^2+bx+a\right)}+\frac{\left(b^2-5ac\right)\left(b^2-2ac\right)}{ax^2}\right)dx}{a(b^2-4ac)}+\frac{20a^2c^2+3bcx(b^2-6ac)-20ab^2c+3b^4}{ax(b^2-4ac)(a+bx+cx^2)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{2ax(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{20a^2c^2+3bcx(b^2-6ac)-20ab^2c+3b^4}{ax(b^2-4ac)(a+bx+cx^2)}+\frac{6\left(\frac{b\left(b^2-4ac\right)^2\log\left(a+bx+cx^2\right)}{2a^2}-\frac{b\log(x)\left(b^2-4ac\right)^2}{a^2}-\frac{\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}\right.}{a(b^2-4ac)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{2ax(b^2-4ac)(a+bx+cx^2)^2}$$

input
```
Int[1/(x^2*(a + b*x + c*x^2)^3),x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(2*a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)^2) + ((3*b^4
- 20*a*b^2*c + 20*a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*x)/(a*(b^2 - 4*a*c)*x*(a +
 b*x + c*x^2)) + (6*(-(((b^2 - 5*a*c)*(b^2 - 2*a*c))/(a*x)) - ((b^6 - 10*a
*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]])/(a^2*Sqrt[b^2 - 4*a*c]) - (b*(b^2 - 4*a*c)^2*Log[x])/a^2 + (b*(b^2 - 4
*a*c)^2*Log[a + b*x + c*x^2])/(2*a^2)))/(a*(b^2 - 4*a*c)))/(2*a*(b^2 - 4*a
*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.74

method	result
default	$-\frac{1}{a^{3} x}-\frac{3 b \ln \left(x\right)}{a^{4}}-\frac{\frac{a \,c^{2} \left(14 a^{2} c^{2}-13 c a \,b^{2}+2 b^{4}\right) x^{3}}{16 a^{2} c^{2}-8 c a \,b^{2}+b^{4}}+\frac{a b c \left(74 a^{2} c^{2}-55 c a \,b^{2}+8 b^{4}\right) x^{2}}{32 a^{2} c^{2}-16 c a \,b^{2}+2 b^{4}}+\frac{a \left(18 a^{3} c^{3}+7 a^{2} b^{2} c^{2}-12 a \,b^{4} c +2 b^{6}\right) x}{16 a^{2} c^{2}-8 c a \,b^{2}+b^{4}}+\frac{a^{2} b \left(58 a^{2} c^{2}-\cdots\right.}{32 a^{2} c^{2}-16\cdots}}{\left(c \,x^{2}+b x +a \right)^{2}}\cdots$
risch	$\frac{-\frac{3 c^{2} \left(10 a^{2} c^{2}-7 c a \,b^{2}+b^{4}\right) x^{4}}{a^{3} \left(16 a^{2} c^{2}-8 c a \,b^{2}+b^{4}\right)}-\frac{3 b c \left(46 a^{2} c^{2}-29 c a \,b^{2}+4 b^{4}\right) x^{3}}{2 \left(16 a^{2} c^{2}-8 c a \,b^{2}+b^{4}\right) a^{3}}-\frac{\left(50 a^{3} c^{3}+7 a^{2} b^{2} c^{2}-18 a \,b^{4} c +3 b^{6}\right) x^{2}}{a^{3} \left(16 a^{2} c^{2}-8 c a \,b^{2}+b^{4}\right)}-\frac{b \left(122 a^{2} c^{2}-68 c a \,b^{2}+9 b^{4}\right) x}{2 a^{2} \left(16 a^{2} c^{2}-8 c a \,b^{2}+b^{4}\right)}-\frac{1}{a}}{x \left(c \,x^{2}+b x +a \right)^{2}}-\frac{3 b \cdots}{\cdots}$

input
```
int(1/x^2/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/a^3/x-3*b*ln(x)/a^4-1/a^4*((a*c^2*(14*a^2*c^2-13*a*b^2*c+2*b^4)/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^3+1/2*a*b*c*(74*a^2*c^2-55*a*b^2*c+8*b^4)/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^2+a*(18*a^3*c^3+7*a^2*b^2*c^2-12*a*b^4*c+2*b^6)/(16*a^2
*c^2-8*a*b^2*c+b^4)*x+1/2*a^2*b*(58*a^2*c^2-36*a*b^2*c+5*b^4)/(16*a^2*c^2-
8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+3/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(-16*a^2
*b*c^3+8*a*b^3*c^2-b^5*c)/c*ln(c*x^2+b*x+a)+2*(10*a^3*c^3-23*a^2*b^2*c^2+9
*a*b^4*c-b^6-1/2*(-16*a^2*b*c^3+8*a*b^3*c^2-b^5*c)*b/c)/(4*a*c-b^2)^(1/2)*
arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1130 vs. $2(222) = 444$.

Time = 0.53 (sec) , antiderivative size = 2280, normalized size of antiderivative = 9.83

$$\int \frac{1}{x^2 (a + bx + cx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
[-1/2*(2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*
c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*x^4 + 3*(4*a*b^7*c - 4
5*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*x^3 + 2*(3*a*b^8 - 30*a^2
*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*x^2 + 3*((b^6*c^2
- 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^5 + 2*(b^7*c - 10*a*b^5*c^
2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^4 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2
 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3
*c^2 - 20*a^4*b*c^3)*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a
^5*c^3)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt
(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (9*a^2*b^7 - 104*a^3*b^5*c
 + 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*x - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^
2*b^3*c^4 - 64*a^3*b*c^5)*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 -
 64*a^3*b^2*c^4)*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3
 - 128*a^4*b*c^4)*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*
b^2*c^3)*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)
*log(c*x^2 + b*x + a) + 6*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a
^3*b*c^5)*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)
*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4
)*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^2 + (
a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)*log(x))/((a^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx + cx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate(1/x**2/(c*x**2+b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (a + bx + cx^2)^3} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^2/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.33

$$\int \frac{1}{x^2 (a + bx + cx^2)^3} \, dx = \frac{3 (b^6 - 10 ab^4c + 30 a^2b^2c^2 - 20 a^3c^3) \arctan \left(\frac{2 cx+b}{\sqrt{-b^2+4 ac}}\right)}{(a^4b^4 - 8 a^5b^2c + 16 a^6c^2)\sqrt{-b^2 + 4 ac}} + \frac{3 b \log (cx^2 + bx + a)}{2 a^4} - \frac{3 b \log (|x|)}{a^4} - \frac{2 a^3b^4 - 16 a^4b^2c + 32 a^5c^2 + 6 (ab^4c^2 - 7 a^2b^2c^3 + 10 a^3c^4)x^4 + 3 (4 ab^5c - 29 a^2b^3c^2 + 46 a^3bc^3)x^3 + 2}{2 (cx^2 + bx + a)^2(b^2 - 4 ac)^2}$$

input
```
integrate(1/x^2/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*x + b)/sqrt
(-b^2 + 4*a*c))/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sqrt(-b^2 + 4*a*c))
+ 3/2*b*log(c*x^2 + b*x + a)/a^4 - 3*b*log(abs(x))/a^4 - 1/2*(2*a^3*b^4 -
16*a^4*b^2*c + 32*a^5*c^2 + 6*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*x^4
 + 3*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3)*x^3 + 2*(3*a*b^6 - 18*a^2
*b^4*c + 7*a^3*b^2*c^2 + 50*a^4*c^3)*x^2 + (9*a^2*b^5 - 68*a^3*b^3*c + 122
*a^4*b*c^2)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c)^2*a^4*x)
```

Mupad [B] (verification not implemented)

Time = 10.62 (sec) , antiderivative size = 1255, normalized size of antiderivative = 5.41

$$\int \frac{1}{x^2 (a + bx + cx^2)^3} \, dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a + b*x + c*x^2)^3),x)
```

output
```
- (1/a + (x^2*(3*b^6 + 50*a^3*c^3 + 7*a^2*b^2*c^2 - 18*a*b^4*c))/(a^3*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(9*b^5 + 122*a^2*b*c^2 - 68*a*b^3*c))/(2*
a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*x^3*(4*b^5*c - 29*a*b^3*c^2 + 46*
a^2*b*c^3))/(2*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*c^2*x^4*(b^4 + 10*
a^2*c^2 - 7*a*b^2*c))/(a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^3*(2*a*c +
b^2) + a^2*x + c^2*x^5 + 2*a*b*x^2 + 2*b*c*x^4) - (3*b*log(x))/a^4 - (3*lo
g(2*a*b^11 + 2*b^12*x + 2*a*b^6*(-(4*a*c - b^2)^5)^(1/2) - 39*a^2*b^9*c -
1696*a^6*b*c^5 + 320*a^6*c^6*x + 2*b^7*x*(-(4*a*c - b^2)^5)^(1/2) + 303*a^
3*b^7*c^2 - 1170*a^4*b^5*c^3 + 2240*a^5*b^3*c^4 - 10*a^4*c^3*(-(4*a*c - b^
2)^5)^(1/2) - 17*a^2*b^4*c*(-(4*a*c - b^2)^5)^(1/2) + 321*a^2*b^8*c^2*x -
1296*a^3*b^6*c^3*x + 2660*a^4*b^4*c^4*x - 2336*a^5*b^2*c^5*x - 40*a*b^10*c
*x + 39*a^3*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 20*a*b^5*c*x*(-(4*a*c - b^2
)^5)^(1/2) - 58*a^3*b*c^3*x*(-(4*a*c - b^2)^5)^(1/2) + 63*a^2*b^3*c^2*x*(-
(4*a*c - b^2)^5)^(1/2))*(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 1024*a^5*b*
c^5 + 160*a^2*b^7*c^2 - 640*a^3*b^5*c^3 + 1280*a^4*b^3*c^4 - 20*a^3*c^3*(-
(4*a*c - b^2)^5)^(1/2) - 20*a*b^9*c + 30*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1
/2) - 10*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2)))/(2*a^4*(4*a*c - b^2)^5) - (3*l
og(2*a*b^11 + 2*b^12*x - 2*a*b^6*(-(4*a*c - b^2)^5)^(1/2) - 39*a^2*b^9*c -
 1696*a^6*b*c^5 + 320*a^6*c^6*x - 2*b^7*x*(-(4*a*c - b^2)^5)^(1/2) + 303*a
^3*b^7*c^2 - 1170*a^4*b^5*c^3 + 2240*a^5*b^3*c^4 + 10*a^4*c^3*(-(4*a*c ...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 2201, normalized size of antiderivative = 9.49

$$\int \frac{1}{x^2 (a + bx + cx^2)^3} \, dx = \text{Too large to display}$$

input
```
int(1/x^2/(c*x^2+b*x+a)^3,x)
```

output
```
( - 120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**5*b*c**
3*x + 180*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b**
3*c**2*x - 240*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**
4*b**2*c**3*x**2 - 240*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b*
*2))*a**4*b*c**4*x**3 - 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
- b**2))*a**3*b**5*c*x + 360*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*
c - b**2))*a**3*b**4*c**2*x**2 + 240*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/s
qrt(4*a*c - b**2))*a**3*b**3*c**3*x**3 - 240*sqrt(4*a*c - b**2)*atan((b +
2*c*x)/sqrt(4*a*c - b**2))*a**3*b**2*c**4*x**4 - 120*sqrt(4*a*c - b**2)*at
an((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**5*x**5 + 6*sqrt(4*a*c - b**2)
*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**7*x - 120*sqrt(4*a*c - b**2)
*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**6*c*x**2 + 60*sqrt(4*a*c - b
**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**5*c**2*x**3 + 360*sqrt(4
*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**4*c**3*x**4 + 18
0*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3*c**4*x
**5 + 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**8*x*
*2 - 48*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**7*c*x
**3 - 120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**6*c
**2*x**4 - 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*
*5*c**3*x**5 + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2)...
```

3.254 $\int \frac{1}{x^3(a+bx+cx^2)^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 280

$$\begin{aligned}\int \frac{1}{x^3(a+bx+cx^2)^3}\,dx &= -\frac{1}{2a^3x^2}+\frac{3b}{a^4x}+\frac{b^4-4ab^2c+2a^2c^2+bc(b^2-3ac)\,x}{2a^3(b^2-4ac)\,(a+bx+cx^2)^2}\\ &\quad+\frac{6b^6-47ab^4c+97a^2b^2c^2-32a^3c^3+6bc(b^4-7ab^2c+11a^2c^2)\,x}{2a^4(b^2-4ac)^2\,(a+bx+cx^2)}\\ &\quad+\frac{3b(2b^6-21ab^4c+70a^2b^2c^2-70a^3c^3)\,\mathrm{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^5(b^2-4ac)^{5/2}}\\ &\quad+\frac{3(2b^2-ac)\log(x)}{a^5}-\frac{3(2b^2-ac)\log(a+bx+cx^2)}{2a^5}\end{aligned}$$

output
```
-1/2/a^3/x^2+3*b/a^4/x+1/2*(b^4-4*a*b^2*c+2*a^2*c^2+b*c*(-3*a*c+b^2)*x)/a^
3/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+1/2*(6*b^6-47*a*b^4*c+97*a^2*b^2*c^2-32*a^3
*c^3+6*b*c*(11*a^2*c^2-7*a*b^2*c+b^4)*x)/a^4/(-4*a*c+b^2)^2/(c*x^2+b*x+a)+
3*b*(-70*a^3*c^3+70*a^2*b^2*c^2-21*a*b^4*c+2*b^6)*arctanh((2*c*x+b)/(-4*a*
c+b^2)^(1/2))/a^5/(-4*a*c+b^2)^(5/2)+3*(-a*c+2*b^2)*ln(x)/a^5-3/2*(-a*c+2*
b^2)*ln(c*x^2+b*x+a)/a^5
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^3 (a+bx+cx^2)^3} \, dx$$

$$= \frac{-\frac{a^2}{x^2}+\frac{6ab}{x}+\frac{a^2(b^4-4ab^2c+2a^2c^2+b^3cx-3abc^2x)}{(b^2-4ac)(a+x(b+cx))^2}+\frac{a(6b^6-47ab^4c+97a^2b^2c^2-32a^3c^3+6b^5cx-42ab^3c^2x+66a^2bc^3x)}{(b^2-4ac)^2(a+x(b+cx))}-\frac{6b(2b^6-21ab^4}{}}{2a^5}$$

input
```
Integrate[1/(x^3*(a + b*x + c*x^2)^3),x]
```

output
```
(-(a^2/x^2) + (6*a*b)/x + (a^2*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x - 3*
a*b*c^2*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (a*(6*b^6 - 47*a*b^4*c +
 97*a^2*b^2*c^2 - 32*a^3*c^3 + 6*b^5*c*x - 42*a*b^3*c^2*x + 66*a^2*b*c^3*x
))/((b^2 - 4*a*c)^2*(a + x*(b + c*x))) - (6*b*(2*b^6 - 21*a*b^4*c + 70*a^2
*b^2*c^2 - 70*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a
*c)^(5/2) + 6*(2*b^2 - a*c)*Log[x] + 3*(-2*b^2 + a*c)*Log[a + x*(b + c*x)]
)/(2*a^5)
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1165, 25, 1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx+cx^2)^3} \, dx$$

$$\Big\downarrow \; 1165$$

$$\frac{-2ac+b^2+bcx}{2ax^2 (b^2-4ac) (a+bx+cx^2)^2}-\frac{\int -\frac{4(b^2-3ac)+5bcx}{x^3(cx^2+bx+a)^2}dx}{2a (b^2-4ac)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{4(b^2-3ac)+5bcx}{x^3(cx^2+bx+a)^2}dx}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{2ax^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\Big\downarrow$ 1235

$$\frac{\frac{24a^2c^2+2bcx(2b^2-11ac)-25ab^2c+4b^4}{ax^2(b^2-4ac)(a+bx+cx^2)}-\frac{\int -\frac{6\left(2b^4-13acb^2+c\left(2b^2-11ac\right)xb+16a^2c^2\right)}{x^3\left(cx^2+bx+a\right)}dx}{a(b^2-4ac)}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{2ax^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{6\int \frac{2b^4-13acb^2+c\left(2b^2-11ac\right)xb+16a^2c^2}{x^3\left(cx^2+bx+a\right)}dx}{a(b^2-4ac)}+\frac{24a^2c^2+2bcx(2b^2-11ac)-25ab^2c+4b^4}{ax^2(b^2-4ac)(a+bx+cx^2)}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{2ax^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\Big\downarrow$ 1200

$$\frac{\frac{6\int \left(-\frac{\left(ac-2b^2\right)\left(4ac-b^2\right)^2}{a^3x}+\frac{-c\left(2b^2-ac\right)x\left(b^2-4ac\right)^2-b\left(2b^6-19acb^4+55a^2c^2b^2-43a^3c^3\right)}{a^3\left(cx^2+bx+a\right)}+\frac{b\left(2b^2-9ac\right)\left(3ac-b^2\right)}{a^2x^2}+\frac{2b^4-13acb^2+16a^2c^2}{ax^3}\right)dx}{a(b^2-4ac)}+\frac{24a^2c\cdots}{\cdots}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{2ax^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\Big\downarrow$ 2009

$$\frac{\frac{24a^2c^2+2bcx(2b^2-11ac)-25ab^2c+4b^4}{ax^2(b^2-4ac)(a+bx+cx^2)}+\frac{6\left(-\frac{\left(2b^2-ac\right)\left(b^2-4ac\right)^2\log\left(a+bx+cx^2\right)}{2a^3}+\frac{\log(x)\left(2b^2-ac\right)\left(b^2-4ac\right)^2}{a^3}+\frac{b\left(2b^2-9ac\right)\left(b^2-3ac\right)}{a^2x}-\frac{16a^2c^2-1\cdots}{2a\cdots}\cdots\right.}{a(b^2-4ac)}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{2ax^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

input
```
Int[1/(x^3*(a + b*x + c*x^2)^3),x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(2*a*(b^2 - 4*a*c)*x^2*(a + b*x + c*x^2)^2) + ((4*b^
4 - 25*a*b^2*c + 24*a^2*c^2 + 2*b*c*(2*b^2 - 11*a*c)*x)/(a*(b^2 - 4*a*c)*x
^2*(a + b*x + c*x^2)) + (6*(-1/2*(2*b^4 - 13*a*b^2*c + 16*a^2*c^2)/(a*x^2)
 + (b*(2*b^2 - 9*a*c)*(b^2 - 3*a*c))/(a^2*x) + (b*(2*b^6 - 21*a*b^4*c + 70
*a^2*b^2*c^2 - 70*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*Sq
rt[b^2 - 4*a*c]) + ((b^2 - 4*a*c)^2*(2*b^2 - a*c)*Log[x])/a^3 - ((b^2 - 4*
a*c)^2*(2*b^2 - a*c)*Log[a + b*x + c*x^2])/(2*a^3)))/(a*(b^2 - 4*a*c)))/(2
*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.65

method	result
default	$-\frac{1}{2a^3x^2}+\frac{(-3ac+6b^2)\ln(x)}{a^5}+\frac{3b}{a^4x}+\frac{\frac{3ab\,c^2\left(11a^2c^2-7ca\,b^2+b^4\right)x^3}{16a^2c^2-8ca\,b^2+b^4}-\frac{ac\left(32a^3c^3-163a^2b^2c^2+89a\,b^4c-12b^6\right)x^2}{2\left(16a^2c^2-8ca\,b^2+b^4\right)}+\frac{ab\left(23a^3c^3+24a^2 \cdots}{16a^2c^2 \cdots}}{\left(c\,x^2+bx+a\right)^2}$
risch	Expression too large to display

input
```
int(1/x^3/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^3/x^2+(-3*a*c+6*b^2)/a^5*ln(x)+3*b/a^4/x+1/a^5*((3*a*b*c^2*(11*a^2*
c^2-7*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*a*c*(32*a^3*c^3-163*
a^2*b^2*c^2+89*a*b^4*c-12*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+a*b*(23*a^3*
c^3+24*a^2*b^2*c^2-20*a*b^4*c+3*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x-1/2*a^2*
(40*a^3*c^3-115*a^2*b^2*c^2+55*a*b^4*c-7*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4))/
(c*x^2+b*x+a)^2+3/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(16*a^3*c^4-40*a^2*b^2*c
^3+17*a*b^4*c^2-2*b^6*c)/c*ln(c*x^2+b*x+a)+2*(43*a^3*b*c^3-55*a^2*b^3*c^2+
19*a*b^5*c-2*b^7-1/2*(16*a^3*c^4-40*a^2*b^2*c^3+17*a*b^4*c^2-2*b^6*c)*b/c)
/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1324 vs. $2(268) = 536$.

Time = 1.09 (sec) , antiderivative size = 2669, normalized size of antiderivative = 9.53

$$\int \frac{1}{x^3 (a + bx + cx^2)^3} \, dx = \text{Too large to display}$$

input

```
integrate(1/x^3/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output

```
[-1/2*(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3 - 6*(2*a*b^7*c
^2 - 23*a^2*b^5*c^3 + 87*a^3*b^3*c^4 - 108*a^4*b*c^5)*x^5 - 3*(8*a*b^8*c -
 94*a^2*b^6*c^2 + 369*a^3*b^4*c^3 - 500*a^4*b^2*c^4 + 64*a^5*c^5)*x^4 - 2*
(6*a*b^9 - 63*a^2*b^7*c + 188*a^3*b^5*c^2 - 25*a^4*b^3*c^3 - 412*a^5*b*c^4
)*x^3 - (18*a^2*b^8 - 217*a^3*b^6*c + 887*a^4*b^4*c^2 - 1300*a^5*b^2*c^3 +
 288*a^6*c^4)*x^2 + 3*((2*b^7*c^2 - 21*a*b^5*c^3 + 70*a^2*b^3*c^4 - 70*a^3
*b*c^5)*x^6 + 2*(2*b^8*c - 21*a*b^6*c^2 + 70*a^2*b^4*c^3 - 70*a^3*b^2*c^4)
*x^5 + (2*b^9 - 17*a*b^7*c + 28*a^2*b^5*c^2 + 70*a^3*b^3*c^3 - 140*a^4*b*c
^4)*x^4 + 2*(2*a*b^8 - 21*a^2*b^6*c + 70*a^3*b^4*c^2 - 70*a^4*b^2*c^3)*x^3
 + (2*a^2*b^7 - 21*a^3*b^5*c + 70*a^4*b^3*c^2 - 70*a^5*b*c^3)*x^2)*sqrt(b^
2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c
*x + b))/(c*x^2 + b*x + a)) - 4*(a^3*b^7 - 12*a^4*b^5*c + 48*a^5*b^3*c^2 -
 64*a^6*b*c^3)*x + 3*((2*b^8*c^2 - 25*a*b^6*c^3 + 108*a^2*b^4*c^4 - 176*a^
3*b^2*c^5 + 64*a^4*c^6)*x^6 + 2*(2*b^9*c - 25*a*b^7*c^2 + 108*a^2*b^5*c^3
- 176*a^3*b^3*c^4 + 64*a^4*b*c^5)*x^5 + (2*b^10 - 21*a*b^8*c + 58*a^2*b^6*
c^2 + 40*a^3*b^4*c^3 - 288*a^4*b^2*c^4 + 128*a^5*c^5)*x^4 + 2*(2*a*b^9 - 2
5*a^2*b^7*c + 108*a^3*b^5*c^2 - 176*a^4*b^3*c^3 + 64*a^5*b*c^4)*x^3 + (2*a
^2*b^8 - 25*a^3*b^6*c + 108*a^4*b^4*c^2 - 176*a^5*b^2*c^3 + 64*a^6*c^4)*x^
2)*log(c*x^2 + b*x + a) - 6*((2*b^8*c^2 - 25*a*b^6*c^3 + 108*a^2*b^4*c^4 -
 176*a^3*b^2*c^5 + 64*a^4*c^6)*x^6 + 2*(2*b^9*c - 25*a*b^7*c^2 + 108*a^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + bx + cx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate(1/x**3/(c*x**2+b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (a + bx + cx^2)^3} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^3/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.46

$$\int \frac{1}{x^3 (a + bx + cx^2)^3} \, dx = -\frac{3 (2 b^7 - 21 ab^5 c + 70 a^2 b^3 c^2 - 70 a^3 bc^3) \arctan \left(\frac{2 cx+b}{\sqrt{-b^2+4 ac}}\right)}{(a^5 b^4 - 8 a^6 b^2 c + 16 a^7 c^2)\sqrt{-b^2 + 4 ac}}$$
$$+ \frac{12 b^5 c^2 x^5 - 90 ab^3 c^3 x^5 + 162 a^2 bc^4 x^5 + 24 b^6 cx^4 - 186 ab^4 c^2 x^4 + 363 a^2 b^2 c^3 x^4 - 48 a^3 c^4 x^4 + 12 b^7 x^3 -}{}$$
$$- \frac{3 (2 b^2 - ac) \log (cx^2 + bx + a)}{2 a^5} + \frac{3 (2 b^2 - ac) \log (|x|)}{a^5}$$

input
```
integrate(1/x^3/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
-3*(2*b^7 - 21*a*b^5*c + 70*a^2*b^3*c^2 - 70*a^3*b*c^3)*arctan((2*c*x + b)
/sqrt(-b^2 + 4*a*c))/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt(-b^2 + 4*a
*c)) + 1/2*(12*b^5*c^2*x^5 - 90*a*b^3*c^3*x^5 + 162*a^2*b*c^4*x^5 + 24*b^6
*c*x^4 - 186*a*b^4*c^2*x^4 + 363*a^2*b^2*c^3*x^4 - 48*a^3*c^4*x^4 + 12*b^7
*x^3 - 78*a*b^5*c*x^3 + 64*a^2*b^3*c^2*x^3 + 206*a^3*b*c^3*x^3 + 18*a*b^6*
x^2 - 145*a^2*b^4*c*x^2 + 307*a^3*b^2*c^2*x^2 - 72*a^4*c^3*x^2 + 4*a^2*b^5
*x - 32*a^3*b^3*c*x + 64*a^4*b*c^2*x - a^3*b^4 + 8*a^4*b^2*c - 16*a^5*c^2)
/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*(c*x^3 + b*x^2 + a*x)^2) - 3/2*(2*b
^2 - a*c)*log(c*x^2 + b*x + a)/a^5 + 3*(2*b^2 - a*c)*log(abs(x))/a^5
```

Mupad [B] (verification not implemented)

Time = 10.81 (sec) , antiderivative size = 1404, normalized size of antiderivative = 5.01

$$\int \frac{1}{x^3 (a + bx + cx^2)^3} \, dx = \text{Too large to display}$$

input
```
int(1/(x^3*(a + b*x + c*x^2)^3),x)
```

output
```
((2*b*x)/a^2 - 1/(2*a) + (x^2*(18*b^6 - 72*a^3*c^3 + 307*a^2*b^2*c^2 - 145
*a*b^4*c))/(2*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*x^4*(8*b^6*c - 16*a
^3*c^4 - 62*a*b^4*c^2 + 121*a^2*b^2*c^3))/(2*a^4*(b^4 + 16*a^2*c^2 - 8*a*b
^2*c)) + (b*x^3*(6*b^6 + 103*a^3*c^3 + 32*a^2*b^2*c^2 - 39*a*b^4*c))/(a^4*
(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*c^2*x^5*(2*b^4 + 27*a^2*c^2 - 15*a*
b^2*c))/(a^4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2*x^2
 + c^2*x^6 + 2*a*b*x^3 + 2*b*c*x^5) - (3*log(x)*(a*c - 2*b^2))/a^5 + (3*lo
g(4*a*b^12 + 4*b^13*x + 1536*a^7*c^6 - 4*a*b^7*(-(4*a*c - b^2)^5)^(1/2) -
80*a^2*b^10*c - 4*b^8*x*(-(4*a*c - b^2)^5)^(1/2) + 645*a^3*b^8*c^2 - 2643*
a^4*b^6*c^3 + 5640*a^5*b^4*c^4 - 5552*a^6*b^2*c^5 + 36*a^2*b^5*c*(-(4*a*c
- b^2)^5)^(1/2) + 59*a^4*b*c^3*(-(4*a*c - b^2)^5)^(1/2) + 682*a^2*b^9*c^2*
x - 2913*a^3*b^7*c^3*x + 6606*a^4*b^5*c^4*x - 7232*a^5*b^3*c^5*x - 48*a^4*
c^4*x*(-(4*a*c - b^2)^5)^(1/2) - 82*a*b^11*c*x - 95*a^3*b^3*c^2*(-(4*a*c -
 b^2)^5)^(1/2) + 2656*a^6*b*c^6*x + 42*a*b^6*c*x*(-(4*a*c - b^2)^5)^(1/2)
- 146*a^2*b^4*c^2*x*(-(4*a*c - b^2)^5)^(1/2) + 179*a^3*b^2*c^3*x*(-(4*a*c
- b^2)^5)^(1/2))*(2*b^12 + 1024*a^6*c^6 - 2*b^7*(-(4*a*c - b^2)^5)^(1/2) +
 340*a^2*b^8*c^2 - 1440*a^3*b^6*c^3 + 3200*a^4*b^4*c^4 - 3328*a^5*b^2*c^5
- 41*a*b^10*c + 70*a^3*b*c^3*(-(4*a*c - b^2)^5)^(1/2) - 70*a^2*b^3*c^2*(-(
4*a*c - b^2)^5)^(1/2) + 21*a*b^5*c*(-(4*a*c - b^2)^5)^(1/2)))/(2*a^5*(4*a*
c - b^2)^5) + (3*log(4*a*b^12 + 4*b^13*x + 1536*a^7*c^6 + 4*a*b^7*(-(4*...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 2450, normalized size of antiderivative = 8.75

$$\int \frac{1}{x^3 (a+bx+cx^2)^3} \, dx = \text{Too large to display}$$

input
```
int(1/x^3/(c*x^2+b*x+a)^3,x)
```

output

```
(420*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**5*b*c**3*x
**2 - 420*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b**
3*c**2*x**2 + 840*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*
a**4*b**2*c**3*x**3 + 840*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c -
 b**2))*a**4*b*c**4*x**4 + 126*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*
a*c - b**2))*a**3*b**5*c*x**2 - 840*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sq
rt(4*a*c - b**2))*a**3*b**4*c**2*x**3 - 420*sqrt(4*a*c - b**2)*atan((b + 2
*c*x)/sqrt(4*a*c - b**2))*a**3*b**3*c**3*x**4 + 840*sqrt(4*a*c - b**2)*ata
n((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**2*c**4*x**5 + 420*sqrt(4*a*c - b
**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**5*x**6 - 12*sqrt(4*a*c
 - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**7*x**2 + 252*sqrt(4*
a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**6*c*x**3 - 168*sq
rt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**5*c**2*x**4
- 840*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**4*c*
*3*x**5 - 420*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2
*b**3*c**4*x**6 - 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2
))*a*b**8*x**3 + 102*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2
))*a*b**7*c*x**4 + 252*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b*
*2))*a*b**6*c**2*x**5 + 126*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
 - b**2))*a*b**5*c**3*x**6 - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqr...
```

3.255 $\int \frac{x^8}{(a+bx+cx^2)^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 441

$$
\begin{aligned}
\int \frac{x^8}{(a+bx+cx^2)^4}\,dx = {} & \frac{x}{c^4} \\
& - \frac{ab(b^6-7ab^4c+14a^2b^2c^2-7a^3c^3)+(b^8-8ab^6c+20a^2b^4c^2-16a^3b^2c^3+2a^4c^4)\,x}{3c^7\,(b^2-4ac)\,(a+bx+cx^2)^3} \\
& + \frac{b^9-17ab^7c+95a^2b^5c^2-202a^3b^3c^3+125a^4bc^4-c(7b^8-68ab^6c+212a^2b^4c^2-220a^3b^2c^3+38a^4c^4)\,x}{3c^7\,(b^2-4ac)^2\,(a+bx+cx^2)^2} \\
& + \frac{2(b^9-15ab^7c+83a^2b^5c^2-198a^3b^3c^3+163a^4bc^4-c(3b^8-36ab^6c+146a^2b^4c^2-212a^3b^2c^3+58a^4c^4}{c^6\,(b^2-4ac)^3\,(a+bx+cx^2)} \\
& - \frac{4(b^8-14ab^6c+70a^2b^4c^2-140a^3b^2c^3+70a^4c^4)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5\,(b^2-4ac)^{7/2}} \\
& - \frac{2b\log(a+bx+cx^2)}{c^5}
\end{aligned}
$$

output
```
x/c^4-1/3*(a*b*(-7*a^3*c^3+14*a^2*b^2*c^2-7*a*b^4*c+b^6)+(2*a^4*c^4-16*a^3
*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)*x)/c^7/(-4*a*c+b^2)/(c*x^2+b*x+a)^3
+1/3*(b^9-17*a*b^7*c+95*a^2*b^5*c^2-202*a^3*b^3*c^3+125*a^4*b*c^4-c*(38*a^
4*c^4-220*a^3*b^2*c^3+212*a^2*b^4*c^2-68*a*b^6*c+7*b^8)*x)/c^7/(-4*a*c+b^2
)^2/(c*x^2+b*x+a)^2+2*(b^9-15*a*b^7*c+83*a^2*b^5*c^2-198*a^3*b^3*c^3+163*a
^4*b*c^4-c*(58*a^4*c^4-212*a^3*b^2*c^3+146*a^2*b^4*c^2-36*a*b^6*c+3*b^8)*x
)/c^6/(-4*a*c+b^2)^3/(c*x^2+b*x+a)-4*(70*a^4*c^4-140*a^3*b^2*c^3+70*a^2*b^
4*c^2-14*a*b^6*c+b^8)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^5/(-4*a*c+b^
2)^(7/2)-2*b*ln(c*x^2+b*x+a)/c^5
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 435, normalized size of antiderivative = 0.99

$$\int \frac{x^8}{(a+bx+cx^2)^4}\,dx$$

$$= \frac{3c^3x+\frac{-b^8x+a^2b^4c(7b-20cx)-ab^6(b-8cx)-2a^3b^2c^2(7b-8cx)+a^4c^3(7b-2cx)}{(b^2-4ac)(a+x(b+cx))^3}+\frac{b^9-17ab^7c+95a^2b^5c^2-202a^3b^3c^3+125a^4bc^4-7b^8cx+68}{(b^2-4ac)^2(a+x(b-}}{}$$

input
```
Integrate[x^8/(a + b*x + c*x^2)^4,x]
```

output
```
(3*c^3*x + (-(b^8*x) + a^2*b^4*c*(7*b - 20*c*x) - a*b^6*(b - 8*c*x) - 2*a^
3*b^2*c^2*(7*b - 8*c*x) + a^4*c^3*(7*b - 2*c*x))/((b^2 - 4*a*c)*(a + x*(b
+ c*x))^3) + (b^9 - 17*a*b^7*c + 95*a^2*b^5*c^2 - 202*a^3*b^3*c^3 + 125*a^
4*b*c^4 - 7*b^8*c*x + 68*a*b^6*c^2*x - 212*a^2*b^4*c^3*x + 220*a^3*b^2*c^4
*x - 38*a^4*c^5*x)/((b^2 - 4*a*c)^2*(a + x*(b + c*x))^2) - (6*c*(-b^9 + 15
*a*b^7*c - 83*a^2*b^5*c^2 + 198*a^3*b^3*c^3 - 163*a^4*b*c^4 + 3*b^8*c*x -
36*a*b^6*c^2*x + 146*a^2*b^4*c^3*x - 212*a^3*b^2*c^4*x + 58*a^4*c^5*x))/((
b^2 - 4*a*c)^3*(a + x*(b + c*x))) - (12*c^2*(b^8 - 14*a*b^6*c + 70*a^2*b^4
*c^2 - 140*a^3*b^2*c^3 + 70*a^4*c^4)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]
])/(-b^2 + 4*a*c)^(7/2) - 6*b*c^2*Log[a + x*(b + c*x)])/(3*c^7)
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1164, 27, 1233, 1233, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx+cx^2)^4}dx$$

$$\Big\downarrow \ 1164$$

$$\frac{x^7(2a+bx)}{3(b^2-4ac)(a+bx+cx^2)^3}-\frac{\int \frac{2x^6(7a+bx)}{(cx^2+bx+a)^3}dx}{3(b^2-4ac)}$$

$$\Big\downarrow \ 27$$

$$\frac{x^7(2a+bx)}{3(b^2-4ac)(a+bx+cx^2)^3}-\frac{2\int \frac{x^6(7a+bx)}{(cx^2+bx+a)^3}dx}{3(b^2-4ac)}$$

$$\Big\downarrow \ 1233$$

$$\frac{x^7(2a+bx)}{3(b^2-4ac)(a+bx+cx^2)^3}-\frac{2\left(\frac{\int \frac{x^4\left(5a\left(b^2-14ac\right)+2b\left(2b^2-13ac\right)x\right)}{\left(cx^2+bx+a\right)^2}dx}{2c(b^2-4ac)}-\frac{x^5\left(bx(b^2-9ac)+a(b^2-14ac)\right)}{2c(b^2-4ac)(a+bx+cx^2)^2}\right)}{3(b^2-4ac)}$$

$$\Big\downarrow \ 1233$$

$$\frac{x^7(2a+bx)}{3(b^2-4ac)(a+bx+cx^2)^3}-\frac{2\left(\frac{\frac{\int \frac{12x^2\left(a\left(b^4-9acb^2+35a^2c^2\right)+b\left(b^4-10acb^2+29a^2c^2\right)x\right)}{cx^2+bx+a}dx}{c\left(b^2-4ac\right)}-\frac{x^3\left(bx\left(122a^2c^2-39ab^2c+4b^4\right)+4a\left(35a^2c^2-9ab^2c+b^4\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}}{2c(b^2-4ac)}-\frac{x^5\left(bx(b^2-9ac)+a(b^2-14\ldots\right.}{2c(b^2-4ac)(a+bx+cx^2\ldots}\right.}{3(b^2-4ac)}$$

$$\Big\downarrow \ 27$$

$$\frac{x^7(2a+bx)}{3(b^2-4ac)(a+bx+cx^2)^3}-\frac{2\left(\frac{\frac{12\int\frac{x^2\left(a\left(b^4-9acb^2+35a^2c^2\right)+b\left(b^4-10acb^2+29a^2c^2\right)x\right)}{cx^2+bx+a}dx}{c\left(b^2-4ac\right)}-\frac{x^3\left(bx\left(122a^2c^2-39ab^2c+4b^4\right)+4a\left(35a^2c^2-9ab^2c+b^4\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}}{2c(b^2-4ac)}-\frac{x^5(bx(b^2-9ac)+a(b^2-14}{2c(b^2-4ac)(a+bx+cx^2}\right.}{3(b^2-4ac)}$$

$\downarrow$ 1200

$$\frac{x^7(2a+bx)}{3(b^2-4ac)(a+bx+cx^2)^3}-\frac{2\left(\frac{\frac{12\int\left(-\frac{b^6}{c^2}+\frac{11ab^4}{c}-38a^2b^2+\frac{\left(b^4-10acb^2+29a^2c^2\right)xb}{c}+35a^3c+\frac{bx\left(b^2-4ac\right)^3+a\left(b^6-11acb^4+38a^2c^2b^2-35a^3c^3\right)}{c^2\left(cx^2+bx+a\right)}\right)dx}{c\left(b^2-4ac\right)}-\frac{x^3\left(bx\left(122a^2c^2-39ab^2c+4b^4\right)+\right.}{c\left(b^2-4ac\right)\left(a+b\right.}}{2c(b^2-4ac)}\right.}{3(b^2-4ac)}$$

$\downarrow$ 2009

$$\frac{x^7(2a+bx)}{3(b^2-4ac)(a+bx+cx^2)^3}-\frac{2\left(\frac{\frac{12\left(\frac{bx^2\left(29a^2c^2-10ab^2c+b^4\right)}{2c}-x\left(-35a^3c+38a^2b^2-\frac{11ab^4}{c}+\frac{b^6}{c^2}\right)+\frac{\left(70a^4c^4-140a^3b^2c^3+70a^2b^4c^2-14ab^6c+b^8\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{b\left(b^2-4ac\right)^3 1}{2}\right.}{c\left(b^2-4ac\right)}}{2c(b^2-4ac)}\right.}{3(b^2-4ac)}$$

input
```
Int[x^8/(a + b*x + c*x^2)^4,x]
```

output
```
(x^7*(2*a + b*x))/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^3) - (2*(-1/2*(x^5*(a
*(b^2 - 14*a*c) + b*(b^2 - 9*a*c)*x))/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2
) + (-((x^3*(4*a*(b^4 - 9*a*b^2*c + 35*a^2*c^2) + b*(4*b^4 - 39*a*b^2*c +
122*a^2*c^2)*x))/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2))) + (12*(-((38*a^2*b^2
 + b^6/c^2 - (11*a*b^4)/c - 35*a^3*c)*x) + (b*(b^4 - 10*a*b^2*c + 29*a^2*c
^2)*x^2)/(2*c) + ((b^8 - 14*a*b^6*c + 70*a^2*b^4*c^2 - 140*a^3*b^2*c^3 + 7
0*a^4*c^4)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c])
 + (b*(b^2 - 4*a*c)^3*Log[a + b*x + c*x^2])/(2*c^3)))/(c*(b^2 - 4*a*c)))/(
2*c*(b^2 - 4*a*c))))/(3*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 730, normalized size of antiderivative = 1.66

method	result
default	$\frac{x}{c^4} - \frac{-\frac{2c\left(58a^4c^4-212a^3b^2c^3+146a^2b^4c^2-36a\,b^6c+3b^8\right)x^5}{64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6}+\frac{2b\left(47a^4c^4+226a^3b^2c^3-209a^2b^4c^2+57a\,b^6c-5b^8\right)x^4}{64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6}-\frac{\left(544a^5c^5-3234a^4b^2c^4\right.}{3c\left(64a\right.}}{}$
risch	Expression too large to display

input
```
int(x^8/(c*x^2+b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
x/c^4-1/c^4*((-2*c*(58*a^4*c^4-212*a^3*b^2*c^3+146*a^2*b^4*c^2-36*a*b^6*c+
3*b^8)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^5+2*b*(47*a^4*c^4+226*
a^3*b^2*c^3-209*a^2*b^4*c^2+57*a*b^6*c-5*b^8)/(64*a^3*c^3-48*a^2*b^2*c^2+1
2*a*b^4*c-b^6)*x^4-1/3*(544*a^5*c^5-3234*a^4*b^2*c^4+1788*a^3*b^4*c^3-68*a
^2*b^6*c^2-96*a*b^8*c+13*b^10)/c/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6
)*x^3+a*b*(304*a^4*c^4+387*a^3*b^2*c^3-486*a^2*b^4*c^2+143*a*b^6*c-13*b^8)
/c/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^2-a^2*(76*a^4*c^4-694*a^3*
b^2*c^3+567*a^2*b^4*c^2-150*a*b^6*c+13*b^8)/c/(64*a^3*c^3-48*a^2*b^2*c^2+1
2*a*b^4*c-b^6)*x+1/3*(590*a^3*c^3-535*a^2*b^2*c^2+147*a*b^4*c-13*b^6)*a^3*
b/c/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6))/(c*x^2+b*x+a)^3+4/(64*a^3*
c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*(1/2*(64*a^3*b*c^3-48*a^2*b^3*c^2+12*a*
b^5*c-b^7)/c*ln(c*x^2+b*x+a)+2*(35*a^4*c^3-38*a^3*b^2*c^2+11*a^2*c*b^4-a*b
^6-1/2*(64*a^3*b*c^3-48*a^2*b^3*c^2+12*a*b^5*c-b^7)*b/c)/(4*a*c-b^2)^(1/2)
*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1647 vs. 2(433) = 866.

Time = 0.15 (sec) , antiderivative size = 3314, normalized size of antiderivative = 7.51

$$\int \frac{x^8}{(a+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^8/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2769 vs. 2(466) = 932.

Time = 5.66 (sec) , antiderivative size = 2769, normalized size of antiderivative = 6.28

$$\int \frac{x^8}{(a+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x**8/(c*x**2+b*x+a)**4,x)
```

output
```
(-2*b/c**5 - 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3
 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(c**5*(16384*a**7*c**7 - 28672*
a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b*
*8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14)))*log(x + (-372*a**4
*b*c**3 - 256*a**4*c**8*(-2*b/c**5 - 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c
**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(c**5*(
16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*
b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**
14))) + 232*a**3*b**3*c**2 + 256*a**3*b**2*c**7*(-2*b/c**5 - 2*sqrt(-(4*a*
c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*
a*b**6*c + b**8)/(c**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**
5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c
**2 + 28*a*b**12*c - b**14))) - 52*a**2*b**5*c - 96*a**2*b**4*c**6*(-2*b/c
**5 - 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a
**2*b**4*c**2 - 14*a*b**6*c + b**8)/(c**5*(16384*a**7*c**7 - 28672*a**6*b*
*2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3
 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14))) + 4*a*b**7 + 16*a*b**6*c*
*5*(-2*b/c**5 - 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c
**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(c**5*(16384*a**7*c**7 - 286
72*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(a+bx+cx^2)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.06

$$\int \frac{x^8}{(a+bx+cx^2)^4}\,dx = \frac{4\,(b^8-14\,ab^6c+70\,a^2b^4c^2-140\,a^3b^2c^3+70\,a^4c^4)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^6c^5-12\,ab^4c^6+48\,a^2b^2c^7-64\,a^3c^8)\sqrt{-b^2+4\,ac}} + \frac{x}{c^4} - \frac{2\,b\log(cx^2+bx+a)}{c^5} - \frac{13\,a^3b^7-147\,a^4b^5c+535\,a^5b^3c^2-590\,a^6bc^3+6\,(3\,b^8c^2-36\,ab^6c^3+146\,a^2b^4c^4-212\,a^3b^2c^5+58\,a^4c^6}{}$$

input
```
integrate(x^8/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

output
```
4*(b^8 - 14*a*b^6*c + 70*a^2*b^4*c^2 - 140*a^3*b^2*c^3 + 70*a^4*c^4)*arcta
n((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^
7 - 64*a^3*c^8)*sqrt(-b^2 + 4*a*c)) + x/c^4 - 2*b*log(c*x^2 + b*x + a)/c^5
 - 1/3*(13*a^3*b^7 - 147*a^4*b^5*c + 535*a^5*b^3*c^2 - 590*a^6*b*c^3 + 6*(
3*b^8*c^2 - 36*a*b^6*c^3 + 146*a^2*b^4*c^4 - 212*a^3*b^2*c^5 + 58*a^4*c^6)
*x^5 + 6*(5*b^9*c - 57*a*b^7*c^2 + 209*a^2*b^5*c^3 - 226*a^3*b^3*c^4 - 47*
a^4*b*c^5)*x^4 + (13*b^10 - 96*a*b^8*c - 68*a^2*b^6*c^2 + 1788*a^3*b^4*c^3
 - 3234*a^4*b^2*c^4 + 544*a^5*c^5)*x^3 + 3*(13*a*b^9 - 143*a^2*b^7*c + 486
*a^3*b^5*c^2 - 387*a^4*b^3*c^3 - 304*a^5*b*c^4)*x^2 + 3*(13*a^2*b^8 - 150*
a^3*b^6*c + 567*a^4*b^4*c^2 - 694*a^5*b^2*c^3 + 76*a^6*c^4)*x)/((c*x^2 + b
*x + a)^3*(b^2 - 4*a*c)^3*c^5)
```

Mupad [B] (verification not implemented)

Time = 10.39 (sec) , antiderivative size = 1159, normalized size of antiderivative = 2.63

$$\int \frac{x^8}{(a+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
int(x^8/(a + b*x + c*x^2)^4,x)
```

output
```
x/c^4 - ((2*x^5*(3*b^8*c + 58*a^4*c^5 - 36*a*b^6*c^2 + 146*a^2*b^4*c^3 - 2
12*a^3*b^2*c^4))/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (2*x^4
*(47*a^4*b*c^4 - 5*b^9 - 209*a^2*b^5*c^2 + 226*a^3*b^3*c^3 + 57*a*b^7*c))/
(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (x^3*(13*b^10 + 544*a^5
*c^5 - 68*a^2*b^6*c^2 + 1788*a^3*b^4*c^3 - 3234*a^4*b^2*c^4 - 96*a*b^8*c))
/(3*c*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x^2*(143*a^2*b^
7*c - 13*a*b^9 + 304*a^5*b*c^4 - 486*a^3*b^5*c^2 + 387*a^4*b^3*c^3))/(c*(b
^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (a^2*(13*a*b^7 - 147*a^2
*b^5*c - 590*a^4*b*c^3 + 535*a^3*b^3*c^2))/(3*c*(b^6 - 64*a^3*c^3 + 48*a^2
*b^2*c^2 - 12*a*b^4*c)) + (a*x*(13*a*b^8 + 76*a^5*c^4 - 150*a^2*b^6*c + 56
7*a^3*b^4*c^2 - 694*a^4*b^2*c^3))/(c*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)))/(x^3*(b^3*c^4 + 6*a*b*c^5) + x^2*(3*a^2*c^5 + 3*a*b^2*c^4) +
 a^3*c^4 + c^7*x^6 + x^4*(3*a*c^6 + 3*b^2*c^5) + 3*b*c^6*x^5 + 3*a^2*b*c^4
*x) + (log(a + b*x + c*x^2)*(4*b^15 - 65536*a^7*b*c^7 + 1344*a^2*b^11*c^2
- 8960*a^3*b^9*c^3 + 35840*a^4*b^7*c^4 - 86016*a^5*b^5*c^5 + 114688*a^6*b^
3*c^6 - 112*a*b^13*c))/(2*(16384*a^7*c^12 - b^14*c^5 + 28*a*b^12*c^6 - 336
*a^2*b^10*c^7 + 2240*a^3*b^8*c^8 - 8960*a^4*b^6*c^9 + 21504*a^5*b^4*c^10 -
 28672*a^6*b^2*c^11)) - (4*atan((((4*x*(b^8 + 70*a^4*c^4 + 70*a^2*b^4*c^2
- 140*a^3*b^2*c^3 - 14*a*b^6*c))/(c^4*(4*a*c - b^2)^7) + (2*(b^7*c^4 - 12*
a*b^5*c^5 - 64*a^3*b*c^7 + 48*a^2*b^3*c^6)*(b^8 + 70*a^4*c^4 + 70*a^2*b...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 3413, normalized size of antiderivative = 7.74

$$\int \frac{x^8}{(a+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
int(x^8/(c*x^2+b*x+a)^4,x)
```

output
```
( - 840*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**7*b*c**
4 + 1680*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**6*b**3
*c**3 - 2520*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**6*
b**2*c**4*x - 2520*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))
*a**6*b*c**5*x**2 - 840*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b
**2))*a**5*b**5*c**2 + 5040*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
 - b**2))*a**5*b**4*c**3*x + 2520*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt
(4*a*c - b**2))*a**5*b**3*c**4*x**2 - 5040*sqrt(4*a*c - b**2)*atan((b + 2*
c*x)/sqrt(4*a*c - b**2))*a**5*b**2*c**5*x**3 - 2520*sqrt(4*a*c - b**2)*ata
n((b + 2*c*x)/sqrt(4*a*c - b**2))*a**5*b*c**6*x**4 + 168*sqrt(4*a*c - b**2
)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b**7*c - 2520*sqrt(4*a*c - b**
2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b**6*c**2*x + 2520*sqrt(4*a*c
 - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b**5*c**3*x**2 + 9240*s
qrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b**4*c**4*x**3
 + 2520*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b**3*
c**5*x**4 - 2520*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a
**4*b**2*c**6*x**5 - 840*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c -
b**2))*a**4*b*c**7*x**6 - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*
c - b**2))*a**3*b**9 + 504*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
- b**2))*a**3*b**8*c*x - 2016*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(...
```

3.256 $\int \frac{x^7}{(a+bx+cx^2)^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 342

$$\int \frac{x^7}{(a+bx+cx^2)^4}\,dx = -\frac{bx}{3c^3(b^2-4ac)}+\frac{x^6(2a+bx)}{3(b^2-4ac)(a+bx+cx^2)^3}$$
$$+\frac{a(b^6-17ab^4c+53a^2b^2c^2-24a^3c^3)+b(b^6-18ab^4c+69a^2b^2c^2-62a^3c^3)x}{6c^5(b^2-4ac)^2(a+bx+cx^2)^2}$$
$$+\frac{26ab^6-\frac{b^8}{c}-193a^2b^4c+546a^3b^2c^2-384a^4c^3+6b(b^6-16ab^4c+77a^2b^2c^2-106a^3c^3)x}{6c^4(b^2-4ac)^3(a+bx+cx^2)}$$
$$+\frac{b(b^6-14ab^4c+70a^2b^2c^2-140a^3c^3)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4(b^2-4ac)^{7/2}}+\frac{\log(a+bx+cx^2)}{2c^4}$$

output
```
-1/3*b*x/c^3/(-4*a*c+b^2)+1/3*x^6*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^3+1
/6*(a*(-24*a^3*c^3+53*a^2*b^2*c^2-17*a*b^4*c+b^6)+b*(-62*a^3*c^3+69*a^2*b^
2*c^2-18*a*b^4*c+b^6)*x)/c^5/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^2+1/6*(26*a*b^6-
b^8/c-193*a^2*b^4*c+546*a^3*b^2*c^2-384*a^4*c^3+6*b*(-106*a^3*c^3+77*a^2*b
^2*c^2-16*a*b^4*c+b^6)*x)/c^4/(-4*a*c+b^2)^3/(c*x^2+b*x+a)+b*(-140*a^3*c^3
+70*a^2*b^2*c^2-14*a*b^4*c+b^6)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^4/
(-4*a*c+b^2)^(7/2)+1/2*ln(c*x^2+b*x+a)/c^4
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.13

$$\int \frac{x^7}{(a+bx+cx^2)^4}\,dx$$

$$= \frac{\frac{-2b^8+29ab^6c-139a^2b^4c^2+233a^3b^2c^3-72a^4c^4+11b^7cx-98ab^5c^2x+259a^2b^3c^3x-182a^3bc^4x}{(b^2-4ac)^2(a+x(b+cx))^2} - \frac{3c(3b^8-40ab^6c+191a^2b^4c^2-374a^3b^2c^3+192c}{(b^2-4ac)^3}}{}$$

input
```
Integrate[x^7/(a + b*x + c*x^2)^4,x]
```

output
```
((-2*b^8 + 29*a*b^6*c - 139*a^2*b^4*c^2 + 233*a^3*b^2*c^3 - 72*a^4*c^4 + 1
1*b^7*c*x - 98*a*b^5*c^2*x + 259*a^2*b^3*c^3*x - 182*a^3*b*c^4*x)/((b^2 -
4*a*c)^2*(a + x*(b + c*x))^2) - (3*c*(3*b^8 - 40*a*b^6*c + 191*a^2*b^4*c^2
 - 374*a^3*b^2*c^3 + 192*a^4*c^4 - 6*b^7*c*x + 70*a*b^5*c^2*x - 266*a^2*b^
3*c^3*x + 308*a^3*b*c^4*x))/((b^2 - 4*a*c)^3*(a + x*(b + c*x))) + (2*(-2*a
^4*c^3 + b^7*x + a*b^5*(b - 7*c*x) + a^3*b*c^2*(9*b - 7*c*x) + 2*a^2*b^3*c
*(-3*b + 7*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))^3) + (6*b*c^2*(b^6 - 14
*a*b^4*c + 70*a^2*b^2*c^2 - 140*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*
a*c]])/(-b^2 + 4*a*c)^(7/2) + 3*c^2*Log[a + x*(b + c*x)])/(6*c^6)
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1164, 1233, 1233, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx+cx^2)^4}\,dx$$

$$\Big\downarrow \; 1164$$

$$\frac{x^6(2a+bx)}{3(b^2-4ac)(a+bx+cx^2)^3} - \frac{\int \frac{x^5(12a+bx)}{(cx^2+bx+a)^3}dx}{3(b^2-4ac)}$$

$\Big\downarrow$ 1233

$$\frac{x^6(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}-\frac{\frac{\int \frac{x^3\left(4a\left(b^2-24ac\right)+b\left(3b^2-22ac\right)x\right)}{\left(cx^2+bx+a\right)^2}dx}{2c(b^2-4ac)}-\frac{x^4(bx(b^2-14ac)+a(b^2-24ac))}{2c(b^2-4ac)(a+bx+cx^2)^2}}{3\left(b^2-4ac\right)}$$

$\Big\downarrow$ 1233

$$\frac{x^6(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}-\frac{\frac{\frac{\int \frac{6x\left(a\left(b^4-10acb^2+64a^2c^2\right)+b\left(b^4-11acb^2+38a^2c^2\right)x\right)}{cx^2+bx+a}dx}{c\left(b^2-4ac\right)}-\frac{x^2\left(bx\left(140a^2c^2-32ab^2c+3b^4\right)+3a\left(64a^2c^2-10ab^2c+b^4\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}}{2c(b^2-4ac)}-\frac{x^4(bx(b^2-14ac)+a(b^2-24ac)\cdots}{2c(b^2-4ac)(a+bx+cx^2)^2}}{3\left(b^2-4ac\right)}$$

$\Big\downarrow$ 27

$$\frac{x^6(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}-\frac{\frac{\frac{6\int \frac{x\left(a\left(b^4-10acb^2+64a^2c^2\right)+b\left(b^4-11acb^2+38a^2c^2\right)x\right)}{cx^2+bx+a}dx}{c\left(b^2-4ac\right)}-\frac{x^2\left(bx\left(140a^2c^2-32ab^2c+3b^4\right)+3a\left(64a^2c^2-10ab^2c+b^4\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}}{2c(b^2-4ac)}-\frac{x^4(bx(b^2-14ac)+a(b^2-24ac\cdots}{2c(b^2-4ac)(a+bx+cx^2)^2}}{3\left(b^2-4ac\right)}$$

$\Big\downarrow$ 1200

$$\frac{x^6(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}-\frac{\frac{\frac{6\int \left(-b\left(-\frac{b^4}{c}+11ab^2-38a^2c\right)-\frac{x\left(b^2-4ac\right)^3+ab\left(b^4-11acb^2+38a^2c^2\right)}{c\left(cx^2+bx+a\right)}\right)dx}{c\left(b^2-4ac\right)}-\frac{x^2\left(bx\left(140a^2c^2-32ab^2c+3b^4\right)+3a\left(64a^2c^2-10ab^2c+b^4\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}}{2c(b^2-4ac)}-\frac{x^4(bx(b^2-14ac\cdots}{2c(b^2-4ac)(\cdots}}{3\left(b^2-4ac\right)}$$

$\Big\downarrow$ 2009

$$\frac{x^6(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}-\frac{\frac{\frac{6\left(-bx\left(-38a^2c+11ab^2-\frac{b^4}{c}\right)-\frac{b\left(-140a^3c^3+70a^2b^2c^2-14ab^4c+b^6\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{\left(b^2-4ac\right)^3\log\left(a+bx+cx^2\right)}{2c^2}\right)}{c\left(b^2-4ac\right)}-\frac{x^2\left(bx\left(140a^2c^2-32ab^2c+3b^4\right)\cdots}{c\left(b^2-4ac\right)\left(\cdots\right.}}{2c(b^2-4ac)}\cdots}{3\left(b^2-4ac\right)}$$

input

```
Int[x^7/(a + b*x + c*x^2)^4,x]
```

output
```
(x^6*(2*a + b*x))/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^3) - (-1/2*(x^4*(a*(b
^2 - 24*a*c) + b*(b^2 - 14*a*c)*x))/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2)
+ (-((x^2*(3*a*(b^4 - 10*a*b^2*c + 64*a^2*c^2) + b*(3*b^4 - 32*a*b^2*c + 1
40*a^2*c^2)*x))/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2))) + (6*(-(b*(11*a*b^2 -
 b^4/c - 38*a^2*c)*x) - (b*(b^6 - 14*a*b^4*c + 70*a^2*b^2*c^2 - 140*a^3*c^
3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) - ((b^2
 - 4*a*c)^3*Log[a + b*x + c*x^2])/(2*c^2)))/(c*(b^2 - 4*a*c)))/(2*c*(b^2 -
 4*a*c)))/(3*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 680 vs. $2(328) = 656$.

Time = 0.83 (sec) , antiderivative size = 681, normalized size of antiderivative = 1.99

method	result
default	$\frac{\frac{\left(154a^3c^3-133a^2b^2c^2+35a\,b^4c-3b^6\right)b\,x^5}{c^2\left(64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6\right)}+\frac{\left(192a^4c^4+242a^3b^2c^3-341a^2b^4c^2+100a\,b^6c-9b^8\right)x^4}{2\left(64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6\right)c^3}+\frac{b\left(2272a^4c^4-1698a^3b^2c^3+117a^2b^4c^2+76a\,b\right.}{6c^4\left(64a^3c^3-48a^2b^2c^2+12a\,b^4c-b\right.}}{}$
risch	Expression too large to display

input
```
int(x^7/(c*x^2+b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
((154*a^3*c^3-133*a^2*b^2*c^2+35*a*b^4*c-3*b^6)*b/c^2/(64*a^3*c^3-48*a^2*b
^2*c^2+12*a*b^4*c-b^6)*x^5+1/2*(192*a^4*c^4+242*a^3*b^2*c^3-341*a^2*b^4*c^
2+100*a*b^6*c-9*b^8)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/c^3*x^4+1/
6*b/c^4*(2272*a^4*c^4-1698*a^3*b^2*c^3+117*a^2*b^4*c^2+76*a*b^6*c-11*b^8)/
(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^3+1/2/c^4*a*(288*a^4*c^4+152*
a^3*b^2*c^3-381*a^2*b^4*c^2+119*a*b^6*c-11*b^8)/(64*a^3*c^3-48*a^2*b^2*c^2
+12*a*b^4*c-b^6)*x^2+1/2*a^2*b*(428*a^3*c^3-460*a^2*b^2*c^2+126*a*b^4*c-11
*b^6)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/c^4*x+1/6*(352*a^3*c^3-43
8*a^2*b^2*c^2+124*a*b^4*c-11*b^6)*a^3/c^4/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*
b^4*c-b^6))/(c*x^2+b*x+a)^3+1/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/c
^3*(1/2*(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/c*ln(c*x^2+b*x+a)+2*(-3
8*a^3*b*c^2+11*a^2*b^3*c-a*b^5-1/2*(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b
^6)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1432 vs. $2(328) = 656$.

Time = 0.14 (sec) , antiderivative size = 2884, normalized size of antiderivative = 8.43

$$\int \frac{x^7}{(a+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^7/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

output
```
[1/6*(11*a^3*b^8 - 168*a^4*b^6*c + 934*a^5*b^4*c^2 - 2104*a^6*b^2*c^3 + 14
08*a^7*c^4 + 6*(3*b^9*c^2 - 47*a*b^7*c^3 + 273*a^2*b^5*c^4 - 686*a^3*b^3*c
^5 + 616*a^4*b*c^6)*x^5 + 3*(9*b^10*c - 136*a*b^8*c^2 + 741*a^2*b^6*c^3 -
1606*a^3*b^4*c^4 + 776*a^4*b^2*c^5 + 768*a^5*c^6)*x^4 + (11*b^11 - 120*a*b
^9*c + 187*a^2*b^7*c^2 + 2166*a^3*b^5*c^3 - 9064*a^4*b^3*c^4 + 9088*a^5*b*
c^5)*x^3 + 3*(11*a*b^10 - 163*a^2*b^8*c + 857*a^3*b^6*c^2 - 1676*a^4*b^4*c
^3 + 320*a^5*b^2*c^4 + 1152*a^6*c^5)*x^2 + 3*(a^3*b^7 - 14*a^4*b^5*c + 70*
a^5*b^3*c^2 - 140*a^6*b*c^3 + (b^7*c^3 - 14*a*b^5*c^4 + 70*a^2*b^3*c^5 - 1
40*a^3*b*c^6)*x^6 + 3*(b^8*c^2 - 14*a*b^6*c^3 + 70*a^2*b^4*c^4 - 140*a^3*b
^2*c^5)*x^5 + 3*(b^9*c - 13*a*b^7*c^2 + 56*a^2*b^5*c^3 - 70*a^3*b^3*c^4 -
140*a^4*b*c^5)*x^4 + (b^10 - 8*a*b^8*c - 14*a^2*b^6*c^2 + 280*a^3*b^4*c^3
- 840*a^4*b^2*c^4)*x^3 + 3*(a*b^9 - 13*a^2*b^7*c + 56*a^3*b^5*c^2 - 70*a^4
*b^3*c^3 - 140*a^5*b*c^4)*x^2 + 3*(a^2*b^8 - 14*a^3*b^6*c + 70*a^4*b^4*c^2
 - 140*a^5*b^2*c^3)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 -
2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 3*(11*a^2*b^9
- 170*a^3*b^7*c + 964*a^4*b^5*c^2 - 2268*a^5*b^3*c^3 + 1712*a^6*b*c^4)*x +
 3*(a^3*b^8 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3 + 256*a^7*c^
4 + (b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6 + 256*a^4*c
^7)*x^6 + 3*(b^9*c^2 - 16*a*b^7*c^3 + 96*a^2*b^5*c^4 - 256*a^3*b^3*c^5 + 2
56*a^4*b*c^6)*x^5 + 3*(b^10*c - 15*a*b^8*c^2 + 80*a^2*b^6*c^3 - 160*a^3...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2565 vs. $2(340) = 680$.

Time = 4.24 (sec) , antiderivative size = 2565, normalized size of antiderivative = 7.50

$$\int \frac{x^7}{(a+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x**7/(c*x**2+b*x+a)**4,x)
```

output
```
(-b*sqrt(-(4*a*c - b**2)**7)*(140*a**3*c**3 - 70*a**2*b**2*c**2 + 14*a*b**
4*c - b**6)/(2*c**4*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b
**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2
 + 28*a*b**12*c - b**14)) + 1/(2*c**4))*log(x + (-256*a**4*c**7*(-b*sqrt(-
(4*a*c - b**2)**7)*(140*a**3*c**3 - 70*a**2*b**2*c**2 + 14*a*b**4*c - b**6
)/(2*c**4*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 -
 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b*
*12*c - b**14)) + 1/(2*c**4)) + 128*a**4*c**3 + 256*a**3*b**2*c**6*(-b*sqr
t(-(4*a*c - b**2)**7)*(140*a**3*c**3 - 70*a**2*b**2*c**2 + 14*a*b**4*c - b
**6)/(2*c**4*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**
5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a
*b**12*c - b**14)) + 1/(2*c**4)) - 58*a**3*b**2*c**2 - 96*a**2*b**4*c**5*(
-b*sqrt(-(4*a*c - b**2)**7)*(140*a**3*c**3 - 70*a**2*b**2*c**2 + 14*a*b**4
*c - b**6)/(2*c**4*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b*
*4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2
+ 28*a*b**12*c - b**14)) + 1/(2*c**4)) + 13*a**2*b**4*c + 16*a*b**6*c**4*(
-b*sqrt(-(4*a*c - b**2)**7)*(140*a**3*c**3 - 70*a**2*b**2*c**2 + 14*a*b**4
*c - b**6)/(2*c**4*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b*
*4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2
+ 28*a*b**12*c - b**14)) + 1/(2*c**4)) - a*b**6 - b**8*c**3*(-b*sqrt(-(...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{(a+bx+cx^2)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.22

$$\int \frac{x^7}{(a+bx+cx^2)^4}\,dx$$
$$= -\frac{(b^7-14\,ab^5c+70\,a^2b^3c^2-140\,a^3bc^3)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^6c^4-12\,ab^4c^5+48\,a^2b^2c^6-64\,a^3c^7)\sqrt{-b^2+4\,ac}}+\frac{\log\left(cx^2+bx+a\right)}{2\,c^4}$$
$$+\frac{11\,a^3b^6-124\,a^4b^4c+438\,a^5b^2c^2-352\,a^6c^3+6\,(3\,b^7c^2-35\,ab^5c^3+133\,a^2b^3c^4-154\,a^3bc^5)x^5+3\,(9\,b^{8}$$

input
```
integrate(x^7/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

output
```
-(b^7 - 14*a*b^5*c + 70*a^2*b^3*c^2 - 140*a^3*b*c^3)*arctan((2*c*x + b)/sq
rt(-b^2 + 4*a*c))/((b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*
sqrt(-b^2 + 4*a*c)) + 1/2*log(c*x^2 + b*x + a)/c^4 + 1/6*(11*a^3*b^6 - 124
*a^4*b^4*c + 438*a^5*b^2*c^2 - 352*a^6*c^3 + 6*(3*b^7*c^2 - 35*a*b^5*c^3 +
 133*a^2*b^3*c^4 - 154*a^3*b*c^5)*x^5 + 3*(9*b^8*c - 100*a*b^6*c^2 + 341*a
^2*b^4*c^3 - 242*a^3*b^2*c^4 - 192*a^4*c^5)*x^4 + (11*b^9 - 76*a*b^7*c - 1
17*a^2*b^5*c^2 + 1698*a^3*b^3*c^3 - 2272*a^4*b*c^4)*x^3 + 3*(11*a*b^8 - 11
9*a^2*b^6*c + 381*a^3*b^4*c^2 - 152*a^4*b^2*c^3 - 288*a^5*c^4)*x^2 + 3*(11
*a^2*b^7 - 126*a^3*b^5*c + 460*a^4*b^3*c^2 - 428*a^5*b*c^3)*x)/((c*x^2 + b
*x + a)^3*(b^2 - 4*a*c)^3*c^4)
```

Mupad [B] (verification not implemented)

Time = 10.34 (sec) , antiderivative size = 1055, normalized size of antiderivative = 3.08

$$\int \frac{x^7}{(a+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
int(x^7/(a + b*x + c*x^2)^4,x)
```

output
```
(b*atan((((b*x*(b^6 - 140*a^3*c^3 + 70*a^2*b^2*c^2 - 14*a*b^4*c))/(c^3*(4*
a*c - b^2)^7) - (b^2*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5
)*(b^6 - 140*a^3*c^3 + 70*a^2*b^2*c^2 - 14*a*b^4*c))/(2*c^7*(4*a*c - b^2)^
7*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(128*a^3*c^7*(4*a*c -
 b^2)^(7/2) - 2*b^6*c^4*(4*a*c - b^2)^(7/2) + 24*a*b^4*c^5*(4*a*c - b^2)^(
7/2) - 96*a^2*b^2*c^6*(4*a*c - b^2)^(7/2)))/(b^7 - 140*a^3*b*c^3 + 70*a^2*
b^3*c^2 - 14*a*b^5*c))*(b^6 - 140*a^3*c^3 + 70*a^2*b^2*c^2 - 14*a*b^4*c))/
(c^4*(4*a*c - b^2)^(7/2)) - (log(a + b*x + c*x^2)*(b^14 - 16384*a^7*c^7 +
336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5
 + 28672*a^6*b^2*c^6 - 28*a*b^12*c))/(2*(16384*a^7*c^11 - b^14*c^4 + 28*a*
b^12*c^5 - 336*a^2*b^10*c^6 + 2240*a^3*b^8*c^7 - 8960*a^4*b^6*c^8 + 21504*
a^5*b^4*c^9 - 28672*a^6*b^2*c^10)) - ((x^4*(192*a^4*c^4 - 9*b^8 - 341*a^2*
b^4*c^2 + 242*a^3*b^2*c^3 + 100*a*b^6*c))/(2*c^3*(b^6 - 64*a^3*c^3 + 48*a^
2*b^2*c^2 - 12*a*b^4*c)) - (a^3*(11*b^6 - 352*a^3*c^3 + 438*a^2*b^2*c^2 -
124*a*b^4*c))/(6*c^4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (
b*x^5*(3*b^6 - 154*a^3*c^3 + 133*a^2*b^2*c^2 - 35*a*b^4*c))/(c^2*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (a*x^2*(288*a^4*c^4 - 11*b^8 -
381*a^2*b^4*c^2 + 152*a^3*b^2*c^3 + 119*a*b^6*c))/(2*c^4*(b^6 - 64*a^3*c^3
 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (b*x^3*(2272*a^4*c^4 - 11*b^8 + 117*a^2
*b^4*c^2 - 1698*a^3*b^2*c^3 + 76*a*b^6*c))/(6*c^4*(b^6 - 64*a^3*c^3 + 4...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 2933, normalized size of antiderivative = 8.58

$$\int \frac{x^7}{(a+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
int(x^7/(c*x^2+b*x+a)^4,x)
```

output
```
( - 840*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**6*b*c**
3 + 420*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**5*b**3*
c**2 - 2520*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**5*b
**2*c**3*x - 2520*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*
a**5*b*c**4*x**2 - 84*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**
2))*a**4*b**5*c + 1260*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b*
*2))*a**4*b**4*c**2*x - 1260*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*
c - b**2))*a**4*b**3*c**3*x**2 - 5040*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/
sqrt(4*a*c - b**2))*a**4*b**2*c**4*x**3 - 2520*sqrt(4*a*c - b**2)*atan((b
+ 2*c*x)/sqrt(4*a*c - b**2))*a**4*b*c**5*x**4 + 6*sqrt(4*a*c - b**2)*atan(
(b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**7 - 252*sqrt(4*a*c - b**2)*atan((b
 + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**6*c*x + 1008*sqrt(4*a*c - b**2)*atan
((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**5*c**2*x**2 + 1680*sqrt(4*a*c - b
**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**4*c**3*x**3 - 1260*sqrt(
4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**3*c**4*x**4 - 2
520*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**2*c**5
*x**5 - 840*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b
*c**6*x**6 + 18*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*
*2*b**8*x - 234*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*
*2*b**7*c*x**2 - 84*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b*...
```

3.257 $\int \frac{x^6}{(a+bx+cx^2)^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 155

$$\int \frac{x^6}{(a+bx+cx^2)^4}\,dx = \frac{x^5(2a+bx)}{3(b^2-4ac)(a+bx+cx^2)^3} - \frac{5ax^3(2a+bx)}{3(b^2-4ac)^2(a+bx+cx^2)^2} - \frac{10a^2(ab+(b^2-2ac)x)}{c(b^2-4ac)^3(a+bx+cx^2)} + \frac{40a^3\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{7/2}}$$

output
```
1/3*x^5*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^3-5/3*a*x^3*(b*x+2*a)/(-4*a*c
+b^2)^2/(c*x^2+b*x+a)^2-10*a^2*(a*b+(-2*a*c+b^2)*x)/c/(-4*a*c+b^2)^3/(c*x^
2+b*x+a)+40*a^3*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(7/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 314 vs. 2(155) = 310.

Time = 0.16 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.03

$$\int \frac{x^6}{(a+bx+cx^2)^4}\,dx = \frac{b^7-12ab^5c+48a^2b^3c^2-59a^3bc^3-4b^6cx+33ab^4c^2x-72a^2b^2c^3x+26a^3c^4x}{3c^5(b^2-4ac)^2(a+x(b+cx))^2} + \frac{-b^7+12ab^5c-48a^2b^3c^2+74a^3bc^3+b^6cx-12ab^4c^2x+48a^2b^2c^3x-44a^3c^4x}{c^4(-b^2+4ac)^3(a+x(b+cx))} + \frac{b^6x+ab^4(b-6cx)+a^3c^2(5b-2cx)+a^2b^2c(-5b+9cx)}{3c^5(-b^2+4ac)(a+x(b+cx))^3} + \frac{40a^3\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{7/2}}$$

input
```
Integrate[x^6/(a + b*x + c*x^2)^4,x]
```

output
```
(b^7 - 12*a*b^5*c + 48*a^2*b^3*c^2 - 59*a^3*b*c^3 - 4*b^6*c*x + 33*a*b^4*c
^2*x - 72*a^2*b^2*c^3*x + 26*a^3*c^4*x)/(3*c^5*(b^2 - 4*a*c)^2*(a + x*(b +
 c*x))^2) + (-b^7 + 12*a*b^5*c - 48*a^2*b^3*c^2 + 74*a^3*b*c^3 + b^6*c*x -
 12*a*b^4*c^2*x + 48*a^2*b^2*c^3*x - 44*a^3*c^4*x)/(c^4*(-b^2 + 4*a*c)^3*(
a + x*(b + c*x))) + (b^6*x + a*b^4*(b - 6*c*x) + a^3*c^2*(5*b - 2*c*x) + a
^2*b^2*c*(-5*b + 9*c*x))/(3*c^5*(-b^2 + 4*a*c)*(a + x*(b + c*x))^3) + (40*
a^3*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(7/2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1153, 1153, 1153, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx+cx^2)^4}\,dx$$

$$\Big\downarrow \; 1153$$

$$\frac{x^5(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}-\frac{10a\int\frac{x^4}{\left(cx^2+bx+a\right)^3}dx}{3\left(b^2-4ac\right)}$$

$\Big\downarrow$ 1153

$$\frac{x^5(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}-\frac{10a\left(\frac{x^3(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{3a\int\frac{x^2}{\left(cx^2+bx+a\right)^2}dx}{b^2-4ac}\right)}{3\left(b^2-4ac\right)}$$

$\Big\downarrow$ 1153

$$\frac{x^5(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}-\frac{10a\left(\frac{x^3(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{3a\left(\frac{x(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{2a\int\frac{1}{cx^2+bx+a}dx}{b^2-4ac}\right)}{b^2-4ac}\right)}{3\left(b^2-4ac\right)}$$

$\Big\downarrow$ 1083

$$\frac{x^5(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}-\frac{10a\left(\frac{x^3(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{3a\left(\frac{4a\int\frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}+\frac{x(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}\right)}{b^2-4ac}\right)}{3\left(b^2-4ac\right)}$$

$\Big\downarrow$ 219

$$\frac{x^5(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}-\frac{10a\left(\frac{x^3(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{3a\left(\frac{4a\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{x(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}\right)}{b^2-4ac}\right)}{3\left(b^2-4ac\right)}$$

input
```
Int[x^6/(a + b*x + c*x^2)^4,x]
```

output
```
(x^5*(2*a + b*x))/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^3) - (10*a*((x^3*(2*a
 + b*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (3*a*((x*(2*a + b*x))/((b
^2 - 4*a*c)*(a + b*x + c*x^2)) + (4*a*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]])/(b^2 - 4*a*c)^(3/2)))/(b^2 - 4*a*c)))/(3*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1153
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m - 2)*(a + b*x +
c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
&& LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(147) = 294$.

Time = 0.84 (sec) , antiderivative size = 531, normalized size of antiderivative = 3.43

method	result
default	$\frac{-\frac{\left(44a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6\right)x^5}{c\left(64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6\right)}-\frac{b\left(14a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6\right)x^4}{\left(64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6\right)c^2}-\frac{\left(160a^4c^4-286a^3b^2c^3+12a^2b^4c^2+7a\,b^6c-b^8\right)x^3}{3c^3\left(64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6\right)}+\frac{ba\left(16a^3c^3\cdots\right.}{c^3\left(64a^3c^3\cdots\right.}}{\left(c\,x^2+bx+a\right)^3}$
risch	$\frac{-\frac{\left(44a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6\right)x^5}{c\left(64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6\right)}-\frac{b\left(14a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6\right)x^4}{\left(64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6\right)c^2}-\frac{\left(160a^4c^4-286a^3b^2c^3+12a^2b^4c^2+7a\,b^6c-b^8\right)x^3}{3c^3\left(64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6\right)}+\frac{ba\left(16a^3c^3\cdots\right.}{c^3\left(64a^3c^3\cdots\right.}}{\left(c\,x^2+bx+a\right)^3}$

input
```
int(x^6/(c*x^2+b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(-(44*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/c/(64*a^3*c^3-48*a^2*b^2*c^2+
12*a*b^4*c-b^6)*x^5-b*(14*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(64*a^3*c
^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/c^2*x^4-1/3/c^3*(160*a^4*c^4-286*a^3*b^2
*c^3+12*a^2*b^4*c^2+7*a*b^6*c-b^8)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b
^6)*x^3+b/c^3*a*(16*a^3*c^3+53*a^2*b^2*c^2-12*a*b^4*c+b^6)/(64*a^3*c^3-48*
a^2*b^2*c^2+12*a*b^4*c-b^6)*x^2-a^2*(20*a^3*c^3-66*a^2*b^2*c^2+13*a*b^4*c-
b^6)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/c^3*x+1/3*(66*a^2*c^2-13*a
*b^2*c+b^4)*a^3*b/c^3/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6))/(c*x^2+b
*x+a)^3+40*a^3/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2
)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 827 vs. 2(147) = 294.

Time = 0.12 (sec) , antiderivative size = 1675, normalized size of antiderivative = 10.81

$$\int \frac{x^6}{(a+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

output
```
[-1/3*(a^3*b^7 - 17*a^4*b^5*c + 118*a^5*b^3*c^2 - 264*a^6*b*c^3 + 3*(b^8*c
^2 - 16*a*b^6*c^3 + 96*a^2*b^4*c^4 - 236*a^3*b^2*c^5 + 176*a^4*c^6)*x^5 +
3*(b^9*c - 16*a*b^7*c^2 + 96*a^2*b^5*c^3 - 206*a^3*b^3*c^4 + 56*a^4*b*c^5)
*x^4 + (b^10 - 11*a*b^8*c + 16*a^2*b^6*c^2 + 334*a^3*b^4*c^3 - 1304*a^4*b^
2*c^4 + 640*a^5*c^5)*x^3 + 3*(a*b^9 - 16*a^2*b^7*c + 101*a^3*b^5*c^2 - 196
*a^4*b^3*c^3 - 64*a^5*b*c^4)*x^2 + 60*(a^3*c^6*x^6 + 3*a^3*b*c^5*x^5 + 3*a
^5*b*c^3*x + a^6*c^3 + 3*(a^3*b^2*c^4 + a^4*c^5)*x^4 + (a^3*b^3*c^3 + 6*a^
4*b*c^4)*x^3 + 3*(a^4*b^2*c^3 + a^5*c^4)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2
*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
 + a)) + 3*(a^2*b^8 - 17*a^3*b^6*c + 118*a^4*b^4*c^2 - 284*a^5*b^2*c^3 + 8
0*a^6*c^4)*x)/(a^3*b^8*c^3 - 16*a^4*b^6*c^4 + 96*a^5*b^4*c^5 - 256*a^6*b^2
*c^6 + 256*a^7*c^7 + (b^8*c^6 - 16*a*b^6*c^7 + 96*a^2*b^4*c^8 - 256*a^3*b^
2*c^9 + 256*a^4*c^10)*x^6 + 3*(b^9*c^5 - 16*a*b^7*c^6 + 96*a^2*b^5*c^7 - 2
56*a^3*b^3*c^8 + 256*a^4*b*c^9)*x^5 + 3*(b^10*c^4 - 15*a*b^8*c^5 + 80*a^2*
b^6*c^6 - 160*a^3*b^4*c^7 + 256*a^5*c^9)*x^4 + (b^11*c^3 - 10*a*b^9*c^4 +
320*a^3*b^5*c^6 - 1280*a^4*b^3*c^7 + 1536*a^5*b*c^8)*x^3 + 3*(a*b^10*c^3 -
 15*a^2*b^8*c^4 + 80*a^3*b^6*c^5 - 160*a^4*b^4*c^6 + 256*a^6*c^8)*x^2 + 3*
(a^2*b^9*c^3 - 16*a^3*b^7*c^4 + 96*a^4*b^5*c^5 - 256*a^5*b^3*c^6 + 256*a^6
*b*c^7)*x), -1/3*(a^3*b^7 - 17*a^4*b^5*c + 118*a^5*b^3*c^2 - 264*a^6*b*c^3
 + 3*(b^8*c^2 - 16*a*b^6*c^3 + 96*a^2*b^4*c^4 - 236*a^3*b^2*c^5 + 176*a...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 938 vs. $2(146) = 292$.

Time = 1.72 (sec) , antiderivative size = 938, normalized size of antiderivative = 6.05

$$\int \frac{x^6}{(a+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x**6/(c*x**2+b*x+a)**4,x)
```

output
```
-20*a**3*sqrt(-1/(4*a*c - b**2)**7)*log(x + (-5120*a**7*c**4*sqrt(-1/(4*a*
c - b**2)**7) + 5120*a**6*b**2*c**3*sqrt(-1/(4*a*c - b**2)**7) - 1920*a**5
*b**4*c**2*sqrt(-1/(4*a*c - b**2)**7) + 320*a**4*b**6*c*sqrt(-1/(4*a*c - b
**2)**7) - 20*a**3*b**8*sqrt(-1/(4*a*c - b**2)**7) + 20*a**3*b)/(40*a**3*c
)) + 20*a**3*sqrt(-1/(4*a*c - b**2)**7)*log(x + (5120*a**7*c**4*sqrt(-1/(4
*a*c - b**2)**7) - 5120*a**6*b**2*c**3*sqrt(-1/(4*a*c - b**2)**7) + 1920*a
**5*b**4*c**2*sqrt(-1/(4*a*c - b**2)**7) - 320*a**4*b**6*c*sqrt(-1/(4*a*c
- b**2)**7) + 20*a**3*b**8*sqrt(-1/(4*a*c - b**2)**7) + 20*a**3*b)/(40*a**
3*c)) + (66*a**5*b*c**2 - 13*a**4*b**3*c + a**3*b**5 + x**5*(-132*a**3*c**
5 + 144*a**2*b**2*c**4 - 36*a*b**4*c**3 + 3*b**6*c**2) + x**4*(-42*a**3*b*
c**4 + 144*a**2*b**3*c**3 - 36*a*b**5*c**2 + 3*b**7*c) + x**3*(-160*a**4*c
**4 + 286*a**3*b**2*c**3 - 12*a**2*b**4*c**2 - 7*a*b**6*c + b**8) + x**2*(
48*a**4*b*c**3 + 159*a**3*b**3*c**2 - 36*a**2*b**5*c + 3*a*b**7) + x*(-60*
a**5*c**3 + 198*a**4*b**2*c**2 - 39*a**3*b**4*c + 3*a**2*b**6))/(192*a**6*
c**6 - 144*a**5*b**2*c**5 + 36*a**4*b**4*c**4 - 3*a**3*b**6*c**3 + x**6*(1
92*a**3*c**9 - 144*a**2*b**2*c**8 + 36*a*b**4*c**7 - 3*b**6*c**6) + x**5*(
576*a**3*b*c**8 - 432*a**2*b**3*c**7 + 108*a*b**5*c**6 - 9*b**7*c**5) + x*
*4*(576*a**4*c**8 + 144*a**3*b**2*c**7 - 324*a**2*b**4*c**6 + 99*a*b**6*c*
*5 - 9*b**8*c**4) + x**3*(1152*a**4*b*c**7 - 672*a**3*b**3*c**6 + 72*a**2*
b**5*c**5 + 18*a*b**7*c**4 - 3*b**9*c**3) + x**2*(576*a**5*c**7 + 144*a...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6}{(a+bx+cx^2)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^6/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(147) = 294$.

Time = 0.22 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.49

$$\int \frac{x^6}{(a+bx+cx^2)^4}\,dx = -\frac{40\,a^3\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3)\sqrt{-b^2+4\,ac}} - \frac{3\,b^6c^2x^5-36\,ab^4c^3x^5+144\,a^2b^2c^4x^5-132\,a^3c^5x^5+3\,b^7cx^4-36\,ab^5c^2x^4+144\,a^2b^3c^3x^4-42\,a^3bc^4x^4}{}$$

input
```
integrate(x^6/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

output
```
-40*a^3*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^6 - 12*a*b^4*c + 48*a^2
*b^2*c^2 - 64*a^3*c^3)*sqrt(-b^2 + 4*a*c)) - 1/3*(3*b^6*c^2*x^5 - 36*a*b^4
*c^3*x^5 + 144*a^2*b^2*c^4*x^5 - 132*a^3*c^5*x^5 + 3*b^7*c*x^4 - 36*a*b^5*
c^2*x^4 + 144*a^2*b^3*c^3*x^4 - 42*a^3*b*c^4*x^4 + b^8*x^3 - 7*a*b^6*c*x^3
 - 12*a^2*b^4*c^2*x^3 + 286*a^3*b^2*c^3*x^3 - 160*a^4*c^4*x^3 + 3*a*b^7*x^
2 - 36*a^2*b^5*c*x^2 + 159*a^3*b^3*c^2*x^2 + 48*a^4*b*c^3*x^2 + 3*a^2*b^6*
x - 39*a^3*b^4*c*x + 198*a^4*b^2*c^2*x - 60*a^5*c^3*x + a^3*b^5 - 13*a^4*b
^3*c + 66*a^5*b*c^2)/((b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^
6)*(c*x^2 + b*x + a)^3)
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 656, normalized size of antiderivative = 4.23

$$\int \frac{x^6}{(a+bx+cx^2)^4}\,dx = -\frac{\frac{a^3\,(66\,a^2\,b\,c^2-13\,a\,b^3\,c+b^5)}{3\,c^3\,(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}+\frac{x^5\,(-44\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}{c\,(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}-\frac{x^3\,(160\,a^4\,c^4-286\,a^3\,b^2\,c^3+12\,a^2\,b^4\,c^2+7\,a\,b^6\,c-}{3\,c^3\,(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}}{x^2\,(3\,c\,a^2+3\,a\,b^2)+x^4\,(3\,b^2\,c+3\,a\,c^2)+a^3} - \frac{40\,a^3\,\mathrm{atan}\left(\frac{\left(\frac{20\,a^3\,(-64\,a^3\,b\,c^3+48\,a^2\,b^3\,c^2-12\,a\,b^5\,c+b^7)}{(4\,a\,c-b^2)^{7/2}\,(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}+\frac{40\,a^3\,c\,x}{(4\,a\,c-b^2)^{7/2}}\right)(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}{20\,a^3}\right)}{(4\,a\,c-b^2)^{7/2}}$$

input
```
int(x^6/(a + b*x + c*x^2)^4,x)
```

output
```
- ((a^3*(b^5 + 66*a^2*b*c^2 - 13*a*b^3*c))/(3*c^3*(b^6 - 64*a^3*c^3 + 48*a
^2*b^2*c^2 - 12*a*b^4*c)) + (x^5*(b^6 - 44*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a
*b^4*c))/(c*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x^3*(160*
a^4*c^4 - b^8 + 12*a^2*b^4*c^2 - 286*a^3*b^2*c^3 + 7*a*b^6*c))/(3*c^3*(b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x^4*(b^7 - 14*a^3*b*c^3 +
 48*a^2*b^3*c^2 - 12*a*b^5*c))/(c^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 1
2*a*b^4*c)) + (a*x^2*(b^7 + 16*a^3*b*c^3 + 53*a^2*b^3*c^2 - 12*a*b^5*c))/(
c^3*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (a^2*x*(b^6 - 20*a
^3*c^3 + 66*a^2*b^2*c^2 - 13*a*b^4*c))/(c^3*(b^6 - 64*a^3*c^3 + 48*a^2*b^2
*c^2 - 12*a*b^4*c)))/(x^2*(3*a*b^2 + 3*a^2*c) + x^4*(3*a*c^2 + 3*b^2*c) +
a^3 + x^3*(b^3 + 6*a*b*c) + c^3*x^6 + 3*b*c^2*x^5 + 3*a^2*b*x) - (40*a^3*a
tan((((20*a^3*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/((4*a*c
- b^2)^(7/2)*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (40*a^3*c
*x)/(4*a*c - b^2)^(7/2))*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
/(20*a^3)))/(4*a*c - b^2)^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1181, normalized size of antiderivative = 7.62

$$\int \frac{x^6}{(a+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
int(x^6/(c*x^2+b*x+a)^4,x)
```

output

```
(120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**6*b*c**2 +
 360*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**5*b**2*c**
2*x + 360*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**5*b*c
**3*x**2 + 360*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**
4*b**3*c**2*x**2 + 720*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b*
*2))*a**4*b**2*c**3*x**3 + 360*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*
a*c - b**2))*a**4*b*c**4*x**4 + 120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sq
rt(4*a*c - b**2))*a**3*b**4*c**2*x**3 + 360*sqrt(4*a*c - b**2)*atan((b + 2
*c*x)/sqrt(4*a*c - b**2))*a**3*b**3*c**3*x**4 + 360*sqrt(4*a*c - b**2)*ata
n((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**2*c**4*x**5 + 120*sqrt(4*a*c - b
**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**5*x**6 + 176*a**7*c**3
 + 28*a**6*b**2*c**2 + 288*a**6*b*c**3*x + 528*a**6*c**4*x**2 - 22*a**5*b*
*4*c + 144*a**5*b**3*c**2*x + 12*a**5*b**2*c**3*x**2 + 416*a**5*b*c**4*x**
3 + 528*a**5*c**5*x**4 + a**4*b**6 - 66*a**4*b**5*c*x + 168*a**4*b**4*c**2
*x**2 + 64*a**4*b**3*c**3*x**3 - 348*a**4*b**2*c**4*x**4 + 176*a**4*c**6*x
**6 + 3*a**3*b**7*x - 63*a**3*b**6*c*x**2 + 6*a**3*b**5*c**2*x**3 + 198*a*
*3*b**4*c**3*x**4 - 236*a**3*b**2*c**5*x**6 + 3*a**2*b**8*x**2 - 16*a**2*b
**7*c*x**3 - 48*a**2*b**6*c**2*x**4 + 96*a**2*b**4*c**4*x**6 + a*b**9*x**3
 + 3*a*b**8*c*x**4 - 16*a*b**6*c**3*x**6 + b**8*c**2*x**6)/(3*b*c**2*(256*
a**7*c**4 - 256*a**6*b**2*c**3 + 768*a**6*b*c**4*x + 768*a**6*c**5*x**2...
```

3.258 $\int \frac{x^5}{(a+bx+cx^2)^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 154

$$\int \frac{x^5}{(a+bx+cx^2)^4}\,dx = -\frac{x^5(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^3} + \frac{5bx^3(2a+bx)}{6(b^2-4ac)^2(a+bx+cx^2)^2} + \frac{5ab(ab+(b^2-2ac)x)}{c(b^2-4ac)^3(a+bx+cx^2)} - \frac{20a^2b\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{7/2}}$$

output
```
-1/3*x^5*(2*c*x+b)/(-4*a*c+b^2)/(c*x^2+b*x+a)^3+5/6*b*x^3*(b*x+2*a)/(-4*a*
c+b^2)^2/(c*x^2+b*x+a)^2+5*a*b*(a*b+(-2*a*c+b^2)*x)/c/(-4*a*c+b^2)^3/(c*x^
2+b*x+a)-20*a^2*b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(7/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.73

$$\int \frac{x^5}{\left(a+bx+cx^2\right)^4}\,dx = \frac{1}{6}\left(\frac{-2b^6+19ab^4c-61a^2b^2c^2+48a^3c^3+5b^5cx-40ab^3c^2x+70a^2bc^3x}{c^4\left(b^2-4ac\right)^2(a+x(b+cx))^2} + \frac{3(b^6-12ab^4c+38a^2b^2c^2-64a^3c^3-20a^2bc^3x)}{c^3\left(-b^2+4ac\right)^3(a+x(b+cx))} - \frac{2(2a^3c^2+b^5x+ab^3(b-5cx)+a^2bc(-4b+5cx))}{c^4\left(-b^2+4ac\right)(a+x(b+cx))^3} - \frac{120a^2b\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{7/2}}\right)$$

input
```
Integrate[x^5/(a + b*x + c*x^2)^4,x]
```

output
```
((-2*b^6 + 19*a*b^4*c - 61*a^2*b^2*c^2 + 48*a^3*c^3 + 5*b^5*c*x - 40*a*b^3
*c^2*x + 70*a^2*b*c^3*x)/(c^4*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^2) + (3*(b
^6 - 12*a*b^4*c + 38*a^2*b^2*c^2 - 64*a^3*c^3 - 20*a^2*b*c^3*x))/(c^3*(-b^
2 + 4*a*c)^3*(a + x*(b + c*x))) - (2*(2*a^3*c^2 + b^5*x + a*b^3*(b - 5*c*x
) + a^2*b*c*(-4*b + 5*c*x)))/(c^4*(-b^2 + 4*a*c)*(a + x*(b + c*x))^3) - (1
20*a^2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(7/2))/6
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1156, 1153, 1153, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\left(a+bx+cx^2\right)^4}\,dx$$

$$\Big\downarrow \text{ 1156}$$

$$\frac{5b\int \frac{x^4}{(cx^2+bx+a)^3}dx}{3(b^2-4ac)}-\frac{x^5(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^3}$$

$$\Big\downarrow \; 1153$$

$$\frac{5b\left(\frac{x^3(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{3a\int \frac{x^2}{(cx^2+bx+a)^2}dx}{b^2-4ac}\right)}{3(b^2-4ac)}-\frac{x^5(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^3}$$

$$\Big\downarrow \; 1153$$

$$\frac{5b\left(\frac{x^3(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{3a\left(\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{2a\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}\right)}{b^2-4ac}\right)}{3(b^2-4ac)}-\frac{x^5(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^3}$$

$$\Big\downarrow \; 1083$$

$$\frac{5b\left(\frac{x^3(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{3a\left(\frac{4a\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}+\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}\right)}{3(b^2-4ac)}-\frac{x^5(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^3}$$

$$\Big\downarrow \; 219$$

$$\frac{5b\left(\frac{x^3(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{3a\left(\frac{4a\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}\right)}{3(b^2-4ac)}-\frac{x^5(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^3}$$

input
```
Int[x^5/(a + b*x + c*x^2)^4,x]
```

output
```
-1/3*(x^5*(b + 2*c*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)^3) + (5*b*((x^3*(2
*a + b*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (3*a*((x*(2*a + b*x))/(
(b^2 - 4*a*c)*(a + b*x + c*x^2)) + (4*a*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a
*c]])/(b^2 - 4*a*c)^(3/2)))/(b^2 - 4*a*c)))/(3*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1153
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m - 2)*(a + b*x +
c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
&& LtQ[p, -1]
```

rule 1156
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] + Simp[m*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*c)))   Int[
(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 485 vs. $2(146) = 292$.

Time = 0.78 (sec) , antiderivative size = 486, normalized size of antiderivative = 3.16

method	result
default	$\frac{-\frac{10a^2bc^2x^5}{64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6}-\frac{\left(64a^3c^3+2a^2b^2c^2+12a\,b^4c-b^6\right)x^4}{2\left(64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6\right)c}-\frac{b\left(224a^3c^3+62a^2b^2c^2+12a\,b^4c-b^6\right)x^3}{6\left(64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6\right)c^2}-\frac{a\left(64a^3c^3+32a^2b^2c^2+17a\,b\right.}{2\left(64a^3c^3-48a^2b^2c^2+12a\,b\right.}}{\left(c\,x^2+bx+a\right)^3}$
risch	$\frac{-\frac{10a^2bc^2x^5}{64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6}-\frac{\left(64a^3c^3+2a^2b^2c^2+12a\,b^4c-b^6\right)x^4}{2\left(64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6\right)c}-\frac{b\left(224a^3c^3+62a^2b^2c^2+12a\,b^4c-b^6\right)x^3}{6\left(64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6\right)c^2}-\frac{a\left(64a^3c^3+32a^2b^2c^2+17a\,b\right.}{2\left(64a^3c^3-48a^2b^2c^2+12a\,b\right.}}{\left(c\,x^2+bx+a\right)^3}$

input
```
int(x^5/(c*x^2+b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(-10*a^2*b/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*c^2*x^5-1/2*(64*a^3*
c^3+2*a^2*b^2*c^2+12*a*b^4*c-b^6)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^
6)/c*x^4-1/6*b*(224*a^3*c^3+62*a^2*b^2*c^2+12*a*b^4*c-b^6)/(64*a^3*c^3-48*
a^2*b^2*c^2+12*a*b^4*c-b^6)/c^2*x^3-1/2*a*(64*a^3*c^3+32*a^2*b^2*c^2+17*a*
b^4*c-b^6)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/c^2*x^2-1/2*a^2*b*(4
4*a^2*c^2+18*a*b^2*c-b^4)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/c^2*x
-1/6*a^3*(64*a^2*c^2+18*a*b^2*c-b^4)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c
-b^6)/c^2)/(c*x^2+b*x+a)^3-20*a^2*b/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-
b^6)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 787 vs. $2(146) = 292$.

Time = 0.11 (sec) , antiderivative size = 1594, normalized size of antiderivative = 10.35

$$\int \frac{x^5}{(a+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

output
```
[-1/6*(a^3*b^6 - 22*a^4*b^4*c + 8*a^5*b^2*c^2 + 256*a^6*c^3 - 60*(a^2*b^3*
c^4 - 4*a^3*b*c^5)*x^5 + 3*(b^8*c - 16*a*b^6*c^2 + 46*a^2*b^4*c^3 - 56*a^3
*b^2*c^4 + 256*a^4*c^5)*x^4 + (b^9 - 16*a*b^7*c - 14*a^2*b^5*c^2 + 24*a^3*
b^3*c^3 + 896*a^4*b*c^4)*x^3 + 3*(a*b^8 - 21*a^2*b^6*c + 36*a^3*b^4*c^2 +
64*a^4*b^2*c^3 + 256*a^5*c^4)*x^2 + 60*(a^2*b*c^5*x^6 + 3*a^2*b^2*c^4*x^5
+ 3*a^4*b^2*c^2*x + a^5*b*c^2 + 3*(a^2*b^3*c^3 + a^3*b*c^4)*x^4 + (a^2*b^4
*c^2 + 6*a^3*b^2*c^3)*x^3 + 3*(a^3*b^3*c^2 + a^4*b*c^3)*x^2)*sqrt(b^2 - 4*
a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b
))/(c*x^2 + b*x + a)) + 3*(a^2*b^7 - 22*a^3*b^5*c + 28*a^4*b^3*c^2 + 176*a
^5*b*c^3)*x)/(a^3*b^8*c^2 - 16*a^4*b^6*c^3 + 96*a^5*b^4*c^4 - 256*a^6*b^2*
c^5 + 256*a^7*c^6 + (b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2
*c^8 + 256*a^4*c^9)*x^6 + 3*(b^9*c^4 - 16*a*b^7*c^5 + 96*a^2*b^5*c^6 - 256
*a^3*b^3*c^7 + 256*a^4*b*c^8)*x^5 + 3*(b^10*c^3 - 15*a*b^8*c^4 + 80*a^2*b^
6*c^5 - 160*a^3*b^4*c^6 + 256*a^5*c^8)*x^4 + (b^11*c^2 - 10*a*b^9*c^3 + 32
0*a^3*b^5*c^5 - 1280*a^4*b^3*c^6 + 1536*a^5*b*c^7)*x^3 + 3*(a*b^10*c^2 - 1
5*a^2*b^8*c^3 + 80*a^3*b^6*c^4 - 160*a^4*b^4*c^5 + 256*a^6*c^7)*x^2 + 3*(a
^2*b^9*c^2 - 16*a^3*b^7*c^3 + 96*a^4*b^5*c^4 - 256*a^5*b^3*c^5 + 256*a^6*b
*c^6)*x), -1/6*(a^3*b^6 - 22*a^4*b^4*c + 8*a^5*b^2*c^2 + 256*a^6*c^3 - 60*
(a^2*b^3*c^4 - 4*a^3*b*c^5)*x^5 + 3*(b^8*c - 16*a*b^6*c^2 + 46*a^2*b^4*c^3
 - 56*a^3*b^2*c^4 + 256*a^4*c^5)*x^4 + (b^9 - 16*a*b^7*c - 14*a^2*b^5*c...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(148) = 296$.

Time = 1.30 (sec) , antiderivative size = 898, normalized size of antiderivative = 5.83

$$\int \frac{x^5}{(a + bx + cx^2)^4}\, dx = \text{Too large to display}$$

input
```
integrate(x**5/(c*x**2+b*x+a)**4,x)
```

output
```
10*a**2*b*sqrt(-1/(4*a*c - b**2)**7)*log(x + (-2560*a**6*b*c**4*sqrt(-1/(4
*a*c - b**2)**7) + 2560*a**5*b**3*c**3*sqrt(-1/(4*a*c - b**2)**7) - 960*a*
*4*b**5*c**2*sqrt(-1/(4*a*c - b**2)**7) + 160*a**3*b**7*c*sqrt(-1/(4*a*c -
 b**2)**7) - 10*a**2*b**9*sqrt(-1/(4*a*c - b**2)**7) + 10*a**2*b**2)/(20*a
**2*b*c)) - 10*a**2*b*sqrt(-1/(4*a*c - b**2)**7)*log(x + (2560*a**6*b*c**4
*sqrt(-1/(4*a*c - b**2)**7) - 2560*a**5*b**3*c**3*sqrt(-1/(4*a*c - b**2)**
7) + 960*a**4*b**5*c**2*sqrt(-1/(4*a*c - b**2)**7) - 160*a**3*b**7*c*sqrt(
-1/(4*a*c - b**2)**7) + 10*a**2*b**9*sqrt(-1/(4*a*c - b**2)**7) + 10*a**2*
b**2)/(20*a**2*b*c)) + (-64*a**5*c**2 - 18*a**4*b**2*c + a**3*b**4 - 60*a*
*2*b*c**4*x**5 + x**4*(-192*a**3*c**4 - 6*a**2*b**2*c**3 - 36*a*b**4*c**2
+ 3*b**6*c) + x**3*(-224*a**3*b*c**3 - 62*a**2*b**3*c**2 - 12*a*b**5*c + b
**7) + x**2*(-192*a**4*c**3 - 96*a**3*b**2*c**2 - 51*a**2*b**4*c + 3*a*b**
6) + x*(-132*a**4*b*c**2 - 54*a**3*b**3*c + 3*a**2*b**5))/(384*a**6*c**5 -
 288*a**5*b**2*c**4 + 72*a**4*b**4*c**3 - 6*a**3*b**6*c**2 + x**6*(384*a**
3*c**8 - 288*a**2*b**2*c**7 + 72*a*b**4*c**6 - 6*b**6*c**5) + x**5*(1152*a
**3*b*c**7 - 864*a**2*b**3*c**6 + 216*a*b**5*c**5 - 18*b**7*c**4) + x**4*(
1152*a**4*c**7 + 288*a**3*b**2*c**6 - 648*a**2*b**4*c**5 + 198*a*b**6*c**4
 - 18*b**8*c**3) + x**3*(2304*a**4*b*c**6 - 1344*a**3*b**3*c**5 + 144*a**2
*b**5*c**4 + 36*a*b**7*c**3 - 6*b**9*c**2) + x**2*(1152*a**5*c**6 + 288*a*
*4*b**2*c**5 - 648*a**3*b**4*c**4 + 198*a**2*b**6*c**3 - 18*a*b**8*c**2...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx+cx^2)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(146) = 292$.

Time = 0.24 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.12

$$\int \frac{x^5}{(a+bx+cx^2)^4}\,dx = \frac{20\,a^2b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3)\sqrt{-b^2+4\,ac}} + \frac{60\,a^2bc^4x^5-3\,b^6cx^4+36\,ab^4c^2x^4+6\,a^2b^2c^3x^4+192\,a^3c^4x^4-b^7x^3+12\,ab^5cx^3+62\,a^2b^3c^2x^3+224\,a^3}{6\,(b^6c^2-12\,ab^4c^3}$$

input
```
integrate(x^5/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

output
```
20*a^2*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^6 - 12*a*b^4*c + 48*a^
2*b^2*c^2 - 64*a^3*c^3)*sqrt(-b^2 + 4*a*c)) + 1/6*(60*a^2*b*c^4*x^5 - 3*b^
6*c*x^4 + 36*a*b^4*c^2*x^4 + 6*a^2*b^2*c^3*x^4 + 192*a^3*c^4*x^4 - b^7*x^3
 + 12*a*b^5*c*x^3 + 62*a^2*b^3*c^2*x^3 + 224*a^3*b*c^3*x^3 - 3*a*b^6*x^2 +
 51*a^2*b^4*c*x^2 + 96*a^3*b^2*c^2*x^2 + 192*a^4*c^3*x^2 - 3*a^2*b^5*x + 5
4*a^3*b^3*c*x + 132*a^4*b*c^2*x - a^3*b^4 + 18*a^4*b^2*c + 64*a^5*c^2)/((b
^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*(c*x^2 + b*x + a)^3)
```

Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 563, normalized size of antiderivative = 3.66

$$\int \frac{x^5}{(a+bx+cx^2)^4}\,dx = \frac{\frac{a^3\,(64\,a^2\,c^2+18\,a\,b^2\,c-b^4)}{6\,c^2\,(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}+\frac{x^4\,(64\,a^3\,c^3+2\,a^2\,b^2\,c^2+12\,a\,b^4\,c-b^6)}{2\,c\,(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}+\frac{10\,a^2\,b\,c^2\,x^5}{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}+\frac{a\,x^2\,(64\,a^3}{2\,c^2\,(-64\,a^3}}{x^2\,(3\,c\,a^2+3\,a\,b^2)+x^4\,(3\,b^2\,c+3\,a\,c^2)+a^3+x^3\,(b^3} + \frac{20\,a^2\,b\,\mathrm{atan}\left(\frac{\left(\frac{10\,a^2\,b^2}{(4\,a\,c-b^2)^{7/2}}+\frac{20\,a^2\,b\,c\,x}{(4\,a\,c-b^2)^{7/2}}\right)(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}{10\,a^2\,b}\right)}{(4\,a\,c-b^2)^{7/2}}$$

input
```
int(x^5/(a + b*x + c*x^2)^4,x)
```

output
```
((a^3*(64*a^2*c^2 - b^4 + 18*a*b^2*c))/(6*c^2*(b^6 - 64*a^3*c^3 + 48*a^2*b
^2*c^2 - 12*a*b^4*c)) + (x^4*(64*a^3*c^3 - b^6 + 2*a^2*b^2*c^2 + 12*a*b^4*
c))/(2*c*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (10*a^2*b*c^2
*x^5)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (a*x^2*(64*a^3*c^
3 - b^6 + 32*a^2*b^2*c^2 + 17*a*b^4*c))/(2*c^2*(b^6 - 64*a^3*c^3 + 48*a^2*
b^2*c^2 - 12*a*b^4*c)) + (b*x^3*(224*a^3*c^3 - b^6 + 62*a^2*b^2*c^2 + 12*a
*b^4*c))/(6*c^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (a^2*b
*x*(44*a^2*c^2 - b^4 + 18*a*b^2*c))/(2*c^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*
c^2 - 12*a*b^4*c)))/(x^2*(3*a*b^2 + 3*a^2*c) + x^4*(3*a*c^2 + 3*b^2*c) + a
^3 + x^3*(b^3 + 6*a*b*c) + c^3*x^6 + 3*b*c^2*x^5 + 3*a^2*b*x) + (20*a^2*b*
atan((((10*a^2*b^2)/(4*a*c - b^2)^(7/2) + (20*a^2*b*c*x)/(4*a*c - b^2)^(7/
2))*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))/(10*a^2*b)))/(4*a*c
- b^2)^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1136, normalized size of antiderivative = 7.38

$$\int \frac{x^5}{(a+bx+cx^2)^4} \, dx = \text{Too large to display}$$

input
```
int(x^5/(c*x^2+b*x+a)^4,x)
```

output
```
( - 120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**5*b*c**
2 - 360*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b**2*
c**2*x - 360*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*
b*c**3*x**2 - 360*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*
a**3*b**3*c**2*x**2 - 720*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c -
 b**2))*a**3*b**2*c**3*x**3 - 360*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt
(4*a*c - b**2))*a**3*b*c**4*x**4 - 120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)
/sqrt(4*a*c - b**2))*a**2*b**4*c**2*x**3 - 360*sqrt(4*a*c - b**2)*atan((b
+ 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3*c**3*x**4 - 360*sqrt(4*a*c - b**2)*
atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2*c**4*x**5 - 120*sqrt(4*a*c
- b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c**5*x**6 - 176*a**6*c
**3 - 28*a**5*b**2*c**2 - 288*a**5*b*c**3*x - 528*a**5*c**4*x**2 + 22*a**4
*b**4*c - 144*a**4*b**3*c**2*x - 12*a**4*b**2*c**3*x**2 - 416*a**4*b*c**4*
x**3 - 528*a**4*c**5*x**4 - a**3*b**6 + 66*a**3*b**5*c*x - 168*a**3*b**4*c
**2*x**2 - 64*a**3*b**3*c**3*x**3 + 348*a**3*b**2*c**4*x**4 + 80*a**3*c**6
*x**6 - 3*a**2*b**7*x + 63*a**2*b**6*c*x**2 - 6*a**2*b**5*c**2*x**3 - 198*
a**2*b**4*c**3*x**4 - 20*a**2*b**2*c**5*x**6 - 3*a*b**8*x**2 + 16*a*b**7*c
*x**3 + 48*a*b**6*c**2*x**4 - b**9*x**3 - 3*b**8*c*x**4)/(6*c**2*(256*a**7
*c**4 - 256*a**6*b**2*c**3 + 768*a**6*b*c**4*x + 768*a**6*c**5*x**2 + 96*a
**5*b**4*c**2 - 768*a**5*b**3*c**3*x + 1536*a**5*b*c**5*x**3 + 768*a**5...
```

3.259 $\int \frac{x^4}{(a+bx+cx^2)^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 182

$$\int \frac{x^4}{(a+bx+cx^2)^4}\,dx = \frac{x^3(2a+bx)}{3(b^2-4ac)(a+bx+cx^2)^3} - \frac{x(a(b^2+6ac)+b(b^2+ac)x)}{3c(b^2-4ac)^2(a+bx+cx^2)^2} - \frac{a(b(b^2+26ac)+12c(b^2+ac)x)}{3c(b^2-4ac)^3(a+bx+cx^2)} + \frac{8a(b^2+ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{7/2}}$$

output
```
1/3*x^3*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^3-1/3*x*(a*(6*a*c+b^2)+b*(a*c
+b^2)*x)/c/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^2-1/3*a*(b*(26*a*c+b^2)+12*c*(a*c+
b^2)*x)/c/(-4*a*c+b^2)^3/(c*x^2+b*x+a)+8*a*(a*c+b^2)*arctanh((2*c*x+b)/(-4
*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(7/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.21

$$\int \frac{x^4}{(a+bx+cx^2)^4}\,dx = \frac{1}{3}\left(\frac{b^5-7ab^3c+17a^2bc^2-b^4cx+10ab^2c^2x-14a^2c^3x}{c^3(b^2-4ac)^2(a+x(b+cx))^2} + \frac{6a(b^2+ac)(b+2cx)}{c(-b^2+4ac)^3(a+x(b+cx))} + \frac{b^4x+ab^2(b-4cx)+a^2c(-3b+2cx)}{c^3(-b^2+4ac)(a+x(b+cx))^3} + \frac{24a(b^2+ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{7/2}}\right)$$

input
```
Integrate[x^4/(a + b*x + c*x^2)^4,x]
```

output
```
((b^5 - 7*a*b^3*c + 17*a^2*b*c^2 - b^4*c*x + 10*a*b^2*c^2*x - 14*a^2*c^3*x
)/(c^3*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^2) + (6*a*(b^2 + a*c)*(b + 2*c*x)
)/(c*(-b^2 + 4*a*c)^3*(a + x*(b + c*x))) + (b^4*x + a*b^2*(b - 4*c*x) + a^
2*c*(-3*b + 2*c*x))/(c^3*(-b^2 + 4*a*c)*(a + x*(b + c*x))^3) + (24*a*(b^2
+ a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(7/2))/3
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1163, 27, 1227, 1153, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx+cx^2)^4}\,dx$$

$$\Big\downarrow \; 1163$$

$$\frac{\int \frac{2x^3(2b-cx)}{(cx^2+bx+a)^3}dx}{3(b^2-4ac)}-\frac{x^4(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^3}$$

$\Big\downarrow$ 27

$$\frac{2\int \frac{x^3(2b-cx)}{(cx^2+bx+a)^3}dx}{3(b^2-4ac)}-\frac{x^4(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^3}$$

$\Big\downarrow$ 1227

$$\frac{2\left(\frac{3(ac+b^2)\int \frac{x^2}{(cx^2+bx+a)^2}dx}{b^2-4ac}-\frac{x^3(2(ac+b^2)+5bcx)}{2(b^2-4ac)(a+bx+cx^2)^2}\right)}{3(b^2-4ac)}-\frac{x^4(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^3}$$

$\Big\downarrow$ 1153

$$\frac{2\left(\frac{3(ac+b^2)\left(\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{2a\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}\right)}{b^2-4ac}-\frac{x^3(2(ac+b^2)+5bcx)}{2(b^2-4ac)(a+bx+cx^2)^2}\right)}{3(b^2-4ac)}-\frac{x^4(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^3}$$

$\Big\downarrow$ 1083

$$\frac{2\left(\frac{3(ac+b^2)\left(\frac{4a\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}+\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}-\frac{x^3(2(ac+b^2)+5bcx)}{2(b^2-4ac)(a+bx+cx^2)^2}\right)}{3(b^2-4ac)}-\frac{x^4(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^3}$$

$\Big\downarrow$ 219

$$\frac{2\left(\frac{3(ac+b^2)\left(\frac{4a\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}-\frac{x^3(2(ac+b^2)+5bcx)}{2(b^2-4ac)(a+bx+cx^2)^2}\right)}{3(b^2-4ac)}-\frac{x^4(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^3}$$

input
```
Int[x^4/(a + b*x + c*x^2)^4,x]
```

output
```
-1/3*(x^4*(b + 2*c*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)^3) + (2*(-1/2*(x^3
*(2*(b^2 + a*c) + 5*b*c*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*(b^2
+ a*c)*((x*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (4*a*ArcTanh[(
b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/(b^2 - 4*a*c)))/(3*(b
^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1153
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m - 2)*(a + b*x +
c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
&& LtQ[p, -1]
```

rule 1163
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] - Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1
)*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p +
1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 0] && (LtQ[
m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c, d,
e, m, p, x]
```

rule 1227
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(
(b*f - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[m*((b*(
e*f + d*g) - 2*(c*d*f + a*e*g))/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m
 - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
&& EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(172) = 344$.

Time = 0.79 (sec) , antiderivative size = 438, normalized size of antiderivative = 2.41

method	result
default	$\frac{\frac{4a\left(ac+b^{2}\right)c^{2}x^{5}}{64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12a\,b^{4}c-b^{6}}+\frac{10a\left(ac+b^{2}\right)bc\,x^{4}}{64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12a\,b^{4}c-b^{6}}-\frac{\left(32a^{3}c^{3}-102a^{2}b^{2}c^{2}-10a\,b^{4}c-b^{6}\right)x^{3}}{3c\left(64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12a\,b^{4}c-b^{6}\right)}+\frac{b\left(16ac+b^{2}\right)a\left(ac+b^{2}\right)x^{2}}{c\left(64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12a\,b^{4}c-b^{6}\right)}}{\left(c\,x^{2}+bx+a\right)^{3}}$
risch	$\frac{\frac{4a\left(ac+b^{2}\right)c^{2}x^{5}}{64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12a\,b^{4}c-b^{6}}+\frac{10a\left(ac+b^{2}\right)bc\,x^{4}}{64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12a\,b^{4}c-b^{6}}-\frac{\left(32a^{3}c^{3}-102a^{2}b^{2}c^{2}-10a\,b^{4}c-b^{6}\right)x^{3}}{3c\left(64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12a\,b^{4}c-b^{6}\right)}+\frac{b\left(16ac+b^{2}\right)a\left(ac+b^{2}\right)x^{2}}{c\left(64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12a\,b^{4}c-b^{6}\right)}}{\left(c\,x^{2}+bx+a\right)^{3}}$

input
```
int(x^4/(c*x^2+b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(4*a*(a*c+b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*c^2*x^5+10*a*(a*
c+b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*b*c*x^4-1/3*(32*a^3*c^3-
102*a^2*b^2*c^2-10*a*b^4*c-b^6)/c/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^
6)*x^3+b*(16*a*c+b^2)*a*(a*c+b^2)/c/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-
b^6)*x^2-a^2*(4*a^2*c^2-22*a*b^2*c-b^4)/c/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*
b^4*c-b^6)*x+1/3*(26*a*c+b^2)*a^3*b/c/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*
c-b^6))/(c*x^2+b*x+a)^3+8*a*(a*c+b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*
c-b^6)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 802 vs. $2(172) = 344$.

Time = 0.11 (sec) , antiderivative size = 1625, normalized size of antiderivative = 8.93

$$\int \frac{x^4}{(a+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

output
```
[-1/3*(a^3*b^5 + 22*a^4*b^3*c - 104*a^5*b*c^2 + 12*(a*b^4*c^3 - 3*a^2*b^2*
c^4 - 4*a^3*c^5)*x^5 + 30*(a*b^5*c^2 - 3*a^2*b^3*c^3 - 4*a^3*b*c^4)*x^4 +
(b^8 + 6*a*b^6*c + 62*a^2*b^4*c^2 - 440*a^3*b^2*c^3 + 128*a^4*c^4)*x^3 + 3
*(a*b^7 + 13*a^2*b^5*c - 52*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2 + 12*(a^4*b^2*
c + a^5*c^2 + (a*b^2*c^4 + a^2*c^5)*x^6 + 3*(a*b^3*c^3 + a^2*b*c^4)*x^5 +
3*(a*b^4*c^2 + 2*a^2*b^2*c^3 + a^3*c^4)*x^4 + (a*b^5*c + 7*a^2*b^3*c^2 + 6
*a^3*b*c^3)*x^3 + 3*(a^2*b^4*c + 2*a^3*b^2*c^2 + a^4*c^3)*x^2 + 3*(a^3*b^3
*c + a^4*b*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*
c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 3*(a^2*b^6 + 18*a^
3*b^4*c - 92*a^4*b^2*c^2 + 16*a^5*c^3)*x)/(a^3*b^8*c - 16*a^4*b^6*c^2 + 96
*a^5*b^4*c^3 - 256*a^6*b^2*c^4 + 256*a^7*c^5 + (b^8*c^4 - 16*a*b^6*c^5 + 9
6*a^2*b^4*c^6 - 256*a^3*b^2*c^7 + 256*a^4*c^8)*x^6 + 3*(b^9*c^3 - 16*a*b^7
*c^4 + 96*a^2*b^5*c^5 - 256*a^3*b^3*c^6 + 256*a^4*b*c^7)*x^5 + 3*(b^10*c^2
 - 15*a*b^8*c^3 + 80*a^2*b^6*c^4 - 160*a^3*b^4*c^5 + 256*a^5*c^7)*x^4 + (b
^11*c - 10*a*b^9*c^2 + 320*a^3*b^5*c^4 - 1280*a^4*b^3*c^5 + 1536*a^5*b*c^6
)*x^3 + 3*(a*b^10*c - 15*a^2*b^8*c^2 + 80*a^3*b^6*c^3 - 160*a^4*b^4*c^4 +
256*a^6*c^6)*x^2 + 3*(a^2*b^9*c - 16*a^3*b^7*c^2 + 96*a^4*b^5*c^3 - 256*a^
5*b^3*c^4 + 256*a^6*b*c^5)*x), -1/3*(a^3*b^5 + 22*a^4*b^3*c - 104*a^5*b*c^
2 + 12*(a*b^4*c^3 - 3*a^2*b^2*c^4 - 4*a^3*c^5)*x^5 + 30*(a*b^5*c^2 - 3*a^2
*b^3*c^3 - 4*a^3*b*c^4)*x^4 + (b^8 + 6*a*b^6*c + 62*a^2*b^4*c^2 - 440*a...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 957 vs. $2(168) = 336$.

Time = 1.24 (sec) , antiderivative size = 957, normalized size of antiderivative = 5.26

$$\int \frac{x^4}{(a+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x**4/(c*x**2+b*x+a)**4,x)
```

output
```
-4*a*sqrt(-1/(4*a*c - b**2)**7)*(a*c + b**2)*log(x + (-1024*a**5*c**4*sqrt
(-1/(4*a*c - b**2)**7)*(a*c + b**2) + 1024*a**4*b**2*c**3*sqrt(-1/(4*a*c -
 b**2)**7)*(a*c + b**2) - 384*a**3*b**4*c**2*sqrt(-1/(4*a*c - b**2)**7)*(a
*c + b**2) + 64*a**2*b**6*c*sqrt(-1/(4*a*c - b**2)**7)*(a*c + b**2) + 4*a*
*2*b*c - 4*a*b**8*sqrt(-1/(4*a*c - b**2)**7)*(a*c + b**2) + 4*a*b**3)/(8*a
**2*c**2 + 8*a*b**2*c)) + 4*a*sqrt(-1/(4*a*c - b**2)**7)*(a*c + b**2)*log(
x + (1024*a**5*c**4*sqrt(-1/(4*a*c - b**2)**7)*(a*c + b**2) - 1024*a**4*b*
*2*c**3*sqrt(-1/(4*a*c - b**2)**7)*(a*c + b**2) + 384*a**3*b**4*c**2*sqrt(
-1/(4*a*c - b**2)**7)*(a*c + b**2) - 64*a**2*b**6*c*sqrt(-1/(4*a*c - b**2)
**7)*(a*c + b**2) + 4*a**2*b*c + 4*a*b**8*sqrt(-1/(4*a*c - b**2)**7)*(a*c
+ b**2) + 4*a*b**3)/(8*a**2*c**2 + 8*a*b**2*c)) + (26*a**4*b*c + a**3*b**3
 + x**5*(12*a**2*c**4 + 12*a*b**2*c**3) + x**4*(30*a**2*b*c**3 + 30*a*b**3
*c**2) + x**3*(-32*a**3*c**3 + 102*a**2*b**2*c**2 + 10*a*b**4*c + b**6) +
x**2*(48*a**3*b*c**2 + 51*a**2*b**3*c + 3*a*b**5) + x*(-12*a**4*c**2 + 66*
a**3*b**2*c + 3*a**2*b**4))/(192*a**6*c**4 - 144*a**5*b**2*c**3 + 36*a**4*
b**4*c**2 - 3*a**3*b**6*c + x**6*(192*a**3*c**7 - 144*a**2*b**2*c**6 + 36*
a*b**4*c**5 - 3*b**6*c**4) + x**5*(576*a**3*b*c**6 - 432*a**2*b**3*c**5 +
108*a*b**5*c**4 - 9*b**7*c**3) + x**4*(576*a**4*c**6 + 144*a**3*b**2*c**5
- 324*a**2*b**4*c**4 + 99*a*b**6*c**3 - 9*b**8*c**2) + x**3*(1152*a**4*b*c
**5 - 672*a**3*b**3*c**4 + 72*a**2*b**5*c**3 + 18*a*b**7*c**2 - 3*b**9*...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(a+bx+cx^2)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.61

$$\int \frac{x^4}{(a+bx+cx^2)^4}\,dx = -\frac{8\,(ab^2+a^2c)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3)\sqrt{-b^2+4\,ac}} - \frac{12\,ab^2c^3x^5+12\,a^2c^4x^5+30\,ab^3c^2x^4+30\,a^2bc^3x^4+b^6x^3+10\,ab^4cx^3+102\,a^2b^2c^2x^3-32\,a^3c^3x^3+3\,a}{3\,(b^6c-12\,ab^4c^2+48\,a^2b^2c^3-64\,a^3c}$$

input
```
integrate(x^4/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

output
```
-8*(a*b^2 + a^2*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^6 - 12*a*b^4
*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-b^2 + 4*a*c)) - 1/3*(12*a*b^2*c^3*
x^5 + 12*a^2*c^4*x^5 + 30*a*b^3*c^2*x^4 + 30*a^2*b*c^3*x^4 + b^6*x^3 + 10*
a*b^4*c*x^3 + 102*a^2*b^2*c^2*x^3 - 32*a^3*c^3*x^3 + 3*a*b^5*x^2 + 51*a^2*
b^3*c*x^2 + 48*a^3*b*c^2*x^2 + 3*a^2*b^4*x + 66*a^3*b^2*c*x - 12*a^4*c^2*x
 + a^3*b^3 + 26*a^4*b*c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*
c^4)*(c*x^2 + b*x + a)^3)
```

Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 588, normalized size of antiderivative = 3.23

$$\int \frac{x^4}{(a+bx+cx^2)^4}\,dx = -\frac{\frac{x^3\,(-32\,a^3\,c^3+102\,a^2\,b^2\,c^2+10\,a\,b^4\,c+b^6)}{3\,c\,(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}+\frac{a^3\,(b^3+26\,a\,c\,b)}{3\,c\,(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}+\frac{a^2\,x\,(-4\,a^2\,c^2+22\,a\,b^2\,c+b^4)}{c\,(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}+\frac{}{-64}}{x^2\,(3\,c\,a^2+3\,a\,b^2)+x^4\,(3\,b^2\,c+3\,a\,c^2)+a^3+x^3\,(b^3}$$
$$-\frac{8\,a\,\mathrm{atan}\left(\frac{\left(\frac{4\,a\,(b^2+a\,c)\,(-64\,a^3\,b\,c^3+48\,a^2\,b^3\,c^2-12\,a\,b^5\,c+b^7)}{(4\,a\,c-b^2)^{7/2}\,(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}+\frac{8\,a\,c\,x\,(b^2+a\,c)}{(4\,a\,c-b^2)^{7/2}}\right)(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}{4\,c\,a^2+4\,a\,b^2}\right)(b^2+a\,c)}{(4\,a\,c-b^2)^{7/2}}$$

input
```
int(x^4/(a + b*x + c*x^2)^4,x)
```

output
```
- ((x^3*(b^6 - 32*a^3*c^3 + 102*a^2*b^2*c^2 + 10*a*b^4*c))/(3*c*(b^6 - 64*
a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (a^3*(b^3 + 26*a*b*c))/(3*c*(b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (a^2*x*(b^4 - 4*a^2*c^2 +
22*a*b^2*c))/(c*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (4*a*c
^2*x^5*(a*c + b^2))/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (a*
x^2*(a*c + b^2)*(b^3 + 16*a*b*c))/(c*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)) + (10*a*b*c*x^4*(a*c + b^2))/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c
^2 - 12*a*b^4*c))/(x^2*(3*a*b^2 + 3*a^2*c) + x^4*(3*a*c^2 + 3*b^2*c) + a^3
 + x^3*(b^3 + 6*a*b*c) + c^3*x^6 + 3*b*c^2*x^5 + 3*a^2*b*x) - (8*a*atan(((
(4*a*(a*c + b^2)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/((4*a
*c - b^2)^(7/2)*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (8*a*c
*x*(a*c + b^2))/(4*a*c - b^2)^(7/2))*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c))/(4*a*b^2 + 4*a^2*c))*(a*c + b^2))/(4*a*c - b^2)^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1443, normalized size of antiderivative = 7.93

$$\int \frac{x^4}{(a+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
int(x^4/(c*x^2+b*x+a)^4,x)
```

output

```
(24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**5*b*c**2 +
24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b**3*c + 7
2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b**2*c**2*x
 + 72*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b*c**3*
x**2 + 72*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**
4*c*x + 144*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b
**3*c**2*x**2 + 144*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2)
)*a**3*b**2*c**3*x**3 + 72*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
- b**2))*a**3*b*c**4*x**4 + 72*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*
a*c - b**2))*a**2*b**5*c*x**2 + 168*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sq
rt(4*a*c - b**2))*a**2*b**4*c**2*x**3 + 144*sqrt(4*a*c - b**2)*atan((b + 2
*c*x)/sqrt(4*a*c - b**2))*a**2*b**3*c**3*x**4 + 72*sqrt(4*a*c - b**2)*atan
((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2*c**4*x**5 + 24*sqrt(4*a*c - b**
2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c**5*x**6 + 24*sqrt(4*a*c -
 b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**6*c*x**3 + 72*sqrt(4*a*c
- b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**5*c**2*x**4 + 72*sqrt(4*
a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**4*c**3*x**5 + 24*sqr
t(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3*c**4*x**6 - 16
*a**6*c**3 + 92*a**5*b**2*c**2 - 96*a**5*b*c**3*x - 48*a**5*c**4*x**2 - 18
*a**4*b**4*c + 240*a**4*b**3*c**2*x + 108*a**4*b**2*c**3*x**2 - 224*a**...
```

3.260 $\int \frac{x^3}{(a+bx+cx^2)^4} dx$

Optimal result

Integrand size = 16, antiderivative size = 175

$$\int \frac{x^3}{(a+bx+cx^2)^4} dx = \frac{x^2(2a+bx)}{3(b^2-4ac)(a+bx+cx^2)^3} + \frac{x(10ab+(3b^2+8ac)x)}{6(b^2-4ac)^2(a+bx+cx^2)^2} + \frac{a(11b^2+16ac)+3b(b^2+6ac)x}{3(b^2-4ac)^3(a+bx+cx^2)} - \frac{2b(b^2+6ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{7/2}}$$

output
```
1/3*x^2*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^3+1/6*x*(10*a*b+(8*a*c+3*b^2)
*x)/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^2+1/3*(a*(16*a*c+11*b^2)+3*b*(6*a*c+b^2)*
x)/(-4*a*c+b^2)^3/(c*x^2+b*x+a)-2*b*(6*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+
b^2)^(1/2))/(-4*a*c+b^2)^(7/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.18

$$\int \frac{x^3}{(a+bx+cx^2)^4}\,dx = \frac{1}{6}\left(\frac{4a^2c-2b^3x-2ab(b-3cx)}{c^2(-b^2+4ac)(a+x(b+cx))^3} - \frac{2b^4-9ab^2c+24a^2c^2+b^3cx+6abc^2x}{c^2(b^2-4ac)^2(a+x(b+cx))^2} - \frac{3b(b^2+6ac)(b+2cx)}{c(-b^2+4ac)^3(a+x(b+cx))} - \frac{12b(b^2+6ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{7/2}}\right)$$

input
```
Integrate[x^3/(a + b*x + c*x^2)^4,x]
```

output
```
((4*a^2*c - 2*b^3*x - 2*a*b*(b - 3*c*x))/(c^2*(-b^2 + 4*a*c)*(a + x*(b + c
*x))^3) - (2*b^4 - 9*a*b^2*c + 24*a^2*c^2 + b^3*c*x + 6*a*b*c^2*x)/(c^2*(b
^2 - 4*a*c)^2*(a + x*(b + c*x))^2) - (3*b*(b^2 + 6*a*c)*(b + 2*c*x))/(c*(-
b^2 + 4*a*c)^3*(a + x*(b + c*x))) - (12*b*(b^2 + 6*a*c)*ArcTan[(b + 2*c*x)
/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(7/2))/6
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1163, 1234, 27, 1224, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx+cx^2)^4}\,dx$$

$$\Big\downarrow \; 1163$$

$$\frac{\int \frac{x^2(3b-4cx)}{(cx^2+bx+a)^3}dx}{3(b^2-4ac)}-\frac{x^3(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^3}$$

$$\Big\downarrow \text{1234}$$

$$\frac{-\frac{\int -\frac{2x(3b^2-5cxb+8ac)}{(cx^2+bx+a)^2}dx}{2(b^2-4ac)}-\frac{x^2(8ac+3b^2+10bcx)}{2(b^2-4ac)(a+bx+cx^2)^2}}{3(b^2-4ac)}-\frac{x^3(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{x(3b^2-5cxb+8ac)}{(cx^2+bx+a)^2}dx}{b^2-4ac}-\frac{x^2(8ac+3b^2+10bcx)}{2(b^2-4ac)(a+bx+cx^2)^2}}{3(b^2-4ac)}-\frac{x^3(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^3}$$

$$\Big\downarrow \text{1224}$$

$$\frac{\frac{\frac{3b(6ac+b^2)\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}+\frac{2bx(4b^2-ac)+a(16ac+11b^2)}{(b^2-4ac)(a+bx+cx^2)}}{b^2-4ac}-\frac{x^2(8ac+3b^2+10bcx)}{2(b^2-4ac)(a+bx+cx^2)^2}}{3(b^2-4ac)}-\frac{x^3(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^3}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{\frac{2bx(4b^2-ac)+a(16ac+11b^2)}{(b^2-4ac)(a+bx+cx^2)}-\frac{6b(6ac+b^2)\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}}{b^2-4ac}-\frac{x^2(8ac+3b^2+10bcx)}{2(b^2-4ac)(a+bx+cx^2)^2}}{3(b^2-4ac)}-\frac{x^3(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{2bx(4b^2-ac)+a(16ac+11b^2)}{(b^2-4ac)(a+bx+cx^2)}-\frac{6b(6ac+b^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}}{b^2-4ac}-\frac{x^2(8ac+3b^2+10bcx)}{2(b^2-4ac)(a+bx+cx^2)^2}}{3(b^2-4ac)}-\frac{x^3(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^3}$$

input
```
Int[x^3/(a + b*x + c*x^2)^4,x]
```

output
```
-1/3*(x^3*(b + 2*c*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)^3) + (-1/2*(x^2*(3
*b^2 + 8*a*c + 10*b*c*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + ((a*(11*b^
2 + 16*a*c) + 2*b*(4*b^2 - a*c)*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (6*
b*(b^2 + 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2
))/(b^2 - 4*a*c))/(3*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1163
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] - Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1
)*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p +
1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 0] && (LtQ[
m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c, d,
e, m, p, x]
```

rule 1224
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c))   Int[(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -
1] &&  !(IntegerQ[p] && NeQ[a, 0] && NiceSqrtQ[b^2 - 4*a*c])
```

rule 1234
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(
(f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p +
 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*Simp[g
*(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*
(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1
] && GtQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(165) = 330$.

Time = 0.84 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.35

method	result
default	$\frac{-\frac{b\left(6ac+b^{2}\right)c^{2}x^{5}}{64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12a\,b^{4}c-b^{6}}-\frac{5b^{2}\left(6ac+b^{2}\right)c\,x^{4}}{2\left(64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12a\,b^{4}c-b^{6}\right)}-\frac{\left(16ac+11b^{2}\right)b\left(6ac+b^{2}\right)x^{3}}{6\left(64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12a\,b^{4}c-b^{6}\right)}-\frac{a\left(32a^{2}c^{2}+24ca\,b^{2}+17b^{4}\right)x^{2}}{2\left(64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12a\,b^{4}c-b\right.}}{\left(c\,x^{2}+bx+a\right)^{3}}$
risch	$\frac{-\frac{b\left(6ac+b^{2}\right)c^{2}x^{5}}{64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12a\,b^{4}c-b^{6}}-\frac{5b^{2}\left(6ac+b^{2}\right)c\,x^{4}}{2\left(64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12a\,b^{4}c-b^{6}\right)}-\frac{\left(16ac+11b^{2}\right)b\left(6ac+b^{2}\right)x^{3}}{6\left(64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12a\,b^{4}c-b^{6}\right)}-\frac{a\left(32a^{2}c^{2}+24ca\,b^{2}+17b^{4}\right)x^{2}}{2\left(64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12a\,b^{4}c-b\right.}}{\left(c\,x^{2}+bx+a\right)^{3}}$

input
```
int(x^3/(c*x^2+b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(-b*(6*a*c+b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*c^2*x^5-5/2*b^2
*(6*a*c+b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*c*x^4-1/6*(16*a*c+
11*b^2)*b*(6*a*c+b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^3-1/2*a
*(32*a^2*c^2+24*a*b^2*c+17*b^4)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)
*x^2-10*a^2*b*(a*c+b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x-1/3*(
16*a*c+11*b^2)*a^3/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6))/(c*x^2+b*x+
a)^3-2*b*(6*a*c+b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(165) = 330$.

Time = 0.12 (sec) , antiderivative size = 1522, normalized size of antiderivative = 8.70

$$\int \frac{x^3}{(a+bx+cx^2)^4} \, dx = \text{Too large to display}$$

input
```
integrate(x^3/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

output
```
[1/6*(22*a^3*b^4 - 56*a^4*b^2*c - 128*a^5*c^2 + 6*(b^5*c^2 + 2*a*b^3*c^3 -
 24*a^2*b*c^4)*x^5 + 15*(b^6*c + 2*a*b^4*c^2 - 24*a^2*b^2*c^3)*x^4 + (11*b
^7 + 38*a*b^5*c - 232*a^2*b^3*c^2 - 384*a^3*b*c^3)*x^3 + 3*(17*a*b^6 - 44*
a^2*b^4*c - 64*a^3*b^2*c^2 - 128*a^4*c^3)*x^2 - 6*((b^3*c^3 + 6*a*b*c^4)*x
^6 + a^3*b^3 + 6*a^4*b*c + 3*(b^4*c^2 + 6*a*b^2*c^3)*x^5 + 3*(b^5*c + 7*a*
b^3*c^2 + 6*a^2*b*c^3)*x^4 + (b^6 + 12*a*b^4*c + 36*a^2*b^2*c^2)*x^3 + 3*(
a*b^5 + 7*a^2*b^3*c + 6*a^3*b*c^2)*x^2 + 3*(a^2*b^4 + 6*a^3*b^2*c)*x)*sqrt
(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(
2*c*x + b))/(c*x^2 + b*x + a)) + 60*(a^2*b^5 - 3*a^3*b^3*c - 4*a^4*b*c^2)*
x)/(a^3*b^8 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3 + 256*a^7*c^
4 + (b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6 + 256*a^4*c
^7)*x^6 + 3*(b^9*c^2 - 16*a*b^7*c^3 + 96*a^2*b^5*c^4 - 256*a^3*b^3*c^5 + 2
56*a^4*b*c^6)*x^5 + 3*(b^10*c - 15*a*b^8*c^2 + 80*a^2*b^6*c^3 - 160*a^3*b^
4*c^4 + 256*a^5*c^6)*x^4 + (b^11 - 10*a*b^9*c + 320*a^3*b^5*c^3 - 1280*a^4
*b^3*c^4 + 1536*a^5*b*c^5)*x^3 + 3*(a*b^10 - 15*a^2*b^8*c + 80*a^3*b^6*c^2
 - 160*a^4*b^4*c^3 + 256*a^6*c^5)*x^2 + 3*(a^2*b^9 - 16*a^3*b^7*c + 96*a^4
*b^5*c^2 - 256*a^5*b^3*c^3 + 256*a^6*b*c^4)*x), 1/6*(22*a^3*b^4 - 56*a^4*b
^2*c - 128*a^5*c^2 + 6*(b^5*c^2 + 2*a*b^3*c^3 - 24*a^2*b*c^4)*x^5 + 15*(b^
6*c + 2*a*b^4*c^2 - 24*a^2*b^2*c^3)*x^4 + (11*b^7 + 38*a*b^5*c - 232*a^2*b
^3*c^2 - 384*a^3*b*c^3)*x^3 + 3*(17*a*b^6 - 44*a^2*b^4*c - 64*a^3*b^2*c...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(163) = 326$.

Time = 1.11 (sec) , antiderivative size = 925, normalized size of antiderivative = 5.29

$$\int \frac{x^3}{(a+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x**3/(c*x**2+b*x+a)**4,x)
```

output
```
b*sqrt(-1/(4*a*c - b**2)**7)*(6*a*c + b**2)*log(x + (-256*a**4*b*c**4*sqrt
(-1/(4*a*c - b**2)**7)*(6*a*c + b**2) + 256*a**3*b**3*c**3*sqrt(-1/(4*a*c
- b**2)**7)*(6*a*c + b**2) - 96*a**2*b**5*c**2*sqrt(-1/(4*a*c - b**2)**7)*
(6*a*c + b**2) + 16*a*b**7*c*sqrt(-1/(4*a*c - b**2)**7)*(6*a*c + b**2) + 6
*a*b**2*c - b**9*sqrt(-1/(4*a*c - b**2)**7)*(6*a*c + b**2) + b**4)/(12*a*b
*c**2 + 2*b**3*c)) - b*sqrt(-1/(4*a*c - b**2)**7)*(6*a*c + b**2)*log(x + (
256*a**4*b*c**4*sqrt(-1/(4*a*c - b**2)**7)*(6*a*c + b**2) - 256*a**3*b**3*
c**3*sqrt(-1/(4*a*c - b**2)**7)*(6*a*c + b**2) + 96*a**2*b**5*c**2*sqrt(-1
/(4*a*c - b**2)**7)*(6*a*c + b**2) - 16*a*b**7*c*sqrt(-1/(4*a*c - b**2)**7
)*(6*a*c + b**2) + 6*a*b**2*c + b**9*sqrt(-1/(4*a*c - b**2)**7)*(6*a*c + b
**2) + b**4)/(12*a*b*c**2 + 2*b**3*c)) + (-32*a**4*c - 22*a**3*b**2 + x**5
*(-36*a*b*c**3 - 6*b**3*c**2) + x**4*(-90*a*b**2*c**2 - 15*b**4*c) + x**3*
(-96*a**2*b*c**2 - 82*a*b**3*c - 11*b**5) + x**2*(-96*a**3*c**2 - 72*a**2*
b**2*c - 51*a*b**4) + x*(-60*a**3*b*c - 60*a**2*b**3))/(384*a**6*c**3 - 28
8*a**5*b**2*c**2 + 72*a**4*b**4*c - 6*a**3*b**6 + x**6*(384*a**3*c**6 - 28
8*a**2*b**2*c**5 + 72*a*b**4*c**4 - 6*b**6*c**3) + x**5*(1152*a**3*b*c**5
- 864*a**2*b**3*c**4 + 216*a*b**5*c**3 - 18*b**7*c**2) + x**4*(1152*a**4*c
**5 + 288*a**3*b**2*c**4 - 648*a**2*b**4*c**3 + 198*a*b**6*c**2 - 18*b**8*
c) + x**3*(2304*a**4*b*c**4 - 1344*a**3*b**3*c**3 + 144*a**2*b**5*c**2 + 3
6*a*b**7*c - 6*b**9) + x**2*(1152*a**5*c**4 + 288*a**4*b**2*c**3 - 648*...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx+cx^2)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.47

$$\int \frac{x^3}{(a+bx+cx^2)^4}\,dx = \frac{2\,(b^3+6\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3)\sqrt{-b^2+4\,ac}} + \frac{6\,b^3c^2x^5+36\,abc^3x^5+15\,b^4cx^4+90\,ab^2c^2x^4+11\,b^5x^3+82\,ab^3cx^3+96\,a^2bc^2x^3+51\,ab^4x^2+72\,a^2b^2c}{6\,(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3)(cx^2+bx+}$$

input
```
integrate(x^3/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

output
```
2*(b^3 + 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^6 - 12*a*b^4*
c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-b^2 + 4*a*c)) + 1/6*(6*b^3*c^2*x^5
+ 36*a*b*c^3*x^5 + 15*b^4*c*x^4 + 90*a*b^2*c^2*x^4 + 11*b^5*x^3 + 82*a*b^3
*c*x^3 + 96*a^2*b*c^2*x^3 + 51*a*b^4*x^2 + 72*a^2*b^2*c*x^2 + 96*a^3*c^2*x
^2 + 60*a^2*b^3*x + 60*a^3*b*c*x + 22*a^3*b^2 + 32*a^4*c)/((b^6 - 12*a*b^4
*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*(c*x^2 + b*x + a)^3)
```

Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.93

$$\int \frac{x^3}{(a+bx+cx^2)^4}\,dx$$
$$= \frac{\frac{16\,c\,a^4+11\,a^3\,b^2}{3\,(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}+\frac{a\,x^2\,(32\,a^2\,c^2+24\,a\,b^2\,c+17\,b^4)}{2\,(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}+\frac{10\,b\,x\,(c\,a^3+a^2\,b^2)}{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}+\frac{b\,x^3\,(b^2+6}{6\,(-64\,a^3\,c^3+4}}{x^2\,(3\,c\,a^2+3\,a\,b^2)+x^4\,(3\,b^2\,c+3\,a\,c^2)+a^3+x^3\,(b^3+6\,c}$$
$$+\frac{2\,b\,\mathrm{atan}\left(\frac{\left(\frac{b^2\,(b^2+6\,a\,c)}{(4\,a\,c-b^2)^{7/2}}+\frac{2\,b\,c\,x\,(b^2+6\,a\,c)}{(4\,a\,c-b^2)^{7/2}}\right)(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}{b\,(b^2+6\,a\,c)}\right)(b^2+6\,a\,c)}{(4\,a\,c-b^2)^{7/2}}$$

input
```
int(x^3/(a + b*x + c*x^2)^4,x)
```

output
```
((16*a^4*c + 11*a^3*b^2)/(3*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*
c)) + (a*x^2*(17*b^4 + 32*a^2*c^2 + 24*a*b^2*c))/(2*(b^6 - 64*a^3*c^3 + 48
*a^2*b^2*c^2 - 12*a*b^4*c)) + (10*b*x*(a^3*c + a^2*b^2))/(b^6 - 64*a^3*c^3
 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (b*x^3*(6*a*c + b^2)*(16*a*c + 11*b^2))/
(6*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (5*b^2*c*x^4*(6*a*c
 + b^2))/(2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (b*c^2*x^5
*(6*a*c + b^2))/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))/(x^2*(3*
a*b^2 + 3*a^2*c) + x^4*(3*a*c^2 + 3*b^2*c) + a^3 + x^3*(b^3 + 6*a*b*c) + c
^3*x^6 + 3*b*c^2*x^5 + 3*a^2*b*x) + (2*b*atan((((b^2*(6*a*c + b^2))/(4*a*c
 - b^2)^(7/2) + (2*b*c*x*(6*a*c + b^2))/(4*a*c - b^2)^(7/2))*(b^6 - 64*a^3
*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))/(b*(6*a*c + b^2)))*(6*a*c + b^2))/(4*
a*c - b^2)^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1352, normalized size of antiderivative = 7.73

$$\int \frac{x^3}{(a+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
int(x^3/(c*x^2+b*x+a)^4,x)
```

output
```
( - 72*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b*c -
12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**3 - 216
*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**2*c*x - 2
16*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**2*x**
2 - 36*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**4*x
 - 252*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3*c
*x**2 - 432*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b
**2*c**2*x**3 - 216*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2)
)*a**2*b*c**3*x**4 - 36*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b
**2))*a*b**5*x**2 - 144*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b
**2))*a*b**4*c*x**3 - 252*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c -
 b**2))*a*b**3*c**2*x**4 - 216*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*
a*c - b**2))*a*b**2*c**3*x**5 - 72*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqr
t(4*a*c - b**2))*a*b*c**4*x**6 - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sq
rt(4*a*c - b**2))*b**6*x**3 - 36*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(
4*a*c - b**2))*b**5*c*x**4 - 36*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4
*a*c - b**2))*b**4*c**2*x**5 - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt
(4*a*c - b**2))*b**3*c**3*x**6 - 80*a**5*c**2 - 60*a**4*b**2*c - 96*a**4*b
*c**2*x - 240*a**4*c**3*x**2 + 20*a**3*b**4 - 192*a**3*b**3*c*x - 60*a**3*
b**2*c**2*x**2 - 96*a**3*b*c**3*x**3 + 144*a**3*c**4*x**4 + 54*a**2*b**...
```

3.261 $\int \frac{x^2}{(a+bx+cx^2)^4} dx$

Optimal result

Integrand size = 16, antiderivative size = 165

$$\int \frac{x^2}{(a+bx+cx^2)^4} dx = -\frac{ab+(b^2-2ac)x}{3c(b^2-4ac)(a+bx+cx^2)^3} + \frac{(b^2+ac)(b+2cx)}{3c(b^2-4ac)^2(a+bx+cx^2)^2} - \frac{2(b^2+ac)(b+2cx)}{(b^2-4ac)^3(a+bx+cx^2)} + \frac{8c(b^2+ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{7/2}}$$

output

```
-1/3*(a*b+(-2*a*c+b^2)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)^3+1/3*(a*c+b^2)*(2*
c*x+b)/c/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^2-2*(a*c+b^2)*(2*c*x+b)/(-4*a*c+b^2)
^3/(c*x^2+b*x+a)+8*c*(a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a
*c+b^2)^(7/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.01

$$\int \frac{x^2}{(a+bx+cx^2)^4}\,dx = \frac{1}{3}\left(\frac{b^2x+a(b-2cx)}{c(-b^2+4ac)\,(a+x(b+cx))^3} + \frac{(b^2+ac)\,(b+2cx)}{c\,(b^2-4ac)^2\,(a+x(b+cx))^2} - \frac{6(b^2+ac)\,(b+2cx)}{(b^2-4ac)^3\,(a+x(b+cx))} + \frac{24c(b^2+ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{7/2}}\right)$$

input
```
Integrate[x^2/(a + b*x + c*x^2)^4,x]
```

output
```
((b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))^3) + ((b^2 +
a*c)*(b + 2*c*x))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^2) - (6*(b^2 + a*c)
*(b + 2*c*x))/((b^2 - 4*a*c)^3*(a + x*(b + c*x))) + (24*c*(b^2 + a*c)*ArcT
an[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(7/2))/3
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1164, 27, 1159, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx+cx^2)^4}\,dx$$

$$\Big\downarrow \; 1164$$

$$\frac{x(2a+bx)}{3\,(b^2-4ac)\,(a+bx+cx^2)^3} - \frac{\int \frac{2(a-2bx)}{(cx^2+bx+a)^3}dx}{3\,(b^2-4ac)}$$

$$\Big\downarrow \; 27$$

$$\frac{x(2a+bx)}{3(b^2-4ac)(a+bx+cx^2)^3}-\frac{2\int\frac{a-2bx}{(cx^2+bx+a)^3}dx}{3(b^2-4ac)}$$

$\Big\downarrow$ 1159

$$\frac{x(2a+bx)}{3(b^2-4ac)(a+bx+cx^2)^3}-\frac{2\left(-\frac{3(ac+b^2)\int\frac{1}{(cx^2+bx+a)^2}dx}{b^2-4ac}-\frac{2x(ac+b^2)+5ab}{2(b^2-4ac)(a+bx+cx^2)^2}\right)}{3(b^2-4ac)}$$

$\Big\downarrow$ 1086

$$\frac{x(2a+bx)}{3(b^2-4ac)(a+bx+cx^2)^3}-\frac{2\left(-\frac{3(ac+b^2)\left(-\frac{2c\int\frac{1}{cx^2+bx+a}dx}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}-\frac{2x(ac+b^2)+5ab}{2(b^2-4ac)(a+bx+cx^2)^2}\right)}{3(b^2-4ac)}$$

$\Big\downarrow$ 1083

$$\frac{x(2a+bx)}{3(b^2-4ac)(a+bx+cx^2)^3}-\frac{2\left(-\frac{3(ac+b^2)\left(\frac{4c\int\frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}-\frac{2x(ac+b^2)+5ab}{2(b^2-4ac)(a+bx+cx^2)^2}\right)}{3(b^2-4ac)}$$

$\Big\downarrow$ 219

$$\frac{x(2a+bx)}{3(b^2-4ac)(a+bx+cx^2)^3}-\frac{2\left(-\frac{3(ac+b^2)\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}-\frac{2x(ac+b^2)+5ab}{2(b^2-4ac)(a+bx+cx^2)^2}\right)}{3(b^2-4ac)}$$

input
```
Int[x^2/(a + b*x + c*x^2)^4,x]
```

output
```
(x*(2*a + b*x))/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^3) - (2*(-1/2*(5*a*b +
2*(b^2 + a*c)*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (3*(b^2 + a*c)*(-((
b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)/S
qrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/(b^2 - 4*a*c)))/(3*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(157) = 314$.

Time = 0.81 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.48

method	result
default	$\frac{\frac{4c^3(ac+b^2)x^5}{64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6}+\frac{10c^2(ac+b^2)b\,x^4}{64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6}+\frac{2(16ac+11b^2)c(ac+b^2)x^3}{3(64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6)}+\frac{b(16ac+b^2)(ac+b^2)x^2}{64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6}-\cdots}{(c\,x^2+bx+a)^3}$
risch	$\frac{\frac{4c^3(ac+b^2)x^5}{64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6}+\frac{10c^2(ac+b^2)b\,x^4}{64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6}+\frac{2(16ac+11b^2)c(ac+b^2)x^3}{3(64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6)}+\frac{b(16ac+b^2)(ac+b^2)x^2}{64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6}-\cdots}{(c\,x^2+bx+a)^3}$

input
```
int(x^2/(c*x^2+b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(4*c^3*(a*c+b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^5+10*c^2*(a*
c+b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*b*x^4+2/3*(16*a*c+11*b^2
)*c*(a*c+b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^3+b*(16*a*c+b^2
)*(a*c+b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^2-a*(4*a^2*c^2-22
*a*b^2*c-b^4)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x+1/3*a^2*b*(26*a
*c+b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6))/(c*x^2+b*x+a)^3+8*c*(a
*c+b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*arcta
n((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 756 vs. $2(157) = 314$.

Time = 0.12 (sec) , antiderivative size = 1533, normalized size of antiderivative = 9.29

$$\int \frac{x^2}{(a+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

output
```
[-1/3*(a^2*b^5 + 22*a^3*b^3*c - 104*a^4*b*c^2 + 12*(b^4*c^3 - 3*a*b^2*c^4
- 4*a^2*c^5)*x^5 + 30*(b^5*c^2 - 3*a*b^3*c^3 - 4*a^2*b*c^4)*x^4 + 2*(11*b^
6*c - 17*a*b^4*c^2 - 92*a^2*b^2*c^3 - 64*a^3*c^4)*x^3 + 3*(b^7 + 13*a*b^5*
c - 52*a^2*b^3*c^2 - 64*a^3*b*c^3)*x^2 + 12*((b^2*c^4 + a*c^5)*x^6 + a^3*b
^2*c + a^4*c^2 + 3*(b^3*c^3 + a*b*c^4)*x^5 + 3*(b^4*c^2 + 2*a*b^2*c^3 + a^
2*c^4)*x^4 + (b^5*c + 7*a*b^3*c^2 + 6*a^2*b*c^3)*x^3 + 3*(a*b^4*c + 2*a^2*
b^2*c^2 + a^3*c^3)*x^2 + 3*(a^2*b^3*c + a^3*b*c^2)*x)*sqrt(b^2 - 4*a*c)*lo
g((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x
^2 + b*x + a)) + 3*(a*b^6 + 18*a^2*b^4*c - 92*a^3*b^2*c^2 + 16*a^4*c^3)*x)
/(a^3*b^8 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3 + 256*a^7*c^4
+ (b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6 + 256*a^4*c^7
)*x^6 + 3*(b^9*c^2 - 16*a*b^7*c^3 + 96*a^2*b^5*c^4 - 256*a^3*b^3*c^5 + 256
*a^4*b*c^6)*x^5 + 3*(b^10*c - 15*a*b^8*c^2 + 80*a^2*b^6*c^3 - 160*a^3*b^4*
c^4 + 256*a^5*c^6)*x^4 + (b^11 - 10*a*b^9*c + 320*a^3*b^5*c^3 - 1280*a^4*b
^3*c^4 + 1536*a^5*b*c^5)*x^3 + 3*(a*b^10 - 15*a^2*b^8*c + 80*a^3*b^6*c^2 -
 160*a^4*b^4*c^3 + 256*a^6*c^5)*x^2 + 3*(a^2*b^9 - 16*a^3*b^7*c + 96*a^4*b
^5*c^2 - 256*a^5*b^3*c^3 + 256*a^6*b*c^4)*x), -1/3*(a^2*b^5 + 22*a^3*b^3*c
 - 104*a^4*b*c^2 + 12*(b^4*c^3 - 3*a*b^2*c^4 - 4*a^2*c^5)*x^5 + 30*(b^5*c^
2 - 3*a*b^3*c^3 - 4*a^2*b*c^4)*x^4 + 2*(11*b^6*c - 17*a*b^4*c^2 - 92*a^2*b
^2*c^3 - 64*a^3*c^4)*x^3 + 3*(b^7 + 13*a*b^5*c - 52*a^2*b^3*c^2 - 64*a^...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(160) = 320$.

Time = 1.06 (sec) , antiderivative size = 920, normalized size of antiderivative = 5.58

$$\int \frac{x^2}{(a+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x**2/(c*x**2+b*x+a)**4,x)
```

output
```
-4*c*sqrt(-1/(4*a*c - b**2)**7)*(a*c + b**2)*log(x + (-1024*a**4*c**5*sqrt
(-1/(4*a*c - b**2)**7)*(a*c + b**2) + 1024*a**3*b**2*c**4*sqrt(-1/(4*a*c -
 b**2)**7)*(a*c + b**2) - 384*a**2*b**4*c**3*sqrt(-1/(4*a*c - b**2)**7)*(a
*c + b**2) + 64*a*b**6*c**2*sqrt(-1/(4*a*c - b**2)**7)*(a*c + b**2) + 4*a*
b*c**2 - 4*b**8*c*sqrt(-1/(4*a*c - b**2)**7)*(a*c + b**2) + 4*b**3*c)/(8*a
*c**3 + 8*b**2*c**2)) + 4*c*sqrt(-1/(4*a*c - b**2)**7)*(a*c + b**2)*log(x
+ (1024*a**4*c**5*sqrt(-1/(4*a*c - b**2)**7)*(a*c + b**2) - 1024*a**3*b**2
*c**4*sqrt(-1/(4*a*c - b**2)**7)*(a*c + b**2) + 384*a**2*b**4*c**3*sqrt(-1
/(4*a*c - b**2)**7)*(a*c + b**2) - 64*a*b**6*c**2*sqrt(-1/(4*a*c - b**2)**
7)*(a*c + b**2) + 4*a*b*c**2 + 4*b**8*c*sqrt(-1/(4*a*c - b**2)**7)*(a*c +
b**2) + 4*b**3*c)/(8*a*c**3 + 8*b**2*c**2)) + (26*a**3*b*c + a**2*b**3 + x
**5*(12*a*c**4 + 12*b**2*c**3) + x**4*(30*a*b*c**3 + 30*b**3*c**2) + x**3*
(32*a**2*c**3 + 54*a*b**2*c**2 + 22*b**4*c) + x**2*(48*a**2*b*c**2 + 51*a*
b**3*c + 3*b**5) + x*(-12*a**3*c**2 + 66*a**2*b**2*c + 3*a*b**4))/(192*a**
6*c**3 - 144*a**5*b**2*c**2 + 36*a**4*b**4*c - 3*a**3*b**6 + x**6*(192*a**
3*c**6 - 144*a**2*b**2*c**5 + 36*a*b**4*c**4 - 3*b**6*c**3) + x**5*(576*a*
*3*b*c**5 - 432*a**2*b**3*c**4 + 108*a*b**5*c**3 - 9*b**7*c**2) + x**4*(57
6*a**4*c**5 + 144*a**3*b**2*c**4 - 324*a**2*b**4*c**3 + 99*a*b**6*c**2 - 9
*b**8*c) + x**3*(1152*a**4*b*c**4 - 672*a**3*b**3*c**3 + 72*a**2*b**5*c**2
 + 18*a*b**7*c - 3*b**9) + x**2*(576*a**5*c**4 + 144*a**4*b**2*c**3 - 3...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx+cx^2)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.62

$$\int \frac{x^2}{(a+bx+cx^2)^4}\,dx = -\frac{8\,(b^2c+ac^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3)\sqrt{-b^2+4\,ac}} - \frac{12\,b^2c^3x^5+12\,ac^4x^5+30\,b^3c^2x^4+30\,abc^3x^4+22\,b^4cx^3+54\,ab^2c^2x^3+32\,a^2c^3x^3+3\,b^5x^2+51\,ab^3cx^2}{3\,(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3)(cx^2+b}$$

input
```
integrate(x^2/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

output
```
-8*(b^2*c + a*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^6 - 12*a*b^4
*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-b^2 + 4*a*c)) - 1/3*(12*b^2*c^3*x^
5 + 12*a*c^4*x^5 + 30*b^3*c^2*x^4 + 30*a*b*c^3*x^4 + 22*b^4*c*x^3 + 54*a*b
^2*c^2*x^3 + 32*a^2*c^3*x^3 + 3*b^5*x^2 + 51*a*b^3*c*x^2 + 48*a^2*b*c^2*x^
2 + 3*a*b^4*x + 66*a^2*b^2*c*x - 12*a^3*c^2*x + a^2*b^3 + 26*a^3*b*c)/((b^
6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*(c*x^2 + b*x + a)^3)
```

Mupad [B] (verification not implemented)

Time = 9.31 (sec) , antiderivative size = 574, normalized size of antiderivative = 3.48

$$\int \frac{x^2}{(a+bx+cx^2)^4}\,dx = -\frac{\frac{26\,c\,a^3\,b+a^2\,b^3}{3\,(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}+\frac{2\,x^3\,(16\,a^2\,c^3+27\,a\,b^2\,c^2+11\,b^4\,c)}{3\,(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}+\frac{a\,x\,(-4\,a^2\,c^2+22\,a\,b^2\,c+b^4)}{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}+\frac{x^2\,(b^2\cdots}{-64\,a^3\,c^3+\cdots}}{x^2\,(3\,c\,a^2+3\,a\,b^2)+x^4\,(3\,b^2\,c+3\,a\,c^2)+a^3+x^3\,(b^3+6\,\cdots} - \frac{8\,c\,\mathrm{atan}\left(\frac{\left(\frac{8\,c^2\,x\,(b^2+a\,c)}{(4\,a\,c-b^2)^{7/2}}+\frac{4\,c\,(b^2+a\,c)\,(-64\,a^3\,b\,c^3+48\,a^2\,b^3\,c^2-12\,a\,b^5\,c+b^7)}{(4\,a\,c-b^2)^{7/2}\,(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}\right)(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}{4\,b^2\,c+4\,a\,c^2}\right)(b^2+a\,c)}{(4\,a\,c-b^2)^{7/2}}$$

input
```
int(x^2/(a + b*x + c*x^2)^4,x)
```

output
```
- ((a^2*b^3 + 26*a^3*b*c)/(3*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4
*c)) + (2*x^3*(11*b^4*c + 16*a^2*c^3 + 27*a*b^2*c^2))/(3*(b^6 - 64*a^3*c^3
 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (a*x*(b^4 - 4*a^2*c^2 + 22*a*b^2*c))/(b
^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (x^2*(a*c + b^2)*(b^3 + 1
6*a*b*c))/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (4*c^3*x^5*(a
*c + b^2))/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (10*b*c^2*x^
4*(a*c + b^2))/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))/(x^2*(3*a
*b^2 + 3*a^2*c) + x^4*(3*a*c^2 + 3*b^2*c) + a^3 + x^3*(b^3 + 6*a*b*c) + c^
3*x^6 + 3*b*c^2*x^5 + 3*a^2*b*x) - (8*c*atan((((8*c^2*x*(a*c + b^2))/(4*a*
c - b^2)^(7/2) + (4*c*(a*c + b^2)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 1
2*a*b^5*c))/((4*a*c - b^2)^(7/2)*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a
*b^4*c)))*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))/(4*a*c^2 + 4*b
^2*c))*(a*c + b^2))/(4*a*c - b^2)^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1404, normalized size of antiderivative = 8.51

$$\int \frac{x^2}{(a+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
int(x^2/(c*x^2+b*x+a)^4,x)
```

output
```
(24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b*c**2 +
24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**3*c + 7
2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**2*c**2*x
 + 72*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**3*
x**2 + 72*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**
4*c*x + 144*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b
**3*c**2*x**2 + 144*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2)
)*a**2*b**2*c**3*x**3 + 72*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
- b**2))*a**2*b*c**4*x**4 + 72*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*
a*c - b**2))*a*b**5*c*x**2 + 168*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(
4*a*c - b**2))*a*b**4*c**2*x**3 + 144*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/
sqrt(4*a*c - b**2))*a*b**3*c**3*x**4 + 72*sqrt(4*a*c - b**2)*atan((b + 2*c
*x)/sqrt(4*a*c - b**2))*a*b**2*c**4*x**5 + 24*sqrt(4*a*c - b**2)*atan((b +
 2*c*x)/sqrt(4*a*c - b**2))*a*b*c**5*x**6 + 24*sqrt(4*a*c - b**2)*atan((b
+ 2*c*x)/sqrt(4*a*c - b**2))*b**6*c*x**3 + 72*sqrt(4*a*c - b**2)*atan((b +
 2*c*x)/sqrt(4*a*c - b**2))*b**5*c**2*x**4 + 72*sqrt(4*a*c - b**2)*atan((b
 + 2*c*x)/sqrt(4*a*c - b**2))*b**4*c**3*x**5 + 24*sqrt(4*a*c - b**2)*atan(
(b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*c**4*x**6 - 16*a**5*c**3 + 92*a**4*b*
*2*c**2 - 96*a**4*b*c**3*x - 48*a**4*c**4*x**2 - 18*a**3*b**4*c + 240*a**3
*b**3*c**2*x + 108*a**3*b**2*c**3*x**2 + 32*a**3*b*c**4*x**3 - 48*a**3*...
```

3.262 $\int \frac{x}{(a+bx+cx^2)^4} dx$

Optimal result

Integrand size = 14, antiderivative size = 137

$$\int \frac{x}{(a+bx+cx^2)^4} dx = \frac{2a+bx}{3(b^2-4ac)(a+bx+cx^2)^3} - \frac{5b(b+2cx)}{6(b^2-4ac)^2(a+bx+cx^2)^2} + \frac{5bc(b+2cx)}{(b^2-4ac)^3(a+bx+cx^2)} - \frac{20bc^2\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{7/2}}$$

output
```
1/3*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^3-5/6*b*(2*c*x+b)/(-4*a*c+b^2)^2/
(c*x^2+b*x+a)^2+5*b*c*(2*c*x+b)/(-4*a*c+b^2)^3/(c*x^2+b*x+a)-20*b*c^2*arct
anh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(7/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

$$\int \frac{x}{(a+bx+cx^2)^4} dx = \frac{\frac{2(b^2-4ac)^2(2a+bx)}{(a+x(b+cx))^3} - \frac{5b(b^2-4ac)(b+2cx)}{(a+x(b+cx))^2} + \frac{30bc(b+2cx)}{a+x(b+cx)} + \frac{120bc^2\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{6(b^2-4ac)^3}$$

input
```
Integrate[x/(a + b*x + c*x^2)^4,x]
```

output
```
((2*(b^2 - 4*a*c)^2*(2*a + b*x))/(a + x*(b + c*x))^3 - (5*b*(b^2 - 4*a*c)*
(b + 2*c*x))/(a + x*(b + c*x))^2 + (30*b*c*(b + 2*c*x))/(a + x*(b + c*x))
+ (120*b*c^2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(
6*(b^2 - 4*a*c)^3)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1159, 1086, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx+cx^2)^4}dx$$

$$\Big\downarrow \text{1159}$$

$$\frac{5b\int \frac{1}{(cx^2+bx+a)^3}dx}{3(b^2-4ac)}+\frac{2a+bx}{3(b^2-4ac)(a+bx+cx^2)^3}$$

$$\Big\downarrow \text{1086}$$

$$\frac{5b\left(-\frac{3c\int \frac{1}{(cx^2+bx+a)^2}dx}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}\right)}{3(b^2-4ac)}+\frac{2a+bx}{3(b^2-4ac)(a+bx+cx^2)^3}$$

$$\Big\downarrow \text{1086}$$

$$\frac{5b\left(-\frac{3c\left(-\frac{2c\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}\right)}{3(b^2-4ac)}+\frac{2a+bx}{3(b^2-4ac)(a+bx+cx^2)^3}$$

$$\Big\downarrow \text{1083}$$

$$\frac{5b\left(-\frac{3c\left(\frac{4c\int\frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}\right)}{3(b^2-4ac)}+\frac{2a+bx}{3(b^2-4ac)(a+bx+cx^2)^3}$$

$$\Big\downarrow \; 219$$

$$\frac{5b\left(-\frac{3c\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}\right)}{3(b^2-4ac)}+\frac{2a+bx}{3(b^2-4ac)(a+bx+cx^2)^3}$$

input
```
Int[x/(a + b*x + c*x^2)^4,x]
```

output
```
(2*a + b*x)/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^3) + (5*b*(-1/2*(b + 2*c*x)
/((b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (3*c*(-((b + 2*c*x)/((b^2 - 4*a*c)*
(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 -
4*a*c)^(3/2)))/(b^2 - 4*a*c)))/(3*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.21

method	result
default	$\frac{-bx-2a}{3(4ac-b^2)(cx^2+bx+a)^3}-\frac{5b\left(\frac{2cx+b}{2\left(4ac-b^2\right)\left(cx^2+bx+a\right)^2}+\frac{3c\left(\frac{2cx+b}{\left(4ac-b^2\right)\left(cx^2+bx+a\right)}+\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}\right)}{4ac-b^2}\right)}{3(4ac-b^2)}$
risch	$\frac{-\frac{10b\,c^4x^5}{64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6}-\frac{25b^2c^3x^4}{64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6}-\frac{5\left(16ac+11b^2\right)b\,c^2x^3}{3\left(64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6\right)}-\frac{5b^2\left(16ac+b^2\right)c\,x^2}{2\left(64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6\right)}-}{\left(cx^2+bx+a\right)^3}$

input
```
int(x/(c*x^2+b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)^3-5/3*b/(4*a*c-b^2)*(1/2*(2*c*x+b
)/(4*a*c-b^2)/(c*x^2+b*x+a)^2+3*c/(4*a*c-b^2)*((2*c*x+b)/(4*a*c-b^2)/(c*x^
2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(129) = 258$.

Time = 0.12 (sec) , antiderivative size = 1364, normalized size of antiderivative = 9.96

$$\int \frac{x}{(a+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

output
```
[-1/6*(a*b^6 - 22*a^2*b^4*c + 8*a^3*b^2*c^2 + 256*a^4*c^3 - 60*(b^3*c^4 -
4*a*b*c^5)*x^5 - 150*(b^4*c^3 - 4*a*b^2*c^4)*x^4 - 10*(11*b^5*c^2 - 28*a*b
^3*c^3 - 64*a^2*b*c^4)*x^3 - 15*(b^6*c + 12*a*b^4*c^2 - 64*a^2*b^2*c^3)*x^
2 + 60*(b*c^5*x^6 + 3*b^2*c^4*x^5 + 3*a^2*b^2*c^2*x + a^3*b*c^2 + 3*(b^3*c
^3 + a*b*c^4)*x^4 + (b^4*c^2 + 6*a*b^2*c^3)*x^3 + 3*(a*b^3*c^2 + a^2*b*c^3
)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 3*(b^7 - 22*a*b^5*c + 28*a^2*b
^3*c^2 + 176*a^3*b*c^3)*x)/(a^3*b^8 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*
a^6*b^2*c^3 + 256*a^7*c^4 + (b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256
*a^3*b^2*c^6 + 256*a^4*c^7)*x^6 + 3*(b^9*c^2 - 16*a*b^7*c^3 + 96*a^2*b^5*c
^4 - 256*a^3*b^3*c^5 + 256*a^4*b*c^6)*x^5 + 3*(b^10*c - 15*a*b^8*c^2 + 80*
a^2*b^6*c^3 - 160*a^3*b^4*c^4 + 256*a^5*c^6)*x^4 + (b^11 - 10*a*b^9*c + 32
0*a^3*b^5*c^3 - 1280*a^4*b^3*c^4 + 1536*a^5*b*c^5)*x^3 + 3*(a*b^10 - 15*a^
2*b^8*c + 80*a^3*b^6*c^2 - 160*a^4*b^4*c^3 + 256*a^6*c^5)*x^2 + 3*(a^2*b^9
 - 16*a^3*b^7*c + 96*a^4*b^5*c^2 - 256*a^5*b^3*c^3 + 256*a^6*b*c^4)*x), -1
/6*(a*b^6 - 22*a^2*b^4*c + 8*a^3*b^2*c^2 + 256*a^4*c^3 - 60*(b^3*c^4 - 4*a
*b*c^5)*x^5 - 150*(b^4*c^3 - 4*a*b^2*c^4)*x^4 - 10*(11*b^5*c^2 - 28*a*b^3*
c^3 - 64*a^2*b*c^4)*x^3 - 15*(b^6*c + 12*a*b^4*c^2 - 64*a^2*b^2*c^3)*x^2 +
 120*(b*c^5*x^6 + 3*b^2*c^4*x^5 + 3*a^2*b^2*c^2*x + a^3*b*c^2 + 3*(b^3*c^3
 + a*b*c^4)*x^4 + (b^4*c^2 + 6*a*b^2*c^3)*x^3 + 3*(a*b^3*c^2 + a^2*b*c^...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 799 vs. $2(134) = 268$.

Time = 0.88 (sec) , antiderivative size = 799, normalized size of antiderivative = 5.83

$$\int \frac{x}{(a+bx+cx^2)^4}\,dx$$

$$= 10bc^2\sqrt{-\frac{1}{(4ac-b^2)^7}}\log\left(x+\frac{-2560a^4bc^6\sqrt{-\frac{1}{(4ac-b^2)^7}}+2560a^3b^3c^5\sqrt{-\frac{1}{(4ac-b^2)^7}}-960a^2b^5c^4\sqrt{-\frac{1}{(4ac-b^2)^7}}}{20bc^3}\right.$$

$$-10bc^2\sqrt{-\frac{1}{(4ac-b^2)^7}}\log\left(x+\frac{2560a^4bc^6\sqrt{-\frac{1}{(4ac-b^2)^7}}-2560a^3b^3c^5\sqrt{-\frac{1}{(4ac-b^2)^7}}+960a^2b^5c^4\sqrt{-\frac{1}{(4ac-b^2)^7}}}{20bc^3}\right.$$

$$+\frac{}{384a^6c^3-288a^5b^2c^2+72a^4b^4c-6a^3b^6+x^6\cdot(384a^3c^6-288a^2b^2c^5+72ab^4c^4-6b^6c^3)+x^5\cdot(1152a^3}$$

input
```
integrate(x/(c*x**2+b*x+a)**4,x)
```

output
```
10*b*c**2*sqrt(-1/(4*a*c - b**2)**7)*log(x + (-2560*a**4*b*c**6*sqrt(-1/(4
*a*c - b**2)**7) + 2560*a**3*b**3*c**5*sqrt(-1/(4*a*c - b**2)**7) - 960*a*
*2*b**5*c**4*sqrt(-1/(4*a*c - b**2)**7) + 160*a*b**7*c**3*sqrt(-1/(4*a*c -
 b**2)**7) - 10*b**9*c**2*sqrt(-1/(4*a*c - b**2)**7) + 10*b**2*c**2)/(20*b
*c**3)) - 10*b*c**2*sqrt(-1/(4*a*c - b**2)**7)*log(x + (2560*a**4*b*c**6*s
qrt(-1/(4*a*c - b**2)**7) - 2560*a**3*b**3*c**5*sqrt(-1/(4*a*c - b**2)**7)
 + 960*a**2*b**5*c**4*sqrt(-1/(4*a*c - b**2)**7) - 160*a*b**7*c**3*sqrt(-1
/(4*a*c - b**2)**7) + 10*b**9*c**2*sqrt(-1/(4*a*c - b**2)**7) + 10*b**2*c*
*2)/(20*b*c**3)) + (-64*a**3*c**2 - 18*a**2*b**2*c + a*b**4 - 150*b**2*c**
3*x**4 - 60*b*c**4*x**5 + x**3*(-160*a*b*c**3 - 110*b**3*c**2) + x**2*(-24
0*a*b**2*c**2 - 15*b**4*c) + x*(-132*a**2*b*c**2 - 54*a*b**3*c + 3*b**5))/
(384*a**6*c**3 - 288*a**5*b**2*c**2 + 72*a**4*b**4*c - 6*a**3*b**6 + x**6*
(384*a**3*c**6 - 288*a**2*b**2*c**5 + 72*a*b**4*c**4 - 6*b**6*c**3) + x**5
*(1152*a**3*b*c**5 - 864*a**2*b**3*c**4 + 216*a*b**5*c**3 - 18*b**7*c**2)
+ x**4*(1152*a**4*c**5 + 288*a**3*b**2*c**4 - 648*a**2*b**4*c**3 + 198*a*b
**6*c**2 - 18*b**8*c) + x**3*(2304*a**4*b*c**4 - 1344*a**3*b**3*c**3 + 144
*a**2*b**5*c**2 + 36*a*b**7*c - 6*b**9) + x**2*(1152*a**5*c**4 + 288*a**4*
b**2*c**3 - 648*a**3*b**4*c**2 + 198*a**2*b**6*c - 18*a*b**8) + x*(1152*a*
*5*b*c**3 - 864*a**4*b**3*c**2 + 216*a**3*b**5*c - 18*a**2*b**7))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\left(a+bx+cx^2\right)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.66

$$\int \frac{x}{\left(a+bx+cx^2\right)^4}\,dx = \frac{20\,bc^2\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3\right)\sqrt{-b^2+4\,ac}} + \frac{60\,bc^4x^5+150\,b^2c^3x^4+110\,b^3c^2x^3+160\,abc^3x^3+15\,b^4cx^2+240\,ab^2c^2x^2-3\,b^5x+54\,ab^3cx+132\,a^2b}{6\left(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3\right)\left(cx^2+bx+a\right)^3}$$

input
```
integrate(x/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

output
```
20*b*c^2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^6 - 12*a*b^4*c + 48*a^
2*b^2*c^2 - 64*a^3*c^3)*sqrt(-b^2 + 4*a*c)) + 1/6*(60*b*c^4*x^5 + 150*b^2*
c^3*x^4 + 110*b^3*c^2*x^3 + 160*a*b*c^3*x^3 + 15*b^4*c*x^2 + 240*a*b^2*c^2
*x^2 - 3*b^5*x + 54*a*b^3*c*x + 132*a^2*b*c^2*x - a*b^4 + 18*a^2*b^2*c + 6
4*a^3*c^2)/((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*(c*x^2 + b*x
+ a)^3)
```

Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 482, normalized size of antiderivative = 3.52

$$\int \frac{x}{(a+bx+cx^2)^4}\,dx$$
$$= \frac{\frac{64\,a^3\,c^2+18\,a^2\,b^2\,c-a\,b^4}{6\,(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}+\frac{10\,b\,c^4\,x^5}{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}+\frac{b\,x\,(44\,a^2\,c^2+18\,a\,b^2\,c-b^4)}{2\,(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}+\frac{5\,c\,x^2}{2\,(-64\,a^3\,c^3+4}}{x^2\,(3\,c\,a^2+3\,a\,b^2)+x^4\,(3\,b^2\,c+3\,a\,c^2)+a^3+x^3\,(b^3+6\,a}$$
$$+\frac{20\,b\,c^2\,\mathrm{atan}\left(\frac{\left(\frac{10\,b^2\,c^2}{(4\,a\,c-b^2)^{7/2}}+\frac{20\,b\,c^3\,x}{(4\,a\,c-b^2)^{7/2}}\right)(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}{10\,b\,c^2}\right)}{(4\,a\,c-b^2)^{7/2}}$$

input
```
int(x/(a + b*x + c*x^2)^4,x)
```

output
```
((64*a^3*c^2 - a*b^4 + 18*a^2*b^2*c)/(6*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2
 - 12*a*b^4*c)) + (10*b*c^4*x^5)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a
*b^4*c) + (b*x*(44*a^2*c^2 - b^4 + 18*a*b^2*c))/(2*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c)) + (5*c*x^2*(b^4 + 16*a*b^2*c))/(2*(b^6 - 64*a^3
*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (25*b^2*c^3*x^4)/(b^6 - 64*a^3*c^3
+ 48*a^2*b^2*c^2 - 12*a*b^4*c) + (5*b*c*x^3*(16*a*c^2 + 11*b^2*c))/(3*(b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))/(x^2*(3*a*b^2 + 3*a^2*c) +
x^4*(3*a*c^2 + 3*b^2*c) + a^3 + x^3*(b^3 + 6*a*b*c) + c^3*x^6 + 3*b*c^2*x^
5 + 3*a^2*b*x) + (20*b*c^2*atan((((10*b^2*c^2)/(4*a*c - b^2)^(7/2) + (20*b
*c^3*x)/(4*a*c - b^2)^(7/2))*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4
*c))/(10*b*c^2)))/(4*a*c - b^2)^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1042, normalized size of antiderivative = 7.61

$$\int \frac{x}{(a+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
int(x/(c*x^2+b*x+a)^4,x)
```

output
```
( - 120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**
2 - 360*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2*
c**2*x - 360*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*
b*c**3*x**2 - 360*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*
a*b**3*c**2*x**2 - 720*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b*
*2))*a*b**2*c**3*x**3 - 360*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
 - b**2))*a*b*c**4*x**4 - 120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a
*c - b**2))*b**4*c**2*x**3 - 360*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(
4*a*c - b**2))*b**3*c**3*x**4 - 360*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sq
rt(4*a*c - b**2))*b**2*c**4*x**5 - 120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)
/sqrt(4*a*c - b**2))*b*c**5*x**6 - 176*a**4*c**3 - 28*a**3*b**2*c**2 - 288
*a**3*b*c**3*x + 240*a**3*c**4*x**2 + 22*a**2*b**4*c - 144*a**2*b**3*c**2*
x - 780*a**2*b**2*c**3*x**2 - 160*a**2*b*c**4*x**3 + 240*a**2*c**5*x**4 -
a*b**6 + 66*a*b**5*c*x + 120*a*b**4*c**2*x**2 - 320*a*b**3*c**3*x**3 - 420
*a*b**2*c**4*x**4 + 80*a*c**6*x**6 - 3*b**7*x + 15*b**6*c*x**2 + 90*b**5*c
**2*x**3 + 90*b**4*c**3*x**4 - 20*b**2*c**5*x**6)/(6*(256*a**7*c**4 - 256*
a**6*b**2*c**3 + 768*a**6*b*c**4*x + 768*a**6*c**5*x**2 + 96*a**5*b**4*c**
2 - 768*a**5*b**3*c**3*x + 1536*a**5*b*c**5*x**3 + 768*a**5*c**6*x**4 - 16
*a**4*b**6*c + 288*a**4*b**5*c**2*x - 480*a**4*b**4*c**3*x**2 - 1280*a**4*
b**3*c**4*x**3 + 768*a**4*b*c**6*x**5 + 256*a**4*c**7*x**6 + a**3*b**8 ...
```

3.263 $\int \frac{1}{(a+bx+cx^2)^4} dx$

Optimal result

Integrand size = 12, antiderivative size = 136

$$\int \frac{1}{(a+bx+cx^2)^4} dx = -\frac{b+2cx}{3(b^2-4ac)(a+bx+cx^2)^3} + \frac{5c(b+2cx)}{3(b^2-4ac)^2(a+bx+cx^2)^2} - \frac{10c^2(b+2cx)}{(b^2-4ac)^3(a+bx+cx^2)} + \frac{40c^3\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{7/2}}$$

output
```
-1/3*(2*c*x+b)/(-4*a*c+b^2)/(c*x^2+b*x+a)^3+5/3*c*(2*c*x+b)/(-4*a*c+b^2)^2
/(c*x^2+b*x+a)^2-10*c^2*(2*c*x+b)/(-4*a*c+b^2)^3/(c*x^2+b*x+a)+40*c^3*arct
anh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(7/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.99

$$\int \frac{1}{(a+bx+cx^2)^4} dx = -\frac{\frac{(b^2-4ac)^2(b+2cx)}{(a+x(b+cx))^3} - \frac{5c(b^2-4ac)(b+2cx)}{(a+x(b+cx))^2} + \frac{30c^2(b+2cx)}{a+x(b+cx)} + \frac{120c^3\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{3(b^2-4ac)^3}$$

input
```
Integrate[(a + b*x + c*x^2)^(-4),x]
```

output
```
-1/3*(((b^2 - 4*a*c)^2*(b + 2*c*x))/(a + x*(b + c*x))^3 - (5*c*(b^2 - 4*a*
c)*(b + 2*c*x))/(a + x*(b + c*x))^2 + (30*c^2*(b + 2*c*x))/(a + x*(b + c*x
)) + (120*c^3*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/
(b^2 - 4*a*c)^3
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1086, 1086, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx+cx^2)^4}dx$$

$$\Big\downarrow \text{1086}$$

$$-\frac{10c\int \frac{1}{(cx^2+bx+a)^3}dx}{3(b^2-4ac)}-\frac{b+2cx}{3(b^2-4ac)(a+bx+cx^2)^3}$$

$$\Big\downarrow \text{1086}$$

$$-\frac{10c\left(-\frac{3c\int \frac{1}{(cx^2+bx+a)^2}dx}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}\right)}{3(b^2-4ac)}-\frac{b+2cx}{3(b^2-4ac)(a+bx+cx^2)^3}$$

$$\Big\downarrow \text{1086}$$

$$-\frac{10c\left(-\frac{3c\left(-\frac{2c\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}\right)}{3(b^2-4ac)}-\frac{b+2cx}{3(b^2-4ac)(a+bx+cx^2)^3}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{10c\left(-\frac{3c\left(\frac{4c\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}\right)}{3(b^2-4ac)}-\frac{b+2cx}{3(b^2-4ac)(a+bx+cx^2)^3}$$

$$\Big\downarrow \; 219$$

$$-\frac{10c\left(-\frac{3c\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}\right)}{3(b^2-4ac)}-\frac{b+2cx}{3(b^2-4ac)(a+bx+cx^2)^3}$$

input
```
Int[(a + b*x + c*x^2)^(-4),x]
```

output
```
-1/3*(b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)^3) - (10*c*(-1/2*(b + 2*
c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (3*c*(-((b + 2*c*x)/((b^2 - 4*a
*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^
2 - 4*a*c)^(3/2)))/(b^2 - 4*a*c)))/(3*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.21

method	result
default	$\frac{2cx+b}{3(4ac-b^2)(cx^2+bx+a)^3}+\frac{10c\left(\frac{2cx+b}{2\left(4ac-b^2\right)\left(cx^2+bx+a\right)^2}+\frac{3c\left(\frac{2cx+b}{\left(4ac-b^2\right)\left(cx^2+bx+a\right)}+\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}\right)}{4ac-b^2}\right)}{3(4ac-b^2)}$
risch	$\frac{\frac{20c^5x^5}{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}+\frac{50bc^4x^4}{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}+\frac{10\left(16ac+11b^2\right)c^3x^3}{3\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{5b\left(16ac+b^2\right)c^2x^2}{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}+\cdots}{(cx^2+bx+a)^3}$

input
```
int(1/(c*x^2+b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^3+10/3*c/(4*a*c-b^2)*(1/2*(2*c*x+b
)/(4*a*c-b^2)/(c*x^2+b*x+a)^2+3*c/(4*a*c-b^2)*((2*c*x+b)/(4*a*c-b^2)/(c*x^
2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(128) = 256$.

Time = 0.10 (sec) , antiderivative size = 1337, normalized size of antiderivative = 9.83

$$\int \frac{1}{(a+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

output
```
[-1/3*(b^7 - 17*a*b^5*c + 118*a^2*b^3*c^2 - 264*a^3*b*c^3 + 60*(b^2*c^5 -
4*a*c^6)*x^5 + 150*(b^3*c^4 - 4*a*b*c^5)*x^4 + 10*(11*b^4*c^3 - 28*a*b^2*c
^4 - 64*a^2*c^5)*x^3 + 15*(b^5*c^2 + 12*a*b^3*c^3 - 64*a^2*b*c^4)*x^2 + 60
*(c^6*x^6 + 3*b*c^5*x^5 + 3*a^2*b*c^3*x + a^3*c^3 + 3*(b^2*c^4 + a*c^5)*x^
4 + (b^3*c^3 + 6*a*b*c^4)*x^3 + 3*(a*b^2*c^3 + a^2*c^4)*x^2)*sqrt(b^2 - 4*
a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b
))/(c*x^2 + b*x + a)) - 3*(b^6*c - 22*a*b^4*c^2 + 28*a^2*b^2*c^3 + 176*a^3
*c^4)*x)/(a^3*b^8 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3 + 256*
a^7*c^4 + (b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6 + 256
*a^4*c^7)*x^6 + 3*(b^9*c^2 - 16*a*b^7*c^3 + 96*a^2*b^5*c^4 - 256*a^3*b^3*c
^5 + 256*a^4*b*c^6)*x^5 + 3*(b^10*c - 15*a*b^8*c^2 + 80*a^2*b^6*c^3 - 160*
a^3*b^4*c^4 + 256*a^5*c^6)*x^4 + (b^11 - 10*a*b^9*c + 320*a^3*b^5*c^3 - 12
80*a^4*b^3*c^4 + 1536*a^5*b*c^5)*x^3 + 3*(a*b^10 - 15*a^2*b^8*c + 80*a^3*b
^6*c^2 - 160*a^4*b^4*c^3 + 256*a^6*c^5)*x^2 + 3*(a^2*b^9 - 16*a^3*b^7*c +
96*a^4*b^5*c^2 - 256*a^5*b^3*c^3 + 256*a^6*b*c^4)*x), -1/3*(b^7 - 17*a*b^5
*c + 118*a^2*b^3*c^2 - 264*a^3*b*c^3 + 60*(b^2*c^5 - 4*a*c^6)*x^5 + 150*(b
^3*c^4 - 4*a*b*c^5)*x^4 + 10*(11*b^4*c^3 - 28*a*b^2*c^4 - 64*a^2*c^5)*x^3
+ 15*(b^5*c^2 + 12*a*b^3*c^3 - 64*a^2*b*c^4)*x^2 - 120*(c^6*x^6 + 3*b*c^5*
x^5 + 3*a^2*b*c^3*x + a^3*c^3 + 3*(b^2*c^4 + a*c^5)*x^4 + (b^3*c^3 + 6*a*b
*c^4)*x^3 + 3*(a*b^2*c^3 + a^2*c^4)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqr...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 777 vs. $2(134) = 268$.

Time = 0.90 (sec) , antiderivative size = 777, normalized size of antiderivative = 5.71

$$\int \frac{1}{(a+bx+cx^2)^4}\,dx =$$

$$-20c^3\sqrt{-\frac{1}{(4ac-b^2)^7}}\log\left(x+\frac{-5120a^4c^7\sqrt{-\frac{1}{(4ac-b^2)^7}}+5120a^3b^2c^6\sqrt{-\frac{1}{(4ac-b^2)^7}}-1920a^2b^4c^5\sqrt{-\frac{1}{(4ac}}}{40c^4}\right.$$

$$+20c^3\sqrt{-\frac{1}{(4ac-b^2)^7}}\log\left(x+\frac{5120a^4c^7\sqrt{-\frac{1}{(4ac-b^2)^7}}-5120a^3b^2c^6\sqrt{-\frac{1}{(4ac-b^2)^7}}+1920a^2b^4c^5\sqrt{-\frac{1}{(4ac-b}}}{40c^4}\right.$$

$$+\frac{}{192a^6c^3-144a^5b^2c^2+36a^4b^4c-3a^3b^6+x^6\cdot(192a^3c^6-144a^2b^2c^5+36ab^4c^4-3b^6c^3)+x^5\cdot(576a^3b}$$

input
```
integrate(1/(c*x**2+b*x+a)**4,x)
```

output
```
-20*c**3*sqrt(-1/(4*a*c - b**2)**7)*log(x + (-5120*a**4*c**7*sqrt(-1/(4*a*
c - b**2)**7) + 5120*a**3*b**2*c**6*sqrt(-1/(4*a*c - b**2)**7) - 1920*a**2
*b**4*c**5*sqrt(-1/(4*a*c - b**2)**7) + 320*a*b**6*c**4*sqrt(-1/(4*a*c - b
**2)**7) - 20*b**8*c**3*sqrt(-1/(4*a*c - b**2)**7) + 20*b*c**3)/(40*c**4))
 + 20*c**3*sqrt(-1/(4*a*c - b**2)**7)*log(x + (5120*a**4*c**7*sqrt(-1/(4*a
*c - b**2)**7) - 5120*a**3*b**2*c**6*sqrt(-1/(4*a*c - b**2)**7) + 1920*a**
2*b**4*c**5*sqrt(-1/(4*a*c - b**2)**7) - 320*a*b**6*c**4*sqrt(-1/(4*a*c -
b**2)**7) + 20*b**8*c**3*sqrt(-1/(4*a*c - b**2)**7) + 20*b*c**3)/(40*c**4)
) + (66*a**2*b*c**2 - 13*a*b**3*c + b**5 + 150*b*c**4*x**4 + 60*c**5*x**5
+ x**3*(160*a*c**4 + 110*b**2*c**3) + x**2*(240*a*b*c**3 + 15*b**3*c**2) +
 x*(132*a**2*c**3 + 54*a*b**2*c**2 - 3*b**4*c))/(192*a**6*c**3 - 144*a**5*
b**2*c**2 + 36*a**4*b**4*c - 3*a**3*b**6 + x**6*(192*a**3*c**6 - 144*a**2*
b**2*c**5 + 36*a*b**4*c**4 - 3*b**6*c**3) + x**5*(576*a**3*b*c**5 - 432*a*
*2*b**3*c**4 + 108*a*b**5*c**3 - 9*b**7*c**2) + x**4*(576*a**4*c**5 + 144*
a**3*b**2*c**4 - 324*a**2*b**4*c**3 + 99*a*b**6*c**2 - 9*b**8*c) + x**3*(1
152*a**4*b*c**4 - 672*a**3*b**3*c**3 + 72*a**2*b**5*c**2 + 18*a*b**7*c - 3
*b**9) + x**2*(576*a**5*c**4 + 144*a**4*b**2*c**3 - 324*a**3*b**4*c**2 + 9
9*a**2*b**6*c - 9*a*b**8) + x*(576*a**5*b*c**3 - 432*a**4*b**3*c**2 + 108*
a**3*b**5*c - 9*a**2*b**7))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a + bx + cx^2)^4} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.62

$$\int \frac{1}{(a+bx+cx^2)^4}\,dx = -\frac{40\,c^3\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3)\sqrt{-b^2+4\,ac}} - \frac{60\,c^5x^5+150\,bc^4x^4+110\,b^2c^3x^3+160\,ac^4x^3+15\,b^3c^2x^2+240\,abc^3x^2-3\,b^4cx+54\,ab^2c^2x+132\,a^2c^{}}{3\,(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3)(cx^2+bx+a)^3}$$

input
```
integrate(1/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

output
```
-40*c^3*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^6 - 12*a*b^4*c + 48*a^2
*b^2*c^2 - 64*a^3*c^3)*sqrt(-b^2 + 4*a*c)) - 1/3*(60*c^5*x^5 + 150*b*c^4*x
^4 + 110*b^2*c^3*x^3 + 160*a*c^4*x^3 + 15*b^3*c^2*x^2 + 240*a*b*c^3*x^2 -
3*b^4*c*x + 54*a*b^2*c^2*x + 132*a^2*c^3*x + b^5 - 13*a*b^3*c + 66*a^2*b*c
^2)/((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*(c*x^2 + b*x + a)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx+cx^2)^4}\,dx = \begin{cases} \frac{20\left(\frac{b}{2}+c\,x\right)\left(\frac{c^2}{6\left(4\,a\,c-b^2\right)^2\left(c\,x^2+b\,x+a\right)^2}+\frac{c^3}{\left(4\,a\,c-b^2\right)^3\left(c\,x^2+b\,x+a\right)}+\frac{c}{30\left(4\,a\,c-b^2\right)\left(c\,x^2+b\,x+a\right)^3}\right)}{c} - \frac{20\,c^3\,\ln\left(\frac{\frac{b}{2}-\sqrt{\frac{b^2}{4}-a\,c}+c\,x}{\frac{b}{2}+\sqrt{\frac{b^2}{4}-a\,c}+c\,x}\right)}{\left(b^2-4\,a\,c\right)^{7/2}} & \text{if} \\ \frac{20\left(\frac{b}{2}+c\,x\right)\left(\frac{c^2}{6\left(4\,a\,c-b^2\right)^2\left(c\,x^2+b\,x+a\right)^2}+\frac{c^3}{\left(4\,a\,c-b^2\right)^3\left(c\,x^2+b\,x+a\right)}+\frac{c}{30\left(4\,a\,c-b^2\right)\left(c\,x^2+b\,x+a\right)^3}\right)}{c} + \frac{20\,c^3\,\mathrm{atan}\left(\frac{\frac{b}{2}+c\,x}{\sqrt{a\,c-\frac{b^2}{4}}}\right)}{\sqrt{a\,c-\frac{b^2}{4}}\,(4\,a\,c-b^2)^3} & \text{if} \\ \int \frac{1}{\left(c\,x^2+b\,x+a\right)^4}\,dx & \text{if} \end{cases}$$

input
```
int(1/(a + b*x + c*x^2)^4,x)
```

output
```
piecewise(0 < - 4*a*c + b^2, - (20*c^3*log((b/2 - (- a*c + b^2/4)^(1/2) +
c*x)/(b/2 + (- a*c + b^2/4)^(1/2) + c*x)))/(- 4*a*c + b^2)^(7/2) + (20*(b/
2 + c*x)*(c^2/(6*(4*a*c - b^2)^2*(a + b*x + c*x^2)^2) + c^3/((4*a*c - b^2)
^3*(a + b*x + c*x^2)) + c/(30*(4*a*c - b^2)*(a + b*x + c*x^2)^3)))/c, - 4*
a*c + b^2 < 0, (20*(b/2 + c*x)*(c^2/(6*(4*a*c - b^2)^2*(a + b*x + c*x^2)^2
) + c^3/((4*a*c - b^2)^3*(a + b*x + c*x^2)) + c/(30*(4*a*c - b^2)*(a + b*x
 + c*x^2)^3)))/c + (20*c^3*atan((b/2 + c*x)/(a*c - b^2/4)^(1/2)))/((a*c -
b^2/4)^(1/2)*(4*a*c - b^2)^3), ~in(- 4*a*c + b^2, 'real') | b^2 == 4*a*c,
int(1/(a + b*x + c*x^2)^4, x))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1056, normalized size of antiderivative = 7.76

$$\int \frac{1}{(a+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
int(1/(c*x^2+b*x+a)^4,x)
```

output

```
(120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**3 +
 360*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2*c**
3*x + 360*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c
**4*x**2 + 360*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b
**3*c**3*x**2 + 720*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2)
)*a*b**2*c**4*x**3 + 360*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c -
b**2))*a*b*c**5*x**4 + 120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
- b**2))*b**4*c**3*x**3 + 360*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a
*c - b**2))*b**3*c**4*x**4 + 360*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(
4*a*c - b**2))*b**2*c**5*x**5 + 120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sq
rt(4*a*c - b**2))*b*c**6*x**6 - 80*a**4*c**4 + 284*a**3*b**2*c**3 + 288*a*
*3*b*c**4*x - 240*a**3*c**5*x**2 - 118*a**2*b**4*c**2 + 144*a**2*b**3*c**3
*x + 780*a**2*b**2*c**4*x**2 + 160*a**2*b*c**5*x**3 - 240*a**2*c**6*x**4 +
 17*a*b**6*c - 66*a*b**5*c**2*x - 120*a*b**4*c**3*x**2 + 320*a*b**3*c**4*x
**3 + 420*a*b**2*c**5*x**4 - 80*a*c**7*x**6 - b**8 + 3*b**7*c*x - 15*b**6*
c**2*x**2 - 90*b**5*c**3*x**3 - 90*b**4*c**4*x**4 + 20*b**2*c**6*x**6)/(3*
b*(256*a**7*c**4 - 256*a**6*b**2*c**3 + 768*a**6*b*c**4*x + 768*a**6*c**5*
x**2 + 96*a**5*b**4*c**2 - 768*a**5*b**3*c**3*x + 1536*a**5*b*c**5*x**3 +
768*a**5*c**6*x**4 - 16*a**4*b**6*c + 288*a**4*b**5*c**2*x - 480*a**4*b**4
*c**3*x**2 - 1280*a**4*b**3*c**4*x**3 + 768*a**4*b*c**6*x**5 + 256*a**4...
```

3.264 $\int \frac{1}{x(a+bx+cx^2)^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 282

$$\begin{aligned}\int \frac{1}{x(a+bx+cx^2)^4}\,dx &= \frac{b^2-2ac+bcx}{3a(b^2-4ac)(a+bx+cx^2)^3}+\frac{3b^4-23ab^2c+24a^2c^2+bc(3b^2-22ac)x}{6a^2(b^2-4ac)^2(a+bx+cx^2)^2}\\&+\frac{2b^6-23ab^4c+86a^2b^2c^2-64a^3c^3+2bc(b^4-11ab^2c+38a^2c^2)x}{2a^3(b^2-4ac)^3(a+bx+cx^2)}\\&+\frac{b(b^6-14ab^4c+70a^2b^2c^2-140a^3c^3)\,\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4(b^2-4ac)^{7/2}}\\&+\frac{\log(x)}{a^4}-\frac{\log(a+bx+cx^2)}{2a^4}\end{aligned}$$

output
```
1/3*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^2+b*x+a)^3+1/6*(3*b^4-23*a*b^2*c
+24*a^2*c^2+b*c*(-22*a*c+3*b^2)*x)/a^2/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^2+1/2*
(2*b^6-23*a*b^4*c+86*a^2*b^2*c^2-64*a^3*c^3+2*b*c*(38*a^2*c^2-11*a*b^2*c+b
^4)*x)/a^3/(-4*a*c+b^2)^3/(c*x^2+b*x+a)+b*(-140*a^3*c^3+70*a^2*b^2*c^2-14*
a*b^4*c+b^6)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(7/2)+
ln(x)/a^4-1/2*ln(c*x^2+b*x+a)/a^4
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.99

$$\int \frac{1}{x\left(a+bx+cx^2\right)^4}\,dx$$
$$=\frac{-\frac{2a^3(-b^2+2ac-bcx)}{(b^2-4ac)(a+x(b+cx))^3}+\frac{a^2(3b^4-23ab^2c+24a^2c^2+3b^3cx-22abc^2x)}{(b^2-4ac)^2(a+x(b+cx))^2}-\frac{3a(-2b^6+23ab^4c-86a^2b^2c^2+64a^3c^3-2b^5cx+22ab^3c^2x-76a^2bc}{(b^2-4ac)^3(a+x(b+cx))}}{6a^4}$$

input
```
Integrate[1/(x*(a + b*x + c*x^2)^4),x]
```

output
```
((-2*a^3*(-b^2 + 2*a*c - b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))^3) + (a^
2*(3*b^4 - 23*a*b^2*c + 24*a^2*c^2 + 3*b^3*c*x - 22*a*b*c^2*x))/((b^2 - 4*
a*c)^2*(a + x*(b + c*x))^2) - (3*a*(-2*b^6 + 23*a*b^4*c - 86*a^2*b^2*c^2 +
 64*a^3*c^3 - 2*b^5*c*x + 22*a*b^3*c^2*x - 76*a^2*b*c^3*x))/((b^2 - 4*a*c)
^3*(a + x*(b + c*x))) + (6*b*(b^6 - 14*a*b^4*c + 70*a^2*b^2*c^2 - 140*a^3*
c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(7/2) + 6*Log[
x] - 3*Log[a + x*(b + c*x)])/(6*a^4)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1165, 25, 1235, 27, 1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx+cx^2\right)^4}\,dx$$
$$\Big\downarrow \text{1165}$$
$$\frac{-2ac+b^2+bcx}{3a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}-\frac{\int -\frac{3(b^2-4ac)+5bcx}{x(cx^2+bx+a)^3}dx}{3a\left(b^2-4ac\right)}$$
$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3(b^2-4ac)+5bcx}{x(cx^2+bx+a)^3}dx}{3a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{3a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}$$

$$\Big\downarrow \; 1235$$

$$\frac{\frac{24a^2c^2+bcx(3b^2-22ac)-23ab^2c+3b^4}{2a(b^2-4ac)(a+bx+cx^2)^2}-\frac{\int -\frac{3\left(2\left(b^2-4ac\right)^2+bc\left(3b^2-22ac\right)x\right)}{x\left(cx^2+bx+a\right)^2}dx}{2a(b^2-4ac)}}{3a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{3a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\int \frac{2\left(b^2-4ac\right)^2+bc\left(3b^2-22ac\right)x}{x\left(cx^2+bx+a\right)^2}dx}{2a(b^2-4ac)}+\frac{24a^2c^2+bcx(3b^2-22ac)-23ab^2c+3b^4}{2a(b^2-4ac)(a+bx+cx^2)^2}}{3a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{3a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}$$

$$\Big\downarrow \; 1235$$

$$\frac{\frac{3\left(\frac{-64a^3c^3+86a^2b^2c^2+2bcx\left(38a^2c^2-11ab^2c+b^4\right)-23ab^4c+2b^6}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int -\frac{2\left(\left(b^2-4ac\right)^3+bc\left(b^4-11acb^2+38a^2c^2\right)x\right)}{x\left(cx^2+bx+a\right)}dx}{a\left(b^2-4ac\right)}\right)}{2a(b^2-4ac)}+\frac{24a^2c^2+bcx(3b^2-22ac)-23ab}{2a(b^2-4ac)(a+bx+cx^2)}}{3a\left(b^2-4ac\right)}$$

$$\frac{-2ac+b^2+bcx}{3a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\left(\frac{2\int \frac{\left(b^2-4ac\right)^3+bc\left(b^4-11acb^2+38a^2c^2\right)x}{x\left(cx^2+bx+a\right)}dx}{a\left(b^2-4ac\right)}+\frac{-64a^3c^3+86a^2b^2c^2+2bcx\left(38a^2c^2-11ab^2c+b^4\right)-23ab^4c+2b^6}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}\right)}{2a(b^2-4ac)}+\frac{24a^2c^2+bcx(3b^2-22ac)-23ab^2c+}{2a(b^2-4ac)(a+bx+cx^2)^2}}{3a\left(b^2-4ac\right)}$$

$$\frac{-2ac+b^2+bcx}{3a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}$$

$$\Big\downarrow \; 1200$$

$$\frac{\frac{3\left(\frac{2\int \left(\frac{-cx\left(b^2-4ac\right)^3-b\left(b^2-6ac\right)\left(b^4-7acb^2+17a^2c^2\right)}{a\left(cx^2+bx+a\right)}-\frac{\left(4ac-b^2\right)^3}{ax}\right)dx}{a\left(b^2-4ac\right)}+\frac{-64a^3c^3+86a^2b^2c^2+2bcx\left(38a^2c^2-11ab^2c+b^4\right)-23ab^4c+2b^6}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}\right)}{2a(b^2-4ac)}+\frac{24a^2c^2+b\ldots}{2a(\ldots}}{3a\left(b^2-4ac\right)}$$

$$\frac{-2ac+b^2+bcx}{3a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}$$

$\Big\downarrow$ 2009

$$\frac{\frac{24a^2c^2+bcx(3b^2-22ac)-23ab^2c+3b^4}{2a(b^2-4ac)(a+bx+cx^2)^2}+\frac{3\left(\frac{2\left(\frac{b\left(-140a^3c^3+70a^2b^2c^2-14ab^4c+b^6\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{\left(b^2-4ac\right)^3\log\left(a+bx+cx^2\right)}{2a}+\frac{\log(x)\left(b^2-\ldots\right.}{a}\ldots\right.}{a\left(b^2-4ac\right)}\ldots\right)}{2a(b^2-4ac)}}{3a\left(b^2-4ac\right)}$$

$$\frac{-2ac+b^2+bcx}{3a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}$$

input
```
Int[1/(x*(a + b*x + c*x^2)^4),x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(3*a*(b^2 - 4*a*c)*(a + b*x + c*x^2)^3) + ((3*b^4 -
23*a*b^2*c + 24*a^2*c^2 + b*c*(3*b^2 - 22*a*c)*x)/(2*a*(b^2 - 4*a*c)*(a +
b*x + c*x^2)^2) + (3*((2*b^6 - 23*a*b^4*c + 86*a^2*b^2*c^2 - 64*a^3*c^3 +
2*b*c*(b^4 - 11*a*b^2*c + 38*a^2*c^2)*x)/(a*(b^2 - 4*a*c)*(a + b*x + c*x^2
)) + (2*((b*(b^6 - 14*a*b^4*c + 70*a^2*b^2*c^2 - 140*a^3*c^3)*ArcTanh[(b +
 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + ((b^2 - 4*a*c)^3*Log[x
])/a - ((b^2 - 4*a*c)^3*Log[a + b*x + c*x^2])/(2*a)))/(a*(b^2 - 4*a*c))))/
(2*a*(b^2 - 4*a*c)))/(3*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(270) = 540$.

Time = 0.83 (sec) , antiderivative size = 661, normalized size of antiderivative = 2.34

method	result
default	$\frac{\ln(x)}{a^4} - \frac{\frac{b\,c^3 a\left(38a^2c^2-11ca\,b^2+b^4\right)x^5}{64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6} - \frac{c^2 a\left(64a^3c^3-238a^2b^2c^2+67a\,b^4c-6b^6\right)x^4}{2\left(64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6\right)} + \frac{bac\left(160a^3c^3+578a^2b^2c^2-189a\,b^4c+18b^6\right)x^3}{384a^3c^3-288a^2b^2c^2+72a\,b^4c-6b^6} - \frac{a\left(160\right.}{}}{\left(c\,x^2+\right.}$
risch	Expression too large to display

input
```
int(1/x/(c*x^2+b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a^4-1/a^4*((b*c^3*a*(38*a^2*c^2-11*a*b^2*c+b^4)/(64*a^3*c^3-48*a^2*b
^2*c^2+12*a*b^4*c-b^6)*x^5-1/2*c^2*a*(64*a^3*c^3-238*a^2*b^2*c^2+67*a*b^4*
c-6*b^6)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^4+1/6*b*a*c*(160*a^3
*c^3+578*a^2*b^2*c^2-189*a*b^4*c+18*b^6)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b
^4*c-b^6)*x^3-1/2*a*(160*a^4*c^4-328*a^3*b^2*c^3+27*a^2*b^4*c^2+13*a*b^6*c
-2*b^8)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^2-1/2*a^2*b*(44*a^3*c
^3-172*a^2*b^2*c^2+54*a*b^4*c-5*b^6)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c
-b^6)*x-1/6*(352*a^3*c^3-438*a^2*b^2*c^2+124*a*b^4*c-11*b^6)*a^3/(64*a^3*c
^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6))/(c*x^2+b*x+a)^3+1/(64*a^3*c^3-48*a^2*b^
2*c^2+12*a*b^4*c-b^6)*(1/2*(64*a^3*c^4-48*a^2*b^2*c^3+12*a*b^4*c^2-b^6*c)/
c*ln(c*x^2+b*x+a)+2*(102*a^3*b*c^3-59*a^2*b^3*c^2+13*a*b^5*c-b^7-1/2*(64*a
^3*c^4-48*a^2*b^2*c^3+12*a*b^4*c^2-b^6*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1764 vs. $2(270) = 540$.

Time = 0.95 (sec) , antiderivative size = 3548, normalized size of antiderivative = 12.58

$$\int \frac{1}{x\left(a+bx+cx^2\right)^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx+cx^2\right)^4}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(c*x**2+b*x+a)**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(a+bx+cx^2\right)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.44

$$\int \frac{1}{x\left(a+bx+cx^2\right)^4}\,dx = -\frac{\left(b^7-14\,ab^5c+70\,a^2b^3c^2-140\,a^3bc^3\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^4b^6-12\,a^5b^4c+48\,a^6b^2c^2-64\,a^7c^3\right)\sqrt{-b^2+4\,ac}} - \frac{\log\left(cx^2+bx+a\right)}{2\,a^4} + \frac{\log\left(|x|\right)}{a^4} + \frac{11\,a^3b^6-124\,a^4b^4c+438\,a^5b^2c^2-352\,a^6c^3+6\left(ab^5c^3-11\,a^2b^3c^4+38\,a^3bc^5\right)x^5+3\left(6\,ab^6c^2-67\,a^2b^4\right.}{}$$

input
```
integrate(1/x/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

output
```
-(b^7 - 14*a*b^5*c + 70*a^2*b^3*c^2 - 140*a^3*b*c^3)*arctan((2*c*x + b)/sq
rt(-b^2 + 4*a*c))/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*
sqrt(-b^2 + 4*a*c)) - 1/2*log(c*x^2 + b*x + a)/a^4 + log(abs(x))/a^4 + 1/6
*(11*a^3*b^6 - 124*a^4*b^4*c + 438*a^5*b^2*c^2 - 352*a^6*c^3 + 6*(a*b^5*c^
3 - 11*a^2*b^3*c^4 + 38*a^3*b*c^5)*x^5 + 3*(6*a*b^6*c^2 - 67*a^2*b^4*c^3 +
 238*a^3*b^2*c^4 - 64*a^4*c^5)*x^4 + (18*a*b^7*c - 189*a^2*b^5*c^2 + 578*a
^3*b^3*c^3 + 160*a^4*b*c^4)*x^3 + 3*(2*a*b^8 - 13*a^2*b^6*c - 27*a^3*b^4*c
^2 + 328*a^4*b^2*c^3 - 160*a^5*c^4)*x^2 + 3*(5*a^2*b^7 - 54*a^3*b^5*c + 17
2*a^4*b^3*c^2 - 44*a^5*b*c^3)*x)/((c*x^2 + b*x + a)^3*(b^2 - 4*a*c)^3*a^4)
```

Mupad [B] (verification not implemented)

Time = 11.04 (sec) , antiderivative size = 1680, normalized size of antiderivative = 5.96

$$\int \frac{1}{x\left(a+bx+cx^2\right)^4}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(a + b*x + c*x^2)^4),x)
```

output
```
log(x)/a^4 + ((11*b^6 - 352*a^3*c^3 + 438*a^2*b^2*c^2 - 124*a*b^4*c)/(6*a*
(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x^2*(160*a^4*c^4 - 2*
b^8 + 27*a^2*b^4*c^2 - 328*a^3*b^2*c^3 + 13*a*b^6*c))/(2*a^3*(b^6 - 64*a^3
*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(5*b^7 - 44*a^3*b*c^3 + 172*a^2*
b^3*c^2 - 54*a*b^5*c))/(2*a^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^
4*c)) - (x^4*(64*a^3*c^5 - 6*b^6*c^2 + 67*a*b^4*c^3 - 238*a^2*b^2*c^4))/(2
*a^3*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x^3*(18*b^7*c -
189*a*b^5*c^2 + 160*a^3*b*c^4 + 578*a^2*b^3*c^3))/(6*a^3*(b^6 - 64*a^3*c^3
 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (b*c^2*x^5*(b^4*c + 38*a^2*c^3 - 11*a*b
^2*c^2))/(a^3*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))/(x^2*(3*a
*b^2 + 3*a^2*c) + x^4*(3*a*c^2 + 3*b^2*c) + a^3 + x^3*(b^3 + 6*a*b*c) + c^
3*x^6 + 3*b*c^2*x^5 + 3*a^2*b*x) + (log(2*a*b^14 + 2*b^15*x - 24576*a^8*c^
7 + 2*a*b^7*(-(4*a*c - b^2)^7)^(1/2) - 55*a^2*b^12*c + 2*b^8*x*(-(4*a*c -
b^2)^7)^(1/2) + 647*a^3*b^10*c^2 - 4218*a^4*b^8*c^3 + 16408*a^5*b^6*c^4 -
37856*a^6*b^4*c^5 + 47488*a^7*b^2*c^6 - 25*a^2*b^5*c*(-(4*a*c - b^2)^7)^(1
/2) - 166*a^4*b*c^3*(-(4*a*c - b^2)^7)^(1/2) + 673*a^2*b^11*c^2*x - 4504*a
^3*b^9*c^3*x + 18124*a^4*b^7*c^4*x - 43792*a^5*b^5*c^5*x + 58688*a^6*b^3*c
^6*x + 192*a^4*c^4*x*(-(4*a*c - b^2)^7)^(1/2) - 56*a*b^13*c*x + 107*a^3*b^
3*c^2*(-(4*a*c - b^2)^7)^(1/2) - 33536*a^7*b*c^7*x - 28*a*b^6*c*x*(-(4*a*c
 - b^2)^7)^(1/2) + 143*a^2*b^4*c^2*x*(-(4*a*c - b^2)^7)^(1/2) - 310*a^3...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 3378, normalized size of antiderivative = 11.98

$$\int \frac{1}{x\left(a+bx+cx^2\right)^4}\,dx = \text{Too large to display}$$

input
```
int(1/x/(c*x^2+b*x+a)^4,x)
```

output
```
( - 840*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**6*b*c**
3 + 420*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**5*b**3*
c**2 - 2520*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**5*b
**2*c**3*x - 2520*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*
a**5*b*c**4*x**2 - 84*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**
2))*a**4*b**5*c + 1260*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b*
*2))*a**4*b**4*c**2*x - 1260*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*
c - b**2))*a**4*b**3*c**3*x**2 - 5040*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/
sqrt(4*a*c - b**2))*a**4*b**2*c**4*x**3 - 2520*sqrt(4*a*c - b**2)*atan((b
+ 2*c*x)/sqrt(4*a*c - b**2))*a**4*b*c**5*x**4 + 6*sqrt(4*a*c - b**2)*atan(
(b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**7 - 252*sqrt(4*a*c - b**2)*atan((b
 + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**6*c*x + 1008*sqrt(4*a*c - b**2)*atan
((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**5*c**2*x**2 + 1680*sqrt(4*a*c - b
**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**4*c**3*x**3 - 1260*sqrt(
4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**3*c**4*x**4 - 2
520*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**2*c**5
*x**5 - 840*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b
*c**6*x**6 + 18*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*
*2*b**8*x - 234*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*
*2*b**7*c*x**2 - 84*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b*...
```

3.265 $\int \frac{1}{x^2(a+bx+cx^2)^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 334

$$\begin{aligned}\int \frac{1}{x^2(a+bx+cx^2)^4}\,dx &= -\frac{1}{a^4x}-\frac{b(b^2-3ac)+c(b^2-2ac)x}{3a^2(b^2-4ac)(a+bx+cx^2)^3}\\
&\quad-\frac{b(3b^2-7ac)(b^2-5ac)+c(3b^4-20ab^2c+22a^2c^2)x}{3a^3(b^2-4ac)^2(a+bx+cx^2)^2}\\
&\quad-\frac{b(3b^6-34ab^4c+124a^2b^2c^2-134a^3c^3)+c(3b^6-32ab^4c+104a^2b^2c^2-76a^3c^3)x}{a^4(b^2-4ac)^3(a+bx+cx^2)}\\
&\quad-\frac{4(b^8-14ab^6c+70a^2b^4c^2-140a^3b^2c^3+70a^4c^4)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^5(b^2-4ac)^{7/2}}\\
&\quad-\frac{4b\log(x)}{a^5}+\frac{2b\log(a+bx+cx^2)}{a^5}\end{aligned}$$

output
```
-1/a^4/x-1/3*(b*(-3*a*c+b^2)+c*(-2*a*c+b^2)*x)/a^2/(-4*a*c+b^2)/(c*x^2+b*x
+a)^3-1/3*(b*(-7*a*c+3*b^2)*(-5*a*c+b^2)+c*(22*a^2*c^2-20*a*b^2*c+3*b^4)*x
)/a^3/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^2-(b*(-134*a^3*c^3+124*a^2*b^2*c^2-34*a
*b^4*c+3*b^6)+c*(-76*a^3*c^3+104*a^2*b^2*c^2-32*a*b^4*c+3*b^6)*x)/a^4/(-4*
a*c+b^2)^3/(c*x^2+b*x+a)-4*(70*a^4*c^4-140*a^3*b^2*c^3+70*a^2*b^4*c^2-14*a
*b^6*c+b^8)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^5/(-4*a*c+b^2)^(7/2)-4
*b*ln(x)/a^5+2*b*ln(c*x^2+b*x+a)/a^5
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^2 (a+bx+cx^2)^4}\,dx$$

$$= \frac{-\frac{3a}{x} + \frac{a^3(b^3-3abc+b^2cx-2ac^2x)}{(-b^2+4ac)(a+x(b+cx))^3} - \frac{a^2(3b^5-22ab^3c+35a^2bc^2+3b^4cx-20ab^2c^2x+22a^2c^3x)}{(b^2-4ac)^2(a+x(b+cx))^2} + \frac{3a(-3b^7+34ab^5c-124a^2b^3c^2+134a^3bc^3-}{(b^2-4ac)^3(}}{}$$

input
```
Integrate[1/(x^2*(a + b*x + c*x^2)^4),x]
```

output
```
((-3*a)/x + (a^3*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((-b^2 + 4*a*c)*(a
 + x*(b + c*x))^3) - (a^2*(3*b^5 - 22*a*b^3*c + 35*a^2*b*c^2 + 3*b^4*c*x -
 20*a*b^2*c^2*x + 22*a^2*c^3*x))/((b^2 - 4*a*c)^2*(a + x*(b + c*x))^2) + (
3*a*(-3*b^7 + 34*a*b^5*c - 124*a^2*b^3*c^2 + 134*a^3*b*c^3 - 3*b^6*c*x + 3
2*a*b^4*c^2*x - 104*a^2*b^2*c^3*x + 76*a^3*c^4*x))/((b^2 - 4*a*c)^3*(a + x
*(b + c*x))) - (12*(b^8 - 14*a*b^6*c + 70*a^2*b^4*c^2 - 140*a^3*b^2*c^3 +
70*a^4*c^4)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(7/2) -
 12*b*Log[x] + 6*b*Log[a + x*(b + c*x)])/(3*a^5)
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1165, 27, 1235, 27, 1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx+cx^2)^4} \, dx$$

$$\Big\downarrow \text{1165}$$

$$\frac{-2ac+b^2+bcx}{3ax (b^2-4ac) (a+bx+cx^2)^3}-\frac{\int -\frac{2(2b^2+3cxb-7ac)}{x^2(cx^2+bx+a)^3}dx}{3a (b^2-4ac)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{2b^2+3cxb-7ac}{x^2(cx^2+bx+a)^3}dx}{3a (b^2-4ac)}+\frac{-2ac+b^2+bcx}{3ax (b^2-4ac) (a+bx+cx^2)^3}$$

$$\Big\downarrow \text{1235}$$

$$\frac{2\left(\frac{2(7a^2c^2-7ab^2c+b^4)+bcx(2b^2-13ac)}{2ax(b^2-4ac)(a+bx+cx^2)^2}-\frac{\int -\frac{2\left(\left(3b^2-7ac\right)\left(b^2-5ac\right)+2bc\left(2b^2-13ac\right)x\right)}{x^2\left(cx^2+bx+a\right)^2}dx}{2a(b^2-4ac)}\right)}{3a (b^2-4ac)}+\frac{-2ac+b^2+bcx}{3ax (b^2-4ac) (a+bx+cx^2)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{\int \frac{\left(3b^2-7ac\right)\left(b^2-5ac\right)+2bc\left(2b^2-13ac\right)x}{x^2\left(cx^2+bx+a\right)^2}dx}{a(b^2-4ac)}+\frac{2(7a^2c^2-7ab^2c+b^4)+bcx(2b^2-13ac)}{2ax(b^2-4ac)(a+bx+cx^2)^2}\right)}{3a (b^2-4ac)}+\frac{-2ac+b^2+bcx}{3ax (b^2-4ac) (a+bx+cx^2)^3}$$

$$\Big\downarrow \text{1235}$$

$$2\left(\frac{\frac{-70a^3c^3+105a^2b^2c^2+3bcx\left(29a^2c^2-10ab^2c+b^4\right)-32ab^4c+3b^6}{ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int -\frac{6\left(b^6-11acb^4+38a^2c^2b^2+c\left(b^4-10acb^2+29a^2c^2\right)xb-35a^3c^3\right)}{x^2\left(cx^2+bx+a\right)}dx}{a\left(b^2-4ac\right)}}{a(b^2-4ac)}+\frac{2(7a^2c^2-7ab}{2ax(b^2-}\right.$$

$$\overline{\qquad 3a\left(b^2-4ac\right)}$$

$$\frac{-2ac+b^2+bcx}{3ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{\frac{6\int \frac{b^6-11acb^4+38a^2c^2b^2+c\left(b^4-10acb^2+29a^2c^2\right)xb-35a^3c^3}{x^2\left(cx^2+bx+a\right)}dx}{a\left(b^2-4ac\right)}+\frac{-70a^3c^3+105a^2b^2c^2+3bcx\left(29a^2c^2-10ab^2c+b^4\right)-32ab^4c+3b^6}{ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)}}{a(b^2-4ac)}+\frac{2(7a^2c^2-7ab^2c+}{2ax(b^2-4ac}\right.$$

$$\overline{\qquad 3a\left(b^2-4ac\right)}$$

$$\frac{-2ac+b^2+bcx}{3ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}$$

$$\Big\downarrow \; 1200$$

$$2\left(\frac{\frac{6\int \left(\frac{b\left(4ac-b^2\right)^3}{a^2x}+\frac{bcx\left(b^2-4ac\right)^3+\left(b^2-5ac\right)\left(b^6-8acb^4+19a^2c^2b^2-7a^3c^3\right)}{a^2\left(cx^2+bx+a\right)}+\frac{b^6-11acb^4+38a^2c^2b^2-35a^3c^3}{ax^2}\right)dx}{a\left(b^2-4ac\right)}+\frac{-70a^3c^3+105a^2b^2c^2+3bcx\left(29a^2c^2-\right.}{ax\left(b^2-4ac\right)\left(a+b\right.}}{a(b^2-4ac)}\right.$$

$$\overline{\qquad 3a\left(b^2-4ac\right)}$$

$$\frac{-2ac+b^2+bcx}{3ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}$$

$$\Big\downarrow \; 2009$$

$$2\left(\frac{2(7a^2c^2-7ab^2c+b^4)+bcx(2b^2-13ac)}{2ax(b^2-4ac)(a+bx+cx^2)^2}+\frac{\frac{-70a^3c^3+105a^2b^2c^2+3bcx\left(29a^2c^2-10ab^2c+b^4\right)-32ab^4c+3b^6}{ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{6\left(\frac{b\left(b^2-4ac\right)^3\log\left(a+bx+cx^2\right)}{2a^2}-\frac{b\log(x)}{}\right.}{}}{}\right.$$

$$\overline{\qquad 3a\left(b^2-4ac\right)}$$

$$\frac{-2ac+b^2+bcx}{3ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}$$

input
```
Int[1/(x^2*(a + b*x + c*x^2)^4),x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(3*a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)^3) + (2*((2*(
b^4 - 7*a*b^2*c + 7*a^2*c^2) + b*c*(2*b^2 - 13*a*c)*x)/(2*a*(b^2 - 4*a*c)*
x*(a + b*x + c*x^2)^2) + ((3*b^6 - 32*a*b^4*c + 105*a^2*b^2*c^2 - 70*a^3*c
^3 + 3*b*c*(b^4 - 10*a*b^2*c + 29*a^2*c^2)*x)/(a*(b^2 - 4*a*c)*x*(a + b*x
+ c*x^2)) + (6*(-((b^6 - 11*a*b^4*c + 38*a^2*b^2*c^2 - 35*a^3*c^3)/(a*x))
- ((b^8 - 14*a*b^6*c + 70*a^2*b^4*c^2 - 140*a^3*b^2*c^3 + 70*a^4*c^4)*ArcT
anh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - (b*(b^2 - 4*
a*c)^3*Log[x])/a^2 + (b*(b^2 - 4*a*c)^3*Log[a + b*x + c*x^2])/(2*a^2)))/(a
*(b^2 - 4*a*c)))/(a*(b^2 - 4*a*c))))/(3*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 711 vs. 2(326) = 652.

Time = 0.84 (sec) , antiderivative size = 712, normalized size of antiderivative = 2.13

method	result
default	$-\frac{1}{a^4 x}-\frac{4b\ln(x)}{a^5}-\frac{\frac{c^3 a\left(76a^3c^3-104a^2b^2c^2+32a\,b^4c-3b^6\right)x^5}{64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6}+\frac{b\,c^2 a\left(286a^3c^3-332a^2b^2c^2+98a\,b^4c-9b^6\right)x^4}{64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6}+\frac{ac\left(544a^4c^4+306a^3b^2c^3-\right.}{192a^3c^3-144a}}{}$
risch	Expression too large to display

input
```
int(1/x^2/(c*x^2+b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/a^4/x-4*b*ln(x)/a^5-1/a^5*((c^3*a*(76*a^3*c^3-104*a^2*b^2*c^2+32*a*b^4*
c-3*b^6)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^5+b*c^2*a*(286*a^3*c
^3-332*a^2*b^2*c^2+98*a*b^4*c-9*b^6)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c
-b^6)*x^4+1/3*a*c*(544*a^4*c^4+306*a^3*b^2*c^3-832*a^2*b^4*c^2+279*a*b^6*c
-27*b^8)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^3+b*a*(496*a^4*c^4-3
97*a^3*b^2*c^3+30*a^2*b^4*c^2+20*a*b^6*c-3*b^8)/(64*a^3*c^3-48*a^2*b^2*c^2
+12*a*b^4*c-b^6)*x^2+a^2*(116*a^4*c^4+166*a^3*b^2*c^3-243*a^2*b^4*c^2+75*a
*b^6*c-7*b^8)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x+1/3*(590*a^3*c^
3-535*a^2*b^2*c^2+147*a*b^4*c-13*b^6)*a^3*b/(64*a^3*c^3-48*a^2*b^2*c^2+12*
a*b^4*c-b^6))/(c*x^2+b*x+a)^3+4/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)
*(1/2*(-64*a^3*b*c^4+48*a^2*b^3*c^3-12*a*b^5*c^2+b^7*c)/c*ln(c*x^2+b*x+a)+
2*(35*a^4*c^4-102*a^3*b^2*c^3+59*a^2*b^4*c^2-13*a*b^6*c+b^8-1/2*(-64*a^3*b
*c^4+48*a^2*b^3*c^3-12*a*b^5*c^2+b^7*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1957 vs. $2(326) = 652$.

Time = 1.52 (sec) , antiderivative size = 3934, normalized size of antiderivative = 11.78

$$\int \frac{1}{x^2 (a + bx + cx^2)^4} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx + cx^2)^4} \, dx = \text{Timed out}$$

input
```
integrate(1/x**2/(c*x**2+b*x+a)**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (a + bx + cx^2)^4} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^2/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.49

$$\int \frac{1}{x^2 (a + bx + cx^2)^4} \, dx$$
$$= \frac{4 \left(b^8 - 14 \, ab^6 c + 70 \, a^2 b^4 c^2 - 140 \, a^3 b^2 c^3 + 70 \, a^4 c^4\right) \arctan\left(\frac{2 \, cx + b}{\sqrt{-b^2 + 4 \, ac}}\right)}{\left(a^5 b^6 - 12 \, a^6 b^4 c + 48 \, a^7 b^2 c^2 - 64 \, a^8 c^3\right) \sqrt{-b^2 + 4 \, ac}} + \frac{2 \, b \log\left(cx^2 + bx + a\right)}{a^5} - \frac{4 \, b \log\left(|x|\right)}{a^5} - \frac{3 \, a^4 b^6 - 36 \, a^5 b^4 c + 144 \, a^6 b^2 c^2 - 192 \, a^7 c^3 + 12 \left(ab^6 c^3 - 11 \, a^2 b^4 c^4 + 38 \, a^3 b^2 c^5 - 35 \, a^4 c^6\right) x^6 + 6 \left(6 \, ab^7 c^2\right.}{}$$

input
```
integrate(1/x^2/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

output
```
4*(b^8 - 14*a*b^6*c + 70*a^2*b^4*c^2 - 140*a^3*b^2*c^3 + 70*a^4*c^4)*arcta
n((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^
2 - 64*a^8*c^3)*sqrt(-b^2 + 4*a*c)) + 2*b*log(c*x^2 + b*x + a)/a^5 - 4*b*l
og(abs(x))/a^5 - 1/3*(3*a^4*b^6 - 36*a^5*b^4*c + 144*a^6*b^2*c^2 - 192*a^7
*c^3 + 12*(a*b^6*c^3 - 11*a^2*b^4*c^4 + 38*a^3*b^2*c^5 - 35*a^4*c^6)*x^6 +
 6*(6*a*b^7*c^2 - 67*a^2*b^5*c^3 + 238*a^3*b^3*c^4 - 239*a^4*b*c^5)*x^5 +
2*(18*a*b^8*c - 189*a^2*b^6*c^2 + 578*a^3*b^4*c^3 - 225*a^4*b^2*c^4 - 560*
a^5*c^5)*x^4 + 3*(4*a*b^9 - 26*a^2*b^7*c - 54*a^3*b^5*c^2 + 621*a^4*b^3*c^
3 - 880*a^5*b*c^4)*x^3 + 3*(10*a^2*b^8 - 108*a^3*b^6*c + 351*a^4*b^4*c^2 -
 214*a^5*b^2*c^3 - 308*a^6*c^4)*x^2 + (22*a^3*b^7 - 255*a^4*b^5*c + 967*a^
5*b^3*c^2 - 1166*a^6*b*c^3)*x)/((c*x^2 + b*x + a)^3*(b^2 - 4*a*c)^3*a^5*x)
```

Mupad [B] (verification not implemented)

Time = 11.34 (sec) , antiderivative size = 1856, normalized size of antiderivative = 5.56

$$\int \frac{1}{x^2 (a+bx+cx^2)^4} \, dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a + b*x + c*x^2)^4),x)
```

output
```
(2*log(2*a*b^8*(-(4*a*c - b^2)^7)^(1/2) - 2*b^16*x - 2*a*b^15 + 55*a^2*b^1
3*c + 26816*a^8*b*c^7 - 4480*a^8*c^8*x + 2*b^9*x*(-(4*a*c - b^2)^7)^(1/2)
- 647*a^3*b^11*c^2 + 4218*a^4*b^9*c^3 - 16443*a^5*b^7*c^4 + 38276*a^6*b^5*
c^5 - 49168*a^7*b^3*c^6 + 35*a^5*c^4*(-(4*a*c - b^2)^7)^(1/2) - 25*a^2*b^6
*c*(-(4*a*c - b^2)^7)^(1/2) - 673*a^2*b^12*c^2*x + 4504*a^3*b^10*c^3*x - 1
8159*a^4*b^8*c^4*x + 44282*a^5*b^6*c^5*x - 61208*a^6*b^4*c^6*x + 39136*a^7
*b^2*c^7*x + 56*a*b^14*c*x + 107*a^3*b^4*c^2*(-(4*a*c - b^2)^7)^(1/2) - 16
6*a^4*b^2*c^3*(-(4*a*c - b^2)^7)^(1/2) - 28*a*b^7*c*x*(-(4*a*c - b^2)^7)^(
1/2) + 227*a^4*b*c^4*x*(-(4*a*c - b^2)^7)^(1/2) + 143*a^2*b^5*c^2*x*(-(4*a
*c - b^2)^7)^(1/2) - 310*a^3*b^3*c^3*x*(-(4*a*c - b^2)^7)^(1/2))*(b^8*(-(4
*a*c - b^2)^7)^(1/2) - b^15 + 16384*a^7*b*c^7 - 336*a^2*b^11*c^2 + 2240*a^
3*b^9*c^3 - 8960*a^4*b^7*c^4 + 21504*a^5*b^5*c^5 - 28672*a^6*b^3*c^6 + 70*
a^4*c^4*(-(4*a*c - b^2)^7)^(1/2) + 28*a*b^13*c + 70*a^2*b^4*c^2*(-(4*a*c -
 b^2)^7)^(1/2) - 140*a^3*b^2*c^3*(-(4*a*c - b^2)^7)^(1/2) - 14*a*b^6*c*(-(
4*a*c - b^2)^7)^(1/2)))/(a^5*(4*a*c - b^2)^7) - (4*b*log(x))/a^5 - (2*log(
2*a*b^15 + 2*b^16*x + 2*a*b^8*(-(4*a*c - b^2)^7)^(1/2) - 55*a^2*b^13*c - 2
6816*a^8*b*c^7 + 4480*a^8*c^8*x + 2*b^9*x*(-(4*a*c - b^2)^7)^(1/2) + 647*a
^3*b^11*c^2 - 4218*a^4*b^9*c^3 + 16443*a^5*b^7*c^4 - 38276*a^6*b^5*c^5 + 4
9168*a^7*b^3*c^6 + 35*a^5*c^4*(-(4*a*c - b^2)^7)^(1/2) - 25*a^2*b^6*c*(-(4
*a*c - b^2)^7)^(1/2) + 673*a^2*b^12*c^2*x - 4504*a^3*b^10*c^3*x + 18159...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 3927, normalized size of antiderivative = 11.76

$$\int \frac{1}{x^2 (a+bx+cx^2)^4} \, dx = \text{Too large to display}$$

input
```
int(1/x^2/(c*x^2+b*x+a)^4,x)
```

output
```
( - 840*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**7*b*c**
4*x + 1680*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**6*b*
*3*c**3*x - 2520*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a
**6*b**2*c**4*x**2 - 2520*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c -
 b**2))*a**6*b*c**5*x**3 - 840*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*
a*c - b**2))*a**5*b**5*c**2*x + 5040*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/s
qrt(4*a*c - b**2))*a**5*b**4*c**3*x**2 + 2520*sqrt(4*a*c - b**2)*atan((b +
 2*c*x)/sqrt(4*a*c - b**2))*a**5*b**3*c**4*x**3 - 5040*sqrt(4*a*c - b**2)*
atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**5*b**2*c**5*x**4 - 2520*sqrt(4*a*c
 - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**5*b*c**6*x**5 + 168*sqrt(
4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b**7*c*x - 2520*sq
rt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b**6*c**2*x**2
+ 2520*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b**5*c
**3*x**3 + 9240*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*
*4*b**4*c**4*x**4 + 2520*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c -
b**2))*a**4*b**3*c**5*x**5 - 2520*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt
(4*a*c - b**2))*a**4*b**2*c**6*x**6 - 840*sqrt(4*a*c - b**2)*atan((b + 2*c
*x)/sqrt(4*a*c - b**2))*a**4*b*c**7*x**7 - 12*sqrt(4*a*c - b**2)*atan((b +
 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**9*x + 504*sqrt(4*a*c - b**2)*atan((b +
 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**8*c*x**2 - 2016*sqrt(4*a*c - b**2)*...
```

3.266 $\int \frac{x^4}{1+x+x^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 45

$$\int \frac{x^4}{1+x+x^2} \, dx = -\frac{x^2}{2}+\frac{x^3}{3}-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1+x+x^2\right)$$

output
```
-1/2*x^2+1/3*x^3-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/2*ln(x^2+x+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{x^4}{1+x+x^2} \, dx = \frac{1}{6}\left(x^2(-3+2x)-2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+3\log\left(1+x+x^2\right)\right)$$

input
```
Integrate[x^4/(1 + x + x^2),x]
```

output
```
(x^2*(-3 + 2*x) - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 3*Log[1 + x + x^2]
)/6
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^2+x+1} \, dx$$

$$\Big\downarrow \ 1143$$

$$\int \left(x^2+\frac{x}{x^2+x+1}-x\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{x^3}{3}-\frac{x^2}{2}+\frac{1}{2}\log\left(x^2+x+1\right)$$

input
```
Int[x^4/(1 + x + x^2),x]
```

output
```
-1/2*x^2 + x^3/3 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 + x + x^2]/2
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{x^{2}}{2}+\frac{x^{3}}{3}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln\left(x^{2}+x+1\right)}{2}$	37
risch	$\frac{x^{3}}{3}-\frac{x^{2}}{2}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln\left(4x^{2}+4x+4\right)}{2}$	41

input
```
int(x^4/(x^2+x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2+1/3*x^3-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/2*ln(x^2+x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \frac{x^4}{1+x+x^2}\,dx = \frac{1}{3}\,x^{3} - \frac{1}{2}\,x^{2} - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{2}\,\log\left(x^{2}+x+1\right)$$

input
```
integrate(x^4/(x^2+x+1),x, algorithm="fricas")
```

output
```
1/3*x^3 - 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2*log(x^
2 + x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{x^4}{1+x+x^2}\,dx = \frac{x^{3}}{3} - \frac{x^{2}}{2} + \frac{\log\left(x^{2}+x+1\right)}{2} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate(x**4/(x**2+x+1),x)
```

output
```
x**3/3 - x**2/2 + log(x**2 + x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(
3)/3)/3
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \frac{x^4}{1+x+x^2}\,dx = \frac{1}{3}\,x^3 - \frac{1}{2}\,x^2 - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{2}\,\log\left(x^2+x+1\right)$$

input
```
integrate(x^4/(x^2+x+1),x, algorithm="maxima")
```

output
```
1/3*x^3 - 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2*log(x^
2 + x + 1)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \frac{x^4}{1+x+x^2}\,dx = \frac{1}{3}\,x^3 - \frac{1}{2}\,x^2 - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{2}\,\log\left(x^2+x+1\right)$$

input
```
integrate(x^4/(x^2+x+1),x, algorithm="giac")
```

output
```
1/3*x^3 - 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2*log(x^
2 + x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{x^4}{1+x+x^2}\,dx = \frac{\ln\left(x^2+x+1\right)}{2} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{3} - \frac{x^2}{2} + \frac{x^3}{3}$$

input
```
int(x^4/(x + x^2 + 1),x)
```

output
```
log(x + x^2 + 1)/2 - (3^(1/2)*atan((2*3^(1/2)*x)/3 + 3^(1/2)/3))/3 - x^2/2
 + x^3/3
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{1+x+x^2}\,dx = -\frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{3} + \frac{\log(x^2+x+1)}{2} + \frac{x^3}{3} - \frac{x^2}{2}$$

input
```
int(x^4/(x^2+x+1),x)
```

output
```
( - 2*sqrt(3)*atan((2*x + 1)/sqrt(3)) + 3*log(x**2 + x + 1) + 2*x**3 - 3*x
**2)/6
```

3.267 $\int \frac{x^3}{1+x+x^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 30

$$\int \frac{x^3}{1+x+x^2} \, dx = -x + \frac{x^2}{2} + \frac{2 \arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
-x+1/2*x^2+2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{1+x+x^2} \, dx = -x + \frac{x^2}{2} + \frac{2 \arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Integrate[x^3/(1 + x + x^2),x]
```

output
```
-x + x^2/2 + (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^2+x+1} \, dx$$
$$\Big\downarrow \text{1143}$$
$$\int \left(\frac{1}{x^2+x+1}+x-1\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{x^2}{2}-x$$

input
```
Int[x^3/(1 + x + x^2),x]
```

output
```
-x + x^2/2 + (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

method	result	size
default	$-x+\frac{x^2}{2}+\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	26
risch	$-x+\frac{x^2}{2}+\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	26

input
```
int(x^3/(x^2+x+1),x,method=_RETURNVERBOSE)
```

output
```
-x+1/2*x^2+2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{1+x+x^2}\,dx=\frac{1}{2}\,x^2+\frac{2}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-x$$

input
```
integrate(x^3/(x^2+x+1),x, algorithm="fricas")
```

output
```
1/2*x^2 + 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{1+x+x^2}\,dx=\frac{x^2}{2}-x+\frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate(x**3/(x**2+x+1),x)
```

output
```
x**2/2 - x + 2*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{1+x+x^2}\,dx = \frac{1}{2}\,x^2 + \frac{2}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - x$$

input
```
integrate(x^3/(x^2+x+1),x, algorithm="maxima")
```

output
```
1/2*x^2 + 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - x
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{1+x+x^2}\,dx = \frac{1}{2}\,x^2 + \frac{2}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - x$$

input
```
integrate(x^3/(x^2+x+1),x, algorithm="giac")
```

output
```
1/2*x^2 + 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - x
```

Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{1+x+x^2}\,dx = \frac{2\,\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{3} - x + \frac{x^2}{2}$$

input
```
int(x^3/(x + x^2 + 1),x)
```

output
```
(2*3^(1/2)*atan((2*3^(1/2)*x)/3 + 3^(1/2)/3))/3 - x + x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{x^3}{1+x+x^2}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{3} + \frac{x^2}{2} - x$$

input
```
int(x^3/(x^2+x+1),x)
```

output
```
(4*sqrt(3)*atan((2*x + 1)/sqrt(3)) + 3*x**2 - 6*x)/6
```

3.268 $\int \frac{x^2}{1+x+x^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \frac{x^2}{1+x+x^2} dx = x - \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{2}\log\left(1+x+x^2\right)$$

output
```
x-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/2*ln(x^2+x+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{1+x+x^2} dx = x - \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{2}\log\left(1+x+x^2\right)$$

input
```
Integrate[x^2/(1 + x + x^2),x]
```

output
```
x - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x + x^2]/2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^2+x+1} \, dx$$
$$\Big\downarrow \ 1143$$
$$\int \left(1-\frac{x+1}{x^2+x+1}\right)dx$$
$$\Big\downarrow \ 2009$$
$$-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(x^2+x+1\right)+x$$

input
```
Int[x^2/(1 + x + x^2),x]
```

output
```
x - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x + x^2]/2
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

method	result	size
default	$x-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\frac{\ln\left(x^{2}+x+1\right)}{2}$	28
risch	$x-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\frac{\ln\left(4x^{2}+4x+4\right)}{2}$	32

input
```
int(x^2/(x^2+x+1),x,method=_RETURNVERBOSE)
```

output
```
x-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/2*ln(x^2+x+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{1+x+x^2}\,dx=-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x-\frac{1}{2}\log\left(x^2+x+1\right)$$

input
```
integrate(x^2/(x^2+x+1),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x - 1/2*log(x^2 + x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{1+x+x^2}\,dx=x-\frac{\log\left(x^2+x+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate(x**2/(x**2+x+1),x)
```

output
```
x - log(x**2 + x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{1+x+x^2} dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + x - \frac{1}{2}\log\left(x^2+x+1\right)$$

input
```
integrate(x^2/(x^2+x+1),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x - 1/2*log(x^2 + x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{1+x+x^2} dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + x - \frac{1}{2}\log\left(x^2+x+1\right)$$

input
```
integrate(x^2/(x^2+x+1),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x - 1/2*log(x^2 + x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{1+x+x^2} dx = x - \frac{\ln\left(x^2+x+1\right)}{2} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
int(x^2/(x + x^2 + 1),x)
```

output
```
x - log(x + x^2 + 1)/2 - (3^(1/2)*atan((2*3^(1/2)*x)/3 + 3^(1/2)/3))/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{1+x+x^2}\,dx = -\frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{3} - \frac{\log(x^2+x+1)}{2} + x$$

input
```
int(x^2/(x^2+x+1),x)
```

output
```
( - 2*sqrt(3)*atan((2*x + 1)/sqrt(3)) - 3*log(x**2 + x + 1) + 6*x)/6
```

3.269 $\int \frac{x}{1+x+x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 31

$$\int \frac{x}{1+x+x^2} dx = -\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}\log\left(1+x+x^2\right)$$

output
```
-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/2*ln(x^2+x+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+x+x^2} dx = -\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}\log\left(1+x+x^2\right)$$

input
```
Integrate[x/(1 + x + x^2),x]
```

output
```
-(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 + x + x^2]/2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^2+x+1}dx$$

$$\Big\downarrow \ 1142$$

$$\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-\frac{1}{2}\int \frac{1}{x^2+x+1}dx$$

$$\Big\downarrow \ 1083$$

$$\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx+\int \frac{1}{-(2x+1)^2-3}d(2x+1)$$

$$\Big\downarrow \ 217$$

$$\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{2}\log\left(x^2+x+1\right)-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[x/(1 + x + x^2),x]
```

output
```
-(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 + x + x^2]/2
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln(x^2+x+1)}{2}$	27
risch	$-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln(4x^2+4x+4)}{2}$	31

input
```
int(x/(x^2+x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/2*ln(x^2+x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x}{1+x+x^2}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{2}\log\left(x^2+x+1\right)$$

input
```
integrate(x/(x^2+x+1),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2*log(x^2 + x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{x}{1+x+x^2}\,dx = \frac{\log\left(x^2+x+1\right)}{2} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate(x/(x**2+x+1),x)
```

output
```
log(x**2 + x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x}{1+x+x^2}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{2}\log\left(x^2+x+1\right)$$

input
```
integrate(x/(x^2+x+1),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2*log(x^2 + x + 1)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x}{1+x+x^2}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{2}\log\left(x^2+x+1\right)$$

input
```
integrate(x/(x^2+x+1),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2*log(x^2 + x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{x}{1+x+x^2}\,dx = \frac{\ln\left(x^2+x+1\right)}{2} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
int(x/(x + x^2 + 1),x)
```

output
```
log(x + x^2 + 1)/2 - (3^(1/2)*atan((2*3^(1/2)*x)/3 + 3^(1/2)/3))/3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{x}{1+x+x^2}\,dx = -\frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{3} + \frac{\log(x^2+x+1)}{2}$$

input
```
int(x/(x^2+x+1),x)
```

output
```
( - 2*sqrt(3)*atan((2*x + 1)/sqrt(3)) + 3*log(x**2 + x + 1))/6
```

3.270 $\int \frac{1}{1+x+x^2} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 19

$$\int \frac{1}{1+x+x^2} \, dx = \frac{2 \arctan \left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x+x^2} \, dx = \frac{2 \arctan \left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Integrate[(1 + x + x^2)^(-1),x]
```

output
```
(2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2+x+1} dx$$
$$\Big\downarrow \text{ 1083}$$
$$-2\int \frac{1}{-(2x+1)^2-3}d(2x+1)$$
$$\Big\downarrow \text{ 217}$$
$$\frac{2\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[(1 + x + x^2)^(-1),x]
```

output
```
(2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	17
risch	$\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	17

input
```
int(1/(x^2+x+1),x,method=_RETURNVERBOSE)
```

output
```
2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{1+x+x^2}\,dx = \frac{2}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)$$

input
```
integrate(1/(x^2+x+1),x, algorithm="fricas")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{1}{1+x+x^2}\,dx = \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate(1/(x**2+x+1),x)
```

output
```
2*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{1+x+x^2}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

input
```
integrate(1/(x^2+x+1),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{1+x+x^2}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

input
```
integrate(1/(x^2+x+1),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{1+x+x^2}\,dx = \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,(2x+1)}{3}\right)}{3}$$

input
```
int(1/(x + x^2 + 1),x)
```

output
```
(2*3^(1/2)*atan((3^(1/2)*(2*x + 1))/3))/3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{1+x+x^2}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{3}$$

input
```
int(1/(x^2+x+1),x)
```

output
```
(2*sqrt(3)*atan((2*x + 1)/sqrt(3)))/3
```

3.271 $\int \frac{1}{x(1+x+x^2)} dx$

Optimal result

Integrand size = 12, antiderivative size = 33

$$\int \frac{1}{x(1+x+x^2)} dx = -\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \log(x) - \frac{1}{2}\log\left(1+x+x^2\right)$$

output
```
-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+ln(x)-1/2*ln(x^2+x+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+x+x^2)} dx = -\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \log(x) - \frac{1}{2}\log\left(1+x+x^2\right)$$

input
```
Integrate[1/(x*(1 + x + x^2)),x]
```

output
```
-(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 + x + x^2]/2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1144, 25, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (x^2+x+1)} \, dx$$

$$\Big\downarrow \text{1144}$$

$$\int -\frac{x+1}{x^2+x+1}dx+\log (x)$$

$$\Big\downarrow \text{25}$$

$$\log (x)-\int \frac{x+1}{x^2+x+1}dx$$

$$\Big\downarrow \text{1142}$$

$$-\frac{1}{2} \int \frac{1}{x^2+x+1}dx-\frac{1}{2} \int \frac{2 x+1}{x^2+x+1}dx+\log (x)$$

$$\Big\downarrow \text{1083}$$

$$-\frac{1}{2} \int \frac{2 x+1}{x^2+x+1}dx+\int \frac{1}{-(2 x+1)^2-3}d(2 x+1)+\log (x)$$

$$\Big\downarrow \text{217}$$

$$-\frac{1}{2} \int \frac{2 x+1}{x^2+x+1}dx-\frac{\arctan \left(\frac{2 x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\log (x)$$

$$\Big\downarrow \text{1103}$$

$$-\frac{\arctan \left(\frac{2 x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2} \log \left(x^2+x+1\right)+\log (x)$$

input
```
Int[1/(x*(1 + x + x^2)),x]
```

output
```
-(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 + x + x^2]/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\ln\left(x\right)-\frac{\ln\left(x^{2}+x+1\right)}{2}$	29
risch	$-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\frac{\ln\left(4x^{2}+4x+4\right)}{2}+\ln\left(x\right)$	33

input
```
int(1/x/(x^2+x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+ln(x)-1/2*ln(x^2+x+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(1+x+x^2)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{2}\log\left(x^2+x+1\right) + \log(x)$$

input
```
integrate(1/x/(x^2+x+1),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{1}{x(1+x+x^2)}\,dx = \log(x) - \frac{\log(x^2+x+1)}{2} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate(1/x/(x**2+x+1),x)
```

output
```
log(x) - log(x**2 + x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(1+x+x^2)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{2}\log\left(x^2+x+1\right) + \log(x)$$

input
```
integrate(1/x/(x^2+x+1),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{x(1+x+x^2)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{2}\log\left(x^2+x+1\right) + \log(|x|)$$

input
```
integrate(1/x/(x^2+x+1),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1) + log(ab
s(x))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(1+x+x^2)}\,dx = \ln(x) - \frac{\ln\left(x^2+x+1\right)}{2} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,(2x+1)}{3}\right)}{3}$$

input
```
int(1/(x*(x + x^2 + 1)),x)
```

output
```
log(x) - log(x + x^2 + 1)/2 - (3^(1/2)*atan((3^(1/2)*(2*x + 1))/3))/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{1}{x(1+x+x^2)}\,dx = -\frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{3} - \frac{\log(x^2+x+1)}{2} + \log(x)$$

input
```
int(1/x/(x^2+x+1),x)
```

output
```
( - 2*sqrt(3)*atan((2*x + 1)/sqrt(3)) - 3*log(x**2 + x + 1) + 6*log(x))/6
```

3.272 $\int \frac{1}{x^2(1+x+x^2)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 40

$$\int \frac{1}{x^2(1+x+x^2)}\,dx = -\frac{1}{x}-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\log(x)+\frac{1}{2}\log\left(1+x+x^2\right)$$

output
```
-1/x-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-ln(x)+1/2*ln(x^2+x+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(1+x+x^2)}\,dx = -\frac{1}{x}-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\log(x)+\frac{1}{2}\log\left(1+x+x^2\right)$$

input
```
Integrate[1/(x^2*(1 + x + x^2)),x]
```

output
```
-x^(-1) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[x] + Log[1 + x + x^2]/2
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (x^2+x+1)}dx$$

$$\Big\downarrow \text{1145}$$

$$\int -\frac{x+1}{x (x^2+x+1)}dx-\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{x+1}{x (x^2+x+1)}dx-\frac{1}{x}$$

$$\Big\downarrow \text{1200}$$

$$-\int \left(\frac{1}{x}-\frac{x}{x^2+x+1}\right)dx-\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(x^2+x+1\right)-\frac{1}{x}-\log(x)$$

input
```
Int[1/(x^2*(1 + x + x^2)),x]
```

output
```
-x^(-1) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[x] + Log[1 + x + x^2]/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 4.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{1}{x}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\ln\left(x\right)+\frac{\ln\left(x^{2}+x+1\right)}{2}$	36
risch	$-\frac{1}{x}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln\left(4x^{2}+4x+4\right)}{2}-\ln\left(x\right)$	40

input
```
int(1/x^2/(x^2+x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/x-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-ln(x)+1/2*ln(x^2+x+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^2(1+x+x^2)}\,dx = -\frac{2\sqrt{3}x\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-3x\log\left(x^2+x+1\right)+6x\log(x)+6}{6x}$$

input
```
integrate(1/x^2/(x^2+x+1),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*x*arctan(1/3*sqrt(3)*(2*x + 1)) - 3*x*log(x^2 + x + 1) + 6
*x*log(x) + 6)/x
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^2(1+x+x^2)}\,dx = -\log(x)+\frac{\log\left(x^2+x+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}-\frac{1}{x}$$

input
```
integrate(1/x**2/(x**2+x+1),x)
```

output
```
-log(x) + log(x**2 + x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
- 1/x
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2(1+x+x^2)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{x}+\frac{1}{2}\log\left(x^2+x+1\right)-\log(x)$$

input
```
integrate(1/x^2/(x^2+x+1),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/x + 1/2*log(x^2 + x + 1) -
log(x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2(1+x+x^2)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{x} + \frac{1}{2}\log\left(x^2+x+1\right) - \log\left(|x|\right)$$

input
```
integrate(1/x^2/(x^2+x+1),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/x + 1/2*log(x^2 + x + 1) -
log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 8.80 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2(1+x+x^2)}\,dx = \frac{\ln\left(x^2+x+1\right)}{2} - \ln\left(x\right) - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,(2\,x+1)}{3}\right)}{3} - \frac{1}{x}$$

input
```
int(1/(x^2*(x + x^2 + 1)),x)
```

output
```
log(x + x^2 + 1)/2 - log(x) - (3^(1/2)*atan((3^(1/2)*(2*x + 1))/3))/3 - 1/
x
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2\left(1+x+x^2\right)}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)x+3\log\left(x^2+x+1\right)x-6\log\left(x\right)x-6}{6x}$$

input
```
int(1/x^2/(x^2+x+1),x)
```

output
```
( - 2*sqrt(3)*atan((2*x + 1)/sqrt(3))*x + 3*log(x**2 + x + 1)*x - 6*log(x)
*x - 6)/(6*x)
```

3.273 $\int \frac{1}{x^3(1+x+x^2)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 30

$$\int \frac{1}{x^3(1+x+x^2)}\,dx = -\frac{1}{2x^2}+\frac{1}{x}+\frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
-1/2/x^2+1/x+2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3(1+x+x^2)}\,dx = -\frac{1}{2x^2}+\frac{1}{x}+\frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Integrate[1/(x^3*(1 + x + x^2)),x]
```

output
```
-1/2*1/x^2 + x^(-1) + (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (x^2+x+1)} \, dx$$

$$\Big\downarrow \text{ 1145}$$

$$\int -\frac{x+1}{x^2 (x^2+x+1)}dx-\frac{1}{2 x^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{x+1}{x^2 (x^2+x+1)}dx-\frac{1}{2 x^2}$$

$$\Big\downarrow \text{ 1200}$$

$$-\int \left(\frac{1}{x^2}+\frac{1}{-x^2-x-1}\right)dx-\frac{1}{2 x^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2 \arctan \left(\frac{2 x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2 x^2}+\frac{1}{x}$$

input
```
Int[1/(x^3*(1 + x + x^2)),x]
```

output
```
-1/2*1/x^2 + x^(-1) + (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{x-\frac{1}{2}}{x^{2}}+\frac{2 \arctan \left(\frac{(1+2 x) \sqrt{3}}{3}\right) \sqrt{3}}{3}$	25
default	$-\frac{1}{2 x^{2}}+\frac{1}{x}+\frac{2 \arctan \left(\frac{(1+2 x) \sqrt{3}}{3}\right) \sqrt{3}}{3}$	26

input
```
int(1/x^3/(x^2+x+1),x,method=_RETURNVERBOSE)
```

output
```
(x-1/2)/x^2+2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^3\left(1+x+x^2\right)}\,dx = \frac{4\sqrt{3}x^2\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+6x-3}{6x^2}$$

input
```
integrate(1/x^3/(x^2+x+1),x, algorithm="fricas")
```

output
```
1/6*(4*sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x + 1)) + 6*x - 3)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^3\left(1+x+x^2\right)}\,dx = \frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}+\frac{2x-1}{2x^2}$$

input
```
integrate(1/x**3/(x**2+x+1),x)
```

output
```
2*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3 + (2*x - 1)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^3\left(1+x+x^2\right)}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{2x-1}{2x^2}$$

input
```
integrate(1/x^3/(x^2+x+1),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2*(2*x - 1)/x^2
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^3(1+x+x^2)}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{2x-1}{2x^2}$$

input
```
integrate(1/x^3/(x^2+x+1),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2*(2*x - 1)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3(1+x+x^2)}\,dx = \frac{x-\frac{1}{2}}{x^2} + \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
int(1/(x^3*(x + x^2 + 1)),x)
```

output
```
(x - 1/2)/x^2 + (2*3^(1/2)*atan((2*3^(1/2)*x)/3 + 3^(1/2)/3))/3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^3(1+x+x^2)}\,dx = \frac{4\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)x^2+6x-3}{6x^2}$$

input
```
int(1/x^3/(x^2+x+1),x)
```

output
```
(4*sqrt(3)*atan((2*x + 1)/sqrt(3))*x**2 + 6*x - 3)/(6*x**2)
```

3.274 $\int \frac{1}{x^4(1+x+x^2)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 47

$$\int \frac{1}{x^4(1+x+x^2)}\,dx = -\frac{1}{3x^3}+\frac{1}{2x^2}-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{1}{2}\log\left(1+x+x^2\right)$$

output
```
-1/3/x^3+1/2/x^2-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+ln(x)-1/2*ln(x^2+
x+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4(1+x+x^2)}\,dx = -\frac{1}{3x^3}+\frac{1}{2x^2}-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{1}{2}\log\left(1+x+x^2\right)$$

input
```
Integrate[1/(x^4*(1 + x + x^2)),x]
```

output
```
-1/3*1/x^3 + 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[x] - Log[
1 + x + x^2]/2
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (x^2+x+1)} \, dx$$

$$\Big\downarrow \text{1145}$$

$$\int -\frac{x+1}{x^3 (x^2+x+1)}dx-\frac{1}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{x+1}{x^3 (x^2+x+1)}dx-\frac{1}{3x^3}$$

$$\Big\downarrow \text{1200}$$

$$-\int \left(\frac{x+1}{x^2+x+1}-\frac{1}{x}+\frac{1}{x^3}\right)dx-\frac{1}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3x^3}+\frac{1}{2x^2}-\frac{1}{2}\log\left(x^2+x+1\right)+\log(x)$$

input
```
Int[1/(x^4*(1 + x + x^2)),x]
```

output
```
-1/3*1/x^3 + 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[x] - Log[
1 + x + x^2]/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{1}{3x^{3}}+\frac{1}{2x^{2}}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\ln\left(x\right)-\frac{\ln\left(x^{2}+x+1\right)}{2}$	39
risch	$\frac{\frac{x}{2}-\frac{1}{3}}{x^{3}}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\frac{\ln\left(4x^{2}+4x+4\right)}{2}+\ln\left(x\right)$	42

input
```
int(1/x^4/(x^2+x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^3+1/2/x^2-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+ln(x)-1/2*ln(x^2+
x+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^4(1+x+x^2)}\,dx = -\frac{2\sqrt{3}x^3\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+3x^3\log\left(x^2+x+1\right)-6x^3\log(x)-3x+2}{6x^3}$$

input
```
integrate(1/x^4/(x^2+x+1),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x + 1)) + 3*x^3*log(x^2 + x + 1)
 - 6*x^3*log(x) - 3*x + 2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^4(1+x+x^2)}\,dx = \log(x)-\frac{\log\left(x^2+x+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}+\frac{3x-2}{6x^3}$$

input
```
integrate(1/x**4/(x**2+x+1),x)
```

output
```
log(x) - log(x**2 + x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3 +
 (3*x - 2)/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^4(1+x+x^2)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{3x-2}{6x^3}-\frac{1}{2}\log\left(x^2+x+1\right)+\log(x)$$

input
```
integrate(1/x^4/(x^2+x+1),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*(3*x - 2)/x^3 - 1/2*log(x
^2 + x + 1) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^4\left(1+x+x^2\right)}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{3\,x-2}{6\,x^3} - \frac{1}{2}\,\log\left(x^2+x+1\right) + \log\left(|x|\right)$$

input
```
integrate(1/x^4/(x^2+x+1),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*(3*x - 2)/x^3 - 1/2*log(x
^2 + x + 1) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 8.78 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4\left(1+x+x^2\right)}\,dx = \ln\left(x\right) + \frac{\frac{x}{2}-\frac{1}{3}}{x^3} + \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

input
```
int(1/(x^4*(x + x^2 + 1)),x)
```

output
```
log(x) + (x/2 - 1/3)/x^3 + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 -
 1/2) - log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (1 + x + x^2)} \, dx$$
$$= \frac{-2\sqrt{3}\, atan\left(\frac{2x+1}{\sqrt{3}}\right) x^3 - 3\log(x^2 + x + 1)\, x^3 + 6\log(x)\, x^3 + 3x - 2}{6x^3}$$

input
```
int(1/x^4/(x^2+x+1),x)
```

output
```
( - 2*sqrt(3)*atan((2*x + 1)/sqrt(3))*x**3 - 3*log(x**2 + x + 1)*x**3 + 6*
log(x)*x**3 + 3*x - 2)/(6*x**3)
```

3.275 $\int \frac{x^2}{(2+2x+x^2)^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 15

$$\int \frac{x^2}{(2+2x+x^2)^2} dx = \frac{1}{2+2x+x^2} + \arctan(1+x)$$

output
```
1/(x^2+2*x+2)+arctan(1+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(2+2x+x^2)^2} dx = \frac{1}{2+2x+x^2} + \arctan(1+x)$$

input
```
Integrate[x^2/(2 + 2*x + x^2)^2,x]
```

output
```
(2 + 2*x + x^2)^(-1) + ArcTan[1 + x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1153, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x^2+2x+2)^2}dx$$

$$\Big\downarrow \text{1153}$$

$$\int \frac{1}{x^2+2x+2}dx-\frac{x(x+2)}{2(x^2+2x+2)}$$

$$\Big\downarrow \text{1082}$$

$$-\int \frac{1}{-(x+1)^2-1}d(x+1)-\frac{x(x+2)}{2(x^2+2x+2)}$$

$$\Big\downarrow \text{217}$$

$$\arctan(x+1)-\frac{x(x+2)}{2(x^2+2x+2)}$$

input
```
Int[x^2/(2 + 2*x + x^2)^2,x]
```

output
```
-1/2*(x*(2 + x))/(2 + 2*x + x^2) + ArcTan[1 + x]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1153
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m - 2)*(a + b*x +
c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
&& LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{1}{x^2+2x+2}+\arctan\left(x+1\right)$	16
risch	$\frac{1}{x^2+2x+2}+\arctan\left(x+1\right)$	16
parallelrisch	$-\frac{i\ln(x+1-i)x^2-i\ln(x+1+i)x^2+2i\ln(x+1-i)x-2i\ln(x+1+i)x-2+2i\ln(x+1-i)-2i\ln(x+1+i)}{2(x^2+2x+2)}$	77

input
```
int(x^2/(x^2+2*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(x^2+2*x+2)+arctan(x+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \frac{x^2}{(2+2x+x^2)^2}\,dx=\frac{(x^2+2\,x+2)\arctan\left(x+1\right)+1}{x^2+2\,x+2}$$

input
```
integrate(x^2/(x^2+2*x+2)^2,x, algorithm="fricas")
```

output
```
((x^2 + 2*x + 2)*arctan(x + 1) + 1)/(x^2 + 2*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{(2+2x+x^2)^2}\,dx = \operatorname{atan}(x+1) + \frac{1}{x^2+2x+2}$$

input
```
integrate(x**2/(x**2+2*x+2)**2,x)
```

output
```
atan(x + 1) + 1/(x**2 + 2*x + 2)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(2+2x+x^2)^2}\,dx = \frac{1}{x^2+2\,x+2} + \arctan(x+1)$$

input
```
integrate(x^2/(x^2+2*x+2)^2,x, algorithm="maxima")
```

output
```
1/(x^2 + 2*x + 2) + arctan(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(2+2x+x^2)^2}\,dx = \frac{1}{x^2+2\,x+2} + \arctan(x+1)$$

input
```
integrate(x^2/(x^2+2*x+2)^2,x, algorithm="giac")
```

output
```
1/(x^2 + 2*x + 2) + arctan(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(2+2x+x^2)^2} \, dx = \operatorname{atan}(x+1) + \frac{1}{x^2+2\,x+2}$$

input
```
int(x^2/(2*x + x^2 + 2)^2,x)
```

output
```
atan(x + 1) + 1/(2*x + x^2 + 2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.27

$$\int \frac{x^2}{(2+2x+x^2)^2} \, dx = \frac{atan(x+1)\,x^2+2atan(x+1)\,x+2atan(x+1)+1}{x^2+2x+2}$$

input
```
int(x^2/(x^2+2*x+2)^2,x)
```

output
```
(atan(x + 1)*x**2 + 2*atan(x + 1)*x + 2*atan(x + 1) + 1)/(x**2 + 2*x + 2)
```

3.276 $\int \frac{x}{(2+2x+x^2)^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 26

$$\int \frac{x}{(2+2x+x^2)^2} dx = -\frac{2+x}{2(2+2x+x^2)} - \frac{1}{2}\arctan(1+x)$$

output
```
-1/2*(2+x)/(x^2+2*x+2)-1/2*arctan(1+x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x}{(2+2x+x^2)^2} dx = \frac{-2-x}{2(2+2x+x^2)} - \frac{1}{2}\arctan(1+x)$$

input
```
Integrate[x/(2 + 2*x + x^2)^2,x]
```

output
```
(-2 - x)/(2*(2 + 2*x + x^2)) - ArcTan[1 + x]/2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1159, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x^2+2x+2)^2}dx$$

$$\Big\downarrow \text{1159}$$

$$-\frac{1}{2}\int \frac{1}{x^2+2x+2}dx-\frac{x+2}{2(x^2+2x+2)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\int \frac{1}{-(x+1)^2-1}d(x+1)-\frac{x+2}{2(x^2+2x+2)}$$

$$\Big\downarrow \text{217}$$

$$-\frac{1}{2}\arctan(x+1)-\frac{x+2}{2(x^2+2x+2)}$$

input
```
Int[x/(2 + 2*x + x^2)^2,x]
```

output
```
-1/2*(2 + x)/(2 + 2*x + x^2) - ArcTan[1 + x]/2
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{-\frac{x}{2}-1}{x^2+2x+2}-\frac{\arctan(x+1)}{2}$	24
default	$\frac{-2x-4}{4x^2+8x+8}-\frac{\arctan(x+1)}{2}$	25
parallelrisch	$\frac{i\ln(x+1-i)x^2-i\ln(x+1+i)x^2+2i\ln(x+1-i)x-2i\ln(x+1+i)x-2+2i\ln(x+1-i)-2i\ln(x+1+i)+x^2}{4x^2+8x+8}$	80

input
```
int(x/(x^2+2*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*x-1)/(x^2+2*x+2)-1/2*arctan(x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x}{(2+2x+x^2)^2}\,dx = -\frac{(x^2+2\,x+2)\arctan(x+1)+x+2}{2\,(x^2+2\,x+2)}$$

input
```
integrate(x/(x^2+2*x+2)^2,x, algorithm="fricas")
```

output
```
-1/2*((x^2 + 2*x + 2)*arctan(x + 1) + x + 2)/(x^2 + 2*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x}{(2+2x+x^2)^2}\,dx = \frac{-x-2}{2x^2+4x+4} - \frac{\operatorname{atan}(x+1)}{2}$$

input
```
integrate(x/(x**2+2*x+2)**2,x)
```

output
```
(-x - 2)/(2*x**2 + 4*x + 4) - atan(x + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x}{(2+2x+x^2)^2}\,dx = -\frac{x+2}{2\,(x^2+2\,x+2)} - \frac{1}{2}\arctan(x+1)$$

input
```
integrate(x/(x^2+2*x+2)^2,x, algorithm="maxima")
```

output
```
-1/2*(x + 2)/(x^2 + 2*x + 2) - 1/2*arctan(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x}{(2+2x+x^2)^2}\,dx = -\frac{x+2}{2\,(x^2+2\,x+2)} - \frac{1}{2}\arctan(x+1)$$

input
```
integrate(x/(x^2+2*x+2)^2,x, algorithm="giac")
```

output
```
-1/2*(x + 2)/(x^2 + 2*x + 2) - 1/2*arctan(x + 1)
```

Mupad [B] (verification not implemented)

Time = 8.84 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x}{(2+2x+x^2)^2}\,dx = -\frac{\mathrm{atan}(x+1)}{2} - \frac{\frac{x}{2}+1}{x^2+2\,x+2}$$

input
```
int(x/(2*x + x^2 + 2)^2,x)
```

output
```
- atan(x + 1)/2 - (x/2 + 1)/(2*x + x^2 + 2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.54

$$\int \frac{x}{(2+2x+x^2)^2}\,dx = \frac{-2atan(x+1)\,x^2 - 4atan(x+1)\,x - 4atan(x+1) + x^2 - 2}{4x^2+8x+8}$$

input
```
int(x/(x^2+2*x+2)^2,x)
```

output
```
( - 2*atan(x + 1)*x**2 - 4*atan(x + 1)*x - 4*atan(x + 1) + x**2 - 2)/(4*(x
**2 + 2*x + 2))
```

3.277 $\int \frac{x}{5+2x+x^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 26

$$\int \frac{x}{5+2x+x^2} \, dx = -\frac{1}{2}\arctan\left(\frac{1+x}{2}\right)+\frac{1}{2}\log\left(5+2x+x^2\right)$$

output
```
-1/2*arctan(1/2+1/2*x)+1/2*ln(x^2+2*x+5)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x}{5+2x+x^2} \, dx = -\frac{1}{2}\arctan\left(\frac{1+x}{2}\right)+\frac{1}{2}\log\left(5+2x+x^2\right)$$

input
```
Integrate[x/(5 + 2*x + x^2),x]
```

output
```
-1/2*ArcTan[(1 + x)/2] + Log[5 + 2*x + x^2]/2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^2+2x+5}dx$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\int \frac{2(x+1)}{x^2+2x+5}dx-\int \frac{1}{x^2+2x+5}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x+1}{x^2+2x+5}dx-\int \frac{1}{x^2+2x+5}dx$$

$$\Big\downarrow \text{1083}$$

$$\int \frac{x+1}{x^2+2x+5}dx+2\int \frac{1}{-(2x+2)^2-16}d(2x+2)$$

$$\Big\downarrow \text{217}$$

$$\int \frac{x+1}{x^2+2x+5}dx-\frac{1}{2}\arctan\left(\frac{1}{4}(2x+2)\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\log\left(x^2+2x+5\right)-\frac{1}{2}\arctan\left(\frac{1}{4}(2x+2)\right)$$

input
```
Int[x/(5 + 2*x + x^2),x]
```

output
```
-1/2*ArcTan[(2 + 2*x)/4] + Log[5 + 2*x + x^2]/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\arctan\left(\frac{1}{2}+\frac{x}{2}\right)}{2}+\frac{\ln\left(x^{2}+2x+5\right)}{2}$	21
risch	$-\frac{\arctan\left(\frac{1}{2}+\frac{x}{2}\right)}{2}+\frac{\ln\left(x^{2}+2x+5\right)}{2}$	21
parallelrisch	$\frac{\ln(x+1-2i)}{2}+\frac{i\ln(x+1-2i)}{4}+\frac{\ln(x+1+2i)}{2}-\frac{i\ln(x+1+2i)}{4}$	36

input
```
int(x/(x^2+2*x+5),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctan(1/2+1/2*x)+1/2*ln(x^2+2*x+5)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x}{5+2x+x^2}\,dx = -\frac{1}{2}\,\arctan\left(\frac{1}{2}\,x+\frac{1}{2}\right)+\frac{1}{2}\,\log\left(x^2+2\,x+5\right)$$

input
```
integrate(x/(x^2+2*x+5),x, algorithm="fricas")
```

output
```
-1/2*arctan(1/2*x + 1/2) + 1/2*log(x^2 + 2*x + 5)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x}{5+2x+x^2}\,dx = \frac{\log\left(x^2+2x+5\right)}{2}-\frac{\operatorname{atan}\left(\frac{x}{2}+\frac{1}{2}\right)}{2}$$

input
```
integrate(x/(x**2+2*x+5),x)
```

output
```
log(x**2 + 2*x + 5)/2 - atan(x/2 + 1/2)/2
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x}{5+2x+x^2}\,dx = -\frac{1}{2}\,\arctan\left(\frac{1}{2}\,x+\frac{1}{2}\right)+\frac{1}{2}\,\log\left(x^2+2\,x+5\right)$$

input
```
integrate(x/(x^2+2*x+5),x, algorithm="maxima")
```

output
```
-1/2*arctan(1/2*x + 1/2) + 1/2*log(x^2 + 2*x + 5)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x}{5+2x+x^2}\,dx = -\frac{1}{2}\arctan\left(\frac{1}{2}\,x+\frac{1}{2}\right)+\frac{1}{2}\log\left(x^2+2\,x+5\right)$$

input
```
integrate(x/(x^2+2*x+5),x, algorithm="giac")
```

output
```
-1/2*arctan(1/2*x + 1/2) + 1/2*log(x^2 + 2*x + 5)
```

Mupad [B] (verification not implemented)

Time = 8.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x}{5+2x+x^2}\,dx = \frac{\ln\left(x^2+2\,x+5\right)}{2}-\frac{\mathrm{atan}\left(\frac{x}{2}+\frac{1}{2}\right)}{2}$$

input
```
int(x/(2*x + x^2 + 5),x)
```

output
```
log(2*x + x^2 + 5)/2 - atan(x/2 + 1/2)/2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x}{5+2x+x^2}\,dx = -\frac{atan\left(\frac{x}{2}+\frac{1}{2}\right)}{2}+\frac{\log(x^2+2x+5)}{2}$$

input
```
int(x/(x^2+2*x+5),x)
```

output
```
( - atan((x + 1)/2) + log(x**2 + 2*x + 5))/2
```

3.278 $\int \frac{x}{(1+x+x^2)^3} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{x}{(1+x+x^2)^3} \, dx = -\frac{2+x}{6(1+x+x^2)^2} - \frac{1+2x}{6(1+x+x^2)} - \frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

output
```
-1/6*(2+x)/(x^2+x+1)^2-(1+2*x)/(6*x^2+6*x+6)-2/9*arctan(1/3*(1+2*x)*3^(1/2
))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{x}{(1+x+x^2)^3} \, dx = \frac{1}{18}\left(-\frac{3(3+4x+3x^2+2x^3)}{(1+x+x^2)^2} - 4\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)\right)$$

input
```
Integrate[x/(1 + x + x^2)^3,x]
```

output
```
((-3*(3 + 4*x + 3*x^2 + 2*x^3))/(1 + x + x^2)^2 - 4*Sqrt[3]*ArcTan[(1 + 2*
x)/Sqrt[3]])/18
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1159, 1086, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(x^2+x+1\right)^3}dx$$

$$\Big\downarrow \text{1159}$$

$$-\frac{1}{2}\int \frac{1}{\left(x^2+x+1\right)^2}dx-\frac{x+2}{6\left(x^2+x+1\right)^2}$$

$$\Big\downarrow \text{1086}$$

$$\frac{1}{2}\left(-\frac{2}{3}\int \frac{1}{x^2+x+1}dx-\frac{2x+1}{3\left(x^2+x+1\right)}\right)-\frac{x+2}{6\left(x^2+x+1\right)^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{4}{3}\int \frac{1}{-(2x+1)^2-3}d(2x+1)-\frac{2x+1}{3\left(x^2+x+1\right)}\right)-\frac{x+2}{6\left(x^2+x+1\right)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(-\frac{4\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{2x+1}{3\left(x^2+x+1\right)}\right)-\frac{x+2}{6\left(x^2+x+1\right)^2}$$

input
```
Int[x/(1 + x + x^2)^3,x]
```

output
```
-1/6*(2 + x)/(1 + x + x^2)^2 + (-1/3*(1 + 2*x)/(1 + x + x^2) - (4*ArcTan[(
1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]))/2
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{-\frac{1}{3}x^3-\frac{1}{2}x^2-\frac{2}{3}x-\frac{1}{2}}{(x^2+x+1)^2}-\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}$	42
default	$\frac{-2-x}{6(x^2+x+1)^2}-\frac{1+2x}{6(x^2+x+1)}-\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}$	48

input
```
int(x/(x^2+x+1)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*x^3-1/2*x^2-2/3*x-1/2)/(x^2+x+1)^2-2/9*arctan(1/3*(1+2*x)*3^(1/2))*3
^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.31

$$\int \frac{x}{(1+x+x^2)^3}\,dx$$
$$= -\frac{6\,x^3+4\,\sqrt{3}(x^4+2\,x^3+3\,x^2+2\,x+1)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+9\,x^2+12\,x+9}{18\,(x^4+2\,x^3+3\,x^2+2\,x+1)}$$

input
```
integrate(x/(x^2+x+1)^3,x, algorithm="fricas")
```

output
```
-1/18*(6*x^3 + 4*sqrt(3)*(x^4 + 2*x^3 + 3*x^2 + 2*x + 1)*arctan(1/3*sqrt(3
)*(2*x + 1)) + 9*x^2 + 12*x + 9)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

$$\int \frac{x}{(1+x+x^2)^3}\,dx = \frac{-2x^3-3x^2-4x-3}{6x^4+12x^3+18x^2+12x+6} - \frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(x/(x**2+x+1)**3,x)
```

output
```
(-2*x**3 - 3*x**2 - 4*x - 3)/(6*x**4 + 12*x**3 + 18*x**2 + 12*x + 6) - 2*s
qrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x+x^2)^3}\,dx = -\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{2\,x^3+3\,x^2+4\,x+3}{6\,(x^4+2\,x^3+3\,x^2+2\,x+1)}$$

input
```
integrate(x/(x^2+x+1)^3,x, algorithm="maxima")
```

output
```
-2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*(2*x^3 + 3*x^2 + 4*x + 3)
/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{x}{(1+x+x^2)^3}\,dx = -\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{2\,x^3+3\,x^2+4\,x+3}{6\,(x^2+x+1)^2}$$

input
```
integrate(x/(x^2+x+1)^3,x, algorithm="giac")
```

output
```
-2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*(2*x^3 + 3*x^2 + 4*x + 3)
/(x^2 + x + 1)^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{x}{(1+x+x^2)^3}\,dx = -\frac{2\,\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{9} - \frac{\frac{x^3}{3}+\frac{x^2}{2}+\frac{2\,x}{3}+\frac{1}{2}}{x^4+2\,x^3+3\,x^2+2\,x+1}$$

input
```
int(x/(x + x^2 + 1)^3,x)
```

output
```
- (2*3^(1/2)*atan((2*3^(1/2)*x)/3 + 3^(1/2)/3))/9 - ((2*x)/3 + x^2/2 + x^3
/3 + 1/2)/(2*x + 3*x^2 + 2*x^3 + x^4 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.19

$$\int \frac{x}{(1+x+x^2)^3}\,dx$$
$$= \frac{-4\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)x^4 - 8\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)x^3 - 12\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)x^2 - 8\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)x - 4\sqrt{3}\,atan\left(2\right.}{18x^4+36x^3+54x^2+36x+18}$$

input
```
int(x/(x^2+x+1)^3,x)
```

output
```
( - 4*sqrt(3)*atan((2*x + 1)/sqrt(3))*x**4 - 8*sqrt(3)*atan((2*x + 1)/sqrt
(3))*x**3 - 12*sqrt(3)*atan((2*x + 1)/sqrt(3))*x**2 - 8*sqrt(3)*atan((2*x
+ 1)/sqrt(3))*x - 4*sqrt(3)*atan((2*x + 1)/sqrt(3)) + 3*x**4 - 6*x - 6)/(1
8*(x**4 + 2*x**3 + 3*x**2 + 2*x + 1))
```

3.279 $\int \frac{1}{x^{5/2}(a+bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 360

$$\begin{aligned}\int \frac{1}{x^{5/2}(a+bx+cx^2)^2}\,dx &= -\frac{5b^2-14ac}{3a^2(b^2-4ac)\,x^{3/2}} \\ &+ \frac{b(5b^2-19ac)}{a^3(b^2-4ac)\sqrt{x}} + \frac{b^2-2ac+bcx}{a(b^2-4ac)\,x^{3/2}(a+bx+cx^2)} \\ &+ \frac{\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2+b(5b^2-19ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^3(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} \\ &- \frac{\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2-b(5b^2-19ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^3(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

output
```
-1/3*(-14*a*c+5*b^2)/a^2/(-4*a*c+b^2)/x^(3/2)+b*(-19*a*c+5*b^2)/a^3/(-4*a*
c+b^2)/x^(1/2)+(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x^(3/2)/(c*x^2+b*x+a)+1/2*
c^(1/2)*(5*b^4-29*a*b^2*c+28*a^2*c^2+b*(-19*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))
*arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^3/
(-4*a*c+b^2)^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*c^(1/2)*(5*b^4-29*a*b^
2*c+28*a^2*c^2-b*(-19*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2
)*x^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^3/(-4*a*c+b^2)^(3/2)/(b+
(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 2.28 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^{5/2}(a+bx+cx^2)^2}\,dx = \frac{-\frac{2(8a^3c+15b^3x^2(b+cx)+abx(10b^2-62bcx-57c^2x^2)-2a^2(b^2+20bcx-7c^2x^2))}{x^{3/2}(a+x(b+cx))} - \frac{3\sqrt{2}\sqrt{c}\left(5b^4-29ab^2c-\right.}{}}{}$$

input
```
Integrate[1/(x^(5/2)*(a + b*x + c*x^2)^2),x]
```

output
```
((-2*(8*a^3*c + 15*b^3*x^2*(b + c*x) + a*b*x*(10*b^2 - 62*b*c*x - 57*c^2*x
^2) - 2*a^2*(b^2 + 20*b*c*x - 7*c^2*x^2)))/(x^(3/2)*(a + x*(b + c*x))) - (
3*Sqrt[2]*Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*
c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3
*Sqrt[2]*Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - 5*b^3*Sqrt[b^2 - 4*a*c
] + 19*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*a
^3*(-b^2 + 4*a*c))
```

Rubi [A] (verified)

Time = 1.97 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1165, 27, 1198, 25, 1198, 25, 1197, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \text{1165}$$

$$\frac{-2ac+b^2+bcx}{ax^{3/2}(b^2-4ac)(a+bx+cx^2)} - \frac{\int -\frac{5b^2+5cxb-14ac}{2x^{5/2}(cx^2+bx+a)}dx}{a(b^2-4ac)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5b^2+5cxb-14ac}{x^{5/2}(cx^2+bx+a)}dx}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{ax^{3/2}\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \; 1198$$

$$\frac{\frac{\int -\frac{b\left(5b^2-19ac\right)+c\left(5b^2-14ac\right)x}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2\left(5b^2-14ac\right)}{3ax^{3/2}}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{ax^{3/2}\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int \frac{b\left(5b^2-19ac\right)+c\left(5b^2-14ac\right)x}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2\left(5b^2-14ac\right)}{3ax^{3/2}}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{ax^{3/2}\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \; 1198$$

$$\frac{-\frac{\frac{\int -\frac{5b^4-24acb^2+c\left(5b^2-19ac\right)xb+14a^2c^2}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2b\left(5b^2-19ac\right)}{a\sqrt{x}}}{a}-\frac{2\left(5b^2-14ac\right)}{3ax^{3/2}}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{ax^{3/2}\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{-\frac{\int \frac{5b^4-24acb^2+c\left(5b^2-19ac\right)xb+14a^2c^2}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2b\left(5b^2-19ac\right)}{a\sqrt{x}}}{a}-\frac{2\left(5b^2-14ac\right)}{3ax^{3/2}}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{ax^{3/2}\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \; 1197$$

$$\frac{-\frac{-\frac{2\int \frac{5b^4-24acb^2+c\left(5b^2-19ac\right)xb+14a^2c^2}{cx^2+bx+a}d\sqrt{x}}{a}-\frac{2b\left(5b^2-19ac\right)}{a\sqrt{x}}}{a}-\frac{2\left(5b^2-14ac\right)}{3ax^{3/2}}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{ax^{3/2}\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \; 1480$$

$$\frac{-\frac{-\frac{2\left(\frac{c\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\int \frac{1}{\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)+cx}d\sqrt{x}}{2\sqrt{b^2-4ac}}-\frac{c\left(28a^2c^2-29ab^2c-b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\int \frac{1}{\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)+cx}}{2\sqrt{b^2-4ac}}\right)}{a}}{a}}{2a\left(b^2-4ac\right)}$$

$$\frac{-2ac+b^2+bcx}{ax^{3/2}\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\downarrow 218$$

$$-\frac{-\frac{2\left(\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}\right.}{a}}{a}}{2a\left(b^2-4ac\right)}$$

$$\frac{-2ac+b^2+bcx}{ax^{3/2}\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

input
```
Int[1/(x^(5/2)*(a + b*x + c*x^2)^2),x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^(3/2)*(a + b*x + c*x^2)) + ((-2*(
5*b^2 - 14*a*c))/(3*a*x^(3/2)) - ((-2*b*(5*b^2 - 19*a*c))/(a*Sqrt[x]) - (2
*((Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + b*(5*b^2 - 19*a*c)*Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(5*b^4 -
 29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 -
4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/a)/a)/(2*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1198
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c
*d^2 - b*d*e + a*e^2))), x] + Simp[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x
)^(m + 1)*(Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]/(a + b*x + c*x^
2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] && LtQ[m, -1
]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 335, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{2(-6bx+a)}{3a^3x^{\frac{3}{2}}}+\frac{\frac{2bc(3ac-b^2)x^{\frac{3}{2}}}{8ac-2b^2}-\frac{(2a^2c^2-4ca\,b^2+b^4)\sqrt{x}}{4ac-b^2}}{c\,x^2+bx+a}+\frac{4c\left(-\frac{\left(19abc\sqrt{-4ac+b^2}-5b^3\sqrt{-4ac+b^2}-28a^2c^2+29ca\,b^2-5b\cdots\right.}{8\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4\cdots\right.}}\cdots\right.}{\cdots}$
derivativedivides	$-\frac{2\left(\frac{-\frac{bc(3ac-b^2)x^{\frac{3}{2}}}{2(4ac-b^2)}+\frac{(2a^2c^2-4ca\,b^2+b^4)\sqrt{x}}{8ac-2b^2}}{c\,x^2+bx+a}+\frac{2c\left(-\frac{\left(-19abc\sqrt{-4ac+b^2}+5b^3\sqrt{-4ac+b^2}+28a^2c^2-29ca\,b^2+5b^4\right)\sqrt{2}\,\operatorname{arctan}\cdots}{8\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\cdots\right.}{\cdots}\right.}{\cdots}$
default	$-\frac{2\left(\frac{-\frac{bc(3ac-b^2)x^{\frac{3}{2}}}{2(4ac-b^2)}+\frac{(2a^2c^2-4ca\,b^2+b^4)\sqrt{x}}{8ac-2b^2}}{c\,x^2+bx+a}+\frac{2c\left(-\frac{\left(-19abc\sqrt{-4ac+b^2}+5b^3\sqrt{-4ac+b^2}+28a^2c^2-29ca\,b^2+5b^4\right)\sqrt{2}\,\operatorname{arctan}\cdots}{8\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\cdots\right.}{\cdots}\right.}{\cdots}$

input
```
int(1/x^(5/2)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-6*b*x+a)/a^3/x^(3/2)+1/a^3*(2*(1/2*b*c*(3*a*c-b^2)/(4*a*c-b^2)*x^(3
/2)-1/2*(2*a^2*c^2-4*a*b^2*c+b^4)/(4*a*c-b^2)*x^(1/2))/(c*x^2+b*x+a)+4/(4*
a*c-b^2)*c*(-1/8*(19*a*b*c*(-4*a*c+b^2)^(1/2)-5*b^3*(-4*a*c+b^2)^(1/2)-28*
a^2*c^2+29*c*a*b^2-5*b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctanh(c*x^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+
1/8*(19*a*b*c*(-4*a*c+b^2)^(1/2)-5*b^3*(-4*a*c+b^2)^(1/2)+28*a^2*c^2-29*c*
a*b^2+5*b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(c*x^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3449 vs. $2(309) = 618$.

Time = 1.31 (sec) , antiderivative size = 3449, normalized size of antiderivative = 9.58

$$\int \frac{1}{x^{5/2}\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^(5/2)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}\left(a+bx+cx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(5/2)/(c*x**2+b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^{5/2}\left(a+bx+cx^2\right)^2}\,dx = \int \frac{1}{\left(cx^2+bx+a\right)^2 x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
1/3*(3*(5*b^4*c - 24*a*b^2*c^2 + 14*a^2*c^3)*x^(5/2) + 3*(5*b^5 - 19*a*b^3
*c - 5*a^2*b*c^2)*x^(3/2) + 2*(15*a*b^4 - 67*a^2*b^2*c + 28*a^3*c^2)*sqrt(
x) + 10*(a^2*b^3 - 4*a^3*b*c)/sqrt(x) - 2*(a^3*b^2 - 4*a^4*c)/x^(3/2))/(a^
5*b^2 - 4*a^6*c + (a^4*b^2*c - 4*a^5*c^2)*x^2 + (a^4*b^3 - 4*a^5*b*c)*x) +
 integrate(-1/2*((5*b^4*c - 24*a*b^2*c^2 + 14*a^2*c^3)*x^(3/2) + (5*b^5 -
29*a*b^3*c + 33*a^2*b*c^2)*sqrt(x))/(a^5*b^2 - 4*a^6*c + (a^4*b^2*c - 4*a^
5*c^2)*x^2 + (a^4*b^3 - 4*a^5*b*c)*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3656 vs. $2(309) = 618$.

Time = 0.50 (sec) , antiderivative size = 3656, normalized size of antiderivative = 10.16

$$\int \frac{1}{x^{5/2} (a + bx + cx^2)^2} dx = \text{Too large to display}$$

input
```
integrate(1/x^(5/2)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
(b^3*c*x^(3/2) - 3*a*b*c^2*x^(3/2) + b^4*sqrt(x) - 4*a*b^2*c*sqrt(x) + 2*a
^2*c^2*sqrt(x))/((a^3*b^2 - 4*a^4*c)*(c*x^2 + b*x + a)) - 1/8*(10*a^6*b^9*
c^2 - 138*a^7*b^7*c^3 + 680*a^8*b^5*c^4 - 1376*a^9*b^3*c^5 + 896*a^10*b*c^
6 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^9 +
69*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c + 1
0*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c - 34
0*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 -
98*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 -
 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^2 +
 688*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3
 + 288*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c
^3 + 49*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^5*
c^3 - 448*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b
*c^4 - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b
^2*c^4 - 144*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8
*b^3*c^4 + 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^9*b*c^5 - 10*(b^2 - 4*a*c)*a^6*b^7*c^2 + 98*(b^2 - 4*a*c)*a^7*b^5*c^3 - 2
88*(b^2 - 4*a*c)*a^8*b^3*c^4 + 224*(b^2 - 4*a*c)*a^9*b*c^5 + (10*b^5*c^2 -
 78*a*b^3*c^3 + 152*a^2*b*c^4 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^5 + 39*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^...
```

Mupad [B] (verification not implemented)

Time = 11.77 (sec) , antiderivative size = 8768, normalized size of antiderivative = 24.36

$$\int \frac{1}{x^{5/2}\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x^(5/2)*(a + b*x + c*x^2)^2),x)
```

output
```
atan((((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 63
66*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*
c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^1
3*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^
2)^9)^(1/2))/(8*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^
2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(x^
(1/2)*(-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 636
6*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c
^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13
*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2
)^9)^(1/2))/(8*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2
 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(327
68*a^21*b*c^8 + 8*a^15*b^13*c^2 - 192*a^16*b^11*c^3 + 1920*a^17*b^9*c^4 -
10240*a^18*b^7*c^5 + 30720*a^19*b^5*c^6 - 49152*a^20*b^3*c^7) - 57344*a^19
*c^9 + 20*a^12*b^14*c^2 - 496*a^13*b^12*c^3 + 5176*a^14*b^10*c^4 - 29280*a
^15*b^8*c^5 + 96000*a^16*b^6*c^6 - 179200*a^17*b^4*c^7 + 169984*a^18*b^2*c
^8) - x^(1/2)*(50176*a^16*c^10 - 50*a^9*b^14*c^3 + 1180*a^10*b^12*c^4 - 11
602*a^11*b^10*c^5 + 61012*a^12*b^8*c^6 - 182336*a^13*b^6*c^7 + 300160*a^14
*b^4*c^8 - 233984*a^15*b^2*c^9))*(-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1
/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*...
```

Reduce [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 3914, normalized size of antiderivative = 10.87

$$\int \frac{1}{x^{5/2}\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/x^(5/2)/(c*x^2+b*x+a)^2,x)
```

output
```
( - 312*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*s
qrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c**2*
x + 204*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*s
qrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**3*c*
x - 312*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*s
qrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c*
*2*x**2 - 312*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqr
t(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b
*c**3*x**3 - 30*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*
*5*x + 204*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c
)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**4*c*
x**2 + 204*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c
)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**3*c*
*2*x**3 - 30*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt
(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b**6*x*
*2 - 30*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*s
qrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b**5*c*x**3
+ 168*sqrt(x)*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqr
t(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*c**2*x...
```

3.280 $\int \frac{x^{9/2}}{(a+bx+cx^2)^3} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 392

$$\int \frac{x^{9/2}}{(a+bx+cx^2)^3} \, dx = -\frac{b\sqrt{x}}{2c^2(b^2-4ac)} + \frac{x^{7/2}(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2} - \frac{\sqrt{x}(ab(b^2-16ac)+(b^4-17ab^2c+28a^2c^2)x)}{4c^2(b^2-4ac)^2(a+bx+cx^2)} + \frac{3\left(b^4-9ab^2c+28a^2c^2-\frac{b(b^4-11ab^2c+44a^2c^2)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}c^{5/2}(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}} + \frac{3\left(b^4-9ab^2c+28a^2c^2+\frac{b(b^4-11ab^2c+44a^2c^2)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}c^{5/2}(b^2-4ac)^2\sqrt{b+\sqrt{b^2-4ac}}}$$

output

```
-1/2*b*x^(1/2)/c^2/(-4*a*c+b^2)+1/2*x^(7/2)*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+
b*x+a)^2-1/4*x^(1/2)*(a*b*(-16*a*c+b^2)+(28*a^2*c^2-17*a*b^2*c+b^4)*x)/c^2
/(-4*a*c+b^2)^2/(c*x^2+b*x+a)+3/8*(b^4-9*a*b^2*c+28*a^2*c^2-b*(44*a^2*c^2-
11*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b-(-4*
a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(5/2)/(-4*a*c+b^2)^2/(b-(-4*a*c+b^2)^(1/2
))^(1/2)+3/8*(b^4-9*a*b^2*c+28*a^2*c^2+b*(44*a^2*c^2-11*a*b^2*c+b^4)/(-4*a
*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
)*2^(1/2)/c^(5/2)/(-4*a*c+b^2)^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 4.51 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.10

$$\int \frac{x^{9/2}}{\left(a+bx+cx^2\right)^3}\,dx = \frac{-\frac{2\sqrt{c}\sqrt{x}\left(b^4x^2(3b+5cx)+4a^3c(-6b+7cx)+ab^2x\left(6b^2-20bcx-37c^2x^2\right)+a^2\left(3b^3-49b^2cx-4bc^2x^2+44c^3x^3\right)\right)}{\left(b^2-4ac\right)^2(a+x(b+cx))^2}+}{}$$

input
```
Integrate[x^(9/2)/(a + b*x + c*x^2)^3,x]
```

output
```
((-2*Sqrt[c]*Sqrt[x]*(b^4*x^2*(3*b + 5*c*x) + 4*a^3*c*(-6*b + 7*c*x) + a*b
^2*x*(6*b^2 - 20*b*c*x - 37*c^2*x^2) + a^2*(3*b^3 - 49*b^2*c*x - 4*b*c^2*x
^2 + 44*c^3*x^3)))/((b^2 - 4*a*c)^2*(a + x*(b + c*x))^2) + (3*Sqrt[2]*(-b^
5 + 11*a*b^3*c - 44*a^2*b*c^2 + b^4*Sqrt[b^2 - 4*a*c] - 9*a*b^2*c*Sqrt[b^2
 - 4*a*c] + 28*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) + (3*Sqrt[2]*(b^5 - 11*a*b^3*c + 44*a^2*b*c^2 + b^4*Sqrt[b^2 - 4*a*
c] - 9*a*b^2*c*Sqrt[b^2 - 4*a*c] + 28*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)
*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(8*c^(5/2))
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1164, 27, 1233, 27, 1196, 25, 1197, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}}{\left(a+bx+cx^2\right)^3}dx$$

$$\Big\downarrow \text{1164}$$

$$\frac{x^{7/2}(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{\int \frac{x^{5/2}(14a+bx)}{2\left(cx^2+bx+a\right)^2}dx}{2\left(b^2-4ac\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{x^{7/2}(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\int \frac{x^{5/2}(14a+bx)}{(cx^2+bx+a)^2}dx}{4(b^2-4ac)}$$

$$\Big\downarrow \ 1233$$

$$\frac{x^{7/2}(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{\int \frac{3\sqrt{x}\left(a\left(b^2-28ac\right)+b\left(b^2-8ac\right)x\right)}{2\left(cx^2+bx+a\right)}dx}{c(b^2-4ac)}-\frac{x^{3/2}\left(bx\left(b^2-16ac\right)+a\left(b^2-28ac\right)\right)}{c(b^2-4ac)(a+bx+cx^2)}}{4(b^2-4ac)}$$

$$\Big\downarrow \ 27$$

$$\frac{x^{7/2}(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{3\int \frac{\sqrt{x}\left(a\left(b^2-28ac\right)+b\left(b^2-8ac\right)x\right)}{cx^2+bx+a}dx}{2c(b^2-4ac)}-\frac{x^{3/2}\left(bx\left(b^2-16ac\right)+a\left(b^2-28ac\right)\right)}{c(b^2-4ac)(a+bx+cx^2)}}{4(b^2-4ac)}$$

$$\Big\downarrow \ 1196$$

$$\frac{x^{7/2}(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{3\left(\frac{\int -\frac{ab\left(b^2-8ac\right)+\left(b^4-9acb^2+28a^2c^2\right)x}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{c}+\frac{2b\sqrt{x}\left(b^2-8ac\right)}{c}\right)}{2c(b^2-4ac)}-\frac{x^{3/2}\left(bx\left(b^2-16ac\right)+a\left(b^2-28ac\right)\right)}{c(b^2-4ac)(a+bx+cx^2)}}{4(b^2-4ac)}$$

$$\Big\downarrow \ 25$$

$$\frac{x^{7/2}(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{3\left(\frac{2b\sqrt{x}\left(b^2-8ac\right)}{c}-\frac{\int \frac{ab\left(b^2-8ac\right)+\left(b^4-9acb^2+28a^2c^2\right)x}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{c}\right)}{2c(b^2-4ac)}-\frac{x^{3/2}\left(bx\left(b^2-16ac\right)+a\left(b^2-28ac\right)\right)}{c(b^2-4ac)(a+bx+cx^2)}}{4(b^2-4ac)}$$

$$\Big\downarrow \ 1197$$

$$\frac{x^{7/2}(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{3\left(\frac{2b\sqrt{x}\left(b^2-8ac\right)}{c}-\frac{2\int \frac{ab\left(b^2-8ac\right)+\left(b^4-9acb^2+28a^2c^2\right)x}{cx^2+bx+a}d\sqrt{x}}{c}\right)}{2c(b^2-4ac)}-\frac{x^{3/2}\left(bx\left(b^2-16ac\right)+a\left(b^2-28ac\right)\right)}{c(b^2-4ac)(a+bx+cx^2)}}{4(b^2-4ac)}$$

$\Big\downarrow$ 1480

$$\frac{x^{7/2}(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{3\left(\frac{2b\sqrt{x}(b^2-8ac)}{c}-\frac{2\left(\frac{1}{2}\left(-\frac{b(44a^2c^2-11ab^2c+b^4)}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\int\frac{1}{\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)+cx}d\sqrt{x}+\frac{1}{2}\left(\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\int\frac{1}{\frac{1}{2}(b\ldots}}{c}\right)}{2c(b^2-4ac)}}{4(b^2-4ac)}$$

$\Big\downarrow$ 218

$$\frac{x^{7/2}(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{3\left(\frac{2b\sqrt{x}(b^2-8ac)}{c}-\frac{2\left(\frac{\left(-\frac{b(44a^2c^2-11ab^2c+b^4)}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\arctan\left(\frac{\ldots}{\sqrt{\ldots}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{c}\right)}{2c(b^2-4ac)}}{4(b^2-4ac)}$$

input

```
Int[x^(9/2)/(a + b*x + c*x^2)^3,x]
```

output

```
(x^(7/2)*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (-((x^(3/2)*
(a*(b^2 - 28*a*c) + b*(b^2 - 16*a*c)*x))/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2
))) + (3*((2*b*(b^2 - 8*a*c)*Sqrt[x])/c - (2*(((b^4 - 9*a*b^2*c + 28*a^2*c
^2 - (b*(b^4 - 11*a*b^2*c + 44*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b -
 Sqrt[b^2 - 4*a*c]]) + ((b^4 - 9*a*b^2*c + 28*a^2*c^2 + (b^5 - 11*a*b^3*c
+ 44*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/c)
)/(2*c*(b^2 - 4*a*c)))/(4*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1196
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int
[(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] &
& GtQ[m, 0]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{-\frac{\left(44a^2c^2-37ca\,b^2+5b^4\right)x^{\frac{7}{2}}}{4\left(16a^2c^2-8ca\,b^2+b^4\right)c}+\frac{b\left(4a^2c^2+20ca\,b^2-3b^4\right)x^{\frac{5}{2}}}{4c^2\left(16a^2c^2-8ca\,b^2+b^4\right)}-\frac{a\left(28a^2c^2-49ca\,b^2+6b^4\right)x^{\frac{3}{2}}}{4c^2\left(16a^2c^2-8ca\,b^2+b^4\right)}+\frac{3a^2b\left(8ac-b^2\right)\sqrt{x}}{4c^2\left(16a^2c^2-8ca\,b^2+b^4\right)}}{\left(c\,x^2+bx+a\right)^2}+\frac{3\left(28a\right.}{}$
default	$\frac{-\frac{\left(44a^2c^2-37ca\,b^2+5b^4\right)x^{\frac{7}{2}}}{4\left(16a^2c^2-8ca\,b^2+b^4\right)c}+\frac{b\left(4a^2c^2+20ca\,b^2-3b^4\right)x^{\frac{5}{2}}}{4c^2\left(16a^2c^2-8ca\,b^2+b^4\right)}-\frac{a\left(28a^2c^2-49ca\,b^2+6b^4\right)x^{\frac{3}{2}}}{4c^2\left(16a^2c^2-8ca\,b^2+b^4\right)}+\frac{3a^2b\left(8ac-b^2\right)\sqrt{x}}{4c^2\left(16a^2c^2-8ca\,b^2+b^4\right)}}{\left(c\,x^2+bx+a\right)^2}+\frac{3\left(28a\right.}{}$

input
```
int(x^(9/2)/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(-1/8*(44*a^2*c^2-37*a*b^2*c+5*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c*x^(7/2)
+1/8*b*(4*a^2*c^2+20*a*b^2*c-3*b^4)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)
-1/8*a/c^2*(28*a^2*c^2-49*a*b^2*c+6*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(3/2
)+3/8*a^2*b*(8*a*c-b^2)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2))/(c*x^2+b*x
+a)^2+3/c/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/8*(28*a^2*c^2*(-4*a*c+b^2)^(1/2)-9
*c*a*b^2*(-4*a*c+b^2)^(1/2)+b^4*(-4*a*c+b^2)^(1/2)+44*a^2*b*c^2-11*a*b^3*c
+b^5)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(c*x^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(28*a^2*c^2*(-4*a
*c+b^2)^(1/2)-9*c*a*b^2*(-4*a*c+b^2)^(1/2)+b^4*(-4*a*c+b^2)^(1/2)-44*a^2*b
*c^2+11*a*b^3*c-b^5)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctanh(c*x^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4275 vs. $2(336) = 672$.

Time = 0.77 (sec) , antiderivative size = 4275, normalized size of antiderivative = 10.91

$$\int \frac{x^{9/2}}{(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(9/2)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{9/2}}{(a+bx+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(9/2)/(c*x**2+b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{9/2}}{(a+bx+cx^2)^3}\,dx = \int \frac{x^{\frac{9}{2}}}{(cx^2+bx+a)^3}\,dx$$

input
```
integrate(x^(9/2)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*(3*(b^3*c - 8*a*b*c^2)*x^(9/2) + (b^4 - 11*a*b^2*c - 44*a^2*c^2)*x^(7/
2) + 2*(a*b^3 - 22*a^2*b*c)*x^(5/2) + (a^2*b^2 - 28*a^3*c)*x^(3/2))/(a^2*b
^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x
^4 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^3 + (b^6*c - 6*a*b^4*c^2 +
 32*a^3*c^4)*x^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x) + integra
te(-3/8*((b^3 - 8*a*b*c)*x^(3/2) + (a*b^2 - 28*a^2*c)*sqrt(x))/(a*b^4*c -
8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b
^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2436 vs. 2(336) = 672.

Time = 1.84 (sec) , antiderivative size = 2436, normalized size of antiderivative = 6.21

$$\int \frac{x^{9/2}}{(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(9/2)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
3/16*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 - 16*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^6*c - 2*b^7*c + 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 +
24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 32*a*b^5*c^2 - 2*b^6*c^2 - 128*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b^2*c^3 - 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^
3 - 160*a^2*b^3*c^3 + 28*a*b^4*c^3 + 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b*c^4 + 256*a^3*b*c^4 - 192*a^2*b^2*c^4 + 448*a^3*c^5 + sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 14*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 96*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 20*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 2*(b^2 - 4*a*c)*b^5*c - 24*(b^2 - 4*a*c
)*a*b^3*c^2 + 2*(b^2 - 4*a*c)*b^4*c^2 + 64*(b^2 - 4*a*c)*a^2*b*c^3 - 20...
```

Mupad [B] (verification not implemented)

Time = 13.80 (sec) , antiderivative size = 10944, normalized size of antiderivative = 27.92

$$\int \frac{x^{9/2}}{(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(9/2)/(a + b*x + c*x^2)^3,x)
```

output
```
- ((x^(3/2)*(6*a*b^4 + 28*a^3*c^2 - 49*a^2*b^2*c))/(4*c^2*(b^4 + 16*a^2*c^
2 - 8*a*b^2*c)) + (x^(7/2)*(5*b^4 + 44*a^2*c^2 - 37*a*b^2*c))/(4*c*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)) - (b*x^(5/2)*(4*a^2*c^2 - 3*b^4 + 20*a*b^2*c))/(4
*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (3*a^2*b*x^(1/2)*(8*a*c - b^2))/(4*
c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 +
2*a*b*x + 2*b*c*x^3) - atan(((((3*(64*a*b^13*c^3 + 524288*a^7*b*c^9 - 1792
*a^2*b^11*c^4 + 20480*a^3*b^9*c^5 - 122880*a^4*b^7*c^6 + 409600*a^5*b^5*c^
7 - 720896*a^6*b^3*c^8))/(64*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 24
0*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) -
 (x^(1/2)*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 -
769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9
*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 4
9*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c -
b^2)^15)^(1/2)))/(128*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*
a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^
10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2
621440*a^9*b^2*c^14)))^(1/2)*(64*b^11*c^5 - 1280*a*b^9*c^6 - 65536*a^5*b*c
^10 + 10240*a^2*b^7*c^7 - 40960*a^3*b^5*c^8 + 81920*a^4*b^3*c^9))/(8*(256*
a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*((9
*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^...
```

Reduce [B] (verification not implemented)

Time = 4.39 (sec) , antiderivative size = 6759, normalized size of antiderivative = 17.24

$$\int \frac{x^{9/2}}{(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(9/2)/(c*x^2+b*x+a)^3,x)
```

output

```
( - 336*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
 b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*c**3 + 60*sqrt(
a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(
x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**2*c**2 - 672*sqrt(a)*sqrt
(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt
(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c**3*x - 672*sqrt(a)*sqrt(2*sqrt(
c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqr
t(2*sqrt(c)*sqrt(a) + b))*a**3*c**4*x**2 - 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a
) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(
c)*sqrt(a) + b))*a**2*b**4*c + 120*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*ata
n((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a)
 + b))*a**2*b**3*c**2*x - 216*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sq
rt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b)
)*a**2*b**2*c**3*x**2 - 672*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt
(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*
a**2*b*c**4*x**3 - 336*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*
c**5*x**4 - 12*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sq
rt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**5*c*x +
48*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b...
```

3.281 $\int \frac{1}{x^{3/2}(a+bx+cx^2)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 437

$$\int \frac{1}{x^{3/2}(a+bx+cx^2)^3}\,dx = -\frac{3(5b^2-12ac)(b^2-5ac)}{4a^3(b^2-4ac)^2\sqrt{x}} + \frac{b^2-2ac+bcx}{2a(b^2-4ac)\sqrt{x}(a+bx+cx^2)^2} + \frac{5b^4-35ab^2c+36a^2c^2+bc(5b^2-32ac)x}{4a^2(b^2-4ac)^2\sqrt{x}(a+bx+cx^2)} - \frac{3\sqrt{c}\left(5b^5-47ab^3c+124a^2bc^2+\sqrt{b^2-4ac}(5b^4-37ab^2c+60a^2c^2)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}a^3(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{3\sqrt{c}\left(5b^5-47ab^3c+124a^2bc^2-\sqrt{b^2-4ac}(5b^4-37ab^2c+60a^2c^2)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}a^3(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output

```
-3/4*(-12*a*c+5*b^2)*(-5*a*c+b^2)/a^3/(-4*a*c+b^2)^2/x^(1/2)+1/2*(b*c*x-2*
a*c+b^2)/a/(-4*a*c+b^2)/x^(1/2)/(c*x^2+b*x+a)^2+1/4*(5*b^4-35*a*b^2*c+36*a
^2*c^2+b*c*(-32*a*c+5*b^2)*x)/a^2/(-4*a*c+b^2)^2/x^(1/2)/(c*x^2+b*x+a)-3/8
*c^(1/2)*(5*b^5-47*a*b^3*c+124*a^2*b*c^2+(-4*a*c+b^2)^(1/2)*(60*a^2*c^2-37
*a*b^2*c+5*b^4))*arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2))*2^(1/2)/a^3/(-4*a*c+b^2)^(5/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+3/8*c^(1/2
)*(5*b^5-47*a*b^3*c+124*a^2*b*c^2-(-4*a*c+b^2)^(1/2)*(60*a^2*c^2-37*a*b^2*
c+5*b^4))*arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(
1/2)/a^3/(-4*a*c+b^2)^(5/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 6.56 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^{3/2} \left(a+bx+cx^2\right)^3} \, dx =$$

$$-\frac{\frac{2\left(128a^4c^2+15b^4x^2(b+cx)^2+4a^3c\left(-16b^2+91bcx+81c^2x^2\right)+ab^2x\left(25b^3-91b^2cx-227bc^2x^2-111c^3x^3\right)+a^2\left(8b^4-194b^3cx+25b^2c^2x^2+392bc^3x^3-\right.\right.}{\sqrt{x}(a+x(b+cx))^2}}{}$$

input
```
Integrate[1/(x^(3/2)*(a + b*x + c*x^2)^3),x]
```

output
```
-1/8*((2*(128*a^4*c^2 + 15*b^4*x^2*(b + c*x)^2 + 4*a^3*c*(-16*b^2 + 91*b*c
*x + 81*c^2*x^2) + a*b^2*x*(25*b^3 - 91*b^2*c*x - 227*b*c^2*x^2 - 111*c^3*
x^3) + a^2*(8*b^4 - 194*b^3*c*x + 25*b^2*c^2*x^2 + 392*b*c^3*x^3 + 180*c^4
*x^4)))/(Sqrt[x]*(a + x*(b + c*x))^2) + (3*Sqrt[2]*Sqrt[c]*(5*b^5 - 47*a*b
^3*c + 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt[b^2 - 4*a
*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
 (3*Sqrt[2]*Sqrt[c]*(-5*b^5 + 47*a*b^3*c - 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2
- 4*a*c] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4
*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(a^3*(b^2 - 4*a*c)^2)
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 426, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1165, 27, 1235, 27, 1198, 25, 1197, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2} \left(a+bx+cx^2\right)^3} \, dx$$

$$\Big\downarrow \; 1165$$

$$\frac{-2ac+b^2+bcx}{2a\sqrt{x}\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{\int -\frac{5b^2+7cxb-18ac}{2x^{3/2}\left(cx^2+bx+a\right)^2}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{5b^2+7cxb-18ac}{x^{3/2}\left(cx^2+bx+a\right)^2}dx}{4a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{2a\sqrt{x}\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$$\Big\downarrow \ 1235$$

$$\frac{\frac{36a^2c^2+bcx\left(5b^2-32ac\right)-35ab^2c+5b^4}{a\sqrt{x}\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int -\frac{3\left(\left(5b^2-12ac\right)\left(b^2-5ac\right)+bc\left(5b^2-32ac\right)x\right)}{2x^{3/2}\left(cx^2+bx+a\right)}dx}{a\left(b^2-4ac\right)}}{4a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{2a\sqrt{x}\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{3\int \frac{\left(5b^2-12ac\right)\left(b^2-5ac\right)+bc\left(5b^2-32ac\right)x}{x^{3/2}\left(cx^2+bx+a\right)}dx}{2a\left(b^2-4ac\right)}+\frac{36a^2c^2+bcx\left(5b^2-32ac\right)-35ab^2c+5b^4}{a\sqrt{x}\left(b^2-4ac\right)\left(a+bx+cx^2\right)}}{4a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{2a\sqrt{x}\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$$\Big\downarrow \ 1198$$

$$\frac{\frac{3\left(\frac{\int -\frac{b\left(5b^4-42acb^2+92a^2c^2\right)+c\left(5b^2-12ac\right)\left(b^2-5ac\right)x}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2\left(5b^2-12ac\right)\left(b^2-5ac\right)}{a\sqrt{x}}\right)}{2a\left(b^2-4ac\right)}+\frac{36a^2c^2+bcx\left(5b^2-32ac\right)-35ab^2c+5b^4}{a\sqrt{x}\left(b^2-4ac\right)\left(a+bx+cx^2\right)}}{4a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{2a\sqrt{x}\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{3\left(-\frac{\int \frac{b\left(5b^4-42acb^2+92a^2c^2\right)+c\left(5b^2-12ac\right)\left(b^2-5ac\right)x}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2\left(5b^2-12ac\right)\left(b^2-5ac\right)}{a\sqrt{x}}\right)}{2a\left(b^2-4ac\right)}+\frac{36a^2c^2+bcx\left(5b^2-32ac\right)-35ab^2c+5b^4}{a\sqrt{x}\left(b^2-4ac\right)\left(a+bx+cx^2\right)}}{4a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{2a\sqrt{x}\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\Big\downarrow$ 1197

$$\frac{\frac{3\left(-\frac{2\int \frac{b\left(5b^4-42acb^2+92a^2c^2\right)+c\left(5b^2-12ac\right)\left(b^2-5ac\right)x}{cx^2+bx+a}d\sqrt{x}}{a}-\frac{2\left(5b^2-12ac\right)\left(b^2-5ac\right)}{a\sqrt{x}}\right)}{2a(b^2-4ac)}+\frac{36a^2c^2+bcx(5b^2-32ac)-35ab^2c+5b^4}{a\sqrt{x}(b^2-4ac)(a+bx+cx^2)}}{4a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{2a\sqrt{x}\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\Big\downarrow$ 1480

$$\frac{\frac{3\left(-\frac{2\left(\frac{1}{2}c\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)\int\frac{1}{\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)+cx}d\sqrt{x}+\frac{1}{2}c\left(\left(5b^2-12ac\right)\left(b^2-5ac\right)-\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\frac{1}{2}\left(b\cdots\right.}}{a}\right.}{2a(b^2-4ac)}}{4a\left(b^2-4ac\right)}$$

$$\frac{-2ac+b^2+bcx}{2a\sqrt{x}\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\Big\downarrow$ 218

$$\frac{\frac{3\left(-\frac{2\left(\frac{\sqrt{c}\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(\left(5b^2-12ac\right)\left(b^2-5ac\right)-\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}\right)\arctan\left(\cdots\right.}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}\right.}{a}\right.}{2a(b^2-4ac)}}{4a\left(b^2-4ac\right)}$$

$$\frac{-2ac+b^2+bcx}{2a\sqrt{x}\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

input

```
Int[1/(x^(3/2)*(a + b*x + c*x^2)^3),x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(2*a*(b^2 - 4*a*c)*Sqrt[x]*(a + b*x + c*x^2)^2) + ((
5*b^4 - 35*a*b^2*c + 36*a^2*c^2 + b*c*(5*b^2 - 32*a*c)*x)/(a*(b^2 - 4*a*c)
*Sqrt[x]*(a + b*x + c*x^2)) + (3*((-2*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(a*S
qrt[x]) - (2*((Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) + (b*(5*b^4 - 47*a*
b^2*c + 124*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqr
t[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) - (b*(5*b^4 - 47*a*b^2*c + 124*a^2*c^
2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2
- 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/a))/(2*a*(b^2 - 4*a*c)
))/(4*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1198
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c
*d^2 - b*d*e + a*e^2))), x] + Simp[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x
)^(m + 1)*(Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]/(a + b*x + c*x^
2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] && LtQ[m, -1
]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.19

method	result
derivativedivides	$-\frac{2}{a^{3}\sqrt{x}}-\frac{2\left(\frac{\frac{c^{2}\left(52a^{2}c^{2}-47ca\,b^{2}+7b^{4}\right)x^{\frac{7}{2}}}{128a^{2}c^{2}-64ca\,b^{2}+8b^{4}}+\frac{cb\left(136a^{2}c^{2}-99ca\,b^{2}+14b^{4}\right)x^{\frac{5}{2}}}{128a^{2}c^{2}-64ca\,b^{2}+8b^{4}}+\frac{\left(68a^{3}c^{3}+25a^{2}b^{2}c^{2}-43a\,b^{4}c+7b^{6}\right)x^{\frac{3}{2}}}{128a^{2}c^{2}-64ca\,b^{2}+8b^{4}}+\frac{3ab(\cdots}{8\cdots}}{\left(c\,x^{2}+bx+a\right)^{2}}\cdots\right.}{\cdots}$
default	$-\frac{2}{a^{3}\sqrt{x}}-\frac{2\left(\frac{\frac{c^{2}\left(52a^{2}c^{2}-47ca\,b^{2}+7b^{4}\right)x^{\frac{7}{2}}}{128a^{2}c^{2}-64ca\,b^{2}+8b^{4}}+\frac{cb\left(136a^{2}c^{2}-99ca\,b^{2}+14b^{4}\right)x^{\frac{5}{2}}}{128a^{2}c^{2}-64ca\,b^{2}+8b^{4}}+\frac{\left(68a^{3}c^{3}+25a^{2}b^{2}c^{2}-43a\,b^{4}c+7b^{6}\right)x^{\frac{3}{2}}}{128a^{2}c^{2}-64ca\,b^{2}+8b^{4}}+\frac{3ab(\cdots}{8\cdots}}{\left(c\,x^{2}+bx+a\right)^{2}}\cdots\right.}{\cdots}$
risch	$-\frac{2}{a^{3}\sqrt{x}}-\frac{\frac{\frac{2c^{2}\left(52a^{2}c^{2}-47ca\,b^{2}+7b^{4}\right)x^{\frac{7}{2}}}{128a^{2}c^{2}-64ca\,b^{2}+8b^{4}}+\frac{2cb\left(136a^{2}c^{2}-99ca\,b^{2}+14b^{4}\right)x^{\frac{5}{2}}}{128a^{2}c^{2}-64ca\,b^{2}+8b^{4}}+\frac{2\left(68a^{3}c^{3}+25a^{2}b^{2}c^{2}-43a\,b^{4}c+7b^{6}\right)x^{\frac{3}{2}}}{128a^{2}c^{2}-64ca\,b^{2}+8b^{4}}+\frac{3ab(\cdots}{4\cdots}}{\left(c\,x^{2}+bx+a\right)^{2}}\cdots}{\cdots}$

input
```
int(1/x^(3/2)/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/a^3/x^(1/2)-2/a^3*((1/8*c^2*(52*a^2*c^2-47*a*b^2*c+7*b^4)/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^(7/2)+1/8*c*b*(136*a^2*c^2-99*a*b^2*c+14*b^4)/(16*a^2*c^2-
8*a*b^2*c+b^4)*x^(5/2)+1/8*(68*a^3*c^3+25*a^2*b^2*c^2-43*a*b^4*c+7*b^6)/(1
6*a^2*c^2-8*a*b^2*c+b^4)*x^(3/2)+3/8*a*b*(36*a^2*c^2-22*a*b^2*c+3*b^4)/(16
*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2))/(c*x^2+b*x+a)^2+3/2/(16*a^2*c^2-8*a*b^2*c
+b^4)*c*(1/8*(60*a^2*c^2*(-4*a*c+b^2)^(1/2)-37*c*a*b^2*(-4*a*c+b^2)^(1/2)+
5*b^4*(-4*a*c+b^2)^(1/2)-124*a^2*b*c^2+47*a*b^3*c-5*b^5)/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^(1/2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(60*a^2*c^2*(-4*a*c+b^2)^(1/2)-37*c*a*b^2*
(-4*a*c+b^2)^(1/2)+5*b^4*(-4*a*c+b^2)^(1/2)+124*a^2*b*c^2-47*a*b^3*c+5*b^5
)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x
^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4933 vs. $2(377) = 754$.

Time = 2.73 (sec) , antiderivative size = 4933, normalized size of antiderivative = 11.29

$$\int \frac{1}{x^{3/2}\left(a+bx+cx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^(3/2)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\left(a+bx+cx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(3/2)/(c*x**2+b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^{3/2}\left(a+bx+cx^2\right)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^(3/2)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5281 vs. $2(377) = 754$.

Time = 0.59 (sec) , antiderivative size = 5281, normalized size of antiderivative = 12.08

$$\int \frac{1}{x^{3/2}\left(a+bx+cx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^(3/2)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
-3/32*(10*a^6*b^14*c^2 - 254*a^7*b^12*c^3 + 2712*a^8*b^10*c^4 - 15552*a^9*
b^8*c^5 + 50432*a^10*b^6*c^6 - 87552*a^11*b^4*c^7 + 63488*a^12*b^2*c^8 - 5
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^14 + 127*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^12*c + 10*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^13*c - 135
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^10*c^2 -
 214*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^11*c^
2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^12*c
^2 + 7776*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^
8*c^3 + 1856*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8
*b^9*c^3 + 107*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^7*b^10*c^3 - 25216*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^10*b^6*c^4 - 8128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^9*b^7*c^4 - 928*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^8*b^8*c^4 + 43776*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^11*b^4*c^5 + 17920*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^10*b^5*c^5 + 4064*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^9*b^6*c^5 - 31744*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^12*b^2*c^6 - 15872*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^11*b^3*c^6 - 8960*sqrt(2)*sqrt(b^2 - ...
```

Mupad [B] (verification not implemented)

Time = 14.01 (sec) , antiderivative size = 12164, normalized size of antiderivative = 27.84

$$\int \frac{1}{x^{3/2} \left(a+bx+cx^2\right)^3} \, dx = \text{Too large to display}$$

input
```
int(1/(x^(3/2)*(a + b*x + c*x^2)^3),x)
```

output
```
- (2/a + (x^2*(15*b^6 + 324*a^3*c^3 + 25*a^2*b^2*c^2 - 91*a*b^4*c))/(4*a^3
*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*x*(25*b^4 + 364*a^2*c^2 - 194*a*b^2*
c))/(4*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*x^3*(30*b^4*c + 392*a^2*c^
3 - 227*a*b^2*c^2))/(4*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*c*x^4*(5*b
^4*c + 60*a^2*c^3 - 37*a*b^2*c^2))/(4*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))
/(x^(5/2)*(2*a*c + b^2) + a^2*x^(1/2) + c^2*x^(9/2) + 2*a*b*x^(3/2) + 2*b*
c*x^(7/2)) - atan((((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 189
23520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4
*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7
*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c
- b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2
) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(128*(a^7*b^20 + 1048576*a^17*
c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*
b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c
^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(x^(1/2)*(-(9*(2
5*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a
^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^1
1*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8
 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^
19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c...
```

Reduce [B] (verification not implemented)

Time = 2.90 (sec) , antiderivative size = 8137, normalized size of antiderivative = 18.62

$$\int \frac{1}{x^{3/2} (a + bx + cx^2)^3} \, dx = \text{Too large to display}$$

input
```
int(1/x^(3/2)/(c*x^2+b*x+a)^3,x)
```

output

```
(720*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt
(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**5*c**3 - 996
*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a)
- b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*b**2*c**2 + 14
40*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a
) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*b*c**3*x + 1
440*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(
a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*c**4*x**2 +
 312*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt
(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**4*c - 1
992*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(
a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**3*c**2*x
 - 1272*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*s
qrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**2*c*
*3*x**2 + 1440*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*
b*c**4*x**3 + 720*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*
*3*c**5*x**4 - 30*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))...
```

3.282 $\int \frac{2\sqrt{x}}{1+2x-x^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 66

$$\int \frac{2\sqrt{x}}{1+2x-x^2}\,dx = -\sqrt{2\left(-1+\sqrt{2}\right)}\arctan\left(\frac{\sqrt{x}}{\sqrt{-1+\sqrt{2}}}\right) + \sqrt{2\left(1+\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{x}}{\sqrt{1+\sqrt{2}}}\right)$$

output
```
-(-2+2*2^(1/2))^(1/2)*arctan(x^(1/2)/(2^(1/2)-1)^(1/2))+(2+2*2^(1/2))^(1/2
)*arctanh(x^(1/2)/(1+2^(1/2))^(1/2))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09

$$\int \frac{2\sqrt{x}}{1+2x-x^2}\,dx = 2\left(-\sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\arctan\left(\sqrt{1+\sqrt{2}}\sqrt{x}\right) + \sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\operatorname{arctanh}\left(\sqrt{-1+\sqrt{2}}\sqrt{x}\right)\right)$$

input
```
Integrate[(2*Sqrt[x])/(1 + 2*x - x^2),x]
```

output
```
2*(-(Sqrt[(-1 + Sqrt[2])/2]*ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x]]) + Sqrt[(1 +
 Sqrt[2])/2]*ArcTanh[Sqrt[-1 + Sqrt[2]]*Sqrt[x]])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {27, 1148, 1450, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2\sqrt{x}}{-x^2+2x+1}\,dx$$
$$\Big\downarrow \text{27}$$
$$2\int \frac{\sqrt{x}}{-x^2+2x+1}dx$$
$$\Big\downarrow \text{1148}$$
$$4\int \frac{x}{-x^2+2x+1}d\sqrt{x}$$
$$\Big\downarrow \text{1450}$$
$$4\left(\frac{1}{4}\left(2-\sqrt{2}\right)\int \frac{1}{-x-\sqrt{2}+1}d\sqrt{x}+\frac{1}{4}\left(2+\sqrt{2}\right)\int \frac{1}{-x+\sqrt{2}+1}d\sqrt{x}\right)$$
$$\Big\downarrow \text{217}$$
$$4\left(\frac{1}{4}\left(2+\sqrt{2}\right)\int \frac{1}{-x+\sqrt{2}+1}d\sqrt{x}-\frac{\left(2-\sqrt{2}\right)\arctan\left(\frac{\sqrt{x}}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}}\right)$$
$$\Big\downarrow \text{219}$$
$$4\left(\frac{\left(2+\sqrt{2}\right)\text{arctanh}\left(\frac{\sqrt{x}}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}-\frac{\left(2-\sqrt{2}\right)\arctan\left(\frac{\sqrt{x}}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}}\right)$$

input
```
Int[(2*Sqrt[x])/(1 + 2*x - x^2),x]
```

output
```
4*(-1/4*((2 - Sqrt[2])*ArcTan[Sqrt[x]/Sqrt[-1 + Sqrt[2]]])/Sqrt[-1 + Sqrt[
2]] + ((2 + Sqrt[2])*ArcTanh[Sqrt[x]/Sqrt[1 + Sqrt[2]]])/(4*Sqrt[1 + Sqrt[
2]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1148
```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[2*e   Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c
*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

method	result
derivativedivides	$-\sqrt{\sqrt{2}-1}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{x}}{\sqrt{\sqrt{2}-1}}\right)+\sqrt{1+\sqrt{2}}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{x}}{\sqrt{1+\sqrt{2}}}\right)$
default	$-\sqrt{\sqrt{2}-1}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{x}}{\sqrt{\sqrt{2}-1}}\right)+\sqrt{1+\sqrt{2}}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{x}}{\sqrt{1+\sqrt{2}}}\right)$
trager	$2\,\mathrm{RootOf}\left(64\_Z^4-16\_Z^2-1\right)\ln\left(\frac{-8\,\mathrm{RootOf}\left(64\_Z^4-16\_Z^2-1\right)^3 x+8\,\mathrm{RootOf}\left(64\_Z^4-16\_Z\right.}{8\,\mathrm{RootOf}\left(64\_Z^4-16\_Z^2-1\right.}\right.$

input
```
int(2*x^(1/2)/(-x^2+2*x+1),x,method=_RETURNVERBOSE)
```

output
```
-(2^(1/2)-1)^(1/2)*2^(1/2)*arctan(x^(1/2)/(2^(1/2)-1)^(1/2))+(1+2^(1/2))^(
1/2)*2^(1/2)*arctanh(x^(1/2)/(1+2^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.35

$$\int \frac{2\sqrt{x}}{1+2x-x^2}\,dx = -2\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}\arctan\left(\sqrt{x}\left(\sqrt{2}+2\right)\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}\right) + \sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}+\sqrt{x}\right) - \sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\log\left(-\sqrt{2}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}+\sqrt{x}\right)$$

input
```
integrate(2*x^(1/2)/(-x^2+2*x+1),x, algorithm="fricas")
```

output
```
-2*sqrt(1/2*sqrt(2) - 1/2)*arctan(sqrt(x)*(sqrt(2) + 2)*sqrt(1/2*sqrt(2) -
 1/2)) + sqrt(1/2*sqrt(2) + 1/2)*log(sqrt(2)*sqrt(1/2*sqrt(2) + 1/2) + sqr
t(x)) - sqrt(1/2*sqrt(2) + 1/2)*log(-sqrt(2)*sqrt(1/2*sqrt(2) + 1/2) + sqr
t(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(56) = 112$.

Time = 1.95 (sec) , antiderivative size = 340, normalized size of antiderivative = 5.15

$$\int \frac{2\sqrt{x}}{1+2x-x^2}\,dx = \frac{4\log\left(\sqrt{x}-\sqrt{1+\sqrt{2}}\right)}{-12\sqrt{1+\sqrt{2}}+8\sqrt{2}\sqrt{1+\sqrt{2}}} - \frac{2\sqrt{2}\log\left(\sqrt{x}-\sqrt{1+\sqrt{2}}\right)}{-12\sqrt{1+\sqrt{2}}+8\sqrt{2}\sqrt{1+\sqrt{2}}} + \frac{2\sqrt{2}\log\left(\sqrt{x}+\sqrt{1+\sqrt{2}}\right)}{-12\sqrt{1+\sqrt{2}}+8\sqrt{2}\sqrt{1+\sqrt{2}}} - \frac{4\log\left(\sqrt{x}+\sqrt{1+\sqrt{2}}\right)}{-12\sqrt{1+\sqrt{2}}+8\sqrt{2}\sqrt{1+\sqrt{2}}} + \frac{12\sqrt{2}\sqrt{-1+\sqrt{2}}\sqrt{1+\sqrt{2}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt{-1+\sqrt{2}}}\right)}{-12\sqrt{1+\sqrt{2}}+8\sqrt{2}\sqrt{1+\sqrt{2}}} - \frac{16\sqrt{-1+\sqrt{2}}\sqrt{1+\sqrt{2}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt{-1+\sqrt{2}}}\right)}{-12\sqrt{1+\sqrt{2}}+8\sqrt{2}\sqrt{1+\sqrt{2}}}$$

input
```
integrate(2*x**(1/2)/(-x**2+2*x+1),x)
```

output
```
4*log(sqrt(x) - sqrt(1 + sqrt(2)))/(-12*sqrt(1 + sqrt(2)) + 8*sqrt(2)*sqrt
(1 + sqrt(2))) - 2*sqrt(2)*log(sqrt(x) - sqrt(1 + sqrt(2)))/(-12*sqrt(1 +
sqrt(2)) + 8*sqrt(2)*sqrt(1 + sqrt(2))) + 2*sqrt(2)*log(sqrt(x) + sqrt(1 +
 sqrt(2)))/(-12*sqrt(1 + sqrt(2)) + 8*sqrt(2)*sqrt(1 + sqrt(2))) - 4*log(s
qrt(x) + sqrt(1 + sqrt(2)))/(-12*sqrt(1 + sqrt(2)) + 8*sqrt(2)*sqrt(1 + sq
rt(2))) + 12*sqrt(2)*sqrt(-1 + sqrt(2))*sqrt(1 + sqrt(2))*atan(sqrt(x)/sqr
t(-1 + sqrt(2)))/(-12*sqrt(1 + sqrt(2)) + 8*sqrt(2)*sqrt(1 + sqrt(2))) - 1
6*sqrt(-1 + sqrt(2))*sqrt(1 + sqrt(2))*atan(sqrt(x)/sqrt(-1 + sqrt(2)))/(-
12*sqrt(1 + sqrt(2)) + 8*sqrt(2)*sqrt(1 + sqrt(2)))
```

Maxima [F]

$$\int \frac{2\sqrt{x}}{1+2x-x^2}\,dx = \int -\frac{2\sqrt{x}}{x^2-2x-1}\,dx$$

input
```
integrate(2*x^(1/2)/(-x^2+2*x+1),x, algorithm="maxima")
```

output
```
-2*integrate(sqrt(x)/(x^2 - 2*x - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.11

$$\int \frac{2\sqrt{x}}{1+2x-x^2}\,dx = -\sqrt{2\sqrt{2}-2}\arctan\left(\frac{\sqrt{x}}{\sqrt{\sqrt{2}-1}}\right) + \frac{1}{2}\sqrt{2\sqrt{2}+2}\log\left(\sqrt{x}+\sqrt{\sqrt{2}+1}\right) - \frac{1}{2}\sqrt{2\sqrt{2}+2}\log\left(\left|\sqrt{x}-\sqrt{\sqrt{2}+1}\right|\right)$$

input
```
integrate(2*x^(1/2)/(-x^2+2*x+1),x, algorithm="giac")
```

output
```
-sqrt(2*sqrt(2) - 2)*arctan(sqrt(x)/sqrt(sqrt(2) - 1)) + 1/2*sqrt(2*sqrt(2
) + 2)*log(sqrt(x) + sqrt(sqrt(2) + 1)) - 1/2*sqrt(2*sqrt(2) + 2)*log(abs(
sqrt(x) - sqrt(sqrt(2) + 1)))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.30

$$\int \frac{2\sqrt{x}}{1+2x-x^2}\,dx = \sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\sqrt{1-\sqrt{2}}\left(\sqrt{x}\left(\frac{\sqrt{2}}{2}-\frac{1}{2}\right)+\frac{3\sqrt{x}}{2}\right)\right)\sqrt{1-\sqrt{2}} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{-2\sqrt{2}-2}\left(\sqrt{x}\left(\frac{\sqrt{2}}{2}+\frac{1}{2}\right)-\frac{3\sqrt{x}}{2}\right)\right)\sqrt{\sqrt{2}+1}\,1i$$

input
```
int((2*x^(1/2))/(2*x - x^2 + 1),x)
```

output
```
2^(1/2)*atan((- 2*2^(1/2) - 2)^(1/2)*(x^(1/2)*(2^(1/2)/2 + 1/2) - (3*x^(1/
2))/2))*(2^(1/2) + 1)^(1/2)*1i + 2^(1/2)*atanh(2^(1/2)*(1 - 2^(1/2))^(1/2)
*(x^(1/2)*(2^(1/2)/2 - 1/2) + (3*x^(1/2))/2))*(1 - 2^(1/2))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int \frac{2\sqrt{x}}{1+2x-x^2}\,dx = \frac{\sqrt{2}\left(-2\sqrt{\sqrt{2}-1}\,atan\left(\frac{\sqrt{x}}{\sqrt{\sqrt{2}-1}}\right)-\sqrt{\sqrt{2}+1}\log\left(-\sqrt{\sqrt{2}+1}+\sqrt{x}\right)+\sqrt{\sqrt{2}+1}\log\left(\sqrt{\sqrt{2}+1}+\sqrt{x}\right)\right)}{2}$$

input
```
int(2*x^(1/2)/(-x^2+2*x+1),x)
```

output
```
(sqrt(2)*( - 2*sqrt(sqrt(2) - 1)*atan(sqrt(x)/sqrt(sqrt(2) - 1)) - sqrt(sq
rt(2) + 1)*log( - sqrt(sqrt(2) + 1) + sqrt(x)) + sqrt(sqrt(2) + 1)*log(sqr
t(sqrt(2) + 1) + sqrt(x))))/2
```

3.283 $\int \frac{1}{\sqrt{x}\left(1+\frac{1-x^2}{2x}\right)} dx$

Optimal result

Integrand size = 24, antiderivative size = 66

$$\int \frac{1}{\sqrt{x}\left(1+\frac{1-x^2}{2x}\right)} dx = -\sqrt{2\left(-1+\sqrt{2}\right)}\arctan\left(\frac{\sqrt{x}}{\sqrt{-1+\sqrt{2}}}\right) + \sqrt{2\left(1+\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{x}}{\sqrt{1+\sqrt{2}}}\right)$$

output
```
-(-2+2*2^(1/2))^(1/2)*arctan(x^(1/2)/(2^(1/2)-1)^(1/2))+(2+2*2^(1/2))^(1/2
)*arctanh(x^(1/2)/(1+2^(1/2))^(1/2))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{x}\left(1+\frac{1-x^2}{2x}\right)} dx = 2\left(-\sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\arctan\left(\sqrt{1+\sqrt{2}}\sqrt{x}\right) + \sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\operatorname{arctanh}\left(\sqrt{-1+\sqrt{2}}\sqrt{x}\right)\right)$$

input
```
Integrate[1/(Sqrt[x]*(1 + (1 - x^2)/(2*x))),x]
```

output
```
2*(-(Sqrt[(-1 + Sqrt[2])/2]*ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x]]) + Sqrt[(1 +
 Sqrt[2])/2]*ArcTanh[Sqrt[-1 + Sqrt[2]]*Sqrt[x]])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\left(\frac{1-x^2}{2x}+1\right)}dx$$

$$\Big\downarrow \text{2035}$$

$$2\int \frac{1}{\frac{1-x^2}{2x}+1}d\sqrt{x}$$

$$\Big\downarrow \text{7299}$$

$$2\int \frac{1}{\frac{1-x^2}{2x}+1}d\sqrt{x}$$

input
```
Int[1/(Sqrt[x]*(1 + (1 - x^2)/(2*x))),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

method	result
derivativedivides	$-\sqrt{\sqrt{2}-1}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{x}}{\sqrt{\sqrt{2}-1}}\right)+\sqrt{1+\sqrt{2}}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{x}}{\sqrt{1+\sqrt{2}}}\right)$
default	$-\sqrt{\sqrt{2}-1}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{x}}{\sqrt{\sqrt{2}-1}}\right)+\sqrt{1+\sqrt{2}}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{x}}{\sqrt{1+\sqrt{2}}}\right)$
trager	$-2\,\mathrm{RootOf}\left(64\_Z^4-16\_Z^2-1\right)\ln\left(\frac{8\,\mathrm{RootOf}\left(64\_Z^4-16\_Z^2-1\right)^3x-8\,\mathrm{RootOf}\left(64\_Z^4-16\_Z\right.}{8\,\mathrm{RootOf}\left(64\_Z^4-16\_Z^2-\right.}\right.$

input
```
int(1/x^(1/2)/(1+1/2*(-x^2+1)/x),x,method=_RETURNVERBOSE)
```

output
```
-(2^(1/2)-1)^(1/2)*2^(1/2)*arctan(x^(1/2)/(2^(1/2)-1)^(1/2))+(1+2^(1/2))^(
1/2)*2^(1/2)*arctanh(x^(1/2)/(1+2^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.35

$$\begin{aligned}\int \frac{1}{\sqrt{x}\left(1+\frac{1-x^2}{2x}\right)}\,dx &= -2\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}\arctan\left(\sqrt{x}\left(\sqrt{2}+2\right)\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}\right)\\ &\quad+\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}+\sqrt{x}\right)\\ &\quad-\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\log\left(-\sqrt{2}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}+\sqrt{x}\right)\end{aligned}$$

input
```
integrate(1/x^(1/2)/(1+1/2*(-x^2+1)/x),x, algorithm="fricas")
```

output
```
-2*sqrt(1/2*sqrt(2) - 1/2)*arctan(sqrt(x)*(sqrt(2) + 2)*sqrt(1/2*sqrt(2) -
 1/2)) + sqrt(1/2*sqrt(2) + 1/2)*log(sqrt(2)*sqrt(1/2*sqrt(2) + 1/2) + sqr
t(x)) - sqrt(1/2*sqrt(2) + 1/2)*log(-sqrt(2)*sqrt(1/2*sqrt(2) + 1/2) + sqr
t(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(56) = 112$.

Time = 3.56 (sec) , antiderivative size = 337, normalized size of antiderivative = 5.11

$$\int \frac{1}{\sqrt{x}\left(1+\frac{1-x^2}{2x}\right)}\,dx = \frac{2\log\left(\sqrt{x}-\sqrt{1+\sqrt{2}}\right)}{-6\sqrt{1+\sqrt{2}}+4\sqrt{2}\sqrt{1+\sqrt{2}}} - \frac{\sqrt{2}\log\left(\sqrt{x}-\sqrt{1+\sqrt{2}}\right)}{-6\sqrt{1+\sqrt{2}}+4\sqrt{2}\sqrt{1+\sqrt{2}}} + \frac{\sqrt{2}\log\left(\sqrt{x}+\sqrt{1+\sqrt{2}}\right)}{-6\sqrt{1+\sqrt{2}}+4\sqrt{2}\sqrt{1+\sqrt{2}}} - \frac{2\log\left(\sqrt{x}+\sqrt{1+\sqrt{2}}\right)}{-6\sqrt{1+\sqrt{2}}+4\sqrt{2}\sqrt{1+\sqrt{2}}} + \frac{6\sqrt{2}\sqrt{-1+\sqrt{2}}\sqrt{1+\sqrt{2}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt{-1+\sqrt{2}}}\right)}{-6\sqrt{1+\sqrt{2}}+4\sqrt{2}\sqrt{1+\sqrt{2}}} - \frac{8\sqrt{-1+\sqrt{2}}\sqrt{1+\sqrt{2}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt{-1+\sqrt{2}}}\right)}{-6\sqrt{1+\sqrt{2}}+4\sqrt{2}\sqrt{1+\sqrt{2}}}$$

input
```
integrate(1/x**(1/2)/(1+1/2*(-x**2+1)/x),x)
```

output
```
2*log(sqrt(x) - sqrt(1 + sqrt(2)))/(-6*sqrt(1 + sqrt(2)) + 4*sqrt(2)*sqrt(
1 + sqrt(2))) - sqrt(2)*log(sqrt(x) - sqrt(1 + sqrt(2)))/(-6*sqrt(1 + sqrt
(2)) + 4*sqrt(2)*sqrt(1 + sqrt(2))) + sqrt(2)*log(sqrt(x) + sqrt(1 + sqrt(
2)))/(-6*sqrt(1 + sqrt(2)) + 4*sqrt(2)*sqrt(1 + sqrt(2))) - 2*log(sqrt(x)
+ sqrt(1 + sqrt(2)))/(-6*sqrt(1 + sqrt(2)) + 4*sqrt(2)*sqrt(1 + sqrt(2)))
+ 6*sqrt(2)*sqrt(-1 + sqrt(2))*sqrt(1 + sqrt(2))*atan(sqrt(x)/sqrt(-1 + sq
rt(2)))/(-6*sqrt(1 + sqrt(2)) + 4*sqrt(2)*sqrt(1 + sqrt(2))) - 8*sqrt(-1 +
 sqrt(2))*sqrt(1 + sqrt(2))*atan(sqrt(x)/sqrt(-1 + sqrt(2)))/(-6*sqrt(1 +
sqrt(2)) + 4*sqrt(2)*sqrt(1 + sqrt(2)))
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\left(1+\frac{1-x^2}{2x}\right)}\,dx = \int -\frac{2}{\sqrt{x}\left(\frac{x^2-1}{x}-2\right)}\,dx$$

input
```
integrate(1/x^(1/2)/(1+1/2*(-x^2+1)/x),x, algorithm="maxima")
```

output
```
-2*integrate(1/(sqrt(x)*((x^2 - 1)/x - 2)), x)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{x}\left(1+\frac{1-x^2}{2x}\right)}\,dx = -\sqrt{2\sqrt{2}-2}\arctan\left(\frac{\sqrt{x}}{\sqrt{\sqrt{2}-1}}\right) + \frac{1}{2}\sqrt{2\sqrt{2}+2}\log\left(\sqrt{x}+\sqrt{\sqrt{2}+1}\right) - \frac{1}{2}\sqrt{2\sqrt{2}+2}\log\left(\left|\sqrt{x}-\sqrt{\sqrt{2}+1}\right|\right)$$

input
```
integrate(1/x^(1/2)/(1+1/2*(-x^2+1)/x),x, algorithm="giac")
```

output
```
-sqrt(2*sqrt(2) - 2)*arctan(sqrt(x)/sqrt(sqrt(2) - 1)) + 1/2*sqrt(2*sqrt(2
) + 2)*log(sqrt(x) + sqrt(sqrt(2) + 1)) - 1/2*sqrt(2*sqrt(2) + 2)*log(abs(
sqrt(x) - sqrt(sqrt(2) + 1)))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt{x}\left(1+\frac{1-x^2}{2x}\right)}\,dx = \sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\sqrt{1-\sqrt{2}}\left(\sqrt{x}\left(\frac{\sqrt{2}}{2}-\frac{1}{2}\right)+\frac{3\sqrt{x}}{2}\right)\right)\sqrt{1-\sqrt{2}} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{-2\sqrt{2}-2}\left(\sqrt{x}\left(\frac{\sqrt{2}}{2}+\frac{1}{2}\right) - \frac{3\sqrt{x}}{2}\right)\right)\sqrt{\sqrt{2}+1}\,1\mathrm{i}$$

input
```
int(-1/(x^(1/2)*((x^2/2 - 1/2)/x - 1)),x)
```

output
```
2^(1/2)*atan((- 2*2^(1/2) - 2)^(1/2)*(x^(1/2)*(2^(1/2)/2 + 1/2) - (3*x^(1/
2))/2))*(2^(1/2) + 1)^(1/2)*1i + 2^(1/2)*atanh(2^(1/2)*(1 - 2^(1/2))^(1/2)
*(x^(1/2)*(2^(1/2)/2 - 1/2) + (3*x^(1/2))/2))*(1 - 2^(1/2))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{x}\left(1+\frac{1-x^2}{2x}\right)}\,dx = \frac{\sqrt{2}\left(-2\sqrt{\sqrt{2}-1}\,atan\left(\frac{\sqrt{x}}{\sqrt{\sqrt{2}-1}}\right)-\sqrt{\sqrt{2}+1}\log\left(-\sqrt{\sqrt{2}+1}+\sqrt{x}\right)+\sqrt{\sqrt{2}+1}\log\left(\sqrt{\sqrt{2}+1}+\sqrt{x}\right)\right)}{2}$$

input
```
int(1/x^(1/2)/(1+1/2*(-x^2+1)/x),x)
```

output
```
(sqrt(2)*( - 2*sqrt(sqrt(2) - 1)*atan(sqrt(x)/sqrt(sqrt(2) - 1)) - sqrt(sq
rt(2) + 1)*log( - sqrt(sqrt(2) + 1) + sqrt(x)) + sqrt(sqrt(2) + 1)*log(sqr
t(sqrt(2) + 1) + sqrt(x))))/2
```

3.284 $\int \frac{3-x+x^2}{\sqrt[3]{x}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 28

$$\int \frac{3-x+x^2}{\sqrt[3]{x}}\,dx=\frac{9x^{2/3}}{2}-\frac{3x^{5/3}}{5}+\frac{3x^{8/3}}{8}$$

output
```
9/2*x^(2/3)-3/5*x^(5/3)+3/8*x^(8/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{3-x+x^2}{\sqrt[3]{x}}\,dx=\frac{3}{40}x^{2/3}\left(60-8x+5x^2\right)$$

input
```
Integrate[(3 - x + x^2)/x^(1/3),x]
```

output
```
(3*x^(2/3)*(60 - 8*x + 5*x^2))/40
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-x+3}{\sqrt[3]{x}}\,dx$$

$$\Big\downarrow \ 1140$$

$$\int \left(x^{5/3}-x^{2/3}+\frac{3}{\sqrt[3]{x}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{3x^{8/3}}{8}-\frac{3x^{5/3}}{5}+\frac{9x^{2/3}}{2}$$

input
```
Int[(3 - x + x^2)/x^(1/3),x]
```

output
```
(9*x^(2/3))/2 - (3*x^(5/3))/5 + (3*x^(8/3))/8
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.54

method	result	size
trager	$\left(\frac{3}{8}x^{2}-\frac{3}{5}x+\frac{9}{2}\right)x^{\frac{2}{3}}$	15
gosper	$\frac{3x^{\frac{2}{3}}\left(5x^{2}-8x+60\right)}{40}$	16
risch	$\frac{3x^{\frac{2}{3}}\left(5x^{2}-8x+60\right)}{40}$	16
orering	$\frac{3x^{\frac{2}{3}}\left(5x^{2}-8x+60\right)}{40}$	16
derivativedivides	$\frac{9x^{\frac{2}{3}}}{2}-\frac{3x^{\frac{5}{3}}}{5}+\frac{3x^{\frac{8}{3}}}{8}$	17
default	$\frac{9x^{\frac{2}{3}}}{2}-\frac{3x^{\frac{5}{3}}}{5}+\frac{3x^{\frac{8}{3}}}{8}$	17

input
```
int((x^2-x+3)/x^(1/3),x,method=_RETURNVERBOSE)
```

output
```
(3/8*x^2-3/5*x+9/2)*x^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.54

$$\int \frac{3-x+x^2}{\sqrt[3]{x}}\,dx=\frac{3}{40}\left(5\,x^2-8\,x+60\right)x^{\frac{2}{3}}$$

input
```
integrate((x^2-x+3)/x^(1/3),x, algorithm="fricas")
```

output
```
3/40*(5*x^2 - 8*x + 60)*x^(2/3)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{3-x+x^2}{\sqrt[3]{x}}\,dx = \frac{3x^{\frac{8}{3}}}{8} - \frac{3x^{\frac{5}{3}}}{5} + \frac{9x^{\frac{2}{3}}}{2}$$

input
```
integrate((x**2-x+3)/x**(1/3),x)
```

output
```
3*x**(8/3)/8 - 3*x**(5/3)/5 + 9*x**(2/3)/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.57

$$\int \frac{3-x+x^2}{\sqrt[3]{x}}\,dx = \frac{3}{8}\,x^{\frac{8}{3}} - \frac{3}{5}\,x^{\frac{5}{3}} + \frac{9}{2}\,x^{\frac{2}{3}}$$

input
```
integrate((x^2-x+3)/x^(1/3),x, algorithm="maxima")
```

output
```
3/8*x^(8/3) - 3/5*x^(5/3) + 9/2*x^(2/3)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.57

$$\int \frac{3-x+x^2}{\sqrt[3]{x}}\,dx = \frac{3}{8}\,x^{\frac{8}{3}} - \frac{3}{5}\,x^{\frac{5}{3}} + \frac{9}{2}\,x^{\frac{2}{3}}$$

input
```
integrate((x^2-x+3)/x^(1/3),x, algorithm="giac")
```

output
```
3/8*x^(8/3) - 3/5*x^(5/3) + 9/2*x^(2/3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.54

$$\int \frac{3-x+x^2}{\sqrt[3]{x}}\,dx=\frac{3\,x^{2/3}\,\left(5\,x^2-8\,x+60\right)}{40}$$

input
```
int((x^2 - x + 3)/x^(1/3),x)
```

output
```
(3*x^(2/3)*(5*x^2 - 8*x + 60))/40
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.54

$$\int \frac{3-x+x^2}{\sqrt[3]{x}}\,dx=\frac{3x^{\frac{2}{3}}(5x^2-8x+60)}{40}$$

input
```
int((x^2-x+3)/x^(1/3),x)
```

output
```
(3*x**(2/3)*(5*x**2 - 8*x + 60))/40
```

3.285 $\int x\sqrt{3-2x-x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 52

$$\int x\sqrt{3-2x-x^2}\,dx=-\frac{1}{2}(1+x)\sqrt{3-2x-x^2}-\frac{1}{3}\left(3-2x-x^2\right)^{3/2}+2\arcsin\left(\frac{1}{2}(-1-x)\right)$$

output
```
-1/2*(1+x)*(-x^2-2*x+3)^(1/2)-1/3*(-x^2-2*x+3)^(3/2)-2*arcsin(1/2+1/2*x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int x\sqrt{3-2x-x^2}\,dx=\frac{1}{6}\sqrt{3-2x-x^2}\left(-9+x+2x^2\right)+4\arctan\left(\frac{\sqrt{3-2x-x^2}}{3+x}\right)$$

input
```
Integrate[x*Sqrt[3 - 2*x - x^2],x]
```

output
```
(Sqrt[3 - 2*x - x^2]*(-9 + x + 2*x^2))/6 + 4*ArcTan[Sqrt[3 - 2*x - x^2]/(3
 + x)]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1160, 1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{-x^2-2x+3}\,dx$$

$$\Big\downarrow \text{1160}$$

$$-\int \sqrt{-x^2-2x+3}dx-\frac{1}{3}\left(-x^2-2x+3\right)^{3/2}$$

$$\Big\downarrow \text{1087}$$

$$-2\int \frac{1}{\sqrt{-x^2-2x+3}}dx-\frac{1}{3}\left(-x^2-2x+3\right)^{3/2}-\frac{1}{2}(x+1)\sqrt{-x^2-2x+3}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{1-\frac{1}{16}(-2x-2)^2}}d(-2x-2)-\frac{1}{3}\left(-x^2-2x+3\right)^{3/2}-\frac{1}{2}(x+1)\sqrt{-x^2-2x+3}$$

$$\Big\downarrow \text{223}$$

$$2\arcsin\left(\frac{1}{4}(-2x-2)\right)-\frac{1}{3}\left(-x^2-2x+3\right)^{3/2}-\frac{1}{2}(x+1)\sqrt{-x^2-2x+3}$$

input
```
Int[x*Sqrt[3 - 2*x - x^2],x]
```

output
```
-1/2*((1 + x)*Sqrt[3 - 2*x - x^2]) - (3 - 2*x - x^2)^(3/2)/3 + 2*ArcSin[(-
2 - 2*x)/4]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{(2x^2+x-9)(x^2+2x-3)}{6\sqrt{-x^2-2x+3}}-2\arcsin\left(\frac{1}{2}+\frac{x}{2}\right)$
default	$-\frac{(-x^2-2x+3)^{\frac{3}{2}}}{3}+\frac{(-2x-2)\sqrt{-x^2-2x+3}}{4}-2\arcsin\left(\frac{1}{2}+\frac{x}{2}\right)$
trager	$\left(\frac{1}{3}x^2+\frac{1}{6}x-\frac{3}{2}\right)\sqrt{-x^2-2x+3}+2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2-2x\right.$

input
```
int(x*(-x^2-2*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*x^2+x-9)*(x^2+2*x-3)/(-x^2-2*x+3)^(1/2)-2*arcsin(1/2+1/2*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int x\sqrt{3-2x-x^2}\,dx = \frac{1}{6}\left(2\,x^2+x-9\right)\sqrt{-x^2-2\,x+3} + 2\,\arctan\left(\frac{\sqrt{-x^2-2\,x+3}(x+1)}{x^2+2\,x-3}\right)$$

input
```
integrate(x*(-x^2-2*x+3)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(2*x^2 + x - 9)*sqrt(-x^2 - 2*x + 3) + 2*arctan(sqrt(-x^2 - 2*x + 3)*(
x + 1)/(x^2 + 2*x - 3))
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.62

$$\int x\sqrt{3-2x-x^2}\,dx = \sqrt{-x^2-2x+3}\left(\frac{x^2}{3}+\frac{x}{6}-\frac{3}{2}\right) - 2\,\mathrm{asin}\left(\frac{x}{2}+\frac{1}{2}\right)$$

input
```
integrate(x*(-x**2-2*x+3)**(1/2),x)
```

output
```
sqrt(-x**2 - 2*x + 3)*(x**2/3 + x/6 - 3/2) - 2*asin(x/2 + 1/2)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int x\sqrt{3-2x-x^2}\,dx = -\frac{1}{3}\left(-x^2-2\,x+3\right)^{\frac{3}{2}} - \frac{1}{2}\sqrt{-x^2-2\,x+3}x - \frac{1}{2}\sqrt{-x^2-2\,x+3} + 2\,\arcsin\left(-\frac{1}{2}\,x-\frac{1}{2}\right)$$

input
```
integrate(x*(-x^2-2*x+3)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(-x^2 - 2*x + 3)^(3/2) - 1/2*sqrt(-x^2 - 2*x + 3)*x - 1/2*sqrt(-x^2 -
 2*x + 3) + 2*arcsin(-1/2*x - 1/2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.62

$$\int x\sqrt{3-2x-x^2}\,dx = \frac{1}{6}\left((2\,x+1)x-9\right)\sqrt{-x^2-2\,x+3} - 2\,\arcsin\left(\frac{1}{2}\,x+\frac{1}{2}\right)$$

input
```
integrate(x*(-x^2-2*x+3)^(1/2),x, algorithm="giac")
```

output
```
1/6*((2*x + 1)*x - 9)*sqrt(-x^2 - 2*x + 3) - 2*arcsin(1/2*x + 1/2)
```

Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

$$\int x\sqrt{3-2x-x^2}\,dx = \frac{\sqrt{-x^2-2\,x+3}\,(8\,x^2+4\,x-36)}{24} + \ln\left(x+1-\sqrt{-x^2-2\,x+3}\,1\mathrm{i}\right)\,2\mathrm{i}$$

input
```
int(x*(3 - x^2 - 2*x)^(1/2),x)
```

output
```
log(x - (3 - x^2 - 2*x)^(1/2)*1i + 1)*2i + ((3 - x^2 - 2*x)^(1/2)*(4*x + 8
*x^2 - 36))/24
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int x\sqrt{3-2x-x^2}\,dx = -2asin\left(\frac{x}{2}+\frac{1}{2}\right)+\frac{\sqrt{-x^2-2x+3}\,x^2}{3} + \frac{\sqrt{-x^2-2x+3}\,x}{6}-\frac{3\sqrt{-x^2-2x+3}}{2}+\frac{8}{3}$$

input
```
int(x*(-x^2-2*x+3)^(1/2),x)
```

output
```
( - 12*asin((x + 1)/2) + 2*sqrt( - x**2 - 2*x + 3)*x**2 + sqrt( - x**2 - 2
*x + 3)*x - 9*sqrt( - x**2 - 2*x + 3) + 16)/6
```

3.286 $\int x\sqrt{8+2x-x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 56

$$\int x\sqrt{8+2x-x^2}\,dx=-\frac{1}{2}(1-x)\sqrt{8+2x-x^2}-\frac{1}{3}\left(8+2x-x^2\right)^{3/2}-\frac{9}{2}\arcsin\left(\frac{1-x}{3}\right)$$

output
```
-1/2*(1-x)*(-x^2+2*x+8)^(1/2)-1/3*(-x^2+2*x+8)^(3/2)+9/2*arcsin(-1/3+1/3*x
)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int x\sqrt{8+2x-x^2}\,dx=\frac{1}{6}\sqrt{8+2x-x^2}\left(-19-x+2x^2\right)-9\arctan\left(\frac{\sqrt{8+2x-x^2}}{2+x}\right)$$

input
```
Integrate[x*Sqrt[8 + 2*x - x^2],x]
```

output
```
(Sqrt[8 + 2*x - x^2]*(-19 - x + 2*x^2))/6 - 9*ArcTan[Sqrt[8 + 2*x - x^2]/(
2 + x)]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1160, 1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{-x^2+2x+8}\,dx$$

$$\Big\downarrow \text{1160}$$

$$\int \sqrt{-x^2+2x+8}dx-\frac{1}{3}\left(-x^2+2x+8\right)^{3/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{9}{2}\int \frac{1}{\sqrt{-x^2+2x+8}}dx-\frac{1}{3}\left(-x^2+2x+8\right)^{3/2}-\frac{1}{2}(1-x)\sqrt{-x^2+2x+8}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{3}{4}\int \frac{1}{\sqrt{1-\frac{1}{36}(2-2x)^2}}d(2-2x)-\frac{1}{3}\left(-x^2+2x+8\right)^{3/2}-\frac{1}{2}(1-x)\sqrt{-x^2+2x+8}$$

$$\Big\downarrow \text{223}$$

$$-\frac{9}{2}\arcsin\left(\frac{1}{6}(2-2x)\right)-\frac{1}{3}\left(-x^2+2x+8\right)^{3/2}-\frac{1}{2}(1-x)\sqrt{-x^2+2x+8}$$

input
```
Int[x*Sqrt[8 + 2*x - x^2],x]
```

output
```
-1/2*((1 - x)*Sqrt[8 + 2*x - x^2]) - (8 + 2*x - x^2)^(3/2)/3 - (9*ArcSin[(
2 - 2*x)/6])/2
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{(2x^2-x-19)(x^2-2x-8)}{6\sqrt{-x^2+2x+8}}+\frac{9\arcsin\left(-\frac{1}{3}+\frac{x}{3}\right)}{2}$
default	$-\frac{(-x^2+2x+8)^{\frac{3}{2}}}{3}-\frac{(-2x+2)\sqrt{-x^2+2x+8}}{4}+\frac{9\arcsin\left(-\frac{1}{3}+\frac{x}{3}\right)}{2}$
trager	$\left(\frac{1}{3}x^2-\frac{1}{6}x-\frac{19}{6}\right)\sqrt{-x^2+2x+8}+\frac{9\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2+2x+8}+\mathrm{RootOf}\left(\_Z^2+\right.\right.}{2}$

input
```
int(x*(-x^2+2*x+8)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*x^2-x-19)*(x^2-2*x-8)/(-x^2+2*x+8)^(1/2)+9/2*arcsin(-1/3+1/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int x\sqrt{8+2x-x^2}\,dx = \frac{1}{6}\left(2\,x^2-x-19\right)\sqrt{-x^2+2\,x+8} - \frac{9}{2}\arctan\left(\frac{\sqrt{-x^2+2\,x+8}(x-1)}{x^2-2\,x-8}\right)$$

input
```
integrate(x*(-x^2+2*x+8)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(2*x^2 - x - 19)*sqrt(-x^2 + 2*x + 8) - 9/2*arctan(sqrt(-x^2 + 2*x + 8
)*(x - 1)/(x^2 - 2*x - 8))
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.61

$$\int x\sqrt{8+2x-x^2}\,dx = \sqrt{-x^2+2x+8}\left(\frac{x^2}{3}-\frac{x}{6}-\frac{19}{6}\right)+\frac{9\operatorname{asin}\left(\frac{x}{3}-\frac{1}{3}\right)}{2}$$

input
```
integrate(x*(-x**2+2*x+8)**(1/2),x)
```

output
```
sqrt(-x**2 + 2*x + 8)*(x**2/3 - x/6 - 19/6) + 9*asin(x/3 - 1/3)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int x\sqrt{8+2x-x^2}\,dx = -\frac{1}{3}\left(-x^2+2\,x+8\right)^{\frac{3}{2}}+\frac{1}{2}\sqrt{-x^2+2\,x+8}x - \frac{1}{2}\sqrt{-x^2+2\,x+8}-\frac{9}{2}\arcsin\left(-\frac{1}{3}\,x+\frac{1}{3}\right)$$

input
```
integrate(x*(-x^2+2*x+8)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(-x^2 + 2*x + 8)^(3/2) + 1/2*sqrt(-x^2 + 2*x + 8)*x - 1/2*sqrt(-x^2 +
 2*x + 8) - 9/2*arcsin(-1/3*x + 1/3)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.57

$$\int x\sqrt{8+2x-x^2}\,dx = \frac{1}{6}\left((2\,x-1)x-19\right)\sqrt{-x^2+2\,x+8}+\frac{9}{2}\,\arcsin\left(\frac{1}{3}\,x-\frac{1}{3}\right)$$

input
```
integrate(x*(-x^2+2*x+8)^(1/2),x, algorithm="giac")
```

output
```
1/6*((2*x - 1)*x - 19)*sqrt(-x^2 + 2*x + 8) + 9/2*arcsin(1/3*x - 1/3)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

$$\int x\sqrt{8+2x-x^2}\,dx = -\frac{\sqrt{-x^2+2\,x+8}\,(-8\,x^2+4\,x+76)}{24} - \frac{\ln\left(x-1-\sqrt{-x^2+2\,x+8}\,1\mathrm{i}\right)\,9\mathrm{i}}{2}$$

input
```
int(x*(2*x - x^2 + 8)^(1/2),x)
```

output
```
- (log(x - (2*x - x^2 + 8)^(1/2)*1i - 1)*9i)/2 - ((2*x - x^2 + 8)^(1/2)*(4
*x - 8*x^2 + 76))/24
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int x\sqrt{8+2x-x^2}\,dx = \frac{9asin\left(\frac{x}{3}-\frac{1}{3}\right)}{2}+\frac{\sqrt{-x^2+2x+8}\,x^2}{3} - \frac{\sqrt{-x^2+2x+8}\,x}{6}-\frac{19\sqrt{-x^2+2x+8}}{6}+9$$

input
```
int(x*(-x^2+2*x+8)^(1/2),x)
```

output
```
(27*asin((x - 1)/3) + 2*sqrt( - x**2 + 2*x + 8)*x**2 - sqrt( - x**2 + 2*x
+ 8)*x - 19*sqrt( - x**2 + 2*x + 8) + 54)/6
```

3.287 $\int x\sqrt{4+2x+x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int x\sqrt{4+2x+x^2}\,dx=-\frac{1}{2}(1+x)\sqrt{4+2x+x^2}+\frac{1}{3}\left(4+2x+x^2\right)^{3/2}-\frac{3}{2}\text{arcsinh}\left(\frac{1+x}{\sqrt{3}}\right)$$

output
```
-1/2*(1+x)*(x^2+2*x+4)^(1/2)+1/3*(x^2+2*x+4)^(3/2)-3/2*arcsinh(1/3*(1+x)*3
^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int x\sqrt{4+2x+x^2}\,dx=\frac{1}{6}\sqrt{4+2x+x^2}\left(5+x+2x^2\right)+\frac{3}{2}\log\left(-1-x+\sqrt{4+2x+x^2}\right)$$

input
```
Integrate[x*Sqrt[4 + 2*x + x^2],x]
```

output
```
(Sqrt[4 + 2*x + x^2]*(5 + x + 2*x^2))/6 + (3*Log[-1 - x + Sqrt[4 + 2*x + x
^2]])/2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1160, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{x^2+2x+4}\,dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{3}\left(x^2+2x+4\right)^{3/2}-\int \sqrt{x^2+2x+4}dx$$

$$\Big\downarrow \text{1087}$$

$$-\frac{3}{2}\int \frac{1}{\sqrt{x^2+2x+4}}dx+\frac{1}{3}\left(x^2+2x+4\right)^{3/2}-\frac{1}{2}(x+1)\sqrt{x^2+2x+4}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{1}{4}\sqrt{3}\int \frac{1}{\sqrt{\frac{1}{12}(2x+2)^2+1}}d(2x+2)+\frac{1}{3}\left(x^2+2x+4\right)^{3/2}-\frac{1}{2}(x+1)\sqrt{x^2+2x+4}$$

$$\Big\downarrow \text{222}$$

$$-\frac{3}{2}\text{arcsinh}\left(\frac{2x+2}{2\sqrt{3}}\right)+\frac{1}{3}\left(x^2+2x+4\right)^{3/2}-\frac{1}{2}(x+1)\sqrt{x^2+2x+4}$$

input
```
Int[x*Sqrt[4 + 2*x + x^2],x]
```

output
```
-1/2*((1 + x)*Sqrt[4 + 2*x + x^2]) + (4 + 2*x + x^2)^(3/2)/3 - (3*ArcSinh[
(2 + 2*x)/(2*Sqrt[3])])/2
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{(2x^2+x+5)\sqrt{x^2+2x+4}}{6}-\frac{3\,\operatorname{arcsinh}\left(\frac{(x+1)\sqrt{3}}{3}\right)}{2}$	33
trager	$\left(\frac{1}{3}x^2+\frac{1}{6}x+\frac{5}{6}\right)\sqrt{x^2+2x+4}-\frac{3\ln\left(x+1+\sqrt{x^2+2x+4}\right)}{2}$	39
default	$\frac{(x^2+2x+4)^{\frac{3}{2}}}{3}-\frac{(2+2x)\sqrt{x^2+2x+4}}{4}-\frac{3\,\operatorname{arcsinh}\left(\frac{(x+1)\sqrt{3}}{3}\right)}{2}$	42

input
```
int(x*(x^2+2*x+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*x^2+x+5)*(x^2+2*x+4)^(1/2)-3/2*arcsinh(1/3*(x+1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int x\sqrt{4+2x+x^2}\,dx = \frac{1}{6}\left(2\,x^2+x+5\right)\sqrt{x^2+2\,x+4}+\frac{3}{2}\,\log\left(-x+\sqrt{x^2+2\,x+4}-1\right)$$

input
```
integrate(x*(x^2+2*x+4)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(2*x^2 + x + 5)*sqrt(x^2 + 2*x + 4) + 3/2*log(-x + sqrt(x^2 + 2*x + 4)
 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.74

$$\int x\sqrt{4+2x+x^2}\,dx = \left(\frac{x^2}{3}+\frac{x}{6}+\frac{5}{6}\right)\sqrt{x^2+2x+4}-\frac{3\,\mathrm{asinh}\left(\frac{\sqrt{3}(x+1)}{3}\right)}{2}$$

input
```
integrate(x*(x**2+2*x+4)**(1/2),x)
```

output
```
(x**2/3 + x/6 + 5/6)*sqrt(x**2 + 2*x + 4) - 3*asinh(sqrt(3)*(x + 1)/3)/2
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int x\sqrt{4+2x+x^2}\,dx = \frac{1}{3}\left(x^2+2\,x+4\right)^{\frac{3}{2}}-\frac{1}{2}\,\sqrt{x^2+2\,x+4}x-\frac{1}{2}\,\sqrt{x^2+2\,x+4}-\frac{3}{2}\,\mathrm{arsinh}\left(\frac{1}{3}\,\sqrt{3}(x+1)\right)$$

input
```
integrate(x*(x^2+2*x+4)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(x^2 + 2*x + 4)^(3/2) - 1/2*sqrt(x^2 + 2*x + 4)*x - 1/2*sqrt(x^2 + 2*x
 + 4) - 3/2*arcsinh(1/3*sqrt(3)*(x + 1))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int x\sqrt{4+2x+x^2}\,dx = \frac{1}{6}\left((2\,x+1)x+5\right)\sqrt{x^2+2\,x+4} + \frac{3}{2}\log\left(-x+\sqrt{x^2+2\,x+4}-1\right)$$

input
```
integrate(x*(x^2+2*x+4)^(1/2),x, algorithm="giac")
```

output
```
1/6*((2*x + 1)*x + 5)*sqrt(x^2 + 2*x + 4) + 3/2*log(-x + sqrt(x^2 + 2*x +
4) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int x\sqrt{4+2x+x^2}\,dx = \frac{\sqrt{x^2+2\,x+4}\,(8\,x^2+4\,x+20)}{24} - \frac{3\,\ln\left(x+\sqrt{x^2+2\,x+4}+1\right)}{2}$$

input
```
int(x*(2*x + x^2 + 4)^(1/2),x)
```

output
```
((2*x + x^2 + 4)^(1/2)*(4*x + 8*x^2 + 20))/24 - (3*log(x + (2*x + x^2 + 4)
^(1/2) + 1))/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.16

$$\int x\sqrt{4+2x+x^2}\,dx = \frac{\sqrt{x^2+2x+4}\,x^2}{3} + \frac{\sqrt{x^2+2x+4}\,x}{6} + \frac{5\sqrt{x^2+2x+4}}{6} - \frac{3\log\left(\frac{\sqrt{x^2+2x+4}+x+1}{\sqrt{3}}\right)}{2}$$

input
```
int(x*(x^2+2*x+4)^(1/2),x)
```

output
```
(2*sqrt(x**2 + 2*x + 4)*x**2 + sqrt(x**2 + 2*x + 4)*x + 5*sqrt(x**2 + 2*x
+ 4) - 9*log((sqrt(x**2 + 2*x + 4) + x + 1)/sqrt(3)))/6
```

3.288 $\int \frac{\sqrt{2-x-x^2}}{x^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 62

$$\int \frac{\sqrt{2-x-x^2}}{x^2} \, dx = -\frac{\sqrt{2-x-x^2}}{x} + \arcsin\left(\frac{1}{3}(-1-2x)\right) + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}(1-x)}{\sqrt{2-x-x^2}}\right)}{\sqrt{2}}$$

output
```
-(-x^2-x+2)^(1/2)/x-arcsin(1/3+2/3*x)+1/2*arctanh(2^(1/2)*(1-x)/(-x^2-x+2)
^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{2-x-x^2}}{x^2} \, dx = -\frac{\sqrt{2-x-x^2}}{x} + 2\arctan\left(\frac{\sqrt{2-x-x^2}}{2+x}\right) - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2-x-x^2}}{\sqrt{2}(-1+x)}\right)}{\sqrt{2}}$$

input
```
Integrate[Sqrt[2 - x - x^2]/x^2,x]
```

output
```
-(Sqrt[2 - x - x^2]/x) + 2*ArcTan[Sqrt[2 - x - x^2]/(2 + x)] - ArcTanh[Sqr
t[2 - x - x^2]/(Sqrt[2]*(-1 + x))]/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1161, 25, 1269, 1090, 223, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-x^2-x+2}}{x^2} \, dx$$

$$\Big\downarrow \text{1161}$$

$$\frac{1}{2} \int -\frac{2x+1}{x\sqrt{-x^2-x+2}}dx-\frac{\sqrt{-x^2-x+2}}{x}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2} \int \frac{2x+1}{x\sqrt{-x^2-x+2}}dx-\frac{\sqrt{-x^2-x+2}}{x}$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2} \left(-2 \int \frac{1}{\sqrt{-x^2-x+2}}dx-\int \frac{1}{x\sqrt{-x^2-x+2}}dx\right)-\frac{\sqrt{-x^2-x+2}}{x}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{2} \left(\frac{2}{3} \int \frac{1}{\sqrt{1-\frac{1}{9}(-2x-1)^2}}d(-2x-1)-\int \frac{1}{x\sqrt{-x^2-x+2}}dx\right)-\frac{\sqrt{-x^2-x+2}}{x}$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{2} \left(2 \arcsin \left(\frac{1}{3}(-2x-1)\right)-\int \frac{1}{x\sqrt{-x^2-x+2}}dx\right)-\frac{\sqrt{-x^2-x+2}}{x}$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2} \left(2 \int \frac{1}{8-\frac{(4-x)^2}{-x^2-x+2}}d\frac{4-x}{\sqrt{-x^2-x+2}}+2 \arcsin \left(\frac{1}{3}(-2x-1)\right)\right)-\frac{\sqrt{-x^2-x+2}}{x}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2} \left(2 \arcsin \left(\frac{1}{3}(-2x-1)\right)+\frac{\text{arctanh}\left(\frac{4-x}{2\sqrt{2}\sqrt{-x^2-x+2}}\right)}{\sqrt{2}}\right)-\frac{\sqrt{-x^2-x+2}}{x}$$

input
```
Int[Sqrt[2 - x - x^2]/x^2,x]
```

output
```
-(Sqrt[2 - x - x^2]/x) + (2*ArcSin[(-1 - 2*x)/3] + ArcTanh[(4 - x)/(2*Sqrt
[2]*Sqrt[2 - x - x^2])]/Sqrt[2])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

method	result
risch	$\frac{x^2+x-2}{x\sqrt{-x^2-x+2}}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(4-x)\sqrt{2}}{4\sqrt{-x^2-x+2}}\right)}{4}-\arcsin\left(\frac{1}{3}+\frac{2x}{3}\right)$
default	$-\frac{(-x^2-x+2)^{\frac{3}{2}}}{2x}-\frac{\sqrt{-x^2-x+2}}{4}-\arcsin\left(\frac{1}{3}+\frac{2x}{3}\right)+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(4-x)\sqrt{2}}{4\sqrt{-x^2-x+2}}\right)}{4}+\frac{(-1-2x)\sqrt{-x^2-x+2}}{4}$
trager	$-\frac{\sqrt{-x^2-x+2}}{x}+\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(2\operatorname{RootOf}\left(\_Z^2+1\right)x+2\sqrt{-x^2-x+2}+\operatorname{RootOf}\left(\_Z^2\right.\right.$

input
```
int((-x^2-x+2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
(x^2+x-2)/x/(-x^2-x+2)^(1/2)+1/4*2^(1/2)*arctanh(1/4*(4-x)*2^(1/2)/(-x^2-x
+2)^(1/2))-arcsin(1/3+2/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{2-x-x^2}}{x^2}\,dx = \frac{\sqrt{2}x\log\left(-\frac{4\sqrt{2}\sqrt{-x^2-x+2}(x-4)+7x^2+16x-32}{x^2}\right)+8x\arctan\left(\frac{\sqrt{-x^2-x+2}(2x+1)}{2(x^2+x-2)}\right)-8\sqrt{-x^2-x+2}}{8x}$$

input
```
integrate((-x^2-x+2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
1/8*(sqrt(2)*x*log(-(4*sqrt(2)*sqrt(-x^2 - x + 2)*(x - 4) + 7*x^2 + 16*x -
 32)/x^2) + 8*x*arctan(1/2*sqrt(-x^2 - x + 2)*(2*x + 1)/(x^2 + x - 2)) - 8
*sqrt(-x^2 - x + 2))/x
```

Sympy [F]

$$\int \frac{\sqrt{2-x-x^2}}{x^2}\,dx = \int \frac{\sqrt{-(x-1)(x+2)}}{x^2}\,dx$$

input
```
integrate((-x**2-x+2)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 2))/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{2-x-x^2}}{x^2}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{2\sqrt{2}\sqrt{-x^2-x+2}}{|x|}+\frac{4}{|x|}-1\right) - \frac{\sqrt{-x^2-x+2}}{x} + \arcsin\left(-\frac{2}{3}x-\frac{1}{3}\right)$$

input
```
integrate((-x^2-x+2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*log(2*sqrt(2)*sqrt(-x^2 - x + 2)/abs(x) + 4/abs(x) - 1) - sqrt
(-x^2 - x + 2)/x + arcsin(-2/3*x - 1/3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(52) = 104$.

Time = 0.21 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.71

$$\int \frac{\sqrt{2-x-x^2}}{x^2}\,dx = -\frac{1}{4}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+\frac{2\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+6\right|}{\left|4\sqrt{2}+\frac{2\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+6\right|}\right) + \frac{6\left(\frac{3\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+1\right)}{\frac{6\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+\frac{\left(2\sqrt{-x^2-x+2}-3\right)^2}{(2x+1)^2}+1} - \arcsin\left(\frac{2}{3}x+\frac{1}{3}\right)$$

input

```
integrate((-x^2-x+2)^(1/2)/x^2,x, algorithm="giac")
```

output

```
-1/4*sqrt(2)*log(abs(-4*sqrt(2) + 2*(2*sqrt(-x^2 - x + 2) - 3)/(2*x + 1) +
 6)/abs(4*sqrt(2) + 2*(2*sqrt(-x^2 - x + 2) - 3)/(2*x + 1) + 6)) + 6*(3*(2
*sqrt(-x^2 - x + 2) - 3)/(2*x + 1) + 1)/(6*(2*sqrt(-x^2 - x + 2) - 3)/(2*x
 + 1) + (2*sqrt(-x^2 - x + 2) - 3)^2/(2*x + 1)^2 + 1) - arcsin(2/3*x + 1/3
)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{2-x-x^2}}{x^2}\,dx = \frac{\sqrt{2}\ln\left(\frac{2}{x}+\frac{\sqrt{2}\sqrt{-x^2-x+2}}{x}-\frac{1}{2}\right)}{4} - \frac{\sqrt{-x^2-x+2}}{x} + \ln\left(x\,1\mathrm{i}+\sqrt{-x^2-x+2}+\frac{1}{2}\mathrm{i}\right)1\mathrm{i}$$

input

```
int((2 - x^2 - x)^(1/2)/x^2,x)
```

output

```
log(x*1i + (2 - x^2 - x)^(1/2) + 1i/2)*1i - (2 - x^2 - x)^(1/2)/x + (2^(1/
2)*log(2/x + (2^(1/2)*(2 - x^2 - x)^(1/2))/x - 1/2))/4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{2-x-x^2}}{x^2}\,dx$$
$$= \frac{-4asin\left(\frac{2x}{3}+\frac{1}{3}\right)x - 4\sqrt{-x^2-x+2} + \sqrt{2}\log\left(-2\sqrt{2}+\tan\left(\frac{asin\left(\frac{2x}{3}+\frac{1}{3}\right)}{2}\right)-3\right)x - \sqrt{2}\log\left(2\sqrt{2}+\tan\left(\right.\right.}{4x}$$

input
```
int((-x^2-x+2)^(1/2)/x^2,x)
```

output
```
( - 4*asin((2*x + 1)/3)*x - 4*sqrt( - x**2 - x + 2) + sqrt(2)*log( - 2*sqr
t(2) + tan(asin((2*x + 1)/3)/2) - 3)*x - sqrt(2)*log(2*sqrt(2) + tan(asin(
(2*x + 1)/3)/2) - 3)*x)/(4*x)
```

3.289 $\int \frac{\sqrt{-2-3x+5x^2}}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int \frac{\sqrt{-2-3x+5x^2}}{x}\,dx = \sqrt{-2-3x+5x^2}+2\sqrt{2}\arctan\left(\frac{\sqrt{2}(1-x)}{\sqrt{-2-3x+5x^2}}\right)+\frac{3\text{arctanh}\left(\frac{\sqrt{5}(1-x)}{\sqrt{-2-3x+5x^2}}\right)}{\sqrt{5}}$$

output
```
(5*x^2-3*x-2)^(1/2)+2*2^(1/2)*arctan(2^(1/2)*(1-x)/(5*x^2-3*x-2)^(1/2))+3/
5*arctanh(5^(1/2)*(1-x)/(5*x^2-3*x-2)^(1/2))*5^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{-2-3x+5x^2}}{x}\,dx = \sqrt{-2-3x+5x^2}+2\sqrt{2}\arctan\left(\frac{\sqrt{-2-3x+5x^2}}{\sqrt{2}(-1+x)}\right)-\frac{3\text{arctanh}\left(\frac{\sqrt{5}\sqrt{-2-3x+5x^2}}{2+5x}\right)}{\sqrt{5}}$$

input
```
Integrate[Sqrt[-2 - 3*x + 5*x^2]/x,x]
```

output
```
Sqrt[-2 - 3*x + 5*x^2] + 2*Sqrt[2]*ArcTan[Sqrt[-2 - 3*x + 5*x^2]/(Sqrt[2]*
(-1 + x))] - (3*ArcTanh[(Sqrt[5]*Sqrt[-2 - 3*x + 5*x^2])/(2 + 5*x)])/Sqrt[
5]
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1162, 1269, 1092, 219, 1154, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{5x^2-3x-2}}{x}dx$$

$$\Big\downarrow \text{1162}$$

$$\sqrt{5x^2-3x-2}-\frac{1}{2}\int \frac{3x+4}{x\sqrt{5x^2-3x-2}}dx$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2}\left(-3\int \frac{1}{\sqrt{5x^2-3x-2}}dx-4\int \frac{1}{x\sqrt{5x^2-3x-2}}dx\right)+\sqrt{5x^2-3x-2}$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(-4\int \frac{1}{x\sqrt{5x^2-3x-2}}dx-6\int \frac{1}{20-\frac{(3-10x)^2}{5x^2-3x-2}}d\left(-\frac{3-10x}{\sqrt{5x^2-3x-2}}\right)\right)+\sqrt{5x^2-3x-2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{3\text{arctanh}\left(\frac{3-10x}{2\sqrt{5}\sqrt{5x^2-3x-2}}\right)}{\sqrt{5}}-4\int \frac{1}{x\sqrt{5x^2-3x-2}}dx\right)+\sqrt{5x^2-3x-2}$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(8\int \frac{1}{-\frac{(3x+4)^2}{5x^2-3x-2}-8}d\left(-\frac{3x+4}{\sqrt{5x^2-3x-2}}\right)+\frac{3\text{arctanh}\left(\frac{3-10x}{2\sqrt{5}\sqrt{5x^2-3x-2}}\right)}{\sqrt{5}}\right)+\sqrt{5x^2-3x-2}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(2\sqrt{2}\arctan\left(\frac{3x+4}{2\sqrt{2}\sqrt{5x^2-3x-2}}\right)+\frac{3\text{arctanh}\left(\frac{3-10x}{2\sqrt{5}\sqrt{5x^2-3x-2}}\right)}{\sqrt{5}}\right)+\sqrt{5x^2-3x-2}$$

input
```
Int[Sqrt[-2 - 3*x + 5*x^2]/x,x]
```

output
```
Sqrt[-2 - 3*x + 5*x^2] + (2*Sqrt[2]*ArcTan[(4 + 3*x)/(2*Sqrt[2]*Sqrt[-2 -
3*x + 5*x^2])] + (3*ArcTanh[(3 - 10*x)/(2*Sqrt[5]*Sqrt[-2 - 3*x + 5*x^2])]
)/Sqrt[5])/2
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

method	result
default	$\sqrt{5x^2-3x-2}-\frac{3\ln\left(\frac{\left(-\frac{3}{2}+5x\right)\sqrt{5}}{5}+\sqrt{5x^2-3x-2}\right)\sqrt{5}}{10}-\sqrt{2}\,\arctan\left(\frac{(-3x-4)\sqrt{2}}{4\sqrt{5x^2-3x-2}}\right)$
trager	$\sqrt{5x^2-3x-2}+\frac{3\,\mathrm{RootOf}\left(\_Z^2-5\right)\ln\left(-10\,\mathrm{RootOf}\left(\_Z^2-5\right)x+10\sqrt{5x^2-3x-2}+3\,\mathrm{RootOf}\left(\_Z^2-5\right)\right)}{10}+\mathrm{RootOf}$

input
```
int((5*x^2-3*x-2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(5*x^2-3*x-2)^(1/2)-3/10*ln(1/5*(-3/2+5*x)*5^(1/2)+(5*x^2-3*x-2)^(1/2))*5^
(1/2)-2^(1/2)*arctan(1/4*(-3*x-4)*2^(1/2)/(5*x^2-3*x-2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{-2-3x+5x^2}}{x}\,dx = \sqrt{2}\arctan\left(\frac{\sqrt{2}(3\,x+4)}{4\,\sqrt{5\,x^2-3\,x-2}}\right) + \frac{3}{20}\,\sqrt{5}\log\left(-4\,\sqrt{5}\sqrt{5\,x^2-3\,x-2}(10\,x-3)+200\,x^2 - 120\,x-31\right) + \sqrt{5\,x^2-3\,x-2}$$

input
```
integrate((5*x^2-3*x-2)^(1/2)/x,x, algorithm="fricas")
```

output
```
sqrt(2)*arctan(1/4*sqrt(2)*(3*x + 4)/sqrt(5*x^2 - 3*x - 2)) + 3/20*sqrt(5)
*log(-4*sqrt(5)*sqrt(5*x^2 - 3*x - 2)*(10*x - 3) + 200*x^2 - 120*x - 31) +
 sqrt(5*x^2 - 3*x - 2)
```

Sympy [F]

$$\int \frac{\sqrt{-2-3x+5x^2}}{x}\,dx = \int \frac{\sqrt{(x-1)\,(5x+2)}}{x}\,dx$$

input
```
integrate((5*x**2-3*x-2)**(1/2)/x,x)
```

output
```
Integral(sqrt((x - 1)*(5*x + 2))/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{-2-3x+5x^2}}{x}\,dx = \sqrt{2}\arcsin\left(\frac{3\,x}{7\,|x|}+\frac{4}{7\,|x|}\right) - \frac{3}{10}\,\sqrt{5}\log\left(2\,\sqrt{5}\sqrt{5\,x^2-3\,x-2}+10\,x-3\right) + \sqrt{5\,x^2-3\,x-2}$$

input
```
integrate((5*x^2-3*x-2)^(1/2)/x,x, algorithm="maxima")
```

output
```
sqrt(2)*arcsin(3/7*x/abs(x) + 4/7/abs(x)) - 3/10*sqrt(5)*log(2*sqrt(5)*sqr
t(5*x^2 - 3*x - 2) + 10*x - 3) + sqrt(5*x^2 - 3*x - 2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{-2-3x+5x^2}}{x}\,dx = -2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{5}x-\sqrt{5\,x^2-3\,x-2}\right)\right) + \frac{3}{10}\sqrt{5}\log\left(\left|-10\sqrt{5}x+3\sqrt{5}+10\sqrt{5\,x^2-3\,x-2}\right|\right) + \sqrt{5\,x^2-3\,x-2}$$

input
```
integrate((5*x^2-3*x-2)^(1/2)/x,x, algorithm="giac")
```

output
```
-2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(5)*x - sqrt(5*x^2 - 3*x - 2))) + 3/10
*sqrt(5)*log(abs(-10*sqrt(5)*x + 3*sqrt(5) + 10*sqrt(5*x^2 - 3*x - 2))) +
sqrt(5*x^2 - 3*x - 2)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{-2-3x+5x^2}}{x}\,dx = \sqrt{5\,x^2-3\,x-2} - \frac{3\sqrt{5}\,\ln\left(\sqrt{5\,x^2-3\,x-2}+\frac{\sqrt{5}\,\left(5\,x-\frac{3}{2}\right)}{5}\right)}{10} - \sqrt{2}\,\ln\left(-\frac{2}{x}-\frac{3}{2}+\frac{\sqrt{2}\,\sqrt{5\,x^2-3\,x-2}\,1\mathrm{i}}{x}\right)\,1\mathrm{i}$$

input
```
int((5*x^2 - 3*x - 2)^(1/2)/x,x)
```

output
```
(5*x^2 - 3*x - 2)^(1/2) - (3*5^(1/2)*log((5*x^2 - 3*x - 2)^(1/2) + (5^(1/2
)*(5*x - 3/2))/5))/10 - 2^(1/2)*log((2^(1/2)*(5*x^2 - 3*x - 2)^(1/2)*1i)/x
 - 2/x - 3/2)*1i
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{-2-3x+5x^2}}{x}\,dx = -2\sqrt{2}\,atan\left(\frac{\sqrt{5x^2-3x-2}\,\sqrt{5}+5x}{\sqrt{10}}\right) + \sqrt{5x^2-3x-2} - \frac{3\sqrt{5}\log\left(\frac{2\sqrt{5x^2-3x-2}\,\sqrt{5}}{7}+\frac{10x}{7}-\frac{3}{7}\right)}{10}$$

input
```
int((5*x^2-3*x-2)^(1/2)/x,x)
```

output
```
( - 20*sqrt(2)*atan((sqrt(5*x**2 - 3*x - 2)*sqrt(5) + 5*x)/sqrt(10)) + 10*
sqrt(5*x**2 - 3*x - 2) - 3*sqrt(5)*log((2*sqrt(5*x**2 - 3*x - 2)*sqrt(5) +
 10*x - 3)/7))/10
```

3.290 $\int \frac{x}{\sqrt{2+4x+3x^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 40

$$\int \frac{x}{\sqrt{2+4x+3x^2}}\,dx = \frac{1}{3}\sqrt{2+4x+3x^2} - \frac{2\text{arcsinh}\left(\frac{2+3x}{\sqrt{2}}\right)}{3\sqrt{3}}$$

output
```
1/3*(3*x^2+4*x+2)^(1/2)-2/9*arcsinh(1/2*(2+3*x)*2^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{x}{\sqrt{2+4x+3x^2}}\,dx = \frac{1}{9}\left(3\sqrt{2+4x+3x^2} + 2\sqrt{3}\log\left(-2-3x+\sqrt{6+12x+9x^2}\right)\right)$$

input
```
Integrate[x/Sqrt[2 + 4*x + 3*x^2],x]
```

output
```
(3*Sqrt[2 + 4*x + 3*x^2] + 2*Sqrt[3]*Log[-2 - 3*x + Sqrt[6 + 12*x + 9*x^2]
])/9
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{3x^2+4x+2}}dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{3}\sqrt{3x^2+4x+2}-\frac{2}{3}\int \frac{1}{\sqrt{3x^2+4x+2}}dx$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{3}\sqrt{3x^2+4x+2}-\frac{\int \frac{1}{\sqrt{\frac{1}{8}(6x+4)^2+1}}d(6x+4)}{3\sqrt{6}}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{3}\sqrt{3x^2+4x+2}-\frac{2\text{arcsinh}\left(\frac{6x+4}{2\sqrt{2}}\right)}{3\sqrt{3}}$$

input
```
Int[x/Sqrt[2 + 4*x + 3*x^2],x]
```

output
```
Sqrt[2 + 4*x + 3*x^2]/3 - (2*ArcSinh[(4 + 6*x)/(2*Sqrt[2])])/(3*Sqrt[3])
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\sqrt{3x^2+4x+2}}{3}-\frac{2\sqrt{3}\,\operatorname{arcsinh}\left(\frac{3\sqrt{2}\,\left(x+\frac{2}{3}\right)}{2}\right)}{9}$	30
risch	$\frac{\sqrt{3x^2+4x+2}}{3}-\frac{2\sqrt{3}\,\operatorname{arcsinh}\left(\frac{3\sqrt{2}\,\left(x+\frac{2}{3}\right)}{2}\right)}{9}$	30
trager	$\frac{\sqrt{3x^2+4x+2}}{3}+\frac{2\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-3\,\mathrm{RootOf}\left(\_Z^2-3\right)x-2\,\mathrm{RootOf}\left(\_Z^2-3\right)+3\sqrt{3x^2+4x+2}\right)}{9}$	57

input
```
int(x/(3*x^2+4*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*x^2+4*x+2)^(1/2)-2/9*3^(1/2)*arcsinh(3/2*2^(1/2)*(x+2/3))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.30

$$\int \frac{x}{\sqrt{2+4x+3x^2}}\,dx=\frac{1}{9}\,\sqrt{3}\log\left(\sqrt{3}\sqrt{3\,x^2+4\,x+2}(3\,x+2)-9\,x^2-12\,x-5\right)+\frac{1}{3}\,\sqrt{3\,x^2+4\,x+2}$$

input
```
integrate(x/(3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/9*sqrt(3)*log(sqrt(3)*sqrt(3*x^2 + 4*x + 2)*(3*x + 2) - 9*x^2 - 12*x - 5
) + 1/3*sqrt(3*x^2 + 4*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{x}{\sqrt{2+4x+3x^2}}\,dx = \frac{\sqrt{3x^2+4x+2}}{3} - \frac{2\sqrt{3}\operatorname{asinh}\left(\frac{3\sqrt{2}\left(x+\frac{2}{3}\right)}{2}\right)}{9}$$

input
```
integrate(x/(3*x**2+4*x+2)**(1/2),x)
```

output
```
sqrt(3*x**2 + 4*x + 2)/3 - 2*sqrt(3)*asinh(3*sqrt(2)*(x + 2/3)/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{x}{\sqrt{2+4x+3x^2}}\,dx = -\frac{2}{9}\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{2}\sqrt{2}(3x+2)\right) + \frac{1}{3}\sqrt{3x^2+4x+2}$$

input
```
integrate(x/(3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-2/9*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x + 2)) + 1/3*sqrt(3*x^2 + 4*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{x}{\sqrt{2+4x+3x^2}}\,dx = \frac{2}{9}\sqrt{3}\log\left(-\sqrt{3}\left(\sqrt{3}x - \sqrt{3x^2+4x+2}\right) - 2\right) + \frac{1}{3}\sqrt{3x^2+4x+2}$$

input
```
integrate(x/(3*x^2+4*x+2)^(1/2),x, algorithm="giac")
```

output
```
2/9*sqrt(3)*log(-sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 4*x + 2)) - 2) + 1/3*sq
rt(3*x^2 + 4*x + 2)
```

Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \frac{x}{\sqrt{2+4x+3x^2}}\,dx = \frac{\sqrt{3\,x^2+4\,x+2}}{3} - \frac{2\,\sqrt{3}\,\ln\left(\sqrt{3\,x^2+4\,x+2}+\frac{\sqrt{3}\,(3\,x+2)}{3}\right)}{9}$$

input
```
int(x/(4*x + 3*x^2 + 2)^(1/2),x)
```

output
```
(4*x + 3*x^2 + 2)^(1/2)/3 - (2*3^(1/2)*log((4*x + 3*x^2 + 2)^(1/2) + (3^(1
/2)*(3*x + 2))/3))/9
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int \frac{x}{\sqrt{2+4x+3x^2}}\,dx = \frac{\sqrt{3x^2+4x+2}}{3} - \frac{2\sqrt{3}\log\left(\frac{\sqrt{3x^2+4x+2}\,\sqrt{3}+3x+2}{\sqrt{2}}\right)}{9}$$

input
```
int(x/(3*x^2+4*x+2)^(1/2),x)
```

output
```
(3*sqrt(3*x**2 + 4*x + 2) - 2*sqrt(3)*log((sqrt(3*x**2 + 4*x + 2)*sqrt(3)
+ 3*x + 2)/sqrt(2)))/9
```

3.291 $\int \frac{x}{\sqrt{2+4x-3x^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 40

$$\int \frac{x}{\sqrt{2+4x-3x^2}}\,dx = -\frac{1}{3}\sqrt{2+4x-3x^2}-\frac{2\arcsin\left(\frac{2-3x}{\sqrt{10}}\right)}{3\sqrt{3}}$$

output
```
-1/3*(-3*x^2+4*x+2)^(1/2)-2/9*arcsin(1/10*(2-3*x)*10^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.50

$$\int \frac{x}{\sqrt{2+4x-3x^2}}\,dx = -\frac{1}{3}\sqrt{2+4x-3x^2}-\frac{4\arctan\left(\frac{\sqrt{3}x}{\sqrt{2}-\sqrt{2+4x-3x^2}}\right)}{3\sqrt{3}}$$

input
```
Integrate[x/Sqrt[2 + 4*x - 3*x^2],x]
```

output
```
-1/3*Sqrt[2 + 4*x - 3*x^2] - (4*ArcTan[(Sqrt[3]*x)/(Sqrt[2] - Sqrt[2 + 4*x
 - 3*x^2])])/(3*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1160, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{-3x^2+4x+2}}dx$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{2}{3}\int \frac{1}{\sqrt{-3x^2+4x+2}}dx-\frac{1}{3}\sqrt{-3x^2+4x+2}$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{\int \frac{1}{\sqrt{1-\frac{1}{40}(4-6x)^2}}d(4-6x)}{3\sqrt{30}}-\frac{1}{3}\sqrt{-3x^2+4x+2}$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{2\arcsin\left(\frac{4-6x}{2\sqrt{10}}\right)}{3\sqrt{3}}-\frac{1}{3}\sqrt{-3x^2+4x+2}$$

input
```
Int[x/Sqrt[2 + 4*x - 3*x^2],x]
```

output
```
-1/3*Sqrt[2 + 4*x - 3*x^2] - (2*ArcSin[(4 - 6*x)/(2*Sqrt[10])])/(3*Sqrt[3]
)
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{\sqrt{-3x^2+4x+2}}{3}+\frac{2\sqrt{3}\,\arcsin\left(\frac{3\sqrt{10}\,\left(x-\frac{2}{3}\right)}{10}\right)}{9}$	30
risch	$\frac{3x^2-4x-2}{3\sqrt{-3x^2+4x+2}}+\frac{2\sqrt{3}\,\arcsin\left(\frac{3\sqrt{10}\,\left(x-\frac{2}{3}\right)}{10}\right)}{9}$	40
trager	$-\frac{\sqrt{-3x^2+4x+2}}{3}-\frac{2\,\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(3\,\mathrm{RootOf}\left(\_Z^2+3\right)x-2\,\mathrm{RootOf}\left(\_Z^2+3\right)+3\sqrt{-3x^2+4x+2}\right)}{9}$	57

input
```
int(x/(-3*x^2+4*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-3*x^2+4*x+2)^(1/2)+2/9*3^(1/2)*arcsin(3/10*10^(1/2)*(x-2/3))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\int \frac{x}{\sqrt{2+4x-3x^2}}\,dx=-\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2+4\,x+2}(3\,x-2)}{3\,(3\,x^2-4\,x-2)}\right)-\frac{1}{3}\,\sqrt{-3\,x^2+4\,x+2}$$

input
```
integrate(x/(-3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
```

output
```
-2/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(-3*x^2 + 4*x + 2)*(3*x - 2)/(3*x^2 -
4*x - 2)) - 1/3*sqrt(-3*x^2 + 4*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{x}{\sqrt{2+4x-3x^2}}\,dx = -\frac{\sqrt{-3x^2+4x+2}}{3} + \frac{2\sqrt{3}\operatorname{asin}\left(\frac{3\sqrt{10}\left(x-\frac{2}{3}\right)}{10}\right)}{9}$$

input
```
integrate(x/(-3*x**2+4*x+2)**(1/2),x)
```

output
```
-sqrt(-3*x**2 + 4*x + 2)/3 + 2*sqrt(3)*asin(3*sqrt(10)*(x - 2/3)/10)/9
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{x}{\sqrt{2+4x-3x^2}}\,dx = -\frac{2}{9}\sqrt{3}\arcsin\left(-\frac{1}{10}\sqrt{10}(3x-2)\right) - \frac{1}{3}\sqrt{-3x^2+4x+2}$$

input
```
integrate(x/(-3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-2/9*sqrt(3)*arcsin(-1/10*sqrt(10)*(3*x - 2)) - 1/3*sqrt(-3*x^2 + 4*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{x}{\sqrt{2+4x-3x^2}}\,dx = \frac{2}{9}\sqrt{3}\arcsin\left(\frac{1}{10}\sqrt{10}(3x-2)\right) - \frac{1}{3}\sqrt{-3x^2+4x+2}$$

input
```
integrate(x/(-3*x^2+4*x+2)^(1/2),x, algorithm="giac")
```

output
```
2/9*sqrt(3)*arcsin(1/10*sqrt(10)*(3*x - 2)) - 1/3*sqrt(-3*x^2 + 4*x + 2)
```

Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{x}{\sqrt{2+4x-3x^2}}\,dx = -\frac{\sqrt{-3x^2+4x+2}}{3} - \frac{\sqrt{3}\ln\left(\sqrt{-3x^2+4x+2}+\frac{\sqrt{3}(3x-2)\,1\mathrm{i}}{3}\right)2\mathrm{i}}{9}$$

input
```
int(x/(4*x - 3*x^2 + 2)^(1/2),x)
```

output
```
- (3^(1/2)*log((4*x - 3*x^2 + 2)^(1/2) + (3^(1/2)*(3*x - 2)*1i)/3)*2i)/9 -
 (4*x - 3*x^2 + 2)^(1/2)/3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int \frac{x}{\sqrt{2+4x-3x^2}}\,dx = \frac{2\sqrt{3}\,asin\left(\frac{3x-2}{\sqrt{10}}\right)}{9} - \frac{\sqrt{-3x^2+4x+2}}{3}$$

input
```
int(x/(-3*x^2+4*x+2)^(1/2),x)
```

output
```
(2*sqrt(3)*asin((3*x - 2)/sqrt(10)) - 3*sqrt( - 3*x**2 + 4*x + 2))/9
```

3.292 $\int \frac{x}{\sqrt{2+5x+3x^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 52

$$\int \frac{x}{\sqrt{2+5x+3x^2}}\,dx=\frac{1}{3}\sqrt{2+5x+3x^2}-\frac{5\text{arctanh}\left(\frac{\sqrt{3}(1+x)}{\sqrt{2+5x+3x^2}}\right)}{3\sqrt{3}}$$

output
```
1/3*(3*x^2+5*x+2)^(1/2)-5/9*arctanh(3^(1/2)*(1+x)/(3*x^2+5*x+2)^(1/2))*3^(
1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int \frac{x}{\sqrt{2+5x+3x^2}}\,dx=\frac{1}{9}\left(3\sqrt{2+5x+3x^2}-5\sqrt{3}\text{arctanh}\left(\frac{\sqrt{\frac{2}{3}+\frac{5x}{3}+x^2}}{1+x}\right)\right)$$

input
```
Integrate[x/Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
(3*Sqrt[2 + 5*x + 3*x^2] - 5*Sqrt[3]*ArcTanh[Sqrt[2/3 + (5*x)/3 + x^2]/(1
+ x)])/9
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{3x^2+5x+2}}dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{3}\sqrt{3x^2+5x+2}-\frac{5}{6}\int \frac{1}{\sqrt{3x^2+5x+2}}dx$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\sqrt{3x^2+5x+2}-\frac{5}{3}\int \frac{1}{12-\frac{(6x+5)^2}{3x^2+5x+2}}d\frac{6x+5}{\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\sqrt{3x^2+5x+2}-\frac{5\text{arctanh}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{6\sqrt{3}}$$

input
```
Int[x/Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
Sqrt[2 + 5*x + 3*x^2]/3 - (5*ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[2 + 5*x + 3
*x^2])])/(6*Sqrt[3])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\sqrt{3x^2+5x+2}}{3}-\frac{5\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^2+5x+2}\right)\sqrt{3}}{18}$	45
risch	$\frac{\sqrt{3x^2+5x+2}}{3}-\frac{5\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^2+5x+2}\right)\sqrt{3}}{18}$	45
trager	$\frac{\sqrt{3x^2+5x+2}}{3}-\frac{5\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+5\,\mathrm{RootOf}\left(\_Z^2-3\right)+6\sqrt{3x^2+5x+2}\right)}{18}$	57

input
```
int(x/(3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*x^2+5*x+2)^(1/2)-5/18*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x+2)^(1/2))
*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{x}{\sqrt{2+5x+3x^2}}\,dx = \frac{5}{36}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3\,x^2+5\,x+2}(6\,x+5)+72\,x^2+120\,x+49\right) + \frac{1}{3}\sqrt{3\,x^2+5\,x+2}$$

input
```
integrate(x/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

output
```
5/36*sqrt(3)*log(-4*sqrt(3)*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 72*x^2 + 120
*x + 49) + 1/3*sqrt(3*x^2 + 5*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{x}{\sqrt{2+5x+3x^2}}\,dx = \frac{\sqrt{3x^2+5x+2}}{3} - \frac{5\sqrt{3}\log\left(6x+2\sqrt{3}\sqrt{3x^2+5x+2}+5\right)}{18}$$

input
```
integrate(x/(3*x**2+5*x+2)**(1/2),x)
```

output
```
sqrt(3*x**2 + 5*x + 2)/3 - 5*sqrt(3)*log(6*x + 2*sqrt(3)*sqrt(3*x**2 + 5*x
 + 2) + 5)/18
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \frac{x}{\sqrt{2+5x+3x^2}}\,dx = -\frac{5}{18}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+5x+2}+6x+5\right)+\frac{1}{3}\sqrt{3x^2+5x+2}$$

input
```
integrate(x/(3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-5/18*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x + 2) + 6*x + 5) + 1/3*sqrt(3*
x^2 + 5*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{x}{\sqrt{2+5x+3x^2}}\,dx = \frac{5}{18}\sqrt{3}\log\left(\left|-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+5x+2}\right)-5\right|\right) + \frac{1}{3}\sqrt{3x^2+5x+2}$$

input
```
integrate(x/(3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

output
```
5/18*sqrt(3)*log(abs(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 5*x + 2)) - 5))
+ 1/3*sqrt(3*x^2 + 5*x + 2)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{2+5x+3x^2}}\,dx = \frac{\sqrt{3x^2+5x+2}}{3} - \frac{5\sqrt{3}\ln\left(\sqrt{3x^2+5x+2}+\frac{\sqrt{3}\left(3x+\frac{5}{2}\right)}{3}\right)}{18}$$

input
```
int(x/(5*x + 3*x^2 + 2)^(1/2),x)
```

output
```
(5*x + 3*x^2 + 2)^(1/2)/3 - (5*3^(1/2)*log((5*x + 3*x^2 + 2)^(1/2) + (3^(1
/2)*(3*x + 5/2))/3))/18
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

$$\int \frac{x}{\sqrt{2+5x+3x^2}}\,dx = \frac{\sqrt{3x^2+5x+2}}{3} - \frac{5\sqrt{3}\log\left(2\sqrt{3x^2+5x+2}\,\sqrt{3}+6x+5\right)}{18}$$

input
```
int(x/(3*x^2+5*x+2)^(1/2),x)
```

output
```
(6*sqrt(3*x**2 + 5*x + 2) - 5*sqrt(3)*log(2*sqrt(3*x**2 + 5*x + 2)*sqrt(3)
 + 6*x + 5))/18
```

3.293 $\int \frac{x}{\sqrt{2+5x-3x^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 38

$$\int \frac{x}{\sqrt{2+5x-3x^2}}\,dx = -\frac{1}{3}\sqrt{2+5x-3x^2} - \frac{5\arcsin\left(\frac{1}{7}(5-6x)\right)}{6\sqrt{3}}$$

output
```
-1/3*(-3*x^2+5*x+2)^(1/2)+5/18*arcsin(-5/7+6/7*x)*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \frac{x}{\sqrt{2+5x-3x^2}}\,dx = \frac{1}{9}\left(-3\sqrt{2+5x-3x^2} - 5\sqrt{3}\arctan\left(\frac{\sqrt{6+15x-9x^2}}{1+3x}\right)\right)$$

input
```
Integrate[x/Sqrt[2 + 5*x - 3*x^2],x]
```

output
```
(-3*Sqrt[2 + 5*x - 3*x^2] - 5*Sqrt[3]*ArcTan[Sqrt[6 + 15*x - 9*x^2]/(1 + 3
*x)])/9
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1160, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{-3x^2+5x+2}}dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{5}{6}\int \frac{1}{\sqrt{-3x^2+5x+2}}dx-\frac{1}{3}\sqrt{-3x^2+5x+2}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{5\int \frac{1}{\sqrt{1-\frac{1}{49}(5-6x)^2}}d(5-6x)}{42\sqrt{3}}-\frac{1}{3}\sqrt{-3x^2+5x+2}$$

$$\Big\downarrow \text{223}$$

$$-\frac{5\arcsin\left(\frac{1}{7}(5-6x)\right)}{6\sqrt{3}}-\frac{1}{3}\sqrt{-3x^2+5x+2}$$

input
```
Int[x/Sqrt[2 + 5*x - 3*x^2],x]
```

output
```
-1/3*Sqrt[2 + 5*x - 3*x^2] - (5*ArcSin[(5 - 6*x)/7])/(6*Sqrt[3])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

method	result	size
default	$-\frac{\sqrt{-3x^{2}+5x+2}}{3}+\frac{5\arcsin\left(-\frac{5}{7}+\frac{6x}{7}\right)\sqrt{3}}{18}$	27
risch	$\frac{3x^{2}-5x-2}{3\sqrt{-3x^{2}+5x+2}}+\frac{5\arcsin\left(-\frac{5}{7}+\frac{6x}{7}\right)\sqrt{3}}{18}$	37
trager	$-\frac{\sqrt{-3x^{2}+5x+2}}{3}-\frac{5\,\mathrm{RootOf}\left(\_Z^{2}+3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^{2}+3\right)x-5\,\mathrm{RootOf}\left(\_Z^{2}+3\right)+6\sqrt{-3x^{2}+5x+2}\right)}{18}$	57

input
```
int(x/(-3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-3*x^2+5*x+2)^(1/2)+5/18*arcsin(-5/7+6/7*x)*3^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(26) = 52$.

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

$$\int \frac{x}{\sqrt{2+5x-3x^2}}\,dx=-\frac{5}{18}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^{2}+5\,x+2}(6\,x-5)}{6\,(3\,x^{2}-5\,x-2)}\right)-\frac{1}{3}\sqrt{-3\,x^{2}+5\,x+2}$$

input
```
integrate(x/(-3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

output
```
-5/18*sqrt(3)*arctan(1/6*sqrt(3)*sqrt(-3*x^2 + 5*x + 2)*(6*x - 5)/(3*x^2 -
 5*x - 2)) - 1/3*sqrt(-3*x^2 + 5*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{x}{\sqrt{2+5x-3x^2}}\,dx = -\frac{\sqrt{-3x^2+5x+2}}{3}+\frac{5\sqrt{3}\,\mathrm{asin}\left(\frac{6x}{7}-\frac{5}{7}\right)}{18}$$

input
```
integrate(x/(-3*x**2+5*x+2)**(1/2),x)
```

output
```
-sqrt(-3*x**2 + 5*x + 2)/3 + 5*sqrt(3)*asin(6*x/7 - 5/7)/18
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \frac{x}{\sqrt{2+5x-3x^2}}\,dx = -\frac{5}{18}\sqrt{3}\arcsin\left(-\frac{6}{7}x+\frac{5}{7}\right)-\frac{1}{3}\sqrt{-3x^2+5x+2}$$

input
```
integrate(x/(-3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-5/18*sqrt(3)*arcsin(-6/7*x + 5/7) - 1/3*sqrt(-3*x^2 + 5*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \frac{x}{\sqrt{2+5x-3x^2}}\,dx = \frac{5}{18}\sqrt{3}\arcsin\left(\frac{6}{7}x-\frac{5}{7}\right)-\frac{1}{3}\sqrt{-3x^2+5x+2}$$

input
```
integrate(x/(-3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

output
```
5/18*sqrt(3)*arcsin(6/7*x - 5/7) - 1/3*sqrt(-3*x^2 + 5*x + 2)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \frac{x}{\sqrt{2+5x-3x^2}}\,dx = -\frac{\sqrt{-3\,x^2+5\,x+2}}{3}-\frac{\sqrt{3}\,\ln\left(\sqrt{-3\,x^2+5\,x+2}+\frac{\sqrt{3}\,\left(3\,x-\frac{5}{2}\right)\,1{}\mathrm{i}}{3}\right)\,5{}\mathrm{i}}{18}$$

input
```
int(x/(5*x - 3*x^2 + 2)^(1/2),x)
```

output
```
- (3^(1/2)*log((5*x - 3*x^2 + 2)^(1/2) + (3^(1/2)*(3*x - 5/2)*1i)/3)*5i)/1
8 - (5*x - 3*x^2 + 2)^(1/2)/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{x}{\sqrt{2+5x-3x^2}}\,dx = \frac{5\sqrt{3}\,asin\left(\frac{6x}{7}-\frac{5}{7}\right)}{18}-\frac{\sqrt{-3x^2+5x+2}}{3}$$

input
```
int(x/(-3*x^2+5*x+2)^(1/2),x)
```

output
```
(5*sqrt(3)*asin((6*x - 5)/7) - 6*sqrt( - 3*x**2 + 5*x + 2))/18
```

3.294 $\int \frac{x}{\sqrt{-2+4x+3x^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 54

$$\int \frac{x}{\sqrt{-2+4x+3x^2}}\,dx=\frac{1}{3}\sqrt{-2+4x+3x^2}-\frac{2\text{arctanh}\left(\frac{2+3x}{\sqrt{3}\sqrt{-2+4x+3x^2}}\right)}{3\sqrt{3}}$$

output
```
1/3*(3*x^2+4*x-2)^(1/2)-2/9*arctanh(1/3*(2+3*x)*3^(1/2)/(3*x^2+4*x-2)^(1/2
))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{x}{\sqrt{-2+4x+3x^2}}\,dx=\frac{1}{9}\left(3\sqrt{-2+4x+3x^2}+2\sqrt{3}\log\left(-2-3x+\sqrt{-6+12x+9x^2}\right)\right)$$

input
```
Integrate[x/Sqrt[-2 + 4*x + 3*x^2],x]
```

output
```
(3*Sqrt[-2 + 4*x + 3*x^2] + 2*Sqrt[3]*Log[-2 - 3*x + Sqrt[-6 + 12*x + 9*x^
2]])/9
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{3x^2+4x-2}}dx$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{1}{3}\sqrt{3x^2+4x-2}-\frac{2}{3}\int \frac{1}{\sqrt{3x^2+4x-2}}dx$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{1}{3}\sqrt{3x^2+4x-2}-\frac{4}{3}\int \frac{1}{12-\frac{4(3x+2)^2}{3x^2+4x-2}}d\frac{2(3x+2)}{\sqrt{3x^2+4x-2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}\sqrt{3x^2+4x-2}-\frac{2\text{arctanh}\left(\frac{3x+2}{\sqrt{3}\sqrt{3x^2+4x-2}}\right)}{3\sqrt{3}}$$

input
```
Int[x/Sqrt[-2 + 4*x + 3*x^2],x]
```

output
```
Sqrt[-2 + 4*x + 3*x^2]/3 - (2*ArcTanh[(2 + 3*x)/(Sqrt[3]*Sqrt[-2 + 4*x + 3
*x^2])])/(3*Sqrt[3])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\sqrt{3x^2+4x-2}}{3}-\frac{2\ln\left(\frac{(3x+2)\sqrt{3}}{3}+\sqrt{3x^2+4x-2}\right)\sqrt{3}}{9}$	45
risch	$\frac{\sqrt{3x^2+4x-2}}{3}-\frac{2\ln\left(\frac{(3x+2)\sqrt{3}}{3}+\sqrt{3x^2+4x-2}\right)\sqrt{3}}{9}$	45
trager	$\frac{\sqrt{3x^2+4x-2}}{3}-\frac{2\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(3\operatorname{RootOf}\left(\_Z^2-3\right)x+2\operatorname{RootOf}\left(\_Z^2-3\right)+3\sqrt{3x^2+4x-2}\right)}{9}$	57

input
```
int(x/(3*x^2+4*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*x^2+4*x-2)^(1/2)-2/9*ln(1/3*(3*x+2)*3^(1/2)+(3*x^2+4*x-2)^(1/2))*3^
(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{x}{\sqrt{-2+4x+3x^2}}\,dx = \frac{1}{9}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+4\,x-2}(3\,x+2)+9\,x^2+12\,x-1\right) + \frac{1}{3}\sqrt{3\,x^2+4\,x-2}$$

input
```
integrate(x/(3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
```

output
```
1/9*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 4*x - 2)*(3*x + 2) + 9*x^2 + 12*x -
1) + 1/3*sqrt(3*x^2 + 4*x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{x}{\sqrt{-2+4x+3x^2}}\,dx = \frac{\sqrt{3x^2+4x-2}}{3} - \frac{2\sqrt{3}\log\left(6x+2\sqrt{3}\sqrt{3x^2+4x-2}+4\right)}{9}$$

input
```
integrate(x/(3*x**2+4*x-2)**(1/2),x)
```

output
```
sqrt(3*x**2 + 4*x - 2)/3 - 2*sqrt(3)*log(6*x + 2*sqrt(3)*sqrt(3*x**2 + 4*x
 - 2) + 4)/9
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \frac{x}{\sqrt{-2+4x+3x^2}}\,dx = -\frac{2}{9}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+4x-2}+6x+4\right)+\frac{1}{3}\sqrt{3x^2+4x-2}$$

input
```
integrate(x/(3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
```

output
```
-2/9*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 4*x - 2) + 6*x + 4) + 1/3*sqrt(3*x
^2 + 4*x - 2)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{x}{\sqrt{-2+4x+3x^2}}\,dx = \frac{2}{9}\sqrt{3}\log\left(\left|-\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+4\,x-2}\right)-2\right|\right) + \frac{1}{3}\sqrt{3\,x^2+4\,x-2}$$

input
```
integrate(x/(3*x^2+4*x-2)^(1/2),x, algorithm="giac")
```

output
```
2/9*sqrt(3)*log(abs(-sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 4*x - 2)) - 2)) + 1
/3*sqrt(3*x^2 + 4*x - 2)
```

Mupad [B] (verification not implemented)

Time = 8.85 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{x}{\sqrt{-2+4x+3x^2}}\,dx = \frac{\sqrt{3\,x^2+4\,x-2}}{3} - \frac{2\sqrt{3}\,\ln\left(\sqrt{3\,x^2+4\,x-2}+\frac{\sqrt{3}\,(3\,x+2)}{3}\right)}{9}$$

input
```
int(x/(4*x + 3*x^2 - 2)^(1/2),x)
```

output
```
(4*x + 3*x^2 - 2)^(1/2)/3 - (2*3^(1/2)*log((4*x + 3*x^2 - 2)^(1/2) + (3^(1
/2)*(3*x + 2))/3))/9
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \frac{x}{\sqrt{-2+4x+3x^2}}\,dx = \frac{\sqrt{3x^2+4x-2}}{3} - \frac{2\sqrt{3}\log\left(\frac{\sqrt{3x^2+4x-2}\,\sqrt{3}+3x+2}{\sqrt{10}}\right)}{9}$$

input
```
int(x/(3*x^2+4*x-2)^(1/2),x)
```

output
```
(3*sqrt(3*x**2 + 4*x - 2) - 2*sqrt(3)*log((sqrt(3*x**2 + 4*x - 2)*sqrt(3)
+ 3*x + 2)/sqrt(10)))/9
```

3.295 $\int \frac{x}{\sqrt{-2+4x-3x^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 54

$$\int \frac{x}{\sqrt{-2+4x-3x^2}}\,dx=-\frac{1}{3}\sqrt{-2+4x-3x^2}-\frac{2\arctan\left(\frac{2-3x}{\sqrt{3}\sqrt{-2+4x-3x^2}}\right)}{3\sqrt{3}}$$

output
```
-1/3*(-3*x^2+4*x-2)^(1/2)-2/9*arctan(1/3*(2-3*x)*3^(1/2)/(-3*x^2+4*x-2)^(1
/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.96

$$\int \frac{x}{\sqrt{-2+4x-3x^2}}\,dx=\frac{1}{9}\left(\frac{3\left(-2\left(i+\sqrt{2}\right)+3\sqrt{2}x\right)\sqrt{-2+4x-3x^2}}{2\sqrt{1+2i\sqrt{2}-3\sqrt{2}x}}+4\sqrt{3}\arctan\left(\frac{-2-i\sqrt{2}+3x}{\sqrt{-6+12x-9x^2}}\right)\right)$$

input
```
Integrate[x/Sqrt[-2 + 4*x - 3*x^2],x]
```

output
```
((3*(-2*(I + Sqrt[2]) + 3*Sqrt[2]*x)*Sqrt[-2 + 4*x - 3*x^2])/(2*Sqrt[1 + (
2*I)*Sqrt[2]] - 3*Sqrt[2]*x) + 4*Sqrt[3]*ArcTan[(-2 - I*Sqrt[2] + 3*x)/Sqr
t[-6 + 12*x - 9*x^2]])/9
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1160, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{-3x^2+4x-2}}dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{2}{3}\int \frac{1}{\sqrt{-3x^2+4x-2}}dx-\frac{1}{3}\sqrt{-3x^2+4x-2}$$

$$\Big\downarrow \text{1092}$$

$$\frac{4}{3}\int \frac{1}{-\frac{4(2-3x)^2}{-3x^2+4x-2}-12}d\frac{2(2-3x)}{\sqrt{-3x^2+4x-2}}-\frac{1}{3}\sqrt{-3x^2+4x-2}$$

$$\Big\downarrow \text{217}$$

$$-\frac{2\arctan\left(\frac{2-3x}{\sqrt{3}\sqrt{-3x^2+4x-2}}\right)}{3\sqrt{3}}-\frac{1}{3}\sqrt{-3x^2+4x-2}$$

input
```
Int[x/Sqrt[-2 + 4*x - 3*x^2],x]
```

output
```
-1/3*Sqrt[-2 + 4*x - 3*x^2] - (2*ArcTan[(2 - 3*x)/(Sqrt[3]*Sqrt[-2 + 4*x -
 3*x^2])])/(3*Sqrt[3])
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{\sqrt{-3x^2+4x-2}}{3}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x-\frac{2}{3}\right)}{\sqrt{-3x^2+4x-2}}\right)}{9}$	41
risch	$\frac{3x^2-4x+2}{3\sqrt{-3x^2+4x-2}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x-\frac{2}{3}\right)}{\sqrt{-3x^2+4x-2}}\right)}{9}$	51
trager	$-\frac{\sqrt{-3x^2+4x-2}}{3}-\frac{2\,\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(3\,\mathrm{RootOf}\left(\_Z^2+3\right)x-2\,\mathrm{RootOf}\left(\_Z^2+3\right)+3\sqrt{-3x^2+4x-2}\right)}{9}$	57

input
```
int(x/(-3*x^2+4*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-3*x^2+4*x-2)^(1/2)+2/9*3^(1/2)*arctan(3^(1/2)*(x-2/3)/(-3*x^2+4*x-2
)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.50

$$\int \frac{x}{\sqrt{-2+4x-3x^2}}\,dx = \frac{1}{3}\sqrt{-\frac{1}{3}}\log\left(-\frac{2\left(3\sqrt{-\frac{1}{3}}\sqrt{-3\,x^2+4\,x-2}+3\,x-2\right)}{x}\right) - \frac{1}{3}\sqrt{-\frac{1}{3}}\log\left(\frac{2\left(3\sqrt{-\frac{1}{3}}\sqrt{-3\,x^2+4\,x-2}-3\,x+2\right)}{x}\right) - \frac{1}{3}\sqrt{-3\,x^2+4\,x-2}$$

input
```
integrate(x/(-3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(-1/3)*log(-2*(3*sqrt(-1/3)*sqrt(-3*x^2 + 4*x - 2) + 3*x - 2)/x) -
 1/3*sqrt(-1/3)*log(2*(3*sqrt(-1/3)*sqrt(-3*x^2 + 4*x - 2) - 3*x + 2)/x) -
 1/3*sqrt(-3*x^2 + 4*x - 2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{-2+4x-3x^2}}\,dx = -\frac{\sqrt{-3x^2+4x-2}}{3} - \frac{2\sqrt{3}i\log\left(-6x+2\sqrt{3}i\sqrt{-3x^2+4x-2}+4\right)}{9}$$

input
```
integrate(x/(-3*x**2+4*x-2)**(1/2),x)
```

output
```
-sqrt(-3*x**2 + 4*x - 2)/3 - 2*sqrt(3)*I*log(-6*x + 2*sqrt(3)*I*sqrt(-3*x*
*2 + 4*x - 2) + 4)/9
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.57

$$\int \frac{x}{\sqrt{-2+4x-3x^2}}\,dx = -\frac{2}{9}i\sqrt{3}\,\text{arsinh}\left(\frac{1}{2}\sqrt{2}(3x-2)\right) - \frac{1}{3}\sqrt{-3x^2+4x-2}$$

input
```
integrate(x/(-3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
```

output
```
-2/9*I*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x - 2)) - 1/3*sqrt(-3*x^2 + 4*x - 2)
```

Giac [F]

$$\int \frac{x}{\sqrt{-2+4x-3x^2}}\,dx = \int \frac{x}{\sqrt{-3x^2+4x-2}}\,dx$$

input
```
integrate(x/(-3*x^2+4*x-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(-3*x^2 + 4*x - 2), x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{-2+4x-3x^2}}\,dx = -\frac{\sqrt{-3x^2+4x-2}}{3} - \frac{\sqrt{3}\,\ln\left(\sqrt{-3x^2+4x-2}+\frac{\sqrt{3}\,(3x-2)\,1\mathrm{i}}{3}\right)\,2\mathrm{i}}{9}$$

input
```
int(x/(4*x - 3*x^2 - 2)^(1/2),x)
```

output
```
- (3^(1/2)*log((4*x - 3*x^2 - 2)^(1/2) + (3^(1/2)*(3*x - 2)*1i)/3)*2i)/9 -
 (4*x - 3*x^2 - 2)^(1/2)/3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.56

$$\int \frac{x}{\sqrt{-2+4x-3x^2}}\,dx = -\frac{2\sqrt{3}\,asinh\left(\frac{3x-2}{\sqrt{2}}\right)i}{9} + \frac{\sqrt{-3x^2+4x-2}}{3}$$

input
```
int(x/(-3*x^2+4*x-2)^(1/2),x)
```

output
```
( - 2*sqrt(3)*asinh((3*x - 2)/sqrt(2))*i + 3*sqrt( - 3*x**2 + 4*x - 2))/9
```

3.296 $\int \frac{x}{\sqrt{-2+5x+3x^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 52

$$\int \frac{x}{\sqrt{-2+5x+3x^2}}\,dx = \frac{1}{3}\sqrt{-2+5x+3x^2} - \frac{5\text{arctanh}\left(\frac{\sqrt{3}(2+x)}{\sqrt{-2+5x+3x^2}}\right)}{3\sqrt{3}}$$

output
```
1/3*(3*x^2+5*x-2)^(1/2)-5/9*arctanh(3^(1/2)*(2+x)/(3*x^2+5*x-2)^(1/2))*3^(
1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int \frac{x}{\sqrt{-2+5x+3x^2}}\,dx = \frac{1}{9}\left(3\sqrt{-2+5x+3x^2} - 5\sqrt{3}\text{arctanh}\left(\frac{\sqrt{-\frac{2}{3}+\frac{5x}{3}+x^2}}{2+x}\right)\right)$$

input
```
Integrate[x/Sqrt[-2 + 5*x + 3*x^2],x]
```

output
```
(3*Sqrt[-2 + 5*x + 3*x^2] - 5*Sqrt[3]*ArcTanh[Sqrt[-2/3 + (5*x)/3 + x^2]/(
2 + x)])/9
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{3x^2+5x-2}}dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{3}\sqrt{3x^2+5x-2}-\frac{5}{6}\int \frac{1}{\sqrt{3x^2+5x-2}}dx$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\sqrt{3x^2+5x-2}-\frac{5}{3}\int \frac{1}{12-\frac{(6x+5)^2}{3x^2+5x-2}}d\frac{6x+5}{\sqrt{3x^2+5x-2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\sqrt{3x^2+5x-2}-\frac{5\text{arctanh}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x-2}}\right)}{6\sqrt{3}}$$

input
```
Int[x/Sqrt[-2 + 5*x + 3*x^2],x]
```

output
```
Sqrt[-2 + 5*x + 3*x^2]/3 - (5*ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[-2 + 5*x +
 3*x^2])])/(6*Sqrt[3])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\sqrt{3x^2+5x-2}}{3}-\frac{5\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^2+5x-2}\right)\sqrt{3}}{18}$	45
risch	$\frac{\sqrt{3x^2+5x-2}}{3}-\frac{5\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^2+5x-2}\right)\sqrt{3}}{18}$	45
trager	$\frac{\sqrt{3x^2+5x-2}}{3}-\frac{5\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2+5x-2}+5\,\mathrm{RootOf}\left(\_Z^2-3\right)\right)}{18}$	57

input
```
int(x/(3*x^2+5*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*x^2+5*x-2)^(1/2)-5/18*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x-2)^(1/2))
*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{x}{\sqrt{-2+5x+3x^2}}\,dx = \frac{5}{36}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3x^2+5x-2}(6x+5)+72x^2+120x+1\right) + \frac{1}{3}\sqrt{3x^2+5x-2}$$

input
```
integrate(x/(3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

output
```
5/36*sqrt(3)*log(-4*sqrt(3)*sqrt(3*x^2 + 5*x - 2)*(6*x + 5) + 72*x^2 + 120
*x + 1) + 1/3*sqrt(3*x^2 + 5*x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{x}{\sqrt{-2+5x+3x^2}}\,dx = \frac{\sqrt{3x^2+5x-2}}{3} - \frac{5\sqrt{3}\log\left(6x+2\sqrt{3}\sqrt{3x^2+5x-2}+5\right)}{18}$$

input
```
integrate(x/(3*x**2+5*x-2)**(1/2),x)
```

output
```
sqrt(3*x**2 + 5*x - 2)/3 - 5*sqrt(3)*log(6*x + 2*sqrt(3)*sqrt(3*x**2 + 5*x
 - 2) + 5)/18
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \frac{x}{\sqrt{-2+5x+3x^2}}\,dx = -\frac{5}{18}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+5x-2}+6x+5\right)+\frac{1}{3}\sqrt{3x^2+5x-2}$$

input
```
integrate(x/(3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
```

output
```
-5/18*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x - 2) + 6*x + 5) + 1/3*sqrt(3*
x^2 + 5*x - 2)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{x}{\sqrt{-2+5x+3x^2}}\,dx = \frac{5}{18}\sqrt{3}\log\left(\left|-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+5\,x-2}\right)-5\right|\right) + \frac{1}{3}\sqrt{3\,x^2+5\,x-2}$$

input
```
integrate(x/(3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

output
```
5/18*sqrt(3)*log(abs(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 5*x - 2)) - 5))
+ 1/3*sqrt(3*x^2 + 5*x - 2)
```

Mupad [B] (verification not implemented)

Time = 8.79 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{-2+5x+3x^2}}\,dx = \frac{\sqrt{3\,x^2+5\,x-2}}{3} - \frac{5\sqrt{3}\,\ln\left(\sqrt{3\,x^2+5\,x-2}+\frac{\sqrt{3}\,\left(3\,x+\frac{5}{2}\right)}{3}\right)}{18}$$

input
```
int(x/(5*x + 3*x^2 - 2)^(1/2),x)
```

output
```
(5*x + 3*x^2 - 2)^(1/2)/3 - (5*3^(1/2)*log((5*x + 3*x^2 - 2)^(1/2) + (3^(1
/2)*(3*x + 5/2))/3))/18
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

$$\int \frac{x}{\sqrt{-2+5x+3x^2}}\,dx = \frac{\sqrt{3x^2+5x-2}}{3} - \frac{5\sqrt{3}\log\left(\frac{2\sqrt{3x^2+5x-2}\,\sqrt{3}}{7}+\frac{6x}{7}+\frac{5}{7}\right)}{18}$$

input
```
int(x/(3*x^2+5*x-2)^(1/2),x)
```

output
```
(6*sqrt(3*x**2 + 5*x - 2) - 5*sqrt(3)*log((2*sqrt(3*x**2 + 5*x - 2)*sqrt(3
) + 6*x + 5)/7))/18
```

3.297 $\int \frac{x}{\sqrt{-2+5x-3x^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 34

$$\int \frac{x}{\sqrt{-2+5x-3x^2}}\,dx = -\frac{1}{3}\sqrt{-2+5x-3x^2}-\frac{5\arcsin(5-6x)}{6\sqrt{3}}$$

output
```
-1/3*(-3*x^2+5*x-2)^(1/2)+5/18*arcsin(-5+6*x)*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.50

$$\int \frac{x}{\sqrt{-2+5x-3x^2}}\,dx = \frac{1}{9}\left(-3\sqrt{-2+5x-3x^2}-5\sqrt{3}\arctan\left(\frac{\sqrt{-6+15x-9x^2}}{-2+3x}\right)\right)$$

input
```
Integrate[x/Sqrt[-2 + 5*x - 3*x^2],x]
```

output
```
(-3*Sqrt[-2 + 5*x - 3*x^2] - 5*Sqrt[3]*ArcTan[Sqrt[-6 + 15*x - 9*x^2]/(-2
+ 3*x)])/9
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1160, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{-3x^2+5x-2}}dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{5}{6}\int \frac{1}{\sqrt{-3x^2+5x-2}}dx-\frac{1}{3}\sqrt{-3x^2+5x-2}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{5\int \frac{1}{\sqrt{1-(5-6x)^2}}d(5-6x)}{6\sqrt{3}}-\frac{1}{3}\sqrt{-3x^2+5x-2}$$

$$\Big\downarrow \text{223}$$

$$-\frac{5\arcsin(5-6x)}{6\sqrt{3}}-\frac{1}{3}\sqrt{-3x^2+5x-2}$$

input
```
Int[x/Sqrt[-2 + 5*x - 3*x^2],x]
```

output
```
-1/3*Sqrt[-2 + 5*x - 3*x^2] - (5*ArcSin[5 - 6*x])/(6*Sqrt[3])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{\sqrt{-3x^2+5x-2}}{3}+\frac{5\arcsin(-5+6x)\sqrt{3}}{18}$	27
risch	$\frac{3x^2-5x+2}{3\sqrt{-3x^2+5x-2}}+\frac{5\arcsin(-5+6x)\sqrt{3}}{18}$	37
trager	$-\frac{\sqrt{-3x^2+5x-2}}{3}-\frac{5\,\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2+3\right)x-5\,\mathrm{RootOf}\left(\_Z^2+3\right)+6\sqrt{-3x^2+5x-2}\right)}{18}$	57

input
```
int(x/(-3*x^2+5*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-3*x^2+5*x-2)^(1/2)+5/18*arcsin(-5+6*x)*3^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(26) = 52.

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.62

$$\int \frac{x}{\sqrt{-2+5x-3x^2}}\,dx = -\frac{5}{18}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2+5\,x-2}(6\,x-5)}{6\,(3\,x^2-5\,x+2)}\right) - \frac{1}{3}\sqrt{-3\,x^2+5\,x-2}$$

input
```
integrate(x/(-3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

output
```
-5/18*sqrt(3)*arctan(1/6*sqrt(3)*sqrt(-3*x^2 + 5*x - 2)*(6*x - 5)/(3*x^2 -
 5*x + 2)) - 1/3*sqrt(-3*x^2 + 5*x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{-2+5x-3x^2}}\,dx = -\frac{\sqrt{-3x^2+5x-2}}{3} + \frac{5\sqrt{3}\,\mathrm{asin}\,(6x-5)}{18}$$

input
```
integrate(x/(-3*x**2+5*x-2)**(1/2),x)
```

output
```
-sqrt(-3*x**2 + 5*x - 2)/3 + 5*sqrt(3)*asin(6*x - 5)/18
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{x}{\sqrt{-2+5x-3x^2}}\,dx = \frac{5}{18}\sqrt{3}\arcsin\left(6\,x-5\right) - \frac{1}{3}\sqrt{-3\,x^2+5\,x-2}$$

input
```
integrate(x/(-3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
```

output
```
5/18*sqrt(3)*arcsin(6*x - 5) - 1/3*sqrt(-3*x^2 + 5*x - 2)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{x}{\sqrt{-2+5x-3x^2}}\,dx = \frac{5}{18}\sqrt{3}\arcsin\left(6\,x-5\right) - \frac{1}{3}\sqrt{-3\,x^2+5\,x-2}$$

input
```
integrate(x/(-3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

output
```
5/18*sqrt(3)*arcsin(6*x - 5) - 1/3*sqrt(-3*x^2 + 5*x - 2)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{x}{\sqrt{-2+5x-3x^2}}\,dx = -\frac{\sqrt{-3\,x^2+5\,x-2}}{3} - \frac{\sqrt{3}\,\ln\left(\sqrt{-3\,x^2+5\,x-2}+\frac{\sqrt{3}\,\left(3\,x-\frac{5}{2}\right)\,1\mathrm{i}}{3}\right)\,5\mathrm{i}}{18}$$

input
```
int(x/(5*x - 3*x^2 - 2)^(1/2),x)
```

output
```
- (3^(1/2)*log((5*x - 3*x^2 - 2)^(1/2) + (3^(1/2)*(3*x - 5/2)*1i)/3)*5i)/1
8 - (5*x - 3*x^2 - 2)^(1/2)/3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{x}{\sqrt{-2+5x-3x^2}}\,dx = \frac{5\sqrt{3}\,asin(6x-5)}{18} - \frac{\sqrt{-3x^2+5x-2}}{3}$$

input
```
int(x/(-3*x^2+5*x-2)^(1/2),x)
```

output
```
(5*sqrt(3)*asin(6*x - 5) - 6*sqrt( - 3*x**2 + 5*x - 2))/18
```

3.298 $\int \frac{1}{x\sqrt{2+4x+3x^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \frac{1}{x\sqrt{2+4x+3x^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}(1+x)}{\sqrt{2+4x+3x^2}}\right)}{\sqrt{2}}$$

output
```
-1/2*arctanh(2^(1/2)*(1+x)/(3*x^2+4*x+2)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{1}{x\sqrt{2+4x+3x^2}}\,dx = \sqrt{2}\operatorname{arctanh}\left(\sqrt{\frac{3}{2}}x - \frac{\sqrt{2+4x+3x^2}}{\sqrt{2}}\right)$$

input
```
Integrate[1/(x*Sqrt[2 + 4*x + 3*x^2]),x]
```

output
```
Sqrt[2]*ArcTanh[Sqrt[3/2]*x - Sqrt[2 + 4*x + 3*x^2]/Sqrt[2]]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{3x^2+4x+2}}dx$$

$$\Big\downarrow \; 1154$$

$$-2\int \frac{1}{8-\frac{16(x+1)^2}{3x^2+4x+2}}d\frac{4(x+1)}{\sqrt{3x^2+4x+2}}$$

$$\Big\downarrow \; 219$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{2}(x+1)}{\sqrt{3x^2+4x+2}}\right)}{\sqrt{2}}$$

input
```
Int[1/(x*Sqrt[2 + 4*x + 3*x^2]),x]
```

output
```
-(ArcTanh[(Sqrt[2]*(1 + x))/Sqrt[2 + 4*x + 3*x^2]]/Sqrt[2])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(4+4x)\sqrt{2}}{4\sqrt{3x^{2}+4x+2}}\right)}{2}$	29
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)x+\sqrt{3x^{2}+4x+2}+\operatorname{RootOf}\left(\_Z^{2}-2\right)}{x}\right)}{2}$	41

input
```
int(1/x/(3*x^2+4*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*2^(1/2)*arctanh(1/4*(4+4*x)*2^(1/2)/(3*x^2+4*x+2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{1}{x\sqrt{2+4x+3x^2}}\,dx = \frac{1}{4}\,\sqrt{2}\log\left(\frac{2\,\sqrt{2}\sqrt{3\,x^{2}+4\,x+2}(x+1)-5\,x^{2}-8\,x-4}{x^{2}}\right)$$

input
```
integrate(1/x/(3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log((2*sqrt(2)*sqrt(3*x^2 + 4*x + 2)*(x + 1) - 5*x^2 - 8*x - 4
)/x^2)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{2+4x+3x^2}}\,dx = \int \frac{1}{x\sqrt{3x^2+4x+2}}\,dx$$

input
```
integrate(1/x/(3*x**2+4*x+2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(3*x**2 + 4*x + 2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1}{x\sqrt{2+4x+3x^2}}\,dx = -\frac{1}{2}\sqrt{2}\,\operatorname{arsinh}\left(\frac{\sqrt{2}x}{|x|}+\frac{\sqrt{2}}{|x|}\right)$$

input
```
integrate(1/x/(3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*arcsinh(sqrt(2)*x/abs(x) + sqrt(2)/abs(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(25) = 50.

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

$$\int \frac{1}{x\sqrt{2+4x+3x^2}}\,dx = -\frac{1}{2}\sqrt{2}\log\left(-\sqrt{3}x+\sqrt{2}+\sqrt{3\,x^2+4\,x+2}\right) + \frac{1}{2}\sqrt{2}\log\left(\left|-\sqrt{3}x-\sqrt{2}+\sqrt{3\,x^2+4\,x+2}\right|\right)$$

input
```
integrate(1/x/(3*x^2+4*x+2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*log(-sqrt(3)*x + sqrt(2) + sqrt(3*x^2 + 4*x + 2)) + 1/2*sqrt(
2)*log(abs(-sqrt(3)*x - sqrt(2) + sqrt(3*x^2 + 4*x + 2)))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\sqrt{2+4x+3x^2}}\,dx = -\frac{\sqrt{2}\,\ln\left(\frac{2\,x+\sqrt{6\,x^2+8\,x+4}+2}{x}\right)}{2}$$

input
```
int(1/(x*(4*x + 3*x^2 + 2)^(1/2)),x)
```

output
```
-(2^(1/2)*log((2*x + (8*x + 6*x^2 + 4)^(1/2) + 2)/x))/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.74

$$\int \frac{1}{x\sqrt{2+4x+3x^2}}\,dx = \frac{\sqrt{2}\left(2atan\left(\frac{\sqrt{3x^2+4x+2}\,\sqrt{3}\,i+3ix+2i}{\sqrt{6}-2}\right)i-\log\left(3\sqrt{3x^2+4x+2}\,\sqrt{3}\,x+2\sqrt{3x^2+4x+2}\,\sqrt{3}+2\sqrt{6}+9x^2+1\right.\right.}{4}$$

input
```
int(1/x/(3*x^2+4*x+2)^(1/2),x)
```

output
```
(sqrt(2)*(2*atan((sqrt(3*x**2 + 4*x + 2)*sqrt(3)*i + 3*i*x + 2*i)/(sqrt(6)
 - 2))*i - log(3*sqrt(3*x**2 + 4*x + 2)*sqrt(3)*x + 2*sqrt(3*x**2 + 4*x +
2)*sqrt(3) + 2*sqrt(6) + 9*x**2 + 12*x) + 2*log((sqrt(3*x**2 + 4*x + 2)*sq
rt(3) - sqrt(6) + 3*x)/sqrt(2))))/4
```

3.299 $\int \frac{1}{x\sqrt{2+4x-3x^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \frac{1}{x\sqrt{2+4x-3x^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}(1+x)}{\sqrt{2+4x-3x^2}}\right)}{\sqrt{2}}$$

output
```
-1/2*arctanh(2^(1/2)*(1+x)/(-3*x^2+4*x+2)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{2+4x-3x^2}}\,dx = \frac{-\log(x)+\log\left(-2-2x+\sqrt{4+8x-6x^2}\right)}{\sqrt{2}}$$

input
```
Integrate[1/(x*Sqrt[2 + 4*x - 3*x^2]),x]
```

output
```
(-Log[x] + Log[-2 - 2*x + Sqrt[4 + 8*x - 6*x^2]])/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{-3x^2+4x+2}}dx$$
$$\Big\downarrow \text{1154}$$
$$-2\int \frac{1}{8-\frac{16(x+1)^2}{-3x^2+4x+2}}d\frac{4(x+1)}{\sqrt{-3x^2+4x+2}}$$
$$\Big\downarrow \text{219}$$
$$-\frac{\text{arctanh}\left(\frac{\sqrt{2}(x+1)}{\sqrt{-3x^2+4x+2}}\right)}{\sqrt{2}}$$

input
```
Int[1/(x*Sqrt[2 + 4*x - 3*x^2]),x]
```

output
```
-(ArcTanh[(Sqrt[2]*(1 + x))/Sqrt[2 + 4*x - 3*x^2]]/Sqrt[2])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(4+4x)\sqrt{2}}{4\sqrt{-3x^{2}+4x+2}}\right)}{2}$	29
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)x+\operatorname{RootOf}\left(\_Z^{2}-2\right)+\sqrt{-3x^{2}+4x+2}}{x}\right)}{2}$	41

input
```
int(1/x/(-3*x^2+4*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*2^(1/2)*arctanh(1/4*(4+4*x)*2^(1/2)/(-3*x^2+4*x+2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \frac{1}{x\sqrt{2+4x-3x^2}}\,dx = \frac{1}{4}\,\sqrt{2}\log\left(-\frac{2\,\sqrt{2}\sqrt{-3\,x^{2}+4\,x+2}(x+1)+x^{2}-8\,x-4}{x^{2}}\right)$$

input
```
integrate(1/x/(-3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-(2*sqrt(2)*sqrt(-3*x^2 + 4*x + 2)*(x + 1) + x^2 - 8*x - 4
)/x^2)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{2+4x-3x^2}}\,dx = \int \frac{1}{x\sqrt{-3x^2+4x+2}}\,dx$$

input
```
integrate(1/x/(-3*x**2+4*x+2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(-3*x**2 + 4*x + 2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \frac{1}{x\sqrt{2+4x-3x^2}}\,dx = -\frac{1}{2}\sqrt{2}\log\left(\frac{2\sqrt{2}\sqrt{-3x^2+4x+2}}{|x|}+\frac{4}{|x|}+4\right)$$

input
```
integrate(1/x/(-3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*log(2*sqrt(2)*sqrt(-3*x^2 + 4*x + 2)/abs(x) + 4/abs(x) + 4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(25) = 50.

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.16

$$\int \frac{1}{x\sqrt{2+4x-3x^2}}\,dx = -\frac{1}{6}\sqrt{6}\sqrt{3}\log\left(\frac{\left|-14\sqrt{10}-14\sqrt{6}+\frac{28\left(\sqrt{3}\sqrt{-3x^2+4x+2}-\sqrt{10}\right)}{3x-2}\right|}{\left|-14\sqrt{10}+14\sqrt{6}+\frac{28\left(\sqrt{3}\sqrt{-3x^2+4x+2}-\sqrt{10}\right)}{3x-2}\right|}\right)$$

input
```
integrate(1/x/(-3*x^2+4*x+2)^(1/2),x, algorithm="giac")
```

output
```
-1/6*sqrt(6)*sqrt(3)*log(abs(-14*sqrt(10) - 14*sqrt(6) + 28*(sqrt(3)*sqrt(
-3*x^2 + 4*x + 2) - sqrt(10))/(3*x - 2))/abs(-14*sqrt(10) + 14*sqrt(6) + 2
8*(sqrt(3)*sqrt(-3*x^2 + 4*x + 2) - sqrt(10))/(3*x - 2)))
```

Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\sqrt{2+4x-3x^2}}\,dx = -\frac{\sqrt{2}\,\ln\left(\frac{2\,x+\sqrt{-6\,x^2+8\,x+4}+2}{x}\right)}{2}$$

input
```
int(1/(x*(4*x - 3*x^2 + 2)^(1/2)),x)
```

output
```
-(2^(1/2)*log((2*x + (8*x - 6*x^2 + 4)^(1/2) + 2)/x))/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.58

$$\int \frac{1}{x\sqrt{2+4x-3x^2}}\,dx = \frac{\sqrt{2}\left(2atanh\left(\frac{2\tan\left(\frac{asin\left(\frac{3x-2}{\sqrt{10}}\right)}{2}\right)}{\sqrt{10}-\sqrt{6}}\right)+\log\left(\sqrt{15}+\tan\left(\frac{asin\left(\frac{3x-2}{\sqrt{10}}\right)}{2}\right)^2-4\right)-2\log\left(\frac{\sqrt{10}}{2}+\frac{\sqrt{6}}{2}+\tan\left(\frac{asin(}{}\right.\right.\right.}{4}$$

input
```
int(1/x/(-3*x^2+4*x+2)^(1/2),x)
```

output
```
(sqrt(2)*(2*atanh((2*tan(asin((3*x - 2)/sqrt(10))/2))/(sqrt(10) - sqrt(6))
) + log(sqrt(15) + tan(asin((3*x - 2)/sqrt(10))/2)**2 - 4) - 2*log((sqrt(1
0) + sqrt(6) + 2*tan(asin((3*x - 2)/sqrt(10))/2))/2)))/4
```

3.300 $\int \frac{1}{x\sqrt{2+5x+3x^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \frac{1}{x\sqrt{2+5x+3x^2}}\,dx = -\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}(1+x)}{\sqrt{2+5x+3x^2}}\right)$$

output
```
-2^(1/2)*arctanh(2^(1/2)*(1+x)/(3*x^2+5*x+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\sqrt{2+5x+3x^2}}\,dx = -\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2+5x+3x^2}}{\sqrt{2}(1+x)}\right)$$

input
```
Integrate[1/(x*Sqrt[2 + 5*x + 3*x^2]),x]
```

output
```
-(Sqrt[2]*ArcTanh[Sqrt[2 + 5*x + 3*x^2]/(Sqrt[2]*(1 + x))])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{3x^2+5x+2}}dx$$

$$\Big\downarrow \text{1154}$$

$$-2\int \frac{1}{8-\frac{(5x+4)^2}{3x^2+5x+2}}d\frac{5x+4}{\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{5x+4}{2\sqrt{2}\sqrt{3x^2+5x+2}}\right)}{\sqrt{2}}$$

input
```
Int[1/(x*Sqrt[2 + 5*x + 3*x^2]),x]
```

output
```
-(ArcTanh[(4 + 5*x)/(2*Sqrt[2]*Sqrt[2 + 5*x + 3*x^2])]/Sqrt[2])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{(5x+4)\sqrt{2}}{4\sqrt{3x^{2}+5x+2}}\right)}{2}$	29
trager	$\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right) \ln\left(-\frac{5 \operatorname{RootOf}\left(\_Z^{2}-2\right) x+4 \operatorname{RootOf}\left(\_Z^{2}-2\right)-4\sqrt{3x^{2}+5x+2}}{x}\right)}{2}$	47

input
```
int(1/x/(3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*2^(1/2)*arctanh(1/4*(5*x+4)*2^(1/2)/(3*x^2+5*x+2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.39

$$\int \frac{1}{x\sqrt{2+5x+3x^2}}\, dx = \frac{1}{4}\, \sqrt{2} \log\left(-\frac{4\, \sqrt{2}\sqrt{3\, x^{2} + 5\, x + 2}(5\, x + 4) - 49\, x^{2} - 80\, x - 32}{x^{2}}\right)$$

input
```
integrate(1/x/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-(4*sqrt(2)*sqrt(3*x^2 + 5*x + 2)*(5*x + 4) - 49*x^2 - 80*
x - 32)/x^2)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{2+5x+3x^2}}\,dx = \int \frac{1}{x\sqrt{(x+1)\,(3x+2)}}\,dx$$

input
```
integrate(1/x/(3*x**2+5*x+2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt((x + 1)*(3*x + 2))), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \frac{1}{x\sqrt{2+5x+3x^2}}\,dx = -\frac{1}{2}\,\sqrt{2}\log\left(\frac{2\,\sqrt{2}\sqrt{3\,x^2+5\,x+2}}{|x|}+\frac{4}{|x|}+5\right)$$

input
```
integrate(1/x/(3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*log(2*sqrt(2)*sqrt(3*x^2 + 5*x + 2)/abs(x) + 4/abs(x) + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(25) = 50.

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.97

$$\begin{aligned}\int \frac{1}{x\sqrt{2+5x+3x^2}}\,dx = & -\frac{1}{2}\,\sqrt{2}\log\left(\left|-\sqrt{3}x+\sqrt{2}+\sqrt{3\,x^2+5\,x+2}\right|\right)\\ & +\frac{1}{2}\,\sqrt{2}\log\left(\left|-\sqrt{3}x-\sqrt{2}+\sqrt{3\,x^2+5\,x+2}\right|\right)\end{aligned}$$

input
```
integrate(1/x/(3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*log(abs(-sqrt(3)*x + sqrt(2) + sqrt(3*x^2 + 5*x + 2))) + 1/2*
sqrt(2)*log(abs(-sqrt(3)*x - sqrt(2) + sqrt(3*x^2 + 5*x + 2)))
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\sqrt{2+5x+3x^2}}\,dx = -\frac{\sqrt{2}\,\ln\left(\frac{5\,x+2\,\sqrt{6\,x^2+10\,x+4}+4}{x}\right)}{2}$$

input
```
int(1/(x*(5*x + 3*x^2 + 2)^(1/2)),x)
```

output
```
-(2^(1/2)*log((5*x + 2*(10*x + 6*x^2 + 4)^(1/2) + 4)/x))/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.77

$$\int \frac{1}{x\sqrt{2+5x+3x^2}}\,dx = \frac{\sqrt{2}\,\left(\log\left(2\sqrt{3x^2+5x+2}\,\sqrt{3}-2\sqrt{6}+6x\right)-\log\left(2\sqrt{3x^2+5x+2}\,\sqrt{3}+2\sqrt{6}+6x\right)\right)}{2}$$

input
```
int(1/x/(3*x^2+5*x+2)^(1/2),x)
```

output
```
(sqrt(2)*(log(2*sqrt(3*x**2 + 5*x + 2)*sqrt(3) - 2*sqrt(6) + 6*x) - log(2*
sqrt(3*x**2 + 5*x + 2)*sqrt(3) + 2*sqrt(6) + 6*x)))/2
```

3.301 $\int \frac{1}{x\sqrt{2+5x-3x^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 33

$$\int \frac{1}{x\sqrt{2+5x-3x^2}}\,dx = -\sqrt{2}\text{arctanh}\left(\frac{2-x}{\sqrt{2}\sqrt{2+5x-3x^2}}\right)$$

output
```
-2^(1/2)*arctanh(1/2*(2-x)*2^(1/2)/(-3*x^2+5*x+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\sqrt{2+5x-3x^2}}\,dx = \sqrt{2}\text{arctanh}\left(\frac{\sqrt{4+10x-6x^2}}{-2+x}\right)$$

input
```
Integrate[1/(x*Sqrt[2 + 5*x - 3*x^2]),x]
```

output
```
Sqrt[2]*ArcTanh[Sqrt[4 + 10*x - 6*x^2]/(-2 + x)]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{-3x^2+5x+2}}\,dx$$
$$\Big\downarrow \; 1154$$
$$-2\int \frac{1}{8-\frac{(5x+4)^2}{-3x^2+5x+2}}d\frac{5x+4}{\sqrt{-3x^2+5x+2}}$$
$$\Big\downarrow \; 219$$
$$-\frac{\operatorname{arctanh}\left(\frac{5x+4}{2\sqrt{2}\sqrt{-3x^2+5x+2}}\right)}{\sqrt{2}}$$

input
```
Int[1/(x*Sqrt[2 + 5*x - 3*x^2]),x]
```

output
```
-(ArcTanh[(4 + 5*x)/(2*Sqrt[2]*Sqrt[2 + 5*x - 3*x^2])]/Sqrt[2])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(5x+4)\sqrt{2}}{4\sqrt{-3x^{2}+5x+2}}\right)}{2}$	29
trager	$\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)\ln\left(\frac{-5\operatorname{RootOf}\left(\_Z^{2}-2\right)x+4\sqrt{-3x^{2}+5x+2}-4\operatorname{RootOf}\left(\_Z^{2}-2\right)}{x}\right)}{2}$	46

input
```
int(1/x/(-3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*2^(1/2)*arctanh(1/4*(5*x+4)*2^(1/2)/(-3*x^2+5*x+2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

$$\int \frac{1}{x\sqrt{2+5x-3x^2}}\,dx = \frac{1}{4}\,\sqrt{2}\log\left(-\frac{4\,\sqrt{2}\sqrt{-3\,x^{2}+5\,x+2}(5\,x+4)-x^{2}-80\,x-32}{x^{2}}\right)$$

input
```
integrate(1/x/(-3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-(4*sqrt(2)*sqrt(-3*x^2 + 5*x + 2)*(5*x + 4) - x^2 - 80*x
- 32)/x^2)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{2+5x-3x^2}}\,dx = \int \frac{1}{x\sqrt{-(x-2)(3x+1)}}\,dx$$

input
```
integrate(1/x/(-3*x**2+5*x+2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(-(x - 2)*(3*x + 1))), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\sqrt{2+5x-3x^2}}\,dx = -\frac{1}{2}\sqrt{2}\log\left(\frac{2\sqrt{2}\sqrt{-3x^2+5x+2}}{|x|}+\frac{4}{|x|}+5\right)$$

input
```
integrate(1/x/(-3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*log(2*sqrt(2)*sqrt(-3*x^2 + 5*x + 2)/abs(x) + 4/abs(x) + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(26) = 52.

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.55

$$\int \frac{1}{x\sqrt{2+5x-3x^2}}\,dx = -\frac{1}{6}\sqrt{6}\sqrt{3}\log\left(\frac{\left|-4\sqrt{6}+\frac{10\left(2\sqrt{3}\sqrt{-3x^2+5x+2}-7\right)}{6x-5}-14\right|}{\left|4\sqrt{6}+\frac{10\left(2\sqrt{3}\sqrt{-3x^2+5x+2}-7\right)}{6x-5}-14\right|}\right)$$

input
```
integrate(1/x/(-3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

output
```
-1/6*sqrt(6)*sqrt(3)*log(abs(-4*sqrt(6) + 10*(2*sqrt(3)*sqrt(-3*x^2 + 5*x
+ 2) - 7)/(6*x - 5) - 14)/abs(4*sqrt(6) + 10*(2*sqrt(3)*sqrt(-3*x^2 + 5*x
+ 2) - 7)/(6*x - 5) - 14))
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\sqrt{2+5x-3x^2}}\,dx = -\frac{\sqrt{2}\,\ln\left(\frac{5\,x+2\,\sqrt{-6\,x^2+10\,x+4}+4}{x}\right)}{2}$$

input
```
int(1/(x*(5*x - 3*x^2 + 2)^(1/2)),x)
```

output
```
-(2^(1/2)*log((5*x + 2*(10*x - 6*x^2 + 4)^(1/2) + 4)/x))/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

$$\int \frac{1}{x\sqrt{2+5x-3x^2}}\,dx = \frac{\sqrt{2}\left(\log\left(-2\sqrt{6}+5\tan\left(\frac{asin\left(\frac{6x}{7}-\frac{5}{7}\right)}{2}\right)+7\right)-\log\left(2\sqrt{6}+5\tan\left(\frac{asin\left(\frac{6x}{7}-\frac{5}{7}\right)}{2}\right)+7\right)\right)}{2}$$

input
```
int(1/x/(-3*x^2+5*x+2)^(1/2),x)
```

output
```
(sqrt(2)*(log( - 2*sqrt(6) + 5*tan(asin((6*x - 5)/7)/2) + 7) - log(2*sqrt(
6) + 5*tan(asin((6*x - 5)/7)/2) + 7)))/2
```

3.302 $\int \frac{1}{x\sqrt{-2+4x+3x^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 33

$$\int \frac{1}{x\sqrt{-2+4x+3x^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{2}(1-x)}{\sqrt{-2+4x+3x^2}}\right)}{\sqrt{2}}$$

output
```
-1/2*arctan(2^(1/2)*(1-x)/(3*x^2+4*x-2)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \frac{1}{x\sqrt{-2+4x+3x^2}}\,dx = -\sqrt{2}\arctan\left(\sqrt{\frac{3}{2}}x - \frac{\sqrt{-2+4x+3x^2}}{\sqrt{2}}\right)$$

input
```
Integrate[1/(x*Sqrt[-2 + 4*x + 3*x^2]),x]
```

output
```
-(Sqrt[2]*ArcTan[Sqrt[3/2]*x - Sqrt[-2 + 4*x + 3*x^2]/Sqrt[2]])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1154, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{3x^2+4x-2}}dx$$
$$\Big\downarrow \text{1154}$$
$$-2\int \frac{1}{-\frac{16(1-x)^2}{3x^2+4x-2}-8}d\left(-\frac{4(1-x)}{\sqrt{3x^2+4x-2}}\right)$$
$$\Big\downarrow \text{217}$$
$$-\frac{\arctan\left(\frac{\sqrt{2}(1-x)}{\sqrt{3x^2+4x-2}}\right)}{\sqrt{2}}$$

input
```
Int[1/(x*Sqrt[-2 + 4*x + 3*x^2]),x]
```

output
```
-(ArcTan[(Sqrt[2]*(1 - x))/Sqrt[-2 + 4*x + 3*x^2]]/Sqrt[2])
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\sqrt{2}\,\arctan\left(\frac{(-4+4x)\sqrt{2}}{4\sqrt{3x^2+4x-2}}\right)}{2}$	29
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\sqrt{3x^2+4x-2}-2x+2}{x}\right)}{2}$	39

input
```
int(1/x/(3*x^2+4*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*arctan(1/4*(-4+4*x)*2^(1/2)/(3*x^2+4*x-2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\sqrt{-2+4x+3x^2}}\,dx = \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}(x-1)}{\sqrt{3\,x^2+4\,x-2}}\right)$$

input
```
integrate(1/x/(3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*arctan(sqrt(2)*(x - 1)/sqrt(3*x^2 + 4*x - 2))
```

Sympy [F]

$$\int \frac{1}{x\sqrt{-2+4x+3x^2}}\,dx = \int \frac{1}{x\sqrt{3x^2+4x-2}}\,dx$$

input
```
integrate(1/x/(3*x**2+4*x-2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(3*x**2 + 4*x - 2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{1}{x\sqrt{-2+4x+3x^2}}\,dx = \frac{1}{2}\sqrt{2}\arcsin\left(\frac{\sqrt{10}x}{5|x|}-\frac{\sqrt{10}}{5|x|}\right)$$

input
```
integrate(1/x/(3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arcsin(1/5*sqrt(10)*x/abs(x) - 1/5*sqrt(10)/abs(x))
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{-2+4x+3x^2}}\,dx = \sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{3}x-\sqrt{3x^2+4x-2}\right)\right)$$

input
```
integrate(1/x/(3*x^2+4*x-2)^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(3)*x - sqrt(3*x^2 + 4*x - 2)))
```

Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{1}{x\sqrt{-2+4x+3x^2}}\,dx = \frac{\sqrt{2}\ln\left(\frac{2x-2+\sqrt{2}\sqrt{3x^2+4x-2}\,1i}{x}\right)1i}{2}$$

input
```
int(1/(x*(4*x + 3*x^2 - 2)^(1/2)),x)
```

output
```
(2^(1/2)*log((2*x + 2^(1/2)*(4*x + 3*x^2 - 2)^(1/2)*1i - 2)/x)*1i)/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\sqrt{-2+4x+3x^2}}\,dx = \sqrt{2}\,atan\left(\frac{\sqrt{3x^2+4x-2}\,\sqrt{3}+3x}{\sqrt{6}}\right)$$

input
```
int(1/x/(3*x^2+4*x-2)^(1/2),x)
```

output
```
sqrt(2)*atan((sqrt(3*x**2 + 4*x - 2)*sqrt(3) + 3*x)/sqrt(6))
```

3.303 $\int \frac{1}{x\sqrt{-2+4x-3x^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 33

$$\int \frac{1}{x\sqrt{-2+4x-3x^2}}\,dx=-\frac{\arctan\left(\frac{\sqrt{2}(1-x)}{\sqrt{-2+4x-3x^2}}\right)}{\sqrt{2}}$$

output
```
-1/2*arctan(2^(1/2)*(1-x)/(-3*x^2+4*x-2)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\sqrt{-2+4x-3x^2}}\,dx=\frac{\arctan\left(\frac{-1+x}{\sqrt{-1+2x-\frac{3x^2}{2}}}\right)}{\sqrt{2}}$$

input
```
Integrate[1/(x*Sqrt[-2 + 4*x - 3*x^2]),x]
```

output
```
ArcTan[(-1 + x)/Sqrt[-1 + 2*x - (3*x^2)/2]]/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1154, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{-3x^2+4x-2}}dx$$

$$\Big\downarrow \text{1154}$$

$$-2\int \frac{1}{-\frac{16(1-x)^2}{-3x^2+4x-2}-8}d\left(-\frac{4(1-x)}{\sqrt{-3x^2+4x-2}}\right)$$

$$\Big\downarrow \text{217}$$

$$-\frac{\arctan\left(\frac{\sqrt{2}(1-x)}{\sqrt{-3x^2+4x-2}}\right)}{\sqrt{2}}$$

input
```
Int[1/(x*Sqrt[-2 + 4*x - 3*x^2]),x]
```

output
```
-(ArcTan[(Sqrt[2]*(1 - x))/Sqrt[-2 + 4*x - 3*x^2]]/Sqrt[2])
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\sqrt{2}\, \arctan\left(\frac{(-4+4x)\sqrt{2}}{4\sqrt{-3x^2+4x-2}}\right)}{2}$	29
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+2\right) \ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\sqrt{-3x^2+4x-2}-2x+2}{x}\right)}{2}$	39

input
```
int(1/x/(-3*x^2+4*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*arctan(1/4*(-4+4*x)*2^(1/2)/(-3*x^2+4*x-2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(26) = 52.

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.94

$$\int \frac{1}{x\sqrt{-2+4x-3x^2}}\, dx = \frac{1}{4}\,\sqrt{-2}\log\left(\frac{\sqrt{-2}\sqrt{-3\,x^2+4\,x-2}+2\,x-2}{x}\right) - \frac{1}{4}\,\sqrt{-2}\log\left(-\frac{\sqrt{-2}\sqrt{-3\,x^2+4\,x-2}-2\,x+2}{x}\right)$$

input
```
integrate(1/x/(-3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(-2)*log((sqrt(-2)*sqrt(-3*x^2 + 4*x - 2) + 2*x - 2)/x) - 1/4*sqrt
(-2)*log(-(sqrt(-2)*sqrt(-3*x^2 + 4*x - 2) - 2*x + 2)/x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{-2+4x-3x^2}}\,dx = \int \frac{1}{x\sqrt{-3x^2+4x-2}}\,dx$$

input
```
integrate(1/x/(-3*x**2+4*x-2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(-3*x**2 + 4*x - 2)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\sqrt{-2+4x-3x^2}}\,dx = \frac{1}{2}i\,\sqrt{2}\,\text{arsinh}\left(\frac{\sqrt{2}x}{|x|}-\frac{\sqrt{2}}{|x|}\right)$$

input
```
integrate(1/x/(-3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*I*sqrt(2)*arcsinh(sqrt(2)*x/abs(x) - sqrt(2)/abs(x))
```

Giac [F]

$$\int \frac{1}{x\sqrt{-2+4x-3x^2}}\,dx = \int \frac{1}{\sqrt{-3\,x^2+4\,x-2}x}\,dx$$

input
```
integrate(1/x/(-3*x^2+4*x-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-3*x^2 + 4*x - 2)*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{1}{x\sqrt{-2+4x-3x^2}}\,dx = \frac{\sqrt{2}\,\ln\left(\frac{2\,x-2+\sqrt{2}\,\sqrt{-3\,x^2+4\,x-2}\,1\mathrm{i}}{x}\right)\,1\mathrm{i}}{2}$$

input
```
int(1/(x*(4*x - 3*x^2 - 2)^(1/2)),x)
```

output
```
(2^(1/2)*log((2*x + 2^(1/2)*(4*x - 3*x^2 - 2)^(1/2)*1i - 2)/x)*1i)/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.55

$$\int \frac{1}{x\sqrt{-2+4x-3x^2}}\,dx = \frac{\sqrt{2}\left(-2atan\left(\frac{\sqrt{-3x^2+4x-2}\,\sqrt{3}-3ix+2i}{\sqrt{6}-2}\right)-\log\left(3\sqrt{-3x^2+4x-2}\,\sqrt{3}\,ix-2\sqrt{-3x^2+4x-2}\,\sqrt{3}\,i+2\sqrt{6}+9\right.\right.}{4}$$

input
```
int(1/x/(-3*x^2+4*x-2)^(1/2),x)
```

output
```
(sqrt(2)*( - 2*atan((sqrt( - 3*x**2 + 4*x - 2)*sqrt(3) - 3*i*x + 2*i)/(sqr
t(6) - 2)) - log(3*sqrt( - 3*x**2 + 4*x - 2)*sqrt(3)*i*x - 2*sqrt( - 3*x**
2 + 4*x - 2)*sqrt(3)*i + 2*sqrt(6) + 9*x**2 - 12*x)*i + 2*log((sqrt( - 3*x
**2 + 4*x - 2)*sqrt(3)*i + sqrt(6) + 3*x)/sqrt(2))*i))/4
```

3.304 $\int \frac{1}{x\sqrt{-2+5x+3x^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \frac{1}{x\sqrt{-2+5x+3x^2}}\,dx = -\sqrt{2}\arctan\left(\frac{2+x}{\sqrt{2}\sqrt{-2+5x+3x^2}}\right)$$

output
```
-2^(1/2)*arctan(1/2*(2+x)*2^(1/2)/(3*x^2+5*x-2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \frac{1}{x\sqrt{-2+5x+3x^2}}\,dx = -\sqrt{2}\arctan\left(\frac{\sqrt{-2+5x+3x^2}}{\sqrt{2}(-1+3x)}\right)$$

input
```
Integrate[1/(x*Sqrt[-2 + 5*x + 3*x^2]),x]
```

output
```
-(Sqrt[2]*ArcTan[Sqrt[-2 + 5*x + 3*x^2]/(Sqrt[2]*(-1 + 3*x))])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1154, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{3x^2+5x-2}}dx$$

$$\Big\downarrow \text{ 1154}$$

$$-2\int \frac{1}{-\frac{(4-5x)^2}{3x^2+5x-2}-8}d\left(-\frac{4-5x}{\sqrt{3x^2+5x-2}}\right)$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{\arctan\left(\frac{4-5x}{2\sqrt{2}\sqrt{3x^2+5x-2}}\right)}{\sqrt{2}}$$

input
```
Int[1/(x*Sqrt[-2 + 5*x + 3*x^2]),x]
```

output
```
-(ArcTan[(4 - 5*x)/(2*Sqrt[2]*Sqrt[-2 + 5*x + 3*x^2])]/Sqrt[2])
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\sqrt{2}\,\arctan\left(\frac{(5x-4)\sqrt{2}}{4\sqrt{3x^2+5x-2}}\right)}{2}$	29
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{5x\operatorname{RootOf}\left(\_Z^2+2\right)+4\sqrt{3x^2+5x-2}-4\operatorname{RootOf}\left(\_Z^2+2\right)}{x}\right)}{2}$	46

input
```
int(1/x/(3*x^2+5*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*arctan(1/4*(5*x-4)*2^(1/2)/(3*x^2+5*x-2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\sqrt{-2+5x+3x^2}}\,dx = \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}(5\,x-4)}{4\,\sqrt{3\,x^2+5\,x-2}}\right)$$

input
```
integrate(1/x/(3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*arctan(1/4*sqrt(2)*(5*x - 4)/sqrt(3*x^2 + 5*x - 2))
```

Sympy [F]

$$\int \frac{1}{x\sqrt{-2+5x+3x^2}}\,dx = \int \frac{1}{x\sqrt{(x+2)\,(3x-1)}}\,dx$$

input
```
integrate(1/x/(3*x**2+5*x-2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt((x + 2)*(3*x - 1))), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{1}{x\sqrt{-2+5x+3x^2}}\,dx = \frac{1}{2}\,\sqrt{2}\arcsin\left(\frac{5\,x}{7\,|x|}-\frac{4}{7\,|x|}\right)$$

input
```
integrate(1/x/(3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arcsin(5/7*x/abs(x) - 4/7/abs(x))
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{x\sqrt{-2+5x+3x^2}}\,dx = \sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{3}x-\sqrt{3\,x^2+5\,x-2}\right)\right)$$

input
```
integrate(1/x/(3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(3)*x - sqrt(3*x^2 + 5*x - 2)))
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\sqrt{-2+5x+3x^2}}\,dx = \frac{\sqrt{2}\,\ln\left(\frac{5\,x-4+\sqrt{2}\,\sqrt{3\,x^2+5\,x-2}\,2\mathrm{i}}{x}\right)\,1\mathrm{i}}{2}$$

input
```
int(1/(x*(5*x + 3*x^2 - 2)^(1/2)),x)
```

output
```
(2^(1/2)*log((5*x + 2^(1/2)*(5*x + 3*x^2 - 2)^(1/2)*2i - 4)/x)*1i)/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\sqrt{-2+5x+3x^2}}\,dx = \sqrt{2}\,atan\left(\frac{\sqrt{3x^2+5x-2}\,\sqrt{3}+3x}{\sqrt{6}}\right)$$

input
```
int(1/x/(3*x^2+5*x-2)^(1/2),x)
```

output
```
sqrt(2)*atan((sqrt(3*x**2 + 5*x - 2)*sqrt(3) + 3*x)/sqrt(6))
```

3.305 $\int \frac{1}{x\sqrt{-2+5x-3x^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 33

$$\int \frac{1}{x\sqrt{-2+5x-3x^2}}\,dx = -\sqrt{2}\arctan\left(\frac{\sqrt{2}(1-x)}{\sqrt{-2+5x-3x^2}}\right)$$

output
```
-2^(1/2)*arctan(2^(1/2)*(1-x)/(-3*x^2+5*x-2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{-2+5x-3x^2}}\,dx = -\sqrt{2}\arctan\left(\frac{\sqrt{-4+10x-6x^2}}{-2+3x}\right)$$

input
```
Integrate[1/(x*Sqrt[-2 + 5*x - 3*x^2]),x]
```

output
```
-(Sqrt[2]*ArcTan[Sqrt[-4 + 10*x - 6*x^2]/(-2 + 3*x)])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1154, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{-3x^2+5x-2}}dx$$

$$\Big\downarrow \text{1154}$$

$$-2\int \frac{1}{-\frac{(4-5x)^2}{-3x^2+5x-2}-8}d\left(-\frac{4-5x}{\sqrt{-3x^2+5x-2}}\right)$$

$$\Big\downarrow \text{217}$$

$$-\frac{\arctan\left(\frac{4-5x}{2\sqrt{2}\sqrt{-3x^2+5x-2}}\right)}{\sqrt{2}}$$

input
```
Int[1/(x*Sqrt[-2 + 5*x - 3*x^2]),x]
```

output
```
-(ArcTan[(4 - 5*x)/(2*Sqrt[2]*Sqrt[-2 + 5*x - 3*x^2])]/Sqrt[2])
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\sqrt{2}\,\arctan\left(\frac{(5x-4)\sqrt{2}}{4\sqrt{-3x^{2}+5x-2}}\right)}{2}$	29
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}+2\right)\ln\left(\frac{5x\operatorname{RootOf}\left(\_Z^{2}+2\right)+4\sqrt{-3x^{2}+5x-2}-4\operatorname{RootOf}\left(\_Z^{2}+2\right)}{x}\right)}{2}$	46

input
```
int(1/x/(-3*x^2+5*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*arctan(1/4*(5*x-4)*2^(1/2)/(-3*x^2+5*x-2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

$$\int \frac{1}{x\sqrt{-2+5x-3x^2}}\,dx = -\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-3\,x^2+5\,x-2}(5\,x-4)}{4\,(3\,x^2-5\,x+2)}\right)$$

input
```
integrate(1/x/(-3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*arctan(1/4*sqrt(2)*sqrt(-3*x^2 + 5*x - 2)*(5*x - 4)/(3*x^2 -
5*x + 2))
```

Sympy [F]

$$\int \frac{1}{x\sqrt{-2+5x-3x^2}}\,dx = \int \frac{1}{x\sqrt{-(x-1)(3x-2)}}\,dx$$

input
```
integrate(1/x/(-3*x**2+5*x-2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(-(x - 1)*(3*x - 2))), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

$$\int \frac{1}{x\sqrt{-2+5x-3x^2}}\,dx = \frac{1}{2}\sqrt{2}\arcsin\left(\frac{5x}{|x|}-\frac{4}{|x|}\right)$$

input
```
integrate(1/x/(-3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arcsin(5*x/abs(x) - 4/abs(x))
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{1}{x\sqrt{-2+5x-3x^2}}\,dx = -\frac{1}{3}\sqrt{6}\sqrt{3}\arctan\left(\frac{1}{12}\sqrt{6}\left(\frac{5\left(2\sqrt{3}\sqrt{-3x^2+5x-2}-1\right)}{6x-5}-1\right)\right)$$

input
```
integrate(1/x/(-3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

output
```
-1/3*sqrt(6)*sqrt(3)*arctan(1/12*sqrt(6)*(5*(2*sqrt(3)*sqrt(-3*x^2 + 5*x -
 2) - 1)/(6*x - 5) - 1))
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{1}{x\sqrt{-2+5x-3x^2}}\,dx = \frac{\sqrt{2}\,\ln\left(\frac{5\,x-4+\sqrt{2}\,\sqrt{-3\,x^2+5\,x-2}\,2\mathrm{i}}{x}\right)\,1\mathrm{i}}{2}$$

input
```
int(1/(x*(5*x - 3*x^2 - 2)^(1/2)),x)
```

output
```
(2^(1/2)*log((5*x + 2^(1/2)*(5*x - 3*x^2 - 2)^(1/2)*2i - 4)/x)*1i)/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{1}{x\sqrt{-2+5x-3x^2}}\,dx = \sqrt{2}\,atan\left(\frac{5\tan\left(\frac{asin(6x-5)}{2}\right)+1}{2\sqrt{6}}\right)$$

input
```
int(1/x/(-3*x^2+5*x-2)^(1/2),x)
```

output
```
sqrt(2)*atan((5*tan(asin(6*x - 5)/2) + 1)/(2*sqrt(6)))
```

3.306 $\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 57

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx = -\frac{\sqrt{1+x+x^2}}{2x^2}+\frac{3\sqrt{1+x+x^2}}{4x}+\frac{1}{8}\text{arctanh}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)$$

output
```
-1/2*(x^2+x+1)^(1/2)/x^2+3/4*(x^2+x+1)^(1/2)/x+1/8*arctanh(1/2*(2+x)/(x^2+
x+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx = \frac{(-2+3x)\sqrt{1+x+x^2}}{4x^2}-\frac{1}{4}\text{arctanh}\left(x-\sqrt{1+x+x^2}\right)$$

input
```
Integrate[1/(x^3*Sqrt[1 + x + x^2]),x]
```

output
```
((-2 + 3*x)*Sqrt[1 + x + x^2])/(4*x^2) - ArcTanh[x - Sqrt[1 + x + x^2]]/4
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1167, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{x^2+x+1}}dx$$

$$\Big\downarrow \text{1167}$$

$$-\frac{1}{2}\int \frac{2x+3}{2x^2\sqrt{x^2+x+1}}dx-\frac{\sqrt{x^2+x+1}}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{4}\int \frac{2x+3}{x^2\sqrt{x^2+x+1}}dx-\frac{\sqrt{x^2+x+1}}{2x^2}$$

$$\Big\downarrow \text{1228}$$

$$\frac{1}{4}\left(\frac{3\sqrt{x^2+x+1}}{x}-\frac{1}{2}\int \frac{1}{x\sqrt{x^2+x+1}}dx\right)-\frac{\sqrt{x^2+x+1}}{2x^2}$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{4}\left(\int \frac{1}{4-\frac{(x+2)^2}{x^2+x+1}}d\frac{x+2}{\sqrt{x^2+x+1}}+\frac{3\sqrt{x^2+x+1}}{x}\right)-\frac{\sqrt{x^2+x+1}}{2x^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{1}{2}\text{arctanh}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)+\frac{3\sqrt{x^2+x+1}}{x}\right)-\frac{\sqrt{x^2+x+1}}{2x^2}$$

input
```
Int[1/(x^3*Sqrt[1 + x + x^2]),x]
```

output
```
-1/2*Sqrt[1 + x + x^2]/x^2 + ((3*Sqrt[1 + x + x^2])/x + ArcTanh[(2 + x)/(2
*Sqrt[1 + x + x^2])]/2)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.70

method	result	size
trager	$\frac{(-2+3x)\sqrt{x^2+x+1}}{4x^2}+\frac{\ln\left(\frac{2\sqrt{x^2+x+1}+2+x}{x}\right)}{8}$	40
risch	$\frac{3x^3+x^2+x-2}{4x^2\sqrt{x^2+x+1}}+\frac{\operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{8}$	42
default	$-\frac{\sqrt{x^2+x+1}}{2x^2}+\frac{3\sqrt{x^2+x+1}}{4x}+\frac{\operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{8}$	44

input
```
int(1/x^3/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-2+3*x)/x^2*(x^2+x+1)^(1/2)+1/8*ln((2*(x^2+x+1)^(1/2)+2+x)/x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx = \frac{x^2\log\left(-x+\sqrt{x^2+x+1}+1\right)-x^2\log\left(-x+\sqrt{x^2+x+1}-1\right)+6\,x^2+2\,\sqrt{x^2+x+1}(3\,x-2)}{8\,x^2}$$

input
```
integrate(1/x^3/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(x^2*log(-x + sqrt(x^2 + x + 1) + 1) - x^2*log(-x + sqrt(x^2 + x + 1)
- 1) + 6*x^2 + 2*sqrt(x^2 + x + 1)*(3*x - 2))/x^2
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx = \int \frac{1}{x^3\sqrt{x^2+x+1}}\,dx$$

input
```
integrate(1/x**3/(x**2+x+1)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(x**2 + x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx = \frac{3\sqrt{x^2+x+1}}{4\,x} - \frac{\sqrt{x^2+x+1}}{2\,x^2} + \frac{1}{8}\,\operatorname{arsinh}\left(\frac{\sqrt{3}x}{3\,|x|} + \frac{2\,\sqrt{3}}{3\,|x|}\right)$$

input
```
integrate(1/x^3/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

output
```
3/4*sqrt(x^2 + x + 1)/x - 1/2*sqrt(x^2 + x + 1)/x^2 + 1/8*arcsinh(1/3*sqrt
(3)*x/abs(x) + 2/3*sqrt(3)/abs(x))
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.47

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx = \frac{\left(x-\sqrt{x^2+x+1}\right)^3 + 9\,x - 9\,\sqrt{x^2+x+1} + 8}{4\left(\left(x-\sqrt{x^2+x+1}\right)^2 - 1\right)^2} + \frac{1}{8}\,\log\left(\left|-x+\sqrt{x^2+x+1}+1\right|\right) - \frac{1}{8}\,\log\left(\left|-x+\sqrt{x^2+x+1}-1\right|\right)$$

input
```
integrate(1/x^3/(x^2+x+1)^(1/2),x, algorithm="giac")
```

output
```
1/4*((x - sqrt(x^2 + x + 1))^3 + 9*x - 9*sqrt(x^2 + x + 1) + 8)/((x - sqrt
(x^2 + x + 1))^2 - 1)^2 + 1/8*log(abs(-x + sqrt(x^2 + x + 1) + 1)) - 1/8*l
og(abs(-x + sqrt(x^2 + x + 1) - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx = \int \frac{1}{x^3\,\sqrt{x^2+x+1}}\,dx$$

input
```
int(1/(x^3*(x + x^2 + 1)^(1/2)),x)
```

output
```
int(1/(x^3*(x + x^2 + 1)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx = \frac{6\sqrt{x^2+x+1}\,x-4\sqrt{x^2+x+1}+\log\left(-2\sqrt{x^2+x+1}-x-2\right)x^2-\log(x)\,x^2}{8x^2}$$

input
```
int(1/x^3/(x^2+x+1)^(1/2),x)
```

output
```
(6*sqrt(x**2 + x + 1)*x - 4*sqrt(x**2 + x + 1) + log( - 2*sqrt(x**2 + x +
1) - x - 2)*x**2 - log(x)*x**2)/(8*x**2)
```

3.307 $\int \left(\frac{1}{x}-\frac{1}{x\sqrt{1+bx+cx^2}}\right) dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \left(\frac{1}{x}-\frac{1}{x\sqrt{1+bx+cx^2}}\right) dx = \log\left(-2-bx-2\sqrt{1+bx+cx^2}\right)$$

output
```
ln(-2-b*x-2*(c*x^2+b*x+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \left(\frac{1}{x}-\frac{1}{x\sqrt{1+bx+cx^2}}\right) dx = 2\log(x)-\log\left(-2-bx+2\sqrt{1+bx+cx^2}\right)$$

input
```
Integrate[x^(-1) - 1/(x*Sqrt[1 + b*x + c*x^2]),x]
```

output
```
2*Log[x] - Log[-2 - b*x + 2*Sqrt[1 + b*x + c*x^2]]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{1}{x}-\frac{1}{x\sqrt{bx+cx^2+1}}\right) dx$$
$$\Big\downarrow \text{2009}$$
$$\text{arctanh}\left(\frac{bx+2}{2\sqrt{bx+cx^2+1}}\right)+\log(x)$$

input
```
Int[x^(-1) - 1/(x*Sqrt[1 + b*x + c*x^2]),x]
```

output
```
ArcTanh[(2 + b*x)/(2*Sqrt[1 + b*x + c*x^2])] + Log[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
default	$\ln(x)+\text{arctanh}\left(\frac{bx+2}{2\sqrt{c\,x^2+bx+1}}\right)$	24

input
```
int(1/x-1/x/(c*x^2+b*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(x)+arctanh(1/2*(b*x+2)/(c*x^2+b*x+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \left(\frac{1}{x}-\frac{1}{x\sqrt{1+bx+cx^2}}\right)\,dx=\log\left(x\right)-\log\left(-\frac{bx-2\sqrt{cx^2+bx+1}+2}{x}\right)$$

input
```
integrate(1/x-1/x/(c*x^2+b*x+1)^(1/2),x, algorithm="fricas")
```

output
```
log(x) - log(-(b*x - 2*sqrt(c*x^2 + b*x + 1) + 2)/x)
```

Sympy [F]

$$\int \left(\frac{1}{x}-\frac{1}{x\sqrt{1+bx+cx^2}}\right)\,dx=\int \frac{\sqrt{bx+cx^2+1}-1}{x\sqrt{bx+cx^2+1}}\,dx$$

input
```
integrate(1/x-1/x/(c*x**2+b*x+1)**(1/2),x)
```

output
```
Integral((sqrt(b*x + c*x**2 + 1) - 1)/(x*sqrt(b*x + c*x**2 + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \left(\frac{1}{x}-\frac{1}{x\sqrt{1+bx+cx^2}}\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/x-1/x/(c*x^2+b*x+1)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c-b^2>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(21) = 42$.

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.17

$$\int \left(\frac{1}{x}-\frac{1}{x\sqrt{1+bx+cx^2}}\right) dx = \log\left(\left|-\sqrt{c}x+\sqrt{cx^2+bx+1}+1\right|\right) - \log\left(\left|-\sqrt{c}x+\sqrt{cx^2+bx+1}-1\right|\right) + \log\left(|x|\right)$$

input
```
integrate(1/x-1/x/(c*x^2+b*x+1)^(1/2),x, algorithm="giac")
```

output
```
log(abs(-sqrt(c)*x + sqrt(c*x^2 + b*x + 1) + 1)) - log(abs(-sqrt(c)*x + sq
rt(c*x^2 + b*x + 1) - 1)) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \left(\frac{1}{x}-\frac{1}{x\sqrt{1+bx+cx^2}}\right) dx = \ln\left(\frac{b}{2}+\frac{\sqrt{c\,x^2+b\,x+1}}{x}+\frac{1}{x}\right)+\ln\left(x\right)$$

input
```
int(1/x - 1/(x*(b*x + c*x^2 + 1)^(1/2)),x)
```

output
```
log(b/2 + (b*x + c*x^2 + 1)^(1/2)/x + 1/x) + log(x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 125, normalized size of antiderivative = 5.43

$$\int \left(\frac{1}{x}-\frac{1}{x\sqrt{1+bx+cx^2}}\right)\,dx$$
$$= -atan\left(\frac{2\sqrt{c}\,\sqrt{c\,x^2+bx+1}\,i+bi+2cix}{2\sqrt{c}-b}\right)i$$
$$+\frac{\log\left(4\sqrt{c}\,\sqrt{c\,x^2+bx+1}\,b+8\sqrt{c}\,\sqrt{c\,x^2+bx+1}\,cx+4\sqrt{c}\,b+8bcx+8c^2x^2\right)}{2}$$
$$-\log\left(2\sqrt{c}\,\sqrt{c\,x^2+bx+1}-2\sqrt{c}+2cx\right)+\log(x)$$

input
```
int(1/x-1/x/(c*x^2+b*x+1)^(1/2),x)
```

output
```
( - 2*atan((2*sqrt(c)*sqrt(b*x + c*x**2 + 1)*i + b*i + 2*c*i*x)/(2*sqrt(c)
 - b))*i + log(4*sqrt(c)*sqrt(b*x + c*x**2 + 1)*b + 8*sqrt(c)*sqrt(b*x + c
*x**2 + 1)*c*x + 4*sqrt(c)*b + 8*b*c*x + 8*c**2*x**2) - 2*log(2*sqrt(c)*sq
rt(b*x + c*x**2 + 1) - 2*sqrt(c) + 2*c*x) + 2*log(x))/2
```

3.308 $\int \frac{x}{(5-4x-x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 23

$$\int \frac{x}{(5-4x-x^2)^{3/2}}\,dx=\frac{5-2x}{9\sqrt{5-4x-x^2}}$$

output
```
1/9*(5-2*x)/(-x^2-4*x+5)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{x}{(5-4x-x^2)^{3/2}}\,dx=\frac{(-5+2x)\sqrt{5-4x-x^2}}{9(-1+x)(5+x)}$$

input
```
Integrate[x/(5 - 4*x - x^2)^(3/2),x]
```

output
```
((-5 + 2*x)*Sqrt[5 - 4*x - x^2])/(9*(-1 + x)*(5 + x))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(-x^2-4x+5)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1158}$$

$$\frac{5-2x}{9\sqrt{-x^2-4x+5}}$$

input
```
Int[x/(5 - 4*x - x^2)^(3/2),x]
```

output
```
(5 - 2*x)/(9*Sqrt[5 - 4*x - x^2])
```

Defintions of rubi rules used

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
risch	$-\frac{-5+2x}{9\sqrt{-x^{2}-4x+5}}$	20
gosper	$\frac{(x+5)(x-1)(-5+2x)}{9(-x^{2}-4x+5)^{\frac{3}{2}}}$	26
orering	$\frac{(x+5)(x-1)(-5+2x)}{9(-x^{2}-4x+5)^{\frac{3}{2}}}$	26
trager	$\frac{(-5+2x)\sqrt{-x^{2}-4x+5}}{9x^{2}+36x-45}$	30
default	$\frac{1}{\sqrt{-x^{2}-4x+5}}+\frac{-2x-4}{9\sqrt{-x^{2}-4x+5}}$	33

input
```
int(x/(-x^2-4*x+5)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*(-5+2*x)/(-x^2-4*x+5)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{x}{(5-4x-x^2)^{3/2}}\,dx=\frac{\sqrt{-x^2-4\,x+5}(2\,x-5)}{9\,(x^2+4\,x-5)}$$

input
```
integrate(x/(-x^2-4*x+5)^(3/2),x, algorithm="fricas")
```

output
```
1/9*sqrt(-x^2 - 4*x + 5)*(2*x - 5)/(x^2 + 4*x - 5)
```

Sympy [F]

$$\int \frac{x}{(5-4x-x^2)^{3/2}}\,dx = \int \frac{x}{(-(x-1)(x+5))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(-x**2-4*x+5)**(3/2),x)
```

output
```
Integral(x/(-(x - 1)*(x + 5))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \frac{x}{(5-4x-x^2)^{3/2}}\,dx = -\frac{2\,x}{9\,\sqrt{-x^2-4\,x+5}} + \frac{5}{9\,\sqrt{-x^2-4\,x+5}}$$

input
```
integrate(x/(-x^2-4*x+5)^(3/2),x, algorithm="maxima")
```

output
```
-2/9*x/sqrt(-x^2 - 4*x + 5) + 5/9/sqrt(-x^2 - 4*x + 5)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{x}{(5-4x-x^2)^{3/2}}\,dx = \frac{\sqrt{-x^2-4\,x+5}(2\,x-5)}{9\,(x^2+4\,x-5)}$$

input
```
integrate(x/(-x^2-4*x+5)^(3/2),x, algorithm="giac")
```

output
```
1/9*sqrt(-x^2 - 4*x + 5)*(2*x - 5)/(x^2 + 4*x - 5)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x}{(5-4x-x^2)^{3/2}}\,dx = -\frac{2\,x-5}{9\,\sqrt{-x^2-4\,x+5}}$$

input
```
int(x/(5 - x^2 - 4*x)^(3/2),x)
```

output
```
-(2*x - 5)/(9*(5 - x^2 - 4*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x}{(5-4x-x^2)^{3/2}}\,dx = \frac{-2x+5}{9\sqrt{-x^2-4x+5}}$$

input
```
int(x/(-x^2-4*x+5)^(3/2),x)
```

output
```
( - 2*x + 5)/(9*sqrt( - x**2 - 4*x + 5))
```

3.309 $\int (dx)^{5/2}\sqrt{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 575

$$\int (dx)^{5/2}\sqrt{a+bx+cx^2}\,dx = \frac{2b(8b^2-27ac)\,d^2\sqrt{dx}\sqrt{a+bx+cx^2}}{315c^3}$$
$$-\frac{2d(dx)^{3/2}\left(2b^2+7ac+10bcx\right)\sqrt{a+bx+cx^2}}{105c^2}+\frac{2d(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9c}$$
$$-\frac{\sqrt{2}(8b^4-36ab^2c+21a^2c^2)\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)d^{5/2}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\Big(\text{ar}}{315c^{9/2}\sqrt{a+x(b+cx)}}$$
$$+\frac{\sqrt{2}\sqrt{-b+\sqrt{b^2-4ac}}\left(8b^5-44ab^3c+48a^2bc^2+\sqrt{b^2-4ac}(8b^4-36ab^2c+21a^2c^2)\right)d^{5/2}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}}{315c^{9/2}\sqrt{a+x(b+cx)}}$$

output
```
2/315*b*(-27*a*c+8*b^2)*d^2*(d*x)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^3-2/105*d*(d
*x)^(3/2)*(10*b*c*x+7*a*c+2*b^2)*(c*x^2+b*x+a)^(1/2)/c^2+2/9*d*(d*x)^(3/2)
*(c*x^2+b*x+a)^(3/2)/c-1/315*2^(1/2)*(21*a^2*c^2-36*a*b^2*c+8*b^4)*(-b+(-4
*a*c+b^2)^(1/2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*d^(5/2)*(1+2*c*x/(b-(-4*a*c+
b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2^(1/2
)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^
2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))/c^(9/2)/(a+x*(c*x+b))^(1/2)+1/315
*2^(1/2)*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(8*b^5-44*a*b^3*c+48*a^2*b*c^2+(-4*
a*c+b^2)^(1/2)*(21*a^2*c^2-36*a*b^2*c+8*b^4))*d^(5/2)*(1+2*c*x/(b-(-4*a*c+
b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(2^(1/2
)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^
2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))/c^(9/2)/(a+x*(c*x+b))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.67 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.04

$$\int (dx)^{5/2}\sqrt{a+bx+cx^2}\,dx = \frac{(dx)^{5/2}\left(\frac{2\sqrt{x}(a+x(b+cx))\left(8b^3-6b^2cx+bc\left(-27a+5cx^2\right)+7c^2x\left(2a+5cx^2\right)\right)}{c^3}+\frac{x\left(-\frac{4\left(8b^4-36ab^2c+\cdots\right.}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}$$

input
```
Integrate[(d*x)^(5/2)*Sqrt[a + b*x + c*x^2],x]
```

output
```
((d*x)^(5/2)*((2*Sqrt[x]*(a + x*(b + c*x))*(8*b^3 - 6*b^2*c*x + b*c*(-27*a
 + 5*c*x^2) + 7*c^2*x*(2*a + 5*c*x^2)))/c^3 + (x*((-4*(8*b^4 - 36*a*b^2*c
+ 21*a^2*c^2)*(a + x*(b + c*x)))/x^(3/2) + (I*(8*b^4 - 36*a*b^2*c + 21*a^2
*c^2)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[b^2 - 4*a*c])*x)]
*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*Ellip
ticE[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sq
rt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]
 - (I*(-8*b^5 + 44*a*b^3*c - 48*a^2*b*c^2 + 8*b^4*Sqrt[b^2 - 4*a*c] - 36*a
*b^2*c*Sqrt[b^2 - 4*a*c] + 21*a^2*c^2*Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4*a)/((
b + Sqrt[b^2 - 4*a*c])*x)]*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - S
qrt[b^2 - 4*a*c]*x)]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4
*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[
a/(b + Sqrt[b^2 - 4*a*c])]))/c^4))/(315*x^(5/2)*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 485, normalized size of antiderivative = 0.84, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {1166, 27, 1236, 27, 1231, 27, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{5/2}\sqrt{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{2\int -\frac{3}{2}d^2\sqrt{dx}(a+2bx)\sqrt{cx^2+bx+a}dx}{9c}+\frac{2d(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9c}$$

$$\Big\downarrow \text{27}$$

$$\frac{2d(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9c}-\frac{d^2\int \sqrt{dx}(a+2bx)\sqrt{cx^2+bx+a}dx}{3c}$$

$$\Big\downarrow \text{1236}$$

$$\frac{2d(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9c}-\frac{d^2\left(\frac{2\int -\frac{d\left(2ab+\left(8b^2-7ac\right)x\right)\sqrt{cx^2+bx+a}}{2\sqrt{dx}}dx}{7c}+\frac{4b\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}\right)}{3c}$$

$$\Big\downarrow \text{27}$$

$$\frac{2d(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9c}-\frac{d^2\left(\frac{4b\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{d\int\frac{\left(2ab+\left(8b^2-7ac\right)x\right)\sqrt{cx^2+bx+a}}{\sqrt{dx}}dx}{7c}\right)}{3c}$$

$$\Big\downarrow \text{1231}$$

$$\frac{2d(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9c}-\frac{d^2\left(\frac{4b\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{d\left(\frac{2\sqrt{dx}\left(3cx\left(8b^2-7ac\right)+b\left(3ac+8b^2\right)\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\int\frac{d^2\left(ab\left(8b^2-27ac\right)+2\left(8b^4-36acb^2+21a^2c^2\right)x\right)}{2\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{15cd^2}\right)}{7c}\right)}{3c}$$

$$\Big\downarrow \text{27}$$

$$\frac{2d(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9c}-\frac{d^2\left(\frac{4b\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{d\left(\frac{2\sqrt{dx}\left(3cx\left(8b^2-7ac\right)+b\left(3ac+8b^2\right)\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{\int\frac{ab\left(8b^2-27ac\right)+2\left(8b^4-36acb^2+21a^2c^2\right)x}{\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{15c}\right)}{7c}\right)}{3c}$$

$$\Big\downarrow \text{1241}$$

$$\frac{2d(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9c}-\frac{d^2\left(\frac{4b\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{d\left(\frac{2\sqrt{dx}\left(3cx\left(8b^2-7ac\right)+b\left(3ac+8b^2\right)\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{\sqrt{x}\int\frac{ab\left(8b^2-27ac\right)+2\left(8b^4-36acb^2+21a^2c^2\right)x}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{15c\sqrt{dx}}\right)}{7c}\right)}{3c}$$

$$\Big\downarrow \text{1240}$$

$$\frac{2d(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9c}-\frac{d^2\left(\frac{4b\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{d\left(\frac{2\sqrt{dx}\left(3cx\left(8b^2-7ac\right)+b\left(3ac+8b^2\right)\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\int\frac{ab\left(8b^2-27ac\right)+2\left(8b^4-36acb^2+21a^2c^2\right)x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{15c\sqrt{dx}}\right)}{7c}\right)}{3c}$$

$$\Big\downarrow \text{ 1511}$$

$$\frac{2d(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9c}-\frac{d^2\left(\frac{4b\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{d\left(\frac{2\sqrt{dx}\left(3cx\left(8b^2-7ac\right)+b\left(3ac+8b^2\right)\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\left(\sqrt{a}\left(\frac{2\left(21a^2c^2-36ab^2c+8b^4\right)}{\sqrt{c}}+\sqrt{a}b\left(8b^2-27ac\right)\right)\int\frac{1}{\sqrt{cx^2+b}}\right.}{15c\sqrt{d}}}{7c}}{3c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2d(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9c}-\frac{d^2\left(\frac{4b\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{d\left(\frac{2\sqrt{dx}\left(3cx\left(8b^2-7ac\right)+b\left(3ac+8b^2\right)\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\left(\sqrt{a}\left(\frac{2\left(21a^2c^2-36ab^2c+8b^4\right)}{\sqrt{c}}+\sqrt{a}b\left(8b^2-27ac\right)\right)\int\frac{1}{\sqrt{cx^2+b}}\right.}{15c\sqrt{dx}}}{7c}}{3c}$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{2d(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9c}-$$

$$\frac{d^2\left(\frac{4b\sqrt{dx}(a+bx+cx^2)^{3/2}}{7c}-\frac{d\left(\frac{2\sqrt{dx}\left(3cx\left(8b^2-7ac\right)+b\left(3ac+8b^2\right)\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\frac{2\left(21a^2c^2-36ab^2c+8b^4\right)}{\sqrt{c}}+\sqrt{a}b\left(8b^2-27ac\right)\right)(\sqrt{a}+\sqrt{c}}{2\sqrt[4]{c}\surd}\right.}{}\right.}{7c}\right.}{3c}$$

$\downarrow$ 1509

$$\frac{2d(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9c}-$$

$$d^2\left(\frac{4b\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{\left(d\left(\frac{2\sqrt{dx}\left(3cx\left(8b^2-7ac\right)+b\left(3ac+8b^2\right)\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\frac{2\left(21a^2c^2-36ab^2c+8b^4\right)}{\sqrt{c}}+\sqrt{a}b\left(8b^2-27ac\right)\right)\left(\sqrt{a}+\sqrt{c}\right.}{2\sqrt[4]{c}\sqrt{}}\right.}{}\right.\right.}{}\right.$$

input
```
Int[(d*x)^(5/2)*Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*d*(d*x)^(3/2)*(a + b*x + c*x^2)^(3/2))/(9*c) - (d^2*((4*b*Sqrt[d*x]*(a
+ b*x + c*x^2)^(3/2))/(7*c) - (d*((2*Sqrt[d*x]*(b*(8*b^2 + 3*a*c) + 3*c*(8
*b^2 - 7*a*c)*x)*Sqrt[a + b*x + c*x^2])/(15*c*d) - (2*Sqrt[x]*((-2*(8*b^4
- 36*a*b^2*c + 21*a^2*c^2)*(-((Sqrt[x]*Sqrt[a + b*x + c*x^2])/(Sqrt[a] + S
qrt[c]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a
] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(S
qrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x + c*x^2])))/Sqrt[c] + (a^(1/4)*
(Sqrt[a]*b*(8*b^2 - 27*a*c) + (2*(8*b^4 - 36*a*b^2*c + 21*a^2*c^2))/Sqrt[c
])*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*E
llipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]
)/(2*c^(1/4)*Sqrt[a + b*x + c*x^2])))/(15*c*Sqrt[d*x])))/(7*c)))/(3*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 989 vs. $2(479) = 958$.

Time = 2.61 (sec) , antiderivative size = 990, normalized size of antiderivative = 1.72

method	result
elliptic	$\frac{\sqrt{dx}\,\sqrt{dx(c\,x^2+bx+a)}\left(\frac{2d^2x^3\sqrt{cd\,x^3+bd\,x^2+adx}}{9}+\frac{2d^2b\,x^2\sqrt{cd\,x^3+bd\,x^2+adx}}{63c}+\frac{2\left(\frac{2a\,d^3}{9}-\frac{2d^3b^2}{21c}\right)x\sqrt{cd\,x^3+bd\,x^2+adx}}{5cd}+\frac{2\left(-\frac{5d^3ba}{63c}-\frac{4(\ldots}{\ldots}\right.}{\ldots}\right.}{\ldots}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((d*x)^(5/2)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/x*(d*x)^(1/2)/(c*x^2+b*x+a)^(1/2)*(d*x*(c*x^2+b*x+a))^(1/2)*(2/9*d^2*x
^3*(c*d*x^3+b*d*x^2+a*d*x)^(1/2)+2/63*d^2*b/c*x^2*(c*d*x^3+b*d*x^2+a*d*x)^
(1/2)+2/5*(2/9*a*d^3-2/21*d^3*b^2/c)/c/d*x*(c*d*x^3+b*d*x^2+a*d*x)^(1/2)+2
/3*(-5/63*d^3*b/c*a-4/5*(2/9*a*d^3-2/21*d^3*b^2/c)/c*b)/c/d*(c*d*x^3+b*d*x
^2+a*d*x)^(1/2)-1/3*(-5/63*d^3*b/c*a-4/5*(2/9*a*d^3-2/21*d^3*b^2/c)/c*b)/c
^2*a*(b+(-4*a*c+b^2)^(1/2))*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-
4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a
*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)*EllipticF(2^(1/2)*((x+
1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(
-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)
^(1/2))))^(1/2))+(-3/5*(2/9*a*d^3-2/21*d^3*b^2/c)/c*a-2/3*(-5/63*d^3*b/c*a
-4/5*(2/9*a*d^3-2/21*d^3*b^2/c)/c*b)/c*b)*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)
*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2
/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a
*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*
x^2+a*d*x)^(1/2)*((-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1
/2)))*EllipticE(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^
(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/
2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.46

$$\int (dx)^{5/2}\sqrt{a+bx+cx^2}\,dx = \frac{2\left(\left(16\,b^5-96\,ab^3c+123\,a^2bc^2\right)\sqrt{cd}d^2\text{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3}{27}\right.\right.}{}$$

input
```
integrate((d*x)^(5/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/945*((16*b^5 - 96*a*b^3*c + 123*a^2*b*c^2)*sqrt(c*d)*d^2*weierstrassPInv
erse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c
) + 6*(8*b^4*c - 36*a*b^2*c^2 + 21*a^2*c^3)*sqrt(c*d)*d^2*weierstrassZeta(
4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, weierstrassPInverse(4/
3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c)) + 3*
(35*c^5*d^2*x^3 + 5*b*c^4*d^2*x^2 - 2*(3*b^2*c^3 - 7*a*c^4)*d^2*x + (8*b^3
*c^2 - 27*a*b*c^3)*d^2)*sqrt(c*x^2 + b*x + a)*sqrt(d*x))/c^5
```

Sympy [F]

$$\int (dx)^{5/2}\sqrt{a+bx+cx^2}\,dx=\int (dx)^{\frac{5}{2}}\sqrt{a+bx+cx^2}\,dx$$

input
```
integrate((d*x)**(5/2)*(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d*x)**(5/2)*sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int (dx)^{5/2}\sqrt{a+bx+cx^2}\,dx=\int \sqrt{cx^2+bx+a}(dx)^{\frac{5}{2}}\,dx$$

input
```
integrate((d*x)^(5/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(d*x)^(5/2), x)
```

Giac [F]

$$\int (dx)^{5/2}\sqrt{a+bx+cx^2}\,dx=\int \sqrt{cx^2+bx+a}(dx)^{\frac{5}{2}}\,dx$$

input
```
integrate((d*x)^(5/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(d*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{5/2}\sqrt{a+bx+cx^2}\,dx = \int (d\,x)^{5/2}\,\sqrt{c\,x^2+b\,x+a}\,dx$$

input
```
int((d*x)^(5/2)*(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((d*x)^(5/2)*(a + b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int (dx)^{5/2}\sqrt{a+bx+cx^2}\,dx = \frac{\sqrt{d}\,d^2\Big(-42\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,a^2c+18\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,a\,b^2+28\sqrt{x}\,\sqrt{c\,x}$$

input
```
int((d*x)^(5/2)*(c*x^2+b*x+a)^(1/2),x)
```

output
```
(sqrt(d)*d**2*( - 42*sqrt(x)*sqrt(a + b*x + c*x**2)*a**2*c + 18*sqrt(x)*sq
rt(a + b*x + c*x**2)*a*b**2 + 28*sqrt(x)*sqrt(a + b*x + c*x**2)*a*b*c*x -
12*sqrt(x)*sqrt(a + b*x + c*x**2)*b**3*x + 10*sqrt(x)*sqrt(a + b*x + c*x**
2)*b**2*c*x**2 + 70*sqrt(x)*sqrt(a + b*x + c*x**2)*b*c**2*x**3 + 63*int((s
qrt(x)*sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*x**2),x)*a**2*c**2 - 108*int
((sqrt(x)*sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*x**2),x)*a*b**2*c + 24*in
t((sqrt(x)*sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*x**2),x)*b**4 + 21*int((
sqrt(x)*sqrt(a + b*x + c*x**2))/(a*x + b*x**2 + c*x**3),x)*a**3*c - 9*int(
(sqrt(x)*sqrt(a + b*x + c*x**2))/(a*x + b*x**2 + c*x**3),x)*a**2*b**2))/(3
15*b*c**2)
```

3.310 $\int (dx)^{3/2}\sqrt{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 512

$$\int (dx)^{3/2}\sqrt{a+bx+cx^2}\,dx = -\frac{2d\sqrt{dx}(4b^2+5ac+12bcx)\sqrt{a+bx+cx^2}}{105c^2}+\frac{2d\sqrt{dx}(a+bx+cx^2)^{3/2}}{7c}$$
$$+\frac{b(8b^2-29ac)\sqrt{-b+\sqrt{b^2-4ac}}(b+\sqrt{b^2-4ac})\,d^{3/2}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\cdots}{\sqrt{-b+\sqrt{b^2}\cdots}}\right.\right.\cdots}{105\sqrt{2}c^{7/2}\sqrt{a+x(b+cx)}}$$
$$-\frac{\sqrt{-b+\sqrt{b^2-4ac}}(8b^4-37ab^2c+20a^2c^2+b(8b^2-29ac)\sqrt{b^2-4ac})\,d^{3/2}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-\cdots}}}\cdots}{105\sqrt{2}c^{7/2}\sqrt{a+x(b+cx)}}$$

output
```
-2/105*d*(d*x)^(1/2)*(12*b*c*x+5*a*c+4*b^2)*(c*x^2+b*x+a)^(1/2)/c^2+2/7*d*
(d*x)^(1/2)*(c*x^2+b*x+a)^(3/2)/c+1/210*b*(-29*a*c+8*b^2)*(-b+(-4*a*c+b^2)
^(1/2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*d^(3/2)*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2
)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2^(1/2)*c^(1/2)
*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))
/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/c^(7/2)/(a+x*(c*x+b))^(1/2)-1/210*
(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(8*b^4-37*a*b^2*c+20*a^2*c^2+b*(-29*a*c+8*b^
2)*(-4*a*c+b^2)^(1/2))*d^(3/2)*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2
*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(2^(1/2)*c^(1/2)*(d*x)^(1/2)/(
-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^
2)^(1/2)))^(1/2))*2^(1/2)/c^(7/2)/(a+x*(c*x+b))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.08 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.05

$$\int (dx)^{3/2}\sqrt{a+bx+cx^2}\,dx = \frac{(dx)^{3/2}\left(\frac{4(a+x(b+cx))(-4b^2+3bcx+5c(2a+3cx^2))}{c^2x}+\frac{4b(8b^2-29ac)\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}(a+x(b+cx))+}{}\right.}{}$$

input
```
Integrate[(d*x)^(3/2)*Sqrt[a + b*x + c*x^2],x]
```

output
```
((d*x)^(3/2)*((4*(a + x*(b + c*x))*(-4*b^2 + 3*b*c*x + 5*c*(2*a + 3*c*x^2)
))/(c^2*x) + (4*b*(8*b^2 - 29*a*c)*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*(a + x*
(b + c*x)) + I*b*(8*b^2 - 29*a*c)*(b - Sqrt[b^2 - 4*a*c])*Sqrt[1 + (2*a)/(
(b + Sqrt[b^2 - 4*a*c])*x)]*x^(3/2)*Sqrt[(4*a + 2*b*x - 2*Sqrt[b^2 - 4*a*c
]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b +
 Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*
a*c])] - I*(8*b^4 - 37*a*b^2*c + 20*a^2*c^2 - 8*b^3*Sqrt[b^2 - 4*a*c] + 29
*a*b*c*Sqrt[b^2 - 4*a*c])*Sqrt[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(3
/2)*Sqrt[(4*a + 2*b*x - 2*Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)
]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]],
(b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(c^3*Sqrt[a/(b + Sqrt[b^
2 - 4*a*c])]*x^2)))/(210*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 414, normalized size of antiderivative = 0.81, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {1166, 27, 1231, 27, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2}\sqrt{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{2\int -\frac{d^2(a+4bx)\sqrt{cx^2+bx+a}}{2\sqrt{dx}}dx}{7c}+\frac{2d\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}$$

$$\Big\downarrow \text{27}$$

$$\frac{2d\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{d^2\int \frac{(a+4bx)\sqrt{cx^2+bx+a}}{\sqrt{dx}}dx}{7c}$$

$$\Big\downarrow \text{1231}$$

$$\frac{2d\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{d^2\left(\frac{2\sqrt{dx}(5ac+4b^2+12bcx)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\int \frac{d^2\left(2a\left(2b^2-5ac\right)+b\left(8b^2-29ac\right)x\right)}{2\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{15cd^2}\right)}{7c}$$

$$\Big\downarrow \text{27}$$

$$\frac{2d\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{d^2\left(\frac{2\sqrt{dx}(5ac+4b^2+12bcx)\sqrt{a+bx+cx^2}}{15cd}-\frac{\int \frac{2a\left(2b^2-5ac\right)+b\left(8b^2-29ac\right)x}{\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{15c}\right)}{7c}$$

$$\Big\downarrow \text{1241}$$

$$\frac{2d\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{d^2\left(\frac{2\sqrt{dx}(5ac+4b^2+12bcx)\sqrt{a+bx+cx^2}}{15cd}-\frac{\sqrt{x}\int \frac{2a\left(2b^2-5ac\right)+b\left(8b^2-29ac\right)x}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{15c\sqrt{dx}}\right)}{7c}$$

$$\Big\downarrow \text{1240}$$

$$\frac{2d\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{d^2\left(\frac{2\sqrt{dx}(5ac+4b^2+12bcx)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\int \frac{2a\left(2b^2-5ac\right)+b\left(8b^2-29ac\right)x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{15c\sqrt{dx}}\right)}{7c}$$

$$\Big\downarrow \text{1511}$$

$$\frac{\frac{2d\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}-d^2\left(\frac{2\sqrt{dx}\left(5ac+4b^2+12bcx\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\left(\sqrt{a}\left(\frac{b\left(8b^2-29ac\right)}{\sqrt{c}}+2\sqrt{a}\left(2b^2-5ac\right)\right)\int\frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{\sqrt{a}b\left(8b^2-29ac\right)\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\cdots}{\sqrt{c}}\cdots}{15c\sqrt{dx}}\cdots\right.}{7c}$$

$\downarrow$ 27

$$\frac{\frac{2d\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}-d^2\left(\frac{2\sqrt{dx}\left(5ac+4b^2+12bcx\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\left(\sqrt{a}\left(\frac{b\left(8b^2-29ac\right)}{\sqrt{c}}+2\sqrt{a}\left(2b^2-5ac\right)\right)\int\frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{b\left(8b^2-29ac\right)\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{15c\sqrt{dx}}\cdots\right.}{7c}$$

$\downarrow$ 1416

$$\frac{\frac{2d\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}-d^2\left(\frac{2\sqrt{dx}\left(5ac+4b^2+12bcx\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\frac{b\left(8b^2-29ac\right)}{\sqrt{c}}+2\sqrt{a}\left(2b^2-5ac\right)\right)\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\cdots\right.}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}\cdots\right.}{15c\sqrt{dx}}\cdots\right.}{7c}$$

$\downarrow$ 1509

$$\frac{2d\sqrt{dx}(a+bx+cx^2)^{3/2}}{7c}-\frac{d^2\left(\frac{2\sqrt{dx}(5ac+4b^2+12bcx)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\frac{b(8b^2-29ac)}{\sqrt{c}}+2\sqrt{a}(2b^2-5ac)\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(\right.\right.}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}\right.}{}\right)}{7c}$$

input
```
Int[(d*x)^(3/2)*Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*d*Sqrt[d*x]*(a + b*x + c*x^2)^(3/2))/(7*c) - (d^2*((2*Sqrt[d*x]*(4*b^2
+ 5*a*c + 12*b*c*x)*Sqrt[a + b*x + c*x^2])/(15*c*d) - (2*Sqrt[x]*(-((b*(8*
b^2 - 29*a*c)*(-((Sqrt[x]*Sqrt[a + b*x + c*x^2])/(Sqrt[a] + Sqrt[c]*x)) +
(a^(1/4)*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x
)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c
]))/4])/(c^(1/4)*Sqrt[a + b*x + c*x^2])))/Sqrt[c]) + (a^(1/4)*((b*(8*b^2 -
 29*a*c))/Sqrt[c] + 2*Sqrt[a]*(2*b^2 - 5*a*c))*(Sqrt[a] + Sqrt[c]*x)*Sqrt[
(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqr
t[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x + c*
x^2])))/(15*c*Sqrt[d*x])))/(7*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 872 vs. 2(422) = 844.

Time = 1.85 (sec) , antiderivative size = 873, normalized size of antiderivative = 1.71

method	result
elliptic	$\frac{\sqrt{dx}\,\sqrt{dx(c\,x^2+bx+a)}\left(\frac{2d\,x^2\sqrt{cd\,x^3+bd\,x^2+adx}}{7}+\frac{2dbx\sqrt{cd\,x^3+bd\,x^2+adx}}{35c}+\frac{2\left(\frac{2a\,d^2}{7}-\frac{4d^2b^2}{35c}\right)\sqrt{cd\,x^3+bd\,x^2+adx}}{3cd}-\frac{\left(\frac{2a\,d^2}{7}-\frac{4d^2b^2}{35c}\right)a\left(\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((d*x)^(3/2)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/x*(d*x)^(1/2)/(c*x^2+b*x+a)^(1/2)*(d*x*(c*x^2+b*x+a))^(1/2)*(2/7*d*x^2
*(c*d*x^3+b*d*x^2+a*d*x)^(1/2)+2/35/c*d*b*x*(c*d*x^3+b*d*x^2+a*d*x)^(1/2)+
2/3*(2/7*a*d^2-4/35/c*d^2*b^2)/c/d*(c*d*x^3+b*d*x^2+a*d*x)^(1/2)-1/3*(2/7*
a*d^2-4/35/c*d^2*b^2)/c^2*a*(b+(-4*a*c+b^2)^(1/2))*2^(1/2)*((x+1/2*(b+(-4*
a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b
^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))
^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)
*EllipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c
-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+(-3/35/c*d^2*b*a-2/3*(2/7*a*d^2-4/
35/c*d^2*b^2)/c*b)*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2
)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/
2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*
(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)*((-1/2
*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(2^(1/2)
*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2
*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c
+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(2^(1/2)*((x+
1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(
-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.44

$$\int (dx)^{3/2}\sqrt{a+bx+cx^2}\,dx =$$

$$-\frac{2\left(\left(8\,b^4-41\,ab^2c+30\,a^2c^2\right)\sqrt{cd}d\text{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\frac{3\,cx+b}{3\,c}\right)+3\,(8\,b^3c-29\,abc\right.}{}$$

input
```
integrate((d*x)^(3/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/315*((8*b^4 - 41*a*b^2*c + 30*a^2*c^2)*sqrt(c*d)*d*weierstrassPInverse(
4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c) + 3
*(8*b^3*c - 29*a*b*c^2)*sqrt(c*d)*d*weierstrassZeta(4/3*(b^2 - 3*a*c)/c^2,
 -4/27*(2*b^3 - 9*a*b*c)/c^3, weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -
4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c)) - 3*(15*c^4*d*x^2 + 3*b*c^
3*d*x - 2*(2*b^2*c^2 - 5*a*c^3)*d)*sqrt(c*x^2 + b*x + a)*sqrt(d*x))/c^4
```

Sympy [F]

$$\int (dx)^{3/2}\sqrt{a+bx+cx^2}\,dx=\int (dx)^{\frac{3}{2}}\sqrt{a+bx+cx^2}\,dx$$

input
```
integrate((d*x)**(3/2)*(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d*x)**(3/2)*sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int (dx)^{3/2}\sqrt{a+bx+cx^2}\,dx=\int \sqrt{cx^2+bx+a}(dx)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*x)^(3/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(d*x)^(3/2), x)
```

Giac [F]

$$\int (dx)^{3/2}\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(dx)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*x)^(3/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(d*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2}\sqrt{a+bx+cx^2}\,dx = \int (d\,x)^{3/2}\,\sqrt{c\,x^2+b\,x+a}\,dx$$

input
```
int((d*x)^(3/2)*(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((d*x)^(3/2)*(a + b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int (dx)^{3/2}\sqrt{a+bx+cx^2}\,dx = \frac{\sqrt{d}\,d\Big(-6\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,a+4\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,bx+20\sqrt{x}\,\sqrt{c\,x^2+bx}$$

input
```
int((d*x)^(3/2)*(c*x^2+b*x+a)^(1/2),x)
```

output
```
(sqrt(d)*d*( - 6*sqrt(x)*sqrt(a + b*x + c*x**2)*a + 4*sqrt(x)*sqrt(a + b*x
 + c*x**2)*b*x + 20*sqrt(x)*sqrt(a + b*x + c*x**2)*c*x**2 + 29*int((sqrt(x
)*sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*x**2),x)*a*c - 8*int((sqrt(x)*sqr
t(a + b*x + c*x**2)*x)/(a + b*x + c*x**2),x)*b**2 + 3*int((sqrt(x)*sqrt(a
+ b*x + c*x**2))/(a*x + b*x**2 + c*x**3),x)*a**2))/(70*c)
```

3.311 $\int \sqrt{dx}\sqrt{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 478

$$\int \sqrt{dx}\sqrt{a+bx+cx^2}\,dx = \frac{2b\sqrt{dx}\sqrt{a+bx+cx^2}}{15c}+\frac{2(dx)^{3/2}\sqrt{a+bx+cx^2}}{5d}$$
$$-\frac{\sqrt{2}(b^2-3ac)\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\sqrt{d}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\left(\arcsin\left(\frac{\sqrt{2}}{\sqrt{-b+\sqrt{}}}\right.\right.}{15c^{5/2}\sqrt{a+x(b+cx)}}$$
$$+\frac{\sqrt{2}\sqrt{-b+\sqrt{b^2-4ac}}\left(b^3-4abc+\sqrt{b^2-4ac}(b^2-3ac)\right)\sqrt{d}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\,\text{EllipticF}}{15c^{5/2}\sqrt{a+x(b+cx)}}$$

output
```
2/15*b*(d*x)^(1/2)*(c*x^2+b*x+a)^(1/2)/c+2/5*(d*x)^(3/2)*(c*x^2+b*x+a)^(1/
2)/d-1/15*2^(1/2)*(-3*a*c+b^2)*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(b+(-4*a*c+b^
2)^(1/2))*d^(1/2)*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a
*c+b^2)^(1/2)))^(1/2)*EllipticE(2^(1/2)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^
2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1
/2))/c^(5/2)/(a+x*(c*x+b))^(1/2)+1/15*2^(1/2)*(-b+(-4*a*c+b^2)^(1/2))^(1/2
)*(b^3-4*a*b*c+(-4*a*c+b^2)^(1/2)*(-3*a*c+b^2))*d^(1/2)*(1+2*c*x/(b-(-4*a*
c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(2^(1
/2)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+
b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))/c^(5/2)/(a+x*(c*x+b))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.89 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.03

$$\int \sqrt{dx}\sqrt{a+bx+cx^2}\,dx$$

$$= \frac{d\left(-4(b^2-3ac)\left(a+x(b+cx)\right)+2cx(b+3cx)(a+x(b+cx))+\frac{i(b^2-3ac)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{2+\frac{4a}{\left(b+\sqrt{b^2-4ac}\right)x}}x^{3/}}{}\right.}{}$$

input
```
Integrate[Sqrt[d*x]*Sqrt[a + b*x + c*x^2],x]
```

output
```
(d*(-4*(b^2 - 3*a*c)*(a + x*(b + c*x)) + 2*c*x*(b + 3*c*x)*(a + x*(b + c*x
)) + (I*(b^2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[b
^2 - 4*a*c])*x)]*x^(3/2)*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqr
t[b^2 - 4*a*c]*x)]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a
*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a/
(b + Sqrt[b^2 - 4*a*c])] + (I*(b^3 - 4*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 3*a
*c*Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(3/2)*
Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*Ellipt
icF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqr
t[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])
)/(15*c^2*Sqrt[d*x]*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.79, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1162, 25, 27, 1236, 27, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\sqrt{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{1162}$$

$$\frac{2(dx)^{3/2}\sqrt{a+bx+cx^2}}{5d}-\frac{\int -\frac{d\sqrt{dx}(2a+bx)}{\sqrt{cx^2+bx+a}}dx}{5d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{d\sqrt{dx}(2a+bx)}{\sqrt{cx^2+bx+a}}dx}{5d}+\frac{2(dx)^{3/2}\sqrt{a+bx+cx^2}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{\sqrt{dx}(2a+bx)}{\sqrt{cx^2+bx+a}}dx+\frac{2(dx)^{3/2}\sqrt{a+bx+cx^2}}{5d}$$

$$\Big\downarrow \text{1236}$$

$$\frac{1}{5}\left(\frac{2\int -\frac{d(ab+2(b^2-3ac)x)}{2\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{3c}+\frac{2b\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}\right)+\frac{2(dx)^{3/2}\sqrt{a+bx+cx^2}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\frac{2b\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{d\int \frac{ab+2(b^2-3ac)x}{\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{3c}\right)+\frac{2(dx)^{3/2}\sqrt{a+bx+cx^2}}{5d}$$

$$\Big\downarrow \text{1241}$$

$$\frac{1}{5}\left(\frac{2b\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{d\sqrt{x}\int \frac{ab+2(b^2-3ac)x}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{3c\sqrt{dx}}\right)+\frac{2(dx)^{3/2}\sqrt{a+bx+cx^2}}{5d}$$

$$\Big\downarrow \text{1240}$$

$$\frac{1}{5}\left(\frac{2b\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{2d\sqrt{x}\int \frac{ab+2(b^2-3ac)x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{3c\sqrt{dx}}\right)+\frac{2(dx)^{3/2}\sqrt{a+bx+cx^2}}{5d}$$

$$\Big\downarrow \text{1511}$$

$$\frac{1}{5}\left(\frac{2b\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{2d\sqrt{x}\left(\sqrt{a}\left(\frac{2(b^2-3ac)}{\sqrt{c}}+\sqrt{a}b\right)\int\frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{2\sqrt{a}(b^2-3ac)\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{3c\sqrt{dx}}\right)+$$

$$\frac{2(dx)^{3/2}\sqrt{a+bx+cx^2}}{5d}$$

$\downarrow$ 27

$$\frac{1}{5}\left(\frac{2b\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{2d\sqrt{x}\left(\sqrt{a}\left(\frac{2(b^2-3ac)}{\sqrt{c}}+\sqrt{a}b\right)\int\frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{2(b^2-3ac)\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{3c\sqrt{dx}}\right)+$$

$$\frac{2(dx)^{3/2}\sqrt{a+bx+cx^2}}{5d}$$

$\downarrow$ 1416

$$\frac{1}{5}\left(\frac{2b\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{2d\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\frac{2(b^2-3ac)}{\sqrt{c}}+\sqrt{a}b\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}\right.}{3c\sqrt{dx}}\right.$$

$$\frac{2(dx)^{3/2}\sqrt{a+bx+cx^2}}{5d}$$

$\downarrow$ 1509

$$\frac{1}{5}\left(\frac{2b\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{2d\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\frac{2(b^2-3ac)}{\sqrt{c}}+\sqrt{a}b\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}\right.}{3c\sqrt{}}\right.$$

$$\frac{2(dx)^{3/2}\sqrt{a+bx+cx^2}}{5d}$$

input
```
Int[Sqrt[d*x]*Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*(d*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*d) + ((2*b*Sqrt[d*x]*Sqrt[a + b*x
 + c*x^2])/(3*c) - (2*d*Sqrt[x]*((-2*(b^2 - 3*a*c)*(-((Sqrt[x]*Sqrt[a + b*
x + c*x^2])/(Sqrt[a] + Sqrt[c]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(
a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt
[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x + c*x^2
])))/Sqrt[c] + (a^(1/4)*(Sqrt[a]*b + (2*(b^2 - 3*a*c))/Sqrt[c])*(Sqrt[a] +
 Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*Ar
cTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*
Sqrt[a + b*x + c*x^2])))/(3*c*Sqrt[d*x]))/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 764, normalized size of antiderivative = 1.60

method	result
risch	$\frac{2(3cx+b)x\sqrt{c\,x^2+bx+a}\,d}{15c\sqrt{dx}}-\frac{\left(-\frac{\left(6ac-2b^2\right)\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{-\frac{2c\cdots}{b+\sqrt{-4\cdots}}}\cdots}{\cdots}\cdots\right.}{\cdots}$
elliptic	$\frac{\sqrt{dx}\,\sqrt{dx(c\,x^2+bx+a)}\left(\frac{2x\sqrt{cd\,x^3+bd\,x^2+adx}}{5}+\frac{2b\sqrt{cd\,x^3+bd\,x^2+adx}}{15c}-\frac{adb\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{x-\frac{-b+\cdots}{\cdots}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}\cdots}}\cdots}{\cdots}\cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int((d*x)^(1/2)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(3*c*x+b)*x*(c*x^2+b*x+a)^(1/2)/c*d/(d*x)^(1/2)-1/15/c*(-(6*a*c-2*b^2
)*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4
*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*
c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)*((-1/2*(b+(-4*a*c+b^2)^
(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(2^(1/2)*((x+1/2*(b+(-4*a
*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^
(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1
/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^
2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2)
)/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))
+a*b*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+
(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4
*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)*EllipticF(2^(1/2)*((
x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b
+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^
2)^(1/2))))^(1/2)))*d*(d*x*(c*x^2+b*x+a))^(1/2)/(d*x)^(1/2)/(c*x^2+b*x+a)^
(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.39

$$\int \sqrt{dx}\sqrt{a+bx+cx^2}\,dx = \frac{2\left((2\,b^3-9\,abc)\sqrt{cd}\text{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\frac{3\,cx+b}{3\,c}\right)+6\,(b^2c-3\,ac^2)\sqrt{cd}\text{weierstras}\right.}{}$$

input
```
integrate((d*x)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/45*((2*b^3 - 9*a*b*c)*sqrt(c*d)*weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^
2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c) + 6*(b^2*c - 3*a*c^2)*s
qrt(c*d)*weierstrassZeta(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^
3, weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3,
 1/3*(3*c*x + b)/c)) + 3*(3*c^3*x + b*c^2)*sqrt(c*x^2 + b*x + a)*sqrt(d*x)
)/c^3
```

Sympy [F]

$$\int \sqrt{dx}\sqrt{a+bx+cx^2}\,dx=\int \sqrt{dx}\sqrt{a+bx+cx^2}\,dx$$

input
```
integrate((d*x)**(1/2)*(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(d*x)*sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \sqrt{dx}\sqrt{a+bx+cx^2}\,dx=\int \sqrt{cx^2+bx+a}\sqrt{dx}\,dx$$

input
```
integrate((d*x)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*sqrt(d*x), x)
```

Giac [F]

$$\int \sqrt{dx}\sqrt{a+bx+cx^2}\,dx=\int \sqrt{cx^2+bx+a}\sqrt{dx}\,dx$$

input
```
integrate((d*x)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*sqrt(d*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}\sqrt{a+bx+cx^2}\,dx=\int \sqrt{d\,x}\,\sqrt{c\,x^2+b\,x+a}\,dx$$

input
```
int((d*x)^(1/2)*(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((d*x)^(1/2)*(a + b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{dx}\sqrt{a+bx+cx^2}\,dx = \frac{\sqrt{d}\left(2\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,a+2\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,bx-3\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,x}{c\,x^2+bx+a}dx\right)ac+\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,x}{c\,x^2+bx+a}dx\right.}{5b}$$

input
```
int((d*x)^(1/2)*(c*x^2+b*x+a)^(1/2),x)
```

output
```
(sqrt(d)*(2*sqrt(x)*sqrt(a + b*x + c*x**2)*a + 2*sqrt(x)*sqrt(a + b*x + c*
x**2)*b*x - 3*int((sqrt(x)*sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*x**2),x)
*a*c + int((sqrt(x)*sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*x**2),x)*b**2 -
 int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a*x + b*x**2 + c*x**3),x)*a**2))/(5
*b)
```

3.312 $\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{dx}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 434

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{dx}}\,dx = \frac{2\sqrt{dx}\sqrt{a+bx+cx^2}}{3d} + \frac{b\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\,E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{dx}}{\sqrt{-b+\sqrt{b^2-4ac}}\sqrt{d}}\right)\middle|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{2}c^{3/2}\sqrt{d}\sqrt{a+x(b+cx)}} - \frac{\sqrt{-b+\sqrt{b^2-4ac}}\left(b^2-4ac+b\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{dx}}{\sqrt{-b+\sqrt{b^2-4ac}}\sqrt{d}}\right),\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{2}c^{3/2}\sqrt{d}\sqrt{a+x(b+cx)}}$$

output
```
2/3*(d*x)^(1/2)*(c*x^2+b*x+a)^(1/2)/d+1/6*b*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*
(b+(-4*a*c+b^2)^(1/2))*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+
(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2^(1/2)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a
*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)
))^(1/2))*2^(1/2)/c^(3/2)/d^(1/2)/(a+x*(c*x+b))^(1/2)-1/6*(-b+(-4*a*c+b^2)
^(1/2))^(1/2)*(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))*(1+2*c*x/(b-(-4*a*c+b^2)^(1
/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(2^(1/2)*c^(1/
2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2
))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/c^(3/2)/d^(1/2)/(a+x*(c*x+b))^(1
/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.39 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{dx}}\,dx$$
$$= \frac{2\left(ab+b^2x+acx+2bcx^2+c^2x^3+i\sqrt{2}bc\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2a}{\left(b+\sqrt{b^2-4ac}\right)x}}x^{3/2}\sqrt{\frac{2a+bx-\sqrt{b^2-4ac}x}{bx-\sqrt{b^2-4ac}x}}E\left(i\text{arcsi}\right.\right.}{}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/Sqrt[d*x],x]
```

output
```
(2*(a*b + b^2*x + a*c*x + 2*b*c*x^2 + c^2*x^3 + I*Sqrt[2]*b*c*Sqrt[a/(b +
Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(3/2)*Sq
rt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*Elliptic
E[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[
b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*Sqrt[2]*c*Sqrt[b^2 - 4*a*c]*Sqr
t[a/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x
^(3/2)*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]
*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (
b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(3*c*Sqrt[d*x]*Sqrt[a +
x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.74, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1162, 25, 27, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{dx}}\,dx$$
$$\Big\downarrow \text{1162}$$

$$\frac{2\sqrt{dx}\sqrt{a+bx+cx^2}}{3d}-\frac{\int -\frac{d(2a+bx)}{\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{3d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{d(2a+bx)}{\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{3d}+\frac{2\sqrt{dx}\sqrt{a+bx+cx^2}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{2a+bx}{\sqrt{dx}\sqrt{cx^2+bx+a}}dx+\frac{2\sqrt{dx}\sqrt{a+bx+cx^2}}{3d}$$

$$\Big\downarrow \text{1241}$$

$$\frac{\sqrt{x}\int \frac{2a+bx}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{3\sqrt{dx}}+\frac{2\sqrt{dx}\sqrt{a+bx+cx^2}}{3d}$$

$$\Big\downarrow \text{1240}$$

$$\frac{2\sqrt{x}\int \frac{2a+bx}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{3\sqrt{dx}}+\frac{2\sqrt{dx}\sqrt{a+bx+cx^2}}{3d}$$

$$\Big\downarrow \text{1511}$$

$$\frac{2\sqrt{x}\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{\sqrt{a}b\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{3\sqrt{dx}}+\frac{2\sqrt{dx}\sqrt{a+bx+cx^2}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sqrt{x}\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{b\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{3\sqrt{dx}}+\frac{2\sqrt{dx}\sqrt{a+bx+cx^2}}{3d}$$

$$\Big\downarrow \text{1416}$$

$$\frac{2\sqrt{x}\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}-\frac{b\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{3\sqrt{dx}}+\frac{2\sqrt{dx}\sqrt{a+bx+cx^2}}{3d}$$

$$\Big\downarrow \text{1509}$$

$$\frac{2\sqrt{x}\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}} - \frac{b\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\,E\left(2\arctan\right.}{\sqrt[4]{c}\sqrt{a+bx+c}}\right.}{}\right.}{3\sqrt{dx}}$$

$$\frac{2\sqrt{dx}\sqrt{a+bx+cx^2}}{3d}$$

input
```
Int[Sqrt[a + b*x + c*x^2]/Sqrt[d*x],x]
```

output
```
(2*Sqrt[d*x]*Sqrt[a + b*x + c*x^2])/(3*d) + (2*Sqrt[x]*(-((b*(-((Sqrt[x]*S
qrt[a + b*x + c*x^2])/(Sqrt[a] + Sqrt[c]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]
*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^
(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b
*x + c*x^2])))/Sqrt[c]) + (a^(1/4)*(2*Sqrt[a] + b/Sqrt[c])*(Sqrt[a] + Sqrt
[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[
(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[
a + b*x + c*x^2])))/(3*Sqrt[d*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 679, normalized size of antiderivative = 1.56

method	result
default	$\frac{2\sqrt{-4ac+b^2}\,\sqrt{\frac{2cx+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}}\,\sqrt{\frac{-2cx+\sqrt{-4ac+b^2}-b}{\sqrt{-4ac+b^2}}}\,\sqrt{-\frac{cx}{b+\sqrt{-4ac+b^2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{2cx+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}},\frac{\sqrt{2}\sqrt{\frac{b+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2}\right)ac-}{}$
risch	$\frac{2x\sqrt{c\,x^2+bx+a}}{3\sqrt{dx}}+\frac{\left(\frac{b\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\,\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\,\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\,\sqrt{-\frac{2cx}{b+\sqrt{-4ac+b^2}}}\left(\left(-\frac{b+\sqrt{-4ac}}{2c}\right.\right.}{}\right.}{}$
elliptic	$\frac{\sqrt{dx(c\,x^2+bx+a)}\left(\frac{2\sqrt{cd\,x^3+bd\,x^2+adx}}{3d}+\frac{b\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\,\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\,\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\,\sqrt{-\frac{2cx}{b+\sqrt{-4ac}}}}{}\right.}{}$

input
```
int((c*x^2+b*x+a)^(1/2)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*(-4*a*c+b^2)^(1/2)*((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1
/2)))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x
/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+
(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
)^(1/2))^(1/2))*a*c+4*((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2))
)^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+
(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*
a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1
/2))^(1/2))*a*b*c-((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1
/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+(-4*
a*c+b^2)^(1/2)))^(1/2)*EllipticE(((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+
b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))
^(1/2))*b^3-(-4*a*c+b^2)^(1/2)*((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^
2)^(1/2)))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2)^(1/2))^(1/2)*
(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(((2*c*x+(-4*a*c+b^2)^(1/2)+b
)/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*
c+b^2)^(1/2))^(1/2))*b^2+2*c^3*x^3+2*b*c^2*x^2+2*a*c^2*x)/(c*x^2+b*x+a)^(1
/2)/(d*x)^(1/2)/c^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{dx}}\,dx = -\frac{2\left(3\sqrt{cd}bc\,\text{weierstrassZeta}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\text{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\frac{3\,cx+}{3\,c}\right.\right.\right.}{9\,c}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/9*(3*sqrt(c*d)*b*c*weierstrassZeta(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3
- 9*a*b*c)/c^3, weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 -
9*a*b*c)/c^3, 1/3*(3*c*x + b)/c)) - 3*sqrt(c*x^2 + b*x + a)*sqrt(d*x)*c^2
+ (b^2 - 6*a*c)*sqrt(c*d)*weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27
*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c))/(c^2*d)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{dx}}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{\sqrt{dx}}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/(d*x)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)/sqrt(d*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{dx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{\sqrt{dx}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/sqrt(d*x), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{dx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{\sqrt{dx}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(d*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/sqrt(d*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{dx}}\,dx=\int \frac{\sqrt{c\,x^2+b\,x+a}}{\sqrt{d\,x}}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/(d*x)^(1/2),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/(d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{dx}}\,dx = \frac{\sqrt{d}\left(2\sqrt{x}\,\sqrt{c\,x^2+bx+a}-\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,x}{c\,x^2+bx+a}dx\right)c+\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+bx+a}}{c\,x^3+b\,x^2+ax}dx\right)a\right)}{2d}$$

input
```
int((c*x^2+b*x+a)^(1/2)/(d*x)^(1/2),x)
```

output
```
(sqrt(d)*(2*sqrt(x)*sqrt(a + b*x + c*x**2) - int((sqrt(x)*sqrt(a + b*x + c
*x**2)*x)/(a + b*x + c*x**2),x)*c + int((sqrt(x)*sqrt(a + b*x + c*x**2))/(
a*x + b*x**2 + c*x**3),x)*a))/(2*d)
```

3.313 $\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 416

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{3/2}}\,dx = -\frac{2\sqrt{a+bx+cx^2}}{d\sqrt{dx}}$$

$$+\frac{\sqrt{2}\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{dx}}{\sqrt{-b+\sqrt{b^2-4ac}}\sqrt{d}}\right)|\frac{b-\sqrt{b^2\ldots}}{b+\sqrt{b^2\ldots}}\ldots\right.}{\sqrt{c}d^{3/2}\sqrt{a+x(b+cx)}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-b+\sqrt{b^2-4ac}}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{dx}}{\sqrt{-b+\sqrt{b^2-4ac}}\sqrt{d}}\right),\frac{b-\ldots}{b+\ldots}\ldots\right.}{\sqrt{c}d^{3/2}\sqrt{a+x(b+cx)}}$$

output
```
-2*(c*x^2+b*x+a)^(1/2)/d/(d*x)^(1/2)+2^(1/2)*(-b+(-4*a*c+b^2)^(1/2))^(1/2)
*(b+(-4*a*c+b^2)^(1/2))*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b
+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2^(1/2)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*
a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2
)))^(1/2))/c^(1/2)/d^(3/2)/(a+x*(c*x+b))^(1/2)-2^(1/2)*(-4*a*c+b^2)^(1/2)*
(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*
c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(2^(1/2)*c^(1/2)*(d*x)^(1/2)/(-
b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2
)^(1/2)))^(1/2))/c^(1/2)/d^(3/2)/(a+x*(c*x+b))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.43 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{3/2}}\,dx = \frac{x\left(2\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}(a+x(b+cx))-i\left(-b+\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2a}{\left(b+\sqrt{b^2-4ac}\right)x}}x^{3/2}\sqrt{\frac{4a+\cdots}{\cdots}}\right.}{}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/(d*x)^(3/2),x]
```

output
```
(x*(2*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*(a + x*(b + c*x)) - I*(-b + Sqrt[b^2
 - 4*a*c])*Sqrt[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(3/2)*Sqrt[(4*a +
 2*b*x - 2*Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticE[I*A
rcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 -
 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[1 + (
2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(3/2)*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a
*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b
 + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 -
4*a*c])]))/(Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*(d*x)^(3/2)*Sqrt[a + x*(b + c*
x)])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.77, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1161, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{3/2}}\,dx$$

$$\Big\downarrow \; 1161$$

$$\frac{\int \frac{b+2cx}{\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{d}-\frac{2\sqrt{a+bx+cx^2}}{d\sqrt{dx}}$$

$$\Big\downarrow \; 1241$$

$$\frac{\sqrt{x}\int \frac{b+2cx}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{d\sqrt{dx}}-\frac{2\sqrt{a+bx+cx^2}}{d\sqrt{dx}}$$

$$\Big\downarrow \; 1240$$

$$\frac{2\sqrt{x}\int \frac{b+2cx}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{d\sqrt{dx}}-\frac{2\sqrt{a+bx+cx^2}}{d\sqrt{dx}}$$

$$\Big\downarrow \; 1511$$

$$\frac{2\sqrt{x}\left(\left(2\sqrt{a}\sqrt{c}+b\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-2\sqrt{a}\sqrt{c}\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\sqrt{x}\right)}{d\sqrt{dx}}-\frac{2\sqrt{a+bx+cx^2}}{d\sqrt{dx}}$$

$$\Big\downarrow \; 27$$

$$\frac{2\sqrt{x}\left(\left(2\sqrt{a}\sqrt{c}+b\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-2\sqrt{c}\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}\right)}{d\sqrt{dx}}-\frac{2\sqrt{a+bx+cx^2}}{d\sqrt{dx}}$$

$$\Big\downarrow \; 1416$$

$$\frac{2\sqrt{x}\left(\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx+cx^2}}-2\sqrt{c}\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}\right)}{d\sqrt{dx}}-\frac{2\sqrt{a+bx+cx^2}}{d\sqrt{dx}}$$

$$\Big\downarrow \; 1509$$

$$\frac{2\sqrt{x}\left(\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx+cx^2}}-2\sqrt{c}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}E\left(2\text{ arc}\right.}{\sqrt[4]{c}\sqrt{a+bx}}\right.\right.}{d\sqrt{dx}}$$

$$\frac{2\sqrt{a+bx+cx^2}}{d\sqrt{dx}}$$

input
```
Int[Sqrt[a + b*x + c*x^2]/(d*x)^(3/2),x]
```

output
```
(-2*Sqrt[a + b*x + c*x^2])/(d*Sqrt[d*x]) + (2*Sqrt[x]*(-2*Sqrt[c]*(-((Sqrt
[x]*Sqrt[a + b*x + c*x^2])/(Sqrt[a] + Sqrt[c]*x)) + (a^(1/4)*(Sqrt[a] + Sq
rt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticE[2*ArcTa
n[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[
a + b*x + c*x^2])) + ((b + 2*Sqrt[a]*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(
a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt
[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a + b
*x + c*x^2])))/(d*Sqrt[d*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 739 vs. $2(338) = 676$.

Time = 1.63 (sec) , antiderivative size = 740, normalized size of antiderivative = 1.78

method	result
risch	$-\frac{2\sqrt{c\,x^2+bx+a}}{d\sqrt{dx}}+\frac{\left(\frac{b\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{-\frac{2cx}{b+\sqrt{-4ac+b^2}}}\,\mathrm{EllipticF}\left(\sqrt{2}\ldots\right.}{c\sqrt{cd\,x^3+bd\,x^2+adx}}\ldots\right.}{\ldots}$
elliptic	$\frac{\sqrt{dx(c\,x^2+bx+a)}\left(-\frac{2\left(cd\,x^2+bdx+ad\right)}{d^2\sqrt{x\left(cd\,x^2+bdx+ad\right)}}+\frac{b\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{-\frac{\ldots}{b+\sqrt{\ldots}}}\ldots}{dc\sqrt{cd\,x^3\ldots}}\ldots\right.}{\ldots}$
default	$\frac{-4\sqrt{\frac{2cx+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{-2cx+\sqrt{-4ac+b^2}-b}{\sqrt{-4ac+b^2}}}\sqrt{-\frac{cx}{b+\sqrt{-4ac+b^2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{2cx+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}},\frac{\sqrt{2}\sqrt{\frac{b+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2}\right)ac+\sqrt{\frac{2cx+\sqrt{\ldots}}{b+\sqrt{\ldots}}}\ldots}{\ldots}$

input
```
int((c*x^2+b*x+a)^(1/2)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(c*x^2+b*x+a)^(1/2)/d/(d*x)^(1/2)+(b*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*(
(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c
*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c
+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^
2+a*d*x)^(1/2)*EllipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*
a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c
+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(b+(-4*a*c+b^2)^(1
/2))*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2
/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(
c*d*x^3+b*d*x^2+a*d*x)^(1/2)*((-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4
*a*c+b^2)^(1/2)))*EllipticE(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(
-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*
a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c
+b^2)^(1/2))*EllipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*
c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b
^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))/d*(d*x*(c*x^2+b*x+a)
)^(1/2)/(d*x)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{3/2}}\,dx = \frac{2\left(\sqrt{cd}bx\text{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2}, -\frac{4\,(2\,b^3-9\,abc)}{27\,c^3}, \frac{3\,cx+b}{3\,c}\right) - 6\,\sqrt{cd}cx\text{weierstras}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(sqrt(c*d)*b*x*weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3
 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c) - 6*sqrt(c*d)*c*x*weierstrassZeta(4/3*
(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, weierstrassPInverse(4/3*(b
^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c)) - 3*sqrt
(c*x^2 + b*x + a)*sqrt(d*x)*c)/(c*d^2*x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{3/2}}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/(d*x)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)/(d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{3/2}}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/(d*x)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{3/2}}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(d*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/(d*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{3/2}}\,dx=\int \frac{\sqrt{c\,x^2+b\,x+a}}{(d\,x)^{3/2}}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/(d*x)^(3/2),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/(d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{3/2}}\,dx=\frac{\sqrt{d}\left(-2\sqrt{c\,x^2+bx+a}+\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+bx+a}}{c\,x^3+b\,x^2+ax}dx\right)b+2\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+bx+a}}{c\,x^2+bx+a}dx\right)c\right)}{\sqrt{x}\,d^2}$$

input
```
int((c*x^2+b*x+a)^(1/2)/(d*x)^(3/2),x)
```

output
```
(sqrt(d)*( - 2*sqrt(a + b*x + c*x**2) + sqrt(x)*int((sqrt(x)*sqrt(a + b*x
+ c*x**2))/(a*x + b*x**2 + c*x**3),x)*b + 2*sqrt(x)*int((sqrt(x)*sqrt(a +
b*x + c*x**2))/(a + b*x + c*x**2),x)*c))/(sqrt(x)*d**2)
```

3.314 $\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 448

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{5/2}}\,dx = -\frac{2(a+bx)\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}} + \frac{b\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{dx}}{\sqrt{-b+\sqrt{b^2-4ac}}\sqrt{d}}\right)|\frac{b-\sqrt{b^2-4}}{b+\sqrt{b^2-4}}\right.}{3\sqrt{2}a\sqrt{c}d^{5/2}\sqrt{a+x(b+cx)}} - \frac{\sqrt{-b+\sqrt{b^2-4ac}}\left(b^2-4ac+b\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{}}{\sqrt{-b+\sqrt{b^2-}}}\right.\right.}{3\sqrt{2}a\sqrt{c}d^{5/2}\sqrt{a+x(b+cx)}}$$

output
```
-2/3*(b*x+a)*(c*x^2+b*x+a)^(1/2)/a/d/(d*x)^(3/2)+1/6*b*(-b+(-4*a*c+b^2)^(1
/2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2^(1/2)*c^(1/2)*(d*x)^(1/2
)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c
+b^2)^(1/2)))^(1/2))*2^(1/2)/a/c^(1/2)/d^(5/2)/(a+x*(c*x+b))^(1/2)-1/6*(-b
+(-4*a*c+b^2)^(1/2))^(1/2)*(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))*(1+2*c*x/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(
2^(1/2)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*
a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/a/c^(1/2)/d^(5/2)/(
a+x*(c*x+b))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.08 (sec) , antiderivative size = 441, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{5/2}}\,dx = \frac{ix\left(4ia\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}(a+x(b+cx))-b\left(-b+\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2a}{\left(b+\sqrt{b^2-4ac}\right)x}}\,x^{5/2}\sqrt{\cdots}\right.}{\cdots}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/(d*x)^(5/2),x]
```

output
```
((I/6)*x*((4*I)*a*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*(a + x*(b + c*x)) - b*(-
b + Sqrt[b^2 - 4*a*c])*Sqrt[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(5/2)
*Sqrt[(4*a + 2*b*x - 2*Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*E
llipticE[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b
+ Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + (-b^2 + 4*a*c + b*Sqrt[b^2
 - 4*a*c])*Sqrt[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(5/2)*Sqrt[(4*a +
 2*b*x - 2*Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticF[I*A
rcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 -
 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(a*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*(d*
x)^(5/2)*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.82, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1161, 1237, 27, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{5/2}}\,dx$$

$$\Big\downarrow \; 1161$$

$$\frac{\int \frac{b+2cx}{(dx)^{3/2}\sqrt{cx^2+bx+a}}dx}{3d}-\frac{2\sqrt{a+bx+cx^2}}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{ 1237}$$

$$\frac{-\frac{2\int -\frac{cd(2a+bx)}{2\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{ad^2}-\frac{2b\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{3d}-\frac{2\sqrt{a+bx+cx^2}}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{c\int \frac{2a+bx}{\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{ad}-\frac{2b\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{3d}-\frac{2\sqrt{a+bx+cx^2}}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{ 1241}$$

$$\frac{\frac{c\sqrt{x}\int \frac{2a+bx}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{ad\sqrt{dx}}-\frac{2b\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{3d}-\frac{2\sqrt{a+bx+cx^2}}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{ 1240}$$

$$\frac{\frac{2c\sqrt{x}\int \frac{2a+bx}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{ad\sqrt{dx}}-\frac{2b\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{3d}-\frac{2\sqrt{a+bx+cx^2}}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{ 1511}$$

$$\frac{\frac{2c\sqrt{x}\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{\sqrt{a}b\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{ad\sqrt{dx}}-\frac{2b\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{3d}-\frac{2\sqrt{a+bx+cx^2}}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{2c\sqrt{x}\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{b\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{ad\sqrt{dx}}-\frac{2b\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{3d}-\frac{2\sqrt{a+bx+cx^2}}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{\frac{2c\sqrt{x}\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}-\frac{b\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{ad\sqrt{dx}}-\frac{2b\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{3d}-\frac{2\sqrt{a+bx+cx^2}}{3d(dx)^{3/2}}$$

$$\Big\downarrow \; 1509$$

$$\frac{\frac{2c\sqrt{x}\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}-\frac{b\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right.\right.}{\sqrt[4]{c}\sqrt{a+bx+cx^2}}\right.}{\sqrt{c}}\right)}{ad\sqrt{dx}}}{3d}$$

$$\frac{2\sqrt{a+bx+cx^2}}{3d(dx)^{3/2}}$$

input
```
Int[Sqrt[a + b*x + c*x^2]/(d*x)^(5/2),x]
```

output
```
(-2*Sqrt[a + b*x + c*x^2])/(3*d*(d*x)^(3/2)) + ((-2*b*Sqrt[a + b*x + c*x^2
])/(a*d*Sqrt[d*x]) + (2*c*Sqrt[x]*(-((b*(-((Sqrt[x]*Sqrt[a + b*x + c*x^2])
/(Sqrt[a] + Sqrt[c]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c
*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4
)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x + c*x^2])))/Sqrt[c
]) + (a^(1/4)*(2*Sqrt[a] + b/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x
+ c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(
1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x + c*x^2])))/(
a*d*Sqrt[d*x]))/(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 707, normalized size of antiderivative = 1.58

method	result
default	$\frac{2\sqrt{\frac{-2cx+\sqrt{-4ac+b^2}-b}{\sqrt{-4ac+b^2}}}\sqrt{-\frac{cx}{b+\sqrt{-4ac+b^2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{2cx+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}},\frac{\sqrt{2}\sqrt{\frac{b+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2}\right)\sqrt{-4ac+b^2}\sqrt{\frac{2cx+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}}\,acx}{}$
risch	$-\frac{2\sqrt{c\,x^2+bx+a}\,(bx+a)}{3xa\,d^2\sqrt{dx}}+\frac{c\left(\frac{b\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{-\frac{2cx}{b+\sqrt{-4ac+b^2}}}\left(\left(-\right.\right.}{}\right)}{}$
elliptic	$\frac{\sqrt{dx(c\,x^2+bx+a)}\left(-\frac{2\sqrt{cd\,x^3+bd\,x^2+adx}}{3d^3x^2}-\frac{2\left(cd\,x^2+bdx+ad\right)b}{3d^3a\sqrt{x\left(cd\,x^2+bdx+ad\right)}}+\frac{2\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{x-\frac{-b+\sqrt{-}}{2}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}-}}}{}\right.}{}$

input

```
int((c*x^2+b*x+a)^(1/2)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+(
-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a
*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/
2))^(1/2))*(-4*a*c+b^2)^(1/2)*((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2
)^(1/2)))^(1/2)*a*c*x+4*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2)^(1/2))
^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(((2*c*x+(-4*a*c+b^2)^
(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))
/(-4*a*c+b^2)^(1/2))^(1/2))*((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^
(1/2)))^(1/2)*a*b*c*x-((-2*c*x+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2)^(1/2))^(
1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(((2*c*x+(-4*a*c+b^2)^(1
/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(
-4*a*c+b^2)^(1/2))^(1/2))*((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1
/2)))^(1/2)*b^3*x-((-2*c*x+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2)^(1/2))^(1/2)
*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(((2*c*x+(-4*a*c+b^2)^(1/2)+
b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a
*c+b^2)^(1/2))^(1/2))*(-4*a*c+b^2)^(1/2)*((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+
(-4*a*c+b^2)^(1/2)))^(1/2)*b^2*x-2*b*c^2*x^3-2*a*c^2*x^2-2*b^2*c*x^2-4*a*b
*c*x-2*a^2*c)/x/(c*x^2+b*x+a)^(1/2)/d^2/(d*x)^(1/2)/a/c
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{5/2}}\,dx =$$

$$-\frac{2\left(3\sqrt{cd}bcx^2\text{weierstrassZeta}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\text{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\frac{3\,cx+b}{3\,c}\right.\right.\right.}{9}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/9*(3*sqrt(c*d)*b*c*x^2*weierstrassZeta(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*
b^3 - 9*a*b*c)/c^3, weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^
3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c)) + (b^2 - 6*a*c)*sqrt(c*d)*x^2*weiers
trassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c
*x + b)/c) + 3*(b*c*x + a*c)*sqrt(c*x^2 + b*x + a)*sqrt(d*x))/(a*c*d^3*x^2
)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{5/2}}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/(d*x)**(5/2),x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)/(d*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{5/2}}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/(d*x)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{5/2}}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(d*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/(d*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{5/2}}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{(d\,x)^{5/2}}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/(d*x)^(5/2),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/(d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{5/2}}\,dx = \frac{\sqrt{d}\left(-2\sqrt{c\,x^2+bx+a}-\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+bx+a}}{c\,x^5+b\,x^4+a\,x^3}dx\right)ax+\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+bx+a}}{c\,x^3+b\,x^2+ax}dx\right)ca}{2\sqrt{x}\,d^3x}$$

input
```
int((c*x^2+b*x+a)^(1/2)/(d*x)^(5/2),x)
```

output
```
(sqrt(d)*( - 2*sqrt(a + b*x + c*x**2) - sqrt(x)*int((sqrt(x)*sqrt(a + b*x
+ c*x**2))/(a*x**3 + b*x**4 + c*x**5),x)*a*x + sqrt(x)*int((sqrt(x)*sqrt(a
 + b*x + c*x**2))/(a*x + b*x**2 + c*x**3),x)*c*x))/(2*sqrt(x)*d**3*x)
```

3.315 $\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 504

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{7/2}}\,dx = \frac{4(b^2-3ac)\sqrt{a+bx+cx^2}}{15a^2d^3\sqrt{dx}} - \frac{2(3a+bx)\sqrt{a+bx+cx^2}}{15ad(dx)^{5/2}}$$
$$- \frac{\sqrt{2}(b^2-3ac)\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{dx}}{\sqrt{-b+\sqrt{b^2-4ac}}\cdots}\right.\right.}{15a^2\sqrt{c}d^{7/2}\sqrt{a+x(b+cx)}}$$
$$+ \frac{\sqrt{2}\sqrt{-b+\sqrt{b^2-4ac}}\left(b^3-4abc+\sqrt{b^2-4ac}(b^2-3ac)\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\operatorname{EllipticF}\left(\text{arcs}\cdots\right.}{15a^2\sqrt{c}d^{7/2}\sqrt{a+x(b+cx)}}$$

output
```
4/15*(-3*a*c+b^2)*(c*x^2+b*x+a)^(1/2)/a^2/d^3/(d*x)^(1/2)-2/15*(b*x+3*a)*(
c*x^2+b*x+a)^(1/2)/a/d/(d*x)^(5/2)-1/15*2^(1/2)*(-3*a*c+b^2)*(-b+(-4*a*c+b
^2)^(1/2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(
1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2^(1/2)*c^(1/2)*(d*x
)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(
-4*a*c+b^2)^(1/2)))^(1/2))/a^2/c^(1/2)/d^(7/2)/(a+x*(c*x+b))^(1/2)+1/15*2^
(1/2)*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(b^3-4*a*b*c+(-4*a*c+b^2)^(1/2)*(-3*a*
c+b^2))*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1
/2)))^(1/2)*EllipticF(2^(1/2)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^
(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))/a^2/c
^(1/2)/d^(7/2)/(a+x*(c*x+b))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.81 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{7/2}}\,dx = \frac{x\left(-4(b^2-3ac)\,x^2(a+x(b+cx))-2(a+x(b+cx))\,(3a^2-2b^2x^2+ax(b+6cx)\right.}{}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/(d*x)^(7/2),x]
```

output
```
(x*(-4*(b^2 - 3*a*c)*x^2*(a + x*(b + c*x)) - 2*(a + x*(b + c*x))*(3*a^2 -
2*b^2*x^2 + a*x*(b + 6*c*x)) + (I*(b^2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c])*S
qrt[2 + (4*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(7/2)*Sqrt[(2*a + b*x - Sqrt[
b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticE[I*ArcSinh[(Sqrt[2]*
Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sq
rt[b^2 - 4*a*c])])/Sqrt[a/(b + Sqrt[b^2 - 4*a*c])] + (I*(b^3 - 4*a*b*c - b
^2*Sqrt[b^2 - 4*a*c] + 3*a*c*Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[
b^2 - 4*a*c])*x)]*x^(7/2)*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sq
rt[b^2 - 4*a*c]*x)]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*
a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a
/(b + Sqrt[b^2 - 4*a*c])]))/(15*a^2*(d*x)^(7/2)*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 431, normalized size of antiderivative = 0.86, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1161, 1237, 27, 1237, 27, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{7/2}}\,dx$$

$$\Big\downarrow \text{1161}$$

$$\frac{\int \frac{b+2cx}{(dx)^{5/2}\sqrt{cx^2+bx+a}}dx}{5d}-\frac{2\sqrt{a+bx+cx^2}}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{1237}$$

$$\frac{-\frac{2\int \frac{d\left(2\left(b^2-3ac\right)+bcx\right)}{2(dx)^{3/2}\sqrt{cx^2+bx+a}}dx}{3ad^2}-\frac{2b\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}}{5d}-\frac{2\sqrt{a+bx+cx^2}}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{2\left(b^2-3ac\right)+bcx}{(dx)^{3/2}\sqrt{cx^2+bx+a}}dx}{3ad}-\frac{2b\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}}{5d}-\frac{2\sqrt{a+bx+cx^2}}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{1237}$$

$$\frac{-\frac{-\frac{2\int -\frac{cd\left(ab+2\left(b^2-3ac\right)x\right)}{2\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{ad^2}-\frac{4\left(b^2-3ac\right)\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{3ad}-\frac{2b\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}}{5d}-\frac{2\sqrt{a+bx+cx^2}}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\frac{c\int \frac{ab+2\left(b^2-3ac\right)x}{\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{ad}-\frac{4\left(b^2-3ac\right)\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{3ad}-\frac{2b\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}}{5d}-\frac{2\sqrt{a+bx+cx^2}}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{1241}$$

$$\frac{-\frac{\frac{c\sqrt{x}\int \frac{ab+2\left(b^2-3ac\right)x}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{ad\sqrt{dx}}-\frac{4\left(b^2-3ac\right)\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{3ad}-\frac{2b\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}}{5d}-\frac{2\sqrt{a+bx+cx^2}}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{1240}$$

$$\frac{-\frac{\frac{2c\sqrt{x}\int \frac{ab+2\left(b^2-3ac\right)x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{ad\sqrt{dx}}-\frac{4\left(b^2-3ac\right)\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{3ad}-\frac{2b\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}}{5d}-\frac{2\sqrt{a+bx+cx^2}}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{1511}$$

$$-\frac{\frac{\frac{2c\sqrt{x}\left(\sqrt{a}\left(\frac{2\left(b^2-3ac\right)}{\sqrt{c}}+\sqrt{a}b\right)\int\frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{2\sqrt{a}\left(b^2-3ac\right)\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{ad\sqrt{dx}}-\frac{4\left(b^2-3ac\right)\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{3ad}-\frac{2b\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}}{5d}-\frac{2\sqrt{a+bx+cx^2}}{5d(dx)^{5/2}}$$

$\big\downarrow$ 27

$$-\frac{\frac{\frac{2c\sqrt{x}\left(\sqrt{a}\left(\frac{2\left(b^2-3ac\right)}{\sqrt{c}}+\sqrt{a}b\right)\int\frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{2\left(b^2-3ac\right)\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{ad\sqrt{dx}}-\frac{4\left(b^2-3ac\right)\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{3ad}-\frac{2b\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}}{5d}-\frac{2\sqrt{a+bx+cx^2}}{5d(dx)^{5/2}}$$

$\big\downarrow$ 1416

$$-\frac{\frac{\frac{2c\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\frac{2\left(b^2-3ac\right)}{\sqrt{c}}+\sqrt{a}b\right)\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}-\frac{2\left(b^2-3ac\right)\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{ad\sqrt{dx}}-\frac{4\left(b^2-3ac\cdots\right.}{a\cdots}}{3ad}\cdots}{5d}-\frac{2\sqrt{a+bx+cx^2}}{5d(dx)^{5/2}}$$

$\big\downarrow$ 1509

$$-\frac{\frac{\frac{2c\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\frac{2\left(b^2-3ac\right)}{\sqrt{c}}+\sqrt{a}b\right)\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}-\frac{2\left(b^2-3ac\right)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}E\cdots}{\sqrt[4]{c}\cdots}\cdots\right.}{\cdots}\cdots\right.}{ad\sqrt{dx}}\cdots}{3ad}\cdots}{5d}-\frac{2\sqrt{a+bx+cx^2}}{5d(dx)^{5/2}}$$

input
```
Int[Sqrt[a + b*x + c*x^2]/(d*x)^(7/2),x]
```

output
```
(-2*Sqrt[a + b*x + c*x^2])/(5*d*(d*x)^(5/2)) + ((-2*b*Sqrt[a + b*x + c*x^2
])/(3*a*d*(d*x)^(3/2)) - ((-4*(b^2 - 3*a*c)*Sqrt[a + b*x + c*x^2])/(a*d*Sq
rt[d*x]) + (2*c*Sqrt[x]*((-2*(b^2 - 3*a*c)*(-((Sqrt[x]*Sqrt[a + b*x + c*x^
2])/(Sqrt[a] + Sqrt[c]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x
+ c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(
1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x + c*x^2])))/Sqr
t[c] + (a^(1/4)*(Sqrt[a]*b + (2*(b^2 - 3*a*c))/Sqrt[c])*(Sqrt[a] + Sqrt[c]
*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^
(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a +
 b*x + c*x^2])))/(a*d*Sqrt[d*x]))/(3*a*d))/(5*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 790, normalized size of antiderivative = 1.57

method	result
risch	$-\frac{2\sqrt{c\,x^2+bx+a}\,(6ac\,x^2-2b^2x^2+abx+3a^2)}{15x^2a^2d^3\sqrt{dx}}-\frac{c\left(-\frac{(6ac-2b^2)\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}-\frac{-b+\sqrt{\cdots}}{\cdots}}}\cdots}{\cdots}\cdots\right)}{\cdots}$
elliptic	$\frac{\sqrt{dx(c\,x^2+bx+a)}\left(-\frac{2\sqrt{cd\,x^3+bd\,x^2+adx}}{5d^4x^3}-\frac{2b\sqrt{cd\,x^3+bd\,x^2+adx}}{15d^4a\,x^2}-\frac{4\left(cd\,x^2+bdx+ad\right)\left(3ac-b^2\right)}{15d^4a^2\sqrt{x\left(cd\,x^2+bdx+ad\right)}}-\frac{b\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+\cdots}}{2c}\right)\cdots}{b+\sqrt{-4ac+b\cdots}}}\cdots}{\cdots}\cdots\right)}{\cdots}$
default	Expression too large to display

input
```
int((c*x^2+b*x+a)^(1/2)/(d*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(c*x^2+b*x+a)^(1/2)*(6*a*c*x^2-2*b^2*x^2+a*b*x+3*a^2)/x^2/a^2/d^3/(d
*x)^(1/2)-1/15*c/a^2*(-(6*a*c-2*b^2)*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+
1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-
b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^
2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a
*d*x)^(1/2)*((-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))
*EllipticE(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c
-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*Elli
pticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^
(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/
c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+a*b*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*(
(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c
*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c
+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^
2+a*d*x)^(1/2)*EllipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*
a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c
+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))/d^3*(d*x*(c*x^2+b*x
+a))^(1/2)/(d*x)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{7/2}}\,dx = \frac{2\left(\left(2\,b^3-9\,abc\right)\sqrt{cd}x^3\text{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\frac{3\,cx+b}{3\,c}\right)+6\,(b^2\cdots\right.}{\cdots}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(d*x)^(7/2),x, algorithm="fricas")
```

output
```
2/45*((2*b^3 - 9*a*b*c)*sqrt(c*d)*x^3*weierstrassPInverse(4/3*(b^2 - 3*a*c
)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c) + 6*(b^2*c - 3*a*c^
2)*sqrt(c*d)*x^3*weierstrassZeta(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a
*b*c)/c^3, weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b
*c)/c^3, 1/3*(3*c*x + b)/c)) - 3*(a*b*c*x + 3*a^2*c - 2*(b^2*c - 3*a*c^2)*
x^2)*sqrt(c*x^2 + b*x + a)*sqrt(d*x))/(a^2*c*d^4*x^3)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{7/2}}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{(dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/(d*x)**(7/2),x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)/(d*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{7/2}}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(d*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/(d*x)^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{7/2}}\,dx=\int \frac{\sqrt{cx^2+bx+a}}{(dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(d*x)^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/(d*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{7/2}}\,dx=\int \frac{\sqrt{c\,x^2+b\,x+a}}{{(d\,x)}^{7/2}}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/(d*x)^(7/2),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/(d*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(dx)^{7/2}}\,dx=\frac{\sqrt{d}\left(-8\sqrt{c\,x^2+bx+a}\,a-16\sqrt{c\,x^2+bx+a}\,c\,x^2+4\sqrt{x}\left(\int \frac{\sqrt{c\,x^2+bx+a}}{\sqrt{x}\,a\,x^2+\sqrt{x}\,b\,x^3+\sqrt{x}\,c\,x^4}dx\right.\right.}{20\sqrt{x}\,a\,d}$$

input
```
int((c*x^2+b*x+a)^(1/2)/(d*x)^(7/2),x)
```

output
```
(sqrt(d)*( - 8*sqrt(a + b*x + c*x**2)*a - 16*sqrt(a + b*x + c*x**2)*c*x**2
 + 4*sqrt(x)*int(sqrt(a + b*x + c*x**2)/(sqrt(x)*a*x**2 + sqrt(x)*b*x**3 +
 sqrt(x)*c*x**4),x)*a*b*x**2 + 3*sqrt(x)*int((sqrt(a + b*x + c*x**2)*x)/(s
qrt(x)*a + sqrt(x)*b*x + sqrt(x)*c*x**2),x)*c**2*x**2 + 5*sqrt(x)*int((sqr
t(x)*sqrt(a + b*x + c*x**2))/(a + b*x + c*x**2),x)*c**2*x**2))/(20*sqrt(x)
*a*d**4*x**2)
```

3.316 $\int (dx)^{3/2} \left(a+bx+cx^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 619

$$\int (dx)^{3/2} \left(a+bx+cx^2\right)^{3/2} \, dx = \frac{2d\sqrt{dx}(8b^4-21ab^2c-30a^2c^2+3bc(8b^2-31ac)\,x)\sqrt{a+bx+cx^2}}{1155c^3} - \frac{2d\sqrt{dx}(3(2b^2+ac)+14bcx)\left(a+bx+cx^2\right)^{3/2}}{231c^2} + \frac{2d\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11c} - \frac{4\sqrt{2}b(2b^2-9ac)\left(b^2-3ac\right)\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)d^{3/2}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\Big(\mathrm{ar}}{1155c^{9/2}\sqrt{a+x(b+cx)}} + \frac{\sqrt{2}\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\left(4b(2b^2-9ac)\left(b^2-3ac\right)-\frac{ac(8b^4-51ab^2c+60a^2c^2)}{b+\sqrt{b^2-4ac}}\right)d^{3/2}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-}}}}{1155c^{9/2}\sqrt{a+x(b+cx)}}$$

output
```
2/1155*d*(d*x)^(1/2)*(8*b^4-21*a*b^2*c-30*a^2*c^2+3*b*c*(-31*a*c+8*b^2)*x)
*(c*x^2+b*x+a)^(1/2)/c^3-2/231*d*(d*x)^(1/2)*(14*b*c*x+3*a*c+6*b^2)*(c*x^2
+b*x+a)^(3/2)/c^2+2/11*d*(d*x)^(1/2)*(c*x^2+b*x+a)^(5/2)/c-4/1155*2^(1/2)*
b*(-9*a*c+2*b^2)*(-3*a*c+b^2)*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(b+(-4*a*c+b^2
)^(1/2))*d^(3/2)*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*
c+b^2)^(1/2)))^(1/2)*EllipticE(2^(1/2)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2
)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/
2))/c^(9/2)/(a+x*(c*x+b))^(1/2)+1/1155*2^(1/2)*(-b+(-4*a*c+b^2)^(1/2))^(1/
2)*(b+(-4*a*c+b^2)^(1/2))*(4*b*(-9*a*c+2*b^2)*(-3*a*c+b^2)-a*c*(60*a^2*c^2
-51*a*b^2*c+8*b^4)/(b+(-4*a*c+b^2)^(1/2)))*d^(3/2)*(1+2*c*x/(b-(-4*a*c+b^2
)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(2^(1/2)*c
^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^
(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))/c^(9/2)/(a+x*(c*x+b))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.84 (sec) , antiderivative size = 633, normalized size of antiderivative = 1.02

$$\int (dx)^{3/2} \left(a+bx+cx^2\right)^{3/2} dx = \frac{(dx)^{3/2} \left(-16b(2b^4-15ab^2c+27a^2c^2)\left(a+x(b+cx)\right)+2cx(a+x(b+cx))\left(8b^4-6b^3cx+\right.\right.}{}$$

input
```
Integrate[(d*x)^(3/2)*(a + b*x + c*x^2)^(3/2),x]
```

output
```
((d*x)^(3/2)*(-16*b*(2*b^4 - 15*a*b^2*c + 27*a^2*c^2)*(a + x*(b + c*x)) +
2*c*x*(a + x*(b + c*x))*(8*b^4 - 6*b^3*c*x + b^2*c*(-51*a + 5*c*x^2) + 4*b
*c^2*x*(8*a + 35*c*x^2) + 15*c^2*(4*a^2 + 13*a*c*x^2 + 7*c^2*x^4)) + ((4*I
)*b*(2*b^4 - 15*a*b^2*c + 27*a^2*c^2)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4
*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(3/2)*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*
c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b
+ Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4
*a*c])])/Sqrt[a/(b + Sqrt[b^2 - 4*a*c])] - (I*(-8*b^6 + 68*a*b^4*c - 159*a
^2*b^2*c^2 + 60*a^3*c^3 + 8*b^5*Sqrt[b^2 - 4*a*c] - 60*a*b^3*c*Sqrt[b^2 -
4*a*c] + 108*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[b^2 -
4*a*c])*x)]*x^(3/2)*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2
 - 4*a*c]*x)]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]
)/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a/(b +
Sqrt[b^2 - 4*a*c])]))/(1155*c^4*x^2*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 518, normalized size of antiderivative = 0.84, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {1166, 27, 1231, 27, 1231, 27, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2} \left(a+bx+cx^2\right)^{3/2} \, dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{2\int -\frac{d^2(a+6bx)\left(cx^2+bx+a\right)^{3/2}}{2\sqrt{dx}}dx}{11c}+\frac{2d\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11c}$$

$$\Big\downarrow \text{27}$$

$$\frac{2d\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11c}-\frac{d^2\int \frac{(a+6bx)\left(cx^2+bx+a\right)^{3/2}}{\sqrt{dx}}dx}{11c}$$

$$\Big\downarrow \text{1231}$$

$$\frac{2d\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11c}-\frac{d^2\left(\frac{2\sqrt{dx}(3(ac+2b^2)+14bcx)\left(a+bx+cx^2\right)^{3/2}}{21cd}-\frac{2\int\frac{3d^2\left(2a\left(b^2-3ac\right)+b\left(8b^2-31ac\right)x\right)\sqrt{cx^2+bx+a}}{2\sqrt{dx}}dx}{21cd^2}\right)}{11c}$$

$\downarrow$ 27

$$\frac{2d\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11c}-\frac{d^2\left(\frac{2\sqrt{dx}(3(ac+2b^2)+14bcx)\left(a+bx+cx^2\right)^{3/2}}{21cd}-\frac{\int\frac{\left(2a\left(b^2-3ac\right)+b\left(8b^2-31ac\right)x\right)\sqrt{cx^2+bx+a}}{\sqrt{dx}}dx}{7c}\right)}{11c}$$

$\downarrow$ 1231

$$\frac{2d\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11c}-\frac{d^2\left(\frac{2\sqrt{dx}(3(ac+2b^2)+14bcx)\left(a+bx+cx^2\right)^{3/2}}{21cd}-\frac{\frac{2\sqrt{dx}\left(-30a^2c^2+3bcx\left(8b^2-31ac\right)-21ab^2c+8b^4\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\int\frac{d^2\left(a\left(8b^4-51acb^2+60a^2c^2\right)+8b\left(\right.\right.}{2\sqrt{dx}\sqrt{cx^2+b}}}{15cd^2}}{7c}\right.}{11c}$$

$\downarrow$ 27

$$\frac{2d\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11c}-\frac{d^2\left(\frac{2\sqrt{dx}(3(ac+2b^2)+14bcx)\left(a+bx+cx^2\right)^{3/2}}{21cd}-\frac{\frac{2\sqrt{dx}\left(-30a^2c^2+3bcx\left(8b^2-31ac\right)-21ab^2c+8b^4\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{\int\frac{a\left(8b^4-51acb^2+60a^2c^2\right)+8b\left(2b^2-\right.}{\sqrt{dx}\sqrt{cx^2+bx+a}}}{15c}}{7c}\right.}{11c}$$

$\downarrow$ 1241

$$\frac{2d\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11c}-\frac{d^2\left(\frac{2\sqrt{dx}(3(ac+2b^2)+14bcx)\left(a+bx+cx^2\right)^{3/2}}{21cd}-\frac{\frac{2\sqrt{dx}\left(-30a^2c^2+3bcx\left(8b^2-31ac\right)-21ab^2c+8b^4\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{\sqrt{x}\int\frac{a\left(8b^4-51acb^2+60a^2c^2\right)+8b\left(2\right.}{\sqrt{x}\sqrt{cx^2+bx-}}}{15c\sqrt{dx}}}{7c}\right.}{11c}$$

$\downarrow$ 1240

$$\frac{2d\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11c}-$$
$$\frac{d^2\left(\frac{2\sqrt{dx}\left(3\left(ac+2b^2\right)+14bcx\right)\left(a+bx+cx^2\right)^{3/2}}{21cd}-\frac{\frac{2\sqrt{dx}\left(-30a^2c^2+3bcx\left(8b^2-31ac\right)-21ab^2c+8b^4\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\int\frac{a\left(8b^4-51acb^2+60a^2c^2\right)+8b\left(\right.}{\sqrt{cx^2+bx+}}}{15c\sqrt{dx}}}{7c}\right.}{11c}$$

$\downarrow$ 1511

$$\frac{2d\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11c}-$$
$$\frac{d^2\left(\frac{2\sqrt{dx}\left(3\left(ac+2b^2\right)+14bcx\right)\left(a+bx+cx^2\right)^{3/2}}{21cd}-\frac{\frac{2\sqrt{dx}\left(-30a^2c^2+3bcx\left(8b^2-31ac\right)-21ab^2c+8b^4\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\left(\sqrt{a}\left(\sqrt{a}\left(60a^2c^2-51ab^2c+8b^4\right)\right.\right.}{}}{7}\right.}{11c}$$

$\downarrow$ 27

$$\frac{2d\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11c}-$$
$$\frac{d^2\left(\frac{2\sqrt{dx}\left(3\left(ac+2b^2\right)+14bcx\right)\left(a+bx+cx^2\right)^{3/2}}{21cd}-\frac{\frac{2\sqrt{dx}\left(-30a^2c^2+3bcx\left(8b^2-31ac\right)-21ab^2c+8b^4\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\left(\sqrt{a}\left(\sqrt{a}\left(60a^2c^2-51ab^2c+8b^4\right)\right.\right.}{}}{7c}\right.}{11c}$$

$\downarrow$ 1416

$$\frac{2d\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11c}-$$
$$\frac{d^2\left(\frac{2\sqrt{dx}\left(3\left(ac+2b^2\right)+14bcx\right)\left(a+bx+cx^2\right)^{3/2}}{21cd}-\frac{\frac{2\sqrt{dx}\left(-30a^2c^2+3bcx\left(8b^2-31ac\right)-21ab^2c+8b^4\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}\left(60a^2c^2-51ab^2c+8b\right.\right.}{}\right.}{}}{}\right.}{11c}$$

$\downarrow$ 1509

$$\frac{2d\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11c}-$$

$$d^2\left(\frac{2\sqrt{dx}\left(3\left(ac+2b^2\right)+14bcx\right)\left(a+bx+cx^2\right)^{3/2}}{21cd}-\frac{\frac{2\sqrt{dx}\left(-30a^2c^2+3bcx\left(8b^2-31ac\right)-21ab^2c+8b^4\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}\left(60a^2c^2-51ab^2c+8b\right.\right.}{}\right.}{}}{}\right.$$

input

```
Int[(d*x)^(3/2)*(a + b*x + c*x^2)^(3/2),x]
```

output

```
(2*d*Sqrt[d*x]*(a + b*x + c*x^2)^(5/2))/(11*c) - (d^2*((2*Sqrt[d*x]*(3*(2*
b^2 + a*c) + 14*b*c*x)*(a + b*x + c*x^2)^(3/2))/(21*c*d) - ((2*Sqrt[d*x]*(
8*b^4 - 21*a*b^2*c - 30*a^2*c^2 + 3*b*c*(8*b^2 - 31*a*c)*x)*Sqrt[a + b*x +
 c*x^2])/(15*c*d) - (2*Sqrt[x]*((-8*b*(2*b^2 - 9*a*c)*(b^2 - 3*a*c)*(-((Sq
rt[x]*Sqrt[a + b*x + c*x^2])/(Sqrt[a] + Sqrt[c]*x)) + (a^(1/4)*(Sqrt[a] +
Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticE[2*Arc
Tan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqr
t[a + b*x + c*x^2])))/Sqrt[c] + (a^(1/4)*((8*b*(2*b^2 - 9*a*c)*(b^2 - 3*a*
c))/Sqrt[c] + Sqrt[a]*(8*b^4 - 51*a*b^2*c + 60*a^2*c^2))*(Sqrt[a] + Sqrt[c
]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c
^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a
+ b*x + c*x^2])))/(15*c*Sqrt[d*x]))/(7*c)))/(11*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1254 vs. $2(519) = 1038$.

Time = 2.81 (sec) , antiderivative size = 1255, normalized size of antiderivative = 2.03

method	result	size
elliptic	Expression too large to display	1255
risch	Expression too large to display	1376
default	Expression too large to display	2278

input
```
int((d*x)^(3/2)*(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/x*(d*x)^(1/2)/(c*x^2+b*x+a)^(1/2)*(d*x*(c*x^2+b*x+a))^(1/2)*(2/11*c*d*
x^4*(c*d*x^3+b*d*x^2+a*d*x)^(1/2)+8/33*b*d*x^3*(c*d*x^3+b*d*x^2+a*d*x)^(1/
2)+2/7*((2*a*c+b^2)*d^2-9/11*a*d^2*c-32/33*b^2*d^2)/c/d*x^2*(c*d*x^3+b*d*x
^2+a*d*x)^(1/2)+2/5*(38/33*a*b*d^2-6/7*((2*a*c+b^2)*d^2-9/11*a*d^2*c-32/33
*b^2*d^2)/c*b)/c/d*x*(c*d*x^3+b*d*x^2+a*d*x)^(1/2)+2/3*(a^2*d^2-5/7*((2*a*
c+b^2)*d^2-9/11*a*d^2*c-32/33*b^2*d^2)/c*a-4/5*(38/33*a*b*d^2-6/7*((2*a*c+
b^2)*d^2-9/11*a*d^2*c-32/33*b^2*d^2)/c*b)/c*b)/c/d*(c*d*x^3+b*d*x^2+a*d*x)
^(1/2)-1/3*(a^2*d^2-5/7*((2*a*c+b^2)*d^2-9/11*a*d^2*c-32/33*b^2*d^2)/c*a-4
/5*(38/33*a*b*d^2-6/7*((2*a*c+b^2)*d^2-9/11*a*d^2*c-32/33*b^2*d^2)/c*b)/c*
b)/c^2*a*(b+(-4*a*c+b^2)^(1/2))*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(
b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(
-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)*EllipticF(2^(1/2)*
((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*
(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+
b^2)^(1/2))))^(1/2))+(-3/5*(38/33*a*b*d^2-6/7*((2*a*c+b^2)*d^2-9/11*a*d^2*
c-32/33*b^2*d^2)/c*b)/c*a-2/3*(a^2*d^2-5/7*((2*a*c+b^2)*d^2-9/11*a*d^2*c-3
2/33*b^2*d^2)/c*a-4/5*(38/33*a*b*d^2-6/7*((2*a*c+b^2)*d^2-9/11*a*d^2*c-32/
33*b^2*d^2)/c*b)/c*b)/c*b)*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4
*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.48

$$\int (dx)^{3/2} \left(a + bx + cx^2\right)^{3/2} \, dx = \frac{2 \left(\left(16 b^6 - 144 a b^4 c + 369 a^2 b^2 c^2 - 180 a^3 c^3\right) \sqrt{cd} d \, \text{weierstrassPInverse}\left(\frac{4 \left(b^2 - 3 a c\right)}{3 c^2}, -\frac{4 \left(2 b^3 - 9 a \right.}{27 c^3}\right.\right.}{}$$

input
```
integrate((d*x)^(3/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3465*((16*b^6 - 144*a*b^4*c + 369*a^2*b^2*c^2 - 180*a^3*c^3)*sqrt(c*d)*d
*weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1
/3*(3*c*x + b)/c) + 24*(2*b^5*c - 15*a*b^3*c^2 + 27*a^2*b*c^3)*sqrt(c*d)*d
*weierstrassZeta(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, weier
strassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*
c*x + b)/c)) + 3*(105*c^6*d*x^4 + 140*b*c^5*d*x^3 + 5*(b^2*c^4 + 39*a*c^5)
*d*x^2 - 2*(3*b^3*c^3 - 16*a*b*c^4)*d*x + (8*b^4*c^2 - 51*a*b^2*c^3 + 60*a
^2*c^4)*d)*sqrt(c*x^2 + b*x + a)*sqrt(d*x))/c^5
```

Sympy [F]

$$\int (dx)^{3/2} \left(a+bx+cx^2\right)^{3/2} \, dx = \int (dx)^{\frac{3}{2}} \left(a+bx+cx^2\right)^{\frac{3}{2}} \, dx$$

input
```
integrate((d*x)**(3/2)*(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((d*x)**(3/2)*(a + b*x + c*x**2)**(3/2), x)
```

Maxima [F]

$$\int (dx)^{3/2} \left(a+bx+cx^2\right)^{3/2} \, dx = \int \left(cx^2+bx+a\right)^{\frac{3}{2}} (dx)^{\frac{3}{2}} \, dx$$

input
```
integrate((d*x)^(3/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)*(d*x)^(3/2), x)
```

Giac [F]

$$\int (dx)^{3/2} \left(a+bx+cx^2\right)^{3/2} \, dx = \int \left(cx^2+bx+a\right)^{\frac{3}{2}} (dx)^{\frac{3}{2}} \, dx$$

input
```
integrate((d*x)^(3/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)*(d*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2} \left(a+bx+cx^2\right)^{3/2} \, dx = \int (d\,x)^{3/2} \left(c\,x^2+b\,x+a\right)^{3/2} \, dx$$

input
```
int((d*x)^(3/2)*(a + b*x + c*x^2)^(3/2),x)
```

output
```
int((d*x)^(3/2)*(a + b*x + c*x^2)^(3/2), x)
```

Reduce [F]

$$\int (dx)^{3/2} (a+bx +cx^2)^{3/2} \, dx = \frac{\sqrt{d}\,d\Big(-96\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,a^2c+18\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,a\,b^2+64\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,abcx}{}$$

input
```
int((d*x)^(3/2)*(c*x^2+b*x+a)^(3/2),x)
```

output

```
(sqrt(d)*d*( - 96*sqrt(x)*sqrt(a + b*x + c*x**2)*a**2*c + 18*sqrt(x)*sqrt(
a + b*x + c*x**2)*a*b**2 + 64*sqrt(x)*sqrt(a + b*x + c*x**2)*a*b*c*x + 390
*sqrt(x)*sqrt(a + b*x + c*x**2)*a*c**2*x**2 - 12*sqrt(x)*sqrt(a + b*x + c*
x**2)*b**3*x + 10*sqrt(x)*sqrt(a + b*x + c*x**2)*b**2*c*x**2 + 280*sqrt(x)
*sqrt(a + b*x + c*x**2)*b*c**2*x**3 + 210*sqrt(x)*sqrt(a + b*x + c*x**2)*c
**3*x**4 + 324*int((sqrt(x)*sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*x**2),x
)*a**2*c**2 - 180*int((sqrt(x)*sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*x**2
),x)*a*b**2*c + 24*int((sqrt(x)*sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*x**
2),x)*b**4 + 48*int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a*x + b*x**2 + c*x**
3),x)*a**3*c - 9*int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a*x + b*x**2 + c*x*
*3),x)*a**2*b**2))/(1155*c**2)
```

3.317 $\int \sqrt{dx}(a+bx+cx^2)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 569

$$\int \sqrt{dx}\left(a+bx+cx^2\right)^{3/2}\,dx=-\frac{8b(b^2-6ac)\sqrt{dx}\sqrt{a+bx+cx^2}}{315c^2}$$
$$+\frac{2(dx)^{3/2}\left(b^2+14ac+5bcx\right)\sqrt{a+bx+cx^2}}{105cd}+\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9d}$$
$$+\frac{(8b^4-57ab^2c+84a^2c^2)\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\sqrt{d}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\Big(\arcsin}{315\sqrt{2}c^{7/2}\sqrt{a+x(b+cx)}}$$
$$-\frac{\sqrt{-b+\sqrt{b^2-4ac}}\left(8b^5-65ab^3c+132a^2bc^2+\sqrt{b^2-4ac}(8b^4-57ab^2c+84a^2c^2)\right)\sqrt{d}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{}}{315\sqrt{2}c^{7/2}\sqrt{a+x(b+cx)}}$$

output
```
-8/315*b*(-6*a*c+b^2)*(d*x)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2+2/105*(d*x)^(3/2
)*(5*b*c*x+14*a*c+b^2)*(c*x^2+b*x+a)^(1/2)/c/d+2/9*(d*x)^(3/2)*(c*x^2+b*x+
a)^(3/2)/d+1/630*(84*a^2*c^2-57*a*b^2*c+8*b^4)*(-b+(-4*a*c+b^2)^(1/2))^(1/
2)*(b+(-4*a*c+b^2)^(1/2))*d^(1/2)*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2^(1/2)*c^(1/2)*(d*x)^(1/2
)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c
+b^2)^(1/2)))^(1/2))*2^(1/2)/c^(7/2)/(a+x*(c*x+b))^(1/2)-1/630*(-b+(-4*a*c
+b^2)^(1/2))^(1/2)*(8*b^5-65*a*b^3*c+132*a^2*b*c^2+(-4*a*c+b^2)^(1/2)*(84*
a^2*c^2-57*a*b^2*c+8*b^4))*d^(1/2)*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*
(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(2^(1/2)*c^(1/2)*(d*x)^(1/
2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*
c+b^2)^(1/2)))^(1/2))*2^(1/2)/c^(7/2)/(a+x*(c*x+b))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.89 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.05

$$\int \sqrt{dx}\left(a+bx+cx^2\right)^{3/2}\,dx=\frac{\sqrt{dx}\left(\frac{4\sqrt{x}(a+x(b+cx))\left(-4b^3+3b^2cx+7c^2x\left(11a+5cx^2\right)+2bc\left(12a+25cx^2\right)\right)}{c^2}+\frac{x\left(\frac{4\left(8b^4-57ab^2c+84a^2c^2\right)(a+x(b+cx))}{x^{3/2}}-\frac{i(\cdots}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}$$

input
```
Integrate[Sqrt[d*x]*(a + b*x + c*x^2)^(3/2),x]
```

output
```
(Sqrt[d*x]*((4*Sqrt[x]*(a + x*(b + c*x))*(-4*b^3 + 3*b^2*c*x + 7*c^2*x*(11
*a + 5*c*x^2) + 2*b*c*(12*a + 25*c*x^2)))/c^2 + (x*((4*(8*b^4 - 57*a*b^2*c
 + 84*a^2*c^2)*(a + x*(b + c*x)))/x^(3/2) - (I*(8*b^4 - 57*a*b^2*c + 84*a^
2*c^2)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[b^2 - 4*a*c])*x)
]*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*Elli
pticE[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + S
qrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a/(b + Sqrt[b^2 - 4*a*c])
] + (I*(-8*b^5 + 65*a*b^3*c - 132*a^2*b*c^2 + 8*b^4*Sqrt[b^2 - 4*a*c] - 57
*a*b^2*c*Sqrt[b^2 - 4*a*c] + 84*a^2*c^2*Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4*a)/
((b + Sqrt[b^2 - 4*a*c])*x)]*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x -
 Sqrt[b^2 - 4*a*c]*x)]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 -
 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqr
t[a/(b + Sqrt[b^2 - 4*a*c])]))/c^3))/(630*Sqrt[x]*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 474, normalized size of antiderivative = 0.83, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {1162, 25, 27, 1236, 27, 1231, 27, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\left(a+bx+cx^2\right)^{3/2}dx$$

$$\Big\downarrow \text{1162}$$

$$\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9d}-\frac{\int -d\sqrt{dx}(2a+bx)\sqrt{cx^2+bx+a}dx}{3d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int d\sqrt{dx}(2a+bx)\sqrt{cx^2+bx+a}dx}{3d}+\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \sqrt{dx}(2a+bx)\sqrt{cx^2+bx+a}dx+\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9d}$$

$$\Big\downarrow \text{1236}$$

$$\frac{1}{3}\left(\frac{2\int -\frac{d(ab+2(2b^2-7ac)x)\sqrt{cx^2+bx+a}}{2\sqrt{dx}}dx}{7c}+\frac{2b\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}\right)+\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9d}$$

$\Big\downarrow$ 27

$$\frac{1}{3}\left(\frac{2b\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{d\int \frac{(ab+2(2b^2-7ac)x)\sqrt{cx^2+bx+a}}{\sqrt{dx}}dx}{7c}\right)+\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9d}$$

$\Big\downarrow$ 1231

$$\frac{1}{3}\left(\frac{2b\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{d\left(\frac{2\sqrt{dx}\left(6cx\left(2b^2-7ac\right)+b\left(4b^2-9ac\right)\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\int \frac{d^2\left(4ab\left(b^2-6ac\right)+\left(8b^4-57acb^2+84a^2c^2\right)x\right)}{2\sqrt{dx}\sqrt{cx^2+bx+a}}d\cdots}{15cd^2}\cdots\right)}{7c}\cdots\right)+\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9d}$$

$\Big\downarrow$ 27

$$\frac{1}{3}\left(\frac{2b\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{d\left(\frac{2\sqrt{dx}\left(6cx\left(2b^2-7ac\right)+b\left(4b^2-9ac\right)\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{\int \frac{4ab\left(b^2-6ac\right)+\left(8b^4-57acb^2+84a^2c^2\right)x}{\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{15c}\right)}{7c}\right)+\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9d}$$

$\Big\downarrow$ 1241

$$\frac{1}{3}\left(\frac{2b\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{d\left(\frac{2\sqrt{dx}\left(6cx\left(2b^2-7ac\right)+b\left(4b^2-9ac\right)\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{\sqrt{x}\int \frac{4ab\left(b^2-6ac\right)+\left(8b^4-57acb^2+84a^2c^2\right)x}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{15c\sqrt{dx}}\cdots\right)}{7c}\cdots\right)+\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9d}$$

$$\Big\downarrow \; 1240$$

$$\frac{1}{3}\left(\frac{2b\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{d\left(\frac{2\sqrt{dx}\left(6cx\left(2b^2-7ac\right)+b\left(4b^2-9ac\right)\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\int\frac{4ab\left(b^2-6ac\right)+\left(8b^4-57acb^2+84a^2c^2\right)x}{\sqrt{cx^2+bx+a}}d\sqrt{}}{15c\sqrt{dx}}\right.}{7c}\right.$$

$$\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9d}$$

$$\Big\downarrow \; 1511$$

$$\frac{1}{3}\left(\frac{2b\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{d\left(\frac{2\sqrt{dx}\left(6cx\left(2b^2-7ac\right)+b\left(4b^2-9ac\right)\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\left(\frac{\sqrt{a}\left(84a^2c^2-57ab^2c+4\sqrt{a}b\sqrt{c}\left(b^2-6ac\right)+8b\right.}{\sqrt{c}}\right.}{}\right.}{7c}\right.$$

$$\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\frac{2b\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{d\left(\frac{2\sqrt{dx}\left(6cx\left(2b^2-7ac\right)+b\left(4b^2-9ac\right)\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\left(\frac{\sqrt{a}\left(84a^2c^2-57ab^2c+4\sqrt{a}b\sqrt{c}\left(b^2-6ac\right)+8b\right.}{\sqrt{c}}\right.}{}\right.}{7c}\right.$$

$$\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9d}$$

$$\Big\downarrow \; 1416$$

$$\frac{1}{3}\left(\frac{2b\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{d\left(\frac{2\sqrt{dx}\left(6cx\left(2b^2-7ac\right)+b\left(4b^2-9ac\right)\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\left(\frac{\sqrt[4]{a}\left(84a^2c^2-57ab^2c+4\sqrt{a}b\sqrt{c}\left(b^2-6ac\right)+\right.}{}\right.}{}\right.}{}\right.$$

$$\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9d}$$

$\downarrow$ 1509

$$\frac{1}{3}\left(\frac{2b\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{d\left(\frac{2\sqrt{dx}\left(6cx\left(2b^2-7ac\right)+b\left(4b^2-9ac\right)\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\left(\frac{\sqrt[4]{a}\left(84a^2c^2-57ab^2c+4\sqrt{a}b\sqrt{c}\left(b^2-6ac\right)+\right.}{}\right.}{}\right.}{}\right.$$

$$\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9d}$$

input

```
Int[Sqrt[d*x]*(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*(d*x)^(3/2)*(a + b*x + c*x^2)^(3/2))/(9*d) + ((2*b*Sqrt[d*x]*(a + b*x +
 c*x^2)^(3/2))/(7*c) - (d*((2*Sqrt[d*x]*(b*(4*b^2 - 9*a*c) + 6*c*(2*b^2 -
7*a*c)*x)*Sqrt[a + b*x + c*x^2])/(15*c*d) - (2*Sqrt[x]*(-(((8*b^4 - 57*a*b
^2*c + 84*a^2*c^2)*(-((Sqrt[x]*Sqrt[a + b*x + c*x^2])/(Sqrt[a] + Sqrt[c]*x
)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt
[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*S
qrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x + c*x^2])))/Sqrt[c]) + (a^(1/4)*(8*b^4
- 57*a*b^2*c + 84*a^2*c^2 + 4*Sqrt[a]*b*Sqrt[c]*(b^2 - 6*a*c))*(Sqrt[a] +
Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*Arc
Tan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*S
qrt[a + b*x + c*x^2])))/(15*c*Sqrt[d*x])))/(7*c))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 991 vs. 2(473) = 946.

Time = 2.68 (sec) , antiderivative size = 992, normalized size of antiderivative = 1.74

method	result
elliptic	$\frac{\sqrt{dx}\,\sqrt{dx(c\,x^2+bx+a)}\left(\frac{2c\,x^3\sqrt{cd\,x^3+bd\,x^2+adx}}{9}+\frac{20b\,x^2\sqrt{cd\,x^3+bd\,x^2+adx}}{63}+\frac{2\left(\left(2ac+b^2\right)d-\frac{7acd}{9}-\frac{20b^2d}{21}\right)x\sqrt{cd\,x^3+bd\,x^2+adx}}{5cd}+\frac{2\left(\frac{76}{\ldots}\right.}{\ldots}\right.}{\ldots}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((d*x)^(1/2)*(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/x*(d*x)^(1/2)/(c*x^2+b*x+a)^(1/2)*(d*x*(c*x^2+b*x+a))^(1/2)*(2/9*c*x^3
*(c*d*x^3+b*d*x^2+a*d*x)^(1/2)+20/63*b*x^2*(c*d*x^3+b*d*x^2+a*d*x)^(1/2)+2
/5*((2*a*c+b^2)*d-7/9*a*c*d-20/21*b^2*d)/c/d*x*(c*d*x^3+b*d*x^2+a*d*x)^(1/
2)+2/3*(76/63*d*a*b-4/5*((2*a*c+b^2)*d-7/9*a*c*d-20/21*b^2*d)/c*b)/c/d*(c*
d*x^3+b*d*x^2+a*d*x)^(1/2)-1/3*(76/63*d*a*b-4/5*((2*a*c+b^2)*d-7/9*a*c*d-2
0/21*b^2*d)/c*b)/c^2*a*(b+(-4*a*c+b^2)^(1/2))*2^(1/2)*((x+1/2*(b+(-4*a*c+b
^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(
1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2
)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)*Elli
pticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^
(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/
c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+(a^2*d-3/5*((2*a*c+b^2)*d-7/9*a*c*d-20/
21*b^2*d)/c*a-2/3*(76/63*d*a*b-4/5*((2*a*c+b^2)*d-7/9*a*c*d-20/21*b^2*d)/c
*b)/c*b)*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c
)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2
*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b
+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)*((-1/2*(b+(-4*a*
c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(2^(1/2)*((x+1/2*(
b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*
c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.43

$$\int \sqrt{dx}\left(a+bx+cx^2\right)^{3/2}\,dx =$$

$$-\frac{2\left(\left(8\,b^5-69\,ab^3c+156\,a^2bc^2\right)\sqrt{cd}\,\text{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\frac{3\,cx+b}{3\,c}\right)+3\,(8\,b^4c-57\,ab\right.}{}$$

input
```
integrate((d*x)^(1/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/945*((8*b^5 - 69*a*b^3*c + 156*a^2*b*c^2)*sqrt(c*d)*weierstrassPInverse
(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c) +
3*(8*b^4*c - 57*a*b^2*c^2 + 84*a^2*c^3)*sqrt(c*d)*weierstrassZeta(4/3*(b^2
 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, weierstrassPInverse(4/3*(b^2 -
 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c)) - 3*(35*c^5*
x^3 + 50*b*c^4*x^2 - 4*b^3*c^2 + 24*a*b*c^3 + (3*b^2*c^3 + 77*a*c^4)*x)*sq
rt(c*x^2 + b*x + a)*sqrt(d*x))/c^4
```

Sympy [F]

$$\int \sqrt{dx}\,(a+bx+cx^2)^{3/2}\,dx=\int \sqrt{dx}\,(a+bx+cx^2)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*x)**(1/2)*(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral(sqrt(d*x)*(a + b*x + c*x**2)**(3/2), x)
```

Maxima [F]

$$\int \sqrt{dx}\,(a+bx+cx^2)^{3/2}\,dx=\int (cx^2+bx+a)^{\frac{3}{2}}\sqrt{dx}\,dx$$

input
```
integrate((d*x)^(1/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)*sqrt(d*x), x)
```

Giac [F]

$$\int \sqrt{dx}\left(a+bx+cx^2\right)^{3/2}\,dx=\int \left(cx^2+bx+a\right)^{\frac{3}{2}}\sqrt{dx}\,dx$$

input
```
integrate((d*x)^(1/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)*sqrt(d*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}\left(a+bx+cx^2\right)^{3/2}\,dx=\int \sqrt{d\,x}\,\left(c\,x^2+b\,x+a\right)^{3/2}\,dx$$

input
```
int((d*x)^(1/2)*(a + b*x + c*x^2)^(3/2),x)
```

output
```
int((d*x)^(1/2)*(a + b*x + c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \sqrt{dx}(a+bx+cx^2)^{3/2}\,dx=\frac{\sqrt{d}\left(168\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,a^2c-18\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,a\,b^2+308\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,abcx+\right.}{}$$

input
```
int((d*x)^(1/2)*(c*x^2+b*x+a)^(3/2),x)
```

output
```
(sqrt(d)*(168*sqrt(x)*sqrt(a + b*x + c*x**2)*a**2*c - 18*sqrt(x)*sqrt(a +
b*x + c*x**2)*a*b**2 + 308*sqrt(x)*sqrt(a + b*x + c*x**2)*a*b*c*x + 12*sqr
t(x)*sqrt(a + b*x + c*x**2)*b**3*x + 200*sqrt(x)*sqrt(a + b*x + c*x**2)*b*
*2*c*x**2 + 140*sqrt(x)*sqrt(a + b*x + c*x**2)*b*c**2*x**3 - 252*int((sqrt
(x)*sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*x**2),x)*a**2*c**2 + 171*int((s
qrt(x)*sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*x**2),x)*a*b**2*c - 24*int((
sqrt(x)*sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*x**2),x)*b**4 - 84*int((sqr
t(x)*sqrt(a + b*x + c*x**2))/(a*x + b*x**2 + c*x**3),x)*a**3*c + 9*int((sq
rt(x)*sqrt(a + b*x + c*x**2))/(a*x + b*x**2 + c*x**3),x)*a**2*b**2))/(630*
b*c)
```

3.318 $\int \frac{(a+bx+cx^2)^{3/2}}{\sqrt{dx}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 505

$$\int \frac{(a+bx+cx^2)^{3/2}}{\sqrt{dx}}\,dx = \frac{2\sqrt{dx}(b^2+10ac+3bcx)\sqrt{a+bx+cx^2}}{35cd}$$

$$+\frac{2\sqrt{dx}(a+bx+cx^2)^{3/2}}{7d}$$

$$-\frac{\sqrt{2}b(b^2-8ac)\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{dx}}{\sqrt{-b+\sqrt{b^2-4ac}}\cdots}\right.\right.}{35c^{5/2}\sqrt{d}\sqrt{a+x(b+cx)}}$$

$$+\frac{\sqrt{2}\sqrt{-b+\sqrt{b^2-4ac}}\left(b^4-9ab^2c+20a^2c^2+b(b^2-8ac)\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\,\text{Elli}\cdots}{35c^{5/2}\sqrt{d}\sqrt{a+x(b+cx)}}$$

output
```
2/35*(d*x)^(1/2)*(3*b*c*x+10*a*c+b^2)*(c*x^2+b*x+a)^(1/2)/c/d+2/7*(d*x)^(1
/2)*(c*x^2+b*x+a)^(3/2)/d-1/35*2^(1/2)*b*(-8*a*c+b^2)*(-b+(-4*a*c+b^2)^(1/
2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1
+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2^(1/2)*c^(1/2)*(d*x)^(1/2)
/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+
b^2)^(1/2)))^(1/2))/c^(5/2)/d^(1/2)/(a+x*(c*x+b))^(1/2)+1/35*2^(1/2)*(-b+(
-4*a*c+b^2)^(1/2))^(1/2)*(b^4-9*a*b^2*c+20*a^2*c^2+b*(-8*a*c+b^2)*(-4*a*c+
b^2)^(1/2))*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2
)^(1/2)))^(1/2)*EllipticF(2^(1/2)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/
2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))/c
^(5/2)/d^(1/2)/(a+x*(c*x+b))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.12 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx+cx^2)^{3/2}}{\sqrt{dx}}\,dx = \frac{\sqrt{x}\left(\frac{2\sqrt{x}(a+x(b+cx))\left(b^2+8bcx+5c(3a+cx^2)\right)}{c}+\frac{x\left(-\frac{4b\left(b^2-8ac\right)(a+x(b+cx))}{x^{3/2}}+\frac{ib\left(b^2-8ac\right)\left(-b+\sqrt{b^2-4ac}\right.\cdots}{\cdots}\right.\cdots}{\cdots}\right.\cdots}{\cdots}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/Sqrt[d*x],x]
```

output
```
(Sqrt[x]*((2*Sqrt[x]*(a + x*(b + c*x))*(b^2 + 8*b*c*x + 5*c*(3*a + c*x^2))
)/c + (x*((-4*b*(b^2 - 8*a*c)*(a + x*(b + c*x)))/x^(3/2) + (I*b*(b^2 - 8*a
*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*S
qrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*Ellipti
cE[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt
[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a/(b + Sqrt[b^2 - 4*a*c])] +
 (I*(b^4 - 9*a*b^2*c + 20*a^2*c^2 - b^3*Sqrt[b^2 - 4*a*c] + 8*a*b*c*Sqrt[b
^2 - 4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*Sqrt[(2*a + b*x -
 Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticF[I*ArcSinh[(Sq
rt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(
b - Sqrt[b^2 - 4*a*c])])/Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]))/c^2))/(35*Sqrt[
d*x]*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 399, normalized size of antiderivative = 0.79, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1162, 25, 27, 1231, 27, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{1162}$$

$$\frac{2\sqrt{dx}(a+bx+cx^2)^{3/2}}{7d}-\frac{3\int -\frac{d(2a+bx)\sqrt{cx^2+bx+a}}{\sqrt{dx}}dx}{7d}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\int \frac{d(2a+bx)\sqrt{cx^2+bx+a}}{\sqrt{dx}}dx}{7d}+\frac{2\sqrt{dx}(a+bx+cx^2)^{3/2}}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{7}\int \frac{(2a+bx)\sqrt{cx^2+bx+a}}{\sqrt{dx}}dx+\frac{2\sqrt{dx}(a+bx+cx^2)^{3/2}}{7d}$$

$$\Big\downarrow \text{1231}$$

$$\frac{3}{7}\left(\frac{2\sqrt{dx}\left(10ac+b^2+3bcx\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\int \frac{d^2\left(a\left(b^2-20ac\right)+2b\left(b^2-8ac\right)x\right)}{2\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{15cd^2}\right)+\frac{2\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7d}$$

$\downarrow$ 27

$$\frac{3}{7}\left(\frac{2\sqrt{dx}\left(10ac+b^2+3bcx\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{\int \frac{a\left(b^2-20ac\right)+2b\left(b^2-8ac\right)x}{\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{15c}\right)+\frac{2\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7d}$$

$\downarrow$ 1241

$$\frac{3}{7}\left(\frac{2\sqrt{dx}\left(10ac+b^2+3bcx\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{\sqrt{x}\int \frac{a\left(b^2-20ac\right)+2b\left(b^2-8ac\right)x}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{15c\sqrt{dx}}\right)+\frac{2\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7d}$$

$\downarrow$ 1240

$$\frac{3}{7}\left(\frac{2\sqrt{dx}\left(10ac+b^2+3bcx\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\int \frac{a\left(b^2-20ac\right)+2b\left(b^2-8ac\right)x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{15c\sqrt{dx}}\right)+\frac{2\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7d}$$

$\downarrow$ 1511

$$\frac{3}{7}\left(\frac{2\sqrt{dx}\left(10ac+b^2+3bcx\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\left(\sqrt{a}\left(\sqrt{a}\left(b^2-20ac\right)+\frac{2b\left(b^2-8ac\right)}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{2\sqrt{a}b(\cdots}{\cdots}\right.}{15c\sqrt{dx}}\right.$$
$$+\frac{2\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7d}$$

$\downarrow$ 27

$$\frac{3}{7}\left(\frac{2\sqrt{dx}\left(10ac+b^2+3bcx\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\left(\sqrt{a}\left(\sqrt{a}\left(b^2-20ac\right)+\frac{2b\left(b^2-8ac\right)}{\sqrt{c}}\right)\int\frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{2b(b^2-\right.}{15c\sqrt{dx}}\right.$$

$$\frac{2\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7d}$$

$\Big\downarrow$ 1416

$$\frac{3}{7}\left(\frac{2\sqrt{dx}\left(10ac+b^2+3bcx\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}\left(b^2-20ac\right)+\frac{2b\left(b^2-8ac\right)}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2a\right.}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}\right.}{15c\sqrt{dx}}\right.$$

$$\frac{2\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7d}$$

$\Big\downarrow$ 1509

$$\frac{3}{7}\left(\frac{2\sqrt{dx}\left(10ac+b^2+3bcx\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}\left(b^2-20ac\right)+\frac{2b\left(b^2-8ac\right)}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2a\right.}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}\right.}{}\right.$$

$$\frac{2\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{7d}$$

input
```
Int[(a + b*x + c*x^2)^(3/2)/Sqrt[d*x],x]
```

output
```
(2*Sqrt[d*x]*(a + b*x + c*x^2)^(3/2))/(7*d) + (3*((2*Sqrt[d*x]*(b^2 + 10*a
*c + 3*b*c*x)*Sqrt[a + b*x + c*x^2])/(15*c*d) - (2*Sqrt[x]*((-2*b*(b^2 - 8
*a*c)*(-((Sqrt[x]*Sqrt[a + b*x + c*x^2])/(Sqrt[a] + Sqrt[c]*x)) + (a^(1/4)
*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*Ell
ipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/
(c^(1/4)*Sqrt[a + b*x + c*x^2])))/Sqrt[c] + (a^(1/4)*(Sqrt[a]*(b^2 - 20*a*
c) + (2*b*(b^2 - 8*a*c))/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*
x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)
], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x + c*x^2])))/(15*c
*Sqrt[d*x])))/7
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 846 vs. $2(415) = 830$.

Time = 2.28 (sec) , antiderivative size = 847, normalized size of antiderivative = 1.68

method	result
elliptic	$\frac{\sqrt{dx(c\,x^2+bx+a)}\left(\frac{2c\,x^2\sqrt{cd\,x^3+bd\,x^2+adx}}{7d}+\frac{16bx\sqrt{cd\,x^3+bd\,x^2+adx}}{35d}+\frac{2\left(\frac{9ac}{7}+\frac{3b^2}{35}\right)\sqrt{cd\,x^3+bd\,x^2+adx}}{3cd}+\frac{\left(a^2-\frac{a\left(\frac{9ac}{7}+\frac{3b^2}{35}\right)}{3c}\right)\left(b+\sqrt{-\right.}}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((c*x^2+b*x+a)^(3/2)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(d*x*(c*x^2+b*x+a))^(1/2)/(d*x)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/7*c/d*x^2*(c*
d*x^3+b*d*x^2+a*d*x)^(1/2)+16/35*b/d*x*(c*d*x^3+b*d*x^2+a*d*x)^(1/2)+2/3*(
9/7*a*c+3/35*b^2)/c/d*(c*d*x^3+b*d*x^2+a*d*x)^(1/2)+(a^2-1/3*a/c*(9/7*a*c+
3/35*b^2))*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))
/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1
/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)*EllipticF(2^(1
/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*
(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*
a*c+b^2)^(1/2))))^(1/2))+(46/35*a*b-2/3*b/c*(9/7*a*c+3/35*b^2))*(b+(-4*a*c
+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1
/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)
))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)*((-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2
/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2
))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1
/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(
-b+(-4*a*c+b^2)^(1/2))*EllipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b
+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.43

$$\int \frac{(a+bx+cx^2)^{3/2}}{\sqrt{dx}}\,dx = \frac{2\left((2\,b^4-19\,ab^2c+60\,a^2c^2)\sqrt{cd}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\frac{3\,c\cdots}{3\cdots}\right.\right.}{}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
2/105*((2*b^4 - 19*a*b^2*c + 60*a^2*c^2)*sqrt(c*d)*weierstrassPInverse(4/3
*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c) + 6*(b
^3*c - 8*a*b*c^2)*sqrt(c*d)*weierstrassZeta(4/3*(b^2 - 3*a*c)/c^2, -4/27*(
2*b^3 - 9*a*b*c)/c^3, weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*
b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c)) + 3*(5*c^4*x^2 + 8*b*c^3*x + b^2*c
^2 + 15*a*c^3)*sqrt(c*x^2 + b*x + a)*sqrt(d*x))/(c^3*d)
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{\sqrt{dx}}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}}{\sqrt{dx}}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/(d*x)**(1/2),x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)/sqrt(d*x), x)
```

Maxima [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{\sqrt{dx}}\,dx = \int \frac{(cx^2+bx+a)^{\frac{3}{2}}}{\sqrt{dx}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)/sqrt(d*x), x)
```

Giac [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{\sqrt{dx}}\,dx = \int \frac{(cx^2+bx+a)^{\frac{3}{2}}}{\sqrt{dx}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(d*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)/sqrt(d*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{\sqrt{dx}}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}}{\sqrt{d\,x}}\,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)/(d*x)^(1/2),x)
```

output
```
int((a + b*x + c*x^2)^(3/2)/(d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{\sqrt{dx}}\,dx = \frac{\sqrt{d}\,\Big(46\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,a+16\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,bx+10\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,c}{}$$

input
```
int((c*x^2+b*x+a)^(3/2)/(d*x)^(1/2),x)
```

output
```
(sqrt(d)*(46*sqrt(x)*sqrt(a + b*x + c*x**2)*a + 16*sqrt(x)*sqrt(a + b*x +
c*x**2)*b*x + 10*sqrt(x)*sqrt(a + b*x + c*x**2)*c*x**2 - 24*int((sqrt(x)*s
qrt(a + b*x + c*x**2)*x)/(a + b*x + c*x**2),x)*a*c + 3*int((sqrt(x)*sqrt(a
 + b*x + c*x**2)*x)/(a + b*x + c*x**2),x)*b**2 + 12*int((sqrt(x)*sqrt(a +
b*x + c*x**2))/(a*x + b*x**2 + c*x**3),x)*a**2))/(35*d)
```

3.319 $\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 483

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{3/2}}\,dx = \frac{2\sqrt{dx}(7b+6cx)\sqrt{a+bx+cx^2}}{5d^2} - \frac{2(a+bx+cx^2)^{3/2}}{d\sqrt{dx}} + \frac{(b^2+12ac)\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\,E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{dx}}{\sqrt{-b+\sqrt{b^2-4ac}}\sqrt{d}}\right.\right.}{5\sqrt{2}c^{3/2}d^{3/2}\sqrt{a+x(b+cx)}} - \frac{\sqrt{-b+\sqrt{b^2-4ac}}\left(b^3-4abc+\sqrt{b^2-4ac}(b^2+12ac)\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(\arcsin\right.}{5\sqrt{2}c^{3/2}d^{3/2}\sqrt{a+x(b+cx)}}$$

output

```
2/5*(d*x)^(1/2)*(6*c*x+7*b)*(c*x^2+b*x+a)^(1/2)/d^2-2*(c*x^2+b*x+a)^(3/2)/
d/(d*x)^(1/2)+1/10*(12*a*c+b^2)*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(b+(-4*a*c+b
^2)^(1/2))*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)
^(1/2)))^(1/2)*EllipticE(2^(1/2)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2
))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^
(1/2)/c^(3/2)/d^(3/2)/(a+x*(c*x+b))^(1/2)-1/10*(-b+(-4*a*c+b^2)^(1/2))^(1/
2)*(b^3-4*a*b*c+(-4*a*c+b^2)^(1/2)*(12*a*c+b^2))*(1+2*c*x/(b-(-4*a*c+b^2)^
(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(2^(1/2)*c^(
1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1
/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/c^(3/2)/d^(3/2)/(a+x*(c*x+b))^
(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.58 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{3/2}}\,dx = \frac{x^{3/2}\left(\frac{4(b^2+12ac)(a+x(b+cx))}{c\sqrt{x}} + \frac{4(a+x(b+cx))(-5a+x(2b+cx))}{\sqrt{x}} - \frac{i(b^2+12ac)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{2+\cdots}}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/(d*x)^(3/2),x]
```

output
```
(x^(3/2)*((4*(b^2 + 12*a*c)*(a + x*(b + c*x)))/(c*Sqrt[x]) + (4*(a + x*(b
+ c*x))*(-5*a + x*(2*b + c*x)))/Sqrt[x] - (I*(b^2 + 12*a*c)*(-b + Sqrt[b^2
 - 4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x*Sqrt[(2*a + b*x -
 Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticE[I*ArcSinh[(Sq
rt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(
b - Sqrt[b^2 - 4*a*c])])/(c*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]) + (I*(-b^3 +
4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 12*a*c*Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4*a)
/((b + Sqrt[b^2 - 4*a*c])*x)]*x*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*
x - Sqrt[b^2 - 4*a*c]*x)]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^
2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/
(c*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])))/(10*(d*x)^(3/2)*Sqrt[a + x*(b + c*x)
])
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.78, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1161, 1231, 27, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(dx)^{3/2}}dx$$

$$\Big\downarrow \ \text{1161}$$

$$\frac{3\int \frac{(b+2cx)\sqrt{cx^2+bx+a}}{\sqrt{dx}}dx}{d}-\frac{2\left(a+bx+cx^2\right)^{3/2}}{d\sqrt{dx}}$$

$$\Big\downarrow \ \text{1231}$$

$$\frac{3\left(\frac{2\sqrt{dx}(7b+6cx)\sqrt{a+bx+cx^2}}{15d}-\frac{2\int -\frac{cd^2\left(8ab+\left(b^2+12ac\right)x\right)}{2\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{15cd^2}\right)}{d}-\frac{2\left(a+bx+cx^2\right)^{3/2}}{d\sqrt{dx}}$$

$$\Big\downarrow \ \text{27}$$

$$\frac{3\left(\frac{1}{15}\int \frac{8ab+\left(b^2+12ac\right)x}{\sqrt{dx}\sqrt{cx^2+bx+a}}dx+\frac{2\sqrt{dx}(7b+6cx)\sqrt{a+bx+cx^2}}{15d}\right)}{d}-\frac{2\left(a+bx+cx^2\right)^{3/2}}{d\sqrt{dx}}$$

$$\Big\downarrow \ \text{1241}$$

$$\frac{3\left(\frac{\sqrt{x}\int \frac{8ab+\left(b^2+12ac\right)x}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{15\sqrt{dx}}+\frac{2\sqrt{dx}(7b+6cx)\sqrt{a+bx+cx^2}}{15d}\right)}{d}-\frac{2\left(a+bx+cx^2\right)^{3/2}}{d\sqrt{dx}}$$

$$\Big\downarrow \ \text{1240}$$

$$\frac{3\left(\frac{2\sqrt{x}\int \frac{8ab+\left(b^2+12ac\right)x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{15\sqrt{dx}}+\frac{2\sqrt{dx}(7b+6cx)\sqrt{a+bx+cx^2}}{15d}\right)}{d}-\frac{2\left(a+bx+cx^2\right)^{3/2}}{d\sqrt{dx}}$$

$$\Big\downarrow \ \text{1511}$$

$$\frac{3\left(\frac{2\sqrt{x}\left(\frac{\sqrt{a}\left(8\sqrt{a}b\sqrt{c}+12ac+b^2\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{a}\left(12ac+b^2\right)\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{15\sqrt{dx}}+\frac{2\sqrt{dx}(7b+6cx)\sqrt{a+bx+cx^2}}{15d}\right)}{d}-\frac{2\left(a+bx+cx^2\right)^{3/2}}{d\sqrt{dx}}$$

$$\Big\downarrow \ \text{27}$$

$$\frac{3\left(\frac{2\sqrt{x}\left(\frac{\sqrt{a}\left(8\sqrt{a}b\sqrt{c}+12ac+b^2\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}-\frac{\left(12ac+b^2\right)\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{15\sqrt{dx}}+\frac{2\sqrt{dx}(7b+6cx)\sqrt{a+bx+cx^2}}{15d}\right)}{d}-\frac{2\left(a+bx+cx^2\right)^{3/2}}{d\sqrt{dx}}$$

$\big\downarrow$ 1416

$$\frac{3\left(\frac{2\sqrt{x}\left(\frac{\sqrt[4]{a}\left(8\sqrt{a}b\sqrt{c}+12ac+b^2\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx+cx^2}}-\frac{\left(12ac+b^2\right)\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{15\sqrt{dx}}+\frac{2\sqrt{dx}\cdots}{\cdots}\right)}{d}-\frac{2\left(a+bx+cx^2\right)^{3/2}}{d\sqrt{dx}}$$

$\big\downarrow$ 1509

$$\frac{3\left(\frac{2\sqrt{x}\left(\frac{\sqrt[4]{a}\left(8\sqrt{a}b\sqrt{c}+12ac+b^2\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx+cx^2}}-\frac{\left(12ac+b^2\right)\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\cdots}{\sqrt[4]{c}\cdots}\cdots\right)}{\cdots}\right)}{15\sqrt{dx}}\cdots\right)}{d}-\frac{2\left(a+bx+cx^2\right)^{3/2}}{d\sqrt{dx}}$$

input
```
Int[(a + b*x + c*x^2)^(3/2)/(d*x)^(3/2),x]
```

output
```
(-2*(a + b*x + c*x^2)^(3/2))/(d*Sqrt[d*x]) + (3*((2*Sqrt[d*x]*(7*b + 6*c*x
)*Sqrt[a + b*x + c*x^2])/(15*d) + (2*Sqrt[x]*(-(((b^2 + 12*a*c)*(-((Sqrt[x
]*Sqrt[a + b*x + c*x^2])/(Sqrt[a] + Sqrt[c]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt
[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[
(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a
+ b*x + c*x^2])))/Sqrt[c]) + (a^(1/4)*(b^2 + 8*Sqrt[a]*b*Sqrt[c] + 12*a*c)
*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*Ell
ipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/
(2*c^(3/4)*Sqrt[a + b*x + c*x^2])))/(15*Sqrt[d*x])))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 767, normalized size of antiderivative = 1.59

method	result
risch	$-\frac{2\sqrt{c\,x^2+bx+a}\,\left(-c\,x^2-2bx+5a\right)}{5d\sqrt{dx}}+\frac{\left(12ac+b^2\right)\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{-}}{}$
elliptic	$\frac{\sqrt{dx(c\,x^2+bx+a)}\left(-\frac{2\left(cd\,x^2+bdx+ad\right)a}{d^2\sqrt{x\left(cd\,x^2+bdx+ad\right)}}+\frac{2cx\sqrt{cd\,x^3+bd\,x^2+adx}}{5d^2}+\frac{4b\sqrt{cd\,x^3+bd\,x^2+adx}}{5d^2}+\frac{8ab\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b}}{2c}\right)}{b+\sqrt{-4ac+b^2}}}}{}\right.}{}$
default	Expression too large to display

input
```
int((c*x^2+b*x+a)^(3/2)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(c*x^2+b*x+a)^(1/2)*(-c*x^2-2*b*x+5*a)/d/(d*x)^(1/2)+(1/5*(12*a*c+b^2
)*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4
*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*
c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)*((-1/2*(b+(-4*a*c+b^2)^
(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(2^(1/2)*((x+1/2*(b+(-4*a
*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^
(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1
/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^
2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2)
)/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))
+8/5*a*b*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c
)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2
*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b
+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)*EllipticF(2^(1/2
)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-
2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*
c+b^2)^(1/2))))^(1/2)))/d*(d*x*(c*x^2+b*x+a))^(1/2)/(d*x)^(1/2)/(c*x^2+b*x
+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.41

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{3/2}}\,dx =$$

$$-\frac{2\left((b^3-12\,abc)\sqrt{cdx}\text{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\frac{3\,cx+b}{3\,c}\right)+3\,(b^2c+12\,ac^2)\sqrt{cdx}\text{weierst}\right.}{}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/15*((b^3 - 12*a*b*c)*sqrt(c*d)*x*weierstrassPInverse(4/3*(b^2 - 3*a*c)/
c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c) + 3*(b^2*c + 12*a*c^2
)*sqrt(c*d)*x*weierstrassZeta(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*
c)/c^3, weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)
/c^3, 1/3*(3*c*x + b)/c)) - 3*(c^3*x^2 + 2*b*c^2*x - 5*a*c^2)*sqrt(c*x^2 +
 b*x + a)*sqrt(d*x))/(c^2*d^2*x)
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{3/2}}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/(d*x)**(3/2),x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)/(d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{3/2}}\,dx = \int \frac{(cx^2+bx+a)^{\frac{3}{2}}}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)/(d*x)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{3/2}}\,dx = \int \frac{(cx^2+bx+a)^{\frac{3}{2}}}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(d*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)/(d*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{3/2}}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}}{(d\,x)^{3/2}}\,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)/(d*x)^(3/2),x)
```

output
```
int((a + b*x + c*x^2)^(3/2)/(d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{3/2}}\,dx = \frac{\sqrt{d}\left(-10\sqrt{c\,x^2+bx+a}\,a+4\sqrt{c\,x^2+bx+a}\,bx+2\sqrt{c\,x^2+bx+a}\,c\,x^2+8\sqrt{x}\,\right.}{5}$$

input
```
int((c*x^2+b*x+a)^(3/2)/(d*x)^(3/2),x)
```

output
```
(sqrt(d)*( - 10*sqrt(a + b*x + c*x**2)*a + 4*sqrt(a + b*x + c*x**2)*b*x +
2*sqrt(a + b*x + c*x**2)*c*x**2 + 8*sqrt(x)*int((sqrt(x)*sqrt(a + b*x + c*
x**2))/(a*x + b*x**2 + c*x**3),x)*a*b + 12*sqrt(x)*int((sqrt(x)*sqrt(a + b
*x + c*x**2))/(a + b*x + c*x**2),x)*a*c + sqrt(x)*int((sqrt(x)*sqrt(a + b*
x + c*x**2))/(a + b*x + c*x**2),x)*b**2))/(5*sqrt(x)*d**2)
```

3.320 $\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 471

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{5/2}}\,dx = -\frac{2(3b-2cx)\sqrt{a+bx+cx^2}}{3d^2\sqrt{dx}} - \frac{2(a+bx+cx^2)^{3/2}}{3d(dx)^{3/2}} + \frac{4\sqrt{2}b\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{dx}}{\sqrt{-b+\sqrt{b^2-4ac}}\sqrt{d}}\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{c}d^{5/2}\sqrt{a+x(b+cx)}} - \frac{\sqrt{2}\sqrt{-b+\sqrt{b^2-4ac}}\left(b^2-4ac+4b\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{dx}}{\sqrt{-b+\sqrt{b^2-4ac}}\sqrt{d}}\right),\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{c}d^{5/2}\sqrt{a+x(b+cx)}}$$

output

```
-2/3*(-2*c*x+3*b)*(c*x^2+b*x+a)^(1/2)/d^2/(d*x)^(1/2)-2/3*(c*x^2+b*x+a)^(3
/2)/d/(d*x)^(3/2)+4/3*2^(1/2)*b*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(b+(-4*a*c+b
^2)^(1/2))*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)
^(1/2)))^(1/2)*EllipticE(2^(1/2)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2
))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))/c^
(1/2)/d^(5/2)/(a+x*(c*x+b))^(1/2)-1/3*2^(1/2)*(-b+(-4*a*c+b^2)^(1/2))^(1/2
)*(b^2-4*a*c+4*b*(-4*a*c+b^2)^(1/2))*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2
)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(2^(1/2)*c^(1/2)*(d*x)^(
1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*
a*c+b^2)^(1/2)))^(1/2))/c^(1/2)/d^(5/2)/(a+x*(c*x+b))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.89 (sec) , antiderivative size = 463, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{5/2}}\,dx = \frac{x\left(2\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}\left(-a^2+3abx+x^2(4b^2+5bcx+c^2x^2)\right)-4ib\left(-b+\sqrt{b^2-4ac}\right)\right.}{}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/(d*x)^(5/2),x]
```

output
```
(x*(2*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*(-a^2 + 3*a*b*x + x^2*(4*b^2 + 5*b*c
*x + c^2*x^2)) - (4*I)*b*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[1 + (2*a)/((b + Sqr
t[b^2 - 4*a*c])*x)]*x^(5/2)*Sqrt[(4*a + 2*b*x - 2*Sqrt[b^2 - 4*a*c]*x)/(b*
x - Sqrt[b^2 - 4*a*c]*x)]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^
2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] +
 I*(-b^2 + 4*a*c + 4*b*Sqrt[b^2 - 4*a*c])*Sqrt[1 + (2*a)/((b + Sqrt[b^2 -
4*a*c])*x)]*x^(5/2)*Sqrt[(4*a + 2*b*x - 2*Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt
[b^2 - 4*a*c]*x)]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*
c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(3*Sqrt
[a/(b + Sqrt[b^2 - 4*a*c])]*(d*x)^(5/2)*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.80, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1161, 1230, 27, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{5/2}}\,dx$$

$$\Big\downarrow \; 1161$$

$$\frac{\int \frac{(b+2cx)\sqrt{cx^2+bx+a}}{(dx)^{3/2}}dx}{d}-\frac{2\left(a+bx+cx^2\right)^{3/2}}{3d(dx)^{3/2}}$$

$\Big\downarrow$ 1230

$$\frac{-\frac{2\int -\frac{d\left(3b^2+8cxb+4ac\right)}{2\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{3d^2}-\frac{2(3b-2cx)\sqrt{a+bx+cx^2}}{3d\sqrt{dx}}}{d}-\frac{2\left(a+bx+cx^2\right)^{3/2}}{3d(dx)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{3b^2+8cxb+4ac}{\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{3d}-\frac{2(3b-2cx)\sqrt{a+bx+cx^2}}{3d\sqrt{dx}}}{d}-\frac{2\left(a+bx+cx^2\right)^{3/2}}{3d(dx)^{3/2}}$$

$\Big\downarrow$ 1241

$$\frac{\frac{\sqrt{x}\int \frac{3b^2+8cxb+4ac}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{3d\sqrt{dx}}-\frac{2(3b-2cx)\sqrt{a+bx+cx^2}}{3d\sqrt{dx}}}{d}-\frac{2\left(a+bx+cx^2\right)^{3/2}}{3d(dx)^{3/2}}$$

$\Big\downarrow$ 1240

$$\frac{\frac{2\sqrt{x}\int \frac{3b^2+8cxb+4ac}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{3d\sqrt{dx}}-\frac{2(3b-2cx)\sqrt{a+bx+cx^2}}{3d\sqrt{dx}}}{d}-\frac{2\left(a+bx+cx^2\right)^{3/2}}{3d(dx)^{3/2}}$$

$\Big\downarrow$ 1511

$$\frac{\frac{2\sqrt{x}\left(\left(8\sqrt{a}b\sqrt{c}+4ac+3b^2\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-8\sqrt{a}b\sqrt{c}\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\sqrt{x}\right)}{3d\sqrt{dx}}-\frac{2(3b-2cx)\sqrt{a+bx+cx^2}}{3d\sqrt{dx}}}{d}-\frac{2\left(a+bx+cx^2\right)^{3/2}}{3d(dx)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{2\sqrt{x}\left(\left(8\sqrt{a}b\sqrt{c}+4ac+3b^2\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-8b\sqrt{c}\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}\right)}{3d\sqrt{dx}}-\frac{2(3b-2cx)\sqrt{a+bx+cx^2}}{3d\sqrt{dx}}}{d}-\frac{2\left(a+bx+cx^2\right)^{3/2}}{3d(dx)^{3/2}}$$

$\Big\downarrow$ 1416

$$\frac{\frac{2\sqrt{x}\left(\frac{\left(8\sqrt{a}b\sqrt{c}+4ac+3b^2\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx+cx^2}}-8b\sqrt{c}\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}\right)}{3d\sqrt{dx}}-\frac{2(3b-2cx)\sqrt{a+bx-}}{3d\sqrt{dx}}}{d}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{1509}$$

$$\frac{\frac{2\sqrt{x}\left(\frac{\left(8\sqrt{a}b\sqrt{c}+4ac+3b^2\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx+cx^2}}-8b\sqrt{c}\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{}}{}\right.\right.}{\sqrt[4]{c}\sqrt{a+bx+cx^2}}\right.\right.}{3d\sqrt{dx}}}{d}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}}{3d(dx)^{3/2}}$$

input
```
Int[(a + b*x + c*x^2)^(3/2)/(d*x)^(5/2),x]
```

output
```
(-2*(a + b*x + c*x^2)^(3/2))/(3*d*(d*x)^(3/2)) + ((-2*(3*b - 2*c*x)*Sqrt[a
 + b*x + c*x^2])/(3*d*Sqrt[d*x]) + (2*Sqrt[x]*(-8*b*Sqrt[c]*(-((Sqrt[x]*Sq
rt[a + b*x + c*x^2])/(Sqrt[a] + Sqrt[c]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*
x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(
1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*
x + c*x^2])) + ((3*b^2 + 8*Sqrt[a]*b*Sqrt[c] + 4*a*c)*(Sqrt[a] + Sqrt[c]*x
)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1
/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*Sq
rt[a + b*x + c*x^2])))/(3*d*Sqrt[d*x]))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 827 vs. $2(381) = 762$.

Time = 2.74 (sec) , antiderivative size = 828, normalized size of antiderivative = 1.76

method	result
elliptic	$\sqrt{dx(c\,x^2+bx+a)}\left(-\frac{2a\sqrt{cd\,x^3+bd\,x^2+adx}}{3d^3x^2}-\frac{8\left(cd\,x^2+bdx+ad\right)b}{3d^3\sqrt{x\left(cd\,x^2+bdx+ad\right)}}+\frac{2c\sqrt{cd\,x^3+bd\,x^2+adx}}{3d^3}+\frac{\left(\frac{2ac+b^2}{d^2}-\frac{2ca}{3d^2}\right)\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x\cdots\right.}{\cdots}}}{\cdots}\right.$
risch	Expression too large to display
default	Expression too large to display

input
```
int((c*x^2+b*x+a)^(3/2)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(d*x*(c*x^2+b*x+a))^(1/2)/(d*x)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2/3/d^3*a*(c*d
*x^3+b*d*x^2+a*d*x)^(1/2)/x^2-8/3*(c*d*x^2+b*d*x+a*d)/d^3*b/(x*(c*d*x^2+b*
d*x+a*d))^(1/2)+2/3*c/d^3*(c*d*x^3+b*d*x^2+a*d*x)^(1/2)+((2*a*c+b^2)/d^2-2
/3*c/d^2*a)*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2)
)/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-
1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x
/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)*EllipticF(2^(
1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2
*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4
*a*c+b^2)^(1/2))))^(1/2))+8/3*b/d^2*(b+(-4*a*c+b^2)^(1/2))*2^(1/2)*((x+1/2
*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(
-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^
(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*
x)^(1/2)*((-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*El
lipticE(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c
)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/
2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*Ellipti
cF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/
2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(
-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.40

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{5/2}}\,dx =$$

$$-\frac{2\left(24\sqrt{cd}bcx^2\text{weierstrassZeta}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\text{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\frac{3\,cx+}{3\,c}\right.\right.\right.}{}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/9*(24*sqrt(c*d)*b*c*x^2*weierstrassZeta(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2
*b^3 - 9*a*b*c)/c^3, weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b
^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c)) - (b^2 + 12*a*c)*sqrt(c*d)*x^2*weie
rstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3
*c*x + b)/c) - 3*(c^2*x^2 - 4*b*c*x - a*c)*sqrt(c*x^2 + b*x + a)*sqrt(d*x)
)/(c*d^3*x^2)
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{5/2}}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/(d*x)**(5/2),x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)/(d*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{5/2}}\,dx = \int \frac{(cx^2+bx+a)^{\frac{3}{2}}}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)/(d*x)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{5/2}}\,dx = \int \frac{(cx^2+bx+a)^{\frac{3}{2}}}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(d*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)/(d*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{5/2}}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}}{(d\,x)^{5/2}}\,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)/(d*x)^(5/2),x)
```

output
```
int((a + b*x + c*x^2)^(3/2)/(d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{5/2}}\,dx = \frac{\sqrt{d}\left(-10\sqrt{c\,x^2+bx+a}\,a+8\sqrt{c\,x^2+bx+a}\,bx+2\sqrt{c\,x^2+bx+a}\,c\,x^2-12\sqrt{x}\right.}{3\sqrt{x}\,d^3x}$$

input
```
int((c*x^2+b*x+a)^(3/2)/(d*x)^(5/2),x)
```

output
```
(sqrt(d)*( - 10*sqrt(a + b*x + c*x**2)*a + 8*sqrt(a + b*x + c*x**2)*b*x +
2*sqrt(a + b*x + c*x**2)*c*x**2 - 12*sqrt(x)*int((sqrt(x)*sqrt(a + b*x + c
*x**2))/(a*x**3 + b*x**4 + c*x**5),x)*a**2*x + 3*sqrt(x)*int((sqrt(x)*sqrt
(a + b*x + c*x**2))/(a*x + b*x**2 + c*x**3),x)*b**2*x))/(3*sqrt(x)*d**3*x)
```

3.321 $\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 503

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{7/2}}\,dx = -\frac{2c(6a-bx)\sqrt{a+bx+cx^2}}{5ad^3\sqrt{dx}} - \frac{2(a+bx)\left(a+bx+cx^2\right)^{3/2}}{5ad(dx)^{5/2}}$$
$$+\frac{(b^2+12ac)\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{dx}}{\sqrt{-b+\sqrt{b^2-4ac}}\sqrt{d}}\right.\right.}{5\sqrt{2}a\sqrt{c}d^{7/2}\sqrt{a+x(b+cx)}}$$
$$-\frac{\sqrt{-b+\sqrt{b^2-4ac}}\left(b^3-4abc+\sqrt{b^2-4ac}(b^2+12ac)\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(\arcsin\right.}{5\sqrt{2}a\sqrt{c}d^{7/2}\sqrt{a+x(b+cx)}}$$

output
```
-2/5*c*(-b*x+6*a)*(c*x^2+b*x+a)^(1/2)/a/d^3/(d*x)^(1/2)-2/5*(b*x+a)*(c*x^2
+b*x+a)^(3/2)/a/d/(d*x)^(5/2)+1/10*(12*a*c+b^2)*(-b+(-4*a*c+b^2)^(1/2))^(1
/2)*(b+(-4*a*c+b^2)^(1/2))*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x
/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2^(1/2)*c^(1/2)*(d*x)^(1/2)/(-b+(
-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(
1/2)))^(1/2))*2^(1/2)/a/c^(1/2)/d^(7/2)/(a+x*(c*x+b))^(1/2)-1/10*(-b+(-4*a
*c+b^2)^(1/2))^(1/2)*(b^3-4*a*b*c+(-4*a*c+b^2)^(1/2)*(12*a*c+b^2))*(1+2*c*
x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*Ell
ipticF(2^(1/2)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),(
(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/a/c^(1/2)/d^
(7/2)/(a+x*(c*x+b))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.63 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{7/2}}\,dx = \frac{x\left(-4(a^2+2abx+(b^2+7ac)\,x^2)\,(a+x(b+cx)) + \frac{x^2\left(4(b^2+12ac)\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}(a+x(\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/(d*x)^(7/2),x]
```

output
```
(x*(-4*(a^2 + 2*a*b*x + (b^2 + 7*a*c)*x^2)*(a + x*(b + c*x)) + (x^2*(4*(b^
2 + 12*a*c)*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*(a + x*(b + c*x)) + I*(b^2 + 1
2*a*c)*(b - Sqrt[b^2 - 4*a*c])*Sqrt[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]
*x^(3/2)*Sqrt[(4*a + 2*b*x - 2*Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*
c]*x)]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[
x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^3 + 4*a*b*c
+ b^2*Sqrt[b^2 - 4*a*c] + 12*a*c*Sqrt[b^2 - 4*a*c])*Sqrt[1 + (2*a)/((b + S
qrt[b^2 - 4*a*c])*x)]*x^(3/2)*Sqrt[(4*a + 2*b*x - 2*Sqrt[b^2 - 4*a*c]*x)/(
b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[
b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]
))/Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]))/(10*a*(d*x)^(7/2)*Sqrt[a + x*(b + c*x
)])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 396, normalized size of antiderivative = 0.79, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1161, 1229, 27, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(dx)^{7/2}}\,dx$$

$$\Big\downarrow \ 1161$$

$$\frac{3\int \frac{(b+2cx)\sqrt{cx^2+bx+a}}{(dx)^{5/2}}dx}{5d}-\frac{2\left(a+bx+cx^2\right)^{3/2}}{5d(dx)^{5/2}}$$

$$\Big\downarrow \ 1229$$

$$\frac{3\left(-\frac{2\int -\frac{cd^2\left(8ab+\left(b^2+12ac\right)x\right)}{2\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{3ad^4}-\frac{2\sqrt{a+bx+cx^2}\left(x\left(6ac+b^2\right)+ab\right)}{3ad(dx)^{3/2}}\right)}{5d}-\frac{2\left(a+bx+cx^2\right)^{3/2}}{5d(dx)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{3\left(\frac{c\int \frac{8ab+\left(b^2+12ac\right)x}{\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{3ad^2}-\frac{2\left(x\left(6ac+b^2\right)+ab\right)\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}\right)}{5d}-\frac{2\left(a+bx+cx^2\right)^{3/2}}{5d(dx)^{5/2}}$$

$$\Big\downarrow \ 1241$$

$$\frac{3\left(\frac{c\sqrt{x}\int \frac{8ab+\left(b^2+12ac\right)x}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{3ad^2\sqrt{dx}}-\frac{2\left(x\left(6ac+b^2\right)+ab\right)\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}\right)}{5d}-\frac{2\left(a+bx+cx^2\right)^{3/2}}{5d(dx)^{5/2}}$$

$$\Big\downarrow \ 1240$$

$$\frac{3\left(\frac{2c\sqrt{x}\int \frac{8ab+\left(b^2+12ac\right)x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{3ad^2\sqrt{dx}}-\frac{2\left(x\left(6ac+b^2\right)+ab\right)\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}\right)}{5d}-\frac{2\left(a+bx+cx^2\right)^{3/2}}{5d(dx)^{5/2}}$$

$$\Big\downarrow \ 1511$$

$$\frac{3\left(\frac{2c\sqrt{x}\left(\frac{\sqrt{a}\left(8\sqrt{a}b\sqrt{c}+12ac+b^2\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{a}\left(12ac+b^2\right)\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{3ad^2\sqrt{dx}}-\frac{2\left(x\left(6ac+b^2\right)+ab\right)\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}\right)}{5d}-\frac{2\left(a+bx+cx^2\right)^{3/2}}{5d(dx)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{3\left(\frac{2c\sqrt{x}\left(\frac{\sqrt{a}\left(8\sqrt{a}b\sqrt{c}+12ac+b^2\right)\int\frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}-\frac{\left(12ac+b^2\right)\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{3ad^2\sqrt{dx}}-\frac{2\left(x\left(6ac+b^2\right)+ab\right)\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}\right)}{5d}-\frac{2\left(a+bx+cx^2\right)^{3/2}}{5d(dx)^{5/2}}$$

$$\Big\downarrow \; 1416$$

$$\frac{3\left(\frac{2c\sqrt{x}\left(\frac{\sqrt[4]{a}\left(8\sqrt{a}b\sqrt{c}+12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx+cx^2}}-\frac{\left(12ac+b^2\right)\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{3ad^2\sqrt{dx}}-\frac{2(x(\cdots}{\cdots}\right)}{5d}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}}{5d(dx)^{5/2}}$$

$$\Big\downarrow \; 1509$$

$$\frac{3\left(\frac{2c\sqrt{x}\left(\frac{\sqrt[4]{a}\left(8\sqrt{a}b\sqrt{c}+12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx+cx^2}}-\frac{\left(12ac+b^2\right)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}\cdots}{\sqrt[4]{\cdots}}\cdots\right)}{\cdots}\right)}{3ad^2\sqrt{dx}}\cdots\right)}{5d}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}}{5d(dx)^{5/2}}$$

input
```
Int[(a + b*x + c*x^2)^(3/2)/(d*x)^(7/2),x]
```

output
```
(-2*(a + b*x + c*x^2)^(3/2))/(5*d*(d*x)^(5/2)) + (3*((-2*(a*b + (b^2 + 6*a
*c)*x)*Sqrt[a + b*x + c*x^2])/(3*a*d*(d*x)^(3/2)) + (2*c*Sqrt[x]*(-(((b^2
+ 12*a*c)*(-((Sqrt[x]*Sqrt[a + b*x + c*x^2])/(Sqrt[a] + Sqrt[c]*x)) + (a^(
1/4)*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]
*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/
4])/(c^(1/4)*Sqrt[a + b*x + c*x^2])))/Sqrt[c]) + (a^(1/4)*(b^2 + 8*Sqrt[a]
*b*Sqrt[c] + 12*a*c)*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a]
 + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sq
rt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x + c*x^2])))/(3*a*d^2*Sqrt[d*x]
)))/(5*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 786, normalized size of antiderivative = 1.56

method	result
risch	$-\frac{2\sqrt{c\,x^{2}+bx+a}\,\left(7ac\,x^{2}+b^{2}x^{2}+2abx+a^{2}\right)}{5x^{2}a\,d^{3}\sqrt{dx}}+\frac{c\left(\frac{\left(12ac+b^{2}\right)\left(b+\sqrt{-4ac+b^{2}}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^{2}}}{2c}\right)c}{b+\sqrt{-4ac+b^{2}}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^{2}}}{2c}}{-\frac{b+\sqrt{-4ac+b^{2}}}{2c}-\frac{-b+\sqrt{-4a}}{2c}}}}{}\right)}{}$
elliptic	$\frac{\sqrt{dx\left(c\,x^{2}+bx+a\right)}\left(-\frac{2a\sqrt{cd\,x^{3}+bd\,x^{2}+adx}}{5d^{4}x^{3}}-\frac{4b\sqrt{cd\,x^{3}+bd\,x^{2}+adx}}{5d^{4}x^{2}}-\frac{2\left(cd\,x^{2}+bdx+ad\right)\left(7ac+b^{2}\right)}{5d^{4}a\sqrt{x\left(cd\,x^{2}+bdx+ad\right)}}+\frac{8b\left(b+\sqrt{-4ac+b^{2}}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac}}{2c}\right)}{b+\sqrt{-4ac}}}}{}\right)}{}$
default	Expression too large to display

input
```
int((c*x^2+b*x+a)^(3/2)/(d*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(c*x^2+b*x+a)^(1/2)*(7*a*c*x^2+b^2*x^2+2*a*b*x+a^2)/x^2/a/d^3/(d*x)^(
1/2)+1/5*c/a*((12*a*c+b^2)*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4
*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+
b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))
)^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2
)*((-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE
(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
,1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b
+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(2^(1
/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*
(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*
a*c+b^2)^(1/2))))^(1/2)))+8*a*b*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(
b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4
*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1
/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)
^(1/2)*EllipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)
^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1
/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))/d^3*(d*x*(c*x^2+b*x+a))^(1/
2)/(d*x)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.43

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{7/2}}\,dx =$$

$$-\frac{2\left((b^3-12\,abc)\sqrt{cd}x^3\text{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\frac{3\,cx+b}{3\,c}\right)+3\,(b^2c+12\,ac^2)\sqrt{cd}x^3\text{weier}\right.}{}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(d*x)^(7/2),x, algorithm="fricas")
```

output
```
-2/15*((b^3 - 12*a*b*c)*sqrt(c*d)*x^3*weierstrassPInverse(4/3*(b^2 - 3*a*c
)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c) + 3*(b^2*c + 12*a*c
^2)*sqrt(c*d)*x^3*weierstrassZeta(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*
a*b*c)/c^3, weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*
b*c)/c^3, 1/3*(3*c*x + b)/c)) + 3*(2*a*b*c*x + a^2*c + (b^2*c + 7*a*c^2)*x
^2)*sqrt(c*x^2 + b*x + a)*sqrt(d*x))/(a*c*d^4*x^3)
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{7/2}}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}}{(dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/(d*x)**(7/2),x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)/(d*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{7/2}}\,dx = \int \frac{(cx^2+bx+a)^{\frac{3}{2}}}{(dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(d*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)/(d*x)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{7/2}}\,dx = \int \frac{(cx^2+bx+a)^{\frac{3}{2}}}{(dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(d*x)^(7/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)/(d*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{7/2}}\,dx=\int \frac{(c\,x^2+b\,x+a)^{3/2}}{(d\,x)^{7/2}}\,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)/(d*x)^(7/2),x)
```

output
```
int((a + b*x + c*x^2)^(3/2)/(d*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{7/2}}\,dx=\frac{\sqrt{d}\left(-2\sqrt{c\,x^2+bx+a}\,a^2-20\sqrt{c\,x^2+bx+a}\,abx-14\sqrt{c\,x^2+bx+a}\,ac\,x^2+3\right.}{}$$

input
```
int((c*x^2+b*x+a)^(3/2)/(d*x)^(7/2),x)
```

output
```
(sqrt(d)*( - 2*sqrt(a + b*x + c*x**2)*a**2 - 20*sqrt(a + b*x + c*x**2)*a*b
*x - 14*sqrt(a + b*x + c*x**2)*a*c*x**2 + 30*sqrt(a + b*x + c*x**2)*b**2*x
**2 - 24*sqrt(x)*int(sqrt(a + b*x + c*x**2)/(sqrt(x)*a*x**2 + sqrt(x)*b*x*
*3 + sqrt(x)*c*x**4),x)*a**2*b*x**2 - 18*sqrt(x)*int((sqrt(a + b*x + c*x**
2)*x)/(sqrt(x)*a + sqrt(x)*b*x + sqrt(x)*c*x**2),x)*a*c**2*x**2 + 30*sqrt(
x)*int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a + b*x + c*x**2),x)*a*c**2*x**2
- 15*sqrt(x)*int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a + b*x + c*x**2),x)*b*
*2*c*x**2))/(5*sqrt(x)*a*d**4*x**2)
```

3.322 $\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{9/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 534

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{9/2}} \, dx = \frac{2(2a(b^2-5ac)+b(2b^2-13ac)\,x)\sqrt{a+bx+cx^2}}{35a^2d^3(dx)^{3/2}}$$

$$-\frac{2(5a+3bx)\,(a+bx+cx^2)^{3/2}}{35ad(dx)^{7/2}}$$

$$-\frac{\sqrt{2}b(b^2-8ac)\sqrt{-b+\sqrt{b^2-4ac}}\,(b+\sqrt{b^2-4ac})\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\,E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{dx}}{\sqrt{-b+\sqrt{b^2-4ac}}\ldots}\right.\right.}{35a^2\sqrt{c}d^{9/2}\sqrt{a+x(b+cx)}}$$

$$+\frac{\sqrt{2}\sqrt{-b+\sqrt{b^2-4ac}}\,(b^4-9ab^2c+20a^2c^2+b(b^2-8ac)\sqrt{b^2-4ac})\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\,\text{Elli}\ldots}{35a^2\sqrt{c}d^{9/2}\sqrt{a+x(b+cx)}}$$

output
```
2/35*(2*a*(-5*a*c+b^2)+b*(-13*a*c+2*b^2)*x)*(c*x^2+b*x+a)^(1/2)/a^2/d^3/(d
*x)^(3/2)-2/35*(3*b*x+5*a)*(c*x^2+b*x+a)^(3/2)/a/d/(d*x)^(7/2)-1/35*2^(1/2
)*b*(-8*a*c+b^2)*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*(1+2
*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*
EllipticE(2^(1/2)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2
),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))/a^2/c^(1/2)/d^(9/
2)/(a+x*(c*x+b))^(1/2)+1/35*2^(1/2)*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(b^4-9*a
*b^2*c+20*a^2*c^2+b*(-8*a*c+b^2)*(-4*a*c+b^2)^(1/2))*(1+2*c*x/(b-(-4*a*c+b
^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(2^(1/2)
*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2
)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))/a^2/c^(1/2)/d^(9/2)/(a+x*(c*x+b))^
(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.33 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{9/2}}\,dx = \frac{x\left(-4b(b^2-8ac)\,x^3(a+x(b+cx))-2(a+x(b+cx))\,(5a^3-2b^3x^3+a^2x(8b+\right.}{}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/(d*x)^(9/2),x]
```

output
```
(x*(-4*b*(b^2 - 8*a*c)*x^3*(a + x*(b + c*x)) - 2*(a + x*(b + c*x))*(5*a^3
- 2*b^3*x^3 + a^2*x*(8*b + 15*c*x) + a*b*x^2*(b + 16*c*x)) + (I*b*(b^2 - 8
*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[b^2 - 4*a*c])*x)]
*x^(9/2)*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x
)]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]],
 (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a/(b + Sqrt[b^2 -
4*a*c])] + (I*(b^4 - 9*a*b^2*c + 20*a^2*c^2 - b^3*Sqrt[b^2 - 4*a*c] + 8*a*
b*c*Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(9/2)
*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*Ellip
ticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sq
rt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]
))/(35*a^2*(d*x)^(9/2)*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 455, normalized size of antiderivative = 0.85, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1161, 1229, 27, 1237, 27, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{9/2}}\,dx$$

$$\Big\downarrow \text{1161}$$

$$\frac{3\int \frac{(b+2cx)\sqrt{cx^2+bx+a}}{(dx)^{7/2}}dx}{7d}-\frac{2(a+bx+cx^2)^{3/2}}{7d(dx)^{7/2}}$$

$$\Big\downarrow \text{1229}$$

$$\frac{3\left(-\frac{2\int \frac{d^2\left(2b\left(b^2-8ac\right)+c\left(b^2-20ac\right)x\right)}{2(dx)^{3/2}\sqrt{cx^2+bx+a}}dx}{15ad^4}-\frac{2\sqrt{a+bx+cx^2}\left(x\left(10ac+b^2\right)+3ab\right)}{15ad(dx)^{5/2}}\right)}{7d}-\frac{2(a+bx+cx^2)^{3/2}}{7d(dx)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(-\frac{\int \frac{2b(b^2-8ac)+c(b^2-20ac)x}{(dx)^{3/2}\sqrt{cx^2+bx+a}}dx}{15ad^2}-\frac{2\sqrt{a+bx+cx^2}(x(10ac+b^2)+3ab)}{15ad(dx)^{5/2}}\right)}{7d}-\frac{2(a+bx+cx^2)^{3/2}}{7d(dx)^{7/2}}$$

$$\Big\downarrow \text{1237}$$

$$\frac{3\left(-\frac{-\frac{2\int -\frac{cd(a(b^2-20ac)+2b(b^2-8ac)x)}{2\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{ad^2}-\frac{4b(b^2-8ac)\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{15ad^2}-\frac{2\sqrt{a+bx+cx^2}(x(10ac+b^2)+3ab)}{15ad(dx)^{5/2}}\right)}{7d}-\frac{2(a+bx+cx^2)^{3/2}}{7d(dx)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(-\frac{\frac{c\int \frac{a(b^2-20ac)+2b(b^2-8ac)x}{\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{ad}-\frac{4b(b^2-8ac)\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{15ad^2}-\frac{2\sqrt{a+bx+cx^2}(x(10ac+b^2)+3ab)}{15ad(dx)^{5/2}}\right)}{7d}-\frac{2(a+bx+cx^2)^{3/2}}{7d(dx)^{7/2}}$$

$$\Big\downarrow \text{1241}$$

$$\frac{3\left(-\frac{\frac{c\sqrt{x}\int \frac{a(b^2-20ac)+2b(b^2-8ac)x}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{ad\sqrt{dx}}-\frac{4b(b^2-8ac)\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{15ad^2}-\frac{2\sqrt{a+bx+cx^2}(x(10ac+b^2)+3ab)}{15ad(dx)^{5/2}}\right)}{7d}-\frac{2(a+bx+cx^2)^{3/2}}{7d(dx)^{7/2}}$$

$$\Big\downarrow \text{1240}$$

$$\frac{3\left(-\frac{\frac{2c\sqrt{x}\int \frac{a(b^2-20ac)+2b(b^2-8ac)x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{ad\sqrt{dx}}-\frac{4b(b^2-8ac)\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{15ad^2}-\frac{2\sqrt{a+bx+cx^2}(x(10ac+b^2)+3ab)}{15ad(dx)^{5/2}}\right)}{7d}-\frac{2(a+bx+cx^2)^{3/2}}{7d(dx)^{7/2}}$$

$$\Big\downarrow \text{1511}$$

$$\frac{3\left(-\frac{\frac{2c\sqrt{x}\left(\sqrt{a}\left(\sqrt{a}\left(b^2-20ac\right)+\frac{2b\left(b^2-8ac\right)}{\sqrt{c}}\right)\int\frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{2\sqrt{a}b\left(b^2-8ac\right)\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{ad\sqrt{dx}}-\frac{4b\left(b^2-8ac\right)\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{15ad^2}-\frac{2\sqrt{a+bx+cx}}{15}\right.}{7d}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}}{7d(dx)^{7/2}}$$

$\downarrow$ 27

$$\frac{3\left(-\frac{\frac{2c\sqrt{x}\left(\sqrt{a}\left(\sqrt{a}\left(b^2-20ac\right)+\frac{2b\left(b^2-8ac\right)}{\sqrt{c}}\right)\int\frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{2b\left(b^2-8ac\right)\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{ad\sqrt{dx}}-\frac{4b\left(b^2-8ac\right)\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{15ad^2}-\frac{2\sqrt{a+bx+cx^2}(x(1}{15ad(dx}\right.}{7d}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}}{7d(dx)^{7/2}}$$

$\downarrow$ 1416

$$\frac{3\left(-\frac{\frac{2c\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}\left(b^2-20ac\right)+\frac{2b\left(b^2-8ac\right)}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}-\frac{2b\left(b^2-8ac\right)\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{}}{\sqrt{c}}\right.}{ad\sqrt{dx}}}{15ad^2}\right.}{7d}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}}{7d(dx)^{7/2}}$$

$\downarrow$ 1509

$$3\left(-\frac{\frac{2c\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}\left(b^2-20ac\right)+\frac{2b\left(b^2-8ac\right)}{\sqrt{c}}\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}-\frac{2b\left(b^2-8ac\right)\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x)}{}\right.}{}\right.}{ad\sqrt{dx}}}{15ad^2}\right.$$

$$\overline{\hspace{30em}7d}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}}{7d(dx)^{7/2}}$$

input

```
Int[(a + b*x + c*x^2)^(3/2)/(d*x)^(9/2),x]
```

output

```
(-2*(a + b*x + c*x^2)^(3/2))/(7*d*(d*x)^(7/2)) + (3*((-2*(3*a*b + (b^2 + 1
0*a*c)*x)*Sqrt[a + b*x + c*x^2])/(15*a*d*(d*x)^(5/2)) - ((-4*b*(b^2 - 8*a*
c)*Sqrt[a + b*x + c*x^2])/(a*d*Sqrt[d*x]) + (2*c*Sqrt[x]*((-2*b*(b^2 - 8*a
*c)*(-((Sqrt[x]*Sqrt[a + b*x + c*x^2])/(Sqrt[a] + Sqrt[c]*x)) + (a^(1/4)*(
Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*Ellip
ticE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c
^(1/4)*Sqrt[a + b*x + c*x^2])))/Sqrt[c] + (a^(1/4)*(Sqrt[a]*(b^2 - 20*a*c)
 + (2*b*(b^2 - 8*a*c))/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^
2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)],
 (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x + c*x^2])))/(a*d*Sq
rt[d*x]))/(15*a*d^2)))/(7*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(444) = 888$.

Time = 3.93 (sec) , antiderivative size = 900, normalized size of antiderivative = 1.69

method	result
elliptic	$\frac{\sqrt{dx\left(c\,x^2+bx+a\right)}\left(-\frac{2a\sqrt{cd\,x^3+bd\,x^2+adx}}{7d^5x^4}-\frac{16b\sqrt{cd\,x^3+bd\,x^2+adx}}{35d^5x^3}-\frac{2\left(15ac+b^2\right)\sqrt{cd\,x^3+bd\,x^2+adx}}{35d^5a\,x^2}-\frac{4\left(cd\,x^2+bdx+ad\right)b\left(8ac-b^2\right)}{35d^5a^2\sqrt{x\left(cd\,x^2+bdx+ad\right)}}+\cdots\right.}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((c*x^2+b*x+a)^(3/2)/(d*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
(d*x*(c*x^2+b*x+a))^(1/2)/(d*x)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2/7/d^5*a*(c*d
*x^3+b*d*x^2+a*d*x)^(1/2)/x^4-16/35/d^5*b*(c*d*x^3+b*d*x^2+a*d*x)^(1/2)/x^
3-2/35/d^5/a*(15*a*c+b^2)*(c*d*x^3+b*d*x^2+a*d*x)^(1/2)/x^2-4/35*(c*d*x^2+
b*d*x+a*d)/d^5/a^2*b*(8*a*c-b^2)/(x*(c*d*x^2+b*d*x+a*d))^(1/2)+(c^2/d^4-1/
35*c*(15*a*c+b^2)/a/d^4)*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a
*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^
2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^
(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)*
EllipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))
*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-
1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2/35*b*(8*a*c-b^2)/a^2/d^4*(b+(-4*a
*c+b^2)^(1/2))*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1
/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)
))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)*((-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2
/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2
))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1
/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(
-b+(-4*a*c+b^2)^(1/2))*EllipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.47

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{9/2}}\,dx = \frac{2\left(\left(2\,b^4-19\,ab^2c+60\,a^2c^2\right)\sqrt{cd}x^4\text{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\right.\right.}{}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(d*x)^(9/2),x, algorithm="fricas")
```

output
```
2/105*((2*b^4 - 19*a*b^2*c + 60*a^2*c^2)*sqrt(c*d)*x^4*weierstrassPInverse
(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c) +
6*(b^3*c - 8*a*b*c^2)*sqrt(c*d)*x^4*weierstrassZeta(4/3*(b^2 - 3*a*c)/c^2,
 -4/27*(2*b^3 - 9*a*b*c)/c^3, weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -
4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c)) - 3*(8*a^2*b*c*x + 5*a^3*c
 - 2*(b^3*c - 8*a*b*c^2)*x^3 + (a*b^2*c + 15*a^2*c^2)*x^2)*sqrt(c*x^2 + b*
x + a)*sqrt(d*x))/(a^2*c*d^5*x^4)
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{9/2}}\,dx=\int \frac{(a+bx+cx^2)^{\frac{3}{2}}}{(dx)^{\frac{9}{2}}}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/(d*x)**(9/2),x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)/(d*x)**(9/2), x)
```

Maxima [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{9/2}}\,dx=\int \frac{(cx^2+bx+a)^{\frac{3}{2}}}{(dx)^{\frac{9}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(d*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)/(d*x)^(9/2), x)
```

Giac [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{9/2}}\,dx=\int \frac{(cx^2+bx+a)^{\frac{3}{2}}}{(dx)^{\frac{9}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(d*x)^(9/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)/(d*x)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{9/2}}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}}{(d\,x)^{9/2}}\,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)/(d*x)^(9/2),x)
```

output
```
int((a + b*x + c*x^2)^(3/2)/(d*x)^(9/2), x)
```

Reduce [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(dx)^{9/2}}\,dx = \frac{\sqrt{d}\left(-6\sqrt{c\,x^2+bx+a}\,a-42\sqrt{c\,x^2+bx+a}\,c\,x^2+24\sqrt{x}\left(\int \frac{\sqrt{c\,x^2+bx+a}}{\sqrt{x}\,a\,x^3+\sqrt{x}\,b\,x^4+\sqrt{x}\,c}\right.\right.}{}$$

input
```
int((c*x^2+b*x+a)^(3/2)/(d*x)^(9/2),x)
```

output
```
(sqrt(d)*( - 6*sqrt(a + b*x + c*x**2)*a - 42*sqrt(a + b*x + c*x**2)*c*x**2
 + 24*sqrt(x)*int(sqrt(a + b*x + c*x**2)/(sqrt(x)*a*x**3 + sqrt(x)*b*x**4
+ sqrt(x)*c*x**5),x)*a*b*x**3 + 20*sqrt(x)*int(sqrt(a + b*x + c*x**2)/(sqr
t(x)*a*x**2 + sqrt(x)*b*x**3 + sqrt(x)*c*x**4),x)*a*c*x**3 - 56*sqrt(x)*in
t((sqrt(x)*sqrt(a + b*x + c*x**2))/(a*x**3 + b*x**4 + c*x**5),x)*a*c*x**3
+ 21*sqrt(x)*int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a*x**3 + b*x**4 + c*x**
5),x)*b**2*x**3))/(21*sqrt(x)*d**5*x**3)
```

3.323 $\int \sqrt{dx}\left(a+bx+cx^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 694

$$\int \sqrt{dx}\left(a+bx+cx^2\right)^{5/2}\,dx = \frac{2b(24b^4-241ab^2c+708a^2c^2)\sqrt{dx}\sqrt{a+bx+cx^2}}{9009c^3}$$

$$-\frac{2(dx)^{3/2}\left(6b^4-19ab^2c-308a^2c^2+10bc(3b^2-20ac)\,x\right)\sqrt{a+bx+cx^2}}{3003c^2d}$$

$$+\frac{10(dx)^{3/2}\left(3b^2+22ac+9bcx\right)\left(a+bx+cx^2\right)^{3/2}}{1287cd}+\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{5/2}}{13d}$$

$$-\frac{\sqrt{2}(24b^6-268ab^4c+951a^2b^2c^2-924a^3c^3)\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\sqrt{d}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\cdots}}{9009c^{9/2}\sqrt{a+x(b+cx)}}$$

$$+\frac{\sqrt{2}\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\left(24b^6-268ab^4c+951a^2b^2c^2-924a^3c^3-\frac{abc\left(24b^4-241ab^2c+708a^2c^2\right)}{b+\sqrt{b^2-4ac}}\right)\cdots}{9009c^{9/2}\sqrt{a+x(b+c\cdots}}$$

output
```
2/9009*b*(708*a^2*c^2-241*a*b^2*c+24*b^4)*(d*x)^(1/2)*(c*x^2+b*x+a)^(1/2)/
c^3-2/3003*(d*x)^(3/2)*(6*b^4-19*a*b^2*c-308*a^2*c^2+10*b*c*(-20*a*c+3*b^2
)*x)*(c*x^2+b*x+a)^(1/2)/c^2/d+10/1287*(d*x)^(3/2)*(9*b*c*x+22*a*c+3*b^2)*
(c*x^2+b*x+a)^(3/2)/c/d+2/13*(d*x)^(3/2)*(c*x^2+b*x+a)^(5/2)/d-1/9009*2^(1
/2)*(-924*a^3*c^3+951*a^2*b^2*c^2-268*a*b^4*c+24*b^6)*(-b+(-4*a*c+b^2)^(1/
2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*d^(1/2)*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^
(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2^(1/2)*c^(1/2)*(d*
x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+
(-4*a*c+b^2)^(1/2)))^(1/2))/c^(9/2)/(a+x*(c*x+b))^(1/2)+1/9009*2^(1/2)*(-b
+(-4*a*c+b^2)^(1/2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*(24*b^6-268*a*b^4*c+951*
a^2*b^2*c^2-924*a^3*c^3-a*b*c*(708*a^2*c^2-241*a*b^2*c+24*b^4)/(b+(-4*a*c+
b^2)^(1/2)))*d^(1/2)*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-
4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(2^(1/2)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c
+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))
^(1/2))/c^(9/2)/(a+x*(c*x+b))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.45 (sec) , antiderivative size = 708, normalized size of antiderivative = 1.02

$$\int \sqrt{dx}\left(a+bx+cx^2\right)^{5/2}\,dx = \frac{\sqrt{dx}\left(-\frac{4\left(24b^6-268ab^4c+951a^2b^2c^2-924a^3c^3\right)(a+x(b+cx))}{\sqrt{x}}+2c\sqrt{x}(a+x(b+cx))\left(24b^5-18b^4cx+b^3\right.\right.}{}$$

input
```
Integrate[Sqrt[d*x]*(a + b*x + c*x^2)^(5/2),x]
```

output
```
(Sqrt[d*x]*((-4*(24*b^6 - 268*a*b^4*c + 951*a^2*b^2*c^2 - 924*a^3*c^3)*(a
+ x*(b + c*x)))/Sqrt[x] + 2*c*Sqrt[x]*(a + x*(b + c*x))*(24*b^5 - 18*b^4*c
*x + b^3*c*(-241*a + 15*c*x^2) + 3*b^2*c^2*x*(54*a + 371*c*x^2) + 77*c^3*x
*(31*a^2 + 28*a*c*x^2 + 9*c^2*x^4) + b*c^2*(708*a^2 + 3071*a*c*x^2 + 1701*
c^2*x^4)) + (I*(24*b^6 - 268*a*b^4*c + 951*a^2*b^2*c^2 - 924*a^3*c^3)*(-b
+ Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x*Sqrt[(2
*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticE[I*A
rcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 -
 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a/(b + Sqrt[b^2 - 4*a*c])] + (I*(2
4*b^7 - 292*a*b^5*c + 1192*a^2*b^3*c^2 - 1632*a^3*b*c^3 - 24*b^6*Sqrt[b^2
- 4*a*c] + 268*a*b^4*c*Sqrt[b^2 - 4*a*c] - 951*a^2*b^2*c^2*Sqrt[b^2 - 4*a*
c] + 924*a^3*c^3*Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[b^2 - 4*a*c]
)*x)]*x*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)
]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]],
(b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a/(b + Sqrt[b^2 - 4
*a*c])]))/(9009*c^4*Sqrt[x]*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 600, normalized size of antiderivative = 0.86, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {1162, 25, 27, 1236, 27, 1231, 27, 1231, 27, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\left(a+bx+cx^2\right)^{5/2}dx$$

$$\Big\downarrow \text{1162}$$

$$\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{5/2}}{13d}-\frac{5\int -d\sqrt{dx}(2a+bx)\left(cx^2+bx+a\right)^{3/2}dx}{13d}$$

$$\Big\downarrow \text{25}$$

$$\frac{5\int d\sqrt{dx}(2a+bx)\left(cx^2+bx+a\right)^{3/2}dx}{13d}+\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{5/2}}{13d}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{13}\int \sqrt{dx}(2a+bx)\left(cx^2+bx+a\right)^{3/2}dx+\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{5/2}}{13d}$$

$$\Big\downarrow \text{1236}$$

$$\frac{5}{13}\left(\frac{2\int -\frac{d\left(ab+2\left(3b^2-11ac\right)x\right)\left(cx^2+bx+a\right)^{3/2}}{2\sqrt{dx}}dx}{11c}+\frac{2b\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11c}\right)+$$
$$\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{5/2}}{13d}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{13}\left(\frac{2b\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11c}-\frac{d\int \frac{\left(ab+2\left(3b^2-11ac\right)x\right)\left(cx^2+bx+a\right)^{3/2}}{\sqrt{dx}}dx}{11c}\right)+$$
$$\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{5/2}}{13d}$$

$$\Big\downarrow \text{1231}$$

$$\frac{5}{13}\left(\frac{2b\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11c}-\frac{d\left(\frac{2\sqrt{dx}\left(14cx\left(3b^2-11ac\right)+3b\left(6b^2-19ac\right)\right)\left(a+bx+cx^2\right)^{3/2}}{63cd}-\frac{2\int \frac{d^2\left(2ab\left(3b^2-20ac\right)+\left(24b^4-181acb^2\right.\right.}{2\sqrt{dx}}}{21cd^2}\right.}{11c}\right.$$
$$\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{5/2}}{13d}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{13}\left(\frac{2b\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11c}-\frac{d\left(\frac{2\sqrt{dx}\left(14cx\left(3b^2-11ac\right)+3b\left(6b^2-19ac\right)\right)\left(a+bx+cx^2\right)^{3/2}}{63cd}-\frac{\int \frac{\left(2ab\left(3b^2-20ac\right)+\left(24b^4-181acb^2+30\right.\right.}{\sqrt{dx}}}{21c}\right.}{11c}\right.$$
$$\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{5/2}}{13d}$$

$$\Big\downarrow \text{1231}$$

$$\frac{5}{13}\left(\frac{2b\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11c}-\frac{d\left(\frac{2\sqrt{dx}\left(14cx\left(3b^2-11ac\right)+3b\left(6b^2-19ac\right)\right)\left(a+bx+cx^2\right)^{3/2}}{63cd}-\frac{2\sqrt{dx}\left(3cx\left(308a^2c^2-181ab^2c+24b^4\right)+b\right.}{15}\right.}{}\right.$$

$$\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{5/2}}{13d}$$

$\downarrow$ 27

$$\frac{5}{13}\left(\frac{2b\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11c}-\frac{d\left(\frac{2\sqrt{dx}\left(14cx\left(3b^2-11ac\right)+3b\left(6b^2-19ac\right)\right)\left(a+bx+cx^2\right)^{3/2}}{63cd}-\frac{2\sqrt{dx}\left(3cx\left(308a^2c^2-181ab^2c+24b^4\right)+b\right.}{15}\right.}{}\right.$$

$$\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{5/2}}{13d}$$

$\downarrow$ 1241

$$\frac{5}{13}\left(\frac{2b\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11c}-\frac{d\left(\frac{2\sqrt{dx}\left(14cx\left(3b^2-11ac\right)+3b\left(6b^2-19ac\right)\right)\left(a+bx+cx^2\right)^{3/2}}{63cd}-\frac{2\sqrt{dx}\left(3cx\left(308a^2c^2-181ab^2c+24b^4\right)+b\right.}{15}\right.}{}\right.$$

$$\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{5/2}}{13d}$$

$\downarrow$ 1240

$$\frac{5}{13}\left(\frac{2b\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11c}-\frac{d\left(\frac{2\sqrt{dx}\left(14cx\left(3b^2-11ac\right)+3b\left(6b^2-19ac\right)\right)\left(a+bx+cx^2\right)^{3/2}}{63cd}-\frac{2\sqrt{dx}\left(3cx\left(308a^2c^2-181ab^2c+24b^4\right)+b\right.}{15}\right.}{}\right.$$

$$\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{5/2}}{13d}$$

$\downarrow$ 1511

$$\frac{5}{13}\left(\frac{2b\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11c}-\frac{d\left(\frac{2\sqrt{dx}\left(14cx\left(3b^2-11ac\right)+3b\left(6b^2-19ac\right)\right)\left(a+bx+cx^2\right)^{3/2}}{63cd}-\frac{2\sqrt{dx}\left(3cx\left(308a^2c^2-181ab^2c+24b^4\right)+b\right.}{15}\right.}{}\right.$$

$$\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{5/2}}{13d}$$

$\downarrow$ 27

$$\frac{5}{13}\left(\frac{2b\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11c}-\frac{d\left(\frac{2\sqrt{dx}\left(14cx\left(3b^2-11ac\right)+3b\left(6b^2-19ac\right)\right)\left(a+bx+cx^2\right)^{3/2}}{63cd}-\frac{2\sqrt{dx}\left(3cx\left(308a^2c^2-181ab^2c+24b^4\right)+b\right.}{15}\right.}{}\right.$$

$$\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{5/2}}{13d}$$

$\downarrow$ 1416

$$\frac{5}{13}\left(\frac{2b\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11c}-\frac{d\left(\frac{2\sqrt{dx}(14cx(3b^2-11ac)+3b(6b^2-19ac))\left(a+bx+cx^2\right)^{3/2}}{63cd}-\frac{2\sqrt{dx}\left(3cx\left(308a^2c^2-181ab^2c+24b^4\right)+b\right.}{15}\right.}{}\right.$$

$$\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{5/2}}{13d}$$

$\Big\downarrow$ 1509

$$\frac{5}{13}\left(\frac{2b\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11c}-\frac{d\left(\frac{2\sqrt{dx}\left(14cx\left(3b^2-11ac\right)+3b\left(6b^2-19ac\right)\right)\left(a+bx+cx^2\right)^{3/2}}{63cd}-\frac{2\sqrt{dx}\left(3cx\left(308a^2c^2-181ab^2c+24b^4\right)+b\right.}{15}\right.}{}\right.$$

$$\frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{5/2}}{13d}$$

input
```
Int[Sqrt[d*x]*(a + b*x + c*x^2)^(5/2),x]
```

output
```
(2*(d*x)^(3/2)*(a + b*x + c*x^2)^(5/2))/(13*d) + (5*((2*b*Sqrt[d*x]*(a + b
*x + c*x^2)^(5/2))/(11*c) - (d*((2*Sqrt[d*x]*(3*b*(6*b^2 - 19*a*c) + 14*c*
(3*b^2 - 11*a*c)*x)*(a + b*x + c*x^2)^(3/2))/(63*c*d) - ((2*Sqrt[d*x]*(b*(
24*b^4 - 151*a*b^2*c + 108*a^2*c^2) + 3*c*(24*b^4 - 181*a*b^2*c + 308*a^2*
c^2)*x)*Sqrt[a + b*x + c*x^2])/(15*c*d) - (2*Sqrt[x]*((-2*(24*b^6 - 268*a*
b^4*c + 951*a^2*b^2*c^2 - 924*a^3*c^3)*(-((Sqrt[x]*Sqrt[a + b*x + c*x^2])/
(Sqrt[a] + Sqrt[c]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*
x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)
], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x + c*x^2])))/Sqrt[c]
 + (a^(1/4)*(Sqrt[a]*b*(24*b^4 - 241*a*b^2*c + 708*a^2*c^2) + (2*(24*b^6 -
 268*a*b^4*c + 951*a^2*b^2*c^2 - 924*a^3*c^3))/Sqrt[c])*(Sqrt[a] + Sqrt[c]
*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^
(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a +
 b*x + c*x^2])))/(15*c*Sqrt[d*x]))/(21*c)))/(11*c)))/13
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1415 vs. $2(590) = 1180$.

Time = 4.09 (sec) , antiderivative size = 1416, normalized size of antiderivative = 2.04

method	result	size
risch	Expression too large to display	1416
elliptic	Expression too large to display	1695
default	Expression too large to display	2810

input
```
int((d*x)^(1/2)*(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9009/c^3*(693*c^5*x^5+1701*b*c^4*x^4+2156*a*c^4*x^3+1113*b^2*c^3*x^3+307
1*a*b*c^3*x^2+15*b^3*c^2*x^2+2387*a^2*c^3*x+162*a*b^2*c^2*x-18*b^4*c*x+708
*a^2*b*c^2-241*a*b^3*c+24*b^5)*x*(c*x^2+b*x+a)^(1/2)*d/(d*x)^(1/2)-1/9009/
c^3*(-(1848*a^3*c^3-1902*a^2*b^2*c^2+536*a*b^4*c-48*b^6)*(b+(-4*a*c+b^2)^(
1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-
1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2
)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)*((-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+
(-4*a*c+b^2)^(1/2)))*EllipticE(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(
b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(
-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*
a*c+b^2)^(1/2))*EllipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4
*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*
c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))+24*a*b^5*(b+(-4*a*
c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(
1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2
)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)*EllipticF(2^(1/2)*((x+1/2*(b+(-4*a
*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^
(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.48

$$\int \sqrt{dx}(a+bx+cx^2)^{5/2}\,dx = \frac{2\left((48\,b^7-608\,ab^5c+2625\,a^2b^3c^2-3972\,a^3bc^3)\sqrt{cd}\,\text{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-}{27}\right.\right.}{}$$

input
```
integrate((d*x)^(1/2)*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
2/27027*((48*b^7 - 608*a*b^5*c + 2625*a^2*b^3*c^2 - 3972*a^3*b*c^3)*sqrt(c
*d)*weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3
, 1/3*(3*c*x + b)/c) + 6*(24*b^6*c - 268*a*b^4*c^2 + 951*a^2*b^2*c^3 - 924
*a^3*c^4)*sqrt(c*d)*weierstrassZeta(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 -
9*a*b*c)/c^3, weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*
a*b*c)/c^3, 1/3*(3*c*x + b)/c)) + 3*(693*c^7*x^5 + 1701*b*c^6*x^4 + 24*b^5
*c^2 - 241*a*b^3*c^3 + 708*a^2*b*c^4 + 7*(159*b^2*c^5 + 308*a*c^6)*x^3 + (
15*b^3*c^4 + 3071*a*b*c^5)*x^2 - (18*b^4*c^3 - 162*a*b^2*c^4 - 2387*a^2*c^
5)*x)*sqrt(c*x^2 + b*x + a)*sqrt(d*x))/c^5
```

Sympy [F]

$$\int \sqrt{dx}\left(a+bx+cx^2\right)^{5/2}\,dx=\int \sqrt{dx}\left(a+bx+cx^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((d*x)**(1/2)*(c*x**2+b*x+a)**(5/2),x)
```

output
```
Integral(sqrt(d*x)*(a + b*x + c*x**2)**(5/2), x)
```

Maxima [F]

$$\int \sqrt{dx}\left(a+bx+cx^2\right)^{5/2}\,dx=\int \left(cx^2+bx+a\right)^{\frac{5}{2}}\sqrt{dx}\,dx$$

input
```
integrate((d*x)^(1/2)*(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(5/2)*sqrt(d*x), x)
```

Giac [F]

$$\int \sqrt{dx}(a+bx+cx^2)^{5/2}\,dx=\int (cx^2+bx+a)^{\frac{5}{2}}\sqrt{dx}\,dx$$

input
```
integrate((d*x)^(1/2)*(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^(5/2)*sqrt(d*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}(a+bx+cx^2)^{5/2}\,dx=\int \sqrt{d\,x}\,\left(c\,x^2+b\,x+a\right)^{5/2}\,dx$$

input
```
int((d*x)^(1/2)*(a + b*x + c*x^2)^(5/2),x)
```

output
```
int((d*x)^(1/2)*(a + b*x + c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \sqrt{dx}(a+bx+cx^2)^{5/2}\,dx=\frac{\sqrt{d}\left(1848\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,a^3c^2-486\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,a^2b^2c+4774\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,a\right.}{}$$

input
```
int((d*x)^(1/2)*(c*x^2+b*x+a)^(5/2),x)
```

output
```
(sqrt(d)*(1848*sqrt(x)*sqrt(a + b*x + c*x**2)*a**3*c**2 - 486*sqrt(x)*sqrt
(a + b*x + c*x**2)*a**2*b**2*c + 4774*sqrt(x)*sqrt(a + b*x + c*x**2)*a**2*
b*c**2*x + 54*sqrt(x)*sqrt(a + b*x + c*x**2)*a*b**4 + 324*sqrt(x)*sqrt(a +
 b*x + c*x**2)*a*b**3*c*x + 6142*sqrt(x)*sqrt(a + b*x + c*x**2)*a*b**2*c**
2*x**2 + 4312*sqrt(x)*sqrt(a + b*x + c*x**2)*a*b*c**3*x**3 - 36*sqrt(x)*sq
rt(a + b*x + c*x**2)*b**5*x + 30*sqrt(x)*sqrt(a + b*x + c*x**2)*b**4*c*x**
2 + 2226*sqrt(x)*sqrt(a + b*x + c*x**2)*b**3*c**2*x**3 + 3402*sqrt(x)*sqrt
(a + b*x + c*x**2)*b**2*c**3*x**4 + 1386*sqrt(x)*sqrt(a + b*x + c*x**2)*b*
c**4*x**5 - 2772*int((sqrt(x)*sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*x**2)
,x)*a**3*c**3 + 2853*int((sqrt(x)*sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*x
**2),x)*a**2*b**2*c**2 - 804*int((sqrt(x)*sqrt(a + b*x + c*x**2)*x)/(a + b
*x + c*x**2),x)*a*b**4*c + 72*int((sqrt(x)*sqrt(a + b*x + c*x**2)*x)/(a +
b*x + c*x**2),x)*b**6 - 924*int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a*x + b*
x**2 + c*x**3),x)*a**4*c**2 + 243*int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a*
x + b*x**2 + c*x**3),x)*a**3*b**2*c - 27*int((sqrt(x)*sqrt(a + b*x + c*x**
2))/(a*x + b*x**2 + c*x**3),x)*a**2*b**4))/(9009*b*c**2)
```

3.324 $\int \frac{(a+bx+cx^2)^{5/2}}{\sqrt{dx}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 612

$$\int \frac{(a+bx+cx^2)^{5/2}}{\sqrt{dx}} \, dx =$$
$$-\frac{2\sqrt{dx}(4b^4-27ab^2c-180a^2c^2+12bc(b^2-8ac)\,x)\sqrt{a+bx+cx^2}}{693c^2d}$$
$$+\frac{10\sqrt{dx}(3(b^2+6ac)+7bcx)\,(a+bx+cx^2)^{3/2}}{693cd}+\frac{2\sqrt{dx}(a+bx+cx^2)^{5/2}}{11d}$$
$$+\frac{b(8b^4-93ab^2c+372a^2c^2)\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\Big(\arcsin\Big(}{693\sqrt{2}c^{7/2}\sqrt{d}\sqrt{a+x(b+cx)}}$$
$$-\frac{\sqrt{-b+\sqrt{b^2-4ac}}\left(8b^6-101ab^4c+456a^2b^2c^2-720a^3c^3+b\sqrt{b^2-4ac}(8b^4-93ab^2c+372a^2c^2)\right)\sqrt{1+}}{693\sqrt{2}c^{7/2}\sqrt{d}\sqrt{a+x(b+cx)}}$$

output
```
-2/693*(d*x)^(1/2)*(4*b^4-27*a*b^2*c-180*a^2*c^2+12*b*c*(-8*a*c+b^2)*x)*(c
*x^2+b*x+a)^(1/2)/c^2/d+10/693*(d*x)^(1/2)*(7*b*c*x+18*a*c+3*b^2)*(c*x^2+b
*x+a)^(3/2)/c/d+2/11*(d*x)^(1/2)*(c*x^2+b*x+a)^(5/2)/d+1/1386*b*(372*a^2*c
^2-93*a*b^2*c+8*b^4)*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*
(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1
/2)*EllipticE(2^(1/2)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^
(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/c^(7/
2)/d^(1/2)/(a+x*(c*x+b))^(1/2)-1/1386*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(8*b^6
-101*a*b^4*c+456*a^2*b^2*c^2-720*a^3*c^3+b*(-4*a*c+b^2)^(1/2)*(372*a^2*c^2
-93*a*b^2*c+8*b^4))*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4
*a*c+b^2)^(1/2)))^(1/2)*EllipticF(2^(1/2)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+
b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^
(1/2))*2^(1/2)/c^(7/2)/d^(1/2)/(a+x*(c*x+b))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.15 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx+cx^2)^{5/2}}{\sqrt{dx}}\,dx = \frac{\sqrt{x}\left(\frac{4\sqrt{x}(a+x(b+cx))\left(-4b^4+3b^3cx+b^2c(42a+113cx^2)+bc^2x(347a+161cx^2)+9c^2(37a^2+24acx^2+7c^2x^4)\right)}{c^2}\right.}{}$$

input
```
Integrate[(a + b*x + c*x^2)^(5/2)/Sqrt[d*x],x]
```

output
```
(Sqrt[x]*((4*Sqrt[x]*(a + x*(b + c*x))*(-4*b^4 + 3*b^3*c*x + b^2*c*(42*a +
 113*c*x^2) + b*c^2*x*(347*a + 161*c*x^2) + 9*c^2*(37*a^2 + 24*a*c*x^2 + 7
*c^2*x^4)))/c^2 + (x*((4*b*(8*b^4 - 93*a*b^2*c + 372*a^2*c^2)*(a + x*(b +
c*x)))/x^(3/2) - (I*b*(8*b^4 - 93*a*b^2*c + 372*a^2*c^2)*(-b + Sqrt[b^2 -
4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*Sqrt[(2*a + b*x - Sqrt
[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticE[I*ArcSinh[(Sqrt[2]
*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - S
qrt[b^2 - 4*a*c])])/Sqrt[a/(b + Sqrt[b^2 - 4*a*c])] + (I*(-8*b^6 + 101*a*b
^4*c - 456*a^2*b^2*c^2 + 720*a^3*c^3 + 8*b^5*Sqrt[b^2 - 4*a*c] - 93*a*b^3*
c*Sqrt[b^2 - 4*a*c] + 372*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4*a)/((b
+ Sqrt[b^2 - 4*a*c])*x)]*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqr
t[b^2 - 4*a*c]*x)]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a
*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a/
(b + Sqrt[b^2 - 4*a*c])]))/c^3))/(1386*Sqrt[d*x]*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 514, normalized size of antiderivative = 0.84, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {1162, 25, 27, 1231, 27, 1231, 27, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{5/2}}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{1162}$$

$$\frac{2\sqrt{dx}(a+bx+cx^2)^{5/2}}{11d}-\frac{5\int -\frac{d(2a+bx)(cx^2+bx+a)^{3/2}}{\sqrt{dx}}dx}{11d}$$

$$\Big\downarrow \text{25}$$

$$\frac{5\int \frac{d(2a+bx)(cx^2+bx+a)^{3/2}}{\sqrt{dx}}dx}{11d}+\frac{2\sqrt{dx}(a+bx+cx^2)^{5/2}}{11d}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{11}\int \frac{(2a+bx)(cx^2+bx+a)^{3/2}}{\sqrt{dx}}dx+\frac{2\sqrt{dx}(a+bx+cx^2)^{5/2}}{11d}$$

$$\Big\downarrow \; 1231$$

$$\frac{5}{11}\left(\frac{2\sqrt{dx}\left(3\left(6ac+b^2\right)+7bcx\right)\left(a+bx+cx^2\right)^{3/2}}{63cd}-\frac{2\int \frac{d^2\left(a\left(b^2-36ac\right)+4b\left(b^2-8ac\right)x\right)\sqrt{cx^2+bx+a}}{2\sqrt{dx}}dx}{21cd^2}\right)+\frac{2\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11d}$$

$$\Big\downarrow \; 27$$

$$\frac{5}{11}\left(\frac{2\sqrt{dx}\left(3\left(6ac+b^2\right)+7bcx\right)\left(a+bx+cx^2\right)^{3/2}}{63cd}-\frac{\int \frac{\left(a\left(b^2-36ac\right)+4b\left(b^2-8ac\right)x\right)\sqrt{cx^2+bx+a}}{\sqrt{dx}}dx}{21c}\right)+\frac{2\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11d}$$

$$\Big\downarrow \; 1231$$

$$\frac{5}{11}\left(\frac{2\sqrt{dx}\left(3\left(6ac+b^2\right)+7bcx\right)\left(a+bx+cx^2\right)^{3/2}}{63cd}-\frac{\frac{2\sqrt{dx}\left(-180a^2c^2+12bcx\left(b^2-8ac\right)-27ab^2c+4b^4\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\int \frac{d^2\left(2a\ldots\right.}{\ldots}}{\ldots}}{21c}\ldots\right.$$

$$\frac{2\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11d}$$

$$\Big\downarrow \; 27$$

$$\frac{5}{11}\left(\frac{2\sqrt{dx}\left(3\left(6ac+b^2\right)+7bcx\right)\left(a+bx+cx^2\right)^{3/2}}{63cd}-\frac{\frac{2\sqrt{dx}\left(-180a^2c^2+12bcx\left(b^2-8ac\right)-27ab^2c+4b^4\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{\int \frac{2a\left(2b^4\ldots\right.}{\ldots}}{\ldots}}{21c}\ldots\right.$$

$$\frac{2\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11d}$$

$$\Big\downarrow \; 1241$$

$$\frac{5}{11}\left(\frac{2\sqrt{dx}\left(3\left(6ac+b^2\right)+7bcx\right)\left(a+bx+cx^2\right)^{3/2}}{63cd}-\frac{\frac{2\sqrt{dx}\left(-180a^2c^2+12bcx\left(b^2-8ac\right)-27ab^2c+4b^4\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{\sqrt{x}\int \frac{2a\left(\ldots\right.}{\ldots}}{\ldots}}{21c}\ldots\right.$$

$$\frac{2\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11d}$$

$$\Big\downarrow 1240$$

$$\frac{5}{11}\left(\frac{2\sqrt{dx}\left(3\left(6ac+b^2\right)+7bcx\right)\left(a+bx+cx^2\right)^{3/2}}{63cd}-\frac{\frac{2\sqrt{dx}\left(-180a^2c^2+12bcx\left(b^2-8ac\right)-27ab^2c+4b^4\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\int\frac{2a}{}}{}}{21c}\right.$$

$$\frac{2\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11d}$$

$$\Big\downarrow 1511$$

$$\frac{5}{11}\left(\frac{2\sqrt{dx}\left(3\left(6ac+b^2\right)+7bcx\right)\left(a+bx+cx^2\right)^{3/2}}{63cd}-\frac{\frac{2\sqrt{dx}\left(-180a^2c^2+12bcx\left(b^2-8ac\right)-27ab^2c+4b^4\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\left(\sqrt{c}\right.}{}}{}\right.$$

$$\frac{2\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11d}$$

$$\Big\downarrow 27$$

$$\frac{5}{11}\left(\frac{2\sqrt{dx}\left(3\left(6ac+b^2\right)+7bcx\right)\left(a+bx+cx^2\right)^{3/2}}{63cd}-\frac{\frac{2\sqrt{dx}\left(-180a^2c^2+12bcx\left(b^2-8ac\right)-27ab^2c+4b^4\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\left(\sqrt{c}\right.}{}}{}\right.$$

$$\frac{2\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11d}$$

$$\Big\downarrow 1416$$

$$\frac{5}{11}\left(\frac{2\sqrt{dx}\left(3\left(6ac+b^2\right)+7bcx\right)\left(a+bx+cx^2\right)^{3/2}}{63cd}-\frac{\frac{2\sqrt{dx}\left(-180a^2c^2+12bcx\left(b^2-8ac\right)-27ab^2c+4b^4\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\left(\sqrt[4]{\ldots}\right.}{\ldots}}{\ldots}\right.$$

$$\frac{2\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11d}$$

$\downarrow$ 1509

$$\frac{5}{11}\left(\frac{2\sqrt{dx}\left(3\left(6ac+b^2\right)+7bcx\right)\left(a+bx+cx^2\right)^{3/2}}{63cd}-\frac{\frac{2\sqrt{dx}\left(-180a^2c^2+12bcx\left(b^2-8ac\right)-27ab^2c+4b^4\right)\sqrt{a+bx+cx^2}}{15cd}-\frac{2\sqrt{x}\left(\sqrt[4]{\ldots}\right.}{\ldots}}{\ldots}\right.$$

$$\frac{2\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{11d}$$

input
```
Int[(a + b*x + c*x^2)^(5/2)/Sqrt[d*x],x]
```

output
```
(2*Sqrt[d*x]*(a + b*x + c*x^2)^(5/2))/(11*d) + (5*((2*Sqrt[d*x]*(3*(b^2 +
6*a*c) + 7*b*c*x)*(a + b*x + c*x^2)^(3/2))/(63*c*d) - ((2*Sqrt[d*x]*(4*b^4
 - 27*a*b^2*c - 180*a^2*c^2 + 12*b*c*(b^2 - 8*a*c)*x)*Sqrt[a + b*x + c*x^2
])/(15*c*d) - (2*Sqrt[x]*(-((b*(8*b^4 - 93*a*b^2*c + 372*a^2*c^2)*(-((Sqrt
[x]*Sqrt[a + b*x + c*x^2])/(Sqrt[a] + Sqrt[c]*x)) + (a^(1/4)*(Sqrt[a] + Sq
rt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticE[2*ArcTa
n[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[
a + b*x + c*x^2])))/Sqrt[c]) + (a^(1/4)*(2*Sqrt[a]*(2*b^4 - 21*a*b^2*c + 1
80*a^2*c^2) + (b*(8*b^4 - 93*a*b^2*c + 372*a^2*c^2))/Sqrt[c])*(Sqrt[a] + S
qrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*ArcT
an[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sq
rt[a + b*x + c*x^2])))/(15*c*Sqrt[d*x]))/(21*c)))/11
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1154 vs. $2(515) = 1030$.

Time = 3.45 (sec) , antiderivative size = 1155, normalized size of antiderivative = 1.89

method	result	size
elliptic	Expression too large to display	1155
risch	Expression too large to display	1369
default	Expression too large to display	2274

input
```
int((c*x^2+b*x+a)^(5/2)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(d*x*(c*x^2+b*x+a))^(1/2)/(d*x)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/11*c^2/d*x^4*
(c*d*x^3+b*d*x^2+a*d*x)^(1/2)+46/99*b*c/d*x^3*(c*d*x^3+b*d*x^2+a*d*x)^(1/2
)+2/7*(24/11*a*c^2+113/99*b^2*c)/c/d*x^2*(c*d*x^3+b*d*x^2+a*d*x)^(1/2)+2/5
*(433/99*a*b*c+b^3-6/7*b/c*(24/11*a*c^2+113/99*b^2*c))/c/d*x*(c*d*x^3+b*d*
x^2+a*d*x)^(1/2)+2/3*(3*a^2*c+3*a*b^2-4/5*b/c*(433/99*a*b*c+b^3-6/7*b/c*(2
4/11*a*c^2+113/99*b^2*c))-5/7*a/c*(24/11*a*c^2+113/99*b^2*c))/c/d*(c*d*x^3
+b*d*x^2+a*d*x)^(1/2)+(a^3-1/3*a/c*(3*a^2*c+3*a*b^2-4/5*b/c*(433/99*a*b*c+
b^3-6/7*b/c*(24/11*a*c^2+113/99*b^2*c))-5/7*a/c*(24/11*a*c^2+113/99*b^2*c)
))*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-
4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a
*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)*EllipticF(2^(1/2)*((x+
1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(
-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)
^(1/2))))^(1/2))+(3*a^2*b-3/5*a/c*(433/99*a*b*c+b^3-6/7*b/c*(24/11*a*c^2+1
13/99*b^2*c))-2/3*b/c*(3*a^2*c+3*a*b^2-4/5*b/c*(433/99*a*b*c+b^3-6/7*b/c*(
24/11*a*c^2+113/99*b^2*c))-5/7*a/c*(24/11*a*c^2+113/99*b^2*c)))*(b+(-4*a*c
+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1
/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.47

$$\int \frac{(a+bx+cx^2)^{5/2}}{\sqrt{dx}}\,dx =$$

$$-\frac{2\left(\left(8\,b^6-105\,ab^4c+498\,a^2b^2c^2-1080\,a^3c^3\right)\sqrt{cd}\,\text{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\frac{3\,cx+b}{3\,c}\right)+\cdots\right.}{}$$

input
```
integrate((c*x^2+b*x+a)^(5/2)/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/2079*((8*b^6 - 105*a*b^4*c + 498*a^2*b^2*c^2 - 1080*a^3*c^3)*sqrt(c*d)*
weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/
3*(3*c*x + b)/c) + 3*(8*b^5*c - 93*a*b^3*c^2 + 372*a^2*b*c^3)*sqrt(c*d)*we
ierstrassZeta(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, weierstr
assPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x
 + b)/c)) - 3*(63*c^6*x^4 + 161*b*c^5*x^3 - 4*b^4*c^2 + 42*a*b^2*c^3 + 333
*a^2*c^4 + (113*b^2*c^4 + 216*a*c^5)*x^2 + (3*b^3*c^3 + 347*a*b*c^4)*x)*sq
rt(c*x^2 + b*x + a)*sqrt(d*x))/(c^4*d)
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{5/2}}{\sqrt{dx}}\,dx = \int \frac{(a+bx+cx^2)^{\frac{5}{2}}}{\sqrt{dx}}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(5/2)/(d*x)**(1/2),x)
```

output
```
Integral((a + b*x + c*x**2)**(5/2)/sqrt(d*x), x)
```

Maxima [F]

$$\int \frac{(a+bx+cx^2)^{5/2}}{\sqrt{dx}}\,dx = \int \frac{(cx^2+bx+a)^{\frac{5}{2}}}{\sqrt{dx}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(5/2)/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(5/2)/sqrt(d*x), x)
```

Giac [F]

$$\int \frac{(a+bx+cx^2)^{5/2}}{\sqrt{dx}}\,dx = \int \frac{(cx^2+bx+a)^{\frac{5}{2}}}{\sqrt{dx}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(5/2)/(d*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^(5/2)/sqrt(d*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{5/2}}{\sqrt{dx}}\,dx = \int \frac{(c\,x^2+b\,x+a)^{5/2}}{\sqrt{d\,x}}\,dx$$

input
```
int((a + b*x + c*x^2)^(5/2)/(d*x)^(1/2),x)
```

output
```
int((a + b*x + c*x^2)^(5/2)/(d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx+cx^2)^{5/2}}{\sqrt{dx}}\,dx = \frac{\sqrt{d}\left(2076\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,a^2c-18\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,a\,b^2+1388\sqrt{x}\,\sqrt{c\,x^2+\right.}{}$$

input
```
int((c*x^2+b*x+a)^(5/2)/(d*x)^(1/2),x)
```

output
```
(sqrt(d)*(2076*sqrt(x)*sqrt(a + b*x + c*x**2)*a**2*c - 18*sqrt(x)*sqrt(a +
 b*x + c*x**2)*a*b**2 + 1388*sqrt(x)*sqrt(a + b*x + c*x**2)*a*b*c*x + 864*
sqrt(x)*sqrt(a + b*x + c*x**2)*a*c**2*x**2 + 12*sqrt(x)*sqrt(a + b*x + c*x
**2)*b**3*x + 452*sqrt(x)*sqrt(a + b*x + c*x**2)*b**2*c*x**2 + 644*sqrt(x)
*sqrt(a + b*x + c*x**2)*b*c**2*x**3 + 252*sqrt(x)*sqrt(a + b*x + c*x**2)*c
**3*x**4 - 1116*int((sqrt(x)*sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*x**2),
x)*a**2*c**2 + 279*int((sqrt(x)*sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*x**
2),x)*a*b**2*c - 24*int((sqrt(x)*sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*x*
*2),x)*b**4 + 348*int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a*x + b*x**2 + c*x
**3),x)*a**3*c + 9*int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a*x + b*x**2 + c*
x**3),x)*a**2*b**2))/(1386*c*d)
```

3.325 $\int \frac{(dx)^{7/2}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 542

$$\int \frac{(dx)^{7/2}}{\sqrt{a+bx+cx^2}}\,dx = \frac{2(24b^2-25ac)\,d^3\sqrt{dx}\sqrt{a+bx+cx^2}}{105c^3}$$
$$-\frac{12bd^2(dx)^{3/2}\sqrt{a+bx+cx^2}}{35c^2}+\frac{2d(dx)^{5/2}\sqrt{a+bx+cx^2}}{7c}$$
$$-\frac{4\sqrt{2}b(6b^2-13ac)\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)d^{7/2}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\left(\arcsin\left(\frac{}{\sqrt{-}}\right.\right.}{105c^{9/2}\sqrt{a+x(b+cx)}}$$
$$+\frac{\sqrt{2}\sqrt{-b+\sqrt{b^2-4ac}}\left(24b^4-76ab^2c+25a^2c^2+4b(6b^2-13ac)\sqrt{b^2-4ac}\right)d^{7/2}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{}{b+}}}{105c^{9/2}\sqrt{a+x(b+cx)}}$$

output
```
2/105*(-25*a*c+24*b^2)*d^3*(d*x)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^3-12/35*b*d^2
*(d*x)^(3/2)*(c*x^2+b*x+a)^(1/2)/c^2+2/7*d*(d*x)^(5/2)*(c*x^2+b*x+a)^(1/2)
/c-4/105*2^(1/2)*b*(-13*a*c+6*b^2)*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(b+(-4*a*
c+b^2)^(1/2))*d^(7/2)*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(
-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2^(1/2)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*
c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))
)^(1/2))/c^(9/2)/(a+x*(c*x+b))^(1/2)+1/105*2^(1/2)*(-b+(-4*a*c+b^2)^(1/2))
^(1/2)*(24*b^4-76*a*b^2*c+25*a^2*c^2+4*b*(-13*a*c+6*b^2)*(-4*a*c+b^2)^(1/2
))*d^(7/2)*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)
^(1/2)))^(1/2)*EllipticF(2^(1/2)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2
))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))/c^
(9/2)/(a+x*(c*x+b))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.14 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.99

$$\int \frac{(dx)^{7/2}}{\sqrt{a+bx+cx^2}}\,dx = \frac{(dx)^{7/2}\left(\frac{2\sqrt{x}(a+x(b+cx))\left(24b^2-18bcx+5c\left(-5a+3cx^2\right)\right)}{c^3}+\frac{x\left(-\frac{16b\left(6b^2-13ac\right)(a+x(b+cx))}{x^{3/2}}+\frac{4ib\left(6b^2-13ac\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(d*x)^(7/2)/Sqrt[a + b*x + c*x^2],x]
```

output
```
((d*x)^(7/2)*((2*Sqrt[x]*(a + x*(b + c*x))*(24*b^2 - 18*b*c*x + 5*c*(-5*a
+ 3*c*x^2)))/c^3 + (x*((-16*b*(6*b^2 - 13*a*c)*(a + x*(b + c*x)))/x^(3/2)
+ ((4*I)*b*(6*b^2 - 13*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4*a)/((b +
Sqrt[b^2 - 4*a*c])*x)]*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[
b^2 - 4*a*c]*x)]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c
])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a/(b
 + Sqrt[b^2 - 4*a*c])] + (I*(24*b^4 - 76*a*b^2*c + 25*a^2*c^2 - 24*b^3*Sqr
t[b^2 - 4*a*c] + 52*a*b*c*Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[b^2
 - 4*a*c])*x)]*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*
a*c]*x)]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqr
t[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a/(b + Sqrt[
b^2 - 4*a*c])]))/c^4))/(105*x^(7/2)*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 445, normalized size of antiderivative = 0.82, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {1166, 27, 1236, 27, 1236, 27, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{7/2}}{\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{2\int -\frac{d^2(dx)^{3/2}(5a+6bx)}{2\sqrt{cx^2+bx+a}}dx}{7c}+\frac{2d(dx)^{5/2}\sqrt{a+bx+cx^2}}{7c}$$

$$\Big\downarrow \text{27}$$

$$\frac{2d(dx)^{5/2}\sqrt{a+bx+cx^2}}{7c}-\frac{d^2\int \frac{(dx)^{3/2}(5a+6bx)}{\sqrt{cx^2+bx+a}}dx}{7c}$$

$$\Big\downarrow \text{1236}$$

$$\frac{2d(dx)^{5/2}\sqrt{a+bx+cx^2}}{7c}-\frac{d^2\left(\frac{2\int -\frac{d\sqrt{dx}\left(18ab+\left(24b^2-25ac\right)x\right)}{2\sqrt{cx^2+bx+a}}dx}{5c}+\frac{12b(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c}\right)}{7c}$$

$$\Big\downarrow \text{27}$$

$$\frac{2d(dx)^{5/2}\sqrt{a+bx+cx^2}}{7c}-\frac{d^2\left(\frac{12b(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c}-\frac{d\int\frac{\sqrt{dx}\left(18ab+\left(24b^2-25ac\right)x\right)}{\sqrt{cx^2+bx+a}}dx}{5c}\right)}{7c}$$

$\Big\downarrow$ 1236

$$\frac{2d(dx)^{5/2}\sqrt{a+bx+cx^2}}{7c}-\frac{d^2\left(\frac{12b(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c}-\frac{d\left(\frac{2\int-\frac{d\left(a\left(24b^2-25ac\right)+8b\left(6b^2-13ac\right)x\right)}{2\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{3c}+\frac{2\sqrt{dx}\left(24b^2-25ac\right)\sqrt{a+bx+cx^2}}{3c}\right)}{5c}\right)}{7c}$$

$\Big\downarrow$ 27

$$\frac{2d(dx)^{5/2}\sqrt{a+bx+cx^2}}{7c}-\frac{d^2\left(\frac{12b(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c}-\frac{d\left(\frac{2\sqrt{dx}\left(24b^2-25ac\right)\sqrt{a+bx+cx^2}}{3c}-\frac{d\int\frac{a\left(24b^2-25ac\right)+8b\left(6b^2-13ac\right)x}{\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{3c}\right)}{5c}\right)}{7c}$$

$\Big\downarrow$ 1241

$$\frac{2d(dx)^{5/2}\sqrt{a+bx+cx^2}}{7c}-\frac{d^2\left(\frac{12b(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c}-\frac{d\left(\frac{2\sqrt{dx}\left(24b^2-25ac\right)\sqrt{a+bx+cx^2}}{3c}-\frac{d\sqrt{x}\int\frac{a\left(24b^2-25ac\right)+8b\left(6b^2-13ac\right)x}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{3c\sqrt{dx}}\right)}{5c}\right)}{7c}$$

$\Big\downarrow$ 1240

$$\frac{\frac{2d(dx)^{5/2}\sqrt{a+bx+cx^2}}{7c}-\frac{d^2\left(\frac{12b(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c}-\frac{d\left(\frac{2\sqrt{dx}\left(24b^2-25ac\right)\sqrt{a+bx+cx^2}}{3c}-\frac{2d\sqrt{x}\int\frac{a\left(24b^2-25ac\right)+8b\left(6b^2-13ac\right)x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{3c\sqrt{dx}}\right)}{5c}\right)}{7c}}{}$$

$\Big\downarrow$ 1511

$$\frac{2d(dx)^{5/2}\sqrt{a+bx+cx^2}}{7c}-\frac{d^2\left(\frac{12b(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c}-\frac{d\left(\frac{2\sqrt{dx}\left(24b^2-25ac\right)\sqrt{a+bx+cx^2}}{3c}-\frac{2d\sqrt{x}\left(\sqrt{a}\left(\sqrt{a}\left(24b^2-25ac\right)+\frac{8b\left(6b^2-13ac\right)}{\sqrt{c}}\right)\int\frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{8\sqrt{a}b\left(6b^2-13a\right.}{}\right.}{3c\sqrt{dx}}\right.}{5c}\right.}{7c}$$

$\Big\downarrow$ 27

$$\frac{2d(dx)^{5/2}\sqrt{a+bx+cx^2}}{7c}-\frac{d^2\left(\frac{12b(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c}-\frac{d\left(\frac{2\sqrt{dx}\left(24b^2-25ac\right)\sqrt{a+bx+cx^2}}{3c}-\frac{2d\sqrt{x}\left(\sqrt{a}\left(\sqrt{a}\left(24b^2-25ac\right)+\frac{8b\left(6b^2-13ac\right)}{\sqrt{c}}\right)\int\frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{8b\left(6b^2-13ac\right)}{}\right.}{3c\sqrt{dx}}\right.}{5c}\right.}{7c}$$

$\Big\downarrow$ 1416

$$\frac{2d(dx)^{5/2}\sqrt{a+bx+cx^2}}{7c} - \frac{d^2\left(\frac{12b(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c} - \frac{d\left(\frac{2\sqrt{dx}\left(24b^2-25ac\right)\sqrt{a+bx+cx^2}}{3c} - \frac{2d\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}\left(24b^2-25ac\right)+\frac{8b\left(6b^2-13ac\right)}{\sqrt{c}}\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\,\text{Elliptic}\cdots}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}\cdots\right.}{3c\sqrt{dx}\cdots}\right.}{5c}\cdots\right.}{7c}$$

$\downarrow$ 1509

$$\frac{2d(dx)^{5/2}\sqrt{a+bx+cx^2}}{7c} -$$

$$d^2\left(\frac{12b(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c} - \frac{d\left(\frac{2\sqrt{dx}\left(24b^2-25ac\right)\sqrt{a+bx+cx^2}}{3c} - \frac{2d\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}\left(24b^2-25ac\right)+\frac{8b\left(6b^2-13ac\right)}{\sqrt{c}}\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\text{Elliptic}}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}\right.}{}\right)}{}\right.$$

7

input
```
Int[(d*x)^(7/2)/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*d*(d*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(7*c) - (d^2*((12*b*(d*x)^(3/2)*Sq
rt[a + b*x + c*x^2])/(5*c) - (d*((2*(24*b^2 - 25*a*c)*Sqrt[d*x]*Sqrt[a + b
*x + c*x^2])/(3*c) - (2*d*Sqrt[x]*((-8*b*(6*b^2 - 13*a*c)*(-((Sqrt[x]*Sqrt
[a + b*x + c*x^2])/(Sqrt[a] + Sqrt[c]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x)
*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/
4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x
+ c*x^2])))/Sqrt[c] + (a^(1/4)*(Sqrt[a]*(24*b^2 - 25*a*c) + (8*b*(6*b^2 -
13*a*c))/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] +
Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[
a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x + c*x^2])))/(3*c*Sqrt[d*x])))/(5*
c)))/(7*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 889, normalized size of antiderivative = 1.64

method	result
elliptic	$\frac{\sqrt{dx}\,\sqrt{dx(c\,x^2+bx+a)}\left(\frac{2d^3x^2\sqrt{cd\,x^3+bd\,x^2+adx}}{7c}-\frac{12d^3bx\sqrt{cd\,x^3+bd\,x^2+adx}}{35c^2}+\frac{2\left(-\frac{5d^4a}{7c}+\frac{24d^4b^2}{35c^2}\right)\sqrt{cd\,x^3+bd\,x^2+adx}}{3cd}-\frac{\left(-\frac{5d^4a}{7c}+\frac{24d}{35}\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((d*x)^(7/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/x*(d*x)^(1/2)/(c*x^2+b*x+a)^(1/2)*(d*x*(c*x^2+b*x+a))^(1/2)*(2/7*d^3/c
*x^2*(c*d*x^3+b*d*x^2+a*d*x)^(1/2)-12/35*d^3/c^2*b*x*(c*d*x^3+b*d*x^2+a*d*
x)^(1/2)+2/3*(-5/7*d^4/c*a+24/35*d^4/c^2*b^2)/c/d*(c*d*x^3+b*d*x^2+a*d*x)^
(1/2)-1/3*(-5/7*d^4/c*a+24/35*d^4/c^2*b^2)/c^2*a*(b+(-4*a*c+b^2)^(1/2))*2^
(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((
x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+
(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3
+b*d*x^2+a*d*x)^(1/2)*EllipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+
(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+(18/35*d^4/c^
2*b*a-2/3*(-5/7*d^4/c*a+24/35*d^4/c^2*b^2)/c*b)*(b+(-4*a*c+b^2)^(1/2))/c*2
^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*(
(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b
+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^
3+b*d*x^2+a*d*x)^(1/2)*((-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b
^2)^(1/2)))*EllipticE(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c
+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^
2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^
(1/2))*EllipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)
^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.43

$$\int \frac{(dx)^{7/2}}{\sqrt{a+bx+cx^2}}\,dx = \frac{2\left(\left(48\,b^4-176\,ab^2c+75\,a^2c^2\right)\sqrt{cd}d^3\text{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\right.\right.}{}$$

input
```
integrate((d*x)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/315*((48*b^4 - 176*a*b^2*c + 75*a^2*c^2)*sqrt(c*d)*d^3*weierstrassPInver
se(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c)
+ 24*(6*b^3*c - 13*a*b*c^2)*sqrt(c*d)*d^3*weierstrassZeta(4/3*(b^2 - 3*a*c
)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, weierstrassPInverse(4/3*(b^2 - 3*a*c)/
c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c)) + 3*(15*c^4*d^3*x^2
- 18*b*c^3*d^3*x + (24*b^2*c^2 - 25*a*c^3)*d^3)*sqrt(c*x^2 + b*x + a)*sqrt
(d*x))/c^5
```

Sympy [F]

$$\int \frac{(dx)^{7/2}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(dx)^{\frac{7}{2}}}{\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((d*x)**(7/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d*x)**(7/2)/sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{(dx)^{7/2}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(dx)^{\frac{7}{2}}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((d*x)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^(7/2)/sqrt(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{(dx)^{7/2}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(dx)^{\frac{7}{2}}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((d*x)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x)^(7/2)/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{7/2}}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(d\,x)^{7/2}}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d*x)^(7/2)/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((d*x)^(7/2)/(a + b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(dx)^{7/2}}{\sqrt{a+bx+cx^2}}\,dx=\frac{\sqrt{d}\,d^3\Big(18\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,a-12\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,bx+10\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,c}{}$$

input
```
int((d*x)^(7/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
(sqrt(d)*d**3*(18*sqrt(x)*sqrt(a + b*x + c*x**2)*a - 12*sqrt(x)*sqrt(a + b
*x + c*x**2)*b*x + 10*sqrt(x)*sqrt(a + b*x + c*x**2)*c*x**2 - 52*int((sqrt
(x)*sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*x**2),x)*a*c + 24*int((sqrt(x)*
sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*x**2),x)*b**2 - 9*int((sqrt(x)*sqrt
(a + b*x + c*x**2))/(a*x + b*x**2 + c*x**3),x)*a**2))/(35*c**2)
```

3.326 $\int \frac{(dx)^{5/2}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 488

$$\int \frac{(dx)^{5/2}}{\sqrt{a+bx+cx^2}}\,dx = -\frac{8bd^2\sqrt{dx}\sqrt{a+bx+cx^2}}{15c^2}+\frac{2d(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c}$$
$$+\frac{(8b^2-9ac)\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)d^{5/2}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}}{\sqrt{-b+\sqrt{b^2-4}}}\right.\right.}{15\sqrt{2}c^{7/2}\sqrt{a+x(b+cx)}}$$
$$-\frac{\sqrt{-b+\sqrt{b^2-4ac}}\left(8b^3-17abc+(8b^2-9ac)\sqrt{b^2-4ac}\right)d^{5/2}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\,\text{EllipticF}}{15\sqrt{2}c^{7/2}\sqrt{a+x(b+cx)}}$$

output

```
-8/15*b*d^2*(d*x)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2+2/5*d*(d*x)^(3/2)*(c*x^2+b
*x+a)^(1/2)/c+1/30*(-9*a*c+8*b^2)*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(b+(-4*a*c
+b^2)^(1/2))*d^(5/2)*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-
4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2^(1/2)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c
+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))
^(1/2))*2^(1/2)/c^(7/2)/(a+x*(c*x+b))^(1/2)-1/30*(-b+(-4*a*c+b^2)^(1/2))^(
1/2)*(8*b^3-17*a*b*c+(-9*a*c+8*b^2)*(-4*a*c+b^2)^(1/2))*d^(5/2)*(1+2*c*x/(
b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*Ellipt
icF(2^(1/2)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-
(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/c^(7/2)/(a+x*(c
*x+b))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.89 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.03

$$\int \frac{(dx)^{5/2}}{\sqrt{a+bx+cx^2}}\,dx = \frac{(dx)^{5/2}\left(4(8b^2-9ac)\,(a+x(b+cx))+4cx(-4b+3cx)(a+x(b+cx))-\frac{i(8b^2-9a}{}\right.}{}$$

input
```
Integrate[(d*x)^(5/2)/Sqrt[a + b*x + c*x^2],x]
```

output
```
((d*x)^(5/2)*(4*(8*b^2 - 9*a*c)*(a + x*(b + c*x)) + 4*c*x*(-4*b + 3*c*x)*(
a + x*(b + c*x)) - (I*(8*b^2 - 9*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4
*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(3/2)*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*
c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b
+ Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4
*a*c])])/Sqrt[a/(b + Sqrt[b^2 - 4*a*c])] + (I*(-8*b^3 + 17*a*b*c + 8*b^2*S
qrt[b^2 - 4*a*c] - 9*a*c*Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[b^2
- 4*a*c])*x)]*x^(3/2)*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b
^2 - 4*a*c]*x)]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c]
)])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a/(b
+ Sqrt[b^2 - 4*a*c])]))/(30*c^3*x^3*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.79, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {1166, 27, 1236, 27, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{5/2}}{\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{2\int -\frac{d^2\sqrt{dx}(3a+4bx)}{2\sqrt{cx^2+bx+a}}dx}{5c}+\frac{2d(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c}$$

$$\Big\downarrow \text{27}$$

$$\frac{2d(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c}-\frac{d^2\int \frac{\sqrt{dx}(3a+4bx)}{\sqrt{cx^2+bx+a}}dx}{5c}$$

$$\Big\downarrow \text{1236}$$

$$\frac{2d(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c}-\frac{d^2\left(\frac{2\int -\frac{d\left(4ab+\left(8b^2-9ac\right)x\right)}{2\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{3c}+\frac{8b\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}\right)}{5c}$$

$$\Big\downarrow \text{27}$$

$$\frac{2d(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c}-\frac{d^2\left(\frac{8b\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{d\int \frac{4ab+\left(8b^2-9ac\right)x}{\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{3c}\right)}{5c}$$

$$\Big\downarrow \text{1241}$$

$$\frac{2d(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c}-\frac{d^2\left(\frac{8b\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{d\sqrt{x}\int \frac{4ab+\left(8b^2-9ac\right)x}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{3c\sqrt{dx}}\right)}{5c}$$

$$\Big\downarrow \text{1240}$$

$$\frac{2d(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c}-\frac{d^2\left(\frac{8b\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{2d\sqrt{x}\int \frac{4ab+\left(8b^2-9ac\right)x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{3c\sqrt{dx}}\right)}{5c}$$

$$\Big\downarrow \text{1511}$$

$$\frac{2d(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c}-\frac{d^2\left(\frac{8b\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{2d\sqrt{x}\left(\frac{\sqrt{a}\left(4\sqrt{a}b\sqrt{c}-9ac+8b^2\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{a}\left(8b^2-9ac\right)\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{3c\sqrt{dx}}\right)}{5c}$$

$$\Big\downarrow \text{27}$$

$$\frac{2d(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c}-\frac{d^2\left(\frac{8b\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{2d\sqrt{x}\left(\frac{\sqrt{a}\left(4\sqrt{a}b\sqrt{c}-9ac+8b^2\right)\int\frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}-\frac{\left(8b^2-9ac\right)\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{3c\sqrt{dx}}\right)}{5c}$$

$\downarrow$ 1416

$$\frac{2d(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c}-\frac{d^2\left(\frac{8b\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{2d\sqrt{x}\left(\frac{\sqrt[4]{a}\left(4\sqrt{a}b\sqrt{c}-9ac+8b^2\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx+cx^2}}-\frac{\left(8b^2-9ac\right)\int\frac{\ldots}{\ldots}}{\ldots}\right)}{3c\sqrt{dx}}\right)}{5c}$$

$\downarrow$ 1509

$$\frac{2d(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c}-\frac{d^2\left(\frac{8b\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{2d\sqrt{x}\left(\frac{\sqrt[4]{a}\left(4\sqrt{a}b\sqrt{c}-9ac+8b^2\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx+cx^2}}-\frac{\left(8b^2-9ac\right)\left(\ldots\right)}{\ldots}\right)}{3c\sqrt{dx}}\right)}{5c}$$

input

```
Int[(d*x)^(5/2)/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*d*(d*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*c) - (d^2*((8*b*Sqrt[d*x]*Sqrt[
a + b*x + c*x^2])/(3*c) - (2*d*Sqrt[x]*(-(((8*b^2 - 9*a*c)*(-((Sqrt[x]*Sqr
t[a + b*x + c*x^2])/(Sqrt[a] + Sqrt[c]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x
)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1
/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x
 + c*x^2])))/Sqrt[c]) + (a^(1/4)*(8*b^2 + 4*Sqrt[a]*b*Sqrt[c] - 9*a*c)*(Sq
rt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*Ellipti
cF[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c
^(3/4)*Sqrt[a + b*x + c*x^2])))/(3*c*Sqrt[d*x])))/(5*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 2.53 (sec) , antiderivative size = 771, normalized size of antiderivative = 1.58

method	result
risch	$-\frac{2(-3cx+4b)x\sqrt{c\,x^2+bx+a}\,d^3}{15c^2\sqrt{dx}}+\frac{\left(-\frac{\left(9ac-8b^2\right)\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{-}}{}\right.}{}$
elliptic	$\frac{\sqrt{dx}\,\sqrt{dx(c\,x^2+bx+a)}\left(\frac{2d^2x\sqrt{cd\,x^3+bd\,x^2+adx}}{5c}-\frac{8d^2b\sqrt{cd\,x^3+bd\,x^2+adx}}{15c^2}+\frac{4d^3ba\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{x}{-\frac{b+\sqrt{-}}{}}}}{}\right.}{}$
default	Expression too large to display

input
```
int((d*x)^(5/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(-3*c*x+4*b)*x*(c*x^2+b*x+a)^(1/2)/c^2*d^3/(d*x)^(1/2)+1/15/c^2*(-(9
*a*c-8*b^2)*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2)
)/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-
1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x
/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)*((-1/2*(b+(-4
*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(2^(1/2)*((x+1/
2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4
*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(
1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(2^(1/2)*((x+1/2*(b+
(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+
b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))
))^(1/2)))+4*a*b*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^
(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)
))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-
2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)*Elliptic
F(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2
),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-
b+(-4*a*c+b^2)^(1/2))))^(1/2)))*d^3*(d*x*(c*x^2+b*x+a))^(1/2)/(d*x)^(1/2)/
(c*x^2+b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.41

$$\int \frac{(dx)^{5/2}}{\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{2\left((8\,b^3-21\,abc)\sqrt{cd}d^2\text{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\frac{3\,cx+b}{3\,c}\right)+3\,(8\,b^2c-9\,ac^2)\sqrt{cd}d^2\text{wei}\right.}{}$$

input
```
integrate((d*x)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/45*((8*b^3 - 21*a*b*c)*sqrt(c*d)*d^2*weierstrassPInverse(4/3*(b^2 - 3*a
*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c) + 3*(8*b^2*c - 9*
a*c^2)*sqrt(c*d)*d^2*weierstrassZeta(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 -
 9*a*b*c)/c^3, weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9
*a*b*c)/c^3, 1/3*(3*c*x + b)/c)) - 3*(3*c^3*d^2*x - 4*b*c^2*d^2)*sqrt(c*x^
2 + b*x + a)*sqrt(d*x))/c^4
```

Sympy [F]

$$\int \frac{(dx)^{5/2}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(dx)^{\frac{5}{2}}}{\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((d*x)**(5/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d*x)**(5/2)/sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{(dx)^{5/2}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(dx)^{\frac{5}{2}}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((d*x)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^(5/2)/sqrt(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{(dx)^{5/2}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(dx)^{\frac{5}{2}}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((d*x)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x)^(5/2)/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{5/2}}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{{\left(d\,x\right)}^{5/2}}{\sqrt{c\,x^2+b\,x+a}} \,d x$$

input
```
int((d*x)^(5/2)/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((d*x)^(5/2)/(a + b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(dx)^{5/2}}{\sqrt{a+bx+cx^2}}\,dx=\frac{\sqrt{d}\,d^2\left(-6\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,a+4\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,bx+9\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,x}{c\,x^2+bx+a}dx\right)\right.}{10bc}$$

input
```
int((d*x)^(5/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
(sqrt(d)*d**2*( - 6*sqrt(x)*sqrt(a + b*x + c*x**2)*a + 4*sqrt(x)*sqrt(a +
b*x + c*x**2)*b*x + 9*int((sqrt(x)*sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*
x**2),x)*a*c - 8*int((sqrt(x)*sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*x**2)
,x)*b**2 + 3*int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a*x + b*x**2 + c*x**3),
x)*a**2))/(10*b*c)
```

3.327 $\int \frac{(dx)^{3/2}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 435

$$\int \frac{(dx)^{3/2}}{\sqrt{a+bx+cx^2}}\,dx = \frac{2d\sqrt{dx}\sqrt{a+bx+cx^2}}{3c} - \frac{\sqrt{2}b\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)d^{3/2}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{dx}}{\sqrt{-b+\sqrt{b^2-4ac}}\sqrt{d}}\right)\right|}{3c^{5/2}\sqrt{a+x(b+cx)}} + \frac{\sqrt{2}\sqrt{-b+\sqrt{b^2-4ac}}\left(b^2-ac+b\sqrt{b^2-4ac}\right)d^{3/2}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(\arcsin\left(\frac{}{\sqrt{-b}}\right.\right.}{3c^{5/2}\sqrt{a+x(b+cx)}}$$

output
```
2/3*d*(d*x)^(1/2)*(c*x^2+b*x+a)^(1/2)/c-1/3*2^(1/2)*b*(-b+(-4*a*c+b^2)^(1/
2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*d^(3/2)*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^
(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2^(1/2)*c^(1/2)*(d*
x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+
(-4*a*c+b^2)^(1/2)))^(1/2))/c^(5/2)/(a+x*(c*x+b))^(1/2)+1/3*2^(1/2)*(-b+(-
4*a*c+b^2)^(1/2))^(1/2)*(b^2-a*c+b*(-4*a*c+b^2)^(1/2))*d^(3/2)*(1+2*c*x/(b
-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*Ellipti
cF(2^(1/2)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(
-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))/c^(5/2)/(a+x*(c*x+b))^(1
/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.78 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{3/2}}{\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{(dx)^{3/2}\left(2(2b-cx)(a+x(b+cx))+4i\sqrt{2}bc\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2a}{\left(b+\sqrt{b^2-4ac}\right)x}}\,x^{3/2}\sqrt{\frac{2a+bx-\sqrt{b^2-4acx}}{bx-\sqrt{b^2-4acx}}}\,E\left(i\mathrm{arc}\right.\right.}{}$$

input
```
Integrate[(d*x)^(3/2)/Sqrt[a + b*x + c*x^2],x]
```

output
```
-1/3*((d*x)^(3/2)*(2*(2*b - c*x)*(a + x*(b + c*x)) + (4*I)*Sqrt[2]*b*c*Sqr
t[a/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x
^(3/2)*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]
*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (
b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*Sqrt[2]*c*(-3*b + Sqrt
[b^2 - 4*a*c])*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*a)/((b + Sqrt[b
^2 - 4*a*c])*x)]*x^(3/2)*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqr
t[b^2 - 4*a*c]*x)]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a
*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(c^2*x
^2*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.76, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1166, 27, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{3/2}}{\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \; 1166$$

$$\frac{2\int -\frac{d^2(a+2bx)}{2\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{3c}+\frac{2d\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}$$

$$\Big\downarrow \text{27}$$

$$\frac{2d\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{d^2\int \frac{a+2bx}{\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{3c}$$

$$\Big\downarrow \text{1241}$$

$$\frac{2d\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{d^2\sqrt{x}\int \frac{a+2bx}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{3c\sqrt{dx}}$$

$$\Big\downarrow \text{1240}$$

$$\frac{2d\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{2d^2\sqrt{x}\int \frac{a+2bx}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{3c\sqrt{dx}}$$

$$\Big\downarrow \text{1511}$$

$$\frac{2d\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{2d^2\sqrt{x}\left(\sqrt{a}\left(\sqrt{a}+\frac{2b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{2\sqrt{a}b\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{3c\sqrt{dx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2d\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{2d^2\sqrt{x}\left(\sqrt{a}\left(\sqrt{a}+\frac{2b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{2b\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{3c\sqrt{dx}}$$

$$\Big\downarrow \text{1416}$$

$$\frac{2d\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{2d^2\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\frac{2b}{\sqrt{c}}\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}-\frac{2b\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{3c\sqrt{dx}}$$

$$\Big\downarrow \text{1509}$$

$$\frac{2d\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{2d^2\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\frac{2b}{\sqrt{c}}\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}-\frac{2b\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}E\left(2\operatorname{arc}\right.}{\sqrt[4]{c}\sqrt{a+bx}}\right.}{}\right.}{3c\sqrt{dx}}$$

input
```
Int[(d*x)^(3/2)/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*d*Sqrt[d*x]*Sqrt[a + b*x + c*x^2])/(3*c) - (2*d^2*Sqrt[x]*((-2*b*(-((Sq
rt[x]*Sqrt[a + b*x + c*x^2])/(Sqrt[a] + Sqrt[c]*x)) + (a^(1/4)*(Sqrt[a] +
Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticE[2*Arc
Tan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqr
t[a + b*x + c*x^2])))/Sqrt[c] + (a^(1/4)*(Sqrt[a] + (2*b)/Sqrt[c])*(Sqrt[a
] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2
*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/
4)*Sqrt[a + b*x + c*x^2])))/(3*c*Sqrt[d*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(351) = 702$.

Time $= 1.69$ (sec) , antiderivative size $= 752$, normalized size of antiderivative $= 1.73$

method	result
risch	$\frac{2x\sqrt{c\,x^2+bx+a}\,d^2}{3c\sqrt{dx}} - \frac{\left(\frac{a\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{-\frac{2cx}{b+\sqrt{-4ac+b^2}}}\,\mathrm{EllipticF}\left(\sqrt{\right.}{c\sqrt{cd\,x^3+bd\,x^2+adx}}$
elliptic	$\frac{\sqrt{dx}\,\sqrt{dx(c\,x^2+bx+a)}\left(\frac{2d\sqrt{cd\,x^3+bd\,x^2+adx}}{3c}-\frac{d^2a\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{-}}{3c^2\sqrt{c}}\right.}{}$
default	$\frac{d\sqrt{dx}\left(3\sqrt{\frac{2cx+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{-2cx+\sqrt{-4ac+b^2}-b}{\sqrt{-4ac+b^2}}}\sqrt{-\frac{cx}{b+\sqrt{-4ac+b^2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{2cx+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}},\frac{\sqrt{2}\sqrt{\frac{b+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2}\right)abc-\sqrt{}\right.}{}$

input
```
int((d*x)^(3/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/c*x*(c*x^2+b*x+a)^(1/2)*d^2/(d*x)^(1/2)-1/3/c*(a*(b+(-4*a*c+b^2)^(1/2)
)/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/
c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c
*d*x^3+b*d*x^2+a*d*x)^(1/2)*EllipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2
))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1
/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*b*(b+
(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+
b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2
)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)*((-1/2*(b+(-4*a*c+b^2)^(1/2)
)/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(2^(1/2)*((x+1/2*(b+(-4*a*c+b^
2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2)
)/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+
1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1
/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(
-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))*d^2
*(d*x*(c*x^2+b*x+a))^(1/2)/(d*x)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.39

$$\int \frac{(dx)^{3/2}}{\sqrt{a+bx+cx^2}}\,dx = \frac{2\left(6\sqrt{cd}bcd\text{weierstrassZeta}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\text{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac}{3\,c^2}\right.\right.\right.}{}$$

input
```
integrate((d*x)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(6*sqrt(c*d)*b*c*d*weierstrassZeta(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3
 - 9*a*b*c)/c^3, weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 -
 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c)) + 3*sqrt(c*x^2 + b*x + a)*sqrt(d*x)*c^2
*d + (2*b^2 - 3*a*c)*sqrt(c*d)*d*weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2
, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c))/c^3
```

Sympy [F]

$$\int \frac{(dx)^{3/2}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((d*x)**(3/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d*x)**(3/2)/sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{(dx)^{3/2}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((d*x)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^(3/2)/sqrt(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{(dx)^{3/2}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((d*x)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x)^(3/2)/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{3/2}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d\,x)^{3/2}}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d*x)^(3/2)/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((d*x)^(3/2)/(a + b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(dx)^{3/2}}{\sqrt{a+bx+cx^2}}\,dx = \sqrt{d}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,x}{c\,x^2+bx+a}dx\right)d$$

input
```
int((d*x)^(3/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
sqrt(d)*int((sqrt(x)*sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*x**2),x)*d
```

3.328 $\int \frac{\sqrt{dx}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 156

$$\int \frac{\sqrt{dx}}{\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{dx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{c\sqrt{-\frac{cx}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx+cx^2}}$$

output
```
2^(1/2)*(-4*a*c+b^2)^(1/2)*(d*x)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/
2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((
-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)))^(1/2))/c/(-c*x/(b+(-4*a*c+b^2)^(
1/2)))^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.22 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.85

$$\int \frac{\sqrt{dx}}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{i\left(-b+\sqrt{b^2-4ac}\right)\sqrt{dx}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\left(E\left(i\operatorname{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{x}\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)\right.}{\sqrt{2}c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{x}\sqrt{a+x(b+cx)}}$$

input
```
Integrate[Sqrt[d*x]/Sqrt[a + b*x + c*x^2],x]
```

output
```
(I*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[d*x]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c])]*(Ellip
ticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x]], (b + Sqrt
[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt
[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^
2 - 4*a*c])]))/(Sqrt[2]*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x]*Sqrt[a +
 x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.77, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1171, 1170, 1459, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx}}{\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{ 1171}$$

$$\frac{\sqrt{dx}\int \frac{\sqrt{x}}{\sqrt{cx^2+bx+a}}dx}{\sqrt{x}}$$

$$\Big\downarrow \text{ 1170}$$

$$\frac{2\sqrt{dx}\int \frac{x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{x}}$$

$$\Big\downarrow \; 1459$$

$$\frac{2\sqrt{dx}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{\sqrt{x}}$$

$$\Big\downarrow \; 27$$

$$\frac{2\sqrt{dx}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{\sqrt{x}}$$

$$\Big\downarrow \; 1416$$

$$\frac{2\sqrt{dx}\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx+cx^2}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{\sqrt{x}}$$

$$\Big\downarrow \; 1509$$

$$\frac{2\sqrt{dx}\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx+cx^2}}-\frac{\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{4}\right.}{\sqrt[4]{c}\sqrt{a+bx+cx^2}}}{\sqrt{c}}\right.}{\sqrt{x}}$$

input
```
Int[Sqrt[d*x]/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*Sqrt[d*x]*(-((-((Sqrt[x]*Sqrt[a + b*x + c*x^2])/(Sqrt[a] + Sqrt[c]*x))
 + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]
*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt
[c]))/4])/(c^(1/4)*Sqrt[a + b*x + c*x^2]))/Sqrt[c]) + (a^(1/4)*(Sqrt[a] +
Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*Arc
Tan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*S
qrt[a + b*x + c*x^2])))/Sqrt[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1170
```
Int[(x_)^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2
 Subst[Int[x^(2*m + 1)/Sqrt[a + b*x^2 + c*x^4], x], x, Sqrt[x]], x] /; Free
Q[{a, b, c}, x] && EqQ[m^2, 1/4]
```

rule 1171
```
Int[((e_)*(x_))^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> S
imp[(e*x)^m/x^m   Int[x^m/Sqrt[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c,
e}, x] && EqQ[m^2, 1/4]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1459
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[1/q   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[1/q
 Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(137) = 274$.

Time = 1.16 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.19

method	result
default	$-\frac{\sqrt{dx}\left(b+\sqrt{-4ac+b^2}\right)\sqrt{\frac{2cx+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{-2cx+\sqrt{-4ac+b^2}-b}{\sqrt{-4ac+b^2}}}\sqrt{-\frac{cx}{b+\sqrt{-4ac+b^2}}}\left(2\sqrt{-4ac+b^2}\,\text{EllipticE}\left(\sqrt{\frac{2cx+\sqrt{-4ac+b^2}-}{b+\sqrt{-4ac+b^2}}}\right.\right.}{}$
elliptic	$\frac{\sqrt{dx}\,\sqrt{dx(c\,x^2+bx+a)}\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{-\frac{2cx}{b+\sqrt{-4ac+b^2}}}\left(\left(-\frac{b+\sqrt{}}{}\right.\right.}{}$

input
```
int((d*x)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(d*x)^(1/2)/(c*x^2+b*x+a)^(1/2)*(b+(-4*a*c+b^2)^(1/2))*((2*c*x+(-4*a*
c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-
b)/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(2*(-4*a*
c+b^2)^(1/2)*EllipticE(((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)
))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2))-El
lipticF(((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^
(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2))*(-4*a*c+b^2)^(1/2
)+EllipticF(((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/
2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2))*b)/x/c^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{dx}}{\sqrt{a+bx+cx^2}}\,dx = -\frac{2\left(\sqrt{cd}b\text{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\frac{3\,cx+b}{3\,c}\right)+3\,\sqrt{cd}c\text{weierstrassZeta}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(\right.\right.}{3\,c^2}$$

input
```
integrate((d*x)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(c*d)*b*weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3
- 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c) + 3*sqrt(c*d)*c*weierstrassZeta(4/3*(b^
2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, weierstrassPInverse(4/3*(b^2
- 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c)))/c^2
```

Sympy [F]

$$\int \frac{\sqrt{dx}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{dx}}{\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((d*x)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(d*x)/sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{dx}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{dx}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((d*x)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x)/sqrt(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{\sqrt{dx}}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{\sqrt{dx}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((d*x)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x)/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{dx}}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{\sqrt{d\,x}}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d*x)^(1/2)/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((d*x)^(1/2)/(a + b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{dx}}{\sqrt{a+bx+cx^2}}\,dx=\sqrt{d}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+bx+a}}{c\,x^2+bx+a}dx\right)$$

input
```
int((d*x)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
sqrt(d)*int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a + b*x + c*x**2),x)
```

3.329 $\int \frac{1}{\sqrt{dx}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 157

$$\int \frac{1}{\sqrt{dx}\sqrt{a+bx+cx^2}}\,dx = \frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{cx}{b+\sqrt{b^2-4ac}}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{c\sqrt{dx}\sqrt{a+bx+cx^2}}$$

output
```
2*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-c*(c*x^
2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2)
)^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)))^(1/2))
/c/(d*x)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.22 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt{dx}\sqrt{a+bx+cx^2}}\,dx = \frac{i\sqrt{1+\frac{2a}{\left(b+\sqrt{b^2-4ac}\right)x}}\,x^{3/2}\sqrt{2+\frac{4a}{bx-\sqrt{b^2-4ac}x}}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\frac{\sqrt{2}\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}}{\sqrt{x}}\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}\sqrt{dx}\sqrt{a+x(b+cx)}}$$

input
```
Integrate[1/(Sqrt[d*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(I*Sqrt[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(3/2)*Sqrt[2 + (4*a)/(b*x
 - Sqrt[b^2 - 4*a*c]*x)]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2
 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(
Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[d*x]*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1171, 1170, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx}\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \ 1171$$

$$\frac{\sqrt{x}\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{\sqrt{dx}}$$

$$\Big\downarrow \ 1170$$

$$\frac{2\sqrt{x}\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{dx}}$$

$$\Big\downarrow \ 1416$$

$$\frac{\sqrt{x}(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{a}\sqrt[4]{c}\sqrt{dx}\sqrt{a+bx+cx^2}}$$

input
```
Int[1/(Sqrt[d*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(Sqrt[x]*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x
)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c
]))/4])/(a^(1/4)*c^(1/4)*Sqrt[d*x]*Sqrt[a + b*x + c*x^2])
```

Defintions of rubi rules used

rule 1170
```
Int[(x_)^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2
 Subst[Int[x^(2*m + 1)/Sqrt[a + b*x^2 + c*x^4], x], x, Sqrt[x]], x] /; Free
Q[{a, b, c}, x] && EqQ[m^2, 1/4]
```

rule 1171
```
Int[((e_)*(x_))^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> S
imp[(e*x)^m/x^m   Int[x^m/Sqrt[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c,
e}, x] && EqQ[m^2, 1/4]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.16

method	result
default	$\frac{\left(b+\sqrt{-4ac+b^2}\right)\sqrt{\frac{2cx+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{-2cx+\sqrt{-4ac+b^2}-b}{\sqrt{-4ac+b^2}}}\sqrt{-\frac{cx}{b+\sqrt{-4ac+b^2}}}\operatorname{EllipticF}\left(\sqrt{\frac{2cx+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}},\frac{\sqrt{2}\sqrt{\frac{b+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2}\right)}{\sqrt{c\,x^2+bx+a}\,c\sqrt{dx}}$
elliptic	$\frac{\sqrt{dx(c\,x^2+bx+a)}\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{-\frac{2cx}{b+\sqrt{-4ac+b^2}}}\operatorname{EllipticF}\left(\sqrt{2}\right.}{\sqrt{dx}\,\sqrt{c\,x^2+bx+a}\,c\sqrt{cd\,x^3+bd\,x^2+adx}}$

input

```
int(1/(d*x)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/(c*x^2+b*x+a)^(1/2)*(b+(-4*a*c+b^2)^(1/2))*((2*c*x+(-4*a*c+b^2)^(1/2)+b)
/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2)
^(1/2))^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(((2*c*x+(-4*a*
c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)
^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2))/c/(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.34

$$\int \frac{1}{\sqrt{dx}\sqrt{a+bx+cx^2}}\,dx = \frac{2\sqrt{cd}\text{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\frac{3\,cx+b}{3\,c}\right)}{cd}$$

input

```
integrate(1/(d*x)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output

```
2*sqrt(c*d)*weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*
b*c)/c^3, 1/3*(3*c*x + b)/c)/(c*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{dx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{dx}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/(d*x)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(d*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{dx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(d*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{dx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(d*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{dx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{d\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/((d*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/((d*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{dx}\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+bx+a}}{c\,x^3+b\,x^2+ax}dx\right)}{d}$$

input
```
int(1/(d*x)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a*x + b*x**2 + c*x**3),x))/
d
```

3.330 $\int \frac{1}{(dx)^{3/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 189

$$\int \frac{1}{(dx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2\sqrt{a+bx+cx^2}}{ad\sqrt{dx}} + \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{dx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{ad^2\sqrt{-\frac{cx}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx+cx^2}}$$

output
```
-2*(c*x^2+b*x+a)^(1/2)/a/d/(d*x)^(1/2)+2^(1/2)*(-4*a*c+b^2)^(1/2)*(d*x)^(1
/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a
*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(
1/2)))^(1/2))/a/d^2/(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^2+b*x+a)^(1/2
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.39 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.62

$$\int \frac{1}{(dx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \frac{x\left(-4(a+x(b+cx))+\frac{i\sqrt{2}\left(-b+\sqrt{b^2-4ac}\right)\sqrt{x}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\left(E\left(i\text{ar}\cdots\right.\right.}{\cdots}\right.}{2a(dx)^{3/2}\cdots}$$

input
```
Integrate[1/((d*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(x*(-4*(a + x*(b + c*x)) + (I*Sqrt[2]*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[x]*Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b
^2 - 4*a*c])]*Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] -
 EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x]], (b
+ Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/Sqrt[c/(b + Sqrt[b^2 - 4*a
*c])]))/(2*a*(d*x)^(3/2)*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.66, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1167, 8, 27, 1171, 1170, 1459, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{3/2}\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \; 1167$$

$$-\frac{2\int -\frac{cdx}{2\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{ad^2}-\frac{2\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}$$

$$\Big\downarrow \; 8$$

$$-\frac{2\int -\frac{cd\sqrt{dx}}{2\sqrt{cx^2+bx+a}}dx}{ad^3}-\frac{2\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{c\int \frac{\sqrt{dx}}{\sqrt{cx^2+bx+a}}dx}{ad^2}-\frac{2\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}$$

$$\Big\downarrow \textbf{1171}$$

$$\frac{c\sqrt{dx}\int \frac{\sqrt{x}}{\sqrt{cx^2+bx+a}}dx}{ad^2\sqrt{x}}-\frac{2\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}$$

$$\Big\downarrow \textbf{1170}$$

$$\frac{2c\sqrt{dx}\int \frac{x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{ad^2\sqrt{x}}-\frac{2\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}$$

$$\Big\downarrow \textbf{1459}$$

$$\frac{2c\sqrt{dx}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{ad^2\sqrt{x}}-\frac{2\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{2c\sqrt{dx}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{ad^2\sqrt{x}}-\frac{2\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}$$

$$\Big\downarrow \textbf{1416}$$

$$\frac{2c\sqrt{dx}\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx+cx^2}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{ad^2\sqrt{x}}-\frac{2\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}$$

$$\Big\downarrow \textbf{1509}$$

$$\frac{2c\sqrt{dx}\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx+cx^2}} - \frac{\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{4}\right.}{\sqrt[4]{c}\sqrt{a+bx+cx^2}}}{\sqrt{c}}\right.}{ad^2\sqrt{x}}$$

$$\frac{2\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}$$

input
```
Int[1/((d*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(-2*Sqrt[a + b*x + c*x^2])/(a*d*Sqrt[d*x]) + (2*c*Sqrt[d*x]*(-((-((Sqrt[x]
*Sqrt[a + b*x + c*x^2])/(Sqrt[a] + Sqrt[c]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[
c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(
c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a +
 b*x + c*x^2]))/Sqrt[c]) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x +
c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x + c*x^2])))/(a*
d^2*Sqrt[x])
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1170
```
Int[(x_)^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2
 Subst[Int[x^(2*m + 1)/Sqrt[a + b*x^2 + c*x^4], x], x, Sqrt[x]], x] /; Free
Q[{a, b, c}, x] && EqQ[m^2, 1/4]
```

rule 1171
```
Int[((e_)*(x_))^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> S
imp[(e*x)^m/x^m   Int[x^m/Sqrt[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c,
e}, x] && EqQ[m^2, 1/4]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1459
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[1/q   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[1/q
 Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(166) = 332$.

Time = 2.40 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.55

method	result
risch	$-\frac{2\sqrt{c\,x^2+bx+a}}{a d\sqrt{dx}}+\frac{\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{-\frac{2cx}{b+\sqrt{-4ac+b^2}}}\left(\left(-\frac{b+\sqrt{-\ldots}}{2\ldots}\right.\right.}{\ldots}$
elliptic	$\frac{\sqrt{dx(c\,x^2+bx+a)}\left(-\frac{2\left(c d\,x^2+b d x+a d\right)}{d^2 a\sqrt{x\left(c d\,x^2+b d x+a d\right)}}+\frac{\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{-\frac{\ldots}{b+\sqrt{\ldots}}}\ldots}{\ldots}\right.}{\ldots}$
default	$\frac{-2\sqrt{\frac{2cx+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{-2cx+\sqrt{-4ac+b^2}-b}{\sqrt{-4ac+b^2}}}\sqrt{-\frac{cx}{b+\sqrt{-4ac+b^2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{2cx+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}},\frac{\sqrt{2}\sqrt{\frac{b+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2}\right)ac+4\sqrt{\frac{2cx+\ldots}{b+\ldots}}\ldots}{\ldots}$

input
```
int(1/(d*x)^(3/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(c*x^2+b*x+a)^(1/2)/a/d/(d*x)^(1/2)+1/a*(b+(-4*a*c+b^2)^(1/2))*2^(1/2)*
((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/
c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*
c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x
^2+a*d*x)^(1/2)*((-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/
2)))*EllipticE(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(
1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2
))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*
EllipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))
*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-
1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))/d*(d*x*(c*x^2+b*x+a))^(1/2)/(d*x)^
(1/2)/(c*x^2+b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.87

$$\int \frac{1}{(dx)^{3/2}\sqrt{a+bx+cx^2}}\,dx =$$
$$-\frac{2\left(\sqrt{cd}bx\text{weierstrassPInverse}\left(\frac{4(b^2-3ac)}{3c^2},-\frac{4(2b^3-9abc)}{27c^3},\frac{3cx+b}{3c}\right)+3\sqrt{cd}cx\text{weierstrassZeta}\left(\frac{4(b^2-3ac)}{3c^2},-\frac{4(\cdots}{\cdots}\right.\cdots\right.}{3acd^2x}$$

input
```
integrate(1/(d*x)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(c*d)*b*x*weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^
3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c) + 3*sqrt(c*d)*c*x*weierstrassZeta(4/3
*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, weierstrassPInverse(4/3*(
b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c)) + 3*sqr
t(c*x^2 + b*x + a)*sqrt(d*x)*c)/(a*c*d^2*x)
```

Sympy [F]

$$\int \frac{1}{(dx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(dx)^{\frac{3}{2}}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/(d*x)**(3/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/((d*x)**(3/2)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(dx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\,(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(d*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(dx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\,(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(d*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(d\,x)^{3/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/((d*x)^(3/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/((d*x)^(3/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(dx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{d}\left(-2\sqrt{x}\,\sqrt{c\,x^2+bx+a}+\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+bx+a}}{c\,x^2+bx+a}dx\right)cx\right)}{a\,d^2x}$$

input
```
int(1/(d*x)^(3/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
(sqrt(d)*( - 2*sqrt(x)*sqrt(a + b*x + c*x**2) + int((sqrt(x)*sqrt(a + b*x
+ c*x**2))/(a + b*x + c*x**2),x)*c*x))/(a*d**2*x)
```

3.331 $\int \frac{1}{(dx)^{5/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 475

$$\int \frac{1}{(dx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}} + \frac{4b\sqrt{a+bx+cx^2}}{3a^2d^2\sqrt{dx}} - \frac{\sqrt{2}b\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{dx}}{\sqrt{-b+\sqrt{b^2-4ac}}\sqrt{d}}\right)|\frac{b-\sqrt{}}{b+\sqrt{}}\right.}{3a^2\sqrt{c}d^{5/2}\sqrt{a+x(b+cx)}} + \frac{\sqrt{2}\sqrt{-b+\sqrt{b^2-4ac}}\left(b^2-ac+b\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{-b+\sqrt{b}}}\right.\right.}{3a^2\sqrt{c}d^{5/2}\sqrt{a+x(b+cx)}}$$

output
```
-2/3*(c*x^2+b*x+a)^(1/2)/a/d/(d*x)^(3/2)+4/3*b*(c*x^2+b*x+a)^(1/2)/a^2/d^2
/(d*x)^(1/2)-1/3*2^(1/2)*b*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(b+(-4*a*c+b^2)^(
1/2))*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2
)))^(1/2)*EllipticE(2^(1/2)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1
/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))/a^2/c^(
1/2)/d^(5/2)/(a+x*(c*x+b))^(1/2)+1/3*2^(1/2)*(-b+(-4*a*c+b^2)^(1/2))^(1/2)
*(b^2-a*c+b*(-4*a*c+b^2)^(1/2))*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+
2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(2^(1/2)*c^(1/2)*(d*x)^(1/2)/
(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b
^2)^(1/2)))^(1/2))/a^2/c^(1/2)/d^(5/2)/(a+x*(c*x+b))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.13 (sec) , antiderivative size = 441, normalized size of antiderivative = 0.93

$$\int \frac{1}{(dx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \frac{x\left(-2a\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}(a+x(b+cx))+ib\left(-b+\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2a}{\left(b+\sqrt{b^2-4ac}\right.}}\right.}{}$$

input
```
Integrate[1/((d*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(x*(-2*a*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*(a + x*(b + c*x)) + I*b*(-b + Sqr
t[b^2 - 4*a*c])*Sqrt[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(5/2)*Sqrt[(
4*a + 2*b*x - 2*Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*Elliptic
E[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[
b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + a*c + b*Sqrt[b^2 - 4*a*
c])*Sqrt[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(5/2)*Sqrt[(4*a + 2*b*x
- 2*Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticF[I*ArcSinh[
(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c]
)/(b - Sqrt[b^2 - 4*a*c])]))/(3*a^2*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*(d*x)^
(5/2)*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.78, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {1167, 27, 1237, 27, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{5/2}\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{ 1167}$$

$$-\frac{2\int \frac{d(2b+cx)}{2(dx)^{3/2}\sqrt{cx^2+bx+a}}dx}{3ad^2}-\frac{2\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{2b+cx}{(dx)^{3/2}\sqrt{cx^2+bx+a}}dx}{3ad}-\frac{2\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}$$

$$\Big\downarrow \ 1237$$

$$-\frac{-\frac{2\int -\frac{cd(a+2bx)}{2\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{ad^2}-\frac{4b\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{3ad}-\frac{2\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{c\int \frac{a+2bx}{\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{ad}-\frac{4b\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{3ad}-\frac{2\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}$$

$$\Big\downarrow \ 1241$$

$$-\frac{\frac{c\sqrt{x}\int \frac{a+2bx}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{ad\sqrt{dx}}-\frac{4b\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{3ad}-\frac{2\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}$$

$$\Big\downarrow \ 1240$$

$$-\frac{\frac{2c\sqrt{x}\int \frac{a+2bx}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{ad\sqrt{dx}}-\frac{4b\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{3ad}-\frac{2\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}$$

$$\Big\downarrow \ 1511$$

$$-\frac{\frac{2c\sqrt{x}\left(\sqrt{a}\left(\sqrt{a}+\frac{2b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{2\sqrt{a}b\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{ad\sqrt{dx}}-\frac{4b\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{3ad}-\frac{2\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{2c\sqrt{x}\left(\sqrt{a}\left(\sqrt{a}+\frac{2b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{2b\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{ad\sqrt{dx}}-\frac{4b\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{3ad}-\frac{2\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}$$

$$\Big\downarrow \ 1416$$

$$-\frac{\frac{2c\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\frac{2b}{\sqrt{c}}\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}-\frac{2b\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{ad\sqrt{dx}}-\frac{4b\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{3ad}-\frac{2\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}$$

$$\Big\downarrow \text{ 1509}$$

$$-\frac{\frac{2c\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\frac{2b}{\sqrt{c}}\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}-\frac{2b\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{\cdots}}{\sqrt[4]{a}}\right.\right.}{\sqrt[4]{c}\sqrt{a+bx+cx^2}}\right.}{\sqrt{c}}\cdots}{ad\sqrt{dx}}\cdots}{3ad}$$

$$\frac{2\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}$$

input
```
Int[1/((d*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(-2*Sqrt[a + b*x + c*x^2])/(3*a*d*(d*x)^(3/2)) - ((-4*b*Sqrt[a + b*x + c*x
^2])/(a*d*Sqrt[d*x]) + (2*c*Sqrt[x]*((-2*b*(-((Sqrt[x]*Sqrt[a + b*x + c*x^
2])/(Sqrt[a] + Sqrt[c]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x
+ c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(
1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x + c*x^2])))/Sqr
t[c] + (a^(1/4)*(Sqrt[a] + (2*b)/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a +
b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])
/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x + c*x^2])
))/(a*d*Sqrt[d*x]))/(3*a*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 3.68 (sec) , antiderivative size = 761, normalized size of antiderivative = 1.60

method	result
risch	$-\frac{2\sqrt{c\,x^{2}+bx+a}\,(-2bx+a)}{3a^{2}x\,d^{2}\sqrt{dx}}-\frac{c\left(\frac{a\left(b+\sqrt{-4ac+b^{2}}\right)\sqrt{2}\,\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^{2}}}{2c}\right)c}{b+\sqrt{-4ac+b^{2}}}}\,\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^{2}}}{2c}}{-\frac{b+\sqrt{-4ac+b^{2}}}{2c}-\frac{-b+\sqrt{-4ac+b^{2}}}{2c}}}\,\sqrt{-\frac{2cx}{b+\sqrt{-4ac+b^{2}}}}\,\mathrm{F}}{c\sqrt{cd\,x^{3}+bd\,x^{2}+adx}}\right.}{}$
elliptic	$\frac{\sqrt{dx(c\,x^{2}+bx+a)}\left(-\frac{2\sqrt{cd\,x^{3}+bd\,x^{2}+adx}}{3d^{3}a\,x^{2}}+\frac{4\left(cd\,x^{2}+bdx+ad\right)b}{3d^{3}a^{2}\sqrt{x\left(cd\,x^{2}+bdx+ad\right)}}-\frac{\left(b+\sqrt{-4ac+b^{2}}\right)\sqrt{2}\,\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^{2}}}{2c}\right)c}{b+\sqrt{-4ac+b^{2}}}}\,\sqrt{\frac{x-\frac{-b+\sqrt{-}}{2}}{-\frac{b+\sqrt{-4ac+b^{2}}}{2c}-}}}{}\right.}{}$
default	$\frac{3\sqrt{\frac{2cx+\sqrt{-4ac+b^{2}}+b}{b+\sqrt{-4ac+b^{2}}}}\,\sqrt{\frac{-2cx+\sqrt{-4ac+b^{2}}-b}{\sqrt{-4ac+b^{2}}}}\,\sqrt{-\frac{cx}{b+\sqrt{-4ac+b^{2}}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{2cx+\sqrt{-4ac+b^{2}}+b}{b+\sqrt{-4ac+b^{2}}}},\frac{\sqrt{2}\sqrt{\frac{b+\sqrt{-4ac+b^{2}}}{\sqrt{-4ac+b^{2}}}}}{2}\right)abcx-\sqrt{\frac{-2cx}{\sqrt{}}}}{}$

input
```
int(1/(d*x)^(5/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(c*x^2+b*x+a)^(1/2)*(-2*b*x+a)/a^2/x/d^2/(d*x)^(1/2)-1/3*c/a^2*(a*(b+
(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+
b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2
)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)*EllipticF(2^(1/2)*((x+1/2*(b
+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c
+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2)
)))^(1/2))+2*b*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1
/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))
/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*
c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)*((-1/2*(b+
(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(2^(1/2)*((x
+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+
(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2
)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(2^(1/2)*((x+1/2*
(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a
*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/
2))))^(1/2))))/d^2*(d*x*(c*x^2+b*x+a))^(1/2)/(d*x)^(1/2)/(c*x^2+b*x+a)^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.39

$$\int \frac{1}{(dx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \frac{2\left(6\sqrt{cd}bcx^2\text{weierstrassZeta}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\text{weierstrassPInverse}\left(\right.\right.\right.}{}$$

input
```
integrate(1/(d*x)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(6*sqrt(c*d)*b*c*x^2*weierstrassZeta(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b
^3 - 9*a*b*c)/c^3, weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3
 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c)) + (2*b^2 - 3*a*c)*sqrt(c*d)*x^2*weier
strassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*
c*x + b)/c) + 3*(2*b*c*x - a*c)*sqrt(c*x^2 + b*x + a)*sqrt(d*x))/(a^2*c*d^
3*x^2)
```

Sympy [F]

$$\int \frac{1}{(dx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(dx)^{\frac{5}{2}}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/(d*x)**(5/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/((d*x)**(5/2)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(dx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\,(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(d*x)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(dx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\,(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(d*x)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(d\,x)^{5/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/((d*x)^(5/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/((d*x)^(5/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(dx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+bx+a}}{c\,x^5+b\,x^4+a\,x^3}dx\right)}{d^3}$$

input
```
int(1/(d*x)^(5/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a*x**3 + b*x**4 + c*x**5),x
))/d**3
```

3.332 $\int \frac{(dx)^{7/2}}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 591

$$\int \frac{(dx)^{7/2}}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2d(dx)^{5/2}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}$$
$$+\frac{4(2b^2-5ac)\,d^3\sqrt{dx}\sqrt{a+bx+cx^2}}{3c^2\,(b^2-4ac)}-\frac{2bd^2(dx)^{3/2}\sqrt{a+bx+cx^2}}{c\,(b^2-4ac)}$$
$$-\frac{b(8b^2-29ac)\,\sqrt{-b+\sqrt{b^2-4ac}}\big(b+\sqrt{b^2-4ac}\big)\,d^{7/2}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{-b+\sqrt{b}}}\right.\right.}{3\sqrt{2}c^{7/2}\,(b^2-4ac)\,\sqrt{a+x(b+cx)}}$$
$$+\frac{\sqrt{-b+\sqrt{b^2-4ac}}\big(8b^4-37ab^2c+20a^2c^2+b(8b^2-29ac)\,\sqrt{b^2-4ac}\big)\,d^{7/2}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4}}}}{3\sqrt{2}c^{7/2}\,(b^2-4ac)\,\sqrt{a+x(b+cx)}}$$

output
```
2*d*(d*x)^(5/2)*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/2)+4/3*(-5*a*c+2*b
^2)*d^3*(d*x)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/(-4*a*c+b^2)-2*b*d^2*(d*x)^(3/
2)*(c*x^2+b*x+a)^(1/2)/c/(-4*a*c+b^2)-1/6*b*(-29*a*c+8*b^2)*(-b+(-4*a*c+b^
2)^(1/2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*d^(7/2)*(1+2*c*x/(b-(-4*a*c+b^2)^(1
/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2^(1/2)*c^(1/
2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2
))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/c^(7/2)/(-4*a*c+b^2)/(a+x*(c*x+b
))^(1/2)+1/6*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(8*b^4-37*a*b^2*c+20*a^2*c^2+b*
(-29*a*c+8*b^2)*(-4*a*c+b^2)^(1/2))*d^(7/2)*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)
))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(2^(1/2)*c^(1/2)*
(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/
(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/c^(7/2)/(-4*a*c+b^2)/(a+x*(c*x+b))^
(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.96 (sec) , antiderivative size = 572, normalized size of antiderivative = 0.97

$$\int \frac{(dx)^{7/2}}{(a+bx+cx^2)^{3/2}}\,dx = \frac{(dx)^{7/2}\left(-4b(8b^2-29ac)\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}(a+x(b+cx))-4c\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}x(10a^2c-\right.}{}$$

input
```
Integrate[(d*x)^(7/2)/(a + b*x + c*x^2)^(3/2),x]
```

output
```
((d*x)^(7/2)*(-4*b*(8*b^2 - 29*a*c)*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*(a + x
*(b + c*x)) - 4*c*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*x*(10*a^2*c - b^2*x*(4*b
 + c*x) + a*(-4*b^2 + 13*b*c*x + 4*c^2*x^2)) + I*b*(8*b^2 - 29*a*c)*(-b +
Sqrt[b^2 - 4*a*c])*Sqrt[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(3/2)*Sqr
t[(4*a + 2*b*x - 2*Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*Ellip
ticE[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sq
rt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(8*b^4 - 37*a*b^2*c + 20*a^2
*c^2 - 8*b^3*Sqrt[b^2 - 4*a*c] + 29*a*b*c*Sqrt[b^2 - 4*a*c])*Sqrt[1 + (2*a
)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(3/2)*Sqrt[(4*a + 2*b*x - 2*Sqrt[b^2 - 4*
a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(
b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 -
 4*a*c])]))/(6*c^3*(b^2 - 4*a*c)*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*x^4*Sqrt[
a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 462, normalized size of antiderivative = 0.78, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {1164, 27, 1236, 27, 1236, 27, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{7/2}}{(a+bx+cx^2)^{3/2}}dx$$

$$\Big\downarrow \ 1164$$

$$\frac{2d(dx)^{5/2}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{2\int \frac{5d^2(dx)^{3/2}(2a+bx)}{2\sqrt{cx^2+bx+a}}dx}{b^2-4ac}$$

$$\Big\downarrow \ 27$$

$$\frac{2d(dx)^{5/2}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{5d^2\int \frac{(dx)^{3/2}(2a+bx)}{\sqrt{cx^2+bx+a}}dx}{b^2-4ac}$$

$$\Big\downarrow \ 1236$$

$$\frac{2d(dx)^{5/2}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{5d^2\left(\frac{2\int -\frac{d\sqrt{dx}\left(3ab+2\left(2b^2-5ac\right)x\right)}{2\sqrt{cx^2+bx+a}}dx}{5c}+\frac{2b(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c}\right)}{b^2-4ac}$$

$$\downarrow \text{27}$$

$$\frac{2d(dx)^{5/2}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{5d^2\left(\frac{2b(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c}-\frac{d\int \frac{\sqrt{dx}\left(3ab+2\left(2b^2-5ac\right)x\right)}{\sqrt{cx^2+bx+a}}dx}{5c}\right)}{b^2-4ac}$$

$$\downarrow \text{1236}$$

$$\frac{2d(dx)^{5/2}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{5d^2\left(\frac{2b(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c}-\frac{d\left(\frac{2\int -\frac{d\left(2a\left(2b^2-5ac\right)+b\left(8b^2-29ac\right)x\right)}{2\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{3c}+\frac{4\sqrt{dx}\left(2b^2-5ac\right)\sqrt{a+bx+cx^2}}{3c}\right)}{5c}\right)}{b^2-4ac}$$

$$\downarrow \text{27}$$

$$\frac{2d(dx)^{5/2}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{5d^2\left(\frac{2b(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c}-\frac{d\left(\frac{4\sqrt{dx}\left(2b^2-5ac\right)\sqrt{a+bx+cx^2}}{3c}-\frac{d\int \frac{2a\left(2b^2-5ac\right)+b\left(8b^2-29ac\right)x}{\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{3c}\right)}{5c}\right)}{b^2-4ac}$$

$$\downarrow \text{1241}$$

$$\frac{2d(dx)^{5/2}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{5d^2\left(\frac{2b(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c}-\frac{d\left(\frac{4\sqrt{dx}\left(2b^2-5ac\right)\sqrt{a+bx+cx^2}}{3c}-\frac{d\sqrt{x}\int \frac{2a\left(2b^2-5ac\right)+b\left(8b^2-29ac\right)x}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{3c\sqrt{dx}}\right)}{5c}\right)}{b^2-4ac}$$

$$\Big\downarrow \ 1240$$

$$\frac{2d(dx)^{5/2}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}} - \frac{5d^2\left(\frac{2b(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c} - \frac{d\left(\frac{4\sqrt{dx}\left(2b^2-5ac\right)\sqrt{a+bx+cx^2}}{3c} - \frac{2d\sqrt{x}\int\frac{2a\left(2b^2-5ac\right)+b\left(8b^2-29ac\right)x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{3c\sqrt{dx}}\right)}{5c}\right)}{b^2-4ac}$$

$$\Big\downarrow \ 1511$$

$$\frac{2d(dx)^{5/2}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}} - \frac{5d^2\left(\frac{2b(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c} - \frac{d\left(\frac{4\sqrt{dx}\left(2b^2-5ac\right)\sqrt{a+bx+cx^2}}{3c} - \frac{2d\sqrt{x}\left(\sqrt{a}\left(\frac{b\left(8b^2-29ac\right)}{\sqrt{c}}+2\sqrt{a}\left(2b^2-5ac\right)\right)\int\frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x} - \frac{\sqrt{a}b\left(8b^2-29ac\right)\int \cdots}{\cdots}\right.}{3c\sqrt{dx}} \cdots\right)}{5c}\cdots\right)}{b^2-4ac}$$

$$\Big\downarrow \ 27$$

$$\frac{2d(dx)^{5/2}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}} - \frac{5d^2\left(\frac{2b(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c} - \frac{d\left(\frac{4\sqrt{dx}\left(2b^2-5ac\right)\sqrt{a+bx+cx^2}}{3c} - \frac{2d\sqrt{x}\left(\sqrt{a}\left(\frac{b\left(8b^2-29ac\right)}{\sqrt{c}}+2\sqrt{a}\left(2b^2-5ac\right)\right)\int\frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x} - \frac{b\left(8b^2-29ac\right)\int \frac{-\sqrt{\cdots}}{\sqrt{c}\cdots}}{\sqrt{c}\cdots}\right.}{3c\sqrt{dx}} \cdots\right)}{5c}\cdots\right)}{b^2-4ac}$$

$$\Big\downarrow \ 1416$$

$$\frac{\frac{2d(dx)^{5/2}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}-5d^2\left(\frac{2b(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c}-\frac{d\left(\frac{4\sqrt{dx}\left(2b^2-5ac\right)\sqrt{a+bx+cx^2}}{3c}-\frac{2d\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\frac{b\left(8b^2-29ac\right)}{\sqrt{c}}+2\sqrt{a}\left(2b^2-5ac\right)\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\right.}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}\right.}{3c\sqrt{dx}}\right.}{5c}\right.}{b^2-4ac}$$

$\big\downarrow$ 1509

$$\frac{2d(dx)^{5/2}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}} - \frac{5d^2\left(\frac{2b(dx)^{3/2}\sqrt{a+bx+cx^2}}{5c} - \frac{d\left(\frac{4\sqrt{dx}\left(2b^2-5ac\right)\sqrt{a+bx+cx^2}}{3c} - \frac{2d\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\frac{b\left(8b^2-29ac\right)}{\sqrt{c}}+2\sqrt{a}\left(2b^2-5ac\right)\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\operatorname{EllipticF}\left(2\right.}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}\right.}{}\right)}{}\right)}{b^2-4}$$

input
```
Int[(d*x)^(7/2)/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*d*(d*x)^(5/2)*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) - (5*d
^2*((2*b*(d*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*c) - (d*((4*(2*b^2 - 5*a*c)
*Sqrt[d*x]*Sqrt[a + b*x + c*x^2])/(3*c) - (2*d*Sqrt[x]*(-((b*(8*b^2 - 29*a
*c)*(-((Sqrt[x]*Sqrt[a + b*x + c*x^2])/(Sqrt[a] + Sqrt[c]*x)) + (a^(1/4)*(
Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*Ellip
ticE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c
^(1/4)*Sqrt[a + b*x + c*x^2])))/Sqrt[c]) + (a^(1/4)*((b*(8*b^2 - 29*a*c))/
Sqrt[c] + 2*Sqrt[a]*(2*b^2 - 5*a*c))*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x +
 c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1
/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x + c*x^2])))/(3
*c*Sqrt[d*x]))/(5*c)))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 5.75 (sec) , antiderivative size = 920, normalized size of antiderivative = 1.56

method	result
elliptic	$\frac{\sqrt{dx}\,\sqrt{dx(c\,x^2+bx+a)}\left(-\frac{2cdx\left(-\frac{b\,d^3\left(3ac-b^2\right)x}{c^3\left(4ac-b^2\right)}-\frac{\left(2ac-b^2\right)d^3 a}{c^3\left(4ac-b^2\right)}\right)}{\sqrt{\left(\frac{a}{c}+\frac{bx}{c}+x^2\right)cdx}}+\frac{2d^3\sqrt{cd\,x^3+bd\,x^2+adx}}{3c^2}+\frac{\left(-\frac{d^4\left(2ac-b^2\right)a}{c^2\left(4ac-b^2\right)}-\frac{a\,d^4}{3c^2}\right)\left(b+\sqrt{-4ac+b^2}\right)}{}\right.}{}$
default	Expression too large to display
risch	Expression too large to display

input
```
int((d*x)^(7/2)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/x*(d*x)^(1/2)/(c*x^2+b*x+a)^(1/2)*(d*x*(c*x^2+b*x+a))^(1/2)*(-2*c*d*x*
(-b*d^3/c^3*(3*a*c-b^2)/(4*a*c-b^2)*x-(2*a*c-b^2)*d^3*a/c^3/(4*a*c-b^2))/(
(a/c+b/c*x+x^2)*c*d*x)^(1/2)+2/3/c^2*d^3*(c*d*x^3+b*d*x^2+a*d*x)^(1/2)+(-1
/c^2*d^4*(2*a*c-b^2)*a/(4*a*c-b^2)-1/3*a*d^4/c^2)*(b+(-4*a*c+b^2)^(1/2))/c
*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(
-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*
x^3+b*d*x^2+a*d*x)^(1/2)*EllipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/
c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*
(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+(-5/3*b/c^
2*d^4-b*d^4/c^2*(3*a*c-b^2)/(4*a*c-b^2))*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*
((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/
c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*
c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x
^2+a*d*x)^(1/2)*((-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/
2)))*EllipticE(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(
1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2
))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*
EllipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))
*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 420, normalized size of antiderivative = 0.71

$$\int \frac{(dx)^{7/2}}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2\left(((8\,b^4c-41\,ab^2c^2+30\,a^2c^3)d^3x^2+(8\,b^5-41\,ab^3c+30\,a^2bc^2)d^3x+(8\,ab^4-\right.}{}$$

input
```
integrate((d*x)^(7/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
2/9*(((8*b^4*c - 41*a*b^2*c^2 + 30*a^2*c^3)*d^3*x^2 + (8*b^5 - 41*a*b^3*c
+ 30*a^2*b*c^2)*d^3*x + (8*a*b^4 - 41*a^2*b^2*c + 30*a^3*c^2)*d^3)*sqrt(c*
d)*weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3,
 1/3*(3*c*x + b)/c) + 3*((8*b^3*c^2 - 29*a*b*c^3)*d^3*x^2 + (8*b^4*c - 29*
a*b^2*c^2)*d^3*x + (8*a*b^3*c - 29*a^2*b*c^2)*d^3)*sqrt(c*d)*weierstrassZe
ta(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, weierstrassPInverse
(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c)) +
 3*((b^2*c^3 - 4*a*c^4)*d^3*x^2 + (4*b^3*c^2 - 13*a*b*c^3)*d^3*x + 2*(2*a*
b^2*c^2 - 5*a^2*c^3)*d^3)*sqrt(c*x^2 + b*x + a)*sqrt(d*x))/(a*b^2*c^4 - 4*
a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^2 + (b^3*c^4 - 4*a*b*c^5)*x)
```

Sympy [F]

$$\int \frac{(dx)^{7/2}}{(a+bx+cx^2)^{3/2}}\,dx=\int \frac{(dx)^{\frac{7}{2}}}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)**(7/2)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((d*x)**(7/2)/(a + b*x + c*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(dx)^{7/2}}{(a+bx+cx^2)^{3/2}}\,dx=\int \frac{(dx)^{\frac{7}{2}}}{(cx^2+bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^(7/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^(7/2)/(c*x^2 + b*x + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(dx)^{7/2}}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(dx)^{\frac{7}{2}}}{(cx^2+bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^(7/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x)^(7/2)/(c*x^2 + b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{7/2}}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(d\,x)^{7/2}}{(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int((d*x)^(7/2)/(a + b*x + c*x^2)^(3/2),x)
```

output
```
int((d*x)^(7/2)/(a + b*x + c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(dx)^{7/2}}{(a+bx+cx^2)^{3/2}}\,dx = \sqrt{d}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,x^3}{c^2x^4+2bc\,x^3+2ac\,x^2+b^2x^2+2abx+a^2}dx\right)d^3$$

input
```
int((d*x)^(7/2)/(c*x^2+b*x+a)^(3/2),x)
```

output
```
sqrt(d)*int((sqrt(x)*sqrt(a + b*x + c*x**2)*x**3)/(a**2 + 2*a*b*x + 2*a*c*
x**2 + b**2*x**2 + 2*b*c*x**3 + c**2*x**4),x)*d**3
```

3.333 $\int \frac{(dx)^{5/2}}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 517

$$\int \frac{(dx)^{5/2}}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2d(dx)^{3/2}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}} - \frac{2bd^2\sqrt{dx}\sqrt{a+bx+cx^2}}{c(b^2-4ac)} + \frac{\sqrt{2}(b^2-3ac)\sqrt{-b+\sqrt{b^2-4ac}}(b+\sqrt{b^2-4ac})d^{5/2}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\ldots}{\sqrt{-b+\sqrt{b^2\ldots}}}\right.\right.}{c^{5/2}(b^2-4ac)\sqrt{a+x(b+cx)}} - \frac{\sqrt{2}\sqrt{-b+\sqrt{b^2-4ac}}(b^3-4abc+\sqrt{b^2-4ac}(b^2-3ac))d^{5/2}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\operatorname{EllipticF}\left(\ldots\right.}{c^{5/2}(b^2-4ac)\sqrt{a+x(b+cx)}}$$

output
```
2*d*(d*x)^(3/2)*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/2)-2*b*d^2*(d*x)^(
1/2)*(c*x^2+b*x+a)^(1/2)/c/(-4*a*c+b^2)+2^(1/2)*(-3*a*c+b^2)*(-b+(-4*a*c+b
^2)^(1/2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*d^(5/2)*(1+2*c*x/(b-(-4*a*c+b^2)^(
1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2^(1/2)*c^(1
/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/
2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))/c^(5/2)/(-4*a*c+b^2)/(a+x*(c*x+b))^(1/2
)-2^(1/2)*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(b^3-4*a*b*c+(-4*a*c+b^2)^(1/2)*(-
3*a*c+b^2))*d^(5/2)*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4
*a*c+b^2)^(1/2)))^(1/2)*EllipticF(2^(1/2)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+
b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^
(1/2))/c^(5/2)/(-4*a*c+b^2)/(a+x*(c*x+b))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.02 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.97

$$\int \frac{(dx)^{5/2}}{(a+bx+cx^2)^{3/2}}\,dx = \frac{(dx)^{5/2}\left(2c\sqrt{x}(b^2x+a(b-2cx)) - \frac{4(b^2-3ac)(a+x(b+cx))}{\sqrt{x}} + \frac{i(b^2-3ac)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\cdots}}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(d*x)^(5/2)/(a + b*x + c*x^2)^(3/2),x]
```

output
```
((d*x)^(5/2)*(2*c*Sqrt[x]*(b^2*x + a*(b - 2*c*x)) - (4*(b^2 - 3*a*c)*(a +
x*(b + c*x)))/Sqrt[x] + (I*(b^2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[2 +
 (4*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*
x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + S
qrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*
c])])/Sqrt[a/(b + Sqrt[b^2 - 4*a*c])] + (I*(b^3 - 4*a*b*c - b^2*Sqrt[b^2 -
 4*a*c] + 3*a*c*Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[b^2 - 4*a*c])
*x)]*x*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]
*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (
b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a/(b + Sqrt[b^2 - 4*
a*c])]))/(c^2*(-b^2 + 4*a*c)*x^(5/2)*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.77, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {1164, 27, 1236, 27, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{5/2}}{(a+bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 1164$$

$$\frac{2d(dx)^{3/2}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{2\int \frac{3d^2\sqrt{dx}(2a+bx)}{2\sqrt{cx^2+bx+a}}dx}{b^2-4ac}$$

$$\Big\downarrow \ 27$$

$$\frac{2d(dx)^{3/2}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{3d^2\int \frac{\sqrt{dx}(2a+bx)}{\sqrt{cx^2+bx+a}}dx}{b^2-4ac}$$

$$\Big\downarrow \ 1236$$

$$\frac{2d(dx)^{3/2}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{3d^2\left(\frac{2\int -\frac{d\left(ab+2\left(b^2-3ac\right)x\right)}{2\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{3c}+\frac{2b\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}\right)}{b^2-4ac}$$

$$\Big\downarrow \ 27$$

$$\frac{2d(dx)^{3/2}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{3d^2\left(\frac{2b\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{d\int \frac{ab+2\left(b^2-3ac\right)x}{\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{3c}\right)}{b^2-4ac}$$

$$\Big\downarrow \ 1241$$

$$\frac{2d(dx)^{3/2}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{3d^2\left(\frac{2b\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{d\sqrt{x}\int \frac{ab+2\left(b^2-3ac\right)x}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{3c\sqrt{dx}}\right)}{b^2-4ac}$$

$$\Big\downarrow \ 1240$$

$$\frac{2d(dx)^{3/2}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{3d^2\left(\frac{2b\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{2d\sqrt{x}\int \frac{ab+2\left(b^2-3ac\right)x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{3c\sqrt{dx}}\right)}{b^2-4ac}$$

$$\Big\downarrow \ 1511$$

$$\frac{2d(dx)^{3/2}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}-$$

$$\frac{3d^2\left(\frac{2b\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{2d\sqrt{x}\left(\sqrt{a}\left(\frac{2\left(b^2-3ac\right)}{\sqrt{c}}+\sqrt{a}b\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{2\sqrt{a}\left(b^2-3ac\right)\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{3c\sqrt{dx}}\right)}{b^2-4ac}$$

$\Big\downarrow$ 27

$$\frac{2d(dx)^{3/2}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{3d^2\left(\frac{2b\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{2d\sqrt{x}\left(\sqrt{a}\left(\frac{2\left(b^2-3ac\right)}{\sqrt{c}}+\sqrt{a}b\right)\int\frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{2\left(b^2-3ac\right)\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{3c\sqrt{dx}}\right)}{b^2-4ac}$$

$\Big\downarrow$ 1416

$$\frac{2d(dx)^{3/2}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{3d^2\left(\frac{2b\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{2d\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\frac{2\left(b^2-3ac\right)}{\sqrt{c}}+\sqrt{a}b\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}-\frac{2\left(b^2-3ac\right)\int\cdots}{\cdots}\right.}{3c\sqrt{dx}\cdots}\cdots}{b^2-4ac\cdots}$$

$\Big\downarrow$ 1509

$$\frac{2d(dx)^{3/2}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{3d^2\left(\frac{2b\sqrt{dx}\sqrt{a+bx+cx^2}}{3c}-\frac{2d\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\frac{2\left(b^2-3ac\right)}{\sqrt{c}}+\sqrt{a}b\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}-\frac{2\left(b^2-3ac\right)\left(\cdots\right.}{\cdots}\right.}{3c\sqrt{dx}\cdots}\cdots}{b^2-4ac\cdots}$$

input
```
Int[(d*x)^(5/2)/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*d*(d*x)^(3/2)*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) - (3*d
^2*((2*b*Sqrt[d*x]*Sqrt[a + b*x + c*x^2])/(3*c) - (2*d*Sqrt[x]*((-2*(b^2 -
 3*a*c)*(-((Sqrt[x]*Sqrt[a + b*x + c*x^2])/(Sqrt[a] + Sqrt[c]*x)) + (a^(1/
4)*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*E
llipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]
)/(c^(1/4)*Sqrt[a + b*x + c*x^2])))/Sqrt[c] + (a^(1/4)*(Sqrt[a]*b + (2*(b^
2 - 3*a*c))/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a]
 + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sq
rt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x + c*x^2])))/(3*c*Sqrt[d*x])))/
(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 855, normalized size of antiderivative = 1.65

method	result
elliptic	$\sqrt{dx}\,\sqrt{dx(c\,x^2+bx+a)}\left(-\frac{2cdx\left(\frac{d^2\left(2ac-b^2\right)x}{c^2\left(4ac-b^2\right)}-\frac{ab\,d^2}{c^2\left(4ac-b^2\right)}\right)}{\sqrt{\left(\frac{a}{c}+\frac{bx}{c}+x^2\right)cdx}}-\frac{d^3ab\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\,\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\,\sqrt{\frac{x-\frac{-b+\sqrt{-4\cdots}}{2c}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}-\frac{-b\cdots}{}}}\cdots}{}\right.$
default	Expression too large to display

input
```
int((d*x)^(5/2)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/x*(d*x)^(1/2)/(c*x^2+b*x+a)^(1/2)*(d*x*(c*x^2+b*x+a))^(1/2)*(-2*c*d*x*
(d^2/c^2*(2*a*c-b^2)/(4*a*c-b^2)*x-a*b/c^2*d^2/(4*a*c-b^2))/((a/c+b/c*x+x^
2)*c*d*x)^(1/2)-1/c^2*d^3*a*b/(4*a*c-b^2)*(b+(-4*a*c+b^2)^(1/2))*2^(1/2)*(
(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c
*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c
+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^
2+a*d*x)^(1/2)*EllipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*
a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c
+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+(1/c*d^3+d^3/c*(2*a*
c-b^2)/(4*a*c-b^2))*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^
2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1
/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)
*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)*((-1/
2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(2^(1/2
)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-
2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*
c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(2^(1/2)*((x
+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+
(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2
)^(1/2))))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.67

$$\int \frac{(dx)^{5/2}}{(a+bx+cx^2)^{3/2}}\,dx =$$

$$-\frac{2\left(\left((2\,b^3c-9\,abc^2)d^2x^2+(2\,b^4-9\,ab^2c)d^2x+(2\,ab^3-9\,a^2bc)d^2\right)\sqrt{cd}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4}{}\right.}{}$$

input
```
integrate((d*x)^(5/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(((2*b^3*c - 9*a*b*c^2)*d^2*x^2 + (2*b^4 - 9*a*b^2*c)*d^2*x + (2*a*b^
3 - 9*a^2*b*c)*d^2)*sqrt(c*d)*weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -
4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c) + 6*((b^2*c^2 - 3*a*c^3)*d^
2*x^2 + (b^3*c - 3*a*b*c^2)*d^2*x + (a*b^2*c - 3*a^2*c^2)*d^2)*sqrt(c*d)*w
eierstrassZeta(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, weierst
rassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*
x + b)/c)) + 3*(a*b*c^2*d^2 + (b^2*c^2 - 2*a*c^3)*d^2*x)*sqrt(c*x^2 + b*x
+ a)*sqrt(d*x))/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^2 + (b^3*c^
3 - 4*a*b*c^4)*x)
```

Sympy [F]

$$\int \frac{(dx)^{5/2}}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(dx)^{\frac{5}{2}}}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)**(5/2)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((d*x)**(5/2)/(a + b*x + c*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(dx)^{5/2}}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(dx)^{\frac{5}{2}}}{(cx^2+bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^(5/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^(5/2)/(c*x^2 + b*x + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(dx)^{5/2}}{(a+bx+cx^2)^{3/2}}\,dx=\int \frac{(dx)^{\frac{5}{2}}}{(cx^2+bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^(5/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x)^(5/2)/(c*x^2 + b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{5/2}}{(a+bx+cx^2)^{3/2}}\,dx=\int \frac{(d\,x)^{5/2}}{(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int((d*x)^(5/2)/(a + b*x + c*x^2)^(3/2),x)
```

output
```
int((d*x)^(5/2)/(a + b*x + c*x^2)^(3/2), x)
```

Reduce [F]

$\int \frac{(dx)^{5/2}}{(a+bx+cx^2)^{3/2}}\,dx=$ [numerator, cut off at the page edge:] $\sqrt{d}\,d^2\Big(10\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,a+2\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,c\,x^2-5\Big(\int$ [inner fraction's denominator, cut off:] $\sqrt{x}\,a^2+2\sqrt{x}\,abx+2\sqrt{\cdots}$

input
```
int((d*x)^(5/2)/(c*x^2+b*x+a)^(3/2),x)
```

output
```
(sqrt(d)*d**2*(10*sqrt(x)*sqrt(a + b*x + c*x**2)*a + 2*sqrt(x)*sqrt(a + b*
x + c*x**2)*c*x**2 - 5*int(sqrt(a + b*x + c*x**2)/(sqrt(x)*a**2 + 2*sqrt(x
)*a*b*x + 2*sqrt(x)*a*c*x**2 + sqrt(x)*b**2*x**2 + 2*sqrt(x)*b*c*x**3 + sq
rt(x)*c**2*x**4),x)*a**3 - 5*int(sqrt(a + b*x + c*x**2)/(sqrt(x)*a**2 + 2*
sqrt(x)*a*b*x + 2*sqrt(x)*a*c*x**2 + sqrt(x)*b**2*x**2 + 2*sqrt(x)*b*c*x**
3 + sqrt(x)*c**2*x**4),x)*a**2*b*x - 5*int(sqrt(a + b*x + c*x**2)/(sqrt(x)
*a**2 + 2*sqrt(x)*a*b*x + 2*sqrt(x)*a*c*x**2 + sqrt(x)*b**2*x**2 + 2*sqrt(
x)*b*c*x**3 + sqrt(x)*c**2*x**4),x)*a**2*c*x**2 - 3*int((sqrt(x)*sqrt(a +
b*x + c*x**2)*x**3)/(a**2 + 2*a*b*x + 2*a*c*x**2 + b**2*x**2 + 2*b*c*x**3
+ c**2*x**4),x)*a*c**2 - 3*int((sqrt(x)*sqrt(a + b*x + c*x**2)*x**3)/(a**2
 + 2*a*b*x + 2*a*c*x**2 + b**2*x**2 + 2*b*c*x**3 + c**2*x**4),x)*b*c**2*x
- 3*int((sqrt(x)*sqrt(a + b*x + c*x**2)*x**3)/(a**2 + 2*a*b*x + 2*a*c*x**2
 + b**2*x**2 + 2*b*c*x**3 + c**2*x**4),x)*c**3*x**2))/(4*b*c*(a + b*x + c*
x**2))
```

3.334 $\int \frac{(dx)^{3/2}}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 462

$$\int \frac{(dx)^{3/2}}{\left(a+bx+cx^2\right)^{3/2}}\,dx=\frac{2d\sqrt{dx}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}$$
$$-\frac{b\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)d^{3/2}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{dx}}{\sqrt{-b+\sqrt{b^2-4ac}}\sqrt{d}}\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{a+x(b+cx)}}$$
$$+\frac{\sqrt{-b+\sqrt{b^2-4ac}}\left(b^2-4ac+b\sqrt{b^2-4ac}\right)d^{3/2}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{dx}}{\sqrt{-b+\sqrt{b^2-4ac}}\sqrt{d}}\right),\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{a+x(b+cx)}}$$

output
```
2*d*(d*x)^(1/2)*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/2)-1/2*b*(-b+(-4*a
*c+b^2)^(1/2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*d^(3/2)*(1+2*c*x/(b-(-4*a*c+b^
2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2^(1/2)*
c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)
^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/c^(3/2)/(-4*a*c+b^2)/(a+x*(
c*x+b))^(1/2)+1/2*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(b^2-4*a*c+b*(-4*a*c+b^2)^
(1/2))*d^(3/2)*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+
b^2)^(1/2)))^(1/2)*EllipticF(2^(1/2)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^
(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
)*2^(1/2)/c^(3/2)/(-4*a*c+b^2)/(a+x*(c*x+b))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.22 (sec) , antiderivative size = 458, normalized size of antiderivative = 0.99

$$\int \frac{(dx)^{3/2}}{(a+bx+cx^2)^{3/2}}\,dx = \frac{d^2\left(-4\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}(b^2x+a(b-2cx))+ib\left(-b+\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2a}{\left(b+\sqrt{b^2-4ac}\right)x}}\right.}{}$$

input
```
Integrate[(d*x)^(3/2)/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(d^2*(-4*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*(b^2*x + a*(b - 2*c*x)) + I*b*(-b
 + Sqrt[b^2 - 4*a*c])*Sqrt[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(3/2)*
Sqrt[(4*a + 2*b*x - 2*Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*El
lipticE[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b +
 Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt[b^
2 - 4*a*c])*Sqrt[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(3/2)*Sqrt[(4*a
+ 2*b*x - 2*Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticF[I*
ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2
- 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(2*c*(b^2 - 4*a*c)*Sqrt[a/(b + Sqrt[b
^2 - 4*a*c])]*Sqrt[d*x]*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.75, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1164, 27, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{3/2}}{(a+bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 1164$$

$$\frac{2d\sqrt{dx}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{2\int \frac{d^2(2a+bx)}{2\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{b^2-4ac}$$

$\big\downarrow$ 27

$$\frac{2d\sqrt{dx}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{d^2\int \frac{2a+bx}{\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{b^2-4ac}$$

$\big\downarrow$ 1241

$$\frac{2d\sqrt{dx}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{d^2\sqrt{x}\int \frac{2a+bx}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{\sqrt{dx}\,(b^2-4ac)}$$

$\big\downarrow$ 1240

$$\frac{2d\sqrt{dx}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{2d^2\sqrt{x}\int \frac{2a+bx}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{dx}\,(b^2-4ac)}$$

$\big\downarrow$ 1511

$$\frac{2d\sqrt{dx}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{2d^2\sqrt{x}\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{\sqrt{a}b\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{\sqrt{dx}\,(b^2-4ac)}$$

$\big\downarrow$ 27

$$\frac{2d\sqrt{dx}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{2d^2\sqrt{x}\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{b\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{\sqrt{dx}\,(b^2-4ac)}$$

$\big\downarrow$ 1416

$$\frac{2d\sqrt{dx}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{2d^2\sqrt{x}\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}-\frac{b\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{\sqrt{dx}\,(b^2-4ac)}$$

$\big\downarrow$ 1509

$$\frac{2d\sqrt{dx}(2a+bx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{2d^2\sqrt{x}\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}-\frac{b\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}E\left(2\text{arc}\right.}{\sqrt[4]{c}\sqrt{a+bx}}\right.}{}\right.}{\sqrt{dx}\,(b^2-4ac)}$$

input
```
Int[(d*x)^(3/2)/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*d*Sqrt[d*x]*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) - (2*d^2
*Sqrt[x]*(-((b*(-((Sqrt[x]*Sqrt[a + b*x + c*x^2])/(Sqrt[a] + Sqrt[c]*x)) +
 (a^(1/4)*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*
x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[
c]))/4])/(c^(1/4)*Sqrt[a + b*x + c*x^2])))/Sqrt[c]) + (a^(1/4)*(2*Sqrt[a]
+ b/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[
c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sq
rt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x + c*x^2])))/((b^2 - 4*a*c)*Sqrt[d*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 686, normalized size of antiderivative = 1.48

method	result
default	$\frac{d\sqrt{dx}\left(2\sqrt{-4ac+b^2}\sqrt{\frac{2cx+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{-2cx+\sqrt{-4ac+b^2}-b}{\sqrt{-4ac+b^2}}}\sqrt{-\frac{cx}{b+\sqrt{-4ac+b^2}}}\,\text{EllipticF}\left(\sqrt{\frac{2cx+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}},\frac{\sqrt{2}\sqrt{\frac{b+\sqrt{-4ac+}}{\sqrt{-4ac+b}}}}{2}\right.\right.}{}$
elliptic	$\frac{\sqrt{dx}\sqrt{dx(cx^2+bx+a)}\left(-\frac{2cdx\left(\frac{bdx}{c(4ac-b^2)}+\frac{2ad}{c(4ac-b^2)}\right)}{\sqrt{\left(\frac{a}{c}+\frac{bx}{c}+x^2\right)cdx}}+\frac{2d^2a\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+}}{2c}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}-\frac{-b+\sqrt{}}{}}}}{}\right.}{}$

input
```
int((d*x)^(3/2)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
d*(d*x)^(1/2)*(2*(-4*a*c+b^2)^(1/2)*((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a
*c+b^2)^(1/2)))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2)^(1/2))^(
1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(((2*c*x+(-4*a*c+b^2)^(1
/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(
-4*a*c+b^2)^(1/2))^(1/2))*a*c+4*((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b
^2)^(1/2)))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2)^(1/2))^(1/2)
*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(((2*c*x+(-4*a*c+b^2)^(1/2)+
b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a
*c+b^2)^(1/2))^(1/2))*a*b*c-((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^
(1/2)))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2)^(1/2))^(1/2)*(-c
*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(
b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(1/2))^(1/2))*b^3-(-4*a*c+b^2)^(1/2)*((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+
(-4*a*c+b^2)^(1/2)))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2)^(1/
2))^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(((2*c*x+(-4*a*c+b^
2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(1/2))^(1/2))*b^2-2*b*c^2*x^2-4*a*c^2*x)/x/(c*x^2+b*x+a)^
(1/2)/c^2/(4*a*c-b^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.61

$$\int \frac{(dx)^{3/2}}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2\left(((b^2c-6\,ac^2)dx^2+(b^3-6\,abc)dx+(ab^2-6\,a^2c)d)\sqrt{cd}\text{weierstrassPInverse}(\right.}{}$$

input
```
integrate((d*x)^(3/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(((b^2*c - 6*a*c^2)*d*x^2 + (b^3 - 6*a*b*c)*d*x + (a*b^2 - 6*a^2*c)*d)
*sqrt(c*d)*weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b
*c)/c^3, 1/3*(3*c*x + b)/c) + 3*(b*c^2*d*x^2 + b^2*c*d*x + a*b*c*d)*sqrt(c
*d)*weierstrassZeta(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, we
ierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*
(3*c*x + b)/c)) + 3*(b*c^2*d*x + 2*a*c^2*d)*sqrt(c*x^2 + b*x + a)*sqrt(d*x
))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3
)*x)
```

Sympy [F]

$$\int \frac{(dx)^{3/2}}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)**(3/2)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((d*x)**(3/2)/(a + b*x + c*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(dx)^{3/2}}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{(cx^2+bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^(3/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^(3/2)/(c*x^2 + b*x + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(dx)^{3/2}}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{(cx^2+bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^(3/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x)^(3/2)/(c*x^2 + b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{3/2}}{(a+bx+cx^2)^{3/2}}\,dx=\int \frac{(d\,x)^{3/2}}{(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int((d*x)^(3/2)/(a + b*x + c*x^2)^(3/2),x)
```

output
```
int((d*x)^(3/2)/(a + b*x + c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(dx)^{3/2}}{(a+bx+cx^2)^{3/2}}\,dx=\frac{\sqrt{d}\,d\Big(-10\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,a+8\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,bx-2\sqrt{x}\,\sqrt{c\,x^2+bx+a}$$

input
```
int((d*x)^(3/2)/(c*x^2+b*x+a)^(3/2),x)
```

output
```
(sqrt(d)*d*( - 10*sqrt(x)*sqrt(a + b*x + c*x**2)*a + 8*sqrt(x)*sqrt(a + b*
x + c*x**2)*b*x - 2*sqrt(x)*sqrt(a + b*x + c*x**2)*c*x**2 + 3*int((sqrt(x)
*sqrt(a + b*x + c*x**2)*x**3)/(a**2 + 2*a*b*x + 2*a*c*x**2 + b**2*x**2 + 2
*b*c*x**3 + c**2*x**4),x)*a*c**2 + 3*int((sqrt(x)*sqrt(a + b*x + c*x**2)*x
**3)/(a**2 + 2*a*b*x + 2*a*c*x**2 + b**2*x**2 + 2*b*c*x**3 + c**2*x**4),x)
*b*c**2*x + 3*int((sqrt(x)*sqrt(a + b*x + c*x**2)*x**3)/(a**2 + 2*a*b*x +
2*a*c*x**2 + b**2*x**2 + 2*b*c*x**3 + c**2*x**4),x)*c**3*x**2 - 12*int((sq
rt(x)*sqrt(a + b*x + c*x**2))/(a**2 + 2*a*b*x + 2*a*c*x**2 + b**2*x**2 + 2
*b*c*x**3 + c**2*x**4),x)*a**2*b - 12*int((sqrt(x)*sqrt(a + b*x + c*x**2))
/(a**2 + 2*a*b*x + 2*a*c*x**2 + b**2*x**2 + 2*b*c*x**3 + c**2*x**4),x)*a*b
**2*x - 12*int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a**2 + 2*a*b*x + 2*a*c*x*
*2 + b**2*x**2 + 2*b*c*x**3 + c**2*x**4),x)*a*b*c*x**2 + 5*int((sqrt(x)*sq
rt(a + b*x + c*x**2))/(a**2*x + 2*a*b*x**2 + 2*a*c*x**3 + b**2*x**3 + 2*b*
c*x**4 + c**2*x**5),x)*a**3 + 5*int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a**2
*x + 2*a*b*x**2 + 2*a*c*x**3 + b**2*x**3 + 2*b*c*x**4 + c**2*x**5),x)*a**2
*b*x + 5*int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a**2*x + 2*a*b*x**2 + 2*a*c
*x**3 + b**2*x**3 + 2*b*c*x**4 + c**2*x**5),x)*a**2*c*x**2))/(8*b**2*(a +
b*x + c*x**2))
```

3.335 $\int \frac{\sqrt{dx}}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 439

$$\int \frac{\sqrt{dx}}{(a+bx+cx^2)^{3/2}}\,dx = -\frac{2\sqrt{dx}(b+2cx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}$$
$$+\frac{\sqrt{2}\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\sqrt{d}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{dx}}{\sqrt{-b+\sqrt{b^2-4ac}}\sqrt{d}}\right)|\frac{b-}{b+}\right.}{\sqrt{c}\,(b^2-4ac)\sqrt{a+x(b+cx)}}$$
$$-\frac{\sqrt{2}\sqrt{-b+\sqrt{b^2-4ac}}\sqrt{d}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{dx}}{\sqrt{-b+\sqrt{b^2-4ac}}\sqrt{d}}\right),\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{c}\sqrt{b^2-4ac}\sqrt{a+x(b+cx)}}$$

output
```
-2*(d*x)^(1/2)*(2*c*x+b)/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/2)+2^(1/2)*(-b+(-4*
a*c+b^2)^(1/2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*d^(1/2)*(1+2*c*x/(b-(-4*a*c+b
^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2^(1/2)
*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2
)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))/c^(1/2)/(-4*a*c+b^2)/(a+x*(c*x+b))
^(1/2)-2^(1/2)*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*d^(1/2)*(1+2*c*x/(b-(-4*a*c+b
^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(2^(1/2)
*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2
)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))/c^(1/2)/(-4*a*c+b^2)^(1/2)/(a+x*(c
*x+b))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.02 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{dx}}{(a+bx+cx^2)^{3/2}}\,dx = \frac{d\left(2\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}(2a+bx)-i\left(-b+\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2a}{\left(b+\sqrt{b^2-4ac}\right)x}}\,x^{3/2}\sqrt{\frac{4a+2bx-}{bx-}}\right.}{}$$

input
```
Integrate[Sqrt[d*x]/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(d*(2*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*(2*a + b*x) - I*(-b + Sqrt[b^2 - 4*a
*c])*Sqrt[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(3/2)*Sqrt[(4*a + 2*b*x
 - 2*Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticE[I*ArcSinh
[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c
])/(b - Sqrt[b^2 - 4*a*c])] + I*Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[1 + (2*a)/(
(b + Sqrt[b^2 - 4*a*c])*x)]*x^(3/2)*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)
/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqr
t[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]
)]))/((b^2 - 4*a*c)*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[d*x]*Sqrt[a + x*(
b + c*x)])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.78, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1163, 27, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx}}{(a+bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1163}$$

$$\frac{2\int \frac{d(b+2cx)}{2\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{b^2-4ac}-\frac{2\sqrt{dx}(b+2cx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d\int \frac{b+2cx}{\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{b^2-4ac}-\frac{2\sqrt{dx}(b+2cx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{ 1241}$$

$$\frac{d\sqrt{x}\int \frac{b+2cx}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{\sqrt{dx}\,(b^2-4ac)}-\frac{2\sqrt{dx}(b+2cx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{ 1240}$$

$$\frac{2d\sqrt{x}\int \frac{b+2cx}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{dx}\,(b^2-4ac)}-\frac{2\sqrt{dx}(b+2cx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{ 1511}$$

$$\frac{2d\sqrt{x}\left(\left(2\sqrt{a}\sqrt{c}+b\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-2\sqrt{a}\sqrt{c}\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\sqrt{x}\right)}{\sqrt{dx}\,(b^2-4ac)}-\frac{2\sqrt{dx}(b+2cx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2d\sqrt{x}\left(\left(2\sqrt{a}\sqrt{c}+b\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-2\sqrt{c}\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}\right)}{\sqrt{dx}\,(b^2-4ac)}-\frac{2\sqrt{dx}(b+2cx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{2d\sqrt{x}\left(\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx+cx^2}}-2\sqrt{c}\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}\right)}{\sqrt{dx}\,(b^2-4ac)}-\frac{2\sqrt{dx}(b+2cx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{ 1509}$$

$$\frac{2d\sqrt{x}\left(\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx+cx^2}}-2\sqrt{c}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}\,E\left(2\,\mathrm{a}\right.}{\sqrt[4]{c}\sqrt{a+b}}\right.\right.}{\sqrt{dx}\left(b^2-4ac\right)}$$

$$\frac{2\sqrt{dx}(b+2cx)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

input
```
Int[Sqrt[d*x]/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(-2*Sqrt[d*x]*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (2*d*Sq
rt[x]*(-2*Sqrt[c]*(-((Sqrt[x]*Sqrt[a + b*x + c*x^2])/(Sqrt[a] + Sqrt[c]*x)
) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[
c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sq
rt[c]))/4])/(c^(1/4)*Sqrt[a + b*x + c*x^2])) + ((b + 2*Sqrt[a]*Sqrt[c])*(S
qrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*
a^(1/4)*c^(1/4)*Sqrt[a + b*x + c*x^2])))/((b^2 - 4*a*c)*Sqrt[d*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1163
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] - Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1
)*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p +
1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 0] && (LtQ[
m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c, d,
e, m, p, x]
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
 + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(361) = 722$.

Time = 1.52 (sec) , antiderivative size = 807, normalized size of antiderivative = 1.84

method	result
elliptic	$\frac{\sqrt{dx}\,\sqrt{dx(c\,x^2+bx+a)}\left(-\frac{2cdx\left(-\frac{2x}{4ac-b^2}-\frac{b}{\left(4ac-b^2\right)c}\right)}{\sqrt{\left(\frac{a}{c}+\frac{bx}{c}+x^2\right)cdx}}-\frac{bd\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}-\frac{-b+\sqrt{-4a}}{2c}}}}{}\right.}{}$
default	$\frac{\sqrt{dx}\left(4\sqrt{\frac{2cx+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{-2cx+\sqrt{-4ac+b^2}-b}{\sqrt{-4ac+b^2}}}\sqrt{-\frac{cx}{b+\sqrt{-4ac+b^2}}}\,\text{EllipticF}\left(\sqrt{\frac{2cx+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}},\frac{\sqrt{2}\sqrt{\frac{b+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2}\right)ac-\sqrt{2}\right.}{}$

input

```
int((d*x)^(1/2)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/x*(d*x)^(1/2)/(c*x^2+b*x+a)^(1/2)*(d*x*(c*x^2+b*x+a))^(1/2)*(-2*c*d*x*
(-2/(4*a*c-b^2)*x-b/(4*a*c-b^2)/c)/((a/c+b/c*x+x^2)*c*d*x)^(1/2)-b*d/(4*a*
c-b^2)*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(
b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(
-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)*EllipticF(2^(1/2)*
((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*
(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+
b^2)^(1/2))))^(1/2))-2*d/(4*a*c-b^2)*(b+(-4*a*c+b^2)^(1/2))*2^(1/2)*((x+1/
2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+
(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)
^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d
*x)^(1/2)*((-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*E
llipticE(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*
c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1
/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*Ellipt
icF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1
/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*
(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{dx}}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2\left((bcx^2+b^2x+ab)\sqrt{cd}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2}, -\frac{4\,(2\,b^3-9\,abc)}{27\,c^3}, \frac{3\,cx+b}{3\,c}\right) - \cdots\right.}{}$$

input
```
integrate((d*x)^(1/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3*((b*c*x^2 + b^2*x + a*b)*sqrt(c*d)*weierstrassPInverse(4/3*(b^2 - 3*a*
c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c) - 6*(c^2*x^2 + b*c
*x + a*c)*sqrt(c*d)*weierstrassZeta(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 -
9*a*b*c)/c^3, weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*
a*b*c)/c^3, 1/3*(3*c*x + b)/c)) - 3*(2*c^2*x + b*c)*sqrt(c*x^2 + b*x + a)*
sqrt(d*x))/(a*b^2*c - 4*a^2*c^2 + (b^2*c^2 - 4*a*c^3)*x^2 + (b^3*c - 4*a*b
*c^2)*x)
```

Sympy [F]

$$\int \frac{\sqrt{dx}}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx}}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)**(1/2)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral(sqrt(d*x)/(a + b*x + c*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{dx}}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx}}{(cx^2+bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^(1/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x)/(c*x^2 + b*x + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{dx}}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx}}{(cx^2+bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^(1/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x)/(c*x^2 + b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{dx}}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{\sqrt{d\,x}}{\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

input
```
int((d*x)^(1/2)/(a + b*x + c*x^2)^(3/2),x)
```

output
```
int((d*x)^(1/2)/(a + b*x + c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{dx}}{(a+bx+cx^2)^{3/2}}\,dx = \sqrt{d}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+bx+a}}{c^2x^4+2bc\,x^3+2ac\,x^2+b^2x^2+2abx+a^2}dx\right)$$

input
```
int((d*x)^(1/2)/(c*x^2+b*x+a)^(3/2),x)
```

output
```
sqrt(d)*int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a**2 + 2*a*b*x + 2*a*c*x**2
+ b**2*x**2 + 2*b*c*x**3 + c**2*x**4),x)
```

3.336 $\int \frac{1}{\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 478

$$\int \frac{1}{\sqrt{dx}\,(a+bx+cx^2)^{3/2}}\,dx = \frac{2\sqrt{dx}(b^2-2ac+bcx)}{a\,(b^2-4ac)\,d\sqrt{a+bx+cx^2}} - \frac{b\sqrt{-b+\sqrt{b^2-4ac}}(b+\sqrt{b^2-4ac})\,\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{dx}}{\sqrt{-b+\sqrt{b^2-4ac}}\sqrt{d}}\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}a\sqrt{c}\,(b^2-4ac)\,\sqrt{d}\sqrt{a+x(b+cx)}} + \frac{\sqrt{-b+\sqrt{b^2-4ac}}(b^2-4ac+b\sqrt{b^2-4ac})\,\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{dx}}{\sqrt{-b+\sqrt{b^2-4ac}}\sqrt{d}}\right),\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}a\sqrt{c}\,(b^2-4ac)\,\sqrt{d}\sqrt{a+x(b+cx)}}$$

output
```
2*(d*x)^(1/2)*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/d/(c*x^2+b*x+a)^(1/2)-1/2*b
*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*(1+2*c*x/(b-(-4*a*c+
b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2^(1/2
)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^
2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/a/c^(1/2)/(-4*a*c+b^2)/d^
(1/2)/(a+x*(c*x+b))^(1/2)+1/2*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(b^2-4*a*c+b*(
-4*a*c+b^2)^(1/2))*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*
a*c+b^2)^(1/2)))^(1/2)*EllipticF(2^(1/2)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b
^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(
1/2))*2^(1/2)/a/c^(1/2)/(-4*a*c+b^2)/d^(1/2)/(a+x*(c*x+b))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.94 (sec) , antiderivative size = 450, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{4a\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}(b+2cx)-ib\left(-b+\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2a}{\left(b+\sqrt{b^2-4ac}\right)x}}\,x^{3/2}\sqrt{\frac{4a-\cdots}{\cdots}}\cdots}{\cdots}$$

input
```
Integrate[1/(Sqrt[d*x]*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(4*a*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*(b + 2*c*x) - I*b*(-b + Sqrt[b^2 - 4*
a*c])*Sqrt[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(3/2)*Sqrt[(4*a + 2*b*
x - 2*Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticE[I*ArcSin
h[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*
c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*Sqrt
[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(3/2)*Sqrt[(4*a + 2*b*x - 2*Sqrt
[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticF[I*ArcSinh[(Sqrt[2]
*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - S
qrt[b^2 - 4*a*c])])/(2*a*(-b^2 + 4*a*c)*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*Sq
rt[d*x]*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.75, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1165, 27, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 1165$$

$$\frac{2\sqrt{dx}\left(-2ac+b^2+bcx\right)}{ad\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\int \frac{cd^2(2a+bx)}{2\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{ad^2\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\sqrt{dx}\left(-2ac+b^2+bcx\right)}{ad\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{c\int \frac{2a+bx}{\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 1241}$$

$$\frac{2\sqrt{dx}\left(-2ac+b^2+bcx\right)}{ad\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{c\sqrt{x}\int \frac{2a+bx}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{a\sqrt{dx}\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 1240}$$

$$\frac{2\sqrt{dx}\left(-2ac+b^2+bcx\right)}{ad\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2c\sqrt{x}\int \frac{2a+bx}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{a\sqrt{dx}\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 1511}$$

$$\frac{2\sqrt{dx}\left(-2ac+b^2+bcx\right)}{ad\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2c\sqrt{x}\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{\sqrt{a}b\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{a\sqrt{dx}\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\sqrt{dx}\left(-2ac+b^2+bcx\right)}{ad\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2c\sqrt{x}\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{b\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{a\sqrt{dx}\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{2\sqrt{dx}\left(-2ac+b^2+bcx\right)}{ad\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2c\sqrt{x}\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}-\frac{b\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{a\sqrt{dx}\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 1509}$$

$$\frac{2\sqrt{dx}\left(-2ac+b^2+bcx\right)}{ad\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2c\sqrt{x}\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}-\frac{b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}E\left(2\arctan\right.}{\sqrt[4]{c}\sqrt{a+bx+}}\right.}{}\right.}{a\sqrt{dx}\left(b^2-4ac\right)}$$

input
```
Int[1/(Sqrt[d*x]*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(2*Sqrt[d*x]*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x + c*x^
2]) - (2*c*Sqrt[x]*(-((b*(-((Sqrt[x]*Sqrt[a + b*x + c*x^2])/(Sqrt[a] + Sqr
t[c]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a]
+ Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqr
t[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x + c*x^2])))/Sqrt[c]) + (a^(1/4)*(
2*Sqrt[a] + b/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[
a] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(
Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x + c*x^2])))/(a*(b^2 - 4*a*c)
*Sqrt[d*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.45

method	result
default	$\frac{2\sqrt{-4ac+b^2}\sqrt{\frac{2cx+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{-2cx+\sqrt{-4ac+b^2}-b}{\sqrt{-4ac+b^2}}}\sqrt{-\frac{cx}{b+\sqrt{-4ac+b^2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{2cx+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}},\frac{\sqrt{2}\sqrt{\frac{b+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2}\right)ac-}{}$
elliptic	$\frac{\sqrt{dx(cx^2+bx+a)}\left(-\frac{2cdx\left(\frac{bx}{(4ac-b^2)ad}-\frac{2ac-b^2}{a(4ac-b^2)dc}\right)}{\sqrt{\left(\frac{a}{c}+\frac{bx}{c}+x^2\right)cdx}}+\frac{\left(\frac{1}{a}-\frac{2ac-b^2}{a(4ac-b^2)}\right)\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{x-}{-\frac{b+\sqrt{-4ac}}{2c}}}}{}\right.}{}$

input
```
int(1/(d*x)^(1/2)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(2*(-4*a*c+b^2)^(1/2)*((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2))
)^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+
(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*
a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1
/2))^(1/2))*a*c+4*((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1
/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+(-4*
a*c+b^2)^(1/2)))^(1/2)*EllipticE(((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+
b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))
^(1/2))*a*b*c-((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*
((-2*c*x+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+(-4*a*c+
b^2)^(1/2)))^(1/2)*EllipticE(((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)
^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/
2))*b^3-(-4*a*c+b^2)^(1/2)*((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(
1/2)))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*
x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b
+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^
2)^(1/2))^(1/2))*b^2-2*b*c^2*x^2+4*a*c^2*x-2*b^2*c*x)/(c*x^2+b*x+a)^(1/2)/
a/c/(d*x)^(1/2)/(4*a*c-b^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{dx}\,(a+bx+cx^2)^{3/2}}\,dx = \frac{2\left((ab^2-6\,a^2c+(b^2c-6\,ac^2)x^2+(b^3-6\,abc)x)\sqrt{cd}\text{weierstrassPInverse}\right(}{}$$

input
```
integrate(1/(d*x)^(1/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3*((a*b^2 - 6*a^2*c + (b^2*c - 6*a*c^2)*x^2 + (b^3 - 6*a*b*c)*x)*sqrt(c*
d)*weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3,
 1/3*(3*c*x + b)/c) + 3*(b*c^2*x^2 + b^2*c*x + a*b*c)*sqrt(c*d)*weierstras
sZeta(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, weierstrassPInve
rse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c)
) + 3*(b*c^2*x + b^2*c - 2*a*c^2)*sqrt(c*x^2 + b*x + a)*sqrt(d*x))/((a*b^2
*c^2 - 4*a^2*c^3)*d*x^2 + (a*b^3*c - 4*a^2*b*c^2)*d*x + (a^2*b^2*c - 4*a^3
*c^2)*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{dx}\,(a+bx+cx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{dx}\,(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x)**(1/2)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral(1/(sqrt(d*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{dx}\,(a+bx+cx^2)^{3/2}}\,dx = \int \frac{1}{(cx^2+bx+a)^{\frac{3}{2}}\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + b*x + a)^(3/2)*sqrt(d*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{dx}\,(a+bx+cx^2)^{3/2}}\,dx = \int \frac{1}{(cx^2+bx+a)^{\frac{3}{2}}\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^2 + b*x + a)^(3/2)*sqrt(d*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{dx}\,(a+bx+cx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{d\,x}\,(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int(1/((d*x)^(1/2)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int(1/((d*x)^(1/2)*(a + b*x + c*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{dx}\,(a+bx+cx^2)^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{c\,x^2+bx+a}}{\sqrt{x}\,a^2+2\sqrt{x}\,abx+2\sqrt{x}\,ac\,x^2+\sqrt{x}\,b^2x^2+2\sqrt{x}\,bc\,x^3+\sqrt{x}\,c^2x^4}dx\right)}{d}$$

input
```
int(1/(d*x)^(1/2)/(c*x^2+b*x+a)^(3/2),x)
```

output
```
(sqrt(d)*int(sqrt(a + b*x + c*x**2)/(sqrt(x)*a**2 + 2*sqrt(x)*a*b*x + 2*sq
rt(x)*a*c*x**2 + sqrt(x)*b**2*x**2 + 2*sqrt(x)*b*c*x**3 + sqrt(x)*c**2*x**
4),x))/d
```

3.337 $\int \frac{1}{(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 535

$$\int \frac{1}{(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2}{ad\sqrt{dx}\sqrt{a+bx+cx^2}}$$
$$-\frac{2\sqrt{dx}(b(2b^2-7ac)+2c(b^2-3ac)\,x)}{a^2\left(b^2-4ac\right)d^2\sqrt{a+bx+cx^2}}$$
$$+\frac{\sqrt{2}(b^2-3ac)\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{dx}}{\sqrt{-b+\sqrt{b^2-4ac}}}\right.\right.}{a^2\sqrt{c}\left(b^2-4ac\right)d^{3/2}\sqrt{a+x(b+cx)}}$$
$$-\frac{\sqrt{2}\sqrt{-b+\sqrt{b^2-4ac}}\left(b^3-4abc+\sqrt{b^2-4ac}(b^2-3ac)\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(\text{arcs}\right.}{a^2\sqrt{c}\left(b^2-4ac\right)d^{3/2}\sqrt{a+x(b+cx)}}$$

output
```
-2/a/d/(d*x)^(1/2)/(c*x^2+b*x+a)^(1/2)-2*(d*x)^(1/2)*(b*(-7*a*c+2*b^2)+2*c
*(-3*a*c+b^2)*x)/a^2/(-4*a*c+b^2)/d^2/(c*x^2+b*x+a)^(1/2)+2^(1/2)*(-3*a*c+
b^2)*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*(1+2*c*x/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2^
(1/2)*c^(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*
c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))/a^2/c^(1/2)/(-4*a*c+b^2)/d^(3
/2)/(a+x*(c*x+b))^(1/2)-2^(1/2)*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(b^3-4*a*b*c
+(-4*a*c+b^2)^(1/2)*(-3*a*c+b^2))*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(2^(1/2)*c^(1/2)*(d*x)^(1/2
)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c
+b^2)^(1/2)))^(1/2))/a^2/c^(1/2)/(-4*a*c+b^2)/d^(3/2)/(a+x*(c*x+b))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.85 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.02

$$\int \frac{1}{(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{x\left(4(b^2-3ac)\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}(a+x(b+cx))+2\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}(4a^2c-2b^2x(b\right.}{}$$

input
```
Integrate[1/((d*x)^(3/2)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(x*(4*(b^2 - 3*a*c)*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*(a + x*(b + c*x)) + 2*
Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*(4*a^2*c - 2*b^2*x*(b + c*x) + a*(-b^2 + 7
*b*c*x + 6*c^2*x^2)) + I*(b^2 - 3*a*c)*(b - Sqrt[b^2 - 4*a*c])*Sqrt[1 + (2
*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(3/2)*Sqrt[(4*a + 2*b*x - 2*Sqrt[b^2 -
4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[a
/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
 - 4*a*c])] - I*(b^3 - 4*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 3*a*c*Sqrt[b^2 -
4*a*c])*Sqrt[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(3/2)*Sqrt[(4*a + 2*
b*x - 2*Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticF[I*ArcS
inh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*
a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(a^2*(b^2 - 4*a*c)*Sqrt[a/(b + Sqrt[b^2 -
 4*a*c])]*(d*x)^(3/2)*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 418, normalized size of antiderivative = 0.78, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {1165, 27, 1237, 27, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1165}$$

$$\frac{2\left(-2ac+b^2+bcx\right)}{ad\sqrt{dx}\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{d^2\left(2\left(b^2-3ac\right)+bcx\right)}{2(dx)^{3/2}\sqrt{cx^2+bx+a}}dx}{ad^2\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{2\left(b^2-3ac\right)+bcx}{(dx)^{3/2}\sqrt{cx^2+bx+a}}dx}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ad\sqrt{dx}\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{ 1237}$$

$$\frac{-\frac{2\int -\frac{cd\left(ab+2\left(b^2-3ac\right)x\right)}{2\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{ad^2}-\frac{4\left(b^2-3ac\right)\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ad\sqrt{dx}\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{c\int \frac{ab+2\left(b^2-3ac\right)x}{\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{ad}-\frac{4\left(b^2-3ac\right)\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ad\sqrt{dx}\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{ 1241}$$

$$\frac{\frac{c\sqrt{x}\int \frac{ab+2\left(b^2-3ac\right)x}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{ad\sqrt{dx}}-\frac{4\left(b^2-3ac\right)\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ad\sqrt{dx}\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{ 1240}$$

$$\frac{\frac{2c\sqrt{x}\int \frac{ab+2\left(b^2-3ac\right)x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{ad\sqrt{dx}}-\frac{4\left(b^2-3ac\right)\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ad\sqrt{dx}\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$\downarrow$ 1511

$$\frac{\frac{2c\sqrt{x}\left(\sqrt{a}\left(\frac{2\left(b^2-3ac\right)}{\sqrt{c}}+\sqrt{a}b\right)\int\frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{2\sqrt{a}\left(b^2-3ac\right)\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{ad\sqrt{dx}}-\frac{4\left(b^2-3ac\right)\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ad\sqrt{dx}\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$\downarrow$ 27

$$\frac{\frac{2c\sqrt{x}\left(\sqrt{a}\left(\frac{2\left(b^2-3ac\right)}{\sqrt{c}}+\sqrt{a}b\right)\int\frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{2\left(b^2-3ac\right)\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{ad\sqrt{dx}}-\frac{4\left(b^2-3ac\right)\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ad\sqrt{dx}\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$\downarrow$ 1416

$$\frac{\frac{2c\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\frac{2\left(b^2-3ac\right)}{\sqrt{c}}+\sqrt{a}b\right)\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}-\frac{2\left(b^2-3ac\right)\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{ad\sqrt{dx}}-\frac{4\left(b^2-3a\right.}{}}{a\left(b^2-4ac\right)}$$

$$\frac{2\left(-2ac+b^2+bcx\right)}{ad\sqrt{dx}\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$\downarrow$ 1509

$$\frac{\frac{2c\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\frac{2\left(b^2-3ac\right)}{\sqrt{c}}+\sqrt{a}b\right)\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}-\frac{2\left(b^2-3ac\right)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}E\left(\right.}{\sqrt[4]{c}\sqrt{}}\right.}{}\right.}{ad\sqrt{dx}}}{a\left(b^2-4ac\right)}$$

$$\frac{2\left(-2ac+b^2+bcx\right)}{ad\sqrt{dx}\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

input
```
Int[1/((d*x)^(3/2)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*Sqrt[d*x]*Sqrt[a + b*x + c*x^
2]) + ((-4*(b^2 - 3*a*c)*Sqrt[a + b*x + c*x^2])/(a*d*Sqrt[d*x]) + (2*c*Sqr
t[x]*((-2*(b^2 - 3*a*c)*(-((Sqrt[x]*Sqrt[a + b*x + c*x^2])/(Sqrt[a] + Sqrt
[c]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] +
 Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt
[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x + c*x^2])))/Sqrt[c] + (a^(1/4)*(Sq
rt[a]*b + (2*(b^2 - 3*a*c))/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x +
 c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1
/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x + c*x^2])))/(a
*d*Sqrt[d*x]))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 923 vs. 2(453) = 906.

Time = 5.83 (sec) , antiderivative size = 924, normalized size of antiderivative = 1.73

method	result
elliptic	$\frac{\sqrt{dx(c\,x^2+bx+a)}\left(-\frac{2\left(cd\,x^2+bdx+ad\right)}{d^2a^2\sqrt{x\left(cd\,x^2+bdx+ad\right)}}-\frac{2cdx\left(\frac{\left(2ac-b^2\right)x}{a^2\left(4ac-b^2\right)d^2}+\frac{b\left(3ac-b^2\right)}{a^2\left(4ac-b^2\right)d^2c}\right)}{\sqrt{\left(\frac{a}{c}+\frac{bx}{c}+x^2\right)cdx}}+\frac{\left(-\frac{b}{a^2d}+\frac{b\left(3ac-b^2\right)}{d\,a^2\left(4ac-b^2\right)}\right)\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\cdots}{\cdots}\right.}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

input
```
int(1/(d*x)^(3/2)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(d*x*(c*x^2+b*x+a))^(1/2)/(d*x)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2*(c*d*x^2+b*d
*x+a*d)/d^2/a^2/(x*(c*d*x^2+b*d*x+a*d))^(1/2)-2*c*d*x*((2*a*c-b^2)/a^2/(4*
a*c-b^2)/d^2*x+b*(3*a*c-b^2)/a^2/(4*a*c-b^2)/d^2/c)/((a/c+b/c*x+x^2)*c*d*x
)^(1/2)+(-b/a^2/d+1/d*b*(3*a*c-b^2)/a^2/(4*a*c-b^2))*(b+(-4*a*c+b^2)^(1/2)
)/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/
c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c
*d*x^3+b*d*x^2+a*d*x)^(1/2)*EllipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2
))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1
/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+(c/a^2/
d+c*(2*a*c-b^2)/a^2/(4*a*c-b^2)/d)*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/
2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+
(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)
^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d
*x)^(1/2)*((-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*E
llipticE(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*
c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1
/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*Ellipt
icF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1
/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 378, normalized size of antiderivative = 0.71

$$\int \frac{1}{(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}\,dx =$$

$$-\frac{2\left(((2\,b^3c-9\,abc^2)x^3+(2\,b^4-9\,ab^2c)x^2+(2\,ab^3-9\,a^2bc)x)\sqrt{cd}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3\cdots}{27\cdots}\right.\right.}{\cdots}$$

input
```
integrate(1/(d*x)^(3/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(((2*b^3*c - 9*a*b*c^2)*x^3 + (2*b^4 - 9*a*b^2*c)*x^2 + (2*a*b^3 - 9*
a^2*b*c)*x)*sqrt(c*d)*weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*
b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c) + 6*((b^2*c^2 - 3*a*c^3)*x^3 + (b^3
*c - 3*a*b*c^2)*x^2 + (a*b^2*c - 3*a^2*c^2)*x)*sqrt(c*d)*weierstrassZeta(4
/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, weierstrassPInverse(4/3
*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c)) + 3*(
a*b^2*c - 4*a^2*c^2 + 2*(b^2*c^2 - 3*a*c^3)*x^2 + (2*b^3*c - 7*a*b*c^2)*x)
*sqrt(c*x^2 + b*x + a)*sqrt(d*x))/((a^2*b^2*c^2 - 4*a^3*c^3)*d^2*x^3 + (a^
2*b^3*c - 4*a^3*b*c^2)*d^2*x^2 + (a^3*b^2*c - 4*a^4*c^2)*d^2*x)
```

Sympy [F]

$$\int \frac{1}{(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{1}{(dx)^{\frac{3}{2}}\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x)**(3/2)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral(1/((d*x)**(3/2)*(a + b*x + c*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{1}{(cx^2+bx+a)^{\frac{3}{2}}\,(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(3/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + b*x + a)^(3/2)*(d*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(dx)^{3/2} \left(a+bx+cx^2\right)^{3/2}} \, dx = \int \frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{2}} (dx)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/(d*x)^(3/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^2 + b*x + a)^(3/2)*(d*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{3/2} \left(a+bx+cx^2\right)^{3/2}} \, dx = \int \frac{1}{(d\,x)^{3/2}\,\left(c\,x^2+b\,x+a\right)^{3/2}} \, dx$$

input
```
int(1/((d*x)^(3/2)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int(1/((d*x)^(3/2)*(a + b*x + c*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(dx)^{3/2} \left(a+bx+cx^2\right)^{3/2}} \, dx = \frac{\sqrt{d}\left(-2\sqrt{x}\,\sqrt{c\,x^2+bx+a}-2\left(\int \frac{\sqrt{c\,x^2+bx+a}}{\sqrt{x}\,a^2+2\sqrt{x}\,abx+2\sqrt{x}\,ac\,x^2+\sqrt{x}\,b^2x^2+2\sqrt{x}\,bc\,x^3+}\right.\right.}{}$$

input
```
int(1/(d*x)^(3/2)/(c*x^2+b*x+a)^(3/2),x)
```

output

```
(sqrt(d)*( - 2*sqrt(x)*sqrt(a + b*x + c*x**2) - 2*int(sqrt(a + b*x + c*x**
2)/(sqrt(x)*a**2 + 2*sqrt(x)*a*b*x + 2*sqrt(x)*a*c*x**2 + sqrt(x)*b**2*x**
2 + 2*sqrt(x)*b*c*x**3 + sqrt(x)*c**2*x**4),x)*a*b*x - 2*int(sqrt(a + b*x
+ c*x**2)/(sqrt(x)*a**2 + 2*sqrt(x)*a*b*x + 2*sqrt(x)*a*c*x**2 + sqrt(x)*b
**2*x**2 + 2*sqrt(x)*b*c*x**3 + sqrt(x)*c**2*x**4),x)*b**2*x**2 - 2*int(sq
rt(a + b*x + c*x**2)/(sqrt(x)*a**2 + 2*sqrt(x)*a*b*x + 2*sqrt(x)*a*c*x**2
+ sqrt(x)*b**2*x**2 + 2*sqrt(x)*b*c*x**3 + sqrt(x)*c**2*x**4),x)*b*c*x**3
- 3*int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a**2 + 2*a*b*x + 2*a*c*x**2 + b*
*2*x**2 + 2*b*c*x**3 + c**2*x**4),x)*a*c*x - 3*int((sqrt(x)*sqrt(a + b*x +
 c*x**2))/(a**2 + 2*a*b*x + 2*a*c*x**2 + b**2*x**2 + 2*b*c*x**3 + c**2*x**
4),x)*b*c*x**2 - 3*int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a**2 + 2*a*b*x +
2*a*c*x**2 + b**2*x**2 + 2*b*c*x**3 + c**2*x**4),x)*c**2*x**3))/(a*d**2*x*
(a + b*x + c*x**2))
```

3.338 $\int \frac{1}{(dx)^{5/2}(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 604

$$\int \frac{1}{(dx)^{5/2}\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2}{3ad(dx)^{3/2}\sqrt{a+bx+cx^2}} + \frac{8b}{3a^2d^2\sqrt{dx}\sqrt{a+bx+cx^2}} + \frac{2\sqrt{dx}(8b^4-33ab^2c+10a^2c^2+bc(8b^2-29ac)\,x)}{3a^3\,(b^2-4ac)\,d^3\sqrt{a+bx+cx^2}} - \frac{b(8b^2-29ac)\,\sqrt{-b+\sqrt{b^2-4ac}}(b+\sqrt{b^2-4ac})\,\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{dx}}{\sqrt{-b+\sqrt{b^2-4a}}}\right.\right.}{3\sqrt{2}a^3\sqrt{c}\,(b^2-4ac)\,d^{5/2}\sqrt{a+x(b+cx)}} + \frac{\sqrt{-b+\sqrt{b^2-4ac}}(8b^4-37ab^2c+20a^2c^2+b(8b^2-29ac)\,\sqrt{b^2-4ac})\,\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\,\mathrm{E}}{3\sqrt{2}a^3\sqrt{c}\,(b^2-4ac)\,d^{5/2}\sqrt{a+x(b+cx)}}$$

output
```
-2/3/a/d/(d*x)^(3/2)/(c*x^2+b*x+a)^(1/2)+8/3*b/a^2/d^2/(d*x)^(1/2)/(c*x^2+
b*x+a)^(1/2)+2/3*(d*x)^(1/2)*(8*b^4-33*a*b^2*c+10*a^2*c^2+b*c*(-29*a*c+8*b
^2)*x)/a^3/(-4*a*c+b^2)/d^3/(c*x^2+b*x+a)^(1/2)-1/6*b*(-29*a*c+8*b^2)*(-b+
(-4*a*c+b^2)^(1/2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*(1+2*c*x/(b-(-4*a*c+b^2)^
(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2^(1/2)*c^(
1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(1
/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/a^3/c^(1/2)/(-4*a*c+b^2)/d^(5/
2)/(a+x*(c*x+b))^(1/2)+1/6*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(8*b^4-37*a*b^2*c
+20*a^2*c^2+b*(-29*a*c+8*b^2)*(-4*a*c+b^2)^(1/2))*(1+2*c*x/(b-(-4*a*c+b^2)
^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(2^(1/2)*c^
(1/2)*(d*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/d^(1/2),((b-(-4*a*c+b^2)^(
1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/a^3/c^(1/2)/(-4*a*c+b^2)/d^(5
/2)/(a+x*(c*x+b))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.36 (sec) , antiderivative size = 581, normalized size of antiderivative = 0.96

$$\int \frac{1}{(dx)^{5/2}\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{x\left(12x^2(b^4-4ab^2c+2a^2c^2+b^3cx-3abc^2x)-4a(b^2-4ac)\left(a+x(b+\right.\right.}{}$$

input
```
Integrate[1/((d*x)^(5/2)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(x*(12*x^2*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x - 3*a*b*c^2*x) - 4*a*(b^
2 - 4*a*c)*(a + x*(b + c*x)) - 4*b*(8*b^2 - 29*a*c)*x*(a + x*(b + c*x)) +
20*b*(b^2 - 4*a*c)*x*(a + x*(b + c*x)) + (I*b*(8*b^2 - 29*a*c)*(-b + Sqrt[
b^2 - 4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(5/2)*Sqrt[(2*
a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticE[I*Ar
cSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 -
4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a/(b + Sqrt[b^2 - 4*a*c])] + (I*(8*
b^4 - 37*a*b^2*c + 20*a^2*c^2 - 8*b^3*Sqrt[b^2 - 4*a*c] + 29*a*b*c*Sqrt[b^
2 - 4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(5/2)*Sqrt[(2*a
+ b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticF[I*ArcS
inh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*
a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]))/(6*a^3*(
b^2 - 4*a*c)*(d*x)^(5/2)*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 488, normalized size of antiderivative = 0.81, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {1165, 27, 1237, 27, 1237, 27, 1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{5/2}\left(a+bx+cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1165}$$

$$\frac{2\left(-2ac+b^2+bcx\right)}{ad(dx)^{3/2}\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{d^2\left(2\left(2b^2-5ac\right)+3bcx\right)}{2(dx)^{5/2}\sqrt{cx^2+bx+a}}dx}{ad^2\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2\left(2b^2-5ac\right)+3bcx}{(dx)^{5/2}\sqrt{cx^2+bx+a}}dx}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ad(dx)^{3/2}\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{1237}$$

$$\frac{-\frac{2\int \frac{d\left(b\left(8b^2-29ac\right)+2c\left(2b^2-5ac\right)x\right)}{2(dx)^{3/2}\sqrt{cx^2+bx+a}}dx}{3ad^2}-\frac{4\left(2b^2-5ac\right)\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ad(dx)^{3/2}\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{\int \frac{b(8b^2-29ac)+2c(2b^2-5ac)x}{(dx)^{3/2}\sqrt{cx^2+bx+a}}dx}{3ad}-\frac{4(2b^2-5ac)\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{ad(dx)^{3/2}(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \ 1237$$

$$\frac{-\frac{-\frac{2\int -\frac{cd(2a(2b^2-5ac)+b(8b^2-29ac)x)}{2\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{ad^2}-\frac{2b(8b^2-29ac)\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{3ad}-\frac{4(2b^2-5ac)\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{ad(dx)^{3/2}(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{\frac{c\int \frac{2a(2b^2-5ac)+b(8b^2-29ac)x}{\sqrt{dx}\sqrt{cx^2+bx+a}}dx}{ad}-\frac{2b(8b^2-29ac)\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{3ad}-\frac{4(2b^2-5ac)\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{ad(dx)^{3/2}(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \ 1241$$

$$\frac{-\frac{\frac{c\sqrt{x}\int \frac{2a(2b^2-5ac)+b(8b^2-29ac)x}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{ad\sqrt{dx}}-\frac{2b(8b^2-29ac)\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{3ad}-\frac{4(2b^2-5ac)\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{ad(dx)^{3/2}(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \ 1240$$

$$\frac{-\frac{\frac{2c\sqrt{x}\int \frac{2a(2b^2-5ac)+b(8b^2-29ac)x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{ad\sqrt{dx}}-\frac{2b(8b^2-29ac)\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{3ad}-\frac{4(2b^2-5ac)\sqrt{a+bx+cx^2}}{3ad(dx)^{3/2}}}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{ad(dx)^{3/2}(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \ 1511$$

$$-\frac{\frac{\frac{2c\sqrt{x}\left(\sqrt{a}\left(\frac{b\left(8b^2-29ac\right)}{\sqrt{c}}+2\sqrt{a}\left(2b^2-5ac\right)\right)\int\frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{\sqrt{a}b\left(8b^2-29ac\right)\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{ad\sqrt{dx}}-\frac{2b\left(8b^2-29ac\right)\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{3ad}-\frac{4(2b^2-5ac)}{3ad(}}{a\left(b^2-4ac\right)}$$

$$\frac{2\left(-2ac+b^2+bcx\right)}{ad(dx)^{3/2}\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{2c\sqrt{x}\left(\sqrt{a}\left(\frac{b\left(8b^2-29ac\right)}{\sqrt{c}}+2\sqrt{a}\left(2b^2-5ac\right)\right)\int\frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{b\left(8b^2-29ac\right)\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{ad\sqrt{dx}}-\frac{2b\left(8b^2-29ac\right)\sqrt{a+bx+cx^2}}{ad\sqrt{dx}}}{3ad}-\frac{4(2b^2-5ac)\sqrt{a+}}{3ad(dx)^{3/}}}{a\left(b^2-4ac\right)}$$

$$\frac{2\left(-2ac+b^2+bcx\right)}{ad(dx)^{3/2}\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$\downarrow$ 1416

$$-\frac{\frac{2c\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\frac{b\left(8b^2-29ac\right)}{\sqrt{c}}+2\sqrt{a}\left(2b^2-5ac\right)\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}-\frac{b\left(8b^2-29ac\right)\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{}}{\sqrt{c}}\right.}{ad\sqrt{dx}}}{3ad}}{a\left(b^2-4ac\right)}$$

$$\frac{2\left(-2ac+b^2+bcx\right)}{ad(dx)^{3/2}\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$\downarrow$ 1509

$$-\frac{\frac{2c\sqrt{x}\left(\frac{\sqrt[4]{a}\left(\frac{b\left(8b^2-29ac\right)}{\sqrt{c}}+2\sqrt{a}\left(2b^2-5ac\right)\right)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}-\frac{b\left(8b^2-29ac\right)\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x)}{}\right.}{}\right.}{ad\sqrt{dx}}}{3ad}}{a\left(b^2-4ac\right)}$$

$$\frac{2\left(-2ac+b^2+bcx\right)}{ad(dx)^{3/2}\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

input
```
Int[1/((d*x)^(5/2)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*(d*x)^(3/2)*Sqrt[a + b*x + c*
x^2]) + ((-4*(2*b^2 - 5*a*c)*Sqrt[a + b*x + c*x^2])/(3*a*d*(d*x)^(3/2)) -
((-2*b*(8*b^2 - 29*a*c)*Sqrt[a + b*x + c*x^2])/(a*d*Sqrt[d*x]) + (2*c*Sqrt
[x]*(-((b*(8*b^2 - 29*a*c)*(-((Sqrt[x]*Sqrt[a + b*x + c*x^2])/(Sqrt[a] + S
qrt[c]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a
] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(S
qrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x + c*x^2])))/Sqrt[c]) + (a^(1/4)
*((b*(8*b^2 - 29*a*c))/Sqrt[c] + 2*Sqrt[a]*(2*b^2 - 5*a*c))*(Sqrt[a] + Sqr
t[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan
[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt
[a + b*x + c*x^2])))/(a*d*Sqrt[d*x]))/(3*a*d))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
 + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
 + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 7.52 (sec) , antiderivative size = 994, normalized size of antiderivative = 1.65

method	result
elliptic	$\frac{\sqrt{dx(c\,x^2+bx+a)}\left(-\frac{2\sqrt{cd\,x^3+bd\,x^2+adx}}{3a^2d^3x^2}+\frac{10\left(cd\,x^2+bdx+ad\right)b}{3a^3d^3\sqrt{x\left(cd\,x^2+bdx+ad\right)}}-\frac{2cdx\left(-\frac{b\left(3ac-b^2\right)x}{\left(4ac-b^2\right)a^3d^3}+\frac{2a^2c^2-4ca\,b^2+b^4}{a^3\left(4ac-b^2\right)d^3c}\right)}{\sqrt{\left(\frac{a}{c}+\frac{bx}{c}+x^2\right)cdx}}+\frac{\left(-\frac{c}{3a^2d^2}-\frac{ac-b}{a^3d^2}\right.}{}\right.}{}$
default	Expression too large to display
risch	Expression too large to display

input
```
int(1/(d*x)^(5/2)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(d*x*(c*x^2+b*x+a))^(1/2)/(d*x)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2/3/a^2/d^3*(c
*d*x^3+b*d*x^2+a*d*x)^(1/2)/x^2+10/3*(c*d*x^2+b*d*x+a*d)/a^3/d^3*b/(x*(c*d
*x^2+b*d*x+a*d))^(1/2)-2*c*d*x*(-b*(3*a*c-b^2)/(4*a*c-b^2)/a^3/d^3*x+(2*a^
2*c^2-4*a*b^2*c+b^4)/a^3/(4*a*c-b^2)/d^3/c)/((a/c+b/c*x+x^2)*c*d*x)^(1/2)+
(-1/3*c/a^2/d^2-(a*c-b^2)/a^3/d^2+1/d^2*(2*a^2*c^2-4*a*b^2*c+b^4)/a^3/(4*a
*c-b^2))*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c
)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2
*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b
+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)*EllipticF(2^(1/2
)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-
2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*
c+b^2)^(1/2))))^(1/2))+(-5/3*b/a^3*c/d^2-b*c*(3*a*c-b^2)/(4*a*c-b^2)/a^3/d
^2)*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(
-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*
a*c+b^2)^(1/2)))^(1/2)/(c*d*x^3+b*d*x^2+a*d*x)^(1/2)*((-1/2*(b+(-4*a*c+b^2
)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(2^(1/2)*((x+1/2*(b+(-4
*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2
)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^
(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(2^(1/2)*((x+1/2*(b+(-4*a...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 456, normalized size of antiderivative = 0.75

$$\int \frac{1}{(dx)^{5/2}\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(((8\,b^4c-41\,ab^2c^2+30\,a^2c^3)x^4+(8\,b^5-41\,ab^3c+30\,a^2bc^2)x^3+(8\,ab\right.}{}$$

input
```
integrate(1/(d*x)^(5/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
2/9*(((8*b^4*c - 41*a*b^2*c^2 + 30*a^2*c^3)*x^4 + (8*b^5 - 41*a*b^3*c + 30
*a^2*b*c^2)*x^3 + (8*a*b^4 - 41*a^2*b^2*c + 30*a^3*c^2)*x^2)*sqrt(c*d)*wei
erstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(
3*c*x + b)/c) + 3*((8*b^3*c^2 - 29*a*b*c^3)*x^4 + (8*b^4*c - 29*a*b^2*c^2)
*x^3 + (8*a*b^3*c - 29*a^2*b*c^2)*x^2)*sqrt(c*d)*weierstrassZeta(4/3*(b^2
- 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, weierstrassPInverse(4/3*(b^2 -
3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c)) - 3*(a^2*b^2*
c - 4*a^3*c^2 - (8*b^3*c^2 - 29*a*b*c^3)*x^3 - (8*b^4*c - 33*a*b^2*c^2 + 1
0*a^2*c^3)*x^2 - 4*(a*b^3*c - 4*a^2*b*c^2)*x)*sqrt(c*x^2 + b*x + a)*sqrt(d
*x))/((a^3*b^2*c^2 - 4*a^4*c^3)*d^3*x^4 + (a^3*b^3*c - 4*a^4*b*c^2)*d^3*x^
3 + (a^4*b^2*c - 4*a^5*c^2)*d^3*x^2)
```

Sympy [F]

$$\int \frac{1}{(dx)^{5/2} (a+bx+cx^2)^{3/2}}\,dx = \int \frac{1}{(dx)^{\frac{5}{2}} (a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x)**(5/2)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral(1/((d*x)**(5/2)*(a + b*x + c*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(dx)^{5/2} (a+bx+cx^2)^{3/2}}\,dx = \int \frac{1}{(cx^2+bx+a)^{\frac{3}{2}} (dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(5/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + b*x + a)^(3/2)*(d*x)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(dx)^{5/2} (a+bx+cx^2)^{3/2}} \, dx = \int \frac{1}{(cx^2+bx+a)^{\frac{3}{2}} (dx)^{\frac{5}{2}}} \, dx$$

input
```
integrate(1/(d*x)^(5/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^2 + b*x + a)^(3/2)*(d*x)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{5/2} (a+bx+cx^2)^{3/2}} \, dx = \int \frac{1}{(d\,x)^{5/2}\,(c\,x^2+b\,x+a)^{3/2}} \, dx$$

input
```
int(1/((d*x)^(5/2)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int(1/((d*x)^(5/2)*(a + b*x + c*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(dx)^{5/2} (a+bx+cx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
int(1/(d*x)^(5/2)/(c*x^2+b*x+a)^(3/2),x)
```

output

```
(sqrt(d)*( - 4*sqrt(a + b*x + c*x**2)*a*b + 16*sqrt(a + b*x + c*x**2)*b**2
*x - 20*sqrt(a + b*x + c*x**2)*b*c*x**2 - 10*sqrt(a + b*x + c*x**2)*c**2*x
**3 + 16*sqrt(x)*int(sqrt(a + b*x + c*x**2)/(sqrt(x)*a**2 + 2*sqrt(x)*a*b*
x + 2*sqrt(x)*a*c*x**2 + sqrt(x)*b**2*x**2 + 2*sqrt(x)*b*c*x**3 + sqrt(x)*
c**2*x**4),x)*a*b**3*x + 16*sqrt(x)*int(sqrt(a + b*x + c*x**2)/(sqrt(x)*a*
*2 + 2*sqrt(x)*a*b*x + 2*sqrt(x)*a*c*x**2 + sqrt(x)*b**2*x**2 + 2*sqrt(x)*
b*c*x**3 + sqrt(x)*c**2*x**4),x)*b**4*x**2 + 16*sqrt(x)*int(sqrt(a + b*x +
 c*x**2)/(sqrt(x)*a**2 + 2*sqrt(x)*a*b*x + 2*sqrt(x)*a*c*x**2 + sqrt(x)*b*
*2*x**2 + 2*sqrt(x)*b*c*x**3 + sqrt(x)*c**2*x**4),x)*b**3*c*x**3 + 24*sqrt
(x)*int((sqrt(a + b*x + c*x**2)*x)/(sqrt(x)*a**2 + 2*sqrt(x)*a*b*x + 2*sqr
t(x)*a*c*x**2 + sqrt(x)*b**2*x**2 + 2*sqrt(x)*b*c*x**3 + sqrt(x)*c**2*x**4
),x)*a*b**2*c*x + 24*sqrt(x)*int((sqrt(a + b*x + c*x**2)*x)/(sqrt(x)*a**2
+ 2*sqrt(x)*a*b*x + 2*sqrt(x)*a*c*x**2 + sqrt(x)*b**2*x**2 + 2*sqrt(x)*b*c
*x**3 + sqrt(x)*c**2*x**4),x)*b**3*c*x**2 + 24*sqrt(x)*int((sqrt(a + b*x +
 c*x**2)*x)/(sqrt(x)*a**2 + 2*sqrt(x)*a*b*x + 2*sqrt(x)*a*c*x**2 + sqrt(x)
*b**2*x**2 + 2*sqrt(x)*b*c*x**3 + sqrt(x)*c**2*x**4),x)*b**2*c**2*x**3 + 5
*sqrt(x)*int((sqrt(x)*sqrt(a + b*x + c*x**2)*x**2)/(a**2 + 2*a*b*x + 2*a*c
*x**2 + b**2*x**2 + 2*b*c*x**3 + c**2*x**4),x)*a*c**3*x + 5*sqrt(x)*int((s
qrt(x)*sqrt(a + b*x + c*x**2)*x**2)/(a**2 + 2*a*b*x + 2*a*c*x**2 + b**2*x*
*2 + 2*b*c*x**3 + c**2*x**4),x)*b*c**3*x**2 + 5*sqrt(x)*int((sqrt(x)*sq...
```

3.339 $\int (dx)^m \left(a+bx+cx^2\right)^p \, dx$

Optimal result

Integrand size = 18, antiderivative size = 137

$$\int (dx)^m \left(a+bx+cx^2\right)^p \, dx = \frac{(dx)^{1+m}\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx+cx^2\right)^p \operatorname{AppellF1}\left(1+m,-p,-p,2+m,-\frac{2c}{b-\sqrt{b^2}}\right.}{d(1+m)}$$

output
```
(d*x)^(1+m)*(c*x^2+b*x+a)^p*AppellF1(1+m,-p,-p,2+m,-2*c*x/(b-(-4*a*c+b^2)^
(1/2)),-2*c*x/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)/((1+2*c*x/(b-(-4*a*c+b^2)^(1
/2)))^p)/((1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.17

$$\int (dx)^m \left(a+bx+cx^2\right)^p \, dx = \frac{x(dx)^m\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x(b+cx))^p \operatorname{AppellF1}\left(1+m,-p,-p,2+m,-\frac{2cx}{b+\sqrt{b^2}}\right.}{1+m}$$

input
```
Integrate[(d*x)^m*(a + b*x + c*x^2)^p,x]
```

output
```
(x*(d*x)^m*(a + x*(b + c*x))^p*AppellF1[1 + m, -p, -p, 2 + m, (-2*c*x)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])])/((1 + m)*((b - Sq
rt[b^2 - 4*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c
] + 2*c*x)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx+cx^2\right)^p \, dx$$

$$\Big\downarrow \; 1179$$

$$\frac{\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)^{-p} \left(a+bx+cx^2\right)^p \int (dx)^m \left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^p \left(\frac{2cx}{b+\sqrt{b^2-4ac}}+1\right)^p d(dx)}{d}$$

$$\Big\downarrow \; 150$$

$$\frac{(dx)^{m+1} \left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)^{-p} \left(a+bx+cx^2\right)^p \text{AppellF1}\left(m+1,-p,-p,m+2,-\frac{2cx}{b-\sqrt{b^2-4ac}},\right.}{d(m+1)}$$

input
```
Int[(d*x)^m*(a + b*x + c*x^2)^p,x]
```

output
```
((d*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (-2*c*x)
/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(d*(1 + m)*(1
 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x)/(b + Sqrt[b^2 - 4*a*c])
)^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

Maple [F]

$$\int (dx)^m \left(c\,x^2+bx+a\right)^p dx$$

input
```
int((d*x)^m*(c*x^2+b*x+a)^p,x)
```

output
```
int((d*x)^m*(c*x^2+b*x+a)^p,x)
```

Fricas [F]

$$\int (dx)^m \left(a+bx+cx^2\right)^p \, dx = \int \left(cx^2+bx+a\right)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)^p*(d*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(a+bx+cx^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d*x)**m*(c*x**2+b*x+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (dx)^m \left(a+bx+cx^2\right)^p \, dx = \int (cx^2+bx+a)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^p*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m \left(a+bx+cx^2\right)^p \, dx = \int (cx^2+bx+a)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^p*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+bx+cx^2\right)^p \, dx = \int (d\,x)^m \left(c\,x^2+b\,x+a\right)^p \,dx$$

input
```
int((d*x)^m*(a + b*x + c*x^2)^p,x)
```

output
```
int((d*x)^m*(a + b*x + c*x^2)^p, x)
```

Reduce [F]

$$\int (dx)^m \left(a+bx+cx^2\right)^p \, dx = \text{too large to display}$$

input
```
int((d*x)^m*(c*x^2+b*x+a)^p,x)
```

output

```
(d**m*(2*x**m*(a + b*x + c*x**2)**p*a*p + x**m*(a + b*x + c*x**2)**p*b*m*x
 + x**m*(a + b*x + c*x**2)**p*b*p*x - 2*int((x**m*(a + b*x + c*x**2)**p*x)
/(a*m**2 + 3*a*m*p + a*m + 2*a*p**2 + a*p + b*m**2*x + 3*b*m*p*x + b*m*x +
 2*b*p**2*x + b*p*x + c*m**2*x**2 + 3*c*m*p*x**2 + c*m*x**2 + 2*c*p**2*x**
2 + c*p*x**2),x)*a*c*m**3*p - 10*int((x**m*(a + b*x + c*x**2)**p*x)/(a*m**
2 + 3*a*m*p + a*m + 2*a*p**2 + a*p + b*m**2*x + 3*b*m*p*x + b*m*x + 2*b*p*
*2*x + b*p*x + c*m**2*x**2 + 3*c*m*p*x**2 + c*m*x**2 + 2*c*p**2*x**2 + c*p
*x**2),x)*a*c*m**2*p**2 - 2*int((x**m*(a + b*x + c*x**2)**p*x)/(a*m**2 + 3
*a*m*p + a*m + 2*a*p**2 + a*p + b*m**2*x + 3*b*m*p*x + b*m*x + 2*b*p**2*x
+ b*p*x + c*m**2*x**2 + 3*c*m*p*x**2 + c*m*x**2 + 2*c*p**2*x**2 + c*p*x**2
),x)*a*c*m**2*p - 16*int((x**m*(a + b*x + c*x**2)**p*x)/(a*m**2 + 3*a*m*p
+ a*m + 2*a*p**2 + a*p + b*m**2*x + 3*b*m*p*x + b*m*x + 2*b*p**2*x + b*p*x
 + c*m**2*x**2 + 3*c*m*p*x**2 + c*m*x**2 + 2*c*p**2*x**2 + c*p*x**2),x)*a*
c*m*p**3 - 6*int((x**m*(a + b*x + c*x**2)**p*x)/(a*m**2 + 3*a*m*p + a*m +
2*a*p**2 + a*p + b*m**2*x + 3*b*m*p*x + b*m*x + 2*b*p**2*x + b*p*x + c*m**
2*x**2 + 3*c*m*p*x**2 + c*m*x**2 + 2*c*p**2*x**2 + c*p*x**2),x)*a*c*m*p**2
 - 8*int((x**m*(a + b*x + c*x**2)**p*x)/(a*m**2 + 3*a*m*p + a*m + 2*a*p**2
 + a*p + b*m**2*x + 3*b*m*p*x + b*m*x + 2*b*p**2*x + b*p*x + c*m**2*x**2 +
 3*c*m*p*x**2 + c*m*x**2 + 2*c*p**2*x**2 + c*p*x**2),x)*a*c*p**4 - 4*int((
x**m*(a + b*x + c*x**2)**p*x)/(a*m**2 + 3*a*m*p + a*m + 2*a*p**2 + a*p ...
```

3.340 $\int x^2(a+bx+cx^2)^p\,dx$

Optimal result

Integrand size = 16, antiderivative size = 153

$$\int x^2\left(a+bx+cx^2\right)^p\,dx=-\frac{(b(2+p)-2c(1+p)x)\left(a+bx+cx^2\right)^{1+p}}{2c^2(1+p)(3+2p)}-\frac{2^{-2(1+p)}\left(2ac-b^2(2+p)\right)(b+2cx)\left(a+bx+cx^2\right)^p\left(-\frac{c(a+bx+cx^2)}{b^2-4ac}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{(\ldots}{\ldots}\right.}{c^3(3+2p)}$$

output
```
-1/2*(b*(2+p)-2*c*(p+1)*x)*(c*x^2+b*x+a)^(p+1)/c^2/(p+1)/(3+2*p)-(2*a*c-b^
2*(2+p))*(2*c*x+b)*(c*x^2+b*x+a)^p*hypergeom([1/2, -p],[3/2],(2*c*x+b)^2/(
-4*a*c+b^2))/(2^(2*p+2))/c^3/(3+2*p)/((-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.36 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.99

$$\int x^2\left(a+bx+cx^2\right)^p\,dx=\frac{1}{3}x^3\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x(b+cx))^p\,\text{AppellF1}\left(3,-p,-p,4,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[x^2*(a + b*x + c*x^2)^p,x]
```

output
```
(x^3*(a + x*(b + c*x))^p*AppellF1[3, -p, -p, 4, (-2*c*x)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])])/(3*((b - Sqrt[b^2 - 4*a*c] + 2*
c*x)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(b + Sqrt
[b^2 - 4*a*c]))^p)
```

Rubi [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.35, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1166, 25, 1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+bx+cx^2\right)^p\,dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{\int -\left((a+b(p+2)x)\left(cx^2+bx+a\right)^p\right)dx}{c(2p+3)}+\frac{x\left(a+bx+cx^2\right)^{p+1}}{c(2p+3)}$$

$$\Big\downarrow \text{25}$$

$$\frac{x\left(a+bx+cx^2\right)^{p+1}}{c(2p+3)}-\frac{\int (a+b(p+2)x)\left(cx^2+bx+a\right)^p\,dx}{c(2p+3)}$$

$$\Big\downarrow \text{1160}$$

$$\frac{x\left(a+bx+cx^2\right)^{p+1}}{c(2p+3)}-\frac{\frac{\left(2ac-b^2(p+2)\right)\int \left(cx^2+bx+a\right)^p dx}{2c}+\frac{b(p+2)\left(a+bx+cx^2\right)^{p+1}}{2c(p+1)}}{c(2p+3)}$$

$$\Big\downarrow \text{1096}$$

$$\frac{x\left(a+bx+cx^2\right)^{p+1}}{c(2p+3)}-\frac{\frac{b(p+2)\left(a+bx+cx^2\right)^{p+1}}{2c(p+1)}-\frac{2^p\left(2ac-b^2(p+2)\right)\left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx+cx^2\right)^{p+1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c(p+1)\sqrt{b^2-4ac}}}{c(2p+3)}$$

input
```
Int[x^2*(a + b*x + c*x^2)^p,x]
```

output
```
(x*(a + b*x + c*x^2)^(1 + p))/(c*(3 + 2*p)) - ((b*(2 + p)*(a + b*x + c*x^2
)^(1 + p))/(2*c*(1 + p)) - (2^p*(2*a*c - b^2*(2 + p))*(-((b - Sqrt[b^2 - 4
*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x + c*x^2)^(1 + p)*Hype
rgeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^
2 - 4*a*c])])/(c*Sqrt[b^2 - 4*a*c]*(1 + p)))/(c*(3 + 2*p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && !IntegerQ[4*p] && !IntegerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

Maple [F]

$$\int x^2 (c x^2 + bx + a)^p \, dx$$

input
```
int(x^2*(c*x^2+b*x+a)^p,x)
```

output
```
int(x^2*(c*x^2+b*x+a)^p,x)
```

Fricas [F]

$$\int x^2 (a + bx + cx^2)^p \, dx = \int (cx^2 + bx + a)^p x^2 \, dx$$

input
```
integrate(x^2*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)^p*x^2, x)
```

Sympy [F]

$$\int x^2 (a + bx + cx^2)^p \, dx = \int x^2 (a + bx + cx^2)^p \, dx$$

input
```
integrate(x**2*(c*x**2+b*x+a)**p,x)
```

output
```
Integral(x**2*(a + b*x + c*x**2)**p, x)
```

Maxima [F]

$$\int x^2(a+bx+cx^2)^p\,dx=\int (cx^2+bx+a)^px^2\,dx$$

input
```
integrate(x^2*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^p*x^2, x)
```

Giac [F]

$$\int x^2(a+bx+cx^2)^p\,dx=\int (cx^2+bx+a)^px^2\,dx$$

input
```
integrate(x^2*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+bx+cx^2)^p\,dx=\int x^2\,(c\,x^2+b\,x+a)^p\,dx$$

input
```
int(x^2*(a + b*x + c*x^2)^p,x)
```

output
```
int(x^2*(a + b*x + c*x^2)^p, x)
```

Reduce [F]

$$\int x^2 (a + bx + cx^2)^p \, dx = \text{Too large to display}$$

input
```
int(x^2*(c*x^2+b*x+a)^p,x)
```

output
```
( - 4*(a + b*x + c*x**2)**p*a**2*c*p - 4*(a + b*x + c*x**2)**p*a**2*c + (a
 + b*x + c*x**2)**p*a*b**2*p + 2*(a + b*x + c*x**2)**p*a*b**2 + 4*(a + b*x
 + c*x**2)**p*a*b*c*p**2*x + 4*(a + b*x + c*x**2)**p*a*b*c*p*x - (a + b*x
+ c*x**2)**p*b**3*p**2*x - 2*(a + b*x + c*x**2)**p*b**3*p*x + 2*(a + b*x +
 c*x**2)**p*b**2*c*p**2*x**2 + (a + b*x + c*x**2)**p*b**2*c*p*x**2 + 4*(a
+ b*x + c*x**2)**p*b*c**2*p**2*x**3 + 6*(a + b*x + c*x**2)**p*b*c**2*p*x**
3 + 2*(a + b*x + c*x**2)**p*b*c**2*x**3 + 32*int(((a + b*x + c*x**2)**p*x)
/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x + 8*b*p*x + 3*b*x + 4*c*p**2*x**2 +
8*c*p*x**2 + 3*c*x**2),x)*a**2*c**2*p**4 + 96*int(((a + b*x + c*x**2)**p*x
)/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x + 8*b*p*x + 3*b*x + 4*c*p**2*x**2 +
 8*c*p*x**2 + 3*c*x**2),x)*a**2*c**2*p**3 + 88*int(((a + b*x + c*x**2)**p*
x)/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x + 8*b*p*x + 3*b*x + 4*c*p**2*x**2
+ 8*c*p*x**2 + 3*c*x**2),x)*a**2*c**2*p**2 + 24*int(((a + b*x + c*x**2)**p
*x)/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x + 8*b*p*x + 3*b*x + 4*c*p**2*x**2
 + 8*c*p*x**2 + 3*c*x**2),x)*a**2*c**2*p - 16*int(((a + b*x + c*x**2)**p*x
)/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x + 8*b*p*x + 3*b*x + 4*c*p**2*x**2 +
 8*c*p*x**2 + 3*c*x**2),x)*a*b**2*c*p**5 - 88*int(((a + b*x + c*x**2)**p*x
)/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x + 8*b*p*x + 3*b*x + 4*c*p**2*x**2 +
 8*c*p*x**2 + 3*c*x**2),x)*a*b**2*c*p**4 - 164*int(((a + b*x + c*x**2)**p*
x)/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x + 8*b*p*x + 3*b*x + 4*c*p**2*x*...
```

3.341 $\int x(a+bx+cx^2)^p \, dx$

Optimal result

Integrand size = 14, antiderivative size = 114

$$\int x(a+bx+cx^2)^p \, dx = \frac{(a+bx+cx^2)^{1+p}}{2c(1+p)} - \frac{2^{-2(1+p)}b(b+2cx)(a+bx+cx^2)^p\left(-\frac{c(a+bx+cx^2)}{b^2-4ac}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{(b+2cx)^2}{b^2-4ac}\right)}{c^2}$$

output
```
1/2*(c*x^2+b*x+a)^(p+1)/c/(p+1)-b*(2*c*x+b)*(c*x^2+b*x+a)^p*hypergeom([1/2
, -p],[3/2],(2*c*x+b)^2/(-4*a*c+b^2))/(2^(2*p+2))/c^2/((-c*(c*x^2+b*x+a)/(
-4*a*c+b^2))^p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.24 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.32

$$\int x(a+bx+cx^2)^p \, dx = \frac{1}{2}x^2\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x(b+cx))^p\,\text{AppellF1}\left(2,-p,-p,3,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[x*(a + b*x + c*x^2)^p,x]
```

output
```
(x^2*(a + x*(b + c*x))^p*AppellF1[2, -p, -p, 3, (-2*c*x)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])])/(2*((b - Sqrt[b^2 - 4*a*c] + 2*
c*x)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(b + Sqrt
[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx+cx^2\right)^p\,dx$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{\left(a+bx+cx^2\right)^{p+1}}{2c(p+1)}-\frac{b\int \left(cx^2+bx+a\right)^p dx}{2c}$$

$$\Big\downarrow \text{ 1096}$$

$$\frac{b2^p\left(a+bx+cx^2\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p-1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c(p+1)\sqrt{b^2-4ac}}+\frac{\left(a+bx+cx^2\right)^{p+1}}{2c(p+1)}$$

input
```
Int[x*(a + b*x + c*x^2)^p,x]
```

output
```
(a + b*x + c*x^2)^(1 + p)/(2*c*(1 + p)) + (2^p*b*(-((b - Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x + c*x^2)^(1 + p)*Hypergeom
etric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4
*a*c])])/(c*Sqrt[b^2 - 4*a*c]*(1 + p))
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && !IntegerQ[4*p] && !IntegerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [F]

$$\int x(c\,x^{2}+bx+a)^{p}\,dx$$

input
```
int(x*(c*x^2+b*x+a)^p,x)
```

output
```
int(x*(c*x^2+b*x+a)^p,x)
```

Fricas [F]

$$\int x\left(a+bx+cx^2\right)^p\,dx=\int\left(cx^2+bx+a\right)^p x\,dx$$

input
```
integrate(x*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)^p*x, x)
```

Sympy [F]

$$\int x(a+bx+cx^2)^p\,dx=\int x(a+bx+cx^2)^p\,dx$$

input
```
integrate(x*(c*x**2+b*x+a)**p,x)
```

output
```
Integral(x*(a + b*x + c*x**2)**p, x)
```

Maxima [F]

$$\int x(a+bx+cx^2)^p\,dx=\int (cx^2+bx+a)^p x\,dx$$

input
```
integrate(x*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^p*x, x)
```

Giac [F]

$$\int x(a+bx+cx^2)^p\,dx=\int (cx^2+bx+a)^p x\,dx$$

input
```
integrate(x*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+bx+cx^2)^p\,dx=\int x\,(c\,x^2+b\,x+a)^p\,dx$$

input
```
int(x*(a + b*x + c*x^2)^p,x)
```

output
```
int(x*(a + b*x + c*x^2)^p, x)
```

Reduce [F]

$$\int x(a+bx+cx^2)^p\,dx$$
$$=\frac{-(c\,x^2+bx+a)^p\,a+(c\,x^2+bx+a)^p\,bpx+2(c\,x^2+bx+a)^p\,cp\,x^2+(c\,x^2+bx+a)^p\,c\,x^2+8\Big(\int\frac{}{2cp\,x^2+}$$

input
```
int(x*(c*x^2+b*x+a)^p,x)
```

output
```
( - (a + b*x + c*x**2)**p*a + (a + b*x + c*x**2)**p*b*p*x + 2*(a + b*x + c
*x**2)**p*c*p*x**2 + (a + b*x + c*x**2)**p*c*x**2 + 8*int(((a + b*x + c*x*
*2)**p*x)/(2*a*p + a + 2*b*p*x + b*x + 2*c*p*x**2 + c*x**2),x)*a*c*p**3 +
12*int(((a + b*x + c*x**2)**p*x)/(2*a*p + a + 2*b*p*x + b*x + 2*c*p*x**2 +
 c*x**2),x)*a*c*p**2 + 4*int(((a + b*x + c*x**2)**p*x)/(2*a*p + a + 2*b*p*
x + b*x + 2*c*p*x**2 + c*x**2),x)*a*c*p - 2*int(((a + b*x + c*x**2)**p*x)/
(2*a*p + a + 2*b*p*x + b*x + 2*c*p*x**2 + c*x**2),x)*b**2*p**3 - 3*int(((a
 + b*x + c*x**2)**p*x)/(2*a*p + a + 2*b*p*x + b*x + 2*c*p*x**2 + c*x**2),x
)*b**2*p**2 - int(((a + b*x + c*x**2)**p*x)/(2*a*p + a + 2*b*p*x + b*x + 2
*c*p*x**2 + c*x**2),x)*b**2*p)/(2*c*(2*p**2 + 3*p + 1))
```

3.342 $\int (a+bx+cx^2)^p\, dx$

Optimal result

Integrand size = 12, antiderivative size = 85

$$\int (a+bx+cx^2)^p\, dx = \frac{2^{-1-2p}(b+2cx)(a+bx+cx^2)^p\left(-\frac{c(a+bx+cx^2)}{b^2-4ac}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{(b+2cx)^2}{b^2-4ac}\right)}{c}$$

output
```
2^(-1-2*p)*(2*c*x+b)*(c*x^2+b*x+a)^p*hypergeom([1/2, -p],[3/2],(2*c*x+b)^2
/(-4*a*c+b^2))/c/((-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^p)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.48

$$\int (a+bx+cx^2)^p\, dx = \frac{2^{-1+p}\left(b-\sqrt{b^2-4ac}+2cx\right)\left(\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}\right)^{-p}(a+x(b+cx))^p\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,\right.}{c(1+p)}$$

input
```
Integrate[(a + b*x + c*x^2)^p,x]
```

output
```
(2^(-1 + p)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*(a + x*(b + c*x))^p*Hypergeome
tric2F1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/(2*Sqrt[b^2 - 4
*a*c])])/(c*(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c])^p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.44, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{1096\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx+cx^2)^p\,dx$$

$$\Big\downarrow \text{1096}$$

$$-\frac{2^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p-1}(a+bx+cx^2)^{p+1}\,\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{(p+1)\sqrt{b^2-4ac}}$$

input
```
Int[(a + b*x + c*x^2)^p,x]
```

output
```
-((2^(1 + p)*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]))^(-1 -
p)*(a + b*x + c*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt
[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(1 + p))
)
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

Maple [F]

$$\int \left(c\,x^{2}+b x +a \right)^{p}d x$$

input
```
int((c*x^2+b*x+a)^p,x)
```

output
```
int((c*x^2+b*x+a)^p,x)
```

Fricas [F]

$$\int \left(a+bx+cx^2\right)^p\,dx=\int \left(cx^2+bx+a\right)^p\,dx$$

input
```
integrate((c*x^2+b*x+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)^p, x)
```

Sympy [F]

$$\int \left(a+bx+cx^2\right)^p\,dx=\int \left(a+bx+cx^2\right)^p\,dx$$

input
```
integrate((c*x**2+b*x+a)**p,x)
```

output
```
Integral((a + b*x + c*x**2)**p, x)
```

Maxima [F]

$$\int \left(a+bx+cx^2\right)^p\,dx=\int \left(cx^2+bx+a\right)^p\,dx$$

input
```
integrate((c*x^2+b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^p, x)
```

Giac [F]

$$\int \left(a+bx+cx^2\right)^p\,dx=\int \left(cx^2+bx+a\right)^p\,dx$$

input
```
integrate((c*x^2+b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2\right)^p\,dx=\int \left(c\,x^2+b\,x+a\right)^p\,dx$$

input
```
int((a + b*x + c*x^2)^p,x)
```

output
```
int((a + b*x + c*x^2)^p, x)
```

Reduce [F]

$$\int \left(a+bx+cx^2\right)^p\,dx$$
$$=\frac{2(c\,x^2+bx+a)^p\,a+(c\,x^2+bx+a)^p\,bx-8\left(\int\frac{(c\,x^2+bx+a)^p\,x}{2cp\,x^2+2bpx+c\,x^2+2ap+bx+a}dx\right)ac\,p^2-4\left(\int\frac{(c\,x^2+bx+a)^p}{2cp\,x^2+2bpx+c\,x^2+2}\right.}{b\,(2p+1)}$$

input
```
int((c*x^2+b*x+a)^p,x)
```

output
```
(2*(a + b*x + c*x**2)**p*a + (a + b*x + c*x**2)**p*b*x - 8*int(((a + b*x +
 c*x**2)**p*x)/(2*a*p + a + 2*b*p*x + b*x + 2*c*p*x**2 + c*x**2),x)*a*c*p*
*2 - 4*int(((a + b*x + c*x**2)**p*x)/(2*a*p + a + 2*b*p*x + b*x + 2*c*p*x*
*2 + c*x**2),x)*a*c*p + 2*int(((a + b*x + c*x**2)**p*x)/(2*a*p + a + 2*b*p
*x + b*x + 2*c*p*x**2 + c*x**2),x)*b**2*p**2 + int(((a + b*x + c*x**2)**p*
x)/(2*a*p + a + 2*b*p*x + b*x + 2*c*p*x**2 + c*x**2),x)*b**2*p)/(b*(2*p +
1))
```

3.343 $\int \frac{(a+bx+cx^2)^p}{x} dx$

Optimal result

Integrand size = 16, antiderivative size = 148

$$\int \frac{(a+bx+cx^2)^p}{x} dx = \frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx}{cx}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx}{cx}\right)^{-p}(a+bx+cx^2)^p \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\sqrt{b^2-4ac}}{2cx}\right.}{p}$$

output
```
2^(-1+2*p)*(c*x^2+b*x+a)^p*AppellF1(-2*p,-p,-p,1-2*p,-1/2*(b-(-4*a*c+b^2)^
(1/2))/c/x,-1/2*(b+(-4*a*c+b^2)^(1/2))/c/x)/p/(((b-(-4*a*c+b^2)^(1/2)+2*c*
x)/c/x)^p)/(((b+(-4*a*c+b^2)^(1/2)+2*c*x)/c/x)^p)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx+cx^2)^p}{x} dx = \frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx}{cx}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx}{cx}\right)^{-p}(a+x(b+cx))^p \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b+\sqrt{b^2-4a}}{2cx}\right.}{p}$$

input
```
Integrate[(a + b*x + c*x^2)^p/x,x]
```

output
```
(2^(-1 + 2*p)*(a + x*(b + c*x))^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*(b
+ Sqrt[b^2 - 4*a*c])/(c*x), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x)])/(p*((b - Sq
rt[b^2 - 4*a*c] + 2*c*x)/(c*x))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(c*x))^
p)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^p}{x}\,dx$$

$$\Big\downarrow \text{1178}$$

$$-4^p\left(\frac{1}{x}\right)^{2p}\left(\frac{-\sqrt{b^2-4ac}+b+2cx}{cx}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx}{cx}\right)^{-p}(a+bx+cx^2)^p\int\left(\frac{b-\sqrt{b^2-4ac}}{2cx}+1\right)^p$$

$$\Big\downarrow \text{150}$$

$$\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx}{cx}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx}{cx}\right)^{-p}(a+bx+cx^2)^p\,\text{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\sqrt{b^2-4ac}}{2cx},-\frac{b+}{}\right.}{p}$$

input
```
Int[(a + b*x + c*x^2)^p/x,x]
```

output
```
(2^(-1 + 2*p)*(a + b*x + c*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*(b
- Sqrt[b^2 - 4*a*c])/(c*x), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x)])/(p*((b -
Sqrt[b^2 - 4*a*c] + 2*c*x)/(c*x))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(c*x)
)^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))    Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

Maple [F]

$$\int \frac{(c\,x^2+bx+a)^p}{x}dx$$

input
```
int((c*x^2+b*x+a)^p/x,x)
```

output
```
int((c*x^2+b*x+a)^p/x,x)
```

Fricas [F]

$$\int \frac{(a+bx+cx^2)^p}{x}\,dx=\int \frac{(cx^2+bx+a)^p}{x}\,dx$$

input
```
integrate((c*x^2+b*x+a)^p/x,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)^p/x, x)
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^p}{x}\,dx = \int \frac{(a+bx+cx^2)^p}{x}\,dx$$

input
```
integrate((c*x**2+b*x+a)**p/x,x)
```

output
```
Integral((a + b*x + c*x**2)**p/x, x)
```

Maxima [F]

$$\int \frac{(a+bx+cx^2)^p}{x}\,dx = \int \frac{(cx^2+bx+a)^p}{x}\,dx$$

input
```
integrate((c*x^2+b*x+a)^p/x,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^p/x, x)
```

Giac [F]

$$\int \frac{(a+bx+cx^2)^p}{x}\,dx = \int \frac{(cx^2+bx+a)^p}{x}\,dx$$

input
```
integrate((c*x^2+b*x+a)^p/x,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^p}{x}\,dx = \int \frac{(c\,x^2+b\,x+a)^p}{x}\,dx$$

input
```
int((a + b*x + c*x^2)^p/x,x)
```

output
```
int((a + b*x + c*x^2)^p/x, x)
```

Reduce [F]

$$\int \frac{(a+bx+cx^2)^p}{x}\,dx = \frac{(c\,x^2+bx+a)^p+\left(\int \frac{(c\,x^2+bx+a)^p}{c\,x^3+b\,x^2+ax}dx\right)ap-\left(\int \frac{(c\,x^2+bx+a)^p x}{c\,x^2+bx+a}dx\right)cp}{p}$$

input
```
int((c*x^2+b*x+a)^p/x,x)
```

output
```
((a + b*x + c*x**2)**p + int((a + b*x + c*x**2)**p/(a*x + b*x**2 + c*x**3)
,x)*a*p - int(((a + b*x + c*x**2)**p*x)/(a + b*x + c*x**2),x)*c*p)/p
```

3.344 $\int \frac{(a+bx+cx^2)^p}{x^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 156

$$\int \frac{(a+bx+cx^2)^p}{x^2} \, dx = -\frac{4^p \left(\frac{b-\sqrt{b^2-4ac}+2cx}{cx}\right)^{-p} \left(\frac{b+\sqrt{b^2-4ac}+2cx}{cx}\right)^{-p} (a+bx+cx^2)^p \operatorname{AppellF1}\left(1-2p,-p,-p,2(1-p),-\frac{b-\sqrt{b^2-4}}{2cx}\right.}{(1-2p)x}$$

output
```
-4^p*(c*x^2+b*x+a)^p*AppellF1(1-2*p,-p,-p,2-2*p,-1/2*(b-(-4*a*c+b^2)^(1/2)
)/c/x,-1/2*(b+(-4*a*c+b^2)^(1/2))/c/x)/(1-2*p)/x/(((b-(-4*a*c+b^2)^(1/2)+2
*c*x)/c/x)^p)/(((b+(-4*a*c+b^2)^(1/2)+2*c*x)/c/x)^p)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx+cx^2)^p}{x^2} \, dx = \frac{4^p \left(\frac{b-\sqrt{b^2-4ac}+2cx}{cx}\right)^{-p} \left(\frac{b+\sqrt{b^2-4ac}+2cx}{cx}\right)^{-p} (a+x(b+cx))^p \operatorname{AppellF1}\left(1-2p,-p,-p,2-2p,-\frac{b+\sqrt{b^2-4ac}}{2cx},\right.}{(-1+2p)x}$$

input
```
Integrate[(a + b*x + c*x^2)^p/x^2,x]
```

output
```
(4^p*(a + x*(b + c*x))^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, -1/2*(b + Sqrt
[b^2 - 4*a*c])/(c*x), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x)])/((-1 + 2*p)*x*((b
 - Sqrt[b^2 - 4*a*c] + 2*c*x)/(c*x))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(c
*x))^p)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^p}{x^2} dx$$

$$\Big\downarrow \text{1178}$$

$$-4^p\left(\frac{1}{x}\right)^{2p}\left(\frac{-\sqrt{b^2-4ac}+b+2cx}{cx}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx}{cx}\right)^{-p}(a+bx+cx^2)^p\int\left(\frac{b-\sqrt{b^2-4ac}}{2cx}+1\right)^p$$

$$\Big\downarrow \text{150}$$

$$-\frac{4^p\left(\frac{-\sqrt{b^2-4ac}+b+2cx}{cx}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx}{cx}\right)^{-p}(a+bx+cx^2)^p \text{AppellF1}\left(1-2p,-p,-p,2-2p,-\frac{b-\sqrt{b^2-4ac}}{2cx},-\frac{b}{}\right.}{(1-2p)x}$$

input
```
Int[(a + b*x + c*x^2)^p/x^2,x]
```

output
```
-((4^p*(a + b*x + c*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, -1/2*(b - Sq
rt[b^2 - 4*a*c])/(c*x), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x)])/((1 - 2*p)*x*
((b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(c*x))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)
/(c*x))^p))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

Maple [F]

$$\int \frac{(c\,x^2+bx+a)^p}{x^2}dx$$

input
```
int((c*x^2+b*x+a)^p/x^2,x)
```

output
```
int((c*x^2+b*x+a)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(a+bx+cx^2)^p}{x^2}\,dx=\int \frac{(cx^2+bx+a)^p}{x^2}\,dx$$

input
```
integrate((c*x^2+b*x+a)^p/x^2,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)^p/x^2, x)
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^p}{x^2}\,dx=\int \frac{(a+bx+cx^2)^p}{x^2}\,dx$$

input
```
integrate((c*x**2+b*x+a)**p/x**2,x)
```

output
```
Integral((a + b*x + c*x**2)**p/x**2, x)
```

Maxima [F]

$$\int \frac{(a+bx+cx^2)^p}{x^2}\,dx=\int \frac{(cx^2+bx+a)^p}{x^2}\,dx$$

input
```
integrate((c*x^2+b*x+a)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx+cx^2)^p}{x^2}\,dx=\int \frac{(cx^2+bx+a)^p}{x^2}\,dx$$

input
```
integrate((c*x^2+b*x+a)^p/x^2,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^p}{x^2}\,dx = \int \frac{(c\,x^2+b\,x+a)^p}{x^2}\,dx$$

input
```
int((a + b*x + c*x^2)^p/x^2,x)
```

output
```
int((a + b*x + c*x^2)^p/x^2, x)
```

Reduce [F]

$$\int \frac{(a+bx+cx^2)^p}{x^2}\,dx = \frac{-(c\,x^2+bx+a)^p\,bp+(c\,x^2+bx+a)^p\,b-(c\,x^2+bx+a)^p\,cx+\left(\int \frac{(c\,x^2+bx+a)^p}{cp\,x^3+bp\,x^2-c\,x^3+apx-b\,x^2-ax}dx\right)b^2p^3x}{}$$

input
```
int((c*x^2+b*x+a)^p/x^2,x)
```

output
```
( - (a + b*x + c*x**2)**p*b*p + (a + b*x + c*x**2)**p*b - (a + b*x + c*x**
2)**p*c*x + int((a + b*x + c*x**2)**p/(a*p*x - a*x + b*p*x**2 - b*x**2 + c
*p*x**3 - c*x**3),x)*b**2*p**3*x - 2*int((a + b*x + c*x**2)**p/(a*p*x - a*
x + b*p*x**2 - b*x**2 + c*p*x**3 - c*x**3),x)*b**2*p**2*x + int((a + b*x +
 c*x**2)**p/(a*p*x - a*x + b*p*x**2 - b*x**2 + c*p*x**3 - c*x**3),x)*b**2*
p*x + 2*int((a + b*x + c*x**2)**p/(a*p - a + b*p*x - b*x + c*p*x**2 - c*x*
*2),x)*b*c*p**3*x - 3*int((a + b*x + c*x**2)**p/(a*p - a + b*p*x - b*x + c
*p*x**2 - c*x**2),x)*b*c*p**2*x + int((a + b*x + c*x**2)**p/(a*p - a + b*p
*x - b*x + c*p*x**2 - c*x**2),x)*b*c*p*x + 2*int(((a + b*x + c*x**2)**p*x)
/(a*p - a + b*p*x - b*x + c*p*x**2 - c*x**2),x)*c**2*p**2*x - 2*int(((a +
b*x + c*x**2)**p*x)/(a*p - a + b*p*x - b*x + c*p*x**2 - c*x**2),x)*c**2*p*
x)/(b*x*(p - 1))
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                   ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file